Some Studies in Violation of Symmetry Principles in Particle Physics

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submitted by Mr. Rishikesh D. Vaidya, was carried out by the candidate under my guidance at the Physical Research Laboratory, Ahmedabad. The work presented in this thesis is original and has not formed basis for the award of any degree or diploma by any university or institution.

Dr. Subhendra Mohanty
(Supervisor)

To...

...those authors, whose pedagogic books and reviews have immensely influenced my understanding of Physics. The view from the window they provided, was just too beautiful.

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Contents

Chapter 1

Introduction

1.1 Particle Physics: A Paradigm in Reductionist Approach

The physics curriculum in the 1898-99, University of Chicago catalogue begins with a very Victorian preface [1]:

> "*While it is never safe to affirm that the future of the Physical Sciences has no marvels in store even more astonishing than those of the past, it seems probable that most of the grand underlying principles have been firmly established and that further advances are to be sought chiefly in rigorous applications of these principles to all the phenomena which come under our notice ... An eminent physicist has remarked that the future truths of the Physical Sciences are to be looked for in the sixth place of decimals.*"

Perhaps today, a century after it was written, we are closer to the truth of this quote. Looking at the complexity and vastness of the Universe, both in time and space, it is indeed remarkable how well we have understood its nature at the smallest and the largest possible space-time scales. This speaks for the power of reductionist approach, which aims to understand the most complex, in terms of the properties and inter-relations of the simplest basic units. The methodology of Science is a paradigm in reductionist approach and Particle Physics claims to be the science of ultimate reduction: the basic building blocks, their properties, inter-relations and interactions which bind them to give complex structures such as observed in nature.

It is interesting to note how our conception of the basic building blocks of universe has changed over the period of last few centuries. Newton thought that both matter and light have particle nature. Young's famous double slit interference experiment established that Light possesses wave nature. Birth of Quantum Mechanics erased the *mutually exclusive* wave or particle nature in favour of wave-particle duality. The era of Quantum Field Theory introduced 'fields' as basic entities and defined particles as quanta of field excitations. Impressive contact of Quantum Field Theories (in the garb of Standard Model) with experiments dominated Particle Physics for last more than half a century.

In spite of its resounding success in predicting experimental results with high accuracy, Standard Model (SM) is not palatable to theorists as a complete fundamental theory of particles and interactions. Among other reasons (to be discussed in section 1.3), it has a large number of unexplained free parameters and does not incorporate gravity. The reason being, gravitational coupling has a negative $(mass)^2$ dimension and hence does not satisfy the criterion of renormalizability. A way out to this problem, is to cure the root of ultra-violet infinities which are intrinsic to interacting quantum field theories when excitations are point particles and interactions are local. With the neglect of gravity we had luckily encountered theories without negative mass dimension couplings, where these infinities can be absorbed into bare parameters by finite number of redefinitions. But in theories with negative mass dimension couplings, the degree of divergence grows with each loop and one needs infinite number of redefinitions and thus theory loses predictivity. This disease was remedied by the theory where fundamental objects are extended excitations called strings (generalized to encompass even more extended structures like membranes and so on), which is the first finite theory of quantum gravity. So from Newton's corpuscles to strings and membranes, the quest for fundamental entities is still far from being resolved. Nevertheless, we can justifiably subscribe to a pragmatic phenomenologist approach based on the quantum fields paradigm which has served us so well in the last century. We shall treat SM as an 'effective field theory' which is a low energy approximation to some deeper theory which may not even be a field theory. The reason that it describes physics so well at accessible energies is that the Quantum Field Theory is the only way to reconcile with the principles of quantum mechanics and relativity at sufficiently low energies (say around 100 GeV) [2].

1.2 The Standard Model: A Dictation of Symmetries and Violations

The Standard Model of Particle Physics summarizes our current understanding of fundamental particles and their interactions. It's beauty lies in the economy of a few symmetry principles which completely dictate the Lagrangian of SM[1]. The rudimentary classification of the fundamental particles is in terms of their kinematic properties, mass and spin(a purely quantum attribute). Being kinematic properties these should follow as invariants of space-time symmetries. Indeed mass and spin turn out to be two quadratic Casimir invariants of rank two Poincare Group [2] [4] . If Poincare Group were the only group of

[1]To this one should also add the requirement of renormalizability.

[2]To be precise, all physical states in Quantum Field Theory can be labeled by the eigenvalues of two Casimir operators (P_μ^2, W_μ^2), where P_μ is the momentum four vector and W_μ is the Pauli-Lubanski

symmetries, it would make a very dull kinematic universe of free streaming massive/massless scalars, spinors and vector particles devoid of any dynamics. For dynamics to set in the particles must interact. It turns out that interactions follow naturally as a consequence of certain internal symmetry groups which allow you to transform at your will at different space-time points. The sole purpose of dynamics (interactions) is to 'undo' the effects of these space-time dependent transformations of internal symmetry groups and hence maintain the invariance of the lagrangian. Identification of such internal symmetry groups operative in nature, completely dictates the nature of interactions and assignment of particles to specific representations of internal and space-time symmetry groups. Symmetries, along with the requirement of renormalizability then complete the prescription for the Lagrangian. A fundamental particle is then defined as the one whose field appears in the Lagrangian.

1.2.1 Space-time Symmetries: Building the Blocks

If at all there are some space-time symmetry groups operative in nature, then the fundamental particles necessarily exist as specific irreducible representations of these groups, labeled by invariant kinematic attributes like mass and spin[3]. The question is: Once they exist, what are they supposed to do? In the absence of any internal symmetries, space-time symmetries require them to blindly follow the initial conditions, irrespective of place, time, direction or the choice of inertial frame for measurement. So the existence of space-time symmetries in the absence of internal symmetries (dynamics) completely determines the kinematics. But such a purely kinematic Universe would be dull and uninteresting and we won't be here to comprehend it if complex structures were not to form. The very existence of complexity compels us to look for symmetries which make dynamics possible.

1.2.2 Internal Symmetries: Gluing the Blocks

Having got the basic building blocks from space-time symmetries, let us look for the recipe by which the building blocks communicate with each other via the well known strong,

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pseudovector

[3]In trying to build the argument from the first principles, we propound a point of view that particles follow as a consequence of symmetry principles. One can turn the entire argument around: In the vacuous Universe devoid of particles, do symmetry principles make any sense ? The question of primacy between particles and principles properly belongs to the realm of metaphysics and hence any epistemological answer cannot unambiguously settle the question of premises. The point of view of author is that conceptualization proceeds its actualization and hence the guiding principles (group symmetries) necessarily precede over the building blocks (group representations). We stress the 'independence of principles' from our 'cognition of principles' which follows empirical observations.

weak and elecromagnetic interactions. Note that the quantal description of elementary particles makes use of complex numbers. But physical quantities are real so complex phases can be changed at will without affecting the physical content. This invariance under phase redefinition, called the *gauge symmetry* leads to charge conservation and is the reason behind the existence of interactions [5]. Every fundamental interaction in the nature follows a gauge symmetry principle.

§ *Gauge Theory: An Abelian Example*

The lagrangian describing free fermion of mass m is $\mathcal{L}_{free} = \bar{\psi}(i \not\partial - m)\psi$. It is invariant under the global phase change $\psi \rightarrow exp(i\alpha)\psi$. Now consider independent phase changes at each space-time point x:

$$\psi \rightarrow \psi' \equiv exp[i\alpha(x)]\psi. \tag{1.1}$$

Because of the derivative, the Lagrangian then acquires an additional phase change at each space-time point: $\delta\mathcal{L} = \bar{\psi}i\gamma^{\mu}[i\partial_{\mu}\alpha(x)]\psi$. The free Lagrangian is not invariant under such changes of phase, known as *local gauge transformations.* Local gauge invariance can be restored if we make the replacement $\partial_{\mu} \rightarrow D_{\mu} \equiv \partial_{\mu} + ieA_{\mu}$ in the free fermion Lagrangian, which now is

$$\mathcal{L} = \bar{\psi}(i \not{D} - m)\psi = \bar{\psi}(i \not\partial - m)\psi - e\bar{\psi} \not{A}(x)\psi. \tag{1.2}$$

Here A_{μ} is a vector field. The effect of a local phase in ψ can be compensated if we allow the *vector potential* A_{μ} to change by a total divergence, which does not change the electromagnectic field strength

$$F_{\mu\nu} \equiv \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}. \tag{1.3}$$

Indeed, under the transformation $\psi \rightarrow \psi'$ and with $A \rightarrow A'$ with A' yet to be determined, we have

$$\mathcal{L}' = \bar{\psi}'(i \not\partial - m)\psi' - e\bar{\psi}' \not{A}'\psi' = \bar{\psi}(i \not\partial - m)\psi - \bar{\psi}[\not\partial\alpha(x)]\psi - e\bar{\psi} \not{A}'\psi. \tag{1.4}$$

This will be same as $\mathcal{L}$ if

$$A'_{\mu}(x) = A_{\mu}(x) - \frac{1}{e}\partial_{\mu}\alpha(x). \tag{1.5}$$

The derivative D_{μ} is known as covariant derivative. It is easy to check that under the local gauge transformation, $D_{\mu}\psi \rightarrow e^{i\alpha(x)}D_{\mu}\psi$. Thus we see that interaction term $e\bar{\psi} \not{A}\psi$ naturally comes from an imposition of local internal symmetry. In a similar manner one can understand Strong, and Weak interactions as following from imposition of certain local non-abelian internal symmetries. The nature of interaction is dictated by the structure of the symmetry group. The salient features of gauge theories are:

- Interactions follow naturally when we require invariance under local transformations of internal symmetry groups.

- Interactions among matter particles are understood as being mediated by vector gauge particles. They transform as adjoint representation of the gauge group. Thus vector particles naturally emerge as mediators of Interactions.

- Last but the not the least. Quite independent of experimental indications, one can ask: Is there any compelling theoretical argument which makes gauge invariance an inevitable requirement? Yes. Gauge invariance is the only way of reconciling unitarity with renormalizability in theories with vector particles [6].

1.2.3 The Standard Model Lagrangian

It is clear that space-time and internal symmetry groups completely dictate the classification and interactions of the fundamental particles. Electromagnetic interaction is the result of invariance under the Unitary symmetry group $U(1)_{EM}$. The so called Weak interaction follows from the invariance under Unitary Symmetry Group $SU(2)_L$, which also satisfies a special condition that its determinant is unity. The subscript 'L' stands for the fact that it acts only on the left handed components of a four component Dirac spinor. All too familiar nuclear force responsible for holding nucleons inside a nucleus, is actually a residual strong interaction among the constituents of nucleons called quarks. Such a strong interaction is a consequence of the invariance under $SU(3)_C$ symmetry group. Subscript 'C' implies that it acts only on those particles which carry strong charge called the 'colour charge'. The Standard Model of particle interactions is built upon the gauge group $G_0 = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Here $U(1)_Y$ corresponds to an abelian symmetry group acting over particles carrying non-zero hypercharge Y. The symmetry group G_0 is not exact and is spontaneously broken down to the invariant subgroups $SU(3)_c \otimes U(1)_{EM}$ [3].

§ *Dramatis Personae*

All elementary fermions neatly classify into two broad categories 'quarks' and 'leptons' . They transform under the SM gauge group G_0 as shown in the table 1.2.3. The assignment in the table 1.2.3 must be repeated three times since we know of the existence of three families of quarks and leptons. Such a variety of quarks and leptons is called flavour. Apart from the matter particles, there are force carrier gauge bosons, which are massless in the limit of exact gauge symmetry. Strong force is mediated by 8 gluons, whereas weak force is mediated by massive charged W^+ and W^- bosons (linear combinations of W_1

	$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L$	u_R	d_R	$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	e_R
$SU(3)_C$	3	3	3	1	1
$SU(2)_L$	2	1	1	2	1
$U(1)_Y$	1/6	-2/3	1/3	-1/2	1

Table 1.1: Classification of elementary fermions under SM gauge group

and W_2) and a neutral massive Z^0 boson (a linear combination of W_3 and B_μ -the gauge boson corresponding to $U(1)_Y$). They are massive because they correspond to broken symmetries. The familiar electrically neutral and massless Photon (A_μ), is then obtained as an another linear combination of W_3 and B_μ, corresponding to the unbroken generator Q of an invariant subgroup $U(1)_{EM}$. But how does one account for the mass of fermions and gauge bosons, as these mass-terms violate gauge invariance and also render the theory non-renormalizable ? *Higgs mechanism* is a way out to this problem. It uses the concept of *spontaneous symmetry breaking* by introducing a complex scalar doublet 'Higgs', whereby the Lagrangian retains the gauge symmetry but the vacuum does not. When the real component of neutral scalar Higgs acquires 'vacuum expectation value' (*vev*), it gives mass to the fermions and gauge bosons corresponding to the broken symmetry generators without affecting the renormalizability.

§ *The Weinberg-Salam Lagrangian*

Having known the *dramatis personae*, the final Lagrangian can be written as:

$$\mathcal{L}_{SM} = \mathcal{L}_F + \mathcal{L}_{YM} + \mathcal{L}_S + \mathcal{L}_{yuk} \ . \tag{1.6}$$

The fermion part, $\mathcal{L}_F$ is given as,

$$\mathcal{L}_F = i\bar{\Psi}\gamma^\mu \mathcal{D}_\mu \Psi \ , \tag{1.7}$$

with

$$\Psi = (Q_i \ (u_R)_i, \ (d_R)_i, \ L_i, \ (e_R)_i) \ , \tag{1.8}$$

where Q_i and L_i represent quark and lepton iso-doublets and $(u_R)_i, (d_R)_i$ and $(e_R)_i$ are the up type, down type quark and charged lepton iso-singlets respectively. $\mathcal{D}_\mu$ represents the covariant derivative of the field and is given as:

$$\mathcal{D}_\mu = \partial_\mu - ig_s G^A_\mu \lambda^A - i\frac{g}{2} W^I_\mu \tau^I - ig' B_\mu Y \,. \tag{1.9}$$

Here $A = 1, ..., 8$ with G^A_μ representing the $SU(3)_C$ gauge bosons, $I = 1, 2, 3$ with W^I_μ representing the $SU(2)_L$ gauge bosons and B_μ representing $U(1)_Y$ gauge boson. The self interactions of the gauge fields are given as:

$$\mathcal{L}_{YM} = -\frac{1}{4} G^{\mu\nu A} G^A_{\mu\nu} - \frac{1}{4} W^{\mu\nu I} W^I_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \,, \tag{1.10}$$

with

$$\begin{aligned} G^A_{\mu\nu} &= \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_s\, f_{ABC} G^B_\mu G^C_\nu \\ W^I_{\mu\nu} &= \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g\, f_{IJK} W^J_\mu W^K_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \tag{1.11}$$

where $f_{ABC(IJK)}$ represent the structure constants of the $SU(3)(SU(2))$ group. The scalar part of the Lagrangian is given as:

$$\mathcal{L}_S = (\mathcal{D}_\mu H)^\dagger \mathcal{D}_\mu H - V(H), \tag{1.12}$$

where

$$V(H) = m_H^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2 \,, \tag{1.13}$$

and

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \sim \left(1,\ 2,\ \frac{1}{2}\right) \,. \tag{1.14}$$

The numbers in braces represent the transformation properties of Higgs under SM gauge group. Finally the Yukawa part is given by,

$$\mathcal{L}_{yuk} = h^u \bar{Q} u_R \tilde{H} + h^d \bar{Q} d_R H + h^e \bar{L} e_R H + h.c \,, \tag{1.15}$$

where $\tilde{H} = i\sigma^2 H^\star$. Here we have suppressed the generation indices.

Symmetry breaking is induced by:

$$\langle H^0 \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \,. \tag{1.16}$$

After symmetry breaking, the fields W_μ^a and B_μ recombine and re-emerge as physical photon field A_μ, a neutral massive vector particle Z_μ, and a charged doublet of massive vector particles $W_\mu^\pm$:

$$\begin{aligned} Z_\mu &= \frac{gW_\mu^3 + g'B_\mu}{(g^2+g'^2)^{1/2}} \equiv cos\theta_W W_\mu^3 + sin\theta_W B_\mu \\ A_\mu &= \frac{-g'W_\mu^3 + gB_\mu}{(g^2+g'^2)^{1/2}} \equiv -sin\theta_W W_\mu^3 + cos\theta_W B_\mu \\ W_\mu^\pm &= \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2), \end{aligned} \tag{1.17}$$

where the Weinberg angle θ_W is defined as $tan\theta_w = g'/g$. By examining the mass sector one can read off the masses of the resulting vector particles.

1.3 SM: Vices and Virtues

Having discussed the basic tenets of SM, let us probe a little further into what it actually achieved for us and where it fell short as a complete theory of fundamental particles [7]. On a positive note let us first list successes of SM.

- With the identification of a few symmetry principles and requirement of renormalizability, all particle types and interactions are rigidly determined. The spectrum and the assignment of the fermions under G_0, renders the theory anomaly free [8].

- SM is renormalizable [4] [9].

- The most striking aspect of SM is its exceptional agreement with all the charged and neutral current data. With the recent discovery of ν_τ, all the particles of SM except Higgs boson have been experimentally detected. Also one can fit all the low energy neutral current (NC) data just in terms of two parameters, $\sin^2\theta_W$ and ρ defined as:
$$\rho = \frac{M_W^2}{M_Z^2 cos^2\theta_W}\,. \tag{1.18}$$
Taking all the NC data into account, ρ is required to be close to unity. This is guaranteed by SM.

- In SM weak and electromagnetic interactions are mixed up together, although it is incorrect to speak of a true unified picture of these two interactions.

[4]This virtue of SM is due the happy coincidence that interactions treated by SM do not have couplings with negative mass dimensions.

- Extremely economical prescription of a single Higgs doublet correctly accounts for the symmetry breaking and masses for all the fermions and gauge bosons, without affecting the renormalizability.

- At the tree level, no flavour changing neutral currents (FCNCs) arise (GIM mechanism). With atleast three generations CP violation is predicted. The recent discovery of CP violation in B system by the BaBar [10] and Belle [11] in accordance with the predictions of KM theory is a triumph of the SM. Also It is possible to arrange the parameters in the Yukawa sector in order to get phenomenologically acceptable values of the mixing angle in the KM matrix.

- Baryon(B) and Lepton(L) numbers are accidental global symmetries, atleast at the purturbative level. This is in concordance with extremely strong bounds on B and L violating processes.

At present SM is extraordinary successful. The achieved accuracy of its predictions corresponds to the experimental data within 5% [12]. In spite of its stupendous success, there are strong indications from theory as well as experiments, which require extensions/modifications of SM [13]. We shall enlist a few of them and then discuss at length only those which are central to our work.

§ *Theoretical Shortcomings*

- The major shortcoming of the SM is the so called *hierarchy problem* [4]. Higgs particle gives mass to the entire spectrum of SM particles without affecting the renormalizability of SM. The introduction of scalar Higgs being *ad hoc*, its mass is not protected by any symmetry against quadratically divergent quantum corrections. This drives the Higgs mass to the highest possible mass scale (M_X), where new physics sets in. This can spoil the entire SM spectrum which critically depends on the stability of Higgs mass.

- The SM cannot explain quantization of electric charge.

- The SM does not represent a unified description of the fundamental interactions. Apart from the fact that gravity is completely left out of picture, strong force is not unified with the weak and the electromagnetic forces. Strictly speaking, not even weak and electromagnetic forces are unified in SM. The couplings g_s, g and g' have quite different values whilst in a unified picture we would have expected an equality of the strength of different forces.

- The SM contains 21 *a priori* free parameters (3 coupling constants, 12 fermion masses, 4 fermion mixing parameters, 1 Higgs mass, 1 independent gauge boson mass). Any fundamental theory should be able to explain experiments in terms of a few basic parameters.

- What is usually referred to as 'fermion mass problem' remains a complete mystery. Why is there a replication of families for atleast three times? Why do fermion masses show a strange geometrical hierarchy among families, when the mass ratios are expressed as powers of cabibo angle ? Fermion masses even within a family are completely unpredictable. Also the parameters of the KM matrix are inputs of the theory, instead of being predicted by the model.

- The SM cannot explain the generation of matter-antimatter asymmetry. Also it cannot provide a candidate for the dark matter.

§ *Experimental Indication*

- Within SM, neutrinos are massless by design which does not allow for the right handed neutrinos or the triplet Higgs. Recent experiments at Kamiokande, Super-Kamiokande and Sudbury which measure neutrino flux from Atmosphere and Sun, have reported a deficit in the neutrino flux as due to neutrino flavour oscillations the [1, 2, 30, 30, 32]. Neutrino oscillations are considered to be unambiguous signatures of non-degenerate neutrino masses. This calls for a drastic revision of SM to incorporate massive neutrinos [27]. At present, positive evidence for massive neutrinos is the only experimental indication for Physics beyond SM.

It is clear that even if one ignores aesthetic quest for unification, the above mentioned experimental result provides compelling reasons, to look for Physics beyond the Standard Model. Models of Physics beyond SM tend to fall into two broad categories. (a) Models which radically differ from SM in their basic premises like the nature of fundamental entities (Strings, branes etc) and/or the dimensionality of space-time (models with large extra dimensions). (b) Models which retain the basic premises of SM but augment the space-time and/or internal symmetries of SM. Grand Unified Theories, left-right symmetric theories and supersymmetry are examples of the latter class of models. It should be noted that models in class (a) and (b) are not mutually exclusive and indeed there exists attractive variants which combine the properties of class (a) and (b) both. As already mentioned in section 1.1, all these models are severely constrained by the requirement of reproducing SM as a low energy limit which is vindicated brilliantly by the experiments. Since the problems addressed in this thesis are the so called low energy phenomena accessible to present and future experiments, we shall retain the basic premises of SM and make extensions only in the symmetry principles of SM. One such attractive possibility is the existence of supersymmetry (SUSY). Though invented as an exercise in group theory, it has the potential of solving the infamous hierarchy problem, the quest for Unification and the dark matter puzzle. More importantly it contains natural solutions to the above mentioned discrepancy in theory and experiments, namely Solar and Atmospheric neutrino deficits.

1.3.1 Naturalness: An Excuse for Supersymmetry ?

The guiding philosophy behind 'naturalness' is as follows: the effective interactions at a large length scale, corresponding to a low energy scale μ_1, should follow from the properties at a much smaller length scale, or higher energy scale μ_2, without the requirement that various different parameters at the energy scale μ_2 match with an accuracy of the order of μ_1/μ_2. That would be unnatural[4]. On the other hand, if at the energy scale μ_2 some parameters would be very small, say $\alpha(\mu_2) = O(\mu_1/\mu_2)$ then this may still be natural provided that this property would not be spoilt by any higher order effects. That is, at any energy scale μ, a physical parameter or a set of parameters $\alpha_i(\mu)$ is allowed to be very small only if the replacement $\alpha_i(\mu) = 0$ would increase the symmetry of the system. For instance, at a mass scale $\mu = 50$ GeV, the electron mass m_e is 10^{-5} GeV. This is a small parameter and is acceptable because $m_e = 0$ would imply an additional chiral symmetry corresponding to separate conservation of left-right chiral electron like leptons. This guarantees that all renormalizations of m_e are proportional to m_e itself. Similarly gauge coupling constants and other sets of interaction parameters may be small because putting them equal to zero would turn the gauge bosons or other particles into free particles so that they are separately conserved.

The difficulties with unnatural mass parameters only occur in theories with fundamental scalar fields and hence SM with *adhoc* scalar Higgs is plagued with the problem of "naturalness". The Higgs mass-squared, m_H^2, is small at energy scale $\mu \gg m_H$. Is there an approximate symmetry if $m_H \to 0$? One might consider a Goldstone type symmetry:

$$H(x) \to H(x) + const. \tag{1.19}$$

However we also have the local gauge transformations:

$$H(x) \to \Omega(x) H(x)\,. \tag{1.20}$$

Above transformations only form a closed group if we also have invariance under

$$H(x) \to H(x) + c(x)\,. \tag{1.21}$$

But then it becomes possible to transform Higgs away completely which makes Higgs unphysical. Alternatively, we could have that eq.(1.19) is an approximate symmetry only, and it is broken by all interactions that have to do with the eq.(1.20) which are the gauge field interactions that have the strength of $O(1/137)$. So at best we can have that the symmetry is broken by $O(1/137)$ effects. Therefore,

$$\frac{m_H^2}{\mu^2} \geq O(1/137). \tag{1.22}$$

Also the self interaction term of Higgs field breaks this symmetry. Therefore,

$$\frac{m_H^2}{\mu^2} \geq O(\lambda) \geq O(1/137). \tag{1.23}$$

Now $m_H^2 \sim O(\lambda v^2)$ where $v = 174$ GeV is the *vev* of Higgs. This implies

$$\mu \leq O(v) = O(174GeV). \tag{1.24}$$

This means that at energy scales much beyond v, our model becomes more and more unnatural. This is reflected in the instability of Higgs mass under quantum corrections which diverge quadratically. The renormalized Higgs mass is given as,

$$m_{ren.}^2 \sim m_{bare}^2 + \frac{1}{16\pi^2}\Lambda^2, \tag{1.25}$$

for all dimensionless couplings of order one. Λ is the cut off scale for the quadratically divergent integral, where new Physics sets in to alter the high energy behaviour. Thus in SM, the Higgs mass gets driven to the highest possible scale such as M_{planck}. For correct electroweak (EW) breaking we require $m_{ren.}^2 \sim (100GeV)^2$. This can be achieved by cancellations between m_{bare}^2 and the quantum corrections, which is of the order of one part in $\Lambda^2/(100GeV)^2$. For $\Lambda = M_{planck}$ this calls for enormous fine tuning in the bare Higgs mass parameter and the quantum corrections, which is unnatural. Such a problem does not occur for dimensionless couplings, since the quantum corrections are proportional to the logarithm of the cut-off scale or for the fermions masses which are protected by chiral symmetries. Supersymmetry (SUSY) provides a solution to the gauge hierarchy problem as it relates fermions and bosons, by putting them in the same representations of SUSY transformation. Now the Higgs mass gets self energy corrections from fermionic and bosonic loops which have a relative negative sign and hence give rise to cancellations required to maintain the correct hierarchy between the EW scale and the Planck scale.

1.4 Supersymmetry

The generators of space-time and internal symmetries commute with each other and hence internal symmetries relate particles with same mass and spin. Supersymmetric field theories are based on the supersymmetry algebra, a graded extension of the Poincare algebra, obtained from the latter by adding generators of fermionic character, obeying anti-commutation relations [21]. The basic anti-commutation relations of the supersymmetry algebra are, in two component notation:

$$\left\{Q_\alpha, \bar{Q}_{\dot\alpha}\right\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu \ , \quad \{Q_\alpha, Q_\beta\} = \left\{\bar{Q}_\alpha, \bar{Q}_{\dot\beta}\right\} = 0 \, . \tag{1.26}$$

The most convenient way to classify the representations of SUSY and to construct actions invariant under the SUSY, is to make use of superfields formalism given by Salam and stathdee [22]. Superspace is defined via the generalized coordinates $z = (x, \theta, \bar\theta)$, where x are the usual space-time coordinates, and θ, $\bar\theta$ are the two component anti-commuting coordinates. A superfield is a function in superspace, and can be expanded in ordinary fields as follows:

$$\begin{aligned} \phi(z) &= f(x) + \theta\chi(x) + \bar\theta\bar\chi(x) + \theta\theta m(x) + \bar\theta\bar\theta n(x) + \\ &+ \theta\sigma^\mu\bar\theta v_\mu(x) + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\psi(x) + \bar\theta\bar\theta\theta\theta d(x). \end{aligned} \tag{1.27}$$

To obtain irreducible linear representations of supersymmetry, suitable constraints must be imposed on the generic superfields. The two types of supermultiplets used in the construction of globally supersymmetric extensions of the SM, are the chiral and vector superfields. In a convenient basis for the superspace coordinates $(y = x + i\theta\sigma^\mu\bar\theta)$, chiral superfields have the following simple expansion:

$$\Phi(y, \theta) = \varphi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y), \tag{1.28}$$

where φ is a complex spin-0 field, ψ is a left handed two component spinor and F a complex scalar, corresponding to an auxiliary non-propagating field. In the Wess-Zumino gauge, vector superfields can be expanded as

$$V(x, \theta, \bar\theta) = -\theta\sigma^\mu\bar\theta V_\mu(x) + i\theta\theta\bar\theta\bar\lambda(x) - i\bar\theta\bar\theta\theta\lambda(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(x) \, , \tag{1.29}$$

where V_μ is a real spin-1 field, λ and $\bar\lambda$ are two component spinors of opposite chiralities, and D is a real scalar auxiliary field.

1.4.1 Minimal Supersymmetric Standard Model (MSSM)

The construction of chiral and vector superfield then provides a simple recipe for supersymmetrization of SM: promote all fermionic fields to chiral superfields and all gauge fields to vector superfields . This implies a fermionic (bosonic) degree of freedom for every SM bosonic (fermion) degree of freedom. What about the Higgs field ? It turns

Table 1.2: *Dramatis Super-personae*: In this table we list the spectrum of SM particles along with their superpartners. Here $a = 1, 2, ..., 8$ and $k = 1, 2, 3$ are $SU(3)$ and $SU(2)$ indices, respectively, and $i = 1, 2, 3$ is the generation index.

Superfield	Bosons		Fermions		$SU_c(3)$	$SU_L(2)$	$U_Y(1)$
Gauge							
$\mathbf{G^a}$	gluon	g^a	gluino	$\tilde{g}^a$	8	0	0
$\mathbf{V^k}$	Weak	W^k $(W^\pm, Z)$	wino, zino	$\tilde{w}^k$ $(\tilde{w}^\pm, \tilde{z})$	1	3	0
$\mathbf{V'}$	Hypercharge	B (γ)	bino	$\tilde{b}(\tilde{\gamma})$	1	1	0
Matter							
$\mathbf{L_i}$	sleptons	$\tilde{L}_i = (\tilde{\nu}, \tilde{e})_L$	leptons	$L_i = (\nu, e)_L$	1	2	-1
$\mathbf{E_i^c}$	sleptons	$\tilde{E}_i^c = \tilde{e}_R^c$	leptons	$E_i^c = e_R^c$	1	1	2
$\mathbf{Q_i}$	squarks	$\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$	quarks	$Q_i = (u, d)_L$	3	2	1/3
$\mathbf{U_i^c}$	squarks	$\tilde{U}_i^c = \tilde{u}_R^c$	quarks	$U_i^c = u_R^c$	3^*	1	$-4/3$
$\mathbf{D_i^c}$	squarks	$\tilde{D}_i^c = \tilde{d}_R^c$	quarks	$D_i^c = d_R^c$	3^*	1	2/3
Higgs							
$\mathbf{H_1}$	Higgses	H_1	higgsinos	$\tilde{H}_1$	1	2	-1
$\mathbf{H_2}$	Higgses	H_2	higgsinos	$\tilde{H}_2$	1	2	1

out that in the framework of supersymmetry, we need two Higgs doublets instead of one Higgs doublet of SM. There are two reasons for this. The super-potential is a holomorphic function of chiral superfields and hence one Higgs doublet cannot give masses to both up and down type of quarks. Moreover there exist triangle anomalies which miraculously canceled within SM owing to its particle content. So with the enlarged particle content of supersymmetry, we need two Higgs doublet with opposite hypercharge for these triangle anomalies to cancel. With this the particle content of the MSSM is as given in the table 1.2.

If supersymmetry is exact, superpartners of ordinary particles should have the same mass as their ordinary partners and should have been observed in accelerators by now. The fact that they have not been observed till now, implies that they are much heavy and hence SUSY must be a broken symmetry. Thus it is clear that the complete Lagrangian must contain SUSY conserving as well as SUSY breaking parts.

§ *The MSSM Lagrangian*

The Lagrangian of the MSSM consists of two parts, the first part is SUSY generalization of the Standard Model, while the second part represents SUSY breaking as mentioned above.

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}, \tag{1.30}$$

where,

$$\mathcal{L}_{SUSY} = \mathcal{L}_{Gauge} + \mathcal{L}_{Yukawa}, \tag{1.31}$$

and

$$\begin{aligned} \mathcal{L}_{Gauge} &= \sum_{SU(3),SU(2),U(1)} \frac{1}{4}\left(\int d^2\theta\ TrW^{\alpha}W_{\alpha} + \int d^2\bar{\theta}\ Tr\bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}}\right) \\ &+ \sum_{Matter} \int d^2\theta d^2\bar{\theta}\ \Phi_i^{\dagger} e^{(g_3V_3+g_2V_2+g_1V_1)}\Phi_i, \qquad (1.32) \\ \mathcal{L}_{Yukawa} &= \int d^2\theta\ (W_R + W_{\not R}) + h.c.. \qquad (1.33) \end{aligned}$$

Here W_α is the field strength tensor which is needed to construct gauge invariant Lagrangian. The index R in a superpotential refers to the so-called R-parity symmetry which assigns a "+" charge to all the ordinary particles and a "−" charge to their superpartners [23]. The first part of W is R-symmetric, given as:

$$W_R = y_{ij}^U Q_i U_j^c H_2 + y_{ij}^D Q_i D_j^c H_1 + y_{ij}^L L_i E_j^c H_1 + \mu H_1 H_2, \tag{1.34}$$

where $i, j = 1, 2, 3$ are are the generation indices. We have suppressed the $SU(2)$ indices. This part of the Lagrangian almost exactly repeats that of the SM, except that the fields are now superfields rather than the ordinary fields of the SM. The only difference is the last term which describes the Higgs mixing. It is absent in the SM since we have only one Higgs field there.

The second part is R-nonsymmetric

$$W_{\not R} = \lambda_{ijk}^L L_i L_j E_k^c + \lambda_{ijk}^{L\prime} L_i Q_j D_k^c + \epsilon_i L_i H_2 + \lambda_{ijk}^B U_i^c D_j^c D_k^c. \tag{1.35}$$

These terms are absent in the SM. The reason is very simple: one can not replace the superfields in eq.(1.35) by the ordinary fields like in eq.(1.34) because of the Lorentz invariance. These terms violate either lepton (the first line in eq.(1.35)) or baryon number (the second line). Since both effects are not observed in Nature, these terms must be suppressed or be excluded. In the minimal version of the MSSM these terms are not included, they are forbidden by R-parity conservation.

§ *Breaking it Softly*

Since superpartners are not yet found in accelerators, SUSY must be a broken symmetry. How supersymmetry is actually broken is still debated, but it certainly cannot happen

in the standard manner of spontaneous symmetry breaking within MSSM, due to certain phenomenological problems. The standard lore is that SUSY is broken spontaneously, but in some hidden sector, which contains a set of fields that do not interact with the MSSM fields (which is also called observable sector). The SUSY breaking in the hidden sector is then communicated to the observable sector via a messenger sector, which contains fields interacting with both the sectors. One possibility is that gravity serves as a messenger sector (as it is a universal interaction). This generates SUSY breaking terms in the potential at a high scale such as Grand Unification scale (GUT). These terms are called "soft terms" as they do not (re)introduce quadratic divergences in the theory [24]. These soft terms are basically (i) the mass terms for scalars, (ii) the mass terms for the superpartners of gauge bosons (gauginos) and (iii) the bilinear and trilinear scalar couplings. Addition of these terms not only introduces many new parameters in the theory but can also lead to dangerously fast FCNCs (Flavour Changing Neutral Currents) which have stringent constraints from experiments. The following assumptions are made at the high unification scale, so as to have phenomenologically acceptable spectrum of sparticles at the weak scale. These assumptions also drastically reduce the number of free parameters.
a). All the scalar masses (m) are equal at the GUT scale .
b). All the gaugino masses (M) are equal at the GUT scale.
c). All the trilinear couplings (A) are equal at the GUT scale.

A typical phenomenological approach parameterizes the ignorance of mode of SUSY breaking in the form of $\mathcal{L}_{soft}$, by adding it explicitly to the total Lagrangian, which is now given by,

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}\,. \tag{1.36}$$

Any underlying theory of SUSY breaking should naturally give the terms in $\mathcal{L}_{soft}$ and explain their 'universality'. Indeed it is possible to obtain these terms via super-gravity mechanism (mSUGRA) and are usually introduced at the GUTscale [25].

Another attractive alternative for mediation of SUSY breaking is via gauge interactions; the so called gauge mediated SUSY breaking (GMSB) [26]. The basic idea is to introduce some new chiral supermultiplets , called messengers, which couple to ultimate source of SUSY breaking, and which also couple indirectly to MSSM fields through the ordinary gauge interactions. The soft terms obtained in this manner will be flavour diagonal as they are generated by flavour blind gauge interactions. The minimal version of this model, also known as Minimal Messenger Model (MMM), has the advantage of economy as the entire soft spectrum gets determined in terms of essentially one parameter (characterizing the messenger scale). SUSY breaking scale in GMSB models can be as low as 100 TeV. In both, mSUGRA as well as GMSB case, the low energy parameters get determined by Renormalization Group (RG) scaling of soft parameters which are evolved from high scale boundary conditions.

§ *When $SU(2) \otimes U(1)$ must break*

Within SM, the breaking of $SU(2) \otimes U(1)$ is somewhat *ad hoc* by putting a -ve sign for the scalar mass term. In MSSM, the evolution of the up type Higgs mass term from high GUT scale is such that it attains -ve value around EW scale. Thus it naturally leads to the breaking of electroweak symmetry radiatively. Imposing the condition that the electroweak symmetry is broken at the correct scale would lead to minimization conditions for μ and B_μ. B_μ is the soft term corresponding to the bilinear coupling μ in the superpotential. The conditions are given as:

$$\begin{aligned} \frac{1}{2}M_Z^2 &= \frac{m_{H_1}^2 - tan^2\beta m_{H_2}^2}{tan^2\beta - 1} - \mu^2 \\ sin2\beta &= \frac{2B_\mu\mu}{m_{H_2}^2 + m_{H_1}^2 + 2\mu^2}. \end{aligned} \tag{1.37}$$

The remaining parameters of the MSSM are given by m, M, A,the sign of μ and the ratio of vacuum expectation values of the up and down type Higgs, given as tanβ.

1.5 Motivation and Outline of Thesis

In the wake of recent experimental evidence for massive neutrinos, SM needs drastic revisions/extensions in its frame-work which is based on the assumption of massless neutrinos. These indications have come from the measurement of the flux of Atmospheric and Solar neutrinos, which not only have reported the deficit in the flux but also have reported possibility of neutrino oscillations. Neutrino oscillations are an unambiguous signature of massive (non-degenerate) neutrinos. In this thesis we have focused on the problem of neutrino masses and mixing. The distinctive traits of neutrinos are: extremely tiny masses and large mixing angles, in contrast with the quarks which have large masses and small mixing in the family space. This hints towards a different origin for neutrino masses. Supersymmetry without R-parity is an extension of SM, which can naturally accommodate small neutrino masses and large mixing. We work in such a frame-work and give simultaneous solutions for Atmospheric and Solar neutrino anomalies [12, 13].

In models with R-violation, neutrino mixing is largely determined by ratios of various R-violating parameters whereas the scale of neutrino masses and hierarchy is determined by SUSY breaking soft parameters. This immediately relates the important question of universality in soft sector, to the various possible solutions to the solar and atmospheric neutrino anomalies. We make a model independent investigation of departure from universality, required to obtain the experimentally favoured, two large mixing angle solutions to neutrino anomalies [13].

Having discussed the implications of R-violating couplings for neutrino anomalies, it is important to understand the origin of these couplings which have been treated as free parameters in the problem of neutrino anomalies. In another investigation, we systematically study the structure and magnitude of these couplings, following an abelian family symmetry, which is spontaneously broken at high (Planck) scale. Such a symmetry has proved to be an attractive way of understanding the geometrical hierarchy and small mixing in the quark sector. We study its implications for R-violation, complying with necessary phenomenological constraints [29].

The organization of the thesis is as follows: In chapter 2 we briefly discuss the Renormalization Group Equations in MSSM with R-parity Violation. In chapter 3 we briefly discuss the physics of massive neutrinos and the experimental status of neutrino oscillations. In chapter 4 we very briefly describe the introduction of R-parity and its connection with R-symmetries. In chapters 5, 6 and 7 we discuss the problems addressed in thesis work. In chapter 5, we address the possibility of two large mixing angle solution to atmospheric and solar neutrino anomalies in the framework of bilinear R-parity violation. In Chapter 6, we describe the simultaneous solution to neutrino anomalies with trilinear R violating couplings, in the framework of Gauge Mediated Supersymmetry breaking. In chapter 7 we discuss the patterns and magnitude of R-violation following, from a spontaneously broken Abelian Family symmetry. Chapter 8 summarizes the thesis.

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Chapter 2

Renomalization Group Equation

2.1 The basic idea

Any classical field theory, in the limit of vanishing particle masses (more generally, vanishing dimensionful couplings) will be invariant under scale transformations or dilatations. This could mean that at sufficiently high energies, particle masses do not play an important role, and by studying these approximate symmetries one can infer about the asymptotic regime where generally perturbative techniques fail in theories like QED, which are not asymptotically free. It turns out that such innocent scaling behaviour of classical theory becomes complicated, when one studies the corresponding quantum theory *even in the massless limit* [1]. This is because scale invariance is anomalous at quantum level, due to the fact that any regularization scheme necessarily introduces a mass scale into the theory, thereby violating scale invariance [1].

The momentum scale μ at which we define renormalized masses and couplings, is called the *subtraction point* or the *renormalization point*. The subtraction point μ is introduced purely as a mathematical device, to begin the process of renormalization, and that no physical consequences could emerge from it. If we change the subtraction point μ, other parameters, such as masses and coupling constants must also change in order to compensate for this effect. This can be very easily seen from the point of view of multiplicative renormalization, which expresses a multiplicative relation between the vertex functions of unrenormalized theory $\Gamma_0^{(n)}$, and the vertex functions of the renormalized theory $\Gamma^{(n)}$. However, since the unrenormalized vertex function $\Gamma_0^{(n)}$ is totally independent of the subtraction point μ (since subtractions are computed only for the renormalized vertex); we have,

$$\frac{\partial}{\partial\mu}\Gamma_0^{(n)} = 0. \tag{2.1}$$

Thus, in order to keep the unrenormalized vertex function $\Gamma_0^{(n)}$ independent of μ, there exists a nontrivial relation between the renormalized $\Gamma^{(n)}$ and Z (the wavefunction renor-

[1]Existence of anomalies is not peculiar to quantum field theory but can also be seen in elementary quantum mechanics as beautifully demonstrated in [2].

malization), which is expressed mathematically as Renormalization Group (RG) equations. This can be easily seen from the following relation between $\Gamma_0^{(n)}$ and $\Gamma^{(n)}$ in ϕ^4 theory.

$$\Gamma_0^{(n)}(p_i, g_o, m_0) = Z_\phi^{-n/2}\Gamma^{(n)}(p_i, g, m, \mu), \tag{2.2}$$

where p_i are the momenta of the external lines, $g(g_0)$, $m(m_0)$ are the renormalized (unrenormalized) coupling constant and mass. We assume that the theory has been regulated using dimensional regularization by working in d dimensions ($d = 4 - \epsilon$). We choose as our independent variables μ, g, m and differentiate with respect to dimensionless derivative $\mu(d/d\mu)$. This gives (in the limit $\epsilon \to 0$):

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - n\gamma(g) + m\gamma_m(g)\frac{\partial}{\partial m}\right)\Gamma^{(n)}(p_i, g, m, \mu) = 0, \tag{2.3}$$

where,

$$\beta(g) \equiv \mu\frac{\partial g}{\partial\mu} \quad ; \quad \gamma(g) \equiv \mu\frac{\partial}{\partial\mu}\log\sqrt{Z_\phi} \tag{2.4}$$

$$m\gamma_m(g) \equiv \mu\frac{\partial m}{\partial\mu}. \tag{2.5}$$

These are the Renormalization Group equations and they express how the renormalized vertex function changes, when we make a change in the subtraction point μ. Our goal, however, is to analyze the behaviour of the theory at high energies, so let us make the following scale transformation to obtain a slightly different constraint on the vertex functions. If we scale the momenta as $p_i \to tp_i$, then using the dimensional arguments one can show that the vertex function behaves as:

$$\Gamma^{(n)}(tp_i, g, m, \mu) = \mu^D F\left(g, \frac{t^2p_i^2}{m\mu}\right), \tag{2.6}$$

where D is the mass dimension of the vertex function. This in turn implies that the vertex function obeys the following equation:

$$\left(t\frac{\partial}{\partial t} + m\frac{\partial}{\partial m} + \mu\frac{\partial}{\partial\mu} - D\right)\Gamma^{(n)} = 0. \tag{2.7}$$

Now eliminating the term $\mu(\partial/\partial\mu)\Gamma$ from this equation using eq. (2.3). we find:

$$\left[-t\frac{\partial}{\partial t} + \beta(g)\frac{\partial}{\partial g} - n\gamma(g) + m(\gamma_m(g) - 1)\frac{\partial}{\partial m} + D\right]\Gamma^{(n)}(tp, g, m, \mu) = 0. \tag{2.8}$$

Note that if $\beta(g) = \gamma(g) = \gamma_m(g) = 0$, the scaling of $\Gamma^{(n)}$ is simply given by canonical dimension D, as would be expected from naive scaling argument. It is the effects of *interactions* which give rise to need for renormalization and therefore non-vanishing functions

$\beta(g), \gamma(g), \gamma_m(g)$ and hence departure from pure scaling behaviour of vertex functions. If we start with a *massless* theory, the Lagrangian is *scale invariant*, but the vertex functions are not, because $\beta(g)$ and $\gamma(g)$ are non-vanishing. They contribute to so called "anomalous dimensions" [2]. Hence their origin lies in renormalization, which inevitably introduces a *scale*, in the shape of an arbitrary mass μ in dimensional regularization, or in the shape of momentum cut-off Λ in a cut-off regularization, so even a scale invariant classical theory does not give rise to scale invariant quantum theory.

We now wish to find the solution of eq. (2.8). The equation expresses the fact that a change in t may be compensated by a change in m and g, and an overall factor. So we expect that there should be functions $g(t), m(t)$, and $f(t)$ such that,

$$\Gamma^{(n)}(tp, m, g, \mu) = f(t)\Gamma^{(n)}(p, m(t), g(t), \mu). \tag{2.9}$$

Differentiating with respect to t and comparing with eq. (2.8), the coefficient of $\partial/\partial m$ is given as:

$$t\frac{\partial g(t)}{\partial t} = \beta(g); \tag{2.10}$$

where $g(t)$ is called the *running coupling constant.* Knowledge of the function $\beta(g)$ enables us to find $g(t)$; and of particular interest is the asymptotic limit of $g(t)$ as $t \to \infty$. Just like eq. (2.10), one can obtain the differential equation for m and f as:

$$t\frac{\partial m}{\partial t} = m[\gamma_m(g) - 1], \tag{2.11}$$

$$\frac{t}{f}\frac{df}{dt} = D - n\gamma(g). \tag{2.12}$$

This equation can be integrated to give

$$f(t) = t^D\ exp\left[-\int_0^t \frac{n\gamma(g(t))dt}{t}\right], \tag{2.13}$$

which on substitution into (2.9) gives (using (2.3) and taking the limit $\epsilon \to 0$)

$$\Gamma^{(n)}(tp, m, g, \mu) = t^{4-n}exp\left[-n\int_0^t \frac{\gamma(g(t))dt}{t}\right]\Gamma^{(n)}(p, m(t), g(t), \mu). \tag{2.14}$$

This is the solution to the RG equation (2.8), in terms of running coupling constant $g(t)$ and running mass $m(t)$. The exponential term is the 'anomalous dimension'. The physics

[2] It is interesting to note that when one naively derives the ward identities for broken scale invariance in renormalized perturbation theory it leads to disastrous behavior for renormalized vertex functions in deep Euclidean region (absence of logarithmic factors in every order of perturbation theory), due to the bounds on the behaviour of vertex functions derived by Weinberg [3]. Coleman and Jackiw [1] could repair the damage by changing the scale dimensions of the fields (hence the term "anomalous dimension"), but the method failed at higher orders. Eq. (2.8) clearly demonstrates that *ad hoc* change of scale dimensions cannot solve the problem and that one needs to know the form of β and γ.

at large momentum is governed by $m(t)$ and $g(t)$, and a particular use of the RG equation is to study the large (or even the small) momentum behaviour of quantum field theories. These equations were first derived in the context of QED by Gellman and Low in [4] (for a pedagogic derivation please see [5, 6]). For the RG study of the standard model, see [7].

Almost all extensions of SM (such as unification models, supersymmetry, large extra dimension) probe physics at higher energy scale, and hence RG equations are an inevitable tool to extrapolate physics at higher energy scale using experimental inputs at the weak scale. In this thesis, we are interested in the study of neutrino masses and mixing as implied by R-parity violating SUSY. We will see in chapter 5 and 6, that the typical scale for neutrino masses is determined by SUSY breaking soft parameters (at weak scale), whereas the mixing among neutrinos is determined by ratios of various R-violating couplings. Thus we need to know the values of these parameters at the weak scale. Typically, these parameters are specified as boundary conditions at the GUT scale, and from there they are evolved to the weak scale using Renormalization Group Equations. These are coupled differential equations and can be solved semi-analytically only in the limit of low $\tan\beta$. Our study is largely confined to models with gauge mediated SUSY breaking which tends to favour large $\tan\beta$ values. So our analysis will be completely numerical and we will not discuss analytical and semi-analytical solutions, which are discussed in detail in [8]. Semi-analytical solutions involving R-violation are discussed at length in [9]. For numerical evaluation we will broadly follow the algorithm developed in [9] with some modifications.

In the next section, we compile the necessary RG equations for the various coupling constants and masses ,in a supersymmetric theory with or without R-parity (see, [9, 10, 11, 12]).

2.2 RG Flow in MSSM

The MSSM RG equations are given in Refs [13, 35, 15]; they are now known for the gauge couplings and superpotential parameters up to 3-loop order, and for the soft parameters at 2-loop order. However for many purposes it suffices to work at 1-loop order.

§ *Gauge Couplings: The (Non)supersymmetric (Non)unification*

Consider the RG equations for the gauge couplings in SM.

$$\dot{\tilde{\alpha}}_i = b_i \tilde{\alpha}_i, \tag{2.15}$$

where $\tilde{\alpha}_i = \frac{g_i^2}{16\pi^2}$ and $t = 2ln(\frac{M_X}{Q})$. M_X corresponds to high scale (could be a GUT scale for mSUGRA models and as low as $\sim 100\ TeV$ for Gauge mediated SUSY breaking) and

Q corresponds to weak scale. Here 'dot' represents a derivative with respect to 't'. The SM β functions b_i are $(-7,\ -19/6,\ 41/10)$. These equations can be trivially integrated and one finds the values at the high scale, in terms of experimentally measured inputs at weak scale. The solutions can be written as:

$$\tilde{\alpha_i}(0) = \frac{\tilde{\alpha_i}(t_z)}{(1 - b_i\tilde{\alpha_i}(t_z))}, \tag{2.16}$$

where t_z is the value of t at the weak scale, defined as the mass of Z boson M_Z. Now from the deep inelastic scattering of electron from proton, we know that gauge couplings for non-abelian theories *exclusively* enjoy the property of *asymptotic freedom*, in contrast to the abelian theories, which are not asymptotically free. This is amply suggestive of a possible unification of the three couplings at high scale, as shown by the classic work of Weinberg, Georgi and Quinn [16]. This opens an interesting possibility where the three different gauge groups (G_i for $i = 1, 2, 3$) at weak scale correspond to three different branches, embedded into a simple unifying gauge group (G) when extrapolated to high scale. So at the unification scale($t = 0$) one should expect equality of the gauge couplings, (i.e., $\frac{5}{3}\alpha_1(0) = \alpha_2(0) = \alpha_3(0)$) [3]. The gauge couplings though come fairly close to each other at the GUT scale, meeting at a single point is impossible. It is excluded by more than 8 standard deviations [17]. This means that unification can be achieved only if new physics enters between the electroweak and Planck scale. Introduction of supersymmetry (and hence superpartners) drastically changes the SM β functions and hence the evolution of gauge couplings in such a way, that they actually meet at a point. Thus, unification of gauge couplings provides strong reasons to take supersymmetric models seriously.

§ *Yukawa Couplings*

Along with the gauge coupling unification, certain grand unified theories also predict bottom and tau Yukawa coupling unification at the GUT scale. Renormalization Group studies of MSSM Yukawa couplings show $Y_b - Y_\tau$ unification for large range of $tan\beta$. Here $Y_{b.\tau} = \frac{h^2_{b,\tau}}{16\pi^2}$ and $h_{b,\tau}$ etc. are the Yukawa couplings. Here, we will use the approximation that only third generation Yukawa couplings are significant. Then the RG equations for Yukawa couplings are given as:

$$\dot{Y_t}(t) = Y_t\left(\frac{16}{3}\tilde{\alpha_3}(t) + 3\tilde{\alpha_2}(t) + \frac{13}{15}\tilde{\alpha_1}(t) - 6Y_t(t) - 6Y_b(t)\right) \tag{2.17}$$

$$\dot{Y_b}(t) = Y_b(t)\left(\frac{16}{3}\tilde{\alpha_3}(t) + 3\tilde{\alpha_2}(t) + \frac{7}{15}\tilde{\alpha_1}(t) - Y_t(t) - 6Y_b(t) - Y_\tau(t)\right) \tag{2.18}$$

[3] Since at the unification scale all different gauge groups must merge into a simple unification gauge group, all the low energy generators (T_a) must be normalized in the same way, satisfying the condition, $Tr(T_aT_b) = 2\delta_{ab}$. The factor of (5/3) for the $U(1)$ gauge group is this normalization factor [16].

$$\dot{Y}_\tau = Y_\tau(t)\left(3\tilde{\alpha}_2(t) + \frac{9}{5}\tilde{\alpha}_1(t) - 3Y_b(t) - 4Y_\tau(t)\right). \tag{2.19}$$

Note that the β functions for each superpotential parameter are proportional to itself. This is actually a consequence of a general and powerful *supersymmetric non-renormalization theorem*[18]. This theorem implies that the logarithmically divergent contributions to a given process can always be written in the form of wave-function renormalization, without any vertex renormalization. This is true for any supersymmetric theory, not just the MSSM and holds to all orders in perturbation theory.

§ *Soft Parameters*

In chapter 1 we saw that SUSY must be broken softly in order that the breaking does not reintroduce quadratic divergences. We also briefly discussed the nature of soft terms. The soft terms in the Lagrangian are given as:

$$\begin{aligned}
\mathcal{L}_{soft} = & \frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B} + c.c.\right) \\
& \left(A_u Y_u \tilde{U}^c\tilde{Q}H_2 + A_d Y_d \tilde{D}^c\tilde{Q}H_1 + A_e Y_e \tilde{E}^c\tilde{L}H_1 + c.c.\right) \\
& \tilde{Q}^\dagger m_Q^2 \tilde{Q} + \tilde{L}^\dagger m_L^2 \tilde{L} + \tilde{U}^c m_U^2 \tilde{U}^{c\dagger} + \tilde{D}^c m_D^2 \tilde{D}^{c\dagger} + \tilde{E}^c m_E^2 \tilde{E}^{c\dagger} \\
& + m_{H_2}^2 H_2^* H_2 + m_{H_1}^2 H_1^* H_1 - (B_\mu H_1 H_2 + c.c.) \\
& + A^{\lambda'}\tilde{L}\tilde{Q}\tilde{D}^c + A^{\lambda}_{ijk}\tilde{L}\tilde{L}\tilde{E}^c - \left(B_\epsilon \tilde{L}H_2 + c.c.\right).
\end{aligned} \tag{2.20}$$

Here we have suppressed the generation indices for simplicity. In the above, first line corresponds to gaugino masses, second line corresponds to trilinear soft terms, third and fourth line are the soft scalar masses and the last line corresponds to the lepton number violating trilinear and bilinear soft terms.

The 1-loop RG equations for gaugino mass parameters in the MSSM are determined by the same β functions, which appear in the gauge coupling RG equations:

$$\dot{M}_i = b_i\tilde{\alpha}_i M_i \quad (b_i = 33/5,\ 1,\ -3), \tag{2.21}$$

for $i = 1, 2, 3$. It is easy to see show that the three ratios M_i/g_i^2 are a constant upto small two loop corrections. In mSUGRA models we can therefore write:

$$M_i(Q) = \frac{g_i^2(Q)}{g_i^2(Q_0)} m_{1/2}, \tag{2.22}$$

(where $m_{1/2}$ is the common gaugino mass parameter at the high scale) at any RG scale $Q < Q_0$, where Q_0 is the input scale which is presumably nearly equal to Planck scale $M_P = 10^{19} GeV$. Since the gauge couplings are observed to unify at $M_U = 0.01 M_P$, one expects that all gauge couplings are equal at that scale. Therefore, one finds that

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2}, \tag{2.23}$$

at any RG scale, up to small two loop corrections. The common value in eq. (2.23) is also equal to $(m_{1/2})/g_U^2$ in mSUGRA models, where g_U is the unified gauge coupling at the input scale. Interestingly, eq. (2.23) is also the solution to the 1-loop RG equations in the case of gauge mediated boundary conditions, applied at the messenger scale. This is true even though there is no such thing as unified gaugino mass $m_{1/2}$ in the gauge mediated case, because of the fact that the gaugino masses at the high scale are proportional to g_a^2 times a constant. Thus eq. (2.23) is theoretically well motivated in both the frameworks of SUSY breaking. The prediction of eq. (2.23) is particularly useful since the gauge couplings g_i^2 are already quite well known at the electroweak scale from the experiment. Therefore, they can be extrapolated upto at least M_U, assuming that the apparent unification of gauge couplings is not a fake. The gaugino mass parameters feed into the RG equations for all other soft parameters as we will see.

The trilinear soft parameters A_u, A_d, A_e are matrices in generation space. In the approximation of neglecting first and second generation Yukawa couplings, one can therefore write, at any RG scale,

$$A_u \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_t \end{pmatrix}, \quad A_d \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_b \end{pmatrix}, \quad A_e \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_\tau \end{pmatrix}, \tag{2.24}$$

which defines running parameters A_t, A_b and A_τ. The RG equations for these parameters and the bilinear parameter B are given as:

$$\begin{aligned} \dot{A}_t &= -(GA)_t - 6A_tY_t - A_bY_b \\ \dot{A}_b &= -(GA)_b - 6A_bY_b - A_tY_t - A_\tau Y_\tau \\ \dot{A}_\tau &= -(GA)_\tau - 4A_\tau Y_\tau - 3A_bY_b \\ \dot{B}_\mu &= -4\left\{C_2\tilde{\alpha}_2M_2 + \frac{3}{5}\left(\frac{1}{2}\right)^2\tilde{\alpha}_1M_1\right\} - (A_\tau Y_\tau + 3A_bY_b + 3A_tY_t), \end{aligned} \tag{2.25}$$

where, $C_3 = 4/3, 0$ for triplets and singlets of $SU(3)_C$, $C_2 = 3/4, 0$ for doublets and singlets of $SU(2)_L$ and $C_i^Y = (13/36), (7/36), (3/4)$ for $i = t, b, \tau$. GA_i are given as:

$$(GA)_i = 4\left\{C_3\tilde{\alpha}_3M_3 + C_2\tilde{\alpha}_2M_2 + \frac{3}{5}C_i^Y\tilde{\alpha}_1M_1\right\}. \tag{2.26}$$

Note that the β function for each parameter is not proportional to the parameter itself; as these couplings violate supersymmetry and hence are not protected by supersymmetric non-renormalization theorem. In particular, if they vanish at the input scale (as for example in the gauge mediated supersymmetry breaking), the RG corrections proportional to the gaugino masses appearing in the eq. (2.25) ensure that they will still be non-zero at the electroweak scale.

§ *Scalar Masses and the Radiative Electroweak Symmetry Breaking*

Let us now consider the RG equations for the scalar masses in the MSSM. In the approximation of neglecting first two families yukawa couplings and eq. (2.24), the squarks and sleptons of first two families have only gauge interactions. Now as discussed in the chapter 1, scalar masses are constrained to be universal at input scale so that they do not produce large contributions FCNCs at electroweak scale which are already severely restricted. This means that if scalar masses are universal at input scale, then when renormalized to any other RG scale, they will still be almost diagonal, with the approximate form

$$m_Q^2 \approx \begin{pmatrix} m_{Q_1}^2 & 0 & 0 \\ 0 & m_{Q_1}^2 & 0 \\ 0 & 0 & m_{Q_3}^2 \end{pmatrix}; \quad m_{U^c}^2 \approx \begin{pmatrix} m_{U_1^c}^2 & 0 & 0 \\ 0 & m_{U_2^c}^2 & 0 \\ 0 & 0 & m_{U_3^c}^2 \end{pmatrix}; \tag{2.27}$$

etc. The first and second family squarks and sleptons with given gauge quantum numbers remain very nearly degenerate, but the third family squarks and sleptons feel the effect of larger yukawa couplings and so get renormalized differently. The 1-loop RG equations of first and second family squarks and sleptons squared masses can be written as:

$$(\dot{m_i^2})_{11,22} = 4\left\{C_3\tilde{\alpha}_3 M_3^2 + C_2\tilde{\alpha}_2 M_2^2 + \frac{3}{5}Y^2\tilde{\alpha}_1 M_1^2\right\}, \tag{2.28}$$

where $i = \tilde{Q}, \tilde{U^c}, \tilde{D^c}, \tilde{L}, \tilde{E^c}$. The 1-loop RG equations for the third generation sfermions and up and down type Higgs mass are given as:

$$\begin{aligned}
(\dot{m}_{\tilde{Q}}^2)_{33} &= (\dot{m}_{\tilde{Q}}^2)_{11,22} - Y_t(SS)_t - Y_b(SS)_b \\
(\dot{m}_{\tilde{U}^c}^2)_{33} &= (\dot{m}_{\tilde{U}^c}^2)_{11,22} - 2Y_t(SS)_t \\
(\dot{m}_{\tilde{D}^c}^2)_{33} &= (\dot{m}_{\tilde{D}^c}^2)_{11,22} - 2Y_b(SS)_b \\
(\dot{m}_{\tilde{L}}^2)_{33} &= (\dot{m}_{\tilde{L}}^2)_{11,22} - Y_\tau(SS)_\tau \\
(\dot{m}_{\tilde{E}^c}^2)_{33} &= (\dot{m}_{\tilde{E}^c}^2)_{11,22} - 2Y_\tau(SS)_\tau \\
(\dot{m}_{H_d}^2) &= (\dot{m}_{\tilde{L}}^2)_{11,22} - 3Y_b(SS)_b - Y_\tau(SS)_\tau \\
(\dot{m}_{H_u}^2) &= (\dot{m}_{\tilde{L}}^2)_{11,22} - 3Y_t(SS)_t,
\end{aligned} \tag{2.29}$$

where,

$$(SS)_t = (m_{\tilde{Q}}^2 + m_{\tilde{U}^c}^2)_{33} + m_{H_u}^2 + A_t^2$$

$$\begin{aligned}(SS)_b &= (m_{\tilde{Q}}^2 + m_{\tilde{D}^c}^2)_{33} + m_{H_d}^2 + A_b^2 \\ (SS)_\tau &= (m_{\tilde{L}}^2 + m_{\tilde{E}^c}^2)_{33} + m_{H_d}^2 + A_\tau^2.\end{aligned} \tag{2.30}$$

One can see that $(SS)_{t,b,\tau}$ are always positive, so their effect is always to decrease the Higgs masses as one evolves the RG equations downwards from the input scale to the electroweak scale. Top quark being heavier than other quarks and leptons, it can cause the RG evolved $m_{H_u}^2$ to run to negative value near the electroweak scale, thus generating a nonzero Higgs *vev* resulting in the electroweak symmetry breaking. Thus in supersymmetric theories, electroweak symmetry is broken naturally in a radiative manner by RG evolution.

Before discussing the RG equations for R-violating parameters, we give below the RG equation for the supersymmetric μ parameter.

$$(\dot{\mu})^2 = \left[4\left(C_2\tilde{\alpha_2} + \frac{3}{5}\left(\frac{1}{2}\right)^2\tilde{\alpha_1}\right) - (3Y_t + 3Y_b + Y_\tau)\right]\mu^2. \tag{2.31}$$

The low energy parameter μ is obtained from the minimization conditions of the scalar Higgs potential as discussed in chapter 1. This equation may help to trace back the high-energy value of this parameter.

2.3 RG Scaling and R-parity Violation

Introduction of R-violating couplings has already been discussed in chapter 1. As discussed there, we shall retain only L violating couplings which can naturally accommodate hierarchical neutrino masses and one or two large mixing angles as required by experiments. Though *a priori* arbitrary these couplings are severely constrained by dangerous contributions to FCNCs [17, 20]. They are also required to be suppressed ($\sim O(10^{-4})$) in order to generate small neutrino masses. Thus they do not significantly alter the MSSM RG equations [21]. Since our goal is to study neutrino masses from R-violation, we need to solve the RG equations for these couplings. Being large in number, we have studied the effects of bilinear and trilinear R-violation on neutrino masses separately for simplicity. The necessary RG equations have been derived using the general formulae given in [35]. Below we give the the RG equations for R-violating couplings. The equations presented here agree with the ones given in [17]. For other useful references, please see [22].

§ *Bilinear R-violation*

$$\begin{aligned}\dot{\epsilon_i}(t) &= \epsilon_i(t)\left(\frac{3}{2}\tilde{\alpha}_2(t) + \frac{3}{10}\tilde{\alpha}_1(t) - \frac{1}{2}Y_{ii}^E(t) - \frac{3}{2}Y_t(t)\right) \\ \dot{B_{\epsilon_i}}(t) &= B_{\epsilon_i}(t)\left(-\frac{1}{2}Y_{ii}^E(t) - \frac{3}{2}Y_t(t) + \frac{3}{2}\tilde{\alpha}_2(t) + \frac{3}{10}\tilde{\alpha}_1(t)\right) - \epsilon_i\left(3\tilde{\alpha}_2(t)M_2(t)\right.\end{aligned} \tag{2.32}$$

$$+ \quad \frac{3}{5}\tilde{\alpha}_1(t)M_1(t) + 3A_t(t)Y_t(t) + A^E_{ii}Y^E_{ii}(t)\Big) . \qquad (2.33)$$

§ *Trilinear R-violation* λ'

$$\begin{aligned}
\dot{\lambda}'_{ijk}(t) &= \lambda'_{ijk}(t)\left(-Y^E_{ii}(t) - Y^D_{jj}(t) - Y^U_{jj}(t) - 2Y^D_{kk}(t) - 3Y^D_{jj}(t)\delta_{jk} + \frac{8}{3}\tilde{\alpha}_3(t)\right.\\
&+ \left. 3\tilde{\alpha}_2(t) + \frac{7}{30}\tilde{\alpha}_1(t)\right) - \frac{6}{32\pi^2}\ h^D_{uu}(t)\lambda'_{iuu}(t)h^D_{jk}(t)\delta_{jk} \qquad (2.34)\\
\dot{B}_{\epsilon_i}(t) &= B_{\epsilon_i}(t)\left(-\frac{3}{2}Y_b(t) - \frac{1}{2}Y_\tau(t) + \frac{3}{2}\tilde{\alpha}_2(t) + \frac{3}{10}\tilde{\alpha}_1(t)\right)\\
&- \frac{3}{16\pi^2}\mu(t)\lambda'_{ijj}(t)h^d_{jj}(t)\left(\frac{1}{2}B_\mu(t) + A^{\lambda'}_{ijj}(t)\right) \qquad (2.35)\\
\dot{m^2_{\nu_i H_1}}(t) &= m^2_{\nu_i H_1}(t)\left(-\frac{3}{2}Y_b(t) - \frac{1}{2}Y_\tau(t)\right) - \frac{3}{32\pi^2}\lambda'_{ipp}(t)h^d_{pp}(t)\left(m^2_{H_1}(t)\right.\\
&+ \left. m^2_{L_i}(t) + 2m^2_{Q_p}(t) + 2A^{\lambda'}_{ipp}(t)A^D_{pp}(t) + 2m^2_{D_p}(t)\right) \qquad (2.36)\\
\dot{A}^{\lambda'}_{ijj}(t) &= -\frac{9}{2}A^D_{jj}(t)Y^Djj(t) - \frac{3}{2}A^{\lambda'}_{ijj}(t)Y^D_{jj}(t) - A^U_{jj}(t)Y^U_{jj}(t)\\
&- A^E_{ii}(t)Y^E_{ii}(t) - \frac{7}{30}M_1(t)\tilde{\alpha}_1(t) - 2M_2(t)\tilde{\alpha}_2(t) - \frac{16}{3}M_3(t)\tilde{\alpha}_3(t). \qquad (2.37)
\end{aligned}$$

§ *Trilinear R-violation* λ

$$\begin{aligned}
\dot{\lambda}_{ijk}(t) &= \lambda_{ijk}(t)\left(\frac{-1}{2}Y^E_{ii}(t) - \frac{1}{2}Y^E_{jj}(t) - \frac{1}{2}Y^E_{kk}(t) + \frac{3}{2}\tilde{\alpha}_2(t) + \tilde{\alpha}_1(t)\right)\\
&- \frac{1}{2}\left(\lambda_{ijj}(t)Y^E_{jj}(t)\delta_{j3}\delta_{jk} + \lambda_{jii}(t)Y^E_{ii}(t)\delta_{i3}\delta_{ik}\right) \qquad (2.38)\\
\dot{B}_{\epsilon_i}(t) &= B_{\epsilon_i}(t)\left(-\frac{3}{2}Y_t(t) - \frac{1}{2}Y^E_i(t) + \frac{3}{2}\tilde{\alpha}_2(t) + \frac{1}{2}\tilde{\alpha}_1(t)\right)\\
&- \frac{1}{32\pi^2}\mu(t)\lambda_{idd}(t)h^E_{dd}(t)\left(A^\lambda_{idd}(t) + \frac{1}{2}B_\mu(t)\right) \qquad (2.39)\\
\dot{m^2_{\nu_i H_1}}(t) &= m^2_{\nu_i H_1}(t)\left(-\frac{3}{2}Y^E_i(t) - \frac{1}{2}Y_\tau(t) - \frac{3}{2}Y_b(t)\right) - \frac{1}{32\pi^2}\lambda_{ijj}(t)h^E_{jj}(t)\left(m^2_{H_1}(t)\right.\\
&+ \left. m^2_{L_i}(t)2m^2_{L_j}(t) + 2A^\lambda_{ijj}(t)A^E_{jj}(t) + 2m^2_{E_j}\right) \qquad (2.40)\\
\dot{A}^\lambda_{ijj}(t) &= -3M_1(t)\tilde{\alpha}_1(t) - 3M_2(t)\tilde{\alpha}_2(t) - \frac{7}{2}A^E_{jj}(t)Y^E_{jj}(t) - \frac{3}{2}A^\lambda_{ijj}(t)Y^E_{jj}(t). \qquad (2.41)
\end{aligned}$$

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Chapter 3

Neutrino Anomalies

3.1 Neutrino Mass: A Harbinger for New Physics

Almost massless, chargeless and weakly interacting fermions, neutrinos have remained a mystery ever since they were introduced to understand the continuous spectrum of β decay. Neutrinos played a very instrumental role in establishing the $V - A$ nature of weak interactions, when it was found that only left-handed neutrinos and right-handed antineutrinos are produced in weak decays. This led Pauli to conjecture that neutrinos have very small mass. Fermi [1] and Perrin [2] proposed a kinematic method to determine the neutrino mass through investigation of β spectrum near its end-point. End-point of β spectrum being poorly known, it could put only upper bounds on the mass of the neutrinos. Therefore, it became evident that the neutrino mass (if at all non-zero) is much smaller than the electron mass. This was the main reason that in 1957, after the discovery of parity violation in β decay, the authors of the two component theory of the neutrino (Landau, Lee and Yang and Salam [3]) assumed that the neutrino is a massless particle, the field of which is either a left-handed field ν_L, or a right-handed field ν_R. In 1958, Goldhaber *et al.* [4] measured the helicity of the neutrino. The result of this experiment was in agreement with the two component neutrino theory and it was established that neutrino field is ν_L. The result of the experiment of Goldhaber *et al* could not exclude, however, the possibility of a small neutrino mass. Recent experiments measuring the flux of Solar and Atmospheric neutrinos have given strong hints in favour of neutrino masses. This would call for a drastic revision of SM as well as have seminal implications in areas of Astrophysics and Cosmology. Before we discuss the experimental indications, it would be instructive to discuss theoretical issues related to neutrino masses.

§ *Theoretical Motivations*

Theoretically there is no compelling reason for neutrinos to be massless within the framework of SM. The masslessness of neutrinos within SM is due to its particle content which does not have right handed neutrinos or triplet Higgs.

The theoretical question is not only how one can extend the SM to find models with massive neutrinos, but also how one can understand the smallness of neutrino masses compared to the masses of other charged fermions. One might argue that even the charged fermion masses vary widely. For example, the top quark mass is more than five orders of magnitude larger than the electron mass. This requires an understanding of the family structure of SM and the hierarchy in the family space of fermions. Within the family, quark masses agree to within an order or so. This is in sharp contrast with the lepton sector where for example ν_e is five orders lighter than the electron mass. Thus, the smallness of neutrino mass is indeed an acute problem and perhaps suggesting an altogether different origin.

§ *Motivations from Astrophysics and Cosmology*

Neutrinos are produced inside the core of the Sun in thermonuclear reactions. Since their interaction cross-section is very less, they carry information about the core of the Sun, which is inaccessible to direct optical observation. The flux of solar electron neutrinos detected on earth, is only about one half to one third of the flux calculated using using Nuclear Physics and Standard Solar Model. This is the infamous *Solar Neutrino Problem.* It can be resolved if neutrinos have mass. With massive neutrinos, the interaction eigenstates need not be same as the propagating mass eigenstates. This results in neutrino mixing analogous to quark mixing and the phenomena of *neutrino oscillations* (if neutrino spectrum is non-degenerate), whereby one neutrino flavour converts into another flavour, thus explaining deficit in electron neutrino flux from the Sun. If neutrinos are massive, they can also possess magnetic moment. This can lead to helicity flip in the Solar magnetic field thus turning it into a right-handed non interacting neutrino, and could explain the deficit of left-handed ν_e. A similar deficit of muon neutrinos was also reported for neutrinos coming from the Atmosphere, where they are produced in high energy cosmic ray interactions. This is the so called *Atmospheric Neutrino Anomaly*, and can be understood in terms of oscillations of massive non-degenerate neutrinos.

For gravitationally bound systems of stars, like galaxies, clusters, super-clusters etc, it is known for quite sometime that the mass to light ratio for these different systems increases as one goes to larger and larger systems. This problem of missing light can also be resolved if neutrinos have masses of the order of a few eV. They can be gravitationally bound to these systems and provide them with the non-luminous halo. However neutrinos with this kind of mass can contribute to a huge amount of energy density to the universe and thus affect the evolution of the universe as a whole[1] .

The big-bang nucleosynthesis depends sensitively on neutrino interactions and number

[1]Number density of background neutrinos is about 8 orders of magnitude larger than the average number density of baryons in the universe.

of light neutrino species. Neutrinos may also play an important role in baryogenesis: the observed access of baryons over anti-baryons in the universe may be related to the decays of heavy Majorana neutrinos [2].

3.2 Theoretical Issues in Neutrino Masses and Mixing

Before discussing the experimental evidence for neutrino masses, let us take a look at the theoretical revision necessary to accommodate massive neutrinos. Since neutrinos do not carry electric charge, it is possible that they are their own antiparticle, and hence possess Majorana nature. The Majorana condition puts an additional constraint of self-conjugacy and hence reduces half the degrees of freedom. Thus in the absence of right-handed neutrinos, it is possible to write down a Majorana mass term for the neutrinos. Such a mass term violates lepton number by two units. Lepton number being an accidental symmetry of SM its conservation is not sacrosanct. However, it turns out that it is not possible to add Majorana mass term without augmenting the particle content of SM. The Majorana mass term being a component of an isotriplet operator, the only way it can couple in a gauge invariant, renormalizable and Lorentz invariant way, is with a triplet Higgs, whose neutral component would attain a *vev* and thus generating Majorana mass term. This would also spontaneously break lepton number, giving rise to a massless goldstone boson called Majoron. From the inferred value of number of neutrino species from invisible Z_0 decay width at LEP, triplet Majoron models are ruled out as these would contribute sizeably to the invisible decay width of Z boson [7]. However, a simple extension of these models with introduction of an extra singlet scalar along with the triplet scalar, the LEP constraints on Z_0 can be satisfied [8].

Another simplest and natural extension of SM is just to add three $SU(2)_L$ singlet neutrinos ν_R, one per generation in analogy with other charged fermions. ν_R can have Yukawa couplings to lepton doublets. But they can have "bare" Majorana mass terms which are invariant under $SU(2)_L \times U(1)_Y$. If one assigns lepton number $L = 1$ to ν_R, such a Majorana mass term breaks lepton number by two units. Alternatively if we assign zero lepton number to ν_R, the Yukawa couplings of ν_R break the lepton number. Therefore, an introduction of ν_R in SM, leads to a qualitatively new situation: lepton number is no longer an automatic symmetry of the Lagrangian, following from gauge and Lorentz invariance and renormalizability.

[2] In one of our work [6] we discuss the question of leptogenesis [5](and hence baryogenssis) via decay of heavy right handed Majorana neutrinos. Since the nature of that problem is very different from the ones addressed in the thesis, we have not included it in this thesis.

3.2.1 General Dirac + Majorana Mass term

Let us now consider the most general neutrino mass terms for the case of n species of left handed and right handed neutrinos. It includes not only the Dirac mass m_D and Majorana mass m_R, but also the Majorana mass m_L for the left handed neutrinos. The neutrino mass term can be written as

$$-\mathcal{L}_m = \frac{1}{2}\nu_L^T C m_L \nu_L + \bar{\nu}_L m_D \nu_R + \frac{1}{2}\nu_R^T C m_R \nu_R + h.c. = \frac{1}{2} n_L^T C \mathcal{M} n_L + h.c.. \quad (3.1)$$

Here $n_L = (\nu_L, (\nu_R)^c) = (\nu_L, \nu_L^c)$ is the vector of $2n$ left handed fields, (which we have written as a line rather than column), m_L and m_R are complex symmetric $n \times n$ matrices, m_D is a complex $n \times n$ matrix. The matrix $\mathcal{M}$ has a form

$$\mathcal{M} = \begin{pmatrix} m_L & m_D \\ m_D^T & m_R \end{pmatrix}. \quad (3.2)$$

It is instructive to consider first the one generation case in which m_L, m_R and m_D are just numbers, and $\mathcal{M}$ is a 2×2 matrix. Assuming all the parameters to be real, $\mathcal{M}$ can be diagonalized by the transformation $U^T \mathcal{M} U = \mathcal{M}_d$, where U is an orthogonal matrix and $\mathcal{M}_d = diag(m_1, m_2)$. We introduce the fields χ_L through $n_L = U\chi_L$, or

$$n_L = \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix}. \quad (3.3)$$

Here χ_{1L} and χ_{2L} are the left handed components of the neutrino mass eigenstates. The mixing angle θ is given by

$$\tan 2\theta = \frac{2m_D}{m_R - m_L}, \quad (3.4)$$

and the corresponding mass eigenvalues are

$$m_{1,2} = \frac{m_R + m_L}{2} \mp \sqrt{\left(\frac{m_R - m_L}{2}\right)^2 + m_D^2}. \quad (3.5)$$

They are real but can be of either sign. The mass term can now be rewritten as

$$\begin{aligned} -\mathcal{L}_m &= \frac{1}{2} n_L^T C \mathcal{M} n_L + h.c. = \frac{1}{2}\chi_L^T C M_d \chi_L + h.c. \quad (3.6) \\ &= \frac{1}{2}(m_1 \chi_{1L}^T C \chi_{1L} + m_2 \chi_{2L}^T C \chi_{2L}) + h.c. = \frac{1}{2}(|m_1|\bar{\chi}_1\chi_1 + |m_2|\bar{\chi}_2\chi_2). \end{aligned}$$

Here we have defined

$$\chi_1 = \chi_{1L} + \eta_1(\chi_{1L})^c, \quad \chi_2 = \chi_{2L} + \eta_2(\chi_{2L})^c, \quad (3.7)$$

with $\eta_{1,2} = 1$ or -1 for $m_{1,2} > 0$ or < 0 respectively. It follows from above equation that mass eigenstates are Majorana neutrinos. The relative signs of the mass eigenvalues (η_1 and η_2) determine the relative CP parities of χ_1 and χ_2; physical masses $|m_1|$ and $|m_2|$ are positive, as they should be.

3.2.2 Seesaw Mechanism and Small Neutrino Mass

We have already discussed in section 1.1, that except for neutrinos, the masses of all fermions in each of the three generations are within 1-2 orders of magnitude of each other. Thus the smallness of neutrino mass is a disturbing feature and any complete theory of neutrino masses should also explain the exceptionally small mass for neutrinos.

The seesaw mechanism provides a very simple and attractive explanation of the smallness of neutrino mass, by relating it to the suppression from a very large mass scale. Although the seesaw mechanism is most natural in the framework of the Grand Unified Theories (such as $SO(10)$) or left-right symmetric models, it also operates in the standard model extended to include right handed neutrinos ν_R. The most general mass term for n generations of left-handed and right-handed neutrinos is written in eq. (3.1). In SM, there is no Majorana mass term of left-handed neutrinos as there are no triplet Higgs scalars; however, m_L is different from zero in many extensions of SM, so we shall keep it for generality. The right handed neutrino ν_R is an electroweak singlet and hence its mass is not protected by the electroweak symmetry. One can therefore expect it to be very large, possibly at the Planck scale or at the intermediate scale $M_I \sim \sqrt{vM_{Pl}} \sim 10^{10} - 10^{12}$ GeV which may be relevant for the physics of parity breaking.

Let us first consider the limit $m_L \ll m_D \ll m_R$ of the simple one-generation case discussed in the previous section. In this limit

$$\theta \simeq \frac{m_D}{m_R} \ll 1, \quad m_1 \simeq m_L - \frac{m_D^2}{m_R}, \quad m_2 \simeq m_R \tag{3.8}$$

$$\chi_1 \simeq \nu_L + \eta_1 (\nu_L)^c, \quad \chi_2 \simeq (\nu_R)^c + \eta_2 \nu_R. \tag{3.9}$$

Thus we have a very light Majorana mass eigenstate χ_1 predominantly composed of ν_L and a heavy eigenstate χ_2 mainly composed of ν_R.

Consider now the full n-generation case. We want to block diagonalize the matrix $\mathcal{M}$ is eq. (3.2) so as to decouple the light and heavy neutrino degrees of freedom:

$$n_L = U\chi_L, \quad U^T \mathcal{M} U = U^T \begin{pmatrix} m_L & m_D \\ m_D^T & M_R \end{pmatrix} U = \begin{pmatrix} \tilde{m}_L & 0 \\ 0 & \tilde{M}_R \end{pmatrix}, \tag{3.10}$$

where U is a unitary $2n \times 2n$ matrix, and we have changed the notation $m_R \to M_R$. We shall be looking for the matrix U of the form

$$U = \begin{pmatrix} 1 & \rho \\ -\rho^\dagger & 1 \end{pmatrix}, \quad U^\dagger U = 1 + \mathcal{O}(\rho^2), \tag{3.11}$$

where the elements are $n \times n$ matrices, and ρ will be treated as a perturbation and we shall treat m_L, m_D and m_R as real. The matrix ρ can then be chosen to be real. Block diagonalization of $\mathcal{M}$ gives

$$\rho \simeq m_D M_R^{-1}, \;\; \tilde{m}_L \simeq m_L - m_D M_R^{-1} m_D^T, \;\; \tilde{M}_R \simeq M_R. \tag{3.12}$$

The diagonalization of effective mass matrix $\tilde{m}_L$ yields, n light Majorana neutrinos. Diagonalization of $\tilde{M}_R$ produces n, heavy Majorana neutrinos. It is important that the active neutrinos get Majorana masses $\tilde{m}_L$ even if they have no "direct" mass, i.e. $m_L = 0$ as it is in the Standard Model. The masses of active neutrinos are then of the order of m_D^2/M_R. It is interesting that with the largest Dirac mass eigenvalues of the order of electroweak scale, $m_D \sim 200$ GeV, the right handed scale $M_R \sim 10^{15}$ GeV which is close to the typical GUT scales, and assuming that the direct mass term $m_L \leq m_D^2/M_R$, one obtains the mass of the heaviest of the light neutrinos $m_\nu \sim (10^{-2} - 10^{-1})$ eV, which is just of the right order of magnitude for the neutrino oscillation solution of the atmospheric neutrino anomaly.

We shall see in the next chapters that seesaw like mechanism is also operative in Supersymmetric models with R-parity violation, and these models very naturally accommodate the solutions to Atmospheric and Solar neutrino anomalies. There are many other models proposed for neutrino masses and we shall not go into the details of these models. Instead we refer the interested reader the to the pedagogic description in [7, 9].

3.2.3 Radiative Mechanisms

In the above we have introduced additional fermions with a heavy mass scale to generate small neutrino masses. One can instead modify the scalar sector of the model, which is anyway not well understood. Neutrino masses are now generated radiatively and thus are naturally small.

Within the Standard Model neutrinos can attain Majorana masses if one modifies the scalar sector. This can be seen by considering the operator, $\epsilon_{ab} L_i^a \; C \; L_j^b$, where C is the charge conjugation matrix and a, b are the $SU(2)$ indices and i, j are the generation indices. This can couple to a field, transforming either as a singlet or a triplet under $SU(2)$. Models with triplet Higgs are considered unattractive as they contribute to ρ parameter [11]. Instead here we consider models with a singlet field h^+. The coupling of the lepton fields to this field is given as, $f^{ij} \epsilon_{ab} L_i^a \; C \; L_j^b h^+$.

Though the above coupling violates lepton number, one can always conserve it by defining h^+ to have a lepton number of -2. However if one introduces an additional scalar field ϕ_2 (in-addition to the already existing ϕ_1), a new coupling of the form $M_{\alpha\beta}\phi_\alpha\phi_\beta h^+$

is possible which violates lepton number exactly by two units as required for neutrino mass generation. In this model which is named as $\{\phi_1\phi_2 h\}$ model, neutrinos attain mass at the one-loop level and hence the masses are naturally suppressed. One interesting feature about this model is that the couplings f_{ij} are antisymmetric due to the $SU(2)$ metric. This would lead to an interesting texture of the neutrino mass matrix whose diagonal elements are zero. Instead of adding an additional doublet, one can as well add a doubly charged singlet in to the model, k^{++}. In this case, neutrinos attain mass at the two-loop level. This is popularly known as Babu model in literature [12]. Including both the additional doublet as well as the doubly charged singlet would lead to neutrino mass both at the 1-loop level as well as at the 2-loop level. Such a scenario may be required to understand neutrino anomalies in these models with symmetries like $L_e - L_\mu - L_\tau$ [13].

In this thesis, we consider an alternative method to generate neutrino masses. In these models neutrinos attain mass employing both the 'see-saw type' mechanism as well as radiative mechanisms. We will discuss them in detail in further chapters.

3.3 Neutrino Oscillations in Vacuum and Matter

As we have already mentioned in section 1.1, if neutrinos are massive, then in general mass eigenstates are super-position of interacting flavour eigenstates. This leads to the phenomena of neutrino oscillations, whereby one neutrino flavour converts into another flavour as it propagates, thus explaining the deficit in the Solar and Atmospheric neutrino flux, if there is a correct mass hierarchy and mixing angle among neutrinos. If $|\nu_\alpha\rangle$ are flavour eigenstates ($\alpha = e, \mu, \tau$) and $|\nu_k\rangle$ are the mass eigenstates ($k = 1, 2, 3$) then the mixing is expressed as:

$$|\nu_\alpha\rangle = \sum_{k=1}^{3} U_{\alpha k} |\nu_k\rangle. \tag{3.13}$$

By straight-forward application of Quantum Mechanics and the assumption of ultra-relativistic neutrinos, one can show that the survival probability for ν_e after it has propagated for time t, is given as [3]

$$P_{\nu_e\nu_e} = 1 - 4\sum_{i>j} U_{ei}^2 U_{ej}^2 \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right), \tag{3.14}$$

where Δm_{ij}^2 represents the mass squared difference between ith and jth neutrino mass eigenstates and L represents the distance traveled. If there are only two generations involved, this formula reduces to,

$$P_{\nu_e\nu_e} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right), \tag{3.15}$$

[3] For the quantum field theoretical treatment of the neutrino oscillation formula, please see [14].

where now the mixing matrix is represented as a rotation by an angle θ. The solar and atmospheric neutrino anomalies can be understood in terms of neutrino oscillations, if the mass squared differences of the neutrinos and the mixing angles are of the right order. The experimental data now constrains regions in parameter space of the neutrino oscillation formula. The deficit in the data of a particular anomaly can be caused either by oscillations in two generations or in three generations. Another possibility is that neutrinos oscillate in to a sterile neutrino, which does not even have weak interaction.

In this thesis, we consider that there are only three active neutrinos and the solar and atmospheric neutrino anomalies can be resolved in terms of oscillations among neutrinos in the family space. In general this would require a three generation analysis of the entire solar and atmospheric neutrino data. However, assuming a hierarchical pattern for the neutrino masses [4] an important constraint on the neutrino mixing comes from the CHOOZ experiment [6] (to be discussed in next section) which simplifies such an analysis [17]. With the help of CHOOZ constraint, which requires the mixing matrix element $|U_{e3}| \leq 0.15$, the three flavour analysis reduces to two flavour analysis. Now the survival probability for electrons and muons is given by

$$P_{\nu_e\nu_e} = 1 - 4U_{e1}^2 U_{e2}^2 \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right) - 2U_{e3}^2(1 - U_{e3}^2) \tag{3.16}$$

$$P_{\nu_\mu\nu_\mu} = 1 - 4U_{\mu 2}^2 U_{\mu 3}^2 \sin^2\left(\frac{\Delta m_{32}^2 L}{4E}\right). \tag{3.17}$$

The mixing matrix element U_{e3} is constrained by the CHOOZ experiment to be small [6]. The above formulae are identical to the 2-flavour oscillation formula, eq.(3.15), in the limit of small U_{e3}. In this case, we can concentrate only on two flavour solutions to the neutrino anomalies.

The standard solutions for the the solar neutrino anomaly comprise of three 'regions' in the oscillation parameter space of Δm^2 and $sin^2 2\theta$. One of the regions is called the 'Vacuum Oscillations' or 'just so'. The Δm^2 required is very small $\sim 10^{-11}\text{eV}^2$ and the mixing angle $\sim \frac{\pi}{4}$. The other two solutions require a much larger Δm^2, typically of $O(10^{-6}\text{eV}^2)$. These two solutions consider matter effects on neutrino propagation whilst the neutrino traverses the distance between the core of the Sun and its surface. They allow for matter enhanced resonant conversion (MSW mechanism) [18] of the neutrinos from electron species to another in specific density regions of the Sun. The question is, how does the matter affect the neutrino propagation ? Neutrinos can be absorbed by the matter constituents, or scattered off them, changing their momentum and energy.

[4] For a discussion on the neutrino mass matrix allowed by the experiments, including degenerate spectra please see [15].

However the probabilities of these processes, being proportional to the square of the Fermi constant G_F, are typically very small. Neutrinos can also experience forward scattering, an elastic scattering in which their momentum is not changed. This process is coherent, and it creates mean potentials V_a for neutrinos which are proportional to the number densities of the scatterers. These potentials are of the first order in G_F, but one could still expect them to be too small and of no practical interest. This expectation, however, would be wrong. To assess the importance of matter effects on neutrino oscillations, one has to compare the matter induced potentials of neutrinos V_a with the characteristic neutrino kinetic energy differences $\Delta m^2/2E$. Although the potentials V_a are typically very small, so are $\Delta m^2/2E$; if V_a are comparable to or larger than $\Delta m^2/2E$, matter can strongly affect the neutrino oscillations, even if the mixing angle in vacuum is very small. Such a resonance enhancement or suppression in neutrino oscillation in certain regions of matter with favourable density, is called the MSW effect and offers two different oscillation solutions to the solar neutrino anomaly. One of the solutions allows for a large mixing angle, $\theta \sim \frac{\pi}{4}$ - Large Angle MSW. The other region allows for a small mixing angle $\theta \sim 10^{-3}$ and is called Small Angle MSW [19]. However, from the recent results of solar neutrino experiment at Sudbury Neutrino Observatory, not all the solutions are favourable. We shall discuss these solutions in next section where we discuss the recent experimental results. In the case of atmospheric neutrino anomaly, the results are much more constrained. The analysis of super-Kamiokande results [20] allow solutions for regions in $\Delta m^2 \sim 10^{-3}$ and $\sin^2 2\theta \sim 1$.

In the above we have used $\sin^2 2\theta$ to express the range of mixing angles. However it is a common practise now, to express the mixing angles for solar neutrinos in $\tan^2\theta$ and we shall follow this recent practise in next section when we discuss various solutions. The use of the variable $\tan^2\theta$ instead of the usual $\sin^2 2\theta$ is worth a comment. The probability of 2-flavour neutrino oscillations in vacuum is invariant under the substitutions $\theta \to \pi/2 - \theta$ or $\Delta m^2 \to -\Delta m^2$, but the oscillation probability in matter is not. It is, however, invariant under the combined action of these substitutions. To cover the full parameter space, it is sufficient to assume $0 \leq \theta \leq \pi/4$ and allow for both signs of Δm^2, or to assume that Δm^2 is always positive (which can always be achieved by renaming the mass eigenstates $\nu_1 \leftrightarrow \nu_2$) and let θ be in the full domain $[0, \pi/2]$. Usually, the first approach was adopted; however, the solutions of the solar neutrino problem in the region $\Delta m^2 < 0$ have not been studied (except in the 3-neutrino [21] and 4-neutrino [22] frameworks). This was motivated by the fact that there is no MSW enhancement for neutrinos in this region of parameters. However, in [23] it has been emphasized that if one allows for large enough confidence levels, or treats the solar ^{8}B neutrino flux as a free parameter, or leaves the Homestake result out, solutions in this "dark side" of the parameter space exist, provided that the mixing angle is close to the maximal one. It is convenient to assume $\Delta m^2 > 0$ and plot the allowed regions of the parameter space in the plane $(\tan^2\theta,\, \Delta m^2)$ with $0 \le \theta \le \pi/2$; in the conventional approach one would need two separate plots for $\Delta m^2 > 0$ and $\Delta m^2 < 0$.

It should be noted that from recent results from Super-Kamiokande and SNO experiments, only certain solutions are favourable. We shall discuss these issues in the next section where we also discuss the solar and atmospheric neutrino anomalies in detail.

3.4 Experimental Indications

3.4.1 The Solar Neutrino Problem

As we have already discussed in section 1, Solar Neutrino Problem (SNP) is basically the discrepancy between theoretically calculated and experimentally measured flux of neutrinos coming from Sun. Neutrinos are produced inside the Sun during nuclear fusion reaction. 98% of the energy released, is carried by the photons and the remaining 2% by neutrinos. Neutrinos are mainly produced in proton-proton (pp) reactions and some (less than 2%) are also produced in Carbon-Nitrogen-Oxygen (CNO) cycle. Rates for these reactions are known from application of the principles of Nuclear and Particle physics. Chemical abundances of various elements are known from the Standard Solar Model (SSM) [24]. There are about 20 different models proposed by 10 different groups, all of which agree with each other and with the data from Helio-Seismology very well. These models are based on reasonable assumptions like (a) Hydrostatic equilibrium inside the star (b) energy transport by means of conduction and convection only and (c) change in the chemical composition of Sun is only through the nuclear reactions. Using Nuclear Physics, one can predict energy spectrum of neutrinos produced in various reactions and using SSM, one can predict the flux of neutrinos. Since neutrinos interact very less with matter, the flux reaching earth remains the same.

The earliest measurement of flux was done in around 1968 by Davis and his collaborators in the famous HOMESTAKE chlorine experiment through the following charged current (CC) reaction [26].

$$^{37}Cl + \nu_e \to \, ^{37}Ar + e^-. \tag{3.18}$$

Radioactive Argon atoms are extracted by chemical methods and counted in proportional counter. The energy threshold of the reaction is 0.814 MeV so only ^8B and ^7Be and pep neutrinos (see fig3.1) could be detected. The results showed almost one third reduction in the flux compared to the predictions of SSM. Similar measurements using radiochemical methods, were carried out at SAGE and GALLEX experiments, where they used the CC

Table 3.1: SSM predictions: solar neutrino fluxes and neutrino capture rates for the different experiments, with 1σ uncertainties. The neutrino fluxes are the same as in the original BP00 model except for the ^{8}B flux, which is increased because of the larger adopted value of $S_{17}(0)$ (see [25])

Source	Flux $\left(10^{10}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}\right)$	Cl (SNU)	Ga (SNU)	SK $\left(10^{6}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}\right)$	SNO(CC) $\left(10^{6}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}\right)$
pp	$5.95\left(1.00^{+0.01}_{-0.01}\right)$	0.0	69.7	0.0	0.0
pep	$1.40 \times 10^{-2}\left(1.00^{+0.015}_{-0.015}\right)$	0.22	2.8	0.0	0.0
hep	9.3×10^{-7}	0.04	0.1	0.0093	0.0093
^{7}Be	$4.77 \times 10^{-1}\left(1.00^{+0.10}_{-0.10}\right)$	1.15	34.2	0.0	0.0
^{8}B	$5.93 \times 10^{-4}\left(1.00^{+0.14}_{-0.15}\right)$	6.76	14.2	5.93	5.93
^{13}N	$5.48 \times 10^{-2}\left(1.00^{+0.21}_{-0.17}\right)$	0.09	3.4	0.0	0.0
^{15}O	$4.80 \times 10^{-2}\left(1.00^{+0.25}_{-0.19}\right)$	0.33	5.5	0.0	0.0
^{17}F	$5.63 \times 10^{-4}\left(1.00^{+0.25}_{-0.25}\right)$	0.0	0.1	0.0	0.0
Total		$8.6^{+1.1}_{-1.2}$	130^{+9}_{-7}	$5.93^{+0.89}_{-0.83}$	$5.93^{+0.89}_{-0.83}$
Measured		2.56 ± 0.226	75.6 ± 4.8	2.32 ± 0.085	1.75 ± 0.148
$\frac{\text{Measured}}{\text{SSM}}$		0.298 ± 0.049	0.581 ± 0.055	0.391 ± 0.060	0.295 ± 0.051

reaction

$$^{71}Ga + \nu_e \rightarrow\ ^{71}Ge + e^-. \tag{3.19}$$

The energy threshold was about 0.234 Mev, so they could also detect the lowest energy pp neutrinos. They also reported reduction in the flux.

Recent experiments at Kamiomande ($E > 7.5MeV$) and its upgraded version Super-Kamiokande ($E > 5.5MeV$) employed water cherenkov detectors and use the electron-neutrino scattering reaction,

$$\nu_e + e^- \rightarrow \nu_e + e^-. \tag{3.20}$$

This reaction has zero threshold but to remove the background these energy cuts are used. Owing to their high energy cuts, they are sensitive to only ^8B neutrinos. Above reaction has a very interesting feature that for $E \gg m_e$, the angular distribution of the recoil electrons is forward peaked. For neutrinos detected at Super-Kamiokande (SK) it points 180^o opposite to Sun, thus confirming the Solar origin of these neutrinos.

In the recent experiment at Sudbury Neutrino Observatory, solar neutrinos were detected in real time through charged current, neutral current and elastic scattering experiments. It has not only confirmed the neutrino oscillation hypothesis but also severely constrained the possible solutions. In the next section we discuss its results in detail.

In all the six experiments fewer neutrinos than expected were detected (see table 3.1), thus pointing towards solar neutrino anomaly.

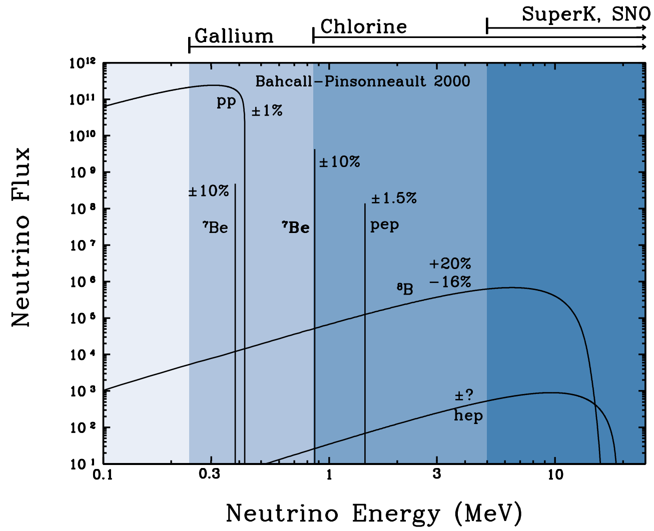

Figure 3.1: Solar neutrino spectrum and estimated theoretical errors of fluxes. The thresholds of solar neutrino experiments are indicated above the figure. From [24]

§ *Is the Solar Neutrino Anomaly for Real ?*

Suppose there is no solar neutrino anomaly and something has gone wrong somewhere. What could possibly be the problem ?

- *Experiments are wrong*: This is most unlikely because six experiments, all using different techniques of detection, cannot simultaneously show a systematic trend of reduction in the flux.

- *Flux calculations are wrong*: This is also ruled out as 20 different models, from 10 different groups agree very well with each other and their predictions have been beautifully vindicated by Helio-seismological measurements. Moreover we will show below the existence of Solar neutrino problem independent of the inputs of SSM.

- *Calculation of energy spectrum is wrong*: This is where possibly the problem is. Calculation of neutrino energy spectrum are done on the assumption of massless neutrinos which is an intuitive input from speculation and does not have empirical evidence. This points towards possible particle physics solutions to the problem, which calls for the introduction of massive neutrinos in the theory.

The existence of Solar Neutrino Problem independent of Standard Solar Model can be demonstrated by comparing ^8B neutrino flux from different experiments. Such a comparision proves that not only the experiments show reduction in the flux, results of different experiments are inconsistent with each other. For example, one can infer the flux of ^8B neutrino from SK. This can be used as an input to the Homestake experiment and one can subtract it from the total flux and obtain the flux of ^7Be neutrinos. This flux turns out to be negative. Forcing all fluxes to be positive, the hypothesis of no-new-physics was rejected at the effective 2.6σ level (99% C.L.) (see Gonzalez-Garcia in [27]). So the oscillation of ν_e to ν_μ or ν_τ is the most likely possible solution to the SNP. Energy of solar neutrino being small, the ν_μ or ν_τ cannot be detected at the charged current reactions in Chlorine or Gallium experiments. And the cross section for NC interaction in Water cherenkov detectors is a factor of 6 smaller than the CC interaction channel and so the deficit in the neutrino flux observed in SK can be explained. The probabilities of neutrino oscillations depend on neutrino energy, and the distortion of the energy spectra of the experimentally detected Solar neutrinos, which is necessary to reconcile with the data of different experiments, is readily obtained. For a pedagogic discussion of Solar neutrino problem please refer to the reviews in [27]

Now let us take a look at the Atmospheric Neutrino Problem which again is a discrepancy between theoretical prediction and experimentally measured neutrino flux.

3.4.2 Atmospheric Neutrino problem

Atmospheric neutrinos are electron and muon neutrinos and their anti-neutrinos which are produced in the hadronic showers induced by cosmic rays in the earth's atmosphere. The main mechanism of production of the atmospheric neutrinos can be summarized as follows:

1. Primary cosmic rays interact with the air molecules producing kaons ($K^\pm$) and pions ($\pi^\pm$).

2. These pions then decay to form a part of the neutrino ($\bar{\nu}_\mu$) flux and the muons ($\mu^\pm$).

3. Lastly muons decay to give rest of the neutrinos ($\bar{\nu}_\mu$) flux and the ($\bar{\nu}_e$) flux.

Atmospheric neutrinos can be detected directly in large mass underground detectors, predominantly by means of their charged current (CC) interactions. Calculation of the atmospheric neutrino fluxes predict the ν_μ/ν_e ratio that depends on neutrino energy and the zenith angle of neutrino trajectory, approaching 2 for low energy neutrinos and horizontal trajectories but exceeding this value for higher energy neutrinos and for trajectories close to vertical. The overall uncertainty of the calculated atmospheric neutrino fluxes is large, and the total fluxes calculated by different authors differ by as much as $20-30\%$. At the same time the ratio of the muon to electron neutrino fluxes is fairly insensitive to this uncertainty, and different calculations yield the ratios of muon-like to electron-like contained events, which agree to about 5%. This ratio has been measured in a number of experiments, and the Kamiokande and IMB Collaborations reported smaller than expected ratio in their contained events, with the ratio of the ratio given as:

$$R_{DATA(MC)} = \left[\frac{(\nu_\mu + \bar{\nu}_\mu)}{(\nu_e + \bar{\nu}_e)}\right]_{DATA(MC)} \; ; \; R(\mu/e) \equiv \frac{R_{DATA}}{R_{MC}} \simeq 0.6, \qquad (3.21)$$

where MC stands for Mote-Carlo simulations. The discrepancy between the observed and predicted atmospheric neutrino fluxes was called the Atmospheric neutrino anomaly. The existence of this anomaly was subsequently confirmed by Soudan 2, MACRO and Super-Kamiokande (SK) experiments.

3.5 Evidence for Massive Neutrinos

3.5.1 The SK Results for Atmospheric Neutrinos

Super-Kamiokande obtained a very convincing evidence of the up-down asymmetry and zenith angle dependent deficiency for the flux of muon neutrinos, which has been interpreted as an evidence for neutrino oscillations. We shall now discuss the SK data and their interpretation.

From eq. (3.14) it is clear that the oscillation probability depends on the distance traveled by the neutrino and its energy. The distances L traveled by the neutrinos before they reach the detector vary in a wide range: for vertically downwards going neutrinos (neutrino zenith angle $\Theta_\nu = 0^o$) $L \sim 15$ km; for horizontal neutrino trajectories ($\Theta_\nu = 90^o$) $L \sim 500$ km; and vertically up going neutrinos ($\Theta_\nu = 180^o$) cross the earth along its diameter and for them $L \sim 13,000$ km.

In fig. 3.2 the zenith angle distribution of the SK e-like and μ like events are shown separately for sub-GeV (visible energy less than 1.33 GeV) and multi-GeV (visible energy greater than 1.33 GeV) contained events. One can see that for e-like events, the measured zenith angle distributions agree very well with the MC predictions (shown by bars), both in the sub-GeV and multi-GeV samples, while for μ-like events both samples show zenith-angle dependent deficiency of event numbers compared to expectations. The deficit of muon neutrinos is stronger for upward going neutrinos which have larger pathlengths. In the multi-GeV sample, there is practically no deficit of events caused by muon neutrinos coming from the upper hemisphere ($\cos\Theta > 0$), whereas in the sub-GeV sample, all μ-like events exhibit a deficit which decreases with $\cos\Theta$. This pattern is perfectly consistent with oscillations $\nu_\mu \leftrightarrow \nu_\tau$ or $\nu_\mu \leftrightarrow \nu_s$ where ν_s is a sterile neutrino. Muon neutrinos responsible for the multi-GeV sample are depleted by the oscillations

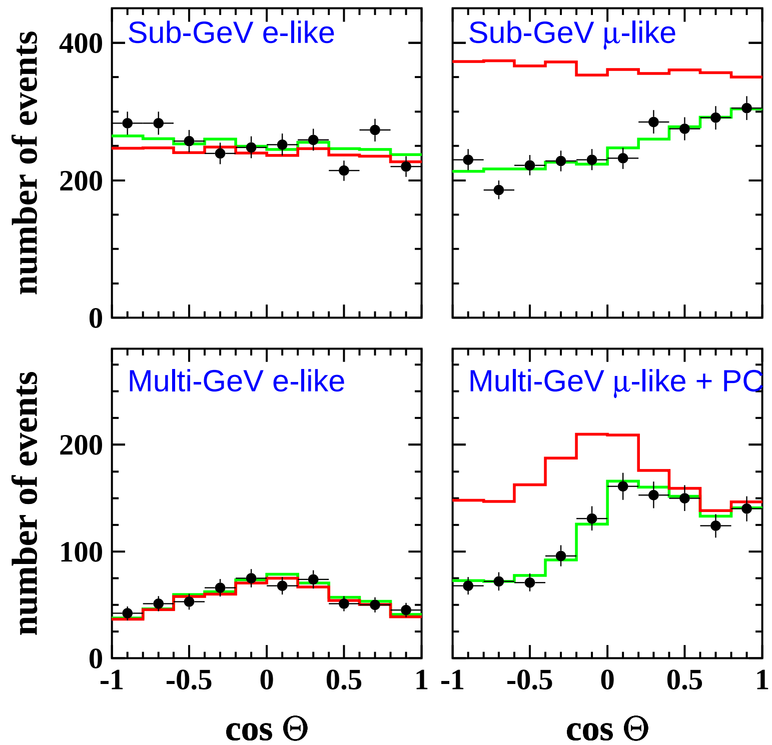

Figure 3.2: Zenith angle distributions for sub-GeV and multi-GeV e-like and μ-like events at SK (1144 live days). The dark-hatched lines show the (no-oscillations) Monte Carlo predictions; light-hatched lines show the predictions for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with the best-fit parameters $\Delta m^2 = 3.2 \times 10^{-3}$ eV^2, $\sin^2 2\theta = 1.0$ from [28]

when their pathlength is large enough; the depletion becomes less pronounced as the pathlength decreases ($\cos\Theta$ increases); for neutrinos coming from the upper hemisphere, the pathlengths are too short and there are practically no oscillations. Neutrinos responsible for the sub-GeV μ-like events have smaller energies, and so their oscillation lengths are smaller; therefore even neutrinos coming from the upper hemisphere experience sizeable depletion due to the oscillations. For up-going sub-GeV neutrinos the oscillation length is much smaller than the pathlength and they experience averaged oscillations. The solid line in fig. 3.2 obtained with the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation parameters in the 2-flavour scheme $\Delta m^2 = 3.2 \times 10^{-3}$ eV^2, $\sin^2 2\theta = 1.0$ gives an excellent fit of the data.

An informative parameter characterizing the distortion of the zenith angle distribution is the up-down event ratio U/D, where up corresponds to the events with $\cos\Theta < -0.2$ and down to those with $\cos\Theta > 0.2$. The flux of atmospheric neutrinos is expected to be nearly up-down symmetric for neutrino energies $E \geq 1$ GeV, with minor deviations coming from geomagnetic effects which are well understood and can be accurately taken into account. In particular, at the geographical location of the SK detector, small upward asymmetry is expected, i.e. U/D should be slightly bigger than 1. Any significant deviation of the up-down asymmetry of neutrino induced events from the asymmetry due to the geomagnetic effects is an indication of neutrino oscillations or some other new neutrino physics. The U/D ratio measured for the SK multi-GeV μ-like events is [28, 29]

$$U/D = 0.54 \pm 0.04\,(stat.) \pm 0.01\,(syst.)\,, \tag{3.22}$$

i.e. is below unity by about 9σ! Thus, this is a concrete evidence for oscillations of atmospheric neutrinos pointing towards massive neutrinos.

3.5.2 SK Results for Solar neutrinos

Recent SK results on solar neutrino data [1], have significantly constrained the possible solutions to solar neutrino anomaly. Information on the different oscillation regimes can be obtained, from the analysis of the energy and time dependence data from SK which is currently presented in the form of observed day-night spectrum. The observed day-night spectrum is essentially undistorted in comparision to the SSM expectation and shows no significant differences between day and night periods. In fig 3.3 we show the SK spectrum corresponding to 1258 days of data relative to the Oritiz *et. al* [31] spectrum normalized to BP00, together with the expectations from the best fit points for the LMA, SMA and LOW solutions. The various solutions give different predictions for the day

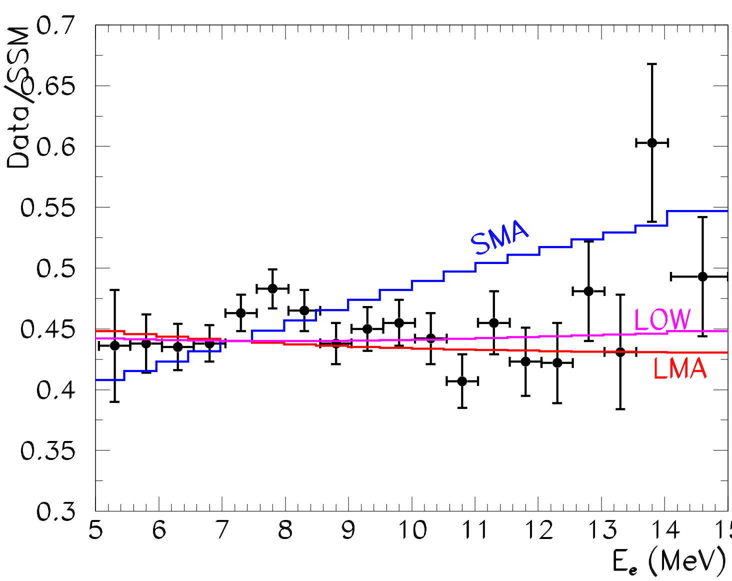

Figure 3.3: The electron recoil energy spectrum measured in SK normalized to the SSM prediction, and the expectations for the best fit points for the LMA, SMA and LOW solutions.

night spectrum. For LMA and LOW, the expected spectrum is very little distorted. For SMA, a positive slope is expected, with larger slope for larger mixing angle within SMA. For VAC, large distortions are expected. The main consequences of adding day-night spectrum information to the analysis of the total event rates are:

- Active SMA: Within this region, the part with larger mixing angle fails to comply with the observed energy spectrum, while the part with smaller mixing angles gives a bad fit to the total rates.
- VAC (either active or sterile): The observed flat spectrum cannot be accommodated.
- Active LMA and active LOW: The small Δm^2 part of LMA and the large Δm^2 part of LOW are reduced because they predict a day-night variation that is larger than observed. Both active LMA and active LOW solutions predict a flat spectrum in agreement with the observation.

In June 2001, the Sudbury Neutrino Observatory (SNO) also announced their results conforming not only the oscillation hypothesis for Solar neutrinos but also severely constrained the possible solutions. We discuss these results in the next section.

3.5.3 The SNO results

In the SNO experiment, located 6800 feet below the ground in the Creighton mine, near Sudbury in Ontario (Canada), a Cherenkov detector containing 1 Kton of heavy water ($D_2 0$), detected neutrinos in real-time through the charged current (CC), the neutral current (NC) and the elastic scattering (ES) reactions

$$\nu_e + d \rightarrow e^- + p + p \quad (CC) \tag{3.23}$$

$$\nu_l + d \rightarrow \nu_l + p + n \quad (NC) \tag{3.24}$$

$$\nu_l + e^- \rightarrow \nu_l + e^- \quad (ES), \tag{3.25}$$

where $l = e, \mu, \tau$. Since energy threshold for the observation of the recoil electron in the CC and ES processes is about 5 MeV, and the neutrino energy threshold for the NC reaction is 2.2 MeV, only ^8B neutrinos can be observed. The CC reaction is sensitive to exclusively electron type neutrinos whereas NC is sensitive to all active flavours. The elastic scattering ES reaction is sensitive to all flavours as well, but with reduced sensitivity to ν_μ and ν_τ. The reaction of ν_e is very well suited for measuring the solar neutrino spectrum: unlike the scattering in the SK detector in which the energy of incoming neutrino is shared between two light particles in the final state, the final state of CC reaction in SNO, contains only one light particle electron, and a heavy 2p system whose kinetic energy is relatively small. Therefore the electron energy is strongly correlated with the energy of the incoming neutrino. But the latest results from SNO, from the analysis of CC events, show no evidence for deviation of a spectral shape from the predicted shape under the no-oscillation hypothesis [30]. Recently SNO also published the analysis of their neutral current data [31, 32]. The measured ^8B flux with each reaction in SNO assuming the standard spectrum shape [31] is given as (in units of $10^6 cm^{-2}s^{-1}$):

$$\begin{aligned} \phi_{CC}^{SNO}(\nu_e) &= 1.75 \pm 0.07(stat.)^{+0.12}_{-0.11}(syst.) \\ \phi_{ES}^{SNO}(\nu_l) &= 2.39 \pm 0.34(stat.)^{+0.16}_{-0.14}(syst.) \\ \phi_{NC}^{SNO}(\nu_l) &= 5.09^{+0.44}_{-0.43}(stat.)^{+0.46}_{-0.46}(syst.). \end{aligned} \tag{3.26}$$

The difference between ^8B flux deduced from ES rate and that deduced from CC rate in SNO is $0.64 \pm 0.40 \times 10^6 cm^{-2}s^{-1}$ or 1.6σ. SNO's ES measurement is consistent with the precision measurement by SK of the ^8B flux using the same ES reaction [1].

$$\phi_{SK}^{ES}(\nu_l) = 2.34 \pm 0.03\ (stat.)^{+0.08}_{-0.07}\ (sys.) \times 10^6 cm^2 s^{-1} \tag{3.27}$$

$$\phi_{SK}^{ES}(\nu_l) - \phi_{SNO}^{CC}(\nu_e) = 0.57 \pm 0.17 \times 10^6 cm^2 s^{-1}. \tag{3.28}$$

The difference is obtained assuming that errors are normally distributed. If oscillation solely to a sterile neutrino is occurring, the SNO derived ^8B flux should be consistent with the Super-Kamiokande ES derived ^8B flux. These data are therefore evidence of a non-electron *active* flavour component in the the solar neutrino flux. These data are also inconsistent with the "Just-so" parameters of neutrino oscillations [35]. Also the excess of NC flux over the CC and ES fluxes implies neutrino flavour transformation. Removing the constraint that solar neutrino energy spectrum is undistorted, the total flux of ^8B neutrinos measured with the NC reaction is

$$\phi_{NC}^{SNO} = 6.42^{+1.57}_{-1.57}(stat.)^{+0.55}_{-0.58}(syst.). \tag{3.29}$$

which is in agreement with the shape constrained value above and with the standard solar model prediction [24] for ^8B, $\phi_{SSM} = 5.05^{+1.01}_{-0.81}$.

In the light of above results, the limits on allowed values of Δm^2 and mixing angle $\tan^2\theta$ have considerably changed [33, 37]. In the units of eV^2, the range for Δm^2 for LMA solution at 3σ level is

$$2.3 \times 10^{-5} < \Delta m^2 < 3.7 \times 10^{-4}. \tag{3.30}$$

For the LOW solution only the following small mass range is allowed,

$$3.5 \times 10^{-8} < \Delta m^2 < 1.2 \times 10^{-7}. \tag{3.31}$$

Exact bi-maximal solution is disfavoured at the 3.3σ C.L. for LMA, at the 3.2σ C.L. for the LOW solution, and at the 2.8σ C.L. for the VAC solutions. But the approximate maximal mixing is heavily favoured. At the three sigma, the following range for LMA is allowed

$$0.24 \ < \ \tan^2\theta \ < \ 0.89, \tag{3.32}$$

and for the LOW solution

$$0.43 \ < \ \tan^2\theta \ < \ 0.86. \tag{3.33}$$

3.5.4 The Borexino Experiment

The Borexino experiment is scheduled to start taking data in 2002. It will detect solar neutrinos through the $\nu_l e$ scattering with a very low energy threshold, and will be able to detect the ^{7}Be neutrino line. Different solutions of the solar neutrino problem predict different degree of suppression of ^{7}Be neutrinos, and their detection could help discriminate between these solutions. Observation of the ^{7}Be neutrino line would be especially important in the case of the VO solution. Due to the eccentricity of the earth's orbit, the distance between the sun and the earth varies by about 3.5% during the year, and this should lead to varying oscillation phase (and therefore varying solar neutrino signal) in the case of vacuum neutrino oscillations. This seasonal variation can in principle be separated from the trivial 7% variation due to the $1/L^2$ law which is not related to neutrino oscillations. Since the oscillation phase depends on neutrino energy, integration over significant energy intervals may make it difficult to observe the seasonal variations of the solar neutrino flux due to VO. The ^{7}Be neutrinos are monochromatic, which should facilitate the observation of the seasonal variations at Borexino.

Borexino will also be capable of confirming or refuting the LOW solution: a strong day/night effect predicted for ^{7}Be neutrinos by this solution should be clearly detectable at Borexino [38]. One can hope that the combined data of the currently operating and forthcoming experiments will allow to finally resolve the solar neutrino problem.

3.5.5 Reactor and Accelarator Experiments

In reactor neutrino experiments oscillations of electron antineutrinos into another neutrino species are searched for by studying possible depletion of the $\bar{\nu}_e$ flux beyond the usual geometrical one. These are the disappearance experiments, because the energies of the reactor $\bar{\nu}_e$'s ($\langle E\rangle \simeq 3$ MeV) are too small to allow the detection of muon or tauon antineutrinos in CC experiments. Small $\bar{\nu}_e$ energy makes the reactor neutrino experiments sensitive to oscillations with rather small values of Δm^2.

Up to now, no evidence for neutrino oscillations has been found in the reactor neutrino experiments, which allowed to exclude certain regions in the neutrino parameter space. The best constraints were obtained by the CHOOZ experiment in France [6]. For the values of $\Delta m^2_{31} \equiv \Delta m^2_{atm}$ in the SK allowed region $(1.5-5)\times 10^{-3}$ eV^2, the CHOOZ results give the following constraint on the element U_{e3} of the lepton mixing matrix: $|U_{e3}|^2(1-|U_{e3}|^2) < 0.055 - 0.015$ at 90% c.l., i.e. $|U_{e3}|$ is either small or close to unity. The latter possibility is excluded by solar and atmospheric neutrino observations, and one finally obtains

$$\sin^2\theta_{13} \equiv |U_{e3}|^2 \leq (0.06-0.018) \quad \text{for} \quad \Delta m^2_{31} = (1.5-5)\times 10^{-3}\ \text{eV}^2\,. \tag{3.34}$$

This is the most stringent constraint on $|U_{e3}|$ to date.

From January 2002, a long baseline reactor experiment KamLAND in Japan has become operational. It is a large liquid scintillator detector experiment using the former Kamiokande site. KamLAND will detect electron antineutrinos coming from several Japanese and Korean power reactors at an average distance of about 180 km. It will be sensitive to values of Δm^2 as low as 4×10^{-6} eV^2, i.e. in the range relevant for the solar neutrino oscillations. It is expected to be able to probe the LMA solution of the solar neutrino problem. It may also be able to directly detect solar ^{8}B and ^{7}Be neutrinos after its liquid scintillator has been purified to ultra high purity level by recirculating through purification.

There have been a number of accelerator experiments looking for neutrino oscillations. In all but one no evidence for oscillations was found and constraints on oscillation parameters were obtained. The LSND Collaboration have obtained an evidence for $\bar\nu_\mu \to \bar\nu_e$ and $\nu_\mu \to \nu_e$ oscillations [39]. The LSND result is the only indication for neutrino oscillations that is a signal and not a deficiency. The KARMEN experiment [40] is looking for neutrino oscillations in $\bar\nu_\mu \to \bar\nu_e$ channel. No evidence for oscillations has been obtained, and part of the LSND allowed region has been excluded. In fig. 3.4 the results from LSND and KARMEN experiments are shown along with the relevant constraints from the BNL E776, CCFR, CHOOZ and Bugey experiments. One can see that the only domain of the LSND allowed region which is presently not excluded is a narrow strip with $\sin^2 2\theta \simeq 1\times 10^{-3} - 4\times 10^{-2}$ and $\Delta m^2 \simeq 0.2 - 2$ eV^2.

The existing neutrino anomalies (solar neutrino problem, atmospheric neutrino anomaly and the LSND result), if all interpreted in terms of neutrino oscillations, require three different scales of mass squared differences: $\Delta m^2_\odot \leq 10^{-4}$ eV^2, $\Delta m^2_{atm} \sim 10^{-3}$ eV^2 and $\Delta m^2_{\rm LSND} \geq 0.2$ eV^2. This is only possible with four (or more) light neutrino species. The fourth light neutrino cannot be just the 4th generation neutrino similar to ν_e, ν_μ and ν_τ because this would be in conflict with the experimentally measured width of Z^0 boson.

It can only be an electroweak singlet (sterile) neutrino. Therefore the LSND result, if correct, would imply the existence of a light sterile neutrino. But such a possibility is disfavoured by the recent results of SNO, as we have already discussed.

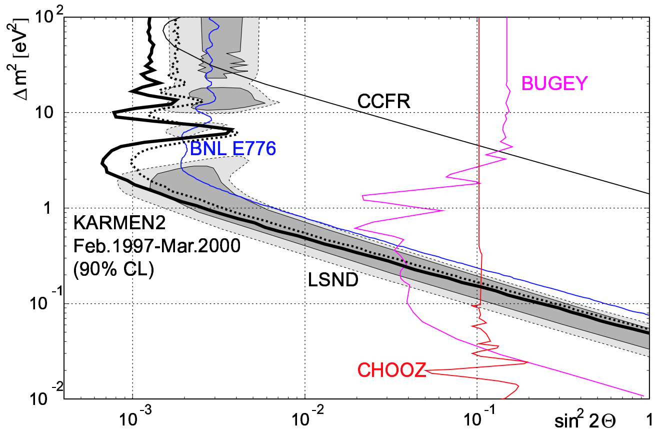

Figure 3.4: LSND allowed parameter region for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations (shaded areas) along with KARMEN, BNL E776, CCFR, CHOOZ and Bugey exclusion regions [40].

Out of all experimental evidences for neutrino oscillations, the LSND result is the only one that has not yet been confirmed by other experiments. It is therefore very important to have it independently checked. This will be done by the MiniBooNE (first phase of BooNE) experiment at Fermilab [41]. MiniBooNE will be capable of observing both $\nu_\mu \to \nu_e$ appearance and ν_μ disappearance. If the LSND signal is due to $\nu_\mu \to \nu_e$ oscillations, MiniBooNE is expected to detect an excess of several hundred of ν_e events during its first year of operation, establishing the oscillation signal at 8σ to 10σ level. If this happens, the second detector will be installed, with the goal to accurately measure the oscillation parameters. MiniBooNE will begin taking data sometime in 2002.

A number of long baseline accelerator neutrino experiments have been proposed to date. They are designed to independently test the oscillation interpretation of the results of the atmospheric neutrino experiments, accurately measure the oscillation parameters and to (possibly) identify the oscillation channel. The first of these experiments, K2K (KEK to Super-Kamiokande), started taking data in 1999. It has a baseline of 250 km, average neutrino energy $\langle E \rangle \simeq 1.4$ GeV and is looking for ν_μ disappearance. K2K should be able to test practically the whole region of oscillation parameters allowed by the SK atmospheric neutrino data except perhaps the lowest-Δm^2 part of it. The data collected from April 1999 to April 2001 have been reported [42]. A total of 44 neutrinos from KEK have been identified in the Super-Kamiokande detector. Based on a wide variety of measurements made at KEK, the number of events expected in the absence of neutrino oscillations would be 64, with error margins conservatively estimated as approximately 10%. Thus the K2K results are statistically inconsistent with the no-oscillations hypothesis (i.e., Standard Model assumption of massless neutrinos) at about the 97% confidence level. K2K has also studied the energy distribution of these events compared to the expectation based on the pion monitor data and a Monte-Carlo simulation, but the statistics to test the neutrino energy spectrum is very poor[5].

Two other long baseline projects, NuMI - MINOS (Fermilab to Soudan mine in the US) [43] and CNGS (CERN to Gran Sasso in Europe) [44], each with the baseline of 730 km, will be sensitive to smaller values of Δm^2 and should be able to test the whole allowed region of SK . MINOS will look for ν_μ disappearance and spectrum distortions due to $\nu_\mu \to \nu_x$ oscillations. It may run in three different energy regimes – high, medium and low energy ($\langle E \rangle \simeq 12$, 6 and 3 GeV, respectively). MINOS is scheduled to start taking data in 2003. CERN to Gran Sasso ($\langle E \rangle \simeq 17$ GeV) will be an appearance experiment looking specifically for $\nu_\mu \to \nu_\tau$ oscillations. It will also probe ν_μ disappearance and $\nu_\mu \to \nu_e$ appearance. At the moment, two detectors have been approved for participation in the experiment – OPERA and ICARUS. The whole project was approved in December of 1999 and the data taking is planned to begin in 2005 [45].

Among widely discussed now future projects are neutrino factories – muon storage rings producing intense beams of high energy neutrinos. In addition to high statistics studies of neutrino interactions, experiments at neutrino factories should be capable of measuring neutrino oscillation parameters with high precision and probing the subdominant neutrino oscillation channels, matter effects and CP violation effects in neutrino oscillations [46].

[5]K2K experiment has suffered a severe setback due to an unfortunate accident in the SK detector on 12th Nov 2001, blowing away the photo-multiplier tubes. However the plans of rebuilding the detector are underway.

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Chapter 4

A Brief Interlude on R-parity

In the previous chapters we briefly discussed the Minimal Supersymmetric Standard Model, RG equations and neutrino anomalies. The purpose of this thesis is to address the question of neutrino anomalies in the framework of supersymmetric standard model without R parity. Before we dwell into those problems it is worth-while to discuss certain issues connected with R-parity that are central to our problem. In this chapter we briefly discuss these issues.

4.1 From R-symmetry to R-parity

The Yukawa interactions of the standard model are extracted from the superpotential:

$$W_Y = \epsilon_{ab}\left(y^U_{ij}Q^a_iU^c_jH^b_2 + y^D_{ij}Q^a_iD^c_jH^b_1 + y^L_{ij}L^a_iE^c_jH^b_1\right). \tag{4.1}$$

This cubic superpotential contains no mass term for the Higgs and has many global symmetries, some nefarious to the phenomenology. Let us identify them. Global transformations on the superfields appear as

$$\Phi_f \to e^{in_f\eta}\Phi_f, \tag{4.2}$$

where f denotes the species: L, E^c, U^c, D^c or Q. The transformations which preserve SUSY obey the relations,

$$\begin{aligned} n_{L_i} + n_{E^c_i} + n_{H_1} &= 0 \quad i = e, \mu, \tau, \\ n_Q + n_{U^c} + n_{H_2} &= 0 \quad \textit{any flavour}, \\ \nu_Q + n_{D^c} + n_{H_1} &= 0 \quad \textit{any flavour}. \end{aligned} \tag{4.3}$$

With only one family, there are seven fields, with seven independent phases, obeying three relations from the couplings, leaving four independent symmetries namely, the lepton number, baryon number, hyper-charge and the Peccei-Quinn symmetry (PQ) [1]. The cubic super-potential is invariant under global $U(1)$ transformations

$$H_{1(2)} \to e^{i\alpha}H_{1(2)}\ ;\ Q(L) \to e^{-i\alpha}Q(L)\ ;\ U^c \to U^c\ ;\ D^c \to D^c\ ;\ E^c \to E^c. \tag{4.4}$$

This is PQ symmetry and is preserved as along as the coefficient of the bilinear term in superpotential μ is zero. Note that the condition $\mu = 0$ would imply that the corresponding soft parameter $B_\mu = 0$. This gives rise to two cases: (a) the *vev* of H_1 or H_2 is zero, that is some quarks/charged leptons are massless ,(b) the mass of CP-odd neutral Higgs $m_A = 0$. Both the cases are experimentally ruled out. Hence μ cannot be zero.

In the absence of μ term, the cubic superpotential enjoys one more global $U(1)$ symmetry, namely the R-symmetry [2, 3]. Under this symmetry $W \to W' = e^{2i\alpha}W$. This can be arranged by the following choice of R-charge:

$$H_{1(2)} \to e^{i\alpha} H_{1(2)} \ ; \ Q(L) \to e^{i\alpha} Q(L) \ ; \ U^c \to U^c \ ; \ D^c \to D^c \ ; \ E^c \to E^c. \tag{4.5}$$

Note that $\mathcal{L} = \int d^2\theta W$ is invariant under this transformation. Therefore $d\theta \to e^{-i\alpha} d\theta$ and $\theta \to e^{i\alpha}\theta$ (to ensure $\int d\theta\theta = 1$). As mentioned in chapter 1, a chiral superfield Φ can be written in terms of its scalar, fermionic and auxiliary components, $\Phi = \phi + \psi\theta + \theta^2 F$. Thus it is clear that ϕ carries same charge as Φ, and ψ carries one unit less. Also since the vector superfield V is real, $R(V) = 0$, which means that $R(V_\mu) = R(D) = 0$ and $R(\lambda) = 1$. Thus gauginos are massless in the limit of exact R-symmetry. In the MSSM, the majorana mass terms for the gauginos are generated by the soft SUSY breaking. Since these terms are quadratic in λ, the majorana gaugino mass terms break the continuous $U(1)_R$ symmetry down to a discrete Z_2 symmetry called R-parity. R-parity is defined as:

$$R_p = (-1)^{3B+L+2S}, \tag{4.6}$$

where B, L and S are the baryon, lepton and spin quantum numbers. R-parity is generally invoked to forbid the baryon and lepton number violating terms which are present when one writes down the most general superpotential allowed by supersymmetry, gauge invariance and renormalizability as we already discussed in chapter 1.

It is clear that the imposition of R_p (henceforth we will remove the subscript 'p') is only for the phenomenological purpose and does not follow as some sacrosanct symmetry principle. So, as far as the R violating couplings respect the constraints from phenomenology, they are not only harmless but give rise to rich phenomenology. For example, it is only the simultaneous presence of B and L violation that is in conflict with proton lifetime. Removing one of them would render proton stable. The popular approach is to invoke an alternative to R-parity, called the baryon parity which does not allow the B violating terms. We shall discuss in the next chapters, how retaining L violation leads to natural solutions for neutrino anomalies.

4.2 Distinguishing leptons and Higgs: Breaking an $SU(4)$

There are certain subtle issues connected with the freedom of rotation of leptons and Higgs. This freedom of rotation plays an important role when we discuss the neutrino masses from R violation in next chapters and hence we briefly discuss them here.

Let us assume for the time being, there are no superpotential terms and no soft SUSY breaking terms, but only gauge interactions. In such a scenario, since leptons and a down-type Higgs H_1 superfields carry the same gauge quantum numbers, there is no way we can distinguish them. The theory possesses an additional $SU(4)$ rotation symmetry in the L_i, H_1 space. Now let us introduce the MSSM superpotential and soft terms. This spoils $SU(4)$ symmetry as lepton and Higgs superfields have different interactions. At this stage there are two possibilities: (a) If the superpotential and the soft terms conserve the lepton number as a global symmetry, the lepton numbers are fixed so the lepton numbers make sense. In fact this along with the imposition of baryon number conservation is the MSSM with R-parity imposed . (b) If they are switched on, but violate the lepton number, although the $SU(4)$ symmetry is lost, a freedom to define the three lepton superfields and the down-type Higgs is still there. For example, if all the terms in the superpotential and the soft potential undergo an SU(4) rotation to redefine the lepton and Higgs superfield of the model, then superficially the *vevs* of the sneutrinos, the mass matrices and the relevant couplings are changed accordingly, whereas the physics is not changed [4, 5, 12, 7, 13, 9].

Because of the freedom of $SU(4)$ rotation, it is possible to rotate away the bilinear R violating terms from the superpotential. This led many authors to conclude that these terms are unphysical. It was pointed out by the work in [7], that regardless of whether the bilinear terms are present in the superpotential at the high scale or not, the corresponding soft bilinear terms are always generated at the weak scale. This is because the removal of bilinear terms by rotation, generates a subclass of trilinear L violating terms in the superpotential and these contribute to RG scaling of the soft bilinear parameters. These issues will be discussed at length in the next chapters. Suffice it to say that the freedom of $SU(4)$ rotation should be used carefully and that different choices for rotation suited for different problems can change *vevs* and the couplings, but the final physics is basis independent as it should be.

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Chapter 5

Bilinear R Violation and Bimaximal Mixing

5.1 Introduction

We discussed at length in chapter 2, the experimental results of deficits in the solar [1, 2] and atmospheric[1] neutrino fluxes. This has provided concrete ground to believe in neutrino oscillations. The experimental results are consistent with a simple picture of three active neutrinos mixing with each other [4]. Within this picture, two independent (mass)^2 differences $(\Delta_{\odot}, \Delta_{atm})$ among three neutrinos govern the oscillations of the solar and atmospheric neutrinos respectively. One needs $\Delta_{\odot}/\Delta_{atm} \leq 10^{-2}$. Two of the mixing angles determining the amplitudes of these oscillations are required to be large [5]. The third mixing angle measured by the survival probability of the electron neutrinos in laboratory experiments such as CHOOZ is found to be much smaller ≤ 0.1 [6].

So one is led to believe that like any other fermion, neutrinos are also massive. Nevertheless, extremely tiny masses and one or two large mixing angles in the neutrino sector are suggestive of a different origin for neutrino masses. Any theory of neutrino masses should naturally explain these distinctive features. In order to do so, the candidate theory should possess following desirable features:

- Extra fermions with same gauge quantum numbers as neutrinos, so that neutrinos can mix with them.
- Some of these fermions be several orders of magnitude heavier than neutrinos so that see-saw like mechanism is possible.
- Since neutrinos do not carry electromagnetic or colour charge, it is very much likely that they are Majorana fermions. This means that the theory should contain lepton number violating couplings, which can conjure to give radiative Majorana mass to neutrino.

We already discussed in chapter 2, the merits and demerits of extending Higgs and/or fermionic sector of SM in the framework of both seesaw as well as radiative mechanisms to explain the neutrino masses. In this chapter we pursue one of the most attractive extension to SM, the so called Minimal Supersymmetric Standard Model, without $R-parity$ which naturally contains all the above attributes of a candidate theory for neutrino masses. In such a framework the virtues of seesaw as well as radiative mechanisms are naturally operative.

One potentially interesting possibility in this regard is supersymmetric standard model containing bilinear R parity and lepton number violation [5, 8, 23, 7, 8, 9, 11, 10, 15]. This assumption is theoretically well motivated. Spontaneous breaking of lepton number [16] could normally result in such a term. This approach however leads to a low scale massless mode called majoron [17] which cannot be reconciled with phenomenology [1]. Alternatively one could imagine generalized Peccei-Quinn (PQ) symmetry whose spontaneous breaking leads to μ and ϵ_i at the weak scale through dimension 5 operators. [19]. Moreover it is possible to choose the PQ charges of different fields in such a way that the generation of effective trilinear operators is enormously suppressed. We follow an approach where lepton number violating couplings are present explicitly in the theory. Two features of this model make it an ideal candidate for the description of neutrino masses. (1) The lepton number violations and hence neutrino masses and mixing are described in this model in terms of only three parameters. Ratios of these parameters control neutrino mixing which can be naturally large. (2) The mechanism for suppression of neutrino masses compared to other fermion masses is automatically built-in for two of the most popular supersymmetry breaking scenario namely the minimal supergravity model (mSUGRA) and models with gauge mediated supersymmetry breaking (GMSB).

Extensive studies of these models have been carried out in the literature [8, 23, 7, 8]. In this chapter, which is based on our work [13], we wish to discuss the conditions under which the bilinear model can lead to two large mixing angle among neutrinos. We discuss this issue analytically and in the process show that the two scenarios mentioned above cannot lead to two large mixing angles although small angle mixing solution to the solar neutrino problem is possible[2].

In order to understand the suppression of neutrino masses, let us briefly discuss, how neutrinos obtain mass in these models. In these modes, the scalar soft potential contains terms linear in sneutrino field (see eq. 6.4). The stationary value conditions for soft potential gives rise to sneutrinos *vevs*. This leads to neutrino-neutralino mixing given by a 7×7 matrix. Neutralinos being very heavy ($\sim$ TeV) the mass matrix has a seesaw type structure. Such a matrix can be block diagonalized to give an effective 3×3 neutrino

[1] See [18] for the solution to this problem.

[2] Feasibility of only small mixing angle solution was pointed out also in [15] . Our analysis considerably differs from theirs.

mass matrix, which upon diagonalization gives neutrino mass eigenvalues (for details, see appendix A). So the neutrino masses are quadratic in sneutrino *vevs*. Sneutrino *vevs* in turn are derived in terms of the differences in the bilinear soft parameters at the weak scale. To be specific, it depends upon the differences between soft parameters of one of the Higgs fields ($\equiv H_1'$), with the corresponding parameters of the leptonic doublets having the same quantum numbers as H_1'. Small differences arise in these parameters at the weak scale due to RG scaling. For example, one finds in case of mSUGRA

$$\Delta m_i^2 \equiv (m_{\tilde{\nu}_i'}^2 - m_{H_1'}^2) \approx \frac{3h_b^2}{4\pi^2} \ln \frac{M_X}{M_Z} m_{susy}^2 \approx 2 \cdot 10^{-3} \; m_{susy}^2, \tag{5.1}$$

where $m_{\nu_i'}^2 (i = 1, 2, 3), m_{H_1'}^2$ respectively denote the weak scale values of the soft SUSY breaking masses of the sneutrino and H_1' respectively and m_{SUSY} is the typical SUSY breaking scale $\sim \mathcal{O}(100 \text{ GeV})$. The h_b in the above equation refers to the b-quark Yukawa coupling. The neutrino masses in this model involve the above and similar differences among B parameters. The suppression in these differences leads to suppression in neutrino masses. Thus the smallness of neutrino masses is linked to near universality of the *Higgs* (H_1') and sneutrino soft parameters. As we will discuss in this chapter, the solar neutrino mixing angle is directly linked to *flavour* universality violation, *i.e,* to differences in sneutrino mass parameters themselves. More specifically, the solar neutrino mixing angle involves the parameter

$$\delta = \frac{m_{\nu_2'}^2 - m_{\nu_1'}^2}{\Delta m_1^2 + \Delta m_2^2}, \tag{5.2}$$

which is required to be $\mathcal{O}(1)$ implying that the weak scale universality violation among different flavours are required to be as strong as the corresponding Higgs-slepton universality violations. This is in sharp contrast with the expectations based on mSUGRA and GMSB where the former violations are mainly controlled by the muon Yukawa coupling while the latter by the b or τ Yukawa couplings. Thus δ in eq.(5.2) is of $\mathcal{O}(10^{-4})$ instead of being one.

Link between universality violation and large mixing was brought out in the numerical study of [10]. In contrast to their work, our analytical study allows us to determine specific pattern of universality violation and also allows us to quantify the amount of violation needed to obtain the LMA solution for the solar neutrino problem.

We present our results in the following manner. The next section outlines general formalism we adopt and our assumptions. It also contains analytic discussion of neutrino mixing and masses in this scheme. The close link between large angle solar neutrino solution and flavour violation is emphasized in section (3) which also contains results based on numerical analysis. The last section contains a summary. Some of the technical aspects relevant to discussions in the text are elaborated in the appendices.

5.2 Sources of neutrino masses

We consider supersymmetric extension of the standard model with the following superpotential:

$$W = h^u_{ij} Q_i u^c_j H_2 + h^d_i Q_i d^c_i H'_1 + h^e_i L'_i e^c_i H'_1 + \mu' H'_1 H_2 + \epsilon_i L'_i H_2. \tag{5.3}$$

Without loss of generality, we have chosen above the basis in which the down quarks and charged lepton masses are diagonal. The ϵ_i characterize lepton number violation in this basis.

We assume the following soft supersymmetry breaking terms:

$$\begin{aligned} V_{soft} &= m^2_{H_1'^0} |H_1'^0|^2 + m^2_{H_2^0} |H_2^0|^2 + m^2_{\tilde{\nu}_i'} |\tilde{\nu}_i'|^2 \\ &- \left(B_\mu \mu' H_1'^0 H_2^0 + c.c \right) - B_i \epsilon_i \left(\tilde{\nu}_i' H_2^0 + c.c. \right) + \ldots . \end{aligned} \tag{5.4}$$

Note that the above equation refers to soft terms at the weak scale. For simplicity we have displayed only the terms involving neutral fields in the above equation. The following comments are needed in connection with eq.(6.4):
(i) Although we have allowed for arbitrary diagonal sneutrino masses, we have not included off-diagonal sneutrino masses in this primed basis since such off-diagonal masses are severely constrained by flavour violating processes, e.g. $\mu \to e\gamma$ [21].
(ii) V_{soft} does not contain sneutrino-Higgs mixing terms of the form $m^2_{\tilde{\nu}'_i H'_1} \tilde{\nu}_i'^* H'_1$ although they are allowed by the gauge symmetry. Such terms are not present in the minimal supergravity theory at high scale. The renormalization group (RG) equations for $m^2_{\tilde{\nu}_i' H'_1}$ given in the appendix, eq.(5.49) show that these terms cannot get generated even at the weak scale if they are not present at high scale. Thus it is meaningful to omit these terms. *We should emphasize that this statement is very specific to the particular basis in which bilinear terms are not rotated away from the superpotential until the weak scale and neglect of such terms would not be justified in any other basis.* In our case, the $\tilde{\nu}_i^* H_1$ term would make its appearance when we go to the basis with no bilinear R violating terms in the superpotential at the weak scale.

The neutrino masses arise from several sources in this model. Discussion of these sources becomes transparent if we re-express eq.(5.3) in the new basis in which bilinear terms are rotated away from W[3]:

$$\begin{aligned} H_1 &= \frac{\mu' H'_1 + \sum_i \epsilon_i L'_i}{\mu'}, \\ L_i &= \frac{\mu' L'_i - \epsilon_i H'_1}{\mu'}. \end{aligned} \tag{5.5}$$

[3]Note that this definition of a new basis is same as that of Ref.[17]. However in the present work, this rotation is done only at the weak scale in contrast to [17] where it is scale-dependent.

This basis are simple but are orthonormal only up to $\mathcal{O}(\frac{\epsilon^2}{\mu'^2})$. This approximation is sufficient for most of our discussions since ϵ_i are required to be much smaller than the typical SUSY scale μ' in order to reproduce the scale of neutrino masses correctly. Generalization of eq.(5.5) valid to higher order in ϵ_i and its consequences are discussed in the appendix C. Eq.(5.3) takes the following form in the unprimed basis:

$$W = h^u_{ij} Q_i u^c_j H_2 + h^d_i Q_i d^c_i H_1 + h^e_i L_i e^c_i H_1 - \lambda'_{ijk} L_i Q_j d^c_k - \lambda_{ijk} L_i L_j e^c_k + \mu H_1 H_2, \quad (5.6)$$

where

$$\begin{aligned} \mu^2 &= \mu'^2 + \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 \approx \mu'^2 \ , \\ \lambda'_{ijk} &= \frac{\epsilon_i}{\mu'} \ h^d_j \delta_{jk} \ , \\ \lambda_{ijk} &= (\delta_{ik} h^e_i \frac{\epsilon_j}{\mu'} - \delta_{jk} h^e_j \frac{\epsilon_i}{\mu'}) \ . \end{aligned} \quad (5.7)$$

Similarly, after rotating primed terms in eq.(6.4) and adding the contribution of the supersymmetric part, we get the following expression for the full scalar potential in the unprimed basis:

$$\begin{aligned} V_{scalar} &= (m^2_{H_1'^0} + \mu^2)|H_1^0|^2 + (m^2_{H_2^0} + \mu^2)|H_2^0|^2 + m^2_{\tilde{\nu}_i'}|\tilde{\nu}_i|^2 + \Delta m_i^2 \frac{\epsilon_i}{\mu} \left(\tilde{\nu}_i^{\star} H_1^0 + c.c\right) \\ &- \left(B_\mu \mu H_1^0 H_2^0 + c.c\right) - \Delta B_i \epsilon_i \left(\tilde{\nu}_i H_2^0 + c.c\right) \\ &+ \frac{1}{8}(g^2 + g'^2) \left(|H_1^0|^2 + |\tilde{\nu}_i|^2 - |H_2^0|^2\right)^2 \ , \end{aligned} \quad (5.8)$$

where

$$\Delta m_i^2 \equiv m^2_{\tilde{\nu}_i'} - m^2_{H_1'^0} \quad ; \quad \Delta B_i \equiv B_i - B_\mu \ . \quad (5.9)$$

Two major sources of neutrino masses arise from eqs.(5.7,5.8). Minimization of eq.(5.8) generates sneutrino *vev*:

$$< \nu_i > \equiv \epsilon_i k_i, \quad (5.10)$$

where

$$k_i \approx \frac{v_1}{\mu} \frac{(-\Delta m_i^2 + \tan\beta \ \mu \Delta B_i)}{(m^2_{\tilde{\nu}_i} + 1/2 M_Z^2 \cos\ 2\beta)}, \quad (5.11)$$

$v_1 =< H_1^0 >$ and M_Z represents the Z boson mass. Sneutrino *vevs* lead to neutrino masses through their mixing with neutral-gauginos:

$$\mathcal{M}_{\mathbf{tree}} \equiv A_0 < \tilde{\nu}_i >< \tilde{\nu}_j >= A_0 \epsilon_i \epsilon_j k_i k_j \ . \quad (5.12)$$

A_0 is obtained by diagonalizing the 7×7 neutrino-neutralino mass matrix in the standard way [21] and is demonstrated in the appendix A of this chapter:

$$A_0 = \frac{\mu\,(g^2 + g'^2)}{2\,(-c\mu M_2 + M_W^2 \sin 2\beta\,(c + \tan^2\theta_W))} \ , \quad (5.13)$$

where θ_W represents the Weinberg angle and M_W represents the W-boson mass. c is given by $5g^2/3g'^{\,2} \sim 0.5$ with M_2 representing the standard gaugino mass parameter.

The trilinear terms in eq.(5.7) lead to the second contribution to neutrino masses at 1-loop level for the λ' couplings. Since these couplings are proportional to the Yukawa couplings, the dominant contributions arise due to exchanges of the b-quark-squark and τ-lepton-slepton in the loops. The loop induced mass matrix is of the form :

$$(\mathcal{M}_{loop})_{ij} = \epsilon_i \epsilon_j \left(A_b + A_\tau \left(1 - \delta_{i3}\right)\left(1 - \delta_{j3}\right)\right), \tag{5.14}$$

where

$$A_b = \frac{3}{16\pi^2}\frac{v_1}{\mu^2} h_b^3 \sin\phi_b \cos\phi_b ln\left(\frac{M_{2b}^2}{M_{1b}^2}\right), \tag{5.15}$$

$$A_\tau = \frac{1}{16\pi^2}\frac{v_1}{\mu^2} h_\tau^3 \sin\phi_\tau \cos\phi_\tau ln\left(\frac{M_{2\tau}^2}{M_{1\tau}^2}\right). \tag{5.16}$$

Here $\phi_{b,(\tau)}$ denotes mixing between the left and the right handed squark (sneutrino) fields. These mixing angles are proportional to the b and τ Yukawa couplings. Approximating them by $\frac{m_{b,\tau}}{m_{\mathrm{susy}}}$ we get the following numerical values

$$\begin{aligned} A_0 &\approx 5\cdot 10^{-3}\,\mathrm{GeV}^{-1}, \\ A_b &\approx 3\cdot 10^{-10}\,\mathrm{GeV}^{-1}, \\ A_\tau &\approx 4\cdot 10^{-12}\,\mathrm{GeV}^{-1}. \end{aligned} \tag{5.17}$$

for $m_{susy} \sim 100\,\mathrm{GeV}$. There are other loop contributions to neutrino masses and a complete discussion is given in [8, 10, 22]. We have retained here only those contributions which are known [22] to be dominant in case of mSUGRA and GMSB. The additional contributions not included in the text come from, (a) R parity violating mixing of the charged leptons with Higgs fields, (b) sneutrino (chargino) exchange diagrams with off-diagonal sneutrino (chargino) mass insertion and (c) loop contribution to the tree level neutrino neutralino mixing. These contributions are sub-dominant as long as the parameters $\Delta m_i^2, \Delta B_i$ are suppressed [22]. Such suppression is required purely from the phenomenological point as we argue below. It is then consistent to omit these sub-dominant terms for the analytical discussion that follows. We however discuss these additional contributions in the appendix C.

The total neutrino mass matrix is given by

$$\left(\mathcal{M}_{tot}\right)_{ij} = A_0 \epsilon_i \epsilon_j k_i k_j + \epsilon_i \epsilon_j \left(A_b + A_\tau \left(1 - \delta_{i3}\right)\left(1 - \delta_{j3}\right)\right) \ . \tag{5.18}$$

The desired hierarchy among neutrino masses is automatically built in the above equations in view of typical numerical values of the parameters $A_{0,b,\tau}$. The tree contribution dominates over the rest (unless k_i are enormously suppressed) but it leads to only one massive neutrino. Switching on the b-quark contribution gives mass to the other neutrino, one neutrino still remaining massless at this stage. The latter obtains its mass from somewhat less dominant contribution due to A_τ. Note that hierarchy among the first two neutrino masses need not be very strong due to similar magnitudes of $A_{b,\tau}$. The above statements are made explicit below which also contains discussion on neutrino mixing.

5.2.1 Neutrino masses and mixing

The tree-level neutrino mass matrix can be easily diagonalized:

$$U_0\ \mathcal{M}_{tree} U_0^T = diag\{0, 0, m_{\nu_3}\}\ , \tag{5.19}$$

where

$$U_0^T = \begin{pmatrix} c_2 & s_2 c_3 & s_2 s_3 \\ -s_2 & c_2 c_3 & c_2 s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}\ , \tag{5.20}$$

with $s_{2,3} = \sin\theta_{2,3}$ and

$$\tan\theta_2 = \epsilon_1 \kappa_1 / \epsilon_2 k_2\ ; \quad \tan\theta_3 = \sqrt{\epsilon_1^2 k_1^2 + \epsilon_2^2 k_2^2} / \epsilon_3 k_3\quad . \tag{5.21}$$

The total mass matrix eq.(5.18), assumes the following form in basis with diagonal tree mass matrix:

$$U_0\ \mathcal{M}_{\mathbf{tot}}\ U_0^T = \begin{pmatrix} a_1^2(A_b + A_\tau) & a_1(A_b a_2 + A_\tau b_2) & a_1(A_b a_3 + A_\tau b_3) \\ a_1(A_b a_2 + A_\tau b_2) & A_b a_2^2 + A_\tau b_2^2 & A_b a_2 a_3 + A_\tau b_2 b_3 \\ a_1(A_b a_3 + A_\tau b_3) & A_b a_2 a_3 + A_\tau b_2 b_3 & A_0 \omega^2 + A_b a_3^2 + A_\tau b_3^2 \end{pmatrix}, \tag{5.22}$$

where

$$\begin{aligned} a_1 &= \frac{\epsilon_1 \epsilon_2}{\omega_\perp}(k_1 - k_2), \\ a_2 &= \frac{\epsilon_3}{\omega_\perp \omega}(\epsilon_1^2 k_1 (k_1 - k_3) + \epsilon_2^2 k_2 (k_2 - k_3)), \\ a_3 &= -\frac{1}{\omega}(\epsilon_1^2 k_1 + \epsilon_2^2 k_2 + \epsilon_3^2 k_3), \\ b_2 &= \frac{\epsilon_3 k_3}{\omega_\perp}\ b_3, \\ b_3 &= -\frac{1}{\omega}(\epsilon_1^2 k_1 + \epsilon_2^2 k_2)\ , \end{aligned} \tag{5.23}$$

with

$$\begin{aligned} \omega &= (\epsilon_1^2 k_1^2 + \epsilon_2^2 k_2^2 + \epsilon_3^2 k_3^2)^{1/2}, \\ \omega_\perp &= (\epsilon_1^2 k_1^2 + \epsilon_2^2 k_2^2)^{1/2} \,. \end{aligned} \tag{5.24}$$

The subsequent diagonalization can be approximately done if we neglect terms of $\mathcal{O}(\frac{A_{b,\tau}}{A_0})$. Let

$$U_1^T = \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \,, \tag{5.25}$$

where

$$\tan 2\theta_1 = \frac{2a_1\,(A_b a_2 + A_\tau b_2)}{A_b\,(a_2^2 - a_1^2) + A_\tau\,(b_2^2 - a_1^2)}. \tag{5.26}$$

We then have

$$U_1 U_0 \mathcal{M}_{tot} U_0^T U_1^T = \begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_3} \end{pmatrix} + \mathcal{O}(\frac{A_{b,\tau}}{A_0}). \tag{5.27}$$

The eigenvalues are approximately given by

$$\begin{aligned} m_{\nu_1} &\approx A_\tau\,\frac{a_1^2(a_2-b_2)^2}{(a_1^2+a_2^2)}, \\ m_{\nu_2} &\approx A_b\,(a_1^2+a_2^2), \\ m_{\nu_3} &\approx A_0\,\omega^2. \end{aligned} \tag{5.28}$$

The mixing among neutrinos is described by

$$U \equiv U_0^T\;U_1^T = \begin{pmatrix} c_2c_1 - s_1s_2c_3 & c_2s_1 + c_1s_2c_3 & s_2s_3 \\ -s_2c_1 - s_1c_2c_3 & -s_2s_1 + c_1c_2c_3 & c_2s_3 \\ s_1s_3 & -c_1s_3 & c_3 \end{pmatrix} \,. \tag{5.29}$$

Let us now discuss consequences of the above algebraic results.

(1) It follows from eqs.(5.17,5.28) that the neutrino masses obey the desired hierarchy:

$$m_{\nu_1} \lesssim m_{\nu_2} \ll m_{\nu_3}.$$

(2) The neutrino masses relevant for the solar and atmospheric scales are respectively given by $A_0\epsilon^2k^2$ and $A_b\epsilon^2$ leading to

$$\frac{\Delta_\odot}{\Delta_{atm}} \approx \left(\frac{A_b}{A_0}\right)^2 \frac{1}{k^4} \,,$$

where ϵ, k represent typical values of ϵ_i, k_i. It follows that the ratio of the solar to atmospheric scales is *independent* of the R violating parameters ϵ_i and depends upon the values of the soft parameters represented by k. One typically needs

$$\epsilon \sim 10^{-1}\ \text{GeV} \quad ; \quad k \sim 10^{-3} - 10^{-4} \tag{5.30}$$

in order to reproduce the scales correctly. This shows in particular that irrespective of details of the SUSY breaking the Higgs-slepton universality (corresponding to very small values of k) is unavoidable in this model if neutrino masses are to be correctly reproduced. (3) If exact flavour universality were to hold between the first two generations then $k_1 = k_2$ (see eq.(5.11)). In this case a_1 as defined in eq.(5.23) would be zero leading to $s_1 = 0$ in eq.(6.44) . The s_1 is required to be large in order to obtain the large mixing angle solution and obtaining this solution would need very sizable departures from the flavour universality among the first two generations. We quantify these remarks in the next section.

5.3 Neutrino mixing and departure from flavour universality

We derived approximate expressions for the neutrino masses and mixing without any specific assumption on the soft symmetry breaking sector. The entire neutrino spectrum can be parameterized in terms of three ϵ_i and three k_i of which k_i depend upon the soft SUSY breaking parameters. We now quantify the amount of flavour universality violations needed for obtaining the most preferred large angle solution to the solar neutrino problem. The following two parameters are introduced as a measure of universality violation:

$$x = (k_1 - k_3)/(k_1 + k_3) \quad ; \quad y = (k_1 - k_2)/(k_1 + k_2). \tag{5.31}$$

We regard x and y as independent parameters but restrict their variation to values between (-1,1) in the numerical analysis that follows.

The neutrino mixing is determined by the matrix U in eq.(5.29). Due to hierarchical mass spectrum, the survival probabilities for the solar and atmospheric neutrinos approximately assume two generation form. The corresponding mixing angles $\theta_\odot$ and $\theta_{\mathbf{atm}}$ are given in terms of elements of the mixing matrix U as follows:

$$\begin{aligned}
\sin^2 2\theta_{atm} &\sim 4\ U_{\mu 3}^2 (1 - U_{\mu 3})^2 \approx 0.8 - 1.0\ , \\
\sin^2 2\theta_\odot &\sim 4\ U_{e2}^2 U_{e1}^2 \approx 0.75 - 1.0\ , \\
\sin^2 \theta_{CHOOZ} &\sim U_{e3}^2 \leq 0.01\ ,
\end{aligned} \tag{5.32}$$

where numbers on the RHS correspond to the required values for these parameters based on two generation analysis of the experimental data [5].

We can convert the above restrictions on $\theta_\odot, \theta_{\mathbf{atm}}$ to restrictions on the mixing angles $s_{1,2,3}$ entering the definition of U. The CHOOZ result requires $|s_2 s_3| \leq 0.1$ and the nearly maximal atmospheric mixing is obtained with $|c_2 s_3| \approx \frac{1}{\sqrt{2}}$. This requires small s_2 and large s_3. The solar mixing angle defined in eq.(5.32) coincides with s_1 in this limit. We thus need $\sin^2 2\theta_1 \sim 0.75 - 1$. Large value of s_1 in turn needs sizable departure from flavour universality as argued in the last subsection.

The expressions for mixing angles and masses obtained in the last section can be used to approximately determine the allowed ranges of parameters k_i, ϵ_i which explain the solar and atmospheric neutrino anomalies. We approximately need $|s_2| \leq \sqrt{2} U_{e3}$ and $|s_3| \approx \frac{1}{\sqrt{2}}$. This implies:

$$\begin{aligned} \epsilon_2^2 k_2^2 &\approx \epsilon_3^3 k_3^2 \ , \\ |\epsilon_1 k_1| &\approx \sqrt{2} |U_{e3} \epsilon_2 k_2|. \end{aligned} \tag{5.33}$$

The magnitude of $\epsilon_3 k_3$ is then approximately fixed by the atmospheric mass scale:

$$\epsilon_3^2 k_3^2 \approx \frac{m_{\nu_3}}{2A_0} \approx \frac{\sqrt{\Delta_{atm}}}{2A_0} \ , \tag{5.34}$$

while the solar scale and mixing angle determines ϵ_3^2:

$$\epsilon_3^2 \approx \left(\frac{\sqrt{\Delta_\odot}}{2A_b \cos^2 \theta_\odot} \right) \frac{(1+x)^2 (1-y)^2}{(x-y)^2} \ . \tag{5.35}$$

Eqs.(5.33,5.34,5.35) allow us to express magnitudes of all ϵ_i, k_i in terms of x, y , approximately known $A_{0,b}$ and the experimentally measurable quantities.

The solar mixing angle following from eq.(5.26) is given in the limit $A_\tau \ll A_b$ by

$$\tan^2 \theta_\odot \approx \tan^2 \theta_1 \approx \frac{4 U_{e3}^2 y^2 (1-x)^2}{(x-y)^2} \ . \tag{5.36}$$

We have used eq.(5.33) in deriving the above relation. It is clear that large θ_1 requires sizable departure from flavour universality, i.e. sizable y. Moreover, one typically needs $|x - y| \approx 2 |U_{e3} y (1-x)|$ in order to obtain a sizable solar angle.

We now numerically determine the region in the x, y plane needed to reproduce the required ranges in mixing angle and masses. We make use of eqs.(5.33-5.35) to determine the approximate input values of ϵ_i, k_i in terms of the $\Delta_\odot, \Delta_{atm}$ and x, y. We allow input values to vary by varying $\Delta_\odot, \Delta_{atm}$ over the experimentally allowed ranges. We also randomly choose x, y between -1 and 1. Through this procedure, we choose a set of 1.5×10^5 different values for the input parameters ϵ_i, k_i. Then we numerically diagonalize the total neutrino mass matrix, eq.(5.18) for each of these values of ϵ_i, k_i and determine a set of x, y values which correctly reproduces the allowed ranges of the solar and atmospheric neutrino parameters and lead to $|U_{e3}| \leq 0.1$. We obtain about 2024 x, y values leading to the correct description of neutrino anomalies. These points in the x, y plane are displayed in Fig.(1). This figure, based on the complete diagonalization clearly shows the features obtained through approximate formulas. All the allowed values of x and y are in the range -0.9 to -0.6 and sizable departure from universality is clearly seen. Also most points satisfy approximate equality $|x - y| \sim 2U_{e3}y$ needed to obtain large solar angle. As an illustration, we give below a typical set of ϵ_i, k_i which correctly reproduces all the parameters:

$$\begin{aligned} \epsilon_3 &\sim 0.1\,\mathrm{GeV} \quad ; \; k_3 \sim 1.1 \; \cdot 10^{-3} \\ \epsilon_2 &\sim 0.031\,\mathrm{GeV} \quad ; \; k_2 \sim 3.5 \; \cdot 10^{-3} \\ \epsilon_1 &\sim 0.087\,\mathrm{GeV} \quad ; \; k_1 \sim 9.1 \; \cdot 10^{-4}. \end{aligned} \tag{5.37}$$

Typically, one needs $e_i \sim \mathcal{O}(10^{-1}\ GeV)$ and $k_i \sim 10^{-3}$ as argued before.

Let us now compare above phenomenological restrictions with expectations based on specific framework like mSUGRA. In order to obtain correct neutrino masses one needs parameters k_i (5.11) to be suppressed, typically $k \sim 10^{-3} - 10^{-4}$ as in eq.(5.30). The other constraint is that y should be $\mathcal{O}(1)$. The k_i provide a measure of the Higgs-slepton universality violation. Typical value of k_i obtained in mSUGRA follows from eq.(1) and is in the range required from phenomenology. Thus mSUGRA provides a very good framework to understand neutrino mass hierarchy as has been demonstrated in number of papers through detailed numerical calculations [8, 8, 10]. However mSUGRA would not be able to provide the required value of y. This can be seen as follows. Theoretically, y can be approximately written using eq.(5.11) as follows:

$$y \approx \frac{\mu \tan\beta(B_1 - B_2) - (m^2_{\tilde{\nu'}_1} - m^2_{\tilde{\nu'}_2})}{\mu \tan\beta(\Delta B_1 + \Delta B_2) - (\Delta m_1^2 + \Delta m_2^2)} \;, \tag{5.38}$$

where we have neglected terms of order $(\Delta m_i^2)^2, (\Delta B_i)^2$ etc. Within mSUGRA, y is identically zero at the high scale as $B_1 = B_2$ and $m^2_{\tilde{\nu}_1} = m^2_{\tilde{\nu}_2}$ due to the universal boundary conditions. At the weak scale, this universality condition is broken solely by RG evolution. In the limit of neglecting first two generation Yukawa couplings, y is identically zero even at the weak scale. A rough estimate of parameters appearing in y can be obtained by approximately integrating the RG equations, eqs.(5.50,5.51) given in appendix B. We see that

$$\frac{m^2_{\tilde{\nu'}_1} - m^2_{\tilde{\nu'}_2}}{\Delta m_1^2 + \Delta m_2^2} \approx \frac{1}{6}\left(\frac{m_\mu}{m_b}\right)^2 \approx 10^{-4} \;,$$

$$\frac{(B_1 - B_2)}{\Delta B_1 + \Delta B_2} \approx \frac{1}{6}\left(\frac{m_\mu}{m_b}\right)^2 \approx 10^{-4} \ . \tag{5.39}$$

Together they would imply very small value for $y \sim 0$ instead of the required value of $\mathcal{O}(1)$. Thus universal boundary conditions of mSUGRA cannot lead to a large mixing angle solution to the solar neutrino problem.

It is clear from the forgoing discussion that one needs small Higgs-slepton universality violation as well as flavour violation of similar magnitude. While mSUGRA cannot give this pattern, such pattern can be incorporated in non-minimal models of GMSB [18]. The flavour universality violations needed to obtain large solar angle can come either from non-universal mass terms or from non-universal B parameters or both. Identical gauge quantum numbers of all sneutrinos assure almost universal sneutrino masses at the weak scale as in the case of mSUGRA. In contrast, there is no natural reason within these models for the flavour universal B parameters. In fact, the B parameters are assumed to vanish in the minimal version of the scheme [20, 27]. Thus the universality of B parameters at supersymmetry breaking scale holds by default. It is possible to choose non-universal and non-zero $B_{1,2}$ terms to start with in this model. This does not significantly influence the conventional phenomenology of the minimal version as long as the parameters ϵ_i are much smaller than the μ-parameter in the superpotential. But it allows the LMA solution as has been demonstrated through a detailed numerical work [7].

Knowing the value of x and y required for a correct neutrino spectrum at the weak scale, it is possible to estimate the amount of non-universality required at the high scale. For example, using y, we have in the limit of neglecting contributions from ΔB terms, the required slepton flavour universality violations to be of order:

$$m^2_{\tilde{\nu}'_2}(0) - m^2_{\tilde{\nu}'_1}(0) \approx y(m^2_{\tilde{\nu'}_2}(0) + m^2_{\tilde{\nu'}_1}(0)) + 2\ y\ \frac{m^2_{\tilde{\nu'}_2}(0)\ m^2_{\tilde{\nu'}_1}(0)}{m^2_{H_1^0}(0) + \delta m^2_{H_1^0}}, \tag{5.40}$$

where $\delta m^2_{H_1^0}$ represents the correction to the high scale Higgs mass due to RG scaling. From the above we see that for a large negative y, $m^2_{\tilde{\nu}_1}(0)$ should be at least a factor of 3 times larger than $m^2_{\tilde{\nu}_2}(0)$. However these estimates should not be taken very seriously. As in the realm of non-universal soft parameters, the sub-dominant contributions which have been neglected in the present analysis can possibly become dominantly contributing depending on the choice of parameters.

5.4 Summary

Supersymmetric model with bilinear R parity violations provides a potentially interesting framework to study neutrino masses and mixing. The dominant sources of neutrino masses can be parameterized in this scenario in terms of three dimensionful parameters ϵ_i and three dimensional parameters k_i. The k_i depend on the structure of soft supersymmetry breaking terms at the weak scale. We have tried to obtain phenomenological restrictions on ϵ_i and k_i without making specific assumptions on the values of the soft supersymmetry breaking parameters. While neutrino masses can be suppressed by lowering the overall scale ϵ_i of R parity violation, phenomenologically preferred hierarchy in neutrino masses require that both ϵ_i and k_i are suppressed, see eq.(5.30). k_i provide a measure of the Higgs-slepton universality and suppression in their values indicate very small amount of this violation. Such violation of universality is already built in the mSUGRA and GMSB scenario.

A large solar neutrino mixing angle can be obtained consistently within these scenarios only if flavour universality violations in the soft parameters of the first two generations are almost as large as the violation of Higgs-slepton universality. This feature does not emerge in models where these universality violations are generated solely by RG scaling as in the case of mSUGRA. Thus mSUGRA seems more suitable to describe the less preferred small mixing angle solution to the solar neutrino problem.

We concentrated throughout on the most dominant sources of neutrino masses in this theory. This is a good assumption in case of small universality violation. The other sources of neutrino masses would become important in case of large universality violation. It is not unlikely that these contributions could also lead to a large solar neutrino mixing angle in such scenarios.

5.5 Appendix A

We present here the derivation of the tree level mass matrix, eq.(5.12) [4]. In the presence of R-violating couplings neutrinos mix with neutralinos. In the bilinear R-parity violating scenarios, this mixing takes place with ϵ_i couplings and the sneutrino *vevs*. In the Weyl basis, the Lagrangian describing the neutrino-neutralino mass matrix is given by,

$$\mathcal{L}_{mass} = -\frac{1}{2}\Psi_0^T \mathcal{M}_0 \Psi_0 + h.c., \tag{5.41}$$

where in the two component notation, Ψ_0 is a column vector of neutrinos and neutralinos,

$$\Psi_0^T = \left(\nu_e, \nu_\mu, \nu_\tau, -i\lambda_1, -i\lambda_3, \psi^0_{H_1}, \psi^0_{H_2}\right). \tag{5.42}$$

[4]Here we work in the un-rotated basis, retaining the bilinear couplings ϵ_i for sake of generality. The tree level mass matrix of of eq. (5.12) can be obtained by putting $\epsilon_i = 0$.

The mass matrix has the following general structure which is of see-saw type:

$$\mathcal{M}_{tree} = \begin{pmatrix} 0 & m \\ m^T & M_4 \end{pmatrix}. \tag{5.43}$$

Here the sub-matrix m is of dimension 3×4 and has the following structure:

$$m = \begin{pmatrix} -\frac{g_1}{\sqrt{2}}\omega_1 & \frac{g_2}{\sqrt{2}}\omega_1 & 0 & \epsilon_1 \\ -\frac{g_1}{\sqrt{2}}\omega_2 & \frac{g_2}{\sqrt{2}}\omega_2 & 0 & \epsilon_2 \\ -\frac{g_1}{\sqrt{2}}\omega_3 & \frac{g_2}{\sqrt{2}}\omega_3 & 0 & \epsilon_3 \end{pmatrix}, \tag{5.44}$$

with the ω_i representing the sneutrino *vevs*. M_4 is the standard 4×4 neutralino mass matrix of the MSSM which has the following form:

$$M_4 = \begin{pmatrix} M_1 & 0 & -\frac{g_1}{\sqrt{2}}v_1 & \frac{g_1}{\sqrt{2}}v_2 \\ 0 & M_2 & -\frac{g_2}{\sqrt{2}}v_1 & \frac{g_2}{\sqrt{2}}v_2 \\ -\frac{g_1}{\sqrt{2}}v_1 & \frac{g_1}{\sqrt{2}}v_1 & 0 & -\mu \\ -\frac{g_2}{\sqrt{2}}v_2 & \frac{g_2}{\sqrt{2}}v_2 & -\mu & 0 \end{pmatrix}. \tag{5.45}$$

The effective 3×3 neutrino mass matrix is obtained by block diagonalizing the above matrix. It has the form :

$$\begin{aligned} m_{eff} &= -mM_4^{-1}m^T \\ &= \frac{\mu(M_1 g^2 + M_2 g'^2)}{D} \begin{pmatrix} \Lambda_1^2 & \Lambda_1\Lambda_2 & \Lambda_1\Lambda_3 \\ \Lambda_1\Lambda_2 & \Lambda_2^2 & \Lambda_2\Lambda_3 \\ \Lambda_1\Lambda_3 & \Lambda_2\Lambda_3 & \Lambda_3^2 \end{pmatrix}. \end{aligned} \tag{5.46}$$

The vector $\vec{\Lambda}$ is defined as,

$$\vec{\Lambda} = \mu\vec{\omega} - v_1\vec{\epsilon}. \tag{5.47}$$

D is given by,

$$D = 2(-\mu\ M_1\ M_2 + 2\ M_W^2 c_\beta s_\beta (M_1 + M_2\ tan^2\theta_W)). \tag{5.48}$$

Here, the matrix m_{eff} is written in the basis where ϵ_i are not rotated away from the superpotential. The rotated form of this matrix is given in the text and takes the form $\mathcal{M}_{tree}$, eq.(5.12), in the charged lepton mass basis. From m_{eff} we can easily see that it has only one eigenvalue, even in the presence of the first generation sneutrino *vev*. This is a generic result of all the R-parity violating models.

5.6 Appendix B

The renormalization group equations for various parameters appearing in the soft scalar potential are basis dependent. We have chosen a specific basis in which bilinear terms in the potential are kept in the superpotential till the weak scale. These terms are rotated only after evolving to weak scale. We collect here RG equations for relevant parameters with this specific choice. They differ for example from the ones derived in [22] where relevant rotation is performed at each scale. The following equations follow in a straightforward manner from the formalism given by Falck[35]:

$$\frac{d}{dt} m^2_{\nu'_i H'_1} = m^2_{\nu'_i H'_1}(-\frac{1}{2}Y^E_i - \frac{1}{2}Y_\tau - \frac{3}{2}Y_b) \; , \tag{5.49}$$

$$\begin{aligned} \frac{d}{dt}(\Delta m_i^2) &= 3Y_b(m^2_{Q_3} + m^2_{D_3} + m^2_{H'_1} + \tilde{A}_b^2) \\ &- Y^E_i(m^2_{L_i} + m^2_{E_i} + m^2_{H'_1} + \tilde{A}^{E\ 2}_i), \end{aligned} \tag{5.50}$$

$$\frac{d}{dt}(\Delta B_i) = \tilde{A}_\tau Y_\tau + 3Y_b\tilde{A}_b - 3Y^E_i\tilde{A}^E_i. \tag{5.51}$$

In the above, we have used standard notation for all the soft parameters appearing in the equations.

5.7 Appendix C

In this appendix we justify the neglect of additional contributions to neutrino masses not included in the main text. We also discuss flavour violating processes $\mu \to e\gamma$ and show that the corresponding branching ratio is very small in the present context.

Detailed analysis of the additional 1-loop diagrams contributing to neutrino mass matrix has been done in [8, 10, 22]. While Refs. [8, 10] calculate all the 1-loop self-energy diagrams to the 7×7 neutrino-neutralino mass matrix and re-diagonalize it, Ref.[22] follows the effective mixing matrix approach. In addition to the contributions considered in the text, large contributions are also expected from diagrams which are not Yukawa suppressed, thus involving only gauge vertices. These can be visualized as diagrams with two R-parity violating mass insertions proportional to Δm_i^2, ΔB_i as given in eq.(5.8), with neutralino (chargino), sneutrino (charged slepton) and neutral Higgs (charged Higgs) in the loops [11, 22]. Typical magnitude of these diagrams is given by

$$\mathcal{M}^\lambda_{ij} \approx \frac{g^2}{16\pi^2}\epsilon_i\epsilon_j k'_i k'_j m^{-1}_{susy} \tag{5.52}$$

with

$$k'_i \approx \frac{c_1\Delta m_i^2 + c_2\Delta B_i^2}{m^2_{\tilde{\nu}_i}} \; .$$

m_{susy} is a typical supersymmetry breaking scale and $c_{1,2}$ are coefficients of order one following from the scalar mass matrices of the model. k'_i are similar to parameters k_i defined in eq.(5.11). It is natural then to choose $k'_i \sim \frac{\mu}{v_1} k_i$ for order of magnitude estimates. Comparing the 1-loop gaugino contribution with the b-quark contribution (eq.(5.14)) $\mathcal{M}^b$ we obtain

$$\frac{\mathcal{M}^{\lambda}_{ij}}{\mathcal{M}^{b}_{ij}} \approx \frac{g^2}{16\pi^2 A_b} \left(\frac{\mu}{v_1}\right)^2 \frac{k_i k_j}{m_{susy}}. \tag{5.53}$$

The numerical value of A_b is given in eq.(5.15). As argued above, we typically need $k_i \sim 10^{-3} \doteq 10^{-4}$. It is seen that the $b - quark$ contribution retained in the main text dominates over the gaugino contribution in this case and it is consistent to neglect the latter. The other contributions to neutrino masses are even less dominant than the gaugino contribution[5]. They come from 1-loop diagrams with two Yukawa vertices. These can be seen as a) diagrams with λ and h_τ vertices with a R-parity violating mass insertion in the internal line connecting charged slepton and charged Higgs, and b) diagrams with h_τ couplings at both the vertices with two R-parity violating mass insertions proportional to the sneutrino *vev.* Both these sets of diagrams are suppressed by the τ-Yukawa coupling. They have been analyzed in detail in Ref.[22] where it has been shown that they can become comparable in magnitude to A_τ in large tan β regions. However as we have seen earlier this contribution is always sub-dominant compared to the contribution from bottom Yukawa couplings, A_b. Thus it is justified to neglect these contributions within the present analysis.

Effects of Basis Rotation up to higher order in ϵ : We now generalize the basis (5.5) to higher order in ϵ and discuss its consequences. Such generalization becomes necessary for discussion of flavour violating transitions such as $\mu \to e\gamma$. Eq.(5.5) can be re-rewritten as follows:

$$\begin{pmatrix} H_1 \\ L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\,\hat{\epsilon}^2 & \hat{\epsilon}_1 & \hat{\epsilon}_2 & \hat{\epsilon}_3 \\ -\hat{\epsilon}_1 & 1-\frac{1}{2}\,\hat{\epsilon}_1^2 & -\frac{1}{2}\,\hat{\epsilon}_1\hat{\epsilon}_2 & -\frac{1}{2}\,\hat{\epsilon}_1\hat{\epsilon}_3 \\ -\hat{\epsilon}_2 & -\frac{1}{2}\,\hat{\epsilon}_1\hat{\epsilon}_2 & 1-\frac{1}{2}\,\hat{\epsilon}_2^2 & -\frac{1}{2}\,\hat{\epsilon}_2\hat{\epsilon}_3 \\ -\hat{\epsilon}_3 & -\frac{1}{2}\,\hat{\epsilon}_1\hat{\epsilon}_3 & -\frac{1}{2}\,\hat{\epsilon}_2\hat{\epsilon}_3 & 1-\frac{1}{2}\,\hat{\epsilon}_3^2 \end{pmatrix} \begin{pmatrix} H'_1 \\ L'_1 \\ L'_2 \\ L'_3 \end{pmatrix} + \mathcal{O}(\hat{\epsilon}^3), \tag{5.54}$$

where $\hat{\epsilon}_i = (\epsilon_i)/\mu, \hat{\epsilon}^2 = \hat{\epsilon}_1^2 + \hat{\epsilon}_2^2 + \hat{\epsilon}_3^2$. The V_{soft} in eq.(6.4) assumes the form

$$\begin{aligned} V_{soft} &= m^2_{H_1}|H_1^0|^2 + m^2_{H_2}|H_2^0|^2 + m^2_{\tilde{\nu}_i}|\tilde{\nu}_i|^2 + \Delta m_i^2 \hat{\epsilon}_i \left(\tilde{\nu}_i{}^* H_1^0 + c.c\right) + (-\mu B H_1^0 H_2^0 + c.c) \\ &- \epsilon_i \Delta B_i \left(\tilde{\nu}_i H_2^0 + c.c\right) - \frac{1}{2}\sum_{i<j} \hat{\epsilon}_i \hat{\epsilon}_j (\Delta m_i^2 + \Delta m_j^2)(\tilde{\nu}_i{}^* \tilde{\nu}_j + c.c.)\ , \end{aligned} \tag{5.55}$$

[5]For a detailed discussion of the various diagrams in mass insertion approximation, see [29].

where

$$\begin{aligned} m^2_{H_1} &= m^2_{H_1'}(1-\hat{\epsilon}^2) + m^2_{\tilde{\nu}_i'}\hat{\epsilon}_i^2, \\ m^2_{\tilde{\nu}_i} &= m^2_{H_1'}\hat{\epsilon}_i^2 + m^2_{\tilde{\nu}_i'}(1-\hat{\epsilon}_i^2), \\ B &= B_\mu(1-\hat{\epsilon}^2) + B_i\hat{\epsilon}_i^2. \end{aligned} \tag{5.56}$$

The rotation has generated off-diagonal flavour violating sneutrino mixing terms at $\mathcal{O}(\epsilon^2)$. Since these terms conserve lepton number, they do not directly contribute to the neutrino masses but lead to flavour violating transitions such as $\mu \to e\gamma$.

The rotation in eq.(5.54) induces mixing among the charged leptons which were diagonal to start with. Define the charged lepton mass matrix as

$$Ł_i \ M_l \ e^c \ ,$$

then

$$M_l = \begin{pmatrix} h_1 d_1 & \hat{\epsilon}_1\hat{\epsilon}_2 h_2 f_2 & \hat{\epsilon}_1\hat{\epsilon}_3 h_3 f_3 \\ \hat{\epsilon}_1\hat{\epsilon}_2 h_1 f_1 & h_2 d_2 & \hat{\epsilon}_1\hat{\epsilon}_3 h_3 f_3 \\ \hat{\epsilon}_1\hat{\epsilon}_3 h_1 f_1 & \hat{\epsilon}_2\hat{\epsilon}_3 h_2 f_2 & h_3 d_3 \end{pmatrix} , \tag{5.57}$$

where

$$\begin{aligned} d_i &\equiv v_1(1+\frac{1}{2}(\hat{\epsilon}_i^2 - |\hat{\epsilon}|^2) + \hat{\epsilon}_i^2 k_i - \hat{\epsilon}_l^2 k_l \ , \\ f_i &\equiv \frac{1}{2}v_1 + k_i \ . \end{aligned}$$

The k_i appearing in above are defined in eq.(5.11) and they signify sneutrino *vev* contribution to the charged lepton mass matrix. As argued in the text, k_i are required to be small $\sim (10^{-3} - 10^{-4})$ in order to account for the correct neutrino masses. It then follows that sneutrino *vev* contribution to each element in M_l is suppressed compared to the corresponding contribution of v_1. Thus this contribution can be neglected while diagonalizing M_l in any realistic theory. Even after neglecting it, the $\mathcal{O}(\hat{\epsilon}^2)$ contribution does produce additional mixing among charged leptons that is not *Yukawa suppressed.* This is easily seen in the simplified case of two generation. The 2×2 version of the charged lepton mass matrix is obtained from eq.(5.57) by setting $\epsilon_3 = 0$. The following rotation on the basis (e_1, e_2) is needed to diagonalize the charged lepton masses:

$$\begin{pmatrix} e \\ \mu \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}\hat{\epsilon}_1\hat{\epsilon}_2 \\ -\frac{1}{2}\hat{\epsilon}_1\hat{\epsilon}_2 & 1 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} . \tag{5.58}$$

(e,μ) here refers to the flavour basis. This additional rotation affects the neutrino mixing terms in eq.(5.55) which can be re-written in the flavour basis as

$$\begin{aligned} V_{soft} &= m_{H_1}^2|H_1^0|^2 + m_{H_2}^2|H_2^0|^2 + m_{\tilde{\nu}_1}^2|\tilde{\nu}_e|^2 + m_{\tilde{\nu}_2}^2|\tilde{\nu}_\mu|^2 \\ &+ \left(\Delta m_1^2\hat{\epsilon}_1\tilde{\nu}_e^\star H_1^0 + \Delta m_2^2\hat{\epsilon}_2\tilde{\nu}_\mu^\star H_1^0 + c.c\right) \\ &- \left(\mu B H_1^0 H_2^0 + c.c\right) - \left(\epsilon_1\Delta B_1\tilde{\nu}_e H_2^0 + \epsilon_2\Delta B_2\tilde{\nu}_\mu H_2^0 + c.c\right) \\ &- \hat{\epsilon}_1\hat{\epsilon}_2\frac{1}{2}(\Delta m_1^2 + \Delta m_2^2 - m_{\tilde{\nu}_2}^2 + m_{\tilde{\nu}_1}^2)(\tilde{\nu}_e^*\tilde{\nu}_\mu + c.c.). \end{aligned} \tag{5.59}$$

One sees that there are no additional lepton number violating mass terms other than present at $\mathcal{O}(\hat{\epsilon})$. Thus discussion on additional contribution to neutrino masses just given remains unchanged. However, eq.(5.59) contains lepton conserving but flavour violating contribution proportional to $\tilde{\nu}_e^*\tilde{\nu}_\mu$. This can lead to process such as $\mu\rightarrow e\gamma$. The branching ratio for this process is given by

$$BR(\mu\rightarrow e\gamma) = \frac{12\pi^2}{G_F^2 m_\mu^2}|B|^2\ . \tag{5.60}$$

In the present case, the amplitude B arises due to insertion of the flavour violating sneutrino mass term given in the last term in eq.(5.59). This is approximately given by [30]

$$|B| \sim \frac{e^3}{16\pi^2}\frac{m_\mu}{m_{\tilde{\nu}}^2}\frac{1}{2}\frac{\epsilon_1\epsilon_2}{\mu^2}k\ , \tag{5.61}$$

where k is a typical magnitude of k_i and $m_{\tilde{\nu}}^2$ is sneutrino (mass)^2. As already argued, we need $\epsilon_i\sim 0.1$ GeV and $k\sim 10^{-3}$. Given this, last equation is seen to give very small contribution to $BR(\mu\rightarrow e\gamma)\ \sim\mathcal{O}(10^{-13}\ |\epsilon|^4|k|^2)$ which makes it unobservable in both present [31] and future [32] experiments.

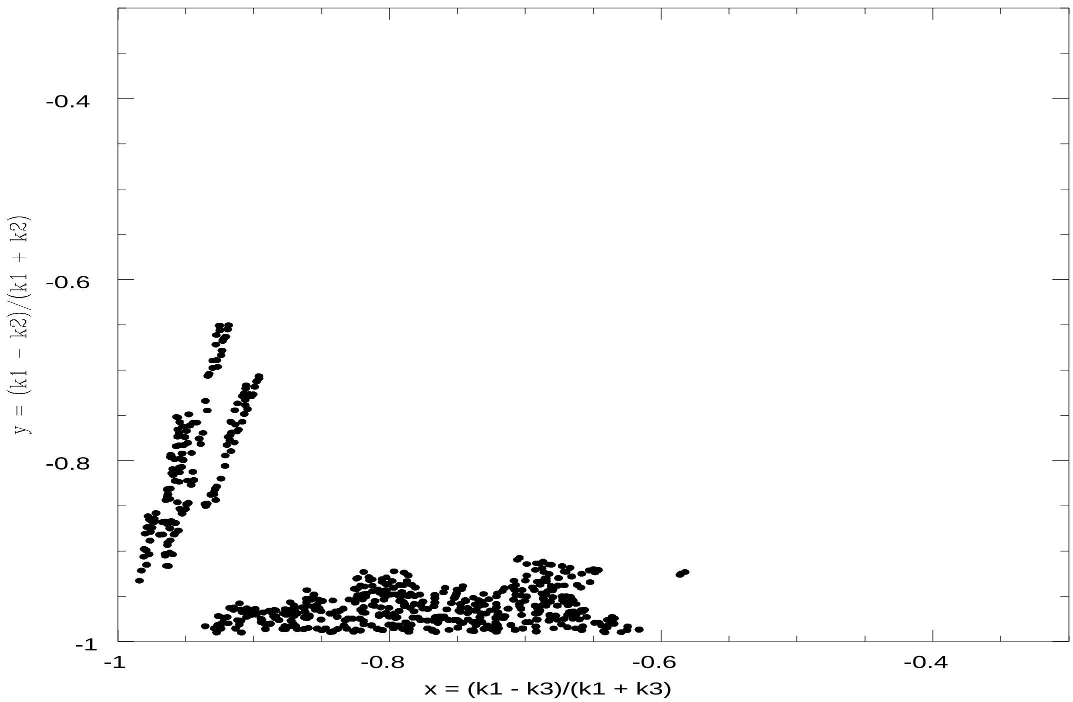

Figure 5.1: Allowed values of x and y for which all the neutrino oscillation constraints are satisfied. The input values of parameters are chosen in a way described in the text.

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Chapter 6

Trilinear R Violation and Neutrino Anomalies

6.1 Introduction

In chapter 3, we discussed in detail the problem of atmospheric and solar neutrino anomalies. It was concluded that the oscillations among three active neutrinos are likely to be responsible for the observed features of the data and that neutrino mass spectrum is characterized by hierarchical masses and one or two large mixing angle. Many mechanisms have been advanced to understand these features of the neutrino spectrum [4]. One of these is provided by supersymmetric theory which contains several features to make it attractive for the description of the neutrino spectrum. The lepton number violation needed to understand neutrino masses is in-built in this theory through the presence of the R parity violating couplings [5]. Moreover, it is possible to understand the hierarchical neutrino masses and large mixing among them within this framework without fine tuning of parameters or without postulating ad hoc textures for the neutrino mass matrices [23, 7].

The supersymmetrized version of the standard model contains the following lepton number violating couplings:

$$W_{\not{L}} = \epsilon_i L_i H_2 + \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k, \tag{6.1}$$

where L, Q, H_2 represent the leptonic, quark and one of the Higgs doublets (up-type) respectively and E^c, D^c represent the leptonic and down quark singlets. Each of these couplings is a potential source for neutrino masses. There have been detailed studies of the effects of these couplings on neutrino masses under different assumptions [23, 7, 8, 9, 10, 11]. We briefly recapitulate the relevant gross features of these studies and motivate additional work that we are going to present in this chapter which is based on our work [12].

The most studied effect is that of the three bilinear mass parameters ϵ_i, particularly in the context of the supersymmetry breaking with universal boundary conditions at a high scale [23, 7, 8, 10, 11, 13]. In the last chapter, we discussed the effect of bilinear couplings on the neutrino masses in a model independent way, by considering the form of the scalar potential at the weak scale. We found that the hierarchy among various generations of neutrinos is related to an approximate Higgs - slepton universality at the weak scale. The solar mixing angle was shown to be related to non-universality in slepton mass terms, specifically to differences in soft parameters of the first two leptonic generations. It was shown that this flavour universality violation should be as strong as the Higgs-slepton universality violation if solar neutrino mixing angle is to be large. The standard supergravity models with universal boundary conditions at a high scale lead to the required Higgs-slepton universality violations but the predicted violation of flavour universality among the first two generations is much smaller than required. This model therefore cannot provide an explanation of large solar mixing angle unless some universality violations in soft supersymmetry breaking parameters are introduced at a high scale itself.

In contrast to the bilinear case, the presence of trilinear interactions can allow two large angles without conflicting with the CHOOZ result. There have been several studies to determine possible set of trilinear couplings which can reproduce the observed features of neutrino masses and mixing [14]. It is not surprising that one could 'fit' the neutrino spectrum in these cases due to very large number of trilinear couplings. But it was realized [15, 16] that gross features of the neutrino spectrum can be understood without making specific assumptions on the trilinear couplings other than requiring them to be similar in magnitude. This makes the R violation with trilinear interaction 'predictive' in spite of the presence of very large number of couplings.

In addition to the R violating parameters, the neutrino spectrum in these models also depends upon the nature of the supersymmetry breaking. This spectrum has been studied in the standard supergravity case, with bilinear as well as trilinear couplings, and in the case of gauge mediated supersymmetry breaking when R violation is only through the bilinear terms in eq.(6.1). The supersymmetry breaking generically introduces two different types of contributions to neutrino masses. The presence of terms linear in the sneutrino field in the scalar potential induces a vacuum expectation value (*vev*) for the former which mix neutrinos with neutralinos and lead to neutrino masses. In addition to this 'tree level' contribution, the trilinear terms in the superpotential also lead to neutrino masses at the 1-loop level. Both the types of contributions are present in models with purely bilinear or purely trilinear terms in the superpotential at a high scale. In the case of the trilinear couplings, the running of the couplings to lower scale generates in the scalar potential couplings linear in the sneutrino *vev* and lead to a tree level contribution which is often neglected in the literature [23, 7] .

The relative importance of the tree level and loop induced contributions to neutrino masses in case of trilinear interactions was studied in [16, 17] in the context of the standard supergravity (mSUGRA) models with universal boundary conditions at a high scale. It was concluded that the tree level contribution dominates over the loop for large ranges in the parameters of the model. This results [16] in the following hierarchy in neutrino masses if all the trilinear λ' couplings are assumed to be similar in magnitudes:

$$\frac{m_{\nu_2}}{m_{\nu_3}} \approx \frac{m_{\text{loop}}}{m_0 + m_{\text{loop}}} \frac{m_s}{m_b} \ . \tag{6.2}$$

The parameters m_{loop} and m_0 characterize the strength of the 1-loop and the RG-induced tree level contributions respectively and are determined by soft SUSY breaking parameters. m_s and m_b denote the strange quark and the bottom quark masses respectively. It is possible to obtain the vacuum or the MSW solution (large angle) to solar neutrino problem in this context by choosing the SUSY parameters in appropriate range [16].

An attractive alternative to the standard supergravity induced SUSY breaking is provided by the gauge mediated SUSY breaking [18]. Neutrino mass spectrum has been studied in gauge mediated models with trilinear R-violation by Choi *et al.* in [10]. Their study has been confined to non-minimal models of this category. The minimal model in this category called the Minimal Messenger Model(MMM) [20] has only two free parameters and is more predictive than the standard SUGRA based models and the models studied in [10]. The two free parameters of the model determine all the soft terms at the high scale $\sim$ a few hundred TeV, where SUSY breaking occurs. Thus this model implies very constrained spectrum for neutrino masses. This constrained spectrum has been shown [7] to be inadequate for simultaneous solution of the solar and atmospheric neutrino anomalies in the case of purely bilinear R violation. In this chapter, we wish to study neutrino masses in the minimal messenger model in the presence of purely trilinear R violation. This would mean that both, the scale at which SUSY breaking occurs as well as the boundary conditions at the high scale would be sufficiently different from the mSUGRA scenario which has been studied in [16]. We have studied the neutrino mass spectrum in the MMM for two separate cases, namely purely λ' couplings and purely λ couplings. Such a choice has been made for simplicity. The λ' couplings with comparable magnitude are argued to describe neutrino spectrum well. In contrast, we find that if all the λ couplings are of similar strength, then one cannot describe the neutrino spectrum well and one needs to postulate somewhat inverse hierarchy among them. We give a specific example with hierarchical λ which reproduces the observed features of the neutrino spectrum.

Within the Minimal Messenger Model, the soft supersymmetry breaking terms are decided by the gauge quantum numbers of the fields. As we will demonstrate later, this significantly alters the hierarchy within the neutrino mass states. In particular, we find that the m_0 dominates over m_{loop} in case of the λ' couplings but the situation is reversed when the R violation occurs through λ couplings. This feature is characteristic of the gauge mediated scenario and is quiet distinct from all the earlier studies [12].

We discuss the basic formalism in the next section which also contains analysis of the effect of the trilinear λ' couplings. The third section has detailed study of the λ couplings and we end with a discussion in the last section.

6.2 Formalism

We consider the following trilinear interaction in this section:

$$W_{\not R_p} = \lambda'_{ijk} L_i Q_j d^c_k \ , \tag{6.3}$$

where i, j, k are generation indices. In spite of very large number of these couplings, one could determine the neutrino masses and mixing in terms of small number of parameters if one assumes that all the trilinear couplings are similar in magnitude. The basic formalism was developed in [16] and we recapitulate here the relevant parts.

The neutrinos obtain their masses from two different contributions in this case. The λ' couplings generate radiative masses through exchange of the down squarks at the 1-loop level. In addition to this, the trilinear interactions also radiatively generate soft SUSY breaking terms which are linear in the sneutrino fields. These terms lead to additional contribution to neutrino masses which can dominate over the the first contribution. The second contribution follows from the RG improved scalar potential [16, 17]:

$$\begin{aligned} V_{scalar} &= m^2_{\tilde{\nu}_i} \mid \tilde{\nu}_i \mid^2 + m^2_{H_1} \mid H^0_1 \mid^2 + m^2_{H_2} \mid H^0_2 \mid^2 + \Big[m^2_{\nu_i H_1} \tilde{\nu}^\star_i H^0_1 \\ &\quad - \mu \ B_\mu H^0_1 H^0_2 - B_{\epsilon_i} \tilde{\nu}_i H^0_2 + H.c \Big] + \frac{1}{8}(g^2_1 + g^2_2)(\mid H^0_1 \mid^2 - \mid H^0_2 \mid^2)^2 + ... \end{aligned} \tag{6.4}$$

where we have used the standard notation for the SM fields and their masses, with B_{ϵ_i} and $m^2_{\nu_i H_1}$ representing the bilinear lepton number violating soft terms. Minimization of the above potential leads to the sneutrino *vevs*:

$$< \tilde{\nu}_i > \ = \frac{B_{\epsilon_i} v_2 - m^2_{\nu_i H_1} v_1}{m^2_{L_i} + \frac{1}{2} M^2_Z \ \cos 2\beta} \ , \tag{6.5}$$

where v_1 and v_2 stand for the *vevs* of the Higgs fields H^0_1 and H^0_2 respectively [1].

[1] These sneutrino *vevs* are derived from the tree level scalar potential. Corrections from the one-loop effective potential can significantly shift these naive tree level values [21]. For neutrino phenomenology these corrections would be important in regions in the parameter space where two contributions to the sneutrino *vev* cancel each other [22]. Such regions are not encountered in MMM parameter space in which we are interested. Moreover, we are approximately including the effect of 1-loop corrections by dynamically choosing soft parameters at appropriate scale in the manner discussed in Refs.[21, 23].

These *vevs* vanish at a high scale since we are assuming only trilinear L violating interactions. They however get generated at the weak scale. The magnitudes of the parameters B_{ϵ_i} and $m^2_{\nu_i H_1}$ and hence the sneutrino *vevs* at the weak scale are determined by solving the renormalization group (RG) equations satisfied by them. These RG equations are presented in Appendix A. The general solution of these equations can be parameterized as

$$\begin{aligned} B_{\epsilon_i} &= \lambda'_{ipp} h^D_p \kappa_{ip} \ , \\ m^2_{\nu_i H_1} &= \lambda'_{ipp} h^D_p \kappa'_{ip} \ , \end{aligned} \tag{6.6}$$

where κ, κ' are dependent on the soft terms appearing in the RHS of the respective RG equations and h^D are down type quark yukawa.

The sneutrino *vevs* break R parity and lead to mixing of neutrinos with neutralinos. This in turn leads to neutrino masses. For small sneutrino *vevs*, the neutrino mass matrix follows from the seesaw approximation and is given by [21]:

$$\mathcal{M}^0_{ij} = \frac{\mu(cg^2 + g'^2) \ <\tilde{\nu}_i> \ <\tilde{\nu}_j>}{2(-c\mu M_2 + 2\ M^2_W c_\beta s_\beta (c + \tan\theta^2_W))} \ , \tag{6.7}$$

where $c = 5g'^2/3g^2 \sim 0.5$, M_W is the W boson mass, μ is the mass term for Higgs/Higgsino, M_2 is mass term for a gaugino, and the Weinberg angle is represented by θ_W. Assuming generation independence of the terms κ, κ' which was found to be a very good approximation in [16], we can rewrite the above mass matrix as

$$\mathcal{M}^0_{ij} \equiv m_0\ \lambda'_{ipp}\ h^D_p\ \lambda'_{jmm}\ h^D_m \ , \tag{6.8}$$

where the parameters p and m are summed over the three generations and m_0 now contains the dependence of the tree level mass on the soft SUSY breaking parameters. Only one neutrino attains mass through this mechanism. The other neutrinos attain mass at the 1-loop level. The complete 1-loop structure of the neutrino masses has been discussed in [22]. In the present case, the most dominant contributions are from diagrams having λ' couplings at both the vertices. The mass matrix generated by these diagrams is given by

$$\mathcal{M}^l_{ij} = \frac{3}{16\pi^2} \lambda'_{ilk}\lambda'_{jkl}\ v_1\ h^D_k\ \sin\phi_l\ \cos\phi_l\ \ln\frac{M^2_{2l}}{M^2_{1l}} \ . \tag{6.9}$$

In the above, $\sin\phi_l\ \cos\phi_l$ determines the mixing of the squark-antisquark pairs and M^2_{1l} and M^2_{2l} represent the eigenvalues of the standard $2\times\ 2$ mass matrix of the down squark system. The indices l and k are summed over. As the mixing $\sin\phi_l\ \cos\phi_l$ is proportional to h^D_l, we rewrite the 1-loop contribution as,

$$\mathcal{M}^l_{ij} = m_{\rm loop}\ \lambda'_{ilk}\ \lambda'_{jkl}\ h^D_k\ h^D_l , \tag{6.10}$$

where $m_{\rm loop}$ is independent of the R violation and solely depends on the MSSM parameters.

The total neutrino mass matrix is given by,

$$\mathcal{M}^\nu = \mathcal{M}^0 + \mathcal{M}^l, \tag{6.11}$$

which can be rewritten in the following form when $\mathcal{O}(h_1^{D^2}, h_2^{D^2})$ terms are neglected:

$$\mathcal{M}^\nu_{ij} \approx (m_0 + m_{\rm loop})\ a_i a_j + m_{\rm loop}\ h_2^D\ h_3^D\ A_{ij}, \tag{6.12}$$

where $a_i = \lambda'_{ipp} h_p^D$ (p summed over generations) and

$$A_{ij} = \lambda'_{i23}\lambda'_{j32} + \lambda'_{i32}\lambda'_{j23} - \lambda'_{i22}\lambda'_{j33} - \lambda'_{i33}\lambda'_{j22}\ \ . \tag{6.13}$$

To derive the eigenvalues of the total matrix $\mathcal{M}^\nu$, we recognize that a) The first matrix on the RHS of eq.(6.12) has only one non-zero eigenvalue ; b) The dominant terms in the total matrix $\mathcal{M}^\nu$ of $\mathcal{O}(h_3^{D\ 2})$ are present only in the first matrix . Moreover, as we will show below $m_{\rm loop} \ll m_0$ in the Minimal Messenger Model in the purely λ' case. Hence approximate eigenvalues can be derived up to $\mathcal{O}(\frac{m_s m_{\rm loop}}{m_b(m_0+m_{\rm loop})})$, neglecting the high order corrections. The detailed derivation of the eigenvalues and the mixing matrix has been presented in [16]. These eigenvalues are given as,

$$\begin{aligned} m_{\nu_1} &\sim m_{\rm loop} h_2^D h_3^D\ \delta_1 \\ m_{\nu_2} &\sim m_{\rm loop} h_2^D h_3^D\ \delta_2 \\ m_{\nu_3} &\sim (m_0 + m_{\rm loop}) \sum_i^3 a_i^2 \\ &\sim (m_0 + m_{\rm loop}) h_3^{D^2} \sum_i^3 {\lambda'_{i33}}^2, \end{aligned} \tag{6.14}$$

where

$$\begin{aligned} \delta_1 &= (c_1^2\ A'_{11} - 2c_1 s_1 A'_{12} + s_1^2 A'_{22})\ \ , \\ \delta_2 &= (s_1^2\ A'_{11} + 2c_1 s_1 A'_{12} + c_1^2 A'_{22})\ \ . \end{aligned} \tag{6.15}$$

The entries A'_{ij} are the elements of the matrix $A' = U_{\lambda'}^T A U_{\lambda'}$ where

$$U_{\lambda'} = \begin{pmatrix} c_2 & s_2 c_3 & s_2 s_3 \\ -s_2 & c_2 c_3 & c_2 s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}\ \ , \tag{6.16}$$

with

$$s_2 = \frac{a_1}{\sqrt{a_1^2 + a_2^2}} \quad ; \quad s_3 = \sqrt{\frac{a_1^2 + a_2^2}{a_1^2 + a_2^2 + a_3^2}} \ . \tag{6.17}$$

The total mixing is given as [16],

$$\begin{aligned} K' &= U_{\lambda'} \, U'_{\lambda'} \\ &= \begin{pmatrix} c_1c_2 - s_1s_2c_3 & s_1c_2 + c_1s_2c_3 & s_2s_3 \\ -s_2c_1 - s_1c_2c_3 & -s_1s_2 + c_1c_2c_3 & c_2s_3 \\ s_1s_3 & -s_3c_1 & c_3 \end{pmatrix} , \end{aligned} \tag{6.18}$$

where the 1-2 mixing angle θ_1 is given by,

$$\tan 2\theta_1 = \frac{2A'_{12}}{A'_{22} - A'_{11}} \ . \tag{6.19}$$

From eq.(6.13), we see that in the limit of exact degeneracy of the λ' couplings, the parameters A_{ij} would be zero. In this case, only one neutrino becomes massive in spite of the inclusion of the loop corrections. The 1-2 mixing also remains undetermined in this case. There is no reason *a priori* for the exact equality of λ' and non-zero but similar value for these parameters determine the 1-2 mixing to be large (see eq.(6.19)) and also generates mass for the other two neutrinos.

6.2.1 MMM and neutrino anomalies

The parameters $m_{\rm loop}$ and m_0 appearing in eq.(6.14) are independent of the details of the R violation and get determined by the soft SUSY breaking terms. We assume throughout that SUSY breaking is mediated by the standard gauge interactions [18]. We work in the so-called minimal messenger model [20]. It is characterized by a messenger sector with a pair of superfields which transform vector-like under a gauge group chosen to be $SU(5)$ for unification purposes. SUSY breaking is characterized by a singlet chiral superfield whose scalar and the auxiliary components acquire *vevs* breaking supersymmetry. This breaking is communicated to the visible sector by loop diagrams. The gauginos acquire masses at the 1-loop level which are given as,

$$M_i(X) = \tilde{\alpha}_i(X) \ \Lambda \ g(x) \ , \tag{6.20}$$

where $X = \lambda < S >$ is the supersymmetric mass of the scalar and fermionic components of the singlet superfield, Λ is the ratio $\frac{F_S}{<S>}$, F_S being the *vev* of the auxiliary field of the singlet. The parameter x is defined as $\frac{\Lambda}{X}$. The scalars acquire masses at the two loop level. They are given by

$$m_i^2 = 2\Lambda^2 \left(C_3^i \tilde{\alpha}_3^2(X) + C_2^i \tilde{\alpha}_2^2(X) + \frac{3}{5} Y_i^2 \tilde{\alpha}_1^2(X) \right) f(x) \ , \tag{6.21}$$

where C_3, C_2 are the quadratic casimirs of the gauge groups $SU(3)$ and $SU(2)$ respectively and Y_i being the hypercharge of the scalar field i , with $i = \{Q_j,\ d_j^c,\ u_j^c,\ L_j,\ e_j^c,\ H_1,\ H_2\}$, where $j = 1, 2, 3$ is the generation index. The functions $f(x), g(x)$ are given in [25] and the dependence of the soft masses on these functions is minimal. In the present analysis, we follow [23] and choose $x = \frac{1}{2}$. Since the dependence of the soft masses on x is minimal, a different choice of x would not significantly modify the results presented here.

The major feature characterizing the model is the absence of A -terms and the B terms in the soft potential at the scale X.

$$A(X) = 0 \ \ , \ \ B(X) = 0 \ . \tag{6.22}$$

Thus, the entire soft spectrum of this model gets essentially determined by one parameter Λ. The parameters tanβ and μ are fixed at the weak scale by requiring the breaking of the $SU(2) \times U(1)$ symmetry. The relevant equations following from the tree level potential are given by

$$\begin{aligned} \sin 2\beta &= \frac{2B_\mu \mu}{m_{H_1}^2 + m_{H_2}^2 + 2\mu^2} \\ \mu^2 &= \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2} M_Z^2, \end{aligned} \tag{6.23}$$

where all the parameters on the RHS of the above equations are evolved to the weak scale using the MSSM RGE. Because of the boundary condition eq.(6.22), the value of the B parameter at the weak scale remains small. This pushes tanβ to very large values in this model [23, 26]. The above equations are strictly valid in case of the MSSM and R violating trilinear couplings can give corrections to these. The smallness of neutrino masses however require very tiny trilinear couplings of O($10^{-4} - 10^{-5}$) which would contribute negligible to eqs.(6.23). We thus continue to use eqs.(6.23).

The neutrino masses in eq.(6.14) are strongly hierarchical in the limit $m_{\text{loop}} \ll m_0$. Specifically, one obtains from eq.(6.14),

$$\frac{m_{\nu_2}}{m_{\nu_3}} \sim \frac{m_{\text{loop}}}{m_0} \frac{m_s}{m_b} \left(\frac{\delta_2}{\sum_i (\lambda'_{i33})^2} \right), \tag{6.24}$$

where m_s and m_b represent the strange and the bottom quark masses. For all the λ' of similar magnitudes, this ratio is completely determined by the parameter Λ.

We can determine the above ratio by exactly solving the RG equations, (6.63) . Before doing this, it is instructive to study the approximate expressions obtained when one neglects the Q^2 dependence of the parameters appearing on the RHS of the RGE. In other words, we neglect the effect of running of the soft masses (from high scale, X to weak scale M_Z) appearing in the expressions of the RGE as well as for neutrino masses. Instead, we take them to be their high scale values given by eqs.(6.21). Noting that more dominant contribution to sneutrino *vev* comes from the $m^2_{\nu_i H_1}$ term in eq.(6.5) and integrating the corresponding RGE for $m^2_{\nu_i H_1}$ in the above approximation, one finds in this simplifying case :

$$m_o \sim \left(\frac{2\cos\beta}{3\pi^2}\right)^2 \frac{M_W^2}{\Lambda}\;\frac{\tilde{\alpha}_3^4(X)}{\tilde{\alpha}_2^5(X)}\;\left(\ln\frac{X^2}{M_Z^2}\right)^2 . \tag{6.25}$$

The $m_{\rm loop}$ defined in eq.(6.9) has the following approximate form in the same approximation as above where we neglect the running of the soft masses.

$$m_{\rm loop} \sim \left(\frac{v^2\mu}{\Lambda^2}\right)\frac{\cos\beta\;\sin\beta}{8\;\pi^2\;\tilde{\alpha}_2^2(X)}. \tag{6.26}$$

Eq.(6.25) clearly demonstrates that the often neglected [14, 15] RG induced contribution dominates over the loop contribution in the present case of MMM just as in the case of the supergravity induced breaking [16, 27]. One would have naively thought that this will not be the case in gauge mediated model since the running of masses in this case (signified by $t \equiv \ln(\frac{X^2}{M_Z^2}) \sim 11$ in eq.(6.25)) is over much smaller range than in the supergravity case where $t \equiv \ln(\frac{M_{GUT}^2}{M_Z^2}) \sim 66$. But smallness of t in MMM is compensated by the largeness of the ratio $\frac{m_{\tilde{Q}}^2}{m_{\tilde{L}}^2}$ (signified by the factor) $\frac{\tilde{\alpha}_3^4}{\tilde{\alpha}_2^4}$. As a result of which the value of m_0 here can be comparable to the corresponding value [16] in supergravity case. From the expressions we see that dependence on the Λ is more severe for the 1-loop mass, $m_{\rm loop}$ compared to the tree level contribution, m_0. However, the μ parameter in the numerator increases approximately linearly with Λ [23]. This makes the Λ dependence of the both the contributions essentially the same. The ratio $\frac{m_{\rm loop}}{m_0}$ following from eqs.(6.25,6.26) is given as:

$$\frac{m_{\rm loop}}{m_0} \sim \left(\frac{\pi^2}{3}\right)\left(\frac{\mu\;\tan\beta\;v^2}{M_W^2\;\Lambda\;t^2}\right)\left(\frac{\tilde{\alpha}_2^3(X)}{\tilde{\alpha}_3^4(X)}\right). \tag{6.27}$$

From the above we see that the dependence of the ratio on Λ is essentially determined by the way the μ parameter scales with respect to Λ. This leads to a very mild dependence of the ratio on Λ. Such Λ 'independence' has also been seen in the case of bilinear R-violating models in MMM [7]. For $\Lambda = 100$ TeV, $t = 2\;\ln\left(\frac{X^2}{M_Z^2}\right) = 10.6$, $\tan\beta = 46$ and

Table 6.1: In the following we present the allowed ranges for mass-squared differences and mixing angles for various solutions to solar and atmospheric neutrino anomalies. The ranges for LMA and LOW solutions are at 3σ level following the recent analysis of SNO neutral current data [28, 29, 30, 31, 32, 33]. Vacuum solutions and SMA are absent at 3σ level, but we retain them here as this work was done prior to publication of SNO data.

Anomaly	Solution	$\Delta m^2 (eV^2)$	$\tan^2\theta$
Solar	MSW-SMA	$(2-10)\times 10^{-6}$	$(1-20)\times 10^{-4}$
	MSW-LMA	$2.3\times 10^{-5} - 3.7\times 10^{-}4$	0.2 - 4.
	LOW-QVO	$3.5\times 10^{-8} - 1.2\times 10^{-7}$	0.1 -8.
	Vacuum (Just-So)	$(4-12)\times\ 10^{-12}$	0.1 -7.
Anomaly		$\Delta m^2 (eV^2)$	$\sin^2 2\theta$
Atmosphere		$(1-8)\times\ 10^{-3}$	0.83 - 1.

$\mu = 400$ GeV, from the above we see that the ratio is 0.39. From eqs.(6.25,6.26) we see that the typical order of magnitude for the ratio of the mass eigenvalues, eq.(6.24) is :

$$\frac{m_{\nu_2}}{m_{\nu_3}} \sim 10^{-3}. \tag{6.28}$$

However the above expressions are approximate. We have determined this ratio exactly by solving the relevant RG equations numerically. The numerical procedure along with the flow diagram is presented in appendix B of this chapter. The ratio $\frac{m_{\nu_2}}{m_{\nu_3}}$, determined following the numerical procedure is plotted in Figure 1 (at the end of this chapter) for Λ varying from $(50-150)$ TeV.

From the figure we see that the ratio of the eigenvalues,

$$\frac{m_{\nu_2}}{m_{\nu_3}} \approx (1-2)\times 10^{-4}\left(\frac{\delta_2}{\sum_i (\lambda'_{i33})^2}\right), \tag{6.29}$$

is typically around the expected value, eq.(6.28). While the ratio shown in Fig. 1 is completely fixed by the value of Λ, the neutrino mass ratio is uncertain by a number of $\mathcal{O}(1)$ which is related to the trilinear parameters.

We now turn to discussing feasibility of the model for the simultaneous description of the solar and atmospheric data. The two generation analysis of each of these experiments constrain the value of the relevant $(mass)^2$ difference and mixing angle.

At present, global analysis including the recent results from the day/night recoil electron energy spectrum and charged and neutral current rates from SNO and of the solar neutrinos from super-Kamiokande [2] favours the MSW-LMA solution. At 3σ vacuum and SMA are absent [33]. In table 6.1 we have retained all solutions as this work was published prior to publication of SNO data. From the table it is clear that the most natural solution for the solar neutrino problem is through the vacuum oscillations but quasi-vacuum solution can also be obtained, if the λ' dependent factor in eq.(6.29) is somewhat large (e.g. ~ 5) instead of exactly being one. The most preferred LMA solution cannot however be obtained.

The mixing among neutrinos is essentially controlled by ratios of trilinear couplings. This mixing is given by eq.(6.18). It is seen from this equation that a choice of angles $s_{1,2,3}$ is possible which reproduces two large and one small mixing angles as required by the present data. As an example, consider the choice

$$c_1 = c_3 = s_1 = s_3 = \frac{1}{\sqrt{2}} \; ; s_2 \sim .13 \; . \tag{6.30}$$

This gives

$$\begin{aligned} \sin^2 2\theta_A &\equiv 4K'^{\,2}_{\mu 3}(1 - K'^{\,2}_{\mu 3}) \approx 0.99 \; , \\ \sin^2 2\theta_S &\equiv 4K'^{\,2}_{e1} \; K'^{\,2}_{e2} \approx 0.95 \; , \\ K'_{e3} &= 0.09 \; . \end{aligned} \tag{6.31}$$

This choice reproduces the required mixing angles and also satisfies the CHOOZ constraint.

6.3 Models with λ_{ijk}

In this section, we discuss the structure of neutrino masses and mixing in the presence of only trilinear λ couplings. The lepton number violating part of the superpotential is given as,

$$W_{\not L} = \lambda_{ijk} L_i L_j e^c_k \; . \tag{6.32}$$

There are two basic changes here compared to the last section. Firstly, the λ_{ijk} are antisymmetric in the first two indices restricting their total number to nine. This strongly restricts neutrino mass structure and one does not get phenomenologically consistent spectrum when all the trilinear couplings are assumed to be similar in magnitudes. Secondly, unlike in the λ' case, the loop induced contribution dominates over the tree level SUSY breaking in the minimal messenger model. Such dominant loop contribution has been earlier seen in some particular regions of the parameter space in the mSUGRA framework [16, 10]. Here this dominance follows generically.

We present here a formalism which is similar to the spirit of the previous section in order to understand the basic features of the neutrino mass matrix . As before, the RG improved effective soft potential contains terms B_{ϵ_i} and $m^2_{\nu_i H_1}$ which break the lepton number. These terms are generated due to the presence of λ couplings in the superpotential. At the weak scale, the magnitude of these depends on the respective RGE, which we have presented in Appendix A. The solutions of eqs.(6.64) can be written as,

$$B_{\epsilon_i} = \lambda_{ipp} h^E_{pp} \tilde{\kappa}_{ip}, \tag{6.33}$$

$$m^2_{\nu_i H_1} = \lambda_{ipp} h^E_{pp} \tilde{\kappa}'_{ip} \ , \tag{6.34}$$

where $(i \neq j)$ due to the anti-symmetric nature of the λ couplings and $\tilde{\kappa}$ and $\tilde{\kappa}'$ represent the dependence on the soft masses in the RGE and h^E are the charged lepton yukawa. Following similar arguments as in Section 2, the presence of these terms in the scalar potential would lead to a tree level neutrino mass matrix of the following form:

$$\mathcal{M}^0_{ij} = m_0 b_i b_j \ , \tag{6.35}$$

where m_0 contains the dependence on the soft terms and b_i are given as, $b_i = \lambda_{ipp} h^E_{pp} \ \ (i \neq p)$. In addition to the tree level mass, the presence of λ couplings also gives rise to contributions at the 1-loop level. Assuming only canonical 1-loop contributions to be the most dominant contributions the 1-loop level mass matrix has the form:

$$\mathcal{M}^l_{ij} = \frac{1}{16\pi^2} \lambda_{ilk} \lambda_{jkl} \ v_1 \ h^E_k \ \ \sin\phi_l \cos\phi_l ln \frac{{}^2_{2l}}{M^2_{1l}}, \tag{6.36}$$

where $\sin\phi_l \cos\phi_l$ and M_{1l}, M_{2l} represent the mixing and the eigenvalues respectively of the standard 2×2 stau slepton mass matrix. Following the previous section, we rewrite the above as,

$$\mathcal{M}^l_{ij} = m_{\rm loop} \ \lambda_{ilk} \lambda_{jkl} h^E_l h^E_k. \tag{6.37}$$

In writing the above, we have implicitly assumed the anti-symmetric nature of the couplings. The total neutrino mass matrix is given as,

$$\mathcal{M} = \mathcal{M}^0 + \mathcal{M}^l. \tag{6.38}$$

The above can be rewritten in the form:

$$\mathcal{M}_{ij} = (m_0 + m_{\rm loop}) b_i b_j + m_{\rm loop} h^E_3 h^E_2 B_{ij} + \mathcal{O}({h^E_2}^2, h^E_2 h^E_1), \tag{6.39}$$

where we have neglected $\mathcal{O}({h^E_2}^2, h^E_2 h^E_1)$ contributions to the mass matrix. The matrix B is given as

$$B = \begin{pmatrix} \lambda_{132}\lambda_{123} - \lambda_{133}\lambda_{122} & \lambda_{123}\lambda_{232} - \lambda_{122}\lambda_{233} & \lambda_{132}\lambda_{323} - \lambda_{133}\lambda_{322} \\ \lambda_{123}\lambda_{232} - \lambda_{233}\lambda_{122} & 0 & 0 \\ \lambda_{132}\lambda_{323} - \lambda_{133}\lambda_{322} & 0 & 0 \end{pmatrix}. \tag{6.40}$$

We diagonalise the total matrix $\mathcal{M}$ in the same manner as for the λ' case and as described in [16]. However we do not make any assumption on the relative magnitude of m_0 and $m_{\rm loop}$. The approximate eigenvalues correct up to $\mathcal{O}(h_2^{E\ 2})$ are derived as,

$$\begin{aligned}
m_{\nu_1} &\approx m_{\rm loop} h_3^E h_2^E \eta_1 + \mathcal{O}(h_2^{E\ 2}) \\
m_{\nu_2} &\approx m_{\rm loop} h_3^E h_2^E \eta_2 + \mathcal{O}(h_2^{E\ 2}) \\
m_{\nu_3} &\approx (m_0 + m_{\rm loop}) \sum_i^3 b_i^2 \\
&\approx (m_0 + m_{\rm loop}) h_3^{E\ 2} \eta_3 + \mathcal{O}(h_3^E h_2^E).
\end{aligned} \tag{6.41}$$

The parameters η_1, η_2, η_3 are given by,

$$\begin{aligned}
\eta_1 &= (c_1^2\ B'_{11} - 2c_1 s_1 B'_{12} + s_1^2 B'_{22}) \\
\eta_2 &= (s_1^2\ B'_{11} + 2c_1 s_1 B'_{12} + c_1^2 B'_{22}) \\
\eta_3 &= \sum_{i=1,2} \lambda_{i33}^2.
\end{aligned} \tag{6.42}$$

The parameters B'_{ij} are elements of the matrix, $B' = U_\lambda^T B U_\lambda$ where, U_λ has the same form as $U_{\lambda'}$ in eq.(6.16) with the angles now given by,

$$s_2 = \frac{b_1}{\sqrt{b_1^2 + b_2^2}} \quad , \quad s_3 = \sqrt{\frac{b_1^2 + b_2^2}{b_1^2 + b_2^2 + b_3^2}}. \tag{6.43}$$

The total mixing matrix is given as in eq.(6.18) with the $1-2$ mixing angle given by,

$$\tan 2\theta_1 = \frac{2B'_{12}}{B'_{22} - B'_{11}}. \tag{6.44}$$

In the above, we have made no assumption on the relative magnitudes of m_0 and $m_{\rm loop}$. Only assumption is almost equality of all the λ couplings. Most definitions are formally the same as in case of the λ' couplings but the physical consequences are quite different. This follows from the expressions of mixing angles $s_{2,3}$ in eq.(6.43). If no hierarchy is assumed among the λ couplings then due to antisymmetry of λ we get,

$$\frac{b_2}{b_3} \approx \frac{b_1}{b_3} \approx \mathcal{O}(\frac{h_2^{E\ 2}}{h_3^{E\ 2}}).$$

As a result, eq.(6.43) implies $c_3 \sim \mathcal{O}(\frac{m_\mu}{m_\tau})$ and the mixing matrix, eq.(6.18) can be written as

$$\begin{aligned}
K &= U_{\ \lambda} U'_\lambda \\
&= \begin{pmatrix} c_1 c_2 & s_1 c_2 & s_2 \\ -s_2 c_1 & -s_1 s_2 & c_2 \\ s_1 & c_1 & 0 \end{pmatrix} + \mathcal{O}(\frac{m_\mu}{m_\tau}).
\end{aligned} \tag{6.45}$$

The equations for η_1, η_2 and $\tan 2\theta_1$ take the following approximate forms in this case:

$$\begin{aligned}\eta_1 &\approx s_1^2 c_2^2 B_{11} + 2B_{12} c_2 s_1 (c_1 - s_1 s_2)\\ \eta_2 &\approx c_1^2 c_2^2 B_{11} - 2B_{12} c_1 s_2 (s_1 + c_1 s_2)\end{aligned} \tag{6.46}$$

$$\tan 2\theta_1 \approx \frac{-2B_{12}}{B_{11} c_2 - 2 s_2 B_{12}}. \tag{6.47}$$

With the hierarchical masses, the effective mixing angles θ_A and θ_{CHOOZ} probed by the atmospheric data and the CHOOZ experiment respectively are given by

$$\sin^2 2\theta_A = 4K_{\mu 3}^2 (1 - K_{\mu 3}^2) \quad ; \quad \sin^2 2\theta_{CHOOZ} \approx 4K_{e3}^2 (1 - K_{e3}^2) \ .$$

Eq.(6.45) implies that these two mixing angles are equal in conflict with the observation.

The above derivation has not assumed any specific mechanism for the supersymmetry breaking and thus the conclusions are valid in supergravity scenario as well as in the MMM case considered here. It is quite interesting that in spite of the presence of nine independent λ parameters, one cannot explain solar and atmospheric neutrino anomaly as long as these parameters are of similar magnitudes. Departure from equalities of λ can lead to explanations of these anomalies and we shall present a specific example in the next section.

6.3.1 Models with λ_{ijk} and Neutrino Anomalies

In this sub-section, we first determine the numerical values of the parameters $m_{\rm loop}$ and m_0 which enter the neutrino mass. The dependence on trilinear couplings is factored out in defining these parameters. But their numerical values are quite different here compared to the λ' case studied in the earlier section. This follows since the soft sfermion masses which determine these parameters are quite different in these two cases (see RG equations in the appendix A).

In models with gauge mediated SUSY breaking, the soft masses are proportional to the gauge quantum numbers they carry. Thus particles with strong interactions have much larger soft masses compared to the weakly interacting particles, as is evident from the eqs.(6.20,6.21). The effect of the gauge couplings also trickles down to the parameters $B_{\epsilon_i}, m_{\nu_i H_1}^2$ through the corresponding soft masses present in their respective renormalization group equations. In the presence of purely λ' interactions, the strong coupling determines the magnitudes of $B_{\epsilon_i}, m_{\nu_i H_1}^2$ at the weak scale and in turn the tree level mass as is evident from eq.(6.25). The loop contribution is still however determined by the weak coupling, eq.(6.26). It is thus the interplay between the strong coupling and the weak coupling which leads to a suppressed loop mass in the case of purely λ' couplings.

In the case of pure λ couplings, the squark masses do not enter the definition of m_0 and $m_{\rm loop}$ and both of these are determined by the weak coupling. However the dependence on the power of the weak coupling is different. Following the same method as described in the λ' case (above eq.(6.25)), an estimate of the parameter m_0 is given by,

$$m_0 = \left(\frac{\cos\beta}{8\pi^2}\right)^2 \frac{M_W^2}{\Lambda} \frac{1}{\tilde{\alpha}_2(X)} \left(\ln \frac{X^2}{M_Z^2}\right)^2 . \tag{6.48}$$

The 1-loop contribution $m_{\rm loop}$ can also be estimated in the similar manner as,

$$m_{\rm loop} = \left(\frac{v^2\mu}{\Lambda^2}\right) \frac{\cos\beta\sin\beta}{24\ \pi^2\tilde{\alpha}_2^2(X)}. \tag{6.49}$$

The ratio $\frac{m_{\rm loop}}{m_0}$ is then given by,

$$\frac{m_{\rm loop}}{m_0} \approx \left(\frac{8\pi^2}{3}\right)\left(\frac{v^2\ \mu\ \tan\beta}{t^2\ \Lambda\ M_W^2}\right)\frac{1}{\tilde{\alpha}_2(X)}, \tag{6.50}$$

where $t = \ln\left(\frac{X^2}{M_Z^2}\right)$. The above is typically of $\mathcal{O}(10^2)$ for $\Lambda = 100$ TeV, $\mu = 400$ GeV, $\tan\beta = 46$ which shows that the tree level mass is much suppressed compared to the 1-loop mass. Comparing the above equation with that of the corresponding one for the λ' case, eq.(6.27), we see that the absence of strong-weak interplay in this case leads to a much larger ratio.

In a general mSUGRA inspired scenario, the tree level mass is much larger compared to the 1-loop mass [16] for large range in MSSM parameters, irrespective of the nature of R parity breaking. In the present case, the relative importance of loop and the tree level contributions is sensitive to the nature of R violation as demonstrated above. This feature arises not as a consequence of running of soft masses but due to difference in the relevant RG equations in case of λ and λ' couplings and the boundary conditions themselves which strongly depend on the gauge couplings in these models.

We have determined the ratio $\frac{m_{\rm loop}}{m_0}$ by solving the relevant RG equations numerically in the manner described in section (3). This ratio is plotted versus Λ in Fig.2 (at the end of this chapter) for $\Lambda = (50 - 150)$ TeV. In this range,

$$\frac{m_{\rm loop}}{m_0} = 25 - 45, \tag{6.51}$$

as expected from eq.(6.50).

The dominance of $m_{\rm loop}$ has the following important implication. The neutrino mass ratio following from eq.(6.41) is given by

$$\frac{m_{\nu_2}}{m_{\nu_3}} = \frac{m_{\rm loop}}{m_0 + m_{\rm loop}} \frac{m_\mu}{m_\tau}\frac{\eta_2}{\eta_3}. \tag{6.52}$$

In the previous case, the hierarchy in $m_{\rm loop}$ and m_0 resulted in strong hierarchy between neutrino masses. As a result, one could only obtain vacuum or quasi-vacuum solution for the solar neutrino. Here due to $m_{\rm loop} \gg m_0$, hierarchy in neutrino mass is much weaker, and we have,

$$\frac{m_{\nu_2}}{m_{\nu_3}} \approx \frac{m_\mu}{m_\tau} \sim 5 \times 10^{-2}. \tag{6.53}$$

We can thus easily get the scale relevant for the LMA solution of the solar neutrino. As already argued the mixing pattern is not appropriate if all the λ couplings are similar. This is no longer true if λ obey some specific hierarchy as we discuss now.

6.3.2 Illustrative Model

We assume that the couplings $\lambda_{123}, \lambda_{233}, \lambda_{322}$ dominate over the rest and neglect the latter. Moreover we assume that non-zero couplings satisfy the following hierarchy,

$$\frac{\lambda_{233}}{\lambda_{322}} \approx \mathcal{O}\left(\frac{h_2^E}{h_3^E}\right) \;\; ; \;\; \frac{\lambda_{123}}{\lambda_{322}} \leq \mathcal{O}\left(\frac{h_2^E}{h_3^E}\right). \tag{6.54}$$

We do not have strong theoretical reasons to assume the above hierarchy. The following considerations should therefore be viewed as an example which leads to the successful explanation of the neutrino anomalies.

The tree level mass matrix in the presence of these couplings is given by,

$$\mathcal{M}_s^0 = m_0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda_{233}^2 h_3^{E\;2} & \lambda_{233} h_3^E \lambda_{322} h_2^E \\ 0 & \lambda_{233} h_3^E \lambda_{322} h_2^E & \lambda_{322}^2 h_2^{E\;2} \end{pmatrix}. \tag{6.55}$$

The 1-loop level mass matrix is given by,

$$\mathcal{M}_s^l = m_{\rm loop} \begin{pmatrix} 0 & \lambda_{123} h_2^E \lambda_{232} h_3^E & 0 \\ \lambda_{123} h_2^E \lambda_{232} h_3^E & \lambda_{233}^2 h_3^{E\;2} & \lambda_{232} h_2^E \lambda_{323} h_3^E \\ 0 & \lambda_{232} h_2^E \lambda_{323} h_3^E & \lambda_{322}^2 h_2^{E\;2} \end{pmatrix}. \tag{6.56}$$

In view of the hierarchy in eq.(6.54), the total mass matrix has the following simple form

$$\mathcal{M}_s \approx \begin{pmatrix} 0 & x & 0 \\ x & A & A \\ 0 & A & A \end{pmatrix}, \tag{6.57}$$

where

$$
\begin{aligned}
A &\equiv (m_0 + m_{\rm loop})\lambda_{233}^2 h_3^{E\ 2} \ , \\
x &\equiv -m_{\rm loop}\lambda_{322}\lambda_{123}h_2^E h_3^E \ . \qquad (6.58)
\end{aligned}
$$

We shall assume $x \leq A$ which is consistent with the hierarchy in eq.(6.54). One can diagonalise the above matrix:

$$
R_{12}(\theta_{12})\ R_{13}(\theta_{13})\ R_{23}(\pi/4)\ \mathcal{M}_s\,[R_{12}(\theta_{12})\ R_{13}(\theta_{13})\ R_{23}(\pi/4)]^T \approx Diag.(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \ .
$$

Here, R_{ij} denotes rotation in the ij^{th} plane with angle θ_{ij}. We have neglected a small contribution of $\mathcal{O}(\frac{x}{2\sqrt{2}A})$ to the $2-3$ mixing angle in the above derivation. The mixing angles are given by

$$
\begin{aligned}
\tan 2\theta_{12} &\approx \frac{4\sqrt{2}A}{x} \ , \\
\tan 2\theta_{13} &\approx \frac{x}{A\sqrt{2}} \ . \qquad (6.59)
\end{aligned}
$$

The eigenvalues can be approximately written as

$$
\begin{aligned}
m_{\nu_1} &\approx \frac{x}{\sqrt{2}} - \frac{x^2}{8A} \ , \\
m_{\nu_2} &\approx -\frac{x}{\sqrt{2}} - \frac{x^2}{8A} \ , \\
m_{\nu_3} &\approx 2A + \frac{x^2}{2A} \ . \qquad (6.60)
\end{aligned}
$$

It is seen from the last two equations that all the mixing angles and the masses are predicted in terms of only two parameters namely, x and A. The atmospheric mixing angle is predicted to be around $\pi/4$ and the other two mixing angles can be expressed in terms of the solar and atmospheric scales. Using eqs.(6.59,6.60) we find,

$$
\begin{aligned}
\tan 2\theta_{solar} &\approx \frac{4\sqrt{2}A}{x} \approx 4\sqrt{2}\left(\frac{8\sqrt{2}\Delta_{solar}}{\Delta_A}\right)^{-1/3} \ , \\
\tan 2\theta_{CHOOZ} &\approx \frac{x}{A\sqrt{2}} \approx \frac{1}{\sqrt{2}}\left(\frac{8\sqrt{2}\Delta_{solar}}{\Delta_A}\right)^{1/3} \ . \qquad (6.61)
\end{aligned}
$$

Choosing $\Delta_A \sim 3\cdot 10^{-3}$ eV^2 and $\Delta_{solar} \sim (2.3-37)\times 10^{-5}$ eV^2 following the recent analysis of SNO neutral current data we get,

$$
\begin{aligned}
\frac{x}{A} &\approx (0.44 - 1.12) \ , \\
\tan^2\theta_{solar} &\approx (0.68 - 0.86) \ , \\
U_{e3}(CHOOZ) &\approx (0.15 - 0.32) \ . \qquad (6.62)
\end{aligned}
$$

The predictions for the mixing angles are in very good agreement with the observations which prefer large mixing angle solution for the solar neutrino. The required value of $\frac{x}{A}$ is also consistent with the assumed hierarchy in eq.(6.54) among the trilinear couplings.

6.4 Discussion

We have discussed the structure of neutrino masses and mixing in the Minimal messenger model (MMM) of gauge mediated supersymmetric breaking with purely trilinear R violating interactions. We considered two specific cases of purely λ' interactions and purely λ interactions for simplicity. The model contains very large number of parameters even under this simplifying assumptions. Remarkably, it is possible to make meaningful statement on the neutrino spectrum in spite of the presence of many unknown parameters if all these parameters are assumed similar in magnitude. This is a natural assumption in the absence of any specific symmetry to restrict the trilinear R parity violation. It is not always easy to justify this specific choice, e.g use of a $U(1)$ symmetry which uses Froggatt -Nielsen [FN] mechanism to obtain quark and lepton masses tend to forbid all the trilinear terms altogether [34].

In the case where only λ' couplings are present, one naturally gets large mixing between the neutrino states. Further, the MMM offers a very constrained structure giving rise to a large hierarchy between the masses $\sim O(10^{-2})$ for all the parameter space. The model is suitable for obtaining simultaneously solutions for atmospheric neutrino problem and quasi-vacuum oscillations.

Assumption of approximate equality of λ couplings in case with only λ couplings, leads to very constrained and phenomenologically inconsistent pattern for neutrino mixing. This conclusion follows on general grounds and it is true even if SUSY breaking is induced by supergravity interactions. It is quite interesting that one can arrive at this strong conclusions in spite of the presence of many unknown parameters by simply assuming them to be of similar magnitude.

One can obtain consistent picture of neutrino anomalies if λ couplings are assumed to be hierarchical. We provided an example which leads to two large and one small mixing and correct hierarchy between the solar and atmospheric neutrino scales.

One interesting result of this analysis is the interplay between the sneutrino *vev* induced contribution and the loop induced contribution to neutrino masses. In the context of supergravity induced SUSY breaking, it has been shown that the large logarithmic factors induced due to RG scaling enhance the sneutrino *vev* induced contribution compared to the loop contribution. We showed that this remains true even in the gauge mediated models of SUSY breaking in case of the trilinear λ' couplings. In the mSUGRA model, the dominance of tree level mass follows simply from the large factor $t = ln\frac{M_{GUT}^2}{M_Z^2}$ in sneutrino *vev* generated by running of soft parameters. In the present case, the tree level dominance occurs essentially due to boundary conditions. In case of λ' couplings, m_0 is determined by squark masses which depend upon $\alpha_3(X)$. m_0 dominates over loop contribution in this case. For λ couplings, m_0 is determined by sleptons rather than by squarks masses. Due to their dependence on weak couplings, slepton masses are much smaller than squark masses. As a consequence, m_0 is suppressed compared to λ' case. This results in loop dominance if R is violated by λ couplings.

The neutrino mass hierarchy strongly depends on the ratio $\frac{m_{\rm loop}}{m_0}$. In case of the tree level dominance (purely λ' couplings) one obtains strong hierarchy and vacuum solution while the case with loop mass dominating corresponds to milder hierarchy and the LMA MSW solution. This analysis along with other similar analysis [16, 17, 10] therefore underlines the need of including both the contributions to neutrino masses in a proper way.

6.5 Appendix A: Relevant RG Equations

Here we present the Renormalization Group Equations (RGE) for the soft parameters B_{ϵ_i} and $m^2_{\nu_i H_1}$ for the two cases considered in this work: either purely λ' couplings or purely λ couplings are the sources of lepton number violation in the superpotential. These equations have been derived using the general formulae given in [35]. These equations can also be found in [17], whereas the equations for the standard soft parameters appearing in the RHS of the equations can be found in many papers such as [36]. In writing the below eqs.(6.63)(eqs.(6.64)), we have neglected $\mathcal{O}(\lambda'^{\,2})$ ($\mathcal{O}(\lambda^2)$) corrections. The notation is as described in the text.

λ' couplings in the superpotential:

$$\begin{aligned}
\frac{dB_{\epsilon_i}(t)}{dt} &= B_{\epsilon_i}(t)\left(-\frac{3}{2}Y_t(t) - \frac{1}{2}Y_i^E(t) + \frac{3}{2}\tilde{\alpha}_2(t) + \frac{3}{10}\tilde{\alpha}_1(t)\right) \\
&- \frac{3}{16\pi^2}\mu(t)\lambda'_{ijj}(t)h^d_{jj}(t)\left(\frac{1}{2}B_\mu(t) + A^{\lambda'}_{ijj}(t)\right) \\
\frac{dm^2_{\nu_i H_1}(t)}{dt} &= m^2_{\nu_i H_1}(t)\left(-\frac{1}{2}Y_i^E(t) - \frac{3}{2}Y_b(t) - \frac{1}{2}Y_\tau(t)\right) - \frac{3}{32\pi^2}\lambda'_{ipp}(t)h^d_{pp}(t)\left(m^2_{H_1}(t)\right. \\
&+ \left. m^2_{L_i}(t) + 2\,m^2_{Q_p}(t) + 2\,A^{\lambda'}_{ipp}(t)A^D_{pp}(t) + 2\,m^2_{D_p}(t)\right) \qquad (6.63)
\end{aligned}$$

λ couplings in the superpotential:

$$\frac{dB_{\epsilon_i}(t)}{dt} = B_{\epsilon_i}(t)\left(-\frac{3}{2}Y_t(t) - \frac{1}{2}Y_i^E(t) + \frac{3}{2}\tilde{\alpha}_2(t) + \frac{1}{2}\tilde{\alpha}_1(t)\right)$$

$$
\begin{aligned}
& - \frac{1}{16\pi^2}\mu(t)\lambda_{idd}(t)h^E_{dd}(t)\left(A^{\lambda}_{idd}(t)+\frac{1}{2}B_{\mu}(t)\right) \\
\frac{dm^2_{\nu_i H_1}(t)}{dt} &= m^2_{\nu_i H_1}(t)\left(-\frac{1}{2}Y^E_i(t)-\frac{1}{2}Y_{\tau}(t)-\frac{3}{2}Y_b(t)\right)-\frac{1}{32\pi^2}\lambda_{ijj}(t)h^E_{jj}(t)\left(m^2_{H_1}(t)\right. \\
& + \left. m^2_{L_i}(t)+2\ m^2_{L_j}(t)+2\ A^{\lambda}_{ijj}(t)A^E_{jj}(t)+2\ m^2_{E_j}(t)\right) \qquad (6.64)
\end{aligned}
$$

6.6 Appendix B: Comments on Numerical Evaluation of RG Equations

We have seen in this chapter that neutrino masses can be expressed as a product of soft parameters and R-violating couplings. So we need to know the values of these parameters at weak scale. Here we briefly discuss the numerical approach used to obtain neutrino masses. In numerical evaluation of relavent RG equations, we closely follow the work of [23] where two loop RG equation for the B parameter were used for fixing the sign of μ parameter at the weak scale. In our calculation we do not use complete 1-loop effective potential. Instead we use an approximately equivalent method in which we define a decoupling scale Q_0 as the geometric mean of $m^2_Q(X)$ and $m^2_U(X)$. The high scale $X \sim (60-150)\ TeV$. To begin with Q_0 is approximately taken as $100 GeV$. Then the RG equations for gauge couplings are evolved from electroweak scale to Q_0 scale using standard model β functions and from Q_0 to high scale X using MSSM β functions. At scale X then one finds the scalar masses in terms of gauge couplings, as defined by gauge mediated boundary conditions. A geometric mean of $m^2_Q(X)$ and $m^2_U(X)$ then gives the improved value of decoupling scale Q_0. This process is iteratively repeated until one finds self-consistent Q_0. Having found Q_0 one runs yukawa from scale M_z to Q_0 using SM β functions for the gauge couplings, and then from Q_0 to X using $MSSM$ β functions for the gauge couplings. Having known yukawas at X all the soft parameters are evolved from X to Q_0. Knowing scalar masses at Q_0, one can find μ and B_μ requiring correct electroweak breaking. Evolving μ and B_μ from Q_0 to X we get their values at X. Now all the soft and supersymmetric parameters are evolved from scale X to Q_0. Trilinear and bilinear soft parameters like A and B are zero at the scale X, which is a typical feature of GMSB models. But they develop non-zero values at Q_0 due to RG scaling. Knowing all the soft parameters at Q_0 one can find neutrino masses. Fig (1.1) illustrates the flow-chart for numerical evaluation of neutrino masses.

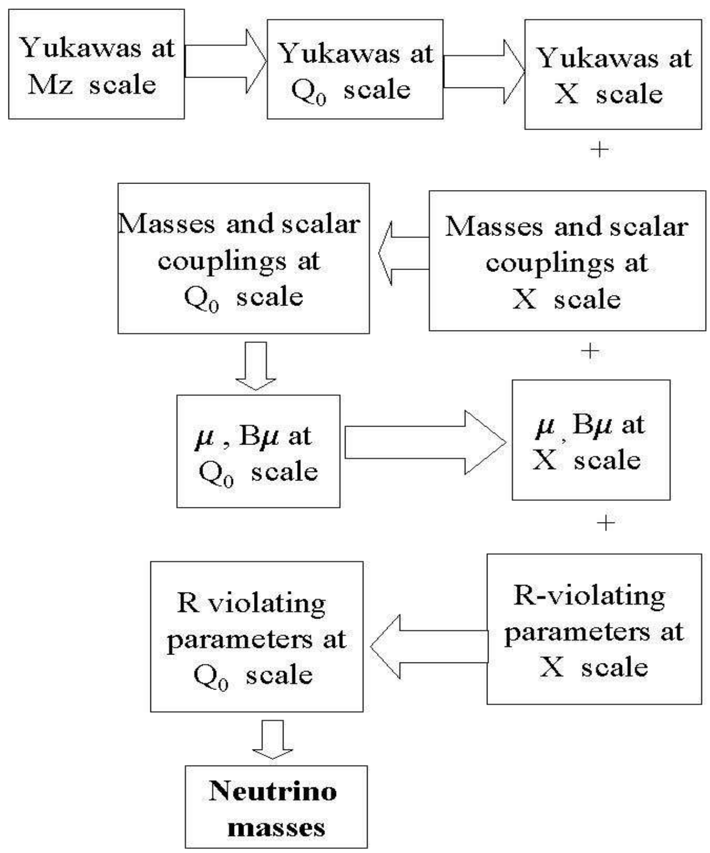

Figure 6.1: Flowchart for numerical evaluation of RG equations.

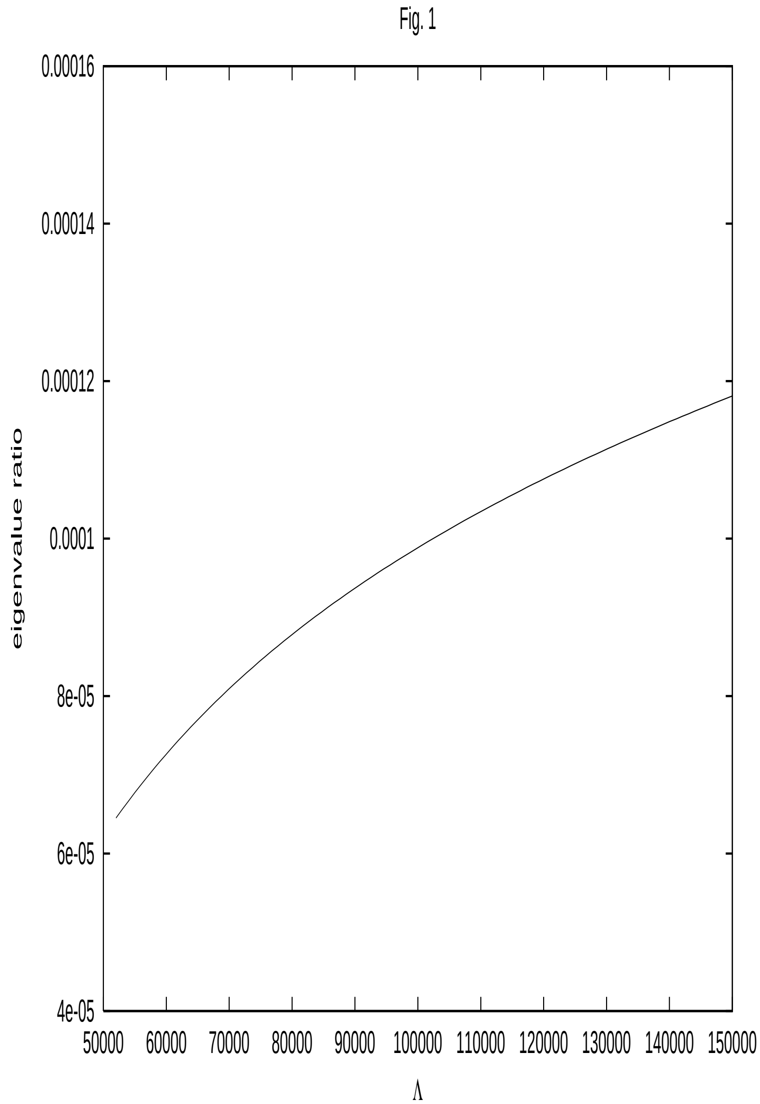

Figure 6.2: Neutrino mass eigen value ratio $\frac{m_{\nu_2}}{m_{n_3}}$ plotted versus Λ (GeV) assuming trilinear λ' couplings of similar strength to be the only source of R violation. Λ is defined in the text.

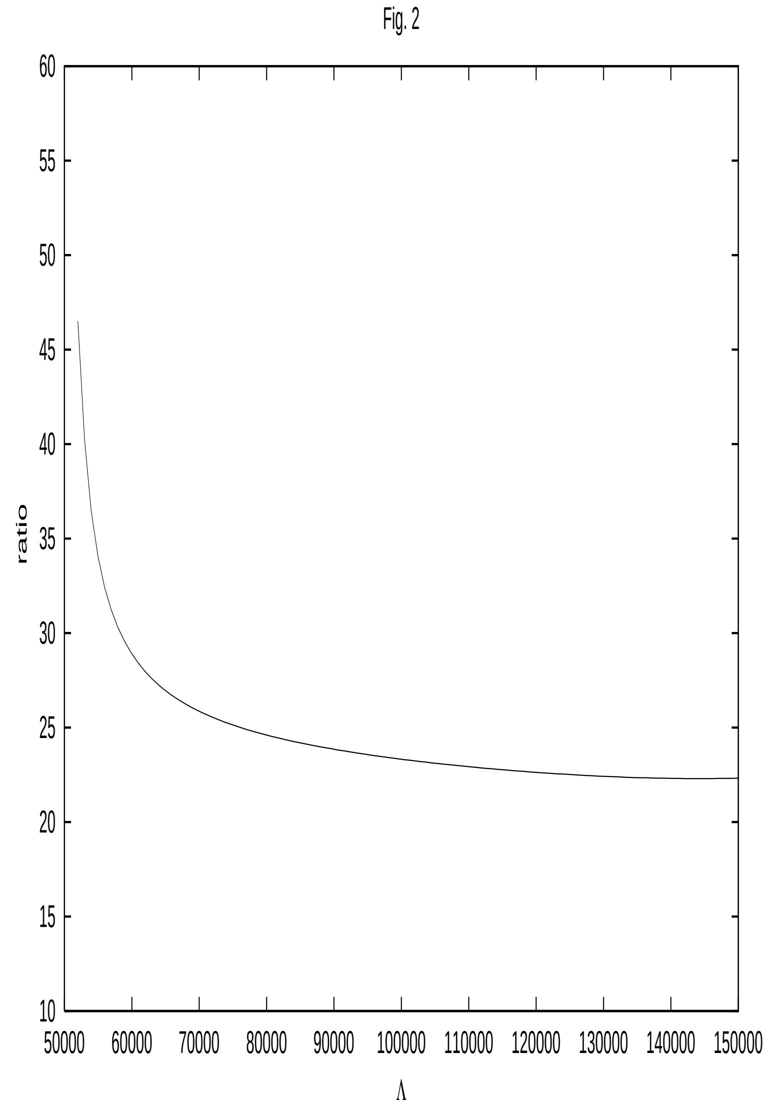

Figure 6.3: The neutrino mass ratio $\frac{m_{loop}}{m_0}$ plotted versus Λ (GeV) assuming trilinear λ couplings of similar strength to be the only source of R violation.

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Chapter 7

$U(1)$ Symmetry and R Violation

7.1 Introduction

In the last chapters we have seen that there are theoretical motivations and experimental evidences for neutrinos to be massive. We described in detail, how one can obtain hierarchical masses and one or two large mixing angles for neutrinos, as suggested by experiments, in the framework of R violating SUSY. Although such a model is very appealing, introduction of a large number of *a priori* arbitrary R violating couplings sounds very *ad hoc*. Being large in number, any phenomenological analysis with these couplings is a daunting task and one is compelled to resort to certain assumptions like, they are similar in magnitude, or possess some kind of hierarchy among them. Is it possible to restrict the number of R violating couplings from some symmetry principle which gives some insight into the pattern and magnitude of these couplings ? It turns out that one does not have to invoke very exotic extensions and a simplest possibility of an anomalous Abelian family symmetry, when used in conjunction with phenomenological restrictions, puts stringent constraints on the possible patterns and magnitudes of R violating couplings. For example, we find that all the trilinear λ'_{ijk} couplings vanish identically, and all but at most two trilinear λ_{ijk} couplings vanish or are enormously suppressed [1].

Such an Abelian family symmetry was considered first by Froggatt and Nielsen [2] in order to explain the large mass ratios of quarks and leptons of different families. Such mass ratios when viewed as powers of Cabibo angle $\lambda \sim 0.22$ show strange geometrical hierarchy. This is also true of charged lepton sector as shown below.

$$\begin{aligned} \frac{m_u}{m_t} &\sim \lambda^8 \quad ; \quad \frac{m_c}{m_t} \sim \lambda^4 \\ \frac{m_d}{m_b} &\sim \lambda^4 \quad ; \quad \frac{m_s}{m_b} \sim \lambda^2 \\ \frac{m_e}{m_\tau} &\sim \lambda^4 \quad ; \quad \frac{m_\mu}{m_\tau} \sim \lambda^2. \end{aligned} \tag{7.1}$$

Froggatt and Nielsen first tried to explain the large mass ratios as due to Renormalization Group evolution of the effective masses ("bare" Higgs couplings) from a more fundamental high energy scale, where they are all supposed to be of the same order of magnitude. They found that there can be a large overall renormalization of the top (bottom) quark mass matrix by many orders of magnitude but there is only a finite renormalization of the mass ratios. It is only possible to obtain a large mass renormalization of the mass ratios between a top type quark and a bottom type quark (or a lepton) [3], where the difference in gauge quantum numbers ensures that they are treated differently by the dynamics. The question is, how to obtain the hierarchy along the family space of fermions ? May be one needs to invoke a gauge symmetry that assigns different gauge quantum numbers to different families. Such a horizontal Abelian symmetry along family space is precisely what Froggatt and Nielsen suggested to solve the problem of mass hierarchy among the fermion families. Such an approach also confers with the beautiful argument given by 't Hooft in [4], that small dimensionless numbers should have a dynamical origin[1]. Although this mechanism is quite general, it becomes quite attractive to combine the virtues of this U(1) symmetry with that of the minimal supersymmetric standard model (MSSM) [5, 6, 7, 8, 9, 10, 11]. In this case, the U(1) can give information not only on the quark spectrum but also on the R parity violating couplings which can determine the neutrino masses through the pattern of the R violation it dictates [9, 10, 12, 13, 14]. The superfields charged under U(1), transform as chiral representation of SUSY [2]. Such a symmetry if un-gauged, would lead to problematic massless Goldstone bosons. If gauged, it would lead to anomalies. It turns out that the pathological anomalies, if cancelled via 4-D version of Green-Schwarz (GS) mechanism [16], are blessings in disguise. In the next subsection, we point out a stringy origin of U(1) symmetry and the positive role of its anomalies. With a Stringy origin U(1), could prove to be a well motivated simple extension, connecting the *hypothetical* Physics at the high scale to its verifiable low energy predictions of fermion masses and mixings. As we will see, it can also predict the Weinberg angle at the string scale. Before we discuss patterns of R violation and $U(1)$ symmetry, it would be instructive to look into motivations for taking $U(1)$ symmetry so seriously.

7.1.1 A brief detour on GS Mechanism

String theory, though beautiful in its own right, suffers a lack of contact with the SM gauge group. Closest thing achieved so far, being models with gauge group $SU(3) \times SU(2) \times$

[1] See the discussion on Naturalness in chapter 1.

[2] The reason for their being chiral, is that most often a U(1) symmetry is a relic of the compactification of String gauge group. As pointed out in [15, 16] it is not easy to obtain an effective four-dimensional theory containing chiral fermions by spontaneous compactification of a theory in $D > 4$ dimensions. It may be necessary that the D-dimensional theory itself contains chiral fermions and elementary Yang-Mills gauge fields.

$U(1)_Y \times U(1)^n \times G$, where G is some "hidden sector" gauge group not coupling to the SM particles. So the Stringy models upon compactification always leave anomalous $U(1)$ gauge groups. Since the original String model is anomaly free, the anomalies must cancel, via Green Schwarz mechanism as follows. For an anomalous gauge theory, the low energy Lagrangian is not invariant under gauge transformation and gets additional anomalous contributions proportional to $F\tilde{F}$, where F are the field strengths corresponding to $SU(2)$ and $U(1)$ gauge groups, and $\tilde{F}$ are dual to F. To restore gauge invariance we can add to the Lagrangian a Green-Schwarz term, which is essentially a coupling of a pseudo-scalar axion η to the anomaly term. An axion couples universally to all gauge groups. The quadratic gauge piece of the Lagrangian has the form

$$\frac{1}{g^2(M)} \sum_{i=1,2,3,X} k_i F_i^2 + \frac{i\eta(x)}{M} \sum_{i=1,2,3,X} k_i F_i \tilde{F}_i, \tag{7.2}$$

where g is the gauge coupling constant at the string scale M, and index i runs over the three gauge groups $SU(3), SU(2), U(1)$ and the 'anomalous' gauge group $U(1)_X$ (we shall use the subscript 'X' here to distinguish 'stringy' $U(1)$ from the $U(1)_Y$) . The coefficients k_i are the Kac-Moody levels of the corresponding gauge algebra [17]. For the case of non-Abelian groups like $SU(3)$ and $SU(2)$ these levels are integer and in practically all models constructed up to now, one had $k_2 = k_3 = 1$. In the case of an Abelian group like $U(1)$ hypercharge, k_1 is a normalization factor (not necessarily integer) and is model dependent. Under a $U(1)_X$ gauge transformation, axion transforms non-trivially in order to cancel the anomalous contribution:

$$\begin{aligned} A_X^\mu &\rightarrow A_X^\mu + + \partial^\mu \theta(x) \\ \eta &\rightarrow \eta - \theta(x)\delta_{GS}, \end{aligned} \tag{7.3}$$

where δ_{GS} is a constant. If the coefficients A_3, A_2 and A_1 of the mixed anomalies of $U(1)_X$ with the SM gauge groups $SU(3), SU(2)$ and $U(1)_Y$ respectively are in the ratio

$$\frac{A_1}{k_1} = \frac{A_2}{k_2} = \frac{A_3}{k_3} = \delta_{GS}, \tag{7.4}$$

those mixed anomalies will be cancelled by the gauge variation of the second term in eq. (7.2). Since there may be in the spectrum extra singlet particles with $U(1)_X$ quantum numbers but no SM gauge interactions, we will not consider here the equivalent conditions involving the $U(1)_X$ anomaly coefficient, since those singlets can always be chosen so that anomaly is cancelled. For the same reason we will not consider the mixed $U(1)_X$ gravitational anomalies. On the other hand, to be consistent, one has to impose that the mixed $U(1)_Y - U(1)_X^2$ anomaly vanishes identically since it only involves the standard model fermions and cannot be cancelled by a GS mechanism.

Before leaving this section, it would be worthwhile to comment upon, one of the important prediction of this U(1) model, and that is the Weinberg angle at the string scale [18]. From eqs, (7.2) and (7.4) one obtains for the tree level weak angle at the string scale

$$sin^2\theta_W = \frac{k_1}{k_1+k_2} = \frac{A_1}{A_1+A_2}. \tag{7.5}$$

The above expression shows that, for each given 'anomalous' $U(1)_X$, the cancellation of the anomalies through a GS mechanism gives a definite prediction for the weak mixing angle in terms of the coefficients of anomaly. The latter may be computed in terms of the $U(1)_X$ charges of the massless fermions of the theory. This mechanism gives us an alternative to GUTs concerning the derivation of $sin^2\theta = 3/8$ in perfect agreement with data when extrapolated to the infra-red. It should be noted that the $U(1)_X$ may be made anomaly free through the GS mechanism *if and only if the normalization of the coupling constants is the canonical one* $g_3^2 = g_2^2 = \frac{5}{3}g_1^2$. The success of that prediction would be an indication of the existence of a 4-D string with the gauge group of the form

$$SU(3) \times SU(2) \times U(1)_Y \times U(1)_X \times G, \tag{7.6}$$

and with the mixed $U(1)_X$ anomalies in the ratio $A_2/(A_1+A_2) = 3/8$.

Of late, anomalous $U(1)_X$ models with anomalies cancelled through GS mechanism have become popular because they provide an attractive mechanism for SUSY breaking. In order to obtain supersymmetric version of Green-Schwarz term, we have to add a dilaton field S to the axion to make a complex chiral superfield. When the dilaton attains a *vev* it generates Fayet-Illiopoulose tade-pole D-term and thus breaking supersymmetry [19]. In the simplest model it turns out that gaugino masses may be too low and one must seek ways around this. However, the A and B terms are also likely to be small in this model and that may provide certain advantages.

So we have seen that $U(1)_X$ symmetry is very well motivated and provides an attractive simple extension to look for the origin of R violation. In the next sections we describe in detail how such a scenario leads to severely constrained patterns of R violation. In the following sections we will remove the subscript 'X' from the $U(1)$ as the meaning is clear from the context.

7.2 $U(1)$ Symmetry and R Violation

As we have already pointed out in the introduction that one of the attractive ways to understand the mysterious hierarchy among quark and lepton masses is to postulate the existence of a U(1) symmetry broken spontaneously at a scale much larger than that of weak interactions [2]. Most fermion masses and the entire Cabibbo Kobayashi Maskawa (CKM) matrix arise in this approach due to the breaking of $U(1)$ and are determined in terms of a parameter $\lambda \sim \frac{<\theta>}{M}$ and the $U(1)$ charges of the fermions. Here $<\theta>$ determines the scale of $U(1)$ breaking and M is some higher scale which could be the Planck scale M_P or the string scale if $U(1)$ arises from an underlying string theory. The λ is usually identified with the Cabibbo angle ~ 0.22 and all the fermion mass matrices are represented as powers of λ. It would be interesting to combine this idea with the minimal supersymmetric standard model. In this case, the $U(1)$ can give information not only on the quark spectrum but also on the R parity violating couplings which can determine the neutrino masses through the pattern of the R violation it dictates [9, 10, 12, 13, 14].

The lepton number violation in the MSSM is generated due to the presence of the supersymmetric partners of quarks and leptons. This can be characterized by the following R violating terms in the superpotential of the model:

$$W_{\not R_p} = \lambda'_{ijk} L_i Q_j D_k^c + \lambda_{ijk} L_i L_j E_k^c + \epsilon_i L_i H_2. \tag{7.7}$$

A priori, this involves 39 independent parameters. Each of this can contribute to the mass matrix for the three light neutrinos. It is desirable to restrict the number of the allowed couplings from some symmetry principle and the $U(1)$ symmetry can play a crucial role. By requiring that the $U(1)$ charges of the MSSM field should be such that it leads to correct quark and charge lepton masses as well as the CKM matrix, one could considerably reduce the freedom in choosing the $U(1)$ charges. Set of charges so determined would lead to definite patterns of the R violating couplings appearing in eq.(7.7). This in turn leads to specific structure for neutrino masses.

We start in the next section with a discussion of our framework and the basic assumptions and highlight the problem of generation of the large ϵ_i parameters within this framework. In the next section, we discuss the structure of trilinear interactions and their consistency with phenomenology in models which can explain the quark spectrum. Section 6.5 contains specific discussion of the consequences of models allowed on phenomenological ground and we summarize the main results in the last section.

7.3 $U(1)$ symmetry and ϵ problem

Let us consider the MSSM augmented with a gauged horizontal $U(1)$ symmetry. The standard superfields (L_i, Q_i, D_i^c, U_i^c, E_i^c, H_1, H_2) are assumed to carry the charges (l_i, q_i, d_i, u_i, e_i, h_1, h_2) respectively with i running from 1 to 3. The $U(1)$ symmetry is assumed to be broken at a high scale by the vacuum expectation value (*vev*) of one gauge singlet superfield θ with the $U(1)$ charge normalized to -1 or with two such fields

θ and $\bar{\theta}$, with charges -1 and 1 respectively. It is normally assumed that only the third generation of fermions have renormalizable couplings invariant under $U(1)$. The rest of the couplings arise in the effective theory from the higher dimensional terms [2]:

$$\Psi_i \Psi_j H \left(\frac{\theta}{M}\right)^{n_{ij}} ,$$

where Ψ_i is a chiral superfield, H is the Higgs doublet and M is some higher mass scale which could be the Planck scale M_p and $n_{ij} = \psi_i + \psi_j + h$ are positive numbers representing the charges of Ψ_i, Ψ_j and H under $U(1)$ respectively. Similar term is absent in case of a negative n_{ij} due to holomorphic nature of W [5]. For positive n_{ij}, one gets an ij^{th} entry of order $\lambda^{n_{ij}}$ in the mass matrix for the field Ψ. Identification $\lambda \sim 0.22$ and proper choice of the $U(1)$ charges leads to successful quark mass matrices [6, 7, 8].

A priori, the model has 15 independent $U(1)$ charges for matter and 2 charges for Higgs fields. Of these, all but four can be fixed from different requirements discussed in the literature which we list below [8].

(1) The fermions in the third generation are assumed to have the following couplings invariant under $U(1)$

$$W_Y = \beta_t Q_3 U_3^c H_2 + \beta_b Q_3 D_3^c H_1 \left(\frac{\theta}{M}\right)^x + \beta_\tau L_3 E_3^c H_1 \left(\frac{\theta}{M}\right)^x , \tag{7.8}$$

where $\beta_{t,b,\tau}$ are assumed to be of $\mathcal{O}(1)$. This is possible if,

$$q_3 + u_3 + h_2 = 0; \qquad q_3 + d_3 + h_1 = \quad l_3 + e_3 + h_1 = x. \tag{7.9}$$

This determines $h_2 = -q_3 - u_3$ and $h_1 = -q_3 - d_3 + x$ with $\tan\beta \sim \lambda^x(m_t/m_b)$. The phenomenological requirement of $\tan\beta \geq O(1)$ implies $0 \leq x \leq 2$. Here $b - \tau$ unification has been implicitly assumed in writing eq.(7.9).

(2) The charge differences $q_{i3} \equiv q_i - q_3$, $u_{i3} \equiv u_i - u_3$ and $d_{i3} \equiv d_i - d_3$ $(i = 1, 2)$ are determined by requiring that the quark masses and the CKM matrix come out to be exactly or approximately correct. Various possible values for these differences have been classified in [8] and we shall use these results.

(3) The $U(1)$ symmetry being gauged is required to be anomaly free. It has been shown [7] that all the relevant $U(1)$ anomalies cannot be zero in models with a single θ if one is to require the correct structure for the quark and lepton masses. These anomalies

then need to be cancelled by the Green-Schwarz mechanism [16] as discussed in the introduction. This requirement imposes three non-trivial relations among the $U(1)$ charges.

(4) The prediction of approximately correct hierarchy among the charged lepton masses requires

$$l_{13} + e_{13} = 4 \text{ OR } 5 \ ; \quad l_{23} + e_{23} = 2. \tag{7.10}$$

After imposing the above listed requirements, the successful model is fixed in terms of the 4 independent charges. Each choice of these charges would imply different patterns for R violation. Since the $U(1)$ is capable of predicting orders of magnitudes of various couplings, it is not guaranteed that all the patterns of R violation predicted in this way would be phenomenologically consistent. In fact very few can meet the constraints from phenomenology. The most stringent constraint on possible choice of $U(1)$ charges is provided by the parameters ϵ_i. The $U(1)$ symmetry can lead to the following term in W:

$$M\ L_i\ H_2 \left(\frac{\theta}{M}\right)^{l_i+h_2} . \tag{7.11}$$

This leads to

$$\epsilon_i \sim M \left(\frac{<\theta>}{M}\right)^{l_i+h_2} \sim M\lambda^{l_i+h_2}. \tag{7.12}$$

Unless the charges $l_i + h_2$ are appropriately chosen, the predicted value for ϵ_i can grossly conflict with (a) the scale of $SU(2) \times U(1)$ breaking which would require sneutrino *vev* $\leq O(M_W)$ and (b) neutrino masses. A bilinear parameter ϵ would imply a neutrino mass [20] of order [21]:

$$m_\nu \sim \left(\frac{\epsilon}{\mu}\right)^2 \frac{M_Z^2}{M_{SUSY}} \sin^2\phi. \tag{7.13}$$

Here, $\sin^2\phi$ is O(1) if SUSY breaking is not characterized by the universal boundary conditions at a high scale. In the converse case, this factor gets enormously suppressed due to the fact that ϵ_i can be rotated away from the full Lagrangian in the limit of vanishing down quark and charged lepton couplings. This issue is discussed in number of papers [22]. Typical order of magnitude estimate of $\sin^2\phi$ is [23]

$$\sin^2\phi \sim \left(\frac{3h_b^2 ln\frac{m_X^2}{m_Z^2}}{16\pi^2}\right)^2 \sim 10^{-7}. \tag{7.14}$$

These equations are very rough estimates. The exact values depend upon the MSSM parameters. But these rough estimates are sufficient to show that phenomenologically required ϵ_i are grossly in disagreement with the typical predictions, for e.g, even with $sin\phi^2 \sim 10^{-7}$, $m_\nu < eV$ would need $\epsilon \sim$ GeV for $\mu \sim M_{SUSY} \sim 100$ GeV.

In order to prevent very large ϵ_i being generated, one must ensure one of the following:

(a) $l_i + h_2$ is bounded by

$$l_i + h_2 \gtrsim 24. \qquad (7.15)$$

This can lead to ϵ_i in GeV range and neutrinos with mass in the eV range in case of models with universal boundary conditions and $M \sim 10^{16}$ GeV. In models without the universal boundary conditions, the required magnitude for $l_i + h_2$ would be even larger.

(b) $U(1)$ is broken by only one superfield θ and $l_i + h_2$ is negative. The terms in eq.(7.12) are then not allowed in W by the $U(1)$ symmetry and by the analyticity of W.

(c) $l_i + h_2$ is fractional, forbidding coupling of bilinear term to θ.

(d) Impose some additional symmetry, e.g. modular invariance which may prevent occurrence of dangerous terms [24].

Note that models containing two θ-like fields with opposite $U(1)$ charges would lead to large ϵ_i independent of the sign of $l_i + h_2$. Thus these models can be made phenomenologically consistent only by choosing fractional or unnaturally high values for $|l_i + h_2|$. We shall therefore not consider these models and concentrate only on models with a single θ and also assume only integer $U(1)$ charges. Then ϵ_i can be suppressed either through (a) or through (b) if no other symmetry is imposed.

Although the structure of R violating interactions following from a $U(1)$ symmetry alone has been discussed in a number of papers [7, 10, 12, 13, 14], the requirement that the $U(1)$ symmetry should not generate large ϵ_i has not always been imposed [7, 10, 13]. It is argued customarily that ϵ_i are unphysical as they can be rotated away by redefining the new H_1 as a linear combination of the original H_1 and L_i appearing in eq.(7.7). This however changes the original μ parameter to $(\mu^2 + \epsilon_i^2)^{1/2}$. Thus if the models do allow large ϵ_i then rotating them away generates equally large μ which is also phenomenologically inconsistent. One must therefore allow only the $U(1)$ charge assignments corresponding to zero or suppressed ϵ_i in W.

7.4 Structures of trilinear couplings

In this section, we shall enumerate possible $U(1)$ models leading to correct quark mass spectrum and investigate structures for the trilinear couplings in these models keeping the phenomenological constraints in mind.

After imposing eqs.(7.9), the quark mass ratios and the CKM mixing angles are determined in terms of the quark charge differences. Systematic search for the possible charge differences led to the eight models [8, 10] reproduced in the table 1.

The model I exactly reproduces the quark mass ratios and all the three CKM mixing angles. Since the predictions of the $U(1)$ symmetry are exact only up to coefficients of O(1), one has to allow for models which may deviate from the exact predictions by small amount. The charge differences in model II, III, and IV represent the models which deviate from the exact predictions by O(λ) [8]. The leptonic mixing analogous to the CKM matrix is still arbitrary in these models but the charged lepton masses are required to satisfy $\frac{m_e}{m_\tau} \sim \lambda^4, \frac{m_\mu}{m_\tau} \sim \lambda^2$ in models (A) and $\frac{m_e}{m_\tau} \sim \lambda^5, \frac{m_\mu}{m_\tau} \sim \lambda^2$ in models (B).

Models

Models	$\mathbf{l_{13}+e_{13}}$	$\mathbf{l_{23}+e_{23}}$	$\mathbf{q_{13}}$	$\mathbf{q_{23}}$	$\mathbf{u_{13}}$	$\mathbf{u_{23}}$	$\mathbf{d_{13}}$	$\mathbf{d_{23}}$
IA	4	2	3	2	5	2	1	0
IIA	4	2	4	3	4	1	1	-1
IIIA	4	2	4	3	4	1	-1	-1
IVA	4	2	-2	-3	10	7	6	5
IB	5	2	3	2	5	2	1	0
IIB	5	2	4	3	4	1	1	-1
IIIB	5	2	4	3	4	1	-1	-1
IVB	5	2	-2	-3	10	7	6	5

Table 1 : We present here all the possible models which generate correct quark and lepton mass hierarchies as well as the CKM matrix.

The $U(1)$ charges are still subject to the anomaly constraint. The anomalies generated due to the presence of the extra $U(1)$ are as follows:

$$[SU(3)]^2U(1)_X: \quad A_3 = \sum_{i=1}^{3}(2q_i + u_i + d_i)$$

$$[SU(2)]^2U(1)_X: \quad A_2 = \sum_{i=1}^{3}(3q_i + l_i) + h_1 + h_2$$

$$[U(1)_Y]^2U(1)_X: \quad A_1 = \sum_{i=1}^{3}(\frac{1}{3}q_i + \frac{8}{3}u_i + \frac{2}{3}d_i + l_i + 2e_i) + h_1 + h_2$$

$$U(1)_Y[U(1)]_X^2 : \quad A_1' = \sum_{i=1}^{3}(q_i^2 - 2u_i^2 + d_i^2 - l_i^2 + e_i^2) - h_1^2 + h_2^2. \tag{7.16}$$

These can be cancelled in string theory through the Green-Schwartz mechanism [16] by requiring

$$A_2 = A_3 = \frac{3}{5}A_1; \quad A_1' = 0. \tag{7.17}$$

The above constraints on A_1, A_2, A_3 can be solved to give:

$$\begin{aligned} h \equiv h_1 + h_2 &= \sum_{i=1}^{3}(q_{i3} + d_{i3}) - \sum_{i=1}^{3}(l_{i3} + e_{i3}), \\ l_2 &= m - (l_1 + l_3 + 9q_3 + 4h - 3x), \end{aligned} \tag{7.18}$$

where

$$m = \sum_{i=1}^{3}(u_{i3} + d_{i3} - q_{i3}). \tag{7.19}$$

Also from eqs.(7.9),

$$u_3 = x - 2q_3 - d_3 - h. \tag{7.20}$$

Note that the parameter h determines whether the μ term is allowed in W. Positive h will result in too large μ unless h is also correspondingly large [3]. Negative h does not allow the μ term in W but phenomenologically consistent value can be generated through GM mechanism in this case. $h = 0$ allows arbitrary μ in W. The anomaly constraints determines h completely in terms of the charge differences fixed by the models in Table 1 and is insensitive to the overall redefinition of the $U(1)$ charges. It is seen that all except model (IIA) lead to zero or negative h and thus are phenomenologically consistent.

The magnitudes and structure of the trilinear couplings is determined by the following equation:

$$\begin{aligned} \lambda'_{ijk} &= \theta(c_i + n^d_{jk})\lambda^{c_i + n^d_{jk}} \\ \lambda_{ijk} &= \theta(c_i + n^l_{jk})\lambda^{c_i + n^l_{jk}}, \end{aligned} \tag{7.21}$$

where $c_i = l_i + x + h_2 - h$; $n^d_{jk} = q_{j3} + d_{k3}$; $n^l_{jk} = l_{j3} + e_{k3}$ with n^d_{jk}, n^l_{jk} being completely fixed for a given model displayed in Table 1. Note that some of the trilinear couplings may be zero if the corresponding exponent is negative. They may still be generated due to non-minimal contribution to the kinetic energy term of different fields [8, 9, 10]. Such contributions do not however affect the order of magnitudes of those couplings which are non-zero to start with [9].

After imposing the constraints of eqs.(7.17), one is still left with four independent parameters including x. One would thus expect considerable freedom in the choice of

[3] see however ref. [24] which imposes additional modular invariance

$\lambda'_{ijk}, \lambda_{ijk}$. Typically, more than one such couplings are allowed to be non-zero simultaneously in various models. Thus they lead to flavour violating transitions which are known to be enormously suppressed. It is these constraints on the product of trilinear couplings which lead to stringent restrictions on the allowed $U(1)$ charges. It turns out that constraint following from the $K^0 - \bar{K}^0$ mass difference alone is sufficient to rule out the presence of non-zero trilinear couplings in most models. The $K^0 - \bar{K}^0$ mass difference constrains the product $\lambda'_{i12}\lambda'_{i21}$ to be $\leq 10^{-9}$ [25] for the slepton masses of O(100 GeV). Allowing for some variation in these masses, we shall use the following conservative limit

$$\lambda'_{i12}\lambda'_{i21} \leq \lambda^{12} \sim 1.3 \cdot 10^{-8}. \tag{7.22}$$

We now analyze the magnitudes of the product in eq.(7.22) predicted by models of Table 1, when one imposes the additional requirement that the $l_i + h_2$ is negative or has a large value given in eq.(7.15). These requirements result in zero or suppressed ϵ_i respectively. But they would also lead to zero or suppressed trilinear interactions as we now discuss. Let us consider these two cases separately.

7.4.1 $l_i + h_2 \gtrsim 24$

In this case, ϵ_i are artificially forced to be small by choosing very large value of $l_i + h_2$ as in eq.(7.15). But the large value of these charges also results in the enormous suppression in the allowed magnitudes of the trilinear couplings. This is easily seen from eqs.(7.21). Since h is zero or negative for all the allowed models, and $x \leq 2$, it follows that

$$c_i = l_i + h_2 + x - h \ \geq \ l_i + h_2 \ \geq \ 22 \ .$$

It follows from Table 1 that the $n^{d,l}_{jk}$ are positive or small negative numbers in all the models. As a consequence, all the trilinear couplings are $\leq \lambda^{19} \sim 10^{-12}$ in this case. This value is too small to have any phenomenological consequence.

7.4.2 $l_i + h_2 < 0$

We shall first show that the most preferred model IA can be phenomenologically consistent in this case only when all λ'_{ijk} are zero and then generalize this result to other cases. The λ'_{ijk} are explicitly given as follows in this model:

$$\lambda'_{ijk} = \lambda^{l_i + h_2 + x} \begin{bmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{bmatrix}, \tag{7.23}$$

where it is implicit that some element is zero if corresponding exponent is negative [5]. The matrix in the above eq. (7.23) coincides with $\epsilon^{-x}\,(M_d)_{jk}$. Hence for negative l_i+h_2, it follows that the λ'_{ijk} is either larger than the matrix element $(M_d)_{jk}$ or is zero for every i. In the former case, one cannot easily meet the phenomenological requirement in eq.(7.22). Specifically, equation for the c_i gets translated to

$$\begin{aligned} c_i \equiv l_i + h_2 + x \quad &< \quad -3 \qquad OR \\ &\geq \quad 3. \end{aligned} \tag{7.24}$$

This condition ensures that $\lambda'_{i12}\lambda'_{i21}$ either satisfies eq.(7.22) (when $c_i > 3$) or is identically zero when $c_i < -3$. But $c_i \geq 3$ is untenable since $l_i + h_2 \leq 0$ and $\tan\beta \sim \lambda^x (m_t/m_b) \geq O(1)$ needs $x \leq 2$ leading to $c_i \leq 2$. As a result one must restrict c_i to less than -3 for *all* i. It can be easily seen that $c_i = -4$ is also ruled out. As follows from eq.(7.23), all the λ'_{ijk} except λ'_{i11} are zero in this case to start with. But the mixing of superfields in kinetic terms can regenerate other λ'_{ijk}. Specifically, one gets

$$\begin{aligned} \lambda'_{i12} &= V^D_{12}\lambda'_{i11} \sim \lambda \\ \lambda'_{i21} &= V^Q_{12}\lambda'_{i11} \sim \lambda \\ \lambda'_{i12}\lambda'_{i21} &\sim \lambda^2, \end{aligned} \tag{7.25}$$

where V^ψ rotates the matter field Ψ_i to bring kinetic terms to canonical form [9]

$$\begin{aligned} \Psi_i &\rightarrow V^\psi_{ij}\Psi_j \\ V^\psi_{ij} &\sim \left(\frac{<\theta>}{M}\right)^{|\psi_i-\psi_j|}. \end{aligned} \tag{7.26}$$

It follows from the above that one must require $c_i < -4$ for all i. One concludes from eq.(7.23) that only phenomenologically viable possibility in model IA is to require vanishing λ'_{ijk} for all values of i, j, k. We emphasize that a non-trivial role is played in the above argument by the requirement of zero or negative $l_i + h_2$ and by the value of h determined from the anomaly constraints.

The above argument also serves to restrict the trilinear couplings λ_{ijk}. Defining the antisymmetric matrices $(\Lambda_k)_{ij} \equiv \lambda_{ijk}$, one could rewrite the Λ_k as follows:

$$(\Lambda_1)_{ij} = \lambda^4 \begin{pmatrix} 0 & \lambda^{c_2} & \lambda^{c_3} \\ -\lambda^{c_2} & 0 & \lambda^{c_3+l_2-l_1} \\ -\lambda^{c_3} & -\lambda^{c_3+l_2-l_1} & 0 \end{pmatrix}$$

$$(\Lambda_2)_{ij} = \lambda^2 \begin{pmatrix} 0 & \lambda^{c_1} & \lambda^{c_3+l_1-l_2} \\ -\lambda^{c_1} & 0 & \lambda^{c_3} \\ -\lambda^{c_3+l_1-l_2} & -\lambda^{c_3} & 0 \end{pmatrix}$$

$$(\Lambda_3)_{ij} = \begin{pmatrix} 0 & \lambda^{c_2+l_1-l_3} & \lambda^{c_1} \\ -\lambda^{c_2+l_1-l_3} & 0 & \lambda^{c_2} \\ -\lambda^{c_1} & -\lambda^{c_2} & 0 \end{pmatrix}, \tag{7.27}$$

where c_i are the same coefficients defined in the context of the λ' and are required to be < -4 as argued above. It then immediately follows from Table 1 that all the λ_{ijk} except $\lambda_{123}, \lambda_{231}$ and λ_{312} are forced to be zero. Moreover, λ_{312} and λ_{231} cannot simultaneously be zero. Thus one reaches an important conclusion that Model IA can be consistent with phenomenology only if all λ'_{ijk} and all λ_{ijk} except at most two are zero. We have not made use of one of the anomaly equation namely, $A'_1 = 0$. Use of this does not allow even one λ_{ijk} to be non-zero in large number of models.

Essentially the same argument can be repeated also in case of other models. The structure of the λ'_{ijk} is determined in these models by

$$\lambda'_{ijk} \sim \lambda^{c_i+q_{j3}+d_{k3}}, \tag{7.28}$$

where $c_i \equiv l_i + h_2 + x - h$; The main difference compared to earlier model is that the h appearing in c_i is not forced to be zero but is given by eq.(7.18) and can take values -1 (Model IB, Model IIIA, Model IVB) or -2 (Model IIIB). The $h = 0$ for model IIB and the above argument made in the case of model IA also remains valid in this case. Because, $h \leq 0$ in these models, they allow somewhat larger values for c_i compared to $c_i \leq 2$ in case of model IA. These larger values of c_i result in extreme case corresponding to $l_i + h_2 = 0$ and $x = 2$. It is possible to satisfy constraint coming from Δm_K in these extreme cases e.g for model IB $l_i + h_2 = 0, x = 2$ leads [4] to

$$(\lambda'_i)_{jk} \approx \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^4 & \lambda^3 & \lambda^3 \end{pmatrix}. \tag{7.29}$$

This structure is consistent with eq.(7.22) as well as all other constraints on λ'_{ijk}. This possibility cannot be therefore ruled out purely on phenomenological grounds. But as we will show, $A'_1 = 0$ plays an important role and does not allow these marginal cases.

7.5 Models

Let us now discuss specific models which successfully meet all the phenomenological constraints. An important role is played in categorizing these models by the anomaly con-

[4]similar marginal cases are also found for models, IIIB,IVB.

straint $A_1' = 0$ which has been not yet imposed. Imposition of this further constraints the model.

It is possible to give a general solution of all the anomaly constraints for all the models listed in Table 1. We outline the solution for $A_1' = 0$ condition in the appendix. We have numerically looked for integer solutions of the anomaly constraints satisfying the criteria (1) $l_i + h_2 \leq 0$ (2) c_i are chosen to satisfy the constraint eq.(7.22) e.g. $c_i < -4$ in case of Model IA (3) The absolute values of $q_3, u_3, d_3, l_1, l_2, l_3$ are restricted to be less than or equal to 10. The last requirement is imposed for simplicity. Moreover in practice, higher values of these charges will generically result in suppressed R violating couplings which may not be of phenomenological interest. Although, all the $U(1)$ couplings can be specified using only four parameters, we have displayed values of x, q_3, u_3, d_3, l_i and $l_i + h_2$ in tables **2A - 2G** (all the tables are put after the appendix). We draw the following conclusions from the tables:

(1) None of the models displayed allow the value $l_i + h_2 = 0$ ruling out the marginal models displayed in eqs.(7.29) at least for the ranges of parameters considered here.

(2) While all the λ'_{ijk} are forced to be zero, some of the models allow one or two non-zero λ_{ijk}. We have shown this in the last column which also gives the order of magnitude for the allowed λ_{ijk}. This need not always be compatible with phenomenology particularly after taking care of the mixing of kinetic energy terms. Thus some of the models displayed in tables would not be allowed.

(3) Although the term $L_i H_2$ is not directly allowed, it can be generated from the Kahler potential through the mechanism proposed by GM [26] in order to explain the μ parameter. The order of magnitudes of the ϵ_i is given in this case by

$$\epsilon_i \sim m_{3/2} \lambda^{|l_i + h_2|}, \tag{7.30}$$

where $m_{3/2}$ is the gravitino mass. This can be read off from the table in all the cases. Uniformly large magnitudes of $l_i + h_2$ found in tables implies that the R violation through effective bilinear term is also quite suppressed but it can still be of phenomenological relevance.

(4) We did not impose baryon parity in the above analysis. The look at the solutions presented in the table however shows that the operator $U_i^c D_j^c D_k^c$ carries large negative charge in all the models. Thus baryon number violating terms are automatically forbidden from the superpotential. These terms will be generated from the effective $U(1)$ violating

D term

$$\frac{1}{M_P}\left(\frac{\theta^*}{M}\right)^{|q_{ijk}|}(U_i^c D_j^c D_k^c),$$

where q_{ijk} is the negative $U(1)$ charge of the combination $U_i^c D_j^c D_k^c$. This leads to baryon number violating couplings

$$\lambda''_{ijk} \sim \frac{m_{3/2}}{M_P}\lambda^{|q_{ijk}|},$$

which are extremely suppressed, $\leq O(10^{-15})$ for $m_{3/2} \sim$ TeV. Thus proton stability gets automatically explained in all the models.

(5) The trilinear lepton number violating terms are not allowed in the superpotential from analyticity. But they will be effectively generated in the same way as λ'' discussed above. Their magnitudes will also be enormously suppressed $\leq 10^{-15}$ depending upon the model.

It follows from the forgoing discussions that consistently implemented $U(1)$ symmetry allows very simple R violating interactions namely three bilinear terms and at most two trilinear coupling λ_{ijk}. The constraints coming from the $K^0 - \bar{K^0}$ mass difference were instrumental in arriving at this conclusion. It is worth emphasizing that the effective bilinear interactions generated from GM mechanism in this case are not subject to such stringent constraint from the flavour violating process. A priori, the bilinear terms can be rotated away in favour of trilinear λ' and λ interactions. It turns out that one does not generate dangerous flavour violating terms in the process. Specifically, one finds for the flavour structure [23],

$$W = -\frac{\tan\theta_3}{< H_1 >}[(O_L^T)_{3\alpha}L_\alpha]\left(m_\beta^l L_\beta e_\beta^c + m_i^D Q_i d_i^c\right). \tag{7.31}$$

where all the fields are in the physical i.e, the mass basis. (O_L^T) represents a mixing matrix determined solely by the ratios of ϵ_i and $\tan\theta_3 = \sqrt{(\sum_i \epsilon_i^2)}/\mu$ and α, β run over e, μ, τ. It is seen that the resulting trilinear interactions are flavour diagonal and thus the parameter ϵ_i are not severely constrained [5]. The major effect of the bilinear terms is to generate the neutrino masses and leptonic Kobayashi Maskawa matrix.

The neutrino masses in the presence of bilinear terms alone, have been discussed in many papers [22]. Large number of these concentrated on universal boundary conditions since they provide natural means to understand smallness of neutrino masses even when the bilinear parameters are not suppressed [22, 23]. The soft SUSY breaking terms are also subject to the $U(1)$ symmetry and need not follow the universal structure [24]. But the smallness of neutrino masses follows here from the $U(1)$ symmetry itself without invoking universal boundary conditions since the allowed values of $|l_i + h_2|$ in various

[5] The same conclusion was also drawn in ref. [8] by using different leptonic basis.

tables are large leading to suppressed $\frac{\epsilon}{\mu}$ and hence neutrino masses, eq.(7.13). The detailed structure of neutrino masses and mixing will be more model dependent here than in case of the universal boundary conditions. It seems possible to obtain reasonable mixing and masses in some of the models. As an example, consider model 2 in table **2 A**. This is characterized by three bilinear terms of equal magnitudes. Thus in the absence of any fine tuning one can expect to get large mixing angles naturally. The heaviest neutrino would have mass of the order

$$m_\nu \sim \lambda^{18} \frac{M_Z^2}{M_{SUSY}} \sim 10^{-1}\,\mathrm{eV},$$

which is in the right range for solving the atmospheric neutrino anomaly. The other mass gets generated radiatively through eq.(7.31) and would be suppressed compared to the above mass. The detailed predictions of the neutrino spectrum would depend upon the structures of soft symmetry breaking terms which themselves would be determined by the $U(1)$ symmetry. We shall not discuss it here.

7.6 Summary

The supersymmetric standard model allows 39 lepton number violating parameters which are not constrained theoretically. We have shown in this chapter that the $U(1)$ symmetry invoked to understand fermion masses can play an important role in constraining these parameters. We restricted ourselves to integer $U(1)$ charges and considered different $U(1)$ charge assignments compatible with fermion spectrum. We have shown that only phenomenologically consistent possibility in this context is that all the trilinear λ'_{ijk} and all but two λ_{ijk} couplings to be zero or extremely small of $O(10^{-15})$. While the patterns of R violation have been earlier discussed in the presence of $U(1)$ symmetry the systematic confrontation of these pattern with phenomenology leading to this important conclusion was not made to the best of our knowledge. In fact, some works [14] which neglected important constraint of $l_i + h_2 \leq 0$ concluded to the contrary that it is possible to obtain phenomenologically consistent and non-zero trilinear couplings.

Our work is restricted to only $U(1)$ symmetry which is by far most popular and to integer $U(1)$ charges. Use of other horizontal symmetries can allow non-zero trilinear interactions and still be consistent with phenomenology. An example of this can be found in [27]. Our work is closely related to and compliments the analysis presented in [12]. It was assumed in this paper that bilinear R violating interactions come from the GM mechanism and are absent in the superpotential. Assuming that there are no trilinear interactions in the superpotential it was shown that flavor violating transitions in the model are adequately suppressed. We have systematically shown that this is the only allowed possibility except for the occurrence of one or two trilinear λ_{ijk} couplings. This way, $U(1)$ symmetry is shown to require that only four or five of the total 39 lepton number violating couplings could have magnitudes in the phenomenologically interesting range!

7.7 Appendix

Here we give the most general solutions for the Green-Schwarz anomaly conditions in terms of the four independent charges. The constraints $A_3 = A_2$ and $A_3 = \frac{3}{5}\ A_1$ gave us eq.(7.18). The condition $A_1' = 0$ can be solved to give,

$$l_3 = A\ d_3 + B\ q_3 + C\ l_1 + D\ x + E, \tag{7.32}$$

where

$$\begin{aligned}
A &= \frac{-1}{k_2}\left(\sum_i (d_{i3} + 2u_{i3}) - h + k_1 + k_2 - m + 3x\right) \\
B &= \frac{-1}{k_2}\left(\sum_i (q_{i3} + 4u_{i3}) - 7h + k_1 + 10k_2 - m + 9x\right) \\
C &= \frac{-1}{k_2}\,(k_2 - k_1) \\
D &= \frac{-1}{k_2}\left(5h - 4\sum_i (u_{i3}) - 3(k_2 + x)\right) \\
E &= \left(\sum_i (d_{i3}^2 + q_{i3}^2 - 2u_{i3}^2 + k_i^2) - 5h^2 + 2k_2(4h - m)\right),
\end{aligned} \tag{7.33}$$

and

$$\begin{aligned}
k_1 &= l_{13} + e_{13} \\
k_2 &= l_{23} + e_{23}.
\end{aligned} \tag{7.34}$$

In the above we have taken q_3, d_3, l_1 and x as four independent parameters and l_3 has been expressed in terms of them. m and u_3 are respectively given by eqs.(7.19,7.20) of the text and remaining charges by the Table 1 defining the models. This way all the $U(1)$ charges get fixed in terms of $q_3, d_3, l_1$and x once a model displayed in the table is chosen.

Model IA

No.	**x**	$\mathbf{q_3}$	$\mathbf{u_3}$	$\mathbf{d_3}$	$\mathbf{l_1}$	$\mathbf{l_2}$	$\mathbf{l_3}$	$\mathbf{f_1}$	$\mathbf{f_2}$	$\mathbf{f_3}$	**If $\lambda_{\mathbf{ijk}}$ allowed**
1	0	2	1	-5	-6	-3	-6	-9	-6	-9	No
2	0	2	1	-5	-5	-5	-5	-8	-8	-8	No
3	0	2	1	-5	-4	-7	-4	-7	-10	-7	No
4	0	2	1	-5	-3	-9	-3	-6	-12	-6	$\lambda_{132} \sim 4.8 \times 10^{-2}$
5	0	2	2	-6	-10	-4	-1	-14	-8	-5	$\lambda_{231} \sim 5.1 \times 10^{-4}$
6	0	3	2	-8	-10	-4	-10	-15	-9	-15	No
7	0	3	2	-8	-9	-6	-9	-14	-11	-14	No
8	0	3	2	-8	-8	-8	-8	-13	-13	-13	No
9	0	3	2	-8	-7	-10	-7	-12	-15	-12	No
10	2	3	2	-6	-7	-3	-8	-12	-8	-13	No
11	2	3	2	-6	-6	-5	-7	-11	-10	-12	No
12	2	3	2	-6	-5	-7	-6	-10	-12	-11	No
13	2	3	2	-6	-4	-9	-5	-9	-14	-10	No
14	2	4	3	-9	-9	-8	-10	-16	-15	-17	No
15	2	4	3	-9	-8	-10	-9	-15	-17	-16	No

Table 2A: Here we display the allowed models where the following constraints have been imposed : a) requirement of correct quark and lepton mass hierarchies as per Model IA in table I b) GS anomaly cancellations c) $f_i = l_i + h_2 \leq 0$ d) phenomenological constraints from $K^0 - \bar{K}^0$ mixing on λ'_{ijk} couplings and (e) $|q_3, u_3, d_3, l_i| \leq 10$.

Model IB

No.	x	q_3	u_3	d_3	l_1	l_2	l_3	f_1	f_2	f_3	If λ_{ijk} allowed
1	0	2	2	-5	-6	-3	-2	-10	-7	-6	$\lambda_{131} \sim 1.0, \lambda_{231} \sim 10^{-2}$
2	0	3	2	-7	-4	-6	-10	-9	-11	-15	No
3	1	3	2	-6	-3	-5	-9	-8	-10	-14	No
4	0	3	3	-8	-10	-1	-9	-16	-7	-15	$\lambda_{231} \sim 1.0$
5	0	3	3	-8	-8	-6	-6	-14	-12	-12	No
6	1	3	3	-7	-8	-4	-5	-14	-10	-11	$\lambda_{231} \sim 1.0$
7	1	3	3	-7	-6	-9	-2	-12	-15	-8	No
8	2	3	3	-6	-8	-2	-4	-14	-8	-10	$\lambda_{121} \sim 1.0, \lambda_{231} \sim 2.3 \times 10^{-3}$
9	1	4	4	-10	-10	-7	-9	-18	-15	-17	No
10	2	4	4	-9	-10	-5	-8	-18	-13	-16	No
11	2	4	4	-9	-8	-10	-5	-16	-18	-13	No

Table 2B: Same as above, but for values given by Model IB.

Model IIB

No.	x	q_3	u_3	d_3	l_1	l_2	l_3	f_1	f_2	f_3	If λ_{ijk} allowed
1	0	2	2	-6	-3	-8	-9	-7	-12	-13	No
2	0	2	3	-7	-8	-5	-7	-13	-10	-12	No
3	0	2	3	-7	-6	-10	-4	-11	-15	-9	No
4	1	2	3	-6	-8	-2	-7	-13	-7	-12	$\lambda_{231} \sim 1.0$
5	1	2	3	-6	-6	-7	-4	-11	-12	-9	No
6	2	2	3	-5	-6	-4	-4	-11	-9	-9	$\lambda_{231} \sim 1.0$
7	1	3	4	-9	-9	-10	-7	-16	-17	-14	No
8	2	3	4	-8	-9	-7	-7	-16	-14	-14	No

Table 2C: Same as above, but for values given by Model IIB.

Model IIIA

No.	x	q_3	u_3	d_3	l_1	l_2	l_3	f_1	f_2	f_3	If λ_{ijk} allowed
1	0	2	3	-6	-7	-2	-9	-12	-7	-14	No
2	0	2	3	-6	-6	-4	-8	-11	-9	-13	No
3	0	2	3	-6	-5	-6	-7	-10	-11	-12	No
4	0	2	3	-6	-4	-8	-6	-9	-13	-11	No
5	0	2	3	-6	-3	-10	-5	-8	-15	-10	$\lambda_{132} \sim 1.0$
6	1	2	3	-5	-6	-2	-7	-11	-7	-12	No
7	1	2	3	-5	-5	-4	-6	-10	-9	-11	No
8	1	2	3	-5	-4	-6	-5	-9	-11	-10	No
9	1	2	3	-5	-3	-8	-4	-18	-13	-9	$\lambda_{132} \sim 1.0$
10	1	2	3	-5	-2	-10	-3	-7	-15	-8	$\lambda_{132} \sim 2.3 \times 10^{-3}$
11	2	2	3	-4	-4	-4	-4	-9	-9	-9	No
12	2	2	3	-4	-3	-6	-3	-8	-11	-8	$\lambda_{132} \sim 1.0$

Table 2D: Same as above, but for values given by Model IIIA.

Model IIIB

No.	x	q_3	u_3	d_3	l_1	l_2	l_3	f_1	f_2	f_3	If λ_{ijk} allowed
1	0	2	3	-5	-2	-5	-7	-7	-10	-12	No
2	0	2	4	-6	-7	-3	-4	-13	-9	-10	$\lambda_{231} \sim 0.22$
3	0	2	4	-6	-5	-8	-1	-11	-14	-7	$\lambda_{131} \sim 1.0, \lambda_{132} \sim 1.0$
4	2	3	4	-6	-3	-4	-10	-10	-11	-17	$\lambda_{123} \sim 1.0$
5	0	3	5	-9	-8	-6	-9	-16	-14	-17	No
6	1	3	5	-8	-9	-3	-8	-17	-11	-16	No
7	1	3	5	-8	-7	-8	-5	-15	-16	-13	No
8	2	3	5	-7	-8	-5	-4	-16	-13	-12	$\lambda_{231} \sim 1.0$
9	2	3	5	-7	-6	-10	-1	-14	-18	-9	$\lambda_{131} \sim 1.0, \lambda_{132} \sim 0.22$
10	2	4	6	-10	-9	-8	-9	-19	-18	-19	No

Table 2E: Same as above, but for values given by Model IIIB.

Model IVA

No.	x	q_3	u_3	d_3	l_1	l_2	l_3	f_1	f_2	f_3	If λ_{ijk} allowed
1	0	6	-3	-9	-10	-4	-7	-13	-7	-10	$\lambda_{231} \sim 1.0$
2	0	6	-3	-9	-9	-6	-6	-12	-9	-9	No
3	0	6	-3	-9	-8	-8	-5	-11	-11	-8	No
4	0	6	-3	-9	-7	-10	-4	-10	-13	-7	No
5	2	7	-2	-10	-8	-6	-10	-13	-11	-15	No
6	2	7	-2	-10	-7	-8	-9	-12	-13	-14	No
7	2	7	-2	-10	-6	-10	-8	-11	-15	-13	No

Table 2F: Same as above, but for values given by Model IVA.

Model IVB

No.	x	q_3	u_3	d_3	l_1	l_2	l_3	f_1	f_2	f_3	If λ_{ijk} allowed
1	0	6	-2	-9	-8	-5	-4	-12	-9	-8	$\lambda_{231} \sim 0.22$
2	2	7	-1	-10	-8	-5	-7	-14	-11	-13	No
3	2	7	-1	-10	-6	-10	-4	-12	-16	-10	No

Table 2G: Same as above, but for values given by Model IVB.

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Chapter 8

Epilogue

SM has successfully passed the test of precision measurements of physical parameters as its predictions match with the experimental measurements with amazing accuracy. As of now there are no indications from experiments that point towards fundamental deficiency of SM. Exception must be made here as regards recent experimental evidence for neutrino oscillations which point towards massive neutrinos and hence the need to modify/extend SM. We briefly discussed, the theoretical issues connected with the massive neutrinos in chapter 3. In this thesis we mainly concentrated on neutrino mass models in the framework of R parity violating supersymmetry. It was argued that such a model naturally accommodates the hierarchical mass spectrum and one or two large mixing angles.

Supersymmetric model with bilinear R parity violations provides a potentially interesting framework to study neutrino masses and mixing. The dominant sources of neutrino masses can be parameterized in this scenario in terms of three dimensionful parameters ϵ_i and three dimensional parameters k_i which depend on the structure of soft supersymmetry breaking terms at the weak scale. We have tried to obtain phenomenological restrictions on ϵ_i and k_i without making specific assumptions on the values of the soft supersymmetry breaking parameters. While neutrino masses can be suppressed by lowering the overall scale ϵ_i of R parity violation, phenomenologically preferred hierarchy in neutrino masses require that both ϵ_i and k_i are suppressed. k_i provide a measure of the Higgs-slepton universality and suppression in their values indicate very small amount of this violation. Such violation of universality is already built in the popular models of SUSY breaking namely, mSUGRA and GMSB scenario.

A large solar neutrino mixing angle can be obtained consistently within these scenarios only if flavour universality violations in the soft parameters of the first two generations are almost as large as the violation of Higgs-slepton universality. This feature does not emerge in models where these universality violations are generated solely by RG scaling as in the case of mSUGRA. Thus mSUGRA seems more suitable to describe the less preferred small mixing angle solution to the solar neutrino problem.

In another investigation we discussed the structure of neutrino masses and mixing in the Minimal Messenger Model (MMM) of gauge mediated supersymmetry breaking with purely trilinear R violating interactions. We considered two specific cases of purely λ' interactions and purely λ interactions for simplicity. The model contains very large number of parameters even under this simplifying assumptions. Remarkably, it is possible to make meaningful statement on the neutrino spectrum in spite of the presence of many unknown parameters if all these parameters are assumed similar in magnitude. This is a natural assumption in the absence of any specific symmetry to restrict the trilinear R parity violation.

In the case where only λ' couplings are present, one naturally gets large mixing between the neutrino states. Further, the MMM offers a very constrained structure giving rise to a large hierarchy between the masses $\sim O(10^{-2})$ for all the parameter space. The model is suitable for obtaining simultaneously solutions for atmospheric neutrino problem and quasi-vacuum oscillations for solar neutrino problem.

Assumption of approximate equality of λ couplings in case with only λ couplings, leads to very constrained and phenomenologically inconsistent pattern for neutrino mixing. This conclusion follows on general grounds and it is true even if SUSY breaking is induced by supergravity interactions. It is quite interesting that one can arrive at this strong conclusions in spite of the presence of many unknown parameters by simply assuming them to be of similar magnitude. One can obtain consistent picture of neutrino anomalies if λ couplings are assumed to be hierarchical. We provided an example which leads to two large and one small mixing and correct hierarchy between the solar and atmospheric neutrino scales.

Though R violating SUSY is an attractive model to understand neutrino masses and mixing, the magnitude of these couplings, apart from phenomenological constraints, is *apriori* arbitrary. It would be indeed interesting if these couplings follow from some symmetry principle. In chapter 7 we discussed in detail, that the $U(1)$ family symmetry invoked to understand fermion masses can play an important role in constraining these parameters. We restricted ourselves to integer $U(1)$ charges and considered different $U(1)$ charge assignments compatible with fermion spectrum. We have shown that only phenomenologically consistent possibility in this context is that all the trilinear λ'_{ijk} and all but two λ_{ijk} couplings to be zero or extremely small of $O(10^{-15})$. The problem of generation of phenemenologically inconsistent bilinear terms ϵ_i in these models was pointed out. Requiring that ϵ_i are suppressed, played an important role in obtaining the constrained pattern of R violation. While the patterns of R violation have been earlier discussed in the presence of $U(1)$ symmetry the systematic confrontation of these pattern with phenomenology leading to this important conclusion was not made.