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A NOTE ON STEPWISE REGRESSION ANALYSIS PROGRAMMING SYSTEM

By

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CONTENTS

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ABSTRACT

This note describes a general programming system - 'RACSRS' for stepwise regression analysis, which has been developed in FORTRAN IV. The system has been thoroughly tested using IBM 360/44. The package can also handle any nonlinear model that can be transformed to linear.

Key words:

1. Stepwise multiple regression analysis
2. Best curve fitting
3. Model building

I. Introduction:-

In Ordinary Multiple Regression Analysis (OMRA) all variables under consideration will be in the model. Situations often arise where a number of predictor (independent) variables are available (sometimes more than the number of observations) and the model builder wishes to select a few of the best predictors (statistically significant or otherwise) to be in the model.

The note describes the Stepwise Multiple Regression Analysis (SMRA) which is a very useful statistical tool for this purpose. The principle underlying it is that off least squares. This differes from OMRA in that only statistically significant variables are introduced in the model.

The technique introduces one variable at a time into the model provided it is statistically significant. After entering a variable, all variables which are in the model are tested statistically for possible removal and removed if not found significant. The mathematical description of the procedure is presented in Section II, while Section III contains the algorithm. Section IV contains details of the programming system. Control card information is provided in Section V, while section VI contains a sample problem alongwith its input and output.

II. Mathematical description:

The stepwise multiple regression analysis is used to obtain a linear relationship between a dependent variable Y and a set of L (unknown, $L \geq 0$) independent variables which are significant statistically from a set of M given variables X_1, X_2..... X_M, when a sample of 'n' observations on them are provided.

Let Y_i = ith observation on the dependent variable for $i = 1$ to n

X_{ij} = ith observation on the jth independent variable for $i=1,...n$ and $j=1,...M$

w_i = Weighting factor for the ith observation for $i = 1$ to n

Now define

$$W_i^{(1)} = n.w_i / \sum w_i$$

$$y_i = Y_i - \bar{Y}$$

$$x_{ij} = X_{ij} - \bar{X}_j \text{ for } i = 1, \ldots n \text{ and } j = 1, \ldots M$$

where

$$\bar{Y} = \sum_{i=1}^{n} W_i Y_i \Big/ \sum_{i=1}^{n} W_i$$

$$\bar{X}_i = \sum_{i=1}^{n} W_i X_{ij} \Big/ \sum_{i=1}^{n} W_i$$

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1) The Weighting factors w_i are usually converted into weights W_i so that $\sum W_i = n$, the number of observations. Where this is not done, then the standard error of the regression coefficients and the criterion of significance will be distorted.

The parameters A in the regression model,

$$Y = a_0 \mathbf{1} + XA + \epsilon \quad \ldots\ldots\ldots \qquad (1)$$

are given by

$$\hat{A} = (x' W x)^{-1} \; x' W y \quad \ldots\ldots\ldots \qquad (2)$$

and

$$\hat{a}_0 = \bar{Y} - \sum_{j=1}^{M} \hat{a}_j \bar{X}_j \quad \ldots\ldots \qquad (3)$$

where

Y is a $n \times 1$ vector of deviations y_i

$\mathbf{1}$ is $n \times 1$ vector of 1's

X is a $n \times M$ matrix of deviations x_{ij}

A is a $M \times 1$ vector of coefficients

$\hat{A}$ is a $M \times 1$ vector of estimates of coefficients

ϵ is a $n \times 1$ vector of disturbances

and W is a $n \times n$ diagonal matrix of weights

The procedure of stepwise regression analysis starts with the calculation of correlation matrix R whose elements r_{jk} are given by

$$r_{jk} = S_{jk} \Big/ \sqrt{S_{jj}} \sqrt{S_{kk}} \quad \text{for } j = 1, \ldots. M \text{ and } k=1, \ldots. M$$

where

$$S_{jk} = \sum_{i=1}^{n} W_i x_{ij} x_{ik} \quad \text{for } j = 1, \ldots. M \text{ and } k=1, \ldots. M$$

and the vector T of correlation coefficients of the dependent variable with the M independent variables whose elements $t_{1,j}$ are given by

$$t_{1j} = S1_{1,j} \Big/ \sqrt{S2} \sqrt{S_{jj}}$$

where $S1_{1,j} = \sum_{i=1}^{n} W_k x_{ij} y_i \quad \text{for } j = 1, \ldots. M$

$$S2 = \sum_{i=1}^{n} W_i y_i y_i$$

After the completion of calculation of correlation matrix the stepwise procedure described in the following four steps is carried out.

1. Select the variable, whose absolute value of the correlation with the dependent variable (d.v) is maximum. If this correlation is significant then introduce the variable into the model. If not significant then the variation in the d.v cannot be explained by any of the given set of M independent variables and hence stop the procedure.
2. Calculate the partial correlations controlling all the variables in the model at that stage.
3. Remove the variable, whose absolute value off the partial cprrelation with the d.v. is least from the model if found not significant and then go to step 2. If found significant proceed to step 4 without removing the variable.
4. Introduce the variable, whose absolute value of the partial correlation with the d.v. is highest into the model if found significant statistically and then go to step 2. If found not significant stop the procedure without introducing the variable.

III. Algorithm:

The algorithm given by Efroymson (1960) has been used for the development of this system. His procedure is based on the gaussian elimination as described by Orden (1960) to solve a system of simultaneous equations (which are referred to in statistical terminology as normal equations).

The regression coefficients are obtained by applying linear transformations to the following partitioned matrix.

$$B = (b_{jk}) = \begin{pmatrix} R & T' \\ T & Z \end{pmatrix}$$

where R is M x M correlation matrix of independent variables (i.v's), T is a 1 x M correlation vector of i.v's with d.v, and Z a scalar, is the correlation of d.v with itself. Thus

$$b_{jk} = r_{jk} \text{ for } j = 1,\ldots M \text{ and } k = 1,\ldots M$$

$$b_{k,M1} = b_{M1,k} = t_{1,k} \text{ for } k = 1,\ldots M$$

$$b_{M1,M1} = Z. \quad \text{where } M1 = M + 1$$

The following four steps depict the algorithm completely.

Step 1:

Calculate the statistic V_j for $j = 1,\ldots M$; if $b_{jj} > T$[1],

where V_j is given by

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(1) T is the tolerance limit which is a small positive quantity. The tolerance on b_{jj} is meant for reducing the possibility of degeneracy when an independent variable is approximately equal to a linear combination of the independent variables.

$V_j = b_{j,M1}\, b_{M1,j} / b_{jj}$

Calculate standard error of $Y = St_y = \left(\frac{S2 . b_{M1,M1}}{D}\right)^{0.5}$

where $D^{(1)}$ stands for degrees of freedom.

Step 2:

For all those V_j's which are positive make a_j's the regression coefficients zero, as the corresponding x_j's are not in the model, and for those V_j 0, calculate the coefficients $a_j = b_{j,M1}\sqrt{(S2/S_{jj})}$ since they are in the model. Also calculate std. error of $a_j = St_y\,(a_{jj}/S_{jj})^{0.5}$

and $a_0 = \bar{Y} - \sum_{j=1}^{M} a_j \bar{X}_j$

Let $|V_p|$ be the minimum among all $V_j < 0$. Remove the variable X_p from the model provided $|V_p| . D / b_{M1,M1} < F_2$ (where F_2[2] is the value for removing a variable) and then go to step 4 after increasing D by 1, otherwise go to step 3.

---

(1) Initially D takes the value $\sum_{i=1}^{n} W_i - 1.0$

(2) The sequential partial F-value which is F(1, N-L-1, 0.95) or F(1,n-L-1, 0.90) where L represents the number of variables in the model at that stage.

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(2) The sequential partial F-value which is F(1, N-L-1, 0.95) or F(1,n-L-1, 0.90) where L represents the number of variables in the model at that stage.

Step 3:

Let V_p be the maximum among all $V_j > 0$. Introduce the variavle X_p into the model provided $V_p \cdot D/(b_{M1,M1} - V_p) > F_1$ (Where $F_1^{(1)}$ is the value for allowing a variable to enter into the model) and then go to step 4 after decreasing D by 1, otherwise stop the procedure.

Step 4:

Let b_{pp} be the diagonal element where p corresponds to the variable entered or removed from the model. Update the matrix elements b_{jk} to get the new matrix elements b^u_{jk} where b^u_{jk} are given by

$$b^u_{jk} \begin{cases} = b_{jk} - b_{jp} b^u_{pj} & \text{for } j \neq p,\ k \neq p \\ = b_{pk} b^u_{pp} & \text{for } j = p,\ k \neq p \\ = -b_{jp} b^u_{pp} & \text{for } j \neq p,\ k = p \\ = 1/b_{pp} & \text{for } j = p,\ k = p \end{cases}$$

where superfix 'u' indicates updated. After this step go to step 1.

When once the model is fitted the other calculations like F-value, R^2, $\bar{R}^2$, RSS, TSS, ESS, Durbin-Watson statistic 'd', first order auto correlation, expected values of d.v are performed as in the case of OMRA.

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(1) The sequential partial F value which is $F(1, n-L-1, 0.95)$ where L represents the number of variables in the model at that stage.

IV. Programming System:-

The system contains a main program which calls the 'STCSR' subroutine. The main program aims at supplying object time dimensions to the subroutine 'STCSR'.

The program is quite versatile and one can fit any model[1] which is linear in parameters to a given data. 21 transformations are provided to make the non-linear models (in variables) linear. Facilities for addition, subtraction, multiplication and division of variables are provided to generate new variables which are finally used in the regression model.

The system performs weighted or unweighted (i.e. $w_i = 1$ for $i = 1$ to n) stepwise regression analysis and it can also be used to do frequency analysis ($\sum_{i=1}^{n} w_i \neq n$). This package handles models with or without constants. One can do OMRA using this package by simply supplying F-values as 0. However it is advisable not to use this package for OMRA as it involves additional computer time.

The system requires F-values to be provided for introducing a variable into and for removing a variable from the model. They should actually be calculated as degrees of freedom vary depending on the number of variables in the model, but F-values are supplied at fixed degrees of freedom (1, n-2), where n stands for number of observations.

The system performs all computations in double precision. The input data is however, accepted in single precision.

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1. The model can be linear or nonlinear in variables.

V. Control Card Information:

This section has to be read alongwith the Control Card Information provided in Section IV of a PRL Technical Note on 'A Programming System on Multiple Regression Analysis.' (PRL Technical Note, TN-83-33).

In order to maintain a close proximity with the programming system 'RACSR' (PRL TN-83-33), the control cards of this system are kept similar to that of 'RACSR'. Thus control card Nos 1, 3, 5 6, 7, 8 and 9 of both the systems are identical. Control card Nos 10 and 11 of 'RACSR' system are not present in this system, hence control card No.10 of this system is similar to that of 12th control card of 'RACSR'. However some control variables in control card 2, 4 and 10 (12th in 'RACSR') are different. The first 8 control variables of control card 4 are same but the next 4 control variables differ widely. Though the control cards and the control variables of them are kept similar to that off 'RACSR', but for a few changes so that one can run both the programs with the same control cards, the following points are worth to be noted.

1) The method of calculation differ in both the systems.

2) Certain control variables which are dummy in one may not be so in other.

3) Certain features(like providing confidence limits on the regression coefficients)which are present in one may be absent in another.

Having seen broadly the similarities and differences of this system with 'RACSR' Programming system, let us go into the details of those control cards which differ from 'RACSR' program. For control cards similar to 'RACSR' program see PRL Technical Note TN-83-33.

Control card 2:-

This control card is most essential in the sense that 1 to 7 additional control cards have to be supplied based on the values taken by some control variables of this card. It has 25 control variables which are read in 25I2 format. All control variables are identical with 'RACSR' program but for 22nd and 23rd . However some of these control variables are dummy in the present system, i.e they won't serve the purpose for which they are used in 'RACSR' program. The details of all control variables are described below. Default options for these control variables are generally zero and they are mentioned otherwise.

Control variables	Value taken	Function/remarks
1	0	–
	1	To read control card No.8 To form new variables involving multiplication and/or division of variables
2 (used alongwith control variable 1 of this card)	0	When variables take positive values, or when the power to which a variable raised is a fraction
	1	When all or few variables take negative values

..12

Control variables	Value taken	Function/remarks
3	0	Input is card reader
	Others	Input unit from which data is read
4	Any integer value ≤ 50	Number of independent variables
5	Any integer value ≤ 45	Number of dependent variables
6	Any integer value ≤ 50	Number of coefficients to be estimated (excluding constant) + No. of dep.variables
7	0	-
	1	To read control card No.4
8	0	-
	1	To read control card No.6
9	0	-
	1	To read control card No.7
10	0	-
	1	To read control card No.8
11*	Always 0	Dummy variable
12**	Always 0	Dummy variable (other than 0 Control card 3 to be provided which is also dummy)
13*	Always 0	Dummy variable
14	0	Input data not printed
	1	Input data printed
15**	Always 0	Dummy variable (other than zero, Control card 3 to be supplied which is dummy)
16*	Always 0	Dummy variable

..13

Control variables	Value taken	Purpose/remarks
17**	Always 0	Dummy variable (3rd control card to be provided which is dummy when value is other than zero)
18	0	Observed and calculated values of the dependent variables are provided in their original form even if transformations are used on dependent variables.
	1	They are given in transformed form of the dependent variables
19**	Always 0	Dummy variable. (Other than 0 third control card which is dummy for this program to be supplied)
20*	Always 0	Dummy variable
21	1	Control card information printed
	0	Control card information not printed
22 Used alongwith con.var 23 ~~of~~ and con.var 1 of con card 4	0	-
	1	Inverse of the matrix of significant variables written on unit specified by con.var 23 of con.card No.2, con.var 1 of con.card 4 should also be different from zero for this to happen.

* Dummy variable for this program

** Dummy variable for this program but control card 3 should be supplied

Control variables	Valuetaken	Purpose/remarks
23	0	-
Used alongwith con.var 22	Other than 0	Output unit to write the inverse of the matrix of significant variables. Control card 3 to be supplied.
24	0	-
Used alongwith con.var 25	Other than 0	Total number of variables read. Control Card No.5 will be read.
25	0	-
	Other than 0	Number of new variables to be generated. Generates by combining variables given by con.var 24.

Control card No.4:-

If the value taken by control variable No.7 of control card No.2 is 1, then this card will be present and hence to be supplied. If one is running either (1) stepwise regression or (2) weighted regression or (3) regression using frequencies as weights, then one must supply this card. If one is running OMRA using this system then this control card is optional. From this control card 13 control variables are read in 11I2, 2F10.4 format. Unless specified, they take zero values by default. The details of the control variables are furnished below in tabular form. The first eight control variables of this card are same as those of 'RACSR' package.

Control variables	Value taken	Purpose/remarks
1	0, 2	Constant included in the regression model
	1	No constant term in the model
	3	Constant is present as a variable
2	0	Moment matrix not printed
	1	Moment matrix printed
3	Always 0	Dummy variable
4	0	-
	1	To calculate expected values of dependent variables
5	0	-
	1	Durbin-Watson 'd' statistic will be printed, provided con.var 4 of this con.card takes 1
6	0	-
	1	Correlation matrix printed
7	0	-
	1	Means & Standard deviations are printed
8 Used alongwith 4th cont.var of this card	0, 2	Observed, expected of dependent variables, their differences, percentage difference printed
	3	Above will not be printed

Control variables	Value taken	Purpose/remark
9	0	-
	1	Stepwise intermediate output will be given. Generally advised not to take
10	0,2	Least square analysis without weights carried out
	1	Performs weighted least square analysis(Weight to be read alongwith each observation)
11	0	-
	1	Weights are treated as frequencies
12	real value	F-value to allow a variable to enter the model
13	real value	F-value to remove a variable from the model

Control card No.10:-

6 control variables are read from this control card in I5, I2, I1, I5, I3, A64 format, and they are described below in tabular form. All control variables are same as those in 12th control card of 'RACSR' program except some change in second control variable.

Control variable	Value taken	Purpose/remarks
1	0	Provide end of file card. Reads as many data points, as supplied
	Any integer	Reads actual number of observations
2	0,1	Branch to 1st control card
	$2 \leq i \leq 10$	Branch to ith control card
	11, 12	Branch to 10th control card
	13	Direct data
3	0	-
	1	Rewounds input unit
4	Any integer	Number of records to be bypassed
5	0	No. of sets processed=1, if con.var 2 of con.card 10 is 13. If con.var 2 of con.card 10 is between 1 to 12 then any number of sets
	Any integer	Number of sets to be processed
6	Any alpha-numeric data	Title of the problem

For processing first data set control cards 1, 2, 10 have to be supplied. The necessarity of other control cards namely 3 to 9 depends on the values taken by the control variables of control card 2. For processing subsequent sets (in the same run) the number and type of control cards depend on the values taken by control variables of control cards 2 and 10.

..18

After the control cards data is provided in the format specified by the first control card. Each observation (data point) accompanied by weighting factor has to be punched on separate card(s). If $w_i = 1$ for all $i = 1$ to n then weighting factor need not be punched.

VI Example:-

Fit a linear mathematical model to the following data (A. Hald-1952)

X_1	X_2	X_3	X_4	Y
7	26	6	60	78.5
1	29	15	52	74.3
11	56	8	20	104.3
11	31	8	47	87.6
7	52	6	33	95.9
11	55	9	22	109.2
3	71	17	6	102.7
1	31	22	44	72.5
2	54	18	22	93.1
21	47	4	26	115.9
1	40	23	34	83.8
11	66	9	12	113.3
10	68	8	12	109.4

Suppose the data is punched in 5F10.1 format. The number of given data points are 13. Required output is averages, standard deviations, correlation matrix, calculated value of the d.v, difference and percentage difference between observed and calculated values, Durbin Watson 'd' statistic and an estimate of the auto-correlation. Here weights are not used, hence w_i will be taken as 1 for i=1 to 13. The F-values for entering and removing a variable are to be taken as 4.0 and 3.8 respectively. Given the above details the input to the program is prepared as follows:

<u>Input:</u>

First control card:

1.....	Column number
(5F10.1) followed by a blank card	Value

Second Control Card:

00000000011111 12345678901234	Column number " "
bbbbbb0401 0501	Values

Fourth Control Card:

000000000111111111122222 123456789012345678901234		333333 012345	Column number " "
bbbbbb01 01 01 01	4.0	3.8	Values

Tenth Control Card:

0000000 1234567	Column number " "
bbb13bb	Values

...20

After these control cards, the given data set is supplied in the format specified by first control card. Each card contains an observation having the values of X_1, X_2, X_3, X_4, Y punched in that order. As weighting factors are not used W_i are not necessary to punch alongwith each observation.

Output

The minimum output that the package provides is the following. The coefficients corresponding to the variables in the model, their standard error and T-test. Regression sum of squares Mean error sum of squares, Standard error of Y, Multiple correlation square (R^2), corrected R-square ($\bar{R}^2$) and F-test.

The output for the sample problem is given below:

```
OUTPUT OF SET 1
OUTPUT CODE

NUMBER OF INDEPENDENT VARIABLES   4
NUMBER OF   DEPENDENT VARIABLES   1
NUMBER OF TERMS                   5
NUMBER OF OBSERVATIONS           13

D.VAR       CONSTANT
  1         0.52577355D 02

D.VAR       I.VAR      COEF.              STD.               T.TEST
  1           1        0.14683055D 01     0.12130097D 00     0.12104543D 02
              2        0.66225041D 00     0.45854739D-01     0.14442355D 02

D.VAR      R.S.S.             E.S.S.             E.S.S.M
  1        0.26578578D 04     0.57904526D 02     0.57904526D 01

S.E.Y.             M.COR.SQ.          CO.M.COR.SQ.       F.TEST
0.24063359D 01     0.97867835D 00     0.97441402D 00     0.22950349D 03

D.O.FR.
2   10
```

OBSERVED		CALCULATED		DIFFERENCE		% DIFFERENCE TO OBSERVED	
0.78500000D	02	0.80074004D	02	-.15740038D	01	-.20050955D	-01
0.74300003D	02	0.73250922D	02	0.10190810D	01	0.14119515D	01
0.10430000D	03	0.10581474D	03	-.15147350D	01	-.14522868D	-01
0.87600006D	02	0.89258478D	02	-.16584717D	01	-.18932306D	01
0.95899994D	02	0.97292514D	02	-.18925205D	01	-.19734307D	-01
0.10920000D	03	0.10515249D	03	0.40475098D	01	0.37065110D	01
0.10270000D	03	0.10400205D	03	-.13020532D	01	-.12678220D	-01
0.72500000D	02	0.74575423D	02	-.20754229D	01	-.28626523D	-01
0.93100006D	02	0.91275488D	02	0.18245134D	01	0.19597405D	-01
0.11589999D	03	0.11453754D	03	0.13624545D	01	0.11755431D	01
0.83800003D	02	0.80535677D	02	0.32643265D	01	0.38953777D	-01
0.11330000D	03	0.11243724D	03	0.86276092D	00	0.76148350D	-00
0.10939999D	03	0.11229344D	03	-.28934436D	01	0.26448296D	-01

Durbin-Watson 'D' = 0.19216410D 01, Serial Correlation= 0.391794D-01

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