# Studies in Topics Going Beyond The Standard Electroweak Model

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*Sudhir Vempati*

## CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution. Any material which has been obtained from other sources has been duly acknowledged in this thesis.


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# Chapter 1

# Introduction

"What lies beyond the Standard Model ?" is the question which intrigues many particle physicists today. It is surprising that such a question should arise when most of the existing data agree very well with the predictions of the Standard Model. However, there are strong theoretical arguments suggesting existence of new physics beyond the Standard Model. Independent of these arguments, there are some recent experimental results which may be considered as the first indications of some new physics. Put together, these arguments constitute a 'strong evidence' for the presence of physics beyond the Standard Model and thus the above question.

At present, the question has not been answered. Future experiments like LHC (Large Hadron Collider) are likely to provide an answer. To satisfy some of the theoretical arguments against it, the Standard Model is usually extended by assuming an additional symmetry called supersymmetry. One of the most popular of such models is the Minimal Supersymmetric Standard Model (MSSM). The popularity of this model lies in the fact that it not only satisfies one of the major theoretical prejudices, namely the hierarchy problem, but is also testable in the future colliders. On the other hand, the experimental 'evidence' for physics beyond the Standard Model comes from the recent results of the atmospheric neutrino experiments which indicate that the neutrinos may have small masses and non-zero mixing among them. The Standard Model which has no provision for neutrino masses thus has to be extended in some sector *viz.*, with additional symmetries, additional particles etc. These extensions are achieved in many cases, independent of the theoretical prejudices, thus leading to a different set of extensions of the Standard Model.

In this thesis, we assume supersymmetry as the physics beyond the standard model. We then study the generation of neutrino masses within supersymmetric standard models and implications from the results of the solar and atmospheric neutrino experiments on these models. But, before proceeding further, we try to motivate our work in this chapter, along with a summary of salient features of the MSSM.

## 1.1 The Standard Model

As a starting point, we here briefly review the salient features of the Standard Model. The Standard Model (SM) [1] is a spontaneously broken Yang-Mills quantum field theory describing the strong and electroweak interactions. The theoretical assumption on which the Standard Model rests on is the principle of local gauge invariance with the gauge group given by,

$$G_{SM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y \tag{1.1}$$

The particle spectrum and their transformation properties under these gauge groups are given as,

$$\begin{array}{ll} Q_i \equiv \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix} \sim \left(3,\ 2,\ \frac{1}{6}\right) & U_i \equiv u_{Ri} \sim \left(3,\ 1,\ \frac{2}{3}\right) \\ & D_i \equiv d_{Ri} \sim \left(3,\ 1,\ -\frac{1}{3}\right) \\ L_i \equiv \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix} \sim \left(1,\ 2,\ -\frac{1}{2}\right) & E_i \equiv e_{Ri} \sim (1,\ 1,\ -1) \end{array} \tag{1.2}$$

In the above $i = 1, 2, 3$ stands for the generation index. $Q_i$ represents the left handed quark doublets, $L_i$ represents left handed lepton doublet, $U_i$, $D_i$, $E_i$ represent right handed up-quark, down-quark and charged lepton singlets respectively. The numbers in the parenthesis represent the transformation properties of the particles under $G_{SM}$ in the order given in eq.(1.1). In addition to the fermion spectra represented above, there is also a fundamental scalar called Higgs whose transformation properties are given as,

$$H \equiv \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \sim \left(1,\ 2,\ \frac{1}{2}\right) \tag{1.3}$$

There are also gauge boson fields which enter the spectrum through the requirement of local gauge invariance. The total lagrangian with the above particle spectrum and gauge group can be represented as,

$$\mathcal{L}_{SM} = \mathcal{L}_F + \mathcal{L}_{YM} + \mathcal{L}_S + \mathcal{L}_{yuk}. \tag{1.4}$$

The fermion part, $\mathcal{L}_F$ is given as,

$$\mathcal{L}_F = i\bar{\Psi}\gamma^\mu \mathcal{D}_\mu \Psi \tag{1.5}$$

with

$$\Psi = (Q_i\ U_i,\ D_i,\ L_i,\ E_i) \tag{1.6}$$

where $\mathcal{D}_\mu$ represents the covariant derivative of the field given as,

$$\mathcal{D}_\mu = \partial\mu - ig_s G^A_\mu \lambda^A - i\frac{g}{2}W^I_\mu \tau^I - ig' B_\mu Y \tag{1.7}$$

Here $A = 1, .., 8$ with $G^A_\mu$ representing the $SU(3)_c$ gauge bosons, $I = 1, 2, 3$ with $W^I_\mu$ representing the $SU(2)_L$ gauge bosons. The $U(1)_Y$ gauge field is represented by $B_\mu$. The self interactions of the gauge fields are given by,

$$\mathcal{L}_{YM} = -\frac{1}{4}G^{\mu\nu A}G^A_{\mu\nu} - \frac{1}{4}W^{\mu\nu I}W^I_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \tag{1.8}$$

stocks by foreign investors which in turn has implications for extending the CAPM to a multi-country context.

The purpose of this paper is to investigate the inter-relationship between the volatilities of the Indian stock market and the rupee-dollar exchange rate. It also addresses the question of whether this spillover effect is asymmetric, i.e. whether "good" and "bad" news from the stock market has differential impact on the exchange rate and *vice-versa*. In keeping with the current literature it of course includes ARCH-GARCH effects within each market. In Section II, we describe the data used for the empirical analysis. Section III reports the results of a cointegration analysis of stock prices and exchange rates.[6] In Section IV the model which incorporates volatility spillovers between stock and foreign exchange markets is specified. Section V presents the results of estimation of this model and section VI contains the concluding remarks.

## Section II
## The Data

The study uses daily closing data on two stock market indices, *viz.* BSE-30 (SENSEX), and the NIFTY-50 and the daily closing USD/INR exchange rate. The period covered by the data is from January 2, 1991 to April 24, 2000. The main limitation of the data is the fact that during the early part of the data series, there are sometimes long gaps due to the stock markets having been closed for several days at a stretch.

Foreign institutional investors were permitted to directly invest in the Indian stock market only after 1997. Since this can be expected to have significant implications for the inter-relationship between the stock and forex markets, we have carried out a separate estimation exercise for the sub-sample covering the period March 1998 to April 2000. The stock market data were obtained from the respective exchanges – BSE and NSE – while the historical exchange rate data were supplied by HDFC Bank.

## Section III
## Preliminary Data Analysis

Table 1 presents some descriptive statistics for the two stock indices and the USD/INR exchange rate. Table 2 presents the same statistics for the stock returns and exchange rate returns series. In all the cases daily returns are computed as log differences of successive observations, *viz.* ($\ln X_t - \ln X_{t-1}$). Because of the data gaps mentioned above, these are not always "daily" returns and hence may impoundinformation which may have arrived during the interval between two successive trading days.[7]

6. As usual, the variables are taken in log form.

7. To the extent such information affected both the forex and the stock market and only one of them was closed while the other was open, this would distort the estimated volatility linkages between the two.

related to the masses of the quarks and leptons ( nine fermion masses, three mixing angles and one phase of the CKM matrix). All these masses which are generated by the Yukawa part of the lagrangian, $\mathcal{L}_{yuk}$ (eq. 1.12) are hierarchical in nature in the generation space. At present, we have no understanding of the origin of this hierarchical nature. Similarly, we have no understanding of why there are only three generations within the SM. All these are collectively known as the 'flavour problem' of the SM. As a solution to this problem, one typically extends the SM with an additional 'flavour symmetry'. The quantum numbers of the SM fields under this symmetry would now decide the magnitude of the masses these particles take.

Whereas the above arguments require an extension of the SM for a deeper understanding of the Standard Model, recent results from the atmospheric neutrino experiments make this requirement almost a necessity. The atmospheric neutrino experiments measure the flux of the electron and muon neutrinos produced in the atmosphere due to cosmic ray collisions with the air molecules. These flux measurements (or rather the ratio of the fluxes ) are anomalous with respect to the predicted flux from various calculations. This reduction of the measured flux is generally understood in terms of 'neutrino oscillations', which require neutrinos to have small masses. Recently, super-Kamiokande experiment has reported evidence for the existence of neutrino oscillations [10]. If taken seriously, these results would imply existence of neutrino masses. In addition to these signals from the atmospheric neutrinos, there are other neutrino experiments like the solar neutrino experiments which also suggest neutrinos to have small masses. As we have seen earlier, the SM does not have right handed neutrinos in its spectrum eq.(1.2), thus denying neutrinos any mass through $\mathcal{L}_{yuk}$ [2]. Thus one has to extend the SM in some sector (symmetries, particles or both) to generate mass for the neutrinos. One of the most standard methods to generate neutrino mass is to add right-handed neutrinos in to the Standard Model particle spectrum. Gauge invariance would allow not only Dirac mass terms but also Majorana mass terms for the right handed neutrino fields. The interplay between these terms gives rise to a 'see-saw' mechanism, in which the left handed neutrinos attain small majorana masses.

Thus we have seen that the SM has to be extended in order to satisfy requirements coming from neutrino experiments as well as for a deeper theoretical understanding. There is also a search for a Grand Unified Theory (GUT) [11] which would unify the strong and the electroweak forces with a single coupling constant. It is interesting to note that in-addition to unification of forces, neutrino masses can also be naturally incorporated within the Grand Unified theories [12] through the 'see-saw' mechanism discussed above. The Standard Model would then be an effective theory valid up to some scale relevant to the weak scale. The scale at which the GUT theories would take over from the SM is estimated from the life time of proton and 'Renormalisation Group' running of the three coupling constants to be $\sim 10^{16}$ GeV [13] [3]. Inclusion of the gravitational forces would push this scale further up to the Planck scale, $M_{planck} \sim 10^{19}$ GeV. This would imply that the SM has to be valid right from the weak scale up to the GUT-scale typically, fourteen orders of magnitude. This would create a conceptual problem within these theories called the

[2] Neutrinos cannot attain majorana masses either, as lepton number is 'accidentally' conserved in the SM.

[3] eV-scale neutrino masses in some GUT theories would also require a scale of same order.

hierarchy problem.

The hierarchy problem arises due to the existence of a fundamental scalar within the Standard Model [14]. A typical Grand Unified Theory uses a single gauge group and a single coupling constant to describe the physics at the GUT scale, $M_{GUT} \sim 10^{16}$ GeV. This symmetry is then spontaneously broken at those scales to the symmetry of the Standard Model, $G_{SM}$ given in eq.(1.1). As a consequence of this breaking, the gauge bosons mediating the grand unified interactions acquire heavy masses of the order of $M_{GUT}$ and thus would not be relevant for the physics at the weak scale. The gauge bosons of the Standard Model remain massless as $G_{SM}$ remains unbroken. The fermion masses also would not be affected by this breaking as they are protected by chiral symmetries. But, the scalar mass is unprotected by any symmetry. There is no reason to assume the scalar would not gain mass of the order of $M_{GUT}$ due to the spontaneous breaking of $G_{GUT} \to G_{SM}$ [14]. Such a large mass for the scalar is disastrous phenomenologically as the observed masses of the gauge bosons typically require a scalar mass of $O(100)$ GeV. To solve this 'hierarchy' problem, one can either bring down the scale of the unification or introduce an extra symmetry to protect the Higgs mass. The former would require introduction of extra space time dimensions [15] whereas the symmetries which would protect the Higgs mass are called supersymmetries. There is also one more approach in the literature which assumes that the Higgs boson is a composite particle of fermions thus ruling out the existence of a fundamental scalar in the theory [16].

In the present thesis, we follow the approach of supersymmetry. Supersymmetry in addition to protecting the Higgs mass from attaining large values has other attractive features. A minimal supersymmetric version of the standard model can be built which is predictable and testable at present and future colliders. The unification of gauge coupling constants at high scales is exact in this case and so, Grand Unified Theories also prefer the existence of supersymmetry. More fundamental theories like string theories may also require supersymmetry to be present as low energy effective theories [17]. Thus supersymmetric standard models make an attractive framework as the physics beyond the Standard Model. Below, we review some salient features of the supersymmetric version of the Standard Model.

## 1.2 The Minimal Supersymmetric Standard Model

Supersymmetry is a symmetry that transforms a fermion in to a boson and vice versa. To understand how it protects the Higgs mass, let us consider the hierarchy problem once again. The Higgs mass enters as a bare mass parameter in the Standard Model lagrangian, eq.(1.10). Contributions from the self energy diagrams of the Higgs are quadratically divergent pushing the Higgs mass up to cut-off scale [18]. In the absence of any other new physics at intermediate energies, the cut-off scale is typically $M_{GUT}$ or $M_{planck}$. Cancellation of these divergences with the bare mass parameter would require fine-tuning of order one part in $10^{-36}$ rendering the theory 'unnatural' [19]. On the other hand, if one has an additional contribution from a fermionic loop, with the fermion carrying the same mass as the scalar, the contribution from this diagram would now cancel the quadratically divergent part, with the total contribution now being only logarithmically divergent [20]. This would require a symmetry which would relate a fermion and a boson with same mass. Such symmetries are known as supersymmetries.

Supersymmetries were first introduced in the context of string theories by Ramond [21]. In quantum field theories, this symmetry is realised through fermionic generators [22] thus escaping the no-go theorems of Coleman and Mandula [23]. The simplest Lagrangian realising this symmetry in four dimensions was built by Wess and Zumino [24] which contains a spin $\frac{1}{2}$ fermion and a scalar. To build interaction lagrangians one normally resorts to the superfield formalism of Salam and Strathdee [25], as superfields simplify addition and multiplication of the representations [26]. It should be noted however that the component fields may always be recovered from superfields by a power series expansion in grassman variable.

A chiral superfield contains a weyl fermion, a scalar and and an auxiliary scalar field generally denoted by F. A vector superfield contains a spin 1 boson, a spin 1/2 fermion and an auxiliary scalar field called D. A minimal supersymmetric extension of the Standard Model [27] is built by replacing every standard model matter field by a chiral superfield and every vector field by a vector superfield. Thus the existing particle spectrum of the Standard Model is doubled. The particle spectrum of the MSSM [4] and their transformation properties under $G_{SM}$ is given by,

$$\begin{array}{ll}
Q_i \equiv \begin{pmatrix} u_{L_i} & \tilde{u}_{L_i} \\ d_{L_i} & \tilde{d}_{L_i} \end{pmatrix} \sim \left(3,\ 2,\ \frac{1}{6}\right) & U_i^c \equiv \begin{pmatrix} u_{R_i}{}^c & \tilde{u}_i^c \end{pmatrix} \sim \left(3,\ 1,\ \frac{2}{3}\right) \\
 & D_i \equiv \begin{pmatrix} d_{R_i}{}^c & \tilde{d}_i^c \end{pmatrix} \sim \left(3,\ 1,\ -\frac{1}{3}\right) \\
L_i \equiv \begin{pmatrix} \nu_{L_i} & \tilde{\nu}_{L_i} \\ e_{L_i} & \tilde{e}_{L_i} \end{pmatrix} \sim \left(1,\ 2,\ -\frac{1}{2}\right) & E_i \equiv \begin{pmatrix} e_{R_i}{}^c & \tilde{e}_i^c \end{pmatrix} \sim (1,\ 1,\ -1)
\end{array} \tag{1.13}$$

The scalar partners of the quarks and the leptons are typically named as 's'quarks and 's'leptons. For example, the scalar partner of the top quark is known as the 'stop'. In the above, these are represented by a 'tilde' on their standard counterparts. A distinct feature of the supersymmetric standard models is that they require two Higgs fields with opposite hypercharges to cancel the anomalies and to give mass to both up-type and the down-type quarks [5] These transform as

$$\begin{aligned}
H_1 &\equiv \begin{pmatrix} H_1^- & \tilde{H}_1^- \\ H_1^0 & \tilde{H}_1^0 \end{pmatrix} \sim \left(1,\ 2,\ -\frac{1}{2}\right) \\
H_2 &\equiv \begin{pmatrix} H_2^+ & \tilde{H}_2^+ \\ H_2^0 & \tilde{H}_2^0 \end{pmatrix} \sim \left(1,\ 2,\ \frac{1}{2}\right)
\end{aligned} \tag{1.14}$$

[4] The same notation is followed in the entire thesis.

[5] The hermitian conjugate of the superfield is not allowed into the superpotential due to its holomorphic nature.

The fermionic parts of the Higgs superfields are known as Higgsinos. The gauge bosons and their fermionic partners, gauginos are represented as,

$$\begin{aligned} V_s^A &: \begin{pmatrix} G^{\mu A} & \tilde{G}^A \end{pmatrix} \\ V_w^I &: \begin{pmatrix} W^{\mu I} & \tilde{W}^I \end{pmatrix} \\ V_y &: \begin{pmatrix} B^{\mu} & \tilde{B} \end{pmatrix} \end{aligned} \tag{1.15}$$

In the above, the fermionic (susy) counter parts of the Higgs and the gauge bosons have been represented by a 'tilde' on their bosonic notations. The total Lagrangian of the MSSM is of the form :

$$\mathcal{L}_{MSSM} = \int d\theta^2 \; W(\Phi) + \int d\theta^2 \; d\bar{\theta}^2 \; \Phi_i^\dagger \; e^{gV} \; \Phi_i + \int d\theta^2 \; \mathcal{W}^\alpha \mathcal{W}_\alpha . \tag{1.16}$$

In the above, $W(\Phi)$ is a function of the chiral superfields called the superpotential, $\Phi$ representing a generic superfield. The renormalisable superpotential is of dimension three or less. For the MSSM, the superpotential invariant under $G_{SM}$ is given as,

$$W = W_1 + W_2, \tag{1.17}$$

where

$$W_1 = h^u_{ij} Q_i U^c_j H_2 + h^d_{ij} Q_i D^c_j H_1 + h^e_{ij} L_i E^c_j H_1 + \mu H_1 H_2 \tag{1.18}$$

$$W_2 = \epsilon_i L_i H_2 + \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \lambda''_{ijk} U^c_i D^c_j D^c_k . \tag{1.19}$$

The second term in the RHS of eq.(1.16) represents the matter-gauge boson coupling, with $V = \left(V_s^A, V_w^I, V_y\right)$, with the appropriate coupling constants. The last term in the RHS of the eq.(1.16) represents the self interaction of the gauge fields. The Lagrangian in component form can be found by expanding the superfields as noted above. The scalar potential can be derived by eliminating the auxiliary fields $D$ and $F$ which appear in the definitions of the vector superfield and chiral superfield respectively.

## 1.3 R-parity

In the above, we have seen the minimal supersymmetric version of the Standard Model. Comparing eq.(1.17) with the Standard Model Lagrangian, we see that in additional to doubling of the particle spectrum, new interactions which violate baryon number and individual lepton number are now allowed in the lagrangian. This is because the Higgs superfield, $H_1$ and the lepton superfields $L_i$ have the same quantum numbers under $G_{SM}$. Though they carry the same quantum numbers in the SM also, these interactions are not possible as the fermions and the Higgs transform as different representations under Lorentz Group. Within the MSSM the distinction is lost since the superfields corresponding to the leptons and the Higgs transform identically under supersymmetry and gauge symmetry and these interactions appear as they are gauge invariant.

The first three terms of the second part of the superpotential $W_2$ (eq.(1.19)), are lepton number violating whereas the last term is baryon number violating. The simultaneous presence of both these interactions can lead to proton decay for example, through a squark exchange. Stringent limits can be placed on the products of these couplings from the life time of proton [28]. To avoid proton decay, one can either remove both these couplings or assume one set (either baryon number violating or lepton number violating couplings) to be zero. The former is normally arrived at by imposing a discrete symmetry on the lagrangian called R-parity. R-parity has been originally introduced as a discrete R-symmetry [6] by Ferrar and Fayet [29] and then later realised to be of the following form by Ferrar and Weinberg [30]:

$$R_p = (-1)^{3(B-L)+F}, \tag{1.20}$$

where B and L represent the Baryon and Lepton number respectively and F represents the Fermion parity given as -1 for fermions and + 1 for bosons. Under R-parity the transformation properties of various superfields can be summarised as:

$$\begin{aligned}
\{V_s^A, V_w^I, V_y\} &\rightarrow \{V_s^A, V_w^I, V_y\} \\
\theta &\rightarrow -\theta^\star \\
\{Q_i, U_i^c, D_i^c, L_i, E_i^c\} &\rightarrow -\{Q_i, U_i^c, D_i^c, L_i, E_i^c\} \\
\{H_1, H_2\} &\rightarrow \{H_1, H_2\}
\end{aligned} \tag{1.21}$$

Imposing R-parity has an advantage that it provides a natural candidate for dark matter. This can be seen by observing that R-parity distinguishes a particle from its superpartner. Thus the lightest supersymmetric particle (LSP) cannot decay and remains stable [31]. R-parity has also been motivated as a remnant symmetry in a large class of supersymmetric theories, especially in theories with an additional $U(1)$ symmetry. It has also been pointed out that though R-parity constraints baryon and lepton number violating couplings of dimension four, dim 6 operators which violate baryon and lepton numbers are still allowed by R-parity [32].

## 1.4 Supersymmetry breaking

The MSSM lagrangian built so far is invariant under supersymmetry. Supersymmetry breaking has to be incorporated in the MSSM to make it realistic. In a general lagrangian, supersymmetry can be broken spontaneously if the auxiliary fields D or F appearing in the definitions of the chiral and vector superfields respectively attain a vacuum expectation value (*vev*). Incorporation of this kind of breaking in MSSM using the MSSM superfields leads to phenomenologically inconsistent scenarios, like for example existence of a very

[6] R-symmetries are symmetries under which the $\theta$ parameter transform non-trivially.

light scalar [7]. An alternative approach to incorporate supersymmetry breaking is using the 'hidden-sector' scenarios.

The hidden-sector is a sector of superfields which do not carry any quantum numbers under ordinary gauge interactions, i.e, $G_{SM}$. The only interactions they share with the visible (MSSM) sector superfields is through gravity. Supersymmetry can then be broken spontaneously in the hidden-sector and this information is then passed on to the MSSM sector through gravitational interactions. Since gravitational interactions play an important role only at very high energies, $M_p \sim O(10^{19})$ GeV, the breaking information is passed on to the visible sector only at those scales. The end effect of this mechanism is that explicitly supersymmetry breaking terms are now added in to the lagrangian. These 'soft' terms do not reintroduce quadratic divergences back into the theory. The strength of the soft terms is characterised by, $M_S^2/M_{planck}$, where $M_S$ is the typical scale of supersymmetry breaking. These masses can be comparable to weak scale for $M_S \sim 10^{10}$ GeV [34]. The 'soft' supersymmetric breaking part of the lagrangian can be classified to contain the following terms [35]:

- a) Mass terms for the gauginos, $M_1, M_2, M_3$.
- b) Mass terms for the scalar particles, $m^2_{\phi_i}$ with $\phi_i$ representing the scalar partner of chiral superfields of the MSSM.
- c) Trilinear scalar interactions, $A_{ijk}\phi_i\phi_j\phi_k$ corresponding to the cubic terms in the superpotential.
- d) Bilinear scalar interactions, $B_{ij}\phi_i\phi_j$ corresponding to the bilinear terms in the superpotential.

The total MSSM lagrangian is thus equal to

$$\mathcal{L}_{total} = \mathcal{L}_{MSSM} + \mathcal{L}_{soft} \tag{1.22}$$

with $\mathcal{L}_{MSSM}$ given in eq.(1.16).

### 1.4.1 Universality and CMSSM

The above mechanism of supersymmetry breaking is called minimal supergravity (mSUGRA) inspired supersymmetry breaking. As we have seen above this type of breaking introduces several new soft parameters in to the theory. Typically the number of these parameters is large $\sim 105$. Moreover, large flavour changing neutral currents (FCNC) are also introduced by this kind of breaking [36]. To reduce the number of parameters as well as remove the large FCNC contributing terms, an ansatz is usually followed at the high scale where the soft terms are introduced in to the theory. This ansatz is called universality and it assumes the following:

[7] This can be seen from the mass sum rules which appear as a consequence of spontaneous supersymmetry breaking [33].

- All the gaugino mass terms are equal at the high scale.

$$M_1 = M_2 = M_3 = M$$

- All the scalar mass terms at the high scale are equal.

$$m^2_{\phi_i} = m^2_0$$

- All the trilinear scalar interactions are equal at the high scale.

$$A_{ijk} = A$$

- All bilinear scalar interactions are equal at the high scale.

$$B_{ij} = B$$

It is possible to construct supergravity models which can give rise to such kind of strong universality [37]. This approximation now drastically reduces the number of parameters of the theory to about five, $m_0, M$ (or equivalently $M_2$), ratio of the *vevs* of the two Higgs, tan$\beta$, $A$, $B$. Thus, these models are also known as 'Constrained' MSSM [38] in literature. The low energy mass spectrum of the soft terms is now determined by the Renormalisation Group scaling of those parameters. The supersymmetric mass spectrum of these models has been extensively studied in literature [39]. The Lightest Supersymmetric Particle (LSP) is mostly a neutralino in this case.

### 1.4.2 Gauge Mediated Supersymmetry breaking

In the above we have seen that supersymmetry is broken at a high scale and is communicated through gravity to the normal particle sector. It induces large FCNC's and a large number of parameters which can be corrected by assuming a 'strong' universality at the $M_{GUT}$. An alternative mechanism has been proposed which tries to avoid these problems in a natural way. The key idea is to use gauge interactions instead of gravity to mediate the supersymmetry breaking from the hidden (also called secluded sector sometimes) to the visible MSSM sector [40]. In this case supersymmetry breaking can be communicated at much lower energies $\sim$ 100 TeV.

A typical model would contain a susy breaking sector called 'messenger sector' which contains a set of superfields transforming under a gauge group which 'contains' $G_{SM}$. Supersymmetry is broken spontaneously in this sector and this breaking information is passed on to the ordinary sector through gauge bosons and their fermionic partners in loops. The end-effect of this mechanism also is to add the soft terms in to the lagrangian. But now these soft terms are flavour diagonal as they are generated by gauge interactions. The soft terms at the messenger scale also have simple expressions in terms of the susy breaking parameters. In addition, in some models of gauge mediated supersymmetry breaking, only one parameter can essentially determine the entire soft spectrum.

In a similar manner as in the above, the low energy susy spectrum is determined by the RG scaling of the soft parameters. But now the high scale is around 100 TeV instead of $M_{GUT}$ as in the previous case. The mass spectrum of these models has been studied in many papers [41]. The lightest supersymmetric particle in this case is mostly the gravitino in contrast to the mSUGRA case.

### 1.4.3 $SU(2) \times U(1)$ breaking

As we have seen earlier, the supersymmetric version of the Standard Model is a two Higgs doublet model. A consistent incorporation of the $SU(2)_L \times U(1)_Y$ breaking puts constraints relating various parameters of the model. To see this, consider the neutral Higgs part of the scalar potential. It is given as

$$\begin{aligned} V_{scalar} &= (m^2_{H_1} + \mu^2)|H_1^0|^2 + (m^2_{H_2} + \mu^2)|H_2^0|^2 - (B_\mu \mu H_1^0 H_2^0 + H.c) \\ &+ \frac{1}{8}(g^2 + g'^2)(H_2^{0^2} - H_1^{0^2})^2 + \ldots, \end{aligned} \tag{1.23}$$

where $H_1^0, H_2^0$ stand for the neutral Higgs scalars and we have parameterised the bilinear soft term $B \equiv B_\mu \mu$. The existence of a minima for the above potential would require at least one of the Higgs mass squared to be negative. In both gravity mediated as well as gauge mediated supersymmetry breaking models, such a condition at low energies can be naturally incorporated. The soft parameters which appear in the above potential are generally positive at the high scale. But radiative corrections significantly modify the low scale values of these parameters, making one of the Higgs mass to be negative at the weak scale. This mechanism is called radiative electroweak symmetry breaking. In addition to ensuring the existence of a minima, one would also require that the minima should be able to reproduce the standard model relations i.e, correct gauge boson masses. This would give rise to constraints on the parameters known as the minimisation conditions. These are given as

$$\frac{1}{2}M_Z^2 = \frac{m^2_{H_1} - \tan^2\beta\, m^2_{H_2}}{\tan^2\beta - 1} - \mu^2 \tag{1.24}$$

$$\mathrm{Sin}2\beta = \frac{2B_\mu\, \mu}{m^2_{H_2} + m^2_{H_1} + 2\mu^2}, \tag{1.25}$$

where $\tan\beta = v_2/v_1$ is the ratio of the vacuum expectation values of $H_2^0$ and $H_1^0$ respectively [8]. In addition to the above conditions care should also be taken such that charge and color breaking minima are absent.

[8] The above minimisation conditions are given for the 'tree level' potential only. The minimisation conditions for the one-loop effective potential are given in [42].

## 1.5 Motivation and Outline of thesis

So far in this chapter we have seen the reasons for believing in the existence of physics beyond the Standard Model. A strong signal comes from the recent neutrino oscillation experiments. On the other hand, extension scenarios with unification ideas are generally plagued by the hierarchy problem. A solution to this problem is assuming supersymmetry just above the weak scale. Supersymmetric standard models are built which can incorporate $SU(2) \times U(1)$ breaking naturally. The question then remains is whether one can incorporate neutrino masses and mixing within these theories in a natural way.

Supersymmetric Standard Model, unlike the Standard electroweak model has a natural source of lepton number violation. Since, there is also baryon number violation in these theories, one imposes R-parity to remove both these set of couplings. But R-parity is ad-hoc. One can always assume symmetries other than R-parity like for example baryon parity [43] which can remove the baryon number violating couplings, leaving us with lepton number violating couplings only. In the presence of these lepton number violating couplings, neutrinos attain majorana masses [9]. The generation of the neutrino masses in these theories and whether they are of the correct order to satisfy the solar and atmospheric neutrino masses is the main subject of this thesis. In addition to neutrino masses R-parity violating theories have a different set of experimental signatures in complete contrast to the standard MSSM signatures [44]. We also study some such experimental signatures in the context of some recent experimental results of HERA detector.

The outline of the thesis is as follows. In chapter 2, we discuss the various neutrino experiments and some standard neutrino mass models. In chapter 3, we study the standard Renormalisation group analysis of the MSSM and then try to understand the effect of RG analysis on the neutrino mass spectrum. In chapter 4, we discuss a specific model of R-violation namely bilinear R-violation within the framework of gauge mediated models supersymmetry breaking. In chapter 5, we discuss models with trilinear lepton number violation and the feasibility of simultaneous solutions to solar and atmospheric neutrino problems. In chapter 6, we derive bounds on the trilinear $\lambda'_{1jk}$ couplings from neutrino mass and study its implications for the HERA anomalies. We end with conclusions and future outlook in chapter 7.

[9]Even with R-conservation, higher dimensional R-parity violating operators would give rise to small neutrino masses [32].

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Chapter 2

# Neutrino Anomalies and Mass Models

Neutrinos are one of the most abundant Standard Model particles in the universe. In addition to the various natural radioactive sources, neutrinos are produced in the hydrogen burning process in the stars as well as when a star dies in supernova explosions. They are also produced when energetic cosmic rays collide with the air molecules. There is also a cosmic neutrino background. Though neutrinos are abundant in the universe, it is an irony that they are the least understood. In the Standard Model, neutrinos have no mass, spin $\frac{1}{2}$ and carry no electric charge. They take part only in weak interactions, which makes them extremely difficult to detect as their cross-sections are much smaller compared to electro-magnetic cross-sections [1].

Of the above properties of the neutrinos, the spin and charge of the neutrinos are experimentally well known. The mass of the neutrino is relatively unknown. Experiments which put kinematic limits on the neutrino mass directly are difficult to conduct and put weak limits [2]. However, the abundant sources of neutrinos, the stars and the atmosphere help us in understanding the properties of neutrinos further. As mentioned earlier, neutrino cross-sections are extremely small. Thus to detect them one would need in addition to large fluxes (which are naturally provided by these sources), large detectors too. These detectors measure the number of neutrinos produced for example, in the cosmic ray showers in the atmosphere. The results from these experiments over the years consistently pointed towards the phenomena of 'neutrino oscillations', which in turn indicates existence of neutrino masses.

The above neutrino oscillation experiments can be broadly divided as a) solar neutrino experiments, b) atmospheric neutrino experiments and c) laboratory experiments, depending on the source of neutrinos. Below, we consider some details of these experiments and how they determine the pattern of the neutrino mass matrix.

## 2.1 Neutrinos from the Sun and the Solar Neutrino Anomaly

As we have seen above, neutrino are produced in the stars which burn hydrogen fuel. In the Sun, this process produces as a byproduct electron neutrinos, $\nu_e$. Since the $\nu_e$ interact weakly with the solar atmosphere, they can escape the Sun without much changes in their flux or energy and thus making it possible to measure their flux and energy on the earth. For over thirty years starting from 1967 [3], this flux of $\nu_e$ has been measured on the earth. It is found that the measured flux is only about one-third of the standard expectation. This discrepancy constitutes the *solar neutrino problem.* Several other experiments like Kamiokande, SAGE, GALLEX and super-Kamiokande have since then confirmed the existence of this problem, the last one with very high statistics.

| Source | Flux ($cm^{-2}s^{-1}$) | Average Neutrino Energy (MeV) | Maximum Neutrino Energy (MeV) |
|---|---|---|---|
| $pp$ | $(5.94 \pm 0.06) \times 10^{10}$ | 0.2668 | $0.423 \pm 0.03$ |
| $pep$ | $(1.39 \pm 0.01) \times 10^{8}$ | 1.445 | 1.445 |
| $^7Be$ | $(4.80 \pm 0.43) \times 10^{9}$ | 0.3855 | 0.3855 |
| | | 0.8631 | 0.8631 |
| $^8B$ | $(5.15^{+0.98}_{-0.72}) \times 10^{6}$ | $6.735 \pm 0.036$ | $\sim 15$ |
| $hep$ | $2.10 \times 10^{3}$ | 9.628 | 18.778 |
| $^{13}N$ | $(6.05^{+1.15}_{-0.77}) \times 10^{8}$ | 0.7063 | $1.1982 \pm 0.0003$ |
| $^{15}O$ | $(5.32^{+1.17}_{-0.80}) \times 10^{8}$ | 0.9964 | $1.7317 \pm 0.0005$ |
| $^{17}F$ | $(6.33^{+0.76}_{-0.70}) \times 10^{6}$ | 0.9977 | $1.7364 \pm 0.0003$ |

**Table 2.1** *In the above, we present the total flux of the neutrinos coming from various interactions along with their average and maximum energies [5].*

In the Sun, the main reactions which produce the energy can be grouped as the **pp** cycle and the **CNO** cycle. The final outcome of both these cycles can be given as,

$$4\,p + 2\,e^- \rightarrow 2\,\nu_e +^4 \mathrm{He}_2 + Q \tag{2.1}$$

Here $Q = 26.72$ MeV represents the energy released in this process. A major part of the energy is in the form of photons. A small part ($\sim 2\%$) is taken by the neutrinos. Neutrinos are produced in various intermediate reactions within the **pp** and the **CNO** cycles. In the **pp** cycle, these intermediate reactions are named ( after their reactants ) as *pp, pep,* $^7Be$, $^8B$, *hep*. In the CNO cycle the corresponding reactions involve $^{13}N$, $^{15}O$,$^{17}F$. The cross-sections of these reactions are determined by the standard model weak interaction physics. But the total flux of the neutrinos emanating from these reactions depends on the chemical abundances in the solar interior which in turn is dependent on the solar physics.

These total fluxes are determined by a Standard Solar Model (SSM) developed by J. Bahcall and his collaborators [4]. The model assumes that a) the Sun is in Hydrostatic equilibrium, b) Energy transport happens in Sun by photons or by convective motions, c) energy generation is done by nuclear reactions and d) changes in the chemical abundances are solely due to nuclear reactions. With these assumptions, the model then predicts the flux of the neutrinos to be observed at various experiments. It should be however noted that the flux of the neutrinos also changes with the neutrino energy. Infact, neutrinos produced from different reactions would have a different energy spectrum. In the **Table 2.1** we present, the total fluxes of the neutrinos from various reactions as predicted by the SSM and their corresponding average and maximum energies.

| Experiment | Result | Theory | $\frac{\text{Result}}{\text{Theory}}$ |
|---|---|---|---|
| HOMESTAKE | $2.56 \pm 0.16 \pm 0.16$ | $7.7^{+1.2}_{-1.0}$ | $0.33^{+0.06}_{-0.05}$ |
| GALLEX | $77.5 \pm 6.2^{+4.3}_{-4.7}$ | $129^{+8}_{-6}$ | $0.60 \pm 0.07$ |
| SAGE | $75.4^{+7.0}_{-6.8}\ ^{+3.5}_{-3.0}$ | $147^{+8}_{-6}$ | $0.51 \pm 0.07$ |
| KAMIOKANDE | $2.80 \pm 0.19 \pm 0.33$ | $5.15^{+1.0}_{-0.7}$ | $0.54 \pm 0.07$ |
| SUPER-KAMIOKANDE | $2.40 \pm 0.03^{+0.08}_{-0.07}$ | $5.15^{+1.0}_{-0.7}$ | $0.465 \pm 0.005$ |

**Table 2.2:** *The results of solar neutrino experiments confronted with the corresponding theoretical predictions. The results of Homestake, GALLEX and SAGE experiments are expressed in terms of event rates in SNU units (1 SNU* $\equiv 10^{-36}$ *events* $atom^{-1}s^{-1}$*), whereas the results of the Kamiokande and Super-Kamiokande are expressed in terms of the* $^8B$ *neutrino flux in the units of* $10^6 cm^{-2}s^{-1}$. *The first experimental error is statistical and the second is systematic [5, 6].*

The first experiment to measure the flux of the neutrinos coming from the Sun was the chlorine HOMESTAKE experiment which measured about one-third of the standard expectation [3]. This experiment has a high threshold ($\sim$ MeV) and hence was able to measure flux of neutrinos coming from $^8B$ and $^7Be$ only. It reported a deficit of about one-third of the SSM prediction. In 1988, the KAMIOKANDE experiment using a real time water cherenkov detector could point out the directionality of the incoming neutrinos and confirmed that neutrinos are indeed coming from the Sun. This experiment too had a high threshold ($\sim$ 6.5 MeV) and hence was able to see only the $^8B$ neutrino flux. This experiment reported a deficit about one-half of the SSM prediction. In 1992, GALLEX and SAGE with thresholds of order $\sim$ 200 KeV were able to measure neutrinos coming from the *pp* reactions also, confirming that the source of Sun's energy is indeed through thermonuclear fusion reactions. They too measured a deficit of about one-half. Recently, SUPER-KAMIOKANDE experiment announced results with very high statistics a deficit of one-half in the Boron neutrino flux to which the experiment is sensitive. In **Table 2.2** we have listed the latest results from various experiments along with the corresponding SSM predictions. It should be however noted that the SSM is also being constantly updated over the years with latest inputs from various experimental results and computational techniques. The theoretical predictions presented in **Table 2.2** are based on the latest version of the model called BP98 [7]. An independent confirmation of the Standard Solar Model was recently achieved by the helioseismological data which confirms the predictions of the SSM for sound velocities in the solar interior to a very high accuracy [8].

In the above, we have seen that the measured solar neutrino flux is about half the expectations based on the Standard Solar Model (SSM). One of the possible reasons for this deficit could be the expectations themselves which are based on a model. However, it can be shown that independent of the SSM, one still faces problem to explain the data from various experimental results. Also model independent constraints like luminosity constraint etc, which when used in conjunction with the experimental data would lead to scenarios of the Sun which are unphysical or severely constrained by other observations like helioseismology. For example, using the luminosity constraint one can show that data from GALLEX or SAGE would fit well if all the neutrinos are coming from *pp* reactions only. Such a scenario which suppresses other sources of the neutrinos is severely constrained by helioseismological data. Similarly, a simultaneous fit for SUPER-KAMIOKANDE data and HOMESTAKE data would leave no room for the flux of neutrinos coming from $^7Be$. This result is again severely constrained by helioseismological data [5, 9]. Thus it looks like independent of Standard Solar Model, one still has a problem in explaining the data coming from solar neutrino experiments. In the present thesis, we believe that the SSM is correct and there exists a solar neutrino problem with respect to the expectations based on the SSM.

## 2.2 Neutrinos from the Skies and the Atmospheric Neutrino Anomaly

The other natural source of neutrinos comes from the cosmic rays. Cosmic rays which have led to many fundamental discoveries in particle physics early this century, have now contributed to our knowledge of neutrinos significantly. Neutrinos are produced when intergalactic cosmic rays interact with the air molecules in the atmosphere. The production mechanism can be summarised as follows:

(1). Primary cosmic rays interact with the air molecules producing kaons ($K^{\pm}$) and pions ($\pi^{\pm}$).

(2). These pions then decay to form a part of the neutrino ($\overset{(-)}{\nu_\mu}$) flux and muons ($\mu^{\pm}$).

(3). Lastly muons decay to give rest of the neutrino ($\overset{(-)}{\nu_\mu}$) flux and the ($\overset{(-)}{\nu_e}$) flux.

The actual flux which reaches the surface of the earth depends on a number of other factors like the properties and composition of the primary cosmic rays, modulations due to Solar wind and the geomagnetic field cut-off. These fluxes can be calculated within some uncertainties. Whereas the modulations due to Solar wind and the earth's geomagnetic field can be incorporated within these calculations, the major uncertainties comes from the large errors in primary cosmic ray flux measurements [10]. However, one can still make strong predictions of the neutrino flux which can be summarised as follows :

(a) Both $\nu_e, \nu_\mu$ and $\bar{\nu}_e, \bar{\nu}_\mu$ are produced in these showers.

(b). One can also predict with very less uncertainties the ratio of $\nu_\mu$ to $\nu_e$ in these fluxes. This is typically of order:

$$\frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e} \approx 2 \tag{2.2}$$

However, this would vary from place to place due to the geomagnetic cut-off and with energy of the neutrinos.

(c). Flux of neutrinos with energy greater than 5 GeV is not affected either by geomagnetic cut-off or by solar modulation.

(d). One can also predict fairly well the angular dependence of the neutrino flux at a given experimental site. Taking the direction of the neutrino to be the direction of the incoming cosmic ray [1], the angular dependence of the neutrino flux is characterised by two angles, the zenith angle ($\theta$) and the azimuth angle ($\phi$). The zenith angle dependence is caused by the increase in the length of the air column (more probability of decay and thus enhanced flux) at angles $\theta \sim \frac{\pi}{2}$. But since this dependence is symmetric over $\pi \to \pi - \theta$, this dependence is characterised only by $|cos\theta|$ and would not induce an up-down asymmetry. The geomagnetic field can induce both zenith and azimuth angle dependence on the neutrino flux. But, as we have seen earlier for neutrino energies $\gtrsim$ 5 GeV, this effect is negligible. Thus these effects would not be able to induce an up-down asymmetry in the neutrino flux.

Though neutrinos from the atmosphere have been detected long time ago at experiments at Kolar Gold Fields, India and in South Africa, detailed measurements of these fluxes have started only about a decade ago. At the energies of the atmospheric neutrinos, the typical detection process is through DIS (Deep Inelastic Scattering):

$$\overset{(-)}{\nu_l} + N \to l^\pm + X, \tag{2.3}$$

where $l$ is the corresponding charged lepton, $N$ is the nucleon and $X$ is the remnant of the scattering. The lepton is then detected either through water cherenkov detectors as in IMB, Kamiokande, super-Kamiokande or through iron calorimeters like Baksan and MACRO. As we have seen earlier that even though there are large uncertainties in the primary cosmic ray fluxes ($\sim$ 20%), the predictions for the ratio, eq.(2.2) are free from such uncertainties. Thus the experimental results are usually presented in the form of 'ratio of ratios' given as,

$$R = \frac{\tilde{R}_{exp}}{\tilde{R}_{MC}}, \tag{2.4}$$

where

$$R_{exp(MC)} = \left(\frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e}\right)_{exp(MC)} \tag{2.5}$$

One would expect this ratio to be one. In the following Table, we present the results from various experiments.

[1] This is a good approximation for neutrinos with energy $\gtrsim$ 100 KeV.

| Experiment | Ratio | Energy |
|---|---|---|
| Kamiokande | $0.60^{+0.07}_{-0.06} \pm 0.05$ | sub-GeV |
| Kamiokande | $0.57^{+0.08}_{-0.07} \pm 0.07$ | multi-GeV |
| super-Kamiokande | $0.652 \pm 0.019 \pm 0.051$ | sub-GeV |
| super-Kamiokande | $0.668 \pm 0.034 \pm 0.079$ | multi-GeV |
| IMB | $0.54 \pm 0.05 \pm 0.11$ | |
| Soudan-2 | $0.61 \pm 0.15 \pm 0.05$ | |

**Table 2.3** In the above we present results from the various atmospheric neutrino experiments [5, 11].
From the above we see that the measured ratios are much different from unity. This constitutes the *atmospheric neutrino anomaly.*

## 2.3 Neutrino Oscillations

In the above we have seen that both solar and atmospheric neutrino experiments measure a deficit in the neutrino flux. This deficit can be understood in terms of neutrino oscillations, requiring neutrinos to have small masses. The main idea is that similar to $K^0 - \bar{K}^0$ oscillations, the current eigenstates of neutrinos are not its mass eigenstates. This is expressed as,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{2.6}$$

where $\nu_i$ are the mass eigenstates and $U$ is the mixing matrix. The neutrinos are produced and detected in their current basis, but they propagate in the physical mass basis. Thus a neutrino produced can now change its flavour as it propagates a distance L where it is detected. Making simplifying assumptions about relativistic nature of the neutrino, a simple survival probability formula can be derived using the schrödinger equation [12]. For example, for the electron neutrino survival probability, this is given as [2],

$$P_{\nu_e\nu_e} = 1 - 4 \sum_{\substack{i,j \\ i>j}} U_{ei}^2 U_{ej}^2 Sin^2 \left( \frac{\Delta m_{ij}^2 L}{4E} \right), \tag{2.7}$$

where the LHS represents the survival probability. $\Delta m_{ij}^2$ represents the mass squared difference between $i, j$ neutrino mass eigenvalues and L represents the distance traveled. If there are only two generations involved, this formula reduces to,

[2]For intricacies involving the neutrino oscillation formula, please see [13].

$$P_{\nu_e\nu_e} = 1 - Sin^2 2\theta Sin^2\left(\frac{\Delta m^2 L}{4E}\right), \tag{2.8}$$

where now the mixing matrix is represented as a rotation by an angle $\theta$. The solar and atmospheric neutrino anomalies can be understood in terms of neutrino oscillations, if the mass squared differences of the neutrinos and the mixing angles are of the right order. The experimental data now constraints regions in the parameter space of the neutrino oscillation formula. The deficit in the data of a particular anomaly can be caused either by oscillations in two generations or in three generations. Another possibility is that the neutrinos oscillate in to a sterile neutrino.

*CHOOZ and simultaneous solutions*

In the present thesis, we consider that there are only three active neutrinos and the solar and atmospheric neutrino anomalies have solutions through oscillations among these neutrinos. This would generally need a three flavour analysis of the entire solar and atmospheric neutrino data. However, assuming a hierarchical pattern for the neutrino masses [3], an important constraint on the neutrino mixing matrix comes from the CHOOZ experiment which simplifies such an analysis [15].

The standard two flavour oscillation solutions for the solar and atmospheric neutrino data show an hierarchy in $\Delta m^2$. Assuming neutrino masses also have an hierarchy [4] the general three flavour oscillation formulae for the solar and the atmospheric neutrino problems decouple and reduce to two flavour oscillation formulae as the mass squared differences are not linearly independent [5]. These are given as,

$$\begin{aligned} P_{\nu_e\nu_e} &= 1 - 4U_{e1}^2 U_{e2}^2 Sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right) - 2U_{e3}^2(1 - U_{e3}^2) && (2.9)\\ P_{\nu_\mu\nu_\mu} &= 1 - 4U_{\mu 2}^2 U_{\mu 3}^2 Sin^2\left(\frac{\Delta m_{32}^2 L}{4E}\right). && (2.10)\end{aligned}$$

The mixing matrix element $U_{e3}$ is constrained by the CHOOZ experiment to be small. The CHOOZ experiment which falls in the category of laboratory neutrino experiments [16], rules out oscillations for the electron neutrino in the mass squared difference range, $\Delta m^2 \gtrsim 10^{-3}$ with a large mixing $\sim 1$. For hierarchical pattern of neutrino masses, this in turn constraints $U_{e3} \leq 0.22$. The above formulae are identical to the 2-flavour oscillation formula, eq.(2.8), in the limit of small $U_{e3}$. In this case, we can concentrate only on two flavour solutions to the neutrino anomalies.

The standard solutions for the the solar neutrino anomaly comprise of three 'regions' in the oscillation parameter space of $\Delta$m$^2$ and $Sin^2 2\theta$. One of the regions is called the 'Vacuum

[3]For a discussion on the patterns of neutrino mass matrix allowed by the experiments, including degenerate spectra please see [14].

[4]Either $m_{\nu_1} \sim m_{\nu_2} \ll m_{\nu_3}$ or $m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3}$.

[5]$\Delta m_{21}^2 + \Delta m_{32}^2 + \Delta m_{13}^2 = 0$.

Oscillations' or 'just so'. The $\Delta m^2$ required is very small $\sim 10^{-11}eV^2$ and the mixing angle $\sim \frac{\pi}{4}$. The other two solutions require a much larger $\Delta m^2$, typically of $O(10^{-6}eV^2)$. These two solutions consider matter effects on neutrino propagation whilst the neutrino traverses the distance between the core of the Sun and its surface. They allow for matter enhanced resonant conversion (MSW mechanism) [17] of the neutrinos from electron species to another in specific density regions of the Sun. One of the solutions allows for a large mixing angle, $\theta \sim \frac{\pi}{4}$ - Large Angle MSW. The other region allows for a small mixing angle $\theta \sim 10^{-3}$ and is called Small Angle MSW [18]. In the case of atmospheric neutrino anomaly, the results are much more constrained. The analysis of super-Kamiokande results [19] allow solutions for regions in $\Delta m^2 \sim 10^{-3}$ and $Sin^2 2\theta \sim 1$. These results are summarised in the table below:

| Anomaly | Solution | $\Delta m^2$ $eV^2$ | $Sin^2 2\theta$ |
|---|---|---|---|
| Solar | SAMSW | $(4-12)\ 10^{-6}$ | $(3-11)\ 10^{-3}$ |
| | LAMSW | $(0.8-3)\ 10^{-5}$ | 0.7-1. |
| | VO | $(6-11)\ 10^{-11}$ | 0.47-1. |
| Atmosphere | | $(4-6)\ 10^{-3}$ | 1 |

**Table 2.4** In the above we present constraints on 2 flavour oscillation solutions for solar and atmospheric neutrino anomalies.

Whereas the above results have been valid during the period where much of the thesis work has been done, recent results from the super-Kamiokande experiment do not favour some regions of the parameter space presented above. These results have been systematically analysed recently by [20] and we summarise them in the following table [6]:

| Anomaly | Solution | $\Delta m^2$ $eV^2$ | $tan^2\theta$ |
|---|---|---|---|
| Solar | SAMSW | $(1-10) \times 10^{-6}$ | $(1-20) \times 10^{-4}$ |
| | LAMSW | $2 \times 10^{-5} - 10^{-3}$ | 0.2 - 3. |
| | LOW-QVO | $6 \times 10^{-7} - 5 \times 10^{-10}$ | 0.1 - 7. |
| Atmosphere | | $(1.6-5.4) \times 10^{-3}$ | 0.43 - 2.7 |
| CHOOZ | | | $\lesssim 0.043$ |

**Table 2.5** Constraints on oscillation parameters in the light of recent super-Kamiokande data are presented above [20].

[6] These results are presented in terms of $tan^2\theta$ as a convenience for the 'dark side' of the parameter space [21].

As we have see from the above Table, the vacuum oscillation solution is no longer favoured by the latest solar neutrino data. This is because of an independent observable called day-night spectrum which aids in distinguishing different possible oscillation scenarios for the solar neutrinos. A 'flat' day-night spectrum as such observed would be difficult to reconcile with the vacuum oscillation solutions. However, there are other solutions of the type 'Quasi-Vacuum Oscillations (QVO)' which contain small matter effects are allowed by the recent data. Most of the work in this thesis was done much before the recent results from super-Kamiokande were announced. We will comment on the implications due to the new data at relevant places.

### 2.3.1 Evidence for neutrino oscillations

Till now we have been assuming neutrino oscillations as a solution to the solar and atmospheric neutrino anomalies. This assumption can be verified in the experiments both with laboratory sources as well as natural sources. One of the first indirect experimental observation of neutrino oscillations has been reported by the super-Kamiokande experiment recently [22]. As we have seen earlier, the incoming atmospheric neutrino flux is independent of the zenith angle, for energies $\gtrsim$ 5 GeV. This is because the geomagnetic field would not affect the primary cosmic ray fluxes at these energies. Zenith angle dependence can also be detected if the involved neutrinos are undergoing oscillations. From eq.(2.7), we see that oscillation probability varies with the distance traveled by the neutrino $L$ and the energy of the neutrino $E$. Neutrinos coming from the atmosphere to the detector in the super-Kamiokande experiment travel distances within a range of 15 Kms - 13,000 Kms. The first number is for the neutrinos which are entering the detector vertically (zenith angle $\sim 0$ ) whereas the latter number is for neutrinos which travel vertically upwards (zenith angle $\sim \pi$). If there are no neutrino oscillations, the number of neutrinos entering the detector vertically downwards should be equal to the number of neutrinos traveling upwards. This is quantified by the up-down asymmetry given as

$$A_\mu = \frac{U - D}{U + D}, \tag{2.11}$$

where $U$ represents the number of upward moving neutrinos and $D$ the number of downward neutrinos. Super-Kamiokande experiment has measured this number for the case of muon neutrinos and found it significantly different from zero. The actual experimental number is [22],

$$A_\mu = -0.296 \pm 0.048 \pm 0.01 \tag{2.12}$$

The non-zero up-down asymmetry provides first indirect evidence for the existence of neutrino oscillations.

At present, we do not have a strong experimental signature for the existence of neutrino oscillations for solar neutrino experiments. The major reason being that different experiments measure different regions in energy of the solar neutrino spectrum. While SUPER-KAMIOKANDE has had extremely high statistics, it has been sensitive only to high energy neutrinos from $^8B$ and *hep* neutrinos only. Most of the neutrinos ($\sim$ 97%) from the Sun are at very low energies (*pp* neutrinos ) which have not been measured with high statistics. While efforts are on for the measurement of such neutrinos, results are awaited from the neutrino experiments like SNO and BOREXINO which would help us to point at a single oscillation solution to the solar neutrino problem and clear the ambiguities [23]. The SNO experiment has a facility to accurately measure the neutral current interactions of the neutrinos also. Thus, if the electron neutrinos from the Sun are converted to some other type of neutrinos (active), then the flux of the other type of neutrinos can be detected by neutral current interactions. This way, we would get a measurement of the total $^8B$ neutrino flux and of course a clear signal of neutrino oscillations. The BOREXINO experiment would be able to measure $^7Be$ flux accurately. This would help to conduct a model independent analysis of the solar neutrino flux. Thus, in the next few years we hope to have a single solution to the solar neutrino problem with accurate values of $\Delta \mathrm{m}^2$ and $Sin^2 2\theta$.

*LSND and KARMEN*

Till now we have concentrated only on solar and atmospheric neutrino anomalies. In addition to these anomalies, there also has been an laboratory experiment called the Liquid Scintillator Neutrino Detector (LSND) which has reported evidence for neutrino oscillations. One characteristic feature which differentiates the LSND from the experiments discussed so far is that it is an 'appearance' experiment. The LSND has reported evidence for $\bar{\nu}_\mu \to \bar{\nu}_e$ transitions by observing an *excess* of $\nu_e$ events [24] and thus a direct evidence for the evidence of neutrino oscillations. In terms of oscillation parameters, these results require $\Delta m^2 \sim 1eV^2$, $Sin^2 2\theta \sim 10^{-3}$. Whereas the above results have been from experiments with $\mu$ decay at rest, later experiments with $\mu$ decay in flight have been consistent with their earlier results. Incorporation of the LSND results would require an additional neutrino species [7], as it has a characteristically different mass-squared difference which cannot be incorporated within three neutrino species which allow only two mass-squared differences.

The KARMEN experiment was originally built to verify the LSND results. The KARMEN experiment found negative results for the some of the allowed parameter space of the LSND results. However, it was realised that the KARMEN experiment would not be able to verify the entire parameter space allowed by the LSND experiment [5]. In this thesis where we consider only three active neutrino species, we do not consider the results from LSND.

## 2.4 Mechanisms of Neutrino Mass Generation

We have seen that neutrino anomalies can be understood in terms of neutrino oscillations if the neutrinos have masses [8]. In the hierarchical scenario, typically there are two distinct mass scales needed for simultaneous solutions of solar and atmospheric neutrino anomalies. The atmospheric mass scale $\sqrt{\Delta_A}$ is $\sim 0.1$ eV, whereas the solar scale, $\sqrt{\Delta_S}$ can be either $\sim 10^{-3}$ eV or $\sim 10^{-5}$ eV. The mixing matrix has atleast one large mixing in the 2-3 sector.

[7] See however, Barenboim and Scheck [25] for scheme involving only three neutrinos.

[8] Alternative scenarios are also considered in literature [26]. We do not consider them here.

Thus we see that neutrino mass spectrum is characteristically different from the quark mass spectrum.

As we have seen in Chapter 1, the Standard Model has to be extended in some sector to incorporate neutrino masses. In addition to explaining the smallness of the neutrino masses, these models should also incorporate large mixing as required by the solutions to the neutrino anomalies. Some of the extensions may also have experimental signatures which may be stringently constrained. Thus a consistent model of neutrino masses should be able satisfy all the above requirements. Many such models have been proposed in literature [27]. Here instead of considering specific models, we consider two of the most popular mechanisms which are often used in literature to build models.

### 2.4.1 See-Saw Mechanism

The most natural way of generating neutrino masses in the Standard Model is to add right handed neutrinos in to the model and demand neutrinos attain Dirac masses in the same spirit as other fermions of the SM. But, in this case, it becomes difficult to justify the smallness of the neutrino mass. A natural way of generating small neutrino masses in this manner was proposed by Gellman, Ramond, Slansky [28] and Mohapatra, Senjanovic [29] and is called the see-saw mechanism. The key idea in to use to the majorana masses for the right handed neutrinos, which are naturally allowed by the gauge symmetry to suppress the masses for the neutrinos.

Representing the three left handed fields by a column vector $\nu_L$ and the three right handed fields by $\nu_R$, the Dirac mass terms are given by,

$$-\mathcal{L}^D = \bar{\nu}_L \mathcal{M}_D \nu_R + H.c \tag{2.13}$$

where $\mathcal{M}^D$ represents the Dirac mass matrix. The majorana masses for the right handed neutrinos are given by,

$$-\mathcal{L}^R = \frac{1}{2}\bar{\nu}_R^c \mathcal{M}_R \nu_R + H.c \tag{2.14}$$

The total mass matrix is given as,

$$-\mathcal{L}^{total} = \frac{1}{2}\bar{\nu}_p \mathcal{M} \nu_p \tag{2.15}$$

where the column vector $\nu_p$ is given as,

$$\nu_p = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \tag{2.16}$$

The matrix $\mathcal{M}$ is given as,

$$\mathcal{M} = \begin{pmatrix} 0 & \mathcal{M}_D^T \\ \mathcal{M}_D & \mathcal{M}_R \end{pmatrix} \tag{2.17}$$

Diagonalising the above matrix, one sees that the left handed neutrinos attain majorana masses of order,

$$\mathcal{M}^\nu = -\mathcal{M}_D^T \, \mathcal{M}_R^{-1} \mathcal{M}_D \tag{2.18}$$

This is called the seesaw mechanism. Choosing for example the Dirac mass of the neutrinos to be typically of the order of charged lepton masses or down quark masses, we see that for a heavy right handed neutrino mass scale, (left handed) neutrinos masses are suppressed. This way the smallness of the neutrino masses can be explained naturally in this mechanism. The see-saw mechanism can be naturally incorporated in Grand Unified Theories like SO(10) and also in left-right symmetric models [29]. Though the actual scenarios are model dependent, broadly for a range of $M_R \sim 10^5 - 10^{15}$ GeV, one attains light neutrinos in these models. Large mixing [30] and degenerate spectra [31] can also be realised in these models. Recently an extensive analysis has been reported which studies proton decay and neutrino masses in SO(10) GUT theories [32].

### 2.4.2 Radiative Mechanisms

In the above we have introduced additional fermions with a heavy mass scale to generate small neutrino masses. One can instead modify the scalar sector of the model, which is anyway not well understood. Neutrino masses are now generated radiatively and thus are naturally small. This model is called the Zee model after A. Zee who first proposed it [33].

Within the Standard Model neutrinos can attain majorana masses by modifying the scalar sector. This can be seen by considering the operator, $\epsilon_{ab} L_i^a \; C \; L_j^b$, where $C$ is the charge conjugation matrix and $a, b$ are the $SU(2)$ indices and $i, j$ are the generation indices. This can couple to a field transforming either as a singlet or a triplet under $SU(2)$. Models with triplet Higgs are considered unattractive as they contribute to $\rho$ parameter [34]. Instead here we consider models with a singlet field $h^+$. The coupling of the lepton fields to this field is given as, $f^{ab} \epsilon_{ab} L_i^a \; C \; L_j^b h^+$.

Though the above coupling violates lepton number one can always conserve it by defining $h^+$ to have a lepton number of -2. However if one introduces an additional scalar $\phi_2$ (in-addition to the already existing $\phi_1$), a new coupling of the form $M_{\alpha\beta}\phi_\alpha\phi_\beta h^+$ is possible which violates lepton number exactly by two units as required for neutrino mass generation. In this model which is named as $\{\phi_1\phi_2 h\}$ model, neutrino attain masses at the one-loop level and thus are naturally suppressed. One interesting fact about this model is that the couplings $f_{ab}$ are antisymmetric due to the $SU(2)$ metric. This would lead to an interesting texture of the neutrino mass matrix whose diagonal elements are zero. Instead of adding an additional doublet one can as well add a doubly charged singlet in to the model, $k^{++}$. In this case, neutrinos attain masses at the two-loop level. This is popularly known as Babu model in literature [35]. Including both the additional doublet as well as the doubly charged singlet would lead to neutrino masses both at the 1-loop level as well as at the 2-loop level. Such a scenario may be required to understand neutrino anomalies in these models with discrete symmetries like $L_e - L_\mu - L_\tau$ [36].

In this thesis, we consider an alternative method to generate neutrino masses. In these models neutrinos attain masses employing both the 'see-saw type' mechanism as well as radiative mechanisms. We will discuss them in detail in further chapters.

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## Chapter 3

# RG scaling and R violation

Renormalisation Group (RG) Methods have been introduced by Gellman and Low [1] to tackle the limitations of perturbation theory in calculating processes at high momenta. Since then they have been applied widely in particle physics and in statistical mechanics [2]. The main idea is that a change in the renormalised 1PI function $\Gamma_r$ [1] due to a change in scale of momentum can be compensated by an appropriate change in the value of the coupling constant [3]. This would require the coupling constants to be scale dependent. In a general regularisation process one introduces an arbitrary mass scale $\mu$. Using this scale, the coupling constants are now defined on a 'sliding renormalisation scale'. But, the unrenormalised function $\Gamma$ is still independent of this arbitrary mass parameter $\mu$ [4, 5]. Requiring the renormalised $\Gamma_r$ to be invariant under a change of scale (in momenta) and the unrenormalised $\Gamma$ to be invariant under a change of $\mu$ would lead to a differential equation called the 'Renormalisation Group Equation'[4]. This equation now depicts the fact that a change in scale can be compensated with a change in the coupling constant. The end product of this analysis are differential equations for the coupling constants with respect to the momentum scale which are typically of the form:

$$t\frac{dg}{dt} = \beta(g) \tag{3.1}$$

where t represents the scale change in momenta. The functions $\beta$ are known as the beta functions of the theory. Thus, knowing the coupling constant at one particular scale, the value of the coupling constant at another scale can be known by integrating the above equation.

Renormalisation group methods are used in field theory to study the asymptotic limits of the theory. For example, asymptotically free field theories are those theories for which the $\beta$-function vanishes at a high-scale ($E \to \infty$). Various other features of the renormalisation group equations (RGE) like fixed points, singularities, sign of the $\beta$-functions would help to understand the nature of the theory at high energies. In particle physics, renormalisation group studies have been used to mainly probe the nature of physics beyond the Standard Model. Used in conjunction with the Grand Unified models, renormalisation group equations can probe coupling constant unification at high energies [6]. The renormalisation

[1] We denote the renormalised function by $\Gamma_r(E, x, g, \mu)$ where $E$ is the scale of energy of the process, $x$ collectively denotes the various momenta, ratios of momenta etc, $g$ are the coupling constants involved and $\mu$ is the arbitrary scale introduced during regularisation like dimensional regularisation.

group studies of the Standard Model have been recently described in [7]. In the present thesis, we concentrate on Renormalisation Group Studies in the context of supersymmetric Standard Model (MSSM) [8] with and without R-parity violation.

## 3.1 Renormalisation Group and MSSM

As we have seen renormalisation group studies allow us to probe the scale of grand unification. Assuming only Standard Model fields to be present up to $M_{GUT}$, detailed analysis have shown that the gauge couplings do not 'meet' at the same point in the momentum scale, where the new theory can take over [7]. The MSSM doubles the particle spectrum of the SM. This significantly modifies the evolution of the gauge couplings leading to unification of the gauge couplings at a scale, $M_{GUT} \approx 10^{16}$ GeV. This can be easily verified from the renormalisation group equations of the gauge couplings which are presented in Appendix for the MSSM case. The solutions of these equations are given as,

$$\tilde{\alpha}_i(0) = \frac{\tilde{\alpha}_i(tz)}{(1 - b_i\tilde{\alpha}_i(tz))} \tag{3.2}$$

where the parameters appearing in the above are defined in the Appendix. Choosing approximate values for the gauge couplings at $M_Z$ as,

$$\alpha_1(M_Z) \approx \frac{1}{99}; \;\; \alpha_2(M_Z) \approx \frac{1}{30}; \;\; \alpha_3(M_Z) \approx \frac{1}{8.5} \tag{3.3}$$

A simple algebra leads us to see that [2],

$$\frac{5}{3}\alpha_1(0) = \alpha_2(0) = \alpha_3(0) \approx \frac{1}{24} \tag{3.4}$$

for $M_{GUT} = 3\times10^{16}$ GeV. The actual analysis, including two loop RGE and threshold effects predicts $\alpha_3(tz) = 0.129$ which is slightly higher than the observed value. The couplings meet at the value $M_X = 2 \times 10^{16}$ GeV [9]. The 'exact' unification of the gauge couplings within the MSSM may or may not be an accident. But it provides enough reasons to consider supersymmetric standard models seriously as it links supersymmetry and grand unification in an inseparable manner [10].

Along with the gauge coupling unification, some grand unified theories also predict bottom quark Yukawa $Y_b$ and tau lepton Yukawa, $Y_\tau$ unification at $M_{GUT}$. Renormalisation Group studies of the MSSM Yukawa couplings show $Y_b - Y_\tau$ unification [11] for large ranges in $\tan\beta$. In the Appendix, we have given the RGE for $Y_t, Y_b$ and $Y_\tau$. We have not presented here the RGE for the first two generation Yukawa couplings. Since the masses of these particles are small, the effect of their Yukawa couplings in the RG studies is generally small compared to the third generation Yukawa couplings. We have neglected them in most of the analyses presented in this thesis except in the chapter 5 (chapter 6) where the second (first) generation Yukawa couplings play an important role.

[2] The factor $\frac{5}{3}$ is required for normalisation of the $U(1)$ group. See [10].

Renormalisation Group studies also play an important role in determining the weak scale spectra of the MSSM soft sector. As we have seen in chapter 1, supersymmetry is generally broken in a hidden sector. The breaking information is then transferred to the visible sector through gravitational interactions (mSUGRA). The result of this mechanism is that supersymmetry breaking soft terms are added in to the Lagrangian at a high scale $\sim M_{GUT}$. At the high scale 'strong' universality is assumed with the respective soft parameters being equal. The weak scale spectrum is modified due to radiative corrections involving Yukawa couplings and gauge parameters. The corresponding renormalisation group equations of the parameters reflect this phenomena. Using the RGE, one can determine the entire weak scale spectrum in terms of the five basic parameters of the model, $\tan\beta, m_0$, $M$, $A$ and $B$. Some salient features of these renormalisation group equations are :

(a). The gaugino masses evolve in the same manner as the gauge couplings. Thus the relation :

$$\frac{M_1}{g_1} = \frac{M_2}{g_2} = \frac{M_3}{g_3} \tag{3.5}$$

holds true for most of the scales up to small two-loop effects [12].

(b). The superpotential parameters are constrained by non-renormalisable theorems and thus the RGE of these parameters are proportional to themselves [12].

(c). The soft parameters are unconstrained unlike the superpotential parameters. Thus, even if the soft parameters are small or zero at the high scale, they can acquire large values at the weak scale due to RG scaling.

*Radiative electroweak symmetry breaking:*

As we have seen in Chapter 1, the MSSM weak scale scalar potential requires at-least one of the Higgs mass squared to be negative to generate vacuum expectation values to the Higgs. Starting with a positive mass squared for the Higgs at the high scale, large radiative corrections from the top quark Yukawa can turn the Higgs mass squared negative at the weak scale. This can be seen from the solution to the RGE for the $m^2_{H_2}$ which can be approximately written as [3],

$$m^2_{H_2}(tz) \approx m^2_{H_2}(0) - \frac{3h_t^2}{4\pi^2} M^2_{SUSY} \ln\left(\frac{M^2_{GUT}}{M^2_Z}\right) \tag{3.6}$$

where the weak scale is characterised by the mass of the $Z$ boson and $M_{SUSY}$ characterises the typical SUSY breaking mass scale $\sim$ 1 TeV which appears in the equations ( for example, squark masses ). This scale together with the large logarithmic factor $\approx$ 66 appears with a negative sign in the above solution. Thus even with a positive mass squared at the high energies, the Higgs mass squared can turn negative at low energies. This mechanism is called radiative electroweak symmetry breaking [13]. Within the mSUGRA inspired MSSM, with universal boundary conditions, radiative $SU(2) \times U(1)$ breaking helps in reducing the

[3] $t = 0$ characterises the high scale in our notation.

number of parameters of the theory. Consider the minimisation conditions which we have already seen in chapter 1:

$$\mu^2 = \frac{m^2_{H_1} - \tan^2\beta\, m^2_{H_2}}{\tan^2\beta - 1} - \frac{1}{2}M_Z^2 \tag{3.7}$$

$$B_\mu = \frac{\mathrm{Sin}2\beta\left(m^2_{H_2} + m^2_{H_1} + 2\mu^2\right)}{2\,\mu} \tag{3.8}$$

The parameters $m^2_{H_2}, m^2_{H_1}$ are determined at the weak scale by $\tan\beta$, $m_0$, $M$ (or equivalently $M_2$) $A$. Using the above equations one can determine $\mu$ and $B_\mu$ at the weak scale. Only $Sign(\mu)$ remains as a free parameter. On the other hand, one can trade $B_\mu$ to determine $\tan\beta$ or $\mu$ to determine $m_0$ etc. In the analyses presented in the thesis we have used both the sets of parameters. This mechanism is effective for large ranges in $\tan\beta$ and other MSSM parameters. Radiative electroweak symmetry breaking can also be incorporated in models with gauge mediated supersymmetry breaking. In this case, the smallness of the logarithmic factor is compensated by the largeness of the squark masses [14].

### 3.1.1 Analytical and semi-analytical solutions

In a typical model of supersymmetry breaking, the soft terms are added at a high scale. The soft masses and the couplings at the high scale are the input parameters of the model. RG evolution would change these masses and couplings at the weak scale significantly. However, one would like to express the weak scale soft masses and couplings also in terms of the basic input parameters of the model. This would require to have solutions for the RGE for the soft parameters analytically. From the RGE given in the appendix we see that except for the third generation sfermion masses, rest of the differential equations can be solved analytically. The third generation sfermion masses are in general coupled and have to be solved numerically. However, Ibanez and Lopez [15] have shown that the solutions of these equations can be expressed in terms of 'semi-analytical' formulae, thus making the dependence on the basic parameters transparent. For example, in mSUGRA inspired MSSM with universality at a high scale, the solutions for the sfermion masses would be typically of the form:

$$m^2_{\tilde{f}}(t) = D_{m_0} m_0^2 + D_M\, M^2 + D_{mMA}\; m\; M\; A + D_{mA}\; m^2\; A^2 \tag{3.9}$$

where $m_0$, $M$, $A$, stand for the standard mSUGRA inspired MSSM parameters. The coefficients $D$'s are given in terms integrals which can be solved numerically. In this section, we present the solutions of the RGE systematically.

As we have seen earlier, the RG equations for the gauge coupling constants can be solved analytically. Since the gauginos also follow similar type of equations, their evolution can also be represented with analytical solutions of the RGE. These solutions are given as

$$\begin{aligned} \tilde{\alpha}_i(t) &= \tilde{\alpha}_i(0) z_i(t) \\ M_i(t) &= M_i(0) z_i(t), \end{aligned} \tag{3.10}$$

where $z_i(t)$ is defined as:

$$z_i(t) = \frac{1}{(1 + b_i\ \tilde{\alpha}_i(0)\ t)}$$

with $i = 1, 2, 3$ over the generations and $b_i$ are defined in the Appendix. Similarly, in the limit where the first two generation Yukawa couplings are neglected, the equations for the corresponding soft masses, eqs.(3.55), depend only the gauge couplings and the gauginos. The general solutions of these equations can be represented as:

$$m^2_{\tilde{f}_j}(t) = m^2_{\tilde{f}_j}(0) + \sum_i \left( C^j_i\ k_i(t)\ M_i(0)^2 \right), \tag{3.11}$$

where $j = \left(Q_{1,2}, U^c_{1,2}, D^c_{1,2}, L_{1,2}, E^c_{1,2}\right)$ and $C^j_i$ are the coefficients appearing in the RHS of the respective differential equations. The functions $k_i$ appearing in the above solutions are given as,

$$k_i(t) = \frac{1}{2\ b_i} \left(1 - z_i(t)^2\right) \tag{3.12}$$

As an example, the solution for the RGE of the first two generation left-handed squark masses is given as,

$$m^2_{Q_{1,2}}(t) = m^2_{Q_{1,2}}(0) + \frac{16}{3} M_3(0)^2 k_3(t) + 3M_2(0)^2 k_2(t) + \frac{1}{15} M_1(0)^2 k_1(t) \tag{3.13}$$

Similarly, the solutions for the rest of the first two generation soft masses can be read off from their RGE. Thus we see that the RGE for the first two generation soft masses have analytical solutions as the corresponding Yukawa couplings are small and thus can be neglected. But as mentioned above, such an approximation is not valid for the third generation Yukawa couplings as they are large and can induce large corrections to the soft masses through RG evolution. This makes the set of equations coupled and in general has to be solved numerically. However, one can still find semi-analytical formulae for the set of equations when tan $\beta$ is small [15]. In this limit, $Y_b$ and $Y_\tau$ are small compared to the Top quark Yukawa and thus can be neglected. Neglecting the $Y_b$ and $Y_\tau$ couplings in the equation for the third generation Yukawa couplings, eq.(3.47), we can arrive at the following solution [16] for $Y_t$ :

$$Y_t(t) = \frac{Y_t(0) E_1(t)}{Y_{den}(t)}, \tag{3.14}$$

where the function $E_1(t)$ is given as,

$$E_1(t) = z_3(t)^{-16/3b_3}\ z_2(t)^{-3/b_2}\ z_1(t)^{-13/15b_1} \tag{3.15}$$

with the functions $z_i(t)$ defined above. The function $Y_{den}(t)$ is given as,

$$Y_{den}(t) = 1 + 6\ Y_t(0)\ F_1(t) \tag{3.16}$$

where

$$F_1(t) = \int_0^t E_1(t') dt' \tag{3.17}$$

$Y_t(0)$ is the Top quark Yukawa at the high scale which is determined in-terms of the Top quark mass at $m_t$ [17]. Defining $tmt = 2\ \ln\left(\frac{M_X}{m_t}\right)$, $Y_t(0)$ is given as,

$$\begin{aligned} Y_t(0) &= \frac{Y_t(tmt)}{E_1(tmt) - 6\ Y_t(tmt)\ F_1(tmt)}, \\ Y_t(tmt) &= \frac{m_t(m_t)^2}{16\ \pi^2\ v^2\ sin\beta^2}. \end{aligned} \tag{3.18}$$

In this approximation, the solutions of $Y_b$ and $Y_\tau$ are given as,

$$\begin{aligned} Y_b(t) &= \frac{Y_b(0)E_2(t)}{Y_{den}^{1/6}} \\ Y_\tau(t) &= Y_\tau(0)E_3(t) \end{aligned} \tag{3.19}$$

With $Y_b(0)$ and $Y_\tau(0)$ determined in a similar manner as $Y_t(0)$ in terms of $Y_b(tmb)$ and $Y_\tau(tmtau)$ with $tmb$ and $tmtau$ having analogous definitions of $tmt$. The functions $E_2(t)$ and $E_3(t)$ are given as,

$$\begin{aligned} E_2(t) &= z_1(t)^{2/5b_1} E_1(t), \\ E_3(t) &= z_2(t)^{-3/b_2} z_1(t)^{-9/5b_1}. \end{aligned} \tag{3.20}$$

The integral $F_1$ is generally solved numerically. Thus, we call these solutions as semi-analytical solutions. In a similar manner, in deriving the solutions for the rest of the parameters we completely neglect the effects of bottom and tau-lepton Yukawa couplings. This scheme is sometimes known as 'top Yukawa dominance' approximation. In this case, we neglect the bottom and tau-lepton Yukawas appearing in the equations for the soft A-parameters, eqs.(3.50), $\mu$ and $B_\mu$ terms, eqs.(3.53), the third generation squark and slepton masses, eqs.( 3.60) and the Higgs mass terms, eqs.(3.65). Since $A_b$ and $A_\tau$ are always accompanied by their respective Yukawa couplings in these equations, we will not worry about their solutions here. The solution for $A_t$ is given as,

$$A_t(t) = \frac{1}{Y_{den}(t)}\left(A_t(0) - \int_0^t Y_{den}(t')\kappa(t')dt'\right) \tag{3.21}$$

where the function $\kappa$ is given as,

$$\kappa(t) = \frac{16}{3}\ \tilde{\alpha}_3(0)\ M_3(0)z_3(t)^2 + 3\ \tilde{\alpha}_2(0)\ M_2(0)z_2(t)^2 + \frac{13}{15}\ \tilde{\alpha}_1(0)\ M_1(0)z_1(t)^2. \tag{3.22}$$

The integral on the RHS of the above formula is solved numerically. In the case of third generation sfermion masses, the observation that only $m^2_{Q_3}$, $m^2_{U_3}$ and $m^2_{H_2}$ depend on the top quark Yukawa, reduces the task significantly. The RGE for $m^2_{D_3}$, $m^2_{L_3}$ and $m^2_{E_3}$ now have the same form as those for the first two generation sfermion masses. Thus their solutions too have the same form as given in eq.(3.11). The coefficients on the RHS of these equations would now form the constants $C_i^j$. $m_{H_1}^2$ evolves in the same manner as $m_{L_3}^2$ in this approximation and thus has the same solution.

To find the solutions for the remaining third generation sfermion masses, ie, $m_{H_2}^2(t)$, $m_{Q_3}^2(t)$ and $m_{U_3}^2(t)$ we observe that though independently they cannot be solved analytically, there are combinations of these masses which can be solved analytically. These combinations do not contain the top quark Yukawa [4].

$$\begin{aligned} m_4^2(t) &= m_{U_3}^2(t) - 2m_{Q_3}^2(t) \\ m_5^2(t) &= m_{H_2}^2(t) - 3m_{Q_3}^2(t) \\ 2\, m_6^2(t) &= 2\, m_{H_2}^2(t) - 3m_{U_3}^2(t) \end{aligned} \tag{3.23}$$

Using the RGE for these three mass parameters, eqs.(3.60), on the RHS of the above, these functions are determined as,

$$\begin{aligned} m_4^2(t) &= m_{U_3}^2(0) - 2\, m_{Q_3}^2(0) - \frac{16}{3}M_3^2(0)k_3(t) - 3M_2^2(0)k_2(t) - \frac{14}{15}M_1^2(0)k_1(t) \\ m_5^2(t) &= m_{U_3}^2(0) - 2m_{Q_3}^2(0) - 16M_3^2(0)k_3(t) - 9M_2^2(0)k_2(t) - \frac{1}{5}M_1^2(0)k_1(t) \\ 2\, m_6^2(t) &= 2\, m_{H_2}^2(0) - 3m_{U_3}^2(0) - 16M_3^2(0)k_3(t) - \frac{16}{5}M_1^2(0)k_1(t) \end{aligned} \tag{3.24}$$

Using the above relations the differential equations for $m_{H_2}^2(t)$, $m_{Q_3}^2(t)$, $m_{U_3}^2(t)$ parameters can be rewritten in the approximation of top Yukawa dominance as,

$$\begin{aligned} \frac{dm_{Q_3}^2(t)}{dt} + 6Y_t(t)m_{Q_3}^2(t) &= L_1(t) \\ \frac{dm_{U_3}^2(t)}{dt} + 6Y_t(t)m_{U_3}^2(t) &= L_2(t) \\ \frac{dm_{H_2}^2(t)}{dt} + 6Y_t(t)m_{H_2}^2(t) &= L_3(t) \end{aligned} \tag{3.25}$$

with the functions $L_i$ are given by,

$$\begin{aligned} L_1(t) &= \frac{16}{3}\tilde{\alpha}_3(t)\ M_3(t)^2 + 3\tilde{\alpha}_2(t)\ M_2(t)^2 + \frac{1}{15}\tilde{\alpha}_1(t)\ M_1(t)^2 \\ &- Y_t(t)\left(-\frac{2}{3}m_6^2(t) + \frac{5}{3}m_5^2(t) + A_t(t)^2\right) \end{aligned} \tag{3.26}$$

$$\begin{aligned} L_2(t) &= \frac{16}{3}\tilde{\alpha}_3(t)\ M_3(t)^2 + \frac{16}{15}\tilde{\alpha}_1(t)\ M_1(t)^2 \\ &- 2\, Y_t(t)\left(-\frac{1}{2}m_4^2(t) + m_6^2(t) + A_t(t)^2\right) \end{aligned} \tag{3.27}$$

$$\begin{aligned} L_3(t) &= 3\tilde{\alpha}_2(t)\ M_2(t)^2 + \frac{3}{5}\tilde{\alpha}_1(t)\ M_1(t)^2 \\ &- 3\, Y_t(t)\left(m_4^2(t) - m_5^2(t) + A_t(t)^2\right) \end{aligned} \tag{3.28}$$

[4] Please see Ibanez-Lopez [15] for a different set of combinations which are valid when all the three third generation Yukawa couplings are considered.

Solving any one of the equations in eqs.(3.25), one can get the solutions of the other parameters from the relations given in eqs.(3.23,3.24). For example, the solution for the equation for $m_{H_2}^2$ is given as,

$$m_{H_2}^2(t) = \frac{1}{Y_{den}(t)}\left(m_{H_2}^2(0) + \int_0^t L_3(t')Y_{den}(t')dt'\right) \tag{3.29}$$

The integral appearing on the RHS of the above equation can be solved numerically. Expanding the function $L_3(t)$ as given in eqs.(3.26), we see that the solution, $m_{H_2}^2(t)$ has the desired form of eq.(3.9). Using this solution and the solutions in eqs.(3.24), we can derive the solutions for $m_{Q_3}^2(t)$ and $m_{U_3}^2(t)$ as per the combinations given in eqs.(3.23). Analogously, we can start with solutions of either $m_{Q_3}^2$ or $m_{U_3}^2$ and arrive at the other two solutions using the combinations listed above.

The parameters $\mu$ and $B_\mu$ are generally determined at the weak scale. Using the values at the weak scale, one finds the values of these parameters at the $M_{GUT}$. The solutions for these parameters at any scale $t$ using weak scale boundary conditions are given as,

$$\mu(t) = \mu(tz)\left(\frac{Y_{den}(t)}{Y_{den}(tz)}\right)^{-1/4}\left(\frac{z_1(t)}{z_1(tz)}\right)^{3/10b1}\left(\frac{z_2(t)}{z_2(tz)}\right)^{3/2b2} \tag{3.30}$$

The functions $z_i$ and $Y_{den}$ have been defined earlier. In the same notation the equation for $B_\mu$ takes the form:

$$\begin{aligned} B_\mu(t) &= B_\mu(tz) - 3\int_{tz}^t A_t(t')Y_t(t')dt' - 3\frac{M_2(0)}{b_2}\left(z_2(tz) - z_2(t)\right) \\ &- \frac{3}{5}\frac{M_1(0)}{b_1}\left(z_1(tz) - z_1(t)\right) \end{aligned} \tag{3.31}$$

As mentioned earlier, in deriving the above solutions, we have followed the approach of Ibanez and Lopez [15]. An alternative approach has been followed by Barbieri et al [18]. Recently, these formulae were also given by Carena et al [19]. All the above solutions hold good only in the top-quark Yukawa dominance approximation. This corresponds to regions in the tan $\beta$ parameter space around $2 - 20$ approximately. For larger values of tan $\beta$ Yukawa couplings of bottom and the tau leptons would also become comparable and subsequently would have to be taken in to account. Recently, Kazakov and collaborators have presented analytical solutions to these RGE even in the limit of large tan $\beta$ [20].

## 3.2 R violation and RG evolution

As we have seen in chapter 1, R-violating schemes in supersymmetric theories have a rich phenomenology of their own. In these models, additional couplings which violate lepton and baryon number are present in the superpotential. The RGE presented earlier for the case of MSSM would be significantly modified to take in to consideration these additional couplings. The presence of these couplings leads in general to processes which are constrained severely by experiments. These experimental limits can be converted into bounds on lepton number and baryon number violating couplings at the low scale. Using the RGE the bounds at the low scales can be converted to bounds at the high scale [21]. Moreover, since these couplings also contain flavour violation, the presence of these couplings at the high scale can generate additional flavour violating contributions through RG scaling [22, 23]. Recently, infrared fixed point studies have also been conducted in the presence of these couplings [24].

In this thesis, we concentrate our studies to neutrino mass structure in the presence of these couplings. As we will see later, the typical magnitude of the R-violating parameters required in this case is very small, for example, $\sim 10^{-4}$ for the dimensionless $\lambda'$ couplings. The presence of such small R-violating parameter would not modify the RG evolution of the MSSM parameters significantly [25]. Thus in this thesis, we consider that the RG evolution of standard MSSM parameters would not be effected by the presence of R-parity violating couplings.

However the RG evolution of these couplings can significantly affect the neutrino mass spectrum in these models. To study the effect of RG evolution on neutrino mass spectrum, we divide the R-parity violating couplings as,
(a). Bilinear lepton number violation (dimensionful $\epsilon_i$ terms)
(b). Trilinear lepton number violation (dimensionless $\lambda'_{ijk}, \lambda_{ijk}$ terms).
We study the structure of neutrino mass spectrum in both of these cases separately. Assuming only one set of the parameters to be present at a time *i.e* either only bilinear or only trilinear couplings, we derive the RGE for the R-parity violating parameters present both in the superpotential and the soft potential. To derive these equations, we have used general formulae given by Falck [26]. In this work, the general formulae were derived by using effective potential method. Martin and Vaughn [27] presented general formulae up to 2-loop order. They used both diagrammatic and effective potential method to arrive at their results. We have used their results in chap.4 where we have derived two loop RGE for R-violating parameters. The equations for the R-violating superpotential and soft potential parameters are presented in the Appendix for each case separately. The equations presented here agree with those presented by Carlos and White [22]. We have also checked our equations with the equations presented in other papers [28].

## 3.3 RG scaling of dimensionless L-violation and soft sector

As we have seen earlier, one of the important characteristics of the RG evolution of the soft sector is that, even though these parameters are small at the high scale, they can acquire large values at the weak scale. This characteristic plays an important role in supersymmetry breaking models like Minimal Messenger Model of Gauge Mediated Supersymmetry Breaking, where the soft parameters $B_\mu$ and $A$ vanish at the high scale. However, these parameters acquire non-zero values at the weak scale due to RG scaling thus facilitating electroweak symmetry breaking [14]. This can be clearly seen from the RG equation for the $B_\mu$ parameter, eq.(3.53). In the presence of non-zero gaugino masses and Yukawa couplings, a non-zero $B_\mu$ can be generated at the weak scale. Similarly even if the bilinear lepton number violating soft parameters are absent at the high scale, they can be generated at the weak scale through RG scaling. This would require non-zero lepton number violating trilinear couplings at the high scale [22, 29, 30].

As mentioned earlier, we would divide the R-violation studies interms of dimensionful $\epsilon_i$ terms and dimensionless $\lambda', \lambda$ couplings. Consider an R-violating model with only dimensionless trilinear $\lambda'$ or $\lambda$ couplings. In this case, the soft potential at the high scale would not contain any bilinear lepton number violating couplings. However, these couplings can be generated at the weak scale due to RG scaling. To see this consider the RG evolution of the bilinear lepton number violating soft terms. These are represented as $B_{\epsilon_i}$ and $m^2_{\nu H_1}$. The corresponding equations for these terms in the presence of only $\lambda'$ or $\lambda$ couplings are given in eqs.(3.69,3.73). From these equations we can see that even if these couplings vanish at the high scale, non-zero values can be acquired at the weak scale in the presence of non-zero $\lambda'$ or $\lambda$ couplings.

The above generation of bilinear soft lepton number violating terms at the weak scale has important consequences for neutrino phenomenology. In general, the presence of dimensionless $\lambda'$, $\lambda$ couplings in the superpotential is believed to give rise to neutrino masses only at the 1-loop level. But, RG scaling can generate additional contribution to neutrino masses which is much larger. This happens as follows. The bilinear soft lepton number violating terms generated at the weak scale lead to generation of *vevs* for the sneutrinos. This in-turn leads to a non-zero mixing between neutrinos and neutralinos leading to a 'tree level' mass for the neutrino. Thus, RG scaling can induce a 'tree level' mass to the neutrino which could be significantly larger than the 1-loop level mass. The effect of this RG scaling in a realistic supersymmetric breaking model and its consequences for the neutrino mass spectrum have been worked out in chapters 5 and 6.

### 3.3.1 Semi-analytical solutions

In the case of MSSM without R-violation, we have seen that the weak scale soft spectrum can be expressed as semi-analytical formulae in terms of the four basic parameters of the mSUGRA model, $m_0$, $M_2, A$ and tan $\beta$. The coefficients of these parameters involve simple integral which can be solved numerically in the limit of low tan $\beta$ where only the top quark Yukawa dominates. A similar analysis can be done for the R-parity violating couplings discussed so far. As an example, we present here analytical and semi-analytical solutions for the case where only trilinear $\lambda'_{ijk}$ couplings are present in the superpotential. Moreover, we choose there is only one coupling of the form $\lambda'_{ijk}$ present in the superpotential. The analysis can be extended for the presence of additional couplings but would be cumbersome.

In this limit of neglecting $Y_b, Y_\tau$, the solutions for the $\lambda'$ parameters take the following simple form,

$$\lambda'_{ijk}(t) = \lambda'_{ijk}(0) z_3(t)^{-8/3b_3} z_2(t)^{-3/b_2} z_1(t)^{-7/30b_1} \tag{3.32}$$

As mentioned above the soft bilinear lepton number violating parameters, $B_{\epsilon_i}$ and $m^2_{\nu_i H_1}$ are generated due to RG scaling in the presence of trilinear $\lambda'$ couplings. The solutions for the RGE for these parameters have the generic form of that of the soft masses. The solution for the RGE for $B_{\epsilon_i}$ in this case is given by,

$$B_{\epsilon_i}(t) = p(t)\left(B_{\epsilon_i}(0) + \frac{3}{16\pi^2}\int_0^t q_{ij}(t')\frac{1}{p(t')}dt'\right) \tag{3.33}$$

The functions $p(t)$ and $q_{ij}(t)$ are given as,

$$\begin{aligned} p(t) &= z_2(t)^{3/2b_2} z_1(t)^{-3/10b_1} \\ q_{ij}(t) &= \mu(t)\lambda'_{ijj}(t)h^d_{jj}(t)\left(\frac{1}{2}B_\mu(t) + A^{\lambda'}_{ijj}(t)\right) \end{aligned} \tag{3.34}$$

For the $m^2_{\nu_i H_1}$, the RGE leads to the following solution:

$$m^2_{\nu_i H_1}(t) = m^2_{\nu_i H_1}(0) - \frac{3}{32\pi^2}\int_0^t r_{ij}(t')dt' \tag{3.35}$$

The function $r_{ij}(t)$ is given by,

$$r_{ij}(t) = m^2_{H_1}(t) + m^2_{L_i}(t) + 2m^2_{Q_j}(t) + 2A^{\lambda'}_{ijj}(t)A^D_{jj}(t) + 2m^2_{D_j}(t) \tag{3.36}$$

The solution for the trilinear lepton number violating coupling, $A^{\lambda'}_{ijj}$ which appears in the above solutions is given as,

$$\begin{aligned} A^{\lambda'}_{ijj}(t) &= A^{\lambda'}_{ijj}(0) - \frac{3}{2}\int_0^t A_t(t')Y_t(t')\delta_{j3}dt' - \frac{7}{30}M_1(0)\tilde{\alpha}_1(0)\xi_1(t) \\ &- 2M_2(0)\tilde{\alpha}_2(0)\xi_2(t) - \frac{16}{3}M_3(0)\tilde{\alpha}_3(0)\xi_3(t) \end{aligned} \tag{3.37}$$

The functions $\xi(t)$ are given as,

$$\xi(t)_i = \frac{1}{2b_i}\left(1 - z_i(t)\right) \tag{3.38}$$

Using the solutions for the various soft parameters in the above, one can determine the weak scale values of these parameters in terms of the supersymmetry breaking soft parameters at the high scale which are input parameters of the model. Using these expressions the entire neutrino spectrum can now be expressed in terms of the few supersymmetry breaking parameters. In all the analyses in this thesis, we have solved these integrals numerically using MATHEMATICA© software.

### 3.3.2 Numerical Methods

The above semi-analytical formulae hold true only in the limit of small tan $\beta$. In calculations involving models which allow large tan $\beta$, the RGE have to be solved numerically. This set of first order differential equations is coupled and non-linear and thus have to be solved simultaneously. We have used the standard Rung-Kutta algorithm [5] [31, 32] which is generally recommended for this purpose.

[5] We have employed fourth order RK method. Sometimes we have also employed standard RK routines provided by IMSL.

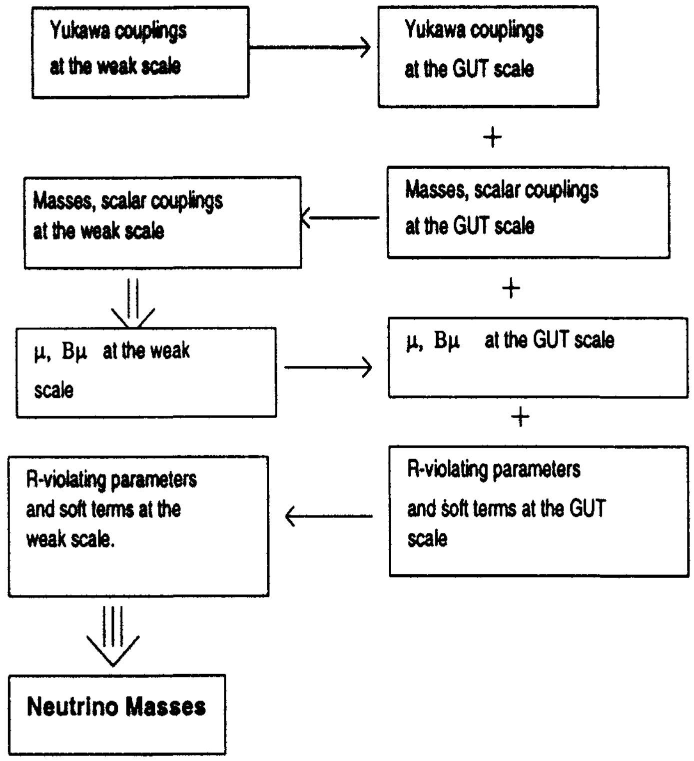


Figure 3.1: Flow chart for numerical analysis of RG equations.

The methodology employed for the numerical analysis is presented in a pictorial manner in Fig.(3.3.2). Masses of the top quark, bottom quark and the tau-lepton determine the input parameters for the third generation Yukawa couplings at the weak scale (characterised by the Z boson mass ) for a given tan $\beta$. These will be then used to determine the Yukawas at the high scale. At the high scale, the masses and the scalar couplings are given as input parameters. Running them down along with the Yukawas determine the weak scale values of those parameters. Using the weak scale values of $m_{H_1}^2$ and $m_{H_2}^2$ one can determine the $\mu$ and $B_\mu$ parameters at the weak scale. In turn we use these parameters to determine their high scale values. The R-parity violating soft terms contain $\mu$ and $B_\mu$ in their RGE. The values of these parameters are known at $M_{GUT}$. Running down the entire set of RGE to the weak scale one finds the weak scale values of the R-parity violating terms. These are then used to find out the neutrino masses.

### 3.3.3 Appendix

**Standard MSSM Equations:** The MSSM RGE are presented in many papers [33]. In writing down the below set of RGE we follow [34].

Here we use the notation $t = 2ln(\frac{M_X}{Q})$ where $M_X$ corresponds to the high scale and $Q$

corresponds to the low scale. In mSUGRA inspired MSSM $M_X$ is taken to be $M_{GUT} \equiv 3 \times 10^{16}$ GeV in our calculations of chapters 5 and 6 whereas $M_X$ varies with $\Lambda$ in Gauge Mediated Models discussed in chapter 4. $tz$ always corresponds to $2ln(\frac{M_X}{M_Z})$, where $M_Z$ is the $Z$ boson mass.

*Definitions*
We also use the following definitions with $g_i$ representing the gauge coupling and $h_f$ representing the Yukawa coupling of the fermion $f$.

$$\tilde{\alpha}_i = \frac{g_i^2}{16\pi^2} \tag{3.39}$$

$$Y_f = \frac{h_f^2}{16\pi^2} \tag{3.40}$$

*Gauge couplings*

$$\frac{d\tilde{\alpha}_3(t)}{dt} = 3\tilde{\alpha}_3(t) \tag{3.41}$$

$$\frac{d\tilde{\alpha}_2(t)}{dt} = -\tilde{\alpha}_2(t) \tag{3.42}$$

$$\frac{d\tilde{\alpha}_1(t)}{dt} = -\frac{33}{5}\tilde{\alpha}_1(t) \tag{3.43}$$

*Gauginos*

$$\frac{dM_3(t)}{dt} = 3\tilde{\alpha}_3(t)M_3(t) \tag{3.44}$$

$$\frac{dM_2(t)}{dt} = -\tilde{\alpha}_2(t)M_2(t) \tag{3.45}$$

$$\frac{dM_1(t)}{dt} = -\frac{33}{5}\tilde{\alpha}_1(t)M_1(t) \tag{3.46}$$

*Yukawa couplings*

$$\frac{dY_t(t)}{dt} = Y_t(t)\left(\frac{16}{3}\tilde{\alpha}_3(t) + 3\tilde{\alpha}_2(t) + \frac{13}{15}\tilde{\alpha}_1(t) - 6Y_t(t) - Y_b(t)\right) \tag{3.47}$$

$$\frac{dY_b(t)}{dt} = Y_b(t)\left(\frac{16}{3}\tilde{\alpha}_3(t) + 3\tilde{\alpha}_2(t) + \frac{7}{15}\tilde{\alpha}_1(t) - Y_t(t) - 6Y_b(t) - Y_\tau(t)\right) \tag{3.48}$$

$$\frac{dY_\tau(t)}{dt} = Y_\tau(t)\left(3\tilde{\alpha}_2(t) + \frac{9}{5}\tilde{\alpha}_1(t) - 3Y_b(t) - 4Y_\tau(t)\right) \tag{3.49}$$

*A-parameters*

$$\begin{aligned}\frac{dA_t(t)}{dt} &= -\frac{16}{3}\tilde{\alpha}_3(t)\ M_3(t) - 3\tilde{\alpha}_2(t)\ M_2(t) - \frac{13}{15}\tilde{\alpha}_1(t)\ M_1(t) \\ &- 6Y_t(t)\ A_t(t) - Y_b(t)\ A_b(t)\end{aligned} \tag{3.50}$$

$$\frac{dA_b(t)}{dt} = -\frac{16}{3}\tilde{\alpha}_3(t)\ M_3(t) - 3\tilde{\alpha}_2(t)\ M_2(t) - \frac{7}{15}\tilde{\alpha}_1(t)\ M_1(t) - Y_t(t)\ A_t(t) - 6Y_b(t)\ A_b(t) - Y_\tau(t)\ A_\tau(t) \quad (3.51)$$

$$\frac{dA_\tau(t)}{dt} = -3\tilde{\alpha}_2(t)\ M_2(t) - \frac{9}{5}\tilde{\alpha}_1(t)\ M_1(t) - 3Y_b(t)\ A_b(t) - 4Y_\tau(t)\ A_\tau(t) \quad (3.52)$$

*$\mu$ and $B_\mu$ parameters*

$$\frac{d\mu(t)}{dt} = \mu(t)\left(-\frac{3}{2}Y_t(t) - \frac{3}{2}Y_b(t) - \frac{1}{2}Y_\tau(t) + \frac{3}{2}\tilde{\alpha}_2(t) + \frac{3}{10}\tilde{\alpha}_1(t)\right) \quad (3.53)$$

$$\frac{dB_\mu(t)}{dt} = -3\tilde{\alpha}_2(t)\ M_2(t) - \frac{3}{5}\tilde{\alpha}_1\ M_1(t) - A_\tau\ Y_\tau - 3A_b\ Y_b - 3A_t\ Y_t \quad (3.54)$$

*First two generation squark and slepton masses*

$$\frac{dm^2_{Q_{1,2}}(t)}{dt} = \frac{16}{3}\tilde{\alpha}_3(t)\ M_3^2(t) + 3\tilde{\alpha}_2(t)\ M_2^2(t) + \frac{1}{15}\tilde{\alpha}_1(t)\ M_1^2(t) \quad (3.55)$$

$$\frac{dm^2_{U_{1,2}}(t)}{dt} = \frac{16}{3}\tilde{\alpha}_3(t)\ M_3^2(t) + \frac{16}{15}\tilde{\alpha}_1(t)\ M_1^2(t) \quad (3.56)$$

$$\frac{dm^2_{D_{1,2}}(t)}{dt} = \frac{16}{3}\tilde{\alpha}_3(t)\ M_3^2(t) + \frac{4}{15}\tilde{\alpha}_1(t)\ M_1^2(t) \quad (3.57)$$

$$\frac{dm^2_{L_{1,2}}(t)}{dt} = 3\tilde{\alpha}_2(t)\ M_2^2(t) + \frac{3}{5}\tilde{\alpha}_1(t)\ M_1^2(t) \quad (3.58)$$

$$\frac{dm^2_{E_{1,2}}(t)}{dt} = \frac{12}{5}\tilde{\alpha}_1\ M_1^2(t) \quad (3.59)$$

*Third generation sfermion masses*

$$\begin{aligned}\frac{dm^2_{Q_3}(t)}{dt} &= \frac{16}{3}\tilde{\alpha}_3(t)\ M_3^2(t) + 3\tilde{\alpha}_2(t)\ M_2^2(t) + \frac{1}{15}\tilde{\alpha}_1(t)\ M_1^2(t)\\ &- Y_t(t)\left(m^2_{Q_3}(t) + m^2_{U_3}(t) + m^2_{H_2}(t) + A_t(t)^2\right)\\ &- Y_b(t)\left(m^2_{Q_3}(t) + m^2_{D_3}(t) + m^2_{H_1}(t) + A_b(t)^2\right) \quad (3.60)\end{aligned}$$

$$\begin{aligned}\frac{dm^2_{U_3}(t)}{dt} &= \frac{16}{3}\tilde{\alpha}_3(t)\ M_3^2(t) + \frac{16}{15}\tilde{\alpha}_1(t)\ M_1^2(t)\\ &- 2Y_t(t)\left(m^2_{Q_3}(t) + m^2_{U_3}(t) + m^2_{H_2}(t) + A_t(t)^2\right) \quad (3.61)\end{aligned}$$

$$\begin{aligned}\frac{dm^2_{D_3}(t)}{dt} &= \frac{16}{3}\tilde{\alpha}_3(t)\ M_3^2(t) + \frac{4}{15}\tilde{\alpha}_1(t)\ M_1^2(t)\\ &- 2Y_b(t)\left(m^2_{Q_3}(t) + m^2_{D_3}(t) + m^2_{H_1}(t) + A_b(t)^2\right) \quad (3.62)\end{aligned}$$

$$\begin{aligned}\frac{dm^2_{L_3}(t)}{dt} &= 3\tilde{\alpha}_2(t)\ M_2^2(t) + \frac{3}{5}\tilde{\alpha}_1(t)\ M_1^2(t)\\ &- Y_\tau\left(m^2_{L_3}(t) + m^2_{E_3}(t) + m^2_{H_1}(t) + A_\tau^2(t)\right) \quad (3.63)\end{aligned}$$

$$\frac{dm^2_{E_3}(t)}{dt} = \frac{12}{5}\tilde{\alpha}_1(t)\ M_1^2(t) - Y_\tau(t)\left(m^2_{L_3}(t) + m^2_{E_3}(t) + m^2_{H_1}(t) + A_\tau^2(t)\right) \quad (3.64)$$

*Higgs mass parameters*

$$\begin{aligned}
\frac{dm^2_{H_1}(t)}{dt} &= 3\tilde{\alpha}_2(t)\ M_2^2(t) + \frac{3}{5}\tilde{\alpha}_1(t)\ M_1^2(t) \\
&- 3Y_b(t)\left(m^2_{Q_3}(t) + m^2_{D_3}(t) + m^2_{H_1}(t) + A_b(t)^2\right) \\
&- Y_\tau(t)\left(m^2_{L_3}(t) + m^2_{E_3}(t) + m^2_{H_1}(t) + A^2_\tau(t)\right) && (3.65) \\
\frac{dm^2_{H_2}(t)}{dt} &= 3\tilde{\alpha}_2(t)\ M_2^2(t) + \frac{3}{5}\tilde{\alpha}_1(t)\ M_1^2(t) \\
&- 3Y_t(t)\left(m^2_{Q_3}(t) + m^2_{U_3}(t) + m^2_{H_2}(t) + A_t(t)^2\right) && (3.66)
\end{aligned}$$

*Additional RGE for MSSM with lepton number violation*
Below we present RGE for the lepton number violating superpotential and soft potential parameters. These are presented separately for the case of dimensionful and dimensionless parameters. In writing them, we have neglected terms higher order in the L-violating parameter. This is permissible as we are interested only in small L-violation as required by neutrino masses.

*Bilinear R-violation*

$$\begin{aligned}
\frac{d\epsilon_i(t)}{dt} &= \epsilon_i(t)\left(\frac{3}{2}\tilde{\alpha}_2(t) + \frac{3}{10}\tilde{\alpha}_1(t) - \frac{1}{2}Y^E_{ii}(t) - \frac{3}{2}Y_t(t)\right) && (3.67) \\
\frac{dB_{\epsilon_i}(t)}{dt} &= B_{\epsilon_i}(t)\left(-\frac{1}{2}Y^E_i i(t) - \frac{3}{2}Y_t(t) + \frac{3}{2}\tilde{\alpha}_2(t) + \frac{1}{2}\tilde{\alpha}_1(t)\right) - \epsilon_i\left(3\tilde{\alpha}_2(t)M_2(t)\right. \\
&+ \left.\frac{3}{5}\tilde{\alpha}_1(t)M_1(t) + 3A_t(t)Y_t(t) + Y^E_{ii}(t)\right) && (3.68)
\end{aligned}$$

*Trilinear R-violation ($\lambda'$)*

$$\begin{aligned}
\frac{d\lambda'_{ijk}(t)}{dt} &= \lambda'_{ijk}(t)\left(-Y^E_{ii}(t) - Y^D_{jj}(t) - Y^U_{jj}(t) - 2Y^D_{kk}(t) - 3Y^D_{jj}(t)\delta_{jk} + \frac{8}{3}\tilde{\alpha}_3(t)\right. \\
&+ \left. 3\tilde{\alpha}_2(t) + \frac{7}{30}\tilde{\alpha}_1(t)\right) - \frac{6}{32\pi^2}\ h^D_{uu}(t)\lambda'_{iuu}(t)h^D_{jk}(t)\delta_{jk} && (3.69) \\
\frac{dB_{\epsilon_i}(t)}{dt} &= B_{\epsilon_i}(t)\left(-\frac{3}{2}Y_b(t) - \frac{1}{2}Y_\tau(t) + \frac{3}{2}\tilde{\alpha}_2(t) + \frac{3}{10}\tilde{\alpha}_1(t)\right) \\
&- \frac{3}{16\pi^2}\mu(t)\lambda'_{ijj}(t)h^d_{jj}(t)\left(\frac{1}{2}B_\mu(t) + A^{\lambda'}_{ijj}(t)\right) && (3.70) \\
\frac{dm^2_{\nu_i H_1}(t)}{dt} &= m^2_{\nu_i H_1}(t)\left(-\frac{3}{2}Y_b(t) - \frac{1}{2}Y_\tau(t)\right) - \frac{3}{32\pi^2}\lambda'_{ipp}(t)h^d_{pp}(t)\left(m^2_{H_1}(t)\right.
\end{aligned}$$

$$
\begin{aligned}
&+ \quad m^2_{L_i}(t) + 2m^2_{Q_p}(t) + 2A^{\lambda'}_{ipp}(t)A^D_{pp}(t) + 2m^2_{D_p}(t)\Big) && (3.71)\\
\frac{dA^{\lambda'}_{ijj}(t)}{dt} &= -\frac{9}{2}A^D_{jj}(t)Y^D jj(t) - \frac{3}{2}A^{\lambda'}_{ijj}(t)Y^D_{jj}(t) - A^U_{jj}(t)Y^U_{jj}(t)\\
&- \quad A^E_{ii}(t)Y^E_{ii}(t) - \frac{7}{30}M_1(t)\tilde{\alpha}_1(t) - 2M_2(t)\tilde{\alpha}_2(t) - \frac{16}{3}M_3(t)\tilde{\alpha}_3(t) && (3.72)
\end{aligned}
$$

*Trilinear R-violation ($\lambda$)*

$$
\begin{aligned}
\frac{d\lambda_{ijk}(t)}{dt} &= \lambda_{ijk}(t)\left(\frac{-1}{2}Y^E_{ii}(t) - \frac{1}{2}Y^E_{jj}(t) - \frac{1}{2}Y^E_{kk}(t) + \frac{3}{2}\tilde{\alpha}_2(t) + \tilde{\alpha}_1(t)\right)\\
&- \quad \frac{1}{2}\left(\lambda_{ijj}(t)Y^E_{jj}(t)\delta_{j3}\delta_{jk} + \lambda_{jii}(t)Y^E_{ii}(t)\delta_{i3}\delta_{ik}\right) && (3.73)\\
\frac{dB_{\epsilon_i}(t)}{dt} &= B_{\epsilon_i}(t)\left(-\frac{3}{2}Y_t(t) - \frac{1}{2}Y^E_i(t) + \frac{3}{2}\tilde{\alpha}_2(t) + \frac{1}{2}\tilde{\alpha}_1(t)\right)\\
&- \quad \frac{1}{32\pi^2}\mu(t)\lambda_{idd}(t)h^E_{dd}(t)\left(A^\lambda_{idd}(t) + \frac{1}{2}B_\mu(t)\right) && (3.74)\\
\frac{dm^2_{\nu_i H_1}(t)}{dt} &= m^2_{\nu_i H_1}(t)\left(-\frac{3}{2}Y^E_i(t) - \frac{1}{2}Y_\tau(t) - \frac{3}{2}Y_b(t)\right) - \frac{1}{32\pi^2}\lambda_{ijj}(t)h^E_{jj}(t)\Big(m^2_{H_1}(t)\\
&+ \quad m^2_{L_i}(t)2m^2_{L_j}(t) + 2A^\lambda_{ijj}(t)A^E_{jj}(t) + 2m^2_{E_j}\Big) && (3.75)\\
\frac{dA^\lambda_{ijj}(t)}{dt} &= -3M_1(t)\tilde{\alpha}_1(t) - 3M_2(t)\tilde{\alpha}_2(t) - \frac{7}{2}A^E_{jj}(t)Y^E_{jj}(t) - \frac{3}{2}A^\lambda_{ijj}(t)Y^E_{jj}(t) && (3.76)
\end{aligned}
$$

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# Chapter 4

# Bilinear R violation and Neutrino Anomalies

## 4.1 Introduction:

Among the Standard Model fermions, neutrinos are the only ones which do not carry any electric charge. This leads to the possibility that these particles may be of majorana nature. Majorana masses to the neutrinos would violate lepton number by two units. To generate majorana masses to the neutrinos within the Standard Model, one has to modify significantly its scalar sector. On the other hand, Supersymmetric Standard Model naturally allows for lepton number violation. However, these couplings are generally removed by imposing R-parity. To allow the lepton number violating couplings back in to the superpotential, one can discard R-parity and impose other symmetries like baryon parity. The R-breaking couplings now present in the superpotential would give rise to neutrino masses [1]. Another approach is to assume R-parity is spontaneously broken [2]. This approach however leads to a low scale massless mode called majoron [3] which can be cured [4]. In this thesis, we follow the earlier approach where explicit lepton number violating couplings are present in the superpotential.

In the present chapter, we look at a specific model of R-parity violation and study the neutrino mass spectrum in these models. As has been noted earlier, lepton number violation is characterised either by dimensionful $\epsilon_i$ terms or the dimensionless $\lambda, \lambda'$ terms in the superpotential. In the present chapter, we consider that only dimensionful $\epsilon_i$ terms are the source of lepton number violation in the superpotential. Such a scenario is sometimes known as bilinear R-parity violation in the literature.

$$W_{\not{L}} = \epsilon_i L'_i H_2 \tag{4.1}$$

There are several theoretical motivations to consider such a scenario. Theories with spontaneous breaking of lepton number can be closely identified with these models if one assumes $\epsilon = h\nu_R$ where $h$ is the Yukawa coupling of the gauge singlet and the singlet field is represented by $\nu_R$ [4]. The $\epsilon$ couplings can be identified as the *vevs* of the singlet fields setting the scale of the lepton number violation. Alternatively one could imagine a generalised Peccei-Quinn symmetry whose spontaneous breaking leads to $\mu$ and $\epsilon_i$ terms at the weak scale through dim 5 operators [5]. Moreover, it is possible to choose the PQ charges of the different fields in such a way that the generation of the effective trilinear operators is suppressed but bilinear terms are allowed [7]. A study of neutrino masses in bilinear R violating models is thus theoretically well motivated. Several works have been done to this extent in the recent past [1, 7, 8, 9].

As we have seen in chapter 2, with the advent of super-Kamiokande results on the atmospheric neutrinos with very high statistics, a definitive and clear indication for neutrino masses and mixing has been achieved. Though there are still ambiguities associated with the solar neutrino experiments, the structure of neutrino mass matrix has achieved more clarity with one large mixing as a necessary ingredient if one would like to have simultaneous solutions for solar and atmospheric neutrino problems with three active neutrinos. It has been known for sometime that neutrino mass matrix and mixing are completely calculable in terms of standard supersymmetry breaking parameters and R violating parameters in supersymmetry models with R violation. The question then arises is, whether it is possible to have a neutrino mass matrix which would be able to provide simultaneous solutions to both solar and atmospheric neutrino problems within bilinear R violating models which have a minimality of only three R violating couplings. Solutions to neutrino anomalies have been extensively studied by Hempfling [8] and Joshipura and Babu [7] within these models though they had not concentrated on simultaneous solutions. These studies have been done within the framework of standard supergravity inspired MSSM with universal boundary conditions at the high scale $\sim M_{GUT}$. In the present work we extend the analysis and look for simultaneous solutions to solar and atmospheric neutrino problems. We also confine our study to models where supersymmetry breaking is communicated through gauge interactions, in particular to the popular Minimal Messenger Model (MMM) [10]. We conduct our study [6] within the framework of Joshipura and Babu [7] which is more analytical.

## 4.2 Structure of Neutrino Masses:

The bilinear lepton number violating couplings have a unique feature associated with them. By a simple redefinition of the fields one can remove these dimensionful couplings from the superpotential. This is possible because the superfields $L_i, H_1$ carry the same quantum numbers under $SU(2)_L \times U(1)_Y$ and such a redefinition would leave the action invariant [11]. For example, considering only one dimensionful coupling in the superpotential $\epsilon_3$, a suitable redefinition of the type:

$$L_3 \to \frac{\epsilon_3 L_3 + \mu H_1}{\sqrt{\epsilon_3^2 + \mu^2}} \quad H_1 \to \frac{-\mu H_1 + \epsilon_3 L_3}{\sqrt{\epsilon_3^2 + \mu^2}} \tag{4.2}$$

would lead to absence of dimensionful lepton number violation in the superpotential. It should be however noted that such a redefinition is not possible in the Standard Model as the leptonic fields and the Higgs fields transform as two different representations under the Lorentz group. This distinction is removed in the MSSM.

The above feature of bilinear R violation had led to the general belief that these couplings would not be of interest as, by a general redefinition one can transform the dimensionful lepton number violation to the dimensionless lepton number violation in the superpotential. A crucial feature which works against the above arguments is the presence of supersymmetry breaking soft terms. These terms do not preserve the above redefinition in general [1]. This complication can completely change the phenomenology of the supersymmetry spectrum and neutrino masses in particular. To understand these issues in a better manner we follow the framework given by Joshipura and Babu. In this framework most of the features of the bilinear R parity violating models are transparent unlike the other studies which were more numerical.

*The framework:*

Here we consider a redefinition of the fields at the weak scale. A similar redefinition of the fields can be done at the high scale. In models with universal soft terms at the high scale, such a redefinition would rotate away the soft bilinear terms too. The superpotential with bilinear R-violation is written as :

$$W = \lambda_i L_i' E_i^c H_1' + \mu' H_1' H_2 + \epsilon_i L_i' H_2 + \lambda_i^D Q_i D_i^c H_1' + \lambda_{ij}^U Q_i U_j^c H_2 \ , \tag{4.3}$$

where $i, j = 1, 2, 3$ represents the generation index. A specific choice of the basis is also made such that these fields denote the mass eigenstates of the charged lepton (down quarks) in the absence of the $\epsilon_i$ terms. We now redefine the leptonic and the Higgs fields in such a way that the superpotential (4.3) does not contain bilinear $\epsilon_i$-dependent terms. The redefinition is an orthogonal transformation which we define as follows in the unprimed basis:

$$\begin{aligned} H_1 &= c_3 H_1' + s_3(s_2(c_1 L_2' + s_1 L_1') + c_2 L_3'), \\ L_1 &= -s_1 L_2' + c_1 L_1', \\ L_2 &= c_2(c_1 L_2' + s_1 L_1') - s_2 L_3', \\ L_3 &= -s_3 H_1' + c_3(s_2(c_1 L_2' + s_1 L_1') + c_2 L_3'); \end{aligned} \tag{4.4}$$

where

$$\begin{aligned} \epsilon_1 &= \mu s_1 s_2 s_3, & \epsilon_2 &= \mu c_1 s_2 s_3, \\ \epsilon_3 &= \mu s_3 c_2, & \mu' &= \mu c_3, \end{aligned} \tag{4.5}$$

and $\mu \equiv (\mu'^2 + \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2)^{1/2}$. The consequences of the above redefinition are two fold : a) The lepton number violation now reappears as trilinear lepton number violation in the superpotential. b)The originally diagonal charged lepton mass matrix $M_l$ now acquires [7] non-diagonal parts given by [1]:

$$\frac{M_l}{< H_1 >} = \begin{pmatrix} \lambda_1 c_1 c_3 & -\lambda_2 s_1 c_3 & 0 \\ \lambda_1 s_1 c_2 c_3 & \lambda_2 c_1 c_2 c_3 & -\lambda_3 s_2 c_3 \\ \lambda_1 s_1 s_2 & \lambda_2 c_1 s_2 & \lambda_3 c_2 \end{pmatrix} \tag{4.6}$$

We denote the mass basis for the charged leptons in the presence of non-zero $\epsilon_i$ as $\alpha = e, \mu, \tau$ and are defined as:

$$L_i = (O_L^T)_{i\alpha} L_\alpha, \quad e_i^c = (O_R^T)_{i\alpha} e_\alpha^c, \tag{4.7}$$

[1]Note that we have neglected here a sub-dominant contribution to $\mathcal{M}_l$ arising due to sneutrino vevs.

where

$$O_L M_l O_R^T = diagonal. \tag{4.8}$$

Note that the parameters $\lambda_i$ which denote charged lepton masses when $\epsilon_i = 0$ still need to be hierarchical if $M_l$ in (4.6) is to reproduce the charged lepton masses [2]. One can determine [7] $O_L$ by assuming $\lambda_1 \ll \lambda_2 \ll \lambda_3$ and neglecting $\lambda_1$:

$$O_L^T \approx N_1 \begin{pmatrix} c_1 & -s_1 N_2 & 0 \\ s_1 c_2 & c_1 c_2 N_2^{-1} & -s_2 c_3 N_1^{-1} N_2^{-1} \\ s_1 s_2 c_3 & c_1 s_2 c_3 N_2^{-1} & c_2 N_1^{-1} N_2^{-1} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & -\sin\theta_{23} \\ 0 & \sin\theta_{23} & \cos\theta_{23} \end{pmatrix}. \tag{4.9}$$

where

$$\begin{aligned} N_1 &= (1 - s_1^2 s_2^2 s_3^2)^{(-1/2)}, \\ N_2 &= (1 - s_2^2 s_3^2)^{(1/2)}, \\ \theta_{23} &\approx N_1 c_1 c_2 s_2 \left(\frac{s_3 m_\mu}{c_3 m_\tau}\right)^2. \end{aligned} \tag{4.10}$$

$\theta_{23}$ is small even for $s_{1,2,3} \sim O(1)$. We shall therefore neglect it. The trilinear terms generated in the superpotential due to rotation (4.4) assume the following form [7] in the physical mass eigenstate basis of charged leptons:

$$W = -\frac{\tan\theta_3}{< H_1 >}[(O_L^T)_{3\alpha} L_\alpha] \left(m_\beta^l L_\beta e_\beta^c + m_i^D Q_i d_i^c\right). \tag{4.11}$$

It should be noted here that the trilinear lepton number violating interactions generated by a rotation of bilinear couplings form only a part of the most general set of trilinear couplings, especially these couplings are flavour diagonal [7]. The above trilinear terms lead to neutrino masses at 1-loop [12]. The other contribution to neutrino mass is generated by the soft supersymmetry breaking terms in a manner discussed below.

### 4.2.1 Sneutrino vevs, neutrino masses and RG scaling

The soft supersymmetry breaking part of the scalar potential can be written as follows in the primed basis:

$$\begin{aligned} V_{soft} &= m_{H_1'^2} |H_1^{0\prime}|^2 + m_{H_2^2} |H_2^0|^2 + m_{\tilde{\nu}_i'^2} |\tilde{\nu}_i'|^2 \\ &- \left[H_2^0 \left(\mu' B_\mu H_1'^0 + \epsilon_i B_i \tilde{\nu'}_i\right) + H.c.\right]. \end{aligned} \tag{4.12}$$

The above soft potential contains bilinear lepton number violating terms which are linear in the sneutrino field. The presence of such terms can lead to the generation of vacuum expectation values for the sneutrinos. The generation of the sneutrino *vev* in these models should be however contrasted with models where lepton number is broken spontaneously

[2] This is because $\mathcal{M}_l$ is still diagonal even in the presence of non-zero $\epsilon_i$.

[2]. The generation of sneutrino *vev* in these models is a consequence of the explicit lepton number violating terms present in the superpotential. In the limit these couplings tend to zero there are no sneutrino *vevs* in these models. Hence these theories are not plagued by majoron problems.

Once the sneutrinos attain *vevs*, neutrinos mix with neutralinos through the kinetic terms in the kahler potential leading to a tree level neutrino mass. The neutrino mass so generated is proportional to the soft lepton number violating coupling, $B_{\epsilon_i}$ which can be $\sim O(M_{susy})$. Thus very large neutrino masses can be generated within these models which can be problematic phenomenologically. It has been found that a natural way of suppressing the neutrino mass would be to incorporate the theory in models where supersymmetry is broken at a high scale and the low energy supersymmetry spectrum is determined by renormalisation group evolution. As we have seen earlier, in these cases, universality between various soft parameters at the high scale is achievable like for example in supergravity inspired models. One can extend this universality to the lepton number violating soft terms which can lead to a suppressed sneutrino *vev* at the weak scale.

The above can be more transparently seen by observing that it is always possible to choose a minimum with zero sneutrino *vev* if $B_\mu = B_i$ and $m_{H_1'^2} = m^2_{\tilde{\nu}_i'}$. These equalities can be imposed at the high scale through universal conditions. But these are not generally satisfied at the weak scale. This is due to the presence of Yukawa couplings which distinguish between Higgs and leptons. Thus the RG equations which determine the low scale values of these parameters depend on the Yukawa couplings. If one neglects the Yukawa couplings of the first two generations then the soft mass parameters related to the first two leptonic generations evolve in the same way. Thus, one would have the following non-zero differences among low energy parameters:

$$\Delta m_{L,H} \equiv (m_{\tilde{\nu}_3'^2} - m^2_{\tilde{\nu}_2', H_1'}) \ , \quad \Delta B_{L,H} \equiv (B_3 - B_{2,\mu}). \tag{4.13}$$

These differences would now determine the sneutrino *vev*. The latter are obtained by minimizing the scalar potential expressed in the redefined basis of eqs.(4.4). In this basis, one finds:

$$\begin{aligned} V &= (m^2_{H_1} + \mu^2)|H_1^0|^2 + (m^2_{H_2} + \mu^2)|H_2^0|^2 + m^2_{\tilde{\nu}_3}|\tilde{\nu}_3|^2 + m^2_{\tilde{\nu}_2'}|\tilde{\nu}_2|^2 + m^2_{\tilde{\nu}_1'}|\tilde{\nu}_1|^2 \\ &- \left[\mu H_2^0 \left(B H_1^0 + c_3 s_3 \tilde{\nu}_3 (\Delta B_H - s_2^2 \Delta B_L) - c_2 s_2 s_3 \tilde{\nu}_2 \Delta B_L\right) \ + H.c.\right] \\ &+ \left[-c_2 s_2 \Delta m_L (s_3 H_1^0 + c_3 \tilde{\nu}_3)\tilde{\nu}_2^* + s_3 c_3 \tilde{\nu}_3 H_1^{0*}(\Delta m_H - s_2^2 \Delta m_L) \ + \ H.c.\right] \\ &+ \frac{1}{8}(g^2 + g'^2)(|H_1^0|^2 + |\tilde{\nu}_1|^2 + |\tilde{\nu}_2|^2 + |\tilde{\nu}_3|^2 - |H_2^0|^2)^2, \end{aligned} \tag{4.14}$$

where

$$\begin{aligned} m^2_{H_1} &= m^2_{H_1'} + s_3^2 \Delta m_H - s_2^2 s_3^2 \Delta m_L, \\ m^2_{\tilde{\nu}_3} &= m^2_{\tilde{\nu}_3'} - s_3^2 \Delta m_H - s_2^2 c_3^2 \Delta m_L, \\ m^2_{\tilde{\nu}_2} &= m^2_{\tilde{\nu}_2'} + s_2^2 \Delta m_L, \\ B &= B_\mu + s_3^2 \Delta B_H - s_2^2 s_3^2 \Delta B_L. \end{aligned} \tag{4.15}$$

The additional terms do not significantly affect the *vev* for the standard Higgs since the sneutrino *vevs* and $\Delta m_{H,L}$, $\Delta B_{H,L}$ are suppressed compared to $m^2_{SUSY}$. This remains true even for $s_3 \sim O(1)$. The effect of these terms is to induce *vevs* for the sneutrinos. These can be determined by the stationary value conditions of the above potential as:

$$
\begin{aligned}
< \tilde{\nu}_2 > &\approx \frac{c_2 s_2 s_3}{(m^2_{\tilde{\nu}_2} + D)}(v_1 \Delta m_L - \mu v_2 \Delta B_L) \; , \\
< \tilde{\nu}_3 > &\approx \frac{c_3 s_3}{(m^2_{\tilde{\nu}_3} + D)}(v_1(-\Delta m_H + s_2^2 \Delta m_L) - \mu v_2(-\Delta B_H + s_2^2 \Delta B_L)) \; .
\end{aligned}
\tag{4.16}
$$

We have neglected terms higher order in $\Delta m_{L,H}, \Delta B_{H,L}$ while writing the above equations. Note that one of the sneutrino field ($\equiv \tilde{\nu}_1$) does not acquire a *vev* in this basis. This *vev* would arise if Yukawa couplings of the first two generations neglected here are turned on. It should be noted however that the above sneutrino *vevs* have been derived from the RG improved 'tree level' scalar potential. Minimisation of the full one-loop effective potential can significantly modify these values [13]. For the sneutrino *vevs*, one loop corrections have been presented recently by Chun *et. al* [14] and Hirsch *et al* [15]. Chun *et. al* use effective potential method whereas, Hirsch *et. al* use diagrammatic method. In the limit of small R violation, as is in our case, Chun *et. al* have found that the one-loop corrections are significant only in the regions of parameter space where the tree level mass is suppressed and the loop mass to the neutrinos dominates. We do not consider these corrections here. However, as we will see below, the effect of these corrections would be negligible on our results as in the present model, we do not encounter such a scenario in the parameter space.

The sneutrino *vevs* are zero at the boundary scale $M_X$ corresponding to the universal masses. Their weak scale values are determined by solving the relevant RG equations. These RG equations can be determined by the general set of RGE presented in chapter 3. They are given as,

$$
\begin{aligned}
\frac{d\,\Delta m_H}{d\,t} &= 3Y_b(t)(m^2_{H_1'} + m^2_{\tilde{b}} + m^2_{\tilde{b}^c} + A_b^2) \; , \\
\frac{d\,\Delta m_L}{d\,t} &= -Y_\tau(t)(m^2_{H_1'} + m^2_{\tilde{\tau}} + m^2_{\tilde{\tau}^c} + A_\tau^2) \; , \\
\frac{d\,\Delta B_H}{d\,t} &= 3Y_b(t)A_b(t) \; , \\
\frac{d\,\Delta B_L}{d\,t} &= -Y_\tau(t)A_\tau \; ,
\end{aligned}
\tag{4.17}
$$

where we follow the same notation as earlier with $Y_f \equiv \frac{\lambda_f^2}{(4\pi)^2}$, $m_{\tilde{f}}$ is the mass of the sfermion concerned, $A_f$ are the trilinear soft susy breaking terms and $t = 2\,ln(M_X/Q)$.

The tree level mass matrix generated due to these *vev* [17] [3] can be written in the

[3] In the Appendix we present the details of this derivation

physical basis $\nu_\alpha$ as:

$$M_0 = m_0 O_L \begin{pmatrix} 0 & 0 & 0 \\ 0 & s_\phi^2 & s_\phi c_\phi \\ 0 & s_\phi c_\phi & c_\phi^2 \end{pmatrix} O_L^T, \tag{4.18}$$

where

$$\tan\phi = \frac{<\tilde{\nu}_2>}{<\tilde{\nu}_3>} \ . \tag{4.19}$$

$O_L$ is defined by eqs.(4.8) and,

$$m_0 = \frac{\mu(cg^2 + g'^2)(<\tilde{\nu}_2>^2 + <\tilde{\nu}_3>^2)}{2(-c\mu M + 2M_W^2 c_\beta s_\beta (c + tan^2\theta_W))}, \tag{4.20}$$

with $M_2$ now representing the weak scale value of the gaugino mass and $c = 0.49$ [4].

### 4.2.2 1-loop mass

The trilinear interactions in eq. (4.11) lead to diagrams involving squarks and sleptons in the loop and generate the neutrino masses at the 1-loop level[12]. These contributions depend upon the masses as well as mixing between the left and the right handed squarks as well as the sleptons. These are however fixed in terms of the basic parameters of supersymmetry breaking. In the present case, the trilinear couplings are not independent and are controlled by the fermion masses. As a consequence, the dominant contribution arises when the b-squark or $\tau$ slepton are exchanged in the loop. We shall retain only this contribution.

Let us define:

$$\begin{aligned} \tilde{b} &= \tilde{b}_1 \cos\phi_b + \tilde{b}_2 \sin\phi_b \ , \\ \tilde{b^c}^\dagger &= \tilde{b}_2 \cos\phi_b - \tilde{b}_1 \sin\phi_b \ . \end{aligned} \tag{4.21}$$

Where, $\tilde{b}_{1,2}$ are the mass eigenstates with masses $M_{b_1,b_2}$ respectively. The mixing angles $\phi_\tau$ and masses $M_{\tau_1,\tau_2}$ are defined analogously in case of the tau slepton. The exchange of b-squark produces the following mass matrix for the neutrinos:

$$(M_{1b})_{\alpha\beta} = m_{1b}(O_L)_{\alpha 3}(O_L)_{\beta 3}. \tag{4.22}$$

Due to the antisymmetry of the leptonic couplings in eq. (4.11), the exchange of the $\tau$ slepton leads to the following contribution:

$$M_{1\tau} = m_{1\tau} \begin{pmatrix} O_{L_{13}}^2 & O_{L_{13}} O_{L_{23}} & 0 \\ O_{L_{13}} O_{L_{23}} & O_{L_{23}}^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4.23}$$

[4] $M_1 = cM_2$ is used in deriving the above [18].

The mixing induced by these contributions is completely fixed by the matrix $O_L$ while the overall scale of both these contributions is set by,

$$m_{1b,1\tau} = N_c \frac{m_{b,\tau}^3}{16\pi^2 v_1^2} \tan\theta_3^2 \sin\phi_{b,\tau} \cos\phi_{b,\tau} \; ln \left( \frac{M^2_{\tilde{b}_2,\tilde{\tau}_2}}{M^2_{\tilde{b}_1,\tilde{\tau}_1}} \right), \tag{4.24}$$

where $N_c = 3, 1$ for the $\tilde{b}$ and $\tilde{\tau}$ contribution respectively.

In the above we have considered one loop corrections to be present only through the combinations of $\lambda\ \lambda$, $\lambda'\ \lambda'$ couplings. But, to derive the one loop neutrino masses, one has to consider the complete 1-loop corrections to the $7 \times 7$ neutrino-neutralino mass matrix [5]. This approach has been followed by [8] and recently by M. Hirsch *et.al* [15] where the complete 1-loop mass matrix is written in the tree level mass basis and re-diagonalised. However, the approximation we have made in this work can be justified following the work of Chun and collaborators [14]. This analysis is based on effective mixing matrix approach. Using this method, one can analytically understand the dominant contributions to the neutrino sector. The 1-loop corrections to the neutralino sector are of order $\sim 6\%$ and thus can be neglected. The corrections to the neutrino-neutralino mixing (Dirac-type) part are sub-dominant to the corrections induced in the neutrino mass matrix. 1-loop corrections to the neutrino mass matrix are through combinations of the couplings $\lambda\ \lambda$, $\lambda'\ \lambda'$, $h_\tau\ h_\tau$, $h_\tau\ \lambda$ [14]. However the most dominant contribution is through the combination of couplings $\lambda\ \lambda$, $\lambda'\ \lambda'$ [6].

The total mass matrix including the 1-loop corrections is given by,

$$\mathcal{M}_\nu = M_0 + M_{1b} + M_{1\tau}. \tag{4.25}$$

We stress that the above $\mathcal{M}_\nu$ is in the physical basis with diagonal charged lepton masses. This matrix assumes particularly simple form when rotated by the matrix $O_L$:

$$O_L^T \mathcal{M}_\nu O_L \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_0 s_\phi^2 + m_{1\tau} N_2^{-4} c_2^2 s_2^2 c_3^2 & m_0 s_\phi c_\phi + m_{1\tau} N_2^{-4} c_2 s_2^3 c_3^3 \\ 0 & m_0 s_\phi c_\phi + m_{1\tau} N_2^{-4} c_2 s_2^3 c_3^3 & m_0 c_\phi^2 + m_{1b} + m_{1\tau} N_2^{-4} s_2^4 c_3^4 \end{pmatrix} \tag{4.26}$$

This explicitly shows that one of the neutrinos is massless in our approximation of neglecting Yukawa couplings of the first two generations. The full mixing matrix analogous to the KM matrix is given by,

$$U = O_L O_\nu^T. \tag{4.27}$$

where $O_\nu$ is the matrix diagonalising the RHS of eq.(4.26). As we will show the mixing angle appearing in $O_\nu$ is small due to hierarchy in neutrino masses while, the $O_L$ can contain large mixing. Hence, the neutrino masses are determined by the matrix (4.26) and mixing among neutrinos is essentially fixed by eq.(4.9).

[5] Please see the appendix.

[6] Recently an additional diagram has been reported in the literature [16]. We do not consider it here.

The above formalism shows that the neutrino masses are greatly suppressed compared to the typical supersymmetry breaking scale if $\Delta m_{H,L}, \Delta B_{H,L}$ vanish at some scale $X$. The weak scale values of sneutrino *vev* and hence neutrino masses follow from evolution of these parameters. It is clear from eq.(4.17) that the $b$ and $\tau$ Yukawa couplings control the evolution of sneutrino *vev*. Similarly, the 1-loop masses following from eq. (4.24) are also controlled by the same couplings. As a result, all the effects of lepton number violating parameters $\epsilon_i$ can be rotated away from the full Lagrangian when the down quark and the charged lepton Yukawa couplings vanish. Neutrino masses also vanish in this limit.

This formalism has been applied to the case of MSSM with universal boundary conditions in [7, 8]. Before studying this model in the case of gauge mediated models which is the work of this thesis, we here review the main results of the works of Hempfling and Joshipura-Babu.

*Review of results from MSSM:*

As mentioned earlier, universality at the high scale for the soft parameters leads naturally to small neutrino masses at the weak scale. Constrained MSSM i.e, minimal Supergravity with universal boundary conditions at $M_{GUT}$ provides an appropriate framework from this point of view. The total parameters of the above model in the CMSSM framework are the standard CMSSM parameters $m$, $M_2$, $\tan\beta$, $A$ and the sign($\mu$) along with the R-parity violating parameters $\epsilon_i$ or the three angles $s_1, s_2, s_3$.

It is found that the loop mass $m_{1\tau}$ contributes negligibly to the neutrino mass spectrum, through out the CMSSM parameter space [7, 8]. In the limit $m_{1\tau} \ll m_{1b}, m_0$ approximate expressions for the neutrino masses can be given as [7]

$$\begin{aligned} m_{\nu_3} &\sim m_0 + m_{1b} \\ m_{\nu_2} &\sim \frac{m_0 m_{1b} s_\phi^2}{m_0 + m_{1b}}, \end{aligned} \tag{4.28}$$

where the approximation $s_\phi^2 \ll 1$ is used in obtaining the second line of the above. One of the neutrinos remains massless in this approximation. The main features of the neutrino mass spectrum in this framework can be summarized as follows :

- For most of the CMSSM parameter space, the tree level contribution dominates over the loop contribution giving rise to the hierarchical pattern in the mass spectrum [7, 8].
- But, there also exist regions in the CMSSM parameter space, where the two contributions to the sneutrino *vevs*, eq.(4.16) cancel each other for one particular sign of $\mu$. In these regions, the loop contribution, $m_{1b}$ can become comparable to the tree level mass, $m_0$ and cancellations among $m_0$ and $m_{1b}$ can take place. Here, the two neutrinos form a pseudo-Dirac pair with a common mass $m_0 s_\phi$ relevant for solutions of the neutrino anomalies [7].

- The mixing matrix is given by the product $O_L O_\nu^T$. Even in the case where $O_\nu$ allows only small mixing, large mixing can be generated from $O_L$, as it depends only on the ratios of the R-parity violating parameters [7].
- Even though the neutrino masses are suppressed in these scenarios, they are typically of the $O$(MeV). Thus, the R-parity violating parameters have to be chosen to be much suppressed compared to the typical order of the supersymmetry breaking scale if neutrino masses are to be of the right order to solve the neutrino anomalies [7, 8].
- Numerical results from Hempfling [8] show that solutions for solar (either with MSW conversion or vacuum oscillations) and the atmospheric neutrino anomalies can be accommodated naturally within these models.
- Recently an extensive analysis has been reported by M. Hirsch *al* within the framework mSUGRA inspired MSSM. They have reported to have found no solutions for the bi-maximal mixing scenarios within these models with universal boundary conditions. However, with non-universal boundary conditions one can achieve the required [15].

## 4.3 Gauge mediated models and neutrino masses

The suppression of neutrino masses due to universality of the supersymmetry breaking parameters at some scale not only happens in the MSSM with universal boundary conditions (CMSSM), but also in models where supersymmetry breaking is mediated through ordinary gauge interactions, which have been introduced in chapter 1. To understand how it happens, we consider here in some detail the minimal version of gauge mediated supersymmetry breaking [19]. In this case the messenger sector contains only one pair of superfields $\Phi, \bar{\Phi}$ transforming as $5+\bar{5}$ representation of the $SU(5)$ group. They couple to a field $S$ which is a gauge singlet. Both the scalar and the auxiliary components of $S$ attain *vevs*, thus introducing a supersymmetric mass scale $X \equiv \lambda < S >$ as well as a SUSY breaking (mass)$^2$ differences of order $F_S$. Models with minimal messenger sector are thus characterized by two parameters $\Lambda \equiv \frac{F_S}{X}$ and $x \equiv \frac{\Lambda}{X}$.

All the soft parameters related to MSSM fields are fixed at $X$ in terms of $\Lambda, x$ and the gauge couplings. The gauginos attain their masses at the 1-loop level whereas the sparticles attain their masses at the two-loop level. These masses have the following simple form [20]:

$$\begin{aligned} m_i^2(X) &= 2\Lambda^2 \left\{ C_3\tilde{\alpha}_3^2(X) + C_2\tilde{\alpha}_2^2(X) + \frac{3}{5}Y^2\tilde{\alpha}_1^2(X) \right\} f(x), \\ M_j(X) &= \tilde{\alpha}_j(X)\,\Lambda\, g(x), \end{aligned} \tag{4.29}$$

where $m_i^2$ represents the scalar masses with $i$ running over all the scalars, whereas, $M_j$ represents the gaugino masses with $j$ representing the three gauge couplings. The functions $f(x)$ and $g(x)$ have been derived in [21]. Here,

$$\tilde{\alpha}_j(X) = \frac{\alpha_j(X)}{(4\pi)}; \tag{4.30}$$

$C_3$ = 4/3,0 for triplets and singlets of $SU(3)_C$, $C_2$ = 3/4,0 for doublets and singlets of $SU(2)_L$ and Y = Q - $T_3$ is the hypercharge. Since the Higgs field $H_1$ and the lepton fields $L_i$ carry the same quantum numbers, $m^2_{H_1}(X) = m^2_{\nu_i}(X)$. The minimal messenger model (MMM) is further characterised by the assumption of the vanishing bilinear (B) and trilinear (A) soft parameters at scale $X$.

The phenomenology of the Minimal Messenger Model has been extensively studied in various papers [10, 21, 22, 23, 20, 24]. In this thesis, we follow the work of Borzumati [24]. The salient features of this model would not change much even in the presence of the non-zero $\epsilon_i$. These can be summarised as follows: Only one parameter essentially determines the entire soft spectrum as the dependence of the boundary conditions in eq.(4.29) on $x$ is very mild. In the present analysis we choose $x = \frac{1}{2}$ following [24]. The MMM is attractive in view of the very restricted structure it offers. But as we will show it turns out to be too restrictive if one wants to solve the solar and atmospheric neutrino problems simultaneously. We shall thus consider an alternative version on phenomenological grounds in which the boundary conditions (4.29) are still imposed but the value of $B_\mu$ at $\Lambda$ is not taken to be zero. This we call as non-minimal model of Gauge Mediated SUSY breaking.

*Framework for gauge mediated models*:

The neutrino mass framework developed in the earlier section holds good for the Gauge Mediated Models too except that the boundary conditions are now defined at the low scale, $X$. The formulae for the neutrino mass matrix remain unchanged. The value of the $B_\mu$ parameter at the weak scale gets fixed through its running. This in turn determines both $\mu$ as well as $\tan\beta$ through the minimisation equations presented in chapter 1:

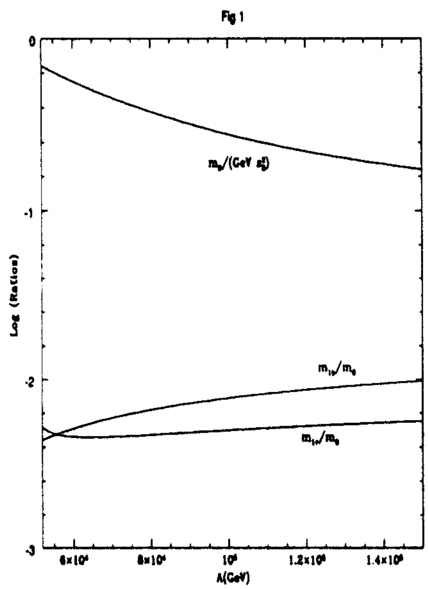
$$\begin{aligned} \mu^2 &= \frac{m^2_{H_1} - m^2_{H_2}\tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}M_Z^2, \\ \text{Sin}2\beta &= \frac{2B_\mu\mu}{m^2_{H_2} + m^2_{H_1} + 2\mu^2}. \end{aligned} \tag{4.31}$$

The presence of $\epsilon_i$ induces corrections to these equations, but they are very small as discussed below eq.(4.15). The eq.(4.31) therefore holds to a very good approximation.

In spite of the restricted structure, it is possible to self consistently solve the above equations [22, 20, 24] and implement breaking of the $SU(2)\times U(1)$ symmetry at low energy. Vanishing of the soft $B_\mu$ parameter at $X$ makes the analysis of this breaking little more involved than in the case of the supergravity induced breaking. One needs to include two loop corrections to the evolution of the $B_\mu$ parameter and also needs to use fully one loop corrected effective potential. Details of this analysis are presented in [22, 20, 24]. We follow the treatment given in [24]. The smallness of the $B_\mu$ at the weak scale results in this scheme, in relatively large value of $\tan\beta$ and its sign fixes the sign of $\mu$ to be positive. The full 1-loop corrected potential was employed in the analysis of [24] but it was found that working with RG improved tree level potential also gives similar results provided one evolves soft parameters of the supersymmetric partners up to a scale $Q_0^2 \equiv (m^2_{\tilde{Q}}(X)m^2_{\tilde{U}}(X))^{\frac{1}{2}}$. We prefer to follow this approach and use the RG improved tree level potential of eq.(4.14) in order to determine the low energy parameters at the minimum. We have however included two loop corrections to the RG equations [25] for $B_\mu$ and $\Delta B_{L,H}$ in determining their values at the weak scale. Use of RG improved tree level potential allows us to analytically understand the structure of neutrino masses and mixing in a transparent way.

*Neutrino Mass structure*:

The three mass parameters $m_{0,1b,1\tau}$, eqs.(4.20, 4.24) with appropriate definitions control the neutrino masses. $m_0$ is determined by solving RG equations (4.17) along with similar ones for parameters occurring in them. We have numerically solved them imposing eq.(4.29) as boundary conditions at $X$. We evolved these equations self consistently up to the scale $Q_0$ defined above. The $m_0$ determined in this manner depends upon $\mu$ as well as $\tan\beta$, both of which are fixed in terms of $\Lambda$.



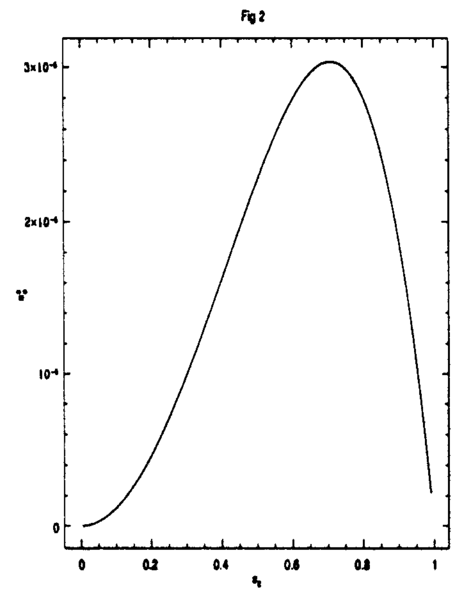




**Fig. 1.** The variations of $\frac{m_0}{(\mathrm{GeV} s_3^2)}$, $\frac{m_{1b}}{m_0}$, $\frac{m_{1\tau}}{m_0}$ are shown here with respect to $\Lambda$. $m_0$ mildly depends upon $s_2$ and the displayed curve is for $s_2 = 0.8$.
**Fig. 2.** The function $s_\phi^2$ is plotted here with respect to $s_2$.

The loop contributions are fixed in terms of the squark and slepton masses and mixings defined in eq. (4.21). These are determined from the standard $2\times2$ matrices involving left and right squarks and slepton mixing. The elements in these matrices are also completely fixed in terms of $\Lambda$. All the three parameters $m_0, m_{1b}, m_{1\tau}$ depend upon an overall scale $s_3$ of the $R$ breaking. For small $s_3^2$ they roughly scale as $s_3^2$. The ratios $m_0/s_3^2$, $m_{1b}/s_3^2$, $m_{1\tau}/s_3^2$ are thus determined by $\Lambda$ alone [7]. We have displayed in Fig. 1 variations of $\frac{m_0}{\mathrm{GeV} s_3^2}$, $\frac{m_{1b}}{m_0}$ and $\frac{m_{1\tau}}{m_0}$ with $\Lambda$. One clearly sees hierarchy in the loop and sneutrino *vev* induced contributions. This hierarchy gets reflected in the neutrino masses and one obtains hierarchical neutrino masses independent of the overall strength of the $R$ violating parameter $s_3$. The mass ratio

[7] $m_0$ also depends on $s_2$ very mildly through the *vev* $\langle\tilde{\nu}_2\rangle$ which is greatly suppressed compared to $\langle\tilde{\nu}_3\rangle$.

and hence the hierarchy among neutrino masses are seen to be less sensitive to $\Lambda$. The $m_0$ roughly scales linearly with $\Lambda$. But since the over all scale of $m_0$ is set by $s_3^2$ which is also unknown, a change in $\Lambda$ is equivalent to changing $s_3$. Thus, we may use one specific value of $\Lambda$ and neutrino mass spectrum is then completely fixed by three angles $s_{1,2,3}$ or equivalently by the three $R$ violating parameters $\epsilon_i$.

The $\Delta m_{H,L}, \Delta B_{H,L}$ entering $m_0$ are determined from the RG equations (4.17) and are fixed in terms of $\Lambda$. For example,

$$\begin{aligned} \mu &\sim 397.0\,\text{GeV}\,, & \tan\beta &\sim 46.39\,, \\ \Delta m_H &\sim 192661.23\,\text{GeV}^2\,, & \Delta m_L &\sim -2392.35\,\text{GeV}^2\,, \\ \Delta B_H &\sim -14.07\ \text{GeV}\,, & \Delta B_L &\sim 0.12\ \text{GeV}, \end{aligned} \tag{4.32}$$

when $\Lambda = 100$ TeV. The suppression in $\Delta m_L, \Delta B_L$ is due to color factors and larger squark masses compared to the slepton masses in the model. It follows that the ratio $\tan\phi$ of the sneutrino *vev*, eq. (4.19) gets considerably suppressed even when the angle $s_2$ is large. We show in Fig 2. the value of $s_\phi^2$ as function of $s_2$ for $\Lambda = 100$ TeV. Note that this ratio is independent of the values of the other $R$ violating parameters when $s_3$ is small. The small value of $s_\phi$ leads to very simple expression for neutrino masses. The neutrino mass matrix in eq.(4.26) is almost diagonal and one finds:

$$\begin{aligned} m_{\nu_3} &\sim m_0 + m_{1b}\,, \\ \frac{m_{\nu_2}}{m_{\nu_3}} &\sim c_2^2 s_2^2 \frac{m_{1\tau}}{(m_0 + m_{1b})} \sim c_2^2 s_2^2\ (7.1\ \times\ 10^{-3} - 5.6\ \times\ 10^{-3})\,, \\ \theta_{23}^\nu &\sim \frac{m_{1\tau} c_2 s_2^3}{(m_0 + m_{1b})} \sim \tan\theta_2 \frac{m_{\nu_2}}{m_{\nu_3}}. \end{aligned} \tag{4.33}$$

The masses are fixed in terms of $m_{0,1b,1\tau}$ which are determined in terms of $\Lambda$ and $s_3$. The mass ratio is fixed in terms of $s_2$. The range indicated on the RHS in above equation corresponds to variation in $\Lambda$ from (51 TeV- 150 TeV) and $\theta_{23}^\nu$ represents the angle diagonalising the matrix in eq.(4.26).

## 4.4 Neutrino masses: Phenomenology

As discussed in the last section, the model considered here implies hierarchical masses and large mixing without any fine tuning of the parameters. We now try to see if the predicted spectrum can be used to simultaneously reconcile both the solar and the atmospheric neutrino anomalies. The model is quite constrained. Three neutrino masses and three mixing angles get completely determined in the model in terms of four parameters namely, $\Lambda$ and three $R$ violating angles $s_{1,2,3}$. In particular, the angle $s_1$ characterizing the electron number violation does not enter the muon and tau neutrino masses, see eq.(4.26). The mixing between neutrinos is largely fixed by the matrix $O_L$ with a small correction coming from the angle $\theta_{23}^\nu$ in eq.(4.33). Thus one has approximately,

$$\nu_e \approx N_1(c_1\nu_1 - s_1c_2\nu_2 + s_1s_2c_3\nu_3)$$

$$
\begin{aligned}
\nu_\mu &\approx N_1(-s_1 N_2 \nu_1 + c_1 c_2 N_2^{-1} \nu_2 + c_1 s_2 c_3 N_2^{-1} \nu_3) \\
\nu_\tau &\approx N_1^{-1} N_2^{-1}(-s_2 c_3 \nu_2 s_1 + c_2 \nu_3)
\end{aligned} \tag{4.34}
$$

Note that $s_1$ ($s_2$) determines $\nu_e - \nu_\mu$ ($\nu_\mu - \nu_\tau$) mixing. We must thus require $s_2$ to be large in order to account for the atmospheric muon neutrino deficit. The $s_1$ should be small for the small angle MSW solution and large for the vacuum oscillation solution to the solar neutrino problem. As we now demonstrate these constraints are too tight and one does not obtain parameter space in case of the MMM, allowing simultaneous solution for both these problems.

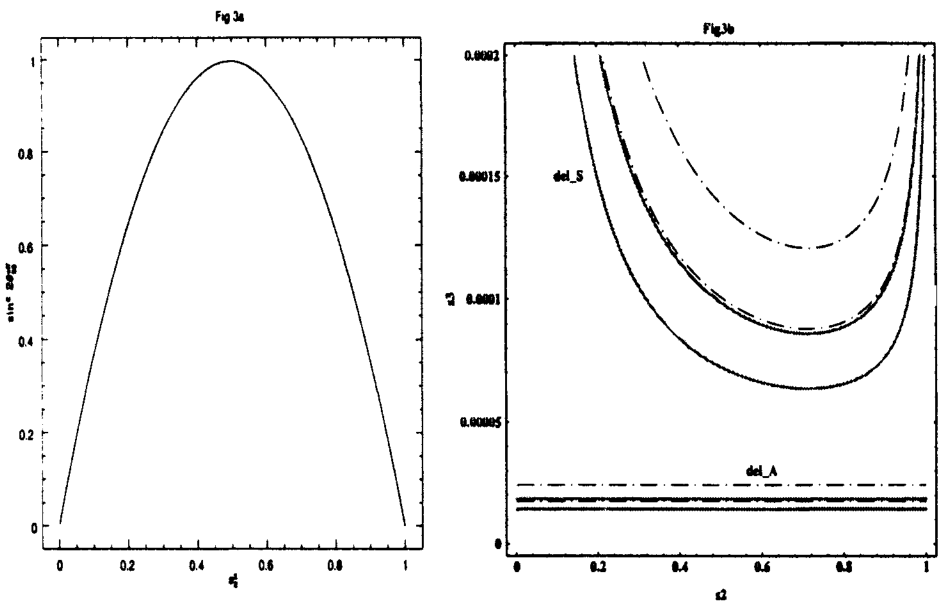



**Fig. 3a.** The effective $\nu_\mu$-$\nu_\tau$ mixing angle is plotted here with respect to $s_2^2$ for $\Lambda = 100\,\text{TeV}$ in the case of minimal messenger model. **Fig. 3b.** Contours of $\Delta m^2$ are plotted in MMM case, for $\Lambda = 70$ TeV (continuous lines) and $\Lambda = 150$ TeV (dash-dot). For $\Delta_A$, the upper (lower) lines correspond to $3 \times 10^{-3}\,\text{eV}^2$ ($0.3 \times 10^{-3}\,\text{eV}^2$). For $\Delta_S$, the upper(lower) lines correspond to $12 \times 10^{-6}\,\text{eV}^2$ ($3 \times 10^{-6}\,\text{eV}^2$).

### 4.4.1 MSW and atmospheric neutrino problem in MMM

The angle $s_1$ can be appropriately chosen to fix the required mixing for the small angle MSW conversion. The angle $s_3$ which determines the overall scale of neutrino masses is also required to be small. In such a case, the survival probability for the atmospheric $\nu_\mu$ assumes two generation form and one can take the restrictions on relevant parameters from the standard analysis as reported in chapter 2. We have determined the effective $\nu_\mu - \nu_\tau$ mixing and neutrino masses following from eq.(4.27) by the procedure outlined in the last section. We show this mixing in Fig.(3a). In Fig.(3b), we show the masses for two values of $\Lambda = 70\,\text{TeV}, 150\,\text{TeV}$. As seen from Fig.(3a), the $s_2 = 0.3 - 0.75$ leads to the required $\sin^2 2\theta_{\mu\tau} = 0.8-1$. Fig.(3b) displays the contours corresponding to $\Delta_S \sim (3.-12.)\ 10^{-6}\,\text{eV}^2$ and $\Delta_A = (0.3-3.)\ 10^{-3}\,\text{eV}^2$ in the $s_2 - s_3$ plane. It is seen that hierarchy among neutrino masses obtained in the required region is stronger than needed for a simultaneous solution of the solar and atmospheric neutrino problems and there is no overlapping region in the $s_2 - s_3$ plane for a combined solution. It is of course possible to solve each of this problem separately and get the required amount of mixing as well.

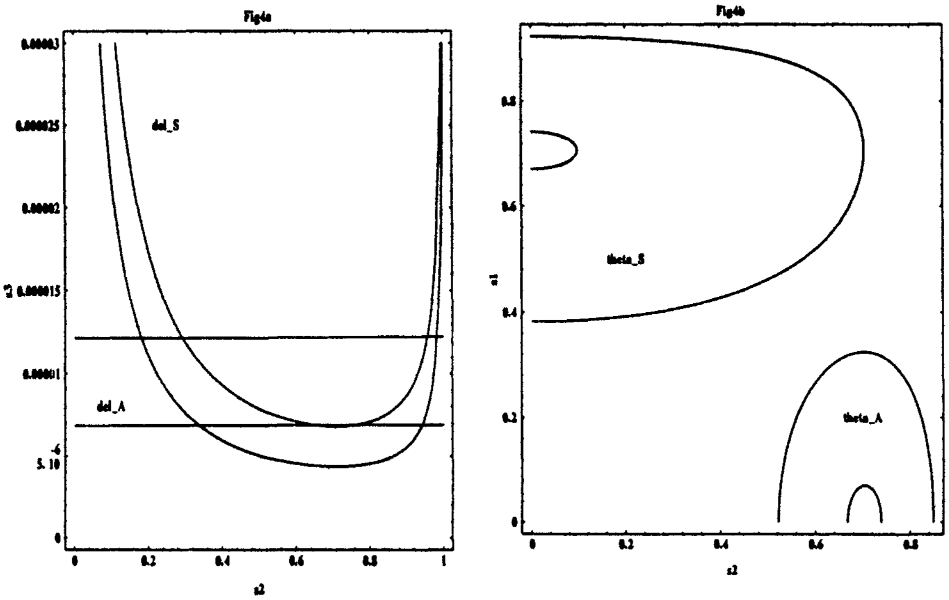


**Fig. 4a.** Contours of $\Delta m^2$ in MMM are plotted for $\Lambda = 100$ TeV. For $\Delta_A$, the upper (lower) line corresponds to $3 \times 10^{-3}\,\text{eV}^2$ $(0.3 \times 10^{-3}\,\text{eV}^2)$. For $\Delta_S$, the upper (lower) line corresponds to $3 \times 10^{-10}\,\text{eV}^2$ $(0.5 \times 10^{-10}\,\text{eV}^2)$. **Fig. 4b.** The effective $Sin^2 2\theta_S$ and $Sin^2 2\theta_A$ are plotted in the case of the minimal messenger model. The inner lines represent contours for 0.9 in both the cases whereas, the outer lines correspond to contours for 0.5 (0.8) for $Sin^2 2\theta_S$ $(Sin^2 2\theta_A)$.

### 4.4.2 Vacuum oscillations and atmospheric neutrino problem in MMM

Unlike in the case of the MSW interpretation, the model can nicely account for the hierarchies required for solving the solar and atmospheric neutrino problems through vacuum oscillations. This is displayed in Fig.(4a) where we show contours corresponding to $\Delta_S = (0.5-3)\ 10^{-10}\,\text{eV}^2$ and $\Delta_A = (0.3-3)\ 10^{-3}\,\text{eV}^2$ in the $s_2 - s_3$ plane. Unlike in case of the MSW conversion, here there is a large overlap region in $s_2 - s_3$ plane which leads to the required values for $\Delta_{S,A}$. Despite this one unfortunately cannot explain both the problems simultaneously in a phenomenologically consistent way. This is due to the very restricted mixing structure displayed in eqs.(4.27). As discussed in chapter 2, the vacuum oscillation probability in the present case is given by,

$$P_e = 1 - 4U_{e1}^2 U_{e2}^2 \sin^2\left(\frac{\Delta_S t}{4E}\right) - 2U_{e3}^2(1 - U_{e3}^2) \;, \tag{4.35}$$

where the last term comes from the averaged oscillations corresponding to the atmospheric neutrino scale. Likewise, the muon neutrino survival probability which determines the atmospheric neutrino flux is given by,

$$P_\mu = 1 - 4U_{\mu 2}^2 U_{\mu 3}^2 \sin^2\left(\frac{\Delta_A t}{4E}\right) . \tag{4.36}$$

The amplitude of oscillations is controlled by two effective angles:

$$\sin^2 2\theta_S = 4U_{e1}^2 U_{e2}^2 \;, \quad \sin^2 2\theta_A = 4U_{\mu 2}^2 U_{\mu 3}^2 . \tag{4.37}$$

The matrix U appearing above is given by eq.(4.27). Restrictions on these angles required for a combined solution of the solar and atmospheric anomaly are worked out in [26] for different values of $U_{e3}$. Independent of the chosen values for $U_{e3}$ one requires,

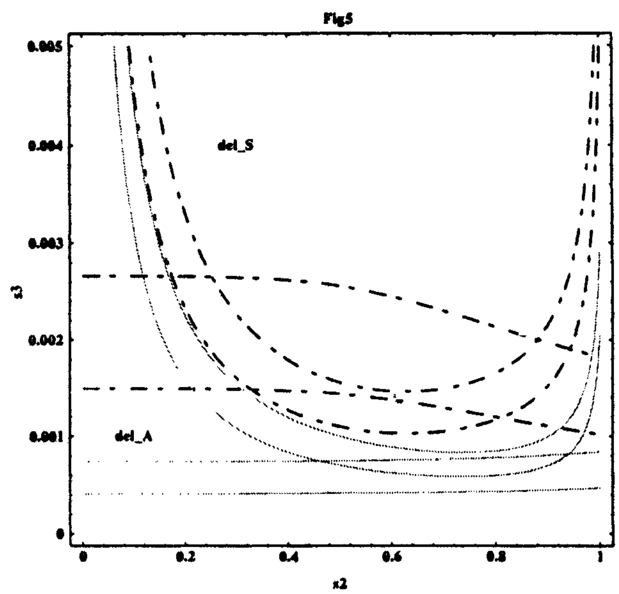
$$\sin^2 2\theta_S = 0.5 - 1 \;, \quad \sin^2 2\theta_A = 0.8 - 1 \;. \tag{4.38}$$

It is possible to choose these angles independently and satisfy above equations in a generic three generation case. In our case, the mixings are also determined in terms of $s_{1,2}$ through eq.(4.9). We have plotted the contours corresponding to restrictions in eq.(4.38) in Fig. 4b. *It is seen that there is no region in $s_1 - s_2$ plane for which the solar and vacuum mixing angles can be simultaneously large ruling out the possibility of reconciling atmospheric anomaly with vacuum solution in the case of the MMM .*

## 4.5 Non-minimal model and neutrino anomalies

We had restricted our analysis so far to the MMM which is characterized by eq.(4.29) and the vanishing of the $B$ and $A$ parameters at $\Lambda$. Apart from predictivity, there are no strong theoretical arguments in favour of this minimal choice. Such a choice would lead to positive $B_\mu$ at the weak scale which in turn assures $\mu$ to be positive as noted above. One could consider variations of the MMM which in general result in introduction of additional low energy parameters. A class of non-minimal models could contain more complicated messenger sector which would influence boundary conditions in eq. (4.29). Alternatively, one may keep the same messenger sector but introduce some direct coupling between messenger and matter fields. This could result in non-zero $B_\mu$ values at $X$. In fact, $B_\mu$ gets generated [19, 27] in models which try to understand origin of $\mu$ term in gauge mediated scenario [28]. $B_\mu$ may be generated in the absence of messenger-matter coupling if MSSM itself is extended.

**Fig. 5.** Contours of $\Delta m^2$ are plotted in **Non- MMM** case with $\mu < 0$, for $\tan\beta = 50$ (continuous lines) and $\tan\beta = 40$ (dash-dot) with $\Lambda = 100\,\mathrm{TeV}$. For $\Delta_A$, the upper




(lower) lines correspond to $3 \times 10^{-3}\,\mathrm{eV}^2$ ($0.3 \times 10^{-3}\,\mathrm{eV}^2$). For $\Delta_S$, the upper (lower) lines correspond to $12 \times 10^{-6}\,\mathrm{eV}^2$ ($3 \times 10^{-6}\,\mathrm{eV}^2$).

We shall not consider any specific model here, but would adopt a purely phenomenological attitude to point out possible ways which can allow simultaneous solutions of the solar and atmospheric neutrino anomalies. It turns out that prediction of $m_0$ is quite sensitive to the sign of $\mu$ term which is fixed to be positive in MMM. This follows from eq.(4.16) which shows that two contributions to $\langle\tilde{\nu}\rangle$ add or cancel depending on the sign of $\mu$. We may thus consider a slightly less restrictive form of the MMM in which we regard the value and sign of $B_\mu$ as independent parameters to be determined phenomenologically. Typically, the $B_\mu$ value remains positive due to the strong Yukawa sector coming from the higher loop terms in the RGE for $B_\mu$. One should thus generate $B_\mu$ sufficiently negative at the $\Lambda$ scale, so that, it remains negative even at the weak scale. The typical value of $B_\mu$ at the $\Lambda$ scale should be around,

$$B_\mu(X) \approx -12\ \tilde{\alpha}_3\ (Q_0)\ M_3(Q_0)\ Y_t\ t \approx -7\ 10^{-4}\ M_3(Q_0)\ t \tag{4.39}$$

where, the magnitude of RHS denotes the dominant positive contribution to $B_\mu$ with $Y_t$ denoting the top Yukawa coupling. A negative $B_\mu$ of this order at the $X$ scale can now generate a negative $\mu$ at the weak scale. We still assume that the mechanism responsible for generation of $B$ parameters does not distinguish between leptonic and the Higgs doublet $H_1$ and hence $B_\mu$ and $B_i$ coincide at the scale $X$. Due to this, sneutrino *vevs* are still characterized by the differences in eq.(4.13) and hence are suppressed. The boundary conditions on soft masses are still assumed to be given by eq.(4.29). This particular scenario is now characterized by parameters $\Lambda$ and $B_\mu$. As follows from the minimum equation, (4.31), one may regard the value of $\tan\beta$ and sign of $\mu$ as independent parameters instead of $B_\mu$. The magnitude of $\mu$ is determined in terms of these parameters by eq.(4.31). It is now possible to simultaneously account for the atmospheric neutrino deficit and have the MSW conversion for the solar neutrinos. This is depicted in Fig. (5) in which we show contours (in $s_2 - s_3$ plane) corresponding to $\Delta_S = 3-12\ 10^{-6}\,\mathrm{eV}^2, \Delta_A = 0.3-3\ 10^{-3}\,\mathrm{eV}^2$ for negative $\mu$ and two representative values of $\tan\beta = 40, 50$. The magnitude of $\mu$ gets fixed by eq. (4.31) to 379.28 GeV and 382.03 GeV in the respective cases. The corresponding $B_\mu$ value at the $X$ scale is approximately given as $B_\mu(X) \approx -13.10$ GeV ($\tan\beta = 40$); $-7.06$ GeV ($\tan\beta = 50$). It is seen that now there is a considerable overlap where two mass scales arise simultaneously. As mentioned before, these masses are independent of the value of $s_1$ which can be chosen in the required range namely,

$$s_1 = 0.0225 - 0.071$$

to allow MSW conversion. The angle $\theta_A$ relevant for the atmospheric anomaly coincides roughly with $s_2$ and as follows from Fig.(5), one can simultaneously account for mixing as well as masses needed to solve the atmospheric and the solar neutrino problems.

## 4.6 Conclusions

The supersymmetric standard model contains natural source of lepton number violation and hence of neutrino masses. The resulting neutrino mass pattern is quite constrained if source of lepton number violation is provided by soft bilinear operators and if the SUSY breaking is introduced through gauge mediated interactions. This scenario has the virtue that one can obtain hierarchical masses and large mixing in the neutrino sector. The hierarchy in masses results from hierarchy in the two different sources of neutrino masses while large mixing can be linked to ratio of $R$ violating parameters $\epsilon_i$. Overall scale of neutrino mass is set by $s_3$ and by the Yukawa couplings of the $b$ and $\tau$ . Neutrino masses are thus naturally suppressed and hierarchical. One however needs to assume relatively suppressed $R$ violation, i.e. $s_3 \sim 10^{-3}$ in order to obtain the mass scale relevant for the atmospheric neutrino anomaly. This requires that $\epsilon_2 \sim \epsilon_3 \sim 1\,\mathrm{GeV}$ when $\mu \sim 1\,\mathrm{TeV}$.

In case of the minimal gauge mediated model, the three neutrino masses and three mixing angles are controlled by five parameters $\Lambda$, $x$ and $s_{1,2,3}$. This proves to be quite constraining and does not allow one to obtain simultaneous solution of the solar and atmospheric neutrino anomalies. In the actual analysis, we chose $x = \frac{1}{2}$. A smaller value of $x$ would not change this conclusion in view of the mild dependence of the boundary conditions, eq.(4.29) on $x$. However, a non-minimal version which allows negative $\mu$ parameter is capable of accommodating the MSW effect and atmospheric neutrino anomaly. The number of parameters needed are still less than in the models based on the minimal supergravity scenario.

### 4.6.1 RG equations

In this subsection we present the two-loop parts of the RGE for the parameters $B_\mu, \Delta B_H, \Delta B_L$ used in the calculations in the MMM model.

$$\begin{aligned}
\frac{dB_\mu^{2-loop}}{dt} &= 30\tilde{\alpha}_2(t)M_2(t) + \frac{9}{5}\tilde{\alpha}_2(t)\tilde{\alpha}_1(t)\left(M_1(t)+M_2(t)\right) + \frac{207}{25}\tilde{\alpha}_1^2(t)M_1(t) \\
&+ 16\tilde{\alpha}_3(t)M_3(t)Y_t(t) + \frac{12}{15}\tilde{\alpha}_1(t)M_1(t)Y_t(t) + 16\tilde{\alpha}_3(t)M_3(t)Y_b(t) \\
&- \frac{12}{30}\tilde{\alpha}_1(t)M_1(t)Y_b(t) + \frac{12}{10}\tilde{\alpha}_1(t)M_1(t)Y_\tau(t) - 16\tilde{\alpha}_3(t)A_t(t)Y_t(t) \\
&- \frac{12}{15}\tilde{\alpha}_1(t)A_t(t)Y_t(t) - 16\tilde{\alpha}_3(t)A_b(t)Y_b(t) + \frac{12}{30}\tilde{\alpha}_1(t)Y_b(t)A_b(t) \\
&- \frac{12}{10}\tilde{\alpha}_1(t)A_\tau(t)Y_\tau(t) + 6A_\tau(t)Y_\tau^2(t) + 6\left(A_t(t)+A_b(t)\right)Y_t(t)Y_b(t) \\
&+ 18A_t(t)Y_t(t)^2 + 18Y_b(t)^2A_b(t)
\end{aligned} \tag{4.40}$$

$$\begin{aligned}
\frac{d\Delta B_L^{2-loop}}{dt} &= 6Y_\tau^2(t)A_\tau(t) + 3Y_\tau(t)Y_b(t)\left(A_b(t)+A_\tau(t)\right) \\
&- \frac{6}{5}\tilde{\alpha}_1(t)A_\tau(t)Y_\tau(t) + \frac{6}{5}\tilde{\alpha}_1(t)M_1(t)Y_\tau(t)
\end{aligned} \tag{4.41}$$

$$\begin{aligned}
\frac{d\Delta B_H^{2-loop}}{dt} &= 3Y_\tau(t)Y_b(t)\left(A_\tau(t)+A_b(t)\right) - 18Y_b^2(t)A_b(t) \\
&- 3Y_b(t)Y_t(t)\left(A_t(t)+A_b(t)\right) - 6Y_b(t)A_b(t)\left(\frac{8}{3}\tilde{\alpha}_3(t) - \frac{1}{15}\tilde{\alpha}_1(t)\right) \\
&- 6Y_b(t)\left(\frac{8}{3}\tilde{\alpha}_3(t)M_3(t) - \frac{1}{15}\tilde{\alpha}_1(t)M_1(t)\right)
\end{aligned} \tag{4.42}$$

## 4.7 Appendix

We present here the derivation of the tree level mass matrix, eq.(4.18). In the presence of R-violating couplings neutrinos mix with neutralinos. In the bilinear R-parity violating scenarios, this mixing takes place with $\epsilon_i$ couplings and the sneutrino *vevs*. In the Weyl basis, the Lagrangian describing the neutrino-neutralino mass matrix is given by,

$$\mathcal{L}_{mass} = -\frac{1}{2}\Psi_0^T \mathcal{M}_0 \Psi_0 + H.c \tag{4.43}$$

where in the two component notation, $\Psi_0$ is a column vector of neutrinos and neutralinos,

$$\Psi_0^T = \left(\nu_e, \nu_\mu, \nu_\tau, -i\lambda_1, -i\lambda_3, \psi_{H_1}^0, \psi_{H_2}^0\right) \tag{4.44}$$

The mass matrix has the following general structure which is of see-saw type:

$$\mathcal{M}_0 = \begin{pmatrix} 0 & m \\ m^T & M_4 \end{pmatrix} \tag{4.45}$$

Here the sub-matrix $m$ is of dimension $3 \times 4$ and has the following structure:

$$m = \begin{pmatrix} -\frac{g_1}{\sqrt{2}}\omega_1 & \frac{g_2}{\sqrt{2}}\omega_1 & 0 & \epsilon_1 \\ -\frac{g_1}{\sqrt{2}}\omega_2 & \frac{g_2}{\sqrt{2}}\omega_2 & 0 & \epsilon_2 \\ -\frac{g_1}{\sqrt{2}}\omega_3 & \frac{g_2}{\sqrt{2}}\omega_3 & 0 & \epsilon_3 \end{pmatrix} \tag{4.46}$$

with the $\omega_i$ representing the sneutrino *vevs*. $M_4$ is the standard $4 \times 4$ neutralino mass matrix of the MSSM which has the following form:

$$M_4 = \begin{pmatrix} M_1 & 0 & -\frac{g_1}{\sqrt{2}}v_1 & \frac{g_1}{\sqrt{2}}v_2 \\ 0 & M_2 & -\frac{g_2}{\sqrt{2}}v_1 & \frac{g_2}{\sqrt{2}}v_2 \\ -\frac{g_1}{\sqrt{2}}v_1 & \frac{g_1}{\sqrt{2}}v_1 & 0 & -\mu \\ -\frac{g_2}{\sqrt{2}}v_2 & \frac{g_2}{\sqrt{2}}v_2 & -\mu & 0 \end{pmatrix} \tag{4.47}$$

The effective $3 \times 3$ neutrino mass matrix is obtained by block diagonalising the above matrix. It has the form :

$$\begin{aligned} m_{eff} &= -mM_4^{-1}m^T \\ &= \frac{\mu(M_1 g^2 + M_2 g'^2)}{D} \begin{pmatrix} \Lambda_1^2 & \Lambda_1\Lambda_2 & \Lambda_1\Lambda_3 \\ \Lambda_1\Lambda_2 & \Lambda_2^2 & \Lambda_2\Lambda_3 \\ \Lambda_1\Lambda_3 & \Lambda_2\Lambda_3 & \Lambda_3^2 \end{pmatrix} \end{aligned} \tag{4.48}$$

The vector $\vec{\Lambda}$ is defined as,

$$\vec{\Lambda} = \mu\vec{\omega} - v_1\vec{\epsilon}. \tag{4.49}$$

$D$ is given by,

$$D = 2(-\mu\ M_1\ M_2 + 2\ M_W^2 c_\beta s_\beta (M_1 + M_2\ tan^2\theta_W)). \tag{4.50}$$

Here, the matrix $m_{eff}$ is written in the basis where $\epsilon_i$ are not rotated away from the superpotential. The rotated form of this matrix is given in the text and takes the form $M_0$, eq.(4.18), in the charged lepton mass basis. From $m_{eff}$ we can easily see that it has only one eigenvalue, even in the presence of the first generation sneutrino *vev*. This is a generic result of all the R-parity violating models [17, 29].

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Chapter 5

# Trilinear R violation and Neutrino Masses

## 5.1 Introduction

In the previous chapter we have been concerned with neutrino masses in the presence of only bilinear R-parity violating couplings. As we have seen bilinear R violation provides a very constrained framework to accommodate simultaneous solutions for both solar and atmospheric neutrino problems. In this chapter we pursue an alternative framework where we consider trilinear R parity violation in the superpotential. We will study the structure of neutrino mass matrix in this framework and the existence of simultaneous solutions for the solar and atmospheric neutrino problems within these models.

One can immediately see that these models definitely allow more freedom compared to the bilinear case as the $\lambda'$ and $\lambda$ are large in number. The $\lambda'$ couplings are 27 in number whereas the $\lambda$ couplings are 9 in number. Thus due to the large freedom they offer we would expect that simultaneous solutions for solar and atmospheric neutrino problems could be accommodated in these models for some definite choice of the parameters. But, in addition to bringing in larger freedom these models also bring in large amount of arbitrariness with them which would make calculations cumbersome. To overcome this arbitrariness and make these models predictive, the following choices are generally made in literature:
1). Assume $\lambda'$, $\lambda$ couplings to be hierarchical. This choice is generally made by motivating that $\lambda'$ couplings if present in the superpotential are likely to have the same pattern as the standard Yukawa couplings which give masses to the fermions [1].
2). Assume only a sub-set of the $\lambda'$, $\lambda$ couplings is non-zero. Fix the values of these couplings numerically, requiring that the neutrino mass matrix determined by this sub-set of couplings would give rise to the required pattern. This analysis can then be extended to all possible sub-sets [2].
In this thesis we do not make any of the above choices. We instead make the assumption that all the trilinear couplings are present in the superpotential are free parameters, but they are typically of the same magnitude. This is a valid assumption as the R-parity violating couplings are required to be suppressed compared to the regular Yukawa couplings to give correct order of neutrino masses to understand neutrino anomalies. This assumption has been earlier followed by Drees et.al [3] in the same context. In their work, neutrinos attain masses at the 1-loop level due to the presence of trilinear interactions. But, as we have seen in chapter 3, in any realistic model of supersymmetry breaking, trilinear interactions would also modify the weak scale soft potential through RG scaling. This would give

additional 'tree level' mass to the neutrinos. In this chapter [4], we reanalyse the neutrino mass spectrum in these models taking in to consideration the 'tree level' contribution to the neutrino masses also. This would significantly modify the neutrino mass spectrum as we will see below.

## 5.2 Sneutrino vevs and Neutrino Masses

For definiteness, we would consider only trilinear $\lambda'$ couplings to be present in the superpotential. Such an assumption is made for simplicity and we would comment on the inclusion of $\lambda$ couplings later on. The superpotential in this case is given as,

$$W = h^u_{ij} Q_i U^c_j H_2 + h^d_{ii} Q_i d^c_i H_1 + h^e_{ii} L_i e^c_i H_1 + \mu H_1 H_2 + \lambda'_{ijk} L_i Q_j D^c_k, \tag{5.1}$$

where we have used the standard notation specified in chapter 1. As we have mentioned earlier, the presence of non-zero $\lambda'$ couplings would induce two separate contributions to the neutrino mass. One contribution arises due to a soft term linear in the sneutrino *vev*. This soft term gets generated through the loops but leads to a sneutrino *vev* in the 'tree level' of the renormalisation group improved low energy effective potential. This contribution indirectly generates neutrino masses through their mixing with the gauginos. A majorana mass term for the light neutrinos is also generated directly by the loop diagrams involving squarks and sleptons [5, 6]. We shall refer to these two contributions as RG induced tree level (or simply tree level) and loop level masses respectively.

*The framework*

In the present chapter, we choose to work in the mSUGRA inspired MSSM. In this case the soft terms are added at the high scale $\sim M_{GUT}$. In the limit all $\lambda'$ couplings are chosen to be similar in magnitude, the analysis of the neutrino mass matrix would become simpler. It is now possible to study the neutrino mass matrix analytically. The two contributions to the neutrino mass matrix are given as below.

### 5.2.1 RG induced Tree Level Mass:

In the mSUGRA inspired MSSM framework, the structure of the superpotential dictates the structure of the soft SUSY breaking terms. Thus, with only trilinear L-violating interactions, the soft terms do not contain bilinear terms at a high scale. But as we have seen in chapter 3, they are nevertheless generated at the weak scale and should be retained in the scalar potential at this scale. The relevant part of the soft scalar potential is now given as:

$$\begin{aligned} V_{soft} &= m^2_{L_i} \mid \tilde{\nu}_i \mid^2 + m^2_{H_1} \mid H^0_1 \mid^2 + m^2_{H_2} \mid H^0_2 \mid^2 + \Big[ m^2_{\nu_i H_1} \tilde{\nu}^\star_i H^0_1 \\ & \quad - u B_\mu H^0_1 H^0_2 - B_{\epsilon_i} \tilde{\nu}_i H^0_2 + h.c \Big] + \dots\ , \end{aligned} \tag{5.2}$$

where, we have retained only neutral fields and used standard notation with $B_{\epsilon_i}$ and $m^2_{\nu_i H_1}$ representing the bilinear lepton number violating soft terms. The weak scale value of the lepton number violating soft parameters is determined by the RGE given in chapter 3. We reproduce them here:

$$\begin{aligned}
\frac{dB_{\epsilon_i}}{dt} &= B_{\epsilon_i}\left(-\frac{1}{2}Y^\tau - \frac{3}{2}Y^t + \frac{3}{2}\tilde{\alpha}_2 + \frac{3}{10}\tilde{\alpha}_1\right) - \frac{3}{16\pi^2}\ \mu\ h_k^D\ \lambda'_{ikk}\left(\frac{1}{2}B_\mu + A^{\lambda'}_{ikk}\right), \\
\frac{dm^2_{\nu_i H_1}}{dt} &= m^2_{\nu_i H_1}\left(-2Y^\tau - \frac{3}{2}Y^b\right) - \left(\frac{3}{32\pi^2}\right)\ h_k^D\ \lambda'_{ikk}\left(m^2_{H_1} + m^2_{L_i}\right. \qquad (5.3) \\
&+ \left. 2\ m^{2Q}_{kk} + 2\ A^{\lambda'}_{ikk}A^D_{kk} + 2\ m^{2D^c}_{kk}\right),
\end{aligned}$$

and the standard MSSM RGE for the parameters on the RHS [1]. In the above, we have confined ourselves with the same notation as in chapter 3. Since we allow only trilinear interactions in $W_{\not{L}}$, $m^2_{\nu_i H_1} = B_{\epsilon_i} = 0$ at high scale. In the presence of non-zero $\lambda'_{ijk}$ and with the above boundary conditions, the parameters $m^2_{\nu_i H_1}$ and $B_{\epsilon_i}$ have low energy solutions of the form given in chapter 3. Using the solutions for $\lambda'_{ijj}$ and $h^D_{jj}$, it is convenient to parameterise these solutions as [2],

$$\begin{aligned}
B_{\epsilon_i} &= \lambda'_{ipp}h^D_p\kappa_{ip}, \\
m^2_{\nu_i H_1} &= \lambda'_{ipp}h^D_p\kappa'_{ip} \ . \qquad (5.4)
\end{aligned}$$

Here $p$ is summed over generations. The parameters $\kappa$ and $\kappa'$ represent the running of the parameters present in the RGE's from the high scale to the weak scale.

The above soft potential would now give rise to sneutrino *vevs*, which are described by the stationary value conditions for the above soft potential. These are given as,

$$< \tilde{\nu}_i > \ = \frac{B_{\epsilon_i}v_2 - m^2_{\nu_i H_1}v_1}{m^2_{L_i} + \frac{1}{2}m^2_Z\ cos2\beta}\ , \qquad (5.5)$$

where $v_1$ and $v_2$ stand for the *vevs* of the Higgs fields $H_1^0$ and $H_2^0$ respectively. The sneutrino *vevs* so generated will now mix the neutrinos with the neutralinos thus giving rise to a tree level neutrino mass matrix as has been described in the previous chapter. This matrix has the structure:

$$\mathcal{M}^0_{ij} = \frac{\mu(cg^2 + g'^2)\ < \tilde{\nu}_i >\ < \tilde{\nu}_j >}{2(-c\mu M_2 + 2M^2_w c_\beta s_\beta(c + tan\theta^2_w))}\ , \qquad (5.6)$$

where $c = \frac{5g'^2}{3g^2}$ and $g(g')$ denotes $SU(2)(U(1))$ gauge coupling.

[1] The standard RGE for the soft parameters appearing on the RHS of the above equations do get modified in the presence of the $\lambda'$ couplings. But as mentioned in chapter 3, in the limit of very small $\lambda'$ couplings, as will be required by our model, the presence of these couplings would not modify the running of the soft parameters appreciably.

[2] The assumption that all $\lambda'$ couplings are similar in magnitude is made at the weak scale. Such an assumption at the high scale would in general lead to hierarchical $\lambda'$ at the weak scale.

### 5.2.2 Loop Level Mass

As we have seen earlier, the trilinear couplings in the superpotential would also give rise to a loop induced neutrino mass with the down squark and anti-squark pairs being exchanged in the loops along with their ordinary partners [5, 7]. This mass can be written as,

$$\mathcal{M}^l_{ij} = \frac{3}{16\pi^2}\lambda'_{ipk}\lambda'_{jkp}\ v_1\ h^D_k\ sin\phi_p\ cos\phi_p\ ln\frac{M^2_{2p}}{M^2_{1p}} \tag{5.7}$$

In the above, $sin\phi_p\ cos\phi_p$ determines the mixing of the squark-antisquark pairs and $M^2_{1p}$ and $M^2_{2p}$ represent the eigenvalues of the standard $2\times\ 2$ mass matrix of the down squark system [8]. The indices $p$ and $k$ are summed over. The mixing $sin\phi_p\ cos\phi_p$ is proportional to $h^D_p$ and thus one can write the loop mass as,

$$\mathcal{M}^l_{ij} = \lambda'_{ipk}\ \lambda'_{jkp}\ h^D_k\ h^D_p\ m_{loop}\ . \tag{5.8}$$

$$\text{Explicitly,}\quad m_{loop} \equiv \frac{3\ v_1}{16\pi^2}\ \frac{sin\phi_p\ cos\phi_p}{h^D_p}\ ln\frac{M^2_{2p}}{M^2_{1p}} \sim \frac{3\ v_1^2}{16\pi^2}\ \frac{1}{M_{SUSY}}, \tag{5.9}$$

with $M_{SUSY} \sim 1$ TeV referring to the typical scale of SUSY breaking. Note that $m_{loop}$ defined above is independent of the R violating couplings and is solely determined by the parameters of the minimal supersymmetric standard model (MSSM).

## 5.3 Neutrino Masses and Mixing

We now make a simplifying approximation which allows us to discuss neutrino masses and mixing analytically. It is seen from the RG eqs.(5.3) that the parameters $\kappa_{ik}$, $\kappa'_{ik}$, defined in eq. (5.4) are independent of generation structure in the limit in which generation dependence of the scalar masses $m^2_{L_i}$, $m^2_{Q_i}$ and soft parameters $A^{\lambda'}_{ikk}$ and $A^D_{ikk}$ is neglected. Since we are assuming the universal boundary conditions, this is true in the leading order in which the $Q^2$ dependence of the parameters multiplying $\lambda'_{ikk}h^D_k$ in eq.(5.3) is neglected. $Q^2$ dependence in these parameters generated through the gauge couplings will also be flavour blind though Yukawa couplings will lead to some generation dependent corrections. But their impact on the the conclusions based on the analytic approximation below is not expected to be significant [3]. The neglect of the generation dependence of $\kappa_{ik}, \kappa'_{ik}$ allows us to rewrite eq.(5.6) as,

$$\mathcal{M}^0_{ij} \equiv m_0 a_i a_j\ , \tag{5.10}$$

where $a_i \equiv \lambda'_{ikk}\ h^D_k$ and $m_0$ is now completely determined by the standard MSSM parameters and the dependence of the R-violating parameters gets factored out as in eq.(5.8). $m_0$ can be determined by solving the relevant RGE. Roughly, $m_0$ is given by,

$$m_0 \sim \left(\frac{3}{4\pi^2}\right)^2 \frac{v^2}{M_{SUSY}} \left(ln\frac{M^2_{GUT}}{M^2_Z}\right)^2\ , \tag{5.11}$$

[3] A rough estimate gives $\sim 20\%$ variation in the parameter space due to this approximation.

where $M_{GUT} = 3 \cdot 10^{16}\,\mathrm{GeV}$.
Let us rewrite the loop induced mass matrix as,

$$\begin{aligned} \mathcal{M}^l_{ij} &= m_{loop}\ \lambda'_{ilk}\ \lambda'_{ikl}\ h^D_k\ h^D_l \\ &= m_{loop}\ a_i\ a_j + m_{loop}\ h^D_2\ h^D_3\ A_{ij} + O(h^D_1, h^{D^2}_2)\ , \end{aligned} \tag{5.12}$$

where $A_{ij} = \lambda'_{i23}\lambda'_{j32} + \lambda'_{i32}\lambda'_{j23} - \lambda'_{i22}\lambda'_{j33} - \lambda'_{i33}\lambda'_{j22}$. Neglecting $O(h^D_1, h^{D\,2}_2)$ corrections to the loop induced mass matrix, the total mass matrix is given by,

$$\begin{aligned} \mathcal{M}^\nu_{ij} &\approx (m_0 + m_{loop})\ a_i a_j + m_{loop}\ h^D_2\ h^D_3\ A_{ij} \\ &\approx \mathcal{M}'_{ij} + m_{loop}\ h^D_2\ h^D_3\ A_{ij}\ . \end{aligned} \tag{5.13}$$

The matrix $\mathcal{M}'$ can be easily diagonalised using a unitary transformation,

$$U^T \mathcal{M}' U = diag\ (0, 0, m_3)\ , \tag{5.14}$$

$$\text{where } m_3 \sim (m_0 + m_{loop})\ (\ \lambda'^2_{333} + \lambda'^2_{233} + \lambda'^2_{133})\ h^{D^2}_3, \tag{5.15}$$

$$\text{and } U = \begin{pmatrix} c_2 & s_2 c_3 & s_2 s_3 \\ -s_2 & c_2 c_3 & c_2 s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}\ , \tag{5.16}$$

with $s_2 = \frac{a_1}{\sqrt{a_1^2 + a_2^2}}$ and $s_3 = \frac{(a_1^2 + a_2^2)^{\frac{1}{2}}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$ .
The total mass matrix is now given by,

$$U^T \mathcal{M}^\nu U \approx m_3\ (\ diag\ (0, 0, 1) + \epsilon A') \tag{5.17}$$

where

$$A' = U^T A\ U \text{ and } \quad \epsilon\ A'_{ij} \approx \frac{m_{loop}}{m_3}\ h^D_2\ h^D_3\ A'_{ij} \approx \frac{m_{loop}}{m_0}\ \frac{h^D_2}{h^D_3}\ . \tag{5.18}$$

The last equality follows under the assumption that $\lambda'_{ijk}$ are similar in magnitude and $m_{loop} \ll m_0$.
If $U'$ is a rotation in the 12 plane with an angle $\theta_1$ defined by $\tan 2\theta_1 = \frac{2A'_{12}}{A'_{22} - A'_{11}}$ , then,

$$\begin{aligned} U'^T U^T \mathcal{M}^\nu U U' &= m_3 \begin{pmatrix} \epsilon\delta_1 & 0 & c_1 \epsilon A'_{13} - s_1 \epsilon A'_{23} \\ 0 & \epsilon\delta_2 & s_1 \epsilon A'_{13} - c_1 \epsilon A'_{23} \\ c_1 \epsilon A'_{13} - s_1 \epsilon A'_{23} & s_1 \epsilon A'_{13} - c_1 \epsilon A'_{23} & 1 + \epsilon A'_{33} \end{pmatrix}\ , \\ &\approx m_3\ diag(\epsilon\delta_1, \epsilon\delta_2, 1)\ . \end{aligned} \tag{5.19}$$

where $\delta_1$ and $\delta_2$ are parameters generically of $O(\lambda'^2)$ if all $\lambda'_{ijk}$ are assumed to be similar in magnitude. The off-diagonal elements will generate additional mixing in the model. But, as $\epsilon A' \ll 1$, we can neglect these off-diagonal elements. The eigenvalues in this approximation are given as,

$$m_{\nu_1} \sim \epsilon\ m_3\ \delta_1 \quad ; \quad m_{\nu_2} \sim \epsilon\ m_3\ \delta_2 \quad ; \quad m_{\nu_3} \sim m_3\ , \tag{5.20}$$

As a consequence,

$$\frac{m_{\nu_2}}{m_{\nu_3}} \sim \frac{m_s}{m_b}\frac{m_{loop}}{m_0}\left(\frac{\delta_2}{\Sigma_i \lambda'^2_{i33}}\right) \tag{5.21}$$

The last factor in the above is of O(1) and the remaining part is controlled completely by the standard parameters of the MSSM. Eq.(5.21) may be regarded as a generic prediction of the model. It is seen from eqs. (5.9,5.11) that typically,

$$\frac{m_{loop}}{m_0} \sim \frac{\pi^2}{3\left(ln\ \frac{M_{GUT}^2}{M_Z^2}\right)^2} \sim 10^{-3} \tag{5.22}$$

Thus the neutrino mass ratio in eq.(5.21) is suppressed considerably compared to the hierarchy obtained when sneutrino *vev* contribution is completely neglected. As we have mentioned earlier, the authors of [3] have not considered the tree level contribution to the neutrinos and thus have achieved a hierarchy of neutrino masses proportional to the Yukawa couplings:

$$\frac{m_{\nu_2}}{m_{\nu_3}} = \frac{m_s}{m_b}, \tag{5.23}$$

where $m_s(m_b)$ represents the strange(bottom) quark mass. But, as we have seen in the above, the RG induced tree level mass alters the hierarchy drastically as the tree level mass is almost thousand times larger than the loop mass for most of the parameter space. The exact value of the additional suppression factor due to the tree level mass is dependent on MSSM parameters and we will calculate it in the next section.

The mixing among neutrinos is governed by $K = U\ U'$. The angles appearing in $K$ are determined by the ratios of the trilinear couplings and hence can be naturally large. Thus, as in supersymmetric model with purely bilinear $R$ violation which we have studied in the previous chapter, one gets hierarchical masses and large mixing without fine tuning in any parameters.

## 5.4 Neutrino Masses : Phenomenology:

We now discuss the phenomenological implications of neutrino masses, eq.(5.20) and mixing. Due to hierarchy in masses, one could simultaneously solve the solar and atmospheric $\nu$ problems provided, $m_{\nu_1} \sim m_{\nu_2} \sim 10^{-5}$ eV and $m_{\nu_3} \sim 10^{-1} - 10^{-2}$ eV.

In order to determine these masses exactly, we have numerically integrated the RGE eq.(5.3) along with similar equations for the parameters appearing in them. We have imposed the standard universal boundary condition and required radiative breaking of the $SU(2) \times U(1)$ symmetry. Solution of the RGE determines both $m_{loop}$ (eq.(5.8)) and $m_0$ (eq.(5.10)). We display these in Fig.1 as a function of $\mu$ for positive (negative) $\mu$ and $\tan\beta = 2.1$, $M_2$ =400 GeV ( $M_2$ =200 GeV ). The ratio $\frac{m_{loop}}{m_0}$ is quite sensitive to the sign of $\mu$. For $\mu > 0$, this ratio is found to be rather small, typically $\sim 10^{-2} - 10^{-3}$, while it can be much larger for $\mu < 0$. There exists a region with negative $\mu$ in which $\frac{m_{loop}}{m_0} \geq 1$. In this region, two contributions to the sneutrino *vev* in eq. (5.5) cancel and $m_0$ gets suppressed. Barring this region, the $\frac{m_{loop}}{m_0}$ is seen to be around $\sim 10^{-1} - 10^{-2}$ for negative $\mu$ leading to

$$\frac{m_{\nu_2}}{m_{\nu_3}} \sim \frac{m_s}{m_b}\frac{m_{loop}}{m_0} \sim 2\ (10^{-3} - 10^{-4}) \tag{5.24}$$

For $m_{\nu_3} \sim 10^{-1} - 10^{-2}\,\mathrm{eV}$, one thus obtains $m_{\nu_2} \sim m_{\nu_1} \sim 2\ (10^{-4} - 10^{-6})\,\mathrm{eV}$ which is in the right range required to solve the solar neutrino problem through vacuum oscillations. The typical value of $m_0 \sim$ GeV found in Fig.1 implies through eq.(5.15), $\lambda' \sim 10^{-4}$. Thus, one needs to choose all $\lambda'_{ijk}$ of this order. Once this is done, one automatically obtains solar neutrino scale for some range in the MSSM parameters.

While, hierarchy needed for the vacuum solution follows more naturally, one could also obtain scales relevant to the MSW conversion. This happens for very specific region of parameters with negative $\mu$ in which two contributions to sneutrino *vev*, eq. (5.5), cancel. As already mentioned, $\frac{m_{loop}}{m_0}$ can be in large this region. One then recovers the result of [3], namely, eq.(5.21) which allows MSW solution for the solar neutrino problem.

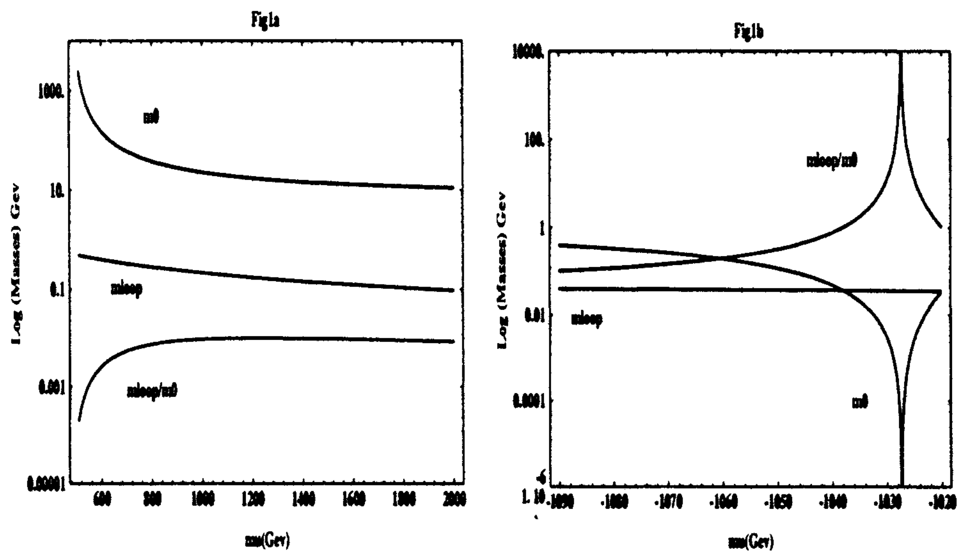

**Fig. 1a.**The absolute values of tree level contribution, $m_0$, the loop level contribution, $m_{loop}$ and their ratio $\frac{m_{loop}}{m_0}$ are plotted with respect to $\mu$ (positive) for $M_2 = 200\,GeV$, $A$=0 and $\tan\beta = 2.1$ . The $m_0$ and $m_{loop}$ are defined in the text.
**Figure 1b.**Same as in Fig. (1a) but for $M_2 = 400\,GeV$ and $\mu$ (negative).

We showed in Figures **1a, 1b** neutrino mass ratio for specific values of $M_2$ and $\tan\beta$. Qualitatively similar results follow for other values of these parameters. We have displayed in Table.1 values for the MSSM parameters and what they imply for $\frac{m_{\nu_2}}{m_{\nu_3}}$. We have shown illustrative values of the parameters which lead to the vacuum as well as MSW solution.

The latter arise only for limited parameter range corresponding to cancellations in eq.(5.5). The former is a more generic possibility which arise for larger region with both positive and negative values of $\mu$. The MSW solution in the present context will have to be restricted to the large angle solution if one does not want to impose any discrete symmetries or fine tune $\lambda$'s. It should be noted that this solution is preferred by the present data from super-Kamiokande experiments as we have seen chapter 2.

| m GeV | $M_2$ GeV | $\mu$ GeV | $m_0$ GeV | $m_{loop}$ GeV | *ratio* $\frac{m_{\nu_2}}{m_{\nu_3}}$ |
|---|---|---|---|---|---|
| 1312 | 200 | 1225 | -16.94 | -0.1689 | 1.8 $10^{-4}$ |
| 3898 | 300 | -3400 | -3.238 | 0.0394 | 2.3 $10^{-4}$ |
| 3921 | 350 | -3450 | -2.655 | 0.0376 | 2.6 $10^{-4}$ |
| 157.2 | 300 | -777.5 | -0.0470 | 0.0487 | $-1.9$ $10^{-2}$ |
| 225.7 | 400 | -1038 | -0.0368 | 0.0369 | $-1.8$ $10^{-2}$ |

**Table1:** *The values of $m_0$, $m_{loop}$ and ratio of the eigenvalues, $\frac{m_{\nu_2}}{m_{\nu_3}}$ for various values of the standard MSSM parameters m, $M_2$ and $\mu$ for $\tan\beta = 2.1$ , A=0.*

The constraints on mixing matrix K, implied by the experimental results are also easy to satisfy keeping all the $\lambda'_{ijk}$ similar in magnitude. As we have seen in chapter 2, the solar and atmospheric neutrino problems can be approximately treated in terms of mixing among two generations (12) and (23) respectively as long as $K_{e3}$ is small (as implied by CHOOZ) and $m_{\nu_3}$ and $m_{\nu_2}$ are hierarchical. The constraint on relevant mixing following from the experiments can then be translated to constraints on elements of K and are given as follows:

$$\begin{aligned} 0.6 \le 4\,K_{\mu 3}^2\,(1-K_{\mu 3}^2) &= s_3^4 \sin^2 2\theta_2 + c_2^2 \sin^2 2\theta_3 \le 1. \\ K_{e3} &\le 0.18 \\ 0.8 \le 4\,K_{e1}^2\,K_{e2}^2 &= 4(c_1c_2 - s_1s_2c_3)^2\ (s_1c_2+s_2c_1c_3)^2 \le 1. \end{aligned} \tag{5.25}$$

It is possible to satisfy all these constraints by choosing for example,

$$c_3 = s_3 = s_1 = c_1 = \frac{1}{\sqrt{2}}\ ;\quad s_2 = 0.254$$

The relative smallness of $s_2$ required here does not imply significant fine tuning and can be easily obtained, e.g. by choosing,

$$\frac{\lambda'_{133}}{\lambda'_{233}} \sim \frac{1}{4}.$$

It is to be emphasized that much smaller value of $K_{e3}$ than at the present level will force fine tuning and cannot be accommodated in the model naturally.

We have so far concentrated on the $\lambda'_{ijk}$ couplings alone. The analogous discussion can be carried out for $\lambda_{ijk}$ couplings appearing in the eq.(5.1). Here also, the tree level contribution to neutrino masses will dominate over the loop contribution although the structure of mixing matrix will differ slightly due to the anti-symmetry of the couplings $\lambda_{ijk}$ in indices $i$ and $j$.

## 5.5 Conclusions

We have discussed in detail the structure of neutrino masses and mixing in MSSM in the presence of trilinear R-violating couplings, specifically $\lambda'_{ijk}$. Noteworthy feature of the present analysis is that it is possible to obtain the required neutrino mass pattern under fairly general assumption of all the $\lambda'_{ijk}$ being of equal magnitudes. It is quite interesting that hierarchy among neutrino masses is controlled by few parameters in MSSM and is largely independent of the trilinear $R$ violating couplings under the assumption that all the trilinear couplings are equal in magnitude. Thus one could understand the required neutrino mass ratio without being specific about the exact values of large number of the trilinear couplings. This 'model-independence' is an attractive feature of the scenario discussed here.

The key difference of the present work compared to many of the other works is proper inclusion of the sneutrino *vev* contribution. While we had to resort to specific case of the minimal supergravity model for calculational purpose, the sneutrino *vev* contribution would arise in any other scheme with $\lambda'_{ikk} \neq 0$ at a high scale such as $M_{GUT}$. Such contribution thus cannot be neglected a priori. On the contrary, the inclusion of this contribution makes the model more interesting and fairly predictive in spite of the presence of large number of unknown couplings. The model prefers simultaneous solutions for solar and atmospheric neutrino problems with vacuum oscillations for the solar neutrino problem. However, Large Angle MSW also can be accommodated in regions of the parameter space where sneutrino *vev* is suppressed.

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Chapter 6

# Neutrino Mass constraints on R violation and HERA anomaly

## 6.1 Introduction

In the last two chapters we were interested in studying the structure of neutrino mass matrix in the presence of explicit lepton number violation and the possibilities of having simultaneous solutions to solar and atmospheric neutrino anomalies. We have motivated such a study by claiming that since lepton number violation naturally occurs in a supersymmetric extension of Standard Model [1], it would be the right framework to generate neutrino masses, as required for solutions of neutrino anomalies. But lepton number violation has not been observed in nature ( except for a possible neutrino mass ), whereas the presence of lepton number violating couplings can lead to various interesting processes like $K^+ \to \pi^+ \nu\bar{\nu}$, the violation of weak universality, neutrinoless double beta decay etc. Most of these processes have not been observed yet in nature and there exist stringent experimental limits on these processes [1]. These experimental limits can be converted into limits on lepton number violating couplings themselves. The limits on various couplings so obtained have been reviewed in [1, 2].

The limits discussed above also have an interesting feature associated with them, which is the so called 'single coupling scheme'. In obtaining the limit on a particular coupling, one usually assumes that the particular coupling is the most dominant one compared to the other lepton number violating couplings [3]. This is to facilitate a meaningful analysis. But the single coupling scheme has its own drawbacks. The single coupling scheme makes the analysis basis dependent [4]. This basis dependence comes due to the CKM matrix in the quark sector. In such a case, one coupling in one basis may correspond to several couplings in another basis.

We have also seen that the presence of trilinear lepton number violating couplings in the superpotential would also lead to majorana masses for the neutrinos. The neutrino masses are themselves constrained by kinematic limits on them. These limits can be used to constrain the lepton number violating couplings present in the superpotential. Thus in the present chapter, we do a sort of 'reverse analysis' to what has been done so far in this thesis. We choose $\lambda'_{1ij}$ couplings for our analysis and study the limits on these couplings from neutrino mass constraints [2] .

[1] When we say natural here, we mean it is not disallowed by gauge symmetries as in SM.

[2] This is because the electron neutrino mass is most stringently constrained compared to other neutrino

As has been mentioned in the earlier chapter, neutrinos attain mass both at the tree level as well as at the 1-loop level in models with trilinear R violation. The present analysis in the literature [5, 6] of the constraints arriving from neutrino masses has neglected the important and dominant tree level contribution to the neutrino masses which would occur when any of the lepton number violating term is present in the superpotential. But as we have seen in chapter 5, in any realistic model where supersymmetry is broken at a high scale, RG evolution strongly modifies the neutrino mass by giving rise to a tree level mass to the neutrino, which we call 'RG induced tree level mass'. Incorporation of this additional contribution can change the already obtained [5, 6] limits significantly. The aim of this chapter is to systematically derive these constraints and discuss its implications. The restriction to only $\lambda'_{1ij}$ couplings is also interesting because these couplings have been relevant in attempts to understand the HERA anomalous results, which we will discuss later in the chapter.

## 6.2 Basis choice and definition of $\lambda'_{ijk}$ :

Since we consider only trilinear R-violation, the lepton number violating part of the superpotential is given as follows:

$$W_{\not{L}} = \lambda'_{ijk}\ L'_i Q'_j D'^c_k \tag{6.1}$$

The above is written in the current basis of the quarks. But, as has been mentioned above, in order to meaningfully constrain the trilinear coupling, it is usually assumed that only a single coupling is non-zero at a time. While the physics implied by these couplings is basis independent, the said assumption makes the constraints on $\lambda'_{ijk}$ basis dependent since a non-zero $\lambda'$ in one basis correspond to several non-zero $\lambda'$'s in the other. The trilinear couplings in eq. (6.1) can be rewritten [4] in the quark mass basis as follows:

$$W_{\not{R}} = \lambda'_{ijk}(-\nu_i d_l K_{lj} + e_i u_j) d^c_k, \tag{6.2}$$

where the above terms are now in the quark mass basis and $K$ denotes the Kobayashi-Maskawa matrix. Even in the mass basis one could choose a different definition for the trilinear couplings:

$$\lambda^{\nu}_{ijk} \equiv K_{jl}\lambda'_{ilk} \tag{6.3}$$

and rewrite (6.2) as,

$$W_{\not{R}} = \lambda^{\nu}_{ijk}(-\nu_i d_j + e_i K^{\dagger}_{lj} u_l) d^c_k \tag{6.4}$$

With the first choice, a single non-zero $\lambda'_{ijk}$ can lead to tree level flavour violations in the neutral sector [4] while this is not so if only one $\lambda^{\nu}_{ijk}$ $(j \neq k)$ is non-zero. Clearly, there are other equivalent definitions of trilinear couplings which are between the two extreme cases given in eqs.(6.2) and (6.4). The basis dependence can be clearly seen here as the first coupling $\lambda'_{121}$ is constrained severely by the neutrino mass but the coupling $\lambda^{\nu}_{121}$ is not.

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masses.

The generation of RG induced tree level mass in the presence of trilinear lepton number violating couplings has already been discussed in chapter 5. However, for the sake of completeness we repeat it here again. The crucial terms to our analysis are the soft supersymmetry breaking terms in the scalar potential. The lepton number violating parts of these terms can be written as:

$$V_{soft} = A^{\nu}_{ij}\lambda^{\nu}_{ij}\ D_i D^c_j \tilde{\nu} - B_\nu\ \tilde{\nu} H^0_2 + m^2_{\nu H^0_1}\ \tilde{\nu}^* H^0_1 + c.c + \cdots\ , \tag{6.5}$$

where we have explicitly written only those terms which are responsible for generating the electron neutrino ($\nu_1 \equiv \nu$) mass and $\lambda^{\nu}_{ij} \equiv \lambda^{\nu}_{1ij}$. Note that the presence of only trilinear terms in the superpotential generate only the first terms in eq.(6.5) at the high scale in conventional supergravity based scenario. The remaining terms are however generated at the weak scale and should therefore be retained. In this basis, the general RGE presented in Chapter 3 become,

$$\begin{aligned}
\frac{dB_\nu}{dt} &= -\frac{3}{2}B_\nu\left(Y^U_t - \tilde{\alpha}_2 - \frac{1}{5}\tilde{\alpha}_1\right) - \frac{3\mu}{16\pi^2}\lambda^{\nu}_{kk}h^D_k\left(A^{\nu}_{kk} + \frac{1}{2}B_\mu\right), && (6.6)\\
\frac{dm^2_{\nu H_1}}{dt} &= -\frac{1}{2}m^2_{\nu H_1}\left(3Y^D_l + Y^E_l\right) - \frac{3}{32\pi^2}\lambda^{\nu}_{kk}h^D_k\left(m^2_{H_1} + m^2_{\tilde{\nu}}\right. \\
&\quad \left. +2\ A^{\nu}_{kk}A^D_k + 2\ m^{Q2}_k + \ 2\ m^{D^c2}_k\right) && (6.7)\\
\frac{d\lambda^{\nu}_{kk}}{dt} &= \lambda^{\nu}_{kk}\left(-3Y^D_k - \frac{1}{2}Y^U_k + \frac{7}{30}\tilde{\alpha}_1 + \frac{3}{2}\tilde{\alpha}_2 + \frac{8}{3}\tilde{\alpha}_3\right), && (6.8)\\
\frac{dA^{\nu}_{kk}}{dt} &= -\frac{3}{2}A^{\nu}_{kk}Y^D_k - \frac{9}{2}A^D_k Y^D_k - \frac{1}{2}A^U_k Y^U_k \\
&\quad - \frac{16}{3}\tilde{\alpha}_3 M_3 - 3\tilde{\alpha}_2 M_2 - \frac{7}{15}\tilde{\alpha}_1 M_1\ . && (6.9)
\end{aligned}$$

The rest of the parameters carry the same meaning as in chapter 3. We have kept only leading order terms in $\lambda^{\nu}_{kk}$ in writing eqs.(6.6). The other parameters appearing in eqs.(6.6) satisfy the standard RG equations to this order in $\lambda^{\nu}_{ij}$. The second terms in eqs. (6.6) generate non-zero $B_\nu$ and $m^2_{\nu H_1}$ at $Q_0$. These terms in eq.(6.5) generate a non-zero *vev* $\langle\tilde{\nu}\rangle$ which can be determined by minimizing the full scalar potential. These are given by,

$$\langle\tilde{\nu}\rangle \sim \frac{B_\nu v_2 - m^2_{\nu H_1}v_1}{m^2_{\tilde{\nu}} + \frac{1}{2}M^2_z\cos 2\beta}\ , \tag{6.10}$$

$m^2_{\tilde{\nu}}$ is the soft SUSY breaking sneutrino mass. This in turn leads to the following neutrino mass:

$$(m_{\nu_e})_{tree} = \frac{\mu(cg^2 + g'^2)(<\tilde{\nu}>^2)}{2(-c\mu M_2 + 2M^2_W c_\beta s_\beta\ (c + tan^2\theta_W))}. \tag{6.11}$$

where $c \simeq 5g'^2/3g^2$, $g^2$ and $g'^2$ are gauge couplings and $\tan\beta = \langle H^0_2\rangle/\langle H^0_1\rangle = v_2/v_1$. It follows from the eqs. (6.6, 6.10) that the $(m_{\nu_e})_{tree}$ involves the combination $(\lambda^{\nu}_{kk}h^D_k)^2$. The trilinear interactions in eq.(6.2) lead also to the following $m_{\nu_e}$ at the one-loop level:

$$(m_{\nu_e})_{loop} \simeq -\frac{(\lambda^{\nu}_{kk})^2}{16\pi^2}m^D_k\sin\phi_k\cos\phi_k\ ln\frac{M^2_{2k}}{M^2_{1k}}\ , \tag{6.12}$$

where we have implicitly assumed that only one $\lambda'_{1jk}$ is non-zero at a time. $\phi_k$ and $M^2_{2,1\ k}$ respectively denote the mixing among squarks $\tilde{d}_k, \tilde{d}^c_k$ and their masses. The mixing $\phi_k$ is proportional to $m^D_k$. As a result just like the tree level mass, the $(m_{\nu_e})_{loop}$ also scales as $(\lambda^{\nu}_{kk} h^D_k)^2$. $m_{\nu_e}$ therefore provides a bound on $\lambda^{\nu}_{kk}$ which can be converted to a bound on $\lambda'_{1lk}$ from eq.(6.3). The resulting bounds become stronger with increase in $\tan\beta$ due to the fact that $B_\nu, m^2_{\nu H_1}$ involve $h^D_k = m^D_k / v\cos\beta$ . For the same reason, bounds display strong hierarchy, typically,

$$\frac{(\lambda^{\nu}_{kk})_{max}}{(\lambda^{\nu}_{ll})_{max}} \sim \frac{m^D_l}{m^D_k} \ . \tag{6.13}$$

It also follows from eq.(6.3) that,

$$\frac{(\lambda'_{1kk})_{max}}{(\lambda'_{1lk})_{max}} \sim \frac{K_{lk}}{K_{kk}} \ . \tag{6.14}$$

It is seen from last two equations that the $\lambda'_{133}$, ($\lambda'_{131}$) is constrained most (least) by $m_{\nu_e}$.

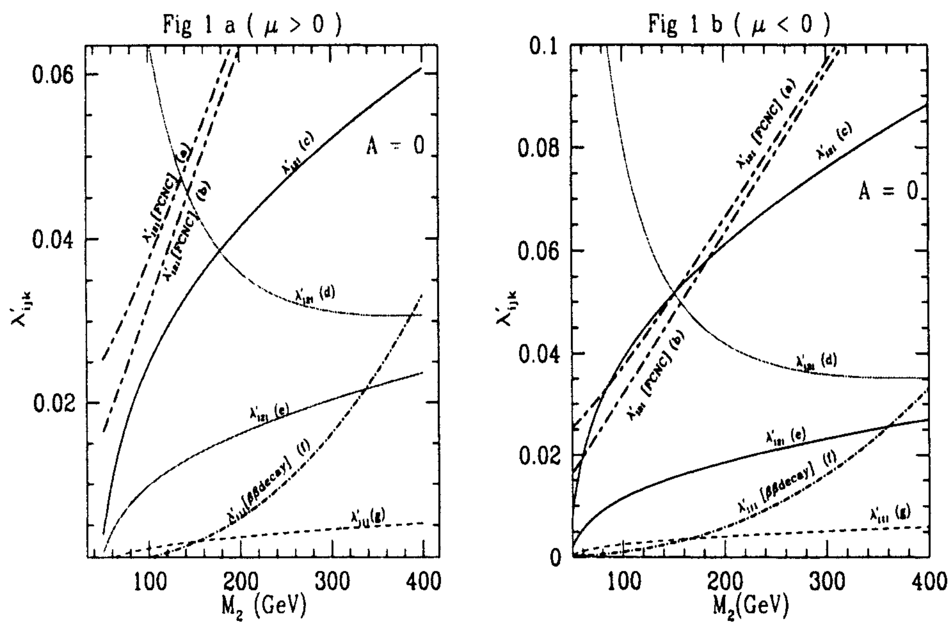


**Figure 1a.** FCNC constraints on $\lambda'_{121}$ for a) $m = 200$ GeV and b) $m = 50$ GeV for $tan\beta = 40$. Neutrino mass constraints on $\lambda'_{121}$ for c) $m = 50$ GeV, $tan\beta = 15$; d) $m = 200$ GeV, $tan\beta = 40$ and e) $m =$ 50GeV, $tan\beta = 40$. f) $\lambda'_{111}$ from neutrino less $\beta\beta$ decay g) $\lambda'_{111}$ from neutrino mass constraints for $m = 50$ GeV and $\tan\beta = 40$. **Figure 1b.** Same as of Figure **1a**) but with $\mu < 0$

## 6.3 Bounds on trilinear couplings :

We now present bounds on the couplings $\lambda'_{1jk}$ obtained from the experimental limit on the neutrino mass $m_{\nu_e}$. In the present framework, the neutrino obtains a majorana mass and hence stringent constraint on $m_{\nu_e}$ would follow from the non-observation of neutrinoless $\beta\beta$ decay. We shall use somewhat conservative value of $m_{\nu_e} \leq 2.0$ eV in numerically discussing the bounds on different couplings which lead to $m_{\nu_e}$. In order to obtain bound on the relevant coupling, we have numerically solved eqs.(6.6) along with the other standard equations for the parameters appearing on the RHS of eqs. (6.6) as discussed in Chapter 3.

We work with R violating version of the MSSM with a standard set of soft supersymmetry breaking terms specified at a high scale near $M_{GUT} = 3 \times 10^{16}$ GeV. We confine ourselves to the scenario with radiative breaking of the $SU(2) \times U(1)$ symmetry. All parameters in eq.(6.10) are specified at a low scale $Q_0$ . We have chosen $Q_0$ to be $M_Z$ [3]. The parameters $B_\nu, m^2_{\nu H_1}$ are assumed to be zero at $M_{GUT}$ . Their values at $Q_0$ depend upon the standard parameters of MSSM which we take to be gaugino (gravitino) mass, $M_2$ ($m$), $\tan\beta$ and universal trilinear strength $A$. $B_\nu$ and $m^2_{\nu H_1}$ determine $(m_\nu)_{tree}$ in terms of these parameters and $\lambda^\nu_{kk}(tz)$, where, $tz = 2\ \ln(\frac{M_{GUT}}{M_Z})$. The 1-loop contribution also gets fixed in terms of these parameters since $\phi_k$ and $M^2_{2,1\ k}$ appearing in eq.(12) are determined using the standard $2 \times 2$ mixing matrix for $\tilde{d}_k, \tilde{d}^c_k$ system.

| $\lambda'_{ijk}$ | Previous limits | $m = 50\ GeV$ <br> $M_2 = 150\ GeV$ | $m = 50\ GeV$ <br> $M_2 = 175\ GeV$ |
|---|---|---|---|
| 111 | 0.021[7] | 0.0017 | 0.0018 |
| 121 | 0.06[4] | 0.0077 | 0.0085 |
| 131 | 1.3[8] | 0.48 | 0.54 |
| 112 | 0.15[8] | $4\ 10^{-4}$ | $4.5\ 10^{-4}$ |
| 122 | 0.06[4] | $8.9\ 10^{-5}$ | $9.9\ 10^{-5}$ |
| 132 | 1.05[9] | $2.1\ 10^{-3}$ | $2.4\ 10^{-3}$ |
| 113 | 0.15[8] | $3.1\ 10^{-4}$ | $3.4\ 10^{-4}$ |
| 123 | 0.06[4] | $7\ 10^{-5}$ | $7.7\ 10^{-5}$ |
| 133 | 0.0029[5] | $2.8\ 10^{-6}$ | $3.1\ 10^{-6}$ |

**Table 1.** Limits on single $\lambda'_{ijk}$ following from the electron neutrino mass for $A = 0$, $\tan\beta = 30$ and $\mu > 0$. The limits become stronger for larger $\tan\beta$ and weaker for negative $\mu$. The existing limits mentioned in the table are for the relevant squark mass $\sim 500$ GeV and gluino mass $M_3 = 500$ GeV in case of $\lambda'_{111}$.

The bounds on different couplings are displayed in Figs. 1 and 2. Apart from being dependent on SUSY parameters, these bounds are quite sensitive to the chosen sign of the $\mu$ parameter. This is due to the fact that for one (namely negative) sign of $\mu$, two terms appearing in the sneutrino *vev*, eq.(6.10) cancel while they add for positive sign. The suppression in sneutrino *vev* occurring in the first case weakens the bound on $\lambda'$. Fig.

[3]Note that change in $Q_0$ can alter some of the bounds obtained by minimization of the tree level potential and more detailed treatment should include 1-loop corrected potential, see G. Gamberini, G. Ridolfi and F. Zwirner, *Nucl. Phys.* **B331** (1990) 331

1 displays bounds on couplings $\lambda'_{111}$ and $\lambda'_{121}$ obtained by demanding $m_{\nu_e} \equiv (m_{\nu_e})_{tree} + (m_{\nu_e})_{loop} \leq 2.0\,\mathrm{eV}$. Curves for three representative values of $\tan\beta$ and universal gravitino mass $m$ are shown. For comparison, we also display in the same figure the existing bounds on these couplings. The most stringent bound on $\lambda'_{111}$ is derived from neutrinoless double beta decay [7] and on $\lambda'_{121}$ from the process $K^+ \to \pi^+\nu\bar{\nu}$ [4] . These are shown as function of $M_2$ in the same figure using MSSM expressions for the relevant squark masses. It is seen that the bounds derived here are comparable or better (in case of larger values of $M_2$ ) than the already existing ones.

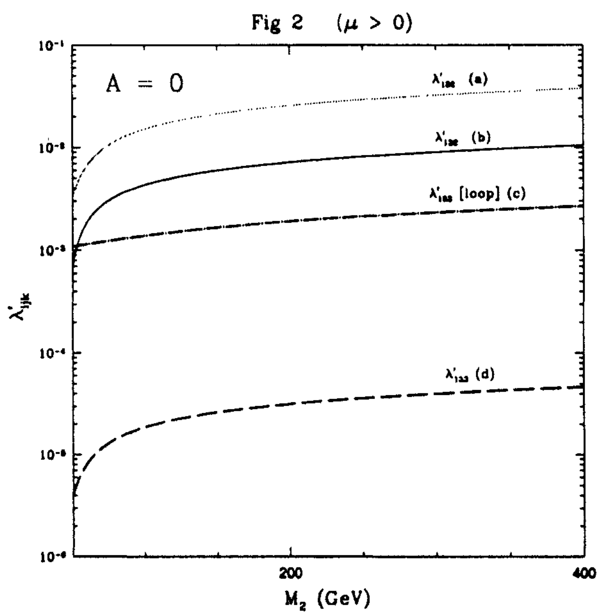


**Figure 2.** Neutrino mass constraints on $\lambda'_{132}$ for a) $tan\beta = 5$ and b) $tan\beta = 25$; on $\lambda'_{133}$ for $tan\beta = 5$ c) considering only loop contributions and d) loop as well as sneutrino *vev* contributions are shown for $m = 50\ GeV$.

Fig 2 shows the bounds on $\lambda'_{133}$ and $\lambda'_{132}$ for $\mu > 0$. The $\lambda'_{133}$ is constrained most and is required to be as low as $3 \times 10^{-6}$ even for $\tan\beta = 5$. For comparison we also show the limit on $\lambda'_{133}$ which would follow if sneutrino *vev* is completely neglected as in previous works [5, 6]. It is clear that inclusion of this *vev* drastically alters the bound. We also display limit on $\lambda'_{132}$ which was poorly constrained in the previous analysis [9]. The bounds on other couplings $\lambda'_{1jk}$ not explicitly displayed in above figures can be read off from eq.(6.14). These constraints are listed in a table for two representative values $m = 50\,\mathrm{GeV}, M_2 = 150\,\mathrm{GeV}$ and $m = 50\,\mathrm{GeV}, M_2 = 175\,\mathrm{GeV}$ and $\tan\beta = 30$. These respectively correspond in MSSM to $m_{\tilde{d}_R} \sim 474, 553\,\mathrm{GeV}$. It is seen that the neutrino mass considerably improves on the existing constraints. This table is given to illustrate the importance of the bounds following from the neutrino mass. It should however be kept in mind that these bounds are sensitive to the choice of the MSSM parameters a fact which becomes evident through figures (1-2).

Thus, we have investigated the restrictions on the $R$ parity violating trilinear couplings, specifically the $\lambda'_{1jk}$ implied by neutrino mass. These restrictions have been discussed earlier [5, 6] but earlier discussions either neglected very important and dominant contribution due to the induced sneutrino *vev* or did not incorporate the effect of the quark mixing. We have attempted here a complete analysis which incorporates these and also pays careful attention to the dependence of the bounds on the choice of basis. Unlike the earlier works, we have also numerically discussed the bounds in terms of parameters of the MSSM.

Many of the discussions and in particular the generation of sneutrino *vev* in the presence of a single trilinear coupling would be true in a more general set up, *e.g.* in case when soft terms arise from the gauge mediated supersymmetry breaking. The main point following from our analysis is that the electron neutrino mass provides much stronger constraints on $R$ violation and associated phenomenology than has been hitherto realized.

## 6.4 HERA anomalies

The HERA experiments at DESY, collide electrons (or positrons) with protons. The electrons typically carry an energy of $\sim$ 30 GeV, whereas the protons carry an energy of around $\sim$ 820(920) GeV. In addition to measuring the structure functions of the protons through the DIS (Deep Inelastic Process), these experiments are also ideal to verify R-violating interactions. The experiment consists of two general purpose detectors, H1 and ZEUS. In 1997, anomalous events have been reported by the H1 and the ZEUS detectors [10] at HERA in the deep inelastic $e^+p$ scattering. They claimed to have found a resonance for a leptoquark of mass $\sim$ 200 GeV. This has generated considerable excitement within the community [11] as, if substantiated these results could form strong evidence for physics beyond the SM and may be for supersymmetry. The work presented here is based on ref.[12] which was motivated from the results discussed above. However, this evidence for the leptoquark did not stand the test of time and the subsequent data from HERA [13] were consistent with the Standard Model expectations. We describe here the work in ref.[12] for the sake of completeness, though it is now only of academic interest. To this extent, we consider here data reported by HERA in 1997.

The available data when taken seriously allowed for two possible interpretations: (*i*) The presence of some lepton number violating contact interaction [14] or (*ii*) production of a resonance in the $e^+q$ channel. MSSM with R-violation [11, 15, 16] provides a natural theoretical framework to incorporate the second possibility although an alternative in terms of a scalar leptoquark [17] is open.

The supersymmetric interpretation of the HERA events assumes that the excess events seen at HERA are due to resonant production and subsequent decay of the squark to $e^+q$. Three possibilities have been considered in this context[11, 15, 16]: $e^+_R d_R \to \tilde{c}_L, e^+_R d_R \to \tilde{t}_L, e^+_R s_R \to \tilde{t}_L$. In analyzing these scenarios [11, 15, 16] it has been implicitly assumed that the squark masses are free parameters of the model. While this would be true in the most general situation, specific model dependence can alter some of the conclusions. Our aim is to show that the very minimal model dependent assumption on the charm squark mass necessarily requires large $\lambda'_{121}$ to understand HERA events and this large coupling by itself is ruled out from other constraints like neutrino mass bounds which we have obtained in the previous section.

The specific assumption that we make and which leads to the above conclusion is that the charm squark mass squared is positive at the unification scale. This assumption is true in the radiative electro-weak breaking scenario with universal boundary conditions at the GUT scale, but it can also be true in a much more general context. We shall first assume that the gaugino masses are unified at $M_{GUT}$ but demonstrate later that the removal of this assumption does not significantly change the basic conclusion. The argument leading to the above conclusion is largely insensitive to the details of the radiative $SU(2) \times U(1)$ breaking in the MSSM and runs as follows.

Consider the following $R$ violating couplings:

$$W_R = \lambda'_{ijk}(-\nu_i d_l K_{lj} + e_i u_j) d^c_k \tag{6.15}$$

The above terms are defined in the quark mass basis and $K$ denotes the Kobayashi-Maskawa matrix. The charm squark interpretation of the HERA anomaly requires $\lambda'_{121}$ to be non-zero. The HERA data can be explained provided

$$\lambda'_{121} \sim \frac{0.025 - 0.034}{B^{1/2}} \tag{6.16}$$

The number in the numerator of eq.(2) is indicative of the required range and depends upon the weightage given to the different experiments as well as on the next to leading order QCD corrections [18]. In the following, we shall take [4] the value 0.025 for the numerator in the RHS of eq.(2). $B$ refers to the branching ratio for the squark decay to $qe^+$. This decay would take place through the coupling in eq.(1) itself. $B$ is also influenced by the $R$ conserving decays of the charm squark to an $s$ ($c$) and a chargino (neutralino) . The $\lambda'_{121}$ and the parameters $\mu, M_2, \tan\beta$ determine $B$ in the MSSM. HERA data can be reconciled if for a region in these parameters (*i*) eq.(2) is satisfied, (*ii*) $\lambda'_{121}$ is consistent with other constraints [4, 7, 19] due to $R$ breaking and (*iii*) charm squark has a mass around 180- 220 GeV.

In supergravity based models, the charm squark mass at the weak scale is governed by the gauge couplings and the gaugino masses. Its value at $Q_0 = 200\,\text{GeV}$ is given in the limit of neglecting Kobayashi-Maskawa and $\tilde{c}_L - \tilde{c}_R$ mixing by [5]

$$m^2_{\tilde{c}_L}(Q_0) \approx m^2_{\tilde{c}_L}(M_{GUT}) + 8.83 M_2^2 + 1/2\ M_Z^2\ \cos 2\beta\ (1 - 4/3 \sin^2\theta_W) \tag{6.17}$$

where we have assumed that the gauge couplings and the gaugino masses are unified at the GUT scale, $M_{GUT} = 3 \times 10^{16}\,\text{GeV}$ and chosen $\alpha_s(M_Z) = 0.12$. The $M_2$ in eq.(3) is the value of the wino mass at the weak scale identified here with $M_Z$. The last term in the

[4]The lower limit is obtained when H1 and ZEUS data from 1997 run are also included while the upper limit corresponds to inclusion of H1 data alone. In both cases, 30% increase in the relevant cross section due to next to leading order corrections [18] is assumed.

[5]We also neglect the effect of additional trilinear $R$ violating couplings on the running of the charm squark mass. Their inclusion does not significantly alter the charm squark mass even when they are large, see e.g. K. Cheung, D. Dicus and B. Dutta [20].

above equation is a (-ve ) contribution from the D-term. It follows that the charm squark mass provides strong upper bound on $M_2$ as long as $m^2_{\tilde{c}_L}(M_{GUT}) > 0$:

$$M_2 \leq 74.04\,\mathrm{GeV}\left(\frac{m_{\tilde{c}_L}}{220\,\mathrm{GeV}}\right)\left(1 - 0.06\cos 2\beta\left(\frac{220\,\mathrm{GeV}}{m_{\tilde{c}_L}}\right)^2\right)^{1/2} \tag{6.18}$$

The branching ratio $B$ is determined in the MSSM by the following widths [21]:

$$\begin{aligned}
\Gamma(\tilde{c}_L \to e^+ c) &= \frac{\lambda'^2_{121}}{16\pi} m_{\tilde{c}_L} & (6.19)\\
\Gamma(\tilde{c}_L \to \chi^0_i c) &= \frac{\alpha}{2\, m^3_{\tilde{c}_L}} \lambda^{\frac{1}{2}}(m^2_{\tilde{c}_L}, m^2_c, m^2_{\chi^0_i}) & \\
& \left[\, (|\, F_L\,|^2 + |\, F_R\,|^2)(m^2_{\tilde{c}_L} - m^2_c - m^2_{\chi^0_i}) - 4 m_c m_{\chi^0_i} Re(F_R F^*_L)\,\right] & (6.20)\\
\Gamma(\tilde{c}_L \to \chi^+_i s) &= \frac{\alpha}{4 sin^2\theta_W m^3_{\tilde{c}_L}} \lambda^{\frac{1}{2}}(m^2_{\tilde{c}_L}, m^2_s, m^2_{\chi^+_i}) & \\
& \left[\, (|\, G_L\,|^2 + |\, G_R\,|^2)(m^2_{\tilde{c}_L} - m^2_s - m^2_{\chi^+_i}) - 4 m_s m_{\chi^+_i} Re(G_R G^*_L)\,\right] & (6.21)\\
where, F_L &= \frac{m_c N'^*_{i4}}{2\; m_W\; sin\theta_W\; sin\beta}, & \\
F_R &= e_c N'_{i1} + \frac{\frac{1}{2} - e_c sin^2\theta_W}{cos\theta_W\; sin\theta_W} N'_{i2}, & \\
G_L &= -\frac{m_s U^*_{k2}}{\sqrt{2}\; m_W\; cos\beta}, & \\
G_R &= V_{k1}. &
\end{aligned}$$

We have adopted the same notation as in [21]. From the expression for B in terms of the above decay widths, and the HERA constraint, eq.(2), one can solve for the allowed $\lambda'_{121}$. The contours in the $\mu - M_2$ plane for different values of $\lambda'_{121}$ are displayed in fig 1a ($\tan\beta$=1) and fig 1b ($\tan\beta$ = 40). The horizontal lines in these figures show the upper bound on $M_2$, eq.(4). We also display, the curves corresponding to two representative values of the chargino masses namely 45 and 85 GeV. The later is the present experimental bound obtained assuming $R$ conservation. This need not hold in the presence of $R$ violation. It is seen from fig.1b that for chargino mass around 85 GeV, the bound on $M_2$ by itself rules out charm squark interpretation for large tan $\beta$ independent of the value of $\lambda'_{121}$ [6]. But irrespective of the value of $\tan\beta$ and the chargino mass one needs very large $\lambda'_{121} \geq 0.13$ in order to satisfy the bound on $M_2$ coming from the charm squark mass. This strong bound on $\lambda'_{121}$ arises because of the following reason. For $M_2 \leq 74\,\mathrm{GeV}$, at least one of the charginos is sufficiently light and contributes dominantly to the $\tilde{c}_L$ decay. This reduces $B$ [7] and results in large value for $\lambda'_{121}$ due to eq.(2). In contrast, the chargino decay is

[6] Specifically, the upper bound on $M_2$ can be reconciled with the chargino mass of 85 GeV or more only if $\tan\beta \leq 2.5$.

[7] Reduction in the branching ratio for charm squark decay in case of the minimal model was also noticed in [16].

suppressed kinematically for $\tan\beta \sim 1$ if $M_2 > 200\,\text{GeV}$. This results in smaller allowed value as seen from the figure. But these are in conflict with the charm squark mass.

Let us now see if one could make large $\lambda'_{121}$ consistent with other constraints. The strong constraints come from atomic parity violation [19], the decay $K^+ \to \pi\nu\bar{\nu}$ [4] and the electron neutrino mass which we have derived above. The data from Cs on the relevant weak charge have been argued [11, 22] to imply

$$\lambda'_{121} \leq 0.074 \tag{6.22}$$

at $2\sigma$ level in conflict with the large value required here. In principle, the extra contribution due to charm squark to atomic parity violation can be canceled by a similar contribution from the scalar bottom or strange squark but the existing constraints on the relevant couplings make this cancelation difficult [22]. Thus, one cannot easily avoid the atomic parity violation constraint strictly in the MSSM but this can be done by postulating new physics, e.g. the presence of an extra $Z$ [22].

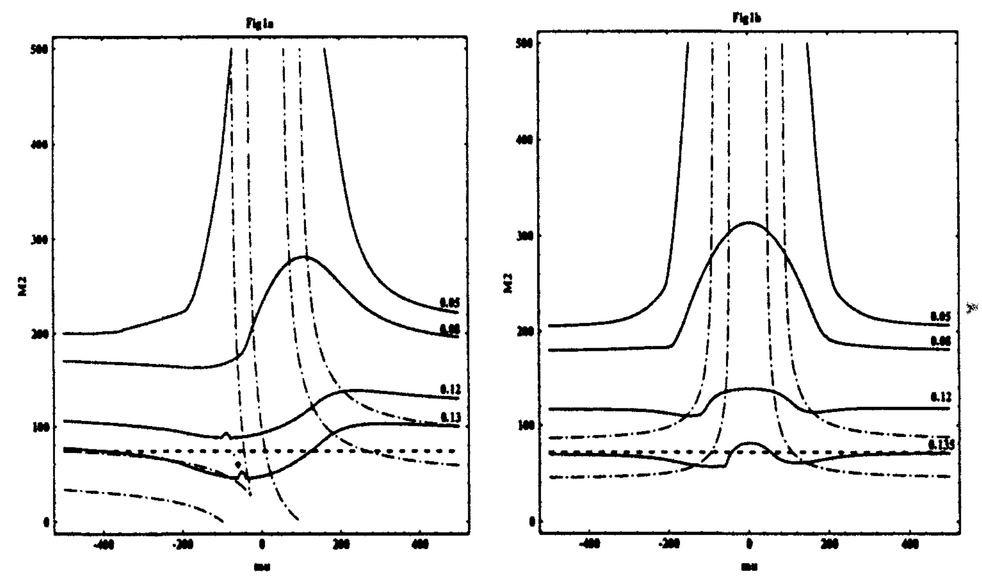

**Fig 1 a:** The contours (continuous lines ) of constant $\lambda'_{121}$ obtained by imposing HERA constraint, eq.(2). The contours are for values 0.05, 0.08, 0.12, and 0.13. The horizontal dashed line represents the bound on $M_2$ coming from requiring $m_{\tilde{c}_L} = 220\,\text{GeV}$. The vertical dash-dot lines represent the bounds on the chargino mass, the upper one for a mass of 85 GeVand the lower one for a mass of 45 GeV. All the above are computed for $\tan\beta = 1$. **Fig 1 b:** Same as fig 1a but for $\tan\beta = 40$ and $\lambda'_{121}$=0.05, 0.08, 0.12, and 0.135.

The other significant constraint comes from the decay $K^+ \to \pi\nu\bar{\nu}$ which implies [4]

$$\lambda'_{121} \leq 0.02 \left(\frac{m_{\tilde{d}_R}}{200\,\text{GeV}}\right). \tag{6.23}$$

The electron neutrino mass also gives similar constraint in the same parameter range. The question of choice of the basis becomes relevant in the discussion of these constraints. This is particularly so when one assumes only one $\lambda'_{ijk}$ to be non-zero. For $\lambda'_{121}$ defined in the mass basis as in eq.(1) the above constraint is unavoidable if rest of the couplings are zero. This basis choice is natural from the point of view of interpreting HERA results but is not unique. One may redefine the couplings as

$$\bar{\lambda}'_{ijk} \equiv K_{jl}\lambda'_{ilk}$$

and rewrite eq.(1) as follows:

$$W_R = \bar{\lambda}'_{ijk}(-\nu_i d_j + e_i K^{\dagger}_{lj} u_l) d^c_k \tag{6.24}$$

HERA result would now require $\bar{\lambda}'_{121}$ to be large. If this is the only non-zero $\bar{\lambda}'_{ijk}$ then there will not be any constraint on $\bar{\lambda}'_{121}$ from the neutrino mass or from the $K^+ \to \pi\nu\bar{\nu}$ decay [4]. But eq.(10) will now generate a contribution to the neutrinoless double beta decay which is also severely constrained. Specifically, one has [7]

$$K^{\dagger}_{12}\bar{\lambda}'_{121} \leq 2.2 \times 10^{-3} \left(\frac{m_{\tilde{u}_L}}{200\,\mathrm{GeV}}\right)^2 \left(\frac{m_{\tilde{g}}}{200\,\mathrm{GeV}}\right)^{1/2} \tag{6.25}$$

This clearly does not allow $\bar{\lambda}'_{121}$ of O(0.1). Thus, notwithstanding basis dependence one has problem in accommodating large value for the relevant coupling. An alternative is to allow more than one non-zero $\bar{\lambda}'_{ijk}$. It is seen from eq.(10) that $\bar{\lambda}'_{1j1}$ (j=1,2,3) contribute to the neutrinoless $\beta\beta$ decay and simultaneous presence of these may lead to cancellations. Eq.(11) now gets replaced by

$$(\bar{\lambda}'_{111} + K^{\dagger}_{12}\bar{\lambda}'_{121} + K^{\dagger}_{13}\bar{\lambda}'_{131}) \leq 2.2 \times 10^{-3} \left(\frac{m_{\tilde{u}_L}}{200\,\mathrm{GeV}}\right)^2 \left(\frac{m_{\tilde{g}}}{200\,\mathrm{GeV}}\right)^{1/2} \tag{6.26}$$

With $\bar{\lambda}'_{121} \sim 0.13$, cancellation between the last two terms is unlikely as it requires $\bar{\lambda}'_{131} \sim 2$. The first two terms can cancel but the $\bar{\lambda}'_{111}$ is independently constrained from the neutrino mass. Its presence generates a large contribution to the electron neutrino mass induced through neutrino-gaugino mixing which can be given as,

$$m_\nu \sim \frac{g^2}{m_{SUSY}} < \tilde{\nu} >^2 \tag{6.27}$$

The value of the induced sneutrino *vev* is sensitive to the MSSM parameters as we have seen above, but can be approximately written as,

$$< \tilde{\nu} > \sim \frac{9\,\bar{\lambda}_{111}}{16\,\pi^2}\, m_d\ ln\left(\frac{M^2_{GUT}}{M^2_Z}\right) \tag{6.28}$$

Requiring $m_\nu \leq 2eV$ leads for $m_{SUSY} \sim 100\,\mathrm{GeV}$ to

$$\bar{\lambda}_{111} \leq .04$$

It is seen that cancellations between the first two terms in eq.(12) are feasible and can allow $\bar{\lambda}_{121} \sim 0.13$ if this fine tuning is accepted. It must be added that the bound in the previous equation is quite sensitive to the MSSM parameters and for a large range in these parameters, the actual bound can be stronger than the generic bound displayed above.

While wino and zino control the decay of the charm squark, its mass is mainly controlled by the large radiative corrections driven by the gluino mass. The unification of the gaugino mass parameters relates the two and leads to the above difficulty. Thus giving up this unification may open up a possibility of reconciling HERA events . Let us treat the gaugino masses $M_{1,2,3}$ at $M_Z$ as independent parameters. Then integration of the RG equation for the charm-squark from $M_{GUT}$ to $Q_0 = 200\,\text{GeV}$ leads to

$$\begin{aligned} m^2_{\tilde{c}_L}(Q_0) &\approx m^2_{\tilde{c}_L}(M_{GUT}) + 0.77M_3^2 + 0.70M_2^2 + 0.024M_1^2 \\ &+ 1/2\ M_Z^2\ \cos 2\beta\ (1 - 4/3\sin^2\theta_W) \end{aligned} \tag{6.29}$$

If gaugino masses were to be unified at $M_{GUT}$ then $M_3 \sim 3.25M_2$ and $M_1 \sim 0.5M_2$. Even in the absence of such unification, the physical gluino mass $m_{\tilde{g}} \sim (1 + 4.2\frac{\alpha_s}{\pi})M_3$ must be greater than the charm squark mass if large $\lambda'_{121}$ is to be avoided. This follows since in the converse case, the charm squark would predominantly decay to a gluino and a quark. This decay being governed by strong coupling, would dominate the other decays and would reduce $B$. The later is given in case of $m_{\tilde{c}_L} \gg m_{\tilde{g}}$ by

$$B \sim \frac{3\ \lambda'^2_{121}}{32\ \pi\ \alpha_s} \sim 2.5 \times\ 10^{-3} \left(\frac{\lambda'_{121}}{0.1}\right)^2$$

Such a tiny value of $B$ would need unacceptably large $\lambda'_{121}$. It therefore follows that one must suppress the squark decay to gluino kinematically by requiring $m_{\tilde{g}} \geq m_{\tilde{c}_L}$. Given this bound on $M_3$ it follows from eq.(15) that

$$M_2 \leq 170\,\text{GeV} \tag{6.30}$$

if $m_{\tilde{c}_L} \sim 220\,\text{GeV}$. This bound on $M_2$ is weaker than the one in the case of the gaugino mass unification, eq.(4). But it nevertheless cannot suppress the decay of squarks to chargino kinematically. It follows [8] from Fig. 1 that one now approximately needs $\lambda'_{121} \geq 0.08$. This value is close to the $2\sigma$ limit coming from the atomic parity violation but one would still need some cancellations to satisfy other constraints as discussed above. Thus giving up unification helps only partially.

An alternative possibility is to allow for a -ve (mass)$^2$ for the charm squark at the unification scale. In view of the large positive contribution induced by the gluino mass such negative (mass)$^2$ need not lead to colour breaking and may be consistent phenomenologically. In fact a -ve (mass)$^2$ for top squark has been considered in the literature in a different context. The universality is a simplifying feature of MSSM but it does not follow from any general principle. It does not hold in a large class of string based models which may allow

[8] Note that fig.1 is based on the assumption of $M_1 = 0.5M_2$ but does not use any relation between $M_2$ and $M_3$.

negative (mass)$^2$ for some sfermions as well [24]. Such masses can also arise when SUSY is broken by an anomalous $U(1)$ [25] with some of the sparticles having -ve charge under this $U(1)$.

The large radiative corrections induced through the running in squark masses from a high $\sim M_{GUT}$ to the weak scale has played an important role in this analysis. In contrast to the supergravity based models, this running is over a much smaller range in models with gauge mediated supersymmetric breaking. But in these models, the initial value of the charm squark (mass)$^2$ is positive and large with the result that these models are incompatible with the charm squark interpretation of HERA anomaly even without the radiative corrections [9].

The interpretation of HERA events in terms of stop may not suffer from the above mentioned difficulty encountered for the charm interpretation for two reasons. Firstly, the stop mass is reduced compared to the charm squark mass due to the possible large $\tilde{t}_L - \tilde{t}_R$ mixing as well due to the large top coupling. Secondly, this mass also involves one more parameter (the trilinear couplings $A$) compared to the charm squark mass. Thus while this is a less constrained possibility, imposition of the requirement that $m_{\tilde{t}} \sim 200\,\mathrm{GeV}$ would certainly lead to more a constrained parameter space than considered in model independent studies [15].

In summary, we have shown that the charm squark interpretation of HERA events is possible only for large $\lambda'_{121} \sim O(0.1)$ in a large class of supersymmetric standard models characterized by a positive charm squark (mass)$^2$ at the GUT scale. The simplest and the most popular minimal supergravity model with universal boundary condition falls in this class. The required large value of $R$ violating parameter is difficult to admit without postulating new physics and /or fine tuned cancellations due to constraints coming from the R-violating processes like neutrino mass, $K^+ \to \pi^+ \nu\bar{\nu}$ etc. However, the later data from HERA experiments did not substantiate their claim for a resonance. Treating the deviations at 200 GeV as statistical fluctuations, the HERA collaborations have now put constraints on the mass and generic couplings of leptoquarks. These bounds are now compatible with the results obtained from Tevatron [13].

[9] See, K. Cheung, D. Dicus and B. Dutta [20].

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Chapter 7

# Conclusions

The highly successful Standard Model has to be incorporated in larger theories to have a deeper understanding of Nature. Supersymmetric Standard Models have been built to solve the hierarchy problem which is encountered whenever Standard Model is being incorporated into a larger theory. Experimentally, one of the first signals for physics beyond Standard Model is provided by the atmospheric neutrino experiments which indicate the existence of atleast one non-zero neutrino mass. An interpretation of both solar and atmospheric neutrino anomalies interms of neutrino oscillations would lead to a specific pattern of the neutrino mass matrix. In this thesis, we studied the possibilities of generating such a pattern within the context of Supersymmetric Standard Models.

Neutrino masses can be generated in the Supersymmetric Standard Model as it naturally allows for lepton number violation. Models which violate lepton number are classified into bilinear and trilinear lepton number violating models based on the dimensionality of the couplings in the superpotential. Firstly, we studied in detail models which violate lepton number only through bilinear (dimensionful) couplings. In these models, only three R-parity violating couplings determine the scale of the neutrino masses. The masses are naturally small due to Yukawa suppression if universal boundary conditions are assumed at the high scale. After presenting a general analytical structure of the neutrino mass matrix in these models, we have studied them in detail in the framework the Minimal Messenger Model (MMM) of gauge mediated supersymmetry breaking. The bilinear R-violating model within MMM, is very economical with essentially only four parameters, the three R-parity violating couplings and the supersymmetry breaking scale $\Lambda$, determining the three masses and three mixing angles of the neutrinos. We have shown that these models are too restrictive to provide simultaneous solutions for solar and atmospheric neutrino problems. For simultaneous solutions with MSW conversion, we have numerically shown that these models do not provide the required mass hierarchy in the neutrino mass eigenvalues. On the other hand, for simultaneous solutions with vacuum oscillations, we have shown that these models cannot accommodate the two large mixings required. Thus, extensions of the minimal messenger model are inevitable for existence of simultaneous solutions for solar and atmospheric neutrino anomalies within these models. A simple extension which would allow for a negative $\mu$ parameter would be able to provide the required framework for simultaneous solutions.

The other class of models we have considered are models with trilinear lepton number violation. These models, unlike the bilinear case, offer much larger freedom due to the larger number of parameters present. One of the major results of this thesis is the effect of Renormalisation Group Evolution on the neutrino mass spectrum in the presence of only trilinear (dimensionless) lepton number violating couplings in the model. RG evolution generates non-zero soft bilinear lepton number violating couplings at the weak scale though these couplings are absent at the high scale. These would generate vacuum expectation values (*vevs*) to the sneutrinos leading to non-zero mixing between neutralinos and neutrinos. A 'tree level' neutrino mass is thus generated, which we call the 'RG induced tree level' mass. Analyses of the neutrino mass matrix in these models presented in the literature have been neglecting this important contribution which can drastically alter the neutrino mass spectrum in these models. We have analysed the neutrino mass spectrum taking into consideration RG induced tree level mass contribution in addition to the standard 1-loop induced masses within the framework of mSUGRA inspired MSSM. Our results show that the model favours vacuum solution to the solar neutrino problem while simultaneously allowing solutions for the atmospheric neutrino problem. However, there exist regions in the parameter space where the hierarchy in the neutrino mass eigenvalues is reduced due to a suppression of the tree level mass. In these regions, Large Angle MSW solution is preferred which is favoured by the latest data from super-Kamiokande.

In addition to studying the neutrino mass spectrum in the R-parity violating scenarios, one can instead consider bounds on the lepton number violating couplings due to direct experimental limits on the neutrino masses. Since neutrinos attain majorana masses in this case, the most stringent limit on them comes from the non-observation of neutrinoless double beta decay. Using this experimental limit, bounds on the R-parity violating couplings can be derived. The existing limits do not consider the RG effects on the neutrino mass. But, as we mentioned above, the RG induced tree level mass would alter the neutrino mass spectrum drastically. We have included this effect and re-derived bounds on the lepton number violating couplings. In addition to this, we have also considered the effect of CKM matrix and discussed the basis dependence of these bounds. We have subsequently used these bounds in our discussion of the charm squark interpretation of the now defunct HERA anomalies. However, our analysis is of general nature and can be redone if a relevant case arises in the future colliders.

## General properties of neutrino masses in R-violating theories

So far, in this thesis, we have concentrated on specific models of R-violation and supersymmetry breaking to study the structure of neutrino mass matrix. Though some of the properties of the neutrino mass matrix (essentially related to solutions of solar and atmospheric neutrino problems) depend on specific models of R-violation and type of supersymmetry breaking, most of the properties hold true in any of the R-violating scenarios. We summarise them as follows.

### *(i) R -violating couplings*

Majorana masses for the neutrinos are produced in these models due to the presence of lepton number violating couplings. The neutrino masses are proportional to the square of these couplings. In the limit R-violating couplings are set to zero, there are no neutrino masses in these models and they represent the standard MSSM with R-parity.

*(ii) Hierarchical masses*

Neutrinos attain masses both at the tree level as well as at the 1-loop level. The tree level mass is generated indirectly through the bilinear soft $L$ violating terms in the scalar potential. They are either present 'originally' in the scalar potential or generated at the weak scale through RG scaling. The total neutrino mass matrix is sum of the tree level and 1-loop contributions. At the tree level only one combination of the neutrinos attains mass. The other combinations attain mass at the 1-loop level only. The tree level mass is much larger compared to the 1-loop contribution for most of the R-violating theories. The hierarchy in the neutrino mass spectrum is typically characterised by $\frac{m_{1-loop}}{m_{tree}}$. Since this factor is much smaller than one, large hierarchy in the neutrino mass spectrum is natural in these models. In some theories it is possible to find regions in parameter space where the tree level mass is suppressed and becomes comparable to the 1-loop level mass.

*(iii) Large Mixing*

In most of these models, by suitable analytical approximations one can decouple the neutrino mass matrix into R-violation dependent part and R-violation independent part. The R-violation dependent part determines the mixing in most of the cases. The mixing angles are dependent on ratios of the R-violating parameters. When R-violating parameters are equal in magnitude large mixing is natural in the model [1]. But this does not guarantee us that two large mixing angles can be simultaneously accommodated in these models.

*(iv) Role of Yukawas*

The R-violating parameters which determine the neutrino masses are always accompanied by down quark or charged lepton Yukawa couplings. Thus in the limit of vanishing charged lepton and down quark Yukawa couplings, neutrino masses also vanish in these theories (In models with only dimensionful R violation this is true only in the case of universal boundary conditions at the high scale). This dependence on the Yukawa couplings can be understood in terms of a $U(1)$ symmetry. In the absence of down quark, charged lepton Yukawas and the $\mu$ term, the superpotential is invariant under the $U(1)$ with charges for the superfields:

$$L = l \;; H_2 = -l \;; E^c = -2\,l; \; D^c = -l; \; U^c = l \tag{7.1}$$

and the rest of the superfields carry zero charge. $l$ is an integer. The neutrino remains massless in this case. The Yukawas and the $\mu$ term break this symmetry and thus accompany the R-violating terms in the neutrino mass formulae.

*(v) Small R violation*

[1]This may not hold true in some models with only $\lambda$ couplings.

Neutrinos are the only standard model fermions which attain mass through the soft sector in these theories. Whereas the loop induced masses are suppressed by the Yukawa couplings and are small, the tree level neutrino mass can be in general very large $\sim O(M_{susy})$ in these models. These large neutrino masses have to be suppressed to be phenomenologically meaningful. These masses can be suppressed either in a natural way or by a choice of suppression of the parameters:

(a) By a suitable choice of the boundary conditions for bilinear lepton number violating soft terms at the high scale where supersymmetry is broken, one can bring down the neutrino masses to O(100 MeV) in these models [2]. For example, by choosing these terms to be zero at the high scale (as naturally happens in models with only dimensionless lepton number violation) or to be of the same magnitude as the other bilinear terms like $B_\mu$ at the high scale (as happens in models with universal boundary conditions) leads to bilinear lepton number violating soft terms which are only Yukawa suppressed at the weak scale due to RG scaling. This kind of suppression can generally lead to O(100 MeV) neutrino masses.

(b) One can always 'choose' the two contributions to the sneutrino *vev* to cancel each other. This naturally happens for some parameter space in mSUGRA inspired MSSM. But, in model independent analyses, which we discuss below the two parameters which contribute to the sneutrino *vev* or the sneutrino *vev* in general itself are free parameters of the model. In these cases, there is no natural choice of the sneutrino *vev* and it has to be chosen small in order to have a small neutrino mass.

Both the above methods would not be able to suppress the neutrino mass sufficiently to give the right scale of $\sim$ O(eV). Thus one has to choose extremely small R-violation to generate O(eV) neutrino masses in these models. This is possible as the neutrino masses are directly proportional to the R-violating couplings in these models. For example, to accommodate solutions of solar and atmospheric neutrino problems, bilinear R-parity violating parameters are required to be $O(10^{-5}) \times M_{susy}$. Similarly the dimensionless $\lambda'$ couplings are typically required to be of $O(10^{-4})$ in these models. Some models have been explored in literature where a natural way of such small R-violation can be achieved [1, 2, 3, 4].

### *(vi) Experimental Signatures*

R-violating theories allow for completely different experimental signatures in comparison to the standard MSSM. The additional L violating couplings have characteristic signatures some of which we have already seen in chapter 6. A comprehensive study of experimental signatures of R-violating theories has been recently presented in [5]. The models presented in this thesis have restricted the R-violating parameter space as well as the standard supersymmetry soft parameters so as to provide simultaneous solutions to solar and atmospheric neutrino problems. Thus these models would have different mass spectrum and decay signatures. An analysis of this type combining neutrino masses and experimental signatures have been done in case of some models recently [6, 7]. The other most important consequence of R-violation is that the LSP is no longer stable. One has to look for a new cold dark matter

[2] This may not generally hold true for some models with only trilinear $\lambda$ couplings.

candidate in these models.

Thus we see that R-parity violating theories provide a natural framework where small neutrino masses can be realised. In these models neutrino masses are calculable in terms of few basic parameters and thus can predict the existence of solutions for solar and atmospheric neutrino masses within specific models of supersymmetry breaking. Both bilinear and trilinear R-violating models have various interesting features associated with them making studies within these theories important and essential.

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