Finite-temperature dynamics of vortices in Bose-Einstein condensates

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We study the dynamics of a single vortex and a pair of vortices in quasi-two-dimensional Bose-Einstein condensates at finite temperatures. To this end, we use the stochastic Gross–Pitaevskii equation, which is the Langevin equation for the Bose–Einstein condensate. For a pair of vortices, we study the dynamics of both the vortex-vortex and vortex-antivortex pairs, which are generated by rotating the trap and moving the Gaussian obstacle potential, respectively. Due to thermal fluctuations, the constituent vortices are not symmetrically generated with respect to each other at finite temperatures. This initial asymmetry coupled with the presence of random thermal fluctuations in the system can lead to different decay rates for the component vortices of the pair, especially in the case of two corotating vortices.

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I. INTRODUCTION

In the spin-zero or scalar Bose–Einstein condensates (BECs), vortices carry integral angular momentum and serve as the unambiguous proof of superfluidity of these systems. The various methods which have been employed by experimentalists to generate vortices in BECs include manipulating the interconversion between the internal spin states of an isotope [1], stirring the BEC with a laser beam [2,3], rotating the BEC [4], phase imprinting [5], etc. Vortex dipoles, consisting of vortex-antivortex pairs, have also been experimentally realized in BECs by moving the condensate across the Gaussian obstacle potential [6]. Vortex dipoles are also formed as the decay product of solitons in quasi-two-dimensional condensates [7]. More recently, formation of the vortex dipoles during the rapid quench through the condensation temperature has also been observed [8,9]. The dynamics of a single vortex and a vortex-antivortex pair has also been observed experimentally [9]. On the theoretical front, the dynamics of vortex dipoles has also been analyzed at zero temperature [10,11]. Recently, the dynamics of small vortex clusters with 2 to 4 corotating vortices was also studied both experimentally and theoretically [12]. The decay of an off-centered vortex at finite temperature has been studied by using a pure classical field treatment [13], the dissipative Gross–Pitaevskii equation (DGPE) [14], the Zaremba-Nikuni-Griffin (ZNG) formalism [15,16], the projected Gross–Pitaevskii equation (PGPE) [17], and the stochastic projected Gross–Pitaevskii equation (SPGPE) [18].

We investigate theoretically the finite-temperature dynamics of a pair of vortices, along with a study of the dynamics of a single vortex. The method we employ is the stochastic Gross–Pitaevskii equation (SGPE) [20–22]. For the case of a single vortex in a condensate with a Thomas-Fermi density profile, the present work employs the SGPE, which is a better method than ZNG in low-dimensional systems near critical temperatures. In the weakly interacting domain, where the condensate has a Gaussian density profile, the stochastic equations of motion for the position of the vortex core have also been derived using a variational approach to the SGPE [23]. The SGPE has been successfully used to study finite-temperature scalar BECs in quasi-one- and quasi-two-dimensional geometries [24–29].

The paper is organized as follows: In Sec. II, we describe the SGPE method. We then discuss the finite-temperature dynamics of a single vortex in Sec. III. This is followed by the investigation of the dynamics of a vortex pair in Sec. IV. We conclude by providing the summary and conclusions in the last section.

II. STOCHASTIC LANGEVIN EQUATION FOR BEC

At $T = 0$ K, a scalar BEC is well described by the Gross–Pitaevskii (GP) equation [30]

$$i\hbar \frac{\partial \Phi(x,t)}{\partial t} = \left( -\frac{\hbar^2 \nabla^2}{2m} + V(x) + g|\Phi(x,t)|^2 \right) \Phi(x,t),$$

where $\Phi(x,t)$ is the wave function of the BEC. The trapping potential $V(x) = (\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)/(2m)$.

The interaction between atoms with mass $m$ is characterized by the interaction strength $g = 4\pi \hbar^2 a/\sqrt{m}$, where $a$ is the s-wave scattering length. The GP equation conserves the total number of atoms and total energy. In order to study the finite-temperature scalar BEC, we use the stochastic Gross–Pitaevskii equation (SGPE) [25]

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = \left( 1 - i\gamma \right) \left( -\frac{\hbar^2 \nabla^2}{2m} + V + g|\Psi(x,t)|^2 - \mu \right) \times \Psi(x,t) + \eta(x,t),$$

where $\gamma$ is the dissipation, $\mu$ is the chemical potential, and $\eta(x,t)$ is the random fluctuation which satisfies the following noise-noise correlation:

$$\langle \eta(x,t)\eta^*(x',t') \rangle = 2\nu k_B T \delta(x - x')\delta(t - t'),$$

according to the fluctuation-dissipation theorem required for equilibration, where $k_B$ and $T$ are the Boltzmann constant and temperature respectively, and $\langle \cdots \rangle$ denotes the averaging over different noise realizations. The system described by the SGPE is assumed to be divided into two parts: The first consists...