A STUDY OF PION CAPTURE IN THREE-BODY SYSTEMS, ^3He AND ^6Li

by

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Submitted to the Gujarat University

for the degree of

DOCTOR OF PHILOSOPHY

1969

PHYSICAL RESEARCH LABORATORY
AHMEDABAD

PREFACE

In recent years there has been considerable interest in reactions involving capture of π^- from an atomic orbit into a nucleus resulting in emission of a pair of nucleons. Such studies are expected to provide a considerable insight into short range correlations of nucleons in nuclei. This is due to the massive nature and the boson character of the pion. Also the pion is more or less at rest when it is captured. The crucial feature of the reaction is the presence in the nucleus of two nucleons within a very short range of each other. It is expected that the detailed nature of the short range correlations in wave function of a pair of nucleons in nuclei would strongly influence the cross sections and the angular distributions of outgoing nucleons. This is the main theme of the present studies.

Usually this nucleon-nucleon correlation is either studied using a phenomenological πNN interaction (five point vortex), or by modifying the wave-function by introducing the 'Jastrow' cut off factor in an adhoc way. Here we use the exact wavefunctions obtained by solving the Schrodinger equation which includes a realistic nucleon-nucleon potential. Thus the correlations built into the wave functions would be clearly related to the potential. For this purpose we study the three-body systems ^{3}He and ^{6}Li (regarded as a+n+p). Largely owing to the work of Faddeev, reasonably accurate solutions (albeit numerical) of the Schrodinger equation for three particle systems are being available. We here follow the method of Mitra and his coworkers who use the non-local factorable potential to study the different properties of three nucleon system. We also discuss the importance of the rescattering term in the πN interaction, although in the present work we deal with only the standard ps-pv interaction (in the static limit). With this we have studied fully the pion capture by these two nuclei.

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COUNTERSIGNED

ACKNOWLEDGEMENTS

I wish to express my sincere gratitude to Professor K.R. Ramanathan and Professor V.A. Sarabhai for giving me an opportunity to work in the Physical Research Laboratory.

I am indebted to Professor S.P. Pandya for introducing me to the subject and valuable guidance throughout the work. I am also very grateful to him for his critical examination of the manuscript and help in its preparation. I am indebted to Professor A.N. Mitra and Dr. K.H. Bhatt who helped me to understand different phases of this work at an early stage. The encouragement given to me by Professor Ramanathan during my work is gratefully acknowledged.

I am thankful to the entire staff of the IBM group for helping and running my computer programmes. Thanks are due to Miss Y.D. Patel for typing the manuscript very carefully.

Finally I extend my thanks to the Ministry of Education, Government of India, for partial financial support during the period of the work.

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1. INTRODUCTION

Since it was pointed out by Ericson (E62a) and Jean (J64a, J64b) that pion interactions with nuclei could be used as a powerful tool to derive a considerable amount of useful information (otherwise difficult or impossible to obtain) on nuclear structure, the subject has received a lot of attention from theoreticians as well as experimentalists. The absorption of pion by nuclei can, in particular, yield information on momentum distributions of nucleons, as well as on two-hole excitation states in nuclei. The advantage of pion as a nuclear probe (over the photon which is also a boson) lies in its large rest mass and three possible charge states. The π^- can easily be absorbed from a bound orbit (virtually at rest) and can thus deliver a large amount of its rest mass energy to nucleons inside the nucleus, without bringing in any momentum at all. Its various charge states enable the exploration of isobaric states in nuclei, for example through the double-charge-exchange reaction $(\pi^{\pm}, \pi^{\mp})$ one can reach states with $\Delta T = 2$. Experimentally, the advantage of pion lies in the possibility of obtaining beams of pions of sufficient intensity with very high energy resolution. As against this photon beams are generally obtained from bremstrahlung and hence are not monochromatic. With pions one can deliver precise amounts of charge, energy and momentum in the nucleus. Although reactions of nuclei with protons such as (p, pn), (p, 2p), (p, p') etc. can also be useful probes for nuclear properties, in these processes the incident proton interacts in general with a single nucleon in the nucleus. On the other hand the pion can disappear in the nucleus, ejecting a pair of nucleons, thus making the analysis of the experimental data on the final states much easier.

In this work we shall only be concerned with pion absorption in nuclei. Whereas both π^+ and π^- beams are available, and both can be captured in flight by nuclei, the π^- can also be slowed down and captured in an atomic orbit by nuclei. The capture generally takes place in a large n orbit, but the π^- rapidly cascades down to lower orbits through successive x-ray emission and is finally captured by nuclei. In light nuclei the nuclear absorption takes place mainly from 1S or 2P orbits. From the theoretical point of view it is obviously simpler to analyse the capture of stopped and bound negative pions, since the pion momentum is then practically zero. Finite pion momentum in initial state can only make the kinematics of final state more complicated, and most of the theoretical studies have been concerned with the capture of bound π^-. The absorption of a π^- at rest in the nucleus releases the entire rest mass energy of 139.2 Mev as kinetic energy. Further the pseudoscalar nature of pion requires a change of parity from the initial nuclear configuration to the final one. If now the entire kinetic energy is given to a single nucleon, it must acquire a momentum of about 500 MeV/c. Since the pion brings in no momentum, conservation of momentum requires that the nucleon must have initially such a large momentum. The highly successful independent particle nuclear model and the nuclear potential well that is its characteristic feature yield a Fermi momentum of no more than about a couple of hundred MeV/c. Hence the pion capture mechanism in which all the energy is taken up by a single nucleon appears very unlikely, and in actual experience is indeed highly suppressed.

It has been proposed and is now commonly accepted that the absorption of π^- would occur on a proton in a strongly correlated pair of nucleons inside the nucleus. If the motion of the centre of mass of the pair in the nucleus be neglected as a first approximation, it is clear that the two nucleons share the energy equally and will move away from each other with a relative momentum $(\underset{\sim}{p}_1 - \underset{\sim}{p}_2)/2$ of about 350 MeV/c. A relative momentum of this magnitude would be provided by their mutual scattering caused by the strong short-range nucleon-nucleon force. An elementary argument based on the uncertainty principle shows that the pair of nucleons must be correlated i.e. within a relative distance of about 0.65 F. This simple schematic model for pion capture suggests that the capture really takes place on a pair of nucleons which are close together and interacting strongly. The capture process probes in a selective fashion only pairs of highly correlated nucleons. We have thus in the study of this process a very useful instrument for studying short-range correlations in nuclei, and the properties of nuclear forces at distances of $\sim$ 0.7 F.* In practice the picture may be somewhat blurred by the centre-of-mass motion of the pair as well as the interactions of nucleons in the final state. Actually most of the experiments in this field confirm the ejection from the nucleus of two high energy nucleons back-to-back, leaving the residual nucleus in the ground or low excited state.

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* We note that the healing distance for the two-nucleon wave function in the nucleus is of the order of a Fermi, and the average inter-nucleon distance in the nucleus is about 1.4 F.

Of course, there remains the possibility that the pion capture may take place on a cluster of more-than two nucleons. In such a case several fragments may emerge. Rabin et.al. (R62a) conclude from the measurements of masses of charged particles in emulsion that π^- stars contain a sizable proportion of heavier-than-proton particles. One can then obtain also information on multi-nucleon correlations in nuclei.

The study of pion capture with ejection of two nucleons can thus yield interesting information on the momentum distribution of nucleons, their correlations, the validity of shell model wavefunctions, as well as on the properties of two-hole states in nuclei that may not be easily accessible otherwise. The study of reactions like (p, 3N) can also yield similar information, but here the analysis of observations is very complicated due to the presence of three nucleons in the final state. Other reactions which need to be studied to supplement the information from π^- capture may be (p, t), (p, He^3), (p, pd) etc.

The simplest nuclear systems with which one can study pion interactions are the proton and the deuteron. The earliest works in this field were concerned mainly with elucidating the properties of the pion and the nature of the pion-nucleon interaction. Pion capture in hydrogen and deuterium showed the boson character of pion, the existence of π^0 and its decay scheme $\pi^0 \rightarrow 2\gamma$, the odd parity and zero spin etc. The pioneering work was by Bruecker, Serber and Watson (B51a) who analysed the experimental data of Panofsky et. al. (P51a). Most of the arguments here related the known rates of pion production in processes like $p + \gamma \rightarrow n + \pi^+$ and $p + p \rightarrow d + \pi^+$ to $\pi^- + d \rightarrow n + n$ and $\pi^- + d \rightarrow 2n + \gamma$ using time reversal symmetry, conservation of isotopic spin and specific assumptions regarding the nature of the pion. Later calculations of Geffen (G55a), Woodruff (W60a), Eckstein (E63a), Koltun and Reitan (K66a) etc. deal with the problem of pion production near threshold in p-p collisions and the inverse process of pion absorption in deuteron. But the main emphasis is again on the nature of the pion-nucleon interaction, rescattering effects etc. In the next chapter we shall discuss these aspects in more detail.

Studies of pion capture in more complex nuclei naturally devote more attention to extracting information on nuclear structure, and particularly nuclear correlations, This idea was also explored by Brueckner et.al. (B51b) who however did not consider in any detail the mechanism of pion absorption or nuclear forces but simply tried to relate the crosssection for star formation,

(π^- + A → star) to the crosssection (π^- + d → 2n). The fundamental concept that the pion capture primarily involves a pair of strongly scattering nucleons was clearly formulated.

The nuclear systems next in complexity to the deuteron are ^{3}He and ^{4}He. Messiah (M52a) considered the absorption of π^- on ^{3}He using a wavefunction for ^{3}He obtained from a simple variational calculation. At that time even the knowledge of nuclear forces was very uncertain, and this may have resulted in large uncertainties in estimates of high momentum components for nucleon motion. Since Tamor (T51a) had earlier used the simple ps-pv theory of pion-nucleon interaction to obtain reasonably good results for Panofsky ratio in deuteron, Messiah's interest was also to check if the same interaction would give good results for ^{3}He. His results were undoubtedly crude and should only be considered as order of magnitude results. A really extensive calculation, elaborating the ideas of Brueckner et. al. was carried out by Diwakaran (D65a) for ^{3}He and Eckstein (E63a) for ^{4}He. Their calculations are very similar, and we shall describe in some detail the work of Diwakaran only. It is argued that since the capture takes place essentially on a closely correlated (spatially) pair of nucleons, the separation of the capturing pair is very small compared to the average inter nucleon distance, and the remaining nucleons in the nucleus may be regarded as inert spectators influencing only the long-range part of the wavefunction of the pair, through the average nuclear potential. Also, since the kinetic energies of the pair are quite large, one may in a first approximation regard them as practically free nucleons. In that case, the amplitude for the capture process in nuclei can be directly related to that for the basic reactions for free nucleons

$$n + p \rightarrow \pi^{-} + p + p \; ; \quad p + p \rightarrow \pi^{+} + n + p$$

the latter reaction being identical with

$$n + n \rightarrow \pi^{-} + n + p$$

by virtue of iso-spin symmetry of strong interactions. Thus all explicit details of short-range nucleon-nucleon forces and correlations in nuclei (and all corresponding uncertainties) may be avoided, and only the amplitudes or interaction matrix elements for pion production in free nucleon collisions enter the calculations. A further, and rather strong, assumption made by Diwakaran and Eckstein is that of considering the capture only on a pair of nucleons in close contact, i.e. the matrix element for the capture process projects out only the components in spatial wavefunction of the pair of nucleons which are of the form $\delta(\vec{r}_1 - \vec{r}_2)$.

Although the above model appears to be quite elegant, it is not quite clear how far the results would be quantitatively affected by the assumptions of zero-range absorption, and of obtaining the effects of short-range correlations entirely from the observed pion production data. In the first place the nucleon-nucleon correlation distance for absorption is about 0.7F which is small compared to the average internucleon distance of about 2F in He nuclei, but is by no means negligible. The assumption of the δ-function for the relative motion of the pair may be rather drastic and may affect the determination of the coupling constants (g_0 and g_1 in Eckstein - Diwakaran notation) of the production matrix elements from observed data. Since the coupling constants are also assumed to be independent of the relative momentum of the absorbing pair, and are determined from observations on free nucleons and pions of positive finite energy, the errors involved in extrapolation to bound pions and nucleons is not clear. Considerable uncertainty appears to prevail in the literature on this point.

Other assumptions made in Diwakaran's calculations, and by almost all other workers are to neglect capture by three- or- more nucleon clusters, and to consider capture only from the 1S orbit of the pionic atom. The first assumption should be quite valid for the simple system ^{3}He and ^{6}Li considered in our work. As regards the second assumption, Diwakaran shows by an explicit calculation that the capture rate from 2P orbit is only about 4 % of the radiative transition rate from 2P → 1S orbit. Since in Diwakaran - Eckstein model, all short-range correlations are already included in the pion production matrix elements, they assume for the ground-state wavefunctions of ^{3}He and ^{4}He smooth, gaussian wavefunctions fitted to give correct r.m.s. radius. We note once again that in this approach, it is impossible to get any direct information on short-range correlations or nuclear forces.

The calculations of Diwakaran have been recently revised by Figureau and Ericson (F69a), who use essentially the same model. They, however, use revised values for the coupling constants g_o and g_1 obtained with more recent data. Their values for $|g_o|^2$ and $|g_1|^2$ are 0.64 and 0.155 F^8 compared to corresponding Eckstein's values 0.32 and 0.29 F^8. Figureau and Ericson also explore the effects of several different kinds of wavefunctions for ^{3}He viz., gaussian, Irving - Gunn type, modified Irving-Gunn type etc. All these wavefunctions are chosen variationally to fit as best as possible the low energy parameters of ^3He, yet they give considerably different results for the capture rates.

They conclude that the capture rates are strongly affected by the long-range behaviour of the ^3He wavefunctions. It should also be noted that the radiative capture rate (for the reaction $^3He + \pi^- \rightarrow {}^3H + \gamma$) calculated by Figureau and Ericson differs from that obtained by Diwakaran by a factor of four! We shall discuss in detail in a later chapter the results of Diwakaran, and Figureau and Ericson.

Another approach to the problem of π^- absorption in nuclei is exemplified in the work of Cheon who again considers the absorption in ^3He. He also considers the absorption on a correlated pair of nucleons from the 1S orbit of the pionic atom. For the ^3He wavefunction, the space-part is chosen to be fully symmetric and of gaussian type which fits the r.m.s. radius correctly. However, since such simple shell-model type functions contain no correlations, Cheon uses a Jastrow - ansatz to introduce the required spatial correlations in an ad-hoc manner. Thus the gaussian wavefunction is now modified to the form

$$\Psi(\underset{\sim}{r}_1, \underset{\sim}{r}_2, \underset{\sim}{r}_3) = \prod_{i<j} \exp(-\alpha r_{ij}^2)\{1 - \exp(-\beta_0 r_{ij}^2)\}$$

The parameter βo is a measure of the nuclear correlation, $\beta_o = \infty$ indicating no correlation whatever.
Cheon calculated the crosssections for the processes

$$\pi^- + {}^3He \rightarrow p + n + n, \text{ and } \pi^- + {}^3He \rightarrow d + n,$$

called the proton-mode and the deuteron-mode respectively. For the proton-mode capture he also takes into account the final-state-interactions among the three ejected nucleons. The pion-nucleon interaction used by Cheon is the simple primary non-relativistic pseudo-scalar interaction made Galilean invariant by adding a nucleon momentum term, i.e.

$$H_{N\pi} = G\left\{(\underset{\sim}{\sigma}\cdot\underset{\sim}{\nabla}_\pi)(\underline{\tau}\cdot\underline{\varphi}) - \frac{m}{M}(\underline{\tau}\cdot\underline{\varphi})(\underset{\sim}{\sigma}\cdot\underset{\sim}{\nabla}_N)\right\}$$

where $\underline{\tau}$, $\underset{\sim}{\sigma}$ are the iso-spin and spin operators for the nucleon, $\underset{\sim}{\nabla}_\pi$, $\underset{\sim}{\nabla}_N$ operate on the pion and nucleon wave-functions only, and $\underline{\varphi}$ is the pion wavefunction (a vector in I-spin space). The coupling constant G has the standard value $G^2/4\pi = (mc^5/\hbar)\ f^2$ with m = pion mass, and $f^2 = 0.08$.

An interesting result of Cheon (C66a) is that this pseudoscalar interaction with zero-range absorption approximation, and gaussian wavefunctions <u>without</u> correlations ($\beta = \infty$) gives a much smaller capture rate (twenty five time smaller for deuteron-mode) than given by the calculations of Diwakaran. This shows clearly that Diwakaran's matrix elements do contain implicitely sizable correlations. We would normally expect that introduction of nuclear correlations would increase the high relative momentum components, and thus would increase the capture rates. However, curiously enough, the results for capture rates quoted by Cheon as a function of the parameter β_o appear to show that as β_o decreases from infinity, the capture rates decrease. Cheon does not compare his results directly with experimental data, and we shall discuss them in detail in relation to our results in a later section.

Finally, we should mention an interesting calculation by Sakamoto and Tohsaki (S67b) on slow π^+ capture on ^3He so that the final state consists of three protons and is thus simpler to handle and analyse. Only pions of 6MeV kinetic energy are considered. Thus the pion momentum is again rather small, and the capture rate probes the short-range correlations of nucleons. Sakamoto and Tohsaki use the ps-pv interaction with correlated wavefunctions just as done by Cheon, i.e. the correlation is brought in by the ad-hoc Jastrow ansatz through a parameter β_o. They however note that Cheon's wavefunction for ^3He is fitted to give a rather small r.m.s. radius, and thus some of his results may be doubtful. The total capture rate obtained by these authors increases as the parameter β_o decreases from $\beta_o = \infty$ (i.e. as correlation increases); this is in contrast to the result of Cheon, and is as one would expect to happen. In absence of any experimental results to compare with, Sakamoto and Tohsaki also calculate the same crosssections using Diwakaran-Eckstein approach with uncorrelated wavefunctions. With the same values for strengths g_o and g_1 of the absorption matrix elements as taken by Eckstein and Diwakaran, they obtain a total capture rate which corresponds to $\beta_o = 1.39\ F^{-2}$. With the newer values of g_o and g_1 (F69a) β_o turns out to be about 1.2 F^{-2}. This value of the correlation parameter appears to be quite satisfactory with a soft-core nucleon-nucleon potential that would be implied by the choice of the wavefunction.

We shall only briefly mention the present status of π^- capture on ^4He. The experimental data is rather scanty and contradictory, the principle controversy being about the rate for triton mode, $\pi^- + {}^4He \rightarrow {}^3H + n$. Ammiraju and Lederman (A56a) find the relative probability of capture in triton mode to be less than 1/60, whereas Schiff et.al. (S61a) find it to be about 1/3. Early theoretical calculations of Clark (C51a) and of Petschek (P53a) were based on variational wavefunctions (as was also done by Messiah for ^3He) and use direct non-relativistic pseudoscalar interaction. They obtain 3 % and 22 % respectively for the relative probability of capture in triton mode. Later work of Eckstein (E63a) (which we have already described in connection with Diwakaran's work on ^3He) shows that the capture rate depends critically on the relative phase of the strength parameters of the matrix elements g_o and g_1, and the positive sign which seems to be favoured gives results in accord with the observations of Schiff et.al. Finally, a recent calculation by Koltun and Reitan (K68a) uses wavefunctions obtained with Hamada-Johnston potential, thus containing correlations, and the ps-pv interaction modified to include effects of rescattering of pions on nucleons. These authors are primarily interested in establishing the role of the rescattering (or pre-scattering) terms in the pion-nucleon interaction, and calculate only the rates for ejection of two fast moving nucleons (nn or np) (rather than triton mode which got more attention previously) and the momentum-distributions as well as the angular distributions of the ejected pair. The main substance of their results is to show that the additional terms in the pion-nucleon interaction affect strongly the momentum - and angular-distributions of the ejected n-n pair.

There have also been a number of calculations in the literature on capture of pions on nuclei in p-shell. The most interesting one is of course ^{6}Li which is a close analogue of deuteron. It is known now that ^{6}Li is somewhat anomalous in the p-shell. It appears to be a rather open structure, and apart from the standard shell model, it has also been treated as an α-d structure or as a three-body problem $\alpha + n + p$. In view of the very high stability of the α-particle, and the rather loose structure of ^{6}Li, it is possible to treat α-particle as a structureless unit and to neglect the effects of anti-symmetrising the nucleons constituting the α-particle (s-shell-nucleons) with the p-shell neutron and proton. In all calculations of pion capture, the α-particle is assumed to be an inert spectator, and the pion capture is assumed to take place on the p-shell proton or the neutron, according as the pion is negatively or positively charged. If the pion were to be captured on a nucleon-pair in the α-particle or by a pair consisting of a nucleon in s-shell and a nucleon in p-shell, the emerging α-particle would not be in the ground state, and the kinematics of the experimental set up would distinguish such captures.

Indeed the experiments of Davis et.al. (D66a) show that only about 40 % of the capture in ^{6}Li leads to ejection of α-particles in ground state. Almost all theoretical calculations reported to-date, except Alberi and Tafera's work, (A68b) deal only with this mode of capture. Alberi and Tafera do consider pion capture on the quasi - α of ^{6}Li in a specific model. As is the case for capture in ^{4}He nuclei, here also the capture of pion is assumed to take place from the 1S pionic orbit. The available experimental data at present is that of Davis et.al. (D66a) and of Nordberg et.al. (N68a). By time of flight technique they have actually measured the kinetic energies of the neutrons which are emitted at an angle 180- θ, where θ is ranging from 0^{o} to 120^{o} in the Ref. (N68a). The distribution of the energies and the total momentum of neutrons shows that most of the neutrons are emitted with a total energy of ~ 140 MeV and total momentum $| \underset{\sim}{p}_1 + \underset{\sim}{p}_2 | = 0$, i.e. they preferentially share the energy equally, and are ejected with θ = o, when the capture results in α-particle in its ground state. No theoretical studies are made for cases when the final state contains α-particle in an excited state or even gets fragmented*. The

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* Alberi and Tafera (A68b) compare their results of two deuterons comming out in this reaction with unpublished data of Cernigoi et.al.

distribution of total momentum for the total energy in the range 115 MeV to 155 MeV can be easily fitted in terms of a simple α-d model for ^{6}Li.

Sakamoto (S65a) calculated the energy spectrum of the recoiling α-particle using the α-deuteron model for ^{6}Li, and the parameters of Tang and Wildermuth (T61a) for the initial wavefunction. For the π-nucleon interaction he has used the Eckstein-Diwakaran form discussed earlier, and also neglects any final state interactions, presumably since the ejected nucleons have large energies. Davis et.al.'s result is very well fitted by Sakamoto's expression with a mean squared value of about 45 MeV for the average momentum of α-particle relative to the c.m. of the p-shell nucleons. This value implies a separation of about 3.5 - 4.0 F between the α-particle and the p-shell nucleon pair, and is in good agreement with the result of the analysis of Jackson (J60a) of the data of electron scattering on ^{6}Li.

In a further paper Kopaleishvilli and Machabeli (K67a) calculate the same process using for the pion-nucleon interaction the standard non-relativistic pseuoscalar-psuedovector interaction, with the strength parameters as given by Geffen (G55a). For the ground state of ^{6}Li they use the shell model wavefunction (with the p-shell nucleons described by a superposition of $^3S_1$, $^3D_1$ and $^1P_1$ states), the α-deuteron model as well as the three-body model. In the shell model wavefunction there are no explicit short-range correlations, and only the long-range residual interaction is considered in order to obtain the superposition of $^3S_1$, $^3D_1$ and $^1P_1$. In the α-d model also the nucleon-nucleon correlations are only brought in via the quasi-deuteron wavefunction, and depend on a proper choice of the parameter describing this deuteron wavefunction. In the three-body model Kopaleishvilli and Machabeli introduce correlations through an empirical Jastrow-function $\exp(-\nu r^2/2)$ with a suitable value of ν. Thus explicit nucleon-nucleon forces and their short-range character is not considered. Again in the final state wavefunctions, final-state-interactions are not taken into account. Kopaleishvilli and Machabeli find that in the shell-model or three-body model the contribution of the $^1P_1$ state of p-shell nucleons to capture rate is zero. This is what one would expect from selection rules, since the final states can only be $^1S_0$, $^1D_2$ or $^3P_1$, all forbidden from conservation of total angular momentum, and required change of parity. They further find that the contribution of the D-state to the capture rate is barely 7 % . This result is in sharp contradiction to that of Koltun and Reitan (K67a) as we shall discuss below. The authors conclude from the comparison of the results of these three models with experimental data of Davis et.al. that the α-d model gives best results. It is not clear how the parameter ν describing nucleon-nucleon correlations was chosen, nor is it clear how change of this parameter would affect the results.

In a recent paper Alberi and Taffara (A68b) have also calculated π^- capture in ^{6}Li using a simple alpha-deuteron model. They calculate the matrix element in Born approximation using the simplest non-relativistic Feynman diagrams describing the process. The matrix element for $^6\text{Li} \rightarrow \alpha + d$ becomes essentially the form factor for the relative motion of α and deuteron, and the matrix elements for $\pi^- + d \rightarrow 2n$ and $\pi^- + \alpha \rightarrow d + 2n$ are related directly to the elementary capture rates observed for these reactions. Thus both α and deuteron are regarded as elementary structureless particles. This approach is again very similar to that of Brueckner et.al. (B51b), and within the model one can obtain no information on nuclear correlations or pion-nucleon interactions. Since the model only involves simple kinematics and phase-space considerations, it is not surprising that the final results for angular and momentum distribution of two nucleons are in general accord with experimental results. These authors have however additionally also calculated the relative crosssection for $\pi^- + {}^6Li \rightarrow n+n+d+d$ as arising from capture of π^- on α-particle in ^6Li ($\pi^- + \alpha \rightarrow n + n + d$) and obtain good results for the total kinetic energy of two nucleons emitted in this process, as well as the crosssection relative to that for emission of α-particle in ground state.

Koltun and Reitan (K67a) have also calculated π^- capture in ^6Li from a somewhat different viewpoint. As we mentioned earlier in discussion of their work on ^4He, these authors emphasize the need for including in the pion-nucleon interaction not only the usual ps-pv term, but also additional terms (as originally suggested by Woodruff) describing the s-wave scattering of a pion from one nucleon before it is absorbed by the other. The argument is that the presence of such terms is absolutely essential in the understanding of the reactions $p+p \rightarrow d + \pi^+$, $p+p \rightarrow p+p+\pi^0$ etc. They further introduce nucleon-nucleon correlations explicitely in the initial as well as the final wavefunctions by taking into account the Hamada-Johnston potential. The initial state of p-shell nucleons then contains not only the short-range correlations, but also an admixture of $^3S_1$ and $^3D_1$ states. They show that both these factors play a very important role in yielding the final results. In direct contrast to the results of Kopaleishvilli and Machabelli the contribution of the initial $^3D_1$ state of two nucleons to the capture rate is by no means negligible, and actually is much larger than that of the $^3S_1$ state for large relative momenta of two nucleons i.e. small momentum of the α-particle. Since the contribution of $^3D_1 \rightarrow {}^3P_1$ amplitude has opposite sign to that of the $^3S_1 \rightarrow {}^3P_1$ amplitude, the effect of including $^3D_1$ is really severe. The contributions of the rescattering terms in the interaction are also seen to be very important for large relative momentum $\underset{\sim}{k}$ of the two nucleons. Again these factors are more important in OS state of relative motion of the two nucleons than in the 1S state. However, a closer look at their table I appears to show that neglecting both the $^3D_1$ state as well as the rescattering terms of the Hamiltonian may not be too bad an approximation. It is possible that the tensor force in Hamada-Johnston potential gives a much larger D-state probability (7 %) than really required for nuclear physics, and its effects are overestimated. We shall further discuss all these results on ^{6}Li in a later section in relation to our own results.

There are no detailed calculations available of the capture of pions in ^{7}Li. The experiments of Davis et.al. showed that whereas for ^{6}Li the distribution of total momentum of the ejected neutron pair peaks at zero value, the corresponding curve for ^{7}Li shows a dip for $|\underset{\sim}{p}_1 + \underset{\sim}{p}_2| = 0$ and a peak at about 50 MeV. Pellegrini (P68a) has shown how this behaviour can follow from very simple arguments based on assuming shell model wavefunctions $(p_{3/2})^2$ and $(p_{3/2})^3$ for the ground states of ^{6}Li and ^{7}Li. For ^{6}Li the capturing pair is primarily in $^3S_1$ state, so that the wavefunction describing c.m. motion of the pair also has orbital angular momentum $\mathcal{L} = 0$, explaining the peak at zero total momentum. On the other hand an analysis of the ^{7}Li wavefunction (in terms of coefficients of fractional parantages), shows a dominant amplitude in $^3D_1$ and $^3D_2$ states. Further assuming the relative motion of the pair to be largely in $\ell = 0$ state, the c.m. motion of the pair would now be described by an $\mathcal{L} = 2$ state, thus yielding the typical dip at zero total momentum, and a peak at higher values.

A number of experiments have been performed on π^- capture in nuclei heavier than Li, the most common targets being C, O, Al, Cu, Pb etc. Much of the early work was done using emulsions or cloud-chambers, and the data was not really good enough to draw definite conclusions. The first definitive work on pion capture in complex nuclei appears to be that of Ozaki et.al. (O60a) who studied capture on C and Al targets, using scintillation counters to detect neutrons and protons. Later and more extensive measurements using a similar apparatus are reported by Nordberg et. al. (N68a) for Li, Be, B, C, N, O, Al, Cu and Pb targets. An outstanding result of all these measurements is that the relative angular distributions of the emerging nucleons peak very strongly at 180°, and fall off sharply as θ increases from 0°. The two-nucleon capture mechanism postulated by Brueckner, Serber and Watson received a very strong confirmation from this feature. The width of the peak is a measure of the internal motion of the c.m. of the two nucleons in nuclei. Nordberg et.al. point out a rather curious, and as yet unexplained feature that the width of p-n peak is broader than the n-n peak, and appears to be so in almost all cases studied. They conclude that about 40 % of the stopped π^{-} absorption events lead to a 180° - correlated pair of nucleons. However in their experimental arrangement it is not possible to obtain a measure of the single or multiple nucleon ejection mechanism.

Another important aspect of the experimental data that has received considerable attention in the literature is the ratio R of the ejected n-n to n-p pairs of nucleons. We note that capture on an n-p pair (p-p pair) will lead to emission of an n-n (n-p) pair. The measure of this ratio depends upon whether one takes into account all the emitted pairs within the narrow cone of angular resolution at 180^o or only the pairs emitted with 180^o correlation. For p-shell nuclei this ratio is $R = 3.3 \pm 0.9$ for all pairs and $R = 5.1 \pm 1.6$ for pairs with 180^o correlation.

Ozaki et.al. obtain somewhat higher value of $R = 5.0 \pm 1.5$ in C, counting all emitted pairs. (They report that all the pairs are correlated at 180^o but actually their angular resolution is not very good). For heavier nuclei, the ratio R appears to be about the same but the statistical accuracy is really quite poor.

A large number of theoretical calculations have been made to explain the value of the ratio R. A simple counting of p-p and n-p pairs in nuclei with consideration of Pauli principle does give a ratio $R = 3$! But such a simple-minded argument does not take into account several complicating factors, most important being nucleon-nucleon correlations. Eisenberg and Le-Tourneux (E67a) report that the short-range correlations are relatively unimportant, whereas the final state interactions greatly reduce the value of R to less than 1. But estimates of the importance of final-state-interaction vary. There also exist in the literature confusing and conflicting estimates of the relative importance of capture on nucleons in relative s-state and relative p-state. The picture becomes even more blurred when one asks as to from which atomic state (S or P) the pion is captured. Although the S-wave pion capture does give a somewhat reasonable value for R, the experimental facts (C55b, S57a) [The ratio

$W(2P - \pi$ capture$)/W(2P \rightarrow 1S$ ratiative transition) is about 15 to 25]

suggest that 2P pion capture is actually predominent, and this gives a value of R less than 1 (E67a). This result is contradictary to that of Cheon (C68a). To add to all this uncertainty, Koltun and Reitan claim that if the charge-exchange-rescattering terms in their pion-nucleon interaction are taken into account, one can suitably raise the value of R.

In the present work, we consider only π^- capture from atomic orbits in simple nuclei like ^{6}Li and ^{3}He which can be considered as three-body nuclear systems. When this work was begun, two different calculational methods were reported in the literature. One approach was that of Eckstein (E63a), Diwakaran(D65a) and Sakamoto (S65a) (for ^{4}He, ^{3}He and ^{6}Li respectively) in which no explicit correlations are introduced into nuclear wavefunctions. The short-range correlations necessary for pion capture by a nucleon pair are included in the matrix element for pion interaction with a nucleon pair (the interaction taking the form of a five-point vertex). The form of the matrix elements is chosen phenomenologically to satisfy the various conservation conditions, and the strengths were obtained from analysis of the observed inverse reactions $p+p \rightarrow d + \pi^+$, etc. As we discussed earlier, this approach is limited to the kinematical situation where the c.m.of the absorbing pair is at rest, and the absorption amplitude dependence on c.m. energy is unknown. The other limitation is that the absorbing pair is assumed to be always in the relative s-state (because of delta function of $\vec{r}_1 - \vec{r}_2$). Apart from these limitations, this method gives no direct information whatsoever on the short-range nuclear forces and correlations produced by them. The other approach commonly used was to assume simple ps-pv form for pion-nucleon interaction, but to introduce explicit correlations in nuclear wavefunctions in an ad-hoc manner through the Jastrow-type correlation function being multiplied into the independent-particle wavefunction for the absorbing pair. This method contains an arbitrary parameter, the correlation distance, which is adjusted to give the correct capture rates at the end. Here again the origin of the correlations viz, nuclear forces, does not enter the model explicitely, and nothing can be learnt about these.

Our interest is primarily in studying the nuclear correlations as produced by the nuclear forces as exactly as possible via their effect on pion capture rates. For this purpose we would like to use 'exact' nuclear wavefunctions. As is wellknown, exact solution of the nuclear many-body problem with realistic interaction is practically impossible. However, in the past few years considerable advance has been made in obtaining 'exact' solutions of three-body systems with realistic interactions using methods developed by Faddeev and collaborators. We use here an equivalent but somewhat simpler approach developed by Mitra and coworkers, based on choosing for nucleon-nucleon interactions a non-local separable potential consisting of several terms. Such potentials were first used by Yamaguchi for nuclear physics problems. With the use of Mitra's techniques we obtain 'exact' wavefunctions for ^{6}Li and ^{3}He which now directly reflect the nature of the nuclear forces. The relevant parameters for nucleon-nucleon as well as α-nucleon potentials are already obtained by Mitra and coworkers. This use of 'exact' wavefunctions is a distinctive feature of our work. (It should be mentioned that after our work was begun, calculations have been reported by Koltun and Reitan as well as by Cheon in which the effects of realistic nucleon-nucleon forces (such as Hamada-Johnston potential) are directly incorporated into the wavefunctions. We shall be able to compare their results with ours in later chapters.) Our approach also enables us to incorporate in the calculations the effects of final-state-interactions in a straightforward manner.

Another aspect in which our calculations differ from others reported in the literature is that the wavefunctions obtained here are in momentum-representation and hence all our calculations are done in momentum-space. In other calculations the wavefunctions are generally in coordinate representation and hence the calculations involve solving Schrodinger equations in coordinate space for obtaining wavefunctions and then evaluation of radial integrals for calculating capture rates. Thus it is not very easy for us to compare our results step-by-step with those of others. In later chapters we shall emphasize more the differences in our calculations from those of others.

In the next chapter we discuss briefly the pion-nucleon interaction, We choose the simple ps-pv form without inclusion of rescattering terms. We make a few comments on the Eckstein-Diwakaran form of pion-nucleon interaction, as well as on the results of Koltun and Reitan who strongly emphasize the role of rescattering terms. The nature of the nucleon-nucleon non-local separable potentials used in subsequent work is also described. As a simple preliminary application of non-local separable (NLS) potentials, we present a calculation of pion capture rate in deuterium.

In the third chapter we describe the derivation of the detailed nature of the ground state wavefunction for ^{6}Li. For this derivation, the nature of the non-local separable α-N interaction is also specified. The Schrodinger equation for the three-body (α-N-N) problem is solved for different forms of the non-local potentials described in chapter 2. Binding energies for ground state as well as for the exicted 0^+ state (analogue of ^{6}He ground state) are also calculated. Our results are compared with those of Hebach et.al. (H67b) and of Allessandrini et.al. (A69a). We discuss in detail the structure of the wave function for the ground state with special emphasis on the D-state probability, and the interpretation of the different terms in the wavefunction. The first order correction to the binding energy due to the spin-orbit term in α-N interaction is calculated perturbatively.

In the fourth chapter we present a calculation of the total capture rate, momentum spectrum as well as the angular distributions for pion capture on the ground state of ^{6}Li. The effect of S - D cancellation on the capture rate which is so prominent in deuteron is studied here also explicitely. We also emphasize and clarify the role of α-proton correlation terms in the wavefunction, and similar effects arising from the CM momentum term in the pion-nucleon interaction. We compare the results obtained here with the available experimental data and those of other authors.

The fifth chapter contains a calculation of the rates for various capture modes for pion capture in ^{3}He. The calculation makes full use of the permutation symmetries of the three-body system. The pion-nucleon interaction is also expressed directly in terms of three-body operators. The wavefunction of ^{3}He used for this calculation is taken from the work of Bhakar (B65b). It may be noted that this wavefunction contains the effects of N-N tensor force, so that S + D state cancellation effects are actually included in our calculations. This is not done so far by other authors.

In the last chapter we conclude with a summary of our present knowledge of nucleon-nucleon correlations gained from pion capture studies. It is emphasized that wavefunctions obtained with non-local separable potentials do contain a rather large degree of nucleon-nucleon correlations, although they do not include any explicit hard-core.

2. PION NUCLEON INTERACTION AND TWO NUCLEON SYSTEM

2.A Pion-Nucleon Interaction

All calculations for pion interactions with nuclei must start with some knowledge of basic pion-nucleon interactions. Such information is most directly available from elastic and charge-exchange scattering of pions by nucleons, radiative capture of pions (e.g. $\pi^- + p \rightarrow n + \gamma$) or inversly photoproduction of pions etc. Pion-proton interaction can be studied directly experimentally, whereas information on pion-neutron interaction has to be derived indirectly by relying on theoretical concepts such as charge-independence, and by studying experimentally pion interactions with deuterons and then extracting out pion-neutron interaction, usually in the framework of impulse approximation. Pion-nucleon interaction is known to be strongest in P-wave, and hence was most explored in early investigations. The S-wave interaction is relatively weak, although it is of more importance for study of pion capture in nuclei, pion-production near threshold etc., where only very low energy pions are involved. Early studies of S-wave π-N interactions are due to Klein (K55a) and Drell, Friedman and Zachariasen (D56a) among others.

The basic π-N interaction is given by the ps-pv (or ps-ps which yields the same result in non-relativistic limit to first order) meson theory. The relativistic form of the interaction is

$$H_{\pi N} = i\sqrt{4\pi}\,\frac{f}{m}\,\bar{\psi}\gamma^5\gamma^\mu\psi P_\mu(\underline{\tau}\cdot\underline{\varphi}) \qquad (2.1)$$

where m is the pion mass, and in units of $\hbar = c = 1$

$$f^2 = 0.08 \qquad (2.2)$$

Here P_μ is the pion four-momentum.

From this one can obtain a non-relativistic static approximation of the following form

$$H_{\pi N} = G\left\{\underset{\sim}{\sigma}\cdot\underset{\sim}{p}_\pi(\underline{\tau}\cdot\underline{\varphi}) - \frac{m}{M}(\underline{\tau}\cdot\underline{\varphi})\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p}_N\right\} \qquad (2.3)$$

where

$$G = \sqrt{4\pi}\,f/\sqrt{2m^3} = 1.188\,F^{-3/2} \; * \qquad (2.4)$$

Here $\underset{\sim}{\sigma}$, $\underline{\tau}$ are the spin and I-spin operators for the nucleon, and $\underset{\sim}{p}_\pi$, $\underset{\sim}{p}_N$ denote the pion and nucleon momenta respectively. M is the mass of the nucleon. The first term in the bracket is the usual P-wave interaction, whereas the second-term (the nucleon recoil term) preserves

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* We note here that our pion wavefunction is in coordinate-space and hence is normalised differently from the usual field-theoretic definitions. Our normalisation differs from standard definitions (Sec K66a) by $(2m)^{-1/2}$.

the galilean invariance and gives the direct S-wave interaction. Thus, for consideration of capture of pions from atomic s-orbits (where pion is practically at rest, $P_\pi \approx 0$), it is conventional to drop the first term in the bracket.

On the other hand, usually the reduction is carried out by Foldy-Dyson transformation applied to the conventional ps-ps theory. It is shown by Barnhill (B69a) that the "galilean invariant" form of this operator is not an unique one. It depends upon the unitary transformation. If in the reduction one keeps terms to second order in the pion wave function it can be shown (K55a, D56a, W60a etc.) that the Hamiltonian contains additional terms to (2.3)) which describe direct and charge-exchange scattering of S-wave pions on nucleons.

These terms are (W60a)

$$H_1 = 4\pi \frac{\mu}{m} \lambda_1 (f/m)^2 \varphi^2$$

$$H_2 = 4\pi \frac{1}{\mu} \lambda_2 (f/m)^2 \underline{\tau} \cdot (\underline{\varphi} \times \underline{\pi}) \tag{2.5}$$

where $\underline{\pi} = \dot{\underline{\varphi}}$ is the canonically conjugate momentum of the pion-field. λ_1 and λ_2 are empirical constants, and are usually obtained from a fit to S-wave phase shifts for pion-nucleon scattering. Recent values are (K66a)

$$\rho_1 = 0.005, \qquad \rho_2 = 0.58 \qquad (2.6)$$

These empirical values are indeed very far from expected values from meson-theoretic derivations, and indicate the importance of charge-exchange scattering. Most of the authors who have studied pion capture in nuclei do not take into account such second-order terms which would describe the scattering of a pion on one nucleon before being absorbed on the second nucleon of the pair. However, the undoubted importance of these terms in processes like $p + p \rightarrow d + \pi^+$ or $p + p \rightarrow p + p + \pi^0$ has been emphasized by Woodruff (W60a) and more recently by Koltun and Reitan (K66a), and Cheon (C68c). It then follows that they may also be of importance in pion capture processes, such as $\pi^- + d \rightarrow n + n$ and in other nuclei. Unfortunately, evaluation of the contribution of these terms requires some assumptions for S-wave pion scattering matrix off energy shell, and various authors get rather conflicting results. We shall summarise in the following paragraphs the experimental and theoretical status of the simplest process viz, $\pi^- + d \rightarrow n+n$, or equivalently the experimentally more amenable processes like $\pi^+ + d \to p + p$ and $p + p \to \pi^+ + d$.

The experimental data on π^- capture in deuterium is still quite inadequate. The capture rate W_d for the process $\pi^- + d \to n + n$ is not yet directly measured. However the ratio

$$R = \frac{W(\pi^- + d \to n + n)}{W(\pi^- + d \to n + n + \gamma)}$$

has been measured by several workers, and the latest results are 3.16 ± 0.10 (R63a) and 2.89 ± 0.09 (K64a).

A straightforward way to obtain W_d is to invoke the principles of charge-independence and detailed balancing.

Then we can write for the crosssections

$$\sigma(\pi^- d \to nn) = \sigma(\pi^+ d \to pp)$$

$$= \frac{2}{3}(p_p / p_\pi)^2 \sigma(pp \to \pi^+ d)$$

$$W_d = v_{\pi d} |\varphi(0)|^2 \sigma(\pi^- d \to nn) \qquad (2.7)$$

$$= \frac{2}{3} |\varphi(0)|^2 \left(1 + \frac{m_\pi}{2m}\right)\left(1 + \frac{m_\pi}{4m}\right) \frac{m}{m_\pi} \left\{ \frac{\sigma(pp \to \pi^+ d)}{\eta} \right\}_{\eta \to 0}$$

The last reaction in the above line has been measured experimentally for various values of pion momenta ($\eta = p_\pi/mc$) in the range $\eta = 0.38$ to $\eta = 0.58$ by Crawford and Stevenson (C55a) and several other workers. For such large pion momenta, the pion is produced in S- as well as P-wave. Since our interest is primarily in S-wave pions, it is necessary to extract out only the S-wave contributions to the crosssection. It is rather difficult to do this from above experiments, since the P-wave production dominates and the S-wave terms in the crosssection are poorly determined. To extract only S-wave contribution, one must extrapolate the observed crosssections to threshold, and this involves considerable uncertainty. Recently however Rose (R67a) has directly measured the crosssection for $\pi^+ + d \rightarrow p + p$ for $\eta = 0.15$ to $\eta = 0.48$. He also applies Coulomb corrections to extract out the pure nuclear part of the crosssections, and is able to separate out in a reliable manner the S-wave and P-wave contributions to the capture crosssection. With the numbers given in his paper, we obtain for the capture rate W_d,

$$W_d = 1.58 \times 10^{15} \sec^{-1} \qquad (2.8)$$

The results of Rose show an enhancement of S-wave crosssections compared to earlier results.

It is also possible to evaluate W_d alternately from the experimentally known crosssections for $\gamma + p \to n + \pi^+$. One can relate this crosssection to $\gamma + n \to \pi^- + p$, according to (R67a)

$$\frac{\sigma(\gamma n \to \pi^- p)}{\sigma(\gamma p \to \pi^+ n)} = \frac{\sigma(\gamma d \to \pi^- pp)}{\sigma(\gamma d \to \pi^+ nn)} = 1{\cdot}23 \pm 0{\cdot}06 \qquad (2.9)$$

The inverse reaction $\pi^- + p \to \gamma + n$, obtained from detailed balancing arguments can next be related to $\pi^- + d \to 2n + \gamma$, and then using the known ratio R, one can obtain W_d. (For a detailed discussion see Rose (R67a)). In this manner one again obtains

$$W_d = 1.57 \times 10^{15} \text{ sec}^{-1}. \qquad (2.8a)$$

Theoretical investigations on $p + p \to d + \pi^+$ were initiated by Brueckner, Serber and Watson (B51a) who however did not concern themselves very much with the details of the interaction. The process can occur from initial diproton states ^1S_o, $^1D_2$ resulting in π^+ in P-state and from $^3P_1$ resulting in an S-wave pion, near threshold. Since we are concerned with the latter, we shall in this brief survey consider calculations of only the amplitude for pion production where the initial state $^3P_1$, goes into final state $^3S_1 + {}^3D_1$. In one of the early calculations Geffen (G55a) used the πN interaction with only linear terms in φ, but treated the P-wave and S-wave coupling constants as free parameters. However, he evaluates the S-wave pion production amplitude very crudely, omitting the contributions from the final $^3D_1$ state of the deuteron. For a reasonable fit to experimental data Geffen requires the strengths of the coupling constants to be about twice the meson-theoretic values, but the ratio of S-wave to P-wave coupling constant is roughly m/M as would be expected. At very low energies ($E_\pi \sim 10$ MeV) the fit to data requires the above ratio to be only 0.054 which is very small compared to m/M. Although Geffen emphasizes the role of the hard-core in NN interaction to obtain the initial diproton state wavefunction, his results are not conclusive, particularly since he omits the $^3D_1$ deuteron component, which as we shall see below plays a very crucial role.

A subsequent calculation of Woodruff (W60a) illustrates clearly almost all the difficulties of the calculations. He uses the Gammel-Thaler potential for the diproton system, and the Gartenhaus wavefunction for the deuteron, (D-state probability $P_D \approx 6.8$ %).

Rescattering of the pion in both S- and P-waves is taken into account, by the use of second order terms in the interaction. Woodruff shows that the D-state of the deuteron plays a very important role. The amplitudes for transitions $^3P_1 \rightarrow {}^3S_1$ and $^3P_1 \rightarrow {}^3D_1$ substantially cancel each other in the absence of rescattering corrections; the values of these amplitudes (C_1 and C_2 in Woodruff's notation) are -.285 and +.415 respectively. Thus the direct S-wave pion production amplitude is very small. He also shows then that the inclusion of rescattering terms (P-wave rescattering gives no contribution here) is necessary to obtain resonably large values for the pion production. Unfortunately, the evaluation of the contributions from the rescattering terms in the interaction depends very critically on the functional form chosen for the S-wave pion scattering matrix off the energy shell in the intermediate states. This correct functional form is unknown in the absence of a suitable meson theory, and introduces large uncertainties in the calculation. Woodruff makes two different approximations (Born approximation and Linearisation approximation) and finds that the total production amplitude from $^3P_1$ state can be as large as 1.18 or 0.51 (compared to 0.13 without rescattering) respectively. The experimental value for this number is 0.74 ± 0.04.

In a recent work Koltun and Reitan (K66a) have revised Woodruff's calculations for S-wave pion production amplitude only. They use the more modern Hamada-Johnston potential to obtain initial state function at threshold energy by numerical integration of the Schrodinger equation. Similarly for the final state (at zero pion energy), both Hamada-Johnston as well as Gartenhaus potential are used to obtain the deuteron wave-function, again by exact numerical integration of the Schrödinger equation, (D-state probability $P_D \sim 7\ \%$). Their results are essentially similar to those of Woodruff, although in their case the cancellation of the direct production amplitudes ${}^3P_1 \rightarrow {}^3S_1$ and ${}^3P_1 \rightarrow {}^3D_1$ is almost complete, the values being -0.080 and $+0.083$ respectively. Then the entire contribution to production comes from rescattering terms. It is indeed very surprising to find the small D-state playing such a strong role! Koltun and Reitan find the crosssection at threshold for the $p + p \rightarrow d + \pi^+$ (s-state)

$$\sigma = 146\,\eta\ \mu b \qquad (2.10)$$

which agrees well with the older value (138 ± 15) η μb (F58a), but is still smaller than the more recent value of Rose, which is

$$\sigma_{expt.} = (240 \pm 20)\eta \ \mu b \quad (2.11)$$

Similarly the capture rates for π^- by deuteron turn out to be $W_d = 0.22 \times 10^{15}$ sec^{-1} if the D-state of the deuteron as well as prescattering corrections are neglected, $W_d \simeq 0$ if only the prescattering corrections are neglected, and $W_d = 0.84 \times 10^{15}$ sec^{-1} for the complete calculation.

The theoretical treatment of $p + p \rightarrow d + \pi^+$ emphasizes (a) the role of $^3D_1$ final state in reducing the crosssection, and (b) the role of the second-order terms in the interaction in increasing the crosssection. The question remains as to the relative importance of these two features in the inverse process viz., $\pi^- + d \rightarrow n + n$. If the pion capture process depends critically on the presence of nucleon-nucleon short-range correlations, one might expect the capture on the $^3D_1$ component of the deuteron state to play a less destructive role. Further, the importance of scattering-before-absorption may also be reduced when the two nucleons are strognly correlated, i.e. in $^3S_1$ state compared to $^3P_1$ initial state in pion production. It would certainly be very desirable to have a direct experimental measurement of $\pi^- + d \rightarrow n + n$ process.

Finally, we draw attention to a calculation of Cheon and Tohsaki (C68a,b) in which they calculate the $\pi^+ + d \rightarrow p + p$ crosssections for pion energy (E_{lab}) of about 5-10 MeV. The form of the interaction is one derived earlier by Cheon (C68c) and includes second-order rescattering terms. At the pion energies considered, the S-wave capture dominates. The strength of the coupling constant for direct S-wave capture is fitted empirically, and turns out to have a value consistent with that obtained by Geffen. An interesting feature of this calculation is that although these authors also use the Hamada-Johnston wavefunction for the deuteron, there is <u>no</u> cancellation of contributions from $^3S_1$ and $^3D_1$ states, the two relative amplitudes $^3S_1 \rightarrow {}^3P_1$ and $^3D_1 \rightarrow {}^3P_1$ being -0.277 and -0.14 only, so that they actually add up. This is completely contrary to the earlier results of Woodruff, and Koltun and Reitan, and also our result to be reported in the next section. It appears to us that there should be some error in Cheon and Tohsaki's treatment of the deuteron. Further, since the contribution of the direct interaction to the crosssection is now quite large, the contribution of the pre-scattering term is relatively smaller, and turns out to be only $\approx$ 15 % compared to nearly 100 % for Koltun and Reitan.

In the subsequent work in this thesis we choose for the πN interaction the usual first order non-relativistic psuedoscalar form, as has been done by almost all theoretical workers. As mentioned earlier, the evaluation of second order terms requires knowledge of off-energy-shell behaviour of the pion-scattering-matrix, and we feel that at the present stage of available experimental data, such refinements are not necessary since the problem is already quite complicated. We shall only consider S-wave pion interaction, since for the light nuclei considered in our work, the capture is primarily from the atomic 1S orbit. For the case of ^3He, Diwakaran has shown that the probability of capture of π^- from 2P orbit is only about 4 % of the probability for radiative transition from 2P → 1S orbit. A similar result is expected to be valid for ^6Li. However, more recently Figureau and Ericson (F69a) show that the absorption rate from 2P state is about 19 % of the 2P → 1S radiative transition rate in ^3He. The relevant πN interaction is now

$$H = -G\frac{m}{m}(\underline{\tau}\cdot\underline{\varphi})\,\underline{\sigma}\cdot\underline{p}_{N} \qquad (2.12)$$

For capture on a pair of nucleons, the interaction becomes,

$$H = -G\frac{m}{m}\left\{(\underline{\tau}_1\cdot\underline{\varphi}_1)\,\underline{\sigma}_1\cdot\underline{p}_1 + (\underline{\tau}_2\cdot\underline{\varphi}_2)\,\underline{\sigma}_2\cdot\underline{p}_2\right\} \qquad (2.13)$$

where $\underline{\varphi}_1$, $\underline{\varphi}_2$ are the pion wavefunctions at the positions of the two nucleons.

We make two straightforward and common approximations as follows:

(a) The nuclear structure is undisturbed by the presence of the pion in S-orbit, so that nuclear wavefunctions can be calculated by standard techniques, and

(b) the motion of π^- in atomic 1S orbit (from which it is assumed to be captured) can be described by simple Bohr theory.

The pion wavefunction in 1S orbit may be written as

$$\varphi(\pi^-) = (Z^3/\pi a^3)^{1/2} \exp(-Zr/a)$$

$$\text{with } a = \hbar^2/me^2 = 200 \text{ F} \qquad (2.14)$$

Since the pion is captured only over the nuclear volume, i.e. in the region where $r \sim 10^{-13}$ cms, it is convenient to approximate the pion wavefunction by $\varphi(r = 0)$. We thus get for the pion wavefunctions in equation (2.13)

$$\varphi_1(\pi^-) = \varphi_2(\pi^-) = (Z^3/\pi a^3)^{1/2} \qquad (2.15)$$

Actually the value of the pion wavefunction over the nucleus can be distorted by (i) the finite charge extension of the nucleus and (ii) the pion-nucleus optical potential in addition to the Coulomb potential. Figureau and Ericson quote unpublished estimates of Krell (K69a) that the first effect reduces the capture rate by 4 %, and the second effect by about 2 % for negative pions. For positive pions the optical potential correction may be almost 30 % .

We can now further reduce the interaction operator in equation (2.13), and recast it in a form which should be more convenient for evaluating matrix elements between two-nucleon states. We define operators,

$$\boldsymbol{P} = (\boldsymbol{p}_1 + \boldsymbol{p}_2) \qquad \boldsymbol{p} = \tfrac{1}{2}(\boldsymbol{p}_1 - \boldsymbol{p}_2)$$

$$\boldsymbol{S} = \tfrac{1}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \qquad \boldsymbol{\sigma} = (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)$$

$$\underline{T} = \tfrac{1}{2}(\underline{\tau}_1 + \underline{\tau}_2) \qquad \underline{\tau} = (\underline{\tau}_1 - \underline{\tau}_2)$$

$$\underline{\Phi} = \tfrac{1}{2}(\underline{\varphi}_1 + \underline{\varphi}_2) \qquad \underline{\varphi} = (\underline{\varphi}_1 - \underline{\varphi}_2)$$

We then obtain

$$(\underline{\tau}_1 \cdot \underline{\varphi}_1)\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{p}_1 + (\underline{\tau}_2 \cdot \underline{\varphi}_2)\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{p}_2$$

$$= \Big\{ (\underline{T} \cdot \underline{\Phi} + \tfrac{1}{4}\, \underline{\tau} \cdot \underline{\varphi})(\boldsymbol{S} \cdot \boldsymbol{P} + \boldsymbol{\sigma} \cdot \boldsymbol{p})$$

$$+ (\underline{T} \cdot \underline{\varphi} + \underline{\tau} \cdot \underline{\Phi})(\boldsymbol{S} \cdot \boldsymbol{p} + \tfrac{1}{4}\, \boldsymbol{\sigma} \cdot \boldsymbol{P}) \Big\} \qquad (2.16)$$

We note that here, for s-wave pion wavefunction assumed to be constant over the nuclear volume, we get

$$\underset{\sim}{\varphi} = 0$$

Then the interaction operator can be written as

$$H = -G\frac{m}{M}\left\{ \underset{\sim}{P}\cdot\left(\underset{\sim}{S}\,\underset{\sim}{T} + \frac{1}{4}\,\underset{\sim}{\sigma}\,\underset{\sim}{\tau}\right)\cdot\underset{\sim}{\Phi} + \underset{\sim}{P}\cdot\left(\underset{\sim}{\sigma}\,\underset{\sim}{T} + \underset{\sim}{S}\,\underset{\sim}{\tau}\right)\cdot\underset{\sim}{\Phi}\right\} \qquad (2.17)$$

From the above form of the operator in two-nucleon coordinates, it is possible to infer the selection rules for change of state of the capturing nucleon pair. The term containing CM momentum $\underset{\sim}{P}$ will give rise to transitions such as

TS = 11 → TS = 11, TS = 11 ↔ TS = 00, and

TS = 10 ↔ TS = 01. The presence of vector $\underset{\sim}{P}$ will change the angular momentum corresponding to the centre-of-mass motion by one unit, but will not change the angular momentum of the relative motion (no parity change); hence this term corresponds to capture of the pion in p-state relative to the centre-of-mass of two nucleons although it is in s-state relative to the centre-of-mass of the nucleus. This aspect has been discussed in detail by Koltun (K67b). Most authors actually drop the $\underset{\sim}{P}$ -dependent term from the calculations. We do not do this;

it will actually turn out in ^{6}Li that the motion of the c.m. of two nucleon, relative to the c.m. of the nucleus, does contribute in a non-neglible way to the pion capture rate.

The term in the interaction which contains relative momentum $\underset{\sim}{p}$ of the nucleon pair is then the main contributor to the capture rate. It enables capture of S-wave pion by producing appropriate parity change. The centre-of-mass motion remains unchanged, whereas the relative angular-momentum will change by one unit. The spin-Ispin quantum numbers change according to TS = 11 $\leftrightarrow$ TS = 10 ; TS = 11 $\leftrightarrow$ TS = 01. To be more specific, it will produce transitions such as

$$^{31}S_0 \rightarrow {}^{33}P_0, \quad {}^{13}S_1 \rightarrow {}^{33}P_1, \quad {}^{31}D_2 \rightarrow {}^{33}D_2, \quad {}^{13}D_1 \rightarrow {}^{33}P_1 \text{ etc.}$$

We shall see explicit examples of such transitions in later chapters.

We have mentioned in the previous chapter the phenomenological two-nucleon operator for pion-capture used by Eckstein, Diwakaran, Sakamoto etc. This operator has the following form

$$\mathcal{M} = \{ \tfrac{1}{2}(\underline{\tau}_1 - \underline{\tau}_2)\cdot\underline{\Phi}\ \tfrac{1}{2}(\underset{\sim}{\sigma}_1 + \underset{\sim}{\sigma}_2)\cdot\underset{\sim}{p}\,(g_0^t T_{12}^{\tau} + g_0^s S_{12}^{\tau})$$

$$+ \tfrac{1}{2}(\underline{\tau}_1 + \underline{\tau}_2)\cdot\underline{\Phi}\ \tfrac{1}{2}(\underset{\sim}{\sigma}_1 - \underset{\sim}{\sigma}_2)\cdot\underset{\sim}{p}\,(g_1^t T_{12}^{\sigma} + g_1^s S_{12}^{\sigma}) \}$$

$$\times\, \delta(\underset{\sim}{r}_1 - \underset{\sim}{r}_2) \qquad (2.18)$$

The operator involves a δ-function of the coordinates of the nucleon pair $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$, so that the capturing pair is essentially in an S state, (ℓ =0), i.e. a spatially symmetric state. The requirement for antisymmetry of the initial state then removes the amplitudes g_0^t and g_1^t. Here $\underline{\Phi}$ is the pion wave-function at the position $\underset{\sim}{r}_1$ (or $\underset{\sim}{r}_2$), and is the same as defined in equation (2.14); $\underset{\sim}{p}$ denotes as previously the relative momentum of two nucleons. T_{12} and S_{12} are the projection operators for triplet and singlet states respectively. It will be seen that apart from the factor $\delta(\underset{\sim}{r}_1 - \underset{\sim}{r}_2)$, this operator has almost the same form as the second ($\underset{\sim}{p}$-dependent) term of the interaction in equation (2.17). The strength constants

g_o and g_1 have been derived by fitting information on the inverse reactions as discussed earlier. This approach would include not only the effects of the nucleon-nucleon correlations, but also those of pion scattering before absorption in the coefficients g_o and g_1. However, the appearance of the δ-function precludes the possibility of including the $^3D_1$ state of the deuteron, and the severe cancellation effects between $^3S_1 \rightarrow {}^3P_1$ and $^3D_1 \rightarrow {}^3P_1$ so forcefully pointed out by Koltun and Reitan. This may give incorrect values for g_1 and g_o. Further, such a determination of g_o, g_1 will not take into account the dependence of the transition amplitude on the c.m. motion of the pair, since in the reactions determining these amplitudes, the c.m. motion is zero.

We note that in the work of Eckstein and Diwakaran, the adopted values of the strengths g_o and g_1 (which correspond to amplitudes for transitions $^{13}S_1 \rightarrow {}^{33}P_1$ and $^{31}S_o \rightarrow {}^{33}P_o$) are approximately equal. In the meson-theoretic interaction given by equation (2.17), the strengths of these two amplitudes are exactly equal, and they have also the same sign, suggesting that the relative phase of g_o and g_1 should also be positive (Re $g_o g_1 > o$).

Diwakaran (D65a) also concludes from the analysis of pion capture on ^3He that Re $g_o g_1 > 0$. This appears to suggest that the meson-theoretic derivation of the interaction should be reliable. However, recent values of g_o and g_1 obtained by Figureau and Ericson (F69a), from an analysis of more recent data of Rose on $\pi^+ + d \rightarrow p + p$, are very different from each other (differ by almost a factor of 2). In this case the direct comparison of the phenomenological Eckstein interaction with meson-theoretic interaction has no value. The different values of g_o, g_1 may reflect the effects of pion rescattering or omission of $^3D_1$ deuteron state, or different degrees of nucleon.nucleon correlation in states of different I-spin.

2.B Nucleon-Nucleon Interactions.

In the treatment of nuclear systems ^6Li and ^3He to be described in later chapters, we choose the non-local separable form for nucleon-nucleon potentials. The reason for this choice essentially is that such potentials are ideally suited for obtaining 'exact' wavefunctions of three-body systems (M68a) as has been shown in a series of papers by Mitra and coworkers. We do not wish to discuss here the relative merits of non-local separable forms over other more conventional forms such as Yale or Hamada-Johnston potentials. Yamaguchi (Y54a) showed that with non-local separable potentials the two-body problem has an explicit algebraic solution (and we shall use this property in the next section for pion capture calculation in deuteron). Further, such potentials can be made realistic enough to give a reasonably good fit to two-nucleon bound-state and scattering data, and can include spin-orbit as well as tensor forces. Mitra and coworkers have given sets of such non-local separable potential parameters (M59a, M61a, N62a). The feature of this form of potential that is most attractive to us is that they can be plugged into the Schrödinger equation for a three-body system, and with a suitable truncation of the number of terms in the potential, one can obtain 'exact' (although numerical) wavefunctions which can be then used for other calculations such as pion capture rates. Such wavefunctions will then contain directly the nucleon-nucleon correlations due to the N-N interactions. (The alternate approach of Fadeev, Lovelace etc. emphasizes formal properties of scattering amplitudes etc. and hence does not seem suitable for our purpose). It should be noted that the truncated form of the potential we shall use below does not contain an explicit repulsive core term, since inclusion of such terms would make the numerical solution of the problem very difficult, involving solution of several coupled integral equations. Nevertheless, it will be seen from the treatment of pion capture in deuteron to be discussed in the next section that a considerable degree of nuclear correlation is indeed contained in the wave-functions.

We shall briefly describe in this section the N-N potential and the values of the parameters adopted for subsequent calculations. Since the potential is a non-local one, the potential term in the Schrodinger equation is of the form

$$V\Psi = \int d^3r'\, v(\underset{\sim}{r}, \underset{\sim}{r}')\, \Psi(\underset{\sim}{r}') = \int d^3r' \langle \underset{\sim}{r} | v | \underset{\sim}{r}' \rangle \Psi(\underset{\sim}{r}') \tag{2.19}$$

Thus the Schrodinger equation in the coordinate space becomes an integro-differential equation. It is much simpler to handle the potential term in momentum-space, since the wave equation then only reduces to an integral equation. The general form of the potential is conventionally written in the separable form as

$$-M \langle \underset{\sim}{p} | v | \underset{\sim}{p}' \rangle = \sum_{\ell} (2\ell+1)\, \lambda_\ell\, g_\ell(p)\, g_\ell(p') \times P_\ell(\hat{p}\cdot\hat{p}') \tag{2.20}$$

where λ_ℓ is the strength constant, $g_\ell(p)$ is a form factor for the ℓ^{th} partial wave, and $P_\ell(\mu)$ is the Legendre polynomial. With this, the Schrodinger equation for relative motion of two nucleons becomes, for example,

$$(p^2 - mE)\psi(\underset{\sim}{p}) = -m\int d^3p' \langle \underset{\sim}{p}|v|\underset{\sim}{p}'\rangle \psi(\underset{\sim}{p}') \tag{2.21}$$

We note that the potential is defined separately in each of the relative angular momentum states, and in practice one confines the sum in equation (2.20) to only a few terms, say $\ell = 0, 1, 2$. The singlet even state potential is only defined for $\ell = 0$ i.e. for s-states. The contribution of the $^1D_2$ state to problems under consideration here is practically nil. We write for $^1S_0$ state potential

$$-m\langle \underset{\sim}{p}|v_S|\underset{\sim}{p}'\rangle = -\lambda_S f(p) f(p') \;, \; f(p) = (\beta_S^2 + p^2)^{-1} \tag{2.22}$$

The values of the parameters β_S, λ_S are listed in table 1, and are chosen to fit the scattering length $a_S = -23.7F$, and effective range $r_S = 2.15$ F; these values are the same as those of Yamaguchi (Y54a), and are also used by Bhakar (B65b) for calculating ^3He

wavefunctions which we shall use later on.

For interactions in triplet even states, a particularly simple for suggested by Yamaguchi is

$$-M\langle \underset{\sim}{p} | v_t | \underset{\sim}{p}' \rangle = \lambda_t \, g(\underset{\sim}{p}) \, g(\underset{\sim}{p}') \qquad (2.23)$$

where

$$g(\underset{\sim}{p}) = C(p) + \frac{1}{\sqrt{8}} S_{12}(\hat{p}) T(p) \qquad (2.24)$$

$$C(p) = (\beta_t^2 + p^2)^{-1} \; ; \; T(p) = -t p^2 (\gamma_t^2 + p^2)^{-2}$$

$$S_{12}(\hat{p}) = 3 \underset{\sim}{\sigma_1} \cdot \hat{p} \, \underset{\sim}{\sigma_2} \cdot \hat{p} - \underset{\sim}{\sigma_1} \cdot \underset{\sim}{\sigma_2} \qquad (2.25)$$

This potential provides a fairly realistic representation of the triplet even interaction, with the values of the parameters listed in table 1. These values are so chosen as to give the deuteron binding energy E_d = 2.22 MeV, the scattering length a_t = 5.378 F, the effective range r_t = 1.716 F and the quadrupole moment Q = 0.285 F^2. The D-state probability P_D with these parameters turns out to be 4 %. It is also

Table-1

Potential	C_{γ}^{eff}	$(C + T)_{\gamma}$	$(C+T)_{\gamma} + \mathcal{L}S$	S_{γ}^{eff}
$\beta_t (F^{-1})$	1.4487	1.3338	1.3433	$\beta_s = 1.4487$
$\lambda_t (F^{-3})$	0.414	0.249	0.284	$\lambda_s = 0.291$
t	0	1.784	0.9519	-
$\gamma_t (F^{-1})$	-	1.5682	1.3433	-
$\lambda_{\mathcal{L}S} (F^{-3})$	0	0	-1.408	-
$\delta (F^{-1})$	-	-	2.7792	-

Table - 2

$\lambda_{33} = -0.6675F^{-1}$, $\beta_{33} = 1.6212F^{-1}$

E_p(CM) in MeV	110	120	130	140	150	160
$(\text{Cot } \delta)_{cal}$	-2.476	-2.332	-2.211	-2.108	-2.018	-1.940
$(\text{Cot } \delta)_{HJ}$	-2.504	-2.352	-2.221	-2.108	-1.998	-1.902

possible to further add to the triplet even interaction a two-body spin-orbit interaction as was done by Mitra and Narasimham (M59/60a) and Naqvi (N62a). This interaction to be added to equation (2.23), takes the form

$$-m \langle \underset{\sim}{p} | v_{\mathcal{L}S} | \underset{\sim}{p}' \rangle = 4\pi \lambda_{\mathcal{L}S} v_2(p) v_2(p') \times \sum_m Y^*_{2m}(\hat{p}) \underset{\sim}{L} \cdot \underset{\sim}{S} \, Y_{2m}(\hat{p}') \tag{2.26}$$

where

$$v_2(p) = p^2 (\delta^2 + p^2)^{-2}$$

$\underset{\sim}{L}$ is the total angular momentum, and $\underset{\sim}{S} = 1/2 \, (\underset{\sim}{\sigma}_1 + \underset{\sim}{\sigma}_2)$. Table 1 also lists values of $\lambda_{\mathcal{L}S}$ and δ given by Naqvi (N62a).

The only odd state of interest in our calculations is $^{33}P_1$, which will arise from the transitions $^3S_1$ or $^3D_1 \to {}^3P_1$. Since this state will occur as a final state, the relative momentum of the nucleons will be $\sim$ 350 MeV/c. A nonlocal separable potential in ^3P_J states has been given by Mitra and Naqvi (M61a). They assume a purely spin-orbit force in these states, and derive the parameters by fitting nucleon-nucleon scattering phase shifts upto about 100 MeV/c. For the region of our interest (i.e. $\sim$ 350 MeV/c), the phase shifts given by this potential differ greatly from the phase shifts derived from HJ potential (H62a). Hence we have chosen a central potential acting only in $^{33}P_1$ state of the form

$$-m\langle \underset{\sim}{p}|V|\underset{\sim}{p}'\rangle = 3\lambda_{33} v_{33}(p) v_{33}(p') P_1(\hat{p}\cdot\hat{p}') \tag{2.27}$$

and $v_{33}(p) = p\ (\beta_{33}^2 + p^2)^{-1}$. The parameters of this potential are fitted to give a good fit to NN phase shifts in the region of interest i.e. E_p(CM) = 138 MeV. The parameters and the quality of fit to the phase-shifts are shown in table 2.

The α-nucleon interaction has also been given in non-local separable form by Mitra, Bhasin and Bhakar (M62b) but we shall describe it in detail in the next chapter during discussion of ^{6}Li wavefunctions.

2.C Pion capture in deuteron.

The Schrodinger equation for the deuteron in momentum space with non-local separable potentials can be written as (we only write for the relative motion of two nucleons)

$$\left(\frac{p^2}{m} - E\right)\psi(\underset{\sim}{p}) = -\int d^3p' \langle \underset{\sim}{p} | v | \underset{\sim}{p'} \rangle \psi(\underset{\sim}{p'})$$

(2.28)

Substituting for the potential, the equations (2.23-2.26), we get with $\alpha_d^2 = -ME$

$$(p^2 + \alpha_d^2)\psi(\underset{\sim}{p}) = \tag{2.29}$$

$$\lambda_t g(\underset{\sim}{p}) \int d^3p' g(\underset{\sim}{p'}) \psi(\underset{\sim}{p'})$$

$$+ 4\pi \lambda_{LS} v_2(p) \sum_m Y_{2m}^*(\hat{p})$$

$$\times \int d^3p' v_2(p') \underset{\sim}{L} \cdot \underset{\sim}{S}\, Y_{2m}^*(\hat{p}') \psi(\underset{\sim}{p'})$$

The solution of the equation is easily seen to be

$$\psi(\underset{\sim}{p}) = N_0 (p^2 + \alpha_d^2)^{-1} \{ C(p) \mathcal{Y}_{101}^m$$

$$+ T(p) \mathcal{Y}_{121}^m + \frac{N_2}{N_0} v_2(p) \mathcal{Y}_{121}^m \} \zeta_{m_\tau = 0}^{T=0}$$

(2.30)

where $\mathcal{Y}_{jls}^m$ is the composite spin-orbital wavefunction

$$\mathcal{Y}^{m}_{j\ell s=1} = \sum_{m_s} C^{\ell\ 1\ j}_{m-m_s\ m_s\ m}\, Y_{\ell,\, m-m_s}(\hat{p})\, \chi^{1}_{m_s}$$

and $\chi^{1}_{m_s}$ and $\zeta^{T=0}_{\tau_t=0}$ are the functions describing states of two nucleons with spin 1 and I-spin T=0 respectively. The normalisation coefficients N_0 and N_2 satisfy the following coupled equation

$$N_0 = \lambda_t \{ N_0 I_{11} + N_2 I_{12} \}$$

$$N_2 = \lambda_{\ell s} \{ N_0 I_{21} + N_2 I_{22} \}$$

$$I_{11} = \int d^3x \{ C^2(x) + T^2(x) \} (x^2 + \alpha_d^2)^{-1}$$

$$I_{12} = I_{21} = \int d^3x \ T(x) v_2(x) (x^2 + \alpha_d^2)^{-1}$$

$$I_{22} = \int d^3x \ v_2^2(x) (x^2 + \alpha_d^2)^{-1}$$

In a similar way the final scattering state wavefunction for two neutrons can be written as

$$\Psi_f = \psi_f(p)\,\chi^{1}_{m_s}\,\zeta^{T=1}_{m_\tau=-1} \tag{2.31}$$

where with the outgoing wave boundary condition (M68a)

$$\psi_f(p) = \left(\frac{2\pi}{L}\right)^{3/2}\left\{\delta(\underset{\sim}{p}-\underset{\sim}{k}) + \frac{1}{2\pi^2}\,\frac{a(\underset{\sim}{p})}{(p^2-k^2-i\epsilon)}\right\} \tag{2.32}$$

In this expression $k^2 = ME$,

$$a(\underset{\sim}{p}) = -2\pi^2 m \langle \underset{\sim}{p}|v_{33}|\underset{\sim}{k}\rangle \{1-\lambda_{33}J(k)\}^{-1}$$

with

$$J(k) = \int d^3x\, v_{33}^2(x)\,(x^2-k^2-i\epsilon)^{-1} \tag{2.33}$$

The physical scattering amplitude is the value of $a(\underset{\sim}{p})$ on the energy shell i.e. for $p^2 = k^2$; otherwise $a(\underset{\sim}{p})$ gives the off-energy-shell scattering amplitude describing two-nucleon correlations at non-asymptotic distances.

To obtain the capture rate, we now evaluate the matrix element of the interaction between initial and final states.

The simple kinematics of the problem gives

$$k^2/m = m - \alpha_d^2/m \Longrightarrow$$

$$k = 1.82\ F^{-1} = 360\ \text{MeV}/c \tag{2.34}$$

The capture rate is given by the standard expression,

$$W_d = 2\pi \sum_{\substack{\text{spin} \\ \text{averages}}} |\langle f|H|i\rangle|^2 \rho(E_f) \tag{2.35}$$

where $\rho(E_f)$ is the density of final states. The spin and I-spin calculations can be carried out very easily, and we obtain

$$\langle f|H|i\rangle = \left\{-\sqrt{2}\, G \frac{2m}{M} \frac{1}{\sqrt{\pi a^3}}\right\}\left\{\frac{2\pi}{L}\right\}^{3/2} \frac{N_0}{\sqrt{4\pi}}$$

$$\times\Big[2k \left\{ c(k) + \frac{1}{\sqrt{2}} T(k) + \frac{1}{\sqrt{2}} \frac{N_2}{N_0} v_2(k) \right\} (k^2+\alpha_d^2)^{-1}$$

$$+ \frac{2\lambda_{33} v_{33}(k)}{1-\lambda_{33} J(k)} \int d^3x \, \frac{v_{33}(x)}{(x^2-k^2-i\epsilon)} \cdot \frac{x}{(x^2+\alpha_d^2)} \times$$

$$\left\{ c(x) + \frac{1}{\sqrt{2}} T(x) + \frac{1}{\sqrt{2}} \frac{N_2}{N_0} v_2(x) \right\} \Big] \qquad (2.36)$$

Table 3 shows separately the contribution of the central, tensor and spin-orbit terms to the matrix element given in equation (2.36) and also the capture rates without and with the inclusion of the final state interaction (described by a(p) term in equation (2.32)).

It should be first noted that the result for capture rate with effective central Yamaguchi (C_Y^{eff}) potential, which takes into account only the $^3S_1$ state of the deuteron, gives a fairly large value viz., 0.63 (or 0.59) x 10^{15} sec^{-1}. This may be compared with 0.22 x 10^{15} sec^{-1} obtained by Koltun and Reitan (K66a), with potentials that contain hard-

Table - 3

Potential	C_{γ}^{eff}	$(C+T)_{\gamma}$	$(C+T)_N + \mathcal{L}S$
Centralterm	.09984	.10611	.10588
Tensor term	-	-.06780	-.04603
$\mathcal{L}S$ term	-	-	-.00817
W_d(no FSI)	.63	.066	.123
W_d(with FSI)	.589	.123	.182

The capture rate W_d is expressed in units of 10^{15} / sec.

core and hence sizable short range correlations. One may conclude that the nonlocal separable potential brings about quite large high momentum components in the two-body wavefunctions, although it does not contain explicitely a repulsive term. When the $^3D_1$ state of the deuteron arising from tensor force is taken into account, we find (in agreement with Woodruff and Koltun and Reitan, but in sharp disagreement with Cheon and Tohsaki) that there exists a cancellation of the amplitudes, and the capture rate is reduced by almost a factor of ten. However, this cancellation is not as drastic as in the case of Koltun and Reitan, the reason perhaps being the difference in the nature of the NN interaction in two cases. If one considers Naqvi's potential which includes a spin-orbit term also, thus reducing to some extent, the effectiveness of the tensor component, the S-D cancellation becomes less severe. The spin-orbit term thus indirectly enhances the capture rate, although its direct contribution is negligible. It should be noted at this stage that Koltun and Reitan remark that the S-D cancellation is independent of the D-state. But we have seen that the Naqvi's potential which yield the D-state probability 3.22 % (4 % for the Yamaguchi's potential), the S-D cancellation is weakened.

It may also be seen that the final state interaction in all cases enhances the capture rate by about 0.06×10^{15}. Thus its effect is comparatively small (about 10 %) for the central Yamaguchi potential, but is quite large (50-100 %), in the presence of tensor force.

Finally, it may be remarked that the results of above calculations of the capture rate fall considerably short of the latest experimental value viz., $W_d(\text{expt}) = 1.585 \times 10^{15}\ \text{sec}^{-1}$ (R67a).

3. BINDING ENERGY AND WAVEFUNCTIONS FOR ^{6}Li

3.A Introduction

The unusually stable structure of the α-particle has led to many investigations of light nuclei in which this object is taken as a single structureless entity. ^{6}Li and ^{6}He are particularly well suited for such investigations, since their structure consists of only two nucleons in addition to the α-particle. Indeed, quite encouraging results have been obtained for the energy levels of ^{6}Li in a shell model spirit (N62b) with L-S coupling configuration for two nucleons outside the α-particle which is assumed to be an inert core. An apperantly more ''dynamical'' model which regarded this nucleus as an α-d system (Y62a) worked quite well except the quadrupole moment.

The recent advances in three-body techniques through the Faddeev formalism (F61a) has given rise to a renewed interest in the problem of ^{6}Li - like nuclei, looked upon as three-body structures. A variational treatment of ^{6}Li as a three-body problem is already available in the work of Wackman and Austern (W62a) and recently of Barsella, Lovitch and Rosati (B68a). The emphasis in the Faddeev technique is however on an exact rather than a variational solution of the problem, the price for ''exactness'' being a truncation of the two body reaction matrix by one or more (always a finite number of) pole terms. However this approximation can be defended on theoretical grounds (L64a).

A related approach which leads to the same equations, and which is particularly convenient for the bound state problems, is provided through a direct parametrization of the input potentials in a separable form, and their use in conjuction with a three-body Schrodinger equation (M62a, S63a). Such an approach has been extensively used in connection with the three-nucleon problem involving both ^3H and n-d states (M66c), and appears equally suited to the ^6Li problem. Similar methods have also been used for light hypernuclei like $^3_\Lambda H$ (H67a, M66a), $^9_\Lambda He$ (M67a) and $^6_{\Lambda\Lambda}He$ (M66b, D68a), all of which involve unequal mass particles. An important difference between ^6Li and these hypernuclei is in the nature of the configurations. While the Λ - particle is not prevented by the Pauli principle from having mainly S-wave interactions with the other entities in these hypernuclei, the nucleons in ^6Li are constrained by the same principle to interact with the α-particle mainly through the P-wave. This renders the system more delicate, and hence numerically more difficult. Nevertheless the feasibility of such a program had been recognised several years ago (M62a) and separable form of the α-N interaction was devised for the purpose (M62b). Hebach et.al. (H67b) have given a similar treatment of ^{6}He using a different form of the α-N interaction. The treatment in this chapter is however designed to cover both ^{6}Li ground state $J = 1^+$ $T = 0$ and the excited state $J=0^+$ $T=1$.

As given in (M62b), the α-N force consists of an S-wave part and a P-wave part. The S-wave part which is important for the α-N scattering problem is however, prevented by the Pauli principle from entering into the ^{6}Li calculations, and will henceforth be neglected. One word of caution should be added here, Once the α-particle is assumed to be a structureless entity, it may seem unreasonable to invoke the Pauli principle for neglecting the S-wave α-N force. However the fact remains that the nucleons in ^{6}Li are filling the P-shell. This essentially means that the S-wave part does not, so to say, get a chance to play its role in a ^{6}Li type system. (It is worthwhile to notice that in the variational treatment of ^{6}Li, Barsella and his group (B68a) who use the central + Spin-orbit local potential for α-N system get larger binding for the ^{6}He nucleus due to the spurious S-state component present in the A = 6 system. It is rather difficult to remove this spurious S-state. In our case the potential is also separable in the l-state, and hence neglect of S-wave force automatically takes care off the spurious component.) While a completely satisfactory answer to this apparent anomaly must depend on a six-body treatment of ^{6}Li or ^{6}He, at this stage it is enough to neglect the S-wave α-N force in the ^{6}Li calculation.

The P-wave part consists of two terms. (i) a dominant central term and (ii) a small spin orbit term with positive sign, which to-gether fit $P_{1/2}$ and $P_{3/2}$ phase-shifts of Miller and Phillips (M58a) and Chritchfield and Dodder (C49a). A similar force but with a different form is also proposed by Hebach et.al. (H67b). This is an effective central P-wave force tuned exclusively to the $P_{3/2}$ data (P66a), thus completely ignoring the $P_{1/2}$ phase shifts. HHK justified this parametrization on the ground that in the energy region important for ^{6}He (^{6}Li), the partial cross-sections for $P_{1/2}$ are much smaller than the $P_{3/2}$ cross-sections. While this is true, we believe that the larger $P_{3/2}$ phase shifts should be obtained through an attractive Spin-orbit term in conjunction with a central term as in (M62b) rather than through a single effective term as in HHK. This is particularly important in view of the recognition that the effectiveness of a non-central force (like spin-orbit) in three-body binding is much more limited than that of the central force, in much the same way as a central + tensor force in the triplet N-N state gives less binding for ^3H than does an effective central force (B65a). It is to be noted here that Shaneley (S68a) who works in the positive energy region (with Amado's model) for ^6Li system, i.e. α-d scattering problem, also emphasizes the dominance of $P_{3/2}$ resonance, though he treats $P_{3/2}$ and $P_{1/2}$ on equal footing for the problem.

In section B we give the NLS potential for the α-N system in P-state and in section C outline the essential steps for the three-body treatment of ^6Li. Section D gives the numerical results, a comparison with other theoretical results, and a discussion of the three-body wave function. The calculation of the first order correction to the binding energy due to the $\underset{\sim}{L}.\underset{\sim}{\sigma}$ term of the α-N potential is presented in section E.

3.B NLS potential in the α-N system

As given in the reference (M62b), we take the P-wave α-N interaction (still a matrix in spin space) of the form:

$$-2M_R \langle \underset{\sim}{q} | v_{\alpha-N} | \underset{\sim}{q}' \rangle = 4\pi \lambda_{\alpha N} v_1(q) v_1(q')$$
$$\times \sum_m Y^*_{1m}(\hat{q}) \{ 1 + \underset{\sim}{L} \cdot \underset{\sim}{\sigma} / t' \} Y_{1m}(\hat{q}') \tag{3.1}$$

where $v_1(q) = q(q^2 + \beta^2_{\alpha N})^{-1}$

with

$$\lambda_{\alpha N} = 0.0884\,F^{-1}, \ \beta_{\alpha N} = 0.9727\,F^{-1} \ \& \ t' = 21.6 \tag{3.2}$$

M_R is the reduced mass of the α-N system given by

$$M_R = M M_\alpha / (M + M_\alpha) = \eta M$$

$$\text{with } a = M/M_\alpha \text{ and } \eta = (1+a)^{-1} \tag{3.3}$$

This gives an effective strength parameter for the $P_{3/2}$ state as

$$\lambda_{\alpha N}(3/2) = \lambda_{\alpha N}\{1 + 1/t'\} = 0.0925\,F^{-1}$$

These values may be compared with the corresponding parameters of HHK, normalised to the expression (3.1)

$$\lambda_{\alpha N}(3/2) = 0.0793\,F^{-1} \quad \& \quad \beta_{\alpha N} = 0.8\,F^{-1} \tag{3.4}$$

To estimate the strength of the P-wave force of HHK, let us compare the volume strength parameter which is related to $\lambda_{\alpha N}/\beta_{\alpha N}$. This quantity turns out to be 0.0967 for HHK parameters, whereas our parameters yield the value 0.0909. Even with our effective strength parameter, this value is 0.0951. The P-wave force in HHK calculation therefore appears to be somewhat over attractive.

3.C The three-body equations

Now in this section we give the essential formulation of the three-body problem for the ground state 1^+ of ^6Li. We consider the nucleus in its CM system and choose the momenta of the two nucleons as $\underset{\sim}{P}_1$ and $\underset{\sim}{P}_2$, so that the momentum of the α-particle $\underset{\sim}{P}_3$ is $-(\underset{\sim}{P}_1 + \underset{\sim}{P}_2) = -\underset{\sim}{P}$. The relative momenta $\underset{\sim}{q}_{13}$ and $\underset{\sim}{q}_{23}$ for the α-N pair are

$$\underset{\sim}{q}_{13,23} = \{ m_\alpha \underset{\sim}{P}_{1,2} + m(\underset{\sim}{P}_1 + \underset{\sim}{P}_2) \} / (m + m_\alpha)$$

$$= \underset{\sim}{P}_{1,2} + a\eta\, \underset{\sim}{P}_{2,1} \tag{3.5}$$

The potentials in the momentum space of $\underset{\sim}{P}_1$ and $\underset{\sim}{P}_2$ are normalised according to

$$\langle \underset{\sim}{P}_1 \ \underset{\sim}{P}_2 | V_{n-p} | \underset{\sim}{P}_1' \ \underset{\sim}{P}_2' \rangle$$

$$= \langle \underset{\sim}{q}_{12} | v_{n-p} | \underset{\sim}{q}_{12}' \rangle \, \delta(\underset{\sim}{P}_1 + \underset{\sim}{P}_2 - \underset{\sim}{P}_1' - \underset{\sim}{P}_2') \quad (3.6)$$

with

$$\underset{\sim}{q}_{12} = (\underset{\sim}{P}_1 - \underset{\sim}{P}_2)/2$$

and

$$\langle \underset{\sim}{P}_1 \ \underset{\sim}{P}_3 | V_{\alpha-N} | P_1' \ P_3' \rangle$$

$$= \langle \underset{\sim}{q}_{13} | v_{\alpha-N} | q_{13}' \rangle \, \delta(\underset{\sim}{P}_2 - \underset{\sim}{P}_2') \quad (3.7)$$

where the reduced matrix elements in equations (3.6) and (3.7) are given by equations (2.22-2.25) and (3.1 - 3.2) respectively. It is better to mention at this stage that in the reduced matrix element for the α-N pair as given in the equation (3.1), we only incorporate the central part of the P-wave term in the three-body Schrödinger equation and the small spin orbit part is left to be considered at most perturbatively at a later stage. Also in the reduced matrix element for n-p pair the $\mathcal{L}S$ term which was added in case of Naqvi's force is dropped for simplicity. In this respect the $(C + T)_N$ force is an incomplete one as compared to $(C+T)_Y$ force.

The three-particle Schrödinger equation now reads as

$$D_E \Psi(\underset{\sim}{P_1}\,\underset{\sim}{P_2}) = -m\left(V_{n-p} + V_{\alpha-n} + V_{\alpha p}\right)\Psi(\underset{\sim}{P_1}\,\underset{\sim}{P_2}) \qquad (3.8)$$

where

$$D_E = \frac{1}{\alpha}P_1^2 + \frac{1}{\alpha}P_2^2 + \frac{a}{2}P_3^2 - mE$$

$$= (2\eta)^{-1}(P_1^2 + P_2^2) + a\,\underset{\sim}{P_1}\cdot\underset{\sim}{P_2} + \alpha_T^2$$

with

$$E \equiv -E_b = -\alpha_T^2/m$$

With the substitution of the equations (3.6) - (3.7) with the factorable shape of the potential, the equation (3.8) will look like

$$D_E \Psi(\underset{\sim}{P_1}\,\underset{\sim}{P_2})$$

$$= \lambda_t \int d^3q_{12}'\, g(\underset{\sim}{q_{12}})\, g(\underset{\sim}{q_{12}'})\, \Psi\left(\underset{\sim}{P} + \frac{\underset{\sim}{q_{12}'}}{2},\ \underset{\sim}{P} - \frac{\underset{\sim}{q_{12}'}}{2}\right)$$

$$+ \frac{3\lambda_{\alpha N}}{2\eta}\int d^3q_{13}'\, v(q_{13})\, \underset{\sim}{q_{13}}\cdot\underset{\sim}{q_{13}'}\, v(q_{13}')\, \Psi(\underset{\sim}{P_1'}\,\underset{\sim}{P_2})$$

$$+ \frac{3\lambda_{\alpha N}}{2\eta}\int d^3q_{23}'\, v(q_{23})\, \underset{\sim}{q_{23}}\cdot\underset{\sim}{q_{23}'}\, v(q_{23})\, \Psi(\underset{\sim}{P_1}\,\underset{\sim}{P_2'}) \qquad (3.9)$$

where

$$v(q) = q^{-1} v_1(q)$$

When one takes into account the proper symmetry of the wave function, i.e. $\Psi(\underset{\sim}{P}_1\ \underset{\sim}{P}_2) = \Psi(\underset{\sim}{P}_2\ \underset{\sim}{P}_1)$, equation (3.9) reduces to the following expression in terms of three unknown functions G, H and F

$$\Psi(\underset{\sim}{P}_1\ \underset{\sim}{P}_2) = D_E^{-1}\{g(\underset{\sim}{q}_{12})\, G^*(\underset{\sim}{P}) + v(q_{13})\, \underset{\sim}{q}_{13}\cdot P_2\, F(P_2) + v(q_{23})\, \underset{\sim}{q}_{23}\cdot \underset{\sim}{P}_1\, F(P_1)\} |\chi^1_{m_s}\rangle \tag{3.10}$$

where $G^*(\underset{\sim}{P}) = G(P) + \frac{1}{\sqrt{8}}\, S_{12}(\hat{P})\, H(P)$

and $|\chi^1_{m_s}\rangle$ is the spin triplet wave function.

The structure of the wave function suggests that in each of the terms in the equation (3.10), the motion of one of the particles is fully separated. The wave function of this particle does not depend explicitely upon how the other two particles are behaving. Thus apparently this third particle just remains present there as a spectator. However we shall now show in detail that actually all the dynamics is contained into the eigenvalue equations for the spectator functions G, H and F, and thus the solutions do depend implicitely, if not explicitely, on the behaviour of every pair. The first term $D_E^{-1} g(q_{12}) G(P)$ may be interpreted as the disassociation of ^{6}Li as $[\alpha, (NN)]$. The part $D_E^{-1} g(q_{12})$ represents in this case the wave function of the n-p pair, whereas $G(P)$ describes the wave function of the α-particle. Similarly the $D_E^{-1} v(q_{13}) q_{13} \cdot P_2 F(P_2)$ term represents $[(\alpha N), N]$ decomposition of the three-body system. The motion of the α-N pair is fixed by the part $D_E^{-1} v(q_{13}) q_{13}$ whereas $P_2 F(P_2)$ shows the behaviour of the spectator nucleon. In this term the P-wave nature of the α-N pair as well as that of the spectator nucleon is exihibited explicitely.

The calculation of these spectator functions could be carried out in the standard way (M62a). Reading off equation (3.10) from equation (3.9) implies

$$G(P)|\chi'_{m_s}\rangle = \lambda_t \int d^3q'_{12}\, g(q'_{12})\, \Psi\left(P + \frac{q'_{12}}{2},\; P - \frac{q'_{12}}{2}\right)$$

and

$$q_{13} \cdot P_2 F(P_2) |\chi'_{m_s}\rangle$$

$$= 3\lambda_{\alpha N} (2\eta)^{-1} \int d^3q'_{13}\, q_{13} \cdot q'_{13}\, v(q'_{13})\, \Psi(P'_1\; P_2) \quad (3.11)$$

We feed back the equation (3.10) in these equations, which reduces them to the following integral equations:

$$\{\lambda_t^{-1} - h_t(p)\}\, G(p) = (-2\eta)\int d^3q\, A(\underset{\sim}{q}, \underset{\sim}{p})\, F(q)$$

$$\{\lambda_t^{-1} - h_t(p)\}\, H(p) = (-2\eta)\int d^3q\, B(\underset{\sim}{q}, \underset{\sim}{p})\, F(q)$$

$$\frac{1}{3}\{\lambda_{\alpha N}^{-1} - h_{\alpha N}(p)\}\, p^2 F(p)$$

$$= \int d^3q\, K(\underset{\sim}{q}, \underset{\sim}{p})\, F(q)$$

$$-\frac{1}{2\eta^2}\int d^3q\, A(\underset{\sim}{p}, \underset{\sim}{q})\, G(q)$$

$$-\frac{1}{2\eta^2}\int d^3q\, B(\underset{\sim}{p}, \underset{\sim}{q})\, H(q) \qquad (3.12)$$

where

$$h_t(x) = \int d^3y\, \{C^2(y) + T^2(y)\}\{y^2 + \frac{1+2a}{4}x^2 + \alpha_T^2\}^{-1}$$

$$h_{\alpha N}(x) = \int d^3y\; v_1^2(y)\{y^2 + (1+2a)\eta^2 x^2 + 2\eta\alpha_T^2\}^{-1}$$

$$A(\underset{\sim}{x}, \underset{\sim}{y}) = \frac{C(\underset{\sim}{x} + \underset{\sim}{y}/2)\, v(\underset{\sim}{y} + \eta\underset{\sim}{x})\{\eta x^2 + \underset{\sim}{x}\cdot\underset{\sim}{y}\}}{\{x^2 + y^2/2\eta + \underset{\sim}{x}\cdot\underset{\sim}{y} + \alpha_T^2\}}$$

$$B(\underset{\sim}{x}, \underset{\sim}{y})$$

$$= \frac{T(\underset{\sim}{x} + \underset{\sim}{y}/2)\, P_2(\widehat{x + \frac{y}{2}}\cdot\hat{y})\, v(\underset{\sim}{y} + \eta\underset{\sim}{x})\{\eta x^2 + \underset{\sim}{x}\cdot\underset{\sim}{y}\}}{\{x^2 + y^2/2\eta + \underset{\sim}{x}\cdot\underset{\sim}{y} + \alpha_T^2\}}$$

$$K(\underset{\sim}{x},\underset{\sim}{y}) = K(\underset{\sim}{y},\underset{\sim}{x}) =$$
$$\theta(\underset{\sim}{x}+a\eta\underset{\sim}{y})\,\theta(a\eta\underset{\sim}{x}+\underset{\sim}{y})\times$$
$$\{a^2\eta^2x^2y^2+a\eta(x^2+y^2)\underset{\sim}{x}\cdot\underset{\sim}{y}+(\underset{\sim}{x}\cdot\underset{\sim}{y})^2\}\times$$
$$\{x^2+y^2+2a\eta\underset{\sim}{x}\cdot\underset{\sim}{y}+2\eta\alpha_T^2\}^{-1}$$

and $P_2(u)$ is the legendre polenomial of order two.

It looks as if we now have to solve three coupled linear integral equations for the spectator functions G(p), H(p) and F(p). But we note the simplicity of the problem which arises from the permutation symmetry requirement to-gether with first rank nature of the α-N force. Substituting G(p) and H(p) from the first two equations of (3.12) into the third equation of (3.12), the equation for F(p) takes the following form of an eigenvalue integral equation with a symmetric kernal.

$$\frac{1}{3}\{\lambda_{\alpha N}^{-1}-h_{\alpha N}(p)\}p^2F(p) = \int d^3q\, K(\underset{\sim}{q},\underset{\sim}{p})\,F(q) \qquad (3.13)$$

where

$$\mathbb{K}(\underset{\sim}{p},\underset{\sim}{q}) = \mathbb{K}(\underset{\sim}{q},\underset{\sim}{p}) = K(\underset{\sim}{p},\underset{\sim}{q}) +$$
$$\frac{1}{\eta}\int d^3x\,\{\lambda_t^{-1}-h_t(x)\}^{-1}\{A(\underset{\sim}{p},\underset{\sim}{x})\,A(\underset{\sim}{q},\underset{\sim}{x})$$
$$+\,B(\underset{\sim}{p},\underset{\sim}{x})\,B(\underset{\sim}{q},\underset{\sim}{x})\}$$

For the excited state 0^+, the same formulation can be utilised with $t=0$ (absence of tensor force) and λ_t, β_t and $|\chi^1_{m_s}\rangle$ being replaced by λ_s, β_s (as given in table 1) and $|\chi^0_{m_s}\rangle$ respectively.

3.D Results and discussions

The solution of equation (3.13) for the eigenvalue problem of determining the binding energy parameter α_T^2, proceeds on the familiar lines of first assuming an input value of this parameter and then calculating the quantity $\lambda_{\alpha N}$ by standard techniques. This ''three-body'' determination of $\lambda_{\alpha N}$ should of course match ''two-body'' value $\lambda_{\alpha N}$ determined to fit the α-N phaseshifts. Thus, the input value of α_T^2 is adjusted until the matching is sufficiently accurate. The variation of ''three-body'' $\lambda_{\alpha N}$ value with the α_T^2 for the n-p potential C_Y^{eff} as well as $(C + T)_Y$ is shown in the tables 4 and 5 respectively.

It is quite clear from these tables that the binding energy or α_T^2 is very sensitive to the value of $\lambda_{\alpha N}$; a change in $\lambda_{\alpha N}$ of 0.001 F^{-1} produces a change in α_T^2 of 0.01 F^{-2} or in E_b a change of 0.4 MeV. Thus to obtain for the binding energy an

Table - 4

C_y^{eff} as n-p potential.

$\alpha_T^2(F^{-2})$	0.125	0.130	0.135	0.140	0.145
E_b(MeV)	5.18	5.39	5.60	5.80	6.01
$\lambda_{\alpha N}(F^{-1})$ (Three-body)	0.08610	0.08667	0.08722	0.08773	0.08824
G^2term %	52.88	51.53	49.96	49.06	47,90

Table - 5

$(C+T)_y$ as n-p potential

$\alpha_T^2(F^{-2})$	0.100	0.105	0.110	0.115	0.120
E_b(MeV)	4.15	4.35	4.56	4.77	4.97
$\lambda_{\alpha N}(F^{-1})$ (Three-body)	0.08582	0.08651	0.08722	0.08787	0.08847
G^2term %	55.47	53.39	51.60	49.96	48.57
P_D %	2.77	2.73	2.70	2.68	2.66

accuracy of 400KeV, it is desirable to know $\lambda_{\alpha N}$ with an accuracy of $0.001\ F^{-1}$. As we discussed earlier, the two-body value of $\lambda_{\alpha N}$ is rendered somewhat uncertain by the role of spin-orbit force in α-N interaction, even though this interaction is very small. The values of $\lambda_{\alpha N}$ and $\lambda_{\alpha N}(3/2)$ are 0.0884 and 0.0925 F^{-1} respectively, thus the binding energy can change by as much as 1.5 MeV, depending upon the neglect or otherwise of spin-orbit force. It seems strange that the small spin-orbit force should be so important, and we shall see later in a perturbative calculation that indeed the first order correction to binding energy due to the spin-orbit force is negligibly small. For this reason we really prefer to match the three-body value of $\lambda_{\alpha N}$ to the two-body value obtained with a spin-orbit force viz., $\lambda_{\alpha N} = 0.0884\ F^{-1}$ and not with $\lambda_{\alpha N}(3/2)$. When HHK consider an effective $\lambda_{\alpha N}(3/2)$ and also use a somewhat stronger force, they obtain much higher binding energies. In table-6 we compare our results with those of HHK as well as of Alessandrini et.al. (A69a) For comparison we note that the Coulomb corrected experimental value (L66a) for the ground state binding energy is 3.70 + 0.83 = 4.53 MeV, 0.83 MeV being the magnitude of the Coulomb correction.

Table - 6

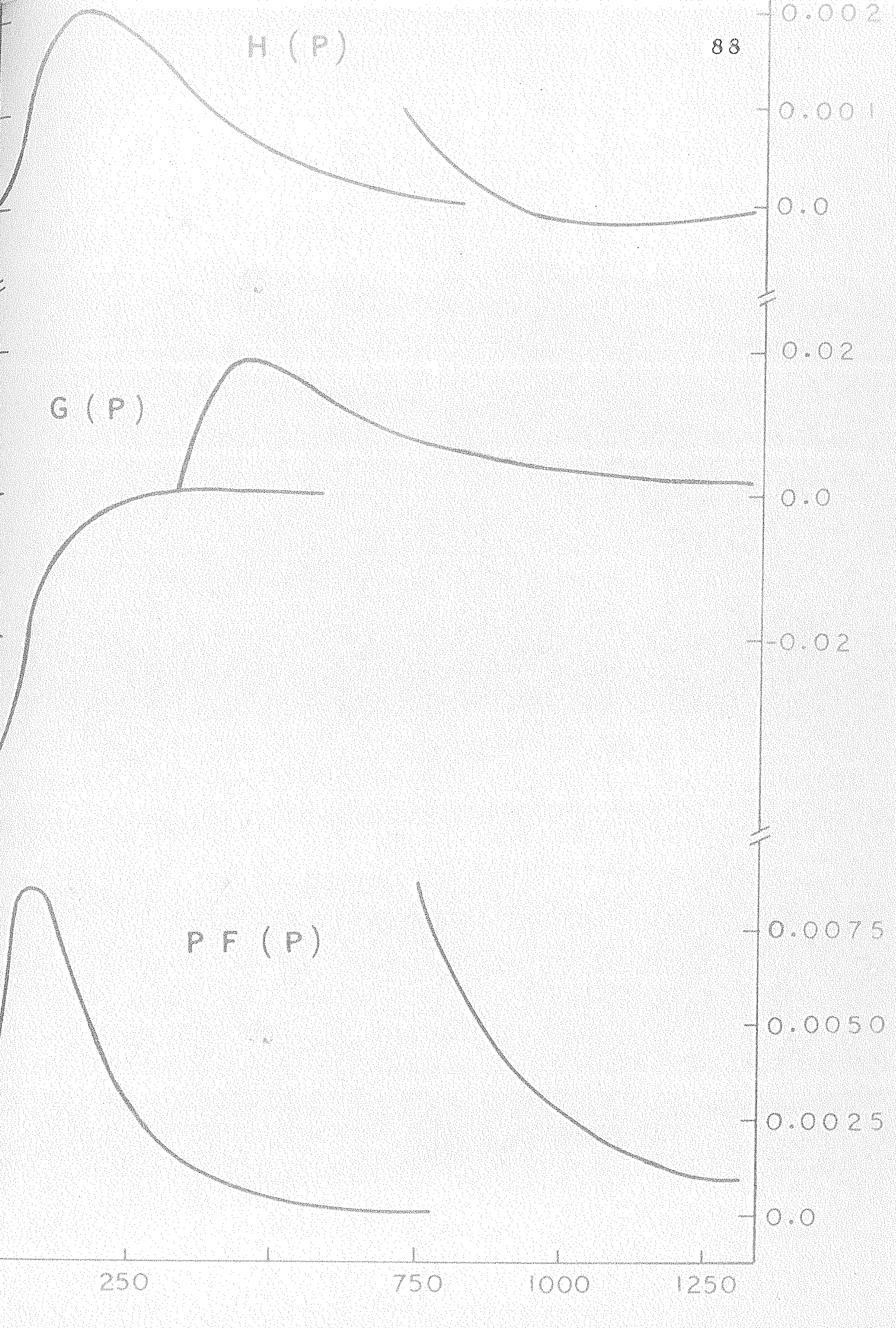
$J^P = 1^+$

$\downarrow V_{\alpha N}$	$V_{n-p} \rightarrow$	C_γ^{eff}	$(C+T)_\gamma$	$(C+T)_N$
	E_b(MeV)	6.011	4.974	4.560
(M62b)	G^2	47.90 %	48.57 %	47.00 %
	P_D	-	2.66 %	1.44 %
HHK	E_b(MeV)	8.7	-	-
(A69a)	E_b(A)	2.725	2.926	-
	E_b(B)	2.875	3.060	-

Table - 6 bears out also the fact that the effective central potential C_{γ}^{eff} is over attractive for the three-body system. The complete Yamaguchi triplet potential including central as well as tensor force does give a better result. The very good agreement of binding energy for Naqvi's set need not be taken too seriously as it is an incomplete force, though one must recognize that the neglected part, being a short range non-central force, may well be unimportant. Thus the result of the binding energy is somewhat larger compared to the Coulomb corrected experimental one. Table-6 also bears out our earlier assertion of an over-attractive ($P_{3/2}$) potential used by HHK which seems to give too large a binding energy with C_{γ}^{eff} for the ground state. It is quite surprising that the results of Alessandrini and coworkers, even with C_{γ}^{eff} potential, give a binding energy which is smaller by 1.7 MeV compared to Coulomb corrected experimental one. It is even more surprising to see that $(C+T)_{\gamma}$ force yields a larger binding energy compared to what C_{γ}^{eff} force yields. This is quite contarary to the usual feeling that (C+T) always give smaller binding compared to C^{eff} in the three-body system. The reason may lie in the method of calculation itself which involves different types of approximations.

For the excited state 0^+ we get a value of 0.2 MeV for the binding energy, which is somewhat lower compared to the coulomb corrected experimental one, viz., (0.13 + 0.83 = 0.96 MeV). It is worth mentioning at this stage that the force S_y^{eff} which is a first rank force does not contain the short range strong repulsion. In this respect we may be overestimating $E_b(0^+)$, but if we look back to the three nucleon system (where each pair is partly interacting through $^1S_0$ force) this short range correlation does not contribute more than 10-15 % as shown in (S67a, D68b). Thus we are somewhat short for the binding energy of the 0^+ state. Our estimate for this $E_b(0^+)$ with HHK parameters appears appreciably larger. The value quoted by HHK, obtained with their parameters is only 1.06 MeV whereas our calculation with the same parameters gives 2.1 MeV.

The spectator functions ''eigenvectors'' of the equation (3.12) have also been evaluated, and are shown in Fig.1 for the input potential $(C+T)_y$ as given in table-1. It may be noted that G(P) has a negative sign for low values of P, and eventually becomes positive small for moderately large momenta. Its dominant part is essentially negative. This merely


H (P)
G (P)
PF (P)
0.002
0.001
0.0
0.02
0.0
-0.02
0.0075
0.0050
0.0025
0.0
250
750
1000
1250

reflects the fact that in the 'polarisation' $[\alpha, (NN)]$ of the ^6Li system which $G(P)$ represents, one has an effective repulsion. This repulsion is of course more than off-set by the attraction in the other two pairs $[(\alpha N), N]$ as represented by the spectator function $PF(P)$, which is clearly seen to be positive. It is also worth noticing that the spectator functions $G(P)$ and $H(P)$ corresponding to the central and tensor part respectively have opposite phases in most of the momentum space, a feature which is quite different from one deduced from their behaviour in the three-nucleon system (B65b). This is possibly due to other pairs being in P-state in the case of ^6Li. The important fact about the spectator function $PF(P)$, which more or less behaves like the form factors $\mathcal{V}_1(P)$ and $C(P)$, is that it has sizeable components for momenta above 300 MeV/c whereas the harmonic oscillator wave function fitted to the gross structure of ^6Li goes practically to zero for large momenta. This feature of the ''exact'' wave-function obtained with reasonably short range potentials tuned to the α-N scattering data should have important consequences on the pion capture in ^6Li.

Once the potential for each pair is chosen, no other assumption or approximation needs to be made to reach the final results. Thus within the limits of the two-body potential truncation, the wave function is an exact function. This accurate wave function will be more of use in the field of investigations of the detailed properties of nuclei where one demands better wave functions. We also need to know the over all normalisation of the complete wave function, as the spectator functions have been derived by some arbitrary condition when eigenvalue equation is solved.

The over all normalisation

$$\sum \iint d^3p_1 \, d^3p_2 \, |\Psi(\underset{\sim}{p_1}, \underset{\sim}{p_2})|^2 = 1$$

directly gives not only the probability of D-state (P_D) but also the percentage of occurance of the different polarisations of the ^{6}Li system. The specific interest here is to know the contribution to the normalisation from the term $|c(q_{12}) G(p)|^2$ which represents the decomposition $[\alpha, (NN)]$. This is nothing but the α-d structure of the ^{6}Li system which has been often invoked in many calculations. The D-state probability is now no more a free parameter to be fitted to the observed data. It is a dynamical output, once the spectator functions are calculated by fixing the potential. As we are working in the total CM frame, it should be clarified that the D-state which we are talking about is of the N-N system explicitely coming from the tensor force in the N-N system and not from the P-wave structure of the α-N system. Let us rewrite the wavefunction showing explicitely the component due to this D-state. We have,

$$\Psi(\underset{\sim}{P_1}\ \underset{\sim}{P_2}) = \mathcal{N}\, D_E^{-1} \Big\{$$

$$\left[C(q_{12})\, G(P) + T(q_{12})\, H(P)\, P_2(\hat{q}_{12}\cdot\hat{P}) \right]$$

$$+ \left[v(q_{13})\, \underset{\sim}{q_{12}}\cdot\underset{\sim}{P_2}\, F(P_2) + v(q_{23})\, \underset{\sim}{q_{23}}\cdot\underset{\sim}{P_1}\, F(P_1) \right]$$

$$+ \Big[\Big\{ C(q_{12}) + \frac{2}{\sqrt{8}}\, T(q_{12}) \Big\} \frac{1}{\sqrt{8}}\, H(P)\, S_{12}(\hat{P})$$

$$+ \Big\{ G(P) + \frac{2}{\sqrt{8}}\, H(P) \Big\} \frac{1}{\sqrt{8}}\, T(q_{12})\, S_{12}(\hat{q}_{12})$$

$$+ \Big\{ i\,\hat{q}_{12}\times\hat{P}\cdot(\underset{\sim}{\sigma_1}+\underset{\sim}{\sigma_2}) - Y_{12}(\hat{q}_{12},\hat{P}) \Big\} \frac{9}{8}\, T(q_{12})\, H(P)\, P_1(\hat{q}_{12}\cdot\hat{P}) \Big]$$

$$\Big\} \left| \chi^1_{m_S} \right\rangle \qquad (3.14)$$

where

$$Y_{12}(\underset{\sim}{a},\underset{\sim}{b}) = \underset{\sim}{\sigma_1}\cdot\underset{\sim}{a}\ \underset{\sim}{\sigma_2}\cdot\underset{\sim}{b} + \underset{\sim}{\sigma_1}\cdot\underset{\sim}{b}\ \underset{\sim}{\sigma_2}\cdot\underset{\sim}{a} - \frac{2}{3}\,\underset{\sim}{\sigma_1}\cdot\underset{\sim}{\sigma_2}\ \underset{\sim}{a}\cdot\underset{\sim}{b}$$

The first square bracket [...] represents the amplitude for the disassociation [α, (NN)], whereas the third bracket explicitely shows the structure of D-state contained in the ground state. As the spectator functions are only obtained in a numerical form, the double integration involved in the normalisation calculation is carried out using the standard gaussian quadrature formula. Tables - 4, 5 and 6 contain the results for P_D as well as G^2 term contribution to the normalisation. The polarisation $[\alpha,(NN)]$ when the n-p system is in the S-state constitutes only 50 % of the total probability with any one of the n-p potentials. Thus it is clear that the total wave function in the CM frame contains a sizeable component of the structure $[(\alpha N), N]$. For the $(C+T)_Y$ case P_D turns out to be 2.66 % whereas it is 1.44 % with $(C+T)_N$ potential. It is obvious that $(C+T)_N$ gives smaller P_D, because of the $\mathcal{L}S$ term (which partly contributes to the deuteron quadrupole moment) in the n-p potential is ignored. This value of P_D does not fit in with the quadrupole moment of ^6Li, which is negative. However a fuller comparison of the quadrupole moment would require a more careful treatment of the $\underset{\sim}{L}\cdot\underset{\sim}{\sigma}$ term in the $V_{\alpha-N}$ potential which is known to be necessary to give the negative sign for the quadrupole moment.

3. E $\underset{\sim}{L}\cdot\underset{\sim}{\sigma}$ Correction

In this section, we present a perturbation calculation of the effect of the $\underset{\sim}{L}\cdot\underset{\sim}{\sigma}$ term on the binding energy of the ground state. This is given as

$$\Delta E_{\underset{\sim}{L}\cdot\underset{\sim}{\sigma}} = \langle \Psi | V_{\underset{\sim}{L}\cdot\underset{\sim}{\sigma}} | \Psi \rangle$$

$$= \left\{ -\frac{3}{2\eta} \frac{\lambda_{\alpha N}}{m} \cdot \frac{1}{t'} \right\} \times \sum \iint d^3 p_1 \, d^3 p_2 \, \Psi^* (\underset{\sim}{p}_1, \underset{\sim}{p}_2)$$

$$\times \iint d^3 p_1' \, d^3 p_2' \left\{ i \, \underset{\sim}{q}_{13} \times \underset{\sim}{q}_{13}' \cdot \underset{\sim}{\sigma}_1 \, v(q_{13}) v(q_{13}') + \right.$$

$$\left. i \, \underset{\sim}{q}_{23} \times \underset{\sim}{q}_{23}' \cdot \underset{\sim}{\sigma}_2 \, v(q_{23}) v(q_{23}') \right\} \Psi (\underset{\sim}{p}_1', \underset{\sim}{p}_2) \qquad (3.15)$$

This turns out to be zero for the excited state (0^+), as any azymuthal $d\varphi$ integrates to zero. It is again zero for the ground state for the input potential C_Y^{eff} which is purely a central force. The correction is proportional to D-state probability P_D. Using the symmetry of the wave function, we see that the non-zero contribution comes from the following terms.

$$\Delta E_{\underset{\sim}{L}\cdot\underset{\sim}{\sigma}} = \left\{ -\frac{3}{2\eta} \frac{\lambda_{\alpha N}}{m} \frac{1}{t'} \right\} |\mathcal{N}|^2 \sum \langle \chi^1_{m_s} | \Big[$$

$$\iiiint d^3x \, d^3y \, d^3x' \, d^3y' \left\{ \left\{ C(x) + \frac{1}{\sqrt{2}} T(x) \right\} \frac{1}{\sqrt{8}} H(y) S_{12}(\hat{y}) \right.$$

$$+ \left\{ G(y) + \frac{1}{\sqrt{2}} H(y) \right\} \frac{1}{\sqrt{8}} T(x) S_{12}(\hat{x})$$

$$- \frac{9}{8} T(x) H(y) P_1(\hat{x}\cdot\hat{y}) Y_{12}(\hat{x},\hat{y}) \Big\} D_E^{-1}(x,y)$$

$$\times \{ v(\xi) \; i \; \xi \times \xi' \cdot (\sigma_1 + \sigma_2) \; v(\xi') \} \; D_E^{-1}(x', y')$$

$$\Big\{ \{ C(x') + \frac{1}{\sqrt{2}} T(x') \} \frac{1}{\sqrt{8}} H(y') S_{12}(\hat{y}')$$

$$+ \{ G(y') + \frac{1}{\sqrt{2}} H(y') \} \frac{1}{\sqrt{8}} T(x') S_{12}(\hat{x}')$$

$$- \frac{9}{8} T(x') H(y') P_1(\hat{x}'\cdot\hat{y}') Y_{12}(\hat{x}', \hat{y}') \Big\} \Big] | \chi^1_{m_s} \rangle \quad (3.16)$$

with $D_E(x,y) = x^2 + (1+2a) y^2/4 + \alpha_T^2$

and $\xi = \eta \{ x + (1+2a) y \}$, $\xi' = \eta \{ x' + (1+2a) y' \}$

This equation (3.16) is simplified using

$$\sum \langle \chi^1_{m_s} | \, Y_{12}(\hat{p}, \hat{q}) \, (\sigma_1 + \sigma_2) \cdot \hat{r} \; Y_{12}(\hat{a}, \hat{b}) \, | \chi^1_{m_s} \rangle$$

$$= \frac{8i}{3} \{ \hat{p}\cdot(\hat{r}\times\hat{a}) \, \hat{q}\cdot\hat{b} + \hat{p}\cdot(\hat{r}\times\hat{b}) \, \hat{q}\cdot\hat{a}$$

$$+ \hat{p}\cdot\hat{b} \, \hat{q}\cdot(\hat{r}\times\hat{a}) + \hat{p}\cdot\hat{a} \, \hat{q}\cdot(\hat{r}\times\hat{b}) \}$$

(3.17)

and remembering that $S_{12}(\hat{r}) = \frac{3}{2} Y_{12}(\hat{r}, \hat{r})$

After substituting equation (3.17) in the equation (3.16), any general term of the (3.16) takes the following form. We only write for the time being the angular integration

$$\Delta E_{\underset{\sim}{L}\cdot\underset{\sim}{\sigma}} \propto \iiiint d\Omega_x d\Omega_y d\Omega_{x'} d\Omega_{y'}\, f(\underset{\sim}{x}\cdot\underset{\sim}{y})$$

$$\times f(\underset{\sim}{x'}\cdot\underset{\sim}{y'})\, \varphi(\hat{x}\cdot\hat{y},\, \hat{x}'\cdot\hat{y}',\, \hat{x}\cdot\hat{x}',\, \hat{x}\cdot\hat{y}',\, \hat{y}\cdot\hat{x}',\, \hat{y}\cdot\hat{y}')$$

where $f(\underset{\sim}{x}\cdot\underset{\sim}{y})$ is of the form $\{a + b\,\underset{\sim}{x}\cdot\underset{\sim}{y}\}^{-1}$ and a and b do not depend on the angular direction, φ is a polynomial function of its argument. The racah algebra helps us to write (keeping in mind the different azymuthal angle integration)

$$\varphi(\hat{x}\cdot\hat{y},\, \hat{x}'\cdot\hat{y}',\, \hat{x}\cdot\hat{x}',\, \hat{x}\cdot\hat{y}',\, \hat{y}\cdot\hat{x}',\, \hat{y}\cdot\hat{y}')$$

$$= \sum_{\ell_1 \ell_2 \ell_3 \ell_4} a(\ell_1 \ell_2 \ell_3 \ell_4)\, P_{\ell_1}(\hat{x})\, P_{\ell_2}(\hat{y})\, P_{\ell_3}(\hat{x}')\, P_{\ell_4}(\hat{y}')$$

With this the angular integration is zero except when either ℓ_1 or ℓ_2, and ℓ_3 or ℓ_4 are zero. Then the angular integration is straight forward. Also x, y are separated from x' y'. Thus we have to evaluate only a double integral for the magnitudes of x and y (or x' and y'). All these evaluations yield a value of 5 KeV (!) as a first order correction to the binding energy. Thus we expect only a very small value of this contribution to the binding energy even in a more adequate treatment. However it is possible that the effect of

$\underset{\sim}{L}\cdot\underset{\sim}{\sigma}$ term on the ^{6}Li wave function as well as its quadrupole moment could be more important. We believe that this accounts largely for the difference of our results from those of HHK who parametrize the $\underset{\sim}{L}\cdot\underset{\sim}{\sigma}$ term through an effective $(P_{3/2})$ force and over-estimate the three-body binding.

It may be worthwhile to make a brief comment on the role of a strong short range repulsion. It would not be adequate to treat such a term in a perturbative way. As far as the ground state is concerned, we are not yet able to include this repulsion term in the potential, since it would now become a third rank interaction (central + tensor + repulsion) and would very much complicate the three-body problem. Recently Tabakin (T68a) has given a first rank central NLS potential for the $^3S_1$ and $^1S_0$ N-N states, which is attractive for small momenta but repulsive for larger momenta. It may be of interest to use this potential in three-body problem to see what is the effect of short range repulsion (present in two-body system) on the three-body system.

In conclusion we can say that the results of our calculations show that this three-body model of ^{6}Li works satisfactorily for the calculation of binding energies. In the next chapter we consider an actual application of this wave function for the pion capture in ^{6}Li

4. PION CAPTURE IN ^{6}LI

In this chapter we present calculations for capture of π^- on ^{6}Li. We consider only the reaction $\pi^- + {}^6Li \rightarrow \alpha nn$ in which the α-particle emerges in ground state (because of our assumption of α being a structureless entity). Experimental evidence shows that about 40 % of the capture results in such a mode. This reaction implies that the pion capture takes place on a valence nucleon, i.e. a P-shell nucleon, and during the reaction α-particle remains as a whole undisturbed entity. Such an approach enables us to use the ^{6}Li wavefunctions derived in the previous chapter. Pion capture on S-wave nucleons which comprise the α-particle core in ^{6}Li would result in an α-particle in excited state or a fragmentation of the α-particle. Alberi and Tafera (A68b) have considered pion capture on the quasi α-particle in ^{6}Li, but they only consider the channel $\pi^- \alpha \rightarrow dnn$. They do not include the possibility of capture on a pair consisting of one valence-nucleon and one core-nucleon.

We shall also assume the pion capture to take place from 1S atomic orbit. Alberi and Tafera quote the experimental data of Backenstass et. al. that the

2P pion capture in ^{6}Li is no more than about 15 % of the radiative (2P→1S) transition rate. We thus use the simple S-wave pion-nucleon interaction operator which is earlier cast into a two-nucleon operator form.

4.A Wave function and kinematics of the reaction.

It was shown in the last chapter that the complete wave function for the ground state of ^{6}Li can be written as

$$\Psi_i = \mathcal{N} D_E^{-1} \{ g(\underset{\sim}{q}_{12}) G(\underset{\sim}{P}) + v(q_{13}) \underset{\sim}{q}_{13} \cdot \underset{\sim}{P}_2 F(P_2) + v(q_{23}) \underset{\sim}{q}_{23} \cdot \underset{\sim}{P}_1 F(P_1) \} \chi^1_{m_s} \zeta^{T=0}_{m_\tau=0}$$

(4.1)

where $\mathcal{N}$ is a normalisation constant, $\chi^1_{m_s}$ and $\zeta^{T=0}_{m_\tau=0}$ are the spin triplet and Ispin singlet states of two valence nucleons, and other terms are defined in chapter 3. We shall discuss here briefly the structure of each of the above terms in the wave function, and its implication for the pion capture process.

The first term viz., the ''g G'' term, describes an α-d type structure with the valence nucleons correlated and α-particle as a spectator. It should however be emphasized that in our wave functions, the α-particle function G(P) contains the dynamics of α-N interaction as well. In the usual α-d model for ^{6}Li, the α-particle wave function does not contain any such dynamic effects, and hence the long range behaviour of G (P) is quite different in our model from that in the usual α-d model. In other words, in our model the spectator α-particle will have larger high-momentum components than in the usual α-d model.

We also note that in the presence of the tensor force the ''g G'' term will also contain a $^3D_1$ state. Thus in the usual shell model notation, the wave function will contain not only the component with ℓ = relative orbital angular momentum of valence nucleons = 0, L = orbital angular momentum of α-particle relative to CM of two valence nucleons = 0 (and $\mathcal{L} = \ell + L = 0$; $J = \mathcal{L} + S = 1$), but also the components with l=2, L=0 and l=0, L=2. We remember that we obtained the D-state probability P_D as a dynamical output when the eigenvalue equations were solved in chapter 3. Sakamato (S65a) used in his calculations the Eckstein form of interaction which precludes the contribution of the D-state of the N-N pair. Kopaleishvilli and Machabelli (K67c) remark that the contribution of the D-state to the capture process is no more than 7 %. However, the nature of their wave function and the D-state probability are not clear. Koltun and Reitan (K67a) have solved the Schrödinger equation for valence nucleons with HJ potential, so that their calculation includes the $\ell = 2$, $L=0$ contribution, but by their assumption of the α-d wave-function the $\ell = 0$ $L=2$ component is excluded.

We now discuss the remaining terms in the wave function which describe explicitely α-nucleon correlation. Our treatment of ^{6}Li differs significantly from that of other authors in that we specifically include such α-nucleon correlations. These terms are generally neglected in pion capture calculations. One of the two terms describes the valence proton correlated with the α-particle. Pion capture through this term can result in the α-particle emerging with a large momentum. It should be noted that we do not include any explicit correlation between a valence nucleon and a core nucleon. Such an correlation would result in a high energy core nucleon with α-particle ending up fragmented or in an excited state. Our model gives a high energy α-particle in ground state. The second term would describe an α-neutron pair with the proton in a ''spectator'' form. Once again it should be noted that the proton, although a spectator in this term, contains the dynamic effects of N-N force, and hence will have sizeable high-momentum components, in contrast to a shell-model-type proton state. In the calculations to be described subsequently, the contributions of different terms in wave function will be shown separately to analyse their importance.

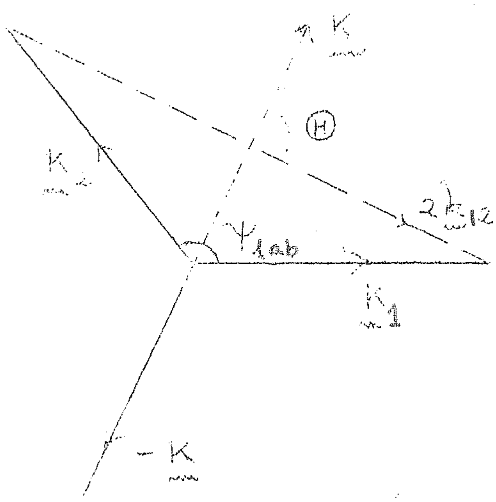
For the pion-nucleon interaction we use the standard S-wave ps-pv interaction given in equation (2.17). It may be noted that the ground state of ^{6}Li is an iso-spin singlet, and hence the $\underline{T} \cdot \underline{\Phi}$ terms in the interaction will not contribute to the capture rate. The transition I = 0 → I = 1 will be brought about by the $\underline{\tau} \cdot \underline{\Phi}$ terms, so that we obtain for the interaction operator,

$$H = \left(-\sqrt{2}\, G \frac{m}{M} \Phi\right) \left\{ \underset{\sim}{S} \cdot \underset{\sim}{p} + \frac{1}{4} \underset{\sim}{\sigma} \cdot \underset{\sim}{P} \right\} \tau_- \qquad (4.2)$$

where we have specialised for negative pion capture $\underline{\tau} \longrightarrow \sqrt{2}\,\tau_-$, and Φ is the pionic Bohr wave function defined in (2.15) where $Z = 3$ for ^{6}Li.

Most of the earlier workers generally also neglect the $\underset{\sim}{\sigma} . \underset{\sim}{P}$ term arising from the CM motion of the nucleon pair. Koltun (K67b) argues that it does not contribute more than a few percent for the pion capture rate. We shall see later that the contribution of this term is by no means negligible, particularly since our model contains explicitely possibility of high momentum components for α-particle through α-N correlations. Looked at from a slightly different view-point this term in the interaction would give an insight into α-d correlation in ^{6}Li.

In the final state the two valence nucleons move away from each other with momenta $\underset{\sim}{K}_1$ and $\underset{\sim}{K}_2$, and the α-particle receives a recoil momentum $-\underset{\sim}{K} = -(\underset{\sim}{K}_1 + \underset{\sim}{K}_2)$. Let Ψ_{lab} and Θ_{CM} be the angles between two nucleons in laboratory and CM frames of reference. A schematic illustration of the kinematics is given below.



The relationship of the lab angle and CM angle is easily seen to be

$$\tan \Psi_{lab} = \frac{K k_{12} \sin \Theta_{cm}}{K^2/4 - k_{12}^2}$$

or with $y = \cos \Psi_{lab}$, $z = \cos \Theta_{cm}$

$$z^2 = 1 - \frac{(K^2/4 - k_{12}^2)^2}{K^2 k_{12}^2} \frac{(1-y^2)}{y^2} \qquad (4.3)$$

For future use we quote.

$$\frac{dz}{dy} = \frac{(K^2/4 - k_{12}^2)^2}{K k_{12} y^2 \left\{ \left(\frac{K^2}{4} + k_{12}^2\right)^2 y^2 - \left(\frac{K^2}{4} - k_{12}^2\right)^2 \right\}^{1/2}} \qquad (4.4)$$

For the final state wave function we assume a simple plane wave. The present formalism can include the final state interaction easily. In the final state the two neutrons can be in either $^{33}P_1$ state ($^{13}S_1 \rightarrow {}^{33}P_1$ via $\underset{\sim}{S} \cdot \underset{\sim}{p}$ term) or $^{31}S_0 + {}^{31}D_2$ (transition taking place through the $\underset{\sim}{\sigma} \cdot \underset{\sim}{P}$ term of the interaction). Unfortunately, as we discussed in the previous chapter, the nonlocal separable potential in ^{33}P states is not accurate enough to give reliable phase shifts over the entire range of final relative momenta of two nucleons.

Similarly the non-local potential in $^1S_0$ state would also have to include a specific repulsive term (making it a second-rank potential) to faithfully reproduce the observed phase shifts. We therefore avoid the final state interaction in the following treatment, although it may be of some importance in considering emission of high momentum α-particles.

The complete final state wave function is written as

$$\Psi_f = \left(\frac{2\pi}{L}\right)^3 \delta(\underset{\sim}{P} - \underset{\sim}{K})\, \delta(\underset{\sim}{q}_{12} - \underset{\sim}{k}_{12})\, \chi^{S_f}_{m_{S'}}\, J^{T=1}_{m_t=-1} \tag{4.5}$$

where the requirements of energy conservation gives

$$\frac{(1+2a)}{4}\frac{K^2}{m} + \frac{k_{12}^2}{m} + \frac{\alpha_T^2}{m} = \mathcal{M} \tag{4.6}$$

4.B Transition amplitude

We now proceed to evaluate the transition matrix element

$$\mathcal{M}_{fi} = \langle \Psi_f | H | \Psi_i \rangle \tag{4.7}$$

Table - 7

$$\mathbf{S}\cdot\mathbf{a}\;\mathbf{S}\cdot\mathbf{b} = \frac{2}{3}\,\mathbf{a}\cdot\mathbf{b}\,P_{\sigma}^{+}(1,2) + \frac{i}{2}\,\mathbf{S}\cdot(\mathbf{a}\times\mathbf{b}) + \frac{1}{4}\,Y_{12}(\mathbf{a},\mathbf{b})$$

$$\boldsymbol{\sigma}\cdot\mathbf{a}\;\mathbf{S}\cdot\mathbf{b} = \frac{i}{2}\,\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b}) + \frac{1}{2}(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot(\mathbf{a}\times\mathbf{b})$$

$$\mathbf{S}\cdot\mathbf{a}\;Y_{12}(\mathbf{r},\mathbf{t}) = (\mathbf{a}\cdot\mathbf{r})\,\mathbf{S}\cdot\mathbf{t} + (\mathbf{a}\cdot\mathbf{t})\,\mathbf{S}\cdot\mathbf{r} - \frac{2}{3}(\mathbf{t}\cdot\mathbf{r})\,\mathbf{S}\cdot\mathbf{a} + \frac{i}{2}\,Y_{12}(\mathbf{a}\times\mathbf{r},\mathbf{t}) + \frac{i}{2}\,Y_{12}(\mathbf{a}\times\mathbf{t},\mathbf{r})$$

$$\boldsymbol{\sigma}\cdot\mathbf{a}\;Y_{12}(\mathbf{r},\mathbf{t}) = -(\mathbf{a}\cdot\mathbf{r})\,\boldsymbol{\sigma}\cdot\mathbf{t} - (\mathbf{a}\cdot\mathbf{t})\,\boldsymbol{\sigma}\cdot\mathbf{r} + \frac{2}{3}(\mathbf{t}\cdot\mathbf{r})\,\boldsymbol{\sigma}\cdot\mathbf{a}$$
$$+ i(\mathbf{a}\cdot\mathbf{r})(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\cdot\mathbf{t}) + i(\mathbf{a}\cdot\mathbf{t})(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\cdot\mathbf{r}) - \frac{2i}{3}(\mathbf{t}\cdot\mathbf{r})(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\cdot\mathbf{a})$$

and $Y_{12}(\mathbf{r},\mathbf{t}) = 2\,X^{2}(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)\cdot W^{2}(\mathbf{t},\mathbf{r})$;

$$X^{2}_{\mu}(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)\{W^{2}_{\mu}(\mathbf{t},\mathbf{r})\} =$$

$$\sum_m C\begin{pmatrix}1 & 1 & 2\\ m & \mu-m & \mu\end{pmatrix}\,\sigma_m(1)\,\sigma_{\mu-m}(2)\,\{t_m\,r_{\mu-m}\}$$

(Table:7 contd.)

Here $\underset{\sim}{a}$, $\underset{\sim}{b}$, $\underset{\sim}{r}$ and $\underset{\sim}{t}$ are arbitary vectors and $P_{\tau}^{+}(1,2)$ is the spin projection operator for the triplet state. We follow the Rose's defination (R57a) for the reduced matrix element i.e.

$$\langle J_2 m_2 | T^K_\mu | J_1 m_1 \rangle = C^{J_1 \; K \; J_2}_{m_1 \; \mu \; m_2} \langle J_2 \| T^K \| J_1 \rangle$$

with this

$$\langle \chi^{S_f} \| S \| \chi^{S_i = 1} \rangle = \sqrt{2}\, \delta_{S_f, 1}$$

$$\langle \chi^{S_f} \| X^2 \| \chi^{S_i = 1} \rangle = \sqrt{20/3}\, \delta_{S_f, 1}$$

$$\langle \chi^{S_f} \| \tau \| \chi^{S_i = 1} \rangle = -2\sqrt{3}\, \delta_{S_f, 0}$$

$$\langle \chi^{S_f} \| i\,\sigma_1 \times \sigma_3 \| \chi^{S_i = 1} \rangle = 2\sqrt{3}\, \delta_{S_f, 0}$$

For this purpose the ''gG'' term in Ψ_i is written explicitely exhibiting the D-state as in equation (3.14). The spin-Ispin integrations are carried out with the use of identities and double-barred-matrix-elements given in Table-7

It was earlier discussed that the major contribution to the pion capture cross section will come from the $\underset{\sim}{S}.\underset{\sim}{p}$ term of the interaction; we thus call the $\underset{\sim}{S}.\underset{\sim}{p}$ and $\underset{\sim}{\sigma}.\underset{\sim}{P}$ terms as the ''major'' and ''minor'' terms of the interaction. With the use of the results of Table-7, we evaluate the effect of the major and minor terms on the initial wave function.

$$\underset{\sim}{S}\cdot\underset{\sim}{q}_{12}\Psi_i = \gamma D_E^{-1}$$

$$\Big[\{v(q_{13})\underset{\sim}{q}_{13}\cdot\underset{\sim}{P}_2F(P_2)+v(q_{23})\underset{\sim}{q}_{23}\cdot\underset{\sim}{P}_1F(P_1)\}\underset{\sim}{S}\cdot\underset{\sim}{q}_{12}$$

$$+\{C(q_{12})+\frac{1}{\sqrt{2}}T(q_{12})\}\{G(P)-\frac{1}{\sqrt{8}}H(P)\}\underset{\sim}{S}\cdot\underset{\sim}{q}_{12}$$

$$+\{C(q_{12})+\frac{1}{\sqrt{2}}T(q_{12})\}\{\frac{3}{\sqrt{8}}\underset{\sim}{q}_{12}\cdot\underset{\sim}{P}\frac{H(P)}{P^2}\}\underset{\sim}{S}\cdot\underset{\sim}{P}$$

$$+\{C(q_{12})+\frac{1}{\sqrt{2}}T(q_{12})\}\frac{3i}{2\sqrt{8}}\frac{H(P)}{P^2}Y_{12}(\underset{\sim}{P},\underset{\sim}{q}_{12}\times\underset{\sim}{P})\Big]$$

$$\chi_{m_S}^{S_i=1}\,\mathcal{J}_{m_\tau=0}^{T=0} \qquad (4.8)$$

and

$$\underset{\sim}{\sigma}\cdot\underset{\sim}{P}\,\Psi_i = \mathcal{N} D_E^{-1} \times$$

$$\Big[\{v(q_{13})\,\underset{\sim}{q}_{13}\cdot\underset{\sim}{P}_2\,F(P_2) + v(q_{23})\,\underset{\sim}{q}_{23}\cdot\underset{\sim}{P}_1\,F(P_1)\}\,\underset{\sim}{\sigma}\cdot\underset{\sim}{P}$$

$$+\{c(q_{12})G(P) + T(q_{12})H(P)P_2(z)\}\,\underset{\sim}{\sigma}\cdot\underset{\sim}{P}$$

$$+\Big\{\frac{1}{\sqrt{8}}T(q_{12})\{G(P)+\frac{1}{\sqrt{2}}H(P)\} - \frac{1}{\sqrt{2}}H(P)\times$$

$$\{c(q_{12}) - \sqrt{2}\,T(q_{12})P_2(z)\}\Big\}\{\underset{\sim}{\sigma}\cdot\underset{\sim}{P} - i\,\underset{\sim}{\sigma}_1\times\underset{\sim}{\sigma}_2\cdot\underset{\sim}{P}\}$$

$$-\frac{3}{\sqrt{8}}\,\underset{\sim}{q}_{12}\cdot\underset{\sim}{P}\,\frac{T(q_{12})}{q_{12}^2}\{G(P)+\frac{1}{\sqrt{2}}H(P)\}\times$$

$$\{\underset{\sim}{\sigma}\cdot\underset{\sim}{q}_{12} - i\,\underset{\sim}{\sigma}_1\times\underset{\sim}{\sigma}_2\cdot\underset{\sim}{q}_{12}\}\Big]\chi_{m_s}^{s_i=1}\,\zeta_{m_\tau=0}^{T=0}$$

(4.9)

It is now possible to write down the complete matrix element,

$$\mathcal{M}_{fi} = \left(-\sqrt{2}\,G\,\frac{m}{M}\,\Phi\right)\left(\frac{2\pi}{L}\right)^{3/2}\mathcal{N}$$

$$\times\Big\{\frac{1+2a}{4}K^2 + k_{12}^2 + \alpha_T^2\Big\}^{-1}\sqrt{2}\,\sum_\mu(-1)^\mu\begin{pmatrix} s_i & \theta & s_f \\ m_s & -\mu & m_s' \end{pmatrix}$$

$$\times\Big\{\sqrt{2}\,\delta_{s_f,1}\,\delta_{e_{n_f},1}\Big\{\Big[v(k_{13})\,\underset{\sim}{k}_{13}\cdot\underset{\sim}{K}_2F(K_2) + v(k_{23})\,\underset{\sim}{k}_{23}\cdot\underset{\sim}{K}_1F(K_1)\Big]\underset{\sim}{k}_{12\mu}$$

$$+\left[C(k_{12})+\frac{1}{\sqrt{2}}T(k_{12})\right]\left[G(K)-\frac{1}{\sqrt{8}}H(K)\right](\underset{\sim}{k}_{12})_\mu$$

$$+\left[C(k_{12})+\frac{1}{\sqrt{2}}T(k_{12})\right]\frac{3}{\sqrt{8}}\,\underset{\sim}{k}_{12}\cdot\underset{\sim}{K}\,\frac{H(K)}{K^2}(\underset{\sim}{K})_\mu\Big\}$$

$$+\sqrt{\frac{15}{2}}\,\delta_{S_f 1}\,\delta_{\theta_\lambda 2}\left[C(k_{12})+\frac{1}{\sqrt{2}}T(k_{12})\right]\frac{H(K)}{K^2}W^2_\mu(\underset{\sim}{K},\underset{\sim}{k}_{12}\times\underset{\sim}{K})$$

$$-\frac{\sqrt{3}}{2}\,\delta_{S_f 0}\,\delta_{\theta_\lambda 1}\Big\{\left[v(k_{13})\,\underset{\sim}{k}_{13}\cdot\underset{\sim}{K}_2F(K_2)+v(k_{23})\,\underset{\sim}{k}_{23}\cdot\underset{\sim}{K}_1F(K_1)\right](\underset{\sim}{K})_\mu$$

$$+\left[C(k_{12})G(K)+T(k_{12})H(K)P_2(z)\right](\underset{\sim}{K})_\mu$$

$$+\frac{1}{\sqrt{8}}T(k_{12})\left[G(K)+\frac{1}{\sqrt{2}}H(K)\right](\underset{\sim}{K})_\mu$$

$$-\left[C(k_{12})-\sqrt{2}\,T(k_{12})P_2(z)\right]\frac{1}{\sqrt{2}}H(K)(\underset{\sim}{K})_\mu$$

$$-\frac{3}{\sqrt{8}}(\underset{\sim}{k}_{12}\cdot\underset{\sim}{K})\frac{T(k_{12})}{k_{12}^2}\left[G(K)+\frac{1}{\sqrt{2}}H(K)\right](\underset{\sim}{k}_{12})_\mu\Big\}\Bigg\}$$

$$(4\cdot10)$$

where

$$\underset{\sim}{K}_{1,2}=\underset{\sim}{K}/2\pm\underset{\sim}{k}_{12}\quad;$$

$$\underset{\sim}{k}_{13,23}=\eta\left\{\frac{1+2a}{2}K\pm k_{12}\right\}$$

$\delta_{r,s}$ = 1 for r=s, and = 0 for $r \neq s$.

Θ_r is the tensorial index of spin operator, 1 for $\underset{\sim}{S}$, $\underset{\sim}{\sigma}$ and ($\underset{\sim}{\sigma}_1 \times \underset{\sim}{\sigma}_2$) and 2 for $X^2(\sigma_1, \sigma_2)$.

W^2_r is defined in the table - 7.

We should next evaluate the square of the matrix element, averaged over the spin magnetic quantum numbers. This quantity directly related to the capture rate, gives all the information about the momentum spectrum and angular distribution, and we shall see how the different terms of the wave function contribute to this in pion capture.

$$\sum |\mathcal{M}_{fi}|^2 = \left(-\sqrt{2}\, G \frac{m}{M} \Phi\right)^2 \left(\frac{2\pi}{L}\right)^6 \left(\mathcal{N}/mM\right)^2$$

$$\times \Bigg[\frac{4}{3} q^2_{k_{12}} \Big\{ \big\{ C(q_{k_{12}}) + \frac{1}{\sqrt{2}} T(q_{k_{12}}) \big\}^2 \times$$

$$\big\{ G^2(K) + H^2(K) + \sqrt{2}\,\big[G(K) - \frac{1}{\sqrt{8}} H(K) \big] H(K)\, P_2(z) \big\}$$

$$+ \big\{ v(q_{k_{13}})\, \underset{\sim}{k}_{13} \cdot \underset{\sim}{K}_2\, F(K_2) + v(q_{k_{23}})\, \underset{\sim}{k}_{23} \cdot \underset{\sim}{K}_1\, F(K_1) \big\}^2$$

$$+ 2 \big\{ v(q_{k_{13}})\, \underset{\sim}{k}_{13} \cdot \underset{\sim}{K}_2\, F(K_2) + v(q_{k_{23}})\, \underset{\sim}{k}_{23} \cdot \underset{\sim}{K}_1\, F(K_1) \big\}$$

$$\times \big\{ C(q_{k_{12}}) + \frac{1}{\sqrt{2}} T(q_{k_{12}}) \big\} \big\{ G(K) + \frac{1}{\sqrt{2}} H(K)\, P_2(z) \big\} \Big\}$$

$$+\frac{1}{6}K^2\Big\{\{c(k_{12})-\sqrt{2}\,T(k_{12})P_2(z)\}^2$$

$$\times\{G(K)-\sqrt{2}\,H(K)\}^2$$

$$+T^2(k_{12})\{G(K)+\frac{1}{\sqrt{2}}H(K)\}^2$$

$$\times\{-2P_2^2(z)+P_2(z)+1\}$$

$$+\{v(k_{13})\,\underset{\sim}{k}_{13}\cdot\underset{\sim}{K}_2F(K_2)+v(k_{23})\,\underset{\sim}{k}_{23}\cdot\underset{\sim}{K}_1F(K_1)\}^2$$

$$+2\{v(k_{13})\,\underset{\sim}{k}_{13}\cdot\underset{\sim}{K}_2F(K_2)+v(k_{23})\,\underset{\sim}{k}_{23}\cdot\underset{\sim}{K}_1F(K_1)\}$$

$$\times\{c(k_{12})-\sqrt{2}\,T(k_{12})\}\{G(K)-\sqrt{2}\,H(K)\}\Big\}\Big]$$

$$=\left(-\sqrt{2}\,G\frac{m}{M}\Phi\right)^2\left(\frac{2\pi}{L}\right)^6(\mathcal{N}/mM)^2\,\mathcal{M}^2(K,z) \tag{4.11}$$

In the final expression above we have separated out the constant factor, and now consider the explicit dependence on the α-particle momentum $\underset{\sim}{K}$ and the CM angle Θ (through z) between $\underset{\sim}{K}$ and $\underset{\sim}{k}_{12}$, contained in $\mathcal{M}^2(K, z)$. The expression in the first large bracket here results from the major component of the interaction, i.e. from the $\underset{\sim}{S}.\underset{\sim}{p}$ term containing relative momentum of two nucleons. In this we have again written separately the contributions from the ''gG'' term of the wave function, the ''νF'' term and the interference term. We have already discussed about the percentage accurance of ''gG'' term in the last chapter. (Table 4, 5). The second large bracket contains the contribution of the ''minor'' term of the interaction, i.e. the $\underset{\sim}{\sigma}.\underset{\sim}{P}$ term or the CM motion term.

It will be in order to discuss at this stage the z-dependence of the squared matrix element. It should be noted that in the absence of tensor force the 'gG'' term (which now reduced to ''αG'') representing the $\alpha + d$ structure contains no z-dependence at all. When the tensor force is included, one obtains a small z-dependence through the $P_2(z)$ term; however such a term always has as its coefficient one of the factors $T(k_{12})$ or $H(K)$. Since the tensor force is rather weak in the

NLS model it is permissible to take (G67a)

$$\left| \frac{H(K)}{G(K)} \right| < \left| \frac{T(k_{12})}{C(k_{12})} \right| < 1$$

so that the z - dependence introduced by the tensor force is rather weak. Koltun and Reitan (K67a) assume, in addition to the α-d structure for ^{6}Li, that the D-state consists only of the $\ell=2$, L=0 term (as previously pointed out), and hence find $\mathcal{M}^2$ to be completely z -independent. On the other hand Kopalieshvilli and Machabelli (K67c) consider only the $\ell=0$, L=2 term in the D-state (ignoring the $\ell=2$, L=0 term) and hence obtain some z -dependence for $\mathcal{M}^2$.

In our model we also have (from α-N interactions) terms such as $|\nabla F|^2$ and interference terms $|\nabla F . gG|$ contributing to the matrix element, and since such interactions are P-wave forces, a rather strong z -dependence in the matrix element arises from such terms which are by no means small. Our result thus differs from that of most other authors in this respect, and gives a fairly strong z -dependence for the cross-sections. We shall discuss this aspect in more detail later on.

4. C Results and Discussion.

In order to compute the total capture rate, momentum spectrum and the angular distributions we need the following equations.

The total capture rate

$$W = 2\pi \sum |\mathcal{M}_{fi}|^2 \rho(E_f)$$

$$= 2\pi \iint \frac{d^3K}{(2\pi)^3} \frac{d^3k_{12}}{(2\pi)^3} (L)^6 \sum |\mathcal{M}_{fi}|^2 \delta(E_K + E_{k_{12}} + \frac{\alpha_T^2}{M} - m)$$

(4.12)

From this it follows

$$d^2W/dK\,dz = 2\pi\left(-\sqrt{2}\,G\frac{m}{M}\Phi\right)^2 (\gamma/mM)^2$$

$$\times \left(\frac{Mk_{12}}{2}\, 8\pi^2 K^2\right) \mathcal{M}^2(K, z)$$

$$= \text{Const.}\ K^2 k_{12} \mathcal{M}^2(K, z)$$

or

$$\frac{d^2w}{dK\,dy} = \text{Const.}\,\frac{(K^2/4 - k_{12}^2)^2 \quad K^2 k_{12} \mathcal{M}^2(K,z)}{K k_{12} y^2 \left\{\left(\frac{K^2}{4} + k_{12}^2\right)^2 y^2 - \left(\frac{K^2}{4} - k_{12}^2\right)^2\right\}^{1/2}}$$

$$\frac{dw}{dK} = \text{Const.}\ K^2 k_{12} \int dz\, \mathcal{M}^2(K, z)$$

and $$dW/dy = \int dK \left(d^2W/dK\,dy\right)$$

We note that the spectator functions to be used in evaluating the matrix element are all obtained only in a numerical form (via numerical integration of the eigenvalue equations) in the chapter 3. For calculational purpose, we fit these numerical forms to analytical expressions of the following type (chosen for convenience in carrying out analytically angular integrations):

$$F(x) = \sum_{i=1}^{3} A_i \left(x^2 + \alpha_i^2\right)^{-1}$$

The six parameters A_i and α_i are chosen to give best fit to the numerically known values of all spectator functions, and we find in every case quite satisfactory fit.

We shall first discuss the total capture rate. The absolute value of the capture rate has not yet been measured experimentally, nor has the value been quoted in any theoretical calculations. Table-8 shows individual contributions to the capture rate from

Table - 8

Different terms of Ψ_i and Interactions	Absolute capture rate W and % contribution. C_Y^{eff} W	%	$(C+T)_Y$ W	%
$\|gG\|^2$	1.34	16.79	0.16	3.26
$\|gG.\nabla F\|$	1.88	23.56	0.54	11.02
$\|\nabla F\|^2$	3.17	39.72	2.79	57.94
major	6.39	80.07	3.49	71.22
minor	1.59	19.93	1.41	28.78
Total	7.98		4.90	

The absolute values of the capture rates are expressed in units of 10^{16} sec^{-1}.

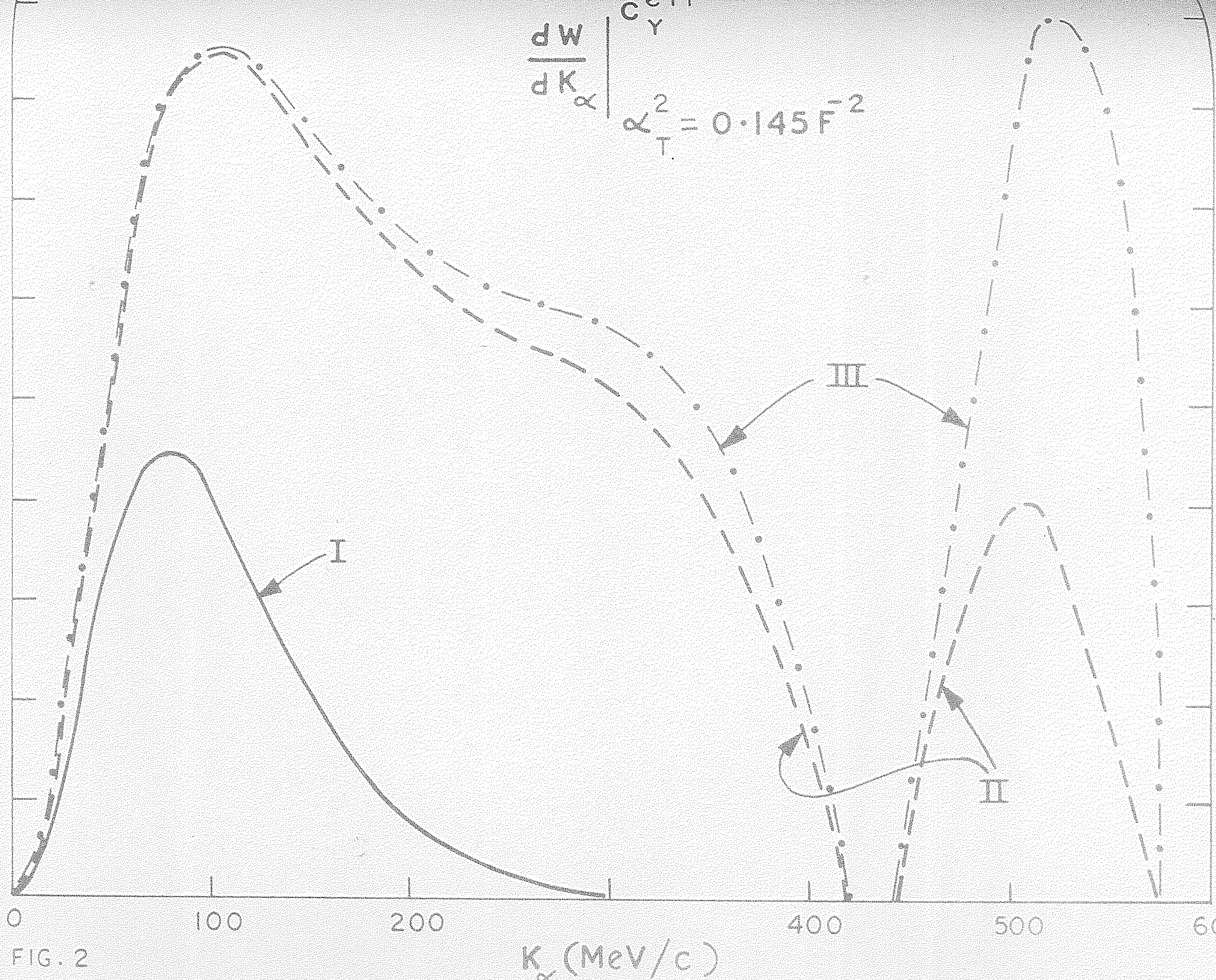
different terms in the wave function as well as from the major and minor terms of the interaction. We compare the results for the effective central force of Yamaguchi (C_Y^{eff}) and the central + tensor force model of Yamaguchi $(C + T)_Y$.

One immediately notices from Table-8 the sizable contribution of the minor ($\sigma . P$) term of the πN interaction to the capture rate. This contribution is about 20 % for the C_Y^{eff} interaction and about 29 % for the $(C + T)_Y$ interaction. If one looks at the origin of these contributions in detail, it becomes clear that they arise from the terms in wave function where one of the nucleons is a spectator, i.e. from terms which occur directly as a result of α-N correlations being taken into account. It is therefore not surprising that earlier theoretical works, which do not consider α-N interactions, do neglect this $\sigma . P$ term from pion-nucleon interaction. In our model such a neglect would not be legitimate. We shall later see the effect of this minor term on angular distributions, momentum spectra etc.

The contribution to the capture rate from the $|gG|^2$ term (which we have interpreted as representing the α-d structure) is about 25 times as large as the capture rate in deuterium (i.e. W_d), calculated in table 3. This factor can easily be seen to arise from the charge ratio of two nuclei occuring in Bohr wave function for pion $\left[(Z_{Li}/Z_D)^3 = 27 \right]$, and thus strengthens our interpretation of the structure of this term.

Next a comparison of the results for the wave functions given by C^{eff} and (C+T) interactions shows the role of S-D cancellation emphasized by Koltun and Reitan (K66a). We see that the reduction in the capture rate brought about by inclusion of D-state (through tensor force) is as much as 88 % in the α-d term, and about 70 % in the interference term. However the term $|\nabla F|^2$ representing α-N correlated pair plus a spectator nucleon is affected by barely 12 %; this is more a dynamic effect of the tensor force rather than a D-state cancellation. Now it is also clear that even for the effective central N-N force, the nucleon-spectator term contributes as much as 50 % of the capture rate due to the ''major'' term, and the contribution of the term giving α-d structure is barely 20 % of the total capture rate. Further the contribution of the ''minor'' term ($\sigma \cdot P$) also remains almost unaffected by the tensor force, since it also arises primarily from nucleon-spectator term of the wave function. We therefore see that the total capture rate is reduced by only about 40 % due to the inclusion of the tensor force, and the S-D cancellation effects are no longer as severe as in the deuteron case. It needs to be emphasized that the inclusion of α-nucleon forces and correlations in our model give very large and qualitative differences for our results as compared to those of previous calculations in literature.

We now discuss in some detail the nature of the momentum distribution for the α-particle, i.e. the center of mass momentum of the valence nucleon pair. From the matrix element $\mathcal{M}^2(K, z)$, integration over $d\varphi$ can be carried out analytically to yield the momentum distribution dW/dK_α. In figures 2 and 3 we have shown the momentum spectrum for wave functions labelled by $C^{eff.}$ and C+T respectively. In each figure the contribution of the $(\alpha+d)$ term in the wave function, the contribution due to the major $(\underset{\sim}{S}.\underset{\sim}{p})$ term of the interaction with the total wave function, and the complete contribution (i.e. including that of the minor term as well) is shown separately. The corresponding curves are labelled I, II, III in fig.2 and IV, V, VI in fig.3.



$\frac{dW}{dK_\alpha}\Big|_{C_Y^{cell}}$
$\alpha_T^2 = 0{\cdot}145\,F^{-2}$
I
II
III
0
100
200
400
500
600
K_α (MeV/c)

FIG. 2

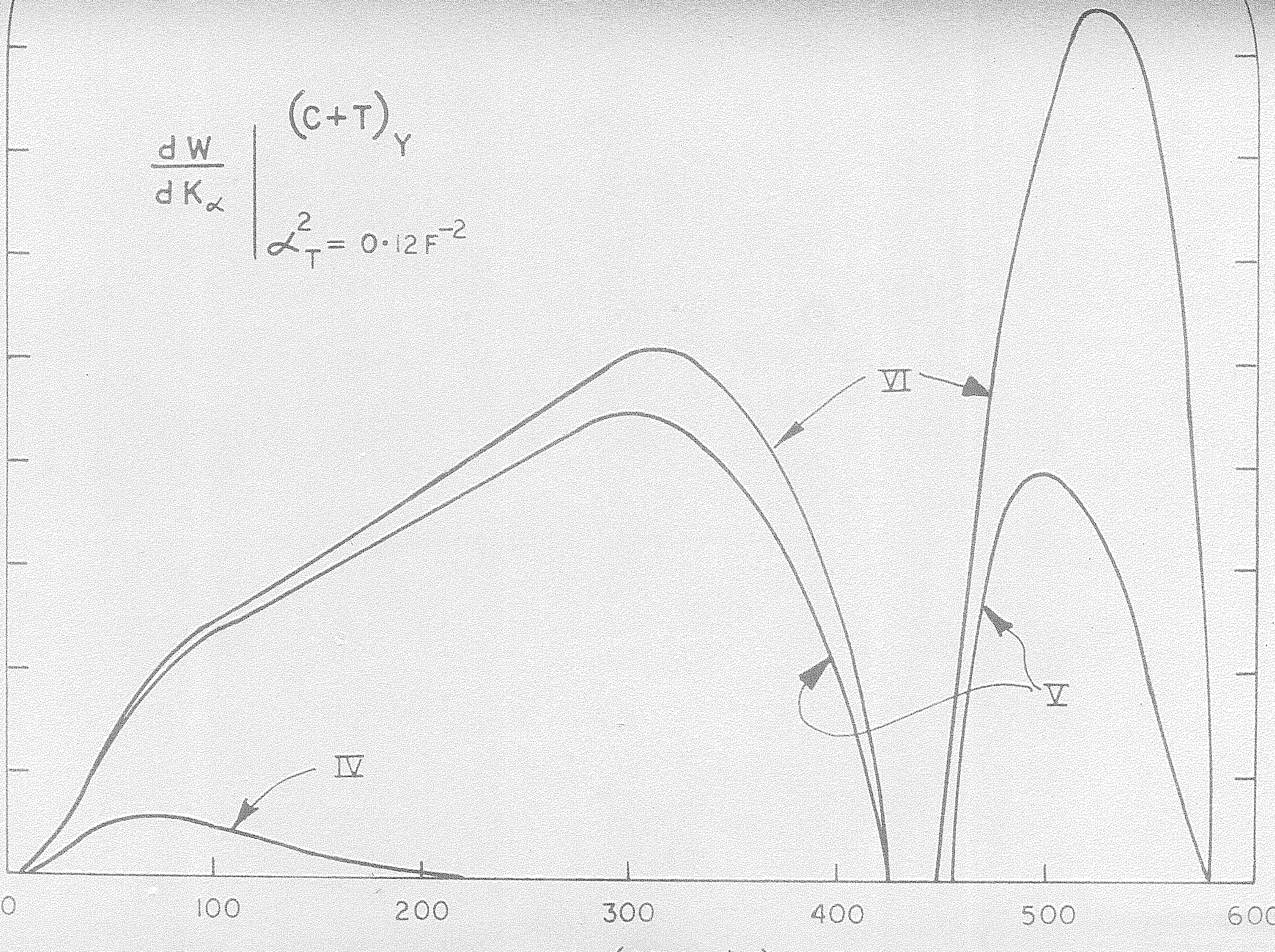


FIG. 3

It is easy to see that if one only considers an (α-d) model for ^{6}Li with pion capture taking place on the valence nucleons, the neutrons would be emitted with large relative momentum, and the spectator α-particle momentum would be rather small, obeying a shell model type distribution. It can be seen from curve I of fig. 2 that the spectrum does show a peak around 75 MeV/c (corresponding to $E_\alpha \sim$ 1MeV). With the addition of α-N correlation terms very drastic changes occur in the character of the spectrum. The α-momentum distribution arising from (α-d) structure itself seems to be enhanced somewhat with the peak shifting to somewhat higher momentum, i.e. $\sim$ 100 MeV/c. At the same time a second very broad distribution arises directly from the $|\nabla F|^2$ type terms, with a peak at about 200 MeV/c, a diffraction minimum at about 425 MeV/c, and a second maximum near 500 MeV/c (see curve II) This second peak is presumably the α-particle recoil when the absorption of π^- takes place on a correlated α-proton pair. When the minor term of the πN interaction is also included, its $\underset{\sim}{\sigma}.\underset{\sim}{P}$ dependence strongly enhances the high momentum component of the distribution, although the low momentum component (K < 400 MeV/c) is only slightly modified.

Fig.3 illustrates the effect of the tensor force which is already partially discussed earlier. The peak in the momentum distribution at 75-100 MeV/c arising from the α-d structure is now very much reduced. The momentum-spectrum is almost completely governed by the α-N correlation terms of the wave function. We now obtain two very prominent peaks for α-particle momenta at ~ 300 and ~ 500 MeV/c. Again note that the tensor force has practically no effect on the high momentum side, as this arises largely from the α-nucleon correlations. For a direct comparison we give in figure 4, the curves I, II and IV, V side by side. Unfortunately it is not possible to compare these results directly with experimental data. We may however compare our results with those of Koltun and Reitan (See their fig.3 (K67a)). They report (for no meson-scattering case) two peaks at about 85 MeV/c and 275 MeV/c, with a diffraction minimum at 170 MeV/c. They do not include the $\sigma.P$ term so that the high momentum peak at ~ 500 MeV/c does not occur in their case. It should however be noted that when second order meson scattering terms are included in interaction, the two peaks merge into only one at about 140 MeV/c. It should be of great interest to check these spectra experimentally, so that relative roles of tensor

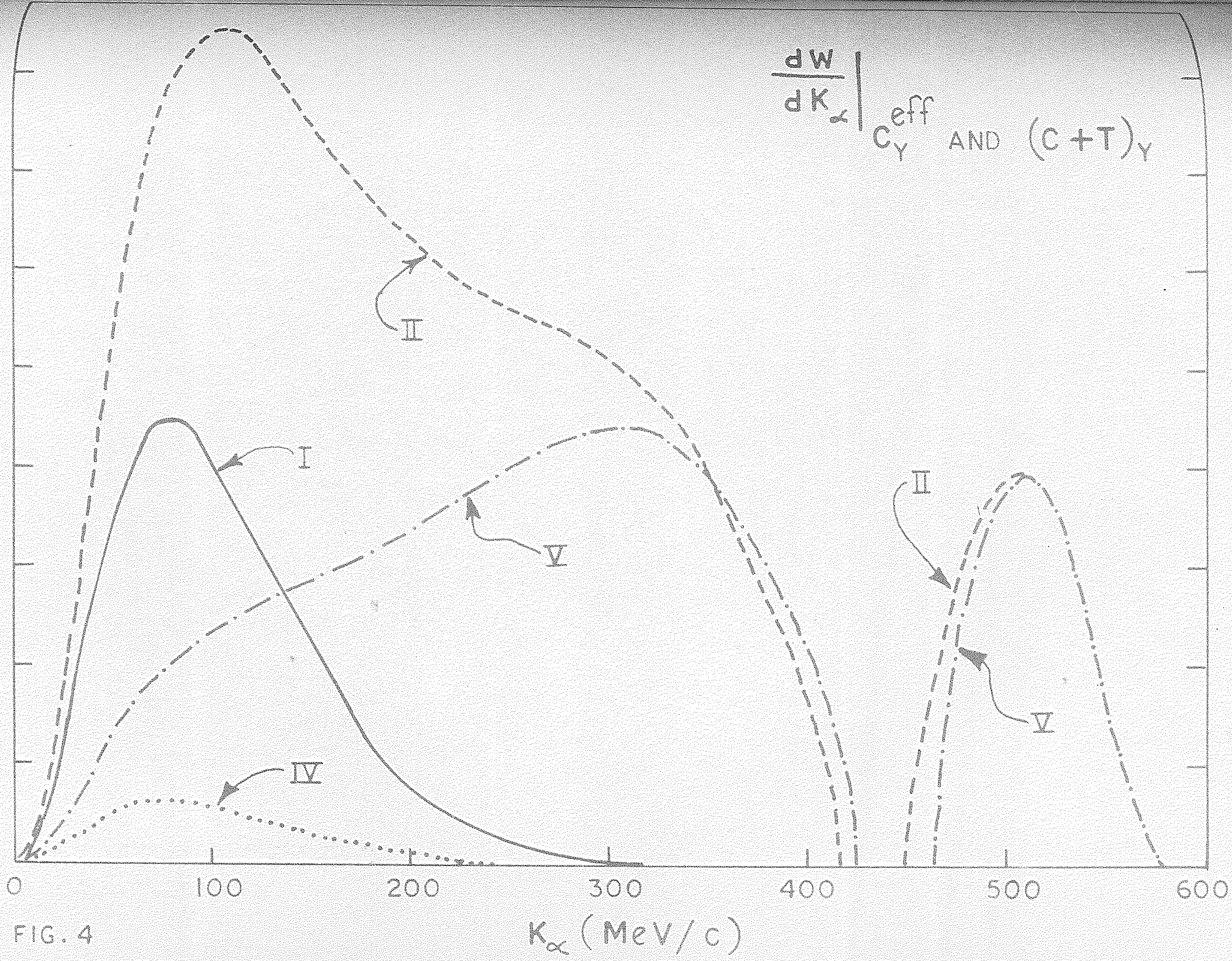
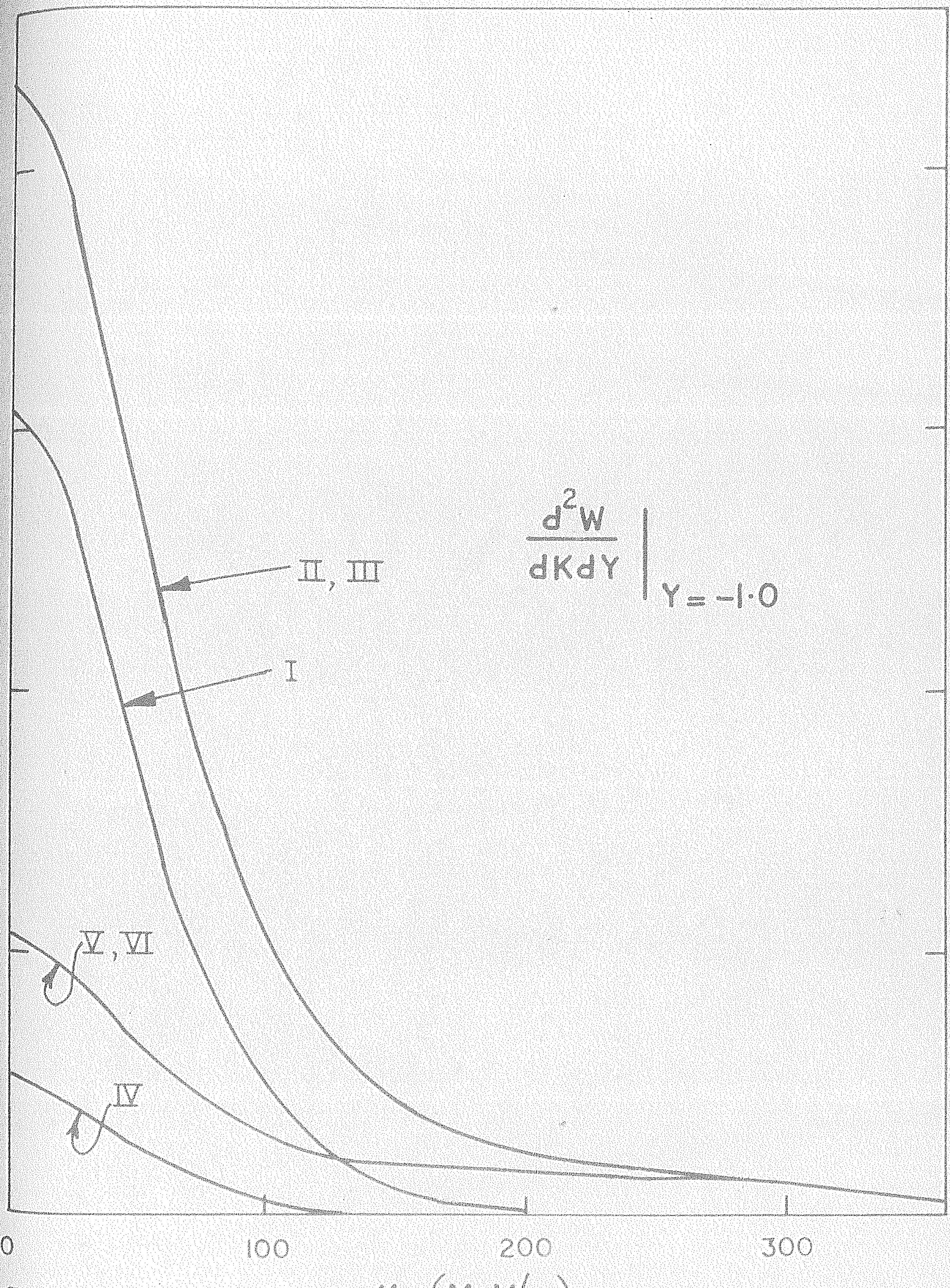

dW/dK_α | C_Y^eff AND (C+T)_Y
I
II
II
IV
V
V
0
100
200
300
400
500
600
K_α (MeV/c)

FIG. 4

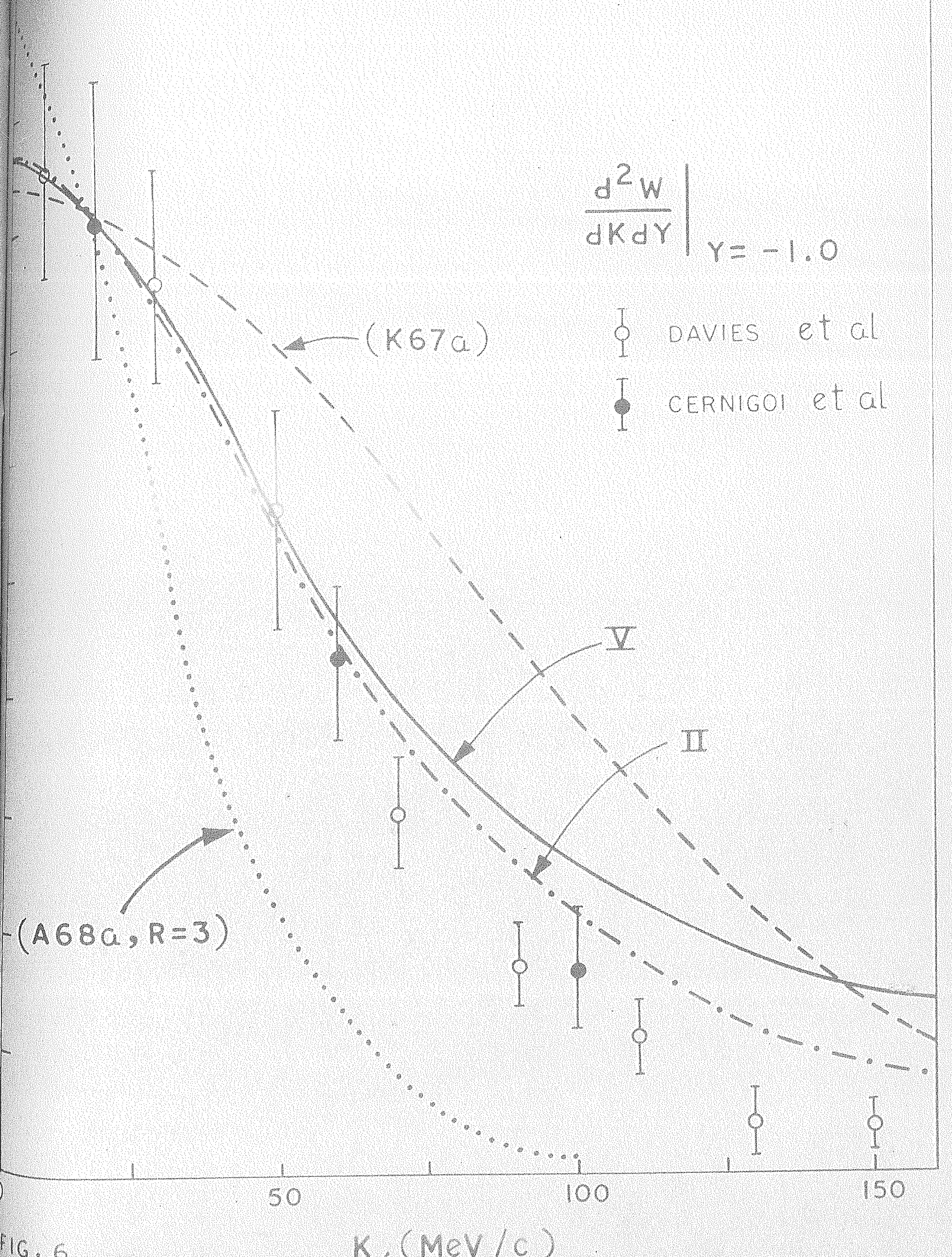
force, α-nucleon correlations, center of mass motion term ($\sigma.P$) etc. can be clarified.

The quantity of interest that can be checked against the available experimental results is the distribution in momentum for the case when the laboratory angle between two emitted nucleons is 180°, i.e. $Y = -1$. We have therefore to calculate $d^2W/dKdY$ at $Y = -1$. The experimental data is due to Davies et.al. (D66a) and Cernigoi et.al. (as quoted in (A68b)). Unfortunately, selection of experimental data for only back to back ejection of neutrons emphasizes only a small part of the theoretical information. Such a process selects out capture by the valence nucleon pair, i.e. the α-d terms of the wave function. The α-particle is left with a low momentum, and contribution of $\sigma.P$ term of πN interaction, as well as of the vF terms in the wave function are de-emphasized. In fig.5 we plot the curves for $d^2W/dKdY$ at $Y = -1$ for the cases I - VI described above. The effect of the tensor force is quite severe here, since the main contribution is from the $|gG|$ terms. In Fig.6, our curves for cases II and V (since the contribution of the minor term $\sigma.P$ is negligible anyway) are compared with the experimental data as



$\frac{d^2W}{dKdY}\Big|_{Y=-1\cdot 0}$
II, III
I
V, VI
IV
0
100
200
300
$\longrightarrow K_\alpha$ (MeV/c) $\longrightarrow$

FIG. 5



$\left.\frac{d^2W}{dKdY}\right|_{Y=-1.0}$
DAVIES et al
CERNIGOI et al
(K67a)
V
II
(A68a, R=3)
50
100
150
K, (MeV/c)

FIG. 6

well as the theoretical results of Koltun and Reitan (K67a) and of Alberi and Tafera (A68b). All the theoretical curves are normalised to the experimental point at K = 20 MeV/c, since they are generally plotted in arbitary units. We show the results of (A68b) for only one value of R = 3.0F which is a more reasonable value for the interaction range between α-particle and the deuteron. It is obvious that our results are in satisfactory agreement with the experimental data. The α-nucleon correlation effects begin to show up for $K \geqslant 100$ MeV/c.

The angular distribution of two neutrons given by dW/dy, i.e. with respect to the laboratory angle ψ_{lab} is given in figure 7. The distribution is expected to peak at $\psi_{lab} = \pi$. Again we find that the ''gG'' term contributes mainly for $\psi_{lab} > 150^{\circ}$, and its contribution is drastically reduced by the tensor force (curves I, IV). The effects of α-N correlations (σF terms) and of the ''minor'' term ($\sigma . P$) are (i) to raise the crosssection in the region near $\psi_{lab} = \pi$, and (ii) to give a small but finite possibility of the two neutrons being emitted with $\psi_{lab} = 30^{\circ} - 60^{\circ}$. Nordberg et.al. (N68a) do seem to observe this later effect. The width of the peak (or the sharpness of the fall of $dW/d\psi$ away from $\psi_{lab} = \pi$) seems to

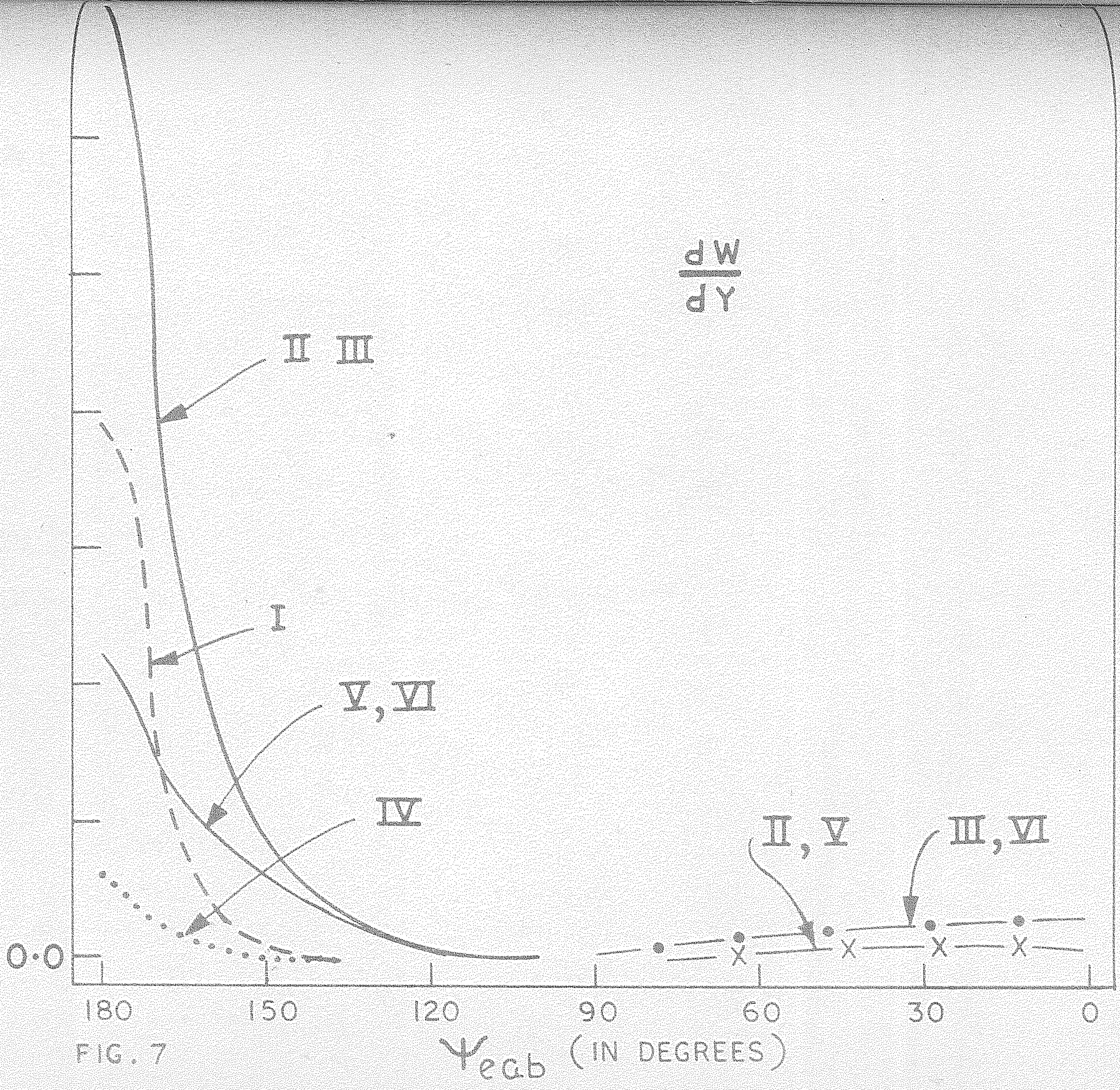

dW/dY
II III
I
V, VI
IV
II, V
III, VI
0·0
180
150
120
90
60
30
0
Ψeab (IN DEGREES)

FIG. 7

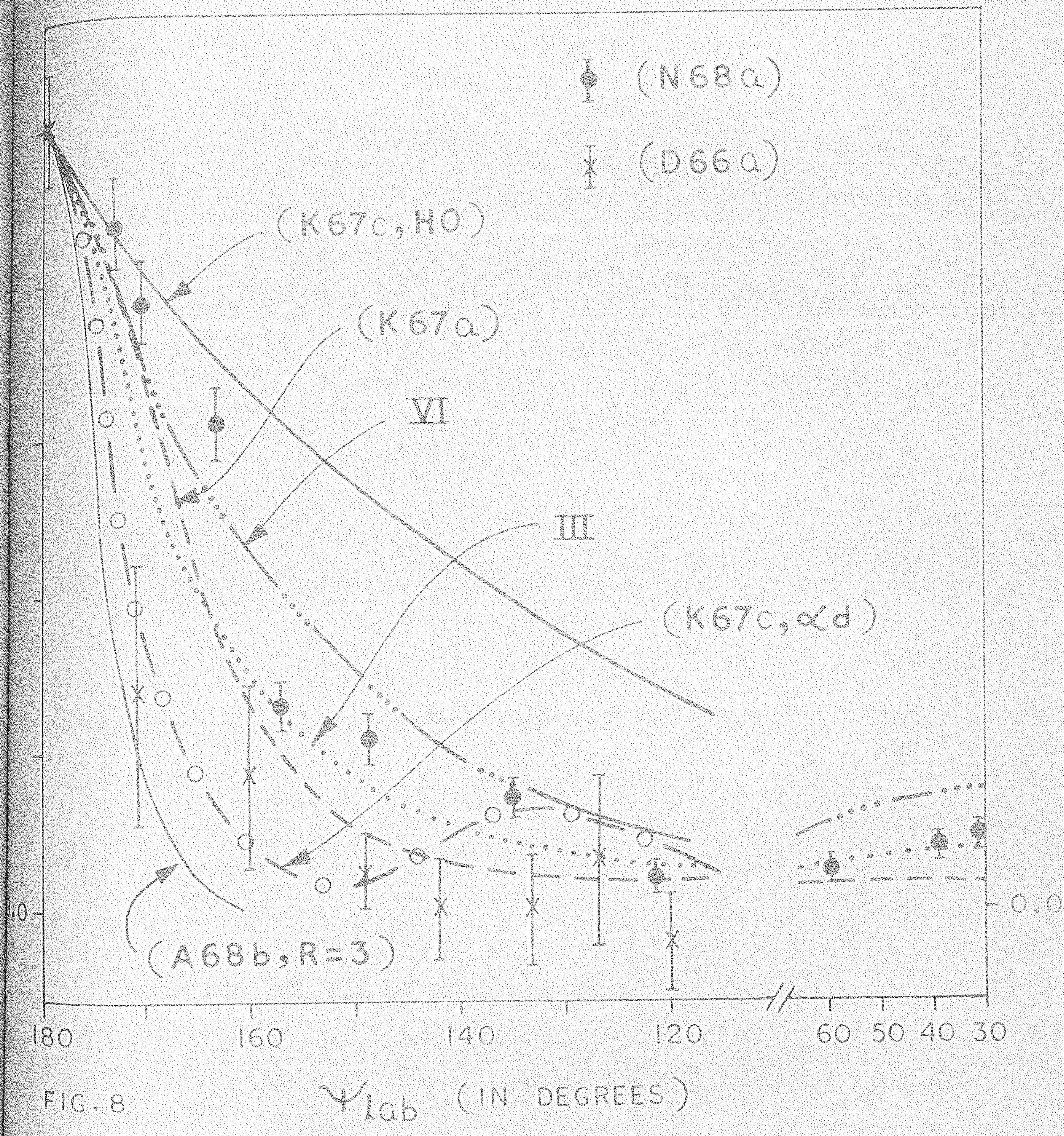

(N68a)
(D66a)
(K67c, HO)
(K67a)
VI
III
(K67c, αd)
(A68b, R=3)
0.0
180
160
140
120
60
50
40
30
Ψlab (IN DEGREES)

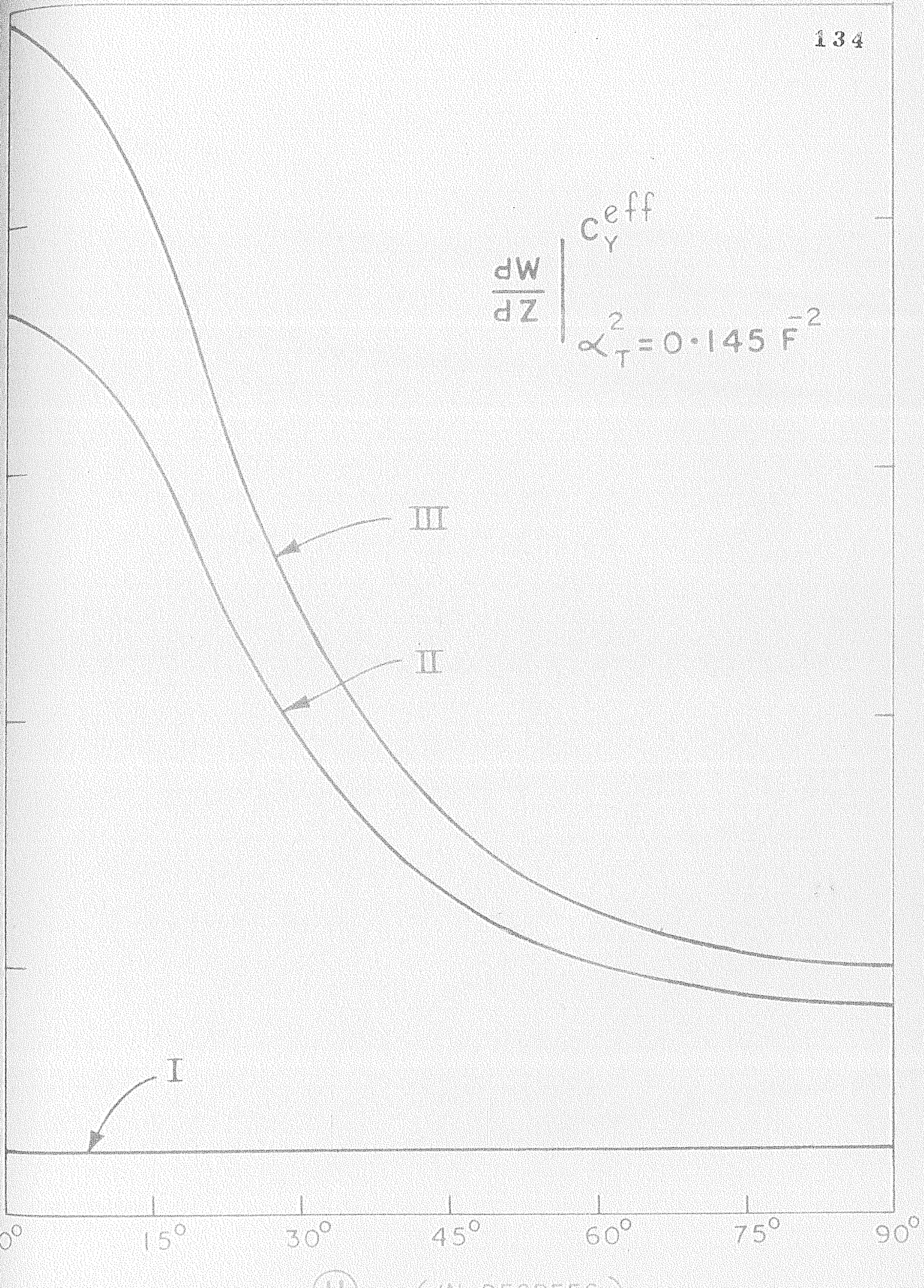
FIG. 8

vary in different cases. In figure 8 we compare our results for cases III and VI with the observations of Nordberg et.al. and also with other previous calculations of Koltum-Reitan (as given in Ref. (N68a)) and Alberi-Taferra. The results of Nordberg et.al. show a somewhat less steep fall away from the peak compared to those of Davies et.al. We have again normalised all the curves at $\psi_{lab} = \pi$. Our results for case III i.e. with central effective potential appear to fit the experiments better than for case VI i.e. with tensor force.

We should here like to make a comment on the role of tensor force and the second order pion rescattering terms of the πN interaction. If we recall the calculation of pion capture in deuterium in chapter 2, we see that the introduction of the tensor force reduces the capture rate by a large factor. As Koltun and Reitan (K66a) pointed out, the inclusion of second order terms in π-N interaction which represent pion scattering before capture can again increase the capture rate to a reasonable value. The two effects thus act in opposite directions. A similar effect also occurs in ^6Li. Since we are not including any pion-scattering terms in our interaction, our results with tensor force are likely to err in one direction. It may be thus more reasonable to consider for comparison with experimental results with central effective N-N force only and drop both the tensor NN force as well as the second order terms in πN interaction. Our earlier discussion actually shows that in general results with C^{eff} alone are in more satisfactory agreement with experiments.

Finally, we give in Fig.9 the angular distribution for two neutrons in CM frame of reference i.e. dW/dZ. The results for the N-N interactions C^{eff} and C+T are quite similar, except for the peak at $\Theta_{CM} = 0$ being smaller for the latter case. We therefore give results only for C^{eff} wave functions. The results confirm our earlier discussion viz., the 'gG' term (curve I) gives Z - independent contribution, whereas the inclusion of α-nucleon correlations and the $\sigma . P$ term introduce a strong Z- dependence. If there does exist such a Z- dependence in the crosssection, the analysis of Nordberg et.al. (N68a) where the theoretical curves are calculated with the assumption of Z - independent matrix elements, should be somewhat doubtful.

To conclude, we summarise the results of this chapter. We used the complete ^{6}Li ground state wave function derived in the previous chapter with the simple psedoscalar pion-nucleon first order interaction to



$\frac{dW}{dZ}\Big|_{\alpha_T^2 = 0\cdot145\,F^{-2}}^{C_Y^{eff}}$
III
II
I
0°
15°
30°
45°
60°
75°
90°
Θ_{CM} (IN DEGREES)

FIG. 9

calculate the capture rates, angular and momentum distributions of two nucleons etc. Our treatment differs from those of other authors in that we have explicitely included in the wave function α-nucleon correlations as well as the CM dependent term $\sigma^{-}.P$ in the pion-nucleon interaction. Both these factors are generally ignored. Our results show that inclusion of these two factors have a significant effect on the capture rates as well as momentum and angular distributions (particularly for large α-particle momenta). It may be that our model somewhat exaggerates these effects, but nevertheless they are quite significant. Unfortunately, the present experimental data are inadequate to check these features, but we suggest that high α-particle momentum events (with α in the ground state) should be carefully investigated to determine precisely the role of these generally ignored terms. This can be done by either looking for a peak in α-particle momentum distribution at $K_{\alpha} \sim 500$ MeV/c or $E_{\alpha} \sim 40$ MeV, and/or the Z-dependence of the two-nucleon angular distribution.

The role of the tensor force in suppressing the capture rate is again quite clearly shown for the α+d type structure in the wave function, but for the overall capture rate this effect is no longer very drastic since the α-nucleon correlation type terms in the wave function are not much affected by the tensor force, and give large contribution to the capture rate. It would be very desirable to have measurements of absolute capture rates in ^{6}Li.

5. PION CAPTURE IN ^{3}HE

In this chapter we shall consider pion capture on ^{3}He nucleus. In this case the capture can take place by either a p-p pair or a p-n pair, so that the process is somewhat more complex than in the case of ^{6}Li. We have discussed already in previous chapter the treatment of the problem by various authors, Diwakaran (D65a), Sakamato and Tohsaki (S67b), Figureau and Ericson (F69a) etc. In these treatments the proton-structure of the wave-function is explicitely exhibited. Our treatment differs from these authors in that we use the fully antisymmetrised wave function in three nucleons (in space, spin and Ispin coordinates) obtained by solving the Schrödinger equation in momentum space with the non-local separable nucleon-nucleon potentials. It will thus be possible to exihibit explicitely the contributions of various symmetry terms in the wave functions. Further, we shall see that in such a three-body treatment, even the apparently uncorrelated or spectator proton can have large momentum components, and hence can capture a pion, whereas in other treatments the uncorrelated nucleon is incapable of capture. This happens since in the three-body treatment (in which we follow essentially the work of Mitra and collaborators (M66c, M68a)), even the spectator function has large dynamic effects built into it. This was already demonstrated in the case of ^6Li. Our calculations are thus more complex and we believe more complete than those of other authors reported previously.

Pion capture in ^3He leads to several possible final states.

$$\begin{aligned} \pi^- + {}^3He \rightarrow\ & n + d\ (d),\quad {}^3H + \gamma\ (\gamma) \\ & n + n + p\ (p),\quad {}^3H + \pi^0\ (CE) \\ & n + n + p + \gamma\ (p\gamma),\quad n + d + \gamma\ (d\gamma) \end{aligned} \qquad (5.1)$$

Our interest here is mainly in the deuteron (d) and proton (p) modes which are purely nuclear pair-absorption processes. The last two modes in which an additional γ -ray is also emitted have very small crosssections because of the small phase-space availibility. The relative crosssection for the $(d\gamma)$ mode is known to be only about (3.6 ± 1.2) % (Z67a). We do not consider them in the following. The γ -mode and the charge-exchange scattering (CE) are single-nucleon processes and are usually calculated in the impulse approximation. Our pion-nucleon interaction is linear in pion-field so that the CE mode can occur only as a second-order process. We do not consider it either. The γ - mode is calculated here (for sake of comparison with d - or p-modes) somewhat crudely using the gauge-invariant interaction ($\underset{\sim}{P} \rightarrow \underset{\sim}{P} - \frac{e}{c}\underset{\sim}{A}$).

5.A π - ^{3}He interaction.

For writing down the wave function of ^{3}He we shall use the full group of permutation symmetries of three nucleons in coordinate, spin and Ispin space. It will then be convenient to express pion-nucleon interaction also in a 3-body form rather than a two-body form as was done in equation (2.16) of chapter 2. The required algebra is given by Verde in (V57a).

For any three operators or functions $\mathcal{O}_1$, $\mathcal{O}_2$ and $\mathcal{O}_3$, one can construct a completely symmetric combination

$$\mathcal{O}^s = \mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3 \qquad (5.2)$$

and two combinations which can form the basis for [2, 1] representation of the symmetry group,

$$\mathcal{O}' = \frac{\sqrt{3}}{2}(\mathcal{O}_3 - \mathcal{O}_2), \qquad \mathcal{O}'' = \mathcal{O}_1 + 1/2(\mathcal{O}_2 + \mathcal{O}_3) \qquad (5.3)$$

The permutation operators P_{ij} leave $\mathcal{O}^s$ invariant and have the following matrix representations in the space of $\mathcal{O}'$, $\mathcal{O}''$.

$$P_{23} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} ; \qquad P_{12,13} = \begin{pmatrix} 1/2 & \pm\sqrt{3}/2 \\ \pm\sqrt{3}/2 & -1/2 \end{pmatrix} \qquad (5.4)$$

We can now write the total pion-nucleon interaction for three nucleons as

$$H_{int} = \left(-\frac{m}{M} G\right) \sum_{i=1}^{3} \left(\underline{\tau}_i \cdot \underline{\varphi}(i)\right)\left(\boldsymbol{\sigma}_i \cdot \boldsymbol{p}_i\right)$$

$$= \left(-\frac{m}{M} G\right) \frac{2}{9} \Big[\underline{\Phi} \cdot \Big\{ \left(\underline{\tau}^{s} \boldsymbol{\sigma}' + \underline{\tau}' \boldsymbol{\sigma}^{s} + \underline{\tau}' \boldsymbol{\sigma}'' + \underline{\tau}'' \boldsymbol{\sigma}'\right) \cdot \boldsymbol{p}'$$

$$+ \left(\underline{\tau}^{s} \boldsymbol{\sigma}'' + \underline{\tau}'' \boldsymbol{\sigma}^{s} + \underline{\tau}' \boldsymbol{\sigma}' - \underline{\tau}'' \boldsymbol{\sigma}''\right) \cdot \boldsymbol{p}'' \Big\} \Big] \tag{5.5}$$

In the above derivation we note that for the pion wave function

$$\varphi(1) = \varphi(2) = \varphi(3) = \Phi ;$$

$$\varphi^{s} = 3\Phi \quad ; \quad \varphi' = \varphi'' = 0 \tag{5.6}$$

Further for S-wave pion capture which alone we consider, the requirement of parity change between initial and final state wave-functions prohibits contributions from terms involving $\boldsymbol{p}^{s}$ in the H_{int} and hence they have been dropped. With the above form it will be very simple to evaluate the matrix elements. For the negative pion capture we shall specialise to each $\underline{\tau} \rightarrow \sqrt{2}\,\tau_{-}$ as in the previous cases.

5.B Wave function for the three-nucleon system:

It is now well established that ^3H - ^3He constitute an Ispin doublet (T = 1/2). The Coulomb energy evaluation for ^3He (A68a, G67b) confirms that nuclear forces are essentially charge-independent. However, the analysis of the charge form factor F_{ch} (^3He) by Griffy (G64a) appears to indicate a small mixing of T = 3/2 state (about 2 % probability) in the ground state of ^3He. In our treatment we only consider T = 1/2 state for the ^3He wave function. Also the magnetic moments of ^3H and ^3He indicate that the two wave-functions are almost symmetric in configuration space, the most important states thus being L = 0, J=S. The inclusion of tensor force. will result in a small probability for $L \neq 0$ (mainly D-state) also being present in the wave function. Fortunately, the choice of a special form of the tensor force as in chapter 2 enables such a force to be included very easily. Thus the major component of the wave function is the $^2S_{1/2}$ state (L = 0, $\mathcal{S}$ = 1/2, T = 1/2). The space-part of the wave function is the symmetric S-state Ψ_S^S with a very small admixture of the mixed symmetric S-states Ψ_S' and Ψ_S''. The solution of the Schrödinger equation with various forces yields the probability of mixed symmetric S-states to be about 1 %.

Let ψ, χ and ζ denote the space, spin and Ispin components of the wave function. For the spin-Ispin part we can construct (T = 1/2, S=1/2) the various combinations (see V57a).

$$\xi^s = \frac{1}{\sqrt{2}}(\chi'\zeta' + \chi''\zeta'') \quad ; \quad \xi^a = \frac{1}{\sqrt{2}}(\chi'\zeta'' - \chi''\zeta')$$

$$\xi' = \frac{1}{\sqrt{2}}(\chi'\zeta'' + \chi''\zeta') \quad ; \quad \xi'' = \frac{1}{\sqrt{2}}(\chi'\zeta' - \chi''\zeta'')$$

$$\chi^s_{m_s=1/2} = \frac{1}{\sqrt{3}}\{(++-) + (+-+) + (-++)\}$$

$$\chi'_{m_s=1/2} = \frac{1}{\sqrt{2}}\{(++-) - (+-+)\}$$

$$\chi''_{m_s=1/2} = \frac{1}{\sqrt{6}}\{-2(-++) + (++-) + (+-+)\}$$

$$(5.7)$$

and then the complete antisymmetric wave function for the three-nucleon system can be written as

$$\Psi = \psi^a\xi^s - \psi^s\xi^a + \psi'\xi'' - \psi''\xi' \quad (5.8)$$

Here the subscript 'a' indicates a completely antisymmetric state. An extensive treatment of the three-body system has been given in a series of papers by Mitra and collaborators (B63a, M63a, M66c), and we shall mostly borrow their results for wave functions with different types of non-local separable interactions. We only consider the following forms of the potentials.

(i) A spin-independent average central S-wave force with $\beta_t = \beta_s = \beta = 1.4487\ F^{-1}$; $\bar{\lambda} = 1/2(\lambda_t + \lambda_s) = 0.353\ F^{-3}$. This force is defined as $C_Y^{avr.}$ in the following text.

(ii) A spin-dependent but central S-state force with parameters denoted by C_Y^{eff} and S_Y^{eff} given in table 1 of chapter 2.

(iii) A central + tensor force in the triplet even states, and a central singlet even S-state force, with parameters that are also defined in table 1 of chapter 2.

Since we only consider even-state potentials, we can immediately drop the $\psi^{\alpha}\xi^{s}$ term of equation (5.8). The space-parts of the wave function ψ^{s} (major term) and ψ', ψ'' (the mixed-symmetric S' state) are obtained by solving the Schrödinger equations. For details we refer to (B65b, M66c). Here we give only the results for the structure of the space-parts of the wave-functions.

(i) The structure of the Schrödinger equations for ψ^S, ψ' and ψ'' shows that in the absence of the spin-dependence of the N-N interaction, the equation for ψ^S is completely uncoupled from ψ' and ψ'', and the ground state has no ψ' or ψ'' components. We have for

$$\psi^S = D_E^{-1} \sum_{ijk} g(q_{ij}) \quad F(P_K) \qquad (5.9)$$

where

$$D_E = q_{ij}^2 + P_K^2 + \alpha_T^2$$

with $\underset{\sim}{q}_{ij} = (\underset{\sim}{P}_i - \underset{\sim}{P}_j)/2, \quad \alpha_T^2 = -ME_b$

Numerical integration of the eigen-value equation for ψ^S yields $F(P_K)$, the spectator function for the K^{th} nucleon, in a numerical form. When the three-body value of $\bar{\lambda}$ is properly matched to the two-body value (0.353 F^{-3}), the binding energy for ^3He turns out to be 12.43 MeV, which is rather large compared to the experimental value viz., 8.48 MeV (the Coulomb-corrected value). However, the correct binding energy is obtained for a value of $\bar{\lambda}$ (three-body) = 0.328 F^{-3} which is only about 7 % decrease from the appropriate two-body value.

(ii) In this case, we obtain

$$\Psi^{s} = D_E^{-1} \sum_{ijk} \{ g(q_{ij}) F(P_k) + f(q_{ij}) G(P_k) \}$$

$$\Psi' = \frac{\sqrt{3}}{2} D_E^{-1} \{ [g(q_{12}) F(P_3) - f(q_{12}) G(P_3)]$$

$$- [(12, 3) \longrightarrow (31, 2)]$$

$$\Psi'' = D_E^{-1} \{ - [g(q_{23}) F(P_1) - f(q_{23}) G(P_1)]$$

$$+ \frac{1}{2} [(23, 1) \longrightarrow (31, 2)]$$

$$+ \frac{1}{2} [(23, 1) \longrightarrow (12, 3)] \}$$

(5.10)

Here $(ij, k) \rightarrow (jk, i)$ represents the similar term with the specified change.

Here F and G are the spectator wave-functions when the correlated pair is interacting via triplet and singlet interactions respectively. The probability for the mixed-symmetry states $P_S' = 2\langle \Psi' | \Psi' \rangle$ turns out to be 0.8 % (B65a). In figure 10, we have plotted the function for F(P) for the case (i), as well as the functions F(P) + G(P) and F(P)- G(P) for case (ii).

The binding energy for this case obtained with Yamaguchi's effective central force defined in table 1 is still quite large, i.e. 12.9 MeV. However Naqvi has constructed a potential model which includes besides a spin-dependent central force, also tensor and spin-orbit forces. The parameters are given by Bhakar (B65a,b). We also use later in this chapter wavefunctions obtained with only the central force of Naqvi, for estimating the sensitivity of the capture rate to different parameters. With this truncated Naqvi force (central part only) the binding energy of ^3He is found to be 8.48 MeV (B65a).

(iii) The introduction of the tensor force brings about the additional complication of a quartet-D ($^4D_{1/2}$) state component in the ground state wave-function. The wave function of equation (5.8) then gets modified to

$$\Psi = \frac{1}{\sqrt{2}}(A'\mathfrak{I}'' - A''\mathfrak{I}') \qquad (5.11)$$

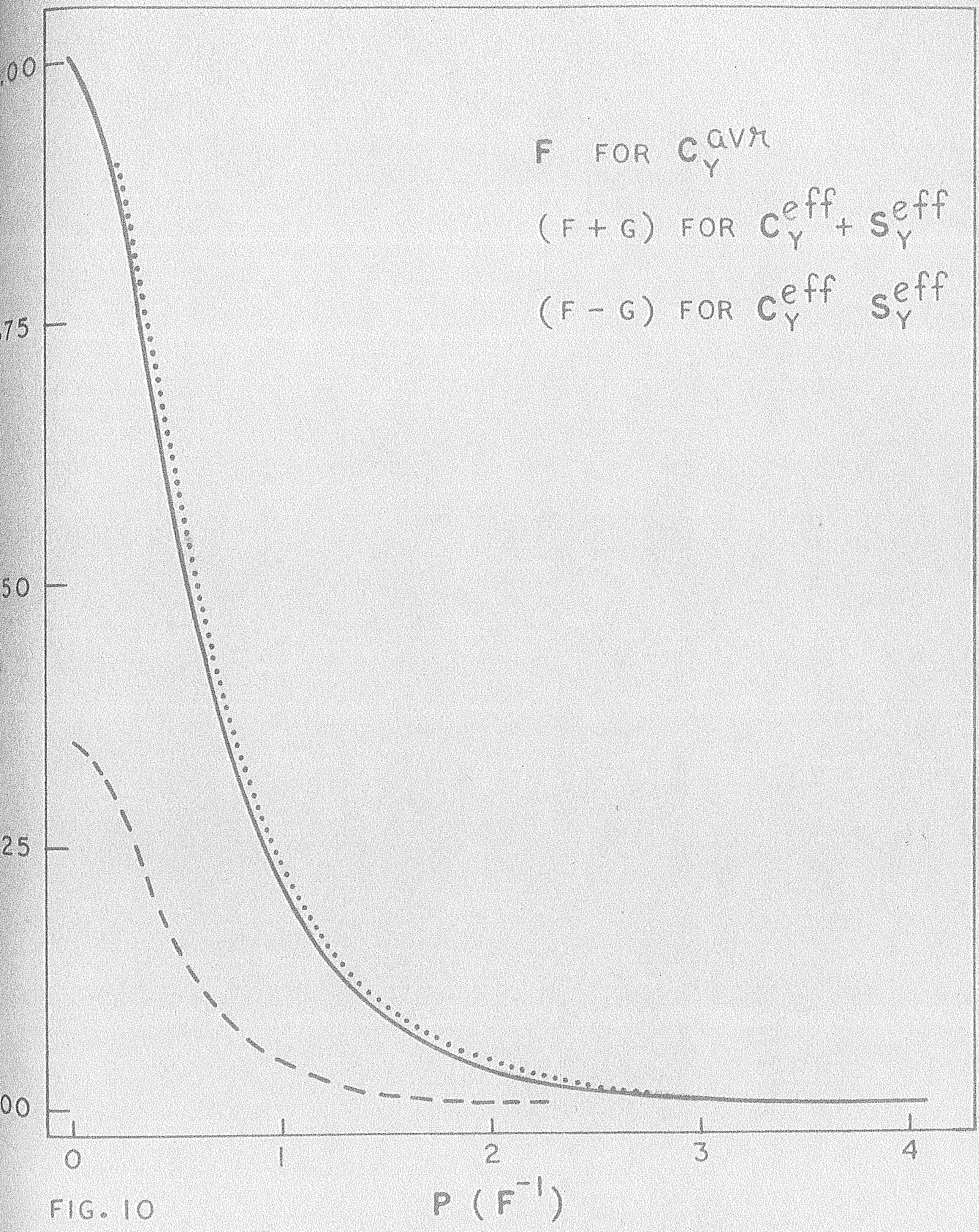

F FOR C_Y^{avr}
(F + G) FOR $C_Y^{eff} + S_Y^{eff}$
(F − G) FOR C_Y^{eff} S_Y^{eff}
.00
.75
.50
.25
.00
0
1
2
3
4
P (F^{-1})

FIG. 10

with A', A'' being mixed-symmetric space-spin composite wave functions, and are expressed as

$$\begin{pmatrix} A' \\ A'' \end{pmatrix} = D_E^{-1}(A_T + A_S)\begin{pmatrix} \chi' \\ \chi'' \end{pmatrix} \tag{5.12}$$

$$A_T = \sum_{ijk} g(q_{ij})\, F_{ij}(P_k)\, P_\sigma^+(ij)$$

$$A_S = \sum_{ijk} f(q_{ij})\, G(P_k)\, P_\sigma^-(ij)$$

$$F_{ij}(P_k) = F(P_k) + \frac{1}{\sqrt{8}} S_{ij}(\hat{P}_k)\, I(P_k)$$

Here $P_\sigma^\pm(ij)$ are projection operators for triplet and singlet spin states respectively. F, G and I are the spectator functions. Our notation here is somewhat different from that of (B65b, M66c) to make the later equations look simpler. The explicit forms of A' and A'' are given in Bhakar's work (B63a), and are not repeated here.

For future reference we also define $\Psi_{\ell m}$ (Ψ_S, Ψ_P, Ψ_D) where the subscript denotes the angular nature of the wave-functions.

$$\Psi_{00}(i) = \Psi_S(i)$$

$$= C(q_{jk})\, F(P_i) + T(q_{jk})\, I(P_i)\, P_2(\hat{q}_{jk}\cdot\hat{P}_i)$$

$$\pm f(q_{jk})\, G(P_i)$$

$$\Psi_{1\mu}(i) = \{\psi_P(i)\}_\mu$$
$$= i\, T(q_{jk})\, I(P_i)\, P_1(\hat{q}_{jk}\cdot\hat{P}_i)(\hat{q}_{jk}\times\hat{P}_i)_\mu$$

$$\Psi_{2\mu}(i) = \{\psi_D(i)\}_\mu$$
$$= \Big[\{C(q_{jk}) + \tfrac{1}{\sqrt{2}} T(q_{jk})\}\, I(P_i)\, W^2_\mu(\hat{P}_i, \hat{P}_i)$$
$$+ T(q_{jk})\{F(P_i) + \tfrac{1}{\sqrt{2}} I(P_i)\}\, W^2_\mu(\hat{q}_{jk}, \hat{q}_{jk})$$
$$- \tfrac{3}{\sqrt{2}} T(q_{jk})\, I(P_i)\, P_1(\hat{q}_{jk}\cdot\hat{P}_i)\, W^2_\mu(\hat{P}_i, \hat{q}_{jk})\Big] \qquad (5.13)$$

where W^2_μ is defined in table 7. One could again define various symmetric combinations of such wave-functions as done in (5.2 - 5.3). For the construction of Ψ^S_5 (Ψ'_5, Ψ''_5) the plus (minus) sign preceding the last term in the expression for $\psi_5(i)$ is the needed one

The final states for three nucleons for cases of our interest would be ^3H or n + d or n + n + p. In the last two states, the total angular momentum J(=1/2) would be a good quantum number, but the total spin or Ispin may not be a good quantum number. Thus the complete final state wave function can be written as

$$\Psi_f = A\,\Psi(S_f = \tfrac{1}{2}, T_f = \tfrac{1}{2}, [3])$$
$$+ B\,\Psi(S_f = \tfrac{1}{2}, T_f = \tfrac{1}{2}, [2,1])$$
$$+ C\,\Psi(S_f = \tfrac{3}{2}, T_f = \tfrac{1}{2})$$
$$+ D\,\Psi(S_f = \tfrac{1}{2}, T_f = \tfrac{3}{2}) \qquad (5.14)$$

where

$$\Psi(S_f = \tfrac{1}{2}, T_f = \tfrac{1}{2}, [3]) = -\varphi^{s}\xi^{a}$$
$$\Psi(S_f = \tfrac{1}{2}, T_f = \tfrac{1}{2}, [2,1]) = \frac{1}{\sqrt{2}}(\varphi'\xi'' - \varphi''\xi')$$
$$\Psi(S_f = \tfrac{3}{2}, T_f = \tfrac{1}{2}) = \frac{1}{\sqrt{2}}(\varphi'\zeta'' - \varphi''\zeta')\chi^{s}$$
$$\Psi(S_f = \tfrac{1}{2}, T_f = \tfrac{3}{2}) = \frac{1}{\sqrt{2}}(\varphi'\chi'' - \varphi''\chi')\zeta^{s}$$

(5.15)

Here φ's * represent space wave functions with

---

* In all subsequent equations the pion wavefunction appear only through $\varphi^{s} = 3\Phi = \varphi_1 + \varphi_2 + \varphi_3$, and hence there should be no confusion between this, and the space functions of three nucleons being defined in equation (5.15).

different symmetries for the three nucleons. They assume different forms for different final states, and their explicit representations will be discussed later as we calculate each capture rate. The values of A, B, C and D would be decided by the statistical availability of different states.

5.C Transition amplitude : $\mathcal{M}_{fi}$

We now calculate the matrix elements for π^- capture from 1S orbit for each component of the final state given in equations (5.14-5.15) with the initial wave-function (5.11). The results for the special cases of N-N forces (ii) and (i) are obtained with suitable approximations viz., for case (ii) $t \to 0$, $I(P_K)$ 0, and in addition for case (i) $F + G \to F$, $F - G \to 0$ etc. To evaluate the spin-Ispin part of the matrix element, we use the double-barred-matrix-element listed in table 9. Only the non-zero matrix elements are shown thown there. We define

$$\mathcal{M}_{fi} = -\frac{m}{M} G \frac{2\sqrt{2}}{q} \Phi \times \sum_{\mu} C^{S_i \; L \; S_f}_{m_s \, -\mu \, m_s'} M_{fi} \tag{5.16}$$

The sum over magnetic quantum numbers lead to a factor $(2S_f+1)\ (2S_i+1)^{-1}\ (2L+1)^{-1}$ upon spin-averaging the matrix element. We shall define L a little later. In the following we write only expression for M_{fi}. Now we give the explicit expressions for the transition matrix element leading to each of the four components with coefficients A,B,C and D in the final state.

$$M^{A}_{fi} = \Big\{ \frac{9\sqrt{3}}{2\sqrt{2}} \Big[\langle \varphi^{S} | \{ p' \otimes \psi'_{P} + p'' \otimes \psi''_{P} \}^{L=1}_{\mu} \rangle \Big] - \frac{9\sqrt{5}}{2} \Big[\langle \varphi^{S} | \{ p' \otimes \psi'_{D} + p'' \otimes \psi''_{D} \}^{L=1}_{\mu} \rangle \Big] \Big\}$$

(5.17)

Clearly, this matrix element becomes zero in the absence of the tensor force.

$$\begin{aligned} M^{B}_{fi} = \Big\{ & -\sqrt{6} \Big[\langle \varphi' | \{ p' \otimes \psi''_{S} + p'' \otimes \psi'_{S} \}^{L=1}_{\mu} \rangle + \langle \varphi'' | \{ p' \otimes \psi'_{S} - p'' \otimes \psi''_{S} \}^{L=1}_{\mu} \rangle \Big] \\ & - \frac{9\sqrt{3}}{4} \Big[\langle \varphi' | \{ p' \otimes \psi''_{P} + p'' \otimes \psi'_{P} \}^{L=1}_{\mu} \rangle + \langle \varphi'' | \{ p' \otimes \psi'_{P} - p'' \otimes \psi''_{P} \}^{L=1}_{\mu} \rangle \Big] \\ & + \frac{3\sqrt{5}}{2} \Big[\langle \varphi' | \{ p' \otimes \psi''_{D} + p'' \otimes \psi'_{D} \}^{L=1}_{\mu} \rangle + \langle \varphi'' | \{ p' \otimes \psi'_{D} - p'' \otimes \psi''_{D} \}^{L=1}_{\mu} \rangle \Big] \end{aligned}$$

(5.18)

$$
\begin{aligned}
M^{C}_{fi} = \Big\{ & \frac{3\sqrt{3}}{2}\Big[\langle\varphi'|\{p'\otimes\psi^{S}_{S}\}^{L=1}_{\mu}\rangle + \langle\varphi''|\{p''\otimes\psi^{S}_{S}\}^{L=1}_{\mu}\rangle\Big] \\
& + \frac{\sqrt{3}}{2}\Big[\langle\varphi'|\{p'\otimes\psi''_{S} + p''\otimes\psi'_{S}\}^{L=1}_{\mu}\rangle \\
& \qquad + \langle\varphi''|\{p'\otimes\psi'_{S} - p''\otimes\psi''_{S}\}^{L=1}_{\mu}\rangle\Big] \\
& + \frac{9\sqrt{3}}{4\sqrt{2}}\Big[\langle\varphi'|\{p'\times\psi^{S}_{P}\}^{L=1}_{\mu}\rangle + \langle\varphi''|\{p''\otimes\psi^{S}_{P}\}^{L=1}_{\mu}\rangle\Big] \\
& - \frac{9\sqrt{3}}{2\sqrt{2}}\Big[\langle\varphi'|\{p'\otimes\psi''_{P} + p''\otimes\psi'_{P}\}^{L=1}_{\mu}\rangle \\
& \qquad + \langle\varphi''|\{p'\otimes\psi'_{P} - p''\otimes\psi''_{P}\}^{L=1}_{\mu}\rangle\Big] \\
& + \frac{9\sqrt{15}}{4\sqrt{2}}\Big[\langle\varphi'|\{p'\otimes\psi^{S}_{P}\}^{L=2}_{\mu}\rangle + \langle\varphi''|\{p''\otimes\psi^{S}_{P}\}^{L=2}_{\mu}\rangle\Big] \\
& + \frac{3\sqrt{5}}{2\sqrt{2}}\Big[\langle\varphi'|\{p'\otimes\psi''_{D} + p''\otimes\psi'_{D}\}^{L=1}_{\mu}\rangle \\
& \qquad + \langle\varphi''|\{p'\otimes\psi'_{D} - p''\otimes\psi''_{D}\}^{L=1}_{\mu}\rangle\Big] \\
& - \frac{9\sqrt{5}}{2\sqrt{2}}\Big[\langle\varphi'|\{p'\otimes\psi''_{D} + p''\otimes\psi'_{D}\}^{L=2}_{\mu}\rangle \\
& \qquad + \langle\varphi''|\{p'\otimes\psi'_{D} - p''\otimes\psi''_{D}\}^{L=2}_{\mu}\rangle\Big]\Big\}
\end{aligned}
\tag{5·19}
$$

Table - 9

$$\langle \chi' \| \sigma^S \| \chi' \rangle = \sqrt{3} \ ; \quad \langle \chi' \| \sigma'' \| \chi' \rangle = -\sqrt{3} \ ;$$

$$\langle \chi' \| \sigma' \| \chi'' \rangle = \langle \chi'' \| \sigma' \| \chi' \rangle = -\sqrt{3} \ ;$$

$$\langle \chi'' \| \sigma^S \| \chi'' \rangle = \sqrt{3} \ ; \quad \langle \chi'' \| \sigma'' \| \chi'' \rangle = -\sqrt{3} \ ;$$

$$\langle \chi'' \| \sigma_1 \cdot \sigma_3 \| \chi' \rangle = -\sqrt{3} \ ;$$

$$\langle \chi'' \| i(\sigma_1 \times \sigma_3) \| \chi' \rangle = -2 \ ; \langle \chi^S \| i(\sigma_1 \times \sigma_3) \| \chi' \rangle = 1 \ ;$$

$$\langle \chi^S \| \sigma' \| \chi' \rangle = -\sqrt{3} \ ; \quad \langle \chi^S \| \sigma'' \| \chi'' \rangle = -\sqrt{3}$$

$$\langle \chi^S \| X^2(\sigma_1, \sigma_3) \| \chi' \rangle = -\sqrt{5/2}$$

—x—x—x—x—x—x—

For defination of X^2 and the double-barred matrix elements see the table 7.

The structure of the matrix element M^D_{fi} is the same as that of M^C_{fi}, except that the numerical coefficients of terms involving Ψ_S^S (Ψ_S', Ψ_S'') are multiplied by $-1/\sqrt{2}$ ($+1/\sqrt{2}$), of terms involving Ψ_P^S (Ψ_D', Ψ_D'') are multiplied by $\sqrt{2}$ ($-\sqrt{2}$), and terms with L=2, as well as those involving Ψ_P', Ψ_P'' turn out to be zero. Table 10 contains various formulae needed for this reduction. $\{p^{(\cdot)} \otimes \Psi_\lambda^{(\cdot)}\}^L_\mu$ is the vector product also defined in table 10.

Table - 10

$$\{p^{(\cdot)} \otimes \psi_l^{(\cdot)}\}^L_\mu = \sum_m C^{1\ l\ L}_{m\ \mu-m\ \mu}\ p^{(\cdot)}_m \psi^{(\cdot)}_{l,\mu-m}$$

$$\{a \otimes I\}^1_\mu = (\underset{\sim}{a})_\mu$$

$$\{a \otimes t\}^1_\mu = \frac{i}{\sqrt{2}}(\underset{\sim}{a} \times \underset{\sim}{b})_\mu$$

$$\{a \otimes t\}^2_\mu = W^2_\mu(\underset{\sim}{a}, \underset{\sim}{t})$$

$$\{a \otimes W^2_\mu(t, r)\}^1_\mu$$

$$= \left(-\sqrt{\frac{3}{20}}\right)\left\{(\underset{\sim}{a}\cdot\underset{\sim}{t})\underset{\sim}{r} + (\underset{\sim}{a}\cdot\underset{\sim}{r})\underset{\sim}{t} - \frac{2}{3}(\underset{\sim}{t}\cdot\underset{\sim}{r})\underset{\sim}{a}\right\}_\mu$$

$$\{a \otimes W^2_\mu(t, r)\}^2_\mu$$

$$= \left(\frac{i}{\sqrt{6}}\right)\left\{W^2_\mu(\underset{\sim}{a}\times\underset{\sim}{t}, \underset{\sim}{r}) + W^2_\mu(\underset{\sim}{a}\times\underset{\sim}{r}, \underset{\sim}{t})\right\}$$

where $\underset{\sim}{a}$, $\underset{\sim}{t}$ and $\underset{\sim}{r}$ are arbitrary vectors.

We note a few selection rules here. The transition operator connects the major component ψ_s^S of the initial state to only the $S_f = 3/2$, $T_f = 1/2$ and $S_f = 1/2$, $T_f = 3/2$ components of the final state. The psedoscalar nature of the pion prohibits transition to the final state with $S_f = 1/2$, $T_f = 1/2$ from initial state ψ_s^S ; this is analogous to the selection rule from equation (2.17) which requires that for S-wave pion capture either S or T or both must change. We also note that the mixed-symmetric S-component of the initial state cannot lead to the (patially) fully symmetric final state.

Before we proceed to actual calculation of the capture rates for different modes, we shall show how different terms occuring in M_{fi} can be expressed in forms which are more useful for numerical evaluation, and also discuss the physical interpretation of such terms. We have normalised φ^S, φ' and φ'' according to the definations of χ^S, χ' and χ'' (see equation (5.7)) rather than as for ψ^S, ψ' and ψ'' as given in equations (5.10). Then the matrix elements contains terms of the following types which can written as,

$$\langle \varphi' | \{ p' \otimes \psi_l^S \}_\mu^L \rangle + \langle \varphi'' | \{ p'' \otimes \psi_l^S \}_\mu^L \rangle$$

$$= \sqrt{\frac{3}{2}} \sum_{ik} \langle \varphi(i) | \{ p_i \otimes \psi_l(k) \}_\mu^L \rangle \qquad (5\cdot20)$$

and similarly

$$\Big[\langle \varphi' | \{ p' \otimes \psi_\ell'' + p'' \otimes \psi_\ell' \}^L_\mu \rangle$$

$$+ \langle \varphi'' | \{ p' \otimes \psi_\ell' - p'' \otimes \psi_\ell'' \}^L_\mu \rangle$$

$$= \sqrt{\tfrac{3}{2}} \sum_i \langle \varphi(i) | \{ p_i \otimes \psi_\ell(i) \qquad (5,21)$$

$$+ p_j \otimes \psi_\ell(k) + p_k \otimes \psi_\ell(j) \}^L_\mu \rangle$$

$$\langle \varphi^s | \{ p' \otimes \psi_\ell' + p'' \otimes \psi_\ell'' \}^L_\mu \rangle$$

$$= \frac{\sqrt{3}}{2} \sum_{ik} \langle \varphi(i) | \{ p_k \otimes \psi_\ell(k) \}^L_\mu \rangle$$

(5,22)

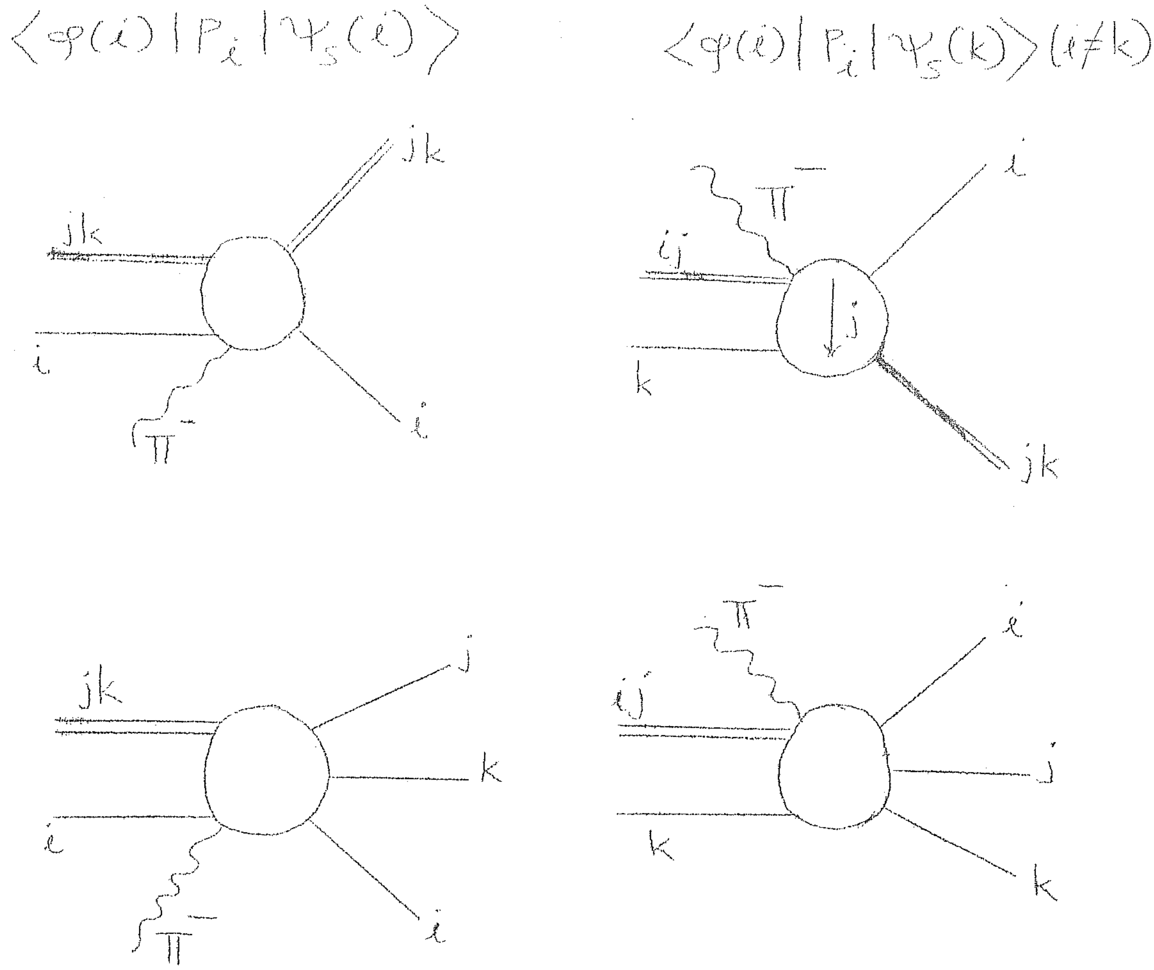
The major contribution to the capture rates comes from the equation (5.20) with $\ell = 0$, $L = 1$. Let us examine its structure in some detail. It reduces to

$$\sum_i \langle \varphi(i) | (\underset{\sim}{p}_i)_\mu | \psi_s(1) + \psi_s(2) + \psi_s(3) \rangle$$

The interaction operator operates upon the i^{th} nucleon which captures the pion. In the sum we shall have following terms;

$\langle \varphi(i) | P_i | \Psi_s(i) \rangle$: This may be interpreted as capture on the uncorrelated i-nucleon, which would imerge with a high-momentum, leaving the remaining n-p pair bound as a deuteron or in a fragmented state. Physical arguments suggest that the contribution of such a term should be negligible. We shall see later that indeed the contribution is small, but not altogether negligible, since this spectator nucleon in our model does have some dynamical interaction effects in its wave-function.

$\langle \varphi(i) | P_i | \Psi_s(k) \rangle$ $(i \neq k)$: In this case the nucleon belongs to an interacting pair, and so this term represents truly a pair-capture. However in the final state the pair may imerge either as a bound state (deuteron) or as free nucleons (np or nn pair) leaving the spectator nucleon K (n or p) with small momentum. Alternatively, one of the nucleons in the absorbing pair may attach itself to the K-nucleon emerging then as a fast deuteron. This of course requires a rather specific structure of the initial state and final states. Schematic representations of processes discussed above are shown below.



5.D Deuteron mode.

Kinematics dictate that in this mode the final state consists of a neutron and a deuteron flying away from each other in a relative P-state. This final state can not contain the component $T_f = 3/2$, i.e. the term with coefficient D in equation (5.14) The space-part of the wave function of the final state (with particle 'i' being a neutron), can be written as

$$\varphi(i) = \left(\frac{2\pi}{L}\right)^{3/2} \delta(\underset{\sim}{P}_i - \underset{\sim}{k})$$
$$\times N_d\, g(\underset{\sim}{q}_{jk})(q_{jk}^2 + \alpha_d^2)^{-1} \qquad (5.23)$$

where we ignore final state-interaction, $N_d = N_o/\sqrt{4\pi}$ as in equation (2.30), K is the relative momentum of the n-d system, and energy conservation requires

$$\frac{3}{4}K^2 - \alpha_d^2 = mM - \alpha_T^2$$

which gives $K \approx 2F^{-1}$ i.e. 400 MeV/c. Such a large relative momentum justifies neglect of the final state interaction.

For calculating the actual numbers for the capture rates (W_d for deuteron mode and W_p for proton mode) we keep the terms ψ_s^s as well as ψ_s' and ψ_s'' in the initial wave function, and also include terms of the order I(P)/F(P) as well as T/C (compared with CF term) (see p.114, and further discussion in (G67a)) in the transition amplitude. The numerical spectator functions are given in Bhakar's thesis (B65b), and analytical forms fitted to these wave functions are given in (G67a) for cases (ii) and (iii). For case (i) we have calculated the relevant spectator function by numerical integration, and then fitted it to a suitable analytical form as was also done in the previous chapter for ^{6}Li. All the angular integrations are facilitated by such analytical representations of the spectator functions, and carried out analytically. The integration over the magnitudes of the vectors are then done numerically.

It should be noted that when the tensor force is included in the initial state as in case (iii), for consistancy it should also be included in the final deuteron state. This brings in some additional terms in the transition amplitude. Some of the important additional terms within our approximation scheme are given below. Writing

$$\varphi(i) = \varphi_S(i) + \varphi_D(i)$$

we get additional terms

$$M_{fi}(\text{C-part}) =$$

$$\left\{ \frac{9\sqrt{5}}{2} \left[\langle \{ \varphi_D' \otimes p' + \varphi_D'' \otimes p'' \}_\mu^{L=1} | \Psi_S^S \rangle \right] + \frac{9}{2\sqrt{2}} \left[\langle \{ \varphi_D' \otimes p' + \varphi_D'' \otimes p'' \}_\mu^{L=2} | \Psi_S^S \rangle \right] \right\}$$

(5.24)

Now we write down explicitely the integrals which have to be evaluated in the momentum space with $\varphi(i)$ as given in (5.23). Apart from the normalisation factor, these integrals have the following forms.

$$\langle \varphi' | \underset{\sim}{p}'_\mu | \Psi_S^S \rangle + \langle \varphi'' | \underset{\sim}{p}''_\mu | \Psi_S^S \rangle$$

$$= \frac{3\sqrt{3}}{\sqrt{2}} (\underset{\sim}{k})_\mu \left[\int d^3x \frac{c(x)}{(x^2+\alpha_d^2)} \cdot \frac{\{c(x)F(K) + f(x)G(K)\}}{(x^2 + \frac{3}{4}K^2 + \alpha_T^2)} \right.$$

$$+ 2\int d^3x \frac{c(\underset{\sim}{x} + \underset{\sim}{K}/2)}{\{(\underset{\sim}{x}+\underset{\sim}{K}/2)^2 + \alpha_d^2\}^2} \{ c(\underset{\sim}{k} + \underset{\sim}{x}/2)F(x) + f(\underset{\sim}{k} + \tfrac{\underset{\sim}{x}}{2})G(x) \}$$

$$\left. \times \{ x^2 + K^2 + \underset{\sim}{x} \cdot \underset{\sim}{k} + \alpha_T^2 \}^{-1} \right]$$

$$= \frac{3\sqrt{3}}{\sqrt{2}} (\underset{\sim}{k})_\mu \left[I_{1+} + 2 I_{2+} \right] \qquad (5.25)$$

$$\Big[\langle \varphi' | \{ \underset{\sim}{p}'_\mu | \psi''_5 \rangle + \underset{\sim}{p}''_\mu | \psi'_5 \rangle \}$$

$$+ \langle \varphi'' | \{ \underset{\sim}{p}'_\mu | \psi'_5 \rangle - \underset{\sim}{p}''_\mu | \psi''_5 \rangle \} \Big]$$

$$= \frac{3\sqrt{3}}{\sqrt{2}} (\underset{\sim}{K})_\mu \Big[\int d^3x \frac{C(x)}{(x^2+\alpha_d^2)} \frac{\{C(x)F(K) - f(x)G(K)\}}{\{x^2 + \frac{3}{4}K^2 + \alpha_T^2\}}$$

$$-2\int d^3x \; C(\underset{\sim}{x}+\underset{\sim}{K}/2)(1+\underset{\sim}{x}\cdot\underset{\sim}{K}/K^2)\{(\underset{\sim}{x}+\underset{\sim}{K}/2)^2+\alpha_d^2\}^{-1}$$

$$\times\{C(\underset{\sim}{K}+\underset{\sim}{x}/2)F(x) - f(\underset{\sim}{K}+\underset{\sim}{x}/2)G(x)\}$$

$$\times\{x^2 + K^2 + \underset{\sim}{x}\cdot\underset{\sim}{K} + \alpha_T^2\}^{-1} \Big]$$

$$= \frac{3\sqrt{3}}{\sqrt{2}} (\underset{\sim}{K})_\mu \, I_{5'} \qquad (5.26)$$

We also write down the general structure of the integral arising from ψ_D as well as φ_D. Taking them all together as I_{T+}, we have

$$I_{T+} = \lambda \Big\{ \int d^3x \; C(x)(x^2+\alpha_d^2)^{-1}(x^2+\tfrac{3}{4}K^2+\alpha_T^2)^{-1}$$

$$\times\{T(x)F(K)\eta_1(x,K) + C(x)I(K)\eta_2(x,K)\}$$

$$+2\int d^3x \; C(\underset{\sim}{x}+\underset{\sim}{K}/2)\{(\underset{\sim}{x}+\underset{\sim}{K}/2)^2+\alpha_d^2\}^{-1}(x^2+K^2+\underset{\sim}{x}\cdot\underset{\sim}{K}+\alpha_T^2)^{-1}$$

$$\times\{T(\underset{\sim}{K}+\underset{\sim}{x}/2)F(x)\eta_3(x,K,\underset{\sim}{x}\cdot\underset{\sim}{K}) + C(\underset{\sim}{K}+\underset{\sim}{x}/2)I(K)\eta_4(x,K,\underset{\sim}{x}\cdot\underset{\sim}{K})\}\Big\}$$

$$+ s \Big\{ \int d^3x \; T(x)\, n_5(x,k) \{ C(x) F(k) + f(x) G(k) \} \times (x^2 + \alpha_d^2)^{-1} (x^2 + \tfrac{3}{4} k^2 + \alpha_T^2)^{-1}$$

$$+ 2 \int d^3x \; T(\underset{\sim}{x} + \underset{\sim}{k}/2)\, n_6(x, k, \underset{\sim}{x}\cdot\underset{\sim}{k}) \{ C(\underset{\sim}{k} + \underset{\sim}{x}/2) F(x) + f(\underset{\sim}{k} + \underset{\sim}{x}/2) G(x) \} \{ (\underset{\sim}{x} + \underset{\sim}{k}/2)^2 + \alpha_d^2 \}^{-1} \times \{ x^2 + k^2 + \underset{\sim}{x}\cdot\underset{\sim}{k} + \alpha_T^2 \}^{-1} \Big\} \qquad (5.27)$$

Here r, s are numerical factors, and n_i's are simple functions of their arguments.

It is useful to remark at this stage that the integral I_{1+} has its origin in the matrix element of the type $\langle \varphi(i) |\, P_i \,|\, \Psi_S(i) \rangle$, i.e. the contribution from the capture on the uncorrelated nucleon, whereas I_{2+} has its origin from an element like $\langle \varphi(i) \,|\, P_i \,|\, \Psi_S(k) \rangle$ ($i \neq k$), i.e. from capture on a correlated pair. In view of the comments made at the end of the previous section, we should thus expect I_{2+} to be large, and I_{1+} small.

The final values for the capture rate W_d is obtained using Fermi's Golden rule. Table 11 gives the results for case (i) for two different values of $\bar{\lambda}$, but the effects of correlation are exihibited

Table - 11.

E_b (MeV)	$\bar{\lambda}$(3-body)F^{-3}	I_{1+}	I_{2+}	$W_d(10^{16}sec^{-1})$
12.43	0.352	.1745	.4642	1.89
8.29	0.328	.1470	.4291	1.21

Table - 12

Potential	E_b(MeV)	I(S)	I(S')	I(T)	$W_d(10^{16}sec^{-1})$
C_Y^{avr}	12.43	1.103	.	.	1.89
$C_Y^{eff}+S_Y^{eff}$	12.9	1.507	-.044	.	1.35
$C_N + S_N$	7.04	1.037	-.0253	.	0.78
$(C+T)_Y+S_Y^{eff}$	10.40	1.270	-.0274	-.134	0.795
$(C+T)_N+S_N$	8.85	1.194	-.0237	-.082	0.66

—x—x—x—x—x—x—

explicitely. We note that although I_{1+} is smaller than I_{2+}, it is by no means negligibly so. This is once again an illustration of the statement made earlier that in our model even the spectator nucleon has some effects of N-N interactions built into it. Table 12 contains the results for a variety of different NLS potentials.

We would like to point out some essential features of the results contained in tables 11 and 12. The capture rates calculated here are rather large compared to value obtained by Diwakaran which is $(1.40 \pm 0.51)\ 10^{15}$ sec^{-1}. However, Figureau and Ericson obtain 0.454, 1.93 and 1.06×10^{16} sec^{-1} for gavssian (G) Irving-Gunn (I-G) and modified Irving-Gunn wavefunctions respectively. As we pointed out earlier Eckstein-Diwakaran version of pion-nucleon interaction contains implicitely the effects of nuclear correlations or short-range nuclear forces. It is obvious that the NLS interaction used here must also introduce quite sizable correlations in the nuclear wavefunction although it does not contain an explicit short-range repulsion term.

The effect of including the S'-states and the D-state (Via tensor force) is to reduce the capture rates as seen from table 12. For Yamaguchi potentials the capture rate is reduced by about 12 % for inclusion of S' state and by about 50 % again for inclusion of tensor force. The Naqvi version of potentials (besides giving better binding energies) gives somewhat smaller capture rates, and the effect of including tensor force is not so severe. We note here that the full Naqvi potential contains also an additional $\underset{\sim}{L}.\underset{\sim}{S}$ potential term which we ignore here and use only a truncated potential. The major contribution to the capture rates comes in all cases from the $I(S)$ term.

Before proceeding ahead, a comment may be made regarding pion capture from atomic P-state. A simple argument shows that this capture rate is of little importance. Crudely speaking,

$$\frac{W(P\text{-wave})}{W(S\text{-wave})} = \left| \frac{p_\pi \varphi_p(\pi)}{\varphi_s(\pi)\, p_N} \frac{M}{m} \right|^2$$

$$\sim \left| \frac{z}{a} \frac{M}{m} \frac{1}{K} \right|^2 \sim 10^{-3} - 10^{-4} \qquad (5.32)$$

Thus P-wave pion capture rate is expected to be of the order of 10^{12}-10^{13} sec^{-1}. This however is comparable to the radiative $2P \rightarrow 1S$ transition rate which is (M52b) known to be 3×10^{12} sec^{-1}. One obtains the total P-wave capture rate more accurately. To do this we use only the ψ_S^S component of the initial state, take only p_π term in the $N\pi$ interaction (equation (2.3)) and then express the interaction in three-body permutation symmetry form, one gets

$$\mathcal{M}_{fi}(P\text{-wave}) = G\frac{2\sqrt{2}}{9} N_P \left(-\frac{9\sqrt{3}}{2}\right) \times C^{\frac{1}{2}\ 1\ \frac{1}{2}}_{m_f\ -m_\pi\ m_s'} \langle \varphi^s | \Psi^s_s \rangle \quad (5.33)$$

The overlap integral is proportional to $(I_{1+} + 2I_{2+})$, and N_P is the normalisation factor for the pion wave function. The integrals I_{1+} and I_{2+} are the same as defined in equation (5.25). We then obtain

W_d(P-wave) $= 2.1 \times 10^{12}\ \text{sec}^{-1}$.

which is quite comparable to the 2P → 1S radiative transition rate. The effect of this factor should be to reduce the absolute value of the pion capture rate from 1S state, since only about half the pion going into atomic orbit reach the 1S state. It seems to us however that since the pion capture rate from 2P-state is so small, the relative ratios for various capture modes from 1S-capture should remain practically unaffected by consideration of P-wave capture. On the other hand, Figureau and Ericson argue that pions captured from 2P-orbit give practically no radiative capture, and hence reduce by 19 % (=W(P-wave)/W(2P→1S)) the ratios of observed capture rates for comparison to calculated values. We do not really understand their argument.

5.E Proton Mode

For the calculation of the proton capture mode we follow the same treatment as outlined in section 5C and 5D. In this case $\varphi(i)$ is defined as

$$\varphi(i) = \left(\frac{2\pi}{L}\right)^3 \delta(\underset{\sim}{P}_i - \underset{\sim}{K})\, \delta(\underset{\sim}{q}_{jk} - \underset{\sim}{k}) \qquad (5.34)$$

where $\underset{\sim}{k}$ is the relative momentum of the (jk)pair of nucleons and $\underset{\sim}{K}$ is the momentum of the 'i' nucleon relative to the c.m. of the (jk) pair. Energy conservation requires

$$k^2 + 3/4\, K^2 + \alpha_T^2 = m\, M \qquad (5.35)$$

We do not include here the final-state-interaction for simplicity. Diwakaran has shown (with the closure approximation) that the final-state-interaction has little effect on the capture rate.

Diwakaran has pointed out that using the closure approximation with three particles in the final state taken as plane wave fields the total capture rate, so that the actual proton capture rate is to be obtained by subtracting from this total rate the deuteron mode capture rate (W_d). This does not appear to be quite valid to us, since the overlap integral of the n-d state with the n-n-p state as defined in the above equation (5.34) turns out to be zero. Thus the use of equation (5.34) for the final state should yield only the proton capture rate. We have thus taken the results of the capture rate obtained for three final nucleons in plane wave states as directly the proton capture rate in the work of Diwakaran and Ericson and Figureau also. This should be kept in mind in comparison of our results with those of the above authors.

The proton capture rate is calculated following the standard procedure as was done earlier say for ^{6}Li.

Table 13 shows the capture rates for different versions of the non-local separable potentials as was done in table 12 for the deuteron mode. The effects of including the S' state and the tensor force (D-state) are also shown explicitely. Again our results are quite large compared to those of Diwakaran, but comparable to those of Ericson and Figureau. The effect of S' state and D-state is also again to reduce the capture rates. The famous S-D cancellation is only about 50 %. Cheon's calculation (as was mentioned in the introduction) uses an incorrect value for the size parameter of the ^{3}He nucleus. his value for the capture rate however is 1.54×10^{17} sec^{-1} in the absence of correlation.

Table - 13

Potential	E_b (MeV)	Capture rate for proton mode (10^{16} sec^{-1})		
		$W_P(S)$	$W_P(S+S')$	$W_P(S+S'+D)$
C_Y^{avr}	12.43	4.56	.	.
$C_Y^{eff} + S_Y^{eff}$	12.90	4.39	4.19	.
$C_N + S_N$	7.04	2.44	2.37	.
$(C+T)_Y + S_Y^{eff}$	10.40	3.15	3.02	2.05
$(C+T)_N + S_N$	8.85	2.15	2.39	1.79

Diwakaran : W_P = .785 (G)

Figureau Ericson : W_p = 1.30 (G)

3.01 (I-G)

1.99 (Modified I-G)

The experiments on pion capture in ^3He generally measure the momentum spectrum of the outgoing proton. In our formulation of the problem full I-spin symmetry of the wavefunctions is used. Thus it is not obvious to identify the outgoing proton. One can obtain the momentum distribution for the i^{th} nucleon, but this would be a coherent superposition of the proton as well as the neutron components of the wave-function. To obtain the momentum-spectrum for the proton, we use the following method. The complete wavefunction given in equation (5.14) is rewritten in terms of the full three-particle antisymmetrisation operator operating on a function in which a proton is separated out. We use the notation where 'u' denotes a proton state and 'ν' a neutron state. Then (5.14) is rewritten as

$$\Psi_f = \frac{1}{3\sqrt{2}} \mathcal{A} \, \Psi_f^{(1)}$$

$$= \frac{1}{3\sqrt{2}} \mathcal{A} \Big[\Big\{ \{A+B+D\} \{ \varphi(1) + \varphi(2) + \varphi(3) \} + 3C \varphi(3) \Big\} \frac{1}{\sqrt{3}} \chi^5 \; +$$

$$+\Big\{A\{\varphi(1)+\varphi(2)+\varphi(3)\}+B\{\varphi(1)+\varphi(2)-2\varphi(3)\}$$

$$+D\{\varphi(1)-2\varphi(2)+\varphi(3)\}\Big\}\frac{1}{\sqrt{6}}\chi''$$

$$+\Big\{A\{\varphi(1)+\varphi(2)+\varphi(3)\}+B\{\varphi(1)-\varphi(2)\}$$

$$+D\{\varphi(1)-\varphi(3)\}\Big\}\frac{1}{\sqrt{2}}\chi'\Big](u_1 v_2 v_3) \quad (5.36)$$

where $\psi_f^{(1)}$ indicates that the particle number 1 is the proton.

$$\mathcal{A} = 1 - P_{12} - P_{13} - P_{23} + P_{123} + P_{132}$$

where P's are well-known permutation operators. Note that $\mathcal{A}$ commentes with an operator fully symmetric in all particles (viz., H_{int}), and $\mathcal{A}\Psi_i = 3!\Psi_i$, we get

$$\mathcal{M}_{fi}(\mathcal{A}) =$$

$$\left\langle \frac{1}{3\sqrt{2}}\mathcal{A}\Psi_f^{(1)} \middle| \sum \underline{\tau}_i \cdot \underline{\varphi}_i(\pi)\, \underset{\sim}{\sigma}_i \cdot \underset{\sim}{p}_i \middle| \Psi_i \right\rangle$$

$$= \sqrt{2}\left\langle \Psi_f^{(1)} \middle| \sum \underline{\tau}_i \cdot \underline{\varphi}_i(\pi)\, \underset{\sim}{\sigma}_i \cdot \underset{\sim}{p}_i \middle| \Psi_i \right\rangle \quad (5.37)$$

and then with the same conventions and approximations as in sections C and D, one can obtain for the transition amplitude

$$M_{fi}(\vartheta) = \Big[\Big\{ (A+B+D)\{\varphi(1)+\varphi(2)+\varphi(3)\} + 3C\varphi(3) \Big\}$$

$$\times \Big\{ \sqrt{6}\{p'\otimes\psi_s^{S}\}_{\mu}^{L=1} + \frac{2\sqrt{6}}{\sqrt{3}}\{p''\otimes\psi_s'\}_{\mu}^{L=1}$$

$$+ \frac{3\sqrt{5}}{2}\{p'\otimes\psi_D''\}_{\mu}^{L=1} + \frac{\sqrt{5}}{2}\{p''\otimes\psi_D'\}_{\mu}^{L=1}$$

$$+ \frac{9\sqrt{5}}{2}\{p'\otimes\psi_D''\}_{\mu}^{L=2} + \frac{3\sqrt{5}}{2}\{p''\otimes\psi_D'\}_{\mu}^{L=2} \Big\} \delta_{S_f, 3/2}$$

$$+ \Big\{ -\frac{3}{2} A\{\varphi(1)+\varphi(2)+\varphi(3)\} \Big\}$$

$$\times \Big\{ \{\frac{1}{\sqrt{3}} p'\otimes\psi_s^{S} - p''\otimes\psi_s^{S}\}_{\mu}^{L=1}$$

$$+ \{ p''\otimes\psi_s'' - p'\otimes\psi_s' - \sqrt{3}\, p'\otimes\psi_s''$$

$$- \frac{1}{3\sqrt{3}}\, p''\otimes\psi_s'\}_{\mu}^{L=1}$$

$$+ \sqrt{10}\{\sqrt{3}\, p''\otimes\psi_D'' + \sqrt{3}\, p'\otimes\psi_D' + p'\otimes\psi_D''$$

$$+ \frac{1}{3} p''\otimes\psi_D'\}_{\mu}^{L=1} \Big\} \delta_{S_f, 1/2} \Big]$$

(5.38)

where all the terms which turn out to be zero upon momentum space integration are already dropped.
The proton momentum spectrum can be calculated in a straightforward manner, and the result is exhibited in fig. 11, for wavefunctions obtained with Yamaguchi's non-local interactions C^{avr}, $C^{eff} + S^{eff}$ and $(C+T)+S^{eff}$, which in turn show the effect of ignoring both the S' and the D-states, including S' states only, and including both S' as well as D-states, respectively.

The most striking feature of the momentum spectra shown in Fig.11 is the absence of a prominent low-momentum peak, which one should expect to arise when the capture takes place on an n-p correlated pair, with the proton a mere spectator. Such a low-momentum peak is indeed observed in experiments (Z67a).
The calculations do give a rather small peak in the low-momentum region, but the correlations dominate to give a much more prominent peak at high momentum, ~ 320 MeV/C. The effect of including the mixed-symmetric S' state appears to be to reduce the high-momentum side of the spectrum somewhat, whereas the inclusion of D-state (tensor force) results in a pronounced reduction of the low-momentum part of the spectrum. In figure 12 we plot the energy-distribution (dW/dE_p) of the outgoing proton, and the results

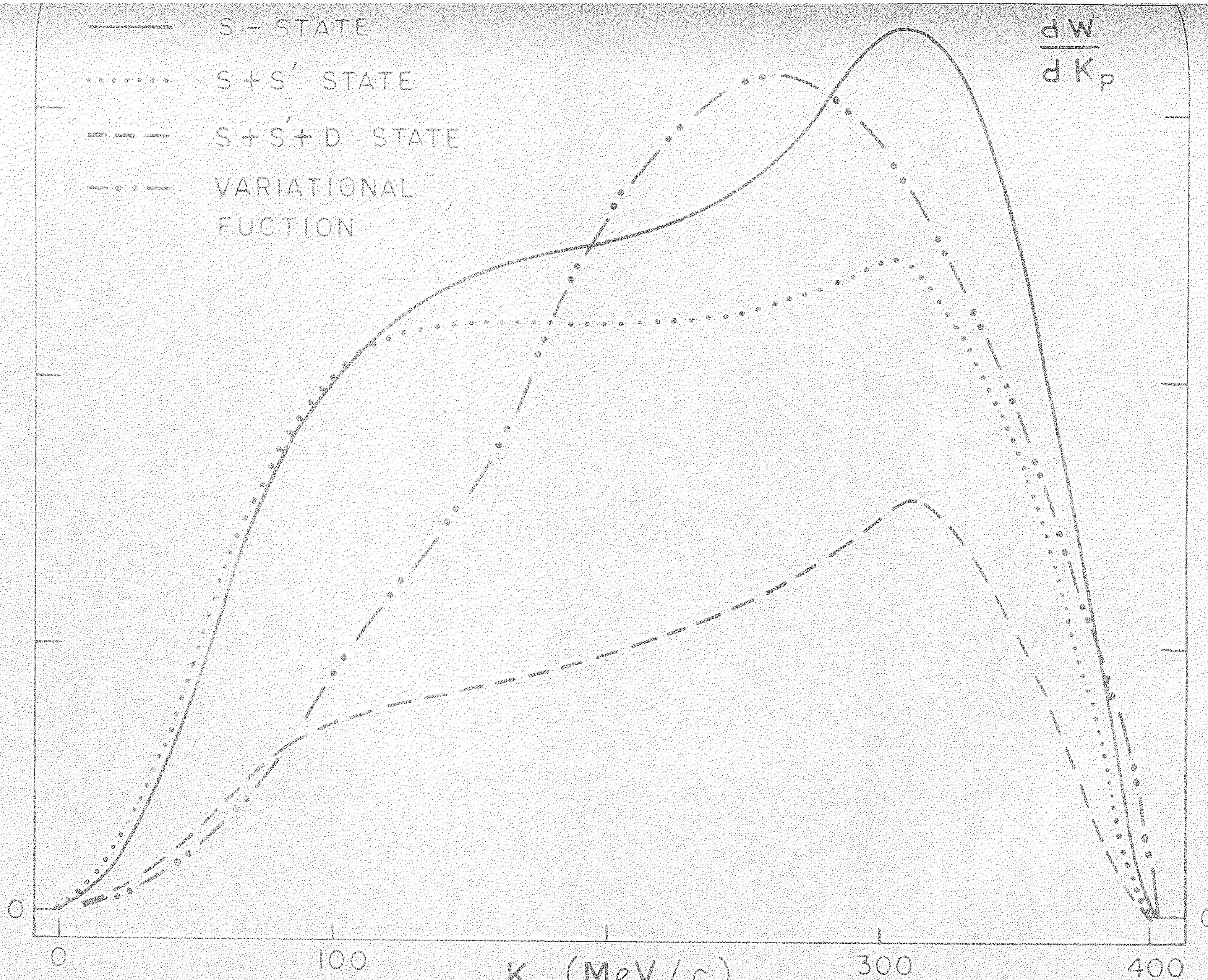

S - STATE
S + S' STATE
S + S' + D STATE
VARIATIONAL FUCTION
dW / dK_P
0
100
300
400
K_P (MeV/c)

FIG. 11

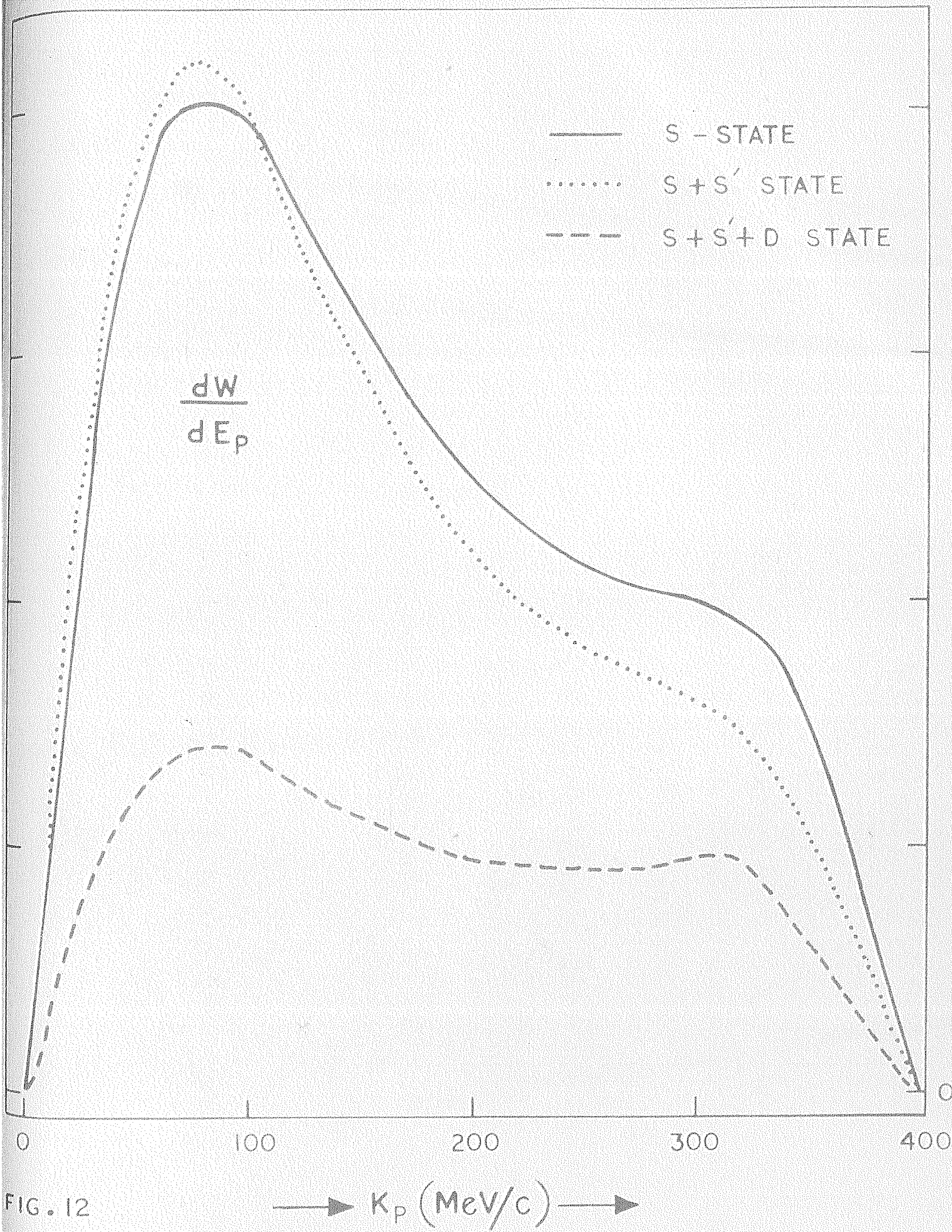


FIG. 12

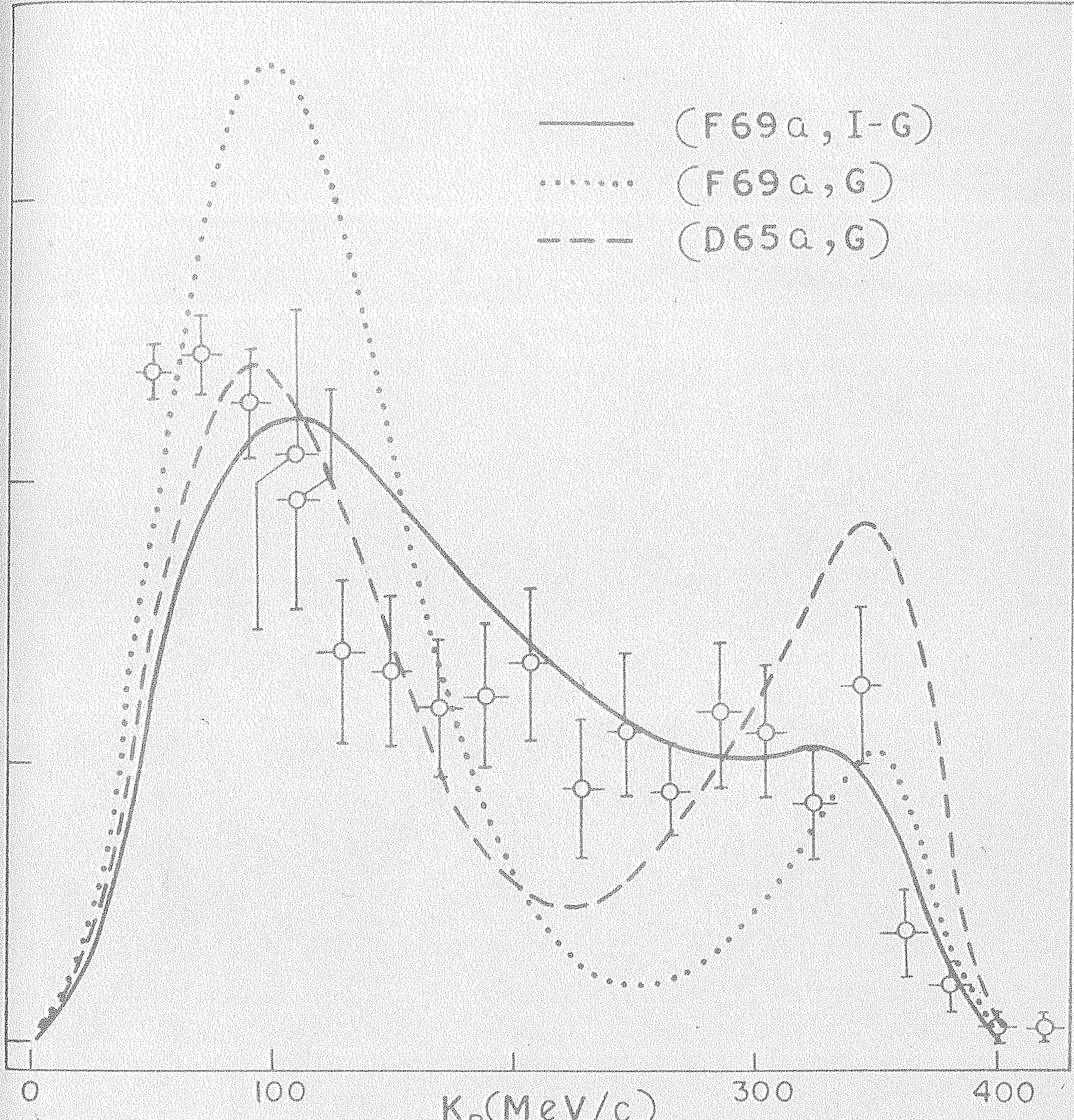


FIG. 13

are similar to those of Fig.11. The experimental results of the Russian group are shown in Fig.13 along with the calculations of Diwakarán (D65a) and Figureau and Ericson (E69a). Clearly the results with uncorrelated wavefunctions and the phenomenological interaction of Eckstein agree much better with experiments than do our calculations. In this connection a brief comment may be made also on the results of Cheon (C66a) and revised calculations of Sakamoto and Tohsaki (S67b). Here again the ^{3}He wavefunctions are Gaussian wavefunctions, with adhoc introduction of correlations via Jastrow factors, and the pion-nucleon forces used are simple ps-pv first order interactions. One sees from Figs. 2 and 3 of reference (C66a) that the momentum distribution shows only one peak in the large momentum region, in the absence of correlations. Even with a suitable value of the correlation length parameter, only a small peak in the low-momentum region (about 1/3 of the peak on high-momentum side) (see the Fig.2 in the ref. (S67b)) is obtained. Perhaps this shows the inadequacy of the simple ps-pv interaction.

5.F γ -mode.

In the previous sections we considered the capture processes in which a pair of nucleons was involved. On the other hand, the photocapture and photoproduction of pions by complex nuclei are generally related to single-nucleon processes e.g. $\pi^- P \to n\gamma$ and $\gamma P \to n\pi^+$ in the impulse approximation; using the CGLN amplitude for the fundamental reactions. Irving and Gunn (G51a) used this method to test various wave-functions for the tri-nucleon systems. A more sophisticated approach has recently been used by Ericson and Figureau (E67a). Here the tri-nucleon system is treated as a single entity with suitable form factors and the capture rate is calculated using the PCAC theory. In the following work we use a rather simple approach to calculate the capture rate in γ-mode. Our primary purpose is to obtain some estimate of the kind of results one can get for single-nucleon processes with non-local-separable potentials, and this test the appropriateness of the wavefunctions obtained for ^{3}He.

Gauge-invariance is invoked to obtain the interaction appropriate for the radiative capture mode. Thus we replace $\underset{\sim}{P}$ by $\underset{\sim}{P} - e\underset{\sim}{A}$ in the expressions for the interaction (equation 2.3), and keeping only the terms relevant to this mode, we obtain

$$H_{\pi\gamma} = G \sum_i \{ \underset{\sim}{\sigma}_i \cdot (-e \underset{\sim}{A}_i(\pi)) \underline{\tau}_i \cdot \underline{\varphi}(i) - \frac{m}{M} \underline{\varphi}(i) \left(\tfrac{1}{2}\right) \cdot \left[\underline{\tau}_i, \frac{1+\tau_3(i)}{2} \right]_+ \underset{\sim}{\sigma}_i \cdot (-e \underset{\sim}{A}_i) \} \tag{5.39}$$

where $[\ ,\ ]_+$ is an anti-commutator, and τ_3 is the Z-component of the I-spin. Since τ_3 does not commute with $\underline{\tau}$, the expression has been symmetrised. Further simplifying the expression, we get,

$$H_{\pi\gamma} = (-eG) \sum_i \{ \underset{\sim}{\sigma}_i \cdot \underset{\sim}{A}_i(\pi)\, \underline{\tau}_i \cdot \varphi(i) - \frac{m}{2M} \underline{\tau}_i \cdot \varphi(i) \underset{\sim}{\sigma}_i \cdot \underset{\sim}{A}_i \} \tag{5.40}$$

where

$$A_i(\underset{\sim}{P}_i, \lambda) = \left(\frac{2\pi}{L}\right)^{3/2} \frac{1}{\sqrt{2W}} \delta(\underset{\sim}{P}_i - \underset{\sim}{k})\, \hat{\epsilon}_\lambda(\underset{\sim}{K}) \tag{5.41}$$

In the above expressions $\underset{\sim}{K}$ and W are the momentum and energy of the photon and $\hat{\epsilon}_\lambda(\underset{\sim}{K})$ denotes its polarisation vector. Including the recoil of the ^3H nucleus, we have

$$\frac{K^2}{2M(^3H)} + K = m, \text{ which gives,}$$

$$K \approx 0.65\ F^{-1} = 130\ \ MeV/c \tag{5.42}$$

It will be noted that the interaction $H_{\pi\gamma}$ is simply the dipole term in the photoproduction process, and in the lowest order theory higher multipole terms will not contribute. Further, since the photon momentum value is far from the resonance region, and (pion being at rest) there is no pionic current, the terms known as ''shaking-off'' and ''photo-electric'' in the usual field-theory jargon are not important in the non-relativistic region under consideration. We are only including in $H_{\pi\gamma}$ here the wellknown ''catastrophic'' term.

The method of calculating the transition amplitude is already described in section 5C. We directly write the matrix element here. For simplicity, we include only the symmetric S and S' terms in the initial and final states (^{3}He and ^{3}H), and ignore the complications of the tensor force. Then,

$$\mathcal{M}_\gamma = \langle \Psi_f | H_{\pi\gamma} | \Psi_i \rangle$$

$$= \left\{ -e G_\pi \left(1 - \frac{m}{2M}\right) \Phi \right\} \sum_\mu C^{1/2\ 1\ 1/2}_{m_s\ -\mu\ m_s'}$$

$$\times \left\{ -\frac{3\sqrt{3}}{2} \langle \psi_s^s | \underset{\sim}{A}{}^s_\mu | \psi^s \rangle + \sqrt{3} \langle \psi_s' | \underset{\sim}{A}{}^s_\mu | \psi_s' \rangle - 8\sqrt{3} \langle \psi_s' | A'_\mu | \psi_s'' \rangle \right\} \qquad (5.43)$$

The capture rate W_γ is calculated using Fermi's golden rule. If the contribution of the fully symmetric S-state alone is taken into account,

$$W_\gamma = 0.58 \times 10^{16}\ \text{sec}^{-1}$$

The contribution of the mixed symmetric states reduces this value to

$$W_\gamma = 0.54 \times 10^{16} \text{ sec}^{-1}$$

These values may be compared with 0.35×10^{16} sec^{-1} obtained by Ericson and Figureau (E67a) using PCAC, 0.10×10^{16} sec^{-1} obtained by Diwakaran and 0.83×10^{16} sec^{-1} of Fujii and Hall (F62a) using the CGLN amplitudes.

5.G Results with Variational Wavefunctions:

The calculations reported in the previous sections have all been carried out in momentum space and with fully antisymmetrised wavefunctions. Thus it was not possible to compare our results at any intermediate steps with those of other authors. In this section we present calculations of capture rates with variational wavefunctions used by other authors, but with different pion-nucleon interaction forms. For example, Figureau and Ericson calculate capture rates with Irving-Gunn (I-G) or modified Irving-Gunn etc. wavefunctions, but use Eckstein form of πNN interaction. It should be interesting to see what results one would obtain with the first order ps-pv πN interaction such as the one we use.

The simplest variational wavefunction viz., a

Gaussian form, can be fitted to the gross properties of ^3He, but fails to give good agreement with the form factors obtained from electron scattering etc. Irving (I51a) and later Irving and Gunn (G51a) have suggested other forms for the wavefunction which fit better the data, expecially for photoproduction of pions. I-G wavefunction gives better agreement with data on electron scattering and radiative neutron capture by deuteron compared to the Irving wavefunction, whereas the latter gives better agreement with muon capture data on ^3He. Since the I-G wavefunction contains a singularity for small inter-nucleon distances, it fits poorly the form factors for momentum transfers larger than $2F^{-1}$. To remedy this Gibson (G67c) has modified the I-G wavefunction at short distances to remove the singularity. Figureau and Ericson (F69a) also consider a somewhat different version of the modified I-G wavefunction.

In view of our result that the inclusion of the mixed-symmetric and D-states in the wavefunctions of ^3He reduces the capture rates, but gives practically unchanged values of W_p/W_d (Table 15), and since our interest is only in comparing the results of different variational wavefunctions with our results, we take for simplicity only the S-state in the variational ^3He wavefunctions.

Since all such wavefunctions are expressed as some functions of $\Sigma r_{ij}^2 = (r_1-r_2)^2 + (r_2-r_3)^2 + (r_3-r_1)^2$ or equivalently functions of $(2R^2 + \frac{3}{2} r^2)$ with $\underset{\sim}{r} = \underset{\sim}{r}_2 - \underset{\sim}{r}_3$ and $\underset{\sim}{R} = \underset{\sim}{r}_1 - 1/2\,(\underset{\sim}{r}_2 + \underset{\sim}{r}_3)$. They become in momentum-space description, simple algebraic functions of $(p_1 - p_2)^2 + (p_2 - p_3)^2 + (p_3 - p_1)^2 = 9/8\, P_i^2 + 3/2\, q_{jk}^2$. Different wavefunctions used in this section are given below both in coordinate as well as momentum-space representation:

Gaussian

$$\Psi_S^S(\underset{\sim}{r}_1, \underset{\sim}{r}_2, \underset{\sim}{r}_3) = \mathcal{N} \exp\left\{-\frac{\lambda}{2}\left(\Sigma r_{ij}^2\right)\right\}$$

$$\Psi_S^S(\underset{\sim}{P}_1, \underset{\sim}{q}_{23}) = \mathcal{N}_G \exp\left\{-\frac{1}{6\lambda}\left(q_{23}^2 + \frac{3}{4} P_1^2\right)\right\}$$

with $\lambda = 0.147\ F^{-2}$

Irving

$$\Psi_S^S(\underset{\sim}{r}_1, \underset{\sim}{r}_2, \underset{\sim}{r}_3) = \mathcal{N} \exp\left\{-\frac{\lambda}{2}\left(\Sigma r_{ij}^2\right)^{1/2}\right\}$$

$$\Psi_S^S(\underset{\sim}{P}_1, \underset{\sim}{q}_{23}) = \mathcal{N}_I \left\{q_{23}^2 + \frac{3}{4} P_1^2 + \frac{3\lambda^2}{8}\right\}^{-7/2}$$

with $\lambda = 1.27\ F^{-1}$

Irving-Gunn

$$\psi_S^S(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \mathcal{N}(\Sigma r_{ij}^2)^{-1/2} \exp\left\{-\frac{\lambda}{2}(\Sigma r_{ij}^2)^{1/2}\right\}$$

$$\psi_S^S(\mathbf{p}_1, \mathbf{q}_{23}) = \mathcal{N}_{I-G}\left\{q_{23}^2 + \frac{3}{4} p_1^2 + \frac{3\lambda^2}{8}\right\}^{-5/2}$$

with $\lambda = 0.771\ F^{-1}$

modified Irving-Gunn (F69a)

$$\psi_S^S(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \mathcal{N}(\Sigma r_{ij}^2)^{-1/2} \times$$

$$\left[\exp\left\{-\frac{\lambda}{2}(\Sigma r_{ij}^2)^{1/2}\right\} - \exp\left\{-\frac{\delta}{2}(\Sigma r_{ij}^2)^{1/2}\right\}\right]$$

$$\psi_S^S(\mathbf{p}_1, \mathbf{q}_{23}) = \mathcal{N}_{\text{modified } I-G}$$

$$\left[\left\{q_{23}^2 + \frac{3}{4} p_1^2 + \frac{3\lambda^2}{8}\right\}^{-5/2} - \left\{q_{23}^2 + \frac{3}{4} p_1^2 + \frac{3\delta^2}{8}\right\}^{-5/2}\right]$$

with $\delta = 2\lambda = 1.874\ F^{-1}$

Now we follow the approach of section 5.C, with the final wavefunctions given by equations (5.23) or (5.34), and the interaction as given in equation (5.5). We obtain for the transition amplitude M_{fi} (initial state ψ^s_s) ;

$$M_{fi} = \frac{3\sqrt{3}}{2}\left[\langle\varphi'|p'_{\mu}|\psi^s_s\rangle + \langle\varphi''|p''_{\mu}|\psi^s_s\rangle\right] \tag{5.44}$$

and the proton momentum spectrum always turns out to be

$$\frac{dW}{dK_1} \sim (8k_{23}^2 + k_{12}^2)\, K_1^2 k_{23} \tag{5.45}$$

where K_1 is the proton-momentum

For each of the above wavefunctions, the capture rates W_d and W_p are evaluated analytically, and the results are summarised in table 14. The Gaussian wavefunction gives not only very low results for the capture rates, but also a rather large ratio for W_p/W_d. This may be due to the lack of high momentum components in this wavefunction. Other wavefunctions also yield rather small values of capture rates compared to the results for non-local separable wavefunctions or Figureau-Ericson calculations. It is obviously necessary to build into these wavefunctions more correlations as in our approach or to modify the πN interaction as in Figureau-Ericson calculation.

The proton momentum distribution is found to be identical for all the wavefunctions since they are all functions of $(3/4\ P_1^2 + q_{23}^2)$ which turns out to be constant as a result of energy conservation. The spectrum is plotted in figure 11, and shows only one peak in the high momentum region.

Table - 14

Wave function	W_d (sec^{-1})	W_p (sec^{-1})	W_p/W_d
Gaussian	2.11×10^{13}	2.72×10^{14}	12.9
Irving	2.25×10^{15}	7.24×10^{15}	3.22
I-G	5.48×10^{15}	1.31×10^{16}	2.38
modified I-G	2.56×10^{15}	7.83×10^{15}	3.06
NLS	1.89×10^{16}	4.57×10^{16}	2.41
experimental			3.6 ± 0.6

5.H Summary

We summarise in table 15 the results for different capture modes and their ratios obtained with different wavefunctions (using Yamaguchi's spin-independent and spin-dependent central forces and the central + tensor force model), as well as the theoretical results of some other authors and the experimental results. It should be noted again that what we call the ''proton capture rate W_p'' is not the same as that quoted in Diwakaran or Ericson-Figureau papers (see comments on p. 169).

We see that although the introduction of spin-dependence and tensor terms in NN interactions reduces the various capture rates, the relative ratios remain practically the same. The results with different potentials and methods of calculations appear to be reasonably well in agreement with each other as well as with experimental results. However, none of them gives a really outstanding agreement with experiments.

Table - 15

Specification and details of the work		Different capture rates (10^{16} sec^{-1}) W_d	W_p	W_γ	W_p/W_d	W_d/W_γ	$\frac{W_p+W_d}{W_\gamma}$
NLS Wave function.	S-state $E_b = 12.43$ Mev.	1.89	4.56	.585	2.41	3.23	11.03
	S+S' state $E_b = 12.9$ Mev.	1.65	4.19	.54	2.54	3.06	10.85
	S+S'+D state $E_b = 10.4$ Mev.	.795	2.05	-	2.58	-	-
Diwakaran - Gaussian wave fn. $g_o^2 = 0.32F^8$, $g_1^2 = 0.29F^8$		.140	.785	.097	5.61	1.44	9.54
Figureau-Ericson $g_o^2 = .64F^8$ $g_1^2 = .155F^8$ $g_o g_1 > 0$	Irving-Gunn	1.93	3.01	.346	1.56	5.58	14.28
	Modified I-G	1.06	1.99	.346	1.88	3.07	8.81
EXPERIMENTAL					3.6 ± 0.6	2.3 ± 0.4	10.7 ± 1.02

Our purpose in this chapter (as also in the previous chapter) was to check what kind of results for pion capture may be obtained with 'exact' wave-functions for ^3He obtained with realistic potentials in the non-local separable form. We find that the wavefunctions obtained with such potentials do contain sufficient correlations to give reasonably good values for capture rates even with the first order ps-pv interactions. We have also explicitely shown the relative effects of mixed-symmetric terms in the wavefunctions and of tensor forces in NN interaction. One result is that the effect of tensor forces (via. S-D cancellation) on capture rates in ^3He is by no means so alarmingly severe as in the Deuteron. The need for including second-order pion-scattering terms is thus not so obvious. On the other hand, the pion momentum spectrum obtained in our calculations appears defective in the sense that the experimentally observed peak for low proton momenta is rather weak, and is further reduced by inclusion of tensor forces. Also the strong interference between the high and low - momentum regions does not allow a clear view of the low momentum peak. We have seen in the last section that even with variational wavefunctions for ^3He which are currently used, there appears only one peak (at high proton momenta) in the proton momentum distribution, when only first order ps-pv interaction is used. On the other hand, Diwakaran and Ericson and Figureau who use the phenomenological pion.nucleon-pair interaction (which presumably already contains the pion-scattering-before-absorption effects) do obtain the low-momentum peak. It is possible that the inclusion of pion-scattering terms is really essential to give the low momentum peak in proton spectrum.

6. CONCLUSION

Our prime interest in this work is to understand the nucleon-nucleon correlations introduced by realistic nuclear forces by studying the pion capture processes. In this respect we have chosen the three-body systems ^{3}He and ^{6}Li. We have solved the three-body equations for the ^{6}Li with the use of non-local-separable potential in the Schrodinger equation. (The solution of 3N system is already available in the literature) The binding energy calculated for the ground state $(J=1^{+})$ is found to be somewhat larger, and for the excited state (0^{+}) somewhat smaller, than the experimental values. The spectator function for the nucleon is shown to contain a sizeable component for large momenta $\gtrsim$ 300 MeV/C. The spin-orbit term in the α-N interaction which was neglected in the Schrodinger equation is treated perturbatively. Its contribution to ground state binding energy is negligible (5 KeV).

Before going to pion capture by complex nuclei, we have first considered the pion capture by the deuteron. For this we have chosen the $N\pi$ interaction to be the ps-pv interaction in the static limit.

(We have discussed the importance of the rescattering term in the interaction, but here the full emphasis is put on the exact wave function.) The Schrodinger equation with central + Tensor + LS non-local-separable potential terms yields an analytical solution for the deuteron. The pion capture rate on the deuteron turns out to be larger compared to one derived from the experimental data for $\pi^+ d \to pp$ (R67a). It should be noted that the NLS potential does not contain explicit strong short range repulsion; yet the wave function does have a large correlation. The inclusion of the tensor force gives very large (though not severe) S-D cancellation, in agreement with Koltun Reitan's result (K66a). In the presence of non central forces the capture rate turns out to be smaller compared to one derived from the experimental data of Rose. The final state interaction contributed equally to the capture rate in the presence of noncentral forces.

In case of ^{6}Li we consider capture only on the valence proton. Here we emphasized the role of the α-p correlation term ('~F') in the nuclear wave-function and the CM momentum term (P) in the pion-nucleon interaction. We have seen that in the presence of '~F' term, P-term of the interaction is no more negligible. This P-term which has no effect on the constrained angular distribution of the α-particle (i,e. $y=-1$), modifies the α-particle momentum spectrum for large momenta. The 'gG' term which represents nothing but the α-d composition contributes only about 20 % to the capture rate. As expected this contribution is reduced quite a lot in the presence of tensor force. But the overall S-D cancellation is not so large compared to deuteron case. This is because of the 'ν-F' term of the wave function. This 'ν-F' term also introduces in the spin averaged squared transition amplitude a large dependence on $\hat{K}.\hat{k}_{12}$ in contrast to the argument of Koltun-Reitan (K67a).

We have carried out the pion capture calculation in ^{3}He by using full three-body permutation symmetry. We have approximated the potential in such a way as to study the effect of S- , S'- and D-states of ^{3}He on the different capture rates. The effect of S'-state is to reduce the capture rate by about 10 % , which is further reduced by 50 % in the presence of tensor force. Thus the infamous S-D cancellation here is not so severe. The capture rates calculated here are comparable to the results of Figureau-Ericson (F69a) where the correlations are indirectly built into the πNN

interaction. This implies again the large correlations built into the NLS wave functions even in the absence of strong short range repulsive term in the potential. Though the capture rates are reduced with the inclusion of S'- and D-state, the ratios for different capture modes are more or less unaffected, and they are in good agreement with the experimental as well as other theoretical results. The main drawback of our results pertains to the proton momentum spectrum where we do not get a low momentum peak which is quite pronounced experimentally. As the NLS wave function (with (C+T) + S potential) for 3N system agrees quite well with the from factor data upto a couple of hundred MeV/C momentum transfer, it may be that the ps-pv interaction is not adequate to give the proton low momentum peak.

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