

A manual prediction of planetary positions in the sky

Introduction:

The night-sky fascinates people. To be able to locate a planet in the night-sky is something that thrills people. Since the planets move with respect to the background stars and continuously change their positions in the sky, locating them in the sky could appear to be a non-trivial task. It is a general notion that calculating the planetary positions is a very tedious task, involving a lot of complicated mathematical equations and computer programmes. However, to be able to locate planets in the sky one does not really need very accurate positions. After all, Kepler's laws, which describe planetary orbits reasonably well, are mathematically simple. Hence, one could use Kepler's laws to predict planetary positions in which mutual influence of planets is not considered. Thereby an accuracy of $\leq 1^\circ$ in planetary positions would be achieved.

Here we utilize a very simple method to calculate the positions of the planets. The technique we use enables us to calculate planetary positions to an accuracy of $\leq 1^\circ$ for ± 50 years from the starting epoch. Moreover, this involves very simple calculations and can be done using a calculator. All we need are the initial specifications of planetary orbital elements for some standard epoch and their time periods of revolution. Although accurate planetary positions could be obtained easily from the internet, yet it is very instructive and much more satisfying to be able to calculate these ourselves, starting from basic principles and using a simple procedure.

Our first step would be to calculate the positions of all the planets (including Earth) in their orbits around the Sun. We initially consider the planets to revolve around the Sun in uniform circular motions. Knowing their original positions for the starting epoch, we calculate their approximate positions for the intended epoch. As a consequence of this approximation there will be an error since the actual orbits are elliptical. To get more accurate positions, we apply some corrections which account for their elliptical motions.

Knowing the positions of the planets around the Sun, we can then use simple coordinate geometry to transform their position with respect to an observer on Earth. Our

task becomes simple since the orbits of all planets more or less lie in the same plane, viz. the ecliptic plane.

Here, we calculate the motion of naked-eye planets only, although the procedure could be applied equally well for the remaining planets also.

Celestial Coordinates:

All celestial bodies in the sky, including stars, planets, Sun, Moon and other objects, appear to lie on the surface of a giant sphere called the Celestial Sphere. Due to Earth's eastward rotation around its axis, the celestial sphere appears to rotate westward around Earth in 24 hours. Infinitely extending the plane of Earth's equator into space it appears to intersect the celestial sphere to form a circle, which is called the Celestial Equator.

As Earth moves around the Sun – as seen from the Earth – the Sun changes its position with respect to the background stars. The path that the Sun takes on the celestial sphere is called the "Ecliptic". The familiar Zodiac constellations are just divisions of the ecliptic into twelve parts. Since all other planets revolve around Sun in nearly the same plane, they also appear to move on the ecliptic.

The celestial equator is inclined to the ecliptic by 23.5° . The points of intersections of these two circles on the

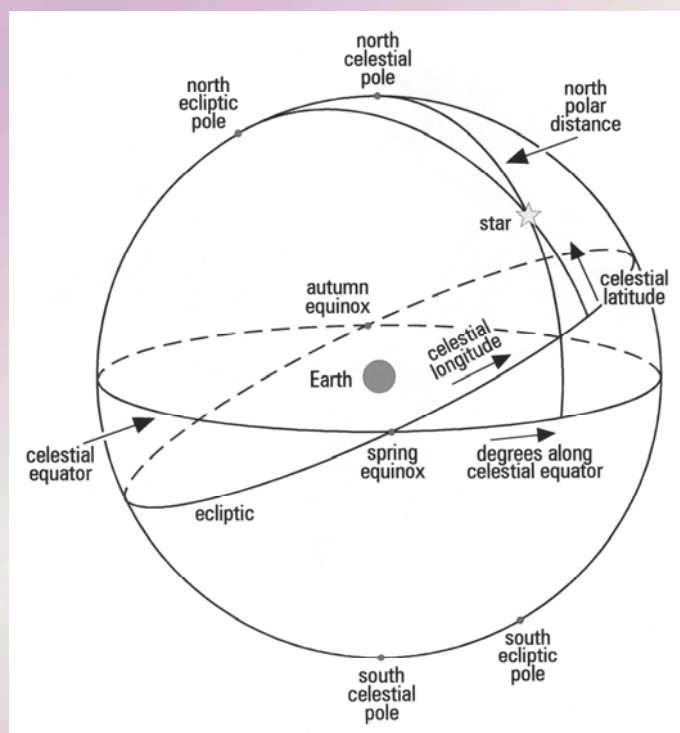


Figure 1: Celestial Sphere

celestial sphere are called the "Vernal Equinox" and the "Autumnal Equinox". The Vernal Equinox, also known as the Spring Equinox, is the point on the celestial sphere that the Sun passes through around 21st of March every year.

In astronomy, an epoch is a moment in time for which celestial coordinates or orbital elements are specified, while a celestial coordinate system is for mapping positions in the sky. There are different celestial coordinate systems each using a different coordinate grid projected on the celestial sphere. The coordinate systems differ only in their choice of the fundamental plane, which divides the sky into two equal hemispheres along a great circle. Each coordinate system is named for its choice of fundamental plane.

The ecliptic coordinate system is a coordinate system that uses the ecliptic for its fundamental plane. The longitudinal angle is called the ecliptic longitude or celestial longitude (denoted λ), measured eastwards from 0° to 360° from the vernal equinox. The latitudinal angle is called the ecliptic latitude or celestial latitude (denoted β), measured positive toward the north. This coordinate system is particularly useful for charting solar system objects.

The Earth's axis of rotation precesses around the ecliptic axis with a time period of about 25800 years. Due to this, the equinoxes shift westwards on the ecliptic. Due to the westward shift of the Vernal Equinox, which is the origin of the ecliptic coordinate system, the ecliptic longitude of the celestial bodies increases by an amount $360^\circ/258 \sim 1.4^\circ$ per century

Most planets, dwarf planets, and many small solar system bodies have orbits with small inclinations to the ecliptic plane, and therefore their ecliptic latitude β is always small. Due to the planets' small deviation from the plane of the ecliptic, the ecliptic longitude may alone suffice to visually locate planets in the sky.

Calculating Planetary Positions:

Heliocentric Circular Orbit:

Here we consider the planets to move around Sun in circular orbits with a uniform angular speed. The initial values of mean longitudes (λ_i) of the planets given in Table 1 are for 1st of January, 2000 A.D., 00:00 UT. In Table 1, we have also listed the period, T (days), of

revolution of the planets. Then, the mean angular speed is given by, $\omega_0 = 360/T$ ($^\circ/\text{day}$). We denote the Mean

Table 1: Mean Longitude λ_0 on 01/01/2007 00:00 UT from the initial value λ_i

Planet	λ_i ($^\circ$)	T (days)	λ_0 ($^\circ/\text{day}$)	λ_0 ($^\circ$)	λ_e ($^\circ$)	Error ($^\circ$)
Mercury	250.2	87.969	4.09235	274.3	268.7	+5.6
Venus	181.2	224.701	1.60213	317.8	317.8	0.0
Earth	100.0	365.256	0.98561	100.2	100.2	0.0
Mars	355.2	686.980	0.52403	255.1	244.5	+10.6
Jupiter	34.3	4332.59	0.08309	246.8	242.6	+4.2
Saturn	50.1	10759.2	0.03346	135.7	140.2	-4.5

Longitude of the planet in the imaginary circular orbit for subsequent dates as λ_0 .

We now demonstrate how to calculate λ_0 for Mars on 1st of January 2007.

λ_0 of Mars on 01.01.2000 at 00:00 UT = 355.2

No. of days b/w 01.01.2000 and 01.01.2007 = 2557 days

Mean angle traversed duration this period = $0.52403 \times 2557 = 1339.9^\circ$

So, λ_0 on 01.01.07 at 00:00 UT = $1339.9^\circ + 355.2 = 255.1^\circ$, where we have taken out the integer number of complete orbits.

In the same way, mean longitudes of all planets have been calculated in Table 1 for the same epoch. For a comparison, we have listed the actual longitude values (λ_e) from Indian Ephemeris for that epoch. Here we see, from the differences in the last column, that one needs to correct for the elliptical shape of the orbits, at least for some of the planets.

Heliocentric Elliptical Orbit:

Before we make corrections for the elliptical shape of the orbit we need to know the orientation of the ellipse within the ecliptic and that can be defined by the longitude of the perihelion which is the point on the elliptical orbit closest to the Sun. Longitudinal distance of the planet from the perihelion along the elliptical orbit is known as its Anomaly (denoted by θ), while angular distance of mean position of planet with respect to the perihelion is called the Mean Anomaly (denoted by θ_0). As has been discussed elsewhere⁵, there is a one-to-one correspondence between θ and θ_0 , and from which we find that the correction $\Delta\theta$ to be added to θ_0 is,

$$\Delta\theta = 2e \sin \theta_0 + \frac{5}{4} e^2 \sin 2\theta_0$$

where e is the eccentricity of the ellipse.

Let's consider Mercury on 01/01/07 at 00:00 UT.

Mean longitude, $\lambda_0 = 274.3^\circ$

Perihelion Longitude, $\lambda_p = 77.5^\circ$

Mean anomaly, $\theta_0 = \lambda_0 - \lambda_p = 196.8^\circ$

1st order correction, $2e \sin\theta_0 = 2 \times 0.2056 \times \sin(196.8^\circ) = -0.11851 \text{ rad} = -6.8^\circ$

2nd order correction, $5/4 e^2 \sin 2\theta_0 = 1.25 \times (0.2056)^2 \times \sin(393.6^\circ) = 0.0294 \text{ rad} = 1.7^\circ$

$\Delta\theta = (\text{1st order correction}) + (\text{2nd order correction}) = -6.8^\circ + 1.7^\circ = -5.1^\circ$

Anomaly $\theta = \theta_0 + \Delta\theta = 196.8^\circ - 5.1^\circ = 191.7^\circ$

Precession of vernal equinox in 7 yrs = $360/25800 \times 7^\circ = 0.1^\circ$

$\lambda = \lambda_0 + \Delta\theta + \text{precession of vernal equinox} = 274.3^\circ - 5.1^\circ +$

Table 2: Corrected Longitude λ at 01/01/07 00:00 UT

Planet	λ_0 (°)	λ_p (°)	λ_0 (°)	e	$\Delta\theta$ (°)	(°)	λ (°)	λ_e (°)	Error (°)
Mercury	274.3	77.5	196.8	0.2056	-5.1	191.7	269.3	268.7	+0.6
Venus	317.8	131.6	186.2	0.0068	-0.1	186.1	317.8	317.8	0.0
Earth	100.2	102.9	357.3	0.0167	-0.1	357.2	100.2	100.2	0.0
Mars	255.1	336.1	279.0	0.0934	-10.7	268.3	244.5	244.5	0.0
Jupiter	246.8	14.3	232.3	0.0485	-4.2	228.2	242.6	242.6	0.0
Saturn	135.7	93.1	42.6	0.0555	4.5	47.1	140.3	140.2	+0.1

$0.10^\circ = 269.3^\circ$.

We can obtain corrections for the elliptical orbits of the remaining planets in the same way. In Table 2, we have listed values of the longitude of perihelion (λ_p) and eccentricity (e) for all planets. Also tabulated are the calculated anomaly (θ) and longitude (λ).

From Table 2, we see that the errors now are indeed smaller than 1° .

Geocentric Perspective:

Until now, we have calculated the longitudes λ of the planets on the celestial sphere centred on the Sun. We can also calculate radii r of their orbits around the sun, giving their positions in polar form (r, λ). To get the positions of planets on the celestial sphere centred on the Earth, we convert the polar coordinates into rectangular form and after shifting the origin from Sun to Earth, we change them back into polar form.

For converting into a rectangular form, we have to decide upon the direction of the X and Y axes. We assume X to be in the positive direction along the line joining the Sun to the Vernal Equinox, and Y to be perpendicular to X in the ecliptic plane in such a way that the longitude is a positive angle.

As an example, this procedure is demonstrated for Mercury's position on 01/01/07 at 00:00 UT.

Heliocentric Coordinates

Distance r of Mercury from Sun can be obtained from its anomaly θ as,

$$\frac{l}{r} = 1 + e \cos \theta$$

where $l = a(1 - e^2)$ is the semi-latus rectum with a as the semi-major axis of its elliptical orbit. From $a = 0.387$ A.U. and $\theta = 191.7^\circ$, we get $r = 0.464$ A.U.

Thus, we get heliocentric polar coordinates of Mercury as $(r, \lambda) = (0.464 \text{ A.U.}, 269.3^\circ)$.

Then we can get heliocentric rectangular coordinates of Mercury as,

$$X_h = r \cos(\lambda) = -0.006 \text{ A.U.}$$

$$Y_h = r \sin(\lambda) = -0.464 \text{ A.U.}$$

Similarly we get heliocentric rectangular coordinates of Earth

as,

$$X_0 = -0.174 \text{ A.U.}$$

$$Y_0 = 0.968 \text{ A.U.}$$

Geocentric Coordinates:

Geocentric rectangular coordinates of Mercury then are,

$$X = X_h - X_0 = 0.168 \text{ A.U.}$$

$$Y = Y_h - Y_0 = -1.432 \text{ A.U.}$$

Converting these into polar form, we get the geocentric distance and longitude as,

$$r_g = \sqrt{X^2 + Y^2} = 1.442 \text{ A.U.}$$

$$\lambda_g = \tan^{-1}(Y/X) = 276.7^\circ$$

We give the calculated geocentric longitudes λ_g on 01.01.07 at 00:00 UT. Comparing with the geocentric longitudes from ephemeris λ_{ge} , it can be seen that the errors are much less than 1° . In the Earth/Sun row in Table 3, r_g , λ_g and λ_{ge} are the geocentric values for the Sun's position. The position of Sun on the celestial sphere, as seen from Earth, is in a direction exactly opposite to that of Earth as seen from the Sun. Therefore the geocentric longitude of Sun is the heliocentric longitude of Earth plus 180° .

We have ignored any perturbations on the motion of a planet due to the effect of other planets which may distort its elliptical path. We are able to get the accuracy of $\leq 1^\circ$ for long periods (± 50 years) because most of the terms ignored in the heliocentric longitude calculations are periodic in nature and do not grow indefinitely with time.

Table 3: Geocentric Longitude λ_g on 01/01/07 00:00 UT

Planet	a (A.U.)	e	r (A.U.)	λ ($^\circ$)	r_g (A.U.)	λ_g ($^\circ$)	λ_{ge} ($^\circ$)	Error ($^\circ$)
Mercury	0.387	0.2056	0.464	269.3	1.44	276.7	276.5	0.2
Venus	0.723	0.0068	0.728	317.8	1.62	296.1	296.1	0.0
Earth/Sun	1.00	0.0167	0.983	100.2	0.983	280.2	280.2	0.0
Mars	1.52	0.0934	1.51	244.5	2.38	258.4	258.4	0.0
Jupiter	5.20	0.0485	5.36	242.6	6.17	248.2	248.2	0.0
Saturn	9.54	0.0555	9.17	140.3	8.45	144.6	144.5	0.1

The other parameters characterizing the elliptical orbit, like the longitude of the perihelion, semi-major axis and eccentricity etc. change so slowly with time that for the accuracy we are interested in these can be considered constant for ± 50 years.

Locating Planets in the Sky:

Now that we have calculated the geocentric longitudes of the planets, we are in a position to locate them in the sky. Anyone familiar with the Zodiac constellations could locate a planet from its position in the constellation in which it lies. The ecliptic is divided into 12 Zodiac signs – Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius, Pisces. The Vernal equinox, at zero ecliptic longitude, is the start of the first Zodiac sign and is also known as the First Point of Aries. But there is a caveat attached. Because of the precession the vernal equinox has shifted westward by almost the full width of a constellation in the last ~ 2000 years since when the Zodiac signs and constellation were perhaps first identified. As a consequence, the First Point of Aries now lies in the constellation Pisces. For example, on 01/01/2007 geocentric longitude 276.7° of Mercury implies it is in the 10th Zodiac sign Capricorn, but actually it lies in the Sagittarius constellation, taking into account the shift by one constellation due to precession. There are further complications. The twelve constellations are not all of equal length of arc along the ecliptic longitude. Moreover there is another constellation, viz. Ophiuchus, through which the ecliptic passes. However these complications are somewhat set aside by the fact that there are only

about half a dozen stars in the Zodiac with an apparent brilliance comparable to the naked-eye planets, therefore with some familiarity of the night-sky, one could locate the planets easily from their geocentric longitude values. It further helps to remember that unlike stars, the planets, because of their large angular sizes, do not twinkle.

For a more precise location of a planet we can calculate its relative angular distance from the Sun along the ecliptic.

The difference between the geocentric positions of a planet and Sun (Table 4) is called the elongation (ψ) of the planet and it tells us about planet's

position in the sky with respect to that of the Sun. The longitude increases eastwards, therefore, if the longitude of the planet is greater than that of the Sun, then the planet lies to the east of the Sun. That means, in the morning the Sun will rise before the planet but in the evening the planet will be setting after the Sun. So the planet will be visible in the evening sky in the west. On the other hand, if the geocentric longitude of the planet is smaller than that of the Sun, it will rise before the Sun and will be visible in the morning in the eastern sky.

Table 4: Elongation ψ of each planet on 01/01/2007

Planet	λ_g ($^\circ$)	ψ ($^\circ$) at 05:30 IST	Rising Time Before Sunrise	ψ ($^\circ$) at 17:30 IST	Setting Time After Sunset
Sun	280.2	–	–	–	–
Mercury	276.7	–3.5	0 ^h 14 ^m	–3.2	–
Venus	296.1	15.9	–	16.0	1 ^h 04 ^m
Mars	258.4	–21.8	1 ^h 27 ^m	–21.9	–
Jupiter	248.2	–32.0	2 ^h 08 ^m	–32.4	–
Saturn	144.6	–135.6	9 ^h 02 ^m	–136.2	–

In Table 4, positions of planets with respect to Sun on 01.01.07 are given for 00:00 UT, which corresponds to 05:30 IST (Indian Standard Time). For example, the geocentric longitude of Mars with respect to Sun is $258.4 - 280.2 = -21.8^\circ$. Thus Mars has a western elongation $\sim 22^\circ$ on 01.01.07, 05:30 IST.

As Earth completes a rotation in 24 hours, the westward motion of the sky is at a rate 15° per hour. This rate is strictly true for the celestial equator, but we can use this

as an approximate rotation rate even for the ecliptic, which is inclined at a 23.5° to the equator. Therefore Mars will rise $22^\circ / 15 \sim$ one and half hour before the Sun. We have also tabulated elongations of the planets for the same date but at 17:30 IST. Note that this corresponds to 12:00 UT and there is a change in the elongation values during this half a day due to shifts in the geocentric longitudes of the planets and the sun. Venus with an eastern elongation $\sim 16^\circ$ on that evening, will be setting a little more than an hour after the sunset. This way, one can easily locate the planets in the sky from their elongations.

The Astronomical Calendar:

In all the examples presented above, the calculations were done manually, at most using a scientific calculator. This procedure is appropriate for quickly getting some occasional planetary positions while locating these objects in the sky. However if one wants to do many computations, say calculate positions for all planets for each day of the year, the process becomes tedious and the chances of a numerical mistake occurring in manual calculations become high. Since the process of computing planetary positions is a repetitive one, it could then be much more convenient to write a simple computer programme using the algorithm described above to carry out the calculations. We have written such a programme to compute positions of all the planets for each day of a specified year and present the results in the form of an astronomical calendar which gives elongations of different planets for all days of that particular year.

The calendar allows us to locate the naked-eye planets in the sky for any time of the corresponding year. The horizontal axis displays the elongation in hours (one hour corresponds to 15°) and is centred around the Sun, which by definition has a zero elongation. The vertical axis marks the day of the year. Thus to locate a planet on any given date of the year, we select that date on the vertical axis and then move in a horizontal direction till we find the planet. From the elongation of the planet we can easily locate it in the sky. We can use the calendar to find which all planets are above the horizon at any time of the day. Suppose for a given date of the year we want to locate all planets visible in the sky at, say, dawn. The

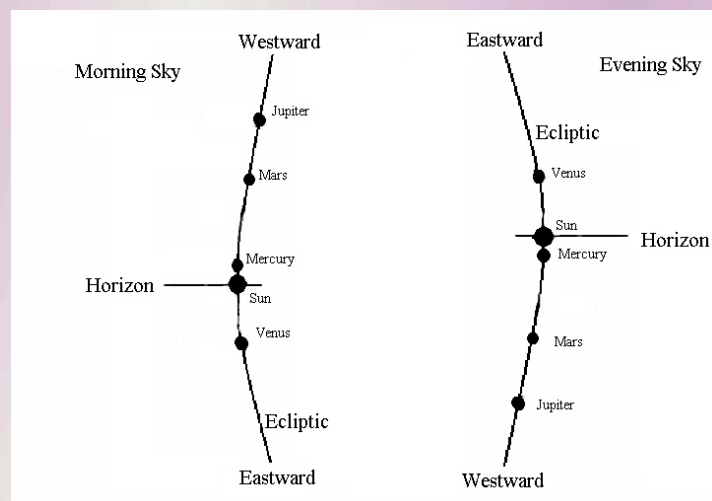


Figure 2: A schematic representation of the elongations of planets at the times of sunrise and sunset on 1st January, 2007

Sun, at 0^h elongation, will at that time be just rising near the eastern horizon and the -12^h elongation point in the calendar will be near the western horizon. The intermediate elongation points will be at in-between positions on the celestial hemisphere, e.g., the -6^h elongation point will be close to the culmination point (the point nearest to the zenith). This way going along the horizontal direction from 0^h to -12^h at the chosen date, we will find the celestial position of all planets visible in the morning sky on that date. At dusk, with sun setting near the western horizon, the visible sky will stretch eastward from 0^h to 12^h elongation on the calendar. Similarly one can locate planets on the celestial sphere at other hours of the time. At midnight, with culmination point being at 12^h (which is the same as -12^h), the sky toward west will stretch from 12^h to 6^h and that toward east will be from -12^h to -6^h elongation, while at 9 p.m., with the culmination point at 9^h , the celestial hemisphere will stretch west to east between 3^h and -9^h elongations.

Conclusions:

It is a general notion that calculation of position of planets in the night sky is a difficult job, which can be accomplished only by complex scientific computations, using fast computers. Here we have tried to bring out the fact that such complex and accurate computations are not always really necessary. One can calculate the position of planets using the method derived here and get the thrill of finding the planet at the predicted position in the night sky.

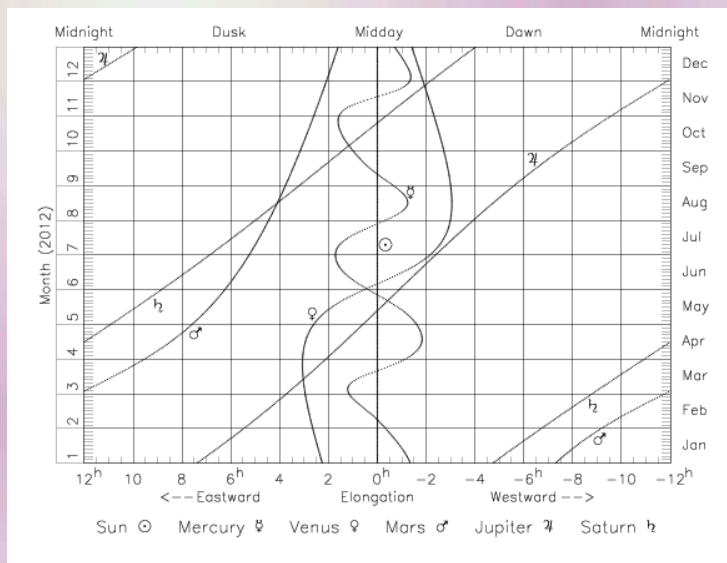


Figure 3: Astronomical calendar for locating planets in the sky for the year 2012.

We have been able to manually obtain the position of planets within an accuracy of $\leq 1^\circ$, at most using a calculator. This method can be used to reckon planetary positions up to ± 50 years of the starting epoch.

Further Reading:

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Katol Meteorite Shower

Meteorites are the remnants of the solar system objects. Their study provides us insight into the formation and evolution processes of their parent bodies and of the solar system. The world collection of meteorites is close to 40000 individual falls and finds (see Box 1 for fall, find definition and Table 1 for distribution on the Earth). After falling on the Earth, the meteorite gets exposed to weathering processes that may alter its original characteristics partially. Hence, it is very important to recover the meteorite pieces from the field immediately after its fall. A fresh meteorite fall is always an important new addition to enhance our knowledge and is well sought after by planetary scientists.

On May 22, 2012, at 14:10 hrs., IST, a large meteorite shower occurred at Katol [Lat. $21^\circ 15.781'$, Long. $78^\circ 35.284'$], near Nagpur. This is the 20th meteorite fall in India in last two decades. A meteorite fall is a spectacular light and sound show in the sky and one should be fortunate enough to witness a fall event. The citizens of Katol town and surrounding villages were a lucky few to experience a meteorite fall, though most were frightened by the thunderous sound on a bright sunny afternoon. Due to the bright sun, the light show was missed, but the sound had created fear and initial rumors predicted crash of an aircraft. Only next day, when the fragments were recovered and examined, it was realized that meteorite fall has taken place in the locality. A team from PRL (Dr. Anil Shukla, Geo-Sciences Division and the authors) visited Katol to recover samples from the meteorite shower and to document the event though the eye witnesses.

Discussions were also held with the scientists from GSI, Nagpur who had collected the first fragments from the local villagers. Here, we provide the eye witness account of the event and also



A fully fusion crusted fragment of Katol as found in an agriculture field

try to convey the excitement of a meteorite search campaign. A detailed scientific investigation is in