Beyond the Standard Model
and
Its Cosmological Consequences

A thesis submitted to

Gujarat University

for the degree of

Doctor of Philosophy in Physics

Submitted by

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Year of submission

2011

To

My Parents, Sister and Teachers

for luring me into science

CERTIFICATE

This is to certify that

(I) the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

(II) the thesis comprises only my original work towards the Ph.D. carried out by me at Physical Research Laboratory.

(III) due acknowledgement has been made in the text to all other material used,

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Acknowledgements

Of all the people I wish to thank, perhaps the most important on the list are my parents. Perhaps words are inadequate to express any kind of acknowledgement for my parents and family members because without their support and encouragement I stand nowhere.

I take this special opportunity to express my deep gratitude and hearty thanks to my thesis supervisor Prof. Utpal Sarkar for his enthusiastic supervision, advice, and guidance from the very early stage of my research work. I have immensely benefited from the discussions with him and his critical evaluations for which I shall always be obliged to him.

Needless to say, however, there are many others who have played a role (big or small) in helping me get over the line. Although I shall not be able to list them all here due to the limited space available, their contribution is very much appreciated and noted. But for those I shall acknowledge here, let me now mention them below.

I would like to express my sincere gratitude to my collaborators Prof. Urjit Yajnik, Dr. Jitesh Bhatt, Prof. Debjyoti Choudhury, Dr. Namit Mahajan, Dr. Anjishnu Sarkar and Debasish Borah for their support, encouragement and very fruitful discussions which helped me to sharpen my understanding in the subject. I am grateful to Prof. Anjan Joshipura, Dr. Raghu Rangarajan, Prof. Saurabh Rindani, Prof. Subhendra Mohanty, and Dr. Srubawati Goswami for various discussions related to Quantum field theory, Neutrino physics and Cosmology through which I gained a lot of insight and understanding of the subject. I accord my thanks to Prof. R.E. Amritkar, Prof. V.K.B. Kota, Prof. J. Banerji, Dr. H. Mishra, Dr. R.P. Singh and Dr. Dilip Angom for their kind encouragement throughout my research work.

I am heartily thankful to my School/College teachers, Damodar Patra, Harihar Panda, Srikanta Samal, Bebi Madam, Swapna Mohapatra, Karmadev Moharana, Rajkishore Mishra and specially, one of my friend cum lecturer, Namita Panda for their encouragement and words of motivation.

I gained a lot of academic knowledge, guidance and sage advice from my seniors; Santosh, Bhavik, Vishal, Subimal, Suman, Soumya, Sreekanth, Ashok, Bhaswar, Sandeep, Amzad, Rabiul and from my friends; Ketan, Moumita, Subrata, Tapas which helped me overcome several problems of academic as well as non-academic nature. I accord my thanks to all other members of Theory division: Purnendu, Brajesh, Manan, Harinder, Tarun, Salman, Zeen, Pankaj, Vimal, Suratna, Pravin, Abhishek, Siddhartha, Tanushree, Shashi for providing a pleasant working atmosphere in the division during the entire course of my research.

I am very much thankful to Lokesh for giving me useful information regarding PhD registration and Thesis submission. I extend my thanks to Aswini, Rajesh, Morthekai, Arvind Singh, Gyana, Sumanta, Rehman, Alok, Jitendra, Naveen Gandhi, Sumita, Jayati, Rohit, Prashant, Srinivas, Satinder, Asish, Arvind Saxena, Rajpurohit, Yogita, Sunil, Arun, Suruchi, Suchita, Iman, Chinmay, Neeraj, Vineet, Naveen Chauhan for very friendly approach enriched with natural and refreshing words.

I extend my deep regards to the director, dean and the Academic committee members of PRL for providing excellent academic environment for students and especially to me. I gratefully acknowledge to the members of the Library, Computer Centre, Dispensary, Administration and Maintenance section, particularly to Dholakiaji, Subhedarji, Jigarbhai, Hitendraji, Alokbhai, for providing support in their respective areas for all their assistance and cooperation during my entire Ph.D. period.

I owe my sincere thanks to my parents (Sri Nityananda Patra and Smt. Madhabilata Patra) and my sisters (Anupama and Sushree Sangita) for their love, care, affection and support during this work. It was their confidence that inspired me the most to complete this work, and hence, I take this opportunity to dedicate this thesis to them.

Sudhanwa Patra
February 2011

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CHAPTER

1

Introduction

1.1 The Standard Model of Particle Physics

According to the current understanding of particle physics, all known particles are made only of fermions and the interaction between the fermions is given by the mediators. To the best of our present knowledge, the nature seems to be equipped with four kinds of interactions (i) strong, (ii) electromagnetic, (iii) weak and (iv) gravitational. Every interaction has its mediator. For example, photon is the mediator of the electromagnetic force, two W's and a Z, are the mediator of the weak force. Then what is the mediator for strong force? There are eight gluon responsible for the strong binding between the quarks. Graviton(yet, to be discovered), presumably the mediator of gravitational interaction. The Standard Model (SM) of particle physics describes the dynamics of the elementary particles [1–4]. It has been constructed to address all the three interactions namely strong, electromagnetic and weak, other than gravity, on one platform. It is a gauge theory of the strong and electroweak interactions based on the gauge symmetry group $G_{ST} = SU(3)_C \times SU(2)_L \times U(1)_Y$. The weak and electromagnetic interactions between the fundamental particles (quarks and leptons) was first proposed by Glashow-Salam-Weinberg [1] which is known as the electroweak theory. The strong interaction is the interaction among the quarks of different colors and flavors and they are mediated by eight gluon. It is best described by the $SU(3)_C$ gauge theory called quantum cromo dynamics (QCD). The color states are confined and hence, only color singlets sates can exist in nature as free particles. The strong nuclear force is the force between the protons and neutrons, which is a manifestation of the underlying $SU(3)_C$ interactions among the quarks. The electromagnetic interaction is the force of all charged particles. It is described by quantum electrodynamics (QED) which is a $U(1)_Q$ gauge theory. The weak interaction describes the nuclear beta decay. The quarks and leptons transform according to left-handed doublets(LH) and right-handed(RH) singlets under $SU(2)_L$ to account for the V-A nature of the charge current weak interactions. The electromagnetic interaction, as like the gravitational interaction, is of infinite range but the ranges of the weak and strong nuclear forces are finite.

Experiments revealed the weak gauge bosons as massive as required by the short range behavior of the weak interaction. But the $SU(2)_L$ gauge symmetry does not permit the mass term for these gauge bosons, and fermions as well, in the Lagrangian. The spontaneous symmetry breaking mechanism is a way out to generate the weak gauge boson and fermion masses in the standard model by introducing an additional weak isodoublet complex scalar field. Weak gauge bosons get masses by absorbing three Goldstone bosons, three components of the scalar field, the remaining degree of freedom corresponds to a physical particle, the Higgs boson, the most wanted member for the present particle physics collider search. Once we choose a ground state, out of infinite possibilities, as the physical one, the electro-weak $SU(2)_L \times U(1)_Y$ symmetry breaks to $U(1)_Q$ symmetry. As a result, via the spontaneous symmetry breaking, the weak gauge bosons and the fermions acquire non-zero masses.

The assignment of weak hypercharge of U(1) group to the various $SU(2)_L$ and $SU(3)_C$ multiplets is

$$Q = T_{3L} + Y \tag{1.1}$$

where Q is the electric charge, T_{3L}, the 3rd component of weak isospin and Y, the weak hypercharge.The particles are represented under the SM gauge group as shown in table[6.2].

1.1.1 Complete Lagrangian for the standard model

The complete Lagrangian of the Standard Model obeying the gauge symmetry is

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm KE} + \mathcal{L}_{\rm Yuk} - \mathcal{V}_{\Phi}$$

where

Table 1.1: Particle content of the Standard Model

	Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Fermions	$Q_L^T \equiv (u,d)_L$	(3, 2, 1/6)
	u_R	(3, 1, 2/3)
	d_R	(3, 1, -1/3)
	$\ell_L^T \equiv (\nu,\ e)_L$	(1, 2, -1/2)
	e_R	(1, 1, -1)
	Φ	(1, 2, +1/2)

$$\begin{aligned}\mathcal{L}_{\text{KE}} = \quad & i\overline{\Psi}_Q\gamma^\mu \mathrm{D}_\mu \Psi_Q + i\overline{\Psi}_{u_R}\gamma^\mu \mathrm{D}_\mu \Psi_{u_R} + i\overline{\Psi}_{d_R}\gamma^\mu \mathrm{D}_\mu \Psi_{d_R} \\ + \quad & i\overline{\Psi}_L\gamma^\mu \mathrm{D}_\mu \Psi_L + i\overline{\Psi}_{e_R}\gamma^\mu \mathrm{D}_\mu \Psi_{e_R} + (\mathrm{D}_\mu \Phi)^\dagger(\mathrm{D}_\mu \Phi) \\ - \quad & \frac{1}{4}G^A_{\mu\nu}G^{\mu\nu}_A - \frac{1}{4}W^i_{\mu\nu}W^{\mu\nu}_i - \frac{1}{4}B_{\mu\nu}B^{\mu\nu},\end{aligned}$$

$$\mathcal{L}_{\text{Yuk}} = \quad y_u\overline{\Psi}_Q\tilde{\Phi}\Psi_{u_R} + y_d\overline{\Psi}_Q\Phi\Psi_{d_R} + y_e\overline{\Psi}_L\Phi\Psi_{e_R} + h.c.$$

$$\mathcal{V}_\Phi = \quad -\mu^2|\Phi|^2 + \lambda|\Phi|^4$$

and the fields are defined by

$$\Psi_Q \equiv \begin{pmatrix}\Psi_u \\ \Psi_d\end{pmatrix}_L, \quad \Psi_L \equiv \begin{pmatrix}\Psi_\nu \\ \Psi_e\end{pmatrix}_L, \quad \Psi_{e_{R,L}} \equiv \frac{1\pm\gamma_5}{2}\Psi_e \text{ and } \tilde{\Phi} \equiv i\tau_2\Phi^* \tag{1.2}$$

$$\begin{aligned}G^A_{\mu\nu} &= \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_3 f^{\mathrm{ABC}} G^B_\mu G^C_\mu \\ W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_L\epsilon^{ijk}W^j_\mu W^k_\mu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu\end{aligned} \tag{1.3}$$

where $A = 1, 2....8$, $i = 1, 2, 3$ and D_μ is the covariant derivative. The theory as written has a total of 18 parameters- the three gauge couplings: g_3, g_L, g_Y; the higgs-sector mass and self-coupling: μ^2, λ and 13 degree of freedom in the Yukawa sector.

1.1.2 Spontaneous Symmetry Breaking and The Higgs Mechanism

Spontaneous symmetry breaking is the idea that the ground state of a system contains only a subset of the symmetries respected by the underlying theory. This idea is not unique to particle physics or the SM, but is prevalent in many areas of physics; for example, ferromagnetism.

In the standard model, Spontaneous Symmetry Breaking is achieved through the spin-0 Higgs boson, Φ. The idea is that the Higgs field acquires a non-zero classical background, called a vacuum expectation value (VEV), and the quantum theory must be written as perturbations around this classical background. The theory still maintains the full symmetry, however the ground state, the one in which the VEV of Φ is nonzero, breaks this symmetry and thus it is not seen in nature.

The local $SU(2)_L \times U(1)_Y$ gauge invariant Lagrangian, thus, can be written as

$$\mathcal{L} = \left[\left(i\partial_\mu - g_L\frac{\vec{\tau}}{2}.\vec{W}_\mu - \frac{g_Y}{2}B_\mu\right)\Phi\right]^\dagger \left[\left(i\partial^\mu - g_L\frac{\vec{\tau}}{2}.\vec{W}^\mu - \frac{g_Y}{2}B^\mu\right)\Phi\right] \\ -V(\Phi^\dagger\Phi) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}\vec{W}_{\mu\nu}\vec{W}^{\mu\nu}, \tag{1.4}$$

where $Y = 1/2$ is used for the Higgs scalar field.

The scalar potential, $V(\Phi^\dagger\Phi)$, is given by

$$V(\Phi^\dagger\Phi) = \mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2. \tag{1.5}$$

Writing the Higgs doublet Φ as

$$\Phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{1.6}$$

where, $\phi^+ = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$ and $\phi^0 = \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4)$. Taking $\phi^+ = 0$ to preserve electric charge conservation and with the non-zero classical background being defined as $\langle\Phi\rangle$ and choosing

$$\langle\Phi\rangle \equiv \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \tag{1.7}$$

The condition for the spontaneous symmetry breaking is $\mu^2 < 0$ and $\lambda > 0$. The minima of the potential are at all those points of ϕ_is which satisfy the following condition

$$\Phi^\dagger\Phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = \frac{v^2}{2} = -\frac{\mu^2}{2\lambda}, \tag{1.8}$$

which implies an infinite number of ground states. The symmetry will spontaneously break once one of it is arbitrarily chosen. Keeping in mind that any unphysical term in the Lagrangian should not be allowed, let us write the scalar field Φ in terms of four fields $\theta_1(x)$, $\theta_2(x)$, $\theta_3(x)$ and $\mathbf{h}(\mathbf{x})$ as:

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \theta_2 - i\theta_1 \\ (v + \mathbf{h}) - i\theta_3 \end{pmatrix} \simeq e^{i\theta_{\mathbf{a}}(\mathbf{x})\tau^{\mathbf{a}}/v} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + \mathbf{h}(\mathbf{x})) \end{pmatrix} \tag{1.9}$$

Once we put this transformed field Φ in the Lagrangian, we will see that there are the three massless unwanted bosons will disappear from the potential. These massless goldstone modes are eaten up by $SU(2)_L$ gauge bosons and hence, $W^{\pm}$, Z become massive. As a result of this SSB. we have now three massive gauge fields $W^{\pm}$ and Z and one massless, the photon field, as needed:

$$\mathrm{m}_W = \frac{1}{2} v\, g_L \ , \ \mathrm{m}_Z = \frac{1}{2} v \sqrt{g_L^2 + g_Y^2} \ , \ \mathrm{m}_A = 0. \tag{1.10}$$

Finally, the shift to the true vacuum gives the fermions of the theory a mass through their yukawa couplings to the Higgs:

$$m_e = \frac{1}{\sqrt{2}} y_e\, v \quad m_u = \frac{1}{\sqrt{2}} y_u\, v \quad m_d = \frac{1}{\sqrt{2}} y_d\, v$$

The remaining Higgs degree of freedom obtains a non-zero, positive mass and should be seen by experiment. Furthermore, the quarks, electron, muon, and tau pick up masses from the yukawa couplings to the higgs while the neutrino remains massless.

1.1.3 Shortcomings of standard model

The Standard Model of elementary particles and interactions is one of the best tested theories in physics. It has been found to be in remarkable agreement with experiment and its validity at the quantum level has been successfully probed in the electroweak sector. It has predicted the masses of W and Z bosons precisely which is excellent agreement with the experiment, made several predictions for testing quantum electroweak corrections, etc. which have all been verified. In SM, weak and electromagnetic interactions are unified and predicts CP violation with at least three generation. In spite of its experimental successes, though, the Standard Model suffers from a number of limitations, and is likely to be an incomplete theory.

Standard model contains many arbitrary parameters; it does not include gravity, the fourth elementary interaction; it does not provide an explanation for the hierarchy between the scale of electroweak interactions and the Planck scale, characteristic of gravitational interactions; and finally, it fails to account for the dark matter and the baryon asymmetry of the universe. It does not represent a unified description of the fundamental interactions. There is no right handed neutrinos in SM and hence enforces the neutrinos to be massless. The most important thing is that the Higgs boson (which is crucial for mass generation through Higgs mechanism) is not found in any of the experiments. Also one can ask why there are only three generations of fermions ? All the fermions and Higgs boson masses and the gauge coupling constants are only parameters in the standard model. The clear evidence for physics beyond the standard model is the small nonzero neutrino mass. This led particle theorists to develop and study various extensions of the Standard Model, such as supersymmetric theories, Grand Unified Theories or theories with extra space-time dimensions; most of which have been proposed well before the experimental verification of the Standard Model. The coming generation of experimental facilities (high-energy colliders, B-physics experiments, neutrino superbeams, as well as astrophysical and cosmological observational facilities) will allow us to test the predictions of these theories and to deepen our understanding of the fundamental laws of nature.

1.2 Beyond Standard Model of Particle Physics

1.2.1 Massive Neutrinos

The Standard Model(SM) in particle physics predicts strictly massless neutrinos and there is neither mixing nor CP violation in the leptonic sector. The experimental observation that neutrinos can oscillate from one flavor to another as they propagate is the strongest indication for nonzero neutrino masses and mixing.

We will briefly discuss the theory of neutrino oscillation [5]. We define the neutrino weak eigenstate ν_α with flavor α (where $\alpha = e, \mu$, or τ) such that it is produced in association with the charged antilepton $\overline{\ell}_\alpha$ in a tree-level interaction with the W boson. These weak eigenstates are in general different from the neutrino mass eigenstates ν_i (with $i = 1, 2,$ and 3), each having a (rest) mass given by m_i. One can relate the weak and mass eigenstates via a unitary transformation and write ν_α as a coherent superposition of the ν_i fields:

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle \ , \tag{1.11}$$

The Unitary mass matrix U given in the above expression is known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [5–8], which is often parametrized as

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\,e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,e^{i\delta} & s_{23}c_{13} \\ -s_{12}s_{23} + c_{12}c_{23}s_{13}\,e^{i\delta} & c_{12}s_{23} + s_{12}c_{23}s_{13}\,e^{i\delta} & -c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.12}$$

where $s_{mn} = \sin\theta_{mn}, c_{mn} = \cos\theta_{mn}$, δ is the CP violating Dirac phase, while α_1 and α_2 denote the two Majorana phases.

To quantify the phenomenon of a neutrino changing from flavor-α to flavor-β as it propagates in vacuum, we are interested in the probability with which this happens, i.e. $\Pr(\nu_\alpha \to \nu_\beta)$, a quantity that depends on how the $|\nu_\alpha\rangle$ state in (1.11) evolves with time. This probability is given by

$$\begin{aligned} \Pr(\nu_\alpha \to \nu_\beta) \equiv |\langle \nu_\beta|\nu_\alpha\rangle|^2 = \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\,\sin^2\left[\frac{\Delta m^2_{ij}\,L}{4E}\right] \\ + 2\sum_{i>j} \mathrm{Im}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\,\sin\left[\frac{\Delta m^2_{ij}\,L}{2E}\right], \end{aligned} \tag{1.13}$$

where $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$. Here E, p_i and m_i are the energy, momentum and mass of ν_i component of neutrino, L is the source-detector distance as measured in the lab-frame. All are related by the Lorentz invariant term $m_i\tau_i$ in terms of laboratory variables as

$$m_i\,\tau_i = E_i\,t - |p_i|\,L\,, \tag{1.14}$$

From this result, it is quite clear that when all neutrino masses m_i's are zero (or nonzero but degenerate) and hence, the second and third term in Eq. (1.13) disappear, neutrino oscillation is not possible. By the same token, the observation that ν_e and ν_μ do change flavor during propagation implies that (at least two of) ν_i's must be massive.

The solar and atmospheric neutrino oscillations determined the values of two large $(\theta_{12}, \theta_{23})$ and one small (θ_{13}) mixing angles , as well as, a small $(\Delta m^2_{\rm sol})$ and a large $(\Delta m^2_{\rm atm})$ squared mass differences. Since the sign of $\Delta m^2_{\rm atm}$ is not known, two arrangements for the neutrino mass spectrum are possible:

Normal hierarchy: $\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0$, which gives $m_1 < m_2 < m_3$ with

$$m_2 = \sqrt{m_1^2 + \Delta m^2_{\rm sol}}\,, \qquad m_3 = \sqrt{m_1^2 + \Delta m^2_{\rm atm}}\,, \tag{1.15}$$

Inverted hierarchy: $\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0$, implying $m_3 < m_1 < m_2$ with

$$m_1 = \sqrt{m_3^2 + \Delta m^2_{\rm atm} - \Delta m^2_{\rm sol}}\,, \qquad m_2 = \sqrt{m_1^2 + \Delta m^2_{\rm sol}}\,. \tag{1.16}$$

Note that in both cases, we have used $\Delta m^2_{\rm sol} \equiv \Delta m^2_{21} > 0$. The best-fit values of the neutrino oscillation parameters at 1σ error level in the three-flavor analysis are summarized as follows [12]:

$$\begin{aligned}
\Delta m^2_{\rm sol} &= 7.65^{+0.23}_{-0.20} \times 10^{-5}\ {\rm eV}^2 \\
|\Delta m^2_{\rm atm}| &= 2.40^{+0.12}_{-0.11} \times 10^{-3}\ {\rm eV}^2 \\
\sin^2\theta_{12} &= 0.304^{+0.022}_{-0.016} \\
\sin^2\theta_{23} &= 0.50^{+0.07}_{-0.06}\,, \\
\sin^2\theta_{13} &\leq 0.01^{+0.016}_{-0.011}
\end{aligned} \tag{1.17}$$

Moreover,until now there is no information about the absolute neutrino masses. One can find the bound on absolute scale of neutrino mass via studies of lepton number (L) violating neutrino less double β-decay (${}^A_Z[{\rm Nucl}] \to {}_{Z+2}^{\ \ A}[{\rm Nucl}'] + 2e^-$), whose observation would imply that neutrinos are Majorana fermions [13]. The effective Majorana neutrino mass found to be

$$m_{\beta\beta} \equiv \left| \sum_{i=1}^{3} U^2_{ei}\, m_i \right| \,. \tag{1.18}$$

Several groups such as the Heidelberg-Moscow [14] and IGEX [15] collaborations conducted experiments with ^{76}Ge, while the more recent CUORICINO experiment [16] used ^{130}Te to test for this. So far there are no confirmed discoveries of the neutrinoless double β-decay, but the best upper bounds on the decay lifetimes are presently provided by CUORICINO (which is still running), whose results are translated to

$$m_\nu \equiv m_{\beta\beta} < 0.19 - 0.68\ {\rm eV} \ \ (90\%\ {\rm C.L.})\,, \tag{1.19}$$

for the neutrino mass.

The strongest bounds on the overall scale for neutrino masses come from cosmology. This is one of the important examples that illustrates the intricate connections between neutrino physics and the evolution of the early universe. The absolute upper bound for each individual neutrino mass coming from cosmology is

$$|m_i| \lesssim 0.2 \text{ eV} \quad (95\% \text{ C.L.}) \quad \text{for all } i \, . \tag{1.20}$$

See-Saw Mechanism

The most natural way to explain the smallness of the neutrino mass is by the use of the seesaw mechanism. The first ingredient in this mechanism is to add a right handed neutrino ν_R(for each generation). Next,in order to implement the seesaw mechanism, a mass scale of ν_R much larger than $v(= 250)$ has to be introduced. This kind of seesaw is known as Type I seesaw.

For one generation case:

$$m_\nu = \frac{m_D^2}{M} \tag{1.21}$$

wherem_D is the Dirac mass with magnitude of same order as those of the known charged fermions and M is the mass scale of ν_R, and is supposed to be substantially larger than m_D.

For one generation case:

$$m_\nu = m_D \frac{1}{M} m_D^T \tag{1.22}$$

The mass matrix that appear in the Lagrangian is now a 6×6 matrix as

$$M(6 \times 6) = \begin{pmatrix} 0 & m_D \\ m_D^T & M^R \end{pmatrix} \tag{1.23}$$

wherem_ν , m_D and M_R are all 3×3 matrices. Like in one generation, here we require that $| M_R | \quad \gg | m_D$.

Neutrino electromagnetic dipole moments

As is well known, the electric neutrality of the neutrino does not preclude its having non-zero dipole moments. And while, naively, the presence of a magnetic dipole moment would seem to call for the presence of a nonzero mass, even this is not strictly necessary [190]. One of the important implications of massive neutrinos is that they can in general possess a nonzero transition magnetic and electric dipole moment (both for Dirac and Majorana neutrinos), regardless of the mechanism by which they gain their mass. If neutrinos are Dirac particles, then they can also have diagonal electromagnetic dipole moments [17–20], unlike their Majorana counterparts. To understand the connection between neutrino mass and neutrino dipole moment, one should consider the generic dipole moment operator:

$$\mathcal{L}^{\rm dm} = \overline{\nu}_j \left(\mu_{jk} + i\gamma^5 d_{jk}\right) \sigma_{\alpha\beta}\, \nu_k\, F^{\alpha\beta} \ , \tag{1.24}$$

where $F^{\alpha\beta}$ denotes the photon field tensor, we see that the magnetic (μ_{jk}) and electric (d_{jk}) dipole moments have dimension of inverse mass. In the SM with massive Dirac neutrinos, the diagonal magnetic dipole moment induced by radiative corrections may be calculated for the mass eigenstate ν_j:

$$\mu_{\nu_j} \simeq \frac{3e\, G_F}{8\pi^2\sqrt{2}}\, m_{\nu_j} \approx 3\times 10^{-19} \left(\frac{m_{\nu_j}}{1 \text{ eV}}\right) \mu_B \ , \tag{1.25}$$

where $\mu_B = e/2m_e$ is the Bohr magneton, G_F is the Fermi constant, m_ν is the mass of light neutrino, μ_ν is the magnetic moment of the neutrino. This contribution is very small in comparison to the experimental bound [21]. So one need to either extend the SM or consider new physics beyond the SM to explain correct magnetic moment of the neutrinos. The current laboratory limits on the magnetic dipole moment are obtained from the low-energy scattering processes and they give a bound of about [22, 23, 23, 24]

$$\mu_\nu \lesssim 0.54 \times 10^{-10} \mu_B \ \ (90\% \text{ C.L.}) \ . \tag{1.26}$$

Moreover, one can estimate of the contribution to neutrino masses from the dipole moment operators, thus gaining important insights into the size of μ_ν in relation to m_ν. Once neutrinos have electromagnetic dipole moments (diagonal or transition), it is clear that new interactions between neutrinos and other fermions are possible. For instance, on top of the usual weak interactions, there can be a new contribution to neutrino-electron scattering due to photon exchange, hence modifying the cross section.

The existence of transition moments can lead to neutrino decays. In particular, if the transition moments between the ordinary LH and heavy RH neutrinos (from the minimally extended SM) are non-vanishing, then the radiative decay of the heavy RH neutrinos can have important implications in the cosmological evolution of matter in the early universe. We will discuss these issues later on how can generate the required lepton asymmetry to explain the matter-antimatter asymmetry of the present Universe via the decay of heavy right handed majorana neutrinos into a light SM lepton and a photon through the dipole moment operators.

1.2.2 Left-Right symmetric theory

While the standard electroweak model based on the spontaneously broken local symmetry has been extremely successful in the description of low-energy weak phenomena, it leaves many question unanswered. One of them has to do with understanding of the origin of the parity violation in low energy weak interaction processes while all other forces in nature are parity conserving. Why are the weak forces apparently not or are they really parity conserving at the fundamental level and we do not see it ? The second one, of a more phenomenological nature but an urgent one, has to do with the origin of neutrino masses, for which now there are convincing evidence from neutrino oscillation experiment. It is found that a theory, which is an extension of the SM, gives answer to both the questions and this theory is known as the Left-Right theory.

The left-right symmetric extension of the standard model is based on the gauge group $\mathcal{G}_{\rm LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [99–101]. The $SU(2)_R \times U(1)_{B-L}$ is broken at some high energy giving our low energy electroweak theory with unbroken $SU(2)_L \times U(1)_Y$. The left-handed fermions are doublets under $SU(2)_L$ while the right handed fermions are doublets under $SU(2)_R$. The electric charge is related to the generators of the group as:

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2} = T_{3L} + Y\,. \tag{1.27}$$

The quarks and leptons transform under the left-right symmetric

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \equiv [3,2,1,\frac{1}{3}], \qquad Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \equiv [3,1,2,\frac{1}{3}],$$

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \equiv [1,2,1,-1], \quad \ell_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \equiv [1,1,2,-1] \tag{1.28}$$

The left-right symmetric models have an interesting feature of breaking parity symmetry spontaneously. The $SU(2)_L$ gauge bosons W_L and the $SU(2)_R$ gauge bosons W_R are not parity eigenstates, but they transform under parity as $W_L \to W_R$. As the left-handed and right-handed fermions are related by parity operation, a discrete (Z_2) symmetry relating the group $SU(2)_L \to SU(2)_R$ can now be identified with the parity operator of the Lorentz group. Hence spontaneous breaking of left-right symmetry will also break parity spontaneously. After the left-right symmetry breaking, the gauge coupling constants for the two $SU(2)$ gauge groups can be different.

The symmetry breaking pattern in left-right models [102, 111] via Higgs scalar is

$$\begin{aligned} SU(3)_c \times SU(2)_L \quad \times & \quad SU(2)_R \times U(1)_{(B-L)} \; [G_{LR}] \\ \stackrel{M_R}{\rightarrow} & \quad SU(3)_c \times SU(2)_L \times U(1)_Y \quad [G_{std}] \\ \stackrel{m_W}{\rightarrow} & \quad SU(3)_c \times U(1)_Q \qquad\qquad\qquad [G_{em}] \, . \end{aligned}$$

The Higgs field which breaks left-right symmetry can give masses to the neutrinos. The $SU(2)_R$ symmetry is broken by a triplet scalar (Δ_R), which transforms under $\mathcal{G}_{\rm LR}$ as (1, 1, 3, -2). The discrete parity symmetry implies there exist another triplet (Δ_L), which transforms under $\mathcal{G}_{\rm LR}$ as (1, 3, 1, -2). The $SU(2)_R$ breaks at high scale via Δ_R and the vev of Δ_L is constrained by the precision experiment to be much less than a m_W. The electroweak symmetry can be broken by a bi-doublet Φ which transforms under $\mathcal{G}_{\rm LR}$ as (1, 2, 2, 0), whose vev can give masses to the charged fermions.

1.2.3 Grand Unified theory

Ultimate unification of all particles and all interactions is the eternal dream of theoretical physicists. The standard model has a grand success in unifying the two fundamental forces at high energies, namely weak and electromagnetism. But the question arises whether there is an another fundamental theory which allows all fundamental forces to unify at higher energies and Standard Model is one of it's subgroup. Such theories are known as Grand Unifies Theories(GUT). It promises to unify the three different gauge coupling constants of the SM. The basic idea is that the three coupling constants vary differently with respect to the energy scale and their renormalization group running shows that they tend to meet at some very high energy scale ($\sim 10^{16}$ GeV) known as the GUT scale. Some new physics is expected to appear at this scale which can be described by a bigger gauge group with single coupling constant, i.e., the grand unified group.

One natural extension of the standard model is to consider a grand unified theory, in which all three groups will be unified [98]. There will be only one unified gauge group with only one coupling constants [25]. At some higher energy, which is the scale of unification (M_U), the grand unified group will break down to the standard model

$$G_U \stackrel{M_U}{\rightarrow} SU(3)_c \times SU(2)_L \times U(1)_Y$$

Another motivation for grand unification is to treat the quarks and leptons in the equal footing at higher energies by putting them in the same representation of the unification group. This quark-lepton unification implies baryon and lepton number violation and hence, predicts proton decay. There are several possible grand unified theories depending on the unification gauge group and the symmetry breaking pattern with different predictions. Some of the GUT models ruled out by present experiment while none of the GUT models has been verified so far.

SU(5) Grand Unified Theories

Our main purpose in constructing the Grand unified theory is to unify the three fundamental forces and the theory only contains only one gauge coupling constant. Georgi and Glashow in 1973 proposed the SU(5) GUT containing the gauge group of rank 4 as the unified group. It gives a beautiful way of unifying all the three standard model gauge couplings. In the standard model the first generation contains fifteen fermions, the left-handed up and down quarks of three flavors, the right-handed up and down quarks, the left-handed neutrinos and left-handed and right-handed electrons. In this $SU(5)$ GUT model there is a unique way to accommodate all the fifteen quarks and leptons in the $\bar{5}$ and 10 representations. The break up of these two multiplets of the $SU(5)$ group in terms of the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ are:

$$5 \equiv (3,1,-\frac{1}{3}) \oplus (1,2,\frac{1}{2}) \quad \text{and} \quad 10 \equiv (1,1,1) \oplus (\bar{3},1,-\frac{2}{3}) \oplus (3,2,\frac{1}{6}). \tag{1.29}$$

The right-handed down quark $d \equiv (d^r, d^g, d^b)$ and right-handed $(e^+, \tilde{\nu_e})$ doublet can preferably be put into the $\bar{5}$ representation respectively. On the other hand the singlet charged left-handed anti-lepton e^+, the left-handed u, d quark doublet and left-handed anti-u quark singlet u^c will be in 10, the antisymmetric part of the product of two 5-plets.

Similarly, $24(= 5^2 - 1)$ gauge bosons associated with the $SU(5)$ gauge group can be decomposed as follows:

$$24 \equiv (8,1,0) + (1,3,0) + (1,1,0) + (3,2,-\frac{5}{6}) + (\bar{3},2,\frac{5}{6}) \tag{1.30}$$

which are the gluons, electro-weak gauge bosons and the new heavy X,Y gauge bosons. These new gauge bosons, X and Y, mediate the proton decay. One can have, for example, for the decay mode,

$$M(p \to e^+\pi^0) \sim \frac{g^2}{\mathrm{m}_X^2}, \tag{1.31}$$

where g is the GUT gauge coupling constant. Hence, the proton lifetime is

$$\tau_p \sim \frac{\mathrm{m}_X^4}{g^4 \mathrm{m}_p^5}. \quad (1.32)$$

Non-observation of proton decay puts a lower limit on these heavy gauge boson masses

$$\mathrm{m}_{X,Y} \;>\; 10^{15}\ \mathrm{GeV} \quad (1.33)$$

Generally, the $SU(5)$ symmetry is broken down to the low energy $SU(3)_C \times U(1)_Q$ by two Higgs scalars Φ_{24} and H_5 which are in the adjoint 24 and 5 of $SU(5)$. The breakdown of these two Higgs multiplets in the $SU(3)_C \times SU(2)_L \times U(1)_Y$ representation are given in eqns. (1.30) and (1.29) respectively.

When the neutral component $(1,1,0)$ of the the Φ_{24} gets a vev at the GUT scale, $SU(5)$ breaks to the SM gauge group while getting a nonzero vev for H_5 at the electro-weak scale breaks the SM down to $SU(3)_C \times U(1)_Q$.

The stepwise breakdown of the gauge symmetry in this case, thus, is

$$SU(5) \stackrel{\Phi_{24}}{\to} SU(3)_C \times SU(2)_L \times U(1)_Y \stackrel{H_5}{\to} SU(3)_C \times U(1)_Q. \quad (1.34)$$

The rank of the $SU(5)$ group is same as the rank of SM gauge group and so it is the smallest GUT gauge group to accommodate SM gauge group. Its non-supersymmetric minimal version, which was initially proposed, has got very tight constraint on parameter space from the negative results of the proton decay experiments and moreover does not unify the three gauge coupling constant. However, several extensions have been studied in literature and we will discuss one interesting scenario later on where one can achieve unification of three fundamental interactions using gravity as a correction to all the three gauge coupling constants.

Gauge hierarchy problem

A major difficulty of the standard model is the gauge hierarchy problem [27]. In order to realize this hierarchy between M_U and M_Z and hence the problem of naturalness let us calculate the quadratic divergence for the Higgs mass due to standard model fermions.

The one loop correction to the Higgs mass m_H is obtained by calculating the two point function:

$$\Pi^f_{hh} = (-1)\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left((\frac{-i\lambda_f}{\sqrt{2}})\frac{i}{\not{k} - \mathrm{m}_f}(\frac{-i\lambda_f}{\sqrt{2}})\frac{i}{\not{k} - \mathrm{m}_f}\right), \quad (1.35)$$

where $_f$ is the fermion-scalar-fermion coupling constant. The loop momentum k can take any value from zero to infinity. This leads to a correction which is infinite and makes the theory ill-defined. So, we assume that our theory is valid upto a cut-off scale . The above integration, thus, becomes

$$\begin{aligned}\Pi^f_{hh} &= -2^2_f \int_0 \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2-\mathrm{m}_f^2}+\frac{2\mathrm{m}_f^2}{(k^2-\mathrm{m}_f^2)^2}\right] \\ &= -\frac{\lambda_f^2}{8\pi^2}\Lambda^2+... \end{aligned} \tag{1.36}$$

Thus the corrected Higgs (mass)^2 is

$$\mathrm{m}_H^2 = \mathrm{m}_{H_0}^2 + \delta\mathrm{m}_H^2 \tag{1.37}$$

where the correction m_H^2 is proportional to the Π^f_{hh}. In GUT we have a new scale at 10^{16} GeV. If there is no new physics before this scale then $\sim 10^{16}$ GeV and to have a Higgs mass of $\mathcal{O}(100$ GeV$)$ a fine-tuning of the co-efficient λ_f to 1 part in 10^{26} is needed.

1.2.4 Renormalization group equations

The renormalization group, in quantum field theory (QFT), tells us how different couplings evolve with energy. But before discussing the renormalization group equations (RGE) an obvious question is: what is renormalization [26] ? In QFT, Green function is a most important thing to be calculated and, in fact, these quantities are divergent in perturbative quantum field theory. The systematic way to remove these divergences is known as renormalization. There are different ways to cancel these infinities. In order to renormalise the theory we need a reference point which is also arbitrary. Different choices of this reference point lead to different sets of parameters for the theory, but physics should not depend on the arbitrary choice of the reference point and be invariant. This invariance leads to the *renormalization group*. In quantum field theory it is a useful method to examine the behavior of physics at a different scale knowing the same at some other scale. Thus, measuring the observables in a low energy experiment one can compare with the values predicted from a theory at a higher scale, *e.g* at the GUT scale and certify about the correctness of the theory.

It was only after the realization of the fact that strength of an interaction is not an absolute concept but varies with the energy scale of the interaction that led to the idea of unification of all the coupling constants. In the standard model, variations of the gauge coupling constants with energy are given by the following renormalization group equations (RGEs)

$$16\pi^2 E\frac{dg_i}{dE} = b_i g_i^3 = \beta_{SM}(g_i) \qquad (1.38)$$

where i stands for $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ and the right-hand-side is known as the β-function of the corresponding coupling. This equation is valid for the lowest one-loop order in perturbations theory. One can write this equation as

$$\frac{d}{d\ln \mathrm{E}}\alpha_i{}^{-1}(E) = -\frac{b_i}{2\pi}. \qquad (1.39)$$

where, $\alpha_i = \frac{g_i^2}{4\pi}$.

Using the measured values of these coupling constants at the scale M_Z as the initial values one can solve these equations as,

$$\alpha_i^{-1}(M_U) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi}\ln\frac{M_U}{M_Z}. \qquad (1.40)$$

In the above equations the co-efficients, b_i, can be calculated for any $SU(N)$ group as

$$b_i = -\frac{11}{3}C_2(G) + \frac{2}{3}n_f C_2(R) + \frac{1}{3}n_s C_2(R) \qquad (1.41)$$

where $C_2(R)$ is the quadratic Casimir operator for the representation R while $C_2(G)$ is that for the adjoint representation. These Casimir operators are discussed below. In the above equation n_f is the number of chiral fermions and n_s is the number of complex scalars contributing to the β-function.

The generators of a gauge group obey the following rules

$$Tr[t_R^a t_R^b] = C(R)\delta^{ab}, \qquad (1.42)$$

and

$$\sum_a t_R^a t_R^a = C_2(R).\mathbf{1} \qquad (1.43)$$

where, the proportionality constant $C_2(R)$ is the quadratic Casimir operator for the particular representation. One can easily show that the quadratic Casimir operator is related with the factor $C(R)$ via

$$C_2(R)d(R) = C(R)r \qquad (1.44)$$

where, r is the number of generators $(= N^2-1)$ of the $SU(N)$ gauge group, equivalent to the dimension of the *adjoint representation*, and $d(R)$ is the dimension of the representation R. The $SU(2)$ generators follow the commutation relation

$$Tr[\frac{\tau^a}{2}\frac{\tau^b}{2}] = \frac{1}{2}\delta^{ab}. \tag{1.45}$$

As stated earlier the bigger GUT $SU(N)$ group will be chosen in such a way that it will contain the $SU(2)$ as a subgroup. The generators of the $SU(N)$ will also follow the same normalization condition – eqn.(1.45) – and, thus, we have $C(R) = \frac{1}{2}$ in the fundamental representation. Immediately eqn.(1.44) implies that for $R = N$, *i.e* for the fundamental representation the quadratic Casimir operator is $C_2(N) = \frac{N^2-1}{2N}$. For the adjoint representation $C_2(G) = N$. For the $U(1)$ gauge group these values will be $C_2(G) = 0$ and $C_2(R) = C(R) = (Y/2)^2$.

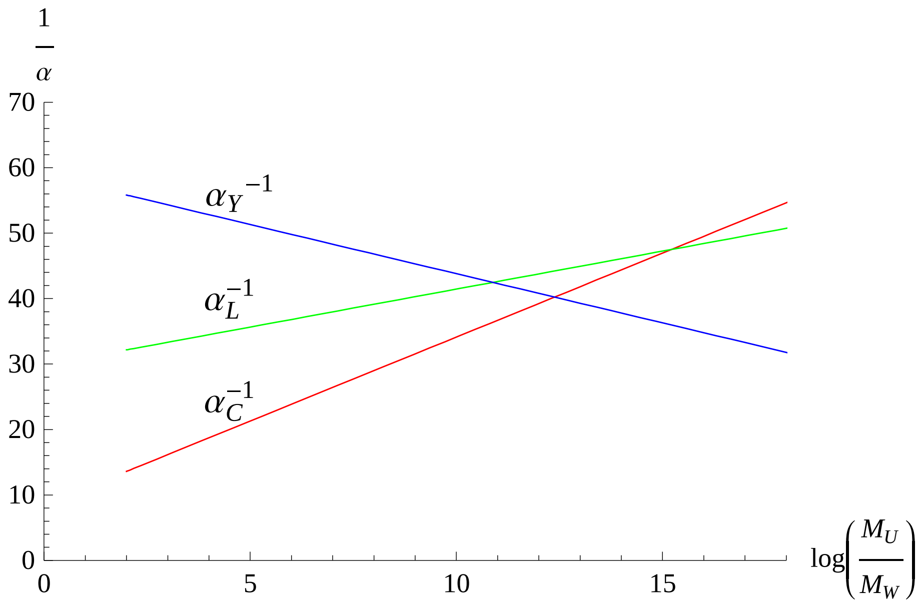


Figure 1.1: Evolution of the gauge couplings in the standard model

So, for the standard model, considering the contribution of all the particles listed in Table [6.2] one has for the three different co-efficients for the gauge groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$

$$\begin{pmatrix} b_Y \\ b_{2L} \\ b_{3C} \end{pmatrix} = \begin{pmatrix} \frac{41}{10} \\ -\frac{19}{6} \\ -7 \end{pmatrix}. \tag{1.46}$$

where, the GUT normalization factor $\frac{3}{5}$ is already multiplied to calculate the co-efficient for the $U(1)_Y$ gauge group. Using these values of 'b' one can find the evolution of the gauge couplings with energy from eqn(1.40) as depicted in Fig. 1.1 up to one loop contribution only.

It shows that all three standard model gauge couplings are trying to unify at some higher scale $\sim 10^{15}$ GeV, comparable to the predicted value of M_G from the proton decay limits. Although in this case they are not unifying exactly, they do so in the supersymmetric scenario. We shall discuss these issues later elaborately.

1.2.5 Supersymmetry

Supersymmetry(SUSY) is a symmetry between fermions and bosons and it unifies the concept of fermions and bosons keeping them in a same supermultiplets. It provides a solution to gauge hierarchy problem. Since supersymmetry has not been observed in nature, it must be broken at some higher energies, if it exists. R-parity invariance is imposed to eliminate fast baryon and lepton number violating terms. One of the main motivations of supersymmetry is that quadratic divergences are absent. Although the fine tuning of parameters required at tree level, there are no loop corrections that may require any fine tuning. This is because the scalars and fermions in the loop contribute quadratic divergences with opposite sign and similar form, so they cancel in the limit of equal masses of fermions and scalars in the loop. Thus, in the limit of exact supersymmetry, there are no quadratic divergence.

In addition to providing a solution to the gauge hierarchy problem and allowing unification of the space-time symmetry with internal symmetries, we now believe that the correct quantum theory of gravity is supersymmetric. The superpartners and their interactions predict interesting phenomenology in the next generation accelerators, which are added attractions of supersymmetry. There are also many cosmological consequences of supersymmetry including its prediction for a natural candidate of cold dark matter.

The minimal supersymmetric standard model (MSSM), is an extension of the standard model where all particles and their interactions are made supersymmetric. The Lagrangian of a SUSY theory is determined my two functions: the Kahler potential ($\mathcal{K}$) and the superpotential ($\mathcal{W}$) as follows

$$\mathcal{L}_{\text{SUSY}} = \frac{1}{2}\int d^4\theta \mathcal{K} + \int d^2\theta \mathcal{W} + h.c.$$

The Kahler potential is a real or vector superfield since $\mathcal{K}^\dagger = \mathcal{K}$ and the superpotential is a chiral superfield.Hence the MSSM potential are as follows

$$\begin{aligned}\mathcal{K} &= Q^{\dagger}e^{\mathcal{G}^{\mathcal{A}}\lambda_{\mathcal{A}}+\mathcal{W}^{\mathcal{A}}\tau_{\mathcal{A}}+\frac{1}{3}\mathcal{B}}Q + U^{c\dagger}e^{\mathcal{G}^{\mathcal{A}}\lambda_{\mathcal{A}}-\frac{2}{3}\mathcal{B}}U^{c} + D^{c\dagger}e^{\mathcal{G}^{\mathcal{A}}\lambda_{\mathcal{A}}+\frac{1}{3}\mathcal{B}}D^{c} \\ &+ L^{\dagger}e^{\mathcal{W}^{\mathcal{A}}\tau_{\mathcal{A}}-\frac{1}{2}\mathcal{B}}L + E^{c\dagger}e^{\mathcal{W}^{\mathcal{A}}\tau_{\mathcal{A}}-\frac{1}{2}\mathcal{B}}E^{c} \\ &+ H_u^{\dagger}e^{\mathcal{W}^{\mathcal{A}}\tau_{\mathcal{A}}+\frac{1}{2}\mathcal{B}}H_u + H_d^{\dagger}e^{\mathcal{W}^{\mathcal{A}}\tau_{\mathcal{A}}-\frac{1}{2}\mathcal{B}}H_d \end{aligned} \tag{1.47}$$

$$\begin{aligned}\mathcal{W} &= y_u Q H_u U^c + y_d L H_d E^c + \mu H_u H_d \\ &+ \frac{1}{4g_Y^2}B^{\alpha}B_{\alpha} + \frac{1}{8g_L^2}Tr\left(W^{\alpha}W_{\alpha}\right) + \frac{1}{12g_C^2}Tr\left(G^{\alpha}G_{\alpha}\right)\end{aligned} \tag{1.48}$$

Supersymmetry must be a broken symmetry, because exact SUSY would dictate that every superpartner is degenerate in mass with its corresponding SM particle, which is clearly ruled out by experiment. Possible ways to achieve a spontaneous breaking of supersymmetry depend on the form of the high energy theory. Supersymmetry may even be explicitly broken without losing its ability to solve the hierarchy problem as long as the breaking is of a certain type known as soft breaking. If supersymmetry is broken softly, the superpartner masses can be lifted to a phenomenologically acceptable range. The scale of the mass splitting between the two partners should be of the order of 100 GeV-1 TeV, because it then can be tied to the scale of electroweak symmetry breaking. In any case, the effective Lagrangian at the electroweak scale is expected to be parameterized by a general set of soft supersymmetry- breaking (SSB) terms if the attractive features of supersymmetry are to be maintained, and the Lagrangian can be separated as MSSM $\mathcal{L} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{Soft}}$ with the supersymmetric part is $\mathcal{L}_{\text{SUSY}}$ and the SUSY violating part is $\mathcal{L}_{\text{Soft}}$.

1.3 Cosmological Consequences of BSM physics

1.3.1 Cosmological baryon asymmetry

Nowadays, one speaks about a "Standard Cosmological Model", in analogy with its very successful counterpart of particle physics. The Standard Cosmological Model tells us that the Universe is in a phase of accelerated expansion and that the total energy in the Universe is shared among at least four components which sum to $\Omega_{\text{tot}} \sim 1$, meaning that the Universe is flat to a good precision. The dominant component (about 73%) is called dark energy, dark matter makes about 23%, ordinary matter (both luminous and dark) only 4% and neutrinos 0.22%, the uncertainty here stemming from the unknown absolute neutrino mass

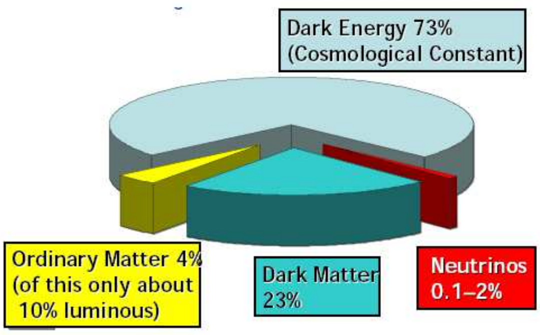


Figure 1.2: The mass-energy budget of the Universe.

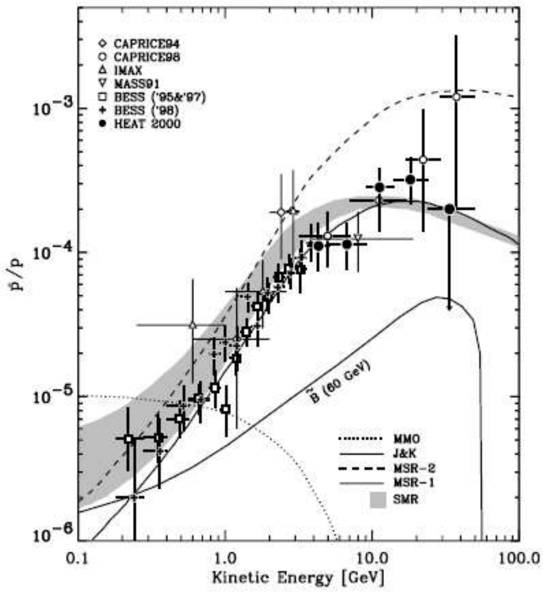


Figure 1.3: The antiproton-to-proton ratio at the top of the atmosphere, as observed (points) and predicted from the models (lines) [49].

scale.

The standard cosmological model has several outstanding questions, the most important ones being the nature of dark matter and dark energy, mechanism of inflation and baryogenesis. The existence of dark matter was originally suggested to explain the galactic rotation curves; it has also become necessary to explain structure formation. The existence of dark matter is generally accepted, but there are many candidates for dark matter particle waiting for experimental confirmation. Dark energy is postulated in order to fit the supernovae data, which suggests that the expansion of the universe has started to accelerate during the late times. Dark energy is becoming more and more accepted as an idea, though there are very few credible candidates for the source of this mysterious energy.

In everyday life, almost everything that we interact with is made of matter. Antimatter is also rare in our local galaxy (Milky Way). Ordinary matter, which constitutes our bodies as well as the Earth and the stars, does not seem at first to introduce any challenge to our understanding. However, this naive perception is wrong because two very puzzling questions remain:

1. Why is antimatter essentially absent in the observable Universe?

2. Why is the number density of baryons so small compared to photons or neutrinos?

These two questions are puzzling because, according to the Standard Big-Bang Theory, matter and antimatter evolved in the same way in the early Universe. On the other hand, today the observable Universe is composed almost exclusively of matter. Antimatter is only seen in particle physics accelerators and in cosmic rays. Moreover, the rates observed in cosmic rays are consistent with the secondary emission of antiprotons, $n_{\bar{p}}/n_p \sim 10^{-4}$ (see Fig. 1.3).

It is difficult to answer why there is an excess of matter over antimatter in the universe today. Recent measurements of the temperature anisotropy of the Cosmic Microwave Background (CMB) radiation by the WMAP probe [29], together with studies of large scale structure [30], have given us a reliable estimate of the baryon-to-photon ratio at the current epoch:

$$\eta_B^{\mathrm{CMB}} \equiv \frac{n_B}{n_\gamma} = (6.1 \pm 0.2) \times 10^{-10} \ , \tag{1.49}$$

where n_B and n_γ denote the number density of baryons and photons respectively. This number is also well agreement with the standard Big Bang Nucleosynthesis (BBN) analysis of the primordial abundances of ^{3}He, ^{4}He, (D)deuterium and standard cosmology. More importantly though, the amount of $\mathcal{O}\left(10^{-10}\right)$ for this ratio signifies that there must have been a primordial baryon asymmetry in the early universe. This is because if the universe was baryon-antibaryon symmetric at $T \simeq \mathcal{O}\,(100)$ MeV, the annihilation process $B + \overline{B} \to 2\,\gamma$ would significantly reduce both the value of n_B/n_γ and $n_{\overline{B}}/n_\gamma$, before they subsequently froze out at $T \sim 22$ MeV when the annihilations became ineffective. By studying the Boltzmann evolution of the number density of the (anti)baryons in this scenario, one can estimate the expected baryon-to-photon ratio for today to be [31]

$$\frac{n_B}{n_\gamma} = \frac{n_{\overline{B}}}{n_\gamma} = \mathcal{O}\left(10^{-18}\right) \ . \tag{1.50}$$

Hence, the apparent discrepancy between (1.49) and (1.50) is a clear indication that during primordial times, the universe must already have been matter-antimatter asymmetric, and the current scarcity of antimatter is just a manifestation of that fact.

Baryogenesis and Sakharov's conditions

Since it is expected that the Universe started with equal amount of baryons and anti-baryons, some interactions of particle physics should have generated this small baryon asymmetry of the Universe before nucleosynthesis. We need a successful theory which will explain the matter content of the present universe. Starting from a baryon symmetric Universe , the process of generating this small amount of baryon asymmetry is called

Baryogenesis. The mechanisms that can lead to this asymmetry, has to satisfy the three basic conditions for baryogenesis as pointed out by Sakharov in 1967 [32]: a dynamical model should contain processes that

1. violate baryon number, B,
2. violate C and CP, and
3. are out of thermal equilibrium.

These are often referred to as the *Sakharov conditions.*

B-number violation: Let us assign a positive number B for baryons while the corresponding antiparticles are given a negative number $\overline{B} \equiv -B$ for their baryon number. The first Sakharov's criterion is obvious as no increase or decrease of baryon number B can happen if all interactions in the model are B conserving.

C and CP violation: For every B violating interaction which involves a baryon, $X \to qq$, there will be a mirror process, $\overline{X} \to \overline{q}\,\overline{q}$, for the corresponding antibaryon that can create an exact negative amount of B and hence, no net B asymmetry may result if both types of processes are equally likely. Hence, Sakharov's second condition demands that C (charge conjugation) and CP (charge conjugation plus parity flip) violations are necessary as they will lead to different rates for the particle and antiparticle processes, i.e. $\Gamma(X \to qq) \neq \Gamma(\overline{X} \to \overline{q}\,\overline{q})$.

Departure from thermal equilibrium: From quantum mechanics, one can show that the thermal expectation value of B vanishes in equilibrium. So, the condition of deviation from thermal equilibrium for these processes is essential.

Sphaleron effect: anomalous B+L violation

In the SM, the baryon number and the lepton number are accidental symmetries. It is thus not possible to violate these symmetries at the classical level. To see how B and L violations come about while at the same time reconciling their apparent conservation at low energies, it is instructive to study the electroweak theory at both the classical and quantum mechanical levels. A well known fact of the classical SM Lagrangian is that it has global $U(1)_B$ and $U(1)_L$ symmetries and is therefore invariant under the following transformations of the quark and lepton fields:

$$U(1)_B: \quad q(x) \to q(x)\, e^{i\theta}\ ; \quad \ell(x) \to \ell(x) \quad , \tag{1.51}$$

$$U(1)_L: \quad q(x) \to q(x) \quad ; \quad \ell(x) \to \ell(x)\, e^{i\phi}\ , \tag{1.52}$$

where θ and ϕ are constants. Noether's theorem then implies that the classical J_μ^B and J_μ^L currents are conserved:

$$\partial^\mu J_\mu^B = \partial^\mu \sum_{\substack{\text{flavors}\\ \text{colors}}} \frac{1}{3}\left(\overline{q}_L\gamma_\mu q_L + \overline{u}_R\gamma_\mu u_R + \overline{d}_R\gamma_\mu d_R\right) = 0\ , \tag{1.53}$$

$$\partial^\mu J_\mu^L = \partial^\mu \sum_{\text{flavors}} \left(\overline{\ell}_L\gamma_\mu \ell_L + \overline{e}_R\gamma_\mu e_R\right) = 0\ , \tag{1.54}$$

where we have conveniently defined the baryon and lepton numbers for quarks and leptons as: $B_{\text{quark}} = 1/3, B_{\text{lepton}} = 0, L_{\text{quark}} = 0$ and $L_{\text{lepton}} = 1$.

In 1969, it was realized [43, 44] that through the Adler-Bell-Jackiw triangle anomaly these symmetries are nevertheless broken and as a result, the baryonic and the leptonic currents are anomalous. Their divergences are then given by

$$\partial^\mu J_\mu^B = \partial^\mu J_\mu^L = \frac{N_f}{32\pi^2}\left(-g^2\mathrm{Tr}[W^a_{\mu\nu}\widetilde{W}^{\mu\nu}_a] + g'^2\mathrm{Tr}[B_{\mu\nu}\widetilde{B}^{\mu\nu}]\right)\ , \tag{1.55}$$

where g and g' are the gauge couplings of $SU(2)_L$ and $U(1)_Y$ respectively, with $W^a_{\mu\nu}$ and $B_{\mu\nu}$ the corresponding field tensors, and N_f denotes the number of generations.

Another important observation from (1.55) is that $\partial^\mu J_\mu^B$ and $\partial^\mu J_\mu^L$ are identical and hence,

$$\partial^\mu\left(J_\mu^B - J_\mu^L\right) = 0\ . \tag{1.56}$$

In other words, the $B-L$ quantum number is strictly conserved in the SM. However, it is also clear from (1.55) that $B+L$ must be violated. To deduce the corresponding change in the $B+L$ quantum number, one must evaluate the Euclidean integral of $\partial^\mu(J_\mu^B + J_\mu^L)$ over d^4x:

$$\Delta(B+L) \equiv \int d^4x\ \partial^\mu J_\mu^{B+L} = \int d^4x\ \frac{2N_f}{32\pi^2}\left(-g^2 W^a_{\mu\nu}\widetilde{W}^{\mu\nu}_a + g'^2 B_{\mu\nu}\widetilde{B}^{\mu\nu}\right)\ , \tag{1.57}$$

$$= 2\,N_f\,\Delta N_{\text{cs}}\ , \tag{1.58}$$

where $\Delta N_{\text{cs}} = \pm 1, \pm 2, \ldots$ is the change in the Chern-Simons number.

In 1976, 'tHooft published an article [50] in which he estimated the rate of these baryon number violating processes. He considered the instanton solution between two separate vacua and calculated the action associated with the saddle point configuration between them. This field configuration is called *Sphaleron*, from the Greek word meaning ready to fall, as the saddle point configuration is inherently unstable. The probability of tunneling between the vacua is approximately

$$\Gamma \sim e^{-\mathrm{S_{inst}}} = e^{-\frac{4\pi}{\alpha}} = \mathcal{O}(10^{-170}).$$

This rate is so infinitely small that the sphaleron process is in no contradiction with the practical observation of the lack of violation of B or L.

The energy of the saddle point configuration can be estimated by the sphaleron configuration. Below the electroweak phase transition temperature ($T < T_{\rm EW}$), the transition rate per unit volume was found to be

$$\frac{\Gamma_{\rm sph}}{V} \sim e^{\frac{M_W}{\alpha T}},$$

which is still very much suppressed. In the symmetric phase ($T > T_{\rm EW}$), however, the transition rate is no longer suppressed, but rather [51]

$$\frac{\Gamma_{\rm sph}}{V} \sim \alpha^5 \ln \alpha^{-1} T^4.$$

Sphaleron processes can be in equilibrium when the sphaleron rate $\Gamma_{\rm sph}$ exceeds the expansion rate of the Universe (H). By comparing the sphaleron rate $\Gamma_{\rm sph}^{\rm T>T_{EW}}$ to $H = 1.67\sqrt{g^*}$ (where g^* is the effective relativistic degrees freedom, $M_{\rm Pl} = 1.22 \times 10^{19}$ GeV, is the Planck mass), one can check that the temperature T lies in the range [52]

$$T_{\rm EW} \leq T \leq 10^{13}\mathrm{GeV}.$$

Candidates for baryogenesis

To explain the cosmic baryon asymmetry, several theories and models have been suggested. The most pleasing alternative has been electroweak baryogenesis, since it requires no physics beyond the standard model, whereas other scenarios require at least some extension to it.

Electroweak baryogenesis: The standard model of particle physics, perhaps surprisingly, fulfills all the Sakharov's conditions. The CP-violation enters through the Cabibbo-Kobayashi-Maskaawa (CKM) matrix. So, in principle at least, the baryogenesis problem may be solved within the framework of the SM. But it is found that the CP violation observed in the quark sector [33] (e.g. in K^0-$\bar{K}^0$ or B^0-$\bar{B}^0$ mesons system) is far too small [34] to give rise to the observed η_B. Moreover, the present empirical lower limit on the Higgs mass, $m_{\text{Higgs}} > 114$ GeV [35], implies that the electroweak phase transition cannot be first order [36], making it difficult for the baryon number violating sphaleron processes in the SM to go out of thermal equilibrium. Since baryogenesis can not be explained within the standard model, the existence of baryons in our universe can be considered as evidence for physics beyond the standard model.

GUT baryogenesis: The standard model describes the interactions of particles by two symmetry groups, $SU(3)_{\text{QCD}}$ and $SU(2)_L \times U(1)_Y$. The motivation for grand unified theories is to explain all these interactions by a single large symmetry group, which includes all these groups as it's subgroups. Since no specific GUT theory has been found, there are many different models tossed around with many common properties. All the Sakharov's condition are easily fulfilled in GUT models. The B-number violation is an unavoidable consequence in grand unified models, as quarks and leptons are unified in the same representation of a single group. Furthermore, sufficient amount of CP violation can be incorporated naturally in GUT models, as there exist many possible complex phases, in addition to those that are present in the SM. The relevant time scales of the decays of heavy gauge bosons or scalars are slow, compared to the expansion rate of the Universe at early epoch of the cosmic evolution. The decays of these heavy particles are thus inherently out-of-equilibrium. But the GUT baryogenesis scenario has difficulties with the non-observation of proton decay, which puts a lower bound on the mass of the decaying boson, and therefore on the reheat temperature after inflation. Simple inflation models do not give such a high reheat temperature, which in addition, might regenerate unwanted relics.

Affleck-Dine baryogenesis: The Affleck-Dine baryogenesis [40, 41]. involves cosmological evolution of scalar fields which carry B charges. It is most naturally implemented in SUSY theories. Nevertheless, this mechanism faces the same challenges as in GUT baryogenesis and in EW baryogenesis.

Leptogenesis: It is another beautiful mechanism put forward by Fukugita and Yanagida [37] where decay of the lightest heavy Majorana neutrino produces a CP violating out-of-equilibrium decay. Our main work focuses on the motivated realization of leptogenesis: electromagnetic leptogenesis via 5D and 6D-dipole moment interactions like standard leptogenesis mediated by Yukawa couplings.

1.3.2 Leptogenesis

Leptogenesis is a mechanism which can generate a lepton asymmetry of the Universe before the electroweak phase transition which can be further converted to the required baryon asymmetry of the Universe in the presence of Sphaleron. The failure of the minimal SM to dynamically generate the correct amount of baryon asymmetry together with the fact that SM sphaleron strictly conserve the $B-L$ quantum number have motivated us to look for new physics that can violate lepton number L when tackling the baryogenesis problem. Indeed, if neutrinos are Majorana, then the induced dim-5 mass term: $y^2\overline{\ell}_L\,\phi\,\phi^T\ell_L^c/\Lambda$, will violate L by two units. Therefore, it is natural to ask whether such lepton violating interactions can actually lead to the observed baryon asymmetry.

The expression for final baryon asymmetry via sphaleron transitions can be written in terms $B-L$ or L [85, 86] is

$$B = \frac{28}{78}\,(B-L) = -\frac{28}{51}\,L\,, \tag{1.59}$$

from which one can conclude that an initial $B-L$ asymmetry can be partially converted into a B asymmetry by sphaleron and other SM processes.

In this work, we are especially interested in the leptogenesis scenario involving type I seesaw models [37] because, in our opinion, it presents the most "elegant" solution to both the smallness of neutrino masses and the observed baryon-to-photon ratio, while it only requires a rather modest extension of the SM. In addition to leptogenesis in type I scenario, it should be added in passing that leptogenesis based on type II [60, 66, 67], type III [68, 69] seesaw are also possible.

Leptogenesis with hierarchical RH neutrinos

The generic leptogenesis scenario of Fukugita and Yanagida [37] involves the type I seesaw Lagrangian of (1.60) with three heavy RH Majorana neutrinos, so that the L violating Yukawa interactions between the RH neutrinos and the ordinary LH leptons can generate a $B-L$ asymmetry during the primordial times. The spectrum of the RH neutrino is assumed to be hierarchical masses in this scenario (i.e, $M_1 < M_2 < M_3$), and therefore the asymmetry created will be dominated by the decays of the lightest RH neutrinos (denoted N_1) due to the efficient washout of any $N_{2,3}$-generated asymmetries by N_1 mediated $\Delta L \neq 0$ scattering processes in equilibrium. Also the Majorana masses of heavy neutrinos are assumed to be GUT scale and this guarantees successful seesaw mechanism producing left-handed light neutrinos with the correct mass scale.

We can write the Lagrangian (1.60) in the mass eigenbasis of the heavy RH neutrinos (denoting the heavy RH Majorana neutrinos with $N \equiv \nu_R{}' + (\nu_R{}')^c$ where $\nu_R{}'$ is the mass eigenstate after the change of basis from ν_R.) as

$$\mathcal{L}_{\rm int} = -y_{\alpha\beta}\,\overline{\ell}_\alpha\,\widetilde{\phi}\,e_\beta - h_{jk}\,\overline{\ell}_j\,\phi\,N_k - \frac{1}{2}\,\overline{N}_k\,M_k\,N_k + \text{h.c.}\ , \tag{1.60}$$

where flavor indices α, β, j can be one of e, μ or τ, and $k = 1, 2, 3$ are labels for the lightest to heaviest RH neutrinos (with mass M_k). The $SU(2)_L$ doublets: $\ell_\alpha = (\nu_L, e_L)^T_\alpha$ and $\phi = (\phi^0, \phi^-)^T$ have their usual meanings, with $\widetilde{\phi} = i\sigma_2\phi^*$ being the charge conjugate Higgs. The Yukawa couplings $h_{jk}\,\overline{\ell}_j\,\phi\,N_k$ in (1.61) can then induce heavy RH neutrino decays via two channels:

$$N_k \rightarrow \begin{cases} \ell_j + \overline{\phi}\ , \\ \overline{\ell}_j + \phi\ , \end{cases} \tag{1.61}$$

which violate lepton number by one unit. All Sakharov's conditions for leptogenesis will be satisfied if these decays also violate CP and go out of equilibrium at some stage during the evolution of the early universe. The requirement for CP violation means that coupling matrix h in (1.60) must be complex and the mass of N_k must be greater than the combined mass of ℓ_j and ϕ, so that interferences between the tree-level process (Fig. 1.4a) and the one-loop corrections (Fig. 1.4b, c) with on-shell intermediate states will be nonzero. Clearly, both of these are possible as type I seesaw mechanism naturally implies a very large M_k in order to induce small LH neutrino masses, while it does not forbid the presence of CP violating phases in the RH neutrino sector. The condition of thermal non-equilibrium is achieved when the expansion rate of the universe exceeds the decay rate of N_k. In practice this requirement is given by

$$\Gamma_D|_{T=M_1}\ <\ H|_{T=M_1}$$

where M_1 is the mass of heavy neutrino.

Now the formula for CP asymmetry in the lepton number production due to N_k decays:

$$\varepsilon_{kj} = \frac{\Gamma(N_k \rightarrow \ell_j\,\overline{\phi}) - \Gamma(N_k \rightarrow \overline{\ell_j}\,\phi)}{\Gamma(N_k \rightarrow \ell_j\,\overline{\phi}) + \Gamma(N_k \rightarrow \overline{\ell_j}\,\phi)}\ . \tag{1.62}$$

Explicit calculation of the interference terms, in case of N_1 dominated scenario, will then

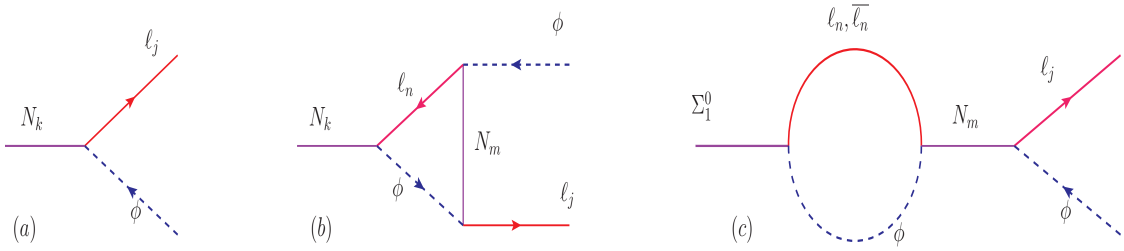


Figure 1.4: The **(a)** tree-level, **(b)** one-loop vertex correction, and **(c)** one-loop self-energy correction graphs for the decay: $N_k \to \ell_j \overline{\phi}$.

result in [38, 39]:

$$\varepsilon_1 = \frac{1}{8\pi} \sum_{m \neq 1} \frac{\mathrm{Im}\left[(h^\dagger h)^2_{1m}\right]}{(h^\dagger h)_{11}} \left\{ f_V\left(\frac{M_m^2}{M_1^2}\right) + f_S\left(\frac{M_m^2}{M_1^2}\right) \right\} , \tag{1.63}$$

where $f_V(x)$ and $f_S(x)$ are given by

$$f_V(x) = \sqrt{x}\left[1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right] \quad \text{and} \quad f_S(x) = \frac{\sqrt{x}}{1-x} \tag{1.64}$$

which denote the vertex and self-energy contributions respectively. The tree-level N_1 decay rate (at $T = 0$) used to calculate the denominator of (1.62) with j summed is given by:

$$\Gamma(N_1 \to \ell\,\overline{\phi}) \equiv \Gamma(N_1 \to \overline{\ell}\,\phi) = \frac{(h^\dagger h)_{11}}{16\pi} M_1 \,. \tag{1.65}$$

Suppose that $|h_{jk}| \leq |h_{33}|$ for all j and k, then in the hierarchical limit of $M_1 \ll M_{2,3}$, the seesaw relation gives:

$$m_3 \simeq \frac{|h_{33}|^2 \langle\phi\rangle^2}{M_3} , \tag{1.66}$$

where m_3 is mass of the heaviest LH neutrino. Assuming these conditions, and using the fact that

$$|f_V(x) + f_S(x)| \simeq \frac{3}{2\sqrt{x}} , \qquad \text{for } x \gg 1 , \tag{1.67}$$

one can estimate the CP asymmetry as

$$|\varepsilon_1| \simeq \frac{3}{16\pi} |h_{33}|^2 \left(\frac{M_1}{M_3}\right) \sin\,\delta_N , \tag{1.68}$$

$$= \frac{3}{16\pi} \frac{m_3 M_1}{\langle\phi\rangle^2} \sin\,\delta_N , \tag{1.69}$$

where in the last line we have used (1.66). The quantity: $\sin\delta_N$, is a measure of the amount of CP violation in the decay with $\delta_N = \arg\left[(h^\dagger h)^2_{13}\right]$ which is in general different from the CP phase appearing in neutrino oscillations. Relation (1.69) implies that the size of $|\varepsilon_1|$ cannot be arbitrarily large for a given M_1. Taking $m_3 \simeq 0.05$ eV and $\langle\phi\rangle \simeq 174$ GeV, one gets a useful ballpark estimate of the maximum CP asymmetry as

$$|\varepsilon_1|^{\max} \approx 10^{-6} \left(\frac{M_1}{10^{10}\ \mathrm{GeV}}\right) . \tag{1.70}$$

Within the type I seesaw paradigm, this result actually holds in general as long as the LH neutrinos are strongly hierarchical [74].

Boltzmann equations for leptogenesis

Leptogenesis is closely related to the classical GUT baryogenesis [31], where the deviation of the distribution function of some heavy particles from its equilibrium distribution provides the necessary departure from thermal equilibrium. The non-equilibrium process of baryogenesis via leptogenesis is usually studied by means of Boltzmann equation [56, 83]. We shall consider the simplest case where the initial temperature is larger than M_1, the mass of the lightest heavy neutrino. In principle, one should take into account all B- and L-violating processes. In this treatise, however, we consider only decays, inverse decays, $\Delta L = 2$ scattering and the sphalerons.

Within this minimal framework, the Boltzmann equations can be written as

$$\frac{dY_{N_1}}{dz} = -(D+S)\left[Y_{N_1} - Y^{eq}_{N_1}\right] \tag{1.71}$$

$$\frac{dY_{\mathcal{B}-\mathcal{L}}}{dz} = -\epsilon_{N_1} D \left[Y_{N_1} - Y^{eq}_{N_1}\right] - W Y_{\mathcal{B}-\mathcal{L}} \tag{1.72}$$

where $z = M_1/T$. There are four classes of processes which contribute to the different terms of the equations: decays, inverse decays, $\Delta L = 1$ scatterings and $\Delta L = 2$ processes mediated by heavy neutrinos. The first three all modify the N_1 abundance and try to push it towards its equilibrium value N_1^{eq}. In this case, we have considered the normalized quantity $Y_{N_1} = N_1/\mathrm{s}$, s is the entropy of the Universe. The term $D = \Gamma_D/(H\,z)$ accounts for decays and inverse decays, while the scattering term $S = \Gamma_S/(H\,z)$ represents the scattering process mediated by the heavy neutrino. Also Decays are the source term for $\mathcal{B}-\mathcal{L}$ asymmetry generation while $W = \Gamma_W/(H\,z)$ is the wash-out term which tries to erase the net $\mathcal{B}-\mathcal{L}$ asymmetry produced by the decay process.

This coupled set of Boltzmann equations may be solved numerically or (semi-)analytically by asymptotic methods. Either way, the conclusion is that for a wide range of seesaw neu-

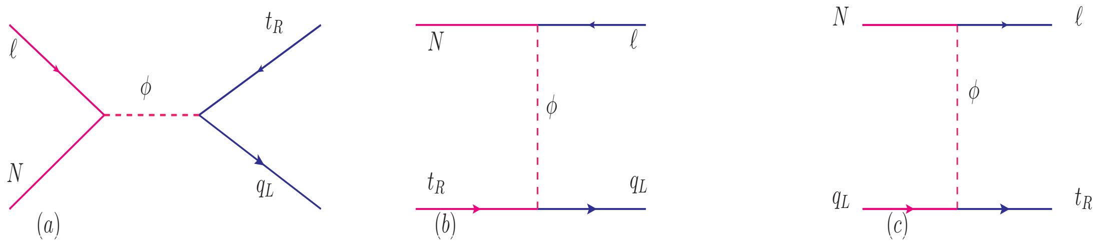


Figure 1.5: The $\Delta L = \pm 1$ processes that can influence n_{N_1} and n_{B-L}: **(a)** s-channel scattering $N\ell \leftrightarrow q_L \bar{t}_R$, **(b)** t-channel scattering $Nt_R \leftrightarrow q_L \bar{\ell}$, **(c)** t-channel scattering $Nq_L \leftrightarrow t_R \ell$. Here q_L denotes the 3rd generation of the quark doublet.

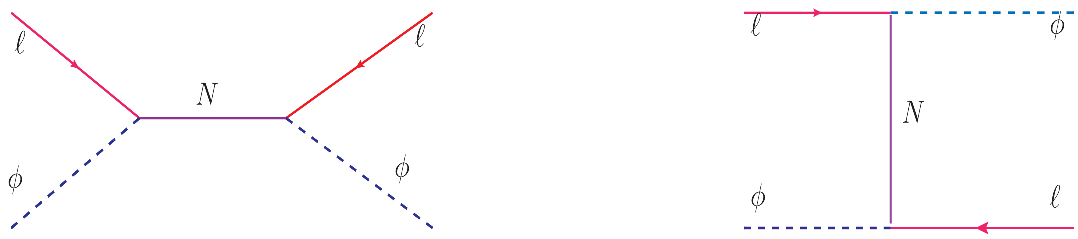


Figure 1.6: The $\Delta L = \pm 2$ s- and t-channel scattering processes mediated by N.

trino parameters, a nonzero excess of $B - L$ can be generated [81–83]. Explicitly, if one expresses the maximum baryon-to-photon ratio generated as

$$\eta_B^{\text{max}} \simeq 0.96 \times 10^{-2}\, |\varepsilon_1|\, \kappa_{\text{f}}^{\text{max}} \,, \tag{1.73}$$

with $\kappa_{\text{f}}^{\text{max}}$ denoting the maximum final efficiency factor obtained from solving the Boltzmann equations, and the pre-factor of 0.96×10^{-2} coming from the dilution due to imperfect sphaleron conversion and photon production before recombination, then one may directly restrict the possible neutrino parameter space for successful baryogenesis via $|\varepsilon_1|$ (and to some degree κ^{f} because the reaction rates depend on the mass of N_1 and the Yukawas). In the best case scenarios where a maximum efficiency factor of about $\kappa_{\text{f}}^{\text{max}} \approx 0.18$ is achieved [81–83], and assuming strongly hierarchical LH neutrinos, then one obtains a lower bound for the heavy RH neutrino mass M_1 as

$$M_1 \gtrsim 3.5 \times 10^9 \text{ GeV} \,, \tag{1.74}$$

where we have used relation (1.70) and taken the value of η_B given by (1.49).

More generally, in many situations with $M_1 \lesssim 10^{14}$ GeV, one has, to good approximation, $\kappa^{\mathrm{f}} \simeq 2 \times 10^{-2}$. This then implies that a raw CP asymmetry of about $|\varepsilon_1| \simeq 3 \times 10^{-6}$ is required for baryogenesis to succeed.

In summary, we have highlighted some of the essential features in quantitatively understanding the classic leptogenesis scenario of [37] which has the type I seesaw setup as its backbone. Specifically, we have discussed the "standard" situation where the heavy RH Majorana neutrinos are strongly hierarchical. As a result, only the lightest of the three RH neutrinos, N_1, is expected to contribute significantly to the final asymmetry. This is because the $B - L$ violating interactions mediated by N_1 would still be in thermal equilibrium when $N_{2,3}$ decayed away, and therefore any excess $B - L$ produced by $N_{2,3}$ would be erased. When the N_1's eventually decay out-of-equilibrium, an excess of $B - L$ is created through CP violating loop effects. Subsequently, this excess is converted into a B asymmetry by SM sphaleron.

The exact amount of B generated in this way depends crucially on the interplay between the decay and washout processes, as well as the raw CP asymmetry the neutrino model under consideration contains. By studying the Boltzmann evolution of the particle species and the explicitly calculating the loop diagrams, both of these crucial ingredients may be conveniently encapsulated into the efficient factor (κ^{f}) and CP asymmetry (ε_1) respectively. Consequently, variations to the standard scenario can be quantified by changes in these values.

Over the years, there has been a dramatic increase in the sophistication of the quantitative analysis of leptogenesis. Many previously neglected effects such as thermal corrections [84], spectator processes [80, 81] and, above all, flavor effects [75, 77, 79] have been considered in recent analyses. Other variations to the general scheme, including asymmetry production dominated by the decays of the second lightest RH neutrino N_2 [76], resonant leptogenesis [59, 88, 89, 91–94] and models with more than three heavy RH neutrinos [78], have also received attention. In the next few subsections, we will briefly mention some of these ideas which go beyond the standard scenario, and hint on how they may broaden the class of neutrino models that will lead to successful leptogenesis.

1.3.3 Resonant leptogenesis

The possibility of quasi-degenerate RH neutrinos are not excluded by any existing experimental data nor they are forbidden by the generic seesaw setup. In this case, the leptogenesis is known as *resonant leptogenesis* [59, 88, 89, 91–94] which can occur when the mass splitting between two RH neutrinos becomes small enough, leading to enhancement of the CP asymmetry ε_j. One need to worry about two main issues namely: the size of the CP asymmetry and the final efficiency factor. When considering the situation of $M_j \simeq M_k$ for $j \neq k$ more closely, we first realize that, qualitatively, the washout rate must increase at $T \simeq M_{j,k}$ because L violating scattering processes mediated by M_j and M_k would both be active, providing more ways to erase the generated asymmetry. Secondly, in the expression for ε_j, we have either employed the approximation of $M_k/M_j \gg 1$ or $M_k/M_j \ll 1$. However, a quasi-degenerate RH neutrino spectrum demands the condition of $M_k/M_j = \mathcal{O}(1)$, and hence the limits on ε_j must be re-studied.

In the expression for ε_j in previous case, we see that the most interesting behavior must come from the self-energy correction term, $f_S(x)$ as $M_j \to M_k$ as

$$\begin{aligned} \lim_{x\to 1} f_S(x) = \lim_{x\to 1} \frac{\sqrt{x}}{1-x} &= \lim_{M_j \to M_k} \frac{M_j\, M_k}{M_j^2 - M_k^2}\,, \qquad \text{with } x \equiv M_k^2/M_j^2\,, \\ &\stackrel{?}{=} \infty\,. \end{aligned} \tag{1.75}$$

This conclusion comes from the fact that, in the calculation of the self energy contribution by Buchmuller and Covi, they do not have to use Pinch mechanism. One may follow the re summation approach of [88, 91, 93] where an additional regulating absorptive term due to the finite decay width of $M_{j,k}$ naturally emerges to overcome such conclusion. The self-energy contribution to the CP violation parameter, near the degenerate case, is then modified to [88, 91, 93]

$$\varepsilon_j \simeq \frac{\mathrm{Im}\left[(h^\dagger h)^2_{jk}\right]}{(h^\dagger h)_{jj}(h^\dagger h)_{kk}}\, \frac{2(M_j^2 - M_k^2)\, M_j\, \Gamma_j}{(M_j^2 - M_k^2)^2 + 4M_j^2\,\Gamma_j^2}\,, \tag{1.76}$$

where $j, k = 1, 2$ or $2, 3$ $(j \neq k)$ and $\Gamma_j = (h^\dagger h)_{jj}\, M_j/16\pi$ is the generalization of the tree-level decay rate as defined in (1.65). From the expression of (1.76), one can see that $\varepsilon_j \to 0$ when $M_j \to M_k$ in accordance with the observation that the RH neutrino running in the loop must be different from the decaying one in order to generate an asymmetry.

More importantly, Eq. (1.76) indicates that the CP asymmetry will be enhanced provided that the mass splitting between the two RH neutrinos coincides with the region of mass parameters about which the ε_j function peaks. Specifically, one requires

$$|M_j - M_k| \sim \Gamma_{j,k}\,, \tag{1.77}$$

to maximize the resonant effect. With this, one can see that if the Yukawa couplings are such that

$$\frac{\mathrm{Im}\left[(h^\dagger h)^2_{jk}\right]}{(h^\dagger h)_{jj}(h^\dagger h)_{kk}} = \mathcal{O}\left(1\right) \ , \tag{1.78}$$

then ε_j can be as large as $\mathcal{O}\left(1\right)$, hence provide a lot more leverage for successful leptogenesis. Indeed, the increase in washout due to the tiny mass gap between N_j's will eventually saturate when the degenerate limit reaches a certain point and the enhancement from resonant effects will be able to dominate the outcome. Consequently, given the substantial enhancement by resonant leptogenesis, some of the stringent constraints on the neutrino properties imposed by the standard hierarchical scenario may be evaded. Most notably, the lower bound (1.74) on M_1 is completely removed, leading to the possibility of TeV scale RH neutrinos and TeV leptogenesis [88, 94]. In SUSY leptogenesis theories, this is particularly advantageous as the upper bound on the reheating temperature ($T_{\rm reh}$) due to BBN constraints on gravitino over-production, is often in conflict with the condition, $T_{\rm reh} \gtrsim M_j$, normally required for the sufficient thermal generation of N_j's which participate in L creation. Furthermore, N_2- and even N_3-leptogenesis are now easily achievable under this scenario, and hence the set of applicable seesaw models is significantly expanded.

Certainly, this particular model and many others that employ resonant leptogenesis can have the RH Majorana neutrinos to be as small as 1 TeV and depending on their couplings to SM particles, collider signatures of them may also accessible in the near future [89, 94]. Recently a very interesting possibility of electromagnetic leptogenesis [193] has been proposed, wherein the source of CP violation has been identified with the electromagnetic dipole moment(s) of the neutrino(s). For a collection of neutrino fields of the same chirality, the most general form of such couplings is given by $\overline{\nu_j^c}(\mu_{jk} + i\gamma_5\mathcal{D}_{jk})\sigma_{\alpha\beta}\nu_k B^{\alpha\beta}$, where $B^{\alpha\beta}$ denotes the $U(1)$ field strength tensor. The magnetic and electric transition moment matrices, μ_{jk} and $\mathcal{D}_{jk}$, each need to be antisymmetric. For two Majorana neutrinos combining to give a Dirac particle, the resultant matrix, clearly, does not not suffer from such restrictions. The aforementioned dimension-five operators are, presumably, generated by some new physics operative beyond the electroweak scale. With CP-violation being encoded in the structure of the dipole moments, the decays of heavier neutrinos to lighter ones and a photon, can, in principle, lead to a lepton asymmetry in the universe. Although the proposal is a very interesting one, thus far it has not been incorporated in any realistic model. We propose a specific model for resonant electromagnetic leptogenesis which will be presented in detail later on. A guiding principle in our quest is that the new physics should be at the TeV scale so as to render the model testable at the LHC or future Linear Colliders

1.4 Outline of our work

The observational evidence for nonzero neutrino masses, origin of parity violation, ultimate unification of fundamental forces and cosmological matter-antimatter asymmetry provides a strong indication for physics beyond the SM. The goal of the thesis is to study several classes of non-susy and supersymmetric models to address these issues like spontaneously parity breaking, neutrino mass via seesaw mechanism and their connection to lepton asymmetry and self consistency with RG running of the coupling constant.

The first part of our work (Chapter-2) is a comprehensive analysis on supersymmetric left-right models in the context of spontaneous parity breaking. We propose a novel implementation of spontaneous parity breaking in supersymmetric left-right symmetric model, avoiding some of the problems encountered in previous studies by including a bitriplet and a singlet, in addition to the bidoublets which extend the Higgs sector of the Minimal Supersymmetric Standard Model (MSSM).

In Chapter (3), we will discuss the different scenarios of spontaneous breaking of D-Parity in both non-Susy and Susy version of left right symmetric models. Main motivation of this work is to explore the possibility of a TeV scale $SU(2)_R$ breaking scale M_R and hence TeV scale right handed neutrinos from both minimization of the scalar potential as well as the coupling constant unification point of view with spontaneous D-parity breaking scheme.

In Chapter (4), we will study the question of parity breaking, neutrino mass and leptogenesis problem in a supersymmetric left-right model, in which the left-right symmetry is broken with Higgs doublets (carrying $B - L = \pm 1$).

In Chapter (5), we analyze the SU(5) gauge coupling unification and argue that the gravitational corrections to gauge coupling constants may not vanish when higher dimensional non-renormalizable terms are included in the problem.

In Chapter (6), we shall discuss the electromagnetic interactions between the LH and RH neutrinos. The inclusion of heavy RH neutrinos to the SM as in type I seesaw then naturally gives rise to new transition electromagnetic moments involving both LH and RH neutrinos. Our main goal is to find a realistic model that will give leptogenesis scenario by explicitly calculating the CP asymmetry coming from the out-of-equilibrium decays of the heavy RH neutrinos via electromagnetic interactions.

Finally, we conclude our entire work in Chapter (7).

CHAPTER

2

Spontaneous Parity breaking in SUSYLR model

The left-right symmetric model has since long received considerable attention as a simple extension of the standard model and it has already been discussed in section (1.2.2) of chapter-[1]. As we know, chirality is an elegant ingredient of nature which prevents unduly large masses for fermions, on the other hand, most of nature is left-right symmetric suggesting the reasonable hypothesis that parity is only spontaneously broken, a principle built into the left-right symmetric models. This class of models also provides a natural embedding of electroweak hypercharge, giving a physical explanation for the required extra $U(1)$ as being generated by the difference between the baryon number (B) and the lepton number (L). Thus, $B - L$, the only exact global symmetry of SM becomes a gauge symmetry, ensuring its exact conservation, in turn leading to several interesting consequences.

One of the attractive features of the supersymmetric models is it's ability to provide a candidate for the cold dark matter of the universe. This however relies on the theory obeying R-parity conservation [112, 113], defined as $R = (-1)^{3(B-L)+2S}$ defined in terms of the gauged $(B - L)$, in order to prevent fast proton decay which we don't want. In MSSM, R-parity is not automatic and is achieved by imposing global baryon and lepton number conservation on the theory as an additional requirements. It is therefore desirable to seek supersymmetric theories where, like the standard model, R-parity conservation (Baryon and lepton number conservation) becomes automatic. The supersymmetric left-right theory can give explanation to all the puzzles of the Standard Model. This kind of theory implements the seesaw mechanism for neutrino masses and gives satisfactory answer to the parity breakdown as seen in low energy electroweak theory [99, 100]. Since right handed neutrino is a automatic consequences of supersymmetric left-right theory, it can explain the tiny neutrino mass and cosmic baryon asymmetry of the present universe.

The minimal supersymmetric left-right theory has its own limitations like other theories and we shall explain some of them in the next section. One need a self-consistent theory which can overcome these drawbacks and we will give such a set up to solve the problem. In this chapter we will discuss another interesting model, which is self consistent and phenomenologically rich, with one copy of bitriplet and parity odd singlet which achieves the goal of spontaneous parity breaking in supersymmetric left-right model.

2.1 Discussion of spontaneous parity breaking in minimal SUSYLR model

We review the particle content of the SUSYLR model in order to show parity can not be spontaneously broken in the minimal model. In the left-right symmetric models, it is assumed that the MSSM gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ is enhanced at some higher energy, when the left-handed and right-handed fermions are treated on equal footing. The minimal supersymmetric left-right (SUSYLR) model has the gauge group $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ which could emerge from a supersymmetric $SO(10)$ grand unified theory.

The quark and lepton superfields in a supersymmetric left-right [99–103] model is given by their transformations are given by,

$$Q \equiv [3, 2, 1, \frac{1}{3}], \qquad Q^c \equiv [3^*, 2, 1, -\frac{1}{3}],$$
$$L \equiv [1, 2, 1, -1], \qquad L^c \equiv [1, 1, 2, 1] \tag{2.1}$$

where, the numbers in the brackets denote the quantum numbers under $SU(3)_C$, $SU(2)_L$, $SU(2)_R$, $U(1)_{B-L}$. We have omitted the generation index for simplicity. The left-right symmetry could be broken by either doublet Higgs scalars or triplet Higgs scalar. It has been argued that for a minimal choice of parameters, it is convenient to break the group with a triplet Higgs scalar. The minimal Higgs superfields required for the symmetry breaking is

$$\begin{aligned}
\Delta &\equiv [1,3,1,2], & \bar{\Delta} &\equiv [1,3,1,-2)], \\
\Delta^c &\equiv [1,1,3,-2)], & \bar{\Delta}^c &\equiv [1,1,3,2)], \\
\Phi_i &\equiv [1,2,2^*,0], & &(i=1,2).
\end{aligned} \tag{2.2}$$

As pointed out in [121], the bidoublets are doubled to achieve a non vanishing Cabibbo-Kobayashi-Maskawa (CKM) quark mixing and the number of triplets is doubled for the sake of anomaly cancellation. The left-right symmetry is implemented in these theories as a discrete parity transformation as

$$\begin{aligned}
&Q \longleftrightarrow Q_c^*, \quad L \longleftrightarrow L_c^*, \quad \Phi \longleftrightarrow \Phi^\dagger \\
&\Delta \longleftrightarrow \Delta^{c*}, \quad \bar{\Delta} \longleftrightarrow \bar{\Delta}^{c*}.
\end{aligned} \tag{2.3}$$

The minimal supersymmetric left-right model however can not break parity spontaneously. To prove this statement, we will follow the discussions of Kuchimanchi and Mohapatra closely [115, 116]. The superpotential for this theory is given by

$$\begin{aligned}
W \quad &= Y^{(i)_q} Q^T \tau_2 \Phi_i \tau_2 Q^c + Y^{(i)_l} L^T \tau_2 \Phi_i \tau_2 L^c \\
&+ \quad i(h L^T \tau_2 \Delta L + h^* L^{cT} \tau_2 \Delta^c L^c) \\
&+ \quad \mu_\Delta \mathrm{Tr}(\Delta\bar{\Delta}) + \mu_\Delta^* \mathrm{Tr}(\Delta^c \bar{\Delta}^c) + \mu_{ij} \mathrm{Tr}(\tau_2 \Phi_i^T \tau_2 \Phi_j)
\end{aligned} \tag{2.4}$$

All couplings $Y^{(i)_{q,l}}$, μ_{ij}, μ_Δ, h in the above potential, are complex with the additional constraint that μ_{ij}, h and h^* are symmetric matrices. It is clear from the above eqn. that the theory has no baryon or lepton number violation terms. The potential obtained from the above superpotential via F and D flat conditions and including the soft-SUSY breaking terms is given by

$$V_{SUSY} = V_F + V_D + V_{\mathrm{soft}} \tag{2.5}$$

where,

$$V_F = \mathrm{Tr}|m_\Delta \overline{\Delta}|^2 + \mathrm{Tr}|m_\Delta \overline{\Delta}^c|^2 + |m_\Delta|^2 \mathrm{Tr}(\Delta^\dagger \Delta + \Delta^{c\dagger}\Delta^c) + 2\mathrm{Tr}|\mu \Phi^T|^2 \tag{2.6}$$

$$V_D = \mathrm{Tr}|m_\Delta \overline{\Delta}|^2 + \mathrm{Tr}|m_\Delta \overline{\Delta}^c|^2 + |m_\Delta|^2 \mathrm{Tr}(\Delta^\dagger \Delta + \Delta^{c\dagger}\Delta^c) + 2\mathrm{Tr}|\mu \Phi^T|^2 \tag{2.7}$$

$$\begin{aligned} V_{\rm soft} &= \left(M_1 - m_\delta^2\right) \mathrm{Tr}[\Delta^\dagger\Delta + \Delta^{c\dagger}\Delta^c + \left(M_2 - m_\delta^2\right) \mathrm{Tr}[\overline{\Delta}^\dagger\overline{\Delta} + \overline{\Delta}^{c\dagger}\overline{\Delta}^c] \\ &+ M'^2\mathrm{Tr}[\Delta\bar{\Delta} + \Delta^c\bar{\Delta}^c] + h.c. \\ &+ (M^2_{\Phi_{ij}} - 4\mu^2)\mathrm{Tr}(\Phi_i^\dagger\Phi_j) + \left[\frac{\mu'_{ij}}{2}\mathrm{Tr}(\tau_2\Phi_i^T\tau_2\Phi_j)\right] + h.c. \end{aligned} \tag{2.8}$$

Here one can choose the mass-squared terms M'^2 and μ^2 positive and real since their phase can be absorbed in redefinition of coupling constants, triplets (Δ's) and bidoublet (Φ's). Here, we have chosen the vevs of quarks and leptons to be zero for the time being.

There are various ranges of vev's of Higgs fields which make the susy potential bounded from below. Demanding that the potential should have a finite ground state, one can generally deduce constraint on the mass parameters depending upon the choice of the vev's of Higgs field. The advantage of doing this is to correlate different mass scales (shown in Table(2.1)) with each other such as:

$$M'^2 = M_1M_2\cos 2\theta, \quad \mu^2 = M^2_{\Phi_{ij}}\sin 2\theta' \tag{2.9}$$

Vev	Constraints
$\langle\Delta\rangle = \langle\Delta^c\rangle = v^2\tau_1$, $\langle\Phi\rangle = 0$, $\langle\overline{\Delta}\rangle = \langle\overline{\Delta}^c\rangle = 0$	$M^2_{1,2} \geq 0$
$\langle\Delta\rangle = \langle\Delta^c\rangle = (v^2/M_1)\tau_1$, $\langle\Phi\rangle = 0$, $\langle\overline{\Delta}\rangle = \langle\overline{\Delta}^c\rangle = -(v^2/M_2)\tau_1$	$M'^2 \leq M_1M_2$
$\langle\Delta\rangle = \langle\Delta^c\rangle = \langle\overline{\Delta}\rangle = \langle\overline{\Delta}^c\rangle = 0$, $\langle\Phi\rangle = 0$ and $k = k' = 0$	$M^2_{\Phi_{ij}} \geq 0$ and $\mu^2 \leq M^2_{\Phi_{ij}}$

Table 2.1: Constraints on mass-squared parameters from ground state of the potential

The Higgs potential with this choice can be written as

$$\begin{aligned} V_{SUSY} &= \cos^2\theta\mathrm{Tr}[(M_1\Delta + M_2\overline{\Delta}^\dagger)^\dagger(M_1\Delta + M_2\overline{\Delta}^\dagger)] \\ &+ \sin^2\theta\mathrm{Tr}[(M_1\Delta - M_2\overline{\Delta}^\dagger)^\dagger(M_1\Delta - M_2\overline{\Delta}^\dagger)] \\ &+ M^2_{\Phi_{ij}}[\cos^2\theta'(k + k'^*)^*(k + k'^*) + \sin^2\theta'(k - k'^*)^*(k - k'^*)] \\ &+ \Delta \to \Delta^c + \overline{\Delta} \to \overline{\Delta}^c + V_D\mathrm{terms} \end{aligned}$$

One of the most important problems is the spontaneous breaking of left-right symmetry [115, 116], viz., all vacuum expectation values breaking $SU(2)_L$ are exactly equal in magnitude to those breaking $SU(2)_R$, making the vacuum parity symmetric. In mathematical language, this can be inferred as: the ground state of the Higgs potential is $V_{SUSY} = 0$ iff

$$\langle\Delta\rangle = \langle\overline{\Delta}\rangle = \langle\Delta^c\rangle = \langle\overline{\Delta}^c\rangle = k = k' = 0$$

From above discussion, It turns out that left-right symmetry imposes rather strong constraints on the ground state of this model. Also that there is no spontaneous parity breaking for this minimal choice of Higgs in the supersymmetric left-right model and as such the ground state remains parity symmetric. There have been suggestions to solve this problem by introducing additional fields, or higher dimensional operators, or by going through a different symmetry breaking chain or breaking the left-right symmetry along with the supersymmetry breaking [115–117, 121, 121, 123, 130].

If parity odd singlets are introduced to break this symmetry [122], then it was shown [115] that the charge breaking vacua have a lower potential than the charge-preserving vacua and as such the ground state does not conserve electric charge. A recent improvement [117] using a parity even singlet may however deviate significantly from MSSM, and remains to be explored fully for its phenomenological consistency. Breaking R parity was another possible solution to this dilemma of breaking parity symmetry. However, if one wants to prevent proton decay, then one must look for alternative solutions. One such possible solution is to add two new triplet superfields $\Omega(1,3,1,0)$, $\Omega_c(1,1,3,0)$ where under parity symmetry $\Omega \leftrightarrow \Omega_c^*$. This field has been explored extensively in [114, 118, 119, 121, 123, 130]. But these models has it's own disadvantage from the cosmological point of view.

We propose an another model to solve the problem of spontaneous parity breaking by adding a bitriplet and parity odd singlet under $SU(2)$ gauge group to the particle content of the minimal supersymmetric left-right model.

2.2 SUSYLR Model including a Bitriplet and a Singlet

We now recapitulate the important features of the minimal left-right symmetric model extended with one bitriplet and parity odd singlet scalar field in the context of spontaneous parity violation and RG running of fermion masses. These extra fields are vector-like and hence do not contribute to anomaly, so we consider only one of these fields.

The gauge group of this model is $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$. The quantum numbers for the superfields including the scalar fields η and σ, under the gauge group considered are given by the table [2.2] as follows

	$SU(3)^c$	$\times$ $SU(2)_L$	$\times$ $SU(2)_R$	$\times$ $U(1)_{B-L}$
Matter Superfiled:				
Q	3	2	1	$+1/3$
Q^c	3	1	2	$-1/3$
L	1	2	1	-1
L^c	1	1	2	$+1$
Higgs Superfiled:				
Φ_a	1	2	2	0
Δ	1	3	1	$+2$
Δ^c	1	1	3	-2
$\bar{\Delta}$	1	3	1	-2
$\bar{\Delta}^c$	1	1	3	$+2$
η	1	3	3	0
σ	1	1	1	0

Table 2.2: This table shows the particle content and their quantum number under the gauge groups $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

The Q and the L are the standard quarks and leptons of the MSSM while the Q^c and L^c contain the corresponding right-handed conjugate fields. In order to keep this model general, we allow for two bidoublets i.e, Φ_a $(a = 1, 2$. The charge is determined by the equation $Q = I_{3L} + I_{3R} + \frac{B-L}{2}$, where I_{3L}, I_{3R} are the 3rd component of isospin of the $SU(2)_L$, $SU(2)_L$ representation of the particle content. The representation of these superfields in matrix form is

$$
\begin{aligned}
Q &= \begin{pmatrix} u \\ d \end{pmatrix} \equiv [3,2,1,\frac{1}{3}] \quad , \quad Q^c = \begin{pmatrix} u^c \\ d^c \end{pmatrix} \equiv [3,1,2,-\frac{1}{3}] \,, \\
L &= \begin{pmatrix} \nu \\ e \end{pmatrix} \equiv [1,2,1,-1] \quad , \quad L^c = \begin{pmatrix} N^c \\ e^c \end{pmatrix} \equiv [1,1,2,-1]
\end{aligned}
\tag{2.10}
$$

Unlike in MSSM, here the Higgs sector consists of the bidoublet and triplet superfields:

$$
\begin{aligned}
\Phi_1 &= \begin{pmatrix} \phi_{11}^0 & \phi_{11}^+ \\ \phi_{12}^- & \phi_{12}^0 \end{pmatrix} \equiv [1,2,2,0], \qquad \Phi_2 = \begin{pmatrix} \phi_{21}^0 & \phi_{21}^+ \\ \phi_{22}^- & \phi_{22}^0 \end{pmatrix} \equiv [1,2,2,0] \,, \\
\Delta &= \begin{pmatrix} \frac{\Delta_L^-}{\sqrt{2}} & \Delta_L^0 \\ \Delta_L^{--} & -\frac{\Delta_L^-}{\sqrt{2}} \end{pmatrix} \equiv [1,3,1,-2], \qquad \bar{\Delta} = \begin{pmatrix} \frac{\delta_L^+}{\sqrt{2}} & \delta_L^{++} \\ \delta_L^0 & -\frac{\delta_L^-}{\sqrt{2}} \end{pmatrix} \equiv [1,2,1,2] \,,
\end{aligned}
\tag{2.11}
$$

$$\Delta_C = \begin{pmatrix} \frac{\Delta_R^-}{\sqrt{2}} & \Delta_R^0 \\ \Delta_R^{--} & -\frac{\Delta_R^-}{\sqrt{2}} \end{pmatrix} \equiv [1,1,3,-2] \quad , \quad \bar{\Delta}_C = \begin{pmatrix} \frac{\delta_R^+}{\sqrt{2}} & \delta_R^{++} \\ \delta_R^0 & -\frac{\delta_R^+}{\sqrt{2}} \end{pmatrix} \equiv [1,1,3,2]\,.$$

These fields transform under $SU(2)$ as

$$\begin{array}{cc} Q \to U_L Q & Q^c \to U_R Q^c \\ L \to U_L L & L^c \to U_R L^c \\ \Delta \to U_L \Delta U_L^\dagger & \Delta^c \to U_R \Delta^c U_R^\dagger \\ \bar{\Delta} \to U_L \bar{\Delta} U_L^\dagger & \bar{\Delta}^c \to U_R \bar{\Delta}^c U_R^\dagger \\ \Phi_a \to U_L \Phi_a U_R^\dagger & \eta \to U_L \eta U_R^\dagger \\ \sigma \to \sigma & \end{array}$$

and under Parity as

$$\begin{array}{cc} Q \to -i\tau_2 Q^{c*} & Q^c \to i\tau_2 Q^* \\ L \to -i\tau_2 L^{c*} & L^c \to i\tau_2 L^* \\ \Delta \to \tau_2 \Delta^{c*} \tau_2 & \Delta^c \to \tau_2 \Delta^* \tau_2 \\ \bar{\Delta} \to \tau_2 \bar{\Delta}^{c*} \tau_2 & \bar{\Delta}^c \to \tau_2 \bar{\Delta}^* \tau_2 \\ \Phi_a \to \Phi_a^\dagger & \eta \to \eta^\dagger \\ \sigma \to -\sigma^* & \end{array}$$

The symmetry breaking pattern in this model is

$$\begin{aligned} SU(2)_L \times SU(2)_R \times \quad & U(1)_{B-L} \times P \\ & \xrightarrow{\langle\sigma\rangle} \quad SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ & \xrightarrow{\langle\Delta_c\rangle} \quad SU(2)_L \times U(1)_Y \\ & \xrightarrow{\langle\Phi\rangle} \quad U(1)_{em} \end{aligned}$$

At high scale ($\geq 10^{15}$ GeV to Planck scale), the parity is broken by a singlet field $\sigma = (1,1,1,0)$ and it leaves the gauge symmetry $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ intact.

2.2.1 Superpotential of the model

The superpotential for the model is written in the more general tensorial notation is

$$\begin{aligned} \mathcal{W} \quad &= \quad \mathcal{W}_1 + \mathcal{W}_2 \qquad (2.12) \\ \text{where} \qquad & \mathcal{W}_1 = i y_{\alpha i}^{a,q} Q_\alpha^T \tau_2 \Phi^a Q_i^c + i y'^{a,\ell} L_\alpha^T \tau_2 \Phi^a L_i^c \\ & + i Y_{\alpha\beta}^{\Delta} L_\alpha^T \tau_2 \Delta L_\beta + i y_{ij}^{\Delta^c} L_i^{cT} \tau_2 \Delta^c L_j^c \end{aligned}$$

$$
\begin{aligned}
\mathcal{W}_2 \;\; &= \;\; f \eta_{\alpha i}\, \Delta_\alpha\, \Delta_i^c + f^* \eta_{\alpha i}\, \bar{\Delta}_\alpha\, \bar{\Delta}_i^c \\
&+ \lambda_1\, \eta_{\alpha i}\, \Phi_{am}\, \Phi_{bn}\, (\tau^\alpha \epsilon)_{ab} \left(\tau^i \epsilon\right)_{mn} + m_\eta\, \eta_{\alpha i}\, \eta_{\alpha i} \\
&+ M \,\left(\Delta_\alpha \bar{\Delta}_\alpha + \Delta_i^c \bar{\Delta}_i^c\right) + \mu\, \epsilon_{ab}\, \Phi_{bm}\, \epsilon_{mn}\, \Phi_{an} \\
&+ m_\sigma\, \sigma^2 + \lambda_2\, \sigma \left(\Delta_\alpha \bar{\Delta}_\alpha - \Delta_i^c \bar{\Delta}_i^c\right), \qquad (2.13)
\end{aligned}
$$

where, $\alpha, \beta = 1, 2, 3$ and $a, b = 1, 2$ are $SU(2)_L$ indices, whereas $i, j = 1, 2, 3$ and $m, n = 1, 2$ are $SU(2)_R$ indices. The summation over repeated index is implied, with the change in basis from numerical $1, 2, 3$ indices to $+, -, 0$ indices as follows,

$$
\begin{aligned}
\Psi_\alpha \Psi_\alpha \;\; &= \;\; \Psi_1 \Psi_1 + \Psi_2 \Psi_2 + \Psi_3 \Psi_3 \\
&= \;\; \Psi_+ \Psi_- + \Psi_- \Psi_+ + \Psi_0 \Psi_0, \qquad (2.14)
\end{aligned}
$$

where, we have defined $\Psi_\pm = (\Psi_1 \pm i \Psi_2)/\sqrt{2}$ and $\Psi_0 = \Psi_3$. The vacuum expectation values (vev) that the neutral components of the Higgs sector acquires are,

$$
\begin{aligned}
&\langle \Delta_- \rangle = \langle \bar{\Delta}_+ \rangle = v_L, \qquad && \langle \Delta_+^c \rangle = \langle \bar{\Delta}_-^c \rangle = v_R, \\
&\langle \Phi_{+-} \rangle = v, && \langle \Phi_{-+} \rangle = v', \\
&\langle \eta_{+-} \rangle = u_1, && \langle \eta_{-+} \rangle = u_2, \\
&\langle \eta_{00} \rangle = u_0. &&
\end{aligned} \qquad (2.15)
$$

Assuming SUSY to be unbroken till the TeV scale implies the F and D flatness conditions for the scalar fields to be,

$$
\begin{aligned}
F_{\Delta_\alpha} \;\; &= \;\; f\, \eta_{\alpha i}\, \Delta_i^c + M \bar{\Delta}_\alpha + \lambda_2\, \sigma\, \bar{\Delta}_\alpha = 0, \\
F_{\bar{\Delta}_\alpha} \;\; &= \;\; f^*\, \eta_{\alpha i}\, \bar{\Delta}_i^c + M \Delta_\alpha + \lambda_2\, \sigma\, \Delta_\alpha = 0, \\
F_{\Delta_i^c} \;\; &= \;\; f\, \eta_{\alpha i}\, \Delta_\alpha + M\, \bar{\Delta}_i^c - \lambda_2\, \sigma\, \bar{\Delta}_i^c = 0, \\
F_{\bar{\Delta}_i^c} \;\; &= \;\; f^*\, \eta_{\alpha i}\, \bar{\Delta}_i + M\, \Delta_i^c - \lambda_2\, \sigma\, \Delta_i^c = 0, \\
F_\sigma \;\; &= \;\; 2 m_\sigma\, \sigma + \lambda_2\, \left(\Delta_\alpha \bar{\Delta}_\alpha - \Delta_i^c \bar{\Delta}_i^c\right) = 0,
\end{aligned}
$$

$$
\begin{aligned}
F_{\eta_{\alpha i}} \;\; &= \;\; f\, \Delta_\alpha\, \Delta_i^c + f^* \bar{\Delta}_\alpha\, \bar{\Delta}_i^c + 2\, m_\eta\, \eta_{\alpha i} \\
&\quad + \lambda_1\, \Phi_{am}\, \Phi_{bn} (\tau^\alpha \epsilon)_{ab} (\tau^i \epsilon)_{mn} = 0,
\end{aligned}
$$

$$
\begin{aligned}
F_{\Phi_{cp}} &= \lambda_1\, \eta_{\alpha i} \Phi_{bn}\, (\tau^\alpha \epsilon)_{cb} \left(\tau^i \epsilon\right)_{pn} \\
&\quad + \lambda_1\, \eta_{\alpha i} \Phi_{am}\, (\tau^\alpha \epsilon)_{ac} (\tau^i \epsilon)_{mp} \\
&\quad + \mu\, \epsilon_{ac}\, \epsilon_{pn}\, \Phi_{an} + \mu\, \epsilon_{cb}\, \Phi_{bm}\, \epsilon_{mp} = 0, \qquad (2.16)
\end{aligned}
$$

$$
\begin{aligned}
D_{R_i} &= 2\Delta^{c\dagger}\tau_i\Delta^c + 2\bar{\Delta}^{c\dagger}\tau_i\bar{\Delta}^c + \eta\tau_i^T\eta^\dagger + \Phi\tau_i^T\Phi^\dagger = 0, \\
D_{L_i} &= 2\Delta^\dagger\tau_i\Delta + 2\bar{\Delta}^\dagger\tau_i\bar{\Delta} + \eta^\dagger\tau_i\eta + \Phi^\dagger\tau_i\Phi = 0, \\
D_{B-L} &= 2\left(\Delta^\dagger\Delta - \bar{\Delta}^\dagger\bar{\Delta}\right) - 2\left(\Delta^{c\dagger}\Delta^c - \bar{\Delta}^{c\dagger}\bar{\Delta}^c\right) = 0. \qquad (2.17)
\end{aligned}
$$

In the above eqns., we have neglected the slepton and squark fields, since they would have zero vev at the scale considered. We have also assumed $v' \ll v$ and hence the terms containing v' can be neglected.

2.3 Phenomenology

An inspection of the minimization conditions obtained at the end of the previous section proves two important statements we have made earlier. First, the electromagnetic charge invariance of this vacuum is automatic for any parameter range of the theory. Secondly, the R-parity, defined as $\mathbf{R} = (-\mathbf{1})^{\mathbf{3(B-L)+2S}}$, is preserved in the present model, since the Δ's are R-parity even whereas the bi-doublet and the bi-triplet Higgs scalars have zero R-parity.

We shall now discuss the conditions that emerge from the vanishing of the various F terms, which after the fields acquire their respective vevs, are given by,

$$
\begin{aligned}
F_\Delta &= f\, u_1 v_R + (M + \lambda_2\langle\sigma\rangle) v_L = 0, && (2.18) \\
F_{\bar{\Delta}} &= f^* u_2 v_R + (M + \lambda_2\langle\sigma\rangle) v_L = 0, && (2.19) \\
F_{\Delta^c} &= f\, u_1 v_L + (M - \lambda_2\langle\sigma\rangle) v_R = 0, && (2.20) \\
F_{\bar{\Delta}^c} &= f^*\, u_2 v_L + (M - \lambda_2\langle\sigma\rangle) v_R = 0, && (2.21) \\
F_\sigma &= m_\sigma\, \langle\sigma\rangle + \lambda_2(v_L^2 - v_R^2) = 0, && (2.22) \\
F_\eta &= f\, v_L v_R + f^*\, v_L v_R + \lambda_1 v^2\ + 2m_\eta(u_1 + u_2 + u_0) = 0, && (2.23) \\
F_\Phi &= -2\lambda_1(u_1 + u_2)v + 2\lambda_1 u_0 v - 2\mu v = 0. && (2.24)
\end{aligned}
$$

At the outset we see that the F_σ flatness condition permits the trivial solution $\langle\sigma\rangle = 0$, which would imply the undesirable solution $v_L = v_R$ and lead to no parity breakdown. But this special point can easily be destabilized once the soft terms are turned on. Away from this special point, we are led to phenomenologically interesting vacuum configurations.

The F flatness conditions for the Δ and $\bar{\Delta}$ fields demand $fu_1 = f^* u_2$ which can be naturally satisfied by choosing

$$f = f^* \qquad \text{and} \qquad u_1 = u_2 \equiv u. \tag{2.25}$$

This is consistent with the relation obtained from the F flatness conditions for the Δ^c and $\bar{\Delta}^c$ fields, which may now be together read as

$$(M - \lambda_2 \langle \sigma \rangle) v_R = -f\, u v_L. \tag{2.26}$$

The first four conditions (2.18)-(2.21) can therefore be used to eliminate the scale u and give a relation

$$\left(\frac{v_L}{v_R} \right)^2 = \frac{M - \lambda_2 \langle \sigma \rangle}{M + \lambda_2 \langle \sigma \rangle}. \tag{2.27}$$

Let us assume the scale of the vev's u_1, u_2 and u_0 to be the same. Then the vanishing of F_η gives a relation

$$2 f v_L v_R \approx -(\lambda_1 v^2 + 6 m_\eta u). \tag{2.28}$$

Finally, the last condition (2.24) has an interesting consequence. While electroweak symmetry is assumed to remain unbroken in the supersymmetric phase, so that v must be chosen to be zero, we see that the factor multiplying v implies a relation

$$\mu \approx -\lambda_1 u. \tag{2.29}$$

That is, taking λ_1 to be order unity, the scale of the μ term determines the scale of u.

We now attempt an interpretation of these relations to obtain reasonable phenomenology. The scale v_R must be higher than the TeV scale. It seems reasonable to assume that the eq. (2.28) provides a see-saw relation between v_L and v_R vev's, and that this product is anchored by the TeV scale. Since bitriplet contributes additional non-doublet Higgs in the Standard Model, it is important that the vacuum expectation value u is much higher or much smaller than the electroweak scale, and we shall explore the latter route. In this case u should be strictly less than 1GeV. The scale m_η determines the masses of triplet majorons and needs to be high compared to the TeV scale. If the above see-saw relation is not to be jeopardized, we must have $m_\eta u \leq m_{EW}^2$. We can avoid proliferation of new mass scales by choosing

$$m_\eta u \approx v^2 = m_{EW}^2. \tag{2.30}$$

This establishes eq. (2.28) as the desired hierarchy equation, with f chosen to be negative.

Now let us examine the consistency of the assumption $u \ll m_{EW}$ in the light of the two equations (2.26) and (4.17). Let us assume that $(v_L/v_R) \ll 1$ as in the non-supersymmetric case. Then eq. (4.17) means that on the right hand side,

$$M - \lambda_2\langle\sigma\rangle \ll M + \lambda_2\langle\sigma\rangle \rightarrow M \sim \lambda_2\langle\sigma\rangle. \tag{2.31}$$

Then eq. (2.18) can be read as

$$\frac{v_L}{v_R} \approx \frac{(-f)u}{2M}. \tag{2.32}$$

We thus see that the required hierarchies of scales can be spontaneously generated, and can be related to each other. Finally, although only the ratios has been related in eq. (2.32) we may choose

$$v_L \approx u, \qquad v_R \approx M. \tag{2.33}$$

We see that through this choice of individual scales and through the see-saw relation (2.28), u and v_R obey a mutual see-saw relation. A small value of u in the eV range would place v_R in the intermediate range as in the traditional proposals for neutrino mass see-saw. A larger range of values close to the GeV scale would lead to v_R and the resulting heavy neutrinos states within the range of collider confirmation.

Finally, returning to eq. (2.29), we can obtain the desirable scale for u by choosing μ to be of that scale, viz., in the sub-GeV range. This solves the μ problem arising in MSSM by relating it to other scales required to keep the v_R high. An interesting consequence of the choices made so far is that using eq.s (2.31) and (2.33) in eq. (4.8) yields

$$|m_\sigma| \approx \lambda_2 \frac{v_R^2}{\langle\sigma\rangle} \sim \lambda_2^2 M. \tag{2.34}$$

To summarize, various phenomenological considerations lead to a natural choice of three of the mass parameters of the superpotential, M, m_σ and m_η to be comparable to each other and large, such as to determine v_R, and in turn the masses of the heavy majorana neutrinos. The scale μ which determines the vacuum expectation value u and in turn the value v_L could be anything less than a GeV. Most importantly we have the see-saw relation eq. (2.28) which relates these scales, and if the v_R scale is to be within a few orders of magnitude of the TeV scale, then μ should be close to though less than a GeV.

We can contemplate two extreme possibilities for the scale M. Keeping in mind the gravitino production and overabundance problem, we can choose the largest value $v_R \leq 10^9$ GeV. If it can be ensured from inflation that this is also the reheat temperature, then the thermalisation of heavy majorana neutrinos required for thermal leptogenesis at a scale somewhat lower than this can be easily accommodated. We can also try to take v_R as low as 10 TeV which is consistent with preserving lepton asymmetry generated by non-thermal mechanisms [124]. Baryogenesis from non-thermal or sleptonic leptogenesis in this kind of setting has been extensively studied [125–128]. This low value of v_R is consistent with neutrino see-saw relation, but will rely critically on the smallness of Yukawa couplings [124] and may be accessible to colliders.

As we have seen, at the large scale, charge conservation also demands conservation of R-parity. The question generally arise as to what happens when heavy fields are integrated out and soft supersymmetry breaking terms are switched on. The analysis done in [121] implies that if M_R is very large (around 10^{10} GeV), the breakdown of R-parity at low energy would give rise to an almost-massless majoron coupled to the Z-bosons, which is ruled out experimentally. This is one of the central aspects of supersymmetric left-right theories with large M_R: R-parity is an exact symmetry of the low energy effective theory. The supersymmetric partners of the neutrinos do not get any *vev* at any scale, which also ensures that the R-parity is conserved.

2.4 RG Running for gauge couplings and Fermion masses

In this section, we will show how coupling and masses parameters evolve with energy. The one loop renormalization group equations (RGEs) [149] for gauge coupling constants in this model can be written as

$$\frac{d\alpha_i}{dt} = b_i \alpha_i^2 \tag{2.35}$$

where, $t = 2\pi \ln(M)$ (M is the varying energy scale), $\alpha_i = \frac{g_i^2}{4\pi}$ is the coupling strength. Also b_i is the one loop beta coefficient. The indices $i, j = (B-L), 2L, 2R, 3C$ refer to the gauge group $U(1)_{B-L}$, $SU(2)_L$, $SU(2)_R$ and $SU(3)_C$ respectively. The beta one loop beta functions for this model are;

★ Below the susy breaking scale M_{SUSY}, the beta functions are same as those of the standard model. So, for $M_{EW} < M < M_{SUSY}$:

$$b_{3C} = -11 + \frac{4}{3}n_F, \quad b_{2L} = -\frac{22}{3} + \frac{4}{3}n_F + \frac{1}{6}n_\phi, \quad b_Y = \frac{4}{3}n_F + \frac{n_\Phi}{10}$$

★ For $M_{susy} < M < M_{B-L}$, the beta functions are same as those of the MSSM

$$b_{3C} = -9 + 2\,n_F, \quad b_{2L} = -6 + 2\,n_F + \frac{n'_\phi}{2}, \quad b_Y = 2\,n_F + \frac{3}{10}n'_\phi$$

★ For $M_R < M < M_{GUT}$:

$$b_{B-L} = 2\,n_F + 9n_\Delta, \quad b_{2L} = -6 + 2\,n_F + \frac{n_\Phi}{2} + 2n_\Delta + 2n_\eta$$

$$b_{2R} = -6 + 2\,n_F + \frac{n_\Phi}{2} + 2n_\Delta + 2n_\eta, \quad b_{3C} = -9 + 2\,n_F$$

.

Where $n_F = 3, n_\Phi = 2, n'_\phi = 2, n_\Delta = 2, n_\eta$ are the number of generations , number of bidoublets, number of doublets in MSSM, number of doublets in SM, number of triplets and number of bitriplets respectively. The detail analysis of RG evolution of gauge coupling constants will be presented in chapter (3).

The renormalization group equations to one-loop order for the mass parameters of the above theory are presented below

$$16\pi^2\frac{d}{dt}m_Q^2 = 2m_Q^2 y_a^q y_a^{q\dagger} + y_a^q\left[2y_a^{q\dagger}m_Q^2 + 4y_b^{q\dagger}m^2_{\Phi ab} + 4m^2_{Q^c}y_a^{q\dagger}\right] \tag{2.36}$$

$$16\pi^2\frac{d}{dt}m_{Q^c}^2 = 2m_{Q^c}^2 y_a'^{q\dagger} y_a'^q + y_a'^{q\dagger}\left[2y_a'^q m_{Q^c}^2 + 4y_b'^q m^2_{\Phi ba} + 4m^2_Q y_a'^q\right] \tag{2.37}$$

$$\begin{aligned} 16\pi^2\frac{d}{dt}m_L^2 &= 6m_L^2\lambda\lambda^\dagger + \lambda\left[6\lambda^\dagger m_L^2 + 12{m_L^2}^T\lambda^\dagger + 12\lambda^\dagger m_\Delta^2\right] \\ &+ 2m_L^2 y_a^\ell y_a^{\ell\dagger} + y_a^\ell\left[2y_a^{\ell\dagger}m_L^2 + 4m^2_{L^c}y_a^{\ell\dagger} + 4y_{bb}^{\ell\dagger}m^2_{\Phi_{ab}}\right] \end{aligned} \tag{2.38}$$

$$\begin{aligned} 16\pi^2\frac{d}{dt}m_{L^c}^2 &= 6m_{L^c}^2\lambda^{*\dagger}\lambda^* + \lambda^*\left[6\lambda^* m_L^2 + 12{m_L^2}^T\lambda^* + 12\lambda^* m_\Delta^2\right] \\ &+ 2m_L^2 y_a'^{\ell\dagger} y_a'^\ell + y_a'^{\ell\dagger}\left[2y_a'^\ell m_L^2 + 4m^2_{L^c}y_a'^\ell + 4y_b'^\ell m^2_{\Phi_{ab}}\right] \end{aligned} \tag{2.39}$$

Fermion Masses	$M = M_Z$ PDG [129]	$M = M_G$ (Bitriplet) ($\|f\| = 0.79$)
m_u(MeV)	$2.33^{+0.42}_{-0.45}$	1.713
m_d(MeV)	$4.69^{+0.60}_{-0.66}$	2.877
m_c(MeV)	677^{+56}_{-61}	401.370
m_s(MeV)	$93.4^{+11.8}_{-13.0}$	57.328
m_t(GeV)	181 ± 13	128.888
m_b(GeV)	3.0 ± 0.11	2.185
m_e(MeV)	$0.48684727 \pm 0.14 \times 10^{-6}$	0.5526
m_μ(MeV)	$102.75138 \pm 3.3 \times 10^{-4}$	116.243
m_τ(GeV)	$1.74669^{+0.00030}_{-0.00027}$	2.070

Table 2.3: RGEs for fermion mass parameters in SUSYLR model with triplets and bitriplet scalar Higgs. For this numerical calculation, we have used $\tan\beta = 10$

$$\begin{aligned} 16\pi^2 \frac{d}{dt} m_\Delta^2 &= \mathrm{Tr}\left[4\lambda^\dagger \lambda m_\Delta^2 + 8\lambda^\dagger m_L^2 \lambda\right] \\ &+ \mu_\Delta^{\alpha *}\left[2\mu_\Delta^\alpha m_\Delta^2 + 2\mu_\Delta^\alpha m_{\bar\Delta}^2 + 2\mu_\Delta^\beta {m_\eta^2}^{\alpha\beta}\right] \end{aligned} \tag{2.40}$$

Similarly, we can write RGEs for all the mass parameters. One can get all the RGEs for all Yukawas, mass parameters in [149], though our result will be slightly different because of extra bitriplet scalar Higgs. The Table: [2.3] gives the running of fermion mass at GUT scale assuming their initial value at the electroweak scale (at 100 GeV).

To summarize the work, we propose an consistent solution to the problem of spontaneous parity breaking, which resembles the non-supersymmetric solution, relating the vacuum expectation values (*vevs*) of the left-handed and right-handed triplet Higgs scalars to the Higgs bi-doublet *vev* through a seesaw relation. The left-right symmetry breaking scale thus becomes inversely proportional to the left-handed triplet Higgs scalar that gives the type II seesaw masses to the neutrinos. The vacuum that preserves both electric charge and R-parity can naturally be the global minimum of the full potential. The most attractive feature of the present model is that generically it does not allow a left-right symmetric vacuum, though the latter appears as a single point within the flat direction of the minima respecting supersymmetry. When the flat direction is lifted all the energy scales

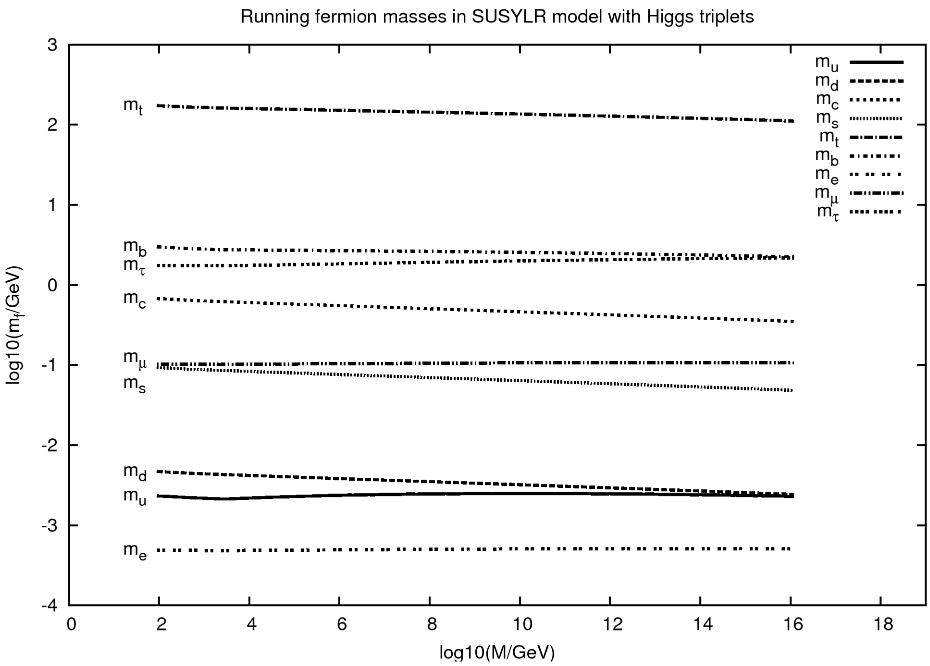


Figure 2.1: RG running of fermion masses in the bitriplet model. $M_{susy} = 500$ GeV, $M_R = 10^{12}$ GeV, $M_\sigma = 10^{16}$ GeV and $|f| = 0.79, \tan\beta = 10$ at $M = M_Z$

required to explain phenomenology result naturally. Also, we have made a complete study of fermion masses and gauge coupling constants in this model including soft-susy breaking effects, but the detail analytical derivation and numerical results will be presented in the next chapter. The original calculation has been carried through by spinner et al [149] and we just modify their results using extra bitriplet Higgs scalar. First we run the fermion masses up to GUT scale (M_G) knowing their initial values at M_Z (at 100 GeV) [129]. In the Fig:(2.1), it has been shown numerically the RG evolution of fermion mass and mixing.

Chapter

3

TeV scale SUSYLR model with spontaneous D-parity breaking

Left-Right symmetric model(LRSM) is a novel extension of the standard model of particle physics [99–103]. In such models the parity is spontaneously broken and the smallness of neutrino masses [104–107] arises in a natural way via seesaw mechanism [108–111]. Incorporating supersymmetry(susy) into such models comes with couple of other advantages in terms of the gauge hierarchy problem, coupling constant unification among many others. Another advantage in such susy models is that they provide a natural candidate for dark matter in terms of the lightest super-particle (LSP). In MSSM, this LSP is stable only if we incorporate an extra symmetry called R-parity $R_p = (-1)^{3(B-L)+2s}$. However in supersymmetric left right (SUSYLR) models [114, 118, 119, 121, 123, 130] based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ this R-parity is a part of the gauge symmetry and hence need not be put by hand. Since $U(1)_{B-L}$ symmetry is broken by a Higgs triplet with even $B - L$ quantum number, R-parity is still preserved at low energy.

Motivation and Outlook: Since there are many discussions exist in the literature studying these aspects of the left-right symmetric models, we summarize here our motivation for this study and how our analysis differs from earlier works. Before the precision measurements of the weak mixing angle and the strong coupling constants, the evolution of the gauge coupling constants could allow low-scale left-right symmetry breaking [141]. This could be achieved with a single stage symmetry breaking. Later it was found that by invoking more intermediate scales, it is possible to have more freedom to adjust the different symmetry breaking scales. However, after the precision electroweak measurements at LEP, it was found that the simplest left-right symmetric models would not allow a left-right symmetry breaking below 10^{12} GeV, in both single stage symmetry breaking as well as multi-stage symmetry breaking [142–144]. $SO(10)$ based models also got constrained with the allowed intermediate scale in the range of $10^9 - 10^{10}$ GeV [145, 146]. Introducing the Pati-Salam symmetry breaking scale would not allow lowering the left-right symmetry breaking scale both in the supersymmetric as well as the non-supersymmetric models. It would be possible to break the $SU(2)_R$ to $U(1)_R$ at a higher scale and then break the group $U(1)_R$ at a lower scale, but the breaking scale of $SU(2)_R$ could not be lowered, keeping the theory consistent with the potential minimization and gauge coupling evolution.

In a recent work of Mohapatra [147], it has been demonstrated that by introducing additional scalars it is possible to lower the scale of left-right symmetry breaking, i.e., break the symmetry group $SU(2)_R$. In this work, we will study the different symmetry breaking patterns to check the consistency with the potential minimization and gauge coupling evolution and see which of these models could allow TeV scale left-right symmetry breaking. We restricted our analysis to only a single stage symmetry breaking, because by introducing the additional symmetry breaking scales it was not found to help lowering the left-right symmetry breaking scales. Of course, our analysis does not rule out other possibilities of lowering the left-right breaking scale by introducing newer symmetry breaking scales and new physics. However, this analysis demonstrates that within the simplest framework of single stage symmetry breaking, which models are consistent with potential minimization, gauge coupling unification, and allows a TeV scale left-right symmetry breaking.

3.1 LR models with spontaneous D-parity breaking

In left-right symmetric models with spontaneous D-parity breaking, the discrete parity symmetry gets broken (by the vev of a parity odd singlet scalar field) much before the $SU(2)_R$ gauge symmetry breaks. The gauge group is effectively $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$, where P is the discrete left-right symmetry which we call D-parity. This D-parity symmetry is different from the Lorentz parity in the sense that Lorentz parity interchanges left handed fermions with the right handed ones but the bosonic fields remain the same. Whereas, the D-parity also interchanges the $SU(2)_L$ Higgs fields with the $SU(2)_R$ Higgs fields. The parity odd singlet field breaks this gauge symmetry at high scale $\sim (10^{16} - 10^{19})$ GeV to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ which further breaks down to the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ at a lower scale. The D-parity breaking introduces an asymmetry between left and right handed Higgs fields and makes the coupling constants of $SU(2)_R$ and $SU(2)_L$ evolve separately under the renormalization group. It should be noted that this D-parity breaking is different from the low energy parity breaking observed in the weak interactions which arises as a result of $SU(2)_R$ gauge symmetry breaking at a scale higher than the electroweak scale. In such D-parity breaking scenario the seesaw relation also gets modified from usual LRSM. Although the type I seesaw term still remains sensitive to the $SU(2)_R$ breaking scale M_R, the other seesaw terms namely type II and type III [135] becomes sensitive to the D-parity breaking scale. A very high value of parity breaking scale therefore leads to type I seesaw dominance. In this section we are going to discuss various such models with different particle contents.

In the usual LRSM, the scale of parity breaking and $SU(2)_R$ gauge symmetry breaking are identical which is not necessary. There have been lots of studies on left-right symmetric models where the parity symmetry gets broken much before the $SU(2)_R$ gauge symmetry breaks by so called spontaneous D-parity breaking [133, 134]. In this work, we will present various types of susy and non-susy left-right models with spontaneous D-parity breaking and check whether the minimization of the scalar potential allows a TeV scale $SU(2)_R$ breaking scale (provided parity breaks at much higher scale) as well as tiny neutrino masses. We then check whether such a choice of intermediate symmetry breaking scales unifies the gauge coupling constants in the SUSYLR framework. We discuss the possible phenomenology of neutrino mass in each cases separately.

3.1.1 LRSM with Higgs doublets

We first study the non-Susy left-right symmetric extension of the standard model with only Higgs doublets. In addition to the usual fermions of the standard model, we require the right-handed neutrinos to complete the representations. One of the important features of the model is that it allows spontaneous parity violation. The Higgs representations then requires a bi-doublet field, which breaks the electroweak symmetry and gives masses to the fermions. But the neutrinos can have a Dirac mass only, which is then expected to be of the order of other fermion masses. To implement the see-saw mechanism and obtain the observed tiny mass of the left-handed neutrinos naturally, one also introduces a singlet fermion plus fermion triplet. However, we shall restrict ourselves to the scalar sector and shall not discuss the implications of the singlet neutrinos and the neutrino masses.

The particle content of the Left-Right symmetric model with Higgs doublet is

$$\begin{aligned}
\text{Fermions}: \; & Q_L \equiv (3,2,1,1/3), \quad Q_R \equiv (3,1,2,1/3), \\
& \Psi_L \equiv (1,2,1,-1), \quad \Psi_R \equiv (1,1,2,-1)
\end{aligned}$$

$$\begin{aligned}
\text{Scalars}: \; & \Phi_a \, (a=1,2) \equiv (1,2,2,0), \quad H_L \equiv (1,2,1,1), \\
& H_R \equiv (1,1,2,1) \quad \rho \equiv (1,1,1,0)
\end{aligned}$$

where the numbers in the brackets are the quantum numbers corresponding to the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In addition to the bi-doublet scalar field Φ, we also introduced two doublet fields H_L and H_R to break the left-right symmetry and contribute to the neutrino masses. Though H_L is not necessary for the desired structure of the symmetry breaking, we introduce it anyway along with H_R so that our model can accommodate left-right symmetric models. The scalar singlet ρ is a D-parity odd field and changes sign under the exchange of $SU(2)_L$ with $SU(2)_R$. Thus the symmetry breaking pattern becomes

$$\begin{aligned}
SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \quad & \xrightarrow{\langle\rho\rangle} \quad SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
& \xrightarrow{\langle H_R\rangle} \quad SU(2)_L \times U(1)_Y \quad \xrightarrow{\langle\Phi\rangle} \quad U(1)_{em}
\end{aligned}$$

We denoted the vacuum expectation values of the neutral components of the Higgs fields as

$$\langle\Phi_1\rangle = v_1, v_2, \quad \langle H_L\rangle = v_L, \quad \langle H_R\rangle = v_R, \quad \langle\rho\rangle = s$$

The scalar potential with all these fields can then be written as

$$V = V_\Phi + V_H + V_{\Phi\,H} + V_\rho \tag{3.1}$$

where

$$\begin{aligned}
V_\Phi = & -\mu_1^2 \,\mathrm{Tr}\left(\Phi_1^\dagger \,\Phi_1\right) - \mu_2^2 \left[\mathrm{Tr}(\Phi_2\Phi_1^\dagger) + \mathrm{Tr}(\Phi_2^\dagger\Phi_1)\right] + \lambda_1 \left[\mathrm{Tr}(\Phi_1^\dagger\Phi_1)\right]^2 \\
& + \lambda_2 \left[[\mathrm{Tr}(\Phi_2\Phi_1^\dagger)^2 + \mathrm{Tr}(\Phi_2^\dagger\Phi_1)]^2 \right] + \lambda_3 \left[\mathrm{Tr}(\Phi_2\,\Phi_1^\dagger)\,\mathrm{Tr}(\Phi_2^\dagger\,\Phi_1)\right] \\
& + \lambda_4 \left[\mathrm{Tr}(\Phi_1^\dagger\Phi_1)[\mathrm{Tr}(\Phi_2\Phi_1^\dagger) + \mathrm{Tr}(\Phi_2^\dagger\Phi_1)]\right] ,
\end{aligned} \tag{3.2}$$

$$V_H = -\mu_h^2 \left(H_L^\dagger H_L + H_R^\dagger H_R\right) + \lambda_5 \left[(H_L^\dagger H_L)^2 + (H_R^\dagger H_R)^2\right] + \lambda_6 \left[(H_L^\dagger H_L)(H_R^\dagger H_R)\right] \quad (3.3)$$

$$\begin{aligned} V_{\Phi H} = {} & \alpha_1 \operatorname{Tr}(\Phi_1^\dagger \Phi_1) \left[H_L^\dagger H_L + H_R^\dagger H_R\right] + \alpha_2 \left[H_L^\dagger \Phi_1 \Phi_1^\dagger H_L + H_R^\dagger \Phi_1^\dagger \Phi_1 H_R\right] \\ & + \alpha_3 \left[H_L^\dagger \Phi_2 \Phi_2^\dagger H_L + H_R^\dagger \Phi_2^\dagger \Phi_2 H_R\right] + \alpha_4 \left[H_L^\dagger \Phi_1 \Phi_2^\dagger H_L + H_R^\dagger \Phi_1^\dagger \Phi_2 H_R\right] \\ & + \alpha_4^* \left[H_L^\dagger \Phi_2 \Phi_1^\dagger H_L + H_R^\dagger \Phi_2^\dagger \Phi_2 H_R\right] + \mu_{h\,\Phi_1} \left[H_L^\dagger \Phi_1 H_R + H_R^\dagger \Phi_1^\dagger H_L\right] \\ & + \mu_{h\,\Phi_2} \left[H_L^\dagger \Phi_2 H_R + H_R^\dagger \Phi_2^\dagger H_L\right] \end{aligned} \quad (3.4)$$

$$\begin{aligned} V_\rho = {} & -\mu_\rho^2 \rho^2 + +\lambda_7 \rho^4 + M \rho \left[H_L^\dagger H_L - H_R^\dagger H_R\right] \\ & + \lambda_8 \rho^2 \left[H_L^\dagger H_L + H_R^\dagger H_R\right] \\ & + \lambda_9 \rho^2 \operatorname{Tr}(\Phi_1^\dagger \Phi_1) + \lambda_{10} \rho^2 \left[\operatorname{Det}(\Phi_1) + \operatorname{Det}(\Phi_1^\dagger)\right] \end{aligned} \quad (3.5)$$

where $\Phi_2 = \tau_2 \Phi_1^* \tau_2$.

To find a consistent solution we now minimize the scalar potential and obtain

$$\frac{\partial V}{\partial v_L} = \mu_L^2 v_L + \lambda_5 v_L^3 + \frac{\lambda_6}{2} v_L v_R^2 + \mu_{h\phi}(v_1 + v_2) v_R = 0 \quad (3.6)$$

$$\frac{\partial V}{\partial v_R} = \mu_R^2 v_R + \lambda_5 v_R^3 + \frac{\lambda_6}{2} v_R v_L^2 + \mu_{h\phi}(v_1 + v_2) v_L = 0 \quad (3.7)$$

where μ_L^2 and μ_R^2 are effective mass terms of H_L and H_R given by

$$\begin{aligned} \mu_L^2 &= \mu_h^2 + Ms + \lambda_8 s^2 + (\alpha_4 + \alpha_4^*) v_1 v_2 + \alpha_1 (v_1^2 + v_2^2) + \alpha_2 v_2^2 + \alpha_3 v_1^2 \\ \mu_R^2 &= \mu_h^2 - Ms + \lambda_8 s^2 + (\alpha_4 + \alpha_4^*) v_1 v_2 + \alpha_1 (v_1^2 + v_2^2) + \alpha_2 v_2^2 + \alpha_3 v_1^2 \end{aligned} \quad (3.8)$$

Thus after the singlet field η gets a vev the left handed Higgs doublet becomes heavy and decouple whereas the right handed Higgs can be much lighter by appropriate fine tuning of the parameters in (3.8). From equations (3.6), (3.7) we get

$$v_L v_R (2Ms) + (\lambda_5 - \frac{\lambda_6}{2})(v_L^2 - v_R^2) v_L v_R + \mu_{h\phi}(v_1 + v_2)(v_R^2 - v_L^2) = 0$$

Thus a non-zero value of $\langle\rho\rangle = s$ does not allow a solution with $v_L = v_R$. The seesaw relation from the above equation is

$$v_L v_R = \frac{\mu_{h\phi}(v_1 + v_2)(v_L^2 - v_R^2)}{2Ms + (\lambda_5 - \frac{\lambda_6}{2})(v_L^2 - v_R^2)}$$

Assuming $v_L \ll v_R \ll s, M$ will give

$$v_L = \frac{-\mu_{h\phi}(v_1 + v_2)v_R}{2Ms} \tag{3.9}$$

Thus we can have small v_L/v_R by appropriately choosing the scales of $M, s, \mu_{h\phi}$ which will account for tiny neutrino masses. In contrast LRSM without D-parity breaking where the right handed scale v_R has to be very high to account for small v_L/v_R, here we can have v_R of TeV scale also. For example, if we set $\mu_{h\phi} = M = s = 10^8$ GeV, and $v_{1,2} \sim M_Z$ then $\frac{v_L}{v_R}$ comes out to be of the order 10^{-6} which is desired for type III seesaw to dominate as we will see when we discuss neutrino masses. The gauge coupling unification has been studied extensively in this model, so we shall not repeat them here. In the absence of D-parity breaking the left-right symmetry breaking scale comes out to be very high, but in D-parity violating models it is possible to lower the scale of left-right symmetry breaking with some amount of fine tuning of parameters. However, for the supersymmetric models restrictions are more stringent, so we shall study them in details.

3.1.2 LRSM with Higgs triplets

In this section we shall study the left-right symmetric models with a different particle contents. The usual fermions, including the right-handed neutrinos, belong to the similar representations as in the previous section. However the scalar sector now contains triplet Higgs scalars in addition to the bi-doublet Higgs scalar to break the left-right symmetry. The triplet Higgs scalars can then give Majorana masses to the neutrinos and allow seesaw mechanism without the need for any additional singlet fermions. The parity odd singlet scalar was originally introduced in this model, so we shall include them in our discussions.

The particle content of LRSM with Higgs triplets is

$$\begin{aligned}
\text{Fermions}: \; & Q_L \equiv (3, 2, 1, 1/3), \quad Q_R \equiv (3, 1, 2, 1/3), \\
& \Psi_L \equiv (1, 2, 1, -1), \quad \Psi_R \equiv (1, 1, 2, -1)
\end{aligned}$$

$$\text{Scalars}: \ \Phi_a\,(a=1,2) \equiv (1,2,2,0), \quad \Delta_L \equiv (1,3,1,2),,$$
$$\Delta_R \equiv (1,1,3,2) \quad \rho \equiv (1,1,1,0)$$

The symmetry breaking pattern in this model remains the same as in the previous model although the structure of neutrino masses changes. In the symmetry breaking pattern, the scalar Δ_c now replaces the role of H_R, but otherwise there is no change. The vacuum expectation values of the neutral components of the Higgs fields are denoted by $\Phi_1, \triangle_L, \triangle_R, \rho$ as

$$\langle \Phi_1 \rangle = v_1, v_2, \quad \langle \triangle_L \rangle = v_L, \quad \langle \triangle_R \rangle = v_R, \quad \langle \rho \rangle = s.$$

The complete scalar potential of this model [103] is given by

$$V = V_\Phi + V_\Delta + V_{\Phi\Delta} + V_\rho \tag{3.10}$$

where

$$\begin{aligned} V_\Phi = & -\mu_1^2\,\mathrm{Tr}\left(\Phi_1^\dagger\,\Phi_1\right) - \mu_2^2\left[\mathrm{Tr}(\Phi_2\Phi_1^\dagger) + \mathrm{Tr}(\Phi_2^\dagger\Phi_1)\right] + \lambda_1\left[\mathrm{Tr}(\Phi_1^\dagger\Phi_1)\right]^2 \\ & + \lambda_2\left[[\mathrm{Tr}(\Phi_2\Phi_1^\dagger)^2 + \mathrm{Tr}(\Phi_2^\dagger\Phi_1)]^2\right] + \lambda_3\left[\mathrm{Tr}(\Phi_2\,\Phi_1^\dagger)\,\mathrm{Tr}(\Phi_2^\dagger\,\Phi_1)\right] \\ & + \lambda_4\left[\mathrm{Tr}(\Phi_1^\dagger\Phi_1)[\mathrm{Tr}(\Phi_2\Phi_1^\dagger) + \mathrm{Tr}(\Phi_2^\dagger\Phi_1)]\right], \end{aligned} \tag{3.11}$$

$$\begin{aligned} V_\Delta = & -\mu_\Delta^2\left[\mathrm{Tr}(\Delta_L^\dagger\,\Delta_L) + \mathrm{Tr}(\Delta_R^\dagger\,\Delta_R)\right] + f_1\left[[\mathrm{Tr}(\Delta_L^\dagger\,\Delta_L)]^2 + [\mathrm{Tr}(\Delta_R^\dagger\,\Delta_R)]^2\right] \\ & + f_2\left[\mathrm{Tr}(\Delta_L\,\Delta_L)\,\mathrm{Tr}(\Delta_L^\dagger\,\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\,\Delta_R)\,\mathrm{Tr}(\Delta_R^\dagger\,\Delta_R^\dagger)\right] \\ & + f_3\left[\mathrm{Tr}(\Delta_L^\dagger\Delta_L)\,\mathrm{Tr}(\Delta_R^\dagger\Delta_R)\right] \\ & + f_4\left[\mathrm{Tr}(\Delta_L\,\Delta_L)\,\mathrm{Tr}(\Delta_R^\dagger\,\Delta_R^\dagger) + \mathrm{Tr}(\Delta_R\,\Delta_R)\,\mathrm{Tr}(\Delta_L^\dagger\,\Delta_L^\dagger)\right], \end{aligned} \tag{3.12}$$

$$\begin{aligned} V_\rho = & -\mu_\rho^2\rho^2 + \lambda_5\,\rho^4 + M\,\rho\left[\mathrm{Tr}(\Delta_L^\dagger\,\Delta_L) - \mathrm{Tr}(\Delta_R^\dagger\,\Delta_R)\right] \\ & + \lambda_6\,\rho^2\left[\mathrm{Tr}(\Delta_L^\dagger\,\Delta_L) + \mathrm{Tr}(\Delta_R^\dagger\,\Delta_R)\right] \\ & + \lambda_7\,\rho^2\,\mathrm{Tr}(\Phi_1^\dagger\,\Phi_1) + \lambda_8\,\rho^2\left[\mathrm{Det}(\Phi_1) + \mathrm{Det}(\Phi_1^\dagger)\right] \end{aligned} \tag{3.13}$$

$$
\begin{aligned}
V_{\Phi\Delta} = &\ \alpha_1 \left[\mathrm{Tr}(\Phi_1^\dagger\,\Phi_1)[\mathrm{Tr}(\Delta_L^\dagger\,\Delta_L) + \mathrm{Tr}(\Delta_R^\dagger\,\Delta_R)]\right] \\
&+ \alpha_2 \left[\mathrm{Tr}(\Phi_2^\dagger\,\Phi_1)\,\mathrm{Tr}(\Delta_R^\dagger\,\Delta_R) + \mathrm{Tr}(\Phi_1^\dagger\,\Phi_2)\,\mathrm{Tr}(\Delta_L^\dagger\,\Delta_L)\right] \\
&+ \alpha_2^* \left[\mathrm{Tr}(\Phi_1^\dagger\,\Phi_2)\,\mathrm{Tr}(\Delta_R^\dagger\,\Delta_R) + \mathrm{Tr}(\Phi_2^\dagger\,\Phi_1)\,\mathrm{Tr}(\Delta_L^\dagger\,\Delta_L)\right] \\
&+ \alpha_3 \left[\mathrm{Tr}(\Phi_1\,\Phi_1^\dagger\,\Delta_L\,\Delta_L^\dagger) + \mathrm{Tr}(\Phi_1^\dagger\,\Phi_1\,\Delta_R\,\Delta_R^\dagger)\right] \\
&+ \beta_1 \left[\mathrm{Tr}(\Phi_1\,\Delta_R\,\Phi_1^\dagger\,\Delta_L^\dagger) + \mathrm{Tr}(\Phi_1^\dagger\,\Delta_L\,\Phi_1\,\Delta_R^\dagger)\right] \\
&+ \beta_2 \left[\mathrm{Tr}(\Phi_2\,\Delta_R\,\Phi_1^\dagger\,\Delta_L^\dagger) + \mathrm{Tr}(\Phi_2^\dagger\,\Delta_L\,\Phi_1\,\Delta_R^\dagger)\right] \\
&+ \beta_3 \left[\mathrm{Tr}(\Phi_1\,\Delta_R\,\Phi_2^\dagger\,\Delta_L^\dagger) + \mathrm{Tr}(\Phi_1^\dagger\,\Delta_L\,\Phi_2\,\Delta_R^\dagger)\right] ,
\end{aligned}
\tag{3.14}
$$

where $\Phi_2 = \tau_2\Phi_1^*\tau_2$. Minimizing the scalar potential we now obtain various conditions

$$
\frac{\partial V}{\partial v_L} = \mu_L^2 v_L + 2f_1 v_L^3 + f_3 v_L v_R^2 + (\beta_1 v_1 v_2 + \beta_2 v_1^2 + \beta_3 v_2^2) v_R = 0 \tag{3.15}
$$

$$
\frac{\partial V}{\partial v_R} = \mu_R^2 v_R + 2f_1 v_R^3 + f_3 v_R v_L^2 + (\beta_1 v_1 v_2 + \beta_2 v_1^2 + \beta_3 v_2^2) v_L = 0 \tag{3.16}
$$

where μ_L^2 and μ_R^2 are effective mass terms of $\triangle_L$ and $\triangle_R$ given by

$$
\begin{aligned}
\mu_L^2 &= \mu_\triangle^2 + Ms + \lambda_6 s^2 + 2(\alpha_2 + \alpha_2^*) v_1 v_2 + \alpha_1 (v_1^2 + v_2^2) + \alpha_3 v_2^2 \\
\mu_R^2 &= \mu_\triangle^2 - Ms + \lambda_6 s^2 + 2(\alpha_2 + \alpha_2^*) v_1 v_2 + \alpha_1 (v_1^2 + v_2^2) + \alpha_3 v_2^2
\end{aligned}
$$

Thus like in the previous case , here also the Higgs triplets $\triangle_L$ become heavier than $\triangle_R$ after the singlet η acquires a vev at the high scale. Equations (3.15), (3.16) gives

$$
(2Ms + (v_R^2 - v_L^2)(f_3 - 2f_1)) v_L v_R = (v_L^2 - v_R^2)(\beta_1 v_1 v_2 + \beta_2 v_1^2 + \beta_3 v_2^2)
$$

Thus a nonzero vev of ρ disallows those solutions for which $v_L = v_R$. Assuming $v_L \ll v_R \ll s, M$ will give

$$
v_L = \frac{-v_R(\beta_1 v_1 v_2 + \beta_2 v_1^2 + \beta_3 v_2^2)}{2Ms} \tag{3.17}
$$

Thus we an have a small $v_L \sim eV$ by appropriately choosing v_R and M, s. Here if we take v_R of TeV scale then the scale of parity breaking M, s should be low ($\sim 10^8 - 10^9$ GeV) so as to give $v_L \sim eV$ needed to account for neutrino masses as we will see later.

3.1.3 SUSYLR model with Higgs doublets

We shall now study the various supersymmetric left-right symmetric models. These models are much more restrictive compared to the non-Susy models. Although the spontaneous parity violation is one of the most important features of the non-Susy version of the left-right symmetric models, in the Susy left-right models with triplet Higgs scalars breaking parity becomes very difficult and one has to extend the model to incorporate any natural mechanism of parity violation. In this section we shall discuss the model where the left-right symmetry is broken by Higgs doublet scalar.

In the particle contents, the fermions belong to the fermion superfields and we denote all the fermions and scalars by their corresponding superfields. We can then write the particle contents of Supersymmetric Left-Right model with Higgs doublet in terms of their superfields as

$$\text{Matter Superfield}: \quad Q_L = (3,2,1,1/3), \quad Q_R = (3,1,2,1/3)$$
$$\Psi_L = (1,2,1,-1), \quad \Psi_R = (1,1,2,-1)$$

$$\text{Higgs Superfield}: \quad \Phi_1 = (1,2,2,0), \quad \Phi_2 = (1,2,2,0)$$
$$H_L = (1,2,1,1), \quad \bar{H}_L = (1,2,1,-1),$$
$$H_R = (1,1,2,-1), \quad \bar{H}_R = (1,1,2,1), \quad \rho = (1,1,1,0)$$

where Higgs particles with “bar" in the notation, helps in anomaly cancellation of the model.

In the model, a singlet scalar field ρ is introduced, which has the special property that it is even under the usual parity of the Lorentz group, but it is odd under the parity that relates the gauge groups $SU(2)_L$ and $SU(2)_R$. This field ρ is thus a scalar and not a pseudo-scalar field, but under the D-parity transformation that interchanges $SU(2)_L$ with $SU(2)_R$, it is odd. This kind of work is proposed in [132, 133, 139]. Although all the scalar fields are even under the parity of the Lorentz group, under the D-parity the Higgs sector transforms as,

$$H_L \leftrightarrow H_R, \qquad \bar{H}_L \leftrightarrow \bar{H}_R,$$
$$\Phi \leftrightarrow \Phi^\dagger, \qquad \rho \leftrightarrow -\rho.$$

The Higgs part of the superpotential relevant in our case is

$$W = \mu_{ij}\mathrm{Tr}[\tau_2\Phi_i^T\tau_2\Phi_j] + M\rho\rho + f_1(H_L^T\Phi_i H_R + \bar{H}_L^T\Phi_i\bar{H}_R)$$

$$+m_h(H_L^T\tau_2\bar{H}_L + H_R^T\tau_2\bar{H}_R) + \lambda_1\rho(H_L^T\tau_2\bar{H}_L - H_R^T\tau_2\bar{H}_R) \tag{3.18}$$

The scalar potential is $V = V_F + V_D + V_{soft}$ where $V_F = |F_i|^2, F_i = -\frac{\partial W}{\partial\phi}$ is the F-term scalar potential, $V_D = D^a D^a/2, D^a = -g(\phi_i^* T_{ij}^a \phi_j)$ is the D-term of the scalar potential and V_{soft} is the soft supersymmetry breaking scalar potential. We introduce the soft Susy breaking terms to check if they alter relations between various mass scales in the model. The soft Susy breaking superpotential in this case is given by

$$V_{soft} = m_H^2 H_L^\dagger H_L + m_H^2\bar{H}_L^\dagger\bar{H}_L + m_H^2 H_R^\dagger H_R + m_H^2\bar{H}_R^\dagger\bar{H}_R + m_{11}^2\Phi_1^\dagger\Phi_1$$

$$+m_{22}^2\Phi_2^\dagger\Phi_2 + m_\rho^2\rho^\dagger\rho + (B_1 H_L^T\tau_2\bar{H}_L + B_2 H_R^T\tau_2\bar{H}_R + B\mu_{ij}\mathrm{Tr}[\tau_2\Phi_i\tau_2\Phi_j] + h.c.)$$

$$+(A_1 H_L^T\Phi_i H_R + A_2\bar{H}_L\Phi_i\bar{H}_R + A_3(\rho H_L^T\tau_2\bar{H}_L - \rho H_R^T\tau_2\bar{H}_R) + h.c.) \tag{3.19}$$

where all the parameters $m_H, m_{11}, m_{22}, B, A$ are of the order of Susy breaking scale $M_{susy} \sim$ TeV. We denote the vev of the neutral components of $\Phi_1, \Phi_2, H_L, \bar{H}_L, H_R, \bar{H}_R$ and ρ as $\langle(\Phi_1)_{11}\rangle = v_1,\ \langle(\Phi_2)_{22}\rangle = v_2,\ \langle H_L, \bar{H}_L\rangle = v_L,\ \langle H_R, \bar{H}_R\rangle = v_R,\ \langle\rho\rangle = s.$

Minimizing the potential with respect to v_L, v_R, we get the relations

$$\frac{\partial V}{\partial v_L} = -\mu_L^2(2v_L) + 2v_L v_R^2 f_1^2 + f_1 v_R(m_h + 4\mu)(v_1 + v_2)$$

$$+(2m_H^2 - m_h^2)v_L + A_1 s v_L + \frac{A_2 v_1 v_R}{2} + \lambda_1^2 v_L(v_R^2 - v_L^2) = 0 \tag{3.20}$$

$$\Rightarrow \frac{v_L}{v_R} = \frac{f_1((m_h + 4\mu)(v_1 + v_2) + \frac{A_1 v_1}{2}}{2\mu_L^2 - 2f_1^2 v_R^2 - \lambda_1^2(v_R^2 - v_L^2) - A_2 s}$$

$$\frac{\partial V}{\partial v_R} = -\mu_R^2(2v_R) + 2v_R v_L^2 f_1^2 + f_1 v_R(m_h + 4\mu)(v_1 + v_2)$$

$$+(2m_H^2 - m_h^2)v_R - A_2 s v_R + \frac{A_1 v_1 v_L}{2} - \lambda_1^2 v_L(v_R^2 - v_L^2) = 0 \tag{3.21}$$

where μ_L^2, μ_R^2 are given by

$$\mu_L^2 = \frac{1}{4}[2(m_h + \lambda_1 s)^2 - 4Ms\lambda_1 - f_1^2(v_1^2 + v_2^2)]$$
$$\mu_R^2 = \frac{1}{4}[2(m_h - \lambda_1 s)^2 + 4Ms\lambda_1 - f_1^2(v_1^2 + v_2^2)] \tag{3.22}$$

From equations (3.20), (3.21) we get

$$(A_1 v_1 + 4(f_1^2 + \lambda_1^2)v_L v_R + 2f_1(v_1 + v_2)(m_h + 4\mu))(v_R^2 - v_L^2)$$
$$+ (4sA_2 + 8\lambda_1 s(M - m_h))v_L v_R = 0 \tag{3.23}$$

which shows that the minimization disallows the solutions where $v_L = v_R$. Assuming $v_L \ll v_{1,2}, \mu, A \ll s, M, m_h$ and $v_L \ll v_R$ the above expression gives rise to

$$v_L = \frac{v_R(2f_1 m_h(v_1 + v_2) + 4(f_1^2 + \lambda_1^2)v_L v_R + A_1 v_1)}{8(m_h - M)s\lambda_1 + 4sA_2} \tag{3.24}$$

Thus by appropriate choice of m_h, M, s we can have TeV scale $SU(2)_R$ breaking scale v_R as well as $v_L/v_R \sim (10^{-6} - 10^{-9})$ which is necessary to account for small neutrino masses as we will see later. For example, if we set

$$m_h \sim M \sim s \sim 10^{16}\,\text{GeV} \quad \text{D-parity breaking scale}$$

and allow $2m_h - M \sim 10^8$ GeV by appropriate fine tuning then the above relation will give rise to the desired ratio $v_L/v_R \sim 10^{-6}$. For such a choice of scales we can fine tune the parameters to get a light H_R having mass $\mu_R \sim v_R \sim$ TeV and a heavy H_L having mass $\mu_L \sim s, M \sim 10^{16}$ GeV. This will be important in the renormalization group running of the couplings as we will see later.

3.1.4 SUSYLR model with Higgs triplets

The particle contents of Supersymmetric Left-Right model with Higgs triplets in terms of their superfields are

$$\text{Matter Superfield}: \quad Q = (3, 2, 1, 1/3), \quad Q^c = (3, 1, 2, 1/3)$$
$$L = (1, 2, 1, -1), \quad L^c = (1, 1, 2, -1)$$

$$
\begin{aligned}
\text{Higgs Superfield}: \quad & \Phi_1 = (1,2,2,0), \quad \Phi_2 = (1,2,2,0) \\
& \Delta = (1,3,1,2), \quad \bar{\Delta} = (1,3,1,-2), \\
& \Delta^c = (1,1,3,-2), \quad \bar{\Delta}^c = (1,1,3,2), \quad \rho = (1,1,1,0)
\end{aligned}
$$

The left-right symmetry could be broken by either doublet Higgs scalars or triplet Higgs scalar. We will show that for a minimal choice of parameters, it is convenient to break the group with a triplet Higgs scalar. As pointed out in [121] the bidoublets are doubled to achieve a non-vanishing Cabibbo-Kobayashi-Maskawa (CKM) quark mixing and the number of triplets is doubled for the sake of anomaly cancellation.

The superpotential for this theory is given by

$$
\begin{aligned}
W = & Y^{(i)_q} Q^T \tau_2 \Phi_i \tau_2 Q^c + Y^{(i)_l} L^T \tau_2 \Phi_i \tau_2 L^c \\
& + \ i(f L^T \tau_2 \Delta L + f^* L^{cT} \tau_2 \Delta^c L^c) + M\rho^2 \\
& + \ m_\Delta \mathrm{Tr}(\Delta\bar{\Delta}) + m_\Delta^* \mathrm{Tr}(\Delta^c \bar{\Delta}^c) + \mu_{ij} \mathrm{Tr}(\tau_2 \Phi_i^T \tau_2 \Phi_j).
\end{aligned} \tag{3.25}
$$

All couplings $Y^{(i)_{q,l}}$, μ_{ij}, μ_Δ, f in the above potential, are complex with the the additional constraint that μ_{ij}, f and f^* are symmetric matrices. The scalar potential is $V = V_F + V_D + V_{soft}$ where $V_F = |F_i|^2, F_i = -\frac{\partial W}{\partial \phi}$ is the F-term scalar potential, $V_D = D^a D^a/2, D^a = -g(\phi_i^* T_{ij}^a \phi_j)$ is the D-term of the scalar potential and V_{soft} is the soft supersymmetry breaking terms in the scalar potential. In the particular model, the soft-susy breaking terms are given by

$$
\begin{aligned}
V_{\text{soft}} = & m_\delta^2 \mathrm{Tr}[(\Delta^\dagger \Delta) + (\bar{\Delta}^\dagger \bar{\Delta})] + m_\delta^2 \mathrm{Tr}[(\Delta^{c\dagger} \Delta^c) + (\bar{\Delta}^{c\dagger} \bar{\Delta}^c)] \\
& + m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + m_\rho^2 \rho^\dagger \rho + B\, \mu_{ij}\, \mathrm{Tr}(\tau_2 \Phi_i \tau_2 \Phi_j) \\
& + A\, \rho\, \left[\mathrm{Tr}(\Delta\bar{\Delta}) - \mathrm{Tr}(\Delta^c \bar{\Delta}^c) + h.c. \right].
\end{aligned} \tag{3.26}
$$

where all the parameters in the soft supersymmetry breaking scalar potential is of the order of supersymmetry breaking scale $M_{susy} \sim$ TeV. We denote the vev of the neutral components of $\Phi_1, \Phi_2, \Delta, \bar{\Delta}, \Delta^c, \bar{\Delta}^c$ and ρ as

$$
\langle (\Phi_1)_{11} \rangle = v_1, \quad \langle (\Phi_2)_{22} \rangle = v_2, \quad \langle \Delta, \bar{\Delta} \rangle = v_L, \quad \langle \Delta^c, \bar{\Delta}^c \rangle = v_R, \quad \langle \rho \rangle = s
$$

Minimizing the scalar potential with respect to v_L, v_R we get

$$
\frac{\partial V}{\partial v_L} = v_L [2(m_\Delta + \lambda_1 s)^2 + 2\lambda_1^2 (v_L^2 - v_R^2) + As + 2m_\delta^2] = 0
$$

$$\Rightarrow v_R^2 - v_L^2 = \frac{2m_\delta^2 + (A + 2\lambda_1 M)s + 2(m_\Delta + \lambda_1 s)^2}{2\lambda_1^2} \tag{3.27}$$

$$\frac{\partial V}{\partial v_R} = v_R[2(m_\Delta - \lambda_1 s)^2 - 2\lambda_1^2(v_L^2 - v_R^2) - As + 2m_\delta^2] = 0$$

$$\Rightarrow v_R^2 - v_L^2 = \frac{-2m_\delta^2 + (A + 2\lambda_1 M)s - 2(m_\Delta - \lambda_1 s)^2}{2\lambda_1^2} \tag{3.28}$$

Also

$$v_R\frac{\partial V}{\partial v_L} - v_L\frac{\partial V}{\partial v_R} = 4v_L v_R[2(Ms + 2m_\Delta s)\lambda_1 + 2\lambda_1^2(v_L^2 - v_R^2) + As] = 0$$

$$\Rightarrow v_R^2 - v_L^2 = \frac{2\lambda_1(Ms + 2m_\Delta) + As}{2\lambda_1^2} \tag{3.29}$$

Thus the minimization conditions disallows solutions with $v_L = v_R$. But from equations (3.27), (3.28), (3.29) it can be seen that it is difficult to adjust the various scales $M, s, m_\triangle$ so as to satisfy them simultaneously and giving rise to a TeV scale v_R and an eV scale v_L. Thus we need to add more particles to the above particle content which can give rise to spontaneous D-parity breaking with a TeV scale v_R. This scenario of minimal SUSYLR model with parity odd singlet was studied long ago and was shown [115] that the charge-breaking vacua have a lower potential than the charge-preserving vacua and as such the ground state does not conserve electric charge

3.1.5 SUSYLR model with Higgs triplets and bitriplet

In minimal left-right supersymmetric models with triplet Higgs bosons leads to several nettlesome obstructions which may be considered to be a guidance towards a unique consistent theory. One of the most important problems is the spontaneous breaking of left-right symmetry and there are many substantial amount of work has been done to cure this problem. This can be cured either by adding some extra fields to the minimal particle content [115] or with the help of non-renormalization operator [118]. There is another solution to the problem, which resembles the non-supersymmetric solution, relating the vacuum expectation values (vevs) of the left-handed and right-handed triplet Higgs scalars to the Higgs bi-doublet vev through a seesaw relation. The novel feature consists in the introduction of a bitriplet Higgs and another Higgs singlet under left-right group [140]. We will try to extremize the full potential of this particular model and see what are the mass scales, different vevs coming out from the extremization.

We now present our model, where we include a bi-triplet and a parity odd singlet fields, in the minimal supersymmetric left-right symmetric model with triplet Higgs discussed in previous subsection [3.1.4]. These fields are vector-like and hence do not contribute to anomaly, so we consider only one of these fields. The quantum numbers for the new scalar fields, bitriplet ($\eta = (1,3,3,0)$) and parity odd singlet ($\rho = (1,1,1,0)$). Under parity, these fields transform as $\eta \leftrightarrow \eta$ and $\rho \leftrightarrow -\rho$. The superpotential for the model is written in the more general tensorial notation [140] as follows

$$
\begin{aligned}
W = & f\eta_{\alpha i}\Delta_\alpha \Delta^c_i + f^*\eta_{\alpha i}\bar{\Delta}_\alpha \bar{\Delta}^c_i + \lambda_1 \eta_{\alpha i}\Phi_{am}\Phi_{bn}(\tau^\alpha \epsilon)_{ab}(\tau^i \epsilon)_{mn} \\
& + m_\eta \eta_{\alpha i}\eta_{\alpha i} + M_\Delta(\Delta_\alpha \bar{\Delta}_\alpha + \Delta^c_i \bar{\Delta}^c_i) + \mu \epsilon_{ab}\Phi_{bm}\epsilon_{mn}\Phi_{an} \\
& + m_\rho \rho^2 + \lambda_2 \rho(\Delta_\alpha \bar{\Delta}_\alpha - \Delta^c_i \bar{\Delta}^c_i)
\end{aligned} \tag{3.30}
$$

where α, a, b are $SU(2)_L$ and i, m, n are $SU(2)_R$ indices. The symmetry breaking pattern in this model is

$$
SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \quad \xrightarrow{\langle\rho\rangle} \quad SU(2)_L \times SU(2)_R \times U(1)_{B-L}
$$

$$
\xrightarrow{\langle\triangle_c\rangle} \quad SU(2)_L \times U(1)_Y \quad \xrightarrow{\langle\Phi\rangle} \quad U(1)_{em}
$$

Denoting the vev's as $\langle\Delta_-\rangle = \langle\bar{\Delta}_+\rangle = v_L,\ \langle\Delta^c_+\rangle = \langle\bar{\Delta}^c_-\rangle = v_R,\ \langle\Phi_{+-}\rangle = v,\ \langle\Phi_{-+}\rangle = v',\ \langle\eta_{+-}\rangle = u_1,\ \langle\eta_{-+}\rangle = u_2,\ \langle\eta_{00}\rangle = u_0$ and $\langle\rho\rangle = s$.

The scalar potential is $V = V_F + V_D + V_{soft}$ where $V_F = |F_i|^2, F_i = -\frac{\partial W}{\partial \phi}$ is the F-term scalar potential, $V_D = D^a D^a/2, D^a = -g(\phi^*_i T^a_{ij}\phi_j)$ is the D-term of the scalar potential and V_{soft} is the soft supersymmetry breaking terms in the scalar potential. In the particular model, the soft-susy breaking terms are given by

$$
\begin{aligned}
V_{soft} = & V_{soft}(\text{containing } \Delta \text{ and } \Phi) + m_{\eta(soft)}\eta^\dagger_{\alpha i}\eta_{\alpha i} \\
& +(A_2\eta_{\alpha i}\Phi_{am}\Phi_{bn}(\tau^\alpha\epsilon)_{ab}(\tau^i\epsilon)_{mn} + A_3(\eta_{\alpha i}\Delta_\alpha\Delta^c_i) + h.c.)
\end{aligned} \tag{3.31}
$$

where V_{soft}(containing Δ and Φ) is given by the eqn: (3.26) in the subsection [3.1.4].

Minimizing the scalar potential with respect to v_L, v_R we get

$$
\begin{aligned}
\frac{\partial V}{\partial v_L} = & \mu_L^2(2\, v_L) + 2\,\lambda_2^2\, v_L\,(v_L^2 - v_R^2) + 2\,(f\, u_1 + f^*\, u_2)\, M_\Delta\, v_R \\
& + v_R\,(f + f^*)\,[2\, m_\eta(u_1 + u_2 + u_3) + \lambda_1\, v^2 + v_L\, v_R\,(f + f^*)] \\
& + 4\, v_L\, m_\delta^2 + 2\, A\, v_L\, s + A_3\, v_R\,(u_1 + u_2 + u_3) = 0
\end{aligned} \tag{3.32}
$$

$$\frac{\partial V}{\partial v_R} = \mu_R^2\,(2\,v_R) - 2\,\lambda_2^2\,v_R\,(v_L^2 - v_R^2) + 2\,(f\,u_1 + f^*\,u_2)\,M_\Delta\,v_L$$

$$+\,v_L\,(f + f^*)\,[2\,m_\eta\,(u_1 + u_2 + u_3) + \lambda_1\,v^2 + v_L\,v_R\,(f + f^*)]$$

$$+\,4\,v_R\,m_\delta^2 - 2\,A\,v_R\,s + A_3\,v_L\,(u_1 + u_2 + u_3) = 0 \qquad (3.33)$$

Where the effective mass terms μ_L^2, μ_R^2 are given by

$$\mu_L^2 = (M_\Delta + \lambda_2 s)^2 + \lambda_2 m_\rho s + \frac{1}{2}(f^2 u_1^2 + f^{*2} u_2^2) \qquad (3.34)$$

$$\mu_R^2 = (M_\Delta - \lambda_2 s)^2 - \lambda_2 m_\rho s + \frac{1}{2}(f^2 u_1^2 + f^{*2} u_2^2) \qquad (3.35)$$

Thus after the singlet field ρ acquires a vev the degeneracy of the Higgs triplets goes away and the left handed triplets being very heavy get decoupled whereas the right handed triplets can be as light as 1 TeV by appropriate fine tuning in the above two expressions. Assuming $v_L \ll v, v', \mu, A \ll m_\rho, s$ and $v_L \ll v_R$ we get from equations (3.32), (3.33):

$$v_L = \frac{-v_R[M_\Delta u_2 f^* + m_\eta(u_2 + u_3)(f + f^*) + u_1(f M_\Delta + m_\eta(f + f^*)]}{2m_\rho s\lambda_2 + 4M_\Delta s\lambda_2 + 2As} \qquad (3.36)$$

Thus we can get a small $v_L(\sim$ eV) and a TeV scale v_R by appropriate choice of $M_\Delta, m_\eta, m_\rho, s$. We take the vev of the bitriplet $u \ll M_Z$. Thus if we want $v_R \sim 1$ TeV then the above relation will give us an eV scale v_L only if the scale of parity breaking is kept low that is, $s \sim m_\rho \sim M_\Delta \sim 10^{10}$ GeV. Thus in such a type II seesaw dominated case, the right handed triplets Δ^c will be as light as $\mu_R \sim v_R \sim 1$ TeV and the left handed triplets Δ as heavy as $\mu_L \sim 10^{10}$ GeV by appropriate fine tuning of the parameters. However as we will see later, such a light Higgs triplet with $B - L$ charge 2 spoils the gauge coupling unification. Hence we are forced to keep the intermediate symmetry breaking scale M_R close to the unification scale.

3.2 Gauge Coupling Unification

Grand unified theories (GUTs) offer the possibility of unifying the three gauge groups viz., $SU(3)$, $SU(2)$ and $U(1)$ of the standard model into one large group at a high energy scale M_U. This scale is determined as the intersection point of the $SU(3)$, $SU(2)$ and $U(1)$ couplings. The particle content of the theory completely determines the variation of the couplings with energy. It is hard to achieve low intermediate scale without taking into account the effect of D-parity breaking in the RGEs. We have seen in the previous section that in spontaneous D-parity breaking models, the minimization of the scalar potential simultaneously allows us to have right handed scale v_R of the order of TeV and tiny neutrino masses from seesaw mechanisms. However the evolution of gauge couplings will be very different in models with Higgs triplets and with Higgs doublets. In this section we study the renormalization group evolution of the gauge couplings and see if unification at a high scale ($\sim 10^{16}$ GeV) allows us to have a TeV scale v_R. Similar analysis were done in [147, 148] for Higgs doublet case. Here we use the $U(1)$ normalization constant $\sqrt{\frac{3}{8}}$ as in [149]. We restrict our study to the supersymmetric case only. The gauge coupling unification in the non-supersymmetric versions of such models were studied before and can be found in [133, 150].

3.2.1 Unification in SUSYLR model with Higgs doublets

We will study the evolution of couplings according to their respective beta functions with the account of spontaneous D-parity breaking. The renormalization group equations(RGEs) for this model cane be written as

$$\frac{d\alpha_i}{dt} = \alpha_i^2[b_i + \alpha_j b_{ij} + O(\alpha^2)] \tag{3.37}$$

where, $t = 2\pi\, ln(M)$ (M is the varying energy scale), $\alpha_i = \frac{g_i^2}{4\pi}$ is the coupling strength. Also b_i and b_{ij} are the one loop and two loop beta coefficients and we will study only the one loop contributions to RGEs [149]. The indices $i, j = 1, 2, 3$ refer to the gauge group $U(1)$, $SU(2)$ and $SU(3)$ respectively.

The particle content of SUSYLR model with Higgs doublets is shown in subsection [3.1.3]. It turns out that the minimal particle content is not enough for proper gauge coupling unification. For required unification purposes we add two copies of $\delta \equiv (1, 1, 1, 2)$, $\bar{\delta} \equiv (1, 1, 1, -2)$ at the $SU(2)_R$ breaking scale. The beta functions are given as

- Below the Susy breaking scale M_{susy} the beta functions are same as those of the standard model
$$b_s = -7, \quad b_{2L} = -\frac{19}{6} \quad b_Y = \frac{41}{10}$$

- For $M_{susy} < M < M_R$, the beta functions are same as those of the MSSM

$$b_s = -9 + 2n_g, \quad b_{2L} = -6 + 2n_g + \frac{n_b}{2}, \quad b_Y = 2n_g + \frac{3}{10}n_b$$

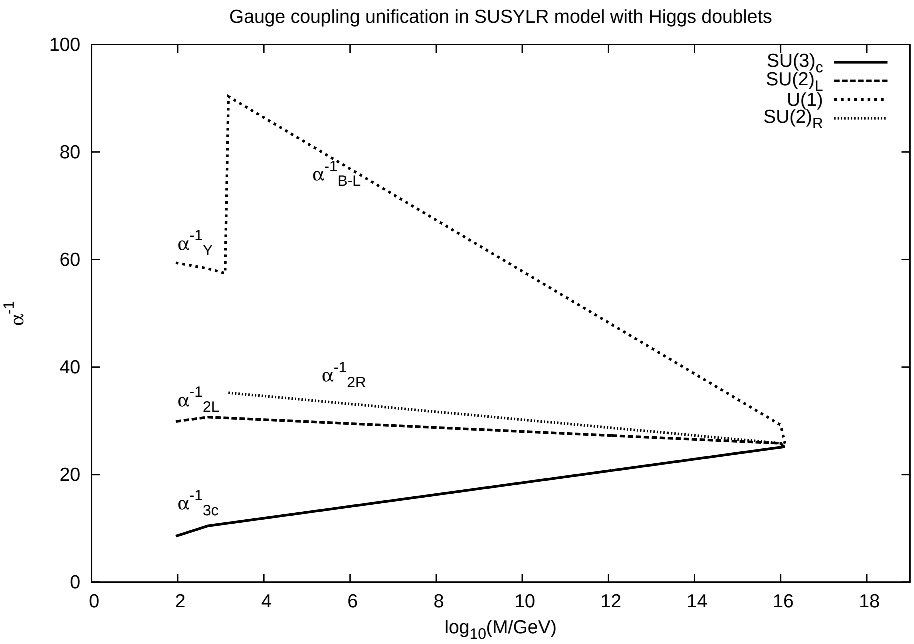


Figure 3.1: Gauge coupling unification with $M_{susy} = 500$ GeV, $M_R = 1.5$ TeV, $M_\rho = 10^{16}$ GeV

- For $M_R < M < \langle\rho\rangle$ the beta functions are

$$b_s = -9 + 2n_g, \quad b_{2L} = -6 + 2n_g + \frac{n_b}{2}$$

$$b_{2R} = -6 + 2n_g + \frac{n_b}{2} + \frac{n_{HR}}{2}, \quad b_{B-L} = 2n_g + 3n_\delta + \frac{3}{4}n_{HR}$$

- For $\langle\rho\rangle < M < M_{GUT}$ the beta functions are

$$b_s = -9 + 2n_g, \quad b_{2L} = -6 + 2n_g + \frac{n_b}{2} + \frac{n_{HL}}{2}$$

$$b_{2R} = -6 + 2n_g + \frac{n_b}{2} + \frac{n_{HR}}{2}, \quad b_{B-L} = 2n_g + 3n_\delta + \frac{3}{4}(n_{HL} + n_{HR})$$

where n_g is the number of fermion generations and number of Higgs bidoublets $n_b = 2$, number of Higgs doublets $n_{HL} = n_{HR} = 2$, number of extra Higgs singlets $n_\delta = 2$. The experimental initial values for the couplings at electroweak scale $M = M_Z$ [129] are

$$\begin{pmatrix} \alpha_s(M_Z) \\ \alpha_{2L}(M_Z) \\ \alpha_{1Y}(M_Z) \end{pmatrix} = \begin{pmatrix} 0.118 \pm 0.003 \\ 0.033493^{+0.000042}_{-0.000038} \\ 0.016829 \pm 0.000017 \end{pmatrix} \tag{3.38}$$

The normalization condition at $M = M_R$ where the $U(1)_Y$ gauge coupling merge with $SU(2)_R \times U(1)_{B-L}$ is $\alpha_{B-L}^{-1} = \frac{5}{2}\alpha_Y^{-1} - \frac{3}{2}\alpha_L^{-1}$. Using all these we arrive at the gauge coupling unification as shown in (3.1). Here we have taken $M_{susy} = 500$ GeV, $M_R = 1.5$ TeV, $M_\rho = 10^{16}$ GeV. The couplings seems to unify at a scale slightly above the D-parity breaking scale. Thus the D-parity breaking scale need not be the same as the GUT scale, but can be lower also. However if we make the D-parity breaking scale arbitrarily lower, the unification wont be possible as can be seen from the figure (3.1). Since both the left handed and right handed Higgs doublets will contribute to the $U(1)_{B-L}$ couplings after the D-parity breaking scale, the α_{BL}^{-1} will come down sharply and meet the other couplings at some energy below the expected GUT scale.

3.2.2 Unification in SUSYLR model with Higgs triplets

The particle content of SUSYLR model with Higgs triplets is shown in subsection [3.1.4]. It is very difficult to achieve unification with low M_R with the minimal particle content. We add a parity odd singlet $\rho(1,1,1,0)$ to achieve spontaneous D-parity breaking. This may change the scale of M_R, but it is found that the M_R remains higher that 10^{10} GeV. For unification purposes, we need in the recent model, one heavy bidoublet $\chi(1,2,2,0)$ has been added which gets mass at the $SU(2)_R$ breaking scale. Below the $SU(2)_R$ breaking scale the beta functions are similar to the MSSM as written above. The beta functions above this scale are

- For $M_R < M < M_\rho$ the beta functions are

$$b_s = -9 + 2n_g, \quad b_{2L} = -6 + 2n_g + \frac{n_b}{2} + \frac{n_\chi}{2}$$

$$b_{2R} = -6 + 2n_g + \frac{n_b}{2} + 2n_\triangle + \frac{n_\chi}{2}, \quad b_{B-L} = 2n_g + \frac{9}{2}n_\triangle$$

- For $\langle\rho\rangle < M < M_{GUT}$ the beta functions are

$$b_s = -9 + 2n_g, \quad b_{2L} = -6 + 2n_g + \frac{n_b}{2} + 2n_\triangle + \frac{n_\chi}{2}$$

$$b_{2R} = -6 + 2n_g + \frac{n_b}{2} + 2n_\triangle + \frac{n_\chi}{2}, \quad b_{B-L} = 2n_g + 9n_\triangle$$

where number of Higgs triplets $n_\triangle = 2$, number of additional Higgs field added for unification $n_\chi = 1$,, number of generations $n_g = 3$, and number of Higgs bidoublets $n_b = 2$. Using the same initial values and normalization relations like before we arrive at the gauge

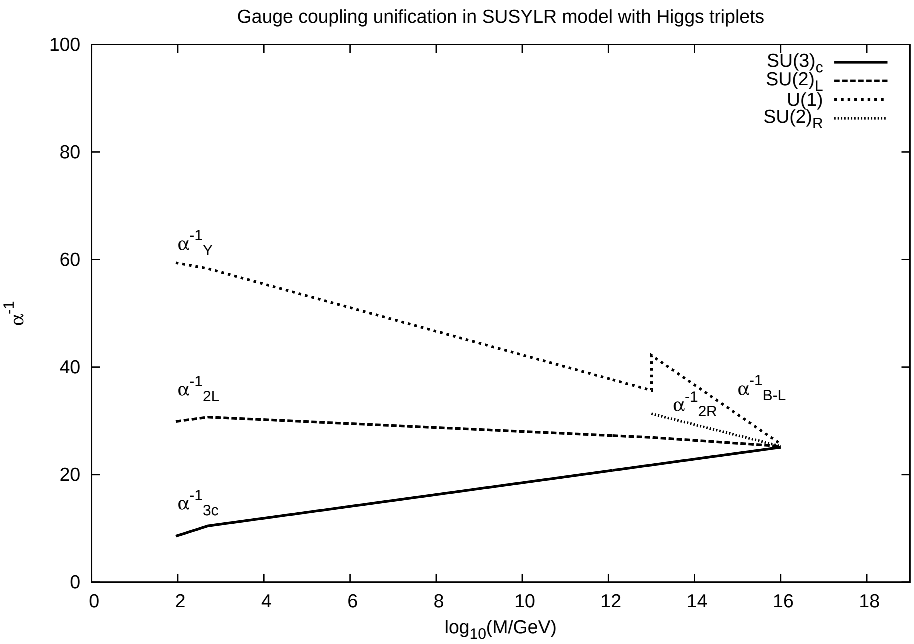


Figure 3.2: Gauge coupling unification with $M_R = 10^{13}$ GeV, $M_\rho = 10^{16}$ GeV

coupling unification as shown in (5.1). Here the unification scale M_{GUT} coincides with the D-parity breaking scale M_ρ. Lower values of M_R will make the unification worse because of the large contributions of triplets to the $U(1)_{B-L}$ beta functions compared to the doublets in the previous case. Thus in the minimal triplet case, both the minimization conditions as well as unification disallow a TeV scale v_R. Although after adding a bitriplet, the minimization conditions allow a TeV scale v_R, it wont make the unification better as we discuss in the next subsection.

3.2.3 Unification in SUSYLR model with Higgs triplets and bitriplet

As we saw before, the minimization of the scalar potential in a SUSYLR model with Higgs triplets with spontaneous D-parity breaking does not allow a TeV scale M_R. The same thing is true from gauge coupling unification point of view as shown in the previous subsection. Now we consider the SUSYLR model with Higgs triplet as well a bitriplet [140]. For unification purposes we add three heavy colored particles $\chi(3,1,1,0)$ which decouple after the $SU(2)_R$ breaking scale M_R. The beta functions above M_R are

- For $M_R < M < M_\rho$ the beta functions are

$$b_s = -9 + 2n_g + \frac{n_\chi}{2}, \quad b_{2L} = -6 + 2n_g + \frac{n_b}{2} + 2n_\eta$$

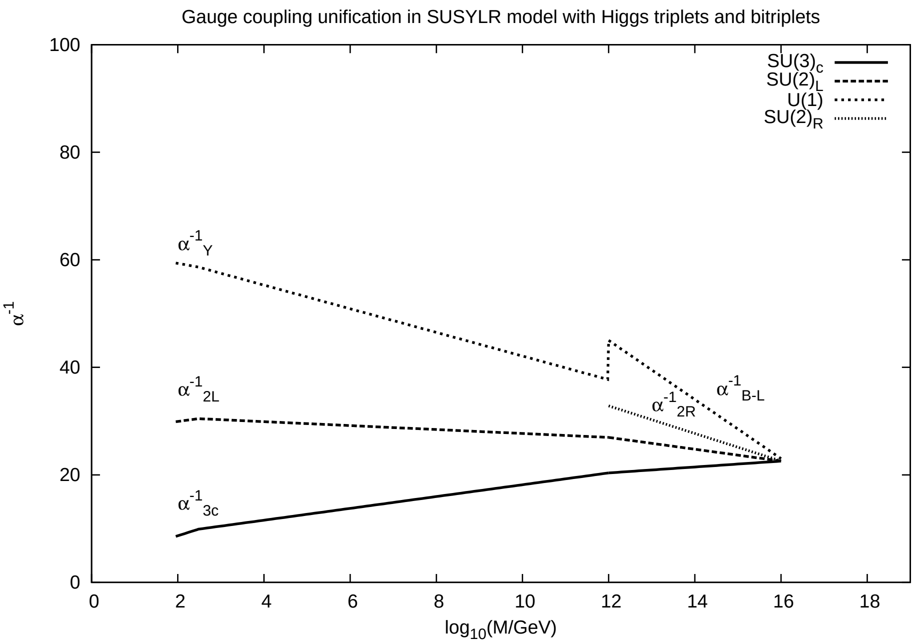


Figure 3.3: Gauge coupling unification with $M_R = 10^{12}$ GeV, $M_\rho = 10^{16}$ GeV

$$b_{2R} = -6 + 2n_g + \frac{n_b}{2} + 2n_\triangle + 2n_\eta, \quad b_{B-L} = 2n_g + \frac{9}{2}n_\triangle$$

- For $\langle\rho\rangle < M < M_{GUT}$ the beta functions are

$$b_s = -9 + 2n_g + \frac{n_\chi}{2}, \quad b_{2L} = -6 + 2n_g + \frac{n_b}{2} + 2n_\triangle + 2n_\eta$$

$$b_{2R} = -6 + 2n_g + \frac{n_b}{2} + 2n_\triangle + 2n_\eta, \quad b_{B-L} = 2n_g + 9n_\triangle$$

where number of Higgs triplets $n_\triangle = 2$, number of colored Higgs $n_\chi = 3$, number of generations $n_g = 3$, number of Higgs bidoublets $n_b = 2$ and number of Higgs bitriplets $n_\eta = 1$. Using the same initial values and normalization relations like before we arrive at the gauge coupling unification as shown in (5.2). Here the unification scale is the same as the D-parity breaking scale. Similar to the case with just Higgs triplets, here also lower value of M_R makes the unification look worse. Thus although minimization of the scalar potential allows the possibility of a TeV scale M_R in this model, the gauge coupling unification criteria rules out such a possibility.

3.3 Neutrino mass in SUSYLR model with Higgs doublets

In left-right symmetric models with only doublet scalar fields, the question of neutrino masses has been discussed in details. We shall try to restrict ourselves as close as possible to these existing non-supersymmetric models, and check the consistency of these solutions when D-parity is broken spontaneously in the present SUSYLR model.

We introduced a singlet fermionic superfield S to the particle content of the model discussed in subsection [3.1.3]. This kind of model has been discussed without the D-parity breaking effect and from the neutrino mass prospective cite. The effect of this singlet field has been accounted in the RGEs shown in subsection [3.2.1]. With the addition of this singlet fermion, the superpotential and resulting neutrino mass matrix become

$$W = \mathcal{M}_{ij} S_i S_j + F_{ij} \Psi_{Li} S_j H_L + F'_{ij} \Psi_{Ri} S_j H_R, \tag{3.39}$$

and

$$W_{\rm neut} = (\nu_i \quad N_i^c \quad S_i) \begin{pmatrix} 0 & (M_N)_{ij} & F_{ij} v_L \\ (M_N)_{ji} & 0 & F'_{ij} v_R \\ F_{ji} v_L & F'_{ji} v_R & \mathcal{M}_{ij} \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j^c \\ S_j \end{pmatrix}. \tag{3.40}$$

where M_N is the general Dirac term coming from the term $(M_N)_{ij} \nu_i N_j^c$. In the above mass matrix, the mass of the singlet $\mathcal{M}_{ij}$ and the vev of the right-handed Higgs doublet v_R are heavy, while M_N and vev of the left-handed Higgs doublet v_L are of low scale.

The resulting light neutrino mass matrix after diagonalizing the above mass matrix is

$$\begin{aligned} M_\nu &= -M_N M_R^{-1} M_N^T - (M_N H + H^T M_N^T) \left(\frac{v_L}{v_R}\right), & (3.41) \\ \text{where,} \qquad H &\equiv \left(F' \cdot F^{-1}\right)^T, & (3.42) \\ M_R &= (F' v_R) \mathcal{M}^{-1} (F'^T v_R). & (3.43) \end{aligned}$$

Here we can see that the first term in eqn (4.23) is the type-I seesaw contribution and the second term gives the type-III seesaw contribution. The type-III contribution to ν mass will dominate over type-I if the elements of the matrix $\mathcal{M}_{ij}$ are small compared to the contribution of H term. It is clear from the eqn (4.25) that the scale of M_R found to be TeV for $\mathcal{M}_{ij}$ =1 TeV, v_R =1 TeV which is automatically comes from the minimization of the potential and consistent with the RG evolutions which has already studied in subsection [3.2.1] and F of the order of unity. With the mass scales and M_N of the order of MeV, we can found neutrino mass to be eV.

Neutrino mass in case of Fermionic triplet:

Let us introduce fermionic triplets (one for each family) order to realize the double seesaw mechanism:

$$\Sigma_L = \frac{1}{2}\begin{pmatrix} \Sigma_L^0 & \sqrt{2}\Sigma_L^+ \\ \sqrt{2}\Sigma_L^- & -\Sigma_L^0 \end{pmatrix} \equiv (3,1,1,0),$$

and

$$\Sigma_R = \frac{1}{2}\begin{pmatrix} \Sigma_R^0 & \sqrt{2}\Sigma_R^+ \\ \sqrt{2}\Sigma_R^- & -\Sigma_R^0 \end{pmatrix} \equiv (1,3,1,0),$$

Under left-right parity transformation one has the following relations

$$\Sigma_L \longleftrightarrow \Sigma_R.$$

In the context of lepton masses, the relevant term in the Lagrangian is

$$\mathcal{L}_\ell = \bar{\ell}_L(Y_1\Phi + Y_2\tilde{\Phi})\ell_R + h.c.$$

where $\tilde{\Phi} = \tau_2\Phi\tau_2$. Once the bidoublet Φ takes vev. i.e $v_1 = \langle\phi_1^0\rangle$ and $v_2 = \langle\phi_2^0\rangle$, the Dirac mass matrix for the neutrinos is

$$m_\nu^D = Y_1 v_1 + Y_2 v_2$$

The relevant Yukawa terms that gives masses (for the *double* seesaw mass matrix) to the three generations of leptons are given by

$$\begin{aligned} \mathcal{L}_\nu^{III} &= h_{ij}\ell_{iL}^T\, C\, i\sigma_2\, \Sigma_{jL}\, H_L\, + g_{ij}\, \ell_{iR}^T\, C\, i\sigma_2\, \Sigma_{jR}\, H_R \\ &+ M_\Sigma\, Tr\left(\Sigma_L^T\, C\, \Sigma_L\, +\, \Sigma_R^T\, C\Sigma_R\right) + h.c. \end{aligned} \tag{3.44}$$

Once the Higgs doublets gets vev i.e,$v_L = \langle H_L^0\rangle$ and $v_R = \langle H_R^0\rangle$, $SU(2)_L \otimes SU(2)_R$ is broken spontaneously. Now the mass matrix in the basis $\left(\nu_L,\ \nu_R,\ \Sigma_R^0\right)$ reads as:

$$M_\nu^{III} = \begin{pmatrix} 0 & m_\nu^D & 0 & hv_L \\ (m_\nu^D)^T & 0 & gv_R & 0 \\ 0 & g^T v_R & M_\Sigma & 0 \\ h^T v_L & 0 & 0 & M_\Sigma \end{pmatrix}. \tag{3.45}$$

As one expects the neutrino masses are generated through the Type I + Type III seesaw mechanisms and one has a *double* seesaw mechanism since the mass of the right-handed neutrinos are generated through the Type III seesaw once we integrate out Σ_R^0.

The neutrino mass formula derived from the above mass matrix is given by

$$m_{\nu_L} = \frac{1}{v_R^2\,(g^T g)}\,[m_\nu^D\, M_\Sigma\,(m_\nu^D)^T - v_R\, v_L\, m_\nu^D\,(g\,h)^T - v_R\, v_L\,(g\,h)\,(m_\nu^D)^T] \tag{3.46}$$

with right handed neutrino masses

$$M_R = v_R^2\, g\ (M_\Sigma)^{-1}\ g^T. \tag{3.47}$$

We take the Dirac mass of the all the three neutrinos to be of MeV order. This fixes the scale of the M_Σ and M_R so as to give rise to eV scale neutrino masses on the left hand side of above relation [3.46]. If we assume that the first term of [3.46] will dominate then the seesaw relations will become $m_\nu = \frac{m_e^2}{M_R}$. As $m_e = 0.5$ MeV, we need the values of the right handed Majorana neutrino as: $M_R = 10^3$ GeV to have 0.1 eV light neutrino mass. We can arrive at the appropriate value of M_R by choosing g and M_Σ. Since we are taking $v_R \sim 1$TeV hence to get $M_R \geq 1$TeV we must have $M_\Sigma \leq 1$TeV. Once the scale of right handed Majorana neutrino gets fixed by the light neutrino mass, we can find the values of M_Σ and v_R. We have taken the Yukawa couplings as $g, h < 1$, $v_R = 10^3$ GeV in Eq. [3.47] and these lead to triplet fermion masses :$M_\Sigma \sim 10^3$ GeV.

If $M_\Sigma << 1$TeV and $v_R \sim 1$TeV, then the first term of the above neutrino mass formula becomes to small to give rise to neutrino masses. In that case the second and the third term in the equation [3.46] can contribute to the neutrino masses if $v_L/v_R \sim 10^{-6}$. And such a ratio can naturally be achieved (even if we have a TeV scale v_R) by choosing various symmetry breaking scales and mass parameters as we discussed in section [3.1].

Role of Σ_L, Σ_R in unification:

The fermion triplets with $U(1)_{B-L}$ charge zero contributes to the $SU(2)_L$ and $SU(2)_R$ gauge coupling running. As discussed above, for the seesaw purposes we have to take low values of $M_\Sigma <= v_R$ which will ruin the gauge coupling unification for a TeV scale $SU(2)_R$ breaking scale v_R. Unification and small neutrino mass are possible only if $SU(2)_R$ breaking scale as well as mass of the triplet fermions are close to the unification scale. However if we add fermion singlet in place of triplets then there is no constraints from unification point of view on v_R and M_Σ. The mass matrix becomes 3×3 in this case. Thus in Supersymmetric left-right model with Higgs doublets, we can achieve unification with TeV scale $SU(2)_R$

breaking scale only if fermion singlet is added in place of triplets as in the conventional type III seesaw.

3.4 Neutrino mass in SUSYLR model with Higgs triplets and bi-triplets

The relevant Yukawa couplings which leads to small non-zero neutrino mass is given by

$$\begin{aligned} \mathcal{L}_\nu^{II} &= y_{ij}\ell_{iL}\Phi\ell_{jR} + y'_{ij}\ell_{iL}\tilde{\Phi}\ell_{jR} + h.c. \\ &+ f'_{ij}\ \left(\ell_{iR}^T\, C\ i\sigma_2\Delta_R\ell_{jR} + (R\leftrightarrow L)\right) + h.c. \end{aligned} \quad (3.48)$$

The Majorana Yukawa couplings f is same for both left and right handed neutrinos because of left-right symmetry. After symmetry breaking, the effective mass matrix of the neutrinos is

$$m_\nu = \frac{-f\, v^2\, v_R}{2\, m_\sigma\, s} - \frac{v^2}{v_R} y\, f^{-1}\, y^T = m_\nu^{II} + m_\nu^{I} \quad (3.49)$$

Consider the values of y, f are of the order of unity, then the relative magnitude of m_ν^{II} and m_ν^{I} depend on the parameters like v_R, m_σ, s. As discussed in section [3.1], the type II term can become dominant (even if $v_R \sim 1$ TeV) if we take $m_\sigma \sim s \sim 10^8 - 10^{10}$ GeV.

3.5 Results and Discussions

Spontaneous breaking of Lorentz parity occurs via Higgs doublet in SUSYLR model with doublet Higgs only and via Higgs triplets/bitriplet in SUSYLR model with Higgs triplets and bitriplet. After taking into account of spontaneous D-parity breaking, the minimization of the scalar potential also allows the possibility of $M_R \sim$ TeV, $v_L \sim$ eV in LRSM with Higgs triplets and SUSYLR models with Higgs triplets and Higgs bitriplet. It also allows $M_R \sim$ TeV, $v_L/v_R \sim 10^{-6}$ in both Susy and non-Susy LR models with Higgs doublets.

In the SUSYLR model with Higgs doublets we can have a TeV scale M_R as well as $v_L/v_R \sim 10^{-6}$ by keeping the D-parity breaking scale very high $\sim 10^{16}$ GeV. The gauge couplings also unify for the same choice of scales although at the cost of adding extra particles which contribute to the beta functions at high energy. However if we add fermion triplets for seesaw, then unification is not possible with TeV scale $SU(2)_R$ breaking scale.

Adding fermion singlet for seesaw purposes can evade this difficulty.

In SUSYLR model with Higgs triplet, the minimization conditions do not allow the possibility of a TeV scale M_R and eV scale v_L simultaneously although gauge couplings unify if we take M_R as high as 10^{13} GeV. Thus we can not have TeV scale M_R, type II seesaw dominance and gauge coupling unification simultaneously.

In SUSYLR model with Higgs triplets and bitriplet, we can have TeV scale M_R and eV scale v_L only if we keep the D-parity breaking scale as low as 10^{10} GeV. However such a choice of parity breaking scale spoils the gauge coupling unification. The gauge couplings unify if we take $M_R = 10^{12}$ GeV and the D-parity breaking scale as 10^{16} GeV with inclusion of three extra colored particles. Thus we can not have a TeV scale M_R and unification simultaneously.

To summarize the work, we have analyzed the different scenarios of spontaneous breaking of D-Parity in both non-Susy and Susy version of left right symmetric models. We have discussed the possibility of obtaining a TeV scale M_R, gauge coupling unification and type II/type III seesaw dominance of neutrino mass within the framework of different SUSYLR models. In all the models where we explore the possibility of a TeV scale M_R, it is difficult to achieve unification with the minimal particle content. We have added some extra scalar particles as well as their superpartners with suitable transformation properties under the gauge group to achieve unification. We have shown that except for the SUSYLR model with Higgs doublets, we can not have a TeV scale M_R and gauge coupling unification. In SUSYLR model with Higgs doublet, type III seesaw can dominate even if the D-parity breaking scale is as high as the GUT scale whereas in SUSYLR model with Higgs triplets and bitriplet, the D-parity breaking scale has to be kept as low as 10^{10} GeV for type II seesaw to dominate. However adding fermion triplets to give rise to seesaw spoils the unification with a TeV scale M_R in the SUSYLR model with Higgs doublet. Adding fermion singlets instead of triplets do not give rise to this problem and can reproduce the necessary seesaw without affecting the RG evolution of the couplings.

CHAPTER

4

Leptogenesis and Neutrino mass in susyLR with Higgs doublet

The existence of massive neutrinos, the unknown origin of parity violation in the Standard Model (SM) and the hierarchy problem are some of the important motivations for physics beyond the SM. The most natural extension of the standard model that addresses these issues is the supersymmetric version of the left-right symmetric extension of the standard model, which will treat the left-handed and right-handed particles on equal footing, and the parity violation we observe at low energies would be due to the spontaneous breaking of the left-right symmetry at some high scale [99–103]. Another interesting feature of the left-right symmetric model is that the difference between the baryon number (B) and the lepton number (L) becomes a gauge symmetry, which leads to several interesting consequences.

In spite of the several virtues of the minimal supersymmetric left-right symmetric models (MSLRM), we are yet to arrive at a fully consistent model, from which we can descend down to the MSSM. One of the most important problems is the spontaneous breaking of left-right symmetry [115, 116]. There has been suggestions to solve this problem by introducing additional fields or higher dimensional operators or by going through a different symmetry breaking chain or breaking the left-right symmetry around the supersymmetry breaking scale [115–117, 121, 121, 123, 130]. In some cases, this problem is cured through the introduction of a parity-odd singlet, but the soft susy breaking terms then lead to breaking of electromagnetic charge invariance. One of the interesting SUSYLR model is the minimal SUSYLR model, which has been studied extensively [115, 116, 121], and it has been found that global minimum of the Higgs potential is either charge violating or R-parity violating. The details of these discussion has been reviewed in second chapter and the simplest solution is to include a bi-triplet field [140] and allow D-parity breaking at some high scale, which may then allow parity violation spontaneously, allowing the scale of $SU(2)_R$ breaking to be different from the $SU(2)_L$ breaking scale. We will now extend this argument to the models involving only doublets.

In this work, we will address the question of parity breaking in a supersymmetric left-right model, in which the left-right symmetry is broken with Higgs doublets (carrying $B - L = \pm 1$). Unlike the left-right symmetric models with triplet Higgs scalars (carrying $B - L = \pm 2$), in this model it is possible to break parity spontaneously by adding a parity odd singlet. We shall also discuss how neutrino mass of type III (as named in the work of Albright) seesaw or, double-seesaw, can be invoked in this model by adding extra fermion singlets. We considered simple forms of the mass matrices that are consistent with the unification scheme and demonstrate how they can reproduce the required neutrino mixing matrix. In this model, the baryon asymmetry of the universe is generated via leptogenesis. The required mass scales in the model is then found to be consistent with the gauge coupling unification.

4.1 SUSYLR with Higgs doublets with odd B-L and parity odd singlet

We consider here a SUSYLR model with only doublet Higgs scalars, which is the simplest extension of the non-supersymmetric LR model. This includes the bi-doublet scalar field that is required to give masses to the charged fermions and also to break the $SU(2)_L$ symmetry after the left-right symmetry is broken. The doubling of the bidoublet Higgs in previous models was to ensure a non-vanishing CKM matrix. For the sake of simplicity of our model we forgo this condition since it doesn't have any bearing on parity breaking. However, extension of the present model via doubling of the bidoublet is fairly trivial.

The superpotential for supersymmetric left-right theory with Higgs doublets which is relevant for us is

$$W = f\Phi\left(\bar{\chi}_L\chi_R + \chi_L\bar{\chi}_R\right) + m_\Phi\Phi\Phi + m_\chi\left(\bar{\chi}_L\chi_L + \bar{\chi}_R\chi_R\right)$$

where f is a dimensionless constant in the theory. The Higgs fields acquire vevs as follows

$\langle\chi_L\rangle = \langle\bar{\chi}_L\rangle = v_L$, $\langle\chi_R\rangle = \langle\bar{\chi}_R\rangle = v_R$ and $\langle\Phi\rangle = v$. From flatness condition, one can easily deduce the relations like

$$\begin{aligned} v_L &= \frac{-m_\Phi v}{f v_R} \\ m_\chi v_L + f v\, v_R &= 0 \\ m_\chi v_R + f v v_L &= 0 \end{aligned} \tag{4.1}$$

The last two relations are not consistent for $v_L \ll v_R$ as we are interested in the case where parity has to be broken spontaneously. That means we need the left-right scales should be different so that we can achieve spontaneous parity breaking. Why the scales of left-right scale should be different is not clear from the above relations. The simplest solution for this problem is to add a parity odd singlet.

We will now present a model which is phenomenologically consistent and explains the neutrino mass, baryon asymmetry via leptogenesis mechanism. This model can give answer to the question of spontaneous parity breaking in the supersymmetric version of the left-right symmetric models, in which all symmetry breaking takes place with only doublet Higgs scalars and a D-parity odd singlet scalar. We will review to the case where the electroweak gauge group is the left-right symmetric group $G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and we will study susy version of this model. The field content of the supersymmetric left right model is given by

$$\begin{aligned} Q &= \begin{pmatrix} u \\ d \end{pmatrix} \equiv [3,2,1,\frac{1}{3}] \quad , \quad Q^c = \begin{pmatrix} u^c \\ d^c \end{pmatrix} \equiv [3,1,2,\frac{1}{3}]\,, \\ L &= \begin{pmatrix} \nu \\ e \end{pmatrix} \equiv [1,2,1,-1] \quad , \quad L^c = \begin{pmatrix} N^c \\ e^c \end{pmatrix} \equiv [1,1,2,-1] \end{aligned} \tag{4.2}$$

where the numbers in the brackets denote the quantum numbers under $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. The right handed neutrino is now required by the gauge group.

Thus, the Higgs sector of our model is given by,

$$\begin{aligned} \chi_L &\equiv (1,2,1,-1), & \overline{\chi}_L &\equiv (1,2,1,1), \\ \chi_R &\equiv (1,1,2,-1), & \overline{\chi}_R &\equiv (1,1,2,1), \\ \Phi_a &= (1,2,2,0), & \sigma &\equiv (1,1,1,0). \end{aligned}$$

where, with usual custom the subscript L and R denotes the left and right handedness of the Higgs particle. The Higgs particles with "bar" in the notation, helps in anomaly cancellation of the model.

The gauge group of this model is $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$. The quantum numbers for the superfields under the gauge group considered are given by the table [4.1] as follows

	$SU(3)^c$	$\times$	$SU(2)_L$	$\times$	$SU(2)_R$	$\times$	$U(1)_{B-L}$
Matter Superfiled:							
Q	3		2		1		$+1/3$
Q^c	3		1		2		$-1/3$
L	1		2		1		-1
L^c	1		1		2		$+1$
Higgs Superfiled:							
Φ_a	1		2		2		0
χ_L	1		2		1		$+1$
χ_R	1		1		2		-1
$\overline{\chi}_L$	1		2		1		-1
$\overline{\chi}_R$	1		1		2		$+1$
σ	1		1		1		0

Table 4.1: This table shows the particle content and their quantum number under the gauge groups $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

We have also included a singlet scalar field σ, which has the special property that it is even under the usual parity of the Lorentz group, but it is odd under the parity that relates the gauge groups $SU(2)_L$ and $SU(2)_R$. This field σ is thus a scalar and not a pseudo-scalar field, but under the D-parity transformation that interchanges $SU(2)_L$ with $SU(2)_R$, it is odd. This kind of work is proposed in [133, 139]. Although all the scalar fields are even under the parity of the Lorentz group, under the D-parity the Higgs sector transforms as,

$$
\begin{aligned}
\chi_L &\leftrightarrow \chi_R, & \bar{\chi}_L &\leftrightarrow \bar{\chi}_R, \\
\Phi &\leftrightarrow \Phi^\dagger, & \sigma &\leftrightarrow -\sigma.
\end{aligned}
$$

The superpotential of the model relevant in the context of parity breaking is given by,

$$
\begin{aligned}
W &= f\Phi\left(\bar{\chi}_L\chi_R + \chi_L\bar{\chi}_R\right) + m_\Phi \Phi\Phi \\
&\quad + m_\chi\left(\bar{\chi}_L\chi_L + \bar{\chi}_R\chi_R\right) \\
&\quad + m_\sigma\,\sigma^2 + \lambda\sigma\left(\bar{\chi}_L\chi_L - \bar{\chi}_R\chi_R\right). \qquad (4.3)
\end{aligned}
$$

Supersymmetry being unbroken, implies the F and D conditions are equal to zero. The F

flatness conditions for the various Higgs fields are given by,

$$
\begin{aligned}
F_\Phi &= f\,(\bar{\chi}_L\chi_R + \chi_L\bar{\chi}_R) + 2\,m_\Phi\Phi = 0, \\
F_{\chi_L} &= f\Phi\bar{\chi}_R + m_\chi\bar{\chi}_L + \lambda\sigma\bar{\chi}_L = 0, \\
F_{\bar{\chi}_L} &= f\Phi\chi_R + m_\chi\chi_L + \lambda\sigma\chi_L = 0, \\
F_{\chi_R} &= f\Phi\bar{\chi}_L + m_\chi\bar{\chi}_R - \lambda\sigma\bar{\chi}_R = 0, \\
F_{\bar{\chi}_R} &= f\Phi\chi_L + m_\chi\chi_R - \lambda\sigma\chi_L = 0, \\
F_\sigma &= 2m_\sigma\sigma + \lambda(\bar{\chi}_L\chi_L - \bar{\chi}_R\chi_R). \qquad (4.4)
\end{aligned}
$$

Similarly, the D flatness conditions, are given by,

$$
\begin{aligned}
D_{R_i} &= \chi_R^\dagger\tau_i\chi_R + \bar{\chi}_R^\dagger\tau_i\bar{\chi}_R = 0, \\
D_{L_i} &= \chi_L^\dagger\tau_i\chi_L + \bar{\chi}_L^\dagger\tau_i\bar{\chi}_L = 0, \\
D_{B-L} &= (\chi_L^\dagger\chi_L - \bar{\chi}_L^\dagger\bar{\chi}_L) - (\chi_R^\dagger\chi_R - \bar{\chi}_R^\dagger\bar{\chi}_R) = 0. \qquad (4.5)
\end{aligned}
$$

In both the F and D flat conditions we have neglected the lepton fields, since they would have a zero vev. The vev's for the scalar fields are given by,

$$
\begin{aligned}
\langle\chi_L\rangle &= \langle\bar{\chi}_L\rangle = v_L, \\
\langle\chi_R\rangle &= \langle\bar{\chi}_R\rangle = v_R, \\
\langle\Phi\rangle &= v, \quad \langle\sigma\rangle = s. \qquad (4.6)
\end{aligned}
$$

Here, for simplicity of the model, we have assumed χ_L and $\bar{\chi}_L$ to have the same vev v_L. Similarly, for the right-handed fields χ_R and $\bar{\chi}_R$.

Here, however, in order to determine the vacuum structure of our model, we minimize the F flat conditions and discuss about the relations that emerge from them. Suppose the field σ takes the vev as $\langle\sigma\rangle = s$. After the scalar fields have acquired their respective vevs, the F flatness conditions are given by,

$$F_\Phi = f(v_Lv_R + v_Rv_L) + 2m_\Phi v = 0, \qquad (4.7)$$

$$F_\sigma = 2m_\sigma s + \lambda(v_L^2 - v_R^2) = 0. \qquad (4.8)$$

$$F_{\chi_L} = fvv_R + \lambda sv_L + m_\chi v_L = 0, \qquad (4.9)$$

$$F_{\bar{\chi}_L} = fvv_R + \lambda sv_L + m_\chi v_L = 0, \qquad (4.10)$$

$$F_{\chi R} = f v v_L - \lambda s v_R + m_\chi v_R = 0, \tag{4.11}$$

$$F_{\bar{\chi} R} = f v v_L - \lambda s v_R + m_\chi v_R = 0, \tag{4.12}$$

Solving the equations we get four relations among the vevs.

$$v_L = \frac{-m_\Phi v}{f v_R} \tag{4.13}$$

$$m_\chi + \lambda s = \frac{f\, v\, v_R}{v_L} \tag{4.14}$$

$$m_\chi - \lambda s = -\frac{f v v_L}{v_R} \tag{4.15}$$

$$s = \frac{\lambda}{2m_\sigma}(v_R^2 - v_L^2) \tag{4.16}$$

The role of D-parity odd singlets σ is uni-important in left-right breaking. This can be understood from eqns. (4.14) and (4.15) as follows:

$$\left(\frac{v_L}{v_R}\right)^2 = \frac{M - \lambda\, s}{M + \lambda\, s} \tag{4.17}$$

If there is no σ field, then $s = 0$. This implies $v_L = v_R$ which is a left-right symmetric solution. Also the F-term conditions (4.9)-(4.12) are not consistent without the inclusion of the parity odd singlet σ in the model. Hence, the parity odd singlet σ is necessary to account for the spontaneous left-right breaking and for the consistency of the model.

We now try to interpret these results to get a working phenomenology. Considering the last of the relations eqn (4.16) we see that $s = 0$ is a trivial solution, and will put v_L and v_R on equal footing thus leading to unbroken parity. However, $s = 0$ is a special solution of eqn (4.16). For $s \neq 0$, we have $v_L \neq v_R$ and parity is violated spontaneously. We will choose $v_R \gg v_L$, as it is usually assumed in model building for phenomenological reasons. Choosing the mass (m_Φ) and vev (v) of Φ to be of electroweak (EW) scale and considering the dimensionless coupling constant λ to be of order unity, we immediately come to the conclusion, from eqn (4.15), that $m_\chi \sim s$.

In order to avoid generic susy problems like over abundance of gravitino, we assume the mass scale of v_R to be $\leq 10^9$ GeV. This together with eqn (4.13) gives the value of $v_L \simeq 10^{-5}$ GeV, where f, another dimensionless quantity, without any fine-tuning is considered to be of order unity. This is also consistent with the assumption that $v_R \gg v_L$. Now using eqn (4.14) and the above derived relation that $m_\chi \sim s$ we get $m_\chi \sim s \simeq 10^{16}$ GeV. Finally, from eqn (4.16) one derives the mass of σ (m_σ) to be of EW scale. If one

Masses/Vevs	Case - I (In GeV)
m_χ, s	10^{16}
v_R	10^9
$m_\Phi, v\ , m_\sigma$	10^2
v_L	10^{-5}

Table 4.2: Mass scales of the model

considers non-thermal leptogenesis, then one can consider the alternative possibility of having a low value of v_R i.e. $\sim \mathcal{O}(10)$ TeV. Then all the mass scales and vevs are reduced by a couple of orders and could be accessible to colliders. The results are summarized in Table (4.2).

4.1.1 Effect of soft susy breaking terms

We introduce the soft susy breaking terms to check if they alter relations between various mass scales in the model. The soft susy breaking Lagrangian is

$$\begin{aligned}
-\mathcal{L}_{soft} &= M^2_{\tilde{Q}}\tilde{Q}^\dagger\tilde{Q} + M^2_{\tilde{Q}^c}\tilde{Q}^{c\dagger}\tilde{Q}^c + M^2_{\tilde{L}}\tilde{L}^\dagger\tilde{L} + M^2_{\tilde{L}^c}\tilde{L}^{c\dagger}\tilde{L}^c \\
&+ m^2_{\chi_L}\chi_L^\dagger\chi_L + m^2_{\chi_R}\chi_R^\dagger\chi_R + m^2_{\bar{\chi}_L}\bar{\chi}_L^\dagger\bar{\chi}_L + m^2_{\bar{\chi}_R}\bar{\chi}_R^\dagger\bar{\chi}_R \\
&+ m^2_{11}\Phi_1^\dagger\Phi_1 + m^2_{22}\Phi_2^\dagger\Phi_2 + (B_1\chi_L^T\tau_2\bar{\chi}_L + B_2\chi_R^T\tau_2\bar{\chi}_R + B\mu_{ij}\mathrm{Tr}\tau_2\Phi_i^T\tau_2\Phi_j \\
&+ C'_{\phi\chi}\chi_L^\dagger\Phi\chi_R + C''_{\phi\chi}\bar{\chi}_L^\dagger\Phi\bar{\chi}_R + D'_{\phi\chi}\chi_L^T\Phi\bar{\chi}_R + D''_{\phi\chi}\bar{\chi}_L^T\Phi\chi_R \\
&+ A_{q\phi}\tilde{Q}^T\tau_2\Phi_i\tau_2\tilde{Q}^c + A_{\ell\phi}\tilde{L}^T\tau_2\Phi_i\tau_2\tilde{L}^c + A_{\phi\chi}\chi_L^T\Phi_i\chi_R + A'_{\phi\chi}\bar{\chi}_L^T\Phi_i\bar{\chi}_R \\
&+ \text{Gaugino mass terms} + h.c.)
\end{aligned} \tag{4.18}$$

Where all the parameters are of the susy breaking scale which is $\sim$ TeV.

The Higgs part of the superpotential is

$$\begin{aligned}
W &= f\Phi\left(\bar{\chi}_L\chi_R + \chi_L\bar{\chi}_R\right) + m_\Phi\Phi\Phi \\
&\quad + m_\chi\left(\bar{\chi}_L\chi_L + \bar{\chi}_R\chi_R\right) \\
&\quad + m_\sigma\sigma^2 + \lambda\sigma(\bar{\chi}_L\chi_L - \bar{\chi}_R\chi_R).
\end{aligned}$$

We write the scalar potential as

$$V = |F|^2 + D^aD^a/2 + V_{soft}$$

where $D^a = -g(\phi_i^* T_{ij}^a \phi_j)$, g is gauge coupling constants, T^a is the generators of the corresponding gauge group and ϕ's are chiral superfields.

The F-term scalar potential is $V = |F_i|^2$ where $F_i = -\frac{\partial W}{\partial \phi}$. We denote the vev of the neutral components of Higgs fields as: $\langle \Phi \rangle = v$, $\langle \chi_L \rangle = \langle \bar{\chi}_L \rangle = v_L$, $\langle \chi_R \rangle = \langle \bar{\chi}_R \rangle = v_R$ and $\langle \sigma \rangle = s$.

Minimizing the scalar potential with respect to v_L, v_R, we get

$$\begin{aligned} \frac{\partial V}{\partial v_L} &= 2\, v_L\, \mu_L^2 + (8\, f\, v\, m_\Phi + 8\, f\, v\, m_\chi) v_R + 2\, (4\, f^2 - 2\, \lambda^2) v_L\, v_R^2 \\ &+ 4\lambda^2\, v_L^3 + (A_{\phi\chi} + C'_{\phi\chi} + D'_{\phi\chi}) v\, v_R = 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial V}{\partial v_R} &= 2\, v_R\, \mu_R^2 + (8\, f\, v\, m_\Phi + 8\, f\, v\, m_\chi) v_L + 2\, (4\, f^2 - 2\, \lambda^2) v_R\, v_L^2 \\ &+ 4\lambda^2\, v_R^3 + (A'_{\phi\chi} + C''_{\phi\chi} + D''_{\phi\chi}) v\, v_L = 0 \end{aligned} \tag{4.19}$$

where

$$\mu_L^2 = m_{\chi_L}^2 - 4\, \lambda\, m_\sigma s + (m_\chi - \lambda s)^2 + 2\, f^2\, v^2\, + B_1\, v_L^2$$

One need some fine tuning to get the value of μ_L from the above relation. If one take λ to be order of one, then allow $m_\chi - \lambda s \sim 10^9$ GeV by appropriate fine tuning. Hence this contribution cancels with the term $-4\, \lambda m_\sigma\, s$ giving μ_L a value of TeV range. From eqn.(4.19), it is clear that $\mu_R = v_R$ as the only relevant dominant terms are $2\, \lambda^2\, v_R^2$. If we take $v_R = 10^9$ GeV, then the value of μ_R is also 10^9 GeV.

From the minimization condition $v_R \frac{\partial V}{\partial v_L} - v_L \frac{\partial V}{\partial v_R}$, we get the relation

$$v_L v_R = \frac{8\, f\, v\, m_\chi\, v_R^2}{4\, \lambda\, m_\chi\, s + 4\, \lambda\, m_\sigma\, s} \tag{4.20}$$

Here we have taken the approximation: $v << v_R << m_\chi, s$. The scales in our model are $s = m_\chi = 10^{16}$ GeV and $v_R = 10^9$ GeV. From the above relation, putting these values we can have VEV of $v_L = 10^{-5}$ GeV. It is clear that the scale of v_L and v_R are consistent with the model which we derive from the minimization condition of the scalar potential. Thus adding the soft terms do not alter the relations between various mass scales of the theory.

For such a choice of scales we can fine tune the parameters to get a light χ_L having mass $\mu_L \sim$ TeV and a heavy χ_R having mass $\mu_R \sim v_R \sim 10^9$ GeV, $M_U \sim 10^{16}$ GeV. This will be important in the renormalization group evolution of the gauge couplings as we will see later.

4.2 Neutrino Mass

In LR models with only doublet scalar fields, the question of neutrino masses and leptogenesis has been discussed in details. We shall try to restrict ourselves as close as possible to these existing non-supersymmetric models, and check the consistency of these solutions when parity is broken in the present SUSYLR model. We shall first discuss the scenario with conserved D-parity, but since LR symmetry cannot be broken without breaking D-parity we shall discuss the D-parity breaking scenario afterwards.

In conventional type I seesaw, neutrino mass can be realized via three right handed neutrinos N_i^c where we have Majorana mass term $(M_R)_{ij}N_i^cN_j^c$ and Dirac masses with the ordinary neutrinos $(M_N)_{ij}\nu_i N_j^c = (Y_N)_{ij}\nu_i N_j^c\langle\Phi\rangle$. After diagonalizing, the resulting neutrino mass is $M_\nu^I = -M_N\, M_R^{-1}\, M_N^T$. Type II seesaw requires a $SU(2)_L$ triplet Higgs field T with mass of order m_T. Integrating out the Higgs triplet T leads to an mass operator $(M_T)_{ij}\nu_i\nu_j$ with $M_T \propto \frac{Y_T\langle\Phi\rangle^2}{m_T} \sim \frac{v^2}{M_G}$. Combination of these neutrino mass are also possible in left-right models which contains both type I and type-II or, type I and type III [60, 120].

In type III neutrino mass [135] three hypercharge neutral fermionic triplets Σ^a $(a = 1, 2, 3)$ are added to explain the ν mass term. In our model, however, we have an extra fermionic superfield which give rise ν mass term which is similar to the conventional type III seesaw mechanism. Thus, it is in this spirit that we can call the seesaw mechanism in our model as type III seesaw. For the review of the standard type III seesaw mechanism we closely follow [153].

Along with the Dirac neutrino mass term $(M_N)_{ij}\nu_i N_j^c$, the relevant superpotential for ν mass term, which is due to the extra fermion singlet (S) is given by,

$$W = M_{ij}S_iS_j + F_{ij}l_{Li}S_j\chi_L + F'_{ij}l_{Ri}S_j\chi_R, \tag{4.21}$$

From the above superpotential one can see that the vev of the left-handed doublet Higgs field which acquires a low scale vev $\langle\chi L\rangle = v_L$ directly couples the left-handed $\nu_i's$ with the singlet S_i. The mass matrix for the neutral leptons has the form,

$$W_{\rm neut} = (\nu_i \quad N_i^c \quad S_i) \begin{pmatrix} 0 & (M_N)_{ij} & F_{ij}v_L \\ (M_N)_{ji} & 0 & F_{ij}v_R \\ F_{ji}v_L & F_{ji}v_R & \mathcal{M}_{ij} \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j^c \\ S_j \end{pmatrix}. \tag{4.22}$$

In the above mass matrix, the mass of the singlet $\mathcal{M}_{ij}$ and the vev of the right-handed Higgs doublet v_R are heavy, while M_N and vev of the left-handed Higgs doublet v_L are of low scale.

Since in our model we have more than one left-handed Higgs doublet $(\chi_L, \bar{\chi}_R)$, the ν mass is given by,

$$M_\nu = -M_N M_R^{-1} M_N^T - (M_N H + H^T M_N^T)\left(\frac{v_L}{v_R}\right), \tag{4.23}$$

$$\text{where,} \quad H \equiv \left(F' \cdot F^{-1}\right)^T, \tag{4.24}$$

$$M_R = (F\, v_R)\mathcal{M}^{-1}(F^T v_R). \tag{4.25}$$

The first term in eqn (4.23) is the type I seesaw contribution and the second term gives the type III seesaw contribution. Type III contribution to ν mass will dominate over type I if the elements of the matrix $\mathcal{M}_{ij}$ are small compared to the contribution of H term.

We will partly follow the formalism and parametrization used in [153, 154] where the elements of the Dirac mass matrix are $M_{N11} = \eta v$, $M_{N33} = v$, $M_{N23} = -M_{N32} = v\epsilon$ and else are zero. Here $\eta = 0.6 \times 10^{-5}$ and $\epsilon \sim 0.14$.

If the elements of F_{ij} and F'_{ij} are considered to be of the order of f, a dimensionless parameter then from eqn. (4.24) we find that $H_{ij} \sim 1$ $(i, j = 1, 2, 3)$. Thus, the ν mass resulting from eqn (4.23) is

$$M_\nu = \begin{pmatrix} \eta & \epsilon & 1 \\ \epsilon & \epsilon & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{v\, v_L}{v_R} \tag{4.26}$$

The neutrino mass as presented above mostly satisfy the observed neutrino mass with a minor fine tuning in the 13 element.

Another set of parameters can be chosen to explain both neutrino mass and leptogenesis where both F_{ij} and F'_{ij} take the form [153]

$$F, F' \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix},$$

where $\lambda \sim \eta/\epsilon$. With this form of F, F' we have from eqns (4.23) and (4.24),

$$H \sim \begin{pmatrix} 1 & \epsilon/\eta & \epsilon/\eta \\ \eta/\epsilon & 1 & 1 \\ \eta/\epsilon & 1 & 1 \end{pmatrix},$$

and

$$M_\nu \sim \begin{pmatrix} \eta & \epsilon & \epsilon \\ \epsilon & \epsilon & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \frac{v\, v_L}{v_R}.$$

For the study of leptogenesis, a diagonal F_{ij} would suffice better. The parameters in this new basis would be represented via a tilde. The right-handed neutrino and the singlet has to be transformed via a unitary transformation to attain the diagonal basis as such $N_i^c = U_{ij}\tilde{N}_j^c$ and $S_i = V_{ij}\tilde{S}_j$. To attain the diagonal form of F_{ij} the unitary matrix U_{ij} can have the form

$$U = \begin{pmatrix} u_{11} & \lambda u_{12} & \lambda u_{13} \\ \lambda u_{21} & u_{22} & u_{23} \\ \lambda u_{31} & u_{32} & u_{33} \end{pmatrix}$$

with V_{ij} having a similar form. Here the u_{ij} elements are of $\mathcal{O}(1)$. For simplicity and numerical computation we will use the particular form of the unitary matrix which is

$$U = \begin{pmatrix} 1 & -\lambda(1+\sqrt{2})i & \lambda \\ -\lambda(1+\sqrt{2})i & 1/\sqrt{2} & i/\sqrt{2} \\ \lambda & i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}.$$

The elements of the diagonalized matrix $\tilde{F}_{ij} v_R = (U_{ki} F_{k\ell} V_{\ell j}) v_R$ can be written

$$\tilde{F} v_R = \mathrm{diag}[\lambda^2 F_1, F_2, F_3] v_R \equiv \mathrm{diag}[M_1, M_2, M_3],$$

where $F_i \sim 1$. In this basis the matrices $\tilde{F}'_{ij}u$ and $\tilde{\mathcal{M}}_{ij}$ can be parametrized as

$$\begin{aligned} \tilde{F}'u &= \begin{pmatrix} \lambda^2 f_{11} & \lambda f_{12} & \lambda f_{13} \\ \lambda f_{21} & f_{22} & f_{23} \\ \lambda f_{31} & f_{32} & f_{33} \end{pmatrix} v, \\ \tilde{\mathcal{M}} &= \begin{pmatrix} \lambda^2 g_{11} & \lambda g_{12} & \lambda g_{13} \\ \lambda g_{21} & g_{22} & g_{23} \\ \lambda g_{31} & g_{32} & g_{33} \end{pmatrix} M_S, \end{aligned} \tag{4.27}$$

where, $f_{ij}, g_{ij} \sim 1$. The assumption here is that the scale of $M_S \ll v_R$. In the new basis, the Dirac neutrino mass matrix M_N transforms as $\tilde{M}_N = M_N U$ and the form of the transformed matrix is

$$\tilde{M}_N \cong \begin{pmatrix} \eta u_{11} & \eta\lambda u_{12} & \eta\lambda u_{13} \\ \epsilon\lambda u_{31} & \epsilon u_{32} & \epsilon u_{33} \\ \lambda u_{31} & u_{32} & u_{33} \end{pmatrix} v \equiv \tilde{Y} v. \tag{4.28}$$

After doing all the parametrization, the type III seesaw contribution to the light neutrino mass matrix (which dominates, since $M_S \ll v_R$) from eqn (4.23) is given by,

$$M_\nu \cong - \begin{bmatrix} 2\eta \left(\frac{u_{11} f_{11}}{F_1}\right) & \frac{\eta}{\lambda}\left(\frac{u_{11} f_{21}}{F_1}\right) & \frac{\eta}{\lambda}\left(\frac{u_{11} f_{31}}{F_1}\right) \\ \frac{\eta}{\lambda}\left(\frac{u_{11} f_{21}}{F_1}\right) & 2\epsilon \sum_j \left(\frac{u_{3j} f_{2j}}{F_j}\right) & \sum_j \left(\frac{u_{3j} f_{2j}}{F_j}\right) \\ \frac{\eta}{\lambda}\left(\frac{u_{11} f_{31}}{F_1}\right) & \sum_j \left(\frac{u_{3j} f_{2j}}{F_j}\right) & 2\sum_j \left(\frac{u_{3j} f_{3j}}{F_j}\right) \end{bmatrix} \left(\frac{v^2}{v_R}\right). \tag{4.29}$$

Now we discuss the leptogenesis scenario in the given form of the neutrino matrix M_N, $\mathcal{M}$, M_S and U [153, 154]. Consider the case where the six super heavy two-component neutrinos have the mass matrix

$$(\tilde{N}_i^c, \tilde{S}_i) \begin{pmatrix} 0 & M_i \delta_{ij} \\ M_i \delta_{ij} & \tilde{\mathcal{M}}_{ij} \end{pmatrix} \begin{pmatrix} \tilde{N}_j^c \\ \tilde{S}_j \end{pmatrix},$$

where, $\tilde{\mathcal{M}}_{ij}$ is given in eqn (4.27). The leptogenesis can be realized by the decays of the lightest pair of these super heavy neutrinos, which have effectively the 2×2 mass matrix

$$(\tilde{N}_1^c, \tilde{S}_1) \begin{pmatrix} 0 & M_1 \\ M_1 & \tilde{\mathcal{M}}_{11} \end{pmatrix} \begin{pmatrix} \tilde{N}_1^c \\ \tilde{S}_1 \end{pmatrix} = (\tilde{N}_1^c, \tilde{S}_1)\, \lambda^2 \begin{pmatrix} 0 & F_1 v_R \\ F_1 v_R & g_{11} M_S \end{pmatrix} \begin{pmatrix} \tilde{N}_1^c \\ \tilde{S}_1 \end{pmatrix}.$$

Consider the scenario where $M_S \ll v_R$, then this results an almost degenerate pseudo-Dirac pair or equivalently two Majorana neutrinos with nearly equal and opposite masses. These Majorana neutrinos are $N_\pm \cong (\tilde{N}_1^c \pm \tilde{S}_1)/\sqrt{2}$, with masses $M_\pm \cong \pm M_1 + \frac{1}{2}\tilde{\mathcal{M}}_{11} = \lambda^2(\pm F_1 v_R + \frac{1}{2}g_{11}M_S)$. These can decay into light neutrino plus Higgs via the term $Y_{i\pm}(N_\pm \nu_i)H$, where

$$Y_{i\pm} \cong (\tilde{Y}_{i1} \pm \tilde{F}'_{i1})/\sqrt{2} \mp \frac{\tilde{\mathcal{M}}_{11}}{4M_1}(\tilde{Y}_{i1} \mp \tilde{F}'_{i1})/\sqrt{2}. \tag{4.30}$$

Here $\tilde{Y}$ is the Dirac Yukawa coupling matrix given in eqn (4.28). It is straightforward to show that the lepton asymmetry produced by the decays of $N_\pm$ [153] is given by

$$\epsilon_1 = \frac{1}{4\pi}\frac{Im[\sum_j (Y_{j+}Y^*_{j-})]^2}{\sum_j [|Y_{j+}|^2 + |Y_{j-}|^2]} I(M_-^2/M_+^2), \tag{4.31}$$

where $f(M_{1+}^2/M_{1-}^2)$ comes from the absorptive part of the decay amplitude of $N_\pm$. This function is given by

$$I(x) = \sqrt{x}\left[\frac{1}{1-x} + 1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right]$$

Making use of eqns (4.30) and (4.31) one obtains

$$\begin{aligned}
\epsilon_1 &= \frac{1}{4\pi}\frac{\sum_j(|\tilde{Y}_{j1}|^2 - |\tilde{F}'_{j1}|^2)\mathrm{Im}(\sum_k \tilde{Y}^*_{k1}\tilde{F}'_{k1})}{\sum_j(|\tilde{Y}_{j1}|^2 + |\tilde{F}'_{j1}|^2)} f(M_{1+}^2/M_{1-}^2), \\
\text{or,} \qquad \epsilon_1 &\cong \frac{\lambda^2}{4\pi}\left[\frac{(|u_{31}|^2 - |f'_{31}|^2)\mathrm{Im}(u^*_{31}f'_{31})}{|u_{31}|^2 + |f'_{31}|^2 + |f'_{21}|^2}\right] f(M_{1+}^2/M_{1-}^2). \qquad (4.32)
\end{aligned}$$

The lepton asymmetry produced by the decay on lightest Majorana neutrino is partially diluted by the lepton number violating decay processes. This decay processes try to wash out the lepton asymmetry already produce before. This wash out factor is given by,

$$k(\tilde{m}_1) \sim 0.3\left(\frac{10^-3\,\mathrm{eV}}{\tilde{m}_1}\right)\left(\log\frac{\tilde{m}_1}{10^-3\,\mathrm{eV}}\right)^{-0.6}$$

The equilibrium mass of the neutrino is given by

$$\tilde{m}_1 \equiv \frac{8\pi v_u^2 \Gamma_{N1\pm}}{M^2_{N1_\pm}} \cong \lambda^2\frac{v_u^2}{M_1}(|u_{31}|^2 + |f'_{31}|^2 + |f'_{21}|^2).$$

Input	Case (III-1)	Case (III-2)	Case (III-3)	Case (III-4)
v_R (GeV)	2.7×10^{14}	2.7×10^{12}	8.8×10^{10}	9.8×10^{8}
F_1	1.0	10.	31	50
F_2	1.0	0.1	0.1	1.0
F_3	1.0	1.0	1.0	1.0
M_S(GeV)	4.3×10^{5}	430	43	10.0
f_{21}	-0.950 + 0.534i	-0.050 + 0.0534 i	-0.950 + 0.11 i	-0.01+0.01 i
f_{22}	-2.279 - 1.537i	-0.227 - 0.154i	-0.228 - 0.154i	-0.225+0.138 i
f_{23}	-0.194 + 1.523i	-0.194 + 1.523i	-0.193 + 0.573 i	-0.195 + 1.23 i
f_{31}	0.6+3.5 i	-0.012 + 0.385 i	-0.46 + 0.42 i	0.04 +0.04 i
f_{32}	-0.354i	-0.035i	-0.035i	0.023 i
f_{33}	0.354	0.354	0.354	0.523

Table 4.3: Type III seesaw and Leptogenesis results for four cases

Output	Case (III-1)	Case (III-2)	Case (III-3)	Case (III-4)
M_1 (GeV)	4.53×10^{5}	4.53×10^{3}	4.58×10^{3}	82.37
M_2 (GeV)	2.70×10^{14}	2.70×10^{12}	8.8×10^{10}	9.8×10^{8}
M_3 (GeV)	2.70×10^{14}	2.70×10^{12}	8.8×10^{10}	9.8×10^{8}
$(M_{1+} + M_{1-})/M_{1+}$	1.6×10^{-9}	1.59×10^{-10}	1.57×10^{-10}	4.08×10^{-9}
ϵ_1	-2.5×10^{-6}	-2.1×10^{-4}	-1.01×10^{-6}	-1.01×10^{-4}
$\tilde{m}_1$ (eV)	0.511	0.569	4.774	0.694
κ_1	5.1×10^{-4}	4.5×10^{-4}	4.5×10^{-5}	3.6×10^{-4}
η_B	1.11×10^{-10}	1.147×10^{-10}	3.911×10^{-10}	1.461×10^{-10}

Table 4.4: Type III seesaw results for four cases

4.2.1 Numerical Result

The lepton asymmetry produced per unit entropy, taking into account decays of Majorana neutrino and their washout factors, is given by

$$\begin{aligned} \frac{n_L}{s} &\cong \frac{k\,\epsilon_1}{s}\frac{g_N\,T^3}{\pi^2} \\ &\cong \frac{45}{2\,\pi^4}\frac{g_N}{g_*}k\,\epsilon_1 \end{aligned}$$

We have used the expression for entropy of the comoving volume, $s = \frac{2}{45}\,g_*\pi^2\,T^3$. Here $g_N = 2$ for Majorana spin degrees freedom and $g_* = 228.75$ is the relativistically spin degrees of freedom for supersymmetry.

The corresponding B-L asymmetry per unit entropy is just the negative of n_L/s, since baryon number is conserved in the right-handed Majorana neutrino decays.While $B-L$ is conserved by the electroweak interaction following those decays, the sphaleron processes violate $B+L$ conservation and convert the $B-L$ asymmetry into a baryon asymmetry.The baryon asymmetry for supersymmetric case is

$$\frac{n_B}{s} = -\frac{28}{79}\frac{n_L}{s}$$

With the entropy density $s = 7.04\ n_\gamma$ in terms of the photon density, the baryon asymmetry(η_B) of the Universe, defined by the ratio n_B of the net baryon number to the photon number, is given in terms of the lepton asymmetry(ϵ_1) and washout parameter (k) by

$$\eta_B = \frac{n_B}{n_\gamma} \cong -0.039\, k\, \epsilon_1. \tag{4.33}$$

Successful Leptogensis will require that the final result for η_B should be order of 10^{10}. where $\lambda = \eta/\epsilon = 4.1 \times 10^{-5}$ as before.

The input parameter given in the table (4.3) which will determine the small neutrino mass, leptogenesis parameter as output given in the table (4.4) of our model.

4.3 Gauge coupling unification

Grand Unified Theories (GUTs) offer the possibility of unifying the three gauge groups viz., $SU(3)$, $SU(2)$ and $U(1)$ of the standard model into one large group at a high energy scale M_U. This scale is determined as the intersection point of the $SU(3)$, $SU(2)$ and $U(1)$ couplings. The particle content of the theory completely determines the dependence of the couplings with energy. Given the particle content of the theory one can evolve the couplings, determined at low energies, to determine whether there is unification or not.

In this section we will discuss how one can obtain $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}(g_L = g_R)(\cong G_{2213})$ intermediate gauge symmetry in R-parity conserving supersymmetric grand unified theory through one-loop unification of gauge couplings. Suppose we want to evolve coupling parameter between the scales M_1 and M_2 (i.e, $M_1 \leq \mu \leq M_2$) corresponding to the two scales of physics, then the RGE's depend on the gauge symmetry and particle content at $\mu = M_1$. For this purpose, we consider the two step breaking of the group G to the minimal supersymmetric standard model (MSSM) through G_{3221}

intermediate gauge symmetry in the so called minimal grand unified theory.

$$
\begin{array}{lll}
G \stackrel{M_U}{\rightarrow} & SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} & [G_{3221}] \\
\stackrel{M_R}{\rightarrow} & SU(3)_c \times SU(2)_L \times U(1)_Y & [G_{321}] \\
\stackrel{M_W}{\rightarrow} & SU(3)_c \times U(1)_Q & [G_{em}].
\end{array}
$$

4.3.1 RGE for SUSYLR model with doublet Higgs

The couplings evolve according to their respective beta functions. The renormalization group equations(RGEs) for this model cane be written as

$$\frac{d\alpha_i}{dt} = \alpha_i^2[b_i + \alpha_j b_{ij} + O(\alpha^2)] \tag{4.34}$$

where, $t = 2\pi\, ln(\mu)$. The indices $i, j = 1, 2, 3$ refer to the gauge group $U(1)$, $SU(2)$ and $SU(3)$ respectively.

Unlike the D-parity breaking case where the intermediate left-right gauge group has four different coupling constants as discussed in [147], in the present case G_{3221} has only three gauge couplings, $g_{2L} = g_{2R}$, g_{3C} , and g_{BL} for $\mu \geq M_R$. We now write down the RG evolution equation of gauge couplings upto one loop order which are given below

$$
\begin{aligned}
\frac{1}{\alpha_Y(M_Z)} &= \frac{1}{\alpha_G} + \frac{a_Y}{2\pi}\ln\frac{M_R}{M_Z} + \frac{1}{10\pi}\left(3a'_{2L} + 2a'_{BL}\right)\ln\frac{M_U}{M_R}, \\
\frac{1}{\alpha_{2L}(M_Z)} &= \frac{1}{\alpha_G} + \frac{a_{2L}}{2\pi}\ln\frac{M_R}{M_Z} + \frac{a'_{2L}}{2\pi}\ln\frac{M_U}{M_R}, \\
\frac{1}{\alpha_{3C}(M_Z)} &= \frac{1}{\alpha_G} + \frac{a_{3C}}{2\pi}\ln\frac{M_R}{M_Z} + \frac{a'_{3C}}{2\pi}\ln\frac{M_U}{M_R}.
\end{aligned} \tag{4.35}
$$

where $\alpha_G = g_G^2/4\pi$ is the GUT fine-structure constant and the beta function coefficients a_i and a'_i are determined by the particle spectrum in the ranges from M_Z to M_R, and from M_R to M_U, respectively.

Here we are using PDG values, $\alpha(M_Z) = 127.9$, $\sin^2\theta_W(M_Z) = 0.2312$, and $\alpha_{3C}(M_Z) = 0.1187$ [156]. Consider the case where $SU(2)_R \times U(1)_{B-L}$ breaks down to $U(1)_Y$. In that case

$$\frac{Y}{2} = I_{3,R} + \frac{B-L}{2} \tag{4.36}$$

The normalized generators are $I_Y = (\frac{3}{5})^{1/2}\frac{Y}{2}$ and $I_{B-L} = (\frac{3}{2})^{1/2}\frac{B-L}{2}$. Using these, one can write

$$I_Y = \sqrt{\frac{3}{5}}I_{3,R} + \sqrt{\frac{2}{5}}I_{B-L} \tag{4.37}$$

Which implies that the matching of the coupling constant at the scale where the left-right symmetry begins to manifest itself is given by

$$\alpha_Y^{-1} = \frac{3}{5}\alpha_{2R}^{-1} + \frac{2}{5}\alpha_{B-L}^{-1} \tag{4.38}$$

At the scale $\mu = M_Z - M_R$, the values of beta coefficients are: $b_Y = 33/5$, $b_{2L} = 1$, $b_{3C} = -3$. Similarly, at the scale $\mu = M_R - M_U$, $b'_{BL} = 16$, $b'_{2L} = b'_{2R} = 4$ $b'_{3C} = b_{3C} = -3$. With these parameters, the evolution of gauge couplings is shown in fig:(4.1).

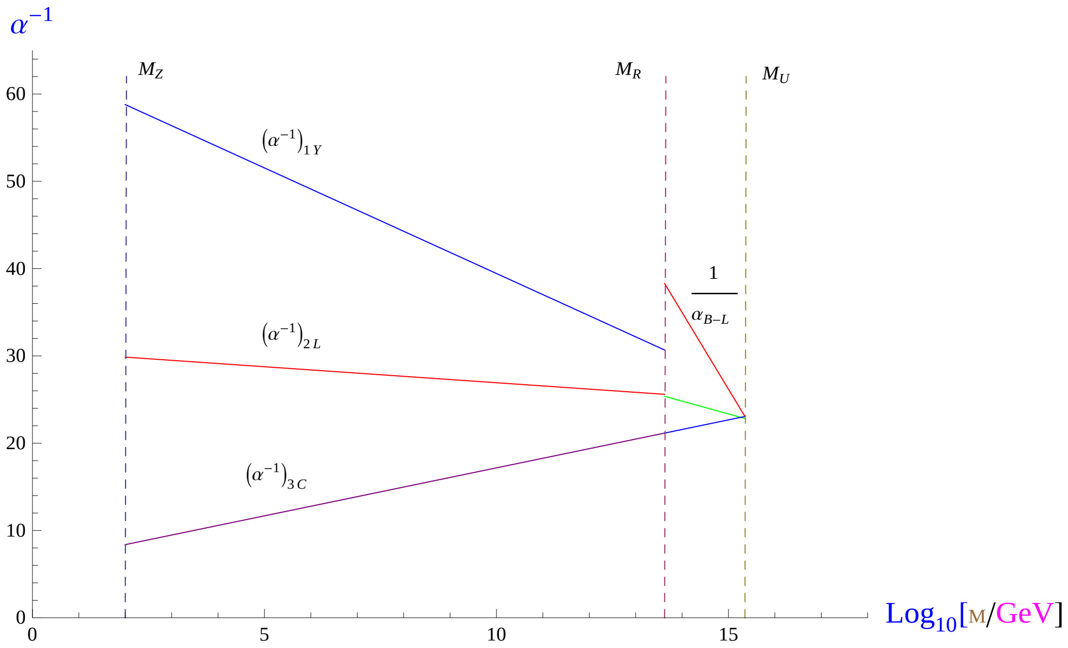


Figure 4.1: Evolution of coupling constants in susylr model with Higgs doublet. The $M_R = 10^{13}$ GeV and Unification scale $M_U = 0.67 \times 10^{16}$ GeV.

This will change once we add contributions coming from extra particle added to the minimal supersymmetric model. Once we fix the values of beta functions, we can achieve lower values of M_R. There are discussion [149, 159, 160], where the Unification is possible at the same energy scale around 10^{16} GeV, but the scale of M_R varies from 10^9 - 10^{12} GeV. We have considered here one scenario where the $M_R = 10^9$ GeV, $M_U = 10^{16}$ GeV and the effect of D-parity breaking is included. This is possible in our model by adding three copies of singlets charged under $U(1)_{B-L}$ which is shown in figure (4.1) to the minimal particle content.

4.3.2 Result

We consider the minimal particle content of SUSYLR model and found that unification is not possible for low scales of M_R. We have given the unification plot shown in figure (4.1)

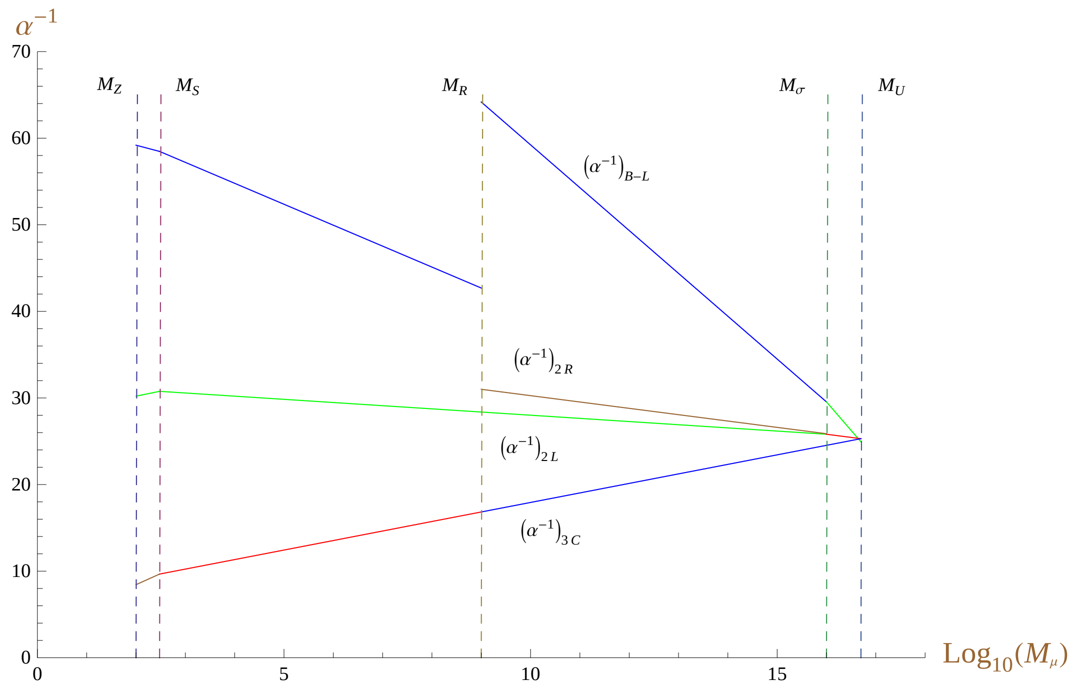


Figure 4.2: Unification plot for SUSYLR model with Higgs doublet+ three copies of singlets charged under the $U(1)_{B-L}$ gauge group. The value of M_R is 10^9 GeV and the unification scale is 5.3×10^{16} GeV.

for higher values of M_R without taking into account the D-parity breaking effect. It is clear from the figure (4.1) that the gauge couplings unify at a scale 0.67×10^{16} GeV. Also the right handed scale M_R is found to be 2.69×10^{13} GeV in our model without including the effect of D-parity. But spontaneous D-parity breaking changes the result and makes $M_R = 10^9$ GeV or even lower for certain choices of parameters as shown in figure (4.2) though three singlet scalar charged under $B - L$ gauge group added to the model. There are models [139,147] where one can achieve unification of all three fundamental interactions in which D-parity is broken at the GUT level. In this work, we have demonstrated that one can achieve unification including D-parity breaking effect and scale of μ_L and μ_R can be low. This result has been found from minimization of the scalar potential of our model including SUSY-breaking effect and also low scale μ_L and μ_R is possible from gauge coupling unification.

CHAPTER

5

Gravity Correction in SU(5) gauge coupling constants

5.1 Introduction

The question of gravitational corrections to the evolution of the gauge coupling constant has attracted some attention in recent times, following the seminal paper of Robinson and Wilczek [163]. They studied the one-loop quantum corrections to the running of the gauge couplings in an effective quantum theory of gravity, which is valid at energies below the Planck scale and found a quadratic divergent behavior. The character of the correction has been arrived at from a general consideration, which has been shown to have important phenomenological consequences in theories with low scale gravity [164]. However, this result has been questioned by some authors and the result has been studied from different approaches. This gravitational correction has been shown to depend on the choice of gauge in an explicit calculation [165]. They studied the abelian theory and used a parameter dependent gauge to arrive at their result. Subsequently a more general result has been obtained using a gauge invariant background field method that the gravitational corrections to the gauge couplings vanishes [166]. Following the doubts raised by these two references on the result of ref. [163], a one-loop diagrammatical calculation has been performed in the full Einstein-Yang-Mills system, which had also confirmed the vanishing of the one-loop contributions of quantum gravity to the gauge coupling evolution [167].

The quantum gravity corrections to the running of gauge couplings were calculated for pure Einstein-Yang-Mills system. It is not clear, however, if there is a spontaneously broken symmetry (let us say in $SU(5)$ grand unified theory) with the scalar field then the results of ref. [163] will remain valid. Recently the gravitational corrections to the gauge coupling evolution has been studied including a cosmological constant and quantum gravity effect has been found to affect the running of the gauge couplings [168]. However, the one-loop contributions in the presence of a cosmological constant differs from that of ref. [163], which was obtained from a general consideration. This raises the question: what are the other factors that would make the quantum gravity effects significant?

In this work, we argue from a phenomenological approach that the quantum gravity effects should be significant when higher dimensional non-renormalizable interactions are taken into consideration. Since quantizing the general theory of relativity for small fluctuations around flat space gives us a non-renormalizable field theory, we need to include an infinite set of higher dimensional counterterms. Since these terms are suppressed by appropriate powers of the Planck mass $M_p \sim 10^{19}$ GeV, at energies well below the Planck scale these higher dimensional terms may be considered as small perturbations in the effective theory of quantum gravity [169]. However, at the scale of grand unification these terms may not be ignored, and hence, in some version of the grand unified theories dimension-5 and dimension-6 gauge invariant terms have been included on phenomenological ground to see if these terms can change any of the conclusions for some reasonable values of the coupling constants [170]. It was found that although the minimal SU(5) grand unified theory fails to satisfy the gauge coupling unification, inclusion of the higher dimensional terms change the boundary conditions and allow gauge coupling unification at a higher scale [170, 171]. Here we point out that if the gravitational contributions to the gauge coupling evolution vanish, then the boundary conditions appearing due to the higher dimensional terms become inconsistent. We then show how the gauge coupling constants evolve from low energy to the GUT scale and satisfy the non-renormalizable operator induced matching condition at the new GUT scale, if we include gravitational corrections to the gauge couplings, which diverge quadratically near the Planck scale.

5.2 Effect of higher dimensional operators in SU(5) unification

Most of the grand unified theories (GUTs) with intermediate symmetry breaking scales can satisfy the experimentally observed constraints on proton lifetime (τ_p) for the p $\rightarrow e^+\pi^0$ mode and the electroweak mixing angle $\sin^2\theta_w$

$$\tau_p \geq 3 \times 10^{32}\ \text{yr}, \qquad \sin^2\theta_w = 0.230 \pm 0.005\,.$$

The minimal SU(5) and other GUTs with no intermediate symmetry breaking scale and no new particles beyond the minimal representations are ruled out as they predict significantly lower values. In other words, with the present range for the $\sin^2\theta_w$, if we evolve the three gauge coupling constants from the electroweak scale to the grand unification scale, they do not meet at a point, and hence, there is no unification. In an interesting proposal it was pointed out that since the grand unification occurs at a scale $M_U \geq 10^{15}$ GeV), which is close to the Planck scale, it is natural to expect that there could be significant modification to the GUT predictions by gravity-induced corrections [170]. These corrections may allow gauge coupling unification, make proton stable, give correct neutrino masses and proper charged fermion mass relations at the GUT scale, even for the minimal SU(5) GUT. In this article we include the higher dimensional terms to study the gauge coupling unification and infer that the evolution of the gauge coupling constants should be modified by the gravitational corrections.

We start with the $SU(5)$ Lagrangian and then the breaking of $SU(5)$ group into the Standard Model group $SU(3)_C \times SU(2)_L \times U(1)_Y$ via the Higgs field ϕ, which transforms under the 24-dimensional adjoint representation of $SU(5)$. We write down the Lagrangian as a combination of the usual four dimensional terms plus the new higher dimensional terms which has been induced by the non-renormalizable interactions of perturbative quantum gravity. Since the couplings of these terms are not known, we cannot make any predictions at this stage, so we look for consistent solutions for a reasonable range of the unknown parameters. The SU(5) gauge invariant Lagrangian, including higher dimensional terms can be written as

$$L = L_0 + \Sigma_{n=1} L^{(n)} \tag{5.1}$$

where

$$L_0 = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \tag{5.2}$$

Where the sum is over the higher dimensional operators. For the present we shall restrict ourselves to only five- and six-dimensional operators, which are:

$$L^{(1)} = -\frac{1}{2}\frac{\eta^{(1)}}{M_{Pl}}\mathrm{Tr}(F_{\mu\nu}\phi F^{\mu\nu}) \tag{5.3}$$

$$\begin{aligned} L^{(2)} &= -\frac{1}{2}\frac{1}{M_{Pl}^2}\left[\eta_a^{(2)}\mathrm{Tr}(F_{\mu\nu}\phi^2F^{\mu\nu}) + \mathrm{Tr}(F_{\mu\nu}\phi F^{\mu\nu}\phi)\right. \\ &\left. +\eta_b^{(2)}\mathrm{Tr}(\phi^2)\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \eta_c^{(3)}\mathrm{Tr}(F_{\mu\nu}\phi)\mathrm{Tr}(F^{\mu\nu}\phi)\right] \end{aligned} \quad (5.4)$$

where

$$F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\,[A_\mu, A_\nu] \quad (5.5)$$

$$(A_\mu)^a{}_b = A_\mu^i \left[\frac{\lambda_i}{2}\right]_b^a \quad (5.6)$$

and

$$\mathrm{Tr}\,(\lambda_i\lambda_j) = \frac{1}{2}\delta_{ij}\,. \quad (5.7)$$

Here A^i is the ith component of the gauge field, λ_i is the corresponding generator and η^n, n=1,2,... are the unknown parameters, induced by gravitational corrections.

When the scalar ϕ acquires a vacuum expectation value (*vev*) and breaks the SU(5) symmetry at the GUT scale, we may replace these fields in the above expressions by its *vev*. This will give us the effective low energy theory with only dimension-4 interactions, but the effective gauge fields will be modified below the GUT scale. We may define the new physical gauge fields below the unification scale to be

$$A_i' = A_i(1+\varepsilon_i)^{1/2} \quad (5.8)$$

and the modified coupling constants including the higher dimensional operators as

$$\widetilde{g}_3^2(M_U) = {g_3}^2(M_U)(1+\varepsilon_C)^{-1} \quad (5.9)$$

$$\widetilde{g}_3^2(M_U) = {g_2}^2(M_U)(1+\varepsilon_L)^{-1} \quad (5.10)$$

$$\widetilde{g}_1^2(M_U) = {g_1}^2(M_U)(1+\varepsilon_Y)^{-1} \quad (5.11)$$

The g_i are the couplings in the absence of higher dimensional operators, whereas $\widetilde{g}_i$ are the physical couplings which evolve down to the lower scales. The value of the ε^n associated with the given operator of dimension n+4 may be expressed in the following way

$$\varepsilon^n = \left[\frac{1}{\sqrt{15}}\frac{\phi_0}{M_{Pl}}\right]^n \eta^{(n)} \quad (5.12)$$

The vev ϕ_0 is related to M_U

$$\phi_0 = \left[\frac{6}{5\pi\alpha_G}\right]^{1/2} M_U \tag{5.13}$$

The change in the coupling constants are then related to the ε^ns through the following equations

$$\varepsilon_C = \varepsilon^{(1)} + {\varepsilon_a}^{(2)} + \frac{15}{2}{\varepsilon_b}^{(2)} + ... \tag{5.14}$$

$$\varepsilon_L = -\frac{3}{2}\varepsilon^{(1)} + \frac{9}{4}{\varepsilon_a}^{(2)} + \frac{15}{2}{\varepsilon_b}^{(2)} + ... \tag{5.15}$$

$$\varepsilon_Y = -\frac{1}{2}\varepsilon^{(1)} + \frac{7}{4}{\varepsilon_a}^{(2)} + \frac{15}{4}{\varepsilon_b}^{(2)} + \frac{7}{8}{\varepsilon_c}^{(2)} + ... \tag{5.16}$$

This shows how the effect of higher dimensional operator modify the gauge coupling constants. The Unification scale, M_U, is now defined through the new boundary condition

$${g_3}^2(1+\varepsilon_C) = {g_2}^2(1+\varepsilon_L) = {g_1}^2(1+\varepsilon_Y) = {g_0}^2\,. \tag{5.17}$$

With this in mind, one may use the standard one loop renormalization group (RG) equations

$${\alpha_i}^{-1}(M_z) = {\alpha_i}^{-1}(M_U) + \frac{b_i}{2\pi}\log\left(\frac{M_U}{M_z}\right) \tag{5.18}$$

with the beta functions $b_1 = \frac{41}{10}$,$b_2 = \frac{-19}{6}$,$b_3 = -7$. We have taken N_f=3 and N_{Higgs}=1.

Solving the RG equations without any higher dimensional contributions yield

$$\log\left(\frac{M_U}{M_z}\right) = \frac{6}{67\alpha}\frac{1}{D}\left[1 - \frac{8}{3}\frac{\alpha}{\alpha_s} + \varepsilon_C - \frac{5\varepsilon_Y + 3\varepsilon_L}{3}\frac{\alpha}{\alpha_s}\right] \tag{5.19}$$

$$\sin^2\theta_w = \frac{1}{D}\left[\sin^2{\theta_w}^{(5)} - \frac{19}{134}\varepsilon_C + \frac{1}{67}\left(21 + \frac{41}{2}\frac{\alpha}{\alpha_s}\right)\varepsilon_L + \frac{95}{402}\frac{\alpha}{\alpha_s}\varepsilon_Y\right] \tag{5.20}$$

$$\frac{1}{\alpha_G} = \frac{3}{67}\frac{1}{D}\left[\frac{11}{3\alpha_s} + \frac{7}{\alpha}\right] \tag{5.21}$$

$$D = 1 + \frac{1}{67}(11\varepsilon_C + 21\varepsilon_L + 35\varepsilon_Y) \tag{5.22}$$

Where the $\sin^2\theta_w{}^{(5)}$ is the usual minimal SU(5) prediction

$$\sin^2\theta_w{}^{(5)} = \frac{23}{134} + \frac{109}{201}\frac{\alpha}{\alpha_s} \tag{5.23}$$

In this case of minimal SU(5), the gauge coupling constants do not meet at a point, and hence, unification is not possible. We now show how this result gets modified by including higher dimensional terms.

We first consider only the following SU(5) invariant non-renormalizable (NR) (dimension five) interaction term

$$L_{NR} = -\frac{1}{2}\left(\frac{\eta}{M_{Pl}}\right)\mathrm{Tr}(F_{\mu\nu}\phi F^{\mu\nu})\,, \tag{5.24}$$

where ϕ_{24} is the Higgs 24-plet, η is a dimensionless parameter and M_{Pl} is the Planck mass. Suppose the Higgs field acquires a vacuum expectation value(vev)

$$\langle\phi\rangle = \frac{1}{\sqrt{15}}\phi_0\mathrm{diag}[1,1,1,-\frac{3}{2},-\frac{3}{2}] \tag{5.25}$$

The SU(5) gauge symmetry breaks to $SU(3)_C \times SU(2)_L \times U(1)_Y$ at this scale because of non-invariance of the Higgs field under the SU(5) symmetry. The presence of non-renormalizable couplings modifies the usual kinetic energy terms of the $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ gauge boson part of the low-energy Lagrangian. The modified Lagrangian becomes

$$-\frac{1}{2}(1+\varepsilon)\mathrm{Tr}(F_{\mu\nu}{}^{(3)}F^{\mu\nu(3)}) - \frac{1}{2}(1-\frac{3}{2}\varepsilon)\mathrm{Tr}(F_{\mu\nu}{}^{(2)}F^{\mu\nu(2)}) - \frac{1}{2}(1-\frac{1}{2}\varepsilon)\mathrm{Tr}(F_{\mu\nu}{}^{(1)}F^{\mu\nu(1)})\,, \tag{5.26}$$

where the superscripts 3,2 and 1 refer to gauge field strengths of $SU(3)$, $SU(2)$ and $U(1)$ respectively and ε is defined as

$$\varepsilon = \left[\frac{1}{\sqrt{15}}\frac{\phi_0}{M_{Pl}}\right]\eta\,. \tag{5.27}$$

We used $\varepsilon^{(2)} = \varepsilon^{(3)} = 0$ and $\varepsilon^{(1)} = \varepsilon = \eta\phi_0/(\sqrt{15}M_U)$, so that $\varepsilon_C = \varepsilon$, $\varepsilon_L = -\frac{3}{2}\varepsilon$, $\varepsilon_Y = -\frac{1}{2}\varepsilon$. Now, using these expressions, we get

$$\frac{1}{\alpha_G} = \frac{11\alpha_s{}^{-1} + 21\alpha^{-1}}{67 - 38\varepsilon}\,, \tag{5.28}$$

$$\log\left(\frac{M_U}{M_z}\right) = \frac{6\pi}{67-38\varepsilon}\left[\alpha^{-1} - \frac{8}{3}\alpha_s{}^{-1} + \left(\frac{7}{3}\alpha_s{}^{-1} + \alpha^{-1}\right)\varepsilon\right] \tag{5.29}$$

$$\sin^2\theta_w = \frac{1}{67-38\varepsilon}\left[\frac{23}{2} + \frac{109}{3}\frac{\alpha}{\alpha_s} - \left(41 + \frac{116}{3}\frac{\alpha}{\alpha_s}\right)\varepsilon\right] \tag{5.30}$$

Taking the experimental values of $\alpha_s = 0.1088$, $\alpha = 1/127.54$, it is possible to obtain a consistent choice of the parameters ε_C, ε_L, ε_Y which satisfy the constraints on $\sin^2\theta_w$ and M_U. But the unification scale remains low and the proton lifetime becomes less than the present experimental bound. For central value of $\sin^2\theta_w (= 0.2333)$, we obtain $\varepsilon^{(1)} = -0.0441$ and $M_U = 3.8 \times 10^{13}$ GeV and the corresponding value of $\alpha_G = 0.0245$. The lifetime of proton (m_p is the mass of the proton)

$$\tau_p = \frac{1}{\alpha_G{}^2}\frac{M_U{}^4}{m_p{}^5} \tag{5.31}$$

then becomes too low to be consistent with experimental limits on τ_p for the given value of M_U. Hence, it is not possible to obtain a consistent solution with the five Dimensional operator.

Table 5.1: Unification in SU(5) using gravity corrections

ϵ_C	ϵ_L	ϵ_Y	M_U
0.04	0.0675	0.24	10^{17} GeV
0.3894	0.44	0.98	10^{18} GeV
1.3894	1.445	1.98	$10^{18.6}$ GeV

If we now include both five and six dimensional terms, then there are whole range of parameters that are consistent with the values of $\sin^2\theta_w$, M_U and proton lifetime. We present a few representative set of values that are consistent with proton lifetime in table 5.1. So, from now on we shall consider both dimension five and dimension six non-renormalizable terms for our discussion.

5.3 Evolution of gauge couplings including gravitational contributions

In the last section we discussed the effect of higher dimensional non-renormalizable interaction on the boundary condition, satisfied by the gauge couplings. In fact, the effective gauge couplings get modified at the time of GUT phase transition, which allows the gauge coupling unification for some parameter range. If we now start evolving the gauge coupling constants from low energy, when the effects due to the higher dimensional terms are negligible, we should be able to reach the new modified boundary condition continuously. In other words, the modified effective gauge couplings should evolve with energy in such a way that at low energy they become the usual gauge couplings. If we now assume that the gravitational corrections to the evolution of the gauge couplings vanishes, then this transition is not possible. On the other hand, if we consider that the gravitational corrections are of the quadratic nature, as recommended in ref. [163], then it is possible to continuously evolve the gauge coupling constant from the modified effective coupling near the GUT phase transition scale to the low energy experimentally observed couplings.

In this section we shall first argue how the non-renormalizable interactions could change the gravitational corrections to the gauge couplings. Then we shall demonstrate how the gauge coupling constants evolve from low energy to the unification scale in the presence of the higher dimensional contributions. Although the modified boundary condition and its effect was studied by many authors, the running of the gauge couplings from low energy to the unification scale could not be studied. This is because the running of the gauge couplings in the presence of gravitational corrections were not considered.

As the gauge boson vertex has strength g and gravity couple to energy momentum with a dimensional coupling $\propto \frac{1}{M_{Pl}}$, dimensional analysis implies that the running of couplings in four dimensions will be governed by a Callan-Symanzik β function of the form

$$\beta(g, E) = \frac{dg}{dlnE} = -\frac{b_0}{4\pi^2} g^3 + a_0 \frac{E^2}{{M_{Pl}}^2} g \tag{5.32}$$

where the first term is the non-gravitational contribution and the 2nd term is the gravitational contribution, as suggested in ref. [163]. This quadratic gravitational correction was then revisited in ref. [165–167] and it was shown that this contribution vanishes. We shall now argue that in the presence of non-renormalizable interactions, this contribution may not vanish.

Following equations 8-11, we write down the effective coupling constant at the GUT scale as

$$\widetilde{g}^{-2} = g^{-2} + C, \tag{5.33}$$

where C is the contribution coming from the non-renormalizable interactions. We shall now argue that although the gauge coupling evolution may not be affected by gravitational corrections (as stated in refs. [165–167]), the evolution of C is dominated by gravitational correction, and hence, it should evolve as suggested in ref. [163].

In the absence of non-renormalizable interactions and gravitational corrections, the three gauge couplings for a particular model evolve as inverse logarithm of E at one loop order. Although unification may not be achieved in case of minimal SU(5), including non-renormalizable terms (i.e., including C) they may get unified at a scale $\approx 10^{17-18}$ GeV. In ref. [163], it was shown that in absence of C, the couplings are unified near the Planck scale and the value of the couplings are zero, as shown in figure 5.1. The negative value of a_0 in the beta function signifies that the gravitational correction works in the direction of asymptotic freedom, i.e. it causes coupling constants to decrease at high energy (above 10^{16} GeV).

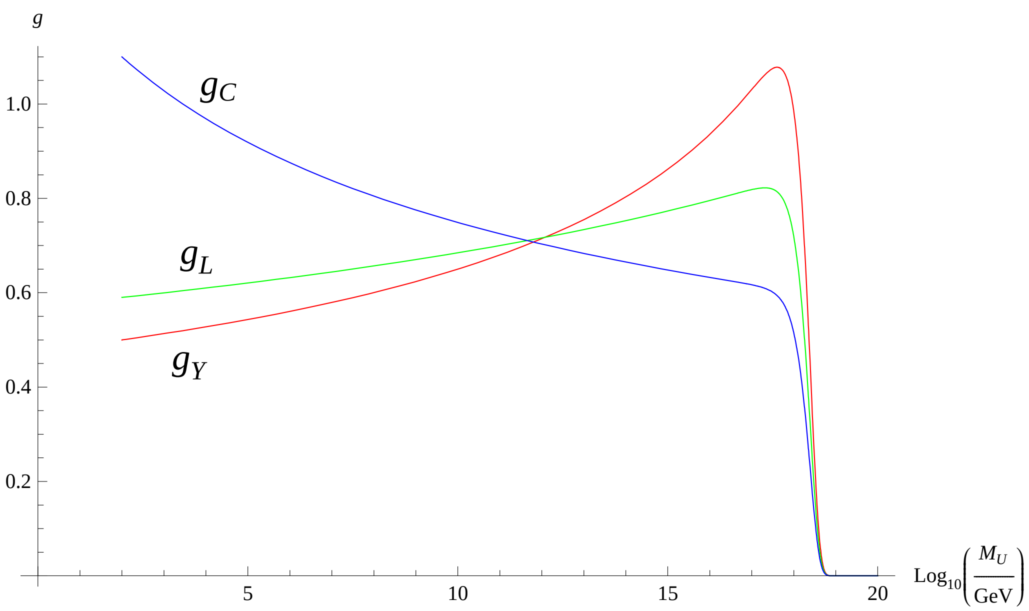


Figure 5.1: Evolution of the gauge coupling constants without higher dimensional terms, but including gravitational corrections [163].

The modifications to the gauge couplings arising due to non-renormalizable terms are symbolically denoted by C in equation 33. To comply with the unification condition described by equation 17, the correction of each of the three coupling constants will have different weights. This would give nonzero contribution to the coupling constants unlike in ref. [165–167]. One can justify this point as follows: For the purpose of a demonstration consider the diagramatic method of ref. [167]. Here one starts with the Einstein-Yang-Mills Lagrangian

$$\mathcal{L}_4 = \frac{2}{\kappa^2}\sqrt{-g}\mathbf{R} - \frac{1}{2}\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}\mathrm{Tr}[F_{\mu\nu}F_{\rho\sigma}]\,, \tag{5.34}$$

with the Ricci scalar $\mathbf{R}$. We then expand the metric in terms of the flat metric $\eta_{\mu\nu}$ and the graviton field $h_{\mu\nu}$ to write

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} - \kappa h_{\mu\nu} + \kappa^2 h_{\mu\beta} h^{\beta}_{\nu} \\ \sqrt{-g} &= 1 + \frac{\kappa}{2} h + \frac{\kappa}{8}\left(h^2 - 2h^{\alpha\beta}h_{\alpha\beta}\right) . \end{aligned} \tag{5.35}$$

It is then possible to write down the propagators for this theory and explicitly calculate the one-loop diagrams to show that the gravitational corrections to the β-functions vanish [165–167]. It should be noted that the term of type $\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}\mathrm{Tr}[F_{\mu\nu}F_{\rho\sigma}]$ (in equation 34) give contribution to the coupling constant that is quadratic in the energy [163].

If we now include the scalar fields Φ in the theory, there will be interactions of the scalar fields with the graviton field, which comes from the Lagrangian

$$\mathcal{L}_S = \sqrt{-g}[D_\mu \Phi D_\nu \Phi] g^{\mu\nu} . \tag{5.36}$$

In this case also there seem to be cancellation of the quadratic divergences (we considered the diagrams to order κ^2 for the abelian case only) and there may not be any gravitational corrections to the gauge coupling evolution.

However, the inclusion of higher dimemsional non-renormalizable terms would completely change the scenario. Such non-renormalizable terms are expected in a theory that incorporates the effect of quantum gravity. In any grand unified theory, where the unification scale is only 2-3 orders of magnitude lower than the Planck scale (the proliferation of particles near the GUT scale could also lower the Planck scale [172]), such non-renormalizable terms may contribute significantly. Consider, for example, the dimension-5 term in presence of the 24-plet scalar ϕ of SU(5)

$$\mathcal{L}_5 = -\frac{1}{2M_{Pl}}\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}\mathrm{Tr}[F_{\mu\nu}F_{\rho\sigma}\phi] . \tag{5.37}$$

For the case when $E \leq M_U$, the scalar ϕ acquires a *vev* ($\langle\phi\rangle \equiv M$ diag$[1,1,1,-3/2,-3/2]$), this term would give contribution to the C term in equation 33 that vary quadratically with the energy. However, to be consistent with the modified boundary condition given by equation 17, the different gauge fields with different weight factors will give nonzero contribution. It ought to be noted that the coupling constants now meet at $E \approx M_U$ which is lower than the Planck scale This supports our earlier inference that the gravitational corrections to the gauge couplings may not vanish when the higher dimensional interactions are included.

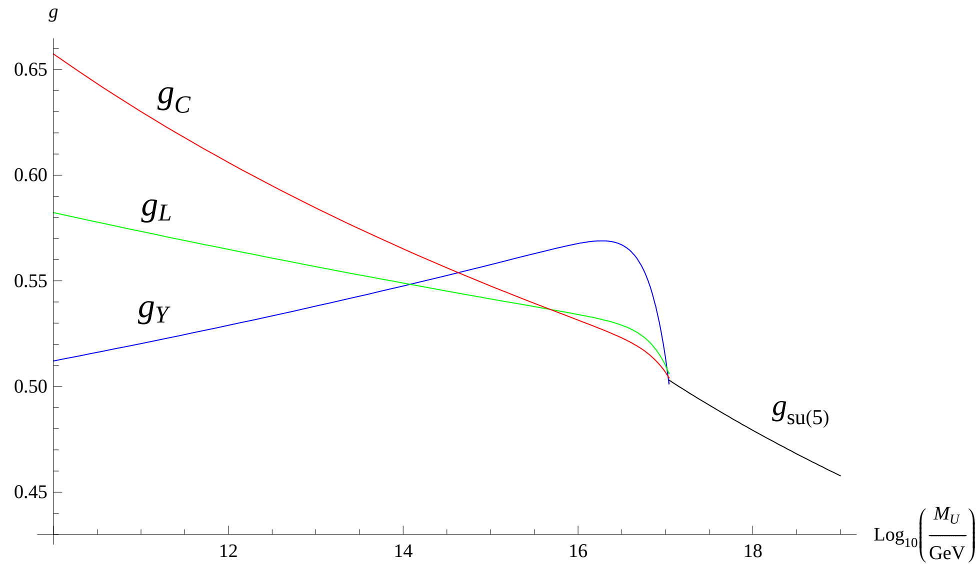


Figure 5.2: Evolution of the gauge coupling constants in the presence of higher dimensional terms and gravitational corrections.

Above the unification scale M_U, the scalar field has not acquired *vev* and SU(5) symmetery is exact. In this regime there will be only one gauge coupling constant for entire SU(5) and it will evolve without any gravitational corrections as if the higher dimensional terms were absent. It is shown in figure (5.2) that how the coupling constants vary with energy in the presence of C terms in the regime $E \leq M_U$. For the regime $E \geq M_U$, there is only one coupling constant as the exact SU(5) symmetry is restored.

The higher dimensional effective contributions has been studied in the literature, whereby the gauge coupling constants get modified near the grand unification scale. These modifications of the boundary conditions allow gauge coupling unification even for the minimal SU(5) GUT. However, the running of the modified gauge couplings have not been studied. We show that this modified gauge couplings should evolve including the gravitational corrections, otherwise the low energy gauge couplings may not be consistent with the modified boundary conditions. From this we infer that the gravitational corrections to the gauge couplings may not vanish when higher dimensional non-renormalizable interactions are included in the Einstein-Yang-Mills system.

Chapter

6

Electromagnetic leptogenesis

The recent neutrino experiments like solar and atmospheric oscillation experiment as well as long baseline accelerator and reactor neutrino experiments gives enough evidence in favor of the existence of non-zero neutrino masses and mixing, and this is also the evidence of new physics beyond the Standard Model (SM). While both could be admitted into the Standard Model (SM) by the simple expedient of adding right-handed neutrino fields (omitted, at the inception of the SM, only on account of the then apparent masslessness of the neutrinos), many theoretical challenges persist. Indeed, some authors have claimed neutrino masses to be the evidence of physics beyond the SM.The couplings of neutrinos with the photons are generic consequences of finite neutrino masses, and are one of the important intrinsic neutrino properties to explore. The study of neutrino EMDM can provide, in principle, a way to distinguish between Dirac and Majorana neutrinos since the Majorana neutrinos can only have flavor changing, transition magnetic moments while the Dirac neutrinos can only have flavor conserving one.

The seesaw mechanism and the associated mechanism of leptogenesis [37] are very attractive means to explain the origin of the small neutrino masses and the baryon asymmetry of the universe. Leptogenesis [37] provides an elegant mechanism to consistently address the observed Baryon Asymmetry in the Universe (BAU) [194] in minimal extensions of the Standard Model (SM) [195]. In standard leptogenesis, there exist heavy right handed neutrino of mass close to GUT scale 10^{15} GeV and it's out of equilibrium decay creates a net lepton asymmetry which get converted into the observed baryon asymmetry via the

$B + L$ violating sphaleron interactions [84, 196]. At the same time, the inclusion of right handed Majorana neutrino can explain the observed smallness of light neutrinos through the so-called seesaw mechanism [197].

Although the aforementioned scheme is theoretically very attractive, it suffers from the lack of direct detectability, e.g. at high-energy colliders, such as the LHC or ILC, or in any other foreseeable experiment. This has, naturally, led to efforts towards alternative routes to leptogenesis. A phenomenologically interesting solution to this problem may be obtained within the framework of resonant leptogenesis (RL) [88–91, 93, 95]. Characterized by the presence of two (or more) nearly degenerate heavy Majorana neutrinos, in such scenarios the corrections to the self-energies play a pivotal role in determining the lepton asymmetry [38]. Indeed, if the mass difference be comparable to their decay widths, the resonant enhancement could render asymmetries to be as large as $\mathcal{O}(1)$ [89, 91].

Recently a very interesting possibility of electromagnetic leptogenesis [193] has been proposed, wherein the source of CP violation has been identified with the electromagnetic dipole moment of the neutrinos. The general form of this dipole moment coupling of the light neutrinos, ν, to the heavy neutrinos, N, is given by $\overline{\nu_j}\,(\mu_{jk} + i\,\gamma_5\,\mathcal{D}_{jk})\sigma_{\alpha\beta}N_k B^{\alpha\beta}$, where μ_{jk} and $\mathcal{D}_{jk}$ are the magnetic and electric transition moments, respectively. The aforementioned dimension-five operators are, presumably, generated by some new physics operative beyond the electroweak scale. With CP-violation being encoded in the structure of the dipole moments, the decays of heavier neutrinos to lighter ones and a photon, can, in principle, lead to matter-antimatter asymmetry in the universe. Although the proposal is a very interesting one, so far it has not been incorporated in any realistic model. In this work, we propose a specific model for resonant electromagnetic leptogenesis. A guiding principle in our quest is that the new physics should be at the TeV scale so as to render the model testable at the LHC or future Linear Colliders.

To implement the idea of electromagnetic leptogenesis, one should first understand the electromagnetic interaction between the left-handed (LH) light neutrino ν and right-handed (RH) heavy neutrino N via the effective transition dipole moment operator and their cosmological implications. In the subsequent discussion , we will investigate whether the lepton number violating radiative decay of the heavy sterile neutrinos ($N \to \nu\gamma$) which can explain the baryon asymmetry, in analogy to the standard leptogenesis scenario where N-decays are mediated by the Yukawa couplings ($N \to \nu\overline{\phi}$). We will present a general properties of EMDM couplings and explicit calculation of the CP-asymmetry induced by the decays of N through such effective dipole moment operator.

6.1 Electromagnetic properties of light and heavy neutrinos

Let us understand the properties of the transition form factors μ_{jk} and $\mathcal{D}_{jk}$ in the generic dipole moment coupling between light (ν) and heavy (N) neutrinos:

$$\mathcal{L}_{\rm EM} = \overline{\nu}_j \left(\mu_{jk} + i\gamma_5 \mathcal{D}_{jk}\right) \sigma_{\alpha\beta}\, N_k\, \mathcal{F}^{\alpha\beta} + \text{h.c.} \tag{6.1}$$

where μ_{jk} is the transition magnetic moment, $\mathcal{D}_{jk}$ is the transition electric moment. The $\nu_j = e^{i\vartheta_j}\,\nu_j^c$ and $N_k = e^{i\varphi_k}\,N_k^c$ (j, k are the mass labels) are Majorana neutrino fields with masses m_j and M_k respectively, while $F^{\alpha\beta}$ denotes the photon field tensor as usual. Here ϑ_j and φ_k are the charge conjugation phase factors associated with the Majorana neutrinos. We use the definition: $\sigma_{\alpha\beta} = \frac{i}{2}\left[\gamma_\alpha, \gamma_\beta\right]$. Rewriting ν_j and N_k using the Majorana condition, we obtain

$$\begin{aligned}\mathcal{L}_{\rm EM} &\equiv \overline{\left(e^{i\vartheta_j}\,\nu_j^c\right)} \left(\mu_{jk} + i\gamma_5\mathcal{D}_{jk}\right) \sigma_{\alpha\beta}\, e^{i\varphi_k}\, N_k^c\, F^{\alpha\beta} + \text{h.c.} \\ &= -e^{-i(\vartheta_j - \varphi_k)} \nu_j^T C^{-1} \left(\mu_{jk} + i\gamma_5\mathcal{D}_{jk}\right) \sigma_{\alpha\beta}\, C\overline{N}_k^T\, F^{\alpha\beta} + \text{h.c.}\end{aligned} \tag{6.2}$$

where C is the charge conjugation operator with the following conventions:

$$\begin{gathered}\psi^c = C\overline{\psi}^T\,,\quad C^\dagger = C^{-1}\,,\quad C^T = -C\,,\quad C^\dagger C^T = C^* C = -I\,,\quad C^{-1}\gamma^5 C = (\gamma^5)^T\,, \\ C^{-1}\gamma^\mu C = (-\gamma^\mu)^T\,,\quad C^{-1}\sigma^{\mu\nu} C = (-\sigma^{\mu\nu})^T\,,\quad C^{-1} P_{R,L} C = (P_{R,L})^T\,,\end{gathered} \tag{6.3}$$

where $P_{R,L} \equiv (1 \pm \gamma^5)/2$. Taking the transpose of the first term in Eqn:(6.2) and using Eqn:(6.3) to simplify the expression, one eventually gets after some algebra

$$\mathcal{L}_{\rm EM} = -e^{-i(\vartheta_j - \varphi_k)} \overline{N}_k \left(\mu_{jk} + i\gamma_5 \mathcal{D}_{jk}^N\right) \sigma_{\alpha\beta}\, \nu_j\, F^{\alpha\beta} + \text{h.c.} \tag{6.4}$$

If we write out the h.c. term of equation (6.1) (which is $\overline{N}_k\,(\mu_{jk}^* + i\gamma_5\mathcal{D}_{jk}^*)\,\sigma_{\alpha\beta}\,\nu_j\,F^{\alpha\beta}$) and compare it with the first term in (6.4), we can conclude that

$$\mu_{jk} = -e^{i(\vartheta_j - \varphi_k)} \mu_{jk}^* \quad \text{ and } \quad \mathcal{D}_{jk} = -e^{i(\vartheta_j - \varphi_k)} \mathcal{D}_{jk}^*\,. \tag{6.5}$$

From this, we get

$$\mu_{jk}{}^2 = |\mu_{jk}|^2\, e^{i(\vartheta_j - \varphi_k + \pi)}\,, \tag{6.6}$$

$$\Rightarrow \quad \mu_{jk} = |\mu_{jk}|\, i\, e^{i(\vartheta_j - \varphi_k)/2}\,. \tag{6.7}$$

Similarly, we have the analogous expression for $\mathcal{D}_{jk}$. An important note on this is that although the relations between μ_{jk} and μ^*_{jk}, as well as $\mathcal{D}_{jk}$ and $\mathcal{D}^*_{jk}$ depends on the choice of the charge conjugation phase factor, once ϑ_j and φ_k are chosen, they are fixed. In particular, when $\vartheta_j = \varphi_k$, we have the situation where μ_{jk} and $\mathcal{D}_{jk}$ must be purely imaginary. Furthermore, it is worth mentioning that if Lagrangian (6.1) is CP invariant, then only one of μ_{jk} and $\mathcal{D}_{jk}$ survives. But in our work here we do not impose such condition and the only assumptions we shall make are Hermiticity and CPT invariance.

In calculations, it is often much simpler to consider the EMDM coupling between the associated chiral components of the ν and N (instead of using the form written in (6.4)) because the resultant Lagrangian contains only one type of electromagnetic dipole moment coupling rather than distinct magnetic (μ_{jk}) and electric ($\gamma^5\mathcal{D}_{jk}$) moment terms as $\gamma^5 P_{R,L} = \pm P_{R,L}$. Letting $\nu_j = \nu_{Lj} + e^{i\vartheta_j}\nu^c_{Lj}$ and $N_k = N_{Rk} + e^{i\varphi_k}N^c_{Rk}$ where ν_L and N_R are the usual LH and RH neutrino states, then (6.4) can be rewritten into

$$\begin{aligned}
\mathcal{L}_{\rm EM} &= \overline{\nu}_{Lj}\,(\mu_{jk} + i\mathcal{D}_{jk})\,\sigma_{\alpha\beta}\,N_{Rk}\,F^{\alpha\beta} + e^{-i(\vartheta_j-\varphi_k)}\overline{(\nu_{Lj})^c}\,(\mu_{jk} - i\mathcal{D}_{jk})\,\sigma_{\alpha\beta}\,N^c_{Rk}\,F^{\alpha\beta} \\
&\quad + e^{i(\vartheta_j-\varphi_k)}\overline{(N_{Rk})^c}\,(\mu^*_{jk} + i\mathcal{D}^*_{jk})\,\sigma_{\alpha\beta}\,\nu^c_{Lj}\,F^{\alpha\beta} + \overline{N}_{Rk}\,(\mu^*_{jk} - i\mathcal{D}^*_{jk})\,\sigma_{\alpha\beta}\,\nu_{Lj}\,F^{\alpha\beta}\ , \\
&= \overline{\nu}_{Lj}\,(\mu_{jk} + i\mathcal{D}_{jk})\,\sigma_{\alpha\beta}\,N_{Rk}\,F^{\alpha\beta} - e^{-i(\vartheta_j-\varphi_k)}\overline{N}_{Rk}\,(\mu_{jk} - i\mathcal{D}_{jk})\,\sigma_{\alpha\beta}\,\nu_{Lj}\,F^{\alpha\beta} \\
&\quad - e^{i(\vartheta_j-\varphi_k)}\overline{\nu}_{Lj}\,(\mu^*_{jk} + i\mathcal{D}^*_{jk})\,\sigma_{\alpha\beta}\,N_{Rk}\,F^{\alpha\beta} + \overline{N}_{Rk}\,(\mu^*_{jk} - i\mathcal{D}^*_{jk})\,\sigma_{\alpha\beta}\,\nu_{Lj}\,F^{\alpha\beta}\ , \qquad (6.8)
\end{aligned}$$

where in the last step we have followed the same procedure as that leading to (6.4). Using (6.7) and the analogous form for $\mathcal{D}_{jk}$, the Lagrangian simplifies to the form (after absorbing the common factor of 2 into the definitions of μ and $\mathcal{D}$):

$$\mathcal{L}'_{\rm EM} = \overline{\nu}_{Lj}\,(\mu_{jk} + i\mathcal{D}_{jk})\,\sigma_{\alpha\beta}\,N_{Rk}\,F^{\alpha\beta} + \text{h.c.}\ , \qquad (6.9)$$

$$= \overline{\nu}_{Lj}\,\lambda_{jk}\,\sigma_{\alpha\beta}\,P_R\,N_k\,F^{\alpha\beta} + \text{h.c.}\ , \qquad (6.10)$$

where we have defined $\lambda_{jk} \equiv \mu_{jk} + i\mathcal{D}_{jk}$ and it is in general a complex matrix. As a result, we can assume that the EMDM coupling matrix λ is completely arbitrary in all of our subsequent analysis. Now we shall investigate the viability of electromagnetic leptogenesis. We must first check that the out-of-equilibrium decay of the RH neutrinos can give rise to a nonzero CP asymmetry under the most general situations. In addition, because of the constraints from other sectors of the theory, it is also necessary to examine whether the parameter space has enough degrees of freedom to produce an asymmetry of the correct magnitude.

6.2 Discussion of electromagnetic leptogenesis with effective dipole operator

There is an alternate scenario where leptogenesis is mediated not by the standard Yukawa couplings, but instead by electromagnetic dipole moment couplings. In this new scenario of leptogenesis, the lepton asymmetry is generated by the CP-violating decays of heavy Majorana neutrinos either to SM lepton, photon in 2-body decay or to SM lepton, Higgs and photon in 3-body decay via electromagnetic dipole moment couplings. In this section, we will review the general discussion of electromagnetic leptogenesis (work of Kayser et al. [193]) and, at the end of this section, will give motivation towards our work.

We construct an effective theory by taking the usual minimally extended SM Lagrangian with three generations of heavy Majorana neutrinos, and augmenting it with EMDM operators. The dimension-5 EMDM operators involving only (the minimally extended) SM fields is of the form of (6.10). We assume that these EMDM couplings are generated by some new physics at an energy scale $\Lambda > M$, where M generically denotes the mass a heavy RH Majorana neutrino, and work with the effective theory that is valid below Λ, obtained after integrating out all new heavy degrees of freedom. The EMDM interaction Lagrangian of interest is:

$$\mathcal{L}_{\rm EM}^{\rm 5D} = -\lambda_{jk}\,\overline{\nu}_{Lj}\,\sigma^{\alpha\beta}\,P_R\,N_k\,F_{\alpha\beta} + {\rm h.c.}\,, \tag{6.11}$$

$$\equiv -\frac{1}{\Lambda}(\lambda_0)_{jk}\,\overline{\nu}_{Lj}\,\sigma^{\alpha\beta}\,P_R\,N_k\,F_{\alpha\beta} + {\rm h.c.}\,, \tag{6.12}$$

where $j = e, \mu, \tau$ and $k = 1, 2, 3$. $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is, as before, the electromagnetic field strength tensor, with A_α being the photon field. We have defined λ_0 as a dimensionless 3×3 matrix of complex coupling constants, and Λ is the cut-off scale of our effective theory, which has dimensions of energy.

An important observation is that the SM gauge symmetry, $SU(2)_L \times U(1)_Y$ is explicitly broken and the model is invariant only under the electromagnetic symmetry $U(1)_Q$. However, one major difficulty is that the theory demands to be valid up to the scale of Λ (i.e above M), hence only $U(1)_Q$ is unbroken, while the SM implies that electroweak symmetry must be restored at that scale since $\Lambda, M \gg \Lambda_{\rm EW} \simeq 10^2$ GeV.

The most economical of such operators involving only (the minimally extended with three heavy right handed Majorana neutrinos) SM fields are of dimension six and the interaction Lagrangian of interest is

$$\mathcal{L}_{\rm EM} = -\overline{\ell}_j \left[\lambda'_{jk}\, \phi\, \sigma^{\alpha\beta}\, B_{\alpha\beta} + \widetilde{\lambda}'_{jk}\, \tau_i\, \phi\, \sigma^{\alpha\beta}\, W^i_{\alpha\beta}\right] P_R\, N_k + \text{h.c.}\,, \tag{6.13}$$

$$\equiv -\frac{1}{\Lambda^2}\, \overline{\ell}_j \left[(\lambda'_0)_{jk}\, \phi\, \sigma^{\alpha\beta}\, B_{\alpha\beta} + (\widetilde{\lambda}'_0)_{jk}\, \tau_i\, \phi\, \sigma^{\alpha\beta}\, W^i_{\alpha\beta}\right] P_R\, N_k + \text{h.c.}\,, \tag{6.14}$$

where the τ_i are the $SU(2)_L$ generators, $\ell_j = (\nu_{Lj}, e_{Lj})^T$ is the lepton doublet, and $\phi = (\phi^0, \phi^-)^T$ is the SM Higgs doublet. The field strength tensors of $U(1)_Y$ and $SU(2)_L$ are given by $B_{\alpha\beta} = \partial_\alpha B_\beta - \partial_\beta B_\alpha$ and $W^i_{\alpha\beta} = \partial_\alpha W^i_\beta - \partial_\beta W^i_\alpha - g\, \epsilon_{imn} W^m_\alpha W^n_\beta$, respectively, where g' and g are the corresponding coupling constants. As before, Λ denotes the high energy cut-off of our effective theory, while the newly defined dimensionless EMDM coupling matrices, λ'_0 and $\widetilde{\lambda}'_0$, are in general complex. Note that λ'_0 and $\widetilde{\lambda}'_0$ play the exact same role as λ_0 in Lagrangian

The higher dimension (non-renormalizable) operators of Eq. (6.13) are assumed to be generated at the energy scale Λ, beyond the electroweak scale. Although the presence of these operators would imply the existence of some new physics at high energies, we shall not speculate on the nature of it here. After spontaneous breaking of $SU(2)_L \otimes U(1)_Y$, these operators will then give rise to the usual transition moments between N_k and ν_j. But, for the purposes of leptogenesis, we are of course interested in the regime above the electroweak symmetry breaking scale.

Electromagnetic Leptogenesis	
with 5-D EMDM operator	$\Gamma_1{}^{\rm em,5D} = \frac{(\lambda_0^\dagger \lambda_0)_{11}}{4\pi} \left(\frac{M_1}{\Lambda}\right)^2 M_1$
$-\frac{1}{\Lambda}(\lambda_0)_{jk}\, \overline{\nu}_{Lj}\, \sigma^{\alpha\beta}\, P_R\, N_k\, F_{\alpha\beta} + \text{h.c.}$	$\lvert\varepsilon_1^{\rm em,\,5D}\rvert \simeq \frac{1}{\pi} \sum_{m\neq 1} \frac{\text{Im}\,[(\lambda_0^\dagger \lambda_0)^2_{1m}]}{(\lambda_0^\dagger \lambda_0)_{11}} \frac{M_1}{M_m} \left(\frac{M_1}{\Lambda}\right)^2$
with 6-D EMDM operator	$\Gamma_1^{\rm em,\,6D} = \frac{1}{4\pi} (\lambda_0'^\dagger \lambda_0')_{11} M_1 \left(\frac{M_1^2}{8\pi\Lambda^2}\right)^2$
$-\frac{1}{\Lambda^2}\, \overline{\ell}_j \left[(\lambda'_0)_{jk}\, \phi\, \sigma^{\alpha\beta}\, B_{\alpha\beta}\right] P_R\, N_k + \text{h.c.}$	$\lvert\varepsilon_1^{\rm em,\,6D}\rvert \simeq \frac{1}{\pi} \sum_{m\neq 1} \frac{\text{Im}\,[(\lambda_0'^\dagger \lambda_0')^2_{1m}]}{(\lambda_0'^\dagger \lambda_0')_{11}} \frac{M_1}{M_m} \left(\frac{M_1^2}{8\pi\Lambda^2}\right)^2$

Table 6.1: Comparison of key quantities in electromagnetic leptogenesis for both 5D and 6D-EMDM operator [193], where λ_0 and λ'_0 denote the dimensionless 5D and 6D-EMDM coupling constants respectively.

We have presented a summary of electromagnetic leptogenesis with both 5D and 6D-EMDM operator given in the table [6.1] from which a couple of general observations for electromagnetic leptogenesis can be made. For our investigation here, we are particularly interested in examining if electromagnetic leptogenesis alone (i.e. when the Yukawa couplings are not taken into account or forbidden by some symmetry) can give rise to the required asymmetry without contradicting any known experimental constraints.

Firstly, we will examine the scenario of electromagnetic leptogenesis with 6D-dipole operator. In the paper of Kayser [193], it is clear that to obtain a reasonable size for the CP asymmetry (e.g. $\mathcal{O}\left(10^{-6}\right)$), the scale for M_1 must be at least $\mathcal{O}\left(10^{12}\right)$ GeV, a result which is similar to that from standard N_1-leptogenesis. The allowed values of the parameters: $\Lambda \simeq 10M_{2,3} \simeq 20M_1$, $\lambda_0' \simeq 35$ are sufficient to the produce an asymmetry of $|\varepsilon_1^{\mathrm{EM}}| \sim 10^{-6}$ (where λ is the effective dimension-5 EMDM coupling and defined as $\lambda = \frac{\lambda_0}{\Lambda}$). It is not well understood how one can get such big number $\lambda_0 > 35$ and what is the effective theory ? So, qualitatively speaking, we expect that successful electromagnetic leptogenesis is achievable with these parameter choices which can in principle realized in a realistic model. Secondly, the 5D-dipole moment operator can not give successful leptogenesis in this choice of parameters.

Are there plausible models in which a sizable amount of EDM which links between light and heavy neutrino occurs ? A natural question we may ask is whether the introduction of CP-violating dipole moment couplings will allow leptogenesis to occur at a lower scale, closer to experimentally accessible energies. Suppose one assumes that CP violation is due to some sort of new physics at the TeV scale. Then one can write the effective low-energy, dimension-5 Lagrangian as $-\frac{1}{\Lambda}(\lambda_0)_{jk}\,\overline{\nu}_{Lj}\,\sigma^{\alpha\beta}\,P_R\,N_k\,F_{\alpha\beta} + \mathrm{h.c.}$. If Λ is $\mathcal{O}\left(1\right)$ TeV, then one need to study the electromagnetic leptogenesis scenario more carefully. We will present a realistic model where the resonant electromagnetic leptogenesis is possible and also explain it's intimate connection to the light neutrino mass.

6.3 Realistic Model for electromagnetic leptogenesis

Now we shall discuss the possibility of generating a lepton asymmetry through the EMDM interactions described earlier. Since we are interested in leptogenesis energy scales above the electroweak phase transition, we shall identify the light neutrino in (6.10) to be a massless LH state (the same ν_L as appears in the SM lepton doublet), while N is assumed to have a large Majorana mass as in type-I seesaw. The simplest model that we are considering contains the minimally extended SM Lagrangian with three heavy RH neutrinos augmented by dimension-5 EMDM operators providing neutrino mass via TeV scale seesaw mechanism. The present model consists of all SM particles plus right-handed Majorana neutrinos (N_R), a singly charged scalar (H^+), two extra Higgs doublets (Σ, D) and one singly charged vector-like fermion with components E_L and E_R. This minimal set of extra fields is shown to lead to (resonant) electromagnetic leptogenesis.

6.3.1 The particle content and symmetry of the model

Retaining the gauge symmetry of the SM, we augment the fermion content by including three right-handed singlet fields N_{iR} and, in addition, a singly charged vector-like fermion E. Also added are a singly charged scalar (H^+) and a pair of Higgs doublets (Σ, D). In keeping with our stated paradigm of only one new scale, all the new masses are assumed to be around a few TeV. While it could be arranged that all these masses arise from the vacuum expectation value of a single scalar field, for simplicity, we incorporate explicit mass terms. The entire particle content, along with the quantum number assignments, is displayed in Table [6.2].

At this stage, we are faced with a problem generic to electromagnetic leptogenesis. While the effective $\bar{N}\,\ell\,\gamma$ coupling has to be allowed (so as to allow the mandatory $N \to \nu + \gamma$), the coupling of the fermion pair to the SM Higgs, viz. $\bar{N}\ell\Phi$ needs to be highly suppressed on two counts, (i) to ensure that the light neutrino mass, accruing from the seesaw mechanism, is not too large and (ii) to prevent the N from decaying dominantly to $\ell + \Phi$. While this could, nominally, be ensured by invoking some symmetry wherein the photon and the Φ transform differently, such an assignment would adversely impact the phenomenology of the charged particles. We rather choose to introduce a discrete Z_2 symmetry. All of the SM particles as well as the charged singlet scalar H^+ are even under this Z_2 symmetry, while all other particles are odd (see Table [6.2]).

Table 6.2: Particle content of the proposed Model

	Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Z_2
Fermions	$Q_L \equiv (u,d)_L^T$	(3, 2, 1/6)	+
	u_R	(3, 1, 2/3)	+
	d_R	(3, 1, -1/3)	+
	$\ell_L \equiv (\nu,\ e)_L^T$	(1, 2, -1/2)	+
	e_R	(1, 1, -1)	+
	E_L	(1, 1, -1)	-
	E_R	(1, 1, -1)	-
	N_R	(1, 1, 0)	-
Scalars	Φ	(1, 2, +1/2)	+
	Σ	(1, 2, +1/2)	-
	D	(1, 2, +1/2)	-
	H^+	(1, 1, +1)	+

The Z_2 symmetry allows both the Majorana mass terms $\nu\,\nu$ and $N\,N$, but the former is precluded if we limit ourselves to a renormalizable Lagrangian. On the other hand, the coupling of the neutrinos with the SM Higgs Φ, namely a term of the form $\bar{N}\ell\Phi$ is prevented. More importantly, the Z_2 symmetry forbids an effective Dirac mass term of the form $\bar{N}\,\nu$ as well as the the magnetic moment $\bar{N}\ell\gamma$. These can be generated only when the Z_2 is broken. Rather than break it spontaneously, and thereby risk domain walls, we choose to break it explicitly, but only through a soft term. While preserving the essential features of the model, this, then, allows the generation of both Dirac neutrino mass terms as well as magnetic moments and, thereby, driving resonant leptogenesis successfully.

While the Yukawa Lagrangian for the quarks remains unchanged from the SM, that for the leptonic sector can be written as

$$\begin{aligned}\mathcal{L}_{\rm Yuk} \ni\ & \Big[y_H\ \overline{N_R}\ E_L H^+ + y_\Sigma \overline{\ell_L}\Sigma E_R + y_D \overline{\ell_L} D E_R \\ &+ h_\Sigma \overline{\ell_L}\tilde{\Sigma} N_R + h_D \overline{\ell_L}\tilde{D} N_R + y_e \overline{\ell_L}\Phi e_R + h.c.\Big] \\ &+ \left[\frac{1}{2}\overline{(N_R)^C} M_N N_R - M_E \overline{E_R} E_L + h.c.\right] \end{aligned} \tag{6.15}$$

where the last two terms (M_N, M_E) represent gauge- and Z_2–invariant bare mass matrices. In the above, $\tilde{\Phi} = i\sigma_2\Phi^*$ (similarly for $\tilde{D}$ and $\tilde{\Sigma}$) and y_H, y_Σ, y_D, h_Σ and h_D are Yukawa coupling matrices.

The full scalar potential in our model with the fields Φ, Σ, D and H^+ is given by

$$\begin{aligned} V =\ & -\mu_\Phi^2|\Phi|^2 + \lambda_1|\Phi|^4 + m_2^2|\Sigma|^2 + \lambda_2|\Sigma|^4 + m_3^2|D|^2 + \lambda_3|D|^4 \\ &+ m_h^2\,|H|^2 + \lambda_h|H|^4 + \lambda_{\Phi H}(\Phi^\dagger\Phi)\,|H|^2 + \lambda_{DH}(D^\dagger D)\,|H|^2 \\ &+ \lambda_{\Sigma H}(\Sigma^\dagger\Sigma)|H|^2 + \lambda_{D\Sigma H}(D^\dagger\Sigma)|H|^2 + \frac{\lambda_{\Phi\Sigma}}{2}\left[(\Phi^\dagger\Sigma)^2 + h.c.\right] \\ &+ f_1\,(\Phi^\dagger\Phi)\,(D^\dagger D) + f_2\,(\Phi^\dagger\Phi)\,(\Sigma^\dagger\Sigma) + \lambda_{D\Phi}(D^\dagger\Sigma)(\Phi^\dagger\Phi) \\ &+ f_4|\Phi^\dagger\,\Sigma|^2 + f_3\,|\Phi^\dagger D|^2 + f_5\,(D^\dagger D)\,(\Sigma^\dagger\Sigma) + f_6|D^\dagger\,\Sigma|^2 \\ &+ \left[\mu_s\,\Sigma\cdot D\,(H^+)^* + h.c.\right]. \end{aligned} \tag{6.16}$$

The parameters are so chosen that only the standard model Higgs scalar doublet acquires a *vev* at this stage. The fields D and Σ do not acquire any *vev* and both of them are heavier than the right-handed neutrinos, so that the right-handed neutrinos can not decay into $\nu + D$ or $\nu + \Sigma$.

We now introduce a soft term to break the Z_2 symmetry, so that it does not affect the other interactions and also does not cause domain wall problem. We introduce the soft term without going into the details of its origin, which is given by:

$$V_{soft} = \mu_{soft}^2 \Phi^\dagger D + ... \tag{6.17}$$

The scale of the soft symmetry breaking μ_{soft} is lower than the electroweak symmetry breaking scale, and we also assume that the mass of the scalar D is of the order of $m_3 \sim$ 10 TeV. Ellipses above denote other allowed soft terms that do not concern us directly here. With suitable choice of the parameters it is possible to arrange $\langle D \rangle << \langle \Phi \rangle$. The same applies to the field Σ. This will then give us the Dirac mass term $\bar{N}\ell$ and the magnetic moment term $\bar{N}\ell\gamma$, as required for the present model. This will also generate the unwanted term $\bar{N}\ell\Phi$ due to the mixing of D and Σ with Φ, but this interaction will be suppressed by a factor of $\langle D \rangle$ / $\langle \Phi \rangle$, which if $\mathcal{O}(10^{-3})$, is consistent with the light neutrino mass as we explain below.

6.3.2 Neutrino mass

The Yukawa term $\bar{\ell}\Phi N$ is not allowed because of the Z_2 assignment in our model. Hence there is no Dirac neutrino mass at the tree level. However after the soft breaking of the Z_2 symmetry after the electroweak phase transition, the field D gets an induced *vev*, which in turn gives a Dirac mass to the neutrinos:

$$M_D = h_D \langle D \rangle = h_D v_d. \tag{6.18}$$

There will be another contribution to the neutrino mass coming from the mixing of D with the SM Higgs Φ, which will be further suppressed by the soft Z_2 breaking scale, so we do not include that contribution. The Dirac mass term together with the heavy right handed Majorana neutrino mass M_N will then give rise to a light neutrino Majorana mass via type-I seesaw mechanism:

$$m_\nu = M_D^{\rm loop} M_N^{-1} M_D^{\rm loop\,T}.$$

For the choice of parameters we are interested, $M_D \sim 10^{-3} h_D v \ \sim 10^{-4}$ GeV, for $\langle \Phi \rangle =$ $v \sim 100$ GeV and $h_D \sim 0.001$, and the right-handed neutrinos are lighter than the SM Higgs scalar, so $M_N \sim$ TeV. This gives the correct magnitude of the light neutrino masses $m_\nu \sim 10^{-10}$ GeV $\ \sim 0.1$ eV. The hierarchy of masses could be obtained because of the different values of the elements of the matrices M_N and h_D.

6.3.3 Estimation of the Dimension-5 EMDM coupling constant

First we present the dipole moment operator between light ν and heavy N neutrinos before deduce the potential implications of the EMDM operator in leptogenesis. Due to the Majorana nature, the diagonal component of the dipole moment of Majorana neutrinos is zero. There is only transition moments for them. The Lagrangian describing the neutrino interaction between light ν and heavy N neutrinos with electromagnetic field due to non-zero anomalous transition moment has the form

$$\mathcal{L}_{\rm EM} = \lambda_{jk}\,\overline{\nu}_j\,\sigma_{\alpha\beta}\,N_k\,B^{\alpha\beta} + \text{h.c.} \tag{6.19}$$

The h.c. term is $\lambda^*_{jk}\overline{N}_k\sigma_{\alpha\beta}\nu_i F^{\alpha\beta}$. In the decay calculations, it is much simpler to consider the EMDM coupling between the associated chiral components of the ν and N. In terms of chiral component, the above expression becomes

$$\mathcal{L}_{\rm EM} = \lambda_{jk}\,\overline{\nu}_j\,\sigma_{\alpha\beta}P_R\,N_k F^{\alpha\beta} + \text{h.c.} \tag{6.20}$$

The coupling λ_{jk} appearing in the EMDM operator is completely arbitrary and hence the matrix λ is complex in general. Now we need to estimate the value of the coupling strength λ in our model.

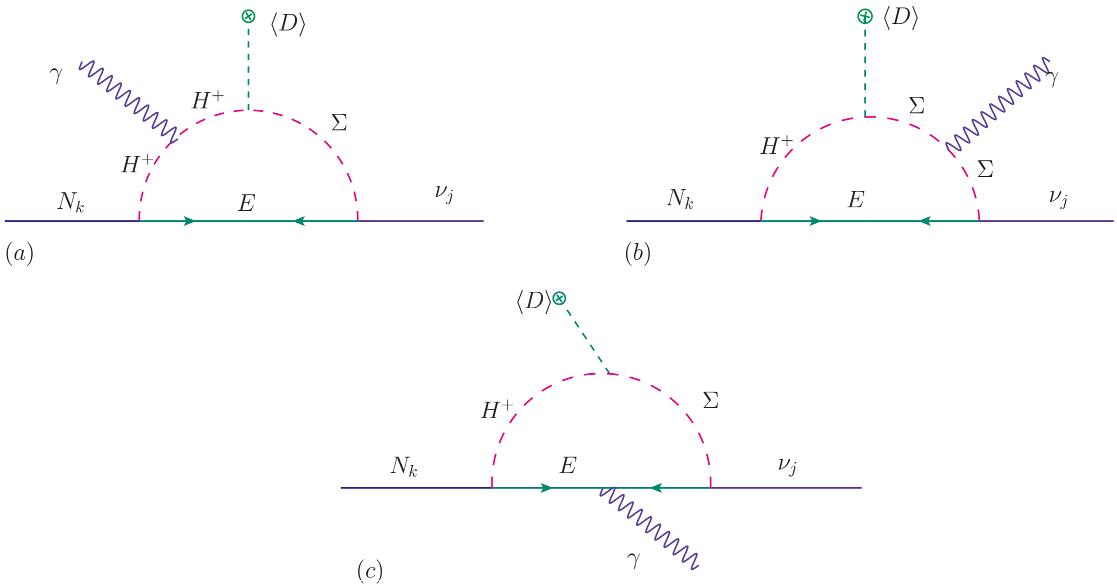


Figure 6.1: Feynman diagrams which estimate the effective EMDM coupling strength between light neutrino ν_j and N_k.

The Feynman diagrams which will quantify the EMDM coupling strength is shown in Fig. (6.3.3). The relevant term which will give the effective operator is $\overline{\nu}_{Lj}\,\lambda_{jk}\,\sigma_{\alpha\beta}\,P_R\,N_k\,B^{\alpha\beta}$. In paper of Kayser [193], the value of λ is $\lambda = \frac{\lambda_0}{\Lambda}$ and the successful leptogenesis requires $\Lambda \sim 10^{10}$ GeV and λ_0 to be > 35. However, they did not construct any explicit model to show how these numbers could arise and, in general, it is extremely difficult to get such large value of λ_0. The main motivation of this work is to show that it is possible to construct a simple extension of the SM, where it will be possible to calculate this effective coupling, which will lead to resonant electromagnetic leptogenesis. It should be noted that without the resonant condition, it is not possible to have correct amount of leptogenesis in these models, when the effective couplings are so small.

The Feynman rules and details of the calculation have been shown in appendix explicitly. Here, we will only give the final form of the EMDM coupling strength in our model. The analytical expression for EMDM coupling strength which is responsible for electromagnetic leptogenesis is given by

$$\lambda = -\frac{(y_\Sigma^*\,y_H\,\mu_s\,v_D)}{16\,\pi^2\,[M_\Sigma^2 - M_H^2]}\Big[\mathcal{I}_a + \mathcal{I}_b + \mathcal{I}_c\Big] \tag{6.21}$$

Where $\mathcal{I}_a$, $\mathcal{I}_b$ and $\mathcal{I}_c$ are contribution coming from three diagrams shown in Fig.(6.3.3). The dominant contribution coming from the diagrams (6.3.3 [a]) and (6.3.3 [b])

$$\mathcal{I}_a + \mathcal{I}_b = 2\,M_E\left(\left[B_1^{(0)} - B_1^{(1)} - C_1^{(1)}\right] - \left[B_2^{(0)} - B_2^{(1)} - C_2^{(1)}\right]\right) \tag{6.22}$$

$$\begin{aligned}
\text{where}\quad B_1^{(n)} &= \int_0^1 dx \int_0^{1-x} dy\, x^n \;\; \omega_1\\
C_1^{(n)} &= \int_0^1 dx \int_0^{1-x} dy\, y^n \;\; \omega_1\\
B_2^{(n)} &= \int_0^1 dx \int_0^{1-x} dy\, x^n \;\; \omega_2\\
C_2^{(n)} &= \int_0^1 dx \int_0^{1-x} dy\, y^n \;\; \omega_2\\
\text{Also,}\quad \omega_1 &= \frac{1}{-y\,M_\Sigma^2 + x(1-M_\Sigma^2) - (1-x-y)M_E^2 - x(x+y)M_N^2}\\
\omega_2 &= \frac{1}{-y\,M_\Sigma^2 + x(1-M_H^2) - (1-x-y)M_E^2 - x(x+y)M_N^2}
\end{aligned} \tag{6.23}$$

where $n = 0, 1, 2, ..$ is an integer.

Similarly, the dominant contribution coming from the diagram (6.3.3 [c])

$$\begin{aligned} \mathcal{I}_c &= M_E \int_0^1 dx \int_0^{1-x} dy\, (y-1)[\Omega_1 - \Omega_2] \\ \Omega_1 &= \frac{1}{(y - y^2 - x\,y)M_N^2 - y\,M_\Sigma^2 - (1-y)M_E^2} \\ \Omega_2 &= \frac{1}{(y - y^2 - x\,y)M_N^2 - y\,M_H^2 - (1-y)M_E^2} \end{aligned} \tag{6.24}$$

The effective dimension-5 coupling constant λ can thus be expressed in a simple form under the assumption of almost equal mass for the particles in the loop ($M_E \sim M_H \sim M_\Sigma \sim M_{eq}$) as:

$$\lambda = -\frac{y_\Sigma^*\, y_H\, \mu_s\, v_D}{64\,\pi^2\, M_{eq}^3} \tag{6.25}$$

For a representative reasonable sets of parameters: $M_N \sim$ TeV, $M_{eq} \sim$ TeV, $y_\Sigma = y_H \sim 1$, $\mu_s \sim$ TeV and $v_D = 0.1$ GeV, the EMDM coupling strength which is responsible for electromagnetic leptogenesis is found to be $\lambda \sim 10^{-11}$. Although the scales are shown to be of the order of TeV, it could range from 1–10 TeV, with the condition, $M_N < M_{eq}$, so that N_R can not decay into $\nu + D$ or $\nu + \Sigma$.

Now we shall investigate the viability of electromagnetic leptogenesis. We must first check that the out-of-equilibrium decay of the RH neutrinos can give rise to a nonzero CP asymmetry under the most general situations. In addition, it is also necessary to examine whether the parameters considered in our model can produce an asymmetry of the correct magnitude via the dimension-five dipole moment operator through the self-energy enhancement.

6.3.4 Resonant Electromagnetic Leptogenesis

As has been described above, leptogenesis, in this scenario is driven by the electromagnetic dipole moment terms appearing in the effective Lagrangian. Specifically, the lepton asymmetry generated by the CP-violating decays of heavy singlet neutrinos to the SM-like light neutrinos and photon. A natural question we may ask is whether the introduction of CP-violating dipole moment couplings will allow leptogenesis to occur keeping the model consistent with neutrino masses and the new physics will be accessible to LHC or ILC. As should be apparent from the discussion in the last section, the size of the EMDM that is generated and the extent of CP-violation in them is inadequate for thermal leptogenesis. Indeed, this is a generic problem for all models of electromagnetic leptogenesis that seeks to be consistent with observed physics and yet be natural. Given this, we investigate the possibility of a resonant enhancement. As is well-known, this mechanism is contingent upon the existence of at least two neutrino species that are very closely degenerate, and this is what we shall assume. Aesthetically, the extent of degeneracy needed may seem uncomfortable. While it can, in principle, be motivated on the imposition of additional global symmetries, it should be noted that, in all models of resonant leptogenesis, the subsequent breaking of the same would, naturally, lead to a lifting of the degeneracy by a degree that negates the conditions for resonant enhancement. Hence, rather than introduce additional symmetries, and a host of fields an additional mechanisms to compress the spectrum adequately, we just assume that the said heavy neutrinos are highly degenerate. In this class of leptogenesis, only self-energy diagrams are important which we will present in the following section.

The key quantity of interest in resonant electromagnetic leptogenesis is to calculate the CP-asymmetry for the decay of N_k to a photon and a light neutrino as shown in fig:(6.3). This quantity is given by

$$\varepsilon^{(5)}_{k,j} = \frac{\Gamma(N_k \to \nu_j\,\gamma) - \overline{\Gamma}(N_k \to \overline{\nu}_j\,\gamma)}{\Gamma(N_k \to \nu\,\gamma) + \overline{\Gamma}(N_k \to \overline{\nu}\,\gamma)}\ , \tag{6.26}$$

where $\Gamma(N_k \to \nu\,\gamma) \equiv \sum_j \Gamma(N_k \to \nu_j\,\gamma)$ denotes the decay rate (summed over final state flavor j). So with this in mind, we begin by calculating the lowest order contribution to the decay rate, $\Gamma(N_k \to \nu_j\,\gamma)$. Since we are interested in leptogenesis energy scales above the electroweak phase transition, we shall identify the SM light neutrino ν to be a massless left-handed state while N assumed to have Majorana mass of around 1 TeV. As it is well known that $\Gamma(N_k \to \nu_j\,\gamma) \equiv \Gamma(N_k \to \overline{\nu}_j\,\gamma)$, the total decay rate is, $\Gamma_{\text{tot}} = 2\Gamma(N_k \to \nu\,\gamma)$, to first order.

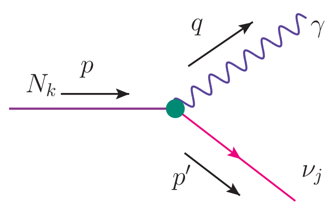


Figure 6.2: The Feynman graph for the lowest order decay, $N_k \to \nu_j\,\gamma$ via the dimension-5 EMDM coupling of Eq. (**??**). Here $q = p - p'$ and $2\lambda_{jk}\,P_R\,\sigma^{\alpha\beta}\,q_\beta$ is the vertex factor.

The Feynman diagram for the lowest order contribution to the process is shown in fig:(6.2). The lowest order decay rate is given by

$$\Gamma(N_k \to \nu\,\gamma) \;=\; \frac{(\lambda^\dagger\lambda)_{kk}}{4\pi}\,M_k^3 \tag{6.27}$$

For effectively creating a lepton asymmetry of the universe, the decays of $N_1 \to \gamma\,\nu$ should be out of equilibrium, which is described by $\Gamma \lesssim H(T)\,|_{T=M_1}$ where $\Gamma = \Gamma\,(N_1 \to \nu + \gamma) = \frac{(\lambda^\dagger\,\lambda)_{11}}{4\pi} M_1^3$ is the total decay width and $H(T) = 1.67\,g_*^{1/2}\,\frac{T^2}{M_{\rm Pl}}$ is the Hubble parameter with the Planck mass $M_{\rm Pl} \simeq 1.2 \times 10^{19}\,{\rm GeV}$ and the relativistic degrees of freedom $g_* \simeq 100$. In order to satisfy the out of equilibrium condition, we should have

$$\begin{aligned} &\Gamma \lesssim H(T = M_1) \\ \Rightarrow\; &\frac{(\lambda^\dagger\,\lambda)}{4\pi}\,M_1^3 \lesssim 1.67\,g_*^{1/2}\,\frac{M_1^2}{M_{\rm Pl}} \end{aligned} \tag{6.28}$$

where M_1 is the mass of the lightest RH heavy neutrino which is taken to be 1 TeV. From this expression, the upper bound on the EMDM couplings reads as

$$\sqrt{\sum_m |\lambda|^2} < 10^{-20}\sqrt{\frac{1}{(M_1/TeV)}}\,. \tag{6.29}$$

This is satisfied by the effective EMDM coupling λ, for the choice of parameters we considered here.

Now the next task is to calculate the interference terms between the tree level process and the one-loop diagrams with on shell intermediate states shown in fig. (6.3). In this particular scenario, the EMDM coupling strength is found to be in the range from 10^{-10} to 10^{-11} from our previous calculation. The usual contributions to lepton asymmetry coming from vertex diagram is found to be very small, i.e, ($\epsilon_1 = \lambda^2/4\pi M_1^3 \sim 10^{-22}\cdot 10^{-1}\cdot M_1^3 \sim 10^{-17}$when M_1 is of the order of TeV scale) and hence, can be neglected. So the self energy contribution will only be considered during the rest of the discussion. The Feynman diagram contributing to the self-energy diagram is shown in fig. (6.3).

For resonant leptogenesis case, the CP-asymmetry [38, 88, 89, 91] in standard Yukawa mediated case is slightly different from the CP-asymmetry in the present case. The CP-asymmetry [193] of N_k decays via the interaction of (6.20) has been calculated for the case of hierarchical RH neutrino. In this work, we have calculated the self-energy diagrams for

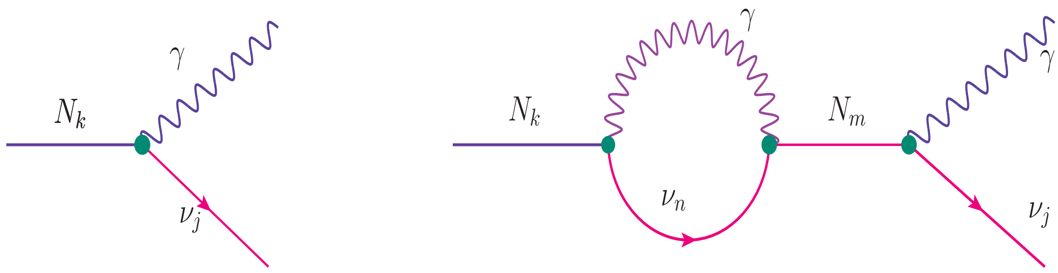


Figure 6.3: Self energy diagrams which contribute to the CP-asymmetry of N_k decays via the interaction of (6.20).

nearly degenerate heavy RH neutrinos and in this case, the CP-asymmetry found to be

$$\varepsilon_k = -\frac{M_k^2}{2\pi}\frac{\sum_{m\neq k}\text{Im}\left[(\lambda^\dagger\lambda)^2_{km}\right]}{(\lambda^\dagger\lambda)_{kk}}\frac{(M_k^2-M_m^2)M_kM_m}{(M_m^2-M_k^2)^2+M_k^2\Gamma_m^2} \tag{6.30}$$

$$= -\frac{\sum_{m\neq k}\text{Im}\left[(\lambda^\dagger\lambda)^2_{km}\right]}{(\lambda^\dagger\lambda)_{kk}\,(\lambda^\dagger\lambda)_{mm}}2\left(\frac{M_k}{M_m}\right)^2\frac{(M_m^2-M_k^2)M_k\Gamma_m}{(M_m^2-M_k^2)^2+M_k^2\Gamma_m^2} \tag{6.31}$$

Consider the case where $M_1 \sim M_2 \ll M_3$. From equation (6.27), it is clear that $\Gamma_1 \sim \Gamma_2$ for nearly degenerate right handed neutrino with mass M_1 and M_2. Hence, we can put the value of $\Gamma_2 \sim \Gamma_1 = \frac{(\lambda^\dagger\lambda)_{22}}{4\pi}M_2^3$ in the numerator of equation (6.30) and the expression for the CP-asymmetry for N_1 dominated case becomes

$$\varepsilon_1 = -\frac{M_1^2}{2\pi}\frac{\sum_{m\neq 1}\text{Im}\left[(\lambda^\dagger\lambda)^2_{1m}\right]}{(\lambda^\dagger\lambda)^2_{11}}\frac{(M_2^2-M_1^2)M_1M_2}{(M_2^2-M_1^2)^2+M_1^2\Gamma_2^2} \tag{6.32}$$

We are interested in the case where $|M_1-M_2| \gg \Gamma_2$. With this condition, the 2nd term in the denominator of equation (6.32) can be neglected in comparison to the first term. So, the the CP-asymmetry for the situation we are interested ($|M_1-M_2| \gg \Gamma_2$) is

$$\varepsilon_1 = -\frac{M_1^2}{2\pi}\frac{\sum_{m\neq 1}\text{Im}\left[(\lambda^\dagger\lambda)^2_{1m}\right]}{(\lambda^\dagger\lambda)^2_{11}}\frac{M_1M_2}{M_2^2-M_1^2} \tag{6.33}$$

The scenario of leptogenesis is different for the case where $M_1 \neq M_2$. But in the almost degenerate case, the asymmetry is resonantly enhanced. The factor in the denominator can be simplified as $M_2^2 - M_1^2 = (M_2-M_1)(M_2+M_1) \sim 2M_2(M_2-M_1) \sim 2M_1(M_2-M_1)$. Under this assumption $M_1 \simeq M_2$ and using equation (6.33), one can write the

CP-asymmetry parameter as

$$\varepsilon_1 = -\frac{M_1^2}{4\pi}\frac{\sum_{m\neq 1}\mathrm{Im}\left[(\lambda^\dagger\lambda)_{1m}^2\right]}{(\lambda^\dagger\lambda)_{11}^2}\mathcal{R} \tag{6.34}$$

where $\mathcal{R} \equiv \frac{M_1}{|M_1 - M_2|}$.

As described above, the CP-violating parameter can give rise to a net lepton number asymmetry in the Universe, provided its expansion rate is larger than the decay rate. The nonperturbative sphaleron interaction may partially convert this lepton number asymmetry into a net baryon number asymmetry [84],

$$Y_B \simeq -2.96 \times 10^{-2}\,\varepsilon_1\,k$$

where k is the efficiency factors measuring the washout effects associated with the out-of-equilibrium decays of N_1. In our model, the k is approximately 10^{-3} in order of magnitude. Hence the formula for baryon asymmetry of the Universe is given by

$$Y_B \simeq -2.96 \times 10^{-5}\,\varepsilon_1 \tag{6.35}$$

So we need $|\varepsilon_1| \sim 10^{-5}$ for successful baryon asymmetry of the Universe. This is easily satisfied from equation (6.34) for $\mathcal{R} = 10^{+10}$ or $|M_2 - M_1| = 10^{-7}$ GeV where the right handed Majorana neutrino are of TeV scale.

In this paper we shall not discuss the origin of the small mass differences between the degenerate right-handed neutrinos, but for completeness we demonstrate that a mass splitting of the order of 10^{-7} GeV is not unnatural for TeV scale right-handed neutrinos. Consider a diagram with a vertex $\lambda_{HD}(D^\dagger D)(H^\dagger H)$ attached to the singly charged scalar H which runs in loop and this kind of digram gives a finite contribution to the mass splitting. A simple calculation gives

$$\Delta M_R \sim \frac{\lambda_{HD}y_H^* y_H}{(4\pi)^2}\frac{\langle D\rangle^2}{4M_E} \tag{6.36}$$

For the mass of the charged lepton to be around 1 TeV (i.e, $M_E \sim 1$ TeV), $\langle D\rangle$=0.1 GeV and $y_H \sim 1$, one can write

$$\Delta M_R \sim \frac{10^{-2}}{64\pi^2 M_E}\lambda_{HD} \tag{6.37}$$

Now one can easily get the mass splitting between two right handed neutrinos of the order of O(10^{-7}) GeV.

If we thus start with a symmetry to get a TeV Scale degenerate right-handed neutrinos, after the symmetry breaking, we get a mass splitting between the companion states of right-handed neutrinos to be in the range of O(10^{-7}) GeV, naturally through radiative corrections.

6.3.5 Numerical estimation for Y_B

In generic leptogenesis scenario, the deviation of the distribution function of some heavy particles from its equilibrium distribution distribution provides the necessary departure from thermal equilibrium. The non-equilibrium process of baryogenesis via leptogenesis is usually studied by means of Boltzmann equation [31, 37, 39, 56, 83]. We shall consider the simplest case where the initial temperature is larger than M_1, the mass of the lightest heavy neutrino. In principle, one should take into account all B- and L-violating processes. In this treatise, however, we consider only decays, inverse decays, $\Delta L = 2$ scattering and the sphalerons.

Within this minimal framework, the Boltzmann equations can be written as

$$\frac{dY_{N_1}}{dz} = -\{D(z) + S(z)\}\left[Y_{N_1} - Y_{N_1}^{eq}\right] \tag{6.38}$$

$$\frac{dY_{\mathcal{B}-\mathcal{L}}}{dz} = -\epsilon_{N_1} D(z)\left[Y_{N_1} - Y_{N_1}^{eq}\right] - W(z) Y_{\mathcal{B}-\mathcal{L}} \tag{6.39}$$

where $z = M_1/T$. There are four classes of processes which contribute to the different terms of the equations: decays, inverse decays, $\Delta L = 1$ scatterings and $\Delta L = 2$ processes mediated by heavy neutrinos. The first three all modify the N_1 abundance and try to push it towards its equilibrium value N_1^{eq}. In this case, we have considered the normalized quantity $Y_{N_1} = N_1/$s, s is the entropy of the Universe.

The term $D(z) = \Gamma_D/(H\, z)$ accounts for decays and inverse decays and can be approximately written as

$$D(z) = z\, K \frac{\mathcal{K}_1(z)}{\mathcal{K}_2(z)} \tag{6.40}$$

where parameter K is a measure of how fast the decay rate is in comparison with the expansion rate of the universe at temperature at $T = M_1$ and defined by the relation

$$K = \frac{\Gamma_1}{H(T = M_1)} \tag{6.41}$$

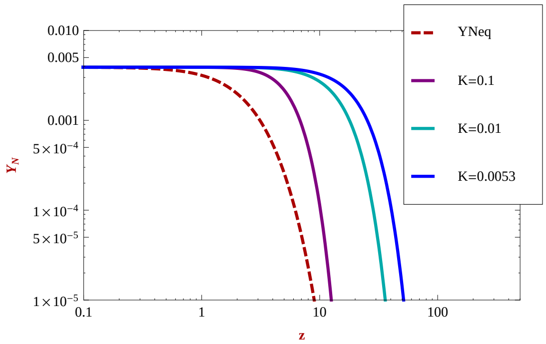


Figure 6.4: Plot of equlibrium number density and abundance of RH neutrino for different values of $K = \Gamma_N/H$.

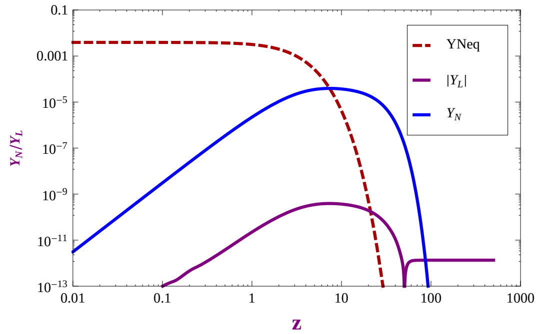


Figure 6.5: Abundance of RH neutrino and lepton asymmetry for different values of $K = \Gamma_N/H$.

The scattering term $S = \Gamma_S/(H\,z)$ represents the scattering process mediated by the heavy neutrino and gauge scattering terms. Also Decays are the source term for $\mathcal{B}-\mathcal{L}$ asymmetry generation while $W = \Gamma_W/(H\,z)$ is the wash-out term which tries to erase the net $\mathcal{B}-\mathcal{L}$ asymmetry produced by the decay process. In our model, only decay and inverse decays are important. Since the $\Delta L = 1, 2$ processes are suppressed, we shall not take into account them while solving the Boltzmann equation. To ignore the $\Delta L = 2$ scattering, we need to replace the washout term W with a washout term with contribution only from the inverse decays. This can be written as

$$W_{\rm ID} = \frac{1}{2}\frac{\Gamma_{\rm ID}}{H\,z}\frac{\mathcal{K}_1(z)}{\mathcal{K}_2(z)}. \tag{6.42}$$

The inverse decay width, $\Gamma_{\rm ID}$, is related to the decay width by the equilibrium number densities of the heavy neutrinos and lepton doublets,

$$\Gamma_{\rm ID} = \Gamma_N \frac{N_N^{\rm eq}(z)}{N_\ell^{\rm eq}}. \tag{6.43}$$

For leptons, $N_\ell^{\rm eq} = 3/4$ at high temperature we are considering, while for heavy neutrinos N_1, the equilibrium number density per comoving volume $Y_{N_1}^{eq}$ is given by

$$Y_{N_1}^{eq} = \frac{N_{N_1}^{eq}}{s} = \frac{45}{2\pi^4 g_*}\frac{3\zeta(3)}{4} z^2\,\mathcal{K}_2(z). \tag{6.44}$$

Combining these results, we get

$$W_{\rm ID} = \frac{1}{4}\,\frac{z\,K_1(z)\,\Gamma_N}{H} = \frac{1}{4}\,z^2\,D(z)\,K_2(z) = \frac{1}{2}\,D(z)\,\frac{N_N^{\rm eq}}{N_\ell^{\rm eq}}. \tag{6.45}$$

Replacing the general washout term W with $W_{\rm ID}$, we arrive at the Boltzmann equations and their solutions with only decays and inverse decays:

$$\frac{d\,Y_{N_1}}{d\,z} = -D(z)\,\left[Y_{N_1} - Y_{N_1}^{eq}\right] \tag{6.46}$$

$$\frac{d\,Y_{\mathcal{B}-\mathcal{L}}}{d\,z} = -\epsilon_{N_1}\,D(z)\,\left[Y_{N_1} - Y_{N_1}^{eq}\right] - W_{\rm ID}(z)\,Y_{\mathcal{B}-\mathcal{L}} \tag{6.47}$$

$$\kappa = -\frac{4}{3}\int_{z_i}^{z} dz'\,\frac{d\,Y_{N_1}}{d\,z'} e^{-\int_{z'}^{z} dz''\,W_{\rm ID}(z'')} \tag{6.48}$$

Using the simplified set of equations, the final baryon asymmetry asymmetry can be solved in terms of only two parameters: ϵ_{N_1}, signifying the amount of CP-violation, and K, signifying the strength of the decay compared to Hubble's expansion of the Universe. The

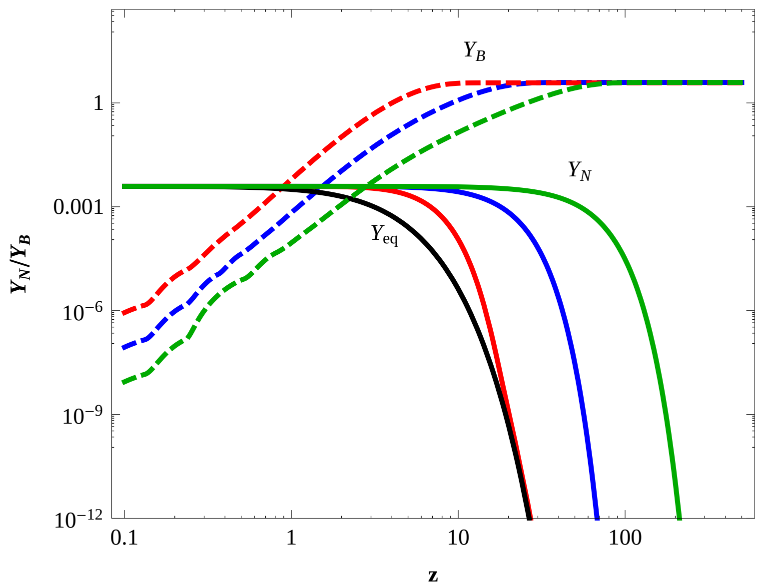


Figure 6.6: Plot of $Y_N = N/s$ and $10^{10} \times Y_B$ as a function of temperature. It is important to note that large baryon asymmetry is generated at $T = M_1$, but it is dissipated by the gauge scattering processes at lower temperatures.

Boltzmann equations are numerically solved to give the present baryon asymmetry of the Universe as shown in figure (6.5) and (6.6).

6.4 Production of right-handed neutrinos through magnetic moment

Magnetic moment of right-handed neutrinos

Due to the Majorana nature, the diagonal component of the magnetic moment of heavy Majorana neutrinos is zero. There is only transition moments for them. First, we will estimate the transition magnetic moment in the model discussed for electromagnetic leptogenesis. The Yukawa interactions of heavy Majorana neutrinos with S and E can be rewritten in the following way

$$\mathcal{L}_N \ni \frac{1}{2}\left[\overline{N^c} H^- Y_H^T P_R E^c + \overline{N} H^+ (Y^\dagger)_H P_L E\right] + \frac{1}{2}\left[\overline{E} Y_H H^- P_R N + \overline{E^c} Y_H^* H^+ P_L N^c\right]. \tag{6.49}$$

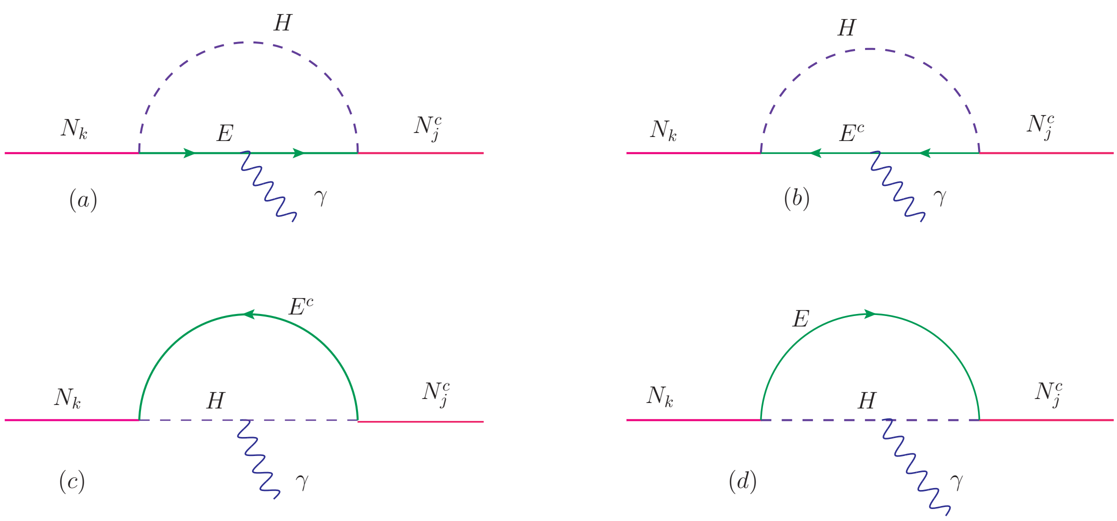


Figure 6.7: Explicit calculation of dipole moments

In the model considered, we have four diagrams contributing to the transitional magnetic moment of heavy right handed neutrino, which are depicted in Fig. [6.7 (a) and (b)]: a loop with the photon line attached to the E and Fig. [6.7 (c) and (d)]: a loop with the photon line attached to the scalar H.

Assuming that heavy Majorana neutrinos are nearly degenerate, i.e., $M_j \approx M_k \approx M$, we derive the expression of $\mu_{N_{jk}}$:

$$\mu_{N_{jk}} = \frac{M}{64\pi^2}\left[(Y_H^\dagger)_{km}(Y_H)_{mj} - (Y_H^T)_{km}(Y_H)^*_{mj}\right] \times \left[\mathcal{I}(M_H^2, M^2, M_E) - \mathcal{I}(M_E, M^2, M_H^2)\right] \tag{6.50}$$

with

$$\mathcal{I}(A,B,C) = \int dx \frac{x(1-x)^2}{(1-x)A + x(x-1)B + xC} ,$$

where M_E and M_H are the mass eigenvalues of heavy vector-like fermion E and singly charged scalar H, respectively. In the equal mass limit ($M_E \sim M_H \sim M$), one can write the transition magnetic moment of heavy right handed neutrino as

$$\mu_{N_{jk}} = \frac{1}{64\pi^2}\left[(Y_H^\dagger)_{km}(Y_H)_{mj} - (Y_H^T)_{km}(Y_H)^*_{mj}\right]\frac{e}{M_E}F(x) \tag{6.51}$$

where the function $F(x)$ is $F(x) = \frac{1}{1-x} + \frac{x}{(1-x)^2}\ln(x)$ and the parameter x is $x = \frac{M_H^2}{M_E^2}$. The non-perturbative limit gives us $[(Y_H^\dagger)_{km}(Y_H)_{mj} - (Y_H^T)_{km}(Y_H)^*_{mj}] \leq 4\pi$. We found that $1/(64\pi^2) \sim 10^{-3}$ and Yukawa couplings can take the value from $0.01 - 1$. With this spectrum, one can get the large magnetic moment of the order of $10^{-10}\mu_B$ for TeV scale right handed neutrinos.

The most dramatic effect of a large EDM of a heavy neutrino will be in the production cross section and angular distribution. A discussion of the differential cross section for a heavy charged lepton can be found in Ref. [198, 199] and we will qualitatively discuss how one can produce RH Majorana neutrinos in near future experiment. In the discussion of Escribano and Masso [200], one can write a $U(1)$ invariant operator as: $\overline{N}_{Rj}\,(\mu_N^{jk} + i\mathcal{D}_N^{jk})\,\sigma_{\alpha\beta}\,N_{Rk}^c\,B^{\alpha\beta}$, where $B^{\alpha\beta}$ is the $U(1)$ field tensor. This gives a coupling to the photon, which we define to be the EDM, as well as a coupling to the Z which is the EDM times $\tan\theta_W$. When we include the effect of Z coupling to N in the differential cross section, it turns out that the contribution has very little effect on the result.

A discussion of the differential cross section for a heavy charged lepton can be found in Ref. [198,199,201]. We are interested in the production of the heavy right handed neutrino using the parameters used in the model. The differential cross-section for the process,

$e^+e^- \to \gamma,\, Z^* \to N_k N_j$ $(k \neq j)$, is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\sqrt{1-\frac{4M^2}{s}}\left(\mathcal{F}_1 + \frac{1}{8\sin^4 2\theta_W} P_{Z\,Z}\,\mathcal{F}_2\right) + \left(\frac{(1-4\sin^2\theta_W)\tan\theta_W}{\sin^2 2\theta_W} P_{\gamma\,Z}\,\mathcal{F}_3\right) \tag{6.52}$$

where the values of $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{F}_3$, P_{ZZ} and $P_{\gamma\,Z}$

$$\begin{aligned}
\mathcal{F}_1 &= \mu_N^2\, s\, \sin^2\theta \left(1+\frac{4M^2}{s}\right)\,, \\
\mathcal{F}_2 &= 1+\cos^2\theta - \frac{4M^2}{s}\sin^2\theta + 8C_V\cos\theta \\
&\quad + \mu_N^2\, s\, \tan^2\theta_W \left[\sin^2\theta + \frac{4M^2}{s}\left(1+\cos^2\theta\right)\right]\,, \\
\mathcal{F}_3 &= 4\mu_N^2\, s\left[\sin^2\theta + \frac{4M^2}{s}\left(1+\cos^2\theta\right)\right]\,, \\
P_{ZZ} &= \frac{s^2}{(s-M_Z^2)^2+\Gamma^2 M_Z^2}\,, \\
P_{\gamma Z} &= \frac{s(s-M_Z^2)}{(s-M_Z^2)^2+\Gamma^2 M_Z^2}.
\end{aligned} \tag{6.53}$$

with μ_N^{kj} is the transition magnetic moment of heavy Majorana neutrino, $C_V = \frac{1}{2} - 2\sin^2\theta_W$, and we have dropped the numerically negligible C_V^2 terms, for simplicity.

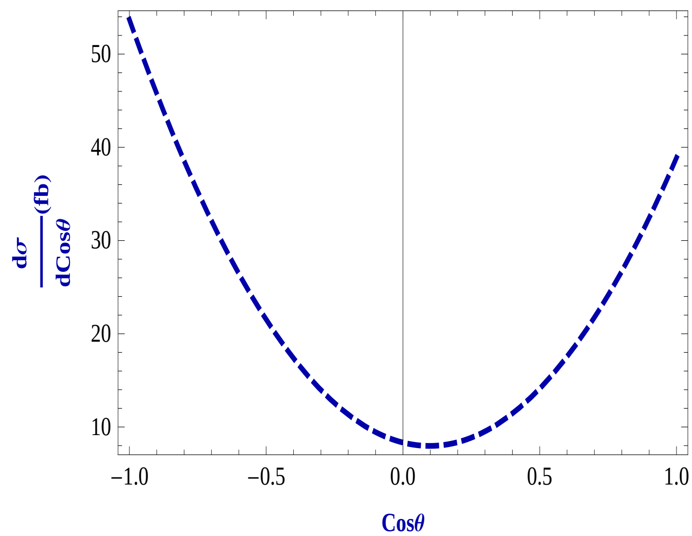


Figure 6.8: The differential cross section for the process $e^+e^- \to \gamma, Z^* \to N_i N_j$ $(i \neq j)$, for a given heavy Majorana mass scale $M = 200$ GeV and a fixed center of collider energy $\sqrt{s} = 500$ GeV as a function of scattering angle $\cos\theta$.

The differential and the total cross sections for the production of heavy right handed

Majorana neutrino are shown in Fig. (6.8), (6.9) and (6.10). In Fig. 6.8, it is shown the differential cross section for the process, $e^+e^- \to \gamma, Z^* \to N_iN_j$ $(i \neq j)$, for a given heavy Majorana mass scale $M = 200$ GeV and a fixed center of collider energy $\sqrt{s} = 500$ GeV as a function of scattering angle $\cos\theta$.

In Fig. 6.9, it is shown the total cross section for the process, $e^+e^- \to \gamma, Z^* \to N_iN_j$ $(i \neq j)$, for varied heavy Majorana mass scales $M = 200,\ 300,\ 400,\ 500$ GeV as a function of center of collider energy $\sqrt{s}$. In Fig. 6.10, it is shown the total cross section for the process, $e^+e^- \to \gamma, Z^* \to N_iN_j$ $(i \neq j)$, for varied center of collider energies $\sqrt{s} = 500,\ 700,\ 800,\ 1000$ GeV as a function of heavy Majorana mass scale M. In these plots, we have used an approximation that the final state right-handed neutrinos have almost the same masses with each other, which is denoted by M. It can be seen that the total cross section for the production of TeV right-handed neutrinos can reach a few fb, $\sigma \sim 5$ fb. After the production of a right-handed neutrino, it decays into a left-handed neutrino (ν_j) and a photon (γ), $N_k \to \nu_j + \gamma$.

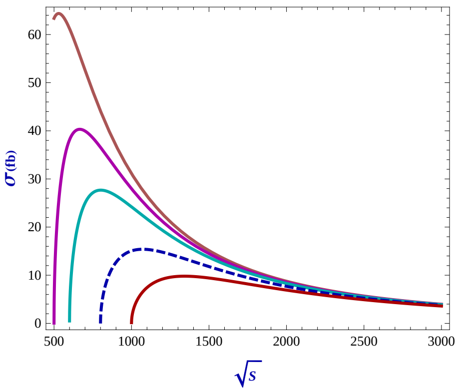


Figure 6.9: The total cross section for heavy right handed neutrino $e^+e^- \to \gamma,\, Z^* \to N_kN_j$ $(k \neq j)$ for various EDMs, in units of Bohr magneton. The cross section is shown as a function of center of collider energy $\sqrt{s}$ and here we have varied the masses of heavy right handed neutrino as $M = 300, 400, 500$ GeV from the top to the bottom curves.

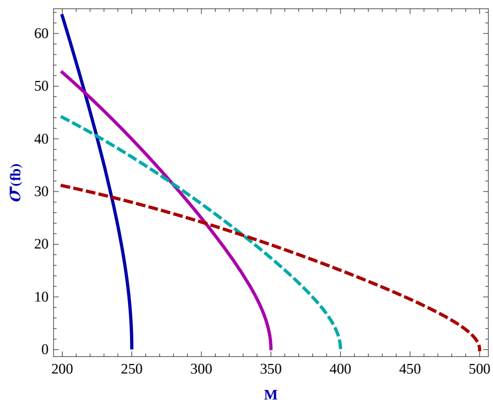


Figure 6.10: The total cross section for heavy right handed neutrino $e^+e^- \to \gamma,\, Z^* \to N_kN_j$ $(k \neq j)$ for various EDMs, in units of Bohr magneton. The cross section is shown as a function of heavy Majorana neutrino mass M and here we have varied the center of collider energy as $\sqrt{s} = 500, 700, 800, 1000$ GeV from left to right.

We have considered the dipole moment interactions between the heavy right handed neutrino and their light counterparts. As a consequences of this, the heavy right handed neutrino decays to photon and light neutrino resulting required amount of lepton asymmetry to explain the matter-antimatter asymmetry of the Universe. We have considered the magnetic moments of right-handed neutrinos, whose masses are set at around TeV scale. Because of the scaling rule of magnetic moment of neutrinos, the heavy right-handed neutrinos can, in general, have a large amount of magnetic moments evading a chiral suppression. Such large magnetic moments can enhance the production cross section of TeV scale right-handed neutrinos though the Drell-Yan process, $e^+e^- \to \gamma, Z^* \to N_i N_j$ $(i \neq j)$, which is within the reach of the future linear collider (ILC).

Chapter

7

Summary of the Thesis

As we have illustrated throughout this chapter, the observational evidence for nonzero neutrino masses, the origin of parity violation at low energy theory and cosmological matter-antimatter asymmetry provides a strong indication for physics beyond the SM. Although many proposals have been suggested, a particularly attractive way (in our opinion) of breaking parity spontaneously in supersymmetric left-right model is possible. With this in mind, our work involves studying several classes of supersymmetric models to have spontaneously parity breaking, neutrino mass via seesaw mechanism and their connection to lepton asymmetry and self cosistency with RG running of the coupling constant.

The first part of our work is a comprehensive analysis on supersymmetric left-right models in the context of spontaneous parity breaking. We propose a novel implementation of spontaneous parity breaking in supersymmetric left-right symmetric model, avoiding some of the problems encountered in previous studies by including a bitriplet and a singlet, in addition to the bidoublets which extend the Higgs sector of the Minimal Supersymmetric Standard Model (MSSM). The supersymmetric vacua of this theory are shown to lead generically to spontaneous violation of parity, while preserving R parity. The model is shown to reproduce the see-saw relation for vacuum expectation values, $v_L v_R \approx m_{EW}^2$ relating the new mass scales v_L, v_R to the electroweak scale m_{EW}, just as in the non-supersymmetric version. The scale v_R determines the mass scale of heavy Majorana neutrinos, which gets related to the observed neutrino masses through type II see-saw relation.

We have discussed the different scenarios of spontaneous breaking of D-Parity in both non-Susy and Susy version of left right symmetric models. We explore the possibility of a TeV scale $SU(2)_R$ breaking scale M_R and hence TeV scale right handed neutrinos from both minimization of the scalar potential as well as the coupling constant unification point of view. We show that although minimization of the scalar potential allows the possibility of a TeV scale MR and tiny neutrino masses in LRSM with spontaneous D-parity breaking, the gauge coupling unification at a high scale $\sim 10^{16}$ GeV does not favor a TeV scale symmetry breaking except in the SUSYLR with Higgs doublet and bidoublet. The phenomenology of neutrino mass is also discussed.

The question of parity breaking in a supersymmetric left-right model, in which the left-right symmetry is broken with Higgs doublets (carrying $B-L = \pm 1$) instead of triplet Higgs scalars (carrying $B-L = \pm 2$) has been presented. Unlike the left-right symmetric models with triplet Higgs scalars (carrying $B-L = \pm 2$), in this model it is possible to break parity spontaneously by adding a parity-odd singlet. We then discussed how neutrino mass of type-III seesaw can be invoked in this model by adding extra fermion singlets. We considered simple forms of the mass matrices that are consistent with the unification scheme and demonstrate how they can reproduce the required neutrino mixing matrix. In this model, the baryon asymmetry of the Universe is generated via leptogenesis. The required mass scales in the model are then found to be consistent with the gauge coupling unification.

We have analyzed the SU(5) gauge coupling unification and argue that the gravitational corrections to gauge coupling constants may not vanish when higher dimensional non-renormalizable terms are included in the problem.

We have constructed an explicit model to implement the idea of electromagnetic leptogenesis, a simple extension of the standard model with few extra scalars and fermions and a discrete symmetry, which can explain non-zero light neutrino mass and generate a baryon asymmetry of the universe through leptogenesis at the TeV scale, where the CP violation comes from the electric dipole moment of the neutrinos. The usual decays of the right-handed neutrinos are forbidden, but there is an effective coupling between the left-handed and right-handed neutrinos, through the electronmagnetic dipole moment, which allows correct leptogenesis with resonant enhancement. In this model light neutrino masses originate from the seesaw mechanism, although the right-handed neutrinos have Majorana masses of the order of TeV. All the new physics introduced are in the TeV scale, so that the model may have detectable signals at LHC or ILC.

CHAPTER

8

Appendix

8.1 Feynman Rules for Majorana Neutrinos

We will discuss the simplest Feynman Rules involved in calculation of various digrams, relevant for electromagnetic leptogenesis, used in Chapter-6. In particular, the simplified set of Feynman rules for Majorana fermions used in our calculations will be discussed. We shall follow the approach outlined in [56] and write down the corresponding rules for Majorana fermions based on a four-component version (rather than the usual two) of the Weyl spinor field, $\Psi \equiv \Psi_R + e^{i\varphi}\Psi_R^c$ (i.e. the Majorana field).

There are basically two types of interactions which are relevant for leptogenesis. Firstly, we have the Yukawa coupling between ℓ_L and N_R, and secondly, we have the electromagnetic dipole interaction between ν_L and ν_R. To be consistent with the notation used in the last section, let us again begin by writing down the interaction Lagrangian in terms of the chiral field ν_R

$$\mathcal{L}_{\rm int} = -\overline{\ell}_L\, Y\, \nu_R\, \phi - \overline{\nu}_L\, \tilde{\lambda}\, \sigma_{\alpha\beta}\, \nu_R\, F^{\alpha\beta} + \text{h.c.}\,, \tag{8.1}$$

where $\ell_L = (\nu_L, e_L)^T$ and $\phi = (\phi^0, \phi^-)^T$ are doublets of $SU(2)_L$.

The interactions which are relevant for electromagnetic leptogenesis are as follows: (i,e. the Yukawa coupling between ℓ_L and N_R and the electromagnetic dipole interaction)

$$\mathcal{L}_{\rm int} = -y_H\, \overline{N_R}\, E_L\, H^+ - y_\Sigma\, \overline{(\ell_L)^c}\, (E_L)^c\, \Sigma - \overline{\nu}_L\, \tilde{\lambda}\, \sigma_{\alpha\beta}\, \nu_R\, F^{\alpha\beta} + \text{h.c.}\,, \tag{8.2}$$

8.1.1 Majorana fermion propagator

Since the Majorana fermion of interest in electromagnetic leptogenesis is the RH neutrino ν_R, let us first discuss the Majorana propagator. To begin with, we write down the theory in terms of the two-component RH neutrino field, $\nu_R = (\nu_{R1}, \nu_{R2}, \nu_{R3})^T$, where the subscripts are indices in flavor space:

$$\mathcal{L}_{\nu_R} = i\,\overline{\nu}_R\,\partial\!\!\!/\,\nu_R - \frac{1}{2}\,\overline{(\nu_R)^c}\,M_R\,\nu_R - \frac{1}{2}\,\overline{\nu}_R\,M_R^*\,(\nu_R)^c\ . \tag{8.3}$$

To diagonalise M_R, we let $\nu_R = \eta^* V^\dagger N_R$, where $\eta = \mathrm{diag}(e^{i\varphi_1/2}, e^{i\varphi_2/2}, e^{i\varphi_3/2})$ and V is a unitary matrix. Note that one can always select V in such a way that the eigenvalues for M_R are all real and positive. We have pulled out the phase φ_k, and will identify it as the charge conjugation phase factor later. So, $\mathcal{L}_{\nu_R}$ becomes

$$\mathcal{L}_{N_R} = i\,\overline{N}_R\,\partial\!\!\!/\,N_R - \frac{1}{2}\,\overline{(N_R)^c}\,D_M\,(\eta^*)^2\,N_R - \frac{1}{2}\,\overline{N}_R\,D_M\,\eta^2\,N_R^c\ , \tag{8.4}$$

where $D_M = \mathrm{diag}(M_1, M_2, M_3)$ is the diagonal mass matrix for the RH neutrinos. At this point, it is convenient to switch to index form and rewrite $\mathcal{L}_{N_R}$ as follows:

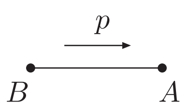
$$\mathcal{L}_{N_R} = \frac{1}{2}\left[i\,\overline{N}_{Rk}\,\partial\!\!\!/\,N_{Rk} + i\,\overline{(N_{Rk})^c}\,\partial\!\!\!/\,N_{Rk}^c - M_k\,e^{-i\varphi_k}\,\overline{(N_{Rk})^c}N_{Rk} - M_k\,e^{i\varphi_k}\,\overline{N}_{Rk}N_{Rk}^c\right]\ ,$$

$$\begin{aligned} &= \frac{1}{2}\Big[i\,(\overline{N}_{Rk} + e^{-i\varphi_k}\overline{(N_{Rk})^c})\,\partial\!\!\!/\,N_{Rk} + i\,(\overline{N}_{Rk} + e^{-i\varphi_k}\,\overline{(N_{Rk})^c})\,\partial\!\!\!/\,e^{i\varphi_k}\,N_{Rk}^c \\ &\qquad - M_k\,(\overline{N}_{Rk} + e^{-i\varphi_k}\overline{(N_{Rk})^c})N_{Rk} - M_k\,(\overline{N}_{Rk} + e^{-i\varphi_k}\overline{(N_{Rk})^c})e^{i\varphi_k}\,N_{Rk}^c\Big]\ , \\ &= \frac{1}{2}\,\left[i\,\overline{N}_k\,\partial\!\!\!/\,N_k - M_k\,\overline{N}_k\,N_k\right]\ , \end{aligned} \tag{8.5}$$

where we have introduced the four-component Majorana field, $N_k = N_{Rk} + e^{i\varphi_k}N_{Rk}^c$ which satisfies $N_k \equiv e^{i\varphi_k}N_k^c$. Using the charge conjugation conventions of (6.3), we note that $\overline{N}_k = \overline{e^{i\varphi_k}N_k^c} = -e^{-i\varphi_k}N_k^T C^\dagger$. Therefore, one may rewrite (8.5) as

$$\mathcal{L}_{N_R} = -\frac{1}{2}\,e^{-i\varphi_k}N_k^T C^\dagger\left[i\,\partial\!\!\!/ - M_k\right]N_k\ . \tag{8.6}$$

From this, the Majorana propagator for N_k can be readily read off as



$$: \qquad [S_{N_k}(p)]_{AB} = \left[\frac{-i\,(p\!\!\!/ + M_k)\,C}{p^2 - M_k^2 + i\epsilon}\right]_{AB}\ , \tag{8.7}$$

where A, B are spinor indices and p is the four-momentum. Note that this is the one and only Majorana fermion propagator arising in this approach.

8.1.2 Vertex factors involving a Majorana fermion

Using $\nu_R = \eta^* V^\dagger N_R$ to write (8.1) in the mass eigenbasis for the RH neutrinos, where all symbols are as defined in the previous section, the Lagrangian becomes

$$\mathcal{L}_{\rm int} = -\eta^*\,\overline{\ell}_L\,h\,N_R\,\phi - \eta^*\,\overline{\nu}_L\,\lambda\,\sigma_{\alpha\beta}\,N_R\,F^{\alpha\beta} + \text{h.c.}\ , \tag{8.8}$$

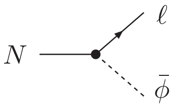
where we have set $h = YV^\dagger$ and $\lambda = \tilde{\lambda}V^\dagger$. Writing this in index form and introducing the four-component Majorana field, $N_k = N_{Rk} + e^{i\varphi_k} N_{Rk}^c$, we then get

$$\begin{aligned}
\mathcal{L}_{\rm int} = & -e^{-i\varphi_k}\,h_{jk}\,\overline{\ell}_{Lj}\,P_R\,N_k\,\phi - e^{i\varphi_k}\,h_{jk}^*\,\overline{N}_k\,P_L\,\ell_{Lj}\,\phi^\dagger \\
& - e^{-i\varphi_k}\,\lambda_{jk}\,\overline{\nu}_{Lj}\,\sigma_{\alpha\beta}\,P_R\,N_k\,F^{\alpha\beta} - e^{i\varphi_k}\,\lambda_{jk}^*\,\overline{N}_k\,\sigma_{\alpha\beta}\,P_L\,\nu_{Lj}\,F^{\alpha\beta}\ , \qquad (8.9)\\
= & \; e^{-i\varphi_k}\Big[-h_{jk}\,\overline{\ell}_{Lj}\,P_R\,N_k\,\phi + h_{jk}^*\,N_k^T\,C^\dagger\,P_L\,\ell_{Lj}\,\phi^\dagger \\
& -2\lambda_{jk}\,\overline{\nu}_{Lj}\,\sigma_{\alpha\beta}\,P_R\,N_k\,\partial^\alpha A^\beta + 2\lambda_{jk}^*\,N_k^T\,C^\dagger\,\sigma_{\alpha\beta}\,P_L\,\nu_{Lj}\,\partial^\alpha A^\beta\Big] \qquad (8.10)
\end{aligned}$$

where in the last step we have used the fact that $F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha$ with A being the photon field, and $\sigma_{\alpha\beta} = -\sigma_{\beta\alpha}$, to simplify the expression. It is important to note that the transition EMDM term displayed in (8.10) has the same form as Eq. (6.10), hence everything that we have discussed regarding the coupling λ_{jk} remains valid.

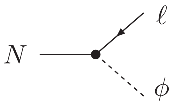
Returning to (8.10), the vertex factors for the four processes are given by :

$N_k \to \ell_{Lj}\,\bar{\phi}$:



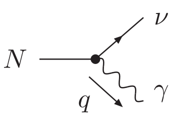
$$= -i\,h_{jk}\,P_R \tag{8.11}$$

$N_k \to \bar{\ell}_{Lj}\,\phi$:



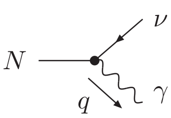
$$= i\,h_{jk}^*\,C^\dagger P_L \tag{8.12}$$

$N_k \to \nu_{Lj}\,A^\rho$:



$$= 2\lambda_{jk}\,P_R\,\sigma^{\alpha\rho}q_\alpha \tag{8.13}$$

$N_k \to \bar{\nu}_{Lj}\,A^\rho$:

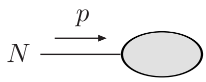


$$= -2\lambda_{jk}^*\,C^\dagger\sigma^{\alpha\rho}q_\alpha P_L \tag{8.14}$$

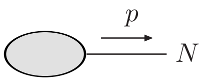
where we have again dropped the phase factor for convenience.

8.1.3 External lines for Majorana fermion

Because of the self-conjugacy of Majorana fermions, there are several possible choices in assigning spinor wave functions to the external lines. We select one convention that is consistent and use it for all diagrams. Specifically, our assignment is as follows

incoming N : 

$$= u^c(p) \tag{8.15}$$

outgoing N : 

$$= u(p) \tag{8.16}$$

8.1.4 Propagators and external fields

scalar particle ϕ : $$D(p) = \frac{i}{p^2 - m_\phi^2 + i\epsilon} \tag{8.17}$$

massless spin-1 particle : $$D_{\mu\nu}(p) = \frac{-ig_{\mu\nu}}{p^2 + i\epsilon} \tag{8.18}$$

Dirac fermion ℓ : $$[S_\ell(p)]_{AB} = \left[\frac{i(\not{p} + m_\ell)}{p^2 - m_\ell^2 + i\epsilon}\right]_{AB} \tag{8.19}$$

external scalar particle : $$1 \tag{8.20}$$

incoming/outgoing photon : $$\varepsilon_\mu(p) \,/\, \varepsilon_\mu^*(p) \tag{8.21}$$

incoming/outgoing Dirac fermion : $$u(p) \,/\, \overline{u}(p) \tag{8.22}$$

incoming/outgoing Dirac anti fermion : $$\overline{v}(p) \,/\, v(p) \tag{8.23}$$

In the above, p denotes four-momentum as usual.

8.1.5 Polarization sums and decay rates

In calculations, the following results are often useful:

$$\begin{aligned}
&\sum_s u\overline{u} = \not{p} + m \,, \quad \sum_s v\overline{v} = \not{p} - m \,, \quad \sum_{\text{pol}} \varepsilon_\mu^* \varepsilon_\nu = -g_{\mu\nu} \,, \\
&C \left[\sum_s u\overline{u}\right]^T C^\dagger = C\left(\not{p}^T + m\right) C^\dagger = -\not{p} + m \,, \\
&(u^c)^T = \overline{u}\, C^T \,, \quad (u^c)^\dagger = -u^T C^\dagger \gamma^0 \,, \quad (u^c)^* = C^* \gamma^0 u \,, \\
&\gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0 \,, \quad \sigma^{\mu\nu\dagger} = \gamma^0 \sigma^{\mu\nu} \gamma^0 \,,
\end{aligned} \tag{8.24}$$

8.2 Decay rate calculation $N_k \to \nu_j \gamma$

The EMDM interaction Lagrangian is

$$\mathcal{L}^{\rm 5D}_{\rm EM} = -\lambda_{jk}\,\overline{\nu}_{Lj}\,\sigma^{\alpha\beta}\,P_R\,N_k\,F_{\alpha\beta} + \text{h.c.}\ , \tag{8.25}$$

where $j = e, \mu, \tau$ and $k = 1, 2, 3$. $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is the electromagnetic field strength tensor, with A_α being the photon field.

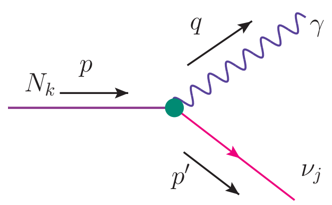


Figure 8.1: The Feynman graph for the lowest order decay, $N_k \to \nu_j\,\gamma$ via the dimension-5 EMDM coupling of Eq. (8.25). Here $q = p - p'$ and $2\lambda_{jk}\,P_R\,\sigma^{\alpha\beta}\,q_\beta$ is the vertex factor.

Now we will calculate the lowest order contribution to the decay rate, $\Gamma_{(N_k \to \nu\,\gamma)}$ and from now onwards we shall call it as tree level diagram. The tree-level diagram for this process is depicted in Fig. 8.1. The amplitude for the tree level process is as follows

$$-i\mathcal{M} = \bar{u}_j(2\lambda_{jk}P_R\sigma^{\alpha\rho}q_\alpha)u^c_k\varepsilon^*_\rho\ , \tag{8.26}$$

$$\begin{aligned}
\Rightarrow\qquad |\mathcal{M}|^2 &= \bar{u}_j(2\lambda_{jk}P_R\sigma^{\alpha\rho}q_\alpha)u^c_k\varepsilon^*_\rho\left[\bar{u}_j(2\lambda_{jk}P_R\sigma^{\beta\sigma}q_\beta)u^c_k\varepsilon^*_\sigma\right]^\dagger\ ,\\
&= 4(\lambda^*_{jk}\lambda_{jk})\bar{u}_jP_R\sigma^{\alpha\rho}q_\alpha u^c_k\varepsilon^*_\rho\varepsilon_\sigma(-u^T_kC^\dagger\gamma^0)\gamma^0\sigma^{\beta\sigma}q_\beta\gamma^0P_R\gamma^0u_j\ ,\\
&= -4(\lambda^*_{jk}\lambda_{jk})\bar{u}_jP_R\,\frac{i}{2}\left[\gamma^\alpha,\gamma^\rho\right]q_\alpha C\bar{u}^T_k u^T_k C^\dagger\,\frac{i}{2}\left[\gamma^\beta,\gamma^\sigma\right]q_\beta P_L u_j\,\varepsilon^*_\rho\varepsilon_\sigma
\end{aligned} \tag{8.27}$$

Averaging initial and summing final polarizations, we obtain

$$\begin{aligned}
|\overline{\mathcal{M}}|^2 &= (\lambda^*_{jk}\lambda_{jk})\,P_R\,(\not{q}\gamma^\rho-\gamma^\rho\not{q})C\left[\frac{1}{2}\sum_s u_k\bar{u}_k\right]^T C^\dagger(\not{q}\gamma^\sigma-\gamma^\sigma\not{q})P_L\sum_{s'}u_j\bar{u}_j\sum_{\rm pol}\varepsilon^*_\rho\varepsilon_\sigma\ ,\\
&= \frac{1}{2}(\lambda^*_{jk}\lambda_{jk})\ \mathrm{Tr}\left[P_R\,(\not{q}\gamma^\rho-\gamma^\rho\not{q})(-\not{p}+M_k)(\not{q}\gamma^\sigma-\gamma^\sigma\not{q})P_L\,\not{q}'(-g_{\rho\sigma})\right]\ ,\\
&\qquad\vdots\\
&= (\lambda^*_{jk}\lambda_{jk})\left[16(p\cdot q)(p'\cdot q)-4(p\cdot p')(q\cdot q)\right]\ .
\end{aligned} \tag{8.28}$$

where we have taken the masses of the light neutrino and photon to be zero. Working in the centre-of-mass frame where

$$p = (M_k\ ,\ \vec{0})\ , \quad p' = (M_k/2\ ,\ -\vec{q})\ , \quad q = (M_k/2\ ,\ \vec{q})\ , \quad |\vec{q}| = M_k/2\ , \qquad \text{and}$$
$$p \cdot p' = p \cdot q = p' \cdot q = M_k^2/2\ , \quad p^2 \equiv (p \cdot p) = M_k^2\ , \quad q^2 = (p')^2 = 0\ , \tag{8.29}$$

Eq. (8.28) becomes

$$|\overline{\mathcal{M}}|^2 = 4\,(\lambda_{jk}^* \lambda_{jk})\, M_k^4\ . \tag{8.30}$$

$$\begin{aligned} \Gamma(N_k \to \nu\,\gamma) &= \frac{|\vec{q}|}{8\pi E_{\rm cm}^2} |\overline{\mathcal{M}}|^2\ , \\ &= \frac{1}{8\pi}\frac{M_k}{2}\frac{1}{M_k^2}\, 4\,(\lambda^\dagger\lambda)_{kk}\, M_k^4\ , \\ &= \frac{(\lambda^\dagger\lambda)_{kk}}{4\pi}\, M_k^3 \end{aligned} \tag{8.31}$$

where we have summed over j. Since we must necessarily have $\Gamma(N_k \to \nu\,\gamma) \equiv \Gamma(N_k \to \overline{\nu}\,\gamma)$, the total decay rate is, $\Gamma_{\rm tot} = 2\Gamma(N_k \to \nu\,\gamma)$, to first order.

8.3 One loop Self-energy calculation for CP asymmetry

In this section, we present the calculational details of the self-energy contributions to the CP asymmetry in standard leptogenesis with the help of simplified Majorana Feynman rules, as well as to confirm that the known results can be obtained this way. Note that there are actually *two* separate self-energy graphs that contribution to the interference term when final state flavor j which is not summed over.

Interference term involving the self-energy correction of Fig. [8.2](a)

Firstly, let us consider the self-energy contributions. Applying the Feynman rules developed for the EMDM couplings, we can write down the interference term of between the tree level diagram [8.1] and the self energy diagram [8.2] as

$$\begin{aligned} I^{5\rm D}_{\text{self-(a)}} = \int \frac{d^4 q_1}{(2\pi)^4}\, (16\lambda_{jk}^*\lambda_{jm}\lambda_{nm}\lambda_{nk}^*)\, [\overline{u}_j]_{1C}\, [P_R\sigma^{\alpha\nu} q_\alpha]_{CA}\, [S_{N_m}(p)]_{AB} \left[P_R \sigma^{\beta\sigma}(-q_{2\beta})\right]_{FB} \\ \times [S_\ell(-q_1)]_{EF} \left[C^\dagger \sigma^{\delta\mu} q_{2\delta} P_L\right]_{DE} [u_k^c]_{D1}\, [D_{\sigma\mu}(q_2)\varepsilon_\nu^*]_{11} \left[-u_k^T C^\dagger \sigma^{\eta\rho} q_\eta P_L u_j \varepsilon_\rho\right]_{11}\ , \end{aligned}$$

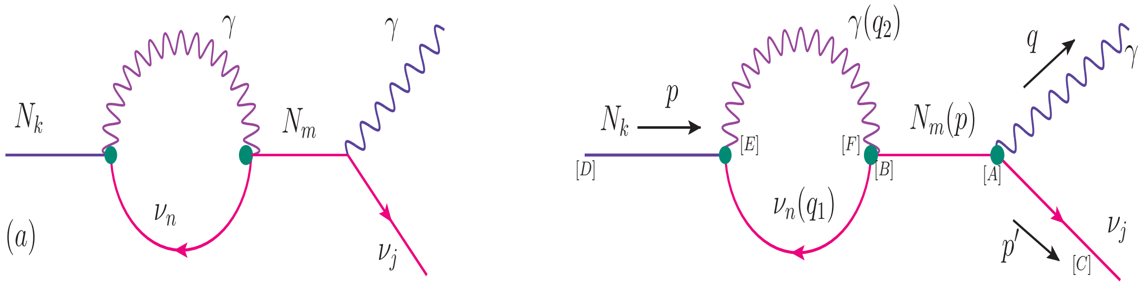


Figure 8.2: Feynman diagram for the self energy contribution to the CP-asymmetry of N_k decay via dim-5 EMDM coupling with $\overline{\nu_n}$ as the intermediate state.

where the $-q_{2\beta}$ in $[\cdots]_{FB}$ comes from the fact that photon momentum, $q_{2\beta}$ is flowing *into* the vertex. Letting $A_\lambda^{(5)} = \lambda^*_{jk}\lambda_{jm}\lambda_{nm}\lambda^*_{nk}$ and using matrix form, we then have

$$\begin{aligned}
I^{5\text{D}}_{\text{self-(a)}} &= 16 A_\lambda^{(5)} \int \frac{d^4 q_1}{(2\pi)^4} \frac{\overline{u}_j\, P_R \sigma^{\alpha\nu} q_\alpha (-i)(\not{p} + M_m) C\, (-\sigma^{\beta\sigma} q_{2\beta})^T P_R^T (i) (-\not{q}_1)^T}{(p^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)} \\
&\qquad \times P_L^T (\sigma^{\delta\mu} q_{2\delta})^T C^* u_k^c (-i) g_{\sigma\mu}\, \varepsilon^*_\nu \varepsilon_\rho (-1) u_k^T C^\dagger \sigma^{\eta\rho} q_\eta P_L u_j \,, \\
&= 8i\, A_\lambda^{(5)} \int \frac{d^4 q_1}{(2\pi)^4} \frac{P_R \sigma^{\alpha\nu} q_\alpha (\not{p} + M_m) \sigma^{\beta\sigma} q_{2\beta} P_R \not{q}_1 P_L \sigma^{\delta\mu} q_{2\delta} C \left[\sum_s u_k \overline{u}_k\right]^T C^\dagger}{(p^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)} \\
&\qquad \times \sigma^{\eta\rho} q_\eta P_L g_{\sigma\mu} \sum_{s'} u_j \overline{u}_j \sum_{\text{pol}} \varepsilon^*_\nu \varepsilon_\rho \,, \\
&\qquad\qquad \vdots \\
&= -\frac{i\, A_\lambda^{(5)} M_m M_k}{2} \int \frac{d^4 q_1}{(2\pi)^4} \frac{1}{(p^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)} \\
&\qquad \times \text{Tr}\left[P_R (\not{q}\gamma^\nu - \gamma^\nu \not{q})(\not{q}_2 \gamma^\sigma - \gamma^\sigma \not{q}_2)\not{q}_1 (\not{q}_2 \gamma_\sigma - \gamma_\sigma \not{q}_2)(\not{q}\gamma_\nu - \gamma_\nu \not{q})\not{p}' \right] \,, \\
&= i\, A_\lambda^{(5)} M_m M_k \int \frac{d^4 q_1}{(2\pi)^4} \frac{(p' \cdot q)\left[-256 (q \cdot q_2)(q_1 \cdot q_2) + 64 (q \cdot q_1) q_2^2\right]}{(p^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)} \,. \qquad (8.32)
\end{aligned}$$

The discontinuity of the integral

$$I^{5\text{D}}_{\text{s-(a)}} \equiv i\, M_m M_k \int \frac{d^4 q_1}{(2\pi)^4} \frac{(p' \cdot q)\left[-256 (q \cdot q_2)(q_1 \cdot q_2) + 64 (q \cdot q_1) q_2^2\right]}{(p^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)} \,, \qquad (8.33)$$

may be determined by the cutting rules as described before, hence

$$\begin{aligned}
\text{Disc}\left[I^{5\text{D}}_{\text{s-(a)}} \right] &= i\, M_k M_m \int \frac{d^4 q_1}{(2\pi)^4} \frac{(-2\pi i)^2 \delta(q_1^2) \delta\left[(p - q_1)^2\right] \Theta(E_1) \Theta(M_k - E_1)}{p^2 - M_m^2} \\
&\qquad \times M_k^2 \left[-128 (q \cdot q_2)(q_1 \cdot q_2) + 32 (q \cdot q_1) q_2^2\right] \,, \qquad (\epsilon \to 0) \,.
\end{aligned}$$

Using $q_1 = (E_1, \vec{q}_1)$, $q_2 = p - q_1$ and (8.29) to simplify, we eventually get

$$\begin{aligned}
\text{Disc}\left[I^{5\text{D}}_{\text{s-(a)}}\right] &= \frac{-i\, M_k^4 M_m}{4\pi^2 (M_k^2 - M_m^2)} \int d^3q_1 dE_1 \,\frac{1}{2|\vec{q}_1|}\delta(E_1 - |\vec{q}_1|)\delta\left[(M_k - E_1)^2 - |\vec{q}_1|^2\right]\Theta(E_1) \\
&\quad \times \Theta(M_k - E_1)\left[-64\,(M_k - E_1 + |\vec{q}_1|\cos\theta)\,(M_k E_1 - E_1^2 + |\vec{q}_1|^2)\right. \\
&\quad \left. +16\,(E_1 - |\vec{q}_1|\cos\theta)\left((M_k - E_1)^2 - |\vec{q}_1|^2\right)\right] ,
\end{aligned}$$

where θ is the smaller angle between $\vec{q}_1$ and $\vec{q}$. Performing the integrals using all the standard tricks, we obtain

$$\begin{aligned}
\text{Disc}\left[I^{5\text{D}}_{\text{s-(a)}}\right] &= \frac{-i\, M_k^4 M_m}{8\pi^2 (M_k^2 - M_m^2)} \int |\vec{q}_1|^2 d|\vec{q}_1| d\Omega\;\delta\left[M_k^2 - 2M_k|\vec{q}_1|\right]\Theta(M_k - |\vec{q}_1|) \\
&\quad \times \left[-64\,(M_k - |\vec{q}_1| + |\vec{q}_1|\cos\theta)\,(M_k) + 16\,(1 - \cos\theta)\left(M_k^2 - 2M_k|\vec{q}_1|\right)\right] , \\
&\quad \vdots \\
&= \frac{-i\, M_k^4 M_m}{16\pi^2 (M_k^2 - M_m^2)} \int d\Omega\,\frac{M_k^2}{4}\left[-64\left(\frac{M_k}{2} + \frac{M_k}{2}\cos\theta\right) + 16\,(1 - \cos\theta)\right] , \\
&= \frac{2i\, M_k^7 M_m}{\pi (M_k^2 - M_m^2)} .
\end{aligned} \tag{8.34}$$

The imaginary part of this interference term and its corresponding phase space, V_φ are given by

$$\text{Im}\left[I^{5\text{D}}_{\text{s-(a)}}\right] = \frac{M_k^7 M_m}{\pi (M_k^2 - m_{Nm}^2)} , \qquad V_\varphi = \frac{|\vec{q}|}{8\pi E_{\text{cm}}^2} = \frac{1}{16\pi M_k} . \tag{8.35}$$

Note that unlike standard leptogenesis, there are no extra factors of 2 in the phase space for this diagram because only one intermediate (and final) state is possible. Putting everything together, the CP asymmetry due to this interference term is

$$\begin{aligned}
\varepsilon^{5\text{D}}_{\text{self-(a)-}k,j} &= -\frac{4}{\Gamma_{\text{tot}}} \sum_{m\neq k}\sum_{n} \text{Im}\left[A_\lambda^{(5)}\right]\,\text{Im}\left[I^{5\text{D}}_{\text{s-(a)}} V_\varphi\right] , \\
&= -\frac{M_k^2}{2\pi (\lambda^\dagger\lambda)_{kk}} \sum_{m\neq k} \text{Im}\left[\lambda_{jk}^* \lambda_{jm} (\lambda^\dagger\lambda)_{km}\right]\frac{\sqrt{z}}{1 - z} ,
\end{aligned} \tag{8.36}$$

where $z \equiv M_m^2/M_k^2$.

Interference term involving the self-energy correction of Fig. [8.3](b)

Now we will calculate the interference term between the tree level diagram [8.1] and the self energy diagram [8.3](b) which is

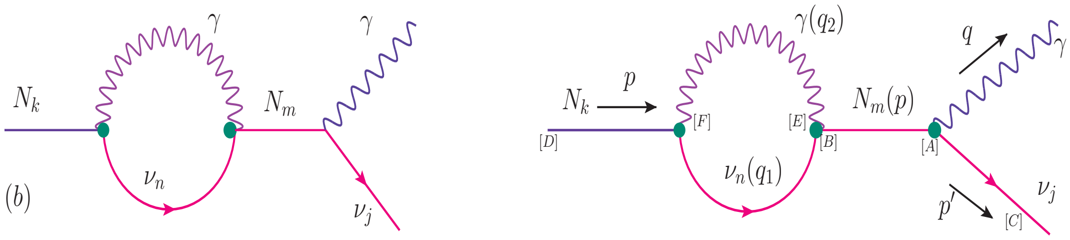


Figure 8.3: Feynman diagram for the self energy contribution to the CP-asymmetry of N_k decay via dim-5 EMDM coupling with ν_n as the intermediate state.

$$
\begin{aligned}
I^{5D}_{\text{self-(b)}} = & \int \frac{d^4 q_1}{(2\pi)^4}\, 16 B^{(5)}_\lambda \, [\overline{u}_j]_{1C}\, [P_R \sigma^{\alpha\nu} q_\alpha]_{CA}\, [S_{N_m}(p)]_{AB} \left[C^\dagger \sigma^{\beta\sigma}(-q_{2\beta}) P_L\right]_{BE} \\
& \times [S_\ell(q_1)]_{EF} \left[P_R \sigma^{\delta\mu} q_{2\delta}\right]_{FD} [u^c_k]_{D1}\, [D_{\sigma\mu}(q_2)\varepsilon^*_\nu]_{11} \left[-u^T_k C^\dagger \sigma^{\eta\rho} q_\eta P_L u_j\, \varepsilon_\rho\right]_{11} \ .
\end{aligned}
$$

where $B^{(5)}_\lambda = \lambda^*_{jk}\lambda_{jm}\lambda^*_{nm}\lambda_{nk}$,

$$
\begin{aligned}
&= 16 B^{(5)}_\lambda \int \frac{d^4 q_1}{(2\pi)^4} \frac{\overline{u}_j\, P_R \sigma^{\alpha\nu} q_\alpha (-i)(\not{p} + M_m) C C^\dagger\, \sigma^{\beta\sigma}(-q_{2\beta}) P_L (i) \not{q}_1 P_R \sigma^{\delta\mu} q_{2\delta} u^c_k}{(p^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)} \\
&\qquad \times (-i) g_{\sigma\mu}\, \varepsilon^*_\nu \varepsilon_\rho (-1) u^T_k C^\dagger \sigma^{\eta\rho} q_\eta P_L u_j \ , \\
&= -8i B^{(5)}_\lambda \int \frac{d^4 q_1}{(2\pi)^4} \frac{P_R \sigma^{\alpha\nu} q_\alpha (\not{p} + M_m) \sigma^{\beta\sigma} q_{2\beta} P_L \not{q}_1 P_R \sigma^{\delta\mu} q_{2\delta} C \left[\sum_s u_k \overline{u}_k\right]^T C^\dagger}{(p^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)} \\
&\qquad \times \sigma^{\eta\rho} q_\eta P_L g_{\sigma\mu} \sum_{s'} u_j \overline{u}_j \sum_{\text{pol}} \varepsilon^*_\nu \varepsilon_\rho \ , \\
&= B^{(5)}_\lambda \int \frac{d^4 q_1}{(2\pi)^4} \frac{-i \mathrm{Tr}[P_R(\not{q}\gamma^\nu - \gamma^\nu \not{q})\not{p}(\not{q}_2 \gamma^\sigma - \gamma^\sigma \not{q}_2)\not{q}_1(\not{q}_2 \gamma_\sigma - \gamma_\sigma \not{q}_2)\not{p}(\not{q}\gamma_\nu - \gamma_\nu \not{q})\not{p}']}{2\,(p^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)}, \\
&= B^{(5)}_\lambda \int \frac{d^4 q_1}{(2\pi)^4} \frac{32 i M_k^4 \left[4(q\cdot q_2)(q_1\cdot q_2) - (q\cdot q_1) q_2^2 - 4(p\cdot q_2)(q_1\cdot q_2) + (p\cdot q_1) q_2^2\right]}{(p^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)},
\end{aligned}
\tag{8.37}
$$

Focusing on the integral:

$$
I^{5D}_{\text{s-(b)}} \equiv 32 i\, M_k^4 \int \frac{d^4 q_1}{(2\pi)^4} \frac{4(q\cdot q_2)(q_1\cdot q_2) - 4(p\cdot q_2)(q_1\cdot q_2) - (q\cdot q_1) q_2^2 + (p\cdot q_1) q_2^2}{(p^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)} \ .
\tag{8.38}
$$

The discontinuity of this integral is determined by cutting through the propagators with momenta q_1 and q_2, which then results in ($q_2 = p - q_1, q_1 \equiv (E_1, \vec{q}_1)$):

$$
\begin{aligned}
\mathrm{Disc}(I^{5D}_{\text{s-(b)}}) = & \frac{32 i\, M_k^4}{M_k^2 - M_m^2} \int \frac{d^4 q_1}{(2\pi)^4} (-2\pi i)^2 \delta(q_1^2) \delta\left[(p - q_1)^2\right] \Theta(E_1)\Theta(M_k - E_1) \\
& \times \left[4(q\cdot q_2)(q_1\cdot q_2) - 4(p\cdot q_2)(q_1\cdot q_2) - (q\cdot q_1) q_2^2 + (p\cdot q_1) q_2^2\right] \ ,
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{32i\, M_k^4}{M_k^2 - M_m^2} \int \frac{d^4 q_1}{(2\pi)^4} \, (-2\pi i)^2 \delta(q_1^2) \delta\left[(p - q_1)^2\right] \Theta(E_1) \Theta(M_k - E_1) \\
&\qquad \times \left[4(q \cdot q_2)(q_1 \cdot q_2) - 4(p \cdot q_2)(q_1 \cdot q_2) - (q \cdot q_1) q_2^2 + (p \cdot q_1) q_2^2\right] \ , \\
&= \frac{-8i\, M_k^4}{\pi^2 (M_k^2 - M_m^2)} \int d^3 q_1 dE_1 \ \delta(E_1^2 - |\vec{q}_1|^2) \delta\left[(M_k - E_1)^2 - |\vec{q}_1|^2\right] \Theta(E_1) \Theta(M_k - E_1) \\
&\qquad \times \Bigg[4(M_k E_1 - E_1^2 + |\vec{q}_1|^2) \left(\frac{M_k^2}{2} - \frac{M_k}{2}(E_1 - |\vec{q}_1| \cos\theta) - M_k^2 + M_k E_1\right) \\
&\qquad \quad + \left(M_k E_1 - \frac{M_k}{2}(E_1 - |\vec{q}_1| \cos\theta)\right) ((M_k - E_1)^2 - |\vec{q}_1|^2)\Bigg] \ ,
\end{aligned}
$$

where θ is the smaller angle between $\vec{q}$ and $\vec{q}_1$,

$$
\begin{aligned}
\mathrm{Disc}(I^{5\mathrm{D}}_{\text{s-(b)}}) &= \frac{-2i\, M_k^5}{\pi^2 (M_k^2 - M_m^2)} \int |\vec{q}_1|^2 d|\vec{q}_1| d\Omega \ \frac{1}{|\vec{q}_1|} \ \delta\left[(M_k - |\vec{q}_1|)^2 - |\vec{q}_1|^2\right] \Theta(M_k - |\vec{q}_1|) \\
&\quad \times \left[4M_k |\vec{q}_1| \left(-M_k + |\vec{q}_1| + |\vec{q}_1| \cos\theta\right) + |\vec{q}_1| \left(1 + \cos\theta\right) \left((M_k - |\vec{q}_1|)^2 - |\vec{q}_1|^2\right)\right] , \\
&= \frac{-2i\, M_k^5}{\pi^2 (M_k^2 - M_m^2)} \int |\vec{q}_1|^2 d|\vec{q}_1| d\Omega \ \frac{1}{|-2M_k|} \ \delta\left[|\vec{q}_1| - \frac{M_k}{2}\right] \Theta(M_k - |\vec{q}_1|) \\
&\quad \times \left[4M_k \left(-M_k + |\vec{q}_1| + |\vec{q}_1| \cos\theta\right) + M_k \left(1 + \cos\theta\right) \left(M_k - 2|\vec{q}_1|\right)\right] \ , \\
&= \frac{-i\, M_k^5}{\pi^2 (M_k^2 - M_m^2)} \int d\phi \int d(\cos\theta) \ \frac{M_k^2}{4} \times 4 \times -\frac{M_k}{2}(1 - \cos\theta) \ , \\
&= \frac{2i\, M_k^8}{\pi (M_k^2 - M_m^2)} \ .
\end{aligned}
\tag{8.39}
$$

So, the the imaginary part is given by

$$
\mathrm{Im}\left[I^{5\mathrm{D}}_{\text{s-(b)}}\right] = \frac{1}{2i} \ \mathrm{Disc}\left[I^{5\mathrm{D}}_{\text{s-(b)}}\right] = \frac{M_k^8}{\pi (M_k^2 - M_m^2)} \ . \tag{8.40}
$$

The total decay rate is given by the twice of (8.31) and phase space is same as for Fig. [8.2](a) with $V_\varphi = 1/16\pi M_k$. Therefore,

$$
\begin{aligned}
\varepsilon^{5\mathrm{D}}_{\text{self-(b)-}k,j} &= -\frac{4}{\Gamma_{\text{tot}}} \ \sum_{m \neq k} \sum_{n} \mathrm{Im}(B_\lambda^{(5)}) \ \mathrm{Im}(I^{5\mathrm{D}}_{\text{s-(b)}} V_\varphi) \ , \\
&= -\frac{M_k^2}{2\pi (\lambda^\dagger \lambda)_{kk}} \sum_{m \neq k} \mathrm{Im}\left[\lambda^*_{jk} \lambda_{jm} (\lambda^\dagger \lambda)_{mk}\right] \frac{1}{1 - z}
\end{aligned}
\tag{8.41}
$$

where we have summed over all heavy Majorana neutrino species $m \neq k$, as well as internal lepton species n. This expression is valid for the hierarchical neutrinos (i.e, for $M_k \ll M_j$).

Resonant enhancement: When two RH neutrinos are nearly degenerate in mass

When we start calculating the imaginary part of the interference term, we did not consider that right handed Majorana neutrinos are unstable and hence can be decay. In this situation, one should take into account the decay width of the heavy Majorana neutrinos in the propagator. So one should write $p^2 - M_m^2 + iM_k\Gamma_m$ instead of $p^2 - M_m^2 + i\epsilon$. With this, the expression for Discontinuity relation becomes

$$\begin{aligned}
\mathrm{Disc}(I^{5\mathrm{D}}_{\text{self-res:(b)}}) &= B^{(5)}_\lambda \int \frac{d^4 q_1}{(2\pi)^4} \\
&\times \frac{32iM_k^4\left[4(q\cdot q_2)(q_1\cdot q_2) - (q\cdot q_1)q_2^2 - 4(p\cdot q_2)(q_1\cdot q_2) + (p\cdot q_1)q_2^2\right]}{(p^2 - M_m^2 + iM_k\Gamma_m)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)} \\
&\vdots \\
&= \frac{i\,M_k^5}{\pi^2(M_k^2 - M_m^2 + iM_k\Gamma_m)} \int d\phi \int d(\cos\theta)\, \frac{M_k^2}{4} \times 4 \times -\frac{M_k}{2}(1-\cos\theta) \\
&= \frac{2i}{\pi}\frac{M_k^8(M_k^2 - M_m^2 - iM_k\Gamma_m)}{(M_k^2 - M_m^2)^2 + M_k^2\Gamma_m^2}
\end{aligned} \tag{8.42}$$

So, the the imaginary part is given by

$$\mathrm{Im}\left[I^{5\mathrm{D}}_{\text{self-res:(b)}}\right] = \frac{1}{2i}\,\mathrm{Disc}\left[I^{5\mathrm{D}}_{\text{self-res:(b)}}\right] = \frac{1}{\pi}\frac{M_k^8(M_k^2 - M_m^2)}{(M_k^2 - M_m^2)^2 + M_k^2\Gamma_m^2}\,. \tag{8.43}$$

The contribution to the CP asymmetry due to this self-energy enhancement is

$$\begin{aligned}
\varepsilon^{5\mathrm{D}}_{\text{self-res:(b)-}k,j} &= -\frac{4}{\Gamma_{\text{tot}}}\sum_{m\neq k}\sum_n \mathrm{Im}(A^{(5)}_\lambda)\,\mathrm{Im}(I^{5\mathrm{D}}_{\text{self-res:(b)}}V_\varphi)\,, \\
&= -\frac{M_k^2}{2\pi(\lambda^\dagger\lambda)_{kk}}\sum_{m\neq k}\mathrm{Im}\left[\lambda^*_{jk}\lambda_{jm}(\lambda^\dagger\lambda)_{mk}\right]\frac{(M_k^2 - M_m^2)\,M_k^2}{(M_k^2 - M_m^2)^2 + M_k^2\Gamma_m^2} \qquad (8.44) \\
&= -\frac{2}{(\lambda^\dagger\lambda)_{kk}(\lambda^\dagger\lambda)_{mm}}\sum_{m\neq k}\mathrm{Im}\left[\lambda^*_{jk}\lambda_{jm}(\lambda^\dagger\lambda)_{mk}\right] \\
&\times \left(\frac{M_k}{M_m}\right)^3\frac{(M_k^2 - M_m^2)\,M_k\,\Gamma_m}{(M_k^2 - M_m^2)^2 + M_k^2\Gamma_m^2} \qquad (8.45)
\end{aligned}$$

Similarly, the the imaginary part coming from the interference diagram [8.2(a)] is given by

$$\mathrm{Im}\left[I^{5\mathrm{D}}_{\text{self-res:(a)}}\right] = \frac{1}{2i}\,\mathrm{Disc}\left[I^{5\mathrm{D}}_{\text{self-res:(a)}}\right] = \frac{1}{\pi}\frac{M_k^7 M_m\,(M_k^2 - M_m^2)}{(M_k^2 - M_m^2)^2 + M_k^2\Gamma_m^2}\,. \tag{8.46}$$

and the contribution to the CP asymmetry due to this self-energy enhancement is

$$\begin{aligned}
\varepsilon^{5\mathrm{D}}_{\text{self-res:(a)-}k,j} &= -\frac{4}{\Gamma_{\mathrm{tot}}} \sum_{m\neq k} \sum_{n} \mathrm{Im}(A^{(5)}_{\lambda})\ \mathrm{Im}(I^{5\mathrm{D}}_{\text{self-res:(a)}} V_{\varphi})\ , \\
&= -\frac{M_k^2}{2\pi(\lambda^\dagger\lambda)_{kk}} \sum_{m\neq k} \mathrm{Im}\left[\lambda^*_{jk}\lambda_{jm}(\lambda^\dagger\lambda)_{km}\right] \frac{(M_k^2 - M_m^2)\, M_k\, M_m}{(M_k^2 - M_m^2)^2 + M_k^2\Gamma_m^2} \qquad (8.47) \\
&= -\frac{2}{(\lambda^\dagger\lambda)_{kk}(\lambda^\dagger\lambda)_{mm}} \sum_{m\neq k} \mathrm{Im}\left[\lambda^*_{jk}\lambda_{jm}(\lambda^\dagger\lambda)_{mk}\right] \\
&\times \left(\frac{M_k}{M_m}\right)^2 \frac{(M_k^2 - M_m^2)\, M_k\, \Gamma_m}{(M_k^2 - M_m^2)^2 + M_k^2\Gamma_m^2} \qquad (8.48)
\end{aligned}$$

where the tree level (lowest order) decay rate is $\Gamma(N_k \to \nu\gamma) = \frac{(\lambda^\dagger\lambda)_{kk}}{4\pi} M_k^3$.

Hence, the total contribution to the CP-asymmetry due to self-energy enhancement, when the heavy right handed neutrinos are nearly degenerate in mass, is the sum of these terms (8.47) and (8.44) and is given by

$$\varepsilon^{5\mathrm{D}}_{\text{self-res-}k,j} = \varepsilon^{5\mathrm{D}}_{\text{self-res:(a)-}k,j} + \varepsilon^{5\mathrm{D}}_{\text{self-res:(b)-}k,j} \qquad (8.49)$$

8.4 Calculation involving 5D-EMDM coupling strength

In this section, we will present the detailed calculation for the five-dimensional dipole moment coupling between light ν and heavy N neutrinos before deduce the potential implications of these EMDM operators for the electromagnetic leptogenesis as discussed in chapter-[6]. The general form of this dipole moment coupling of the light neutrinos, ν, to the heavy neutrinos, N, is given by $\overline{\nu}\lambda\sigma_{\alpha\beta}NB^{\alpha\beta}$, where λ is the five-dimension EMDM coupling constant (λ mainly gives information about the magnetic and electric transition moments). In the subsequent discussion , we will evaluate whether the lepton number violating radiative decay of the heavy sterile neutrinos ($N \to \nu\gamma$) through this 5D-dipole operator which can explain the baryon asymmetry of our present universe. For this calculation, we have considered a minimal extension of the SM with right-handed neutrinos, one vector like charged fermion E^-, a charged scalar H^+, two extra Higgs doublets (Σ, D). The Feynman digram for this 5D-EMDM coupling constant is shown in figure (8.4). The amplitude for this loop diagram is

$$\mathcal{M} = \overline{u_j}(p')\, \Gamma^\mu\, u^c(p)\, A_\mu(q) \qquad (8.50)$$

where the vertex factor Γ contains the contribution from the three diagrams shown in figure (8.4) and can be written in three parts.

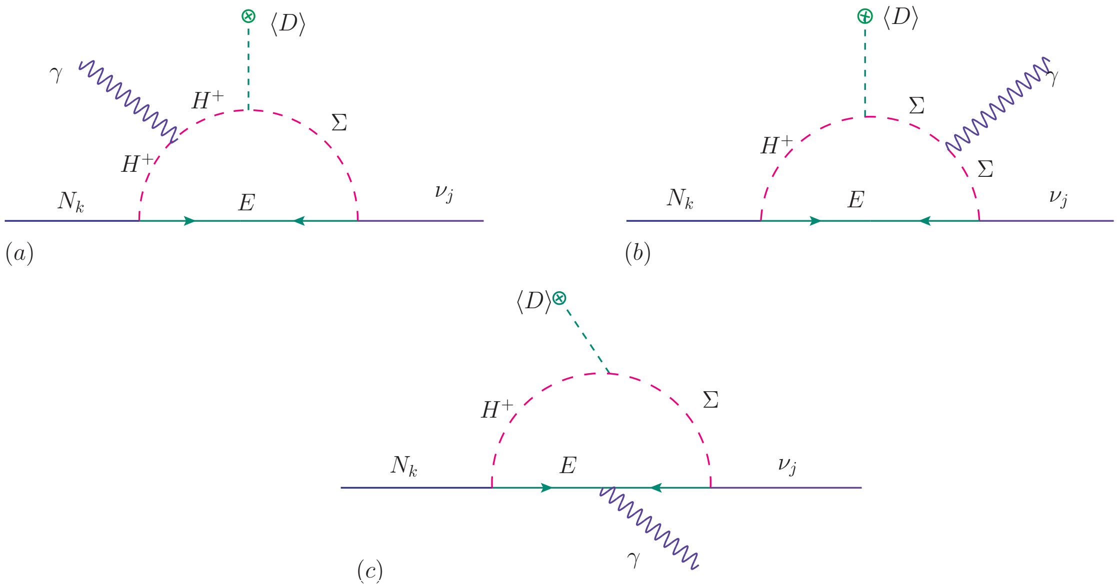


Figure 8.4: Feynman diagrams which estimate the effective EMDM coupling strength between light neutrino ν_j and N_k.

The vertex contribution from the diagram (8.4[a], [b], [c]) is given by

$$\Gamma^{\mu}_{[a]} = \int \frac{d^4k}{(2\pi)^4}\,(i\,y^*_{\Sigma}\,P_R)\,\frac{i}{\not{k}-M_E+i\,\epsilon}\,(-i\,y_H\,P_R)\,\frac{i}{[(p-k)^2-M_H^2+i\,\epsilon]} \times (i\,e\,[p-p']^{\mu})\,\frac{i}{[(p'-k)^2-M_H^2+i\,\epsilon]}\,(i\,\mu_s\,v_D)\,\frac{i}{[(p'-k)^2-M_{\Sigma}^2+i\,\epsilon]} \tag{8.51}$$

$$\Gamma^{\mu}_{[b]} = \int \frac{d^4k}{(2\pi)^4}\,(i\,y^*_{\Sigma}\,P_R)\,\frac{i}{\not{k}-M_E+i\,\epsilon}\,(-i\,y_H\,P_R)\,\frac{i}{[(p-k)^2-M_H^2+i\,\epsilon]} \times (i\,\mu_s\,v_D)\,\frac{i}{[(p-k)^2-M_{\Sigma}^2+i\,\epsilon]}\,(i\,e\,[p-p']^{\mu})\,\frac{i}{[(p'-k)^2-M_{\Sigma}^2+i\,\epsilon]} \tag{8.52}$$

$$\Gamma^{\mu}_{[c]} = \int \frac{d^4k}{(2\pi)^4}\,(i\,y^*_{\Sigma}\,P_R)\,\frac{i}{\not{k}-\not{q}-M_E+i\,\epsilon}\,(i\,e\,\gamma^{\mu})\,\frac{i}{[\not{k}-M_E+i\,\epsilon]}(-i\,y_H\,P_R) \times \frac{i}{[(p-k)^2-M_H^2+i\,\epsilon]}\,(i\,\mu_s\,v_D)\,\frac{i}{[(p'-k)^2-M_{\Sigma}^2+i\,\epsilon]} \tag{8.53}$$

Let us consider the vertex contribution coming from the diagram (8.4[c]). This can be written as

$$\Gamma^{\mu}_{[c]} = (e\,y^*_{\Sigma}\,y_H\,\mu_s\,v_D)\int \frac{d^4k}{(2\pi)^4}\mathcal{D}^{-1}\,P_R[\not{k}-\not{q}+M_E]\gamma^{\mu}\,[\not{k}+M_E]\,P_R \tag{8.54}$$

where

$$\mathcal{D}^{-1} = \left[\{(k-q)^2 - M_E^2]\}\,\{k^2 - M_E^2]\}\,\{(p-k)^2 - M_H^2\}\{(p'-k)^2 - M_\Sigma^2\}\right]^{-1} \tag{8.55}$$

Now, these loop integrals can be calculated using the standard tricks of Feynman parametrization and dimensional regularization scheme.

Some useful formula in dimensional regularization scheme

The standard integrals which will be useful in the calculation are given below

$$\begin{aligned}\frac{1}{a\,b\,c} &= 2\int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{\delta(1-x-y-z)}{[a\,x + b\,y + c\,z]^3} \\ &= 2\int_0^1 dy \int_0^{1-x} dz \frac{1}{[a + (b-a)\,y + (c-a)\,z]^3}\end{aligned} \tag{8.56}$$

$$\frac{1}{a\,b} = \frac{1}{b-a}\int_a^b \frac{d\,t}{t^2} = \int_0^1 \frac{d\,z}{[b + (a-b)\,z]^2} \tag{8.57}$$

$$\frac{1}{a\,b} = \frac{1}{b-a}\int_a^b \frac{d\,t}{t^2} = \frac{1}{b-a}\left(\frac{1}{a} - \frac{1}{b}\right) \tag{8.58}$$

The dimensional regularization modifies the dimensionality of the loop integrals so that the expressions become finite. Firstly, we have to change 4-dimensional integral to D-dimensional integral (where $D = 4 - \eta$ and for $\eta \to 0$, we will revert back to original thing). Corresponding to the standard integrals in 4-dimension, the integral formulas in D-dimension is

$$\int d^D\,k \frac{1}{[k^2 + S + i\epsilon]^n} = i\,\pi^{D/2}\frac{\Gamma(n - D/2)}{\Gamma(n)}\frac{1}{S^{n-D/2}} \tag{8.59}$$

$$\int d^D\,k \frac{k^\mu}{[k^2 + S + i\epsilon]^n} = 0 \tag{8.60}$$

$$\int d^D\,k \frac{k^\mu k^\nu}{[k^2 + S + i\epsilon]^n} = i\,\pi^{D/2}\frac{\Gamma(n - D/2 - 1)}{2\,\Gamma(n)}\frac{g^{\mu\nu}}{S^{n-D/2-1}} \tag{8.61}$$

$$\int d^D\,k \frac{k^2}{[k^2 + S + i\epsilon]^n} = i\,\pi^{D/2}\frac{\Gamma(n - D/2 - 1)}{2\,\Gamma(n)}\frac{D}{S^{n-D/2-1}} \tag{8.62}$$

Similarly, the expression for $g^{\mu\nu}$, trace of gamma matrices and gamma identities will be changed accordingly.

Finally, after doing the simple algebra, the analytical expression for EMDM coupling strength which is responsible for electromagnetic leptogenesis is given by

$$\lambda = -\frac{(y_\Sigma^* \, y_H \, \mu_s \, v_D)}{16\,\pi^2\,[M_\Sigma^2 - M_H^2]}\left[\mathcal{I}_a + \mathcal{I}_b + \mathcal{I}_c\right] \tag{8.63}$$

Where $\mathcal{I}_a$, $\mathcal{I}_b$ and $\mathcal{I}_c$ are contribution coming from three diagrams shown in Fig.(8.4). The dominant contribution coming from the diagrams (8.4 [a]) and (8.4 [b])

$$\mathcal{I}_a + \mathcal{I}_b = 2\,M_E\left(\left[B_1^{(0)} - B_1^{(1)} - C_1^{(1)}\right] - \left[B_2^{(0)} - B_2^{(1)} - C_2^{(1)}\right]\right) \tag{8.64}$$

$$\begin{aligned}
B_1^{(n)} &= \int_0^1 dx \int_0^{1-x} dy\, x^n \;\; \omega_1 \\
C_1^{(n)} &= \int_0^1 dx \int_0^{1-x} dy\, y^n \;\; \omega_1 \\
B_2^{(n)} &= \int_0^1 dx \int_0^{1-x} dy\, x^n \;\; \omega_2 \\
C_2^{(n)} &= \int_0^1 dx \int_0^{1-x} dy\, y^n \;\; \omega_2
\end{aligned} \tag{8.65}$$

where $n = 0, 1, 2, ..$ is an integer and the value of ω_1 and ω_2 is given by

$$\begin{aligned}
\omega_1 &= \frac{1}{-y\,M_\Sigma^2 + x(1 - M_\Sigma^2) - (1-x-y)M_E^2 - x(x+y)M_N^2} \\
\omega_2 &= \frac{1}{-y\,M_\Sigma^2 + x(1 - M_H^2) - (1-x-y)M_E^2 - x(x+y)M_N^2}
\end{aligned} \tag{8.66}$$

Similarly, the dominant contribution coming from the diagram (8.4 [c])

$$\begin{aligned}
\mathcal{I}_c &= M_E \int_0^1 dx \int_0^{1-x} dy\,(y-1)[\Omega_1 - \Omega_2] \\
\Omega_1 &= \frac{1}{(y - y^2 - x\,y)M_N^2 - y\,M_\Sigma^2 - (1-y)M_E^2} \\
\Omega_2 &= \frac{1}{(y - y^2 - x\,y)M_N^2 - y\,M_H^2 - (1-y)M_E^2}
\end{aligned} \tag{8.67}$$

The effective dimension-5 coupling constant λ can thus be expressed in a simple form under the assumption of almost equal mass for the particles in the loop ($M_E \sim M_H \sim M_\Sigma \sim M_{eq}$) as:

$$\lambda = -\frac{y_\Sigma^*\, y_H\, \mu_s\, v_D}{64\,\pi^2\, M_{eq}^3} \tag{8.68}$$

List of Publications

1. **Sudhanwa Patra**, Anjishnu Sarkar, Utpal Sarkar and Urjit A. Yajnik
   Spontaneous Parity Violation in supersymmetric left-right model.
   Phys. Lett. B 679(2009) 386-389.
   e-Print: arXiv:0905.3220 [hep-ph].

2. Debasish Borah, **Sudhanwa Patra** and Utpal Sarkar
   TeV scale Left Right Symmetry with spontaneous D-parity breaking.
   Accepted for publication in Phys. Rev. D.
   e-Print: arXiv:1006.2245 [hep-ph]

3. Jitesh Bhatt, **Sudhanwa Patra** and Utpal Sarkar
   Gravity corrections in SU(5) gauge couplings.
   Mod. Phys. Lett. A 25 (2010) 283-293.
   e-Print: arXiv:0811.3307 [hep-ph].

4. **Sudhanwa Patra**, Anjishnu Sarkar and Utpal Sarkar
   Spontaneous Left-Right Symmetry Breaking in Supersymmetric Models with only Higgs Doublets.
   Phys. Rev. D 82 (2010) 01.
   e-Print: arXiv:1003.5095 [hep-ph].

5. Debajyoti Choudhury, Namit Mahajan, **Sudhanwa Patra** and Utpal Sarkar
   Electromagnetic leptogenesis at TeV scale
   (**will be communicated soon**)

6. **Sudhanwa Patra**
   Testable Leptogenesis in extended Standard Model
   To be appear in Phys. Lett. B.
   e-Print: arXiv:0911.4577 [hep-ph].

7. Debasish Borah and **Sudhanwa Patra**
   Type III seesaw and Dark Matter in Supersymmetric Left-Right Model.

e-Print: arXiv:0910.0146 [hep-ph].

Manuscripts in preparation:

8. **Sudhanwa Patra**
   PAMELA with Fermion triplet using Sommerfeld enhancment and Co-annihilation effect.

9. X. G. He, **Sudhanwa Patra** and Utpal Sarkar.
   Phenomenology of supersymmetric version of inert doublet model.

10. **Sudhanwa Patra.**
    Electromagnetic leptogenesis with dimension-6 dipole moment operator and solution to Boltzman equation.

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