with best compliments to the PRL library
22/12/77

PLASMA INSTABILITIES AND THEIR NONLINEAR STABILIZATION

BY

MUKUL SINHA

A THESIS

SUBMITTED FOR THE DEGREE

DOCTOR OF PHILOSOPHY

OF THE

GUJARAT UNIVERSITY

DECEMBER

1977

PHYSICAL RESEARCH LABORATORY
AHMEDABAD 380009
INDIA

DEDICATED TO
MY FATHER
AND
MY FRIEND JAYARAMA RAO

C E R T I F I C A T E

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

MUKUL SINHA
(Author)

Certified by:

A.K. SUNDARAM

December 17, 1977

ABSTRACT

This thesis is devoted to the study of the dispersive characteristics of low frequency instabilities and their modification due to the presence of strong inhomogeneities, background turbulence and external energy sources. The study of these modifications are often necessary to identify the instabilities observed in actual experiments. The linear dispersion characteristics derived under more simplistic approximations like weak-inhomogeneities quiescent background etc, while explaining certain gross feature of the observations are inadequate for a more detailed identification. A further motivation of this dissertation has been to

study the nonlinear coupling of the low-frequency instabilities with the background turbulence and the study reveals that the turbulence has, in general, a stabilizing influence. Thus, in Chapter I, we study the tearing mode problem in presence of a d.c. electric field directed along the neutral sheet and its subsequent stabilization by a lower hybrid turbulence. Chapter II is devoted to the study of low frequency electrostatic instabilities driven by velocity gradients and the saturation of the Kelvin-Helmholtz mode by the coupling with the lower hybrid turbulence. In Chapters III and IV we study the modifications of the parameteric decay and scattering processes in presence of non-thermal electrons. It has been shown in these two chapters that the efficiency of the energy transfer into the plasma can be improved in presence of an externally injected beam or a small component of cold electrons.

'The time has come', the Walrus said,
'To talk of many things:
Of shoes and ships and tokamaks
Of solenoids and Rings.
And how to keep the plasma hot
And how to stabilize the Kinks -
Through key-holes that one would peep -
And find nothing within the sight,
('t wasn't as if nothing was there)
It was a Black-hole sitting tight !'

ACKNOWLEDGEMENT

Even an Oyester, with it's hard shell of indifference, would find it difficult not to succumb to such temptations; the allurement of science is indeed strong. If one remembers, (ref. Alice in Wonder land, Spring Books) far simpler elusive promises had lured the Oyesters to take the fateful walk with the Walrus. Unlike the Oyesters, (who finished off rather unpleasantly) my walk through the scientific by-lanes, has been an

enjoyable one, some times exciting, the excitement of knowing a little bit more, and finally the satisfaction of an achievement, a triumph, however minor it might be in its absolute evaluation.

I am indebted to Dr. A.K. Sundaram for being my guide and a friend all through this pleasant and exciting period. I am indeed grateful for his helpful and tolerant attitude. Two other persons to whom I would remain indebted are my father and my friend Jayaram. My father who had coaxed me into the scientific world yet had sadly remarked, 'Science is progressing alright, for sure it is leaving us common folks behind', and my friend Jayaram who with his dry sense of humour had said, 'We are pushing the frontiers of science rather too hard, at this rate we run the risk of boring a hole and going out through it'. Said by two different persons, it reflects the same attitude, the pragmatism that has been a valuable asset for me.

There are a host of other fellow travellers to whom I would like to express my gratitude, for just being friends and they are, Gogo, Bhatta, Chatu, Parvez, Therma, Manab, Comrade, Mattoo, KK, Jiten, Dilip, Chaya, Raku, Mohan, Nagesh, Venkat, Avinash, Kamble, Bujar, Soma, Swami, Nanda, Vasanthi, Goyal and Ambastha.

To Gogo I owe a special thanks for associating himself with some of my scientific persuits and also to Bhatta for being philosophic and supplying stimulating literatures. To my mother I owe obviously half of everything and to my brothers and sisters and mamoni for their patience and trust in me. While expressing my gratitude to Nirjhari, words fail me. Atleast, that's how I felt while trying to propose to her.

To Narayanan I owe a deep sense of appreciation. If he does not skip these lines (as he some times does, what with his fantastic speed), I would like him to accept my thanks for having enjoyed typing this manuscript! I thank Ghanshyam Patel in anticipation of the neat cyclostyling that he is famed for.

CONTENTS

CHAPTER I

INTRODUCTION

Plasma waves and instabilities have been the topic of great interest as applied to the processes encountered in astrophysical and laboratory situations. These instabilities are, in fact, invoked as the important mechanisms to explain a variety of processes such as heating, diffusion or potentially explosive effects associated with them. Apparently, depending on the circumstances of a given physical problem, there exist two classes of instabilities which give rise to either desirable or undesirable effects in a plasma. For instance, certain types of instabilities effectively help in turbulent heating of a plasma while some other variety cause the main hindrance to achieving a controlled thermonuclear fusion. The fundamental feature of these instabilities is the fact that they are the manifestations of the collective degrees of freedom which are spontaneously excited in a plasma. Such excitations, in turn, are possible only if the plasma is in a thermodynamically nonequilibrium state. In other words, the excited states occur essentially due to the constant feeding of free energy into the plasma, which is, in general, available through several sources such as non equilibrium particle velocity distribution, the spatial gradients in density or temperature etc. Thus, the plasma instabilities discussed in the existing literature(1,2) can be classified into two frequency regimes, namely, high and low frequencies. From fusion point of view, high frequency instabilities are treated less harmful than the low frequency ones, although they are known to cause enhanced particle diffusion through micro turbulent processes. Instabilities in the low frequency regime to which the present thesis is devoted to, are believed, on the other hand, to be the major obstacle in accomplishing a successful confinement scheme. Besides laboratory interests, low frequency instabilities play a significant role in the magnetospheric plasma dynamics.

The characteristic properties of low frequency unstable waves are quantitatively discussed both from kinetic and macroscopic points of view and are well documented in the standard text books(3,4) and review articles(5). Typical examples of such linear instabilities are the ion acoustic, the ion cyclotron, the lower hybrid instabilities and others. In deriving these instabilities, the theoretical studies are particularly based on such simplifying assumptions as the static equilibrium, the weak spatial inhomogeneity and the absence of external sources etc. Although the linear description under these assumptions reasonably account for the gross features of the observed phenomena, it becomes inadequate for identifying certain types of instabilities. Consequently, a lack of realistic conditions puts a severe constraint in the interpretation of experimental results. In essence, the important factors which deserve the systematic exposition are (a) strong inhomogeneous plasma properties, (b) the background turbulence and (c) the external perturbations triggered due to agencies such as the launching of electromagnetic waves into the plasma or the injection of electron beams or the combination of these factors. The low frequency dispersion characteristics are significantly modified in the presence of these additional parameters which give rise to either stabilization or reduced growth rate of the initially unstable mode or the excitation of new modes.

Plasmas occuring typically in the magnetospheric neutral sheet(6) or in tokamaks(7) are strongly inhomogeneous in certain localized regions and hence the possibility of intense gradients in density, temperature and electric and magnetic fields arises. In the above-cited examples, the magnetic field or the term $k.B_o$, where k is a propagation vector and B_o is an equilibrium magnetic field, vanishes in a specified spatial location. In such cases, the theories developed so far under the 'local' approximation become invalid. Perhaps the inapplicability of the local approach can be more appropriately justified by defining a parameter, ϵ, as the ratio of ion larmor radius to the characteristic plasma dimension. This parameter which controls the degree of inhomogeneity is usually small in many laboratory devices; but it turns out to be large in the regions where strong gradients exist in density or other physical quantities. The well known techniques like WKB(8) and the guiding-centre approximations(9) are no longer applicable under these circumstances. For such intense variations, the solutions and the dispersion relation have to be obtained using the eigen value technique outlined in detail by Chandrasekhar(10) and Lin(11). There are atleast two important features embedded in an eigenvalue approach. One is the development of propagation vector along the direction in which the equilibrium density varies and this additional effect reduces the growth rate by a sizable fraction compared to its local value. The other significant feature depends partly on the density gradient and the direction of electric field perturbation. If the density gradient is perpendicular to the magnetic field and the gradient of the perturbed electrostatic potential, the convective contribution to the charge density due to particle drifts in crossed fields becomes an important effect. Physically, the convective effects manifest themselves by the removal of plasma with one density from a given point and the subsequent replacement by a plasma at the same point with a different density. Thus, a convective process of this type acts as a source of free energy to excite new modes in a plasma.

The turbulent background is another important factor which plays a dominant role in modifying the low frequency dispersion characteristics. Although the plasmas are assumed to be quiescent in most of the theoretical works, it is well known, in reality, that they depart considerably from the equilibrium state and invariably, turbulent fluctuations in density, electric field and other parameters exist. The main reason for such a turbulent state is straightforward to understand. High frequency instabilities which are excited in a plasma saturate at a faster rate than the other unstable modes. This saturation is quickened by their transfer of energy through nonlinear processes to the prevalent stable modes wherein the inherent dissipation mechanisms such as Landau or cyclotron damping take over to extract and feed the energy back to particles. This cyclic process finally leads to the plasma heating or diffusion phenomena. Thus a saturated spectrum of high frequency turbulent waves in the background gives rise to a low frequency ponderomotive force or the radiation pressure which greatly affects the low frequency modes. An extensive treatment of the nonlinear coupling process, described above, is thoroughly discussed by Vedenov et al.[12]. In this situation, the ponderomotive force plays a dual role in either stabilizing the low frequency mode or in exciting quasi modes when the group velocity of the turbulent spectrum matches the phase velocity of the low frequency mode. In laboratory devices, the supplementary plasma heating or the stabilization[13] of potentially dangerous modes has recently been achieved through external sources. Some illustrative methods responsible for these processes are the parametric resonance heating, the injection of cold electron beams and the launching of low frequency waves into the plasma. Presently the parametric heating scheme is acclaimed as an efficient method for additional plasma heating in tokamaks and considerable theoretical work, emphasizing the physics involved in this technique is neatly summarized in the book by Simon and Thompson[14]. The basic requirement for the parametric decay process is the resonance matching conditions for the frequency and wave number. One important by-product of this heating scheme is the excitation of new quasi modes of the macroscopic type, which poses a threat to the plasma confinement. The simultaneous injection of cold electron beams in a RF heated plasma, however, prevents the development of such modes and, in fact, this technique helps in increasing the heating efficiency. A similar injection technique has been successfully applied in 2X||B mirror experiment for suppressing the drift cyclotron loss cone mode(15). Also the presence of a small fraction of impurities or externally created cold electrons(16) in the plasma improves the heating efficiency. Finally enhanced plasma heating is also accomplished by launching undamped low frequency waves into the plasma. For this heating scheme, however, there exists no threshold for the energy transfer process.

Summarizing, the external or self consistent sources in a plasma significantly modify the low frequency modes and they participate efficiently either in plasma heating or instability suppression. In considering different problems of interest, the main emphasis is made on two types of sources, namely, linear and nonlinear. Usage of this terminology is typically dependent on the nature of modification which occurs in the plasma. For instance, the case of background turbulence or parametric coupling is an example of the nonlinear source while sources such as d.c. currents, strong inhomogeneities in velocity or magnetic fields etc. are linear in character. Thus an attempt is made to incorporate these effects in the following chapters and the relevance of the theoretical results to some experimental schemes or observed phenomena in space is pointed out in this dissertation. The plan of the remaining chapters is as follows.

In Chapter II, the tearing mode instability is investigated by including a d.c. electric field which is generally present in the neutral sheet of the geomagnetic tail. Employing the eigen value techniques, the triggering of a new non resonant type of collisionless tearing mode is discussed in detail particularly for growth rates for exceeding the thermal frequency, $k_{\parallel} v_e$. It is found that the growth rates of these newly excited modes are much greater than those of usual tearing modes which invoke the resonant wave-particle interaction process. The presence of a small normal component of magnetic field suppresses the electron tearing mode, exciting concomitantly a turbulent spectrum of lower hybrid waves. The lower hybrid turbulence, in turn, quenches the dormant ion tearing mode which takes over in the absence of electron tearing mode. The results of this theory have applications to the staged development of the magnetospheric substorm process. In fact, the excitation of electron tearing mode identifies with the onset of magnetic substorm.

The effects associated with inhomogeneous electric and magnetic fields are studied in Chapter III. Using the fluid treatment, the possibility of exciting electrostatic instabilities driven primarily by the velocity gradients is examined under various low frequency limits. The velocity gradient driven modes are excited both in convective and non convective regimes and have their typical growth rates a fraction of ion cyclotron frequency. Also, in a different frequency limit, the existence of transverse Kelvin-Helmholtz mode and its suppression by the background lower hybrid turbulence are discussed in this section. The work highlights the dual role of the lower hybrid turbulence which, in addition to preferential heating of ions, can suppress the macroscopic instability. The applicability of these results to Q-machines, auroral ionospheric and magnetospheric phenomena are further emphasized in this chapter.

In Chapter IV, including the effects of externally injected sources, the methods of improving the plasma heating efficiency are outlined. Especially, the parametric heating near lower hybrid resonance is studied under the influence of an injected cold electron beam. The results show that the threshold for the excitation of low frequency quasi-mode can be significantly lowered in the presence of beam electrons. This mechanism not only enables in achieving an efficient transfer of RF energy into the plasma but also helps in preventing the development of macroscopic pump-driven quasi-modes which are detrimental to the plasma confinement scheme. Similarly the effect of a small fraction of cold electrons which are produced either externally or through secondary emissions in a plasma on the dispersion characteristics of Brillouin back scattering process is investigated in an inhomogeneous, unmagnetized plasma. In this case, the threshold for the excitation of back scattering instability is increased.

The fifth chapter discusses an alternate method of supplementary plasma heating by coupling the externally generated magnetosonic waves with a turbulent spectrum of lower hybrid waves. This method does not require any threshold amplitude of the magnetosonic mode and hence very small amplitude waves can deposit their energy into the plasma unlike the conventional parametric heating schemes which require certain minimum threshold power. Using the fluid equations for the magnetosonic modes and wave-kinetic description for the lower hybrid turbulence, the heating of ions is shown to be enhanced by the temporal damping of the magnetosonic waves due to turbulence.

Finally the last chapter focuses on some outstanding problems that have important bearings on the substorm and the laboratory phenomena. Also the main achievements of this dissertation are briefly summarized.

References

1. Mikhailovskii, A.B., Theory of Plasma Instabilities: Vol.I (Consultants Bureau, New York) (1974)

2. Mikhailovskii, A.B., Theory of Plasma Instabilities: Vol.II (Consultants Bureau, New York) (1974)

3. Stix, T.H., Theory of Plasma Waves (McGraw Hill, New York) (1962)

4. Krall, N.A. and Trivelpiece, A.W., Principles of Plasma Physics (McGraw Hill, New York) (1973)

5a. Leontovich, M.A., Reviews of Plasma Physics, Vol.III and V (Consultants Bureau, New York) (1966)

5b. Simon, A. and Thompson, W.B., Vol.I, Article by Krall, N.A., Advances in Plasma Physics (John Wiley and Sons, Inc. New York) (1968)

6. Hess, N.A. and Mead, G.D. (Eds.), Introduction to Space Science (Gordon, Breach Science Publishers, New York) p.399 (1968)

7. Simon, A. and Thompson, W.B., Advances in Plasma Physics, Vol.IV (John Wiley and Sons, Inc., New York) (1971)

8. Morse, P.M. and Feshbach, H., Methods of Theoretical Physics, Vol.II (McGraw Hill, New York) (1953)

9. Northrop, T.G., Adiabatic Motions of Charged Particles (Interscience Publishers) (1963)

10. Chandrasekhar, S., Hydrodynamic and Hydromagnetic Stability (Clarendon, Oxford) (1961)

11. Lin, C.C., The Theory of Hydrodynamic Stability, (Cambridge University Press) (1955)

12. Vedenov, A.A., Gordeev, A.V. and Rudakov, L.I., Plasma Physics 9, 719 (1967)

13. Chu, T.K. and Hendel, H.W., Dynamic Stabilization and Feedback Control, AIP Conference Proceedings No.1, (New York)

14. Simon, A. and Thompson, W.B., Advances in Plasma Physics, Vol.VI (John Wiley and Sons, Inc., New York) (1976)

15. Coensgen, F.H. et al., Proceedings, 6th Int. Conf. on Plasma Physics and Controlled Nuclear Fusion Research held at Berchtesgaden (1976)

16. Goswami, B.N., Satya, Y.S. and Kulkarni, V.H., Pramana 9, 171 (1977)

CHAPTER II

TEARING MODES AND THEIR STABILIZATION BY BACKGROUND TURBULENCE

Introduction

Astrophysical processes such as the occurence of solar flares or the onset of magnetospheric substorms, etc. have been topics of extensive research over a last few decades. Recently the interest in this field is greatly revived, further, by the observation that similar processes do occur in laboratory devices. The emission of hard x-rays during a violent current disruption in tokamaks is a clear example in this direction. In such physical situations, the fundamental process appears to be based on the well known concept of magnetic field line 'merging' (also called the annhilation or reconnection). This field line merging concept, basically reveals two important features of the magnetised plasma. Firstly, in cosmic plasmas (for instance), because of low electrical resistivity of plasma, merging process essentially implies the plasma transport across the separatrix which bifurcates the different classes of magnetic field lines, embedded in the plasma. Consequently the rate of plasma transport across the separatrix surface is a direct measure of the merging rate. Secondly it turns out to be a mechanism which enables the conversion of the stored magnetic energy into kinetic energy of the particles along a x-line (being defined as an intersection of the two branches of a separatrix). Thus a clear understanding of processes such as plasma flow patterns or changes in the magnetic field topology and energy release is closely related to the field-line merging process.

Historically, the possibility of accelerating the charged particles responsible for solar flares and the curora, along the x-type of magnetic neutral line was suggested first by Geovanelli(1,2) (and later by Hoyle(3)). Qualitatively they showed that the magnetic configuration with a neutral line embedded in it is unstable and that the field line merging of x-type occurs. In 1953, Dungey(4) considered a model in which the two branches of the separatrix were almost parallel to each other with a thin sheet of current separating them. Assuming the frozen-in-field line concept, the work describes the process of field line merging and the accompanying plasma flow towards the neutral line. Further works on this topic were more or less the extension of Dungey's model with appropriate modifications. Notable amongst them are the works of Sweet[5], Parker[6] and Petschek[7]. In Sweet's and Parker's models the weak magnetic field on the down stream side of the separatrix was neglected so that the neutral sheet was essentially field free. Their results show that the merging rate crucially depend on the dimensions of the system and the electrical resistivity of plasma and thus the models fall short of the observed energy dissipation rates in solar flare by several orders of magnitude. Petschek's[7] model included the weak magnetic field neglected by Sweet and Parker and the merging rates obtained in this model comes out closer to the real values although the mathematical arguments used in their work were not rigourous enough to justify the results. Sonnerup[8], Yeh and Axford[9] modified the merging rates using a dimensional analysis and achieved a better agreement with the observations. All these models are essentially hydro dynamical models and are devoid of the wave-particle interactions nor they give a quantitative account of the reconnection process as such. A complete review of all these MHD models and their detailed implications on the merging process can be found in an excellent review article by Vasyliunas[10].

In the aforementioned works, greater stress is laid on the particle accleration processes using a fluid treatment and it concentrates less on what possibly triggers the reconnection process. To understand the latter process in detail, the behaviour of the charged particles and the equilibrium characteristics become important factors that govern the dynamics of the neutral sheet configuration. These aspects have been thoroughly studied by Speiser[11a,b,c] and Cowley[12] using the particle picture. Especially the equilibrium properties with a D.C. electric field were discussed by Cowley[12] while the particle trajectories were computed in Speiser's work. Also, within the framework of kinetic theory, the steady state solutions of the magnetic neutral sheet using the Vlasov-Maxwell system of equation was examined by Harris[13] which was later extended for the two dimensional case by Toichi[14]. These equilibrium properties become therefore the necessary ingradients for studying the stability characteristic of such a magnetic field configuration. Although some earlier attempts have been made, in this direction, (For instance, Furth et al.[15], Hazeltine et al.[16]) employing the resistive tearing mode theory as the basic mechanism for the reconnection process, these theories become inadequate since it involves the fluid approach and employs the relation, $E + \frac{V \times B}{c} = \eta J$, where η is the electrical resistivity.) But in realistic systems such as astrophysical or geophysical plasmas (and even laboratory plasmas), collisions are highly infrequent so that the collective effects predominates in the reconnection proces. Thus, taking into account the collective effects, Coppi et al.[17], (hereafter, referred to as Coppi) derived the resonant collisionless electromagnetic mode as the triggering mode for the reconnection process, but it turns out that this mode becomes stabilized for small amplitude of perturbation. So that global rearrangement of the neutral sheet with the onset of instability does not take place. Subsequent investigations by Dobrowolny[20], Galeev and Zelenyi[21] and Coroniti[22], are aimed at understanding the geomagnetic substorm process with appropriate refinements in the tearing mode theory, incorporating the effects of particle trajectories, the magnetic field component normal to the neutral sheet, etc.

In a recent paper, Drake and Lee[23] studied the semi-collisional tearing mode applicable to the tokamak. geometry. The neutral sheet geometry in the tail region, however differs slightly from that of tokomaks. Perhaps it is appropriate here to distinguish between the magnetic field configurations of tokamak and the geomagnetic tail. In Fig.1. we have shown the geomagnetic tail geometry. The normal component of the magnetic field, B_{zo} is perpendicular to the current sheet which lies in the XY plane in the case of the geomagnetic tail configuration whereas

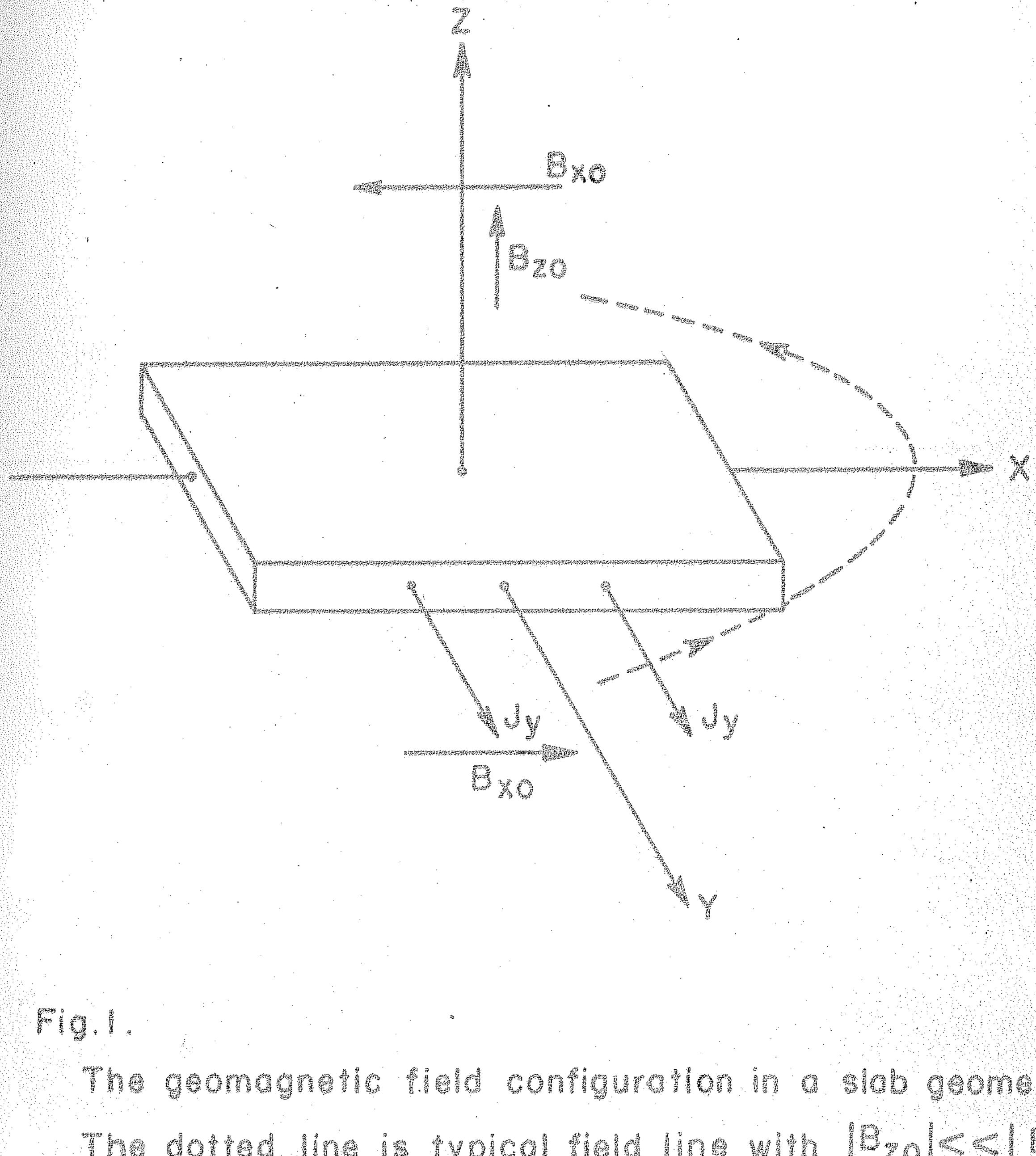

Fig. 1.

The geomagnetic field configuration in a slab geometry. The dotted line is typical field line with $|B_{zo}| << |B_{xo}|$

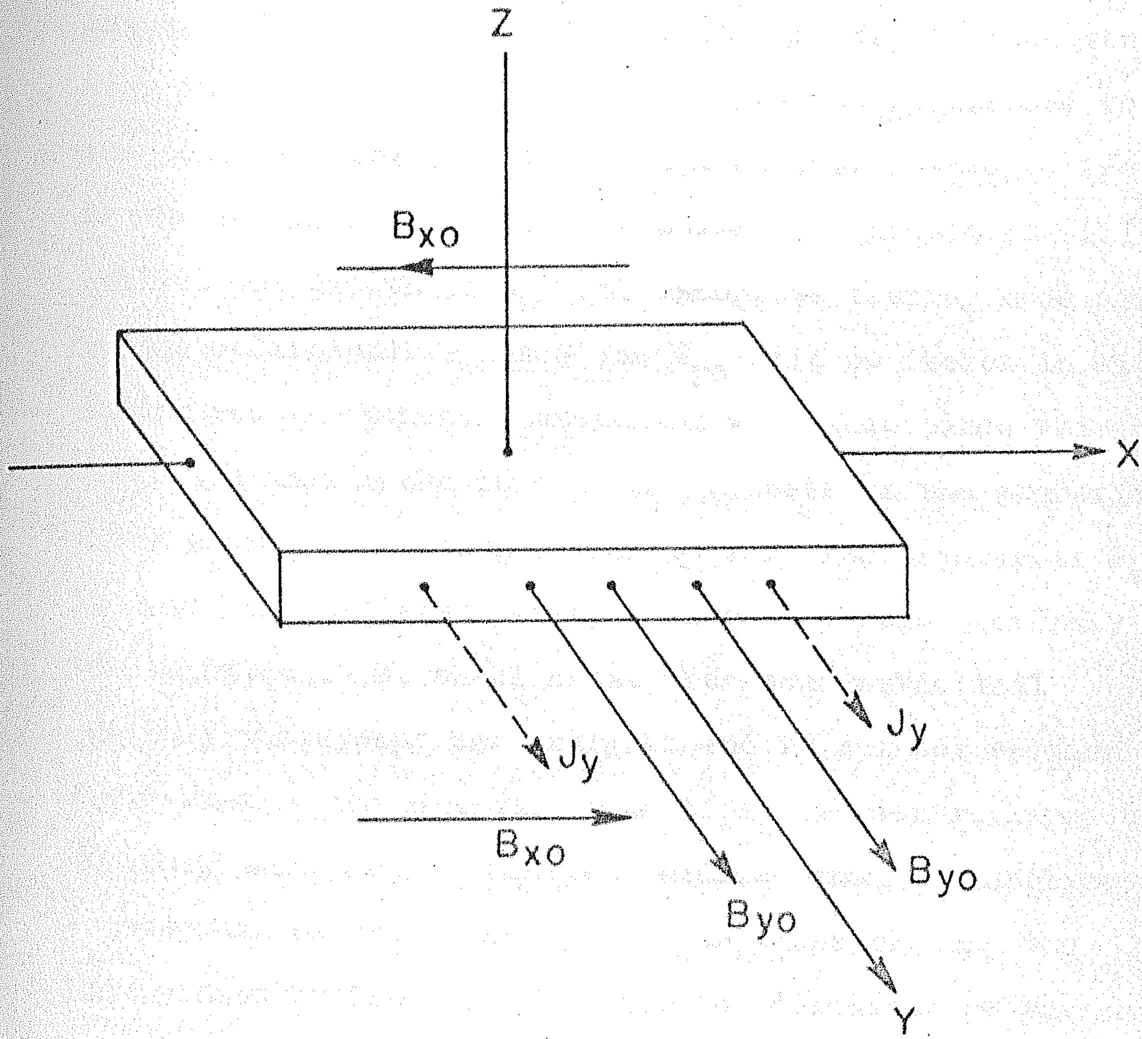

Fig. 2.

The tokamak field configuration. The field B_{yo} is in the same direction as J_y and $|B_{yo}| >> |B_{xo}|$

the toroidal field component B_{yo} is along the current direction for the tokamak (Fig.2). Also $B_{zo} \ll B_{xo}$ in Fig.1, whereas $B_{yo} \gg B_{xo}$ in Fig.2. In Fig.2, representing the tokamak geometry, the x,y,z-axes correspond to the poloidal, toroidal and radial directions respectively. In the tokamaks therefore the strong B_{yo} component will inhibit the growth of any collisionless tearing mode since in the collisionless limit the B_{yo} will be frozen-in with plasma and any pinching mechanism will also pinch this field and this would lead to an increase in the magnetic field energy. In the geomagnetic tail region however B_{zo} is small and its stabilizing influence depends entirely on its magnitude. We shall chose the geomagnetic tail geometry throughout our analysis and in a later section we shall examine the dramatic role of B_{zo} in controlling the substorm mechanism. Perhaps a similar analysis applicable to tokamaks can be investigated in principle and the feasibility of these results will be discussed at the end of this chaper.

All the earlier papers, devoted to the study of the substorm phenomenon have excluded the zero-order electric field from then analysis and the drifts of the charged particles have been taken as the diamagnetic drift satisfying the condition $V_{oe}/T_e = V_{oi}/T_i$ (Harris[13], Galeev and Zelenyi[21]). In absence of this electric field, the drift velocities of the particles are very small ($V_{oj}/V_{thj} << 1$, V_{oj} = drift velocity of the j^{th} species) and only the slowly growing ($\gamma << k V_{thj}$) mode can be excited in this limit. It is not quite clear whether this mode can change the magnetic topology to any great extent. Biskamp et al.(18) had worked out the quasilinear saturation of the Coppi mode and shown that this mode could get saturated at a low level of turbulence. This would mean that the induced electric field due to the instability is too low to account for the fast merging rates. It is quite well known now that atleast for the magnetospheric reconnection process, the plasma sheet as well as the current sheet thins to a very small dimension just before the triggering the reconnection process. Hones et al.(26) (and the references therein) reported such a thinning of the current sheet and this gives a direct evidence of the presence of zero order electric fields along the neutral sheet. Nishida and Nagayama(27) while studying the process of reconnection gives the value of such an electric field. Bowers(24), in his work, has considered the effect of such an electric field localised in the dusk region of the neutral sheet. His theory shows that an electrostatic ion-plasma oscillation can be driven unstable in the presence of the zero-order-electric field. The localised electric field feeds energy into this mode which would finally lead to a turbulent state thereby enhancing the reconnection of the magnetic field. In another paper, Bowers[25] also studied the excitation mechanism for a high frequency wave ($\omega \simeq \omega_{pe}$) and found the threshold condition required for this instability. This gave $e\phi_o \gtrsim kT_{em}$, where ϕ_o is the zero order electric potential and T_{em} is temperature of the magnetosheath electron. Both these instabilities essentially lead to a higher level of turbulence which in turn enhances the reconnection process. The time-scales predicted by these theories, however, do not agree very well with the observed values. The presence of the zero order electric field along the current sheet can therefore have an important role to play in the reconnection processes. The particles in the neutral sheet can be accelerated and then the drift velocities can attain values higher than their thermal velocities. In this limit a new instability can be triggered with a much larger growth rate (than that of Coppi's) and this essentially will be the research topic of this chapter.

We will use a very general formalism in deriving the dispersion relation for the triggering mode, keeping in mind the applicability of the results to the substorm observations. In section 1, the tearing instability in one dimension will be described, using an equilibrium similar to the one derived by Harris[13]. The drift velocities in the Harris equilibrium is replaced by the average drift velocity due to the electric field along the current sheet. In section 2, we shall introduce a small magnetic field component normal to the current sheet and the stabilizing influence of this component on the electron tearing mode will be discussed. We shall also derive the dispersion relation for the Ion-tearing mode which remains unstable even in the presence of the normal component of the magnetic field. Using a phenomenological model, we shall show, in section 3, that the saturation of this instability occurs by a trapping mechanism. Further, the quasilinear effects will be shown to be small for the saturation mechanism. Section 4,is devoted to the suppression of the tearing mode by an external source, the source being a low level of lower hybrid turbulence. In the final section 5, we shall discuss the relevance of our results to the magnetospheric substorm and we will indicate possible applications of our theory to tokamaks.

Section 1 : Linear Electron Tearing Mode:

We choose a slab geometry in which the current sheet is in the xy plane with the drift velocity along the y axis. The zero order electric field is taken along 'y' axis. The current sheet produces the zero order magnetic field given by B_T = Botanh Z/λ and the plasma density given by $n = n_o \operatorname{sech}^2 Z/\lambda$ where λ is the half width of the current sheet along the Z axis and $B_o = \{ 8\pi n_o K(T_{\perp e} + T_{\perp i}) \}^{1/2}$, $\lambda = (cT_{\perp e}/V_j) \times \{ k/2\pi n_o e^2(T_{\perp e} + T_{\perp i}) \}^{1/2}$ where n_o is the density at Z = 0, $T_{e,i}$ are the electron and ion temperature. The details of this equilibrium are given in Appendix 1. 'V_j' denotes the averaged particle drift due to the electric field and its typical magnitude is $(2q_j E_{oL/mj})^{1/2}$, L being the distance in which the electric field becomes effective in accelerating the particles (Cowley[12]). We assume that the plasma is collisionless and use the Vlasov-Maxwell set of equations,

$$\frac{\partial f}{\partial t} + \bar{v}\cdot\bar{\nabla} f + \left(\bar{E} + \frac{\bar{v}\times\bar{B}}{c}\right)\cdot\bar{\nabla}_v f = 0 \qquad (1.1)$$

$$\bar{\nabla}\times\bar{E} = -\frac{1}{c}\frac{\partial B}{\partial t}$$

$$\bar{\nabla}\times\bar{B} = \frac{4\pi}{c}\bar{J} + \frac{1}{c}\frac{\partial E}{\partial t}, \quad J = \sum q_j \int v f \, dv \qquad (1.2)$$

Following the usual procedure of linearizing Eq.(1.1) and integrating over the zero order particle orbits, we obtain the perturbed distribution function as,

$$\tilde{f}_1 = -q_j/m_j \int_{-\infty}^{0} d\tau \left(\bar{E}_1(z') + \frac{\bar{v}\times\bar{B}(z')}{c}\right)\cdot \bar{\nabla}_v f_o \exp(\bar{k}\cdot\bar{x} - \omega\tau)i \qquad (1.3)$$

The orbits are given by

$$\frac{d\bar{v}'}{d\tau} = \frac{q_j}{\bar{m}_j}\,\bar{v}'\times\bar{B}_o\tanh z/\lambda + \frac{q_j}{\bar{m}_j}\bar{E}_o \qquad (1.4)$$

with $v' = v$ and $x' = x$ at $\tau = 0$. The orbits of Eq.(1.4) are indeed complicated and has been worked out by Speiser(11a,b). For our analysis, it is sufficient to work in a region $Z/\lambda \ll 1$ (i.e. close to the neutral sheet) and in this region the magnetic field can be taken as zero (Galeev(21)). In this case, the orbit is essentially straight line orbit with the space-averaged drift (V_j) due to the electric field $\bar{E}_o$. The zero order distribution function can appropriately be taken as

$$f_{oj} = n\left(\frac{\alpha_j}{\pi}\right)^{3/2} \exp\{-\alpha_j v_x^2 - \alpha_j v_z^2 - \alpha_j (v_y - V_j)^2\} \qquad (1.5)$$

The integration over the unperturbed orbits can now be easily performed; we note however that the fields $\bar{E}(Z')$) and $\bar{B}(Z')$ are still functions of the unperturbed variable (Z'). As shown by Speiser(11a,b), the particles in the neutral sheet are essentially trapped around the neutral sheet due to the field reversal and this allows us to expand the field around $Z' = Z$ for $Z/\lambda \ll 1$, i.e.,

$$E(z') = E(z) + (z'-z)\frac{\partial E}{\partial z} + (z'-z)^2\frac{1}{2!}\frac{\partial^2 E}{\partial z^2} + \cdots$$

Since the particles are trapped around $Z = 0$, the average particle excursion, $(Z' - Z)$ over a period is zero (for $\langle Z' - Z\rangle_T \simeq 0$) and therefore we can replace $E(Z')$ by $E(Z)$

in Eq.(1.3). With this replacement, we obtain the perturbed distribution function,

$$f_{1j} = \frac{2\alpha_{\perp j}}{m_j} q_j n_j(z) f_{oj} \left\{ (v_y + V_j)\left(1 + \frac{\alpha_j k_x v_x}{\omega}\right) E_y + V_j \frac{v_z}{i\omega} \frac{\partial E}{\partial z} \right\} / i(kV_x \qquad (1.6)$$
$$/ i(kV_x - \omega) - i\alpha_{\perp j} q_j V_j n_j E_y / m_j \omega .$$

wherein, we have used the relation $\nabla \times E = -1/c \frac{\partial B}{\partial t}$ and retained the electromagnetic term which for propagation in the $\bar{x}$ direction (i.e. $\vec{k} = k_x \hat{e}_x$) gives the electric field in the 'y' direction. To arrive at the dispersion equation, we use the second maxwell's equation:

$$[\nabla \times (\nabla \times E)]_y = (\omega^2/c^2) E_y + 4\pi i \omega J_y / c^2 \qquad (1.7)$$

with $J_y = \sum_j n_j q_j \int v_y f_{1j} \, d\bar{v}$

The typical perturbed quantity, occuring in Eqs. (1.3) to (1.7), is chosen in the form q = q(Z) exp ($ik_x x - i\omega t$). Using Eq.(1.6) and evaluating J_y in Eq. (1.7), we, finally, get the differential equation for E_y:

$$\frac{d^2 E_y}{d\xi^2} + \{ D(\omega, k_x) \operatorname{sech}^2 \xi - k_x^2 \lambda^2 \} E_y = 0 \qquad (1.8)$$

where $\xi = Z/\lambda$ and

$$D(\omega, k_x) = \lambda^2/c^2 \left[- \sum \omega_{pj}^2 + 2 \sum \omega_{pj}^2 \left(\tfrac{1}{2} + \alpha_{\perp j} V_j^2\right) \int_{-\infty}^{\infty} \frac{k_x v_x (1 + \alpha_j k_x v_x/\omega)}{n(z)(k_x v_x - \omega)} f_{oj} \, dv \right]$$

$$\omega_{pj}^2 = 4\pi n_0 e^2 / m_j .$$

Now, we make the transformation, $\eta = \tanh\xi$ and Eq.(1.8) takes the form

$$(1-\eta^2)\frac{d^2E_y}{d\eta^2} - 2\eta\frac{dE_y}{d\eta} + \left\{D(\omega,k_x) - \frac{k_x^2\lambda^2}{1-\eta^2}\right\}E_y = 0 \qquad (1.9)$$

We now assume that the current sheet is thin, i.e. $\lambda/\rho_i \lesssim 1$, ρ_i = gyroradius of the ions in average field B_o. This assumption is quite valid both in the case of geomagnetic neutral sheet (Hones et al.[26]) as well in the tokamaks, Drake and Lee[23]). Since Eq.(1.9) has been derived under the approximation Z/λ << 1, we need to have the boundary condition for some value of Z_o for which Z_o/λ << 1. But if the current sheet is thin we can safely extend the boundary to $Z = \infty$ without making any significant error as the main current, carrying particles, are themselves confined within a narrow region around $Z = 0$. In the case of the thin sheet, therefore the external region (where the particles are magnetized and follow the $\bar{E}_o \times \bar{B}_o$ drift) is extended to $Z \to \infty$ and the solution of the internal zone around $Z = 0$, is to be matched with the solution at $Z \to \infty$. The energy feeding the instability is the free energy associated with the drift of the particles constituting the current sheet. The instability is therefore analagous to the one derived by Momota[28] for a homogeneous plasma. The required boundary condition that can be imposed on the solution of Eq.(1.9) is that 'Ey' vanishes at larger distances ($Z \to \infty$) and 'Ey' remains finite for $Z=0$. The only solution of Eq.(1.9) satisfying these conditions, are the associated Legendre polynomials i.e. $E_y \propto P_m^{\ell}(\eta)$ and this requires

$$D(\omega, k_x) = \ell(\ell+1) \text{ and } k_x \lambda = \pm m \qquad (1.10)$$

where ℓ and m are positive integers. Equation (1.10) immediately gives the dispersion relation for the electromagnetic 'pinching' mode for a thin sheet. The strong inhomogeneities impose the quantised condition for the wavenumber, k_x and restrict it in having values as integral multiples of the sheet width. This indeed is a remarkable result and implies that the field line reconnection leads only to a definite number of loops.

We now proceed to find the growth rate of this mode from Eq.(1.10) and Eq.(1.8). We look for a large growth rate mode, i.e., $\gamma > kV_{the}$ and with this assumption, write $\omega = i\sigma$, where σ is a real quantity. Under the approximation, $\sigma >> kV_{the}$, we obtain an equation for 'σ' given by

$$\sigma^4\{\ell(\ell+1)c^2/\lambda^2 + \sum \omega_{pj}^2\} - \sigma^2 \sum k_x^2 \omega_{pj}^2 (2V_j^2 + 1/2\alpha_{\perp j}) + \frac{3}{2} \sum k_x^4 \omega_{pj}^2 [(2V_j^2 + 1/2\alpha_{\perp j})/2\alpha_{\parallel j}] = 0, \; k_x\lambda = \pm m \qquad (1.11)$$

In deriving this equation, we have expanded the denominator of Eq.(1.8) in powers of $kV_x/\sigma < 1$ and integrated over '$\bar{v}$'. It is evident from equation (1.11) that the electronic term is larger than the ionic term by a ratio m_i/m_e. Eq.(1.11) is a biquatratic in σ and can easily be solved to give

$$\sigma = \pm m \beta_e \omega_{pe} \left[\frac{1}{2} \pm \frac{1}{2} \left(1 - 6 d_{\parallel}^2 / \lambda^2 \beta_e^2 \right)^{1/2} \right]^{1/2} \qquad (1.12)$$

where $d_{\parallel}$ and $d_{\perp}$ are the respective debye lengths parallel and perpendicular to the magnetic field (taken along the 'x' axis) and the quantity β_e is defined by the relation

$$\beta_e \left\{ 1 + l(l+1) c^2/\lambda^2 \omega_{pe}^2 \right\}^{1/2} = \left(\frac{d_\perp}{\lambda} \right) \left(1 + 4\alpha_{\perp e} V_e^2 \right)^{1/2}$$

For σ to be real, the required condition is therefore (from Eq.(1.12)) $V_e^2 > V_{the}^2$. The instability therefore can be triggered when the drift velocity of the electrons exceeds the electron thermal velocity. As we shall see later (Section 5), this condition is easily fulfilled for typical values of the neutral sheet parameter, in the geomagnetic tail, at the onset phase of the magnetic sub-storm. It is worth mentioning here some of the major deviations of our result from the ones derived by Coppi and a host of other research workers. First, the Coppi type of tearing mode feeds on the energy stored in the magnetic configuration in the external region. This magnetic energy is converted into the particle kinetic energy in the internal zone. In our analysis, the instability is driven by the current carrying particles themselves and they essentially 'pinch' the current sheet into filaments. Consequently a change in the magnetic field topology from the straightline oppositely directed fields to closed loops around the Z = 0 line is accomplished. The drift velocity V_e for our case should necessarily be greater than thermal velocity for the mode to become unstable; whereas the Coppi's type is unstable, for $V_e < V_{the}$. Thus the critical V_e required for instability essentially leads to much larger growth rates for our type of instability. The temperature anisotropy can however quench the Coppi's type while it does not play any significant role at all on our instability. Finally, as shown by Biskamp et al., the quasilinear saturation mechanism is operative for the Coppi's type of tearing mode and thus a saturation of this mode occurs at a very low level of turbulence. On the contrary, we will show, in a later section, that the quasi linear mechanism plays a secondary role for our instability and the instability can saturate only by particle trapping in the wave field, which requires a much larger amplitude. A similar conclusion was made by Davidson et al.(31) in their numerical study involving the pinching instability.

Before we end the discussions on the instability developed in this section, it is worthwhile mentioning that all values of l and m do not contribute to the growth rates leading to field-line reconnection. The induced electric field Ey being proportional to the $P_m^l(\eta)$

functions will have maxima at $Z = 0$ only for certain values of 1 and m. For some other values of '1' and 'm', Ey will have maxima only at the edge (i.e. $\eta = 1$) and these modes are spatially damped for $Z = 0$. This would mean that for certain values of 1 and m, the instability will only cause a change in the magnetic field topology at the edge and will therefore cause a 'ripple' on zero order Bx field. Thus we can conclude that the mode with $l = m = 1$ has the maximum electric field at $Z = 0$ and undoubtedly this mode is expected to take part in the reconnection process. For typical values of the geomagnetic tail parameters, $E_o \sim .1$mV, $T_e = 100$ ev, $\lambda \simeq 300$ km. The growth rate 'σ' for the $l = m = 1$ mode turns out to be

$$\sigma \simeq .2 \text{ sec.} \tag{1.13}$$

On the other hand the mode with $m = 1$, $l = 2$ will have the maximum wave electric field at the edge ($\eta = 1$) and will essentially distort the field lines at the edges and it does not obviously participate in the reconnection of field lines.

Section 2 : Ion Tearing Mode:

Next, we shall study the effect of a small component of the magnetic field normal (i.e. along the 'Z'-direction) to the neutral sheet. The total magnetic field can be derived from a vector potential, A(X,Z) i.e. $\bar{B}_T = \bar{\nabla} \times \bar{A}(X,Z)$. Using the steady state Vlasov-Maxwells system of equations, the self consistent solutions in terms of $\bar{A}(X,Z)$ can be derived for a two dimensional neutral sheet geometry, which has been studied in great details by Toichi[14]. For our purpose, however, we shall take the x-dependence to be weak enabling us to write $A(X,Z) = A(Z) + x\frac{\partial A}{\partial x} + \ldots\ldots$, where $\frac{\partial^2 A}{\partial x^2}$ etc. are neglected, assuming $\frac{\partial A}{\partial x}$ to be approximately constant with respect to 'X'. Such an assumption helps in considerable mathematical simplicity and at the same time retains qualitatively the essential features of the normal component B_{zo}. The steady state solution in this case will again be the one derived by Harris[13], namely,

$$B_T = -B_{xo}\tanh(z/\lambda)\hat{e}_x + B_{zo}\hat{e}_z\,, \quad B_{zo} = \frac{\partial A_y}{\partial x}$$
$$\bar{E} = -E_o\hat{e}_y\,, \quad n(z) = n_o\operatorname{sech}^2(z/\lambda) \tag{2.1}$$

$$f_{oj} = n(z)\frac{\alpha_{\perp j}}{\pi}\left(\frac{\alpha_{\parallel j}}{\pi}\right)^{1/2}\exp\{\alpha_{\parallel j}v_x^2 + \alpha_{\perp j}v_z^2 + \alpha_{\perp j}(v_y - V_j)^2\} \tag{2.2}$$

The B_{zo} component is taken to be small such that the separatrix of the two different magnetic topologies are still almost parallel. The magnitude of B_{zo} is such that $k_x \rho_{iz} > 1$ and $k_x \rho_{ez} << 1$, where 'k_x' is the wave number for the unstable mode of section (1) and this assumption, in turn, would imply that $\rho_{ez} << \lambda << \rho_{iz}$, since $k_x \lambda = 1$ for the reconnecting mode as seen in section 1. In this case the electrons get magnetized with respect to the B_{zo} component and clearly they cannot be accelerated by the electric field along the 'y' axis. Under the combined influence of the normal field (B_{zo}) and the electric field (E_o) the electrons gyrate in the neutral sheet (in the xy plane) and simultaneously they drift along the x axis due to the $\bar{E}_o \times \bar{B}_{zo}$ drift. Speiser has studied the particle trajectories in this type of a configuration and the details can be found in his work. The ions, on the other hand, remain unmagnetised and can still accelerate along the y axis by the electric field, attaining drift velocities V_i larger than ion thermal velocity. Under these conditions, the ions will have the same perturbed distribution function as in section (1), (Eq.1.6) and the electron perturbed distribution function is obtained by integrating Eq.(1.3) over the electron unperturbed orbits, namely,

$$V_x = V_\perp \cos(\phi - \Omega_{ze}\tau), \; V_y = V_\perp \sin(\phi - \Omega_{ze}\tau)$$
$$\Omega_{ze} = eB_{zo}/m_e c \; , \; \tau = t' - t \qquad (2.3)$$

The drift velocity 'V_e' is no longer the electron drift used in section (1) and instead it is the diamagnetic drift. As such this drift does not enter in the particle orbits defined by Eq.(2.3). The integration of Eq.(1.3) over the orbits specified in Eq.(2.3) yields

$$f_{1e} = 2\alpha_{\perp e}(q_e/m_e) f_{oe}\Big[\sum_{\ell m} \exp i(n-\ell)\phi\{V_\perp J_\ell' E_y + iV_e J_\ell (-E_y - \tfrac{V_z}{\omega}\tfrac{\partial E_y}{\partial z})\} J_n/(\ell\Omega_{ze}-\omega) - iV_e E_y/2\omega\Big] \tag{2.4}$$

where $J_n(\xi)$ represents the Bessel function of order 'n' and $\xi = k_x V_\perp/\Omega_{ze}$. Following the same procedure as in Section (1), we arrive at equation for Ey with a new $D'(\omega, k_x)$ given by

$$D'(\omega,k_x) = (\lambda^2/c^2)\Big[\frac{2}{n(z)}\alpha_\perp \omega\omega_{pi}^2 \int_{-\infty}^{\infty} v_y f_{oi}\frac{(V_i+v_y)}{k_x v_x-\omega}\,dv + 2\alpha_{\perp i}V_i^2\omega_{pi}^2 + \omega^2\omega_{pe}^2/\Omega_{ze}^2\Big] \tag{2.5}$$

The derivation of the above dielectric function is valid under the approximation, $k_x\rho_{iz} \gg 1$, $k_x\rho_{ez} \ll 1$, (ie, $\rho_{ez} \ll \lambda \ll \rho_{iz}$) and $\Omega_{zi} < |\omega| < \Omega_{ze}$ where in ρ_{jz} is the larmor radius with respect to the field, B_{zo}. For a thin sheet, the boundary conditions are the same ones stipulated in Section (1) to arrive at the dispersion relation, and hence we have $E_y \to 0$ for $Z \to \infty$ and Ey reamining finite at $Z = 0$. This leads to a similar criterion obtained in Section (1) for $D'(\omega, k_x)$, namely,

$$D'(\omega,k_x) = \ell(\ell+1), \quad k_x\lambda = \pm m \tag{2.6}$$

Using Eq.(2.5) and the approximation $|\omega| > k_x V_{thi}$, the dispersion relation for the new mode is given by

$$\left(1 + \omega_{pe}^2/\Omega_{ze}^2\right)\omega^2 - \left[\frac{c^2}{\lambda^2}\ell(\ell+1) + \omega_{pi}^2\right] - \frac{k_x^2 V_{oi}^2 \omega_{pi}^2}{\omega^2} = 0 \qquad (2.7)$$

with $k_x\lambda = \pm m$

In deriving Eq.(2.7), the electron drift is neglected ($V_{oe} \ll V_{oi}$) due to the presence of the normal field B_{zo}. The rotation of the electrons in the field B_{zo} essentially opposes the pinching force on the electron streams by the perturbed magnetic field and inhibits the electron tearing mode. In the next section, we will give an estimate of the magnitude of the normal field that can stabilize the electron mode. Even though the electron-streams are inhibited due to stabilizing mechanism, the ion streams are not affected by the normal field (as $k\rho_{iz} \gg 1$ and $|\omega| > \Omega_{iz}$) and they continue to contribute in the reconnecting process. Solving Eq.(2.7) we find

$$\omega^2 = \frac{\left[c^2/\lambda^2\,\ell(\ell+1) + \omega_{pi}^2\right] \pm \left[\left\{c^2(\ell+1)\ell/\lambda^2 + \omega_{pi}^2\right\}^2 + 4k_x^2 V_{oi}^2 \omega_{pi}^2\left(1+\frac{\omega_{pe}^2}{\Omega_{ze}^2}\right)\right]^{1/2}}{2\left(1 + \omega_{pe}^2/\Omega_{ze}^2\right)} \qquad (2.8)$$

with $k_x\lambda = \pm m$

The solutions of this equation give:

$$\omega_{\ell,m}^1 \simeq \left[\left\{c^2\ell(\ell+1)/\lambda^2 + \omega_{pi}^2\right\}/\left(1+\omega_{pe}^2/\Omega_{ze}^2\right)\right]^{1/2} \qquad (2.9)$$

and

$$\omega^2_{l,m} = \pm i \left[k_x^2 V_{oi}^2 \omega_{pi}^2 / \{ c^2 l(l+1)/\lambda^2 + \omega_{pi}^2 \} \right]^{1/2}, \quad k_x\lambda = \pm m \qquad (2.10)$$

Equation (2.9) and (2.10) are derived under the approximation, $\omega'_{lm} >> |\omega^2_{lm}|$ which is clearly consistent with our earlier approximation, $\Omega_{iz} << |\omega| << \Omega_{ez}$, $(|\gamma| \equiv |\omega^2_{lm}|)$. Thus, we find that instead of the electron tearing mode, two new modes are excited. One is the modified lower hybrid mode (Eq.(2.9)) while the other characterizes the unstable Ion-tearing mode. A comparision of Eqs.(1.12) and (2.10) immediately reveals that the Ion tearing mode has a lower growth rate compared to the electron tearing mode; yet this mode is far more dangerous for the equilibrium than the electron tearing mode. In fact, as we shall see in forthcoming discussions (Sections 3,4,5), it is the Ion tearing mode that plays the crucial role in the reconnection process as well as to act as the triggering mechanism for the magnetic substorms.

Section 3 : Quasilinear Effects and Trapping in Wave Field:

In the last two sections, we have seen that a configuration containing a neutral sheet is inherently unstable and the linear theory developed so far showed that the electron tearing mode could be driven unstable in the one dimensional case ($B_{zo} = 0$) and the Ion tearing mode takes over for $B_{zo} \neq 0$. The question now arises as to which one of the two modes can finally lead to the over all change in the magnetic field topology as such a modification is reportedly observed by Nishida and Nagayama[27] during the substorms for geomagnetic tail region. This question remains unanswered in the discussions made so far and it is, indeed, a formidable task to study the nonlinear tearing mode theory, using rigorous mathematical methods. Even the quasilinear approximations break down for $|\gamma| > k_x V_{th}$ (Shapiro[32], Davidson[31]) and non-linear evolution of the purely growing mode is far from being understood. In this section, we have therefore attempted to explain these aspects rather semi-qualitatively based on certain simple physical considerations.

Davidson et al. had studied a similar case of the electron pinch by a computer simulation method and they had shown that for instabilities having growth rate $|\gamma| > kV_{thj}$, the saturation was primarily due to magnetic trapping of the particles by the wave field. If we assume a maxwellian distribution for the particles, then the major fraction of the particles (particles moving with velocity $V_x < V_{thj}$) would see a coherent unstable mode, since the waves will have grown to a significant level before the particles can move appreciably ($|\gamma| > k_x V_{thj}$). This is the primary reason why a quasilinear approach cannot be used which demands a random set of waves within a definite spectral width Δ. There would of course be a small fraction of particles, lying near the tail of the distribution function, which would have velocities larger than the thermal velocity and would remain untrapped due to their high thermal energies. These particles would obey the quasilinear dispersion mechanism and the theory to this effect will be dealt with in a later paragraph.

Following the same geometry and approximations made in the earlier sections (see Fig.1) (also Sinha and Sundaram[29]), we assume the electron-tearing mode to be unstable and that it saturates at an amplitude given by $B_z = B_{ks}\, P_e^m (\tanh Z/\lambda) \cos kx$. The mode defined by $l = m = 1$ is the most unstable mode and for $Z/\lambda \ll 1$, we have $B_z = P_1^1 (\tan Z/\lambda) \cos kx\, B_{ks} \simeq B_{ks} \cos kx$. This perturbed field can be derived from a vector potential

$$\tilde{A}_{ys} = - B_{ks} \frac{\sin k_x x}{k_x} \tag{3.1}$$

Writing the Hamilton|Jacobi equation for electrons in the presence of a potential given by Eq.(3.1), we get:

$$\frac{1}{2m_e} \left[\frac{\partial S}{\partial r} - (q_e/c)\tilde{A}_{ys} \right]^2 + \frac{\partial S}{\partial t} = 0 \tag{3.2}$$

where 'S' is the Hamilton principal function and the

Hamiltonian is defined by $H = \frac{1}{2m_e}\left[\bar{P} - \frac{q_e}{c}\bar{A}_{ys}\right]^2$
The electric potential is evidentely zero at the saturation point since the mode is purely growing. The generalized moments are given by $P_i = \frac{\partial S}{\partial x_i}$ where $x_1 = x$, $x_2 = y$, $x_3 = z$. For the potential given by Eq.(3.1) the generalised moments in the 'y' direction is conserved i.e., $P_y = mV_y + q/c\ A_{ys} = \alpha_1 = \text{const}$. Also the total energy of the particle is conserved and therefore $\epsilon = \alpha_2 = \text{const}$. These two constants of motion allow us to construct a solution of Eq.(3.2) in the form; $S = W(x) + \alpha_1 y + \alpha_2 t$. (refer[39] Chapter 9). Substituting this solution in Eq. (3.2), we obtain an expression for the momenta in 'x' direction given by

$$P_x = \frac{\partial W}{\partial x} = \left\{2m_e\alpha_2 - \left(\alpha_1 - \frac{q_e}{c}B_{ks}\frac{\sin k_x x}{K_x}\right)^2\right\}^{1/2} \tag{3.3}$$

For trapping of the electron in the field B_z, we therefore need P_x to become imaginary for some value of 'x' which can be interpreted as a reflection of the particle at that point. For a reflection to occur, we must have

$$B_{ks}\sin k_x x > \frac{k_x c}{q_e}\left\{\sqrt{2m_e\alpha_2} - \alpha_1\right\}$$

The minimum value of B_{ks} required for trapping is, therefore, given by

$$B_{ks} \gtrsim \frac{k_x c}{q_e}\left\{\sqrt{2m_e\alpha_2} - \alpha_1\right\} \tag{3.4}$$

where $\alpha_2 \simeq \frac{1}{2} m_e (V_{th}^2 + V_o^2)$ and $\alpha_1 \simeq m_e V_o$. For the typical parameters of the neutral sheet in the tail region, Eq. (3.4) gives a value $B_{ks} \simeq .1\gamma$. This is, in fact, a large value compared to the level of turbulence required to stabilize the Coppi's type of tearing mode which, according to earlier investigations, gives a value

$$B_{ks} \simeq \epsilon^{5/2} = (\rho_e/L)^{5/2} B_{xo} \ll .1\gamma$$

(Galeev and Zelenyi[30], Biskamp et al.[18]). It is evident from Eq.(3.4) that, for $T_i > T_e$ (which is valid in the geomagnetic tail during the substorm, e.g., $T_i > 1$ keV, $T_e \simeq 100$ ev), the magnetic field, B_{ks} required to stabilize the ion tearing mode is much larger and the ratio of B_{ks} for ions and electrons approximately turns out to be

$$\frac{B_{ksi}}{B_{kse}} \simeq \sqrt{\frac{T_i}{T_e}} \left(\frac{m_i}{m_e}\right)^{1/2} \qquad (3.5)$$

Physically, Eq.(3.5) can easily be understood in the following way. The ions, being massive have a larger momentum than the electrons and therefore they require a much larger wave field to be turned around. This argument fully substantiates the reason why an ion tearing mode plays rather a dominant role for the reconnection process. We shall discuss about these properties at a later stage when we deal with the applications of this theory in

Section (5).

At this step, it is of interest to compare the saturation value of B_{ks} having a typical value $\sim 0.1\,\gamma$ for electron tearing mode, derived from Eq.(3.4), with the value of B_{ks} calculated by Davidson et al.[31]. According to their analysis, the expression for the growth rate is given by

$$\gamma_k^2 = \omega_B^2 = (k_x V_{oe})\, e B'_{ks} / m_e c \qquad (3.6)$$

when the saturation occurs. Using the value for k computed from Eq.(1.13) wherein $\frac{2\pi}{\gamma k} \simeq 0.2$ secs. and substituting this value in Eq.(3.6), we get $B_{ks} \simeq .09\,\gamma$. We thus find a very good agreement between our two theoretical estimates. We can therefore conclude that the electron tearing mode would saturate after the amplitude of the mode reaches a level of about $B_k \sim .1\,\gamma$ in the geomagnetic tail region. Even though this value is larger by orders of magnitude than the level required to saturate the Coppi's type of tearing mode, it is still not large enough to change the topology of the tail field where the observed fluctuations in B_{ks} is of the order of a few gammas. The ion-tearing mode would then play an important role since it saturates at a much higher value of B_{ks}. The evolution of the electron tearing mode however would lead to the heating of the particles near the tail of the distribution function. To see this effect, we study the quasilinear relaxation of the distribution function $f_o(\bar{v})$, which is valid for the particles having velocities larger than the average thermal velocity i.e. $V_x > V_{th}$. We have already elaborated on this point, earlier, and for further details, we refer to the work of Shapiro[32]. The quasilinear equation for the space averaged distribution function $< f_o >$ is given by (Shapiro[32])

$$\frac{\partial \langle f_o \rangle}{\partial t} = \left\langle e/m_e \left(\tilde{E}_k + \frac{v \times \tilde{B}_k}{c} \right) \cdot \frac{\partial \tilde{f}_e}{\partial \bar{v}} \right\rangle \qquad (3.7)$$

where the symbole < > stands for the space averaging, $\tilde{f}_e$ is the linear perturbed distribution function as given by Eq.(1.6). Only the electron terms are retained for the electron tearing mode, for the ion-terms are smaller by a mass ratio, m_e/m_i. (refer to section 1). In Eq.(1.6), we write the induced electric field as the time derivative of the perturbed vector potential $\tilde{A}_k$, i.e. $\tilde{E}_k \simeq \frac{i\omega \hat{A}_k}{c}$ and $\tilde{B}_k = ik\tilde{A}_k$. With these substitutions, we get (from Eq.(1.6))

$$\tilde{f}_e = \frac{-e}{m_e c} \hat{A}_k \left(\frac{\partial \langle f_o \rangle}{\partial v_y} - \frac{k v_y}{k v_x - \omega} \frac{\partial \langle f_o \rangle}{\partial v_x} \right) \qquad (3.8)$$

Eliminating $\tilde{f}_e$ between Eqs.(3.8) and (3.7) and using the expressions for $\hat{E}_k$ and $\tilde{B}_k$, in terms of $\tilde{A}_k$ we arrive at the diffusion equation,

$$\frac{1}{v_y^2} \frac{\partial \langle f_o \rangle}{\partial t} = \sum_k \frac{e^2 |A_k|^2 \gamma_k}{m_e^2 c^2} \frac{\partial}{\partial v_x} \frac{1}{v_x^2} \frac{\partial \langle f_o \rangle}{\partial v_x} \qquad (3.9)$$

and

$$\frac{\partial}{\partial t}|A_k|^2 = 2\gamma_k |A_k|^2 \qquad (3.10)$$

Eq.(3.9) is valid for $|\gamma_k|^2 << k^2 V_x^2$ and this would correspond to particles with velocities $V_x >> V_{th}$. This would therefore allow us to find an asymptotic solution for Eq.(3.9). Making the substitution $\tau = 1/2\ \Sigma e^2|A_k|^2/m_e^2c^2$ and $\partial\tau/\partial t = \Sigma e^2|A_k|^2\ \gamma_k/m_e^2c^2$, and writing $V_x^2 = W$, we get the asymptotic equation in the limit $V_x \to \infty$,

$$\frac{1}{V_y^2}\frac{\partial\langle f\rangle}{\partial t} = \frac{\partial^2\langle f\rangle}{\partial W^2} \qquad (3.11)$$

with the initial condition, $\langle f_{\tau=0}\rangle = \left(\frac{\alpha}{\pi}\right)^{3/2} \exp - \alpha\left\{V_x^2 + V_z^2 + (V_y - V_x)^2\right\}$. To solve Eq.(3.11), we express $\langle f\rangle$ as, $\langle f\rangle = \int_{-\infty}^{\infty} f_k(\tau)\, e^{ikW}\, dk$, where $f_k(\tau)$ is the fourier transform. Substitution of this expression in Eq.(3.11) gives,

$$\frac{\partial}{\partial\tau} f_k(\tau) = -k^2 V_y^2 f_k(\tau) \qquad (3.12)$$

with the solution $f_k(\tau) = f_k(0)\, e^{-k^2V_y^2\tau}$. Here $f_k(0)$ is the fourier transform of the initial distribution function at $\tau = 0$. Thus,

$$f_k(0) = \frac{1}{2\pi}\left(\frac{\alpha}{\pi}\right)^{3/2} e^{-\alpha\{V_z^2+(V_y-V_{oe})^2\}} \int_{-\infty}^{\infty} e^{-\alpha|W|}\, e^{-ikW}\, dW$$

ie

$$f_k(0) = \frac{1}{2\pi}\left(\frac{\alpha}{\pi}\right)^{3/2} \exp -\alpha\{V_z^2+(V_y-V_{oe})^2\}\Big/\alpha + ik\ . \qquad (3.13)$$

Finally, from Eq.(3.13) and (3.12) we arrive at the distribution function in the form,

$$\langle f\rangle = \frac{1}{2\pi}\left(\frac{\alpha}{\pi}\right)^{3/2} e^{-\alpha\{V_z^2+(V_y-V_{oe})^2\}} \int_{-\infty}^{\infty} \frac{e^{-k^2V_y^2\tau}\, e^{ikw}}{(\alpha+ik)}\, dk. \tag{3.14a}$$

An explicit analytical expression can be obtained from Eq. (3.14a) for $|k^2V_y^2\tau| \ll 1$, i.e. a low level of turbulence ($\alpha\tau \ll 1$). Also it should be noted here that the major contribution of the integral in Eq.(3.14) comes from a region where $k \to 0$, since W is assumed to be large. Under this approximation therefore,

$$\langle f\rangle = \frac{1}{2\pi}\left(\frac{\alpha}{\pi}\right)^{3/2} e^{-\alpha\{V_z^2+(V_y-V_{oe})^2\}} \int_{-\infty}^{\infty} \frac{dk\, e^{ikw}}{(\alpha+ik)(1+k^2V_y^2\tau)} \tag{3.14b}$$

with, $V_y^2\tau/V_{the}^4 \ll 1$

The integral in Eq.(3.14b) can now be solved by the contour integration method where the countour is chosen as a semi-circle in the upper half of the complex plane since $W > 0$. The poles are at, $k = i\alpha$, $\pm\, i/V_y\, \tau^{1/2}$. The pole $k = -i/V_y\,\tau^{1/2}$ is outside the countour and therefore does not contribute. Also the pole at $k = +i/V_y\,\tau^{1/2} \gg i\alpha$, (since $V_y\,\tau^{1/2}/V_{th}^2 \ll 1$), and therefore gives a small contribution compared to the pole at $k = i\alpha$, and under this approximation we get,

$$\langle f \rangle = \left(\frac{\alpha}{\pi}\right)^{3/2} \exp -\alpha\{v_x^2 + v_z^2 + (v_y - v_{oe})^2\} \Big/ (1 - \alpha^2 v_y^2 \tau) \tag{3.15}$$

The average temperature in the 'x' direction is therefore given by,

$$\frac{1}{2} m_e \int v_x^2 \langle f \rangle \, dv \simeq T_e(1 + \alpha\tau) \tag{3.16}$$

This shows that there is a small amount of heating and the particles with large velocities in the x-direction (confirming to our asymptot·c approximation) gain some energy in the 'x' direction. This gain in energy is at the expense of the streaming energy in the 'y' direction and this quasilinear interaction can be looked upon as a pitch angle scattering process. The streaming energy is essentially randomized and this appears as a small increase in the temperature. This effect would undoubtedly inhibit the growth of the tearing mode, (which grows at the expense of the streaming energy) but this effect is indeed small ($\alpha\tau \ll 1$) and the final saturation of our 'pinching' mode should be by the particle trapping mechanism as mentioned earlier in this section.

Section 4 : Suppression by Lower Hybrid Turbulence:

So far, we have examined the linear propagation characteristics of the reconnecting mode and its intrinsic saturation by the trapping mechanism. In this section, the possible saturation of this mode by an external or self-consistent sources will be studied. In both laboratory as well as astrophysical plasmas, there exists always a certain level of electrostatic noise generated either locally by self consistent fields or by external sources like the parametric heating processes. It therefore becomes important to study the effect of this type of a low level turbulence on the linear mode. Such a turbulence is also reported to be present in the geomagnetic-neutral sheet as mentioned in the work of Scarf et al.[33]. In a magnetised plasma, the lower hybrid turbulence can be generated in the frequency range, $\Omega_i \ll \omega \ll \Omega_e$ because of the presence of currents parallel or perpendicular to the d.c. magnetic field (see, chapters IV and V for details and references) and in fact, this mechanism is advocated as a suitable candidate for the secondary heating scheme applicable to fusion machines. We therefore, choose the lower hybrid turbulence as the external source, and shall study the effect of this turbulence on the linear tearing mode described in earlier sections.

Unlike the previously considered cases, we shall study the dispersion properties in the limits when $V_{oj} >$ or $< V_{thj}$. The latter inequality corresponds to the regime in which the Coppi's results can be recovered. We shall, therefore, derive the general dispersion relation, without any restrictions on the particle drifts, V_{oj}. The same geometry, used in section (2), will be followed in this section also. Further, the effect of the normal magnetic field, will be retained to simulate the saturation of the electron tearing mode since it is the ion tearing mode that leads to the final reconnection, (Coronitti[22]). We shall treat the normal field, B_{zo} to be uniform and we shall further use the approximations of section (2), namely, $q_x \rho_{ze} < 1$, $q_x \rho_{zi} >> 1$ $\Omega_{zi} < |\omega| < \Omega_{ze}$ where $\Omega_z = eB_{zo}/m$, 'q_x' is the wave number for the unstable mode in the x-direction.and $\rho_{zj} = V_{thj}/\Omega_{zj}$. The lower hybrid turbulence will be simply assumed to have the propagation vector along the 'x' axis, since a three-dimensional propagation does not introduce a significant modification in the results. Since the effect of lower hybrid turbulence on the ion tearing mode will be primarily considered in this section, we shall describe the electron dynamics, using the fluid equations while the ions are treated by Vlasov's equation. In the high density limit, $\omega_{pe}^2/\Omega_{ze}^2 \gg 1$, the frequency spectrum of the lower hybrid turbulence is given by $\omega_k \simeq \sqrt{\Omega_{zi}\Omega_{ze}}\left(1+\frac{3}{8}k^2\rho_{ze}^2\right)$ (Sinha and Goswami[34]), where 'k' is the wave number of the turbulent spectra and is taken along the same direction as 'q_x'. The magnetic field perturbation due to the tearing mode is given by the relation $\tilde{B}_1 = \bar{\nabla} \times \bar{A}$, $\bar{A}$ being the vector potential governed by the equation, $-\nabla^2 \bar{A}_1 = 4\pi\bar{J}/c$, $\bar{J} = \bar{J}_e + \bar{J}_i$ where $\bar{J}_i = n_o e \int \bar{V} f_i \, dv$. The electronic current, J_e is defined by

$$\bar{J}_e = -e n_o \tilde{V}_e \qquad (4.1a)$$

where $\tilde{V}_e$ can be computed from the momentum equation,

$$m_e \frac{\partial \tilde{V}_e}{\partial t} + m_e \langle V_{ef} \cdot \nabla V_{ef} \rangle = -\frac{e}{c}\left(-\frac{\partial \tilde{A}_1}{\partial t} + \tilde{V}_e \times \nabla \times \tilde{A}_1\right) \qquad (4.1b)$$

The bracket < > indicates the averaging over the fast time scale of the lower hybrid turbulence (Ref.34) and it gives the ponderomotive force on the electrons. The ion-ponderomotive force enters through the Vlasov equation as a source term and this contribution will be shown to be more important than the electron-ponderomotive force. The ponderomotive force term in Eq.(4.1b) can be expressed as

$$\bar{P}_j = m_j \langle V_{jf} \cdot \nabla V_{jf} \rangle = \sum_k \hat{e}_x \frac{\partial}{\partial x} |V_{jfx}|^2 + \sum_k V_{jfx} \frac{\partial V_{jfy}}{\partial x} \hat{e}_y \qquad (4.2)$$

where 'V_{fj}' is the fast velocity component due to the turbulent field

$$V_{jfx} = -(e/m_j)\frac{\omega_k}{\Omega_j^2-\omega_k^2}\tilde{E}_k \; ; \; V_{jfy} = \left(\frac{e}{m_j}\right)\frac{|\tilde{E}_k \times \Omega_j|}{\Omega_j^2-\omega_k^2} \quad (4.3)$$

In terms of the ponderomotive force, the current $\tilde{J}_e$ can now be computed to give

$$\frac{4\pi \bar{J}_e}{c} = -\frac{\omega_{pe}^2}{\Omega_e^2}\frac{\tilde{A}_y}{c^2} + \frac{4\pi e n P_{xe}}{\Omega_e} - i4\pi e n\left(\frac{\Omega}{\Omega_e^2}\right)P_{ye} \quad (4.4)$$

where $\omega_{pe}^2 = 4\pi n e^2/m_e$ and $n = n_o \operatorname{sech}^2(z/\lambda)$. The linearized Vlasov equation with the ponderomotive force P_i after integration over the unperturbed particle trajectories, enables us to evaluate the perturbed distribution function for ions as

$$\tilde{f}_1 = \frac{2\alpha_i e f_{oi}}{m_i c}\left[\tilde{A}_y V_{oi} - \frac{\Omega \tilde{A}\cdot v}{\Omega - q_x v_x}\right] - i\frac{\bar{P}_i \cdot \nabla_v f_{oi}}{m_i(\Omega - q_x v_x)} \quad (4.5)$$

where 'f_{oi}' is the Harris equilibrium distribution function given by $f_{oi} = (\alpha_i/\pi)^{3/2} n \exp\{-\alpha_i(v_x^2+v_z^2) - \alpha_i(v_y - V_{oi})^2\}$ To complete the set of equations for deriving the modified dispersion relation, in the presence of the turbulent field, we shall use the wave-kinetic equation to describe the turbulence, i.e.,

$$\frac{\partial N_k}{\partial t} + \bar{V}_g \cdot \bar{\nabla} N_k - \frac{\partial \omega_k}{\partial \underline{r}} \cdot \nabla_k N_k = 0 \quad (4.6)$$

where $N_k = |E_k|^2/\omega_k$ is the plasmon occupancy number, $\vec{V}_g$ is the group velocity and $\partial\omega_k/\partial\underline{r}$ is the change in the plasmon frequency due to the linear tearing mode. Linearizing Eq.(4.6) with respect to the tearing mode perturbation and using the dispersion relation for the lower hybrid mode, the perturbed plasmon occupancy number is given by

$$\tilde{n}_k = -i(\Omega_i\Omega_e)^{1/2} q_x^2 \frac{\partial N_{k0}}{\partial k} \tilde{A}_y \Big/ B_{oz}^2 (\Omega - \bar{q}_x \cdot \bar{V}_g) \qquad (4.7)$$

where $\dfrac{\partial \omega_k}{\partial \underline{r}} \simeq \dfrac{\partial}{\partial \underline{r}} (\Omega_i\Omega_e)^{1/2} = -\dfrac{q_x^2 \tilde{A}_y}{B_{oz}^2} (\Omega_i\Omega_e)^{1/2}$

Using the definition of 'N_k' and Eqs.(4.2) and (4.3), the ponderomotive term $\bar{P}_j$ can therefore be expressed in terms of N_k as

$$\bar{P}_e = \sum_k \hat{e}_x \frac{\omega_k^3}{(\Omega_e^2 - \omega_k^2)^2} \frac{e^2}{m_e^2} \frac{\partial \hat{n}_k}{\partial x} + \sum \hat{e}_y \frac{\omega_k^2 \Omega_e}{(\Omega_e^2 - \omega_k^2)^2} \frac{e^2}{m_e^2} \frac{\partial \hat{n}_k}{\partial x}$$

$$\bar{P}_i = \sum_k \hat{e}_x \frac{e^2}{m_i^2 \omega_k} \frac{\partial \hat{n}_k}{\partial x} \quad , (\Omega_i < \omega_k < \Omega_e) \qquad (4.8)$$

Eliminating n_k among Eqs.(4.4), (4.5) and (4.7) with the help of Eq.(4.8), and using the Maxwell's equation

$$\bar{\nabla} \times \bar{\nabla} \times \hat{A}_i = \frac{4\pi e}{c} \int \tilde{f}_i v dv + \frac{4\pi}{c} \bar{J}_e \qquad (4.9)$$

we arrive at the field equation for $\tilde{A}_y$,

$$-\left(\frac{\partial^2}{\partial z^2} - q_x^2\right)\tilde{A}_y = \left[\left\{\frac{4\pi e^2}{m_i c^2}\int 2\alpha_i f_{oi}\left(\hat{A}_y V_{oi} - \frac{\Omega \hat{A}_y v_y}{\Omega - q_x v_x}\right)v_y dv\right\}\right.$$

$$- i\frac{4\pi e^2}{m_i c^2}\frac{(\Omega_i\Omega_e)^{1/2}}{\omega_k B_{oz}}\epsilon q_x^3 \int v_y \frac{\partial f_{oi}}{\partial v_x}/(\Omega - q_x v_x)dv \int dk \frac{\partial N_{ok}}{\partial k}/(\Omega - q_x v_g)\tilde{A}_y$$

$$+ \frac{\Omega^2}{c^2}\tilde{A}_y + \frac{\omega_{pe}^2}{\Omega_e^2}\tilde{A}_y\frac{\Omega^2}{c^2} + \frac{4\pi e n}{\Omega_e} q_x^3\left(\frac{e^2}{m_e^2}\right)\frac{\omega_k^3}{\Omega_e^4}\frac{(\Omega_i\Omega_e)^{1/2}}{B_{oz}}\int dk \frac{\partial N_{ok}/\partial k\, \tilde{A}_y}{(\Omega - q_x v_g)}$$

$$\left. - \tilde{A}_y i \frac{4\pi e n}{\Omega_e^2}\frac{\Omega\omega_k^2}{\Omega_e^3}\left(\frac{e^2}{m_e^2}\right)\frac{(\Omega_i\Omega_e)^{1/2}}{B_{oz}} q^3 \int dk \frac{\partial N_{ko}/\partial k}{(\Omega - q_x v_g)}\right] \quad (4.10)$$

In Eq.(4.10), the electronic ponderomotive term is smaller than the ionic contribution (last two and second term respectively of the right hand side of Eq.(4.10) by a ratio $\Omega / \omega_k \ll 1$. We can, therefore, neglect the last two terms in Eq.(4.10). The field equation for A_y, similar to the one derived in Eq.(1.8) of Section 1, takes the form

$$\frac{\partial^2 A_y}{\partial \xi^2} + \left\{D(\Omega, q_x)\operatorname{sech}^2 z/\lambda - q_x^2\lambda\right\}\tilde{A}_y = 0 \qquad (4.11)$$

where $z/\lambda = \xi$, $\omega_{pi}^2 = 4\pi e^2 n_o/m_i$ and

$$D(\Omega, q_x) = \left[\frac{2\omega_{pi}^2\alpha_i}{c^2 n}\int f_{oi}\left(V_{oi} - \frac{\Omega v_y}{\Omega - q_x v_x}\right)v_y dv\right.$$

$$- i\frac{\omega_{pi}^2}{B_{oz}^2}\frac{\Omega_i}{n}q_x^3\iint dk\,dv \frac{v_y \frac{\partial f_{oi}}{\partial v_x}\frac{\partial N_{ko}}{\partial k}}{(\Omega - q_x v_x)(\Omega - q_x v_g)}$$

$$\left. + \frac{\omega_{pe}^2}{\Omega_e^2}\frac{\Omega^2}{c^2} + \frac{\Omega^2}{c^2}\right]$$

Eq.(4.11) is exactly similar to Eq.(1.8) in the absence of the ponderomotive force term. Using the boundary conditions employed previously, we get, finally the following dispersion relation,

$$D(\Omega, q_x) = l(l+1) \quad , \quad q_x^2 \lambda^2 = m^2 \tag{4.12}$$

From this relation (4.12) we can now derive the modified results of tearing mode due to the background turbulence for the limits when $\gamma \gg qV_{th}$ (our type) or $\gamma \ll q_x V_{th}$ (Coppi et al., Galeev etc.). First, we will solve the case when $\gamma \gg qV_{thi}$. Evaluating the integrals in $D(\Omega, q_x)$ (Eq.4.11) over the velocity space with $|\Omega| \gg q_x V_{thi}$, we find

$$\left(1 + \frac{\omega_{pe}^2}{\Omega_e^2}\right)\Omega^2 - \left\{\frac{c^2}{\lambda^2} l(l+1) + \omega_{pi}^2\right\} - \frac{q^2 V_{oi}^2 \omega_{pi}^2}{\Omega^2}$$
$$+ i \frac{\omega_{pi}^2 \Omega_i q_x^4 V_{oi}}{B_{oz}^2 \Omega^2} \int \frac{\partial N_{ko}/\partial k \, dk}{(\Omega - q_x v_g)} \tag{4.13}$$

For the background turbulence, we can choose the equilibrium distribution function for the plasmons, N_{ko} to be $N_{ko} = N_o (1/\pi\Delta)^{1/2} \exp-(k-k_o)^2/\Delta$ (Sinha and Goswami[34], Guzdar[35]), where Δ is the width of the turbulent spectra and 'k_o' is the peak wave number. The resonant denominator of Eq.(4.13) gives a contribution analogous to the Landau damping. Using $V_g = \frac{\partial \omega_K}{\partial K} \simeq \frac{3}{4} (\Omega_i \Omega_e)^{1/2} k \rho_e^2$ we get,

$$\left(1 + \frac{\omega_{pe}^2}{\Omega_e^2}\right)\Omega^2 - \left\{\frac{c^2}{\Lambda^2} \ell(\ell+1) + \omega_{p_i}^2\right\} - \frac{q_x^2 V_{oi}^2 \omega_{p_i}^2}{\Omega^2} + \frac{\omega_{p_i}^2}{B_{oz}} \frac{\Omega_i}{\Omega^2} q_x^4 V_{oi} \left.\frac{\partial N_{Ko}}{\partial k}\right|_{V_g = \frac{\Omega}{q_x}} = 0 \qquad (4.14)$$

Comparing Eq.(2.7) with Eq.(4.14), we immediately find that the lower hybrid turbulence has a stabilizing influence on the ion-tearing mode. Thus, in the growth-rate expression derived earlier (Eq.(2.10)) $(V_{oi})^2$ has to be replaced now by

$$\left(V_{oi}^2 - \frac{\Omega_i q_x^2 V_{oi}}{B_{oz}^2} \left.\frac{\partial N_{ko}}{\partial k}\right|_{V_g = |\Omega/q_x|} \right)$$

Depending on the various parameters and the strength of turbulence the growth rate can now be reduced or even reversed in sign giving a net damping. For a laboratory plasma, it would be important to see the effect of the turbulence on the Coppi type of tearing mode having a growth rate that satisfies $\gamma \ll qV_{thi}$. A rigorous analysis of the linear mode of this type has recently been studied by Galeev and Zelenyi(30) in which they have shown that the ion-tearing mode can only appear in certain 'gaps' in the magnitude of the normal field component, B_{zo}. Presumably, we can interpret from their remarks that the ion-tearing instability manifests when the electrons are captured by the B_{zo} component; yet it does not imply that the electrons are strongly magnetized (in other words, the typical wavelengths of ion-tearing mode satisfy the relation, $q_x \rho_e < 1$). In these circumstances, both the displacement current as well as the electronic terms are small compared to the resonant contribution of the ionic term (Galeev and Zelenyi[30]). From Eq.(4.11), we can, therefore, drop these two terms and use the approximation, $|\Omega| \ll qV_{thi}$ to get

$$D(\Omega,q_x) = -i\pi^{1/2}\omega_{pi}^2\left(\frac{\Omega}{q_x V_{thi}}\right) - \frac{2i\,\omega_{pi}^2\,\Omega_i\,q^2\alpha_i c^2}{B_{oz}^2}\int \frac{V_{oi}\,\frac{\partial N_{ko}}{\partial k}\,dk}{(\Omega - q_x v_g)}$$

$$= \ell(\ell+1)c^2/\lambda^2 \tag{4.15}$$

Again taking, $N_{ko} = N_o\,(1/\pi\Delta)^{1/2}\exp-(k-k_o)^2/\Delta$, $V_g = \dfrac{\partial\omega_k}{\partial k}$

Eq.(4.15) with 'k' normalized to $\Delta^{1/2}$ can be written in the form

$$-i\pi^{1/2}\omega_{pi}^2\left(\frac{\Omega}{qV_{thi}}\right) + \frac{8}{3}i\,\frac{N_o\,\omega_{pi}^2\,\Omega_i\,q_x^2\,\alpha_i\,c^2\,V_{oi}}{\Delta\,B_{oz}^2\,q_x\,(\eta\,\Omega_i\,\Omega_e)^{1/2}\rho_e^2}\int \frac{\frac{\partial}{\partial k/\Delta^{1/2}}\,e^{-(k-k_o)^2/\Delta}\,dk/\Delta^{1/2}}{\left(\frac{k}{\Delta^{1/2}} - \frac{\Omega}{\frac{3}{4}q_x\rho_e^2(\Omega_i\Omega_e\Delta)^{1/2}}\right)}$$

$$= c^2\,\ell(\ell+1)/\lambda^2$$

or

$$c^2 \ell(\ell+1)/\lambda^2 =$$

$$-i\pi^{1/2}\omega_{p_i}^2 \left(\frac{\Omega}{q_x V_{thi}}\right) + i\,\frac{8 N_o \omega_{p_i}^2 \Omega_i q \alpha_i c^2 V_{oi}}{3 B_{oz}^2 \rho_e^2 \Delta (\pi \Omega_i \Omega_e)^{1/2}} \int \frac{\frac{\partial}{\partial k'} e^{-(k'-k_o')^2}}{(k'-K_o)} \qquad (4.16)$$

where $k' = k/\Delta^{1/2}$, $k_o' = k_o/\Delta^{1/2}$, $K_o = 4\Omega/3 q_x \rho_e^2 (\Omega_i \Omega_e \Delta)^{1/2}$ and $C_1 = 8 N_o \omega_{p_i}^2 \Omega_i q_x \alpha_i c^2 V_{oi} / 3 B_{oz}^2 \rho_e^2 \Delta (\pi \Omega_i \Omega_e)^{1/2}$.
The integral of Eq.(4.16) can be evaluated in terms of the principal part and the pole contribution, under the approximation ($k' >> K_o$, which corresponds to $|\Omega/q_x| << V_g$) to give

$$c^2 \ell(\ell+1)/\lambda^2 =$$

$$-i\pi^{1/2}\omega_{p_i}^2 \left(\frac{\Omega}{V_{thi}}\right) + iC_1 \left[\oint \partial/\partial k' \frac{e^{-(k'-k_o')^2}}{k'-K_o} - i\pi\, \partial/\partial K_o\, e^{-(K_o-k_o')^2} \right] \qquad (4.17)$$

The principal part is unimportant in this problem since it essentially gives a nonlinear shift in the real part of the frequency. We shall, however, drop it from Eq.(4.17) and use $k_o' = k_o/\Delta^{1/2} >> K_o$ to obtain

$$\Omega = i\,\frac{c^2 \ell(\ell+1)}{\lambda^2 \pi^{1/2}} \left(\frac{q_x V_{thi}}{\omega_{p_i}^2}\right) - 2iC_1 \frac{k_o}{\Delta^{1/2}} \left(\frac{q_x V_{thi}}{\pi^{1/2}\omega_{p_i}^2}\right) e^{-k_o^2/\Delta} \qquad (4.18)$$

Writing $N_o \omega_k / n_o T_i$ = W = Energy density of waves normalised to the thermal energy and $\beta = 8\pi n_o T_i/B_{oz}^2$, we have

$$\Omega = i\,\frac{c^2 \ell(\ell+1)}{\lambda^2 \pi^{1/2}} \left(\frac{q_x V_{thi}}{\omega_{p_i}^2}\right) - \frac{i c^2 W \beta}{(3/2) V_{th}^2} \left(\frac{\Omega_i}{\omega_k}\right) \frac{q_x^2 V_{th_i} k_o V_{oi} e^{-k_o^2/\Delta}}{\rho_e^2 \Delta (\Omega_i \Omega_e \Delta)^{1/2}} \qquad (4.19)$$

Finally, we find that the growth of ion tearing mode is retarded by the presence of the damping factor which arises primarily due to the lower hybrid turbulence. For high 'β' plasma the effect is more pronounced and this is indeed the case for the geomagnetic neutral sheet where $\beta >> 1$. The physics of the stabilizing influence of the turbulence can be most easily understood based on the ponderomotive force formalism. The magnetic field perturbation due to the linear tearing mode alters the plasmon frequency (ω_k). The perturbation in 'ω_k' in turn gives rise to a ponderomotive force which inhibits the current streams from pinching any further. The phase matching condition for the perturbation and the ponderomotive force is fulfilled when $V_g = \partial\omega_k/\partial k = \Omega/q_x$, that is, when the resonant interaction occurs. Recent satellite data does seem to indicate the presence of the lower hybrid noise in the tail region as reported by Scarf et al. and might therefore contribute significantly in sustaining the current sheet both at the onset as well as the recovery phase of the substorm. We shall discuss this point further in the concluding section based on their relevance to observational facts.

Section 5 : Applications and Conclusions:

The theory developed in the preceding sections allows us to make certain predictions regarding the reconnection process of the oppositely directed magnetic field lines that can arise out of current sheets present in a given magnetic configuration. We have not incorporated any resistive effects as we expect the magnetospheric plasmas as well as the high temperature fusion plasmas to be virtually collisionless. The resistive theories of the tearing mode (Furth et al.(15), Rutherford(36), Hazeltine et al.(16) etc.) are not applicable to the space plasmas and it has been shown recently by Drake and Lee(23) that the fusion plasma at the most could be semicollisional ($\omega > \nu_c$). Very recently, the importance of collisionless tearing mode theory for tokomak discharges has been realized. In fact, the m = 2 mode, observed independently by Mirnov(37), is identified with the tearing mode in the tokomak experiments. Before we contemplate on the applications of our theory to tokomaks, we must take into cognizance some of the important differences between the tokomak and tail magnetic field configurations. As depicted clearly in figures (1) and (2), the non-zero magnetic field is parallel to the current direction while it is not the case for the geomagnetic tail region. Further, for tokomaks, the parallel field is much stronger than the oppositely directed field (poloidal field) which is in contrast with the requirements in the tail region. Perhaps, the most important difference is the fact that the triggering of tearing mode depends crucially on the parameter $\vec{k}.\vec{B}_0$ which assumes positive and negative values. Thus it becomes immensely difficult to establish a connection between our results and the tokomak disruption. These discrepancies call for a repetition of the above calculations for a tokomak geometry. Under the thin sheet approximation, i.e. $\lambda/\rho_i \lesssim 1$ (ρ_i being the gyroradius in the parallel field), however, our results may be extrapolated to the tokamak observations. It is worthwhile noting here that the Coppi type (and later developed by Galeev and Zelenyi[30]) of tearing mode will totally be inoperative in the tokomak configuration since the resonance interaction of the particles with the waves will be completely removed by the presence of the parallel field, under their approximation $\lambda/\rho_i >> 1$. The elimination of Coppi's type mechanism clearly paves the way for a tearing mode theory of our type, possibly operative in the tokomak regime, since the time scales in which the current sheet disruption occurs are much faster than Coppi's. In the light of our previous arguments, a need arises, therefore, for a re-investigation of our theory tailored to the tokomak conditions. In any case, the reported observations of the laboratory experiments of tearing mode are far too imprecise at present to make any meaningful comparison with a theoretical model. The geomagnetic substorm provides a better ground for the applicability of our theory. The most significant observation that brings into the focus of our theory is the satellite report by Nishida and Nagayama[27] regarding the formation of the x-type of neutral line. It is speculated that the substorm is intimately connected with the formation of the x-type of neutral line in the geomagnetic tail wherein the magnetic energy is dissipated. We shall, first, review the observations of Nishida and Nagayama[7].

The substorm phenomena is divided into three main phases: (1) The growth phase is a period in which the magnetic flux is transported from the day side to the night side giving rise to an induced zero order electric field in the east west direction (aligned with the y-axis in our geometry). This electric field strengthens the tail currents which in turn reduces the normal component of the magnetic field B_{zo}. (2) The expansion phase is signalled as the component, B_{zo} goes to zero, that is, the tail field lines become parallel to each other and there is a rapid dissipation of energy. This phase is often called the explosive phase. During this phase, there is a sharp increase of the B_{zo} field (Relaxation towards a dipolar configuration) for $X < 12\ R_E$ but the field never turns southward (the X-direction is along the sun-earth line and is aligned along the x-axis of our geometry. The North-South direction is along the 'Z' axis). Beyond $X \simeq 12\ R_E$, there is a sharp increase in B_{zo} in the higher latitudes and in the region somewhere between $12\ R_E < X < 35\ R_E$ the component, B_{zo} turns southward for lower latitudes (closer to the neutral sheet taken in the xy plane, $Z = 0$). (3) The Recovery phase, is characterized by the quiet-time cofiguration in which magnetic field topology returns to its original structure that prevailed before the growth phase.

To understand the sequence of events described above, we must visualize the quiet period as the equilibrium configuration in which the current sheet is stable against the tearing mode perturbation. This state is achieved in our case if the drift velocity of the particles is small so that $V_{oj} < V_{thj}$. In this case, only the Copp's type of instability can be operative and, however, this mode will eventually get quenched in the presence of a normal field B_{zo}, which is quite significant (for $B_{zo}/B_{xo} \simeq 0.1$) In the growth phase, the magnetic flux transported to the nightside will give rise to the induced electric field E_{oy} which, according to Nishida and Nagayama is as large as .1 mv/cm and is thus effective along the current direction for a distance over $12\ R_E$. This field will then accelerate the particles and the mean gain in the energy due to this field will be given by $1/2\ m_j\ v_j^2 = q_j\ E_{oy}\ L$. For typical values of $E_o \simeq .1$ mv/cm., $L = 10\ R_E$, the particle drift becomes $v_e \simeq 10^9$ cms/sec. The particle acceleration takes place only along the neutral sheet (i.e. $Z/\lambda << 1$) while for $Z/\lambda > 1$, the particles only execute the $\bar{E}_{oy} \times \bar{B}_{xo}$ motion. This $\bar{E} \times \bar{B}$ motion leads to a thinning of the plasma sheet and this feature is mentioned in the work of Hones et al.[26]. The average temperature of t the electrons during the growth phase is about $T_e = 100$ ev, corresponding to $V_{the} \simeq 10^8$ cm. (Akasofu[32], Hones et. al[26]). As we have seen in section 3, even a small value of B_{zo} can stabilize the electron tearing mode and therefore the expansion phase can start only when B_{xo} falls below this value. (The ion temperature being larger than the electron temperature and the mean gain in the ion energy from the induced field being smaller than the electrons, the condition $V_{oi} > V_{thi}$, cannot be satisfied for the ions in the pre-expansion phase and hence the ion tearing mode will remain stable). As B_{zo} falls below $B_{zo} \simeq 0.1$. (satisfied more easily for distances further down the tail) the electron tearing mode is excited and the reconnection process is triggered. This forms the near-earth neutral line and along this line the ions are further accelerated. The electron tearing mode is however quenched at a low amplitude ($B_{ks} \simeq .1\gamma$). In this process, the ions gain enough drift energy from the field annihilation to excite the ion-tearing mode which ultimately causes a major change in the magnetic field topology. The necessary conditions required for our theory to be valid are therefore easily satisfied during the expansion phase. The plasma sheet thins down to almost the ion-gyroradius limit (Hones et al.[26]) and the electric field along the 'y' axis is large enough to accelerate the particles to the required velocities ($V_{oi} > V_{thi}$). After the stabilization of the electron tearing mode, the possibility of simultaneous excitation of the ion-tearing mode as well as the lower hybrid mode arises and the latter type leading to certain level of turbulence is already reported by Scarf et al. In the presence of the normal field (after the saturation of the electron-tearing mode), the lower hybrid turbulence is generated by the two stream mechanism for $V_{oi} > V_{thi}$ (McBride et al.[33]). This turbulence will continue to exist even after the electron tearing mode is saturated and it could be responsible for preventing the total collapse of the current sheet. As we have seen, in section (4), both our type as well as Coppi's type mode can be stabilized by the lower hybrid turbulence. We expect that such electrostatic noise sustain the current sheet through the expansion phase which will help in a rapid transition to the pre-substorm state in the recovery phase, which is in conformity with the observations by Hones et. al[26], Nishida and Nagayama[27], Akasofu[38] etc. Finally, the predicted high growth rates in our nonresonant model ($2\pi/\gamma \simeq$ .2 sec) agree very well with the fast changes in the magnetic field topology near the neutral sheet.

The suppression mechanism developed in Section (4) could also have some relevance in the tokamak discharges where the tearing mode is observed. The presence of the lower hybrid turbulence would then improve the stability of a system by quenching the tearing mode.

In conclusion, we find that in a magnetic configuration containing a neutral sheet, an electromagnetic instability can be excited with a large growth rate ($\gamma > k V_{thj}$) whenever the drift velocity V_{oj} exceeds the thermal velocity of particles. The electron mode is stabilized at a low value of the field amplitude ($B_{ks} \simeq 0.1 \gamma$) and the subsequent reconnection is achieved by the ion-mode. The quasilinear effects play only a secondary role and leads to a small heating. The presence of a lower hybrid turbulence has a stabilizing effect on the ion-tearing mode and would possibly sustain the current-sheet from the ultimate collapse. The geomagnetic substorm can be interpreted freely well within the framework of our model and the high growth rate of our instability would well account for the explosive nature of the expansion phase of the substorm.

The particle acceleration mechanism still remains an open question and would be an important topic for further research. The exact saturation mechanism of the ion-tearing mode is yet another outstanding problem. It is of utmost importantance to know the saturation levels for this mode in order to make any prediction about the final topology of the magnetic field configuration.

References

1. Geovanelli, R.G., Mon. Notices Roy. Astron. Soc. 107, 338 (1947)

2. Geovanelli, R.G., Mon. Notices Roy. Astron. Soc. 108, 163 (1948)

3. Hoyle, F., Some Recent Observation in Solar Physics, p.92-104 (1949), Cambridge University Press, London

4. Dunjey, J.W., Phil. Mag. Ser. 7, 44, 725 (1953)

5. Sweet, P.A., Annu. Rev. Astron. Astrophysics 7, 147 (1969)

6. Parker, E.N., J. Geophys. Res. 62, 509 (1957)

7. Petschek, H.E., AAS-NASA Symposium on the Physics of Solar Flares, NASA Spec. Publ. SP-50, p.425 (1964)

8. Sonnerup, B.U.O, J. Plasma Phys. 4, 207 (1970)

9. Yeh, T. and Axford, W.I., J. Plasma Phys. 4, 207 (1970)

10. Vasylunas, V.M., Rev. of Geophys. and Sp. Phys. Vol.13, 803 (1975)

11a. Speiser, T.W., JGR 72, 3919 (1967)

11b. Speiser, T.W., JGR 70, 4219 (1965)

11c. Speiser, T.W., Planet Sp. Sci. 18, 613 (1970)

12. Cowley, S.W.H., Cosmic Electrodynamics 3, 448 (1973)

13. Harris, E.G., Nuovo Cimento 23, 115 (1962)

14. Toichi, T., Cosmic Electrodynamics, Vol.3, 81 (1972)

15. Furth, P., Killeen, J.and Rosenbluth, M.N., Phys. Fluids 6, 459 (1963)

16. Hazeltine, R.D., Dobrott, D. and Wong, T.S., Phys. Fluids 16, 1054 (1975)

17. Coppi, B., Phys. Rev. Lett. 16, 1207 (1966)

18. Biskamp, P., Sagdeev, R.Z. and Schindler, K., Cosmic Electrodynamics 1, 297 (1970)

19. Schindler, K., J. Geophys. Res. 79, 2803 (1974)

20. Dohrowolny, M., Nuovo Cimento, B55, 427 (1968)

21. Galeev, A.A. and Zelenyi, L.M., Sov. Phys. JETP 42, 450 (1975)

22. Coroniti, F.V., Phys. Rev. Lett. 38, 1355 (1977)

23. Drake, J.F. and Lee, Y.C., Phys. Fluids 20, 1341 (1977)

24. Bowers, E.C., Astrophysics and Space Science 21, 399 (1973)

25. Bowers, E.C., Astrophysics and Space Science 24, 349, (1973)

26. Hones, Jr. E.W., Asbridge, J.R., Banie, S.J. and Sugei, S., J. Geophy. Res. 78, 109 (1973)

27. Nishida, A.E. and Nagayama, N., JGR 78, 3782 (1973)

28. Momota, H., Progress of Theoretical Physics 35, 380 (1966)

29. Sinha, M. and Sundaram, A.K., Geo. Phys. Res. Lett. 3, 57 (1976)

30. Galeev, A.A. and Zelenyi, L.M., Sov. Phys. JETP 43, 1113 (1976)

31. Davidson, R.C., Hammer, D., Haber, J. and Wagner, C.E., Phys. Fluids 15, 317 (1972)

32. Shapiro, V.D. and Shevchenko, V.I., Sov. Phys. JETP 15, 1053 (1962)

33. Scarf, F.L., Frank, L.A., Ackerson, K.L. and Lepping, R.P., Geophys. Res. Lett. 1, 189 (1974)

34. Sinha, M. and Goswami, B.N., (1977) (To appear in Phys. Fluids)

35. Parvez Guzdar, Phys. Rev. Lett. 35, 1635 (1975)

36. Rutherforth, P.H., Phys. Fluids 16, 1903 (1973)

37. Mirnov, S.V. and Semenov, I.B., Sov. Phys. JETP 33, 1134 (1971)

38. Akasofu, S.I., Hones Jr., E.W., Banie, S.J., Asbridge, J.R. and Lui, A.T.Y., JGR 78, 7527 (1973)

39. Goldstein, H., Classical Mechanics, Addison-Wesley, Chapter 9, p.273 (1959)

APPENDIX - I

The Harris Equilibrium:

The steady state Vlasov equation can be written as

$$\underset{\sim}{v} \cdot \frac{\partial f}{\partial \underset{\sim}{x}} + \frac{e}{M}\left(\underset{\sim}{E} + \frac{1}{c}\underset{\sim}{v} \times \underset{\sim}{B}\right) \cdot \frac{\partial f}{\partial \underset{\sim}{v}} = 0, \tag{I.1}$$

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{E} = 4\pi \sum e \int f \, d^3 v, \tag{I.2}$$

$$\underset{\sim}{\nabla} \times \underset{\sim}{B} = \frac{4\pi}{c} \sum e \int f \underset{\sim}{v} \, d^3 v. \tag{I.3}$$

There is an equation like Eq.I.1 for ions and for electrons with the appropriate values of e and M. The summations are over species of particles (ions and electrons) with appropriate e and f. It is well known that a solution of Eq.I.1 is given by any function of the particles constants of the motion. If we consider $\underset{\sim}{E}$, $\underset{\sim}{B}$, and f which depend only on one co- rdinate (say the z-co-ordinate), then we know that the energy and the momenta conjugate to y and x are constants of the motion. These are

$$W = \frac{1}{2} M (v_x^2 + v_y^2 + v_z^2) + e\Phi(x), \tag{I.4}$$

$$p_y = M v_y + \frac{e}{c} A_y(z) \tag{I.5}$$

$$p_x = M v_x + \frac{e}{c} A_x(z) \tag{I.6}$$

where A(z) is the vector potential. We assume that $\underset{\sim}{E}$ has only an z-component and B has only a x-component. Then $\underset{\sim}{A}$ may be taken to have only a y-component. It is convenient to rearrange Eqs. I.4, I.5 and I.6 to give the constants of the motion

$$\alpha_1^2 = v_z^2 - \frac{2e}{Mc} v_y A_y - \frac{e^2}{M^2c^2} A_y^2 + \frac{2e}{M} \varphi \qquad (I.7)$$

$$\alpha_2 = v_y + \frac{e}{Mc} A_y \qquad (I.8)$$

$$\alpha_3 = v_x \qquad (I.9)$$

There is a set $(\alpha_1, \alpha_2, \alpha_3)$ for the ions and another for the electrons; they differ in the values of e and M they contain. The solution of Eq.I.1 is $f = f(\alpha_1, \alpha_2; \alpha_3)$. Substituting this into Eqs.I.2 and I.3 gives two coupled differential equations for φ and A_y. The nature of the solution will depend on our choice of f and the boundary conditions.

We will assume that at z = 0 the distribution functions are maxwellian centered about some mean velocity in the y-direction. That is

$$f_i = \left(\frac{M}{2\pi\Theta}\right)^{3/2} N \exp\left[-\frac{M}{2\Theta}\left[\alpha_1^2 + (\alpha_2 - V_i)^2 + \alpha_3^2\right]\right] \qquad (I.10)$$

$$f_e = \left(\frac{m}{2\pi\Theta}\right)^{3/2} N \exp\left[-\frac{m}{2\Theta}\left[\alpha_1^2 + (\alpha_2 - V_e)^2 + \alpha_3^2\right]\right] \qquad (I.11)$$

where V_i and V_e are the mean velocities of ions and electrons respectively. If Eqs.I.10 and I.11 are substituted into Eqs.I.2 and I.3 the following equations for the potentials φ and A_y are obtained

$$\frac{d^2\varphi}{dz^2} = -4\pi Ne\left[\exp\left[\left(\frac{e}{\Theta c}V_i A_y - \frac{e}{\Theta}\varphi\right)\right] - \exp\left[\left(-\frac{e}{\Theta c}V_e A_y + \frac{e}{\Theta}\varphi\right)\right]\right] \quad (I.12)$$

$$\frac{d^2 A_y}{dz^2} = -\frac{4\pi Ne}{c}\left[V_i \exp\left[\left(+\frac{e}{\Theta c}V_i A_y - \frac{e}{\Theta}\varphi\right)\right] - V_e \exp\left[\left(\left(-\frac{e}{\Theta c}V_e A_y + \frac{e}{\Theta}\varphi\right)\right]\right\} \quad (I.13)$$

We note in passing that if $V_i = V_e = 0$ then $\varphi = A_y = 0$ and there are no fields and no confinement of the plasma.

We now assume that $V_e = -V_i = -V$. This may always be made true by transforming to a reference moving with the appropriate velocity. The fields found will now be those in the moving reference system. With this assumption Eqs.I.12 and I.13 become

$$\frac{d^2\varphi}{dz^2} = -4\pi Ne \exp\left[\left(\frac{e}{\Theta c}VA_y\right)\right]\left[\exp\left[\left(-\frac{e}{\Theta}\varphi\right)\right] - \exp\left[\left(+\frac{e}{\Theta}\varphi\right)\right]\right] \quad (I.14)$$

$$\frac{d^2 A_y}{dz^2} = -\frac{4\pi Ne}{c}V \exp\left[\left(\frac{e}{\Theta c}VA_y\right)\right]\left[\exp\left[\left(-\frac{e}{\Theta}\varphi\right)\right] + \exp\left[\left(+\frac{e}{\Theta}\varphi\right)\right]\right] \quad (I.15)$$

Eq.I.14 is clearly satisfied by $\varphi = 0$.

Eq.I.15 then becomes

$$\frac{d^2 A_y}{dz^2} = -\frac{8\pi N e V}{c} \exp\left[\left(\frac{e}{\theta c} A V\right)\right] \qquad (I.16)$$

If the boundary conditions are taken to be

$$A_y = 0 \qquad (I.17)$$

and

$$B = \frac{\partial A_y}{\partial z} = 0 \qquad (I.18)$$

at z = 0, then the solution of Eq.I.16 is

$$A_y = -\frac{2\theta c}{eV} \log \cosh\left(\frac{zV}{cL_D}\right) \qquad (I.19)$$

From which we find the magnetic field

$$B = \sqrt{16\pi N\theta} \tanh\left(\frac{Vz}{cL_D}\right) \qquad (I.20)$$

In the above

$$L_D = \left(\frac{\theta}{4\pi N e^2}\right)^{1/2} \text{ Debye length.} \qquad (I.21)$$

The conservation of the total pressure (thermal plus the magnetic pressure) gives the following expression of density 'n',

$$n k T_e + \frac{B^2}{8\pi} = \text{const}, \qquad (I.22)$$

Using Eq.I.20 we get,

$$n = n_0 \operatorname{Sech}^2\left(\frac{z}{x}\right).$$

CHAPTER III

PLASMA INSTABILITIES DRIVEN BY VELOCITY GRADIENTS

Introduction

In the previous chapter the theory revealed that the stability of a system could be affected significantly in the presence of an external source. It was shown that an electric field present along the neutral sheet could freely accelerate the particles, which in turn 'triggers' the tearing mode. Away from the neutral sheet the electric field cannot, however, accelerate the particles and it gives rise to an EXB motion. This velocity is not generally homogeneous and can have a considerable amount of shear. In a theoretical study Kan(1) has proposed the existence of such a shear which accounts for the observed thickening of the plasma sheet in the geomagnetic tail region. This velocity shear in a plasma fluid motion manifests itself as an additional source of free energy and can further alter the stability of the system. Our purpose here, has been to study the low frequency instabilities that can be triggered in presence of such a velocity shear. The analysis is not restricted to a neutral sheet configuration only. since non-uniform electric fields are known to exist in many laboratory experiments (e.g. Q-machines, tokamaks etc.). Further role of spatially non-uniform particle drifts has also been discussed in many laboratory studies, for instance, in pulsed plasma heating (Wharton et al.[2]) and rotating plasma devices (Lehnert[3]). In a bounded plasma, the difference in the diffusion rates of charged species across the confining magnetic field leads to an accumulation of charge near the surface. This produces the non-uniform electric field and we know that the Kelvin-Helmholtz (hereafter referred to as KH) type of instability can be driven unstable in the presence of such non-uniform fields. This type of instability has been already observed in the auroral ionosphere region (Hallinan and Davis[4]). Further, the recent work of Hirose and Alexeff[5] is an illustrative example in which a variety of high frequency ($\omega > \Omega_i$) electrostatic modes are excited in the presence of sheared velocity along the across the magnetic field. Although these high frequency instabilities are directly responsible for the enhancement of effective electron-electron (and electron-ion) collision frequency and the anomalous skin effect they do not contribute to the anomalous resistivity as the ions are unaffected by these instabilities. On the other hand, the low frequency ($\omega < \Omega_i$) plasma instabilities do play an important role in the effective ion-heating. A part of the present work will, therefore, be devoted to studying the low frequency instabilities in detail.

Interfacial plasma instabilities (of KH type) with shear velocity parallel and perpendicular to the magnetic field has been thoroughly investigated by earlier workers (D'Angelo[6], Jassby and Perkins[7], Jassby[8]). At this step, it is important to emphasize the physical process involved in the excitation of different types of modes which depend on the velocity shear. For the development of transverse KH mode in Q-machines, studied in particular by Jassby and Perkins, the main physical process is the centrifugal effect, which causes differential motion between electrons and ions leading to the charge separation and the resultant instability. Although similar types of modes are also excited in a slab geometry (Mikhailoviskii[9] and Chandrasekhar[10]), the excitation mechanisms are distinctly different from the earlier ones. In the latter case, however, the process manifests in terms of varying drift velocity (due to nonuniform electric field or other inhomogeneities) giving rise to spatially separated streams in the plasma, which are possibly responsible for driving the system unstable. Thus the extensive results presented in this chapter are associated with the latter type of physical phenomena. Two important frequency ranges are considerably stressed in this chapter and these frequency domains clearly demarcate the regions in which the low or high frequency modes driven primarily by velocity gradients are triggered.

Further quantitative results included in our analysis comprise of the effect of a turbulent spectrum of lower hybrid waves on the Kelvin-Helmholtz instability wherein the electron dynamics alone plays a decisive role in the triggering process. A coupling of this type is motivated by Jassby's[11] observation that the generation of high frequency electrostatic noise (the frequency spectrum of which is not exactly identified) instantaneously accompanies the suppression of Kelvin-Helmholtz mode for increasingly large values of radial electric field. Possibly this noise could be due to the presence of lower hybrid waves. It has already been shown by Stringer and Schmidt that in the presence of an inhomogenous electric field, a relative drift between electrons and ions can be maintained and this drift, in turn, can generate the lower hybrid mode provided the relative velocity exceeds the ion thermal velocity (McBride, et al.[13]). Therefore, the lower hybrid mode having a much larger growth-rate ($\gamma_{LH} \gg \Omega_i$) than the Kelvin-Helmholtz instability ($\gamma_{KH} \ll \Omega_i$) can quickly go over to the turbulent stage and in that event it would indeed be important to study the effect of this turbulence on the Kelvin-Helmholtz Instability.

The plan for this chapter is therefore as follows:

In section 1 we study the low frequency linear instability driven by the velocity gradient while in section 2 we study the effect of the lower-hybrid turbulence on the low-frequency Kelvin-Helmholtz Instability.

Section 1 : Low frequency electrostatic instability in presence of velocity gradients

We consider an inhomogeneous plasma embedded in a nonuniform magnetic field, B_o along z-axis of a slab geometry. The plasma density and the magnetic field vary along x-direction. We assume a shear drift velocity, $V_o(x)$ for the charged particles directed along y-axis. The velocity $V_o(x)$ is produced by an electric field $E_o(x)$ and can either be a self consistent field or an externally applied field. In the former case, there will be a certain amount of charge imbalance in the equilibrium. The effect of this non-neutrality has a pronounced role only for the low density plasma, i.e. $V_a^2/c^2 >> 1$ (Alfven velocity = V_a c = velocity of light) and the effect becomes insignificant for the high density limit i.e. ($V_a^2/c^2 << 1$) (Stringer and Schmidt). Since we shall be working in the limit ($V_a^2/c^2<<1$) this effect will be ignored in our analysis. This will be rigorously true of course for the case of an external electric field. Since we deal with a general situation, the functional dependence and the nature of particle drift velocity will be kept arbitrary at this stage. We shall now study the stability characteristics of the above equilibrium configuration, using the fluid equations for both electrons and ions. Finite temperature effects (and the stabilizing influence of Larmor radius corrections) will be ignored throughout our analysis for mathematical simplicity. Seeking an electrostatic perturbation of the flute type $\tilde{q}(x)\exp i(ky-\omega t)$ where $\tilde{q}$ represents a typical perturbed quantity, the fluid particle drifts in the presence of small perturbed electric fields are given by

$$\tilde{V}_x = \frac{ie}{m}\left(\omega_0 \frac{d\phi}{dx} - k\phi\Omega\right) \Big/ \left[\Omega\left(\Omega + \frac{dV_0}{dx}\right) - \omega_0^2\right] \quad (1.1)$$

$$\tilde{V}_y = \frac{e}{m}\left[\left(\Omega + \frac{dV_0}{dx}\right)\frac{d\phi}{dx} - k\omega_0\phi\right] \Big/ \left[\Omega\left(\Omega + \frac{dV_0}{dx}\right) - \omega_0^2\right] \quad (1.2)$$

where ϕ is the electrostatic potential, Ω, the cyclotron frequency and ω_0 ($= \omega - kV_0$) is the Doppler shifted frequency. The other symbols have their usual meanings. If n_0 denotes the equilibrium guiding-centre density and δn its perturbed value, then conservation of guiding-centre mass demands that

$$i\omega_0 \frac{\delta n}{n_0} = K_n \tilde{V}_x + ik\tilde{V}_y + \frac{d\tilde{V}_x}{dx} \quad (1.3)$$

Where K_n stands for $(d/dx)(\ln n_0)$. Substituting the values for $\tilde{V}_x$ and $\tilde{V}_y$ (Eqs.(1.1) and (1.2)) and using Eq.(1.3) together with the Poisson's equation $\nabla^2\phi = + 4\pi e$ $(\delta n_e - \delta n_i)$; we finally derive the differential equation for ϕ in the form

$$A\frac{d^2\phi}{dx^2} + B\frac{d\phi}{dx} + C\phi = 0 \tag{1.4}$$

where A, B and C are functions of x and other physical parameters of the problem and are given by the expressions

$$A = 1 + \sum \omega_p^2 \Big/ \left[\Omega\left(\Omega + \frac{dV_0}{dx}\right) - \omega_0^2\right] \tag{1.5}$$

$$B = \sum \omega_p^2 \left[K_n\left\{\Omega\left(\Omega + \frac{dV_0}{dx}\right) - \omega_0^2\right\} - 2k\omega_0\frac{dV_0}{dx}\right.$$

$$\left. - \left(\Omega\frac{d^2V_0}{dx^2} + 2\Omega\frac{d\Omega}{dx} + \frac{dV_0}{dx}\frac{d\Omega}{dx}\right)\right] \Big/ \left[\Omega\left(\Omega + \frac{dV_0}{dx}\right) - \omega_0^2\right]^2 \tag{1.6}$$

$$C = -k^2 + \sum k\omega_p^2\left[K_n\Omega\left\{\omega_0^2 - \Omega\left(\Omega + \frac{dV_0}{dx}\right)\right\}\right.$$

$$+ \left(\omega_0^2 + \Omega^2\right)\frac{d\Omega}{dx} + \Omega^2\frac{d^2V_0}{dx^2}$$

$$\left. + k\omega_0\left\{\omega_0^2 - \Omega\left(\Omega - \frac{dV_0}{dx}\right)\right\}\right] \Big/ \omega_0\left[\Omega\left(\Omega + \frac{dV_0}{dx}\right) - \omega_0^2\right]^2 \tag{1.7}$$

where the summation sign extends over the species and ω_p is the plasma frequency. We note from Eqs.(1.5) to (1.7) that the effect of velocity shear becomes important only if dV_o/dx is comparable to cyclotron frequency. Since in actual experiments, it is difficult to realize dV_o/dx of the same order as Ω_e , hence the discussion in the forthcoming sections will be confined to $dV_o/dx \lesssim \Omega_i$. Further, the application of guiding-centre approximation limits our consideration only to low frequency waves with long wavelengths (greater than ion larmor radius) but still shorter than the plasma dimension, (K_n^{-1}). Thus in the subsequent sections, the dispersive properties of Eq.(1.4) will be discussed in detail for local and nonlocal régimes.

Local Dispersion Relation

To begin with, we shall derive the dispersion relation valid under 'local' approximation (Mikhailovski[9]). For a typical density perturbation, we shall assume that the wavelengths, λ_y (= $2\pi/K$) is small compared to the characteristic distance over which the equilibrium quantities such as density, velocity and magnetic field vary. Further, we shall assume $\lambda_x \gg \lambda_y$, where λ_x is the wavelength along x-axis. Such situations can be realised in magnetospheric plasma sheet regions where the wavelengths along north-south direction could be large compared to the wave-length along the dawn-dusk direction. Thus neglecting the terms involving the derivatives of ϕ

and the electron velocity gradient (compared to Ω_e) in Eq.(1.4) we get the local dispersion relation

$$\frac{-K_n\alpha + K_B}{\alpha^2\,\omega_{oi}} + \frac{k\,\omega_{oi}^2}{\Omega_i^3\,\alpha^2} + \frac{K_n - K_B}{\omega_{oe}} - \frac{k}{\Omega_e} - \frac{k V_A^2}{\Omega_i c^2} = 0 \quad (1.8)$$

where $\alpha = 1 + \beta$ and $\beta = (dV_o/dx)/\Omega_i$. The quantities K_B, V_A ($\ll c$) and c stand respectively for inverse scale length $(d\,\Omega_i/dx)/\Omega_i$, Alfven velocity and the velocity of light. The subscripts i and e identify the ion and electron species. In deriving the dispersion relation (1.8) it is assumed that (dV_o/dx) is constant (linear velocity profile) and $\beta < 1$. Eq.(1.8) being a quartic in ω_{oi}, can be solved in general for the roots using standard numerical methods. However, we shall examine a particular case when $V_{oi}=V_{oe}=V_o$. The particle drift, V_o, in this instance can be visualized as a consequence of EXB motion, E and B being the externally applied nonuniform fields in the system. Furthermore, assuming $1 \gg (\omega_{oi}/\Omega_i)^2 \gg (m/M,\ V_A^2/c^2)$ where m and M are the electron and ion masses, Eq.(1.8) reduces to

$$(\omega_{oi}/\Omega_i)^3 = -\beta\{K_n\alpha - (\alpha+1)K_B\}/k \qquad (1.9)$$

which clearly admits complex roots as solutions leading to either growing or decaying perturbations. Thus we find that the low frequency instability driven by the velocity gradient (either for positive or negative slope) arises in this limit and clearly the magnetic field shear contributes to the stabilization of this instability, provided its gradient has the same sign as the density gradient. Also we note that the growth rate varies directly as $\beta^{1/3}$ and inversely as $(KL_n)^{1/3}$, where $KL_n >> 1$, L_n being the scale length of the density gradient ($= 1/K_n$). Lastly, it must be remarked that the above instability does not arise in the limit $\beta << 1$. For this case, it may easily be verified that the second term [which originates from the expression for C in Eq.(1.7)] in Eq.(1.8) drops out and consequently the relation (1.8) becomes a quadratic in   . This situation will be considered next.

For the limit when $(\omega_{oi}/\Omega_i)^2 << m/M$ (or V_A^2/c^2), Eq.(1.8) can be revised in the form

$$\frac{k_B - k_n\alpha}{\alpha^2\omega_{oi}} + \frac{k_n - k_B}{\omega_{oe}} - \frac{k}{\Omega_i}\left(2 - \alpha + \alpha^2\frac{m}{M} + \frac{V_a^2}{c^2}\alpha^2\right) = 0 \quad (1.10)$$

For $V_{oi} = V_{oe}$, we note that there exists no instability while for $V_{oi} \neq V_{oe}$ Eq.(1.10) can be solved to give the roots

$$\omega = k\left(V_{oe} + V_{oi}\right) + \Omega_i\left(p+q\right) \pm \{k^2\left(V_{oe} - V_{oi}\right)^2 + \Omega_i^2\left(p+q\right)^2 + 2k\Omega_i(V_{oi} - V_{oe})(p-q)\}^{1/2} \quad (1.11)$$

where the quantities p and q are given by the expressions

$$p = (K_B - K_n\alpha)/\left[k\left\{2 - \alpha + \alpha^2\left(\frac{m}{M} + \frac{V_A^2}{c^2}\right)\right\}\right]$$

$$q = (K_n - K_B)\alpha^2/\left[k\left\{2 - \alpha + \alpha^2\left(\frac{m}{M} + \frac{V_A^2}{c^2}\right)\right\}\right]$$

Noting that $|p+q| < |p-q|$ for $\beta < 1$ and assuming that $V_{oi} > V_{oe}$ we find that there exists a low frequency instability, in the absence of magnetic field shear provided $K_n > 0$. In the opposite limit ($K_n < 0$) the low frequency mode becomes stable. In the former case, the ranges of wavelengths for which the low frequency unstable modes occur are defined by

$$p - q > k\left(V_{oi} - V_{oe}\right)/\Omega_i > p + q \tag{1.12}$$

The physical mechanism responsible for this instability does not depend on the velocity gradient. The instability arises due to relative streaming between electrons and ions in an inhomogeneous plasma. As such this instability will be the analogue of Kelvin-Helmholtz instability for transverse streaming. Similarly the same conclusion will hold good for the case when $\beta << 1$.

Non-local Effects

The local theory described above gives a fairly clear idea that the growth rates (Eq.1.9) essentially depend on the finiteness of velocity gradient. We, however, note from Eq.(1.9) that, for $\beta \rightarrow 0$, the doppler shifted frequency also vanishes and thus poses certain difficulties in the applicability of the local theory. Even though a velocity profile given by $V_o = V_o(1 + x/L)$ will never allow ω_o to vanish (since $\frac{dV_o}{dx}$ = constant) this profile is an over-simplification and in practical situations the velocity profile departs considerably from the linear spatial dependence. In such cases, the gradient is no longer constant and may even vanish at certain regions of space. In this region, the differential equation (Eq.1.5) becomes singular differential equation and the local theory breaks down. We now examine the modification of the dispersion characteristics that one encounters in a more realistic velocity profile. For the purpose of our demonstration, we shall choose the profile to be $V = V_o \operatorname{sech}^2 x/L$ which simulates the jet-like profile that commonly arises in Laboratory experiments. In this case, $dV/dx = -2V_o/L \operatorname{sech}^2 x/L \tanh x/L$ and the gradient vanishes at $x = 0$. The maximum value of $|dV/dx|$ occurs when $\operatorname{sech}^2 x/L = 2/3$ and we shall assume that the condition, $(1/\Omega_i)\, dV/dx = (2/\Omega_i) V_o/L \lesssim 1$, is satisfied. At other points in space this approximation will be evidently valid as dV/dx is maximum at $\text{sech}^2 x/L = 2/3$. Let us suppose that $\left|\frac{dV}{dx}\right|$ attains maximum value at $x = \delta$ where $\delta = L\,\text{sech}^{-2}(2/3)$. Along the x-axis, we shall now divide the region in two parts defined by (i) $0 < |x| < \delta$ and (ii) $\delta < |x| < \infty$ which correspond to singular and non-singular regimes of the differential equation (1.5), respectively. To arrive at the modified dispersion relation, we shall solve equation (1.4) in these two regions separately and match the logarithmic derivative of the two solutions at $x/L = \delta$. In the region typified by $\delta \lesssim x < \infty$ the doppler shifted frequency does not vanish and therefore we can obtain the W.K.B. solution of Eq.(1.4). As such, we shall assume that the velocity 'V' changes slowly i.e. $KL > 1$ except at the point where $dv/dx = 0$. In the region $\delta < x < \infty$ this approximation is clearly valid and therefore we can neglect $\frac{d^2\phi}{dx^2}$ compared to $\frac{d\phi}{dx}$ and ϕ. This immediately gives,

$$\frac{d}{dx} \ln \phi_2 = -C_2/B_2 \tag{1.13}$$

where the subscript (2) is used to denote the quantities in the region (2) given by $\delta < x < \infty$ and C_2 and B_2 are the coefficients as given by Eqs.(1.6) and (1.7) respectively. In the region (1), we shall make the following transformation of variables, from x to V, where $V = V_0\,\text{sech}^2 x/L$. Thus, noting $\frac{d}{dx} = \left(\frac{dV}{dx}\right)\frac{d}{dV}$ & $\frac{d^2}{dx^2} = \left(\frac{dV}{dx}\right)^2\frac{d^2}{dV^2} + \frac{d^2V}{dx^2}\frac{d}{dV}$

we can rewrite Eq.(1.4) valid for region (1) as

$$A_1\left(\frac{dv}{dx}\right)^2\frac{d^2\phi_1}{dv^2}+\left\{A_1\left(\frac{d^2v}{dx^2}\right)+B_1\left(\frac{dv}{dx}\right)\right\}\frac{d\phi_1}{dv}+C_1\phi_1=0 \tag{1.14}$$

From Eq.(1.7) we therefore find that the pole $\omega_0 = \omega - kV_0$ is reduced to a regular singular point by this transformation in Eq.(1.14). Near the point $x \simeq 0$ (where the singular point occurs), the first and third terms in Eq.(1.14) is vanishingly small as dv/dx vanishes at this point. Therefore the solution of Eq.(1.14) is governed by the following equation in the neighbourhood of the point x = 0, namely,

$$\frac{d}{dx}\ln\phi_1 = -\frac{C_1}{A_1}\frac{dv/dx}{d^2v/dx^2} \tag{1.15}$$

The solution, ϕ_1, is valid in the region $0 < x < \delta$ and the major contribution arises evidently near $x \simeq 0$. We compute the functions $\frac{dv}{dx}$, $\frac{d^2v}{dx^2}$ around x = 0 to give

$$\frac{d}{dx}\ln\phi \simeq -\frac{k\omega_{p_i}^2}{\Omega_i^2\omega_0}\left(\frac{-2V_0x}{L^2}\right)\Big/\left(1+\omega_{p_i}^2/\Omega_i^2\right) \tag{1.16}$$

Here the values of A_1 and C_1 is substituted from Eq.(1.15) and Eq.(1.7) valid for the region $x \simeq 0$. Matching the logarithmic derivative of ϕ_1 and ϕ_2 at $x/L = \delta$ we obtain the modified dispersion relation as,

$$-C_2/B_2\Big|_{x/L=\delta} = -\frac{2k\omega_{p_i}^2\Omega_i}{\omega_o(\Omega_i^2+\omega_{p_i}^2)}\left[\frac{1}{\Omega_i}\left(-\frac{V_o}{L}\right)\right]\delta$$

or

$$\frac{k\omega_{p_i}^2}{\Omega_i}\left\{\frac{K_n\beta}{\alpha\omega_{oi}}+\frac{k\omega_{oi}^2}{\Omega_i^3\alpha^2}\right\}\Big/\left\{\frac{\omega_{p_i}^2K_n}{\alpha\Omega_i^2}-\frac{2k\omega_{p_i}^2\omega_o\beta}{\Omega_i^3\alpha^2}\right\} = \frac{2k\omega_{p_i}^2\Omega_i\frac{3\sqrt{3}}{4}\beta\delta}{\omega_o(\Omega_i^2+\omega_{p_i}^2)} \qquad (1.17)$$

In deriving Eq.(1.17) we have substituted the values of C_2 and B_2 from Eqs.(1.7) and (1.6) respectively, calculated at $x/L = \delta$. At the point δ where $|dv/dx|$ is maximum the earlier approximations made in the local theory is also valid here, that is $\frac{\omega_o}{\Omega_i}\Big|_{x/L=\delta} \gg \frac{m_e}{m_i}, \frac{V_A^2}{c^2}$ with $\frac{1}{\Omega_i}\frac{dv}{dx}\Big|_{\frac{x}{L}=\delta} = \beta \lesssim 1$, $\alpha = (1+\beta)$ and $\beta = \frac{1}{\Omega_i}\frac{dv_o}{dx}\Big|_{x/L=\delta}$ $= \frac{-4}{3\sqrt{3}}\frac{V_o}{\Omega_i L}$. After some simplification of lengthy algebra, Eq.(1.17) reduces to

$$\left(\frac{\omega_{oi}}{\Omega_i}\right)^3 + 4\delta\beta^2\left(\frac{\omega_o}{\Omega_i}\right) + \frac{k_n}{k}\beta\left(1-\frac{3\sqrt{3}}{2}\delta\right) = 0 \qquad (1.18)$$

Using the Cardan's method we get the complex roots of Eq. (1.18) as $\left(\frac{\omega_o}{\Omega_i}\right) = \omega_1 y_+ + \omega_2 y_-$ and its complex conjugate. Here $\omega_1 = \omega_2^* = \left(-\frac{1}{2} - i\sqrt{\frac{3}{2}}\right)$; $y_\pm = \left[-\frac{k_n}{k}\beta\left(1-\frac{3\sqrt{3}}{2}\delta\right) \pm \left\{\frac{k_n^2}{k^2}\beta^2\left(1-\frac{3\sqrt{3}}{2}\delta\right)^2 - \frac{256}{27}\delta^3\beta^6\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}}$

For $\beta < 1$, $|y_+| \ll |y_-|$, and in this case we have

$$(\omega_0/\Omega_i) \simeq -\tfrac{1}{2}y_- + i\sqrt{3}/2\, y_- \qquad \text{where}$$

$$y_- \simeq \left\{ -\frac{k_n}{k}\beta\left(1 - 3\sqrt{3}\,\delta/2\right) + \frac{256}{27}\delta^3\beta^6 k/k_n\beta\left(1-\frac{3\sqrt{3}}{2}\delta\right) \right\}^{\frac{1}{3}}$$

Comparing this value of $|\omega_0/\Omega_i|$ with the earlier growth rate derived under the realm of local theory Eq.(1.9) we find that for finite values of 'δ', $y_- < (-k_n\beta/k)^{1/3}$ and hence the non-local effects reduces the overall growth rate. It is not difficult to see the reason for this decrease in the growth-rate for the profile $V_o(x) = V_o \operatorname{sech}^2 x/L$; as the growth rate is directly proportional to the velocity gradient, it assumes vanishingly small values for $\beta \to 0$ and this essentially occurs at the maximum value of $V_o(x)$. The waves grow faster in the regions where $V_o(x) < V_{omax}$ and hence derives smaller energies from the streams. In the local theory, however, $\frac{dV_o}{dx}$ being constant the waves derive energy at all points in space from the streams and therefore show apparently a larger growth rate. In the foregoing conclusions we find that the exact analytical solutions of Eq.(1.3) is not necessary to derive the modified dispersion relation, however detail studies depicting the singular behaviour of such differential equations have been thoroughly discussed in the works of Chandrasekhar(10), Drazin and Howard(14) and Case(15). We shall now return to the case of the linear velocity profile and examine the convective effects of unstable modes discussed earlier within the framework of the local theory.

In an inhomogeneous plasma, wave propagation develops in thedirection of varying density and as a result of this property, the dispersion relation is significantly modified. Therefore, in order to account this effect properly, we must solve Eq.(1.4) for its solution with the appropriate boundary conditions. As considered earlier, we shall treat dV_0/dx as constant and β to be less than unity. We shall assume that the scale length of the magnetic field inhomogenity is much longer than L_n (the density scale length). With these approximations, the quantities, A, B and C defined by Eqs.(1.5) to (1.7), simplify to

$$A \simeq \frac{1}{\alpha}\frac{\omega_{pi}^2}{\bar{\Omega}_i^2}\ , \quad B \simeq \frac{\omega_{pi}^2}{\alpha\Omega_i^2}\left(K_n - \frac{2k\omega_{oi}}{\alpha\Omega_i^2}\frac{dV_o}{dx}\right)$$

$$C \simeq -k^2 + k\frac{\omega_{pi}^2}{\bar{\Omega}_i}\left[K_n\left(\frac{1}{\omega_{oe}} - \frac{1}{\alpha\omega_{oi}}\right) + \frac{k}{\Omega_i\alpha^2}\left(\beta - 1 + \frac{\omega_{oi}^2}{\bar{\Omega}_i^2}\right)\right] \tag{1.19}$$

Making a transformation, $\Phi = \psi \exp\left[-\int(B/2A)dx\right]$, Eq.(1.4) can be reduced to the normal form

$$\frac{d^2\psi}{dx^2} + Q^2(x)\psi = 0 \tag{1.20}$$

where $Q^2 = (C/A) - (B^2/4A^2) - \frac{d}{dx}(B/2A)$.

It is clear that the solutions for ψ will depend on the nature and behaviour of function, Q. For our purposes, to illustrate the principal effect of the velocity gradient, we shall choose the density and velocity profiles in the form

$$n = \bar{n}_o \exp(-x/L_n) \; ; \; V_o = \bar{V}_o(1 + x/L_v) \tag{1.21}$$

where $\bar{n}_o$ and $\bar{V}_o$ are constant, L_v is the scale length of the velocity gradient and $(x/L_v) < 1$. With the above choice for the equilibrium parameters, after making a suitable transformations, the differential Eq.(1.14) becomes a parabolic equation of Weber's type

$$\frac{d^2\psi}{dx'^2} + [a^2\alpha/k^2\beta^2 - x'^2]\psi = 0 \tag{1.20a}$$

where β can now be redefined as $\bar{V}_{oi}/L_v\Omega_i$ and the other quantities such as x' and a are given by

$$x' = \frac{k\beta}{\sqrt{\alpha}}\left[x - \frac{\alpha}{(k\beta)^2}\left\{\frac{1}{2L_n} + \frac{k}{\alpha}\beta\frac{\bar{\omega}_{oi}}{\Omega_i} + \frac{\alpha}{2L_n}\frac{\Omega_i^2}{\beta\bar{V}_{oi}}\cdot\left(\frac{\bar{V}_{oi}}{\alpha\bar{\omega}_{oi}^2} - \frac{\bar{V}_{oe}}{\bar{\omega}_{oe}^2}\right)\right\}\right], \; \bar{\omega}_o = \omega - k\bar{V}_o \tag{1.22}$$

$$a^2 = \frac{k^2}{\alpha^2}\left(\beta - 1 + \frac{\bar{\omega}_{oi}^2}{\Omega_i^2}\right) - \frac{k\Omega_i}{L_n}\left(\frac{1}{\bar{\omega}_{oe}} - \frac{1}{\alpha\bar{\omega}_{oi}}\right) - \frac{k^2\beta^2}{\alpha} + \frac{\alpha^2}{4L_n^2\beta^2}\frac{\Omega_i^4}{\bar{V}_{oi}^2}\left(\frac{\bar{V}_{oi}}{\alpha\bar{\omega}_{oi}^2} - \frac{\bar{V}_{oe}}{\bar{\omega}_{oe}^2}\right)^2 + \frac{\alpha}{L_n}\frac{\Omega_i^2}{\beta\bar{V}_{oi}}\left(\frac{\bar{V}_{oi}}{\alpha\bar{\omega}_{oi}^2} - \frac{\bar{V}_{oe}}{\bar{\omega}_{oe}^2}\right)\left(\frac{1}{2L_n} + \frac{k\beta}{\alpha}\frac{\bar{\omega}_{oi}}{\Omega_i}\right) \tag{1.23}$$

From Eq.(1.14a), the solution for the potential ϕ can be expressed in terms of the Weber function, $D_n(x')$ where n is any eigen value. The condition that Eq.(1.14a) represents the Weber's equation identically demands that

$$a^2 = \frac{2n+1}{\alpha} k^2 \beta^2 \tag{1.24}$$

It may be remarked that the relation (1.18) can also be derived directly from Eq.(1.14a) by using the quantization condition in WKB method. Hence Eq.(1.18) defines the dispersion relation which takes into account the convective effects arising due to the changing density. We find that Eq.(1.18) is a polynomial in ω with real coefficients and in general, the roots (real or complex) can be evaluated numerically using the standard methods. Such a procedure helps possibly in delineating the stable and unstable regions for different values of β and KL_n. However, this procedure will not be adopted here. Instead, we shall discuss some special cases in which the dispersion relation (1.18) yields simple analytical solutions revealing the essential characteristic role of the velocity gradient. In the first instance, we shall examine the case when $\overline{V}_{oi} = \overline{V}_{oe} = \overline{V}_o$. With this condition Eq.(1.18) becomes a sixth degree polynomial in $\underline{\delta}$ namely,

$$\underline{\delta}^6 - 2\alpha\beta\underline{\delta}^3/kL_n - \alpha^2\underline{\delta}^2/2k^2L_n^2 + \alpha^2/4k^2L_n^2 = 2(n+1)\alpha\beta^2\underline{\delta}^4 \tag{1.24a}$$

where $\underset{\sim}{\delta} = \bar{\omega}_0/\Omega_i$. Making use of the fact that $\underset{\sim}{\delta} \ll 1$, $\beta \lesssim 1$ and $KL_n \gg 1$ this equation can be reduced to a quartic approximately by dropping the terms involving $\underset{\sim}{\delta}^6$ and $\underset{\sim}{\delta}^2$ (being small compared to the terms retained) in Eq.(1.18a). Further, writing $\underset{\sim}{\delta} = \delta_R + i\delta_I$ where δ_R and δ_I are the real and imaginary parts of $\underset{\sim}{\delta}$ and separately equating to zero the real and imaginary contributions of Eq.(1.18a) we get the coupled equation for δ_R and δ_I in the form

$$\delta_R(\delta_R^2 - 3\delta_I^2)\left[\delta_R + \frac{1}{(n+1)\beta kL_n}\right] - \delta_I^2(3\delta_R^2 - \delta_I^2) - \alpha/[8(n+1)(\beta kL_n)^2] = 0 \tag{1.25}$$

$$\delta_I\left[4\delta_R(\delta_R^2 - \delta_I^2) + (3\delta_R^2 - \delta_I^2)/(n+1)\beta kL_n\right] = 0 \tag{1.26}$$

For $\delta_I \neq 0$ and $\delta_I \gg \delta_R$, Eqs.(1.19) and (1.20) can be solved exactly for δ_R and δ_I which can be expressed as

$$\delta_R = -1/[4(n+1)\beta kL_n]$$
$$\delta_I = \pm\frac{3}{4\sqrt{2}}\left[-1 + \left\{1 + \frac{128}{81}\alpha(n+1)^3\beta^2k^2L_n^2\right\}^{\frac{1}{2}}\right]^{1/2} \Big/ [(n+1)\beta kL_n] \tag{1.27}$$

where the positive sign before the radical within the square parenthesis is chosen so that δ_I remains a real quantity. In Eq.(1.21), the growth rate expression can be further approximated to give $\delta_I \sim (n + 1)^{-1/4}(\beta KL_n)^{-1/2}$ ($\delta_I < 1$ for $\beta KL_n \gg 1$). It may be mentioned here that the excitation of low frequency mode with the same growth rate could occur even for smaller β (unlike the nonconvective results) provided $\beta KL_n > 1$ and n is large (such that $\delta_I < 1$) is satisfied consistently). Hence for the convective case we find that there exists a growing mode driven purely by the velocity gradient. In contrast to the results derived in the earlier section, we observe that the growth rate for the nonlocal case varies inversely as $(\beta KL_n)^{1/2}$. Thus it enable us to conclude that the convective saturation of the low frequency modes occurs with a lesser growth rate than its counterpart in the nonconvective case.

Finally we shall examine the situation in which the ions drift with a velocity $\overline{V}_{oi}$ the electrons being stationary ($\overline{V}_{oe} = 0$). Dropping the bar over the quantities the dispersion relation (1.18) for small β, takes the form

$$\frac{1}{(2kL_n\beta)^2}\left(\frac{\Omega_i}{\omega_{oi}}\right)^4 + \frac{1}{kL_n}\left(\frac{\Omega_i}{\omega_{oi}}\right) + \frac{\Omega_i}{kL_n}\,\frac{kV_o}{\omega_o\omega_{oi}} = \beta_1 \tag{1.28}$$

where $\beta_1 = 1 + 2(n + 1)\,\beta^2$. Instead of discussing the general solution of Eq.(1.22) we shall deal with the particular case when ω is close to KV_o. Therefore, writing $\omega = KV_o + \Delta$ where $|\Delta| \ll KV_o$, Eq.(1.22) transforms into a quartic in Δ given by

$$\left(\frac{\Delta}{\Omega_i}\right)^3\left(\frac{\Delta}{\Omega_i} - \frac{2}{\beta_1 k L_n}\right) - \frac{1}{4\beta_1(kL_n\beta)^2} = 0 \qquad (1.29)$$

Following the same procedure as outlined before, the real and imaginary parts of Δ ($= \Delta_R + i\,\Delta_I$, where $\Delta_R \ll \Delta_I$) can be written in the form

$$\Delta_R = \Omega_i / 2\beta_1 k L_n$$

$$\Delta_I = \pm \frac{3\Omega_i}{kL_n}\left[-1 + \left\{1 + \frac{16}{81}\beta_1^3 k^2 L_n^2/\beta^2\right\}^{1/2}\right]^{1/2} \Big/ 2\sqrt{2}\,\beta_1 \qquad (1.30)$$

The expression for Δ_I can be simplified further to yield a value proportional to $1/(\beta KL_n)^{1/2}$ and this result is similar to the one discussed previously. It follows therefore that in either of the cases described here, the low frequency instabilities with growth rates given by Eq.(1.21) or (1.24) can arise due to the nonlocal effects and small velocity gradient ($\beta < 1$).

Results and Application

We have investigated in general the effect of velocity shear on the low frequency waves in a two-component plasma. We find that some new unstable modes driven primarily by the velocity gradient occur near $\omega_R = KV_0$ for both convective and nonconvective regimes. The growth rates of these modes are larger in the nonconvective limit and they attain the convective saturation with a value proportional to $\Omega_i/(\beta KL_n)^{1/2}$. Of course it must be mentioned that the velocity gradient driven instability in the nonconvective limit triggered only for $\beta \lesssim 1$ and range of frequencies such that $1 \gg (\omega_o/\Omega_i)^2 \gg (m/M, V_A^2/c^2)$. On theother hand, for a frequency range, $(\omega_o/\Omega_i)^2 \ll m/M$ or V_A^2/c^2 transverse KH mode is excited and the effect of velocity shear ($\beta \lesssim 1$) causes a slight modification of its mode structure leading essentially to the localization of KH mode. This latter result is in conformity with an earlier investigation by Rosenbluth and Simon(16) who examine the influence of nonuniform electric fields on KH and Rayleigh-Taylor modes employing FLR time ordering (that is, $\omega/\Omega_i \ll (\rho_i/L)^2 \ll 1$ where ρ_i is the ion larmor radius and L is the typical characteristic length). Thus we conclude that the velocity gradient driven mode becomes operative only for the frequency range satisfying the inequality $1 \gg (\omega_o/\Omega_i)^2 \gg (\frac{m}{M}, \frac{V_A^2}{c^2})$

Perhaps the results of our analysis will be more important to the magnetospheric plasma sheet regions (where the condition $\beta \lesssim 1$ is easily fulfilled) during the pre and post growth periods of the magnetic substorms. In such conditions, it is conceivable that strong velocity gradients could co-exist in the plasma sheet because of the sudden variations in the magnetic field components. Some of the observed features through satellite studies (Akasofu[17] et al. seems to indicate (i) the presence of highly energetic protons and electrons, and (ii) thinning and thickening of the plasma sheet during and after the substorm periods. In fact, a recent theoretical study (Kan[1]) directly correlates the presence of velocity shear to the plasma sheet thickness (showing that increasing velocity shear thickens the sheet). These results might have some interesting applications to the tail plasma sheet.

Of course, in the neutral sheet region, the guiding-centre approach is invalid since adiabatic approximations do not hold good. However, velocity gradients driven instabilities might qualitatively account for plasma heating. A detailed study incorporating the kinetic effects to reveal the features for shorter wavelengths (comparable to larmor radious of ions) and the nonadiabatic effects will be topic for future work.

Section 2 : Suppression of the Kelvin-Helmholtz instability by the Lower-hybrid turbulence

The results of the previous section have conclusively shown, both in laboratory as well as in space plasma that a velocity shear transverse to the magnetic field can give rise to a low frequency instability. In this section, we shall now concentrate on the growth of the KH mode wherein the electrons play dominant role. This type of mode has been observed in auroral regions by Hallinan and Davis.[4] To understand the auroral processes involving the particle energization, we shall therefore consider the possible stabilization of the KH mode through a nonlinear interaction with a high frequency mode. A discussion of this type is stimulated by the reported phenomen-n in Jassby's[11] work. He showed that for large values of the electric field, the low frequency KH mode can be totally suppressed; and in place of this a high frequency mode is excited. The suppression of the KH mode is attributed to the nonlinear damping in presence of the high frequency field, ($\omega > \Omega_i$). In this section, we shall show that the KH instability can also be stabilised by the presence of a weakly turbulent high frequency field. The turbulence will be chosen to be peaked around the lower hybrid frequency since this mode is usually generated by the crossfield drifts beyond a certain threshold velocity ($V_o > V_{thi}$) (McBride et al.[13]).

In space plasma, however, this choice of background turbulence is further supported by the observation of lower hybrid noise in the auroral zone. In any case we shall assume that lower hybrid turbulence exists in the background plasma. Since our primary motivation is to study the effect of this turbulence on the mode, we shall take a simplified version of the linear instability. The calculations can easily be extended to the cases considered in the earlier section. The velocity shear is provided by an idealised electron sheet of half width a. We shall also neglect effects of finite wavelength in the magnetic field direction, collisions and viscosity. Jassby[8] has already given a detailed account of such effects and the criteria for neglecting them in case of KH instability and we shall assume that our linear instability is governed by the mode properties described in Hasegawa's[18] work.

Basic Equations and Their Solutions

Consider the high frequency turbulence in the back ground the energy density of which satisfies the wave kinetic equation, Vedenov et al.[19]

$$\frac{\partial n_k}{\partial t} + \bar{V}_g \cdot \bar{\nabla} n_k - \frac{\partial \omega_k}{\partial \bar{r}} \cdot \frac{\partial n_k}{\partial \bar{k}} = 0 \qquad (2.1)$$

where n_K is the plasmon occupancy number, ω_k and K being the frequency and wave number satisfying the linear dispersion relation for the lower hybrid mode,

$$\omega_k^2 = \omega_{LH}^2 \left(1 + \frac{k_{\parallel}}{k_{\perp}^2} \frac{m_i}{m_e} \right) \tag{2.1a}$$

where $\omega_{LH}^2 = \omega_{pi}^2/(1+\omega_{pe}^2/\Omega_e^2)$, ω_{pj}, Ω_j being the plasma and gyrofrequency respectively for the j^{th} species. The low frequency characteristics of KH mode are governed by the equations of motion and mass conservation for electrons. They are given by

$$\frac{\partial \bar{V}}{\partial t} + \bar{V}\cdot\bar{\nabla}\bar{V} = \frac{q_e}{m_e}\left(\bar{E} + \frac{\bar{V}\times B_0}{c} \right) \tag{2.2a}$$

$$\frac{\partial n}{\partial t} + \bar{V}_0\cdot\bar{\nabla}n_1 = -\tilde{V}_1\cdot\bar{\nabla}n_0 \tag{2.2b}$$

Here the constant magnetic field is taken along the z direction, the $\bar{E}\times\bar{B}$ drift is in the y direction $\{ V_0 = c\bar{E}_0 \times \bar{B}/B_0^2 \}$ and the gradients of V_0 and n_0 are chosen in the x direction. The electron sheet occupies the space between $-a \leq x \leq a$ being parallel to the yz plane. Eqs.(2.2a) and (2.2b) are supplemented by the following boundary conditions (Hasegawa[18]).

$$\hat{n}\cdot[\bar{E}] = (1/\varepsilon_0)\,\rho_s$$
$$\hat{n}\times[\bar{E}] = 0 \tag{2.3}$$

The symbol [ ] denotes the jump in the value of the quantity across the electron sheet. The solutions for the electric field ($\bar{E} = -\bar{\nabla}\phi$) in the respective regions are

$$\phi_{I} = A e^{iqy} e^{-qx} , \quad x > a$$
$$\phi_{II} = B e^{iqy} e^{qx} + C e^{iqy} e^{-qx} , \quad a < x < -a$$
$$\phi_{III} = D e^{iqy} e^{qx} ; \quad x < -a \tag{2.4}$$

Eqs.(2.3) and (2.4) complete the linear description of KH instability and these together with Eqs.(2.2a) and (2.2b) give the dispersion relation. The detailed calculations with respect to the mode characteristics have been carried out by Hasegawa[18] and we have quoted the main equations for the sake of completeness.

The coupling between the low and high frequency modes occurs through the ponderomotive force term. This effect is assumed to be valid in the adiabatic sense i.e. $|\omega_K| \gg |\Omega|$, $|\bar{k}| \gg |\bar{q}|$ where (ω_k, k) and (Ω, q) representing the frequency wave number space, correspond to high and low frequency modes respectively. Writing $\bar{V} = \bar{V}_s + \bar{V}_f$ where subscripts s and f denote slow and fast variations respectively and averaging over an ensemble of a random set of high frequency waves, Eq.(2.2a) becomes

$$\langle \bar{V}_f \cdot \bar{\nabla} \bar{V}_f \rangle = \frac{q_e}{m_e} \left(\bar{E}_s + \bar{V}_s \times \bar{B}_o/c \right) \tag{2.5}$$

Thus, the additional term in Eq.(2.5) provides the necessary coupling between the high and low frequency modes (Sagdeev and Galeev[20]). In Eq.(2.5) we have neglected the inertial term for the slow mode since $\Omega \ll \Omega_e$. Eq.(2.5) can be solved for V_s to give

$$\bar{V}_{s\perp} = c\,\frac{\bar{E}_s \times \bar{B}_o}{\bar{B}_o^2} - \frac{\langle \bar{V}_f \cdot \bar{\nabla} \bar{V}_f \rangle \times \bar{\Omega}_e}{\Omega_e^2}$$

It turns out that $V_{s\parallel}$ is small compared to $V_{s\perp}$ (ie $|V_{s\parallel}| \ll |V_{s\perp}|$) for $\Omega \ll \Omega_e$. Now eliminating $V_{s\perp}$ from Eqs.(2.5a) and (2.2b), we get

$$\frac{\partial n_1}{\partial t} + \bar{V}_o \cdot \bar{\nabla} n_1 = -\left[c\,\frac{\nabla \phi_s}{\bar{B}_o} - \frac{\langle \bar{V}_f \cdot \bar{\nabla} \bar{V}_f \rangle \times \bar{\Omega}_e}{\Omega_e^2} \right] \cdot \bar{\nabla} n_o \quad (2.6)$$

The term $\langle\langle V_f \cdot \nabla V_f \rangle\rangle$ can be expressed in terms of high frequency components as

$$\langle \bar{V}_f \cdot \nabla \bar{V}_f \rangle = \sum_k \Big\langle \left(\frac{1}{2} \frac{\partial V_{f\perp}^2}{\partial x} + V_{f\perp} \frac{\partial V_{fk}}{\partial y} \right) \hat{e}_x + \left(\frac{1}{2} \frac{\partial V_{fk}^2}{\partial y} + V_{fk} \frac{\partial V_{f\perp}}{\partial x} \right) \hat{e}_y \Big\rangle \quad (2.7)$$

where $\bar{V}_{f\perp}$ and V_{fK} are defined in terms of the high frequency field $\bar{E}_{fK}$ by the relations

$$\bar{V}_{f\perp} = -\frac{q_e}{m_e} \bar{E}_{fk} \times \bar{\Omega}_e / (\omega_k^2 - \Omega_e^2)$$

$$V_{fk} = i\frac{q_e}{m_e} \omega_k E_{fk} / (\omega_k^2 - \Omega_e^2) \quad (2.8)$$

where E_{fK} is the Fourier component of the turbulent spectra, K being directed along y axis. It can be seen from Eq.(2.6) that only the 'y' component of $\langle \bar{V}_f \cdot \bar{\nabla} \bar{V}_f \rangle$ in Eq.(2.7) will survive since $\bar{\nabla} n_o$ is in the 'x' direction. Further since the KH mode is excited with propagation vector along the y direction only, $\partial/\partial x \to 0$ and therefore, we get from Eqs. (2.7) and (2.8),

$$\langle \bar{V}_f \cdot \bar{\nabla} \bar{V}_f \rangle = \sum_k \left\{ \frac{1}{2} \frac{\partial}{\partial y} |V_{fk}|^2 \right\} = \sum_k \left[\frac{1}{2} \frac{\partial}{\partial y} \frac{\omega_k^2}{(\omega_k^2 - \Omega_e^2)^2} |E_{fk}|^2 \right] \hat{e}_y \quad (2.9)$$

Defining $n_K = |E_{fK}|^2 / \omega_K$, the term (Eq.(2.9)) can be reexpressed as

$$\langle \bar{V}_f \cdot \bar{\nabla} \bar{V}_f \rangle = \sum_k \left[\frac{1}{2} \omega_k^3 / (\omega_k^2 - \Omega_e^2)^2 \frac{\partial n_k}{\partial y} \right] \hat{e}_y \quad (2.9a)$$

We shall linearize (2.1) by writing $n_K = N_{Ko} + \delta n_K$, $\delta n_K \ll N_{Ko}$. Since the plasmon frequency 'ω_k' is dependent on the density by virtue of Eq.(2.1a), a perturbation of the density by the low frequency instability will change the plasmon frequency and this change is manifested through the term $\partial \omega_k / \partial \bar{r}$ (Sagdeev and Galeev[20]). Instead of calculating this term exactly for a specific case, it can be simply expressed as

$$\frac{\partial \omega_k}{\partial \bar{r}} = \frac{\partial \omega_k}{\partial n} \frac{\partial n}{\partial \bar{r}} \simeq \frac{\partial \omega_k}{\partial n_o} \frac{\partial n_1}{\partial y} \hat{e}_y$$

where $\partial/\partial \bar{r}$ operates on theperturbations of the slow mode and we have used the fact that $\omega_k = \omega_k(n)$ and

$n = n_o + n_1$ ($|n_1| << n_o$). From Eq.(2.1) we, therefore, obtain the relation

$$\delta n_k = \frac{\partial\omega_k/\partial n_o}{\bar{V}_g\cdot\bar{q}-\Omega}\{\bar{q}\cdot\bar{\nabla}_k N_{ko}\}n_1 \qquad (2.10)$$

Where $\partial\omega_k/\partial n_o$ can be computed from the linear dispersion relation defined by Eq.(2.1a).

Finally substituting Eqs.(2.10) and (2.9a) into Eq.(2.6) and using the boundary condition (2.3) and (2.4), we obtain the modified dispersion relation of KH instability as

$$\frac{4\Omega^2}{\omega_o^2} = \left(1 - \frac{qV_o - n_o\Upsilon}{\omega_o}\right)^2 - e^{-4qa} \qquad (2.11)$$

Here Υ is the effective contribution due to the background lower hybrid turbulence and is given by the relation

$$\Upsilon = -\int dk \frac{qc^2}{2aB_o^2}\frac{\Omega_e\omega_k^3}{(\omega_k^2-\Omega_e^2)^2}\frac{\partial\omega_k/\partial n_o}{(\bar{V}_g\cdot\bar{q}-\Omega)}\bar{q}\cdot\frac{\partial N_{ko}}{\partial\bar{k}} \qquad (2.11a)$$

We shall now consider the effect of the turbulence for certain special cases.

Consider a hot plasma (with $\beta = 8\pi n_o T_e/B_o^2 >> \frac{m_e}{m_i}$) in the background of cold plasmons. This implies that the turbulent energy is peaked around certain K_o and we can therefore choose $N_{Ko} = N_o\,\delta(K - K_o)$. Using Eq.(2.1a), the relation (2.11a) can be simplified as

$$n_o\gamma = -\frac{1}{2}\frac{q^2c^2}{aB_o^2}\frac{\omega_k^6}{\bar{\Omega}_e^3}\cdot\frac{q}{\omega_{P_i}^2}\left[\frac{N_o\,\partial V_g/\partial k}{\left(\frac{\partial\omega_k}{\partial k}q-\Omega\right)^2}\right]_{k=k_o} \quad (2.12)$$

where $V_g = \frac{\partial\omega_k}{\partial k} = -2\omega_{LH}k_{\parallel}^2 m_i/k_{\perp}^2 m_e$; $\partial V_g/\partial k = 6\omega_{LH}(k_{\parallel}^2 m_i/k_{\perp}^4 m_e)$

Under the approximations, $V_g \gg \Omega/q$ and $K_{\parallel}^2/K_{\perp}^2 \sim m_e/m_i$ Eq.(2.12) further reduces to

$$n_o\gamma = -\frac{1}{4}\frac{q e^2 N_o}{a B_o^2}\left[\frac{(\omega_{pe}/\Omega_e)^3}{(1+\omega_{pe}^2/\Omega_e^2)^{5/2}}\right]\cdot\frac{3\,(m_e/m_i)^{3/2}}{(k_{\parallel}^2 m_i/k_{\perp}^2 m_e)} \quad (2.12a)$$

We note that '$n_o\gamma$' is strongly dependent on the parameter (ω_{pe}/Ω_e). The functional dependence of $n_o\gamma$ in Eq. (2.12a), in terms of the parameter $X = \omega_{pe}/\Omega_e$ is of the form $X^3/(1 + X^2)^{5/2}$. The maximum of this function occurs for $X^2 = \omega_{pe}^2/\Omega_e^2 = 1.5$. This value is readily attained in the auroral ionosphere. From Eq.(2.11) it is quite evident now that the '$n_o\gamma$' term has a stabilizing effect. To see this clearly, we rewrite Eq.(2.12a) as

$$n_o\gamma = -\frac{3}{32\pi}\frac{q^2e^2}{(qa)\omega_k}\cdot W\beta\left(\frac{m_e}{m_i}\right)^{3/2}\left\{\frac{(\omega_{pe}/\Omega_e)^2}{(1+\omega_{pe}^2/\Omega_e^2)^{5/2}}\right\} \quad (2.12b)$$

where $W = N_o\,\omega_K/n_o T_e$ = the ratio of turbulent energy to the thermal energy, $8\pi n_o T/B_o^2 = \beta$=the ratio of thermal pressure to the magnetic pressure and $(K_{\parallel}^2/K_{\perp}^2)\left(\frac{m_i}{m_e}\right) \sim O(1)$. Since the low frequency KH instability occurs for $V_o < V_{thi} \ll c$ (c = velocity of light) the quantity $n_o\gamma$

can be comparable to the qV_o term in Eq.(2.11) even for $W \sim O(m_e/m_i)$ and $\beta >> m_e/m_i$. The turbulence essentially shifts the unstable region of the KH instability from small 'q' (large growth rates) to the large 'q' regions (with smaller growth rates). Depending on the various parameters in Eq.(2.12a) it can also totally suppress the transverse KH instability.

Next we shall discuss another interesting situation in which the group velocity of the background turbulent waves is small ($(K_{\parallel}^2/K_{\perp}^2)(m_i/m_e) < 1$) and the approximation $q.V_g >> \Omega$ is no longer valid. Since the linear KH instability is a purely growing instability (Ω^2 negative and real) there will not be any resonant interactions but it is still possible to consider a case in which $|V_g|^2 \simeq |\Omega/q|^2$. The denominator of Eq.(2.12) in this situation will be approximately equal to $-2V_g q\Omega$ and this gives a new value for $n_o\gamma$ defined by

$$n_o\gamma = \frac{1}{2}\,\frac{q^2c^2\omega_k^6\,q}{a\beta_o^2\,\bar{\Omega}_e^3\,\bar{\omega}_{p_i}^2}\,N_o\frac{\partial V_g/\partial k}{2V_g q\Omega} = c_2/\Omega \qquad (2.13)$$

where

$$c_2 = \frac{1}{4}\,\frac{q^2c^2\omega_k^6}{a\beta_o^2\Omega_e^3}\,N_o\frac{\partial V_g/\partial k}{\omega_{p_i}^2 V_g}$$

Substituting the value of n_o in Eq.(2.11) we get

$$4\frac{\Omega^2}{\bar{\omega}_0^2} = \left(1 - \frac{qV_0}{\bar{\omega}_0} - \frac{c_2}{\omega_0\Omega}\right)^2 - e^{-4qa} = 4\frac{\Omega_0^2}{\bar{\omega}_0^2} - \frac{2c_2}{\bar{\omega}_0\Omega}\left(1 - \frac{qV_0}{\bar{\omega}_0}\right) \quad (2.13a)$$

where Ω_0^2 the square of the linear growth rate for $c_2 = 0$. We have also assumed $c_2/\omega_0\Omega \ll 1$. Eq.(2.13a) is a cubic equation in ' Ω ' and can be solved by Cardan's method. If the turbulent energy is small or if $(K_{\parallel}^2 / K_{\perp}^2)\left(\frac{m_i}{m_e}\right) \ll 1$, then $c_2/\omega_0\Omega$ can indeed be very small and we can then treat this term as a perturbation and to the first order we can replace Ω by Ω_0 in the denominator of Eq.(2.13a). This gives

$$\Omega^2 = \Omega_0^2 - \left(\frac{\omega_0}{\bar{\Omega}_0}\right)\frac{c_2}{2}\left(1 - qV_0/\omega_0\right)$$

or

$$\Omega \simeq \Omega_0 - \frac{1}{4}\left(\frac{\omega_0}{\Omega_0^2}\right)c_2\left(1 - qV_0/\omega_0\right) \quad (2.14)$$

This immediately shows that the second term has to be real since Ω_0 is imaginary for the KH instability. This essentially gives a nonlinear frequency shift to the originally purely growing mode. The damping of course is nonexistent in this case unlike case 1. This also shows that for $(K_{\parallel}^2 / K_{\perp}^2)(m_i/m_e) \ll 1$ the turbulence cannot be effectively coupled to the KH mode. Physically this is quite clear because the group velocity is essentially proportional to $(K_{\parallel}^2/K_{\perp}^2)(m_i/m_e)$ and in an exact perpendicular propagation ($K_{\parallel} = 0$) the lower hybrid mode becomes dispersionless which decouples the wave kinetic equation from the lower frequency modulation.

Conclusion

Thus, using a wave kinetic approach we have shown that it is possible to suppress the transverse KH instability by lower hybrid turbulence propagating at an angle to the ambient magnetic field. In case of a flute mode ($K_{\parallel} = 0$) the turbulence damping becomes negligible though it can give a nonlinear shift in the frequency of the purely growing KH instability. The damping is shown to be strongly dependent on the ratio (ω_{pe}/Ω_e) and is maximum for (ω_{pe}^2/Ω_e^2) = 1.5. This value of (ω_{pe}/Ω_e) is achieved in the ionosphere. Perhaps this type of mechanism would account for the suppression of the KH instability rather than the nonlinear Landau damping process suggested by Jassby(11) in his theoretical work.

References

1. Kan, J.R., J. Geophys. Res. 78, 3773 (1973)

2. Wharton, C.B., Korn, P. and Robertson, S., Phys. Rev. Lett. 27, 499 (1971)

3. Lehnert, B., Nucl. Fusion 11, 485 (1971)

4. Hallinan, T.J. and Davis, T.N., Planet. Space Sci. 18, 1735 (1970)

5. Hirose, A. and Alexeff, I., Phys. Fluids 16, 1087 (1973)

6. D'Angelo, N., Phys. Fluids 8, 1748 (1965)

7. Jassby, D.L. and Perkins, F.W., Phys. Rev. Lett. 24, 256 (1970)

8. Jassby, D.L., Phys. Fluids 15, 1590 (1972)

9. Mikhailovskii, A.B., Theory of Plasma Instabilities (Consultants Bureau) Vols.1 and 2 (1973)

10. Chandrasekhar, S., Hydrodynamic and Hydromagnetic Stability (Oxford; Clarendon Press) (1961)

11. Jassby, D.L., Phys. Rev. Lett. 25, 1567 (1970)

12. Stringer, T.E. and Schimdt, G., Plasma Physics 9, 53 (1967)

13. McBride, J.B., Ott, E., Bons, J.P. and Orens, J.H., Phys. Fluids 15, 2367 (1972)

14. Drazin, P.G. and Howard, L.N., J. Fluid Mech. 14, 257 (1962)

15. Case, K.M., Phys. Fluids 3, 149 (1960)

16. Rosenbluth, M.N. and Simon, A., Phys. Fluids 8, 1300, (1965)

17. Akasofu, S.I., Perrault, P.D., Yashuhara, F. and Meng, C.I., J. Geophys. Res. 78, 7490 (1973)

18. Hasegawa, A., Plasma Instabilities and Nonlinear Effects, Ch.3., p.129 (Springer, Berlin) (1975)

19. Vedenov, A.A., Gordeev, A.V. and Rudakov, L.I., Plasma Phys. 9, 719 (1967)

20. Sagdeev, R.Z. and Galeev, A.A., Nonlinear Plasma Theory, Ch.1., p.33 (W.A. Benjamin, New York) (1969)

21. Perkins, F.W. and Jassby, D.L., Phys. Fluids 14, 102 (1971)

CHAPTER IV

ABSORPTION AND SCATTERING INSTABILITIES IN PLASMA

Introduction

The investigations carried out so far, in the preceding chapters clearly highlight the processes that are responsible for anomalous diffusion or stabilization leading to the effective control of plasma loss. In this chapter and the following one, the theories will primarily emphasize the effects of external sources on processes which contribute significantly to plasma heating. The basic mechanism involved in heating of plasma differs considerably from the conventional heating concept used generally in a gaseous matter. For instance, in the latter case, the energy transfer from external sources takes place through usual collisional processes, while, in plasmas, anomalous processes like the wave-particle interactions, turbulent heating and parametric processes play a dominant role. The d.c. heating or the absorption of incident energy by the normal collision process becomes in effective for very high temperature plasmas. In this, chapter, the plasma heating due to parametric decay process will be discussed. The absorption of electromagnetic waves in plasmas have extensively been studied by Kaw(1), Liu and Kaw(2) and Nishikawa(3) in connection with the problems of laser fusion and supplementary plasma heating in fusion devices. In the present work, however, our attention will be mainly focussed on the possible methods of improving the heating efficiency of parametric processes. With this goal in mind, the examples considered here can be divided into two sections. Section 1 deals with the effect of a low density cold electron beam on the parametrically excited modes at lower hybrid and ion cyclotron frequencies while the modification of the stimulated Brillouin backscatter process by the introduction of a small component of cold electrons is studied in Section 2.

Section 1: Parametric Decay Instability at Lower Hybrid Frequency in Presence of Electron Beam

The possibility of supplementary plasma heating near LH resonance has gained wide recognition and numerous detailed investigations have been carried out in this direction, both experimentally and theoretically (Ref.4, 5,6). These studies are based on the assumption that the energy transfer process involves the decay of LH wave (lower hybrid wave) into a LH wave and a low frequency wave (e.g. an ion acoustic or an ion cyclotron wave). Although such a decay process agrees well with the computer simulation experiments (Ref.5) in a certain restricted frequency (or wavelength) range, it does not satisfy the usual selection rules (e.g. the frequency or wavenumber matching) expected of parametric processes invoking the dipole approximation. Rather it has been argued by Porkolab[7], in one of his earlier papers, that a new type of quasi-ion modes is parametrically-excited within the plasma for propagation angles such that $(m_i/m_e) \cos^2 \theta < 1$, where θ is the angle subtended by propagation vector with the magnetic field. These new modes, which possibly contribute to plasma heating, oscillate near the ion cyclotron frequency and their growth rates are proportional to the pump intensity; the modes do not exist in the absence of a pump field. With respect to source-induced instabilities, however, two important points should be kept in mind. It should be remembered that these new modes do not cause deleterious effects (detrimental to plasma confinement). Secondly, the parametric instabilities, excited in the peripheral regions wherein the primary energy deposition takes place, cause a decrease in the applied energy density in the centre of the plasma volume, thereby leading to a reduction in heating efficiency or the subsequent non uniform heating. To overcome the disadvantages cited above, we propose in this section, that the dispersion characteristics can be modified significantly by injecting an electron beam in the direction of the magnetic field. An experiment motivated by considerations of this kind has recently been carried out by Gromov et al. and their results apparently indicate enhanced parametric heating. In realistic situations, such modifications in parametric processes can be accomplished either externally by injecting electron beams of given energy and beam width or by the presence of energetic electron beams which are experimentally observed in some fusion devices like tokbmaks or Alcators. (Knoepfel et al.[9]). Thus the effect of an unmodulated electron beam which can also be produced by placing thermionic emitters on the axis of plasma column, lowers the parametric instability threshold considerably and reduces the harmful effects associated with the macroscopic source-induced instabilities. The mathematical details, emphasizing these additional features and the main results are described below.

Beam Effects on Quasi-Modes

Consider a homogeneous, magnetized plasma which is subjected to the combined effect of an RF field and an unmodulated electron beam with density n_b, velocity v_b and thermal spread v_{th}. The electron beam is injected into the plasma in the direction of the magnetic field (i.e. the z-axis) and the applied pump field (in the x-direction) is taken in the form $E_o \cos \omega_o t$ (dipole approximation), when E_o is the uniform pump amplitude and ω_o is the frequency chosen near lower hybrid resonance. In the electrostatic limit, the derivation of the dispersion relation incorporating the effects of the oscillating field and the electron beam is quite straight forward, and, therefore, omitting the detailed steps, (See, Appendix), we can write down the dispersion relation (Porkolab[7]) directly as,

$$\epsilon(\omega) + \frac{1}{4}\mu^2\chi_i(1+\chi_e)\left[\frac{1}{\epsilon(\omega+\omega_o)} + \frac{1}{\epsilon(\omega-\omega_o)}\right] = \alpha \quad (1.1)$$

where $\epsilon(\omega) = 1 + \chi_i(\omega) + \chi_e(\omega) + \chi_{be}(\omega)$, the χ_s are the electrical susceptibilities and μ is the parametric coupling coefficient ($= k_y\, CE_o/B_o\,\omega_o$) which arises purely from the $\bar{E} \times \bar{B}$ - motion of the electrons in the ambient plasma. For the beam electrons, since the predominant motion is the directed particle velocity along the z-axis, the $\bar{E} \times \bar{B}$ - motion is relatively small and therefore the coupling coefficient due to the beam electrons are ignored in Eq.(1). Similarly the modifications due to beam particles in $\epsilon(\omega \pm \omega_o)$ will be neglected since these contributions are small for the frequency range considered in this problem. Thus the response of the beam occurs only through the term $\chi_{be}(\omega)$ in $\epsilon(\omega)$ while the other terms in Eq.(1.1) are identical with those derived by Porkolab[7]. In our analysis, the effects of finite ion temperature and higher order nonlinear coupling terms will not be considered as the gross features of our discussion are not appreciably affected by such considerations. The general expressions for the susceptibilities, χ can be taken as

$$\chi_{be} = \frac{1}{k^2 d_b^2}\left\{1 + \left(\frac{\omega - k_{\parallel} V_b}{k_{\parallel} V_{tb}}\right) Z\left(\frac{\omega - k_{\parallel} V_b}{k_{\parallel} V_{tb}}\right)\right\} \qquad (1.2)$$

$$\chi_j = \frac{1}{k^2 d_{bj}^2}\left\{1 + \frac{\omega}{k_{\parallel} V_{tj}} \sum_j e^{-b_j} I_n(b_j) \times Z\left(\frac{\omega - n\Omega_j}{k_{\parallel} V_{tj}}\right)\right\}, \quad b_j = k_y^2 \rho_j^2 \qquad (1.3)$$

where p_j, $d_{b,j}$, V_{tj} and Ω_j respectively denote the Larmor radius, the Debye length, the thermal velocity and the cyclotron frequency. $I_n(x)$ and $Z(x)$ represent the modified Bessel and plasma dispersion functions, respectively.

We shall now discuss the effect of an electron beam on parametrically induced quasi-ion modes (non resonant modes) which have been extensively treated by Porkolab[7]. In the absence of a beam, there are basically two types of quasi-modes (with their growth rates directly proportional to the amplitude of the pump): (a) a kinetic quasi-ion mode (dissipative type) and (b) non-resonant fluid-like modes. In the former case, the mode is characterized by parallel phase velocities attaining the value of electron thermal velocity or its values close to it and hence the driving mechanism for the instability is primarily due to inverse nonlinear electron Landau damping. On the other hand, the latter situation corresponds to parallel phase velocities far exceeding the electron thermal velocity and, therefore, the resulting growth of the mode is of a macroscopic nature. Firstly, we shall study the instability characteristics of quasi-mode of the kinetic type quasi-mode. In this case, let us consider a non isothermal plasma (Te >> Ti) mode in the frequency range, $|\omega| < \Omega_i, k_{\parallel} v_{te}$. Neglecting the Larmor radius corrections, we obtain for the ion and electron susceptibility functions (defined in Eq.(1.3)),

$$\chi_i(\omega) = -\frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2}\sin^2\theta - \frac{\omega_{pi}^2\cos^2\theta}{\omega^2}$$

$$\chi_e(\omega) = \frac{1}{k^2 d_e^2}\left[1 + \frac{\omega}{k_{\parallel} v_{te}} Z\left(\frac{\omega}{k_{\parallel} v_{te}}\right)\right] \qquad (1.4)$$

Similarly, for the electron beam, we shall treat the injected beam as cold (i.e. $|\omega - k_{\parallel} v_b| \gg k_{\parallel} v_{th}$) throughout this section and so expression (1.2) simplifies to

$$\chi_{be} = -\omega_{pb}^2 \cos^2\theta / (\omega - k_{\parallel} v_b)^2 \qquad (1.5)$$

Finally the dispersion relation for the quasi-mode in the presence of beam can be written as

$$1 - \frac{\omega_{pb}^2\cos^2\theta}{(\omega - k_{\parallel} V_b)^2} + \chi_i + \chi_e + \frac{\mu^2}{4}\chi_i(1 + \chi_e)\left[\frac{1}{\epsilon(\omega+\omega_0)} + \frac{1}{\epsilon(\omega-\omega_0)}\right] = 0 \qquad (1.6)$$

where $\epsilon(\omega \pm \omega_o) \simeq 1 + \chi_i(\omega \pm \omega_o) + \chi_e(\omega \pm \omega_o)$. Eq.(1.1) and (1.5) are used in arriving at the dispersion relation. In the absence of a beam, it may be pertinent to point out that $|\chi_i| >> |\chi_e| >> 1$ in the neighbourhood of frequency $\omega \simeq \Omega_i$ and consequently χ_i is cancelled in Eq.(1.6), leading eventually to the characteristic onset of quasi-modes. In the presence of a beam, however, the ion susceptibility is no longer negligible compared to χ_{be} (for $|\omega| \simeq k_{\parallel} v_b$) and hence it plays an important role in manifesting the new dispersive properties of quasi-modes. Although the dispersion relation (1.6) can be solved for arbitrary values of ω, we shall seek a simplified solution such that $|\omega| \simeq k_{\parallel} v_b \sim \delta$, where δ is the frequency mismatch defined elsewhere. The existence of such solutions of Eq.(1.6) is justified since we expect the combined effects of electron beam and the parametric coupling to be important for the plasma heating process. Neglecting unity compared to χ_i or χ_e in Eq. (1.6) and considering only the Stokes component in the parametric coupling term, we obtain

$$D_R(\omega_o)(\omega + i\Gamma - \delta)\left[1 + \frac{1}{\chi_i}\left\{\chi_e - \frac{\omega_{pb}^2 \cos^2\theta}{(\omega - k_{\parallel} v_b)^2}\right\}\right] + \frac{\mu^2}{4}\frac{1}{k^2 d_e^2}\left[1 + \frac{\omega}{k_{\parallel} v_{te}} Z\left(\frac{\omega}{k_{\parallel} v_{te}}\right)\right] = 0 \tag{1.6a}$$

where we have followed the procedure employed by Kaw[1].

The symbols Γ, δ and $D_R(\omega_o)$ are given by the expressions,

$$\Gamma = \frac{\epsilon_I(\omega_o)}{D_R(\omega_o)}, \quad \delta = \frac{\epsilon_R(\omega_o)}{D_R(\omega_o)}, \quad D_R(\omega_o) = \frac{\partial \epsilon_R(\omega_o)}{\partial \omega_o}$$

The suffixes R and I are used to denote real and imaginary quantities, respectively. Equating the real and imaginary parts of Eq.(1.6a), we can write the expressions for ω_R and γ where $\omega = \omega_R + i\gamma$, in an approximate way:

$$\gamma = \frac{\mu^2}{4\alpha} \frac{1}{k^2 d_e^2} \left(\frac{\omega_R}{k_\parallel v_{te}}\right) \left[Z\left(\frac{\omega_R}{k_\parallel v_{te}}\right)\right]_I - \Gamma. \tag{1.7}$$

$$\omega_R = \delta + \Gamma \left(\frac{k_\parallel v_{te}}{\omega_o}\right) \left\{ 1 + \left(\frac{\omega_o}{k_\parallel v_{te}}\right) \left[Z\left(\frac{\omega_o}{k_\parallel v_{te}}\right)\right]_R \right\} \Big/ \left[Z\left(\frac{\omega_o}{k_\parallel v_{te}}\right)\right]_I \tag{1.8}$$

where $\alpha = 1 - \left(\frac{n_b}{n_o}\right)\left(\frac{m_i}{m_e}\right) \frac{(\Omega_i^2 - \omega_R^2)\cos^2\theta}{(\omega_R - k_\parallel V_b)^2}$. Expression (1.7) clearly shows that the instability driven by the pump power arises when the first term exceeds the linear damping rate of electromagnetic side band modes. The critical threshold field required for the instability is given by the relation

$$\mu_c^2 = \frac{4\Gamma k^2 d_e^2 \alpha}{[Z(\omega_R/k_{\parallel}v_{te})]_I} \left(\frac{k_{\parallel}v_{te}}{\omega_R}\right) \qquad (1.9)$$

Thus, for $\omega_R < \Omega_i$, the threshold defined by Eq.(1.9) is lower than that in the absence of the electron beam. The condition $\omega_R < \Omega_i$ can easily be fulfilled by an appropriate choice of the injected beam velocity and by choosing the pump frequency close to lower hybrid frequency so that $\delta < \Omega_i$ is satisfied. However, for $\omega_R > \Omega_i$, α turns out to be positive and hence threshold value is increased by the injection of beam plasma. Therefore, only for the beam velocity such that $k_{\parallel} v_b < \Omega_i$, the RF heating efficiency can be increased substantially.

Next, we consider the macroscopic type of quasi-mode which arises in the limit $|\omega| \gg k_{\parallel} v_{te}$ and $|\omega| \ll \omega_{LH}$. (lower hybrid frequency). Retaining both Stokes and anti-Stokes components in Eq.(1.6) and assuming that $\Omega_i < |\omega| < \Omega_e$, we can write the reduced dispersion relation in this case:

$$1 - \left(\frac{\omega_{LH}}{\omega}\right)^2\left(1 + \frac{m_i}{m_e}\cos^2\theta\right) - \left(\frac{n_b}{n_0}\right)\left(\frac{m_i}{m_e}\right)\frac{\omega_{LH}^2 \cos^2\theta}{(\omega - k_{\parallel}v_b)^2}$$

$$= \frac{\mu^2}{4}\left(\frac{\omega_{LH}^2}{\omega^2}\right)\frac{\omega_0\delta}{\delta^2-\omega^2}\left[1-\left(\frac{\omega_{LH}}{\omega}\right)^2\left(\frac{m_i}{m_e}\right)\cos^2\theta\right] \qquad (1.10)$$

where $\omega^2_{LH.} = \omega^2_{pi}/(1 + \omega^2_{pe}/\Omega^2_e)$. The discussion of Eq.(1.10) for arbitrary values of the parameters occurring in the equation is too difficult. We shall, therefore, make some simplifications by assuming that $(\omega_{L.H.}/\omega)^2 (m_i/m_e)\cos^2\theta \gg 1$ and $|\omega| \ll \delta$, where $\delta \lesssim \omega_{L.H.}$ (in contrast to the previous case). With these assumptions, the dispersion relation (1.10) becomes

$$1 - \beta(\omega_{LH}/\omega)^2 + \alpha_1 \omega^2/(\omega - k_\parallel V_b)^2 = 0 \quad (1.11)$$

where α_1 and β are constants given by

$$\alpha_1 = \left(\frac{n_b}{n_0}\right)\left(\frac{m_i}{m_e}\right)\cos^2\theta \Big/ \left(1 + \left(\frac{m_i}{m_e}\right)\cos^2\theta\right)$$
$$\beta = \frac{\mu^2}{4 m_e \delta} \; \frac{m_i \omega_0 \cos^2\theta}{1 + \left(\frac{m_i}{m_e}\right)\cos^2\theta} \quad (1.12)$$

Eq.(1.11) can be solved numerically for the preassigned values of the parameters α_1 and β. However, in this text, the solution of Eq.(1.11) will be obtained analytically by introducing an ordering in terms of a smallness parameter ϵ (= me/mi, say) Writing $\omega = k_\parallel V_b + \Delta$ where $|\Delta| \ll k_\parallel V_b$ and choose the order of various parameters, $\nu = k_\parallel V_b/\omega_{L.H.}$, β, α_1, $\omega/(\omega - k_\parallel V_b)$ and $\Delta_1 = \Delta/\omega_{L.H.}$ as $\epsilon^{1/2}$, ϵ, ϵ, $\frac{1}{\epsilon}$ and $\epsilon^{3/2}$, respectively, Eq.(1.11), correct to the terms of order ϵ^2, yields

$$\alpha_1(\nu^4/\Delta_1^2) + 4\nu^3\alpha_1/\Delta_1 = \beta - \nu^2$$

which, as a quadratic equation in Δ_1, can be solved to give the required roots. Thus the analytical solution of Eq.(1.11) can be put in the form

$$\omega = k_{\parallel} V_b \left[1 + \frac{2\alpha_1 k_{\parallel}^2 V_b^2}{k_{\parallel}^2 V_b^2 - \beta \omega_{LH}} \right] \pm i\alpha \frac{k_{\parallel}^3 V_b^3}{k_{\parallel}^2 V_b^2 - \beta \omega_{LH}^2} \left[\frac{k_{\parallel}^2 V_b^2 - \beta \omega_{LH}^2}{\alpha_1 k_{\parallel}^2 V_b^2} - 4 \right]^{1/2} \tag{1.13}$$

wherein we have assumed that

$$k_{\parallel}^2 V_b^2 > \left[\beta / (1 - 4\alpha_1) \right] \omega_{LH}^2 \tag{1.14}$$

for the occurence of a new instability driven primarily by the injected electron beam. In the opposite limit, $k_{\parallel}^2 V_b^2 < \beta \; \omega_{L.H.}^2/(1 - 4\alpha_1)$, Eq.(1.13), however, gives stable modes. In view of the external control of beam parameters, the resultant beam-driven modes are expected to be less harmful for the plasma confinement scheme. Therefore, we find that the electron beams with the velocities such that $k_{\parallel} V_b > \Omega_i$ can significantly alter the characteristics of the macroscopically excited quasi-modes (due to the pump) and further can prevent plasma loss.

Finally to summarize the results, we have shown that the basic structure of pump-driven quasi modes (kinetic as well as fluid types) can be substantially changed by the pressure of unmodulated electron beams. In short, their presence with varying energies and densities, in a

RF-heated plasma, not only increases the efficiency of plasma beating but also hampers the development of deleterious modes driven by the pump. In case of electron beams injected externally by some device, the conditions required for lowering of parametric threshold or the controlled excitation of beam-driven modes (for instance, Eq.(1.14)) are easily fulfilled since beam parameters such as velocity, beam width and density can be manipulated appropriately. Recent experiments (Knoepfel et al.[9]) do observe the presence of runaway electron beams (which form a continuous plateau at a later stage) and these beams have densities which are a few percent of ambient plasma density and energies in the range of 10 Kev to 10 Mev. These parameters fall within the range of interest to our work. Possibly with some modifications in our treatment (e.g. a different choice of the electron beam distribution), our work might also find interesting applications in some fusion devices involving the occurence of beams.

Section 2 : Stimulated Brillouin Back Scatter Instability in Presence of a Small Fraction of Cold Electrons.

In this section, we shall consider another situation in which the increased efficiency of parametric heating occurs. This efficiency can be accomplished by an external injection of cold electrons intermittently in a fusion device and, in fact such a technique extensively planned in magnetically confined laster-heated systems (Dawson et al.[10]) or in long solenoids in order to reduce the harmful effects associated with the heating process. Similarly in a laser plasma, a small fraction of cold electrons can be produced self consistently by the secondary ionization of neutrals by the fast electrons. The primary ionization of the pellet's corona energizes the electrons (upto Kev range) which having significant cross section further ionizes the neutrals to produce the cold electrons in the eV range. Thus the threshold values required for parametrically scattered modes can be enhanced significantly by the presence of such cold electrons and this modification, in turn, can increase the heating efficiency. To demonstrate this effect particularly in a laser plasma, we shall consider a typical back scatter process in an unmagnetized plasma. Although the role of cold electrons in laser-pellet interactions is not considered a serious candidate due to lack of experimental details, their presence by processes described above seem perfectly justified. Apart from its application to laser fusion (Brueckner and Jorna[11]), the analysis is quite general and hence it might find interesting applications to other devices employing the combined operation of parametric heating schemes and the particle-injection methods.

Concerning the stimulated back scatter mechanisms such as Raman (SRBS) and Brillouin (SBBS), both theoretical[12] and computer simulation[13] results predict a large percentage of reflectivity while the experimental findings[14] do not conform to this prediction. To explain this major discrepancy, further theoretical work on scattering processes carried out in the past few years seems to stress on their possible stabilization by such considerations as medium inhomogeneity[15], finite band widths[16], etc. In the former case, for instance, the threshold for SRBS is significantly increased while it remains more or less unaffected for SBBS, the main reason being the fact that the frequency of ion acoustic modes does not depend on the local plasma density. Recently[17], a sizable increase in the threshold value for SBBS is demonstrated by including a weak density dependence of ion acoustic mode, which is achieved through a coupling of background Langmuir turbulence with the ion acoustic wave. The above theory however suffers from a major drawback; the langmuir turbulence is expected to be generated in a laser-pellet interaction by the parametric decay process only at the critical density, $\omega_0 \simeq \omega_{pe}$ whereas the SBBS is excited in the under dense region where $\omega_{pe} << \omega_0$ (ω_0 = pump frequency). Therefore it becomes quite unrealistic to presume that the langmuir turbulence exists in the same spatial locality of the SBBS resonance. Also, since the pump propagates towards higher density, the SBBS is excited much before the pump wave reaches the absorption region. Thus the problem of stimulated Brillouin Back Scatter is reinvestigated taking into account the effect of a small fraction of cold electrons distributed uniformly in the SBBS region. It will be shown that the presence of cold electrons gives a density dependence to the ion acoustic mode and the threshold for SBBS is drastically altered.

We shall take the fluid equations and the maxwell's equations with the usual symbols,

$$\frac{\partial n_j}{\partial t} + \bar{\nabla}\cdot(n_j \bar{V}_j) = 0 \tag{2.1}$$

$$\frac{\partial \bar{V}_j}{\partial t} + \bar{V}_j \cdot \bar{\nabla} \bar{V}_j = \frac{e_j}{m_j}\left(\bar{E} + \bar{V}_j \times \bar{B}/c\right) - \frac{\bar{\nabla} P}{m_j n_j} \tag{2.2}$$

$$\bar{\nabla}\cdot\bar{E} = 4\pi \sum_j n_j e_j \; ; \; \bar{\nabla}\times\bar{E} = -\frac{1}{c}\frac{\partial \bar{B}}{\partial t} , \; \bar{\nabla}\times\bar{B} = \frac{1}{c}\frac{\partial \bar{E}}{\partial t} + \frac{4\pi}{c}\sum_j e_j n_j \bar{V}_j \tag{2.3}$$

We shall assume that the hot electrons have a weak density dependence $n_{he} = n_{ho}(1 + x/L)$ and a small fraction of uniform cold component i.e. n_{co} = constant, $n_{co}/n_{ho} \simeq \frac{m_e}{m_i}$, $T_c/T_h \simeq m_e/m_i$. The cold component is taken to be uniform, since the density of the secondary electrons produced is proportional to the product of the density of the hot electrons and the neutrals and the gradients of the hot electrons and the neutrals are oppositely directed to each other.

Under these approximations, the equation for the ion acoustic mode is given by,

$$\frac{\partial^2 \tilde{n}_e}{\partial t^2} - \left[\frac{T_h}{m_i}\left(\frac{n_{io}}{n_{ho}} + \frac{n_{co}}{n_{ho}}\frac{m_i}{m_e}\right)\right]\nabla^2 \tilde{n}_e = n_{io}\frac{m_e}{m_i}\nabla^2 V_o \cdot \tilde{V}_1 \quad (2.4)$$

where the terms on the r.h.s. arises due to the coupling of the back scatter mode and the initial pump mode. The linear dispersion relation after neglecting the term on the r.h.s. of Eq.(2.4) is

$$\omega_2^2 = k_2^2 \left[\frac{T_h}{m_i}\left(\frac{n_{io}}{n_{ho}} + \frac{n_{co}}{n_{ho}}\frac{m_i}{m_e}\right)\right] \quad (2.5)$$

where $n_{io} = n_{ho} + n_{co} \simeq n_{ho}$, as $n_{ho} >> n_{co}$, the second term on the r.h.s. gives the density dependence through (n_{co}/n_{ho}). The threshold of the SBS in an inhomogeneous plasma is essentially due to the phase mismatch of the interacting waves as they convect out of the resonance zone. The parameter describing this effect is given by $K' = d/dx\ (k_o - k_1 - k_2)$, k_is (i= 0,1,2) are the wave numbers of the incident wave, scattered wave and ion acoustic wave respectively. The W.K.B. steady state equations describing the ion acoustic wave and the scattered electromagnetic wave are[15]

$$\Gamma_a \tilde{n}_e + c_s \frac{\partial \tilde{n}_e}{\partial x} = \frac{i}{2} \frac{m_e}{m_i} n_{io} \frac{k_2}{c_s} (V_o \cdot \tilde{V}_1) e^{iK'x^2/2} \quad (2.6)$$

$$\Gamma_1 \tilde{V}_1 + c \frac{\partial \tilde{V}_1}{\partial x} = -\frac{i}{2} \frac{\omega_{pe}^2}{\omega_o - \omega_2} V_o \frac{\tilde{n}_e}{n_{io}} e^{-iK'x^2/2} \quad (2.7)$$

Assuming that the ionwave is heavily damped, i.e. $\Gamma_a \tilde{n}_e \gg c_s \frac{\partial \tilde{n}_e}{\partial x}$ and the scattered wave is weakly damped, i.e. $\Gamma_1 \tilde{V}_1 \ll c \frac{\partial \tilde{V}_1}{\partial x}$ we get a steady state solution of (2.6) and (2.7) Making the transformation $\sigma = \tilde{n}_e e^{-iK'x^2/2}$ and $S = \tilde{V}_1 e^{-iK'x^2/2}$ and eliminating σ from (2.6) and (2.7), we get 'S' by direct integration[4] as,

$$\log S = -\frac{iK'x^2}{2} - \frac{i\gamma_o^2}{V_1 V_2 K'} \log\left(x + 2i\Gamma_a / K'c_s\right) \quad (2.8)$$

where V_1 and V_2 are the group velocities of the E.M. wave and the ion acoustic wave and γ_o $(= \omega_{pi} V_o / \sqrt{2 c_s c}$ is growth rate for Brillouin scatter in the homogeneous medium. Taking the modulous of 'S' we get the solution with the boundary condition for $x \to -\infty$

$$|S| = e^{(\gamma_o^2 / V_1 V_2 K') \tan^{-1} x K' c_s / 2\Gamma_a} \underset{x \to \infty}{=} e^{-\pi \gamma_o^2 / 2 V_1 V_2 K'}$$

Therefore the threshold for SBBS is

$$\gamma_o^2 / 2 V_1 V_2 K' \gtrsim 1 \quad (2.9)$$

Using the definition of K' and the dispersion relation (Eq.(2.5)), we get the threshold as

$$\left(\frac{V_o}{c}\right)^2 \frac{k_o L}{(k_o\lambda_D)^2} \gtrsim \frac{(n_{co}/n_{ho})(m_i/m_e)}{2\left(n_{io}/n_{ho} + n_{co}/n_{ho}(m_i/m_e)\right)} \qquad (2.10)$$

We note from Eq.(2.10) that even though n_{co}/n_{ho} is a small fraction ($n_{co}/n_{ho} \simeq m_e/m_i$) it can significantly alter the threshold value.

In conclusion we note that the presence of a small fraction of cold electrons can give rise to a significant threshold for SBBS. The principal physics involved in this problem can be explained in the following manner. The response of the hot electrons to the slowly varying electric field which arises from the beating of the pump the scattered modes obeys a Boltzmanian distribution whereas the cold electrons due to their low temperature cannot build up enough pressure to balance the electric field and therefore moves along with the ions and contributes to the total inertia. The perturbed velocity of the electron being (m_i/m_e) times more than that of the ions, a small number of electrons can effectively take part in the charge neutralization due to the slowly varying electric field. This effect is reflected in the second term of the r.h.s. of Eq.(2.5).

Although the presence of cold electrons have not yet been confirmed by the experiments, their coexistence can not be ruled out because the bulk temperature of the laser plasma is determined by the x-ray methods which are insensitive to detect the low energy (< 1 eV) components. Thus, the Brillouin back scatter could be limited by the cold electrons uniformly distributed in the SBBS region. As outlined earlier the mechanism for increasing the threshold for the Brillouin back scatter mode can be applicable to any heating scheme invoking the parametric heating scheme. In a fusion plasma where the temperatures are very high, the introduction of a small amount of cold electrons externally will always inhibit the back scatter mode to grow and therefore it will increase the penetration of the incident wave energy into the plasma.

References

1. Kaw, P.K., in Advances in Plasma Physics, (Eds.), Simon, A. and Thompson, W.B., Volume 6, p.179 (1976).

2. Liu, C.S., and Kaw, P.K., in Advances in Plasma Physics, (Eds.), Simon, A. and Thompson, W.B., Volume 6, p.121 (1976)

3. Nishikawa, K, in Advances in Plasma Physics, (Eds.), Simon, A. and Thompson, W.B., Volume 6, p.3 (1976)

4. Hooke, W.M. and Bernabei, S., Phys. Rev. Lett. 28, 407 (1972); Chu, T.K., Bernabei, S. and Motley, R.W., Phys. Rev. Lett. 31, 211 (1973); Chang, R.P.H. and Porkolab, M., Phys. Rev. Lett. 31, 1241 (1973); Porkolab, M., Bernabei, S., Hooke, W.M., Motley, R.W. and Nagashima, T., Phys. Rev. Lett. 38, 230 (1977)

5. Kindel, J.M. Okuda, H., and Dawson, J.M., Phys. Rev. Lett. 29, 995 (1972)

6. Nishikawa, K., J. Phys. Soc. Japan, 24, 916 (1968)

7. Porkolab, M., Phys. Fluids 17, 1432 (1974)

8. Gromov, S.N., Pasechnik, L.L., and Semenyuk, V.F., Sov. Phys. JETP (lett), 23, 127 (1976)

9. Knoepfel, H., Spong, D.A. and Zweben, S.J., Phys. Fluids 20, 511 (1977)

10. Dawson, J.M., et.al., in Fourth Conf. Proc. Plasma Phys. and Controlled Nuclear Fusion (IAEA, Vienna 1971), Vol.1, p.673

11. Brueckner, K.A., Siebe Jorua, Rev. of Mod. Phys. 46, 325 (1974)

12. Drake, J., Kaw, P., Lee, Y.C., Schmidt, G., Liu, C.S., and Rosenbluth, M.N., Phys. Fluids 17, 778 (1974)

13. Forslund, D., Kindel, J.M., and Lindman, E.L., Phys. Rev. Lett., 30, 739 (1973)

14. Soures, J., Goldman, L.M., and Lubin, M., Nucl. Fusion 13, 829 (1973)

15. Liu, C.S., Rosenbluth, M.N., and White, R.B., Phys. Fluids 17, 1211 (1974)

16. Thomson, J.J., Nucl. Fusion 15, 237 (1975)

17. Guzdar, P.N., Phys. Rev. Lett. 35 1635 (1975)

APPENDIX - II

Consider a homogeneous plasma immersed in a uniform static magnetic field $\underline{\underline{B}} = \underline{\underline{B}}_o \, \underline{\underline{e}}_z$ and an oscillating electric field $\underline{\underline{E}} = \underline{\underline{E}}_o \cos \omega_o t$. We shall make the dipole approximation and assume E_o to be space-independent.

Let $\underline{\underline{R}}_j(t)$ denote the displacement of charged specie (j = e, i for electrons and ions, respectively) under the combined influence of the oscillating electric field and the magnetic field; we have then

$$\underline{\underline{R}}_j(t) = -\frac{e_j}{m_j}\left(\frac{E_{oz}}{\omega_o^2}\hat{e}_z + \frac{\underline{\underline{E}}_{o\perp}}{\omega_o^2 - \Omega_j^2}\right)\cos\omega_o t$$

$$+ \frac{e_j}{m_j}\,\frac{\underline{\underline{E}}_{o\perp} \times \underline{\underline{\Omega}}_j}{\omega_o(\omega_o^2 - \Omega_j^2)}\sin\omega_o t \qquad \text{(II.1)}$$

where $\Omega_j = e_j B_o / m_{jc}$ is the cyclotron frequency for specie j. For each charged specie, Eq.II.1 defines an oscillating frame in which the charged particle does not feel the influence of the external oscillating electric field.

Let us define

$$\hat{N}_j(\underline{\underline{r}}, t) = N_j[\underline{\underline{r}} + R_j(t), t] \qquad \text{(II.2)}$$

as the density fluctuation of charged specie j in its own oscillating frame. The Fourier-Laplace transforms of the two quantities are then related by

$$\tilde{N}_j(\underline{\underline{k}},\omega) = \int \frac{d\omega'}{2\pi} \Delta_j(\underline{\underline{k}},\omega-\omega') N_j(\underline{\underline{k}},\omega') \equiv \Delta_j N_j \qquad \text{(II.3)}$$

The last term is an operator representation of the original. Here

$$\Delta_j(\underline{\underline{k}},\omega) = \int_0^\infty dt \, \exp(-i\underline{\underline{k}}\cdot\underline{\underline{R}}_j + i\omega t) \qquad \text{(II.4)}$$

Eq. II.4 can be inverted to give

$$N_j(\underline{\underline{k}},\omega) = \bar{\Delta}_j \tilde{N}_j \qquad \text{(II.5a)}$$

where

$$\bar{\Delta}_j(\underline{\underline{k}},\omega) = \int_0^\infty dt \, \exp(i\underline{\underline{k}}\cdot\underline{\underline{R}}_j + i\omega t) \qquad \text{(II.5b)}$$

Using Eqs.II.1 and II.4, one can readily obtain an explicit expression for $\Delta_j(\underline{\underline{k}},\omega)$. As a simple example, consider the case when E_{o1} vanishes. We have then

$$\Delta_j(\underline{\underline{k}},\omega) = \sum_n J_n(\underline{\underline{k}}\cdot\underline{\underline{\mathcal{E}}}_j)\, i^{n+1}/(\omega+n\omega_o) \qquad \text{(II.6)}$$

where $\underline{\underline{\mathcal{E}}}_j = e_j \underline{\underline{E}}_o / m_j \omega_o^2$ is the magnitude of excursion of charged specie in the oscillating electric field in the special case when $\underline{\underline{E}}_o \parallel \underline{\underline{B}}_o$.

As mentioned earlier, in the oscillating frame of specie j, the charged particles of specie j do not feel the influence of the externally imposed oscillating electric field and therefore one has the usual linear relation between the density fluctuation and electric field fluctuation, that is,

$$\tilde{N}_j = -\frac{\chi_j}{4\pi e_j} i k \tilde{E}_j = \frac{\chi_j}{4\pi e_j} i k \Delta_j E \qquad \text{(II.7)}$$

where $\hat{E}_j \equiv \Delta_j E$ is the self-consistent electric field as seen in the oscillating frame of specie j and

$$\chi_j(\underline{\underline{k}},\omega) = \sum_{p=-\infty}^{\infty} \frac{\omega_{pj}^2}{k^2} \int_{-\infty}^{\infty} du_{\parallel} \int_0^{\infty} du_\perp u_\perp J_p^2\left(\frac{k_\perp u_\perp}{\Omega_j}\right) \frac{\left(\frac{p\Omega_j}{u_\perp}\right)\frac{\partial f_{oj}}{\partial u_\perp} + k_\parallel \frac{\partial f_{oj}}{\partial u_\parallel}}{(\omega - p\Omega_j - k_\parallel u_\parallel)} \qquad \text{(II.8)}$$

is the usual expression for suceptibility in a magnetized plasma with most symbols having usual meaning and velocity integrations in the appropriate Landau sense.

In the laboratory frame, Poisson equation may be written as

$$i k E = 4\pi e (N_i - N_e)$$

Transforming this to the oscillating electron frame

$$i k \tilde{E}_e = 4\pi e (\Delta_e N_i - \tilde{N}_e)$$

and eliminating E_* with the help of Eq.II.7, one obtaines

$$\tilde{N}_e = \Gamma_e \Delta_e N_i$$

where $\Gamma_e = \chi_e / (1 + \chi_e)$. Similarly, one can show

$$\tilde{N}_i = \Gamma_i \Delta_i N_e$$

Using Eqs.II.3 and II.5a to eliminate N_e from the above two equations, we obtain the final dispersion relation

$$\tilde{N}_i = \Gamma_i \Delta_i \bar{\Delta}_e \Gamma_e \Delta_e \bar{\Delta}_i \tilde{N}_i \qquad \text{(II.9)}$$

In the integral form, Eq.II.9 can be rewritten as

$$\tilde{N}_i(k,\omega) = \iiiint \frac{d\omega_1\, d\omega_2\, d\omega_3\, d\omega_4}{(2\pi)^4} \Delta_i(\underline{k}, \omega - \omega_1)$$
$$\bar{\Delta}_e(\underline{k}, \omega_1 - \omega_2)\, \Gamma_e(\omega_2, \underline{k})$$
$$\Delta_e(\underline{k}, \omega_2 - \omega_3)\, \bar{\Delta}_i(\underline{k}, \omega_3 - \omega_4)\, \tilde{N}_i(\underline{k}, \omega_4)$$

(II.10)

Eqs.II.9 and II.10 have a simple physical interpretation. An ion density fluctuation $\tilde{N}_i$ in the ion oscillating frame appears as an ion fluctuation $\Delta_e \bar{\Delta}_i \tilde{N}_i$ in the electron oscillating frame. This induces an electron density fluctuation $\Gamma_e \Delta_e \bar{\Delta}_i \tilde{N}_i$ which transforms back to the ion frame as an electron fluctuation $\Delta_i \bar{\Delta}_e \Gamma_e \Delta_e \bar{\Delta}_i \tilde{N}_i$. For self-consistency, the new induced ion density fluctuation $\Gamma_i \Delta_i \bar{\Delta}_e \Gamma_e \Delta_e \bar{\Delta}_i \tilde{N}_i$ must be identical to $\tilde{N}_i$, the original ion fluctuation; this gives Eq.II.9 and II.10.

We now wish to express Eq.II.10 in a more conventional form. For simplicity, we illustrate the calculations only for the special case when E_0 is along z-direction and $\Delta_j(\underline{k},\omega)$ is given by Eq.II.6. Using Eq.II.6, we can write

$$\Delta_e \bar{\Delta}_i \tilde{N}_i = -\int \frac{d\omega_3 d\omega_4}{(2\pi)^2} \sum_n \sum_m \frac{J_n(k\cdot\mathcal{E}_e)}{(\omega_2-\omega_3+n\omega_0)} \frac{J_m(-k\cdot\mathcal{E}_i)}{(\omega_3-\omega_4+m\omega_0)} i^{n+m} \tilde{N}_i(\underline{k},\omega_4)$$

$$= \sum_n \sum_m J_n(k\cdot\mathcal{E}_e) J_m(-k\cdot\mathcal{E}_i)\, i^{n+m} \tilde{N}_i(\underline{k},\omega_2+\overline{n+m}\,\omega_0)$$

$$= \sum_p J_p(\mu) i^p \tilde{N}_i(\underline{k},\omega_2+p\omega_0) \qquad \text{(II.11)}$$

where the integrations have been carried out using the residue theorem, we have defined

$$\mu = \underline{\underline{k}} \cdot (\underline{\underline{\mathcal{E}}}_e - \underline{\underline{\mathcal{E}}}_i) \tag{II.12}$$

and have used the Bessel function identity

$$\sum_n J_n(a)\, J_{p-n}(b) = J_p(a+b)$$

Proceeding similarly, we finally obtain the following dispersion relation from Eq.II.10.

$$I^n = \Gamma_i^n \sum_p \sum_q J_{p-n}(\mu)\, J_{p-q}(\mu)\, \Gamma_e^p\, I^q \tag{II.13}$$

Eq.II.13 gives an infinite determinant as the general form of the dispersion relation for electrostatic modes. If the oscillating electric field is arbitrarily oriented with respect to the magnetic field, the only change is in the argument of the Bessel function which takes the more general form

$$\mu = \underline{\underline{k}} \cdot (\underline{\underline{\mathcal{E}}}_e - \underline{\underline{\mathcal{E}}}_i) \equiv \mu_e - \mu_i \tag{II.14}$$

with

$$\mu_j = -\frac{e_j}{m_j}\left[\left(\frac{E_{0z} k_{\parallel}}{\omega_0^2} + \frac{\underline{\underline{E}}_{0\perp} \cdot \underline{\underline{k}}_{\perp}}{\omega_0^2 - \Omega_j^2}\right)^2 + \frac{(\underline{\underline{E}}_{0\perp} \times \underline{\underline{k}}_{\perp})^2\, \Omega_j^2}{\omega_0^2(\omega_0^2 - \Omega_j^2)^2}\right]^{\frac{1}{2}} \tag{II.15}$$

Analysis of Dispersion Relation

We now analyze the dispersion relation Eq.II.13 for the case when one of the excited modes has a frequency ω much less than the driving frequency ω_0.

When the frequency ω_0 has a high value, typical of electron resonant modes, one can ignore the ion response at ω_0 and its multiples. We thus need only retain I^0 which corresponds to ion density fluctuation at frequency (assumed much low than ω_0). Mathematically, the neglect of $I^m (m \neq 0)$ follows from the smallness of $\Gamma_i^m (m \neq 1)$. The dispersion relation now assumes the simple form

$$1 - \Gamma_i^0 \sum_p J_p^2(\mu) \Gamma_e^p = 0 \qquad \text{(II.16)}$$

We restrict our attention to weak pump waves, that is $\mu \ll 1$. In this case we only need retain the terms with $p = 0, \pm 1$. Note that we can also approximate $\mu \simeq \mu_e$ because the displacement of ions in the high-frequency pump field will be negligible. In the weak pump approximation, Eq.II.16 may be rewritten as

$$\frac{1}{\chi_i^0} + \frac{1}{1+\chi_e^0} + \frac{\mu^2}{4}\left(\frac{1}{\epsilon_+^H} + \frac{1}{\epsilon_-^H}\right) = 0 \qquad \text{(II.17)}$$

where $\epsilon_\pm^H = 1 + \chi_e^{\pm 1}$ is the high-frequency dielectric constant at frequencies $|\omega \pm \omega_0| \simeq \omega_0$.

CHAPTER V

DAMPING OF MAGNETOSONIC WAVE IN PRESENCE OF LOWER HYBRID TURBULENCE

Introduction

The discussions in the preceding chapter quantitatively reveal that the combined effects of parametric heating scheme and our appropriate external source enhance the heating efficiency substantially. It is not clear, however, that the harmful effects associated with the macroscopic source-driven models can be completely quenched. Perhaps the nonlinear characteristics of these modes will throw light on their explosive nature or their saturation mechanisms. In this chapter, a new heating method will be described in which the external low frequency electromagnetic waves are damped by a background turbulence in the plasma, thus leading to preferential heating of ions. Since the frequency considered in this problem is lower than the lower hybrid frequency, this mechanism for ion heating has some advantages over the parametric heating scheme near lower hybrid resonance. It is amply demonstrated in the literature that the mechanisms responsible for the excitation of lower hybrid mode are the parametric decay process[1] and the cross-field currents[2-5]. Yet another process is the possibility discussed by Babykin et al.[6]. They essentially show that a large amplitude magnetosonic wave can also excite lower hybrid waves enabling thereby to deposit its energy into the plasma, provided the amplitude of the magnetosonic wave exceeds a certain threshold value. Below this critical value, the small amplitude magnetosonic wave is undamped in a quiescent plasma and wave energy remains uncoupled from the plasma particles. In this chapter, it will be shown that these small amplitude magnetosonic waves can also be damped in the presence of a lower hybrid turbulence. The present work will assume the presence of a lower hybrid turbulence in the plasma background and usual generation mechanisms such as parametric decay process or cross-field currents. It should be noted here that the small amplitude magnetosonic waves themselves do not generate the turbulence and it can be considered as an additional source of energy. The results also demonstrate another interesting aspect of the lower hybrid heating scheme. The non-resonant interaction between the turbulence and the linear perturbation leads to a purely growing instability which essentially bunches the turbulent waves in space. This will lead to an inhomogeneity in the turbulence and can introduce nonuniformity in the heating of the plasma.

Vedenov et al.[7] studied, in great detail, the coupling between a low frequency long wavelegth coherent mode and a high frequency background turbulence. We shall follow the same method and treat the background lower hybrid turbulence as a distribution of quasiparticles governed by the wave kinetic equation and the coherent mode by the linear magnetohydrodynamic equations. The effect of turbulent field on the particles provide a net ponderomotive force on the linear MHD mode and thus it establishes a coupling between background turbulence and the MHD mode. The application of the wave kinetic equation is rather restrictive, as it is limited by the choice of the density perturbation which evidently becomes the coupling parameter[8,9]. In a magnetoplasma, however, we are allowed a wider choice of this parameter and thus we have chosen the magnetic perturbation as the coupling parameter. It should be noted that the choice of density perturbation as the coupling parameter would be inadequate for coupling a magnetosonic wave with a lower hybrid turbulence in a high density plasma. Our proposition is based on the following physical considerations: In a high density plasma ($\omega_{pe}^2/\Omega_e^2 \gg 1$, ω_{pe}, Ω_e being the electron plasma and electron gyrofrequencies, respectively) the frequency of the lower hybrid mode is governed by the magnetic field strength only ($\omega_k \sim (\Omega_e \Omega_i)^{1/2}$, Ω_i being the ion gyrofrequency). The magnetic field fluctuations of the magnetoacoustic mode locally modifies the frequency of the lower hybrid waves. This modification in the frequency of the lower hybrid waves modifies the plasmons distribution. The plasmons then react back on the magnetosonic wave via the ponderomotive force and they modify the propagation characteristics of the magnetosonic wave. This modification turns out to be a damping of the magnetosonic wave.

Modified Dispersion Relation and Discussions

We shall consider a homogeneous collisionless plasma embedded in a constant magnetic field $\vec{B}_o = B_o\hat{e}_z$. The propagation vector of magnetosonic wave will be assumed to be in the x direction. The existence of a stationary lower hybrid turbulence is assumed which is governed by the wave kinetic equation(7).

$$\frac{\partial N_k}{\partial t} + \vec{v}_g \cdot \vec{\nabla} N_k - \frac{\partial \omega_k}{\partial \vec{r}} \cdot \nabla_k N_k = 0 \qquad (1)$$

where $N_k = |E_k|^2/4\pi\omega_k$ is the plasmon distribution function and $\vec{v}_g = \partial\omega_k/\partial\vec{k}$ is the group velocity of the lower hybrid waves. The space and time dependence in Eq.(1) is slow compared with the space and time dependence of the microturbulence itself. This slow dependence, in our case, is provided by the magnetosonic wave (Ω, $\vec{q}$), therefore, we must have $\Omega \ll \omega_k$ and $|\vec{q}| \ll |\vec{k}|$.

In the frequency range $\Omega_i \ll \omega_k \ll \Omega_e$, where $\Omega_{i,e}$ are the ion and electron cyclotron frequencies, the general dispersion relation (See, Appendix) for lower hybrid waves can be written as[2]

$$1 - (2k^2\lambda_D^2)^{-1} \frac{T_e}{T_i} Z'\left(\frac{\omega_k}{\sqrt{2}\, k_\parallel \alpha_i^{1/2}}\right) + \frac{1 - \exp(-\lambda) I_0(\lambda)}{k^2\lambda_D^2}$$
$$- (2k^2\lambda_D^2)^{-1} \exp(-\lambda) I_0(\lambda) Z'\left(\frac{\omega_k}{\sqrt{2}\, k_\parallel \alpha_e^{1/2}}\right) = 0 \tag{2}$$

where $\lambda = k_\perp^2 \rho_e^2$, ρ_e being the electron gyroradius and I_0 is the zeroth order modified Bessel function. Z appearing in Eq.(2) is the usual plasma dispersion function[10] and α_e, α_i are the electron and ion thermal velocities, respectively. In the fluid limit, that is, for $k \rho_e \ll 1$ and $\omega_k/k_\parallel \gg \alpha_e$, Eq.(2) reduces to,

$$\omega_k^2 \simeq \Omega_i \Omega_e \left(1 + \frac{k_\parallel^2}{k_\perp^2} \frac{m_i}{m_e} \right) \tag{2a}$$

where it is assumed that $(k_\parallel^2 / k_\perp^2)(m_i/m_e) \lesssim 1$ and $\omega_{pe}^2/\Omega_e^2 > 1$. On the other hand for an exact perpendicular propagation ($k_\parallel = 0$) but with finite Larmor radius correction Eq.(2) simplifies to,

$$\omega_k^2 \simeq \Omega_e \Omega_i \left(1 + \frac{3}{8} k^2 \rho_e^2\right) \tag{2b}$$

In the deriving Eqs.(2a) and (2b) it has also been assumed that $k \rho_i >> 1$. From Eqs.(2a) and (2b) it is clear that any perturbation on the ambient magnetic field alters the frequency of these waves. In our case such a perturbation is provided by the magnetosonic mode. The magnetosonic wave can be described by the hydromagnetic equations,

$$\rho \frac{\partial \vec{V}}{\partial t} + \rho (\vec{V} \cdot \vec{\nabla}) \vec{V} = \bar{J} \times \bar{B}/c - \bar{\nabla} p \tag{3}$$

$$\vec{E} + (\bar{V} \times \bar{B})/c = 0 \tag{4}$$

$$\bar{\nabla} \times \bar{E} = -1/c \frac{\partial \bar{B}}{\partial t} \quad , \quad \bar{\nabla} \cdot \bar{E} = 0 \tag{5}$$

and

$$\bar{\nabla} \times \bar{B} = 4\pi \bar{J}/c \quad , \quad \bar{\nabla} \cdot \bar{B} = 0 \tag{6}$$

where $\vec{V}$ is the single fluid velocity, ρ is the density, p is the plasma pressure, $\vec{E}$ is the perturbed electric field, $\vec{J}$ is the perturbed current and c is the velocity of light. Now, $\vec{V}$ consists of both slow and fast motions and can be written as $\vec{V} = \vec{V}_f + \vec{V}_s$, $\vec{V}_s$ being the motion of the fluid due to slow magnetosonic mode while $\vec{V}_f$ is due to the fast lower hybrid mode. Assuming a random turbulence, we average over several fast oscillations and from Eqs. (3) to (6), we get

$$\left[\frac{\partial^2}{\partial t^2} - (V_A^2 + C_A^2) \frac{\partial^2}{\partial x^2} \right] B_{1z} = B_o \frac{\partial}{\partial x} \langle (\bar{V}_f \cdot \bar{\nabla}) \bar{V}_f \rangle_x \quad (7)$$

where $V_A = (B_o^2/4\pi\rho_o)^{1/2}$ is the Alfven speed $C_s^2 = (T_e + T_i)/m_i$. The right-hand side of Eq.(7) is essentially the ponderomotive force, which is

$$PF = \langle (\bar{V}_f \cdot \bar{\nabla}) \bar{V}_f \rangle = \left[\left(V_{kx} \frac{\partial}{\partial x} V_{-kx} + V_{ky} \frac{\partial}{\partial y} V_{-kx} \right) \hat{e}_x \right.$$

$$\left. + \left(V_{kx} \frac{\partial}{\partial x} V_{-ky} + V_{ky} \frac{\partial}{\partial y} V_{-ky} \right) \hat{e}_y \right] \quad (8)$$

Using the definition of the fluid velocity, we can write

$$\bar{V}_f = \frac{n_e m_e \bar{V}_{fe} + n_i m_i \bar{V}_{fi}}{n_e m_e + n_i m_i} \quad (9)$$

where the quantities V_{fe} and V_{fi} appearing in Eq.(9) correspond to the electron and ion velocities induced by the lower hybrid mode which has the property that $\bar{k} . \bar{E}_k \gg \bar{k} \times \bar{E}_k$ (i.e. an electrostatic mode). Under the action of this mode, we have

$$\frac{\partial \bar{V}_{fj}}{\partial t} = \frac{e_j}{m_j} \left(\bar{E}_f + \frac{\vec{V}_{fj} \times \vec{B}_o}{c} \right) \quad (10)$$

Eq.(10) yields

$$V_{fxj} = -i \frac{e_j}{m_j} \omega_k E_x (\Omega_j^2 - \omega_k^2)^{-1}$$

and

$$V_{fyj} = \frac{e_j}{m_j} (\bar{E} \times \bar{\Omega}_j)(\Omega_j^2 - \omega_k^2)^{-1} \quad (11)$$

Since the magnetosonic mode is essentially sustained by the ion motion, the ponderomotive force on the ions is the important factor. The ponderomotive force on the electrons will not affect the magnetosonic mode. This can also be seen by taking ratios of velocity components given by Eq. (11), namely, $V_{fxi}/V_{fxe} \approx O(1)$ where as $V_{fyi}/V_{fye} \approx O(m_e/m_i)$. Therefore, the ponderomotive force in the $\hat{e}_y$ direction is unimportant in our problem and hence we have chosen the propagation of the magnetosonic mode to be in the x direction only. Hence, from Eq.(9) we can write $V_f \simeq V_{fi}$, since $V_{fi}/V_{fe} \sim O(1)$ and charge neutrality can be assumed. Now, Eq.(7), under these assumptions, becomes

$$\left[\left(\frac{\partial^2}{\partial t^2} - (V_A^2 + C_s^2)\frac{\partial^2}{\partial x^2}\right] B_{1z} = \frac{B_0 \omega_{pi}^2}{2 n_0 m_i} \frac{\omega_k^2}{(\Omega_i^2 - \omega_k^2)^2} \sum_k \frac{\partial^2 N_k}{\partial x^2} \qquad (12)$$

where use has been made of the relation $N_k = |E_k|^2/4\pi\,\omega_k$.

We now write $N_k = N_k^0 + \tilde{n}_k$, $\tilde{n}_k$ being the perturbed part of the plasmon distribution function and assume that $\tilde{n}_k \sim \tilde{n}_k \exp\left[i(\vec{q}\cdot\vec{r} - \Omega t)\right]$ and $\tilde{E}_k \sim \tilde{E}_k \exp\left[i(\vec{k}\cdot\vec{r} - \omega_k t)\right]$. Thus the linearized version of Eq.(1) gives

$$\tilde{n}_k = \left(\frac{\partial \omega_k}{\partial \vec{r}} \cdot \bar{\nabla}_k N_k^0\right) / \left[i(\bar{V}_g \cdot \bar{q} - \Omega)\right] \qquad (13)$$

Now, from Eq.(2a), we have $\omega_k \simeq (\Omega_e \times \Omega_i)^{1/2}$, and therefore

$$\frac{\partial \omega_k}{\partial \vec{r}} = \left(\frac{e^2}{m_i m_e c^2}\right)^{1/2} \frac{\partial B}{\partial \vec{r}} = i\left(\frac{e^2}{m_i m_e c^2}\right)^{1/2} \vec{q}\, B_{1z} \qquad (14)$$

From Eqs.(13) and (14) it is clear that a perturbation in the magnetic field, $\tilde{B}_{1z}$, due to the slow magnetosonic wave can change the plasmon distribution which in turn couples the turbulence with the magnetosonic mode. Combining Eqs. (12), (13) and (14), we get the modified dispersion relation for the magnetosonic wave under the influence of a lower hybrid turbulence, as

$$\left(q^2 M_A^2 - \Omega^2\right) = \frac{q^2 \omega_{p_i}^2}{2 m_i n_0} \frac{\omega_k^3}{(\Omega_i^2 - \omega_k^2)^2} (\Omega_e \Omega_i)^{1/2} \int \frac{\bar{q}\cdot\bar{\nabla}_k N_k^0}{(\Omega - \bar{q}\cdot\bar{V}_g)} dk \tag{15}$$

where $M_A^2 = V_A^2 + C_s^2$.

Analogous to the usual Landau damping of particles Eq.(15) inherently gives a damping term due to the pole contribution. Assuming the imaginary part γ_q to be much smaller than Ω, we have

$$\gamma_q = \frac{\pi q \omega_{p_i}^2}{4 m_i n_0} \int (\bar{q}\cdot\bar{\nabla}_k) N_k^0 \, \delta(q M_A - \bar{V}_g\cdot\bar{q}) \, d\bar{k} \tag{16}$$

where we have used the fact that $\omega_k \simeq (\Omega_e \times \Omega_i)^{1/2}$. It is obvious from Eq.(16) that the maximum contribution to the integral arises when $qM_A - \bar{v}_g\cdot\vec{q} = 0$.

The short wavelength transverse lower hybrid mode given by Eq.(2b) cannot participate in the resonant interaction.as $V_g = (3\, k\, \rho_e^2/4)\,(\Omega_e \times \Omega_i)^{1/2} = M_A$ would mean $k^2 \rho_e^2 = M_A^2/C_s^2 = (1 + 1/\beta)$ where $\beta = C_s^2/V_A^2 = 8\pi n_o \mathcal{K} T_e/B_o^2$ is the ratio of plasma pressure to magnetic pressure. The requirement violates the condition $k\rho_e < 1$ for $\beta < 1$. For this short wavelength mode, therefore, $V_g/M_A << 1$ and it can interact only nonresonatly. The fluid mode as given by Eq.(2a) can, however, resonate with the magnetosonic mode by virtue of its larger group velocity. The resonance condition along with Eq.(2a) gives,

$$\left[\frac{k_\parallel}{k} \left(\frac{m_i}{m_e} \right)^{1/2} \right]^3 = \left(\frac{M_A}{C_s} \right) \left(\frac{k_\parallel \alpha_e}{\omega_k} \right) \qquad (17)$$

In this case, the condition $[(k_\parallel/k)(m_i/m_e)^{1/2}] < 1$ can easily be satisfied as $M_A/C_s \simeq 1$ for $\beta \lesssim 1$ and $k_\parallel \alpha_e/\omega_k << 1$.

In a high-β plasma, the finite Larmor correction for the transverse lower hybrid mode provides a nonresonant coupling between the magnetosonic perturbation and the background lower hybrid turbulence. It should be noted that this analysis is applicable to any experiment invoking the lower hybrid heating scheme. The magnetosonic perturbation considered in this work being linear, the analysis can predict the stability of the turbulence even in the absence of an externally imposed magnetic perturbation. As shown earlier, the group velocity V_g, as given by Eq.(2b), is small compared to M_A. Hence, the term $(\Omega - \vec{V}_g.\vec{q})^{-1}$ in Eq.(15) can be expanded in powers of qV_g/Ω. The equilibrium solution of Eq.(1) can be written as a

Maxwellian distribution for the plasmons, namely

$$N_k^o = N_o(\Delta/\pi)^{1/2}\exp[-\Delta(k-k_o)^2] \tag{18}$$

Using Eq.(18), Eq.(15) can be simplified to give

$$qM_A^2-\Omega^2 = -\frac{N_o k}{\Omega^2}\left[\frac{q^2\omega_{pi}^2}{2m_i n_o}\cdot\frac{\omega_k^3}{\Omega_i^4}(\Omega_e\Omega_i)^{1/2}\right] \tag{19}$$

where the term $(qV_g/\Omega\ )^3$ and higher order terms are neglected. The roots of Eq.(19) are

$$\Omega^2 = \frac{1}{2}\left[q^2M_A^2 \pm (q^4M_A^4+4K)^{1/2}\right] \tag{20}$$

with $K = (N_o\ kq^2 \times \omega_{pi}^2/2\ m_i\ n_o)(\omega_k^3/\Omega_i^4)$. Thus, there exists a purely gorwing mode with the growth given by

$$\gamma \simeq K/q^2M_A^2 \tag{21}$$

This instability arises primarily due to a nonlinear coupling between the lower hybrid turbulence and the magnetosonic waves injected externally into the system or present as noise in the system. This process would lead to a bunching of the turbulent energy in space. An initially homogeneous turbulence tends to become inhomogeneous in space and it will probably cause nonuniform heating.

In summary, we find that efficient transfer of energy can take place from an externally produced magnetosonic wave into a plasma in the presence of a lower hybrid turbulence. The coupling is achieved by the choice of a magnetic perturbation as the coupling parameter. The long wavelength fluid mode takes part in the resonant interaction whereas the short wavelength transverse lower hybrid mode participates in the nonresonant interaction. The later interaction might lead to an undesirable process for the heating scheme, since the homogeneous turbulence can be bunched in space by this interaction leading to non uniform heating.

References

1. P.K. Kaw, in Advances in Plasma Physics, (Ed. A. Simon and W. Thompson, John Wiley and Sons) Vol.6, p.179 (1976).

2. J.B. McBride, E. Ott, J.P. Boris and J.H. Orens, Phys. Fluids, 15, 2367 (1972)

3. M.S. Ioffe, R.I. Sobolev, V.G. Telkovskii and E.E. Yushmanov, Zh. Eksp. Teor. Fiz 39, 1602 (1960) (Sov. Phys. - JETP 12, 1117 (1961))

4. I. Alexeff, L.A. Berry, J.M. Dudley, K.G. Estabrook, A. Hirose, W.D. Jones, R.V. Neidigh, J.N. Olsen, F.R. Scott, W.L. Stirling, M.N. Winder and W.R. Wing, in Plasma Physics and Controlled Fusion Research (International Atomic Energy Agency, 1971) Vol.2, p.221.

5. I. Alexeff, K. Estabrook, A. Hirose, W.D. Jones, R.V. Neidigh, J.N. Olsen, F.R. Scott, W.L. Stirling and Wing, Phys. Rev. Lett. 25, 848 (1970)

6. M.V. Babykin, P.P. Garvin, E.K. Zavoiskii, L.I. Rudakov, V. Skoryupin and G.V. Sholin, Zh. Eksp. Teor. Fiz. 46, 511 (1964) (Sov. Phys. JETP 19, 349 (1964)).

7. A.A. Vedenov, A.V. Gordeev and L.I. Rudakov, Plasma Phys. 9, 719 (1967)

8. Y.S. Satya and P.K. Kaw, Phys. Rev. Lett. 31, 1453 (1973)

9. P.N. Guzdar, Phys. Rev. Lett. 35, 1635 (1975)

10. B.D. Fried and S.D. Conte, The Plasma Dispersion Function (Academic Press, New York 1961)

APPENDIX - III

Derivation of the Electrostatic Lower Hybrid Mode with Finite Larmor Corrections

The perturbed distribution function $\tilde{f}_{kj}$ can be written down from the linearised Vlasov theory as:

$$\tilde{f}_{kj} = -\frac{q_j}{m_j}\int_{-\infty}^{0} E_k \cdot \nabla_{v'} f_{\alpha o} \exp i(k\cdot x' - \omega_k \tau)\, d\tau \qquad (III.1)$$

where the integration has to be performed over the unperturbed orbits. Since the lower hybrid mode has a frequency given by $\Omega_i \ll \omega_k \ll \Omega_e$ and also $k\rho_i \gg 1 \gg k\rho_e$, (Ω_j and ρ_j are the gyrofrequency and gyroradius of the jth species) we consider the ions to be unmagnetised and the electrons to be magnetised. The unperturbed orbits are therefore,

$$\left.\begin{array}{l} v' = v = \text{constant} \\ x' = x + v\tau \end{array}\right\} \quad \text{for ions}$$

and $V_x' = V_\perp \operatorname{Cos}(\phi - \Omega_e\tau)$, $V_y' = V_\perp \operatorname{Sin}(\phi - \Omega_e\tau)$,

$$X' = X = \frac{V_\perp}{\Omega_e}\operatorname{Sin}(\phi - \Omega_e\tau) + \frac{V_\perp}{\Omega_e}\operatorname{Sin}\phi ,$$

$$y = y + \frac{V_\perp}{\Omega_e}\operatorname{Cos}(\phi - \Omega_e\tau) - \frac{V_\perp}{\Omega_e}\operatorname{Cos}\phi$$

$V_z' = V_\parallel$, $Z' = V_\parallel\tau + Z$, for the electrons.

Integrating over these orbits and using the poisson's equation $\nabla\cdot E = 4\pi\sum q_j \int \tilde{f}_{k_j} d\bar{v}$, we get the desired dispersion relation in the form

$$-k^2 = -\omega_{pi}^2 \int k_{\parallel} \frac{\partial}{\partial v_{\parallel}} f_{oi} / (k_{\parallel} v_{\parallel} - \omega_k)\, dv_{\parallel} + 2\alpha_e \omega_{pe}^2 \Big[1 + + \omega_k \sum_{l} \int J_l^2\left(\frac{k_\perp v_\perp}{\Omega_e}\right) f_{oe} / (k_{\parallel} v_{\parallel} - \omega_k + l\,\Omega_e)\, d\bar{v} \Big] \tag{III.2}$$

where $J_l(k_\perp v_\perp / \Omega_e)$ is the bessel function of the order l and λ_D is the debye length.

For the electronic term the $l = 0$ term contribute maximum to summation as the rest of the terms are smaller by a ratio $k^2 \rho_e^2 << 1$. Therefore integrating over 'v' we find,

$$1 - (2k_2\lambda_D^2)^{-1} \frac{T_e}{T_i} Z'\left(\frac{\omega_k}{k_{\parallel} v_{thi}}\right) + \{1 - \exp(-k_\perp^2 \rho_e^2) I_o(k_\perp^2 \rho_e^2)\}/k^2\lambda_D^2 - (2k^2\lambda_D^2)^{-1} \exp(-k_\perp^2 \rho_e^2) I_o(k_\perp^2 \rho_e^2)\, Z'\left(\frac{\omega_k}{k_{\parallel} v_{the}}\right) \tag{III.3}$$

where Z is the dispersion function and $I_o = \left[\dfrac{\int_0^\infty J_o^2\left(\frac{k v_\perp}{\Omega_e}\right) e^{-\alpha v_\perp^2} d\alpha v_\perp^2}{e^{-k_\perp^2 \rho_e^2}} \right]$

CHAPTER VI

CONCLUDING REMARKS AND FUTURE WORK

The main highlights of this dissertation can be briefly summarized in this concluding chapter. The methods of improving the heating efficiency and the stabilization of some dangerous modes are the fundamental objectives of the preceding chapters. In explaining the physical mechanism responsible for substorm process, the collisionless tearing mode theory has been invoked with the allowance of zero order electric field in the equilibrium configuration. This factor permits the excitation of a new non resonant tearing mode which fairly seems to account for the sattelite observations during the magnetospheric substorms. Another important feature is the excitation of velocity gradient driven instabilities. These instabilities arise only for nonuniform electric and magnetic fields in a certain restricted frequency range. Finally the work also stresses the role of some externally controllable parameters for increasing the heating efficiency in a fusion plasma.

In our work, we have skipped some intricate nonlinear processes responsible for plasma heating in the neutral sheet. Rather than making an indepth study into a particular process, the main emphasis has been made only to assess the roles of different parameters like strong plasma inhomogeneities, external perturbations and the background turbulent fluctuations on a typical laboratory or space plasma process. It is worth while recording some pertinent observations regarding the work that will be attempted in future.

Even though the problems studied in this thesis exhaustively cover varied effects of different physical parameters, they all suffer from certain degrees of incompleteness. The tearing mode problem has been studied with a field configuration closely resembling the earths magnetospheric tail configuration, it would also be important to study similar processes in a tokamak geometry including the toroidal effects in view of the recent observation of this mode in tokamaks. Another aspect which would be interesting topic of future work is the non-linear saturation of the ion-tearing mode. The knowledge of the saturation amplitude of this mode is essential for predicting the final magnetic topology. The exact heating mechanism of the magnetic annhilation process is yet to be identified and still it remains an open topic of research, specially in connection with the substorm process. While dealing with the velocity gradient driven instabilities, we have neglected the finite larmor radius effects. These effects generally give stabilizing influence and it would be worth while extending our theory to the kinetic regime. In the heating aspect of the plasma, we have considered the effect of cold component of electrons. It is quite well known that in parametric heating schemes (specially for laser-pellet interactions) a high energy tail is created for the electrons. These supra thermal electrons can also modify various other parametric processes. We have already emphasised the role of such electrons in case of the absorptive instability, (Chapter IV), and it would be of interest to study their effects on the scattering instabilities. In Chapter V we have studied the turbulence induced damping of the magnetosonic waves in the high density limit (i.e. $\omega_{pe}^2/\Omega_e^2 \gg 1$) which can be true only for Laboratory plasmas. For space plasmas wherein the relation $\omega_{pe}^2/\Omega_e^2 \ll 1$ holds good, a similar damping mechanism can also be studied specially in connection with the heating of the plasma in the inner magnetosphere where both magnetic pulsation as well as the electrical noise (both driven unstable by field aligned currents) have been observed.