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WINDOWS AND DIGITAL FILTERS IN PROCESSING SAMPLED DATA

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WINDOWS AND DIGITAL FILTERS IN PROCESSING SAMPLED DATA

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Abstract

In this report we present a systematic procedure adopted for processing the IPS data obtained at Thaltej. Various data processing operations such as sampling, windowing, filtering, etc. involved in handling the time series data are described with the help of their mathematical forms. The application of these operations in one domain and their effects in the corresponding Fourier transform domain are discussed in detail. A number of different window forms are described and documentation of computer programs to obtain them is given. Furthermore, documentation of computer program to design a non-recursive band pass filter is also given.

Key words: Windows, digital filters, sampling, Interplanetary scintillation data processing.

I. INTRODUCTION

The objective of processing sampled data has generally been to obtain the features of the original continuous signal in a given domain and in its Fourier Transform (FT) domain. The process of sampling, if not done properly, may limit and/or distort the information in the original signal.

Windowing and filtering are important tools in sampling and processing data. Like sampling improper use of these operations may lead to further distortion of the original signal. It is therefore necessary to study apriori the possible effects of these operations on given data. This may give more confidence that the results of processing would closely match the true features of the original signal. In this report we discuss the various operations associated with windowing and filtering such as sampling, truncation, smoothing, sub-sampling, filtering, etc. performed in processing the IPS data.

a) <u>Sampling</u>:

The original signal x(t) to be sampled should be band-limited, so that $X(f) = 0$ for $|f| > f_s$. According to the Nyquist sampling theorem, the sampling interval $\Delta t \leq \frac{1}{2f_s}$. In case of undersampling when $\Delta t > \frac{1}{2f_s}$, the frequencies above $f_1 = \frac{1}{2\Delta t}$ are folded back below f_1.

This folding back is known as aliasing, which tends to transfer the energy from the part of the spectrum above the frequency f_1 to its part below f_1. This results in contaminating the high frequency end of the estimated spectrum. In practice, this may lead to wrong interpretation of high frequency components as low frequency ones.

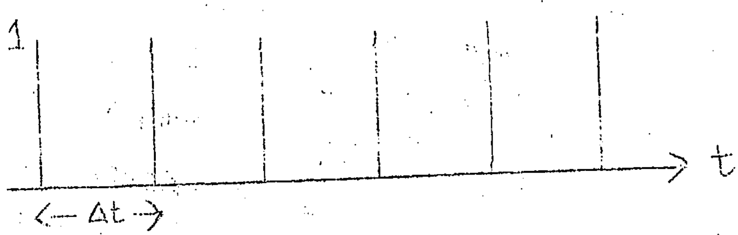
The sampling process:

This process is mathematically expressed as the product of a given continuous function x(t) with a sampling function, e.g.

$$g(t) = x(t).\,Ш(t)$$

where Ш(t) is Shah's sampling function given by

$$Ш(t) = \sum_{n=-\infty}^{\infty} \delta\,(t-n\Delta t)$$



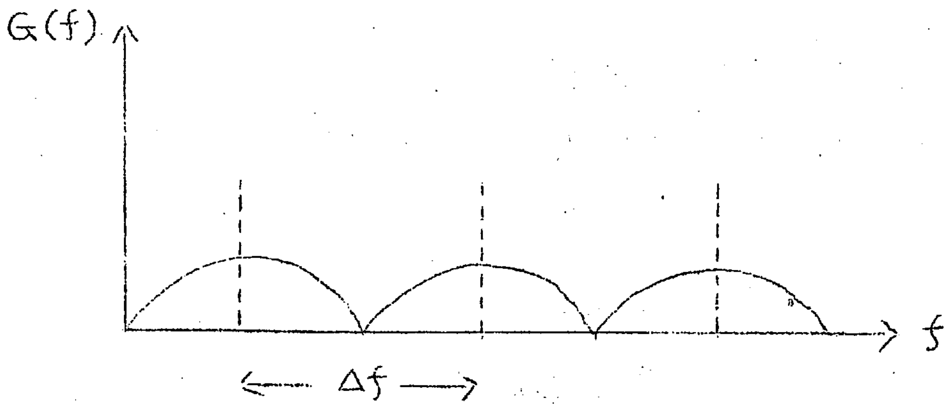
The Shah's function is a series of unit amplitude spikes that occur at time intervals of Δt. The FT of the Shah's function is also a Shah's function expressed as

$$\text{Ш}(f) = \sum_{n=-\infty}^{\infty} \delta(f - n \cdot \Delta f), \quad \text{where} \quad \Delta f = \Delta t^{-1}.$$

Sampling in time domain results in the convolution of their Fourier transforms. Thus a convolved spectrum can be expressed as

$$G(f) = X(f) * \text{Ш}(f)$$

where * denotes the operation of convolution. G(f) will contain multiple copies of the spectrum X(f) at time intervals of Δt^{-1}.



To conclude, the spectrum of any sampled function is periodic with frequency interval of Δt^{-1}.

b) <u>Truncation:</u>

The sampled data is collected over a finite length of time. This is equivalent to the product of x(t) with a rectangular window w(t), which is defined as

$$w(t) = 1, \quad 0 \leqslant t \leqslant T$$
$$\quad = 0, \quad \text{otherwise}$$

This windowing operation in time results in the convolution of X(f) with W(f), where X(f) and W(f) are Fourier transforms of x(t) and w(t) respectively. Since W(f) is of the form of the sinc function $\frac{\text{Sin}f}{f}$, which has an infinite extent in frequency, the resulting convolved function would also have an infinite extent in the frequency domain. Thus, the truncated signal is no more band-limited. Consequently, aliasing results. It should be noted that aliasing due to windowing operation is unavoidable. However, its severity can be reduced in two ways:

i) by choosing a window with large enough width and

ii) by oversampling.

In the first case maximum energy will be included in the main lobe of the sinc function while its side lobes will be damped rapidly towards higher frequencies. As a result, the aliasing is reduced considerably.

In the case of oversampling, the frequency band of the estimated spectrum is extended beyond the range of interest. Thus, the aliasing, if any, tends to contaminate that part of the spectrum which lies beyond the bandwidth of interest.

Besides causing aliasing, the main lobe of the sinc function broadens the non-zero frequency components of the spectrum X(f). The extent of this broadening depends upon the width of the main lobe. This produces smoothing of fine spectral features and gives rise to transition bands at sharp discontinuities.

The side lobes of the sinc function produce "ripples" in the estimated spectrum. This effect is analogous to the Gibbs oscillations observed in a truncated Fourier series.

Choice of window:

A window is chosen such that the width of the main lobe of its FT is narrow enough to resolve the spectral features under consideration. Also the attending side lobes should be relatively small so that they do not give rise to appreciable ripples in the estimated spectrum which in turn may cause uncertainties in the measurement of the power spectrum. In practice, forms of windows are selected such that the amplitude of the side lobes is minimum at the same time the width of the main lobe is maintained as small as possible, keeping the window length (in the domain in which it is applied) constant. The functional forms of some of the most widely used windows will be presented in Section II, which will also describe a computer programme to obtain them.

c) Filters:

Linear operations in data processing such as, smoothing, interpolating, band-limiting, differentiating, etc. are called "filtering operations" and, when used with digital sampled data, are known as digital filters. In general, a digital filter is expressed as

$$y_n = \sum_{k=-N}^{N} C_k \; x_{n-k} + \sum_{k=1}^{N} d_k \; y_{n-k}$$

where x_n denotes the given sampled data points and y_n denotes the filtered data points. When $d_k = 0$ for all k, the above expression contains only the first term, in which case the filter is called a "non-recursive filter". Furthermore, when $C_k = C_{-k}$ for all k, then the filter becomes even symmetric. In this report we will discuss only this type of filters.

Design considerations:

Designing a digital filter amounts to computing the coefficients C_k. This set of coefficients is the response of the filter to an impulse function. Hence, the coefficients C_k are called the "impulse response" of the filter. It may be noted from the filter expression that the filtering operation implies convolution of the filter coefficients C_k with the sampled points x_n. Unlike convolution, the filtering operation becomes complete if and only if all the coefficients C_k as well as the data points are involved in it. Thus as a result of filtering, the filtered output will have a smaller number of points than the original sampled data by an amount which depends on the number of filter coefficients. The length of a filter should, therefore, be small as compared to the data points. Again, to design a filter it may be specified in the domain in which it is applied or in its FT domain. When it is specified in time domain as in the case of the smoothing filter, its transfer function should be properly understood apriori. A band pass filter (BPF) is an example of a filter whose specifications are given in the frequency domain, while it is actually applied in the time domain.

Smoothing filters are designed by fitting polynomials of various degrees to sampled data. A linear fit gives rise to running average formulae in statistics which are commonly used. The transfer function of such smoothing filters using (2 m + 1) points is given by

$$H(\omega) = \frac{\sin\,(m+\frac{1}{2})\,\omega}{(2m+1)\,\sin(\omega/2)}$$

Higher the number of filter points, sharper is the frequency cut-off and more frequent are the wiggles of the transfer function. Smoothing filters designed using higher degree polynomials have flat response over a wider frequency range. In general, smoothing filters work as low pass filters (LPF).

In Section III we describe a non-recursive symmetric BPF and give the details of a computer programme to obtain the same.

Sub-sampling:

Sometimes it is desired to obtain a high resolution spectrum at low frequencies from a signal which may contain high frequency components also. In such a case, a large number of data samples is to be processed. Such data are first filtered to remove the unwanted high frequency components. The filtered data can then be sampled at a lower rate appropriate for obtaining the low frequency spectrum. This process of sampling (at lower rate) a large sample of data is called "Sub-sampling". It is obvious that this process enables determine a high resolution low frequency spectrum using a smaller sub-sample.

In Section IV we present the various data processing operations described in the preceding paragraphs by taking interplanetary scintillation (IPS) data as an example.

II. TYPES OF WINDOWS

Following types of windows are generally used for data processing:

i) Rectangular - This window with (2N + 1) points is given as

$$w(n) = 1 \ , \text{ for } \ 1 \leqslant n \leqslant (2N + 1)$$
$$= 0 \ , \text{ otherwise}$$

The frequency response of this weighting function has a narrow main lobe but significantly large side lobes.

ii) Modified rectangular - This is expressed as

$$w(n) = \tfrac{1}{2} \ , \text{ for } \ n = 1 \ \text{ and } (2N + 1)$$
$$= 1 \ , \text{ for } \ 1 < n < (2N + 1)$$
$$= 0 \ , \text{ otherwise.}$$

The frequency response of this window has a slightly wider main lobe but weaker side lobes which damp faster as compared to the rectangular window.

iii) Triangular or Bartlett - Its weighting function is given as

$$w(n) = \frac{n-1}{N} \ , \text{ for } \ 1 \leqslant n \leqslant (N + 1)$$
$$= 2 - \frac{n-1}{N} \ , \text{ for} (N + 1) \leqslant n \leqslant (2N + 1)$$

Its frequency response has a main lobe twice as wide as that of the rectangular window, but its side lobes are much lower. Such a tapered window has wide applications in radio astronomy antennas.

iv) Hanning window - Its weighting function is given as

$$w(n) = 0.5 + 0.5 \cos\left[\frac{\pi(N + 1 - n)}{N}\right]$$

for $1 \leqslant n \leqslant (2N + 1)$

$= 0$, otherwise.

Its frequency response has a wider (less than double) main lobe but negligible side lobes compared to that of the rectangular window.

v) Hamming window - Its weighting function is given by

$$w(n) = 0.54 + 0.46 \cos\left[\frac{\pi(N + 1-n)}{N}\right],$$

for $1 \leqslant n \leqslant (2N + 1)$

$= 0$, otherwise.

This window is obtained by the weighted sum of modified rectangular and Hanning windows. The weighting coefficients are obtained by minimizing the maxima of side lobes. The width of the main lobe of its frequency response is only slightly increased as compared to that of the Hanning window.

vi) Blackman window - Its weighting function is given by

$$w(n) = 0.42 + 0.5 \cos\left[\frac{\pi (N + 1 - n)}{N}\right]$$

$$= 0.08 \quad \cos\left[\frac{2 \pi (N + 1 - n)}{N}\right]$$

$$\text{for} \quad 1 \leqslant n \leqslant (2N + 1)$$

$$= 0 \text{, otherwise}$$

The main lobe of its frequency response is about 1.13 times that of the Hanning window but its side lobes are negligibly low.

vii) Kaiser window - Its weighting function is given as

$$w(n) = \frac{I_o\left(\beta\sqrt{1- [(N + 1 - n) / N]^2}\right),}{I_o(\beta)}$$

$$\text{for} \quad 1 \leqslant n \leqslant (2N + 1)$$

$$= 0 \text{, otherwise.}$$

Where I_o is the zero order Bessel function and β is a constant that specifies a frequency response trade-off between the maximum of the side lobe and width of the main lobe. This window is an optimum window in that it is a finite duration sequence that has a minimum spectral energy beyond some specified frequency. The range of β is $(4 < \beta < 9)$. By selecting appropriate values of β the width of the main lobe can be varied by simultaneously keeping minimum energy in the side lobes.

It may be noted that although the functional form of a window may be optimized, since the result of its operation is through convolution in the FT domain, its effect as seen in the estimated spectrum is never optimum.

Description of program to obtain the coefficients of windows

A FORTRAN sub-program is written to obtain coefficients of various window forms and has been stored in the library OWNLIB which resides on system disk "PLSYS" on unit 1C0 at the PRL computer center.

A. Definition of the sub-program:

SUBROUTINE WINDOW (NTERM, ITYPE, PARAM, W)

B. General description of the sub-program:

1. Precision - Double
2. Name of the sub-program called - FUNCTION AIO
3. Total memory requirement in bytes - 2320

C. Detailed description of the arguments of the sub-program:

NTERM :- (INTEGER * 4, SCALAR, IN-GOING, INTACT)

This is a positive <u>odd</u> integer (2N + 1) representing the number of window coefficients to be obtained.

ITYPE :- (INTEGER * 4, SCALAR, IN-GOING, INTACT)

This number denotes the type of window as follows:

ITYPE = 1 Rectangular window
= 2 Modified Rectangular window
= 3 Triangular (Bartlett) window
= 4 Hanning window
= 5 Hamming window
= 6 Blackman window
= 7 Kaiser window

PARAM :- (REAL * 8, SCALAR, IN-GOING, INTACT)

This parameter denoted by β in this report is used only for the Kaiser window. Its value should preferably lie between 4 and 9.

W :- (REAL * 8, ARRAY (1), OUT-COMING)

This array represents the NTERM coefficients of a desired window. In general, the window coefficient W(N + 1) has maximum value of unity which tapers down to either side in all the windows except in the types 1 and 2.

Remark :- Array W has been given object time dimensions in the subroutine WINDOW.

III. A NON-RECURSIVE SYMMETRIC BANDPASS FILTER (BPF)

As stated in Section I a non-recursive filter with (2N + 1) points is expressed as

$$y_n = \sum_{k=-N}^{N} C_k \; x_{n-k}$$

where $$C_k = C_{-k}$$

To design a BPF the following specifications are given:

$$H(f) = 1, \text{ for } f_1 \leq |f| \leq f_2$$
$$= 0, \text{ otherwise.}$$

As the filter is symmetric this can be expressed as

$$H(\omega) = C_o + 2 \sum_{k=1}^{\infty} C_k \cos(\omega k)$$

where $$C_k = \frac{1}{\pi} \int_0^{\pi} H(\omega) \cos(\omega k) \, d\omega$$

Changing the variable of integration from ω to f, we get

$$C_k = 2 \int_0^{1/2} H(f) \cos(2\pi fk) \, df$$

In the case of the filter under consideration

$$C_k = 2 \int_{f_1}^{f_2} \cos (2\pi fk)\, df$$

Thus,

$$C_k = \frac{\left[\sin (2\pi f_2 k) - \sin (2\pi f_1 k)\right]}{\pi k},$$

for $k > 0$

$$C_o = 2(f_2 - f_1)$$

.·. the transfer function becomes

$$H(f) = 2(f_2 - f_1) + 2 \sum_{k=1}^{\infty} \left[\sin (2\pi f_2 k) - \sin (2\pi f_1 k)\right] / \pi k \times \cos (2\pi fk)$$

In practice, it is necessary to truncate this infinite series. To design a filter with (2N + 1) points, we truncate this series after N terms. It may be recalled that this truncation amounts to a window operation and therefore an optimum window is used such that tolerable transition bands with minimum side lobes are produced in the desired transfer function. Since these conditions are satisfied by a Kaiser window, we use an appropriately designed Kaiser window by spacifying the tolerable transition bands Δf and half-ripple amplitude δ. Using this δ the attenuation A of sides lobes in decibels is given by

$$A = -20 \log_{10} \delta$$

from which, using Kaiser's empirical formula, the parameter β is obtained from

$$\beta = \begin{cases} 0.1102\,(A-8.7) \text{ for } A > 50 \\ 0.5842\,(A-21)^{0.4} + 0.07886\,(A-21), \\ \qquad\qquad \text{for } 21 < A < 50 \\ 0.0 \quad \text{for } A \leq 21 \end{cases}$$

From the width Δf of the transition band, the number of coefficients N of the filter is given by

$$N = \frac{A - 7.95}{28.72\ \Delta f}, \text{ for } A \geq 21$$

$$= \frac{0.9222}{2\ \Delta f}, \text{ for } A < 21$$

The final filter coefficients $(2N + 1)$ are obtained by multiplying the window coefficients W_k with C_k. It may be noted that the number of filter coefficients is inversely proportional to the transition width and directly proportional to the attenuation.

Description of the computer program:

A FORTRAN Subroutine is developed to obtain the coefficients of a BPF with given specifications. This subroutine has been stored in the private library OWNLIB which resides on system disk PLSYS on the disc 1CO.

A. Definition of the sub-program:-

SUBROUTINE FILTER (F1, F2, DF, FS, DWATT, NTERM, W, C, IPRINT)

B. General description of the subprogram:

1. Precision - Double

2. Name of the program called - FUNCTION AIO

3. Total memory requirement - 2576 bytes

C. Detailed description of the arguments of the subprogram:

F1 :- (REAL * 8, SCALAR, IN-GOING, INTACT)

This is the lower frequency cut-off in Hz of the desired BPF. For a LPF, F1 = 0.

F2 :- (REAL * 8, SCALAR, IN-GOING, INTACT)

This is the high frequency cut-off in Hz of the desired BPF. For a HPF, F2 = maximum frequency component in the sampled data.

DF :- (REAL * 8, SCALAR, IN-GOING, INTACT)

This is the allowable width in Hz of the transition bands.

FS :- (REAL * 8, SCALAR, IN-GOING, INTACT)

This is the sampling frequency in Hz used to sample the data.

DWATT:- (REAL * 8, SCALAR, IN-GOING, INTACT)

This is the allowable ripple in percentage of unity.

NTERM:- (INTEGER * 4, SCALAR, OUT-COMING)

This is an odd integer representing the number of coefficients in the desired filter.

W :- (REAL * 8, ARRAY (1), OUT-COMING)

This array of NTERM elements represents the coefficients of the Kaiser window used to design a filter.

C :- (REAL * 8, ARRAY (1), OUT-COMING)

This array of NTERM elements gives the coefficients of the desired filter.

IPRINT:- (INTEGER * 4, SCALAR, IN-GOING, INTACT)

This parameter controls the printing of the filter coefficients as follows:

If IPRINT = 0, coefficients will not be printed

$\neq$ 0, coefficients will be printed.

Remark:- This sub-program uses object time dimension for the arrays W and C depending on the variable NTERM.

IV. IPS DATA PROCESSING

The fluctuations of intensity of radio emission from a compact radio source are interpreted to be due to the density irregularities in the solar plasma which scatter these radio waves and cause intensity pattern at the observer. These intensity variations, called interplanetary scintillation (IPS), are observed at a frequency of 103 MHz using the IPS radio telescope at Thaltej, near Ahmedabad. Both analog and digital recordings of the intensity variations are made. Since the temporal IPS spectrum is typically prominent in the frequency range of 0.1 to 5 Hz, the data are sampled at a frequency of 20 Hz, which is oversampling by a factor of 2. This minimizes the effects of aliasing which might be caused due to truncation. The sampled data are then subjected to a Hamming window to reduce the distortion of the spectrum that might result due to abrupt cut-off of the data at the commencement and at the end. These windowed data are then passed through a BPF having a pass band of 0.1 to 5 Hz. The filtered data are used to compute scintillation index, autocorrelation function (ACF) and power spectrum. The latter two operations are carried out with the help of the FFTAPP - Package developed by us. The ACF can be used to estimate the scale size of plasma density irregularities provided their velocity is known. In case it is not required to calculate the ACF, the filtered IPS data can be processed using FFT to give power spectrum directly.

The above procedure of data processing is being adopted for the single station IPS data collected at Thaltej. The three-station data which will become available in future can be processed using essentially the same operations to compute cross-correlation functions, cross-power spectra, solar wind velocity, etc.

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