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SOFTWARE FOR PRINCIPAL FACTOR ANALYSIS

BY

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SOFTWARE FOR PRINCIPAL FACTOR ANALYSIS

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ABSTRACT

This note describes briefly the principal factor analysis and principal component analysis. A programming system has been developed in FORTRAN IV language to carry out either of the analyses. Various methods of extracting factor loadings have been provided and the user can select any one of them depending on his requirements. Various options have been provided for estimating communalities. Detailed control card information for execution of the system is provided. A sample problem illustrates the input and output of the system.

Key words : Factor analysis, Principal component analysis

I. Introduction:-

The note describes fitting of factor analysis model to a given set of N observations on M variables, The model fitted must explain the information in the collected data in terms of a small number of reference variables. The model which aims at extracting maximum variance is known as principal component analysis and the one that maximally reproduces the correlations is called principal factor analysis.

The next section presents a brief mathematical description of factor analysis. A FORTRAN IV programming system has been developed to perform the required calculations and was tested on DEC-1091 system. Section III contains the details of the system while section IV contains control card information to use the system. Section V provides a sample problem along with its input and output.

II. Mathematical description:-

The classical factor analysis is used to describe each of a set of M variables in terms of P common factors and a unique factor. Thus it is represented as

$$Z!(j)=a(j,1)F!(1)+\ldots+a(j,P)F!(p)+u(j)Y!(j) \text{ for } j=1 \text{ to } M \quad \ldots(1)$$

For ith observation the model (1) can be written as

$$z(j,i)=a(j,1)f(1,i)+\ldots a(j,P)f(P,i)+u(j)y(j,i)$$

$$\text{for } i=1,\ldots,N \text{ and } j=1,\ldots M \ldots(2)$$

In matrix notation (2) can be written as

$$Z=A.F+U.Y \ldots \qquad \ldots(3)$$

Where

$$Z=\begin{bmatrix} z(1,1) & z(1,2) & \cdots\cdots & z(1,N) \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ z(M,1) & z(M,2) & \cdots\cdots & z(M,N) \end{bmatrix}=\begin{bmatrix} Z!(1) \\ \cdots \\ \cdots \\ \cdots \\ Z!(M) \end{bmatrix}$$

is original set of observations.

$$F=\begin{bmatrix} f(1,1) & f(1,2) & \cdots\cdots & f(1,N) \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ f(P,1) & f(P,2) & \cdots\cdots & f(P,N) \end{bmatrix}=\begin{bmatrix} F!(1) \\ \cdots \\ \cdots \\ \cdots \\ F!(P) \end{bmatrix}$$

are the common factors,

$$Y=\begin{bmatrix} y(1,1) & y(1,2) & \cdots\cdots & y(1,N) \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ y(M,1) & y(M,2) & \cdots\cdots & y(M,N) \end{bmatrix}=\begin{bmatrix} Y!(1) \\ \cdots \\ \cdots \\ \cdots \\ Y!(M) \end{bmatrix}$$

are unique factors

$$A=\begin{bmatrix} a(1,1) & a(1,2) & \cdots\cdots & a(1,p) \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a(M,1) & a(M,2) & \cdots\cdots & a(M,p) \end{bmatrix}=\begin{bmatrix} A!(1) \\ \cdots \\ \cdots \\ \cdots \\ A!(M) \end{bmatrix}$$

are factor loadings or weights or coefficients,

$$\begin{bmatrix} u(1) & 0 & \cdots\cdots\cdots & 0 \\ 0 & u(2) & \cdots\cdots\cdots & 0 \\ 0 & 0 & \cdots\cdots\cdots & 0 \\ 0 & 0 & \cdots\cdots\cdots & 0 \\ 0 & 0 & \cdots\cdots\cdots & U(M) \end{bmatrix} = \begin{bmatrix} u1(1) \\ \cdots \\ \cdots \\ \cdots \\ u1(M) \end{bmatrix}$$

are unique factor coefficients.

The problem is to find the coefficient matrix A known as the pattern matrix for a given Z.

Postmultiplying both sides of equation (3) by Z we have

$$\begin{aligned} ZZ' &= (AF+UY)\ (AF+UY)' \\ &= (AF+UY)\ (F'A'+Y'U') \\ &= AFF'A' + AFY'U' + UYY'U' + UYF'A' \quad \ldots\ldots (4) \end{aligned}$$

Under the assumption that the unique factors are uncorrelated with common factors and also among themselves equation (4) reduces to

$$ZZ' = AFF'A' + UIU' \qquad \ldots\ldots \qquad (5)$$

When Z is normalized the l.h.s. of (5) is a correlation matrix. From (5) we have

$$ZZ' = UIU' = AFF'A' = ASA' \qquad \ldots\ldots \qquad (6)$$

Where S is the correlation matrix of the common factors. The l.h.s. of (6) is the correlation matrix but for the diagonal elements. The diagonal elements are $h^2(i) = 1.0 - u^2(i)$ for $i=1,2,.....M$; where $h^2(i)$ is the communality of ith variable. Now if the factors are uncorrelated i.e. orthognal to one another equation (6) becomes

$$\bar{R} = A\ A' \quad .... (7)$$

where R is the l.h.s. of (6). Equation (7) is known as the fundamental factor theorem. The component analysis model is described as

$$Z1(j)=a(j,1)F1(1)+....+a(j.M)F1(M), \text{ for } j=1,2,...M \quad ...(8)$$

and it is different from (1) as it does not contain a unique factor and in this case equation (7) becomes

$$R = A\ A' \quad ....(9)$$

The factor coefficients are obtained in such a way that the sum of the contributions of ith factor to the total communality is greater than or equal to the j th factor contribution where i j. The sum of the contributions of i th factor is given by

$$v(i) = S\ \ (j,M;a\ (j,i)) \quad ....(10)$$

Where S $(j,M;x(j))$ stands for the summation of the elements $x(j)$ for $j = 1$ to M.

Thus v(i) is to be maximized subject to the condition (7). It can be shown that maximization of (10) subject to (7) is equivalent to the determination of i th characteristic root and vector of the matrix R. Let C be the ith characteristic root and

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$b(1,i)$, $b(2,i)$,.....$b(M,i)$ be the corresponding vector. The characteristic vector is scaled to obtain the coefficients of i th factpr as

$$a(j,i) = b(j,i) \sqrt{(C)} / \sqrt{S(j,M;b\ (j,i)} \quad \text{for } j=1,\ldots,M \qquad ..(11)$$

The principal component analysis follows on similar lines, however, that (7) replaced by (9). The component analysis requires all the M principal components to explain completely the inter correlations between variables. Various options have been provided to extract/retain less than or equal to M principal components. The principal factor analysis however requires apriori either an estimation of communalities or an estimation of the number of common factors.

Estimation of communalities:-

Several options have been provided for estimation of communalities. They are

a) Take the communality of variable i as equal to highest correlation of it from among its correlations with others.

b) Take communality of variable i as $h^2(i)=r(i,j).r(i,k)/r(j,k)$ where $r(i,j)$,$r(i,k)$ are highest correlations of variable i compared to others.

c) Take communality of variable i as

$$h^2(i) + S(j,M,i\neq j;\ r(i,j))/-(M-1)$$

where the notation $S(j,M,i\neq j;x(j))$ stands for the summation of the elements $x(j)$, for $j=1$ to M and $i\neq j$.

d) Take communality of i the variable as equal to squared multiple correlation of it with the remaining variables.

A. Methods based on known communalities:-

After the selection of communality user is provided with a variety of procedures to select from as per his requirements to obtain factor loadings. The various methods that have been provided are

a) Hotelling method.

b) Hotelling iterative scheme.

c) Power method.

d) Power method with rayleigh quotient.

e) Inverse power method.

f) Jacobi method.

g) Givens method.

h) Householders method.

They are described briefly here:

a) Hotelling method:-

This procedure is nothing but power method with matrix powers modified to suit factor analysis. In this method each factor extracts the maximum amount of variance and provides the smallest possible residuals. By this method the correlation matrix is condensed into the smallest number of orthogonal factors. It has the advantage of giving a mathematically unique (in the sense of least squares) solution to a given table of correlations. The algorithm is briegly as follows:

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1) Sum each column of the correlation matrix R into the vector S.

2) Obtain $T = R.S$, where R is the correlation matrix or reduced correlation matrix.

3) Normalize the vectors S and T to make their largest values as one. The ratio of largest value of S to largest value of T is known as normalizing factor.

4) If the maximum absolute difference of the elements of these normalized vectors is less than epsilon, go to step six.

5) Multiply the correlation matrix by itself to obtain R1. Replace R by Ri and S by T and goto step two.

6) Calculate the vector V given by

$$V = R\ T$$

7) Obtain the factor loadings as

$$A1' = V/\ (T\ V)$$

8) Obtain the residual correlation matrix as

$$R1 = R - A1' \cdot A1$$

where R1 denotes the residual matrix and A1 a column vector of A.

9) If residuals are small stop extracting further factors.

10) Estimate the communalities if required and go to step one with R replaced by residual matrix R1.

b) <u>Hotelling iterative scheme:-</u>

1. Do the procedure 'a' to obtain the factor loadings for as many factors as required.

2. Estimate the communalities.

3. Use these communalities and repeat procedure 'a'.

4. Repeat steps one to three till the maximum absolute difference among the communalities of two consecutive iterations is small.

c) Power method:-

Same as method 'a' but for step five replaced by 'replace S by T and go to step two'.

d) Power method with Rayleigh quotient:-

Same as method 'c' except replace normalizing factor by

$$V'(q;)V(q+1;)/ \quad V'(q;)V(q;)$$

where $V(q;)$ is the vector at q th iteration.

e) Inverse power method:-

In this method the set of linear equations

$$[R-K(i+1;)I] \quad Z(i+1;) = V(i;)$$

is solved using partial pivoting, where

$$K(i+1;)=V'(i;)RV(i;)/V'(i;)V(i;) \quad \text{and}$$

$$V(i+1;)=1(i+1;)Z(i+1;)$$

where $1(i+1;)$ is normalizing factor at 'i+1' th iteration such that $\| V(i;) \| = 1$

f) Jacobi method:-

The method involves the diagonalization of the matrix R by performing a sequence of orthogonal transformations on it, in order to reduce one of the off-diagonal element to zero at each stage. Each orthogonal transfermation is of the form B R B'. When an element is reduced to zero it does not, in general remain at* squares of the off-diagonal elements will decrease each time by an amount corresponding to $2r^2(j,k)$.

1. Initialize $j = 1$, $k=j+1$, $m=1$

2. If $d(j,k) < e$, (where $d(j,k)=r(j,k)$ initially and e is a small positive quantity), then goto step six.

3. Compute B, where B is a unit matrix except for the elements $b(j,j)=b(k,k)=Sqrt(0.5+D1)$
   and $b(j,k) = Sqrt(0.5-D1)$; $b(k,j)=-b(j,k)$,
   where $D1 = Sqrt((P^2-4r^2(j,k)P)/2P)$
   and $P = 4r^2(j,k)+h^4(j)+h^4(k)-2h^2(j)h^2(k)$
   where $h^2(j)$ is communality of j th variable.

4. Compute

   $$D(\overline{m}) = B \quad D(\overline{m-1};) \; B'$$

   Where $D(\overline{0})=R$ initially and $D(\overline{m})$ is the matrix at m th stage in 'i' th iteration.

5. Compute

   $$B(\overline{m})=BB(\overline{m-1})$$

   Where $B(\overline{0})=I$ initially and $B(\overline{m})$ is the matrix at m th stage in i th iteration.

6. If $k=M$ go to step 8.

7. Replace k by k+1, m by m+1 and go to step two.

8. If $j=M-1$ goto step 10.

* zero during subsequent transfermation. However, the sum of the

9. Increment j by one and replace k by j+1 and goto step two.

10. Let $b(i;)=B(\bar{m})$ and $D(i;)=D(\bar{m})$, where $B(i;)$ and $D(i;)$ are the matrices at 'i' th iteration. Find trace $D(i;)$. If DIF = $|$ Trace $D(i;)$ - Trace R $| > e1$, then stop after printing DIF, where e1 is a small positive quantity.

11. Compute $D(i;)P(i;)$ and $P(i;)R$, where $P(i;) = B(i;) \ldots B(1;)$.

12. If the maximum absolute difference between elements of $D(i;)P(i;)$ and $P(i;)R > e2$, then stop after printing difference.

13. Let the number of off-diagonal elements $d(j,k)$ whose absolute values are not less than e be k.

14. If k=0 goto step 16.

15. If specified number of iterations are not performed goto step 1, after making B as unit matrix.

16. Determine

$$A = B'\sqrt{D}$$

g) <u>Given's method:-</u>

This method uses orthogonal matrices to tridiagonalize R. Denote the matrix $B(p,q,r)$ whose elements are

$$b(p,p)=b(q,q)=Cd(r,p)$$

$$b(p,q)=b(q,p)=-Cd(r,q)$$

$$b(i,i)=1$$

$$b(i,j)=0$$

Where $C = 1/\left[(d(r,p))^2 + (d(r,q))^2\right]^{1/2}$

Pre and post multiply the matrix R by $B(j,i,j-1)$ for $j=2$ to $n-1$ and $i=j+1$ to n, to get D.

Where $D(0;) = R$ initially.

Get the eigen values of D using any algorithm.

h) <u>Householder's method:-</u>

Reduce the symmetric matrix to a tridiagonal symmetric matrix. Determine the eigen values of the tridiagonal matrix by means of a series of orthoganal rotations such that the resulting matrices remain tridiagonal. Solve for the eigen vectors of the tridiagonal matrix directly as a set of simultaneous equations. In this technique a whole row and column are reduced to zero at a time excluding tridiagonal elements. Choose the vector

$$V(k;k) = \left[0 \quad 0 \; \ldots \quad v(k;k) \; v(k;k+1) \; \ldots \; v(k;M)\right]$$

Where $(v(k;k))^2 = 0.5\left[1+d(k-1,k)/\sqrt{(s1)}\right]$

and $(v(k;j)) = +d(k-1,j) \,/\, \left[2v(k;k)\sqrt{(s1)}\right]$

Where $s1 = S\left[j,M;d(k-1,j)\right]$

Form $P(k;) = I-2V(k;)V'(k;)$
then apply $D(k;) = P'(k;)D(k-1;)P(k;)$
to get the tridiagonal matrix D, where $D(0;)=R$ initially.

B) Methods based on known number of factors:-

When user prefers an estimate of the number of factors, the procedure of minimization of residual correlations popularly known as 'MINERS' method is adapted. Here the off-diagonal elements of the correlation matrix are maximally reproduced- The method gives as a by product the communalities of the variables. The 'MINERS' method is briefly described below.

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MINERS METHOD:-

1. Calculate factor matrix A(O) consisting of the first M1 principal components of the matrix R.

2. Calculate the incremental changes $e(j) = R1 \, !'(0;) A \left(A(j^*)A(j^*)\right)^{-1}$ for the variable j starting with j=1, where $A(j^*)$ stands for the factor matrix with j th row elements replaced by zero. Find the matrix $A!'(j;)$ in which the loadings in the 'j' th row have been replaced by the computed values

$$b(j,p) = a(j,p) + e(p), \text{ for } p=1,\ldots.M.$$

3. Calculate the communality $h^2(j)$. If $h^2(j) < 1$, goto step 11.

4. Compute the matrix W where

$$b'Wb = S(k,M,k\neq j; \left(r(j,k)-S\left[i,P;a(k,i)\,b(j,i)\right]\right)^2)$$

5. Diagnalize matrix W

6. Obtain X, Y, K given by

$$f(j) = S\left[i,P; \left(X(i)-Y(i)\right)^2\right] +K$$

subject to

$$S\left[i,P; \; x^2(i) / L^2(i)\right] < 1$$

Where f(j) is sum of residual correlations of j th variable.

7. Determine

$$x(0;1) = y(1)/ \sqrt{S\left(i,P;(x(i)/L(i))\right)^2}$$

8. Compute

$$x(0;i)=L(i)y(i)x(1)/\left[(L^2(i)-L^2(1))x(1)+L^2(1)y(1)\right]$$

for i=2 to P-1

$$x(0;M)=1-S\left[i,P;X^2(i)/L^2(i)\right]$$

9. If the absolute difference between $x(k;i)$ and $x(k+1;i)$ is not small go to step 7, otherwise step 10.

10. Determine modified matrix $A(i;)$.

11. Compute residual matrix.

12. When $j < M$ go to step two.

13. If maximum change in factor loadings of two consecutive iterations is small go to step 15.

14. If required number of iterations are not performed go to step two.

15. Rotate to canonical form and print the results.

Canonical form presentation:-

The solution can be transfered to a canonical form using

$D=A.T$ --------(12)

Where D is in canonical form of factor matrix and T is an orthoganal transfermation matrix. It can be shown that T is the eigen vectors of the matrix $A'A$.

Stopping criteria:-

1. When 90% of the total variation accounted then no additional factor accounting for less than 2% is retained.

2. The number of common factors should be equal to the number of eigen values greater than one using full correlation matrix.

Testing criteria:-

To test the significance for P factors the chi-value given by

$$CHI=(N-1-(2M+5)/6-2P/3)\left[\ln|AA'+U^2|-\ln|R|+tr\left(R(AA'+U^2)\right)-M\right]$$

is provided which follows a chi-square distribution with

$0.5\left[(M-P)^2+(M-P)\right]$ *degrees of freedom.*

III. Programming system:-

The program does principal component analysis or principal factor analysis with or without iteration. All computations are done in double precision. All floating point variables are in double precision. The program gives orthogonal factor loadings, percent variation and cumulative percent variation, percent contribution of each variable to its percent factor variation. Output is available in non canonical form as well as in canonical form.

The program is generally used along with 'CACSR' program which calculates the correlation matrix. The output of 'CACSR' program forms the input to this program. Correlation matrices calculated using other packages can also be used as input to this program. After the factors are extracted if found necessary one would like to rotate the factors orthogonally or obliquly. The program 'FAROTCSR' can be used for this purpose. Thus one may use three program systems for a complete factor analysis.

IV Control Card Information:-

Input:-

The input to the program consists of 3 to 5 control cards.

Control card 1:-

This control card reads the data format, to read correlation matrix which is input to this program.

Control card 2:-

This control card reads 20 parameters in 20 I 4 format. Default values for these parameters are zero. All these values must be right justified.

Parameter 1:

Specifies order of the correlation matrix.

Parameter 2:

If specified value is 1, residual correlation matrix forms part of output.

Parameter 3:

If specified value is 1 names of variables are supplied and this requires control card(s) 4.

Parameter 4:

This parameter specifies the input unit from which data is read. Default option card reader.

Parameter 5:

This parameter specifies the output unit on which results have to be written. Generally specified when rotation of factor loadings required. For printing need not be specified as it always prints.

Parameter 6:-

This parameter specifies the number of factors to be extracted. By default extracts as many factors as possible using other criteria.

Parameter 7:

This parameter specifies the number of records to be bypassed.

Parameter 8:

When specified value is 1, rewinds the input unit.

Parameter 9:

If unspecified reads upper triangular matrix including diagonal elements row by row. When specified as 1 reads lower triangular matrix excluding diagonal row by row.

Parameter 10: If 1 input printed, otherwise not.

Parameter 11:

This parameter is meant for carrying out different types of analysis as follows:

Value	Type of analysis
0	Principal component analysis
1	Principal factor analysis
2	Principal factor analysis with iteration

Parameter 12:

This parameter is required if the value of 11 th parameter of this card is 2. It specifies the maximum number of iterations one would like to allow for convergence to take place. If convergence does not take place before the specified number of iterations result will be printed after the completion of specified number of iterations.

Parameter 13:

This parameter is used for branching to different parts of the program while processing more than one data set.

Value	next data set from
0,1	Control card 1
2	Control card 2
3	Control card 3
4	Control card 4
5	Control card 5
6	Direct data set

Parameter 14:

This parameter is required when the value of 11 th parameter of this card is 2. This parameter is for intermediate output.

This parameter specifies after how many iterations communalities are to be printed. When the specified value is 10, communalities will be printed after every 10 iterations.

Parameter 15

When specified as 1, third control card will be read.

Parameter 16:

This parameter specifies the method selected. Values 1 to 8 correspond to methods 'a' to 'h'. By default 1. If 9 MINERS method. If 9, parameter 6 must be specified.

Parameter 17:

Required if selected method is 'a'. Decisive parameter. If 1 reestimates communalities after obtaining each factor otherwise not. By default 0.

Parameter 18:

Required if selected method is 'g'. One to five correspond to methods 'a' to 'e'. By default method 'a'.

Parameter 19:

Decisive parameter. If value is 1 canonical form results also printed otherwise not.

Parameter 20:

Decisive parameter. If value is 1, chi-square test tested otherwise not.

Control card 3:-

This control card is necessary if the 15th parameter of control card two is 1. This control card reads seven parameters in 7 F 10.7 format.

Parameter 1:

This parameter specifies the epsilon value for stopping the iterative procedure while extracting each factor. The default option for this parameter is 0.00001.

Parameter 2:

This parameter specifies the maximum value for stopping extracting further factors against cumulative percentage variation. The default option is 99.0.

Parameter 3:

This parameter specifies the check value against zero for column sum. The default option is 0.05.

Parameter 4:

This parameter specifies the check value for stopping the iterative procedure while determining communalities. Default option is 0.00001.

Parameter 5:

When value is 1.0, stopping criteria 1 is used. If value is 2 stopping criteria 2 is used, otherwise extracts as many as possible based on the other parameter values of this control card.

Parameter 6:

When value is 1.0 then percent contribution of each variable to its factor will be calculated and printed otherwise not. Default value is 0.

Parameter 7:

Decisive parameter. Values 1.0 to 4.0 correspond to 'a' to 'd' options. Default value 1.0.

Control card(s) 4:-

This control card(s) will be present if the 3rd parameter of control card two is 1. This control card reads the variable names for printing along with the output. These variables are read in (8(A8,2X) format, that is at the rate of eight variable names per record.

Control card 5:-

This card reads the title for the data to be processed, in 10A8 format.

V. *Sample problem:-*

Do the principal component analysis on the following correlation matrix (refer H.H.Harman), with all parameters at default values.

1.0	0.00975	0.97245	0.43887	0.02241
0.00975	1.0	0.15428	0.69141	0.86307
0.97245	0.15428	1.0	0.51472	0.12193
0.43887	0.69141	0.51472	1.0	0.77765
0.02241	0.86307	0.12193	0.77765	1.0

Input:-

First control card:-

1.......... column number

(5F15.5) ntrol card:-

Second.control card:-

00000 column number

12345 " "

5 value

Fifth control card:-

1.......... column number

SAMPLE PROBLEM value

After these control cards the given data is supplied in the format specified by the first control card.

Output:

The output of the system is factor loadings, along with percent variation and cumulative percent variation.

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Factor 1

0.581 0.767 0.672 0.932 0.791

Variation 2.873

Percent variation 57.5

Cumulative percent variation 57.5

Factor 2

0.806 -0.545 0.726 0.104 0.558

Variation 1.797

percent variation 35.9

Cumulative percent variation 93.4

How to execute the system:-

```
COPY FACSRH.FOR=FACSRH.FOR [211,124]
```

Change the parameter card as per dimensions requirement.

```
COMP FACSRH.FOR
LOAD FACSRH, [211,124] FHCSR,INVCSR
SAVE
RUN FAHCSRH
```

Control card 1
Control card 2
Control card 3
DATA

VI. <u>Acknowledgements:-</u>

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