STUDIES OF LOW FREQUENCY INSTABILITIES IN A COLLISIONLESS PLASMA

BY

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A THESIS
SUBMITTED FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
OF THE

GUJARAT UNIVERSITY

MAY 1975

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CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

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May 15, 1975

DEDICATED

TO

MY RESPECTED MOTHER

(SMT. DARYAW DEVI)

AND

TO THE SACRED MEMORY OF

MY FATHER

(LATE SRI NEELKANTH VYAS)

S T A T E M E N T

The work presented in this thesis is mainly centred around the study of very low frequency wave particle interactions in collisionless plasmas with a view to (i) investigating the generation mechanism for the magnetospheric VLF emissions, and (ii) explaining the favourable triggering of these emissions at half the equatorial gyrofrequency along the field line of their propagation.

Our approach to this study consists in first finding how the waves affect or modify the plasma particle distribution function and then investigating how this modified distribution affects low amplitude perturbations existing in the system.

The thesis starts with a preliminary introduction to the magnetospheric plasma and presents a brief survey of the work done by various investigators in the field.

The effect of the Landau resonance of whistler mode pulses on the particle distribution function in a homogenous collissionless magnetoplasma has been discussed in chapter II and an extension of the model for VLF emissions proposed by Das (1968) has been presented in chapter III. The model is based on gyro resonant interaction and the modification made therein is the inclusion of the effect of increasing the amplitude of a resonant pulse beyond a certain critical limit.

Then the question of the preferential triggering of VLF emissions at half the equatorial electron gyrofrequency has been taken up. The behaviour of whistler mode dispersion relation has been examined carefully and it is found that it exhibits many interesting characteristics at that frequency. The influence of these characteristics on the generation of VLF emissions has been investigated in great detail in chapter IV and V and the results have been found to be quite encouraging.

In both the chapters IV and V, the emphasis has been put on the effects of Landau resonance. The former discusses the Landau resonant diffusion of particles in velocity space and the latter presents a study of the effects of Landau damping on the gyroresonant interaction.

The last chapter presents a synoptic view of the whole work and gives a discussion of the results obtained. The scope for future work has also been pointed out towards the end of the thesis.

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ACKNOWLEDGMENTS

I wish to express my sincere appreciation to Dr. Anath Chandra Das for his guidance, patience, magnanimity, stimulating counsel and ever helpful attitude during all phases of this work.

I have received enormous help and encouragement from Mr. Ram Rattan in completing this work. I am deeply indebted to him.

I acknowledge the helpful discussions that I used to have quite often with Mr. V.H. Kulkarni, Mr. R.K. Jain and Mr. Y.S. Satya.

My sincere acknowledgements are due to all the plasma physicists and other scientists who worked at the Physical Research Laboratory during the course of this work and helped me in some way or other in getting ahead with the work.

The help of Dr.K.S. Rao and Dr.D.R. Kulkarni in solving computational problems was invaluable. I express my sincere thanks to them. I also acknowledge the general cooperation of the personnel of the PRL Computer Centre.

I am very much thankful to Dr. Dinesh Patel for his general helpful attitude.

I wish to express my special appreciation to Mr. D. S. Kamat Mr. H. S. Raina and Mr. N. V. Maslekar who consistently encouraged and consoled me in my efforts and often turned despair into renewed hope.

I acknowledge the neat and prompt typing of the manuscript by Mr. P. P. Narayanan.

It is a pleasure to acknowledge the keen interest taken in the progress of this work by all my friends both at the Physical Research Laboratory and in the Indian Space Research Organisation.

I am also grateful to all others who helped me directly or indirectly in bringing this work to the present shape.

Whatever value this work may have, however, may be attributed in large part to my uncles, brothers and cousins who, right from the beginning, helped me in cultivating an interest for learning.

Nand Kishore Vyas
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CONTENTS

CHAPTER - I

INTRODUCTION

1.1. OBJECTIVE

It is well known that the whole of the universe, but for a trace of it, is composed of matter in plasma state. The Physics of plasma is, therefore, an extremely important discipline for understanding the different physical processes of the universe. Study of plasma processes can be carried out in laboratories through the artificial production of plasmas. However, the study of space plasma phenomena is difficult in a laboratory where the simulation of such plasmas is not always possible because of the dimensions involved.

The direct experimentation, wherever possible, with the naturally occuring space plasmas and the investigation of the associated phenomena seems to be a more plausible approach to the study of plasma physics in particular and to the understanding of nature, in general.

The terrestrial magnetosphere is one of such natural plasma laboratories accessible to man. It contains a fully ionised collisionless plasma immersed in a dipole magnetic field. Observations show that various forms of low frequency waves and emissions are generated in this plasma. The investigation of the generation mechanisms and the explanation of the different features of these waves and emissions would not only be helpful to us in developing a deeper understanding of nature but would also provide us with more detailed information regarding the magnetospheric plasma. The phenomenon of the VLF emissions offers us a new approach to the study of plasma physics in an environment that is free of many of the restrictions of the terrestrial laboratories.

The present effort is aimed at studying from different angles the resonant wave particle interactions in collisionless magnetoplasmas with special reference to the earth's magnetosphere and phenomena occuring there. Emphasis has been given to the relevance of these interactions to the triggering mechanism for VLF emissions. An attempt has also been made to study what processes should contribute to the observed favourable emission at half the equatorial gyrofrequency (along the field line of propagation of the triggering signal) which constitutes one of the most outstanding problems in magnetospheric physics today.

1.2 MAGNETOSPHERIC PLASMA, WHISTLERS AND VLF EMISSIONS: A DESCRIPTION OF THE EARTH'S NEAR SPACE ENVIRONMENT

The space near the earth is not empty but is filled with charged particles which are being ejected out from the sun and are moving at very fast speeds of about 300 to 500 kms/sec. This stream of charged particles constitutes the so called solar wind. Because of its high electrical conductivity, it distorts and drags the solar magnetic field along with it.

When the solar wind meets earth's magnetic field on its way, the latter behaves like an obstacle in its path. The speed of the solar wind is very large compared to that of the hydromagnetic waves that can propagate through it. Therefore, where the solar wind impinges on the magnetic field in earth's environment, a shock wave is set up and the wind turns round and envelopes the field without penetrating it, just as a stream of gas with supersonic speed does while blowing past an obstacle.

During the process the magnetic field in earth's environment behaves like a deformable baloon and gets distorted because of the pressure of the solar wind. The region of this distorted magnetic field is called the magnetosphere.

Magnetic field measurements (Cahill and Amazeen, 1963) show that at a distance of about 10 earth radii on the sunward side from the centre of the earth, the magnetic field suddenly goes down. This forms the boundary of the magnetosphere and is known as magnetopause. Beyond the magnetopause, upto a distance of about 20.5 earth radii from the centre of the earth, the magnetic field fluctuates randomly. This is the region of the shock wave where the wind particles stop to proceed further and turn round the earth. This is known as magnetosheath.

At 20.5 earth radii, the magnetic field again drops down suddenly and becomes more or less smooth and independent of distance. This is the region of the free and the unobstructed motion of the solar wind. The field here is frozen into it and is being carried away with it.

This model of the magnetosphere, however, is based only on the direct interaction of solar wind plasma with the geomagnetic field and does not take the interplanetary magnetic field into account. The model can be improved upon through the introduction of an interplanetary magnetic field into the system. The new system can be described as follows:

i) To start with, there is a dipole magnetic field immersed into a relatively weak but uniform magnetic field.

ii) Then the highly conducting solar wind starts blowing radially outwards from the sun at a tremendous speed and distorts the system.

The geometry of the dual geomagnetic and interplanetary magnetic field systems contain magnetically neutral points. The dynamics of plasmas close to these neutral points is rather involved because of the complex field topology there and has been discussed by

Dungey (1963).

Figure (1.1) gives a view of the magnetosphere as deduced from spacecraft measurements (Ness, 1969).

THE EFFECTS OF THE GEOMAGNETIC FIELD AND THE COLLISIONS ON THE MOTION OF THE CHARGED PARTICLES:

Inside the magnetosphere the movement of charged particles is governed by the earth's magnetic field and the interparticle collisions. The relative importance of the two depends on the ratio of the gyro frequency to the collision frequency. In ionospheric regions (i.e. upto heights of the order of a few hundreds of kilometers) where many collisions take place during a single gyroperiod, the guidance of the particles by the earth's magnetic field is relatively unimportant. At greater heights, one collision may take place only after the particle has made several gyrations. Thus the collisions have little effect on the movement of the particles at higher altitudes and the magnetic field is the predominant factor that governs their motion.

The earth's magnetic field is approximately dipole in character and is therefore large near the

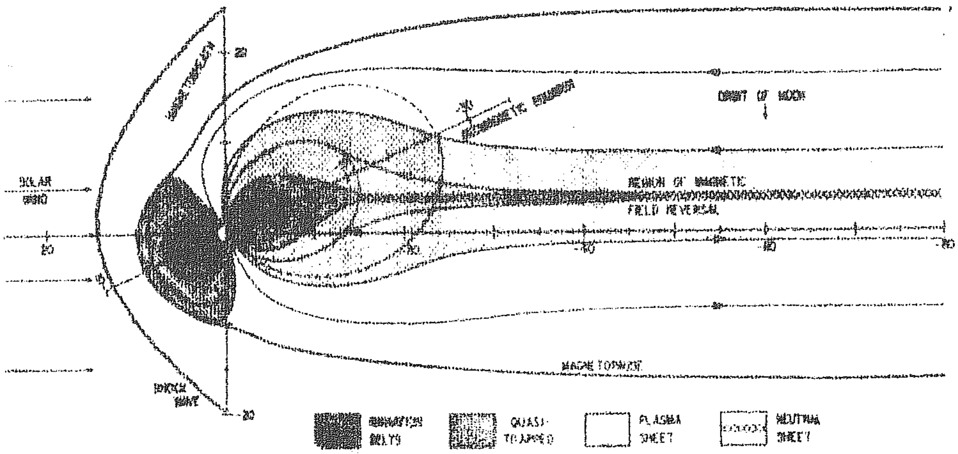

Fig. 1.1 (after Ness, 1969)

A summary view of the magnetosphere and the geomagnetic tail as deduced from spacecraft measurements poles but minimum at the equator along any field line.

The motion of particles under the influence of such a magnetic field, in the absence of inter-particle collisions, will have the following three components: (see fig. 1.2A).

i) A gyration of the particles around the field line

ii) An oscillatory movement of particles parallel to the field line, and

iii) A movement of the particles from one field line to another.

Corresponding to each of these three types of motions, an adiabatic invariant is associated. An adiabatic invariant is an approximate constant of motion and comes into picture when the variation of magnetic field over one gyroradius is small.

The adiabatic invariant associated with the gyration of a particle is its magnetic moment $\frac{1}{2}\frac{mV_{\perp}^2}{B}$. As the particle moves from equator to higher latitudes, B increases and so also its $V_{\perp}$. Since the total energy of the particle is an exact constant of motion, an increase in $V_{\perp}$ would cause a a decrease in $V_{\parallel}$ and for a sufficiently large value of B, $V_{\parallel}$ will become

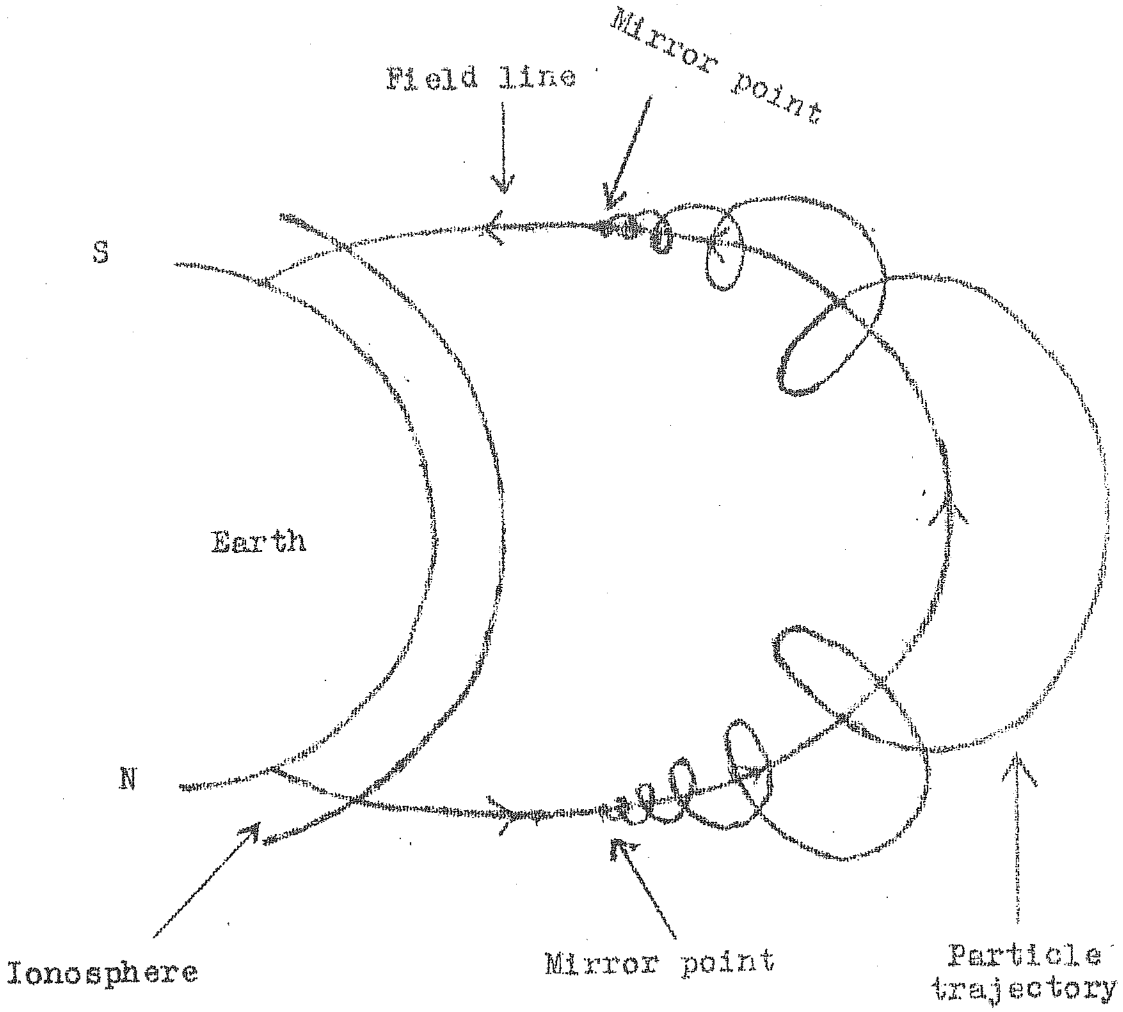

Fig. 1.2 A

Gyration and Mirroring of Particles in Geomagnetic Field. Drift across the field lines is not shown.

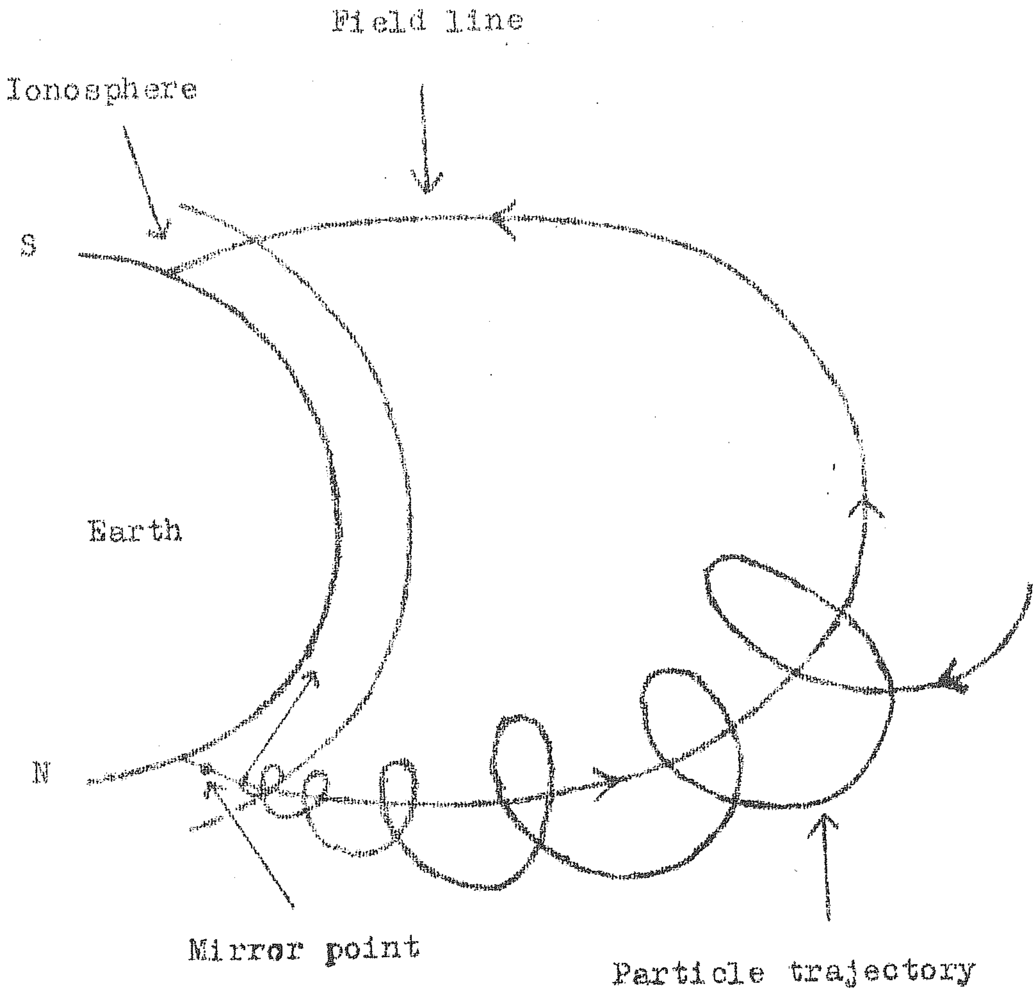

Fig. 1.2 B

Particles with their mirror points deep into the ionosphere are likely to be lost due to collisions before they are reflected from a mirror point zero. Thus particle can not continue moving away from the equator indefinitely and must retrace its path again and again between two fixed points known as mirror points situated one on each side of the equator.

The adiabatic invariant associated with the oscillations of a particle along the field lines is the integral of its $v_{\parallel}$ along that field line between the two mirror points. The particle may go from one field line to another because of the gradient or the curvature drifts but in any case, as long as the adiabatic condition is fulfilled, $\int_{M_1}^{M_2} v_{\parallel}\, ds$ will remain invariant where ds is the line element along a field line and M_1 and M_2 are mirror points.

Finally, the third adiabatic invariant comes from the drift of a particle from one field line to another which is caused by the gradient and the curvature drifts. During this type of motion the particle goes round the globe repetitively and during the process encompasses a certain amount of magnetic flux. This flux would remain invariant as long as the condition of adiabaticity is fulfilled.

The variation of geomagnetic field at magnetospheric heights over one Larmour radius is negligibly small and therefore the conditions of adiabaticity are very well fulfilled there.

High energy particles have been observed to be trapped in the geomagnetic field at altitudes of about 2 to 4 earth radii (Van Allen, 1959). The mechanism behind the trapping of these particles in these regions which are known as Van Allen Belts is their consecutive reflections from the pairs of conjugate mirror points.

The distance of the mirror points for a particle from the equator depends upon its equatorial pitch angle. The pitch angle is defined as arctan ($v_\perp / v_\parallel$) where $v_\perp$ and $v_\parallel$ are the components of the velocity of a particle perpendicular and parallel to the magnetic field Bo respectively. For pitch angles below a certain value which is known as loss cone angle, the corresponding mirror points lie far down in the ionosphere. Near such mirror points the collision frequency is much larger than the gyrofrequency and therefore the guidance by the field is lost and the particle gets dumped into ionosphere. As a result, the pitch angle distribution of particles in Van Allen belts becomes highly anisotropic.

Figure 1.2B shows how a particle is likely to be lost because of the dominance of the collisions if its mirror point is sufficiently deep in the atmosphere.

WHISTLERS:

The electromagnetic signals in the frequency range 300 to 30,000 Hz., generated below the lower edge of the ionosphere during a lightning discharge penetrate the ionosphere and propagate out into the magnetosphere if the angle of propagation is within a narrow cone around the field direction. They are known as whistlers and they suffer partial reflections successively from either hemisphere as evidenced by the traces of their consecutive echoes recorded on spectrograms that display their frequency time structure (Figs. 1.3 and 1.4)

If the time and the hemisphere containing the location of the lightning discharge are considered as reference, the successive whistler echoes are observed at intervals proportional to 2,4,6, ... in the reference hemisphere and to 1,3,5 ..... in the opposite hemisphere at the geomagnetically conjugate points. This indicates that the path of propagation of a whistler is fixed in the magnetosphere, most likely along a geomagnetic field line. The explanation for this has been provided by the duct theory (Smith, 1961) which presumes the existence of the regions of enhanced ionisation along the field lines in the magnetosphere. The variation of the refractive index across a duct confines the propagation of the whistlers to within a very narrow angle from the magnetic field direction (see Figs. 1.4 and 1.5).

In the absence of ducting, the guidance to whistler propagation along the field lines is lost and therefore such whistlers do not reach the ground.

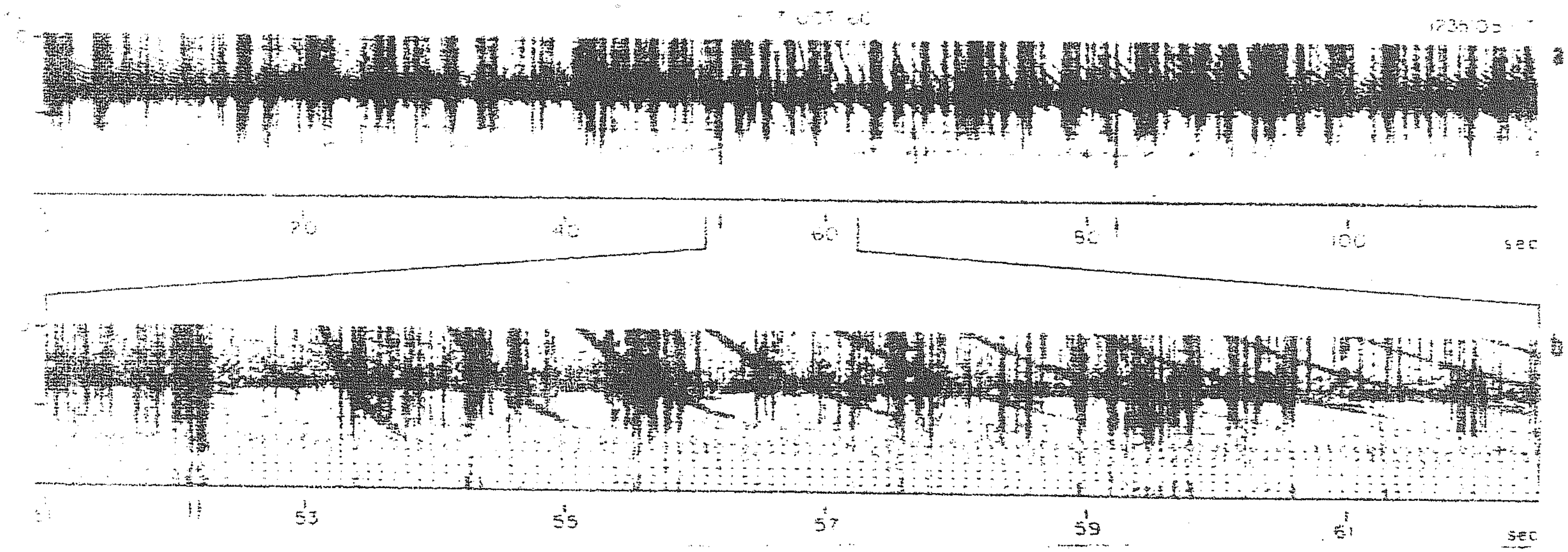

Fig. 1.3 (after Helliwell, 1965)

Long-enduring whistler echo trains. a, Two even-order echo trains, with sources indicated, superimposed on a background of echoes from a previous whistler with associated periodic emission at a frequency of about 6 kc/s. b, Expanded version of a whistler is seen to be of the multiflash type, with two main components.

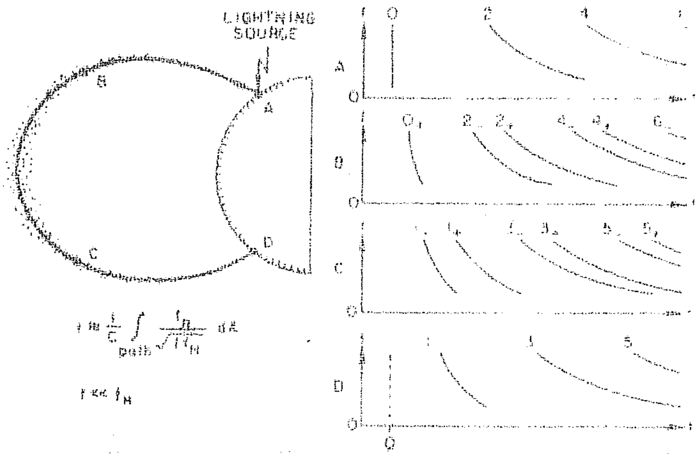

Fig.1.4

(After Helliwell, 1969)

Echoing ducted whistlers as observed on the ground at points A and D and in satellites at points B and C. The lightning source is located at point A, and the Eckersley approximation isused to sketch the spectra. The vertical dashed line in the lowest spectragram represents the attenuated sferic arriving at D via the earth-ionosphere waveguide.

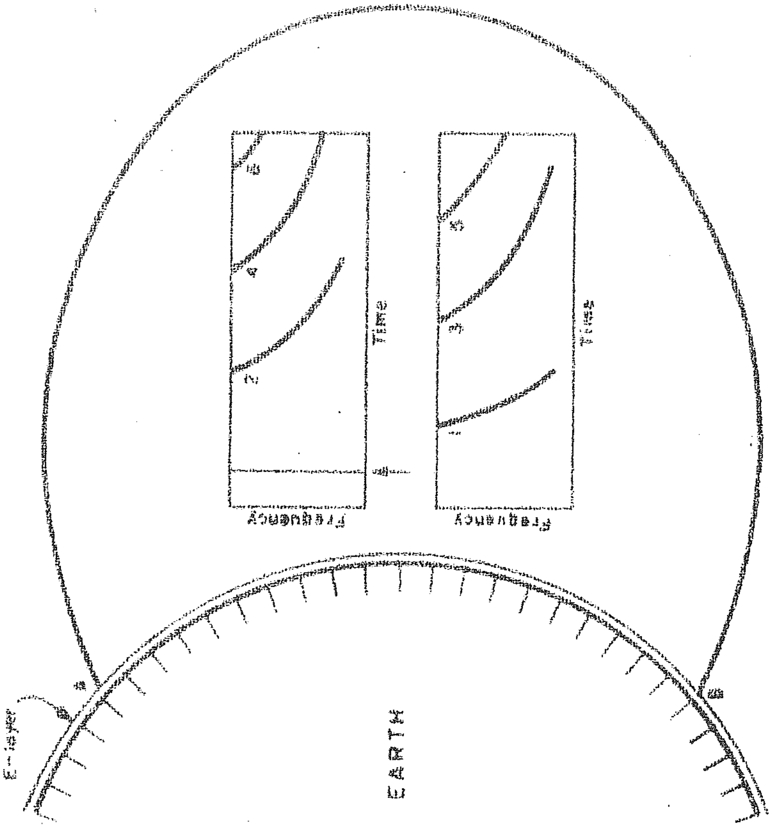

Fig. 1.5

(After Helliwell, 1965)

Field-line path followed by a ducted whistler. Inset diagrams show idealised spectra of whistler echo trains at conjugate points A and B.

However, they have been observed from satellites.

The different frequency components of a whistler propagate with different group velocities and consequently arrive at the observation point at different times. This explains the observed variation of the whistler frequency with time on a spectrogram.

The theoretical expression for the whistler mode dispersion relation for a homogenous cold plasma in a uniform magnetic field B is given by (Stix, 1962):

$$\frac{c^2k^2}{\omega^2} = \frac{\omega_p^2}{\omega(\Omega \cos \theta - \omega)} \qquad \text{---(1.1)}$$

where ω and $\vec{k}$ are the frequency and the wave vector of a whistler mode wave, ω_p and Ω the plasma frequency and the electron gyrofrequency, c the velocity of light, and θ the angle between $\vec{k}$ and B, assumed to be small.

This dispersion relation shows that the group velocity of the waves is maximum at a certain frequency. The waves at this frequency, if contained within the original whistler, will suffer minimum time delay (from the time of their generation) in reaching the observer. Both the higher and the lower frequency components of the whistler will be delayed further. Such a whistler, owing to the shape of its frequency time structure is called a nose whistler and the frequency of minimum time delay is called nose frequency (See Fig. 1.6).

The nose frequency of a whistler is closely related with the minimum gyrofrequency along the path of its propagation and, therefore, a knowledge of it can be used to determine the field line of the whistler propagation.

The time delay at the nose frequency depends upon the electron concentration (the integrated electron density) along the path of propagation and thus helps us in estimating the plasma densities at various distances from the earth. Figure 1.7 gives the details of the different types of whistlers and their spectral forms.

The observations lead us to conclude that at a distance of about four earth radii from the centre

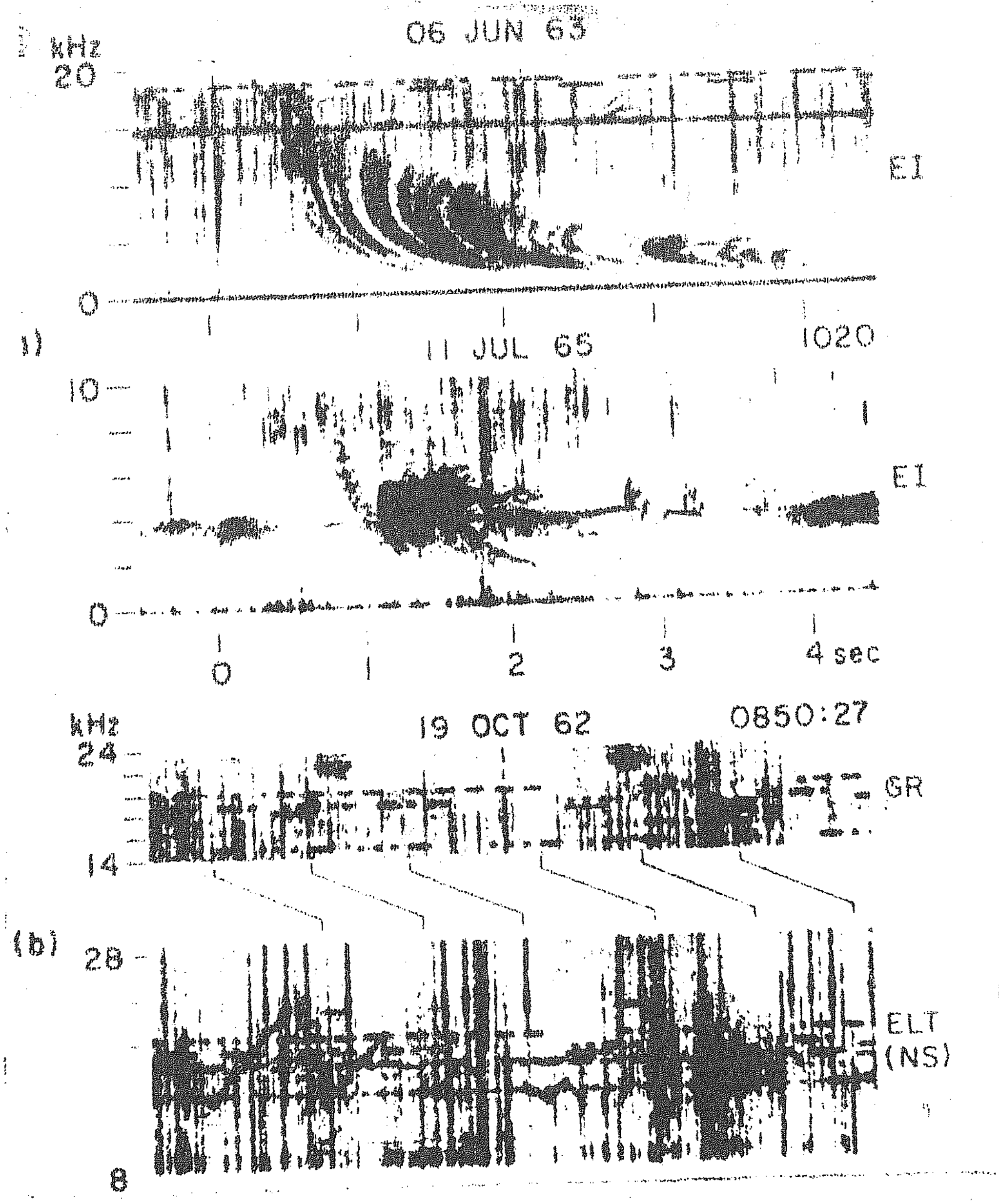

Fig. 1.6

(After Helliwell, 1969)

Triggered emissions: (a) Upper panel shows multipath nose whistler and rising tones triggered by NAA at 14.7 kHz; lower panel shows discrete emissions triggered by nose whistlers at their upper cutoff frequencies (Ω_{eq}/2). (b) Artificially stimulated emissions (ASE's) from the Morse-code dashes transmitted by station NAA.

Whistler Types

Type and Definition	Spectral Form
I. *One-hop (short)* A whistler that has traversed one complete path through the ionosphere.	
II. *Two-hop (long)* A whistler that has traversed in sequence two complete paths through the ionosphere. The two paths may or may not be the same.	
III. *Hybrid* A combination of a one-hop and a two-hop whistler originating in the same source.	
IV. *Echo train* A. Odd order. A succession of echoes of a one-hop whistler. Delays usually in ratio 1:3:5:7, etc. Components called one-hop, three-hop, five-hop, etc. B. Even order: A succession of echoes of a two-hop whistler. Delays usually in ratio 2:4:6:8, etc. Components called two-hop, four-hop, six-hop, etc.	
V. *Multiple-component* A. Multipath. A whistler with two or more components, each of which has traversed a different path through the ionosphere. B. Mixed-path: A multiple whistler of two or more hops in which combinations of the basic one-hop paths occur.	
VI. *Multiple-source (multiflash)* Two or more whistlers closely associated in time, but having different sources.	
VII. *Nose* A whistler whose frequency-time curve exhibits both rising and falling branches. The delay is a minimum at the nose frequency	
VIII. *Fractional-hop* A whistler that has completed only a fraction of a one-hop path (often observed from a probe or satellite).	

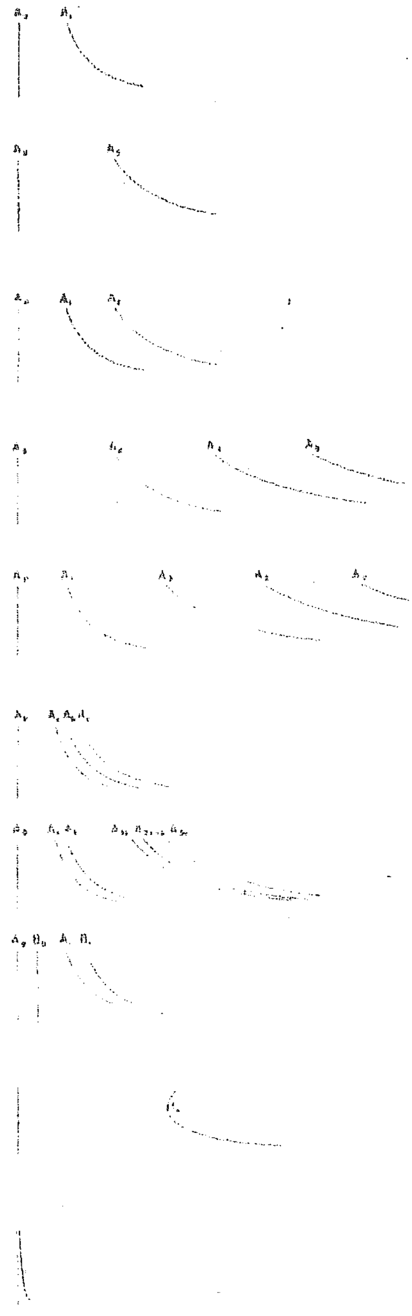

Fig. 1.7

(After Helliwell, 1965)

of the earth, the electron concentration falls by a factor of 10 to 100 (Helliwell, 1969) over a distance of less than one earth radius, thereby separating the high density plasma region, known as plasmasphere from rest of the magnetosphere. The boundary of the plasmasphere is known as plasmapause.

THE VERY LOW FREQUENCY (VLF) EMISSIONS

The electromagnetic signals at frequencies lying in the whistler frequency range but having their source within the magnetosphere are known as very low frequency (VLF) emissions. They may be broadly classified into two groups: continuous and discrete. The continuous emission or hiss which has three subclasses - auroral hiss, midlatitude hiss and the polar chorus, consists of wide band noise which generally lasts for about a few hours.

The discrete emissions are characterised (i) by their comparatively short duration which is of the order of a few seconds and (ii) by their frequency time structure which exhibits a variety of shapes. An emission is called a riser (see fig. 1.8) or a falling tone (see fig. 1.9) depending on whether its frequency is rising or falling with time. Sometimes the frequency oscillates between two values and this is called an oscillating tone. If the spectral form of the emission resembles a hook, it is referred to as a 'hook' (see fig. 1.10).

Figure 1.11 and figure 1.12 show combinations of different types of discrete VLF emissions.

The discrete emissions some times also show a periodicity or a quasiperiodicity (see fig. 1.13) in their occurence. Frequently a succession of overlapping risers is observed which is called dawn chorus or simply chorus. The chorus has got the properties of both the discrete and the continuous VLF emissions.

On the basis of their generation, the discrete emissions can further be divided into two classes: (i) spontaneous emissions, and (ii) triggered emissions (see fig. 1.6). The latter class belongs

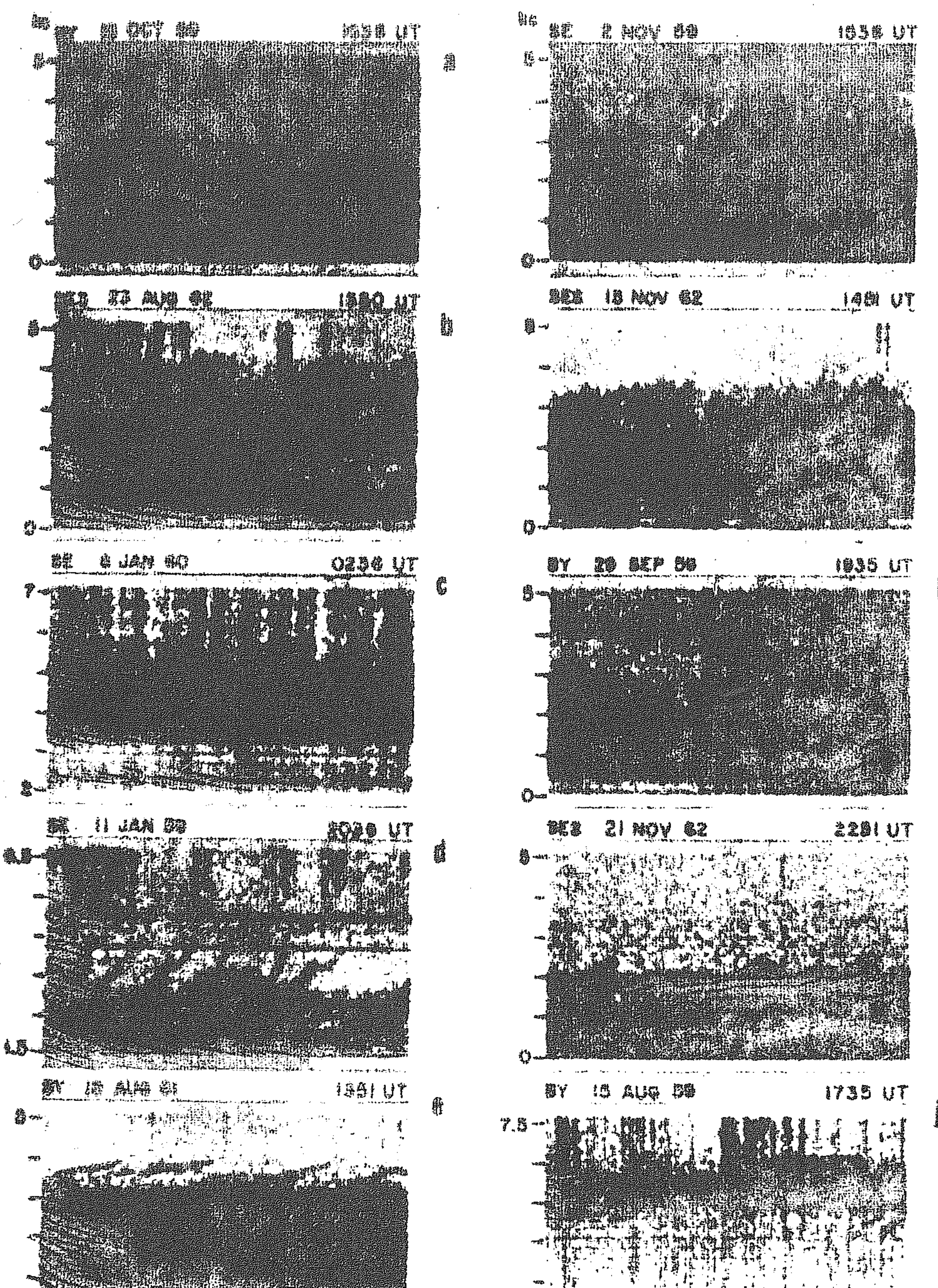

Fig. 1.8 (After Helliwell, 1965)

Risers, Slope decreases from top to bottom.

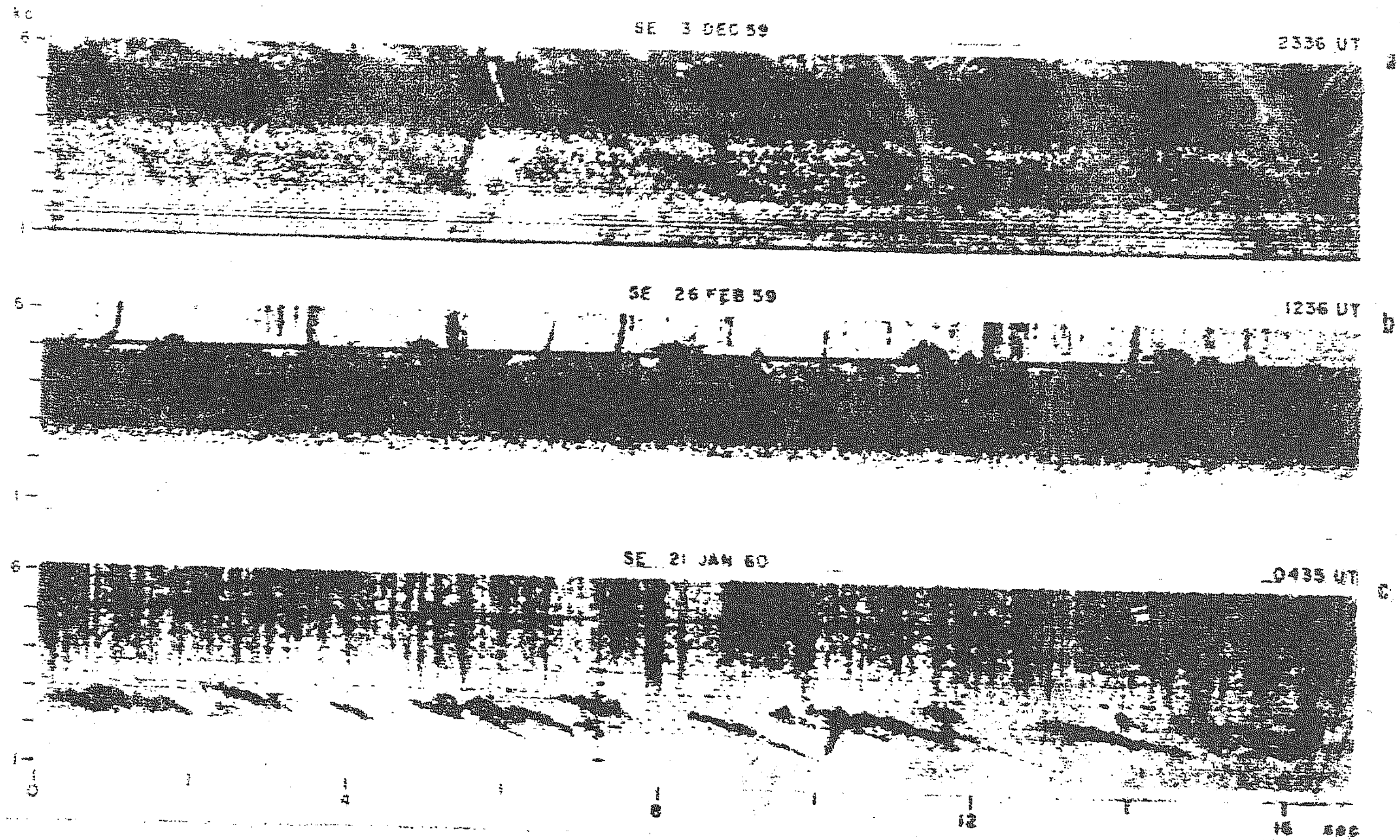

Fig. 1.9 (After Helliwell, 1965)

Falling tones. Each element is a member of a set of dispersive periodic emissions, a, P(3.5 kc/s) = 3.28 sec. b, P(5 kc/s) = 3.22 sec. c, P(1.6 kc/s) = 4.56 sec. Several risers and hooks can be seen

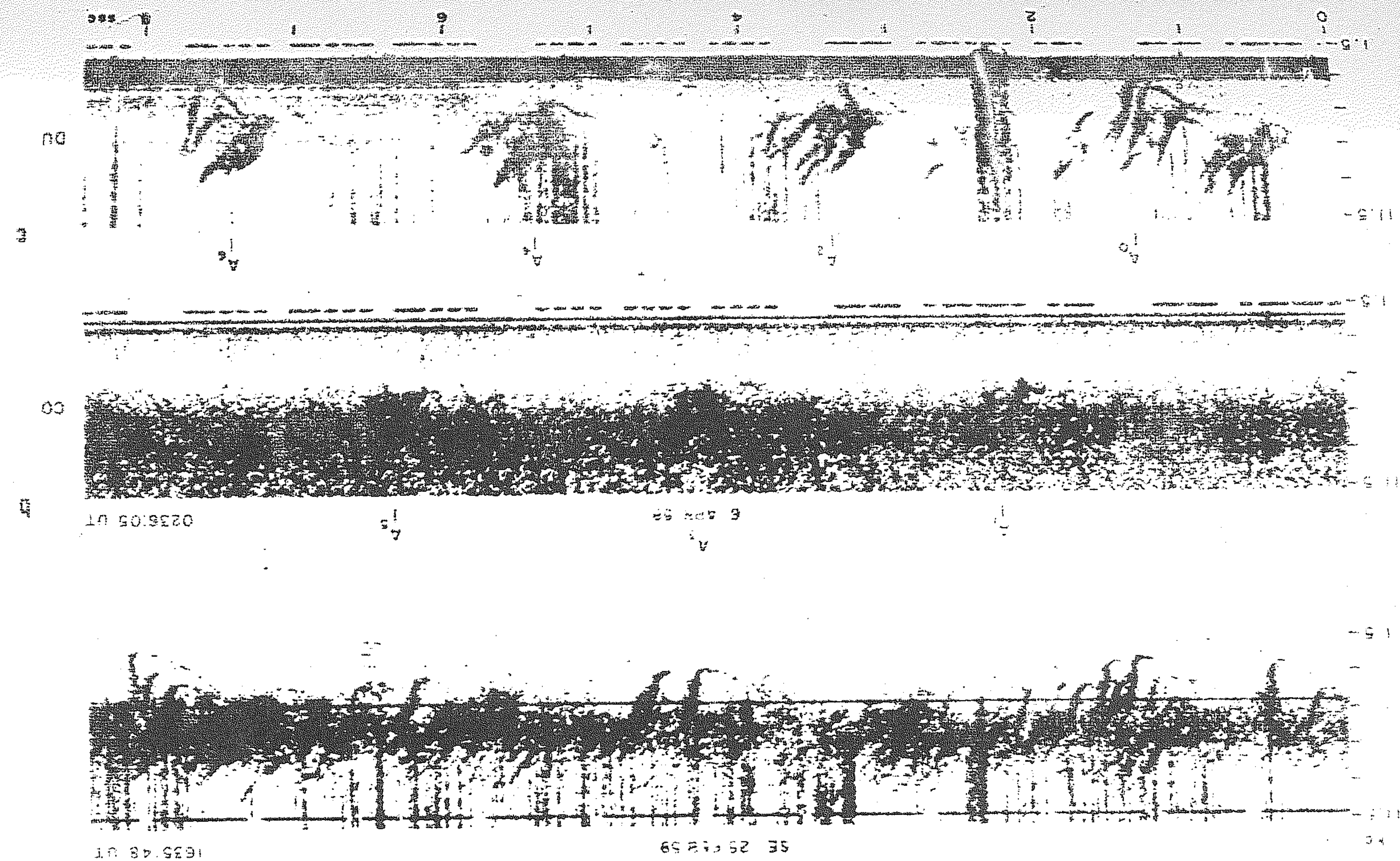

Fig. 1.10 (After Helliwell, 1965)

Hooks. a, Tendency for hooks to occur in groups or clusters of two or more, b,c, Hook clusters that appear to be periodic and in anti-phase at the conjugate points; each cluster considered to be an element of a set, and its estimated center is marked; P(7.3 kc/s) = 2.0 sec. Note absorption band from 4 to 6 kc/s at CO that is not evident at DU.

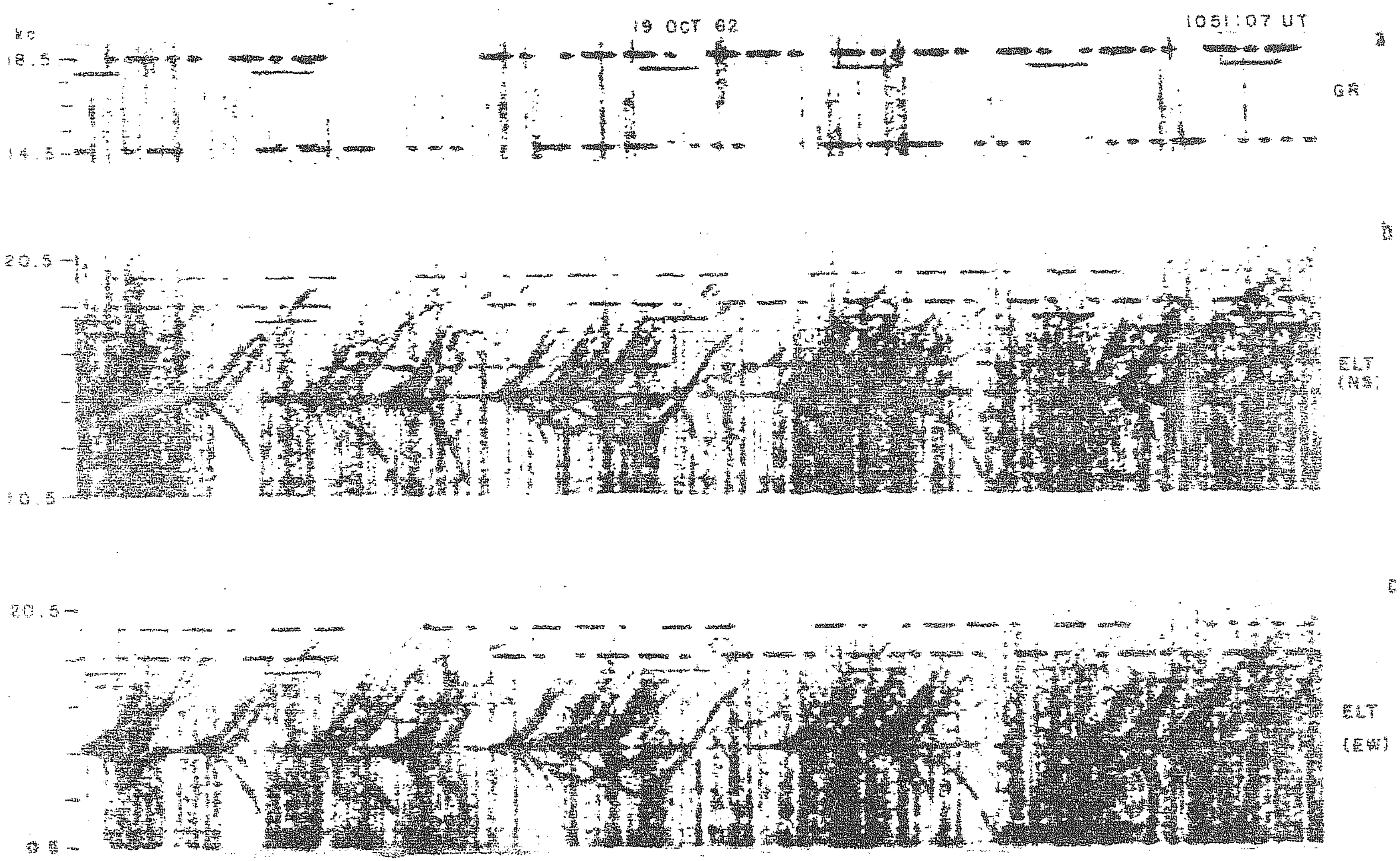

Fig.1.11 (After Helliwell, 1965)
Artificially stimulated emissions from station NAA on 14.7 kc/s b.c. show the rising and falling tones and some hooks

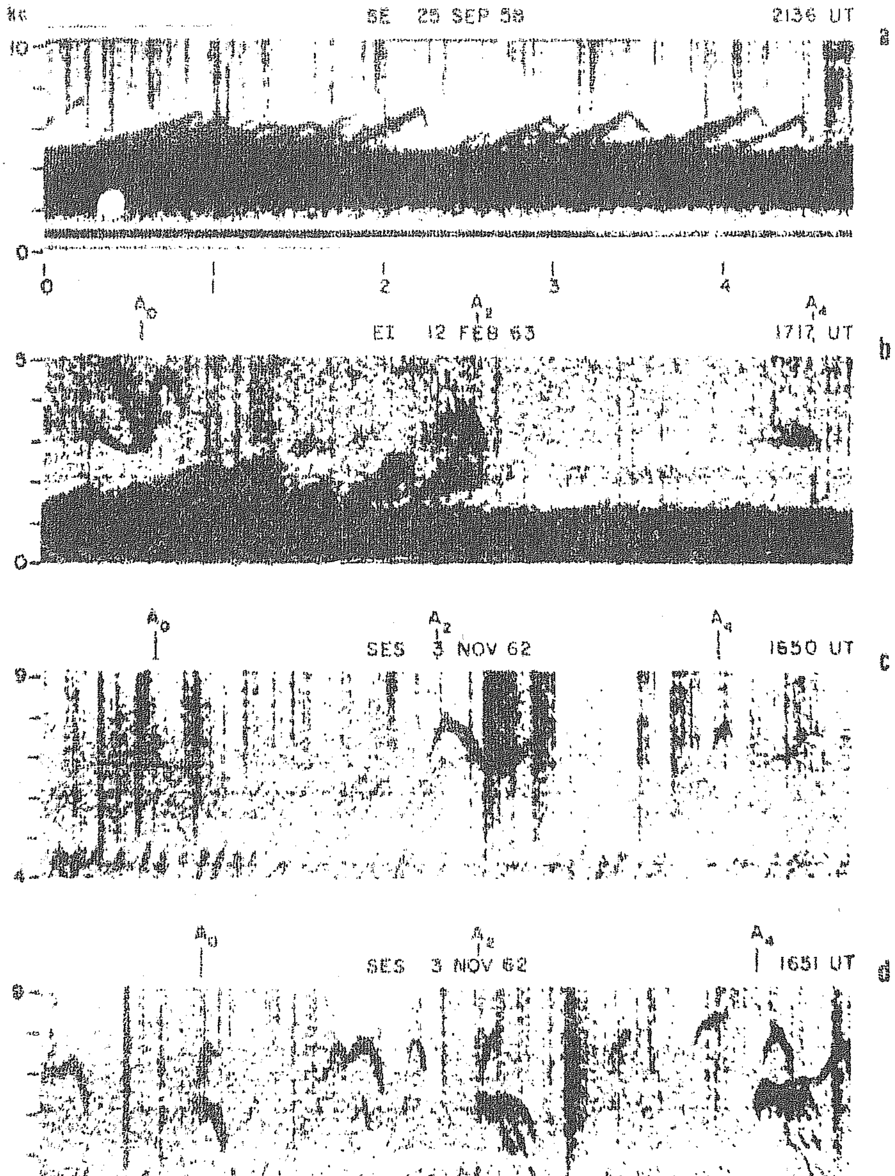

Fig. 1.12 (After Helliwell, 1965)

Combinations of discrete emissions. a, Examples of riser followed by falling tone, sometimes called 'inverted hook,' beginning in band of hiss. b, Hook followed by falling tone; c, Riser followed by hook; d, Complex combinations

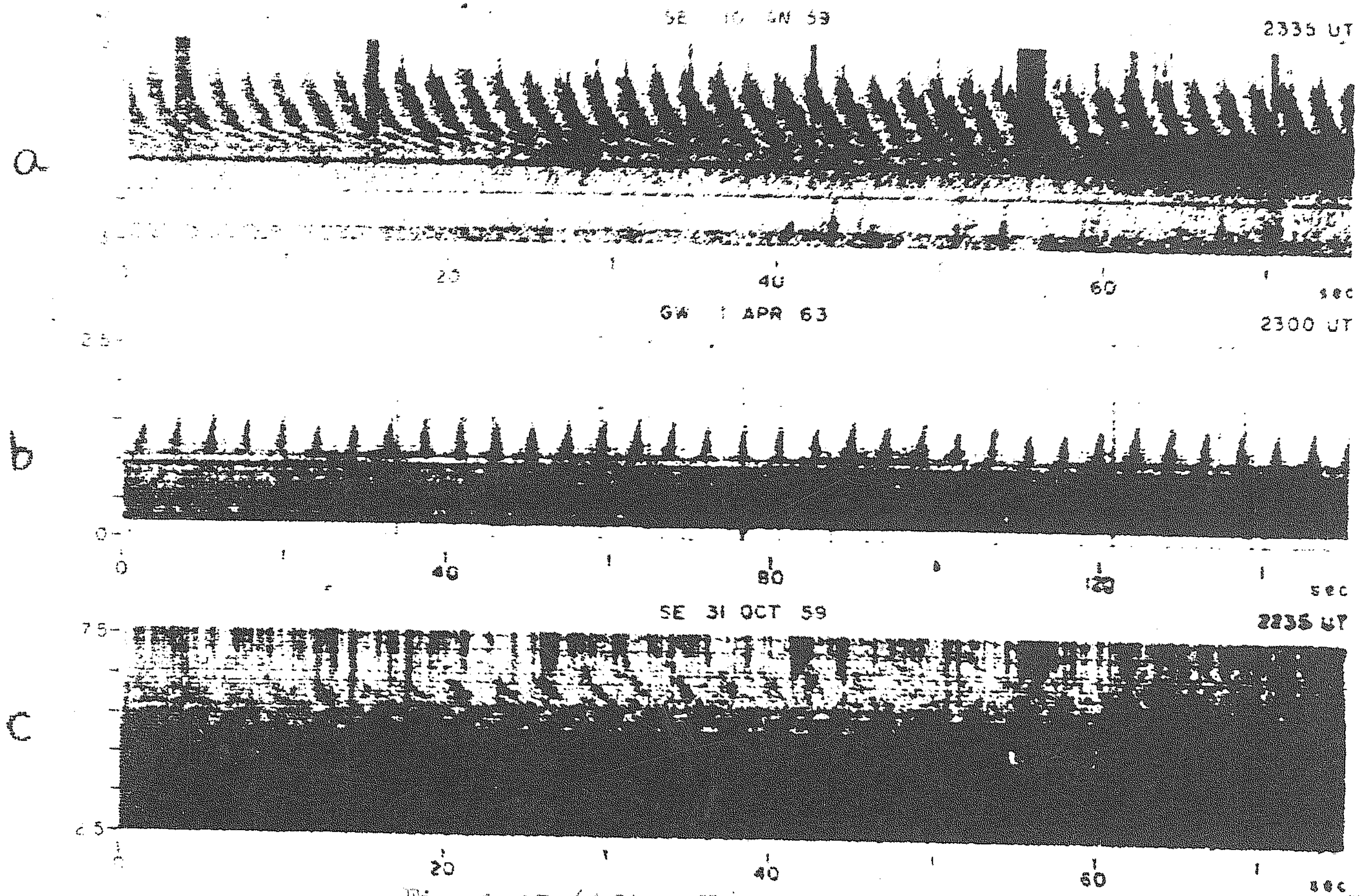

Fig. 1.13 (After Helliwell, 1965)

Periodic emissions a, Elements show negative slope; P(5.7 kc/s) = 1.96 sec, P(7.0 kc/s) = 1.91 sec. b, P(1.5 kc/s)=4.33 sec; c, Strong periodic emissions between 4 and 6 kc/s show positive slope; weaker periodic emissions appearing above 5.7 kc/s show negative slope; P(4.9 kc/s)= 2.83 sec, P(5.5 kc/s) = 2.84 sec, P(6.3 kc/s) = 2.80 sec.

to those emissions which are excited by the action of either a whistler or a spontaneous discrete emission or a man made signal from VLF transmitters.

The discrete emissions have the following outstanding features:

i) They have a very narrow bandwidth of approximately 100 Hz. which is roughly 1 % of the central frequency of the emissions.

ii) The amplitude of the emissions remains almost constt (± 6 dB) over its entire duration.

iii) The central frequency varies significantly with time. The ratio of the maximum to minimum central frequency of an emission may be some times as high as 2 : 1 .

The emissions artificially stimulated through the use of VLF transmitters have been found to show:

i) More frequent triggering by Morse Code dashes (duration 150 msecs.) than by Morse Code dots (duration 50 m secs.).

ii) A time delay of roughly 70 to 130 milliseconds with respect to the triggering signal.

iii) An off set frequency which is slightly higher than the frequency of the triggering signal.

iv) More frequent triggering through VLF signals having a frequency equal to half the minimum gyrofrequency on the geomagnetic field line along which they propagate.

Figure 1.14 shows the frequency time structure of the various types of VLF emissions and figure 1.15 illustrates the properties of discrete emissions. Lower panel of figure 1.6a shows a typical triggered VLF emission at half the equatorial gyrofrequency.

VLF emissions also, like whistlers propagate in both ducted, as well as nonducted mode. The nonducted emissions donot reach down to the surface of the earth and therefore can not be observed at ground stations. However, satellite borne equipments have been able to record such emissions.

Observations of the nonducted chorus from OGO-5 satellite show that the generation frequency of these emissions is very close to half the equatorial gyrofrequency and that their point of origin lies

MODEL SPECTRAL FORMS OF VLF EMISSIONS

Type and Name	Model Spectral Form
I. *Hiss*	
II. *Discrete emissions*	
A. Rising tone	
B. Falling tone	
C. Hook	
D. Combinations	
III. *Periodic emissions*	
A. Dispersive	
B. Non-dispersive	
C. Multiphase	
D. Drifting	
IV. *Chorus*	
V. *Quasi-periodic emissions*	
VI. *Triggered emission*	

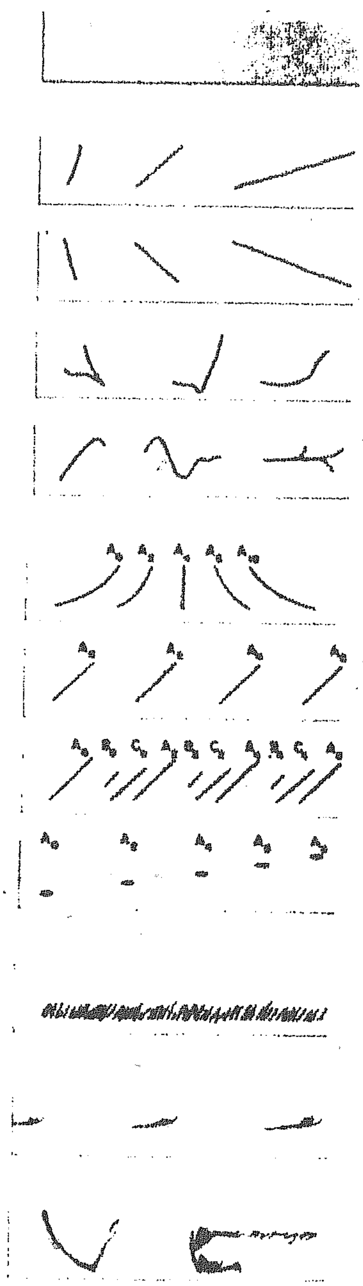

Fig. 1.14

(After Helliwell, 1965)

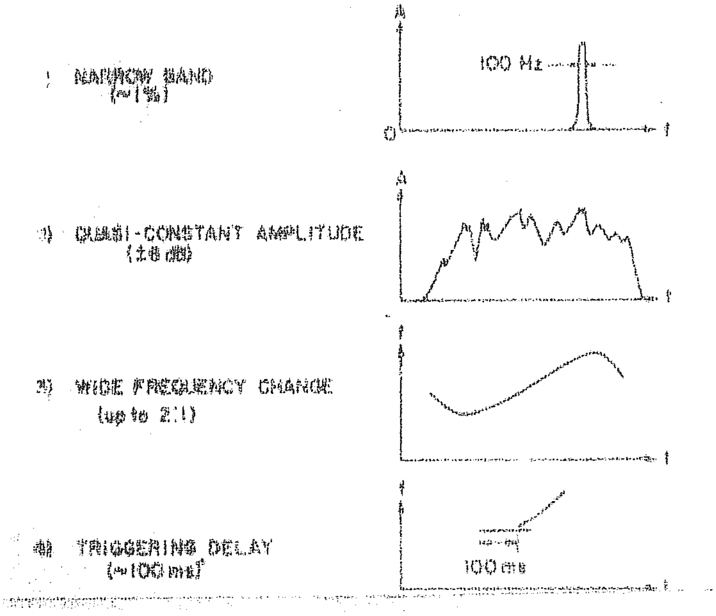

Fig. 1.15

(After Helliwell, 1969)

Properties of discrete emissions

somewhere near the equatorial plane between 25°N and 25°S latitudes in day time and between 2°N and 2°S in night time. The direction of their propagation has always been found to be towards the poles (Burton and Holzer, 1974).

The fact that most of the VLF emissions occur at half the equatorial gyrofrequency is very important and its explanation is expected to be very helpful in understanding the nature of whistler mode propagation in plasmas. Attempts have been made by several workers to explain the various properties of the VLF emissions but no satisfactory explanation has, as yet been given for their preferential triggering at half the equatorial gyrofrequency. To understand this phenomenon is one of the main goals of this work.

1.5 A REVIEW OF THE WORK DONE TOWARDS THE EXPLANATION OF VLF EMISSIONS

Several attempts have been made by various workers in the field to explain the generation mechanism of VLF emissions. Initially, it was thought that the Cerenkov and the cyclotron radiations from the fast moving charged particles are responsible for the observed VLF emissions. Cerenkov radiation is produced by a charged particle through its electromagnetic interaction with other charged particles of medium provided its velocity in that medium is greater than the phase velocity of light. The Cerenkov radiation propagates at an angle $\theta = \cos^{-1}(v_p/v)$ with the direction of motion of the particle. Here v_p is the phase velocity of the electromagnetic waves in the medium and v is the particle velocity.

The charged particles gyrating around the geomagnetic field lines give rise to another type of radiation which is known as cyclotron radiation. The effects of these have been discussed by

Gershman and Ugarov (1961) and Gershman and Trakhtengertz (1966).

However, the incoherent radiations generated by the Cerenkov and the cyclotron mechanisms are too weak to explain the observed intensities of VLF emissions.

Gallet and Helliwell (1959) assumed the existence of bunches of trapped electrons streaming along field lines. They considered the longitudinal resonance between a bunch of particles and the ambient whistler mode noise and showed that it will result into wave amplification over a narrow band of frequencies giving rise to discrete VLF emissions. However, no such electron bunches could be experimentally observed.

Dowden (1962a) also assumed the existence of similar particle bunches spiralling around geomagnetic field lines that give rise to Doppler shifted cyclotron radiation. The frequency of this radiation will change from point to point along a field line.

With his theory, Dowden could explain various types of frequency time structures of the VLF emissions. An electron bunch would produce a riser or a falling tone depending upon whether it is moving in the direction of increasing or decreasing field strength, that is, away from the equator or towards the equator. According to this theory, the emission of hooks is associated with the bunches of electrons crossing the equator.

Dowden (1962b) applied his theory to the phenomenon of periodic discrete emissions also. Since the interval between successive discrete emissions was comparable with the bounce periods of the resonant electrons, he attributed their origin to a bunch of electrons echoing between the mirror points. This was further supported by the fact that, quite often, a series of periodic hooks observed in one of the hemispheres was found to be shifted in time by about half a period with respect to the corresponding series observed at the conjugate point in the other hemisphere.

Dowden's theory was not capable of explaining long enduring quasi-constant tones. Furthermore, the observations of the periodic emissions showed that quite often they were closely associated with whistlers and their periodicity was equal to the whistler mode echoing period for the path. This indicated that the periodic emissions might be generated by the successive echoes of the whistler mode waves and not by the mirroring particle bunches.

Brice (1963) showed, that under the effect of a whistler mode wave which is right circularly polarized, the electrons have a tendency to organise such that their velocity transverse to the static magnetic field becomes antiparallel to the wave magnetic field.

If the phase distribution of the resonance electrons is initially random and their density distribution is uniform, their interaction with the waves would modify their phase distribution leaving the density distribution unaffected. Brice qualitatively showed that such a bunching in phase space would lead to coherent cyclotron radiation.

The directions of motion of the waves and the gyroresonant electrons are opposite to each other. Thus the radiated waves can organise the phases of the incomming particles which is similar to feed back. The whole mechanism may be considered to act as an oscillator.

Hansen (1963) also recognized the importance of the phase organisation of the particles by the waves. She considered a whistler going away from the equator and electrons spiraling in towards the equator, along the same field line. A cyclotron resonance would be possible between the waves and the particles if Doppler shifted wave frequency is equal to the gyrofrequency of the particles. She pointed out that in this case, the resonance condition shall remain satisfied for longer time if the rate of change of particle gyrofrequency is equal to that of the Doppler shifted wave frequency.

Helliwell (1967) constructed his theory for VLF emissions by developing and extending the ideas introduced by Brice (1963) and Hansen (1963) as mentioned above. He showed that the wave amplitude gets saturated at a certain value and any further increase in the input particle flux causes the interaction region to drift downstream. He further showed that the time rate of change of emission frequency depends upon the location of the interaction region and changes from one point to another on a field line.

He attributed the principal emission forms including the hooks and the inverted hooks to the drifting of the interaction region across the equator. The triggering delay and the offset frequency are also explained by the theory. However, the theory is phenomenological and does not provide any satisfactory explanation of the favourable triggering at half the equatorial gyrofrequency. While considering the change in $v_{\parallel}$ along a field line he considered only the change due to the nonuniformity of the field and did not include the effects of the resonance to the same.

The gyroresonant interaction gives rise to pitch angle diffusion of particles which, in turn, tends to isotropize the distribution of particles in pitch angle. Brice (1964) showed that any increase or decrease in particle pitch angle leads to the damping or the growth of the whistlers, respectively. Thus if initially there are more particles with larger pitch angles than with smaller pitch angles, the interaction will lead to the growth of the whistlers. This is equivalent to saying that whistlers would grow if the transverse temperature of a plasma exceeds its longitudinal temperature.

Dungey (1963) and Cornwall (1964) invoked the whistler induced pitch angle diffusion to explain the loss of energetic electrons from the radiation belts. The particles lying at the boundary of the loss cone would, through a small decrease in their pitch angle, be lost permanently to the ionosphere. This will lead to a unidirectional movement of the particles across the loss cone boundary. Furthermore, the particles would be constrained to move only along some diffusion curves in the velocity space. The nature of these diffusion curves is found to be such that a decrease in particle pitch angle would be associated with a decrease in the overall energy of the particles and with an increase in the wave energy in the system. This one way traffic of particles from higher pitch angles to lower pitch angles across the loss cone boundary will cause a growth in the whistler energy.

This reflects that the process of VLF emissions might comprise of the following two steps:

i) An increase in the transverse energy of the system through some unknown mechanism leaving the parallel energy unaffected.

ii) Amplification of whistler mode signals that converts part of the tranverse energy into longitudinal energy and thus lowers the particle mirror points down into the ionosphere where they are lost.

Vedenov et al. (1962) developed quasilinear theory to describe the evolution of a system containing electrostatic oscillations in a collisionless plasma. The essence of the theory lies in considering the effects of the variations (assumed to be small) of the amplitude of the oscillations and those in the plasma distribution function over each other. Any small change in one produces a corresponding change in the other and vice versa. Ultimately the system reaches an equilibrium state when the plasma distribution assumes the form of a ledge and can not be further distorted by the waves. The wave growth or damping rate approaches to zero in this situation.

Kennel and Petschek (1966) developed an analogus theory to explain the observed 40 keV stably trapped particle fluxes in the magnetosphere. They considered the existence of a loss cone in the particle pitch angle distribution which provided an amplification mechanism for waves existing in the system. The source of energy for this growth of waves is constituted by the energetic electrons trapped in the radiation belts of the earth. These electrons diffuse in pitch angle during their gyroresonance with the waves. This diffusion process drives the particles lying close to the loss cone boundary, into the loss cone.

The decrease in the particle pitch angles at the time of their entering into the loss cone is associated with a net decrease in particle energy and net gain in the wave energy. Thus waves cause pitch angle diffusion that amplifies the waves by extracting particle energy and driving them into the loss cone. These amplified waves cause more pitch angle diffusion, further amplification of waves and loss of more particles. However, the rate of amplification is controlled by the number of resonant particles and therefore, it continuously goes down as more and more particles are lost and ultimately reaches to zero.

Kennel and Petschek assumed that the waves get reflected at high latitudes and thus retrace their path again and again. They get amplified at each pass across the equator. They also assumed that the wave growth rate is a function of the resonant electron flux J, and for a critical value of it, the loss of wave energy due to absorption and imperfect reflection is just balanced by the wave growth. If J exceeds this value the waves would start growing and they would drive the excess particles into the loss cone and would thus bring down the flux level back to its critical value. This critical value corresponds to the level of stably trapped particle flux and was found to agree resonably with the observed limitation of 40 keV electron flux levels in the outer zone.

Liemohn (1967) also considered amplification of whistlers in presence of a loss cone during the gyroresonant wave particle interactions. The variation of the complex wave number $\vec{k}$ along the field line due to variations in the ambient magnetic field and the plasma parameters was taken into account and the power transfer function between the input and the output power spectra was determined by integrating the imaginary part of $\vec{k}$ along the path of wave propagation. For his calculations he considered the energy and the pitch angle distributions of the form E^{-n} $\mathrm{Sin}^{m}\,\alpha$ where E and α are particle energy and pitch angle and n and m are integers. He found that the amplification has a maximum followed by a sharp cut off, consistent with some observations.

Das (1968) took a magnetoplasma with a loss cone in the pitch angle distribution of particles and considered its gyroresonant interaction with a narrow band whistler mode wavepacket in presence of a background hiss.

The resonance takes place over a finite range of parallel velocities and results into a diffusion of particles into the loss cone. He then assumes a model for the particle distribution function smeared out after the passage of the wave packet. Using this distribution he calculated the growth rates at different frequencies and found that the amplification at the edges of the original pulse would be substantially large compared to that of the background noise.

Sudan and Ott (1971) showed that a finite length whistler train causes the gyroresonant electrons to become phase correlated with the wave magnetic field over a time of the order of the period of oscillation of a particle in the effective potential well of the wave. The wave acceleration due to the inhomogenity of the magnetic field is considered small so that the particles remain trapped in the potential well. These phase correlated electrons were shown to give rise to an instability in the form of an emitted whistler.

In a nonuniform magnetic field, both the wave vector $\vec{k}$ and the particle speed $v_{\parallel}$ vary with the position of a particle along a field line. Therefore, the condition $\omega - k_{\parallel} v_{\parallel} = \Omega$ gets violeted as the particle moves away from the resonance point thereby making the resonance ineffective after a shortwhile. Dungey (1969), however, pointed out that if the rate of change of the Doppler shifted frequency is equal to the rate of change of electron gyrofrequency along a field line, the resonance will remain effective for a comparatively longer period. He called the phenomenon as the second order resonance.

Nunn (1971) gave importance to the idea of second order resonance. He considered a whistler wave train moving in an inhomogeneous plasma along the static magnetic field direction and found that the effects of the second order resonant particles dominate over those of the resonant particle distribution function, after a time of the order of one or two trapping periods. He calculated the currents produced by these particles and showed that they were large and were capable of giving growth rates large compared to the corresponding linear growth rates. He also showed that the reactive component of the current causes a change in the wave frequency with time. However, a more detailed calculation is still needed in this aspect.

Ashour Abdalla (1972) studied the effect of a succession of narrow band whistler mode pulses on a steady state distribution function of electrons trapped in the radiation belts of the earth. She computed pitch angle and energy diffusion co-efficients for the particles and showed that the particles would be constrained to move on given surfaces in the velocity space. By calculating the diffusion co-efficients along the diffusion curves, she reduced the complexity of the problem and could use one dimensional form of the Fokker Plank equation to study the time evolution of the distribution function. She started with a Kennel and Petschek type of distribution and found that it gradually developed a slot at a velocity corresponding to the frequency of the pulses. She computed growth rates for this modified (or, primed) distribution and found that the growth rate is very large in the slot region. Her mechanism is capable of explaining some of the attributes of the Artificially Stimulated Emissions like the off set frequency and the frequent triggerings of emissions from the equatorial region of the magnetosphere.

1.4. A SHORT DESCRIPTION OF THE PRESENT WORK

The present work consists of a study of the wave particle interactions in collisionless plasmas and makes an attempt towards understanding the factors that are likely to contribute to the favourable triggering of the VLF emissions at half the equatorial gyrofrequency (written as $\frac{1}{2}\Omega_{eq}$ for short) along the field line of their propagation.

The effects of both the Landau and the gyro-resonances of the whistler mode pulses with electrons in a homogeneous collisionless plasma immersed in a magnetic field B_0 have been studied in detail. In chapter II the changes in $v_{\parallel}$ (velocity component along B_0) of the particles due to Landau resonance are computed and are found to exhibit a quasiperiodic structure when plotted against corresponding $v_{\parallel}$.

The consequent evolution of the distribution function f is calculated and, as expected, is found to develop a fine structure close to the central resonant velocity for the pulses. The positive gradients thus exhibited by $\partial f / \partial v_{\parallel}$ at different points of the fine structure would make the plasma unstable for waves at the corresponding frequencies. The perturbations at suitable frequencies will then start growing in such a system and will appear as emissions when the growth becomes substantial.

In chapter III, we discuss the effects of gyroresonance in a plasma with a loss cone.

A whistler mode pulse propagating in a plasma containing a loss cone in its velocity distribution diffuses the gyroresonant particles such that the distribution gets significantly modified in a region which is close to both the loss cone boundary and the resonant $\vartheta_{\parallel}$. The modified distribution amplifies whistler mode noise in narrow spectral bands on either side of the central resonant $\vartheta_{\parallel}$.

This earlier theory by Das has been extended to study the effect of the large amplitude pulses which were not included in the previous theory. It is found that the peaks in the growth rate are greatly enhanced and are capable of giving enough amplificat-ion for VLF emissions to be seen and also explain some of the duplicate traces in the whistler mode observed at a low latitude station. A very wide band wave packet with large amplitude has also been considered and is found to give several peaks in amplification on both sides of the central frequency thereby explaining some of the observed multiple emissions.

The wave particle interactions mentioned so far do not lead to amplification at any preferred frequency. However, the observations show a definite favoured triggering at $\frac{1}{2} \Omega_{eq}$. An attempt has been made to investigate the effects of the following properties shown by the whistler mode dispersion relation at this frequency: (i) the equality of the particle speeds necessary for the Landau and the gyroresonances, (ii) the equality of the group and the phase velocities, and (iii) the occurance of a maximum in the Landau resonant velocity. The results obtained are quite encouraging towards the explanation of the emissions at half the equatorial electron gyrofrequency.

Chapter IV aims at discussing the effects of the equality of the group velocity and the phase velocity at $\omega \approx \frac{1}{2} \Omega$ towards the favourable emissions at half the equatorial gyrofrequency. The theory of the development of a slot in the velocity distribution of the particles during the gyro resonance of the whistler mode pulses with the magnetospheric plasma has been considered for its extension to the case of the Landau resonant interaction. Then Dungey's suggestion for the combined effect of the two resonances leading to the favourable emissions at $\frac{1}{2}\Omega_{eq}$ has been critically examined and it has been shown how the same mechanism turns out to be quite powerful due to the effects of the equality of the phase velocity and the group velocity at the frequency $\frac{1}{2}\Omega_{eq}$ for whistlers.

The simultaneous propagation of the electrostatic and whistler mode waves in opposite directions in a homogeneous collisionless plasma in a uniform magnetic field has been considered in chapter V and it has been shown that the resulting Landau and the gyro resonances affect the progress of each other. The Landau resonance distorts the distribution in such a way that the VLF perturbations at suitable frequencies start growing.

The Landau resonance of the off angle whistler mode noise has been shown to divide the whole velocity distribution into two regions: one disturbed and the other undisturbed. This happens because the Landau resonant velocity exhibits a maximum at

$\omega = \frac{1}{2} \Omega \cos \theta$ (ω being the wave frequency). The resulting sharp change in the distribution at the corresponding $V_{\parallel}$ has been shown to give amplification to VLF perturbations at $\omega = \frac{1}{2} \Omega_{2q_{\parallel}}$.

Towards the end of the thesis in the last chapter the limitations of the theory and the scope for further work have also been discussed.

CHAPTER II

THE LANDAU RESONANT INTERACTION OF OFF ANGLE WHISTLER MODE PULSES

2.1. INTRODUCTION

The Artificial stimulation of Very Low Frequency Emissions through Morse Code pulses suggests that their origin is associated with the interaction of wave packets with the particles. The study of the wave particle interactions is, therefore, very important and is expected to provide clues as to the actual triggering mechanism of the VLF emissions.

The most important wave particle interactions are resonant interactions as they lead to the most efficient exchange of energy between the waves and the particles. The general resonance condition between obliquely propagating whistler mode waves and electrons is given by $\omega - k_{\parallel} V_{\parallel} = n\Omega$ where n is either zero or a positive integer and other symbols have meanings as defined earlier in Chapter I.

The resonance characterised by $n = 1$ is known as gyroresonance and occurs when the rate of change of the phase difference between the rotating velocity vector of the electron and the rotating field vector of the wave becomes zero. Another resonance characterised by $n = 0$ is known as Landau resonance and occurs when the rate of change of the phase of the longitudinal component of the wave electric field as seen by the particle becomes zero.

Brinca (1972) considered non linear Landau wave particle interaction between electrostatic waves and a collisionless plasma and showed that side band growth would occur in this case. Since the obliquely propagating whistlers also have a component of their electric field along the geomagnetic field direction, he suggested that similar effects will be produced by them and the consequent side band growth is likely to result into the generation of VLF emissions.

Here, in this chapter, we shall concentrate our attention on the effects of the Landau resonant interaction of an obliquely propagating whistler mode wave packet with a collisionless plasma.

2.2 MATHEMATICAL FORMULATION

We consider a fully ionised collisionless plasma immersed in a uniform magnetic field B_0 such that the electron plasma frequency ω_p is much greater than the electron gyrofrequency Ω. A whistler mode wave packet containing a narrow range of frequencies denoted by ω and wave vector $\vec{k}$ is assumed to propagate in x-z plane at an angle θ with the ambient magnetic field B_0 acting along z direction (See fig.2.1). The shape of the wavepacket is assumed to be Gaussian.

The equation of motion of a charged particle in such a system can be written as

$$\frac{d\vec{V}}{dt} = \frac{e}{m}\left\{\vec{E} + \frac{\vec{V}\times(\vec{B}_0+\vec{b})}{c}\right\} \quad ---(2.1)$$

where $\vec{V}$ is the particle velocity, $\vec{E}$ and $\vec{b}$ are the wave electric and magnetic fields and e,m and c are the electron charge, electron mass and velocity of light in free space, respectively.

Here we have ignored all the forces on electrons except those of the electromagnetic origin.

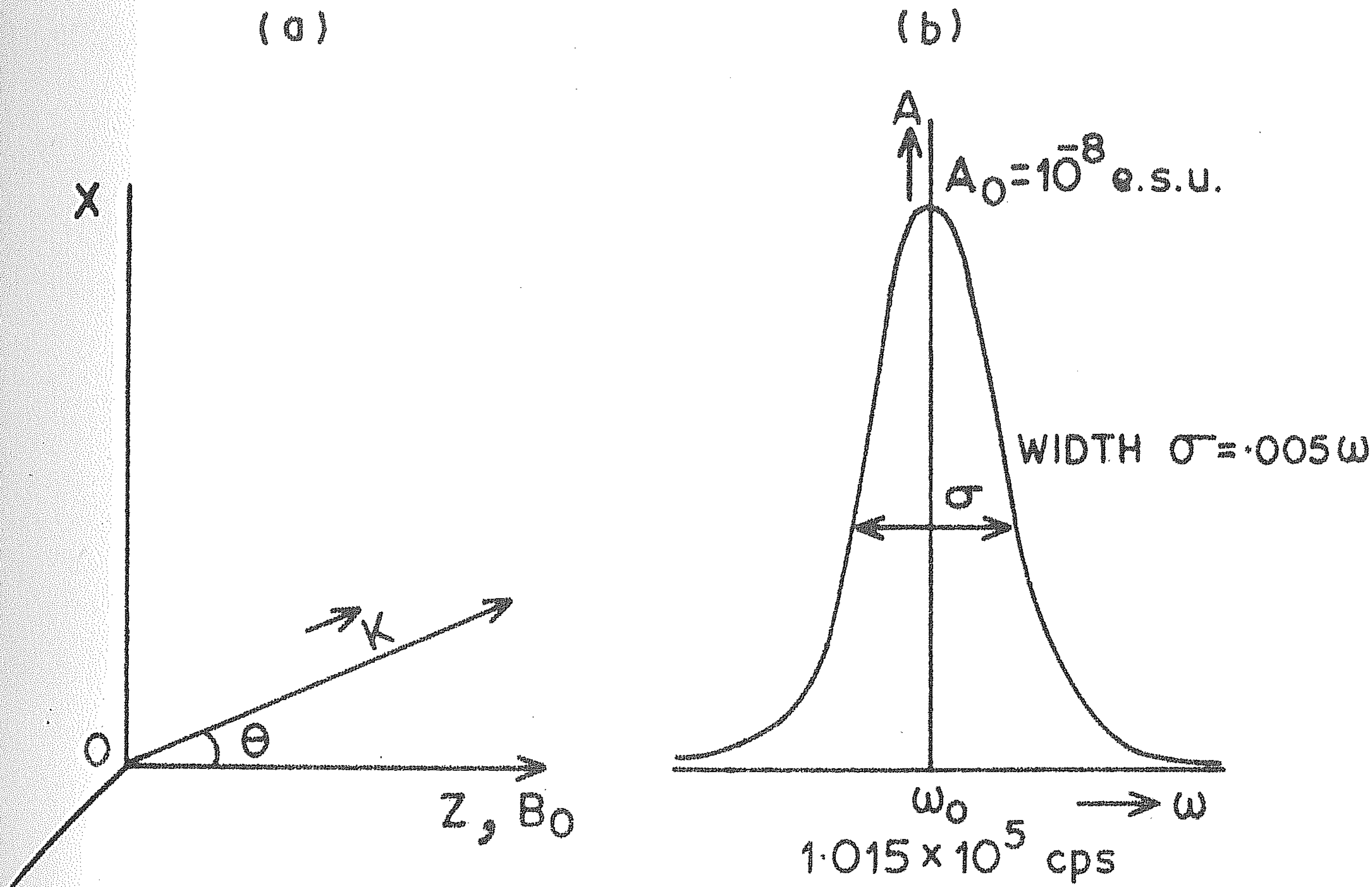

Fig. 2·1

(A) THE PULSE MOVES ALONG $\vec{k}$ MAKING AN ANGLE θ WITH $\vec{B}_0$.

(B) A DESCRIPTIVE SKETCH OF THE PULSE. A : AMPLITUDE, σ : BANDWIDTH, ω_0 : CENTRAL FREQUENCY.

Using Maxwell's equations, equation (2.1) can be reduced to

$$\frac{dv_{\parallel}}{dt} = \frac{e}{m}\left[|E_z| \cos\psi + \frac{k_{\parallel} v_{\perp}}{\omega} \left\{ |E_x| \cos\psi \cos\phi + |E_y| \sin\psi \sin\phi \right\} - \frac{k_{\perp} v_{\perp}}{\omega} |E_z| \cos\psi \cos\phi \right] \quad \text{----} (2.2a)$$

and

$$\frac{dv_{\perp}}{dt} = \frac{e}{m}\left[\left(1 - \frac{k_{\parallel} v_{\parallel}}{\omega}\right)\left(|E_x| \cos\psi \cos\phi + |E_y| \sin\psi \sin\phi\right) + \frac{k_{\perp} v_{\parallel}}{\omega} |E_z| \cos\phi \cos\psi \right] \quad \text{----} (2.2b)$$

where the subscripts ' $\parallel$ ' and ' $\perp$ ' refer to vector components parallel and perpendicular to the ambient magnetic field direction and $\psi \ (= \vec{k} \cdot \vec{r} - \omega t + \psi_0)$ and $\phi (= \Omega t + \phi_0)$ are the wave phase and the particle gyration phase respectively. $|E_x|$, $|E_y|$ and $|E_z|$ represent the components of the amplitude of the wave electric field in x,y and z directions.

We are interested in the consequences of equations (2.2a) and (2.2b) over periods much greater than the gyration period of an electron. For integrating these equations, only variations in ψ and ϕ need be considered as the effects of variations in other quantities would be relatively small. We also impose the condition of Landau resonance $\omega - k_{\parallel} v_{\parallel} = 0$ during integration. We find that under these circumstances $\int \cos \psi \cos \phi \, dt$ goes to zero over an integral number of gyroperiods under the linear assumption and therefore eqn. (2.2b), on integration, gives:

$$\delta v_{\perp} = 0 \qquad \text{---(2.3b)}$$

The integration of equation (2.2a) is rather involved and we drop the terms containing the integral of $\cos \psi \cos \phi$ as their contribution over the time period of interest would be relatively small.

Thus equation (2.2a) may be written as

$$\frac{dv_{\parallel}}{dt} = \frac{e}{m}\left[|E_z|\cos\Psi + \frac{k_{\parallel}v_{\perp}}{\omega}|E_y|\sin\Psi\sin\phi\right]$$

---(2.3)

With the Landau resonance condition one knows that

$$\int_0^{\frac{2\pi n}{\Omega}} \cos\Psi\, dt = \cos\Psi_0 \; \frac{2\pi n}{\Omega} \; J_0\left(\frac{k_\perp v_\perp}{\Omega}\right),$$

and

$$\int_0^{\frac{2\pi n}{\Omega}} \sin\Psi \sin\phi\, dt = \cos\Psi_0 \, \frac{2\pi n}{\Omega} \, J_1\left(\frac{k_\perp v_\perp}{\Omega}\right)$$

under linear approximation.

Since a nonlinear solution of eqn. (2.3) is still complicated, the relative importance of the two terms on R.H.S can be estimated by substituting the results of integration under linear approximation.

We find that the second term on the R.H.S.of (2.3) can be neglected in comparison of its first term if

$$|E_z|\, J_0\left(\frac{k_\perp v_\perp}{\Omega}\right) >> |E_y|\, \frac{v_\perp}{v_\parallel}\, J_1\left(\frac{k_\perp v_\perp}{\Omega}\right)$$

If $|E_z|$ and $|E_y|$ are comparable, and both $\frac{v_\perp}{\Omega}$ and $\frac{v_\perp}{v_\parallel}$ are small, this condition is satisfied and we are left with

$$\frac{dv_\parallel}{dt} = \frac{e|E_z|}{m}\cos\Psi = \frac{e|E_z|}{m}\cos(\vec{k}.\vec{r} - \omega t)$$

This may also be written as

$$\frac{dv_\parallel}{dt} = \frac{e|E_z|}{m}\cos\left[(k_\parallel v_\parallel - \omega)t + \Psi_0 + \frac{k_\perp v_\perp}{\Omega}\sin \Omega t\right]$$

or

$$\frac{dv_{\parallel}}{dt} = \frac{e|E_z|}{m}\Big[\cos\left\{(k_{\parallel}v_{\parallel}-\omega)t+\psi_0\right\}\cos\left\{\frac{k_{\perp}v_{\perp}}{\Omega}\sin\Omega t\right\}$$
$$-\sin\left\{(k_{\parallel}v_{\parallel}-\omega)t+\psi_0\right\}\sin\left\{\frac{k_{\perp}v_{\perp}}{\Omega}\sin\Omega t\right\}\Big]$$

Near the Landau resonance point, the variation in $(k_{\parallel}v_{\parallel}-\omega)t$ is very slow and that in $\frac{k_{\perp}v_{\perp}}{\Omega}\sin\Omega t$ is very fast. Therefore, for a time period large compared to a gyroperiod, we can substitute the average values of $\cos\left(\frac{k_{\perp}v_{\perp}}{\Omega}\sin\Omega t\right)$ and $\sin\left(\frac{k_{\perp}v_{\perp}}{\Omega}\sin\Omega t\right)$ (over an integral number of gyroperiods) which are $J_0\left(\frac{k_{\perp}v_{\perp}}{\Omega}\right)$ and zero respectively. Thus the above equation will be reduced to

$$\frac{dv_{\parallel}}{dt} = \frac{e|E_z|}{m}J_0\left(\frac{k_{\perp}v_{\perp}}{\Omega}\right)\cos\left\{(k_{\parallel}v_{\parallel}-\omega)t+\psi_0\right\}$$

If $\frac{k_{\perp}v_{\perp}}{\Omega}$ is very small compared to unity this equation can be further approximated by

$$\frac{dv_{\parallel}}{dt} = \frac{e|E_z|}{m} \cos\left\{(k_{\parallel}v_{\parallel} - \omega)t + \psi_0\right\} \qquad ---(2.3a)$$

We make use of this equation for computing the changes in the $v_{\parallel}$ of the particles caused by the Landau resonant interaction.

2.3 COMPUTATION OF THE CHANGES IN PARTICLE VELOCITIES

To determine the effect of waves on particles we must find how the distribution function of the particles changes during the resonance. Since the Landau resonance does not significantly affect the $v_{\perp}$ of the particles, we do not expect the particle distribution with respect to $v_{\perp}$ to change with time.

However, $v_{\parallel}$ of the particles changes with time and over long periods, the linear approximation is not valid. Therefore, a numerical solution of equatiion (2.3a) was attempted on computer to see how the particle velocities change with time.

A wave packet with central frequency $\omega_0 = 1.015 \times 10^5$ radians per second was considered. Its band width was taken to be 0.5% of its central frequency and its shape was assumed to be Gaussian.

The electric field E_z corresponding to the central frequency ω_0 in the wave packet was taken to be 10^{-8} e.s.u..

The velocity distribution of particles was assumed to consist of two components:

i) a back ground of low energy particles such that the thermal velocities of the particles are much smaller compared to the phase velocity of the whistler mode waves, and

ii) a thin distribution of high energy particles with their $v_{\parallel}$'s comparable to the phase velocity of the whistler mode waves.

The propagation characteristics of the wave packet would be determined by the back ground component which can be considered as a cold plasma. This makes it possible for us to use the well known cold plasma dispersion relation for whistler mode:

$$\frac{c^2 k^2}{\omega^2} = \frac{\omega_p^2}{\omega(\Omega \cos\theta - \omega)}$$

to compute k for a given value of ω .

The second component contains particles that undergo resonance and are therefore affected most. The distribution of these particles will change with time. To study the time evolution of the distribution function, we must first integrate equation (2.3a) to find how the velocities change with time and then develop a method to compute the effects of these velocity changes on the particle distribution function. We restrict our attention to a narrow range of particle velocities lying in the close proximity of the central resonant $v_{\parallel}$. All the quantities in equation (2.3a) were then normalised by dividing them with suitable constants of the system having dimensions same as the quantities in question. The relations connecting the normalised and the unnormalised quantities are listed below.

$$t' = t/(1/\Omega)$$

$$\omega' = \omega/\Omega$$

$$k' = k/k(\omega_0)$$

$$v' = v/\{2(\omega_0/k(\omega_0))\}$$

$$\left(\frac{eE}{m}\right)' = \left(\frac{eE}{m}\right)/\left\{\frac{\Omega^2}{k(\omega_0)}\right\}$$

Here ω_0 represents the central frequency of the wavepacket and the prime indicates that the concerned quantities are normalised. In the discussion here after, through out this chapter, we would drop the prime for the purpose of clarity of notation as we will be dealing only with the normalised quantities now onwards.

The Runge Kutta method was employed for the integration of equation (2.3a). The time increment Δt between consecutive steps was chosen to be one gyro period and the integration was carried out for a total period of about 1500 gyroperiods. The normalised value of central resonant $\vartheta_{\parallel}$ was 0.5 and the initial values for $\vartheta_{\parallel}$ were given in the range $\vartheta_{\parallel}$ = 0.496 to 0.504 with a uniform spacing of 0.0001. The range of $\vartheta_{\parallel}$ considered included both the trapped and the nearly trapped particles.

If the R.H.S of equation (2.3a) is represented by F($\vartheta_{\parallel}, t$) and if the value of F($\vartheta_{\parallel}, t$) at the end of n'th time interval is known, then $\Delta \vartheta_{\parallel}$ (the change in $\vartheta_{\parallel}$ during the next time interval) can be calculated from the equation given below:

$$\Delta \vartheta_{\parallel} = \frac{1}{6} \left\{ k_1 + 2k_2 + 2k_3 + k_4 \right\}$$

where

$$k_1 = F(t_n, \vartheta_{\parallel n}) \Delta t ,$$

$$k_2 = F\left(t_n + \frac{\Delta t}{2}, \vartheta_{\parallel n} + \frac{k_1}{2}\right) \Delta t,$$

$$k_3 = F\left(t_n + \frac{\Delta t}{2}, v_{\parallel n} + \frac{k_2}{2}\right)\Delta t,$$

and $$k_4 = F\left(t_n + \Delta t, v_{\parallel n} + k_3\right)\Delta t$$

Here, the subscript 'n' indicates the value of the concerned quantity after n'th time interval.

The errors involved in Runge Kutta method are extremely small as they are proportional to the fifth power of the time interval Δt . Taking Δt to be of the order of 3×10^{-5} sec. the error would be approximately of the order of 10^{-23}.

The results of the integration were used to find $\delta v_{\parallel}$ (the total change in $v_{\parallel}$ over the period of integration) which was later plotted against initial $v_{\parallel}$. One such typical plot is shown in fig. (2.2) which corresponds to a time period of about 1200 gyro periods.

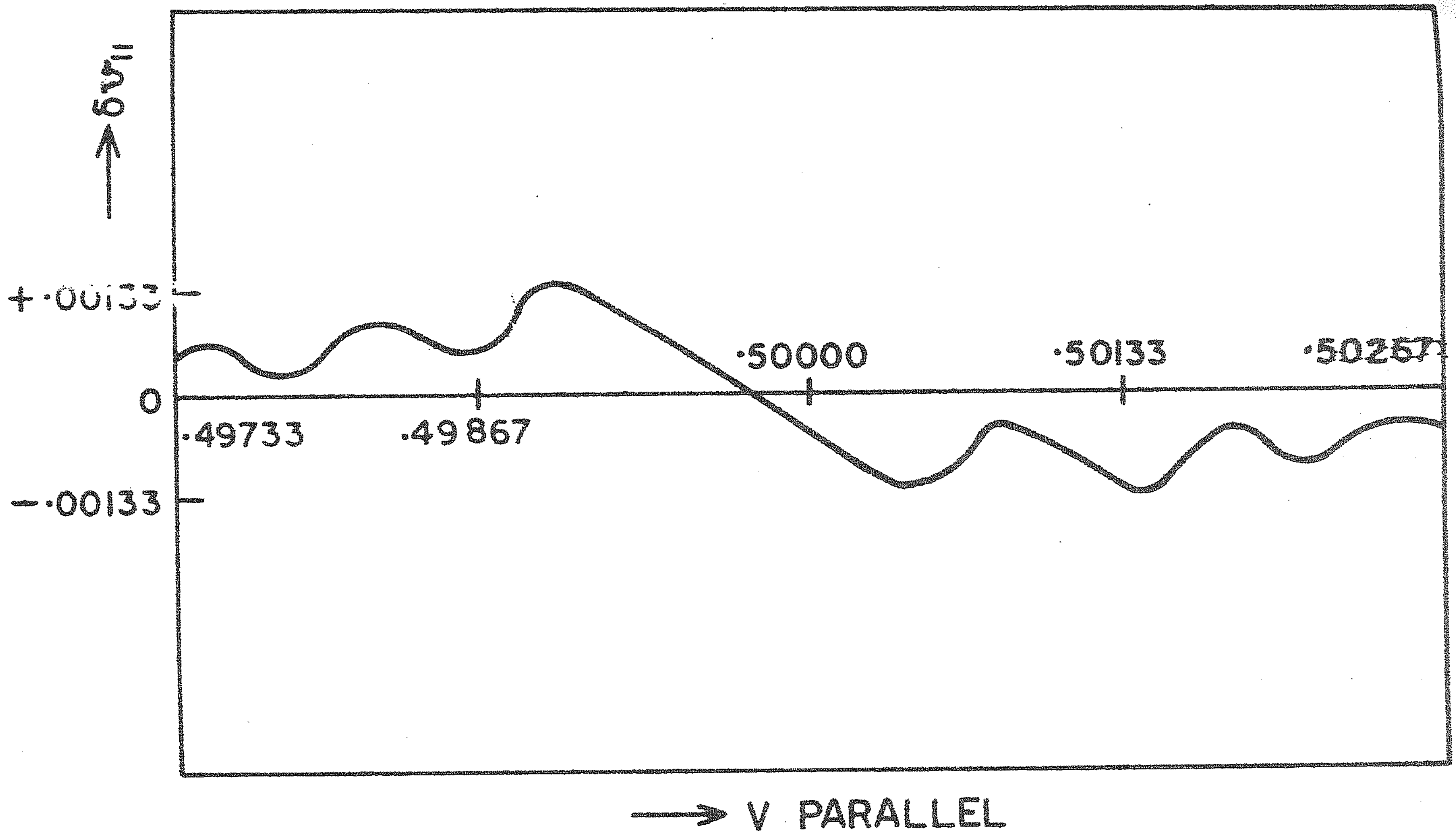

Fig. 2·2

CHANGE IN 'V' PARALLEL, '$\delta V_{\parallel}$', AGAINST '$V_{\parallel}$'
(TIME OF INTERACTION WITH THE WAVE PACKET : 1200 GP)

The figure shows that there is a point on $\vartheta_{\parallel}$ axis slightly below the central resonant ϑ_{H} where the $\delta\vartheta_{\parallel}$ versus $\vartheta_{\parallel}$ curve crosses the $\vartheta_{\parallel}$ axis. This point, which corresponds to a velocity, say, ϑ_{o} , divides the whole axis into two parts. The particles with their $\vartheta_{\parallel}$ less than ϑ_{o} will be accelerated by the waves and will therefore extract energy from them. The particles with their $\vartheta_{\parallel}$ greater than ϑ_{o} are decelerated and thereby they lose their energy to the waves. The plot of $\delta\vartheta_{\parallel}$ versus $\vartheta_{\parallel}$ also shows a very interesting feature as it exhibits fluctuations in $\delta\vartheta_{\parallel}$ on either side of ϑ_{o} . These fluctuations are more or less equally spaced along the $\vartheta_{\parallel}$ axis and their amplitude decreases as we go away from $\vartheta_{\parallel} = \vartheta_{o}$. By looking at the shape of the fluctuations, one can qualitatively infer that the distribution of the particles should develop some sort of bunching or fine structure in the resonant range of the velocity space.

2.4 CALCULATION OF THE NEW DISTRIBUTION FUNCTION

We presume that during the interaction, the particles affected by the waves lie in a narrow range R of $v_{\parallel}$ close to the central resonant parallel velocity. The particles outside this range are considered to be unaffected by the waves and therefore their distribution function should remain unchanged. This leads to a further conclusion that, although the particles are redistributed in the region R, the total number of particles in that region should also remain unchanged. This amounts to saying that there is no diffusion of particles across the boundary of the resonant and the non-resonant regions.

Let the initial and the final velocities of a particle, parallel to B_0 be $v_{\parallel 1}$ and $v'_{\parallel 1}$ and those of another be $v_{\parallel 2}$ and $v'_{\parallel 2}$. Through out this treatment the prime refers to the final value of a quantity.

If $|v_{\parallel 1} - v_{\parallel 2}|$ is very very small compared to the characteristic range of $v_{\parallel}$ over which the initial distribution function f_0 changes by a factor of e , the number of particles having $v_{||}$ between $v_{||1}$ and $v_{||2}$ can be considered proportional to

$$\left| v_{||2} - v_{||1} \right| f \left\{ \frac{v_{||1} + v_{||2}}{2} \right\}$$

Similarly, if $\left| v_{||2}' - v_{||1}' \right|$ is very very small compared to the average range of $v_{||}$, in the $\delta v_{||}$ versus $v_{||}$ graph (shown in fig. 2.2), over which $\delta v_{||}$ keeps on fluctuating, the number of redistributed particles having velocities between $v_{||1}'$ and $v_{||2}'$ can be taken proportional to

$$\left| v_{||2}' - v_{||1}' \right| f' \left\{ \frac{v_{||1}' + v_{||2}'}{2} \right\}$$

From the shape of the $\delta v_{||}$ versus $v_{||}$ curve shown in fig. 2.2, it can be inferred that the number density of particles redistributed along the $v_{||}$ - axis will exhibit alternate compressions and rarefactions in the resonant region.

This shows that if we consider ranges of velocities on $\delta v_{\parallel} - v_{\parallel}$ graph that are very narrow compared to the periodicity of the fluctuations themselves, then as a result of the interaction, they will either contract or expand as illustrated in ~~the~~ figure (2.3).

In this situation, the condition for the conservation of the number of particles in the resonant region combined with the fact that what ever particles existed initially in the range $v_{\parallel 1} - v_{\parallel 2}$ occupy the range $v'_{\parallel 1} - v'_{\parallel 2}$ after the redistribution, gives the following relationship:

$$|v'_{\parallel 2} - v'_{\parallel 1}| \, f' \left\{ \frac{v'_{\parallel 2} + v'_{\parallel 1}}{2} \right\} = |v_{\parallel 2} - v_{\parallel 1}| \, f\left(\frac{v_{\parallel 2} + v_{\parallel 1}}{2} \right)$$

which directly gives

$$f'\left(\frac{v'_{\parallel 2} + v'_{\parallel 1}}{2} \right) = \frac{|v_{\parallel 2} - v_{\parallel 1}|}{|v'_{\parallel 2} - v'_{\parallel 1}|} \, f\left(\frac{v_{\parallel 2} + v_{\parallel 1}}{2} \right) \quad (2.4)$$

from which the final distribution f' can be easily calculated if the initial distribution function f is known.

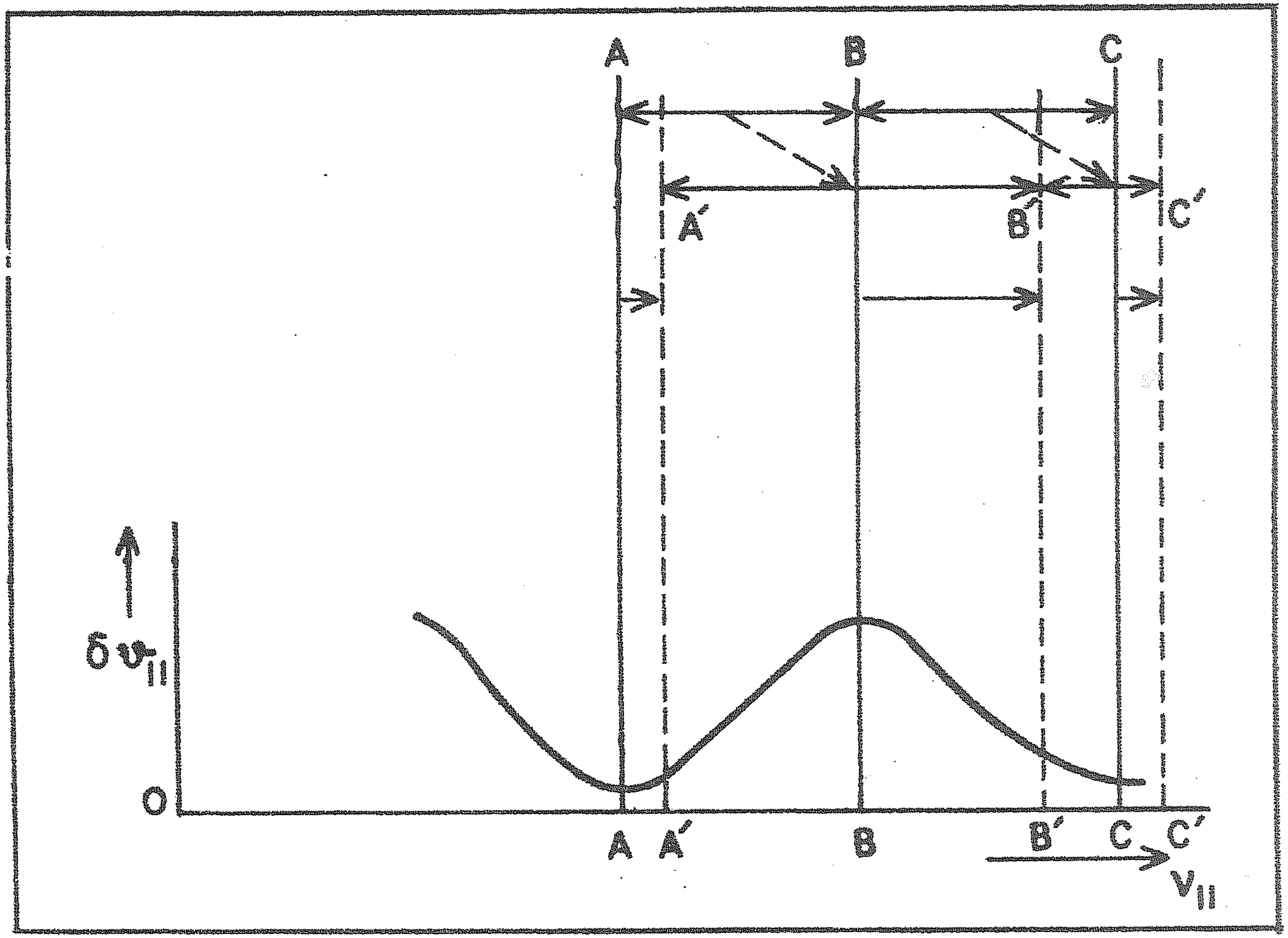

Fig. 2·3

ILLUSTRATION OF THE EXPANSION OF THE RANGE AB INTO A' B' AND THE CONTRACTION OF THE RANGE BC INTO B'C' AS A RESULT OF THE UNEVEN VARIATION IN $V_{\parallel}$ AT A, B AND C

The initial distribution function was taken to be of the following form:

$$f = f_o \, v^{-4} \ln \left(\sin \alpha / \sin \alpha_o \right) \quad \text{---} \quad (2.5)$$

where f_o is a constant and $\alpha = \tan^{-1} (v_\perp / v_\parallel)$ is the particle pitch angle. α_o is the lower limit on the pitch angle of the particles existing in the system.

The numerical values of the different quantities used during computations are as follows:

The electron gyro frequency	Ω	$= 2.03 \times 10^5$ rad/sec.
The electron plasma frequency	ω_p	$= 11.115 \times 10^5$ rad/sec
Propagation angle	θ	$= 30^o$
Pitch angle	α	$= 10^o$
Lowest pitch angle existing in the system (i.e. the loss cone angle)	α_o	$= 6^o$

Figure (2.4) shows the results of the computations for the new distribution function after an interaction lasting about 1200 gyroperiods. The changes in $\vartheta_{\parallel}$ used during the calculations for this new distribution are same as shown in fig. (2.2).

The initial distribution of the particles over the range of resonant velocities has been shown by the broken curve in fig.2.4. It looks like a horizontal straight line because the range of the resonant velocities is so narrow that the change in f over this range is too small to show up itself on the limited scale of the graph. The continuous curve shows how the particles get redistributed and develop a fine structure during the interaction.

This fine structure contains alternate positive and negative gradients of f with $\vartheta_{\parallel}$. These were also computed and are shown in figure (2.5). Positive gradient of f with $\vartheta_{\parallel}$ lead to wave growth and since the graph of $\partial f / \partial \vartheta_{\parallel}$ versus $\vartheta_{\parallel}$ shows many pronounced peaks on either side of the central resonant velocity, the growth of waves at corresponding

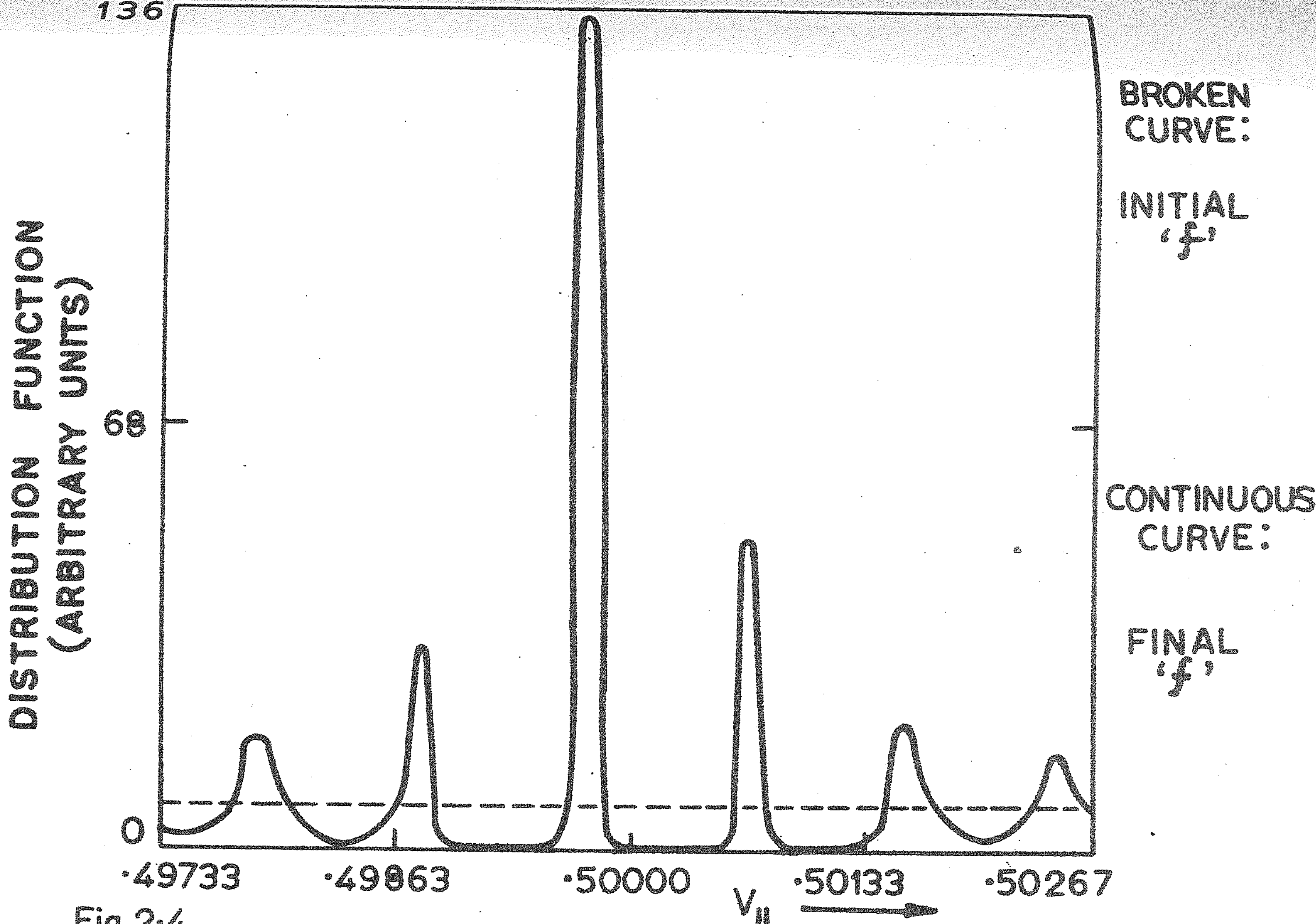

Fig.2·4

INITIAL AND FINAL DISTRIBUTION FUNCTION 'f' AGAINST '$V_{\parallel}$'

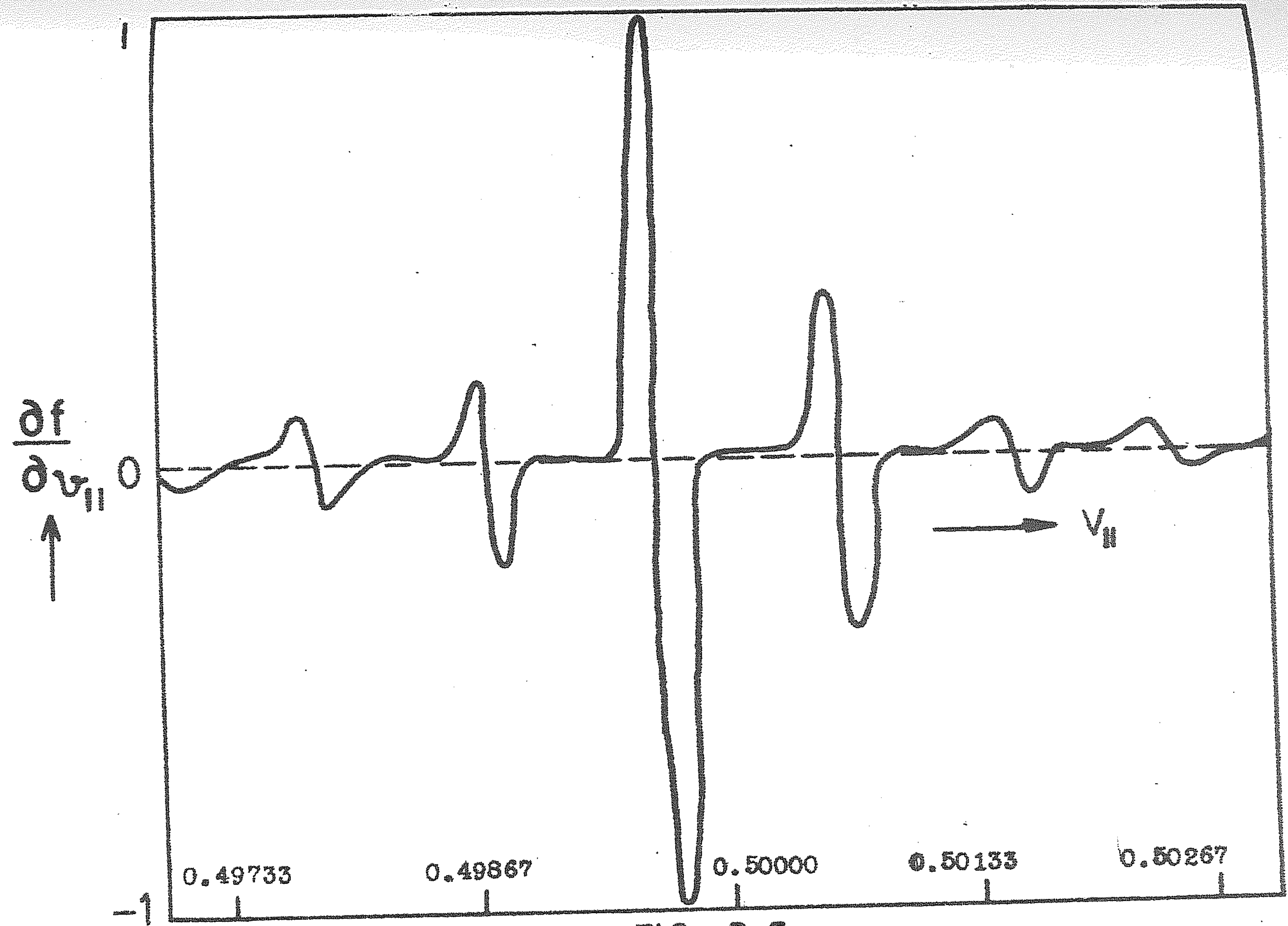

FIG 2.5

A PLOT OF $\partial f/\partial V_\parallel$ AGAINST $V_\parallel$ (UNITS ARBITRARY). BROKEN CURVE SHOWS INITIAL $\partial f/\partial V_\parallel$ WHICH IS SMALL AND DOES NOT SHOW UP. CONTINUOUS CURVE SHOWS $\partial f/\partial V_\parallel$ AFTER 1200 GP frequencies is expected if suitable perturbations exist in the system. It can be seen that there is no growth of waves corresponding to the central resonant velocity.

2.5 APPLICATIONS TO VLF EMISSIONS:

The system that we have studied so far closely resembles the equatorial magnetosphere where the VLF emissions are believed to be generated. The earth's magnetic field is approximately dipole in character and is roughly given by

$$B = (B_o/L^3)(1+3\sin^2\lambda)^{1/2}(\cos\lambda)^{-6} \quad (2.6a)$$

where B is the field strength along field line at a geomagnetic latitude λ , B_0 is the equatorial field strength at the surface of the earth and L is the distance of the equatorial point of a field line from the centre of the earth measured by taking the radius of the earth as the unit of length.

In the vicinity of the equator, the equation (2.6b) can be closely approximated by

$$B = B_0 \left(1 + \frac{9}{2} \frac{z^2}{R_E^2 L^2} \right) \qquad \text{--- (2.6b)}$$

Here R_E is the radius of the earth and z is the distance from the equator, of a point at which the field strength is B.

The most significant changes in the velocities of the particles take place in about a thousand gyroperiods' time as shown by the calculations mentioned in the last section. Considering the particle velocities to be roughly one tenth of the velocity of light and taking $L = 3$ at which a gyroperiod is approximately 3×10^{-5} sec., the distance travelled by the particles in 1000 gyro periods would be of the order of 1000 kilometers. The variation in the field strength over this distance is to the tune of 1% only and therefore the field may be considered to be fairly uniform over the duration and the spatial length of the effective resonance.

The plasma in the magnetosphere is fully ionised and at L = 3, and has a density merely of the order of 400 particles/c.c. It can be considered as collisionless because the collision frequencies are very small compared to the plasma frequency which is of the order of 10^6 radians/sec.

The plasma in radiation belts has two components very similar to those we have considered for our calculations: One is the low temperature back ground plasma and the other consists of magnetically trapped high energy particles. The distribution of the high energy particles gets modified by interaction with the waves. A loss cone has also been included in the previous calculations to make the resemblance complete.

We should consider the effects of the resonant interaction of the pulse with the particles all along the line of propagation. However, the resonances that take place away from the equator do not contribute much to the changes in $v_{\parallel}$ owing to their effectively shorter durations compared to those of the near equator resonances. This difference is caused by the nonuniformity of the static magneticfield in the regions away from the equator where k and $v_{\parallel}$ change relatively faster with z, being functions of B, the magnetic field. (see fig.2.6).

Now if the distribution function is considered to be already modified by an initial off angle VLF whistler mode pulse, it is easy to see that another pulse moving along a field line in the same mode will not be appreciably affected unless it reaches near the equator where the strong resonant effects come into picture and lead to a damping of the central frequencies and to the growth of its side bands. This modified pulse, when reaches again to the regions of highly nonuniform magnetic field on the other side of the equator, does not suffer any further significant changes in its spectrum and reaches the ground observation station in more or less the same shape.

A mechanism, like the one discussed above, leading to the growth of the side bands of a wave packet (Brinca 1972, Vyas 1974) may or may not directly trigger VLF emissions but the underlying physical process is very interesting and important and suggests that the investigation of similar processes might lead

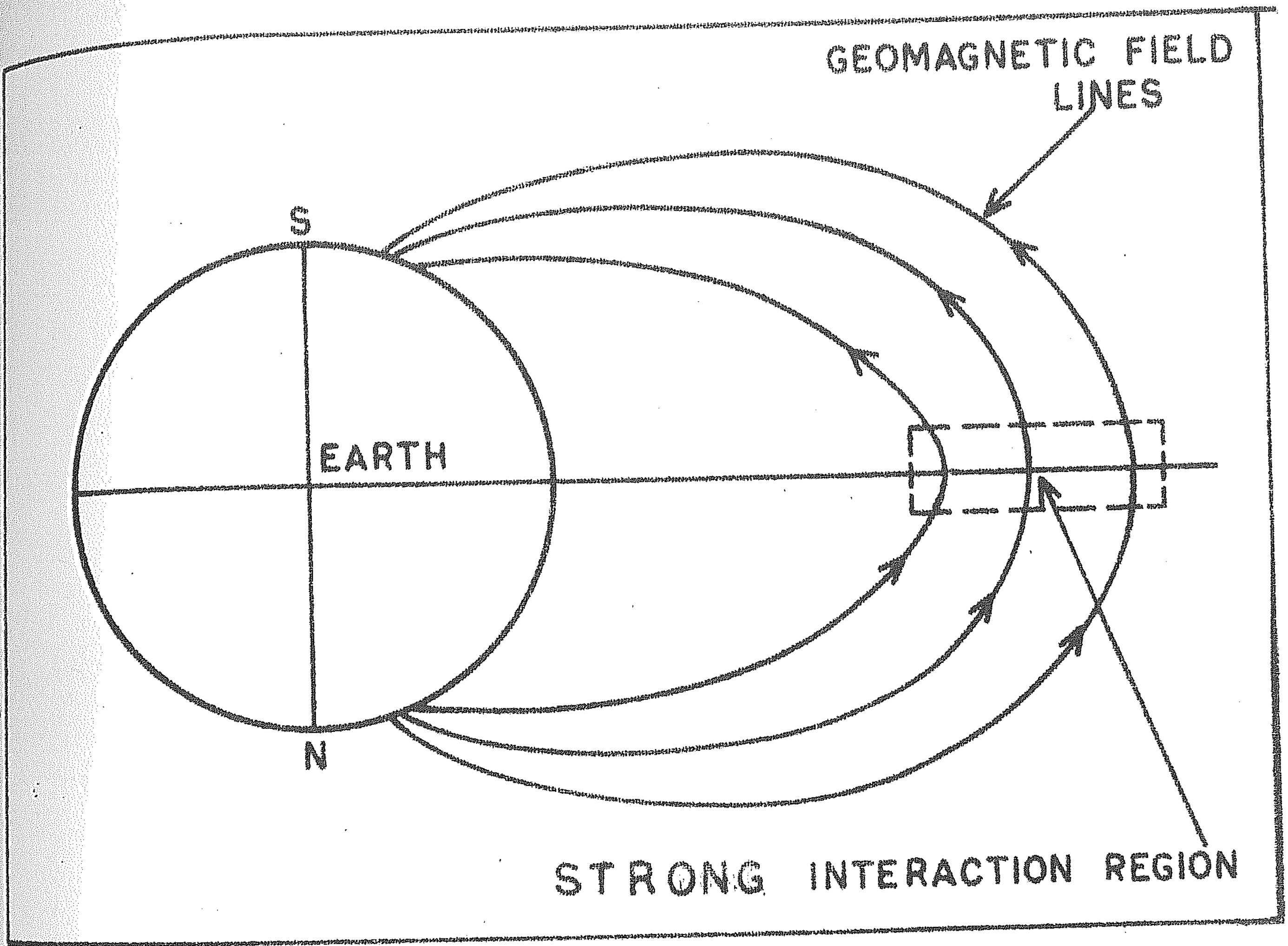

Fig. 2·6

THE REGION OF STRONG INTERACTION LIES CLOSE TO THE EQUATORIAL ZONE OF THE EARTHS MAGNETOSPHERE WHERE THE FIELD IS RELATIVELY UNIFORM us to a better understanding of the phenomenon of VLF emissions from the magnetosphere.

In the next chapter, we will take up the study of the gyroresonance of a wavepacket with the particles and discuss its relevance to VLF emissions.

CHAPTER - III

GYRO RESONANT INTERACTION OF WHISTLER MODE WAVE PACKETS: EXTENSION OF DAS'S MODEL FOR VLF EMISSIONS

3.1 INTRODUCTION:

In the last chapter we discussed the effect of the Landau resonance of the off angle whistler pulses with the electrons in a homogeneous collisionless magnetoplasma. It has already been pointed out earlier that a study of the wave particle interactions is likely to give clues regarding the generation mechanism of the VLF emissions. Therefore, it would be worthwhile to look into the effects caused by other resonances, too.

One of the very important resonances is gyro resonance or cyclotron resonance which occurs when the rate of change of the angle between the rotating wave vector and the velocity vector of the gyrating electron becomes zero. According to linear theory the particle should be accelerated or decelerated indefinitely during a resonant interaction but, in practice, the change in the velocity soon upsets the resonance condition and thus brings the interaction into nonlinear regime.

For off angle whistlers the resonance condition can be mathematically put as

$$\omega - k_{\parallel} v_{\parallel} = N\Omega \quad , \quad N = 0, \pm 1, \pm 2$$

For parallel propagation (i.e. θ = 0), only resonance is the gyroresonance given by

$$\omega - k_{\parallel} v_{\parallel} = \Omega$$

The relative importance of the Landau and the gyro resonances can be assessed by substituting their respective conditions in equations (2.2a) and (2.2b) and then comparing the effects produced by the two.

The rigorous treatment for this is rather involved and therefore here we attempt to have a less rigorous but simpler and more intuitive approach.

Although, both the parallel and the perpendicular components of the wavefields contribute to velocity changes during either type of resonance, the Landau resonance effects are caused mainly by the parallel component of the wave field while the contribution to the cyclotron resonant effects comes primarily from the perpendicular wave fields. This indicates that a comparison of the parallel and the perpendicular components of the wave fields should provide a good measure of the relative importance of the two resonances and should also tell under what conditions one resonance would dominate over the other.

The dispersion relation for a wave propagating at an angle θ to a uniform magnetic field Bo embedded in a cold uniform plasma is given by an equation in which zero is equated to the determinant of the matrix on the L.H.S of the following equation (Stix, 1962)

$$\begin{bmatrix} S - n^2 \cos^2\theta & -iD & n^2 \sin\theta \cos\theta \\ iD & S - n^2 & 0 \\ n^2 \sin\theta \cos\theta & 0 & P - n^2 \sin^2\theta \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{---(3.1)}$$

where

$$S = \frac{1}{2}(R+L) \quad , \quad D = \frac{1}{2}(R-L)$$

$$R = 1 - \frac{\omega_p^2}{\omega^2}\left(\frac{\omega}{\omega - \Omega}\right)$$

$$L = 1 - \frac{\omega_p^2}{\omega^2}\left(\frac{\omega}{\omega + \Omega}\right)$$

$$P = 1 - \frac{\omega_p^2}{\omega^2}$$

$\vec{n} = \frac{c\vec{k}}{\omega}$ = refractive index of the medium for whistler mode waves.

and $\omega_p = \frac{4\pi n e^2}{m}$ = the electron plasma frequency

The equations (3.1) readily give,

$$E_x = C_{xz} E_z$$

and $$E_y = i\, C_{yz} E_z$$

where, $$C_{xz} = \frac{n^2 \sin^2\theta - P}{n^2 \sin\theta \cos\theta}$$

and $$C_{yz} = \frac{D}{n^2 - S} C_{xz}$$ ---(3.2)

Assuming that the wave fields vary as the real part of

$$\exp\left\{ i\left(\vec{k}.\vec{r} - \omega t + \psi_0\right)\right\}$$

we get,

$$E_z = R_e\left\{|E_z|\, e^{i(\vec{k}.\vec{r} - \omega t + \psi_0)}\right\}$$
$$= |E_z| \cos\left(\vec{k}.\vec{r} - \omega t + \psi_0\right)$$

$$E_x = R_e\left\{ C_{xz}|E_z|\, e^{i(\vec{k}.\vec{r} - \omega t + \psi_0)}\right\}$$
$$= C_{xz}|E_z| \cos\left(\vec{k}.\vec{r} - \omega t + \psi_0\right)$$

and,

$$E_y = Re\left\{ i C_{yz}|E_z|\, e^{i(\vec{k}.\vec{r} - \omega t + \psi_0)}\right\}$$
$$= -C_{yz}|E_z| \sin\left(\vec{k}.\vec{r} - \omega t + \psi_0\right) \quad ---(3.3)$$

With the help of equations (3.3), $E_\perp$, the component of the wave field E perpendicular to $\vec{B}o$ may be written as

$$E_{\perp} = (E_x^2 + E_y^2)^{\frac{1}{2}} = |E_z| \left\{ C_{xz}^2 \cos^2\psi + C_{yz}^2 \sin^2\psi \right\}^{1/2}$$

where $\psi = (\vec{k}.\vec{r} - \omega t + \psi_0)$ is the phase of the wave.

It is evident that the wave is elliptically polarised and $E_{\perp}$ varies periodcally between a minimum and a maximum. For comparison purposes it would be convenient to deal with the value of $E_{\perp}$ averaged over one wave period.

$$\overline{E_{\perp}} = \frac{2}{\pi}\int_0^{\pi/2} E_{\perp}\, d\psi$$

$$= \frac{2}{\pi}|E_z| \int_0^{\pi/2} d\psi \left(C_{xz}^2 \cos^2\psi + C_{yz}^2 \sin^2\psi \right)^{1/2}$$

where $\overline{E_{\perp}}$ is the average value of $E_{\perp}$.

This equation can be reduced to

$$\overline{E_{\perp}} = \frac{2}{\pi} C_{xz} |E_z| \, E\left(\frac{\pi}{2}, ecc\right)$$

where $E(\pi/2, ecc)$ represents the elliptic integral of second kind and $ecc = \left(1 - \frac{C_{yz}^2}{C_{xz}^2}\right)^{1/2}$ is the eccentricity of the ellipse of polarisation.

Thus we finally get

$$\frac{\overline{E_{\perp}}}{|E_z|} = \frac{2}{\pi} C_{xz} E\left(\pi/2, ecc\right) \quad \text{---(3.4)}$$

This expression shows that the ratio of $\overline{E_{\perp}}$ and $|E_z|$ is dependent on C_{xz} and C_{yz} and, therefore, it is determined both (i) by the plasma parameters like the electron plasma frequency ω_p and the electron gyro frequency Ω, and (ii) by the wave parameters like the wave frequency ω, the propagation angle θ and the total wave amplitude E, i.e. $(E_x^2 + E_y^2 + E_z^2)^{1/2}$. In majority of the cases this ratio is larger than unity which suggests that the gyro resonance effects should dominate over the Landau resonance effects.

3.2 EXTENSION OF DAS'S MODEL FOR VLF EMISSIONS:

Das's model (1968) is based primarily on the work of Kennel and Petschek (1966) although the concept involved is equivalent to quasilinear theory first developed by Vedenov et al. (1962), Engel (1965) and Andronov and Trakhtengertz (1964). However, the way in which the problem was tackled is different from them. He started with the idea that the energetic particles in the radiation belts should constitute the source of supply of energy to the VLF emissions. He considered the presence of a loss cone in the pitch angle distribution f of the trapped particles and, for simplicity, took $f = f_0$ outside and $f = 0$ inside the loss cone. He also considered existence of a back ground hiss which would grow for a distribution having a step at the loss cone angle.

For the gyroresonant particles the change in $v_{\parallel}$ in nonlinear regime is given by

$$\frac{dv_{\parallel}}{dt} = -\frac{eb}{mc} v_{\perp} \sin(\psi - \phi) \qquad \text{--- (3.5)}$$

where b is the wave magnetic field and $(\psi - \phi)$ is the phase angle between $\vec{b}$ and $\vec{v}_\perp$. It has been assumed here that the propagation vector $\vec{k}$ is parallel to $\vec{B_o}$.

In a nonuniform magnetic field such as that of the earth, the $v_\parallel$ of a particle varies continuously as it moves along a field line

$$\frac{dv_\parallel}{dt} = -\frac{v_\perp^2}{2B}\frac{dB}{dz} \approx \frac{v_\perp^2}{r}$$, r being the gyroradius.

Also, the wave phase velocity $v_p = \frac{\omega}{k}$ changes as the wave moves from one point to another on the field line.

Therefore, the resonance condition for a particle is soon upset and it may go out of resonance. However, a period of effective resonance can be defined and the contributions to the change in the velocity of a particle from the resonance and from the nonuniformity of the magnetic field can be separately calculated and compared.

The effects of resonance will be substantial and worth consideration only if the change in $v_\parallel$ due to the resonance is large compared to that due to the nonuniformity of the field over the period of effective resonance. This condition can be mathematically put as (Das, 1968)

$$\frac{eb}{mc} \sin\psi > \frac{v_\perp^2}{r}$$

where r is the radius of gyration of a particle.

This condition is strictly true at the equator and it is not satisfied at points far away from the equator. It shows that the resonant effects in the earth's geomagnetic field would be strong and important only in the equatorial region. Incidentally, observations of VLF emissions also show that they are generated near the equator and thereby suggest that the resonant interaction might have something to do with the emissions.

A wave packet would resonate with particles over a finite range of velocities owing to the finite width of its spectrum. The whole of the velocity space can be divided into two parts (see fig.3.1): resonant and nonresonant. The nonresonant region can be taken as

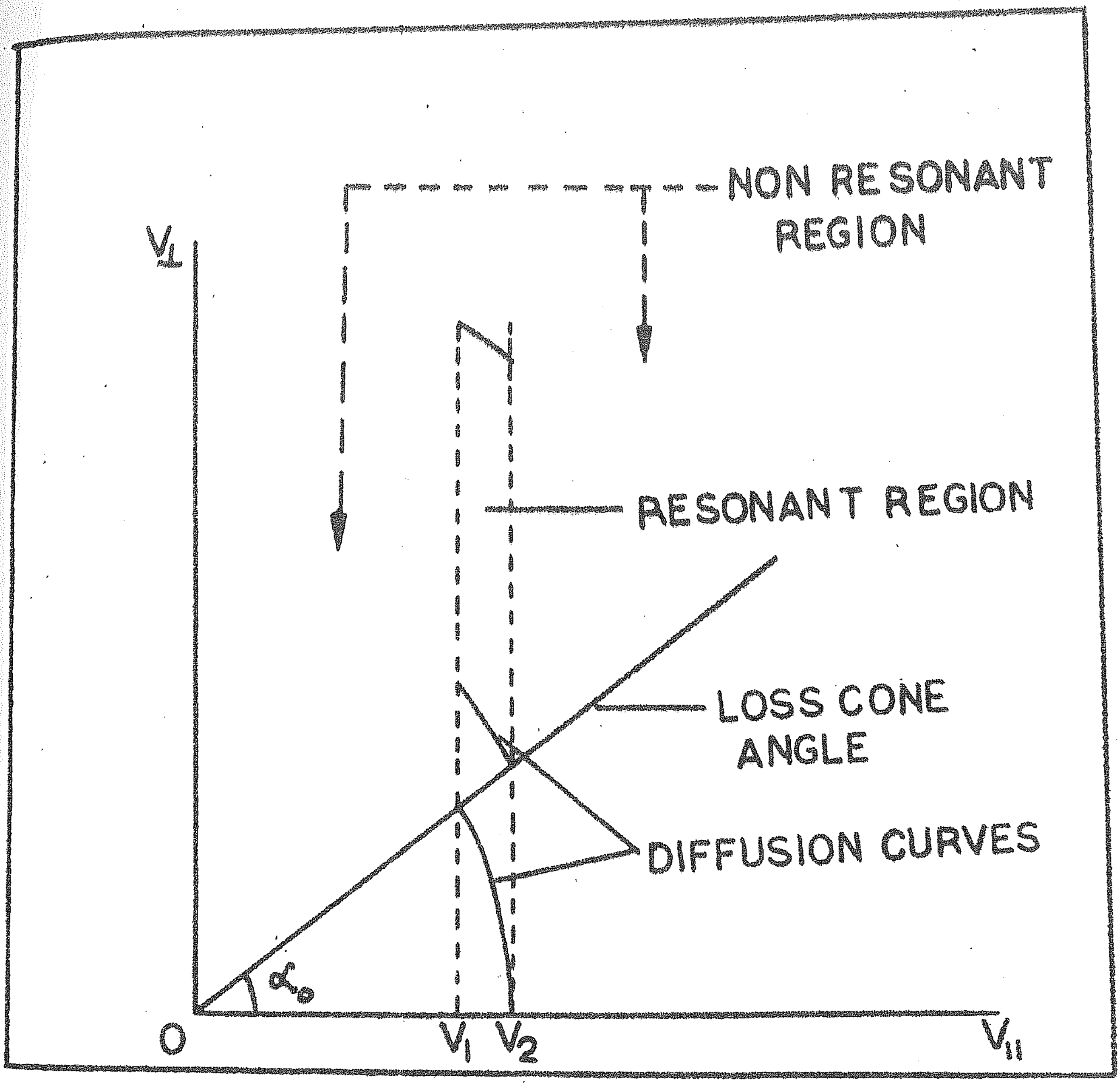

FIGURE 3.1

THE RESONANT AND NONRESONANT REGIONS IN VELOCITY SPACE. THE PARTICLES DIFFUSE ALONG DIFFUSION CURVES THROUGH RESONANCES.

unaffected by the waves and the distribution function therein may be assumed to remain unchanged with time. In the resonant region, the particles, will be trapped and the eddies will be formed in the phase space. The trapping period T is given by

$$\frac{2\pi}{T} = \frac{e}{mc}\left(B b \tan \alpha \right)^{1/2}$$

For $b \sim 1\ m\gamma$ and $L \sim 3$, T comes out to be about 0.1 sec. If a particle remains in a wave packet for a time much larger than T, each eddy will take many turns and the final distribution f along each eddy will be constant. The process is similar to diffusion in velocity space where the particles are constrained to move along certain diffusion curves. It can be shown that f will become constant along these diffusion curves and $\frac{\partial f}{\partial v_{\parallel}}$ will become highly negative at the boundaries of the resonant region but at the central resonant $v_{\parallel}$ it will become zero. The growth rate γ for such a system is given by following equation (Vedenov et al. 1962)

$$\gamma = - \frac{\pi k^2 v_{res}^2}{n\Omega} \int_0^\infty \left[v_\perp \frac{\partial f}{\partial v_\parallel} - (v_\parallel - v_p) \frac{\partial f}{\partial v_\perp} \right]_{v_\parallel = v_{res}} v_\perp^2 \, dv_\perp \quad \text{--- (3.6)}$$

where n is electron concentration and v_{res} is the gyroresonant $v_\parallel$.

It can be shown that $\int_{-\infty}^{\infty} \gamma \, dv_\parallel$ is a constant. Since γ, in the nonresonant region, does not change with time and that in the resonant region is reduced with time because of the redistribution of the particles, it can be inferred that large peaks in γ will occur near the boundaries of the disturbed region.

If the particle remains in the wavepacket for a time much shorter than the trapping period T, the linear theory can be applied to study its motion. The resonant particles situated near the loss cone boundary are redistributed in such a fashion that the loss cone boundary in its modified form looks like a conical screw (see fig. 3.2). When the wave packet has passed, the fine structure is smeared out and the new distribution thus developed is found to give high growth rates

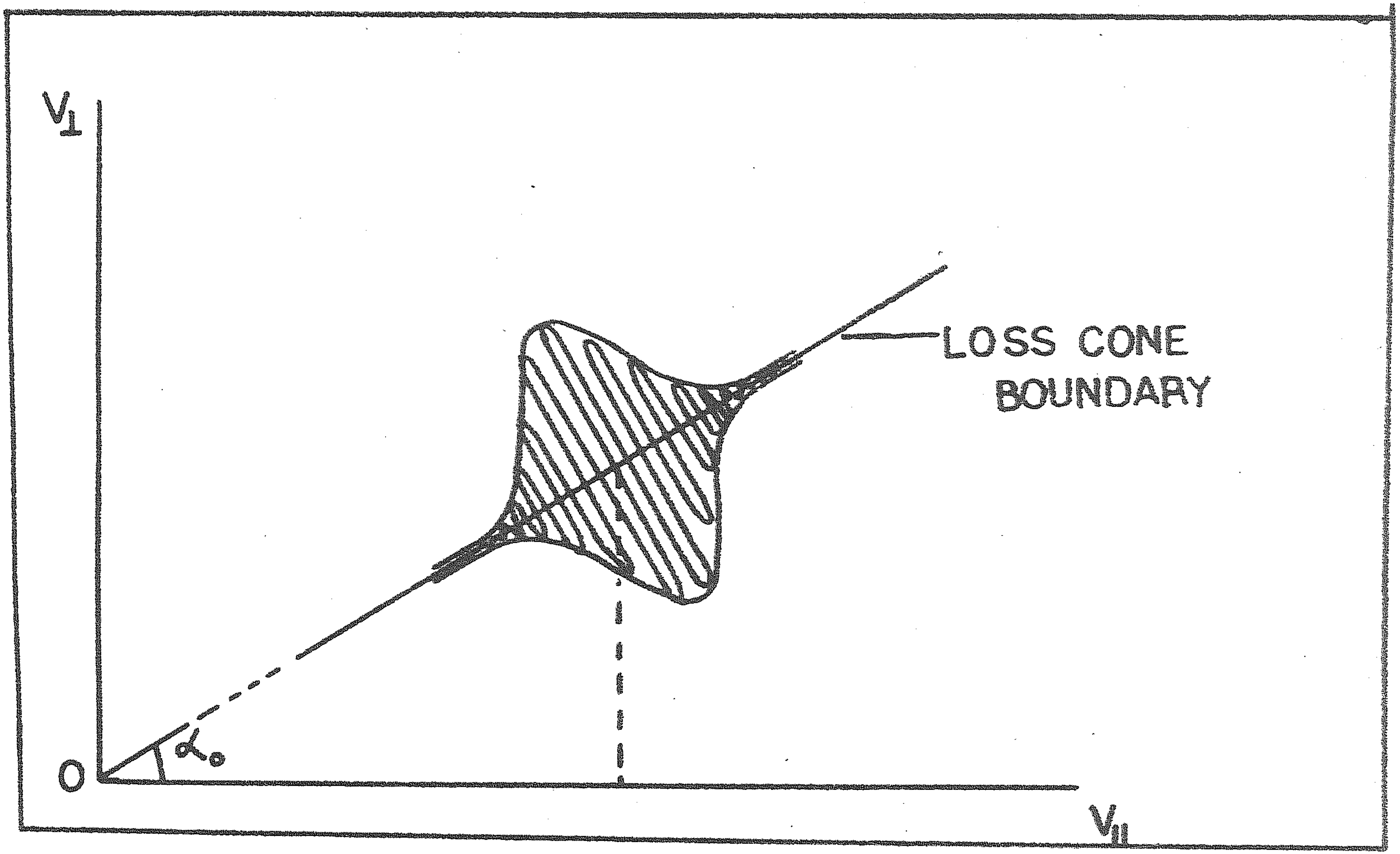

FIGURE 3.2

THE GYRORESONANCE OF A PULSE DISTORTS THE DISTRIBUTION AT LOSS CONE BOUNDARY. THE DISTURBANCE TAKES THE FORM OF A CONICAL SCREW.

at frequencies resonant with slightly higher and slightly lower $v_{\parallel}$ than the central resonant $v_{\parallel}$ for the wave packet (Das, 1968).

Fig. (3.3) shows the region in the velocity space where the distribution function is modified.

The figure shows two gaussian curves symmetrically situated on either side of the loss cone angle α_0 in $v_{\parallel} - v_{\perp}$ space. Before the interaction the distribution function is $f = f_0$ for $\alpha > \alpha_0$ and $f = 0$ for $\alpha \leq \alpha_0$. However, after the wave-packet has passed, the distribution gets modified in the region bounded by the two gaussians while it still remains equal to f_0 above the upper gaussian and equal to zero below the lower gaussian. In the disturbed region, f increases from zero at the lower gaussian as we move upwards and finally becomes f_0 when we reach the upper gaussian. If we draw a straight line perpendicular to the loss cone at Q ($v_{\parallel}'$, $v_{\perp}'$) which intersects upper gaussian at R ($v_{\parallel}''$, $v_{\perp}''$) and the lower gaussian at S ($v_{\parallel}'''$, $v_{\perp}'''$) as shown in the figure, then the distribution f at a point P ($v_{\parallel}$, $v_{\perp}$) on the line would be given by

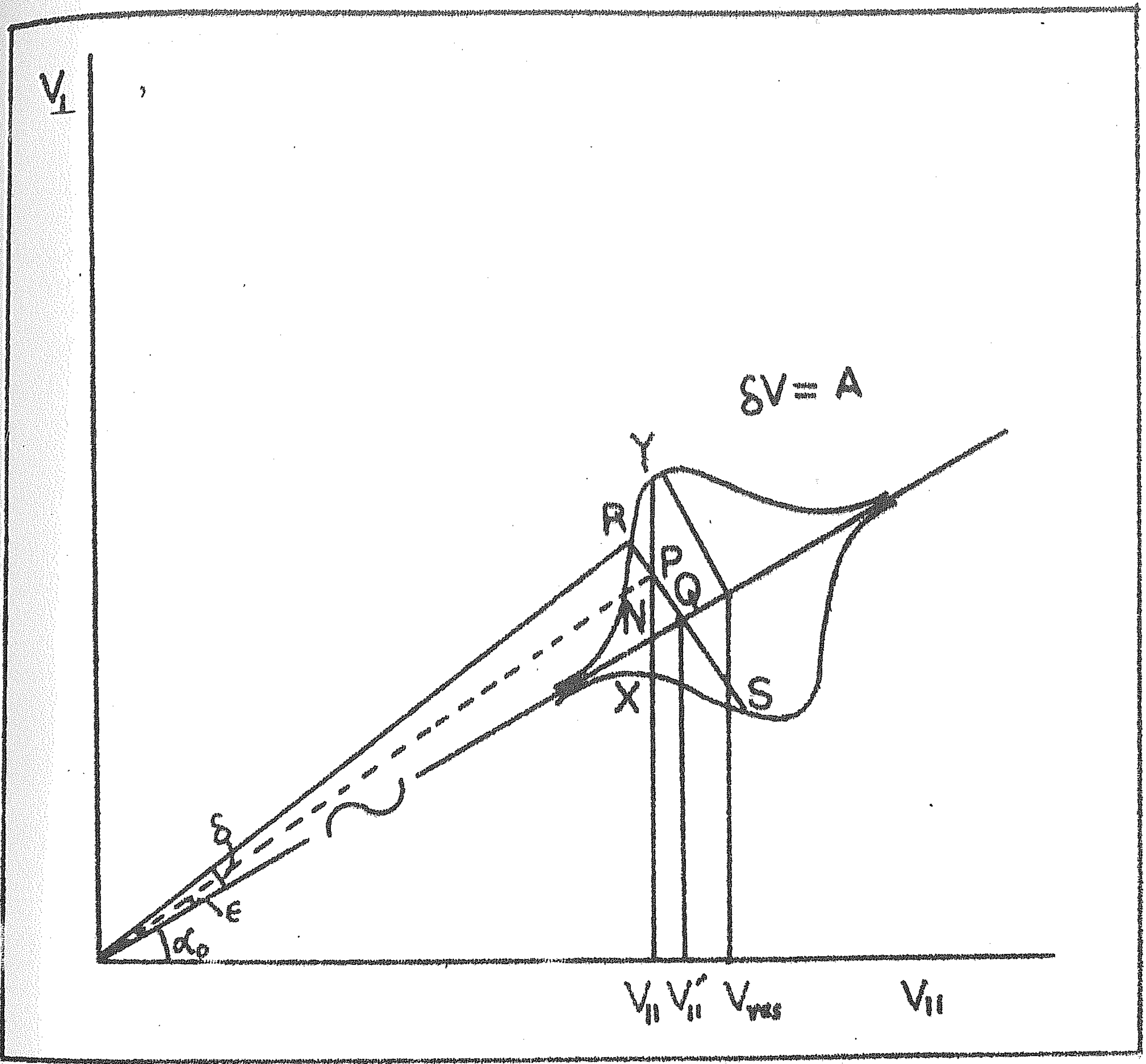

FIGURE 3.3

DISTRIBUTION GETS MODIFIED IN THE REGION BOUNDED BY THE TWO GAUSSIANS. THE COORDINATES OF P, Q, R AND S ARE RESPECTIVELY $(V_\parallel, V_\perp)$, $(V_\parallel', V_\perp')$, $(V_\parallel'', V_\perp'')$ AND $(V_\parallel''', V_\perp''')$

$$f = \frac{1}{2} f_0 (v_\perp'' / v_\perp) \left[1 + \frac{2}{\pi} \sin^{-1}\left(\frac{\epsilon}{\delta}\right) \right] \quad \text{--- (3.7)}$$

where $\epsilon = \angle$ POQ and $\delta = \angle$ ROQ. The growth rate for such a distribution function would be given by

$$\gamma = \int \frac{1}{2} f_0 \; v_\perp'' v_{\parallel}' \sec\alpha_0 \left[\frac{2}{\pi} \frac{v_\perp / \delta v}{\pi\left[1 - (\epsilon/\delta)^2\right]^{1/2}} - \left\{ 1 + \frac{2}{\pi} \sin^{-1}\left(\frac{\epsilon}{\delta}\right) \right\} \cos\alpha_0 \right]$$

--- (3.8)

where the integration for the growth rate at frequency ω has to be carried out from the point of intersection X of the straight line $v_{\parallel} = \frac{|\omega - \Omega|}{k}$ with the lower gaussian to the point of intersection Y of the same straight line with the upper gaussian.

The various relationships between the co-ordinates of P,Q,R and S and the angles α and δ are listed below:

$$v_\perp'' = v_\parallel \tan\alpha_0 + \delta v \sec\alpha_0$$

$$v_\parallel' = v_\parallel + \delta v \sin\alpha_0$$

$$\epsilon = v_\perp \cos\alpha_0 - v_\parallel \sin\alpha_0$$

where $\delta v = v_\parallel \sec\alpha_0 . \delta$
$= A \exp\{-(v_\parallel - v_{\parallel 0})^2 \sec^2\alpha_0/d^2\}$. The last equation determines the form of the envelope of the wave packet such that A and d represent the amplitude and the band width of the wave packet and consequently the dimensions of the disturbed region. $v_{\parallel 0}$ is the value of $v_\parallel$ of the particles which resonate with the central frequency of the wave packet.

The results obtained from the model are very interesting. The growth rate is reduced at the central frequency while enhancements occur at two frequencies slightly above and below the central frequency (see fig. 3.4).

The peaks in the growth rate suggest that the waves inside the wave packet at corresponding frequencies

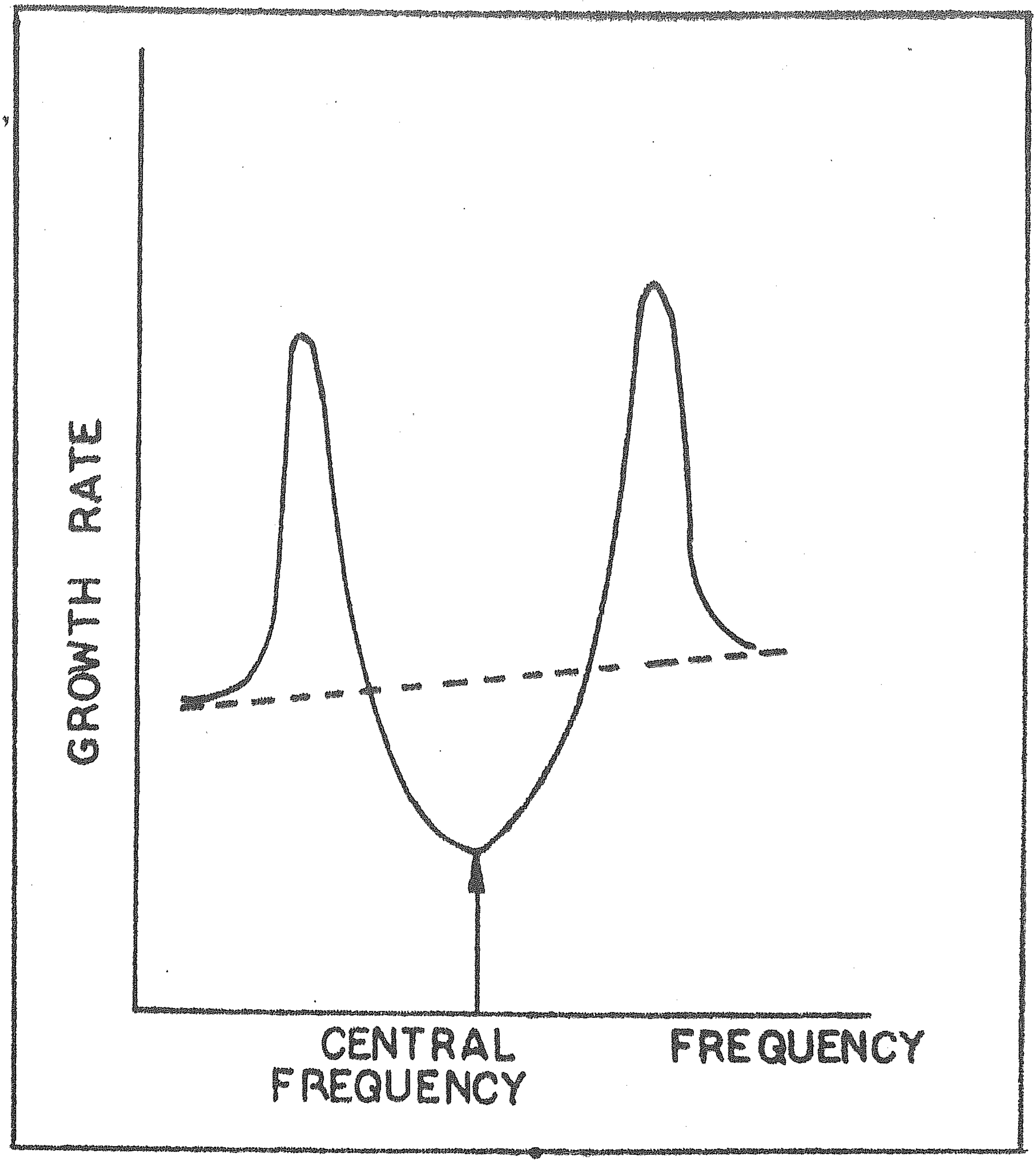

FIGURE 3.4

A TYPICAL GROWTH RATE VERSUS FREQUENCY CURVE FOR A PERTURBATION MOVING THROUGH THE MODIFIED DISTRIBUTION OF PARTICLES shall grow and are likely to be observed as emissions. This furnishes a mechanism for the generation of VLF emissions.

The amplification is essentially due to resonant interaction between the whistler mode wave packet and the energetic particles of the medium. The amplitude of the wave packet ~~and~~ in the previous model is restricted to a critical value to avoid complications in the computational procedure. However, the increase in amplitude beyond the critical value brings into picture two distinct regions in the velocity space contributing to the growth of the waves at the same frequency. It would be interesting to study the behaviour of the growth rate for amplitudes of the wave packet larger than the critical one and compare it with the growth rates obtained from smaller amplitude pulses. In what follows, the details of the computation of growth rate and a suitable computer programme evolved for the purpose have been discussed.

The extended model for the localised disturbed region in velocity space is shown in fig. (3.5). The disturbed region for the small amplitude wave discussed

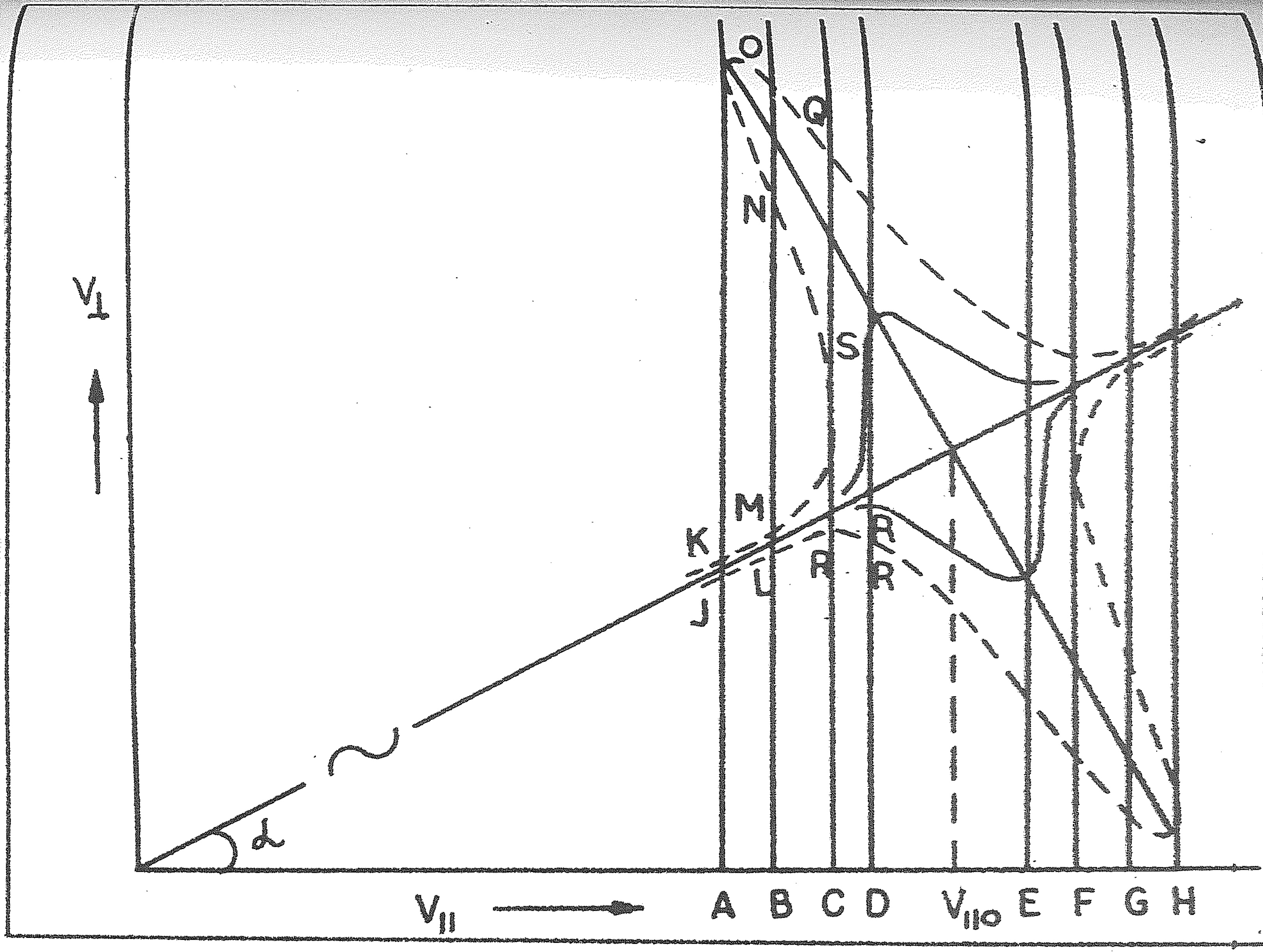

FIGURE 3.5

COMPARISON OF DISTURBED REGIONS DUE TO SMALL AND LARGE AMPLITUDE PULSES.

in the previous model is shown to be bounded by continuous solid Gaussian curves and that for the large amplitude wave considered here has been shown to be bounded by the dashed gaussian curves. The difference between the previous situation and the present one arises from the fact that we meet the disturbed region only once in the previous situation and sometime once and some times twice in the present situation as we move from the lower gaussian to the upper gaussian in the disturbed region along the straight line $v_{\parallel} = |v_{\parallel G}| = |v_{res}| = \frac{|\omega - \Omega|}{k}$. This situation arises only for those gaussian boundaries whose amplitude exceeds a certain critical limit A_c given by

$$A_c = \frac{e\alpha^{3/2}}{2\tan\alpha_0} \qquad \text{--- (3.9)}$$

If the amplitude A of the wave packet is less than A_c, it can be handled by the computation procedure developed for the earlier model. However, for $A > A_c$ that will not be sufficient unless suitably modified.

The contribution to the growth rate in this case at B (see fig.3.5) comes from the region L to M as well as from N to O . Similar things happen at any $\vartheta_{\parallel}$ lying between OA and OC and between OF and OH. The $\vartheta_{\parallel}$'s lying between OC and OF do not show this peculiar behaviour. We presume that the growth rate given by equation (8) still holds good. However, we have to calculate it separately for the two regions that contribute to it and then to add up them together.

The limits of integration are obtained by finding the points of intersection of a straight line parallel to $\vartheta_{\perp}$ axis with the gaussian boundaries of the disturbed region.

The growth rate computations for wavepackets with amplitudes less than the critical amplitude is easy because the line along which the integration is to be carried out does not intersect one gaussian at more than one points. For example, at $\vartheta_{\parallel}$ = OD , the integration is to be carried out only from R to S.

The problem of integration for computing the growth rate for a large amplitude pulse shown by the dashed curves in figure (3.5) is complicated because a vertical line at some values of $\vartheta_{\parallel}$ intersects one of the gaussians at more than one points. The integration procedure for OH < $\vartheta_{\parallel}$ < OA and OC < $\vartheta_{\parallel}$ < OF remains the same while for values of $\vartheta_{\parallel}$ between OA and OC and between OF and OH, the integration has to be carried out separately for different ranges in the disturbed region at the same value of $\vartheta_{\parallel}$, e.g., at $\vartheta_{\parallel}$ = OB, the integration has to be carried out from L to M and then from N to O.

Thus it is necessary to determine the limits of integration by solving the equation of the gaussian and that of the vertical line at $\vartheta_{\parallel}$ under question simultaneously. The computer programme is divided into two parts, the first part being for those values of $\vartheta_{\parallel}$ where the vertical line intersects each curve only at one point, and the second part for those values of $\vartheta_{\parallel}$ where the vertical line intersects one of the curves at three points and the remaining curve at one point only.

The method of false position was used to determine the exact points of intersection which is slightly complicated in the second part. After the limits are obtained, the integration is performed in the following way. The distance between the limits of integration is divided into 25 equal steps. The integrand is calculated at the centre point of each step and is then multiplied by the length of the step. The total value of the integral is given by adding up all the contributions from different steps. In the region where the integration is to be carried out for two different sets of limits at same $\vartheta_{\parallel}$, the individual contributions are added up to give the total value of the growth rate at the given value of $\vartheta_{\parallel}$.

3.3 RESULTS AND DISCUSSION:

The results obtained are shown in fig.(3.6). The continuous curve represents the growth rate at different resonant $\vartheta_{\parallel}$'s in the case of a disturbing pulse having its amplitude less than the critical amplitude A_c . The dashed curve describes the growth rate in the case when amplitude of the disturbing

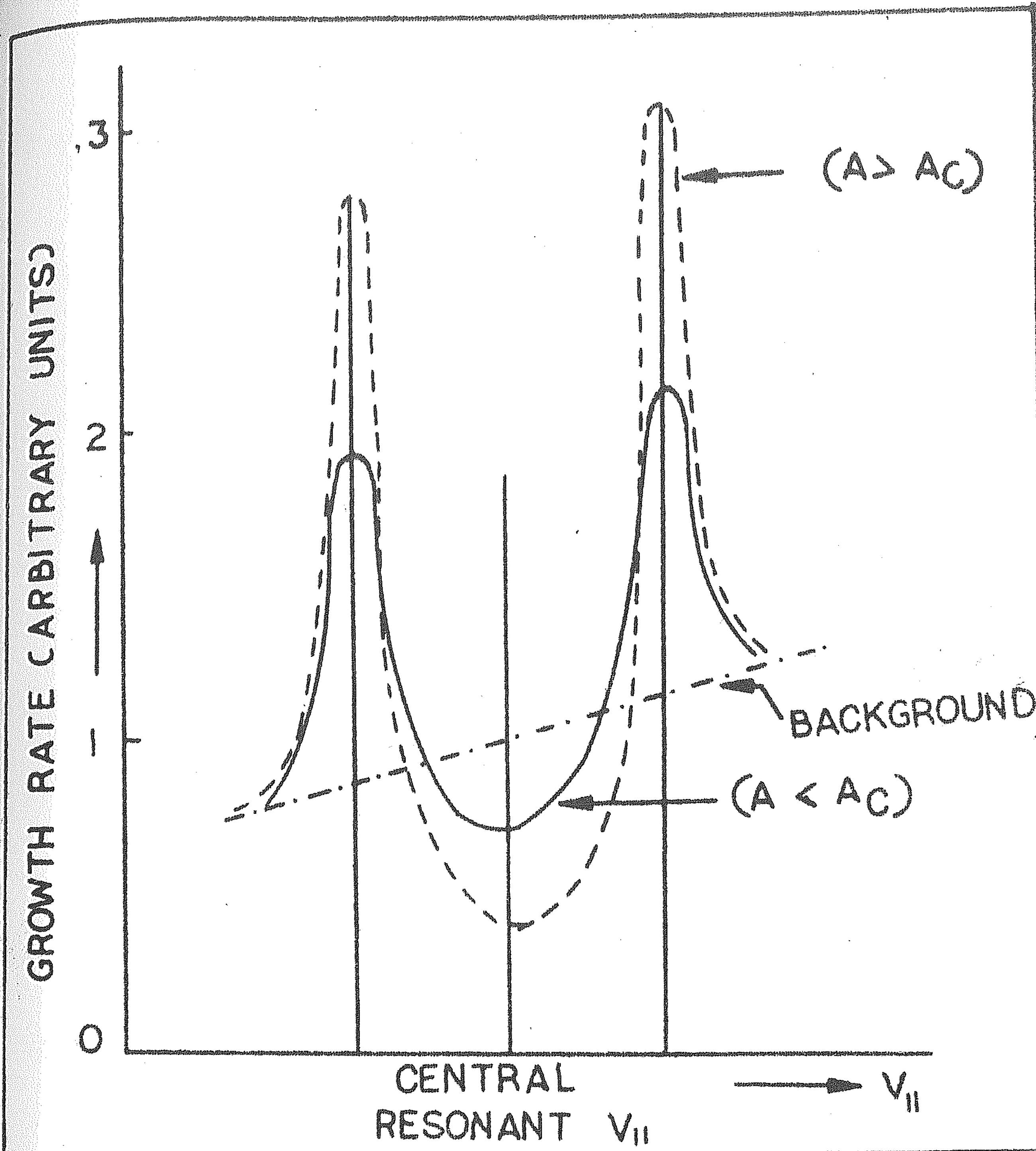

FIGURE 3.6

COMPARISON OF GROWTH RATES OBTAINED FROM THE DISTRIBUTION DISTURBED BY THE LARGE AND THE SMALL AMPLITUDE PULSES.

pulse is large compared to the critical amplitude A_c. In both cases it is seen that the growth rate is reduced at the central frequency range while enhancements occur at the frequencies slightly above and below the central frequency. In order that proper comparisons can be made of the results obtained from the previous and the extended models, care is taken to vary the amplitude A and the width d of the wave-packet in such a way that $\int |A(v_{\parallel} \sec \alpha)|^2 dv_{\parallel}$ remains the same. This amounts to saying that, as we go on increasing the amplitude, we go on decreasing the frequency width of the pulse, or equivalently saying, we go on increasing the duration of the pulse keeping the energy content of the pulse constant.

As figure (3.6) shows, the growth rate of the sidebands of the wave packet is much larger for the extended model when compared with the same for the earlier model (Das and Vyas, 1971).

This sudden change occuring at the critical amplitude (which will correspond to a critical time duration of the pulse) should account for the frequent triggering of VLF emissions by Morse code dashes (duration 150 ms) and fewer triggerings by Morse code dots (duration 50 ms). The effect is probably because of the sudden appearance of two regions contributing to the growth at the same frequency.

Tantry (1970) has recently reported that he has frequently observed duplicate traces of whistlers at a time difference of 15 msec. It is interesting to mention here that the computation of the time lag of perturbations corresponding to these two peaks in the growth rate, to be observed at a ground station in the model shows a difference of the order of 20 msec. Therefore, if these traces are treated as emissions, the model described here is quite adequate to explain these observations.

The critical amplitude A_c is a function of the bandwidth d (see eq. 3.9) and it increases with increase in d. This shows that the critical amplitude will be large for small duration pulses. A pulse should have its amplitude greater than the critical amplitude so as to give large growth rates and therefore, if it has also a large band width, it must have its total energy content, $\int |E(\omega)|^2 d\omega$, large enough to attain this critical amplitude at its central frequency. (Here $E(\omega)$ is the wave amplitude at the frequency ω inside the pulse). We can, in general say that short duration pulses like the Morse code dots are also capable of triggering emissions, if they are sufficiently strong to have their amplitude greater than A_c. This explains how some times dots have also been able to trigger VLF emissions.

Another interesting result was obtained when the growth rate calculations were extended to those pulses for which the amplitude A was much large compared to A_c. It was found that secondary peaks were protruding above the background.. However, the growth rates corresponding to these peaks were not large compared to background growthrate. Therefore, it was decided to consider strong pulses with large bandwidth and large amplitude ($A >> A_c$) and the results obtained are as shown in fig. (3.7). Two peaks in the growth rate appear on each side of the central

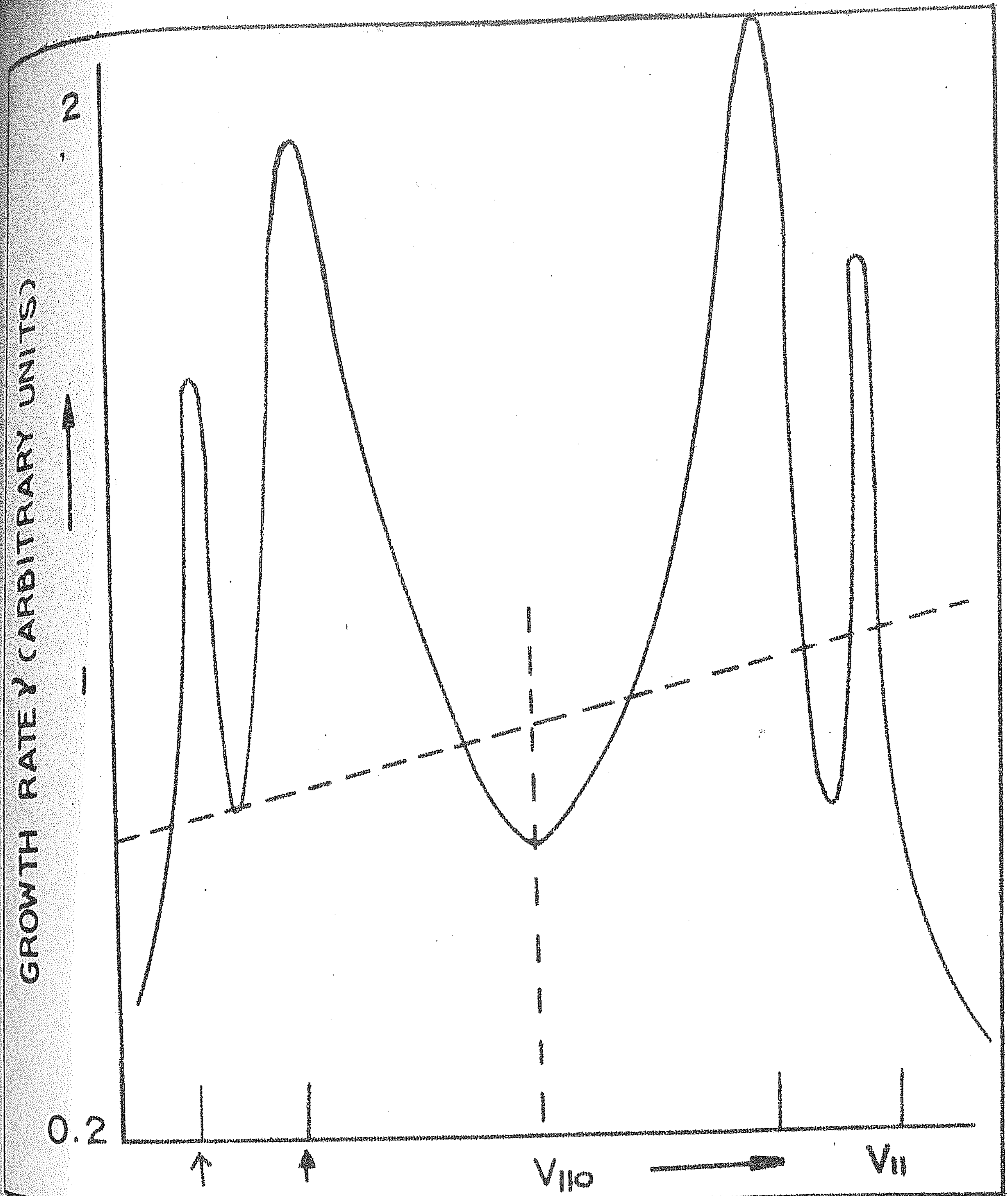

FIGURE 3.7

GROWTH RATE γ VERSUS $V_{\parallel}$

CONTINUOUS CURVE : γ FOR DISTURBED DISTRIBUTION

BROKEN CURVE : γ FOR UNDISTURBED DISTRIBUTION

(CASE OF A STRONG BROAD BAND PULSE)

frequency instead of one seen in fig.(3.6). The growth rate at central frequency is reduced as before. This shows that a very strong wavepacket is capable of producing multiple emissions also.

CHAPTER IV

EMISSIONS AT HALF THE EQUATORIAL GYROFREQUENCY AND THE GROUP RESONANCE OF A WAVE PACKET

4.1 INTRODUCTION

So far we have studied only the effects of the resonant wave particle interactions on particle distribution functions in plasmas with special reference to the magnetospheric conditions and have discussed their possible relevance to the stimulation of VLF emissions. Looking back to the observed properties of VLF emissions, already mentioned in chapter I, we must remember that any mechanism proposed for the generation of these emissions should have the inherent capability to account for their properties.

One of the very outstanding features of VLF emissions is that they tend to occur at a frequency close to half the equatorial gyrofrequency (written as $(1/2)\,\Omega_{eq}$ hereafter) along the field line of propagation. It has also been observed that the artificial stimulation of these emissions is highly favourable if the frequency of the triggering pulse is close to $(1/2)\ \Omega_{eq}$.

This suggests that the wave particle interaction for waves at frequency $(1/2)\ \Omega_{eq}$ has some special features associated with it that favourably contribute to the triggering of emissions near that frequency. Therefore, it is desirable that the resonance condition and the propagation characteristics for the whistler mode waves be examined carefully to understand their behaviour at the frequency $\omega = \frac{1}{2}\Omega_{eq}$. Once this behaviour is understood and its difference from that at other frequencies is brought about, detailed investigation can be carried out to see whether it can really contribute to the favourable emissions at $\omega = (1/2)\ \Omega_{eq}$.

4.2 WHISTLER CHARACTERISTICS AT $\omega \simeq \frac{1}{2}\Omega_{eq}$:

The speeds of the Landau resonant and the gyroresonant particles are same if the wave frequency $\omega = \frac{1}{2}\Omega$.

This can be very easily seen from the general resonance condition:

$$\omega - k_{\parallel} v_{\parallel} = n\Omega$$

For the whistler mode waves, it gives

$$v_{\parallel L} = \frac{\omega}{k_{\parallel}} \quad \text{for } n = 0$$

and $$v_{\parallel G} = \frac{\omega - \Omega}{k_{\parallel}} \quad \text{for } n = 1$$

Where $v_{\parallel L}$ and $v_{\parallel G}$ represent the Landau and the gyro resonant velocities respectively. Since $\omega < \Omega$ for whistlers, $v_{\parallel G}$ is always negative. However, the speeds $|v_{\parallel L}|$ and $|v_{\parallel G}|$ would be equal if $\omega = \Omega - \omega$ or $\omega = \frac{1}{2}\Omega$. This was pointed out by Dungey as an important feature that might contribute to preferential triggering at

$\omega = \frac{1}{2}\Omega_{eq}$. However, we have obtained the following two additional properties which can play a rather important role in triggering these emissions at this particular frequency.

i) The group velocity of a pulse would be roughly equal to the phase velocity at its central frequency provided the pulse is centred at half the electron gyrofrequency in a uniform magnetic field.

This can be easily inferred from the following expressions for the phase and the group velocities of a whistler mode pulse:

$$v_p = \frac{c}{\omega_p}\,\omega^{1/2}(\Omega\cos\theta - \omega)^{1/2}$$

and
$$v_g = \frac{c}{\omega_p}\,\omega^{1/2}(\Omega\cos\theta - \omega)^{1/2}\left\{\frac{\Omega\cos\theta - \omega}{(\Omega\cos\theta)/2}\right\}$$

It is evident that $v_p = v_g$ at $\omega = \frac{1}{2}\Omega\cos\theta$. But θ has to be a small angle for the whistler mode dispersion relation to hold and therefore $\cos\theta$ may be taken as unity. Alternatively, we can say that

$$v_p \simeq v_g \quad \text{at} \quad \omega = \frac{1}{2}\Omega \; .$$

ii) The phase velocity v_p of a whistler mode wave has a maximum at $\omega \simeq \frac{1}{2}\Omega$:

From the expression for v_p for a whistler mode wave quoted above, it is easy to see that it will have a maximum at $\omega = \frac{1}{2}\Omega \cos\theta$, the maximum value of v_p being $c\Omega\cos\theta/(2\omega_p)$. Since θ is small for whistler mode propagation, it turns out that the maximum in the phase velocity occurs for a value of ω close to $\frac{1}{2}\Omega$. The maximum of the Landau resonance velocity $\frac{\omega}{k_{\parallel}}$ would be $\frac{c\Omega}{2\omega_p}$ which is independent of the propagation angle θ. It may also be noted that particles with $v_{\parallel} = \frac{c\Omega}{2\omega_p}$ would gyro resonate with a pulse having frequency $\omega = \frac{1}{2}\Omega$ and wave vector anti parallel to $\vec{B}o$.

Figure (4.1) shows the variation of the phase velocity of a pulse with its frequency ω . The maximum occuring at $\omega = \frac{1}{2}\Omega \cos\theta$ may be noted.

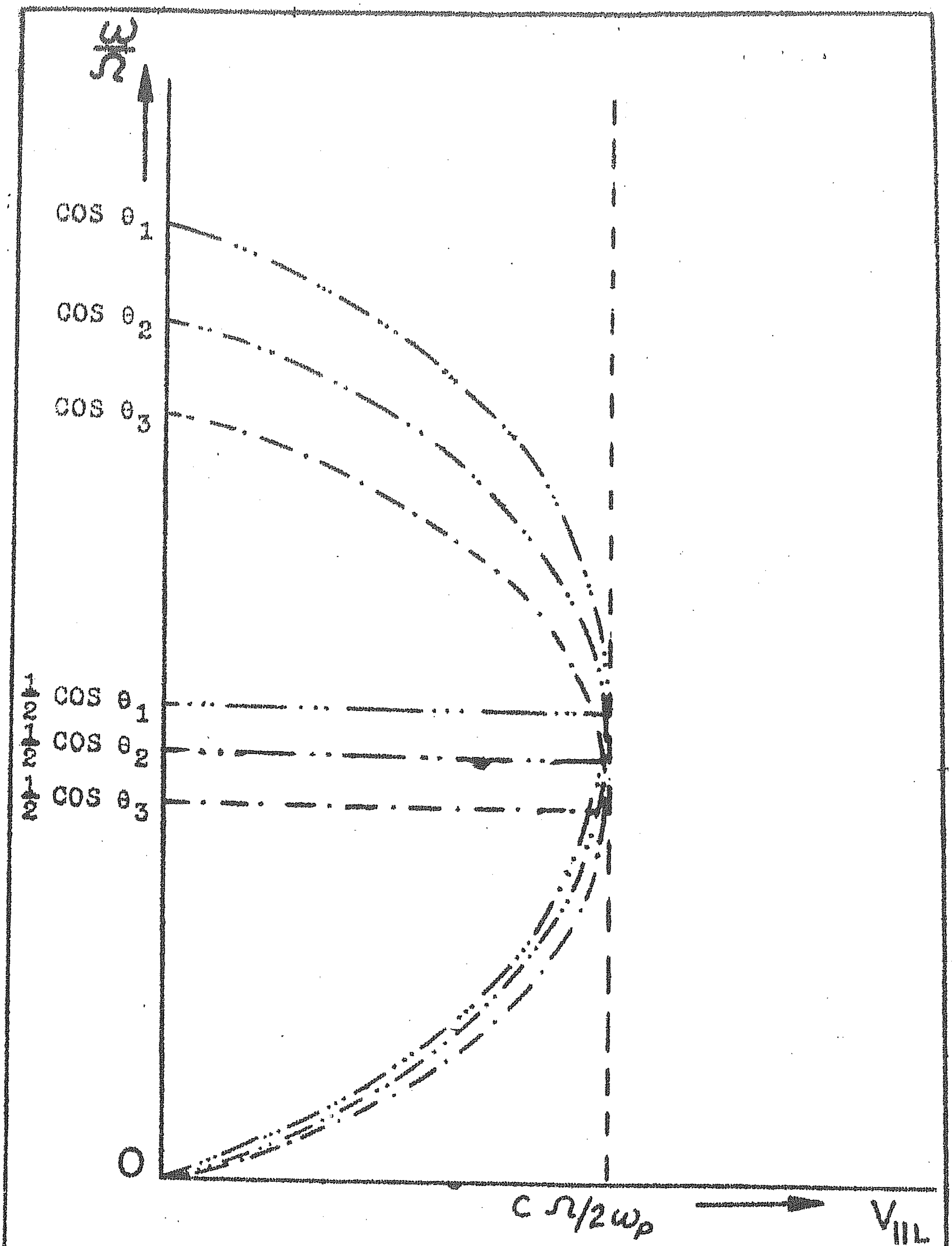

FIG 4.1

VARIATION OF $V_{\parallel L}$ THE LANDAU RESONANT VELOCITY, WITH NORMALISED FREQUENCY ω/Ω, FOR THREE DIFFERENT PROPAGATION ANGLES θ_1, θ_2 AND θ_3.

We would like to see if one or more of the above mentioned characteristics for whistlers at $\omega \approx \frac{1}{2}\Omega$ can contribute to the mechanism for the observed favourable emissions at $\omega \approx \frac{1}{2}\Omega_{eq}$.

4.3 A SHORT DESCRIPTION OF ABDELLA'S MODEL:

It was first pointed out by Dungey that the speeds of the Landau resonant and the gyroresonant particles are same at $\omega = \frac{1}{2}\Omega$ for whistler mode waves. Ashour Abdalla (1972), on this basis, qualitatively argued that her model for VLF emissions which was primarily based on gyroresonant effects would be able to explain the favourable emissions at $\omega = \frac{1}{2}\Omega_{eq}$ if the Landau resonant effects are also included in the theory.

She considered the gyroresonant interaction of a succession of whistler mode pulses with particles mirroring in earth's radiation belts. The process leads to a diffusion of particles in velocity space.

The mirroring particles have their $v_{\parallel}$ maximum at the equator and the $v_{\parallel}$ that satisfies the gyro-resonant condition for a wave has a minimum there. The behaviour of $v_{\parallel}$ and $v_{\parallel G}$ has been shown in fig.4.2 which gives a plot of $v_{\parallel}$ and $|v_{\parallel G}|$ against distance S measured from the equator along a field line. Three curves have been shown for the variation of each of $v_{\parallel}$ and $|v_{\parallel G}|$ to clearly illustrate how they vary with S. The waves with different frequencies will have different curves for the corresponding $|v_{\parallel G}|$ as shown by the lables I, II and III. Similarly the changes in the $v_{\parallel}$ of the particles with S for different values of $v_{\parallel eq}$ will be given by different curves as shown by (i), (ii) and (iii). Here the $v_{\perp eq}$ of all the three particles has been assumed to be the same. If for a given ω, $|v_{\parallel eq}| < |v_{\parallel G eq}|$, no gyroresonance can take place (The subscript 'eq ' refers to the equatorial value). Therefore, the diffusion co-efficients for $|v_{\parallel eq}| < |v_{\parallel G eq}|$ would be negligibly small.

If $v_{\parallel eq} = v_{\parallel G eq}$, the resonance takes

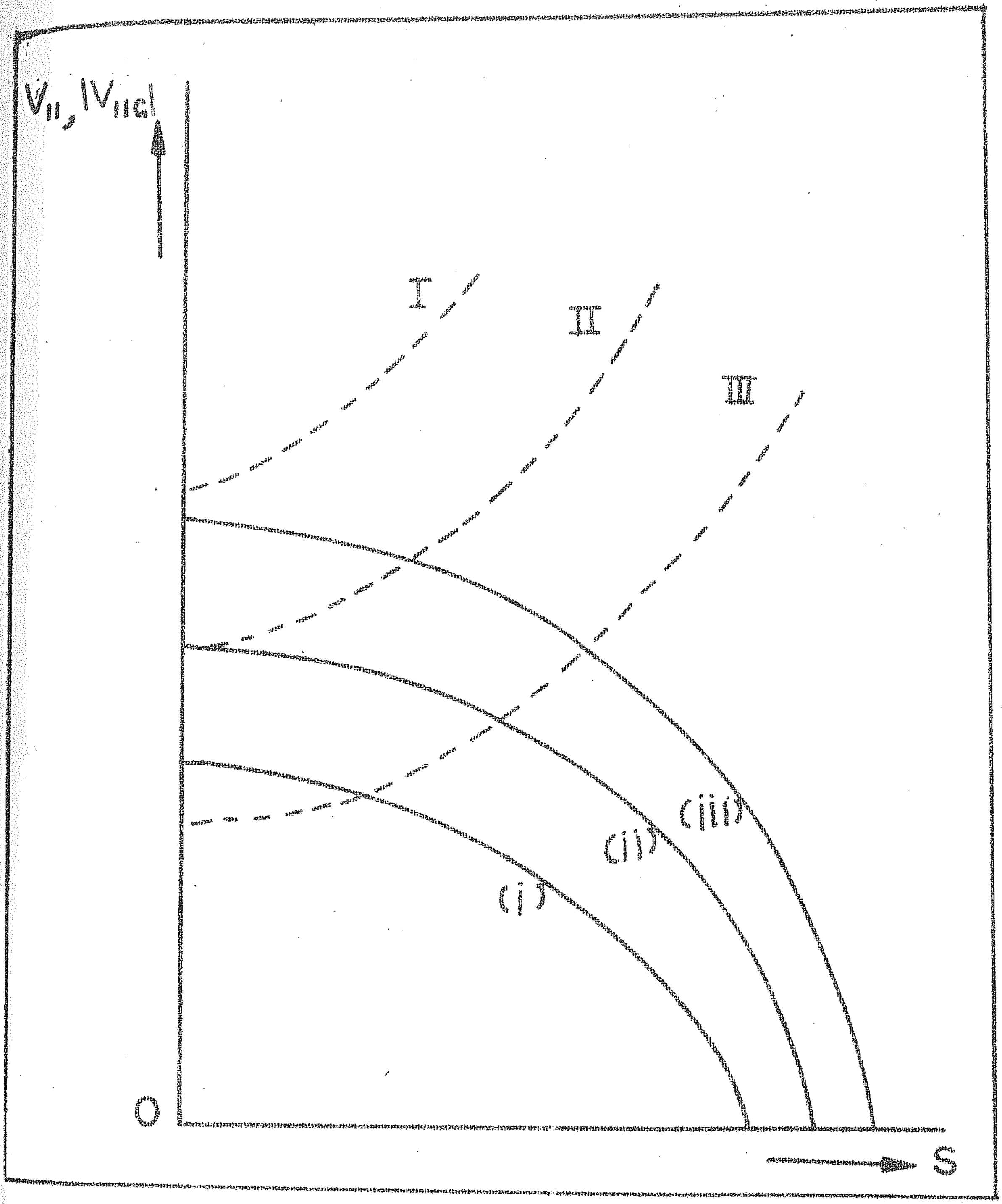

FIG 4.2

VARIATION OF $V_{\parallel}$ (CONTINUOUS CURVES) AND $|V_{\parallel G}|$ (DOTTED CURVES) WITH S, THE DISTANCE ALONG A FIELD LINE MEASURED FROM THE EQUATOR.

place right at the equator and is quite strong and therefore the diffusion co-efficient will suddenly rise at $v_{\parallel eq} = |v_{\parallel G eq}|$. As $v_{\parallel eq}$ exceeds $|v_{\parallel G eq}|$, the resonance point shifts more and more away from the equator where the nonuniformity of the field does not allow the resonance to be quite strong. Therefore the diffusion co-efficient would slowly go on decreasing as $v_{\parallel eq}$ exceeds more and more beyond $|v_{\parallel G eq}|$. Fig. 4.3 shows a typical plot of diffusion co-efficients against $|v_{\parallel eq}|$.

It is found that the changes in $v_{\parallel}$ and $v_{\perp}$ are not independent of each other. In the equatorial region the two changes are related by the following expression:

$$\frac{dv_{\perp eq}}{dv_{\parallel eq}} = - \frac{v_{\parallel eq}}{v_{\perp eq}\left(1 - \omega/\Omega_{eq}\right)} \quad \text{---(4.1)}$$

which, on integration gives diffusion curves given by

$$\frac{v_{\perp eq}^2}{1 - \omega/\Omega_{eq}} + v_{\parallel eq}^2 = \text{constt} \quad \text{---(4.2)}$$

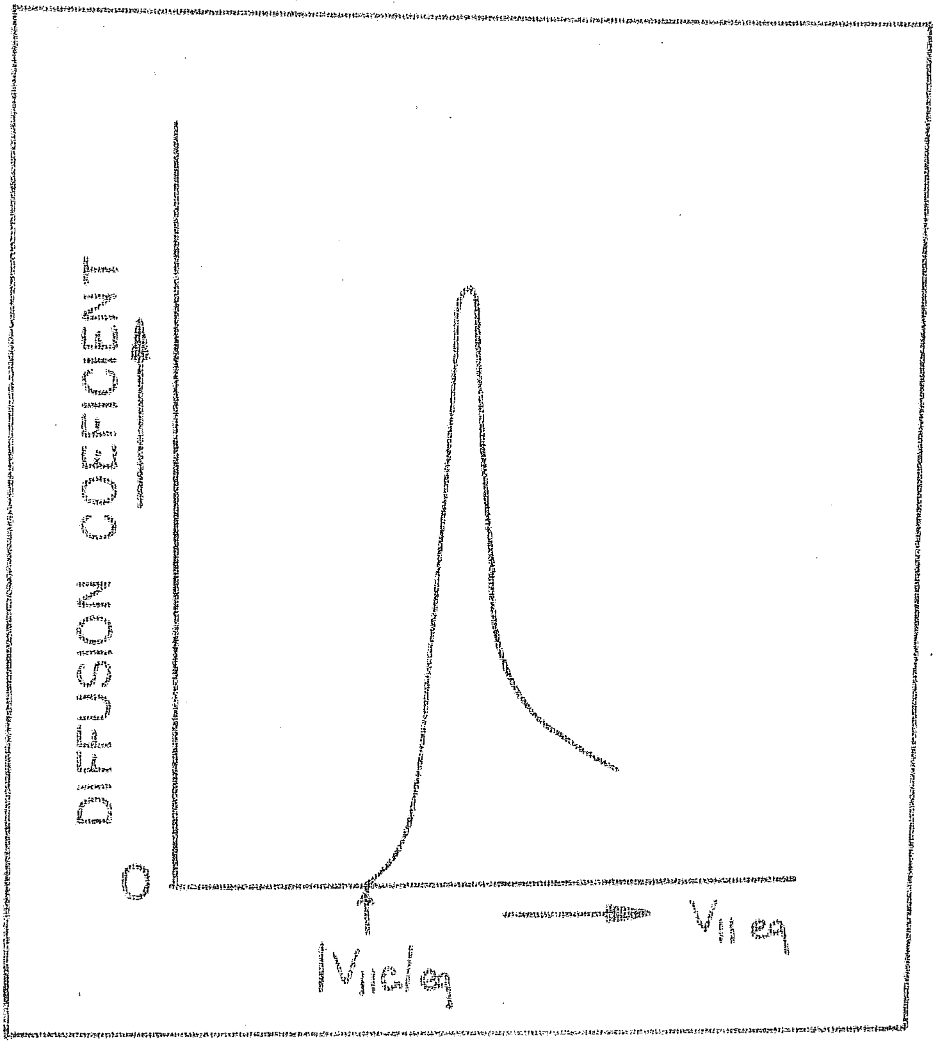

FIG 4.3

NATURE OF VARIATION OF DIFFUSION COEFFICIENT WITH $|V_{\parallel}|_{eq}$

(After Ashour Abdalla, 1972)

which represents a family of ellipses (see fig.4.4). Since a loss cone α_0 exists in the magnetospheric particle pitch angle distribution, the particles lying close to the loss cone boundary would slowly diffuse into the loss cone along the diffusion curves and get lost. Since the diffusion co-efficients are quite large at $v_{\parallel eq} = |v_{\parallel G\, eq}|$ in the velocity space, the most efficient loss of particles would occur in a region which is close to the point of intersection of the lines $\alpha = \alpha_0$ and $v_{\parallel eq} = |v_{\parallel G\, eq}|$. The fast diffusion occuring there will lead to the formation of a slot in the velocity distribution of the particles. The large negative value of the gradient of f with $|v_{\parallel}|$ near the slot leads to large growth rates for resonant waves at corresponding frequencies (fig.4.5).

She considers the effects of resonances to be similar to those of collisions as far as the changes in the energy W and the magnetic moment μ of the particles are concerned. The analogy is further strengthened by the assumption that the distribution of phase angle between the velocity vector and the wave

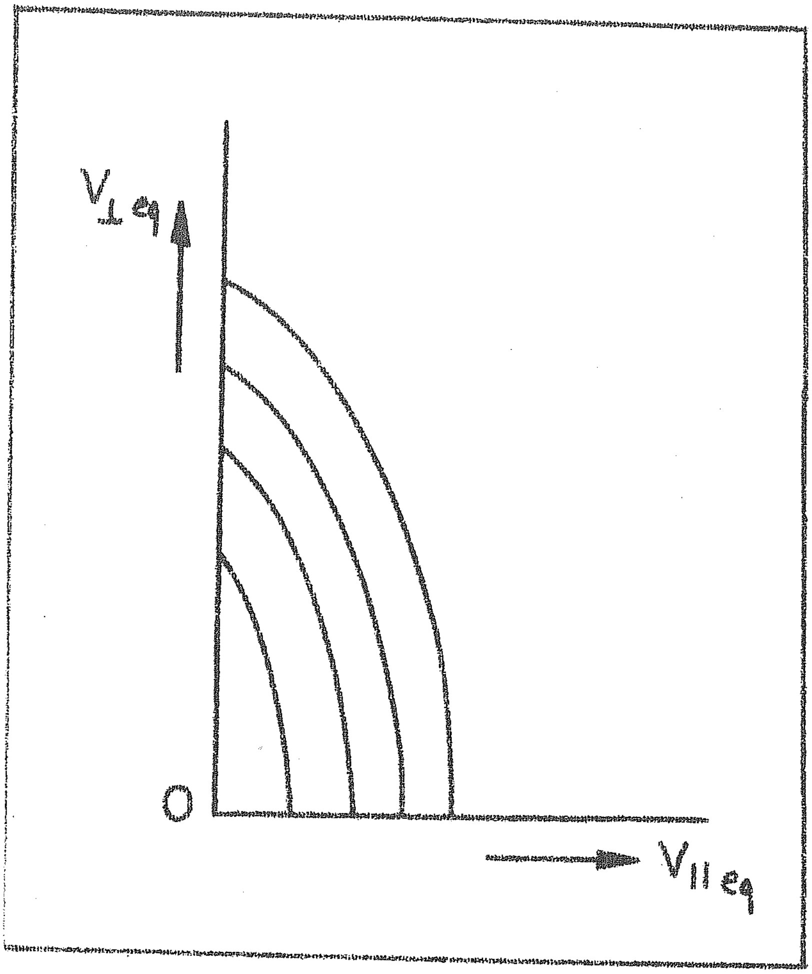

FIG 4.4

NATURE OF THE EQUATORIAL GYRORESONANCE DIFFUSION CURVES IN $V_{\parallel} - V_{\perp}$ SPACE

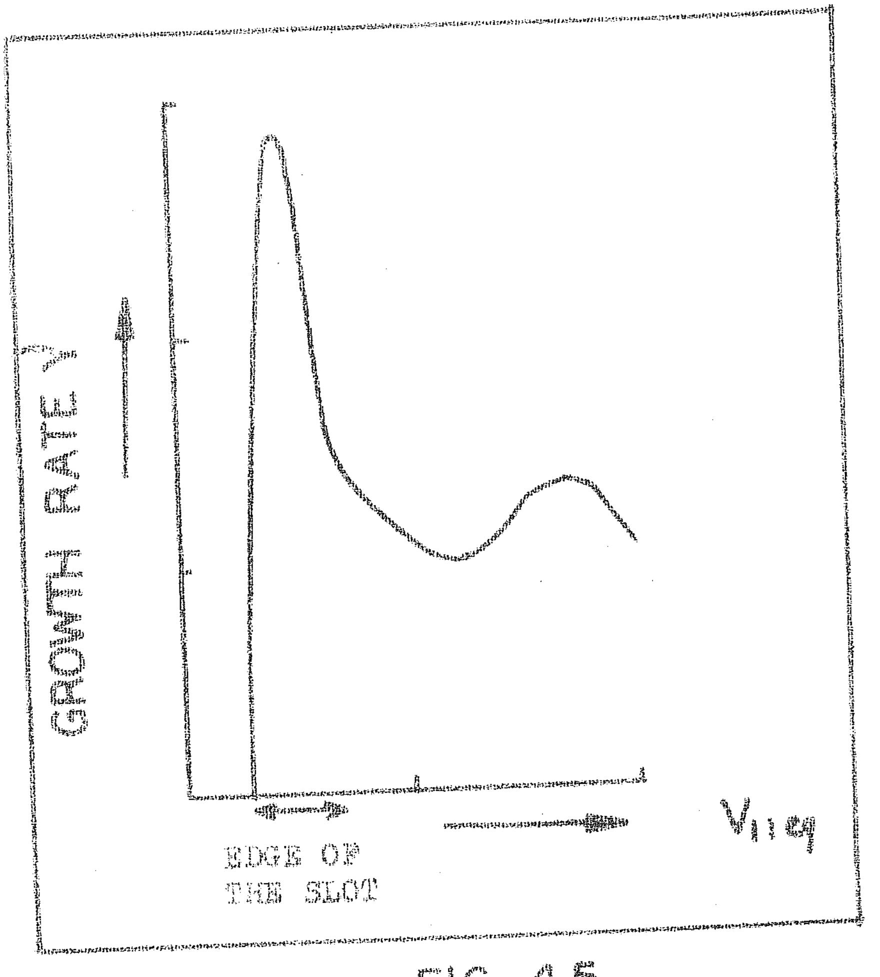

FIG 4.5

TYPICAL BEHAVIOUR OF GROWTH RATE γ WITH VARIATION IN $V_{\parallel\, eq}$

(Based on the calculations of Ashour Abdalla, 1972) field vector is random and therefore changes in W and μ are also random. Thus the analysis of the process can be handled using the one dimensional Fokker Planck equation.

4.4. A PARALLEL MODEL FOR LANDAU RESONANT INTERACTION:

The Landau resonant velocity $v_{\parallel L} = \frac{\omega}{k_{\parallel}}$ increases as we move away from the equator along a field line. On the other hand, particle $v_{\parallel}$ will decrease as it moves away from the equator along the same field line. The behaviour of $v_{\parallel L}$ here, is similar to that of $|v_{\parallel G}|$ illustrated in figure (4.2). It is evident that a particle must have a $v_{\parallel eq}$ greater than $v_{\parallel L eq}$ in order to resonate at some point or other on the field line with a given wave. This would mean that for values of $v_{\parallel eq} < v_{\parallel L eq}$, the diffusion co-efficients in velocity space, caused by the Landau resonant interaction would be practically zero. For $v_{\parallel eq} \geq v_{\parallel L eq}$ the value of the diffusion co-efficient will depend upon the position of the point on the field line at which the resonance takes place. The resonance for $v_{\parallel eq} \simeq v_{\parallel L eq}$ will take place near the equator and would therefore be strong and will last longer giving large diffusion co-efficients. The resonance for $v_{\parallel eq} \gg v_{\parallel L eq}$ would take place quite away from the equator and its effective duration would be small because of the dominance of the nonuniform magnetic field. This would lead to comparatively smaller diffusion co-efficients. Thus the nature of the diffusion co-efficient curve plotted against the distance from equator remains same as in the gyroresonance case (fig.4.3).

It is found that the changes in $v_{\perp}$ during Landau resonance are negligible and $\frac{dv_{\perp eq}}{dv_{\parallel eq}}$ may be considered nearly equal to zero. Thus the resonant diffusion of particles will occur along the straight lines given by $v_{\perp eq}$ = constant. The region of strongest diffusion will lie in the close proximity of the point of intersection of the lines given by $v_{\parallel eq} = v_{\parallel L eq}$ and $\alpha = \alpha_0$ which defines the loss cone. Through a mechanism similar to the one discussed by Abdalla for gyro resonant case, a slot will be developed in the distribution function in velocity space for the Landau resonance case also.

The negative gradients of f with $v_{\parallel}$ developed at the lower $v_{\parallel}$ boundary of the Landau resonant slot will give growth to the perturbations at corresponding frequency.

The two slots are, in general, formed at two different places in the velocity space, one near $v_{\parallel eq} = |v_{\parallel G eq}|$ and the other near $v_{\parallel eq} = v_{\parallel L eq}$. If $v_{\parallel L eq}$ and $|v_{\parallel G eq}|$ are equal, the two slots will overlap and the effect of the slots on the growth rate shall be enhanced. This might cause the favourable emissions at $\omega = \frac{1}{2}\Omega_{eq}$.

4.5 DUNGEY'S SUGGESTION AND ITS CRITICAL APPRECIATION:

Dungey had suggested earlier that for pulses with their central frequency $\omega = \frac{1}{2}\Omega_{eq}$, the effect of the Landau resonant diffusion would reinforce the effect of the gyroresonant diffusion in distorting the distribution function and thus enhancing the growth of the perturbations. This is because the Landau resonant and the gyroresonant particle speeds are same. However, the Landau resonance effects are usually weak compared to the gyroresonant effects mainly because the propagation angle θ for whistler mode waves is small and for small values of θ, $E_{\parallel}$, the parallel component of the wave electric field $\vec{E}$ is rather small compared to $E_{\perp}$, the perpendicular component of $\vec{E}$.

Under such circumstances, contribution of Landau resonant effect to the gyroresonant effect at $\omega = \frac{1}{2} \Omega_{eq}$ may not be able to account for the observed favourable emissions at this frequency. Unless there is some other process which strengthens the Landau resonance effects at $\omega = \frac{1}{2} \Omega_{eq}$. The simple combinations of Landau and the gyro resonances may not be able to explain the preferential triggering of VLF emissions at $\omega = \frac{1}{2} \Omega_{eq}$.

4.6 THE GROUP RESONANCE OF A WAVEPACKET AT $\omega = \frac{1}{2}\Omega$.

The effect of a resonance would be strengthened if the waves at different frequencies inside a wavepacket simultaneously resonate with the same particle and thus reinforce the effect of each other. We would like here to investigate the condition governing the simultaneous resonance of a group of whistler mode waves with the plasma particles.

A wave with frequency ω and wave vector $\vec{k}$ inside a wave packet will experience 'Landau resonance' (hereafter in this section mentioned only as 'resonance') with a particle if

$$\omega - k v_{\parallel} \cos\theta = 0 \qquad \text{---(4.3)}$$

where $v_{\parallel}$ is the particle velocity component along $\vec{B}o$ and θ the angle between $\vec{k}$ and $\vec{B}o$.

Another wave in the same wave packet with frequency $\omega + \Delta\omega$ and wave vector $\vec{k} + \Delta\vec{k}$ will have resonance with the same particle if

$$(\omega + \Delta\omega) - (k + \Delta k)\, v_{11} \cos\theta = 0 \qquad \text{---(4.4)}$$

Assuming $\Delta \vec{k}$ to be in the same direction as $\vec{k}$ and solving equations (4.3) and (4.4) together, we get

$$\Delta\omega = v_{11}\, \Delta k \cos\theta$$

or, $$\frac{\Delta\omega}{\Delta k} = v_{11} \cos\theta \qquad \text{---(4.5)}$$

From equation (4.3),

$$v_{11} \cos\theta = \frac{\omega}{k}$$

Substituting this in eq.(4.5) we get

$$\frac{\Delta\omega}{\Delta k} = \frac{\omega}{k}$$

which in the limit $\Delta\omega \to 0$ and $\Delta k \to 0$ passes over to

$$\frac{d\omega}{dk} = \frac{\omega}{k} \qquad \text{---(4.6)}$$

Equation (4.6) can be interpreted as the following:

'All the waves within a narrow band wavepacket will resonate with the same particle at the same time if the condition

$$v_{\parallel} \cos\theta = v_{po} = v_g \qquad \text{—— (4.7)}$$

is satisfied where v_g is the group velocity of the wavepacket and v_{po}, the phase velocity of the central wave of the wavepacket'.

Effectively speaking, the whole group of waves inside the pulse comes in resonance with the same set of particles thereby making the interaction stronger and more effective.

The condition $v_{po} = v_g$ may not always be satisfied for all types of plasma waves. Nevertheless, for whistlers, a frequency exists at which the condition is fulfilled. The cold plasma whistler mode dispersion relation

$$\frac{c^2 k^2}{\omega^2} = \frac{\omega_p^2}{\omega(\Omega \cos\theta - \omega)}$$

leads to the following expressions for v_{po} and v_g :

$$v_{po} = \frac{c}{\omega_p} \left\{ \omega (\Omega \cos\theta - \omega) \right\}^{1/2} \quad (4.8)$$

and

$$v_g = v_{po} \frac{\Omega \cos\theta - \omega}{\frac{1}{2} \Omega \cos\theta} \quad (4.9)$$

From (4.8) and (4.9), it is clear that $v_{po} = v_g$ at $\omega = \frac{1}{2} \Omega \cos\theta$. Since θ is small, a whistler mode pulse centred around $\omega \simeq \frac{1}{2} \Omega_{eq}$ would undergo most efficient Landau resonance with electrons having appropriate $v_{\parallel}$. We will refer to this phenomenon of the resonance of the whole group of waves inside a narrow band pulse as the 'group resonance' or 'pulse resonance' here after.

4.7 CONCLUSIONS:

Thus we have seen how the group resonance of a wave packet comes into picture when the condition $v_{po} = v_g$ is satisfied. The group resonance essentially means the reinforcement of the effects of different waves by one another and this feature is absent in the ordinary resonance of a wave with particles.

Here the important point is that the range of particle $v_{\parallel}$ affected by the waves is quite narrow. The whole spectrum of waves exerts its effect on particles in this narrow range thereby making the resonant effects quite strong.

Thus, although the Landau resonance of whistlers is, in general, weak, it becomes quite strong at $\omega = \frac{1}{2} \Omega$. In the light of this fact we must reconsider Dungey's suggestion that the joint effect of the Landau and the gyro resonances might account for the favourable triggering at $\omega = \frac{1}{2} \Omega_{eq}$. But before studying the effects of the two together let us consider qualitatively how the group Landau resonance can give rise to emissions.

The Landau resonance will cause diffusion of particles in velocity space along diffusion curves given by $v_\perp$ = constt. where different values of the constant refer to different members of the family (see fig.4.4). The maximum diffusion will take place near the group resonant $v_{\parallel}$. The diffusion at all other places can be neglected in comparison to this. The direction of diffusion will be from the contours of higher f values to those of the lower f values and therefore it will point towards the loss cone boundary (see fig.4.6). The region close to the point of intersection of group resonant $v_{\parallel}$ and the loss cone boundary would be affected to the maximum limit. Particles from this region will migrate into the loss cone and consequently a slot will be developed at that point which will give growth to perturbation at the corresponding frequency.

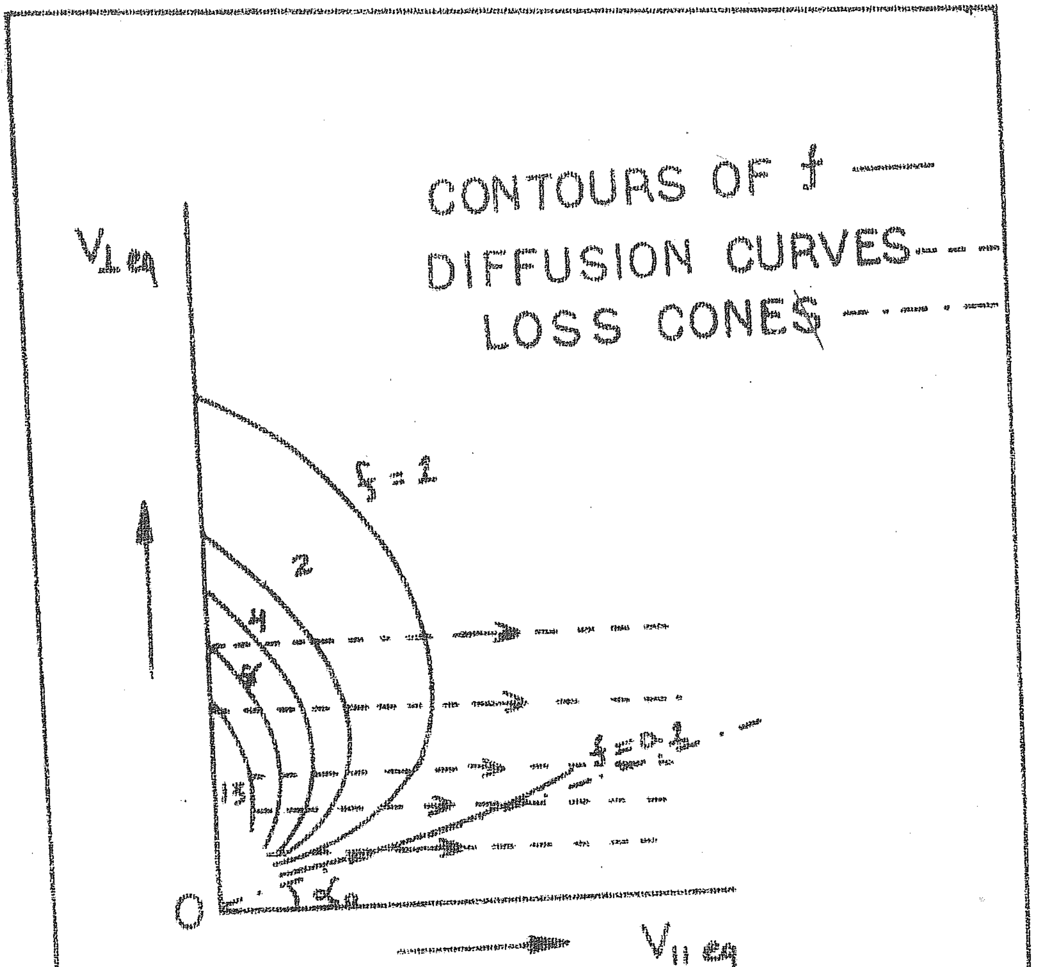

FIG 4.6

CONTOURS OF CONSTANT f AND DIFFUSION CURVES FOR EQUATORIAL LANDAU RESONANCE. THE DIRECTION OF DIFFUSION IS INDICATED BY ARROWS.

(Contours of f are after Ashour Abdalla, 1972)

CALCULATION OF GROWTHRATE:

The slot is formed in the neighbourhood of $v_{\parallel} = v_{\parallel GLR}$ (Group Landau resonant velocity). The distribution $f(v_{\parallel}, v_{\perp})$ for $v_{\perp} \simeq v_{\parallel GLR} \tan \alpha_0$ suffers a steep fall in its value with a slight increase in $v_{\parallel}$ at the lower edge of the slot. This behaviour of f may be approximated by a step of f at $v_{\parallel} = v_{\parallel GLR}$ and therefore the partial derivative $\left.\frac{\partial f}{\partial v_{\parallel}}\right|_{v_{\parallel} = v_{\parallel GLR}}$ may be represented by $-\Delta f_{\parallel}\, \delta(v_{\parallel} - v_{\parallel GLR})$ where $-\Delta f_{\parallel}$ is the step and the symbol δ represents the Dirac δ function. The quantity $\Delta f_{\parallel}$ has been considered to be positive and the negative sign has been attached to it to indicate that the step is negative.

During Landau resonance, the changes in $v_{\perp}$ are very small and therefore the temporal changes in the partial derivative of f with $v_{\perp}$ will be negligible. Again during the gyroresonance the counters of constant f (Ashour Abdalla, 1972, see fig.4.7) at the slot exhibit the gradient direction to be parallel to the $v_{\parallel}$ axis.

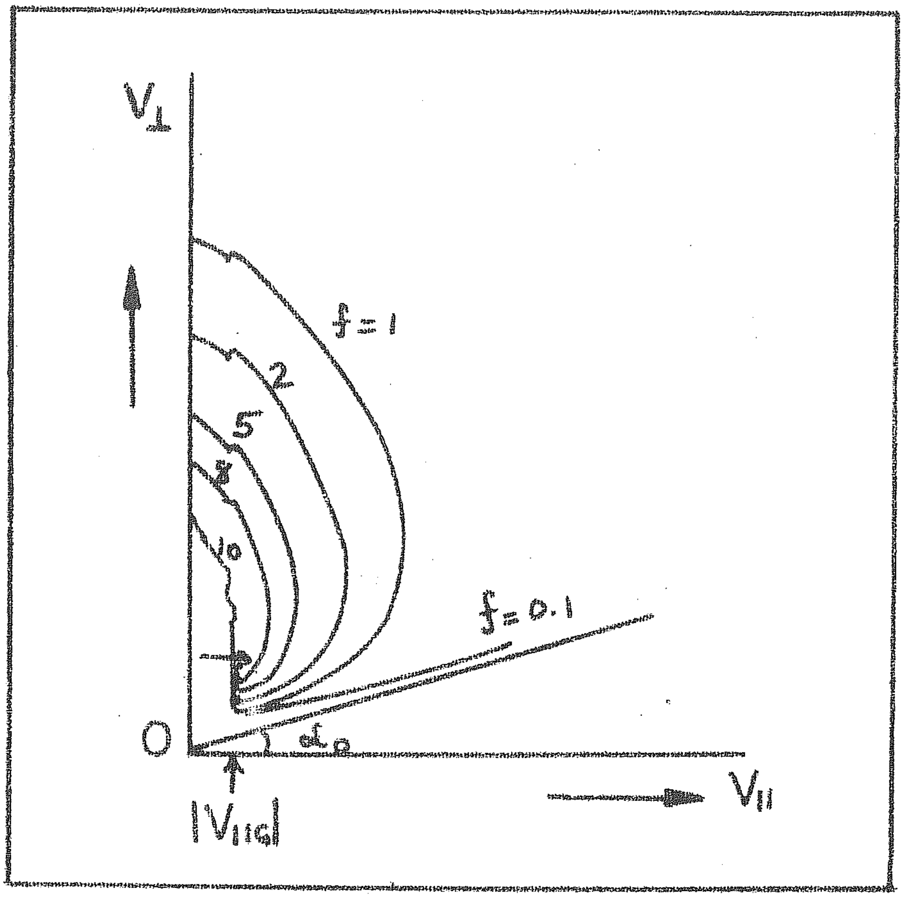

FIG 4.7

TYPICAL CONTOURS OF CONSTANT f IN A DISTRIBUTION MODIFIED BY GYRORESONANT DIFFUSION. VERTICAL ARROW INDICATES THE POSITION OF SLOT AND THE HORIZONTAL ARROW POINTS TO THE DIRECTION OF GRADIENT OF f .

(Contours of f are after Ashour Abdalla, 1972)

However, because of the presence of the loss cone, f will have a step at all $v_\perp$'s given by $v_\perp = v_\parallel \tan\alpha_0$. This step will be positive and we will denote its value at $v_\parallel = v_{\parallel GLR}$ by $+\Delta f_\perp$ and replace $\frac{\partial f}{\partial v_\perp}$ by $\Delta f_\perp \, \delta(v_\perp - v_\parallel \tan\alpha_0)$ for the sake of approximate growth rate calculations.

The gyroresonant growth rate γ at the edge of the slot in the distribution function for a VLF whistler mode perturbation in the system under consideration is given by (Vedenov et al. 1972):

$$\gamma = -\frac{\pi k^2 v_{res}^2}{n\Omega} \int_0^\infty \left[v_\perp \frac{\partial f}{\partial v_\parallel} - (v_\parallel - v_p) \frac{\partial f}{\partial v_\perp} \right]_{v_\parallel = v_{res}} v_\perp^2 \, dv_\perp \quad (4.10)$$

Here n represents the electron concentration and v_{res} the gyroresonant velocity $v_{\parallel G}$ for the perturbation. Since the resonance is taking place at $v_\parallel \simeq v_{\parallel GLR}$, which occurs at $\omega \simeq \frac{1}{2}\Omega$, $v_{\parallel G}$ and $v_{\parallel GLR}$ will be roughly equal to each other. Thus we have the relationship

$$v_{\parallel GLR} \simeq v_{res} \quad \text{---(4.11)}$$

Other symbols in equation (4.10) have their usual meanings as defined earlier.

Substituting $\frac{\partial f}{\partial v_{\parallel}}$ and $\frac{\partial f}{\partial v_{\perp}}$ in eq.(4.10) by their approximately equivalent δ function representations, $-\Delta f_{\parallel}\ \delta(v_{\perp} - v_{\parallel}\tan\alpha_0)$ and $\Delta f_{\perp}\ \delta(v_{\perp} - v_{\parallel}\tan\alpha_0)$ respectively in accordance with our previous discussion, we get:

$$\gamma = \frac{\pi k^2 v_{res}^2}{n\Omega}\int_0^{\infty} v_{\perp}^2 \left[v_{\perp}\Delta f_{\parallel} - (v_{\parallel} - v_p)\Delta f_{\perp}\right]_{v_{\parallel}=v_{res}} \cdot \delta(v_{\perp} - v_{\parallel}\tan\alpha_0)\, dv_{\perp} \quad \text{---(4.12)}$$

which, on integration gives:

$$\gamma = \frac{\pi k^2 v_{res}^2 v_{\perp}^2}{n\Omega}\left[v_{\perp}\Delta f_{\parallel} - (v_{\parallel} - v_p)\Delta f_{\perp}\right]_{v_{\parallel}=v_{res}} \quad \text{---(4.13)}$$

Using eq.(4.11) in conjunction with the following relations

(i) $v_\perp \simeq |v_{res}| \tan\alpha_0$ (at the slot)

and (ii)

$$\omega = \frac{1}{2}\Omega$$

$$K\Big|_{\omega=\frac{1}{2}\Omega} = \frac{\omega_p}{c}$$

$$v_{res}\Big|_{\omega=\frac{1}{2}\Omega} = -\frac{1}{2}\frac{c\Omega}{\omega_p}$$

$$v_p\Big|_{\omega=\frac{1}{2}\Omega} \equiv \frac{1}{2}\frac{c\Omega}{\omega_p}$$

For the gyro-resonant perturbation assumed to be propagating $\parallel$ B_0

we can reduce eq.(4.13) to

$$\gamma = \frac{\pi}{16}\frac{\Omega^4 c^3 \tan^2\alpha_0}{n\omega_p^3}\left\{\frac{1}{2}\tan\alpha_0 . \Delta f_\parallel + \Delta f_\perp\right\} \qquad \text{---(4.14)}$$

which gives the growth rate at the edge of the slot. Evidently, γ increases with $\Delta f_\parallel$, the step in f at the slot which, in turn, depends upon the strength of the slot forming mechanism.

In the light of what has been discussed so far in this chapter, we can make the following concluding remarks:

The slots in the distribution will be formed in the regions in velocity space where the probability for the particles to be lost into the loss cone due to the diffusion by the waves is large compared to the remaining areas. A slot region would be strong if the probability for the loss of the particles is large as is the case with the group Landau resonance or with the gyro resonance. A slot region would be weak if the probability of loss of particles from there is small as is the case with ordinary Landau resonance.

In general, for a whistler mode wave-packet, there are two slot regions, one strong due to the gyroresonance and the other weak due to the Landau resonance. Also, they are located away from each other. However, if $\omega = \frac{1}{2}\Omega$, both the slot regions are strong and at the same time they overlap each other thereby greatly enhancing the probability of the loss of particles into the loss cone. This would lead to the formation of a steep edge and consequently to the development of a highly negative gradient of f with $|v_{\parallel}|$ at the lower $v_{\parallel}$ boundary of the slot. The large negative step thus formed should favour emissions at half the equatorial gyrofrequency (Vyas and Das, 1975b) which is supported by the observations (Helliwell, 1969).

CHAPTER V

THE EFFECT OF LANDAU DAMPING ON CYCLOTRON RESONANCE IN A UNIFORM MAGNETOPLASMA

5.1 INTRODUCTION:

In the last chapter we studied how the effects of the Landau resonance and the gyroresonance would combine together to give enhanced probabilities for emissions at half the equatorial gyrofrequency. A question naturally arises as to whether this is the unique mechanism that supports or favours emissions at this particular frequency, or there are other mechanisms which also contribute to this phenomenon. An attempt has been made in this chapter to answer this question. We take up one of the very important features of the whistler mode dispersion relation already mentioned in the last chapter, namely that the Landau resonant $V_{\parallel}$ has a maximum value that corresponds to a frequency $(1/2)\Omega_{eq}$. Before studying the consequences of this, we would like to investigate the properties of a distribution function distorted through a Landau resonant process by the whistler mode wave. In particular, we would be interested in studying how the gyro resonant perturbations would be affected by such a modified distribution and whether the growth of perturbations is likely under such situations. We would apply the results of our study to the specific case of whistler mode Landau resonance, which exhibits the unusual occurence of a maximum in the value of the resonant $V_{\parallel}$, to see if such a process is likely to contribute to emissions at half the equatorial gyrofrequency.

5.2 LANDAU DAMPING AND CYCLOTRON RESONANCE IN UNIFORM MAGNETOPLASMA:

We consider a homogenous collisionless hot plasma immersed in a uniform static magnetic field $\vec{B}_0$ acting along z direction. We also consider a spectrum of electron plasma waves propagating in the negative z direction. This is our zero order system and our interest lies in finding how a wide band VLF perturbation propagating in the whistler mode in positive z direction would be affected by this system.

The Landau resonance of the electron plasma waves would lead them to damping. The growth rate for such an interaction is given by (Vedenov, 1963)

$$\gamma_L = \frac{\pi}{2} \left\{ \omega_p^3 / k|k| \right\} \left\{ \partial f / \partial v_{\parallel} \right\} \Big|_{V_{\parallel} = V_{res}} \quad \text{---(5.1)}$$

where γ_L is the Landau growth rate, ω_p the electron plasma frequency and, $\vec{k}$, $\vec{v}$ and the subscript '$\parallel$' are used to refer to the wave vector, the particle velocity and the parallel component of a vector respectively, as also used in earlier chapters. However, here we use the symbol V_{res} for the phase velocity $\frac{\omega_p}{k}$ of the electron plasma waves. Since $\vec{k}$ for these waves has been assumed to be negative, V_{res} would also be negative. 'f' represents the normalised distribution function of plasma electrons with respect to $V_{\parallel}$ and $V_{\perp}$, unless otherwise specified through the use of proper arguments, e.g. $f(\alpha, v, t)$. $f(v_{\parallel}, v_{\perp})$ satisfies the following condition:

$$\iint f(v_{\parallel}, v_{\perp})\, v_{\perp}\, dv_{\perp}\, dv_{\parallel} = 1$$

To study the dynamics of the resonant interactions in the system, we would like to apply quasilinear theory. The theory considers that the plasma is in thermal equilibrium and the waves cause only a perturbation to its equilibrium state. Furthermore, it takes into account only the wave particle interactions and neglects both the wave-wave interactions and the particle-particle interactions. Therefore, the validity of the theory is subject to the restriction that the wave energy density in the system is much smaller compared to the thermal energy density of the particles and at the same time it is much larger compared to the energy density associated with the Coulomb interaction of the particles in the system. We will assume that the system under our consideration fulfills this condition.

The basic idea of quasilinear theory is that the growth or damping of plasma waves caused by a particular distribution function would slowly modify the distribution itself and then this modified distribution, in turn, modifies the growth or damping of the waves. The process continues unless some stable equilibrium state between the waves and the plasma is reached.

To apply quasilinear theory we divide the distribution function of the resonant particles into two parts: one rapidly oscillating and another slowly varying with time. The coupling of the mean square of the oscillating part with the slowly varying part leads us to the fact that the behaviour of the slowly varying part, $\bar{f}$, of the distribution can be described by the following equation of diffusion in phase space (Vedenov, 1963)

$$\frac{\partial \bar{f}}{\partial t} = \frac{\partial}{\partial v_{\parallel}} \left\{ \frac{4\pi e^2}{m^2 v_{\parallel}} \frac{E_k^2}{8\pi} \frac{\partial \bar{f}}{\partial v_{\parallel}} \right\} \quad \text{---} (5.2)$$

where t represents time, e and m the electron charge and mass and $E_k^2/8\pi$ the energy density of the waves. The growth rate γ_L of plasma waves will also vary with time as given by equation (5.1) where $\bar{f}$ is considered as the nonoscillating part of the distribution, varying slowly with time.

The growth rate γ_c for the whistler mode perturbations as a result of cyclotron resonance with electrons is given by (Kennel and Petschek, 1966).

$$\gamma_c = \pi\Omega\,(1-\omega/\Omega)^2\,\eta(V_R)\left\{A(V_R)-\omega/(\Omega-\omega)\right\} \qquad (5.3)$$

where ω and Ω are the wave frequency and the electron gyrofrequency, V_R, is the cyclotron resonant velocity which is negative in magnitude and is given by the relationship

$$\omega - kV_R = \Omega$$

also

$$\eta(V_R) = 2\pi\left\{\frac{\Omega-\omega}{k}\right\}\int_0^\infty v_\perp dv_\perp\, f(v_\perp, v_\parallel = V_R) \qquad (5.3a)$$

and

$$A(V_K) = \left[\frac{\left\{ \int_0^\infty v_\perp \, dv_\perp \left(v_\parallel \frac{\partial f}{\partial v_\perp} - v_\perp \frac{\partial f}{\partial v_\parallel} \right) \frac{v_\perp}{v_\parallel} \right\}}{\left\{ 2 \int_0^\infty v_\perp \, dv_\perp f \right\}} \right]_{V_\parallel = V_K} \qquad (5.3b)$$

The cyclotron growth of the perturbation would affect the particle pitch angle distribution function. A quasilinear treatment of this process would lead us to the following expression for the time evolution of the slowly varying part of the distribution function (Kennel and Petschek, 1966).

$$\frac{\partial f(\alpha, v, t)}{\partial t} = \frac{\pi e^2}{m^2 \sin\alpha} \frac{\partial}{\partial \alpha} \left\{ \frac{\sin\alpha \, B_K^2}{|v \cos\alpha|} \frac{\partial f}{\partial \alpha} \right\} \qquad (5.4)$$

Here α is the pitch angle of the particles and $B_K^2/8\pi$ the magnetic energy density of the whistler mode waves. The expression has been derived under the assumption $\omega \ll \Omega$. $v_\parallel$ and $v_\perp$ of the particles are related to V and α through the relations

$$v_{\parallel} = v \cos \alpha$$

and, $$v_{\perp} = v \sin \alpha$$

If, in our system, the range of velocities which the Landau resonance takes place has an overlap with the range of velocities having gyroresonance, the change in the distribution function over the overlapping range of $V_{\parallel}$ would be simultaneously affected by the two types of resonant interactions. Under these circumstances the time evolution of the distribution function can be obtained by solving equations (5.2) and (5.4) together as a pair of simultaneous partial differential equations.

An analytical solution of these two equations is quite complicated and the numerical solution would also be highly time consuming even on a high speed computer. Therefore here we would try to analyse the problem in a somewhat qualitative manner so as to understand how the system would behave as the time passes.

The gyroresonant growth rate γ_C as defined in equation (5.3) is a function of $\partial f / \partial v_{\parallel}$ which itself changes with time as a result of the Landau resonant interaction. This change in γ_C affects the pitch angle distribution of the particle which then also starts varying with time in accordance with equation (5.4). The role of this equation (derived with the assumption $\omega \ll \Omega$) is to isotropize the distribution function in pitch angle. In general, the distribution function becomes constant along the diffusion curves in velocity space. Or, in other words, the diffusion curves become the contours of constant f. Thus the whole pitch angle distribution $f(\alpha)$ is reshaped. Now $f(\alpha)$ is a function of the distribution with respect of $V_{\parallel}$ and distribution with respect to $V_{\perp}$. Thus a change in $f(\alpha)$ and $\frac{\partial f}{\partial \alpha}$ would cause a corresponding change in $f(V_{\parallel})$ and $\frac{\partial f}{\partial v_{\parallel}}$ respectively. This change in $\partial f / \partial v_{\parallel}$ would again modify the Landau resonant growth rate γ_L. Thus a coupling is established between the tow types of growths.

In order to simplify the situation, we will introduce here certain restrictions. We presume that to start with, the whistler mode perturbations have amplitudes sufficiently small such that any change in the particle distribution caused by them is negligible compared to that caused by the Landau resonant interaction. With time the perturbations will grow and after some time they might become strong enough to react back and modify the particle distribution. This situation would be quite complicated to study, and therefore, we will not consider it here. We will restrict our attention only to that stage of interaction where the growth of the perturbations is insufficient to react back on the distribution function and cause any significant modification therein. This restriction will enable us to consider only the effect of the electron plasma waves on the growth of whistler mode perturbations neglecting the reciprocal effect of the whistler mode perturbations on the growth of electron plasma waves.

It is well known that the quasilinear diffusion process due to the Landau resonance of electron plasma waves leads to the formation of a ledge in the electron distribution function with respect to $V_{\parallel}$ over the range of resonant velocities (see fig.5.1). 'k' being negative, the ledge forms over negative values of $V_{\parallel}$ and the distribution f becomes constant over the length of the ledge. Therefore, $\partial f / \partial v_{\parallel}$ would become zero over the entire range of resonant $V_{\parallel}$ except at the boundaries of this region where steep gradients of f with $V_{\parallel}$ will be formed and the value of $\partial f / \partial v_{\parallel}$ shall become very large as shown in figure 5.1.

Under the assumption $\omega \ll \Omega$ equation (5.3) can be rewritten as

$$\gamma_c = \pi \Omega \eta(V_R) A(V_R)$$

where $\eta(V_R)$ and Ω are always positive and only the positive or negative sign of the value of $A(V_R)$ will decide whether the waves are growing or damping. Taking a distribution in which f monotonically decreases with increase in $|v_{\parallel}|$ and $v_{\perp}$, and noting that V_R is negative, we can reduce equation (5.3b) to the following form:

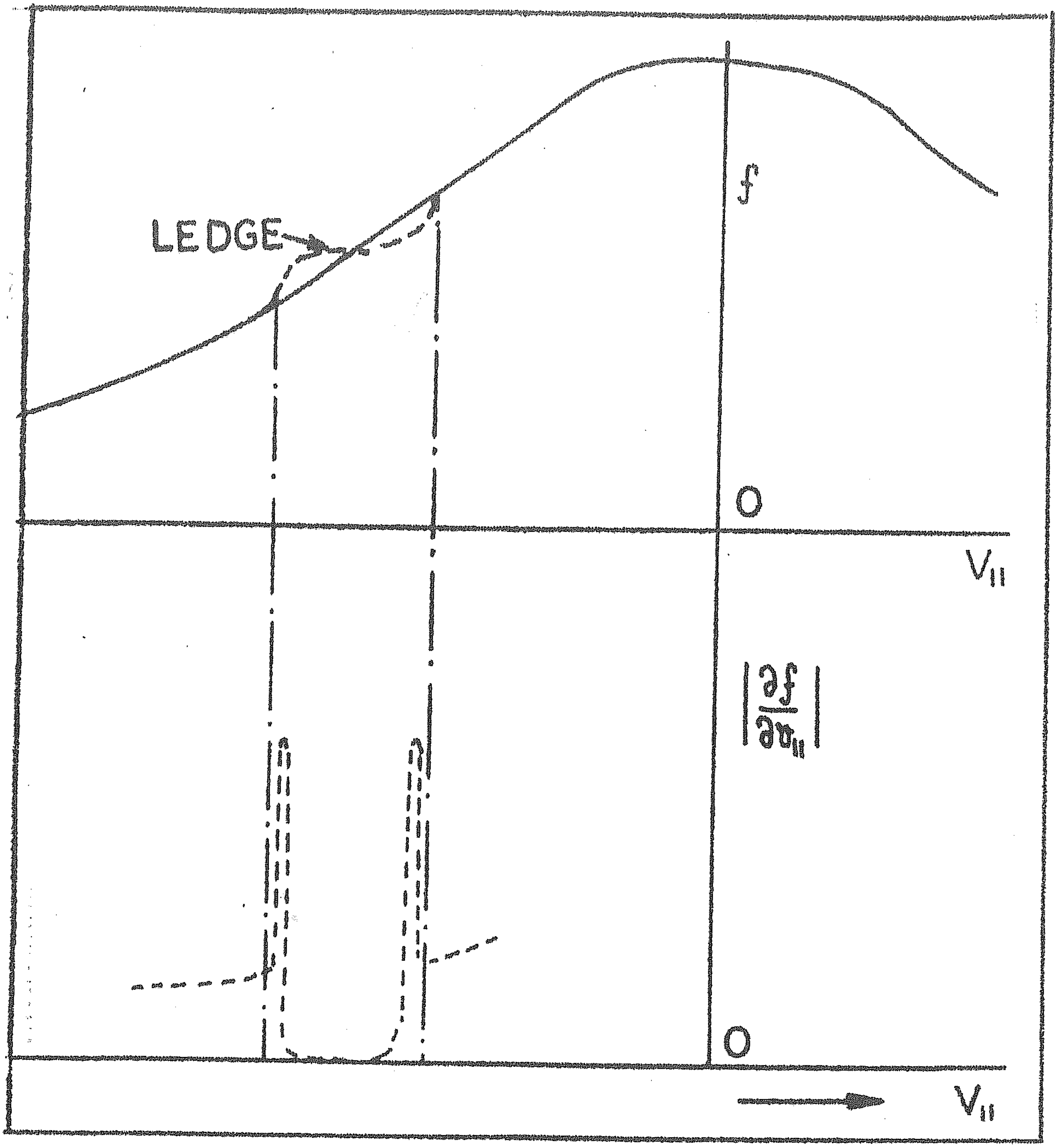

FIG 5.1

f AND $\frac{\partial f}{\partial V_{\parallel}}$ PLOTTED AGAINST $V_{\parallel}$ DURING THE LEDGE FORMATION. STEEP RISE IN f AT THE BOUNDARIES OF THE LEDGE GIVES THE PEAKS IN $\partial f/\partial V_{\parallel}$

$$A(V_R) = \left[\int_0^\infty v_\perp^2 dv_\perp \left\{ \frac{v_\perp}{|v_{\parallel}|} \left|\frac{\partial f}{\partial v_{\parallel}}\right| - \left|\frac{\partial f}{\partial v_\perp}\right| \right\} \Big/ \left\{ 2 \int_0^\infty v_\perp dv_\perp f \right\} \right]_{v_{\parallel} = V_R} \quad \text{---(5.5)}$$

Now, if we assume initial distribution to be isotropic i.e.

$$\left.\frac{\partial f}{\partial \alpha}\right|_{t=0} = 0 \quad \text{or,} \quad v_\perp \left.\frac{\partial f}{\partial v_{\parallel}}\right|_{t=0} = v_{\parallel} \left.\frac{\partial f}{\partial v_\perp}\right|_{t=0}$$

$$\text{or,} \quad \frac{v_\perp}{|v_{\parallel}|} \left|\frac{\partial f}{\partial v_{\parallel}}\right|_{t=0} = \left|\frac{\partial f}{\partial v_\perp}\right|_{t=0}$$

$A(V_R)$ becomes zero at $t = 0$ and therefore, the growth rate would be zero to start with. $\partial f / \partial v_\perp$ will, however, not be affected by the Landau resonant interaction.

Thus

$$\left|\frac{\partial f}{\partial v_\perp}\right|_t = \left|\frac{\partial f}{\partial v_\perp}\right|_0 = \frac{v_\perp}{v_{\parallel}} \left|\frac{\partial f}{\partial v_{\parallel}}\right|_{t=0}$$

Substituting this into equation (5.5), we get

$$A(V_R) = \left. \frac{\int_0^\infty \frac{v_\perp^3}{|v_{\parallel}|} dv_\perp \left\{ \left| \frac{\partial f}{\partial v_\parallel} \right|_t - \left| \frac{\partial f}{\partial v_\parallel} \right|_0 \right\}}{2 \int_0^\infty v_\perp \, dv_\perp \, f} \right|_{v_\parallel = V_R} \qquad (5.6)$$

As time goes, $\left| \frac{\partial f}{\partial v_\parallel} \right|$ goes on diminishing at the centre of the disturbed region and increasing at the boundaries because of the Landau resonant interaction. Hence the value of γ_c would become negative in the central part of the ledge and positive at its boundaries. So the VLF perturbations would damp or grow depending upon whether they resonate with the particles lying at the centre of the ledge or at the boundary of the ledge in the distribution function.

Thus we see that the growth or the damping of the perturbations depends upon how the ledge develops with time in the distribution and consequently it depends upon how $\partial f(v_\parallel, v_\perp, t)/\partial v_\parallel$ would vary with t.

The maxima occuring in $\partial f(v_{\parallel}, v_{\perp}, t)/\partial v_{\parallel}$ would correspond to the frequencies of maximum growth at time t and we would be mainly interested in the growth of the waves at these frequencies.

The evolution of these peaks can be qualitatively studied from fig.(5.2) which shows the different stages of ledge formation.

For a time period over which the assumptions made earlier in this chapter remain valid, the behaviour of the peaks in $\partial f(v_{\parallel}, v_{\perp}, t)/\partial v_{\parallel}$ with $V_{\parallel}$ and t can be stated as below:

i) They grow in height with time.

ii) Their width decreases with time i.e. they become sharper, with time, and

iii) Their position with respect to $V_{\parallel}$ changes with time - they move more and more towards the boundaries of the disturbed region.

Ideally speaking, as time progresses, the peaks would tend to become infinitely narrow and infinitely

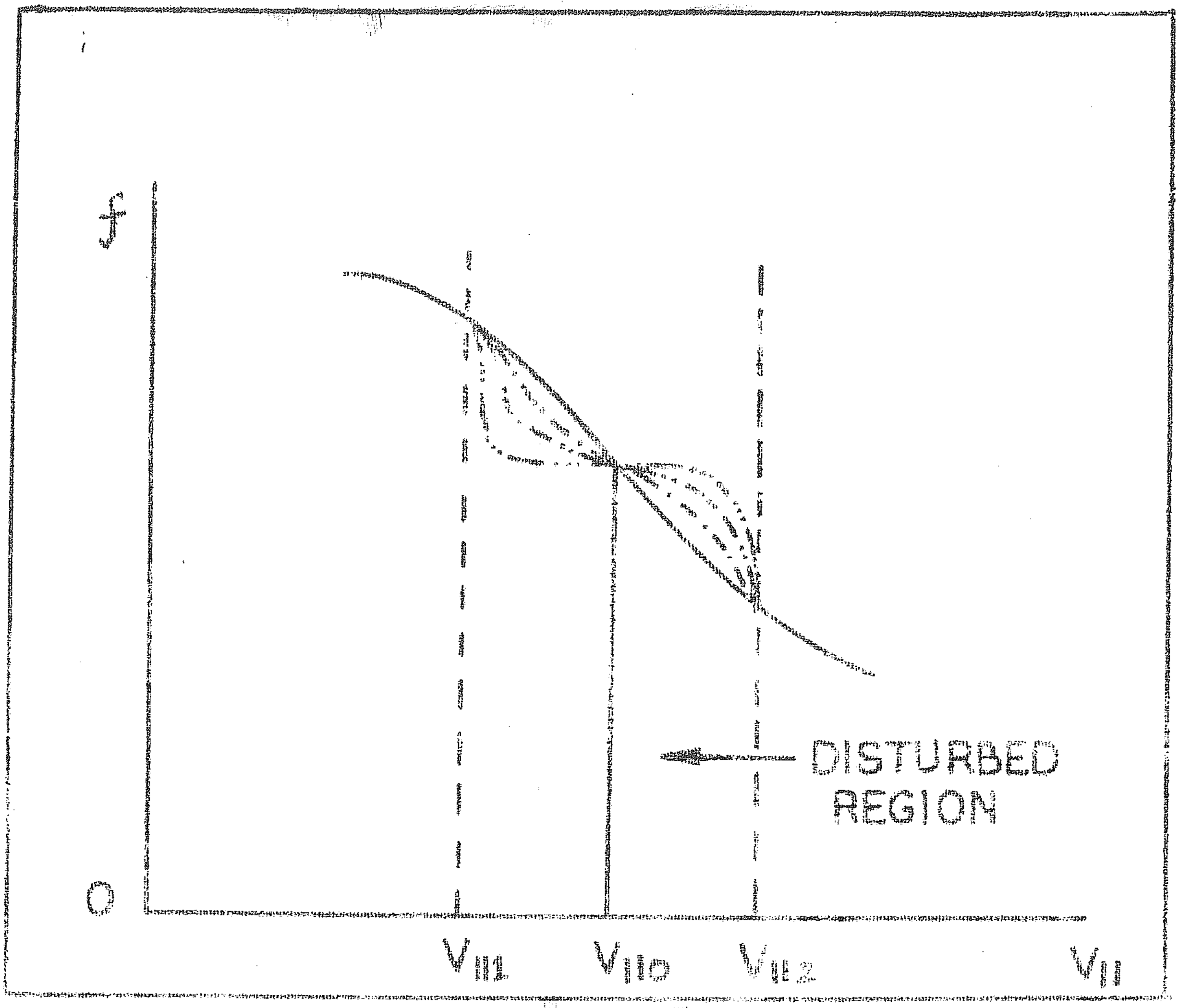

FIG 5.2

THE DIFFERENT STAGES DURING THE DEVELOPMENT OF THE LEDGE IN THE DISTURBED REGION OF THE VELOCITY DISTRIBUTION ARE INDICATED BY SINGLE DOTTED, DOUBLE DOTTED AND TRIPLE DOTTED LINES ARRANGED RESPECTIVELY IN CHRONOLOGICAL ORDER large and their position would tend to stabilise close to the edges of the disturbed region (see fig.5.2 and 5.3).

This shows that the growth of a wave at a certain frequency would be time dependent. Initially, the growth of the VLF perturbations will be at frequencies that correspond to the V_R lying in the central part of the disturbed region. But as time passes, the frequency of maximum growth would drift to values that correspond to velocities closer and closer to the edges of the disturbed region. Simultaneously the width in frequency of the growing waves would go on narrowing down and the magnitude of the growth rate will increase (fig.5.3).

The speculative picture presented above is linear in the sense that the amplitude of VLF wave is assumed not to have grown to an extent that can significantly react back on the particle distribution.

The whole treatment carried out so far is under the assumption that the wave frequency ω is very small compared to the particle gyrofrequency Ω. Under this condition, the initial growth rate becomes

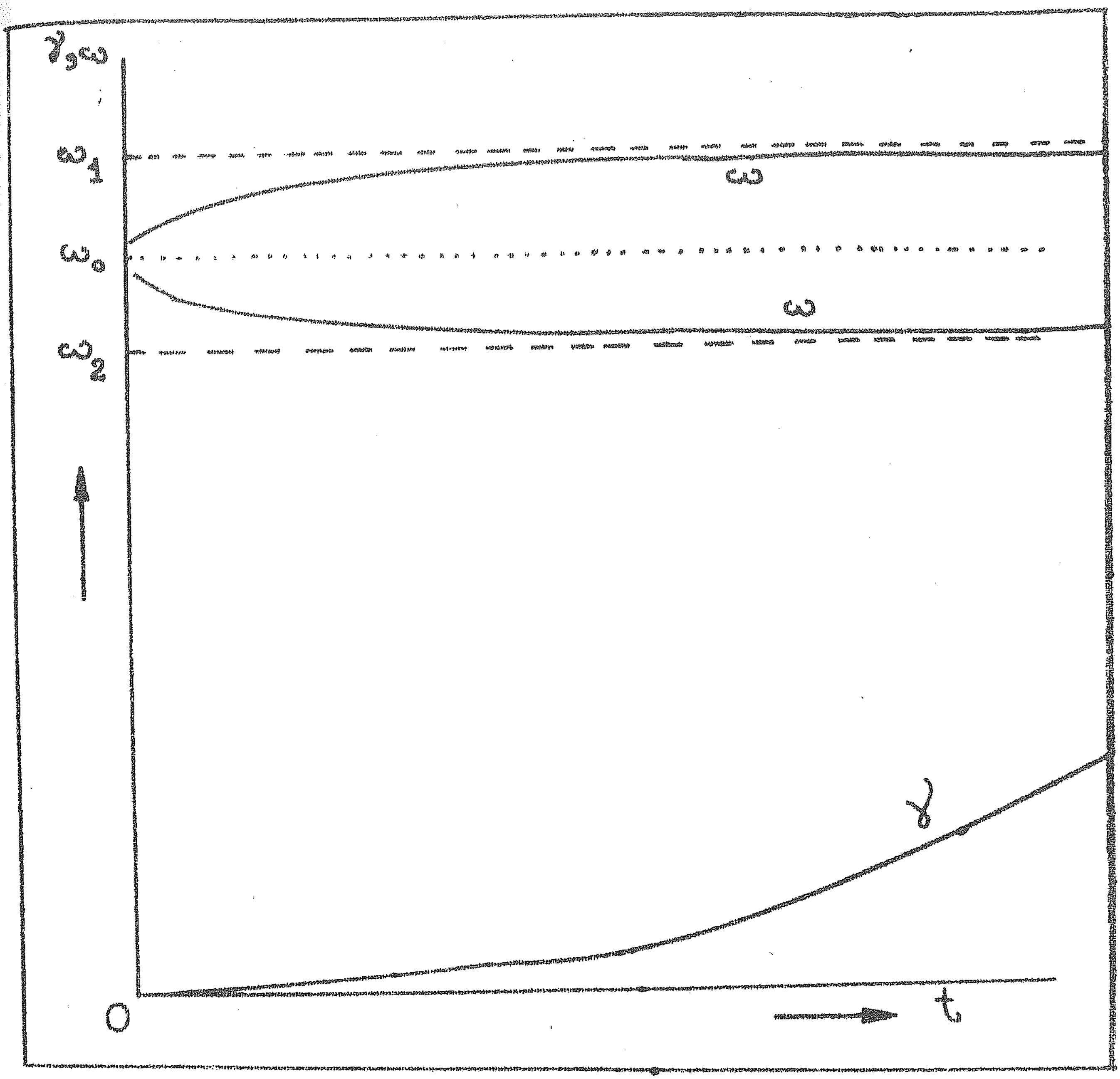

FIG 5.3

BEHAVIOUR OF GROWING WAVES WITH TIME. UPPER PART SHOWS THE TIME VARIATION OF THE FREQUENCY (ω) OF MAXIMUM GROWTH AND THE LOWER PART IS A SKETCH OF THE VARIATION OF CORRESPONDING GROWTH RATE WITH TIME. ω_0 CORRESPONDS TO THE CENTRE OF THE DISTURBED REGION AND ω_1 AND ω_2 TO THE EDGES OF THE LEDGE zero. However, if ω is not very small compared to Ω, the initial growth rate will be negative and the waves would damp in the beginning. This damping will continue unless, as equation (5.3) suggests, $A(V_R)$ exceeds the quantity $\omega/(\Omega-\omega)$ during the evolution of the ledge. This means that in the initial stage of the interaction, Landau damping would be accompanied by cyclotron damping and only after a certain time when the quantity $\{A(V_R) - \omega/(\Omega-\omega)\}$ in equation (5.3) becomes positive, the cyclotron growth would come into picture. This suggests that the growth will occur with a certain time delay.

5.3 CONSIDERATION OF THE EFFECT OF THE OCCURENCE OF A MAXIMUM IN THE LANDAU RESONANT $V_{\parallel}$ OF THE PARTICLES:

Our aim has been to study the possible mechanisms that contribute to the favourable triggering of emissions at half the equatorial gyrofrequency. In the last chapter, we considered the effects of the equality of the Landau resonant and the gyro resonant speeds at $\omega = \frac{1}{2}\Omega$, coupled with the equality of the group velocity of the wavepacket and the phase velocity of the central wave of the wave packet. Here, in this chapter, we started with an idea to study the effects of the maximum occuring in Landau resonant $V_{\parallel}$ of the particles.

Obviously, this maximum Landau resonant $V_{\parallel}$ denoted by $V_{\parallel L\ max}$ hereafter will divide the entire velocity space into two regions - one for which $V_{\parallel} < V_{\parallel L\ max}$ where the particles are susceptible to resonance if the waves of suitable frequencies exist in the system and the other for which $V_{\parallel} > V_{\parallel L\ max}$ where the particles will have no Landau resonance with any whistler mode wave having any frequency whatsoever (see fig. 5.4).

Now, if we assume a wide spectrum of oblique whistler turbulence (Helliwell, 1969) for a period of the order of quasilinear relaxation time for f due to the Landau resonant interaction with the waves, a ledge is likely to form over the entire disturbed region in the velocity space. The distribution in the undisturbed region will however, remain unchanged.

At the boundary of the two regions, f will develop a steep negative gradient with respect to $V_{\parallel}$ (see fig.5.4). The application of the mechanism discussed in this chapter seems to be quite promising to favour growth of perturbations at a frequency corresponding to this $V_{\parallel}$.

The whistler mode dispersion relation

$$\frac{c^2k^2}{\omega^2} \approx \frac{\omega_p^2}{\omega(\Omega\cos\theta - \omega)}$$

shows that the maximum $V_{\parallel}$ that a Landau resonant particle can have is given by

$$V_{\parallel L\,max} = c\,\Omega/2\omega_p$$

It may be noted that a particle with $V_{\parallel} = c\,\Omega/2\omega_p$ would gyroresonate with a whistler mode wave with frequency $\omega = \Omega/2$ propagating antiparallel to B_0 .

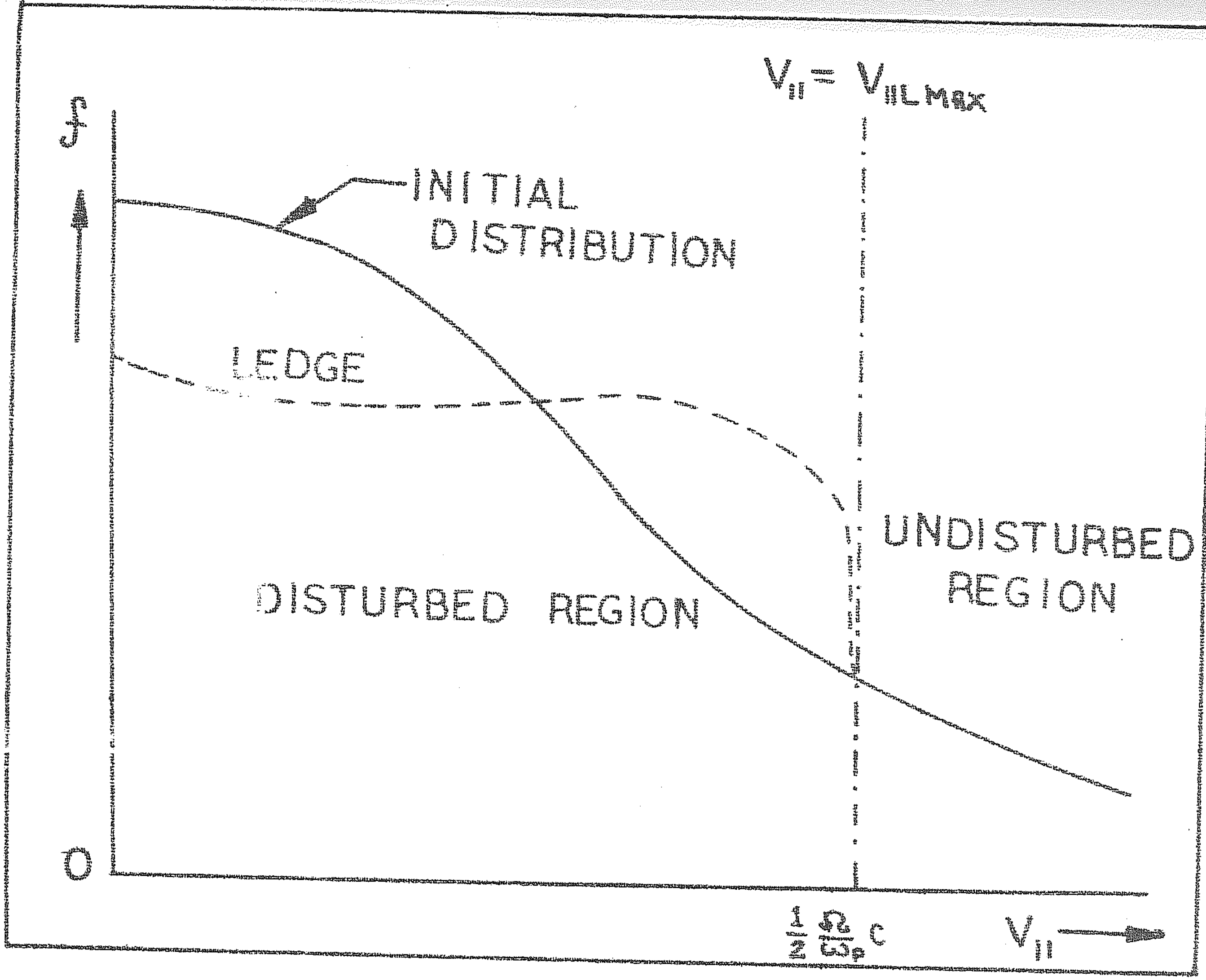

FIG 5.4

THE DISTURBED AND THE UNDISTURBED REGIONS IN VELOCITY SPACE RESULTING FROM THE LANDAU RESONANCE OF WIDE BAND WHISTLER MODE NOISE. THE STEEP GRADIENT OF f WITH $V_{\parallel}$ DEVELOPED NEAR $V_{\parallel} = V_{\parallel LMAX}$ INCREASES PITCH ANGLE ANISOTROPY LEADING TO WHISTLER GROWTH.

5.4 CALCULATIONS OF GROWTH RATE AT $\omega = \frac{1}{2}\Omega$ AND CONCLUSIONS

The steep gradient in $f(V_{\parallel})$ formed at $V_{\parallel} = C\Omega/2\omega_p$ would be unstable to whistler mode perturbations at the frequency $\omega = \frac{1}{2}\Omega$ propagating in the negative z direction as equation (5.6) suggests and the amplification of the waves is expected at this frequency. However, this amplification (see eq. 3) would be delayed as in this case $\omega/(\Omega - \omega)$ is not negligible and so the growth rate is initially negative for a finite period and then it becomes zero and then positive. The nature of these results will not be altered even if the initial distribution were assumed to have an anisotropy in pitch angle.

To calculate the growth rate we use the following equation which is equivalent to equation (5.3) and has already been used in the last chapter for similar calculations:

$$\gamma = -\frac{\pi k^2 V_R^2}{n\Omega}\int_0^{\infty}\left\{ v_{\perp}\frac{\partial f}{\partial v_{\parallel}} - (v_{\parallel} - v_p)\frac{\partial f}{\partial v_{\perp}}\right\}_{V_{\parallel}=V_R} v_{\perp}^2\, dv_{\perp} \quad \text{---(5.7)}$$

When the gradient $\partial f/\partial V_{\parallel}$ is sufficiently steep, the following inequality will hold·

$$\frac{\partial f}{\partial v_{\parallel}} >> \frac{v_{\parallel} - v_p}{v_{\perp}} \frac{\partial f}{\partial v_{\perp}}$$

The steep gradient in f formed at $V_{\parallel} = V_{\parallel L\, max}$ because of the Landau resonant interaction allows to make use of the above inequality and neglect the second term in the integral of eq.(5.7). Thus we get

$$\gamma \doteqdot - \frac{\pi k^2 V_R^2}{n\Omega} \left[\int_0^\infty v_{\perp}^2 \frac{\partial f}{\partial v_{\parallel}} v_{\perp} dv_{\perp} \right]_{V_{\parallel} = V_R}$$

or,

$$\gamma \doteqdot - \frac{\pi k^2 V_R^2}{n\Omega} \frac{2}{m} \left[\frac{\partial}{\partial v_{\parallel}} \int_0^\infty \frac{1}{2} m v_{\perp}^2 f \, v_{\perp} dv_{\perp} \right]_{V_{\parallel} = V_R}$$

replacing $\frac{1}{n}\left[\int_0^\infty \left(\frac{1}{2} m v_\perp^2 f\right) v_\perp \, dv_\perp\right]_{v_{\parallel}=V_R}$ by $T_\perp(V_R)$, the average perpendicular energy per particle at $V_{\parallel} = V_R$, we get

$$\gamma = -\frac{2\pi k^2 V_R^2}{m\Omega} \left.\frac{\partial T_\perp}{\partial v_\parallel}\right|_{V_\parallel = V_R}$$

Approximating $\left(\partial T_\perp / \partial v_\parallel\right)_{V_\parallel = V_R}$ by a delta function $-\Delta T_\perp(V_R)\,\delta(V_\parallel - V_R)$, the above equation leads to:

$$\gamma = \frac{2\pi k^2 V_R^2}{m\Omega} \Delta T_\perp(V_R)\,\delta(v_\parallel - V_R)$$

Integrating γ over a narrow range around $v_\parallel = V_R$,

$$\int_{V_R-\epsilon}^{V_R+\epsilon} \gamma \, dV_R = \left[\frac{2\pi k^2 V_R^2}{m\Omega} \Delta T_\perp(V_R)\right] = \frac{\pi\Omega}{2m} \Delta T_\perp(V_R) \qquad \text{---} (5.8)$$

Since V_R at $\omega = \frac{1}{2}\Omega$ is equal to $-\left.\frac{\Omega}{2k}\right|_{\omega=\frac{1}{2}\Omega}$

To determine the step $\Delta T_{\perp}$ at $V_{\parallel} = V_R$ the following procedure can be adopted if the initial distribution is known (see fig.5.5).

The curves shown in the figure are contours of constant $V_{\perp}$. For each $V_{\perp}$, a separate ledge will be formed because $V_{\perp}$ does not change during the interaction and therefore the height of the ledge, f_c, would be a function of $V_{\perp}$. This f_c can be calculated from the following expression

$$f_c(v_{\perp}) = \frac{1}{v_{\parallel L\,max}} \int_0^{v_{\parallel L\,max}} f(v_{\parallel}, v_{\perp})\, dv_{\parallel}$$

The step Δf at $V_{\parallel} = V_R = V_{\parallel L\,max}$ would be given by

$$\left.\Delta f(v_{\perp})\right|_{V_{\parallel} = V_{\parallel L\,max}} = \left. f_c(v_{\perp}) - f(v_{\perp}) \right|_{V_{\parallel} = V_{\parallel L\,max} + \epsilon}$$

in the limit as $\epsilon \to 0$.

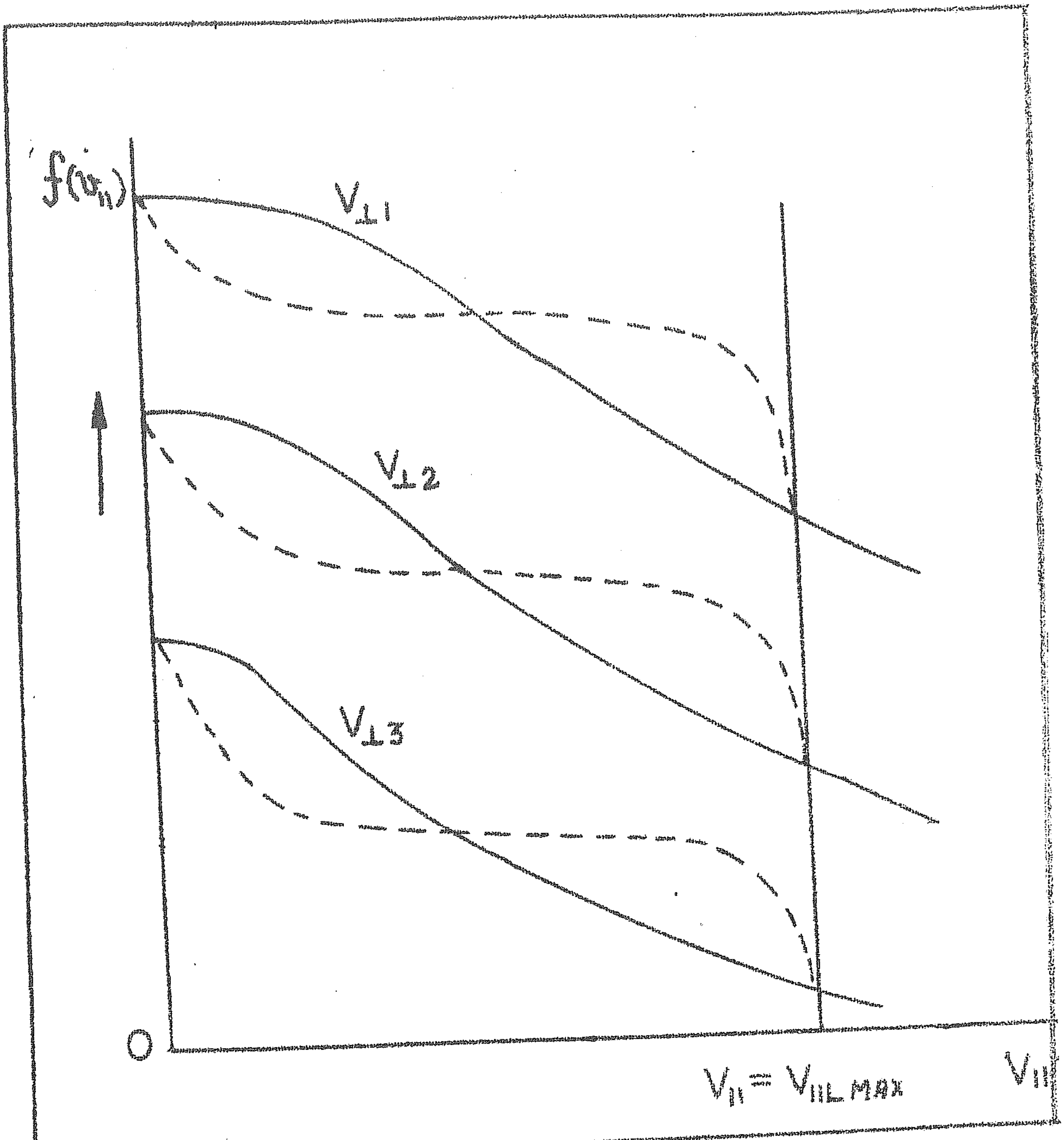

FIG 5.5

AS $V_{\perp}$ DOES NOT CHANGE DURING LANDAU RESONANCE, $f(V_{\parallel})$ FORMS DIFFERENT LEDGES FOR DIFFERENT VALUES OF $V_{\perp}$. HOWEVER, $\int f(V_{\parallel})\, dV_{\parallel}$ FOR A GIVEN $V_{\perp}$, BETWEEN $V_{\parallel} = 0$ AND $V_{\parallel} = V_{\parallel L\,MAX}$ WILL ~~EVALUATE~~ REMAIN UNCHANGED BEFORE AND AFTER THE LEDGE FORMATION DUE TO THE CONSERVATION OF THE NUMBER OF PARTICLES IN THE RESONANT REGION

The step in $T_\perp$ can be easily calculated from this as:

$$\Delta T_\perp \Big|_{v_\parallel = v_{\parallel L max} = V_R} = \int_0^\infty \frac{1}{2} m v_\perp^2 \left\{ \Delta f(v_\perp) \right\}_{v_\parallel = V_R} v_\perp \, dv_\perp \qquad \text{--- (5.9)}$$

The large negative value of $\partial f / \partial v_\parallel$ at $v_\parallel = v_{\parallel L max} = V_R \big|_{\omega = \frac{1}{2}\Omega}$ would give large growth rates for perturbations at $\omega = \frac{1}{2}\Omega$ propagating antiparallel to $\vec{B}_0$.

Throughout this chapter the ambient static magnetic field has been assumed to be uniform. While applying the results obtained here to the emissions from the magnetosphere, we recall that the resonant interactions occuring in the equatorial regions are much more important than those occuring in the non-equatorial regions as explained in the earlier chapters. The field in the equatorial regions can, as a first approximation, be assumed to be uniform and the theory presented here can be applied to this situation. However, we must replace our Ω by Ω_{eq}, the equatorial gyrofrequency and then the mechanism discussed here would exhibit a definite favour to emissions at $\omega = \frac{1}{2} \Omega_{eq}$. (Vyas and Das, 1975).

It has been experimentally observed (Helliwell, 1969) that the VLF emissions tend to occur more often at half the equatorial gyrofrequency and it is very likely that the mechanism described here is one that favours emissions at that frequency.

CHAPTER VI

CONCLUSIONS AND DISCUSSIONS

The main purpose of the present work has been to carry out a study of the low frequency wave particle interactions and their possible consequences. Since the resonant interactions are the strongest wave particle interactions, and there too, the gyro-resonance and the Landau resonance are much more important than any other resonances, we restrict our attention to these two types of interactions only. Our approach to this study consists in first finding how the waves affect or modify the plasma particle distribution function and then seeing how this modified distribution affects the low amplitude perturbations existing in the system.

During the course of this study; special emphasis has been placed upon whether a particular type of interaction studied has some relevance with the generation mechanism of the VLF emissions or with their preferential triggering at half the equatorial electron gyrofrequency.

The study of these interactions has been made from different angles, e.g. the effect of two types of resonances, considered independently, the combined effect of both the resonances and the effect of one type of resonance over the other and each time, attempts have been made to critically examine whether a particular type of interaction is likely to contribute to the generation of very low frequency emissions, and if so, whether it favours emissions at some particular frequency or not.

6.1 A BRIEF SURVEY (SYNOPTIC VIEW) OF THE WORK DONE:

We start with the consideration of the Landau resonance of off angle whistler mode pulses propagating in a predominantly cold collisionless plasma having a high energy tail in the distribution. We find that $V_{\perp}$ of the particles is not affected considerably whereas the $V_{\parallel}$ of these particles is altered in such a way that the changes in $V_{\parallel}$ when plotted against the initial $V_{\parallel}$ exhibit some sort of quasiperiodicity along the $V_{\parallel}$ axis. The consequent evolution of the distribution function $f(v_{\parallel})$ is calculated and, as expected, it is found that the distribution develops a fine structure close to the central resonant velocity of the pulse. The time period for which the changes in $V_{\parallel}$ and the evolution of f have been calculated is small compared to the quasilinear relaxation time of the system. Therefore, the interaction does not lead to the formation of a ledge in the resonant range of the velocity space.

The fine structure developed by f contains alternate negative and positive gradients of f with $V_{\parallel}$. A pulse with a suitable frequency propagating through this modified distribution of particles will undergo growth at the frequencies in its side bands that resonate with the particles lying in the regions of positive gradients in the distribution function.

The results obtained from this study can be applied to the equatorial magnetosphere where the static magnetic field can be crudely considered as uniform and where the particle distribution has a high energy tail because of the high energy particles trapped in the radiation belts.

The time period over which our model calculations have been extended ~~are~~ is of the same order as the periods for which the waves and particles remain in the equatorial region while moving along a field line. If the distribution function gets modified by an initial off angle whistler mode pulse, it is easy to see that another pulse propagating in the same mode will be affected in such a way that its central frequencies get damped whereas its sidebands start growing.

Herein we have not included the effect of the nonequatorial regions of the earth's magnetic field and that of the nonuniformity of the field close to the equatorial magnetosphere. Therefore, it can not be assesed as to how much would be the contribution of this mechanism to VLF emissions, but, it seems that the mechanism might play an important role during the onset of these emissions.

The period during which the wavepacket traverses the equatorial region is the time which is the most significant period for the interaction and is much shorter than the quasilinear relaxation time of the system. Therefore, the process of ledge formation does not come into picture here.

Next, we consider the effect of the gyroresonance of a large amplitude whistler mode pulse. The interaction leads to the diffusion of resonant particles into the velocity space and to the consequent formation of a fine structure in the distribution which is smeared out after the wave packet has passed through. This smeared out distribution is unstable such that the perturbations at frequencies slightly higher and slightly lower than the central frequency of the wave packet grow whereas those close to the central frequency are damped. Similar results were earlier discovered by Das (1968) for interactions of plasma with low amplitude VLF pulses. The present work, however, brings out the following additional important, but subtle features associated with the interaction: As we increase the amplitude $\vec{E}(\omega)$ of the pulse and decrease its band width such that $\int |E(\omega)|^2 d\omega$ remains constant, then at a certain critical amplitude the growth rate γ suddenly increases almost by a factor of three above the background probably because of the sudden appearance of two regions in velocity space contributing to the growth at the same frequency. The value of the critical amplitude depends upon the loss cone angle and on the value of $\int |E(\omega)|^2 d\omega$. As long as we keep $\int |E(\omega)|^2 d\omega$ constant, an increase in the amplitude $\vec{E}(\omega)$ of the pulse means a decrease in its band width. Also, the inverse Fourier transform of the frequency spectrum of a pulse shows that a decrease in the band width of a pulse is equivalent to an increase in its duration T. This indicates that long duration pulses have large amplitude at their central frequency if $\int |E(\omega)|^2 d\omega = \text{constt}$. Thus if the duration of a pulse is increased beyond a certain critical limit the growth rate suddenly increases to a rather high value. This type of interaction may account for the observational fact that the VLF emissions are frequently triggered by Morse Code dashes having a duration of 150 msec. and are rarely triggered by Morese Code dots having only a 50 msec. duration. It is evident that a short duration (large band width) pulse has to be more powerful in order to attain the critical amplitude. This indicates, that, provided they they are strong enough, dots are also capable of triggering VLF emissions.

Another important feature discovered during the course of the present work is that secondary peaks protrude in the growth rate curves on either side of the central resonant frequency. They are more prominent if the pulses under consideration have large amplitude as well as the large band width. This suggests that a strong whistler mode wave packet with a short duration is capable of producing observed multiple emissions.

After studying the effects of the Landau resonance and the gyroresonance separately we attempt to study the simultaneous effect of the two resonances on the particle distribution caused by a succession of whistler mode pulses. Ashour Abdalla has already shown that the gyroresonance of these pulses leads to the formation of a slot in the velocity distribution of the particles in presence of a loss cone. Here, through a parallel model for the Landau resonance interaction, we come to a similar conclusion that the Landau resonance would also cause the formation of a similar slot in the velocity distribution under similar conditions. However, the Landau resonant slot is weak because of the inherent weakness of the effects of the Landau resonance of whistlers as compared to that of their gyroresonance.

The weakness of the Landau resonance slot would not have allowed preferential growth at a frequency $\omega = \frac{1}{2}\Omega_{eq}$ for which both the Landau resonant speed and the gyroresonant speed of the particles are same but for the additional feature of the whistler mode propagation that comes into picture at this frequency. The group velocity of a whistler mode pulse is roughly equal to the phase velocity of the central wave of the pulse at $\omega = \frac{1}{2}\Omega$ for small angles of propagation. In this case the Landau resonance becomes quite strong as all the waves inside the wave packet simultaneously resonate with the same set of particles and thus give a highly pronounced effect. This leads to the formation of a strong Landau resonant slot. It turns out that, for $\omega = \frac{1}{2} \Omega$, both the Landau resonant and the gyroresonant slots are strong and at the same time they occur at the same $|v_{\parallel}|$. Therefore, the two slots overlap each other and form a much deeper slot capable of giving preferential triggering to VLF emissions at the corresponding frequency $\omega = \frac{1}{2} \Omega$.

Next we study how the particle distribution distorted by Landau resonance will affect the gyroresonant growth rate of the perturbations. The distribution f with respect to $V_{\perp}$ is not affected much by Landau resonance but with respect to $V_{\parallel}$, it gets distorted and develops a ledge over the range of resonant velocities. The gradient $\partial f / \partial v_{\parallel}$ at the boundaries of the ledge will assume highly negative values whereas in the central part of the ledge it will approach to zero. The highly negative values of $\partial f / \partial v_{\parallel}$ would impart growth to perturbations moving in a direction opposite to that of the Landau resonant waves and at the same time undergoing gyroresonanc with the particles whose representative points in velocity space lie at the edges of the ledge.

The whole study has been done under the assumption that the growth of the gyroresonant perturbations has not reached the level beyond which they will react back on the distribution of particles.

The expression for growth rate contains two terms. One contributes to the growth and the other to the damping of the waves. If ω is not negligible compared to Ω, the damping term dominates over the growth term in the beginning. However, with time the growth term increases and in due course exceeds the damping term, thus leading to the amplification of the waves. Because of this, there is a finite time lag between the onset of the resonant interaction and the starting of the growth of the waves.

This type of interaction is also capable of contributing to the preferential generation of VLF emissions at $\omega = \frac{1}{2} \Omega_{eq}$. This is because of the association of a special feature of the whistler mode dispersion relation with this frequency. The Landau resonant $V_{\parallel}$ has a maximum value at

$\omega = \frac{1}{2}\Omega$ and this divides the whole distribution into two regions: one, the disturbed region and the other undisturbed. A particle in the undisturbed region can not have Landau resonance with any whistler mode wave having whatever frequency. If a broad spectrum of whistlers is assumed to exist in the system, a ledge is likely to form over the entire disturbed region and a sharp negative gradient would be formed at the boundary of the two regions which corresponds to the gyroresonant frequency $\omega = \frac{1}{2}\Omega$. Therefore, the perturbation at this frequency will be naturally favoured for growth. This model for the VLF emissions also accounts for (or predicts) ~~as~~ initial time delay before the waves start growing.

6.2 CONCLUSIONS:

The different types of wave particle interactions that we have studied so far distort the distribution in such a way that it supports building up of waves at certain frequencies whereas it leads to the dissipation of waves at other frequencies. The sufficiently grown waves are likely to appear as emissions but at this stage it seems as if no mechanism is uniquely responsible for these emissions. On the contrary, it looks that several mechanisms contribute to these emissions. The Landau resonant and the gyroresonant, both types of interactions play significant roles in the generation of these emissions. As far as simple emissions are concerned, the gyroresonance plays a more important role than the Landau resonance in the emission mechanism which is obvious from the fact that the gyroresonance interaction is stronger than the Landau resonant interaction. However, the preference in emissions observed at the frequency $\omega = \frac{1}{2}\Omega$ is because of the reinforcement of the gyroresonant effects by the Landau resonant effects that occur especially at $\omega = \frac{1}{2}\Omega$ and not at any other value of ω, as we have seen in chapter IV and V. Thus VLF pulse at any frequency is capable of generating an emission but if it happens to be centred close to $\omega = \frac{1}{2}\Omega_{e_V}$, its chances of stimulating the emissions increase highly, firstly because the Landau resonance of the background whistler mode noise makes the distribution unstable for perturbations close to this frequency and secondly because a pulse centred at this frequency undergoes the group Landau resonance with the particles which is much stronger than the ordinary Landau resonance occuring for pulses centred at other frequencies.

If the mechanism discussed in chapter III is operative in the system, it is evident that the long duration pulses such as Morse Code dashes stand much better chances to stimulate VLF emissions compared to small duration pulses like the Morse Code dots. This is in agreement with the observations.

In the last chapter, while discussing the preferential generation of $\frac{1}{2}\Omega_{e_V}$ emissions because of the gyroresonant growth of a perturbation caused by the quasilinear Landau resonant relaxation of the distribution we find that the growth starts only after a certain time lag. The emissions observed also show that there is a difference in the time of observation of the triggering pulse and the triggered pulse. Almost all types of interactions discussed show that the onset frequency is different from the frequency of the triggering pulse. This is also the case with the observations. However, none of the interactions discussed above throws any light over the observed variation of the frequency of an emission with time. The theory needs a little modification in this respect.

6.3 SUGGESTED EXPERIMENTS; and SCOPE FOR FURTHERNWORK:

Although this work discusses some aspects of the various types of low frequency wave particle interactions and their possible contribution to the stimulation of VLF emissions, a rigorous mathematical treatment of the whole subject is desirable. While considering the slot development due to both the Landau and the gyroresonances, we have considered the effects of the two resonances separately and then superimposed the effect of one over the other to simplify the situation. An exact treatment of the whole problem in which the two effects are considered to be taking place side by side is also desirable.

A mathematical study applicable to the real physical situations regarding the effectiveness of the group Landau resonance over the ordinary Landau resonance and experimental verification thereof would be quite useful to assess how much contribution to the VLF emissions at $\omega = \frac{1}{2}\Omega_{eq}$ comes from this mechanism.

A study may be carried out as to how the Landau resonant growth rate for an electrostatic first order perturbation would behave in a system containing a spectrum of zero order whistler mode waves tending to make the pitch angle distribution of the system isotropic.

A comparative study of different Planetary Magnetospheres should also be carried out to get a general and broader understanding of the features common to all these magnetospheres. A study of VLF emissions from different planetary atmospheres using artificial satellites which penetrate the atmospheres of such planets may be carried out. It will give a better understanding of the whole phenomenon of whistlers and VLF emissions.

Finally, this type of wave particle interactions may be studied in the laboratory. The controlled laboratory conditions will allow us to change the various parameters of the system and study the phenomena occuring there for all possible aspects. This would give us a much better insight into the magnetospheric Physics in general, and into the phenomenon of VLF emissions, in particular.

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