Aspects of Quantum Field Theory in Phenomenology of Early Universe

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To

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Jethubaba

CERTIFICATE

I feel great pleasure in certifying that the thesis entitled, ***"Aspects of Quantum Field Theory in Phenomenology of Early Universe"*** *embodies a record of the results of investigations carried out by Mrs. Suratna Das under my guidance.*

She has completed the following requirements as per Ph.D. regulations of the University.

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DECLARATION

I, ***Mrs. Suratna Das****, D/O Mr. Saroj Kumar Das, resident of D-108, PRL residences, Navrangpura, Ahmedabad, 380009, hereby declare that the work incorporated in the present thesis entitled,* ***"Aspects of Quantum Field Theory in Phenomenology of Early Universe"*** *is my own and original. This work (in part or in full) has not been submitted to any University for the award of a Degree or a Diploma.*

Date : May 25, 2011

(Suratna Das)

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Abstract

In this thesis, our main focus is on applications of various Quantum Field Theoretic (QFT) treatments in analyzing early Universe phenomena primarily related to the inflationary paradigm [1].

It is shown that non-perturbative QFT technique for calculating two-point correlation in flat space, known as Källén-Lehmann spectral representation, can be used to calculate the power spectrum of interacting scalar fields where the interactions are short-ranged. Decaying inflaton and composite inflaton are two such cases where our method of calculating power spectrum can be applied. Decaying inflaton suppresses the long-distance correlation while the composite inflaton yields some oscillatory features in the low l region of the TT spectrum of CMBR, which may be observed by WMAP or in the future observations with PLANCK.

We investigate whether an exotic quantum field, named the *unparticle*, can play the role of an inflaton and drive inflation. Such exotic fields yields long-range forces due to its anomalous dimension and such anomalous dimension of tensor and vector unparticle is constrained from Mercury's perihelion precession data. Signature of a scalar unparticle inflaton is the suppression of low l modes in the anisotropy spectrum in the CMBR which can be observed by WMAP or PLANCK.

Effects of pre-inflationary radiation era on the primordial non-Gaussianity is also studied using Thermal Field Theory techniques. The bispectrum contribution is enhanced by a factor of 65-90 from that of single-field slow-roll inflationary model. Thermal averaging yields trispectrum non-Gaussianity which does not depend up on the slow-roll parameters and thus can be as large as -42. Signature of such a pre-inflationary radiation era is a large trispectrum non-Gaussianity compared to the bispectrum non-Gaussianity.

[1]**keywords** : Inflation, CMBR anisotropies, Power spectrum, Källén-Lehmann spectral representation, Unparticle, Primordial non-Gaussianity, Thermal Field Theory

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Chapter 1

Introduction

1.1 Macrocosm in the microcosm

Cosmic Microwave Background Radiation (CMBR) representing 93 % of the extragalactic emission of our Universe (while the infrared and visible spectrum contributing up to 5 % and 2 % respectively) [1] has become an important field of research both theoretically and observationally since the last half of a century and discovery of which (by Penzias and Wilson in 1965) establishes the Big Bang Theory as a "Standard Model" of the evolution of our Universe. The presently measured value of this highly uniform temperature of CMBR is 2.725 ± 0.001 K [2] and this electromagnetic spectrum of the Universe is the best-fitted black body spectrum ever found in nature, providing the evidence that matter and radiation were in perfect thermal equilibrium before CMBR formed. In 1992 NASA led COBE (COsmic Background Explorer) mission (which probed the angular power spectrum up to the multipole $l \simeq 26$) clearly detected tiny anisotropies (1 part in 10^5) in the highly uniform temperature distribution of CMBR throughout the sky [3]. WMAP [4], having an angular resolution of $0.23^{\,0}$, probed the TT (Temperature-Temperature) correlation of CMBR up to $l \simeq 783$ and measured these anisotropies more accurately. Besides these two satellite experiments, the balloon borne experiments like Boomerang and Maxima [5] and ground based interferometer like DASI also measure the anisotropy spectrum of CMBR up to $l \simeq 1000$ and provide convincing results of the existence and location of the first peak of the anisotropy spectrum. A new satellite mission PLANCK will achieve an angular resolution of $5'$ in order to probe multipoles up to $l \sim 2160$ and will open new vistas to portray the macrocosm more distinctly. The set of parameters primarily obtained from CMBR measurements like $\Omega_b h^2$ (the present baryon energy density), $\Omega_c h^2$ (the present cold dark matter energy density), Ω_Λ (the present dark energy density), the optical length at reionization, $\Delta_{\mathcal{R}}(k_0)$ (the scale invariant amplitude of curvature perturbations at pivot scale $k_0 = 0.002$ Mpc^{-1}) and n_s (the spectral index of the primordial scalar fluctuations) characterizes our present knowledge about our Universe. In addition to these, other parameters like density of all neutrino species Ω_ν, running of spectral index $\frac{dn_s}{dk}$, the tensor-to-scalar ratio r, the amplitude and spectral index of the isocurvature perturbations, the TE, EE and BB (an important probe of primordial gravitational waves which is yet to be observed) spectrums and amplitudes of the primordial magnetic field, non-linearity parameter f_{NL} and τ_{NL} for non-Gaussianity in CMBR etc. are also extracted from the observational data of CMBR enabling us to predict more precisely about the primordial features of our Universe.

Though the arena of Cosmology is vast enough and mostly the late time dynamics of the Universe is governed by the principles of General Relativity and Newtonian dynamics (treating gravity classically), the early Universe phenomena demand the necessity of application of Quantum Field Theory (QFT) techniques (whose tremendous success comes from collider physics where the predictions of Standard Model of Particle Physics are being tested) in order to achieve a proper evolutionary history of our Universe. Besides CMBR, Big Bang Nucleosynthesis (BBN), due to its precise measurements in recent days, has become another cornerstone of the Big Bang Theory. BBN occurred when the temperature of the Universe was of order 1 MeV (and the age of the Universe was nearly 3 minutes). Due to such high temperature, BBN is such a scenario where the physics of cosmology and particle physics are inter-connected. To calculate the relic abundances of the primordial light nuclei, like He^4, He^3 or H^3, one uses the QFT techniques of calculating decay rates and cross-sections of neutron-proton interactions in the same spirit as one does to analyze the collider phenomena. One can use such techniques of QFT even in curved spacetime as the rates of interactions being much higher than the Hubble expansion rate, these interactions are essentially short-ranged and at small length scales one can ignore the effect of gravity. In a similar spirit several QFT techniques are being used to determine several symmetry breaking phase-transition epochs of the Universe such as QCD (Quantum Chromo Dynamics) phase transition ($\sim 10^{-6}$ sec) or electroweak phase transition ($\sim 10^{-12}$ sec) while treating gravity classically. Moving further beyond in time QFT techniques are also being used to analyze inflationary scenario ($\sim 10^{-35}$ sec) treating gravity semi-classically. Beyond that ($\sim 10^{-42}$ sec) Quantum Gravity effects start dominating the dynamics of the Universe and lack of proper knowledge of Quantum Gravity forbids us to analyze such early time phenomena.

1.2 QFT in Inflationary paradigm: Motivation for the thesis

As by now the cosmological inflationary paradigm has become 30 years old, we will start this section with a brief history of the development of the inflationary paradigm along with the important role played by QFT in its development. Though the discovery of CMBR was one of the cornerstone of Big Bang cosmology and it was quite successful in describing our late-time Universe, it was realized in 1970's that the Big Bang Theory was in grave peril because of several problems arising due to incompatibility with Particle Physics theory such as monopole problem and gravitino problem along with severe fine-tuning problem of initial conditions like horizon problem and flatness problem yielding the existence of our Universe highly improbable. All these problems were simultaneously solved by incorporating the inflationary scenario in the old Big Bang Theory. QFT plays a major role in development of inflationary models as the success of inflationary theory lies in the realization that the energy density of a scalar field can play the role of the vacuum energy which is changing during the cosmological phase transitions.

The first concept of inflation was suggested by A. Starobinsky in [6] which has a problem of having no inflation if the Universe was hot from its very beginning, quite contrary to the hot Big Bang Theory. But Zeldovich in [7] showed that such inflationary scenario could have been created "from nothing", which was not much appreciated by cosmologists of that time (today this idea is very popular and widely accepted in inflationary cosmology and has been named as "supercooled inflation"). Alan Guth in 1981 [8] provided a simple inflationary model, now known as the "old inflation", explaining how inflation can solve the major problems arising in the Big Bang Theory. Though the old inflationary scenario played an important role in the development of modern cosmology, it had major problems related to the rate of bubble formations during inflation [8] and "new inflation" theory was proposed by Linde [9] soon after that to tackle such difficulties. Zeldovich's idea [7] of refuting the assumption of the hot origin of our Universe came back in literature in 1983 in "chaotic inflationary scenario" [10] where the quantum origin of the inflationary Universe was suggested and various initial distributions of the inflaton field, in cases where inflation may occur, were analyzed. Following the same trend of chaotic inflation several other inflationary scenarios emerged in the literature such as power-law inflation [11], extended inflation [12], natural inflation [13], hybrid inflation [14] and many others (ordered chronologically).

It was soon realized after Guth's proposal of inflationary scenario [8], that an additional advantage of having modified Big Bang Cosmology with the inclusion of inflationary scenario is that inflation can generate quantum fluctuations in the early Universe which can be stretched to the astronomical scales providing the seeds for the large scale structures of the Universe [15]. The large scale anisotropy at the Last Scattering Surface (LSS) predicted by slow-roll inflationary model can be described as [16]

$$\left(\Theta_0 + \Psi\right)(k, \tau_*) = -\frac{1}{6}\delta(\tau_*), \tag{1.1}$$

where Θ_0 is the monopole of the Fourier transform of the observed temperature anisotropy $\frac{\delta T}{T}$ at the LSS, Ψ is the perturbation in time component of the background de Sitter metric and δ is the fractional overdensity (in matter or dark matter) at LSS. This equation relates the anisotropy in the temperature to the overdensity in dark matter and shows that an anisotropy in temperature $\left(\frac{\delta T}{T}\right)$ of order 10^{-5} corresponds to an overdensity of 6×10^{-5}. Besides inflation, other models of structure formation predict a coefficient of order unity [16], rather than the above mentioned factor of 6, resulting in a Universe too under-densed to account for the clustering of matter. Linde in his lecture notes on inflationary cosmology [17] has thus quoted correctly about inflation as *"It has broken an umbilical cord connecting it with the old big bang theory, and acquired an independent life of its own"*.

According to inflationary paradigm the primordial fluctuations, generated during inflation, become "frozen" once stretched out of horizon i.e. these superhorizon modes stop evolving along with the evolution of the Universe. The evolution of these modes starts once again when they re-enter the horizon at some later time either during radiation or matter dominated era. This particular nature of primordial fluctuations, of being frozen while being superhorizon, enables us to probe various early Universe phenomena by measuring and properly quantifying such primordial fluctuations. Thus the dynamics of these primordial fluctuations plays a major role in analyzing early Universe phenomena and it is very significant to further probe their possible nature and origin.

The primordial fluctuations, such as the quantum fluctuations of the inflaton field $\delta\phi$, are typically quantified by their power spectrum $\mathcal{P}_{\delta\phi}$ which is the Fourier transform of the two-point correlation function :

$$\langle \delta\phi(\mathbf{k}_1, t)\delta\phi(\mathbf{k}_2, t)\rangle \equiv \frac{2\pi^2}{k^3}\mathcal{P}_{\delta\phi}(k_1)\delta^3(\mathbf{k}_1 + \mathbf{k}_2). \tag{1.2}$$

In Quantum Field Theory language the power spectrum thus defined is nothing but the equal-time Wightman function. The two-point function (time-ordered) of quantum fields is very significant in QFT as they determine the propagators of the fields which gives the probability amplitude for a particle to travel from one place to another and the bosonic propagators mediate force between particles according to the perturbative QFT. In QFT interactions between particles reflect in the potential term of the Lagrangian defining the system. Another way of calculating propagators of interacting fields in QFT is by Källén-Lehmann spectral representation [18, 19] which is a non-perturbative way of calculating propagators and all the interactions of an quantum field is encapsulated in its spectral function $\rho(\sigma^2)$. According to this method the propagator of an interacting field $G^{(\mathrm{int})}(p)$ can be expressed in terms of the free field propagator $G^0(p, \sigma^2)$ and the spectral function $\rho(\sigma^2)$ as

$$G^{(\text{int})}(p) = \int_0^\infty d\sigma^2 \rho(\sigma^2) G^0(p, \sigma^2). \tag{1.3}$$

In inflationary theories, incorporation of interactions of the inflaton field is generally done by modifying inflaton's potential and then determining its effects in the inflaton's power spectrum as similarly done in QFT. Many such possible Particle Physics motivated potentials of the inflaton field have been studied in the literature [20]. But this usual method of incorporating inflaton's interactions by modifying its potential breaks down in several situations, mainly when the inflaton has short-ranged interactions which have no influence in the inflaton's potential. Inflaton's decay width which is smaller than the Hubble expansion rate during inflation or a inflaton which is a condensate of fundamental fermionic fields and whose compositeness scale is smaller than the scale of scalar fluctuations : are a few such scenarios where the inflaton's potential does not reflect such short-ranged interactions like decay width or compositeness of the inflaton field. From our knowledge of QFT, one thus may investigate whether a similar treatment of Källén-Lehmann spectral representation of QFT can be used in a curved background to eventually calculate the power spectrum of interacting scalar fields as

$$P^{(\text{int})}(k) = \int_0^\infty P^{(0)}(k, \sigma^2) \rho\left(\sigma^2\right) d\sigma^2. \tag{1.4}$$

So far we have discussed inflationary scenarios where inflation is driven by slow-roll of a scalar field called inflaton. But, as a fundamental scalar remained unobserved in nature, cosmologists (like particle physicists) look for other scenarios where other quantum fields like vectors fields [21], *classical* and homogeneous spinor fields [22] or condensate of spinor fields [23, 24] plays a roll of inflaton. In 2006 Georgi proposed a new kind of quantum field whose canonical dimension is not like any known particle physics fields [25]. Georgi named such particles as *unparticles*. Due to its anomalous dimension, the unparticle can yield long range forces while exchanged between two systems. Unparticles of tensorial nature can couple to energy-momentum tensor of a system and thus can mimic gravity [26]. But the anomalous dimension of tensor unparticle generates a force deviated from Newtonian inverse square law force. Such a force, which deviates from usual inverse square law force, can result into perihelion precession of planetary orbits. As

Mercury's perihelion precession is very precisely measured, one can constrain unparticle's anomalous dimension if exchange of tensor unparticles results into change in the planetary orbits. On the other hand, as the vector unparticles can couple to baryoinc matter of the planets and the Sun, anomalous dimension of such vector unparticles can also be constrained by looking at Mercury's perihelion precession. Apart from this, as the spectral function of scalar unparticle is known in literature [25], one can vindicate the possibility of having scalar unparticle as inflaton if the Källén-Lehmann spectral representation can be used to determine the nature of power spectrum for interacting scalar fields.

Inflationary paradigm predicts a nearly scale invariant power spectrum with a Gaussian distribution of the primordial fluctuations which are in very good agreement with the present measurement of CMBR data [2]. Power spectrum is a powerful tool to analyze the evolution of the quantum field(s) present during the inflationary era. But as almost all the theoretical inflationary scenarios predict similar power spectrums (nearly scale invariant), it does not play a convincing role in distinguishing interactions of the field(s) present during inflation. To distinctly quantified such interactions one has to look for non-vanishing higher-order correlation functions which indicates a departure from pure Gaussian distribution of primordial fluctuations. The non-linear evolution of primordial perturbations, such as comoving curvature perturbation $\mathcal{R}$, gives rise to non-Gaussian features in the pleasingly simple model of inflation, i.e. the single-field slow roll inflationary model, of the order of slow-roll parameters ϵ. Inflationary scenarios with presence of more than one scalar field during inflation (such as curvaton model [27], multi-field inflationary model) relaxes the condition of slow-rolling of the scalar field responsible for generating curvature perturbations and thus can lead to large primordial non-Gaussianities [28]. Inflationary scenarios with higher derivative interactions of the inflaton field can also give rise to large primordial non-Gaussianity [29]. In inflationary theories, the preferred initial vacuum chosen for the inflaton field is the Bunch-Davies vacuum. It was shown by Gangui et al. in [30] that departure from such an assumption about the inflaton field may lead to large non-Gaussianities in single-field slow-roll inflationary model. One such scenario occurs when the inflation is preceded by a radiation era [31]. Inflation takes place when the energy density of radiation ρ_r drops below the value of the potential of a coherent scalar field. Thermal Field Theory plays a major role in analyzing such a scenario as the inflaton field has an initial thermal distribution which will affect its statistical properties like power spectrum and higher-order correlation functions. One can thus investigate, following Gangui's argument [30], whether such a scenario, where the initial vacuum is a non-Bunch-Davies vacuum for the inflaton, can give rise to larger primordial non-Gaussianities and whether such non-Gaussianities carry any signature of such a pre-inflationary radiation era.

With this introduction we lay out a plot where several QFT techniques can be used to investigate several interesting features of the early Universe such as

- Whether Källén-Lehmann spectral representation of QFT can be used in determining the power spectrum of interacting inflaton and what the imprints of such short-ranged forces (such as inflaton's decay width or the compositeness of the inflaton field) will be on the TT anisotropy spectrum of the CMBR.
- Whether exotic quantum fields like scalar unparticle can play a role of an inflaton and if so what possible signatures it may carry in the CMBR observations.
- Whether a pre-inflationary radiation era, which results in departure from the initial Bunch-Davies vacuum, can give rise to larger non-Gaussianities which can be detected by future experiments and whether such non-Gaussianities will carry any imprints of such a pre-inflationary radiation era so that one can distinguish it from the supercooled inflationary scenario.

1.3 Notations and conventions

All along this thesis we shall use the metric signature $(+ - - -)$. The Greek indices μ, ν will take values 0, 1, 2, 3 whereas the Latin indices i, j will take only the spatial values 1, 2, 3. Also the natural system of units will be adopted throughout this thesis i.e. $\hbar = c = \kappa_B = 1$.

1.4 Scheme of the thesis

We have organized the rest of the thesis as follows :

Chapter 2 focuses on the detailed analysis of several dynamical features of the single-field slow-roll inflationary model where the background metric is quasi-de Sitter. After motivating the necessity of having inflationary era in Big Bang Theory, as inflation solves the severe problems of fine-tuning initial conditions in Big Bang Cosmology, we focus on the issues of the dynamical features of the quantum fluctuations in the inflaton field and how these quantum fluctuations generate perturbations in the matter and radiation after inflation ends. To analyze this, we first discuss the cosmological perturbation theory and deal with the issue of choosing a proper gauge for further calculations. Gauge invariant quantities are also constructed. The solution of the inflaton mode functions (both subhorizon and superhorizon) and the issue of preferred initial vacuum for the inflaton fluctuations are consecutively discussed. The statistical properties of inflaton fluctuations like power spectrum (two-point correlation function) and higher-point correlation functions like bispectrum and trispectrum are also analyzed. Brief discussions on recent and future observation and the current status of the single-field inflationary model have been focused.

In Chapter 3, we formulate a general method of calculating power spectrum for interacting scalar field (where the interactions are being short-ranged) using the method of Källén-Lehmann spectral representation (non-perturbative QFT techniques) and show that the power spectrum for the interacting inflaton field can be expressed in terms of free field power spectrum and the spectral function of the interacting field encapsulating all the features of interactions. We analyze two cases where such a method is applicable : i) inflaton with a decay width and ii) composite inflaton. In both the cases we discuss the imprints of such short-ranged interactions on the TT anisotropy spectrum of CMBR measured by WMAP.

Chapter 4 deals with the exotic properties of an unusual quantum field called the *unparticles*. Firstly, we calculate the perihelion precession of Mercury's orbit due to the exchange of tensor and vector unparticles and thus constrain the anomalous dimension of the tensor and vector unparticle. As the spectral function of the unparticle is known in the literature, we thus vindicate the scenario where scalar unparticle can play the role of an inflaton using the method we formulate in Chapter 3 to calculate the power spectrum of interacting scalar field with the help of Källén-Lehmann spectral function. We also analyze the possible signature of such a scenario in the observed CMBR data.

Chapter 5 deals not with the power spectrum of the inflaton fields, like the previous two chapters, but the higher-order correlation functions of that such as bispectrum and trispectrum. We analyze an inflationary scenario where there is a pre-inflationary radiation era. Due to this radiation era prior to inflation the initial inflaton fluctuations acquires a thermal distribution and applying Thermal Field theory techniques we determine the non-Gaussian features (arising from higher-point correlation functions) in such a scenario and find that the trispectrum non-Gaussianity (arising from four-point correlation function) is larger than bispectrum (three-point correlation function) non-Gaussianity. In a generic single-field slow-roll model the bispectrum contribution is larger by a couple of order of magnitude than that of the trispectrum. One can thus infer that the signature of a pre-inflationary radiation era is a larger trispectrum non-Gaussianity than the bispectrum one.

Chapter 6 concludes by discussing the main results obtained in the thesis. This chapter is followed by several appendices where some of the essential concepts and calculations, required for the clarification of several ideas used in the main chapters, are discussed.

Chapter 2

General framework of Inflation and its $n-$point functions

The highly uniform temperature ($\sim$ 2.73 K) of CMBR in all directions over the sky, having fluctuations only one part in 10^5, indicates that our Universe is homogeneous and isotropic over its large scales. *Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology*, keeping in mind the underlying theory of Big Bang, is quite a successful framework in describing different phases of evolution of our homogeneous and isotropic Universe. The essential features of FLRW cosmology have been briefly discussed in Appendix (A). In spite of its tremendous success as a theory of evolution of our Universe, the Big Bang theory suffers from severe fine-tuning of initial conditions, making our Universe an 'improbable accident'. This problem of fine-tuned initial conditions, which initially plagued the Big Bang theory, can be seen from different aspects. Some of them are :

- **Horizon problem**

  The particle horizon at time t, within which the Universe is causally connected, is defined as

$$R_H(t) \equiv a(t) \int_0^t d\tau = a(t) \int_0^a d\ln a (aH)^{-1}, \tag{2.1}$$

  where τ is the conformal time defined in Eq. (A.7). It can be seen from the above equation that the conformal time τ can also be interpreted as the comoving particle horizon. The comoving Hubble radius $(aH)^{-1}$ of a

Universe dominated by a fluid with equation of state ω can be expressed as $H_0^{-1} a^{\frac{1}{2}(1+3\omega)}$ using Eq. (A.4) and Eq. (A.10). Here H_0 represents the present Hubble parameter. During conventional Big Bang evolution the Universe is dominated by fluids with $\omega > 0$ (such as radiation or dust), showing that the comoving particle horizon τ monotonically increases with time or the scale factor. But any comoving length scale, such as wavelength of perturbations, is independent of time. This implies that any comoving scale, which is now entering the horizon, was far outside the horizon at CMBR formation. But the uniform temperature of CMBR indicates that the regions on the last scattering surface (LSS) should have a priori been in causal contact. This is known as the *Horizon problem* of the Big Bang theory.

- **Flatness problem**

The curvature density $\Omega_k \equiv \Omega - 1 = \frac{k}{(aH)^2}$ (also defined in Eq. (A.5)) varies with the comoving Hubble radius $(aH)^{-1}$. As discussed in the context of Horizon problem, the comoving Hubble radius grows with time, indicating the diverging nature of curvature density with time. Observations show that $-0.0178 < \Omega_k < 0.0063$ for WMAP+BAO+SN$^{\rm a}$ and $-0.0133 < \Omega_k < 0.0084$ for WMAP+BAO+$\rm H_0$ [2]. Thus to have $\Omega_k \sim 0$ today, consistent with observations, one has to fine-tune Ω to extremely close to 1 during the very early Universe. The deviation from flatness during Big Bang nucleosynthesis, in the GUT era and at the Planck scale are respectively [32]

$$\begin{aligned} |\Omega_{\rm BBN} - 1| &\leq \mathcal{O}(10^{-16}) \\ |\Omega_{\rm GUT} - 1| &\leq \mathcal{O}(10^{-55}) \\ |\Omega_{\rm Pl} - 1| &\leq \mathcal{O}(10^{-61}). \end{aligned}$$

This extreme fine-tuning of the curvature density at early Universe, which is required in the Big Bang theory to explain the present observations, is known as the *Flatness problem.*

2.1 Inflation to the rescue of Big Bang Theory

Inflation [8] is one theory which potentially explains these extremely fine-tuned initial conditions of our Universe predicted by Big Bang theory. In this thesis we will restrict ourselves to the pleasingly simple **Single-field slow-roll model** of inflation. The dynamical features of this model has been briefly discussed in Appendix (B). According to this model, one canonical single scalar field, having a state parameter $\omega = -1$ due to its slow-roll, inflates the Universe exponentially. The phase of inflation thus resembles a pure de Sitter like Universe, where the Hubble parameter H remain constant throughout inflation and the scale factor grows exponentially with time as $a(t) \sim e^{Ht}$. This particular nature is quite interesting as it shows that the comoving Hubble radius $(aH)^{-1}$ decreases with time during inflation as H being constant throughout and $a(t)$ growing exponentially with time. This remarkable feature of inflation solves both the Horizon problem and the Flatness problem discussed above :

- **Inflation solving Horizon problem**

  The concept of comoving particle horizon τ and the comoving Hubble radius $(aH)^{-1}$ is very important in discussing Horizon problem. It is important to note that particles separated by length scales greater than comoving particle horizon τ can never communicate with each other, whereas if they are separated by length scales larger than comoving Hubble radius, then they are causally disconnected for that particular time t. It may happen that as τ is much greater than $(aH)^{-1}$ now, so the particles, which cannot communicate today, can be in causal contact in the past if one has decreasing comoving Hubble radius during early times !! This is exactly what happens if one includes an inflationary era in the Big Bang theory. The comoving Hubble radius shrinks during inflation and expands after that according conventional Big Bang Cosmology. Hence the large wavelengths entering the horizon today were causally connected with each other before inflation. This causal physics before inflation also explains the supreme homogeneity of CMBR temperature we observe today.

- **Inflation solving Flatness problem :**

  The shrinking of comoving Hubble radius during inflation also helps solving the Flatness problem as now $\Omega - 1$ shows a converging nature with decreasing aH^{-1} during inflation. Hence Ω which is ~ 1 today, was even closer to 1 in past due to the shrinking of Hubble radius during inflation, solving the Flatness problem of Big Bang.

2.2 Inflation and the quantum origin of structure

Inflation was quite successful in solving the severe fine-tuning problems of Big Bang and now it has become an indispensable part of the conventional Big Bang Cosmology. We have already discussed that homogeneity and isotropy of our Universe on large scales are very well described by the homogeneous and isotropic metric of FLRW cosmology. But the presence of structures in the Universe, like galaxies and clusters, and the tiny fluctuations (of the order of 10^{-5}) in the uniform temperature of CMBR provide evidences that our Universe, from its very beginning, can not be so homogeneous and isotropic. It was realized soon after the concept of inflation being proposed that inflation can actually provide seeds of these large scale structures we see today and can also explain the tiny anisotropy observed in CMBR's temperature today. Quantum fluctuations around inflaton field (see Eq. (B.7)) during inflation help generate the inhomogeneities in a homogeneous and isotropic background. As these observed inhomogeneities are tiny in nature, thus the cosmological perturbations are studied under linear perturbation theory.

2.2.1 Cosmological perturbation theory

Cosmological perturbation theory is very important in study of modern cosmology as it connects the theories of very early universe with present observations. According to this theory, perturbations (which depend on both space and time) of fields or metric are studied on a homogeneous and isotropic background (which are only functions of time). As Einstein field equations (Eq. (A.2)) relate the background metric with the matter content of the universe, the perturbations in the inflaton field generates perturbations in the background metric during inflation. Let us now deal with the cosmological perturbation theory step by step :

- **Perturbations in the metric :**

  The metric perturbations in general can be of three types: scalar, vector and tensor perturbations. Inflation can not generate vector perturbations as there is no rotational velocity fields during the inflationary stage and even though if they are generated by any other mechanism, they decay with the expansion of the Universe. The tensor perturbations generates the primordial gravitational waves, which will be not dealt with in this thesis (for more on vector and tensor perturbations see [33]).

  The scalar perturbations of the background metric generate density fluctuations at the LSS which are the seeds for structure formations in the later phase of Universe. In its most general form, the perturbed metric can be constructed using four scalar perturbations A, B, ψ and E as

  $$\delta g_{\mu\nu} = a^2(\tau) \begin{pmatrix} 2A & -\partial_i B \\ -\partial_i B & 2\psi\delta_{ij} - D_{ij}E \end{pmatrix}, \tag{2.2}$$

  where $D_{ij} = \partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2$ and the most general form of the line element for the background and scalar metric perturbations will be

  $$ds^2 = a^2(\tau)\left[(1+2A)d\tau^2 - 2\partial_i B dx^i d\tau - \{(1-2\psi)\delta_{ij} + D_{ij}E\}\right]. \tag{2.3}$$

- **Choice of Gauge :**

  The choice of background and perturbations in a perturbed Universe is not unique, but depends on the choice of gauges. In General Relativity choosing a gauge implies choosing a coordinate system to define the perturbations. Choosing a spacelike hypersurface with constant cosmological time t is called *slicing*, while choosing a worldline of constant space $\mathbf{x}$ defines the *threading*. A convenient system which completely fixes the coordinates is called the **longitudinal** or **conformal Newtonian gauge** which is defined by $B = E = 0$. Hence in this gauge the metric will become

  $$ds^2 = a^2(\tau)\left[(1+2A)d\tau^2 - (1-2\psi)\delta_{ij}\right]. \tag{2.4}$$

We will continue using this particular gauge throughout this thesis. The perturbation in the Einstein field tensor and in the Klein-Gordon equation of motion for inflaton field due to the perturbed FLRW metric have been discussed in detail in Appendix (C). It is to be noted at this point that absence of stress part in the stress energy-momentum tensor puts further constrain on the scalar perturbation degrees of freedom which essentially implies $A = \psi$ (as has been discussed in Appendix (C) after Eq. (C.26)). Thus we are left with only one scalar metric perturbation ψ.

- **Gauge invariant quantities :**

It has been discussed in Appendix (C) that the scalar metric perturbation ψ is not a gauge invariant quantity but changes with changing coordinate systems as

$$\psi \to \psi + \mathcal{H}\xi^0, \tag{2.5}$$

where ξ^0 is the change in the time coordinate due to change of gauge.

The inflaton perturbations also change with a change of gauge which, following Eq. (C.8), can be written as

$$\delta\phi \to \delta\phi - \phi'\xi^0, \tag{2.6}$$

where $\phi' \equiv \frac{\partial\phi}{\partial\tau}$.

Now, in a *comoving slicing*, which is orthogonal to the worldlines of a comoving observer, there is no flux of energy measured by these observers. It can be seen from Eq. (C.32) that during inflation the flux of energy-momentum tensor is $\delta T^0_i \equiv \frac{1}{a^2}\partial_i\delta\phi\phi'_0$. Thus the comoving observer will measure $\delta\phi_{\rm com} = 0$. If ξ^0 is the time displacement required to go from any generic slicing to a comoving slicing then from Eq. (2.6) one gets

$$\delta\phi_{\rm com} = 0 \quad \Rightarrow \quad \xi^0 = \frac{\delta\phi}{\phi'}. \tag{2.7}$$

Thus the quantity

$$\mathcal{R} = \psi + \mathcal{H}\frac{\delta\phi}{\phi'} = \psi + H\frac{\delta\phi}{\dot\phi}, \tag{2.8}$$

called the *comoving curvature perturbation*, is gauge invariant by construction. This intrinsic curvature of spatial hypersurfaces characterizes the cosmological inhomogeneities. The important feature of $\mathcal{R}$ is that it remain constant on superhorizon scales i.e. when its momentum is smaller than the comoving Hubble horizon ($k < aH$). Because of this feature the amplitude of $\mathcal{R}$ is not affected by the subhorizon physics once it becomes superhorizon during inflation. After inflation the comoving horizon grows and the modes of comoving curvature perturbation re-enters the horizon during the radiation or the matter dominated era and determine the perturbation of the cosmic fluid yielding the CMBR anisotropies and the large scale structures we observe today.

Another notable feature of $\mathcal{R}$ is that in the *spatially flat gauge*, which is defined to be a slicing where there is no curvature i.e. $\psi_{\text{flat}} = 0$, the comoving curvature perturbation is related to the inflaton's fluctuations as

$$\mathcal{R} = H\frac{\delta\phi}{\dot{\phi}}, \tag{2.9}$$

which allows the inflaton's intrinsic features to be reflected in the observed CMBR anisotropies.

Thus, from the above discussion on Cosmological perturbation theory, we note that in conformal Newtonian gauge one is left with only one scalar degree of freedom ψ and this scalar degree of freedom is related to the inflaton's fluctuation $\delta\phi$ (Eq. (2.8)) while constructing a gauge invariant quantity $\mathcal{R}$, the comoving curvature perturbation. We further choose a spatially flat gauge where the comoving curvature perturbation $\mathcal{R}$ is directly related to the inflaton's fluctuation $\delta\phi$. This choice of spatially flat gauge drastically simplifies the situation as now we are left with only one perturbed quantity, the inflaton's fluctuation $\delta\phi$.

2.2.2 Quantum field theory of scalar fields in de Sitter background

The next step is to study the dynamics of the quantum fluctuations $\delta\phi$ around the classical inflaton field ϕ_0 during inflation. The de Sitter spacetime is very important cosmologically as it describes a Universe which is exponentially expanding. Thus the inflationary phase of Universe, which is an exponentially expanding phase, is described by a de Sitter background. A brief introduction to the de Sitter spacetime is given in Appendix (D). The de Sitter spacetime is characterized by a constant Hubble parameter H. But during inflation the Hubble parameter varies slowly with time. Hence at the end we will deal with quasi de Sitter spacetime, also briefly discussed in Appendix (D), where the Hubble parameter H varies with time during inflation. Here we will discuss the equation of motion and the solution of the mode functions of the quantum fluctuations of the inflaton field in de Sitter background.

- **Equation of motion of quantum fluctuations**

  The quantum fluctuations of inflaton field $\delta\phi$, being a function of both space and time, follows the equation of motion, with a mass term as its potential $V(\phi) = \frac{1}{2}m^2\phi^2$, as

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \frac{\nabla^2\delta\phi}{a^2} + m^2\delta\phi = 0, \tag{2.10}$$

  which can be obtained from Eq. (B.2). In conformal FLRW spacetime the above equation can be written as

$$\delta\phi'' + 2\frac{a'}{a}\delta\phi' - \nabla^2\delta\phi + a^2m^2\delta\phi = 0. \tag{2.11}$$

  It is convenient to introduce an auxiliary field $\chi \equiv a(\tau)\delta\phi$ and rewrite the above equation of motion as

$$\chi'' - \nabla^2\chi + \left(m^2a^2 - \frac{a''}{a}\right)\chi = 0. \tag{2.12}$$

  The $\chi(\tau, \mathbf{x})$ has a time-dependent effective mass term

$$m^2_{\text{eff}}(\tau) \equiv m^2a^2 - \frac{a''}{a}. \tag{2.13}$$

  Expanding the field χ in Fourier modes as

$$\chi(\tau, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}}\chi_{\mathbf{k}}(\tau)e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{2.14}$$

the equation of motion of each mode $\chi_{\mathbf{k}}$ can be obtained from Eq. (2.12) as

$$\chi_{\mathbf{k}}'' + \left(k^2 + m^2 a^2 - \frac{a''}{a}\right)\chi_{\mathbf{k}} = 0. \tag{2.15}$$

This is a equation of motion of an harmonic oscillator with a time-dependent frequency $\omega_k^2(\tau) \equiv k^2 + m_{\rm eff}^2$. In de Sitter spacetime we have

$$\frac{a''}{a} = \frac{2}{\tau^2}, \tag{2.16}$$

which follows from Eq. (D.2), and thus the above equation of motion for $\chi_{\mathbf{k}}$ can be written as

$$\chi_{\mathbf{k}}'' + \left(k^2 + m^2 a^2 - \frac{2}{\tau^2}\right)\chi_{\mathbf{k}} = 0. \tag{2.17}$$

- **Mode expansions and quantization of scalar field**

Introducing the creation and annihilation operators for each mode $\chi_{\mathbf{k}}$ as

$$\hat{\chi}_{\mathbf{k}} = u_k(\tau) a_{\mathbf{k}} + u_k^*(\tau) a_{-\mathbf{k}}^\dagger, \tag{2.18}$$

one can promote the scalar field $\chi(\tau, \mathbf{x})$ to an operator by expanding in modes given as

$$\hat{\chi}(\tau, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}} \left[u_k(\tau) a_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + u_k^*(\tau) a_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}}\right]. \tag{2.19}$$

The scalar field $\chi(\tau, \mathbf{x})$ can now be quantized in the standard fashion by introducing the equal-time commutation relations

$$\left[\hat{\chi}(\tau, \mathbf{x}), \hat{\Pi}(\tau, \mathbf{y})\right] = i\delta(\mathbf{x} - \mathbf{y}), \tag{2.20}$$

where $\hat{\Pi} \equiv \hat{\chi}'$ is the canonical momentum and the Hamiltonian for the $\hat{\chi}(\tau, \mathbf{x})$ will be

$$\hat{H}(\tau) = \frac{1}{2}\int d^3\mathbf{x} \left(\hat{\Pi}^2 + (\nabla\hat{\chi})^2 + m_{\rm eff}^2(\tau)\hat{\chi}^2\right). \tag{2.21}$$

Thus the creation and annihilation operators satisfy the commutation relations

$$\left[a_{\mathbf{p}}, a_{\mathbf{q}}^\dagger\right] = \delta(\mathbf{p} - \mathbf{q}), \qquad \left[a_{\mathbf{p}}, a_{\mathbf{q}}\right] = \left[a_{\mathbf{p}}^\dagger, a_{\mathbf{q}}^\dagger\right] = 0. \tag{2.22}$$

The mode function $u_k(\tau)$ obeys the equation of motion

$$u_k'' + \omega_k^2(\tau) u_k = 0 \tag{2.23}$$

as that of $\chi_{\mathbf{k}}$ and the mode expansion of $\hat{\chi}(\tau, \mathbf{x})$ given in Eq. (2.19) requires the normalization condition

$$u_k' u_k^* - u_k u_k^{*\prime} = -i. \tag{2.24}$$

- **Preferred mode function at subhorizon scales and Bunch-Davies vacuum**

The choice of mode functions in mode expansions of a scalar field in de Sitter spacetime, discussed in Eq. (2.19), is not unique. The preferred mode function of a scalar field in a Minkowski background has been discussed in Appendix (E). Unlike in Minkowski spacetime, the frequency of the mode function u_k in de Sitter spacetime depends upon time. Thus the vacuum of minimum-energy depends on some time τ_i at which it is defined. Following the similar arguments as given in Appendix (E), one can determine the vacuum at a given time τ_i which minimizes the expectation value of the Hamiltonian at that moment and the preferred mode function will be

$$u_k(\tau_i) = \frac{e^{-i\omega_k(\tau_i)\tau_i}}{\sqrt{2\omega_k(\tau_i)}}, \qquad u_k'(\tau_i) = i\omega_k(\tau_i)\, u_k(\tau_i). \tag{2.25}$$

This instantaneous minimization of the Hamiltonian is possible if $\omega_k^2(\tau_i) > 0$. The function E_k, given in Eq. (E.12), which accounts for the total energy density can be written in de Sitter spacetime as

$$E_k = r_k'^2 + \frac{1}{4r_k^2} + \omega_k^2(\tau_i) r_k^2, \tag{2.26}$$

which has no minimum for $\omega_k^2(\tau_i) < 0$. This situation occurs as during inflation we require a light scalar field ($m \ll H$) and there always be a small enough k such that $k|\tau_i| \ll 1$ making $\omega_k^2(\tau_i) < 0$. This can be seen from the form of $\omega_k^2(\tau_i)$ in de Sitter spacetime

$$\omega_k^2(\tau_i) \equiv k^2 + \left(\frac{m^2}{H^2} - 2\right) \frac{1}{\tau_i^2}. \tag{2.27}$$

Hence the instantaneous vacuum states can be defined only for modes $\chi_{\mathbf{k}}$ with $k\tau_i \gtrsim 1$.

A wave with a wavenumber $\mathbf{k}$ has the comoving wavelength $\lambda = k^{-1}$ and a physical wavelength $\lambda_P = a(\tau)\lambda$, which in de Sitter spacetime implies

$$k|\tau| = \frac{1}{\lambda}\frac{1}{aH} = \frac{H^{-1}}{\lambda_P}. \tag{2.28}$$

Thus $k|\tau_i| \ll 1$ corresponds to a wave with wavelength much larger than the horizon, which is called the *superhorizon mode*. On the other hand $k|\tau_i| \gg 1$ corresponds to a wave with wavelength much shorter than the horizon, which is called the *subhorizon mode*. Thus at sufficiently early time which corresponds to $\tau \to -\infty$, all the relevant modes $\chi_{\mathbf{k}}$ will satisfy the condition $k|\tau_i| \gg 1$ and hence will remain well inside the horizon. These subhorizon modes at very early times also satisfy the condition $\omega_k^2(\tau_i) < 0$ which minimizes the instantaneous Hamiltonian to get a preferred mode function.

At the early time limit $\tau \to -\infty$, the frequency of mode functions essentially becomes constant $\omega_k^2(\tau_i) \approx k^2$ as in the Minkowski spacetime which we have discussed in Appendix (E). This implies that the effect of gravity on all modes are negligible at sufficiently early times. Thus one can define the mode functions u_k by applying the Minkowski vacuum prescription at early times which yields

$$u_k(\tau) \to \frac{e^{-i\omega_k\tau}}{\sqrt{2\omega_k}}, \qquad \frac{u_k'(\tau)}{u_k(\tau)} \to i\omega_k. \tag{2.29}$$

The vacuum state determining the mode functions which satisfies the above mentioned conditions is called the *Bunch-Davies vacuum*. Thus the *Bunch-Davies vacuum* is essentially defined as the Minkowski vacuum of each mode in the early time limit ($\tau \to -\infty$) and this is the preferred vacuum for the quantum fields in de Sitter spacetime.

- **Mode functions for the superhorizon modes**

We have seen in the previous discussion that for the modes well within the horizon the preferred mode function will be

$$u_k(\tau) \to \frac{e^{-ik\tau}}{\sqrt{2k}} \tag{2.30}$$

where the frequency of the mode $\omega_k \sim k$. On superhorizon scales ($k\tau \ll 1$), the equation of motion of mode function given in Eq. (2.23) will be

$$u_k'' - \left(\frac{a''}{a} - m^2 a^2\right) u_k = 0. \tag{2.31}$$

Considering the case of a massless scalar field for simplicity, the above equation has one solution as

$$u_k = B(k)a. \tag{2.32}$$

Matching the absolute value of this solution to the plane wave solution at the time when a mode with wave number $\mathbf{k}$ leaves the horizon ($k = aH$) (i.e. matching the subhorizon solution and the superhorizon solution at the time of horizon crossing) one gets

$$|B(k)| = \frac{1}{a}\frac{1}{\sqrt{2k}} = \frac{H}{\sqrt{2k^3}}. \tag{2.33}$$

The amplitude of quantum fluctuation $\delta\phi$ of the inflaton field on superhorizon scales thus will be

$$|\delta\phi_k| \equiv \frac{1}{a}|\chi_k| = \frac{1}{a}|u_k| = \frac{H}{\sqrt{2k^3}}. \tag{2.34}$$

This shows that the quantum fluctuations of the inflaton field as well as the comoving curvature perturbations remain constant on superhorizon scales.

Now let us deal with the equation of motion given in Eq. (2.23) for a massive scalar field, in its full glory, which can be rewritten as

$$u_k'' + \left[k^2 - \left(\nu_\phi^2 - \frac{1}{4}\right)\frac{1}{\tau^2}\right] u_k = 0, \tag{2.35}$$

where $\nu_\phi \equiv \frac{9}{4} - \frac{m^2}{H^2}$. This is a Bessel equation whose general solution for real ν_ϕ will be

$$u_k(\tau) = \sqrt{-\tau}\left[c_1(k)H^{(1)}_{\nu_\phi}(-k\tau) + c_2(k)H^{(2)(-k\tau)}_{\nu_\phi}\right], \tag{2.36}$$

where $H^{(1)}_{\nu_\phi}$ and $H^{(2)}_{\nu_\phi}$ are Hankel functions of first and second kind, respectively. For the subhorizon modes ($-k\tau \gg 1$) the above solution should match with the plane wave solution $e^{-ik\tau}/\sqrt{2k}$. In the limit $-k\tau \gg 1$ the Hankel functions have the form

$$\begin{aligned} H^{(1)}_{\nu_\phi}(-k\tau \gg 1) &\sim \sqrt{-\frac{2}{\pi k\tau}} e^{-i\left(k\tau+\frac{\pi}{2}\nu_\phi+\frac{\pi}{4}\right)}, \\ H^{(2)}_{\nu_\phi}(-k\tau \gg 1) &\sim \sqrt{-\frac{2}{\pi k\tau}} e^{i\left(k\tau+\frac{\pi}{2}\nu_\phi+\frac{\pi}{4}\right)}. \end{aligned} \tag{2.37}$$

Hence the requirement of plane wave solution at subhorizon scales fixes the coefficients $c_1(k)$ and $c_2(k)$ as

$$c_1(k) = \frac{\sqrt{\pi}}{2} e^{i\left(\nu_\phi+\frac{1}{2}\right)\frac{\pi}{2}}, \qquad c_2(k) = 0 \tag{2.38}$$

yielding the form of the mode function as

$$u_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\left(\nu_\phi+\frac{1}{2}\right)\frac{\pi}{2}} \sqrt{-\tau} H^{(1)}_{\nu_\phi}(-k\tau). \tag{2.39}$$

On superhorizon scale ($-k\tau \ll 1$) the asymptotic behavior of the Hankel function is given by

$$H^{(1)}_{\nu_\phi}(-k\tau \ll 1) \sim \sqrt{\frac{2}{\pi}} e^{-i\frac{\pi}{2}} 2^{\left(\nu_\phi-\frac{3}{2}\right)} \frac{\Gamma\left(\nu_\phi\right)}{\Gamma\left(\frac{3}{2}\right)} (-k\tau)^{-\nu_\phi}, \tag{2.40}$$

which at the end yields the form of the mode functions on superhorizon scales as

$$u_k(\tau) = e^{i\left(\nu_\phi-\frac{1}{2}\right)\frac{\pi}{2}} 2^{\left(\nu_\phi-\frac{3}{2}\right)} \frac{\Gamma\left(\nu_\phi\right)}{\Gamma\left(\frac{3}{2}\right)} \frac{1}{\sqrt{2k}} (-k\tau)^{\frac{1}{2}-\nu_\phi}. \tag{2.41}$$

Let us consider the case of a light scalar field $m \ll \frac{3}{2}H$ where one can identify the parameter $\eta_\phi = \frac{m^2}{3H^2}$ as the slow-roll parameter introduced in Eq. (B.10). Thus the amplitude of the fluctuations of a light inflaton on superhorizon scales will be

$$|\delta\phi_k| = \frac{H}{\sqrt{2k^3}} \left(\frac{k}{2aH}\right)^{\frac{3}{2}-\nu_\phi}. \tag{2.42}$$

Thus the fluctuations of a light scalar field is not exactly constant on superhorizon scales, but depends upon time.

2.2.3 The Power spectrum and today's observations

It is now remain to characterize the properties of these primordial perturbations of the inflaton field and analyze their signatures in terms of observables. It has already been stated that, as the primordial fluctuations are tiny in nature, their generation and evolution are studied under linear perturbation theory. One major assumption of the study of linear cosmological perturbation theory is that these primordial fluctuations are essentially Gaussian in nature. In that case the two-point correlation function of these primordial fluctuations is the only parameter to determine all the statistical properties of these primordial perturbations. In particular, for a Gaussian distribution, the odd-point correlation functions are zero and the higher even-point correlation functions can be written in terms of powers of two-point correlation function. Thus the two-point correlation function or the Fourier transform of it, named as the *Power spectrum*, is a crucial quantity to determine in order to analyze the properties of primordial fluctuations of the inflaton field.

- **Power spectrum**

  For any given random field $g(t, \mathbf{x})$ which can be expanded in Fourier space as

$$g(t, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}} e^{\mathbf{k}\cdot\mathbf{x}} g_{\mathbf{k}}(t), \tag{2.43}$$

  the power spectrum or the two-point correlation function in Fourier space $\mathcal{P}_g(k)$ is defined as

$$\left\langle g_{\mathbf{k}_1} g^*_{\mathbf{k}_2} \right\rangle \equiv \frac{2\pi^2}{k^3} \mathcal{P}_g(k) \delta^3(\mathbf{k_1} - \mathbf{k_2}). \tag{2.44}$$

  The power spectrum measures the amplitude of the generic random field at a given scale k. In the real space this measures the mean square value of the field as

$$\left\langle g^2(t, \mathbf{x}) \right\rangle = \int \frac{dk}{k} \mathcal{P}_g(k). \tag{2.45}$$

  With this definition, the power spectrum of the inflaton perturbations will be

$$\mathcal{P}_{\delta\phi}(k) \equiv \frac{k^3}{2\pi^2} \left|\delta\phi_k\right|^2 = \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{2aH}\right)^{3-2\nu_\phi}. \tag{2.46}$$

For a heavy inflaton field $m^2 \gg H^2$, the slow-roll condition is strongly violated as $\eta_\phi \equiv \frac{m^2}{3H^2} \gg 1$ and it is shown in [34] that the power spectrum for such a heavy inflaton field will be highly suppressed

$$\mathcal{P}_{\delta\phi}(k)\,|_{\rm heavy} = \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^3 e^{-2m^2/H^2}. \tag{2.47}$$

Since the comoving curvature perturbations defined in Eq. (2.9) in the spatially flat gauge $\mathcal{R}$ approaches a constant value on superhorizon scales, it is useful to compute the *comoving curvature power spectrum* for the comoving curvature perturbations as

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{H^2}{\dot{\phi}^2}\mathcal{P}_{\delta\phi}(k) = \frac{1}{2M_{\rm Pl}^2\epsilon}\left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{2aH}\right)^{3-2\nu_\phi}, \tag{2.48}$$

where we have used the definition of the slow-roll parameter ϵ defined in Eq. (B.12).

- **Connecting with observations**

Now after deriving the power spectrum of comoving curvature perturbation one can connect the theory of inflation with the present day cosmological observations like CMBR and determine the signatures of these early Universe phenomena in the anisotropy spectrum of CMBR.

It has already been discussed that the comoving curvature perturbation $\mathcal{R}$ freezes to a constant once it exists the horizon during inflation at some time, say τ_* and starts evolving again after it re-enters the horizon at some later time, say τ, during the radiation or the matter dominated era of Universe. Thus one should take into account the time evolution of $\mathcal{R}$ once it re-enters the horizon :

$$\mathcal{Q}_{\mathbf{k}}(\tau) = T_{\mathcal{Q}}(k,\tau,\tau_*)\mathcal{R}_{\mathbf{k}}(\tau_*), \tag{2.49}$$

where $T_{\mathcal{Q}}(k,\tau,\tau_*)$ is called the *transfer function* and $\mathcal{Q}_{\mathbf{k}}(\tau)$ is the measure of fluctuations in the radiation field (observed temperature fluctuations in the CMBR) or in the matter field (galaxy density scattered throughout the sky). Here we will discuss how the power spectrum of comoving curvature perturbation $\mathcal{R}$ is related to the observed anisotropies in the CMBR temperature.

Different satellite experiments like COsmic Background Explorer (COBE) or Wilkinson Microwave Background Measurement (WMAP) measure the temperature fluctuations $\Delta T(\hat{n})$ relative to the background uniform temperature $T_0 \sim 2.7$ K of CMBR. As the temperature fluctuation is mapped on a spherical sky surface, the harmonic expansion of these fluctuations are

$$\Theta(\hat{n}) \equiv \frac{\Delta T(\hat{n})}{T_0} = \sum_{lm} a_{lm} Y_{lm}(\hat{n}), \tag{2.50}$$

where $\hat{n}$ is the unit vector denoting the direction in the sky and Y_{lm} are the spherical harmonics with l representing the different multipoles and $m = -l, \cdots, l$. The angular power spectrum of the temperature anisotropy map is determined by the multipole moments a_{lm} as

$$C_l^{TT} = \frac{1}{2l+1} \sum_m \langle a^*_{lm} a_{lm} \rangle. \tag{2.51}$$

As the comoving curvature perturbations generate fluctuations in the radiation field after re-entering the horizon, the temperature fluctuation δT of the CMBR is related to $\mathcal{R}$ by the linear evolution equation as [16]

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} \Delta_{Tl}(k) \mathcal{R}_{\mathbf{k}} Y_{lm}(\hat{\mathbf{k}}), \tag{2.52}$$

where $\Delta_{Tl}(k)$ is the transfer function which connects a_{lm} with $\mathcal{R}$ and generally has to be computed numerically by solving Boltzmann equations using codes like CMBFAST [35] or CAMB [36]. These Boltzmann equations depend on the parameters of the background cosmology. Now, using the above equation, the angular power spectrum for the CMBR fluctuations can be derived as

$$C_l^{TT} = \frac{2}{\pi} \int k^2 dk \; \mathcal{P}_{\mathcal{R}}(k) \; \Delta_{Tl}(k) \Delta_{Tl}(k), \tag{2.53}$$

using the identity of the spherical harmonics

$$\sum_{m=-l}^{l} Y_{lm}(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{k}}') = \frac{2l+1}{4\pi} P_l(\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}'), \tag{2.54}$$

where $P_l(\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}')$ is the Legendre polynomial.

Fig. (2.1) shows the TT angular power spectrum of CMBR from the 7 year data of WMAP [2]. Assuming a fixed background cosmology the shape of the

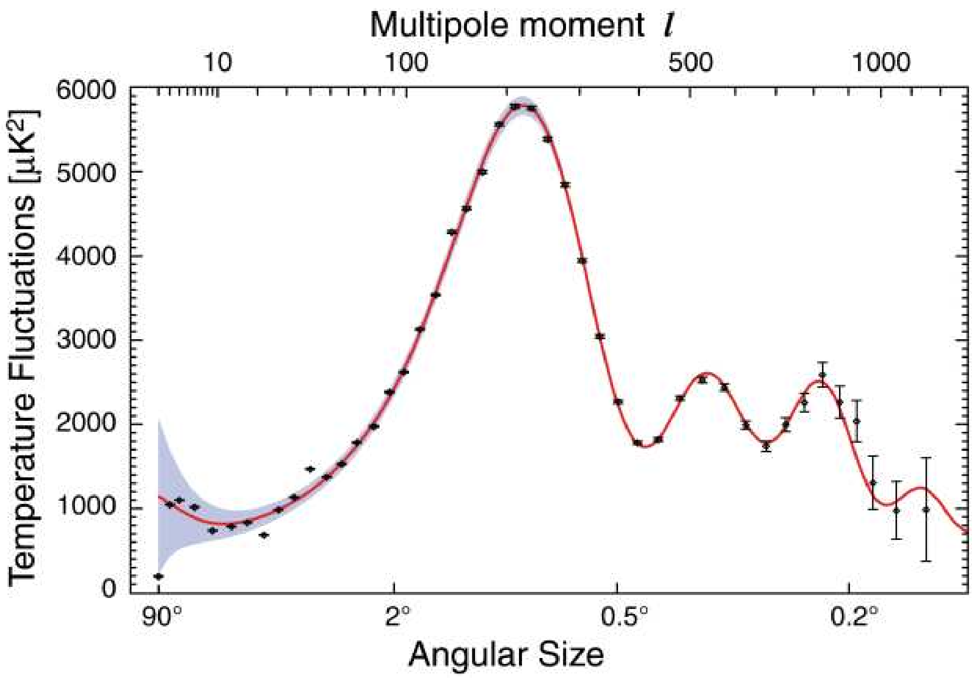

Figure 2.1: The 7-year temperature (TT) power spectrum from WMAP.

angular power spectrum C_l^{TT} contains informations about the initial conditions as described by the comoving curvature power spectrum $\mathcal{P}_{\mathcal{R}}$. The theoretical fit in Fig. (2.1) is for a ΛCDM (Cold Dark Matter) model of the Universe with single field slow-roll model of inflation.

2.3 Primordial Non-Gaussianity and Constraints on slow-roll inflation and its alternatives

We have discussed the single-field slow-roll model of inflation in a great detail in the previous section. It has also been stated that if the primordial fluctuations like comoving curvature perturbations $\mathcal{R}$ are taken to be Gaussian distributed then the power spectrum or the two-point correlation function of these primordial fluctuations contains all the informations about the inflationary dynamics from a statistical point of view. However, if the primordial fluctuations are non-Gaussian in nature, then higher order correlation functions beyond the two-point correlation function will contain additional informations about inflationary dynamics. We will now discuss the non-Gaussian features arising from three-point and four-point correlation functions of the cosmological perturbations in the case of single-field slow-roll inflationary scenario.

2.3.1 Defining Bispectrum and Trispectrum

The non-Gaussianity appearing from the three-point correlation function is quantified in the Fourier space, called the *Bispectrum*, which is defined as [37]

$$\begin{aligned}\langle \mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\rangle &= (2\pi)^{-\frac{3}{2}}\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{6}{5}f_{NL}\left(\frac{P_{\mathcal{R}}(k_1)}{k_1^3}\frac{P_{\mathcal{R}}(k_2)}{k_2^3}+\text{ perms.}\right)\\ &= (2\pi)^{-\frac{3}{2}}\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{6}{5}f_{NL}B(k_1,k_2,k_3), \end{aligned} \tag{2.55}$$

where 'perms.' stands for two permutations of indices and for convenience we have redefine the power spectrum as

$$P_{\mathcal{R}}(k) = (2\pi^2)\mathcal{P}_{\mathcal{R}}(k), \tag{2.56}$$

and the normalization factor of $(2\pi)^{-\frac{3}{2}}$ in Eq. (2.55) has been chosen accordingly. It can be seen from Eq. (2.55) that the bispectrum $B(k_1, k_2, k_3)$ is proportional to the product of two power spectrums and hence the name bispectrum. It follows that the bispectrum represents the lowest-order statistics which is able to distinguish non-Gaussian from Gaussian perturbations. The order of non-Gaussianity arising from bispectrum is characterized by the non-linearity parameter f_{NL}. In general, f_{NL} will be a function of the wave-numbers $\mathbf{k}_i$. The delta function $\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$ in Eq. (2.55) is a consequence of translational invariance of the background and it enforces the three Fourier modes to form a closed triangle due to momentum conservation. Different inflationary models predict maximal non-Gaussianity for different triangle configurations.

The *trispectrum* $T(k_1, k_2, k_3, k_4)$, which is the Fourier counterpart of the 'connected' part of four-point correlation function, is defined as [37]

$$\langle \mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\mathcal{R}(\mathbf{k}_4)\rangle_c = (2\pi)^{-3}\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\frac{1}{2}\tau_{NL}T(k_1,k_2,k_3,k_4), \tag{2.57}$$

where the trispectrum $T(k_1, k_2, k_3, k_4)$ can be written in terms of product of three power spectrums :

$$T(k_1,k_2,k_3,k_4) = \left(\frac{P_{\mathcal{R}}(k_1)}{k_1^3}\frac{P_{\mathcal{R}}(k_2)}{k_2^3}\frac{P_{\mathcal{R}}(k_{13})}{k_{13}^3}+\text{ perms.}\right) \tag{2.58}$$

and hence the name trispectrum. Here $k_{13} \equiv |\mathbf{k}_1 + \mathbf{k}_3|$ and 'perms.' stands for 23 permutations of indices. The order of non-Gaussianity arising from trispectrum is characterized by the non-linearity parameter τ_{NL}.

2.3.2 Non-Gaussianity in a single-field slow-roll inflation model : the Bispectrum and the Trispectrum

Non-Gaussian features, arising from higher order correlation function of cosmological perturbations, in a single-field slow-roll model of inflation can occur from two different aspects :

1. **Self-interactions of inflaton field**

   As the inflaton's fluctuations are quantum in nature (or have a Gaussian distribution as these fluctuations are tiny in nature), the three point correlation function vanishes identically

   $$\langle \delta\phi(\mathbf{k}_1, t)\delta\phi(\mathbf{k}_2, t)\delta\phi(\mathbf{k}_3, t)\rangle = 0. \tag{2.59}$$

   A nontrivial higher-point correlation function appears if the inflaton field has any sort of interaction with itself (or with other fields). The simplest possible interaction that can be present in the inflaton's potential is the cubic self-interaction term ($V(\phi) \sim \lambda\phi^3$). It has been shown in [38] that in the presence of a cubic self-interaction term the three-point correlation function of inflaton fluctuations will be proportional to the coupling of the self-interaction term as

   $$\langle \delta\phi(\mathbf{k}_1, t)\delta\phi(\mathbf{k}_2, t)\delta\phi(\mathbf{k}_3, t)\rangle \propto \frac{\lambda}{H}. \tag{2.60}$$

   It is to be noted that for a cubic self-interaction, the coupling λ will be of the order of the third derivative of the inflaton's potential ($\sim V'''(\phi)$) and in the slow-roll scenario this coupling will be even smaller than the slow-roll parameters ($\lambda < \epsilon, \eta$). It has been estimated in [38] that the three-point correlation function arising from such cubic self-interaction term will be of the order of $\frac{\lambda}{H} \sim 10^{-7}$.

The measure of non-Gaussianity arising from a four-point correlations function comes from its 'connected part' ($\langle \cdots \rangle_c$)

$$\begin{aligned} \langle \delta\phi_1\delta\phi_2\delta\phi_3\delta\phi_4 \rangle &= \langle \delta\phi_1\delta\phi_2 \rangle \langle \delta\phi_3\delta\phi_4 \rangle + \langle \delta\phi_1\delta\phi_3 \rangle \langle \delta\phi_2\delta\phi_4 \rangle \\ &+ \langle \delta\phi_1\delta\phi_4 \rangle \langle \delta\phi_2\delta\phi_3 \rangle + \langle \delta\phi_1\delta\phi_2\delta\phi_3\delta\phi_4 \rangle_c. \end{aligned} \tag{2.61}$$

For a generic scalar field, without self-interaction terms, the four-point correlation function will be equal to the product of two two-point correlation functions

$$\begin{aligned} \langle \delta\phi_1\delta\phi_2\delta\phi_3\delta\phi_4 \rangle &= \langle \delta\phi_1\delta\phi_2 \rangle \langle \delta\phi_3\delta\phi_4 \rangle + \langle \delta\phi_1\delta\phi_3 \rangle \langle \delta\phi_2\delta\phi_4 \rangle \\ &+ \langle \delta\phi_1\delta\phi_4 \rangle \langle \delta\phi_2\delta\phi_3 \rangle. \end{aligned} \tag{2.62}$$

Thus the connected part of the four-point correlation function in this case vanishes identically yielding no non-Gaussian features.

2. **Non-linearities in cosmological perturbations**

Non-linearities in the evolution of the cosmological perturbations, such as $\mathcal{R}$, can also generate primordial non-Gaussian features in CMBR. Presuming that the inflaton fluctuations $\delta\phi$ are initially Gaussian, the comoving curvature perturbations $\mathcal{R}$ given in Eq. (2.9) also obeys Gaussian statistics in the linear order

$$\mathcal{R}_L(t, \mathbf{x}) = \frac{H}{\dot{\phi}} \delta\phi_L(t, \mathbf{x}), \tag{2.63}$$

where $\mathcal{R}_L(t, \mathbf{x})$ can be expanded in Fourier space as

$$\mathcal{R}_L(t, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}} e^{i\mathbf{k}\cdot\mathbf{x}} \mathcal{R}_L(t, \mathbf{k}). \tag{2.64}$$

One can observe that the factor $\frac{H}{\dot{\phi}} \equiv -\frac{1}{m_{\rm Pl}^2}\frac{V(\phi)}{V'(\phi)}$ is a function of ϕ. Hence the comoving curvature perturbations $\mathcal{R}$ can be expanded non-linearly as [39]

$$\mathcal{R}_{NL}(t, \mathbf{x}) = \frac{H}{\dot{\phi}} \delta\phi_L(t, \mathbf{x}) + \frac{1}{2}\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right) \delta\phi_L^2(t, \mathbf{x}) + \mathcal{O}(\delta\phi_L^3), \tag{2.65}$$

and in the Fourier space one gets

$$\mathcal{R}_{NL}(t, \mathbf{k}) = \frac{H}{\dot{\phi}} \delta\phi_L(t, \mathbf{k}) + \frac{1}{2}\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right) \int \frac{d^3\mathbf{p}}{(2\pi)^{\frac{3}{2}}} \delta\phi_L(t, \mathbf{p}) \delta\phi_L(t, \mathbf{k}-\mathbf{p}), \tag{2.66}$$

where the inflaton fluctuations $\delta\phi_L$ are still Gaussian distributed. Using Eq. (2.66) one can compute the bispectrum in this case as follows

$$\langle \mathcal{R}_{NL}(\mathbf{k}_1)\mathcal{R}_{NL}(\mathbf{k}_2)\mathcal{R}_{NL}(\mathbf{k}_3)\rangle = (2\pi)^{-\frac{3}{2}}\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(2M_{\rm Pl}^2\epsilon)\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right) \times\left(\frac{P_{\mathcal{R}}(k_1)}{k_1^3}\frac{P_{\mathcal{R}}(k_2)}{k_2^3}+2\text{ perms.}\right). \quad (2.67)$$

This has been rigorously calculated in Appendix (F). The non-linearity parameter f_{NL} due to non-linear evolution of $\mathcal{R}$ can be quantified using Eq. (2.55) and the above equation as

$$f_{NL} = \frac{5}{3}M_{\rm Pl}^2\epsilon\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right) = -\frac{5\epsilon}{3}\frac{\partial}{\partial\phi}\left(\frac{V(\phi)}{V'(\phi)}\right), \quad (2.68)$$

and can be expressed in terms of the slow-roll parameters given in Eq. (B.10) as

$$f_{NL} = \frac{5}{6}(\delta-\epsilon), \quad (2.69)$$

where $\delta \equiv \eta - \epsilon$. The same form of f_{NL} has been derived in [40]. The non-linearity parameter f_{NL} can also be expressed in terms of the potential $V(\phi)$ using Eq. (2.68) as

$$f_{NL} = -\frac{5}{6}M_{\rm Pl}^2\left(\frac{V'}{V}\right)\frac{\partial}{\partial\phi}\left(\frac{V(\phi)}{V'(\phi)}\right). \quad (2.70)$$

This equation is useful in cases where the form of the potential is known. Such as, for a power-law potential where $V(\phi)\sim\phi^n$ the non-linearity parameter will be [39]

$$f_{NL} = -\left(\frac{5}{6}\right)n\frac{M_{\rm Pl}^2}{\phi^2}. \quad (2.71)$$

The non-linear evolution of $\mathcal{R}$ given in Eq. (2.65) yields a connected part of four-point correlation function or the trispectrum as

$$\langle \mathcal{R}_{NL}(t,\mathbf{k}_1)\mathcal{R}_{NL}(t,\mathbf{k}_2)\mathcal{R}_{NL}(t,\mathbf{k}_3)\mathcal{R}_{NL}(t,\mathbf{k}_4)\rangle_c = 2\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\times (2\pi)^{-3}\left(2M_{\rm Pl}^2\epsilon\right)^2\left(\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right)\right)^2\left[\frac{P_{\mathcal{R}}(k_1)}{k_1^3}\frac{P_{\mathcal{R}}(k_2)}{k_2^3}\frac{P_{\mathcal{R}}(k_{13})}{k_{13}^3}+23\,\text{perm.}\right], \quad (2.72)$$

which has been derived in Appendix (F). It can be seen from the definition of τ_{NL} given in Eq. (2.57) and the form of f_{NL} arising due the non-vanishing three-point correlation function of $\mathcal{R}_{NL}$ given in Eq. (2.68) that the non-linearity parameter τ_{NL} will be [40]

$$\tau_{NL} = \left(2M_{\rm Pl}^2\epsilon\right)^2 \left(\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right)\right)^2 = \left(\frac{6}{5}f_{NL}\right)^2. \tag{2.73}$$

As in the single field slow-roll inflationary scenario the bispectrum non-linear parameter f_{NL} turns out to be of the order of slow-roll parameters (Eq. (2.69)), the trispectrum contribution arising due to non-linear comoving curvature perturbations will be even smaller as τ_{NL}, the trispectrum non-linearity parameter, in this case will be of the order of square of slow-roll parameters ($\mathcal{O}(\epsilon^2)$).

2.3.3 Alternatives to single-field slow-roll inflationary scenario and non-Gaussianity

The physics of inflation provides us an avenue to learn about the evolution and interactions of the quantum fields present in the very early Universe. The power spectrum relates to the evolution of the quantum field (or fields) present during the inflationary era but does not really constrain the interactions of this quantum field (or fields). Inflationary models with different field interactions thus can predict similar power spectrums. On the other hand primordial non-Gaussianity can provide sensitive probes to the interaction of field (or fields) driving inflation. Thus measurement of such primordial non-Gaussianity can reveal vital informations about the fundamental physics driving inflation.

The amount of non-Gaussianity arising from a single-field slow-roll inflationary model is of the order of the slow-roll parameters (in the case of bispectrum) or even smaller (in the case of trispectrum). The essential features of this single-field slow-roll inflationary scenario can be characterized as

1. The inflation is driven by **one canonical scalar field** and this field is responsible for generating primordial seeds for structure.

2. **The canonical kinetic term** of this scalar field yields the propagation of the fluctuations with speed of light.

3. The scalar field **rolls slowly** along its potential during inflation.

4. The initial vacuum state of the inflaton is chosen to be the **Bunch-Davies vacuum**, which is the preferred vacuum for this scalar field.

Other inflationary models violating any of these above mentioned criteria of single-field slow-roll inflationary scenario can generate large primordial non-Gaussianity.

- **Curvaton model of inflation**

  The usual hypothesis, that the inflaton is solely responsible for generating the seeds of anisotropy in matter and radiation we observe today, can be simply extended by having more scalar fields present during inflation. The simplest possible alternative hypothesis is that these density perturbations originate from the perturbations of some other scalar field different from the inflaton [27]. This field is known as the *curvaton.* The energy density of the curvaton field remain sub-dominant during inflation and acquires an almost scale-independent and Gaussian perturbations with spectrum $\left(\frac{H}{2\pi}\right)^2$ [27]. As in this scenario the inflaton field is no longer responsible for the cosmological curvature perturbations, therefore the slow-roll conditions for the curvaton field can be avoided. In a simple curvaton scenario, the curvaton field starts to oscillate during radiation-dominated era after inflation and this oscillation persists for a considerable amount of Hubble time to generate a significant curvature perturbation. The curvaton field decays before neutrino decoupling and the curvature perturbation remains constant till horizon entry. This simple scenario violates two of the above criteria of single-field inflationary model : 1) presence of two scalar fields and 2) the curvaton field, the one responsible for generating curvature perturbations, needs not to slow-roll.

  The curvaton quantum fluctuations during inflation are converted into curvature perturbations ζ after its decay as

$$\zeta \sim r\delta, \tag{2.74}$$

  where δ is the isocurvature fractional density perturbation in the curvaton before decay and r is the fraction of the final radiation that the decay of the curvaton produces. The non-Gaussianity in this scenario will be [28]

$$f_{NL} = \frac{5}{4r}. \tag{2.75}$$

The non-linearity parameter f_{NL} in this case does not depend upon the slow-roll parameters as the curvaton field does not require to slow-roll during inflation and f_{NL} can be very large in the curvaton scenario depending upon the value of r yielding large non-Gaussianity in comparison to the single field slow-roll inflationary scenario.

- **Higher-derivative interactions**

A large non-Gaussianity can be obtained in a single field slow-roll inflationary model if higher derivative terms become important during inflation. This feature violate the criteria of the inflaton field having a canonical kinetic term required in the single field slow-roll model. A general action of the inflaton field can be considered during inflation as

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[R - P(X,\phi)], \tag{2.76}$$

where $X \equiv (\partial\phi)^2$ is the kinetic term and $P(X,\phi)$ is an arbitrary function of X. Thus these models can contain higher-derivative interactions yielding a non-trivial sound speed for the propagation of fluctuations as

$$c_s^2 \equiv \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}. \tag{2.77}$$

In the absence of the higher-derivative terms one has $P = X - V(\phi)$ yielding $c_s = 1$. Thus in the simplest model of single field slow-roll inflationary model with canonical kinetic term the fluctuations propagate with speed of light. For small sound speeds ($c_s^2 \ll 1$), the non-Gaussianity becomes significant and the non-linearity parameter will be [29]

$$f_{NL} = -\frac{35}{108}\left(\frac{1}{c_s^2} - 1\right) + \frac{5}{81}\left(\frac{1}{c_s^2} - 1 - 2\Lambda\right) \tag{2.78}$$

with $\Lambda \equiv \frac{X^2P_{,XX} + \frac{2}{3}X^3P_{,XXX}}{XP_{,X} + 2X^2P_{,XX}}$. String motivated DBI inflationary scenario is such a model where these kind of higher-derivative interactions yielding large primordial non-Gaussianity can be realized [29].

From the above discussion it can be concluded that violating any of the three criteria of single-field slow-roll inflationary model can generating large non-Gaussianity and precise detection of primordial non-Gaussianity can distinguish between different inflationary models. The issue of non-standard initial vacuum state yielding large non-Gaussianity will be covered in Chapter 5.

2.3.4 Observational bounds on primordial non-Gaussianity

The current constraint on CMBR bispectrum from the seven-year data of WMAP is $-214 < f_{NL}^{eq} < 266$ (95% CL) [2]. The single-field slow-roll inflation model prediction of $f_{NL} \sim \mathcal{O}(\epsilon) \sim 10^{-2}$ (Eq. (2.69)) is too small to be detectable in WMAP or the upcoming PLANCK mission where non-Gaussianities at the level of $f_{NL} \sim 5$ can be probed [41]. WMAP constrains the non-Gaussianity from trispectrum at $|\tau_{NL}| \lesssim 10^8$ [42] while PLANCK is expected to reach the sensitivity up to $|\tau_{NL}| \sim 560$ [43], which is still too large compared to the predictions of the single-field slow-roll model of $\tau_{NL} = (\frac{6}{5} f_{NL})^2 \sim \mathcal{O}(\epsilon^2)$ (Eq. (2.73)).

Another source, besides CMBR, is the measurement of 21-cm transition of the neutral hydrogen with which one can constrain the primordial non-Gaussianities. After the decoupling of matter and radiation at the LSS ($z \sim 1100$), the high energy photons produced by the first stars and quasars later reionize the neutral hydrogen ($50 > z > 6$) which is known as *the epoch of reionization*. Also theory predicts that for $200 \geq z \geq 30$, the spin temperature of the neutral hydrogen drops below the temperature of the CMBR and the neutral hydrogen thus can absorb CMBR at the spin flip transition of 21-cm and appear in absorption spectra against the CMBR [44].

The 21-cm background signal contains a wealth of informations about primordial fluctuations. It is to be noted that the CMBR has emerged specifically from one redshift $z \sim 1000$ whereas the 21-cm line occurs from a span of redshift $50 > z > 6$. A full sky map at a single photon frequency (corresponding to one single redshift z), measured up to a maximum mode l_{max}, contains $l_{\max}^2$ independent sample. But as by changing the frequency one can probe different redshifts, an experiment that detects the 21-cm signal over a range of frequency $\Delta\nu$ (corresponding to different redshifts) centered on a frequency ν measures a total of $N_{21\text{cm}} = 3 \times 10^{16}(l_{max}/10^6)^3(\Delta\nu/\nu)(z/100)^{-\frac{1}{2}}$ independent samples, whereas for CMBR one only gets $N_{\text{CMBR}} = 2 \times 10^7(l_{max}/3000)^2$ of independent points, including both temperature and polarization information of CMBR. Thus the number of independent measurements obtained from 21-cm signal is a billion times more than that of CMBR [44].

Thus the most sensitive probe of primordial non-Gaussianities can come from the measurement of anisotropies of the 21-cm background. It has been already mentioned that the neutral hydrogen below the redshift $z \sim 200$ can resonantly absorb the CMBR flux through spin-flip transition. Thus if the CMBR contains informations about primordial non-Gaussianity, then the 21-cm anisotropy will also carry the signature associated with that [45]. The anisotropies of the 21-cm background can constrain the bispectrum to the level $f_{NL} < 0.1$ [45, 46] and the trispectrum of primordial perturbations to the level of $\tau_{NL} \sim 10$ [46]. But still the measurements of f_{NL} or τ_{NL} from 21-cm background anisotropy signal in future will be too large to probe the non-Gaussian features arising from single field slow-roll inflationary scenario.

2.4 Summary

In this chapter we discuss the simplest and well accepted model of inflationary scenario which is the single-field slow-roll inflationary model and our main focus is on the analysis of the power spectrum (two-point correlation function), bispectrum (three-point correlation function) and trispectrum (four-point correlation function) of inflaton perturbations ($\delta\phi$) as well as the comoving curvature perturbations ($\mathcal{R}$) arising in this scenario.

In this analysis the cosmological perturbation theory plays a major role as the primordial fluctuations, being tiny (1 part in 10^5), are studied under linear analysis and cosmological perturbation theory helps us choose the proper gauge to analyze such primordial perturbations in the perturbed background de Sitter metric. We choose the conformal Newtonian gauge in which the metric perturbations have a constraint $B = E$ (defined in Eq. (2.2)). We further choose a spatially flat gauge (for simplification of further analysis) where the gauge invariant quantity $\mathcal{R}$ is directly related to the inflaton's fluctuations (Eq. (2.9)). This specific choice of gauge helps us calculate all the physical quantities in terms of quantum fluctuations of the inflaton field.

The statistical properties of the inflaton's fluctuations, which are directly related to the observations in CMBR, are studied consequently. We start by deriving the preferred initial vacuum (the Bunch-Davies vacuum) and the solution of the superhorizon modes of these fluctuations in the inflaton field. As these fluctuations are tiny, the two-point function of these perturbations is most important statistically. The power spectrum (two-point function in Fourier space) of the inflaton fluctuation and consequently that for the comoving curvature perturbation arising in this inflationary scenario is nearly scale invariant (as shown in Eq. (2.48)) which is in accordance with the CMBR observations.

The observations of CMBR shows that the primordial fluctuations are nearly Gaussian and thus study of only power spectrum determines all the statistical properties of such tiny primordial anisotropies. But the higher-point correlations are more enriched as they can quantify the interactions of the inflaton field and thus very useful in distinguishing between several existing inflationary model which power spectrum alone fails to do. Keeping this in mind we calculate the bispectrum and trispectrum in single-field slow-roll inflationary scenario and show that the non-linearity parameters f_{NL} and τ_{NL} arising from these higher-point correlations can be at best of the order of the slow-roll parameter ϵ (see Eq. (2.69) and Eq. (2.73)). These non-Gaussianities arise due to non-linear evolution of $\mathcal{R}$. This model predicts a larger bispectrum non-Gaussianity than the trispectrum one ($f_{NL} > \tau_{NL}$). Such tiny non-Gaussianity is beyond the range of detectability of WMAP, PLANCK or future 21-cm background experiments measuring such primordial non-Gaussianities imprinted in it.

Chapter 3

Källén-Lehmann representation of QFT and power spectrum of interacting inflaton

3.1 Introduction

We discussed in the previous chapter that in a generic inflation model [8], inflation is caused by a slow-roll of the inflaton scalar field and the perturbations of the inflaton field give rise to density perturbations [47] and CMB anisotropies observed at cosmological scales. The two-point correlation function of the inflaton perturbation during inflation or the power spectrum of this two-point correlation in momentum space determines the CMB anisotropy of the Universe at last scattering which we observe today. Inflation may also be caused by more than one scalar field and these multifield models have interesting consequences in the CMB anisotropy like isocurvature perturbation [48] or large non-gaussianity such as in curvaton models [49]. In models of inflation with elementary scalar fields, the perturbations of the inflaton obey the Klein-Gordon equation (given in Eq. (B.2)) in the quasi de Sitter space [50], whose solutions are used for calculation of the two-point correlation function of inflaton and consequently the comoving curvature power spectrum. The generic slow-roll model of inflation are characterized by the inflaton potential and its derivatives. The simplest way to include interaction of the inflaton field in inflationary models is by modifying the inflaton's potential and thus a large variety of particle physics potential have been studied in the literature [20].

However, it may be possible that the inflaton field is a composite of fermions and we can ask if the compositeness changes the perturbation spectrum which can be observed in the CMB anisotropy. Similarly if the inflaton is unstable with a decay width $\Gamma \lesssim H/N$ (such that the inflaton decays after N e-foldings of inflation are over) then again we can ask if the decay of the inflaton is reflected in the power spectrum and CMB anisotropy. For such situations the standard methods of calculating the power spectrum do not work as not all forms of the short range structure of the scalar field are reflected in the inflaton potential. If the length scale of the scalar perturbation is of the same order as the compositeness scale then the effective theory description of the scalar potential breaks down. Similarly if the inflaton is a resonance with a lifetime of the same order as the duration of inflation $\tau \sim N/H$ i.e. a width $\Gamma \sim H/N$ then there are corrections to the two point correlation that are not reflected in the inflaton potential.

In non-perturbative techniques of Quantum Field Theory, one way to include interactions in the two-point function of any quantum field is by following Källén-Lehmann spectral representation [18, 19] where the interactions are all encapsulated in Källén-Lehmann spectral function. In our work [23], we find a general method for computing the power spectrum of inflaton perturbations using the concepts of this non-perturbative technique of Quantum Field Theory in cases where the inflaton has non-trivial interactions like a decay width or if the inflaton is not an elementary scalar but a composite of fermions. We show in general that the two-point correlations of the interacting field can be written in terms of the two-point function of the free field (in the de Sitter background) by use of the Källén-Lehmann spectral function [18, 19]. The assumptions which are essential for deriving such a general method of calculating power spectrum of interacting scalar fields are :

1. the short wavelength limit of the mode functions are the plane wave states $\frac{1}{\sqrt{2k}}e^{-ik\tau}$, which in the quasi de-Sitter space is enforced by the assumption of

the Bunch-Davis boundary conditions (as has been discussed in the previous chapter)

2. there exists a complete orthonormal set of mode functions of the free theory in curved spacetime which is true in the quasi de-Sitter space relevant for inflation power spectrum calculation.

In Quantum Field Theory the two-point correlation of an interacting theory can be written as a convolution of the free-field correlation function $G^0(p,\sigma^2)$ with a spectral function $\rho(\sigma^2)$

$$G^{(\text{int})}(p) = \int_0^\infty d\sigma^2 \rho(\sigma^2) G^0(p,\sigma^2), \tag{3.1}$$

where $G^{(\text{int})}(p)$ is the two-point correlation of the interacting theory and σ is called the mass parameter. The Källén-Lehmann (KL) representation holds for all two point correlations like the Feynman propagator $\Delta(p,\sigma^2)$ or the equal time Wightman function $W_{\text{ET}}(x-y)$. We will show that this result can be generalized to the curved space if we assume that a complete orthogonal basis set of states of the interacting theory exists in curved spacetime.

In inflationary theory the power spectrum of the inflaton perturbation is related to the equal-time Wightman function of a scalar field in the de Sitter space as

$$W_{\text{ET}}^{\text{dS}}(x) = \langle 0|(\delta\phi(\mathbf{x},t))^2|0\rangle = \int \frac{dk}{k}\mathcal{P}_{\delta\phi}(k). \tag{3.2}$$

The Bunch-Davies boundary condition is that the inflaton perturbations, in the limit where the momentum k is large compared to the inflaton horizon (for a spatially flat de-Sitter space), tend to the free field form $\delta\phi(k,\tau) = \frac{1}{\sqrt{2k}}e^{-ik\tau}$. Assuming the Bunch-Davies boundary conditions, if we have short range interactions which dominate at scales smaller than the inflation horizon, we may be justified in using the flat space form of the spectral function in Eq. (3.2) to compute the two point correlation function for interacting theory. Therefore power spectrum of the interacting scalar field can be expressed as

$$P^{(\text{int})}(k) = \int_0^\infty P^{(0)}(k,\sigma^2)\rho\left(\sigma^2\right) d\sigma^2, \tag{3.3}$$

where $P^{(0)}(k,\sigma^2)$ is the power spectrum of the free scalar field with a mass parameter σ and $\rho(\sigma^2)$ is the KL spectral function which encapsulates all the short distance interactions (like compositeness or resonance) of the scalar field.

3.2 Power spectrum of interacting scalar field - general case

The power spectrum for the inflaton is essentially given by the equal-time Wightman function in de-Sitter space. In this section we will provide a general formalism of calculating power spectrum for interacting scalar field using KL representation. Derivations of the two point correlation functions for interacting real scalar field using KL representation in Minkowski space is given in Appendix (G).

It is assumed in the following derivation that the asymptotic 'in' and 'out' states of an interacting scalar field are free particle states in the curved space. We assume the interactions, being short-ranged, dominate over curvature effects at short distances. Since we assume the Bunch-Davies boundary conditions that the curved space mode functions in the large momentum limit go over to the flat space plane wave form, we can directly use the flat space calculation of spectral function of the interaction theory in the inflation power spectrum formula.

To generalize the KL formalism in de Sitter spacetime it is to be noted that in de Sitter spacetime there is *no translational invariance in the time direction* like Minkowski space. Due to this particular feature of de Sitter spacetime the mode functions given in Eq. (G.5) can be written in a more general form for the inflaton fluctuations as

$$\langle 0|\delta\phi(x)|n\rangle = \left(\sqrt{2p_n^0}\right)\delta\phi(p_n^0,\tau)\mathrm{e}^{i\mathbf{p}_n\cdot\mathbf{x}}\langle 0|\delta\phi(0)|n\rangle, \tag{3.4}$$

where $\delta\phi(p_n^0,\tau)$ are the free field mode functions which obey the Klein-Gordon equation in the curved background and in the flat space limit one has

$$\delta\phi(p_n^0,\tau) = \frac{1}{\sqrt{2p_n^0}}\exp\left(-ip_n^0\tau\right). \tag{3.5}$$

Hence the Wightman function in de-Sitter space can be written as

$$\begin{aligned} W_{\rm ET}^{\rm dS}(x,y) &= \langle 0|\delta\phi(x)\delta\phi(y)|0\rangle \\ &= \sum_n \left(2p_n^0\right)\delta\phi(p_n^0,\tau)\delta\phi(p_n^0,\tau')\mathrm{e}^{i\mathbf{p}_n\cdot(\mathbf{x}-\mathbf{y})}|\langle 0|\delta\phi(0)|n\rangle|^2. \end{aligned} \tag{3.6}$$

Here $\langle 0|\delta\phi(0)|n\rangle$ represents the short-range interactions of the interacting inflaton perturbations and according to our previous assumption this can be replaced by the spectral function $\rho(q^2)$ of Minkowski space defined in Eq. (G.7) as

$$\theta(q^0)\rho(q^2) = (2\pi)^3 \sum_n \delta^4(q - p_n)|\langle 0|\delta\phi(0)|n\rangle|^2. \tag{3.7}$$

With this definition of spectral function Eq. (3.6) can be written as

$$\begin{aligned}\langle 0|\delta\phi(x)\delta\phi(y)|0\rangle &= \int \frac{d^4q}{(2\pi)^3} \int_0^\infty d\sigma^2 \left(2q^0\right) \delta\phi(q^0,\tau)\delta\phi(q^0,\tau')\mathrm{e}^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})} \\ &\quad \times\theta(q^0)\rho(\sigma^2)\delta(q^2+\sigma^2) \\ &= \int_0^\infty d\sigma^2\rho(\sigma^2) \int \frac{d^3q}{(2\pi)^3}\delta\phi(\omega,\tau)\delta\phi(\omega,\tau')\mathrm{e}^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})}. \end{aligned} \tag{3.8}$$

The equivalent form of Wightman function in Minkowski space of the above equation is given in Eq. (G.19). Here $\omega = \sqrt{\mathbf{q}^2 + \sigma^2}$ and in de-Sitter space $\delta\phi(\omega, \eta)$ has the solution given in Eq (2.42) with mass m of the inflaton field replaced by the mass parameter σ. The solution for light scalar field given in Eq. (2.46) will be used in the following derivation because for very massive fields ($m_\phi > H$) the power spectrum is highly damped in superhorizon scales as given in Eq. (2.47) and hence the upper limit of σ^2 integration in the above equation should have a cut-off at m_0^2 where $m_0 \ll H$.

The equal-time Wightman function in de-Sitter space $W_{\mathrm{ET}}^{\mathrm{dS}}(x)$ gives the power spectrum for the inflaton fluctuations

$$\begin{aligned}\langle 0|\left(\delta\phi(x)\right)^2|0\rangle &= \int_0^{m_0^2} d\sigma^2\rho(\sigma^2) \int \frac{dq}{q}\frac{q^3}{2\pi^2}\left|\delta\phi(\omega,\tau)\right|^2 \\ &= \int \frac{dq}{q}\int_0^{m_0^2} d\sigma^2\rho(\sigma^2)\mathcal{P}_{\delta\phi}^{(0)}(q,\sigma^2), \end{aligned} \tag{3.9}$$

where $\mathcal{P}_{\delta\phi}^{(0)}(q,\sigma^2)$ is the power spectrum of the free inflaton field given by Eq. (2.46) with m replaced by σ

$$\mathcal{P}_{\delta\phi}^{(0)}(q,\sigma^2) = \frac{H^2}{4\pi^2}\left(\frac{q}{2aH}\right)^{\frac{2}{3}\frac{\sigma^2}{H^2}}. \tag{3.10}$$

Following Eq. (2.45) the power spectrum for the interacting scalar field is given by

$$\langle 0|(\delta\phi(\mathbf{x},\tau))^2|0\rangle = \int \frac{dq}{q}\mathcal{P}_{\delta\phi}^{(\mathrm{int})}(q). \tag{3.11}$$

From Eq. (3.9) and Eq. (3.11) we get

$$\mathcal{P}_{\delta\phi}^{(\mathrm{int})}(k) = \int_0^{m_0^2} d\sigma^2\rho(\sigma^2)\mathcal{P}_{\delta\phi}^{(0)}(k,\sigma^2), \tag{3.12}$$

and hence the curvature power spectrum (defined in Eq. (2.48)) for interacting inflaton field will be

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{H^2}{\dot{\phi}^2}\mathcal{P}_{\delta\phi}^{(\text{int})}(k) = \frac{1}{2m_{\text{Pl}}^2\epsilon}\int_0^{m_0^2}\mathcal{P}_{\delta\phi}^{(0)}(k,\sigma^2)\rho\left(\sigma^2\right)d\sigma^2, \tag{3.13}$$

where ϵ is the slow roll parameter of the inflaton. This form of curvature spectrum will be used as input in CAMB [36] or CMBFAST [35] to determine the CMB anisotropy spectrum from a given model of inflaton interactions.

3.3 Inflaton with a decay width

From the fact that the inflation must end in reheating we expect that the inflaton has couplings to other particles and it can decay into lighter particles. The inflaton decay width must be smaller than H/N (where $N \simeq 100$ is the number of e-foldings needed to solve the horizon and flatness problems). Since $\Gamma \lesssim 10^{-2}H$, the decay width term is negligible compared to the $H\delta\dot{\phi}$ term in the Klein-Gordon equation given in Eq. (2.10).

To compute the power spectrum of the decaying inflaton, we start with the Breit-Wigner propagator in flat space, of an unstable scalar particle with decay width Γ and mass m

$$\Delta^{(\text{int})}(q^2) = \frac{1}{q^2 - m^2 + im\Gamma}, \tag{3.14}$$

whose spectral function has the form [51]

$$\rho(\sigma^2) = \frac{1}{\pi}\frac{m\Gamma}{\left(\sigma^2 - m^2\right)^2 + m^2\Gamma^2}. \tag{3.15}$$

Using the spectral function from Eq (3.15) in Eq (3.13) the power spectrum for inflaton with a decay width will be

$$\begin{aligned}\mathcal{P}_{\mathcal{R}}(k) &= \frac{H^2}{8M_{\text{Pl}}^2\epsilon\pi^2}\left[\tan^{-1}\left(\frac{m}{\Gamma}\right) - \tan^{-1}\left(\frac{m^2-m_0^2}{m\Gamma}\right)\right] + \frac{m^2}{12M_{\text{Pl}}^2\epsilon\pi^2}\ln\left(\frac{z}{2}\right)\\ &\times\left[\tan^{-1}\left(\frac{m}{\Gamma}\right) - \cot^{-1}\left(\frac{m\Gamma}{m^2-m_0^2}\right) + \frac{\Gamma}{2m}\ln\left(\frac{\left(m^2-m_0^2\right)^2 + m^2\Gamma^2}{m^2\left(m^2+\Gamma^2\right)}\right)\right],\end{aligned} \tag{3.16}$$

where $z = \frac{k}{aH}$ and $m_0 \ll H$ is the cut-off scale for the mass parameter σ.

In Fig (3.1) we plot $\mathcal{P}_{\mathcal{R}}(k)$ vs. k plot for the decaying inflaton. We observe that for the free scalar field (i.e. $\Gamma = 0$) the curvature power spectrum is scale-invariant where for the decaying inflaton the power spectrum gets suppressed at low k and increases at high k with respect to the free inflaton case. We also observe that the higher the decay width Γ, more is the suppression of power at low k and increase of power at high k.

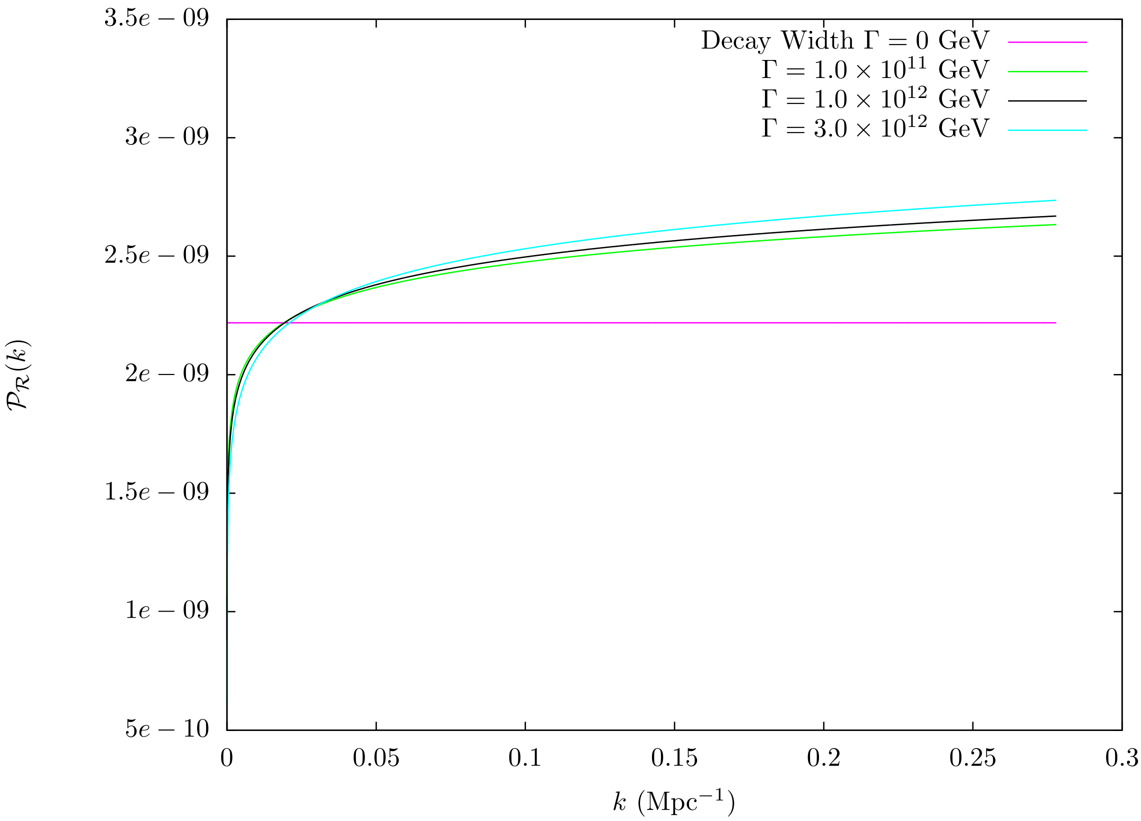

Figure 3.1: $\mathcal{P}_{\mathcal{R}}(k)$ vs. k plot for decaying scalar inflaton

In Fig (3.2) we plot the TT angular spectrum for the inflaton with a decay width. The parameters used for the above plots are $H = 10^{13}$ GeV, $m = 3.5 \times 10^{12}$ GeV, $m_0 = 7.5 \times 10^{12}$ GeV and for $\Gamma = 1.0 \times 10^{11}$ GeV, $\Gamma = 1.0 \times 10^{12}$ GeV and $\Gamma = 3.0 \times 10^{12}$ GeV the values of ϵ used are 1.412×10^{-5}, 1.29×10^{-5} and 1.069×10^{-5} respectively. In previous figure, we find that as the inflaton decay width Γ is increased the power at large distance scales gets suppressed. This results in suppression of the TT spectrum at low l with increasing decay width in this plot. A decay width of the inflaton may be a viable explanation of the WMAP observation of suppression in the TT power spectrum [52, 53].

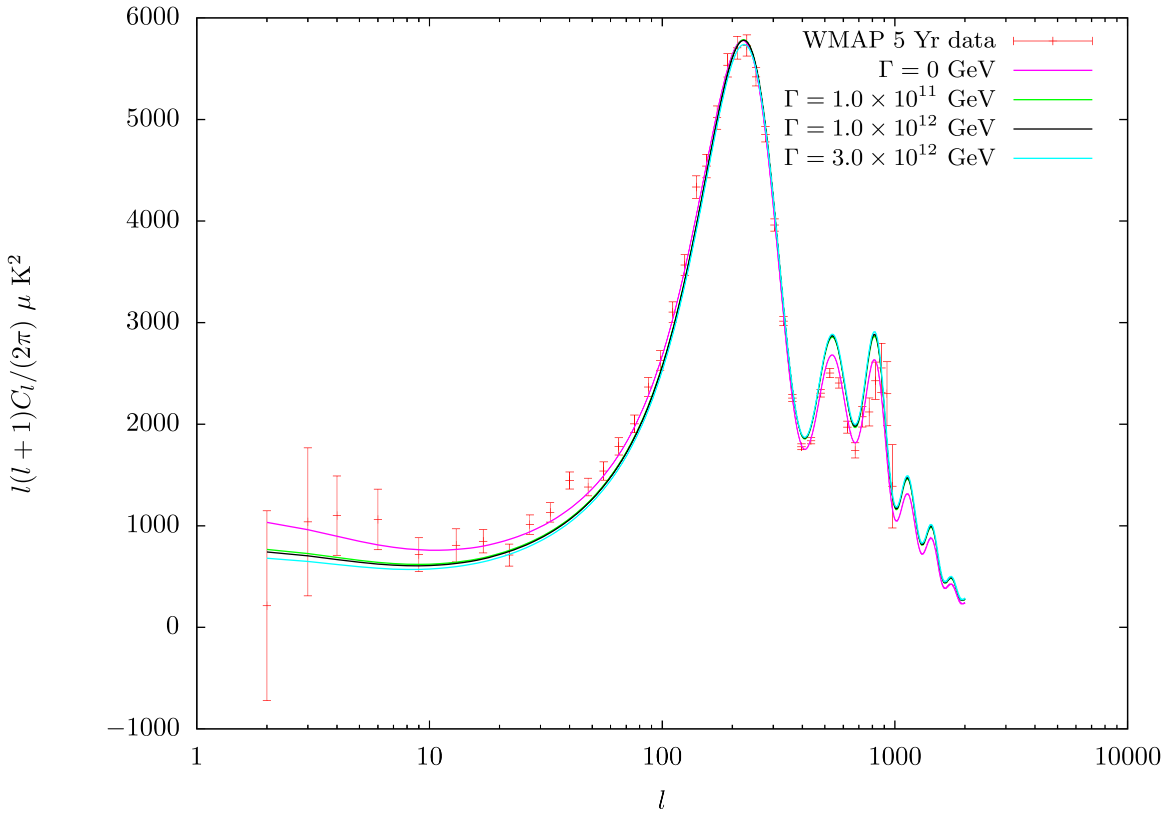

Figure 3.2: The TT angular spectrum for the inflaton with a decay width.

3.4 Inflaton as Composite Particle

An interesting model of inflation can be with the inflaton as a GUT scale techni-pion which arises from a condensate of fermions in a GUT scale SU(N) techni-colour model [54] or the inflaton can be a composite of heavy right handed neutrinos [24]. In such models one may ask in what way the compositeness of the inflaton affects the power spectrum. We use the spectral representation of a composite scalar in deriving the power spectrum in such a case.

The spectral function for a composite scalar can be taken as in QCD models [55] as

$$\rho(\sigma^2) = Z\delta(\sigma^2 - m_\varphi^2) + \frac{1}{f_\varphi^2 m_\varphi^2}\rho_c(\sigma^2)\theta(\sigma^2 - s_0^2), \tag{3.17}$$

where m_φ is the techni-pion mass, f_φ is the symmetry breaking scale and s_0 is the threshold for the onset of a continuum contribution $\rho_c(\sigma^2)$.

The wave function renormalization constant Z can be determined using the following property of the spectral function

$$\int_0^\infty \rho(\sigma^2) d\sigma^2 = 1. \tag{3.18}$$

The spectral function for the continuum is given as [56]

$$\rho_c(\sigma^2) = \frac{N}{8\pi^2}\sigma^2\left(1 - \frac{s_0^2}{\sigma^2}\right)^{\frac{3}{2}}, \tag{3.19}$$

where N is the number of fermion flavours. Using Eq. (3.17), Eq. (3.18) and Eq. (3.19) we get

$$Z = 1 - \frac{N}{8\pi^2}\frac{1}{f_\varphi^2 m_\varphi^2}\left(\frac{1}{2}\Lambda^4 - \frac{3s_0^2}{2}\Lambda^2 + s_0^4\right), \tag{3.20}$$

where Λ is the ultra-violet cut-off of the composite theory.

Now using Eq. (3.17) and Eq. (3.19) in Eq. (3.13) we find the power spectrum for a composite scalar particle as

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{ZH^2}{8\pi^2 M_{\rm Pl}^2\epsilon}\left(\frac{z}{2}\right)^{\frac{2}{3}\frac{m_\varphi^2}{H^2}} + \frac{3NH^4}{256\pi^4 M_{\rm Pl}^2\epsilon\left[\ln\left(\frac{z}{2}\right)\right]^2}\frac{1}{f_\varphi^2 m_\varphi^2}\times$$
$$\left[\left(\frac{z}{2}\right)^{\frac{2}{3}\frac{s_0^2}{H^2}}\left\{3H^2 + s_0^2\ln\left(\frac{z}{2}\right)\right\} + \left(\frac{z}{2}\right)^{\frac{2}{3}\frac{m_0^2}{H^2}}\left\{-3H^2 + \left(2m_0^2 - 3s_0^2\right)\ln\left(\frac{z}{2}\right)\right\}\right]. \tag{3.21}$$

In Fig. (3.3) $\mathcal{P}_{\mathcal{R}}(k)$ vs k plot for composite inflaton is given. We see that though the curvature power is scale invariant for a free scalar field (i.e. $1 - Z = 0$), there is a sharp resonance at $k = 0.002$ Mpc^{-1}, due to compositeness ($1 - Z > 0$) in the inflaton field. The resonances increases as the compositeness of the inflaton increases (smaller Z). Such resonances in curvature power spectrum can lead to oscillatory features in TT angular power of CMBR as seen in other examples where spikes in the power spectrum can arise due a period of fast roll [57] or a bump in the potential [58].

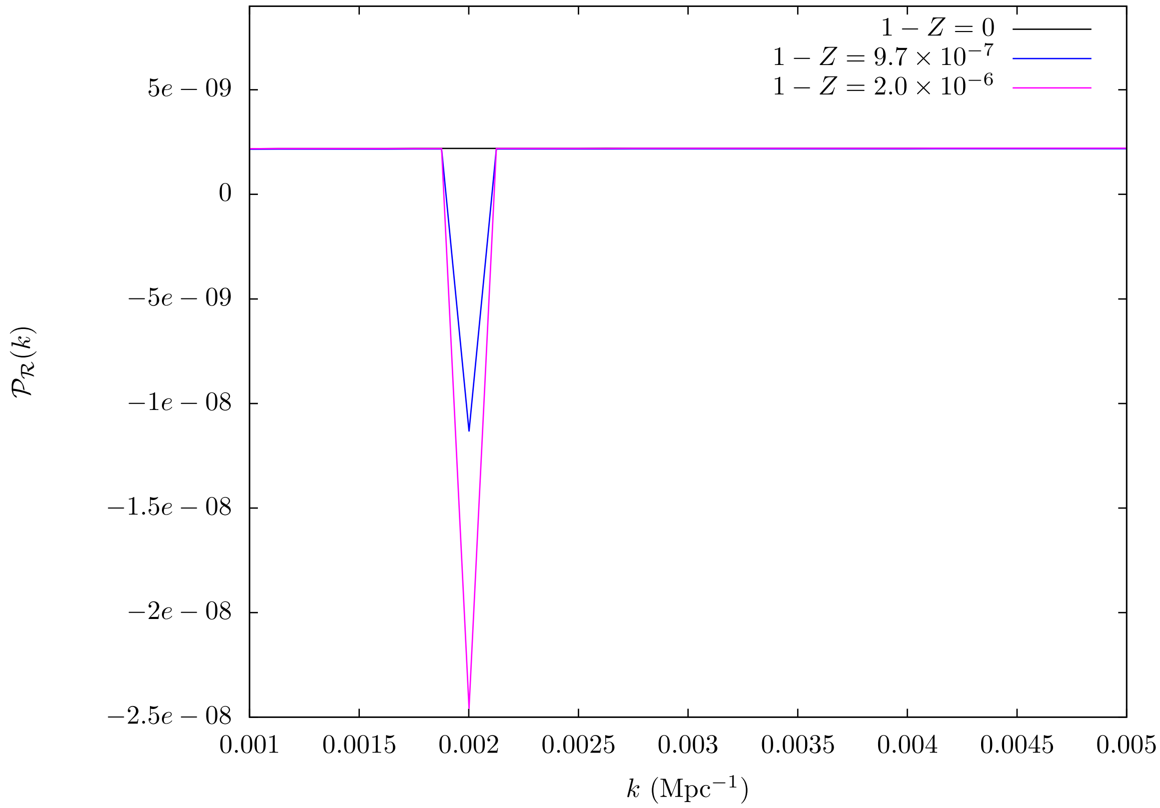

Figure 3.3: $\mathcal{P}_{\mathcal{R}}(k)$ vs k plot for composite inflaton

In Fig (3.4) we plot the TT angular spectrum for the case of a composite inflaton. The parameters used for these plots are $H = 10^{13}$ GeV, $m_\varphi = 1.0 \times 10^{12}$ GeV, $m_0 = 3.0 \times 10^{12}$ GeV, $s_0 = 1.0 \times 10^{11}$ GeV, $\Lambda = 1.0 \times 10^{13}$ GeV, $N = 3$ and for $1 - Z = 9.7 \times 10^{-7}$ and $1 - Z = 2.0 \times 10^{-6}$ we take $f_\varphi = 1.4 \times 10^{16}$ GeV, $\epsilon = 3.92 \times 10^{-6}$ and $f_\varphi = 1.0 \times 10^{16}$ GeV, $\epsilon = 3.87 \times 10^{-6}$ respectively. We find that there are oscillatory features in the power spectrum at $l = 30$.

Analysis of WMAP data by several groups [52] suggests that the power spectrum may have such oscillatory features. We have given the plot for some plausible values of the parameters.

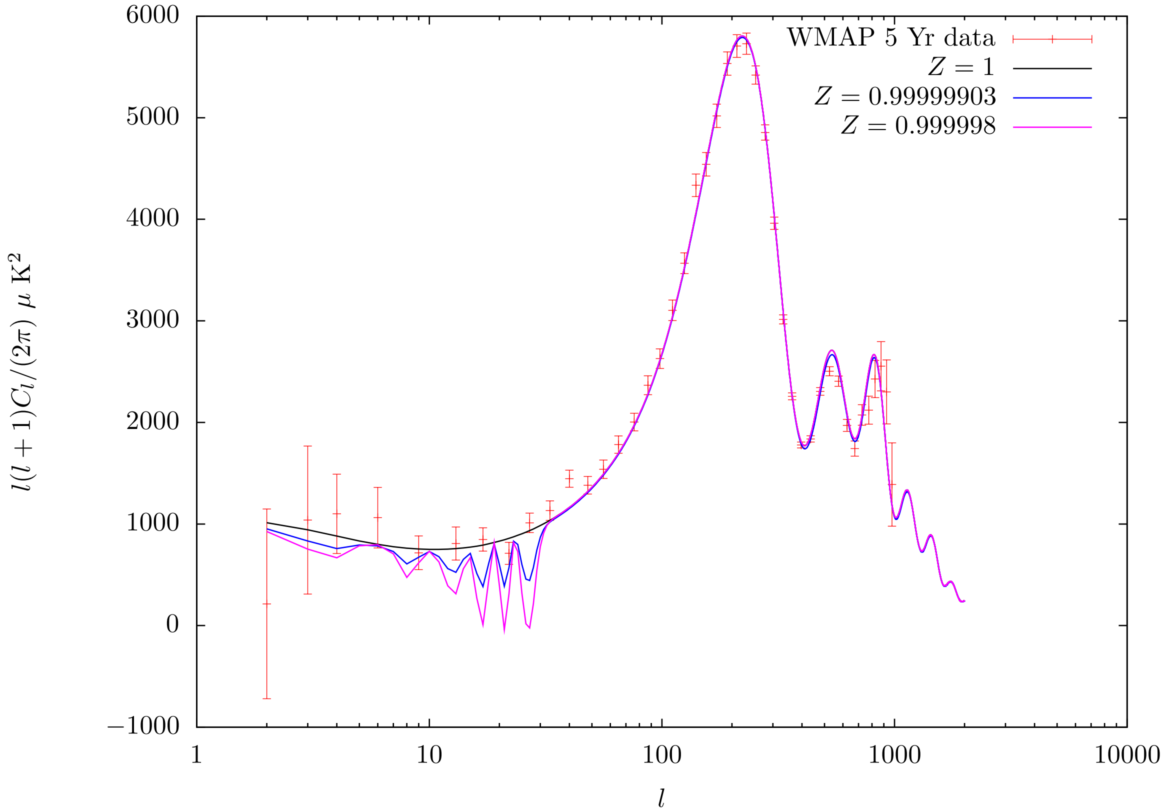

Figure 3.4: The TT angular spectrum for the inflaton as a composite particle.

3.5 Conclusion

We have derived a general formula for incorporating short-range interactions in the two-point correlation functions and the power spectrum by use of the Källén-Lehmann spectral function of flat spacetime. This method is useful if the short wavelength limit of the mode functions are plane wave states $\frac{1}{\sqrt{2k}}e^{-ik\tau}$, follows in the quasi de-Sitter inflation by the assumption of the Bunch-Davis boundary conditions and there exists a complete orthonormal set of mode functions of the free theory in curved spacetime which is true in the quasi de-Sitter space relevant for inflation power spectrum calculation. In interacting inflaton models like the ones studied in this paper we find that there are more interesting variations in the power spectrum due to the modification of the propagators which do not affect the slow roll parameters. We apply our formulation to study inflation with decaying and composite inflatons. We find that the decay of the inflaton results in the suppression of long distance correlations and thereby a loss of the quadrupole anisotropy [53]. This may be related to the observation of low quadrupole power by WMAP [59].

When the inflaton is taken as a composite of two fermions the power spectrum displays even more interesting features like oscillations. An examination of the WMAP data by wavelet analysis and by the cosmic inversion method reveals that the data may have such features [52].

Chapter 4

An exotic quantum field : Unparticle as inflaton

4.1 Introduction

In most of the inflationary scenarios scalar fields are the most favorable candidates to drive inflation as scalar fields are naturally homogeneous and isotropic and can lead to exponential expansion of Universe by slowly rolling along its potential. But the fact that scalar fields still remained unobserved in nature, drives a quest for looking for other quantum fields to act as an inflaton. We discuss few such attempts made in the literature :

- **Inflation driven by vector fields :**

  In [21] an attempt was made to drive inflation with a self-coupled vector field. As the stress tensor $T_{\mu\nu}$ for vector fields is not isotropic, the Universe in this scenario will exit inflation into an anisotropic expansion. The isotropy of inflation is achieved in [60] by considering a non-minimally coupled triplet of orthogonal vector fields. One can also take into account N randomly oriented vector fields where the anisotropy of the order $\frac{1}{\sqrt{N}}$ survives until the end of inflation [60].

- **Inflation driven by spinor fields :**

  In [22] *classical*, homogeneous spinor field has been considered in a flat FLRW

Universe. By *classical* spinor field it is meant a set of four complex-valued spacetime functions that transforms according to the spinor representation of the Lorentz group. It has been observed in [22] that such a field can lead to a blue spectrum of perturbations if it is allowed to drive inflation. Thus this scenario is not much appealing and the authors look for quartic self-interaction of such spinor field which lead to non-singular cyclic Universe.

In [61], fermionic fields are studied in presence of gravity and the Dirac equation is done via the tetrad formalism where the components of the tetrad play the role of the gravitational degrees of freedom. These spinor fields can be a source of accelerated phases of Universe such as inflation and late time acceleration (dark energy dominant Universe).

A new class of spinor fields (Dark spinors or Elko) are also studied in the context of inflation [62] where these unusual spinor fields (of mass dimension 1) can play the role of inflaton and drive inflation.

- **Inflation driven by spinor condensate :**

  In such scenarios the inflaton field is not a fundamental scalar but is considered as a condensate of spinor fields. We have discussed one such case in the previous chapter where the composite inflaton shows some oscillatory features in the low l region of the TT anisotropy spectrum of CMBR [23]. In [24] a condensate of right-handed neutrinos are considered as inflaton.

Recently Georgi in [25] has proposed a new class of particles, called the *unparticles*, which have dimensions different from their canonical scaling dimensions to exist in an effective low energy theory. One assumes that an ultraviolet theory has a IR fixed point at some scale Λ_u where the fields become conformal invariant. The effective coupling of the ultraviolet theory operators O_{UV} of dimension d_{uv} with the standard model operators O_{SM} of dimension n are suppressed by a heavy mass scale M_u and can be written as

$$\frac{1}{M_u^{d_{uv}+n-4}} O_{UV} O_{SM}, \tag{4.1}$$

where d_{uv} is the canonical dimension of the operator O_{UV}. Below the scale Λ_u (conventionally assumed as 1 TeV), the fields of the UV theory become scale invariant and by dimensional transmutation acquire a dimension d_u which is different from their canonical dimension. These conformally coupled unparticle operators O_U will couple to the standard model operators as

$$\left(\frac{\Lambda_u}{M_u}\right)^{d_{uv}+n-4} \frac{1}{\Lambda_u^{d_u+n-4}} O_U \, O_{SM}. \tag{4.2}$$

These exotic particles, along with its quite different anomalous dimensions, have another noble feature of generating long range forces while exchanged between two systems either microscopic or macroscopic. It has been pointed out [63] that the exchange of scalar (pseudoscalar) unparticles can give rise to spin independent (spin-dependent) long range forces. Long range forces from vectors and axial-vectors have been studied in [63, 64]. Tensor unparticles can couple to the energy momentum tensor and mimic gravity as pointed out in [26].

We will now discuss how anomalous dimensions of tensor and vector unparticles can be constrained from Mercury's perihelion precession as exchange of unparticles yields long range forces which deviates from Newtonian inverse-square law force giving rise to perihelion precession in planetary orbits. Gauge invariance protects the tensor and vector unparticle fields from picking up a mass from radiative corrections enabling them to generate long range forces. On the other hand a scalar unparticle will pick up a mass term from radiative corrections due to which a scalar field can give rise to only short-ranged forces. Thus the anomalous dimension of a scalar unparticle can not be constrained from Mercury's perihelion precession. In [65] we study the possibility of scalar unparticle to play the role of inflaton and look for its possible signatures in TT anisotropy spectrum of CMBR. This will be discussed consequently.

4.2 Unparticle long range forces and constraints on its anomalous dimension from Mercury perihelion precession

Unparticle exchange gives rise to long range forces which deviate from the usual inverse square law for massless particles due to the anomalous scaling of the unparticle propagator. In [26, 64] bounds have been put on the unparticle couplings from millimeter scale long range force experiments [66]. It is well known that a deviation from the Newtonian inverse square gravity will result in unclosed orbits which results in a shift in perihelion of planetary orbits. Since exchange of massless unparticles gives rise to long range forces which deviate from the inverse square law, we expect an additional contribution to the perihelion shift of planets in addition to that caused by general relativity. In [67] we consider the effect on the perihelion shift of Mercury due to the coupling of tensor and vector unparticles to SM particles. The perihelion shift due to general relativistic effects has been measured to 0.3% level and thus provides tight constraints on additional long range forces [68]. We find that this gives more stringent bounds on unparticle couplings compared to the one from fifth force search experiments at solar system distances [66]. Some consequences of unparticles in astrophysical phenomena has been explored in [69–72].

4.2.1 Ungravity from tensor unparticles

We take the gravitational coupling of the tensor unparticle (ungravitons [26]) to the stress-energy tensor $T_{\mu\nu}$ to be of the form

$$\kappa_* \frac{1}{\Lambda_u^{d_u-1}} \sqrt{g} T^{\mu\nu} O^U_{\mu\nu}, \tag{4.3}$$

where $\kappa_* = \frac{1}{\Lambda_u}\left(\frac{\Lambda_u}{M_u}\right)^{d_{uv}}$. We impose the gauge symmetry as in the case of gravity,

$$\begin{aligned} x_\mu &\rightarrow x_\mu + \epsilon_\mu \,, \\ O^U_{\mu\nu} &\rightarrow O^U_{\mu\nu} + \frac{\Lambda_u^{d_u-1}}{\kappa_*}\left(\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu\right), \end{aligned} \tag{4.4}$$

which ensures that the ungraviton remains massless below the scale Λ_u. The massless ungraviton results in long range forces which can be probed at solar system length scales.

The ungraviton propagators are [26]

$$\Delta^{\mu\nu\alpha\beta}(P) = B_{d_u} P^{\mu\nu\alpha\beta}(-P^2)^{d_u-2}, \tag{4.5}$$

where the normalization factor B_{d_u} is

$$B_{d_u} \equiv -\left(\frac{8\pi^{\frac{3}{2}}}{(2\pi)^{2d_u}}\right)\frac{\Gamma\left(2-d_u\right)\Gamma\left(d_u+\frac{1}{2}\right)}{\Gamma\left(2d_u\right)}, \tag{4.6}$$

and $P^{\mu\nu\alpha\beta}$ is the projection operator of the form

$$P^{\mu\nu\alpha\beta}(P) \equiv \frac{1}{2}(P^{\mu\alpha}P^{\nu\beta} + P^{\mu\beta}P^{\nu\alpha} - \alpha P^{\mu\nu}P^{\alpha\beta}), \tag{4.7}$$

where $P^{\mu\nu} = \left(-\eta^{\mu\nu} + \frac{P^\mu P^\nu}{P^2}\right)$. For massless ungravitons, obeying the gauge condition of Eq (4.4), $\alpha = 1$.

The ungravitational potential is obtained by taking the Fourier transform of the propagator $\Delta^{\mu\nu\alpha\beta}$ in the static limit ($P^0 = 0$) :

$$V_u(r) = \frac{\kappa_*^2}{\Lambda_u^{2d_u-2}} \int \frac{d^3\mathbf{P}}{(2\pi)^3} T_{\mu\nu}\Delta^{\mu\nu\alpha\beta}(P^0 = 0)T_{\alpha\beta}e^{i\mathbf{P}\cdot\mathbf{x}}, \tag{4.8}$$

where $|\mathbf{x}| = r$. Evaluating the integral gives the potential arising due to ungraviton exchange as

$$\begin{aligned} V_u(r) &= -m_1 m_2 \left(\frac{\kappa_*^2}{\Lambda_u^{2d_u-2}}\right)\left(\frac{2}{\pi^{2d_u-1}}\right)\frac{\Gamma\left(d_u+\frac{1}{2}\right)\Gamma\left(d_u-\frac{1}{2}\right)}{\Gamma\left(2d_u\right)}\left(\frac{1}{r^{2d_u-1}}\right) \\ &= -\frac{G_u m_1 m_2}{r^{2d_u-1}}, \end{aligned} \tag{4.9}$$

where G_u is defined to be

$$G_u \equiv \frac{\kappa_*^2}{\Lambda_u^{2d_u-2}} C(d_u), \tag{4.10}$$

and $C(d_u)$ is

$$C(d_u) \equiv \left(\frac{2}{\pi^{2d_u-1}}\right)\frac{\Gamma\left(d_u+\frac{1}{2}\right)\Gamma\left(d_u-\frac{1}{2}\right)}{\Gamma\left(2d_u\right)}. \tag{4.11}$$

We notice that if the anomalous dimension (d_u) of $O_{\mu\nu}$ is not equal to 1 there are deviations from the inverse square law. So for $d_u \neq 1$ the total potential will be of the form :

$$\begin{aligned} V(r) &= -\frac{Gm_1m_2}{r} - \frac{G_u m_1 m_2}{r^{2d_u-1}}. \\ &= -\frac{Gm_1m_2}{r}\left[1 + \frac{1}{G\Lambda_u^2}\left(\frac{\Lambda_u}{M_u}\right)^{2d_{uv}} \frac{C(d_u)}{\Lambda_u^{2d_u-2}}\frac{1}{r^{2d_u-2}}\right]. \end{aligned} \tag{4.12}$$

We will consider the case $d_u > 1$ as $d_u < 1$ will lead to forces which fall off slower than gravity and can be easily ruled out from fifth force experiments [66].

Perihelion precession of mercury orbit

In polar coordinates (r, θ), the equation of motion of a planet's orbit around the Sun is

$$\ddot{r} - r\dot{\theta}^2 + \frac{V(r)_{,r}}{m_p} = 0, \tag{4.13}$$

where m_p is the mass of the planet and the overdot ($\equiv \dot{}$) represents derivative with respect to time and $V_{,r} \equiv \frac{\partial V(r)}{\partial r}$. The angular momentum of the planet $l = m_p r^2 \dot{\theta}$ is a constant of motion.

Changing variables to $u(\theta) = \frac{1}{r(\theta)}$, Eq (4.13) can be written as

$$u_{,\theta\theta} + u = \alpha + \beta u^{2d_u - 2}. \tag{4.14}$$

Here $\alpha \equiv \frac{M m_p^2 G}{l^2}$ and $\beta \equiv \frac{M m_p^2 G_u (2d_u - 1)}{l^2}$, where M is the mass of the Sun. This is an inhomogeneous second order ordinary differential equation (ODE). Assuming the deviation from the inverse square law to be very small, we have $\beta << \alpha$ and Eq (4.14) can be solved using a perturbation expansion in β. To first order in β we assume the form of the solution to be

$$u(\theta) = u_0(\theta) + \beta u_1(\theta), \tag{4.15}$$

where u_0 is the solution of the ODE

$$u_{0,_{\theta\theta}} + u_0 = \alpha, \tag{4.16}$$

and u_1 is the particular solution of the inhomogeneous equation

$$u_{1,_{\theta\theta}} + u_1 = u_0^{2d_u - 2}. \tag{4.17}$$

The solution of Eq (4.16) is

$$u_0 = \frac{1 - e\cos(\theta)}{a(1 - e^2)}, \tag{4.18}$$

where a is the semi-major axis of the elliptical orbit of the planet, given by

$$a = \frac{l^2}{M m_p^2 G (1 - e^2)} \tag{4.19}$$

and e is the eccentricity of the orbit. As the eccentricity of Mercury's orbit is very small we keep terms only up to $\mathcal{O}(e)$ and neglect the higher order terms in Eq (4.18). Using the above form of u_0, $u_1(\theta)$ obeys the equation

$$u_{1_{\theta\theta}} + u_1 = \frac{1}{a^{2d_u - 2}} - \frac{(2d_u - 2)\, e\cos(\theta)}{a^{2d_u - 2}}. \tag{4.20}$$

This has the particular solution

$$u_1 = \frac{1}{a^{2d_u - 2}} - \frac{(d_u - 1)\, e}{a^{2d_u - 2}} \theta \sin(\theta). \tag{4.21}$$

Thus, from Eq (4.15), the trajectory of the planet to order β is given by

$$u = \frac{1}{a} + \beta\frac{1}{a^{2d_u-2}} - \frac{e}{a}\left[\cos(\theta) + \frac{\beta\,(d_u-1)}{a^{2d_u-3}}\theta\sin(\theta)\right]. \tag{4.22}$$

For small β, Eq (4.22) can be written as

$$u \approx \frac{1}{a} + \beta\frac{1}{a^{2d_u-2}} - \frac{e}{a}\left[\cos\left(\theta - \frac{\beta\,(d_u-1)}{a^{2d_u-3}}\theta\right)\right]. \tag{4.23}$$

To complete one full rotation, with a perihelion shift, the condition is

$$\theta\left(1 - \frac{\beta\,(d_u-1)}{a^{2d_u-3}}\right) = 2\pi, \tag{4.24}$$

which gives

$$\theta \approx 2\pi\left(1 + \frac{\beta\,(d_u-1)}{a^{2d_u-3}}\right), \tag{4.25}$$

keeping only terms linear in β. So the perihelion shift induced by ungraviton couplings is given by

$$\begin{aligned}\delta\theta &= 2\pi\left(\frac{\beta\,(d_u-1)}{a^{2d_u-3}}\right)\\ &= (d_u-1)(2d_u-1)C(d_u)\frac{2\pi}{G\Lambda_u^2}\left(\frac{\Lambda_u}{M_u}\right)^{2d_{uv}}\frac{1}{\Lambda_u^{2d_u-2}}\frac{1}{a^{2d_u-2}}.\end{aligned} \tag{4.26}$$

As expected, the perihelion shift vanishes for $d_u = 1$, as it should since it corresponds to the usual inverse square law case (with a different gravitational constant). Comparing the expression for the unparticle potential Eq (4.12) (for $r = a$) with the expression for perihelion advance we see that they are related as

$$\delta\theta \simeq (d_u-1)(2d_u-1)2\pi\frac{V_u}{V_N}, \tag{4.27}$$

where V_u is the unparticle exchange potential and V_N is the Newtonian potential. The constraint on the ungravity couplings derived from mercury perihelion are more stringent than that from fifth force measurement by testing deviation from Kepler's Law at planetary distances [73],[74]. However at millimeter scales there are stringent tests of deviations of Newton's Law as has been noted in [26, 64].

The observed precession of perihelion of mercury is 43.13 ± 0.14 arcsec/century [75] and the prediction from General Relativity (GR) is 42.98 arcsec/century. This means that at 2-σ the unparticle contribution is $-0.13 < \delta\theta < 0.43$. We derive a limit on unparticle coupling by demanding that the unparticle contribution does not exceed the discrepancy between measurement and GR. From the 2-σ upper bound on the possible contribution from unparticle given by Eq (4.26) we get the limit

$$(d_u - 1)(2d_u - 1)C(d_u)\frac{2\pi}{G\Lambda_u^2}\left(\frac{\Lambda_u}{M_u}\right)^{2d_{uv}}\frac{1}{(a\Lambda_u)^{2d_u-2}}\left(\frac{\text{century}}{T}\right) < 0.43 \text{ arcsec} \tag{4.28}$$

per century, where $T = 87.96$ days is the orbital time period of Mercury.

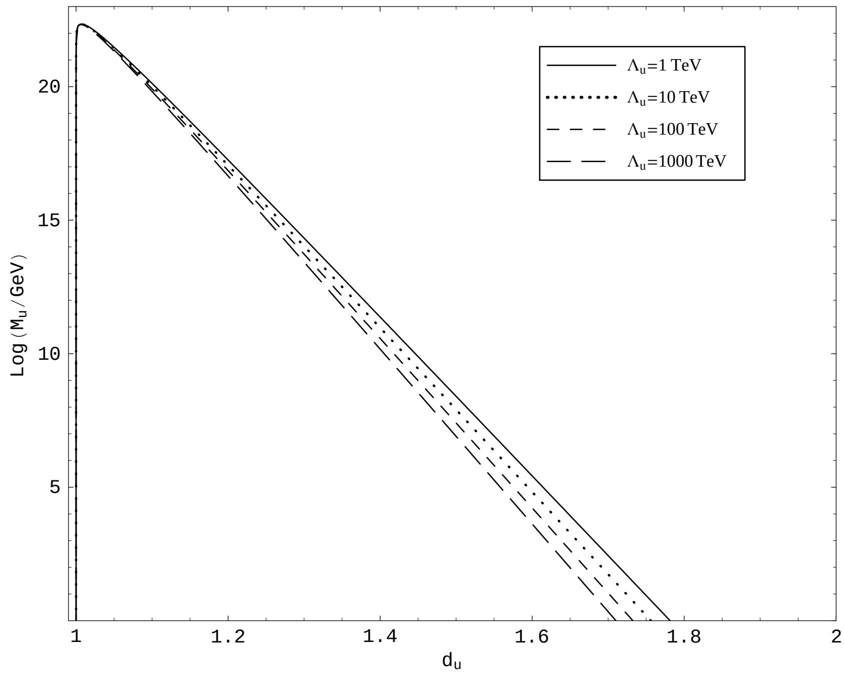

Figure 4.1: $\log\left(\frac{M_u}{\text{GeV}}\right)$ vs. d_u plot for tensor unparticles.

In Fig (4.1) we plot $\log\left(\frac{M_u}{\text{GeV}}\right)$ vs. d_u which gives the tensor unparticle contribution of 0.43 arcsec/century to the perihelion advance of mercury. Here regions above the curves represent the allowed values of $\log\left(\frac{M_u}{\text{GeV}}\right)$ and d_u from observations of Mercury orbit. We have taken $d_{uv} = 1$ and the values of Λ_u from 1 TeV to 1000 TeV. The areas above the curves represent the allowed regions for M_u and d_u at 2-σ for different values of Λ_u .

4.2.2 Long range force from vector unparticles

Now we consider long range forces resulting from the coupling of vector unparticles [63, 64] to baryonic and leptonic currents. The effective coupling is of the form

$$\frac{\lambda}{\Lambda_u^{d_u-1}} J^\mu O_\mu^U, \tag{4.29}$$

where J_μ is the baryonic or leptonic current. As in the tensor case, we assume that the unparticle operator O^U and the fermion fields Ψ obey a gauge symmetry

$$\begin{aligned} \Psi &\rightarrow \exp[i\alpha]\Psi \\ O_\mu^U &\rightarrow O_\mu^U + \frac{\Lambda_u^{d_u-1}}{\lambda}\partial_\mu\alpha. \end{aligned} \tag{4.30}$$

As a result of this $U(1)$ gauge symmetry the vector unparticle remains massless below the scale Λ_u. The gauge unparticle propagator is

$$\Delta^{\mu\nu} = A_{d_u} P^{\mu\nu}(-p^2)^{d_u-2}, \tag{4.31}$$

where

$$A_{d_u} \equiv \frac{16\pi^{\frac{5}{2}}}{(2\pi)^{2d_u}} \frac{\Gamma\left(d_u + \frac{1}{2}\right)}{\Gamma\left(d_u - 1\right)\Gamma\left(2d_u\right)}, \tag{4.32}$$

and

$$P^{\mu\nu}(p) = \eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}. \tag{4.33}$$

As usual, we get the unparticle exchange potential by taking the Fourier transform of the propagator given in Eq (4.31) in the static limit. This gives

$$\begin{aligned} V_u(r) &= \frac{1}{2\pi^{2d_u}} \frac{\lambda^2}{\Lambda_u^{2d_u-2}} \frac{\Gamma\left(d_u+\frac{1}{2}\right)\Gamma\left(d_u-\frac{1}{2}\right)}{\Gamma\left(2d_u\right)} \frac{N_1 N_2}{r^{2d_u-1}} \\ &= \frac{C'\left(d_u\right)\lambda^2 N_1 N_2}{r^{2d_u-1}}, \end{aligned} \tag{4.34}$$

where

$$C'(d_u) \equiv \frac{1}{2\pi^{2d_u}} \frac{1}{\Lambda_u^{2d_u-2}} \frac{\Gamma\left(d_u+\frac{1}{2}\right)\Gamma\left(d_u-\frac{1}{2}\right)}{\Gamma\left(2d_u\right)}, \tag{4.35}$$

is a constant and N_1 and N_2 are the total number of baryons ($N_i = \frac{M_i}{m_n}$,where M_i is the mass of the Sun or the planet and m_n is the nucleon mass) in the Sun and the planet. Hence the total potential is

$$\begin{aligned} V(r) &= V_N(r) + V_u(r) \\ &= -\frac{Gm_1m_2}{r}\left[1 - \frac{C'\left(d_u\right)\lambda^2 N_1 N_2}{Gm_1m_2}\frac{1}{r^{2d_u-2}}\right]. \end{aligned} \tag{4.36}$$

By following the same methodology as in the tensor case we find the perihelion shift, due to exchange of vector unparticles, to be

$$\delta\theta = -2\pi(d_u - 1)(2d_u - 1)\frac{C'(d_u)\,\lambda^2 N_1 N_2}{G m_1 m_2}\frac{1}{a^{2d_u-2}}. \tag{4.37}$$

Vector unparticle exchange would cause a retardation in the perihelion of mercury

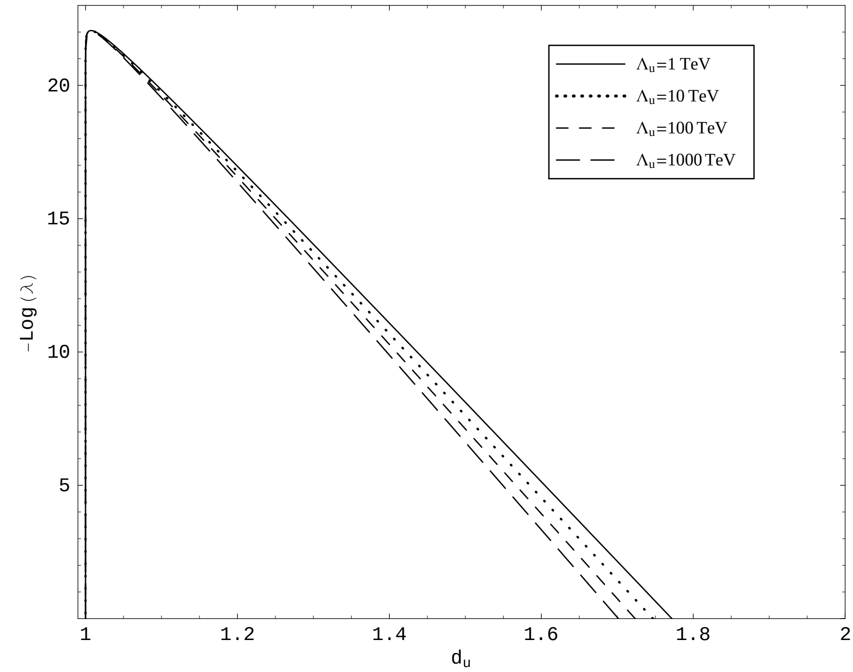

Figure 4.2: $-\log(\lambda)$ vs. d_u plot for vector unparticles

orbit ($\delta\theta < 0$) due to the fact that the force is repulsive. At 1-σ the discrepancy between theory and experiment is still positive ($0.01 < \delta\theta < 0.29$) which means that the vector unparticle force can be ruled out at 1-σ. At $2-\sigma$ the allowed range for a unparticle vector contribution is $-0.13 < \delta\theta < 0.43$. The maximum value of this retardation allowed from observations [75] and the prediction of general relativity is 0.13 arcsec/century at 2-σ. This puts an upper bound on the vector unparticle couplings

$$2\pi(d_u - 1)(2d_u - 1)\frac{C'(d_u)\,\lambda^2 N_1 N_2}{G m_1 m_2}\frac{1}{a^{2d_u-2}}\left(\frac{\text{century}}{T}\right) < 0.13 \text{ arcsec} \tag{4.38}$$

per century, where $T = 87.96$ days is the orbital time period of Mercury as stated before.

In Fig (4.2) we show $-\log(\lambda)$ vs. d_u plot taking $\delta\theta = 0.13$ arcsec/century. Here region above the curve represents the allowed values of $-\log(\lambda)$ and d_u from observations of Mercury orbit. We have taken the values of Λ_u from 1 TeV to 1000 TeV. The areas above the curves represent the allowed values of λ and d_u at 2-σ experimental error for different Λ_u after accounting for the contribution to perihelion shift from general relativity.

4.3 Unparticle as inflaton

There may be large deviations of the scale dimension d_u of a scalar particle from the canonical dimension $d_u = 1$ due to non-perturbative interactions at a high scale M. The scalar propagator of such an unparticle [25] of dimension d_u will be

$$\frac{1}{\left(p^2-\mu^2\right)^{2-d_u}}. \tag{4.39}$$

This gives rise to a deviation from inverse square law, as has been discussed in the previous section, having some interesting astrophysical consequences [67].

Here we will investigate the scenario of a scalar unparticle driving inflation. Such an attempt was also made in earlier works as in [76] and [77]. We will follow the general method of calculating power spectrum for an inflaton field using Källén-Lehmann representation discussed in the previous chapter (see Sec. (3.2)) as the flat space spectral function for unparticle in well known in literature. The spectral function for a scalar unparticle, where the conformal invariance is broken at a low energy μ, is given as [78]

$$\rho(\sigma^2) = A_{d_u}\theta(\sigma^2-\mu^2)(\sigma^2-\mu^2)^{d_u-2}, \tag{4.40}$$

where

$$A_{d_u} = \frac{16\pi^{\frac{5}{2}}}{(2\pi)^{2d_u}}\frac{\Gamma\left(d_u+\frac{1}{2}\right)}{\Gamma\left(d_u-1\right)\Gamma\left(2d_u\right)}. \tag{4.41}$$

In the limit $d_u \to 1$ when the scale dimension approaches the canonical dimension the spectral function in Eq. (4.40) approaches the ordinary massive particle spectral function [25]

$$\lim_{d_u\to 1}\left(A_{d_u}\theta(\sigma^2-\mu^2)(\sigma^2-\mu^2)^{d_u-2}\right) = \delta(\sigma^2-\mu^2). \tag{4.42}$$

If we assume that the inflaton is an unparticle of scale dimension d_u, then to calculate the power spectrum of the comoving density perturbation we assume the form of the potential of this unparticle inflaton as

$$V(\phi) = \frac{1}{2}\frac{\mu^2}{M^{2\Delta_u}}\phi^2, \tag{4.43}$$

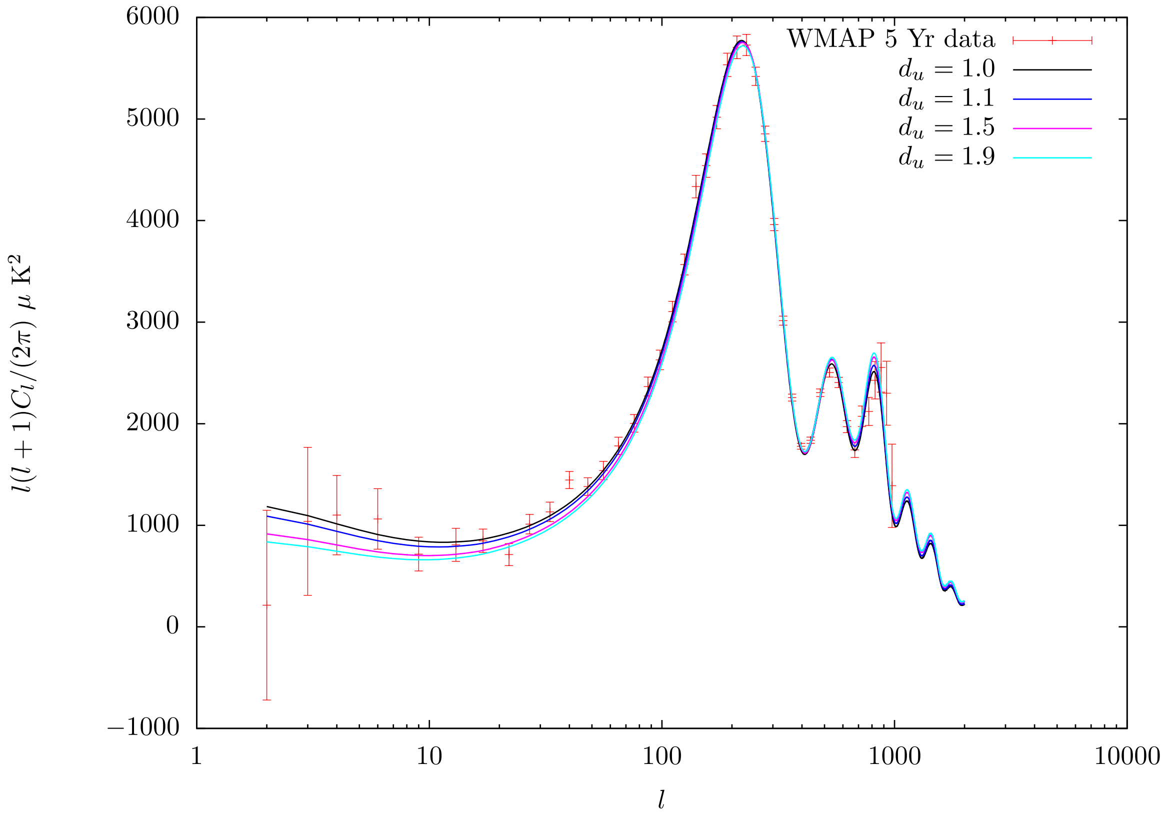

Figure 4.3: The TT angular spectrum for the Unparticle-inflaton

where the mass of the unparticle is taken as μ at which the scale invariance of the unparticle is broken and $\Delta_u = d_u - 1$. Since the scale dimension of the inflaton is different from unity, we introduce a heavy mass scale M such that $\phi \to \frac{\phi}{M^{\Delta_u}}$ and so the potential contains in the denominator $M^{2\Delta}$, compared to the usual quadratic potential $\mu^2\phi^2$. The Klein-Gordon equation of motion gives us

$$\frac{\dot{\phi}^2}{H^2} = \frac{m_{\rm Pl}^2 M^{2\Delta_u}}{4\Delta N}, \tag{4.44}$$

where ΔN is the number of e-foldings during inflation.

Using Eq (4.40) and Eq (4.44) in Eq (3.13) the power spectrum for the case of an unparticle inflaton will be

$$\mathcal{P}_{\mathcal{R}} = \frac{A_{d_u} H^2 \Delta N}{\pi^2 M_{\rm Pl}^2 M^{2\Delta_u}} \left(\frac{z}{2}\right)^{\frac{2}{3}\frac{\mu^2}{H^2}} \left(\frac{3H^2}{2\ln\left(\frac{2}{z}\right)}\right)^{\Delta_u} \left[\Gamma(\Delta_u) - \Gamma\left(\Delta_u, \frac{2}{3}\frac{\Lambda_u^2}{H^2}\ln\left(\frac{2}{z}\right)\right)\right], \tag{4.45}$$

where $\Gamma(.,.)$ is the incomplete gamma function and $\Lambda_u \ll H$ is the energy scale of the unparticle up to which the assumptions for the power spectrum is retained.

In Fig (4.3) we plot the TT angular spectrum for the Unparticle-inflaton with different values of d_u and M. The parameters used for the above plots are $H = 10^{13}$ GeV, $\Delta N = 60$, $\mu = 0.1 \times 10^{13}$ GeV and $\Lambda_u = 0.5 \times 10^{13}$ GeV. The values of M used are 1.158×10^2 GeV, 6.23×10^9 GeV and 3.715×10^{10} GeV for $d_u = 1.1$, $d_u = 1.5$ and $d_u = 1.9$ respectively. We see that as d_u deviates from the canonical value 1 there is a greater suppression of the power at large angular scales.

4.4 Conclusions

In this chapter we first put bounds on the tensor and vector unparticle anomalous dimensions with respect to its coupling to matter from perihelion precession of Mercury and secondly we venture the possibility of scalar unparticle playing role of an inflaton.

There are several bounds on unparticle couplings to standard model particles from collider experiments [79] from the anomalous missing energy spectrum. There are also bounds on such couplings from the cooling rates of supernova and stars [69–72]. If the conformal invariance of unparticles remains unbroken then these particles can give rise to extra long range forces [26, 64] which can be constrained from fifth force experiments [66]. In [67] we have considered unparticle gauge bosons of spin-1 and spin-2. The gauge symmetry ensures that the unparticles remain massless. The main characteristic feature of unparticle long range force which we apply in this paper is a deviation from the inverse square law which leads to a perihelion shift in planetary orbits. The constraints from perihelion shift are more stringent than the constraints from the deviation from the inverse square law at the scale of solar system distances [73, 74]. However at millimeter scales there are stringent tests of deviations of Newton's Law as has been noted in [26, 64]. Comparing our bounds on vector and tensor unparticle couplings with that of [26] and [64] we find our bounds based on perihelion precession are more stringent when $d_u \lesssim 1.4$.

When the inflaton field is taken as a scalar unparticle [65], the TT angular power spectrum of CMBR is suppressed at low l which may be related to the observation of low quadrupole power by WMAP [59].

Chapter 5

Thermal field theory and enhanced non-Gaussianity

5.1 Introduction

We discussed in detail about non-Gaussian features arising in single-field slow-roll inflationary model and importance of looking for primordial non-Gaussianities in Chapter 2. In this chapter we will discuss the case where the initial vacuum for the inflaton fluctuations is not the conventional Bunch-Davies vacuum but a thermal initial state.

It was shown earlier by Gangui et al. [30] and more recently in [29, 80–82] that if the initial state of the inflatons is not the Bunch-Davies vacuum but some excited state then there is an enhancement of the non-Gaussianity from such initial state effects. A natural example of a non-Bunch-Davies initial state arises if there is a radiation era prior to inflation [31]. Inflation takes place when the energy density of radiation ρ_r drops below the value of the potential of a coherent scalar field. In such models it is seen that the power spectrum is enhanced at low k which can be used to put constraint on the comoving temperature at the time of inflation [31]. These kind of inflation scenarios with a pre-radiation era have an interesting prediction that the B-mode polarization spectrum is enhanced at low l due the contribution of thermal gravitons [83, 84].

The scenario of thermal initial condition is very general and would be applicable for any model of inflation if there was a pre-inflationary radiation dominated era. The effects of the initial thermal era to be observable either in the CMB anisotropy spectrum or in the non-Gaussianities the perturbations entering the horizon today should have left the de-Sitter horizon at a temperature T not too small compared to H (the Hubble parameter at the time of inflation). If there were a large number of e-foldings prior to the present perturbation modes leaving the inflation horizon then the effect of the pre-inflationary thermal era would be unobservable. In models where the total number of e-foldings are just enough to solve the flatness and horizon problems, there can be a imprint of the spatial curvature at the time of inflation on the power spectrum [85]. A natural model where inflation commences just as the temperature falls below a critical temperature and is of limited duration is where a fermion pair forms a scalar condensate which acts as the inflaton. Such models have been studied in [23, 24].

In [86] we study non-Gaussianities in the primordial perturbations in single field inflation where there is radiation era prior to inflation. The thermal background of inflaton, gravitons and other fields is decoupled from the actual dynamical evolution of the inflaton unlike in the warm inflation models [87], where there can be large non-Gaussianities [88, 89] due to dissipative coupling between the inflaton and the radiation bath. In this model the temperature of the decoupled radiation bath goes down as $T_{\rm ph} = T/a$ where T is the constant comoving temperature. The thermal distribution functions which depend on the ratio $\frac{k_{\rm ph}}{T_{\rm ph}} = \frac{k}{T}$ (where k is the comoving wavenumber of the perturbations) retain the same form during inflation. We will first discuss the effect of such a pre-radiation era on comoving curvature perturbation and TT anisotropy spectrum of CMBR.

5.2 Thermal average of inflaton power spectra

If there was a radiation era prior to inflation one expects a thermal distribution of inflatons to be present which might have decoupled from other fields prior to inflation. It has been shown in [31] that this thermal distribution of inflaton modifies the power spectrum of inflaton fluctuations and the curvature power spectrum will have an additional temperature depended term. In this section we compute the two point correlation of inflaton perturbations taking this thermal distribution of inflatons into consideration.

The Fourier expansion of inflaton fluctuations in de Sitter space is

$$\delta\phi(\mathbf{x},t) = \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}} \left(b_{\mathbf{k}}\varphi_k(t) + b^{\dagger}_{-\mathbf{k}}\varphi^*_k(t) \right) e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{5.1}$$

where $\varphi_k(t)$ are the mode functions which satisfy the Klein-Gordon equation in Fourier space and $b_{\mathbf{k}}$ and $b^{\dagger}_{\mathbf{k}}$ are the annihilation and creation operators respectively. In Fourier space the inflaton fluctuations can be written as

$$\delta\phi(\mathbf{k},t) = b_{\mathbf{k}}\varphi_k(t) + b^{\dagger}_{-\mathbf{k}}\varphi^*_k(t). \tag{5.2}$$

The canonical commutation relation satisfied by these creation and annihilation operators is

$$\left[b_{\mathbf{k}_1}, b^{\dagger}_{\mathbf{k}_2} \right] = \delta^3(\mathbf{k_1} - \mathbf{k}_2), \tag{5.3}$$

with the vacuum satisfying $b_{\mathbf{k}}|0\rangle = 0$ at zero temperature, which ensures that the vacuum has zero occupation $N_k|0\rangle = 0$ where $N_k \equiv b^{\dagger}_{\mathbf{k}}b_{\mathbf{k}}$ is the number operator and the power spectrum of inflaton fluctuations $P_{\delta\phi}(k)$ will be (as has been defined in Eq. (2.44))

$$P_{\delta\phi}(k) \equiv \frac{k^3}{2\pi^2} \left\langle \delta\phi(k,t)\delta\phi(k,t) \right\rangle, \tag{5.4}$$

where $k \equiv |\mathbf{k}|$.

This scenario changes when there is a radiation era prior to inflation, as in this case the inflaton will have a thermal distribution during inflation. Due to this distribution the thermal vacuum $|\Omega\rangle \equiv |n_{k_1}, n_{k_2}, \cdots\rangle$ will now contain real particles yielding

$$N_k|\Omega\rangle = n_k|\Omega\rangle, \tag{5.5}$$

where n_k is the number of particles with momentum $\mathbf{k}$ present in the thermal vacuum. In general, i.e. for creation-annihilation operators with different momenta, one gets

$$b^{\dagger}_{\mathbf{k}_1}b_{\mathbf{k}_2}|\Omega\rangle = \delta^3(\mathbf{k}_1 - \mathbf{k}_2)n_{k_1}|\Omega\rangle. \tag{5.6}$$

For further discussions we will consider non-interacting real scalar fields for which the chemical potential $\mu = 0$. For a single inflaton with momentum $\mathbf{k}$ the partition function will be

$$z = \sum_{n_k=0}^{\infty} e^{-\beta n_k k} = \frac{1}{1 - e^{-\beta k}}\,, \tag{5.7}$$

where β is the inverse of the comoving temperature T. Due to this thermal distribution of the inflaton fluctuation a thermal statistical average of the two-point correlation function will determine the power spectrum

$$P^{\rm th}_{\delta\phi}(k) = \frac{k^3}{2\pi^2} \langle\Omega|\delta\phi(k,t)\delta\phi(k,t)|\Omega\rangle_\beta = \frac{k^3}{2\pi^2} \sum_{\varepsilon_k} p(\varepsilon_k) \langle\Omega|\delta\phi(k,t)\delta\phi(k,t)|\Omega\rangle\,. \tag{5.8}$$

Here $p(\varepsilon_k)$ is the probability of the system to be in the state $\varepsilon_k \equiv n_k k$ which is defined as

$$p(\varepsilon_k) \equiv \frac{e^{-\beta n_k k}}{\sum_{n_k} e^{-\beta n_k k}} = \frac{e^{-\beta n_k k}}{z}\,, \tag{5.9}$$

where z is given in Eq. (5.7). However, due to the thermal distribution of the inflaton field the inflaton fluctuations will follow the relations given in Eq. (5.3) and Eq. (5.5) which yield

$$\langle\Omega|\delta\phi(k,t)\delta\phi(k,t)|\Omega\rangle = |\varphi_k(t)|^2 \langle\Omega| \left(1 + 2N_k\right) |\Omega\rangle = |\varphi_k(t)|^2 \left(1 + 2n_k\right). \tag{5.10}$$

Hence the power spectrum given in Eq. (5.8) will be

$$P^{\rm th}_{\delta\phi}(k) = \frac{k^3}{2\pi^2} |\varphi_k(t)|^2 \frac{1}{z} \sum_{n_k} e^{-\beta n_k k} \left(1 + 2n_k\right) = \frac{k^3}{2\pi^2} |\varphi_k(t)|^2 \left(1 + 2f_B(k)\right), \tag{5.11}$$

where $f_B(k) \equiv \frac{1}{e^{\beta k}-1}$ is the Bose-Einstein distribution. To get the last equality in the above equation the following relation is used [90]

$$\sum_{n=0}^{\infty} n q^n = \frac{q}{(1-q)^2}. \tag{5.12}$$

Now for a light inflaton ($m \ll H$, m being the mass of the inflaton and H being the Hubble parameter during inflation) the mode function has the solution as given in Eq. (2.42)[50] :

$$|\varphi_k| \simeq \frac{H}{\sqrt{2k^3}} \left(\frac{k}{aH}\right)^{\frac{3}{2}-\nu_\varphi}\,, \tag{5.13}$$

where a is the cosmic scale factor and $\nu_\varphi \simeq \frac{3}{2} - \frac{m_\phi^2}{H^2}$. In a generic single field inflationary model this mode function solution along with the k^3 factor in the power spectrum gives a nearly scale invariant spectra for inflaton fluctuations. But due to the thermal distribution of the inflaton fluctuations, expression for power spectrum in Eq. (5.11) contains an additional temperature dependent factor of $(1+2f_B(k)) = \coth(\beta k/2)$. Thus the thermal power spectrum of inflaton fluctuations is given by

$$P_{\delta\phi}^{\rm th}(k) = \coth(\beta k/2) P_{\delta\phi}(k), \tag{5.14}$$

and hence the thermal average of the power spectrum for comoving curvature perturbations defined in Eq. (2.48) will be

$$\mathcal{P}_{\mathcal{R}}^{\rm th}(k) = \coth(\beta k/2) \mathcal{P}_{\mathcal{R}}(k), \tag{5.15}$$

as has been already stated in [31].

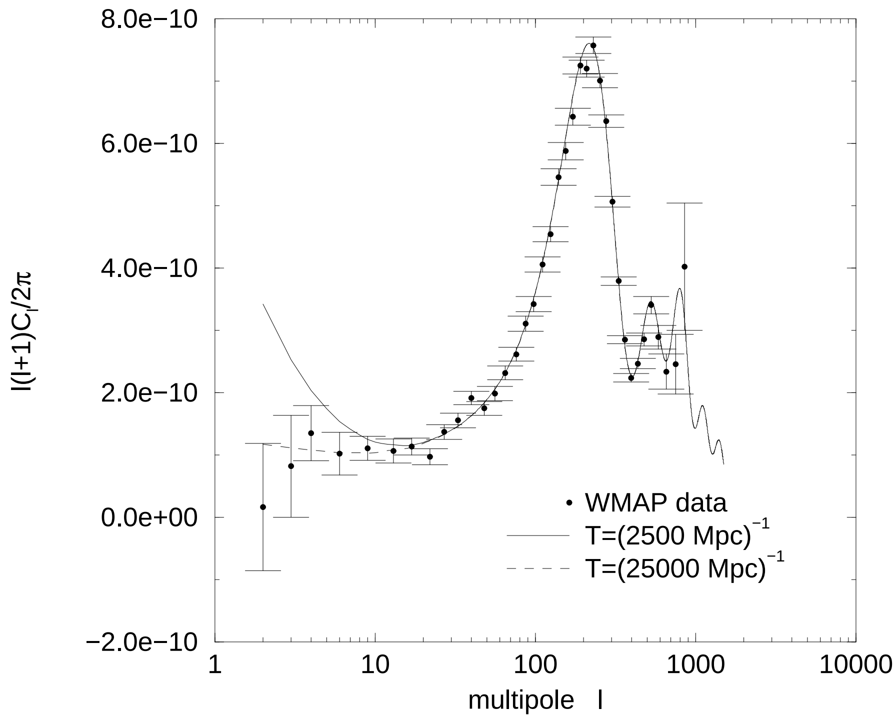

Figure 5.1: The TT anisotropy spectrum of CMBR for a inflationary scenario with prior radiation era, taken from the original paper [31]

In Fig. (5.1) the CMB power spectrum generated using the thermal comoving curvature power spectrum is compared with WMAP data and a constraint on comoving temperature has been put from Fig. (5.2) as $T < 1.0 \times 10^{-3}$ Mpc^{-1}

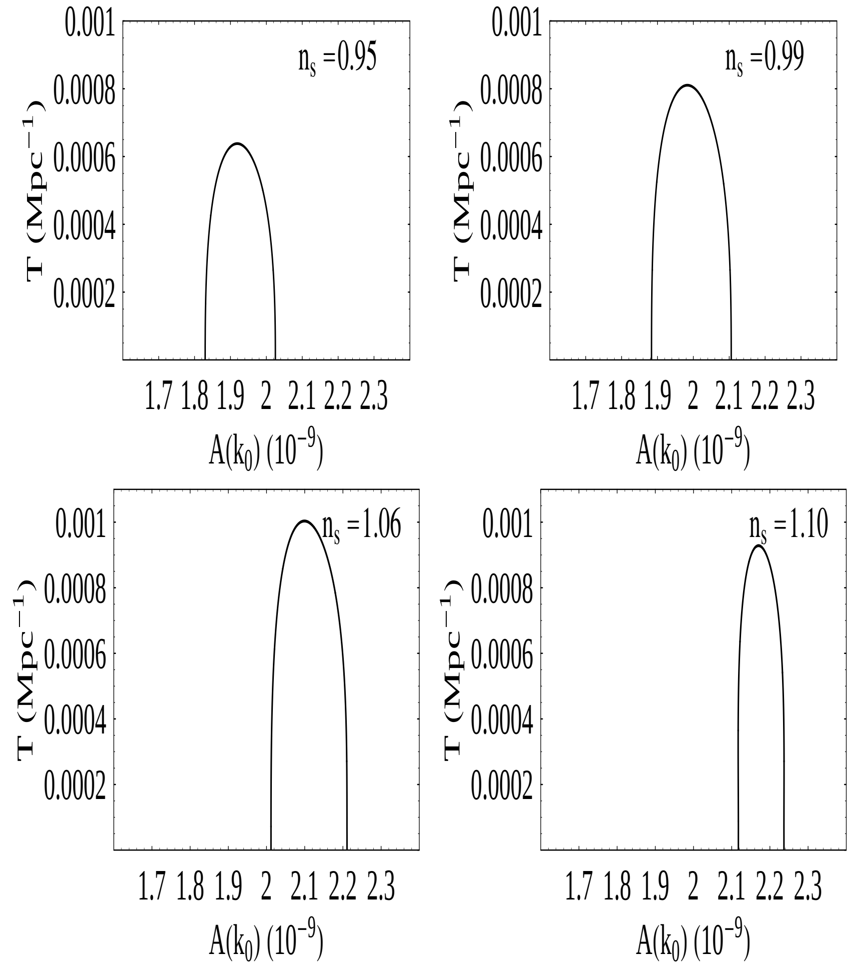

Figure 5.2: The TT anisotropy spectrum of CMBR for a inflationary scenario with prior radiation era, taken from the original paper [31]

(with the convention that the present scale factor $a_0 \equiv 1$). Such a bound is also found in [84] from thermal primordial gravitational waves. Since $T = a_i T_{\rm ph}$ where $T_{\rm ph}$ and a_i are the physical temperature and the scale factor when our current horizon scale crossed the de-Sitter horizon during inflation, this constraint can be rewritten as $T_0 < 4.2H$. As the comoving wavenumber $k = a_i H$ one can put a lower bound on βk as

$$\beta k = \frac{a_i H}{a_i T_{\rm ph}} > 0.238. \qquad (5.16)$$

This lower bound on βk will be used in following sections to quantify the maximum value of non-Gaussianity in thermal bispectrum and thermal trispectrum.

5.3 Non-Gaussianity in bispectrum from thermal distribution of inflaton

The three point correlation function of comoving curvature perturbations $\mathcal{R}$ or the bispectrum is defined in Eq. (2.55) and the non-linear parameter for bispectrum in the case of single field slow-roll model is given in Eq. (2.69). In presence of a pre-inflationary radiation era the bispectrum will also receive a modification as in the case of the power spectrum just discussed in the previous section. Hence in this case the three point correlation function of the non-linear curvature perturbation will be

$$\langle \mathcal{R}_{NL}(\mathbf{k}_1)\mathcal{R}_{NL}(\mathbf{k}_2)\mathcal{R}_{NL}(\mathbf{k}_3)\rangle_\beta = \frac{1}{2}\left(\frac{H}{\dot{\phi}}\right)^2 \frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right)\int \frac{d^3\mathbf{p}}{(2\pi)^{\frac{3}{2}}} \times$$
$$\left[\langle\delta\phi_L(\mathbf{p})\delta\phi_L(\mathbf{k}_1-\mathbf{p})\delta\phi_L(\mathbf{k}_2)\delta\phi_L(\mathbf{k}_3)\rangle_\beta + \text{perms}\right], \tag{5.17}$$

where R.H.S. of the above equation contains the thermal average of four-point correlation functions of the inflaton perturbations.

We will first generalize the case of thermal average of the two-point correlation function to derive the thermal average of the four-point correlation function of scalar perturbations with arbitrary four momenta $\mathbf{k}_i$. The thermal average of any higher order correlation function of inflaton perturbation will be of the form

$$\langle \phi_{k_1}\phi_{k_2}\phi_{k_3}\cdots\rangle_\beta = \sum_{\{n_{k_i}\}} p(k_1,k_2,k_3,\cdots)\,\langle\Omega|\phi_{k_1}\phi_{k_2}\phi_{k_3}\cdots|\Omega\rangle, \tag{5.18}$$

where the thermal probability of the occupancy of different momenta $\mathbf{k}_i$ and $\varepsilon \equiv \sum_{n_{k_r}} n_{k_r}k_r$ is

$$p(k_1,k_2,k_3,\cdots) \equiv \frac{\prod_r e^{-\beta n_{k_r}k_r}}{\prod_r \sum_{n_k} e^{-\beta n_{k_r}k_r}} = \frac{\prod_r e^{-\beta n_{k_r}k_r}}{Z}. \tag{5.19}$$

Here Z is the grand partition function of massless inflatons with energies $E_{k_r} = \sqrt{\mathbf{k}_r{}^2} = k_r$ which is given as

$$Z = \prod_r \sum_{n_{k_r}=0}^{\infty} e^{-\beta n_{k_r}k_r} = \prod_r \frac{1}{1-e^{-\beta k_r}}, \tag{5.20}$$

where r is the index for different energy levels.

The four-point correlation function of inflaton fluctuations with four different momenta contains six different combinations of two creation and two annihilation operators and thermal average of one of these combinations can be derived as follows :

Let us consider the thermal average of $\left(b^{\dagger}_{-\mathbf{k}_1} b_{\mathbf{k}_2} b^{\dagger}_{-\mathbf{k}_3} b_{\mathbf{k}_4}\right)$ which yields

$$\begin{aligned}\left\langle b^{\dagger}_{-\mathbf{k}_1} b_{\mathbf{k}_2} b^{\dagger}_{-\mathbf{k}_3} b_{\mathbf{k}_4}\right\rangle_{\beta} &= \sum_{\varepsilon} p(k_1,k_2,k_3,k_4)\left\langle \Omega \left| b^{\dagger}_{-\mathbf{k}_1} b_{\mathbf{k}_2} b^{\dagger}_{-\mathbf{k}_3} b_{\mathbf{k}_4} \right| \Omega \right\rangle \\ &= \delta^3(\mathbf{k}_1+\mathbf{k}_2)\delta^3(\mathbf{k}_3+\mathbf{k}_4)\frac{1}{Z}\sum_{n_{k_1}}\sum_{n_{k_2}} e^{-\beta\left(n_{k_1}k_1+n_{k_3}k_3\right)} n_{k_1} n_{k_3},\end{aligned} \tag{5.21}$$

where $Z=\left(\frac{1}{1-e^{-\beta k_1}}\right)\left(\frac{1}{1-e^{-\beta k_3}}\right)$. The summations in the above equation yields

$$\left\langle b^{\dagger}_{-\mathbf{k}_1} b_{\mathbf{k}_2} b^{\dagger}_{-\mathbf{k}_3} b_{\mathbf{k}_4}\right\rangle_{\beta} = \delta^3(\mathbf{k}_1+\mathbf{k}_2)\delta^3(\mathbf{k}_3+\mathbf{k}_4)\left[f_B(k_1)f_B(k_3)\right], \tag{5.22}$$

where the identity stated in Eq. (5.12) is used. Similarly the thermal average of other combinations of the two creation and two annihilation operators will be

$$\begin{aligned}\left\langle b_{\mathbf{k}_1} b_{\mathbf{k}_2} b^{\dagger}_{-\mathbf{k}_3} b^{\dagger}_{-\mathbf{k}_4}\right\rangle_{\beta} &= \delta^3(\mathbf{k}_1+\mathbf{k}_4)\delta^3(\mathbf{k}_2+\mathbf{k}_3)\left[1+f_B(k_1)\right]+\delta^3(\mathbf{k}_1+\mathbf{k}_3)\times \\ &\quad \delta^3(\mathbf{k}_2+\mathbf{k}_4)\left[1+f_B(k_1)+f_B(k_2)+f_B(k_1)f_B(k_2)\right], \qquad (5.23)\\ \left\langle b_{\mathbf{k}_1} b^{\dagger}_{-\mathbf{k}_2} b_{\mathbf{k}_3} b^{\dagger}_{-\mathbf{k}_4}\right\rangle_{\beta} &= \delta^3(\mathbf{k}_1+\mathbf{k}_2)\delta^3(\mathbf{k}_3+\mathbf{k}_4)\times \\ &\quad \left[1+f_B(k_1)+f_B(k_3)+f_B(k_1)f_B(k_3)\right], \qquad (5.24)\\ \left\langle b_{\mathbf{k}_1} b^{\dagger}_{-\mathbf{k}_2} b^{\dagger}_{-\mathbf{k}_3} b_{\mathbf{k}_4}\right\rangle_{\beta} &= \delta^3(\mathbf{k}_1+\mathbf{k}_2)\delta^3(\mathbf{k}_3+\mathbf{k}_4)\left[f_B(k_3)+f_B(k_1)f_B(k_3)\right], \qquad (5.25)\\ \left\langle b^{\dagger}_{-\mathbf{k}_1} b_{\mathbf{k}_2} b_{\mathbf{k}_3} b^{\dagger}_{-\mathbf{k}_4}\right\rangle_{\beta} &= \delta^3(\mathbf{k}_1+\mathbf{k}_2)\delta^3(\mathbf{k}_3+\mathbf{k}_4)\left[f_B(k_1)+f_B(k_1)f_B(k_3)\right], \qquad (5.26)\\ \left\langle b^{\dagger}_{-\mathbf{k}_1} b^{\dagger}_{-\mathbf{k}_2} b_{\mathbf{k}_3} b_{\mathbf{k}_4}\right\rangle_{\beta} &= -\delta^3(\mathbf{k}_1+\mathbf{k}_4)\delta^3(\mathbf{k}_2+\mathbf{k}_3)f_B(k_1) \\ &\quad +\delta^3(\mathbf{k}_1+\mathbf{k}_3)\delta^3(\mathbf{k}_2+\mathbf{k}_4)\left[f_B(k_1)f_B(k_2)\right]. \qquad (5.27)\end{aligned}$$

Hence the thermal average of a general four-point correlation function with four different momenta will be

$$\begin{aligned}&\left\langle \delta\phi(\mathbf{k}_1,t)\delta\phi(\mathbf{k}_2,t)\delta\phi(\mathbf{k}_3,t)\delta\phi(\mathbf{k}_4,t)\right\rangle_{\beta} = \left|\varphi_{k_1}(t)\right|^2\left|\varphi_{k_2}(t)\right|^2\times \\ &\left[\delta^3(\mathbf{k}_1+\mathbf{k}_4)\delta^3(\mathbf{k}_2+\mathbf{k}_3)+\delta^3(\mathbf{k}_1+\mathbf{k}_3)\delta^3(\mathbf{k}_2+\mathbf{k}_4)\left\{1+f_B(k_1)+\right.\right. \\ &\left.\left. f_B(k_2)+2f_B(k_1)f_B(k_2)\right\}\right]+\left|\varphi_{k_1}(t)\right|^2\left|\varphi_{k_3}(t)\right|^2\delta^3(\mathbf{k}_1+\mathbf{k}_2)\delta^3(\mathbf{k}_3+\mathbf{k}_4) \\ &\times\left\{1+2f_B(k_1)+2f_B(k_3)+4f_B(k_1)f_B(k_3)\right\}.\end{aligned} \tag{5.28}$$

With this general result one can calculate the thermal average of the three-point correlation function of the comoving curvature perturbations using Eq. (5.17) as

$$\langle \mathcal{R}_{NL}(\mathbf{k}_1)\mathcal{R}_{NL}(\mathbf{k}_2)\mathcal{R}_{NL}(\mathbf{k}_3)\rangle_\beta \simeq (2\pi)^{-\frac{3}{2}}\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(2m_{\rm Pl}^2\epsilon)\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right)$$
$$\times\left[\frac{P_{\mathcal{R}}(k_1)}{k_1^3}\frac{P_{\mathcal{R}}(k_2)}{k_2^3}\left(1+\frac{1}{2}f_B(k_1)+\frac{1}{2}f_B(k_2)+f_B(k_1)f_B(k_2)\right)\right.$$
$$+\frac{P_{\mathcal{R}}(k_2)}{k_2^3}\frac{P_{\mathcal{R}}(k_3)}{k_3^3}\left(1+\frac{1}{2}f_B(k_2)+\frac{1}{2}f_B(k_3)+f_B(k_2)f_B(k_3)\right)$$
$$\left.+\frac{P_{\mathcal{R}}(k_3)}{k_3^3}\frac{P_{\mathcal{R}}(k_1)}{k_1^3}\left(1+\frac{3}{2}f_B(k_3)+\frac{3}{2}f_B(k_1)+3f_B(k_3)f_B(k_1)\right)\right], \tag{5.29}$$

where $P_{\mathcal{R}}(k)$ is defined in Eq. (2.56). The three momenta form a triangle due to the presence of the delta function. The non-linear parameter f_{NL} for these three momenta configurations are discussed below :

- *Squeezed triangle case* : For a "squeezed" triangle the configuration suggests $|\mathbf{k}_1| \approx |\mathbf{k}_2| \approx k \gg |\mathbf{k}_3|$. In this configuration the f_{NL} will be

$$f_{NL}^{\rm th} = \frac{5}{6}(\delta-\epsilon)\left(2+2f_B(k_3)\coth\left(\frac{\beta k}{2}\right)\right). \tag{5.30}$$

  At low temperature $\beta\to\infty$ and $f_B(k_3)\to 0$, yielding the same contribution to the f_{NL} for super-cool inflation. The minimum value k_3 can obtain when the corresponding wavelength is of Hubble size while crossing the horizon such that $\lambda_3 = \frac{1}{k_3} \sim H^{-1}$ which implies $\beta k_3 \sim 0.238$. Hence it yields

$$\begin{aligned} f_{NL}^{\rm th} &= \frac{5}{6}(\delta-\epsilon)\times 2\left(1+3.72\coth\left(\frac{\beta k}{2}\right)\right) \\ &= f_{NL}\times 2\left(1+3.72\coth\left(\frac{\beta k}{2}\right)\right). \end{aligned} \tag{5.31}$$

  A lower bound on βk can be given from thermal power spectrum which is given in Eq. (5.16) and for this constraint f_{NL} will be maximum yielding $f_{NL}^{\rm th} = 64.82 f_{NL} \sim 0.65$.

- *Equilateral triangle case* : For a "equilateral" triangle we have $|\mathbf{k}_1| = |\mathbf{k}_2| = |\mathbf{k}_3| = k$ and in this case the f_{NL} will be

$$\begin{aligned} f_{NL}^{\rm th} &= \frac{5}{6}(\delta-\epsilon)\left(3+\frac{5}{4\sinh^2\left(\frac{\beta k}{2}\right)}\right) \\ &= f_{NL}\left(3+\frac{5}{4\sinh^2\left(\frac{\beta k}{2}\right)}\right). \end{aligned} \tag{5.32}$$

This implies that for the modes corresponding to our present horizon $\beta k > 0.238$ and the $f_{NL}^{\rm th} = 90.85 f_{NL} \sim 0.9$.

- *Folded triangle case* : For "flattened" isosceles triangle or the "folded" triangle case we have $|\mathbf{k}_1| = |\mathbf{k}_3| = \frac{1}{2}|\mathbf{k}_2| = k$ and in this case the f_{NL} will be

$$f_{NL}^{\rm th} = \frac{5}{6}(\delta - \epsilon)\left(3 + \frac{1}{\sinh^2\left(\frac{\beta k}{2}\right)}\right) = f_{NL}\left(3 + \frac{1}{\sinh^2\left(\frac{\beta k}{2}\right)}\right). \quad (5.33)$$

In this configuration the non-linearity will be $f_{NL}^{\rm th} = 73.28 f_{NL} \sim 0.73$ at horizon crossing for the modes corresponding to our current horizon.

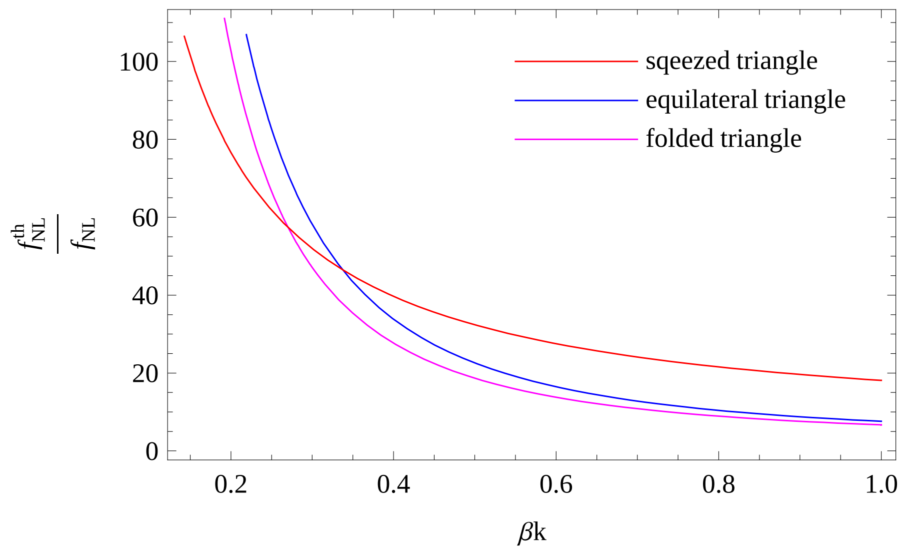

Figure 5.3: $\frac{f_{NL}^{\rm th}}{f_{NL}}$ as a function of βk for different triangle configurations of the three momenta.

In Fig. (5.3) the thermal enhancement factor $\frac{f_{NL}^{\rm th}}{f_{NL}}$ is plotted as a function of βk for three different triangle configurations of the three momenta. From the above discussion it is seen that the maximum contribution for f_{NL} comes from the "equilateral" configuration, though the contribution from the other two configurations are of the same order. Non-Gaussianity in all these three cases may be measurable by the 21-cm background radiation observations [45].

5.4 Non-Gaussianity in trispectrum from thermal distribution of inflaton

In this model of slow-roll inflation with a radiation era prior to inflation, the analysis for trispectrum turns out to be quite different from the super-cool inflationary scenario. In a generic single-field slow-roll inflationary scenario, with no prior radiation era, the trispectrum non-Gaussianity is of the order of ϵ^2 and the non-Gaussianity arising from trispectrum turns out to be smaller than that of from bispectrum. The reason that the non-Gaussianities arising from single-field slow-roll model depends upon the slow-roll parameters is due to the non-linear evolution of $\mathcal{R}$ whose co-efficients are function of slow-roll parameters (see Eq. (2.65)).

In presence of a pre-inflationary radiation era the four point correlation function which contributes to the non-Gaussianity will be thermal averaged as in the case of power spectrum and bispectrum. It is worth pointing out that due to thermal averaging, in the case of slow-roll inflation with prior radiation era, the four-point function of *linear comoving curvature perturbations* is not just the square of the two-point function as that would have been the case at zero temperature. Following Eq. (2.61) the connected part of the four-point correlation function of comoving curvature perturbation will be

$$\begin{aligned}\langle\mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\mathcal{R}(\mathbf{k}_4)\rangle_c &= \langle\mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\mathcal{R}(\mathbf{k}_4)\rangle \\ &- \left(\langle\mathcal{R}_L(\mathbf{k}_1)\mathcal{R}_L(\mathbf{k}_2)\rangle\langle\mathcal{R}_L(\mathbf{k}_3)\mathcal{R}_L(\mathbf{k}_4)\rangle + 2\text{ perm}\right).\end{aligned} \tag{5.34}$$

So in this case, by connected part of the four-point function, as defined in above equation, we will simply mean the excess of the thermal averaged of four-point function than the square of its two-point Gaussian part. We will now define the non-linear parameter τ_{NL} in the following way

$$\begin{aligned}&\langle\mathcal{R}_L(\mathbf{k}_1)\mathcal{R}_L(\mathbf{k}_2)\mathcal{R}_L(\mathbf{k}_3)\mathcal{R}_L(\mathbf{k}_4)\rangle_c \equiv \langle\mathcal{R}_L(\mathbf{k}_1)\mathcal{R}_L(\mathbf{k}_2)\mathcal{R}_L(\mathbf{k}_3)\mathcal{R}_L(\mathbf{k}_4)\rangle_\beta \\ &-\Big(\langle\mathcal{R}_L(\mathbf{k}_1)\mathcal{R}_L(\mathbf{k}_2)\rangle_\beta\langle\mathcal{R}_L(\mathbf{k}_3)\mathcal{R}_L(\mathbf{k}_4)\rangle_\beta + 2\text{ perm.}\Big) \\ &= \tau_{NL}\left[\frac{P_\mathcal{R}(k_1)}{k_1^3}\frac{P_\mathcal{R}(k_2)}{k_2^3}\delta^3(\mathbf{k}_1+\mathbf{k}_3)\delta^3(\mathbf{k}_2+\mathbf{k}_4) + 2\text{ perm}\right].\end{aligned} \tag{5.35}$$

Hence in this case τ_{NL} will not depend upon the slow-roll parameters as the non-linear $\mathcal{R}$ is not contributing to give rise to trispectrum non-Gaussian features. The thermal average of the four-point correlation function of inflaton fluctuation has been calculated in the last section in Eq. (5.28). Using this equation the thermal average of the four-point correlation of curvature perturbation can be derived as

$$\langle \mathcal{R}_L(\mathbf{k}_1)\mathcal{R}_L(\mathbf{k}_2)\mathcal{R}_L(\mathbf{k}_3)\mathcal{R}_L(\mathbf{k}_4)\rangle_\beta = \frac{P_{\mathcal{R}}(k_1)}{k_1^3}\frac{P_{\mathcal{R}}(k_2)}{k_2^3}\left[\delta^3(\mathbf{k}_1+\mathbf{k}_4)\delta^3(\mathbf{k}_2+\mathbf{k}_3)+\right.$$
$$\delta^3(\mathbf{k}_1+\mathbf{k}_3)\delta^3(\mathbf{k}_2+\mathbf{k}_4)\left\{1+f_B(k_1)+f_B(k_2)+2f_B(k_1)f_B(k_2)\right\}\Big]+\frac{P_{\mathcal{R}}(k_1)}{k_1^3}\times$$
$$\frac{P_{\mathcal{R}}(k_3)}{k_3^3}\left[\delta^3(\mathbf{k}_1+\mathbf{k}_2)\delta^3(\mathbf{k}_3+\mathbf{k}_4)\left\{1+2f_B(k_1)+2f_B(k_3)+4f_B(k_1)f_B(k_3)\right\}\right], \tag{5.36}$$

and the thermal average of two-point function can be given in terms of the power spectrum as

$$\langle \mathcal{R}_L(\mathbf{k}_1)\mathcal{R}_L(\mathbf{k}_2)\rangle_\beta = \frac{P_{\mathcal{R}}(k_1)}{k_1^3}\left(1+2f_B(k_1)\right)\delta^3(\mathbf{k}_1+\mathbf{k}_2). \tag{5.37}$$

Hence the connected part will be

$$\langle \mathcal{R}_L(\mathbf{k}_1)\mathcal{R}_L(\mathbf{k}_2)\mathcal{R}_L(\mathbf{k}_3)\mathcal{R}_L(\mathbf{k}_4)\rangle_c = -\frac{P_{\mathcal{R}}(k_1)}{k_1^3}\frac{P_{\mathcal{R}}(k_2)}{k_2^3}\left[\delta^3(\mathbf{k}_1+\mathbf{k}_3)\delta^3(\mathbf{k}_2+\mathbf{k}_4)\right.$$
$$\left\{f_B(k_1)+f_B(k_2)+2f_B(k_1)f_B(k_2)\right\}+2\delta^3(\mathbf{k}_1+\mathbf{k}_4)\delta^3(\mathbf{k}_2+\mathbf{k}_3)\times$$
$$\left.\left\{f_B(k_1)+f_B(k_2)+f_B(k_1)f_B(k_2)\right\}\right] \tag{5.38}$$

The four momenta in this case will not form a quadrilateral as in other trispectrum cases. But due to the presence of two delta functions on the R.H.S. of the above equation the non-linear parameter τ_{NL} can be calculated in the following two cases

1. $\mathbf{k}_1 = -\mathbf{k}_3$, $\mathbf{k}_2 = -\mathbf{k}_4$ and $k_i = k$ $(i = 1, 2, 3, 4)$:
$$\tau_{NL}^{\text{th}} = -\frac{1}{\cosh(\beta k) - 1}. \tag{5.39}$$
The maximum observable value of $\left|\tau_{NL}^{\text{th}}\right|$ can be obtained using the constraint on the comoving temperature as $\beta k > 0.238$. Hence for $\beta k \sim 0.238$ one finds that $\tau_{NL}^{\text{th}} \sim -35.14$.

2. $\mathbf{k}_1 = -\mathbf{k}_4$, $\mathbf{k}_2 = -\mathbf{k}_3$ and $k_i = k$ $(i = 1, 2, 3, 4)$:
$$\tau_{NL}^{\text{th}} = -2\frac{1-2e^{\beta k}}{\left(e^{\beta k}-1\right)^2}. \tag{5.40}$$
The maximum value of τ_{NL}^{th} for this case will be $\tau_{NL}^{\text{th}} \sim -42.58$.

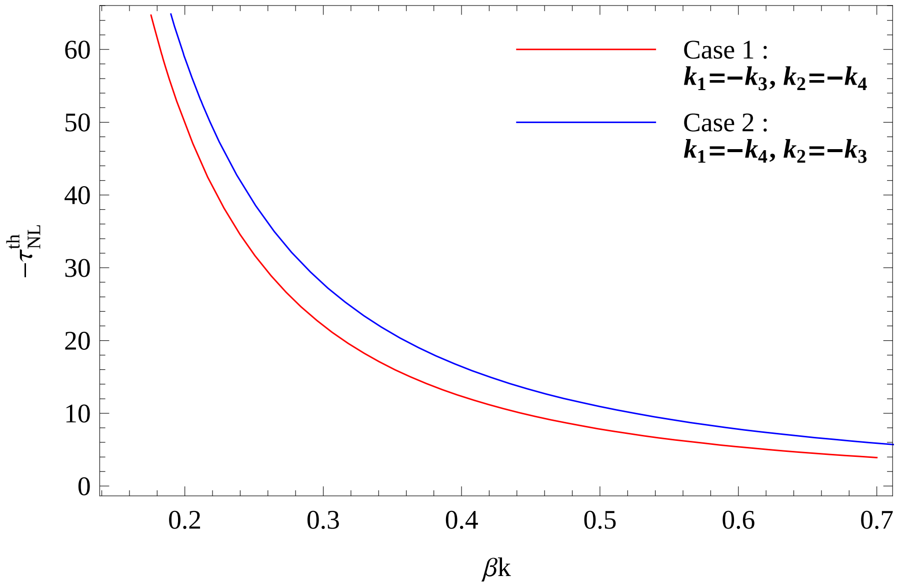

Figure 5.4: Plot of $\tau_{NL}^{\rm th}$ for two different momenta configurations as a function of βk

In Fig. (5.4) we have plotted $\tau_{NL}^{\rm th}$ as a function of βk. We find that the maximum value of non-Gaussianity comes from the configuration when $\mathbf{k}_1 = -\mathbf{k}_4$, $\mathbf{k}_2 = -\mathbf{k}_3$ and $k_i = k$ $(i = 1, 2, 3, 4)$ which is ~ -42. We do not compare this contribution to non-Gaussianity due to thermal initial states with the zero temperate case as there is no contribution from the later at this order and the leading order τ_{NL} in zero temperature is $\mathcal{O}(\epsilon^2)$.

5.5 Conclusion

We studied the effect of a decoupled thermal spectrum of inflatons (which exist in the scenario where the inflation is preceded by a prior thermal era) on the non-Gaussianity of the primordial perturbation. We found that thermal inflatons can enhance the bispectrum non-Gaussianity parameter f_{NL} by a factors of $(65 - 90)$ depending upon the momentum configuration. The zero temperature non-Gaussianity parameter f_{NL} in single field inflation models is proportional to the slow roll parameters and is expected to be of order $\sim 10^{-2}$. Therefore the observed value of f_{NL} in thermal history models will be of ~ 1. This is too small to be measured by WMAP or even the forthcoming PLANCK experiment. Measurements of anisotropies in the Hydrogen 21-cm radiation background can detect non-Gaussianities as low as $f_{NL} \sim 0.1$ [45], and this may be the ideal experiment in which non-Gaussianities with a thermal origin can be observed. The 21-cm observations may also be able to measure the non-Gaussianity in the trispectrum $\tau_{NL} \sim \mathcal{O}(10)$ and for which the prediction from thermal history inflation scenarios is $0 > \tau_{NL} > -43$. We conclude that a signature of thermal inflaton background at the time of inflation is a large trispectrum non-Gaussianity compared to the bispectrum non-Gaussianity.

Chapter 6

Conclusion

Phenomenology of early Universe has become an active field of research in last few decades as several satellite experiments like COBE, WMAP and very recently PLANCK have opened new vistas to look into the dynamics of early Universe by determining their imprints on several features of CMBR. Quantum Field Theory plays a major role in analyzing early Universe phenomena like inflation [8], reheating [91] of Universe after inflation, Baryogenesis [92] and Letpogenesis [93] and Big Bang Nucleosynthesis [94]. We focus mainly in the inflationary dynamics and its signatures in CMBR in this thesis. We have applied several methods of Quantum Field Theory to extend the already existing inflationary scenarios by including interesting features like inflaton's short-ranged interactions of the and thermal distribution of the inflaton due to a radiation era preceded by inflation.

The pleasingly simple and widely accepted single-field slow-roll inflationary model has been extensively discussed in Chapter 2. In such a scenario inflation is driven by one scalar field, with canonical kinetic term, which slowly rolls along its potential driving the Universe to inflate exponentially. The preferred initial vacuum chosen for inflaton field is the Bunch-Davies vacuum. Such a model predicts a nearly scale invariant power spectrum (as given in Eq. (2.48)) and almost Gaussian distribution of the primordial perturbations which are consistent with the CMBR data [2]. The amount of primordial non-Gaussianity predicted by such a model of inflation are tiny : non-Gaussianity arising from the bispectrum is of the order of the slow roll parameter ϵ (Eq. (2.69)) whereas that of from trispectrum is of the order of ϵ^2 (Eq. (2.73)). Such tiny non-Gaussianities are not likely to be measured either by the present experiments like WMAP or PLANCK or by future experiments where the non-Gaussianities imprinted in the 21-cm background will be measured.

Including interactions of the inflaton field is the simplest extension of the above mentioned inflationary scenario and it is being done most generally by modifying the inflaton's potential. Many such Particle Physics motivated potentials have been studied in the literature [20]. In Quantum Field Theory, one way of treating interacting fields is by using methods of Källén-Lehmann spectral representation which is a non-perturbative method of calculating two-point correlation functions of quantum fields such as Feynman propagator, Wightman function etc. In Chapter 3 we apply the same formalism of Källén-Lehmann spectral representation to calculate the power spectrum of an interacting scalar field as power spectrum of an inflaton is nothing but a equal-time Wightman function of inflaton field in a curved background. Though we calculate the power spectrum in quasi-de Sitter background, which is a curved background, one can use the flat space spectral function of interacting scalar field to determine the power spectrum because these kind of interactions, being short-ranged, can avoid the effect of gravity at short wavelength limit. We derive a general method of calculating power spectrum of interacting scalar field using Källén-Lehmann spectral function and the form of the comoving curvature power spectrum for interacting inflaton is given in Eq. (3.13).

In Chapter 3 we analyze two cases where such method can be applied. First we consider the case of a decaying inflaton where the decay width, Γ, of the inflaton is smaller than H/N (H being the Hubble constant during inflation and N being the number of e-foldings inflation lasts for). Such a scenario is natural to take into account as the inflaton must decay at the end of inflation to reheat the Universe. Again as the lifetime of the inflaton is of the same order of the duration of inflation, the correction due to the inflaton's decay width will not reflect into the inflaton's potential. We observe that the decay width of the inflaton yields suppression of long distance correlations and thereby a loss in the quadrupole anisotropy of the TT anisotropy spectrum of CMBR. Secondly we analyze a scenario where the inflaton is not a fundamental scalar but is a composite of fundamental fermionic constituents. The fact that a fundamental scalar is still unobserved in nature motivates such a scenario. If the compositeness scale of the condensate inflaton is of the order of the scale of perturbations then also the features due to the composite nature of inflaton will not reflect into the potential. We observe that due to the composite nature of the inflaton the power spectrum shows more interesting variations yielding oscillatory features in the low l region of the TT anisotropy spectrum of the CMBR. Wavelet analysis of WMAP data reveals that the actual data may have such oscillatory features [52]. These kind of suppression of power or the oscillatory features in the low l region of CMBR anisotropy spectrum may be vindicated in the WMAP data and confirmed in future by PLANCK.

Scalar fields, being naturally homogeneous and isotropic, became the most favorable candidate for inflaton. In spite of that attempts were made in the literature to look for other quantum fields to play the role of an inflaton, such as vectors fields [21], *classical* and homogeneous spinor fields [22], condensate of spinor fields [23, 24]. In Chapter (4) we deal with an exotic quantum field, the *unparticle* [25], which has peculiar properties like its anomalous dimensions due to which it yields long range forces while exchanged between two systems. Tensor unparticle, generating long range forces through the coupling with energy-momentum tensor, can mimic gravity [26] and thus often named as *ungraviton* in literature. Due to its anomalous dimension, the potential yielded due to exchange of such ungravitons gives rise to forces which deviates from usual Newtonian inverse square law force. Thus if such ungravitons are being exchanged between Sun and planets in the Solar system, it can give rise to extra perihelion precession of planetary orbits. Vector unparticles can also be exchanged in such Solar system bodies as it couples to baryonic matter present in the Sun and the planet. We put bounds on the coupling of the unparticles from Mercury's perihelion precession and showed in Chapter (4) that unparticle coupling is more stringent when its anomalous dimension $d_u \lesssim 1.4$. In this chapter we also vindicate the possibility of having scalar unparticle as inflaton. Such a scenario also results in suppression of power in the low multipole region which can be confirmed by WMAP or PLANCK data in future.

We discuss in Chapter (2) that if the number of scalar fields present during inflation is grater than one [28] or the inflaton field has higher derivative interacting terms yielding non-canonical kinetic term for the inflaton [29] then such scenarios yield large primordial non-Gaussianity compared to that of single-field slow-roll inflationary model. In [30] it has been shown that non-Bunch-Davies vacuum can give rise to large non-Gaussianities. Such a scenario can arise when there is a radiation era preceded by inflation. Inflation takes place when the energy density of radiation drops below the value of the potential of a coherent scalar field. Due to the initial thermal vacuum yielding thermal distribution of the inflaton fluctuations, we use the techniques of Thermal Field Theory to perform the thermal averaging of the two, three and four-point correlation functions of inflaton fluctuations. We show that the thermal averaging enhances the f_{NL} by a factor of at most 90 (in the equilateral configuration) from its value obtained from single-field slow-roll inflationary model. This amount of primordial non-Gaussianity arising from bispectrum is in the range of detectability with the 21-cm anisotropy measurements [45]. On the other hand the four-point function in this case appears due to the thermal averaging and the fact that thermal averaging of four-point correlation is not the same as the square of the thermal averaging of the two-point function. Due to this fact τ_{NL} is not proportional to the slow-roll parameters and can be as large as -42. Non-Gaussianities in the four-point correlation of the order of 10 can be detected by 21-cm background observations [46] and thus the trispectrum non-Gaussianity appearing in this case is in the rage of its detectability. As the single-field slow-roll inflationary model predict $f_{NL} > |\tau_{NL}|$, then measurement of larger trispectrum non-Gaussianity than bispectrum can be considered as a signature of such a pre-inflationary radiation era.

To conclude, we have applied several methods of Quantum Field Theory to analyze different dynamics of early Universe. But the features we thus obtained, like suppression of power (due to resonant or unparticle inflaton) or oscillatory features in the low l region of TT anisotropy of CMBR (due to composite inflaton) can be confirmed by more precise measurements of CMBR anisotropy by PLANCK. On the other hand, the primordial non-Gaussianity measurement which can confirm the existence of pre-inflationary radiation era should await till the low-frequency 21-cm observation becomes possible by advent of adequate technology.

Appendix A

Basics of FLRW Cosmology

In this section we briefly describe the essential features of Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology which is described by FLRW metric, considered as the background metric of our Universe. This homogeneous and spatially isotropic (i.e. the 3-surface of constant curvature) metric has its unique line element in spherical spatial coordinates as

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]. \tag{A.1}$$

The 3-dimensional space (4-dimensional space-time) can be elliptical (closed), Euclidean (flat) or hyperbolic (open) depending on whether $k = 1,\ 0,$ or -1 respectively. This metric is able to describe an expanding universe through its dynamical parameter $a(t)$ known as the *scale factor*.

The Einstein field equation in General Relativity connects the background metric with the matter content of the universe as

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{A.2}$$

where the Einstein tensor $G_{\mu\nu}$ is derived from the background metric and $T_{\mu\nu}$ being the energy-momentum tensor of the matter content of the Universe. The perfect fluid, in its rest frame and under the assumption of homogeneity and isotropy, is described by a energy-momentum tensor as

$$T^{\mu}_{\ \nu} = \mathrm{diag}\left(\rho(t), -p(t), -p(t), -p(t)\right), \tag{A.3}$$

ρ and p being the energy density and pressure of the fluid respectively. With the FLRW metric (Eq. A.1) along with a perfect fluid as the content of the Universe, the Einstein field equations given in Eq. (A.2) lead to which is known as the Friedmann equation given as

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}. \tag{A.4}$$

Here the overdot refers to a time derivative $\dot{} \equiv \partial_t$ and H being the *Hubble parameter*. This equation can be re-written in terms of physically measurable quantities as

$$\Omega - 1 = \frac{k}{a^2 H^2}, \tag{A.5}$$

where $\Omega \equiv \frac{\rho}{\rho_{\rm crit}}$ is the ratio of total to the critical density and the critical density of Universe at time t is defined as $\rho_{\rm crit} \equiv \frac{3H^2}{8\pi G}$. The subscript 0 will refer to the current epoch and the curvature density of Universe at present is defined as $\Omega_k = \Omega_0 - 1$ which includes contributions from matter, radiation and any form of energy such as a cosmological constant. Observations show that $-0.0178 < \Omega_k < 0.0063$ for WMAP+BAO+SN[a] and $-0.0133 < \Omega_k < 0.0084$ for WMAP+BAO+$\mathrm{H_0}$ [2] which implies that our present Universe is very flat ($\Omega_k \sim 0$). Hence, for simplicity, we will consider $k = 0$ for the rest of our discussion. In the same spirit the line element for FLRW metric convenient for further discussion will be

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] = dt^2 - a^2(t)d\mathbf{x}^2, \tag{A.6}$$

where the last equality is written in terms of Cartesian coordinates. At this point it is important to introduce the concept conformal time τ as

$$d\tau = \frac{dt}{a}. \tag{A.7}$$

Then line element becomes conformally flat and can be written as

$$ds^2 = a^2(\tau)\left(d\tau^2 - d\mathbf{x}^2\right). \tag{A.8}$$

Now let us go back and discuss the features of Einstein field equations in FLRW cosmology with a perfect fluid as the matter content of the Universe. The conservation of energy-momentum leads to the equation

$$\dot{\rho} = -3H(\rho + p), \tag{A.9}$$

which also yields the evolution of energy density with scale factor as

$$\rho = \rho_0 \left(\frac{a_0}{a}\right)^{3(1+\omega)}, \tag{A.10}$$

defining the state parameter as $\omega \equiv \frac{p}{\rho}$. Eq. (A.9) along with the Friedmann equation (Eq. (A.4)) yields the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{A.11}$$

This equation shows that for an accelerating Universe ($\ddot{a} > 0$), the fluid dominating the energy density of the Universe should have an equation of state as

$$p < -\frac{1}{3}\rho. \tag{A.12}$$

Any conventional physical fluid, having positive pressure, is unable to drive the Universe to accelerate. But inflation is such a scenario during which Universe accelerates and inflates exponentially. Hence to yield inflation Quantum theory comes in to rescue. The existence of scalar fields in Quantum theory allows the Universe to accelerate by slow-rolling of a scalar field through its potential.

Appendix B

Dynamics of single-field slow-roll inflationary model

We will briefly summarize the essential features of inflationary dynamics in terms of its simplest model known as the single-field slow-roll inflation. An inflationary model is defined by specifying inflaton's kinetic and potential term along with its coupling to gravity. In its simplest model the inflaton field is minimally coupled to gravity. In general, the total action of a minimally coupled scalar field $\phi(t, \vec{x})$ can be written as a sum of gravitational Einstein-Hilbert action $S_{\rm EH}$ and the action of the scalar field $S(\phi)$, with canonical kinetic term, as

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\left[-\frac{M_{\rm Pl}^2}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \\ &= S_{\rm EH} + S(\phi). \end{aligned} \tag{B.1}$$

Here $M_{\rm Pl} \equiv \frac{1}{\sqrt{8\pi G}}$ is the *reduced Planck mass* and R is the Ricci curvature scalar. The field equation of motion is obtained by varying the scalar action with field $\frac{\delta S(\phi)}{\delta\phi} = 0$, which yields

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2\phi}{a^2} + V_{,\phi} = 0, \tag{B.2}$$

where $V_{,\phi} \equiv \frac{\partial V}{\partial\phi}$. The energy-momentum of the field is obtained by varying the action with the metric $g^{\mu\nu}$ as

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S\,(\phi)}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial^\alpha\phi\partial_\alpha\phi - V(\phi)\right). \tag{B.3}$$

Under the assumption of a perfect fluid, defined in Eq. (A.3), the energy-momentum tensor of a scalar field takes the form

$$\begin{aligned} \rho_\phi &= \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\frac{(\nabla\phi)^2}{a^2} + V(\phi) \\ p_\phi &= \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}\frac{(\nabla\phi)^2}{a^2} - V(\phi) \end{aligned} \tag{B.4}$$

Hence a scalar field is able to accelerate the Universe if the configuration satisfies :

$$\frac{1}{2}\frac{(\nabla\phi)^2}{a^2} \ll V(\phi), \tag{B.5}$$

$$\frac{1}{2}\dot{\phi}^2 \ll V(\phi), \tag{B.6}$$

which allow a equation of state for the scalar field as $p_\phi = -\rho_\phi$, good enough to drive inflation. Since in the first condition the gradient redshifts as the universe expands, this condition will always be satisfied if it is initially satisfied. Hence the scalar field which drives inflation will be a homogeneous field (i.e. a scalar field varies only with time). In this spirit one can decompose a scalar field in its "classical part" ϕ_0, which is the expectation value of the field and responsible for driving inflation, and quantum fluctuations around ϕ_0 as

$$\phi(t,\vec{x}) = \phi_0(t) + \delta\phi(t,\vec{x}). \tag{B.7}$$

Hence the equation of motion of inflaton field will be

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_{,\phi} = 0, \tag{B.8}$$

which can be obtained from Eq. (B.2).

B.1 Conditions for slow-rolling of the inflaton

The second condition given in Eq. (B.6) tells us that the kinetic term of the inflaton field is negligible with respect to its potential, which signifies that the field is slow-rolling its potential. This condition also demands that $V_{,\phi} \gg \ddot{\phi}$, which yields the equation of motion for a inflaton field during slow-roll as

$$3H\dot{\phi}_0 \sim -V_{,\phi}\,. \tag{B.9}$$

The slow-roll of inflaton depends up on the shape of the potential. Defining the *slow-roll parameters* during inflation as

$$\begin{aligned} \epsilon &\equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2, \\ \eta &\equiv M_{\rm Pl}^2\frac{V_{,\phi\phi}}{V}, \end{aligned} \tag{B.10}$$

the slow-roll of inflaton is ensured by having ϵ, $|\eta| \ll 1$ and inflation ends when the slow-roll conditions are violated i.e. $\epsilon \sim 1$.

As during inflation the energy density of Universe is dominated by the inflaton field, the Friedmann equation given in Eq. (A.4) will take the form

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}V(\phi). \tag{B.11}$$

Another two useful form of the slow-roll parameter ϵ required for further discussions are

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{2M_{\rm Pl}^2}\frac{\dot{\phi}^2}{H^2}, \tag{B.12}$$

which can be derived from the definition of ϵ given in Eq. (B.10) and using Eq. (B.9) and Eq. (B.11).

The duration of inflation is quantified by the number of *e-folds* which is defined as

$$N(\phi) = \ln\frac{a_{\rm end}}{a} \sim \frac{1}{M_{\rm Pl}^2}\int_{\phi_{\rm end}}^{\phi}\frac{V}{V_{,\phi}}d\phi. \tag{B.13}$$

Inflation should last for at least $N_{\rm total} = \ln\frac{a_{\rm end}}{a_{\rm begin}} \sim 60$ to solve the severe pathologies of Big Bang model like the Horizon problem and the Flatness problem.

Appendix C

Perturbations during Inflation

In this section we will discuss the perturbations in the Einstein field tensor and in the Klein-Gordon equation generated during inflation. The scalar perturbations in the FLRW metric given in Eq. (2.4) are not invariant quantities but change under a change of coordinates. Consider a coordinate change as

$$\begin{aligned} \tilde{x}^0 &= x^0 + \xi^0 \\ \tilde{x}^i &= x^i + \delta^{ij}\beta_{,j} \,. \end{aligned} \tag{C.1}$$

The invariance of line element in both the coordinate systems demands

$$ds^2 = \tilde{g}_{\mu\nu} d\tilde{x}^\mu d\tilde{x}^\nu = g_{\mu\nu} dx^\mu dx^\nu, \tag{C.2}$$

where we can relate the two coordinate systems as :

$$\begin{aligned} \tilde{a}^2(\tilde{x}_0) &= a^2(x_0)\left(1 + 2\frac{a'}{a}\xi^{0\prime}\right), \\ d\tilde{x}^0 &= (1+\xi^{0\prime})dx^0 + \partial_i \xi^0 dx^i, \\ d\tilde{x}^i &= dx^i + \partial^i \beta' dx^0 + \partial_j \partial^i \beta dx^j. \end{aligned} \tag{C.3}$$

Thus up to first order in perturbations one gets

$$\begin{aligned} \left(d\tilde{x}^0\right)^2 &\simeq (1+2\xi^{0\prime})\left(dx^0\right)^2 + 2\partial_i\xi^0 dx^0 dx^i, \\ d\tilde{x}^i d\tilde{x}^j &\simeq dx^i dx^j + \partial^i\beta' dx^0 dx^j + \partial^j \beta' dx^0 dx^i + \partial_k \partial^j \beta dx^k dx^i + \partial_k \partial^i \beta dx^k dx^j. \end{aligned} \tag{C.4}$$

In the longitudinal gauge $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ are diagonal yielding vanishing mixed terms like $d\tilde{x}^0 d\tilde{x}^i$. Thus putting these values of $\tilde{a}^2(\tilde{x}_0)$, $\left(d\tilde{x}^0\right)^2$ and $d\tilde{x}^i d\tilde{x}^j$ into Eq. (C.2) and equating one gets

$$\begin{aligned} \tilde{A} &= A - \frac{a'}{a}\xi^0 - \xi^{0\prime}, \\ \tilde{\psi} &= \psi + \frac{a'}{a}\xi^0. \end{aligned} \tag{C.5}$$

These transformations will be required in deriving the gauge-invariant quantities later on in this chapter.

The gauge transformations also changes scalar quantities other than the scalar metric perturbations. Let us consider a background scalar quantity $f(t)$ which is defined independently of the coordinates at a given space-time point. If we only change the slicing then only the time coordinate will be affected as

$$\tilde{\tau}(\tau, \mathbf{x}) = \tau + \xi^0(\tau, \mathbf{x}). \tag{C.6}$$

This change of slicing will not change the form of $f(t)$ at a given spatial position but will change the perturbations around it. This implies

$$f(\tau) + \delta f(\tau, \mathbf{x}) = f(\tilde{\tau}) + \widetilde{\delta f}(\tilde{\tau}, \mathbf{x}) = f(\tilde{\tau}) + \widetilde{\delta f}(\tau, \mathbf{x}), \tag{C.7}$$

where we have used Eq. (C.6) in getting the last equality. Thus one gets

$$\widetilde{\delta f}(\tau, \mathbf{x}) = \delta f(\tau, \mathbf{x}) - (f(\tilde{\tau}) - f(t)) \simeq \delta f(\tau, \mathbf{x}) - f'\xi^0. \tag{C.8}$$

If we consider a change in threading there will be no change in the perturbation as the background value $f(t)$ is independent of position.

C.1 Perturbed Einstein field equations

In the longitudinal gauge the perturbed metric will take the form as

$$g_{\mu\nu} = g^0_{\mu\nu} + \delta g_{\mu\nu} = a^2(\tau) \begin{pmatrix} 1+2A & 0 \\ 0 & -(1-2\psi)\delta_{ij} \end{pmatrix}, \tag{C.9}$$

whose inverse metric will be

$$g^{\mu\nu} = g^{\mu\nu}_0 + \delta g^{\mu\nu} = \frac{1}{a^2(\tau)} \begin{pmatrix} 1-2A & 0 \\ 0 & -(1+2\psi)\delta^{ij} \end{pmatrix}. \tag{C.10}$$

- **Perturbed affine connections :**

  Assuming the affine connections to be symmetric under exchange of its lower indices, the affine connections can be derived from a metric as

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g_0^{\alpha\sigma} \left[g^0_{\sigma\beta,\gamma} + g^0_{\sigma\gamma,\beta} - g^0_{\gamma\beta,\sigma} \right], \tag{C.11}$$

  where the perturbed affine connections will be

$$\delta\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} \delta g^{\alpha\sigma} \left[g^0_{\sigma\beta,\gamma} + g^0_{\sigma\gamma,\beta} - g^0_{\gamma\beta,\sigma} \right] + \frac{1}{2} g_0^{\alpha\sigma} \left[\delta g_{\sigma\beta,\gamma} + \delta g_{\sigma\gamma,\beta} - \delta g_{\gamma\beta,\sigma} \right] \tag{C.12}$$

  Hence for the metric given in Eq. (C.9), the non-zero affine connections will be

$$\begin{array}{lll} \Gamma^0_{00} = \frac{a'}{a}, & \Gamma^i_{0j} = \frac{a'}{a}\delta^i{}_j, & \Gamma^0_{ij} = \frac{a'}{a}\delta_{ij}, \\ \delta\Gamma^0_{00} = A', & \delta\Gamma^0_{0i} = \partial_i A, & \delta\Gamma^0_{ij} = -\left(2\frac{a'}{a}(A+\psi) + \psi'\right)\delta_{ij}, \\ \delta\Gamma^i_{00} = \partial^i A, & \delta\Gamma^i_{0j} = -\psi'\delta^i{}_j, & \delta\Gamma^i_{jk} = \partial^i\psi\delta_{jk} - \partial_k\psi\delta^i{}_j - \partial_j\psi\delta^i{}_k. \end{array} \tag{C.13}$$

- **Perturbed Ricci tensors :**

  Knowing the affine connections of a given metric the Ricci tensors of the space-time can be calculated as:

$$R_{\mu\nu} = \partial_\alpha \Gamma^{\alpha}_{\mu\nu} - \partial_\mu \Gamma^{\alpha}_{\alpha\nu} + \Gamma^{\alpha}_{\sigma\alpha}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\alpha}_{\sigma\nu}\Gamma^{\sigma}_{\mu\alpha}, \tag{C.14}$$

  and perturbing which one gets

$$\delta R_{\mu\nu} = \partial_\alpha \delta\Gamma^{\alpha}_{\mu\nu} - \partial_\mu \delta\Gamma^{\alpha}_{\alpha\nu} + \delta\Gamma^{\alpha}_{\sigma\alpha}\Gamma^{\sigma}_{\mu\nu} + \Gamma^{\alpha}_{\sigma\alpha}\delta\Gamma^{\sigma}_{\mu\nu} - \delta\Gamma^{\alpha}_{\sigma\nu}\Gamma^{\sigma}_{\mu\alpha} - \Gamma^{\alpha}_{\sigma\nu}\delta\Gamma^{\sigma}_{\mu\alpha}. \tag{C.15}$$

  Hence for the perturbed FLRW metric the Ricci tensors will be

$$R_{00} = -3\frac{a''}{a} + 3\left(\frac{a'}{a}\right)^2, \qquad R_{0i} = 0, \qquad R_{ij} = \left[\frac{a''}{a} + \left(\frac{a'}{a}\right)^2\right]\delta_{ij}, \tag{C.16}$$

  where the perturbed ones are

$$\begin{aligned} \delta R_{00} &= \partial_i\partial^i A + 3\psi'' + 3\left(\frac{a'}{a}\right)(\psi' + A'), \\ \delta R_{0i} &= 2\partial_i\psi' + 2\left(\frac{a'}{a}\right)\partial_i A, \\ \delta R_{ij} &= \left[-2\frac{a''}{a}(A+\psi) - 2\left(\frac{a'}{a}\right)^2(A+\psi) - \frac{a'}{a}(A' + 5\psi') - \psi'' + \partial_k\partial^k\psi\right]\delta_{ij} \\ &\quad + \partial_i\partial_j(\psi - A). \end{aligned} \tag{C.17}$$

- **Perturbed Ricci scalars :**

The Ricci scalar or the curvature scalar of a space-time is

$$R = g^{\mu\nu} R_{\mu\nu}, \tag{C.18}$$

and the perturbation of the Ricci scalar is

$$\delta R = \delta g^{\mu\nu} R_{\mu\nu} + g^{\mu\nu} \delta R_{\mu\nu}. \tag{C.19}$$

Hence for the perturbed FLRW metric the Ricci scalar and the perturbed Ricci scalar are

$$\begin{aligned} R &= -\frac{6}{a^2}\left(\frac{a''}{a}\right), \\ \delta R &= \frac{1}{a^2}\left[12\frac{a''}{a}A + 2\partial_i\partial^i A + 6\frac{a'}{a}A' + 6\psi'' + 18\frac{a'}{a}\psi' - 4\partial_i\partial^i\psi\right] \end{aligned} \tag{C.20}$$

respectively.

- **Perturbed Einstein tensor :**

The Einstein tensor $G_{\mu\nu}$ is constructed by the Ricci tensor and Ricci scalar as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}, \tag{C.21}$$

and its linear order perturbation yields

$$\delta G_{\mu\nu} = \delta R_{\mu\nu} - \frac{1}{2}\delta R g_{\mu\nu} - \frac{1}{2} R \delta g_{\mu\nu}. \tag{C.22}$$

In components the Einstein tensor is

$$G_{00} = 3\left(\tfrac{a'}{a}\right)^2, \qquad G_{0i} = 0, \qquad G_{ij} = \left\{-2\tfrac{a''}{a} + \left(\tfrac{a'}{a}\right)^2\right\}\delta_{ij}, \tag{C.23}$$

and the components of the perturbed Einstein tensor are

$$\begin{aligned} \delta G_{00} &= 2\partial_i\partial^i\psi - 6\frac{a'}{a}\psi', \\ \delta G_{0i} &= 2\partial_i\psi' + 2\frac{a'}{a}\partial_i A, \\ \delta G_{ij} &= \left[4\frac{a''}{a}(A+\psi) - 2\left(\frac{a'}{a}\right)^2 (A+\psi) + 2\frac{a'}{a}(A' + 2\psi') + 2\psi'' \right. \\ &\quad \left. + \partial_k\partial^k(A-\psi)\right]\delta_{ij} - \partial_i\partial_j(A-\psi). \end{aligned} \tag{C.24}$$

For requirement, we also compute the linear perturbation of the mixed component Einstein tensor $G^{\mu}_{\ \nu} \equiv g^{\mu\alpha}G_{\alpha\nu}$ as

$$\delta G^{\mu}_{\ \nu} = \delta g^{\mu\alpha}G_{\alpha\nu} + g^{\mu\alpha}\delta G_{\alpha\nu}, \tag{C.25}$$

which can be written in components as

$$\begin{aligned}
\delta G^{0}_{\ 0} &= \frac{2}{a^2}\left[\partial_i\partial^i\psi - 3\frac{a'}{a}\psi' - 3A\left(\frac{a'}{a}\right)^2\right], \\
\delta G^{0}_{\ i} &= \frac{2}{a^2}\left[\partial_i\psi' + \frac{a'}{a}\partial_i A\right], \\
\delta G^{i}_{\ 0} &= -\frac{2}{a^2}\left[\partial^i\psi' + \frac{a'}{a}\partial^i A\right], \\
\delta G^{i}_{\ j} &= -\frac{1}{a^2}\left[4\frac{a''}{a}A - 2\left(\frac{a'}{a}\right)^2 A + 2\frac{a'}{a}(A' + 2\psi') + 2\psi'' + \partial_k\partial^k(A-\psi)\right]\delta^{i}_{\ j} \\
&\quad -\partial^i\partial_j(A-\psi).
\end{aligned} \tag{C.26}$$

It is important to note here that as a perfect fluid has stress part, i.e. $T^{i}_{\ j} \propto \delta^{i}_{\ j}$, thus according the Einstein field equations $\delta G^{i}_{\ j}$ cannot have any stress part implying

$$\partial^i\partial_j(A-\psi) = 0 \quad \Rightarrow \quad A = \psi. \tag{C.27}$$

Hence we are left with only one degree of scalar perturbation ψ in the perturbed FLRW metric.

- **Perturbed stress energy-momentum tensor of inflaton :**

Note that the inflaton field ϕ_0 is a homogeneous field $\phi_0 \equiv \phi_0(\tau)$ and the quantum fluctuations around it is both function of space and time $\delta\phi \equiv \delta\phi(\tau, \mathbf{x})$. Thus the stress energy-momentum tensor, given in Eq. (B.3), for the inflaton filed will be

$$T_{00} = \tfrac{1}{2}\phi_0'^2 + a^2(\tau)V(\phi), \qquad T_{0i} = 0, \qquad T_{ij} = \left(\tfrac{1}{2}\phi_0'^2 - a^2(\tau)V(\phi)\right)\delta_{ij}. \tag{C.28}$$

Perturbing linearly the stress energy-momentum tensor, given in Eq. (B.3), one gets

$$\begin{aligned}
\delta T_{\mu\nu} &= \partial_\mu\delta\phi\partial_\nu\phi + \partial_\mu\phi\partial_\nu\delta\phi - \delta g_{\mu\nu}\left(\frac{1}{2}\partial^\alpha\phi\partial_\alpha\phi - V(\phi)\right) \\
&- g_{\mu\nu}\left[\frac{1}{2}\delta g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \frac{1}{2}g^{\alpha\beta}\partial_\alpha\delta\phi\partial_\beta\phi + \frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\delta\phi - \frac{\partial V(\phi)}{\partial\phi}\delta\phi\right],
\end{aligned} \tag{C.29}$$

which can be written in components as

$$\begin{aligned}
\delta T_{00} &= \delta\phi'\phi_0' + 2Aa^2(\tau)V(\phi) + a^2(\tau)V_{,\phi}\,\delta\phi, \\
\delta T_{0i} &= \partial_i\delta\phi\phi_0', \\
\delta T_{ij} &= \left[\delta\phi'\phi_0' - A\phi_0'^2 - a^2(\tau)V_{,\phi}\,\delta\phi - \psi\phi_0'^2 + 2a^2(\tau)\psi V(\phi)\right]\delta_{ij}.
\end{aligned} \tag{C.30}$$

The linear perturbation of the mixed stress energy-momentum tensor $T^\mu_{\ \nu} \equiv g^{\mu\alpha}T_{\alpha\nu}$ can be written as

$$\delta T^\mu_{\ \nu} = \delta g^{\mu\alpha}T_{\alpha\nu} + g^{\mu\alpha}\delta T_{\alpha\nu}, \tag{C.31}$$

and its components are

$$\begin{aligned}
\delta T^0_{\ 0} &= \frac{1}{a^2}\left[\delta\phi'\phi_0' - A\phi_0'^2 + a^2(\tau)V_{,\phi}\,\delta\phi\right], \\
\delta T^0_{\ i} &= \frac{1}{a^2}\partial_i\delta\phi\phi_0', \\
\delta T^i_{\ 0} &= -\frac{1}{a^2}\partial^i\delta\phi\phi_0', \\
\delta T^i_{\ j} &= -\frac{1}{a^2}\left[\delta\phi'\phi_0' - A\phi_0'^2 - a^2(\tau)V_{,\phi}\,\delta\phi\right]\delta^i_{\ j}.
\end{aligned} \tag{C.32}$$

- **Perturbed Einstein field equations :**

Einstein field equations, described in Eq. (A.2), relates the space-time geometry with the matter content of the Universe. Thus the perturbations in the metric and as well as in the matter are studied by perturbing the Einstein field equations as

$$\delta G_{\mu\nu} = \frac{1}{M_{\rm Pl}^2}\delta T_{\mu\nu}. \tag{C.33}$$

Using Eqs. (C.26) and Eqs. (C.32) and equating the components of the above equations one gets the following field equations as

$$\begin{aligned}
\nabla^2\psi - 3\mathcal{H}\psi' - 3\mathcal{H}^2\psi &= \frac{1}{2M_{\rm Pl}^2}\left[\delta\phi'\phi_0' - \psi\phi_0'^2 + a^2(\tau)V_{,\phi}\,\delta\phi\right], \\
\psi' + \mathcal{H}\psi &= \frac{1}{2M_{\rm Pl}^2}\delta\phi\phi_0', \\
\psi'' + 3\mathcal{H}\psi' + \left(2\mathcal{H}' + \mathcal{H}^2\right)\psi &= \frac{1}{2M_{\rm Pl}^2}\left[\delta\phi'\phi_0' - \psi\phi_0'^2 - a^2(\tau)V_{,\phi}\,\delta\phi\right].
\end{aligned} \tag{C.34}$$

Here $\mathcal{H} \equiv \frac{a'}{a}$ is the conformal Hubble parameter and we have used the constrain $A = \psi$ in deriving the above equations.

C.2 Perturbed Klein-Gordon Equation

The Klein-Gordon equation of a canonical scalar field is obtained by varying the scalar action given in Eq. (B.1) with respect to the field $\frac{\delta S(\phi)}{\delta\phi} = 0$ which yields

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) + V_{,\phi} = 0. \tag{C.35}$$

An equivalent form of this equation is given in Eq. (B.2) where $g_{\mu\nu}$ is the unperturbed FLRW metric. Perturbing the Klein-Gordon equation linearly one gets

$$\begin{aligned}\delta\left(\frac{1}{\sqrt{-g}}\right)\partial_\mu\left(\sqrt{-g}g_0^{\mu\nu}\partial_\nu\phi_0\right) + \frac{1}{\sqrt{-g}}\partial_\mu\left(\delta\left(\sqrt{-g}\right)g_0^{\mu\nu}\partial_\nu\phi_0\right) &+ \\ \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\delta g^{\mu\nu}\partial_\nu\phi_0\right) + \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g_0^{\mu\nu}\partial_\nu\delta\phi\right) + V_{,\phi\phi}\,\delta\phi &= 0.\end{aligned} \tag{C.36}$$

The perturbed terms in the above equation are $\delta\left(\frac{1}{\sqrt{-g}}\right)$, $\delta\left(\sqrt{-g}\right)$, $\delta g_{\mu\nu}$ and $\delta\phi$. Now the derivative of an invertible matrix A can be computed as

$$\frac{d\det A}{d\alpha} = \det(A)\mathrm{tr}\left(A^{-1}\frac{dA}{d\alpha}\right). \tag{C.37}$$

Thus one gets

$$\begin{aligned}\delta\left(\sqrt{-g}\right) &= -\frac{\delta g}{2\sqrt{-g}} = -\frac{g_0 g_0^{\mu\nu}\delta g_{\mu\nu}}{2\sqrt{-g}} = a^4(A-3\psi), \\ \delta\left(\frac{1}{\sqrt{-g}}\right) &= \frac{\delta g}{2\left(-g\right)^{\frac{3}{2}}} = \frac{g_0 g_0^{\mu\nu}\delta g_{\mu\nu}}{2\left(-g\right)^{\frac{3}{2}}} = -\frac{1}{a^4}(A-3\psi).\end{aligned} \tag{C.38}$$

where $g_0 = \det g^0_{\mu\nu}$. Hence the first, second, third and the fourth term of Eq. (C.36) will be

$$\begin{aligned}\delta\left(\frac{1}{\sqrt{-g}}\right)\partial_\mu\left(\sqrt{-g}g_0^{\mu\nu}\partial_\nu\phi_0\right) &= -\frac{1}{a^2}(A-3\psi)\left(\phi_0'' - 2\frac{a'}{a}\phi_0'\right), \\ \frac{1}{\sqrt{-g}}\partial_\mu\left(\delta\left(\sqrt{-g}\right)g_0^{\mu\nu}\partial_\nu\phi_0\right) &= \frac{1}{a^2}(A-3\psi)\left(\phi_0'' - 2\frac{a'}{a}\phi_0'\right) + \frac{1}{a^2}(A'-3\psi')\phi_0', \\ \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\delta g^{\mu\nu}\partial_\nu\phi_0\right) &= -\frac{2}{a^2}\left(A\phi_0'' + 2\frac{a'}{a}A\phi_0' + A'\phi_0'\right), \\ \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g_0^{\mu\nu}\partial_\nu\delta\phi\right) &= \frac{1}{a^2}\left(\delta\phi'' + 2\frac{a'}{a}\delta\phi' - \partial_i\partial^i\delta\phi\right).\end{aligned} \tag{C.39}$$

Now, using the Euler-Lagrange equation of motion for the scalar field in conformal metric

$$\phi_0'' + 2\frac{a'}{a}\phi_0' + a^2 V_{,\phi} = 0, \tag{C.40}$$

one gets from Eq. (C.36) the perturbed Klein-Gordon equation as

$$\delta\phi'' + 2\frac{a'}{a}\delta\phi' - \partial_i\partial^i\delta\phi + a^2V_{,\phi\phi}\,\delta\phi - (A' + 3\psi')\phi_0' + 2a^2AV_{,\phi} = 0, \qquad \text{(C.41)}$$

and putting the constrain $A = \psi$ one gets from the above equation

$$\delta\phi'' + 2\frac{a'}{a}\delta\phi' - \partial_i\partial^i\delta\phi + a^2V_{,\phi\phi}\,\delta\phi = 4\psi'\phi_0' - 2a^2V_{,\phi}\,\psi. \qquad \text{(C.42)}$$

C.3 Equation of motion of metric scalar perturbation ψ

To derive the equation of motion for ψ one can use the perturbed Einstein field equations given in Eqs. (C.34). Adding the first two equations of Eqs. (C.34) one gets

$$\psi'' + 6\mathcal{H}\psi' + 2\left(\mathcal{H}' + 2\mathcal{H}^2\right)\psi - \nabla^2\psi = -\frac{1}{M_{\rm Pl}^2}a^2(\tau)V_{,\phi}\,\delta\phi. \qquad \text{(C.43)}$$

Now the equation of motion for the inflaton field given in Eq. (C.40) yields

$$-\,a^2V_{,\phi} = \phi_0'' + 2\frac{a'}{a}\phi_0' \qquad \text{(C.44)}$$

and the second equation of Eqs. (C.34) yields

$$\frac{1}{2M_{\rm Pl}^2}\delta\phi = \frac{1}{\phi_0'}\left(\psi' + \mathcal{H}\psi\right). \qquad \text{(C.45)}$$

Substituting the above two equations in Eq. (C.43) we gets the equation of motion for ψ as

$$\psi'' + 2\left(\mathcal{H} - \frac{\phi_0''}{\phi_0'}\right)\psi' + 2\left(\mathcal{H}' - \frac{\phi_0''}{\phi_0'}\mathcal{H}\right)\psi - \nabla^2\psi = 0. \qquad \text{(C.46)}$$

Appendix D

Essentials of de Sitter spacetime

Here we will briefly review the essential features of the *de Sitter* spacetime. The de Sitter background is described by the flat FLRW metric given in Eq. (A.6) where the scale factor grows exponentially with time

$$a(t) = a_0 e^{Ht}. \tag{D.1}$$

Here the Hubble parameter $H > 0$ is constant of time. A Universe dominated by fluid with a state parameter $\omega = -1$ (e.g. a scalar field slow-rolling its potential) can lead to such an exponential expansion. For such a fluid the energy density remains constant with time (which can be seen from Eq. (A.10)) and the Friedmann equation given in Eq. (A.4) yields an exponentially expanding scale factor with $H = \sqrt{\frac{8\pi G\rho_0}{3}}$. The conformal time defined in Eq. (A.7) and the corresponding scale factor in de Sitter spacetime are given as

$$\tau = -\frac{1}{H}e^{-Ht}, \qquad a(\tau) = -\frac{1}{H\tau}. \tag{D.2}$$

The cosmic time t goes from $-\infty$ to ∞ implying that the conformal time τ changes from $-\infty$ to 0.

D.1 Horizon in de Sitter spacetime

One important feature of de Sitter spacetime is the presence of horizons. The trajectories of light follow null worldline which is given by $ds^2 = 0$ implying

$$a(t)\dot{\mathbf{x}}(t) = 1. \tag{D.3}$$

This, in de Sitter spacetime, yields a solution

$$|\mathbf{x}(t)| = \frac{1}{H}\left(e^{-Ht_0} - e^{-Ht}\right), \tag{D.4}$$

for trajectories starting at the origin, $\mathbf{x}(t_0) = 0$. This implies that the trajectories started at the origin at t_0 asymptotically approach the sphere with radius $r_{\text{hor}}(t_0) \equiv \frac{1}{H}e^{-Ht_0}$. This sphere is called the *horizon* for the observer at the origin. On the other hand signals originating at t_0 from $|\mathbf{x}| > r_{hor}$ will never reach the observer sitting at the origin.

D.2 Quasi de Sitter spacetime

During inflation, as the inflaton slow-rolls its potential, the Hubble parameter does not remain constant but varies with time

$$\dot{H} = -\epsilon H^2, \tag{D.5}$$

which can be seen from Eq. (B.12). This shows that in quasi de Sitter spacetime $H = \frac{1}{\epsilon t}$ and thus the scale factor $a(\tau)$ will be

$$a(\tau) = -\frac{1}{(1-\epsilon)\,H\tau}. \tag{D.6}$$

Appendix E

Uniqueness of mode functions in Minkowski spacetime

Here we will discuss how different choice of mode functions can be made and how one can choose an unambiguous and thus physical vacuum in Minkowski space time. The derivations here will closely follow [32].

E.1 Bogolyubov transformation

Consider the following mode function

$$v_k(\tau) = \alpha_k u_k(\tau) + \beta_k u_k^*(\tau), \tag{E.1}$$

where α_k and β_k are complex numbers and v_k, thus constructed, also satisfies the equation of motion of mode function given in Eq. (2.23). This transformation between mode functions and the coefficients α_k and β_k are called the *Bogolyubov transformation* and *Bogolyubov coefficients* respectively. The normalization condition of the mode function given in Eq. (2.24) is satisfied if the Bogolyubov coefficients satisfy the condition

$$|\alpha_k|^2 - |\beta_k|^2 = 1. \tag{E.2}$$

In terms of v_k now the mode expansion given in Eq. (2.19) can be written as

$$\hat{\chi}(\tau, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}} \left[v_k(\tau) b_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + v_k^*(\tau) b_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}} \right]. \tag{E.3}$$

Hence one cannot prefer one mode function over the other at this point. The Bogolyubov transformations between the two sets of creation and annihilation operators can be given as

$$a_{\mathbf{k}} = \alpha_k^* b_{\mathbf{k}} + \beta_k b_{\mathbf{k}}^\dagger, \qquad a_{\mathbf{k}}^\dagger = \alpha_k b_{\mathbf{k}}^\dagger + \beta_k^* b_{\mathbf{k}}. \tag{E.4}$$

Both sets of operators generate a basis of states in the Hilbert space:

$$a_{\mathbf{k}}|0\rangle_a = 0, \qquad b_{\mathbf{k}}|0\rangle_b = 0, \tag{E.5}$$

which imply that the b-vacuum contains $a-$particles which can be seen as

$$_b\langle 0|N_{\mathbf{k}}^a|0\rangle_b = |\beta_k|^2\delta(0), \qquad _b\langle 0|N_{\mathbf{k}}^b|0\rangle_b = 0, \tag{E.6}$$

and similarly it can be shown that the a-vacuum contains $b-$particles.

E.2 Preferred mode functions in Minkowski space

In Minkowski spacetime $a(\tau) = 1$ and considering a case of a massless scalar the equation of motion for the mode functions given in Eq. (2.23) becomes

$$u_k'' + k^2 u_k = 0. \tag{E.7}$$

In the Minkowski spacetime a preferable set of mode functions will be such that the expectation value of the Hamiltonian in the vacuum state is minimized. Writing the Hamiltonian given in Eq. (2.21) in terms of the mode function u_k one gets

$$\hat{H}(\tau) = \frac{1}{4}\int d^3\mathbf{k}\left[a_{\mathbf{k}}a_{-\mathbf{k}}F_k^* + a_{\mathbf{k}}^\dagger a_{-\mathbf{k}}^\dagger F_k + (2a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \delta(0))E_k\right], \tag{E.8}$$

where

$$\begin{aligned} E_k &\equiv |u_k'|^2 + k^2|u_k|^2, \\ F_k &\equiv u_k'^2 + k^2 u_k^2. \end{aligned} \tag{E.9}$$

Then the expectation value of the instantaneous Hamiltonian is

$$_u\langle 0|\hat{H}(\tau)|0\rangle_u = \frac{\delta(0)}{4}\int d^3\mathbf{k}E_k. \tag{E.10}$$

Using the parameterization $u_k = r_k e^{i\alpha_k}$, where $r_k(\tau)$ and $\alpha_k(\tau)$ are real, the normalization condition of the mode functions given in Eq. (2.24) yields

$$r_k^2 \alpha_k' = -\frac{1}{2}, \tag{E.11}$$

and we get from Eq. (E.9)

$$E_k = r_k'^2 + \frac{1}{4r_k^2} + k^2 r_k^2. \tag{E.12}$$

Minimizing the energy with respect to $r_k'^2$ and r_k yields that $r_k' = 0$ and $r_k = \frac{1}{\sqrt{2k}}$ and these also give $\alpha_k = -k\tau$. Thus the mode function which minimizes the Hamiltonian will be

$$u_k = \frac{e^{-ik\tau}}{\sqrt{2k}}. \tag{E.13}$$

This defines the preferred mode functions for a scalar field in Minkowski space. For this mode functions $E_k = k \equiv \omega_k$ and $F_k = 0$ which yields the familiar form of the Hamiltonian as

$$\hat{H}(\tau) = \int d^3\mathbf{k}\ \omega_k \left(a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \frac{1}{2}\delta(0) \right). \tag{E.14}$$

Appendix F

Three-point and connected four-point correlation function of $\mathcal{R}_{NL}$ in single field slow-roll model

We will calculate in detail the three-point and connected four-point correlation function of non-linear comoving curvature perturbations $\mathcal{R}_{NL}$ in single field slow-roll model.

F.1 Calculating three-point correlation function of $\mathcal{R}_{NL}$

The non-vanishing three-point correlation function of comoving curvature perturbation $\mathcal{R}$ arises from terms like $\langle \mathcal{R}_L(t, \mathbf{k}_i)\mathcal{R}_L(t, \mathbf{k}_j)\mathcal{R}_{NL}(t, \mathbf{k}_k)\rangle$, which due to the presence of $\mathcal{R}_{NL}$ eventually turns out to be a four-point function of inflaton's fluctuations $\delta\phi_L$. Therefore, one can write the non-vanishing three-point function as

$$\langle \mathcal{R}_{NL}(t, \mathbf{k}_1)\mathcal{R}_{NL}(t, \mathbf{k}_2)\mathcal{R}_{NL}(t, \mathbf{k}_3)\rangle \simeq \langle \mathcal{R}_L(t, \mathbf{k}_1)\mathcal{R}_L(t, \mathbf{k}_2)\mathcal{R}_{NL}(t, \mathbf{k}_3)\rangle$$
$$+\langle \mathcal{R}_L(t, \mathbf{k}_1)\mathcal{R}_{NL}(t, \mathbf{k}_2)\mathcal{R}_L(t, \mathbf{k}_3)\rangle + \langle \mathcal{R}_{NL}(t, \mathbf{k}_1)\mathcal{R}_L(t, \mathbf{k}_2)\mathcal{R}_L(t, \mathbf{k}_3)\rangle. \quad (F.1)$$

The first term on the right hand side of the equation yields

$$
\begin{aligned}
\langle \mathcal{R}_L(t,\mathbf{k}_1)\mathcal{R}_L(t,\mathbf{k}_2)\mathcal{R}_{NL}(t,\mathbf{k}_3)\rangle &= \left(\frac{H}{\dot{\phi}}\right)^2 \frac{1}{2}\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right)\int \frac{d^3\mathbf{p}}{(2\pi)^{\frac{3}{2}}} \\
&\quad \langle \delta\phi_L(t,\mathbf{k}_1)\delta\phi_L(t,\mathbf{k}_2)\delta\phi_L(t,\mathbf{p})\delta\phi_L(t,\mathbf{k}_3-\mathbf{p})\rangle \\
&= \left(\frac{H}{\dot{\phi}}\right)^2 \frac{1}{2}\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right)\int \frac{d^3\mathbf{p}}{(2\pi)^{\frac{3}{2}}} \\
&\quad [\langle \delta\phi_L(t,\mathbf{k}_1)\phi_L(t,\mathbf{p})\rangle\langle \delta\phi_L(t,\mathbf{k}_2)\delta\phi_L(t,\mathbf{k}_3-\mathbf{p})\rangle \\
&+ \langle \delta\phi_L(t,\mathbf{k}_2)\phi_L(t,\mathbf{p})\rangle\langle \delta\phi_L(t,\mathbf{k}_1)\delta\phi_L(t,\mathbf{k}_3-\mathbf{p})\rangle] \,.
\end{aligned}
\tag{F.2}
$$

The last equality in the above equation could be written as the inflaton's fluctuations are Gaussian primordially and the four-point correlation of $\delta\phi_L$ can be written in terms of product of two two-point correlation functions. Now one can use the definition of power spectrum given in Eq. (2.44) and Eq. (2.56) as

$$
\langle \delta\phi_L(t,\mathbf{k}_1)\delta\phi_L(t,\mathbf{k}_2)\rangle \equiv \frac{1}{k_1^3}P_{\delta\phi}(k_1)\delta^3(\mathbf{k_1}+\mathbf{k_2}),
\tag{F.3}
$$

to get from Eq. (F.1)

$$
\begin{aligned}
&\langle \mathcal{R}_L(t,\mathbf{k}_1)\mathcal{R}_L(t,\mathbf{k}_2)\mathcal{R}_{NL}(t,\mathbf{k}_3)\rangle = \left(\frac{H}{\dot{\phi}}\right)^2 \frac{1}{2}\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right)\frac{P_{\delta\phi}(k_1)}{k_1^3}\frac{P_{\delta\phi}(k_2)}{k_2^3}\times \\
&\int \frac{d^3\mathbf{p}}{(2\pi)^{\frac{3}{2}}}\left[\delta^3(\mathbf{k_1}+\mathbf{p})\delta^3(\mathbf{k_2}+\mathbf{k_3}-\mathbf{p})+\delta^3(\mathbf{k_2}+\mathbf{p})\delta^3(\mathbf{k_1}+\mathbf{k_3}-\mathbf{p})\right] \\
&= \left(\frac{H}{\dot{\phi}}\right)^2 \frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right)\frac{1}{(2\pi)^{\frac{3}{2}}}\delta^3(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\frac{P_{\delta\phi}(k_1)}{k_1^3}\frac{P_{\delta\phi}(k_2)}{k_2^3}.
\end{aligned}
\tag{F.4}
$$

Using the relation between the power spectrum of the comoving curvature perturbation $P_{\mathcal{R}}$ and the inflaton's fluctuations $P_{\delta\phi}$ as given in Eq. (2.48) and the definition of the slow-roll parameter ϵ as given in Eq. (B.12) one can rewrite the above equation as

$$
\begin{aligned}
\langle \mathcal{R}_L(t,\mathbf{k}_1)\mathcal{R}_L(t,\mathbf{k}_2)\mathcal{R}_{NL}(t,\mathbf{k}_3)\rangle &= \delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{2M_{\rm Pl}^2\epsilon}{(2\pi)^{\frac{3}{2}}}\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right)\times \\
&\quad \frac{P_{\mathcal{R}}(k_1)}{k_1^3}\frac{P_{\mathcal{R}}(k_2)}{k_2^3}.
\end{aligned}
\tag{F.5}
$$

Therefore, calculating the other two terms on the right hand side of Eq. (F.1) following the same steps as before the three-point correlation function of $\mathcal{R}_{NL}$

turns out to be

$$\langle \mathcal{R}_{NL}(\mathbf{k}_1)\mathcal{R}_{NL}(\mathbf{k}_2)\mathcal{R}_{NL}(\mathbf{k}_3)\rangle = (2\pi)^{-\frac{3}{2}}\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(2M_{\rm Pl}^2\epsilon)\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right) \times\left(\frac{P_{\mathcal{R}}(k_1)}{k_1^3}\frac{P_{\mathcal{R}}(k_2)}{k_2^3}+2\text{ perms.}\right). \tag{F.6}$$

F.2 Calculating connected four-point correlation function of $\mathcal{R}_{NL}$

The non-vanishing connected part of the four-point correlation of the comoving curvature perturbation $\mathcal{R}$ in single field slow-roll inflationary scenario, which yields maximum non-Gaussianity due to non-linear evolution of $\mathcal{R}$, appears from terms like $\langle \mathcal{R}_L(t,\mathbf{k}_i)\mathcal{R}_L(t,\mathbf{k}_j)\mathcal{R}_{NL}(t,\mathbf{k}_k)\mathcal{R}_{NL}(t,\mathbf{k}_l)\rangle$. Thus the four-point correlation function of $\mathcal{R}_{NL}$ can be written as

$$\langle \mathcal{R}_{NL}(t,\mathbf{k}_1)\mathcal{R}_{NL}(t,\mathbf{k}_2)\mathcal{R}_{NL}(t,\mathbf{k}_3)\mathcal{R}_{NL}(t,\mathbf{k}_4)\rangle \simeq \langle \mathcal{R}_{L}(t,\mathbf{k}_1)\mathcal{R}_{L}(t,\mathbf{k}_2)\mathcal{R}_{NL}(t,\mathbf{k}_3)\mathcal{R}_{NL}(t,\mathbf{k}_4)\rangle + 5\,\text{perm.} \tag{F.7}$$

Let us derive the first term of the right hand side of the above equation :

$$\langle \mathcal{R}_{L}(t,\mathbf{k}_1)\mathcal{R}_{L}(t,\mathbf{k}_2)\mathcal{R}_{NL}(t,\mathbf{k}_3)\mathcal{R}_{NL}(t,\mathbf{k}_4)\rangle = \langle \mathcal{R}_{L}(t,\mathbf{k}_1)\mathcal{R}_{L}(t,\mathbf{k}_2)\mathcal{R}_{L}(t,\mathbf{k}_3)\mathcal{R}_{L}(t,\mathbf{k}_4)\rangle + \left(\frac{H}{\dot{\phi}}\right)^2\left(\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right)\right)^2\int\frac{d^3\mathbf{p}}{(2\pi)^{\frac{3}{2}}} \int\frac{d^3\mathbf{q}}{(2\pi)^{\frac{3}{2}}}\langle\delta\phi_L(t,\mathbf{k}_1)\delta\phi_L(t,\mathbf{k}_2)\delta\phi_L(t,\mathbf{p})\delta\phi_L(t,\mathbf{k}_3-\mathbf{p})\delta\phi_L(t,\mathbf{q})\delta\phi_L(t,\mathbf{k}_4-\mathbf{q})\rangle\,. \tag{F.8}$$

As the inflaton's fluctuations $\delta\phi$ are Gaussian initially, the vacuum expectation value of six inflaton's perturbations in the above equation can be written in terms of several combinations of product of three two-point correlation functions as

$$\begin{aligned}
&\langle\delta\phi_L(t,\mathbf{k}_1)\delta\phi_L(t,\mathbf{k}_2)\delta\phi_L(t,\mathbf{p})\delta\phi_L(t,\mathbf{k}_3-\mathbf{p})\delta\phi_L(t,\mathbf{q})\delta\phi_L(t,\mathbf{k}_4-\mathbf{q})\rangle\\
&=\langle\delta\phi_L(t,\mathbf{k}_1)\delta\phi_L(t,\mathbf{p})\rangle\langle\delta\phi_L(t,\mathbf{k}_2)\delta\phi_L(t,\mathbf{q})\rangle\langle\delta\phi_L(t,\mathbf{k}_3-\mathbf{p})\delta\phi_L(t,\mathbf{k}_4-\mathbf{q})\rangle\\
&+\langle\delta\phi_L(t,\mathbf{k}_1)\delta\phi_L(t,\mathbf{p})\rangle\langle\delta\phi_L(t,\mathbf{k}_2)\delta\phi_L(t,\mathbf{k}_4-\mathbf{q})\rangle\langle\delta\phi_L(t,\mathbf{k}_3-\mathbf{p})\delta\phi_L(t,\mathbf{q})\rangle\\
&+\langle\delta\phi_L(t,\mathbf{k}_1)\delta\phi_L(t,\mathbf{q})\rangle\langle\delta\phi_L(t,\mathbf{k}_2)\delta\phi_L(t,\mathbf{p})\rangle\langle\delta\phi_L(t,\mathbf{k}_3-\mathbf{p})\delta\phi_L(t,\mathbf{k}_4-\mathbf{q})\rangle\\
&+\langle\delta\phi_L(t,\mathbf{k}_1)\delta\phi_L(t,\mathbf{q})\rangle\langle\delta\phi_L(t,\mathbf{k}_2)\delta\phi_L(t,\mathbf{k}_3-\mathbf{p})\rangle\langle\delta\phi_L(t,\mathbf{p})\delta\phi_L(t,\mathbf{k}_4-\mathbf{q})\rangle\\
&+\langle\delta\phi_L(t,\mathbf{k}_1)\delta\phi_L(t,\mathbf{k}_3-\mathbf{p})\rangle\langle\delta\phi_L(t,\mathbf{k}_2)\delta\phi_L(t,\mathbf{k}_4-\mathbf{q})\rangle\langle\delta\phi_L(t,\mathbf{p})\delta\phi_L(t,\mathbf{q})\rangle\\
&+\langle\delta\phi_L(t,\mathbf{k}_1)\delta\phi_L(t,\mathbf{k}_3-\mathbf{p})\rangle\langle\delta\phi_L(t,\mathbf{k}_2)\delta\phi_L(t,\mathbf{q})\rangle\langle\delta\phi_L(t,\mathbf{p})\delta\phi_L(t,\mathbf{k}_4-\mathbf{q})\rangle\\
&+\langle\delta\phi_L(t,\mathbf{k}_1)\delta\phi_L(t,\mathbf{k}_4-\mathbf{q})\rangle\langle\delta\phi_L(t,\mathbf{k}_2)\delta\phi_L(t,\mathbf{k}_3-\mathbf{p})\rangle\langle\delta\phi_L(t,\mathbf{p})\delta\phi_L(t,\mathbf{q})\rangle\\
&+\langle\delta\phi_L(t,\mathbf{k}_1)\delta\phi_L(t,\mathbf{k}_4-\mathbf{q})\rangle\langle\delta\phi_L(t,\mathbf{k}_2)\delta\phi_L(t,\mathbf{p})\rangle\langle\delta\phi_L(t,\mathbf{k}_3-\mathbf{p})\delta\phi_L(t,\mathbf{q})\rangle.
\end{aligned} \tag{F.9}$$

Using the form of power spectrum given in Eq. (F.3) and after performing the integrations in Eq. (F.8) one gets

$$\begin{aligned}
&\langle\mathcal{R}_L(t,\mathbf{k}_1)\mathcal{R}_L(t,\mathbf{k}_2)\mathcal{R}_{NL}(t,\mathbf{k}_3)\mathcal{R}_{NL}(t,\mathbf{k}_4)\rangle=\\
&\langle\mathcal{R}_L(t,\mathbf{k}_1)\mathcal{R}_L(t,\mathbf{k}_2)\mathcal{R}_L(t,\mathbf{k}_3)\mathcal{R}_L(t,\mathbf{k}_4)\rangle+\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\frac{2}{(2\pi^3)}\left(\frac{H}{\dot\phi}\right)^2\\
&\left(\frac{\partial}{\partial\phi}\left(\frac{H}{\dot\phi}\right)\right)^2\frac{P_{\delta\phi}(k_1)}{k_1^3}\frac{P_{\delta\phi}(k_2)}{k_2^3}\left(\frac{P_{\delta\phi}(k_{13})}{k_{13}^3}+\frac{P_{\delta\phi}(k_{23})}{k_{23}^3}+\frac{P_{\delta\phi}(k_{14})}{k_{14}^3}+\frac{P_{\delta\phi}(k_{24})}{k_{24}^3}\right),
\end{aligned} \tag{F.10}$$

where $k_{ij}\equiv|\mathbf{k}_i+\mathbf{k}_j|$. Hence each of the other five similar terms on the right hand side of Eq. (F.7) will have four such combinations of product of three power spectrums, yielding 24 of such terms in the final expression. Thus the 'connected' part of the four-point correlation function of $\mathcal{R}_{NL}$ will be

$$\begin{aligned}
&\langle\mathcal{R}_{NL}(t,\mathbf{k}_1)\mathcal{R}_{NL}(t,\mathbf{k}_2)\mathcal{R}_{NL}(t,\mathbf{k}_3)\mathcal{R}_{NL}(t,\mathbf{k}_4)\rangle_c=2\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\times\\
&(2\pi)^{-3}\left(\frac{H}{\dot\phi}\right)^2\left(\frac{\partial}{\partial\phi}\left(\frac{H}{\dot\phi}\right)\right)^2\left[\frac{P_{\delta\phi}(k_1)}{k_1^3}\frac{P_{\delta\phi}(k_2)}{k_2^3}\frac{P_{\delta\phi}(k_{13})}{k_{13}^3}+23\,\text{perm.}\right].
\end{aligned} \tag{F.11}$$

Using the relation between the power spectrum of the comoving curvature perturbation $P_{\mathcal{R}}$ and the inflaton's fluctuations $P_{\delta\phi}$ as given in Eq. (2.48) and the definition of the slow-roll parameter ϵ as given in Eq. (B.12) one can rewrite the above equation as

$$\langle \mathcal{R}_{NL}(t,\mathbf{k}_1)\mathcal{R}_{NL}(t,\mathbf{k}_2)\mathcal{R}_{NL}(t,\mathbf{k}_3)\mathcal{R}_{NL}(t,\mathbf{k}_4)\rangle_c = 2\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\times$$
$$(2\pi)^{-3}\left(2M_{\rm Pl}^2\epsilon\right)^2\left(\frac{\partial}{\partial\phi}\left(\frac{H}{\dot\phi}\right)\right)^2\left[\frac{P_{\mathcal{R}}(k_1)}{k_1^3}\frac{P_{\mathcal{R}}(k_2)}{k_2^3}\frac{P_{\mathcal{R}}(k_{13})}{k_{13}^3}+23\,\text{perm.}\right]. \quad \text{(F.12)}$$

Appendix G

Propagator for interacting scalar field in Minkowski space using KL representation

KL representation is a non-perturbative way to derive propagator for interacting fields. Here a brief description of deriving the Feynman propagator and the Wightman function of interacting real scalar fields is being discussed. A more detailed derivation can be found in [95]. Considering a generic scalar field $\Phi(x)$, the vacuum expectation value of the time-ordered product $\langle 0|\mathcal{T}\{\Phi(x)\Phi(y)\}|0\rangle$ gives the complete Feynman propagator for the scalar field in Fourier space

$$-i\Delta'(p) = \int d^4x \exp[ip\cdot(x-y)]\langle 0|\mathcal{T}\{\Phi(x)\Phi(y)\}|0\rangle, \tag{G.1}$$

while the vacuum expectation value of product of two scalar fields is known as the Wightman function

$$W'(x-y) = \langle 0|\Phi(x)\Phi(y)|0\rangle. \tag{G.2}$$

Inserting a complete set of momentum eigenstates in between the two field operators the vacuum expectation value of $\Phi(x)\Phi(y)$ can be written as

$$\langle 0|\Phi(x)\Phi(y)|0\rangle = \sum_n \langle 0|\Phi(x)|n\rangle\langle n|\Phi(y)|0\rangle. \tag{G.3}$$

Translational invariance in Minkowski space yields

$$\Phi(x) = \exp(ip\cdot x)\Phi(0)\exp(-ip\cdot x), \tag{G.4}$$

where

$$\langle 0|\Phi(x)|n\rangle = \exp(-ip_n\cdot x)\langle 0|\Phi(0)|n\rangle$$
$$\langle n|\Phi(y)|0\rangle = \exp(ip_n\cdot y)\langle n|\Phi(0)|0\rangle. \tag{G.5}$$

In Minkowski space therefore Eq. (G.3) can be written as

$$\langle 0|\Phi(x)\Phi(y)|0\rangle = \sum_n \exp(-ip_n\cdot(x-y))|\langle 0|\Phi(0)|n\rangle|^2. \tag{G.6}$$

$|\langle 0|\Phi(0)|n\rangle|^2$ encapsulating the interacting features of the scalar field can be replaced by a spectral function $\rho(q^2)$ defined as

$$\theta(q^0)\rho(q^2) = (2\pi)^3\sum_n \delta^4(q-p_n)|\langle 0|\Phi(0)|n\rangle|^2. \tag{G.7}$$

The spectral function $\rho(q^2)$ is a function of q^2 due to Lorentz invariance and is real, positive and vanishes for $q^2 < 0$. With this definition of spectral function Eq. (G.3) can be expressed as

$$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int_0^\infty d\sigma^2\rho(\sigma^2)\Delta(x-y;\sigma^2), \tag{G.8}$$

where

$$\Delta(x-y;\sigma^2) = \frac{1}{(2\pi)^3}\int d^4q\,\exp[-iq\cdot(x-y)]\theta(q^0)\delta(q^2-\sigma^2), \tag{G.9}$$

and σ is known as the mass parameter. Similarly one can find

$$\langle 0|\Phi(y)\Phi(x)|0\rangle = \int_0^\infty d\sigma^2\rho(\sigma^2)\Delta(y-x;\sigma^2), \tag{G.10}$$

where

$$\Delta(y-x;\sigma^2) = \frac{1}{(2\pi)^3}\int d^4q\,\exp[-iq\cdot(y-x)]\theta(q^0)\delta(q^2+\sigma^2). \tag{G.11}$$

G.1 Feynman propagator for interacting scalar field

The vacuum expectation value of two time-ordered field operators is

$$\langle 0|\mathcal{T}\{\Phi(x)\Phi(y)\}|0\rangle = \Theta(x_0-y_0)\langle 0|\Phi(x)\Phi(y)|0\rangle + \Theta(y_0-x_0)\langle 0|\Phi(y)\Phi(x)|0\rangle. \tag{G.12}$$

Inserting Eq. (G.12), Eq. (G.8) and Eq. (G.10) in Eq. (G.1) gives the propagator for interacting scalar field as

$$-i\Delta^{(\mathrm{int})}(p) = -i\int d^4x \exp[ip\cdot(x-y)]\int_0^\infty d\sigma^2\rho(\sigma^2)\Delta_F(x-y;\sigma^2), \quad \text{(G.13)}$$

where the Feynman propagator $\Delta_F(x-y;\sigma^2)$ for the scalar field is

$$\begin{aligned} -i\Delta_F(x-y;\sigma^2) &= \Theta(x_0-y_0)\Delta(x-y;\sigma^2)+\Theta(y_0-x_0)\Delta(y-x;\sigma^2) \\ &= \frac{-i}{(2\pi)^4}\int d^4q \exp[-iq\cdot(x-y)]\frac{1}{q^2-\sigma^2-i\varepsilon}. \end{aligned} \quad \text{(G.14)}$$

To derive the last equality the form of the step function

$$\Theta(t) = -\frac{1}{2\pi i}\lim_{\varepsilon\to 0}\int_{-\infty}^{+\infty}\frac{e^{-ist}}{s+i\varepsilon}ds \quad \text{(G.15)}$$

is to be used. This yields the form of the full propagator for the interacting scalar field in terms of the spectral function as

$$\Delta^{(\mathrm{int})}(p) = \int_0^\infty d\sigma^2\rho(\sigma^2)\frac{1}{p^2-\sigma^2+i\varepsilon}. \quad \text{(G.16)}$$

$\frac{1}{p^2-\sigma^2+i\varepsilon}$ can be recognized as the propagator for a free scalar field with the mass m of the scalar field replaced by the mass parameter σ. Hence one can write the above equation as

$$\Delta^{(\mathrm{int})}(p) = \int_0^\infty d\sigma^2\rho(\sigma^2)\Delta^0(p;\sigma^2), \quad \text{(G.17)}$$

where $\Delta^0(p;\sigma^2) \equiv \frac{1}{p^2-\sigma^2+i\varepsilon}$ is the free propagator of the scalar field.

G.2 Wightman function for interacting scalar field

For a free scalar field with mass m the Wightman function defined in Eq. (G.2) is

$$W^0(x-y) = \frac{1}{(2\pi)^3}\int\frac{d^3k}{2\omega_k}e^{-i\omega_k(x_0-y_0)+i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}. \quad \text{(G.18)}$$

where $\omega_k \equiv \sqrt{\mathbf{k}^2+m^2}$. For the interacting scalar field the Wightman function can be derived using Eq. (G.8) which turns out to be

$$W^{(\mathrm{int})}(x-y) = \int_0^\infty d\sigma^2\rho(\sigma^2)\Delta(x-y;\sigma^2), \quad \text{(G.19)}$$

where $\Delta(x-y;\sigma^2)$ given in Eq. (G.9) can be written as

$$\Delta(x-y;\sigma^2) = \frac{1}{(2\pi)^3}\int \frac{d^3q}{2\omega_q} e^{-i\omega_q(x_0-y_0)+i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})}. \tag{G.20}$$

This can be identified as the Wightman function for the free scalar field given in Eq. (G.18) where the mass m of the scalar field is replaced by the mass parameter σ and $\omega_q \equiv \sqrt{\mathbf{q}^2+\sigma^2}$ and hence Eq. (G.19) can be written as

$$W^{(\mathrm{int})}(x-y) = \int_0^\infty d\sigma^2 \rho(\sigma^2) W^0(x-y;\sigma^2). \tag{G.21}$$

The equal-time Wightman function ($x_0 = y_0$) for the interacting scalar field has the form

$$\begin{aligned} W^{(\mathrm{int})}_{\mathrm{ET}}(x-y) &= \frac{1}{(2\pi)^3}\int_0^\infty d\sigma^2\rho(\sigma^2)\int \frac{d^3q}{2\omega_q} e^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})} \\ &= \int_0^\infty d\sigma^2 \rho(\sigma^2) W^0_{\mathrm{ET}}(x-y;\sigma^2). \end{aligned} \tag{G.22}$$

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List of Publication

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1. S. Das, S. Mohanty and K. Rao, "Test of unparticle long range forces from perihelion precession of Mercury," Phys. Rev. D **77**, 076001 (2008) [arXiv:0709.2583 [hep-ph]];

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