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FORTRAN PROGRAMMES FOR SPECTROSCOPIC CALCULATIONS IN (sdg)-BOSON SPACE : THE PACKAGE SDGIBM1

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12. ABSTRACT : This report documents the package SDGIBM1 which enables one to perform nuclear spectroscopic calculations in sdg boson space. All the necessary formulas and equations that are used in the programmes are given. A comprehensive upto date review of the sdg interacting boson model of atomic muclei together with a bibliography on the subject is also included.

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1. Introduction

The Interacting Boson Model of atomic nuclei was introduced by Arima and Iachello [1] in 1975. This model provides an algebraic description of the quadrupole collective properties of low-lying states in Nuclei in terms of a system of interacting bosons. The bosons are assumed to be made up of correlated pairs of valence nucleons (as Iachello puts it the correlations are so strong that the pairs lose the memory of being fermions) and they carry angular momentum ℓ=0 (s boson) or ℓ = 2 (d boson). The bosons are allowed to interact via (1+2) - body forces and the boson number N (which is half the number of valence nucleons (particles or holes)) is assumed to be conserved. This model is remarkably successful in explaining experimental data [2,3]. In the words of Feshbach [4], "IBM indeed has generated a renaissance in the feild of Nuclear Spectroscopy". One of the most important features of IBM is that it has a rich group structure. This model admits three (only three) dynamical symmetries [5] denoted by the groups SU(5), SU(3) and O(6) and they describe the vibrational, axially symmetric deformed and γ - unstable nuclei, respectively. Apart from these limiting situations, the model allows for rapid analysis of experimental data in the transitional nuclei (several of them interpolate the limiting situatiuions [6], for example $_{62}^{}Sm$, $_{76}^{}Os$, $_{44}^{}Ru$ isotopes interpolate SU(5) and SU(3), SU(3) and O(6) and SU(5) and O(6) respectively) as the general IBM Hamiltonian possess only seven free parameters and the model - space dimensions are small ($\sim$ 100, for N $\leq$ 15); Scholten [7] has written a computer code (PHINT) for detailed IBM calculations and

it is widely used. There is also a reasonably good understanding of the microscopic basis of IBM [8].

In the first two workshops (held in 1979 and 1980 respectively [4]) that recorded the success of IBM, in his concluding remarks Feshbach raised the following questions - "the success of the interacting boson model would indicate a whole new method for looking for symmetries. As an example, are there 4^+bosons and if so, what kind of symmetry is implied? In addition to the interaction among the 4^+ bosons, for example the 0^+ and 2^+ subspaces, what kind of broken symmetry is implied by the existence of the 4^+? Comparison with the experimental data would obviously help to determine the impact of the 4^+ boson ------.------. The question we now ask has correspondingly changed. We no longer ask if the s and d boson description is adequate. The question has become: What is the range of phenomena for which only s and d bosons are needed? ----- for even - even nuclei, indicates that including the $\ell = 4^+$ boson can on occasion be advantageous as noted by Barrett, Gelberg and VanIsacker ----. Turning now to microscopic theory, that is theory which attempts to relate the IBM to a more fundamental underlying theory ------ the more fundamental theory is for most part the nuclear shell model -------. In the microscopic theories, will it be necessary to include the $\ell = 4^+$ boson?". Taking these questions and remarks seriously several research groups (Tokyo, Australia, Belgium, Groningen, Ahmedabad etc.) started exploring the extended sdg interacting boson model (we refer to it as gIBM and reserve the name IBM for the sd boson model) . Steady growth of

experimental data in the past few years on the hexadecupole (E4) properties of nuclei across the periodic table, the microscopic theories of IBM which brought out the importance of $G(L^{\pi} = 4^{+})$ pairs and the various specific features ($K^{\pi} = 3^{+}$ band, 1^{+} levels, extension of band cut - off, g - factor variation etc.) that appear in the low - lying levels etc. clearly point out that g - bosons should be included in IBM; see sect.6 ahead for a detailed description of the (direct and indirect) evidence for including g bosons. Broadly speaking three different approaches are adopted in the literature to take into account g - boson effects. The first method, is to use some renormalization technique so that one can still employ (sd) boson space; this method fails for states that lie outside the (sd) space such as the $k^{\pi} = 3^{+}$ band in ^{168}Er, the 4^{+}state in ^{192}Os, the 6^{+}state in ^{20}Ne, etc. The second method is to allow for weak coupling, which is equivalent to performing calculations in a truncated space consisting of $(sd)^{N}$, $(sd)^{N-1}(g)^{1}$, ..., $(sd)^{N-n_g}(g)^{n_g}$ configurations, with the g - boson number n_g taken to be small ($\sim$ 2 - 3). The third method, which corresponds to strong coupling, is to treat g bosons on equal footing with (sd) bosons; the studies based on dynamical symmetries and those involving intrinsic states belong to this category. In Sect.7 we will give a review of the important results obtained in gIBM so far. There is a general beleif [2], derived from the success of IBM, that the second method above, which is clearly numerical in nature, is appropriate for a majority of nuclei; the only exceptions may be heavy deformed nuclei like ^{156}Gd, ^{168}Er, etc. However, the existing weak - coupling

calculations are limited to the cases with $n_g \leq 1$, and there is a clear need to go beyond this. Atleast to verify the adequacy of $n_g = 1$ calculations one should be able to handle $n_g = 2 - 4$ cases. Therefore over the last couple of years we constructed a general code that is suitable for the weak coupling approach and it will enable us to analyse systematically the E4 properties of rare-earth and actinide nuclei.The purpose of this technical report is to give a write up of the programmes. We will now give a preview.

In Section 2 the theory needed for constructing the gIBM Hamiltonian(H) matrices in $(sdg)^N$ spaces is given; we essentially collect all the needed formulas without going into details of their derivatons.In Section 3 some of the gIBM model interactions are discussed and expressions for their two particle matrix elements are given. In Section 4 the long write up of the nine programmes (DIMVTL, VRACAH, DIAGON, BTBME, CFP, MAINHH, SUBGH, MAE2E4 and SUBGM) that enable us to construct and diagonalise the gIBM H-matrices and calculate matrix elements of transition operators, is given. The package consisting of the above nine programmes is called SDGIBM1. For each programme the write up consists of (1) General purpose (2) Subroutines/common blocks, (3) Method,(4) Internal/external parameters,(5) Input,(6) Output, (7) Special features/restrictions if any, and (8) References. Section 5 describes variousways of testing the programmes. In Section 6 we collect the evidence for including g bosons in IBM. In Section 7 we give a brief review of the work done on gIBM till to date and finally in Section 8 some concluding remarks are made.

All the programmes are written in Fortran and they are tested on DEC-1091 system at the Physical Research Laboratory, Ahmedabad. They can be run on any system that admits Fortran-77, we tested them on VAX-8800 machine at the University of Rochester. All the programmes (in SDGIBM1) are on a tape and they are available upon request. The package SDGIBM1 is ~4000 lines long. All the programmes in the package are thoroughly checked (see Sect.5 ahead), however, if any user encounters a problem, he/she should contact:

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Let us mention that the subroutines BLOCKS, DWR3 and DRR3 (used in the programme BTBME) are taken from the SU(3) package of Draayer and Akiyama [9], the programme MCONF of DIMVTL is discussed in detail in ref [10], the programmes TRED2, IMTQL2 of DIAGON are taken from the EISPACK package [11] and the programme VRACAH, developed by Alan Burgess of Cambridge (England), is supplied to us by V.B.Sheorey.

2. Theory

In this section we collect all the relevant expressions (most of them are derived by us) that are needed for performing spectroscopic calculations in sdg - boson space. The weak coupling basis defined by s,d and g boson numbers n_s,n_d and n_g respectively is employed throughout. We should remark that all the expressions given in this report use the fact that the single boson angular momenta (ℓ) are for us, even numbers (ℓ = 0,2 and 4 for s,d and g bosons respectively); therefore the phase factors $(-1)^{\ell}$ are dropped at will. In 2.1 we define the weak coupled basis states. In 2.2 expressions for calculating identicle boson fractional parentage coefficients (CFP) in single ℓ - orbit space are collected. In 2.3 the general gIBM H is defined and in 2.4 the matrix elements in terms of CFP, of the various parts of H in weak coupled basis states are given. Similarly in 2.5 transition operators in gIBM are defined and expressions for their matrix elements in weak coupled basis states are written down.

2.1 Basis States

In order to construct the gIBM Hamiltonian matrix in many particle spaces, we adopt the weak coupled basis,

$$| n_s ; n_d v_d \alpha_d L_d ; n_g v_g \alpha_g L_g ; L > \qquad (1)$$

where v_d and v_g are the d and g boson seniority quantum numbers respectively and α's are multiplicity labels which will give the number of times a certain L value occurs for a given v. Note that the total boson number $N = n_s + n_d + n_g$. The basis (1) corresponds

to the group chain

$$
\begin{array}{ccccc}
U(15) & \supset & U_s(1) & \oplus & U_d(5) \quad \oplus \quad U_g(9) \\
\{N\} & & \{n_s\} & & \{n_d\} \qquad \{n_g\} \\
 & & & & \cup \qquad\quad \cup \\
 & & & & [v_d] \; O_d(5) \quad O_g(9) \; [v_g] \\
 & & & & \cup \qquad\quad \cup \\
 & & & & \alpha_d L_d \; O_d(3) \; \oplus \; O_g(3) \; \alpha_g L_g \\
 & & & & \qquad\quad \cup \\
 & & & & \qquad O_{sdg}(3) \\
 & & & & \qquad\quad \downarrow \\
 & & & & \qquad\quad L
\end{array}
\qquad (2)
$$

The method described in Ref.12 is used to obtain the reductions $v_d \rightarrow L_d$ and $v_g \rightarrow L_g$ and in Table - 1a,1b we give the results for

Table-1a,1b

$v_d \leq 15, L_d \leq 20$ and $v_g \leq 6$, $L_g \leq 20$. In fact Tables - 1a,1b give the number of times (multiplicity) a given L_d occurs for a fixed value of v_d (similarly for L_g in v_g). For example with $v_g = 5$, the multiplicity for $L_g = 6$ is five and this means that the $\alpha_g = 1, 2, 3, 4$ and 5. In Table - 2 we give the Hamiltoniian

Table - 2

dimensionalities D(N,L) in sdg space for various boson numbers and L values ($N \leq 15$ and $L \leq 12$). These results follow easily by repeated use of the reductions $v_d \rightarrow L_d$ and $v_g \rightarrow L_g$. In the weak coupling approximation the value of n_g will not be large; usually it is expected to be $\leq 2-3$ (see the exercise carried out by Yoshinaga in Ref [13]). Therefore we also give in Table - 2 the dimensions for [N,L] with $n_g \leq 0, 1, 2, 3, 4$. For example with 8 - bosons the matrix dimension for L = 2 space is 851, however, with the restrictions $n_g = 0, \leq 2, \leq 4$ the dimensions are 15, 167 and 511 respectively. We see from Table - 2 that D(N,L) grow rapidly

with N. For example, with N = 10, L = 6 one has to diagonalize a 6352 x 6352 matrix and it becomes 487 x 487 matrix if we impose the restriction $n_g \leq 2$.

The numbers D(N,L), to a good accuracy can be quickly estimated using a stastical theory [14]. With M the eigenvalue of the Z - component L_z of L, one can see that fixed (N,M) dimensions will have Gaussian distribution (because of the action of Central Limit Theorem (CLT)). In sdg - space we have,

$$d(N,M) = \binom{N+14}{14} \left\{ \sqrt{2\Pi}\ \sigma_L \right\}^{-1} \exp - \frac{1}{2} \left[\frac{M^2}{\sigma_L^2} \right]$$

where

$$\sigma_L^2 = \langle L_z^2 \rangle^N = \{ \sum_\alpha < N\ \alpha \mid L_z^2 \mid N\ \alpha > \} \ / \ d(N) = \frac{7}{24} N(N+15) \qquad (3)$$

The dimensions D(N,L) follow from (3) by simple subtraction,

$$\begin{aligned} D(N,L) &= d(N,M = L) - d(N,M = L+1) \\ &\simeq - \frac{\partial}{\partial L} d(N,M = L + 1/2) \\ &= \binom{N+14}{14} \frac{(2L+1)}{\sqrt{8\Pi}\ \sigma_L^3} \exp - \frac{(L+1/2)^2}{2\sigma_L^2} \end{aligned} \qquad (4)$$

Using (3), (4) we find D(8,8) = 1405, D(12,10) = 24555 and D(14,4) = 51258 while the exacty values (see Table - 2) are 1460, 25008 and 53748 respectively, the error is $\sim$ 5%. One can improve upon (4) by adding corrections to the Gaussian form in (3). Employing the Edgeworth form [15], the first correction turns out to be $\frac{\gamma_2}{72} \times \left[\frac{M^4}{\sigma_L^4} - \frac{6M^2}{\sigma_L^2} + 3 \right]$ where the excess parameter

$$\gamma_2 = \frac{\langle L_z^4 \rangle}{\sigma_L^4} - 3;$$

$$\langle L_z \rangle^4 = N(N+15)[16.17.18]^{-1}\left\{[14.15 + 6N(N+15)] * \frac{742}{15} + 45(N-1)(N+16)\frac{4900}{225}\right\}$$

It is easy to see that $\gamma_2 \simeq 0$ for almost all N. Therefore one has to include still higher order corrections and this can be done if needed. Using (sd) and (g) orbits as two unitary orbits (#1, #2) one can calculate $\bar{D}_{12}(n_s + n_d, n_g, M) = D_1(n_s + n_d) \otimes D_2(n_g)$ [M] where ⊗ stands for convolution. The other entries (except the last one) in Table - 2 are given by $D_{12}(n_{sd}, n_g^{max}, L)$ where $(n_{sd} = N-n_g)$,

$$D_{12}(n_{sd}, n_g^{max}, L) = \sum_{n_g=0}^{n_g^{max}} \left\{\bar{D}_{12}(n_{sd}, n_g, M=L) - \bar{D}_{12}(n_{sd}, n_g, M=L+1)\right\}$$

$$\bar{D}_{12}(n_{sd}, n_g, M) = \binom{n_{sd}+5}{5}\binom{n_g+8}{8}\left\{2\Pi\sigma^2\right\}^{-1/2} \exp - \frac{M^2}{2\sigma^2}$$

$$\sigma^2 = \sigma_{sd}^2 + \sigma_g^2 = \frac{5}{21} n_{sd}\left[n_{sd}+6\right] + \frac{2}{3} n_g\left[n_g+9\right]$$

(5)

Using (5) we obtain for $(n_{sd}, n_g^{max}, L) = (8,4,6)$, $(10,3,4)$, $(10,2,4)$, $(12,4,6)$ the values of D_{12} to be 834, 1033, 426, 4653 respectively and the corresponding numbers that follow from Table - 2 are 829, 1028, 447, 4589 respectively. As in the case of D(N,L) one can add γ_2 correction for D_{12}; note that γ_2 expressions follow easily from Kota and Potbhare in ref[14].

2.2 Coefficients of Fractional Parentage (Identicle Boson CFP's)

Given identicle bosons each having angular momentum 'ℓ', one can expand (m+1) particle states with definite seniority (v_f) and angular momentum (L_f) in terms of the product of m - particle and one particle states. Then the resulting expansion coefficients are called one particle fractional parentage coefficients (CFP),

$$|\, m+1, v_f, \alpha_f, L_f \,\rangle = \sum_{v_i, \alpha_i, L_i} \langle m, v_i, \alpha_i, L_i, 111\ell \,|\} m+1, v_f, \alpha_f, L_f \,\rangle \; |m, v_i, \alpha_i, L_i; 111\ell; L_f\rangle \tag{6}$$

In (6) $\langle \ldots . |\} \ldots \rangle$ is the m x 1 → m+1 1 - particle CFP. With $b^{\dagger}_{\ell m}$ and $b_{\ell m}$ representing one - boson creation and destruction operators respectively and $\tilde{b}_{\ell m} = (-1)^{\ell - m} b_{\ell - m}$, the CFP in (6) can be written in terms of the reduced matrix elements of $b^{\dagger}$,

$$\langle m, v_i, \alpha_i, L_i, 111\ell \,|\} m+1, v_f, \alpha_f, L_f \,\rangle = \{(2L_f+1)(m+1)\}^{-1/2} \times \langle m+1, v_f, \alpha_f, L_f \,\|\, b^{\dagger}_{\ell} \,\|\, m, v_i, \alpha_i, L_i \,\rangle$$

$$\langle m, v_i, \alpha_i, L_i \,\|\, \tilde{b}_{\ell} \,\|\, m+1, v_f, \alpha_f, L_f \,\rangle = (-1)^{L_i + L_f} \times \langle m+1, v_f, \alpha_f, L_f \,\|\, b^{\dagger}_{\ell} \,\|\, m, v_i, \alpha_i, L_i \,\rangle \tag{7}$$

Calculation of CFP's in (6) follow easily from the recognition that the basis $|m, v, \alpha.L\rangle$ correspond to the group chain

$$\Big| \underset{\{m\}}{U(2\ell+1)} \supset \underset{\{m\}}{SU(2\ell+1)} \supset \underset{[v]}{O(2\ell+1)} \supset \underset{\alpha L}{O(3)} \Big\rangle \tag{8}$$

and a class of reduced Wigner coefficients for the above chain give the 1 - particle CFP. The operators u^k_q defined by,

$$u^k_q = (b^{\dagger}_{\ell} \tilde{b}_{\ell})^k_q \tag{9}$$

form a closed algebra and they generate the group U(2ℓ+1),

$$[u^k_q, u^{k'}_{q'}] = \sqrt{(2k+1)(2k'+1)} \sum_{k"} (-1)^{k"} \langle kqk'q' \| k"q" \rangle * \begin{Bmatrix} k & k' & k" \\ \ell & \ell & \ell \end{Bmatrix} \left\{1-(-1)^{k+k'+k"}\right\} u^{k"}_{q"} \tag{10}$$

It is easy to see from (10) that u^k_q, k=1,2,.....,2ℓ generate SU(2ℓ+1) group, u^k_q with k being odd (1,3,....,2ℓ-1) generate O(2ℓ+1) group and u^1_q generate the O(3) group respectively. The quadratic Casimir operators of the various groups in (8) in terms of the generators u^k_q and their eigenvalues are given in Table - 3.

Table - 3

The 1 - particle CFP follow by diagonalising $aC_2(SU(2\ell+1)) + bC_2(O(2\ell+1))$ in the basis $|m, v_i, \alpha_i, L_i; 111\ell; L\rangle$. The product of m x 1 particle space produces symmetric { m + 1} states and mixed symmetric states { m, 1}. One should choose the constants a and b given above such that the eigenvalues corresponding to { m + 1 } and { m, 1 } should be distinct. Following Bayman and Lande [16] we have,

$$\langle m-1,\nu_f,\alpha_f,L_f;111\ell;L|C_2(SU^B(2\ell+1))|m-1,\nu_i,\alpha_i,L_i;111\ell;L\rangle =$$

$$\delta_{\nu_f,\nu_i}\delta_{\alpha_f,\alpha_i}\delta_{L_f,L_i}\left[\frac{2\ell(m-1)(2\ell+m)}{(2\ell+1)}+\frac{2\ell(2\ell+2)}{(2\ell+1)}-\frac{2(m-1)}{(2\ell+1)}\right]+2(m-1)\sqrt{(2L_i+1)(2L_f+1)}\times$$

$$(-1)^{L_i+L_f}\times\sum_{\nu''\alpha''L''}\begin{Bmatrix}\ell & L_i & L''\\ \ell & L_f & L\end{Bmatrix}\times$$

$$\langle m-2,\nu'',\alpha'',L'';111\ell|\}m-1,\nu_f,\alpha_f,L_f\rangle\times$$

$$\langle m-2,\nu'',\alpha'',L'';111\ell|\}m-1,\nu_i,\alpha_i,L_i\rangle$$

$$\langle m-1,\nu_f,\alpha_f,L_f;111\ell;L|C_2(O^B(2\ell+1))|m-1,\nu_i,\alpha_i,L_i;111\ell;L\rangle =$$

$$\delta_{\nu_f,\nu_i}\delta_{\alpha_f,\alpha_i}\delta_{L_f,L_i}\left[\nu_f(\nu_f+2\ell-1)+2\ell\right]$$

$$+2(m-1)\sqrt{(2L_i+1)(2L_f+1)}\,(-1)^{L_i+L_f}\times$$

$$\left\{\sum_{\nu''\alpha''L''}\left[\begin{Bmatrix}\ell & L_i & L''\\ \ell & L_f & L\end{Bmatrix}\times\right.\right.$$

$$\langle m-2,\nu'',\alpha'',L'';111\ell|\}m-1,\nu_f,\alpha_f,L_f\rangle\times$$

$$\left.\langle m-2,\nu'',\alpha'',L'';111\ell|\}m-1,\nu_i,\alpha_i,L_i\rangle\right]$$

$$-\left[(2L+1)^{-1}\sum_{\substack{\nu''\alpha''L''\\(L''=L)}}\right.$$

$$\langle m-2,\nu'',\alpha'',L'';111\ell|\}m-1,\nu_f,\alpha_f,L_f\rangle\times$$

$$\left.\left.\langle m-2,\nu'',\alpha'',L'';111\ell|\}m-1,\nu_i,\alpha_i,L_i\rangle\right]\right\} \qquad (11)$$

The recursive method defined by (11) is used to generate CFP's on the machine and a programme for the same is described in Sect 4.

2.3 General gIBM Hamiltonian

The general (1+2) - body gIBM Hamiltonian H = h+v that preserves angular momentum (and also conserves the boson number)

contains 35 parameters; 3 single - particle energies (defining h) and 32 2 - body matrix elements (defining V). With $\varepsilon_s, \varepsilon_d$ and ε_g the three single particle energies and $V^L(\text{-----})$ the two - particle matrix elements of V, the explicit form of the gIBM Hamiltonian is,

$$
\begin{aligned}
H_{gIBM} &= \epsilon_s\left[s^+\cdot\tilde{s}\right] + \epsilon_d\left[d^+\cdot\tilde{d}\right] + \epsilon_g\left[g^+\cdot\tilde{g}\right] \\
&+ \tfrac{1}{2}V^0(ssss)\left[(s^+s^+)^0(\tilde{s}\tilde{s})^0\right]^0 + \tfrac{1}{2}V^0(ssdd)\left[(s^+s^+)^0(\tilde{d}\tilde{d})^0 + h\cdot c\cdot\right]^0 \\
&+ \tfrac{1}{2}V^0(ssgg)\left[(s^+s^+)^0(\tilde{g}\tilde{g})^0 + h\cdot c\cdot\right]^0 + \sqrt{5}V^2(sdsd)\left[(s^+d^+)^2(\tilde{s}\tilde{d})^2\right]^0 \\
&+ \sqrt{\tfrac{5}{2}}V^2(sddd)\left[(s^+d^+)^2(\tilde{d}\tilde{d})^2 + h\cdot c\cdot\right]^0 + \sqrt{5}V^2(sddg)\left[(s^+d^+)^2(\tilde{d}\tilde{g})^2 + h\cdot c\cdot\right]^0 \\
&+ \sqrt{\tfrac{5}{2}}V^2(sdgg)\left[(s^+d^+)^2(\tilde{g}\tilde{g})^2 + h\cdot c\cdot\right]^0 + 3V^4(sgsg)\left[(s^+g^+)^4(\tilde{s}\tilde{g})^4\right]^0 \\
&+ \tfrac{3}{\sqrt{2}}V^4(sgdd)\left[(s^+g^+)^4(\tilde{d}\tilde{d})^4 + h\cdot c\cdot\right]^0 + 3V^4(sgdg)\left[(s^+g^+)^4(\tilde{d}\tilde{g})^4 + h\cdot c\cdot\right]^0 \\
&+ \tfrac{3}{\sqrt{2}}V^4(sggg)\left[(s^+g^+)^4(\tilde{g}\tilde{g})^4 + h\cdot c\cdot\right]^0 \\
&+ \sum_{L_0=0,2,4} \tfrac{1}{2}\sqrt{2L_0+1}\, V^{L_0}(dddd)\left[(d^+d^+)^{L_0}(\tilde{d}\tilde{d})^{L_0}\right]^0 \\
&+ \sum_{L_0=2,4} \sqrt{\tfrac{2L_0+1}{2}}\, V^{L_0}(dddg)\left[(d^+d^+)^{L_0}(\tilde{d}\tilde{g})^{L_0} + h\cdot c\cdot\right] \\
&+ \sum_{L_0=0,2,4} \tfrac{1}{2}\sqrt{2L_0+1}\, V^{L_0}(ddgg)\left[(d^+d^+)^{L_0}(\tilde{g}\tilde{g})^{L_0} + h\cdot c\cdot\right]^0 \\
&+ \sum_{L_0=2,3,4,5,6} \sqrt{2L_0+1}\, V^{L_0}(dgdg)\left[(d^+g^+)^{L_0}(\tilde{d}\tilde{g})^{L_0}\right]^0 \\
&+ \sum_{L_0=2,4,6} \sqrt{\tfrac{2L_0+1}{2}}\, V^{L_0}(dggg)\left[(d^+g^+)^{L_0}(\tilde{g}\tilde{g})^{L_0} + h\cdot c\cdot\right]^0 \\
&+ \sum_{L_0=0,2,4,6,8} \tfrac{1}{2}\sqrt{2L_0+1}\, V^{L_0}(gggg)\left[(g^+g^+)^{L_0}(\tilde{g}\tilde{g})^{L_0}\right]^0 \qquad (12)
\end{aligned}
$$

In (12) $V^L(sdgg) = \langle (sd)L|V|(gg)L\rangle$ and similarly all other $V^L(----)$ are defined. In a one g-boson calculation several terms (all those containing $g^\dagger g^\dagger$ or $\tilde{g}\tilde{g}$) in (12) will not appear.

2.4 Many Particle Matrix Elements of gIBM Hamiltonian Operator

Given the general gIBM H (12) and the weak - coupled basis (1), we can derive the expressions for the many - particle matrix elements in terms of the reduced matrix elements of $d^\dagger$, $\tilde{d}$, $d^\dagger d^\dagger$, $d^\dagger\tilde{d}$, $\tilde{d}\tilde{d}$, $g^\dagger$, $\tilde{g}$, $g^\dagger g^\dagger$, $g^\dagger\tilde{g}$ and $\tilde{g}\tilde{g}$ operators. Here the standard angular momentum algebra [17] and the following relations are used,

$$\langle L_f \| (b_\ell^\dagger b_\ell^\dagger)^k \| L_i \rangle = \sqrt{2k+1}\ (-1)^{k+L_i+L_f} \sum_{\{L''\}} \begin{Bmatrix} \ell & \ell & K \\ L_i & L_f & L'' \end{Bmatrix} \langle L_f \| b_\ell^\dagger \| L''\rangle \langle L'' \| b_\ell^\dagger \| L_i \rangle$$

$$\langle L_f \| (\tilde{b}_\ell \tilde{b}_\ell)^k \| L_i \rangle = (-1)^{L_i+L_f} \langle L_i \| (b_\ell^\dagger b_\ell^\dagger)^k \| L_f \rangle$$

and

$$\langle L_f \| (b_\ell^\dagger \tilde{b}_\ell)^k \| L_i \rangle = \sqrt{2k+1}\ (-1)^{k+L_f} \sum_{\{L''\}} (-1)^{L''} \begin{Bmatrix} \ell & \ell & K \\ L_i & L_f & L'' \end{Bmatrix} \langle L_f \| b_\ell^\dagger \| L''\rangle \langle L_i \| b_\ell^\dagger \| L''\rangle \qquad (13)$$

In deriving (13) one uses the relations (6) and (7). Before giving the final expressions, some remarks on notations are in order. We use $\{B^f\}$ to denote the set of quantum numbers $(n_s^f;\ n_d^f,\ v_d^f,\ \alpha_d^f,\ L_d^f;\ n_g^f,\ v_g^f,\ \alpha_g^f,\ L_g^f;\ L^f)$ and similarly $\{B^i\}$. The set of quantum numbers

$(n_d^i, v_d^i, \alpha_d^i)$ are denoted by $\{n_d^i\}$ and similarly $\{n_d^f\}$, $\{n_g^i\}$ and $\{n_g^f\}$ are defined. The symbol $\delta_{\{a^f\}\{a^i\}}$ means that each of the quantum number in $\{a^f\}$ should be equal to the corresponding ones in $\{a^i\}$ for $\delta_{\{a^f\}\{a^i\}} = 1$ and otherwise $\delta_{\{a^f\}\{a^i\}} = 0$. The explicit expressions are,

$$\langle \{B^f\} \mid \varepsilon_s n_s \mid \{B^i\} \rangle = \varepsilon_s n_s^i \delta_{\{B^f\},\{B^i\}} \tag{14.a}$$

$$\langle \{B^f\} \mid \varepsilon_d n_d \mid \{B^i\} \rangle = \varepsilon_d n_d^i \delta_{\{B^f\},\{B^i\}} \tag{14.b}$$

$$\langle \{B^f\} \mid \varepsilon_g n_g \mid \{B^i\} \rangle = \varepsilon_g n_g^i \delta_{\{B^f\},\{B^i\}} \tag{14.c}$$

$$\langle \{B^f\} \mid \tfrac{1}{2} V^0(ssss)\{(s^\dagger s^\dagger)^0(\tilde{s}\tilde{s})^0\}^0 \mid \{B\}^i \rangle =$$

$$V^0(ssss) * \frac{n_s^i(n_s^i-1)}{2} \delta_{ij} \tag{15.V1}$$

$$\langle \{B^f\} \mid \tfrac{1}{2} V^0(ssdd)\{(s^\dagger s^\dagger)^0(\tilde{d}\tilde{d})^0\}^0 \mid \{B^i\} \rangle =$$

$$V^0(ssdd) * \frac{1}{2}\sqrt{\frac{(n_s^i+1)(n_s^i+2)}{(2L_d^i+1)}} *$$

$$\langle \{n_d^f\} L_d^f \| (\tilde{d}\ \tilde{d})^0 \| \{n_d^i\} L_d^i \rangle \delta_{n_s^f, n_s^i+2} *$$

$$\delta_{L_d^f, L_d^i} * \delta_{n_d^f, n_d^i-2} * \delta_{\{n_g^f\},\{n_g^i\}} * \delta_{L_g^f, L_g^i} \tag{15.V2}$$

$$\langle \{ B^f \} \mid \tfrac{1}{2} V^0(ssgg)\{(s^\dagger s^\dagger)^0(\tilde{g}\tilde{g})^0\}^0 \mid \{ B^i \} \rangle =$$

$$V^0(ssgg) * \frac{1}{2}\sqrt{\frac{(n_s^i+1)(n_s^i+2)}{(2L_g^i+1)}} *$$

$$\langle \{ n_g^f \} L_g^f \| (\tilde{g}\ \tilde{g})^0 \| \{ n_g^i \} L_g^i \rangle \delta_{n_s^f, n_s^i+2} *$$

$$\delta_{L_d^f, L_d^i} * \delta_{n_g^f, n_g^i-2} * \delta_{\{ n_d^f \},\{ n_d^i \}} * \delta_{L_g^f, L_g^i}$$

(15.V3)

$$\langle \{ B^f \} \mid \sqrt{5}\ V^2(sdsd)\{(s^\dagger d^\dagger)^2(\tilde{s}\tilde{d})^2\}^0 \mid \{ B^i \} \rangle =$$

$$V^2(sdsd) * n_s\, n_d\, \delta_{\{ B^f \},\{ B^i \}}$$

(15.V4)

$$\langle \{ B^f \} \mid \sqrt{\tfrac{5}{2}}\ V^2(sddd)\{(s^\dagger d^\dagger)^2(\tilde{d}\tilde{d})^2\}^0 \mid \{ B^i \} \rangle =$$

$$V^2(sddd) \left[\sqrt{\frac{n_s^i+1}{2}} \sum_{\{n_d^f-1\}L''} \frac{(-1)^{L''+L_d^i}}{(2L_d^i+1)} \right.$$

$$* \langle \{ n_d^f-1 \} L'' \| (\tilde{d}\ \tilde{d})^2 \| \{ n_d^i \} L_d^i \rangle *$$

$$\left. \langle \{ n_d^f \} L_d^f \| d^\dagger \| \{ n_d^f-1 \} L'' \rangle \right] * \delta_{n_s^f, n_s^i+1} *$$

$$\delta_{L_d^f, L_d^i} * \delta_{n_d^f, n_d^i-1} * \delta_{\{ n_g^f \},\{ n_g^i \}} * \delta_{L_g^f, L_g^i}$$

(15.V5)

$$\langle \{ B^f \} \mid \sqrt{5}\ V^2(sddg)\{(s^\dagger d^\dagger)^2(\tilde{d}\tilde{g})^2\}^0 \mid \{ B^i \} \rangle =$$

$$V^2(sddg) * \left[\sqrt{\frac{5}{9}}\ \sqrt{n_s^i+1}\ (-1)^{L_d^i + L_g^f + L} * \begin{Bmatrix} L & L_g^f & L_d^f \\ 4 & L_d^i & L_g^i \end{Bmatrix} * \right.$$

$$< \{ n_d^f \} L_d^f \| (d^\dagger \tilde{d})^4 \| \{ n^i{}_d \} L_d^i > *$$

$$< \{ n_g^f \} L_g^f \| \tilde{g} \| \{ n_g^i \} L_g^i > \Big] * \delta_{n_s^f, n_s^i+1} *$$

$$\delta_{n_d^f, n_d^i} * \delta_{n_g^f, n_g^i-1}$$

(15.V6)

$$< \{ B^f \} | \sqrt{\frac{5}{2}} \; V^2(sdgg)\{(s^\dagger d^\dagger)^2 (\tilde{g}\tilde{g})^2\}^0 | \{ B^i \} > =$$

$$V^2(sdgg) * \Bigg[\sqrt{\frac{n_s^i+1}{2}} \; (-1)^{L_d^i + L_g^f + L} * \begin{Bmatrix} L & L_g^f & L_d^f \\ 2 & L_d^i & L_g^i \end{Bmatrix} *$$

$$< \{ n_d^f \} L_d^f \| d^\dagger \| \{ n^i{}_d \} L^i{}_d > *$$

$$< \{ n_g^f \} L_g^f \| (\tilde{g}\tilde{g})^2 \| \{ n_g^i \} L_g^i > \Big] * \delta_{n_s^f, n_s^i+1} *$$

$$\delta_{n_d^f, n_d^i+1} * \delta_{n_g^f, n_g^i-2}$$

(15.V7)

$$< \{ B^f \} | \; 3 \; V^4(sgsg)\{(s^\dagger g^\dagger)^4 (\tilde{s}\tilde{g})^4\}^0 | \{ B^i \} > =$$

$$V^4(sgsg) * n_s \, n_g \, \delta_{\{ B^f \}, \{ B^i \}}$$

(15.V8)

$$< \{ B^f \} | \frac{3}{\sqrt{2}} \; V^4(sgdd)\{(s^\dagger g^\dagger)^4 (\tilde{d}\tilde{d})^4\}^0 | \{ B^i \} > =$$

$$V^4(sgdd) * \Bigg[\sqrt{\frac{n_s^i+1}{2}} \; (-1)^{L_d^i + L_g^f + L} * \begin{Bmatrix} L & L_g^f & L_d^f \\ 4 & L_d^i & L_g^i \end{Bmatrix} *$$

$$< \{ n_d^f \} L_d^f \| (\tilde{d}\tilde{d})^4 \| \{ n_d^i \} L_d^i > *$$

$$< \{ n_g^f \} L_g^f \| g^\dagger \| \{ n_g^i \} L_g^i > \Big] * \delta_{n_s^f, n_s^i+1} *$$

$$\delta_{n_g^f, n_g^i+1} * \delta_{n_d^f, n_d^i-2}$$

(15.V9)

$$< \{ B^f \} | 3 V^4(sgdg) \{ (s^\dagger g^\dagger)^4 (\tilde{\tilde{dg}})^4 \}^0 | \{ B^i \} > =$$

$$V^4(sgdg) * \Bigg[\sqrt{\frac{9}{5}} \sqrt{n_s^i+1} \; (-1)^{L_d^i + L_g^f + L} * \begin{Bmatrix} L & L_g^f & L_d^f \\ 2 & L_d^i & L_g^i \end{Bmatrix} *$$

$$< \{ n_d^f \} L_d^f \| \tilde{d} \| \{ n_d^i \} L_d^i > *$$

$$< \{ n_g^f \} L_g^f \| (g^\dagger \tilde{g})^2 \| \{ n_g^i \} L_g^i > \Big] * \delta_{n_s^f, n_s^i+1} *$$

$$\delta_{n_g^f, n_g^i} * \delta_{n_d^f, n_d^i-1}$$

(15.V10)

$$< \{ B^f \} | \frac{3}{\sqrt{2}} V^4(sggg) \{ (s^\dagger g^\dagger)^4 (\tilde{\tilde{gg}})^4 \}^0 | \{ B^i \} > =$$

$$V^4(sggg) \Bigg[\sqrt{\frac{n_s^i+1}{2}} \sum_{\{n_g^i - 2\} L''} \frac{(-1)^{L'' + L_g^i}}{(2L_g^f+1)}$$

$$* < \{ n_g^i - 2 \} L'' \| (\tilde{g} \; \tilde{g})^4 \| \{ n_g^i \} L_g^i > *$$

$$< \{ n_g^f \} L_g^f \| g^\dagger \| \{ n_g^i - 2 \} L'' > \Big] * \delta_{n_s^f, n_s^i+1} *$$

$$\delta_{L_d^f, L_d^i} * \delta_{n_g^f, n_g^i-1} * \delta_{\{ n_d^f \}, \{ n_d^i \}} * \delta_{L_g^f, L_g^i}$$

(15.V11)

$$\langle \{ B^f \} \mid \frac{\sqrt{2L_o+1}}{2} V^{L_o}(dddd)\{(d^\dagger d^\dagger)^{L_o}(\tilde{d}\tilde{d})^{L_o}\}^0 \mid \{ B^i \} \rangle =$$

$$V^{L_o}(dddd) \left[\frac{1}{2(2L_d^i+1)} \sum_{\{n_g^i-2\}L''} (-1)^{L''+L_d^i} \right.$$

$$* \langle \{ n_d^i-2 \} L'' \parallel (\tilde{d}\ \tilde{d})^{L_o} \parallel \{ n_d^i \} L_d^i \rangle *$$

$$\left. \langle \{ n_d^f \} L_d^f \parallel (d^\dagger d^\dagger)^{L_o} \parallel \{ n_d^i-2 \} L'' \rangle \right] * \delta_{n_s^f, n_s^i} *$$

$$\delta_{L_d^f, L_d^i} * \delta_{n_d^f, n_d^i} * \delta_{\{ n_g^f \}, \{ n_g^i \}} * \delta_{L_g^f, L_g^i}$$

for $L_o = 0,2,4$

(15.V12)

$$\langle \{ B^f \} \mid \sqrt{\frac{2L_o+1}{2}} V^{L_o}(dddg)\{(d^\dagger d^\dagger)^{L_o}(\tilde{d}\tilde{g})^{L_o}\}^0 \mid \{ B^i \} \rangle =$$

$$V^{L_o}(dddg) * \left[\sqrt{\frac{2L_o+1}{2}} (-1)^{L_g^f+L_d^f+L} * \begin{Bmatrix} L & L_g^f & L_d^f \\ 4 & L_d^i & L_g^i \end{Bmatrix} * \right.$$

$$\left\{ \sum_{\{n_d^i-1\}L''} \begin{Bmatrix} L_o & 2 & 4 \\ L_d^i & L_d^f & L'' \end{Bmatrix} * \right.$$

$$\langle \{ n_d^f \} L_d^f \parallel (d^\dagger d^\dagger)^{L_o} \parallel \{ n_d^i-1 \} L'' \rangle *$$

$$\left. \left. \langle \{ n_d^i-1 \} L'' \parallel \tilde{d} \parallel \{ n_d^i \} L_d^i \rangle \right\} * \right.$$

$$\left. \langle \{ n_g^f \} L_g^f \parallel \tilde{g} \parallel \{ n_g^i \} L_g^i \rangle \right] *$$

$$\delta_{n_s^f, n_s^i} * \delta_{n_d^f, n_d^i+1} * \delta_{n_g^f, n_g^i-1}$$

for $L_o = 2,4$

(15.V15)

$$\langle \{ B^f \} \| \sqrt{\frac{2L_o+1}{4}}\, V^{L_o}(ddgg)\{(d^\dagger d^\dagger)^{L_o}(\tilde{g}\tilde{g})^{L_o}\}^0 \| \{ B^i \} \rangle =$$

$$V^{L_o}(ddgg) * \left[\frac{1}{2} (-1)^{L_g^f+L_d^i+L} * \begin{Bmatrix} L & L_g^f & L_d^f \\ L_o & L_d^i & L_g^i \end{Bmatrix} * \right.$$

$$\langle \{ n_d^f \} L_d^f \| (d^\dagger d^\dagger)^{L_o} \| \{ n_d^i \} L_d^i \rangle *$$

$$\left. \langle \{ n_g^f \} L_g^f \| (\tilde{g}\tilde{g})^{L_o} \| \{ n_g^i \} L_g^i \rangle \right] *$$

$$\delta_{n_s^f, n_s^i} * \delta_{n_d^f, n_d^i+2} * \delta_{n_g^f, n_g^i-2}$$

for $L_o = 0,2,4$

(15.V17)

$$\langle \{ B^f \} \| \sqrt{2L_o+1}\; V^{L_o}(dgdg)\{(d^\dagger g^\dagger)^{L_o}(\tilde{d}\tilde{g})^{L_o}\}^0 \| \{ B^i \} \rangle =$$

$$V^{L_o}(dgdg) * \left[(2L_o+1)\, (-1)^{L_g^f+L_d^i+L+L_o} * \right.$$

$$\sum_{L'} \begin{Bmatrix} 2 & 4 & L_o \\ 4 & 2 & L' \end{Bmatrix} * \begin{Bmatrix} L & L_g^f & L_d^f \\ L' & L_d^i & L_g^i \end{Bmatrix} *$$

$$\langle \{ n_d^f \} L_d^f \| (d^\dagger \tilde{d})^{L'} \| \{ n_d^i \} L_d^i \rangle *$$

$$\left. \langle \{ n_g^f \} L_g^f \| (g^\dagger \tilde{g})^{L'} \| \{ n_g^i \} L_g^i \rangle \right] *$$

$$\delta_{n_s^f, n_s^i} * \delta_{n_d^f, n_d^i} * \delta_{n_g^f, n_g^i}$$

for $L_o = 2,3,4,5,6$

(15.V20)

$$\langle \{ B^f \} \left| \sqrt{\frac{2L_o+1}{2}}\, V^{L_o(dggg)} \{ (d^\dagger g^\dagger)^{L_o} (\tilde{\tilde{g}}\tilde{\tilde{g}})^{L_o} \}^0 \right| \{ B^i \} \rangle =$$

$$V^{L_o(dggg)} * \left[\sqrt{\frac{2L_o+1}{2}}\, (-1)^{L_g^f + L_d^i + L} * \begin{Bmatrix} L & L_g^f & L_d^f \\ 2 & L_d^i & L_g^i \end{Bmatrix} * \right.$$

$$\langle \{ n_d^f \} L_d^f \| d^\dagger \| \{ n_d^i \} L_d^i \rangle *$$

$$\left[\sum_{\{n_g^i - 2\} L''} \begin{Bmatrix} 4 & L_o & 2 \\ L_g^i & L_g^f & L'' \end{Bmatrix} * \right.$$

$$\langle \{ n_g^i - 2 \} L'' \| (\tilde{g}\,\tilde{g})^{L_o} \| \{ n_g^i \} L_g^i \rangle *$$

$$\left. \left. \langle \{ n_g^f \} L_g^f \| g^\dagger \| \{ n_g^i - 2 \} L'' \rangle \right] \right] *$$

$$\delta_{n_s^f, n_s^i} * \delta_{n_d^f, n_d^i + 1} * \delta_{n_g^f, n_g^i - 1}$$

for $L_o = 2,4,6$

(15.V25)

$$\langle \{ B^f \} \left| \frac{\sqrt{2L_o+1}}{2}\, V^{L_o(gggg)} \{ (g^\dagger g^\dagger)^{L_o} (\tilde{\tilde{g}}\tilde{\tilde{g}})^{L_o} \}^0 \right| \{ B^i \} \rangle =$$

$$V^{L_o(gggg)} \left[\frac{1}{2(2L_g^i+1)} \sum_{\{n_g^i - 2\} L''} (-1)^{L'' + L_g^i} \right.$$

$$* \langle \{ n_g^i - 2 \} L'' \| (\tilde{g} \tilde{g})^{L_o} \| \{ n_g^i \} L_g^i \rangle *$$

$$\langle \{ n_g^f \} L_g^f \| (g^\dagger g^\dagger)^{L_o} \| \{ n_g^i - 2 \} L'' \rangle \Big] * \delta_{n_s^f, n_s^i} *$$

$$\delta_{L_d^f, L_d^i} * \delta_{n_g^f, n_g^i} * \delta_{\{ n_d^f \}, \{ n_d^i \}} * \delta_{L_g^f, L_g^i}$$

for $L_o = 0,2,4,6,8$

(15.V28)

These equations (15.V1 - V28) are programmed in SUBGH (see Section 4).

2.5 Matrix Elements of Transition Operators in gIBM

An eigenfunction ψ_E of gIBM H in the basis $|\{B^i\}\rangle$ defined in (1), can be written in general as

$$|\psi_E\rangle = \sum_{\{B^i\}} C^E_{\langle B^i \rangle} |\{B^i\}\rangle \quad .$$

(16)

The transition strengths for electromagnetic (EM) decay, two-nucleon transfer etc. are given by the reduced matrix elements $\langle \psi_{E_f} \| T \| \psi_{E_i} \rangle$ of the corresponding operator T. From (16) it follows that one needs to evaluate $\langle \{B^f\} \| T \| \{B^i\} \rangle$ in order to produce transiiton strengths. Here below we discuss the form of the operators for EM transitions, occupancies and two nucleon transfer. The expressions for their matrix elements, in terms of 1 - particle CFP, in the $|\{B^i\}\rangle$ basis are also given.

2.5.1 EM Transition Operators

In gIBM the EM transition operators $T^{E\lambda}, T^{M\lambda}$ are taken to be of one-body type. A general k-th rank operator O^k in gIBM is given by

$$O^k_q = \sum_{\substack{\ell,\ell' \\ =0,2,4}} a^{(k)}(\ell,\ell')(b^{\dagger\sim}_{\ell} b_{\ell'})^k_q$$

(17)

In (17) above the parameters $a^{(k)}(\ell,\ell')$ are essentially g-factors for $T^{M\lambda}$ and effective charges for $T^{E\lambda}$. Taking properly into account the transformation properties of $T^{M\lambda}$ and $T^{E\lambda}$ under complex conjugation, one can easily write down the explicit forms for $T^{M\lambda}$ and $T^{E\lambda}$. For M1, M3,E2 and E4 transitions they are,

$$T^{M1} = \sqrt{\frac{3}{4\pi}}\left\{g^{(1)}(2,2)\left[d^{\dagger\sim}d\right]^1_q + g^{(1)}(4,4)\left[g^{\dagger\sim}g\right]^1_q\right\}$$

$$T^{M3} = \sqrt{\frac{7}{4\pi}}\left\{g^{(3)}(2,2)\left[d^{\dagger\sim}d\right]^3_q + g^{(3)}(2,4)\left[g^{\dagger\sim}d+d^{\dagger\sim}g\right]^3_q + g^{(3)}(4,4)\left[g^{\dagger\sim}g\right]^3_q\right\}$$

$$T^{E2} = e^{(2)}(0,2)\left[s^{\dagger\sim}d+d^{\dagger\sim}s\right]^2_q + e^{(2)}(2,2)\left[d^{\dagger\sim}d\right]^2_q +$$

$$e^{(2)}(2,4)\left[g^{\dagger\sim}d+d^{\dagger\sim}g\right]^2_q + e^{(2)}(4,4)\left[g^{\dagger\sim}g\right]^2_q$$

$$T^{E4} = e^{(4)}(0,4)\left[s^{\dagger\sim}g+g^{\dagger\sim}s\right]^4_q + e^{(4)}(2,2)\left[d^{\dagger\sim}d\right]^4_q +$$

$$e^{(4)}(2,4)\left[g^{\dagger\sim}d+d^{\dagger\sim}g\right]^4_q + e^{(4)}(4,4)\left[g^{\dagger\sim}g\right]^4_q$$

(18)

The expressions for static moments (Q_λ ,μ_λ), reduced transition strengths (B(Eλ),B(Mλ)) and the E2/M1 mixing ratios, in terms of reduced matrix elements of $T^{E\lambda}$ and $T^{M\lambda}$ are

$$Q_\lambda(L) = \sqrt{\frac{16\pi}{2\lambda+1}}\begin{pmatrix} L & \lambda & L \\ L & 0 & -L \end{pmatrix}\langle L\|T^{E\lambda}\|L\rangle$$

$$\mu_\lambda(L) = \sqrt{\frac{4\pi}{2\lambda+1}}\begin{pmatrix} L & \lambda & L \\ L & 0 & -L \end{pmatrix}(-)^\lambda\langle L\|T^{M\lambda}\|L\rangle$$

$$B(E\lambda;L_i \longrightarrow L_f) = (2L_i+1)^{-1}\left|\langle L_f\|T^{E\lambda}\|L_i\rangle\right|^2$$

$$B(M\lambda; L_i \longrightarrow L_f) = (2L_i+1)^{-1} \left| \langle L_f \| T^{M\lambda} \| L_i \rangle \right|^2$$

$$\delta(E2/M1; L_i \longrightarrow L_f) = 0.835\, E_\gamma(\text{in MeV}) \; \frac{\langle L_f \| T^{E2} \| L_i \rangle \text{ in e.b.}}{\langle L_f \| T^{M1} \| L_i \rangle \text{ in n.m.}}$$

(19)

In (19) $\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$ is a Wigner 3 - j symbol and E_γ is the γ - ray energy [18]. The expressions for $\langle \{ B^f \} \| (b_\ell^\dagger \tilde{b}_\ell)^k \| \{ B^i \} \rangle$, that determine via (16) and (19) the static moments and transition strengths, are

$$\langle \{ B^f \} \| (d^\dagger \tilde{d})^k \| \{ B^i \} \rangle = \delta_{n_s^f, n_s^i} * \delta_{\{n_g^f\}, \{n_g^i\}} * \delta_{n_d^f, n_d^i} *$$

$$(-1)^{k+L_d^f+L_g^i+L^i} \sqrt{(2L^f+1)(2L^i+1)} \begin{Bmatrix} L_d^f & L^f & L_g^i \\ L^i & L_d^i & k \end{Bmatrix} *$$

$$\langle \{ n_d^f \} L_d^f \| (d^\dagger \tilde{d})^k \| \{ n_d^i \} L_d^i \rangle$$

(20.01)

$$\langle \{ B^f \} \| (g^\dagger \tilde{g})^k \| \{ B^i \} \rangle = \delta_{n_s^f, n_s^i} * \delta_{\{n_d^f\}, \{n_d^i\}} * \delta_{n_g^f, n_g^i} *$$

$$\delta_{L_d^f, L_d^i} * (-1)^{k+L_d^i+L_g^i+L^f} \sqrt{(2L^f+1)(2L^i+1)} \begin{Bmatrix} L_d^f & L^f & L_d^i \\ L^i & L_g^i & k \end{Bmatrix}$$

$$* \langle \{ n_g^f \} L_g^f \| (g^\dagger \tilde{g})^k \| \{ n_g^i \} L_g^i \rangle$$

(20.02)

$$\langle \{ B^f \} \| (s^\dagger \tilde{d})^k \| \{ B^i \} \rangle = \delta_{n_s^f, n_s^i+1} * \delta_{\{n_g^f\},\{n_g^i\}} *$$

$$\delta_{n_d^f, n_d^i-1} * \delta_{L_g^f, L_g^i} * (-1)^{k+L_d^f+L_g^i+L^i} \sqrt{(2L^f+1)(2L^i+1)}$$

$$* \sqrt{(n_s^i+1)} \begin{Bmatrix} L_d^f & L^f & L_g^i \\ L^i & L_d^i & k \end{Bmatrix}$$

$$* \langle \{ n_d^f \} L_d^f \| \tilde{d} \| \{ n_d^i \} L_d^i \rangle$$

(20.031)

$$\langle \{ B^f \} \| (d^\dagger \tilde{s})^k \| \{ B^i \} \rangle = \delta_{n_s^f, n_s^i-1} * \delta_{\{n_g^f\},\{n_g^i\}} *$$

$$\delta_{n_d^f, n_d^i+1} * \delta_{L_g^f, L_g^i} * (-1)^{k+L_d^f+L_g^i+L^i} \sqrt{(2L^f+1)(2L^i+1)}$$

$$* \sqrt{n_s^i} \begin{Bmatrix} L_d^f & L^f & L_g^i \\ L^i & L_d^i & k \end{Bmatrix}$$

$$* \langle \{ n_d^f \} L_d^f \| d^\dagger \| \{ n_d^i \} L_d^i \rangle$$

(20.032)

$$\langle \{ B^f \} \| (s^\dagger \tilde{g})^k \| \{ B^i \} \rangle = \delta_{n_s^f, n_s^i+1} * \delta_{\{n_d^f\},\{n_d^i\}} *$$

$$\delta_{n_g^f, n_g^i-1} * \delta_{L_d^f, L_d^i} * (-1)^{k+L_d^i+L_g^i+L^f} \sqrt{(2L^f+1)(2L^i+1)}$$

$$* \sqrt{(n_s^i+1)} \begin{Bmatrix} L_g^f & L^f & L_d^i \\ L^i & L_g^i & k \end{Bmatrix}$$

$$* \langle \{ n_g^f \} L_g^f \| \tilde{g} \| \{ n_g^i \} L_g^i \rangle$$

(20.041)

$$\langle \{ B^f \} \| (g^\dagger \tilde{s})^k \| \{ B^i \} \rangle = \delta_{n_s^f, n_s^i-1} * \delta_{\{n_d^f\},\{n_d^i\}} *$$

$$\delta_{n_g^f, n_g^i+1} * \delta_{L_d^f, L_d^i} * (-1)^{k+L_d^i+L_g^i+L^f} \sqrt{(2L^f+1)(2L^i+1)}$$

$$* \sqrt{n_s^i} \begin{Bmatrix} L_g^f & L^f & L_d^i \\ L^i & L_g^i & k \end{Bmatrix}$$

$$* \langle \{ n_g^f \} L_g^f \| g^\dagger \| \{ n_g^i \} L_g^i \rangle$$

(20.042)

$$\langle \{ B^f \} \| (d^\dagger \tilde{g})^k \| \{ B^i \} \rangle = \delta_{n_s^f, n_s^i} * \delta_{n_d^f, n_d^i + 1} *$$

$$\delta_{n_g^f, n_g^i - 1} * \sqrt{(2L^f+1)(2L^i+1)(2k+1)} \begin{Bmatrix} L_d^f & L_d^i & 2 \\ L_g^f & L_g^i & 4 \\ L^f & L^i & k \end{Bmatrix}$$

$$* \langle \{ n_d^f \} L_d^f \| d^\dagger \| \{ n_d^i \} L_d^i \rangle *$$

$$* \langle \{ n_g^f \} L_g^f \| \tilde{g} \| \{ n_g^i \} L_g^i \rangle$$

(20.051)

and

$$\langle \{ B^f \} \| (g^\dagger \tilde{d})^k \| \{ B^i \} \rangle = \delta_{n_s^f, n_s^i} * \delta_{n_d^f, n_d^i - 1} *$$

$$\delta_{n_g^f, n_g^i + 1} * \sqrt{(2L^f+1)(2L^i+1)(2k+1)} \begin{Bmatrix} L_d^f & L_d^i & 2 \\ L_g^f & L_g^i & 4 \\ L^f & L^i & k \end{Bmatrix}$$

$$* \langle \{ n_d^f \} L_d^f \| \tilde{d} \| \{ n_d^i \} L_d^i \rangle *$$

$$* \langle \{ n_g^f \} L_g^f \| g^\dagger \| \{ n_g^i \} L_g^i \rangle$$

(20.052)

2.5.2 Occupation Numbers

The occupation numbers, $\langle n_\ell \rangle^E$ for s, d and g orbits, in the H - eigenstates are given by $\langle n_\ell \rangle^E = \langle \psi_E | n_\ell | \psi_E \rangle$; ℓ=0, 2 and 4. They are useful for example to estimate the importance of g - bosons. We can easily write down expressions for $\langle n_\ell \rangle^E$,

$$\langle n_\ell \rangle^E = \sum_{\{B^i\}} |C^E_{\{B^i\}}|^2 n^i_\ell \qquad (21)$$

where C^E_{--} are defined in (16) and $n^i_0 = n^i_s$, $n^i_2 = n^i_d$ and $n^i_4 = n^i_g$ respectively.

2.5.3 Two - nucleon Transfer Strengths

Apart from a possible cut - off factor [2], in the lowest order the L = 0 two nucleon addition operator $P^\dagger(0)$ can be taken to be,

$$P^\dagger(0) = \alpha s^\dagger \qquad (22)$$

The transfer intensities are given by,

$$I(N,L \longrightarrow N+1,L) = |\langle N+1, L \| P^\dagger(0) \| NL \rangle|^2 \qquad (23)$$

With the transition operator given in (22), I's can be calculated easily and here one has to use the expression,

$$\langle \{B^f\} \| s^\dagger \| \{B^i\} \rangle = \sqrt{(n^i_s+1)(2L^i+1)}\ \delta_{n^f_s, n^i_s+1} * \delta_{\{n^f_d\},\{n^i_d\}} * \delta_{L^f_d, L^i_d} * \delta_{\{n^f_g\},\{n^i_g\}} * \delta_{L^f_g, L^i_g} * \delta_{L^f, L^i} \qquad (24)$$

The expressions (20.---), (21) and (24) are programmed in MAE2E4 and SUBGM; see Section 4.7 for details.

3. Model Interactions in gIBM

The gIBM H as given in (12) contains far too many free parameters, 35 in number. So far there are no viable microscopic theories (see however ref [13] and Sect.7) to fix the values of or produce constraints on the 35 parameters and their determination by least square fits is cumbersome. Therefore, one has to use a simplified H based on some phenomenological/model considerations. Some of the model interactions in gIBM are described below.

3.1 Interactions derived from Dynamical symmetries

It is by now well known [12] that gIBM possess seven dynamical symmetry limits, four are the strong coupling limits denoted by the groups SU(3), SU(5), SU(6) and O(15) and three are the weak-coupling limits denoted by the groups U(6) ⊕ U(9), U(1) ⊕ U(14) and U(5) ⊕ U(10) respectively. One simple way to produce interactions is to write H as a linear combination of the linear and quadratic Casimir operators of the various groups (say i_o in number) in the seven dynamical symmetry limits, (i.e.) we can use

$$H_{DYN} = \sum_{i=1}^{i_o} \left\{ a_i C_1(G) + b_i C_2(G) \right\} \qquad (25)$$

in actual calculations; see ref [19] for an example. In Table-4, we collect

Table - 4

some relevant information regarding gIBM dynamical symmetry limits; see ref [12] for further details. To use the model interaction dictated by dynamical symmetries, one should know the single particle energies (ε_ℓ) and two-body matrix elements

$(V^L(\ell_1,\ell_2,\ell_3,\ell_4))$ corresponding to $C_2(G)$ and they follow easily as $C_2(G)$ is made up of terms of the type $O^k.O^k$ where O^k are defined in (17). Given $O^k.O^k$ operators, the corresponding ε_ℓ and $V^L(\ell_1,\ell_2,\ell_3,\ell_4)$ are

$$\varepsilon_\ell = \sum_{\ell'} \left[\frac{2k+1}{2\ell+1}\right] a^k(\ell,\ell')\ a^k(\ell',\ell)$$

$$V^L(\ell_1,\ell_2,\ell_3,\ell_4) = \sqrt{(1+\delta_{\ell_1\ell_2})(1+\delta_{\ell_3\ell_4})}\ (2k+1)\ *$$

$$\left[\left\{ \frac{(2-\delta_{\ell_1\ell_2})(2-\delta_{\ell_3\ell_4})}{1+(1-\delta_{\ell_1\ell_2})(1-\delta_{\ell_3\ell_4})} \right\}\ (-1)^L\ a^k(\ell_1\ell_3)\ * \right.$$

$$a^k(\ell_2\ell_4)\ * \begin{Bmatrix} \ell_1 & \ell_3 & k \\ \ell_4 & \ell_2 & L \end{Bmatrix} + 2\ (1-\delta_{\ell_1\ell_2})\ (1-\delta_{\ell_3\ell_4})\ *$$

$$\left. a^k(\ell_1\ell_4)\ *\ a^k(\ell_3\ell_2)\ * \begin{Bmatrix} \ell_1 & \ell_4 & k \\ \ell_3 & \ell_2 & L \end{Bmatrix} \right]$$

(26)

The $a^k(\ell_1,\ell_2)$ appearing in (26) are already defined in (17). In Tables- 5,6 we give explicitly the ε_ℓ, $V^L(----)$ for some of the

Table - 5,6

group Casimirs.

3.2 Boson Surface Delta Interaction

Recently Chen etal [20] advocated the use of Boson surface delta interaction (BSDI) in extended IBM (with g, s', d' etc -----bosons) studies. The BSDI interaction is constructed out of the SDI,

$$V^{SDI}(1,2) = -4\Pi\ A\ \delta(\vec{r}(1)-\vec{r}(2))\ \delta(r(1)-R_0)$$

(27)

where $\vec{r}(1)$, $\vec{r}(2)$ are the position vectors of the interacting particles, R_0 is the nuclear radius and A is the strength parameter of the delta force. In spherical polar coordinates, $\delta(\vec{r}(1)-\vec{r}(2))$ is given by

$$\delta(\vec{r}(1)-\vec{r}(2)) = \frac{\delta(r(1)-r(2))}{r(1)\ r(2)} \sum_{km} Y^{k}_{m}{}^{*}(\hat{r}(1))\ Y^{k}_{m}(\hat{r}(2)) \tag{28}$$

Here the radial part of the interaction yields constant contribution $C(R_0)$ to the matrix elements, as SDI assumes the interaction to be independent of the shell-model orbits in which the particles move. Putting $A*C(R_0) = 1$, we have the boson surface delta interaction (BSDI),

$$\langle BSDI\rangle^{Symm}_{L} = 4\Pi \langle \ell_a \ell_b | \sum_{k} Y^{k}(r(1)).Y^{k}(r(2)) | \ell_c \ell_d \rangle^{Symm}_{L}$$

$$= 2 \left\{(1+\delta_{\ell_a \ell_b})(1+\delta_{\ell_c \ell_d})\right\}^{-1/2} *$$

$$\sqrt{\frac{(2\ell_a+1)(2\ell_b+1)(2\ell_c+1)(2\ell_d+1)}{(2L+1)}}\ \langle \ell_a 0 \ell_b 0 | L0\rangle\ \langle \ell_c 0 \ell_d 0 | L0\rangle \tag{29}$$

The above interaction can be generalised in two ways, either by making the strength parameter (in (29) we have unit strength) to be L dependent,

$$BSDI(C_L) = \sum_{L} C_L\ \langle BSDI\rangle^{Symm}_{L} \tag{30}$$

or by replacing $\sum_{k} Y^{k}.Y^{k}$ in (29), which has uniform sum w.r.t k, with k-dependent summation $\sum_{k} A_k\ Y^{k}.Y^{k}$,

$$BSDI(A_k) = 4\Pi \langle \ell_a \ell_b | \sum_k A_k \; Y^k . Y^k | \ell_c \ell_d \rangle_L^{Symm}$$

$$= (-1)^{\ell_a+\ell_b} \sqrt{(2\ell_a+1)(2\ell_b+1)(2\ell_c+1)(2\ell_d+1)} \; * \; \frac{\sum_k A_k}{\sqrt{(1+\delta_{\ell_a \ell_b})(1+\delta_{\ell_c \ell_d})}} \left[(-1)^L * \begin{Bmatrix} \ell_a & \ell_b & L \\ \ell_d & \ell_c & k \end{Bmatrix} * \right.$$

$$\langle \ell_a 0 \ell_c 0 | k0 \rangle \langle \ell_b 0 \ell_d 0 | k0 \rangle + \begin{Bmatrix} \ell_a & \ell_b & L \\ \ell_c & \ell_d & k \end{Bmatrix} *$$

$$\left. \langle \ell_a 0 \ell_d 0 | k0 \rangle \langle \ell_b 0 \ell_c 0 | k0 \rangle \right]$$

(31)

we remark that Chen et.al [20] employed BSDI (C_L) in their numerical studies

3.3 Mapped multipole interaction

A third class of interactions follow by using

$$H_{MAP} = \sum_k B_k \; O^k . O^k$$

(32)

where the parameters $a^k(\ell,\ell')$ that define O^k are determined by OAI mapping given in [21] (of course one can also use (32) with all $a^k(\ell,\ell')$and B_k to be free parameters - this then can yield a consistent - Q formalism in gIBM). Given a transition operator Q^k in fermion space, using the correspondence,

$$|j^{2N} \; v = 0 \; J = 0\rangle \longrightarrow |n_s = N, \; L = 0\rangle$$

$$|j^{2N} \; v = 2 \; J = 2\rangle \longrightarrow |n_s = N-1, \; n_d = 1, \; L = 2\rangle$$

$$|j^{2N} \; v = 2 \; J = 4\rangle \longrightarrow |n_s = N-1, \; n_g = 1, \; L = 4\rangle$$

(33)

and following OAI procedure, we obtain (with $\Omega = (2j+1)/2$),

$$a^{(k)}(0,\ell) = a^{(k)}(\ell,0) = \sqrt{\frac{2(\Omega-N)}{\Omega(\Omega-1)(2k+1)}}\ \langle j \| Q^k \| j \rangle\ \delta_{k\ell},$$

$$a^{(k)}(\ell,\ell') = a^{(k)}(\ell',\ell) = -\left(\frac{\Omega-2N}{\Omega-2}\right)\sqrt{\frac{4(2\ell+1)(2\ell'+1)}{(2k+1)}} * (\pm 1) *$$

$$\begin{Bmatrix} \ell & \ell' & k \\ j & j & j \end{Bmatrix} \langle j \| Q^k \| j \rangle \qquad \ell \neq \ell'$$

(34)

where the (+) sign is for particle bosons (fermion number $N_f \leq \Omega$, $N = N_f/2$) and negative sign for hole bosons (fermion number $N_f \geq \Omega$, $N = (2\Omega-N_f)/2$). Using (34) and (26) we can write down the single particle enrgies and the 2 body matrix elements of H_{MAP}.

Interactions defined by symmetries, surface delta model and by the OAI mapping of Q^L operators are programmed and they are described in Section 4.4.

4. Long Write - up

The package SDGIBM1 consists of nine routines, their names and functions are:

DIMVTL	v —> L reductions
VRACAH	3-j, 6-j, 9-j coefficients
DIAGON	Eigenvalues and Eigenvectors of real symmetric matrices
BTBME	Model interactions in gIBM
CFP	One particle fractional parentage coefficients for identical boson systems (d or g)
MAINHH +SUBGH	Construction and diagonalisation of gIBM Hamiltonian matrix
MAE2E4 +SUBGM	Calculation of various EM - transition strengths, static moments, occupation numbers and two - nucleon transfer strengths

In this section we give a long write-up of various routines to enable the users to run them with ease.The writeup consists of (1) General purpose, (2) Subroutines/common blocks, (3) Method, (4) Internal/External parameters, (5) Input, (6) Output, (7)Special features/restrictions and (8) References. First applications of

this package are reported in ref [19].

4.1 DIMVTL

4.1.1 General Purpose

This program consists of two subroutines, one generates configurations and the other produces v ——> L reductions using the former. The routines CFP, MAINHH, MAE2E4 have to be compiled with DIMVTL.

4.1.2. Subroutines/Common blocks

Subroutines:MCONF, IVLDIM

common blocks:None

4.1.3. Method

The programme MCONF generates configurations $\{m_1, m_2, ---, m_r\}$ corresponding to the various distributions of given number of particles (for us bosons) in r - orbits with various specifications; [Kota,1984] gives complete details. The routine IVLDIM uses MCONF to produce v ——> L reductions for identical bosons and the method is described in detail in [Kota,1986].

4.1.4. Internal/External Parameters

(a) Internal parameters

L_o = angular momentum of a single boson (L_o = 2 for d bosons and L_o = 4 for g boson)

NP = # of bosons

(b) External parameters

NA = Max (NP)

NB = MAx (L_o)

NC = 2 * NB + 1

ND = NA * NB

(c) Dimensions

MCONF: JORB(NC), MINO(NC), MAXX(NC), MAXO(NC)
MVAL(NC), MCONFG(ND+1), MTOT(NC), MDIF(NC),
MACT(NC), MZZ(NC)

IVLDIM:MCONFG(ND+1), MINO(NC), MAXO(NC),
MAXX(NC), MVAL(NC), JORB(NC), MZZ(NC),
LDEG(0:NA,0:ND), LVDIM(0:NA,0:ND).

4.1.5. Special Features/Restrictions

The routine IVLDIM as supplied works for $n_d \leq 12$ and $n_g \leq 6$. To go beyond these numbers one has to change dimensionalities and also the following Do loops,

Do 2 I = ... in IVLDIM and Do 9910 I = ... in MCONF.

4.1.6. References

V.K.B.Kota,1984,"Fortran Programmes for stastical spectroscopy calculations" (University of Rochester Report, UR - 887)

V.K.B.Kota,1986,"Tables of group representations for the six limiting symmetries in gIBM" (PRL Technical report, PRL-TN-86-54)

4.2. VRACAH

4.2.1. General Purpose:

This programme consists of subroutines to calculate 3j, 6j and 9j symbols.

4.2.2. Subroutines/Common Blocks

Subroutines:DELTA2, GAMAF, WIG3J, WIG6J, DELTA, UNINEJ

Common Blocks:There is one unlabelled common block

4.2.3. Method

The formulas, for 3j,6j, and 9j symbols, given by Rotenberg et.al.[1959] are programmed. In order to handle factorials of large arguments (say n! = A*10^{B}) the programme keeps track of base A and the exponent B throughout. Note that with n_d > 8 or n_g > 4 a straightforward calculation of n! will fail on DEC1091 system.

4.2.4. Internal/External Parameters

(a) Internal Parameters:None

(b) External parameters:None

(c) Dimensions:

DELTA2:GAM(500),JGAM(500)

GAMAF:GAM(500),JGAM(500)

WIG3J:GAM(500),JGAM(500),K(3),L(2)

WIG6J:GAM(500),JGAM(500),K(4),L(3)

4.2.5. Special Features/Restrictions

The dimensions in the subroutines are set to be large enough values so that they need not be altered at any time. Before ever calling the programmes, CALL GAMAF(500) statement should be given.

4.2.6. References

M.Rotenberg, R. Bivins, N.Metropolis and J.K.Woolen Jr, 1959, (M.I.T. Press, Cambridge, Massachusetts)

4.3. DIAGON

4.3.1. General Purpose:

To obtain eigenvalues and eigenvectors of a real symmetric

matrix. DIAGON is used by the routines CFP and MAINHH.

4.3.2. Subroutines/Common Blocks

Subroutines:TRED2,IMTQL2

4.3.3. Notes

The programmes TRED2 (tridiagonalise a given real symmetric matrix) and IMTQL2 (uses the output of TRED2 to give the eigenvalues and eigenvectors of the given real symmetric matrix) are from the EISPACK package [Smith et.al.1974]. These subroutines are called in the following manner:

```
CALL TRED2 (NM,N,A,D,E,Z)
CALL IMTQL2 (NM,N,D,E,Z,IERR)
```

where

A = given matrix

N = matrix dimension

NM = defined in the calling programme and NM > N.

D = eigenvalues (in IMTQL2)

Z = eigenvectors (in IMTQL2)

All arrays in the programme are object dimensioned, hence need not be changed. Given the I - th eigenvalue D(I), the corresponding eigenvector (as given by IMTQL2) is (Z,(I,J),J = 1,N)

4.3.4. Special Features/Restrictions:

See EISPACK package [Smith et.al.1974]. Here we are using the single precision version of TRED2 and IMTQL2. The machine dependent parameter MACHEP appearing in IMTQL2 is set to 10^{-8}. All arrays in these routines are object dimensioned and hence no alterations are needed at any time

4.3.5. References:

B.T.Smith, J.M.Boyle, B.S.Garbow, Y.Ikebe, V.C.Klema and C.B.Moler, 1974, "Matrix Eigensystem Routines – EISPACK guide", Lecture notes in Computer Science, vol.6 (Springer, Heidelberg)

4.4. BTBME

4.4.1. General Purpose

To generate single particle energies and two-body matrix elements (in the standard form) corresponding to (i) quadratic Casimir operators of various gIBM dynamical symmetry groups,(ii) general $O^L.O^L$ operator (iii) BSDI and (iv) OAI mapped interaction H_{Map}.

4.4.2. Subroutines/Common Blocks

Subroutines:V1, V2, EE, O15O14, BSDI, OAI, BLOCKS, DWR3, DRR3, DELTA

Common Blocks:/BKDF/

4.4.3. Method

In Sect 3 various gIBM interactions are defined and expressions for their matrix elements are given. The Casimir operators of gIBM groups SU(3), SU(5) (orU(5)), O(5), SU(6) (or U(6)), Sp(6), O(15) and O(14) are defined in Table-4 and their matrix elements follow from eq (26) – they are implemented in the MAIN programme and the subroutines V1, V2, EE and O15O14. General O^L operator is defined in (17) and we impose the restriction $a^{(L)}(\ell,\ell') = a^{(L)}(\ell',\ell)$. The matrix elements of the $O^{(L)}.O^{(L)}$ operator (for any L) follow from (26) and it is programmed in MAIN. The subroutine BSDI gives the matrix elements

of the interactions BSDI(C_L) and BSDI(A_k) that are defined in (30) and (31) respectively. The matrix elements of the OAI mapped interaction H_{MAP} defined in (34) can be obtained from the subroutine OAI.

4.4.4. Internal/External Parameters

All dimension statements are fixed and they should not be changed.

4.4.5. Input

The routine BTBME should be run on-line.At every stage there will be a prompt asking for appropriate input.

(a) Unit # 5 (Terminal)

Card 1. NINTER

NINTER = # of intractions to be generated

Card 2. N, PX, PY, KPZ

The correspondence between the values N, PX, PY, KPZ and the interaction it generates is displayed every time (NINTER times);

N=3 PX=0.0 PY=0.0 KPZ=0 - (3/4)*Q(S).Q(S) of SU(3)

N=3 PX=1.0 PY=0.0 KPZ=L General O(L).O(L)

N=5 PX=1.0 PY=0.0 KPZ=0 C_2(U(5))

N=5 PX=1.0 PY=0.0 KPZ=1 C_2(SU(5))

N=5 PX=0.0 PY=1.0 KPZ=0 C_2(O(5))

N=6 PX=1.0 PY=0.0 KPZ=0 C_2(U(6))

N=6 PX=1.0 PY=0.0 KPZ=1 C_2(SU(6))

N=6 PX=0.0 PY=1.0 KPZ=0 C_2(Sp(6))

N=14 PX=0.0 PY=0.0 KPZ=0 C_2(O(14))

N=15 PX=±1.0 PY=0.0 KPZ=0 C_2(O(15))

N=15 PX=±1.0 PY=0.0 KPZ=1 C_2(O(15))-C_2(O(14))

N=20 PX=0.0 PY=0.0 KPZ=0 BSDI with A(K)'s

N=20 PX=0.0 PY=0.0 KPZ=1 BSDI with C(L)'s

N=25 PX=0.0 PY=0.0 KPZ=0 MAPPED INTERACTION (H_{MAP})

For N=20 and N=25 the programme asks for further inputs A(K), C(L) and B(K), where A(K) = A_K, C(L) =C_L and B(K) = B_k. A_k, C_L, and B_K defined in (30),(31) and (32) respectively. For N=3, PX=1.0, KPZ = 0 the programme asks for $E(\ell,\ell') = a^{(L)}(\ell,\ell')$. For N=25 the programme asks for Ω (=(2j+1)/2), N(# of bosons) and index IP (IP = +1 for particles, -1 for holes).

(B) Unit # 50: INPUT.DAT

Cards 1-32 $\ell_1,\ell_2,\ell_3,\ell_4$,L

FORMAT (10X,5I5)

All 32 combinations of $(\ell_1,\ell_2,\ell_3,\ell_4,L)$ in $V^L(\ell_1,\ell_2,\ell_3,\ell_4)$ should be given (see Test Input - 1).

4.4.6. Output

Unit # 38 FILENAME = SDGINP.DAT

(1+2) body matrix elements in standad format.

gIBM Interaction in standard format consists of

(1) Title Card (maximum of 72 characters)

(2) Single Particle Energies ε_s, ε_d and ε_g

(FORMAT:(3F10.5))

(3) L1,L2,L3,L4,L,V^L(L1,L2,L3,L4) for each of the 32 combinations (i.e the # of Cards = 32), FORMAT(10X,5I5,10X,E15.8)

Unit # 39 FILENAME = FOR39.DAT

(1+2) body matrix elements in standard format together with some extra information

about the interactions.

4.5. CFP

4.5.1. General Purpose

To calculate identical boson 1-particle CFP's for d and g boson systems.

4.5.2. Subroutines/Common BLocks:

Subroutines: SPLIT, SSPLIT

4.5.3. Method

Identical boson 1-particle MX1 —>M+1 CFP $\langle m, v_i, \alpha_i, L_i; 111\ell | \} m+1, v_f, \alpha_f, L_f \rangle$ defined in (6) are calculated uisng the iterative procedure given by [Bayman and Lande, 1966]; see section 2.2 for details and especially eq (11). This involves diagonalising the operator $\alpha_o C_2(SU(2\ell+1)) + \beta_o C_2(O(2\ell+1))$ in the basis $|m, v_i, \alpha_i, L_i; 111\ell; L_f \rangle$. In the programmes we choose $\alpha_o = 1$, $\beta_o = 0.055$ and they distinguish the eigenvalues belonging to the $U(2\ell+1)$ partitions $\{m+1\}$ and$\{m,1\}$.

4.5.4. Internal/External Parameters : None

4.5.5. Input

Programmes has to be run online. We will get all the needed prompts.

Card 1 X, NPMAX, KOUT

X=ℓ (X=2 for d bosons, X=4 for g bosons)

NPMAX = N (the programme works for N ≤ 12 with ℓ=2, N ≤ 6 with ℓ=4).

KOUT = Unit # on which formatted output is to be written. (KOUT=5 for terminal, KOUT=45 for output

file).

Card 2 MFINL

If we need CFP for 0X1 —> 1, 1X1 —> 2, 2x1 —> 3,.....,

No-1X1 —> N_o, then MFINL = No. Note that NPMAX = N_o.

FORMAT : *-format

4.5.6. Output

UNit # 30: FOR30.DAT

This is an unformatted file containing all the

1-particle CFP $\langle m-1\ v_i\ \alpha_i\ L_i\ ;\ 11\ell;\frac{1}{2}\}\ m\ v_f\ \alpha_f\ L_f\ \rangle$

for m=1,2,....., MFINL (MFINL is defined above).

This file is used by the routines MAINHH and MAE2E4.

Unit # KOUT

KOUT = 5 Terminal

KOUT = 45 CFPOUT.DAT

This is a formatted output containing all the calculated 1-particle CFP and the eigen-values of $[C_2(SU(2\ell+1)) + 0.055 * C_2(O(2\ell+1))]$.

4.5.7. Special features/Restrictions:

This programme generates CFP's upto 11x1 —> 12 in the case of d bosons and 5x1 —> 6 in the case of g bosons. To remove these restrictions the programme needs modification at several places and they can be figured out easily. To run the routine CFP we need DIMVTL (see 4.1), VRACAH (see 4.2) and DIAGON (see 4.3). The number of non-zero 1-d boson (m-1 —> m) CFP for m = 1-12 are 1,3, 9, 23, 49, 95, 172, 290, 467, 722, 1076 and 1558 respectively. Similarly for 1-g boson CFPs for m = 1-6 the number of non-zero CFPs are 1,5,40, 253, 1271 and 5307 respectively. On DEC1091

system the program takes ~ 45 minutes to generate all d - boson CFPs and ~ 75 minutes to generate all the g - boson CFP. Although the programme need to be run online, if needed changes can be made (easily) so that it can be run in batch mode.

4.5.8. References:

B.F.Bayman and A.Lande,1966, Nucl.Phys.77, 1.

4.6. MAINHH + SUBGH

4.6.1. General Purpose:

This program constructs and diagonalises general gIBM Hamiltonian in sdg space for arbitrary number of bosons.

4.6.2. Subroutines/Common Blocks

MAINHH

Subroutines: HAMILT

Common Blocks: /LVD1/, /LVD2/, /LVD3/, /LVDD/

SUBGH:

SUBROUTINES: TWODG, V2, V3, V5, V6, V7, V9, V10, V11, V12, V15, V17, V20, V25, V28, SPLIT, CFPINP

Common Blocks: /LVD1/, /ALAMOS/, /SRINGR/, /LVD2/

4.6.3. Method

The many particle matrix elements of general gIBM H in the weak-coupling basis $| n_s; n_d, v_d, \alpha_d, L_d; n_g, v_g, \alpha_g, L_g; L >$ are explicitly given in eqns. (15.V.1) - (15.V.28).The basis states are generated in HAMILT (using DIMVTL). The subroutines (in SUBGH) V2, V3, V5, V6, V7, V9, V10, V11, V12, V15, V17, V20, V25 and V28 programme the equations (15-V.2), (15.V.3) ---- (15.V.28). These equations involve reduced matrix elements of the

operators $d^{\dagger}$, $\tilde{d}$, $g^{\dagger}$, $\tilde{g}$, $(d^{\dagger}d^{\dagger})^{k}$, $(\tilde{d}\tilde{d})^{k}$, $(d^{\dagger}\tilde{d})^{k}$, $(g^{\dagger}g^{\dagger})^{k}$, $(\tilde{g}\tilde{g})^{k}$ and $(g^{\dagger}\tilde{g})^{k}$ and they are evaluated in the subprogram TWODG (using the 1-particle CFP output genrated by the routine CFP.FOR) see aslo eqn (13). The calling statement for TWODG is

CALL TWODG (NSR,X,K1,K2,K3)

and X=2 for d bosons, while X=4 for g-bosons. NSR defines the initial and final states $\{n_d^i\}\ L_d^i$, $\{n_d^f\}\ L_d^f$, for X = 2 and $\{n_g^i\}\ L_g^i$, $\{n_g^f\}\ L_g^f$ for X = 4. The correspondence between (X,K1,K2,K3) and the reduced matrix element calculated in TWODG is given in Table - 7.

Table - 7

First applications of MAINHH + SUBGH are reported in [Devi et.al,1989].

4.6.4. Internal/External Parameters

External parameters:

NA = max (# of basis states)

see Table - 2 for the # of basis states

corresponding to a given value of (N, n_g^{max},L)

NO = dimension of the largest matrix we want to diagonalise.

NR = a number larger than NO.

DIMENSIONS:

MAINHH:

MAIN : KA(32), KB(32), KC(32), KD(32), LL(32), ZZV(32), A(72).

Hamilt: /LVD1/ LVDIMD (0:15,0:25), LVDIMG (0:15,0:25)

Dimension : IBASED(NA), IBASEG(NA), LVDIM(0:15,0:25), NSTAI(8), NSTAF(8),IBASE(NA), NDUMY(8), ZV(32), EEEE(NR), H(NR,NR), EIGVAL(NR), EIGVEC (NR,NR)

Note: The dimension of LVDIMD, LVDIMG, LVDIM should be same and they must coincide with dimension given for LVDIM in IVLDIM of DIMVTL.FOR.

SUBGH: Here all dimensions are fixed. They will work for the cases with $n_d \leq 12$ and $n_g \leq 6$.

4.6.5. Input

Unit # KIN (KIN = KINN given in the parameter statement)

KIN=5 data is to be read from the terminal

=45 data is to be read from file IND.DAT

Card 1: KOUT, NGG, ID1, ID2, ID3, ID4, ID5, IDEN1, IDD.

KOUT = Output is written on Unit # KOUT.

NGG = Max. # of g-bosons in the calculation (ng $\leq$ 6 must be satisfied)

ID1 = 1 only basis states will be printed

= 2 only eigenvalues will be printed

= 3 basis states and eigenvalues will be printed

= 4 only eigenvalues and eigenvectors will be printed

= 5 basis states, eigenvalues, eigenvectors will be printed

= 0 SDGOUT.DAT is not created except when there is an error.

ID2 = 1 writes the matrix on Unit # 20 (output file name is MATRIX.DAT).

ID3 = 1 writes eigenvalues and eigenvectors (for calculation of transition probabilities) on Unit # 40 (file name is EIGVEC.DAT)

ID4 = 1 returns after printing Basis States

= 2 returns after printing the H-matrix

ID5 = # of different L-values

IDEN1 = # of eigenvalues to be printed

IDEN1 = 9999 Prints all eigenvalues

IDD = 0 prints the eigenvalues relative to the ground state (i.e. the first $L^{\pi} = 0^{+}$ state).

Card 2. MFINL, LFINL, IDEN2

MFINL = # of bosons ($N = n_s + n_d + n_g$)

LFINL = final angular momentum (L)

IDEN2 = # of eigenvalues and eigen-vectors to be printed and also stored for computing transition probabilities

Repeat Card 2. ID5 times and repeat it once more with MFINL = 999, LFINL = 999, IDEN2 = 999 to stop the programme

Note: If IDD = 0 one of the cards in Card.2 must contain the case with LFINL = 0

Unit # 38: SDGINP.DAT

(1+2) - body matrix elements in standard form as generated for example by BTBME.FOR

Unit # 30: FOR30D.DAT

This is the output generated on unit # 30 by CFP.FOR with the input X = 2, NPMAX = 12 and MFINL=12. .

Unit # 32: FOR30G.DAT

This is the output generated on unit # 30 by CFP.FOR with the input X = 4, NPMAX = 6 and MFINL=6.

4.6.6. Output

Depending upon the values of ID1, ID2, ID3 and ID4, the output will contain (1) Basis states (2) H-matrix, (3) eigen-values and (4) eigenvectors in various combinations (see INPUT).

UNIT # 20:MATRIX.DAT

Basis states and the Hamiltonian matrix.

Unit # 35:SDGOUT.DAT

Basis states, eigenvalues and eigenvectors

UNIT # 40:EIGVEC.DAT

Unformatted output, unlike in the case of SDGOUT.DAT, consists of basis states, eigenvalues and eigenvectors. This file becomes input for MAE2E4.FOR that calculates transition strengths.

4.6.7. Special Features/Restrictons

This program (MAINH.FOR + SUBGH.FOR) has to be compiled with DIMVTL.FOR and DIAGON.FOR. One restriction we have is that $n_d \leq 12$ and $n_g \leq 6$. The diagonalisation routines (TRED2 and IMTQL2) can in principle be replaced by programs that have Lanczos procedure, then we can handle larger matrices). The output EIGVEC.DAT on unit # 40 will be used by MAE2E4 of Section.4.7. In generating EIGVEC.DAT one has to keep in mind that for a given MFINL and LFINL the value of IDEN2 must be less than or equal to the dimensionality of the H matrix for that MFINL and LFINL). Typical run (C.P.U.) times on DEC-1091 for various cases with $N \leq 6$ are shown in Table - 8.

Table - 8

4.6.8. References

Y.D.Devi, V.K.B.Kota and J.A.Sheikh ,1989, Phys.Rev.C39, 2057.

4.7. MAE2E4 + SUBGM

4.7.1. General Purpose:

This programme calculates E2, E4, M1 and M3 transition strengths, static moments, occupation numbers and L=0 two-nucleon transfer intensities using the output generated by (MAINHH + SUBGH).

4.7.2. Subroutines/Common Blocks:

MAE2E4

Subroutines: IWVINP, NSPLIT

Common Blocks: /AUX1/, /AUX2/, /AUX3/, /LVD1/

SUBGM

Subroutines: O1, O2, O3, O4, O5, O6, O7, O8, TWODG, CFPINP

Common Blocks: /LVD1/, /ALAMOS/, /SRINGR/

4.7.3. Method

The electromagnetic transition operators are defined in (18) and the corresponding static moments and reduced transiton strengths (B(Eλ) and B(Mλ)) are defined in (19). The matrix elements of the various pieces in the transition operators are given in eq. (20.01) to (20.052). The subroutines O1, O2, O3, O4, O5, O6, O7 and O8 program the equations (20.01), (20.02), (20.031), (20.032), (20.041), (20.042), (20.051) and (20.052). The needed reduced matrix elements (that enter into ((20.01) - (20.052)) are calculated in TWODG and this subroutine was described in Sect.4.6.3. Occupation numbers given by Eq. (21) and the L =0 transfer amplitude given by (23) are also programmed in the MAIN program. First applications of MAE2E4 + SUBGM are

reported in [Devi etal,1989].

4.7.4. Internal/External Parameters

Internal Parameters

N1 = initial angular momentum (L_i)

N2 = final angular momentum (L_f)

N3 = X (say a given L comes N_L times (i.e.) dimension(N,L) = N_L, then $1 \leq X \leq N_L$).

External Parameters

NA = Max (N1 or N2) that appear in the data

NB = Max (X) that appears in the data

NC = Max matrix dimension (Max (N_L)).

Dimensions

MAE2E4

```
MAIN: SUM(0:NA,0:NA,NB,NB), O(0:4,0:4),
      B(NB,0:NA,NB,0:NA), XNA(0:NA,3,NB),
      XTWON(0:NA,NB,NB), Q(NB,0:NA)
      NSTAI(10), NSTAF(10), LVDIM(0:15,0:25)
      XN(3), A(72)
COMMON /AUX1/ IDM(0:NA,2), EIGNVL(0:NA,2,NB),
              IWAVE1(0:NA,NC,2), IWAVE2(0:NA,NC,2)
COMMON /AUX2/ C(0:NA,2,NC,NB), NYY(2), KOUT
COMMON /AUX3/ ID1(2), LFF(NA+1), ID22(0:NA,2)
COMMON /LVD1/ LVDIMD(0:15,0:25), LVDIMG(0:15,0:25)
```

Note

The dimension of LVDIMD, LVDMIG should be same and they must coincide with the dimenions given for LVDIM in IVLDIM of DIMVTL.FOR.

IWVINP : EIGVEC(NC), A(72)

SUBGM:

The dimensions in all the subroutines are fixed and they work for $n_d^{max} \leq 12$ and $n_g^{max} \leq 6$.

4.7.5. Input

Input data on Unit # KINN (KINN = 5 or 50).

Card 1. KOUT,KIJJ,KID,K

KOUT = output file number

K, KIJJ, KID = for various transition operators, their values are given in Table - 9

Table - 9

Card 2. Title not exceeding 72 characters

Card 3. If KID = 99 give Card 3 containing O(0,2), O(0,4), O(2,2), O(2,4), O(4,4) in * format. Note that O(a,b) are effective charges for E2,E4 operators and g factors of M1,M3 operators(see eqn.(18))

Card 4,5,----

K1,KI,K2,KF

End of file Card

99,99,99,99

Format for Cards 1,3,---- is * format.

The Cards 4,5,---- contain K1, KI, K2, KF values for which the Q(KI,K1), μ(KI,K1), B(K1,KI ——> K2,KF), $\langle n(\ell) \rangle$ and two nucleon transfer strengths are to be printed in the output file. Note that

KI = angular momentum L_i of the initial state

K1 = first, second, third, etc.(example: for 2_1^+ K1 = 1, for 2_3^+ K1 = 3)

KF = angular momentum of the final state

K2 = first, second, third, etc - as above

Input on Unit # 30

FOR30D.DAT (see section 4.6.5).

Input on Unit # 32

FOR30G.DAT (see section 4.6.5)

Input on Unit # 40

EIGVC1.DAT: This is EIGVEC.DAT created by MAINHH (see section 4.6.6).

Input on Unit # 41

EIGVC2.DAT : Note that EIGVC2.DAT is same as EIGVC1.DAT but for a different boson number - needed in two nucleon transfer strength calculations. For $\langle N+1\ L \| s^{\dagger} \| N\ L \rangle$ calculation, EIGVC1.DAT contains data for m=N and EIGVC2.DAT for m=N+1.Note that EIGVC1.DAT and EIGVC2.DAT should contain data for the same L-values and multiplicity lables.

4.7.6. Output

Unit # KOUT (= 5 or 70)

Output file BE2BE4.DAT is created when KOUT = 70.

Depending on the values of (K,KIJJ,KID) given in Table-9, the output will contain B(Eλ), Q(Eλ), or B(Mλ), μ(Mλ) or $\langle n_{\ell} \rangle$ or $\langle N+1, L \| s^{\dagger} \| N, L \rangle$.

4.7.7. (Special Features/Restrictions)

This program (MAE2E4.FOR + SUBGM.FOR) has to be compiled with DIMVTL.FOR and VRACAH.FOR

4.7.8. References

Y.D.Devi, V.K.B.Kota and J.A.Sheikh ,1989, Phys.Rev.C39, 2057.

5. Tests of gIBM Programmes

The package of programmes described in section 4 should be tested thoroughly, for obvious reasons. This can be (and should be) done in various ways. In this section we describe the tests that are carried out using (1) dynamical symmetries SU(3), SU(5) and SU(6) of gIBM, (2) traces of H and its powers and (3) sd IBM calculations published in literature (4) Analytic results in sd and sdg spaces.

5.1. SU(3) Tests

The eigenvalues of the SU(3) quadratic Casimir operator given in table-4 provides a good test of the eigenvalues produced by MAINHH. Here one has to deal with complete sdg space and with our restrictions on g-boson number ($n_g \leq 6$) we can only use the eigenvalues for $N \leq 6$. With N=4 some of the lowlying SU(3) irreps (λ,μ) and the corresponding eigenvalues of the $-\frac{3}{4} \times Q^2(s).Q^2(s)$ operator are given in Table-10 and they are tested in

Table - 10

OUTPUT-5. Using the SU(3) Wigner coefficients [22] $\langle (\lambda\mu)\ kL\ (11)12 \parallel (\lambda\mu)\ k'\ L' \rangle$ one can derive analytic formulas for some of the B(E2) values with $T^{E2} = Q^2(s)$, see Table-4 for the definition of $Q^2(s)$. We can calculate the same employing SU(3) eigenfunctions which can be labelled by $(\lambda\mu)$ and given in OUTPUT-5. The following analytical formulas can be easily derived,

$$B(E2;(\lambda 0)L \longrightarrow (\lambda 0)L-2) = \left[\frac{2L(L-1)(\lambda-L+2)(\lambda+L+1)}{(2L-1)(2L+1)}\right]$$

$$B(E2;(\lambda 2)L \longrightarrow (\lambda 2)L-2) = \left[\frac{2(L-2)(L-3)(L+1)(L+2)(\lambda-L+3)(\lambda+L+2)}{L(L-1)(2L-1)(2L+1)}\right];$$

$$\lambda-L = \text{Odd}$$

$$Q((\lambda 0)L) = -\sqrt{\frac{16\Pi}{15}}\left[\frac{L(2\lambda+3)}{2L+3}\right]$$

$$Q((\lambda 2)L) = -\sqrt{\frac{16\Pi}{15}}\left[\frac{(L-3)(L+4)(2\lambda+5)}{(L+1)(2L+3)}\right];\ \lambda-L = \text{odd}$$

$$Q((\lambda 1)L) = -\sqrt{\frac{16\Pi}{15}}\left[\frac{\{(2\lambda+1)L(L+1)-6(\lambda+2)\}}{(L+1)(2L+3)}\right];\ \lambda-L = \text{odd}$$

(35)

We should remark that $Q^2(s) = 1/\sqrt{3}\,Q^2$(Elliott) [23] and $Q^2(s) = \sqrt{8/3}\,Q^2$ (Arima and Iachello) [5]. The SU(3) selection rule $\Delta\lambda =$, $\Delta\mu = 0$ and the analytical results given in (35) are tested in OUTPUT-8. Similarly with $T^{M1} = \sqrt{\frac{3}{4\Pi}}\,((d^{\dagger}\tilde{d})^1 - \sqrt{6}\,(g^{\dagger}\tilde{g})^1)$, the moments for g.s.band levels are given by,

$$\mu_1(L) = \sqrt{\frac{L}{(L+1)(2L+1)}}\left\{\sqrt{L(L+1)(2L+1)/10} - 2\sqrt{6} * \langle L\|\,(g^{\dagger}\tilde{g})^1\,\|L\rangle_{(4N,0)}\right\}$$

$$\langle L\|\,(g^{\dagger}\tilde{g})^1\,\|L\rangle_{(4N,0)} = -\sqrt{3}\sum_{L''}\begin{Bmatrix}4 & 4 & 1\\ L & L & L''\end{Bmatrix}(2L+1)\,(N) * |\langle(4N-4,0)L''\ (40)4\ \|(4N,0)L\rangle|^2$$

(36)

Eq(36) is tested in OUTPUT - 13.

The SU(3) Wigner coefficients given in Table - 11 (taken from

Table - 11

Vergados [22]) can be used to calculate some of the $\langle n_\ell\rangle^E$ and two-nucleon transfer strengths. For example,

$$\langle\{m\}(4m,0)\ L|n_\ell|\{m\}(4m,0)L\rangle = m\sum_{L''}|\langle(4m-4,0)L''\ (40)\ell\|(4m,0)L\rangle|^2$$

(37)

In particular for the g.s. $|(4m,0)\ L=0\rangle$ the occupation numbers

are $(4m+1)m/5(4m-3)$, $16(m-1)m(4m+1)/7(4m-1)(4m-3)$ and $64m(m-1)(2m-3)/35(4m-3)(4m-1)$ for ℓ = 0(s), 2(d) and 4(g) respectively. Eq.(37) was tested in OUTPUT-14. Similarly the two-nucleon L=0 transfer strengths for (4m,0) ⟶ (4m+4,0) are,

$$|\langle\{m+1\}(4m+4,0)L\|s^{\dagger}\|\{m\}(4m,0)L\rangle| = \sqrt{(m+1)(2L+1)}\left[\frac{(4m-L+2)(4m-L+4)}{5(4m+1)(4m+2)}\times\frac{(4m+L+3)(4m+L+5)}{(4m+3)(4m+4)}\right]^{1/2} \quad (38)$$

The expression in (38) is tested in OUTPUT - 15.

5.2. U(5) Tests

In the U(5) limit using the operator $[-C_2(U(5)) + \frac{1}{2} C_2(O(5))]$ as the Hamiltonian one can write down the eigenvalues using Table-4. For four particles we give in Table-12 some of the low-lying irreps in the U(5) limit and the corresponding eigenvalues. The results given in Table-12 are tested in OUTPUT-6.

Table - 12

with $T^{E2} = G^2$ and $T^{E4} = G^4$, one can derive selection rules in the U(5) limit. Recognizing that $G^{2,4}$ ⟶ {2111}[20]2,4 with respect to the chain U(5) ⊃ O(5) ⊃ O(3), we have the obvious selecton rules that $|\Delta L| \leq 2$ for G^2 , $|\Delta L| \leq 4$ for G^4 and also that the initial and final states should have the same U(5) labels ({f} = {f'}). The later follows from the fact that $G^{2,4}$ are generators of the U(5) group. Some of the O(5) selection rules follow from the Kronecker products given below,

[m] X [2] = [m] ⊕ [m ± 2] ⊕ [m ± 1,1] ⊕ [m,2]

[m,1] X [2] = [m,3] ⊕ [m,2] ⊕ [m ± 1,2] ⊕ [m ± 1,1] ⊕ [m ± 2,1] ⊕

$$[m,1]^2 \oplus [m \pm 1]$$

$$[m,2] \times [2] = [m,4] \oplus [m,3] \oplus [m \pm 1,3] \oplus [m,2]^2 \oplus [m \pm 2,2] \oplus$$
$$[m \pm 1,2] \oplus [m \pm 1,1] \oplus [m,1] \oplus [m] \qquad (39)$$

and they are tested in OUTPUT-9,11. In the U(5) limit, so far we did not have analytical formulas for the E2 and E4 strengths, their derivation (in principle they follow from the work of ref [24]) provides extra checks on the programmes MAINHH and MAE2E4.

5.3. U(6) Tests

In the U(6) limit taking $[-C_2(U(6)) + \frac{1}{2} C_2(SP(6))]$ as the Hamiltonian, we can write down the eigenvalues using Table - 4. For N=4, some of the low-lying eigenvalues and the corresponding U(6), SP(6) and O(3) labels are given in Table-13 and they are tested

Table - 13

in OUTPUT-7. As in Section 5.2, using $T^{E2} = h^2$ and $T^{E4} = h^4$ one can test the selection rules for k = 2, 4 and here one also has {F} = {F'}. The later follow from the fact that h^2, h^4 are genrators of the U(6) group. With resepct to Sp(6), the operators h^2 and h^4 behave as $\langle\lambda_1,\lambda_2,\lambda_3\rangle = \langle 110\rangle$ tensor and we can derive one simple Sp(6) rule,

$$\langle N,N\rangle \times \langle 1,1\rangle = \langle N+1,N+1\rangle \oplus \langle N+1,N,1\rangle \oplus \langle N+1,N-1\rangle \oplus \langle N,N\rangle \oplus$$
$$\langle N,N-1,1\rangle \oplus \langle N-1,N-1\rangle \qquad (40)$$

and they are tested in OUTPUTS - 10,12

5.4. Tests via Averages

Averages of operators over spaces defined by irreps of a symmetry group provide alternative and powerful tests. The subject of fixed symmetry traces, for fermions, was initiated by French and explored further by Parikh, Quesne and others (see ref [14,25] for details) while their extensions to bosons systems was discussed by Kota and others [14,26]. The traces of H and H^2 in fixed (n_s, n_d, n_g) spaces (equivalently with respect to the group structure U(15) $\supset$ U(1) $\oplus$ U(5) $\oplus$ U(9)) is given by

$$\langle H\rangle^{n_s,n_d,n_g} = \sum_{\alpha\in\langle n_s n_d n_g\rangle} \langle \{n_s n_d n_g\}\ \alpha|H|\{n_s n_d n_g\}\ \alpha\rangle/\dim\{n_s n_d n_g\} =$$

$$\sum_{\substack{\ell_1\geq\ell_2\\ =0,2,4}} \frac{m_{\ell_1}(m_{\ell_2}-\delta_{\ell_1\ell_2})}{N_{\ell_1}(N_{\ell_2}+\delta_{\ell_1\ell_2})} V(\ell_1,\ell_2) + \sum_{\ell=0,2,4} m_\ell\,\varepsilon_\ell\ ,$$

$$V(\ell_1,\ell_2) = \sum_L V^L(\ell_1,\ell_2,\ell_1,\ell_2)\quad (2L+1)$$

(41)

Similarly

$$\langle (H-\langle H\rangle^{n_s n_d n_g})^2\rangle^{n_s n_d n_g} = \sum_{\substack{\ell_1\geq\ell_2\\ \ell_3\geq\ell_4}} \frac{m_{\ell_1}(m_{\ell_2}-\delta_{\ell_1\ell_2})(N_{\ell_3}+m_{\ell_3})}{N_{\ell_1}(N_{\ell_2}+\delta_{\ell_1\ell_2})} *$$

$$\frac{(N_{\ell_4}+m_{\ell_4}+\delta_{\ell_3\ell_4})*W(\ell_1,\ell_2,\ell_3,\ell_4)}{(N_{\ell_3}+\delta_{\ell_1\ell_3}+\delta_{\ell_2\ell_3})(N_{\ell_4}+\delta_{\ell_1\ell_4}+\delta_{\ell_2\ell_4}+\delta_{\ell_3\ell_4})},$$

$$W(\ell_1,\ell_2,\ell_3,\ell_4) = \sum_L \Big\{ V^L(\ell_1,\ell_2,\ell_3,\ell_4) - V(\ell_1,\ell_2) \left\{\frac{N_{\ell_1}(N_{\ell_2}+\delta_{\ell_1\ell_2})}{(1+\delta_{\ell_1\ell_2})}\right\}^{-1}$$

$$* \delta_{\ell_1\ell_3}\,\delta_{\ell_2\ell_4}\Big\}^2 * (2L+1) \qquad (42)$$

In the above equations $N_o = 1$, $N_2 = 5$, $N_4 = 9$, $m_o = n_s$, $m_2 = n_d$ and $m_4 = n_g$. Results in (41), (42) are tested in OUTPUT - 17,18; note that here one is really testing the construction of the H - Matrix. In order to generate the OUTPUT - 17, 18 one has to compile SUBGH, DIMVTL, VRACAH with new drivers TRACE and VARNCE (the later two are supplied along with the SDGIBM1 package).

One can go beyond fixed (n_s, n_d, n_g) traces and derive expressions for fixed (n_s, n_d, v_d, n_g, v_g) traces. One can easily produce the following trace equation for H,

$$\langle H\rangle^{n_s n_d v_d n_g v_g} = \sum_{\ell=0,2,4} \varepsilon_\ell \, n_\ell + \bar{V}_{sd}\, n_s n_d + \bar{V}_{sg}\, n_s n_g + \bar{V}_{dg}\, n_d n_g + \binom{n_s}{2}\bar{V}_{ss}$$
$$+ \langle V\rangle^{n_d=v_d=2}\binom{n_d}{2} + \langle V\rangle^{n_g=v_g=2}\binom{n_g}{2}$$
$$+ \left\{\frac{\langle V\rangle^{n_d=2,v_d=0} - \langle V\rangle^{n_d=v_d=2}}{10}\right\}(n_d-v_d)(n_d+v_d+3)$$
$$+ \left\{\frac{\langle V\rangle^{n_g=2,v_g=0} - \langle V\rangle^{n_g=v_g=2}}{18}\right\}(n_g-v_g)(n_g+v_g+7)$$

where

$$\bar{V}_{ij} = \sum_L \frac{\langle (ij)L|V|(ij)L\rangle (2L+1)(1+\delta_{ij})}{2\Omega_i(2\Omega_j+\delta_{ij})},$$

$$\langle V\rangle^{n_i=v_i=2} = \sum_{L \geq 2} \frac{\langle (ii)L|V|(ii)L\rangle (2L+1)}{(\Omega_i+1)(2\Omega_i-1)}$$
$$= \frac{\left[(\Omega_i)(2\Omega_i+1)\bar{V}_{ii} - \langle V\rangle^{n_i=2,v_i=0}\right]}{(\Omega_i+1)(2\Omega_i-1)}$$

$$\langle V \rangle^{n_i=2, v_i=0} = \langle (ii)\ L = 0\ |V|\ (ii)\ L = 0 \rangle$$

$$\Omega_i = (2\ell_i+1)/2;\ i=s,d,g \qquad (43)$$

We tested in OUTPUT - 17 the expression for $\langle H \rangle^{n_s n_d v_d n_g v_g}$. In principle following Quesne [27] one can produce expressions for $\langle H^2 \rangle^{n_s n_d v_d n_g v_g}$.

5.5. sd - IBM Tests

In principle SDGIBM1 package can be used in place of the PHINT code of [7] to perform sdIBM calculation. Here one has to simply put n_g (maximum) = 0 in the programmes. In OUTPUT - 16 our programmes are tested against the published results of Lipas et.al [28] for ^{152}Gd carried out in sd - frame work. They employ the Hamiltonian

$$H = \varepsilon''_d n_d + a_o P^{\dagger} P + a_1 L^2 + a_2 Q^2$$

$$\varepsilon_d = \varepsilon''_d - \frac{(N-1)}{2} a_o + 6a_1 + \left(2N - \frac{17}{4}\right) a_2$$

$$\langle d^2 0 | V | d^2 0 \rangle = 3a_o - 12a_1 - \frac{1}{2} a_2$$

$$\langle d^2 2 | V | d^2 2 \rangle = \frac{1}{2} a_o - 6a_1 - \frac{19}{4} a_2$$

$$\langle d^2 4 | V | d^2 4 \rangle = \frac{1}{2} a_o + 8a_1 - 3a_2$$

$$\langle (ds)2 | V | d^2 2 \rangle = -\sqrt{14}\, a_2$$

$$\langle d^2 0 | V | s^2 0 \rangle = \sqrt{\frac{5}{2}} \left\{ -a_o + 4a_2 \right\}$$

and all other matrix elements are zero (44)

For ^{152}Gd they [28] give ε''_d = 438KeV, a_o = -38.04KeV, a_1 = 1.21KeV and a_2 = -11.36 KeV. Our programme reproduces all the eigenvalues given in Table - III of ref [28]. We calculated the B(E2)'s for

various combinations of L_i and L_f and the results given in OUTPUT - 16 are in confirmity with Tables - VIII, $\underline{X}$ of ref [28]; here the E2 operator is $T^{E2}=0.1430(d^{\dagger}\tilde{s}+s^{\dagger}\tilde{d})^2 - 0.2869(d^{\dagger}\tilde{d})^2$. We have also verified the results given in [28] for ^{154}Gd but they are not shown in the outputs.

In the sd IBM a variety of analytical formulas can be derived [2] and they can be used to test our programmes. For example the analytical expression given below (which appears in the problem of coupling a g boson to O(6) core),

$$\langle n_d,\tau,2\tau \parallel (d^{\dagger}\tilde{d})^3 \parallel n_d,\tau,2\tau\rangle = \langle \tau,\tau,2\tau \parallel (d^{\dagger}\tilde{d})^3 \parallel \tau,\tau,2\tau\rangle \frac{6}{(4\tau-1)!}\left\{\frac{7(4\tau+4)!(4\tau-3)!}{8!}\right\}^{1/2} \tag{45}$$

was tested in OUTPUT - 19 for n_d upto 12. The crucial point about the above matrix element is that it is independent of n_d. In the programmes, to generate $\langle n_d,\tau,2\tau \parallel (d^{\dagger}\tilde{d})^3 \parallel n_d,\tau,2\tau\rangle$ one has to compile SUBGH, DIMVTL, VRACAH with a new driver NDTAU. The routine NDTAU is supplied along with the SDGIBM1 package. We should point out that eq(45) infact verifies the subroutine TWODG that appears in SUBGM.FOR.

6. DIRECT AND INDIRECT EVIDENCE FOR INCLUSION OF g-BOSONS

In order to emphasize the importance of the gIBM model and to make the report more purposeful,in this section we describe the experimental and theoretical results that point out the need (and compel one) to include g-bosons in IBM. Later in Sec.7 ahead we will give a short review of the work done in gIBM till today and the new results it has brought out.

Over the last two decades large amount of experimental data on hexadecupole (E4) deformation parameter β_4 and B(E4) values involving the ground state ($K^\pi = 0^+$) as well as $K^\pi = 2^+,3^+,4^+,$--- bands (or $4^+_1,4^+_2,4^+_3,$--- levels) has accumulated. In order to understand this data in an algebraic (IBM) framework one has to include g-bosons. Just as the s and d bosons represent the pairing and quadrupole degree of freedom, the g-bosons represent the hexadecupole degree of freedom. There is also sacttering data that clearly brings out the hexadecupole nature of certain 4^+ levels in some nuclei. Moreover the presence of odd-K rotational bands ($K^\pi = 3^+,1^+$), which cannot arise in a simple quadrupole model (sd IBM), indicate a possible E4 character of the levels belonging to these bands. In addition some of the $K^\pi = 4^+$ bands also contain E4 collectivity. In section 6.1 the above mentioned data is described in some detail and they constitute direct evidence for the inclusion of g-bosons in IBM. We would like to mention that in section 6.1 all individual experimental papers are not referred to, as it is beyond the scope of this report.

Microscopic theories of IBM, both in spherical and deformed

basis clearly show that the G ($L^{\pi} = 4^{+}$) pairs in addition to the S($L^{\pi} = 0^{+}$) and D ($L^{\pi} = 2^{+}$) pairs are essential for a meaningful description of various observables like intrinsic quadrupole moments, moment of inertia etc. It should be pointed out that the S, D and G pairs are microscopic (fermionic) counterparts of the s, d and g bosons respectively. In section 6.2 a brief description of some of the microscopic theories of IBM are given and shown in particular are the results that clearly point out the need to include g-bosons (G-pairs). Other indirect evidences are collected in section 6.3.

6.1. Experimental Data on E4-Observables:

6.1.1 Rare-earth Nuclei

Large number of scattering experiments [(e,e'), (d,d'), (p,p'), (α,α') etc] that determine the equilibrium deformation in rare-earth nuclei are performed by various research groups over the last two decades. A compilation of all the available data on β_4 values is given by Janecke [29] and they are shown in fig.1; the data points have $\sim$ 10% error in most cases.

Fig.1

The following equation [29] describes the data,

$$\beta_4 = \beta_{40}\ (x)\ (1-x)\ (2-x)\ (y)\ (1-y)\ (2-y)\ (7z^2-14z+4.9),$$

$$\text{where } x = \frac{(Z-50)}{(82-50)},\ y = \frac{(N-82)}{(126-82)},\ z = \frac{x+y}{2},\ \beta_{40} = 0.504 \qquad (46)$$

Ichihara et.al [30] via (p,p') scattering experiments produced experimental information on Y_{42} deformation in rare-earth nuclei.

In fact they produce values for iso-scalar mass transition density B(IS4; $0^+ \longrightarrow 4^+_\gamma$). The results are given in fig.2.

FIG 2

Expressed in single particle units, the B(IS4;$0^+ \longrightarrow 4^+_\gamma$) for ^{152}Sm, ^{154}Sm, ^{160}Gd, ^{164}Dy, ^{166}Er, ^{168}Er, ^{176}Yb, ^{182}W, ^{184}W and ^{192}Os are 3.01, 3.11, 5.69, 6.43, 7.01, 8.20, 3.61, 2.03, 2.82 and 3.36 respectively. Note that the single particle unit (s.p.u) for E4 transition (B(EL↑)) is,

$$B_{s.p.u}(EL\uparrow) = \frac{(2L+1)}{4\Pi}\left[\frac{3}{3+L}\right]^2 R^{2L}\, e^2 fm^{2L};\quad R = 1.2\, A^{1/3} \qquad (47)$$

Recently Govil et.al [31] produced data on B(E4)'s for ^{168}Er and ^{172}Yb by inelastic scattering of α-particles. Their results for B(IS4; $0^+ \longrightarrow 4^+_i$) are given in Table - 14. They also compiled the

TABLE - 14

available experimental data on B(E4;$0^+ \longrightarrow 4^+_k$) for K = 0,2,3 and 4 bands; see fig.9 of the paper on ^{172}Yb [31]. They also include data on Pt isotopes (due to Burke). We should mention that Todd Baker et.al [32], via $(\bar{p},p')$ excitation of 4^+ levels in ^{192}Os, produced transition matrix elements M(E4;$4^+ \longrightarrow 0^+$) where

$$M^4_{04_i} = M(E4;4^+_i \longrightarrow 0^+) = \langle 4^+_i \| T^{E4} \| 0^+ \rangle \qquad (48)$$

For the first three 4^+ levels (580, 910, 1070 KeV)

$$M^4_{04_1} / M^4_{04_2} / M^4_{04_3} = -2000/\pm 1180/\pm 1100\ efm^4$$

Going beyond the data on selected B(E4) values, Goldhoorn

et.al and Wu et.al (see ref [33]) produced results on E4 transition strengths below 3MeV in ^{156}Gd and ^{150}Nd respectively. Thus E4 distributions are known for these nuclei and the results are shown in fig.3.

FIG 3

We remark that these distributions are well reproduced by gIBM, we will return to this in Sect.7.2. Recently [34] E4 distributions (upto 5MeV excitations) in ^{112}Cd are reported: see section 7.2 for more details.

6.1.2. Actinides and (ds) and (fp) shell nuclei

Experimental data on B(E4)'s, though not as exhaustive as in rare-earths, is available for actinides and also for (ds) and (fp) shell nuclei. Zumbro et al [35, and refs. therein] produced high quality data on β_4 values for actinides from the analysis of muonic X-rays. For the nuclei ^{233}U, ^{234}U, ^{235}U, ^{238}U, ^{232}Th, ^{239}Pu, ^{240}Pu and ^{242}Pu the (β_4, Q_4 eb^2) are (0.091,2.55), (0.084,2.49), (0.091, 2.64), (0.067,2.28), (0.095,2.48), (0.090,2.84), (0.070,2.47) and (0.050,2.08) respectively. Endt [36] compiled B(E4) data for (sd) and (fp) shell nuclei and the results are given in Table 15.

TABLE - 15

Scattering Data

Transition densities or form factors determine, in a transparent way the hexadecupole character of the observed 4^+ levels. Burke et.al [39] using (α,α') reaction (Bagnell et.al [40] use (t,α) reaction) showed that the 4^+_3 state at 1.28, 1.163, 1.069

(MeV) in $^{188,190,192}Os$ are predominantly populated by strong (direct) E4 transitions. Thus establishing the hexadecupole vibrational character for these levels (usually they are interpreted as two-phonon excitations). Bagnell et.al [40] reproduce the $^{191,193}Ir(t,\alpha)^{190,192}Os$ reaction cross-sections for exciting the 4^+ level at 1.162 and 1.07 (MeV) in $^{190,192}Os$, by assuming the 4^+ level to be of single-phonon hexadecupole vibration involving [402] 5/2 and [402] 3/2 Nilsson configurations. Borgols et.al [41] gave the form factors of the transitions to the first three 4^+ states at 0.877, 1.293, 1.887 MeV in ^{196}Pt produced via electron scattering. From their analysis they conclude that the 4^+ states should include hexadecupole deformation. In Pd isotopes a similar conclusion was reached by deJegur [42] who analysed the transition densities for $0^+ \longrightarrow 4^+_1$, 4^+_2 measured via electron scattering. Recently Sirota et.al [43] pointed out that the level at 2.367 MeV in ^{100}Ru, populated by inelastic scattering of 16MeV protons, is of hexadecupole type and determined the value of β_4 to be $\simeq$ 0.10 which is one of the largest values known in any region of the periodic table. Finally Zuffi et.al [44] pointed out that the scattering cross-sections, of 2.94 MeV protons to the first five levels of ^{32}S and 40 MeV protons to the first seven levels of ^{24}Mg, can be well reproduced by including E4 effects.

6.1.3. $K^\pi = 1^+, 3^+$ and 4^+ bands:

Nesterenko et.al [45] compiled the energies of $K^\pi = 3^+, 4^+$ band heads in rare-earth nuclei (in most cases L = K, K+1, K+2,

---- etc are observed) and they analyzed these bands in a quasi-particle phonon model. In the nuclei ^{168}Er, ^{170}Er, ^{168}Yb, ^{170}Yb, ^{172}Yb, ^{174}Yb, ^{174}Hf, ^{176}Hf, ^{178}Hf and ^{184}W the $K^{\pi} = 3^{+}$, $L^{\pi} = 4^{+}$ level energies (in MeV) are 1.653, 1.217, 1.452, ~ 1.470, (1.172,1.663), (1.606,2.016), 1.303, 1.578, 1.828, 1.425 respectively. Note that $^{172,174}Yb$ there are two $K^{\pi} = 3^{+}$ bands. In addition in ^{234}U there is a 3^{+} band starting from 1.5 MeV. Similarly in the nuclei ^{158}Gd, ^{160}Gd, ^{158}Dy, ^{160}Dy, ^{162}Dy, ^{164}Dy, ^{166}Er, ^{168}Er, ^{172}Yb, ^{176}Hf, ^{178}Hf, ^{186}Os and ^{188}Os the $K^{\pi} = 4^{+}$, $L^{\pi} = 4^{+}$ level energies (in MeV) are (1.380,1.920), 1.184, 1.895, (1.694,2.096), 1.535, (2.194,2.206), 1.976, 2.055, 2.093, 1.888, 1.514, 1.352 and 1.280 respectively. Nesterenko et.al [45] conclude from their theoretical analysis that $K^{\pi} = 3^{+}$, 4^{+}, $L^{\pi} = 4^{+}$ levels in Gd, Dy isotopes are essentially of two-quasiparticle type, in Er, Yb and Hf, they are mixture of two-quasiparticle and hexadecupole excitations (B(E4) ~ 1 to 2 s.p.u) while in heavy isotopes of Os and in W they are predominantly of E4 type (B(E4) ~ 4 s.p.u). Before turning to 1^{+} levels, let us mention that odd K ritational bands do not appear with quadrupole phonons alone. Recently in large number of nuclei 1^{+} levels (very rarely the 2^{+},3^{+},---- members of the $K^{\pi} = 1^{+}$ band are observed) are identified because of the interest in the so-called (Richter [46] and refs therein) scissor mode. Pittel et.al [47] argued that some of the scissor states may have significant E4 character (hence one may study them via gIBM !). Hartman et.al [48] give the energies (MeV) of the scissor states ($K^{\pi} = 1^{+}$, $L^{\pi} = 1^{+}$ level) in the nuclei

^{46}Ti, ^{110}Pd, ^{154}Sm, ^{154}Gd, ^{156}Gd, ^{158}Gd, ^{164}Dy, ^{168}Er, ^{232}Th, ^{238}U to be 4.3, 3.9, 3.2, 2.9, 3.1, (3.2,3.1), 3.2, 3.8, 2.0 and 2.2 respectively. In ^{172}Yb [49], ^{174}Yb [50], ^{168}Er [51] and ^{176}Hf [52] nuclei 1^+ levels are observed at 1.8, 1.624, (2.13,2.37) and 1.672 (MeV) respectively. See Richter [46] for more data on 1^+ levels (also distribution of 1^+ levels in nuclei Ti, Cr, Gd --- etc) and detailed discussions. We conclude this subsection with a remark taken from Walker et.al [53] which summarises the present situation with regard to data on E4 observables - "Single phonon shape oscillations in deformed nuclei have been the subject of extensive investigations. Much has been learnt about the systematic occurrence of the quadrupole (β,γ) and octupole modes. The situation for the hexadecupole mode is one of comparative ignorance though some nuclei received some attention. ---- We have brought together information on low-lying $K^\pi = 3^+$ bands--- a simple explanation of their properties is possible in terms of hexadecupole shape oscillations, albeit poorly separated from the two quasiparticle degree of freedom". As Akiyama et.al [54] state "we point out that both theoretically and experimentally the $I,K^\pi = 4^+_1$, 3^+_1, state is strongly excited. This fact can be taken as a rather strong evidence for interpreting $K^\pi = 3^+$ band within the gIBM."

6.1.4. Theoretical Approaches for Calculating β_4 and B(E4) Values:

In order to understand the variation of β_4 (also β_2) with mass number in rare-earth nuclei (see fig.1) Bertsch [55] proposed the so-called polar cap model and it was extended recently by Ichihara

et.al [30] to explain the Y_{42} transition strength to the γ-band as a function of mass number (see fig.2). The basic principle in this simple-minded model is that the residual interaction tends to correllate particles spatially. With $\mu = \cos\theta$ (roughly speaking $\mu = \frac{N-m}{N}$, where 'N' is the degeneracy of the valence shells and 'm' is the number of valence nucleons) the λ - th multipole moment will be proportional to

$$\int_{\mu}^{i} P_{\lambda}^{o}(x)\,dx \qquad (49)$$

and the $Y_{\lambda 2}$ transition strength will be proportional to

$$\left[P_{\lambda}^{2}(\mu)\right]^{2} \qquad (50)$$

Where $P_{\lambda}^{r}(x)$ is the assosciated Legendre Polynomial. Equation (50) follows by interpreting the γ - vibration as a superposition of particle-hole states with $K^{\pi} = 2^{+}$ near the Fermi-surface ($\mu \pm \Delta\mu$). In fig.4 the results of

FIG 4

(49) and (50) and also the experimental data for $\lambda = 2$ and 4 are shown.

Broadly speaking one can determine β_4 values by following (1) Strutinsky procedure or (2) Hartree-Fock method. The simplest approach is due to Harada [56] who assumes the value of β_2 and in a deformed potential with some β_4 calculates the correction to the ground state energy and minimises it with respect to β_4. He used this to calculate β_4 values for actinides. In a better calculation

Nilsson et.al [57, references therein] construct a deformed single particle potential with β_2 and β_4 and calculate the ground state energies, they add also the pairing and coulomb corrections. Then they minimize the total energy w.r.t. β_2 and β_4 to determine the equilibrium values (β_2^o, β_4^o). (Here it is not insisted that the ground state masses be produced correctly). They apply this theory with success for rare-earth and actinide nuclei. Moller and Nix [58] in their microscopic-macroscopic approach for the calculation of ground state masses perform more elaborate calculations with β_2 and β_4 and determine their equilibrium values besides studying many other interesting quantities; see also Bengtsson [59]. For a good discussion on the Hartree-Fock method for producing β_4 values see Quentin and Flocard [60]. Finally we remark that Todd Baker [61] used the Davydov and Fillippov asymmetric rotor model for calculating B(E4) values with reasonable success in rare-earth nuclei. We refer the readers to ref.[62] for some work on β_4 deformation in odd-A nuclei.

6.2. G-Pairs in the Microscopic Theories of IBM

The microscopic theories of IBM employing both the spherical and deformed basis (the later in particular) unambiguously brought out the important role played by the pairs of nucleons coupled to $L^\pi = 4^+$ (G-Pairs). Here below we will give a brief description of the results obtained in these two methods.

6.2.1. Spherical Basis:

In the spherical basis one constructs pairs coupled to $L^\pi=0^+\left[S^\dagger=A^{\dagger(0)}\right]$, $2^+\left[D^\dagger=A^{\dagger(2)}\right]$, $4^+\left[G^\dagger=A^{\dagger(4)}\right]$, etc,

$$A^{\dagger(L)} = \sum_{j,j'} \alpha^{(L)}_{jj'} \left[a^{\dagger}_{j} a^{\dagger}_{j'}\right]^{(L)}$$

(51)

and then a one to one correspondence is established between the boson states $(|(s^{\dagger})^{n_s}; (d^{\dagger})^{n_d}...L_d ; (g^{\dagger})^{n_g}...L_g ; L\rangle)$ and the fermionic states built out of $S(A^{\dagger(0)})$, $D(A^{\dagger(2)})$, $G(A^{\dagger(4)})$ pairs ($s \longrightarrow S$, $d \longrightarrow D$, $g \longrightarrow G$, ---). To implement this scheme one needs (i) an appropriate projection operator that generates a one to one correspondence between the above boson and fermion states (ii) a method to fix the coefficients $\alpha^{(L)}_{jj'}$ in (51) and (iii) a method to map the fermion Hamiltonian and here one usually employs the so-called OAI method. This is straight forward for a single j-shell with identical particles [13,21] and it can be extended to the multi-j-shell case as well as for a system of protons and neutrons [8]. We will now describe some results. In one of the earliest calculations Otsuka [63] constructed D and G pairs using the prescription

$$A^{\dagger(L)}_{q} = \left[S^{\dagger}, \sqrt{\frac{4\Pi}{2L+1}}\, Y^{(L)}_{q}(\theta,\phi)\right]_{-}$$

$$S^{\dagger} = \sum_{j} \sqrt{\frac{2j+1}{2}} \left[a^{\dagger}_{j} a^{\dagger}_{j}\right]^{(0)}$$

(52)

and $A^{\dagger(0)}$ are the so-called favoured pairs that arise out of the surface delta interaction. Otsuka considered the case of $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$ and $3s_{1/2}$ shells and performed the calculations with (1) SD truncation (2) Exact and (3) Renormalization due to G-pairs. The results are shown in fig.5a.

FIG 5A

One observes remarkable improvement in the spectra due to inclusion of G-pairs. Van Egmond and Allaart [64] employ the broken pair method to define the S, D and G pairs in proton and neutron spaces and determine the parameters in IBM-2 Hamiltonian

$$H_{IBM2} = \sum_{\rho=\pi,\nu} \varepsilon_\rho n_{d,\rho} + \kappa\, Q_\pi . Q_\nu + \sum_j \xi_j \left[(d^\dagger_\pi d^\dagger_\nu)^{(j)} (\tilde{d}_\pi \tilde{d}_\nu)^{(j)} \right]^{(0)}$$

$$\text{where } Q_\rho = (d^\dagger_\rho s_\rho + s^\dagger_\rho \tilde{d}_\rho)^{(2)} + \chi_\rho (d^\dagger_\rho \tilde{d}_\rho)^{(2)} \qquad (53)$$

by using the OAI mapping. They conclude that states with a collective G-pair are most important and they contribute to about half of the total renormalisation of the parameters as can be seen in fig.5b. Similar studies were also performed by Scholten [65].

FIG 5B

Recently Halse et.al [66] performed exact shell-model calculations in single j = 17/2 shell of identical nucleons and compared the results with those of SD and SDG truncations. They consider a system of six particles interacting through a quadrupole force. They conclude that the levels belonging to the groundstate band can be reproduced reasonably well by SD truncation, however the levels belonging to K = 2 band require G-pairs and even with G-pairs, the moment of inertia is ~ 30% too small (see fig.5c).

FIG 5C

6.2.2. Deformed Basis:

In the deformed basis which is most appropriate for rotational nuclei, one starts with a Nilsson-BCS Wavefunction or its equavalent [67]. Then for example in a single j-shell, the axially symmetric deformed intrinsic state is

$$|\text{Nils} - \text{BCS}\rangle = \prod_{m>0} (u_m + v_m * a_m^\dagger a_{\bar{m}}^\dagger)|0\rangle$$

(54)

where $a_m^\dagger$ creates the time-revised state of a state with 'm', u_m and v_m are the u, v factors (occupation probabilities) of BCS theory. After projecting particle number, we will have for a system of 2N nucleons

$$|2N\rangle = N^{-1} (\Lambda^\dagger)^N |0\rangle$$

$$\Lambda^\dagger = \sum_{m>0} (u_m/v_m)\, a_m^\dagger a_{\bar{m}}^\dagger = \sum_J x_J \frac{1}{\sqrt{2}} (a_j^\dagger a_j^\dagger)^{(J)}$$

(55)

which has a structure analogous to the SU(3) intrinsic state for s, d, g, i ---- boson system. Thus $|x_J|^2$ give the composition of the intrinsic state in terms of S(J = 0), D(J = 2), G(J = 4), ---- pairs in the intrinsic states and they can be calculated easily. One can extend (55) to p-n systems as well as for the multi-j-shell case. We will first discuss Yoshinaga et.al's [68] work which brings out succinctly the important role played by G-pairs in deformed nuclei. The authors consider proton-neutron system in a single j-shell and construct the $K^\pi = 0^+$ intrinsic state as

$$|\psi\rangle = |\phi_\pi\rangle\, |\phi_\nu\rangle$$

$$|\phi_\rho\rangle = (\Lambda_\rho^\dagger)^{N_\rho} |0\rangle$$

$$\Lambda_\rho^\dagger = \frac{1}{\sqrt{2}} \sum_J x_J^\rho (a_j^\dagger a_j^\dagger)_0^{(J)}$$

(56)

where $\rho = \pi$ (proton bosons) or ν (neutron bosons) and $N = N_\pi + N_\nu$. Starting from the Hamiltonian

$$H = -G_o^{\pi}\, S_{\pi}^{\dagger} S_{\pi} - G_o^{\nu}\, S_{\nu}^{\dagger} S_{\nu} - \kappa\, Q_{\pi} \cdot Q_{\nu} \qquad (57)$$

they minimise $\frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}$ by treating x_J^{ρ} as variational parameters. In their calculations they take j=23/2, $N_{\pi} = N_{\nu} = 5$, $G_o^{\pi} = G_o^{\nu} = 0.2$ MeV ($\sim 30/A = 0.2$ for A = 150) and $\kappa = 1$ MeV. Three sets of calculations are performed with (i) all j-pairs (we call it ALL) in $A_{\rho}^{\dagger}$ (ii) SD pairs only and (iii) SDG pairs in (56). The results are shown in the table 16. It is remarkable to note that although

TABLE - 16

the probability of having G-pairs is small (<10%) in the intrinsic states, its effect on intrinsic quadrupole moment (Q_{int}), intrinsic state energy (E_{int}), pairing gap (Δ) and binding energies (the actual binding energy is $\sim$ 24.8 MeV) is substantial. In the case of moment of inertia and B(E2)'s the improvement is significantly large. Moreover replacing the A_{ρ} by $\sum_J \left(x_J^{\rho}\ b_J^{\dagger\rho}\right)$ with the same x_J^{ρ}'s given in the above table and decomposing the resulting boson wavefunction ψ_B (also the fermion wavefunction ψ_F) into eigenstates of good angular momentum,

$$|\psi\rangle_{F \text{ or } B} = \sum_J C_J^{F \text{ or } B}\ |J\rangle_{F \text{ or } B}$$

it is found that the two states are very similar to each other: $C_J^F \simeq C_J^B$. This equality means that $|\psi\rangle_F$ and $|\psi\rangle_B$ respond in the same way to the rotation of the intrinsic frame (see ref [69]).. The conclusion from the study is as stated by Yoshinaga et.al [68], "the inclusion of hexadecupole degree of freedom in addition to the monopole and quadrupole degree of freedom is important and sufficient to reproduce physical quantities". Similar conclusions

are reached by Pannert et.al [70] who use pairing + quadrupole force and employ HFB intrinsic states in place of Λ^{ρ} defined above. They also give a simple prescription to estimate the number of d and g bosons that should be included in a realistic calculation. For the deformed nucleus ^{154}Sm they conclude that one must include many d-bosons and atleast two g-bosons; see fig.5d.

FIG 5D

Dobaczewski and Sskalski [71] employed Skyrme SIII interaction and using the HFB method showed that the inclusion of J = 0, 2 and 4 components is necessary to reproduce equilibrium deformations and deformation energy. Bohr and Mottelson [72] show that by analyzing the intrinsic states of excited 0^+ bands, the G-pairs will have similar effects as above. Finally Cohen [73] constructs a coherent state for pairs of fermions coupled to a definite angular momentum J, writes the classical equations of motion for the coherent state variables and studies their eigenmodes, using the formalism developed (for bosons) by Hatch and Levit [74]. He concludes that the SD space alone is not adequate to describe either the static properties of groundstate or the dynamic properties of low frequency modes of excitation for any well deformed nucleus and with SDG truncation he could qualitatively confirm the importance of G-pairs in the context of dynamics.

6.3. INDIRECT EVIDENCES FOR INCLUSION OF g-BOSONS:

Besides all the direct evidences described in sect 6.1 and 6.2 above, there are a number of indirect evidences for inclusion of g bosons in IBM and some of them are:

(a) g-factor variation with L as seen (say) in ^{166}Er [75]: Unless one includes higher-body terms [76], with sd bosons it is not possible to explain the data. See Sect.7.

(b) Odd-K rotational bands ($K^{\pi} = 1^{+}, 3^{+}$): They will not arise in sd-IBM but they do appear in gIBM. However it is possible that some of the observed $K^{\pi} = 3^{+}$ bands could be two quasiparticle bands while some of the 1^{+} levels could be interpreted by p-n IBM as mixed symmetry states. See Sect.7.

(c) Anharmonicity associated with the $K^{\pi} = 4^{+}$ bands in ^{168}Er: This was first pointed out by Bohr and Mottelson [72]; see Sect.7.

(d) Extension of the sd-IBM band cut-off: In sd-IBM rotational bands terminate at $L_{max} = 2N$ while with g-bosons it gets extended to 4N. This will also effect B(E2) values, see Sect.7. ^{218}Ra provides a good example when the band cut-off extends beyond the sd-IBM value [77].

(e) Certain transitions that are forbidden in sdIBM may be allowed with g-bosons: For example in ^{218}Ra, a 6^{+}_{1} state has $n_d = 3$, while the 5^{-} level is viewed as a fully aligned level of structure $\psi(3^{-}) \times \psi(2^{+})$ and therefore has $n_d = 1$ and $n_f = 1$ (or one octupole phonon). The $L = 7^{-}$ state has $n_d = 2$. Clearly, then, while the later may deexcite to the 6^{+} level by an E1

transition, the $6^+ \longrightarrow 5^-$ E1 transition is forbidden. This contradicts the experimental situation and can be easily remedied by including g-bosons [77]. Similarly in rotational nuclei two nucleon addition experiments populate only two 0^+ levels while with g-bosons one can excite four 0^+ levels, see Sect.7.

(f) B(E2)'s involving γ-band and $K^\pi = 4^+$ band are in general poorly reproduced by the sdIBM. Van Isacker et.al [78] show that they can be improved by adding g-bosons (Todd Baker [61] also arrives at a similar conclusion).

(g) The macroscopic SU(3) scheme of Bhatt et.al [79] provides a quite different approach to probe into the question of the importance of G-pairs. In this method the projection amplitudes of the Hartree-Fock intrinsic state are fitted to the similar coefficients corresponding to an SU(3) representation $(\Lambda,0)$ and then the value of Λ is derived. For a number of deformed nuclei the value of Λ is much larger than the IBM SU(3) value (which is 2N). One can also infer the value of effective boson number from the intrinsic quadrupole moment. Both of them point out that $\Lambda = 4N$ is more appropriate than $\Lambda = 2N$ and hence one has to include g-bosons; see [80].

(h) Careful analysis of data on $K^\pi = 2^+_2$, 0^+_2 band head

energies by Casten et.al [81] point out the need to extend sdIBM: In sdIBM $K^{\pi} = 2^{+}_{\gamma}$, 0^{+}_{β} come from (2N-8,4) and (2N-6,0) irreps. In the SU(3) limit R(2N-8,4) = [E(2N-8,4)-E(2N,0)] /[E(2N-4,2)-E(2N,0)] ——> 2 and similarly R(2N-6,0) ——> 2. The experimental numbers for $E(K=2^{+}_{2})/E(K=2^{+}_{1})$ and $E(K=0^{+}_{2})/E(K=2^{+}_{1})$ are $\simeq$ 1.2. Even detailed IBM calculations fail to reproduce this result. Perhaps one has to then include s', d', g --- bosons.

(i) Levels that clearly lie outside the sd space:
Say the 6^{+}_{1}, 8^{+}_{1} levels in ^{20}Ne, 1^{+} level in ^{24}Mg etc.

(j) In general it appears that to reproduce well M1-properties, one may have to include g-bosons; see Sect.7.

7. sdg-Interacting Boson Model: A Review

Large number of (scattered) papers written on gIBM and the absence of an upto-date data review on the progress made/new physics derived, has prompted us to give a brief account of the work carried out so far in the sdg-model.The present section together with Sect.6 should motivate researchers (experimentalists as well as theoreticians) to carry out furtherwork on gIBM. In Sect.7.1 we describe the renormalization theories to account for g-boson effects. Sect.7.2 deals with numerical (mostly 1-g boson) studies carried out in gIBM. In Sect.7.3 dynamical symmetries in gIBM are described in general and Sect.7.4 describes the SU(3) limit in some detail. Sect.7.5 deals with the 1/N expansions (appropriate for deformed nuclei) in gIBM. In Sect.7.6, we describe the OAI mapped gIBM Hamiltonian and also the BSDI interaction defined in eq (30,31). Finally in Sect.7.7 we describe briefly p-n gIBM, gIBFM and also gIBM-3,4. We point out that Casten and Warner's [3] review on IBM1 has a section on gIBM and it deals mainly with the numerical studies of Sect.7.2 and briefly about the SU(3) limit described ahead in Sect.7.3.

7.1. Renormalization studies

In the situation where the sd-description is satisfactory (at a phenomenological level), it is often found to be adequate to use an effective interaction in sd space i.e renormalized to include g-boson effects. A simple approach, should be good for nuclei near the U(5) (vibrational) limit, is to use second order perturbation theory (PT); see ref [64,65,82,83]. An alternative

approach suggested by Otsuka and Ginocchio (OG) [84] was to employ a unitary transformation that minimizes the coupling between d and g bosons. The OG method may work even for nuclei away from the vibrational region (but not close to SU(3)).

7.1.1. Perturbation Theory

Druce et.al [83] recently demonstrated that the spectroscopic properties of Hg isotopes are well described by taking into account g-boson effects via PT with $|a\rangle$ and $|b\rangle$ denoting states in sd space and $|i\rangle$ states in sdg space with atmost 1-g boson. The effective sd-Hamiltonian $\tilde{H}^{sd}$ is given in second order PT by

$$\langle a|\tilde{H}^{sd}|b\rangle = \langle a|H^{sdg}|b\rangle + \sum_{i} \frac{\langle a|H^{sdg}|i\rangle * \langle i|H^{sdg}|b\rangle}{(E_b - E_i)}$$

$$E_x = \langle x|H^{sdg}|x\rangle \qquad (58)$$

As $\tilde{H}^{sd}$ is in general not hermitian, a hermitized Hamiltonian $\approx{H}^{sd}$ is constructed by a simple averaging prescription.

$$\langle a|\approx{H}^{sd}|b\rangle = \frac{1}{2}\left[\langle a|\tilde{H}^{sd}|b\rangle + \langle b|\tilde{H}^{sd}|a\rangle\right] \qquad (59)$$

Constructing collective S, D, and G pairs and employing the OAI method [21] H^{sdg} is derived and then the renormalized H^{sd} defined by (58) and (59) was produced. Employing semi realistic single particle enrgies and the surface delta interaction with different strengths in pp,nn and pn channels, Druce et.al [83] derived $\approx{H}^{sd}$ and studied the spectroscopic properties (spectra, Q_2, B(E2), B(E2) ratios etc.) of $^{194-202}_{80}Hg$; in fact they employ p-n IBM

throughout. Results for the spectrum are shown in Fig.6a. The agreement with

FIG 6A

experiment is satisfactory. Further improvements can come from the inclusion of renormalisation effects due to non collective pairs (in addition to G pairs); see ref [64,65]. As Druce et.al state "we should reiterate that the analysis carried out in this work is not applicable in strongly deformed regimes. There an alternative formalism ---- is necessary". The OG method which we describe next works even when PT fails.

7.1.2. Otsuka-Ginocchio Unitary Transformation Method

Let us start with the quadrupole operator Q^{ρ} where ρ stands for proton boson (π) or neutron boson (ν),

$$Q^{\rho} = e_1^{\rho} (d_{\rho}^{\dagger}\tilde{s}_{\rho} + s_{\rho}^{\dagger}\tilde{d}_{\rho})^2 + e_2^{\rho} (d_{\rho}^{\dagger}\tilde{d}_{\rho})^2 + e_3^{\rho} (d_{\rho}^{\dagger}\tilde{g}_{\rho} + g_{\rho}^{\dagger}\tilde{d}_{\rho})^2 + e_4^{\rho} (g_{\rho}^{\dagger}\tilde{g}_{\rho})^2 \qquad (60)$$

In the OG method we seek a unitary transformation

$U = \exp(Z)$

with an appropriate choice for the anti - hermitian operator Z that defines a transformed H in which g boson couplings are minimized, then one has

$$H \longrightarrow UHU^{-1} = H^{sd}_{effective} + \underbrace{\boxed{H^{g}_{effective} + V^{sdg}_{effective}}}_{neglected} \qquad (61)$$

In what follows the following theorem is useful,

$$UHU^{-1} = H = [Z,H] + \frac{1}{2!}[Z,[Z,H]] + \cdots$$

if $[Z,[Z,H]] \sim -\eta^2 H$

$$UHU^{-1} = (\cos\ \eta)\ H + \frac{\mathrm{Sin}\eta}{\eta}\ [Z,H]$$

(62)

In the OG paper [84] Z was chosen to be (see [84] for the real motivation),

$$Z = \theta_\nu\ Q^\pi\ E^\nu + \theta_\pi\ Q^\nu\ E^\pi$$
$$E^\rho = \left\{(g^\dagger_\rho \tilde{d}_\rho)^2 - (d^\dagger_\rho \tilde{g}_\rho)^2\right\}$$

(63)

and the parameters θ_π, θ_ν are chosen such that the $V^{sdg}_{effective}$ term in (61) can be neglected and the g-boson energy is pushed up sufficiently high in enrgy. It can be shown easily (using (62), (63)) that,

$$US^\dagger_\alpha U^{-1} = S^\dagger_\alpha$$
$$Ud^\dagger_\alpha U^{-1} = \cos\eta\ d^\dagger_\alpha + \mathrm{Sin}\ \eta\ D^\dagger_\alpha$$
$$Ug^\dagger_\alpha U^{-1} = \cos\eta'\ g^\dagger_\alpha - \mathrm{Sin}\ \eta'\ G^\dagger_\alpha$$
$$\text{where } D^\dagger_{(\pi)\alpha} = \left\{(\overline{Q^\nu})^2\right\}^{-1/2} \left[Q^\nu g^\dagger_\pi\right]^2_\alpha$$
$$G^\dagger_{(\pi)\alpha} = \left\{(\overline{Q^\nu})^2\right\}^{-1/2} \left[Q^\nu d^\dagger_\pi\right]^4_\alpha$$

(64)

One can use (64) to produce $Ud^\dagger\tilde{d}U^{-1}$, $Ug^\dagger\tilde{g}U^{-1}$ etc --- and there by derive H^{sd}_{eff}. Alternately one can work out directly the unitary transformation for various pieces of H^{sdg}. Moreover one can take into account terms that are neglected in deriving (via (64)) $U(d^\dagger\tilde{d})U^{-1}$ etc. Thus there are various ways of producing H^{sd}_{eff}; see Amos et.al [85] for all details.

In the example considered by OG for the nucleus ^{158}Gd, the major effect of the unitary transformation is found to be in

lowering d-boson energy and raising the g-boson energy. This then resulting in good agreement, for the g.s. band, between exact calculation and the one via H^{sd}_{eff} (OG choose $\eta_\nu = -30^o$, $\eta_\pi = -29^o$). In a recent investigation Amos et.al [85] employed $H^{sdg} = - Q^\pi . Q^\nu$ where $Q^\pi = \sqrt{\frac{\chi}{4}} [Q^2(s)]^\pi$ with $Q^2(s)$ defined in Table - 4. Their results with various H^{sd}_{eff} (see discussion above) together with the exact one are shown in Fig.6b

FIG 6B

Firstly the moment of inertia, as found in other calculations [64,66,67,68], is about 3/8 of the exact value. In principle one can bring the ground state moment of inertia to be close to the exact value but only at the expense of spoiling the description of binding energy and that of the excited band levels. As seen in Fig.6c indeed a one g-boson calculation in fact gives better

FIG - 6C

results than sd-approximation resulting from the OG method. As Amos et.al state "The renormalisation model provides a quantitative guide to gross structure of the sdg-boson system but as yet cannot be used as a quantitative tool. In spite of earlier promise, a consistent, systematic and accurate summation (approximation) strategy is difficult to achieve. The explicit inclusion of a few g bosons in any shell model calculation seems less fraught with uncertainities".

7.2 NUMERICAL (ONE g-BOSON) STUDIES

In the earlier exploration of gIBM 1g-boson calculations are performed employing the Hamiltonian

$$H = H_{sd} + \varepsilon_g n_g + H_{\Delta n_g=0} + H_{\Delta n_g=1}$$

$$H_{\Delta n_g=0} \simeq k_g\, Q^2_{sd} \cdot Q^2_g$$

$$H_{\Delta n_g=1} \simeq \eta\, \{(g^\dagger s^\dagger)^4 . (\tilde{d}\tilde{d})^4 + h.c\}$$

(65)

The peice $H_{\Delta n_g=0}$ does not mix the configurations ($n_{sd}=N-1$, $n_g=1$) and ($n_{sd}=N$, $n_g=0$) while $H_{\Delta n_g=1}$ does. The general forms of $H_{\Delta n_g=0}$ and $H_{\Delta n_g=1}$ follow from eq(12). Van Isacker et.al [78,86,87] explored (numerically) the structures that arise in the case where H_{sd} satisfies or approximates one of the sd IBM symmetries or interpolates any two of them; see ref [78,86,87] for details. With the SU(3) core (for sd bosons) they analyzed the spectra and B(E2)'s in ^{156}Gd nucleus and observed that the lowlying levels of the g.s., β, γ bands are little effected by the g-bosons and there appears a $K^\pi = 4^+$ band which is primarily of 1-g boson type ($|(2N-2,0)$; g; $K^\pi = 4$ L>). As shown in Fig 7a

FIG 7A

there is excellent agreement for B(E2)'s. One sees for the B(E2)'s for $k^\pi = 4^+$ band go as $4 \longrightarrow 4 \gg 4 \longrightarrow \gamma \gg 4 \longrightarrow \beta \gg 4 \longrightarrow g$ in agreement with data.

Employing a U(5)-O(6) interpolating Hamiltonian for H_{sd} Heyde et.al [87] studied ^{104}Ru which is known to be a U(5)-O(6) transitional nucleus [6]. In this case the g-boson admixings are not strong as $Q_{sd}.Q_g$ term is much less effective. In their 1g-boson study, choosing an appropriate value for the $e^{(4)}(2,4)$ effective charge (18) they could account for the

large jump in yrast B(E2) at $6^+ \longrightarrow 4^+$ and this has to do with the fact that the band cut-off gets extended with g-bosons. Fig 7b

FIG 7B

shows the B(E2) results.

Todd Baker et.al [32] attempted to describe the nucleus ^{192}Os employing the so-called consistent-Q formalism. With reasonable choice of parameters, so that the 4^+_3 (1070) KeV state is a pure 1g-boson state $(s^\dagger)^{N-1}(g^\dagger)^1|0\rangle$, they could describe the spectra, E2 and E4 properties; see Fig 7c.

FIG 7C

However as there are only three pieces of data on B(E4)'s (see Sect 6.1) they could only derive the $e^{(4)}(\alpha,\beta)$ effective charges (18) and verified that they are not meaningless. Further studies on ^{192}Os are clearly called for.

Recently De Leo etal. [34] meassured E4 distributions upto 5MeV excitations in ^{112}Cd and they showed that a one - g - boson calculation increases the predicted strength (in sd model) from a few upto more than 60% of that observed experimentally. The results are shown in Fig 7d; see [34] for details.

FIG - 7D

Zuffi et.al [44] analysed the proton scattering data (also spectra, B(E2), B(E4)) in ^{24}Mg and ^{32}S employing H similar to the one in (65). For more meaningful studies of (sd) shell nuclei one has to go beyond the 1g-boson approximation [19].

Computer codes for performing detailed numerical calculations (employing weak coupled basis) are documented in Sect 2-5 .

Morrison and Kuyucak (see [114] referred to in detail in Sect 7.5) recently mentioned the availability of a code (SDGBOSON) written in the so called m-scheme. Similarly Yoshinaga mentioned [13] the availability of another code (SDGNH) which can handle even antihermitian operators. However the scope of the codes SDGBOSON and SDGNH is not made clear by the respective authors.

7.3 Dynamical Symmetries

The different ways hexadecupole collectivity manifests itself in the low-lying levels can be studied by the various dynamical symmetries of the gIBM H, they genrate 'bench mark' spactra and E2 and E4 branching ratios and selection rules. Dynamical symmetries are exploited with great success in sd IBM and prompted by this Kota et.al [12,88] classified all the dynamical symmetries of gIBM by extending the sd IBM U(6) group to sdg IBM U(15) group.

Broadly speaking gIBM possess, as already mentioned in Sect.3, seven dynamical symmetries and they correspond (identified by the first subgroup of U(15) in a group chain) to the four strong coupling limits SU(3), SU(5), SU(6) and O(15) and the three weak coupling limits U(6) $\oplus$ U(9), U(5) $\oplus$ U(10), and U(14) respectively. The complete group chains are given in Fig 8. Initial studies on the SU(5), SU(3) and U(14) limits is

Fig 8

due to Sun et.al [89], Ratnaraju [90] and Ling [91] respectively. It should be pointed out that the SU(3) group chain follows trivially from the classic work of Elliott [23], SU(5) group chain

by recognising that a particle with ℓ = 0,2 and 4 can be thought as composed of two pseudo-bosons carrying $\tilde{\ell}$ = 2, similarly SU(6) chain with two pseudo-fermions with $\tilde{j}$ =5/2 and finally the O(15) chain corresponds to seniority in sdg space (with zero coupled pair $S_+ = 15^{-1/2}(s^\dagger.s^\dagger + d^\dagger.d^\dagger + g^\dagger.g^\dagger)$. The remaining three chains U(6) $\oplus$ U(9), U(14) and U(5) $\oplus$ U(10) can be realised by demanding that $\{n_{sd} = n_s + n_d, n_g\}$, $\{n_s, n_{dg} = n_d + n_g\}$ and $\{n_d, n_{sg} = n_s + n_g\}$ respectively to be good quantum numbers. Recently Kota et.al [12] completed the task of determining the generators, the construction of Casimir operators in terms of generators and the plethysm problem (reducing the U(15) irrep {N} into irreps of the various groups in a given chain) for the various group - subgroup chains; see table - 4 for some of these results. Here below we will give the low - lying irreps in each chain.

In the U(6) $\oplus$ U(9) limit, the irreps of U(6) and U(9) are denoted by n_{sd} and n_g where $N = n_{sd} + n_g$; $n_{sd} = n_s + n_d$. The reductions of $\{n_{sd}\}$ of U(6) to irreps of various G and O(3) in $U(6) \supset G \supset O(3)$ are well known [2]. For the chain $U(9) \supset O(9) \supset O(3)$ the basis states are $|n_g\ v_g\ \alpha_g\ L_g\rangle$ where $v_g = n_g, n_g - 2, \cdots$ 0 or 1,

$$v_g \longrightarrow L_g = 4v_g \oplus (4v_g - 2) \oplus (4v_g - 3) \oplus (4v_g - 4)^2 \oplus \cdots \quad \text{for } v_g \geq 4$$

$$v_g = 1 \longrightarrow L_g = 4$$
$$v_g = 2 \longrightarrow L_g = 8,6,4,2$$
$$v_g = 3 \longrightarrow L_g = 12,10,9,8,7,\text{----}. \qquad (66)$$

and α_g is the multiplicity lable for $\nu_g \longrightarrow L_g$.

In the U(14) limit the basis states are $|N,n_{dg},\nu_{dg},\beta[\nu_1\nu_2],\alpha L\rangle$ where the U(14) irrep is $\{n_{dg}\}$ with $n_{dg} = n_d + n_g = N,N-1,---,0$ and the O(14) irrep $\nu_{dg} = N_{dg},N_{dg}-2,---,0$ or 1. The reduction of ν_{dg} to O(5) irreps $[\nu_1\nu_2]$ is,

$$\nu_{dg} \longrightarrow [\nu_1\nu_2] = [2\nu_{dg},0] \oplus [2\nu_{dg}-2,2] \oplus [2\nu_{dg}-2,0] \oplus [2\nu_{dg}-3,1] \oplus [2\nu_{dg}-4,4] \oplus \text{----} \qquad \text{for } \nu_{dg} \geq 4$$

$$\nu_{dg} = 0 \longrightarrow [\nu_1\nu_2] = [0,0]$$

$$\nu_{dg} = 1 \longrightarrow [\nu_1\nu_2] = [2,0]$$

$$\nu_{dg} = 2 \longrightarrow [\nu_1\nu_2] = [4,0] \oplus [2,2] \oplus [2,0]$$

$$\nu_{dg} = 3 \longrightarrow [\nu_1\nu_2] = [6,0] \oplus [4,2] \oplus [4,0] \oplus [3,1] \oplus [2,2] \oplus \text{----} \qquad (67)$$

The reduction of O(5) irrep $[\nu_1\nu_2]$ to L for some lowlying $[\nu_1\nu_2]$ are,

$$[\tau,0] \longrightarrow L = \sum_{r=0}^{\tau/3} (2\tau-6r) \oplus (2\tau-6r-2) \oplus (2\tau-6r-3) \oplus\text{-----}\oplus (\tau-3r);$$

$$[\tau,1] \longrightarrow L = (2\tau+1) \oplus (2\tau) \oplus (2\tau-1)^2 \oplus (2\tau-2)^2 \oplus \text{---} \qquad \text{for } \tau \geq 4$$

$$[1,1] \longrightarrow L = 3,1$$

$$[2,1] \longrightarrow L = 5,4,3,2,1$$

$$[3,1] \longrightarrow L = 7,6,5^2,4,\text{---} \ ;$$

$$[\tau,2] \longrightarrow L = (2\tau+2) \oplus (2\tau+1) \oplus (2\tau)^2 \oplus (2\tau-1)^3 \oplus \text{---} \qquad \text{for } \tau \geq 5$$

$$[2,2] \longrightarrow L = 6,4,3,2,0$$

$$[3,2] \longrightarrow L = 8,7,6,5^2,\text{----}$$

$$[4,2] \longrightarrow L = 10,9,8^2,7^2,6^3,\text{----}. \tag{68}$$

In the U(5) ⊕ U(10) limit U(5) irreps are labelled by d - boson number n_d and the U(10) irreps by $n_{sg} = n_s + n_g$. Reduction of n_d to irreps of O(5) and O(3) in the chain U(5) ⊃ O(5) ⊃ O(3) are well known [2]. In the chain U(10) ⊃ U(9) ⊃ O(9) ⊃ O(3) the U(9) irrep $n_g = n_{sg}, n_{sg}-1, \text{---}, 0$ and reduction of $\{n_g\}$ to O(9) and O(3) irreps was described above (see (66)). In the U(10) ⊃ O(10) ⊃ O(9) ⊃ O(3) chain, the basis states are $|n_{sg},v_{sg},v_g,\alpha\rangle$ and the irreps $v_{sg} = n_{sg}, n_{sg}-2, \text{---}, 0$ or 1 and $v_g = v_{sg}, v_{sg}-1, \text{----}, 0$. The $v_g \longrightarrow L$ reductions are given in (66).

The basis states in the SU(3) limit are labelled by $|N,\alpha(\lambda,\mu),kL\rangle$ and we will deal with this limit in detail in Sect 7.4. In the U(5) limit the basis states are $|N,\{f\},\beta[\tau_1\tau_2],\alpha L\rangle$ where the U(5) irreps $\{f\} = \{2N,0,0,0,0\} \oplus \{2N-2,2,0,0,0\} \oplus \{2N-4,4,0,0,0\} \oplus \{2N-4,2,2,0,0\} \oplus \text{---}$. The reduction of $\{f\}$ into the irreps $[\tau_1\tau_2]$ of O(5) are,

$$\{2N,0,0,0,0\} \longrightarrow [\tau_1\tau_2] = [2N,0] \oplus [2N-2,0] \oplus \text{----} \oplus [0,0]$$

$$\{2n-2,2,0,0,0\} \longrightarrow [\tau_1\tau_2] = [2N-2,0] \oplus [0,0] \oplus \sum_{r=1}^{N-2} [2N-2r-2,0]^2 \oplus$$

$$\sum_{r=1}^{N-2} [2N-2r-1,1] \oplus \sum_{r=1}^{N-1} [2N-2r,2]^2 \tag{69}$$

The reduction of $[\tau_1\tau_2] \longrightarrow L$ was given in (68).

In the O(15) limit the basis states are $|N,\sigma,\tau,\beta[\pi_1\pi_2],\alpha L\rangle$. The O(15) irrep $\sigma = N,N-2,\text{---}0$ or 1, O(14) irrep $\tau = \sigma,\sigma-1,\text{--},0$

and the reduction of $\tau \longrightarrow [\pi_1\pi_2]$ as well as that of $[\pi_1\pi_2] \longrightarrow L$ was already discussed in the context of U(14) limit.

In the U(6) limit the basis states are $|N,\{F\},\beta\langle\lambda_1\lambda_2\lambda_3\rangle,\alpha L\rangle$. The U(6) irrep $\{F\} = \{N,N,0,0,0,0\} \oplus \{N-1,N-1,1,1,0,0\} \oplus \{N-2,N-2,2,2,0,0\} \oplus ---$. Reduction of $\{F\}$ into irreps $\langle\lambda_1\lambda_2\lambda_3\rangle$ of Sp(6) for some lowlying $\{F\}$ are.

$$\{N,N,0,0,0,0\} \longrightarrow \langle\lambda_1\lambda_2\lambda_3\rangle = \langle N,N,0\rangle \oplus \langle N-1,N-1,0\rangle \oplus --- \oplus \langle 0,0,0\rangle$$

$$\{N-1,N-1,1,1,0,0\} \longrightarrow \langle\lambda_1\lambda_2\lambda_3\rangle = \langle N-1,N-1,0\rangle \oplus \langle 0,0,0\rangle \oplus \sum_{r=1}^{N-2} \langle r,r,0\rangle^2 \oplus \sum_{r=1}^{N-1} \langle r,r-1,1\rangle$$

(70)

Finally the irreps of O(3) in $\langle\lambda_1\lambda_2\lambda_3\rangle$ of Sp(6) are, for some of the lowlying $\langle\lambda_1\lambda_2\lambda_3\rangle$,

$\langle N,N,0\rangle \longrightarrow L = (4N) \oplus (4N-2)^2 \oplus (4N-3)^2 \oplus (4N-4)^4 \oplus ---$ for $N \geq 4$

$\langle 1,1,0\rangle \longrightarrow L = 4,2$

$\langle 2,2,0\rangle \longrightarrow L = 8,6^2,5,4^2$

$\langle 3,3,0\rangle \longrightarrow L = 12,10^2,9^2,8^3,----;$

$\langle N,N-1,1\rangle \longrightarrow L = (4N-1) \oplus (4N-2)^2 \oplus (4N-3)^3 \oplus (4n-4)^6 \oplus ----$ for $N \geq 5$

$\langle 2,1,1\rangle \longrightarrow L = 7,6,5,4,---$

$\langle 3,2,1\rangle \longrightarrow L = 11,10^2,9^2,8^4,----$

$\langle 4,3,1\rangle \longrightarrow L = 15,14^2,13^3,12^5,----.$

(71)

Complete tabulations of the plethysm results are given in [12].

In order to apply dynamical symmetries, together with the results given above one should construct the symmetry defined sdg

states and derive expressions for the matrix elements of variious transition operators. This important task is yet to be completed. Some initial attempts in this direction for the SU(5) chain are due to Sun et.al [89] who gave a method for explicit construction of the so called heighest-weight states and Vanthournout et.al [24] gave a complete formalism for calculatong the matrix elements of the U(5) generators G^2,G^4 (see Table-4) in the U(5) irrep {2N}. In the U(9) $\supset$ O(9) $\supset$ O(5) $\supset$ O(3) chain Yu et.al [92] and Sen and Yu [93] gave a formalism for constructing the symmetry defined states by angular momentum projection they use SU(9) $\supset$ O(9) $\supset$ O(5)XSU(2)XSU(2) intrinsic states. See also sect.5 for some selection rules in the U(5) and U(6) limits. Finally, we should mention that in the SU(3) limit, due to the pioneering work of Elliott [23], Hecht [94] and Vergados [22], it is easy to carryout analytic/numerical work.This together with the fact that in deformed nuclei one sees a clear need for inclusion of g bosons, lot of attention was paid to the SU(3) limit and large number of important results are derived; see Sect.7.4 ahead for details. Besides the SU(3) chain (and obviously the U(6) $\oplus$ U(9) chain) only the SU(5),U(14) [91] and the O(15) [95] chains are applied; SU(5) for ^{204}Pb, U(14) for ^{106}Cd and O(15) for ^{218}Ra.

Geometrical aspects of the gIBM dynamical symmetries can be studied via coherent states. This will enable us to relate gIBM to the Bohr Mottelson description of collective states interms of shape variables and thus gain physical insight into the algebraic dynamical symmetries and learn about their applicability in

describing observed spectroscopic properties of various nuclei. In a recent study [96] on geometric shapes in gIBM, the following coherent state is used (it follows from the work of ref [2,97]),

$$|N,\beta_2,\beta_4,\gamma\rangle = \{ N!(1+\beta_2^2+\beta_4^2)^N \}^{-1/2} \{ s_0^\dagger + \beta_2 \mathrm{Cos}\gamma\, d_0^\dagger + \frac{\beta_2 \mathrm{Sin}\gamma}{\sqrt{2}} (d_2^\dagger + d_{-2}^\dagger) + \frac{1}{6}\beta_4 [(5\mathrm{Cos}^2\gamma+1) g_0^\dagger + \sqrt{\frac{15}{2}}\, \mathrm{Sin}2\gamma\, (g_2^\dagger + g_{-2}^\dagger) + \sqrt{\frac{35}{2}}\, \mathrm{Sin}^2\gamma\, (g_4^\dagger + g_{-4}^\dagger)] \}^N |0\rangle \quad (72)$$

where β_2 and β_4 are quadrupole and hexadecupole deformation parameters and γ is the asymmetry angle. Taking H to be $H = a * C_2(G)$ where G is the heighest group in a given chain (see Table-4 for $C_2(G)$), the energy functional $E(\beta_2,\beta_4,\gamma) = \langle N;\beta_2,\beta_4,\gamma|H|N;\beta_2,\beta_4,\gamma\rangle$ is constructed and minimized to obtain the equilibrium values $(\beta_2^0,\beta_4^0,\gamma^0)$ in various cases. Plot of the enrgy functionals are shown in Fig 9. From the solutions it can be

FIG 9

seen that the SU(3) and SU(5) limits have a single absolute minimum while SU(6) and O(15) have multiple minima (SU(6) has 3 minima denoted by I,II and III; O(15) has the same absolute minimum for any value of γ and $\beta_2^2 + \beta_4^2 = 1$). The U(14) and U(5) $\oplus$ U(10) have vibrational character. The shapes in the U(6) $\oplus$ U(9) limit are essentially that of the $U_{sd}(6)$ limit and the later are given in [2]. From these results one can conclude that deformed nuclei can be studied by interpolating [U(6) $\supset$ SU(3) $\supset$ O(3)] $\oplus$ U(9), SU(3) and SU(5) limits, the γ-unstable nuclei by

interpolating [U(6) ⊃ O(6) ⊃ O(3)] ⊕ U(9), SU(6), O(15) chains while the vibrational nuclei can be studied by interpolating [U(6) ⊃ U(5) ⊃ O(5) ⊃ O(3)] ⊕ U(9), U(14), U(5) ⊕ U(10) limits.

7.4 THE SU(3) LIMIT

The SU(3) limit of gIBM is essentially, in structure, same as the SU(3) limit of sd-IBM. However it produces several new results and they are in confirmity with data. Some of them are (1) appearence of low-lying $K^{\pi} = 1^{+},3^{+}$ bands, (2) Extension of band cut-off beyond L = 2N, (3) Two $K = 4^{+}$ bands (one two-phonon, other one-phonon or hexadecupole), (4) M1 - transitions, as in pnIBM, with a one-body operator etc.

7.4.1. SU(3) Irreps

The basis states in the SU(3) limit are labelled by $|\{N\},\alpha,(\lambda,\mu),k,L\rangle$. The SU(3) irreps (λ,μ) for a given N may appear more than once and they are distinguished by the multiplicity lable α. Similarly k is the Elliot's band quantum number and the $(\lambda,\mu) \longrightarrow L$ reductions are well known [23]. Following the general method given in [98] one can, on a machine, generate the $\{N\} \longrightarrow (\lambda,\mu)$ reductions and the results for $N \leq 15$ are tabulated in [12]. Akiyama [99] proposed a novel method to obtain the U(15) ⊃ SU(3) reductions ($\{N\} \longrightarrow (\lambda,\mu)$). In the $U_{\varepsilon}(1) \otimes [SU_{\Lambda}(2) \supset U_{M_{\Lambda}}(1)]$ subgroup of SU(3) one has $\varepsilon = 2\lambda+\mu-3(p+q)$, $\Lambda = \frac{\mu}{2}+\frac{1}{2}(p-q)$, $0 \leq p \leq \lambda$, $0 \leq q \leq \mu$. Using the oscillator orbits (n_z,n_x,n_y) for a single boson we have $\varepsilon_0 = 2n_z-n_x-n_y$, $\Lambda_0 = \frac{n_x-n_y}{2}$, then for the case $n_x+n_y+n_z = 4$ we have $\varepsilon_0 = 8,5,2,-1,-4$ and $\Lambda_0 = 0,\frac{1}{2},1,\frac{3}{2},2$ respectively. For N boson system given a (λ,μ), the minimum

ε-value and the corresponding Λ value are $\varepsilon_{min} = -(\lambda+2\mu)$ and $\Lambda_{min} = \frac{\lambda}{2}$. Therefore all irreps with the same $\varepsilon_{min} = -(\lambda+2\mu) = -4N$ can be projected out from the single intrinsic state $(\Phi(\varepsilon_0 = -4, \Lambda_0 = 2))^N$. Now it is easy to see that all the SU(3) irreps with $4N = \lambda+2\mu$ can be obtained by reducing {N} of U(5) to IRR $L = \frac{\lambda}{2}$ of O(3) and this is a solved problem in the U(5) limit of sd-IBM. The group U(5) has O(5) as a subgroup in the chain U(5) $\supset$ O(5) $\supset$ O(3), with v denoting the O(5) label we have

$N \longrightarrow v = N, N-2, \cdots 0 \text{ or } 1$

with $v = 3n_\Delta + \bar{\lambda}$; $L = \bar{\lambda}, \bar{\lambda}-1, \cdots 2\bar{\lambda}-2, 2\bar{\lambda}$

$v = N \quad \bar{\lambda}=N \; L = 2N, 2N-2, 2N-3, \cdots, 2N-8, \cdots$

$\bar{\lambda}=N-3 \; L = 2N-6, 2N-8, \cdots$

$v = N-2 \; \bar{\lambda}=N-2 \; L = 2N-4, 2N-6, 2N-7, 2N-8, \cdots$

$\bar{\lambda}=N-5 \; L = 2N-10, 2N-12, \cdots$

$v = N-4 \; \bar{\lambda}=N-4 \; L = 2N-8, \cdots,$

(73)

Thus

$$\{N\} \longrightarrow (\lambda,\mu) = (4N,0) \oplus (4N-4,2) \oplus (4N-6,3) \oplus (4N-8,4)^2 \oplus (4N-10,5) \oplus (4N-12,6)^3 \oplus (4N-14,7) \oplus (4N-16,8)^4 \oplus \cdots\cdots\cdots$$

(74)

It should be kept in mind that (74) gives only those SU(3) irreps that satisfy the condition $\lambda + 2\mu = 4N$ while no such restriction exists for the results given in [12]. It is useful to note that multiple occurence of (λ,μ) can be resolved in some cases by the seniority quantum number $\underline{v}$ in (73). For example the (4N-8,4) irrep appears twice and they can be distinguished by $v = N, N-2$

values. Akiyama [99] showed that the appropriate pairing operator, relevent for (73), is $S = [B^{\dagger}(04)\tilde{B}(40)]^{(00)}$; $B^{\dagger}(\lambda,\mu) = [b^{\dagger}_{(40)} b^{\dagger}_{(40)}]^{(04)}$. With the notation $W = \frac{N-v}{2}$

$$\langle S \rangle^{(\lambda,\mu)W} = (375)^{-1/2} W(2N-2W+3) \qquad (75)$$

and W = 0,1 for the two irreps $(4N-8,4)^2$.

7.4.2. Simple analysis of Spectra

In the Su(3) limit one can write down a simple solvable Hamiltonian,

$$H_{SU(3)} = \alpha C_2(SU(3)) + \beta\,(S) + \gamma L.L$$

$$\langle H_{SU(3)} \rangle^{(\lambda,\mu)WL} = \alpha\,[\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)] + \frac{\beta}{\sqrt{375}}\, W(2N-2W+3) + \gamma L(L+1) \qquad (76)$$

Similarly using the SU(3) generator (Table-4) $Q^2(S)$ as the E2 - operator one can derive analytic expressions/selection rules for a number of B(E2)'s; see Sect 5.1. The simplicity of eq(76,35), the occurence of $K^{\pi} = 1^+, 3^+$ bands (from (4N-6,3) irrep) and two $K^{\pi} = 4^+$ bands (from (4N-8,4) with W = 0,1 respectively) lead to simple analysis of several deformed nuclei where one <u>assumes</u> SU(3) to be exact. Wu [100] analyzed the spectrum of ^{176}Hf (it has 1^+ level at 1.672 MeV) by adding $L^2(L+1)^2$ term to (76) and putting $\beta = 0$, Suguna et.al [101] analyzed ^{184}W (it has 3^+ band starting from ~ 1.5 MeV) and ^{234}U (it has 3^+ band starting from ~ 1.3 MeV) by putting $\beta = 0$, Wu and Zhow [102] analyzed ^{168}Er (it has a 2^+ band starting from ~ 1.8 MeV) by adding perturbation corrections to (76) but putting $\beta = 0$ and Akiyama et.al [103] analyzed ^{178}Hf (it

has a $K^\pi = 3^+$ band starting from ~ 1.6 MeV but no $K^\pi = 1^+$ band/level, it has also $K^\pi = 0^+, 2^+, 4^+$ bands occuring twice and hence interpreted interms of $(4N-8,4)_{W=0,1}$ and ^{234}U with non-zero β in (76). Figures 10 a-c show some of the calculated

FIG 10 A-C

spectra. The extension of the band cut-off beyond the sd IBM value ($L_{max} = 2N$) can be seen from (74), with (4N,0) being the lowest irrep $L_{max} = 4N$. With this the B(E2;L+2 ⟶ L)/B(E2;$2^+ \longrightarrow 0^+$) grow and come closer to rotational model values. Wu [100] analyzed the B(E2) data for ^{232}Th, ^{168}Hf, ^{170}Hf, ^{164}Yb while Akiyama et.al [103] analyzed ^{238}U and the data is in closer agreement with the sdg model than the sd model; see fig 10d. Suguna et.al [101],

FIG 10D

employing (35) and it's extension, fitted with a single effective charge the B(E2;$2^+ \longrightarrow 0^+$) values taking $\eta = 2$ (sd bosons) for ^{120}Xe, $\eta = 4$ (sdg) for ^{188}Pt, ^{154}Gd, ^{186}W, ^{158}Gd, ^{184}Pt, ^{156}Gd, ^{158}Dy and ^{160}Gd and $\eta = 6$ (sdgi) for ^{288}Ra, $^{228-232}Th$ and $^{230-236}U$ and obtained good agreement. Note that given a value of η the ground state band corresponds to the SU(3) irrep $(\eta N, 0)$.

Occupation numbers $\langle n_\ell \rangle$ take a simple form in the SU(3) limit and they are studied by Wu and Zhow [102] and Kota [104]. Eq(37) gives the expression for $\langle n_\ell \rangle^{(4N,0)L}$ in terms of SU(3) ⊃ O(3) Wigner Coefficients. In large N limit for the ground state (L = 0) $\langle n_s \rangle \longrightarrow N/6$, $\langle n_d \rangle \longrightarrow 4N/7$ and $\langle n_g \rangle \longrightarrow 8N/35$. Table-17 gives

TABLE - 17

results for various L values in the case N = 20. For most of the low-lying levels the g-boson occupancy is ~ 20%

Recently Ratnaraju [105] proposed that the elementary permissible diagrams (EPD) of Bargmann and Moshinsky [106] can be used to distinguish multiplicity labels $\underline{\alpha}$ Of (λ,μ) and he uses it to write energy formulas for levels belonging to each (λ,μ). The 0^+ levels in ^{170}Yb are studied using this method.

7.4.3. SU(3) fractional Parentage Coefficients and two - nucleon transfer

The SU(3) parentage coefficients (CFP) are essential to go beyond the simple Hamiltonian given in (76) and even in the exact SU(3) limit to derive matrix elements of general transition operators (two nucleon transfer, M1, E4 etc ---). The problem of deriving SU(3) CFP was addressed by Yoshinaga [107], Wu et.al [108], Kota and Devi [109]. Following the procedure developed by Hecht [94] (Wu et.al and Yoshinaga follow a different approach) the SU(3) intrinsic $|(\lambda,\mu)\epsilon_{max}\Lambda_{max}M_{\Lambda_{max}}>$ can be written down easily, for (4N,0),(4N-4,2),(4N-6,3) and $(4N-8,4)^2$ irreps and they are given in Table-18. The states for $(4N-8,4)_{\alpha=0,1}$ differ from

TABLE - 18

that of Wu et.al [108] as they use the W label. By diagnalizing the S - operator defined above (75) in the $(4N-8,4)_{\alpha=0,1}$ basis we get back Wu et.al result; Yoshinaga's [107] states are not orthogonal to each other. The states given in Table - 18 coincide with Wu et.al's and Yoshinaga's in N $\longrightarrow \infty$ limit. Using the intrinsic states given in Table - 18 one can produce SU(3) CFP's which are $<|||\quad|||>$ matrix elements in the following equation,

$$\langle\{N\}\alpha(\lambda,\mu)\varepsilon_{max}\Lambda_{max}M_{\Lambda_{max}}|b^{\dagger}_{(40)\varepsilon_0\Lambda_0M_{\Lambda_0}}|\{N-1\}\alpha'(\lambda',\mu')\varepsilon'_{max}\Lambda'_{max}M'_{\Lambda_{max}}\rangle$$

$$= \langle\{N\}\alpha(\lambda,\mu)|||\ b^{\dagger}_{(40)}\ |||\{N-1\}\alpha'(\lambda',\mu')\rangle\ *$$

$$\langle(\lambda',\mu')\varepsilon'_{max}\Lambda'_{max}M'_{\Lambda_{max}}\ (40)\varepsilon_0\Lambda_0M_{\Lambda_0}|(\lambda,\mu)\varepsilon_{max}\Lambda_{max}M_{\Lambda_{max}}\rangle$$

(77)

and the coefficients $\langle\ |\ \rangle$ are avilable from [94]. The SU(3) CFP for $(\lambda',\mu') = (4N,0)$ are given in Table-19.The results given in

TABLE - 19

the table can be used to write down the intrinsic states for $|(\lambda,\mu)k\rangle_\alpha$ in the N $\longrightarrow$ ∞ limit and there by derive analytic expressions [107] for B(E2), B(E4), B(M1), B(M3) etc. and they are useful in practical applications as N is large ($\geq$ 12) for many deformed nuclei; see Table - 20 for $|(\lambda,\mu)k\rangle_\alpha$ intrinsic states.

TABLE - 20

One good application of the results in Table - 18 and the CFP's given in Table - 19 is in deriving analytic expressions for two nucleon transfer (TNT) matrix elements. In the gIBM the two nucleon addition operators $P^{\dagger(\ell)}$ for ℓ = 0, 2, and 4 transfer are taken to be, $P^{\dagger(0)} = \zeta_0 s^{\dagger}$; $P^{\dagger(2)} = \zeta_2 d^{\dagger}$; $P^{\dagger(4)} = \zeta_4 g^{\dagger}$ where ζ_0, ζ_2 and ζ_4 are free parameters. In the SU(3) limit of gIBM the ground state of the target nucleus is represented by $|N,(4N,0)K=0\ L=0\rangle$ and the final states are $|N+1,\alpha(\lambda,\mu)K',L'\rangle$.The definition of the spectroscopic strength for the transfer from the ground state of the target and the expression for the same in terms of the SU(3) recoupling coefficients is given by (see also (38)),

$$S_\ell(N,(4N,0)K=0L=0 \longrightarrow N+1,\alpha(\lambda,\mu)K'L') = |\langle N+1,\alpha(\lambda,\mu)K'L' \| P^{\dagger(\ell)} \| N,(4N,0)K=0L=0\rangle|^2$$

$$= |\langle N+1,\alpha(\lambda,\mu) \| \| b^\dagger_\ell \| \| N,(4N,0)\rangle|^2 * |\langle (4N,0)K=0L=0\ (40)\ell \| (\lambda,\mu)K'L'\rangle|^2\ \delta_{\ell,L'} \qquad (78)$$

where $b^\dagger_\ell$ with ℓ = 0,2 and 4 correspond to $s^\dagger$,$d^\dagger$ and $g^\dagger$ respectively. The SU(3) selection rules lead to the result (4N,0) X (40) $\longrightarrow$ (λ,μ) = (4N+4,0) $\oplus$ (4N,2) $\oplus$ (4N-2,3) $\oplus$ (4N-4,4). It should be noted that for the nucleus with N+1 bosons the (4N-4,4) irrep in fact appears twice and we label them as α = 0 and 1 respectively; all other irreps appear only once and for them we use α = 0. The CFP's $\langle \| \| b^\dagger \| \| \rangle$ given in Table - 19 and SU(3) $\supset$ O(3) Wigners $\langle \| \rangle$ given by Bijker and Kota [110] produce analytic expressions for S_ℓ and the expressions correct to order 1/N are given in Table - 19. Here are some comments on the results:

(1) For SU(3) (deformed) nuclei the boson number N being large ($\geq$ 12) the 1/N expressions given in Table - 19 are adequate, for example with N=15 the maximum deviation is $\sim$ 3%. Results in Table - 19 allow for rapid analysis of data.

(2) The $(4N-4,4)_{\alpha=0,1}$ intrinsic states are two-phonon and one-g-boson type respectively. From Table-19 one can see that there is selection rule: the transfer to $(4N-4,4)_{\alpha=0}$ is forbidden. Therefore we can infer the nature of the observed (4N-4,4) bands using TNT. This is indeed seen in ^{166}Er(t,p)^{168}Er reaction. With Akiyama's multiplicity lable

(W) the selection rule, except in the large N limit, is not exact; see discussion above (77).

(3) Ref [54] dealt with, for the first time, two nucleon transfer (TNT) in the SU(3) limit of gIBM. However the authors evaluate S_ℓ numerically and they give results for N = 15 appropriate for ^{166}Er(t,p)^{168}Er. In sd IBM one cannot excite $K_f^\pi = 0_3^+, 0_4^+, 2_2^+$, etc --- because (2N,0) X (20) $\longrightarrow$ $(2N+2,0)K^\pi = 0_1^+ \oplus (2N-2,2)K^\pi = 0_2^+, 2_1^+$. The forbidden band levels are observed experimentally [54]. As can be seen from Table-19, the transfer to I $K^\pi = 2\ 2_2^+$ belonging to $(4N-8,4)_{\alpha=1}$ is six times the transfer to I $K^\pi = 2\ 2_1^+$ and it is close to data value. The excitation to 4_3^+ cannot be one step in sd IBM while in gIBM it can be as it belongs to I $K^\pi = 4\ 3_1^+$ and it is strongly populated although the theoretical S_ℓ and experimental do not match very well. Thus 4_3^+ clearly belongs to $K^\pi = 3_1^+$. The transfer to 0_3^+, 0_4^+ is well reproduced by theory. The strength to 0_1^+, 0_2^+ (1.217 MeV), 0_3^+ (1.422 MeV) and 0_4^+ (1.833 MeV) are (100,15,10,2.4), (100,8.3,14.7,0), (100,21.6,3.6,7.6) in experiment, SU(3) and in a mixing calculation respectively.

(4) Recently Burke et.al (private communication) analyzed TNT data and produced for rare-earth nuclei the ratio R, where R $= \sum_I S_\ell\ (0^+_{g.s} \longrightarrow 0^+_{i,exci})/S_\ell\ (0^+_{g.s} \longrightarrow 0^+_{g.s})$. Ignoring fluctuations and shell closure effects, the data value for R is $\sim$ 0.2 - 0.3. The SU(3) limit of gIBM predicts R $\xrightarrow{N\to\infty}$ 4/N (= 2/N for sdIBM). For rare-earth nuclei, with N $\sim$ 15,

we have R ~ 0.25 which is consistenet with data.

(5) The expressions in Table - 19 are best suited for studying the fragmentation of the transfer strength in a given (deformed) nucleus, therefore it is desirable to have individual strengths S_ℓ ($0^+_{g.s} \longrightarrow 0^+_i$) rather than the ratio R.

7.4.4. The nucleus ^{168}Er in the SU(3) limit of gIBM

Davidson et.al [111] using neutron capture gama ray spectroscopy ((n,γ) reaction) produced precise and almost complete information about all the bands below 2MeV excitation energy. As Bohr and Mottelson criticized [72] in ^{168}Er sd-IBM fails to predict correctly (1) the anharmonicity assosciated with the 4^+_1 band $E(4,4^+_1)/E(2,2^+_1) \simeq 2.5$ but not 2 (2) the Mikhailow plots of B(E2)'s, the slope of $\beta \longrightarrow \gamma$ transitions has wrong sign and (3) variation of moment of inertia from band to band. In addition the appearence of low-lying $K^\pi = 3^+$ band, M1 transition involving $\gamma \longrightarrow g$, $\gamma \longrightarrow \gamma$ (in sdIBM one needs two-body M1 operator), the observed two nucleon transfer strengths in ^{166}Er(t,p)^{168}Er etc require the extension of IBM to gIBM. Arima and collaborators [112,51] carried out careful analysis of ^{168}Er data and produced by far the best (and beautiful) example of gIBM. They employ a truncated SU(3) basis with all SU(3) irreps satisfying $\lambda+2\mu = 4N-3r$, r=0,1,2 and $\mu \leq 8,7,6$ respectively (54 SU(3) irreps, 20 distinct ones). The Hamiltonian in (76) was extended to include (1) two pieces (U and Z) that do not change the positions of W = 0 states (2) a piece (H_1) which *does not* connect the states (4N,0)

K=0 L=4 and (4N-4,2) K=0 L=4 and thus H_1 can produce the anharmonicity associated with the $K = 4_1^+$ band (3) a piece H_2 which does not connect (4N-4,2) K = 0 L = 4 and (4N-8,4) K = 0 W = 1 L = 4 so that mainly it lowers 0_2^+, 0_3^+ bands and (4) a piece P that eliminates connection between (4N-6,3) K = 1 L = 4 with other lowlying 4^+ levels so that the K = 1 band can be moved up easily. See ref [51] for the definition of U, Z, H_1, H_2 and P and also for details regarding the actual calculations in the SU(3) basis. The spectra obtained is in excellent agreement with data, see Fig.11.

FIG 11

As can be seen from fig.11 gIBM cures the problem of anharmonicity associated with the 4_1^+ band and it also predicts correctly the posiiton of $K^\pi = 3_1^+$ band. Employing the general four parameter E2 transition operator (see eq(18)) the observed B(E2)'s are reproduced very well. Writing B(E2)'s as,

$$B(E2;K_i\ I_i \longrightarrow K_f\ I_f) = |\langle I_i K_i\ 2K_f - K_i \| I_f\ K_f\rangle|^2 * \{M_0 + M_1[\,I_f(I_f+1) - I_i(I_i+1)]\}^2 * \begin{cases} 2 & K_i=0\ K_f\neq 0 \text{ or } K_f=0\ K_i\neq 0 \\ 1 & \text{otherwise} \end{cases} \tag{79}$$

the Mikhialov plot $B(E2)/|\langle \| \rangle|^2$ vs $[\,I_f(I_f+1) - I_i(I_i+1)]$ for $\gamma \longrightarrow g$, $\beta \longrightarrow g$ and $\beta \longrightarrow \gamma$ shows good agreement with data. For example the M_0, M_1 values in (79) are

	sd IBM		sdg IBM		expt	
	M_0	M_1	M_0	M_1	M_0	M_1
$\gamma \longrightarrow g$	25	0.15	29 (30)	1.1 (1.2)	28	0.5
$\beta \longrightarrow g$	6.5	0.05	3.8 (5.3)	0.02 (0.06)	4.4	0.06
$\beta \longrightarrow \gamma$	19	0.13	11 (12)	-0.05 (-0.45)	12	-0.3

(80)

In (80) for sdg IBM results with two sets of parameters in T^{E2} are shown.

Observed B(E4) values also show that sd IBM is inadequate. For example the experimental $B(E4;0^+_g \longrightarrow 4^+_\gamma)/B(E4;0^+_g \longrightarrow 4^+_g)$ = 4.2 while the sd IBM E4-operator $(d^\dagger\tilde{d})^4$ predicts it to be 0.5. Using the general E4-operator (see eq(18)) and the data for g $\longrightarrow$ g, g $\longrightarrow \gamma$, g $\longrightarrow K^\pi = 4^+$ $B(E4; 4^+ \longrightarrow 0^+)$ values and assuming g $\longrightarrow \beta$ to be zero, the four effective charges $e^{(K)}(\ell_1,\ell_2)$ are fixed and predicted the unknown values. In terms of single particle units the calculated values for $B(E4;0^+ \longrightarrow 4^+)$ in g $\longrightarrow$ g, g $\longrightarrow \gamma$, g $\longrightarrow \beta$, g $\longrightarrow 0^+_3$, g $\longrightarrow 3^+_1$, g $\longrightarrow 4^+_1$, g $\longrightarrow 2^+_2$, g $\longrightarrow 4^+_2$ are 3.9,16.5,0.0,0.1,50.8,0.6,18.9, and 2.8 respectively. Similarly $\langle I^\pi \| T^{E4} \| I^\pi \rangle_{g.s}$ for $I^\pi = 2^+,4^+,6^+,8^+$ are -2.083,-1.311,0.279 and 3.640. Experimental tests of these numbers really establishes gIBM. We already described (below (78)) the two nucleon transfer data in $^{166}Er(t,p)^{168}Er$. Finally about M1 transitions and 1^+ levels. The observed $\gamma \longrightarrow$ g, $\gamma \longrightarrow \gamma$ M1 transition strengths in

^{168}Er are well explained by gIBM as well as by pn-IBM. In these two models the M1 operator and the resulting matrix elements have similarity (in gIBM $T^{M1} = g_d \vec{L}_d + g_g \vec{L}_g$, in IBM-2 $T^{M1} = g_\pi \vec{L}_\pi + g_\nu \vec{L}_\nu$); see [51] for details. Experimnetally there are two 1^+ levels and in these 1^+_1 at 2.7 MeV is a mixture of (58,3) and (56,1) while the 1^+_2 at 3.8 MeV is predominantly (58,3). The $B(M1; 0^+_g \longrightarrow 1^+_2) \sim 1.5\ \mu_\nu^2$ and $E(1^+_2) \simeq 3.8$ MeV and they are consistent with the 1^+ level observed at 3.3 MeV. Thus the isovector M1 state (1^+_2) usually described by pn-IBM is well described by gIBM. However complete B(M1) study will decide which description is correct. In fact the conjecture is that there may be two types of 1^+ levels, one an isovector IBM-2 (or sdg IBM-2) type and another of isoscalar gIBM type. This is being investigated (see Sect 7.5 and 7.7 ahead). The conclusion of Arima et.al study is that "gIBM is very successful in providing a consistent description of deformed nuclei. At the same time the parameters of the phenomenological Hamiltonian must be calculated from the microscopic point of view. This is a problem for the future".

7.4.5. E4 properties in ^{150}Nd and ^{156}Gd

Experimental data on E4 strength distributions (see Fig.3) was described by Wu et.al [33] using gIBM. Firstly in a Hartree Bose + TDA approximation they studied the simple Hamiltonian

$$H = \varepsilon_g n_g - k \left(4\sqrt{\frac{7}{15}}\right)^{-1} Q^2(s).Q^2(s) \qquad (81)$$

where $Q^2(s)$ is the SU(3) quadrupole operator defined in Table - 4.

They find that (for N = 14, K = 0.02 MeV) with $\varepsilon_g \sim 0.8$ MeV the probability of finding g pairs in the ground state condensate is ~ 10% (as seen in Microscopic theories -see Sect.6) and with $\varepsilon_g \geq$ 1.6 MeV the band structure is that of [U(6) ⊃ SU(3)] ⊕ U(9) with g.s, β, γ and 1g - boson K = 1,3,0,2 and 4 bands far removed from the g.s while for $\varepsilon_g < 0.8$ MeV the band structure approaches that of gIBM SU(3). Secondly employing mapped E4 operator (see eq(34)) and SU(3) wave functions Wu et.al [33] showed that the E4 anamoly (β_4 vs A in fig.1) in rare earths can be qualitatively understood. Employing the Hamiltonian

$$H = \varepsilon_g n_g + \varepsilon_d n_d - k_2 Q^2 \cdot Q^2 \qquad (82)$$

and $\varepsilon_g \simeq 1$ MeV so that the band structure is intermediate to SU(3) and [U(6) ⊃ SU(3)] ⊕ U(9), the E4 strength data (shown in fig ■ 3) is well reproduced; they use $\varepsilon_g = 1.2$ MeV, $\varepsilon_d = 0.2$ MeV for ^{156}Gd and $\varepsilon_g = 0.8$ MeV, $\varepsilon_d = 0.3$ MeV for ^{150}Nd and the mapped T^{E4} operator with one free parameter. In ^{150}Nd for example the summed E4 strength (in 10^2 $e^2 fm^4$) for g ⟶ g, g ⟶ β,γ and g ⟶ all higher states are (171,3.3,85.1) and (171,17.7,89.2) in experiment and theory respectively.

7.5 1/N Expansions

For deformed nuclei the boson number being large (N ≥ 10), it is quite satisfactory if one can derive 1/N expansions for physical observables. Kuyucack and Morrison [75,113,114] developed the same by angular momentum projection from intrinsic states corresponding to g.s, β, γ and higher K bands. This approach (and

the formalism) is applicable for any system of bosons s,d,g,--- with ℓ = 0,2,4, ---- as long as N is sufficiently large. First we will describe the formalism and then some of the applications.

Let us define intrinsic bosons $b^{\dagger}_{m}$ with m the projection on an appropriate axis,

$$b^{\dagger}_{m} = \sum_{\ell} x_{\ell m} b^{\dagger}_{\ell m}; \quad \sum_{\ell} (x_{\ell m})^2 = 1$$

$$\rightarrow$$

$$X = (x_{0m}, x_{2m}, x_{4m}, \text{----})$$

(83)

and also the quadrupole operator Q^2_{μ} (in a different notation than in (18)),

$$Q_{\mu} = \sum q_{ij} (b^{\dagger}_{i} \tilde{b}_{j})^2_{\mu} ; \quad q_{ji} = q_{ij}$$

(84)

where i,j,--- denote ℓ-values while m,n,--- the projection quantum number. A general intrinsic state $|\phi\rangle$ can be written as

$$|\phi\rangle = (b^{\dagger}_{0})^{N_0} (b^{\dagger}_{1})^{N_1} (b^{\dagger}_{2})^{N_2} \text{-----} (b^{\dagger}_{r})^{N_r} |0\rangle$$

$$N = N_0 + N_1 + N_2 + \text{---} + N_r$$

and to leading order (adequate for 1/N corrections),

$$|\phi_{g.s}\rangle = (b^{\dagger}_{0})^{N} |0\rangle$$

$$|\beta\rangle = (b^{\dagger}_{0})^{N-1} (b^{\dagger}_{0})^{1} |0\rangle$$

$$|\gamma\rangle = (b^{\dagger}_{0})^{N-1} (b^{\dagger}_{2})^{1} |0\rangle$$

$$|K\rangle = (b^{\dagger}_{0})^{N-1} (b^{\dagger}_{K})^{1} |0\rangle \qquad K = 1,3,4, \text{----}$$

(85)

where $b_0^\dagger$ and $b'^\dagger_0$ have different values for $x_{\ell 0}$ in (83). The parameters $x_{\ell m}$ that characterize the intrinsic bosons are determined by minimizing $\langle H\rangle$ with $H = Q^2.Q^2$ either before projection (VBP) or after projection (VAP),

$$\langle H\rangle = \langle\phi|H|\phi\rangle/\langle\phi|\phi\rangle \qquad \longrightarrow \text{VBP}$$

$$\langle H\rangle_L = \langle\phi|HP^L_{KK}|\phi\rangle/\langle\phi|P^L_{KK}|\phi\rangle \qquad \longrightarrow \text{VAP}$$

$$\text{where } P^L_{MK}(\Omega) = \left(\frac{2L+1}{8\pi^2}\right) \int D^{L*}_{MK}(\Omega)\ R(\Omega)\ d\Omega$$

(86)

Now we will illustrate the method of calculating $\langle H\rangle_L$ for the g.s band and a similar procedure (but tedious) can be used for β,γ and other higher K-bands [114]. Firstly it is easy to see that

$$\langle Q.Q\rangle_L = \frac{(2L+1)}{2N(\phi_g,L)} \int d\beta\ \text{Sin}\ \beta\ d^L_{00}(\beta)\ \langle 0|(b_0)^N(Q.Q)\ e^{-i\beta J_y}(b_0^\dagger)^N\ |0\rangle$$

$$N(\phi_g,L) = \langle\phi_g P^L_{00}\phi_g\rangle = \frac{2L+1}{2} \int d\beta\ \text{Sin}\beta\ d^L_{00}(\beta)\langle 0|(b_0)^N\ e^{-i\beta J_y}(b_0^\dagger)^N\ |0\rangle$$

(87)

using the elementary rules $[b_i, f(b^\dagger)] = \partial f(b^\dagger)/\partial b_i^\dagger$ and $[b^\dagger_i \bar{f}(b)] = \partial\bar{f}(b)/\partial b_i$ where $f,\bar{f}$ are any functions of $b^\dagger$ and b respectively and the rotated intrinsic boson $b_R^\dagger$,

$$b_R^\dagger = e^{-i\beta j_y}\ b_0^\dagger\ e^{i\beta j_y} = \sum_{\ell,m} x_{\ell 0} d^\ell_{m0}(\beta) b^\dagger_{\ell m}$$

(88)

we have

$$\langle 0|(b_0)^N e^{-i\beta J_Y (b_0^\dagger)^N}|0\rangle = N \sum_\ell \alpha_{\ell 0} \left[\frac{\partial b_R^\dagger}{\partial b_{\ell 0}^\dagger}\right] \langle (b_0)^{N-1} e^{-i\beta J_Y (b_0^\dagger)^{N-1}}\rangle$$

$$= \left\{N \sum_\ell (\alpha_{\ell 0})^2 \, d^L_{00}(\beta)\right\} \left\{\langle (b_0)^{N-1} e^{-i\beta J_Y (b_0^\dagger)^{N-1}}\rangle\right\}$$

$\longrightarrow$

$$\langle (b_0)^N \; e^{-i\beta J_Y (b_0^\dagger)^N} \rangle = N! \; [Z(\beta)]^N \tag{89}$$

where $Z(\beta) = \sum_\ell (\alpha_{\ell 0})^2 \, d^L_{00}(\beta)$ (90)

In (89) we used $\langle\longrightarrow\rangle$ for simplicity in place of $\langle 0|\text{—}|0\rangle$. Writing Q.Q in normal order and applying the boson commutation relations one has [114],

$$\langle (b_0)^N (Q.Q) \; e^{-i\beta J_Y (b_0^\dagger)^N} \rangle = (5N)\,(N!)\,[z(\beta)]^{N-2} * \Bigg[\Big\{ (N-1) * \sum_{J,i,i',j,j'} \langle i0i'0|J0\rangle \langle j0j'0|J0\rangle \begin{Bmatrix} i' & i & J \\ j & j' & 2 \end{Bmatrix} q_{ij} \, q_{i'j'} * \alpha_{i0}\alpha_{j0}\alpha_{i'0}\alpha_{j'0} \, d^J_{00}(\beta) \Big\} + \Big\{ Z(\beta) \sum_{I,j} (2j+1)^{-1} (q_{ij}\alpha_j)^2 d^J_{00}(\beta) \Big\} \Bigg] \tag{91}$$

Now one can minimize $\langle\phi_{g.s}|H|\phi_{g.s}\rangle / \langle\phi_{g.s}|\phi_{g.s}\rangle$ (for this put $\beta = 0$ in (89) and (91)) or carry out VAP. Using Gaussian appriximation for $[Z(\beta)]^N$, which is adequate for large N

$$[Z(\beta)]^N = \left\{ \sum_\ell (x_{\ell 0})^2 \right\}^N e^{-\beta^2/\Gamma} \; ; \; \Gamma = \frac{2}{\gamma N} \; ;$$

$$\gamma = \frac{1}{2} \left\{ \sum_\ell \ell(\ell+1) x_{\ell 0}^2 \right\} \Big/ \left\{ \sum_\ell (x_{\ell 0})^2 \right\}$$

(92)

and evaluating the integrals in (87) one finally arrives at the following expansion for $\langle Q.Q \rangle_L$,

$$\langle Q.Q \rangle_L = N^2 \left[\left(\frac{A_{00}}{\vec{X}.\vec{X}} \right)^2 + \frac{1}{N} \left\{ \left(\frac{A_{00}}{\vec{X}.\vec{X}} \right)^2 - \frac{(2A_{10}A_{00} - 3A_{00}^2)}{2\gamma(\vec{X}.\vec{X})^2} + \frac{C_0}{\vec{X}.\vec{X}} \right\} + \right.$$

$$\left. \frac{1}{N^2} \frac{L(L+1)}{\gamma} \left\{ - \left(\frac{A_{00}}{\vec{X}.\vec{X}} \right)^2 + \frac{(2A_{10}A_{00} - 3A_{00}^2)}{4\gamma(\vec{X}.\vec{X})^2} \right\} + \text{----} \right.$$

(93)

where $A_{ab} = \sum_{i,j} \{i(i+1)\}^a \{j(j+1)\}^b x_{i0} x_{j0} \langle i0j0|20\rangle q_{ij}$

$$C_a = \sum_{i,j} \frac{5}{(2j+1)} [j(j+1)]^a (q_{ij} x_j)^2$$

$$\vec{X}.\vec{X} = \sum_\ell x_{\ell 0}^2$$

(94)

The expression in (93) was first given in [113] and the corresponding ones for sd and sdg systems was given in [75] while its extensions to β,γ,--- bands was given in [114]. One can minimize $\langle Q.Q \rangle_L$ and obtain $x_{\ell 0}$. It can be seen that except of sd system in general $x_{\ell 0}$ depend on L, (i.e) in general boson systems exhibit stretching. However for any boson system with SU(3) Q.Q operator $x_{\ell 0}$ will again be independent of L [75]. To lowest order VAP and VBP are equivalent and they lead to the eigenmode

condition

$$[Q_0, b_m^\dagger] = \lambda_m b_m^\dagger$$

$$\sum_j (-1)^m \langle jmi-m|20\rangle\, q_{ij}\, x_{jm} = \lambda_m x_{im}$$

(95)

obtained by putting $\partial/\partial x\ [H-\lambda\ X.X] = 0$. The three corresponding solutions for sdg system define $b_0^\dagger$, $b'^\dagger_0$ and $b''^\dagger_0$ corresponding to g.s, β and β' bands. One can use (93) and go beyond (95) to obtain L dependent x_{00}.

The method described above can be used to derive $\langle Q.Q\rangle_{\beta L}$ and it differs (to order $1/N^2$) from (93) only in the term $\frac{1}{N}\left[\frac{A_{00}}{X.X}\right]^2 \longrightarrow \frac{1}{N}\left[-\left[\frac{A_{00}}{X.X}\right]^2 + 2\left[\frac{A_{00}}{X.X}\right]\left[\frac{A'_{00}}{X'.X'}\right]\right]$ where to obtain A'_{00} replace x_{k0} in (83) by x'_{k0} and note that x'_{k0} (hence X') define $b_0'^\dagger$. Similarly for the γ and other k-bands replace $\frac{1}{N}\left[\frac{A_{00}}{X.X}\right]^2$ in (93) by $\frac{1}{N}\left[\frac{2A_{00}A(K)}{(X.X)(X(K).X(K))} + 2\,\frac{B^2(K)}{[X(K).X(K)]^2} - \left[\frac{A_{00}}{X.X}\right]^2\right]$ where

$$A(K) = \sum_{i,j} (-1)^K \langle iKj-K|20\rangle\, q_{ij}\, x_{iK}\, x_{jK}$$

$$B(K) = \sum_{i,j} \langle iKj0\,|2K\rangle\, q_{ij}\, x_{iK}\, x_{j0}$$

$$X(K).X(K) = \sum_\ell x_{\ell K}^2$$

(96)

The formalism given above can be extended to derive 1/N expansions for reduced matrix elements of multipole transition operators, Kuyucack and Morrison give expressions for E2, M1

matrix elements and g factors; see [114] and [115]. Here we will give the final results (in the following X.X is supressed for convenience) for some E2 matrix elements,

$$\langle L' \| Q \| L \rangle_{g.s} = N \sqrt{2L+1}\ \langle L020 | L'0 \rangle \left[A_{00} + \frac{1}{N}\left(A_{00} - \frac{A_{10}-3A_{00}}{2y} \right) \right.$$

$$- \frac{L(L'+1)}{2yN^2} \left[A_{00} + \left\{ \frac{A_{20}-A_{11}-10A_{10}+12A_{00}}{8y} \right\} \delta_{L',L} \right.$$

$$\left. \left. - \left\{ \frac{A_{20}-A_{11}+6A_{10}-12A_{00}}{24y} \right\} \delta_{L',L+2} \right) \right] \qquad (97)$$

$\langle L+2 \| Q \| L \rangle_{\beta}$ = Replace in (97) the term $\frac{1}{N}\left(A_{00} - \frac{A_{10}-3A_{00}}{2y} \right)$ by

$$\frac{1}{N}\left(A'_{00} - \frac{A_{10}-3A_{00}}{2y} \right)$$

(98)

$$\langle K, L+K \| Q^K \| gL \rangle = \sqrt{2-\delta_{K0}}\ \sqrt{N}\ \sqrt{2L+1}\ \langle L0\ KK | L+K\ K \rangle \ast$$

$$\sum_{i,j} t^K_{ij}\ \langle iKj0 | KK \rangle\ x_{iK}\ x_{j0} \qquad (99)$$

In (99) $Q^K = \sum t^K_{ij}\ (b^{\dagger}_i \tilde{b}_j)^K$ and this equation can be used for g $\longrightarrow$ γ, $\beta \longrightarrow \gamma$ and even g $\longrightarrow \beta$; see [114]. Similarly for some of the M1 transiitons (and g-factors) we have, by adding corrections to the intrinsic states in (85) , for sdg system [115],

$$T^{M1} = g\,\vec{L} + g'(g^{\dagger}\tilde{g})^{1},$$

$$\langle \beta, L \,\|T^{M1}\|\, g\, L\rangle = g'\sqrt{\frac{(2L+1)L(L+1)}{3N}}\;\frac{x_{40}}{2y}\left\{\sqrt{20}\;x'_{40} - \left(\frac{b}{b_1}\right)x_{41}\right\}$$

$$\langle \gamma, L' \,\|T^{M1}\|\, gL\rangle = -\frac{g'}{\sqrt{3N}}\sqrt{(2L+1)(L'(L'+1)-2)}\;\langle L0\;11\;|L'1\rangle \times$$

$$\frac{x_{40}}{2y}\left[\sqrt{18}\;x_{42} - \frac{b_2}{b_1}\,x_{41}\right]$$

$$\langle \gamma, L+1 \,\|T^{M1}\|\, \gamma L\rangle = g'\sqrt{\frac{(L+1)^2 - 4}{(L+1)}}\;2\sqrt{\frac{5}{3}}\left[-\frac{x_{40}^2}{y} + \frac{x_{42}^2}{10}\right]$$

$$B(M1; 0^{+}_{g.s} \longrightarrow 1^{+}_{s}) = \frac{3}{4\pi}(g')^2\,\frac{N}{y}\,(x_{20}x_{40})^2$$

$$g(k,L) = g + g'\sqrt{\frac{5}{3}}\left[\frac{x_{40}^2}{y} + \frac{K^2}{L(L+1)}\left\{-\frac{x_{40}^2}{y} + \frac{x_{4K}^2}{10}\right\}\right]$$

$$b = \sum_{\ell}\ell(\ell+1)\,x_{\ell 0}\,x'_{\ell 0}$$

$$b_1 = \sum_{\ell}\ell(\ell+1)\,x_{\ell 0}\,x_{\ell 1}$$

$$b_2 = \sum_{\ell}\sqrt{\ell(\ell+1)[\ell(\ell+1)-2]}\;x_{\ell 0}\,x_{\ell 2}$$

(100)

Applications

The analytical formulas given above can be applied to analyze E2 and M1 data. Here we will describe some of the important results obtained so far:

(1) Morrison [116] showed that, by extending (99) with A_{00} replaced by $\sum e^{(4)}(\ell,\ell')\,\langle \ell 0\;\ell' 0\,|\,40\rangle\;x_{\ell 0}\;x_{\ell' 0}$ and using for $e^{(4)}(\ell,\ell')$ the OAI mapped values (34), the E4 anomaly (see fig.1) can be explained, (i.e) the trend in β_4 α A

comes out naturally. See Sect 7.4 for a better calculation by Wu et.al [34].

(2) Using exact SU(3) or VBP one will not see [117] any variation in g(K,L) with L. However as said earlier (see discussion below (94)) the $\alpha_{\ell 0}$ depend on L (roughly $\alpha_{\ell 0}$ = a + b/N + cL(L+1)/N^2) in VAP and hence from (100) we see that ground state g factors take the form (in sdg and higher but not in sd) $g(L) = g_0 + g_1 L(L+1)$. It is seen that with g_0 = 0.325, g_1 = -0.0014 the g-factor data (for g.s band) in ^{166}Er is well explained; see Fig.12. Thus the stretching of the sdg system explain the observed g-factor variation [75]. In sd system , with a multibody M1 operator one can explain data [76]; to be meaningful here one must carry out a detailed study as the one in [118].

(3) Using the expressions for B(M1) and B(E2) given in (100) and (99) one has for the E2/M1 mixing ratios,

$$\Delta(E2/M1;\ L_K \longrightarrow L_g) = C_K\ N \sqrt{\frac{2K+1}{(2L-1)(2L+3)}} \qquad \begin{matrix} K=o & \beta \\ k=2 & \gamma \end{matrix}$$

$$\Delta(E2/M1;\ (L+1)_\gamma \longrightarrow L_g) = C_\gamma\ N \sqrt{\frac{1}{L(L+2)}} \qquad (101)$$

In Table - 21a results of eq(101) are compared with data for

TABLE - 21A

^{154}Gd which has most extensive Δ data. The argument [illegible] and the predicted 1/L fall off (which indicate collective nature of these transitions) is seen in data. In the data of various nuclei it is also seen that the N dependence in

(101) is roughly followed (sd IBM has no N dependence). Finally with g′ expected to be negative, the expected decrease (see eq (100)) in g-factors with increasing K is seen in the data for $g(2^+_\gamma)$. See ref [115] for details.

(4) In a recent study Morrison et.al [119], using simple microscopic models, considered the effect of high spin intruder shells vis-a-vis the g bosons for magnetic dipole properties in rare-earth nuclei. They conclude that the low-lying B(M1) strength and the 1^+ level of gIBM (see (100) and also Sect 7.4) was associated in rare-earths with excitation of intruder ($h_{11/2}$, $i_{13/2}$) shells where as the large scissor mode (mixed symmetry 1^+ level in IBM-2 or gIBM-2) was associated with oscillations of normal parity orbits.

(5) As can be seen from the discussion below (95) in the $1/N^2$ limit all the bands have the same moment of inertia (M.I) and this conclusion remains same even with a general $H = \sum \varepsilon_i n_i + \sum_k A_k \, Q^k . Q^k$ [120]. Thus a deviation of about 20% or more in M.I. and in $R_{\beta\gamma}$,

$$R_{\beta\gamma} = \frac{B(E2; 2^+_\beta \longrightarrow 0^+_g)}{B(E2; 2^+_\gamma \longrightarrow 0^+_g)} = \frac{1}{2}\left[\frac{B(0)}{B(2)}\right]^2, \qquad (102)$$

where B(K) is defined in (96), can be taken as a signature for 2q.p character of $0^+_2(\beta)$ band. Analysis of data [120] for (M.I) and $R_{\beta\gamma}$ in rare-earths show that especially in some of the Yb isotopes the β-bands could indeed be 2 q.p.bands.

(6) Going beyond the simple intrinsic states given in (85), accurate expressions (upto 1/N) for $\gamma \longrightarrow g$ E2 transitions are derived recently (considering band mixing) [121]. The Mikhailov plot analysis of the expressions leads to a better understanding of gIBM vis-a-vis the geometric model; see [121] for details.

7.6 Effective interactions in sdg space

In sdg space effective interactions are, as given in (12), characterized by 35 parameters. As discussed in Sect.3 one can construct effective (model) interactions that contain limited number of free parameters. However to understand some general features of gIBM it is profitable to deal with a microscopically derived interaction. Recently Yoshinaga [13,122] using the simple OAI method produced sdg Hamiltonian by mapping in single-j shell approximation. Results of Sect.3.3 differ from Yoshinaga's work in that in eq(34) one is first mapping Q^K and then $H = Q^K \cdot Q^K$ is obtained while in Yoshinaga's work $Q^K . Q^K$ itself is mapped. We will describe briefly the formalism and then some of the interesting results that are obtained. Later we will also describe some results that are obtained using the BSDI interaction defined in (30).

7.6.1. sdg H in seniority scheme

Starting with S, D and G pair states in a single j-shell,

$$S^{\dagger} = A^{\dagger(0)}, \quad D^{\dagger} = P\, A^{\dagger(2)}, \quad G^{\dagger} = P\, A^{\dagger(4)},$$

$$A^{\dagger(J)} = \sqrt{\frac{1}{2}}\,(a_j^{\dagger}\tilde{a}_j)^J$$

(103)

where P is projection operator [21] onto states of maximum seniority, in the zeroth order OAI mapping we need to construct 4-fermion states interms of S,D and G pairs. The states $|DD\rangle$, $|DG\rangle$ and $|GG\rangle$ will not ingeneral be orthogonal to each other, then one can use the normal procedure,

$$|D\tilde{G};\ LM\rangle = \{|DG;\ LM\rangle - a_L\ |DD;\ LM\rangle\}/\sqrt{1-a_L^2}$$

$$|\tilde{G}\tilde{G};\ LM\rangle = N^{-1}\ \{|GG;\ LM\rangle - b_L\ |DG;\ LM\rangle - c_L\ |DD;\ LM\rangle\}$$

(104)

where N is a normalisation factor and the coefficients a_L, b_L and c_L are determined by imposing $\langle DD|D\tilde{G}\rangle = 0$, $\langle DD|\tilde{G}\tilde{G}\rangle = \langle D\tilde{G}|\tilde{G}\tilde{G}\rangle = 0$. The procedure given in (104) can be extended for six, eight etc.--- fermion system (i.e) for 3,4,5,--- bosons), see ref [122] for details. Now establishing the correspondence $|s\rangle \longrightarrow |S\rangle$, $|d\rangle \longrightarrow |D\rangle$, $|g\rangle \longrightarrow |G\rangle$, $|s^2\rangle \longrightarrow |SS\rangle$, $|sd\rangle \longrightarrow |SD\rangle$, $|sg\rangle \longrightarrow |SG\rangle$, $|d^2\rangle \longrightarrow |DD\rangle$, $|dg\rangle \longrightarrow |D\tilde{G}\rangle$, $|g^2\rangle \longrightarrow |\tilde{G}\tilde{G}\rangle$ and equating the fermion space matrix elements with the corresponding ones in the boson space,the mapped interaction is obtained.

The mapping for the pairing operator $P = -A^{\dagger(0)}A^{(0)}$ is trivial and the corresponding boson Hamiltonian is,

$$H_{sdg} = [-s^{\dagger}s + \frac{4}{(2j+1)}\{\frac{1}{2}\, s^{\dagger}s^{\dagger}ss + s^{\dagger}d^{\dagger}.\tilde{d}\tilde{s} + s^{\dagger}g^{\dagger}.\tilde{g}\tilde{s}\}]$$

(105)

Starting from a model fermion operator,

$$H_f = - \chi (A^{\dagger(0)} A^{(0)}) - (1-\chi) :Q^2.Q^2:$$

(106)

where : : means normal ordering and $Q^2 = (a_j^{\dagger} \tilde{a}_j)^2$, Yoshinaga [13] showed that in vibrational case (χ =0.7), in $(j=23/2)^6$ system, the sdg H gives a more complete description of the exact spectrum (several of the levels lie outside the sd space). In the transition class (χ = 0.3) the sdg description (moment of inertia, binding energy etc.---) is quite good. In the rotational case (χ = 0.0) with sufficiently large number of bosons sdg model provides good rotational bands. In Fig • 13a calculations for N=5 (10 fermions) with $n_g \leq 0,1,---,5$ are shown. The striking result is

FIG 13A

that with $n_g \leq 3$ one obtains results close to exact. In fact this calculation provides good support to our view (hence the need for the programmes documented in this report) that often $n_g \leq 4$ calculations are adequate even in deformed nuclei.

In a recent paper [122] Yoshinaga carried out calculations in many fermion spaces with SDG truncation and compared the results in (sdg) boson space obtained with zeroth order OAI mapped interaction. The conclusion of this study is "the SDG - pair approach reproduces well the low-lying states of the ground state band in the deformed limit. In general the zeroth - order approximation in the OAI is good, but in order to reproduce the ground state enrgies, we need higher order interactions".

7.6.2. BSDI interaction

The BSDI interaction defined in (30) was

introduced by Chen et.al [20] and using it they performed gIBM calculations for several nuclei in the mass region A ~ 40 - 100. Results of the calculations for ^{94}Mo (with ^{88}Sr core) are shown in fig.13b. The single particle energies are chosen to be those of

FIG 13B

the neighbouring one boson nucleus ^{90}Zr. The BSDI description is better than (see Fig. 13a) some of the available shell model calculations. Similar, successful, analysis was carried out for ^{52}Cr, ^{96}Pd, $^{76-82}$Kr, ^{76}Se etc.-- [20].

7.7 OTHER TOPICS (P-N gIBM,gIBFM,gIBM-3,4)

The extensions of gIBM to proton-neutron gIBM (p-n gIBM or gIBM-2), gIBFM (interacting boson-fermion model including g-bosons) and gIBM-3,4 (where isospin degree of freedom is included) are described in this section. These topics are yet to be fully explored.

7.7.1. p-n gIBM

The interst in M1 observables and 1^+ states lead to the extension of gIBM to p-n gIBM. Barrett and Halse [123] very early pointed out that the inclusion of g bosons (with p-n) leads to doubling of B(M1) strength (for decay of 1^+_M scissor or mixed symmetry states to g.s) compared to sd space; see eq(109) ahead. Similarly using a meanfield + TDA Pittel et.al [47] showed that, employing a broken SU(3) Hamiltonian, the M1-strength in gIBM-2 is strongly fragmented and the strongest state need not always be the lowest. Wu et.al [124] considered the M1-strength fragmentation in the SU(3) limit of gIBM-2, here the group chain and some of the

low-lying SU(3) irreps are:

$$U^{\pi}(15) \otimes U^{\nu}(15) \supset U^{\pi+\nu}(15) \supset SU^{\pi+\nu}(3) \supset O(3)$$

N_π	N_ν	$\{N_\pi+N_\nu = N\}$	$(4N,0),(4N-4,2)$	L	F-spin N/2
			$\underline{(4N-6,3)},(4N-8,4)^2$---		
		{N-1,1}	$\underline{(4N-2,1)},(4N-4,2)$		N/2-1
			$\underline{(4N-6,3)}^2,(4N-8,4)^2$+---		
		{N-2,2}	(4N-4,2),-----		N/2-2

(107)

In (107) the underlined irreps give $K^{\pi} = 1^{+}$ band and $L^{\pi} = 1^{+}$ levels . They are represented by $(4N-6,3)1^{+}_{S}$, $(4N-2,1)1^{+}_{m}$, $(4N-6,3)1^{+}_{m_1}$, $1^{+}_{m_2}$, where the first one comes from {N} (hence gIBM-1) and later two from {N-1,1}. The 1^{+}_{m} is the usual scissor state. The SU(3) selection rules clearly point out ((4N,0)X(33) $\longrightarrow$ (λ,μ)) that only 1^{+}_{S} , 1^{+}_{m} , $1^{+}_{m_1}$ and $1^{+}_{m_2}$ will be excited by M1-operator. Writing the M1 - operator as,

$$T^{M1} = \sqrt{\frac{3}{4\pi}}\ (g_R \vec{L} + (g_{R,\pi} - g_{R,\nu})\ L^{(1)}_a + \sum_{\rho} L_\rho\ \Lambda^1_\rho)$$

$$\Lambda^1_\rho = \frac{1}{7}\ (4\ L^1_{d\rho} - 3L^1_{g\rho}),\ \rho = \pi \text{ or } \nu$$

$$L^1_a = \frac{1}{N}\ (N_\nu\ L^1_\pi - N_\pi\ L^1_\nu)$$

$$L^1_{d\rho} = \sqrt{10}\ (d^{\dagger}_{\rho}\tilde{d}_{\rho})^1,\ L^1_{g\rho} = \sqrt{60}\ (g^{\dagger}_{\rho}\tilde{g}_{\rho})^1$$

$$\vec{L} = \sum_{\rho} (L_{d\rho} + L_{g\rho}) = L_\pi + L_\nu$$

$$g_R = [N_\pi\ g_{R,\pi} + N_\nu\ g_{R,\nu}]/N$$

(108)

where the second term in (108) has the selection rule $\Delta F = 1$, $\Delta\mu = 1$ while the third term $\Delta F = 0$, 1 and $\Delta\mu = 1$, 3, in the large

N limit it was shown that

$$B(M1;\ 0^+_{g.s} \longrightarrow 1^+_i) = \frac{3}{4\pi}\frac{96}{4N}\frac{(N_\pi h_\pi + N_\nu h_\nu)^2}{N} \qquad \text{for } 1^+_s$$

$$= \frac{3}{4\pi}\frac{8N_\pi N_\nu}{N}(g_{R,\pi} - g_{R,\nu})^2 \qquad \text{for } 1^+_m$$

$$= \frac{3}{4\pi}\frac{192}{49}\frac{N_\pi N_\nu}{N}(h_\pi - h_\nu)^2 \qquad \text{for } 1^+_{m_1}$$

$$\simeq \frac{1}{N} B(M1;\ 0^+_{g.s} \longrightarrow 1^+_{m_1}) \qquad \text{for } 1^+_{m_2}$$

(109)

It can be seen that the 1^+_m strength given in (109) is twice the corresponding value in sd (p-n) - IBM. From microscopic theories one sees that [124] $g_{R,\rho}$ are much larger than h_ρ and hence the 1^+_m has large strength (compared to that of 1^+_s) and it supports the idea that 1^+_m is the scissor state seen experimentally.

Kuyucak et.al [125] addressed the M1 properties in gIBM-2 using the 1/N expansion method described in Sect 7.5 extending it to $\pi - \nu$ intrinsic states,

$$|\phi_g\rangle = (b^\dagger_\pi)^{N_\pi}(b^\dagger_\nu)^{N_\nu}|0\rangle$$

$$|\phi_k\rangle = \left[N_\pi (b^\dagger_\pi)^{N_\pi - 1} b^\dagger_{\pi k}(b^\dagger_\nu)^{N_\nu} + (\pi \longleftrightarrow \nu)\right]|0\rangle$$

$$|\phi_{KM}\rangle = \left[N_\pi (b^\dagger_\pi)^{N_\pi - 1} b^\dagger_{\pi k}(b^\dagger_\nu)^{N_\nu} - (\pi \longleftrightarrow \nu)\right]|0\rangle$$

(110)

where M denotes states with $F = F_{max} - 1$. In (110) replacing $N_\pi \longrightarrow \sqrt{N_\pi}\,\mathrm{Cos}\gamma$ and $N_\nu \longrightarrow \sqrt{N_\nu}\,\mathrm{Sin}\gamma$ one can include F-spin breaking. Using (110), expressions for M1 -strengths are derived

and their application to $\gamma \longrightarrow \gamma$, $\gamma \longrightarrow$ g M1 strengths is shown (for ^{168}Er) in Table - 21b.

TABLE - 21B

They point out that both g-bosons and F-spin breaking are needed for a quantitative description of data. Moreover the experimentally important quantity $B(E2; 2^+_\gamma \longrightarrow 0^+_g)/B(E2; 2^+_g \longrightarrow 0^+_g) = 0.024$ is reproduced by the 1/N theory. A detailed discussion of the 1/N expansion method in p-n gIBM is given very recently [126].

Otsuka and Sugita [127] performed a successful calculation for Sm isotopes, in the framework of gIBM-2, using the Hamiltonian

$$H = \sum_\rho \left[\varepsilon_{d_\rho} n_{d_\rho} + \varepsilon_{g_\rho} n_{g_\rho}\right] - k\, Q_\pi . Q_\nu \qquad (111)$$

where they choose $\varepsilon_{d_\pi} = \varepsilon_{d_\nu} = 1.3$ MeV, $\varepsilon_{g_\pi} = \varepsilon_{g_\nu} = 1.8$ MeV, k = 0.15 MeV and $Q_\rho = \left\{4\sqrt{\frac{7}{15}}\right\}^{-1} Q^2(S)$ where $Q^2(S)$ is defined in Table - 4. The projected HB calculation, with the above parameter set, reproduces the data for ^{158}Sm isotopes rather well; see fig - 14.

FIG 14

A simple projected gIBM - 1 Hamiltonian $H = \varepsilon_d n_d + \varepsilon_g n_g - \frac{1}{4} k\, Q.Q$ where $\varepsilon_d = \varepsilon_{d_\rho}$, $\varepsilon_g = \varepsilon_{g_\rho}$, and k same as in (111) and E2 operator $T^{E2} = \frac{1}{2}(e_\pi + e_\nu)Q^2$ also is capable of reproducing data(see fig-14). Thus in the above calculation the spherical-deformed phase transition occurs as boson number changes although the Hamiltonian

remains unchanged. As Otsuka and Sugita stress "the gIBM-2 should be used for the global description of low-lying collective states".

7.7.2. gIBFM

A preliminary attempt to extend gIBM to gIBFM and apply the same for describing odd-A nuclei was due to Kota [128]. Coupling of an odd particle carrying angular momentum j = 1/2, 3/2, 5/2, 7/2 and 9/2 to an even-even core nucleus described by the SU(3) limit of gIBM, generates the U(15) ⊕ U(30) Bose-Fermi (BF) symmetry scheme. Here one can construct subgroup chains in two different ways (much the same as in the U(6/12) case [129]).

$$U^B(15) \otimes U^F(30) \supset U^B(15) \otimes U^F(15) \otimes SU^F(2)$$
$$\supset \begin{cases} SU^B(3) \otimes SU^F(3) \otimes SU^F(2) & \text{--- I} \\ \text{or } U^{BF}(15) \otimes SU^F(2) \supset SU^{BF}(3) \otimes SU^F(2) & \text{--- II} \end{cases} \quad (112)$$

and they are used to describe the Nilsson amplitudes (with coriolis mixing) in ^{185}W. Instead of $U^B(15) \otimes U^F(30)$ BF - symmetry Yu [130] considered the U(15/30) SUSY and worked out some of the relevent algebra for the chain

$$U(15/30) \supset U^{BF}(15) \otimes U^{BF}(1/2) \supset SU^{BF}(3) \otimes SU^F(2) \otimes U^B(1) \otimes U^F(1) \quad (113)$$

Further explorations of gIBFM/p-n gIBFM are clearly called for.

7.7.3. gIBM-3,4

The IBM-3,4 models of Elliott [131] can be formally extended to include g-bosons. Recently Abdelaziz et.al [132] pointed out that the M1 transition strengths ($4^+_i \longrightarrow 4^+_j$) in Ti isotopes indicate clearly the g-boson character of some of the 4^+

levels. In their gIBM-3 description of ^{46}Ti which includes g-bosons, the first 4^+ is mostly d^2 while the second and third 4^+ levels have large g-boson components. Further exploration of gIBM-3,4 will be interesting.

Inspite of the fact gIBM H has large number of parameters, yet gIBM produced many interesting and important results, as reviewed above. We repeat here the statement made by Casten and Warner [3] — "clear need for g-bosons in this context, perhaps more than any other factor, that suggests that the improved agreement obtained for observables is physically significant as not just an exercise in parameter proliferation".

8 Conclusions

We developed the package SDGIBM1 which allows one to perform spectroscopic calculations in sdg-boson space. The documentation of the package SDGIBM1 together with the first comprehensive upto date review of gIBM given in this report, we hope will lead to extensive and illuminating explorations of gIBM and this may provide new way of understanding hexadecupole collective motion and in uncovering the resulting structures in nuclei.

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Table Captions

Table 1. $v \longrightarrow L$ reductions: (a) $v_d \longrightarrow L_d$ (b) $v_g \longrightarrow L_g$.

Table 2. Hamiltonian matrix dimensions (for various N, L values). For the first row $n_g=0$, second row $n_g \leq 1$, third row $n_g \leq 2$, fourth row $n_g \leq 3$, fifth row $n_g \leq 4$ and for the last row there is no restriction on g - boson number ($n_g \leq N$).

Table 3. Casimir operators and their eigen values for $U(2\ell+1)$, $SU(2\ell+1)$, $O(2\ell+1)$ and $O(3)$ groups.

Table 4. Group Chains, irreps, generators, Casimir operators and their eigenvalues in the four strong coupling limits of gIBM.

Table 5. Two-body matrix elements $V^{LL}(L1,L2,L3,L4)$ corresponding to various model interactions in sdg - space.

Table 6. Single particle energies (ε_s, ε_d and ε_g) for various simple Hamiltonians.

Table 7. Correspondence between the calling parameters (X,K1,K2,K3) of the subroutine TWODG and the operator for which the reduced matrix elements are computed.

Table 8. Some typical running times.

Table 9. Correspondence between the input data (KIJJ,KID,K) that goes into the routine MAE2E4 and the operator that is handled by the routine.

Table 10. Lowlying SU(3) irreps $(\lambda\mu)$ and the corresponding eigenvalues for the operator $-\frac{3}{4} Q^2(s).Q^2(s)$. The

number of bosons is four.

Table 11. SU(3) $\supset$ O(3) Wigner Coefficients $\langle(\lambda 0)L_1;\ (40)\ell \parallel (\lambda+4,0)L\rangle$. Table taken from Vergodos [22]. Note that the factor $[L(L+1)]^{1/2}$ for the case $\ell = 2$, $L_1 = L$ is missing in the original table of Vergodos.

Table 12. Lowlying U(5) and O(5) irreps $\{f_1 f_2\}$ and $[v_1 v_2]$ respectively (in U(15) $\supset$ U(5) chain) and the corresponding eigenvalues for the operator $-C_2(U(5)) + \frac{1}{2} C_2(O(5))$. The number of bosons is four.

Table 13. Lowlying U(6) and Sp(6) irreps $\{F_1F_2F_3F_4F_5F_6\}$ and $\langle\lambda_1\lambda_2\lambda_3\rangle$ respectively (in U(15) $\supset$ U(6) chain) and the corresponding eigenvalues for the operator $[-C_2(U(6)) + \frac{1}{2} C_2(Sp(6)) + 64]$. The number of bosons is four.

Table 14. $B(IS4;0^+ \longrightarrow 4^+)$ in ^{168}Er and ^{172}Yb.

Table 15. $B(E4;J_i^+ \longrightarrow J_f^+)$ in (ds) and (fp) shell nuclei.

Table 16. SD and SDG truncations in the intrinsic states.

Table 17. s,d,g orbit occupancies for N = 20. Taken from [104].

Table.18. Structure of the SU(3) Intrinsic states in gIBM.

Table 19. Analytic expressions for the spectroscopic factors S_ℓ. Taken from [109]. The last column in the Table gives CFP2= $|\langle\{N+1\}(\lambda\mu)\alpha|||\ b^\dagger_{(40)}\ |||\ \{N\}(4N,0)\rangle|^2$.

Table 20. $(\lambda\mu)$ K intrinsic states in N $\longrightarrow \infty$ limit.

Table 21a. Reduced mixing ratios $\Delta(E2/M1)$ for ^{154}Gd. Taken from [115].

Table 21b. Reduced M1 matrix elements for $\gamma \longrightarrow g$ and $\gamma \longrightarrow \gamma$ transitions in ^{168}Er. Taken from [125].

Figure Captions

Fig 1. Experimental deformation parameters β_4. The curves are calculated from eq(46). Figure taken from ref [29].

Fig 2. $B(IS4; 0^+_{g.s} \longrightarrow 4^+_\gamma)$ vs mass number A. Figure taken from ref [30]. Experimental numbers obtained via scattering of 65 MeV protons) are compared with a Random phase approximation (RPA) calculation.

Fig 3. The experimental 4^+ states and corresponding B(E4) strengths of ^{156}Gd and ^{150}Nd. The B(E4) value for the 4^+ state of the g.s band must be multiplied with a factor of 12 and 9, respectively, as is indicated in the figure. Figure taken from ref [33].

Fig 4. (a),(b) The quadrupole (Y_{20}) and hexadecupole (Y_{40}) moments of the deformed optical potential obtained from the coupled channel analysis of inelastic scattering of polarized protons at 65 MeV. (c),(d) The quadrupole (Y_{22}) and hexadecupole (Y_{42}) transition strengths to the γ - vibrational band obtained from the coupled - channel analysis of inelastic scattering of polarized protons at 65 MeV. (e),(f) The quadrupole (Y_{20}) and hexadecupole (Y_{40}) moments of a nucleus when the valence particles are filled from the z - axis to $\mu=\cos\theta$ according to the extension of Bertsch's model.Figure taken from ref [33].

Fig 5a Spectra of $n_\pi = |n_\nu| = 4$: (EXACT) compared with those of the P - N IBM (IBM) and the P - N IBM with renormalization [IBM(ren)]. The shell - model states

with spins in parentheses are "intruders." Figure taken from ref [63].

Fig 5b. Calculated D - pair energies for the proton pair, ε_π (upper figure) and the neutron pair, ε_ν (lower figure). The curves $\tilde{\varepsilon}$ indicate the renormalized values. If only the collective G - pair is considered as a source of renormalization then the curves $\varepsilon_\pi(G)$ and $\varepsilon_\nu(G)$ are obtained. For comparison the results of a phenomenological IBM - fit with $\varepsilon_\pi = \varepsilon_\nu$, are shown. Figure taken from ref [64].

Fig 5c. Energy spectrum for six identical nucleons in a $j=\frac{17}{2}$ shell with $H = -Q.Q$ and spectra corresponding to a restriction to SDG, and to SD, pairs. Figure taken from [66].

Fig 5d. The probabilities $c^2_{\nu 2}$ and $c^2_{\nu 4}$ for finding $\nu 2$ d - bosons or $\nu 4$ g - bosons in the deformed boson state. Full lines correspond to neutrons and dashed lines correspond to protons. Figure taken from [70].

Fig 6a. Energy Spectra for the even Hg isotopes. The spectra labeled CALCULATED were obtained using the renormalization (via second order perturbation theory) due to g - bosons. Figure taken from [83].

Fig 6b. Spectra of the various approximate forms of the transformed, test case s-d-g boson Hamiltonian compared with the exact results. Here OG unitary transformation method is employed. Figure taken from [85].

Fig 6c. Spectra from the many boson, shell - model calculations of the test s-d-g boson Hamiltonian. Figure taken from [85]

Fig 7a. Comparison of calculated (hatched) and emperical (black) B(E2) values for transitions from the $L^{\pi}K = 4^{+}4$ and $5^{+}5$ states to the members of the ground, β and γ bands in ^{156}Gd. From [86].

Fig 7b. Comparison of emperical and calculated intraband B(E2) values for (a) ground band and (b) γ band in ^{104}Ru. The calculations utilize an IBA - 1 Hamiltonian incorporating g - boson excitations. The different solid curves correspond to different strengths of the boson effective charge $e_{gd}^{(2)}$ and are labeled in units of 0.1eb. The empirical results are roughly consistent with the calculations corresponding to $e_{gd}^{(2)} \simeq -1.5$. The dashed curve corresponds to the sd IBA - 1. From [87].

Fig 7c. The lower part of the figure compares the experimental band structure of ^{192}Os with calculations including a g-boson. The leftmost spectrum is for no mixing, so the bands labeled G, γ and "4" are the bands which appear in the usual interacting-boson approximation; the band labeled g is the g-boson band. The right most spectrum has mixing and a lowered g - boson band;also shown, in parentheses, are the percentages of g-boson configurations in each state. The energy scale is 0.5 MeV/division. The upper portion shows E2 branching

ratios for experiment (first number), for the calculation without mixing (second number), and for the calculation with mixing (third number). Figure taken from [32].

Fig 7d. (i) Experimental and calculated excitation energies of positive parity states of ^{112}Cd.The IBA calculations are in the sd and sdg approximations.

(ii) Integrated experimental (full lines) and calculated quadrupole (upper part) and hexadecupole (lower part) strengths in ^{112}Cd. The IBM-1 calculations are in the sd (dotted) and sdg (dot dashed,dashed) approximations. In sdg calculations two different Hamiltonians are used. See [34] for details. Figure taken from [34].

Fig 8. Classification of the dynamical symmetry groups in the U(15) extended IBM and their consecutive decomposition into the physical O(3) group. Taken from [12].

Fig 9. Energy functionals $E_G(N;\beta_2,\beta_4,\gamma)$ as a function of β_2 (β_4,γ being fixed) and β_4 (β_2 and γ being fixed) for G = SU(3), SU(5) and SU(6)-I, II and III. In G = O(15) case $E_{O(15)}(N;\beta_2,\beta_4,\gamma)$ as a function of $(\beta_2^2 + \beta_4^2)^{1/2}$ was shown. From [96].

Fig 10a. Calculated and experimental spectrum of ^{176}Hf. The SU(3) irreps ($\lambda\mu$) are shown in the figure. Figure taken from [100].

Fig 10b. Calculated and experimental spectrum of ^{184}Wu. Figure taken from [101].

Fig 10c. Calculated and experimental spectrum of ^{234}U. Figure taken from [103].

e —— experimental

t —— Theory (SU(3) limit of gIBM)

Fig 10d. B(E2) values in the gsb of ^{238}U, normalized to rotational values. I - sd - SU(3), II - sdg - SU(3), R = [B(E2;I —> I-2)/B(E2;2 —> 0)]/[B(E2;I —> I-2)/B(E2;2 —> $0)_{rot}$]. Boson number N = 15. Figure taken from [103].

Fig 11. Energy levels of all positive - parity bands below 2.4 MeV. Solid lines show theoretical ones and dashed lines show the experimental band head energies. The label below each bandhead represents an SU(3) representation $(\lambda\mu)$ of its main component and K quantum nuymber. States with asterisks cannot be assigned to definite SU(3) representations because the probability of their main components is less than 50%. The SU(3) labels $(56,4)^1$ and $(56,4)^2$ indicate $(56,4)^{w=0}$ and $(56,4)^{w=1}$, respectively. Figure taken from [112].

Fig 12. Comparison of the sdg IBM results for g factors of the ground - state band with experiment. Figure taken from [75].

Fig 13a. The ground band ($K_i^\pi = 0_i^+$) and the gamma band ($K_i^\pi = 2_i^+$) are shown as a function of the number of active g - bosons. The number of g - bosons are indicated below energy levels. Figure taken from [13].

Fig 13b. Calculated and Experimental excitation energies and relative B(E2) values in ^{94}Mo. The data is from [52]. The BSDI parameters are $\varepsilon_d - \varepsilon_s$ = 1 MeV, $\varepsilon_g - \varepsilon_s$ = 1.6MeV and G_L = - 0.05 MeV for all L values. Figure taken from [20].

Fig 14. Properties of the 2_1^+ and 4_1^+ states of Sm isotopes. Points are experiments, solid lines are proton - neutron sdg (gIBM-2) calculations and dashed lines are gIBM-1 results, for (a) energy levels, (b) ratios R = $E_x(4_1^+)/E_x(2_1^+)$, (c) two - nucleon seperation energies, (d) transition moments of $0_1^+ \longrightarrow 2_1^+$ (closed circles), and static moments of 2_1^+ states (open circles). The thin line in (c) indicates the linear term $A_0 + A_1 N_\nu$. Figure taken from [127].

TABLES

TABLE - 1a

$O(5) \supset O(3)$
$\nu_d \quad L_d$

ν/L	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
1	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
2	–	–	1	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
3	1	–	–	1	1	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–
4	–	–	1	–	1	1	1	–	1	–	–	–	–	–	–	–	–	–	–	–	–
5	–	–	1	–	1	1	1	1	1	–	1	–	–	–	–	–	–	–	–	–	–
6	1	–	–	1	1	–	2	1	1	1	1	–	1	–	–	–	–	–	–	–	–
7	–	–	1	–	1	1	1	1	2	1	1	1	1	–	1	–	–	–	–	–	–
8	–	–	1	–	1	1	1	1	2	1	2	1	1	1	1	–	1	–	–	–	–
9	1	–	–	1	1	–	2	1	1	2	2	1	2	1	1	1	1	–	1	–	–
10	–	–	1	–	1	1	1	1	2	1	2	2	2	1	2	1	1	1	1	–	1
11	–	–	1	–	1	1	1	1	2	1	2	2	2	2	2	1	2	1	1	1	1
12	1	–	–	1	1	–	2	1	1	2	2	1	3	2	2	2	2	1	2	1	1
13	–	–	1	–	1	1	1	1	2	1	2	2	2	2	3	2	2	2	2	1	2
14	–	–	1	–	1	1	1	1	2	1	2	2	2	2	3	2	3	2	2	2	2
15	1	–	–	1	1	–	2	1	1	2	2	1	3	2	2	3	3	2	3	2	2

TABLE - 1b

$O(9) \supset O(3)$
$\nu_g \quad L_g$

ν/L	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
1	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
2	–	–	1	–	1	–	1	–	1	–	–	–	–	–	–	–	–	–	–	–	–
3	1	–	1	1	1	1	2	1	1	1	1	–	1	–	–	–	–	–	–	–	–
4	1	–	2	1	3	2	3	2	3	2	3	1	2	1	1	–	1	–	–	–	–
5	1	1	3	2	4	4	5	4	6	4	5	4	4	3	3	2	2	1	1	–	1
6	2	1	4	4	7	5	9	7	9	8	9	7	9	6	7	5	5	3	4	2	2

TABLE - 2

N/L	0	1	2	3	4	5	6	7	8	9	10	11	12
2	2	0	2	0	1	0	0	0	0	0	0	0	0
	2	0	3	1	3	1	1	0	0	0	0	0	0
	3	0	4	1	4	1	2	0	1	0	0	0	0
	3	0	4	1	4	1	2	0	1	0	0	0	0
	3	0	4	1	4	1	2	0	1	0	0	0	0
	3	0	4	1	4	1	2	0	1	0	0	0	0
3	3	0	3	1	2	0	1	0	0	0	0	0	0
	4	1	6	4	7	3	4	1	1	0	0	0	0
	6	2	10	6	11	5	8	3	4	1	1	0	0
	7	2	11	7	13	6	10	4	5	2	2	0	1
	7	2	11	7	13	6	10	4	5	2	2	0	1
	7	2	11	7	13	6	10	4	5	2	2	0	1
4	4	0	5	1	4	1	2	0	1	0	0	0	0
	6	3	12	8	14	8	9	4	4	1	1	0	0
	11	6	23	15	27	16	21	11	13	5	5	1	1
	13	8	29	21	36	24	30	18	20	10	10	4	4
	15	8	32	22	40	26	34	20	24	12	13	5	6
	15	8	32	22	40	26	34	20	24	12	13	5	6
5	5	0	7	2	6	2	4	1	2	0	1	0	0
	9	6	20	15	24	16	18	10	10	4	4	1	1
	18	14	43	34	53	38	46	29	31	16	15	6	5
	24	22	62	54	81	64	75	53	55	34	32	17	15
	29	26	75	65	99	79	95	69	74	48	47	27	25
	31	27	79	68	105	84	102	74	81	53	53	31	30
6	7	0	9	3	9	3	7	2	4	1	2	0	1
	13	10	30	25	38	27	32	20	20	11	10	4	4
	29	26	73	63	95	73	88	61	64	39	36	18	16
	42	48	117	114	161	137	159	123	124	88	80	49	43
	56	63	157	152	219	189	224	178	186	137	130	85	78
	66	73	184	179	261	227	274	221	236	179	175	120	115
7	8	0	12	4	12	5	10	4	7	2	4	1	2
	17	15	43	37	56	43	50	35	36	22	21	11	10
	41	44	114	105	153	127	149	113	117	79	74	44	38
	65	90	203	208	286	260	294	244	246	186	171	117	100
	93	132	296	308	425	395	453	388	399	315	298	216	191
	126	181	406	428	597	566	660	582	614	506	496	383	356
8	10	0	15	5	16	7	14	6	11	4	7	2	4
	22	21	58	52	78	63	74	55	58	39	38	23	21
	58	67	167	160	231	200	235	187	196	142	125	89	79
	98	151	323	346	467	442	498	432	437	349	324	238	206
	152	241	511	556	753	728	829	739	761	630	598	460	410
	246	405	851	949	1292	1287	1490	1381	1460	1275	1257	1040	976
9	12	0	18	7	20	9	19	9	15	7	11	4	7
	28	28	75	70	104	87	104	81	86	63	62	41	39
	77	97	231	231	328	295	347	289	303	234	225	160	145
	141	234	481	535	712	694	784	703	714	597	560	433	383
	231	404	816	926	1229	1227	1392	1283	1318	1137	1084	873	785
	460	860	1696	1991	2647	2743	3158	3048	3216	2943	2917	2537	2404
10	14	0	22	8	25	12	24	12	21	10	16	7	11
	34	36	95	90	134	116	139	113	122	93	94	67	64
	100	132	309	315	447	413	487	419	445	357	350	263	243
	192	340	684	776	1028	1028	1160	1070	1097	945	900	727	652
	336	626	1234	1430	1887	1927	2182	2061	2130	1886	1817	1515	1378
	839	1714	3267	3947	5194	5537	6352	6312	6691	6308	6314	5694	5450
11	16	0	26	10	30	15	30	16	27	14	22	11	16
	41	45	117	113	168	149	180	151	165	131	134	101	98
	125	174	398	416	586	555	656	581	621	517	513	404	379
	254	471	930	1076	1420	1447	1638	1545	1596	1413	1363	1139	1039
	466	919	1770	2096	2745	2858	3236	3121	3238	2938	2854	2451	2255
	1486	3275	6051	7513	9782	10669	12217	12425	13210	12777	12876	11951	11552

TABLE - 2 (Contd)

12	19	0	30	12	36	18	37	20	34	19	29	15	23
	49	55	141	139	206	186	227	195	215	177	183	143	142
	155	221	500	531	748	720	856	774	835	715	719	585	560
	328	627	1221	1437	1891	1956	2225	2136	2223	2015	1966	1688	1567
	631	1284	2438	2932	3829	4041	4589	4495	4692	4341	4256	3739	3488
	2591	6002	10861	13754	17800	19733	22613	23395	24976	24632	25008	23712	23162
13	21	0	35	14	42	22	44	25	42	24	37	21	30
	57	66	168	167	248	228	279	245	273	230	241	195	195
	186	275	614	662	930	911	1085	1001	1087	953	969	813	787
	410	809	1561	1857	2443	2562	2922	2850	2990	2759	2723	2395	2252
	820	1730	3245	3952	5147	5504	6259	6221	6528	6140	6073	5447	5135
	4381	10645	18933	24356	31366	35272	40409	42427	45459	45540	46540	44930	44251
14	24	0	40	16	49	26	52	30	51	30	46	27	39
	66	78	197	198	294	274	337	301	338	291	308	256	259
	222	334	741	807	1135	1125	1346	1260	1379	1232	1266	1087	1067
	504	1017	1947	2340	3078	3263	3736	3692	3900	3657	3646	3270	3115
	1046	2255	4197	5160	6718	7254	8274	8318	8781	8374	8354	7620	7266
	7286	18296	32149	41860	53748	61092	70068	74402	80031	81192	83488	81757	81181
15	27	0	45	19	56	30	61	36	60	37	56	34	49
	76	91	228	232	344	324	401	363	410	360	384	326	334
	260	400	879	969	1360	1364	1637	1554	1709	1554	1610	1411	1400
	610	1251	2379	2887	3796	4061	4669	4665	4958	4716	4743	4325	4170
	1302	2866	5292	6570	8542	9307	10645	10813	11470	11079	11135	10310	9930
	11856	30635	53257	70090	89748	102947	118194	126719	136748	140242	144976	143685	143692

TABLE-3

Group	Casimir Operator	Irrep	Eigenvalues
$U(2\ell+1)$	$\sum_{k=0}^{2\ell} u^k.u^k$	$\{m\}$	$m(2\Omega+m-1)$
$SU(2\ell+1)$	$\sum_{k=1}^{2\ell} u^k.u^k$	$\{m\}$	$\frac{m(2\Omega+m)(2\Omega-1)}{2\Omega}$
$O(2\ell+1)$	$\sum_{k=odd} u^k.u^k$	$[v]$	$v(2\Omega+v-2)$
$O(3)$	$u^1.u^1$	L	L(L+1)

$2\Omega = 2\ell+1$. The eigenvalues for $U(2\ell+1)$ follow easily by putting $u^k.u^k$ in normal order form and identifying the $\binom{n}{2}_\ell$ and n_ℓ operators appropriately; n_ℓ is the number operator. In the case of $O(2\ell+1)$ we use the relation $4S_+S_- = [C_2(U(2\ell+1)) - 2\,C_2(O(2\ell+1)) - n_\ell]$ where the quasispin step up operator $S_+ = \frac{1}{2} \times \sqrt{2\ell+1}(-1)^\ell(b^\dagger_\ell b^\dagger_\ell)^0$ and the step down operator $S_- = \frac{1}{2} \times \sqrt{2\ell+1}(-1)^\ell(\tilde{b}_\ell \tilde{b}_\ell)^0$. The eigenvalues $(m-v)(m+2\Omega+v-2)/4$ of S_+S_- operator follow easily from the quasispin algebra.

Table - 4

GROUP CHAIN	GROUP	IRREP LABELS	GENERATORS	QUADRATIC CASIMIR OPERATOR	EIGEN-VALUES
$U(15) \supset SU(3) \supset O(3)$	$SU(3)$	$(\lambda\ \mu)$	$Q^2_\mu(s) = \left[4\sqrt{\frac{7}{15}}(s^\dagger\tilde{d} + d^\dagger\tilde{s})^2_\mu + \frac{36}{\sqrt{105}}(d^\dagger\tilde{g} + g^\dagger\tilde{d})^2_\mu - 11\sqrt{\frac{2}{21}}(d^\dagger\tilde{d})^2_\mu - 2\sqrt{\frac{33}{7}}(g^\dagger\tilde{g})^2_\mu\right]$	$\frac{3}{4}\left[Q(s)\cdot Q(s) + L\cdot L\right]$	$\lambda^2+\mu^2+3(\lambda+\mu)+\lambda\mu$
	$O(3)$	L	$L'_{q'} = \sqrt{10}(d^\dagger\tilde{d})'_{q'} + \sqrt{60}(g^\dagger\tilde{g})'_{q'}$	$L\cdot L$	$L(L+1)$
$U(15) \supset U(5) \supset SU(5)$ $\supset O(5) \supset O(3)$	$U(5)$	$\{f_1 f_2 f_3 f_4 f_5\}$	$G^{L_0=0-4}_\mu = \sum_{\ell_1,\ell_2=0,2,4} \sqrt{(2\ell_1+1)(2\ell_2+1)}\ \times(-1)^{L_0}\begin{Bmatrix}\ell_1 & \ell_2 & L_0\\ 2 & 2 & 2\end{Bmatrix} \times (b^\dagger_{\ell_1}\tilde{b}_{\ell_2})^{L_0}_\mu$	$4\sum_{L_0=0}^{4} G^{L_0}\cdot G^{L_0}$	$\sum_{i=1}^{5} f_i(f_i+6-2i)$
	$SU(5)$	$\{m_1 m_2 m_3 m_4\}$ $m_i = f_i - f_5$ $i = 1-4$	$G^{L_0=1-4}_\mu$	$4\sum_{L_0=1}^{4} G^{L_0}\cdot G^{L_0}$	$\left[\sum_{i=1}^{4} m_i(m_i+6-2i) - \frac{1}{5}\left(\sum_{i=1}^{4} m_i\right)^2\right]$

Table – 4 (contd)

GROUP CHAIN	GROUP	IRREP LABELS	GENERATORS	QUADRATIC CASIMIR OPERATOR	EIGEN-VALUES
	$O(5)$	ν_1, ν_2	$G_\mu^{L_0 = 1,3}$	$8\sum_{L_0=1,3} G^{L_0} \cdot G^{L_0}$	$[\nu_1(\nu_1+3) + \nu_2(\nu_2+1)]$
$U(15) \supset U(6) \supset SU(6)$ $\supset Sp(6) \supset O(3)$	$U(6)$	$\{F_1 F_2 F_3 F_4 F_5 F_6\}$	$h_\mu^{L_0=0-5} = \sum_{l_1, l_2 = 0,2,4} \sqrt{(2l_1+1)(2l_2+1)} \times (-1)^{L_0} \begin{Bmatrix} l_1 & l_2 & L_0 \\ 5/2 & 5/2 & 5/2 \end{Bmatrix} \times (b^\dagger_{l_1} \tilde{b}_{l_2})^{L_0}_\mu$	$4\sum_{L_0=0}^{5} h^{L_0} \cdot h^{L_0}$	$\sum_{i=1}^{6} F_i (F_i - 2i + 7)$
	$SU(6)$	$\{m_1 m_2 m_3 m_4 m_5\}$ $m_i = F_i - F_6$ $i = 1-5$	$h_\mu^{L_0 = 1-5}$	$4\sum_{L_0=1}^{5} h^{L_0} \cdot h^{L_0}$	$\{\sum_{i=1}^{5} m_i(m_i + 7 - 2i) - \frac{1}{6}(\sum_{i=1}^{5} m_i)^2\}$
	$Sp(6)$	$\langle \lambda_1 \lambda_2 \lambda_3 \rangle$	$h_\mu^{L_0 = 1,3,5}$	$8\sum_{L_0=1,3,5} h^{L_0} \cdot h^{L_0}$	$[\lambda_1(\lambda_1+6) + \lambda_2(\lambda_2+4) + \lambda_3(\lambda_3+2)]$

Table - 4 (contd)

GROUP CHAIN	GROUP	IRREP LABELS	GENERATORS	QUADRATIC CASIMIR OPERATOR	EIGEN-VALUES
$U(15) \supset O(15) \supset O(14) \supset O(5) \supset O(3)$	$O(15)$	$[\sigma]$	$(d^\dagger \tilde{d})^{1,3}_\mu, (q^\dagger \tilde{q})^{1,3,5,7}_\mu,$ $(d^\dagger \tilde{s} + s^\dagger \tilde{d})^2_\mu, (q^\dagger \tilde{s} + s^\dagger \tilde{q})^4_\mu,$ $(d^\dagger \tilde{q} - (-1)^k q^\dagger \tilde{d})^{k=2-6}_\mu$	$P_2(O(15)) = S_+ S_-$ $S_+ = \frac{1}{\sqrt{2}} \sum_{\ell=0,2,4} \Omega_\ell^{1/2} \times (b_\ell^\dagger \tilde{b}_\ell)^0$ $S_- = (S_+)^\dagger$ $\Omega_\ell = \frac{(2\ell+1)}{2}$	$\left[\frac{1}{4}(m-\sigma) \times (m+\sigma+13)\right]$
	$O(14)$	$[\tau]$	$(d^\dagger \tilde{d})^{1,3}_\mu, (q^\dagger \tilde{q})^{1,3,5,7}_\mu$ $(d^\dagger \tilde{q} - (-1)^k q^\dagger \tilde{d})^{k=2-6}_\mu$	$2\{\sum_{k=1,3} (d^\dagger \tilde{d})^k \cdot (d^\dagger \tilde{d})^k + \sum_{k'=1,3,5,7} (q^\dagger \tilde{q})^{k'} \cdot (q^\dagger \tilde{q})^{k'}\}$ $+ \sum_{k=2}^{6} (d^\dagger \tilde{q} - (-1)^k q^\dagger \tilde{d})^k \cdot (d^\dagger \tilde{q} - (-1)^k q^\dagger \tilde{d})^k$	$\left[\tau(\tau+12)\right]$

TABLE-5

					V(L1,L2,L3,L4;LL)					
L1	L2	L3	L4	LL	#1	#2	#3	#4	#5	#6
0	0	0	0	0	0.0000	-1.6000	-1.3333	0.0000	0.0000	-0.3136
0	0	2	2	0	-25.0440	-3.5777	-2.9814	4.4721	2.2361	-1.3023
0	0	4	4	0	0.0000	-4.8000	-4.0000	6.0000	3.0000	-0.9408
0	2	0	2	2	-11.2000	-3.2000	-2.6667	2.0000	1.0000	-0.8960
0	2	2	2	2	19.6774	1.0842	1.3469	0.0000	-1.6903	0.7725
0	2	2	4	2	-14.4000	-2.7429	-2.9692	0.0000	3.2071	-1.0185
0	2	4	4	2	0.0000	-5.0854	-1.8952	0.0000	-2.1622	-0.5329
0	4	0	4	4	0.0000	-3.2000	-2.6667	2.0000	1.0000	-0.6272
0	4	2	2	4	-15.1789	-1.4456	-1.5649	0.0000	1.6903	-0.7190
0	4	2	4	4	13.2665	-5.3605	-1.9977	0.0000	-2.2792	-0.2433
0	4	4	4	4	0.0000	-1.9327	2.7897	0.0000	1.7068	0.4924
2	2	2	2	0	-17.2857	-3.1429	-3.6667	-8.0000	4.0000	-2.4023
2	2	2	2	2	3.7041	-1.8571	-1.6599	2.0000	0.4286	-0.8793
2	2	2	2	4	-4.9388	-1.7143	-1.5408	2.0000	0.4286	0.2992
2	2	2	4	2	7.2284	-2.1684	-0.7498	0.0000	-2.7105	-0.0422
2	2	2	4	4	14.1268	0.0000	0.5862	0.0000	-1.9263	0.4551
2	2	4	4	0	-13.7997	-6.8999	-2.2361	-13.4164	6.7082	-0.9878
2	2	4	4	2	-10.3381	2.0102	-0.4786	0.0000	1.8274	-0.2806
2	2	4	4	4	-3.9291	-3.0559	-0.8186	0.0000	1.4425	-0.3511
2	4	2	4	2	-12.4939	-1.9429	-0.3469	2.0000	4.1429	-2.3688
2	4	2	4	3	-0.5000	2.0000	-2.0000	2.0000	-1.0000	-1.7040
2	4	2	4	4	5.0918	-5.4286	-1.2517	2.0000	1.5974	-0.8759
2	4	2	4	5	13.0000	-4.0000	-2.0000	2.0000	-1.0000	0.1800
2	4	2	4	6	-16.5714	-2.2857	-2.0000	2.0000	2.1469	0.6874
2	4	4	4	2	3.8528	-2.5427	1.0551	0.0000	-3.4673	0.2226
2	4	4	4	4	11.2387	-1.0302	-1.0449	0.0000	-1.9451	0.0309
2	4	4	4	6	11.4998	-2.7105	0.0000	0.0000	-1.9902	0.1920
4	4	4	4	0	-15.7143	-7.2571	-5.0000	-16.0000	8.0000	-5.4721
4	4	4	4	2	-9.0102	-4.0000	-1.3265	2.0000	1.3377	-3.9009
4	4	4	4	4	2.3469	-0.6571	-0.5408	2.0000	0.4565	-2.0111
4	4	4	4	6	8.0714	0.2857	-2.0000	2.0000	0.2587	0.0006
4	4	4	4	8	-8.0000	-4.0000	-2.0000	2.0000	0.8141	2.3920

#1 $(-3/4)Q(S).Q(S)=-C_2(SU(3))+(3/4)L.L$ #4 $C_2(O(15))$

#2 $-C_2(U(5)+(1/2)C_2(O(5))$ #5 BSDI

#3 $-C_2(U(6)+(1/2)C_2(Sp(6))$ #6 0.024*[#1]+0.031*[#2]+0.198[#3]+0.097[L.L]

TABLE-6

Operator	ε_s	ε_d	ε_g
$G^0.G^0$	1/5	1/5	1/5
$G^1.G^1$	0	3/20	1/2
$G^2.G^2$	1	11/28	9/14
$G^3.G^3$	0	11/10	3/4
$G^4.G^4$	9/5	81/70	127/140
$C_2(U(5))$	12	12	12
$C_2(SU(5))$	56/5	56/5	56/5
$C_2(O(5))$	0	10	10
$h^0.h^0$	1/6	1/6	1/6
$h^1.h^1$	0	3/35	2/7
$h^2.h^2$	5/6	11/24	5/24
$h^3.h^3$	0	33/40	23/72
$h^4.h^4$	3/2	3/8	5/8
$h^5.h^5$	0	33/56	451/504
$C_2(U(6))$	10	10	10
$C_2(SU(6))$	28/3	28/3	28/3
$C_2(Sp(6))$	0	12	12
$Q^2(S).Q^2(S)$	112/3	94/3	52/3
L.L	0	6	20
$C_2(O(15))$	14	14	14
$C_2(O(14))$	0	13	13

TABLE-7

X	K1	K2	K3	Operator
2	1	K	0	$(\tilde{d}\tilde{d})^k$
2	2	K	0	$(d^\dagger d^\dagger)^k$
2	3	K	0	$(d^\dagger \tilde{d})^k$
2	1	0	1	$\tilde{d}$
2	2	0	1	$d^\dagger$
4	1	K	0	$(\tilde{g}\tilde{g})^k$
4	2	K	0	$(g^\dagger g^\dagger)^k$
4	3	K	0	$(g^\dagger \tilde{g})^k$
4	1	0	1	$\tilde{g}$
4	2	0	1	$g^\dagger$

TABLE-8

# of Bosons N	Angular Momentum L	Dimension	Time (Minutes)*
5	2	75	~2
5	3	65	~2
5	4	99	~4
5	5	79	$\sim\frac{7}{2}$
5	6	95	~5
6	0	56	$\sim\frac{3}{2}$
6	1	63	$\sim\frac{5}{2}$
6	2	157	~13
6	3	152	$\sim\frac{27}{2}$
6	4	219	~29
6	5	189	~24
6	6	224	~35

1) every where $n_g \leq 4$ and there is no restriction on n_d.

2) for 2-4 particle system the program takes about one minute or less

* For the cases with $n_g = 5$ or 6 and/or $n_d = 10 - 12$ the present version of the codes is some what slow. The programmes can be speeded up by modifying the subroutine CFPINP.FOR that appears in SUBGH and SUBGM.

TABLE-9

Operator	KIJJ	KID	K
T^{M1}	1	99	1
T^{M3}	1	99	3
T^{E2}(SU(3))	1	3	2
T^{E2}(SU(5))	1	5	2
T^{E2}(SU(6))	1	6	2
T^{E2}(general)	1	99	2
T^{E4}(SU(5))	1	5	4
T^{E4}(SU(6))	1	6	4
T^{E4}(general)	1	99	4
Occupation numbers $\langle n_\ell \rangle$	1	100	0
Two nucleon transfer amplitudes	2	100	5

TABLE-10

L^{π}	Irreps (λ,μ)	Eigenvalue
0^{+}_{1}	(16,0)	-304
0^{+}_{2}	(12,2)	-214
0^{+}_{3}	(8,4)	-148
1^{+}_{1}	(10,3)	-176.5
1^{+}_{2}	(7,3)	-107.5
1^{+}_{3}	(8,1)	-98.5
2^{+}_{1}	(16,0)	-299.5
2^{+}_{2}	(12,2)	-209.5
2^{+}_{3}	(12,2)	-209.5
3^{+}_{1}	(12,2)	-205.0
3^{+}_{2}	(10,3)	-169.0
3^{+}_{3}	(10,3)	-169.0
4^{+}_{1}	(16,0)	-289.0
4^{+}_{2}	(12,2)	-199.0
4^{+}_{3}	(12,2)	-199.0
5^{+}_{1}	(12,2)	-181.0
5^{+}_{2}	(10,3)	-155.0
5^{+}_{3}	(10,3)	-155.0
6^{+}_{1}	(16,0)	-272.5
6^{+}_{2}	(12,2)	-182.5
6^{+}_{3}	(12,2)	-182.5

Table-11

$\langle(\lambda 0)L_1;\,(40)l||(\lambda+4,\,0)L\rangle$

$\lambda - L$ = even

l	L_1	$\langle(\lambda 0)\,L_1;\,(40)l\|\|(\lambda+4,\,0)L\rangle$
$l=0$	L	$\left[\frac{(\lambda-L+2)(\lambda-L+4)(\lambda+L+3)(\lambda+L+5)}{5(\lambda+1)(\lambda+2)(\lambda+3)(\lambda+4)}\right]^{\frac{1}{2}}$
$l=2$	$L-2$	$\left[\frac{6(\lambda-L+4)(\lambda+L+1)(\lambda+L+3)(\lambda+L+5)(L-1)L}{7(\lambda+1)(\lambda+2)(\lambda+3)(\lambda+4)(2L-1)(2L+1)}\right]^{\frac{1}{2}}$
	L	$-\left[\frac{4(\lambda-L+2)(\lambda-L+4)(\lambda+L+3)(\lambda+L+5)}{7(\lambda+1)(\lambda+2)(\lambda+3)(\lambda+4)(2L-1)(2L+3)}\right]^{\frac{1}{2}} \times [L(L+1)]^{1/2}$
	$L+2$	$\left[\frac{6(\lambda+L+5)(\lambda-L)(\lambda-L+2)(\lambda-L+4)(L+2)(L+1)}{7(\lambda+1)(\lambda+2)(\lambda+3)(\lambda+4)(2L+1)(2L+3)}\right]^{\frac{1}{2}}$
$l=4$	$L-4$	$\left[\frac{(\lambda+L-1)(\lambda+L+1)(\lambda+L+3)(\lambda+L+5)(L-3)(L-2)(L-1)L}{(\lambda+1)(\lambda+2)(\lambda+3)(\lambda+4)(2L-5)(2L-3)(2L-1)(2L+1)}\right]^{\frac{1}{2}}$
	$L-2$	$-\left[\frac{4(\lambda+L+1)(\lambda+L+3)(\lambda+L+5)(\lambda-L+4)(L-2)(L-1)L(L+1)}{7(\lambda+1)(\lambda+2)(\lambda+3)(\lambda+4)(2L-5)(2L-1)(2L+1)(2L+3)}\right]^{\frac{1}{2}}$
	L	$\left[\frac{18(\lambda-L+2)(\lambda-L+4)(\lambda+L+3)(\lambda+L+5)(L-1)L(L+1)(L+2)}{35(\lambda+1)(\lambda+2)(\lambda+3)(\lambda+4)(2L-3)(2L-1)(2L+3)(2L+5)}\right]^{\frac{1}{2}}$
	$L+2$	$-\left[\frac{4(\lambda-L)(\lambda-L+2)(\lambda-L+4)(\lambda+L+5)(L+3)(L+2)(L+1)L}{7(\lambda+1)(\lambda+2)(\lambda+3)(\lambda+4)(2L-1)(2L+1)(2L+3)(2L+7)}\right]^{\frac{1}{2}}$
	$L+4$	$\left[\frac{(\lambda-L-2)(\lambda-L)(\lambda-L+2)(\lambda-L+4)(L+4)(L+3)(L+2)(L+1)}{(\lambda+1)(\lambda+2)(\lambda+3)(\lambda+4)(2L+1)(2L+3)(2L+5)(2L+7)}\right]^{\frac{1}{2}}$

TABLE-12

L^{π}	Irreps		Eigenvalue
	U(5) $\{f_1,f_2\}$	O(5) $[v_1,v_2]$	
0^+_1	{8}	[0]	-86
0^+_2	{8}	[6]	-69
0^+_3	{6,2}	[0]	-66
1^+_1	{6,2}	[3,1]	-58
1^+_2	{6,2}	[4,2]	-51
1^+_3	{6,2}	[5,1]	-47
2^+_1	{8}	[2]	-91
2^+_2	{8}	[4]	-82
2^+_3	{6,2}	[20]	-63
3^+_1	{8}	[6]	-69
3^+_2	{6,2}	[2,2]	-60
3^+_3	{8,2}	[3,1]	-58
4^+_1	{8}	[2]	-91
4^+_2	{8}	[4]	-82
4^+_3	{8}	[6]	-69
5^+_1	{8}	[4]	-82
5^+_2	{6,2}	[3,1]	-58
5^+_3	{6,2}	[3,1]	-58
6^+_1	{8}	[4]	-82
6^+_2	{8}	[6]	-69
6^+_3	{8}	[6]	-69

TABLE-13

L	Irreps		Eigen-value
	U(6) $\{F_1,F_2,F_3,F_4,F_5,F_6\}$	Sp(6) $\langle\lambda_1,\lambda_2,\lambda_3\rangle$	
0^+	{4,4,0,0,0,0}	<0,0,0>	0
$2^+,4^+$	{4,4,0,0,0,0}	<1,1,0>	6
$0^+,(2^+)^2,3^+,(4^+)^2,5^+$ $(6^+)^2,8^+$	{4,4,0,0,0,0}	<2,2,0>	14
0^+	{3,3,1,1,0,0}	<0,0,0>	20
$(0^+)^2,1^+,(2^+)^3,(3^+)^3$ $(4^+)^5,(5^+)^3,(6^+)^5$	{4,4,0,0,0,0}	<3,3,0>	24
$2^+,4^+$	$\{3,3,1,1,0,0\}^2$	<1,1,0>	26
$1^+,2^+,(3^+)^2,5^+,6^+,7^+$	{3,3,1,1,0,0}	<2,1,1>	32
0^+	{2,2,2,2,0,0}	<0,0,0>	32
$0^+,(2^+)^2,3^+,(4^+)^2,$ $5^+,(6^+)^2,8^+$	$\{2,2,2,2,0,0\}^2$	<2,2,0>	34
$(0^+)^2,(1^+)^2,(2^+)^7,$ $(3^+)^5,(4^+)^{10},(5^+)^8,$ $(6^+)^{10}$	{4,4,0,0,0,0}	<4,4,0>	36
$2^+,4^+$	{2,2,2,2,0,0}	<1,1,0>	38
$0^+,(1^+)^3,(2^+)^5,(3^+)^5,$ $(4^+)^6,(5^+)^7,(6^+)^5$	{3,3,1,1,0,0}	<3,2,1>	41
$(0^+)^2,1^+,(2^+)^3,(3^+)^3,$ $(4^+)^5,(5^+)^3,(6^+)^5$	{3,3,1,1,0,0}	<3,3,0>	44
0^+	{2,2,1,1,1,1}	<0,0,0>	44
$0^+,(2^+)^2,3^+,(4^+)^2,$ $5^+,(6^+)^2,8^+$	{2,2,2,2,0,0}	<2,2,0>	46
$2^+,4^+$	{2,2,1,1,1,1}	<1,1,0>	50

TABLE - 14

Nucleus	Transition	Energy (keV)	B(IS) e^2b^4	B(IS) s.p.u
^{168}Er	$0^+ \longrightarrow 4^+_1$	264	0.004	3.9
	$0^+ \longrightarrow 4^+_\gamma$	944	0.083	16.5
	$0^+ \longrightarrow 4^+_{K=4}$	2055	0.003	0.6
^{172}Yb	$0^+ \longrightarrow 4^+_1$	260	0.0003	0.05
	$0^+ \longrightarrow 4^+_{K=3}$	1263	0.036	6.9
	$0^+ \longrightarrow 4^+_{K=2}$	1658	0.006	1.1
	$0^+ \longrightarrow 4^+_{K=2}$	1803	$\leq$ 0.012	$\leq$ 2.2

Table – 15

Nucleus	Energy E(MeV)	$J_i^\pi \rightarrow J_f^\pi$	S (W.U)
^{20}Ne	4.25	$4^+ \rightarrow 0^+$	$(2.3 \pm 0.5) \times 10$
^{24}Mg	4.12	$4^+ \rightarrow 0^+$	(1.2 ± 0.2)
	6.01	$4^+ \rightarrow 0^+$	$(1.5 \pm 0.4) \times 10$
^{25}Mg	4.06	$\frac{9}{2}^+ \rightarrow \frac{5}{2}^+$	(2.4 ± 0.6)
^{26}Mg	4.90	$4^+ \rightarrow 0^+$	$(1.1 \pm 0.2) \times 10$
	5.72	$4^+ \rightarrow 0^+$	(5.1 ± 1.8)
^{28}Si	4.62	$4^+ \rightarrow 0^+$	(4.4 ± 0.5)
^{44}Sc	0.27	$6^+ \rightarrow 2^+$	(1.1 ± 0.1)
^{52}Cr	2.37	$4^+ \rightarrow 0^+$	(3.4 ± 0.6)
^{52}Mn	0.38	$2^+ \rightarrow 6^+$	$(1.5 \pm 0.1) \times 10^{-1}$

Nucleus	Energy E(MeV)	$J_i^\pi \rightarrow J_f^\pi$	S (W.U)
^{56}Fe	2.09	$4^+ \rightarrow 0^+$	(1.6 ± 0.2)
^{58}Ni	2.46	$4^+ \rightarrow 0^+$	(3.5 ± 1.1)
^{60}Ni	2.51	$4^+ \rightarrow 0^+$	(5.2 ± 0.6)
	3.13	$4^+ \rightarrow 0^+$	(1.0 ± 0.2)
	3.67	$4^+ \rightarrow 0^+$	(1.8 ± 0.4)
^{62}Ni	2.34	$4^+ \rightarrow 0^+$	(3.0 ± 0.3)
^{78}Kr	1.12	$4^+ \rightarrow 0^+$	(5.5 ± 1.1)
^{80}Kr	1.42	$4^+ \rightarrow 0^+$	(2.3 ± 0.5)
^{82}Kr	1.83	$4^+ \rightarrow 0^+$	(1.5 ± 0.3)
^{84}Kr	2.10	$4^+ \rightarrow 0^+$	(2.3 ± 0.6)
^{86}Kr	2.25	$4^+ \rightarrow 0^+$	(3.4 ± 0.87)

Table – 15 (Contd)

Nucleus	Energy E(MeV)	$J_i^\pi \rightarrow J_f^\pi$	S (W.U)
$^{90}_{Zr}$	3.08	$4^+ \rightarrow 0^+$	(3.3 ± 0.9)
	4.32	$4^+ \rightarrow 0^+$	(1.7 ± 0.5)
	4.44	$4^+ \rightarrow 0^+$	(3.9 ± 0.7)

1. All transitions are to the ground state 0^+ except in ^{25}Mg, ^{44}Sc and ^{52}Mn.
2. For (ds) shell nuclei Wildenthal et al. [37] gave large number of E4 Matrix elements derived from shell-model calculations.
3. Data on Q_4 moments derived via polarized proton inelastic scattering for Ni, Zn, Ge, Se, Sr was given by Matsuki et al [38].
4. W.U. $= (2L+1)^{-1} B(EL\uparrow)$; See eqn (47) for the definition $B(EL\uparrow)$.

TABLE - 16

Space	Structure of Λ pair			Q_{int}	E_{int}	Pairing gap	B.E	Moment of Inertia	B(E2)
	X_0	X_2	X_4	%	%	%	%	%	%
SD	0.76	0.65	0	82	88	150	92	33	74
SDG	0.59	0.76	0.28	97	99	110	99	80	96
ALL	0.63	0.75	0.18	100	100	100	100	100	100

Table – 17

L	$\langle n_s \rangle$ (4N, 0)	$\langle n_d \rangle$ (4N, 0)	$\langle n_g \rangle$ (4N, 0)
0	4·21	11·57	4·23
4	4·18	11·55	4·27
12	4·00	11·44	4·56
20	3·67	11·27	4·96
28	3·20	10·81	6·00
36	2·62	10·22	7·16
44	1·98	9·37	8·64
52	1·34	8·19	10·47
60	0·74	6·59	12·67
68	0·28	4·45	15·27
76	0·02	1·67	18·31
80	0·0	0·0	20·0

TABLE -18

Structure of SU(3) intrinsic states $|(\lambda,\mu)\varepsilon_{max},\Lambda_{max},M_{\Lambda_{max}}\rangle_\alpha$

$$|(4N,0)\varepsilon=8N,\Lambda=0,M_\Lambda=0\rangle = \Sigma^N$$

$$|(4N-4,2)\varepsilon=8N-6,\Lambda=1,M_\Lambda=1\rangle = \frac{1}{\sqrt{4N-1}}\left\{-\sqrt{3}\,\Sigma^{N-2}(\Delta_{1/2})^2 + \sqrt{4N-4}\,\Sigma^{N-1}\Pi_1\right\}$$

$$|(4N-6,3)\varepsilon=8N-9,\Lambda=\tfrac{3}{2},M_\Lambda=\tfrac{3}{2}\rangle = \frac{1}{\sqrt{(2N-1)(N-1)}}\left\{\sqrt{3}\,\Sigma^{N-3}(\Delta_{1/2})^3 - \sqrt{3(N-2)}\,\Sigma^{N-2}\Delta_{1/2}\Pi_1 + \sqrt{2(N-2)(N-1)}\,\Sigma^{N-1}\Omega_{3/2}\right\}$$

$$|(4N-8,4)\varepsilon=8N-12,\Lambda=2,M_\Lambda=2\rangle_{\alpha=0} = \frac{1}{\sqrt{16N^2-56N+51}}\left\{-3\sqrt{3}\,\Sigma^{N-4}(\Delta_{1/2})^4 + 2\sqrt{6(N-3)}\,\Sigma^{N-3}(\Delta_{1/2})^2\Pi_1 - 4\sqrt{(N-3)(N-2)}\,\Sigma^{N-2}(\Pi_1)^2\right\}$$

$$|(4N-8,4)\varepsilon=8N-12,\Lambda=2,M_\Lambda=2\rangle_{\alpha=1} = \frac{1}{\sqrt{16N^2-56N+51}}\left\{-12\sqrt{3(N-3)(N-2)}\,*\right.$$

$$\Sigma^{N-4}(\Delta_{1/2})^4 + 8(N-3)\sqrt{6(N-2)}\,\Sigma^{N-3}(\Delta_{1/2})^2\Pi_1 + 3(8N-15)\,\Sigma^{N-2}(\Pi_1)^2 -$$

$$\left.\sqrt{2}\,(16N^2-56N+51)\Sigma^{N-2}\Delta_{1/2}\Omega_{3/2} + \sqrt{2(N-1)}\,(16N^2-56N+51)\Sigma^{N-1}\Gamma_2\right\}$$

Foot Note:

$$\Sigma = \sqrt{\frac{1}{35}}\left(\sqrt{7}\,s_o^\dagger + \sqrt{20}\,d_o^\dagger + \sqrt{8}\,g_o^\dagger\right) \Big\} \varepsilon = 8,\ \Lambda = 0$$

$$\Delta_{\pm1/2} = \pm\sqrt{\frac{1}{7}}\left(\sqrt{3}\,d_{\pm1}^\dagger + \sqrt{4}\,g_{\pm1}^\dagger\right) \Big\} \varepsilon = 5,\ \Lambda = 1/2$$

$$\Pi_{\pm1} = \sqrt{\frac{1}{7}}\left(d_{\pm2}^\dagger + \sqrt{6}g_{\pm2}^\dagger\right)$$

$$\Pi_o = \sqrt{\frac{1}{105}}\left(\sqrt{28}\,s_o^\dagger + \sqrt{5}\,d_o^\dagger - \sqrt{72}\,g_o^\dagger\right) \Big\} \varepsilon = 2,\ \Lambda = 1$$

$$\Omega_{\pm3/2} = \pm g_{\pm3}^\dagger\ ;\ \Omega_{\pm1/2} = \mp\sqrt{\frac{1}{7}}\left(\sqrt{4}\,d_{\pm1}^\dagger - \sqrt{3}\,g_{\pm1}^\dagger\right) \quad \varepsilon = -1,\ \Lambda = 3/2$$

$$\Gamma_{\pm2} = g_{\pm4}^\dagger\ ;\ \Gamma_{\pm1} = \sqrt{\frac{1}{7}}\left(\sqrt{6}\,d_{\pm2}^\dagger - g_{\pm2}^\dagger\right)$$

$$\Gamma_o = \sqrt{\frac{1}{105}}\left(\sqrt{56}\,s_o^\dagger - \sqrt{40}\,d_o^\dagger + \sqrt{9}\,g_o^\dagger\right) \Big\} \varepsilon = -4,\ \Lambda = 2$$

Table – 19

$(\lambda\mu)$ K' α	$S_\ell(N,(4N,0)K=0\ L=0 \rightarrow N+1, \alpha(\lambda\mu)K'\ L')$			CFP2
	$L'=0$	$L'=2$	$L'=4$	
(4N+4,0)0 0	(N/5)(1+2/N)	(N/35)(1+11/N)	(8N/315)(1+9/2N)	(N+1)
(4N ,2)0 0	(4/15)(1+1/N)	(1/105)(1+25/4N)	(8/105)(1+1/N)	(4N+2)/(4N-1)
2 0	0	(2/35)(1-1/4N)	(4/21)(1+3/2N)	
(4N-2,3)1 0	0	(8/35)(1+1/2N)	(2/21)(1-5/4N)	(4N-4)(4N+1)/(4N-1)(4N-2)
3 0	0	0	(2/9)(1-1/4N)	
(4N-4,4)0 0	0	0	0	0
2 0	0	0	0	
4 0	0	0	0	
(4N-4,4)0 1	(8/15)(1-1/2N)	(8/105)(1-5/4N)	(1/105)(1-3/N)	2N(16N²-24N+11)/(2N-1)(4N-1)(4N-3)
2 1	0	(12/35)(1-3/4N)	(2/63)(1-5/2N)	
4 1	0	0	(2/9)(1-1/N)	

TABLE - 20

Intrinsic state with definite K	Structure
$\|(4N,0)\ K=0\rangle$	$\|\Sigma^N\rangle$
$\|(4N-4,2)\ K=0\rangle$	$\|\Sigma^{N-1}\Pi_0\rangle$
$\|(4N-4,2)\ K=2\rangle$	$\frac{1}{\sqrt{2}}\{\|\Sigma^{N-1}\Pi_1\rangle + \|\Sigma^{N-1}\Pi_{-1}\rangle\}$
$\|(4N-6,3)\ K=1\rangle$	$\frac{1}{\sqrt{2}}\{\|\Sigma^{N-1}\Omega_{1/2}\rangle + \|\Sigma^{N-1}\Omega_{-1/2}\rangle\}$
$\|(4N-6,3)\ K=3\rangle$	$\frac{1}{\sqrt{2}}\{\|\Sigma^{N-1}\Omega_{3/2}\rangle + \|\Sigma^{N-1}\Omega_{-3/2}\rangle\}$
$\|(4N-8,4)\ K=0\rangle_{\alpha=0}$	$\|\Sigma^{N-2}\Pi_0^2\rangle$
$\|(4N-8,4)\ K=2\rangle_{\alpha=0}$	$\frac{1}{\sqrt{2}}\{\|\Sigma^{N-2}\Pi_1\Pi_0\rangle + \|\Sigma^{N-2}\Pi_{-1}\Pi_0\rangle\}$
$\|(4N-8,4)\ K=4\rangle_{\alpha=0}$	$\frac{1}{\sqrt{2}}\{\|\Sigma^{N-2}\Pi_1^2\rangle + \|\Sigma^{N-2}\Pi_{-1}^2\rangle\}$
$\|(4N-8,4)\ K=0\rangle_{\alpha=1}$	$\|\Sigma^{N-1}\Gamma_0\rangle$
$\|(4N-8,4)\ K=2\rangle_{\alpha=1}$	$\frac{1}{\sqrt{2}}\{\|\Sigma^{N-1}\Gamma_1\rangle + \|\Sigma^{N-1}\Gamma_{-1}\rangle\}$
$\|(4N-8,4)\ K=4\rangle_{\alpha=1}$	$\frac{1}{\sqrt{2}}\{\|\Sigma^{N-1}\Gamma_2\rangle + \|\Sigma^{N-1}\Gamma_{-2}\rangle\}$

See Table-18 for the description of $\Sigma, \Delta, \Pi, \Omega$ and Γ; see also [107]

Table _ 21a

Transition	$\Delta(e\,b/\mu_N)$	
	experiment[a]	theory
$2_\gamma \to 2_g$	-13.3 ± 0.7	-13.3[b]
$3_\gamma \to 2_g$	-8.9 ± 0.5	-12.4
$3_\gamma \to 4_g$	-8.9 ± 0.3	-9.0
$4_\gamma \to 4_g$	-5.5 ± 0.5	-6.9
$5_\gamma \to 4_g$	$-4.9^{+1.4}_{-2.9}$	-7.1
$5_\gamma \to 6_g$	$-11.6^{+7.1}_{-\infty}$	-5.9
$6_\gamma \to 6_g$	$-4.2^{+1.8}_{-\infty}$	-4.7
$7_\gamma \to 6_g$	$-3.0^{+0.5}_{-0.7}$	-5.0
$7_\gamma \to 8_g$	$-5.7^{+1.2}_{-1.8}$	-4.4
$2_\beta \to 2_g$	$14.4^{+2.6}_{-1.9}$	14.4[b]
$4_\beta \to 4_g$	5.3 ± 2.3	7.5
$6_\beta \to 6_g$	2.9 ± 0.3	5.1
$8_\beta \to 8_g$	$2.3^{+0.8}_{-0.6}$	3.9
$10_\beta \to 10_g$	$2.4^{+1.0}_{-0.7}$	3.1

[b] Fitted.

Table _ 21b

J_i	J_f	$\lvert\langle J_f \lVert T(M1) \rVert J_i\rangle\rvert\ (\mu_N)$	
		experiment[a]	theory
2_γ	2_g		0.028
3_γ	2_g	0.070(15)	0.039
3_γ	4_g	0.049(3)	0.034
4_γ	4_g		0.079
5_γ	4_g	0.08(5)	0.081
6_γ	6_g	0.14(6)	0.14
3_γ	2_γ	0.160(5)	0.14
4_γ	3_γ		0.18
5_γ	4_γ	0.23(1)	0.22
6_γ	5_γ	0.20(4)	0.24
7_γ	6_γ	0.27(6)	0.27

FIGURES

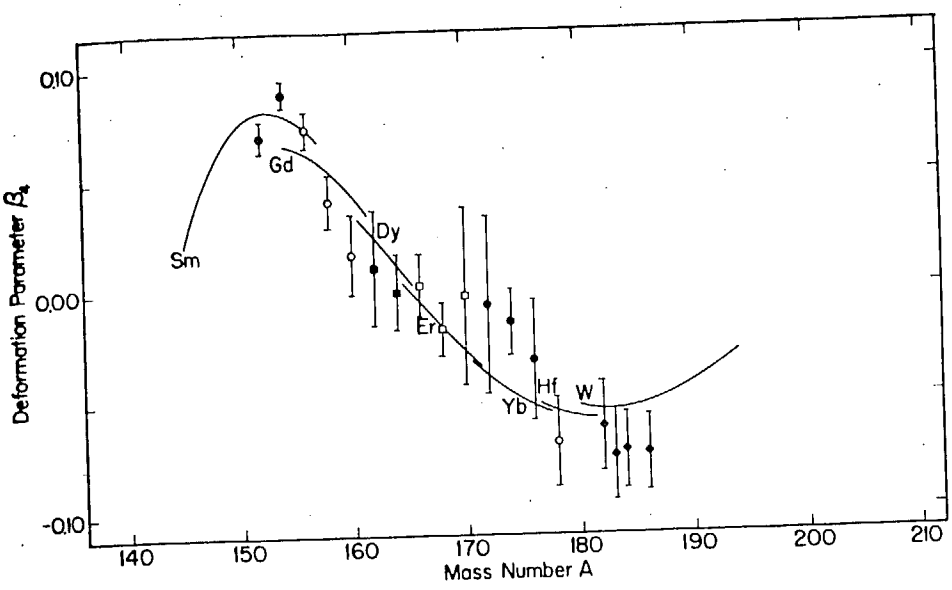

0.10
0.00
-0.10
Deformation Parameter β_4
Sm
Gd
Dy
Er
Yb
Hf
W
140
150
160
170
180
190
200
210
Mass Number A

FIG-1

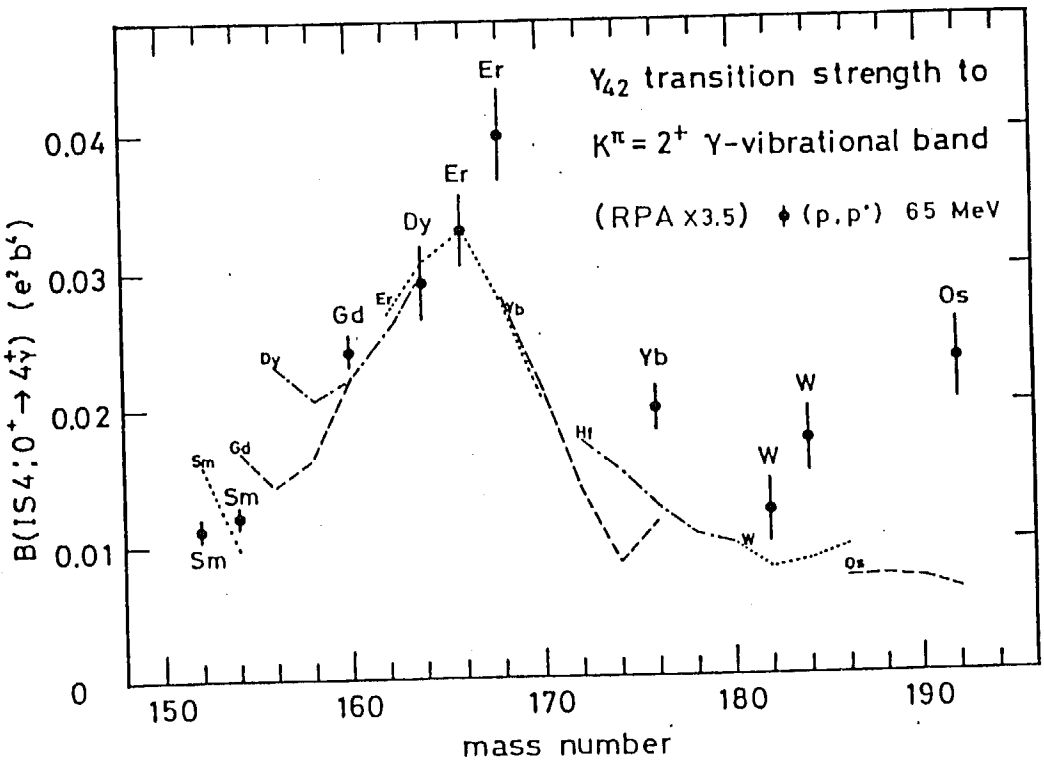

Y_{42} transition strength to
$K^\pi = 2^+$ γ-vibrational band
(RPA x3.5) (p,p') 65 MeV
$B(IS4;0^+ \rightarrow 4^+_\gamma)$ $(e^2 b^4)$
0.04
0.03
0.02
0.01
0
Sm
Gd
Dy
Er
Yb
Hf
W
Os
150
160
170
180
190
mass number

FIG-2

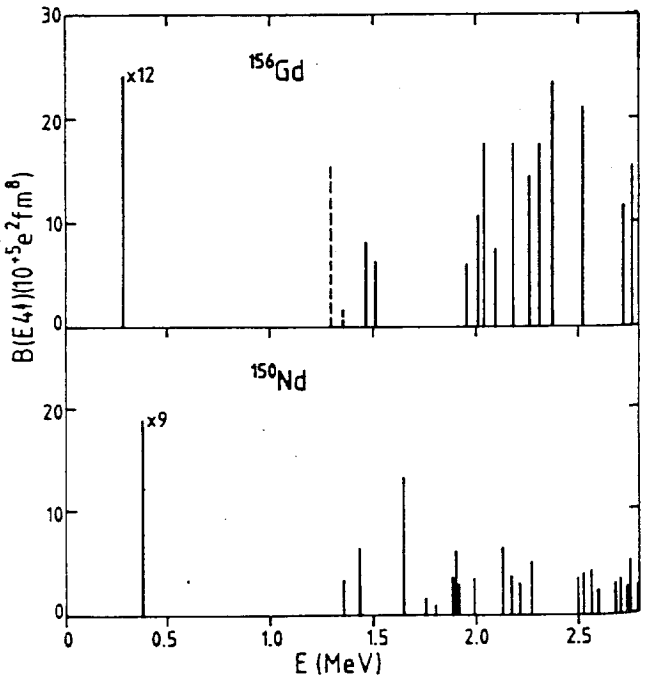

^{156}Gd
x12
^{150}Nd
x9
B(E4)(10^{+5}e^{2}fm^8)
E (MeV)

FiG-3

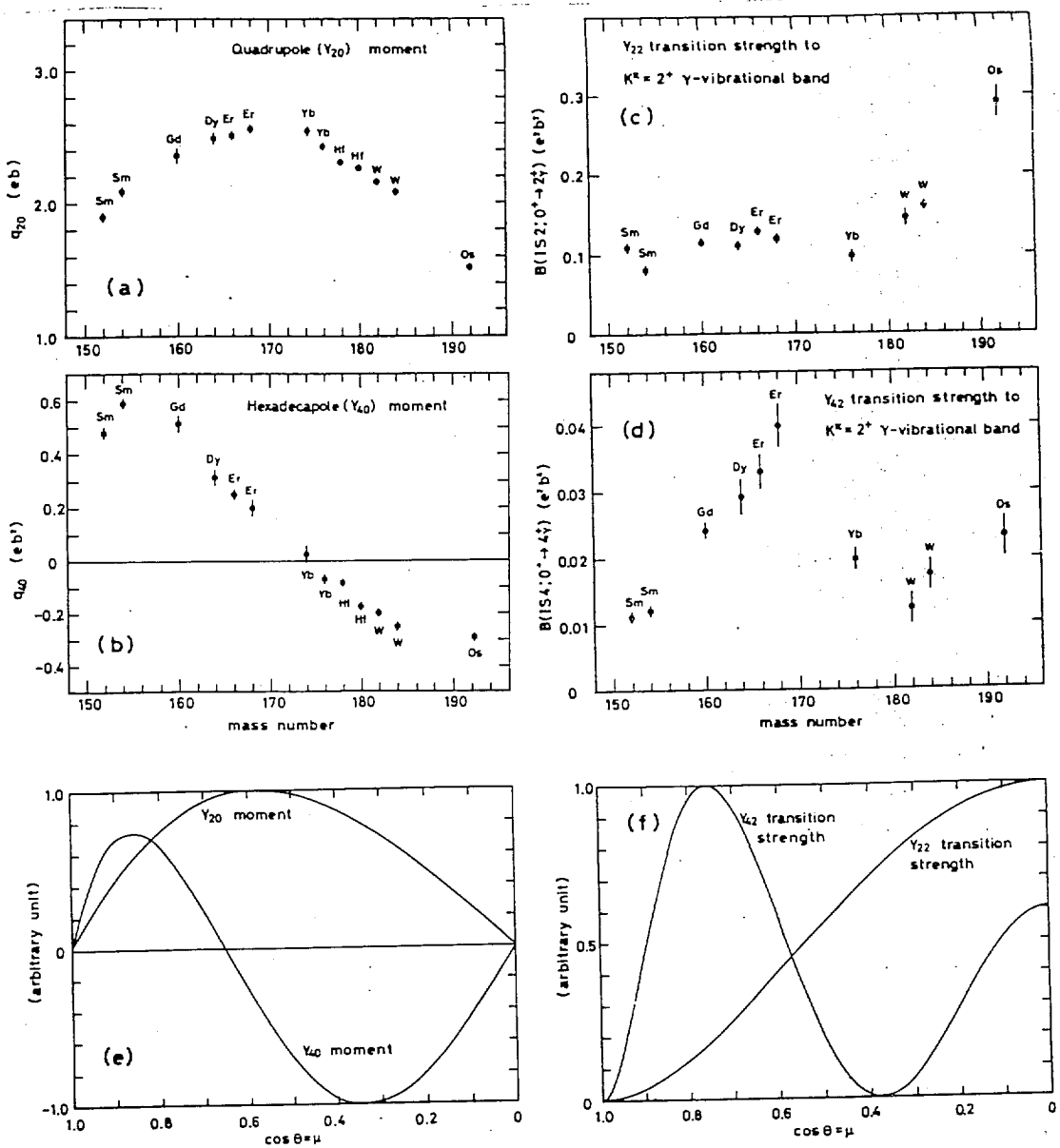

Quadrupole (Y_{20}) moment
q_{20} (eb)
(a)
Y_{22} transition strength to
$K^\pi = 2^+$ γ-vibrational band
(c)
B(IS2;$0^+ \rightarrow 2^+_\gamma$) (e^{2}b^2)
Hexadecapole (Y_{40}) moment
q_{40} (eb^2)
(b)
mass number
Y_{42} transition strength to
$K^\pi = 2^+$ γ-vibrational band
(d)
B(IS4;$0^+ \rightarrow 4^+_\gamma$) (e^{2}b^4)
mass number
Y_{20} moment
Y_{40} moment
(e)
(arbitrary unit)
cos θ = μ
Y_{42} transition strength
Y_{22} transition strength
(f)
(arbitrary unit)
cos θ = μ

FiG-4

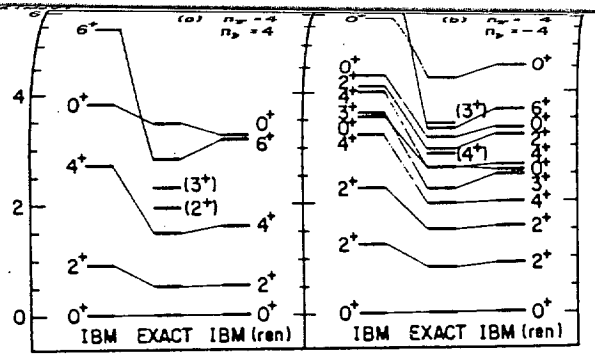


Fig-5a

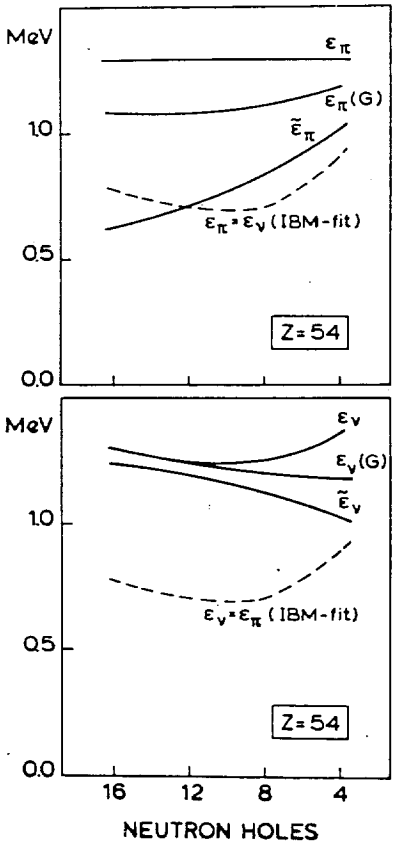


Fig-5b

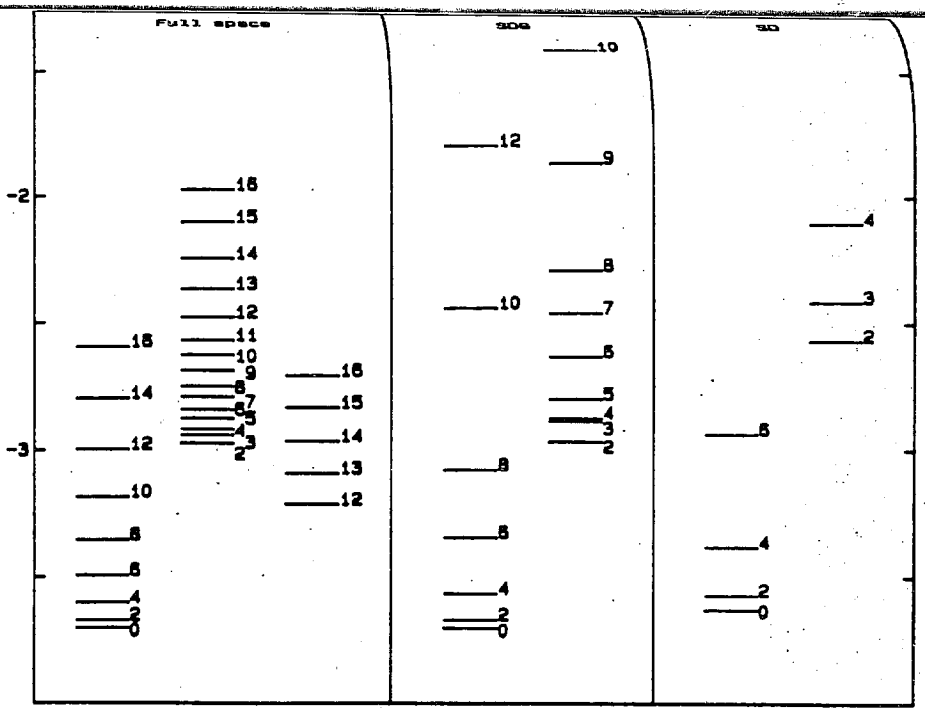


Fig-5c

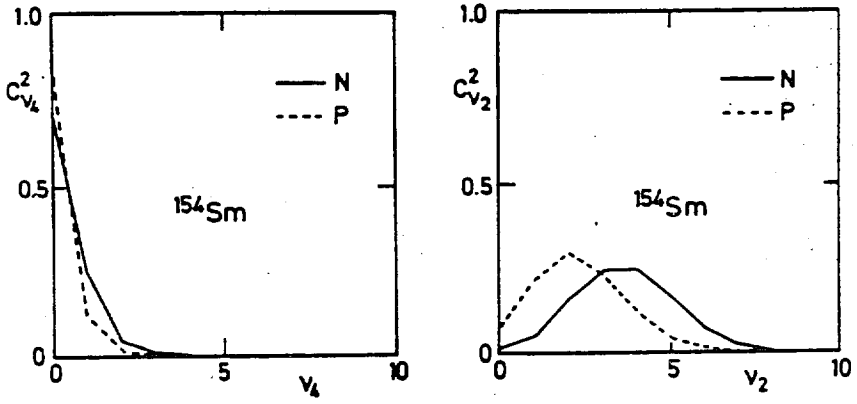


Fig-5d

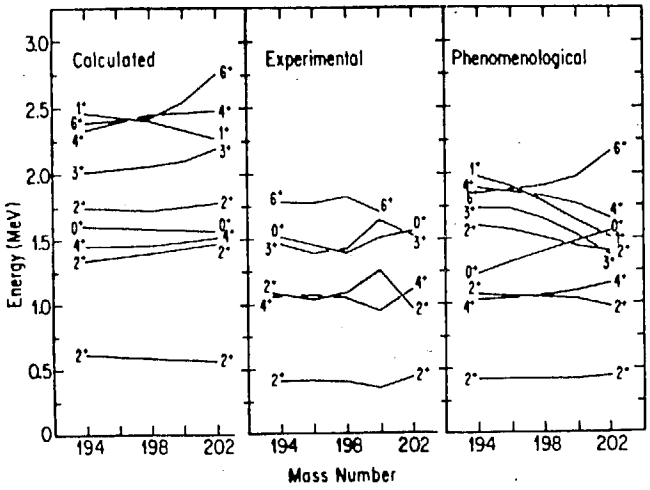


Fig-6a

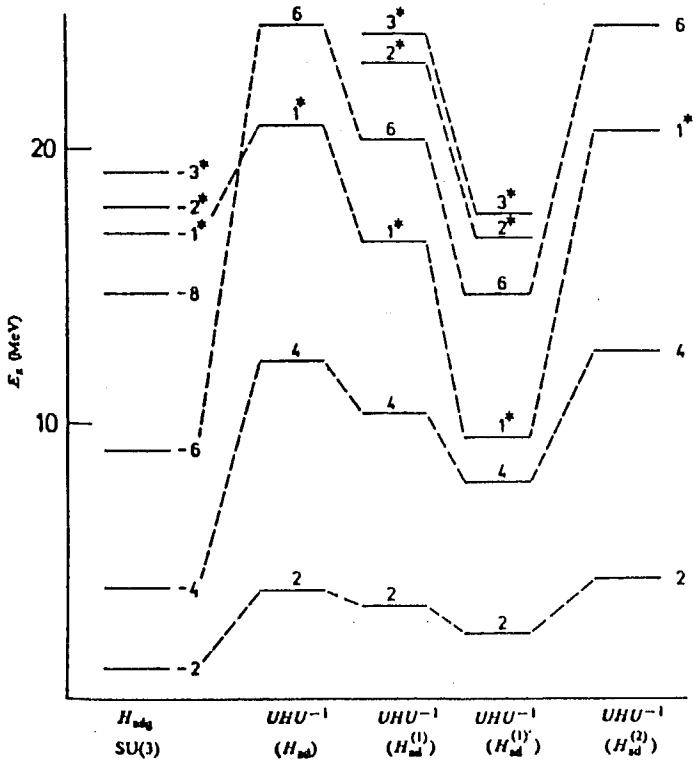


Fig-6b

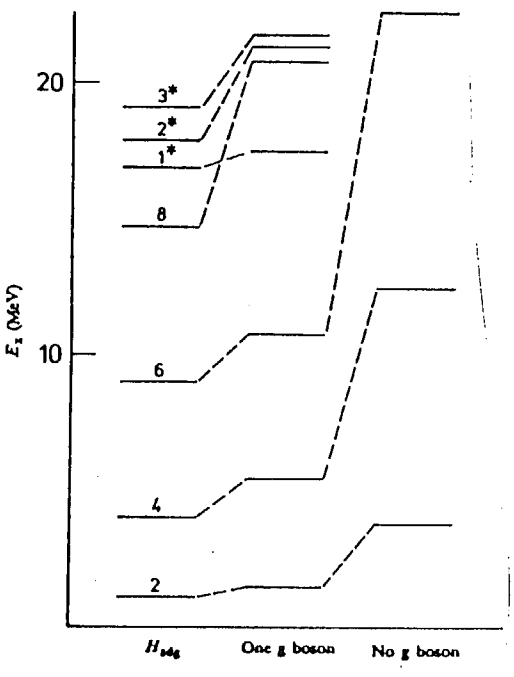


Fig-6c

Fig-7a

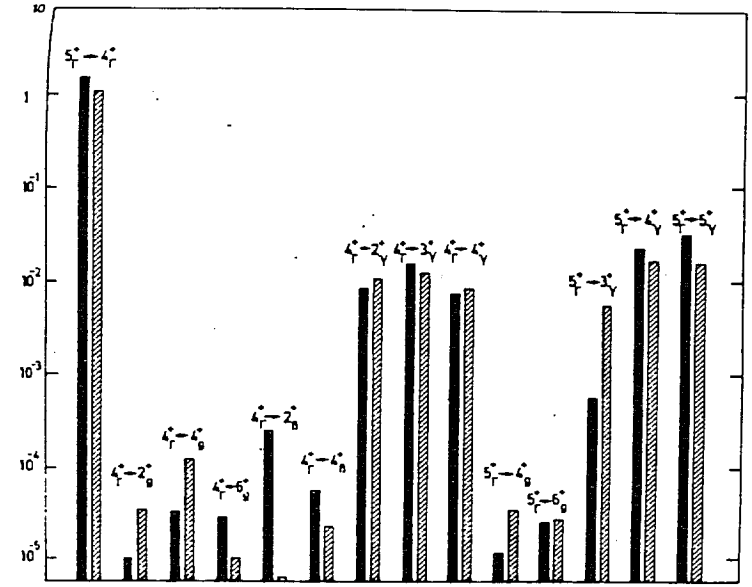
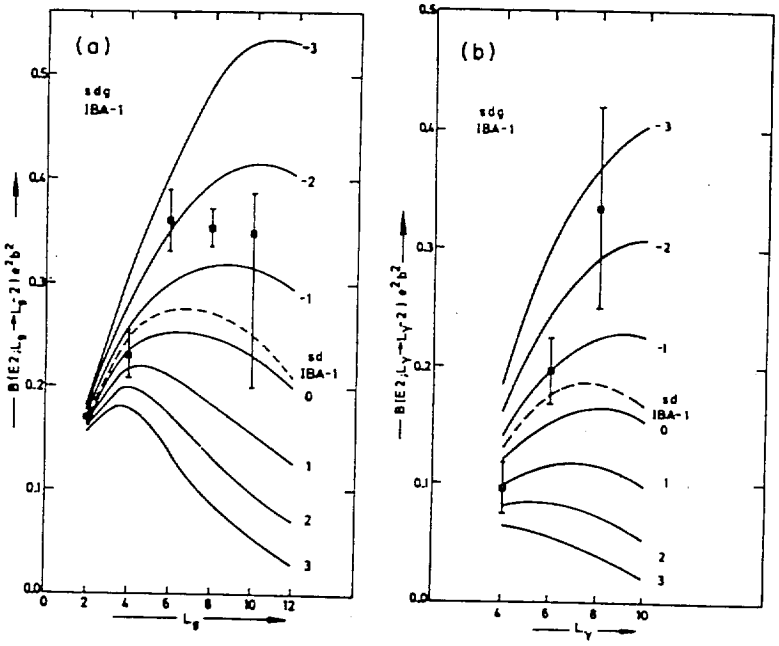


Fig-7b



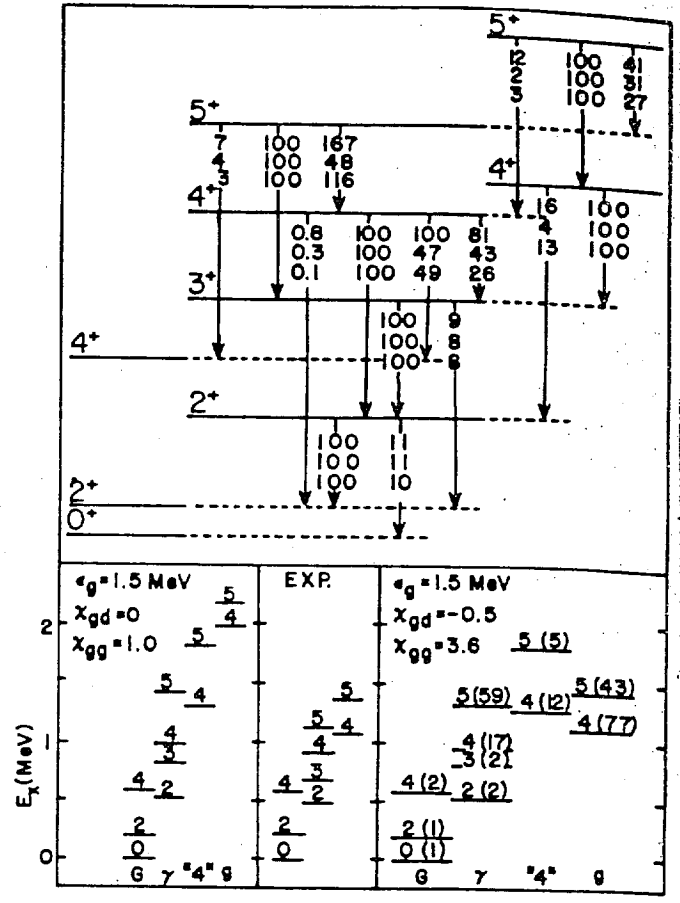
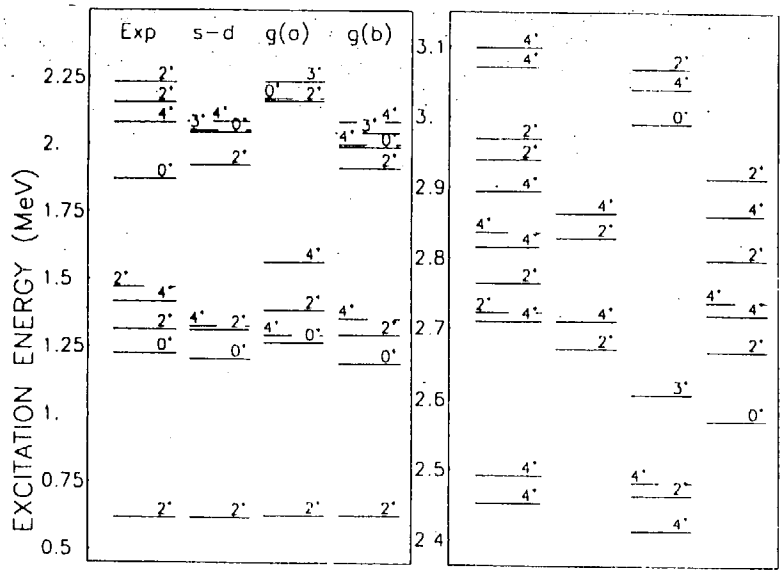
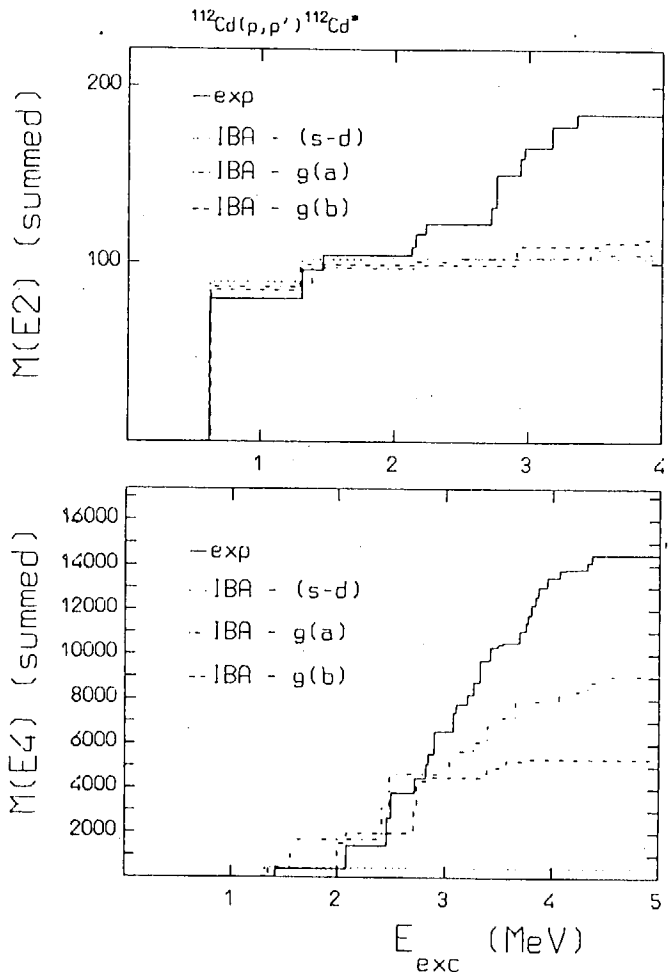


Fig-7c


Exp
s-d
g(a)
g(b)
EXCITATION ENERGY (MeV)
2.25
2.
1.75
1.5
1.25
1.
0.75
0.5
3.1
3.
2.9
2.8
2.7
2.6
2.5
2.4

(i)


$^{112}Cd(p,p')^{112}Cd^*$
M(E2) (summed)
200
100
—exp
IBA - (s-d)
IBA - g(a)
IBA - g(b)
1
2
3
4
M(E4) (summed)
16000
14000
12000
10000
8000
6000
4000
2000
5
E_{exc} (MeV)

(ii)

Fig 7 d

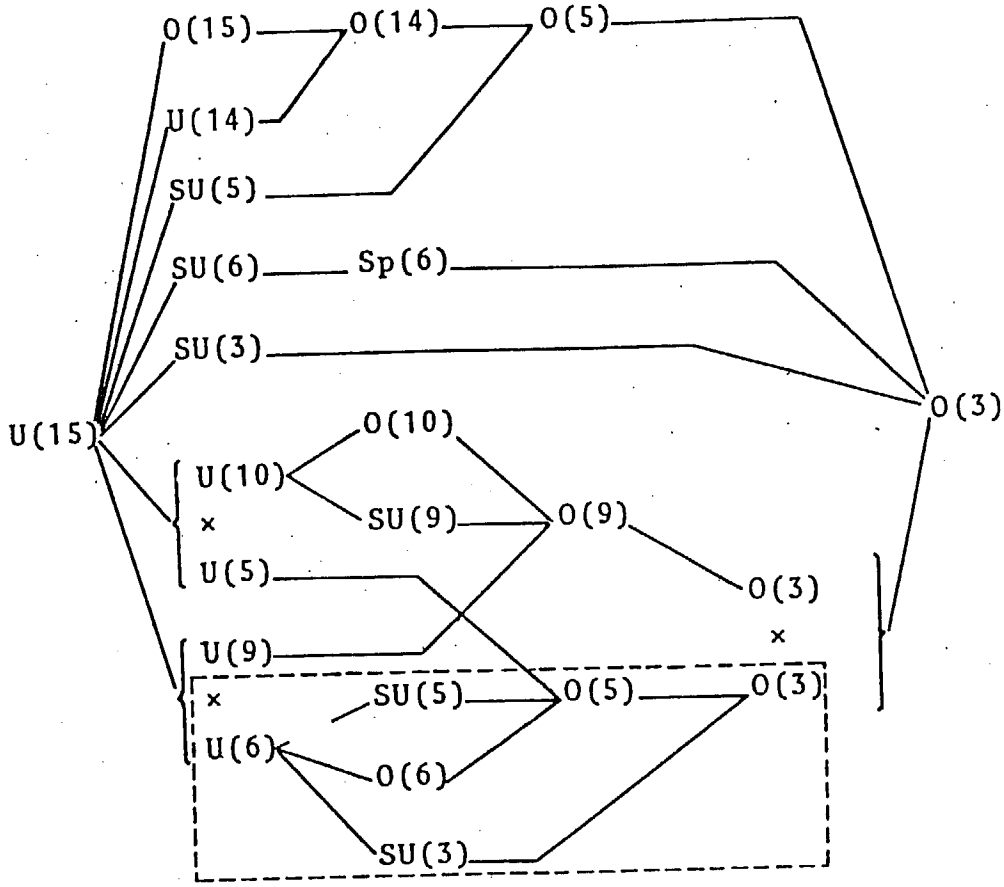

O(15)
O(14)
O(5)
U(14)
SU(5)
SU(6)
Sp(6)
SU(3)
O(3)
U(15)
O(10)
U(10)
×
SU(9)
O(9)
U(5)
O(3)
×
U(9)
×
SU(5)
O(5)
O(3)
U(6)
O(6)
SU(3)

Fig - 8

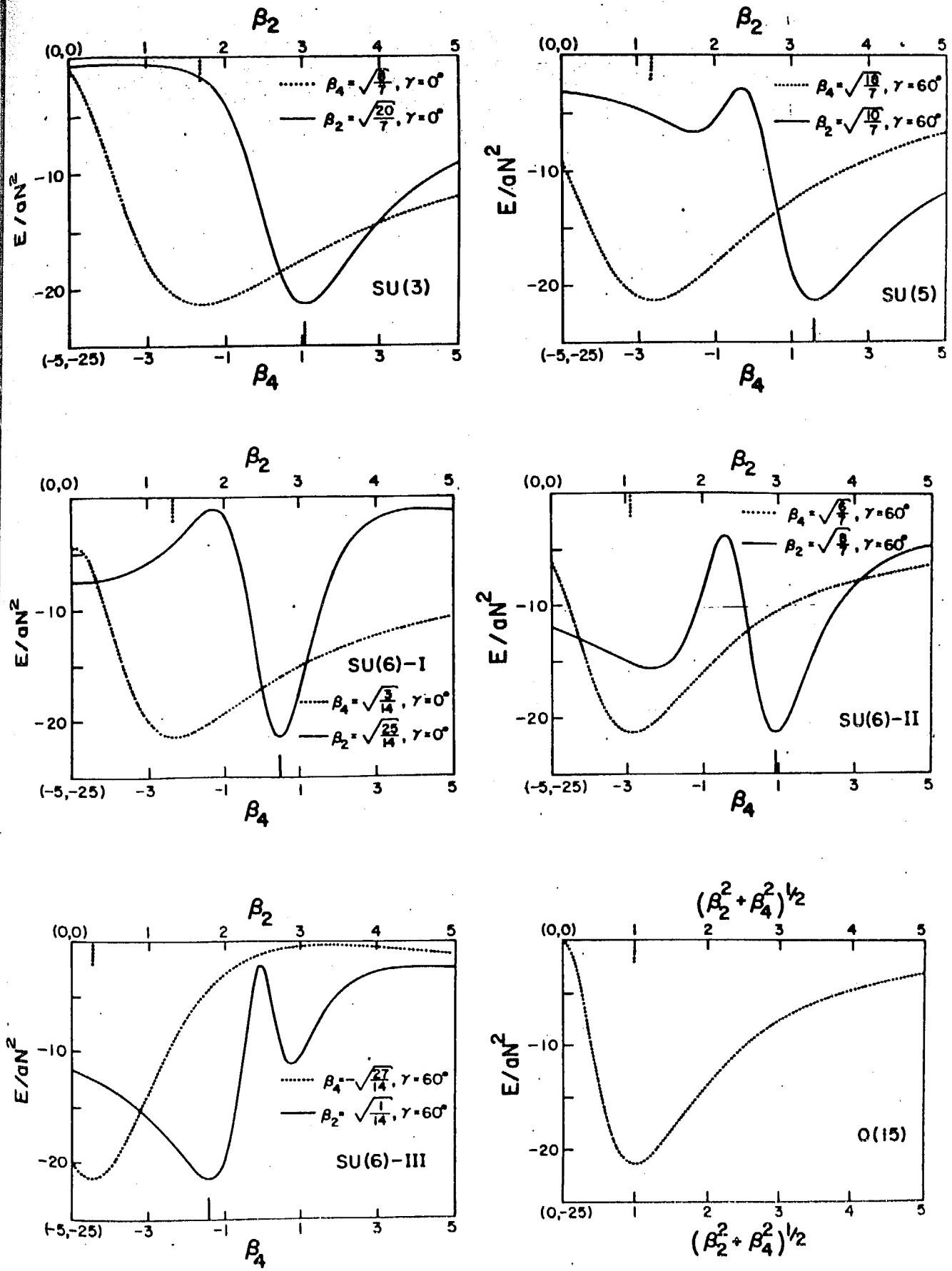
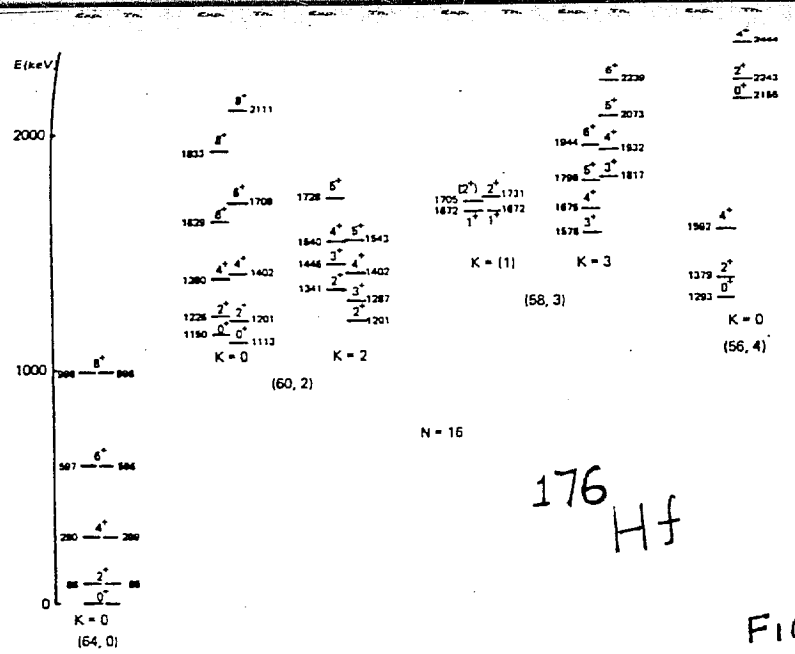


Fig - 9



176 Hf

FIG - 10a

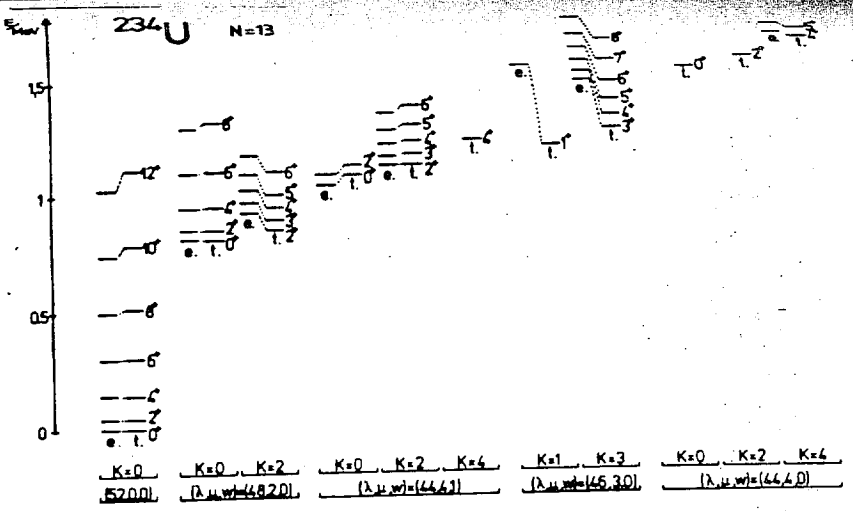
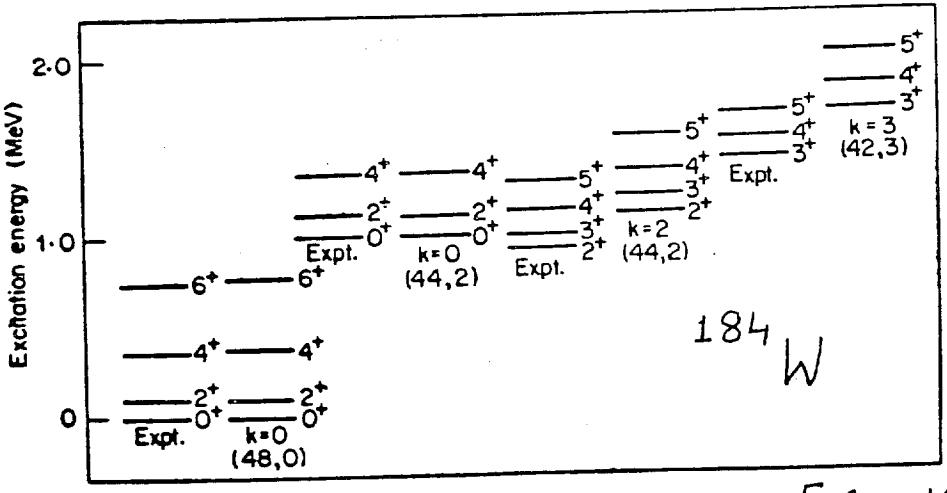


FIG -



184 W

FIG - 10b

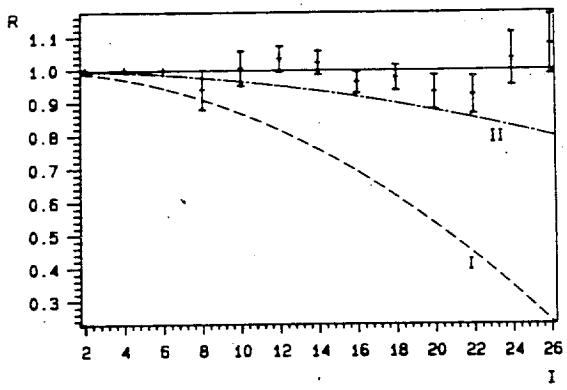


FIG - 10c

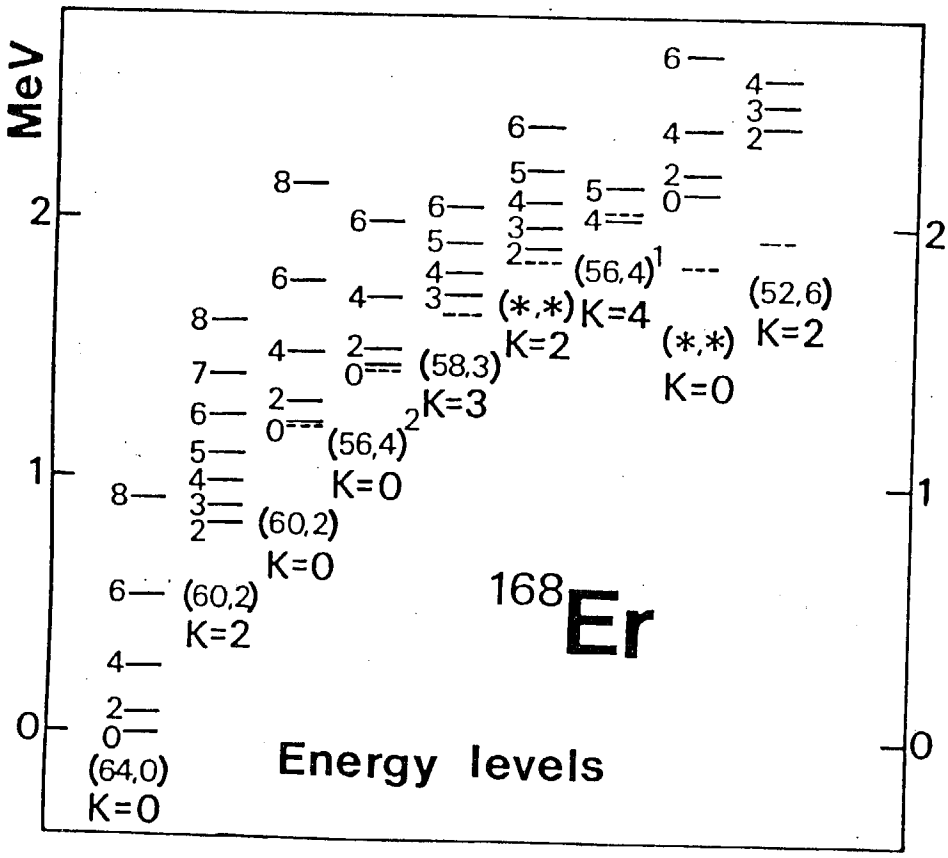

MeV
2
1
0
^{168}Er
Energy levels
(64,0)
K=0
(60,2)
K=0
(60,2)
K=2
(56,4)
K=0
(58,3)
K=3
(*,*)
K=2
(56,4)
K=4
(*,*)
K=0
(52,6)
K=2

FIG - 11

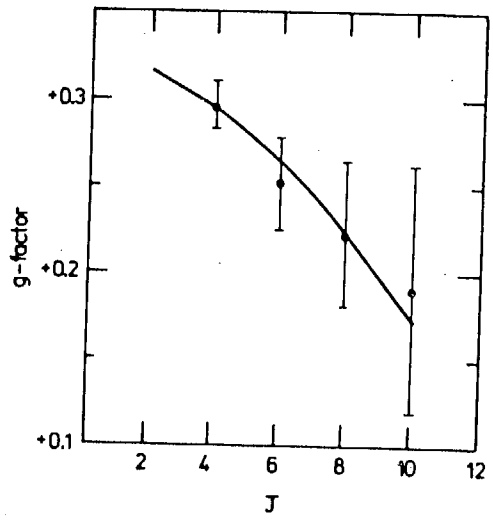

g-factor
+0.3
+0.2
+0.1
2
4
6
8
10
12
J

FIG - 12

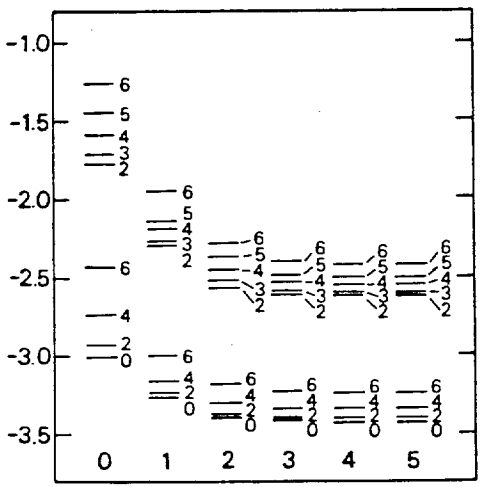


FIG-13a

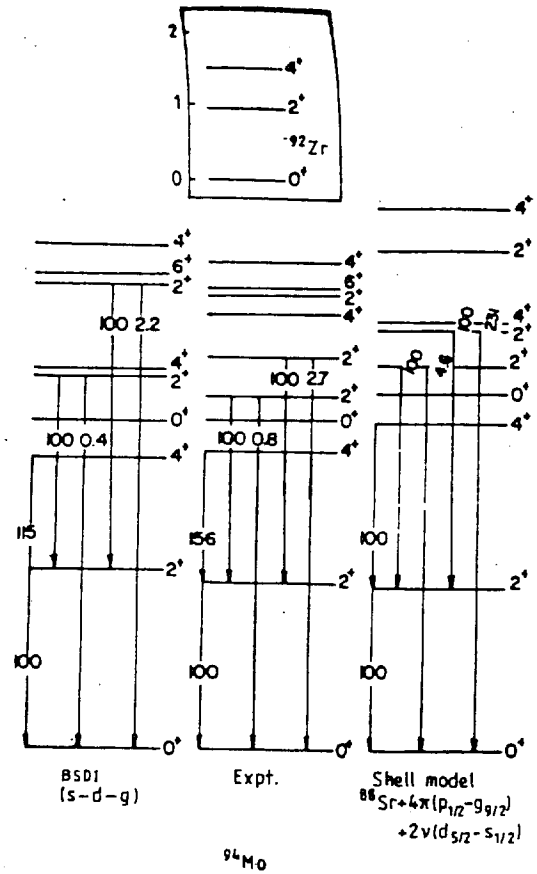


FIG-13b

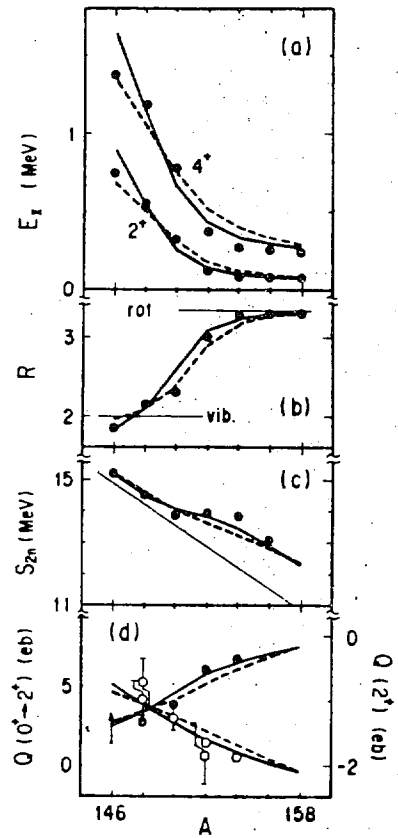


FIG-14

TEST INPUTS

The data is given in four columns and they are top left, top right, bottom left and bottom right respectively.

BTBME

Test Input - 1

on Unit #5 :

7

1.0 0.0 0

f(U(5)) INTERACTION MATRIX ELEMENTS

5 0.0 0.0 0

AT MAPPED INTERACTION FOR Q(2).Q(2) OPERATOR

1 4 1

.0 0.0 1.0 0.0 0.0 0.0 0.0 0.0 0.0

on Unit #50 :

0	0	0	0	0
0	0	2	2	0
0	0	4	4	0
0	2	0	2	2
0	2	2	2	2
0	2	2	4	2
0	2	4	4	2
0	4	0	4	4
0	4	2	2	4
0	4	2	4	4
0	4	4	4	4
2	2	2	2	0
2	2	2	2	2
2	2	2	2	4
2	2	2	4	2
2	2	2	4	4
2	2	4	4	0
2	2	4	4	2
2	2	4	4	4
2	4	2	4	2
2	4	2	4	3
2	4	2	4	4
2	4	2	4	5

2	4	2	4	6
2	4	4	4	2
2	4	4	4	4
2	4	4	4	6
4	4	4	4	0
4	4	4	4	2
4	4	4	4	4
4	4	4	4	6
4	4	4	4	8

CFP

Test Input - 2

Input on Unit #5 :

2 12 45

12

Test Input - 3

Input on Unit #5 :

4 6 45

6

MAINHH

Test Input - 4

Input on Unit #5 :

35 3 5 1 0 0 5 9999 0

3 0 9999

3 1 9999

3 2 9999

3 3 9999

3 4 9999

3 5 9999

3 6 9999

3 7 9999

3 8 9999

3 9 9999

3 10 9999

3 11 9999

3 12 9999

99 999 999

on Unit #38 :

U(3)) INTERACTION

0000 -23.50000 -13.00000

0	0	0	0	0	0.00000000E+00
0	0	2	2	0	-0.25043962E+02
0	0	4	4	0	0.00000000E+00
0	2	0	2	2	-0.11200000E+02
0	2	2	2	2	0.19677398E+02
0	2	2	4	2	-0.14400000E+02
0	2	4	4	2	0.00000000E+00
0	4	0	4	4	0.00000000E+00
0	4	2	2	4	-0.15178933E+02
0	4	2	4	4	0.13266499E+02
0	4	4	4	4	0.00000000E+00
2	2	2	2	0	-0.17285714E+02
2	2	2	2	2	0.37040817E+01
2	2	2	2	4	-0.49387755E+01
2	2	2	4	2	0.72284321E+01
2	2	2	4	4	0.14126813E+02
2	2	4	4	0	-0.13799734E+02
2	2	4	4	2	-0.10338059E+02
2	2	4	4	4	-0.39290671E+01
2	4	2	4	2	-0.12493878E+02
2	4	2	4	3	-0.50000001E+00
2	4	2	4	4	0.50918369E+01
2	4	2	4	5	0.13000000E+02
2	4	2	4	6	-0.16571429E+02
2	4	4	4	2	0.38527672E+01
2	4	4	4	4	0.11238655E+02
2	4	4	4	6	0.11499778E+02
4	4	4	4	0	-0.15714286E+02
4	4	4	4	2	-0.90102041E+01
4	4	4	4	4	0.23469388E+01
4	4	4	4	6	0.80714288E+01
4	4	4	4	8	-0.80000001E+01

Input on Unit # 30

FOR30D.DAT

The CFP program is to be run with the test input - 2

and the output file FOR30.DAT obtained, is to be renamed

as FOR30D.DAT.

Input on Unit # 32

FOR30G.DAT

The CFP program is to be run with the test input - 3

and the output file FOR30.DAT obtained, is to be renamed

as FOR30G.DAT.

Test Input - 5

Input on Unit # 5 :

35 4 3 0 1 0 7 9999 0

4 0 3

4 1 3

4 2 3

4 3 3

```
     4     4     3

     4     5     3

     4     6     3

     999   999   999
```

Input on Unit # 38 :

(Same as in Test Input -4)

Input on Unit # 30 :

(Same as in Test Input -4)

Input on Unit # 32 :

(Same as in Test Input -4)

Test Input - 6

Input on Unit # 5 :

```
     35    4     2     0     1     0     7     9999     0

     4     0     3

     4     1     3

     4     2     3

     4     3     3

     4     4     3

     4     5     3

     4     6     3

     999   999   999
```

Input on Unit # 38 :

-C2(U(5)) + 0.5*C2(O(5)) INTERACTION MATRIX ELEMENTS.

```
     35    4     2     0     1     0     7     9999     0

     4     0     3

     4     1     3

     4     2     3

     4     3     3

     4     4     3

     4     5     3

     4     6     3

     999   999   999
```

Input on Unit # 38 :

-C2(U(6)) + 0.5*C2(Sp(6)) INTERACTION

-10.00000	-4.00000	-4.00000			
0	0	0	0	0	-0.13333334E+01
0	0	2	2	0	-0.29814241E+01
0	0	4	4	0	-0.40000001E+01
0	2	0	2	2	-0.26666667E+01
0	2	2	2	2	0.13468701E+01
0	2	2	4	2	-0.29692300E+01
0	2	4	4	2	-0.18952142E+01
0	4	0	4	4	-0.26666667E+01
0	4	2	2	4	-0.15649216E+01
0	4	2	4	4	-0.19977312E+01
0	4	4	4	4	0.27896797E+01
2	2	2	2	0	-0.36666667E+01
2	2	2	2	2	-0.16598640E+01
2	2	2	2	4	-0.15408164E+01
2	2	2	4	2	-0.74984381E+00
2	2	2	4	4	0.58617985E+00
2	2	4	4	0	-0.22360680E+01
2	2	4	4	2	-0.47861384E+00
2	2	4	4	4	-0.81855567E+00
2	4	2	4	2	-0.34693877E+00
2	4	2	4	3	-0.20000001E+01
2	4	2	4	4	-0.12517007E+01
2	4	2	4	5	-0.20000001E+01
2	4	2	4	6	-0.20000000E+01
2	4	4	4	2	0.10551237E+01
2	4	4	4	4	-0.10449432E+01
2	4	4	4	6	-0.14901161E-07

-12.00000 -7.00000 -7.00000

0	0	0	0	0	-0.16000000E+01
0	0	2	2	0	-0.35777088E+01
0	0	4	4	0	-0.48000000E+01
0	2	0	2	2	-0.32000000E+01
0	2	2	2	2	0.10842095E+01
0	2	2	4	2	-0.27428571E+01
0	2	4	4	2	-0.50853932E+01
0	4	0	4	4	-0.32000000E+01
0	4	2	2	4	-0.14456126E+01
0	4	2	4	4	-0.53604751E+01
0	4	4	4	4	-0.19327468E+01
2	2	2	2	0	-0.31428572E+01
2	2	2	2	2	-0.18571429E+01
2	2	2	4	2	-0.17142857E+01
2	2	2	4	4	-0.21684189E+01
2	2	4	4	0	0.00000000E+00
2	2	4	4	2	-0.68998669E+01
2	2	4	4	4	0.20101782E+01
2	4	2	4	2	-0.30559410E+01
2	4	2	4	3	-0.19428571E+01
2	4	2	4	4	0.20000000E+01
2	4	2	4	5	-0.54285714E+01
2	4	2	4	6	-0.40000001E+01
2	4	4	4	2	-0.22857143E+01
2	4	4	4	4	-0.25426966E+01
2	4	4	4	6	-0.10301575E+01
4	4	4	4	0	-0.27105237E+01
4	4	4	4	2	-0.72571430E+01
4	4	4	4	4	-0.40000000E+01
4	4	4	4	6	-0.65714285E+00
4	4	4	4	8	0.28571427E+00

Input on Unit # 30 :

(Same as in Test Input -4)

Input on Unit # 32 :

(Same as in Test Input -4)

Test Input - 7

Input on Unit # 5 :

4	4	4	4	0	-0.50000000E+01
4	4	4	4	2	-0.13265307E+01
4	4	4	4	4	-0.54081634E+00
4	4	4	4	6	-0.20000000E+01
4	4	4	4	8	-0.20000001E+01

Input on Unit # 30 :

(Same as in Test Input -4)

Input on Unit # 32 :

(Same as in Test Input -4)

MAE2E4

Test Input - 8

Input on Unit # 5 or 50 :

```
     70    1     3     2
```

B(E2)'S AND Q2'S FOR THE SU3 QUADRUPOLE OPERATOR.

Input on Unit # 50 :

```
     6     1     6     1
     6     1     6     2
     6     1     6     3
     6     2     6     1
     6     2     6     2
     6     2     6     3
     6     3     6     1
     6     3     6     2
     6     3     6     3
     6     1     5     1
     6     1     5     2
     6     1     5     3
     6     2     5     1
     6     2     5     2
     6     2     5     3
     6     3     5     1
     6     3     5     2
     6     3     5     3
     6     1     4     1
```

```
6    1    4    2
6    1    4    3
6    2    4    1
6    2    4    2
6    2    4    3
6    3    4    1
6    3    4    2
6    3    4    3
5    1    5    1
5    1    5    2
5    1    5    3
5    2    5    1
5    2    5    2
5    2    5    3
5    3    5    1
5    3    5    2
5    3    5    3
5    1    4    1
5    1    4    2
5    1    4    3
5    2    4    1
5    2    4    2
5    2    4    3
5    3    4    1
5    3    4    2
5    3    4    3
5    1    3    1
5    1    3    2
5    1    3    3
5    2    3    1
5    2    3    2
5    2    3    3
5    3    3    1
5    3    3    2
5    3    3    3
4    1    4    1
4    1    4    2
4    1    4    3
4    2    4    1
4    2    4    2
4    2    4    3
4    3    4    1
4    3    4    2
4    3    4    3
4    1    3    1
4    1    3    2
4    1    3    3
4    2    3    1
4    2    3    2
```

```
4    2    3    3
4    3    3    1
4    3    3    2
4    3    3    3
4    1    2    1
4    1    2    2
4    1    2    3
4    2    2    1
4    2    2    2
4    2    2    3
4    3    2    1
4    3    2    2
4    3    2    3
3    1    3    1
3    1    3    2
3    1    3    3
3    2    3    1
3    2    3    2
3    2    3    3
3    3    3    1
3    3    3    2
3    3    3    3
3    1    2    1
3    1    2    2
3    1    2    3
3    2    2    1
3    2    2    2
3    2    2    3
3    3    2    1
3    3    2    2
3    3    2    3
3    1    1    1
3    1    1    2
3    1    1    3
3    2    1    1
3    2    1    2
3    2    1    3
3    3    1    1
3    3    1    2
3    3    1    3
2    1    2    1
2    1    2    2
2    1    2    3
2    2    2    1
2    2    2    2
2    2    2    3
2    3    2    1
2    3    2    2
2    3    2    3
```

```
2    1    1    1
2    1    1    2
2    1    1    3
2    2    1    1
2    2    1    2
2    2    1    3
2    3    1    1
2    3    1    2
2    3    1    3
2    1    0    1
2    1    0    2
2    1    0    3
2    2    0    1
2    2    0    2
2    2    0    3
2    3    0    1
2    3    0    2
2    3    0    3
1    1    1    1
1    1    1    2
1    1    1    3
1    2    1    1
1    2    1    2
1    2    1    3
1    3    1    1
1    3    1    2
1    3    1    3
99   99   99   99
```

Out on Unit # 30 :

(Same as in Test Input -4)

Out on Unit # 32 :

(Same as in Test Input -4)

Out on Unit # 40 :

The program MAINHH is to be run with test input-5 and the output file EIGVEC.DAT obtained, is to be renamed as EIGVC1.DAT.

Out on Unit # 41 :

Not necessary.

Test Input - 9

Input on Unit # 5 or 50 :

```
70    1    5    2
```

B(E2)'S AND Q2'S FOR THE SU5 QUADRUPOLE OPERATOR.

Input on Unit # 50 :

(Same as in Test Input - 8)

Input on Unit # 30 :

(Same as in Test Input - 4)

Input on Unit # 32 :

(Same as in Test Input - 4)

Input on Unit # 40 :

The program MAINHH is to be run with test input-6 and the output file EIGVEC.DAT obtained, is to be renamed as EIGVC1.DAT.

Input on Unit # 41 :

Not necessary.

Test Input - 10

Input on Unit # 5 or 50 :

```
70    1    6    2
```

B(E2)'S AND Q2'S FOR THE SU6 QUADRUPOLE OPERATOR.

Input on Unit # 50 :

(Same as in Test Input - 8)

Input on Unit # 30 :

(Same as in Test Input - 4)

Input on Unit # 32 :

(Same as in Test Input - 4)

Input on Unit # 40 :

The program MAINHH is to be run with test input-7 and the output file EIGVEC.DAT obtained, is to be renamed as EIGVC1.DAT.

Input on Unit # 41 :

Not necessary.

Test Input - 11

Input on Unit # 5 or 50 :

```
70    1    5    4
```

B(E4)'S AND Q4'S FOR THE SU5 Q(4) OPERATOR.

Input on Unit # 50 :

```
6    1    6    1
6    1    6    2
6    1    6    3
6    2    6    1
6    2    6    2
6    2    6    3
6    3    6    1
6    3    6    2
6    3    6    3
6    1    5    1
6    1    5    2
6    1    5    3
6    2    5    1
6    2    5    2
6    2    5    3
6    3    5    1
6    3    5    2
6    3    5    3
6    1    4    1
6    1    4    2
6    1    4    3
6    2    4    1
6    2    4    2
6    2    4    3
6    3    4    1
6    3    4    2
6    3    4    3
6    1    3    1
6    1    3    2
6    1    3    3
6    2    3    1
6    2    3    2
6    2    3    3
6    3    3    1
6    3    3    2
6    3    3    3
6    1    2    1
6    1    2    2
6    1    2    3
6    2    2    1
6    2    2    2
6    2    2    3
6    3    2    1
6    3    2    2
6    3    2    3
5    1    5    1
5    1    5    2
5    1    5    3
5    2    5    1
5    2    5    2
5    2    5    3
5    3    5    1
5    3    5    2
5    3    5    3
5    1    4    1
5    1    4    2
5    1    4    3
5    2    4    1
5    2    4    2
5    2    4    3
5    3    4    1
5    3    4    2
5    3    4    3
5    1    3    1
5    1    3    2
5    1    3    3
5    2    3    1
5    2    3    2
5    2    3    3
5    3    3    1
5    3    3    2
5    3    3    3
5    1    2    1
5    1    2    2
5    1    2    3
5    2    2    1
5    2    2    2
5    2    2    3
5    3    2    1
5    3    2    2
5    3    2    3
5    1    1    1
5    1    1    2
5    1    1    3
5    2    1    1
5    2    1    2
5    2    1    3
5    3    1    1
5    3    1    2
5    3    1    3
4    1    4    1
4    1    4    2
4    1    4    3
4    2    4    1
4    2    4    2
4    2    4    3
4    3    4    1
4    3    4    2
4    3    4    3
4    1    3    1
4    1    3    2
4    1    3    3
4    2    3    1
4    2    3    2
4    2    3    3
4    3    3    1
4    3    3    2
4    3    3    3
4    1    2    1
4    1    2    2
4    1    2    3
4    2    2    1
4    2    2    2
4    2    2    3
4    3    2    1
4    3    2    2
4    3    2    3
4    1    1    1
4    1    1    2
4    1    1    3
4    2    1    1
4    2    1    2
4    2    1    3
4    3    1    1
4    3    1    2
4    3    1    3
4    1    0    1
4    1    0    2
4    1    0    3
4    2    0    1
4    2    0    2
4    2    0    3
4    3    0    1
4    3    0    2
4    3    0    3
3    1    3    1
3    1    3    2
3    1    3    3
3    2    3    1
3    2    3    2
3    2    3    3
3    3    3    1
3    3    3    2
3    3    3    3
3    1    2    1
3    1    2    2
3    1    2    3
3    2    2    1
3    2    2    2
3    2    2    3
3    3    2    1
3    3    2    2
3    3    2    3
3    1    1    1
3    1    1    2
3    1    1    3
3    2    1    1
3    2    1    2
3    2    1    3
```

```
3     3     1     1
3     3     1     2
3     3     1     3
2     1     2     1
2     1     2     2
2     1     2     3
2     2     2     1
2     2     2     2
2     2     2     3
2     3     2     1
2     3     2     2
2     3     2     3
99    99    99    99
```

put on Unit # 30 :

(Same as in Test Input - 4)

put on Unit # 32 :

(Same as in Test Input - 4)

put on Unit # 40 :

(Same as in Test Input - 9)

put on Unit # 41 :

Not necessary.

Test Input - 12

put on Unit # 5 or 50 :

```
70     1     6     4
```

B(E4)'S AND Q4'S FOR THE SU6 Q(4) OPERATOR.

put on Unit # 50 :

(Same as in Test Input - 11)

put on Unit # 30 :

(Same as in Test Input - 4)

```
5     1     5     3
5     2     5     1
5     2     5     2
5     2     5     3
5     3     5     1
5     3     5     2
5     3     5     3
5     1     4     1
5     1     4     2
5     1     4     3
5     2     4     1
5     2     4     2
5     2     4     3
5     3     4     1
5     3     4     2
5     3     4     3
4     1     4     1
4     1     4     2
4     1     4     3
4     2     4     1
4     2     4     2
4     2     4     3
4     3     4     1
4     3     4     2
4     3     4     3
4     1     3     1
4     1     3     2
4     1     3     3
4     2     3     1
4     2     3     2
4     2     3     3
4     3     3     1
4     3     3     2
4     3     3     3
3     1     3     1
3     1     3     2
3     1     3     3
3     2     3     1
3     2     3     2
3     2     3     3
3     3     3     1
3     3     3     2
3     3     3     3
3     1     2     1
3     1     2     2
3     1     2     3
3     2     2     1
3     2     2     2
3     2     2     3
```

Input on Unit # 32 :

(Same as in Test Input - 4)

Input on Unit # 40 :

(Same as in Test Input - 10)

Input on Unit # 41 :

Not necessary.

Test Input - 13

Input on Unit # 5 or 50 :

```
70     1     99     1
```

B(M1)'s and M1's with the SU3 Hamiltonian

```
0E0    1E0   0E0    0E0   -0.2449490E01
```

Input on Unit # 50 :

```
6     1     6     1
6     1     6     2
6     1     6     3
6     2     6     1
6     2     6     2
6     2     6     3
6     3     6     1
6     3     6     2
6     3     6     3
6     1     5     1
6     1     5     2
6     1     5     3
6     2     5     1
6     2     5     2
6     2     5     3
6     3     5     1
6     3     5     2
6     3     5     3
5     1     5     1
5     1     5     2
```

```
3     3     2     1
3     3     2     2
3     3     2     3
2     1     2     1
2     1     2     2
2     1     2     3
2     2     2     1
2     2     2     2
2     2     2     3
2     3     2     1
2     3     2     2
2     3     2     3
2     1     1     1
2     1     1     2
2     1     1     3
2     2     1     1
2     2     1     2
2     2     1     3
2     3     1     1
2     3     1     2
2     3     1     3
1     1     1     1
1     1     1     2
1     1     1     3
1     2     1     1
1     2     1     2
1     2     1     3
1     3     1     1
1     3     1     2
1     3     1     3
1     1     0     1
1     1     0     2
1     1     0     3
1     2     0     1
1     2     0     2
1     2     0     3
1     3     0     1
1     3     0     2
1     3     0     3
99    99    99    99
```

Input on Unit # 30 :

(Same as in Test Input - 4)

Input on Unit # 32 :

(Same as in Test Input - 4)

Input on Unit # 40 :

(Same as in Test Input - 8)

Input on Unit # 41 :

Not necessary.

Test Input - 14

Input on Unit # 5 or 50 :

70 1 3 0

<n(l)>'s FOR THE SU3 LIMIT.

Input on Unit # 50 :

0	1
0	2
0	3
0	4
0	5
1	1
1	2
2	1
2	2
2	3
2	4
2	5
3	1
3	2
3	3
3	4
3	5
4	1
4	2
4	3
4	4
4	5
5	1
5	2
5	3
5	4
5	5
6	1
6	2
6	3
6	4
6	5
7	1
7	2
7	3
7	4
8	1
8	2
8	3
8	4
8	5
9	1
9	2
10	1
10	2
12	1
99	99

Input on Unit # 30 :

(Same as in Test Input - 4)

Input on Unit # 32 :

(Same as in Test Input - 4)

Input on Unit # 40 :

The program MAINHH is to be run with test input - 4 and the ouput file EIGVEC.DAT obtained, is to be renamed EIGVC1.DAT.

Input on Unit # 41 :

Not necessary.

Test Input - 15

Input on Unit # 5 or 50 :

70 2 3 5

TWO NUCLEON TRANSFER STRENGTHS IN THE SU3 LIMIT(3-->4).

Input on Unit # 50 :

1	0	1	0
2	0	2	0
3	0	3	0
1	1	1	1
2	1	2	1
1	2	1	2
2	2	2	2
3	2	3	2
1	3	1	3
2	3	2	3
3	3	3	3
1	4	1	4
2	4	2	4
3	4	3	4
1	5	1	5
2	5	2	5
3	5	3	5
1	6	1	6
2	6	2	6
3	6	3	6
1	7	1	7
2	7	2	7
3	7	3	7
1	8	1	8
2	8	2	8
3	8	3	8
1	9	1	9
2	9	2	9
1	10	1	10
2	10	2	10
1	12	1	12
99	99	99	99

Input on Unit # 30 :

(Same as in Test Input - 4)

Input on Unit # 32 :

(Same as in Test Input - 4)

Input on Unit # 40 :

(Same as in Test Input - 14)

Input on Unit # 41 :

The program MAINHH is to be run with test input - 5 and the ouput file EIGVEC.DAT obtained, is to be renamed EIGVC2.DAT.

(NOTE:For this program the inputs on unit #s 40,41 should be created for same values of LFINL,IDEN2 ; for LFINL = 1,9,10 IDEN2 = 2 ; LFINL = 11 should not be given ; for LFINL =12 IDEN2 = 1)

MAE2E4

Test Input - 16

AINHH.FOR with the following input:

n Unit #45:

```
     0       5       0       1       4      9999      0
     0       3
     2       3
     3       3
     4       3
   999     999
```

n Unit #38:

```
INTERACTION FOR Gd(152)
0000   0.43752    0.00000
       0     0     0     0     0     0.00000000E+00
       0     0     2     2     0    -0.82734517E-02
       0     0     4     4     0     0.00000000E+00
       0     2     0     2     2     0.00000000E+00
       0     2     2     2     2     0.42505227E-01
       0     2     2     4     2     0.00000000E+00
       0     2     4     4     2     0.00000000E+00
       0     4     0     4     4     0.00000000E+00
       0     4     2     2     4     0.00000000E+00
       0     4     2     4     4     0.00000000E+00
       0     4     4     4     4     0.00000000E+00
       2     2     2     2     0    -0.12296000E+00
       2     2     2     2     2     0.27680000E-01
       2     2     2     2     4     0.24740000E-01
       2     2     2     4     2     0.00000000E+00
       2     2     2     4     4     0.00000000E+00
       2     2     4     4     0     0.00000000E+00
       2     2     4     4     2     0.00000000E+00
       2     2     4     4     4     0.00000000E+00
       2     4     2     4     2     0.00000000E+00
       2     4     2     4     3     0.00000000E+00
       2     4     2     4     4     0.00000000E+00
       2     4     2     4     5     0.00000000E+00
       2     4     2     4     6     0.00000000E+00
       2     4     4     4     2     0.00000000E+00
       2     4     4     4     4     0.00000000E+00
       2     4     4     4     6     0.00000000E+00
       4     4     4     4     0     0.00000000E+00
       4     4     4     4     2     0.00000000E+00
       4     4     4     4     4     0.00000000E+00
       4     4     4     4     6     0.00000000E+00
       4     4     4     4     8     0.00000000E+00
```

```
1      1     2     1
1      1     2     2
1      1     2     3
1      2     2     1
1      2     2     2
1      2     2     3
1      3     2     1
1      3     2     2
1      3     2     3
1      1     2     1
1      1     2     2
1      1     2     3
1      2     2     1
1      2     2     2
1      2     2     3
1      3     2     1
1      3     2     2
1      3     2     3
1      1     0     1
1      1     0     2
1      1     0     3
1      2     0     1
1      2     0     2
1      2     0     3
1      3     0     1
1      3     0     2
1      3     0     3
1     99    99    99
```

Unit #30:

(Same as in Test Input - 4)

Unit #32:

(Same as in Test Input - 4)

Unit #40:

The output file EIGVEC.DAT obtained by running MAINHH.FOR,

should be renamed as EIGVC1.DAT.

Unit #41:

Necessary

Input on Unit #30:

(Same as in Test Input - 4)

Input on Unit #32:

(Same as in Test Input - 4)

2. Run MAE2E4.FOR with the following input:

Input on Unit #50:

```
    70       1       99      2
        Gd(152) B(E2)'s
        0.1430     -0.2869      0.0     0.0     0.0
        4       1       4       1
        4       1       4       2
        4       1       4       3
        4       2       4       1
        4       2       4       2
        4       2       4       3
        4       3       4       1
        4       3       4       2
        4       3       4       3
        4       1       3       1
        4       1       3       2
        4       1       3       3
        4       2       3       1
        4       2       3       2
        4       2       3       3
        4       3       3       1
        4       3       3       2
        4       3       3       3
        4       1       2       1
        4       1       2       2
        4       1       2       3
        4       2       2       1
        4       2       2       2
        4       2       2       3
        4       3       2       1
        4       3       2       2
        4       3       2       3
        3       1       3       1
        3       1       3       2
        3       1       3       3
        3       2       3       1
        3       2       3       2
        3       2       3       3
        3       3       3       1
        3       3       3       2
        3       3       3       3
```

TRACE

Test Input - 17

Input on Unit # 5 or 39:

```
        72
        6       1       1       99      99
        6       2       1       99      99
        6       1       2       99      99
        6       2       2       99      99
        8       1       4       99      99
        8       1       3       99      99
        5       0       5       99      99
        8       3       1       1       1
        8       2       2       2       0
        8       3       3       1       1
        8       5       1       3       1
        99      0       0       0       0
```

Input on Unit #38:

(Same as in Test Input - 4)

Input on Unit #30:

(Same as in Test Input - 4)

Input on Unit #32:

(Same as in Test Input - 4)

NOTE:To obtain Test Output - 17 run the driver routine TRACE.FOR along with SUBGH.FOR, DIMVTL.FOR, VRACAH.FOR.

VARNCE

Test Input - 18

Input on Unit # 5 or 31:

```
        33
        2       2       0
        4       1       1
        4       2       2
        6       2       1
        8       4       2
        99      0       0
```

Input on Unit #38:

(Same as in Test Input - 4)

Input on Unit #30:

(Same as in Test Input - 4)

Input on Unit #32:

(Same as in Test Input - 4)

NOTE:To obtain Test Output - 18 run the driver routine VARNCE.FOR along with SUBGH.FOR, DIMVTL.FOR, VRACAH.FOR .

NDTAU

Test Input - 19

Input on Unit #5:

3

NOTE:To obtain Test Output - 19, run the driver routine NDTAU.FOR with SUBGH.FOR,DIMVTL.FOR,VRACAH.FOR.

TEST OUTPUTS

The data is given in four columns and they are top left, top right, bottom left and bottom right respectively.

Test output - 1
(Unit #38 SDGINP.DAT)

C2(U(5)) INTERACTION MATRIX ELEMENTS
12.00000 12.00000 12.00000

0	0	0	0	0	0.16000000E+01
0	0	2	2	0	0.35777088E+01
0	0	4	4	0	0.48000000E+01
0	2	0	2	2	0.32000000E+01
0	2	2	2	2	-0.10842095E+01
0	2	2	4	2	0.27428571E+01
0	2	4	4	2	0.50853932E+01
0	4	0	4	4	0.32000000E+01
0	4	2	4	4	0.14456126E+01
0	4	2	4	4	0.53604751E+01
0	4	4	4	4	0.19327468E+01
2	2	2	2	0	-0.17142857E+01
2	2	2	2	2	0.33469388E+01
2	2	2	2	4	0.27755102E+01
2	2	2	4	2	0.52661604E+01
2	2	2	4	4	-0.24216158E+01
2	2	4	4	0	0.30666073E+01
2	2	4	4	2	-0.22973465E+01
2	2	4	4	4	0.52387561E+01
2	4	2	4	2	0.15346938E+01
2	4	2	4	3	-0.40000000E+01
2	4	2	4	4	0.18775509E+01
2	4	2	4	5	0.80000000E+01
2	4	2	4	6	0.45714286E+01
2	4	4	4	2	0.72648467E+00
2	4	4	4	4	-0.11773230E+01
2	4	4	4	6	0.54210474E+01
4	4	4	4	0	0.11428550E+00
4	4	4	4	2	-0.81632817E-01
4	4	4	4	4	0.14693872E+00
4	4	4	4	6	-0.57142857E+00
4	4	4	4	8	0.80000001E+01

OAI MAPPED INTERACTION FOR Q(2).Q(2) OPERATOR
-1.21212 -0.44747 -0.13006

0	0	0	0	0	0.00000000E+00
0	0	2	2	0	-0.10841542E+01
0	0	4	4	0	0.00000000E+00
0	2	0	2	2	-0.48484849E+00
0	2	2	2	2	0.37535262E+00
0	2	2	4	2	-0.35830942E+00
0	2	4	4	2	0.00000000E+00
0	4	0	4	4	0.00000000E+00
0	4	2	2	4	-0.37769129E+00
0	4	2	4	4	0.23408299E+00
0	4	4	4	4	0.00000000E+00
2	2	2	2	0	-0.14529239E+00
2	2	2	2	2	0.31134084E-01
2	2	2	2	4	-0.41512111E-01
2	2	2	4	2	0.79254437E-01
2	2	2	4	4	0.15489011E+00
2	2	4	4	0	-0.19736682E+00
2	2	4	4	2	-0.14785719E+00
2	2	4	4	4	-0.56194380E-01
2	4	2	4	2	-0.98098307E-01
2	4	2	4	3	0.90807764E-03
2	4	2	4	4	0.25202097E-01
2	4	2	4	5	0.13464312E+00
2	4	2	4	6	-0.19597962E+00
2	4	4	4	2	0.39074551E-01
2	4	4	4	4	0.11398182E+00
2	4	4	4	6	0.11663011E+00
4	4	4	4	0	-0.11301437E+00
4	4	4	4	2	-0.64799794E-01
4	4	4	4	4	0.16878769E-01
4	4	4	4	6	0.58048288E-01
4	4	4	4	8	-0.57534586E-01

Test output - 1
(Unit #39 FOR39.DAT)

N=5,PX=-,PY=-,KPZ=-, GIVES THE S.P.E AND T.B.M.E OF
PX*C2(U5)+PY*C2(05)

N = 5 PX = 1.0 PY = .00 KPZ = 0

THE S.P.E AND T.B.M.E ARE AS FOLLOWS :

ES= 12.00000 ED= 12.00000 EG= 12.00000

L1	L2	L3	L4	LL	V(L1,L2,L3,L4;LL)
0	0	0	0	0	0.16000000E+01
0	0	2	2	0	0.35777088E+01
0	0	4	4	0	0.48000000E+01
0	2	0	2	2	0.32000000E+01
0	2	2	2	2	-0.10842095E+01
0	2	2	4	2	0.27428571E+01
0	2	4	4	2	0.50853932E+01
0	4	0	4	4	0.32000000E+01
0	4	2	2	4	0.14456126E+01
0	4	2	4	4	0.53604751E+01
0	4	4	4	4	0.19327468E+01
2	2	2	2	0	-0.17142857E+01
2	2	2	2	2	0.33469388E+01
2	2	2	2	4	0.27755102E+01
2	2	2	4	2	0.52661604E+01
2	2	2	4	4	-0.24216158E+01
2	2	4	4	0	0.30666073E+01
2	2	4	4	2	-0.22973465E+01
2	2	4	4	4	0.52387561E+01
2	4	2	4	2	0.15346938E+01
2	4	2	4	3	-0.40000000E+01
2	4	2	4	4	0.18775509E+01
2	4	2	4	5	0.80000000E+01
2	4	2	4	6	0.45714286E+01
2	4	4	4	2	0.72648467E+00
2	4	4	4	4	-0.11773230E+01
2	4	4	4	6	0.54210474E+01
4	4	4	4	0	0.11428550E+00
4	4	4	4	2	-0.81632817E-01
4	4	4	4	4	0.14693872E+00
4	4	4	4	6	-0.57142857E+00
4	4	4	4	8	0.80000001E+01

N=25,PX=-,PY=-,KPZ=-, GIVES THE S.P.E AND T.B.M.E OF
OAI MAPPED INTERACTION

N = 25 PX = 0.0 PY = .00 KPZ = 0

WEIGHT FACTORS : B(0) = 0.0 B(1) = 0.0
B(2) = 1.0 B(3) = 0.0 B(4) = 0.0
B(5) = 0.0 B(6) = 0.0 B(7) = 0.0
B(8) = 0.0
OMEGA = 12 BOSON NUMBER = 4 IP = 1

THE S.P.E AND T.B.M.E ARE AS FOLLOWS :

ES= -1.21212 ED= -0.44747 EG= -0.13006

L1	L2	L3	L4	LL	V(L1,L2,L3,L4;LL)
0	0	0	0	0	0.00000000E+00
0	0	2	2	0	-0.10841542E+01
0	0	4	4	0	0.00000000E+00
0	2	0	2	2	-0.48484849E+00
0	2	2	2	2	0.37535262E+00
0	2	2	4	2	-0.35830942E+00
0	2	4	4	2	0.00000000E+00
0	4	0	4	4	0.00000000E+00
0	4	2	2	4	-0.37769129E+00
0	4	2	4	4	0.23408299E+00
0	4	4	4	4	0.00000000E+00
2	2	2	2	0	-0.14529239E+00
2	2	2	2	2	0.31134084E-01
2	2	2	2	4	-0.41512111E-01
2	2	2	4	2	0.79254437E-01
2	2	2	4	4	0.15489011E+00
2	2	4	4	0	-0.19736682E+00
2	2	4	4	2	-0.14785719E+00
2	2	4	4	4	-0.56194380E-01
2	4	2	4	2	-0.98098307E-01
2	4	2	4	3	0.90807764E-03
2	4	2	4	4	0.25202097E-01
2	4	2	4	5	0.13464312E+00
2	4	2	4	6	-0.19597962E+00
2	4	4	4	2	0.39074551E-01
2	4	4	4	4	0.11398182E+00
2	4	4	4	6	0.11663011E+00
4	4	4	4	0	-0.11301437E+00
4	4	4	4	2	-0.64799794E-01
4	4	4	4	4	0.16878769E-01
4	4	4	4	6	0.58048288E-01
4	4	4	4	8	-0.57534586E-01

Test output - 2
(Unit #45 CFPOUT.DAT)

```
3, 20.19000, 1, 19.42000
DIMENSIONALITY= 1
EIGENVALUES = 20.19000
MTII= 1
DIMENSIONALITY= 1
EIGENVALUES = 13.86000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
DIMENSIONALITY= 3
EIGENVALUES = 13.42000, 13.86000, 19.42000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 4
DIMENSIONALITY= 2
EIGENVALUES = 13.86000, 20.19000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 6
DIMENSIONALITY= 2
EIGENVALUES = 13.86000, 20.19000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 8
DIMENSIONALITY= 1
EIGENVALUES = 13.86000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
DIMENSIONALITY= 1
EIGENVALUES = 20.19000
MTII= 9

    <(M-1)= 2 || M=  3 >

    <V= 2 ALP=1 L= 2 || V= 3 ALP=1 L= 0 >=   1.00000

    <V= 2 ALP=1 L= 2 || V= 1 ALP=1 L= 2 >=  -0.43644

    <V= 2 ALP=1 L= 4 || V= 1 ALP=1 L= 2 >=  -0.58554

    <V= 0 ALP=1 L= 0 || V= 1 ALP=1 L= 2 >=  -0.68313

    <V= 2 ALP=1 L= 2 || V= 3 ALP=1 L= 3 >=   0.84515

    <V= 2 ALP=1 L= 4 || V= 3 ALP=1 L= 3 >=  -0.53452

    <V= 2 ALP=1 L= 2 || V= 3 ALP=1 L= 4 >=   0.72375

    <V= 1 ALP=1 L= 2 || V= 0 ALP=1 L= 0 >=   1.00000

    <V= 3 ALP=1 L= 0 || V= 2 ALP=1 L= 2 >=  -0.18257

    <V= 3 ALP=1 L= 3 || V= 2 ALP=1 L= 2 >=   0.40825

    <V= 3 ALP=1 L= 4 || V= 2 ALP=1 L= 2 >=  -0.39641

    <V= 1 ALP=1 L= 2 || V= 2 ALP=1 L= 2 >=   0.80178

    <V= 3 ALP=1 L= 0 || V= 4 ALP=1 L= 2 >=  -0.60553

    <V= 3 ALP=1 L= 3 || V= 4 ALP=1 L= 2 >=  -0.67700

    <V= 3 ALP=1 L= 4 || V= 4 ALP=1 L= 2 >=  -0.41833

    <V= 1 ALP=1 L= 2 || V= 4 ALP=1 L= 2 >=   0.00000

    <V= 3 ALP=1 L= 3 || V= 2 ALP=1 L= 4 >=  -0.19245

    <V= 3 ALP=1 L= 4 || V= 2 ALP=1 L= 4 >=  -0.28172

    <V= 3 ALP=1 L= 6 || V= 2 ALP=1 L= 4 >=  -0.49065

    <V= 1 ALP=1 L= 2 || V= 2 ALP=1 L= 4 >=   0.80178

    <V= 3 ALP=1 L= 3 || V= 4 ALP=1 L= 4 >=   0.72776

    <V= 3 ALP=1 L= 4 || V= 4 ALP=1 L= 4 >=  -0.67794

    <V= 3 ALP=1 L= 6 || V= 4 ALP=1 L= 4 >=   0.10380

    <V= 1 ALP=1 L= 2 || V= 4 ALP=1 L= 4 >=   0.00000

    <V= 3 ALP=1 L= 3 || V= 4 ALP=1 L= 5 >=  -0.72457

    <V= 3 ALP=1 L= 4 || V= 4 ALP=1 L= 5 >=  -0.40850

    <V= 3 ALP=1 L= 6 || V= 4 ALP=1 L= 5 >=   0.48617

    <V= 3 ALP=1 L= 4 || V= 4 ALP=1 L= 6 >=   0.82572

    <V= 3 ALP=1 L= 6 || V= 4 ALP=1 L= 6 >=   0.56408

    <V= 3 ALP=1 L= 6 || V= 4 ALP=1 L= 8 >=   1.00000

, 42.20000, 3, 40.99000, 1, 40.22000
IMENSIONALITY= 2
```

```
    <V= 2 ALP=1 L= 4 || V= 3 ALP=1 L= 4 >=   0.69007

    <V= 2 ALP=1 L= 4 || V= 3 ALP=1 L= 6 >=   1.00000

4, 30.34000, 2, 29.35000, 0, 28.80000
DIMENSIONALITY= 1
EIGENVALUES = 28.80000
MTII= 1
DIMENSIONALITY= 2
EIGENVALUES = 21.13000, 21.90000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
DIMENSIONALITY= 4
EIGENVALUES = 21.35000, 21.90000, 29.35000, 30.34000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 5
MTII= 9
DIMENSIONALITY= 3
EIGENVALUES = 21.13000, 21.90000, 21.90000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
DIMENSIONALITY= 4
EIGENVALUES = 21.35000, 21.90000, 29.35000, 30.34000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 13
MTII= 17
DIMENSIONALITY= 3
EIGENVALUES = 21.90000, 21.90000, 30.34000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 20
DIMENSIONALITY= 2
EIGENVALUES = 21.90000, 30.34000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 22
DIMENSIONALITY= 1
EIGENVALUES = 21.90000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
DIMENSIONALITY= 1
EIGENVALUES = 30.34000
MTII= 23

    <(M-1)= 3 || M=  4 >

EIGENVALUES = 30.99000, 40.99000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 2
DIMENSIONALITY= 2
EIGENVALUES = 30.66000, 31.65000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
DIMENSIONALITY= 5
EIGENVALUES = 30.22000, 30.66000, 31.65000, 40.22000, 42.20000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 7
MTII= 12
DIMENSIONALITY= 5
EIGENVALUES = 30.66000, 30.99000, 31.65000, 31.65000, 40.99000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 17
DIMENSIONALITY= 6
EIGENVALUES = 30.66000, 30.99000, 31.65000, 31.65000, 40.99000,
42.20000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 23
MTII= 29
DIMENSIONALITY= 4
EIGENVALUES = 30.66000, 31.65000, 31.65000, 42.20000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 33
DIMENSIONALITY= 5
EIGENVALUES = 30.99000, 31.65000, 31.65000, 40.99000, 42.20000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 38
MTII= 43
DIMENSIONALITY= 3
EIGENVALUES = 31.65000, 31.65000, 42.20000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 46
DIMENSIONALITY= 2
```

```
EIGENVALUES = 31.65000, 42.20000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 48
DIMENSIONALITY= 1
EIGENVALUES = 31.65000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
DIMENSIONALITY= 1
EIGENVALUES = 42.20000
MTII= 49

    <(M-1)= 4 |) M=  5 >

    <V= 4 ALP=1 L= 2 |) V= 3 ALP=1 L= 0 >=    0.51640

    <V= 2 ALP=1 L= 2 |) V= 3 ALP=1 L= 0 >=    0.85635

    <V= 4 ALP=1 L= 2 |) V= 1 ALP=1 L= 2 >=    0.00000

    <V= 4 ALP=1 L= 4 |) V= 1 ALP=1 L= 2 >=    0.00000

    <V= 2 ALP=1 L= 2 |) V= 1 ALP=1 L= 2 >=   -0.47809

    <V= 2 ALP=1 L= 4 |) V= 1 ALP=1 L= 2 >=   -0.64143

    <V= 0 ALP=1 L= 0 |) V= 1 ALP=1 L= 2 >=   -0.60000

    <V= 4 ALP=1 L= 2 |) V= 5 ALP=1 L= 2 >=   -0.86189

    <V= 4 ALP=1 L= 4 |) V= 5 ALP=1 L= 2 >=   -0.50709

    <V= 2 ALP=1 L= 2 |) V= 5 ALP=1 L= 2 >=    0.00000

    <V= 2 ALP=1 L= 4 |) V= 5 ALP=1 L= 2 >=    0.00000

    <V= 0 ALP=1 L= 0 |) V= 5 ALP=1 L= 2 >=    0.00000

    <V= 4 ALP=1 L= 2 |) V= 3 ALP=1 L= 3 >=    0.21822

    <V= 4 ALP=1 L= 4 |) V= 3 ALP=1 L= 3 >=   -0.31472

    <V= 4 ALP=1 L= 5 |) V= 3 ALP=1 L= 3 >=   -0.34641

    <V= 2 ALP=1 L= 2 |) V= 3 ALP=1 L= 3 >=   -0.72375

    <V= 2 ALP=1 L= 4 |) V= 3 ALP=1 L= 3 >=    0.45774

    <V= 4 ALP=1 L= 2 |) V= 3 ALP=1 L= 4 >=   -0.11892

    <V= 4 ALP=1 L= 4 |) V= 3 ALP=1 L= 4 >=   -0.25855

    <V= 4 ALP=1 L= 5 |) V= 3 ALP=1 L= 4 >=    0.20597

    <V= 4 ALP=1 L= 6 |) V= 3 ALP=1 L= 4 >=    0.37848

    <V= 2 ALP=1 L= 2 |) V= 3 ALP=1 L= 4 >=   -0.61978

    <V= 2 ALP=1 L= 4 |) V= 3 ALP=1 L= 4 >=   -0.59094

    <V= 4 ALP=1 L= 2 |) V= 5 ALP=1 L= 4 >=   -0.71168

    <V= 4 ALP=1 L= 4 |) V= 5 ALP=1 L= 4 >=    0.28567

    <V= 4 ALP=1 L= 5 |) V= 5 ALP=1 L= 4 >=   -0.57525

    <V= 4 ALP=1 L= 6 |) V= 5 ALP=1 L= 4 >=    0.28459

    <V= 2 ALP=1 L= 2 |) V= 5 ALP=1 L= 4 >=    0.00000

    <V= 2 ALP=1 L= 4 |) V= 5 ALP=1 L= 4 >=    0.00000

    <V= 4 ALP=1 L= 4 |) V= 5 ALP=1 L= 5 >=   -0.82902

    <V= 4 ALP=1 L= 5 |) V= 5 ALP=1 L= 5 >=   -0.48990

    <V= 4 ALP=1 L= 6 |) V= 5 ALP=1 L= 5 >=   -0.26968

    <V= 2 ALP=1 L= 4 |) V= 5 ALP=1 L= 5 >=    0.00000

    <V= 4 ALP=1 L= 4 |) V= 3 ALP=1 L= 6 >=   -0.03294

    <V= 4 ALP=1 L= 5 |) V= 3 ALP=1 L= 6 >=    0.17056

    <V= 4 ALP=1 L= 6 |) V= 3 ALP=1 L= 6 >=   -0.21513

    <V= 4 ALP=1 L= 8 |) V= 3 ALP=1 L= 6 >=   -0.43613

    <V= 2 ALP=1 L= 4 |) V= 3 ALP=1 L= 6 >=    0.85635

    <V= 4 ALP=1 L= 4 |) V= 5 ALP=1 L= 6 >=    0.63915

    <V= 4 ALP=1 L= 5 |) V= 5 ALP=1 L= 6 >=   -0.42027

    <V= 4 ALP=1 L= 6 |) V= 5 ALP=1 L= 6 >=   -0.63610

    <V= 4 ALP=1 L= 8 |) V= 5 ALP=1 L= 6 >=    0.10114

    <V= 2 ALP=1 L= 4 |) V= 5 ALP=1 L= 6 >=    0.00000

    <V= 4 ALP=1 L= 5 |) V= 5 ALP=1 L= 7 >=    0.82808

    <V= 4 ALP=1 L= 6 |) V= 5 ALP=1 L= 7 >=   -0.34641

    <V= 4 ALP=1 L= 8 |) V= 5 ALP=1 L= 7 >=    0.44078

    <V= 4 ALP=1 L= 6 |) V= 5 ALP=1 L= 8 >=    0.87178

    <V= 4 ALP=1 L= 8 |) V= 5 ALP=1 L= 8 >=    0.48990

    <V= 4 ALP=1 L= 8 |) V= 5 ALP=1 L=10 >=    1.00000

| Note: only a part of the output is reproduced here |
```

Test output - 3
(Unit #45 CFPOUT.DAT)

```
3, 33.65000, 1, 32.44000
DIMENSIONALITY= 1
EIGENVALUES = 33.65000
MTII= 1
DIMENSIONALITY= 1
EIGENVALUES = 27.32000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
DIMENSIONALITY= 3
EIGENVALUES = 27.32000, 27.32000, 33.65000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 4
DIMENSIONALITY= 3
EIGENVALUES = 27.32000, 27.32000, 33.65000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 7
DIMENSIONALITY= 5
EIGENVALUES = 26.44000, 27.32000, 27.32000, 32.44000, 33.65000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 12
MTII= 17
DIMENSIONALITY= 4
EIGENVALUES = 27.32000, 27.32000, 27.32000, 33.65000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 21
DIMENSIONALITY= 4
EIGENVALUES = 27.32000, 27.32000, 33.65000, 33.65000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 25
MTII= 29
DIMENSIONALITY= 3
EIGENVALUES = 27.32000, 27.32000, 33.65000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 32
DIMENSIONALITY= 3
EIGENVALUES = 27.32000, 27.32000, 33.65000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 35
DIMENSIONALITY= 2
EIGENVALUES = 27.32000, 33.65000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 37
DIMENSIONALITY= 2
EIGENVALUES = 27.32000, 33.65000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
MTII= 39
DIMENSIONALITY= 1
EIGENVALUES = 27.32000
EIGENVALUES NOT MATCHING  ......... CHECK CHECK
DIMENSIONALITY= 1
EIGENVALUES = 33.65000
MTII= 40

   <(M-1)= 2 |} M=  3 >

   <V= 2 ALP=1 L= 4 |} V= 3 ALP=1 L= 0 >=   1.00000

   <V= 2 ALP=1 L= 2 |} V= 3 ALP=1 L= 2 >=  -0.64242

   <V= 2 ALP=1 L= 4 |} V= 3 ALP=1 L= 2 >=  -0.48349

   <V= 2 ALP=1 L= 6 |} V= 3 ALP=1 L= 2 >=  -0.59459

   <V= 2 ALP=1 L= 2 |} V= 3 ALP=1 L= 3 >=   0.32121

   <V= 2 ALP=1 L= 4 |} V= 3 ALP=1 L= 3 >=  -0.80582

   <V= 2 ALP=1 L= 6 |} V= 3 ALP=1 L= 3 >=   0.49747

   <V= 2 ALP=1 L= 2 |} V= 1 ALP=1 L= 4 >=  -0.25950

   <V= 2 ALP=1 L= 4 |} V= 1 ALP=1 L= 4 >=  -0.34816

   <V= 2 ALP=1 L= 6 |} V= 1 ALP=1 L= 4 >=  -0.41843

   <V= 2 ALP=1 L= 8 |} V= 1 ALP=1 L= 4 >=  -0.47849

   <V= 0 ALP=1 L= 0 |} V= 1 ALP=1 L= 4 >=  -0.63828

   <V= 2 ALP=1 L= 2 |} V= 3 ALP=1 L= 4 >=   0.69163

   <V= 2 ALP=1 L= 4 |} V= 3 ALP=1 L= 4 >=  -0.23793

   <V= 2 ALP=1 L= 6 |} V= 3 ALP=1 L= 4 >=  -0.60050

   <V= 2 ALP=1 L= 8 |} V= 3 ALP=1 L= 4 >=   0.32315

   <V= 0 ALP=1 L= 0 |} V= 3 ALP=1 L= 4 >=   0.00000

   <V= 2 ALP=1 L= 2 |} V= 3 ALP=1 L= 5 >=  -0.72474

   <V= 2 ALP=1 L= 4 |} V= 3 ALP=1 L= 5 >=  -0.37398

   <V= 2 ALP=1 L= 6 |} V= 3 ALP=1 L= 5 >=  -0.04495

   <V= 2 ALP=1 L= 8 |} V= 3 ALP=1 L= 5 >=   0.57695

   <V= 2 ALP=1 L= 2 |} V= 3 ALP=1 L= 6 >=  -0.52143

   <V= 2 ALP=1 L= 4 |} V= 3 ALP=1 L= 6 >=  -0.61356

   <V= 2 ALP=1 L= 6 |} V= 3 ALP=1 L= 6 >=  -0.12646

   <V= 2 ALP=1 L= 8 |} V= 3 ALP=1 L= 6 >=  -0.57937

   <V= 2 ALP=1 L= 2 |} V= 3 ALP=2 L= 6 >=   0.31914

   <V= 2 ALP=1 L= 4 |} V= 3 ALP=2 L= 6 >=  -0.44381

   <V= 2 ALP=1 L= 6 |} V= 3 ALP=2 L= 6 >=   0.83736

   <V= 2 ALP=1 L= 8 |} V= 3 ALP=2 L= 6 >=   0.00000

   <V= 2 ALP=1 L= 4 |} V= 3 ALP=1 L= 7 >=  -0.77098

   <V= 2 ALP=1 L= 6 |} V= 3 ALP=1 L= 7 >=  -0.28427

   <V= 2 ALP=1 L= 8 |} V= 3 ALP=1 L= 7 >=   0.56990

   <V= 2 ALP=1 L= 4 |} V= 3 ALP=1 L= 8 >=  -0.63135

   <V= 2 ALP=1 L= 6 |} V= 3 ALP=1 L= 8 >=  -0.49237

   <V= 2 ALP=1 L= 8 |} V= 3 ALP=1 L= 8 >=  -0.59914

   <V= 2 ALP=1 L= 6 |} V= 3 ALP=1 L= 9 >=   0.85635

   <V= 2 ALP=1 L= 8 |} V= 3 ALP=1 L= 9 >=  -0.51640

   <V= 2 ALP=1 L= 6 |} V= 3 ALP=1 L=10 >=   0.71492

   <V= 2 ALP=1 L= 8 |} V= 3 ALP=1 L=10 >=   0.69921

   <V= 2 ALP=1 L= 8 |} V= 3 ALP=1 L=12 >=   1.00000
```

[Note: only a part of the output is reproduced here]

Test output - 4
(Unit #35 SDGOUT.DAT)

C2(SU(3)) INTERACTION

S.P.E. AND TWO-BODY MATRIX ELEMENTS

ES = -28.00000 ED = -23.50000 EG = -13.00000

L1	L2	L3	L4	LL	V(L1 L2 L3 L4 LL)
0	0	0	0	0	0.00000000E+00
0	0	2	2	0	-0.25043962E+02
0	0	4	4	0	0.00000000E+00
0	2	0	2	2	-0.11200000E+02
0	2	2	2	2	0.19677398E+02
0	2	2	4	2	-0.14400000E+02
0	2	4	4	2	0.00000000E+00
0	4	0	4	4	0.00000000E+00
0	4	2	4	4	-0.15178933E+02
0	4	2	4	4	0.13266499E+02
0	4	4	4	4	0.00000000E+00
2	2	2	2	0	-0.17285714E+02
2	2	2	2	2	0.37040817E+01
2	2	2	2	4	-0.49387755E+01
2	2	2	4	2	0.72284321E+01
2	2	2	4	4	0.14126813E-02
2	2	4	4	0	-0.13799734E+02
2	2	4	4	2	-0.10338059E+02
2	2	4	4	4	-0.39290671E+01
2	4	2	4	2	-0.12493878E+02
2	4	2	4	3	-0.50000001E+00
2	4	2	4	4	0.50918369E+01
2	4	2	4	5	0.13000000E+02
2	4	2	4	6	-0.16571429E+02
2	4	4	4	2	0.38527672E+01
2	4	4	4	4	0.11238655E+02
2	4	4	4	6	0.11499778E+02
4	4	4	4	0	-0.15714286E+02
4	4	4	4	2	-0.90102041E+01
4	4	4	4	4	0.23469388E+01
4	4	4	4	6	0.80714288E+01
4	4	4	4	8	-0.80000001E+01

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 0

DIMENSION = 7

! NS	ND	VD	ALPD	LD	NG	VG	ALPG	LG	L >
3	0	0	1	0	0	0	1	0	0
1	0	0	1	0	2	0	1	0	0
0	0	0	1	0	3	3	1	0	0
0	1	1	1	2	2	2	1	2	0
1	2	0	1	0	0	0	1	0	0
0	2	2	1	4	1	1	1	4	0
0	3	3	1	0	0	0	1	0	0

EIGENVALUES

0.00000000E+00 0.66000003E+02 0.10800000E+03
0.12600000E+03 0.12600000E+03 0.15600000E+03
0.18000000E+03

EIGENVALUE(1) = 0.00000000E+00 EIGENVECTOR FOLLOWS -------

0.32249032E+00 0.21278842E+00 0.52835254E-01
-0.18330890E+00 0.71371406E+00 0.40783662E+00
-0.37230217E+00

EIGENVALUE(2) = 0.66000003E+02 EIGENVECTOR FOLLOWS -------

0.48000002E+00 -0.40909389E+00 -0.19168712E+00
0.44620717E+00 0.33196998E+00 -0.50585909E+00
0.17316908E-01

EIGENVALUE(3) = 0.10800000E+03 EIGENVECTOR FOLLOWS -------

-0.65319721E+00 -0.46467021E+00 -0.11955772E+00
0.19144551E+00 0.18070156E+00 0.70989903E-01
-0.51843700E+00

EIGENVALUE(4) = 0.12600000E+03 EIGENVECTOR FOLLOWS -------

0.40520328E-01 -0.90235628E-01 -0.39117358E+00
0.48086793E+00 -0.28024029E-01 0.67843452E+00
0.38072311E+00

EIGENVALUE(5) = 0.12600000E+03 EIGENVECTOR FOLLOWS -------

-0.31442298E+00 0.70019554E+00 -0.43455802E+00
0.26878316E+00 0.21745623E+00 -0.32014384E+00

0.00000000E+00

EIGENVALUE(6) = 0.15600000E+03 EIGENVECTOR FOLLOWS -------

0.34563955E+00 0.32071350E-01 -0.56937587E+00
-0.15195521E+00 -0.47809147E+00 0.51224064E-01
-0.54866214E+00

EIGENVALUE(7) = 0.18000000E+03 EIGENVECTOR FOLLOWS -------

0.13770607E+00 0.26832816E+00 0.52930451E+00
0.63567419E+00 -0.26666668E+00 0.85714295E-01
-0.38253637E+00

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 1

DIMENSION = 2

! NS	ND	VD	ALPD	LD	NG	VG	ALPG	LG	L >
0	1	1	1	2	2	2	1	2	1
0	2	2	1	4	1	1	1	4	1

EIGENVALUES

0.91500003E+02 0.13650000E+03

EIGENVALUE(1) = 0.91500003E+02 EIGENVECTOR FOLLOWS -------

-0.23904571E+00 0.97100831E+00

EIGENVALUE(2) = 0.13650000E+03 EIGENVECTOR FOLLOWS -------

0.97100831E+00 0.23904571E+00

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 2

DIMENSION = 11

! NS	ND	VD	ALPD	LD	NG	VG	ALPG	LG	L >
1	0	0	1	0	2	2	1	2	2
0	0	0	1	0	3	3	1	2	2
2	1	1	1	2	0	0	1	0	2
1	1	1	1	2	1	1	1	4	2
0	1	1	1	2	2	2	1	2	2
0	1	1	1	2	2	2	1	4	2
0	1	1	1	2	2	0	1	0	2
1	2	2	1	2	0	0	1	0	2
0	2	2	1	2	1	1	1	4	2
0	2	2	1	4	1	1	1	4	2

0 3 1 1 2 0 0 1 0 2

EIGENVALUES

0.44999981E+01 0.70499996E+02 0.70499998E+02
0.94500003E+02 0.11250000E+03 0.11250000E+03
0.13050000E+03 0.13050000E+03 0.13950001E+03
0.16050000E+03 0.16050000E+03

EIGENVALUE(1) = 0.44999981E+01 EIGENVECTOR FOLLOWS -------

0.12124745E+00 -0.62766940E-01 -0.47207746E+00
-0.38149619E+00 -0.10954803E+00 -0.11601434E+00
-0.17983905E+00 0.42652572E+00 0.24372898E+00
0.23238648E+00 0.50979574F+00

EIGENVALUE(2) = 0.70499996E+02 EIGENVECTOR FOLLOWS -------

0.15217108E+00 -0.22571383E+00 0.51951452E+00
-0.28725170E+00 -0.31042009E+00 -0.25045409E+00
-0.43158282E+00 -0.32473934E+00 0.30393518E+00
0.17217599E+00 -0.25689353E-01

EIGENVALUE(3) = 0.70499998E+02 EIGENVECTOR FOLLOWS -------

-0.46581604E+00 0.12903768E-01 -0.20685837E+00
-0.32227080E+00 -0.29267620E+00 0.17939653E+00
0.71758928E-02 -0.33935591E+00 0.14120628E+00
-0.60051978E+00 0.15026770E+00

EIGENVALUE(4) = 0.94500003E+02 EIGENVECTOR FOLLOWS -------

0.11547000E+00 0.11012850E+00 -0.23608482E-07
-0.60553007E+00 0.29808033E+00 -0.11399378E+00
0.48934223E+00 -0.59836075E-07 0.30396062E+00
0.31616163E-01 -0.41614635E+00

EIGENVALUE(5) = 0.11250000E+03 EIGENVECTOR FOLLOWS -------

0.25238648E+00 0.82531705E-01 -0.13890181E+00
0.34574674E-01 0.46081863E+00 -0.14475071E+00
0.75290346E-01 -0.65906604E+00 -0.30783811E-01
0.54034133E-01 0.48144567E+00

EIGENVALUE(6) = 0.11250000E+03 EIGENVECTOR FOLLOWS -------

-0.31073099E+00 0.15681869E+00 0.58857504E+00
0.65695353E-01 0.90164864E-01 0.89046314E-01
0.35618504E+00 0.23042474E+00 0.25923280E+00

0.52180506E-01 0.51202633E+00

EIGENVALUE(7) = 0.13050000E+03 EIGENVECTOR FOLLOWS -------

-0.45505874E+00 -0.34896324E+00 -0.25306861E+00
0.27822971E+00 -0.17446147E+00 -0.42608874E+00
0.32537392E+00 -0.18197651E+00 0.18563253E+00
0.37008861E+00 -0.85402633E-01

EIGENVALUE(8) = 0.13050000E+03 EIGENVECTOR FOLLOWS -------

-0.20814383E+00 0.45022164E+00 -0.23786781E-01
-0.26291985E+00 -0.27787340E+00 0.29326716E+00
0.30582983E-01 -0.22864834E+00 -0.34687721E+00
0.58945831E+00 -0.80272852E-02

EIGENVALUE(9) = 0.13950001E+03 EIGENVECTOR FOLLOWS -------

0.40941305E+00 0.39047380E+00 0.43060682E-08
0.13012004E+00 -0.56146733E+00 -0.40417881E+00
0.35489084E+00 0.30989811E-07 -0.65316857E-01
-0.21485653E+00 0.89424092E-01

EIGENVALUE(10) = 0.16050000E+03 EIGENVECTOR FOLLOWS -------

0.38870704E+00 -0.47350083E+00 -0.29060874E-01
0.57244051E-01 -0.27147204E+00 0.60182206E+00
0.37245271E+00 -0.12362519E+00 0.12187274E+00
0.10133649E+00 0.74627306E-01

EIGENVALUE(11) = 0.16050000E+03 EIGENVECTOR FOLLOWS -------

-0.49576834E-01 -0.44758241E+00 0.18245218E+00
-0.35939384E+00 0.71260519E-02 -0.23800447E+00
0.19462373E+00 0.96516576E-01 -0.70273504E+00
-0.90701569E-01 0.16972136E+00

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 3

DIMENSION = 7

NS	ND	VD	ALPD	LD	NG	VG	ALPG	LG	L
0	0	0	1	0	3	3	1	3	3
1	1	1	1	2	1	1	1	4	3
0	1	1	1	2	2	2	1	2	3
0	1	1	1	2	2	2	1	4	3
0	2	2	1	2	1	1	1	4	3
0	2	2	1	4	1	1	1	4	3
0	3	3	1	3	0	0	1	0	3

EIGENVALUES

0.75000000E+02 0.99000002E+02 0.99000002E+02
0.11700000E+03 0.14400000E+03 0.14400000E+03
0.16500001E+03

EIGENVALUE(1) = 0.75000000E+02 EIGENVECTOR FOLLOWS -------

0.89424107E-01 0.63919596E+00 -0.55623279E+00
-0.17586921E+00 -0.23335276E+00 -0.16686982E+00
0.40100874E+00

EIGENVALUE(2) = 0.99000002E+02 EIGENVECTOR FOLLOWS -------

-0.25720469E+00 0.51854496E+00 0.56610293E+00
-0.11536728E+00 -0.47326604E+00 -0.36919659E-01
-0.32531657E+00

EIGENVALUE(3) = 0.99000002E+02 EIGENVECTOR FOLLOWS -------

0.16116456E+00 0.41416625E-07 0.12029651E+00
0.50046359E+00 -0.48428774E-07 -0.84207537E+00
0.00000000E+00

EIGENVALUE(4) = 0.11700000E+03 EIGENVECTOR FOLLOWS -------

0.15609244E+00 -0.15257920E+00 -0.48297091E+00
-0.21901851E+00 -0.23673545E+00 -0.16928881E+00
-0.76578215E+00

EIGENVALUE(5) = 0.14400000E+03 EIGENVECTOR FOLLOWS -------

-0.44416798E+00 -0.51475583E+00 -0.96816034E-01
-0.80031927E-01 -0.62947483E+00 -0.14640473E+00
0.32293946E+00

EIGENVALUE(6) = 0.14400000E+03 EIGENVECTOR FOLLOWS -------

0.44249406E+00 -0.36194212E-07 -0.50813330E-01
0.61069985E+00 -0.48448577E+00 0.44038193E+00
0.00000000E+00

EIGENVALUE(7) = 0.16500001E+03 EIGENVECTOR FOLLOWS -------

0.69456752E+00 -0.18516401E+00 0.33233275E+00
-0.52722680E+00 -0.18589521E+00 -0.13293308E+00
0.20651609E+00

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 4

DIMENSION = 13

NS	ND	VD	ALPD	LD	NG	VG	ALPG	LG	L
2	0	0	1	0	1	1	1	4	4
1	0	0	1	0	2	2	1	4	4
0	0	0	1	0	3	3	1	4	4
0	0	0	1	0	3	1	1	4	4
1	1	1	1	2	1	1	1	4	4
0	1	1	1	2	2	2	1	2	4
0	1	1	1	2	2	2	1	4	4
0	1	1	1	2	2	2	1	6	4
1	2	2	1	4	0	0	1	0	4
0	2	2	1	2	1	1	1	4	4
0	2	2	1	4	1	1	1	4	4
0	2	0	1	0	1	1	1	4	4
0	3	3	1	4	0	0	1	0	4

EIGENVALUES

0.15000008E+02 0.81000002E+02 0.81000013E+02
0.10500000E+03 0.10500000E+03 0.12300000E+03
0.12300000E+03 0.12300000E+03 0.14100000E+03
0.14100000E+03 0.15000000E+03 0.15000000E+03
0.17100000E+03

EIGENVALUE(1) = 0.15000008E+02 EIGENVECTOR FOLLOWS -------

-0.29015595E+00 -0.12478659E+00 -0.16077259E-01
0.99983213E-01 0.35349402E+00 0.14283315E+00
0.10749878E+00 0.13219927E+00 -0.55612149E+00
-0.22583907E+00 -0.23916967E+00 -0.37074768E+00
0.40082455E+00

EIGENVALUE(2) = 0.81000002E+02 EIGENVECTOR FOLLOWS -------

0.25683312E+00 -0.15030670E+00 0.21171224E+00
-0.30985502E+00 0.27356762E-01 0.14744006E+00
0.26775921E-01 -0.12976925E+00 -0.28730034E+00
0.34989920E+00 -0.25381774E+00 0.57441048E+00
0.36225210E+00

EIGENVALUE(3) = 0.81000013E+02 EIGENVECTOR FOLLOWS -------

-0.23844447E+00 0.32074356E+00 -0.53681212E-01
-0.23498615E+00 -0.13627020E+00 -0.44798860E+00
-0.28959663E+00 -0.26421408E+00 -0.50849418E+00
0.29320751E-01 0.38382221E+00 0.48134323E-01

0.12898677E-01

EIGENVALUE(4) = 0.10500000E+03 EIGENVECTOR FOLLOWS -------

0.22823083E+00 0.35381725E+00 0.16339873E-01
0.37781134E+00 -0.48664244E+00 0.32907479E+00
0.24150821E+00 -0.25118479E+00 -0.10935852E+00
0.13485968E+00 0.16109162E+00 -0.21615262E+00
0.33221321E+00

EIGENVALUE(5) = 0.10500000E+03 EIGENVECTOR FOLLOWS -------

0.53153356E+00 -0.11192686E+00 -0.40121931E+00
0.23711726E+00 -0.17887833E+00 -0.15186335E+00
-0.24151862E+00 0.17141172E+00 -0.25468827E+00
-0.46994143E+00 -0.12027462E+00 0.21164181E+00
0.64696623E-03

EIGENVALUE(6) = 0.12300000E+03 EIGENVECTOR FOLLOWS -------

-0.27178827E+00 0.19904257E-01 -0.29396849E+00
0.51368902E+00 0.29597030E+00 0.17206340E+00
0.21160283E+00 -0.65897297E-01 -0.17045056E+00
0.15803008E+00 0.21601281E+00 0.49805572E+00
-0.24570446E+00

EIGENVALUE(7) = 0.12300000E+03 EIGENVECTOR FOLLOWS -------

-0.76284053E-01 -0.11781129E+00 0.36136689E+00
-0.21893260E+00 -0.30314476E+00 0.43253086E+00
0.15099359E+00 0.97463875E-01 -0.32700013E+00
-0.36376084E+00 0.14495464E+00 0.89737622E-01
-0.47137067E+00

EIGENVALUE(8) = 0.12300000E+03 EIGENVECTOR FOLLOWS -------

0.32396948E+00 0.45321356E+00 0.47459312E-01
-0.25945602E+00 0.46533461E+00 0.13464598E-01
0.35680218E+00 0.98900751E-01 0.10290366E+00
-0.33496317E+00 0.33228173E+00 0.10516954E+00
0.14833560E+00

EIGENVALUE(9) = 0.14100000E+03 EIGENVECTOR FOLLOWS -------

0.18215939E+00 -0.73684106E-01 0.31368000E+00
0.21184635E+00 -0.16077273E-01 -0.22391418E+00
0.13749891E-01 0.70009747E+00 -0.18376864E+00
0.37685904E+00 0.31031932E+00 -0.72735839E-01
0.32960065E-02

EIGENVALUE(10) = 0.14100000E+03 EIGENVECTOR FOLLOWS -------

-0.89665044E-01 0.29352921E+00 -0.15440392E+00
-0.10427794E+00 -0.23425924E+00 -0.36069786E+00
0.58273812E+00 0.17049143E+00 -0.12120383E+00
0.85254885E-01 -0.49789934E+00 0.35803072E-01
-0.21519889E+00

EIGENVALUE(11) = 0.15000000E+03 EIGENVECTOR FOLLOWS -------

0.39397437E+00 -0.14775516E+00 0.35751494E+00
0.24145062E+00 0.29762286E+00 -0.26846482E+00
0.12373772E+00 -0.48916519E+00 -0.18877576E+00
0.52933161E-01 -0.98756581E-01 -0.22874947E+00
-0.34795621E+00

EIGENVALUE(12) = 0.15000000E+03 EIGENVECTOR FOLLOWS -------

-0.13250599E+00 0.53859984E+00 0.46058213E+00
0.31105788E+00 0.72487989E-01 0.33217549E-01
-0.39555242E+00 0.99593283E-01 0.63491229E-01
-0.15956902E+00 -0.37172455E+00 0.20622979E+00
-0.22754434E-01

EIGENVALUE(13) = 0.17100000E+03 EIGENVECTOR FOLLOWS -------

0.24440407E+00 0.31314075E+00 -0.33068024E+00
-0.22332759E+00 0.20470652E+00 0.39436814E+00
-0.28420988E+00 0.10663140E+00 -0.19518001E+00
0.37366340E+00 -0.14289880E+00 -0.26488718E+00
-0.35369962E+00

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 5

DIMENSION = 6

!	NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
	0	0	0	1	0	3	3	1	5	5
	1	1	1	1	2	1	1	1	4	5
	0	1	1	1	2	2	2	1	4	5
	0	1	1	1	2	2	2	1	6	5
	0	2	2	1	2	1	1	1	4	5
	0	2	2	1	4	1	1	1	4	5

EIGENVALUES

0.88500001E+02 0.11250000E+03 0.11250000E+03
0.13050000E+03 0.13050000E+03 0.15750000E+03

EIGENVALUE(1) = 0.88500001E+02 EIGENVECTOR FOLLOWS -------

-0.37408782E+00 -0.43094578E+00 0.30578834E+00
-0.46047021E-01 0.68830300E+00 0.32396951E+00

EIGENVALUE(2) = 0.11250000E+03 EIGENVECTOR FOLLOWS -------

-0.35006517E+00 0.40484489E+00 0.52393093E+00
0.25148164E+00 0.12932289E+00 -0.59923615E+00

EIGENVALUE(3) = 0.11250000E+03 EIGENVECTOR FOLLOWS -------

0.37783653E+00 -0.62047525E+00 0.30881474E+00
0.55212865E+00 -0.19820356E+00 -0.18097723E+00

EIGENVALUE(4) = 0.13050000E+03 EIGENVECTOR FOLLOWS -------

-0.63155833E+00 -0.14354220E+00 0.19836751E+00
0.22784033E-01 -0.66619622E+00 0.31119697E+00

EIGENVALUE(5) = 0.13050000E+03 EIGENVECTOR FOLLOWS -------

-0.44259046E+00 -0.28083184E+00 -0.69228972E+00
0.27884829E+00 0.97372239E-01 -0.39842744E+00

EIGENVALUE(6) = 0.15750000E+03 EIGENVECTOR FOLLOWS -------

-0.18626451E-08 -0.40734009E+00 0.13419633E+00
-0.74264067E+00 -0.13011999E+00 -0.49761337E+00

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 6

DIMENSION = 10

!	NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
	1	0	0	1	0	2	2	1	6	6
	0	0	0	1	0	3	3	1	6	6
	0	0	0	1	0	3	3	2	6	6
	1	1	1	1	2	1	1	1	4	6
	0	1	1	1	2	2	2	1	4	6
	0	1	1	1	2	2	2	1	6	6
	0	1	1	1	2	2	2	1	8	6
	0	2	2	1	2	1	1	1	4	6
	0	2	2	1	4	1	1	1	4	6
	0	3	3	1	6	0	0	1	0	6

EIGENVALUES

0.31500004E+02 0.97500003E+02 0.97500005E+02
0.12150000E+03 0.12150000E+03 0.13950000E+03
0.13950000E+03 0.15750000E+03 0.15750000E+03
0.16650000E+03

EIGENVALUE(1) = 0.31500004E+02 EIGENVECTOR FOLLOWS -------

-0.17877477E+00 0.98473182E-01 -0.36117965E-01
0.59962881E+00 0.18234923E+00 0.15245964E+00
0.23809656E+00 -0.38308879E+00 -0.34264503E+00
0.46918603E+00

EIGENVALUE(2) = 0.97500003E+02 EIGENVECTOR FOLLOWS -------

-0.25629330E+00 0.27798373E+00 -0.14613692E+00
-0.46986983E+00 0.56961908E+00 0.33443142E+00
-0.16534596E+00 -0.63834913E-01 -0.36133894E+00
-0.12889859E+00

EIGENVALUE(3) = 0.97500005E+02 EIGENVECTOR FOLLOWS -------

0.83541473E-01 -0.21964028E+00 0.46554796E-01
-0.11461675E+00 -0.29370465E-02 -0.11161668E+00
-0.70359347E+00 0.11856516E+00 -0.12813427E+00
0.62568992E+00

EIGENVALUE(4) = 0.12150000E+03 EIGENVECTOR FOLLOWS -------

0.37769331E+00 0.32652770E+00 -0.25411048E-01
-0.45054787E+00 0.57089190E-02 -0.98647802E-01
0.49880093E+00 0.64345180E-01 0.68905272E-01
0.52880407E+00

EIGENVALUE(5) = 0.12150000E+03 EIGENVECTOR FOLLOWS -------

-0.50068064E+00 -0.38218672E+00 0.17182653E+00
-0.10168549E+00 0.12884359E-02 -0.13443655E+00
0.36397044E+00 0.55261520E+00 -0.30530514E+00
0.11934741E+00

EIGENVALUE(6) = 0.13950000E+03 EIGENVECTOR FOLLOWS -------

0.12884043E+00 0.16590352E-02 -0.31337180E+00
0.34282076E+00 0.56194117E+00 0.68560882E-01
-0.42227198E-03 0.51218557E+00 0.42360631E+00
0.73582492E-01

EIGENVALUE(7) = 0.13950000E+03 EIGENVECTOR FOLLOWS -------

0.26547118E+00 -0.78236593E+00 -0.16474738E+00
-0.19693997E+00 0.22911330E+00 0.19399818E+00
0.19912301E+00 -0.34619697E+00 0.43255467E-01
0.15604550E-03

EIGENVALUE(8) = 0.15750000E+03 EIGENVECTOR FOLLOWS -------

0.20980540E+00 0.10069925E-07 0.45303535E+00
0.62551863E-01 -0.17035707E+00 0.80802161E+00
-0.15716068E-08 0.25040924E+00 0.50412584E-02
0.46661380E-01

EIGENVALUE(9) = 0.15750000E+03 EIGENVECTOR FOLLOWS -------

-0.54291549E+00 -0.11874363E-07 -0.40817154E+00
-0.16186611E+00 -0.33314456E+00 0.32828192E+00
-0.28172508E-07 -0.10764905E+00 0.47024722E+00
0.24690701E+00

EIGENVALUE(10) = 0.16650000E+03 EIGENVECTOR FOLLOWS -------

0.27602621E+00 -0.36028603E-07 -0.66918868E+00
0.82295104E-01 -0.36600868E+00 0.14712741E+00
0.23993147E-07 0.26288211E+00 -0.48495337E+00
-0.96589044E-01

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 7

DIMENSION = 4

!	NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
	0	0	0	1	0	3	3	1	7	7
	0	1	1	1	2	2	2	1	6	7
	0	1	1	1	2	2	2	1	8	7
	0	2	2	1	4	1	1	1	4	7

EIGENVALUES

0.10800000E+03 0.13200000E+03 0.13200000E+03
0.15000000E+03

EIGENVALUE(1) = 0.10800000E+03 EIGENVECTOR FOLLOWS -------

-0.37408783E+00 0.12193771E+00 -0.63176373E+00
0.66788029E+00

EIGENVALUE(2) = 0.13200000E+03 EIGENVECTOR FOLLOWS -------

0.35731377E+00 0.12151810E-01 0.57156544E+00

```
 0.73857439E+00

EIGENVALUE(  3) =  0.13200000E+03   EIGENVECTOR FOLLOWS -------

 0.37098922E+00      -0.87239713E+00      -0.30945423E+00
 0.74352690E-01

EIGENVALUE(  4) =  0.15000000E+03   EIGENVECTOR FOLLOWS -------

-0.77120177E+00      -0.47318787E+00       0.42240455E+00
 0.53994901E-01

# OF BOSONS =  3 , FINAL ANGULAR MOMENTUM =  8

DIMENSION =    5

!   NS ;  ND   VD   ALPD   LD ;  NG   VG   ALPG   LG   L  >
     1     0    0      1     0     2    2      1    8    8
     0     0    0      1     0     3    3      1    8    8
     0     1    1      1     2     2    2      1    6    8
     0     1    1      1     2     2    2      1    8    8
     0     2    2      1     4     1    1      1    4    8

EIGENVALUES

 0.54000003E+02       0.12000000E+03       0.12000000E+03
 0.14400000E+03       0.16200000E+03

EIGENVALUE(  1) =  0.54000003E+02   EIGENVECTOR FOLLOWS -------

 0.40151230E+00      -0.15790840E+00      -0.32447092E+00
-0.34112113E+00       0.76955039E+00

EIGENVALUE(  2) =  0.12000000E+03   EIGENVECTOR FOLLOWS -------

 0.53881373E+00      -0.31852784E+00      -0.52968067E-01
-0.50443214E+00      -0.59242077E+00

EIGENVALUE(  3) =  0.12000000E+03   EIGENVECTOR FOLLOWS -------

 0.52362385E+00       0.47603344E+00       0.69354830E+00
 0.31957743E-01       0.13107133E+00

EIGENVALUE(  4) =  0.14400000E+03   EIGENVECTOR FOLLOWS -------

 0.39999996E+00       0.49815997E+00      -0.62629459E+00
 0.40355558E+00      -0.19166297E+00

EIGENVALUE(  5) =  0.16200000E+03   EIGENVECTOR FOLLOWS -------
```

```
-0.33806174E+00       0.63153286E+00      -0.13659749E+00
-0.68213353E+00      -0.53994905E-01

# OF BOSONS =  3 , FINAL ANGULAR MOMENTUM =  9

DIMENSION =    2

!   NS ;  ND   VD   ALPD   LD ;  NG   VG   ALPG   LG   L  >
     0     0    0      1     0     3    3      1    9    9
     0     1    1      1     2     2    2      1    8    9

EIGENVALUES

 0.13350000E+03       0.15750000E+03

EIGENVALUE(  1) =  0.13350000E+03   EIGENVECTOR FOLLOWS -------

-0.46291003E+00       0.88640526E+00

EIGENVALUE(  2) =  0.15750000E+03   EIGENVECTOR FOLLOWS -------

 0.88640526E+00       0.46291003E+00

# OF BOSONS =  3 , FINAL ANGULAR MOMENTUM =  10

DIMENSION =    2

!   NS ;  ND   VD   ALPD   LD ;  NG   VG   ALPG   LG   L  >
     0     0    0      1     0     3    3      1   10   10
     0     1    1      1     2     2    2      1    8   10

EIGENVALUES

 0.82500004E+02       0.14850000E+03

EIGENVALUE(  1) =  0.82500004E+02   EIGENVECTOR FOLLOWS -------

-0.32232918E+00       0.94662765E+00

EIGENVALUE(  2) =  0.14850000E+03   EIGENVECTOR FOLLOWS -------

 0.94662765E+00       0.32232918E+00

# OF BOSONS =  3 , FINAL ANGULAR MOMENTUM =  11

DIMENSION =    0

!   NS ;  ND   VD   ALPD   LD ;  NG   VG   ALPG   LG   L  >
```

```
EIGENVALUES


# OF BOSONS =  3 , FINAL ANGULAR MOMENTUM =  12

DIMENSION =    1

!   NS ;  ND   VD   ALPD   LD ;  NG   VG   ALPG   LG   L  >
     0     0    0      1     0     3    3      1   12   12

EIGENVALUES

 0.11700000E+03

EIGENVALUE(  1) =  0.11700000E+03   EIGENVECTOR FOLLOWS -------

 0.10000000E+01
```

Test output - 4
(Unit #20 MATRIX.DAT)

C2(SU(3)) INTERACTION

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 0

DIMENSION = 7

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
3	0	0	1	0	0	0	1	0	0
1	0	0	1	0	2	0	1	0	0
0	0	0	1	0	3	3	1	0	0
0	1	1	1	2	2	2	1	2	0
1	2	0	1	0	0	0	1	0	0
0	2	2	1	4	1	1	1	4	0
0	3	3	1	0	0	0	1	0	0

-84.00000 0.00000 0.00000 0.00000 -43.37742 0.00000
0.00000 0.00000 -69.71429 0.00000 26.53300 -13.79973
-21.46625 0.00000 0.00000 0.00000 -31.95918 27.52897
0.00000 -6.80534 0.00000 0.00000 26.53300 27.52897
-48.32653 0.00000 25.42697 -17.90604 -43.37742 -13.79973
0.00000 0.00000-114.68571 -28.80000 48.19958 0.00000
-21.46625 -6.80534 25.42697 -28.80000 -89.92653 17.70597
0.00000 0.00000 0.00000 -17.90604 48.19958 17.70597
-59.38775

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 1

DIMENSION = 2

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
0	1	1	1	2	2	2	1	2	1
0	2	2	1	4	1	1	1	4	1

-46.07143 10.44519 10.44519 -85.92857

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 2

DIMENSION = 11

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
1	0	0	1	0	2	2	1	2	2
0	0	0	1	0	3	3	1	2	2
2	1	1	1	2	0	0	1	0	2
1	1	1	1	2	1	1	1	4	2
0	1	1	1	2	2	2	1	2	2
0	1	1	1	2	2	2	1	4	2
0	1	1	1	2	2	0	1	0	2
1	2	2	1	2	0	0	1	0	2
0	2	2	1	2	1	1	1	4	2
0	2	2	1	4	1	1	1	4	2
0	3	1	1	2	0	0	1	0	2

-63.01020 0.00000 0.00000 3.85277 -11.74560 16.58522
11.86592 -10.33806 0.00000 -21.46625 0.00000 0.00000
-39.94898 0.00000 0.00000 -5.17027 -10.79887 -16.97551
0.00000 11.50313 3.29034 0.00000 0.00000 0.00000
-101.90000 -20.36468 0.00000 0.00000 0.00000 27.82804
0.00000 0.00000 29.63242 3.85277 0.00000 -20.36468
-88.19388 -11.37128 -5.79824 -6.78823 7.22843 33.73268
4.02036 15.39425 -11.74560 -5.17027 0.00000 -11.37128
-64.05102 -3.74622 -21.63335 0.00000 7.26485 -4.76215
7.81484 16.58522 -10.79887 0.00000 -5.79824 -3.74622
-43.84286 -3.51426 0.00000 14.81744 15.89386 3.98480
11.86592 -16.97551 0.00000 -6.78823 -21.63335 -3.51426
-69.32857 0.00000 3.40752 8.93458 16.32807 -10.33806
0.00000 27.82804 7.22843 0.00000 0.00000 0.00000
-93.69592 -8.22857 -21.57550 -21.03602 0.00000 11.50313
0.00000 33.73268 7.26485 14.81744 3.40752 -8.22857
-52.97755 -3.33907 -17.38692 -21.46625 3.29034 0.00000
4.02036 -4.76215 15.89386 8.93458 -21.57550 -3.33907
-78.88776 -10.13086 0.00000 0.00000 29.63242 15.39425
7.81484 3.98480 16.32807 -21.03602 -17.38692 -10.13086
-97.66327

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 3

DIMENSION = 7

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
0	0	0	1	0	3	3	1	3	3
1	1	1	1	2	1	1	1	4	3
0	1	1	1	2	2	2	1	2	3
0	1	1	1	2	2	2	1	4	3
0	2	2	1	2	1	1	1	4	3
0	2	2	1	4	1	1	1	4	3
0	3	3	1	3	0	0	1	0	3

-31.22449 0.00000 15.99472 -10.64673 -5.75157 5.48390
0.00000 0.00000 -76.20000 8.04071 11.59648 21.08293
8.04071 -14.05296 15.99472 8.04071 -76.39942 -13.05567
-1.28426 -4.04082 15.13337 -10.64673 11.59648 -13.05567
-45.46181 -4.63045 17.21835 -3.63761 -5.75157 21.08293
-1.28426 -4.63045 -50.62391 -4.03623 -6.17245 5.48390
8.04071 -4.04082 17.21835 -4.03623 -70.29446 0.00000
0.00000 -14.05296 15.13337 -3.63761 -6.17245 0.00000
-66.79592

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 4

DIMENSION = 13

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
2	0	0	1	0	1	1	1	4	4
1	0	0	1	0	2	2	1	4	4
0	0	0	1	0	3	3	1	4	4
0	0	0	1	0	3	1	1	4	4
1	1	1	1	2	1	1	1	4	4
0	1	1	1	2	2	2	1	2	4
0	1	1	1	2	2	2	1	4	4
0	1	1	1	2	2	2	1	6	4
1	2	2	1	4	0	0	1	0	4
0	2	2	1	2	1	1	1	4	4
0	2	2	1	4	1	1	1	4	4
0	2	0	1	0	1	1	1	4	4
0	3	1	1	4	0	0	1	0	4

-69.00000 0.00000 0.00000 0.00000 18.76166 0.00000
0.00000 0.00000 -21.46625 0.00000 0.00000 -25.04396
0.00000 0.00000 -51.65306 0.00000 0.00000 11.23865
12.36189 -3.96272 11.44155 -3.92907 0.00000 -21.46625
0.00000 0.00000 0.00000 0.00000 -45.30612 15.23002
0.00000 -11.96644 -3.27497 0.40275 0.00000 -12.38439
1.61919 0.00000 0.00000 0.00000 0.00000 15.23002
-60.42857 0.00000 -11.51020 -4.79218 -5.89330 0.00000
4.64661 2.36932 15.25618 0.00000 18.76166 11.23865
0.00000 0.00000 -70.60816 -2.42437 -9.48803 -11.66814
14.12681 11.37200 11.72752 8.39047 -18.14229 0.00000
12.36189 -11.96644 -11.51020 -2.42437 -58.22449 -19.83430
-2.52570 0.00000 4.25939 10.85937 1.81621 -12.95944
0.00000 -3.96272 -3.27497 -4.79218 -9.48803 -19.83430
-49.26939 -4.95639 0.00000 -2.27313 6.95114 7.10795
-4.69613 0.00000 11.44155 0.40275 -5.89330 -11.66814
-2.52570 -4.95639 -47.57143 0.00000 13.31161 9.34602
8.74118 0.00000 -21.46625 -3.92907 0.00000 0.00000
14.12681 0.00000 0.00000 0.00000-102.33878 -16.08143
-8.19995 -9.60000 34.88430 0.00000 0.00000 -12.38439
4.64661 11.37200 4.25939 -2.27313 13.31161 -16.08143
-61.83674 -3.74622 -21.63335 -3.41511 0.00000 -21.46625
1.61919 2.36932 11.72752 10.85937 6.95114 9.34602
-8.19995 -3.74622 -61.62857 -3.51426 18.57464 -25.04396
0.00000 0.00000 15.25618 8.39047 1.81621 7.10795
8.74118 -9.60000 -21.63335 -3.51426 -81.40000 14.95041
0.00000 0.00000 0.00000 0.00000 -18.14229 -12.95944
-4.69613 0.00000 34.88430 -3.41511 18.57464 14.95041
-71.73469

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 5

DIMENSION = 6

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
0	0	0	1	0	3	3	1	5	5
1	1	1	1	2	1	1	1	4	5
0	1	1	1	2	2	2	1	4	5
0	1	1	1	2	2	2	1	6	5
0	2	2	1	2	1	1	1	4	5
0	2	2	1	4	1	1	1	4	5

-60.15306 0.00000 6.00556 -2.89391 12.97728 2.54505
0.00000 -62.70000 3.68979 11.66814 10.73313 13.68211
6.00556 3.68979 -59.59870 -7.54018 -9.42928 0.69346
-2.89391 11.66814 -7.54018 -41.32375 5.32464 15.11550
12.97728 10.73313 -9.42928 5.32464 -69.94898 -6.86807
2.54505 13.68211 0.69346 15.11550 -6.86807 -54.27551

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 6

DIMENSION = 10

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
1	0	0	1	0	2	2	1	6	6
0	0	0	1	0	3	3	1	6	6
0	0	0	1	0	3	3	2	6	6
1	1	1	1	2	1	1	1	4	6
0	1	1	1	2	2	2	1	4	6
0	1	1	1	2	2	2	1	6	6
0	1	1	1	2	2	2	1	8	6
0	2	2	1	2	1	1	1	4	6
0	2	2	1	4	1	1	1	4	6
0	3	3	1	6	0	0	1	0	6

-45.92857 0.00000 0.00000 11.49978 9.51995 7.33410
5.17509 0.00000 -21.46625 0.00000 0.00000 -51.36735
3.85119 0.00000 -8.64157 -6.90070 -7.52011 9.33668
4.17549 0.00000 0.00000 3.85119 -23.38775 0.00000
11.88337 4.75527 0.39207 -5.71458 3.02030 0.00000
11.49978 0.00000 0.00000 -92.27143 -0.56766 -4.57667
-17.35777 26.83282 12.00000 -26.29068 9.51995 -8.64157
11.88337 -0.56766 -51.58257 -16.90416 -0.87976 6.34663
17.33390 -6.80534 7.33410 -6.90070 4.75527 -4.57667
-16.90416 -34.45510 -3.17986 13.26195 10.42582 0.00000
5.17509 -7.52011 0.39207 -17.35777 -0.87976 -3.12986
-75.42559 8.71316 3.89664 0.00000 0.00000 9.33668
-5.71458 26.83282 6.34663 13.26195 8.71316 -59.47959
-15.88065 13.19725 -21.46625 4.17549 3.02030 12.00000
17.33390 10.42582 3.89664 -15.88065 -50.78571 22.13246

```
  0.00000   0.00000   0.00000 -26.29068  -6.80534   0.00000
  0.00000  13.19725  22.13246 -85.31633
```

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 7

DIMENSION = 4

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
0	0	0	1	0	3	3	1	7	7
0	1	1	1	2	2	2	1	6	7
0	1	1	1	2	2	2	1	8	7
0	2	2	1	4	1	1	1	4	7

```
-40.65306   7.66339 -11.53571   5.24676   7.66339 -44.32653
 -1.74892  -2.41445 -11.53571  -1.74892 -54.36735  10.53716
  5.24676  -2.41445  10.53716 -58.65306
```

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 8

DIMENSION = 5

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
1	0	0	1	0	2	2	1	8	8
0	0	0	1	0	3	3	1	8	8
0	1	1	1	2	2	2	1	6	8
0	1	1	1	2	2	2	1	8	8
0	2	2	1	4	1	1	1	4	8

```
-62.00000   0.00000   4.52548  22.59911 -21.46625   0.00000
-38.93878 -14.49266 -16.82347   4.29655   4.52548 -14.49266
-56.75102  -9.45754  19.67066  22.59911 -16.82347  -9.45754
-44.22857  17.01627 -21.46625   4.29655  19.67066  17.01627
-98.08163
```

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 9

DIMENSION = 2

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
0	0	0	1	0	3	3	1	9	9
0	1	1	1	2	2	2	1	8	9

```
-27.64286   9.84782   9.84782 -41.35714
```

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 10

DIMENSION = 2

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
0	0	0	1	0	3	3	1	10	10
0	1	1	1	2	2	2	1	8	10

```
-38.35714  20.13830  20.13830 -90.64286
```

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 11

DIMENSION = 0

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >

OF BOSONS = 3 , FINAL ANGULAR MOMENTUM = 12

DIMENSION = 1

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
0	0	0	1	0	3	3	1	12	12

```
-63.00000
```

Test output - 5
(Unit #35 SDGOUT.DAT)

C2(SU(3)) INTERACTION

S.P.E. AND TWO-BODY MATRIX ELEMENTS

ES = -28.00000 ED = -23.50000 EG = -13.00000

L1	L2	L3	L4	LL	V(L1 L2 L3 L4 LL)
0	0	0	0	0	0.00000000E+00
0	0	2	2	0	-0.25043962E+02
0	0	4	4	0	0.00000000E+00
0	2	0	2	2	-0.11200000E+02
0	2	2	2	2	0.19677398E+02
0	2	2	4	2	-0.14400000E+02
0	2	4	4	2	0.00000000E+00
0	4	0	4	4	0.00000000E+00
0	4	2	2	4	-0.15178933E+02
0	4	2	4	4	0.13266499E+02
0	4	4	4	4	0.00000000E+00
2	2	2	2	0	-0.17285714E+02
2	2	2	2	2	0.37040817E+01
2	2	2	2	4	-0.49387755E+01
2	2	2	4	2	0.72284321E+01
2	2	2	4	4	0.14126813E+02
2	2	4	4	0	-0.13799734E+02
2	2	4	4	2	-0.10338059E+02
2	2	4	4	4	-0.39290671E+01
2	4	2	4	2	-0.12493878E+02
2	4	2	4	3	-0.50000001E+00
2	4	2	4	4	0.50918369E+01
2	4	2	4	5	0.13000000E+02
2	4	2	4	6	-0.16571429E+02
2	4	4	4	2	0.38527672E+01
2	4	4	4	4	0.11238655E+02
2	4	4	4	6	0.11499778E+02
4	4	4	4	0	-0.15714286E+02
4	4	4	4	2	-0.90102041E+01
4	4	4	4	4	0.23469388E+01
4	4	4	4	6	0.80714288E+01
4	4	4	4	8	-0.80000001E+01

OF BOSONS = 4 , FINAL ANGULAR MOMENTUM = 0

DIMENSION = 15

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
4	0	0	1	0	0	0	1	0	0
2	0	0	1	0	2	0	1	0	0
1	0	0	1	0	3	3	1	0	0
0	0	0	1	0	4	4	1	0	0
0	0	0	1	0	4	0	1	0	0
1	1	1	1	2	2	2	1	2	0
0	1	1	1	2	3	3	1	2	0
2	2	0	1	0	0	0	1	0	0
1	2	2	1	4	1	1	1	4	0
0	2	2	1	2	2	2	1	2	0
0	2	2	1	4	2	2	1	4	0
0	2	0	1	0	2	0	1	0	0
1	3	3	1	0	0	0	1	0	0
0	3	3	1	4	1	1	1	4	0
0	4	0	1	0	0	0	1	0	0

EIGENVALUES

```
 0.00000000E+00   0.90000002E+02   0.15600000E+03
 0.15600001E+03   0.17400000E+03   0.19800000E+03
 0.19800001E+03   0.21600001E+03   0.22800000E+03
 0.22800001E+03   0.25800000E+03   0.25800001E+03
 0.27600000E+03   0.27600000E+03   0.29400001E+03
```

OF BOSONS = 4 , FINAL ANGULAR MOMENTUM = 1

DIMENSION = 8

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
1	1	1	1	2	2	2	1	2	1
0	1	1	1	2	3	3	1	2	1
0	1	1	1	2	3	3	1	3	1
1	2	2	1	4	1	1	1	4	1
0	2	2	1	2	2	2	1	2	1
0	2	2	1	4	2	2	1	4	1
0	3	3	1	3	1	1	1	4	1
0	3	3	1	4	1	1	1	4	1

EIGENVALUES

```
 0.12750001E+03   0.19650001E+03   0.20550001E+03
 0.21750001E+03   0.23250001E+03   0.24750001E+03
```

0.25650001E+03 0.28050001E+03

OF BOSONS = 4 , FINAL ANGULAR MOMENTUM = 2

DIMENSION = 32

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
2	0	0	1	0	2	2	1	2	2
1	0	0	1	0	3	3	1	2	2
0	0	0	1	0	4	4	1	2	2
0	0	0	1	0	4	4	2	2	2
0	0	0	1	0	4	2	1	2	2
3	1	1	1	2	0	0	1	0	2
2	1	1	1	2	1	1	1	4	2
1	1	1	1	2	2	2	1	2	2
1	1	1	1	2	2	2	1	4	2
1	1	1	1	2	2	0	1	0	2
0	1	1	1	2	3	3	1	0	2
0	1	1	1	2	3	3	1	2	2
0	1	1	1	2	3	3	1	3	2
0	1	1	1	2	3	3	1	4	2
0	1	1	1	2	3	1	1	4	2
2	2	2	1	2	0	0	1	0	2
1	2	2	1	2	1	1	1	4	2
1	2	2	1	4	1	1	1	4	2
0	2	2	1	2	2	2	1	2	2
0	2	2	1	2	2	2	1	4	2
0	2	2	1	2	2	0	1	0	2
0	2	2	1	4	2	2	1	2	2
0	2	2	1	4	2	2	1	4	2
0	2	2	1	4	2	2	1	6	2
0	2	0	1	0	2	2	1	2	2
0	3	1	1	2	0	0	1	0	2
0	3	3	1	3	1	1	1	4	2
0	3	3	1	4	1	1	1	4	2
0	3	3	1	6	1	1	1	4	2
0	3	1	1	2	1	1	1	4	2
0	4	4	1	2	0	0	1	0	2
0	4	2	1	2	0	0	1	0	2

EIGENVALUES

0.45000000E+01 0.94499990E+02 0.94500008E+02
0.13050000E+03 0.16049999E+03 0.16050000E+03
0.16050000E+03 0.16050001E+03 0.17850001E+03
0.19950000E+03 0.20250001E+03 0.20250001E+03
0.20250001E+03 0.20250001E+03 0.20850001E+03
0.22050000E+03 0.22050001E+03 0.23250000E+03
0.23250001E+03 0.23250001E+03 0.23750001E+03
0.23550000E+03 0.25050000E+03 0.25950000E+03
0.26250000E+03 0.26250000E+03 0.26250000E+03
0.26250001E+03 0.28050000E+03 0.28050001E+03
0.28350000E+03 0.29850001E+03

OF BOSONS = 4 , FINAL ANGULAR MOMENTUM = 3

DIMENSION = 22

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
1	0	0	1	0	3	3	1	3	3
0	0	0	1	0	4	4	1	3	3
2	1	1	1	2	1	1	1	4	3
1	1	1	1	2	2	2	1	2	3
1	1	1	1	2	2	2	1	4	3
0	1	1	1	2	3	3	1	2	3
0	1	1	1	2	3	3	1	3	3
0	1	1	1	2	3	3	1	4	3
0	1	1	1	2	3	3	1	5	3
0	1	1	1	2	3	1	1	4	3
1	2	2	1	2	1	1	1	4	3
1	2	2	1	4	1	1	1	4	3
0	2	2	1	2	2	2	1	2	3
0	2	2	1	2	2	2	1	4	3
0	2	2	1	4	2	2	1	2	3
0	2	2	1	4	2	2	1	4	3
0	2	2	1	4	2	2	1	6	3
1	3	3	1	3	0	0	1	0	3
0	3	3	1	3	1	1	1	4	3
0	3	3	1	4	1	1	1	4	3
0	3	3	1	6	1	1	1	4	3
0	3	1	1	2	1	1	1	4	3

EIGENVALUES

0.99000006E+02 0.13500000E+03 0.13500000E+03
0.16500000E+03 0.16500001E+03 0.20400001E+03
0.20400001E+03 0.20700000E+03 0.20700000E+03
0.21300001E+03 0.22500001E+03 0.22500001E+03
0.23700000E+03 0.23700001E+03 0.24000001E+03
0.24000001E+03 0.25500000E+03 0.25500001E+03
0.26400001E+03 0.26700001E+03 0.26700001E+03
0.28800001E+03

OF BOSONS = 4 , FINAL ANGULAR MOMENTUM = 4

DIMENSION = 40

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
3	0	0	1	0	1	1	1	4	4
2	0	0	1	0	2	2	1	4	4
1	0	0	1	0	3	3	1	4	4
1	0	0	1	0	3	1	1	4	4
0	0	0	1	0	4	4	1	4	4
0	0	0	1	0	4	4	2	4	4
0	0	0	1	0	4	4	3	4	4
0	0	0	1	0	4	2	1	4	4
2	1	1	1	2	1	1	1	4	4
1	1	1	1	2	2	2	1	2	4
1	1	1	1	2	2	2	1	4	4
1	1	1	1	2	2	2	1	6	4
0	1	1	1	2	3	3	1	2	4
0	1	1	1	2	3	3	1	3	4
0	1	1	1	2	3	3	1	4	4
0	1	1	1	2	3	3	1	5	4
0	1	1	1	2	3	3	1	6	4
0	1	1	1	2	3	3	2	6	4
0	1	1	1	2	3	1	1	4	4
2	2	2	1	4	0	0	1	0	4
1	2	2	1	2	1	1	1	4	4
1	2	2	1	4	1	1	1	4	4
1	2	0	1	0	1	1	1	4	4
0	2	2	1	2	2	2	1	2	4
0	2	2	1	2	2	2	1	4	4
0	2	2	1	2	2	2	1	6	4
0	2	2	1	4	2	2	1	2	4
0	2	2	1	4	2	2	1	4	4
0	2	2	1	4	2	2	1	6	4
0	2	2	1	4	2	2	1	8	4
0	2	2	1	4	2	0	1	0	4
0	2	0	1	0	2	2	1	4	4
1	3	3	1	4	0	0	1	0	4
0	3	3	1	0	1	1	1	4	4
0	3	3	1	3	1	1	1	4	4
0	3	3	1	4	1	1	1	4	4
0	3	3	1	6	1	1	1	4	4
0	3	1	1	2	1	1	1	4	4
0	4	4	1	4	0	0	1	0	4
0	4	2	1	4	0	0	1	0	4

EIGENVALUES

0.15000008E+02 0.10499999E+03 0.10500000E+03
0.14099999E+03 0.14100000E+03 0.17100000E+03
0.17100000E+03 0.17100000E+03 0.17100000E+03
0.17100001E+03 0.17100001E+03 0.18900000E+03
0.21000001E+03 0.21000001E+03 0.21300000E+03
0.21300000E+03 0.21300001E+03 0.21300001E+03
0.21300001E+03 0.21300001E+03 0.21900000E+03
0.23100001E+03 0.23100001E+03 0.23100001E+03
0.24300000E+03 0.24300000E+03 0.24300000E+03
0.24300001E+03 0.24600000E+03 0.24600001E+03
0.26100001E+03 0.26100001E+03 0.27000000E+03
0.27300000E+03 0.27300000E+03 0.27300000E+03
0.27300001E+03 0.29100000E+03 0.29100001E+03
0.29400001E+03

OF BOSONS = 4 , FINAL ANGULAR MOMENTUM = 5

DIMENSION = 26

! NS ;	ND	VD	ALPD	LD ;	NG	VG	ALPG	LG	L >
1	0	0	1	0	3	3	1	5	5
0	0	0	1	0	4	4	1	5	5
0	0	0	1	0	4	4	2	5	5
2	1	1	1	2	1	1	1	4	5
1	1	1	1	2	2	2	1	4	5
1	1	1	1	2	2	2	1	6	5
0	1	1	1	2	3	3	1	3	5
0	1	1	1	2	3	3	1	4	5
0	1	1	1	2	3	3	1	5	5
0	1	1	1	2	3	3	1	6	5
0	1	1	1	2	3	3	2	6	5
0	1	1	1	2	3	3	1	7	5
0	1	1	1	2	3	1	1	4	5
1	2	2	1	2	1	1	1	4	5
1	2	2	1	4	1	1	1	4	5
0	2	2	1	2	2	2	1	4	5
0	2	2	1	2	2	2	1	6	5
0	2	2	1	4	2	2	1	2	5
0	2	2	1	4	2	2	1	4	5
0	2	2	1	4	2	2	1	6	5
0	2	2	1	4	2	2	1	8	5
0	3	3	1	3	1	1	1	4	5
0	3	3	1	4	1	1	1	4	5
0	3	3	1	6	1	1	1	4	5
0	3	1	1	2	1	1	1	4	5
0	4	4	1	5	0	0	1	0	5

EIGENVALUES

0.11250001E+03 0.14850000E+03 0.14850000E+03
0.17850000E+03 0.17850001E+03 0.17850001E+03
0.17850001E+03 0.21750000E+03 0.21750001E+03
0.22050000E+03 0.22050001E+03 0.22050001E+03
0.22050001E+03 0.22650001E+03 0.23850000E+03
0.23850000E+03 0.23850001E+03 0.25050000E+03

0.25050001E+03 0.25350000E+03 0.25350001E+03
0.26850000E+03 0.26850001E+03 0.27750001E+03
0.28050001E+03 0.28050001E+03

OF BOSONS = 4 , FINAL ANGULAR MOMENTUM = 6

DIMENSION = 34

NS	ND	VD	ALPD	LD	NG	VG	ALPG	LG	L
2	0	0	1	0	2	2	1	6	6
1	0	0	1	0	3	3	1	6	6
1	0	0	1	0	3	3	2	6	6
0	0	0	1	0	4	4	1	6	6
0	0	0	1	0	4	4	2	6	6
0	0	0	1	0	4	4	3	6	6
0	0	0	1	0	4	2	1	6	6
2	1	1	1	2	1	1	1	4	6
1	1	1	1	2	2	2	1	4	6
1	1	1	1	2	2	2	1	6	6
1	1	1	1	2	2	2	1	8	6
0	1	1	1	2	3	3	1	4	6
0	1	1	1	2	3	3	1	5	6
0	1	1	1	2	3	3	1	6	6
0	1	1	1	2	3	3	2	6	6
0	1	1	1	2	3	3	1	7	6
0	1	1	1	2	3	3	1	8	6
0	1	1	1	2	3	1	1	4	6
1	2	2	1	2	1	1	1	4	6
1	2	2	1	4	1	1	1	4	6
0	2	2	1	2	2	2	1	4	6
0	2	2	1	2	2	2	1	6	6
0	2	2	1	2	2	2	1	8	6
0	2	2	1	4	2	2	1	2	6
0	2	2	1	4	2	2	1	4	6
0	2	2	1	4	2	2	1	6	6
0	2	2	1	4	2	2	1	8	6
0	2	0	1	0	2	2	1	6	6
1	3	3	1	6	0	0	1	0	6
0	3	3	1	3	1	1	1	4	6
0	3	3	1	4	1	1	1	4	6
0	3	3	1	6	1	1	1	4	6
0	3	1	1	2	1	1	1	4	6
0	4	4	1	6	0	0	1	0	6

EIGENVALUES

0.31500004E+02 0.12150000E+03 0.12150001E+03
0.15749999E+03 0.15750000E+03 0.18750000E+03
0.18750000E+03 0.18750000E+03 0.18750001E+03

0.18750001E+03 0.18750001E+03 0.20550000E+03
0.22650000E+03 0.22650000E+03 0.22950000E+03
0.22950001E+03 0.22950001E+03 0.22950001E+03
0.22950001E+03 0.22950001E+03 0.23550000E+03
0.24750000E+03 0.24750000E+03 0.24750001E+03
0.25950000E+03 0.25950000E+03 0.25950001E+03
0.25950001E+03 0.26250000E+03 0.26250000E+03
0.27750000E+03 0.28650002E+03 0.28950001E+03
0.28950001E+03

Test output - 6
(Unit #35 SDGOUT.DAT)

-C2(U(5)) + 0.5*C2(O(5)) INTERACTION

S.P.E. AND TWO-BODY MATRIX ELEMENTS

ES = -12.00000 ED = -7.00000 EG = -7.00000

L1	L2	L3	L4	LL	V(L1 L2 L3 L4 LL)
0	0	0	0	0	-0.16000000E+01
0	0	2	2	0	-0.35777088E+01
0	0	4	4	0	-0.48000000E+01
0	2	0	2	2	-0.32000000E+01
0	2	2	2	2	0.10842095E+01
0	2	2	4	2	-0.27428571E+01
0	2	4	4	2	-0.50853932E+01
0	4	0	4	4	-0.32000000E+01
0	4	2	2	4	-0.14456126E+01
0	4	2	4	4	-0.53604751E+01
0	4	4	4	4	-0.19327468E+01
2	2	2	2	0	-0.31428572E+01
2	2	2	2	2	-0.18571429E+01
2	2	2	2	4	-0.17142857E+01
2	2	2	4	2	-0.21684189E+01
2	2	2	4	4	0.00000000E+00
2	2	4	4	0	-0.68998669E+01
2	2	4	4	2	0.20101782E+01
2	2	4	4	4	-0.30559410E+01
2	4	2	4	2	-0.19428571E+01
2	4	2	4	3	0.20000000E+01
2	4	2	4	4	-0.54285714E+01
2	4	2	4	5	-0.40000001E+01
2	4	2	4	6	-0.22857143E+01
2	4	4	4	2	-0.25426966E+01
2	4	4	4	4	-0.10301575E+01
2	4	4	4	6	-0.27105237E+01
4	4	4	4	0	-0.72571430E+01
4	4	4	4	2	-0.40000000E+01
4	4	4	4	4	-0.65714285E+00
4	4	4	4	6	0.28571427E+00
4	4	4	4	8	-0.40000001E+01

OF BOSONS = 4 , FINAL ANGULAR MOMENTUM = 0

DIMENSION = 15

EIGENVALUES

0.00000000E+00 0.26999998E+02 0.28000001E+02
0.36000001E+02 0.40000000E+02 0.44999999E+02
0.48000001E+02 0.52000000E+02 0.54999999E+02
0.57000002E+02 0.60000000E+02 0.60000000E+02
0.64000001E+02 0.68999999E+02 0.72000000E+02

OF BOSONS = 4 , FINAL ANGULAR MOMENTUM = 1

DIMENSION = 8

EIGENVALUES

0.38000000E+02 0.44999999E+02 0.49000000E+02
0.56999999E+02 0.58000000E+02 0.62000001E+02
0.64000000E+02 0.69000001E+02

OF BOSONS = 4 , FINAL ANGULAR MOMENTUM = 2

DIMENSION = 37

EIGENVALUES

0.49999971E+01 0.14000001E+02 0.32999999E+02
0.33000000E+02 0.35999996E+02 0.37999998E+02
0.41999999E+02 0.42000000E+02 0.44000000E+02
0.44999999E+02 0.44999999E+02 0.45000000E+02
0.47999999E+02 0.48999999E+02 0.54000000E+02
0.56999999E+02 0.57000000E+02 0.57000000E+02
0.57000000E+02 0.57999999E+02 0.57999999E+02
0.58000000E+02 0.59999998E+02 0.60000000E+02
0.61999999E+02 0.63999997E+02 0.63999999E+02
0.63999999E+02 0.65999998E+02 0.69000000E+02
0.69000000E+02 0.77000000E+02

OF BOSONS = 4 , FINAL ANGULAR MOMENTUM = 3

DIMENSION = 22

EIGENVALUES

0.27000000E+02 0.36000001E+02 0.38000000E+02

```
 0.38000001E+02      0.45000000E+02      0.45000000E+02
 0.48000000E+02      0.48999999E+02      0.49000000E+02
 0.55000000E+02      0.56999998E+02      0.57000000E+02
 0.58000000E+02      0.58000000E+02      0.59999999E+02
 0.60000000E+02      0.62000000E+02      0.62000000E+02
 0.64000001E+02      0.64000001E+02      0.69000001E+02
 0.69000002E+02

# OF BOSONS =  4 , FINAL ANGULAR MOMENTUM =  4

DIMENSION =   40

EIGENVALUES

 0.50000010E+01      0.14000000E+02      0.26999997E+02
 0.32999999E+02      0.33000000E+02      0.36000000E+02
 0.37999998E+02      0.42000000E+02      0.42000002E+02
 0.43999999E+02      0.44999999E+02      0.44999999E+02
 0.45000000E+02      0.45000001E+02      0.47999998E+02
 0.48999999E+02      0.49000000E+02      0.53999999E+02
 0.55000000E+02      0.56999998E+02      0.57000000E+02
 0.57000000E+02      0.57000001E+02      0.57000002E+02
 0.58000000E+02      0.58000000E+02      0.58000000E+02
 0.58000001E+02      0.60000000E+02      0.60000000E+02
 0.61999999E+02      0.63999998E+02      0.64000001E+02
 0.64000002E+02      0.64000002E+02      0.65999998E+02
 0.68999998E+02      0.68999999E+02      0.68999999E+02
 0.77000000E+02

# OF BOSONS =  4 , FINAL ANGULAR MOMENTUM =  5

DIMENSION =   26

EIGENVALUES

 0.14000000E+02      0.37999998E+02      0.38000000E+02
 0.41999999E+02      0.42000000E+02      0.44000000E+02
 0.44999999E+02      0.45000000E+02      0.48999998E+02
 0.49000000E+02      0.49000000E+02      0.54000000E+02
 0.57000000E+02      0.57000000E+02      0.58000000E+02
 0.58000000E+02      0.58000001E+02      0.58000002E+02
 0.61999998E+02      0.62000000E+02      0.63999998E+02
 0.64000000E+02      0.64000000E+02      0.66000001E+02
 0.68999998E+02      0.69000002E+02

# OF BOSONS =  4 , FINAL ANGULAR MOMENTUM =  6

DIMENSION =   34
```

```
EIGENVALUES

 0.14000004E+02      0.27000000E+02      0.27000001E+02
 0.35999999E+02      0.38000002E+02      0.41999996E+02
 0.41999998E+02      0.43999999E+02      0.45000000E+02
 0.45000000E+02      0.45000000E+02      0.47999999E+02
 0.49000000E+02      0.49000001E+02      0.54000000E+02
 0.54999999E+02      0.55000001E+02      0.56999999E+02
 0.57000001E+02      0.57000001E+02      0.58000000E+02
 0.58000000E+02      0.58000001E+02      0.58000002E+02
 0.59999999E+02      0.60000000E+02      0.61999999E+02
 0.63999996E+02      0.63999999E+02      0.64000002E+02
 0.66000000E+02      0.68999999E+02      0.69000000E+02
 0.69000001E+02
```

Test output - 7
(Unit #35 SDGOUT.DAT)

-C2(U(6)) + 0.5*C2(Sp(6)) INTERACTION

S.P.E. AND TWO-BODY MATRIX ELEMENTS

ES = -10.00000 ED = -4.00000 EG = -4.00000

L1	L2	L3	L4	LL	V(L1 L2 L3 L4 LL)
0	0	0	0	0	-0.13333334E+01
0	0	2	2	0	-0.29814241E+01
0	0	4	4	0	-0.40000001E+01
0	2	0	2	2	-0.26666667E+01
0	2	2	2	2	0.13468701E+01
0	2	2	4	2	-0.29692300E+01
0	2	4	4	2	-0.18952142E+01
0	4	0	4	4	-0.26666667E+01
0	4	2	2	4	-0.15649216E+01
0	4	2	4	4	-0.19977312E+01
0	4	4	4	4	0.27896797E+01
2	2	2	2	0	-0.36666667E+01
2	2	2	2	2	-0.16598649E+01
2	2	2	2	4	-0.15408164E+01
2	2	2	4	2	-0.74984381E+00
2	2	2	4	4	0.58617985E+00
2	2	4	4	0	-0.22360680E+01
2	2	4	4	2	-0.47861384E+00
2	2	4	4	4	-0.81855567E+00
2	4	2	4	2	-0.34693877E+00
2	4	2	4	3	-0.20000001E+01
2	4	2	4	4	-0.12517007E+01
2	4	2	4	5	-0.20000001E+01
2	4	2	4	6	-0.20000000E+01
2	4	4	4	2	0.10551237E+01
2	4	4	4	4	-0.10449432E+01
2	4	4	4	6	-0.14901161E-07
4	4	4	4	0	-0.50000000E+01
4	4	4	4	2	-0.13265307E+01
4	4	4	4	4	-0.54081634E+00
4	4	4	4	6	-0.20000000E+01
4	4	4	4	8	-0.20000001E+01

```
# OF BOSONS =  4 , FINAL ANGULAR MOMENTUM =  0

DIMENSION =   15

EIGENVALUES

 0.00000000E+00      0.14000001E+02      0.20000000E+02
 0.23999998E+02      0.24000000E+02      0.32000000E+02
 0.34000000E+02      0.34000001E+02      0.36000001E+02
 0.36000002E+02      0.41000001E+02      0.44000001E+02
 0.44000002E+02      0.44000002E+02      0.46000001E+02

# OF BOSONS =  4 , FINAL ANGULAR MOMENTUM =  1

DIMENSION =    8

EIGENVALUES

 0.24000001E+02      0.32000002E+02      0.36000000E+02
 0.36000001E+02      0.41000001E+02      0.41000002E+02
 0.41000002E+02      0.44000002E+02

# OF BOSONS =  4 , FINAL ANGULAR MOMENTUM =  2

DIMENSION =   32

EIGENVALUES

 0.59999995E+01      0.14000002E+02      0.14000002E+02
 0.24000000E+02      0.24000000E+02      0.24000001E+02
 0.26000001E+02      0.26000002E+02      0.32000001E+02
 0.34000000E+02      0.34000000E+02      0.34000001E+02
 0.34000001E+02      0.35999999E+02      0.36000000E+02
 0.36000000E+02      0.36000000E+02      0.36000001E+02
 0.36000001E+02      0.36000002E+02      0.37999999E+02
 0.41000000E+02      0.41000000E+02      0.41000001E+02
 0.41000001E+02      0.41000001E+02      0.44000000E+02
 0.44000001E+02      0.44000001E+02      0.46000001E+02
 0.46000002E+02      0.50000001E+02

# OF BOSONS =  4 , FINAL ANGULAR MOMENTUM =  3

DIMENSION =   22

EIGENVALUES

 0.14000001E+02      0.23999999E+02      0.24000000E+02
```

```
 0.24000001E+02      0.32000002E+02      0.32000002E+02
 0.34000000E+02      0.34000001E+02      0.36000000E+02
 0.36000001E+02      0.36000002E+02      0.36000002E+02
 0.36000003E+02      0.41000000E+02      0.41000001E+02
 0.41000001E+02      0.41000002E+02      0.41000003E+02
 0.44000001E+02      0.44000001E+02      0.44000001E+02
 0.46000002E+02

# OF BOSONS =  4 , FINAL ANGULAR MOMENTUM =  4

DIMENSION =   40

EIGENVALUES

 0.60000005E+01      0.14000000E+02      0.14000000E+02
 0.24000001E+02      0.24000001E+02      0.24000002E+02
 0.24000002E+02      0.24000002E+02      0.25999999E+02
 0.26000000E+02      0.32000000E+02      0.34000000E+02
 0.34000000E+02      0.34000000E+02      0.34000002E+02
 0.35999999E+02      0.35999999E+02      0.36000000E+02
 0.36000000E+02      0.36000000E+02      0.36000000E+02
 0.36000001E+02      0.36000001E+02      0.36000002E+02
 0.36000003E+02      0.38000000E+02      0.41000000E+02
 0.41000000E+02      0.41000001E+02      0.41000001E+02
 0.41000001E+02      0.41000002E+02      0.44000000E+02
 0.44000001E+02      0.44000001E+02      0.44000001E+02
 0.44000002E+02      0.46000000E+02      0.46000001E+02
 0.50000001E+02

# OF BOSONS =  4 , FINAL ANGULAR MOMENTUM =  5

DIMENSION =   26

EIGENVALUES

 0.14000001E+02      0.24000001E+02      0.24000002E+02
 0.24000002E+02      0.32000000E+02      0.34000000E+02
 0.34000002E+02      0.36000000E+02      0.36000000E+02
 0.36000000E+02      0.36000001E+02      0.36000001E+02
 0.36000001E+02      0.36000002E+02      0.36000002E+02
 0.41000000E+02      0.41000000E+02      0.41000000E+02
 0.41000001E+02      0.41000001E+02      0.41000002E+02
 0.41000002E+02      0.44000000E+02      0.44000001E+02
 0.44000001E+02      0.46000001E+02

# OF BOSONS =  4 , FINAL ANGULAR MOMENTUM =  6

DIMENSION =   34
```

```
EIGENVALUES

 0.13999998E+02      0.14000000E+02      0.24000000E+02
 0.24000001E+02      0.24000001E+02      0.24000001E+02
 0.24000001E+02      0.32000001E+02      0.34000000E+02
 0.34000000E+02      0.34000000E+02      0.34000001E+02
 0.36000000E+02      0.36000000E+02      0.36000000E+02
 0.36000001E+02      0.36000001E+02      0.36000001E+02
 0.36000001E+02      0.36000001E+02      0.36000002E+02
 0.36000002E+02      0.41000000E+02      0.41000001E+02
 0.41000001E+02      0.41000001E+02      0.41000001E+02
 0.43999999E+02      0.44000000E+02      0.44000001E+02
 0.44000002E+02      0.44000003E+02      0.46000002E+02
 0.46000002E+02
```

```
                    Test output - 8
                (Unit #70  BE2BE4.DAT)

B(E2)'S AND Q2'S FOR THE SU3 QUADRUPOLE OPERATOR.
    # OF PARTICLES =  4

Q ( 1 , L =  6 ) = -0.25628155E+02

B ( E 2 ;  1 LI =  6 ->  2 LF =  6 ) = 0.00000000E+00

B ( E 2 ;  1 LI =  6 ->  3 LF =  6 ) = 0.00000000E+00

B ( E 2 ;  2 LI =  6 ->  1 LF =  6 ) = 0.00000000E+00

Q ( 2 , L =  6 ) = -0.13299498E+02

B ( E 2 ;  2 LI =  6 ->  3 LF =  6 ) = 0.56309965E+00

B ( E 2 ;  3 LI =  6 ->  1 LF =  6 ) = 0.00000000E+00

B ( E 2 ;  3 LI =  6 ->  2 LF =  6 ) = 0.56309972E+00

Q ( 3 , L =  6 ) = -0.23102940E+02

B ( E 2 ;  1 LI =  6 ->  1 LF =  5 ) = 0.00000000E+00

B ( E 2 ;  1 LI =  6 ->  2 LF =  5 ) = 0.00000000E+00

B ( E 2 ;  1 LI =  6 ->  3 LF =  5 ) = 0.00000000E+00

B ( E 2 ;  2 LI =  6 ->  1 LF =  5 ) = 0.30762586E+02

B ( E 2 ;  2 LI =  6 ->  2 LF =  5 ) = 0.00000000E+00

B ( E 2 ;  2 LI =  6 ->  3 LF =  5 ) = 0.00000000E+00

B ( E 2 ;  3 LI =  6 ->  1 LF =  5 ) = 0.82117701E+01

B ( E 2 ;  3 LI =  6 ->  2 LF =  5 ) = 0.00000000E+00

B ( E 2 ;  3 LI =  6 ->  3 LF =  5 ) = 0.00000000E+00

B ( E 2 ;  1 LI =  6 ->  1 LF =  4 ) = 0.11580418E+03

B ( E 2 ;  1 LI =  6 ->  2 LF =  4 ) = 0.00000000E+00

B ( E 2 ;  1 LI =  6 ->  3 LF =  4 ) = 0.00000000E+00

B ( E 2 ;  2 LI =  6 ->  1 LF =  4 ) = 0.00000000E+00

B ( E 2 ;  2 LI =  6 ->  2 LF =  4 ) = 0.12196604E+02

B ( E 2 ;  2 LI =  6 ->  3 LF =  4 ) = 0.63195403E+02

B ( E 2 ;  3 LI =  6 ->  1 LF =  4 ) = 0.00000000E+00

B ( E 2 ;  3 LI =  6 ->  2 LF =  4 ) = 0.39530026E+02

B ( E 2 ;  3 LI =  6 ->  3 LF =  4 ) = 0.17945077E+02

Q ( 1 , L =  5 ) = -0.12250822E+02

B ( E 2 ;  1 LI =  5 ->  2 LF =  5 ) = 0.00000000E+00

B ( E 2 ;  1 LI =  5 ->  3 LF =  5 ) = 0.00000000E+00

B ( E 2 ;  2 LI =  5 ->  1 LF =  5 ) = 0.00000000E+00

Q ( 2 , L =  5 ) = -0.56634718E+01

B ( E 2 ;  2 LI =  5 ->  3 LF =  5 ) = 0.60763824E+01

B ( E 2 ;  3 LI =  5 ->  1 LF =  5 ) = 0.00000000E+00

B ( E 2 ;  3 LI =  5 ->  2 LF =  5 ) = 0.60763824E+01

Q ( 3 , L =  5 ) = -0.84179326E+01

B ( E 2 ;  1 LI =  5 ->  1 LF =  4 ) = 0.00000000E+00

B ( E 2 ;  1 LI =  5 ->  2 LF =  4 ) = 0.53059168E+02

B ( E 2 ;  1 LI =  5 ->  3 LF =  4 ) = 0.14135525E+01

B ( E 2 ;  2 LI =  5 ->  1 LF =  4 ) = 0.00000000E+00

B ( E 2 ;  2 LI =  5 ->  2 LF =  4 ) = 0.00000000E+00

B ( E 2 ;  2 LI =  5 ->  3 LF =  4 ) = 0.00000000E+00

B ( E 2 ;  3 LI =  5 ->  1 LF =  4 ) = 0.00000000E+00

B ( E 2 ;  3 LI =  5 ->  2 LF =  4 ) = 0.00000000E+00
```

B (E 2 ; 3 LI = 5 -> 3 LF = 4) = 0.00000000E+00

B (E 2 ; 1 LI = 5 -> 1 LF = 3) = 0.48363625E+02

B (E 2 ; 1 LI = 5 -> 2 LF = 3) = 0.00000000E+00

B (E 2 ; 1 LI = 5 -> 3 LF = 3) = 0.00000000E+00

B (E 2 ; 2 LI = 5 -> 1 LF = 3) = 0.00000000E+00

B (E 2 ; 2 LI = 5 -> 2 LF = 3) = 0.56582345E+02

B (E 2 ; 2 LI = 5 -> 3 LF = 3) = 0.99714556E+00

B (E 2 ; 3 LI = 5 -> 1 LF = 3) = 0.00000000E+00

B (E 2 ; 3 LI = 5 -> 2 LF = 3) = 0.34600100E+01

B (E 2 ; 3 LI = 5 -> 3 LF = 3) = 0.20988765E+02

Q (1 , L = 4) = -0.23298320E+02

B (E 2 ; 1 LI = 4 -> 2 LF = 4) = 0.00000000E+00

B (E 2 ; 1 LI = 4 -> 3 LF = 4) = 0.00000000E+00

B (E 2 ; 2 LI = 4 -> 1 LF = 4) = 0.00000000E+00

Q (2 , L = 4) = -0.11949198E+02

B (E 2 ; 2 LI = 4 -> 3 LF = 4) = 0.88806000E+01

B (E 2 ; 3 LI = 4 -> 1 LF = 4) = 0.00000000E+00

B (E 2 ; 3 LI = 4 -> 2 LF = 4) = 0.88806014E+01

Q (3 , L = 4) = -0.15076856E+02

B (E 2 ; 1 LI = 4 -> 1 LF = 3) = 0.00000000E+00

B (E 2 ; 1 LI = 4 -> 2 LF = 3) = 0.00000000E+00

B (E 2 ; 1 LI = 4 -> 3 LF = 3) = 0.00000000E+00

B (E 2 ; 2 LI = 4 -> 1 LF = 3) = 0.57594965E+02

B (E 2 ; 2 LI = 4 -> 2 LF = 3) = 0.00000000E+00

B (E 2 ; 2 LI = 4 -> 3 LF = 3) = 0.00000000E+00

B (E 2 ; 3 LI = 4 -> 1 LF = 3) = 0.16627236E+02

B (E 2 ; 3 LI = 4 -> 2 LF = 3) = 0.00000000E+00

B (E 2 ; 3 LI = 4 -> 3 LF = 3) = 0.00000000E+00

B (E 2 ; 1 LI = 4 -> 1 LF = 2) = 0.11199998E+03

B (E 2 ; 1 LI = 4 -> 2 LF = 2) = 0.00000000E+00

B (E 2 ; 1 LI = 4 -> 3 LF = 2) = 0.00000000E+00

B (E 2 ; 2 LI = 4 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 2 LI = 4 -> 2 LF = 2) = 0.10863800E+01

B (E 2 ; 2 LI = 4 -> 3 LF = 2) = 0.30306392E+02

B (E 2 ; 3 LI = 4 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 3 LI = 4 -> 2 LF = 2) = 0.76400977E+02

B (E 2 ; 3 LI = 4 -> 3 LF = 2) = 0.79255682E-01

Q (1 , L = 3) = -0.13544198E-06

B (E 2 ; 1 LI = 3 -> 2 LF = 3) = 0.00000000E+00

B (E 2 ; 1 LI = 3 -> 3 LF = 3) = 0.00000000E+00

B (E 2 ; 2 LI = 3 -> 1 LF = 3) = 0.00000000E+00

Q (2 , L = 3) = -0.79303289E+00

B (E 2 ; 2 LI = 3 -> 3 LF = 3) = 0.37654907E+02

B (E 2 ; 3 LI = 3 -> 1 LF = 3) = 0.00000000E+00

B (E 2 ; 3 LI = 3 -> 2 LF = 3) = 0.37654904E+02

Q (3 , L = 3) = 0.12386721E+02

B (E 2 ; 1 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 1 LI = 3 -> 2 LF = 2) = 0.48627734E+00

B (E 2 ; 1 LI = 3 -> 3 LF = 2) = 0.10141846E+03

B (E 2 ; 2 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 2 LI = 3 -> 2 LF = 2) = 0.00000000E+00

B (E 2 ; 2 LI = 3 -> 3 LF = 2) = 0.00000000E+00

B (E 2 ; 3 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 3 LI = 3 -> 2 LF = 2) = 0.00000000E+00

B (E 2 ; 3 LI = 3 -> 3 LF = 2) = 0.00000000E+00

B (E 2 ; 1 LI = 3 -> 1 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 3 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 3 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 3 -> 1 LF = 1) = 0.43522102E+02

B (E 2 ; 2 LI = 3 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 3 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 3 -> 1 LF = 1) = 0.24207554E+01

B (E 2 ; 3 LI = 3 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 3 -> 3 LF = 1) = 0.00000000E+00

Q (1 , L = 2) = -0.18305824E+02

B (E 2 ; 1 LI = 2 -> 2 LF = 2) = 0.00000000E+00

B (E 2 ; 1 LI = 2 -> 3 LF = 2) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 1 LF = 2) = 0.00000000E+00

Q (2 , L = 2) = -0.12313417E+02

B (E 2 ; 2 LI = 2 -> 3 LF = 2) = 0.29594155E+02

B (E 2 ; 3 LI = 2 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 2 LF = 2) = 0.29594152E+02

Q (3 , L = 2) = 0.12313414E+02

B (E 2 ; 1 LI = 2 -> 1 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 2 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 2 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 1 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 1 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 2 -> 1 LF = 0) = 0.81066653E+02

B (E 2 ; 1 LI = 2 -> 2 LF = 0) = 0.00000000E+00

B (E 2 ; 1 LI = 2 -> 3 LF = 0) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 1 LF = 0) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 2 LF = 0) = 0.56794349E+02

B (E 2 ; 2 LI = 2 -> 3 LF = 0) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 1 LF = 0) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 2 LF = 0) = 0.27231529E+00

B (E 2 ; 3 LI = 2 -> 3 LF = 0) = 0.00000000E+00

Q (1 , L = 1) = 0.69562137E+01

B (E 2 ; 1 LI = 1 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 1 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 1 -> 1 LF = 1) = 0.00000000E+00

Q (2 , L = 1) = 0.14644661E+01

B (E 2 ; 2 LI = 1 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 1 -> 1 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 1 -> 2 LF = 1) = 0.00000000E+00

Q (3 , L = 1) = 0.47595148E+01

Test output - 9
(Unit #70 BE2BE4.DAT)

B(E2)'S AND Q2'S FOR THE SU5 QUADRUPOLE OPERATOR.

OF PARTICLES = 4

Q (1 , L = 6) = 0.24639640E+01

B (E 2 ; 1 LI = 6 -> 2 LF = 6) = 0.18415191E+00

B (E 2 ; 1 LI = 6 -> 3 LF = 6) = 0.26399061E+00

B (E 2 ; 2 LI = 6 -> 1 LF = 6) = 0.18415191E+00

Q (2 , L = 6) = 0.17001860E+01

B (E 2 ; 2 LI = 6 -> 3 LF = 6) = 0.41422417E+00

B (E 2 ; 3 LI = 6 -> 1 LF = 6) = 0.26399061E+00

B (E 2 ; 3 LI = 6 -> 2 LF = 6) = 0.41422415E+00

Q (3 , L = 6) = -0.84435545E+00

B (E 2 ; 1 LI = 6 -> 1 LF = 5) = 0.74556197E+00

B (E 2 ; 1 LI = 6 -> 2 LF = 5) = 0.00000000E+00

B (E 2 ; 1 LI = 6 -> 3 LF = 5) = 0.00000000E+00

B (E 2 ; 2 LI = 6 -> 1 LF = 5) = 0.10417748E+01

B (E 2 ; 2 LI = 6 -> 2 LF = 5) = 0.00000000E+00

B (E 2 ; 2 LI = 6 -> 3 LF = 5) = 0.00000000E+00

B (E 2 ; 3 LI = 6 -> 1 LF = 5) = 0.14226787E-02

B (E 2 ; 3 LI = 6 -> 2 LF = 5) = 0.00000000E+00

B (E 2 ; 3 LI = 6 -> 3 LF = 5) = 0.00000000E+00

B (E 2 ; 1 LI = 6 -> 1 LF = 4) = 0.97402560E+00

B (E 2 ; 1 LI = 6 -> 2 LF = 4) = 0.34062992E+00

B (E 2 ; 1 LI = 6 -> 3 LF = 4) = 0.14350145E+00

B (E 2 ; 2 LI = 6 -> 1 LF = 4) = 0.00000000E+00

B (E 2 ; 2 LI = 6 -> 2 LF = 4) = 0.87421964E-01

B (E 2 ; 2 LI = 6 -> 3 LF = 4) = 0.81059663E-01

B (E 2 ; 3 LI = 6 -> 1 LF = 4) = 0.00000000E+00

B (E 2 ; 3 LI = 6 -> 2 LF = 4) = 0.11261662E+01

B (E 2 ; 3 LI = 6 -> 3 LF = 4) = 0.15934879E+00

Q (1 , L = 5) = 0.20533033E+01

B (E 2 ; 1 LI = 5 -> 2 LF = 5) = 0.00000000E+00

B (E 2 ; 1 LI = 5 -> 3 LF = 5) = 0.00000000E+00

B (E 2 ; 2 LI = 5 -> 1 LF = 5) = 0.00000000E+00

Q (2 , L = 5) = 0.15020559E+01

B (E 2 ; 2 LI = 5 -> 3 LF = 5) = 0.19331336E-02

B (E 2 ; 3 LI = 5 -> 1 LF = 5) = 0.00000000E+00

B (E 2 ; 3 LI = 5 -> 2 LF = 5) = 0.19331331E-02

Q (3 , L = 5) = 0.18723238E+00

B (E 2 ; 1 LI = 5 -> 1 LF = 4) = 0.13636365E+01

B (E 2 ; 1 LI = 5 -> 2 LF = 4) = 0.41118850E+00

B (E 2 ; 1 LI = 5 -> 3 LF = 4) = 0.17322671E+00

B (E 2 ; 2 LI = 5 -> 1 LF = 4) = 0.00000000E+00

B (E 2 ; 2 LI = 5 -> 2 LF = 4) = 0.00000000E+00

B (E 2 ; 2 LI = 5 -> 3 LF = 4) = 0.00000000E+00

B (E 2 ; 3 LI = 5 -> 1 LF = 4) = 0.00000000E+00

B (E 2 ; 3 LI = 5 -> 2 LF = 4) = 0.00000000E+00

B (E 2 ; 3 LI = 5 -> 3 LF = 4) = 0.00000000E+00

B (E 2 ; 1 LI = 5 -> 1 LF = 3) = 0.18545455E+00

B (E 2 ; 1 LI = 5 -> 2 LF = 3) = 0.00000000E+00

B (E 2 ; 1 LI = 5 -> 3 LF = 3) = 0.00000000E+00

B (E 2 ; 2 LI = 5 -> 1 LF = 3) = 0.00000000E+00

B (E 2 ; 2 LI = 5 -> 2 LF = 3) = 0.14207719E-01

B (E 2 ; 2 LI = 5 -> 3 LF = 3) = 0.87000575E-02

B (E 2 ; 3 LI = 5 -> 1 LF = 3) = 0.00000000E+00

B (E 2 ; 3 LI = 5 -> 2 LF = 3) = 0.20259353E-01

B (E 2 ; 3 LI = 5 -> 3 LF = 3) = 0.32102995E+00

Q (1 , L = 4) = 0.39545106E+01

B (E 2 ; 1 LI = 4 -> 2 LF = 4) = 0.21438840E+00

B (E 2 ; 1 LI = 4 -> 3 LF = 4) = 0.00000000E+00

B (E 2 ; 2 LI = 4 -> 1 LF = 4) = 0.21438842E+00

Q (2 , L = 4) = -0.11669958E+01

B (E 2 ; 2 LI = 4 -> 3 LF = 4) = 0.63960642E+00

B (E 2 ; 3 LI = 4 -> 1 LF = 4) = 0.00000000E+00

B (E 2 ; 3 LI = 4 -> 2 LF = 4) = 0.63960645E+00

Q (3 , L = 4) = 0.40992860E+00

B (E 2 ; 1 LI = 4 -> 1 LF = 3) = 0.00000000E+00

B (E 2 ; 1 LI = 4 -> 2 LF = 3) = 0.00000000E+00

B (E 2 ; 1 LI = 4 -> 3 LF = 3) = 0.00000000E+00

B (E 2 ; 2 LI = 4 -> 1 LF = 3) = 0.35868132E+00

B (E 2 ; 2 LI = 4 -> 2 LF = 3) = 0.00000000E+00

B (E 2 ; 2 LI = 4 -> 3 LF = 3) = 0.00000000E+00

B (E 2 ; 3 LI = 4 -> 1 LF = 3) = 0.33257922E+00

B (E 2 ; 3 LI = 4 -> 2 LF = 3) = 0.00000000E+00

B (E 2 ; 3 LI = 4 -> 3 LF = 3) = 0.00000000E+00

B (E 2 ; 1 LI = 4 -> 1 LF = 2) = 0.44444408E+00

B (E 2 ; 1 LI = 4 -> 2 LF = 2) = 0.28344667E+00

B (E 2 ; 1 LI = 4 -> 3 LF = 2) = 0.00000000E+00

B (E 2 ; 2 LI = 4 -> 1 LF = 2) = 0.14739231E+01

B (E 2 ; 2 LI = 4 -> 2 LF = 2) = 0.85470064E-01

B (E 2 ; 2 LI = 4 -> 3 LF = 2) = 0.00000000E+00

B (E 2 ; 3 LI = 4 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 3 LI = 4 -> 2 LF = 2) = 0.63516472E+00

B (E 2 ; 3 LI = 4 -> 3 LF = 2) = 0.00000000E+00

Q (1 , L = 3) = 0.94557239E-08

B (E 2 ; 1 LI = 3 -> 2 LF = 3) = 0.00000000E+00

B (E 2 ; 1 LI = 3 -> 3 LF = 3) = 0.00000000E+00

B (E 2 ; 2 LI = 3 -> 1 LF = 3) = 0.00000000E+00

Q (2 , L = 3) = 0.15300186E+01

B (E 2 ; 2 LI = 3 -> 3 LF = 3) = 0.64027659E-01

B (E 2 ; 3 LI = 3 -> 1 LF = 3) = 0.00000000E+00

B (E 2 ; 3 LI = 3 -> 2 LF = 3) = 0.64027664E-01

Q (3 , L = 3) = -0.12376138E+01

B (E 2 ; 1 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 1 LI = 3 -> 2 LF = 2) = 0.10408163E+01

B (E 2 ; 1 LI = 3 -> 3 LF = 2) = 0.00000000E+00

B (E 2 ; 2 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 2 LI = 3 -> 2 LF = 2) = 0.00000000E+00

B (E 2 ; 2 LI = 3 -> 3 LF = 2) = 0.00000000E+00

B (E 2 ; 3 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 3 LI = 3 -> 2 LF = 2) = 0.00000000E+00

B (E 2 ; 3 LI = 3 -> 3 LF = 2) = 0.00000000E+00

B (E 2 ; 1 LI = 3 -> 1 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 3 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 3 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 3 -> 1 LF = 1) = 0.31098134E-01

B (E 2 ; 2 LI = 3 -> 2 LF = 1) = 0.50534490E-01

B (E 2 ; 2 LI = 3 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 3 -> 1 LF = 1) = 0.33120285E+00

B (E 2 ; 3 LI = 3 -> 2 LF = 1) = 0.38947213E-01

B (E 2 ; 3 LI = 3 -> 3 LF = 1) = 0.27673801E-01

Q (1 , L = 2) = -0.84739480E+00

B (E 2 ; 1 LI = 2 -> 2 LF = 2) = 0.16326527E+01

B (E 2 ; 1 LI = 2 -> 3 LF = 2) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 1 LF = 2) = 0.16326529E+01

Q (2 , L = 2) = 0.84739463E+00

B (E 2 ; 2 LI = 2 -> 3 LF = 2) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 2 LF = 2) = 0.00000000E+00

Q (3 , L = 2) = -0.34537705E+00

B (E 2 ; 1 LI = 2 -> 1 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 2 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 2 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 1 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 1 LF = 1) = 0.29869332E-01

B (E 2 ; 3 LI = 2 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 2 -> 1 LF = 0) = 0.14857141E+01

B (E 2 ; 1 LI = 2 -> 2 LF = 0) = 0.00000000E+00

B (E 2 ; 1 LI = 2 -> 3 LF = 0) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 1 LF = 0) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 2 LF = 0) = 0.38857138E+00

B (E 2 ; 2 LI = 2 -> 3 LF = 0) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 1 LF = 0) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 2 LF = 0) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 3 LF = 0) = 0.93770996E+00

Q (1 , L = 1) = 0.15737331E+00

B (E 2 ; 1 LI = 1 -> 2 LF = 1) = 0.30699704E+00

B (E 2 ; 1 LI = 1 -> 3 LF = 1) = 0.79591900E-01

B (E 2 ; 2 LI = 1 -> 1 LF = 1) = 0.30699704E+00

Q (2 , L = 1) = 0.18158475E+00

B (E 2 ; 2 LI = 1 -> 3 LF = 1) = 0.27550997E-01

B (E 2 ; 3 LI = 1 -> 1 LF = 1) = 0.79591900E-01

B (E 2 ; 3 LI = 1 -> 2 LF = 1) = 0.27550998E-01

Q (3 , L = 1) = -0.25421859E+00

Test output - 10
(Unit #70 BE2BE4.DAT)

B(E2)'S AND Q2'S FOR THE SU6 QUADRUPOLE OPERATOR.

OF PARTICLES = 4

Q (1 , L = 6) = -0.96865301E+00

B (E 2 ; 1 LI = 6 -> 2 LF = 6) = 0.00000000E+00

B (E 2 ; 1 LI = 6 -> 3 LF = 6) = 0.22296033E-02

B (E 2 ; 2 LI = 6 -> 1 LF = 6) = 0.00000000E+00

Q (2 , L = 6) = 0.96865234E-01

B (E 2 ; 2 LI = 6 -> 3 LF = 6) = 0.71252245E-01

B (E 2 ; 3 LI = 6 -> 1 LF = 6) = 0.22296033E-02

B (E 2 ; 3 LI = 6 -> 2 LF = 6) = 0.71252262E-01

Q (3 , L = 6) = -0.12552218E-01

B (E 2 ; 1 LI = 6 -> 1 LF = 5) = 0.41236361E+00

B (E 2 ; 1 LI = 6 -> 2 LF = 5) = 0.95803693E-01

B (E 2 ; 1 LI = 6 -> 3 LF = 5) = 0.49294276E-01

B (E 2 ; 2 LI = 6 -> 1 LF = 5) = 0.53030237E-02

B (E 2 ; 2 LI = 6 -> 2 LF = 5) = 0.72572392E-01

B (E 2 ; 2 LI = 6 -> 3 LF = 5) = 0.26776258E+00

B (E 2 ; 3 LI = 6 -> 1 LF = 5) = 0.67542249E-03

B (E 2 ; 3 LI = 6 -> 2 LF = 5) = 0.45552368E-02

B (E 2 ; 3 LI = 6 -> 3 LF = 5) = 0.92724699E-02

B (E 2 ; 1 LI = 6 -> 1 LF = 4) = 0.24107145E+00

B (E 2 ; 1 LI = 6 -> 2 LF = 4) = 0.15371499E+00

B (E 2 ; 1 LI = 6 -> 3 LF = 4) = 0.13771504E+00

B (E 2 ; 2 LI = 6 -> 1 LF = 4) = 0.44642836E+00

B (E 2 ; 2 LI = 6 -> 2 LF = 4) = 0.12766902E+00

B (E 2 ; 2 LI = 6 -> 3 LF = 4) = 0.28107223E+00

B (E 2 ; 3 LI = 6 -> 1 LF = 4) = 0.00000000E+00

B (E 2 ; 3 LI = 6 -> 2 LF = 4) = 0.20848814E+00

B (E 2 ; 3 LI = 6 -> 3 LF = 4) = 0.17341932E+00

Q (1 , L = 5) = -0.72648991E+00

B (E 2 ; 1 LI = 5 -> 2 LF = 5) = 0.23683663E-01

B (E 2 ; 1 LI = 5 -> 3 LF = 5) = 0.69378966E-02

B (E 2 ; 2 LI = 5 -> 1 LF = 5) = 0.23683661E-01

Q (2 , L = 5) = -0.78636762E+00

B (E 2 ; 2 LI = 5 -> 3 LF = 5) = 0.30381591E-01

B (E 2 ; 3 LI = 5 -> 1 LF = 5) = 0.69378985E-02

B (E 2 ; 3 LI = 5 -> 2 LF = 5) = 0.30381595E-01

Q (3 , L = 5) = -0.77523825E+00

B (E 2 ; 1 LI = 5 -> 1 LF = 4) = 0.68750013E+00

B (E 2 ; 1 LI = 5 -> 2 LF = 4) = 0.53165494E-01

B (E 2 ; 1 LI = 5 -> 3 LF = 4) = 0.21856167E+00

B (E 2 ; 2 LI = 5 -> 1 LF = 4) = 0.00000000E+00

B (E 2 ; 2 LI = 5 -> 2 LF = 4) = 0.84653587E-01

B (E 2 ; 2 LI = 5 -> 3 LF = 4) = 0.00000000E+00

B (E 2 ; 3 LI = 5 -> 1 LF = 4) = 0.00000000E+00

B (E 2 ; 3 LI = 5 -> 2 LF = 4) = 0.11145704E+00

B (E 2 ; 3 LI = 5 -> 3 LF = 4) = 0.00000000E+00

B (E 2 ; 1 LI = 5 -> 1 LF = 3) = 0.24499987E-01

B (E 2 ; 1 LI = 5 -> 2 LF = 3) = 0.84620807E-01

B (E 2 ; 1 LI = 5 -> 3 LF = 3) = 0.82391552E-01

B (E 2 ; 2 LI = 5 -> 1 LF = 3) = 0.20547491E-01

B (E 2 ; 2 LI = 5 -> 2 LF = 3) = 0.15535251E+00

B (E 2 ; 2 LI = 5 -> 3 LF = 3) = 0.12958915E-01

B (E 2 ; 3 LI = 5 -> 1 LF = 3) = 0.28806284E+00

B (E 2 ; 3 LI = 5 -> 2 LF = 3) = 0.13160773E+00

B (E 2 ; 3 LI = 5 -> 3 LF = 3) = 0.57818827E-01

Q (1 , L = 4) = -0.12108167E+01

B (E 2 ; 1 LI = 4 -> 2 LF = 4) = 0.44649693E+00

B (E 2 ; 1 LI = 4 -> 3 LF = 4) = 0.48082023E-01

B (E 2 ; 2 LI = 4 -> 1 LF = 4) = 0.44649694E+00

Q (2 , L = 4) = -0.14419392E+01

B (E 2 ; 2 LI = 4 -> 3 LF = 4) = 0.31669294E-03

B (E 2 ; 3 LI = 4 -> 1 LF = 4) = 0.48082019E-01

B (E 2 ; 3 LI = 4 -> 2 LF = 4) = 0.31669303E-03

Q (3 , L = 4) = 0.14243277E+01

B (E 2 ; 1 LI = 4 -> 1 LF = 3) = 0.53472206E+00

B (E 2 ; 1 LI = 4 -> 2 LF = 3) = 0.00000000E+00

B (E 2 ; 1 LI = 4 -> 3 LF = 3) = 0.00000000E+00

B (E 2 ; 2 LI = 4 -> 1 LF = 3) = 0.22971329E+00

B (E 2 ; 2 LI = 4 -> 2 LF = 3) = 0.31071170E-02

B (E 2 ; 2 LI = 4 -> 3 LF = 3) = 0.21964774E-01

B (E 2 ; 3 LI = 4 -> 1 LF = 3) = 0.13050480E-01

B (E 2 ; 3 LI = 4 -> 2 LF = 3) = 0.92351527E-02

B (E 2 ; 3 LI = 4 -> 3 LF = 3) = 0.13886215E+00

B (E 2 ; 1 LI = 4 -> 1 LF = 2) = 0.35156236E+00

B (E 2 ; 1 LI = 4 -> 2 LF = 2) = 0.12537385E+00

B (E 2 ; 1 LI = 4 -> 3 LF = 2) = 0.19804017E-01

B (E 2 ; 2 LI = 4 -> 1 LF = 2) = 0.90718457E-03

B (E 2 ; 2 LI = 4 -> 2 LF = 2) = 0.24684327E-01

B (E 2 ; 2 LI = 4 -> 3 LF = 2) = 0.23860889E+00

B (E 2 ; 3 LI = 4 -> 1 LF = 2) = 0.11373262E+01

B (E 2 ; 3 LI = 4 -> 2 LF = 2) = 0.50737836E-01

B (E 2 ; 3 LI = 4 -> 3 LF = 2) = 0.22809192E-01

Q (1 , L = 3) = -0.90811224E+00

B (E 2 ; 1 LI = 3 -> 2 LF = 3) = 0.95511401E-01

B (E 2 ; 1 LI = 3 -> 3 LF = 3) = 0.15443824E+00

B (E 2 ; 2 LI = 3 -> 1 LF = 3) = 0.95511393E-01

Q (2 , L = 3) = -0.15116341E+00

B (E 2 ; 2 LI = 3 -> 3 LF = 3) = 0.20477754E-01

B (E 2 ; 3 LI = 3 -> 1 LF = 3) = 0.15443824E+00

B (E 2 ; 3 LI = 3 -> 2 LF = 3) = 0.20477753E-01

Q (3 , L = 3) = 0.82673126E+00

B (E 2 ; 1 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 1 LI = 3 -> 2 LF = 2) = 0.33051342E-01

B (E 2 ; 1 LI = 3 -> 3 LF = 2) = 0.30028190E+00

B (E 2 ; 2 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 2 LI = 3 -> 2 LF = 2) = 0.48361612E-01

B (E 2 ; 2 LI = 3 -> 3 LF = 2) = 0.29269458E+00

B (E 2 ; 3 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 2 ; 3 LI = 3 -> 2 LF = 2) = 0.25914799E+00

B (E 2 ; 3 LI = 3 -> 3 LF = 2) = 0.17358314E+00

B (E 2 ; 1 LI = 3 -> 1 LF = 1) = 0.12244898E-01

B (E 2 ; 1 LI = 3 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 3 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 3 -> 1 LF = 1) = 0.21161503E+00

B (E 2 ; 2 LI = 3 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 3 -> 3 LF = 1) = 0.34618984E-01

B (E 2 ; 3 LI = 3 -> 1 LF = 1) = 0.14216934E-03

B (E 2 ; 3 LI = 3 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 3 -> 3 LF = 1) = 0.51387334E-01

Q (1 , L = 2) = 0.86486876E+00

B (E 2 ; 1 LI = 2 -> 2 LF = 2) = 0.72253857E+00

B (E 2 ; 1 LI = 2 -> 3 LF = 2) = 0.47707869E+00

B (E 2 ; 2 LI = 2 -> 1 LF = 2) = 0.72253859E+00

Q (2 , L = 2) = -0.16415429E+00

B (E 2 ; 2 LI = 2 -> 3 LF = 2) = 0.22262840E+00

B (E 2 ; 3 LI = 2 -> 1 LF = 2) = 0.47707868E+00

B (E 2 ; 3 LI = 2 -> 2 LF = 2) = 0.22262840E+00

Q (3 , L = 2) = 0.68307593E+00

B (E 2 ; 1 LI = 2 -> 1 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 2 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 2 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 1 LF = 1) = 0.11775826E+00

B (E 2 ; 2 LI = 2 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 1 LF = 1) = 0.17834559E+00

B (E 2 ; 3 LI = 2 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 2 -> 1 LF = 0) = 0.95238091E+00

B (E 2 ; 1 LI = 2 -> 2 LF = 0) = 0.17678570E+00

B (E 2 ; 1 LI = 2 -> 3 LF = 0) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 1 LF = 0) = 0.00000000E+00

B (E 2 ; 2 LI = 2 -> 2 LF = 0) = 0.18083805E-02

B (E 2 ; 2 LI = 2 -> 3 LF = 0) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 1 LF = 0) = 0.00000000E+00

B (E 2 ; 3 LI = 2 -> 2 LF = 0) = 0.14635831E+00

B (E 2 ; 3 LI = 2 -> 3 LF = 0) = 0.00000000E+00

Q (1 , L = 1) = -0.48432664E+00

B (E 2 ; 1 LI = 1 -> 2 LF = 1) = 0.00000000E+00

B (E 2 ; 1 LI = 1 -> 3 LF = 1) = 0.10135383E-01

B (E 2 ; 2 LI = 1 -> 1 LF = 1) = 0.00000000E+00

Q (2 , L = 1) = -0.84936010E-08

B (E 2 ; 2 LI = 1 -> 3 LF = 1) = 0.00000000E+00

B (E 2 ; 3 LI = 1 -> 1 LF = 1) = 0.10135382E-01

B (E 2 ; 3 LI = 1 -> 2 LF = 1) = 0.00000000E+00

Q (3 , L = 1) = -0.17511795E+00

Test output - 11
(Unit #70 BE2BE4.DAT)

B(E4)'S AND Q4'S FOR THE -C2(U(5)) + 0.5*C2(O(5)) INTERACTION.

OF PARTICLES = 4

Q (1 , L = 6) = 0.13688686E+01

B (E 4 ; 1 LI = 6 -> 2 LF = 6) = 0.97830961E-01

B (E 4 ; 1 LI = 6 -> 3 LF = 6) = 0.27265998E+00

B (E 4 ; 2 LI = 6 -> 1 LF = 6) = 0.97830970E-01

Q (2 , L = 6) = -0.51589130E+00

B (E 4 ; 2 LI = 6 -> 3 LF = 6) = 0.10549707E+01

B (E 4 ; 3 LI = 6 -> 1 LF = 6) = 0.27265999E+00

B (E 4 ; 3 LI = 6 -> 2 LF = 6) = 0.10549707E+01

Q (3 , L = 6) = -0.21193349E+00

B (E 4 ; 1 LI = 6 -> 1 LF = 5) = 0.37278096E-01

B (E 4 ; 1 LI = 6 -> 2 LF = 5) = 0.00000000E+00

B (E 4 ; 1 LI = 6 -> 3 LF = 5) = 0.00000000E+00

B (E 4 ; 2 LI = 6 -> 1 LF = 5) = 0.90431181E-02

B (E 4 ; 2 LI = 6 -> 2 LF = 5) = 0.00000000E+00

B (E 4 ; 2 LI = 6 -> 3 LF = 5) = 0.00000000E+00

B (E 4 ; 3 LI = 6 -> 1 LF = 5) = 0.60206172E+00

B (E 4 ; 3 LI = 6 -> 2 LF = 5) = 0.00000000E+00

B (E 4 ; 3 LI = 6 -> 3 LF = 5) = 0.00000000E+00

B (E 4 ; 1 LI = 6 -> 1 LF = 4) = 0.77922030E+00

B (E 4 ; 1 LI = 6 -> 2 LF = 4) = 0.12305252E

B (E 4 ; 1 LI = 6 -> 3 LF = 4) = 0.28700300E

B (E 4 ; 2 LI = 6 -> 1 LF = 4) = 0.00000000E

B (E 4 ; 2 LI = 6 -> 2 LF = 4) = 0.43710865E

B (E 4 ; 2 LI = 6 -> 3 LF = 4) = 0.20518261E

B (E 4 ; 3 LI = 6 -> 1 LF = 4) = 0.00000000E

B (E 4 ; 3 LI = 6 -> 2 LF = 4) = 0.12898655E

B (E 4 ; 3 LI = 6 -> 3 LF = 4) = 0.20337184E

B (E 4 ; 1 LI = 6 -> 1 LF = 3) = 0.51210336E

B (E 4 ; 1 LI = 6 -> 2 LF = 3) = 0.00000000E

B (E 4 ; 1 LI = 6 -> 3 LF = 3) = 0.00000000E

B (E 4 ; 2 LI = 6 -> 1 LF = 3) = 0.52249710E

B (E 4 ; 2 LI = 6 -> 2 LF = 3) = 0.00000000E

B (E 4 ; 2 LI = 6 -> 3 LF = 3) = 0.00000000E

B (E 4 ; 3 LI = 6 -> 1 LF = 3) = 0.10271355E

B (E 4 ; 3 LI = 6 -> 2 LF = 3) = 0.00000000E

B (E 4 ; 3 LI = 6 -> 3 LF = 3) = 0.00000000E

B (E 4 ; 1 LI = 6 -> 1 LF = 2) = 0.97402593E

B (E 4 ; 1 LI = 6 -> 2 LF = 2) = 0.69190400E

B (E 4 ; 1 LI = 6 -> 3 LF = 2) = 0.00000000E

B (E 4 ; 2 LI = 6 -> 1 LF = 2) = 0.00000000E

B (E 4 ; 2 LI = 6 -> 2 LF = 2) = 0.59684984E

B (E 4 ; 2 LI = 6 -> 3 LF = 2) = 0.00000000E

B (E 4 ; 3 LI = 6 -> 1 LF = 2) = 0.00000000E

B (E 4 ; 3 LI = 6 -> 2 LF = 2) = 0.19011685E

B (E 4 ; 3 LI = 6 -> 3 LF = 2) = 0.00000000E+00

Q (1 , L = 5) = -0.22814480E+00

B (E 4 ; 1 LI = 5 -> 2 LF = 5) = 0.00000000E+00

B (E 4 ; 1 LI = 5 -> 3 LF = 5) = 0.00000000E+00

B (E 4 ; 2 LI = 5 -> 1 LF = 5) = 0.00000000E+00

Q (2 , L = 5) = -0.53800410E-01

B (E 4 ; 2 LI = 5 -> 3 LF = 5) = 0.12138543E+00

B (E 4 ; 3 LI = 5 -> 1 LF = 5) = 0.00000000E+00

B (E 4 ; 3 LI = 5 -> 2 LF = 5) = 0.12138543E+00

Q (3 , L = 5) = -0.66397865E+00

B (E 4 ; 1 LI = 5 -> 1 LF = 4) = 0.00000000E+00

B (E 4 ; 1 LI = 5 -> 2 LF = 4) = 0.28822755E+01

B (E 4 ; 1 LI = 5 -> 3 LF = 4) = 0.23985241E+00

B (E 4 ; 2 LI = 5 -> 1 LF = 4) = 0.00000000E+00

B (E 4 ; 2 LI = 5 -> 2 LF = 4) = 0.00000000E+00

B (E 4 ; 2 LI = 5 -> 3 LF = 4) = 0.00000000E+00

B (E 4 ; 3 LI = 5 -> 1 LF = 4) = 0.00000000E+00

B (E 4 ; 3 LI = 5 -> 2 LF = 4) = 0.00000000E+00

B (E 4 ; 3 LI = 5 -> 3 LF = 4) = 0.00000000E+00

B (E 4 ; 1 LI = 5 -> 1 LF = 3) = 0.47552453E-01

B (E 4 ; 1 LI = 5 -> 2 LF = 3) = 0.00000000E+00

B (E 4 ; 1 LI = 5 -> 3 LF = 3) = 0.00000000E+00

B (E 4 ; 2 LI = 5 -> 1 LF = 3) = 0.00000000E+00

B (E 4 ; 2 LI = 5 -> 2 LF = 3) = 0.52666625E-01

B (E 4 ; 2 LI = 5 -> 3 LF = 3) = 0.20393695E+00

B (E 4 ; 3 LI = 5 -> 1 LF = 3) = 0.00000000E+00

B (E 4 ; 3 LI = 5 -> 2 LF = 3) = 0.44006716E-02

B (E 4 ; 3 LI = 5 -> 3 LF = 3) = 0.29908025E-01

B (E 4 ; 1 LI = 5 -> 1 LF = 2) = 0.13636363E+01

B (E 4 ; 1 LI = 5 -> 2 LF = 2) = 0.83522730E+00

B (E 4 ; 1 LI = 5 -> 3 LF = 2) = 0.00000000E+00

B (E 4 ; 2 LI = 5 -> 1 LF = 2) = 0.00000000E+00

B (E 4 ; 2 LI = 5 -> 2 LF = 2) = 0.00000000E+00

B (E 4 ; 2 LI = 5 -> 3 LF = 2) = 0.27914339E-01

B (E 4 ; 3 LI = 5 -> 1 LF = 2) = 0.00000000E+00

B (E 4 ; 3 LI = 5 -> 2 LF = 2) = 0.00000000E+00

B (E 4 ; 3 LI = 5 -> 3 LF = 2) = 0.11253600E-02

B (E 4 ; 1 LI = 5 -> 1 LF = 1) = 0.00000000E+00

B (E 4 ; 1 LI = 5 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 1 LI = 5 -> 3 LF = 1) = 0.00000000E+00

B (E 4 ; 2 LI = 5 -> 1 LF = 1) = 0.28010714E+00

B (E 4 ; 2 LI = 5 -> 2 LF = 1) = 0.32601910E-02

B (E 4 ; 2 LI = 5 -> 3 LF = 1) = 0.62266944E-03

B (E 4 ; 3 LI = 5 -> 1 LF = 1) = 0.10035303E+00

B (E 4 ; 3 LI = 5 -> 2 LF = 1) = 0.32072212E-01

B (E 4 ; 3 LI = 5 -> 3 LF = 1) = 0.13679927E-01

Q (1 , L = 4) = 0.65908470E+00

B (E 4 ; 1 LI = 4 -> 2 LF = 4) = 0.82457273E+00

B (E 4 ; 1 LI = 4 -> 3 LF = 4) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 1 LF = 4) = 0.82457263E+00

Q (2 , L = 4) = -0.27937519E+00

B (E 4 ; 2 LI = 4 -> 3 LF = 4) = 0.61500638E-02

B (E 4 ; 3 LI = 4 -> 1 LF = 4) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 2 LF = 4) = 0.61500641E-02

Q (3 , L = 4) = 0.10294202E+01

B (E 4 ; 1 LI = 4 -> 1 LF = 3) = 0.00000000E+00

B (E 4 ; 1 LI = 4 -> 2 LF = 3) = 0.00000000E+00

B (E 4 ; 1 LI = 4 -> 3 LF = 3) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 1 LF = 3) = 0.18341658E+00

B (E 4 ; 2 LI = 4 -> 2 LF = 3) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 3 LF = 3) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 1 LF = 3) = 0.17006890E+00

B (E 4 ; 3 LI = 4 -> 2 LF = 3) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 3 LF = 3) = 0.00000000E+00

B (E 4 ; 1 LI = 4 -> 1 LF = 2) = 0.30555556E+01

B (E 4 ; 1 LI = 4 -> 2 LF = 2) = 0.41228733E-01

B (E 4 ; 1 LI = 4 -> 3 LF = 2) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 1 LF = 2) = 0.21438846E+00

B (E 4 ; 2 LI = 4 -> 2 LF = 2) = 0.97921503E+00

B (E 4 ; 2 LI = 4 -> 3 LF = 2) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 1 LF = 2) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 2 LF = 2) = 0.57742243E+00

B (E 4 ; 3 LI = 4 -> 3 LF = 2) = 0.00000000E+00

B (E 4 ; 1 LI = 4 -> 1 LF = 1) = 0.00000000E+00

B (E 4 ; 1 LI = 4 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 1 LI = 4 -> 3 LF = 1) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 1 LF = 1) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 3 LF = 1) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 1 LF = 1) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 3 LF = 1) = 0.00000000E+00

B (E 4 ; 1 LI = 4 -> 1 LF = 0) = 0.14857141E+01

B (E 4 ; 1 LI = 4 -> 2 LF = 0) = 0.00000000E+00

B (E 4 ; 1 LI = 4 -> 3 LF = 0) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 1 LF = 0) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 2 LF = 0) = 0.74725280E-01

B (E 4 ; 2 LI = 4 -> 3 LF = 0) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 1 LF = 0) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 2 LF = 0) = 0.89796378E+00

B (E 4 ; 3 LI = 4 -> 3 LF = 0) = 0.00000000E+00

Q (1 , L = 3) = -0.94369002E+00

B (E 4 ; 1 LI = 3 -> 2 LF = 3) = 0.00000000E+00

B (E 4 ; 1 LI = 3 -> 3 LF = 3) = 0.00000000E+00

B (E 4 ; 2 LI = 3 -> 1 LF = 3) = 0.00000000E+00

Q (2 , L = 3) = -0.10200123E+00

B (E 4 ; 2 LI = 3 -> 3 LF = 3) = 0.10479261E+00

B (E 4 ; 3 LI = 3 -> 1 LF = 3) = 0.00000000E+00

B (E 4 ; 3 LI = 3 -> 2 LF = 3) = 0.10479261E+00

Q (3 , L = 3) = 0.41905883E+00

B (E 4 ; 1 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 4 ; 1 LI = 3 -> 2 LF = 2) = 0.41632646E+00

B (E 4 ; 1 LI = 3 -> 3 LF = 2) = 0.00000000E+00

B (E 4 ; 2 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 4 ; 2 LI = 3 -> 2 LF = 2) = 0.00000000E+00

B (E 4 ; 2 LI = 3 -> 3 LF = 2) = 0.82247933E+00

B (E 4 ; 3 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 4 ; 3 LI = 3 -> 2 LF = 2) = 0.00000000E+00

B (E 4 ; 3 LI = 3 -> 3 LF = 2) = 0.10215373E-01

B (E 4 ; 1 LI = 3 -> 1 LF = 1) = 0.00000000E+00

B (E 4 ; 1 LI = 3 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 1 LI = 3 -> 3 LF = 1) = 0.00000000E+00

B (E 4 ; 2 LI = 3 -> 1 LF = 1) = 0.17638478E+00

B (E 4 ; 2 LI = 3 -> 2 LF = 1) = 0.67379354E-01

B (E 4 ; 2 LI = 3 -> 3 LF = 1) = 0.00000000E+00

B (E 4 ; 3 LI = 3 -> 1 LF = 1) = 0.17846923E+00

B (E 4 ; 3 LI = 3 -> 2 LF = 1) = 0.10621636E+00

B (E 4 ; 3 LI = 3 -> 3 LF = 1) = 0.20214070E-01

Q (1 , L = 2) = 0.18831002E+00

B (E 4 ; 1 LI = 2 -> 2 LF = 2) = 0.51020397E+00

B (E 4 ; 1 LI = 2 -> 3 LF = 2) = 0.00000000E+00

B (E 4 ; 2 LI = 2 -> 1 LF = 2) = 0.51020400E+00

Q (2 , L = 2) = 0.30600370E+00

B (E 4 ; 2 LI = 2 -> 3 LF = 2) = 0.00000000E+00

B (E 4 ; 3 LI = 2 -> 1 LF = 2) = 0.00000000E+00

B (E 4 ; 3 LI = 2 -> 2 LF = 2) = 0.00000000E+00

Q (3 , L = 2) = 0.76750390E-01

Test output - 12
(Unit #70 BE2BE4.DAT)

BE(4)'S AND Q4'S FOR -C2(U(6)) + 0.5*C2(Sp(6)) INTERACTION.

OF PARTICLES = 4

Q (1 , L = 6) = -0.76736815E+00

B (E 4 ; 1 LI = 6 -> 2 LF = 6) = 0.20444280E+00

B (E 4 ; 1 LI = 6 -> 3 LF = 6) = 0.21901986E+00

B (E 4 ; 2 LI = 6 -> 1 LF = 6) = 0.20444279E+00

Q (2 , L = 6) = 0.45473660E+00

B (E 4 ; 2 LI = 6 -> 3 LF = 6) = 0.54388763E-02

B (E 4 ; 3 LI = 6 -> 1 LF = 6) = 0.21901986E+00

B (E 4 ; 3 LI = 6 -> 2 LF = 6) = 0.54388753E-02

Q (3 , L = 6) = -0.21084814E+00

B (E 4 ; 1 LI = 6 -> 1 LF = 5) = 0.16969698E+00

B (E 4 ; 1 LI = 6 -> 2 LF = 5) = 0.53045408E-03

B (E 4 ; 1 LI = 6 -> 3 LF = 5) = 0.11203830E-01

B (E 4 ; 2 LI = 6 -> 1 LF = 5) = 0.38719183E+00

B (E 4 ; 2 LI = 6 -> 2 LF = 5) = 0.63355797E-02

B (E 4 ; 2 LI = 6 -> 3 LF = 5) = 0.25572229E-01

B (E 4 ; 3 LI = 6 -> 1 LF = 5) = 0.22306954E+00

B (E 4 ; 3 LI = 6 -> 2 LF = 5) = 0.67405464E-02

B (E 4 ; 3 LI = 6 -> 3 LF = 5) = 0.12503568E-01

B (E 4 ; 1 LI = 6 -> 1 LF = 4) = 0.89285684E+00

B (E 4 ; 1 LI = 6 -> 2 LF = 4) = 0.16834221E+00

B (E 4 ; 1 LI = 6 -> 3 LF = 4) = 0.25181331E-01

B (E 4 ; 2 LI = 6 -> 1 LF = 4) = 0.48214298E+00

B (E 4 ; 2 LI = 6 -> 2 LF = 4) = 0.22404226E+00

B (E 4 ; 2 LI = 6 -> 3 LF = 4) = 0.16564924E+00

B (E 4 ; 3 LI = 6 -> 1 LF = 4) = 0.00000000E+00

B (E 4 ; 3 LI = 6 -> 2 LF = 4) = 0.47816703E-01

B (E 4 ; 3 LI = 6 -> 3 LF = 4) = 0.37369016E-01

B (E 4 ; 1 LI = 6 -> 1 LF = 3) = 0.25984847E-01

B (E 4 ; 1 LI = 6 -> 2 LF = 3) = 0.74953320E-01

B (E 4 ; 1 LI = 6 -> 3 LF = 3) = 0.12077034E-01

B (E 4 ; 2 LI = 6 -> 1 LF = 3) = 0.21220952E+00

B (E 4 ; 2 LI = 6 -> 2 LF = 3) = 0.40821210E-01

B (E 4 ; 2 LI = 6 -> 3 LF = 3) = 0.65498199E-03

B (E 4 ; 3 LI = 6 -> 1 LF = 3) = 0.36126852E-02

B (E 4 ; 3 LI = 6 -> 2 LF = 3) = 0.18256190E-01

B (E 4 ; 3 LI = 6 -> 3 LF = 3) = 0.19433511E-01

B (E 4 ; 1 LI = 6 -> 1 LF = 2) = 0.24107143E+00

B (E 4 ; 1 LI = 6 -> 2 LF = 2) = 0.22334279E+00

B (E 4 ; 1 LI = 6 -> 3 LF = 2) = 0.76664055E-01

B (E 4 ; 2 LI = 6 -> 1 LF = 2) = 0.44642848E+00

B (E 4 ; 2 LI = 6 -> 2 LF = 2) = 0.51428077E-02

B (E 4 ; 2 LI = 6 -> 3 LF = 2) = 0.10654222E+00

B (E 4 ; 3 LI = 6 -> 1 LF = 2) = 0.00000000E+00

B (E 4 ; 3 LI = 6 -> 2 LF = 2) = 0.13321400E-01

B (E 4 ; 3 LI = 6 -> 3 LF = 2) = 0.19103369E+00

Q (1 , L = 5) = 0.41684195E+00

B (E 4 ; 1 LI = 5 -> 2 LF = 5) = 0.54042486E-01

B (E 4 ; 1 LI = 5 -> 3 LF = 5) = 0.14156107E-01

B (E 4 ; 2 LI = 5 -> 1 LF = 5) = 0.54042481E-01

Q (2 , L = 5) = -0.33910843E+00

B (E 4 ; 2 LI = 5 -> 3 LF = 5) = 0.30809854E-03

B (E 4 ; 3 LI = 5 -> 1 LF = 5) = 0.14156107E-01

B (E 4 ; 3 LI = 5 -> 2 LF = 5) = 0.30809875E-03

Q (3 , L = 5) = 0.30853810E+00

B (E 4 ; 1 LI = 5 -> 1 LF = 4) = 0.00000000E+00

B (E 4 ; 1 LI = 5 -> 2 LF = 4) = 0.89782272E-01

B (E 4 ; 1 LI = 5 -> 3 LF = 4) = 0.22760408E+00

B (E 4 ; 2 LI = 5 -> 1 LF = 4) = 0.00000000E+00

B (E 4 ; 2 LI = 5 -> 2 LF = 4) = 0.33660433E+00

B (E 4 ; 2 LI = 5 -> 3 LF = 4) = 0.69742424E-01

B (E 4 ; 3 LI = 5 -> 1 LF = 4) = 0.00000000E+00

B (E 4 ; 3 LI = 5 -> 2 LF = 4) = 0.52010924E-02

B (E 4 ; 3 LI = 5 -> 3 LF = 4) = 0.17431759E-01

B (E 4 ; 1 LI = 5 -> 1 LF = 3) = 0.44236112E+00

B (E 4 ; 1 LI = 5 -> 2 LF = 3) = 0.10751919E-01

B (E 4 ; 1 LI = 5 -> 3 LF = 3) = 0.26914141E-01

B (E 4 ; 2 LI = 5 -> 1 LF = 3) = 0.23680065E-02

B (E 4 ; 2 LI = 5 -> 2 LF = 3) = 0.24384666E-01

B (E 4 ; 2 LI = 5 -> 3 LF = 3) = 0.86925358E-03

B (E 4 ; 3 LI = 5 -> 1 LF = 3) = 0.29708496E-01

B (E 4 ; 3 LI = 5 -> 2 LF = 3) = 0.17211326E+00

B (E 4 ; 3 LI = 5 -> 3 LF = 3) = 0.12355050E+00

B (E 4 ; 1 LI = 5 -> 1 LF = 2) = 0.68750003E+00

B (E 4 ; 1 LI = 5 -> 2 LF = 2) = 0.97474889E-01

B (E 4 ; 1 LI = 5 -> 3 LF = 2) = 0.27764693E-03

B (E 4 ; 2 LI = 5 -> 1 LF = 2) = 0.00000000E+00

B (E 4 ; 2 LI = 5 -> 2 LF = 2) = 0.13574231E+00

B (E 4 ; 2 LI = 5 -> 3 LF = 2) = 0.21554432E+00

B (E 4 ; 3 LI = 5 -> 1 LF = 2) = 0.00000000E+00

B (E 4 ; 3 LI = 5 -> 2 LF = 2) = 0.20889813E+00

B (E 4 ; 3 LI = 5 -> 3 LF = 2) = 0.19178068E+00

B (E 4 ; 1 LI = 5 -> 1 LF = 1) = 0.89256200E-01

B (E 4 ; 1 LI = 5 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 1 LI = 5 -> 3 LF = 1) = 0.00000000E+00

B (E 4 ; 2 LI = 5 -> 1 LF = 1) = 0.12880383E+00

B (E 4 ; 2 LI = 5 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 2 LI = 5 -> 3 LF = 1) = 0.41922377E-02

B (E 4 ; 3 LI = 5 -> 1 LF = 1) = 0.00000000E+00

B (E 4 ; 3 LI = 5 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 3 LI = 5 -> 3 LF = 1) = 0.38780663E-02

Q (1 , L = 4) = 0.78157876E+00

B (E 4 ; 1 LI = 4 -> 2 LF = 4) = 0.48109891E+00

B (E 4 ; 1 LI = 4 -> 3 LF = 4) = 0.14150945E+00

B (E 4 ; 2 LI = 4 -> 1 LF = 4) = 0.48109887E+00

Q (2 , L = 4) = -0.25311155E+00

B (E 4 ; 2 LI = 4 -> 3 LF = 4) = 0.66994154E-01

B (E 4 ; 3 LI = 4 -> 1 LF = 4) = 0.14150945E+00

B (E 4 ; 3 LI = 4 -> 2 LF = 4) = 0.66994157E-01

Q (3 , L = 4) = -0.15024861E+00

B (E 4 ; 1 LI = 4 -> 1 LF = 3) = 0.00000000E+00

B (E 4 ; 1 LI = 4 -> 2 LF = 3) = 0.00000000E+00

B (E 4 ; 1 LI = 4 -> 3 LF = 3) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 1 LF = 3) = 0.37969426E+00

B (E 4 ; 2 LI = 4 -> 2 LF = 3) = 0.99907268E-01

B (E 4 ; 2 LI = 4 -> 3 LF = 3) = 0.83865307E-01

B (E 4 ; 3 LI = 4 -> 1 LF = 3) = 0.72389003E-01

B (E 4 ; 3 LI = 4 -> 2 LF = 3) = 0.14442294E-02

B (E 4 ; 3 LI = 4 -> 3 LF = 3) = 0.91022284E-02

B (E 4 ; 1 LI = 4 -> 1 LF = 2) = 0.28645839E+00

B (E 4 ; 1 LI = 4 -> 2 LF = 2) = 0.11173097E+00

B (E 4 ; 1 LI = 4 -> 3 LF = 2) = 0.45923703E+00

B (E 4 ; 2 LI = 4 -> 1 LF = 2) = 0.44649685E+00

B (E 4 ; 2 LI = 4 -> 2 LF = 2) = 0.99599463E-01

B (E 4 ; 2 LI = 4 -> 3 LF = 2) = 0.22915358E-01

B (E 4 ; 3 LI = 4 -> 1 LF = 2) = 0.48082049E-01

B (E 4 ; 3 LI = 4 -> 2 LF = 2) = 0.45357364E+00

B (E 4 ; 3 LI = 4 -> 3 LF = 2) = 0.12706164E+00

B (E 4 ; 1 LI = 4 -> 1 LF = 1) = 0.00000000E+00

B (E 4 ; 1 LI = 4 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 1 LI = 4 -> 3 LF = 1) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 1 LF = 1) = 0.10478728E-01

B (E 4 ; 2 LI = 4 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 3 LF = 1) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 1 LF = 1) = 0.35625164E-01

B (E 4 ; 3 LI = 4 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 3 LF = 1) = 0.00000000E+00

B (E 4 ; 1 LI = 4 -> 1 LF = 0) = 0.95238098E+00

B (E 4 ; 1 LI = 4 -> 2 LF = 0) = 0.54563500E-01

B (E 4 ; 1 LI = 4 -> 3 LF = 0) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 1 LF = 0) = 0.00000000E+00

B (E 4 ; 2 LI = 4 -> 2 LF = 0) = 0.81669148E-01

B (E 4 ; 2 LI = 4 -> 3 LF = 0) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 1 LF = 0) = 0.00000000E+00

B (E 4 ; 3 LI = 4 -> 2 LF = 0) = 0.15360860E+00

B (E 4 ; 3 LI = 4 -> 3 LF = 0) = 0.00000000E+00

Q (1 , L = 3) = -0.52105253E-01

B (E 4 ; 1 LI = 3 -> 2 LF = 3) = 0.14603127E+00

B (E 4 ; 1 LI = 3 -> 3 LF = 3) = 0.27870303E-01

B (E 4 ; 2 LI = 3 -> 1 LF = 3) = 0.14603127E+00

Q (2 , L = 3) = -0.44800241E-01

B (E 4 ; 2 LI = 3 -> 3 LF = 3) = 0.17133346E-02

B (E 4 ; 3 LI = 3 -> 1 LF = 3) = 0.27870300E-01

B (E 4 ; 3 LI = 3 -> 2 LF = 3) = 0.17133346E-02

Q (3 , L = 3) = 0.24194011E+00

B (E 4 ; 1 LI = 3 -> 1 LF = 2) = 0.68749990E+00

B (E 4 ; 1 LI = 3 -> 2 LF = 2) = 0.16124362E+00

B (E 4 ; 1 LI = 3 -> 3 LF = 2) = 0.86811871E-01

B (E 4 ; 2 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 4 ; 2 LI = 3 -> 2 LF = 2) = 0.18803081E+00

B (E 4 ; 2 LI = 3 -> 3 LF = 2) = 0.12545032E-01

B (E 4 ; 3 LI = 3 -> 1 LF = 2) = 0.00000000E+00

B (E 4 ; 3 LI = 3 -> 2 LF = 2) = 0.98202450E-01

B (E 4 ; 3 LI = 3 -> 3 LF = 2) = 0.74130201E-01

B (E 4 ; 1 LI = 3 -> 1 LF = 1) = 0.13775507E+00

B (E 4 ; 1 LI = 3 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 1 LI = 3 -> 3 LF = 1) = 0.00000000E+00

B (E 4 ; 2 LI = 3 -> 1 LF = 1) = 0.21251305E-02

B (E 4 ; 2 LI = 3 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 2 LI = 3 -> 3 LF = 1) = 0.25713795E-02

B (E 4 ; 3 LI = 3 -> 1 LF = 1) = 0.47309241E-01

B (E 4 ; 3 LI = 3 -> 2 LF = 1) = 0.00000000E+00

B (E 4 ; 3 LI = 3 -> 3 LF = 1) = 0.43339450E-01

Q (1 , L = 2) = -0.16748114E+00

B (E 4 ; 1 LI = 2 -> 2 LF = 2) = 0.22567284E+00

B (E 4 ; 1 LI = 2 -> 3 LF = 2) = 0.35647257E-01

B (E 4 ; 2 LI = 2 -> 1 LF = 2) = 0.22567287E+00

Q (2 , L = 2) = -0.70087839E-01

B (E 4 ; 2 LI = 2 -> 3 LF = 2) = 0.30169990E+00

B (E 4 ; 3 LI = 2 -> 1 LF = 2) = 0.35647256E-01

B (E 4 ; 3 LI = 2 -> 2 LF = 2) = 0.30169994E+00

Q (3 , L = 2) = 0.10854649E+00

Test output - 13
(Unit #70 BE2BE4.DAT)

B(M1)'S AND M'S WITH SU(3)-R

OF PARTICLES = 4

M (1 , L = 6) = -0.11616621E+00

B (M 1 ; 1 LI = 6 -> 2 LF = 6) = 0.16358599E-01

B (M 1 ; 1 LI = 6 -> 3 LF = 6) = 0.35798904E+00

B (M 1 ; 2 LI = 6 -> 1 LF = 6) = 0.16358599E-01

M (2 , L = 6) = -0.48860645E+00

B (M 1 ; 2 LI = 6 -> 3 LF = 6) = 0.15819268E-01

B (M 1 ; 3 LI = 6 -> 1 LF = 6) = 0.35798904E+00

B (M 1 ; 3 LI = 6 -> 2 LF = 6) = 0.15819268E-01

M (3 , L = 6) = -0.50001013E+00

B (M 1 ; 1 LI = 6 -> 1 LF = 5) = 0.37131712E-01

B (M 1 ; 1 LI = 6 -> 2 LF = 5) = 0.81452392E-01

B (M 1 ; 1 LI = 6 -> 3 LF = 5) = 0.50678380E-01

B (M 1 ; 2 LI = 6 -> 1 LF = 5) = 0.35885097E-02

B (M 1 ; 2 LI = 6 -> 2 LF = 5) = 0.11895594E+00

B (M 1 ; 2 LI = 6 -> 3 LF = 5) = 0.84744584E-04

B (M 1 ; 3 LI = 6 -> 1 LF = 5) = 0.74372988E-03

B (M 1 ; 3 LI = 6 -> 2 LF = 5) = 0.65990933E-02

B (M 1 ; 3 LI = 6 -> 3 LF = 5) = 0.42368863E-02

M (1 , L = 5) = -0.51518226E+00

B (M 1 ; 1 LI = 5 -> 2 LF = 5) = 0.58517791E-03

B (M 1 ; 1 LI = 5 -> 3 LF = 5) = 0.41590557E-01

B (M 1 ; 2 LI = 5 -> 1 LF = 5) = 0.58517791E-03

M (2 , L = 5) = -0.38842225E+00

B (M 1 ; 2 LI = 5 -> 3 LF = 5) = 0.19996479E-01

B (M 1 ; 3 LI = 5 -> 1 LF = 5) = 0.41590557E-01

B (M 1 ; 3 LI = 5 -> 2 LF = 5) = 0.19996479E-01

M (3 , L = 5) = -0.36268678E+00

B (M 1 ; 1 LI = 5 -> 1 LF = 4) = 0.27474517E-01

B (M 1 ; 1 LI = 5 -> 2 LF = 4) = 0.14883009E-02

B (M 1 ; 1 LI = 5 -> 3 LF = 4) = 0.34804090E-02

B (M 1 ; 2 LI = 5 -> 1 LF = 4) = 0.60268309E-01

B (M 1 ; 2 LI = 5 -> 2 LF = 4) = 0.25283774E-01

B (M 1 ; 2 LI = 5 -> 3 LF = 4) = 0.84121898E-01

B (M 1 ; 3 LI = 5 -> 1 LF = 4) = 0.37497975E-01

B (M 1 ; 3 LI = 5 -> 2 LF = 4) = 0.41692727E-02

B (M 1 ; 3 LI = 5 -> 3 LF = 4) = 0.87633980E-02

M (1 , L = 4) = 0.36140382E-01

B (M 1 ; 1 LI = 4 -> 2 LF = 4) = 0.11952908E+00

B (M 1 ; 1 LI = 4 -> 3 LF = 4) = 0.96701741E-01

B (M 1 ; 2 LI = 4 -> 1 LF = 4) = 0.11952908E+00

M (2 , L = 4) = -0.37959751E+00

B (M 1 ; 2 LI = 4 -> 3 LF = 4) = 0.34241558E-02

B (M 1 ; 3 LI = 4 -> 1 LF = 4) = 0.96701756E-01

B (M 1 ; 3 LI = 4 -> 2 LF = 4) = 0.34241558E-02

M (3 , L = 4) = -0.62927090E-01

B (M 1 ; 1 LI = 4 -> 1 LF = 3) = 0.11393177E-01

B (M 1 ; 1 LI = 4 -> 2 LF = 3) = 0.57467539E-01

B (M 1 ; 1 LI = 4 -> 3 LF = 3) = 0.59778627E-01

B (M 1 ; 2 LI = 4 -> 1 LF = 3) = 0.32914488E-02

B (M 1 ; 2 LI = 4 -> 2 LF = 3) = 0.14349570E-02

B (M 1 ; 2 LI = 4 -> 3 LF = 3) = 0.49449299E-01

B (M 1 ; 3 LI = 4 -> 1 LF = 3) = 0.75844664E-03

B (M 1 ; 3 LI = 4 -> 2 LF = 3) = 0.23365293E-01

B (M 1 ; 3 LI = 4 -> 3 LF = 3) = 0.61808180E-01

M (1 , L = 3) = -0.31691167E+00

B (M 1 ; 1 LI = 3 -> 2 LF = 3) = 0.12766311E+00

B (M 1 ; 1 LI = 3 -> 3 LF = 3) = 0.33128707E-03

B (M 1 ; 2 LI = 3 -> 1 LF = 3) = 0.12766311E+00

M (2 , L = 3) = -0.23972613E+00

B (M 1 ; 2 LI = 3 -> 3 LF = 3) = 0.28554473E-01

B (M 1 ; 3 LI = 3 -> 1 LF = 3) = 0.33128707E-03

B (M 1 ; 3 LI = 3 -> 2 LF = 3) = 0.28554473E-01

M (3 , L = 3) = -0.31199294E+00

B (M 1 ; 1 LI = 3 -> 1 LF = 2) = 0.13020770E-01

B (M 1 ; 1 LI = 3 -> 2 LF = 2) = 0.44875871E-03

B (M 1 ; 1 LI = 3 -> 3 LF = 2) = 0.28501812E-02

B (M 1 ; 2 LI = 3 -> 1 LF = 2) = 0.65677166E-01

B (M 1 ; 2 LI = 3 -> 2 LF = 2) = 0.30337580E-01

B (M 1 ; 2 LI = 3 -> 3 LF = 2) = 0.41308012E-01

B (M 1 ; 3 LI = 3 -> 1 LF = 2) = 0.68318427E-01

B (M 1 ; 3 LI = 3 -> 2 LF = 2) = 0.54928720E-01

B (M 1 ; 3 LI = 3 -> 3 LF = 2) = 0.11972920E+00

M (1 , L = 2) = 0.54210350E-01

B (M 1 ; 1 LI = 2 -> 2 LF = 2) = 0.54771479E-01

B (M 1 ; 1 LI = 2 -> 3 LF = 2) = 0.14499086E-01

B (M 1 ; 2 LI = 2 -> 1 LF = 2) = 0.54771479E-01

M (2 , L = 2) = 0.29277565E-01

B (M 1 ; 2 LI = 2 -> 3 LF = 2) = 0.32285767E-04

B (M 1 ; 3 LI = 2 -> 1 LF = 2) = 0.14499090E-01

B (M 1 ; 3 LI = 2 -> 2 LF = 2) = 0.32285770E-04

M (3 , L = 2) = -0.28074726E+00

B (M 1 ; 1 LI = 2 -> 1 LF = 1) = 0.67613326E-01

B (M 1 ; 1 LI = 2 -> 2 LF = 1) = 0.00000000E+00

B (M 1 ; 1 LI = 2 -> 3 LF = 1) = 0.00000000E+00

B (M 1 ; 2 LI = 2 -> 1 LF = 1) = 0.40031258E-01

B (M 1 ; 2 LI = 2 -> 2 LF = 1) = 0.25944260E-02

B (M 1 ; 2 LI = 2 -> 3 LF = 1) = 0.86062793E-02

B (M 1 ; 3 LI = 2 -> 1 LF = 1) = 0.12797551E+00

B (M 1 ; 3 LI = 2 -> 2 LF = 1) = 0.10714749E+00

B (M 1 ; 3 LI = 2 -> 3 LF = 1) = 0.14678601E-01

M (1 , L = 1) = -0.34329531E-07

B (M 1 ; 1 LI = 1 -> 2 LF = 1) = 0.00000000E+00

B (M 1 ; 1 LI = 1 -> 3 LF = 1) = 0.00000000E+00

B (M 1 ; 2 LI = 1 -> 1 LF = 1) = 0.00000000E+00

M (2 , L = 1) = -0.12907322E+00

B (M 1 ; 2 LI = 1 -> 3 LF = 1) = 0.39772340E-02

B (M 1 ; 3 LI = 1 -> 1 LF = 1) = 0.00000000E+00

B (M 1 ; 3 LI = 1 -> 2 LF = 1) = 0.39772340E-02

M (3 , L = 1) = -0.64536266E-01

B (M 1 ; 1 LI = 1 -> 1 LF = 0) = 0.18979202E+00

B (M 1 ; 1 LI = 1 -> 2 LF = 0) = 0.10949925E+00

B (M 1 ; 1 LI = 1 -> 3 LF = 0) = 0.10245356E+00

B (M 1 ; 2 LI = 1 -> 1 LF = 0) = 0.00000000E+00

B (M 1 ; 2 LI = 1 -> 2 LF = 0) = 0.13498359E-01

B (M 1 ; 2 LI = 1 -> 3 LF = 0) = 0.28698582E-02

B (M 1 ; 3 LI = 1 -> 1 LF = 0) = 0.00000000E+00

B (M 1 ; 3 LI = 1 -> 2 LF = 0) = 0.26996845E-01

B (M 1 ; 3 LI = 1 -> 3 LF = 0) = 0.52929940E-02

Test output - 14
(Unit #70 BE2BE4.DAT)

OCCUPATION NUMBERS IN THE SU3 LIMIT.

OF PARTICLES = 3

L	I	<NL(I)|na|NL(I)>	<NL(I)|nd|NL(I)>	<NL(I)|nq|NL(I)>
0	1	0.867E+00	0.180E+01	0.332E+00
0	2	0.969E+00	0.932E+00	0.110E+01
0	3	0.153E+01	0.918E+00	0.553E+00
0	4	0.139E-01	0.159E+01	0.140E+01
0	5	0.834E+00	0.372E+00	0.179E+01
1	1	0.000E+00	0.194E+01	0.106E+01
1	2	0.000E+00	0.106E+01	0.194E+01
2	1	0.788E+00	0.180E+01	0.416E+00
2	2	0.751E+00	0.115E+01	0.109E+01
2	3	0.522E+00	0.132E+01	0.115E+01
2	4	0.380E+00	0.141E+01	0.121E+01
2	5	0.538E+00	0.183E+01	0.631E+00
3	1	0.409E+00	0.140E+01	0.120E+01
3	2	0.269E+00	0.137E+01	0.136E+01
3	3	0.172E-14	0.168E+01	0.132E+01
3	4	0.233E-01	0.223E+01	0.744E+00
3	5	0.265E+00	0.143E+01	0.131E+01
4	1	0.618E+00	0.177E+01	0.616E+00
4	2	0.238E+00	0.163E+01	0.113E+01
4	3	0.494E+00	0.119E+01	0.131E+01
4	4	0.478E+00	0.100E+01	0.152E+01
4	5	0.674E+00	0.833E+00	0.149E+01
5	1	0.186E+00	0.144E+01	0.138E+01
5	2	0.164E+00	0.125E+01	0.158E+01
5	3	0.385E+00	0.929E+00	0.169E+01
5	4	0.206E-01	0.114E+01	0.184E+01
5	5	0.789E-01	0.972E+00	0.195E+01
6	1	0.392E+00	0.166E+01	0.947E+00
6	2	0.286E+00	0.100E+01	0.171E+01
6	3	0.201E-01	0.176E+01	0.122E+01
6	4	0.346E+00	0.132E+01	0.134E+01
6	5	0.261E+00	0.100E+01	0.174E+01
7	1	0.000E+00	0.131E+01	0.169E+01
7	2	0.000E+00	0.142E+01	0.158E+01
7	3	0.000E+00	0.868E+00	0.213E+01
7	4	0.000E+00	0.408E+00	0.259E+01
8	1	0.161E+00	0.141E+01	0.143E+01
8	2	0.290E+00	0.959E+00	0.175E+01
8	3	0.274E+00	0.516E+00	0.221E+01
8	4	0.160E+00	0.629E+00	0.221E+01
8	5	0.114E+00	0.490E+00	0.240E+01
9	1	0.000E+00	0.786E+00	0.221E+01
9	2	0.000E+00	0.214E+00	0.279E+01
10	1	0.000E+00	0.896E+00	0.210E+01
10	2	0.000E+00	0.104E+00	0.290E+01
12	1	0.000E+00	0.000E+00	0.300E+01

Test output - 15
(Unit #70 BE2BE4.DAT)

TWO NUCLEON TRANSFER STRENGTHS FOR N=3 IN THE SU(3) LIMIT.

L	I1	I2	<N+1 L(I1) || a(+)|| N L(I2)>
0	1	1	-0.10228166E+01
0	2	2	-0.87602190E+00
0	3	3	-0.50364886E+00
1	1	1	-0.13627703E+01
1	2	2	0.12127889E+01
2	1	1	0.22291771E+01
2	2	2	0.17408202E+01
2	3	3	-0.17920469E+01
3	1	1	-0.21927336E+01
3	2	2	0.46614821E+00
3	3	3	-0.59024512E+00
4	1	1	-0.28093249E+01
4	2	2	0.22465357E+01
4	3	3	-0.22915781E+01
5	1	1	-0.23962934E+01
5	2	2	0.18164802E+01
5	3	3	0.17458426E+01
6	1	1	-0.30331501E+01
6	2	2	0.24328749E+01
6	3	3	0.21286173E+01

TEST OUTPUT - 16
(UNIT #70 152E20.DAT)

Gd(152) B(E2)'s

OF PARTICLES = 10

Q (1 , L = 4) = -0.18323956E+01
B (E 2 ; 1 LI = 4 -> 2 LF = 4) = 0.78019270E-01
B (E 2 ; 1 LI = 4 -> 3 LF = 4) = 0.26743261E-01
B (E 2 ; 2 LI = 4 -> 1 LF = 4) = 0.78019266E-01
Q (2 , L = 4) = -0.16555602E+01
B (E 2 ; 2 LI = 4 -> 3 LF = 4) = 0.21544537E+00
B (E 2 ; 3 LI = 4 -> 1 LF = 4) = 0.26743262E-01
B (E 2 ; 3 LI = 4 -> 2 LF = 4) = 0.21544537E+00
Q (3 , L = 4) = -0.10687911E+01
B (E 2 ; 1 LI = 4 -> 1 LF = 3) = 0.28447095E-01
B (E 2 ; 1 LI = 4 -> 2 LF = 3) = 0.00000000E+00
B (E 2 ; 1 LI = 4 -> 3 LF = 3) = 0.00000000E+00
B (E 2 ; 2 LI = 4 -> 1 LF = 3) = 0.80093706E-01
B (E 2 ; 2 LI = 4 -> 2 LF = 3) = 0.41755281E-02
B (E 2 ; 2 LI = 4 -> 3 LF = 3) = 0.00000000E+00
B (E 2 ; 3 LI = 4 -> 1 LF = 3) = 0.21234436E+00
B (E 2 ; 3 LI = 4 -> 2 LF = 3) = 0.59205113E-02
B (E 2 ; 3 LI = 4 -> 3 LF = 3) = 0.00000000E+00
B (E 2 ; 1 LI = 4 -> 1 LF = 2) = 0.47904513E+00
B (E 2 ; 1 LI = 4 -> 2 LF = 2) = 0.20277691E+00
B (E 2 ; 1 LI = 4 -> 3 LF = 2) = 0.55949608E-01
B (E 2 ; 2 LI = 4 -> 1 LF = 2) = 0.88380590E-02
B (E 2 ; 2 LI = 4 -> 2 LF = 2) = 0.46584509E+00
B (E 2 ; 2 LI = 4 -> 3 LF = 2) = 0.23298217E+00
B (E 2 ; 3 LI = 4 -> 1 LF = 2) = 0.29380178E-02
B (E 2 ; 3 LI = 4 -> 2 LF = 2) = 0.99575327E-02
B (E 2 ; 3 LI = 4 -> 3 LF = 2) = 0.22606972E+00
Q (1 , L = 3) = -0.71643249E-08
B (E 2 ; 1 LI = 3 -> 2 LF = 3) = 0.00000000E+00
B (E 2 ; 1 LI = 3 -> 3 LF = 3) = 0.00000000E+00
B (E 2 ; 2 LI = 3 -> 1 LF = 3) = 0.00000000E+00
Q (2 , L = 3) = -0.78438421E-08
B (E 2 ; 2 LI = 3 -> 3 LF = 3) = 0.00000000E+00
B (E 2 ; 3 LI = 3 -> 1 LF = 3) = 0.00000000E+00
B (E 2 ; 3 LI = 3 -> 2 LF = 3) = 0.00000000E+00
Q (3 , L = 3) = 0.55345516E-07
B (E 2 ; 1 LI = 3 -> 1 LF = 2) = 0.14200198E-01
B (E 2 ; 1 LI = 3 -> 2 LF = 2) = 0.58523604E-01
B (E 2 ; 1 LI = 3 -> 3 LF = 2) = 0.35517705E+00
B (E 2 ; 2 LI = 3 -> 1 LF = 2) = 0.00000000E+00
B (E 2 ; 2 LI = 3 -> 2 LF = 2) = 0.20167814E-02
B (E 2 ; 2 LI = 3 -> 3 LF = 2) = 0.59423763E-02
B (E 2 ; 3 LI = 3 -> 1 LF = 2) = 0.00000000E+00
B (E 2 ; 3 LI = 3 -> 2 LF = 2) = 0.00000000E+00
B (E 2 ; 3 LI = 3 -> 3 LF = 2) = 0.00000000E+00
Q (1 , L = 2) = -0.10317443E+01
B (E 2 ; 1 LI = 2 -> 2 LF = 2) = 0.16283257E+00
B (E 2 ; 1 LI = 2 -> 3 LF = 2) = 0.41263765E-01
B (E 2 ; 2 LI = 2 -> 1 LF = 2) = 0.16283257E+00
Q (2 , L = 2) = -0.57406086E+00

```
B ( E 2 ;  2 LI =  2 ->  3 LF =  2 ) = 0.47590107E+00
B ( E 2 ;  3 LI =  2 ->  1 LF =  2 ) = 0.41263767E-01
B ( E 2 ;  3 LI =  2 ->  2 LF =  2 ) = 0.47590107E+00
Q ( 3 , L =  2 ) = -0.38873655E+00
B ( E 2 ;  1 LI =  2 ->  1 LF =  0 ) = 0.21408043E+00
B ( E 2 ;  1 LI =  2 ->  2 LF =  0 ) = 0.14717493E+00
B ( E 2 ;  1 LI =  2 ->  3 LF =  0 ) = 0.87597517E-03
B ( E 2 ;  2 LI =  2 ->  1 LF =  0 ) = 0.93870664E-02
B ( E 2 ;  2 LI =  2 ->  2 LF =  0 ) = 0.35015386E+00
B ( E 2 ;  2 LI =  2 ->  3 LF =  0 ) = 0.12984044E+00
B ( E 2 ;  3 LI =  2 ->  1 LF =  0 ) = 0.26159292E-02
B ( E 2 ;  3 LI =  2 ->  2 LF =  0 ) = 0.64743818E-01
B ( E 2 ;  3 LI =  2 ->  3 LF =  0 ) = 0.21290085E+00
```

TEST OUTPUT - 17
(UNIT #72 FTROUT.DAT)

C2(SU(3)) INTERACTION

```
ns = 4 nd = 1 ng = 1
SPECTRAL AVERAGE =-0.19535714E+03
COMPUTED AVERAGE =-0.19535715E+03      0.45000000E+02

ns = 3 nd = 2 ng = 1
SPECTRAL AVERAGE =-0.21819524E+03
COMPUTED AVERAGE =-0.21819524E+03      0.13500000E+03

ns = 3 nd = 1 ng = 2
SPECTRAL AVERAGE =-0.17278572E+03
COMPUTED AVERAGE =-0.17278572E+03      0.22500000E+03

ns = 2 nd = 2 ng = 2
SPECTRAL AVERAGE =-0.18648095E+03
COMPUTED AVERAGE =-0.18648096E+03      0.67500000E+03

ns = 3 nd = 1 ng = 4
SPECTRAL AVERAGE =-0.21075714E+03
COMPUTED AVERAGE =-0.21075715E+03      0.24750000E+04
```

```
ns = 4 nd = 1 ng = 3
SPECTRAL AVERAGE =-0.23018571E+03
COMPUTED AVERAGE =-0.23018572E+03      0.82500000E+03

ns = 0 nd = 0 ng = 5
SPECTRAL AVERAGE =-0.80714286E+02
COMPUTED AVERAGE =-0.80714289E+02      0.12870000E+04

ns = 4 nd = 3 vd = 1 ng = 1 vg = 1
SPECTRAL AVERAGE =-0.36323470E+03
COMPUTED AVERAGE =-0.36323470E+03      0.45000000E+02

ns = 4 nd = 2 vd = 2 ng = 2 vg = 0
SPECTRAL AVERAGE =-0.30039490E+03
COMPUTED AVERAGE =-0.30039490E+03      0.14000000E+02

ns = 2 nd = 3 vd = 1 ng = 3 vg = 1
SPECTRAL AVERAGE =-0.29980613E+03
COMPUTED AVERAGE =-0.29980613E+03      0.45000000E+02

ns = 2 nd = 5 vd = 3 ng = 1 vg = 1
SPECTRAL AVERAGE =-0.36126020E+03
COMPUTED AVERAGE =-0.36126022E+03      0.27000000E+03
```

NOTE:Last column in the above output gives dimensionalitie

TEST OUTPUT - 18
(UNIT #72 FVROUT.DAT)

C2(SU(3)) INTERACTION

```
ns = 0 nd = 2 ng = 0
SPECTRAL VARIANCE = 0.53468822E+03
COMPUTED TRACE = 0.53468821E+03      0.15000000E+02

ns = 2 nd = 1 ng = 1
SPECTRAL VARIANCE = 0.36053340E+04
COMPUTED TRACE = 0.36053339E+04      0.45000000E+02

ns = 0 nd = 2 ng = 2
SPECTRAL VARIANCE = 0.23381053E+04
COMPUTED TRACE = 0.23381053E+04      0.67500000E+03

ns = 3 nd = 2 ng = 1
SPECTRAL VARIANCE = 0.13127638E+05
COMPUTED TRACE = 0.13127638E+05      0.13500000E+03

ns = 2 nd = 4 ng = 2
SPECTRAL VARIANCE = 0.30583084E+05
COMPUTED TRACE = 0.30583083E+05      0.31500000E+04
```

NOTE:Last column in the above output gives dimensionalities .

TEST OUTPUT - 19
(UNIT #70 MDTAU.DAT)

K	MD	NT	<MD|O|ND>	<T|O|T>
3	1	1	0.2646E+01	0.2646E+01
3	3	1	0.2646E+01	0.2646E+01
3	5	1	0.2646E+01	0.2646E+01
3	7	1	0.2646E+01	0.2646E+01
3	9	1	0.2646E+01	0.2646E+01
3	11	1	0.2646E+01	0.2646E+01
3	2	2	0.3761E+01	0.3761E+01
3	4	2	0.3761E+01	0.3761E+01
3	6	2	0.3761E+01	0.3761E+01
3	8	2	0.3761E+01	0.3761E+01
3	10	2	0.3761E+01	0.3761E+01
3	12	2	0.3761E+01	0.3761E+01
3	3	3	0.5457E+01	0.5457E+01
3	5	3	0.5457E+01	0.5457E+01
3	7	3	0.5457E+01	0.5457E+01
3	9	3	0.5457E+01	0.5457E+01
3	11	3	0.5457E+01	0.5457E+01
3	4	4	0.7441E+01	0.7441E+01
3	6	4	0.7441E+01	0.7441E+01
3	8	4	0.7441E+01	0.7441E+01
3	10	4	0.7441E+01	0.7441E+01
3	12	4	0.7441E+01	0.7441E+01
3	5	5	0.9655E+01	0.9655E+01
3	7	5	0.9655E+01	0.9655E+01
3	9	5	0.9655E+01	0.9655E+01
3	11	5	0.9655E+01	0.9655E+01
3	6	6	0.1207E+02	0.1207E+02
3	8	6	0.1207E+02	0.1207E+02
3	10	6	0.1207E+02	0.1207E+02
3	12	6	0.1207E+02	0.1207E+02
3	7	7	0.1467E+02	0.1467E+02
3	9	7	0.1467E+02	0.1467E+02
3	11	7	0.1467E+02	0.1467E+02
3	8	8	0.1744E+02	0.1744E+02
3	10	8	0.1744E+02	0.1744E+02
3	12	8	0.1744E+02	0.1744E+02
3	9	9	0.2036E+02	0.2036E+02
3	11	9	0.2036E+02	0.2036E+02
3	10	10	0.2344E+02	0.2344E+02
3	12	10	0.2344E+02	0.2344E+02
3	11	11	0.2667E+02	0.2667E+02
3	12	12	0.3003E+02	0.3003E+02