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COMPUTER SIMULATION OF I-BIT DIGITAL AUTOCORRELATOR

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COMPUTER SIMULATION OF 1-BIT DIGITAL AUTOCORRELATOR

S.K. Alurkar and D.R. Kulkarni

Abstract

Computer simulation of a 1-bit digital autocorrelator is performed. A comparison study is made of the power spectra of the generated random noise using an ideal filter and another simulated filter having its characteristics identical with those of the actual filter of the correlator. Furthermore, the effect of hard-clipping the noise, followed by the Van Vleck correction, on the power spectra in the case of the two filters is also studied. It is shown that the rms deviation of the power spectrum of the clipped noise is less than 5% as compared with that of the unclipped noise. The amount of departure of the characteristics of the real filter from those of the ideal one determines the shape of the measured power spectrum. It is concluded that this simulation exercise has given a deep insight into the design parameters and overall functioning of the 1-bit autocorrelator.

Key Words

Computer Simulation
1-bit Autocorrelator
Random Noise
Fast Fourier Transofm
Power Spectrum

I. Introduction:

Computer simulation of signals and signal processing systems provides a deep insight into the overall operation of such systems. The simulation exercies enables one to acquire a prior knowledge of the functioning of a system under design since one has the freedom of playing with various values of the system parameters and studying their effect on the operation of the system.

We report here computer simulation of a 1-bit digital autocorrelator that is under development at the Physical Research Laboratory, Ahmedabad. This correlator will form the back-end of an RF spectral line receiver to be used for the detection of intense narrow-band emissions at frequencies of 1665.4 and 1667.4 MHz from interstellar hydroxyl (OH) molecules.

II. The 1-bit autocorrelator:

The overall functioning of the autocorrelator consists of hardware as well as software operations. A block diagram of a 1-bit autocorrelator (Weinreb, 1963) is shown in Figure 1, wherein the hardware units (blocks with solid lines) namely, noise source, filter, clipper, sampler and correlator are enclosed in a rectangle with dashed-lines, while the dashed-line block at the bottom represents software operations namely, Van Vleck correction, smoothing and Fourier Transform. The source is a radio noise which, after band-limitation, is hard-clipped retaining the phase information of a signal. The clipped signal is then sampled at the Nyquist rate and passed through a series of shift registers wherein it undergoes time delays in equal discrete steps. The correlator then compares the signal after each delay with the original one and delivers a pulse to the following counter when the two coincide. These counts, accumulated over a certain time interval, are then converted into autocorrelation functions of the clipped signal. The autocorrelation functions of the original signal are derived by applying the Van Vleck correction (Van Vleck and Middleton, 1966) to the autocorrelation functions of the infinitely clipped signal.

Mathematically, the correlator calculates the autocorrelation function $R'(\Delta t)$ as a function of time lag Δt of the clipped signal $x'(t)$ using the relation

$$R'(\Delta t) = \int x'(t).x'(t + \Delta t)dt \qquad \ldots(1)$$

The autocorrelation function $R(\Delta t)$ of the original signal $x(t)$ is extracted from this by applying the Van Vleck correction. Thus,

$$R(\Delta t) = \text{Sin}\left[\pi/2 .\ R'(\Delta t)\right] \qquad \ldots(2)$$

This then is Fourier transformed to obtain the power spectrum of the signal. The spectral resolution obtainable from the correlator depends upon the number of delays or channels through which the signal is passed.

III. Computer Simulation of the Autocorrelator:

The advantage of computer software simulation over the actual hardware system design lies in the fact that whereas in the case of the former one can vary the values of the various parameters of a system and study their effects on the overall performance of the system, in the latter case, however, this is not easily feasible. In the case of the autocorrelator its performance can be studied by varying the filter characteristics, number of delays to be introduced, etc. In addition, one may also like to study the effect of clipping the signal on its power spectrum. We have simulated the entire system of the autocorrelator for two different filter characteristics and a large number of channels. We have also made a comparative study of the power spectra of clipped and unclipped simulated random noise. It may be mentioned here that such a comparison study is not possible by hardware using simple and inexpensive system configuration.

IV. Simulation of the subsystems of the correlator:

a) Radio noise

It is known that the radio noise emanated from various radio sources is a random noise with Gaussian statistics. We have, therefore, simulated such a noise (IBM, 1970) by generating random numbers with Gaussian distribution of mean zero and standard deviation one.

b) Filter and its operation

The bandpass filter which has a flat response over a frequency range is simulated by a rectangular function which represents an ideal filter having sharp cut-offs at the ends of the flat frequency response. The simulated ideal filter has a flat response over the frequency range of 24 to 44 kHz. The actual filter that is used with the hardware correlator has a flat response from 4 to 40 kHz with rates of fall of 26 db/octave on high frequency and 20 db/octave on low frequency sides.

The filtering operation is simulated by the convolution of the random noise with the transfer function of the filter.

c) <u>Clipper</u>

In the one-bit correlator, the clipping operation is defined as

$$Y(t) = 1 \quad \text{if} \quad x(t) > 0$$
$$Y(t) = -1 \quad \text{if} \quad x(t) < 0 \qquad \ldots(3)$$

where x(t) and Y(t) are the amplitudes of the original and the clipped signals respectively. In simulation, the clipping is performed at the mean level of the convolved signal. It may be noted that the result of convolution is, in general, a complex quantity and hence the clipping operation should be performed separately on its real and imaginary parts so that the phase information of the signal is retained.

d) <u>Autocorrelation</u>

The function of the autocorrelation is simulated by solving the correlation integral (exp.1) numerically. This gives the autocorrelation of the clipped random signal. Thus,

$$R'(\Delta t) = \sum_i x'(t).\ x'(t + i\,\Delta t) \qquad \ldots(4)$$

To get the autocorrelation of the original signal, the Van Vleck correction (exp.2) is applied separately to the real and imaginary parts of the autocorrelation of the clipped signal. This is then normalised, truncated and smoothed before calculating the Fourier Transform to give the spectrum of the signal.

For computational efficiency, the convolution, the correlation and the Fourier Transform integrals (Brigham, 1974) have been evaluated using a Fast Fourier algorithm. A separate package, based on the Cooley-Tukey algorithm (Cooley and Tukey, 1965), of FFT and its various applications (Kulkarni and Alurkar, 1976) is developed by the authors and is used in the present simulation.

V. Procedure of computer simulation:

The simulation is carried out on IBM system 360/44 computer using a sample of 1024 random numbers of Gaussian distribution. A sampling frequency of 64 kHz is used in the case of the ideal filter while the same for the real filter is taken to be 128 kHz. Four sets of simulations were studied as follows using,

a) ideal filter with clipping

b) ideal filter without clipping

c) real filter with clipping

d) real filter without clipping

In order to get statistically reliable estimates, we used 100 sets of samples of 1024 points each in all these four cases. The simulation path in the case of the unclipped signal, after filter, passes through the software operations of autocorrelation, truncation, smoothing and Fourier Transform, as shown on the left-hand side of Figure 1.

VI. Results:

The power spectrum is obtained by Fourier transforming the autocorrelation function with 256 time lags in the case of the ideal filter and 512 lags in the case of the real filter. Figure 2 shows the real and imaginary parts of the power spectrum of the generated random noise obtained using an ideal filter whose flat response ranged from 20 kHz to 44 kHz. The power in the flat portion of the spectrum varied over a range of 0.8 db.

Figure 3 shows the power spectrum of the same noise which was hard-clipped and corrected by the Van Vleck correction using expression (2). In this case, the spread in the power in the flat portion was 1.2 db. Again the root mean square deviation (σ) of the clipped power spectrum with respect to that of the unclipped one can be calculated by using the formula,

$$\sigma = \sqrt{\sum_{i=1}^{100} \left[P_i(f) - P'(f) \right]^2 / 100} \qquad \ldots(5)$$

where $P_i(f)$ and $P_i'(f)$ are the unclipped and clipped power spectra respectively. This r.m.s. deviation turns out to be about 1.0. The higher value of the spread in the power spectrum of the clipped noise is due to the clipping operation which introduces quantization noise. However, the deviation of the clipped power spectrum is less than 5%, indicating that the hard-clipping operation, followed by the Van Vleck correction, almost completely recovers the original power spectrum. This important result has led to the development of a 1-bit digital autocorrelator (Weinreb, 1963). A many-bit autocorrelator, although may give less deviation of the power spectrum, would obviously be more complicated and expensive.

Figure 4 shows the real and imaginary parts of the power spectrum of the random noise obtained using a simulated filter with characteristics identical with those of the real filter designed at the Physical Research Laboratory. The spread in the power values in the flat region of the spectrum is about 1.4 db. The same for the clipped noise (Figure 5) is 1.9 db.

The r.m.s. deviation (σ) of the clipped power spectrum with respect to the unclipped one is about 0.5, or varies between 4% and 6%. It can be seen that generally the spread in the spectrum obtained using a real filter is higher than that obtained using an ideal filter. This may be attributed to the small variations ($\sim$1 db) in the flat region of the characteristics of the real filter.

VII. Conclusion:

It is interesting to see that the hard-clipping operation, followed by the Van Vleck correction, almost completely recovers the original unclipped power spectrum. Also, the simulation has offered an added advantage of using a large number of delays to increase the resolution. The number of channels in the hardware autocorrelator, however, cannot obviously be increased with ease. The effect of filter characteristics over the power spectrum can also be seen through this simulation exercise. Thus, it is seen that this computer simulation has given a good insight into the overall functioning of the 1-bit autocorrelator. This will help us in finalising the detailed design of the autocorrelator.

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Appendix

The computational procedure to carry out the simulation is organized in four broad steps:

1. First a computer file of 100 sets of Gaussian ($\mu = 0$, $\sigma = 1$) random noise samples of 1024 values is created.

2. The file is used to create two other files by convolving each sample with the transfer functions of the ideal and real filters.

3. Each sample in the convolved (band-limited) sets is hard-clipped and the auto-correlation function is obtained both for the unclipped and hard-clipped samples. The four files of auto-correlation functions are thus obtained.

4. The auto-correlation functions for the clipped samples are subjected to Van Vleck correction. The final power spectra for the unclipped and clipped samples using both the filters are obtained by Fourier transforming these autocorrelation functions.

The computer programs for the convolution, autocorrelation and Fast Fourier transform are given in reference (Kulkarni and Alurkar, 1976). We give below the computer programs for the simulation of clipping operation and the Van Vleck correction.

THE FOLLOWING VARIABLES ARE USED IN THE PROGRAMS GIVEN BELOW.
NPT - THE NUMBER OF SAMPLES IN EACH SET.
N - NUMBER OF VALUES IN EACH SAMPLE.
DT - TIME INTERVAL IN SECONDS BETWEEN TWO CONSECUTIVE VALUES IN THE SAMPLE.

PROGRAM I .

THIS PROGRAM OBTAINS THE AUTO-CORRELATION FUNCTION OF A CONVOLVED SAMPLE WHICH IS HARD-CLIPPED.

```
   COMPLEX * 16 X(2048),SUM
   REAL * 8 DT,AM1,AM2,CC1,CC2
   CALL SETBUF(16400)
   NPT = 100
   DT = 1.0D0/128.0D0
   N = 1024
   NN = N + N
   DO 40 IR = 1,NPT
   READ ONE OF THE CONVOLVED SAMPLES FROM THE APPROPRIATE FILE WHICH
   IS ON SYS002.
   READ(2) (X(I),I=1,N)
   FIND THE MEAN OF THE REAL AND IMAGINARY PARTS OF THE SAMPLE.
   SUM = (0.0D0,0.0D0)
   DO 10 I = 1, N
   SUM = SUM + X(I)
10 CONTINUE
   SUM = SUM / DFLOAT(N)
   AM1 = DREAL(SUM)
   AM2 = DIMAG(SUM)
   SIMULATE THE OPERATION OF THE HARD-CLIPPING.
   DO 20 I = 1, N
   CC1 = -1.0D0
   CC2 = -1.0D0
   IF(DREAL(X(I)) .GT. AM1) CC1 = 1.0D0
   IF(DIMAG(X(I)) .GT. AM2) CC2 = 1.0D0
   X(I) = DCMPLX(CC1,CC2)
20 CONTINUE
   FIND THE AUTO-CORRELATION OF THE CLIPPED SAMPLE.
   CALL RCOREL(X,N,DT,X,N,DT,NN,11,1)
   WRITE THE AUTO-CORRELATION FUNCTION ON TAPE ON UNIT SYS  3.
   WRITE(3) (X(I),I=1,N)
   PRINT30,IR
30 FORMAT(1X,'SET NO. = ',I4)
40 CONTINUE
   STOP
   END
```

PROGRAM II .

```
C     THIS PROGRAM NORMALISES THE AUTO-CORRELATION FUNCTION AND OBTAINS
C     THE POWER SPECTRUM AFTER APPLYING THE VAN-VLECK CORRECTION IF THE
C     SAMPLE IS HARD-CLIPPED.

      COMPLEX * 16 X(2048),W(512),AA
      DIMENSION AX(256),AY(256),AZ(256)
      REAL * 8 DT,XX,AM1,AM2,CC1,CC2,PI2,PI
      LOGICAL * 1 STAR
      DATA STAR/'*'/
      DATA W/512*(0.0D0,0.0D0)/
      DATA PI/3.14159265D0/
      PI2 = PI/2.0D0
      CALL SETBUF(16400)
C     READ A CONTROL VARIABLE NVV.
C     IF NVV = 0 VAN-VLECK CORRECTION IS NOT TO BE APPLIED.
C     IF NVV = 1 VAN VLECK CORRECTION IS TO BE APPLIED.
      READ10,NVV
   10 FORMAT(I4)
      NLG = 512
      NPW = 9
      N = 1024
      NPT = 100
      DT = 1.0D0/128.0D0
      DO 70 IR = 1, NPT
C     READ THE AUTO-CORRELATION FUNCTION OF THE CONVOLVED SAMPLE FROM THE
C     FILE ON UNIT SYS002.
      READ(2) (X(I),I=1,N)
C     NORMALISE THE AUTO-CORRELATION FUNCTION.
      AA = X(1)
      DO 20 I = 1,NLG
      X(I) = X(I)/AA
   20 CONTINUE
C     APPLY VAN-VLECK CORRECTION.
      IF(NVV.EQ.0) GO TO 40
      DO 30 I = 1,NLG
      AM1 = DREAL(X(I))
      AM2 = DIMAG(X(I))
      CC1 = DSIN(PI2*AM1)
      CC2 = DSIN(PI2*AM2)
      X(I) = DCMPLX(CC1,CC2)
   30 CONTINUE
   40 CONTINUE
```

```
C     GET THE POWER SPECTRUM.
      CALL RFFT(X,NLG,NPW,DT,1.0D0)
C     WRITE THE POWER SPECTRUM ON THE FILE ON UNIT SYS003.
      WRITE(3) (X(I),I=1,NLG)
      PRINT50,IR
   50 FORMAT(1X,'SET NO = ',I4)
      DO 60 I = 1, NLG
      W(I) = W(I) + X(I)
   60 CONTINUE
   70 CONTINUE
C     FIND THE MEAN POWER SPECTRUM.
      DO 80 I = 1, NLG
      W(I) = W(I)/DFLOAT(NPT)
   80 CONTINUE
      N2 = NLG/2
C     GET THE POWER IN MILLIWATTS FOR REAL,IMAGINARY AND ABSOLUTE VALUES.
      DO 90 I = 1, N2
      AX(I) = DREAL(W(I)) * 1000.0D0
      AY(I) = DIMAG(W(I)) * 1000.0D0
      W(I) = CDABS(W(I)) * 1000.0D0
      AZ(I) = I
   90 CONTINUE
C     PLOT THE ABSOLUTE,REAL AND IMAGINARY VALUES OF THE SPECTRUM.
      CALL GRAPH(W,N2)
      CALL PLOT(AZ,AX,N2,100,60,STAR)
      CALL PLOT(AZ,AY,N2,100,60,STAR)
      STOP
      END
```

Figure Captions

Figure 1: Block diagram showing (i) subsystems of hardware 1-bit autocorrelator (enclosed in the bigger rectangle with dashed lines) followed by a dashed-line rectangle showing the software operations, and (ii) autocorrelator without clipper on the left hand side, the blocks of random noise and filter being common.

Figure 2: Real and imaginary parts of power spectrum of unclipped noise with ideal filter having flat response over 20 to 44 kHz.

Figure 3: Real and imaginary parts of power spectrum of clipped noise with ideal filter having flat response over 20 to 44 kHz.

Figure 4: Real and imaginary parts of power spectrum of unclipped noise with real filter having flat response over 4 to 40 kHz.

Figure 5: Real and imaginary parts of power spectrum of clipped noise with real filter having flat response over 4 to 40 kHz.

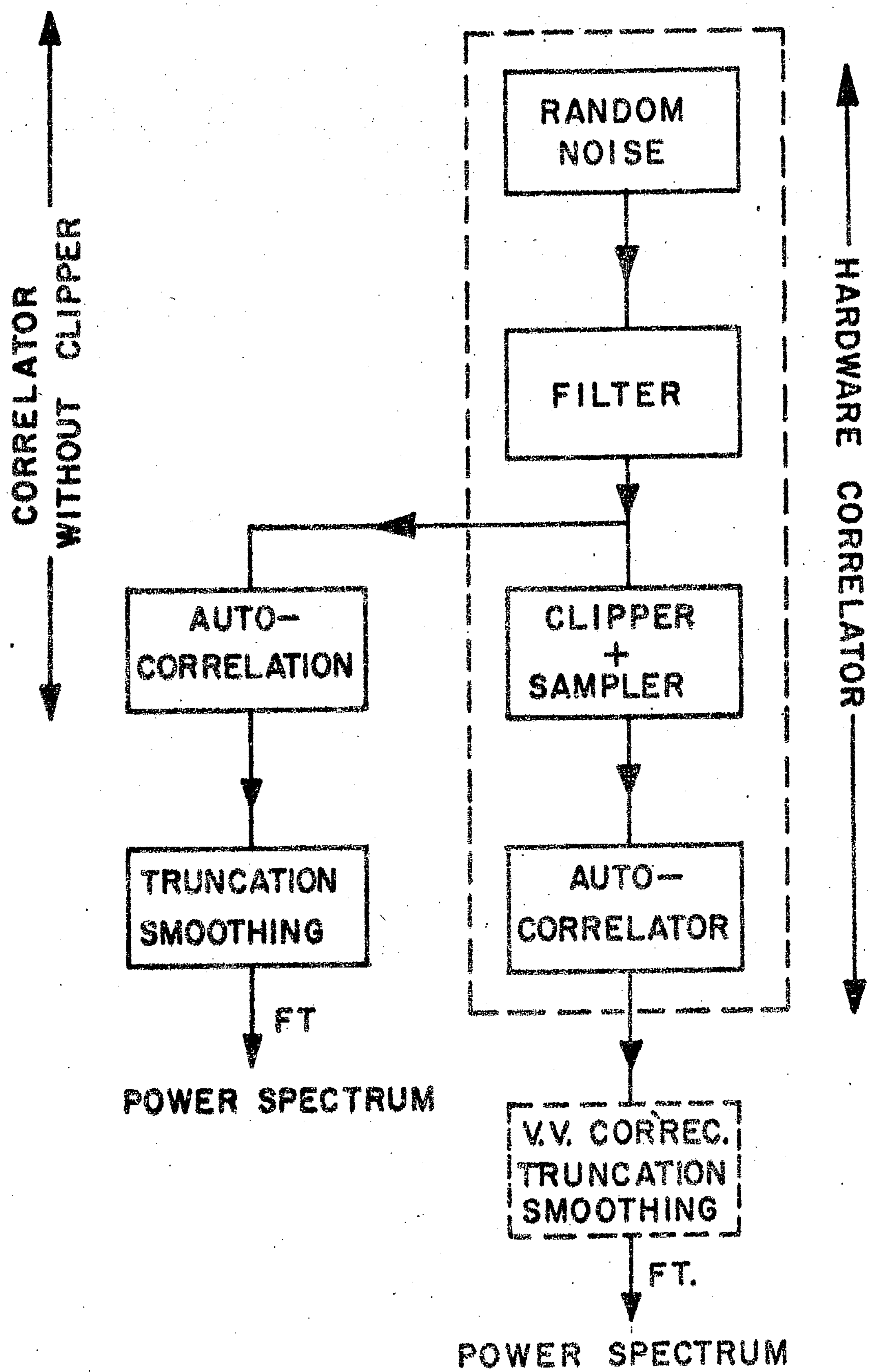

CORRELATOR
WITHOUT CLIPPER
RANDOM
NOISE
FILTER
AUTO-
CORRELATION
CLIPPER
+
SAMPLER
AUTO-
CORRELATOR
HARDWARE CORRELATOR
TRUNCATION
SMOOTHING
FT
POWER SPECTRUM
V.V. CORREC.
TRUNCATION
SMOOTHING
FT.
POWER SPECTRUM

FIG 1

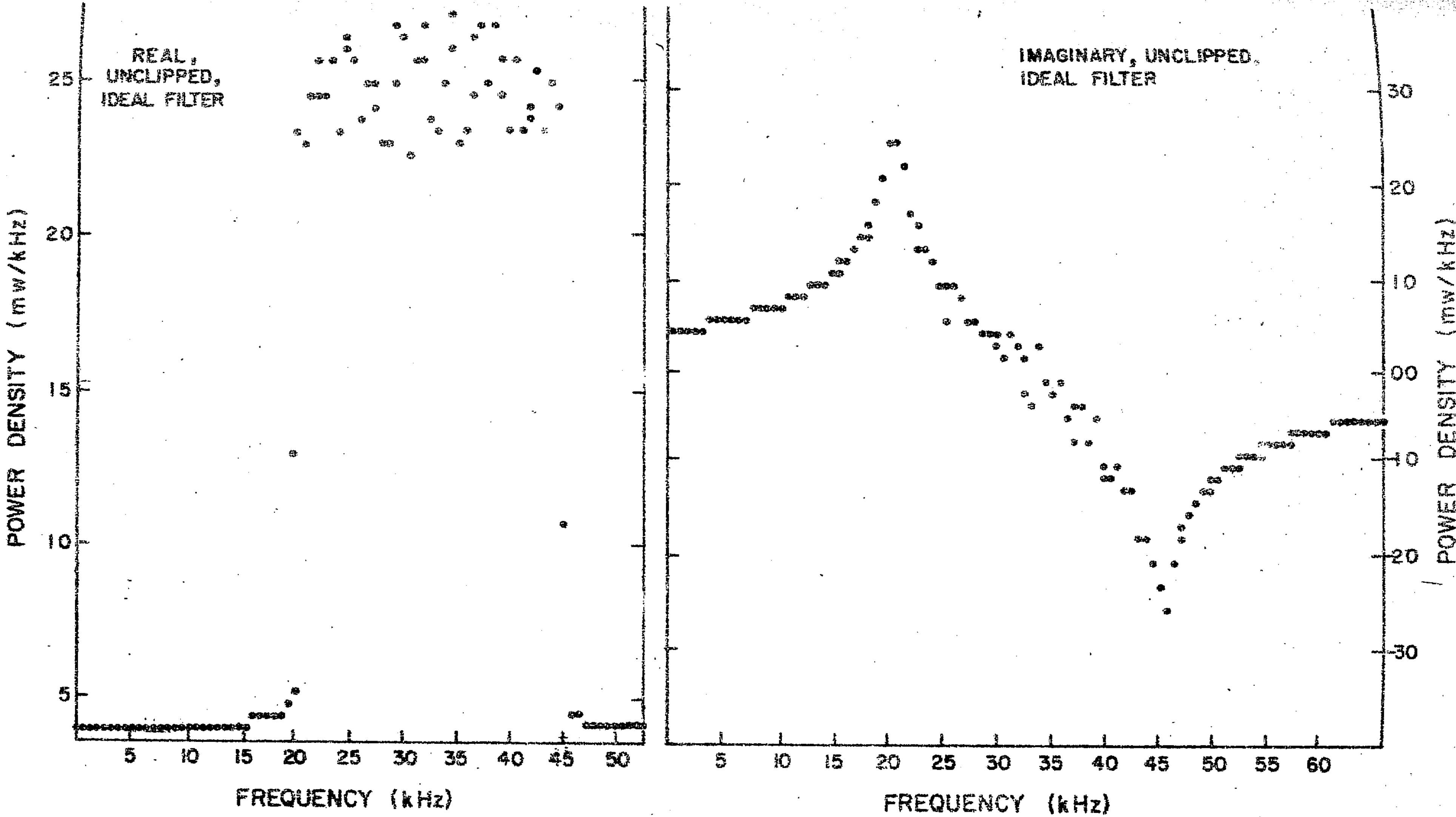
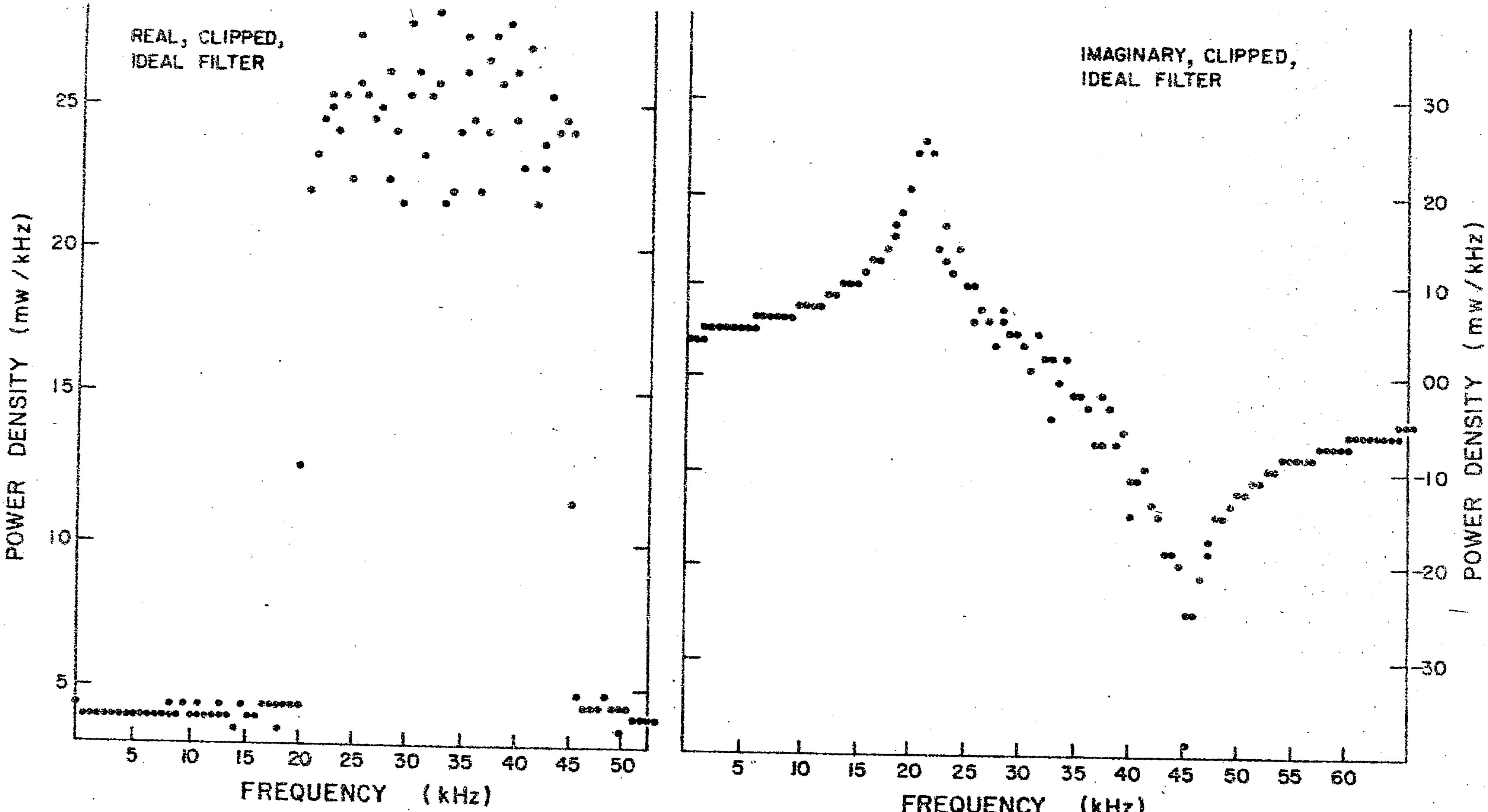
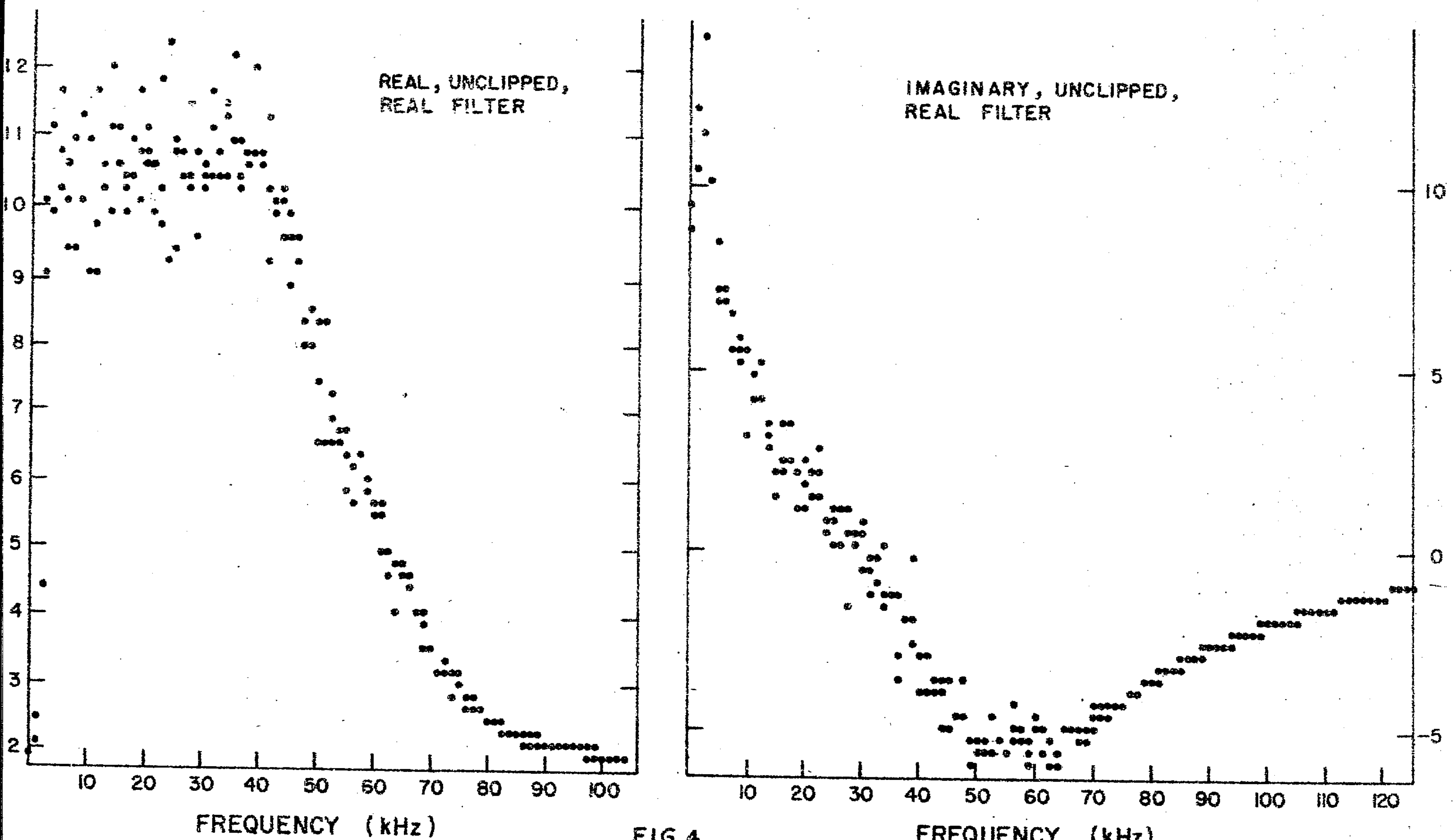

REAL,
UNCLIPPED,
IDEAL FILTER
POWER DENSITY (mw/kHz)
FREQUENCY (kHz)
IMAGINARY, UNCLIPPED,
IDEAL FILTER
POWER DENSITY (mw/kHz)
FREQUENCY (kHz)

FIG 2



REAL, CLIPPED, IDEAL FILTER
POWER DENSITY (mw / kHz)
25
20
15
10
5
5
10
15
20
25
30
35
40
45
50
FREQUENCY (kHz)
IMAGINARY, CLIPPED, IDEAL FILTER
POWER DENSITY (mw / kHz)
30
20
10
00
-10
-20
-30
5
10
15
20
25
30
35
40
45
50
55
60
FREQUENCY (kHz)

FIG 3



REAL, UNCLIPPED,
REAL FILTER
IMAGINARY, UNCLIPPED,
REAL FILTER
FREQUENCY (kHz)
FREQUENCY (kHz)

FIG 4

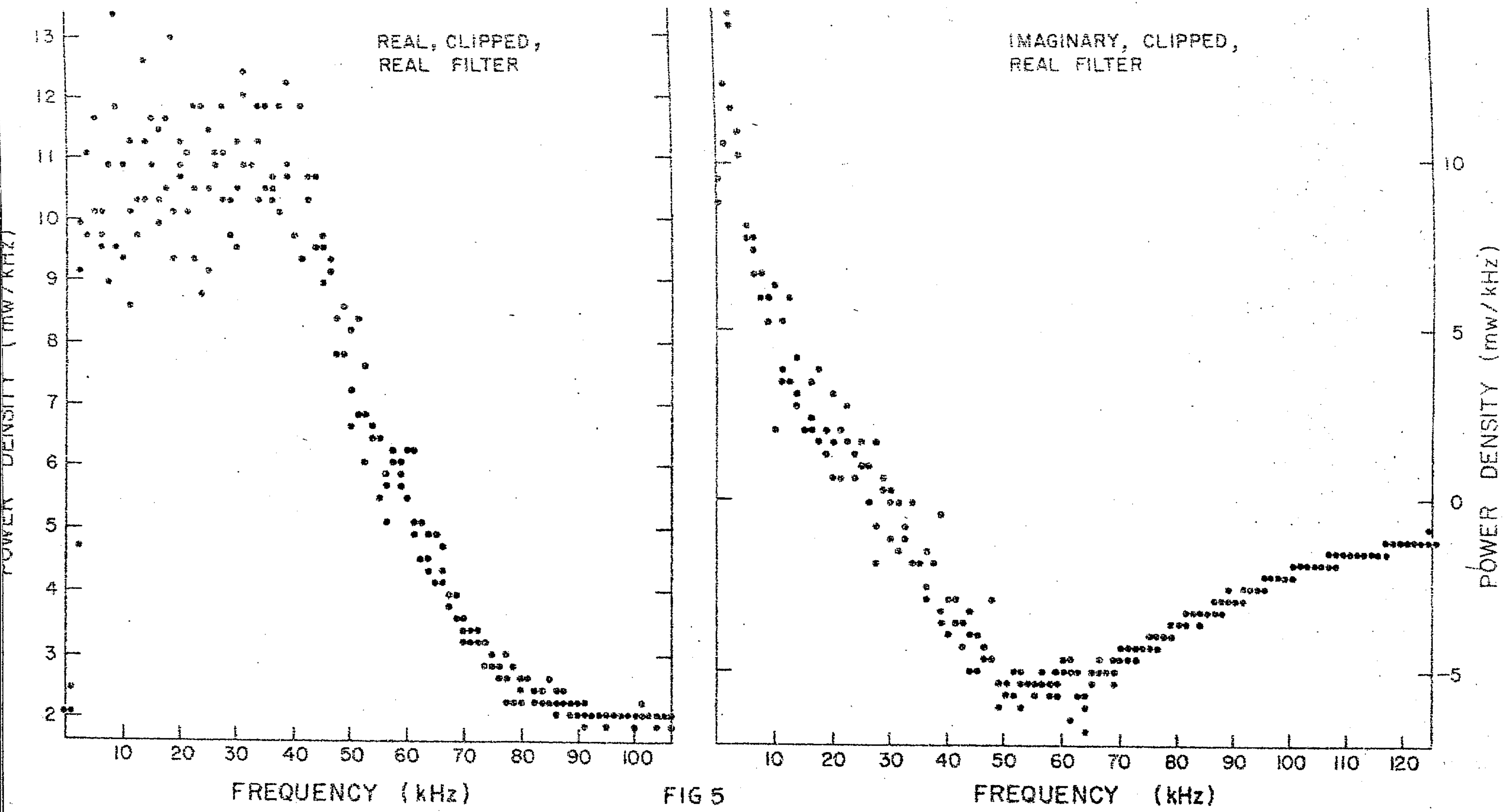

REAL, CLIPPED,
REAL FILTER
POWER DENSITY (mw/kHz)
FREQUENCY (kHz)
IMAGINARY, CLIPPED,
REAL FILTER
POWER DENSITY (mw/kHz)
FREQUENCY (kHz)

FIG 5