MODIFICATION OF LOW FREQUENCY WAVES BY OSCILLATING FIELDS

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C E R T I F I C A T E

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

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TO

MY TEACHER

SHREE BEG RAM SINGH

WAVE INTERACTIONS IN A TURBULENT PLASMA

STATEMENT

It has been widely recognized by now that a plasma generally exists in a turbulent state and the properties of plasma in this state are significantly different from those in a quiet state. In the present work the author examines some effects of turbulence on dispersion and damping characteristics of collective modes in a plasma and points out the relevance of the results to various situations encountered in laboratory and space plasmas.

The thesis starts with an introductory chapter which summarises the work. In the second and third chapters, it has been shown that the low frequency dispersion properties of plasma are significantly affected by high frequency short wavelength turbulence. Not only are the usual modes modified, but also new modes appear. Chapter II studies the electrostatic drift waves in the presence of plasma wave turbulence. In chapter III we go over to the study of a plasma in the presence of whistler turbulence. Magnetosphere is good example of such a system. It has been found that a purely growing instability may arise which may lead to the field aligned irregularities of the kind actually observed in magnetosphere.

Having seen some of the effects of high frequency short wavelength turbulence in the previous chapters, chapter IV is devoted to the influence of low frequency short wavelength turbulence on high frequency waves. An example of lower hybrid wave damping and the resultant heating has been dealt with at length. Lower hybrid hating of thermonuclear plasmas has attracted considerable attention recently. In the present work it has been found that even moderate levels of low frequency short wavelength turbulence in the background can enhance the heating rate significantly. It is worth pointing out that in contrast to wellknown parametric heating effects, this type of heating is a thresholdless process.

It is wellknown that parametric coupling processes may be of importance in widely different situations like laser - driven fusion, pulsar dynamics, artificial modification of ionosphere etc. Chapter V is therefore devoted to the study of parametric coupling process in a turbulent plasma with random density fluctuations. For the sake of illustration detailed calculation has been done for Stimulated Raman Scattering. It has been found that the growth rate can be significantly reduced but that the instability cannot be quenched completely by the turbulence. Large scale length turbulence is found to be more effective in weakening the parametric amplification processes.

Finally, the last chapter has been devoted to the study of parametric excitation of drift waves by ion - ion hybrid fields. For typical fusion plasmas, ion-ion hybrid frequency is low frequency and so sufficient RF power is readily available. There is an added advantage of direct ion heating. Threshold power and growth rate have been calculated. Two interesting results have been found by comparing this work with Nishikawa's general formulation of parametric instability : a) no purely growing instability is possible in inhomogeneous plasma and b) at high pump power (much above the threshold) the maximum growth rate need not follow the cube root law which was supposed to be general enough after Nishikawa. Instead every case needs an individual examination. Examples of square root law, fourth root law etc. have been pointed out.

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C O N T E N T S

CHAPTER I

INTRODUCTION

One of the fundamental characteristic features of a plasma is that it can support a variety of collective modes of oscillations. A plasma consists of various species of charged particles which interact with each other via screened coulomb force. Short range encounters (distance less than the shielding distance viz. Debye length) lead to typical 'collisional' effects like radiation, resistivity etc. whereas long range interactions (over distances much longer than the Debye length) are responsible for collective effects. Collective modes basically involve the organized motion of large number of plasma particles and primarily arise because the plasma strives to maintain macroscopic neutrality.

The conventional theory of the collective modes of oscillations in a plasma is the usual linear theory of wave propagation. In this theory, one starts with a quiescent equilibrium and then considers small perturbations on it. The linearized equations are written for these perturbations. One then Fourier analyses in space and time and the elimination of all variables except one leads to a self-consistent condition which is known as the dispersion relation. The dispersion relation, a relation between the frequency ω and wave number k, describes the characteristic features of small amplitude collective modes in a quiescent equilibrium plasma. This type of linear theory is commonly well developed and is a part of standard text books [1, 2].

Although most theories of collective oscillations start with quiescent equilibrium, it is widely recognized that most plasmas in practice are far from equilibrium state. The plasmas found in nature or created in the laboratory as a rule turn out to be 'turbulent', that is, already contain fluctuations of density, temperature, electric and magnetic fields and other parameters. Actually plasmas can be considered as a nonlinear system of many degrees of freedom (collective oscillations) which is far from equilibrium because they contain many free energy sources like gradients in density, temperature, magnetic field and other kinds of plasma streaming.

The collective oscillations in certain regions of k - space are excited (through an instability mechanism) to trap some of this free energy. The instability is then responsible for turbulence - very much like the case of fluid turbulence [3, 4]. What happens when such an instability is excited is that the wave amplitude grows to such an extent that many nonlinear processes come into play and redistribute the energy over various degrees of freedom. Thus an energy flow in k- space is the result. In particular energy flows to a region of k- space where various dissipative mechanisms are operative. In plasmas, many of the dissipative mechanisms involve interaction with particles, for example Landau and cyclotron dampings. Due to these special damping mechanisms, the energy flow can take place towards smaller k values. The entropy decrease in this manner (reduction in phase space volume) could be compensated for by particle heating effects.

Thus, in conclusion, a thermodynamically nonequilibrium system relaxes by exciting the instabilities in certain collective modes. Then various nonlinear processes transfer energy from the unstable (modes) to the other stable modes wherein the damping mechanisms extract the energy out of stable waves and feed it to the particles resulting in the effects such as strong heating, anomalous diffusion, etc. If these sources and sinks of energy are constantly maintained a quasisteady state may be reached and the turbulence may be maintained at quasi-stationary level.

Granting that most of the plasmas exist in a turbulent state, it seems to be desirable to find out how the known properties of an equilibrium plasma get modified. For example, what is the modified dispersion relation? How are the various damping or growth rates altered? One particular thing of interest from the point of view of applications is to investigate how a given type of turbulence affects some plasma instabilities which are commonly considered to be dangerous for the plasma confinement. These are some of the questions we ask ourselves in the present piece of work.

It is well known that no satisfactory theory of strong turbulence of plasmas or fluids has been developed so far. However plasma turbulence normally involves propagating waves. Unlike the liquid eddies, propagating waves can interact with each other only weakly provided their amplitudes are not too high. This type of turbulence is called a weak turbulence and their theory is facilitated very much by the presence of a small parameter in the system. Perturbation theories thus can be used by expanding various quantities in terms of wave amplitude and retaining only few terms of the series. Under the weak nonlinearities a kinetic wave equation can be derived which includes nonlinear wave particle interaction as well as three wave processes like coalescence of two waves into one and splitting of one wave into two.

Furthermore, it is also possible to separate interactions into resonant and adiabatic interactions. Resonant conditions are required by wave particle interactions and by three wave resonant interactions. While the adiabatic interactions are those where slow wave provides an inhomogeneity in the background and the wave packets of the other waves are deformed and their wavelength changed as they propagate through such an inhomogeneity; as a result of these adiabatic interactions, the wave packets move about in wave number space, which leads to a strong correlation of nearby Fourier components, which now describe essentially one and the same wave-packet.

The first two chapters of the present thesis are devoted to studying the effects of these adiabatic interactions. As we have ignored the resonant interactions (considered to be small comparatively), the kinetic wave equation reduces to a simpler form written by Vedenov et al [5] which is very similar to Vlasov equation. In the first chapter we have studied how the low frequency dispersion relation in an inhomogeneous magnetized plasma is modified due to these adiabatic interactions in the presence of Langmuir turbulence. It has been found that the drift wave dispersion relation are modified significantly. Some of the existing instabilities might be suppressed whereas some new ones may be excited. These low frequency long wavelength modes would be macroscopic in nature and therefore must be taken into account to consider the gross features like confinement. The drift waves in turbulent plasma due to Langmuir turbulence have been previously studied by Krivorutsky et al [6] . Our results reduce to theirs in the corresponding limit. New features of our work are 'kinetic' type of instability, where a resonance is found such that the group velocity of the Langmuir waves matches with the phase velocity of the drift waves. Considerable energy exchange between the turbulence and drift waves is possible in this case. To be more realistic we have also taken the angular dispersion in the group velocity of plasma turbulence into account. The contribution of this term turns out to be more important than the usual thermal dispersion. Our mathematical treatment is also simpler than that of reference [6] . Dobrowolny [7] has studied drift waves in the presence of ion acoustic turbulence and has obtained more or less similar results qualitatively.

In the next chapter, we consider a case of whistler turbulence and again use the wave kinetic equation emphasizing only on the adiabatic part of the nonlinear interactions. It has been found that a filamentation instability may grow in the case of quasi-longitudinal whistlers. This has been speculated to be a possible mechanism of production of ducts in the magnetosphere. Ducts are supposed to be the field aligned electron density irregularities which behave like parallel wave guides for whistlers and constrain whistlers to propagate along the lines of forces. That is how whistlers generated in one hemisphere can be received in another hemisphere. How are these field aligned irregularities produced is an open question and to our knowledge has not been attempted so far. We find that a purely growing instability can break whistlers into field aligned filaments. More and more electrons are accumulated inside these filaments. This density irregularity may thus contribute (at least partly) to the duct formation. Stenzel [8] has recently experimentally investigated the self-ducting of large amplitude whistler waves in a plasma. However, his experiment seems to be restricted to the case when a single large amplitude whistler wave is present and coherent effects like particle trapping etc. can be important. Our picture of duct formation is, however, based on a turbulent spectrum of whistler waves in the background where particle trapping effects are ignorable.

The adiabatic approximation used so far requires very different scale sizes. We have assumed that the frequency and the wave number of drift waves are sufficiently small as compared to those of background turbulence so that the turbulence 'sees' the low frequency long wavelength perturbation as slowly (adiabatically) varying back-ground. Similarly in the case of whistler turbulence, the scale size of filamentation instability is quite slow compared to that of whistlers. Thus only high frequency short wavelength turbulence can be treated in the foregoing manner. There are however other types of turbulence also. For example there may exist a low frequency short wave length turbulence. Our fourth chapter deals with this type of the turbulence.

Heating of plasmas by high frequency a.c. field has been considered as an important auxiliary method of heating. Normally such heating requires a sort of the threshold power and makes use of excitation of parametric processes. Therefore we shall study in the fourth chapter the anomalous plasma heating by a.c. field at lower hybrid frequency assuming a low frequency short wavelength turbulence in the background. We shall find that the power absorption rate by ions will be increased abnormally even if the turbulence is maintained at moderate level. The long wavelength pump power couples with the short wavelength turbulence and generates short wavelength sidebands which then are absorbed anomalously, say, by Landau damping.

Still more interesting point in this type of anomalous heating is that no threshold condition is required. Thus in contrast with normal parametric heating, a thresholdless anomalous heating mechanism is available so far as the background turbulence exists which may either exist already or may be externally generated by suitable means.

In fifth chapter we go over to the study of the effects of a quasi-static random density fluctuations on the three wave resonant interactions like stimulated Raman and Brillouin scattering (SRS and SBS). These processes are of great relevance to laser plasmas. After Fourier analysing the time variable and eliminating the amplitude of one of the stimulated waves, the basic equation describing the parametric process reduces to a fourth order differential equation with coefficients which are random functions of the position co-ordinates. Standard methods from the theory of wave propagation in random media are then used. It is found that in a statistically homogeneous medium, the effect of fluctuations is to enhance the growth length, i.e., to make the growth weaker. The resonance matching conditions $\omega_{incident} = \omega_{scattered} + \omega_{longitudinal}$ must be satisfied. In a random density medium, the plasma frequency ω_p is also a random function of space co-ordinate and therefore the probability of satisfying this matching condition is reduced which effectively reduces the growth rate of the parametric processes.

Finally, in the last chapter, we come to the excitation and suppression of drift waves by a large amplitude long wavelength oscillating field at the ion - ion hybrid resonance. This problem is essentially an example of parametric process. Threshold power for excitation of drift waves has been calculated and the characteristics of dispersion relation near threshold and well above threshold are examined. It has been realized that no purely growing instability is possible for inhomogeneous plasmas. Also at sufficiently large applied powers (much above threshold), the maximum growth rate does not always go as Nishikawa's [9] cube root law. Different such laws for different cases have been found and discussed. In fact for inhomogeneous plasma each and every case of parametric instability should be examined separately as the law turns out to be different in different cases. Precise knowledge of such law may be useful in identifying the instability.

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CHAPTER II

DRIFT WAVES IN TURBULENT PLASMA

2.1 Introduction

In a turbulent plasma, the dispersion relation for various collective modes may be very different from that of a quiescent plasma. As an illustration of this phenomenon, in this chapter we study the influence of high frequency short wavelength electron plasma waves on low frequency long wavelength drift waves. Low frequency long wavelength fluctuations are generally associated with large scale mass motions and anomalous plasma transport in magnetic plasmas. Furthermore, one often introduces short wavelength high frequency turbulence into the plasma to improve its heating. Any influence of the latter on former is therefore of considerable interest in fusion systems and studies of the type carried out in this chapter may be used for this purpose. Although this particular calculation is devoted to plasma waves interacting with drift waves, the method and many results are quite general and there are many other physical situations where it may be applicable. In the next chapter, for example, we shall investigate a similar problem, which is of interest for duct formation in magnetosphere.

Earlier, some studies of this kind have been made by Krivorutsky et al [1] and Dobrowonly [2], both of whom followed a rather cumbersome Vlasov treatment, which was somewhat physically obscure. A much simpler and physically transparent method was developed by Vedenov and Rudakov [3], who used it to study ion - acoustic waves in the presence of Langmuir turbulence. We extend this method to the problem of an inhomogeneous magnetized plasma and the study of drift waves. Our calculation shows that not only is it possible to modify the already existing modes but also that new classes of drift instabilities can arise. In the appropriate limits, our results reduce to those of Krivorutsky et al [1] ; furthermore several new features emerge.

By turbulent plasma, we shall mean one in a quasi-stationary turbulent state. We restrict ourselves to weak turbulence, i.e., turbulence generated by the nonlinear saturation of various modes which closely follow the linear dispersion relation. By the very nature of our problem, we shall be restricting our attention to 'adiabatic' interactions only, i.e., long wavelength perturbations on a short wavelength background of turbulence. These various limitations permit us to use the kinetic wave equation to describe the turbulence and follow the Vedenov - Rudakov approach.

2.2 Derivation of Dispersion Relations

Let us consider a low - β inhomogeneous plasma (gradient along the x-axis) immersed in a uniform magnetic field

$B_o = (0, 0, B_o)$. Let us assume that there is a high frequency short wavelength turbulence in the plasma and that its behaviour is governed by the wave kinetic equation. As has been pointed out earlier in Chapter I, one can separate the resonant and adiabatic interactions and then treat the adiabatic interactions in a quasi-classical way, namely the influence of adiabatic interactions manifests itself effectively in terms of "slowly" varying background parameters (such as density etc.) in space and time. The wave kinetic equation then (after this separation) can be written in the following form

$$\frac{\partial N_k}{\partial t} + v_g \cdot \frac{\partial N_k}{\partial r} - \frac{\partial \omega_k}{\partial r} \cdot \frac{\partial N_k}{\partial k} \simeq C(N_k, N_k) \tag{2.1}$$

where N_k is the quasiparticles (excitons) distribution function defined by

$$N_k \equiv W_k / \omega_k \tag{2.2}$$

where $$W_k = \frac{1}{\omega_k} \frac{\partial}{\partial \omega_k} \left(\omega_k^2 \epsilon_k \right) \frac{|E_k|^2}{8\pi} \tag{2.3}$$

i.e., W_k is the wave energy density, ω_k is the frequency of turbulence in the linear approximation,

$\epsilon_k \simeq 0$ for the electrostatic turbulence, while

$\epsilon_k \simeq \frac{c^2 k^2}{\omega_k^2}$ for electromagnetic turbulence ; N_k is thus the wave energy density per unit frequency interval.

Equation (2.1) looks like a Boltzman equation in which $v_g = \frac{\partial \omega}{\partial k}$ = group velocity plays the role of the excitons velocity (speed of energy flow). The last term on the left hand side is due to the adiabatic interactions. The slowly varying background introduces a kind of force $\equiv - \nabla \omega_k$. The right hand side of Eq. (2.1) is like a collision term and includes various types of resonance processes, e.g., wave particle resonance (both linear and nonlinear Landau damping) and the three wave resonance interactions (both coalescence of two waves into one and spliting of one wave into two). Our purpose in the present chapter is to investigate the effects of adiabatic term only. We shall therefore assume that all the resonance terms are either very small in magnitude ~~are~~ or exactly balance each other. The right hand side of Eq. (2.1) then can be put to zero and one obtains the equation used by Vedenov and Rudakov [3,5]

$$\frac{\partial N_k}{\partial t} + v_g \frac{\partial N_k}{\partial z} - \frac{\partial \omega_k}{\partial z} \cdot \frac{\partial N_k}{\partial k} \simeq 0 \tag{2.4}$$

This is a Vlasov - like equation and implies that the number of plasmons N_k is conserved as they move on the trajectory

$$\frac{dz}{dt} = v_g \quad \text{and} \quad \frac{dk}{dt} = - \frac{\partial \omega_k}{\partial z} \tag{2.5}$$

Space and time variation in equation (2.4) are slow compared to space - time variation of the microturbulence. Thus if this slow dependence is provided by drift waves$(\Omega, \underset{\sim}{q})$ in the medium, we must require $\Omega \ll \omega_k$, $q \ll k$ for the validity of equation (2.4). The presence of a small density fluctuation $\tilde{n}_e$ due to drift waves modifies the plasmon distribution N_k . This is evident from the plasma wave dispersion relation (for an electron-cyclotron frequency $\omega_{ce} \gg \omega_{pe}$ the plasma frequency),

$$\omega_k^2 = (k_z^2/k^2)(\omega_{pe}^2 + k^2 v_e^2) \tag{2.6}$$

which shows that $\dfrac{\partial \omega_k}{\partial \underset{\sim}{k}} \simeq \dfrac{i}{2} \underset{\sim}{q}\, \omega_k \tilde{n}_e / n_0$

Substitution of this into the linearised version of Eq. (2.4) then yields

$$\tilde{N}_k = -\frac{\omega_{pe}}{2} \frac{\tilde{n}_e}{n_0} q \cdot \frac{\partial N_k^0}{\partial k} \Big/ (\Omega - q \cdot v_g) \tag{2.7}$$

Eq. (2.6) shows that fluctuations in electron density due to drift waves directly induce corresponding fluctuation in the density of electron plasma waves.

The modified plasmon distribution reacts back on the drift waves through the averaged ponderomotive force term $\langle (v \cdot \nabla) v \rangle$ in the low frequency electron equation of motion. The parallel component of the linearised equation of motion now takes the form,

$$0 \simeq -\frac{T_e}{m n_0}\frac{\partial \tilde{n}_e}{\partial z} + \frac{e}{m}\frac{\partial \tilde{\phi}}{\partial z} - \frac{e^2}{2m^2}\frac{\partial}{\partial z}\sum_k \tilde{N}_k / \omega_k \tag{2.8}$$

wherein the last term we have used the definition

$$\tilde{N}_k = \widetilde{|E_k|^2} / 4\pi\omega_k$$

Electron inertia has been ignored because $\Omega << q_z v_e$ for drift waves. We have also neglected the electron collisions (normal or "anomalous") to exclude the conventional drift-dissipative instability. Equations (2.7) and (2.8) show that

$$\tilde{n}_e / n_0 \simeq (e\tilde{\phi}/T_e)\left[1 - A\right]^{-1} \tag{2.9}$$

where

$$A = \frac{W}{4 n_0 T_e}\omega_{pe}\int \frac{q \cdot \frac{\partial N_k^0}{\partial k}}{\Omega - q \cdot v_g} d^3k \left[\int N_k^0 (k_z/k)\, d^3k\right]^{-1} \tag{2.10}$$

and $W = C\int N_k^0 \omega_k d^3k$ is the energy density of plasma wave turbulence, C being a normalization constant taking care of dimensions while going from $\sum_k$ to $\int d^3k$.

The plasma wave turbulence leaves the ion dynamics unaltered. We can therefore follow the conventional treatment of drift waves for subsequent analysis. For $T_i \simeq 0$ and

$\Omega \ll \omega_{ci}$, the ion cyclotron frequency, one can show [6] that

$$(\tilde{n}_i / n_o) \simeq (\omega_* / \Omega)(e\tilde{\phi}/T_e) \tag{2.11}$$

where $\omega_* = -\frac{c q_y T_e}{e B_o} \frac{1}{n_o} \frac{d n_o}{dx}$ is the drift frequency. The parallel motion of ions and the corrections due to finite ion Larmour radius are ignored. Using the conditions of quasineutrality, Equations (2.9) and (2.11) yield the dispersion relation

$$\Omega = \omega_* (1 - A) \tag{2.12}$$

Note that A is a complicated function of frequency Ω (Eq. (2.10)).

When the ion temperature is finite $(T_e = T_i = T)$ and a strong temperature gradient exists $(d \ln T / d \ln n \gg 1)$, one has to retain the parallel motion of ions and one thus gets [7],

$$\frac{\tilde{n}_i}{n_o} \simeq \frac{e\tilde{\phi}}{T} \frac{q_z^2 v_s^2}{\Omega^2} \frac{\omega_T^*}{\Omega} \tag{2.13}$$

where $\omega_T^* = -(q_y c / e B_o) \frac{dT}{dx}$ and $v_s = \sqrt{2T/M}$ is the speed of sound. Together with equation (2.9) this yields the modified dispersion relation for the drift temperature gradient instability, viz.

$$\Omega^3 = (q_z^2 v_s^2 \omega_T^*)(1 - A) \tag{2.14}$$

2.3. Analysis of Dispersion Relations

We begin our discussion of Eqs. (2.12) and (2.14) by considering one-dimensional turbulence of electron plasma waves propagating along the magnetic field $(k_z \simeq k)$ and having an equilibrium spectrum

$$N_k^0 = (N_0/\sqrt{2\pi}\,\Delta)\exp\left[-(k-k_0)^2/2\Delta^2\right] \tag{2.15}$$

Equation (2.10) may now be expressed in terms of the plasma dispersion function [4];

$$A = \frac{W}{4n_0 T_e}\frac{\omega_{pe}^2}{v_e^2\Delta^2}\left[1 + \frac{k_0}{q_z}\frac{\Omega - q\cdot v_0}{\Delta v_0} Z\left(\frac{k_0}{q_z}\frac{\Omega - q\cdot v_0}{\Delta v_0}\right)\right] \tag{2.16}$$

where $v_0 = v_g(k = k_0)$. Two limiting cases are of interest.

(i) "Cold" limit :- This corresponds to a sharply peaked spectrum of plasma waves in k-space. Choosing $\Delta \ll k_0(\Omega - q\cdot v_0)/q\, v_0$ we get

$$A \simeq \alpha/(\Omega - q\cdot v_0)^2 \tag{2.17a}$$

with

$$\alpha = -q_z^2 v_e^2 (W/4n_0 T_e) \tag{2.17b}$$

(ii) "Warm" limit :- This corresponds to a broad spectrum of plasma waves satisfying $\Delta \gg k_o(\Omega - q \cdot v_o)/q \cdot v_o$ and here

$$A \simeq \frac{W}{4 n_o T_e} \frac{\omega_{pe}^2}{\Delta^2 v_e^2} \left[1 + i\sqrt{\frac{\pi}{2}} \frac{k_o(\Omega - q \cdot v_o)}{q_z v_o \Delta} - \left(\frac{k_o}{q_z} \frac{\Omega - q \cdot v_o}{v_o \Delta} \right)^2 \right] \tag{2.18}$$

In the cold limit equation (2.12) takes the form

$$(\Omega - \omega_*)(\Omega - q \cdot v_o)^2 + \alpha \omega_* = 0 \tag{2.19}$$

This cubic equation can be readily solved by Cardan's method. For large α , the solution is

$$\Omega \simeq (\alpha \omega_*)^{1/3} \left[-1, \exp\left(\pm \frac{i\pi}{3}\right) \right] ,$$

$$|\alpha| \gg (4/27) |q \cdot v_o - \omega_*|^3 / |\omega_*| , \tag{2.20}$$

an equation similar to (2.20) has been derived by Krivorutsky et al [1] . For small α,

$$\Omega \simeq \omega_* , \; q \cdot v_o \pm i \left[\alpha \omega_* / (\omega_* - q \cdot v_o) \right]^{1/2}$$

$$|\alpha| \ll (4/27) |q \cdot v_o - \omega_*|^3 / |\omega_*| \tag{2.21}$$

Equations (2.20) and (2.21) show that even small magnitudes of plasma turbulence can drastically alter the properties of drift oscillations in a plasma. In particular strong instabilities e.g. with a growth rate $\sim W^{1/3}$ can be driven by the turbulence. Let us examine the conditions required by this instability (i.e. Eq. (2.20)).

We know that the parallel phase velocity of the wave must be less than the electron thermal speed, therefore Eq. (2.20) yields

$$(\alpha\omega_*)^{1/3}/q_z v_e \ll 1 \quad \Rightarrow \quad |\alpha| \ll q_z^3 v_e^3/\omega_*$$

Thus

$$\left|\frac{q_z^3 v_e^3}{\omega_*}\right| \gg \alpha = \frac{q_z^2 v_e^2 W}{4 n_0 T_e} \gg \frac{4}{27}\,\frac{|q\cdot v_0 - \omega_*|^3}{|\omega_*|} . \qquad (2.22)$$

Condition (2.22) can be easily satisfied.

Our analysis gives a simple physical picture for these instabilities. In the event of an instability, the phase relationship between $\tilde{n}_e$ and $\tilde{N}_k$ is such that the plasma waves accumulate in the trough of drift waves. This drives more particles out of the trough through ponderomotive force and thus enhances the density perturbations.

The drift temperature gradient instability is also strongly modified. In this case, one obtains

$$(\Omega^3 - q_z^2 v_s^2 \omega_T^*)(\Omega - q\cdot v_0)^2 + \alpha q_z^2 v_s^2 \omega_T^* = 0 \qquad (2.23)$$

For large and small α, this gives the roots,

$$\Omega \simeq (\alpha q_z^2 v_s^2 \omega_T^*)^{1/5}\left[-1,\ \exp\left(\frac{\pm i\pi}{5}\right),\ \exp\left(\frac{\pm 3i\pi}{5}\right)\right] \tag{2.24}$$

and

$$\Omega \simeq (q_z^2 v_s^2 \omega_T^*)^{1/3}\left(\frac{-1+i\sqrt{3}}{2}\right)+\left[\frac{\alpha\left(1+4a+4a^2+i\sqrt{3}\,(1-a^2)\right)}{1+2a+3a^2+2a^3+a^4}\Big/ (q_z^2 v_s^2 \omega_T^*)^{1/3}\right] \tag{2.25}$$

where

$$a = (q\cdot v_0)\,/\,(q_z^2 v_s^2 \omega_T^*)^{1/3} \tag{2.26}$$

Equation (2.24) shows a stronger dependence of growth rate on the temperature gradient (compared to the case of no turbulence). Since α is negative (Eq. (2.17b)), Eq. (2.25), for $a^2 < 1$ shows a possible suppression of the drift temperature instability. The threshold for this suppression is given by

$$\frac{(1-a^2)\,W}{4\,n_0 T_e} \gtrsim \frac{6m}{M}\left(\frac{\omega_T^*}{q_z v_s}\right)^{2/3} = 3\left(\frac{2m}{M}\right)^{2/3}\left(\frac{\eta\,\omega_n^*}{q_z v_e}\right)^{2/3} \tag{2.27}$$

In the "warm" limit Eq. (2.12) takes the form

$$\Omega\left[1+i\sqrt{\frac{\pi}{2}}\,\frac{W}{n_0 T_e}\,\frac{\omega_n^*}{q_z v_e}\,\frac{\omega_{pe}^3}{\Delta^3 v_e^3}\right] = \omega_+\left\{1-\frac{W}{4\,n_0 T_e}\,\frac{\omega_{pe}^2}{\Delta^2 v_e^2}\left[1-i\sqrt{\frac{\pi}{2}}\,\frac{k_0}{\Delta}\right]\right\} \tag{2.28}$$

This equation can be readily solved; for small $\frac{W}{n_o T_e}$, the solution is,

$$\Omega_R = \mathrm{Re}\,\Omega \simeq \omega_n^* \left[1 - \frac{\omega_{pe}^2}{\Delta^2 v_e^2} \frac{W}{n_o T_e}\right],$$

$$\mathrm{Im}\,\Omega \simeq -\sqrt{\frac{\pi}{2}} \frac{W}{4 n_o T_e} \frac{\omega_{pe}^2}{\Delta^2 v_e^2} \frac{\omega_n^* k_o}{q_z} \left(\frac{\Omega_R - q \cdot v_o}{v_o \Delta}\right) \qquad (2.29)$$

In this limit, resonance interaction between the modulations of the plasma waves (propagating at the group velocity) and the parallel phase velocity of the drift waves assumes an important role. Mode coupling effects thus lead to "emission" and "absorption" of drift waves by propagating plasma waves. Instability may result if $v_o > \frac{\Omega_R}{q_z}$.

To investigate the effects of two - dimensional plasma turbulence (in y-z plane), we choose a spectrum $N_k^o = N_o \delta(k_y - k_{oy}) \delta(k_z - k_{oz})$ for the "cold" limit. Equations (2.12) and (2.14) again take the form of Equations (2.19) and (2.23) with a new definition of α,

$$\alpha = -\frac{W \omega_{pe} k_o}{4 n_o T_e k_z} \left[\frac{q_z v_e^2 q \cdot k_o}{\omega_{pe} k_o} + \frac{\omega_{pe}}{k_o^3}(k_{oy} q_z - k_{oz} q_y)(q_y - 3 k_{oy} k_o \cdot q / k_o^2)\right] \qquad (2.30)$$

The basic new feature of two dimensional turbulence is a modified group velocity. Equation (2.5) shows that in the general case, the group velocity of plasma waves is controlled by two effects:

(a) dispersion due to thermal corrections and

(b) dispersion due to angular dependence.

Equation (2.30) shows that both contribute to the modified α. It is interesting to note that for $k_{oy} \to 0$, we do not recover the one dimensional case discussed above, in fact α takes the form

$$\alpha \simeq (W/4 n_o T_e)\,\omega_{pe}^2\,(q_y^2/k_o^2) \tag{2.31}$$

It has not only a different sign from (2.17b) but also a much larger magnitude. Physically the difference arises because a spectral choice $N_o\,\delta(k_y)\,\delta(k_z - k_{oz})$ is different from the corresponding one-dimensional case with no dependence on k_y. It seems to us for possible application to practical systems in the laboratory, Equation (2.31) is better suited than the idealized one-dimensional limit (2.17b).

2.4 Discussion

In conclusion we have shown that moderate amount of short wavelength electron plasma wave turbulence can drastically alter the dispersion properties of drift waves. Compared to the work of Krivorutsky et al [1] the new features of our calculation include the use of a Gaussian spectrum in k-space for the background turbulence, which enables us to represent our results in terms of plasma dispersion function and this in turn makes it possible to consider the 'cold' and the 'warm' limits leading to many new results. In addition the physical insight into the nonlinear consequences due to the ponderomotive force is made clearer by our method. In many ways (both mathematically and physically) the plasma wave packets behave like a "beam" of quasiparticles giving both "hydrodynamic" and "kinetic" type "beam" instabilities. For example in the "cold" limit we have seen that the plasma wavepackets get collected in the trough of the drift wave which push the electrons further due to the ponderomotive force and thus enhance the instability. In the "warm" limit the resonance interaction between the modulations on plasma waves (propagating at the group velocity) and the parallel phase velocity of the drift waves assumes an important role. In the discussion of two dimensional turbulence we have noticed that the angular dependence of group velocity is quite important. Taking this analogy of "beam" further one can speculate that for "weak beams" this instability will nonlinearly saturate when the beam is trapped by the growing waves, i.e. when a good fraction of the plasma waves are trapped in the density fluctuations created by drift waves. Nonlinear BGK Solutions of this kind have recently been constructed for the unmagnetized plasma problem by Kaw et al [8].

The particular choice of microturbulence and low - frequency waves in this chapter was governed by reasons of simplicity. Future work should look at more realistic problems like the effects of Buneman, ion plasma or lower hybrid turbulence on trapped particle modes etc. At the same time laboratory investigation of this interaction should be quite useful.

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CHAPTER III

FORMATION OF FIELD ALIGNED IRREGULARITIES IN A PLASMA DUE TO WHISTLERS

3.1 Introduction

Whistlers are low frequency electromagnetic waves in the audio frequency range. In nature they are produced by ordinary lightening discharges and can be heard to "whistle" at midlattitudesby a simple amplifier connected with a long antenna. Their study has been found of particular importance in determining the electron density distribution in magnetosphere, acceleration of charged particles to high energy and also in inter-hemisphere communication. Signals at very low frequency can be launched into whistler mode and can be received in other hemisphere [1] . Perhaps, one of the most interesting and puzzling feature of whistlers is their path of propagation. Storey in 1953 [2] showed that for frequencies well below the electron cyclotron frequency and electron plasma frequency, whistlers should travel approximately along the earth's magnetic field. He also observed long trains of echoes from whistlers in which the amplitude decayed very slowly through the train. The guiding effect of ionospheric irregularities was suggested to be responsible for low amplitude decrement. Helliwell et al [3] extended Storey's idea of guiding effect to explain the discrete traces often exhibited in the spectrogram of whistlers. They suggested that each trace resulted from the concentration of energy in a thin column of ionization acting as a duct over a large part of ray trajectory in the magnetosphere. A ray theory of whistler propagation based upon the field oriented irregularities of electron density was presented for the first time by Smith et al in 1960 [4]. It was shown that trapping was possible in both enhancements and depressions of ionization. Smith [5], however, concluded that whistlers should propagate in ducts of enhanced ionization and he could explain that for such a propagation there exists a cut-off frequency at approximately one half the minimum electron cyclotron frequency along the whistler path. Thus an indirect evidence of existence of field aligned irregularities in the magnetosphere had been inferred to be essential to explain the ground observations of whistlers. Recently the evidence of these ducts have been supported by satellite observations of whistlers which could be successfully explained by the duct theory [6] and an attempt has been made to look for an association of magnetospheric whistler dispersion characteristics with changes in local plasma density [7]. In fact the problem of ducted propagation can be divided into two parts.

(a) Assuming the existence of field aligned ducts, how do the whistlers propagate and remain trapped in the duct? How to deduce various whistler characteristics from such a ducted propagation?

(b) What is the mechanism of creating such ducts, which to be consistent with (a), are field aligned electron density enhancements extending all the way from one hemisphere to another hemisphere?

Much of the literature has been devoted to (a) [8]. It has been found from such studies that enhancement factor of the density in the duct should be a few percent ($\sim 5\%$), their scale size across the magnetic field of the order of hundred kms. [6]. The life time of ducts is estimated to be of the order of few hours [9].

As regards the second aspect of the problem, the question of origin of ducts, much has not been done so far. The only attempt, made in this direction, to author's knowledge, is the one by Park and Helliwell [10]. They have discussed the formation of field aligned irregularities by electric fields. The source of this electric field as suggested by them could be thunderstorm electricity. They regard the magnetic lines of force as equipotential lines. A specific case has been discussed in which radial electric field pointing out towards the centre of a convective cell of the size of 600 kms. at equator across magnetic field has been assumed. The centre of this convective cell has been taken at L = 4. The E X B motion rotates the tubes of ionization with different contents azimuthally. For the magnetospheric geometry these rotating ionization tubes get mixed and produce thereby an irregular structure in the electron density profile. It has been shown that under reasonable initial conditions, a 0.1 mv/m electric field (constant in time) in the equatorial plane can produce a whistler duct at L = 4 in about 30 minutes. The small scale irregularities can be produced by a much larger scale electric field. The evolution of irregular structure did not depend upon the amplitude but on the shape of the electric field. Thus doubling the strength of the field will lead to the same structure at twice the speed. The enhancement factor was also independent of the electric field and was decided by the maximum variation in initial tube content across the convective cell.

It should be noted that very complex irregular structures are made to evolve from very simple initial conditions. In more complex situations it is not clear how the irregularities will evolve-in fact one would expect the process to get extremely complicated.

In the present chapter of this thesis, we wish to point out a simple self-consistent mechanism which may, at least partially, be responsible for the formation of field aligned electron density irregularities. We have seen in the previous chapter that the presence of a high frequency turbulence can considerably modify the low frequency properties of a plasma. Therefore we intend here to study the effect of whistler turbulence on the background plasma. We shall see that a purely growing instability may arise which can create the electron density irregularities parallel to the ambient magnetic field. In other words, the electron density remains uniform along the lines of force but becomes nonuniform in a perpendicular direction. The fact that the whistlers are concentrated in the regions of local density enhancements emerges self-consistently in this theory. For whistlers the nature of the ponderomotive force on electrons is such that it increases in the direction of increasing whistler energy; thus it attracts more electrons into the region where more whistlers exist, and the increase in electron density further leads to higher concentration of whistler energy and so on. Thus an instability leading to electron and whistler density concentration develops. Similarly in the neighbouring regions there will develop a trough of electrons and whistlers. The growth rate of this instability has been calculated and the optimum size of the duct has been estimated. A preliminary calculation in this direction was presented in [11].

3.2 Calculations

We start with a uniform plasma embedded in a uniform and constant magnetic field $(0, 0, B_0)$ with a quasi-stationary spectrum of whistler waves distributed uniformly in the background. We also assume the quasi-longitudinal approximation for whistlers such that they obey the following linear dispersion relation

$$\epsilon \equiv \frac{c^2 k^2}{\omega^2} = \frac{\omega_p^2}{\omega(\omega_{ce}\cos\theta - \omega)} \tag{3.1}$$

where ω, k are the whistler frequency and wave number respectively, c the velocity of light, ω_p electron plasma frequency, ω_{ce} the local electron cyclotron frequency while θ is a small angle between the ambient magnetic field B_0 and the wave vector k.

We now introduce a low frequency long wavelength perturbation characterised by $(\Omega, \vec{q})$ such that

$$\Omega \ll \omega \quad ; \quad q \ll k \tag{3.2}$$

the condition of adiabaticity, is satisfied.

Under the influence of such a perturbation whistlers can now be regarded as propagating in an adiabatically varying background. The variation of whistlers is then governed by the wave kinetic equation already discussed in the previous chapter

$$\frac{\partial N_k}{\partial t} + v_g \cdot \frac{\partial N_k}{\partial r} - \frac{\partial \omega_k}{\partial r} \cdot \frac{\partial N_k}{\partial k} = 0 \qquad (3.3)$$

where [12]

$$N_k = \frac{1}{\omega_k^2} \frac{\partial (\omega_k^2 \epsilon_k)}{\partial \omega_k} \frac{|E_k|^2}{8\pi} \qquad (3.4)$$

and ϵ is defined by (3.1). $v_g = \frac{\partial \omega}{\partial k}$ is the group velocity of whistlers and $|E_k|$ is their amplitude.

Equations (3.3) and (3.4) show that the whistler energy per unit frequency, N_k, remains constant as it follows the trajectory

$$\frac{dr}{dt} = v_g \quad , \quad \frac{dk}{dt} = - \frac{\partial \omega_k}{\partial r} \qquad (3.5)$$

The variation in the frequency $(-\nabla_r \omega_k)$ due to slowly varying background is thus equivalent to a force term.

From equation (3.1) we can write

$$\omega = c^2 k^2 \omega_{ce} \cos\theta / (\omega_p^2 + c^2 k^2) \qquad (3.6)$$

which implies

$$\nabla_r \omega_k = -\frac{\nabla \tilde{n}_e}{n_0}\left(\frac{\omega_p^2}{\omega_p^2 + c^2k^2}\right)\omega_k \tag{3.7}$$

where $\tilde{n}_e$ is the electron density perturbation. Substitution of (3.7) in the linearized form of (3.3) leads to

$$\tilde{N}_k = \frac{\tilde{n}_e}{n_0}\,\omega_k\left(\frac{\omega_p^2}{\omega_p^2 + c^2k^2}\right)\frac{q \cdot \partial N/\partial k}{\Omega - q \cdot v_g} \tag{3.8}$$

This equation shows that whistler energy increases in the regions of electron density crests and decreases in electron density troughs provided the multiplicity coefficient turns out to be positive. It is shown later that this is indeed the case.

The low frequency wave is in turn influenced by a ponderomotive force term in the electron equation of motion. This force factor arises from the following nonlinear terms

$$\left\langle (\vec{v}_f \cdot \nabla)\vec{v}_f + \frac{e}{mc}(\vec{v}_f \times \vec{B}_f) \right\rangle$$

where $\vec{v}_f$ refers to the electron velocity under the influence of whistler field, $\vec{B}_f$ to the whistler magnetic field and $\langle\ \ \rangle$ denotes an average over the whistler period. We can express the velocity $\vec{v}_f$ and magnetic field $\vec{B}_f$ in terms of the Fourier components of the whistler electric field $\vec{E}_k$. Expanding the above expression in terms of the $\perp$ & $\parallel$ components we find that it is possible to neglect contributions from the parallel terms in comparison to the $\perp$ terms.

Typically

$$\frac{E_{k\parallel}^2}{E_\perp^2} = \frac{1}{2}\,\frac{\sin^2\theta}{(\cos\theta - \omega/\omega_{ce})^2} \simeq \sin^2\theta \ll 1; \quad \theta \text{ is small}$$

Thus for a small angle θ, almost parallelly propagating whistler waves under the quasi-longitudinal approximation, we can always ignore the contribution of parallel wave electric field to the ponderomotive force. But even if one retains it, the result remains unaffected except for a numerical coefficient which can be accounted in calculating the total wave energy density (to be defined later).

Thus the expression for the ponderomotive force simplifies to

$$((-e^2)/4m^2)\sum_k \frac{|E_k|^2}{\omega_k^2 - \omega_{ce}^2} \qquad (3.9)$$

However the parallel component $E_{k\parallel}$ could play an important role in wave-particle interactions e.g. Landau Damping, which we are not taking into account in this calculation.

It is clear from (3.9) that for whistler frequency $(\omega < \omega_{ce})$, the ponderomotive force term appears like a negative pressure term. Making use of (3.4) and (3.9) we obtain this force as

$$-\nabla \Phi = \frac{1}{2mn_0} \nabla \sum_k \frac{\omega_k^2 (\omega_{ce} \cos\theta_k - \omega_k)^2}{(\omega_{ce}^2 - \omega_k^2)\, \omega_{ce} \cos\theta_k} \widetilde{N}_k \tag{3.10}$$

For $\omega_{ce} > \omega_k$, the ponderomotive force acts on electrons in the direction of enhanced whistler concentration N_k i.e., electrons try to accumulate in high whistler regions and in turn attract more whistlers according to the relation (3.8). Thus an instability builds up leading to crests and troughs of electrons and whistler density. To calculate the growth rate of this instability, we substitute (3.10) in the electron equation of motion and obtain (linearized form)

$$\frac{\partial v_e}{\partial t} = \frac{e}{m} \nabla \phi - v_e \times \omega_{ce} - \nabla \Phi \tag{3.11}$$

We also make use of electron continuity equation

$$\frac{\partial n_e}{\partial t} + n_0 \nabla \cdot v_e = 0 , \tag{3.12}$$

ion continuity equation

$$\frac{\partial n_i}{\partial t} + n_0 \nabla \cdot v_i = 0, \tag{3.13}$$

and ion equation of motion,

$$\frac{\partial v_i}{\partial t} = -\frac{e}{M} \nabla \phi + v_i \times \omega_{ci} \tag{3.14}$$

No ponderomotive force term has been included in (3.14) which is justified as long as we assume the frequency of whistlers to be such that $\omega_{ci} \ll \omega \lesssim \omega_{ce}$. The inclusion of still lower frequency whistler waves may be worth investigating, but we have not done so here.

Equations (3.11) - (3.14) form a complete set and we now carry out a normal mode analysis and assume the condition of quasineutrality, i.e., $n_e \simeq n_i$.

This gives us a dispersion relation (for $\Omega \ll \omega_{ci}$)

$$\chi_i + \chi_e = 0 \tag{3.15}$$

where $\chi_i = -\dfrac{\omega_{pi}^2}{\Omega^2}\cos^2\phi + \dfrac{\omega_{pi}^2}{\omega_{ci}^2}\sin^2\phi$,

$$\chi_e \simeq \left[\frac{\omega_{pe}^2}{\omega_{ce}^2}\sin^2\phi - \frac{\omega_{pe}^2}{\Omega^2}\cos^2\phi\right] \Big/ \left[1 + q^2 \frac{\Phi}{I_0} \times \left(\frac{\cos^2\phi}{\Omega^2} - \frac{\sin^2\phi}{\omega_{ce}^2}\right)\right], \tag{3.16}$$

$$\frac{\Phi}{I_0} = \left(\frac{1}{2mn_0}\right) \sum_k \frac{\omega^3(\omega_{ce}\cos\theta - \omega)^2}{(\omega_{ce}\cos\theta)(\omega_{ce}^2 - \omega^2)} \left(\frac{\omega_p^2}{\omega_p^2 + c^2k^2}\right) \times \tag{3.17}$$

$$\cdot \frac{q \cdot \partial N/\partial k}{\Omega - q \cdot v_g} \tag{3.18}$$

and the angle ϕ has been defined as

$$q_{\parallel} = q\cos\phi , \quad q_{\perp} = q\sin\phi$$

Substitution of (3.16) - (3.17) into (3.15) and making some algebric simplifications, we obtain

$$\Omega^2 - \omega_{ce}\omega_{ci}\cos^2\phi \simeq -q_i^2 \Phi_0 \cos^2\phi \tag{3.19}$$

where $\cos\phi \ll 1$ and $\sin\phi \sim 1$ has been assumed.

Use has also been made of the condition

$$1 \gg \frac{\Omega^2}{\omega_{ci}^2} \gg \cos^2\phi \gg \frac{\Omega^2}{\omega_{ce}^2} \tag{3.20}$$

For $\Phi_0 = 0$, relation (3.19) reduces to

$$\Omega^2 \simeq \omega_{ce}\omega_{ci}\cos^2\phi \tag{3.21}$$

This is a natural mode of low frequency oscillations in a cold homogeneous plasma [14, 15] . The condition $\Omega^2 \ll \omega_{ci}^2$ requires

$$\cos^2\phi \ll m/M \tag{3.22}$$

Thus it propagates in a very narrow cone perpendicular to the ambient magnetic field.

Let us now concentrate on (3.19) for $\Phi_0 \neq 0$. We define the whistler energy density as

$$W = \sum_k \omega_k N_k \tag{3.23}$$

and take the limit $\sum_k \rightarrow \int d\vec{k}$ in (3.18) and (3.23).
The expression for Φ_0 then takes the form

$$\Phi_0 = \frac{W}{2mn_0}\left[\int N_k \omega_k d\vec{k}\right]^{-1} \int \left[\frac{\omega_k^3 (\omega_{ce}\cos\theta_k - \omega_k)^2}{\omega_{ce}\cos\theta_k(\omega_{ce}^2 - \omega_k^2)} \left(\frac{\omega_p^2}{\omega_p^2 + c^2k^2}\right) q \cdot \frac{\partial N}{\partial k} \Big/ (\Omega - q \cdot v_g)\right] d\vec{k} \tag{3.24}$$

It is clear from (3.24) that the general behaviour of dispersion relation (3.19) depends upon the gradient of whistler distribution function N_k with respect to k. In particular the low frequency mode may be driven unstable due to a "Landau type" of resonance between low-frequency waves and the whistlers with a growth rate proportional to $(q \cdot \partial N/\partial k)_{\Omega \simeq q \cdot v_g}$. This type of the resonance has been discussed in the previous chapter for drift waves and plasmons. We note that some new modes are also introduced since Φ_0 is a function of frequency Ω. We concentrate here on those modes which can be treated by "fluid like" behaviour of whistlers. This implies that the phase velocity $|\Omega/q| \gg \Delta v_{gr}$, the spread of group velocities of the whistlers. Therefore the functional dependence of N_k on k can be approximated by a δ-function

$$N_k = N_0\,\delta(k_z - k_{0z})\,\delta(k_y - k_{0y}) \tag{3.25}$$

and $k_{0y} \to 0$; $k_{0z} \to k_0 \cos\theta_0$

Introduction of (3.25) greatly simplifies the integration in (3.24) and Φ_0 can be written as

$$\Phi_0 = -\alpha/\Omega^2 \tag{3.26}$$

where

$$\alpha = \frac{W}{2mn_0}\frac{q^2}{k_0^2}\frac{\omega_p^2}{c^2k^2}\frac{\omega_0^4(\omega_{ce}\cos\theta_0 - 2\omega_0)(\omega_{ce}\cos\theta_0 - \omega_0)^2}{\omega_{ce}^3\cos^3\theta_0\,(\omega_{ce}^2 - \omega_0^2)} \tag{3.27}$$

In evaluating the value of α , we have made use of $\Omega \gg q \cdot v_g$, which is justified for the "fluid like" picture of whistlers. Also we have taken the limit $k_{0\perp} \to 0$ (quasilongitudinal approximation for whistlers) and $q_\parallel \to 0$ because $(q_\parallel^2/q_\perp^2) \ll m/M$.

Substituting (3.26) in the dispersion relation (3.19) we get

$$\Omega^4 - (\Omega^2\omega_{ce}\omega_{ci} + q^2\alpha)\cos^2\phi = 0 \tag{3.28}$$

This is a biquadratic equation in Ω and can be solved easily to give (for small α)

$$\Omega_1^2 \simeq \omega_{ce}\,\omega_{ci}\cos^2\phi + q^2\alpha/\omega_{ce}\omega_{ci} \tag{3.29}$$

and

$$\Omega_2^2 = -\,q^2\alpha\,/\,\omega_{ce}\,\omega_{ci} \tag{3.30}$$

Solution (3.29) is nothing but the modified normal mode (3.21) due to the presence of whistler turbulence. Mode (3.30) is however a new mode and is driven only by the turbulence. For $\alpha > 0$, this results into a purely growing instability with a growth rate Γ :

$$\Gamma^2 = \frac{W(M/m)^2}{2Mn_0c^2}\left(\frac{q}{k_0}\right)^4 \frac{\omega_p^2\,\omega_0^4}{\omega_{ce}^5\cos^3\theta_0}\,\frac{(\omega_{ce}\cos\theta_0 - 2\omega_0)(\omega_{ce}\cos\theta_0 - \omega_0)^2}{(\omega_{ce}^2 - \omega_0^2)} \tag{3.31}$$

We see that the growth rate is proportional to the amplitude of the background whistlers $(\sqrt{W})$ and vanishes for whistler frequencies $\omega_0 = \frac{\omega_{ce}}{2}\cos\theta_0$. Thus only whistlers with frequencies below $\omega_{ce}\cos\theta_0/2$ contribute to the growth of this instability.

Starting with the assumption that the above mentioned instability leads to the duct formation (field aligned irregularities) we can estimate the optimum size of the ducts from the condition (3.20) which implies

$$q^2_{min}\,\alpha \gtrsim \omega_{ce}\,\omega_{ci}^3 \cos^2\phi \tag{3.32}$$

and

$$q^2_{max}\,\alpha \lesssim \omega_{ce}^3\,\omega_{ci} \cos^2\phi \tag{3.33}$$

For a given ϕ, the ratio of the shortest to the longest size turns out to be

$$\frac{L_{min}}{L_{max}} = \frac{q_{min}}{q_{max}} \sim \frac{m}{M} \tag{3.34}$$

Now for $(1-2\omega_0/\omega_{ce}\cos\theta_0) \ll 1$ and $\cos\theta_0 \sim 1$ we get from (3.27)

$$\alpha \simeq (1/9600)\left(\frac{4\pi W}{B_0^2}\right)\frac{q^2}{k_0^4}\,\omega_{ce}^4 \tag{3.35}$$

Substituting this value of α for $q = q_{min}$ in (3.32) we obtain an expression for $L_{max} = 2\pi/q_{min}$ as

$$(L_{max}/\lambda_0)^4 < \frac{4\pi W}{B_0^2}(M/m)^3 / 9600\cos^2\phi \tag{3.36}$$

where $\lambda_0 = 2\pi/k_0$ is whistler wavelength. Taking the typical values of whistler wave amplitude $B \sim 10^{-7}$ gauss and the ambient magnetic field $B_0 \sim 10^{-2}$ gauss near equator at L = 3 [16] we calculate $4\pi W/B_0^2 \sim 10^{-10}$. Thus for $\lambda_0 =$ 2 kilometers and $\cos\phi \sim 10^{-9}$, we get

$L_{max} <$ 6 X 10^3 kilometers. From (3.34) we then obtain $L_{min} >$ 3 kilometers. Since the observed sizes of the magnetospheric ducts (of the order of few hundred kilometers [6]) lie well within this range, our theory can be safely applied for them. We also notice that L_{min} obtained in this manner is consistent with the adiabatic condition (3.2).

3.3 Discussion and Conclusion

In the present chapter we have found a mechanism by which whistlers themselves can lead to the formation of field aligned density irregularities owing to the development of a purely growing instability. It is the characteristic of whistler dispersion relation that the whistler frequency is a function of electron density in such a manner that more and more whistlers are collected in a region of enhanced electron density according to (3.8). It is again for frequencies below electron cyclotron frequencies (whistlers range) that the ponderomotive force acts in the right direction to give rise to the purely growing instability. This instability is not a natural mode of whistler free system and its growth rate is proportional to the whistlers amplitude. Growth rate is also sensitive to whistler frequency and goes as square of small frequencies but suddenly drops down at $\omega = (\omega_{ce}/2)\cos\theta$.

We also notice that the growth rate is stronger for the narrower ducts. The typical size of the duct for the magnetosphere turns out to be well within the range of our theory.

Both the enhancements and depressions in the electron density are created by this mechanism. The enhancement factor should depend upon the saturation level of this instability and a more sophisticated theory would be required to calculate this. Unlike the complicated theory of Park and Helliwell [10] the trapping of whistlers in enhanced electron density region is a natural consequence of our theory.

Recently Stenzel [17] has demonstrated experimentally that a large amplitude whistler wave can manifest the phenomenon of self-ducting. But he has taken a single monochromatic wave and assumes wave-particle interaction to dominate. On the other hand we assume wave-wave interaction to dominate and have used the random phase approximation. Also the adiabatic condition assumed in our theory is not satisfied in his experiment. However, as we have seen, magnetospheric plasma provides a good example for our theory.

Finally it is important to note the difference in the direction of ponderomotive force for $\omega < \omega_{ce}$ and $\omega > \omega_{ce}$. In the former case it is towards the increase in the field energy while in latter case towards the decrease in the field energy. Existence of the instability is therefore possible only if in the former case $(\omega < \omega_{ce})$ both particle and waves will have to bunch together while in the latter case $(\omega > \omega_{ce})$ the field energy will be concentrated in the troughs of particle density and not on the crests. The creation of spikons with high frequency field and the instability discussed in the previous chapter are examples of the case where $\omega > \omega_{ce}$. On the other hand the instability discussed in the present chapter is an example of the case for $\omega < \omega_{ci}$. The high frequency mode of course must satisfy the right kind of dispersion relation in order to satisfy the condition of instability appropriately.

In these two chapters we have seen that the background turbulence can considerably alter the wave properties of the system. In next three chapters we shall consider the effect of background turbulence (of a particular type specified in each chapter) on different aspects of plasma heating by oscillating electric fields.

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CHAPTER IV

ANOMALOUS PLASMA HEATING AT LOWER HYBRID FREQUENCY

4.1 Introduction

It is widely recognized now that in most fusion devices like Tokamaks, stellarators etc., ohmic heating alone cannot raise the plasma temperature to the desired value of ten kilovolts or so. This is so because the electron ion collision frequency (and hence the electrical resistivity) is a rapidly decreasing function of electron temperature. Auxiliary heating methods have therefore assumed considerable importance in recent years. Some of the more widely discussed of these methods are heating by neutral beam injection, linear and nonlinear absorption of oscillating electric fields; relativistic electron beam injection etc. In this chapter we shall be primarily concerned with the second of these methods viz. the anomalous heating of a plasma by oscillating electric fields near the lower hybrid frequency.

Anomalous damping of an externally applied oscillating electric field near some plasma resonant frequency arises because of its conversion into short wavelength, heavily damped electrostatic modes. This conversion may occur (i) either because of linear conversion processes involving plasma inhomogeneity (ii) or through nonlinear parametric effects (iii) or via coupling through low - frequency short wavelength density fluctuations in the background plasma. Anomalous heating takes place preferentially for that species of plasma particles which interact strongly with the excited high - frequency electrostatic modes. Thus if the applied oscillating electric field is near the electron plasma frequency, anomalous heating of electrons takes place. Any excessive heating of ions in this case takes place indirectly and is not therefore very efficient. For direct anomalous heating of ions, one must work with a pump field at a lower frequency such as magnetosonic, ion cyclotron, lower hybrid, ion-ion hybrid etc. It is also of interest to point out that high power is much more readily available at the lower frequencies. That is why for anomalous heating for fusion purposes, a great deal of attention is being given to frequencies like the lower hybrid resonance.

The first two of the three methods of anomalous heating discussed in the previous paragraph, have been widely discussed in the literature especially for the lower hybrid resonance frequency. Stix [1] and Piliya and Federov [2] developed a theory of linear conversion processes around the lower hybrid frequency. According to this theory a long wavelength electromagnetic wave at the lower hybrid frequency can be linearly converted into short wavelength electrostatic modes which are then heavily absorbed by linear Landau damping. Experimental evidence for the heating of plasma electrons and ions has also been reported by several workers [3]. The parametric instability and heating of electrons and ions by lower hybrid pump was first predicted theoretically and observed in computer simulation experiments by Kindel, Okuda and Dawson [4]. This was confirmed by Hooke and Bernabei [5] who stressed that parametric processes are likely to play an important, if not a dominant, role in the plasma heating processes at lower hybrid resonance. The parametric process so far stressed was splitting of a long wavelength lower hybrid wave into a shorter wavelength lower hybrid wave and an ion - acoustic wave and the threshold was found smaller than the corresponding threshold at Langmuir resonance. Chang and Porkolab performed another experiment [6] and found that a parametric instability involving an ion 'quasi-mode' due to nonlinear wave particle scattering is possible and plasma heating associated with this mechanism was measured. They called it a 'quasi-mode' as the presence of pump is necessary to excite such a mode. In the absence of pump this is not a normal mode of the system. Physically the mechanism of this instability is quite similar to nonlinear Landau growth in which a particle absorbs a photon or a high - frequency plasmon and emits another plasmon carrying away the difference of the momentum. On the other hand the mechanism pointed out earlier involves nonlinear wave-wave interaction in which a photon or h.f. plasmonsplits into another photon (or h.f. plasmon) and a l.f. plasmon. The possibility of exciting still another type of parametric instability by a lower hybrid pump was discussed by Kaw and Lee [7] and by Ott et al [8] in a two species plasma. This mechanism involves the excitation of an ion - ion hybrid mode [9]. Recently Chu et al [10] discussed the excitation of ion cyclotron and lower hybrid waves by a lower hybrid pump. Similarly, Sundaram and Kaw [11] have investigated the effects of pump at the lower hybrid resonance on the excitation and suppression of drift instabilities in an inhomogeneous plasma.

It is obvious from the foregoing discussion that plasma heating by oscillating fields at lower hybrid resonance has attracted considerable attention recently. We decided therefore to study the third kind of anomalous heating in a plasma viz. coupling to damped side-band modes via background low frequency turbulence in the plasma. Let us now physically understand the mechanism of such a heating. Imagine a long wavelength lower hybrid pump field imposed on a collisionless plasma. If the phase velocity (ω_0/k_0) (subscript 0 for pump) is more than the ion thermal speed, this mode will remain essentially undamped. Now let us suppose that there is a sufficiently low frequency, short wavelength (ω, k) noise in the background such that $\omega \ll \omega_0$, $k \gg k_0$. Nonlinear interaction will lead to the generation of sideband components at $\omega_0 \pm \omega \simeq \omega_0$, $|k_0 \pm k| \simeq |k|$ which have a considerably smaller phase velocity viz. $|\omega_0 / k|$. When the oscillating field itself is close to the lower hybrid frequency, it drives the natural oscillations resonantly and their amplitude becomes quite large. These large amplitude sideband modes are however naturally heavily damped because of their low phase velocity e.g. they may be strongly damped by ions at their natural damping rate $\sim$ $\nu_{ik} \times |\text{amplitude}|^2$, where ν_{ik} is the Landau decrement at wave number k . This leads to anomalous absorption of pump mode at (ω_0, k_0) . Anomalous absorption of high frequency waves like electron plasma waves and electromagnetic waves near upper hybrid frequency, by this mechanism has been studied earlier by Kruer [12] , Kaw et al [13] and Sen and Kaw [14] .

It should be emphasized that this kind of anomalous absorption is thresholdless as long as presence of density fluctuations is assumed in the background plasma. This is also the important point of distinction from parametric coupling processes. In the latter, the pump field has to be intense enough to parametrically excite the low frequency fluctuations which then cause coupling to damped modes. Thus one has to exceed a minimum threshold field for parametric coupling processes to be operative. The point being made here is that there is often enough background turbulence in a plasma, to cause significant anomalous absorption at certain frequencies, even without the onset of parametric instabilities.

Vis-a-vis the work done in previous chapters we note that earlier we investigated the influence of short wavelength high frequency turbulence on the dispersion relation of low - frequency long wavelength waves whereas now we are looking at a complimentary process viz. the effect of short wavelength low-frequency turbulence on the damping of high frequency long wavelength waves. The former is an example of adiabatic mode coupling where high frequency short wavelength waves interact with each other via low frequency long wavelength wave. The latter is an example of quasi-resonant mode coupling [15] where high frequency long wavelength waves interact with high frequency short wave length wave via low frequency fluctuations; this kind of a coupling is efficient only if the mode - coupling coefficient is large enough to overcome the effects of frequency mismatch in the short and long wavelength high - frequency waves (hence the name quasi-resonant mode coupling).

4.2 Basic Equations

Let us consider a plasma with a significant amount of low frequency short wavelength fluctuations immersed in a uniform magnetic field (0, 0, B_0). An oscillating electric field $E = E_0 \cos \omega_0 t$ at the lower hybrid frequency is applied across the magnetic field. This field could be produced by a normally undamped long wavelength lower hybrid wave ($k_0 \simeq 0$, the dipole approximation) in the medium. We wish to investigate the anomalous heating of ions by this wave.

The external lower hybrid field couples with the ambient low frequency fluctuations (denoted by the superscript s) to produce high frequency short wavelength side band fluctuations in the particle density and space charge fields (denoted by superscript f). These fluctuations obviously contribute to the nonlinear current density $\vec{J} = \sum_\alpha e_\alpha n_\alpha \vec{V}_\alpha$ (α refers to ions and electrons) and the total electric field in the plasma. The rate of energy absorbed by ions can be written as

$n_i e E \cdot V_i$ = the work done by the electric field on the ions

Thus the average rate of energy dissipation into ions per unit volume may be written as

$$\langle \vec{J}_i \cdot \vec{E} \rangle = \langle e n_i \vec{V}_i \cdot \vec{E} \rangle$$

$$= \langle e(n_0 + n_i^f + n_i^s)(\vec{V}_{i0} + \vec{V}_i^f + \vec{V}_i^s) \cdot (\vec{E}_0 + \vec{E}^f + \vec{E}^s) \rangle \quad (4.1)$$

where the subscript 0 refers to the plasma with the driving field but without fluctuations, superscript s to the ambient low frequency short wavelength fluctuations and f to the high frequency produced by the coupling of pump field with the ambient fluctuations. $\langle\ \rangle$ and $\overline{\qquad}$, respectively, denote the spatial and temporal averages. Neglecting the small contributions arising from the slow component of ions perturbed velocity $\vec{V}_i^s$ [because $n^f V^s : n^s V^f \equiv n^f \frac{n^s}{n_o}\frac{\omega}{k} : n^s \frac{n^f}{n_o}\frac{\omega_o}{k} \equiv \omega : \omega_o$] and electric field E^s [$n^f E^s : n^s E^f \equiv n^f \frac{M}{e}\omega V^s : n^s \frac{M}{e}\omega_o V^f \equiv \omega^2 : \omega_o^2$] and averaging over fast space - time variations, the significant average rate of energy dissipation into ions per unit volume is

$$\overline{\langle \vec{J}_i \cdot \vec{E} \rangle} = e\, n_o \langle \vec{V}_{io} \cdot \vec{E}_o \rangle + e\, n_o \overline{\langle \vec{V}_i^f \cdot \vec{E}^f \rangle}$$
$$+ e \overline{\langle n_i^s \vec{V}_{io} \cdot \vec{E}^f \rangle} + e \overline{\langle n_i^s \vec{V}_i^f \cdot \vec{E}_o \rangle} \tag{4.2}$$

We note that the neglect of contributions from $\vec{V}_i^s$ and $\vec{E}^s$ can be justified as we shall assume $\omega \ll \omega_o$. [Also quasineutrality at lower frequency makes E^S almost negligible].

It is instructive to define an "Effective Absorption Coefficient" for ions by using the relation

$$\overline{\langle \vec{J}_i \cdot \vec{E} \rangle} = \nu_E \frac{\omega_{pi}^2}{\omega_o^2} \frac{E_o^2}{8\pi} \tag{4.3}$$

where $\omega_{pi}^2 = 4\pi n_0 e^2/M$ is the ion plasma frequency. ν_E is then a direct measure of the anomalous heating rate of ions. Our aim is now to use equations (4.2), (4.3) and other plasma equations to express ν_E in terms of the amplitude of low - frequency fluctuations $|n_i^s|$ and the plasma parameters.

We work in the simple hydrodynamic limit and start with the basic equations

$$\frac{d}{dt} n_j + n_j \nabla \cdot \vec{V_j} = 0 \tag{4.4}$$

$$\frac{d\vec{V_j}}{dt} = \frac{e_j}{m_j}(\vec{E} + \vec{E_0}\cos\omega_0 t) + \vec{V_j} \times \vec{\omega_{c_j}} - \nu_j \vec{V_j} - \nabla p_j / m_j n_j \tag{4.5}$$

where

$\frac{d}{dt} = \frac{\partial}{\partial t} + V_j \cdot \nabla$ is the time derivative in the Lagrangian sense. Most of the other symbols are self explanatory. j corresponds to the plasma species, i.e., ions or electrons. $\vec{E}$ is the self-consistent field satisfying the Poisson's equation

$$\nabla \cdot \vec{E} = 4\pi \sum_j e_j n_j \tag{4.6}$$

For simplicity we assume that the pressure gradient can be expressed in terms of the density gradient by the conventional equation of state in the form

$$\nabla p_j = \gamma_j T_j \nabla n_j \tag{4.7}$$

where T_j is the temperature and is expressed in the units of electron volts. Temperature has been assumed to be uniform. γ_j is a numerical factor.

$\omega_{cj} = e_j B_o / m_j c$ is the cyclotron frequency. The collision term $\nu_j \vec{V}_j$ could represent damping due to the particle neutral collisions. It may also be used to mock up Landau damping type effects in a phenomenological manner as discussed by Nishikawa [16]. For a more accurate description, of course, one would have to do the calculations by using kinetic theory, but a fairly good estimate can be obtained by using our model, provided we replace ν_j by $\nu_j(k \pm k_o)$, that is, the appropriate expression for Landau damping, in the final result. Since we assume $k \gg k_o$, it suffices to use just one expression $\nu_j(k)$ for the two side bands. We have chosen this hydrodynamical model (discussed above) for the sake of simplicity as it avoids the cumbersome calculations of kinetic theory. Further it is comparatively easier to understand physically. We are also going to restrict our attention to anomalous damping effects at the lower hybrid resonance layer where $k_{\parallel} \to 0$ and the damping is primarily due to ions. An extension to finite values of $k_{\parallel}$, where electron damping can become equally important, can be readily made.

Let us, now, separate the variables into two parts. The zero order part corresponding to the equilibrium including the pump field represented by the subscript o and the first order part corresponding to ambient fluctuations and those produced by the coupling of pump field with the ambient ones.

Thus writing

$$n_j = n_o + n_j \, ; \quad V_j = V_{oj} + V_j$$

we obtain

$$\frac{\partial \vec{V}_{oj}}{\partial t} = \frac{e_j}{m_j} \vec{E}_o \cos \omega_o t + \vec{V}_{oj} \times \vec{\omega}_{cj} - \nu_j \vec{V}_{oj} \qquad (4.8)$$

for the equilibrium velocity $\vec{V}_{oj}$ and

$$\frac{d n_j}{dt} + n_o \nabla \cdot \vec{V}_j = 0 \qquad (4.9)$$

$$\frac{d \vec{V}_j}{dt} = \frac{e_j}{m_j} \vec{E} + \vec{V}_j \times \vec{\omega}_{cj} - \frac{T_j \gamma_j \nabla n_j}{m_j \, n_o} - \nu_j \vec{V}_j \qquad (4.10)$$

for the linearized perturbations. The Lagrangian time derivative is given as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + V_{oj} \cdot \nabla$$

The perturbed quantities are now separated into slow (which vary at the frequency of ambient fluctuations) and fast (varying at the sideband frequencies) parts, that is,

$$n_j = n_j^s + n_j^f \text{ and } V_j = V_j^s + V_j^f$$

As was mentioned earlier the superscript s refers to the

the ambient low frequency fluctuations and f to the driven high frequency sidebands. The sideband parts then are

$$\frac{\partial}{\partial t} n_j^f + (\vec{V}_{oj}\cdot\nabla)\, n_j^s + n_o \nabla\cdot\vec{V}_j^{\,f} = 0 \tag{4.11}$$

$$\frac{\partial \vec{V}_j^{\,f}}{\partial t} + (\vec{V}_{oj}\cdot\nabla)\vec{V}_j^{\,s} = \frac{e_j}{m_j}\vec{E}^f + \vec{V}_j^{\,f}\times\vec{\omega}_{c_j} - \frac{\gamma_j T_j \nabla n_j^f}{m_j\, n_o} - \nu_j \vec{V}_j^{\,f} \tag{4.12}$$

along with the Poisson's equation

$$\nabla\cdot\vec{E}^f = 4\pi \sum_j e_j\, n_j^f \tag{4.13}$$

Equations (4.11) - (4.13) are now Fourier analysed in space. The result is

$$\frac{\partial}{\partial t} n_{jk}^f + i\,\vec{k}\cdot\vec{V}_{oj}\, n_{jk}^s + n_o\, i\,\vec{k}\cdot\vec{V}_{jk}^{\,f} = 0 \tag{4.14}$$

$$\frac{\partial}{\partial t}\vec{V}_{jk}^{\,f} + i\,\vec{k}\cdot\vec{V}_{oj}\,\vec{V}_{jk}^{\,s} = \frac{e_j}{m_j}E_k^f + \vec{V}_{jk}^{\,f}\times\vec{\omega}_{c_j} - \frac{i\gamma_j T_j \vec{k}\, n_{jk}^f}{m_j\, n_o} - \nu_{jk}\vec{V}_{jk}^{\,f} \tag{4.15}$$

and

$$i\,\vec{k}\cdot\vec{E}_k^f = 4\pi e\,(n_{ik}^f - n_{ek}^f) \tag{4.16}$$

Our aim is now to solve these equations for n_{jk}^f, $\vec{V}_{jk}^{\,f}$ and $\vec{E}_k^f$. Let us proceed in the following manner. Let us define, for the present, $D \equiv \frac{\partial}{\partial t}$ and $S^2 = \frac{\gamma T}{M}$ and suppress the subscripts j and k ; equation (4.15) can be written in the form as

$$(D+\nu)\, V^f = \frac{e}{m} E^f + V^f \times \omega_c - i k s^2 \frac{n^f}{n_o} - i k \cdot V_o V^S \tag{a 1}$$

Taking the cross product of (a 1) by ω_c

$$(D+\nu)\, V^f \times \omega_c = \frac{e}{m} E^f \times \omega_c - \omega_c^2 V^f - i k \times \omega_c \frac{s^2 n^f}{n_o} - i k \cdot V_o V^S \times \omega_c$$

for $V^f = V_\perp^f$, $V_{\parallel}^f = 0$. (a 2)

$\perp$ and $\parallel$ refer to perpendicular and parallel components with respect to the magnetic field direction. Operating (a 1) by $(D+\nu)$ and substituting (a 2) and then taking the dot product with k , we get

$$(D+\nu)^2\, k \cdot V^f = \frac{e}{m}(D+\nu)\, k \cdot E^f - \omega_c^2\, k \cdot V^f - i k^2 s^2 (D+\nu) \frac{n^f}{n_o}$$

or making use of equation (4.16) and the condition $k \times E^f \rightarrow 0$ which is justified for longitudinal waves, we get

$$\left[(D+\nu)^2 + \omega_c^2\right] k \cdot V^f = -\frac{ie}{m}(D+\nu)(n_j^f - n_{j'}^f) - \frac{i k^2 s^2 n^f}{n_o} \tag{a 3}$$

Small contribution from V^S terms has been ignored. Equation (4.14) is now operated by $\left[(D+\nu)^2 + \omega_c^2\right]$ and (a 3) is substituted in the result and a little rearrangement gives

$$(D+\nu)^2 D n^f + (D+\nu)^2 i k\cdot V_0 n^S + \omega_p^2 (D+\nu)(n_j^f - n_{j'}^f)$$

$$+ k^2 s^2 (D+\nu) n^f = -\omega_c^2 D n^f - \omega_c^2 i k\cdot V_0 n^S$$

Operating with $(D+\nu)^{-1}$,

$$\left[D^2 + \nu D + \omega_p^2 + k^2 s^2\right] n^f + (D+\nu) i k\cdot V_0 n^S - \omega_{p_j}^2 n_{j'}^f =$$

$$= -\frac{\omega_c^2}{D}\left(1+\frac{\nu}{D}\right)^{-1} (D n^f + i k\cdot V_0 n^S)$$

$$\simeq -\omega_c^2 n^f - \omega_c^2 \int i k\cdot V_0 n^S dt + \frac{\nu}{D}\omega_c^2\left[n^f + \int i k\cdot V_0 n^S dt\right]$$

$$+ \cdots \quad \cdots$$

Now returning back to the original notations and ignoring the terms of the order $(\nu_j \omega_{c_j}^2 / \omega_s^3)$, we get

$$\left[\frac{\partial^2}{\partial t^2} + \nu_j \frac{\partial}{\partial t} + \omega_{R_j}^2\right] n_{jk}^f = \omega_{p_j}^2 n_{j'k}^f - \omega_{c_j}^2 \int i \vec{k}\cdot\vec{V}_{0j} n_{jk}^S dt$$

$$-\left(\frac{\partial}{\partial t} + \nu_j\right) i \vec{k}\cdot\vec{V}_{0j} n_{jk}^S \qquad (4.17)$$

where

$$\omega_{R_j}^2 = \omega_{p_j}^2 + \omega_{c_j}^2 + \frac{k^2 T_j \gamma_j}{m_j}$$

and $\omega_{p_j}^2 = 4\pi n_{0j} e_j^2 / m_j$

and j, j' refer to the two species, electrons and ions.

The nonlinear terms on the right hand side of equation (4.17) describe the excitation of fast fluctuations by an interaction of the equilibrium oscillating field with the ambient low frequency fluctuations. The first linear term on right hand side is due to the space charge produced at the side band frequencies.

Close to the lower hybrid frequency $(\omega_{ci} \ll \omega_0 \ll \omega_{ce})$, we can drop the first and second term for $j \rightarrow e$ in equation (4.17) and the electron equation can be solved quite simply to give

$$n_{ek}^{f} \simeq \frac{\omega_{pe}^2}{\omega_{Re}^2} n_{ik}^{f} - \frac{\omega_{ce}^2}{\omega_{Re}^2} \int i k \cdot v_{oe} n_{ek}^{s} \, dt - \left(\frac{\partial}{\partial t} + \nu_e\right) i k \cdot v_{oe} n_{ek}^{s} / \omega_{Re}^2 \qquad (4.18)$$

The last term of Eq. (4.18) is $O\left(\omega_0(\omega_0 + \nu_e)/\omega_{ce}^2\right)$ and will be deleted.

Substituting this expression for n_{ek}^{f} into ion equation and assuming the quasineutrality for the ambient low frequency fluctuations $(n_{ek}^{s} = n_{ik}^{s} \equiv n_k^{s})$, we obtain

$$\left[\frac{\partial^2}{\partial t^2} + \nu_i \frac{\partial}{\partial t} + \omega_{LH}^2\right] n_{ik}^{f} = -\left[\left(\omega_{ce}^2 \omega_{pi}^2 / \omega_{Re}^2\right) \int i k \cdot v_{oe} n_k^{s} dt + i k \cdot \frac{\partial V_{oi}}{\partial t} n_k^{s}\right] \qquad (4.19)$$

where

$$\omega_{LH} \equiv \left\{\left(\omega_{Ri}^2 \omega_{Re}^2 - \omega_{pi}^2 \omega_{pe}^2\right) / \omega_{Re}^2\right\}^{1/2}$$

denotes the lower hybrid frequency.

In the absence of the background density fluctuations, Eq. (4.19) describes the normal lower hybrid mode (right hand side is zero for $n_k^s = 0$) with $k_{\parallel} = 0$. The right hand side describes the coupling between pump field and the background density fluctuations. We notice that both electron (first term) and ion (second term) excursions in the external field are effective in providing coupling between fast and slow fluctuations.

Let us try to understand the physics of these coupling terms. This can be conveniently done by using the $k \cdot \Delta X$ method developed by Lee and Kaw [15]. This method is fully equivalent to Vlasov - Maxwell equations and yet is simpler and much more physically transparent. Using this method we can write,

$$n_{ik}^f = -i(k \cdot \Delta X_{io}) n_{ik}^s - i(k \cdot \Delta X_i^f) n_o \tag{4.20 a}$$

$$n_{ek}^f = -i(k \cdot \Delta X_{eo}) n_{ek}^s - i(k \cdot \Delta X_e^f) n_o \tag{4.20 b}$$

where ΔX_{jo} is the excursion length of the species j in the external field $E_0 \cos\omega_0 t$ and ΔX_j^f is the excursion length in the space charge field E^f. Taking the second time derivative of Eq. (4.20 a), we obtain

$$\frac{\partial^2 n_{ik}^f}{\partial t^2} = -ik\cdot\frac{\partial V_{oi}}{\partial t} n_{ik}^s - \frac{ie}{m_i} k\cdot E^f$$

$$= -ik\cdot\frac{\partial V_{oi}}{\partial t} n_{ik}^s - \omega_{pi}^2 (n_{ik}^f - n_{ek}^f) \qquad (4.21)$$

where the magnetic field effects are ignored for ions (since $\omega_0 \gg \omega_{ci}$). Expressing ΔX_e^f also in terms of E^f (note that we are mainly interested in the excursion along the electric field E^f), Eq. (4.20 b) can be written in the form

$$n_{ek}^f = -\frac{\omega_{ce}^2}{\omega_{ce}^2+\omega_{pe}^2}\int i(k\cdot v_{oe})dt - \frac{\omega_{pe}^2}{\omega_{ce}^2+\omega_{pe}^2} n_{ik}^f$$

$$(4.22)$$

Substitution of Eq. (4.22) into Eq. (4.21) and use of the quasineutrality condition $n_{ek}^s \simeq n_{ik}^s$, now, lead to Eq. (4.19) (for $\nu_i = 0$ and $T_e = T_i = 0$). Eqs. (4.20 a, b) thus give the physical interpretation of the coupling terms on the right hand side of Eq.(4.19). Particle density fluctuations at the side band frequencies are produced because the external field at ω_0 displaces particles along the direction of density gradient set up by the low frequency wave, the second term in Eqs. (420 a, b) is due to the space charge effects produced by self-consistent fields.

Equation (4.19) can readily be solved for fluctuating components at the frequencies $\omega \pm \omega_0$, the lowest order side bands. The solution is given in the appendix. Formally we can write the solution in the following form,

$$n_{ik}^{f} = \left\{ P(k) \sin \omega_0 t + Q(k) \cos \omega_0 t \right\} n_k^{s} \tag{4.23}$$

where $P(k)$ and $Q(k)$ are complex co-efficients and have been evaluated in the appendix. Equation (4.18) now provides the solution of n_{ek}^{f} which combined with the Poisson's equation

$$i\, k \cdot E_k^{f} = 4\pi e \left(n_{ik}^{f} - n_{ek}^{f} \right)$$

allows us to write the self-consistent field E_k^{f} in the form

$$\vec{E}_k^{f} = - n_k^{s} \frac{4\pi i e \vec{k}}{k^2} \left[R(k) \sin \omega_0 t + S(k) \cos \omega_0 t \right] \tag{4.24}$$

Coefficients $R(k)$ and $S(k)$ are related to $P(k)$ and $Q(k)$ respectively and are defined again in the appendix.

Knowing the value of E_k^{f}, of course, one can solve the equation of motion (see the appendix) and find the velocity V_k^{f} as

$$\vec{V}_k^{f} = \frac{i n_k^{s} \omega_{pi}^{2}}{n_0 \omega_0^{2} k^{2}} \left\{ \vec{k} \left[-(s\omega_0 + \nu_i R) \sin \omega_0 t + (\omega_0 R - \nu_i S) \cos \omega_0 t \right] + \frac{\vec{k} \times \vec{B}_0}{B_0} \omega_{ci} \left[R \sin \omega_0 t + S \cos \omega_0 t \right] \right\} \tag{4.25}$$

4.3 Anomalous Absorption Coefficient

We have already defined the anomalous absorption coefficient in Eq. (4.3). In terms of the Fourier components, Eq. (4.2) can be written as

$$\overline{\langle J_i \cdot E \rangle} = e n_0 \overline{V_{0i} \cdot E_0} + e \sum_k \overline{n_k^s E_0 \cdot V_{ik}^f} + e \sum_k \overline{n_k^s V_{i0} \cdot E_k^f} + e n_0 \sum_k \overline{V_{ik}^f \cdot E_k}$$

Making use of Eqs. (4.24) - (4.25), we can write finally,

$$\overline{\langle J_i \cdot E \rangle} = \left\{ \nu_i(k_0) \frac{\omega_{pi}^2}{\omega_0^2} \frac{E_0^2}{8\pi} - \sum_k \nu_i(k) \frac{i n_0 e}{2} \frac{\omega_{pi}^2}{\omega_0^2} \left| \frac{n_k^s}{n_0} \right|^2 \frac{k \cdot E_0}{k^2} (S_k - S_{-k}) + \sum_k \nu_i(k) \frac{n_0 m_i}{2} \frac{\omega_{pi}^4}{\omega_0^2} \left| \frac{n_k^s}{n_0} \right|^2 \frac{1}{k^2} (R_k R_{-k} + S_k S_{-k}) \right\}$$

This combined with Eq. (4.3), gives the anomalous absorption coefficient (due to ions):

$$\nu_E = \nu_i(k_0) - \sum_k \nu_i(k) (4\pi n_0 i e) \frac{k \cdot E_0}{k^2 E_0^2} (S_k - S_{-k}) \left| \frac{n_k^s}{n_0} \right|^2 + \sum_k \nu_i(k) (4\pi n_0 m_i \omega_{pi}^2) \left(\frac{R_k R_{-k} + S_k S_{-k}}{k^2 E_0^2} \right) \left| \frac{n_k^s}{n_0} \right|^2 \qquad (4.26)$$

In the absence of low frequency fluctuations, we have $\nu_E \simeq \nu_i(k_0)$ which is the normal damping rate of the oscillating electric field at the frequency ω_0 and wave number k_0. If ν_i

denotes Landau damping and $\nu_i(k_0 \sim 0)$ is negligibly small, we can, of course ignore this normal damping. Coupling to shorter wavelength lower hybrid modes enhances the damping rate of the k_0 mode, both because the amplitudes of the sideband modes are resonantly enhanced and also because the damping rate at the shorter wavelength is larger.

The quantities $R(k)$ and $S(k)$ contain terms proportional to both $k \cdot E_0$ and $k \cdot (E_0 \times B_0)$. Thus in general, the coupling due to particle excursions along the electric field as well as across the electric field is effective.

Equation (4.26) for ν_E is rather complicated because of the lengthy expressions for $R(k)$ and $S(k)$. We shall now consider some simple situations where the complexity can be reduced considerably and approximate estimates can be made.

(a) $\vec{k} \parallel \vec{E}_0$:-

Let us consider the case where the wave vectors of the background low frequency fluctuations are oriented along the direction of the oscillating electric field. Assuming $\omega_{pe} \ll \omega_{ce}$, one can show that in the lower hybrid frequency range $\omega_0 \sim \omega_{pi}$ and $\omega_{ci} \ll \omega_0 \ll \omega_{ce}$, the ion excursion along the electric field dominates the coupling process. We then obtain

$$\nu_E = \nu_i(k_0) + \sum_k \frac{\omega_{pi}^2}{4} \nu_i(k) \left| \frac{n_k^s}{n_0} \right|^2 \left\{ \frac{\delta_k^2 + \omega_k^2 + \frac{\nu_{ik}^4}{4}}{\left[(\omega_k+\delta_k)^2 + \frac{\nu_{ik}^2}{4}\right]\left[(\omega_k-\delta_k)^2 + \frac{\nu_{ik}^2}{4}\right]^2} \right\} \tag{4.27}$$

where $\delta_k = \omega_0 - \omega_{LH}(k)$ is the frequency mismatch and only the leading terms have been retained.

This case is evidently very similar to the anomalous damping of electron plasma waves in an unmagnetized plasma with low frequency turbulence. Here the ions are free to move but electrons are held back by the magnetic field (in contrast to plasma waves where ions are frozen and electrons move). For $\omega \ll \delta$ the anomalous damping rate may be estimated to be

$$\nu_E \simeq \nu_i(k_0) + \sum_k \nu_i(k) \frac{\omega_{pi}^2}{4(\delta^2 + \frac{\nu_i^2}{4})} \left| \frac{n_k^s}{n_0} \right|^2 \tag{4.28}$$

Since $\left\{ \omega_{pi} / (\delta^2 + \nu_i^2/4)^{1/2} \right\}$ can be very large, $\nu_E / \nu_i(k_0)$ may assume large values even for moderate values of $|n_k^s/n_0|^2$. Equation (4.27) also shows a maximum with respect to δ - variation. It turns out to be

$$\nu_E^{max} = \nu(k_0) + \sum_k \frac{\nu_i(k)\, \omega_{pi}^2}{4(\omega^2 + \nu_i^2/4)} \left| \frac{n_k^s}{n_0} \right|^2$$

at $\delta_k = 0$. Thus maximum occurs at exact resonance. One should, of course, realise that ν_E can not exceed ω_{pi}, the maximum that any collective effect involving ion motion is able to yield.

(b) $\vec{k} \perp \vec{E}_0$:—

In this case the dominant coupling will be due to the ($E \times B_0$) excursion of electrons and for low density plasma $\omega_0 \sim \omega_{pi}$, we obtain

$$\nu_E = \nu_i(k_0) + \frac{\omega_{pi}^4}{4\omega_{ci}^2} \sum_k \nu_i(k) \left| \frac{n_k^s}{n_0} \right|^2 \left\{ \frac{\delta_k^2 + \omega_k^2 + \frac{\nu_{ik}^2}{4}}{\left[(\delta_k - \omega_k)^2 + \frac{\nu_{ik}^2}{4}\right]\left[(\delta_k + \omega_k)^2 + \frac{\nu_{ik}^2}{4}\right]} \right\}^2 \tag{4.29}$$

The order of magnitude of ν_E, in this case, is larger by a factor $\omega_{pi}^2 / \omega_{ci}^2$ for the same level of initial fluctuations n_k^s / n_0. Thus it appears that those k-values of lower frequency fluctuations will contribute most which are almost perpendicular to the applied field for the anomalous ion heating of a low density plasma ($\omega_{pe}^2 \ll \omega_{ce}^2$).

4.4 Discussion

We have shown that a long wavelength oscillating electric field at lower hybrid frequency can be anomalously absorbed in a plasma with short wavelength low frequency fluctuations because of their coupling to short wavelength lower hybrid modes which are damped. The damping of short wavelength lower hybrid modes could be due to Landau type resonant interaction with ions provided their phase velocity (ω_0/k) is of the order of the ion thermal speed and therefore one could have an efficient method of putting the electric field energy into ions. The typical rate of anomalous absorption is determined by Eqns. (4.27) to (4.29) and ν_E can take values as large as a fraction of ω_{pi}. A noteworthy point is that anomalous absorption of lower hybrid waves can result even if the wave is not intense enough to excite parametric instabilities; the only requirement is that there should be a significant amount of ambient low frequency fluctuations in the medium. The result of thresholdless anomalous absorption may be of some relevance to recent experiments [17].

It is easy to see physically why the anomalous absorption coefficient takes the form shown in Eq. (4.27) to (4.29). The external oscillating field interacting with low frequency fluctuations drives the damped lower hybrid modes in the plasma. When the oscillating field itself is close to the lower hybrid frequency, it drives the natural oscillations resonantly and their amplitude becomes quite large, typically $\left[\omega_0/(\delta_k + \frac{i\nu_{ik}}{2})\right]$ times the driving amplitude, where δ_k is the frequency mismatch. These large amplitude lower hybrid modes are damped at their natural damping rate, i.e., $\nu_{ik} \times |\text{amplitude}|^2$ which effectively produces a large damping of the driving field at a rate $\sim C\nu_i\omega_{pi}^2/(\delta^2+\frac{\nu_i^2}{4})$ where C is a coupling coefficient proportional to $|n_k^s|^2$. This is the physical interpretation of Eqs. (4.27) to (4.29). Note that ν_{ik} is the normal Landau damping rate at wave number k.

We should like to add a word here about the nature and role of the low frequency background noise. For the purpose of the thresholdless anomalous absorption, as shown above, the background waves have only to satisfy the conditions $\omega \ll \omega_0$ and $k \gg k_0$. Thus any natural low frequency short wavelength waves in the medium are suitable. Low frequency drift waves can serve as a typical example in our case. Such waves normally exist in most laboratory plasmas, owing to the existence of free energy sources in the form of density gradient etc. Since the natural damping of these waves is quite low $(\nu_{max} \lesssim \omega)$ they undergo very little energy loss to the plasma and do not contribute significantly to heating. However, the sidebands which are excited by their help in the presence of the pump wave have a large damping rate $(\nu \lesssim \omega_0 \gg \omega)$ and cause considerable heating of the plasma, most of the energy absorbed coming from the pump wave.

Finally, it may be remarked that the use of dipole approximation $(k_0 \sim 0)$ for external oscillating field is justified as long as the low frequency fluctuations have wavelength short compared to the scale length of variation of the primary field. In any event, this is not an essential approximation. Similarly, the $k_{\parallel}=0$ approximation for all the lower hybrid mode can be readily modified.

Recently Okuda, Chu and Dawson [18] have observed a large damping of lower hybrid waves in computer simulation experiment. They attribute it to the <u>turbulent</u> electron diffusion produced by the fields of long wave length low frequency fluctuations like convective cells. Since the turbulent damping rate is proportional to the level of fluctuations, it becomes important in a turbulent plasma. They notice that the large damping due to turbulent diffusion of particles (as they have observed) is significant only for intermediate wavelengths as for short wavelengths normal ion Landau damping dominates while for longer wave lengths ion collisional damping dominates. It is important to point out the difference with our work. We have considered the effect of short wavelength low frequency turbulence in the background on lower hybrid wave damping while they attribute their results to the particle diffusion produced by long wavelength low frequency turbulence.

Appendix

Solution of Eq. (4.19):

Equation (4.19) is

$$\left[\frac{\partial^2}{\partial t^2} + \nu_i(k)\frac{\partial}{\partial t} + \omega_{LH}^2\right] n_{ik}^f = -\left[\frac{\omega_{ce}^2\,\omega_{pi}^2}{\omega_{Re}^2}\int ik\cdot v_{oe}\, n_k^s\, dt + ik\cdot\frac{\partial V_{oi}}{\partial t}\, n_k^s\right]$$

This can be written as

$$f(D)\, n_{ik}^f \equiv \left[D^2 + \nu_i D + \omega_{LH}^2\right] n_{ik}^f$$

$$= i\, n_k^s \left[a_k \sin\omega_0 t + b_k \cos\omega_0 t\right] \quad \text{(A-1)}$$

where $D \equiv \frac{\partial}{\partial t}$ and a_k and b_k will be defined below.

Solution of this equation is straightforward and can be written in the form

$$n_{ik}^f = n_k^s \left\{P_k \sin\omega_0 t + Q_k \cos\omega_0 t\right\} \quad \text{(A-2)}$$

where

$$P_k \equiv \frac{1}{2}\left\{\frac{ia_k - b_k}{f(-i\omega + i\omega_0)} + \frac{ia_k + b_k}{f(-i\omega - i\omega_0)}\right\} \quad \text{(A-3)}$$

and

$$Q_k \equiv \frac{1}{2}\left\{\frac{a_k + i b_k}{f(-i\omega + i\omega_0)} - \frac{a_k - i b_k}{f(-i\omega - i\omega_0)}\right\} \tag{A-4}$$

$$f(-i\omega \pm i\omega_0) \simeq \pm 2\omega_0\left[\omega \mp \delta + \frac{i\nu_i(k)}{2}\right] \tag{A-5}$$

It has been assumed that

$$\begin{aligned}\omega_0^2 - \omega_{LH}^2 &= (\omega_0 + \omega_{LH})(\omega_0 - \omega_{LH}) \\ &\simeq 2\omega_0\delta_k\end{aligned} \tag{A-6}$$

for $\omega_0 \simeq \omega_{LH}$ and $\omega_0 - \omega_{LH} \equiv \delta_k \ll \omega_0$

Substituting (A-5) in (A-3) and (A-4), we get

$$\mathrm{Re}P(k) \equiv \frac{1}{4\omega_0}\left\{\frac{-b_k(\omega_k - \delta_k) + \frac{a_k \nu_i(k)}{2}}{(\omega_k - \delta_k)^2 + \frac{\nu_i^2(k)}{4}} - \frac{b_k(\omega_k + \delta_k) + a_k\nu_i(k)/2}{(\omega_k + \delta_k)^2 + \nu_i(k)^2/4}\right\}$$

$$\mathrm{Im}P(k) \equiv \frac{1}{4\omega_0}\left\{\frac{a_k(\omega_k - \delta_k) + \frac{b_k \nu_{ik}}{2}}{(\omega_k - \delta_k)^2 + \frac{\nu_{ik}^2}{4}} - \frac{a_k(\omega_k + \delta_k) - \frac{b_k \nu_{ik}}{2}}{(\omega_k + \delta_k)^2 + \frac{\nu_{ik}^2}{4}}\right\}$$

$$\operatorname{Re} Q(k) \equiv \frac{1}{4\omega_0}\left\{\frac{a_k(\omega_k-\delta_k)+\frac{b_k\nu_{ik}}{2}}{(\omega_k-\delta_k)^2+\nu_{ik}^2/4}+\frac{a_k(\omega_k+\delta_k)-\frac{b_k\nu_{ik}}{2}}{(\omega_k+\delta_k)^2+\nu_{ik}^2/4}\right\}$$

$$\operatorname{Im} Q(k) \equiv \frac{1}{4\omega_0}\left\{\frac{b_k(\omega_k-\delta_k)-\frac{a_k\nu_{ik}}{2}}{(\omega_k-\delta_k)^2+\frac{\nu_{ik}^2}{4}}-\frac{b_k(\omega_k+\delta_k)+\frac{a_k\nu_{ik}}{2}}{(\omega_k+\delta_k)^2+\frac{\nu_{ik}^2}{4}}\right\}$$

We also note that

$$P(-k)=P^*(k) \quad \text{and} \quad Q(-k)\equiv Q^*(k)$$

where $*$ refers to complex conjugate.

Expression for $R(k)$ and $S(k)$: -

Deleting the small contribution of the order of ω_0^2/ω_{ce}^2 of the last term in Eq. (4.18), we can write

$$n_{ek}^f = \frac{\omega_{pe}^2}{\omega_{Re}^2} n_{ik}^f - i\,\frac{\omega_{ce}^2}{\omega_{Re}^2}\int k\cdot v_{oe}\, n_{ek}^s\, dt$$

Let us define $Y(k)$ and $Z(k)$ such that

$$Y(k)\sin\omega_0 t + Z(k)\cos\omega_0 t = \int k\cdot v_{oe}\, dt,$$

then

$$n_{ek}^f = \frac{\omega_{pe}^2}{\omega_{Re}^2} n_{ik}^f - i n_k^s \frac{\omega_{ce}^2}{\omega_{Re}^2}\left[Y(k)\sin\omega_0 t + Z(k)\cos\omega_0 t\right]$$

$\therefore \vec{E}_k^f = -\frac{4\pi i e \vec{k}}{k^2}(n_{ik}^f - n_{ek}^f)$ can be written in the form

$$\vec{E}_k^f = -\frac{4\pi i e \vec{k}}{k^2}\left[R(k)\sin\omega_0 t + S(k)\cos\omega_0 t\right] n_k^s$$

Thus

$$R(k) = (\omega_{co}^2/\omega_{Re}^2)(P(k) - iY(k))$$

and $$S(k) = Q(k) - iZ(k)$$

Solution of equation of motion for ions; —

$$\frac{\partial V_{ki}^f}{\partial t} = \frac{e}{m_i}E_k^f + V_{ki}^f \times \omega_{ci} - \nu_i(k)V_{ki}^f$$

Treating the last two terms as perturbation

$$V_{ki}^f \simeq -i\,\frac{\omega_{pi}^2}{k^2}\,\frac{n_k^s}{n_o}\,\frac{\vec{k}}{\omega_0}\left[-R(k)\cos\omega_0 t + S(k)\sin\omega_0 t\right]$$

where E_k^f has been substituted in terms of $R(k)$ and $S(k)$.

Using this value of V_k^f in the last two terms of equation of motion, we obtain

$$V_{ki}^f \simeq i\,\frac{n_k^s}{n_o}\,\frac{\omega_{pi}^2}{\omega_0^2}\Big\{\vec{k}\Big[-\big(S(k)\omega_0 + \nu_i(k)R(k)\big)\sin\omega_0 t + \big(\omega_0 R(k) - \nu_i(k)S(k)\big)\cos\omega_0 t\Big] + \frac{\vec{k}\times\vec{\omega}_{ci}}{\omega_{ci}}\,\omega_{ci}\left[R(k)\sin\omega_0 t + S(k)\cos\omega_0 t\right]\Big\}$$

Evaluation of $a(k), b(k), Y(k)$ and $Z(k)$: —

To evaluate these coefficients one has to solve the zero order equation of motion for electrons and ions in the presence of pump field. Making use of the conditions $\omega_{ci} \ll \omega_0 \ll \omega_{ce}$ and assuming $\nu_i(k_0)$ and $\nu_e(k_0)$ to be small, the solutions can easily be written in the following form.

For $k_0 \simeq 0$,

$$V_{oi} = \frac{e}{m_i}\frac{E_0}{\omega_0}\left[\sin\omega_0 t + \frac{\nu_i(k_0)}{\omega_0}\cos\omega_0 t\right] + \frac{e}{m_i}\frac{E_0 \times \omega_{ci}}{\omega_0 \omega_{ci}}\frac{\omega_{ci}}{\omega_0}\left[\frac{2\nu_i(k_0)}{\omega_0}\sin\omega_0 t - \cos\omega_0 t\right]$$

and

$$v_{oe} = \frac{e}{m_e}\frac{E_0}{\omega_0}\frac{\omega_0^2}{\omega_{ce}^2}\left[\sin\omega_0 t - \frac{\nu_e(k_0)}{\omega_0}\cos\omega_0 t\right]$$

$$+\frac{e}{m_e}\frac{E_0 \times \omega_{ce}}{\omega_{ce}}\frac{\omega_0}{\omega_{ce}}\left[\cos\omega_0 t + \frac{2\nu_e(k_0)\omega_0}{\omega_{ce}^2}\sin\omega_0 t\right]$$

Using this expression for v_{oe} we can write

$$\int k\cdot v_{oe}\,dt = \frac{e}{m_e}\left\{\frac{k.E_0}{\omega_{ce}^2}\left(-\cos\omega_0 t - \frac{\nu_e(k_0)}{\omega_0}\sin\omega_0 t\right) + k.\frac{E_0 \times B_0}{B_0\omega_0\omega_{ce}}\left(\sin\omega_0 t - \frac{2\nu_e(k_0)\omega_0}{\omega_{ce}^2}\cos\omega_0 t\right]\right\}$$

$$\equiv Y\sin\omega_0 t + Z\cos\omega_0 t \quad \text{(by definition of Y \& Z)}$$

Therefore we get

$$Y(k) = -\frac{e}{m_i}\frac{\nu_e(k_0)}{\omega_0}\frac{k\cdot E_0}{\omega_{ce}\omega_{ci}} + \frac{e}{m_i}\frac{1}{\omega_0\omega_{ci}}k\cdot\frac{E_0 \times B_0}{B_0}$$

and

$$Z(k) = -\frac{e}{m_i}\frac{k.E_0}{\omega_{ce}\omega_{ci}} - \frac{2e}{m_i}\frac{\nu_e(k_0)}{\omega_{ci}\omega_{ce}^2}k\cdot\frac{E_0 \times B_0}{B_0}$$

Similarly writing

$$k\cdot\frac{\partial v_{0i}}{\partial t} \simeq \frac{e}{m_i}\,k\cdot E_0\left[\cos\omega_0 t - \frac{\nu_i(k_0)}{\omega_0}\sin\omega_0 t\right] + \frac{e}{m_i}\,\frac{\omega_{ci}}{\omega_0}\,k\cdot\frac{E_0 \times B_0}{B_0}\left[\frac{2\nu_i(k_0)}{\omega_0}\cos\omega_0 t + \sin\omega_0 t\right]$$

Substituting these expressions for $k\cdot\frac{\partial v_{0i}}{\partial t}$ and $\int k\cdot v_{0e}\,dt$ in (A-1) we find

$$a(k) = -\frac{\omega_{ce}^2\,\omega_{pi}^2}{\omega_{Re}^2}\,Y(k) + \frac{e}{m_i}\left[\frac{\nu_i(k_0)}{\omega_0}\,k\cdot E_0 - \frac{\omega_{ci}}{\omega_0}\,k\cdot\frac{E_0 \times B_0}{B_0}\right]$$

and

$$b(k) = -\frac{\omega_{ce}^2\,\omega_{pi}^2}{\omega_{Re}^2}\,Z(k) - \frac{e}{m_i}\,k\cdot E_0 - \frac{2e}{m_i}\,\frac{\nu_i(k_0)\,\omega_{ci}}{\omega_0^2}\,k\cdot\frac{E_0 \times B_0}{B_0}$$

Though we have taken $k_0 \simeq 0$, we write $\nu(k_0)$ to distinguish it from $\nu(k)$.

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Chapter V

EFFECT OF RANDOM DENSITY FLUCTUATIONS ON PARAMETRIC INTERACTIONS IN A PLASMA

5.1 Introduction

The interaction of intense electromagnetic waves with plasmas has attracted a great deal of attention recently. The physical situations where such studies can be applied are laser-driven fusion, RF plasma heating, artificial ionospheric modification, pulsar electrodynamics etc. One of the basic processes studied widely is parametric excitation of two waves by the incident (pump) wave. Two types of parametric processes may be distinguished from each other:

(a) Both the excited waves are electrostatic e.g. pump photon $\rightarrow$ electron plasma wave + ion acoustic wave. This type of parametric process leads to enhanced (i.e. anomalous) high frequency resistivity of a plasma and is considered to be a potential mechanism for plasma heating by radio-frequency or laser fields.

(b) One of the excited modes is electromagnetic in nature e.g. Stimulated Raman Scattering (SRS) where pump photon $\rightarrow$ Scattered photon + electron plasma wave or Stimulated Brillouin Scattering (SBS) where pump photon Scattered photon +

ion acoustic wave. In this type of parametric process, the excited photon may propagate out of the plasma and get lost. Thus in experiments like laser heating, this kind of a process is a deleterious process whose ill - effects must be overcome.

The parametric instability calculations in homogeneous and weakly inhomogeneous media have been extensively discussed in the literature e.g. [1 - 3]. A unified treatment of various parametric instabilities in an infinite homogeneous unmagnetized plasma is given in [4] where the pump is assumed to be a plane polarized electromagnetic wave. Recent problems of this nature connected with laser fusion are discussed at length in [5,6]. Finally, a review on parametric phenomena has appeared recently [8] in literature.

In all these calculations the background plasma is assumed to be quiescent. However, the plasma produced by laser irradiation of a pellet, for example, is likely to be turbulent in general. This turbulence could be the one produced by parametric instabilities (in which event the theory we shall discuss below will be like a 'nonlinear' theory for parametric instabilities) or may arise as a result of some other agency promoting plasma fluctuations (e.g. violent mechanism of plasma production, some hydrodynamic instabilities etc.). We have already seen in earlier chapters that the presence of background turbulence can significantly modify the plasma properties. Hence it is natural to expect significant effects of the background turbulence on the parametric coupling process. For example, turbulence produces random density fluctuations in the background plasma which may then influence the propagation characteristics of interacting waves and hence the parametric coupling itself. An important problem, that has perhaps great relevance to laser fusion, is therefore the investigation of parametric coupling processes in a plasma with background turbulence.

In the present chapter, we shall carry out a detailed investigation of the effect of quasi-static random density fluctuations on the parametric coupling process. A preliminary investigation of this type was reported in [9]. For simplicity of presentation we shall discuss in detail only the problem of stimulated scattering of an intense electromagnetic wave off electrostatic plasma waves, i.e. the problem of stimulated Raman Scattering (SRS). The analysis can be readily extended to other parametric processes of interest. We shall ignore the time dependence of background density fluctuations, i.e. we treat them as quasi-static. This is justified if the time period of their variation is long compared to the time scales of interest to us e.g. growth time of instability, wave period etc. The one-dimensional problem now effectively reduces to an investigation of stimulated scattering in a plasma in which the background density is an irregular function of the position variable x. The basic equation for this problem can be put in the form of a fourth order differential equation for the amplitude of electrostatic or electromagnetic wave with coefficients as random functions of x (since the problem is homogeneous in time, the time variable is eliminated by appropriate Fourier analysis). Methods developed by Keller and others [10, 11] for the study of wave propagation in random media can now be adopted for an analysis for this equation. Assuming that the degree of random inhomogeneity, i.e. $\left(n(x)-n_0\right)/n_0 = \epsilon(x)$ is small, one can derive an equation for the ensemble average of the wave amplitude $\langle E \rangle$ which is correct to terms of order $|\epsilon|^2$. The averaging is carried out over an ensemble of similarly prepared systems with the background density as a random function of position.

If the basic plasma is homogeneous, n_0 is independent of x, we can use the equation for $\langle E \rangle$ to derive the modified dispersion relation correct to order $|\epsilon|^2$. This dispersion relation involves the two point correlation function for the turbulence. Assuming a Gaussian correlation function one can solve the dispersion relation in two interesting limits, i.e. the limits of long wavelength and short wavelength turbulence. In either case, there is a weakening of instability but not complete quenching.

5.2 Derivation of Dispersion Relation

In this section we investigate the stimulated Raman Scattering in a statistically homogeneous plasma with background random density fluctuations. The electron density in such a plasma may be written as

$$n(x) = n_0\left(1 + \epsilon(x)\right) \tag{5.1}$$

where n_0 is independent of x and $\epsilon(x)$, a random function, is obviously a quantitative measure of the magnitude of random density fluctuation. ϵ is taken to be time independent because it is assumed that the time variation of background electron density is very slow compared to the time scales of interest to us. We first consider the one-dimensional problem in which a linearly polarised electromagnetic wave

$$\vec{E}_0 = E_0\,\hat{e}_y \cos(k_0 x - \omega_0 t) \tag{5.2}$$

interacts with an electron plasma wave $E_\ell\,\hat{e}_x \exp(-i\omega t)$ and other electromagnetic wave $E_t\,\hat{e}_y \exp(-i(\omega-\omega_0)t)$ both propagating along x-direction. We start with the following set of equations.

The wave equation

$$\frac{\partial^2 E_t}{\partial t^2} - c^2\nabla^2 E_t = -4\pi\frac{\partial J_t}{\partial t} \tag{5.3a}$$

$$j_t = -e n_o v_t - e \tilde{n}_\ell v_o \tag{5.3b}$$

where $\tilde{n}_\ell$ is perturbed electron density and v_o and v_t are the electron velocity under the influence of pump field $\vec{E}_o$ and the field of scattered transverse wave $\vec{E}_t$ respectively. Then

$$\frac{\partial v_t}{\partial t} + (v_\ell \cdot \nabla) v_o + (v_o \cdot \nabla) v_\ell = -\frac{e}{m}\left(E_t + \frac{v_\ell \times B_o}{c}\right) \tag{5.4}$$

and
$$\frac{\partial v_\ell}{\partial t} + (v_t \cdot \nabla) v_o + (v_o \cdot \nabla) v_t = -\frac{e}{m}\left[E_\ell + \frac{v_o \times B_t}{c} + \frac{v_t \times B_o}{c}\right] + \frac{T_e}{m n_o} \tilde{n}_\ell \nabla n - \frac{T_e}{m n} \nabla \tilde{n}_\ell \tag{5.5}$$

are the equations of motion at frequencies $(\omega - \omega_o)$ and ω. The suffixes t and ℓ correspond to the excited transverse and longitudinal modes. B_t and B_o are wave magnetic fields associated with the scattered wave and pump wave respectively and are determined by the Maxwell's eqn.

$$\nabla \times \vec{E}_{o,t} = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}_{o,t} \tag{5.6}$$

We also write the continuity equation for the longitudinal wave as

$$\frac{\partial \tilde{n}_\ell}{\partial t} + n \cdot \nabla v_\ell + v_\ell \cdot \nabla n = 0 \tag{5.7}$$

We can Fourier analyse Eqs. (5.3) - (5.7) in time because the plasma parameters are time - independent. The component of E_t at frequency $\omega+\omega_0$ is off-resonant and hence unimportant for the problem considered here. The equations describing the stimulated scattering process in this situation may then be written by eliminating all other variables from Eqs. (5.3) - (5.7) except E_ℓ and E_t. Under the assumption $\omega^2 \ll \omega_0^2$ we obtain

$$-(\omega-\omega_0)^2 E_t + \omega_p^2(x) E_t - c^2 \frac{d^2 E_t}{dx^2} = \frac{eE_0}{m} \cdot \frac{dE_\ell}{dx} \tag{5.8a}$$

$$-\omega^2 E_\ell + \omega_p^2(x) E_\ell - v_e^2 \frac{d^2 E_\ell}{dx^2} + v_e^2 \frac{d\epsilon}{dx} \cdot \frac{dE_\ell}{dx} = \frac{e}{m} \frac{\omega_{p0}^2}{\omega_0(\omega-\omega_0)} \frac{d}{dx}(E_0 \cdot E_t) \tag{5.8b}$$

where $\omega_p(x) = [4\pi n(x) e^2/m]^{1/2}$ is the local electron plasma frequency, $v_e = (T_e/m)^{1/2}$ is electron thermal speed. It is obvious that equations (5.8) are the two oscillator equations at frequencies $\omega-\omega_0$ and ω respectively and coupled to each other by the pump field $\vec{E}_0$.

Defining $D \equiv d/dx$ and eliminating E_ℓ from Eqs. (5.8a,b) we obtain the operator equation

$$(M-V)\, E_t = 0 \tag{5.9}$$

where

$$M = \{(D-ik_0)^2 + k_1^2\}(D^2+k_2^2) + \frac{v_0^2}{c^2} k_d^2 D^2 \tag{5.10b}$$

is the non-random part of the differential operator and

$$V = k_d^2 \left\{ \epsilon(x) \left[(D - ik_o)^2 + k_1^2 \right] + 2 \frac{d\epsilon}{dx} (D - ik_o) + \frac{d^2\epsilon}{dx^2} \right\} \qquad (5.10b)$$

is the random part of the differential operator. Terms of order v_e^2/c^2 have been neglected and the following definitions have been used

$$v_0 = \frac{e}{m} \frac{E_o}{\omega_o} \; ; \quad k_1^2 = \left[(\omega - \omega_o)^2 - \omega_{po}^2 \right] / c^2$$

$$k_d = \omega_{pe} / v_e \text{ is inverse Debye length} \qquad (5.11)$$

$$k_2^2 = \left[\omega^2 - \omega_{po}^2 \right] / v_e^2$$

Equation (5.9) is a fourth order differential equation with coefficients which are random functions of x.

We shall now follow Keller [10] to obtain ensemble average over a large number of similarly prepared systems of Eq. (5.9)

Let E_{to} be the solution of homogeneous part of Eq. (5.9), i.e.

$$M E_{t_o} = 0 \qquad (5.12)$$

So that we can write Eq. (5.9) as

$$(M - V) E_t = 0 = M E_{t_o}$$

Multiplying this by M^{-1} we get

$$E_t = E_{to} + M^{-1} V \left[E_{to} + M^{-1} V E_t \right] - - \qquad (5.13)$$

Operating (5.13) with M and taking ensemble average of the resultant we obtain

$$\left[M - \langle V M^{-1} V \rangle\right] \langle E_t \rangle = 0$$

(5.14)

where $\langle V \rangle = 0$ has been used and the smaller terms $O|\epsilon|^3$ have been ignored.

Thus Eq. (5.14) is correct upto order $|\epsilon|^2$.

In order to solve Eq. (5.14), let us define a Green function $G(x, x')$ such that

$$M G(x, x') = \delta(x - x')$$

(5.15)

remembering that M is a differential operator.

By Fourier analysing Eq. (5.15) we get

$$G(k, k') = \delta(k + k') / M(k) \tag{5.16}$$

where $M(k)$ is a polynomial in k and is given by

$$M(k) = (k^2 - k_2^2)\left[(k-k_0)^2 - k_1^2\right] - k^2\lambda^2 \qquad (5.17a)$$

and $$\lambda^2 = k_d^2 v_0^2 / c^2 \qquad (5.17b)$$

We also note that any function $f(x)$ can be written as

$$f(x) = (1/\sqrt{2\pi}) \int \delta(x-x') f(x') dx'$$

or, $$f(x) = (1/\sqrt{2\pi}) \int M\, G(x, x') f(x') dx'$$

by making use of Eq. (5.15).

Since M is a differential operator which operates only on the function of x, it can be taken out of the above integral sign, so that

$$f(x) = (M/\sqrt{2\pi}) \int G(x, x') f(x') dx'$$

By operating by M^{-1} from left hand side we get

$$M^{-1} f(x) = \frac{1}{\sqrt{2\pi}} \int G(x, x') f(x') dx' \qquad (5.18)$$

Using this definition we can write

$$\langle VM^{-1}V\rangle\langle E_t(x)\rangle = \left\langle \frac{1}{\sqrt{2\pi}}\int V(x)G(x,x')V(x')\langle E_t(x')\rangle dx'\right\rangle \quad (5.19)$$

We now Fourier analyse Eq. (5.14) and make use of Eqs. (5.16) and (5.19) and obtain the dispersion relation

$$M(k) = \frac{\Phi(k)}{2\pi}\int dk'\rho(k-k')\Phi(k')/M(k') \quad (5.20)$$

where

$$\Phi(k) = [(k-k_0)^2 - k_1^2]\,k_d^2 \quad (5.21)$$

and $M(k)$ is defined in Eq. (5.17). $\rho(k)$ is the Fourier transform of the two point correlation function $\langle\epsilon(x)\epsilon(x')\rangle$ for the random density fluctuations. For statistically homogeneous turbulence with Gaussian correlations, we may write,

$$\rho(k) = \sqrt{2\pi}\,L_T|\epsilon|^2\exp(-k^2L_T^2/2) \quad (5.22)$$

where L_T is the correlation length for the turbulence.

Equation (5.20) is our final dispersion relation. When the background turbulence is absent, i.e. $|\epsilon|^2 = 0$, its roots are given by

$$M(k) = (k^2-k_2^2)[(k-k_0)^2-k_1^2] - k^2\lambda^2 = 0 \quad (5.23)$$

This is the wellknown dispersion relation for Stimulated Raman Scattering in a homogeneous plasma [4]. As an example we look for solutions where the plasma wave vector $k \simeq k_2$ and the stimulated electromagnetic wave vector $k_2 - k_0 \simeq -k_1$ where k_1 and k_2 are defined by Eq. (5.11). When $\omega \sim \omega_{po} \ll \omega_0$ this is satisfied, for example, if $k_2 \sim \frac{\omega_{po}}{c} \ll \frac{\omega_0}{c}$ and $k_1 \simeq k_0$. This corresponds to the case of stimulated forward scattering in a homogeneous plasma. Taking $k = k_2 + iq$ and using the above matching conditions, one obtains

$$q = (\gamma_0^2 / |V_1 V_2|)^{1/2} \tag{5.24}$$

where $\gamma_0 = (v_0/c)(\omega_{po}\omega_0)^{1/2}$ is the homogeneous plasma temporal growth rate and V_1, V_2 are the magnitudes of the group velocities of the electromagnetic wave and plasma wave, respectively. Eqn. (5.14) is the wellknown expression for inverse growth length in a homogeneous plasma. It can also be applied for the case of stimulated backscatter ($k_1 \simeq -k_0$, $k_2 \simeq 2k_0$) convective instability provided that one includes damping terms in Eqn. (5.23) and does not exceed the threshold for absolute instability.

5.3 Solution of the Dispersion Relation

For the case when $|\epsilon| \neq 0$, we may solve the dispersion relation (5.20) iteratively. Since we are looking for solution $k \simeq k_2$, we may approximate $M(k')$ in the integral on right hand side as

$$M(k') \simeq (k'^2 - k_2^2)\, \Phi(k') / k_d^2 \qquad (5.25)$$

The k' integration may now be readily carried out (in the limits $k_2 L_T \ll 1$ and $k_2 L_T \gg 1$) to give

$$M(k) = -\frac{i}{2} \frac{k_d^2}{k_2} \Phi(k) \left[\rho(k-k_2) - \rho(k+k_2) \right] \qquad (5.26)$$

where $\rho(k)$ is defined by Eqn. (5.22). We now investigate this equation in the two limiting cases:

(a) Long Scale length Turbulence

In this case $k_2 L_T \gg 1$ and since $k \simeq k_2$, only the first term in square parenthesis gives a significant contribution. Taking $k \simeq k_2 + iq$ and the resonant matching condition for backscattering, for example, we get

$$4 k_0 k_2 q^2 + 4 k_0^2 \lambda^2 = |\epsilon|^2 L_T k_d^4 q \sqrt{\pi/2} \qquad (5.27)$$

where we have assumed that $q L_T \ll 1$.

This is a quadratic which can be solved for the inverse growth length. The case of interest is when turbulence reduces q significantly, i.e., $|q| \ll (\gamma_0^2 / |V_1 V_2|)^{1/2}$. This corresponds to the neglect of first term and gives

$$q \simeq \sqrt{8\pi}\,(\gamma_0^2/\omega_{po}^2)\,|V_2/V_1|\,/\,|\epsilon|^2 L_T \tag{5.28}$$

Eq. (5.28) agrees with the equation derived in [9] on semi-quantitative grounds within a numerical factor. It may therefore be interpreted as before. Long scale length turbulence is like a series of weak inhomogeneities which introduce random phase mismatch in the three interacting waves. If δK is a change in $K = k_0 - k_1 - k_2$ on encountering a typical plasma density fluctuation, then

$$\Delta \simeq |\langle(\delta K)^2\rangle L_T| \tag{5.29}$$

is like a diffusion coefficient for phase change and the modified inverse growth length is given by

$$q \simeq \gamma_0^2/|V_1 V_2|\,\Delta \tag{5.30}$$

For stimulated Raman Scattering this agrees with the above equation (5.28).

(b) Short Scale length Turbulence

We next consider the limit $k_2 L_T \ll 1$. Taking $k \simeq k_2 + iq$ and expanding the exponential in the expression for $\rho(2k_2)$, one gets a quadratic equation for q as before. In the interesting region of significant reduction of inverse growth one obtains

$$|q| \simeq \sqrt{\frac{8}{\pi}}\,\frac{\gamma_0^{\,2}}{\omega_{p_0}^2}\left|\frac{V_2}{V_1}\right|\left(1/k_2^2 L_T^2\right)/|\epsilon|^2 L_T \tag{5.31}$$

Since $k_2 L_T \ll 1$, this means that short scale length turbulence is less effective in weakening the parametric amplification process. Since the correlation length for turbulence is shorter than wave lengths of interacting waves, the waves essentially suffer damping as a result of scattering off the density discontinuities. Eq. (5.31) is similar to the corresponding expression derived in [9] on qualitative grounds.

5.4 Discussion

We have investigated the influence of background density fluctuations on the parametric coupling processes by taking SRS (Stimulated Raman Scattering) as an example. The results can readily be extended to other laser fusion decay instabilities like SBS (Stimulated Brillouin Scattering), decay into plasma waves and ion acoustic waves etc. We find that density fluctuations can enhance the growth length but cannot quench the instability. The influence of density fluctuations is stronger on SRS than on SBS. This will be because in SBS, density changes leave the ion wave propagation unmodified and only produce a weak influence on the high frequency electromagnetic wave. Other forms of turbulence e.g. velocity fluctuations, may be more effective in modifying SBS.

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CHAPTER VI

PARAMETRIC EXCITATION OF DRIFT WAVES BY ION - ION HYBRID FIELDS

6.1 Introduction

In an earlier chapter we studied the heating of turbulent plasma by a lower hybrid pump and deduced that a thresholdless anomalous heating is possible provided there exists short wavelength low frequency background turbulence. Often however, sufficient power is readily available at low frequencies and it is possible to overcome the threshold in order to excite parametric instability which then leads to effective ion heating. This type of consideration has led to extensive studies of parametric waves excited by low frequency pump waves. One such characteristic resonance is the lower hybrid resonance [1 - 3] and we have already seen in chapter IV that it has been considered widely for this purpose. However, in a typical fusion plasma consisting of two ion species (e.g. D and T), it is possible to exploit an even lower frequency wave by using the ion - ion hybrid resonance. Such a low frequency mode is entirely supported by ions and can be expected to have the added advantage of direct heating of ions.

The ion - ion hybrid resonance was first pointed out by

Buchsbaum [4] who discussed the possibility of ion heating by normal resonance absorption. It has been experimentally observed by Haas [5] and Toyama [6]. Some experiments in ion heating at the ion - ion hybrid frequency have also been reported by Tarasenko et al [7]. The possibility of exciting this mode by applying external oscillating fields at lower hybrid frequency or by means of modulated electron beams were first studied by Kaw and Lee [8] and later by Ott et al [9].

In the present chapter we intend to study the parametric instability excited by a pump (the external wave) at the Buchsbaum frequency. At very low frequencies, the predominant electrostatic waves found are those supported by an inhomogeneous plasma, e.g. drift waves and therefore we study their behaviour under the influence of the pump wave. It should be pointed out that, whereas the parametric excitation of these low frequency modes enhances plasma heating, it may at the same time contribute to anomalous plasma transport and therefore be deleterious to plasma confinement.

Drift waves have been studied earlier under the influence of very high frequency fields (close to plasma frequency) by Fainberg and Shapiro [10] and by Okamoto et al [11]. In both these works, the applied electric vector was along the static magnetic field and hence only a parallel nonlinear coupling was involved. Recently, Sundaram and Kaw [12] have investigated the effects of perpendicular coupling of drift waves and lower hybrid fields. At the even lower ion-ion hybrid frequency considered in this chapter, there is a still larger perpendicular excursion of the ions and hence a stronger perpendicular coupling.

In Section 6.2, we derive a general dispersion relation for the drift waves in the presence of an ion-ion hybrid pump. We also delineate the various conditions that allow us to reduce it to a tractable form. In Section 6.3, we study the behaviour of the low frequency drift waves both in the presence and in the absence of a temperature gradient. In particular, we try to obtain the frequency shifts, growth rates, threshold powers and the dependence of growth rates on applied powers in regions much above threshold. Our results are summarised and discussed in Section 6.4.

6.2 Derivation of Dispersion Relation

We consider a plasma consisting of electrons and two species of ions (e.g. a D - T plasma) embedded in a uniform magnetic field $\vec{B}_0 = (0, 0, B_0)$. The plasma is inhomogeneous with a density gradient along the x-axis, $\nabla n_0 / n_0 = (-K, 0, 0)$ where K is positive. Let an a.c. electric field $\vec{E}_{osc} = \vec{E}_0 \cos \omega_0 t$ be applied transverse to the magnetic field.

The frequency ω_0 is taken close to the Buchsbaum frequency which is given by

$$\omega_B^2 = \left(\omega_{p_1}^2 \omega_{c_2}^2 + \omega_{p_2}^2 \omega_{c_1}^2\right) \Big/ \left(\omega_{p_1}^2 + \omega_{p_2}^2\right) \quad (6.1)$$

ω_{p_j} and ω_{c_j} are the plasma and cyclotron frequencies, respectively, and subscripts 1, 2 refer to the two ion species. We neglect the nonuniformity of the applied electric field (dipole approximation) as well as the 'skin' effect. Thus at equilibrium, we have an inhomogeneous low β (β is the ratio of plasma pressure to magnetic field pressure) plasma, with its electron and ion components oscillating at a frequency ω_0 under the influence of the pump field. We now wish to explore the stability of this equilibrium against low frequency perturbations characterised by (ω, k). We restrict our attention to perturbations propagating in the Y-Z plane, i.e., normal to ∇n_0. To derive our dispersion relation, we shall employ the technique developed by Arnush et al [13] which consists of transforming to the oscillating frames of the various species. Such a technique is strictly valid only for homogeneous plasmas, However, it can also be applied for the present problem since we restrict ourselves to perturbations in the plane perpendicular to density gradient (i.e. within a layer of constant density). A perturbation quantity $Q_j(r_j, t)$ in the laboratory frame is related to the corresponding perturbation quantity $\overline{Q}_j(r_j, t)$

in the oscillating frame of the j -th species $(j \equiv e, 1, 2)$ as follows

$$\overline{Q}_j(\vec{r}\,', t) \equiv Q_j(\vec{r}, t) \equiv Q_j(\vec{r}\,' + \vec{R}_{oj}, t) \tag{6.2}$$

where $\vec{R}_{oj}$ is defined by $d\vec{R}_{oj}/dt = \vec{V}_{oj}$. ($\vec{V}_{oj}$ is equilibrium velocity of the oscillating j -th species). $\vec{R}_{oj}$ is thus the excursion of the j -th species particle under the influence of the applied electric field. We can write,

$$\vec{k}\cdot\vec{R}_{oj} = \lambda_j \sin\omega_o t + \mu_j \cos\omega_o t \tag{6.3}$$

where μ_j and λ_j are the components of the excursion along the wave vector $\vec{k}$; they have respectively, phase differences of zero and $\pi/2$ to the applied field. We now take the Fourier transform of Eq. (6.2) and make use of the identity

$$\exp[-ip\sin(\omega_o t + \alpha)] = \sum_n J_n(p)\exp[-in(\omega_o t + \alpha)]$$

to obtain

$$\overline{Q}_j(\vec{k}, \omega) = \sum_{n,\ell} i^{\ell} J_n(\lambda_j) J_\ell(\mu_j) Q_j(\vec{k}, \omega + (n-\ell)\omega_o) \tag{6.4}$$

Similarly, the inverse relation expressing the fluctuations in the oscillating frame of a given species to the Laboratory frame can be written as

$$Q_j(\vec{k},\omega) = \sum_{n',\ell'} i^{\ell'} J_{n'}(-\lambda_j) J_{\ell'}(-\mu_j) \bar{Q}_j(\vec{k}, \omega + (n'-\ell')\omega_0) \tag{6.5}$$

We define susceptibility χ_j in a given oscillating frame by

$$\bar{N}_j = (-i)\, \vec{k} \cdot \overleftrightarrow{\chi}_j \cdot \overleftrightarrow{E}_j / 4\pi e_j \tag{6.6}$$

where $\bar{N}_j$ and $\overleftrightarrow{E}_j$ are density and electric field fluctuations, respectively, in that frame, e_j is the charge corresponding to the j-th species. Writing Poisson's equation as

$$i\, \vec{k} \cdot \vec{E} = 4\pi e (N_1 + N_2 - N_e) \tag{6.7}$$

we make use of Eq. (6.6) to express the density fluctuations in terms of electric fields. The electric fields are further transformed to the laboratory frame by means of relation (6.4). This gives the relation

$$\frac{1+\chi_1(\omega)}{\chi_1(\omega)} \bar{N}_1(\omega) = \sum_{m=-\infty}^{\infty} J_m(r_{1e}) e^{im\phi_{1e}} \bar{N}_e(\omega + m\omega_0) - \sum_{m=-\infty}^{\infty} J_m(r_{12}) e^{im\phi_{12}} \bar{N}_2(\omega + m\omega_0) \tag{6.8}$$

In Eq. (6.8) the following definitions hold

$$\lambda_j - \lambda_k = r_{jk} \cos \phi_{jk}$$
$$\mu_j - \mu_k = r_{jk} \sin \phi_{jk}$$

Eq.(6.8) gives one relation between the density fluctuations of the different plasma components, calculated in their respective oscillating frames. Two more relations can be immediately written down from symmetry considerations.

$$\frac{1+\chi_2(\omega)}{\chi_2(\omega)}\bar{N}_2(\omega)=\sum_{m=-\infty}^{\infty}J_m(r_{2e})e^{im\phi_{2e}}\bar{N}_e(\omega+m\omega_0)$$
$$-\sum_{-\infty}^{\infty}J_m(r_{21})e^{im\phi_{21}}\bar{N}_1(\omega+m\omega_0) \qquad (6.9)$$

$$\frac{1+\chi_e(\omega)}{\chi_e(\omega)}\bar{N}_e(\omega)=\sum_{-\infty}^{\infty}J_m(r_{e1})e^{im\phi_{e1}}\bar{N}_1(\omega+m\omega_0)$$
$$+\sum_{-\infty}^{\infty}J_m(r_{e2})e^{im\phi_{e2}}\bar{N}_2(\omega+m\omega_0) \qquad (6.10)$$

The coefficients of $\bar{N}_j$ in relations (6.8) - (6.10) form an infinite matrix whose determinant set to zero gives us the dispersion relation. We shall now discuss the appropriate conditions under which the determinant can be truncated to a 7 X 7 one as well as the approximations that will reduce it to a tractable form. At the ion - ion hybrid frequency the electrons are tightly held by the magnetic field (since $\omega_0 \ll \omega_{ce}$) and one can normally neglect their perpendicular motion. Further, the two ion components balance their mutual space charges by moving at opposite phases. Thus, even the parallel motion of the electrons becomes unimportant at the Buchsbaum resonance and we can treat $|\chi_e(\omega \pm n\omega_0)| \ll |\chi_i(\omega \pm n\omega_0)|$ for $n \neq 0$. In fact, such consideration had led Ott et al [9] to scale out electrons from their problem, thus facilitating their theoretical analysis as well as their computer simulations. In our case, however, the situation is slightly different. The ion - ion hybrid frequency is now being employed as a pump frequency and for the low frequency modes that it excites in the plasma (i.e., the drift waves) the two ion species move in phase so that their space charges add up. The electron parallel motion thus becomes important in order to neutralise the space charge and it is not possible to scale the electrons out of the problem. We have therefore retained the electron contributions to obtain a more general dispersion relation. To truncate the determinant we note that for particle excursions much smaller than perturbation wavelengths, the argument of the Bessel functions becomes very small, so that

$$J_{\pm n}(r_{ij}) \ll 1 \text{ for } n \geq 2 \text{; as } |r_{ij}| \ll 1$$

Also for $\omega \ll \omega_0$, higher order sideband terms proportional to $\omega \pm n\omega_0$, $n \geq 2$ are nonresonant and can be justifiably ignored. This reduces the determinant to a 7 X 7 one and we get a dispersion relation as follows

$$\epsilon(\omega) = \chi_1(\omega)\chi_2(\omega)\chi_e(\omega)J_1^2(r_{12}) \left[\frac{1}{\epsilon(\omega+\omega_0)} + \frac{1}{\epsilon(\omega-\omega_0)}\right] \tag{6.11}$$

where

$$\epsilon(\omega+n\omega_0) = 1 + \chi_1(\omega+n\omega_0) + \chi_2(\omega+n\omega_0) + \chi_e(\omega+n\omega_0)$$

and $n = 0, \pm 1$. For $\epsilon(\omega) = 0$ we would recover the normal drift waves in an inhomogeneous plasma and the terms on the right hand side of Eq. (6.11) arise due to parametric coupling to ion - ion hybrid waves. It is of interest to note from Eq. (6.11) that only the relative streaming between two ion - species can lead to coupling to ion - ion hybrid waves (remember $\chi_e(\omega \pm \omega_0) \simeq 0$).

We shall now study the stability properties of dispersion relation (6.11) in various regimes. For $k^2 \lambda_D^2 \ll 1$ (where λ_D is the Debye wavelength) we can introduce further simplification by treating $|\chi_i(\omega \pm \omega_0)| \gg 1$ and $|\chi_{i,e}(\omega)| \gg 1$. We note that Eq. (6.11) is quite general and we can study various phenomena by suitably calculating the χ's from fluid equations or kinetic theory.

6.3 Stability Analysis of Drift Waves

(i) No Temperature Gradient $(\nabla T = 0)$:

First, we consider a simple situation of an inhomogeneous plasma with no temperature gradients. For almost normal propagation $(k_{\parallel}^2 \ll k^2, \; k_{\perp}^2 \simeq k^2)$, we can easily calculate the various susceptibilities from a simple fluid model

$$\chi_j(\omega) = \frac{-\omega_{pj}^2(1 + i\nu_j/\omega)}{(\omega + i\nu_j)^2 - \omega_{cj}^2} - \frac{(k_{\parallel}^2/k^2)\,\omega_{pj}^2}{\omega(\omega + i\nu_j)} + \frac{\omega_*}{\omega}\,\frac{\omega_{cj}^2\,(\chi_j/k^2\lambda_D^2)}{(\omega + i\nu_j)^2 - \omega_{cj}^2} \qquad (6.12a)$$

$$\chi_e(\omega) = 1/(k^2\lambda_D^2) \qquad (6.12b)$$

where $\lambda_D^2 = (Te/4\pi n_o e^2)$; $x_j = \frac{n_{oj}}{n_o}$ = concentration of j-th species of ions, $\omega_* = (k_y c Te/eB_o)\left|\frac{1}{n_o}\frac{dn_o}{dx}\right|$. We have assumed $T_i << Te$, ν_j is the collision frequency of ions with neutrals and / or with ions of different species. It can also be used phenomenologically to mock up Landau damping in a collisionless plasma.

For $|\omega + i\nu_j| << |\omega_o - \omega_{cj}|$ we can write

$$\chi_j(\omega \pm \omega_o) \simeq \frac{\omega_{pj}^2}{\omega_o^2 - \omega_{cj}^2}\left[1 \mp \frac{\omega}{\omega_o - \omega_{cj}} \mp \frac{i\nu_j \omega_{cj}}{\omega_o(\omega_o - \omega_{cj})}\right] \quad (6.13)$$

where we have neglected the small contribution arising from inhomogeneity effects and from parallel motion. Using relation (6.13) and keeping in mind the conditions discussed in the previous section, we can express $\epsilon(\omega \pm \omega_o)$ as

$$\epsilon(\omega \pm \omega_o) = (\delta \pm \omega \pm i\nu_H) D \quad (6.14)$$

where $D = \sum_j \frac{\omega_{pj}^2}{(\omega_o^2 - \omega_{cj}^2)(\omega_o - \omega_{cj})}$

$$\delta = -\frac{(\omega_{p1}^2 + \omega_{p2}^2)(\omega_o^2 - \omega_B^2)}{(\omega_o^2 - \omega_{c1}^2)(\omega_o^2 - \omega_{c2}^2) D}$$

and

$$\nu_H = \sum_j \frac{\nu_j\, \omega_{cj}\, \omega_{pj}^2}{(\omega_o^2 - \omega_{c1}^2)(\omega_o^2 - \omega_{c2}^2) D}.$$

δ is an approximate measure of the deviation of the pump frequency from the Buchsbaum frequency

$(\delta \simeq \omega_0 - \omega_B)$ and ν_H is the damping rate of the Buchsbaum mode. If we represent the damping rate of the drift waves by ν, the dispersion relation assumes the simple form

$$\left[\omega(\omega - \omega_* + i\nu) - k_{\parallel}^2 S^2\right]\left[\delta^2 - (\omega + i\nu_H)^2\right] = K\delta \qquad (6.15)$$

where $K = 2 J_1^2(r_{12})\,\omega_*^2 / (k^2 \lambda_D^2 D)$

$$\nu = \sum_j \frac{\nu_j T_e}{M_j}\left[(k^2/\omega_{c_j}^2) + (k_{\parallel}^2/\omega^2)\right]$$

$$S^2 = T_e/n_0 \sum_j (x_j/M_j),$$

K is proportional to the applied power and can be defined as a threshold parameter. We shall now study the solutions of Eq. (6.15). Before attempting a general analysis it is worthwhile considering a simple situation when $k_{\parallel} = 0$ and $|\delta| \sim |\nu_H| \gg |\omega|$.

In this regime, Eq. (6.15) reduces to

$$\omega(\omega - \omega_* + i\nu) = K\delta/(\delta^2 + \nu_H^2)$$

which can be solved to give

$$\omega = \frac{\omega_* - i\nu}{2} \pm (1/2)\left[(\omega_* - i\nu)^2 + \frac{4K\delta}{\delta^2 + \nu_H^2}\right]^{1/2}$$

For moderate amounts of applied power (so that $4K\delta \ll (\omega_* - i\nu)^2(\delta^2 + \nu_H^2)$), the radical can be expanded to give the following two modes:

$$\omega_1 = \omega_* + \frac{K\delta}{2\omega_*(\delta^2 + \nu_H^2)} - i\nu\left(1 - \frac{K\delta}{2\omega_*^2(\delta^2 + \nu_H^2)}\right) \qquad (6.16a)$$

$$\omega_2 = -\frac{K\delta}{2\omega_*(\delta^2 + \nu_H^2)} - \frac{i\nu K\delta}{2\omega_*^2(\delta^2 + \nu_H^2)} \qquad (6.16b)$$

ω_1, is the usual drift mode with a shifted frequency and a reduced damping rate (for $\delta > 0$). ω_2 is a completely new low - frequency mode arising purely due to the pump wave modification of the dispersion relation. It can get unstable for $\delta < 0$.

In general, for $\omega \sim \delta, \nu_H$, Eq. (6.15) is a fourth degree equation, and it is difficult to extract its exact analytic roots. We can, however, obtain an estimate of the threshold power necessary to excite the drift modes by setting $\omega = x + iy$ in equation (6.15) and separating the real and imaginary parts.

We obtain

$$x(x - \omega_*) - y(y + \nu) - k_\parallel^2 s^2 = \frac{K\delta}{F(x,y)}\left[\delta^2 - x^2 + (y+\nu_H)^2\right] \tag{6.17}$$

$$y(x - \omega_*) + x(y + \nu) = 2x\frac{K\delta(y+\nu_H)}{F(x,y)} \tag{6.18}$$

where

$$F(x,y) = \left[\delta^2 - x^2 + (y+\nu_H)^2\right]^2 + 4x^2(y+\nu_H)^2 \tag{6.19}$$

Equations (6.17) - (6.19) can be investigated separately when $\delta < 0$ and when $\delta > 0$. For $\delta > 0$ one can obtain a threshold condition $(y \to 0)$ with a finite x. The expression for critical power $(K = K_c)$ is then given by

$$K_c \simeq \frac{\nu}{2\delta\nu_H}\left[(\delta^2 - \omega_*^2 + \nu_H^2)^2 + 4\nu_H^2\omega_*^2\right] \tag{6.20}$$

The system oscillates at the critical frequency

$$x_c \simeq \omega_* + k_\parallel^2 s^2/\omega_* + \nu(\delta^2 + \nu_H^2 - \omega_*^2)/2\nu_H \omega_* \tag{6.21}$$

The threshold power is a function of the magnitude of δ and we can minimize it by suitably varying δ. The minimum occurs near $\delta \sim \omega_*$ and is given by

$$K_m \simeq 2\nu\nu_H \omega_*^2 / \delta \tag{6.22a}$$

This can also be expressed in terms of minimum electric field magnitude

$$\frac{E_{om}^2}{4\pi n_0 T} = \frac{\nu\nu_H}{\delta\,\omega_{pe}^2}\,\frac{M_1^2 M_2^2}{(M_1-M_2)^2}\,\frac{\omega_0^4}{m}\sum_j \frac{x_j/M_j}{(\omega_0^2-\omega_{cj}^2)(\omega_0-\omega_{cj})} \tag{6.22b}$$

Just above the minimum threshold $K = K_m + K_1 \simeq K_m$ and $x = x_c + \Delta x$. Let y be the growth rate. Linearizing equations (6.17) - (6.19) in these quantities, we obtain

$$\Delta x \simeq (2\nu^2/x_c)(K-K_m)/K_m \tag{6.23}$$

$$y \simeq \nu(K-K_m)/K_m$$

There is thus a linear dependence of the growth rate and the frequency shift on the power, near the threshold.

For $\delta < 0$, one can find a threshold with $x = 0$, $y = 0$

This is given by

$$K_c \sim \left[(\delta^2 + \nu_H^2)/|\delta|\right] k_{\parallel}^2 s^2 \tag{6.24}$$

which may be minimised with respect to δ as usual to give

$$K_m \sim 2\,\nu_H\, k_{\parallel}^2 s^2 \tag{6.25}$$

It should be noted that, in contrast to the usual parametric instability calculations [14], this mode acquires a real part when $K > K_c$. As a matter of fact, very close to threshold, the real and imaginary parts of ω are given by

$$x \sim (|\delta|/\omega_*)(K - K_c)/(\delta^2 + \nu_H^2)$$
$$y \sim (\nu/\omega_*)(|\delta|/\omega_*)(K - K_c)/(\delta^2 + \nu_H^2) \tag{6.26}$$

Thus there is no excitation of purely growing modes and one always has a real part of frequency with which the mode oscillates.

Continuing with $\delta > 0$ case, as the applied power increases, the linear dependence of Δx, y on $(K - K_m)$ as shown by Eq. (6.23) obviously breaks down and the drift mode is strongly modified. Much beyond threshold we would expect $x, y \gg \omega_*, \nu, \nu_H$. In this limit, Eqs. (6.17) - (6.20 reduce to

$$x^2 - y^2 = (\delta^2 + y^2 - x^2) \tag{6.27}$$
$$K\delta = (\delta^2 + y^2 - x^2)^2 + 4x^2y^2 \tag{6.28}$$

Solving these equations for x^2 and y^2, we obtain

$$x^2 = \sqrt{K\delta}/2 + \delta^2/4$$
$$y^2 = \sqrt{K\delta}/2 - \delta^2/4 \qquad (6.29)$$

The growth rate maximizes for $\delta_m \sim (K/4)^{1/3}$ and goes as

$$y_m \sim (\sqrt{3}/2)(K/4)^{1/3} \qquad (6.30a)$$

at the frequency

$$x_m \sim (\sqrt{5}/2)(K/4)^{1/3} \qquad (6.30b)$$

(ii) Effect of Temperature Gradient $(\nabla T \neq 0)$

In the presence of a weak temperature gradient for an isothermal plasma $(T_e = T_1 = T_2 = T)$, the susceptibilities can be written as [15]

$$\chi_i = (-2)(\omega_{pi}^2\, \omega_T^*\, k_{\parallel}^2)/(\omega^3 k^2),$$
$$\chi_e = 1/(k\lambda_D)^2 \qquad (6.31)$$

where $\omega_T^* = -(ck_y/eB_0)\, dT/dx$

Substituting these values in the dispersion relation (6.11) we obtain

$$\omega^6 - 2k_{\parallel}^2 \lambda_D^2 (\omega_{p_1}^2 + \omega_{p_2}^2)\, \omega_T^*\, \omega^3 = \frac{K_T\, \delta}{[\delta^2 - (\omega + i\nu_H)^2]} \qquad (6.32)$$

where $K_T = 8 J_1^2(r_{12})(\omega_T^*\, \omega_{p_1}\, \omega_{p_2})^2 (k_{\parallel}/k)^4$

is again a measure of applied power. For $K_T = 0$, this is a cubic equation and yields the familiar macroscopic drift temperature gradient instability [15]. In the presence of the pump wave $(K_T \neq 0)$, the general eight degree equation is difficult to analyse. An interesting observation can, however, be made by regarding the dependence of wave frequency and growth rate on the power in regions well above the threshold. For such regions, we have $x, y \gg \nu_H, \omega_T^*$ and the second term in equation (6.32) becomes unimportant. We can thus write

$$\omega^6 (\delta^2 - \omega^2) = K_T \delta \tag{6.33}$$

Using dimensional arguments, we note that the maximum in y is achieved for $\delta_m \sim x_m$. At such values, the dependence of y_m (or x_m) on K_T comes out as

$$y_m \sim x_m \sim (K_T)^{1/7} \tag{6.34}$$

We have also solved Eq. (6.33) numerically and this confirms the seventh root dependence of y_m, x_m on K (See Fig. 1). Note that x_m is the real part of frequency for which y_m is maximum at $\delta = \delta_m$. This peculiar seventh root law seems to be characteristic of the drift temperature mode. It could be

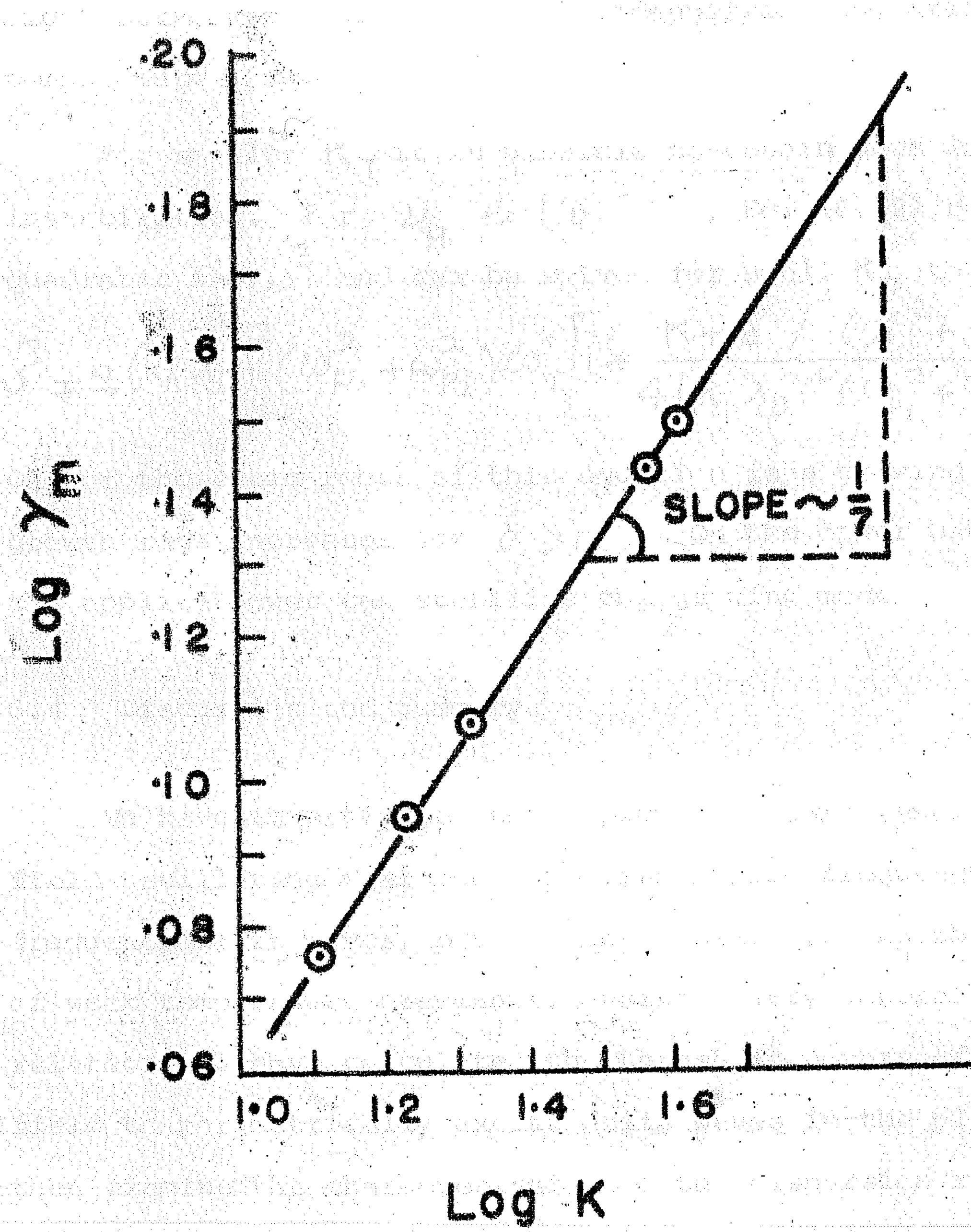

FIG. 1

significant from an experimental point of view in that it might serve as a 'signature' for identifying the drift temperature modes.

For smaller K_T it is possible to obtain much lower frequency instabilities. For $\nu_H \gg |\omega|$, Eq. (6.32) becomes a quadratic in ω^3 and can be solved for small K_T to give

$$\omega^3 = 2(k_{\parallel}\lambda_D)^2(\omega_{p_1}^2+\omega_{p_2}^2)\omega_T^*\left[1+\frac{K_T\delta/(\delta^2+\nu_H^2)}{4(k_{\parallel}\lambda_D)^4(\omega_{p_1}^2+\omega_{p_2}^2)^2\omega_T^*}\right] \quad (6.35$$

One of the cubic roots of this equation is a growing one, whose growth rate increases for $\delta > 0$. On the other hand, for $\delta < 0$, the applied power can stabilize the growing mode.

6.4 Discussion and Summary

We have investigated the effect of a large amplitude electric field oscillating near the ion - ion hybrid frequency on low - frequency drift waves, both in the absence and in the presence of weak temperature gradients. Using a very general dispersion relation, we have calculated the threshold powers of the pump field to parametrically excite drift waves in the plasma. We then examinedthe characteristics of the dispersion relation near the minimum threshold limit as well as in the limit of very large applied powers.

It is interesting to compare our results with those of

Nishikawa [14] who has discussed the parametric pumping of a general coupled oscillators system. His final dispersion relation can be put in a form similar to Eq.(6.11), i.e.,

$$\epsilon(\omega) = c\, \chi_e(\omega)\, \chi_i(\omega) \left[\frac{1}{\epsilon(\omega+\omega_0)} + \frac{1}{\epsilon(\omega-\omega_0)}\right] \quad (6.36)$$

For the cases considered by him, $\epsilon(\omega) = \epsilon(-\omega)$ which is valid for waves like plasma waves, ion acoustic waves, etc. This permits him to obtain a purely growing mode $\omega = i\gamma$ as the solution of dispersion relation. For the drift waves, on the other hand $\epsilon(\omega) \neq \epsilon(-\omega)$. In this case, a purely growing mode $\omega = i\gamma$ does not satisfy the dispersion relation. As a matter of fact, although $\mathrm{Re}\,\omega = 0$ at threshold (for $\delta < 0$), as soon as the pump field exceeds the threshold value, $\mathrm{Re}\,\omega$ acquires a finite value (Eq. (6.26)).

Another interesting feature coming out of our analysis concerns the dependence of the growth rate on pump power well above threshold. We find that the power law depends on the ω-dependence of the co-efficient of the square parenthesis on right hand side of equation (6.32). Thus, for plasma waves-ion wave coupling $\chi_e(\omega)\,\chi_i(\omega) \propto \omega^{-2}$ and we obtain a cube root dependence [14]. For the ion - ion hybrid and drift wave coupling we obtain $\chi_e(\omega)\,\chi_1(\omega)\,\chi_2(\omega) \propto \omega^{-1}$ and we get a square root dependence. Similarly for the temperature gradient drift mode and ion-ion-hybrid mode coupling we get $\chi_e(\omega)\chi_1(\omega)\chi_2(\omega)\propto\omega^{-6}$ and we get a seventh-root-dependence (Eq. (6.34)). Thus the dependence goes as $(K)^{1/(n+1)}$ where the coefficient of right hand side goes as ω^{-n}. This result could prove useful for an experimental identification of drift like modes in a parametric excitation set up.

The expressions for threshold fields and typical growth rates derived above could be used for making some order of magnitude estimates for a fusion plasma. Thus, we may have a typical threshold field as

$$E \simeq (\nu_H \nu / \omega_0 \omega_*)^{1/2} (M \omega_0 S / e) \qquad (6.37)$$

This means that when the directed velocity of ions in the ion ion hybrid field exceeds a small fraction of velocity of sound, the instability could be excited. Thus, for fusion parameters, we obtain $E \sim 1$ esu. We may conclude that moderately strong ion ion hybrid fields in a fusion plasma can lead to significant parametric heating effects. However, we must keep in mind that excitation of such low frequency modes may also lead to enhanced losses from the plasma. Furthermore, in a practical situation, one has to worry about the mechanism of introducing such low-frequency fields in the plasma.

Finally, we must mention that, strictly speaking, in an inhomogeneous plasma, one must include the convective losses from the instability region and also do a normal mode analysis to distinguish between convective and absolute instabilities. However, for the special case with $k \perp \frac{1}{n_0} \nabla n_0$, for perturbations localized along equi-density contours, the approximations used by us are quite justified.

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