SKY-ILLUMINATION AT SUNRISE AND SUNSET.

BY

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Abstract.—Intensities of sky-illumination due to molecular scattering when the sun is on the horizon at a place 2 km. above sea-level are calculated for the wave-lengths 0·45μ, 0·55μ and 0·65μ. The calculated values are in reasonable agreement with the values measured by Dorno at Davos on clear days, but are smaller than those calculated by Gruner. The reason for the discrepancy is explained. The calculations show that the light from the sky when the sun is on the horizon is richer in the longer waves than the normal day-sky. It is argued that we should expect to get a maximum polarisation of sky-light in a direction perpendicular to the sun when he is on the horizon and when we confine ourselves to the red end of the spectrum. Measurements on red light from clear skies at Simla showed values of polarisation as high as 87 per cent. just before sunrise and just after sunset.

INTRODUCTION.

A summary of the principal features of sky-illumination at times of sunrise and sunset as recorded by various observers is given in a convenient form in Pernter's *Meteorologische Optik* and in recent papers by Professors Gruner[1] and Dorno.[2] While a large number of interesting and beautiful phenomena owe their origin to the presence of dust and condensed moisture in the atmosphere, there are other features observable in the clearest weather and at such high levels that the quantity of dust present can be but a very small fraction of that usually present at sea-level. These latter must, from the very circumstances of their occurrence, be capable of explanation by the action of the permanent constituents of the atmosphere.

Some prominent features of sky-illumination in clear weather when the sun is near the horizon.

(1) The sky ceases to have the distinctive blue colour which it has during the day, and becomes grey with a tinge of purple.

(2) A purple counter-glow appears against the sun when he is very near the horizon and gradually rises up as the sun sinks down, being bounded at its lower edge by the greyish-blue earth-shadow. The lower boundary is particularly well-marked when viewed through a red glass. The purple reaches its maximum intensity when the sun is 3° to 4° below the horizon.

(3) The colour of the sky some 5° to 10° above the horizon a few minutes after he sun has gone down, gradually changes as the azimuth of observation is changed, the colours being yellowish white, yellow, ochre, ruddy and steel-blue (colour of the earth-shadow) as we move from the sun to the anti-sun.

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[1] P. Gruner: Beitr. Physik Atmosph., *Leipzig*, **8**, 1919, p. 1 and p. 120.

[2] C. Dorno, Met. Zs., **52**, 1917, p. 153. *See also* F. J. W. Whipple, Article on "Meteorological Optics" in *Dictionary of Applied Physics*, III, p. 522.

Memoirs. Ind.Met. Dept. 25(1), 1–13, 1927.

(4) The polarisation of the sky in a direction perpendicular to the incident light is a maximum when the sun is about 2° below the horizon.

(5) When the light from the vertical sky is analysed into two components, they are of *distinctly* different colours, the stronger component having a smaller proportion of blue than the weaker.

The origin of sky-illumination.

That the blue of the clear day-sky is due to scattering by air-molecules is well-established. The detailed calculations of Professor L. V. King[1] show that most of the sky-illumination on clear days at Mount Whitney and Mount Wilson is due to this cause. We have therefore *a priori* reason to expect that the main features of the twilight sky on clear days at high-level stations should also be due to the same cause. Calculations by Professor Gruner[2] show that scattering by air molecules can account for the changes of colour of the sky as the sun approaches the horizon. Gruner has calculated the intensities of sky-illumination in different directions in the vertical plane containing the sun, as the zenith distance of the sun changes from 90° to 96° for two wave-lengths $0{\cdot}55\mu$ and $0{\cdot}64\mu$ as would be observed from a station 4 km. above sea-level. For convenience of calculation, he divides the atmosphere into four layers 0 to 4 kms., 4 to 12 kms., 12 to 24 kms., and 24 to 84 kms., and assumes a uniform distribution of molecules within each of these layers. The absolute values of the intensities calculated by Gruner are considerably *in excess* of the values observed by Dorno[3] at Davos. The reasons for this appear to be (1) adoption of values of scattering coefficients derived from those of the attenuation coefficients at Mount Whitney, assuming the attenuation to be due solely to molecular scattering and (2) the approximate nature of his method of calculation. In the following paper, a stricter method of calculation is developed and the values of scattering coefficients are calculated by Rayleigh's law; the results show closer agreement with observation, the calculated values being now smaller than the observed; the method also brings out some new points of interest.

Method of calculation.

In the following calculations, atmospheric refraction is neglected, so that the rays are considered horizontal throughout their course. (The effect of refraction is to make an object on the horizon appear to have an altitude of 35′ for an observer at sea-level). Intensities of sky-illumination are calculated for the wave-lengths $0{\cdot}45\mu$, $0{\cdot}55\mu$ and $0{\cdot}65\mu$ both in the vertical plane containing the sun and also in a perpendicular plane. To get the amount of light received in any direction OP we have to integrate the light scattered by all the molecules lying within a cone of small solid angle $d\omega$ in that direction, making allowance for the attenuation of the light in its passage from the molecules to the observer.

Let φ be the angle between the incident and scattered beams of light (Fig. 1).

I the intensity of illumination at P due to the incident light between the wave-lengths λ and $\lambda + d\lambda$

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[1] L. V. King, *Phil. Trans.* A, **212**, 1913, p. 375.

[2] P. Gruner, Beitr. Physik Atmosph., **8**, 1919, p. 120.

[3] C. Dorno, *Veroff. Pruess. Met. Inst.*, 303, **6**, 1919; also *Met. Zs.*, **36**, 1919, p. 109.

l the distance between the place of observation and P,

n the number of molecules per unit volume at P,

$S\,(1 + \cos^2\varphi)$ the scattering coefficient due to a single molecule in a direction making an angle φ with an incident unpolarised beam of light, and nk the attenuation coefficient appropriate to density n. I, S, and k refer to the same wave-length. Then the intensity of light at O between the wave-lengths λ and $\lambda + d\lambda$ due to the scattered light from an element of volume $l^2\, d\omega\, dl$ at P will be

$$n\, S\, I\, (1 + \cos^2\varphi)\, Exp\, [-k \int ndl\,]\, dl\, d\omega\, d\lambda \qquad (1).$$

and from all the particles within the cone, it will be

$$d\omega\, d\lambda \int_0^L n\, S\, I\, (1 + \cos^2\varphi)\, Exp\, [-k \int ndl\,]\, dl \qquad (2).$$

where L is the distance corresponding to the limit of the atmosphere.

Now, I itself varies with the position of the point P, since the original incident light has already been filtered through a certain length of the atmosphere. (*See* fig. 2.)

If ds denotes an element of path of the incident ray in the atmosphere before reaching P, the intensity of illumination at P will be $I_0\, exp\, [-k \int nds\,]$ where I_0 is the intensity outside the atmosphere and the integration is taken from outside the atmosphere to P, and (2) will become $d\omega\, d\lambda \int_0^L n\, S\, I_0\, (1 + \cos^2\varphi)\, Exp\, [-k \int ndl + nds.]\, dl$ (3).

The numerical computation of this quantity involves

(1) a knowledge of the variation of n with height r above sea-level

(2) the conversion of the integrals into appropriate ones involving r as the variable, and

(3) evaluation of the integrals graphically.

(1) The values of n used in the following calculations and given in Table I are, up to a height of 20 km., based on the data of pressure and temperature given by W. H. Dines[1] as average values for the free atmosphere over Europe, and above 20 km., calculated on the assumption of isothermal conditions according to the equation $-dp = p\, dh/h$. Heights above 50 km. are neglected and uniformity of composition is assumed throughout.

TABLE 1.

h in kilometres.	T in degrees absolute.	p in millibars.	$n \times 10^{-19}$.
0	281	1014	2·63
1·0	277	899	2·37
2·0	272	794	2·13
4·0	261	614	1·72
5·0	255	538	1·54
8·0	233	353	$1{\cdot}10_5$

[1] W. H. Dines; Geophys Mem., Met. Office, *London*, **13**, p. 63.

TABLE 1—*contd.*

h in kilometres.	T in degrees absolute.	p in millibars.	$n \times 10^{-19}$.
10·0	222	262	0·860
12·0	219	192	$0{\cdot}64_2$
15·0	219	120	$0{\cdot}40_0$
20·0	219	55	$0{\cdot}18_3$
30·0	219	11·5	0·038
40·0	219	2·4	0·008
50·0	219	0·5	0·0018

(2) dl and ds are expressed in terms of dr in the following way.

In figure 3, C is the centre of the earth, O the position of the observer and P the point at which we want dl and ds. Let R be the radius of the earth, h_1 the height of the observer above sea-level, ξ the zenith distance of P, and θ the angle at the centre of the earth between the vertical through the observer and C P.

Then $\frac{dr}{dl} = \cos(\xi-\theta)$

and $\frac{\sin(\xi-\theta)}{\sin\xi} = \frac{R+h_1}{R+r}$ where r is the height of P above sea-level.

$$\left(\frac{dr}{dl}\right)^2 = 1-\sin^2(\xi-\theta)$$

$$= 1-\frac{(R+h_1)^2}{(R+r)^2}\sin^2\xi$$

$$\therefore\ dl = dr\,\frac{R+r}{[(R+r)^2-(R+h_1)^2\sin^2\xi]^{\frac{1}{2}}} \qquad (4).$$

$$\frac{ds}{dr} = \operatorname{cosec}\theta = \frac{R+r}{s} = \frac{R+r}{[(R+r)^2-(R+h)^2]^{\frac{1}{2}}} \doteq \frac{\sqrt{R}}{\sqrt{2(r-h)}} \qquad (5).$$

where h is the height of Q above the earth.

When P coincides with Q, $r = h$ and the denominator of (5) becomes zero. If, however, we integrate ds in the immediate neighbourhood of Q between the limits $r = h$ and $r = r_1$, we get:

$$S_1^2 = 2R(r_1-h) \text{ or } S_1 = \sqrt{2R(r_1-h)} \qquad (6)$$

The integrals $\int ndl$ and $\int nds$ have to be taken between the appropriate limits. It is convenient to split up $\int_T^P nds$ into two parts, (1) $\int_T^Q nds$ from outside the atmosphere to a point Q vertically overhead and (2) $\int_Q^P nds$ from Q to P. (*See* fig. 2.) These may be distinguished by the symbols $\int nds_1$ and $\int nds_2$ respectively.

(3) We also require the coefficients of scattering and attenuation for different wavelengths. Their values are given in Table 2. The fraction of the incident light scattered by 1 c.c. of air at N. T. P. per unit solid angle in a direction perpendicular to the incident beam is calculated by Rayleigh's formula $\pi^2(\upsilon^2-1)^2/2n\lambda^4$, where υ is the refrac-

tive index of the gas for the particular wave-length, and n the number of molecules per cubic centimetre.

TABLE 2.

λ	S	Attenuation Coefficient.	
		At Mt. Wilson.	At Mt. Whitney.
0·39μ	$2{\cdot}69 \times 10^{-8}$	0·365	0·270
0·45μ	$1{\cdot}515 \times 10^{-8}$	0·223	0·101
0·50μ	$0{\cdot}994 \times 10^{-8}$	0·153	0·105
0·55μ	$0{\cdot}679 \times 10^{-8}$	0·132	0·0866
0·60μ	$0{\cdot}479 \times 10^{-8}$	0·117	0·0683
0·65μ	$0{\cdot}348 \times 10^{-8}$		0·0550

The coefficients of attenuation are obtained from the Smithsonian transmission coefficients measured at Mount Wilson and Mount Whitney.[1] Since the pressures at these two places are different, being 61·7 cm and 44·67 cm respectively, the coefficients of attenuation refer to passage through different thicknesses of atmosphere. They are reduced so as to refer to the same number of molecules by multiplying by $76/(p \times 8{\cdot}0 \times 2{\cdot}705)$ where p is the pressure at the station in centimetres of mercury. Since the height of the "homogeneous" atmosphere is 8·0 kilometres and since each c.c. of air at N. T. P. contains $2{\cdot}705 \times 10^{19}$ molecules, this reduced value corresponds to the passage of a beam of light of unit area of cross section over 10^{24} molecules.[2] If the attenuation is due solely to air molecules, the reduced values for different wave-lengths should be the same when calculated from the transmission coefficient of either station. That this is approximately so will be clear from the following table.

TABLE 3.

λ	Mount Wilson $p_1 = 61{\cdot}7$ cm.		Mount Whitney, $p_2 = 44{\cdot}67$ cm.	
	K_1	$\dfrac{K_1 \times 76}{p_1 \times 2{\cdot}705 \times 8{\cdot}0}$	K_2	$\dfrac{K_2 \times 76}{p_2 \times 2{\cdot}705 \times 8{\cdot}0}$
0·39μ	0·365	0·0208	0·270	0·0212
0·45μ	0·223	0·0127	0·161	0·0127
0·50μ	0·153	$0{\cdot}0087_1$	0·105	$0{\cdot}0082_5$
0·55μ	0·132	$0{\cdot}0075_1$	$0{\cdot}086_6$	0·0068
0·60μ	0·117	$0{\cdot}0066_7$	$0{\cdot}068_3$	0·0054
0·65μ	..	..	0·055	0·0043
0·70μ	0·060	0·0034	0·045	0·0035

[1] L. V. King, *loc. cit.*

[2] This is not strictly correct, as the "effective" temperature of the atmosphere is not exactly 0° C. The approximation is, however, close enough for our present purposes.

In what follows, we shall adopt the average of the values in columns 3 and 5 and calculate the intensities of illumination of the clear sky for wave-lengths $0{\cdot}45\mu$, $0{\cdot}55\mu$ and $0{\cdot}65\mu$ as observed at a place 2 km. above sea-level for azimuths 0°, 90° and 180° and various zenith distances.

The different steps of the calculation are as follow :—

(i) $\int nds_1$ *for different heights.*—A table is prepared showing values of r increasing from $h = 0{\cdot}5$ km. to $h = 50$ km. (column 1), the corresponding values of n (column 2), and the calculated values of $n\sqrt{R}/\sqrt{2(r-h)}$ (column 3). The numbers in column 3 are then plotted on graph paper against those in column 1 and the integral $\int_{h+0\cdot5}^{50} nds_1$ obtained. This may be done in any number of convenient steps. To obtain $\int_{h}^{h+0\cdot5} nds_1$ relation (6) is used, r_1-h being equal to $\frac{1}{2}$ km. and n being taken to be the mean of the values at h and $h+0{\cdot}5$ km.

TABLE 4.

Example.
$h = 12$ km.

r in kilometres.	$n \times 10^{-19}$	$\dfrac{n\sqrt{R}}{\sqrt{2(r-h)}} \times 10^{-19}$
12·5	0·59	47·1
13	0·55	31·0
15	0·40	13·0
18	0·24	5·5
22	0·138	2·46
27	0·063	0·92
32	$0{\cdot}026_2$	0·33
42	0·0060	0·062
52	0·0014	0·0125

From graphs, $\int_{13}^{20} nds = 0{\cdot}75 \times 10^{26}$; $\int_{20}^{50} nds = 0{\cdot}192 \times 10^{26}$ and by calculation $\int_{12}^{12\cdot5} nds = 0{\cdot}49 \times 10^{26}$ and $\int_{12\cdot5}^{13} nds = 0{\cdot}195 \times 10^{26}$. Therefore, $\int_{12}^{50} nds = 1{\cdot}63 \times 10^{26}$. Similar calculations are carried out for different values of h and Table 5 is prepared.

TABLE 5.

$\int nds_1$ *for different heights.*

Height in kilometres.	0	3	5	8	12	20	30	40
$\int nds_1 \times 10^{-26}$ . .	8·43	5·49	4·24	2·93	1·63	0·484	0·094	0·020

(ii) For any chosen value of ξ, $\frac{dl}{dr}$ and $n\frac{dl}{dr}$ are calculated from (4) for various values of r; by plotting $n\frac{dl}{dr}$ against r and integrating graphically between appropriate limits, we get $\int n\frac{dl}{dr}dr$ or $\int ndl$ between the observer and any position of P.

(iii) In figure 3, the height h of Q above sea-level is given by $2R(r-h) = PQ^2 = (h-h_1)^2 \tan^2\xi$. The values of h for different values of r when $\xi = 85°$ are given in table 6. It will be seen that h and r are appreciably different from each other only at large heights where the density is small. We may therefore take the average of the values of n at heights h and r and multiply it by PQ in order to obtain $\int nds_2$ at height h.

TABLE 6.

$\xi = 85°$, $h_1 = 2$ km.

r in kilometres	5	8	10	15	20	30	40	50
h in kilometres	4·9	7·7	9·5	13·7	17·5	24·7	31·2	37·5

(iv) From values obtained as in (i), (ii) and (iii), $Exp\ [-k\int(ndl + nds_1 + nds_2)]$ is calculated for different values of l and hence of r, and the quantity $I_o S (1 + \cos^2\varphi)\, d\omega\, d\lambda \int_2^{50} n\frac{dl}{dr} Exp\ [-k\ (ndl + nds + ndr)]\ dr$ evaluated by graphical integration.

Example

$\xi = 80°$, Azimuth $= 180°$.

TABLE 7.

Height in kilometres.	$\int nds_1$	$\int nds_2$	$\int ndl$	$\int (nds_1 + nds_2 + ndl)$.
2	$6{\cdot}4 \times 10^{26}$	..	..	$6{\cdot}4 \times 10^{26}$
5	$4{\cdot}26 \times 10^{26}$	$0{\cdot}26 \times 10^{26}$	$0{\cdot}31 \times 10^{26}$	$4{\cdot}83 \times 10^{26}$
8	$2{\cdot}95 \times 10^{26}$	$0{\cdot}37 \times 10^{26}$	$0{\cdot}53 \times 10^{26}$	$3{\cdot}85 \times 10^{26}$
10	$2{\cdot}27 \times 10^{26}$	$0{\cdot}38 \times 10^{26}$	$0{\cdot}64 \times 10^{26}$	$3{\cdot}29 \times 10^{26}$
15	$1{\cdot}15 \times 10^{26}$	$0{\cdot}29 \times 10^{26}$	$0{\cdot}82 \times 10^{26}$	$2{\cdot}26 \times 10^{26}$
20	$0{\cdot}53 \times 10^{26}$	$0{\cdot}19 \times 10^{26}$	$0{\cdot}90 \times 10^{26}$	$1{\cdot}62 \times 10^{26}$
30	$0{\cdot}10 \times 10^{26}$	$0{\cdot}07 \times 10^{26}$	$0{\cdot}95 \times 10^{26}$	$1{\cdot}12 \times 10^{26}$
40	$0{\cdot}02 \times 10^{26}$	$0{\cdot}02 \times 10^{26}$	$0{\cdot}96 \times 10^{26}$	$1{\cdot}00 \times 10^{26}$
50	..	..	$0{\cdot}96 \times 10^{26}$	$0{\cdot}96 \times 10^{26}$

TABLE 8.

$\xi = 80^\circ$, azimuth 180°, $\lambda = 0{\cdot}45\mu$.

Height in kilometres.	$Sn\,\frac{dl}{dr}$	$k\int(nds_1 + nds_2 + ndl)$.	$Exp\,[-k\int(nds_1 + nds_2 + ndl)$.	Col. (2) × Col. (4)
2	$6{\cdot}87 \times 10^{-8}$	8·12	0·0003	$\cdot 0021 \times 10^{-8}$
5	$4{\cdot}88 \times 10^{-8}$	6·13	·0022	$\cdot 0108 \times 10^{-8}$
8	$3{\cdot}46 \times 10^{-8}$	4·88	·0076	$\cdot 0264 \times 10^{-8}$
10	$2{\cdot}67 \times 10^{-8}$	4·17	·0155	$\cdot 0415 \times 10^{-8}$
15	$1{\cdot}213 \times 10^{-8}$	2·87	·0567	$\cdot 0640 \times 10^{-8}$
20	$0{\cdot}543 \times 10^{-8}$	2·05	·129	$\cdot 0700 \times 10^{-8}$
30	$0{\cdot}109 \times 10^{-8}$	1·42	·242	$\cdot 0263 \times 10^{-8}$
40	$0{\cdot}022 \times 10^{-8}$	1·27	·281	$\cdot 0061 \times 10^{-8}$
50	$0{\cdot}0048 \times 10^{-8}$	1·22	·295	$\cdot 0014 \times 10^{-8}$

The values in the last column are integrated from 2 to 50 km. and multiplied by $1 + sin^2\xi$, $sin^2\xi$ being equal to $cos^2\varphi$ when the incident rays are horizontal. Similar calculations are carried out for the other two wave-lengths $0{\cdot}55\mu$ and $0{\cdot}65\mu$. For azimuth 0°, $\int(nds_1 + nds_2 + ndl)$ should be replaced by $\int(nds_1 - nds_2 + ndl)$ and for azimuth 90° by $\int(nds_1 + ndl)$.

Results.

The following table gives the values of the intensities of illumination in different directions.

TABLE 9.

λ	Intensity of scattered light per unit solid angle. / Intensity of incident sunlight.				
	$\xi = 0$.	$\xi = 45°$.	$\xi = 60°$.	$\xi = 80°$.	$\xi = 85°$.
Azimuth 0°—					
0·45μ	..	$1{\cdot}81 \times 10^{-3}$	$2{\cdot}65 \times 10^{-3}$	$4{\cdot}94 \times 10^{-3}$	$4{\cdot}20 \times 10^{-3}$
0·55μ	..	$1{\cdot}66 \times 10^{-3}$	$2{\cdot}67 \times 10^{-3}$	$6{\cdot}28 \times 10^{-3}$	$8{\cdot}18 \times 10^{-3}$
0·65μ	..	$1{\cdot}46 \times 10^{-3}$	$2{\cdot}34 \times 10^{-3}$	$6{\cdot}52 \times 10^{-3}$	$9{\cdot}76 \times 10^{-3}$
Azimuth 90°—					
0·45μ	$0{\cdot}90 \times 10^{-3}$	$1{\cdot}16 \times 10^{-3}$	$1{\cdot}40 \times 10^{-3}$	$1{\cdot}89 \times 10^{-3}$	$1{\cdot}12 \times 10^{-3}$
0·55μ	$0{\cdot}79 \times 10^{-3}$	$1{\cdot}07 \times 10^{-3}$	$1{\cdot}42 \times 10^{-3}$	$2{\cdot}70 \times 10^{-3}$	$2{\cdot}80 \times 10^{-3}$
0·65μ	$0{\cdot}69 \times 10^{-3}$	$0{\cdot}95 \times 10^{-3}$	$1{\cdot}28 \times 10^{-3}$	$2{\cdot}87 \times 10^{-3}$	$3{\cdot}89 \times 10^{-3}$
Azimuth 180°—					
0·45μ	..	$1{\cdot}61 \times 10^{-3}$	$2{\cdot}24 \times 10^{-3}$	$2{\cdot}84 \times 10^{-3}$	$1{\cdot}23 \times 10^{-3}$
0·55μ	..	$1{\cdot}55 \times 10^{-3}$	$2{\cdot}30 \times 10^{-3}$	$4{\cdot}39 \times 10^{-3}$	$3{\cdot}93 \times 10^{-3}$
0·65μ	..	$1{\cdot}38 \times 10^{-3}$	$2{\cdot}18 \times 10^{-3}$	$4{\cdot}97 \times 10^{-3}$	$6{\cdot}22 \times 10^{-3}$

It is evident from the above table that the composition of sky-light when the sun is on the horizon is quite different from that of the normal day-light sky. In the zenith, where the relative proportion of short waves is the largest, the ratio of the blue (0·45μ) to the red (0·65μ) is only 1·3 times that in the original light, while according to the inverse fourth power law, the ratio would be 4·3. This accounts for the well marked change of colour which is easily noticeable at times of sunrise and sunset. As we move from the zenith to the horizon the proportion of blue decreases still more, the ratio blue to red becoming 0·66 at a zenith distance of 80° in a direction perpendicular to the sun's rays and 0·57 against the sun. The more marked reddening on increase of azimuth is in agreement with observation.

The purple of the counterglow is a composite colour containing all the colours of the spectrum with a preponderance of red.

The best observational data available for comparison with the above are those obtained by Professor Dorno at Davos (1590 metres above sea-level) in Switzerland. His relative measurements at different parts of the sky were made with Weber's polarisation photometer and measurements at the zenith with the same author's milk-glass photometer.[1] Red and green filters were used and corrections applied for the composite character of the light depending on the ratio green/red. Dorno has given his results[2] in terms of the brightness of an absolutely white, matt surface illuminated with an intensity of one metre-candle, the brightness of the sun outside the earth's atmosphere[3] in terms of the same unit being 7660×10^6. The values given in the following table are the annual means of the brightness of sky in different directions in terms of the brightness of the sun outside the earth's atmosphere.

[1] A description of the instrument is given in the *Dictionary of Applied Physics*, IV, p. 440.
[2] Table 25*a*. "Himmelshelligkeit, etc.", *l. c.*
[3] "Himmelshelligkeit, etc.", p. 54.

TABLE 10.*

$\frac{\text{Brightness of sky}}{\text{Brightness of sun}}$.	Azimuth.	$\xi = 0°$.	$\xi = 45°$.	$\xi = 60°$.	$\xi = 80°$.	$\xi = 85°$.
Calculated—						
·45μ		$0{\cdot}61 \times 10^{-7}$	$1{\cdot}23 \times 10^{-7}$	$1{\cdot}80 \times 10^{-7}$	$3{\cdot}35 \times 10^{-7}$	$2{\cdot}85 \times 10^{-7}$
·55μ		$0{\cdot}54 \times 10^{-7}$	$1{\cdot}12 \times 10^{-7}$	$1{\cdot}81 \times 10^{-7}$	$4{\cdot}25 \times 10^{-7}$	$5{\cdot}5 \times 10^{-7}$
·65μ	0°	$0{\cdot}47 \times 10^{-7}$	$0{\cdot}99 \times 10^{-7}$	$1{\cdot}58 \times 10^{-7}$	$4{\cdot}42 \times 10^{-7}$	$6{\cdot}6 \times 10^{-7}$
Calculated by Gruner—						
·55μ		$1{\cdot}06 \times 10^{-7}$	$2{\cdot}28 \times 10^{-7}$	$3{\cdot}60 \times 10^{-7}$	$9{\cdot}3 \times 10^{-7}$	$14{\cdot}4 \times 10^{-7}$
·64μ		$0{\cdot}97 \times 10^{-7}$	$1{\cdot}99 \times 10^{-7}$	$3{\cdot}40 \times 10^{-7}$	$11{\cdot}2 \times 10^{-7}$	$16{\cdot}7 \times 10^{-7}$
Observed Dorno . .		$0{\cdot}63 \times 10^{-7}$	$1{\cdot}51 \times 10^{-7}$	$3{\cdot}97 \times 10^{-7}$	$7{\cdot}8 \times 10^{-7}$	..
Calculated—						
·45μ		$0{\cdot}61 \times 10^{-7}$	$0{\cdot}77 \times 10^{-7}$	$0{\cdot}95 \times 10^{-7}$	$1{\cdot}28 \times 10^{-7}$	$0{\cdot}76 \times 10^{-7}$
·55μ		$0{\cdot}54 \times 10^{-7}$	$0{\cdot}73 \times 10^{-7}$	$0{\cdot}96 \times 10^{-7}$	$1{\cdot}83 \times 10^{-7}$	$1{\cdot}90 \times 10^{-7}$
·65μ	90°	$0{\cdot}47 \times 10^{-7}$	$0{\cdot}63 \times 10^{-7}$	$0{\cdot}87 \times 10^{-7}$	$1{\cdot}95 \times 10^{-7}$	$2{\cdot}64 \times 10^{-7}$
Calculated by Gruner—						
·55μ		$1{\cdot}06 \times 10^{-7}$	..	..	..	..
·64μ		$0{\cdot}97 \times 10^{-7}$	..	..	..	..
Observed Dorno . .		$0{\cdot}63 \times 10^{-7}$	$1{\cdot}04 \times 10^{-7}$	$1{\cdot}59 \times 10^{-7}$	$2{\cdot}64 \times 10^{-7}$	..
Calculated—						
·45μ		$0{\cdot}61 \times 10^{-7}$	$1{\cdot}09 \times 10^{-7}$	$1{\cdot}52 \times 10^{-7}$	$1{\cdot}92 \times 10^{-7}$	$0{\cdot}83 \times 10^{-7}$
·55μ		$0{\cdot}54 \times 10^{-7}$	$1{\cdot}05 \times 10^{-7}$	$1{\cdot}56 \times 10^{-7}$	$2{\cdot}98 \times 10^{-7}$	$2{\cdot}66 \times 10^{-7}$
·65μ	180°	$0{\cdot}47 \times 10^{-7}$	$0{\cdot}93 \times 10^{-7}$	$1{\cdot}48 \times 10^{-7}$	$3{\cdot}36 \times 10^{-7}$	$4{\cdot}22 \times 10^{-7}$
Calculated by Gruner—						
·55μ		$1{\cdot}06 \times 10^{-7}$	2·12	$3{\cdot}30 \times 10^{-7}$	$7{\cdot}3 \times 10^{-7}$	$7{\cdot}7 \times 10^{-7}$
·64μ		$0{\cdot}97 \times 10^{-7}$	1·98	$3{\cdot}15 \times 10^{-7}$	$8{\cdot}3 \times 10^{-7}$	$11{\cdot}6 \times 10^{-7}$
Observed Dorno . .		$0{\cdot}63 \times 10^{-7}$	$1{\cdot}04 \times 10^{-7}$	$1{\cdot}86 \times 10^{-7}$	$3{\cdot}35 \times 10^{-7}$	..

* The calculated values given in the above table will be raised by a small amount if we add to the polarised scattering contemplated in the Rayleigh law the additional unpolarised scattering due to the anisotropy of the molecules. The expression for the fraction of light scattered per unit angle by 1 c.c. of air in a direction making an angle φ with the incident light, will, if this is included, become $\frac{\pi^2(\upsilon^2-1)^2}{2n\lambda^4}\frac{6(1+\rho)}{6-7\rho}\left(1+\frac{1-\rho}{1+\rho}\cos^2\varphi\right)$ instead of $\frac{\pi^2(\upsilon^2-1)^2}{2n\lambda^4}(1+\cos^2\varphi)$ where ρ is the ratio of the minimum to the maximum intensity when the light scattered perpendicular to the incident beam is examined by means of a nicol prism. The values of ρ for dustfree air determined experimentally is 4·5 per cent. The computed values of sky-brightness will be increased in the following ratios at different zenith distances :

—	0°	45°	60°	80°	85°
Sky-Brightness taking into account unpolarised scattering / Sky-Brightness taking into account only Rayleigh scattering	1·10	1·07	1·06	$1{\cdot}05_5$	$1{\cdot}05_5$

For comparison, the values calculated by Gruner for wave-lengths $5 \cdot 5 \times 10^{-5}$ cm. and $6 \cdot 4 \times 10^{-5}$ cm. are also given, after being reduced to the same units as are here used. It will be noticed that Gruner's values are much higher than those calculated in this paper. The main reason for this difference is the adoption by Gruner of scattering coefficients at Mount Whitney on the assumption that the attenuation is solely due to scattering. Equation (3) shows that the calculated brightness of the sky in any direction will be proportional to the value of S. The following table gives the values of scattering coefficients calculated according to Rayleigh's law and adopted in this paper and the values calculated from the attenuation coefficients at Mount Whitney.

Wave-lengths.	Scattering coefficients.	
	According to Rayleigh's law.	Calculated from the extinction coefficients at Mount Whitney.
$4 \cdot 5 \times 10^{-5}$ cm.	$1 \cdot 52 \times 10^{-8}$	$2 \cdot 05 \times 10^{-8}$
$5 \cdot 5 \times 10^{-5}$ cm.	$0 \cdot 679 \times 10^{-8}$	$1 \cdot 10 \times 10^{-8}$
$6 \cdot 5 \times 10^{-5}$ cm.	$0 \cdot 348 \times 10^{-8}$	$0 \cdot 70 \times 10^{-8}$

It is obvious that the adoption of values in column (3) instead of those in column (2) would lead to much higher values for the calculated brightness.

Other less important reasons for the difference are to be found in (1) that Gruner has calculated the brightness of the sky as seen from a point 4 km. above sea-level and (2) the difference in the methods of calculation.

Taking the calculated values of the intensities near $0 \cdot 55\mu$ (the most luminous part of the spectrum) as a rough guide, the differences from the calculated values are such as can be explained by the added scattering and extinction caused by the presence of dust in the atmosphere.

Dorno states that the more transparent the sky, the darker is the region of the sky on the side of the sun and the brighter the region against the sun. The contrast between the two regions was larger in summer and autumn than in winter and spring; at a distance of 10° to 30° from the sun, the excess of the brightness of the summer sky over the annual mean and the defect of brightness of the spring sky were as much as 50 per cent. The agreement with calculated values would therefore be improved if the values observed in winter and spring are alone considered.

In the above calculations, we have neglected the effect of secondary scattering. There is no doubt that this part is not negligible as is evidenced by the blue colour of the sky in the earth-shadow below the lower limit of the purple counterglow and the absence of colour-match of the two components when the light from the vertical sky is analysed by a double image prism. We should expect the effect of the secondary scattering to be much more serious for the shorter wave-lengths than for the longer ones. Not only is this due to the λ^{-4} law; it is also due to the way in which different layers of the atmosphere contribute to the total illumination. This will be clear from figures 5 and 6. Figure 5 represents the intensities of illumination due to a layer of the atmosphere 1 cm. in thickness and subtending a unit solid angle at the observer's eye for different heights of the layer and for three different wave-lengths. The peak of the curve occurs at a greater height for the shorter wave-lengths. Thus the heights of the layers which contribute the maximum effect for wave-lengths 0·45μ, 0·55μ and 0·65μ are 18 km., 12 km., and 10 km., respeetively. Figure 6 represents the intensities due to a layer 1 cm. in *radial* thickness coming from a direction perpendicular to the sun's rays and at a zenith distance of 80°. The peaks of the curve occur at heights 15 km., 12 km. and 9 km. It will also be noticed that the curve for 0·45μ is flatter than the one for 0·55μ and this again flatter than that for 0·65μ. In a case like this where most of the scattered light has its origin in a comparatively thin layer, the self-illumination will not be so prominent as when the illumination is diffused over a much larger th ckness. As a result of secondary scattering, while the values of intensity in table 10 would be fairly correct for the longest wave-lengths, they will fall short of the actual illumination for the shorter waves.

Polarisation of sky-light in different regions of the spectrum when the sun is just below the horizon.

It is well known that the polarisation of the zenith sky on clear days increases as the sun gets farther from the zenith and that the maximum value of the polarisation is observed when the sun is 1° or 2° below the horizon. This is clearly due to the very small contribution to the sky-illumination made by the lower dusty atmosphere at that time, and also to the diminution of self-illumination and earth-illumination in the atmosphere. From the way in which different layers of atmosphere contribute to the sky-illumination in different regions of the spectrum, it may be expected that the polarisation in a direction perpendicular to the sun's rays should be greatest for those wave-lengths for which the effect of self-illumination is least, that is, for the longest waves. Observations made by the writer during the winter of 1925-26 in the exceptionally favourable atmosphere of Simla confirm this, the polarisation of the zenith sky being distinctly greater for the red than for the blue. In a recent paper, N. N. Kalitin of Pawlawsk,[1] has recorded that under the best conditions of transparency of the atmosphere, the polarisation is always greater in the red than in the blue, and that as the sun sinks down, the former increases more rapidly than the latter. We may expect that the effect of secondary illumination would be a minimum if we take observations about the time of sunrise and sunset in red light at a point in the heavens a few degrees above the horizon perpendicular to the sun's rays where, as the curves in figure 5 indicate, the quantity of red light coming from the sky is greater than that of the blue and is confined to a layer of smaller thickness and nearer the surface of the earth. This conclusion has also been confirmed by observation. The polarisation measured with a double image prism and nicol a few degrees above the southern horizon at Simla with a *deep red* filter, showed that on clear days, the polarisation was distinctly larger than on the zenith, the value on many occasions going up to 87 per cent. (corresponding to a depolarisation of 7 per cent.) thus closely approaching the value of the polarisation of the transversely

[1] N. N. Kalitin; *Met. Zs.*, **43**, 1926, p. 132.

scattered light for pure air. The maximum polarisation (also measured with a deep red filter) near the zenith was about 3 per cent. lower.[1] We can form an approximate estimate of the intensity of self-illumination if we assume that on clear days, the excess of depolarisation over what is to be expected in consequence of the anisotropy of the molecules is due to this cause. Now, in very clear weather, the light from the zenith sky shows a depolarisation of 9 to 11 per cent. when the sun is on or just near the horizon. Let us take 10 per cent. as a mean value. If there were only primary scattering, the depolarisation of the light from the zenith sky will be 4·5 per cent. If we call this ratio B/(A + B) and if we assume the self-illumination to be unpolarised $0{\cdot}10 = \frac{B + X}{A + B + X}$ where 2 X is the intensity due to self-illumination. Hence $\frac{2X}{A + 2B + 2X} = 10$ per cent. Thus the intensity due to self-illumination is nearly 10 per cent. of the total intensity.

SUMMARY.

(1) Some features of sky-illumination at sunrise and sunset which are observable in the clearest weather at high-level stations are pointed out.

(2) The previous work of Gruner on the calculation of sky-illumination when the sun is near the horizon is reviewed and a more rigorous method of calculating the intensities when the sun is on the horizon worked out.

(3) Intensities of sky-illumination are calculated for different portions of the sky when the sun is on the horizon for the wave-lengths $0{\cdot}45\mu$, $0{\cdot}55\mu$, and $0{\cdot}65\mu$.

(4) The results explain the changes of colour of the sky near sunrise and sunset The calculated values are of the same order as the values of brightness observed by Dorno at Davos, but are generally smaller.

(5) It is pointed out that the way in which different layers of the atmosphere contribute towards twilight illumination depends on the wave-lengths, the shorter wavelengths coming more from the higher layers.

(6) The polarisation of sky-light perpendicular to the sun's rays when the sun is 1° to 2° below the horizon is greater for red rays than for the blue. The greatest polarisation is observed in an azimuth of 90° a few degrees above the horizon, isolating red light alone by means of a deep red filter. The results are believed to be due to the smaller influence of self-illumination for the longer rays and to the fact that these rays originate in a comparatively thinner layer and nearer the surface of the earth.

The maximum value of the polarisation observed at Simla in very clear weather during the winter of 1925-26 in red light was 87 per cent. This value was reached on many days.

The writer hopes to extend the computation of sky brightness for positions of the sun below the horizon.

The author's thanks are due to Professor C. V. Raman for his interest in the work. The double image prism and nicol with which sky polarisation measurements were made were kindly lent by him.

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[1] J. J. Tichanowsky (*Met. Zs.*, 43, 1926, p. 154) has recently obtained a maximum value for the polarisation of the zenith sky of 84·7 per cent. at Ai Petri (1180 metres above sea-level).

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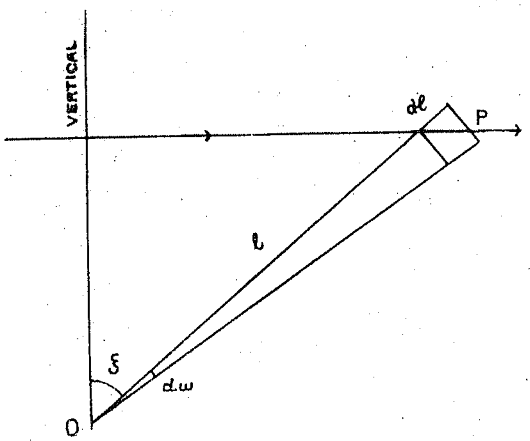

FIG. 1

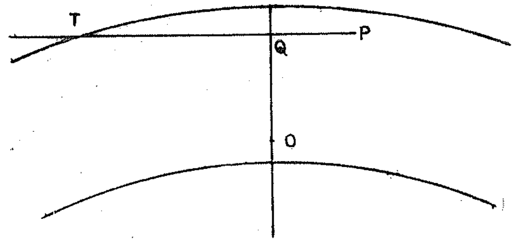

FIG. 2

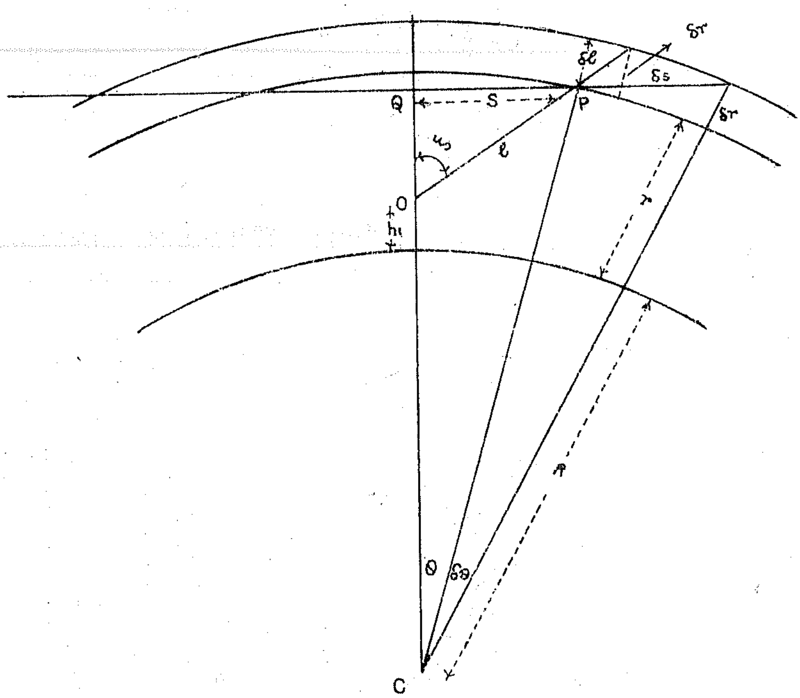

FIG. 3

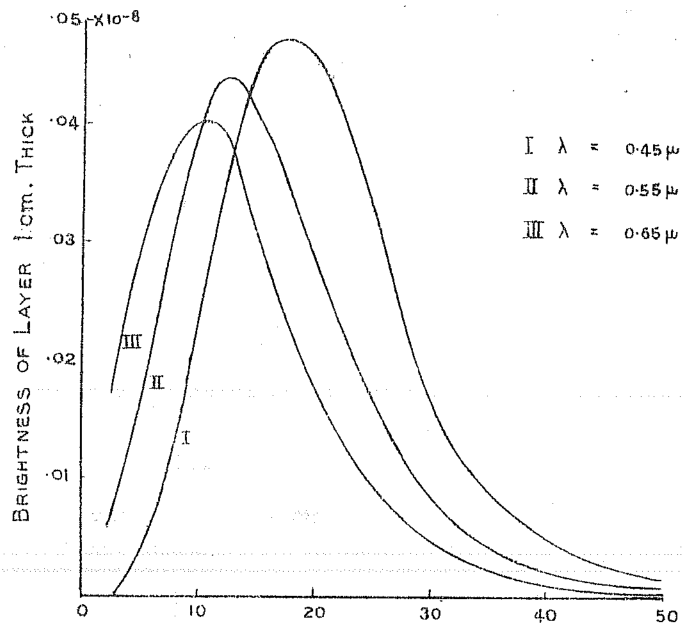

Fig. 4.

Brightness of Different Layers of the Zenith Sky.

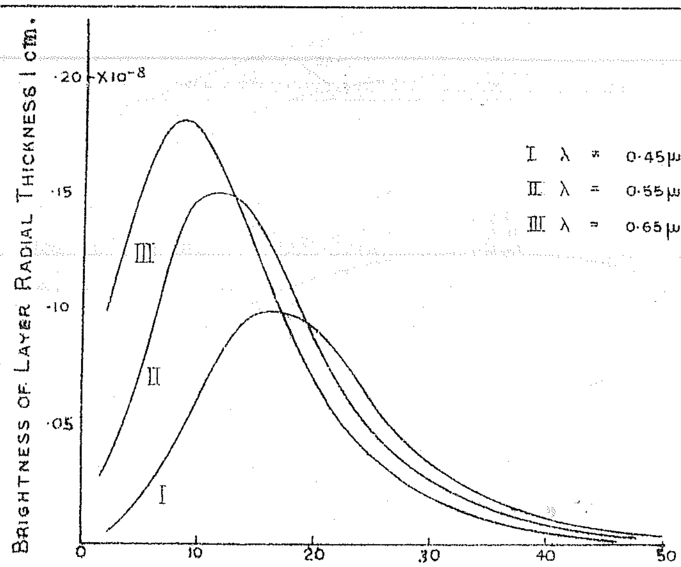

Fig. 5.

Brightness of Different Layers of Atmosphere of Radial thickness 1 cm. in direction $\zeta = 80^\circ, \alpha = 90^\circ$