VACUUM STRUCTURE IN QCD AND HADRONIC CORRELATORS

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by

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Dedicated to

my Parents

CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

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List of Publications/Preprints

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- *Meson correlators in QCD vacuum - is saturation the right approach ?*
  Varun Sheel, Hiranmaya Mishra and Jitendra C. Parikh. (PRL)
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- *Quark propagator and meson correlators in the QCD vacuum*
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- *Hadronic correlators and condensate fluctuations in QCD vacuum*
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Chapter 1

Introduction

It is widely believed that the fundamental theory of strong interactions is *quantum chromodynamics* (QCD) [1, 2]. The theory describes the interaction of the fundamental constituents of matter, quarks via massless vector fields, the gluons. This theory is similar in construct to quantum electrodynamics (QED) and provides a good quantitative description of strong interaction phenomena at small distances.

The simple quark model was initially developed in early 1960s [3] to account for the regularities observed in the hadron spectrum, with hadrons interpreted as bound states of quarks [4]. This view of quarks as the fundamental constituents became more plausible as relations abstracted from the quantum field theory of quarks, i.e. the algebra of quark currents and their divergences were successfully used in the study of hadronic interactions in the late 1960s.

The paradoxes in the simple quark model were overcome by postulating that quarks also have a three-valued quantum number called *colour* [5]. This is analogous to the electric charge in QED. Whereas there is only one kind of electric charge, quarks can have any of the three colours

– say red, blue or green. Just as electrically charged particles interact by exchange of photons, quarks interact by exchange of gluons. In a process like $q \to q + g$, the colour of the quark may change and since colour (like electric charge) is always conserved, gluons must also carry colour. This suggests the existence of coloured gluons. Furthermore, since we know that the hadrons are colour neutral, it suggests that the forces between the coloured quarks must be colour-dependent. The conservation of colour charge implies exact colour symmetry. This idea of exact colour symmetry is strengthened by the agreement with experimental measurements of the anomaly calculation of the $\pi_0 \to 2\gamma$ rate. Then came a series of important experimental measurements, starting with the ones performed by the SLAC-MIT group at the end of the decade of 1960s, on deep inelastic lepton-nucleon scatterings. The cross-sections were revealed to satisfy Bjorken [6] scaling which could be successfully interpreted by Feynman's parton model [7]. This suggested that although the hadron constituents (quarks) are not produced as free particles in the final states of deep inelastic scatterings, they are weakly interacting at short distances.

The above description of hadronic interactions in terms of quarks and gluons follow from the fundamental notion of the non-Abelian gauge symmetry which we now describe. Whereas in global symmetries parameters of the symmetry transformations are independent of space time, a richer class of theories are those where the symmetry transformations are space time dependent. They are called local symmetries or *gauge symmetries* [8].

Such symmetries can be used to generate dynamics, the gauge interactions. It is now believed that all fundamental interactions are described by some form of gauge theory. The prototype gauge theory is quantum electrodynamics (QED). While QED has an Abelian U(1) local symmetry, a fundamentally richer system of gauge theories are those with non-Abelian transformations, namely the Yang-Mills theories [9].

The next question to ask naturally would be that which symmetry of the quark model should be gauged. From the discussions of exact colour symmetry before, it follows naturally that it is the colour symmetry of the quark model that should be gauged. Thus, the strong interaction should be described by an SU(3) colour Yang-Mills theory with quarks transforming as the fundamental triplet representation.

Using the gauge principle we next obtain the Lagrangian of the SU(3) Yang-Mills theory. Let us first consider the Lagrangian for a free colour triplet fermion field $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$ which is given as,

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x)$$

Clearly it has a global SU(3) symmetry corresponding to the invariance of the theory under an SU(3) transformation,

$$\psi(x) \to \psi'(x) = exp(-i\frac{\vec{\lambda}\cdot\vec{\theta}}{2})\psi(x)$$

where $\vec{\lambda} = \lambda^a$ are the usual Gell-Mann matrices satisfying

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = if^{abc}\frac{\lambda^c}{2} \quad tr(\lambda^a\lambda^b) = 2\delta^{ab} \quad a, b = 1\text{–}8$$

and $\vec{\theta} = \theta^a$ are the SU(3) transformation parameters.

We are going to turn this symmetry into a local symmetry, i.e. "to gauge the symmetry". Thus we are going to construct a theory which will be invariant under a space-time dependent set of parameters { θ^i }. However with such a local transformation,

$$\psi(x) \to \psi'(x) = U(\theta)\psi(x)$$

with

$$U(\theta) = exp(-i\frac{\vec{\lambda}\cdot\vec{\theta}(x)}{2}),$$

the free Lagrangian is no longer invariant because the derivative term transforms as

$$\bar{\psi}(x)\partial_\mu\psi(x) \to \bar{\psi}'(x)\partial_\mu\psi'(x) = \bar{\psi}(x)\partial_\mu\psi(x) + \bar{\psi}(x)U^{-1}(\theta)[\partial_\mu U(\theta)]\psi(x)$$

The second term spoils the invariance. We therefore consider a gauge covariant derivative D_μ, to replace ∂_μ, such that $D_\mu\psi(x)$ will have the simple transformation

$$D_\mu\psi(x) \to [D_\mu\psi(x)]' = U(\theta)D_\mu\psi(x)$$

so that the combination $\bar{\psi}(x)D_\mu\psi(x)$ is gauge invariant.

In other words, the action of the covariant derivative on the field will not change the transformation property of the field. This can be realised if we enlarge the theory with a new vector field $W_\mu(x)$, the *gauge field*, and form the covariant derivative as

$$D_\mu\psi = (\partial_\mu - igW_\mu)\,\psi$$

where g is a free parameter which will eventually be identified with the coupling constant and

$$W_\mu = \sum_{a=1}^{8} W_\mu^a \lambda^a/2$$

Then the transformation law for the covariant derivative will be satisfied if the gauge field $W_\mu(x)$ has the transformation property

$$W_\mu(x) \rightarrow \quad W'_\mu(x) = UW_\mu U^{-1} - \frac{i}{g}[\partial_\mu U]U^{-1}$$

From the definition of $W_\mu(x)$, it is clear that they transform like an octet (adjoint) representation under SU(3). Thus the $W^i_\mu s$ carry colour charge (SU(3) quantum number). This is in contrast to the electromagnetic (gauge) fields which do not have electric charge.

To make the gauge field a true dynamical variable we need to add kinetic energy and self interaction terms for gauge fields in the Lagrangian. In analogy with electrodynamics the simplest gauge-invariant term of dimension four is $\mathcal{L}_A = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu}$ where the second rank tensor $G_{\mu\nu}$ is defined as

$$G_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu]$$

The complete gauge invariant Lagrangian which describes the interaction between gauge fields W^a_μ and the SU(3) triplet fields ψ is

$$\mathcal{L}_{QCD} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \sum_k^{n_f} \bar{\psi}_k(i\gamma^\mu D_\mu - m_k)\psi_k$$

The quanta of gauge fields W^a_μ are called *gluons* and the $\psi_k s$ are the quark fields with the subscript k being the flavour index $k = 1, 2, \ldots, n_f(n_f$ is the number of quark flavours) ψ_k : u,d,s,c,b,t.

Some important features ought to be noted here. The pure Yang-Mills term, $-\frac{1}{4}G_{\mu\nu}G^{\mu\nu}$, contains factors that are trilinear and quadrilinear in W^a_μ, which correspond to the self coupling of non-Abelian gauge fields. They are brought about by the nonlinear terms in $G^a_{\mu\nu}$, because the gauge fields W^a_μ themselves transform nontrivially, as members of the adjoint representation. For gauge invariance the fields W_μ have to be massless since a mass term $\sim W_\mu W^\mu$ is not gauge invariant.

A major finding regarding non-Abelian Yang Mills theories came in early 1970s when it was shown that these theories are *asymptotically free* [1, 10], i.e. the coupling constant decreases at short distances. This was the first major indication that colour SU(3) Yang Mills theory may be a candidate theory of strong interactions, because it can explain results of deep inelastic scattering and parton model.

The statement that effective coupling constant vanishes for short distances giving rise to asymptotic freedom also suggests that the coupling increases for long distances. It is believed that these interactions lead to *colour confinement* for quarks. This idea is based on the experimental fact that quarks have not been detected in isolation, but only as constituents of hadrons. It is widely believed that this is a consequence of QCD and numerical studies have lent support to the belief; but as yet no proof exists. In this connection mention may be made of recent work of Seiberg and Witten which claims to prove confinement in N=2 SUSY Yang-Mills [11].

As in any other field theory it is instructive to focus ones attention on the ground state or *vacuum* of the system. There has for a long time been considerable interest in the vacuum structure of interacting quantum field theory. Although in classical field theory the vacuum configuration is usually simple, the situation in quantum field theory is less trivial because the fields are permanently fluctuating. Even in electrodynamics one encounters certain difficulties, namely the energy of zero point oscillations diverges at large frequencies. The problem is solved by putting it equal to zero, since only the difference between the energies of the excited and lowest states is observable. The presence of zero-point oscillations is still important for example as shown up in Lamb shift when the vacuum fields are perturbed by introducing an electron. Similarly, a proper understanding of the QCD vacuum is thought to be vital in the study of low energy QCD processes and in particular the problem of quark confinement. Since at low energies, QCD coupling constant is large, one needs non-perturbative methods to study QCD vacuum. Also it appears that the only fundamental way towards understanding of hadronic structure is to connect it with the underlying theory of the QCD vacuum [12]. Then, the natural question to ask first would be about the structure of the ground state of QCD.

The strength of the vacuum fluctuations, is characterised on an average by a few phenomenological parameters, i.e. vacuum condensates. These are non-vanishing vacuum expectation values of local gauge invariant operators formed from quark and gluon fields. The most important of them are the quark and gluon condensates, $< 0|\bar{q}q|0 >$ and $< 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}|0 >$. More graphically one can say that QCD vacuum is a medium. The vacuum medium consists of two ingredients, one is built from quark fields, the other from gluon fields. From uncertainty principle, it is important that the quark ingredients include only light pairs. A pair of quarks, say $\bar{u}u$, “injected” into the vacuum by an external photon, lives and evolves not in empty space but in the “vacuum medium”. As long as the distance between the quarks in not very large, their dynamics is determined by coarse averaged vacuum characteristics. The condensates thus carry information about the infrared and intermediate range behaviour of the quark and gluon Green functions.

With all the progress in our understanding of interactions at small ($\ll$ 1 fm) distances, we still understand very little about large distance interactions which confine quarks inside hadrons. Experiments tell us much more than just the masses of the lowest collective modes. In a few cases complete correlation functions are known. A set of various hadronic correlation functions plays essentially the same role as that played in nuclear physics by the scattering phase shifts, because in QCD qq or $\bar{q}q$ scattering is not accessible experimentally due to confinement. In view of this a natural rich set of observables is given by point-to-point correlation functions, depending on the distance between points [13]. Correlation functions tell us what the effective forces are between quarks, and how they depend on the distance between them. These forces are much more complicated than just those responsible for confinement and asymptotic freedom.

Different models have been used to describe the ground state structure in QCD [12]. In the "instanton" type vacuum the field is assumed to be concentrated in some localised regions in space time as instantaneous fluctuations [14]. One may also consider the "soliton" type vacuum where nonlinear gauge fields create some particle like clusters, e.g. glueballs [15] or monopoles [16]. The other possibility is the string type vacuum. These configurations correspond to nonhomogeneous structures for the ground state. There are also models based on a homogeneous vacuum which corresponds to a constant magnetic field but then the vacuum is unstable [17].

The main aim of the thesis is to provide a new model of the QCD vacuum and study its consequences in detail. The model includes nonperturbative features but is explicit enough to allow evaluation of various correlation functions By comparing these with correlation functions obtained from experiments, we can improve on the structure of the QCD vacuum. Because the same correlation functions have also been studied on the lattice, through models of the QCD vacuum and by QCD sum rules, there appears a good ground for comparison of their results.

In chapter 2 we shall consider the ground state in QCD with both quark and gluon condensates using a variational calculation. The method is thus nonperturbative. We take an explicit construct for the state with quark and gluon condensates the energy of which is calculated through minimisation. It is found that the perturbative vacuum having no condensates is stable for small $\alpha_s < \alpha_c$. For $\alpha_s > \alpha_c$ the perturbative vacuum is unstable leading to creation of quark and gluon condensates.

The *correlation function* is defined as the vacuum expectation value of time-ordered product of two operators taken at two points x and y :

$$K(x-y) =< 0|TO(x)O(y)|0 >$$

where the operators O can be classified into mesonic and baryonic types.

$$O_{mes}(x) = \bar{\psi}_a \psi_b \delta_{ab} \quad O_{bar}(x) = \psi_a \psi_b \psi_c \epsilon^{abc}$$

where a, b, c are colour indices which we suppress below.

Let us look at the asymptotic behaviour of the correlation function. At small spacelike separation x, due to asymptotic freedom, the quarks and gluons propagate freely, up to small and calculable radiative corrections. Therefore $K(x)$ in the meson (baryonic) case is essentially the square (or cube) of the free quark propagator ($S(x) \sim x^{-3}$) and hence in this limit, $K(x) \sim x^{-6}$ (or $\sim x^{-9}$) for mesons and baryons respectively.

As distance increases, quarks interact more strongly with vacuum fields. Still one could use OPE as long as corrections to free quark theory are not large. But at intermediate distances, description of correlation functions becomes very complicated and one resorts to Lattice or some vacuum models.

Motivated by these reasons correlation functions in various hadronic channels are investigated in chapter 3 using the structure of the QCD vacuum developed in chapter 2.

In the calculation of correlators, quark propagators enter in a direct manner and hence we start the chapter by studying aspects of the interacting propagator in some detail. These features of our propagator are similar to those of the Instanton model of QCD vacuum, though quantitatively there are differences. Having obtained the propagators, we then calculate equal time, point to point spatial ground state correlation functions of hadronic currents. We study the ratio of the physical correlation function to that of massless noninteracting quarks, $R(x)/R_o(x)$. In each channel we associate the current with a physical hadron having quantum numbers identical to that of the current. These include the pseudoscalar channel(π), vector channel (ρ), the scalar and axial vector channels in the meson sector. The study of baryonic channels includes the nucleon and delta.

The behaviour of the ratio $R(x)/R_o(x)$ with the distance x is qualitatively similar to that predicted by phenomenology in the vector and axial vector channels. However in the pseudoscalar channel the peak of the ratio differs from phenomenology by two orders of magnitude. The phenomenological correlators for the vector and axial vector currents are calculated from e^+e^- annihilation into hadrons and τ decay to hadrons respectively, while for the pseudoscalar channel the correlator is related to the mass and decay constant of the particle. This is discussed in detail in Ref. [13] and appendix B. Also the nucleon channel does not show a rise at all while QCD sum rules predict a peak value $R(x)/R_o(x) \approx 5$. The delta correlator is qualitatively similar to predictions of QCD sum rules, the instanton model and lattice calculations.

Motivated by the acute problem in the pseudoscalar channel, we use phenomenological results of the hadronic correlation functions to guide us towards a "true" structure of QCD vacuum in chapter 4. From the study of the pseudoscalar and nucleon channels in chapter 3, we conclude that in our model there ought to be explicit contribution arising from irreducible four point structure of the vacuum. We parameterise such contributions and fix the parameters so that we get good agreement with phenomenology for the pseudoscalar channel. We also obtain the rise in the nucleon channel. The correlators for the strange channels $\phi, K^\star$ and K were also studied by us. Comparison of $R(x)/R_o(x)$ with phenomenology was reasonable. Next, we fit our curves for $R(x)/R_o(x)$ to phenomenologically motivated forms for correlators parameterised in terms of mass, coupling and threshold of the corresponding particle. Our fitted parameters are comparable to those obtained in other models.

Finally, a summary and conclusion of the study carried out in the earlier chapters and a perspective on the future work, is given in chapter 5.

References

[1] D. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343.

[2] S. Weinberg, Phys. Rev. Lett. 31 (1973) 494; H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. 47B (1973) 365.

[3] M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report No. CTSL-20 (1961); Y. Ne'eman, Nucl. Phys. 26 (1961) 222; M. Gell-Mann and Y. Ne'eman, The eightfold way, Benjamin, New York (1964).

[4] M. Gell-Mann, Phys. Lett. 8 (1964) 214; G. Zweig, CERN report No. 8182/TH 401.

[5] O. W. Greenberg, Phys. Rev. Lett. 13 (1964) 598; M. Han and Y. Nambu, Phys. Rev. 139B (1965) 1006; Y. Nambu, In Preludes in theoretical physics (ed. de DeShalit), North-Holland, Amsterdam (1966).

[6] J. D. Bjorken, Phys. Rev. 179 (1969) 1547.

[7] R. P. Feynman, Photon-hadron interaction, Benjamin, Reading, Massachusetts (1972).

[8] H. Weyl, Z. f. Physik. 56 (1929) 330.

[9] C. N. Yang and R. Mills, Phys. Rev. 96 (1954) 191.

[10] D. Gross and F. Wilczek, Phys. Rev. D8 (1973) 3633; H. D. Politzer, Phys. Rev. Lett. 30 (1973) 1346; ibid, Phys. Rep. 14C (1974) 129; A. Zee, Phys. Rev. D7 (1973) 3730; S. Coleman and D.J. Gross, Phys. Rev. Lett. 31 (1973) 851.

[11] N. Seiberg and E. Witten, Nucl. Phys. B426 (1994) 19.

[12] E.V. Shuryak, *The QCD vacuum, hadrons and the superdense matter*, (World Scientific, Singapore, 1988).

[13] E.V. Shuryak, Rev. Mod. Phys. 65 (1993) 1.

[14] D. J. Gross, Phys. Rep. 49 (1979) 143.

[15] T. H. Hansson, K. Johnson, C. Peterson, Phys. Rev. D26 (1982) 2069.

[16] S. Mendelstam, Phys. Rep. C23 (1976) 245; N. P. Nair and C. Rosenzweig, Phys. Rev. D31 (1985) 401.

[17] G. K. Savvidy, Phys. Lett. 71B (1977) 133 ; S. G. Matinyan and G. K. Savvidy, Nucl. Phys. B134 (1978) 539; N. K. Nielsen and P. Olesen, Nucl. Phys. B144 (1978) 376.

Chapter 2

Vacuum Structure in QCD with Quark and Gluon Condensates

2.1 Introduction

It is now believed that quantum chromodynamics (QCD) is the correct theory of strong interaction of quarks and gluons. At low energies, however, the coupling constant in QCD becomes large leading to a breakdown of the perturbative calculations. In this regime, the vacuum structure is also known to be nontrivial [1] with nonzero expectation values for quark and gluon condensates [2]. Instability of QCD vacuum with constant chromomagnetic field or with vortex condensate formation has been studied since quite some time with a semiclassical approach [3]. QCD vacuum has also been studied with gluon or glueball condensates [4, 5] as well as with nonperturbative solutions to Schwinger Dyson equations [6]. Further, a nontrivial vacuum structure with quark condensates in Nambu Jona Lasinio type of models [7] has been seen to be consistent with low energy hadron physics. It is therefore desirable to examine the vacuum structure in QCD with *both* quark and gluon condensates.

A nonperturbative variational method has been proposed earlier with an explicit structure for the QCD vacuum. This has been applied to the case of gluon condensates for vacuum structure in $SU(3)$ Yang-Mills fields demonstrating the instability of perturbative vacuum when coupling is greater than a critical value [8]. The same methods have been applied to study the vacuum structure with quark condensates in QCD motivated phenomenological effective potential models [9] and in Nambu-Jona-Lasinio model [10]. For the ground state or vacuum this has been achieved through a minimisation of energy density. It was seen here that through the variational analysis, one obtains the same gap equation along with other results [9, 10] so that one recognises that the method includes nonperturbative quantum effects. For the chiral symmetry breaking a simple ansatz has been considered of taking the perturbative quarks having a phenomenological Gaussian distribution in the nonperturbative vacuum [11]. This appeared to describe a host of low energy hadronic properties as being related to the vacuum structure associated with chiral symmetry breaking. However, here no energy minimisation has been attempted.

In this chapter, we study the vacuum structure of quantum chromodynamics (QCD) with both quark and gluon condensates using a variational ansatz for the ground state and discuss its stability [12].

As before, we shall take specific forms of condensate functions for quarks and gluons to describe a trial state for vacuum. Such an ansatz necessarily has limited dynamics. We shall circumvent this partially with the constraints that the value of the SVZ parameter $\frac{\alpha_s}{\pi} < G^a_{\mu\nu} G^{a\mu\nu} >$ and the experimental value of the pion decay constant f_π shall be correctly reproduced. The condensate functions still contain two parameters, over which energy is minimised. We then note that for α_s greater than a critical coupling α_c, perturbative vacuum destabilises with non-vanishing condensates in *both* quark and gluon sectors.

The chapter is organised as follows. In section 2.2, we briefly recapitulate quantisation of QCD in Coulomb gauge and give an explicit construct for the nonperturbative vacuum with quark and gluon condensates. The ansatz for the QCD vacuum is similar to BCS ansatz of Cooper pairs in the context of superconductivity. In section 2.3 we consider the stability of such a trial state through an energy minimisation where pion decay constant and SVZ parameter are taken as constraints as mentioned above and then discuss the results. In section 2.4 we summarise our conclusions. We also notice that for $\alpha_s \simeq 1.28$, the ansatz function of Ref. [11] along with the correct charge radius of the pion is reproduced.

The method considered here is nonperturbative and we shall be using the equal time quantum algebra for the interacting field operators, but is limited by the choice of the ansatz functions. The technique has been applied earlier to solvable cases [13] to examine its reliability where, through the present nonperturbative analysis one generates the one loop potential derived originally by Gross and Neveu. It also has been applied to ground state structure for electroweak symmetry breaking and cosmic rays [14] and to nuclear matter, deuteron or quark stars [15].

Although the following calculations are done in Coulomb gauge, we correlate the variational function for the condensates with the known *gauge independent* parameter of Shifman et al [2]

2.2 Vacuum with quark and gluon condensates

The QCD Lagrangian as derived in chapter 1, is given as

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} + \mathcal{L}_{int}, \tag{2.1}$$

where

$$\mathcal{L}_{gauge} = -\frac{1}{4}G^a{}_{\mu\nu}G^{a\mu\nu}, \quad \mathcal{L}_{matter} = \bar{\psi}(i\gamma^\mu\partial_\mu)\psi \quad and\; \mathcal{L}_{int} = g\bar{\psi}\gamma^\mu\frac{\lambda^a}{2}W^a_\mu\psi \tag{2.2}$$

where $W^a{}_\mu$ are the SU(3) colour gauge fields and $\psi's$ are the colour triplet quark spinors in the fundamental representation. γ^μ are the Dirac matrices and λ^a are the Gell-Mann matrices. The second rank tensor $G^a_{\mu\nu}$ is given by

$$G^a_{\mu\nu} = \partial_\mu W^a{}_\nu - \partial_\nu W^a{}_\mu + gf^{abc}W^b{}_\mu W^c_\nu \tag{2.3}$$

We then write the electric fields, $E^a_i = G^a_{0i}$ in terms of the transverse and longitudinal parts as

$$\begin{aligned} G^a{}_{0i} &\equiv \partial_0 W^a_i - \partial_i W^a_0 + gf^{abc}W^b_0 W^c_i \\ &= {}^TG^a{}_{0i} + \partial_i f^a, \end{aligned} \tag{2.4}$$

where the form f^a is to be determined. We shall quantise in Coulomb gauge [16] for which the gauge condition and the equal time algebra for the gauge fields are given as

$$\partial_i W^a_i = 0 \tag{2.5}$$

and

$$\left[W_i^a(\vec{x},t), {}^T G^b{}_{0j}(\vec{y},t)\right] = i\delta^{ab}\left(\delta^{ij} - \frac{\partial_i\partial_j}{\partial^2}\right)\delta(\vec{x}-\vec{y}) \tag{2.6}$$

We note here that a massless field has two degrees of freedom. The gluon fields W_μ^a have four components (W_0^a, W_i^a) of which W_i^a has one longitudinal and two physical transverse degrees of freedom. We see from Eq. (2.5) that Coulomb gauge retains only the physical(transverse) degrees of freedom. The commutation relation in Eq. (2.6) is a reflection of the fact that $G_{0i}^a = \dfrac{\partial\mathcal{L}}{\partial(\partial_0 W_i^a)}$ are the conjugate momenta of W_i^a. The operator $\left(\delta^{ij} - \dfrac{\partial_i\partial_j}{\partial^2}\right)$ takes care of the transverse condition Eq. (2.5).

We take the field expansions for W_i^a and ${}^T G^a{}_{0i}$ at time t=0 as [4, 8]

$$W^a{}_i(\vec{x}) = (2\pi)^{-3/2}\int \frac{d\vec{k}}{\sqrt{2\omega(\vec{k})}}(a^a{}_i(\vec{k}) + a^a{}_i(-\vec{k})^\dagger)\exp(i\vec{k}.\vec{x}) \tag{2.7}$$

and

$${}^T G^a{}_{0i}(\vec{x}) = (2\pi)^{-3/2} i\int d\vec{k}\sqrt{\frac{\omega(\vec{k})}{2}}(-a^a{}_i(\vec{k}) + a^a{}_i(-\vec{k})^\dagger)\exp(i\vec{k}.\vec{x}), \tag{2.8}$$

where, a_i^a and $a_i^{a\dagger}$ are creation and annihilation operators for gluons and $\omega(k)$ is arbitrary [16] and with substitution in Eq. (2.6) for equal time algebra we have

$$\left[a^a{}_i(\vec{k}), a^b{}_j(\vec{k}')^\dagger\right] = \delta^{ab}\Delta_{ij}(\vec{k})\delta(\vec{k}-\vec{k}'), \tag{2.9}$$

with

$$\Delta_{ij}(\vec{k}) = \delta_{ij} - \frac{k_i k_j}{k^2}. \tag{2.10}$$

The equal time quantization condition for the fermionic sector is given as

$$[\psi_\alpha^i(\vec{x},t), \psi_\beta^j(\vec{y},t)^\dagger]_+ = \delta^{ij}\delta_{\alpha\beta}\delta(\vec{x}-\vec{y}), \tag{2.11}$$

where i and j refer to the colour and flavour indices. We now also have the field expansion for fermion field ψ at time t=0 given as

$$\psi^i(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int \left[U_r(\vec{k}) c^i_{Ir}(\vec{k}) + V_s(-\vec{k}) \tilde{c}^i_{Is}(-\vec{k}) \right] e^{i\vec{k}\cdot\vec{x}} d\vec{k}, \tag{2.12}$$

where U and V are given by [17]

$$U_r(\vec{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \vec{\sigma}\cdot\hat{k} \end{pmatrix} u_{Ir}; \quad V_s(-\vec{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \vec{\sigma}.\hat{k} \\ 1 \end{pmatrix} v_{Is}, \tag{2.13}$$

for free chiral fields. The above are consistent with the equal time anti-commutation conditions with [17]

$$[c^i_{Ir}(\vec{k}), c^j_{Is}(\vec{k}')^\dagger]_+ = \delta_{rs}\delta^{ij}\delta(\vec{k}-\vec{k}') = [\tilde{c}^i_{Ir}(\vec{k}), \tilde{c}^j_{Is}(\vec{k}')^\dagger]_+, \tag{2.14}$$

where, c and $\tilde{c}$ are creation operators for a quark and an antiquark.

In Coulomb gauge, the expression for the Hamiltonian density, $\mathcal{T}^{00}$ from Eqs. (2.1-2.2) is given as [16]

$$\begin{aligned} \mathcal{T}^{00} &= :\frac{1}{2}{}^T G^a{}_{0i}{}^T G^a{}_{0i} + \frac{1}{2} W^a{}_i(-\vec{\bigtriangledown}^2) W^a{}_i + g f^{abc} W^a_i W^b_j \partial_i W^c_j \\ &+ \frac{g^2}{4} f^{abc} f^{aef} W^b_i W^c_j W^e_i W^f_j + \frac{1}{2}(\partial_i f^a)(\partial_i f^a) \\ &+ \bar{\psi}(-i\gamma^i\partial_i)\psi - g\bar{\psi}\gamma^i \frac{\lambda^a}{2} W^a_i \psi :, \end{aligned} \tag{2.15}$$

where : : denotes the normal ordering with respect to the perturbative vacuum, say $\mid 0>$, defined through $a^a{}_i(\vec{k}) \mid 0>= 0$, $c^i_{Ir}(\vec{k}) \mid 0>= 0$ and $\tilde{c}^i_{Ir}(\vec{k})^\dagger \mid 0>= 0$.

To examine the stability of the perturbative vacuum, $\mid 0>$, we construct a state with quark and gluon condensates say $\mid vac>$ and calculate its energy density by minimisation of the expectation value of $\mathcal{T}^{00}$ with respect to $\mid vac>$. For this purpose, we have to solve for the operator f^a to be able to take the expectation value of $\mathcal{T}^{00}$ given by Eq. (2.15) with a given ansatz for $\mid vac>$. From the equation of motion for the gauge field W_0^a, we obtain the constraint equation for f^a given as

$$(\vec{\bigtriangledown}^2\delta^{ac} + gf^{abc}W^b{}_i\partial_i)f^c = gf^{abc}W^b{}_i{}^T G^c{}_{0i} - g\bar{\psi}\gamma^0\frac{\lambda^a}{2}\psi \equiv J_0^a \tag{2.16}$$

Since it is not possible to solve the above equation for f^a exactly, we could take a perturbative expansion for f^a in terms of the coupling constant g as

$$f^a = gf_1^a + g^2 f_2^a + \ldots \tag{2.17}$$

However unless we keep a reasonable number of terms in the series on the right hand side of the above equation, the solution will not be nonperturbative. It has been also our objective to attempt variational solution outside the summation of a subset of the perturbative series.

To do this we shall use a version of mean field approximation in the context of condensates. For this purpose let us first take the space divergence of both sides of Eq. (2.4). This gives the relation between f^a and W_0^a as

$$f^a = -W^a{}_0 - g\ f^{abc}\ (\vec{\bigtriangledown}^2)^{-1}(W^b{}_i\ \partial_i W^c{}_0). \tag{2.18}$$

Substituting this expression for f^a in Eq. (2.16), we have

$$\vec{\bigtriangledown}^2 W^a{}_0 + g^2\ f^{abc}f^{cde}\ W^b{}_i\partial_i(\vec{\bigtriangledown}^2)^{-1}(W^d{}_j(\partial_j W^e{}_0))) + 2gf^{abc}W^b{}_i(\partial_i W^c{}_0) = J_0^a \tag{2.19}$$

Similar to Eq. (2.16), it is not possible to solve the above equation for W_0^a. However, we proceed with a mean field type of approximation [8]. What we shall do is to replace the operators in the left hand side of Eq. (2.19) by the corresponding expectation values in the condensate vacuum, $\mid vac>$ for all the fields other than W_0^a. Then Eq. (2.19) gets replaced by

$$\vec{\bigtriangledown}^2 W^a{}_0(\vec{x}) \quad + \quad g^2\ f^{abc}f^{cde}\ < vac \mid W^b{}_i(\vec{x})\partial_i(\vec{\bigtriangledown}^2)^{-1}(W^d{}_j(\vec{x}) \mid vac> \partial_j W^e{}_0(\vec{x}))$$

$$= \quad J^a{}_0(\vec{x}), \tag{2.20}$$

where,

$$J_0^a = gf^{abc}W_i^{bT}G_{0i}^c - g\bar{\psi}\gamma^0\frac{\lambda^a}{2}\psi. \tag{2.21}$$

To evaluate the above expression we first note that

$$< vac \,|: W^a{}_i(\vec{x})W^b{}_j(\vec{y}) :|\, vac >= f_{ij}(\vec{x}-\vec{y})\delta^{ab} \tag{2.22}$$

where

$$f_{ij}(\vec{x}-\vec{y}) = (2\pi)^{-3}\int d\vec{k}e^{i\vec{k}.(\vec{x}-\vec{y})}\; F(\vec{k})\;\Delta_{ij}(\vec{k}), \tag{2.23}$$

The above expression is written down from translational invariance for $|\, vac >$ and the transversality condition for the gluon fields. The function $F(\vec{k})$ will depend upon the particular construct one takes for $|\, vac >$. For calculational convenience let us next take the Fourier transforms

$$\tilde{W}_0^a(\vec{k}) = \int W_0^a(\vec{x})e^{i\vec{k}\cdot\vec{x}}d\vec{x} \tag{2.24}$$

and

$$\tilde{J}_0^a(\vec{k}) = \int J_0^a(\vec{x})e^{i\vec{k}\cdot\vec{x}}d\vec{x} \tag{2.25}$$

Equation (2.20) then reduces to

$$(k^2 + \phi(\vec{k}))\tilde{W}_0^a(\vec{k}) = -\tilde{J}_0^a(\vec{k}), \tag{2.26}$$

where

$$\phi(\vec{k}) = \frac{3g^2}{(2\pi^3)^3}\int d\vec{k}'F(\vec{k}')k^2\frac{(1-(\hat{k}\cdot\hat{k}')^2)}{(\vec{k}-\vec{k}')^2} \tag{2.27}$$

We may assume spherical symmetry for the function $F(\vec{k})$, in which case the integration over angles in the integral for $\phi(\vec{k})$ can be carried out. The equation for $\phi(\vec{k})$ then reduces to, with $k = |\vec{k}|$ and $k' = |\vec{k}'|$,

$$\phi(\vec{k}) = \frac{3g^2}{8\pi^2}\int dk'F(k')\left(k^2 + k'^2 - \frac{(k^2-k'^2)^2}{2kk'}log\left|\frac{k+k'}{k-k'}\right|\right) \tag{2.28}$$

Thus, Eqs. (2.26) and (2.28) give an approximation for W_0^a as modified by the gluon condensates in $\mid vac>$. The solution for $W_0^a(\vec{x})$ will depend on the ansatz for the ground state $|vac>$. We may also note that the mean field type of solution in Eq. (2.20) does not have a perturbative analogy, and is not an exact solution of the problem. As before [8, 10] we consider the same only for low energy phenomenology.

We shall now consider a trial state with gluon as well as quark condensates. We thus explicitly take the ansatz for the above state as [8, 9, 10, 11]

$$|vac>= U_G U_F|0>, \tag{2.29}$$

obtained through the unitary operators U_G and U_F on the perturbative vacuum. For the gluon sector, we have [8]

$$U_G = \exp{(B_G{}^\dagger - B_G)}, \tag{2.30}$$

with the gluon condensate creation operator $B_G{}^\dagger$ as given by [8]

$$B_G{}^\dagger = \frac{1}{2}\int f(\vec{k}) {a^a}_i(\vec{k})^\dagger {a^a}_i(-\vec{k})^\dagger d\vec{k}, \tag{2.31}$$

where $f(\vec{k})$ describes the vacuum structure with gluon condensates. For fermionic sector we have,

$$U_F = \exp{(B_F{}^\dagger - B_F)}, \tag{2.32}$$

with [9, 10, 11]

$$B_F{}^\dagger = \int \left[h(\vec{k}) {c^i}_I(\vec{k})^\dagger (\vec{\sigma}\cdot\hat{k}) \tilde{c}_I^i(-\vec{k}) \right] d\vec{k}, \tag{2.33}$$

Here $h(\vec{k})$ is a trial function associated with quark antiquark condensates. We shall minimise the energy density for $|vac>$ to analyse vacuum stability. For this purpose we first note that with the above transformation the operators, say $b_i^a(\vec{k})$ which annihilate $\mid vac>$ are given as

$$b_i^a(\vec{k}) = U a_i^a(\vec{k}) U^{-1} \tag{2.34}$$

We explicitly obtain relations between the operators corresponding to $| vac >$ and the operators corresponding to $|0 >$ through the Bogoliubov transformations

$$\begin{pmatrix} b_i^a(\vec{k}) \\ b_i^a(-\vec{k})^\dagger \end{pmatrix} = \begin{pmatrix} coshf(\vec{k}) & -sinhf(\vec{k}) \\ -sinhf(\vec{k}) & coshf(\vec{k}) \end{pmatrix} \begin{pmatrix} a_i^a(\vec{k}) \\ a_i^a(-\vec{k})^\dagger \end{pmatrix} \tag{2.35}$$

for the gluon sector and

$$\begin{pmatrix} d_I(\vec{k}) \\ \tilde{d}_I(-\vec{k}) \end{pmatrix} = \begin{pmatrix} cos(h(\vec{k})) & -(\vec{\sigma}\cdot\hat{k})sin(h(\vec{k})) \\ (\vec{\sigma}\cdot\hat{k})sin(h(\vec{k})) & cos(h(\vec{k})) \end{pmatrix} \begin{pmatrix} c_I(\vec{k}) \\ \tilde{c}_I(-\vec{k}) \end{pmatrix}, \tag{2.36}$$

for the quark sector. Using Eqs. (2.9) and (2.35) one obtains the commutation relation for operators b_i^a in the gluon sector

$$\left[b^a{}_i(\vec{k}), b^b{}_j(\vec{k}')^\dagger\right] = \delta^{ab}\Delta_{ij}(\vec{k})\delta(\vec{k}-\vec{k}'), \tag{2.37}$$

which has the same form as that for the operators a_i^a which merely reflects that Bogoliubov transformation is canonical.

Our job now is to evaluate the expectation value of $\mathcal{T}^{00}$ with respect to $| vac >$. For evaluating the same, the following formulae will be useful.

$$< vac |: W^a{}_i(\vec{x})W^b{}_j(\vec{y}) :| vac >= \delta^{ab} \times (2\pi)^{-3}\int d\vec{k}e^{i\vec{k}.(\vec{x}-\vec{y})}\,\frac{F_+(\vec{k})}{\omega(k)}\,\Delta_{ij}(\vec{k}), \tag{2.38}$$

$$< vac | : {}^TG^a_{0i}(\vec{x})^TG^b_{0j}(\vec{y}) : | vac > = \delta^{ab}\times(2\pi)^{-3}\int d\vec{k}e^{i\vec{k}.(\vec{x}-\vec{y})}\Delta_{ij}(\vec{k})\omega(k)F_-(k). \tag{2.39}$$

In the above $F_\pm(k)$ are given as [8]

$$F_\pm(\vec{k}) = \sinh^2 f(k) \pm \frac{\sinh 2f(k)}{2} \tag{2.40}$$

Thus the function $F(\vec{k})$ of Eq. (2.23) is related to the above as $F(\vec{k}) = \frac{F_+(\vec{k})}{\omega(k)}$ and $F(\vec{k}) = \omega(k)F_-(k)$

Similarly, for the quark fields we have the parallel equations given as

$$<:\psi^i_\alpha(\vec{x})^\dagger\psi^j_\beta(\vec{y}):>_{vac}=(2\pi)^{-3}\delta^{ij}\int\Big(\Lambda_-(\vec{k})\Big)_{\beta\alpha}e^{-i\vec{k}.(\vec{x}-\vec{y})}d\vec{k}, \tag{2.41}$$

$$<:\psi^i_\alpha(\vec{x})\psi^j_\beta(\vec{y})^\dagger:>_{vac}=(2\pi)^{-3}\delta^{ij}\int\Big(\Lambda_+(\vec{k})\Big)_{\alpha\beta}e^{i\vec{k}.(\vec{x}-\vec{y})}d\vec{k}, \tag{2.42}$$

where

$$\Lambda_\pm(\vec{k})=\pm\frac{1}{2}(\gamma^0\sin 2h(\vec{k})-2(\vec{\alpha}\cdot\hat{k})\ \sin^2 h(\vec{k})). \tag{2.43}$$

These relations will be used to evaluate the energy expectation value which is carried out in the next section.

2.3 Extremisation of energy functional

We shall here proceed to evaluate the expectation value of the Hamiltonian of Eq. (2.15). However for that we note that we have to know the auxiliary field contribution $(1/2)\partial_i f^a\partial_i f^a$ in Eq. (2.23) or equivalently the contribution arising out of the time like and the longitudinal components of the gauge field.

As stated in the previous section, we take a mean field type of approximation (Eq. (2.20)) with which the solution for W_0^a field is given by Eq. (2.26) :

$$\tilde{W}^a_0(\vec{k})=\frac{{J^a}_0(\vec{k})}{k^2+\phi(\vec{k})} \tag{2.44}$$

where $\tilde{W}^a_0(\vec{k})$ and $\tilde{J}^a_0(\vec{k})$ are Fourier transforms of $W^a_0(\vec{x})$ and $J^a_0(\vec{x})$ and $\phi(\vec{k})$ is given through Eq. (2.28)

$$\phi(k)=\frac{3g^2}{8\pi^2}\int\frac{dk'}{\omega(k')}\ F_+(k')\Big(k^2+k'^2-\frac{(k^2-k'^2)^2}{2kk'}\log\Big|\frac{k+k'}{k-k'}\Big|\Big). \tag{2.45}$$

Using equations (2.15), (2.38), (2.39), (2.41) and (2.42) we then obtain the expectation value of $\mathcal{T}^{00}$ with respect to $|vac>$ as

$$\epsilon_0\ \equiv\ <vac|:\mathcal{T}^{00}:|vac>$$

$$= \quad C_F + C_1 + C_2 + {C_3}^2 + C_4, \qquad (2.46)$$

where

$$\begin{aligned} C_F &= <: \bar{\psi}(-i\gamma^i \partial_i)\psi :>_{vac} \\ &= \frac{12N_f}{(2\pi)^3} \int d\vec{k} |\vec{k}| \sin^2 h(\vec{k}), \end{aligned} \qquad (2.47)$$

$$\begin{aligned} C_1 &= <: \frac{1}{2} {}^T G^a{}_{0i} {}^T G^a{}_{0i} :>_{vac} \\ &= \frac{4}{\pi^2} \int \omega(k) k^2 F_-(k)\, dk, \end{aligned} \qquad (2.48)$$

$$\begin{aligned} C_2 &= <: \frac{1}{2} W^a{}_i (-\vec{\nabla}^2) W^a{}_i :>_{vac} \\ &= \frac{4}{\pi^2} \int \frac{k^4}{\omega(k)}\, F_+(k)\, dk \end{aligned} \qquad (2.49)$$

$$\begin{aligned} {C_3}^2 &= <: \frac{1}{4} g^2 f^{abc} f^{aef} W^b{}_i W^c{}_j W^e{}_i W^f{}_j :>_{vac} \\ &= \left(\frac{2g}{\pi^2} \int \frac{k^2}{\omega(k)}\, F_+(k)\, dk \right)^2, \end{aligned} \qquad (2.50)$$

and

$$C_4 = <: \frac{1}{2} (\partial_i f^a)(\partial_i f^a) :>_{vac} \qquad (2.51)$$

The evaluation of the equations (2.47), (2.48), (2.49) and (2.50) is straightforward. To evaluate Eq. (2.51) we shall first eliminate f^a in favour of W_0^a using Eq. (2.18); write the mean field solution for W_0^a as given by Eq. (2.26)and then evaluate the expectation value. The contribution coming from the first term of Eq. (2.18) is

$$\begin{aligned} C_4^I &= \frac{1}{2} <: W^a{}_0(\vec{x})(-\vec{\nabla}^2) W^a{}_0(\vec{x}) :>_{vac} \\ &= \frac{(2\pi)^{-6}}{2} \int \frac{d\vec{k}' k'^2}{(k'^2 + \phi(k'))^2} <: \tilde{J}_0^a(k) \tilde{J}_0^a(k) :>_{vac} e^{i(\vec{k}+\vec{k}')\cdot\vec{x}} \\ &= 4 \times (2\pi)^{-6} \int d\vec{k} \frac{k^2 G(\vec{k})}{(k^2 + \phi(k))^2}. \end{aligned} \qquad (2.52)$$

where we have substituted

$$<: \tilde{J}^a_0(k)\tilde{J}^a_0(k) :>_{vac} = \delta^{ab}\delta(\vec{k}+\vec{k}')G(\vec{k}). \tag{2.53}$$

In the above $\phi(\vec{k})$ is as given earlier in Eq. (2.28) and $G(\vec{k}) = G_1(\vec{k})+G_2(\vec{k})$ is given as

$$\begin{aligned} G_1(\vec{k}) &= 3g^2 \int d\vec{q} F_+(|\vec{q}\,|)\; F_-(|\vec{k}+\vec{q}|)\; \frac{\omega(|\;\vec{k}+\vec{q}\;|)}{\omega(|\;\vec{q}\;|)} \\ &\times \;\; \Big(1+\frac{(q^2+\vec{k}.\vec{q})^2}{q^2(\vec{k}+\vec{q})^2}\Big), \end{aligned} \tag{2.54}$$

and, the contribution from quarks,

$$\begin{aligned} G_2(\vec{k}) &= -\frac{N_f}{2}g^2 \int d\vec{q} [\sin 2h(\vec{q}) \sin 2h(\vec{k}+\vec{q}) \\ &+ \;\; 4\, \frac{\vec{q}\cdot(\vec{k}+\vec{q})}{|\vec{q}||\vec{k}+\vec{q}|} \sin^2 h(\vec{q}) \sin^2 h(\vec{k}+\vec{q})] \end{aligned} \tag{2.55}$$

Similarly for the second term in Eq. (2.18) one obtains

$$\begin{aligned} C_4^{II} &= \frac{g^2}{2} f^{abc} f^{aef} <: W^c{}_0 W^b_i \partial_i ((\vec{\bigtriangledown}^2)^{-1} W^e{}_j \partial_j W^f{}_0 :>_{vac} \\ &= 4 \times (2\pi)^{-6} \int d\vec{k} \frac{\phi(k)G(\vec{k})}{(k^2+\phi(k))^2}. \end{aligned} \tag{2.56}$$

Thus the expression for C_4 becomes

$$\begin{aligned} C_4 &= C_4^I + C_4^{II} \\ &= 4 \times (2\pi)^{-6} \int d\vec{k} \frac{G(\vec{k})}{k^2+\phi(k)}. \end{aligned} \tag{2.57}$$

As may be noted here, the contributions from the quark condensates to the energy density comes through the auxiliary equation in C_4 as well as from the quark kinetic term in C_F.

We shall now minimise the energy functional ϵ_0 of Eq. (2.46). For the same we shall take $\omega(\vec{k})$ to be of the free field form with an effective mass parameter for the gluon fields given as

$$\omega(\vec{k}) = \sqrt{k^2 + m_G^2}. \tag{2.58}$$

We note that m_G here is a masslike parameter, and not the mass of the free gluon (which does not exist). In principle it may be solved through an extremisation, which is impossible. Instead, we determine it from a self consistency requirement arising from the sum of the single contractions of the quartic gluon field interaction terms of $\mathcal{T}^{00}$ in Eq. (2.15) given as [8, 19]

$$m_G^2 = \frac{2g^2}{\pi^2} \int \frac{k^2}{\omega(k)}\, F_+(k)\, dk\, . \tag{2.59}$$

We note that a masslike parameter for gluons is needed to deal with QCD plasma as has been seen by others [19, 20] and even with Schwinger Dyson equations or lattice QCD, similar conclusions have been drawn [6, 21]. In fact we chose the ansatz of Eq. (2.58) in view of the above results.

The condensate functions $f(\vec{k})$ and $h(\vec{k})$ are to be determined such that the energy density ϵ_0 in Eq. (2.46) is a minimum. In some cases it is possible to solve for the condensate functions through functional differentiation [10, 13], however in general the dependence of the energy density on the condensate functions is highly nonlinear. We therefore adopt here the alternative procedure of taking a reasonably simple ansatz for the condensate functions by parameterizing the same. We parameterize the gluon condensates as, with $k = |\vec{k}|$,

$$\sinh f(\vec{k}) = A e^{-Bk^2/2}, \tag{2.60}$$

which corresponds to taking a Gaussian distribution for the perturbative gluons in the nonperturbative vacuum [8]. However for the function

$h(\vec{k})$ describing the quark antiquark condensates we take a more general ansatz

$$\tan 2h(\vec{k}) = \frac{A'}{(e^{R^2k^2} - 1)^{1/2}}. \tag{2.61}$$

The ansatz of Ref. [11] corresponds to $A' = 1$ and vanishes when $A' = 0$. A' will be determined through energy minimisation. We note that for free massive fermions $\tan 2h(\vec{k}) = m/|\vec{k}|$, so that the above ansatz corresponds to a momentum dependent mass given as

$$m(k) = \frac{kA'}{(e^{R^2k^2} - 1)^{1/2}}. \tag{2.62}$$

We may add here that such a definition of quark mass is the same as that obtained from the pole of the fermion propagator in a condensate vacuum [9, 22]. In the limit of zero momentum we then have the dynamically generated mass for the quarks given as

$$m_q = \frac{A'}{R}. \tag{2.63}$$

The relationship between the decay constant of pion and the quark condensate function has been discussed in appendix A and is given as

$$\frac{N_\pi}{(2\pi)^{3/2}} \int \sin^2 2h(k) d\vec{k} = \frac{f_\pi (m_\pi)^{1/2}}{\sqrt{6}} \tag{2.64}$$

where $N_\pi \sin 2h(k)$ is the wave function for the pion with

$$N_\pi^{-2} = \int \sin^2 2h(k) d\vec{k}. \tag{2.65}$$

With the ansatz of Eq. (2.61) we then have

$$R = \left(\frac{\sqrt{3}}{\pi f_\pi \sqrt{m_\pi}} \right)^{2/3} \cdot \left[\int \frac{A'^2 x^2 dx}{e^{x^2} + 1 - A'^2} \right]^{1/3}. \tag{2.66}$$

The above equation determines R as a function of A' when f_π is known. This will be used when we extremise energy along with a parallel constraint for SVZ parameter as in Eq. (2.78).

With the above ansatz for the condensate functions the energy density ϵ_0 may now be written in terms of the dimensionless quantities $x = Rk$ and $b = B/R^2$ as

$$\begin{aligned}\epsilon_0(A) &= \frac{1}{R^4}(I_F + I_1(A,b) + I_2(A,b) + I_3(A,b)^2 + I_4(A,b)) \\ &\equiv \frac{1}{R^4}F(A,b),\end{aligned} \tag{2.67}$$

where

$$I_F = \frac{3N_f}{\pi^2}\int x^3 dx\Big[1 - \Big(1 - \frac{A'^2}{e^{x^2} - 1 + A'^2}\Big)^{1/2}\Big], \tag{2.68}$$

$$I_1(A,b) = \frac{4}{\pi^2}\int \omega(x,b)x^2 dx\Big(A^2 e^{-bx^2} - Ae^{-bx^2/2}(1 + A^2 e^{-bx^2})^{1/2}\Big), \tag{2.69}$$

$$I_2(A,b) = \frac{4}{\pi^2}\int \frac{x^4 dx}{\omega(x,b)}\Big(A^2 e^{-bx^2} + Ae^{-bx^2/2}(1 + A^2 e^{-bx^2})^{1/2}\Big) \tag{2.70}$$

$$I_3(A,b) = \frac{2g}{\pi^2}\int \frac{x^2 dx}{\omega(x,b)}\Big(A^2 e^{-bx^2} + Ae^{-bx^2/2}(1 + A^2 e^{-bx^2})^{1/2}\Big) \tag{2.71}$$

and

$$I_4(A,b) = 4 \times (2\pi)^{-6}\int d\vec{x}\frac{G_1(\vec{x}) + G_2(\vec{x})}{x^2 + \phi(x)}. \tag{2.72}$$

with

$$\begin{aligned}G_1(\vec{x}) &= 3g^2\int d\vec{x}'\Big(A^2 e^{-bx'^2} + Ae^{-bx'^2/2}(1 + A^2 e^{-bx'^2})^{1/2}\Big) \\ &\times \Big(A^2 e^{-b(\vec{x}+\vec{x}')^2} + Ae^{-b(\vec{x}+\vec{x}')^2/2}(1 + A^2 e^{-b(\vec{x}+\vec{x}')^2})^{1/2}\Big) \\ &\times \frac{\omega(|\vec{x}+\vec{x}'|)}{\omega(x')} \times \Big(1 + \frac{(\vec{x}\,'^2 + \vec{x}\cdot\vec{x}\,')^2}{x\,'^2(\vec{x}+\vec{x}\,')^2}\Big),\end{aligned} \tag{2.73}$$

$$\begin{aligned}G_2(\vec{x}) &= -\frac{N_f}{2}g^2\int d\vec{x}\,'\Big[\frac{A'^4}{(e^{x'^2} - 1 + A'^2)(e^{(\vec{x}+\vec{x}')^2} - 1 + A'^2)} + \frac{\vec{x}\,'\cdot(\vec{x}+\vec{x}\,')}{|\vec{x}\,'||\vec{x}+\vec{x}\,'|} \\ &\times \Big(1 - \Big(1 - \frac{A'^2}{(e^{x'^2} - 1 + A'^2)}\Big)^{1/2}\Big)\Big(1 - \Big(1 - \frac{A'^2}{(e^{(\vec{x}+\vec{x}')^2} - 1 + A'^2)}\Big)^{1/2}\Big)\Big],\end{aligned} \tag{2.74}$$

and

$$\phi(\vec{x}) = \frac{3g^2}{8\pi^2}\int\frac{dx'}{\omega(\vec{x}')}\Big(A^2e^{-bx'^2}+Ae^{-bx'^2/2}(1+A^2e^{-bx'^2})^{1/2}\Big)$$
$$\times \Big(x^2+x'^2-\frac{(x^2-x'^2)^2}{2xx'}log\Big|\frac{x+x'}{x-x'}\Big|\Big). \quad (2.75)$$

In the above, $\omega(x,b)=(x^2+\mu'^2)^{1/2}$, with $\mu'=m_G R$ being the gluon mass. The self consistency condition for gluon mass m_G in Eq. (2.59) yields

$$\mu^2=\frac{2g^2}{\pi^2}\int\frac{x^2dx}{(x^2+\mu^2)^{1/2}}\Big(A^2e^{-x^2}+Ae^{-x^2/2}(1+A^2e^{-x^2})^{1/2}\Big), \quad (2.76)$$

where $\mu=m_G\sqrt{B}$. Clearly μ' and μ are related as $\mu'=\mu/\sqrt{b}$. For given values of A and A', we first determine $R\equiv R(A\,')$ from Eq. (2.65), and then the parameter b from the SVZ parameter for gluon condensates through the equation

$$\frac{g^2}{4\pi^2}<:G^a_{\mu\nu}G^{a\mu\nu}:>_{vac}=0.012\ GeV^4, \quad (2.77)$$

which explicitly gives

$$\frac{1}{R(A\,')^4}\times\frac{g^2}{\pi^2}(-I_1(A,b)+I_2(A,b)+I_3(A,b)^2-I_4(A,b))=0.012\ GeV^4, \quad (2.78)$$

so that b is a function of A and $A\,'$.

2.4 Results and discussions

We now minimise the energy density ϵ_0 with respect to the parameters A and A', with a self consistent determination for the gluon mass, for different values of the QCD coupling constant α_s. The results are plotted in figures 2.1 and 2.2 .

We plot in curve 1 of Fig. 2.1 ϵ_0 as a function of α_s. We note that for $\alpha_s \le 0.62 = \alpha_c$ the condensate functions vanish, so that for the present ansatz, perturbative vacuum is stable, whereas, for $\alpha_s > \alpha_c$, we have a transition to a nonperturbative vacuum. The energy density becomes more negative with increase of coupling and becomes about -55 MeV/fm^3 when $\alpha_s = 1.4$. In curve 2 of the same figure we have plotted gluon condensate parameter A_{min} for different couplings. In contrast to Ref. [8], near $\alpha_s = \alpha_c$ this parameter has a discontinuity. The ansatz function for gluon condensates includes a correlation length proportional to $\sqrt{B}$ which is plotted in curve 3. The length scale here is of the order of a Fermi which appears to be reasonable in the context of confinement. In curve 4, we have plotted the gluon mass parameter m_G in MeV as a function of coupling. This quantity appears to be constant and is around 300 MeV. Such a dynamically generated gluon mass has been anticipated by Cornwall through approximate investigations of Schwinger Dyson equation in continuum QCD [6] or through Monte Carlo calculations of gluon propagator [21] in lattice QCD. We may note that Cornwall had a gluon mass of the order of 300 MeV to 700 MeV [6] corresponding to $\Lambda \simeq 300$ MeV to 700 MeV, the gluon mass being proportional to Λ. In the present context of $\Lambda \simeq 200$ MeV, the mass appears to be generally consistent with his estimates. The lattice QCD generated mass [21] appears to be around 600 MeV, which is associated as usual , with the inverse of lattice spacing as the momentum or mass scale. Our result appears as an average between the two, when $\Lambda \simeq 200$ MeV. For such gluons QGP signals are considered for relativistic heavy ion collisions [19].

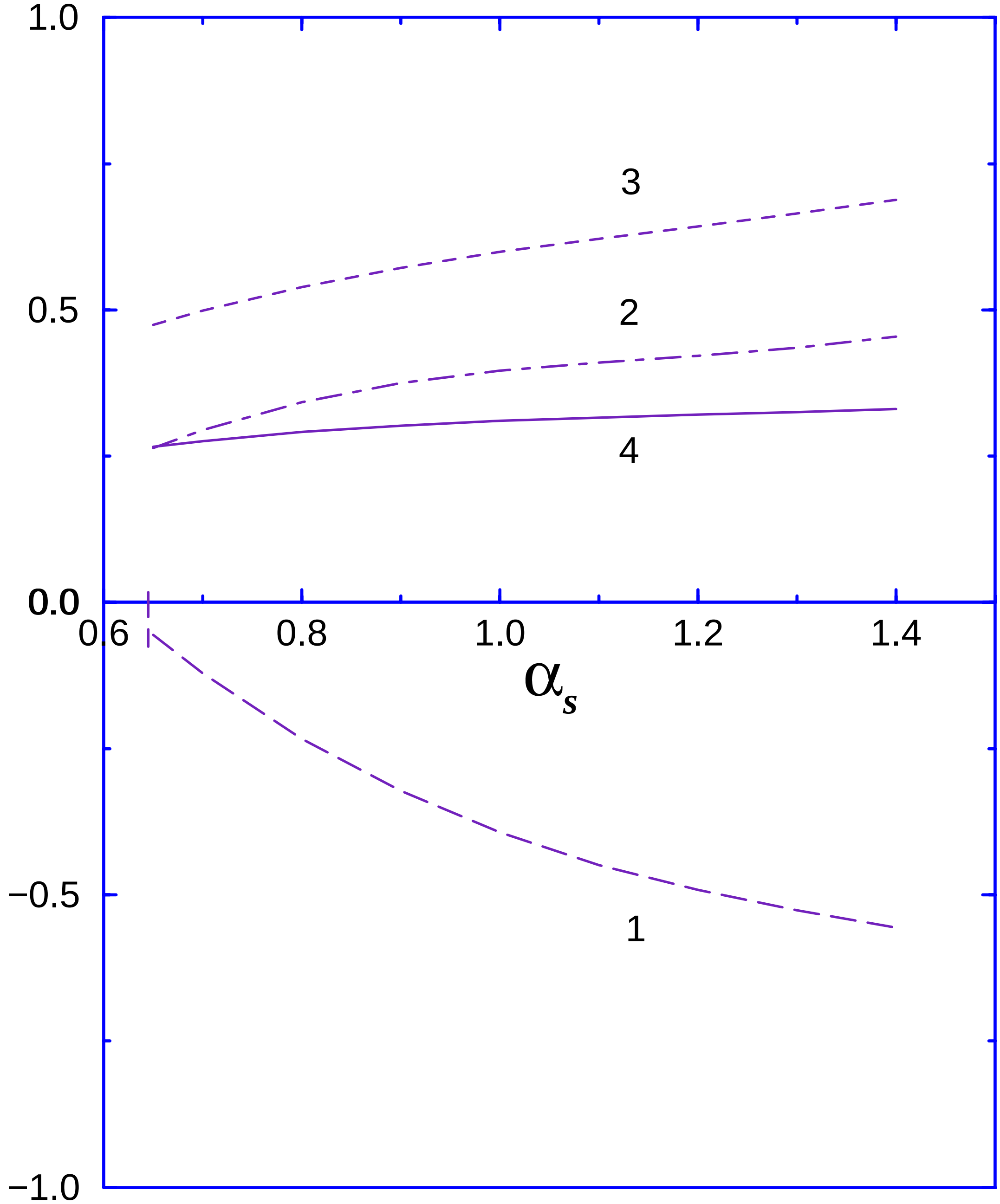

Figure 2.1: In curves 1, 2, 3 and 4, energy density ϵ_0 (in units of 100 MeV/fm^3), A_{min}, $\sqrt{B}$ (in units of fm) and effective gluon mass m_G (in units of GeV) are plotted respectively as functions of strong coupling constant α_s.

In Fig. 2.2 we have plotted different characteristics of the quark condensate functions. In curves 1 and 2 we have plotted the condensate parameters A'_{min} and R respectively. They seem to increase as we approach α_c from above. We have plotted in curve 3 the quark condensate value given as

$$<-\bar{\psi}\psi>^{1/3} = \Big[\frac{12}{(2\pi)^3}\int d\vec{k}\sin 2h(\vec{k})\Big]^{1/3}. \tag{2.79}$$

The condensation effect increases with coupling as expected. In curve 4 we have plotted the dynamically generated quark mass m_q of equation (2.63). We may in fact identify the same as the parallel of constituent mass of quarks as obtained from chiral symmetry breaking. We may remark that for $\alpha_s < \alpha_c$, any nonzero trial functions make ϵ_0 positive, which shows that for the present ansatz the perturbative vacuum is stable.

It shall be amusing to consider the present results in the context of Ref. [11] where *no* extremisation was done. We find that for $\alpha_s = 1.28$, $A'_{min} \simeq 1$, and that the pion charge radius gets correctly reproduced. In fact, with $A' = 1$, we have [11]

$$R_{ch}^2 = R_1^2 + R_2^2, \tag{2.80}$$

where

$$R_1^2 = \frac{N_\pi^2}{4}\int |\vec{\bigtriangledown}\sin 2h(k)|^2 d\vec{k} \tag{2.81}$$

and

$$R_2^2 = \frac{N_\pi^2}{16}\int \sin^2 2h(k)\Big[k^2R^4\tan^2 2h(k) + \frac{4(1-\sin 2h(k))}{k^2}\Big]d\vec{k} \tag{2.82}$$

In the above N_π is given in equation (2.65).

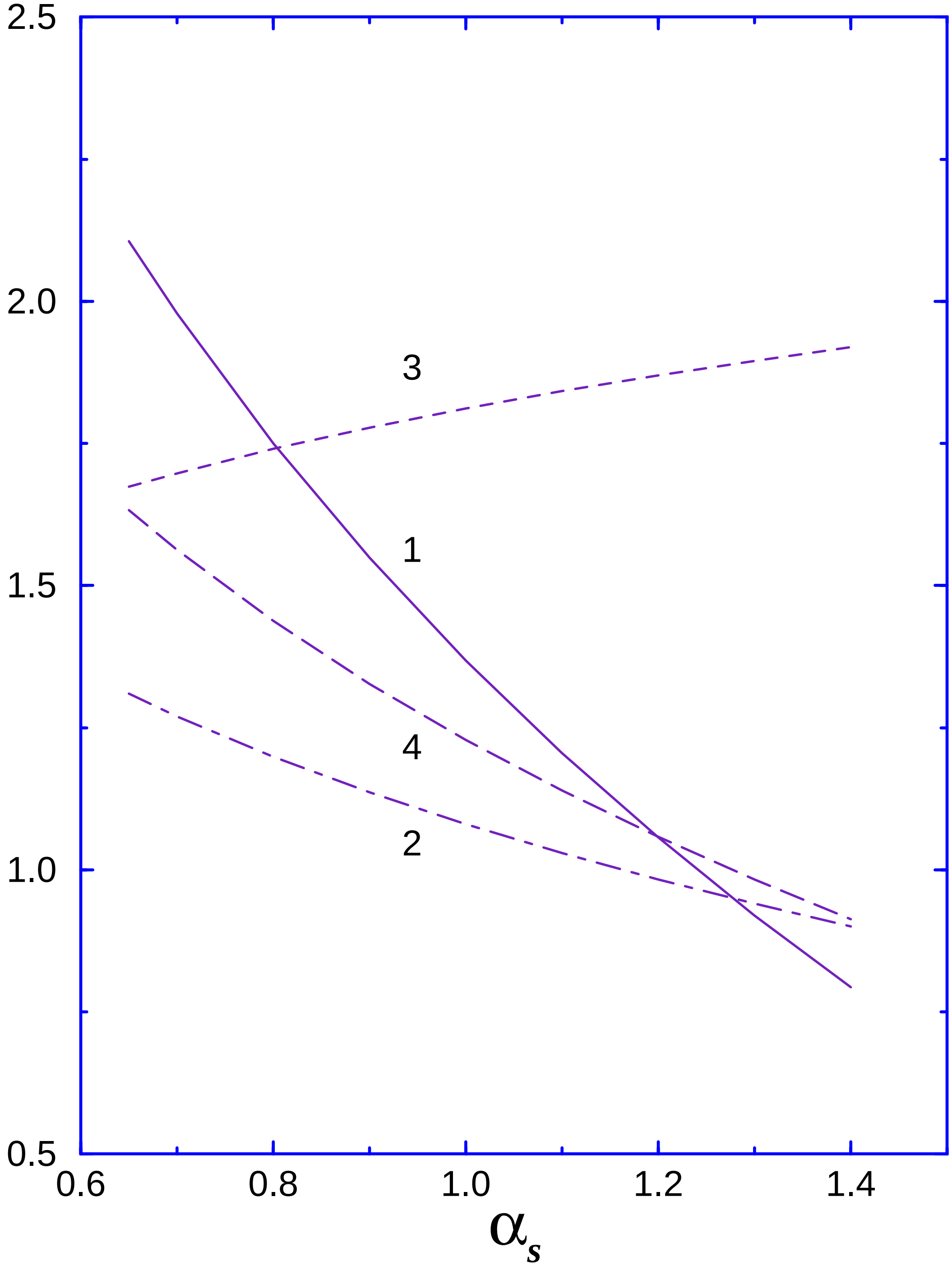

Figure 2.2: In curves 1, 2, 3 and 4, A'_{min}, R (in units of fm), the quark condensate $(<-\bar{\psi}\psi>)^{1/3}$ (in units of 100 MeV) and dynamically generated quark mass m_q (in units of 200 MeV) are plotted respectively as functions of the coupling constant α_s.

With energy minimisation $R \simeq 0.96$ fm, which yields that $R_{ch} \simeq 0.65$ fm, in agreement with the experimental value of 0.66 fm [23]. It is worthwhile to note that the same vacuum structure also yielded other low energy hadron phenomenology [11]. The new feature here is that we are able to obtain the vacuum structure as a function of QCD coupling constant through energy minimisation.

We may note that the average number of perturbative gluons or quarks in such a condensate vacuum is given as

$$N_G = < vac|a_i^a(\vec{z})^\dagger a_i^a(\vec{z})|vac >= 2 \times 8 \times \frac{1}{2\pi^3}\int \sinh^2 f(\vec{k})d\vec{k}. \qquad (2.83)$$

and,

$$\begin{aligned} N_q &= \frac{12}{(2\pi)^3}\int \sin^2 h(\vec{k})d\vec{k} \\ &= \frac{1}{R^3}\frac{3}{\pi^2}\int x^2 dx\Big[1-\Big(1-\frac{A'^2}{e^{x^2}-1+A'^2}\Big)^{1/2}\Big]. \qquad (2.84) \end{aligned}$$

We now take $\alpha_s = 1.28$ which corresponds to the correct charge radius of the pion. This yields that $N_G = 2A^2/(\pi B)^{3/2} \simeq 0.233/\mathrm{fm}^3$ and, $N_q \simeq 0.085/\mathrm{fm}^3$. Also the "bag pressure" is given as $(-\epsilon_0)^{1/4} \equiv B_g^{1/4} = 140$ MeV, which appears to be in general agreement with the phenomenological value of this parameter. We note that $\alpha_s = 1.28$ effectively corresponds to the QCD coupling constant for the vacuum configuration. With "optimised" renormalisation group equations, it has been seen earlier [24] that $\alpha_s(Q)$ *does not* go to infinity as Q decreases below 300 MeV, but freezes to a constant value around unity. Our analysis seems to remind us of a similar situation. The vacuum structure here has also been seen to reproduce the same behaviour for correlators [25] as obtained by others.

The present results are based on (i) the solution of the Gauss's law as in equation (2.20), (ii) introduction of a masslike parameters for gluons as in equation (2.58) with self-consistent determination of the same in (2.59), and (iii) specific ansatz for gluon and quark condensates. In Ref. [13] we had consciously taken an approximate ansatz *different* from the exact solution and then examined the result numerically through a variational parameter. We had then seen that for small couplings, the answer was wrong, whereas for large couplings the answer was almost correct. In analogy our conclusions for QCD could be wrong for small couplings, whereas for larger couplings as here the results could be more reliable. The constraint $\alpha_s > 0.62$ for condensates may thus be the result of the approximation scheme as in Ref. [12]. On the other hand, there could be a genuine dependence on couplings for condensates in vacuum structure as for Nambu Jona-Lasinio model [7, 10]. At present we may not jump to hard conclusions since we feel that our understanding is incomplete, whatever may be the approximation scheme.

We have focussed our attention here on the structure problem. We could therefore as earlier sacrifice explicit Lorentz invariance in favour of simplicity by considering QCD in Coulomb gauge. The vacuum structure we have so determined may thus be reliable in low energy sector. In that spirit we have considered charge radius of the pion as in equation (2.80).

With the structure of QCD vacuum thus fixed from pionic properties and SVZ value we consider quark propagation and study hadron correlation functions in the next chapter.

References

[1] E.V. Shuryak, Rev. Mod. Phys. 65 (1993) 1 ; E.V. Shuryak, *The QCD vacuum, hadrons and the superdense matter*, (World Scientific, Singapore).

[2] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385, 448 and 519 ; R.A. Bertlmann, Acta Physica Austriaca 53 (1981) 305.

[3] G. K. Savvidy, Phys. Lett. 71B (1977) 133 ; S. G. Matinyan and G. K. Savvidy, Nucl. Phys. B134 (1978) 539 ; N. K. Nielsen and P. Olesen, Nucl. Phys. B144 (1978) 376.

[4] D. Schutte, Phys. Rev. D31 (1985) 810.

[5] T. H. Hansson, K. Johnson, C. Peterson, Phys. Rev. D26 (1982) 2069.

[6] J. M. Cornwall, Phys. Rev. D26 (1982) 1453.

[7] Y. Nambu, Phys. Rev. Lett. 4 (1960) 380 ; Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345 ; ibid, 124 (1961) 246 ; J.R. Finger and J.E. Mandula, Nucl. Phys. B199 (1982) 168 ; A. Amer, A. Le Yaouanc, L. Oliver, O. Pene and J.C. Raynal, Phys. Rev. Lett. 50 (1983) 87 ; ibid, Phys. Rev. D28 (1983) 1530 ; S.L. Adler and A.C. Davis, Nucl. Phys. B244 (1984) 469 ; R. Alkofer and P. A. Amundsen,

Nucl. Phys.B306 (1988) 305 ; A.C. Davis and A.M. Matheson, Nucl. Phys. B246 (1984) 203 ; S. Schramm and W. Greiner, Int. Jour. Mod. Phys. E1 (1992) 73.

[8] A. Mishra, H. Mishra, S.P. Misra and S.N. Nayak, Pramana (J. of Phys.) 37 (1991) 59 ; A. Mishra, H. Mishra, S.P. Misra and S.N. Nayak, Z. Phys. C57 (1993) 233 ; A. Mishra, H. Mishra and S.P. Misra, Z. Phys. C59 (1993) 159.

[9] A. Mishra, H. Mishra and S. P. Misra, Z. Phys. C57 (1993) 241.

[10] H.Mishra and S.P. Misra, Phys Rev. D48 (1993) 5376.

[11] A. Mishra and S.P. Misra, Z. Phys. C58 (1993) 325.

[12] A. Mishra, H. Mishra, S.P. Misra, P.K. Panda and Varun Sheel, preprint hep-ph/9404255 (1994), To appear in International Journal of Modern Physics E.

[13] H. Mishra, S.P. Misra and A. Mishra, Int. J. Mod. Phys. A3 (1988) 2331 ; S.P. Misra, Phys. Rev. D35 (1987) 2607.

[14] A. Mishra, H. Mishra, S.P. Misra and S.N. Nayak, Phys. Rev. D44 (1991) 110.

[15] H. Mishra, S.P. Misra, P.K. Panda and B.K. Parida, Int. J. Mod. Phys. E1 (1992) 405 ; P.K. Panda, R. Sahu and S. P. Misra, Phys. Rev. C45 (1993) 2079 ; H. Mishra, S.P. Misra, P.K. Panda and B.K. Parida, Int. J. Mod. Phys. E2 (1993) 547 ; H. Mishra, A. Mishra, S.P. Misra and P.K. Panda, preprint hep-ph/9401344 (to appear in Z. Phys. C).

[16] J. Schwinger, Phys. Rev. 125 (1962) 1043 ; ibid, 127 (1962) 324 ; E. S. Abers and B. W. Lee, Phys. Rep. 9C (1973) 1.

[17] S. P. Misra, Phys. Rev. D18 (1978) 1661, 1673 ; A. Le Youanc et al Phys. Rev. Lett. 54 (1985) 506.

[18] V. N. Gribov, Nucl. Phys. B139 (1978) 1.

[19] Tomas S Biro, Int. J. Mod. Phys. E1 (1992) 39.

[20] S. Nadkarni, Phys. Rev. D33 (1986) 3738 ; Phys. Rev. D34 (1986) 3904.

[21] J.E. Mandula, M. Ogilvie, Phys. Lett. B185 (1987) 127.

[22] M. G. Mitchard, A. C. Davis and A. J. Macfarlane, Nucl. Phys. B325 (1989) 470.

[23] S.R. Amendolia *et al*, Nucl. Phys. B277 (1986) 168.

[24] A.C. Mattingly and P.M. Stevenson, Rice University preprint DE-FG05-92ER40717-7.

[25] Varun Sheel, H. Mishra and J.C. Parikh, preprint PRL-TH/94-36.

Chapter 3

Quark Propagator and Hadronic Correlators in the QCD Vacuum

3.1 Introduction

An interesting quantity to study with the nontrivial structure of QCD vacuum discussed in the previous chapter, is the behaviour of current-current correlators illustrating different physics involved at different spatial distances. This has recently been emphasized in a review by Shuryak [1] and studied through lattice simulations [2]. The basic point is that the correlators can be used to study the interquark interaction — its dependence on distance. In fact they complement bound state hadron properties in the same way that scattering phase shifts provide information about the nucleon-nucleon force complementary to that provided by the properties of the deuteron [2].

We have considered the structure of QCD vacuum with both quark and gluon condensates using a variational ansatz in chapter 2. We shall use here such an explicit construct of QCD vacuum obtained through energy minimisation to evaluate the hadron correlators.

One can study space like correlations between two points, taken at the same instant of time. Alternatively the two points can be considered as two events separated by some interval in imaginary or Euclidean time. We use the former method to calculate equal time, point to point correlation functions for spatially separated hadronic currents, with respect to the vacuum state of chapter 2. Given such an ansatz for the vacuum state, we make no further approximations in the evaluation of the correlators. We compare our results with correlation functions extracted from data (see Ref. [1] and appendix B) and discuss the consequences.

Organisation of this chapter is as follows. We discuss in section 2 the quark propagation in our model of the QCD vacuum. In section 3 we define and calculate meson correlation functions. A similar study is done for baryon correlation functions in section 4. In section 5 we quote the results. Section 6 is devoted to the study of the exceptional case of pseudoscalar correlator and a similar treatment for the nucleon channel is discussed in section 7. Finally we discuss the consequences of the study of hadron correlators in section 8.

3.2 Quark propagation in the vacuum

In the calculation of correlators, quark propagators enter in a direct manner and hence it is instructive to study aspects of the interacting propagator in some detail [3]. The reason for doing this is two folds. We wish to know how it differs from a free massive propagator i.e. how good it is to have a "constituent quark" picture and further to compare and contrast with other approaches such as instanton liquid model or vacuum dominance model based on operator product expansion.

The equal time interacting quark Feynman propagator in the condensate vacuum is given as [4]

$$S_{\alpha\beta}(\vec{x}) = \left\langle \frac{1}{2} \left[\psi^i_\alpha(\vec{x}), \bar{\psi}^i_\beta(0) \right] \right\rangle \tag{3.1}$$

In our model this reduces to

$$S(\vec{x}) = \frac{1}{2}\frac{1}{(2\pi)^3} \int e^{i\vec{k}.\vec{x}} d\vec{k} \left[\sin 2h(\vec{k}) - (\vec{\gamma}\cdot\hat{k}) \ \cos 2h(\vec{k}) \right] \tag{3.2}$$

$$= -\frac{i}{2\pi^2}\frac{\vec{\gamma}\cdot\vec{x}}{x^4} + \frac{1}{(2\pi)^{3/2}}\frac{1}{2R^3} e^{-x^2/(2R^2)} - \frac{i}{(2\pi)^2}\frac{\vec{\gamma}\cdot\vec{x}}{x^2} I(x) \tag{3.3}$$

where

$$I(x) = \int_0^\infty \left(\cos kx - \frac{\sin kx}{kx} \right) \frac{k e^{-R^2k^2}}{1 + (1 - e^{-R^2k^2})^{1/2}} dk \tag{3.4}$$

Clearly, the free massless propagator is given by $S_0(x) = -\frac{i}{2\pi^2}\frac{\vec{\gamma}\cdot\vec{x}}{x^4}$ which can be derived independently or from (Eq. (3.3)) in the limit $R \to \infty$ i.e. in the limit that the condensate functions vanish. It may be useful to note that the free massive propagator is given as

$$S_0(m_q, \vec{x}) = \frac{1}{(2\pi)^2}\frac{m_q^2}{x} \left[K_1(m_q x) - \ i\,\vec{\gamma}\cdot\hat{x} K_2(m_q x) \right] \tag{3.5}$$

where $K_1(m_q x)$ and $K_2(m_q x)$ are the first and second order Bessel functions respectively.

In fact, the present propagator (Eq. 3.3 above) with the condensate structure corresponds to a specific ansatz for the quark self energy (see e.g. Adler and Davis [5], Eq. 2.7 and Alkofer and Amundsen [6], Eq. 3.3 ; see also Shuryak's comments in Ref. [1], pp. 2)

In Fig. 3.1 we plot the two components $Tr[S(\vec{x})]$ and $Tr[(\gamma \cdot \hat{x})S(\vec{x})]$ of the propagator for massless interacting quarks given by Eq. (3.3) corresponding to the chirality flip and non-flip components considered by Shuryak and Verbaarschot [3].

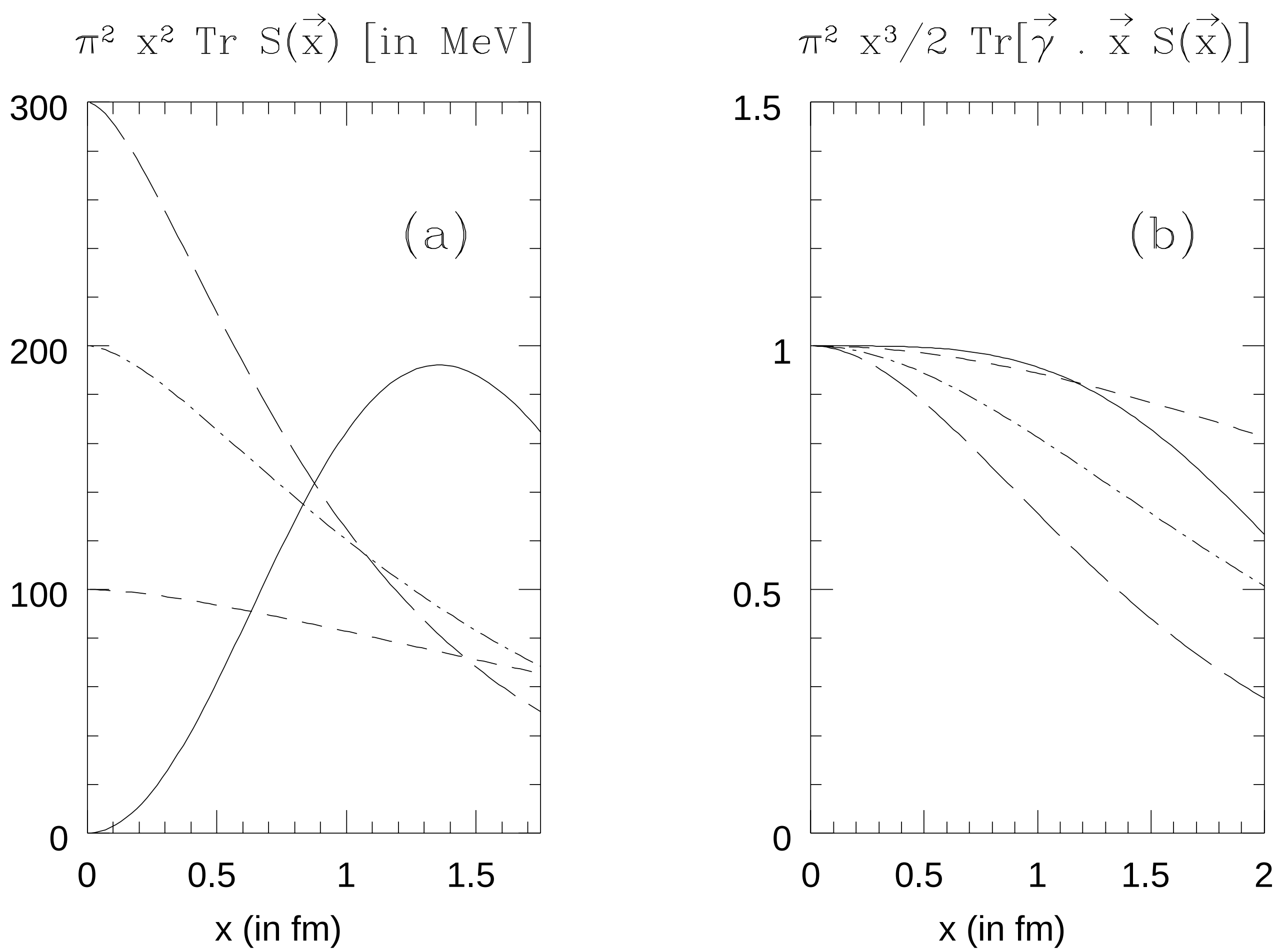

Figure 3.1: The two components of the quark propagator, $Tr[S]$ plotted in (a) and $Tr[(\vec{\gamma} \cdot \hat{x})S]$ in (b) versus the distance x (in fm). The normalisation, indicated in the figure, have been explained in the text. The solid line corresponds to massless quark interacting propagator $S(x)$. The three lines, short dashed, dot-short dashed and long dashed correspond to a massive free propagator with a mass of 100, 200 and 300 MeV, respectively.

The first trace is normalised to the short distance limit of the massive free quark propagator $Tr\ [S_0(m_q,\vec{x})/m_q]$ which from (Eq. (3.5)) is $1/(\pi^2 x^2)$. The second trace is normalised to the free quark propagator which is $Tr\ [(\gamma\cdot\hat{x})S_0(x)] = 2\ i\ /(\pi^2 x^3)$.

To compare with the constituent quark models with an effective constituent mass, we have also plotted the behaviour of free massive quark propagator with masses 100 MeV, 200 MeV and 300MeV. In the chirality flip part, the propagator in the condensate medium starts from zero, consistent with zero quark mass at small distances, attains a maximum value of about 200 MeV at a distance of about 1.3 fm and then falls off gradually. Further the interacting propagator overshoots the massive propagators after about 0.8 fm. These features are qualitatively similar to that of the instanton liquid model of QCD vacuum considered in Ref. [3]

In the chirality non flip part, the interacting propagator starts from 1, again consistent with the behaviour expected from asymptotic freedom, but at larger separation it falls faster than a power law indicative of an effective mass of the order of 150 MeV. Again these features are similar to that of Ref. [3], though quantitatively there are differences.

It may be amusing to consider the leading behaviour of the propagator as $x \to 0$. In this limit, the first term of Eq. (3.2) is given by

$$T_1 = \frac{1}{2}\frac{1}{(2\pi)^3}\int d\vec{k}\sin 2h(\vec{k}) + O(x^2) \tag{3.6}$$

$$= \frac{1}{24} <-\bar{\psi}\psi> + O(x^2) \tag{3.7}$$

where $<-\bar{\psi}\psi>=<-(\bar{u}u+\bar{d}d)>$ for two flavours. Similarly the leading contribution from the second term of Eq. (3.2) is

$$T_2 = -\frac{i}{2\pi^2}\frac{\vec{\gamma}\cdot\vec{x}}{x^4} + O(x) \tag{3.8}$$

Thus in the small x limit the interacting propagator of Eq. (3.2) reduces to

$$S(\vec{x}) = -\frac{i}{2\pi^2}\frac{\vec{\gamma}\cdot\vec{x}}{x^4} + \frac{1}{24} <-\bar{\psi}\psi> \tag{3.9}$$

which is exactly the result of the vacuum dominance model [3] based on operator product expansion.

3.3 Meson correlation functions

Consider a generic meson current of the form

$$J(x) = \bar{\psi}^i_\alpha(x)\Gamma_{\alpha\beta}\psi^j_\beta(x) \tag{3.10}$$

where x is a four vector; α and β are spinor indices; i and j are flavour indices; Γ is a 4×4 matrix $(\mathbb{1}, \gamma_5, \gamma_\mu$ or $\gamma_\mu\gamma_5)$

Because of the homogeneity of the vacuum we define the conjugate current to the above at the origin as,

$$\bar{J}(0) = \bar{\psi}^j_\lambda(0)\Gamma^{'}_{\lambda\delta}\psi^i_\delta(0) \tag{3.11}$$

with $\Gamma^{'} = \gamma_0\Gamma^\dagger\gamma_0$

The meson correlation function for the above currents is defined as,

$$R(x) =< TJ(x)\bar{J}(0) >_{vac} \tag{3.12}$$

From now on we assume that expectation values are always with respect to the nonperturbative vacuum of our model, hence we drop the subscript vac.

Hence with Eqs. (3.10), (3.11) and (3.12) we have

$$R(x) = \Gamma_{\alpha\beta}\Gamma'_{\lambda\delta} < T\bar{\psi}^i_\alpha(x)\psi^j_\beta(x)\bar{\psi}^j_\lambda(0)\psi^i_\delta(0) > \tag{3.13}$$

This reduces to the identity

$$R(x) = \Gamma_{\alpha\beta}\Gamma'_{\lambda\delta} < T\psi^j_\beta(x)\bar{\psi}^j_\lambda(0) >< T\bar{\psi}^i_\alpha(x)\psi^i_\delta(0) > \tag{3.14}$$

The above definition of $R(x)$ is exact since the four point function does not contribute. In fact, in the evaluation of Eq. (3.13) we shall have a sum of two terms. The first is equivalent to the product of two point functions which is Eq. (3.14). The second term arises from contraction of operators at the same spatial point, related to disconnected diagrams and thus can be discarded.

In Eq. (3.14) the first term can be identified as the interacting quark propagator

$$S(x) =< T\psi^j(x)\bar{\psi}^j(0) >$$

It can be shown using the CPT invariance of the vacuum [7] that the second term is given as

$$\begin{aligned} < T\bar{\psi}^i(x)\psi^i(0) > &= -\gamma_5 S(x)\gamma_5 \qquad (3.15) \\ &= -S(-x) \end{aligned}$$

Hence the correlation function of Eq. (3.13) becomes

$$R(x) = -Tr\left[S(x)\Gamma' S(-x)\Gamma\right] \tag{3.16}$$

Similarly the correlator for massless noninteracting quarks can be given as

$$R_0(x) = -Tr\left[S_0(x)\Gamma' S_0(-x)\Gamma\right] \tag{3.17}$$

Our task is now to evaluate the expression (3.16) with the ansatz for QCD vacuum as given in chapter 2. Further we shall be interested in evaluating the equal time point to point correlation functions.

Having obtained the propagators in the earlier section, we can calculate the correlation function, Eq. (3.16) for a generic current of the form as in Eqs. (3.10) and (3.11). For convenience, we will consider the ratio of the physical correlation function to that of massless noninteracting quarks

$$\frac{R(x)}{R_0(x)} = \left(1 + \frac{1}{2}x^2 I(x)\right)^2 + \frac{\pi}{8}\frac{x^6}{R^6}e^{-x^2/R^2}\frac{x^2 Tr\left[\Gamma'\Gamma\right]}{x^i x^j Tr\left[\gamma^i \Gamma' \gamma^j \Gamma\right]} \tag{3.18}$$

which is then evaluated in different channels with the corresponding Dirac structure for the currents. The representations of the Gamma matrices used by us and some trace theorems needed for the above calculations can be found in appendix C.

3.4 Baryon correlation functions

We take the standard Ioffe current for the nucleon [8]

$$J^N(x) = \epsilon_{abc}\left[\tilde{u}^a(x) C \gamma_\mu u^b(x)\right]\gamma^\mu \gamma_5 d^c(x) \tag{3.19}$$

where x is a four vector, μ is a spinor index, a,b and c are colour indices, ϵ is the Levi-Civita symbol and u and d are the quark field operators for the u and d quark, $\tilde{u}$ being the transpose of u. C is the charge conjugation matrix.

Because of the homogeneity of the vacuum, we define the conjugate current to the above at the origin as,

$$\bar{J}^N(0) = \epsilon_{edf}\left[\bar{u}^d(x)\gamma_\rho C^\dagger \bar{\tilde{u}}^e(x)\right]\bar{d}^f(x)\gamma_5\gamma^\rho \tag{3.20}$$

The nucleon correlation function for the above currents is defined as [2],

$$R(x) = \frac{1}{4} Tr \left[< T J^N(x) \bar{J}^N(0) >_{vac} \, x_\nu \gamma_\nu \right] \tag{3.21}$$

With the usual contractions as described in the calculation of meson correlation functions (see Eq.(3.13) to Eq.(3.16)), the vacuum expectation value above reduces to

$$\begin{aligned} R'(x) &= < T J^N(x) \bar{J}^N(0) > \\ &= 2\epsilon_{abc}\epsilon_{edf} \left(C\gamma_\mu S^{bd}(x)\gamma_\rho C^\dagger \tilde{S}^{ae}(x) \right) \gamma^\mu \gamma_5 S^{cf}(x) \gamma_5 \gamma^\rho \end{aligned} \tag{3.22}$$

where the factor 2 is the number of possible contractions. The tilde over S(x) denotes tranpose as defined earlier.

Further, using $\gamma_5 S(x) \gamma_5 = S(-x)$, Eq. (3.22) can be written as,

$$\begin{aligned} R'(x) &= 12 \; Tr \left(\gamma_\mu S(x) \gamma_\rho C^\dagger \tilde{S}(x) C \right) \gamma^\mu S(-x) \gamma^\rho \\ &= 12 \; Tr \left(\gamma_\mu S(x) \gamma_\rho S(-x) \right) \gamma^\mu S(-x) \gamma^\rho \end{aligned} \tag{3.23}$$

Similarly the correlator for massless noninteracting quarks can be obtained by calculating the vacuum expectation value $R'_0(x)$ and then taking its trace with $\vec{\gamma} \cdot \vec{x}$ as in Eq. (3.21).

$$\begin{aligned} R'_0(x) &= 12 \; Tr \left(\gamma_\mu S_0(x) \gamma_\rho C^\dagger \tilde{S}_0(x) C \right) \gamma^\mu S_0(-x) \gamma^\rho \\ &= 12 \; Tr \left(\gamma_\mu S_0(x) \gamma_\rho S_0(-x) \right) \gamma^\mu S_0(-x) \gamma^\rho \end{aligned} \tag{3.24}$$

We substitute our expression for the equal time propagators $S(x)$ and $S_0(x)$ to calculate the expectation values Eq. (3.23) and Eq. (3.24), which after taking trace with $\vec{\gamma} \cdot \vec{x}$ (Eq. (3.21)), gives us the the interacting and free correlators R(x) and $R_0(x)$ respectively. For convenience, we will consider the ratio

$$\frac{R(x)}{R_0(x)} = \left[1 + \frac{1}{2}x^2 I(x)\right]^3 + \frac{\pi}{16}\frac{x^6}{R^6}e^{-x^2/R^2}\left[1 + \frac{1}{2}x^2 I(x)\right] \tag{3.25}$$

The standard current having quantum numbers of the Delta particle is [8]

$$J_\mu^\Delta(x) = \epsilon_{abc}\left[\tilde{u}^a(x)C\gamma_\mu u^b(x)\right]u^c(x) \tag{3.26}$$

In a completely analogous manner to the nucleon channel, the Delta correlation function is defined as

$$R(x) = \frac{1}{4}Tr\left[< TJ^\Delta(x)\bar{J}^\Delta(0) >_{vac} x_\nu\gamma_\nu\right] \tag{3.27}$$

where,

$$< TJ^\Delta(x)\bar{J}^\Delta(0) >_{vac} = 18\; Tr\left(\gamma_\mu S(x)\gamma^\mu S(-x)\right)S(x) \tag{3.28}$$

The ratio of the physical correlator to the free one is

$$\frac{R(x)}{R_0(x)} = \left[1 + \frac{1}{2}x^2 I(x)\right]^3 + \frac{\pi}{4}\frac{x^6}{R^6}e^{-x^2/R^2}\left[1 + \frac{1}{2}x^2 I(x)\right] \tag{3.29}$$

3.5 Results

We have studied the ratio of the physical correlators to that of massless non-interacting quarks for four meson channels and two baryon channels. In each channel we associate the current with a physical hadron having quantum numbers identical to that of the current.

The results for the mesons are shown in Fig 3.2 and Table 3.1 .

We may notice some general features and relationships among the correlators. The pseudoscalar correlator is always greater than the scalar correlator and vector correlator is greater than the axial vector correlator. We may emphasize here that these relations are rather general in the sense that they do not depend on the *explicit* form of the condensate function and arise due to the different Dirac structure of the currents which is reflected in the generic expression for the correlation functions as in Eq. (3.18). The behaviour of each channel is consistent with that predicted by phenomenology except in the pseudoscalar case where the ratio is two orders of magnitude smaller (for $x \sim 0.5$ fm) compared to that from phenomenology. We examine this in the next chapter.

The results for the baryons are plotted in Fig 3.3 . We notice that the delta correlator is always greater than the nucleon correlator. The behaviour of delta channel is consistent with the predicted behaviour [9] but in case of the nucleon channel it does not rise at all [9]. We examine this in chapter 4.

TABLE 3.1. Meson currents and correlation functions

CHANNEL	CURRENT	PARTICLE (J^P,MASS in MeV)	CORRELATOR [a] $\left[\dfrac{R(x)}{R_0(x)}\right]$
Pseudoscalar	$J^p = \bar{u}\gamma_5 d$	$\pi^0(0^-, 135)$	$\left[1+\frac{1}{2}x^2 I(x)\right]^2 + \frac{\pi}{8}\frac{x^6}{R^6}e^{-x^2/R^2}$
Scalar	$J^s = \bar{u}d$	$none(0^+)$	$\left[1+\frac{1}{2}x^2 I(x)\right]^2 - \frac{\pi}{8}\frac{x^6}{R^6}e^{-x^2/R^2}$
Vector	$J_\mu = \bar{u}\gamma_\mu d$	$\rho^\pm(1^-, 770)$	$\left[1+\frac{1}{2}x^2 I(x)\right]^2 + \frac{\pi}{4}\frac{x^6}{R^6}e^{-x^2/R^2}$
Axial	$J^5_\mu = \bar{u}\gamma_\mu\gamma_5 d$	$A_1(1^+, 1100)$	$\left[1+\frac{1}{2}x^2 I(x)\right]^2 - \frac{\pi}{4}\frac{x^6}{R^6}e^{-x^2/R^2}$

[a]The integral $I(x) = \displaystyle\int_0^\infty \left(\cos kx - \frac{\sin kx}{kx}\right)\frac{ke^{-R^2k^2}}{1+(1-e^{-R^2k^2})^{1/2}}dk$

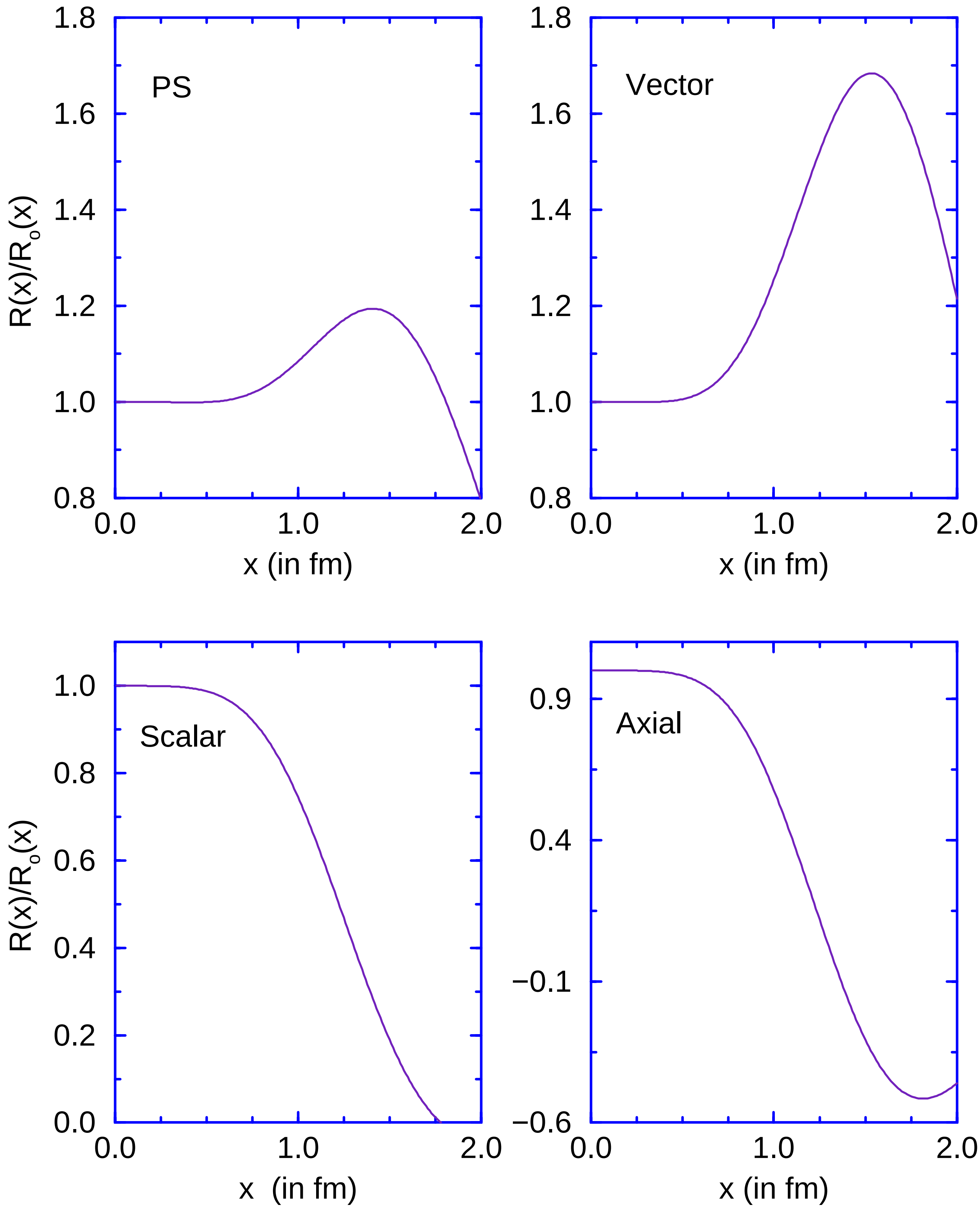

Figure 3.2: The ratio of the meson correlation functions in QCD vacuum to the correlation functions for noninteracting massless quarks, $\frac{R(x)}{R_0(x)}$, plotted vs. distance x (in fm).

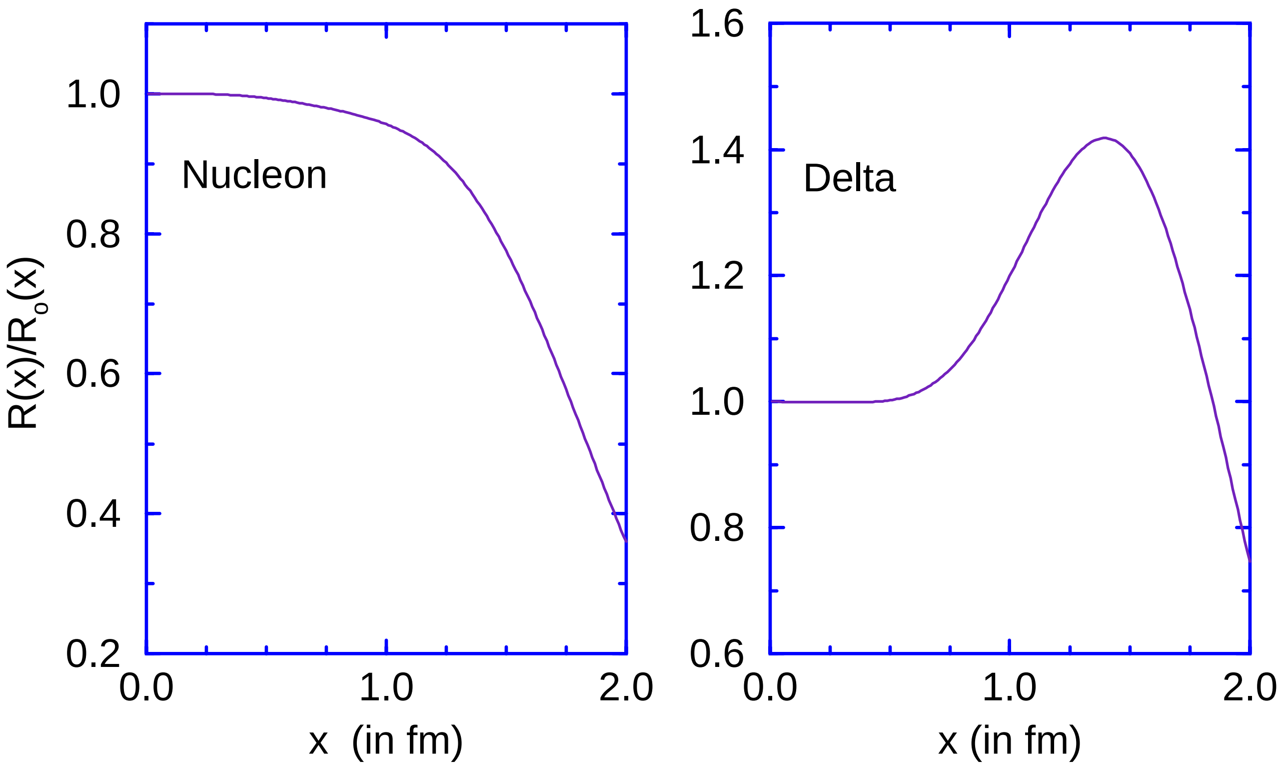

Figure 3.3: The ratio of the baryon correlation functions to the correlation functions for noninteracting massless quarks, $\frac{R(x)}{R_0(x)}$, plotted vs. distance x (in fm).

3.6 Saturation of the pseudoscalar channel

The explicit evaluation of the pseudoscalar correlator gives, using Eq. (3.18)

$$\frac{R(x)}{R_0(x)} = \left[1 + \frac{1}{2}x^2 I(x)\right]^2 + \frac{\pi}{8}\frac{x^6}{R^6}e^{-x^2/R^2} \tag{3.30}$$

which may also be read off from column 4 of Table 3.1 . This is plotted as a function of x in (Fig. 3.2). As may be seen from (Fig. 3.2) this ratio has a maximum of ~ 1.2 at $x \sim 1.3$ fm . Phenomenologically [1] the peak is at ~ 100 at $x \sim 0.5$. In order to compare our results with other calculations we evaluate the same correlator approximately by saturating intermediate states with one pion states.

With our definition of the correlation function (Eq. (3.12)) we have

$$\begin{aligned} R(x) &= < T J^p(x) \bar{J}^p(0) > \\ &= \frac{1}{2}\left(< J^p(\vec{x}) \bar{J}^p(0) > + < \bar{J}^p(0) J^p(\vec{x}) >\right) \end{aligned} \tag{3.31}$$

which comes from the definition of time ordered product for equal time algebra.

We now insert a complete set of intermediate states between the two currents but retain only the one pion state in the sum for the four point function. Thus,

$$\begin{aligned} R(x) &= \frac{1}{2}\int (< vac \mid J^p(\vec{x}) \mid \pi^a(\vec{p}) >< \pi^a(\vec{p}) \mid \bar{J}^p(0) \mid vac > \\ &\quad + < vac \mid \bar{J}^p(0) \mid \pi^a(\vec{p}) >< \pi^a(\vec{p}) \mid J^p(\vec{x}) \mid vac >) d\vec{p} \end{aligned} \tag{3.32}$$

Using translational invariance and the fact that for the pseudoscalar current $J^p = \bar{u}\gamma_5 d$ and $\bar{J}^p = -J^p$, the correlator may be written as

$$R(x) = \frac{1}{2}\int < vac \mid J^p(0) \mid \pi^a(\vec{p}) >< \pi^a(\vec{p}) \mid \bar{J}^p(0) \mid vac > \left(e^{i\vec{p}.\vec{x}} + e^{-i\vec{p}.\vec{x}}\right) d\vec{p} \tag{3.33}$$

We may evaluate the above matrix element using the definition of the pion decay constant given as [10]

$$< vac \mid J_5^{\mu a}(x) \mid \pi^a(p) >= \frac{i f_\pi p^\mu}{(2\pi)^{3/2}(2p_0)^{1/2}} e^{ip\cdot x} \tag{3.34}$$

where $J_5^{\mu a} = [\bar{\psi}\gamma^\mu\gamma^5\tau^a\psi]$ is the axial current.

It can be shown [11] that the divergence of the axial current gives the pseudoscalar current

$$\partial_\mu[\bar{\psi}\gamma^\mu\gamma^5\tau^a\psi] = 2\ i\ m_q[\bar{\psi}\gamma^5\tau^a\psi] \tag{3.35}$$

where m_q is the current quark mass. Thus taking divergence of both sides of Eq. (3.34) and using Eq. (3.35) we get,

$$2\ m_q < vac \mid iJ^{pa}(x) \mid \pi^a(p) >= \frac{-f_\pi m_\pi^2}{(2\pi)^{3/2}(2p_0)^{1/2}} e^{ip\cdot x} \tag{3.36}$$

where we have used $p^2 = m_\pi^2$.

In an earlier paper [12] within our vacuum model and using the fact that pion is an approximate Goldstone mode it was demonstrated that saturating with pion states, gives the familiar current algebra result

$$m_\pi^2 = -\frac{m_q}{f_\pi^2} <\bar{\psi}\psi> \tag{3.37}$$

With this result we eliminate quark mass m_q in Eq. (3.36) in favour of the quark condensate to get the relation

$$< vac \mid J^{pa}(\vec{x}) \mid \pi^a(\vec{p}) >= \frac{i}{2(2\pi)^{3/2}(2p_0)^{1/2}} \frac{<\bar{\psi}\psi>}{f_\pi} e^{i\vec{p}\cdot\vec{x}} \tag{3.38}$$

The expression for the pseudoscalar correlator now becomes

$$R(x) = \frac{1}{64\pi^3}\left(\frac{<\bar{\psi}\psi>}{f_\pi}\right)^2 \int \frac{1}{(p^2+m_\pi^2)^{1/2}}\left(e^{i\vec{p}.\vec{x}} + e^{-i\vec{p}.\vec{x}}\right) d\vec{p} \tag{3.39}$$

The above integral can be evaluated using the standard integral [13]

$$\int_0^\infty p(p^2+\beta^2)^{\nu-1/2} sin(\alpha p)dp = \frac{\beta}{\sqrt{\pi}}\left(\frac{2\beta}{\alpha}\right)^\nu cos(\nu\pi)\Gamma(\nu+\frac{1}{2})K_{\nu+1}(\alpha\beta)$$

for $\alpha > 0, Re\beta > 0$ and in the limit $\nu \to 0$. We then finally get for the correlator (using saturation of pion states)

$$R(x) = \frac{1}{16\pi^2}\left(\frac{<\bar{\psi}\psi>}{f_\pi}\right)^2 \frac{m_\pi K_1(m_\pi x)}{x} \tag{3.40}$$

The correlator for free massless quarks as calculated in the earlier section for the pseudoscalar current is

$$R_0(x) = \frac{1}{\pi^4 x^6} \tag{3.41}$$

Hence the normalised correlation function can be written as

$$\frac{R(x)}{R_0(x)} = \frac{\pi^2}{16}\left(\frac{<\bar{\psi}\psi>}{f_\pi}\right)^2 x^5 m_\pi K_1(m_\pi x) \tag{3.42}$$

We have plotted in Fig. 3.4 this ratio for our value of $<-\bar{\psi}\psi>$ and that used by Shuryak [1, 14].

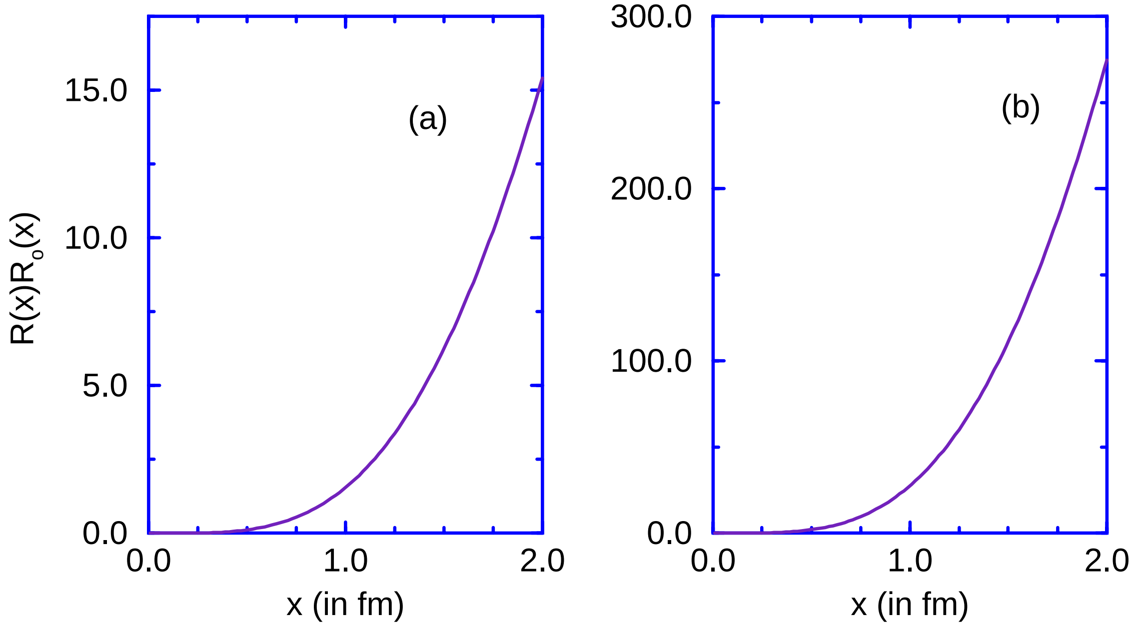

Figure 3.4: The pseudoscalar correlator calculated by saturation with intermediate pion states plotted for our value of $<-\bar{\psi}\psi>=$ (190 MeV)^3 in (a) and that of Shuryak $<-\bar{\psi}\psi>=$ (307.4 MeV)^3 in (b).

Note that our value of $<-\bar{\psi}\psi>$ is an output of the variational calculation of chapter 2 consistent with low energy hadronic properties. We thus observe that the approximate calculation of the pseudoscalar correlator due to saturation with one pion states (Fig. 3.4 (a)) yields higher values ($\simeq$ 15 times more) as compared to the calculations without saturation as an approximation (Fig. 3.2). Thus the fermionic condensate model for QCD vacuum studied in chapter 2, does not give as high values for the pseudoscalar correlator as required by phenomenological results. The value we have used for $<-\bar{\psi}\psi>\simeq$ (190 MeV)^3 is smaller than Shuryak's value of (307.4 MeV)^3 [1] which appears in the parameterisation of the physical spectral density through the coupling constant [14]. With his value of $<-\bar{\psi}\psi>$ the ratio $R(x)/R_0(x)$ is shown in (Fig. 3.4 (b)) which agrees with phenomenology.

3.7 Saturation of the nucleon channel

As in the case of the pseudoscalar channel we evaluate the nucleon correlator approximately by saturating intermediate states with one nucleon states.

With our definition of the correlation function (Eq. (3.21)) we have

$$\begin{aligned} R(x) &= \frac{1}{4} Tr\left[< TJ^N(x)\bar{J}^N(0) > x_\nu\gamma_\nu\right] \\ &= \frac{1}{4}\,\frac{1}{2} Tr\left[(< J^N(\vec{x})\bar{J}^N(0) > + < \bar{J}^N(0)J^N(\vec{x}) >)x_\nu\gamma_\nu\right] \end{aligned} \quad (3.43)$$

which comes from the definition of time ordered product for equal time algebra.

We now insert a complete set of intermediate states between the two currents but retain only the one nucleon state in the sum for the four point function.

[1] Our definition of the condensate value differs from the standard one by a factor of $2^{1/2}$

$$R'(x) = \int (< vac \mid J^N(\vec{x}) \mid N(\vec{p}) >< N(\vec{p}) \mid \bar{J}^N(0) \mid vac > + < vac \mid \bar{J}^N(0) \mid N(\vec{p}) >< N(\vec{p}) \mid J^N(\vec{x}) \mid vac >) d^4p \; \delta(p^2 - M_N^2) \tag{3.44}$$

Now $< vac \mid \bar{J}^N(0) \mid N(\vec{p}) >=< N(\vec{p}) \mid J^N(0) \mid vac >$. From the field expansions the nucleon current will contain particle annihilation and anti particle creation operators. So $J^N(0) \mid vac >$ will give a state with anti particles. Since $< N(\vec{p}) \mid$ is a state with particles, these two states are orthogonal and hence the above expectation value is zero. Hence we are left with one term,

$$R'(x) = \int \Big(< vac \mid J^N(\vec{x}) \mid N(\vec{p}) >< N(\vec{p}) \mid \bar{J}^N(0) \mid vac >\Big) \, d^4p \; \delta(p^2 - M_N^2) \tag{3.45}$$

We may evaluate the above matrix element using the definition of the coupling of the nucleon current to the vacuum

$$< vac \mid J^N(0) \mid N(p) >= \lambda^N \sqrt{\frac{2M_N}{(2\pi)^3}} u(p) e^{ip\cdot x} \tag{3.46}$$

with the normalisation $u(p)\;\bar{u}(p) = (p - M_N)/2M_N$ and then carry out the p_0 integration.

$$\begin{aligned} R'(x) &= \int \frac{\lambda_N^2}{(2\pi)^3}(p - M_N) e^{ip\,x} d^4p \; \delta(p^2 - M_N^2) \\ &= -\int \frac{d^3p}{(2\pi)^3} \lambda_N^2 \frac{\vec{\gamma}\cdot\vec{p}}{\sqrt{p^2 + M_N^2}} e^{i\vec{p}.\vec{x}} \end{aligned} \tag{3.47}$$

where we have retained those terms in $R'(x)$ that will survive on taking trace with $\vec{\gamma}\cdot\vec{x}$.

The above integral can be evaluated as follows [13],

$$\begin{aligned} R'(x) &= \frac{\lambda_N^2}{(2\pi)^3}\, i\, \gamma^i \partial_i \int d^3p \frac{e^{i\vec{p}.\vec{x}}}{\sqrt{p^2+M_N^2}} \\ &= \frac{i}{2\pi^2}\, M_N\, \lambda_N^2\, \gamma^i \partial_i \left[\frac{K_1(M_N x)}{x}\right] \\ &= \frac{i}{2\pi^2}\, M_N\, \lambda_N^2 \frac{\vec{\gamma}\cdot\vec{x}}{x^3}\left[M_N\, x\, K_1'(M_N x) - K_1(M_N x)\right] \end{aligned} \tag{3.48}$$

Taking the trace of the above with $(\vec{\gamma}\cdot\vec{x})$ as defined in Eq. (3.43) we finally get the nucleon correlation function as

$$\begin{aligned} R(x) &= \frac{i}{4\pi^2}\, \lambda_N^2\, \frac{M_N}{x}\left[K_1(M_N x) - M_N x K_1'(M_N x)\right] \\ &= \frac{i}{4\pi^2}\, \lambda_N^2\, M_N^2 K_2(M_N x) \end{aligned} \tag{3.49}$$

As usual this is normalised to the free nucleon correlator $R_0(x) = \frac{24\, i}{\pi^6 x^8}$ and the ratio

$$\frac{R(x)}{R_0(x)} = \frac{\pi^4}{96}\lambda_N^2\, M_N^2\, x^8\, K_2(M_N x) \tag{3.50}$$

is plotted in Fig. 3.5 .

We thus observe that the approximate calculation of the nucleon correlator due to saturation with one nucleon states (Fig. 3.5) shows the behaviour as predicted.

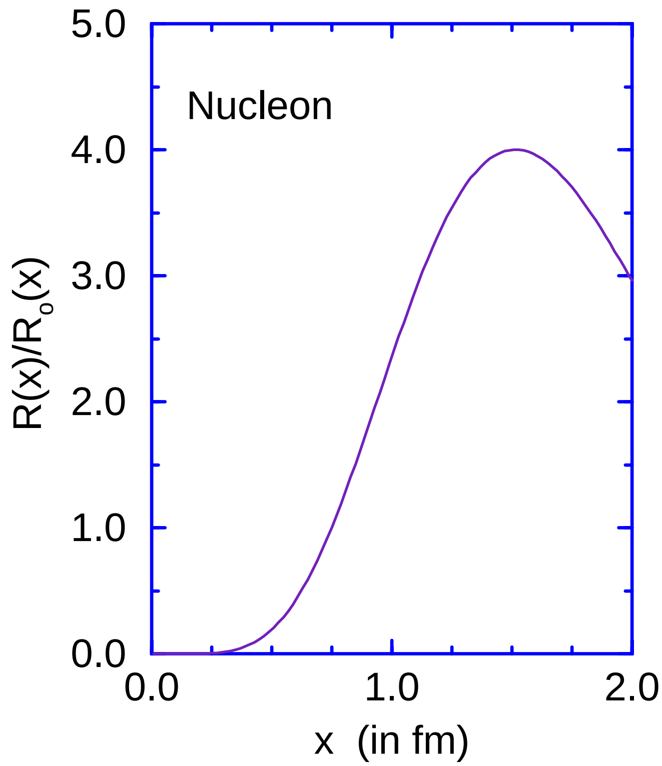

Figure 3.5: The nucleon correlator calculated by saturation with intermediate nucleon states, plotted vs distance x (in fm).

3.8 Summary and discussions

We have evaluated hadronic correlators using a variational construct for the QCD vacuum. The vector, axial and scalar channels show qualitative agreement with phenomenological results [1]. The delta channel also follows the predicted curve. However the pseudoscalar and nucleon channels show large departures.

We have also shown that the quark propagator in our construct for the QCD vacuum is almost identical to that in the instanton model. If one believes that the correlation functions are just the square (or cube) for the meson (baryon) channels [15, 9], then it is not clear why the instanton model should give remarkably good agreement compared to our model.

Following Shuryak [1, 14], we also see that using current algebra approach the pseudoscalar correlator rises sharply with spatial separation by saturation of the correlator with intermediate states. Let us recall that the current algebra result also follows from the approximation of saturating by one pion states in the normalisation of the pion state [12].

It might appear that by suitably changing the value of $<-\bar{\psi}\psi>$ in our calculations without saturation one might be able to reproduce all the phenomenological results. Actually we find that it is not so. In fact, it adversely affects the correlators in the other channel which can be seen in the expressions given in column 4 of Table 3.1 .

In view of these findings, it is not clear whether saturation of intermediate states by one pion states only in the evaluation of the correlator, is sufficiently well justified. This was indicative of the fact that some crucial physics is missing in the construct of the vacuum state in chapter 2. We discuss this in the next chapter.

References

[1] E.V. Shuryak, Rev. Mod. Phys. 65 (1993) 1.

[2] M.-C. Chu, J. M. Grandy, S. Huang and J. W. Negele, Phys. Rev. D 48 (1993) 3340 ; ibid, Phys. Rev. D 49 (1994) 6039.

[3] E. V. Shuryak and J. J. M. Verbaarschot, Nucl. Phys. B 410 (1993) 37.

[4] M. G. Mitchard, A. C. Davis and A. J. Macfarlane, Nucl. Phys. B 325 (1989) 470.

[5] S. L. Adler and and A. C. Davis, Nuc. Phys. B244 (1984) 469.

[6] R. Alkofer and P. A. Amundsen, Nuc. Phys. B306 (1988) 305.

[7] See e.g. in J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965) p. 155 and 213.

[8] B. L. Ioffe, Nucl. Phys. B188 (1981) 317.

[9] T. Schäfer, E. V. Shuryak and J. J. M. Verbaarschot, Nucl. Phys. B412 (1994) 143.

[10] T. D. Lee, *Particle Physics and introduction to Field Theory* (Harwood Academic, 1982) p. 791.

[11] J. J. Sakurai, *Currents and Mesons* (Chicago Lectures in Physics, 1969) p. 18.

[12] A. Mishra, H. Mishra and S. P. Misra, Z. Phys. C57 (1993) 241 ; H.Mishra and S.P. Misra, Phys Rev. D 48 (1993) 5376 ; A. Mishra and S.P. Misra, Z. Phys. C58 (1993) 325.

[13] *Table of Integrals, Series and Products* edited by I. S. Gradshteyn and I.M. Ryzhik (Academic Press,1980) p. 427.

[14] E.V. Shuryak, Nucl. Phys. B 319 (1989) 541.

[15] E. V. Shuryak and J. J. M. Verbaarschot, Nucl. Phys. B 410 (1993) 55.

Chapter 4

Four Point Structure in QCD Vacuum

4.1 Introduction

Correlation functions of hadronic currents were calculated in chapter 3 with respect to the vacuum state proposed in chapter 2. It turned out that without any approximation, the condensate structure uniquely determined the interacting quark propagator and the mesonic (baryonic) correlators are essentially squares (cubes) of the propagator. With a Gaussian ansatz for the quark condensate function $h(\vec{k})$, namely $\sin 2h(\vec{k}) = exp(-R^2k^2/2)$ we find that the behaviour of the quark propagator is similar to that obtained by Shuryak and Verbaarschott [1]. Further, the mesonic correlation functions were also qualitatively similar to phenomenological correlation functions in all channels except for the pseudoscalar (PS) channel. In this channel we did not get the strong attraction at intermediate ranges seen in the data. All these results were dependent very weakly on the choice of the functional form of $h(\vec{k})$.

In view of this outcome, it is obvious that some crucial physics is missing from our model, and hence the vacuum structure considered by us has to be supplemented by additional effects. In our framework, this means that the quark propagators alone do not determine the hadronic correlators. This implies that there ought to be explicit contribution arising from irreducible four point structure of the vacuum. Alternatively we can describe the irreducible four point vacuum structure as a manifestation of condensate fluctuations. This could be an effective way of including explicit gluon condensates [3] which otherwise cannot be done in our model.

In this chapter, we adopt a phenomenological approach to determine the salient features of the QCD vacuum. More precisely, we use phenomenological results of equal time, point to point spatial ground state correlation functions of hadronic currents to guide us towards a “true” structure of QCD vacuum.

4.2 Four point structure

In order to proceed further with this idea of four point vacuum structure we suggest that the actual ground state of QCD could be more complicated than the one defined in chapter 2 (see Eq. (2.29))

$$|vac>= \exp{({B_F}^\dagger - B_F)} \exp{({B_G}^\dagger - B_G)} |0> \tag{4.1}$$

where $|0>$ is the perturbative vacuum and ${B_F}^\dagger$ and ${B_G}^\dagger$ are the Bogoliubov operators corresponding to creation of quark antiquark pairs and gluon pairs respectively. In analogy to the above, we define the “new improved” QCD ground state including the four point vacuum structure,

$|\Omega>$ through a unitary transformation on the $|vac>$ state (Eq. (4.1))

$$|\Omega>= U_\Sigma|vac> \tag{4.2}$$

where

$$U_\Sigma = \exp\left(B_\Sigma{}^\dagger - B_\Sigma\right) \tag{4.3}$$

$$B_\Sigma^\dagger = \int g(y-z)\Sigma_{\alpha\beta}(y)\Sigma_{\beta\alpha}(z)d\vec{y}d\vec{z} \tag{4.4}$$

The operator $B_\Sigma^\dagger$ is a pair creation operator similar to $B_F^\dagger$ and $B_G^\dagger$, with the creation part of the operators Σ contributing in Eq. (4.4) above.

We suggest that the normal ordered operators with respect to $|vac>$ do not annihilate the actual ground state of QCD $|\Omega>$. In this case, we have a more general equation

$$T\bar{\psi}_\alpha(\vec{x})\psi_\beta(\vec{x})\bar{\psi}_\gamma(0)\psi_\delta(0) = S_{\beta\gamma}(\vec{x})S_{\delta\alpha}(-\vec{x}) + :\bar{\psi}_\alpha(\vec{x})\psi_\beta(\vec{x})\bar{\psi}_\gamma(0)\psi_\delta(0): \tag{4.5}$$

where the : denotes normal ordering with respect to the vacuum of Eq.(4.1). To include the effect of four point structure we may write

$$:\bar{\psi}_\alpha(\vec{x})\psi_\beta(\vec{x})\bar{\psi}_\gamma(0)\psi_\delta(0):= \Sigma_{\beta\gamma}(\vec{x})\Sigma_{\delta\alpha}(-\vec{x}) \tag{4.6}$$

such that $<\Omega|\Sigma|\Omega>= 0$ but $<\Omega|\Sigma\Sigma|\Omega>\neq 0$. With such a structure for the ground state of QCD the correlator takes the form

$$R_M(\vec{x}) = Tr\left[S(\vec{x})\Gamma' S(-\vec{x})\Gamma\right] + Tr\left[\Sigma(\vec{x})\Gamma'\Sigma(-\vec{x})\Gamma\right] \tag{4.7}$$

for mesons. Similarly for the baryons the correlator is defined as

$$R_{N,\Delta}(x) = \frac{1}{4}Tr\left[R'_{N,\Delta}(x)\vec{\gamma}\cdot\vec{x}\right] \tag{4.8}$$

where we have

$$R'_N(x) = Tr\left[\gamma_\mu S(x)\gamma_\rho S(-x)\right]\gamma^\mu S(-x)\gamma^\rho + Tr\left[\gamma_\mu \Sigma(x)\gamma_\rho \Sigma(-x)\right]\gamma^\mu \Sigma(-x)\gamma^\rho \tag{4.9}$$

$$R'_{\Delta}(x) = Tr\left[\gamma_\mu S(x)\gamma^\mu S(-x)\right] S(x) + Tr\left[\gamma_\mu \Sigma(x)\gamma^\mu \Sigma(-x)\right]\Sigma(x) \tag{4.10}$$

with the extra terms in Eqs. (4.7), (4.9) and (4.10) arising from the four point vacuum structure. The structure for the field Σ is arbitrary so far with all possible Dirac matrix structures (i.e. $\mathbb{1}$, γ_5, γ_μ, $\gamma_\mu\gamma_5$, $\gamma_{\mu\nu}$; $\mu \neq \nu$)

The experimental data demands that we choose the condensate fluctuation field Σ such that it contributes maximally in the PS channel and should not affect the other channels very much. It should be pointed out [3] that the vector fluctuations ($\sim \gamma_\mu$) contribute only to the PS and nucleon channels, while not contributing to the other physical channels. On the other hand the scalar fluctuations ($\sim \mathbb{1}$) contribute to all channels. Such conditions together with simplicity restrict the choice for the fluctuating field to a structure of the type

$$\Sigma_{\alpha\beta}(\vec{x}) = \Sigma^V_{\alpha\beta}(\vec{x}) + \Sigma^S_{\alpha\beta}(\vec{x}) \quad = \mu_1^2\, (\gamma^i\gamma^j)_{\alpha\beta}\, \epsilon_{ijk}\, \phi^k(\vec{x}) + \mu_2^2\, \delta_{\alpha\beta}\, \phi(\vec{x}) \tag{4.11}$$

where the first term corresponds to vector fluctuations and the second to scalar. μ_1 and μ_2 in the above equations are dimensional parameters which give the strength of the fluctuations and $\phi^k(\vec{x})$ and $\phi(\vec{x})$ are vector and scalar fields such that

$$< \Omega|\phi^k(\vec{x})|\Omega >= 0; \quad < \Omega|\phi(\vec{x})|\Omega >= 0 \tag{4.12}$$

and

$$< \Omega|\phi^i(\vec{x})\phi^j(0)|\Omega >= \delta^{ij} g_V(\vec{x}); \quad < \Omega|\phi(\vec{x})\phi(0)|\Omega >= g_S(\vec{x}) \tag{4.13}$$

This implies an approximation that the ground state of QCD is also a condensate in the fields Σ. The functions $g_{S,V}(x)$ are at this stage arbitrary.

With the structure of the fields $\Sigma_{\alpha\beta}(\vec{x})$ given by (4.11) we can calculate the contribution to the correlator $R(x)$ arising from the second terms in Eqs. (4.7), (4.9) and (4.10). It is then normalised to the correlator for free massless quarks $R_0(x)$. The contributions of the propagator and the fields Σ to mesonic and baryonic channels are shown separately in cols. 3 and 4 of Table 4.1 .

For the vector fields $\Sigma^V_{\alpha\beta}(\vec{x})$ the extra contribution is denoted by F^V and for the scalar fields $\Sigma^S_{\alpha\beta}(\vec{x})$ this contribution is denoted by F^S in column 3. The currents for the particles in column 2 have been defined in chapter 3.

TABLE 4.1. Contribution of four point structure to correlation functions

CHANNEL	PARTICLE	CORRELATION FUNCTIONS $\left[\frac{R(x)}{R_0(x)}\right]$	
		Without fluctuations[a]	Fluctuation contribution (Vector(F^V) and Scalar (F^S))
Vector	ρ	$[F(x)]^2 + \frac{\pi}{4}\frac{x^6}{R^6}e^{-x^2/R^2}$	$F^V = 0$
			$F^S = 8\pi^4 x^6 g_S(2x)$
Pseudoscalar	π^0	$[F(x)]^2 + \frac{\pi}{8}\frac{x^6}{R^6}e^{-x^2/R^2}$	$F^V = -48\pi^4 x^6 g_V(2x)$
			$F^S = 4\pi^4 x^6 g_S(2x)$
Nucleon	N	$[F(x)]^3 + \frac{\pi}{16}\frac{x^6}{R^6}e^{-x^2/R^2}F(x)$	$F^V = -4\pi^4 x^6 g_V(2x)F(x)$
			$F^S = 2\pi^4 x^6 g_S(2x)F(x)$
Delta	Δ	$[F(x)]^3 + \frac{\pi}{4}\frac{x^6}{R^6}e^{-x^2/R^2}F(x)$	$F^V = 0$
			$F^S = 8\pi^4 x^6 g_S(2x)F(x)$

[a] $F(x) = \left[1 + \frac{1}{2}x^2 I(x)\right]$ where $I(x) = \int_0^\infty \left(\cos kx - \frac{\sin kx}{kx}\right)\frac{k e^{-R^2k^2}}{1+(1-e^{-R^2k^2})^{1/2}}dk$

Since we do not expect effects of the Σ fields to be important for $\vec{x} \to 0$ and for large x , we want $g(\vec{x})$ to vanish in these two limits. With these properties in mind we take the function $g(\vec{x})$ as

$$g_V(\vec{x}) = \frac{1}{2\pi^2 x}\left[\mu_1 K_1(\mu_1 x) - \mu_3 K_1(\mu_3 x)\right] \tag{4.14}$$

$$g_S(\vec{x}) = \frac{1}{2\pi^2 x}\left[\mu_2 K_1(\mu_4 x) - \mu_5 K_1(\mu_6 x)\right] \tag{4.15}$$

so that the small x behaviour of the correlator is the same as expected from the asymptotic freedom. $g_V(\vec{x})$ corresponds to taking the Σ field condensate function as difference of two propagators with different masses. Such a structure for the condensate function arises naturally e.g. in the ϕ^4 field theory in the Gaussian effective potential calculations [2] where μ_1 corresponds to the Lagrangian mass parameter and μ_3 corresponds to the variational parameter associated with the Gaussian ansatz. For $g_S(\vec{x})$ we had to consider a more general form (with extra parameters) because the contributions of the propagators and of the scalar fluctuations are comparable, requiring delicate cancellations.

It ought to be noted that to be consistent with phenomenology of correlators, the contribution of vector fluctuations should be greater than that of scalar fluctuations.

4.3 Results and discussions

The parameters of the quark condensate function $h(\vec{k})$ and the of fluctuating fields $g_{S,V}$ are chosen so that our correlation functions are similar to those obtained from phenomenology. We have taken $R = 0.60 fm$ corresponding to $<-(\bar{u}u + \bar{d}d)>= (304.45\ MeV)^3$. Vector fluctuations dominate in the PS channel beyond $x = 0.5 fm$. This behaviour is reproduced by choosing $\mu_1 = 164.8$ and $\mu_3 = 69$. In the vector channel the propagator and fluctuation contributions are of the same order. If we choose $\mu_2 = 245, \mu_4 = 400, \mu_5 = 1280.77, \mu_6 = 518$ we get reasonable agreement with phenomenological curves of correlators. The parameters $\mu_{1...6}$ are in units of MeV. The resulting correlators are plotted in Fig. 4.1 .

We see that we do get the high rise in the pseudoscalar (PS) channel. The behaviour of $R(x)/R_0(x)$ is good for small distances and excellent for large distance. Also the ratio for the nucleon correlator has a fairly good agreement with instanton model calculations. The vector and delta correlators have reasonable agreement with predictions from phenomenology and lattice calculations respectively, except that our curves fall faster for large distances. This may be a reflection of the Gaussian ansatz for our condensate function (chapter 2). It may be noted here that this feature is not true for the PS and nucleon channels where the main contribution comes from the four point structure.

We fit our results (solid curves of Fig. 4.1) to phenomenologically motivated forms for correlators parameterised in terms of mass, coupling and threshold of the corresponding particle [4] (also see end of appendix B). We used the Marquardt method for non-linear least square fit [5]. The method was very stable and the goodness of fit estimated from χ^2 was reasonable for small and intermediate x, which is the region characterising the mass, coupling and threshold of a general correlator [4]. Our fitted parameters and those similarly obtained in other models are tabulated in Table 4.2 . The mass of the ρ and π are extremely close to the experimental masses. The baryon masses are lower (for nucleon) and higher (for delta) than the experimental masses. The coupling is in the range of the phenomenological values for all channels. Our threshold for all channels are consistently on the higher side compared to other values.

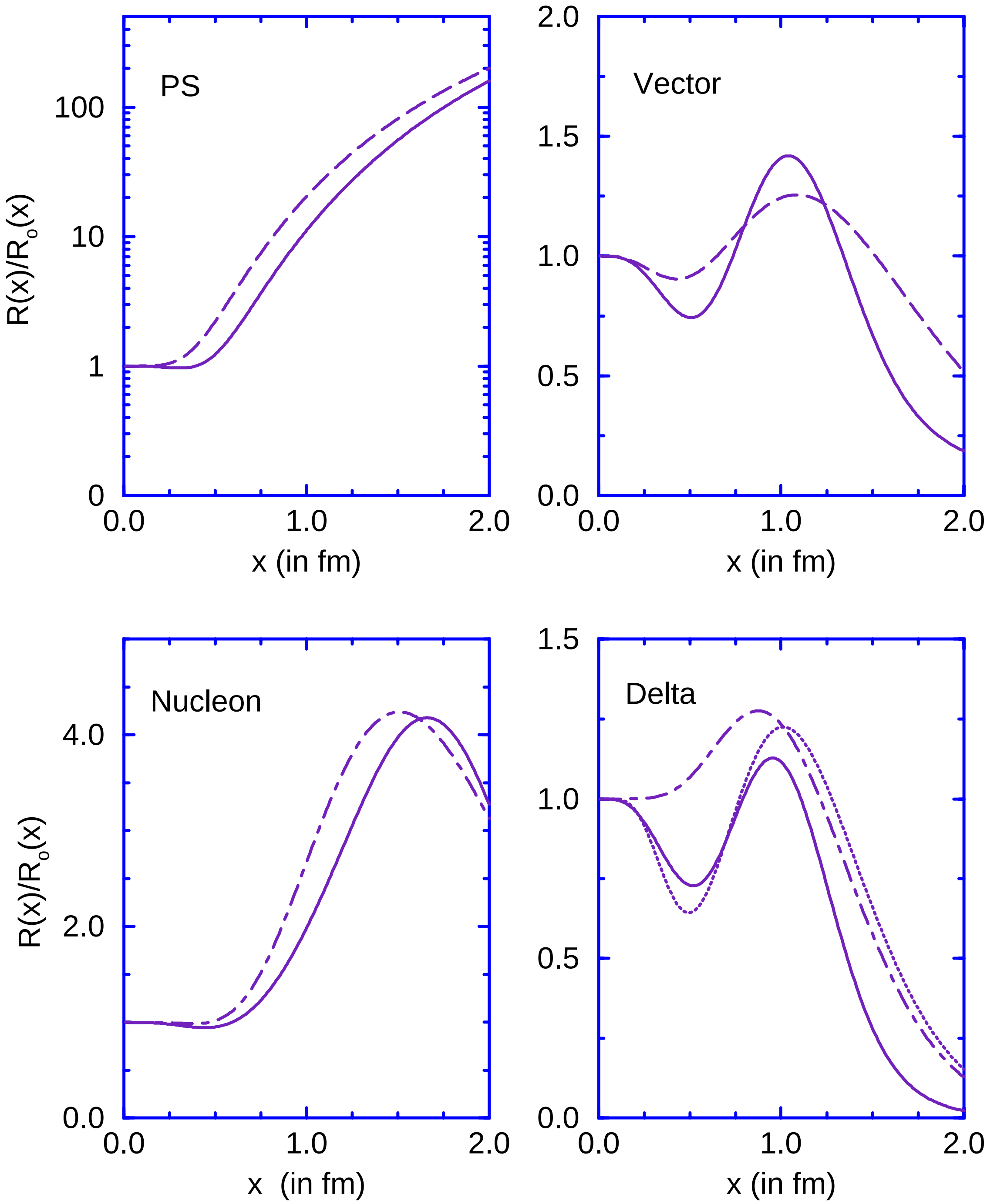

Figure 4.1: The normalised hadron correlation functions $\frac{R(x)}{R_0(x)}$, vs. distance x (in fm). Our results are given by the solid curves. The empirical results determined by dispersion analysis of experimental data in Ref.[3] are shown by long dashed lines. The results from lattice calculations and instanton model are denoted by dotted and dot-dashed lines respectively.

TABLE 4.2. Fitted parameters (non strange channels)

CHANNEL	SOURCE	M (GeV)	λ	$\sqrt{s_0}$(GeV)
Vector	Ours	0.78 ± 0.005	$(0.42 \pm 0.041 \text{ GeV})^2$	2.07 ± 0.02
	Lattice	0.72 ± 0.06	$(0.41 \pm 0.02 \text{ GeV})^2$	1.62 ± 0.23
	Instanton	0.95 ± 0.10	$(0.39 \pm 0.02 \text{ GeV})^2$	1.50 ± 0.10
	Phenomenology	0.78	$(0.409 \pm 0.005 \text{ GeV})^2$	1.59 ± 0.02
Pseudoscalar	Ours	0.137 ± 0.0001	$(0.475 \pm 0.015 \text{ GeV})^2$	2.12 ± 0.083
	Lattice	0.156 ± 0.01	$(0.44 \pm 0.01 \text{ GeV})^2$	< 1.0
	Instanton	0.142 ± 0.014	$(0.51 \pm 0.02 \text{ GeV})^2$	1.36 ± 0.10
	Phenomenology	0.138	$(0.480 \text{ GeV})^2$	1.30 ± 0.10
Nucleon	Ours	0.87 ± 0.005	$(0.286 \pm 0.041 \text{ GeV})^3$	1.91 ± 0.02
	Lattice	0.95 ± 0.05	$(0.293 \pm 0.015 \text{ GeV})^3$	< 1.4
	Instanton	0.96 ± 0.03	$(0.317 \pm 0.004 \text{ GeV})^3$	1.92 ± 0.05
	Phenomenology	0.939	?	1.44 ± 0.04
Delta	Ours	1.52 ± 0.003	$(0.341 \pm 0.041 \text{ GeV})^3$	3.10 ± 0.008
	Lattice	1.43 ± 0.08	$(0.326 \pm 0.020 \text{ GeV})^3$	3.21 ± 0.34
	Instanton	1.44 ± 0.07	$(0.321 \pm 0.016 \text{ GeV})^3$	1.96 ± 0.10
	Phenomenology	1.232	?	1.96 ± 0.10

As is evident from Fig. 4.1 and Table 4.2 our model of the vacuum gives results for the hadronic correlators that are comparable to those in the instanton model [6, 7] and lattice calculations [4] and to phenomenological results.

4.4 Strange channels

We have also studied the strange channels $\phi, K^\star$ and K using the method of chapter 3. For a massive quark the condensate function, as compared to that for massless quarks in chapter 2, is chosen as

$$\tan 2h(\vec{k}) = \frac{m}{k} + \frac{1}{(e^{R^2k^2} - 1)^{1/2}} \tag{4.16}$$

The above ansatz has the proper limits for small and large k. For small k we see that $\tan 2h(\vec{k}) = \dfrac{m + 1/R}{k}$ which corresponds to the identification for a free massive fermion in chapter 2, where $1/R$ is the mass effect due to condensates. For large k (small distance), the second term in the above ansatz falls faster than the first so that we recover the free massless propagator in this limit. With such an ansatz, we calculate the interacting quark propagator which from chapter 3 is given as

$$S(\vec{x}) = \frac{1}{2}\frac{1}{(2\pi)^3}\int e^{i\vec{k}.\vec{x}} d\vec{k} \left[\sin 2h(\vec{k}) - (\vec{\gamma}\cdot\hat{k})\ \cos 2h(\vec{k})\right] \tag{4.17}$$

In chapter 3, the above propagator was calculated for a Gaussian ansatz for the massless u and d quarks. For a general ansatz function as above, the quark propagator has the form

$$S(\vec{x}) = -\frac{i}{2\pi^2}\frac{\vec{\gamma}\cdot\vec{x}}{x^4} + I_1(x) - \vec{\gamma}\cdot\vec{x} I_2(x) \tag{4.18}$$

where the integrals $I_1(x)$ and $I_2(x)$ are given as

$$I_1(x) = \frac{1}{4\pi^2}\int_0^\infty k\frac{sinkx}{x} sin2h(k)\ dk \tag{4.19}$$

$$I_2(x) = \frac{1}{4\pi^2}\frac{1}{x^2}\int_0^\infty \left(\cos kx - \frac{\sin kx}{kx}\right) k\ 2sin^2h(k)\ dk \tag{4.20}$$

Having obtained the massive quark propagator, we can calculate the correlation functions as

$$R(x) = Tr\left[S^{u,s}(x)\Gamma' S^s(-x)\Gamma\right] \tag{4.21}$$

where we use a massive s quark propagator $S^s(x)$ or a massless u quark propagator $S^u(x)$ depending on the current of the relevant channel :

$$\begin{array}{ll} K^\star & \bar{u}\gamma_\mu s \\ \Phi & \bar{s}\gamma_\mu s \\ K & \bar{u}\gamma_5 s \end{array}$$

As before we study the ratio of the physical correlator $R(x)$ to that for massless non-interacting quarks $R_0(x)$. This is done to compare with phenomenology [3].

We chose for the strange quark, $M_s = 230$ MeV and $R = 0.55 fm$ corresponding to $<-\bar{s}s>=$ (228.3 MeV)^3 Comparison of $R(x)/R_o(x)$ with phenomenology was reasonable for a choice of $\mu_1 = 300$ MeV and $\mu_3 = 325$ MeV for the pseudoscalar channel K and with no four point structure for vector channels ϕ and $K^\star$. The correlation functions for the strange channels are plotted in Fig. 4.2 . A good agreement with phenomenology is obtained for the pseudoscalar (K) channel and for the vector channels, the small distance behaviour agrees well while at large distances, our curves fall more rapidly than the phenomenological ones as for the non-strange channels above. In the lower right panel of Fig. 4.2 , we show our curves corresponding to the ρ, $K^\star$ and ϕ correlators. The splitting between theses correlators at large distances is similar to that predicted by phenomenology.

As for the non strange channels, the same method of fitting is used to obtain the mass, coupling and threshold for the strange particles. The results are shown in Table 4.3 . The results are in good agreement with phenomenology and the instanton model. The mass of the K is in particular in excellent agreement with phenomenology. The threshold for all particles are larger than the phenomenological values just as in the non-strange channels.

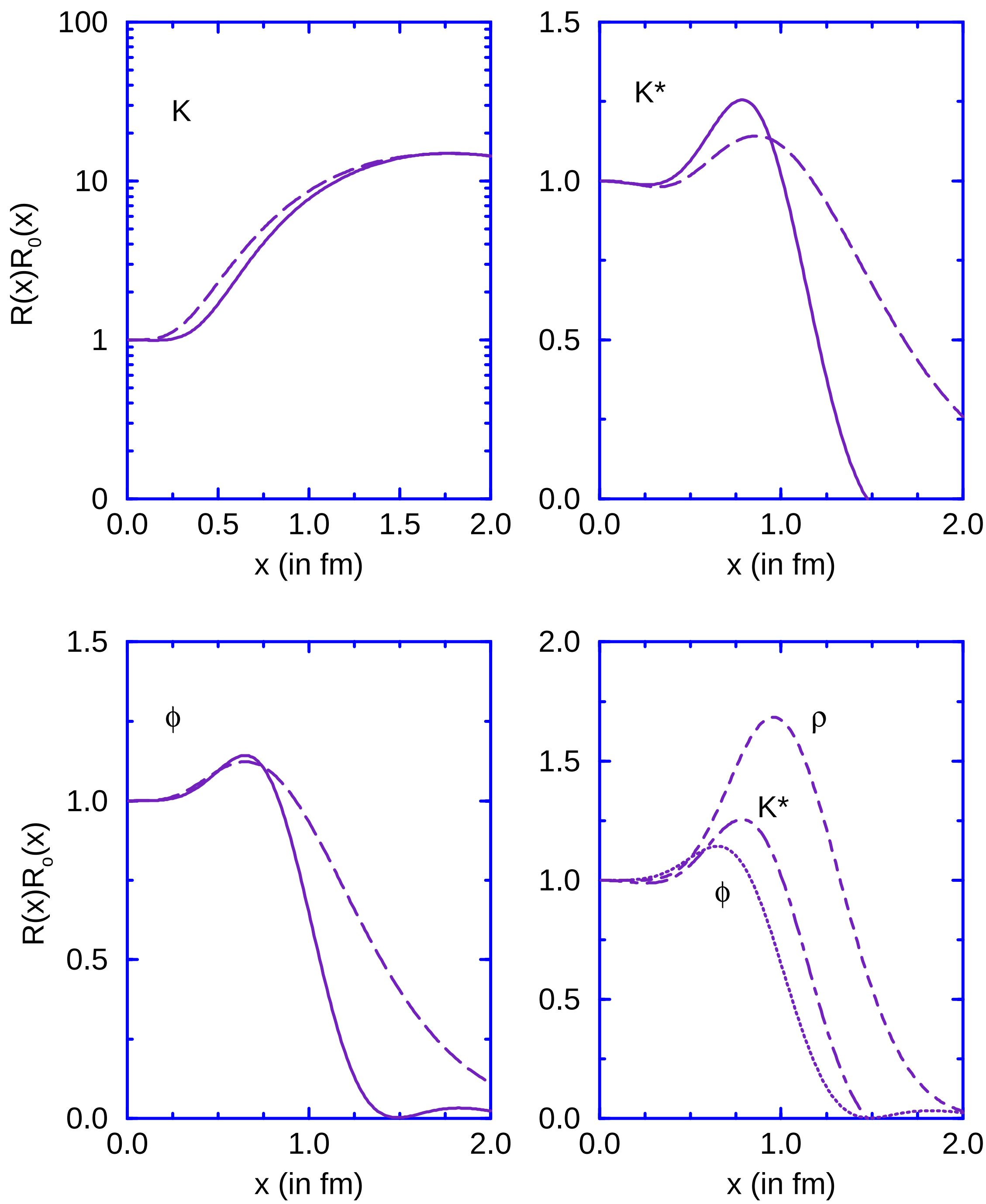

Figure 4.2: The normalised correlation functions $\frac{R(x)}{R_0(x)}$ for the strange pseudoscalar and vector channels, vs. distance x . Our results are given by the solid curves. The empirical results determined by dispersion analysis of experimental data in Ref.[3] are shown by long dashed lines. The figure on the lower right panel compares our results for the different strange vector channels.

TABLE 4.3. Fitted parameters (strange channels)

	Source	M (GeV)	λ	$\sqrt{s_0}$(GeV)
$K^\star$	Ours	0.928 ± 0.003	$(0.473 \pm 0.038\ \mathrm{GeV})^2$	1.562 ± 0.001
	Instanton	0.86 ± 0.015	$(0.341 \pm 0.02\ \mathrm{GeV})^2$	1.30 ± 0.05
	Phen.	0.892	$(0.448 \pm 0.025\ \mathrm{GeV})^2$	?
ϕ	Ours	1.038 ± 0.004	$(0.498 \pm 0.038\ \mathrm{GeV})^2$	1.497 ± 0.001
	Instanton	0.850 ± 0.05	$(0.280 \pm 0.02\ \mathrm{GeV})^2$	1.00 ± 0.04
	Phen.	1.020	$(0.492 \pm 0.015\ \mathrm{GeV})^2$	?
K	Ours	0.492 ± 0.001	$(0.534 \pm 0.039\ \mathrm{GeV})^2$	1.911 ± 0.015
	Instanton	0.482 ± 0.012	$(0.467 \pm 0.020\ \mathrm{GeV})^2$	1.350 ± 0.05
	Phen.	0.495	$(0.534\ \mathrm{GeV})^2$	0.595

References

[1] E. V. Shuryak and J. J. M. Verbaarschot, Nucl. Phys. B410 (1993) 37.

[2] P.M. Stevenson, Phys. Rev. D32 (1985) 1389 ; H. Mishra and A.R. Panda, J. Phys. G (Part. and Nucl. Phys.) 18 (1992) 1301.

[3] E.V. Shuryak, Rev. Mod. Phys. 65 (1993) 1.

[4] M.-C. Chu, J. M. Grandy, S. Huang and J. W. Negele, Phys. Rev. D48 (1993) 3340 ; ibid, Phys. Rev. D49 (1994) 6039.

[5] W.H. Press, S.A. Teukolsky, W.T. Vetterling and B.P. Flannery, *Numerical Recipes (in Fortran),* (Cambridge University Press, 1992).

[6] E. V. Shuryak and J. J. M. Verbaarschot, Nucl. Phys. B410 (1993) 55.

[7] T. Schäfer, E. V. Shuryak and J. J. M. Verbaarschot, Nucl. Phys. B412 (1994) 143.

Chapter 5

Summary and Conclusions

In this thesis, we have examined some low energy non-perturbative aspects of QCD, and the way in which they may be connected to the nature of the vacuum state. Through an explicit construct of the vacuum state we have studied hadron correlation functions. Phenomenological considerations of these correlators revealed the importance of an explicit four point structure in our ansatz for the vacuum state.

After a brief introduction to the investigations under consideration, in chapter 2, a variational approach has been considered with both quark and gluon condensates which is a generalisation of the earlier calculations involving either gluon or quark condensates. This makes the analysis more complete while improving its reliability, and demonstrates the link between the vacuum structure of QCD and many hadronic properties in a direct manner. More precisely, we took an ansatz for the nonperturbative vacuum with trial functions both for quark and gluon condensates. With the quark and gluon operators satisfying equal time algebra, we then do a constrained minimisation of the expectation value of the QCD Hamiltonian such that the pion decay constant and the SVZ parameters are correctly reproduced. Such a constrained energy minimisation led to both gluon and quark condensates with appropriate chiral symmetry breaking. The vacuum with condensates is preferred when α_s is greater than $\alpha_c = 0.62$. Also, we then can have simultaneously the pion decay constant, charge radius of the pion, the phenomenological bag pressure as well as some other hadronic properties as outputs, *starting from the QCD Lagrangian*.

We have considered here chiral symmetry breaking with light quark sector. Also, chiral symmetry breaking is important in the strange quark sector, and calculations adding condensates of the same would be of interest. However, besides some technical difficulties, this has the additional feature of Lagrangian mass being of the same order as dynamical mass and may therefore need nontrivial extension of the method. Work in this direction is of interest for the future.

In chapter 3, we first studied the equal time interacting quark propagator in the condensate vacuum developed in chapter 2. The general features of our propagator were similar to those of the quark propagator in the instanton model of the QCD vacuum. With such a propagator we could calculate equal time, point to point correlation functions for spatially separated hadron currents with respect to the vacuum state of chapter 2. The quantity of interest to study was the ratio of the physical correlation function to that of massless non-interacting quarks, $R(x)/R_0(x)$. This was calculated for the ρ, π, A_1, N and Δ particles and also for the scalar channel. The behaviour of $R(x)/R_0(x)$ with the distance x was in general agreement with predictions from phenomenology, lattice calculations and results of the random instanton liquid model in all channels except the pseudoscalar channel where the peak of this ratio was two orders of magnitude lower. However the approximate calculation of the nucleon and pion correlators by saturation with one particle states gave better results.

As a possibility for future work it would be interesting to study the hadronic correlation functions with a quark propagator which has proper confinement properties, i.e it falls faster than the free quark propagator for large distances.

It was found in chapter 3 that a model with only quark condensate was not adequate to explain the observations of hadronic correlators in all the channels. Motivated by the acute problem in the pseudoscalar channel, in chapter 4, we introduced irreducible four point structure in the vacuum. The objective was to obtain the high rise seen in the observations at ~ 0.5 fm.

With a proper choice of the parameters, this led to excellent overall agreement with the phenomenological curves and parameters (mass, coupling and threshold) in various channels. A similar investigation was done for the strange channels with an appropriate ansatz for the strange quark condensate, with results in good agreement with phenomenology. Thus the contribution of an explicit four point structure is most important and in some ways is related to the “hidden contribution” discussed by Shuryak. It is worth noting that such a structure could take care of the effects of gluon condensates which we are not able to explicitly include in our evaluation of the hadronic correlators in the present work.

As mentioned above, a variational calculation of chapter 2 has not yet been attempted with strange quark condensate and it would be interesting to do so including the four point structure developed in chapter 4. We have not attempted a study of strange baryons here, which would also be of interest as phenomenological information is not available for them.

As a possibility for future work, the effect of temperature and/or density on the condensate vacuum structure can also be examined which could be of relevance to QGP. Once the structure of the 'ground state' at finite temperature and density is known, the correlators at finite temperatures can also be calculated. The vanishing of the quark condensates and gluon condensates at high temperatures might indicate the relationship of chiral phase transition and confinement-deconfinement phase transitions.

Appendix A

One can relate the pion decay constant to the ansatz function for quark condensates. This has been discussed in Ref. [10, 11] of chapter 2. Let us recapitulate the same here.

When chiral symmetry remains good, we have conservation of the axial current

$$J_5^{\mu a}(x) = \bar{\psi}(x)\frac{\lambda_a}{2}\gamma^5\gamma^\mu\psi(x) \tag{A.1}$$

By Noether's theorem, the charge associated with this current is the chiral charge operator

$$\begin{aligned} Q_5^a &= \int J_5^{0a}(x)d\vec{x} \\ &= \int \psi(\vec{x})^\dagger\frac{\lambda^a}{2}\gamma^5\psi(\vec{x})d\vec{x} \end{aligned} \tag{A.2}$$

For chiral symmetry we have

$$Q_5^a \mid 0 >= 0 \quad [Q_5^a, H] = 0 \tag{A.3}$$

However when chiral symmetry is broken then $Q_5^a \mid vac >\neq 0 \ and \quad [Q_5^a, H] = 0$. It can be shown that such a state has zero momentum and zero energy, corresponding to the pion state as a Goldstone mode.

$$\mid \pi^a(\vec{0}) >= N_\pi \ Q_5^a \mid vac > \tag{A.4}$$

Here N_π is the normalisation constant. Such a pion state has been explicitly calculated in Ref. [11] of chapter 2 as

$$| \pi^a(\vec{0}) >= N_\pi \int q_I(\vec{k}) \frac{\lambda_a}{2} q_I(-\vec{k}) sin2h(\vec{k}) d\vec{k} \mid vac > \tag{A.5}$$

In this notation the wave function of pion is by definition

$$\tilde{u}(\vec{k}) \equiv sin2f(\vec{k}) \tag{A.6}$$

With the help of the normalisation condition

$$< \pi^a(\vec{0}) \mid \pi^b(\vec{k}) >= \delta^{ab}\delta(\vec{k}) \tag{A.7}$$

we can immediately see that

$$N_\pi^{-2} = \int sin^2 2h(\vec{k}) d\vec{k} \tag{A.8}$$

To relate N_π to the pion decay constant we start with the definition of pion decay constant

$$< 0 \mid J_5^{\mu a}(x) \mid \pi^a(p) >= \frac{i f_\pi p^\mu}{(2\pi)^{3/2}(2p_0)^{1/2}} e^{ip\cdot x} \tag{A.9}$$

We can thus write the matrix element

$$< 0 \mid J_5^{0a}(x) \mid \pi^a(p) >= \frac{i f_\pi p^0}{(2\pi)^{3/2}(2p_0)^{1/2}} e^{ip\cdot x} \tag{A.10}$$

The normalisation constant N_π is given by using Eqs. (A.4) and (A.7) as

$$\begin{aligned} N_\pi^{-2}\delta^{aa}\delta(0) &= < vac \mid Q_5^a Q_5^a \mid vac > \\ &= \int < vac \mid Q_5^a \mid \pi^b(\vec{k}) > d\vec{p} < \pi^b(\vec{k}) \mid Q_5^a \mid vac > \end{aligned} \tag{A.11}$$

which can be evaluated using Eq. (A.10) and the definition of Q_5^a in Eq. (A.2). We finally get a relation between the pion decay constant and N_π.

$$N_\pi^{-2} = \frac{1}{2} \cdot (2\pi)^3 \cdot m_\pi {f_\pi}^2 \tag{A.12}$$

Using Eqs. (A.8) and (A.12) we get the required relationship between the pion decay constant and the condensate function. This relation is needed to constrain one of the parameters in the ansatz for the condensate function in chapter 2.

Appendix B

The phenomenology of correlation functions has been discussed in detail by Shuryak (E.V. Shuryak, Rev. Mod. Phys. 65 (1993) 1). In order to extract the hadronic correlators from experiments one proceeds in few steps. The correlators are first expressed in terms of spectral density functions (Eq. (B.5)) as discussed below. Next one relates the spectral density function to experimental observed quantities depending on the particular hadron under consideration. For the vector currents, for example, the spectral density function, is related to the cross section of e^+e^- annihilation into hadrons. In the axial vector channel it is related to the decay of τ into hadrons. For the pseudoscalar channel the spectral density function is parameterised in terms of the mass of the pseudoscalar meson, the threshold and coupling constant of the meson to the current. For the sake of completeness we show below how the correlators are related through the spectral density function to the experimental observables.

The correlators are defined by the time ordered product

$$\Pi(x) \equiv < 0|TJ(x)J^{\dagger}(0)|0 > \tag{B.1}$$

where the Dirac indices have been suppressed. If we use completeness relation, PCT and translational invariance of the vacuum state, the correlator can be written in terms of a spectral representation as [see in J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), Page 138]

$$\Pi(x) = \int \frac{d^4q}{(2\pi)^3} \left(\theta(x_0)e^{-iq\cdot x} + \theta(-x_0)e^{iq\cdot x}\right) \delta(q^2 - \sigma^2)\rho(\sigma^2)d\sigma^2 \qquad \text{(B.2)}$$

where $\rho(\sigma^2)$ is the spectral function defined by

$$\rho(\sigma^2) = (2\pi)^3 \sum_n \delta^4(\sigma - q_n) < 0 \mid J(0) \mid n >< n \mid J^\dagger(0) \mid 0 > \qquad \text{(B.3)}$$

The integral over d^4q yields a Feynman propagator of mass $m^2 = \sigma^2$, which for spacelike distances is given by

$$D(\sqrt{(s)}, x) = \frac{\sigma}{4\pi^2 x} K_1(\sigma x) \qquad \text{(B.4)}$$

where $K_1(\sigma x)$ is a modified Bessel function. The integral in (B.2) can be rewritten as

$$\Pi(x) = \int_0^\infty d\sigma^2 D(\sigma, x)\rho(\sigma^2) \qquad \text{(B.5)}$$

This relates a correlator to the spectral density function. For the vector channel, contact with phenomenology is made by writing the correlator in Eq. (B.5) in the form

$$\Pi_{\mu\mu}(x) = \int_0^\infty ds D(\sqrt{s}, x)\ s\ R_i(s)$$

where

$$R_i(s) = \frac{\sigma(e^+e^- \to i)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

is the ratio of the cross section of e^+e^- annihilating to the relevant hadron to that for muon pair production. The details of the data available for such experiments and how $R_i(s)$ is extracted from such data is given in E.V. Shuryak, Rev. Mod. Phys. 65 (1993) 1. It may be worthwhile here to mention that vector correlators are distinguished experimentally from a knowledge of the final state i in $e^+e^- \to i$. For the ρ particle the final state has even number of pions, for ω it has odd number of pions and for ϕ they contain a pair of K mesons.

For the axial vector channel information of the cross section comes from the decay of τ to hadrons ($\tau \to \nu_\tau$ + hadrons).

For the pseudoscalar correlator, the spectral density function is parameterised as follows. One uses a phenomenological parameterisation of the form

$$\rho(s) = \lambda^2 \delta(s - M^2) + f_c(s)\theta(s - s_0) \tag{B.6}$$

where $s = \sigma^2$. M is the bound state mass, λ denotes the coupling of the current to the bound state, and s_0 is the threshold for the onset of a continuum contribution $f_c(s)$. The continuum contribution $f_c(s)$ follows from asymptotic freedom. At short distances the correlator approaches the free correlator which is known, hence left hand side of Eq. (B.5) is known. The right hand side can be obtained by explicit evaluation of the integral for the second (continuum) term in Eq. (B.5), in large s limit. The spectral density function and the corresponding normalised correlation functions are given below for various channels (see Chu et. al., Phys. Rev. D 48 (1993) 3340).

1. Pseudoscalar

$$\rho^P(s) = \lambda_\pi^2 \delta(s - M_\pi^2) + \frac{3s}{8\pi^2}\theta(s - s_o)$$

$$R^P(x)/R_0^P(x) = \frac{\pi^2}{12}\left(\frac{\lambda_\pi}{M_\pi^2}\right)^2 (M_\pi x)^5 K_1(M_\pi x) + \frac{1}{16}\int_{\sqrt{s_0}x}^{\infty} d\alpha\ \alpha^4 K_1(\alpha)$$

2. Vector

$$\rho^V(s) = 3\lambda_\rho^2 \delta(s - M_\rho^2) + \frac{3s}{4\pi^2}\theta(s - s_o)$$

$$R^V(x)/R_0^V(x) = \frac{\pi^2}{8}\left(\frac{\lambda_\rho}{M_\rho^2}\right)^2 (M_\rho x)^5 K_1(M_\rho x) + \frac{1}{16}\int_{\sqrt{s_0}x}^{\infty} d\alpha\ \alpha^4 K_1(\alpha)$$

3. Axial Vector

$$\rho^A(s) = -f_\pi^2 m_\pi^2 \delta(s - M_\pi^2) + 3\lambda_a^2 \delta(s - M_a^2) + \frac{3s}{4\pi^2}\theta(s - s_{oa})$$

$$R^A(x)/R_0^A(x) = \frac{\pi^2}{8}\left(\frac{\lambda_a}{M_a^2}\right)^2 (M_a x)^5 K_1(M_a x) \quad + \quad \frac{1}{16}\int_{\sqrt{s_0}x}^{\infty} d\alpha \; \alpha^4 K_1(\alpha)$$
$$- \quad \frac{\pi^2}{24}\left(\frac{f_\pi}{M_\pi^2}\right)^2 (M_\pi x)^5 K_1(M_\pi x)$$

4. Nucleon

$$R^N(x)/R_0^N(x) = \frac{\pi^4}{96}\left(\frac{\lambda_N}{M_N^3}\right)^2 (M_N x)^8 K_2(M_N x) + \frac{1}{3072}\int_{\sqrt{s_0}x}^{\infty} d\alpha \; \alpha^7 K_2(\alpha)$$

5. Delta

$$R^\Delta(x)/R_0^\Delta(x) = \frac{\pi^4}{36}\left(\frac{\lambda_\Delta}{M_\Delta^3}\right)^2 (M_\Delta x)^8 K_2(M_\Delta x) + \frac{1}{3072}\int_{\sqrt{s_0}x}^{\infty} d\alpha \; \alpha^7 K_2(\alpha)$$

The phenomenological pseudoscalar correlator is calculated by using the experimental values of M_π, s_o and $\lambda_\pi = f_\pi K$ in the expression in (1) above.

Appendix C

In this work the Dirac Pauli representation was used for the Dirac matrices as follows. We take the definition

$$\gamma^\mu = (\beta, \beta\vec{\alpha}) \quad = (\gamma^0, \vec{\gamma}^i)$$

where

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \vec{\sigma}_i \\ -\vec{\sigma}_i & 0 \end{pmatrix}$$

where

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Hence with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

$$\gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

We work in Minkowski space, therefore the metric we use is

$$g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

such that $g^{\mu\nu} g_{\mu\nu} = 4$ and a dot product is given as

$$a \cdot b = a^\mu b^\nu g_{\mu\nu} \quad = a^0 b^0 - \vec{a} \cdot \vec{b}$$

The conjugate quark spinors and hadron currents are defined as

$$\bar{\psi} = \psi^\dagger \gamma^0 \quad and \;\; \bar{J}(x) = J(x)^\dagger$$

We have used the following properties of the Dirac matrices

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \quad \gamma^5 \gamma^\mu + \gamma^\mu \gamma^5 = 0 \quad \gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0$$

In the evaluation of correlation functions in chapter 3, following trace theorems were useful :

1. $Tr\,[\vec{\gamma} \cdot \vec{x}\, \vec{\gamma} \cdot \vec{x}] = -4\; x^2$

2. $Tr\,[\gamma^\mu \gamma_\mu] = 16$

3. $Tr\,[\gamma^\mu\; \vec{\gamma} \cdot \vec{x}\; \gamma_\mu\; \vec{\gamma} \cdot \vec{x}] = 8\; x^2$

4. $Tr\,[\gamma_\mu \gamma_\nu]\, \gamma^\mu\; \vec{\gamma} \cdot \vec{x}\; \gamma^\nu = -8\; \vec{\gamma} \cdot \vec{x}$

5. $Tr\,[\gamma_\mu\; \vec{\gamma} \cdot \vec{x}\; \gamma_\nu \vec{\gamma} \cdot \vec{x}]\, \gamma^\mu\; \vec{\gamma} \cdot \vec{x}\; \gamma^\nu = -16\; x^2\; \vec{\gamma} \cdot \vec{x}$