Maximum Entropy Spectral Analysis of Some Artificial Samples

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Using simple sinusoids as inputs, the Burg spectra are studied with a view to deciding the appropriate lengths of the prediction error filter (LPEF). It is observed that the higher the frequency (vis a vis the total data length representing the fundamental period), the lower the LPEF needed. In the case of mixed samples, frequencies exceeding the fifth harmonic are resolved with LPEF of about 50% (half-data length) and higher LPEF are likely to give spurious peaks. For lower frequencies, higher LPEF may be needed, sometimes even approaching 90% of the data length. Thus a mixed sample needs the study of Burg spectra from low as well as high LPEF simultaneously.

1. Introduction

For studying periodicities in geophysical parameters the traditional methods were Fourier analysis and Blackman and Tukey [1959] power spectrum analysis. Recently Burg evolved the maximum entropy method (MEM), which is considered superior for locating frequencies (see Burg [1972] and the review by Ulrych and Bishop [1975]). Before applying it to physical samples, it would be of interest to see how the method reacts to artificial samples. Ulrych [1972] showed that for truncated sinusoids, the frequency was correctly revealed even when the sample length was as small as 0.57T, where T is the period searched. However, Chen and Stegan [1974] showed that for truncated sinusoids, the spectral maxima showed frequency shifts sometimes as large as 20%, depending upon the initial phase and the length of the sample. Since MEM is gaining considerable popularity, we present here results of MEM for a variety of artificial sinusoids, which could prove useful while interpreting results for physical samples.

2. Analysis and Results

We confine our analysis to samples where the data length is larger than the largest period in any sample. We consider 100 consecutive data points, say 100 consecutive yearly values, of data samples composed of various periodicities, less than T = 100.

Single Sinusoids

We examined samples having only one period. Using sin(2πt/T + φ) as the input with t = 1, 2, ..., 100, we examined periods T = 2.5, 5.0, 10.0, 20.0, 40.0, and 70.0 separately and obtained Burg spectra for LPEF (length of the prediction error filter) = 2, 4, 8, 20, 40, 60, and 80 separately for phase φ = 0°, 60°, and 120°. The sinusoids with T = 2.5, 5.0, and 10.0 were located with an accuracy of ±1% for all phases and even for LPEF as low as 2. This is quite satisfactory.

For periods T = 20.0, 40.0, and 70.0 we found that depending upon the phase, the errors could be 20 ± 2, 40 ± 10, and 70 ± 20 for any LPEF.

Multiple Sinusoids

Sinusoids with T = 2.5, 5.0, 10.0, 20.0, 40.0, and 70.0 all with phase 0° were given as input simultaneously, first without any noise and later when a noise of 30% standard deviation (variance about 0.1) was added. For the noiseless sample, T = 2.5, 5.0, and 10.0 were resolved even for LPEF as low as 8. To resolve T = 20.0, 40.0, and 70.0, LPEF = 40 was needed. In the sample with noise, LPEF = 50 was needed to resolve all the periodicities up to T = 40. But for T = 70 the LPEF needed was 60 or more. However, at such high LPEF, the lower periodicities showed peak splitting and spurious peaks. This has been reported earlier also [Chen and Stegan, 1974], and recently, Fougere [1977] has evolved a modification of Burg's method for avoiding peak splitting. However, the general prescription given by Ulrych and Bishop [1975], viz., using LPEF roughly equal to half the data length (in our case LPEF = 50) does not seem to be appropriate. Unless one wants to use the somewhat complicated extra computation evolved by Fougere [1977], we suggest that an easier procedure would be to calculate Burg spectra for several LPEF (from 10% to 90%) and search for low periodicities (up to T of about 20 in our case) in LPEF up to 50 and for high periodicities in LPEF exceeding 50, keeping in mind that the high periodicities indicated by high LPEF are liable to have an error of about 10-20%.

Spacing between Successive Peaks

Ulrych and Bishop [1975] have mentioned that to resolve two frequencies f1 and f2, the data length needed should exceed 1/(f1 − f2). In the mixed sample mentioned above, this criterion was more or less satisfied for all pairs. We now examine samples with different spacings. A sample was prepared with periodicities which were all harmonics of a fundamental period T = 96 up to the 48th harmonic. Thus T = 96, 48, 32, ..., 2 were involved. It was noticed that LPEF up to 50 gave no worthwhile resolution at all. Even for LPEF = 60, only 10 of the 48 peaks were revealed. Only for LPEF = 80 or more, all the peaks were revealed. This was true for all the three phase angles 0°, 60°, and 120°. The analysis was repeated by introducing a noise of 30% standard deviation. Surprisingly, the resolution was now possible at lower LPEF. Thus at LPEF = 60, 23 peaks could be resolved instead of 10 in the noiseless sample. However, the inaccuracy of frequency location was about half a harmonic in both the noiseless and the noisy samples. That is, any periodicity could be mistaken for the one lower or higher harmonic. Interestingly, the data length of 96 points was good enough to resolve all these peaks by the criterion 1/(f1 − f2) as applied to this case.

We also examined a sample of 96 data points composed of groups of five nearby periodicities at five different broad regions, viz., T = 3.5, 4.0, 4.5, 5.0, 5.5; T = 10.0, 10.5, 11.0, 11.5, 12.0; T = 20, 21, 22, 23, 24; T = 40, 42, 44, 46, 48; and T = 70, 75, 80, 85, 90; all existing simultaneously. Only the T = 3.5-5.5 and T = 10.0-12.0 bands were resolved for LPEF exceeding...
60. Thus the criterion of $1/(f_1 - f_2)$ seemed to be obeyed. Addition of noise did not make any significant difference.

Another sample of 96 data points having $T = 4.75, 5.00, 5.25; T = 9, 10, 11; T = 20, 24; T = 40, 50; and T = 70, 90; simultaneously was tested. Here, the $1/(f_1 - f_2)$ criterion was satisfied for the first three bands. The analysis did show resolution for these three bands for LPEF of about 50, but for the other bands only some resolution was obtained for higher LPEF.

3. CONCLUSIONS

From this study, the following could be concluded.

1. For Burg spectra too the criterion that to resolve two frequencies $f_1$ and $f_2$ the data length should be at least $1/(f_1 - f_2)$ is roughly obeyed. But the general prescription given by Ulrych and Bishop [1975] to use LPEF of half-data length is not proper.

2. For peaks in the high-frequency (low periodicity) region, LPEF near half-data length is adequate. But for low frequencies (high periods, exceeding about $1/5$ of the fundamental period, i.e., the data length), LPEF needed are much higher than half-data length, though often pairs of peaks are reasonably resolved even when the $1/(f_1 - f_2)$ criterion is not strictly satisfied.

3. The presence of noise up to 20% standard deviation does not alter these conclusions. However, for larger noise levels, resolution is inferior.

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REFERENCES


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