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Some Studies In Standard Model and Beyond

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in

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Dedicated to

My Parents

and

My Teacher
Dr. P.C. Nayak
who initiated me
to Physics

CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

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Research Publications

List of Papers on which this thesis is based:

- 17 keV nondegenerate Majorana neutrino and neutrino mixing
  Manoj K. Samal and Utpal Sarkar,
  Phys. Letts **B 267** (1991) 243.

- Symmetric CKM matrix and top quark mass
  Manoj K. Samal, D.Choudhury, U.Sarkar and R.Mann,
  Phys. Rev.**D44** (1991) 2860.

- Symmetric CKM matrix and quark mass matrices
  Manoj K. Samal and Utpal Sarkar,
  Phys. Rev. **D45** (1992) 2421.

- Rank-one mass matrix and Phenomenological constraints
  Manoj K. Samal,
  Mod. Phys. Letts. **A7** 757 (1992).

- Potential minimization in left-right symmetric models
  B. Brahmachari, **Manoj K. Samal**, Utpal Sarkar
  (PRL Preprint No: PRL-TH/93-12).

Synopsis

In recent times, the predictions of the Standard model (SM) of particle physics have been probed to finer level of detail by precession measurements at LEP, by higher luminosity runs at Tevatron. So far no discrepancies emerged between experiments and the predictions of the SM. Despite this spectacular success of the SM in explaining all the low-energy phenomena, there is a hurdle of theoretical short comings of the model which strongly suggest that SM is only an important intermediate step toward the knowledge of fundamental interactions and that, at best, it is an effective theory, valid upto the scale M_X. The SM falls short of a complete theory in an aesthetic sense that the number of parameters required to describe it is **nineteen**, six quark and three lepton masses, three quark mixing angles and a phase parametrising CP violation, three gauge couplings and two boson mass scales M_W and M_ϕ and θ_{QCD} parameter that describes potential strong violation of CP.

Understanding the fermion mass hierarchy and the origin of the quark mixing is one of the outstanding problems of present day particle physics. This thesis is based on the studies related to fermion masses and mixing within the SM and beyond. The motivation for this study is to understand the possible extensions of the SM that are allowed phenomenologically and to study their predictions.

In the first part, we study the quark masses within the framework of the SM given the CKM quark mixing matrix [5] to be symmetric [21, 23, 34]. Present experimental limits [17] on the various elements of the CKM matrix indicate that it is symmetric or approximately symmetric. The elements $|V_{12}|$ and $|V_{21}|$,are quoted to be same modulo the errors and both of them lie between 0.217 and 0.223. Although only very weak bounds for $|V_{13}|$ element is known through the bound $.05 \le q(= \frac{|V_{13}|}{|V_{23}|}) \le .13$ at present, the decay constant $f_{B_d} \approx 220$ MeV could lead to $|V_{13}| \approx |V_{31}|$, in the case of $m_t > 100$GeV. Harris and Rosner[36] showed the possibility of $f_{B_d} \approx 220$ MeV by analyzing the $B_d^0 - \overline{B_d^0}$ mixing and the ϵ_K parameter of K meson system, and this was also suggested in the framework of lattice calculation [37]. It should be noted that for three generations, the assumption that V has symmetric moduli implies a single constraint on the matrix V because the unitarity requirement alone yields

$$A \equiv |V_{12}|^2 - |V_{21}|^2 = |V_{31}|^2 - |V_{13}|^2 = |V_{23}|^2 - |V_{32}|^2$$

for three generations and experimentally the asymmetry parameter A is, in general, small i.e.$A < 10^{-4}$. Thus all the presently available data is consistent with having symmetric moduli for CKM matrix. It has been shown [21] that if V has symmetric modulus, then it is always possible to choose the phases of the quark fields so that V is also symmetric.

We have shown [22] that if the CKM matrix is symmetric then the top quark mass has to be heavier than 180 GeV, to be consistent with the experimental results of ϵ_K, the parameter describing the indirect CP violation in the interactions changing strangeness by two units ($\Delta S = 2$), and the measurement on B_d–$\overline{B_d}$ mixing parameter x_d (which gives the time-integrated probability of a $\overline{B_d}$ appearing in a B_d beam) for the Bag constant $B_K = 1, 2/3$; if the Bag constant $B_K = 1/3$ then $m_t > 275$ GeV. The parameters q and δ(CP violating phase) are constrained to be in the range

$$.113 \leq q \leq .130 \qquad 8.0^{\circ} \leq \delta \leq 31.1^{\circ}$$

for the symmetric CKM matrix over the allowed range of the top quark mass $80GeV \leq m_t \leq 270GeV$. To get a comparative idea it should be noted that accurate measurements, especially at LEP of the properties of Z^0, together with the collider and ν data yield an indirect value for m_t [8] :

$$m_t = 164^{+16}_{-17}\ {}^{+18}_{-21} GeV.$$

Given the fact that CKM matrix is symmetric, one is urged to ask the important question of how to derive the symmetric quark mixing starting from the Yukawa couplings in a natural way. In this regard we tried to find the constraints [29]on the quark mass matrices for the symmetric CKM matrix. Since mass ratios of the up-quark sector and that of the down-quark one are quite different from each other, it is very difficult to get the symmetric CKM matrix being consistent with experiments in the framework of the 'calculable' quark mass matrix, where 'calculable' means that the CKM matrix is given in terms of the quark mass ratios, namely, the number of mass matrix parameters is less than ten observable quantities i.e. quark masses and CKM matrix elements. The symmetry constraint was written as an equation involving the parameters of the mass matrices using flavour projection operators in a basis where M_u is diagonal. The numerical ranges for the mod elements of M_d were given in this basis. This procedure was repeated in the basis where M_d is diagonal. Then, the necessary condition for having a symmetric V in terms of the matrices U and D was derived. A particularly interesting basis was chosen where $U = D^*P$; P being a phase matrix and the ranges for the mod elements of M_u, M_d in that basis was found out using a convenient parametrisation for V. It was noticed that none of the off-diagonal elements of M_u and M_d is consistent with zero for a symmetric V, which means such forms for mass matrices cannot be obtained from any symmetry. But, in principle there exists infinite number of other bases related to each other by similarity transformations. So it is apparent that the numbers provided for the allowed ranges of the mod elements of mass matrices are not basis independent. Finally the symmetry constraint was presented in a basis-independent form.

Recently, there was an attempt made by Tanimoto[35] to obtain an approximately symmetric CKM matrix starting from mass matrices of the type $M_{U,D} = \kappa_{U,D} M_0 + X_{U,D}$ where $\kappa_{U,D}$ are numerical constants; M_0 is a real $3X3$, rank-one matrix and the matrices X_U and X_D are correction terms that have to be added to M_0 to obtain the non-zero masses of the light two generation quarks since the rank-one mass matrix M_0 has only one non-vanishing eigenvalue. The phenomenological validity of this scheme was checked and we have shown[31] that out of the three interesting solutions of the symmetric CKM matrix discussed in this scheme one

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The second part of the thesis deals with the physics of massive neutrinos. The question of whether or not the neutrinos have non-zero masses concerns one of the most important issues in both particle physics and astrophysics. In the minimal SM the neutrinos are predicted to be massless due to the absence of right-handed neutrinos as well as the lepton number violating processes. However, they can acquire mass in extensions of SM involving either new $SU(2)$ singlet neutral fermions (to generate Dirac mass) or new Higgs representations (to generate Majorana mass). There exists an attractive scheme called 'see-saw mechanism' that explains naturally the smallness of the neutrino mass compared to the charged leptons. This mechanism can be embedded[54] naturally in the left-right symmetric extension of the SM. We have made an analysis[53] of the spontaneous symmetry breaking for the Higgs sector taking various Higgs representations in the context of generalised ($g_L \neq g_R$) left-right symmetric model, including the Higgs choice that predicts the correct low energy ratio of $\frac{m_b}{m_\tau}$. The minimisation of these potentials was carried out explicitly and its phenomenological consequences regarding neutrino masses had been studied. A modification in the see-saw relationship between the vacuum expectation values v_L and v_R was found out when left-right parity is broken spontaneously. The mass spectrum for different Higgs chioces consistent with potential minimisation had been used to study the evolution of the couplings g_L and g_R according to the Renormalistion Group equation.

Although none of the observational claims[45] of a 17 keV neutrino (with 1 % mixing with the electron neutrino), which was first observed by Simpson[44] in 1985, seems definitive at this time, the renewed possibility that a heavy mass eigenstate is mixed with the electron neutrino is sufficiently surprising to warrant an examination of its consequences. We studied the constraints[43] put by the existence of a 17 keV mass eigenstate on the neutrino mixing matrix from the various oscillation data, neutrinoless double beta decay and the limit on the ν_e, ν_μ and ν_τ masses. In the limit when all the three eigenvalues are nondegenerate and the mass differences are larger than $100 eV^2$, we identify ν_τ with the 17 keV neutrino and vary the mixing probability between 3% and 0.3%. We find a very narrow allowed region for the various mixing angles. The allowed values of m_{ν_μ} lie between 145 keV and 205 keV for 1% mixing and between 135 keV and 240 keV for 3% mixing (our result differs from the earlier similar works with 3% mixing, where some approximations were made). The $\nu_e \to \nu_\mu$ oscillation probability is found to lie between .001 and .002 for consistency. We then considered the allowed amount of the symmetry breaking when ν_μ and ν_τ form a pseudo-Dirac particle. We found the mass difference to be less than $9 \times 10^{-8} eV$, which puts stringent limits on the symmetry breaking effect on the neutrino mass matrix.

To summarise, the phenomenological bounds on top quark mass and the elements of the mass matrices were studied given the ansatz for quark mixing is symmetric. We found that none of the moduli of the off-diagonal elements of the possible forms of quark mass matrices M_u and M_d that lead to the symmetric CKM matrix, are consistent with zero for these ansätze, which means that such forms for mass matrices are difficult to obtain from any symmetry. The phenomenological consistency of a particular scheme for mass matrices that claims to be obtaining a symmetric quark mixing was checked. The potential minimisation of the most general Higgs representations in the context of generalised ($g_L \neq g_R$) left-right symmetric model was done explicitly and its phenomenological consequences regarding neutrino masses and the see-saw relationship was explored. We studied the limits on the elements of the neutrino mixing matrix consistent with neutrinoless double beta decay and the neutrino oscillation experiments as a function of the mixing probability (x) of ν_e with the $17keV$ neutrino. Stringent limits on m_{ν_μ} (when $m_{\nu_\mu} \gg m_{\nu_\tau}$) and on the mixing matrix were found.

Chapter 1

Introduction

Understanding the microscopic structure of the physical universe, in general, can be thought of consisting of three steps; first, an identification of the basic particles that constitute matter; second, gaining a knowledge of what forces the particles experience; third, finding a quantitative description of the particles behaviour under the influence of the forces given the initial boundary conditions. According to the present understanding, the basic particles can be put into two catagories, namely, matter particles and gauge particles. The basic matter particles are fundamental spin 1/2 fermions called the leptons and quarks whereas the gauge particles are bosons that are exchanged between matter particles during their interactions. These fundamental particles undergo four known types of gauge interactions–gravitation (too weak to be of interest to particle physics phenomenology), electromagnetism, the strong and weak nuclear forces–and also interact with higgs boson.

Historically, the study of weak interactions began with the discovery of radioactivity by Becquerel (1896) and subsequent observation that the decaying nucleus emits electrons (i.e. nuclear β decay). Chadwick's observation (1914) that the electrons in β decay are emitted with a continuous spectrum of energies and subsequent calorimetric measurements of β decay (1920) seemed to suggest the violation of energy and momentum conservation laws in β decay if one assumes a two-body final state. In order to save the fundamental conservation laws, W. Pauli proposed[1] a three-body final state for β decay with an extra neutral particle of near-vanishing or zero rest mass and half-interger spin (later named *neutrino* by Fermi) being emitted along with the electron and it escapes observation because of its feeble interaction with the surrounding matter.

Soon thereafter, Fermi proposed[2] his theory of β-decay in close analogy to quantum electrodynamics by writing a current-current effective interaction Lagrangian density :

$$\mathcal{L}_F = -(2)^{-1/2} G_F(\overline{\psi}_p \gamma_\mu \psi_n)(\overline{\psi}_e \gamma^\mu \psi_{\nu_e}) \tag{1.1}$$

The above Lagrangian density describes a 4-fermion zero-range (pointlike) interaction with a universal coupling, G_F, between the fermion pairs (e, ν_e), (p, n). $\mathcal{L}_F$, being a density, has dimension of (length)$^{-4}$, or, in energy units, dimension 4, while a fermion field has dimension 3/2 because a fermion mass term occurs in the Lagrangian in the form $m\overline{\psi}\psi$. Thus, the 4-

fermion current-current interaction is a dimension-6 operator and so G_F, the Fermi coupling constant, has units (energy)$^{-2}$. Empirically

$$G_F = 1.16639(2) \times 10^{-5} GeV^{-2} = 10^{-5} {m_p}^{-2} \quad (1.2)$$

where m_p is the proton mass. After the discovery of parity violation in weak interactions (1956), it was realised that the structure of the weak Noether current is vector-axial vector (V-A) type. That culminated in the change of γ_μ into $\gamma_\mu(1-\gamma_5)$. Subsequent discovery of many other weak interaction processes and the recognition of the universality of weak interaction led to the (V-A) form[3] of the weak interaction Lagrangian given by Marshak & Sudarshan and Gell-Mann & Feynman :

$$\mathcal{L}_W = -(2)^{-1/2}\frac{G_F}{2}(J^{\mu-}J^+_\mu + J^{\mu+}J^-_\mu), \quad (1.3)$$

where the charge-raising and charge-lowering currents are given respectively by

$$\begin{aligned} J^+_\mu &= \frac{1}{2}\overline{u}\gamma_\mu(1-\gamma_5)d + \frac{1}{2}\overline{\nu}\gamma_\mu(1-\gamma_5)e + ..., \\ J^-_\mu &= \frac{1}{2}\overline{d}\gamma_\mu(1-\gamma_5)u + \frac{1}{2}\overline{e}\gamma_\mu(1-\gamma_5)\nu + ... \end{aligned} \quad (1.4)$$

Since weak interactions distinguish the handedness, we will deal with **Weyl** two-component spinors ψ_L and ψ_R (chiral decomposed states of 4-component Dirac spinors),

$$\psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\psi,$$

each of which represents two physical degrees of freedom. The field ψ_L annihilates a LH particle or creates a RH antiparticle, while $\psi_L^\dagger$ creates an LH particle or annihilates a RH antiparticle. For a ψ_R field the role of LH and RH are reversed. Weyl fermions having no distinct partners of opposite chirality correspond to particles that are either massless or carry no conserved quantum numbers. The charged-currents can be rewritten in terms of the 2-component fields as

$$\begin{aligned} J^+_\mu &= \overline{u_L}\gamma_\mu d_L + \overline{\nu_L}\gamma_\mu e_L + ..., \\ J^-_\mu &= \overline{d_L}\gamma_\mu u_L + \overline{e_L}\gamma_\mu \nu_L + ... \end{aligned} \quad (1.5)$$

At the lowest order (tree level), $\mathcal{L}_F$ gives a very successful description of low-energy weak interaction processes. But, there are quite a few problems associated with $\mathcal{L}_F$.

At low energy, the total cross-section for neutrino-electron scattering comes out to be proportional to $G_F^2 s$, where s is the Mandelstam variable defined by $s = (p_{\nu_e} + p_e)^2$ in terms of the momenta of the incoming ν and e. With increasing energy this cross-section grows without limit. On the otherhand, since ν e scattering occurs in the s-wave, the amplitudes for this process should obey the s-wave unitarity bound, viz.,

$$\sigma^s_{tot} \leq \frac{16\pi}{s}. \quad (1.6)$$

This leads to a contradiction, implying that the 4-fermion description must breakdown above a certain energy called the weak interaction cut-off Λ_{Wk} which was found to vary between 4 GeV to 300 GeV depending upon the weak interaction process under consideration.

Secondly, as one calculates higher order corrections (loops) to any lowest order weak process described by $\mathcal{L}_F$, one finds an infinite sequence of interactions of higher and higher dimension, with increasingly divergent integrals, which would require more and more arbitrary constants to render them finite. The divergence at the L-th loop goes as Λ^{2L}, Λ being the cut-off for the theory. Therefore, in a strict sense, the lowest order calculations are not reliable.

The introduction of massive vector bosons $W^{\pm}$, which play a role analogous to that of photon in QED, improved the situation. The basic interaction is then of the form

$$\mathcal{L}_W = -(2)^{-1/2} g(J^{\mu -} W_\mu^+ + J^{\mu +} W_\mu^-), \tag{1.7}$$

where the coupling constant, g, is now dimensionless. The square of $\mathcal{L}_W$ involves the propagator of the massive W boson and yields

$$\frac{g^2}{2} J^\mu \left(\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2}\right) J^{\nu\dagger}. \tag{1.8}$$

In order to reproduce the successful low-energy ($q^2 << M_W^2$) results of the effective Lagrangian $\mathcal{L}_F$, the coupling g must be related to G_F as

$$\frac{g^2}{2M_W^2} = \frac{4G_F}{\sqrt{2}}. \tag{1.9}$$

Although there is no dimension-6 operator in $\mathcal{L}_W$, the theory is still non-renormalizable. This is due to the fact that the longitudinally polarized W bosons are described by the polarisation vectors (ϵ_μ^L) behaving as

$$\epsilon_\mu^L \longrightarrow \frac{q_\mu}{M_W} \quad as \ q \longrightarrow \infty, \tag{1.10}$$

Consequently, the W propagator approaches a constant as $q \longrightarrow \infty$ rather than decreasing as q^{-2}, and so the "canonical" dimension of the field is 2 instead of 1. Similar problem does not arise from the longitudinally polarised virtual photons in QED because the amplitudes are unchanged under the transformation

$$\epsilon_\mu \longrightarrow \epsilon_\mu + a q_\mu \tag{1.11}$$

due to gauge invariance and so the components of the photon's polarization ϵ_μ that are proportional to q_μ does not contribute to physical processes. Hence, it is quite natural to think whether or not a gauge theory can provide the remedy for non-renormalisability of $\mathcal{L}_W$.

In fact, the only renormalisable theory that accomodates the vector bosons in a fundamental way is the local gauge theory. But, then the assumption that there exists a local gauge theory associated with the weak interactions led to problems; because, in contrast to the photon, the W_μ's had to be charged, parity-violating and massive, and it was not known how to construct a self-consistent renormalisable theory for such fields. This problem was solved in three stages : first, the Yang-Mills gauge theory provided a natural method of introducing charges for the vector mesons; second, the discovery of spontaneous symmetry breaking (SSB) provided a mechanism for introducing mass without violating the gauge symmetry explicitly; finally, the technical problem of proving the renormalisability was solved by using dimensional regularisation and functional integration. Thus was the birth of the electroweak[4] theory which has been successfully explaining all the physical processes involving energies upto at least $\sim$ O(100 GeV).

Despite the immense success of the SM in explaining all the low-energy phenomena, there is a hurdle of theoretical short comings of the model which strongly suggest that SM is only an important intermediate step toward the knowledge of fundamental interactions and that, at best, it is an effective theory, valid upto the scale M_X. The SM falls short of a complete theory in an aesthetic sense that the number of parameters required to describe it is **nineteen** i.e. six quark and three lepton masses, three mixing angles and a phase parametrising CP violation, three gauge couplings and two boson mass scales M_W and M_ϕ and θ_{QCD} parameter that describes potential strong violation of CP. Of course, the predictions of the SM have been probed to finer level of detail by precision measurements at LEP, by higher luminosity runs at the Tevatron and no discrepancies emerged between experiments and the predictions of the SM.

This thesis is based on the studies related to fermion masses and mixing within SM and beyond. As it is well-known, all the masses in the SM arise out of the spontaneous symmetry breaking of the $SU(2)_L \otimes U(1)_Y \longrightarrow U(1)_{em}$. Hence they are necessarily proportional to the order parameter v of the symmetry breakdown. However, in the SM, only the masses of the gauge bosons are predicted since the constant of proportionality between their masses and v is the electric charge (and $\sin^2\theta_W$). On the other hand, both the higgs mass and those of the fermions do not have predictable values since the higgs self interaction constant λ and the Yukawa couplings h_f for each fermion are unknown parameters. In general, the process of generating mass for the fermions is non-diagonal in flavour. Consequently, some off-diagonal mass terms will ensue. This, in turn, forces a mixing between the physical states and the flavour states since the physical basis for the weak charged current interactions of quarks is not flavour diagonal. Then one wonders what fixes the observed weak mixing angles and how these angles are related to the spectrum of masses and the quark mixings and its consequences regarding the top quark mass and the quark mass matrices.

In the first half of this thesis, we study the quark masses within the framework of the SM given the CKM quark mixing matrix [5] to be symmetric. The phenomenological bounds on top quark mass and the elements of the mass matrices were studied in this ansatz for quark mixing. We found out that none of the moduli of the off-diagonal elements of the possible forms of quark mass matrices M_u and M_d that lead to the symmetric CKM matrix, are consistent with zero for these ansätze, which means that such forms for mass matrices are difficult to obtain from any symmetry. The phenomenological consistency of a particular scheme for mass matrices (based on rank one matrices) that claims to be obtaining a symmetric quark mixing was checked. And it was shown[31] that out of the three interesting solutions of the symmetric CKM matrix discussed in this scheme one is inconsistent with experiments, whereas another one requires a very heavy top quark mass ($m_t \approx 255 GeV$) to be consistent.

In the second half, we deal with the physics of massive neutrinos and its consequences. In the SM, neutrinos are massless because one excludes RH neutrinos and lepton number violating processes. Although, they can acquire mass in extensions of SM, the smallness of the neutrino mass compared to that of the charged leptons challenges our understanding. There exists an attractive scheme called 'see-saw mechanism' that explains naturally the smallness of the neutrino mass compared to the charged leptons. This mechanism can be embedded naturally in the left-right symmetric extension of the SM. The potential minimization of the most general higgs representations in the context of generalised ($g_L \neq g_R$) left-right symmetric model was done explicitly and its phenomenological consequences regarding neutrino masses and the see-saw relationship was explored.

We studied the limits on the elements of the neutrino mixing matrix consistent with neutrinoless double beta decay and the neutrino oscillation experiments as a function of the mixing probability (x) of ν_e with the 17 keV neutrino. Stringent limits on m_{ν_μ} (when $m_{\nu_\mu} \gg m_{\nu_\tau}$) and on the mixing matrix were found.

The plan of the thesis is as follows : chapter 2 consists of a review of some relevant topics in SM model and CP violation that provides the necessary background for chapter 3 in which the symmetric quark mixings and its consequences are discussed. In first half of chapter 4, we review the formalism for having massive neutrinos and their mixing and then, discuss how to accomodate the 17 KeV massive neutrino in the SM. The second half of chapter 4 consists of a brief review of the relevant part of the Left-Right symmetric extensions of SM followed by the potential minimization of the most general higgs representations in the context of generalised ($g_L \neq g_R$) left-right symmetric model and subsequent discussions of its phenomenological consequences regarding neutrino masses and the see-saw relationship. We state our conclusions in chapter 5.

Chapter 2

Review of Electroweak Model

The relevant topics of the Standard Model are briefly reviewed in this chapter along with the discussions of $\mathcal{CP}$ violation in the neutral K and B meson systems. This review provides the background for our work that will be reported in the next chapter.

2.1 The $SU(2) \otimes U(1)$ gauge theory

The electroweak model, also known as Glashow-Salam-Weinberg (GSW) model, is based on the gauge group $SU(2) \otimes U(1)$. The observation of weak neutral currents (1973), followed by the discovery of the gauge bosons themselves ($W^{\pm}$ and Z) (1983) constitute the major experimental support for the model, which has proved over the years to be very successful phenomenologically and is in detailed agreement with all observed electroweak phenomena so far.

The gauge sector of this model consists of 4 vector bosons, three denoted by $W^i_\mu, i = 1, 2, 3$ are associated with the adjoint representation of $SU(2)$ and one with $U(1)$ is denoted by B_μ. The fermion sector of the model is such that the charged weak interactions couple the left-handed component of the charged lepton to the associated (left-handed) neutrino. Parity violation is incorporated by assigning all the left handed fermions to transform as doublets under $SU(2)$, while the right-handed fermions are singlets.

The Lagrangian is made gauge-invariant by replacing ∂_μ in the fermion kinetic energy terms by the gauge covariant derivative D_μ i.e.

$$\partial_\mu \longrightarrow D_\mu \equiv \partial_\mu + igT_i W^i_\mu + ig' \frac{Y}{2} B_\mu, \tag{2.1}$$

where g, g' and $T_i, \frac{1}{2}Y$ are the $SU(2), U(1)$ couplings and group generators respectively. The T_i's satisfy the $SU(2)$ algebra

$$[T_i, T_j] = i\epsilon_{ijk} T_k \tag{2.2}$$

and act on the fermion fields as follows:

$$T_i \psi_L = \frac{1}{2} \tau_i \psi_L, \;\; T_i \psi_R = 0 \tag{2.3}$$

where the τ_i are the 2×2 Pauli matrices. The assignments for the other two generations are just replica of this.

Since the weak interaction involves electrically charged $W^{\pm}$ bosons it must be related to electromagnetism, and to incorporate QED in the model, some linear combination of the weak generators has to be identified with the electric charge operator Q corresponding to the group $U(1)_{em}$. The clue comes from the fact that the adjacent members of an isospin multiplet are eigenstates of T_3 with eigenvalues that differ by one unit of electric charge (in units of e). Therefore, we may write

$$Q = T_3 + \frac{Y}{2} \tag{2.4}$$

where T_i and Y are referred to as the weak isospin and weak hypercharge generators respectively. The above relation (called Gell-Mann–Nishijima relation) can be used to specify the eigenvalues of the $U(1)$ generator, $\frac{1}{2}Y$ where the factor $\frac{1}{2}$ is purely a matter of convention. Thus the fermions transform under the full symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ of the standard model as follows:

$$\begin{aligned}
leptons : l_{iL} &= \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L \quad (1, 2, -1); \\
e_{iR} & \quad (1, 1, -2); \\
quarks : q_{iL} &= \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \quad (3, 2, 1/3); \\
u_{iR} & \quad (3, 1, 4/3); \\
d_{iR} & \quad (3, 1, -2/3)
\end{aligned} \tag{2.5}$$

where i denotes the fermion generation. The group structure permits an arbitrary hypercharge assignment for each left-handed doublet and each right-handed singlet, and so we have chosen Y to give the correct electric charges. Apparently, charge quantization must be put by hand in $SU(2)_L \otimes U(1)_Y$ theory.

Now, the $SU(2) \otimes U(1)$ invariant Lagrangian that consists of the kinetic energy terms for massless fermions and gauge bosons and the fermion-fermion-gauge boson couplings takes the form

$$\begin{aligned}
\mathcal{L} &= \sum_f [\overline{f_L} \gamma^\mu (i\partial_\mu - g\frac{\tau_i}{2} W_\mu^i - g'\frac{Y}{2} B_\mu) f_L + \\
& \overline{f_R} \gamma^\mu (i\partial_\mu - g'\frac{Y}{2} B_\mu) f_R] - \frac{1}{4} W_{\mu\nu}^i W_i^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu},
\end{aligned} \tag{2.6}$$

where the sum is over all left- and right-handed fermion fields and the field strength tensors of the $SU(2)$ and $U(1)$ gauge fields are given by

$$\begin{aligned}
W_{\mu\nu}^i &= \partial_\mu W_\nu^i - \partial_\nu W_\mu^i - g\epsilon_{ijk} W_\mu^j W_\nu^k, \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu.
\end{aligned} \tag{2.7}$$

The term bilinear in $W_{\mu\nu}$ generates the trilinear and quadrilinear self-couplings of the W_μ fields that are a characteristic of non-Abelian gauge theory.

2.2 Giving mass to the particles

Since the gauge fields transform under the gauge group $SU(2)\otimes U(1)$ as

$$
\begin{aligned}
W_\mu \rightarrow W_\mu^{'} &= U^{-1}W_\mu U + \frac{i}{g}U^{-1}\partial_\mu U, \\
B_\mu \rightarrow B_\mu^{'} &= B_\mu + \frac{i}{g}U^{-1}\partial_\mu U;
\end{aligned}
\tag{2.8}
$$

it is evident that an explicit gauge-boson mass term is not gauge-invariant. To see the possible mass terms for fermions, consider two left-handed spinors ψ_L and χ_L that transform as $(1/2, 0)$ under Lorentz transformation (LT). The quantity $\chi_L^T\sigma^2\psi_L$ is invariant under LT. With $\chi_L = \sigma^2\psi_R^*$ this invariant is

$$
(\sigma^2\psi_R^*)^T\sigma^2\psi_L = -\psi_R{}^\dagger\psi_L \tag{2.9}
$$

and in 4 component notation, is the Dirac mass term

$$
m\overline{\psi}\psi \equiv m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L) \tag{2.10}
$$

which is excluded because ψ_L, ψ_R transform under $SU(2)$ as doublet and singlet respectively, so that this term too manifestly breaks the gauge invariance.

The other possibility for a mass term is given by the chioce $\chi_L = \psi_L$. Then we have

$$
\frac{m}{2}(\psi_L^T\sigma^2\psi_L + \psi_L^\dagger\sigma^2\psi_L^*). \tag{2.11}
$$

This mass term is called Majorana mass term and it is not invariant under $U(1)$. Consequently, any additive quantum number carried by ψ_L, such as charge, lepton number, etc. is not conserved if ψ_L has a Majorana mass. Since the gauge structure of the SM conserves the lepton number such a mass term for the fermions is not allowed.

Now we discuss how to generate gauge-boson and fermion masses without destroying the renormalizability of the theory, which depends so critically on the gauge symmetry of the interactions. As the low energy symmetry observed in nature is $SU(3)_C \otimes U(1)_{em}$, the gauge symmetry $SU(2)_L \otimes U(1)_Y$ must break down to $U(1)_{em}$. This is achieved through the spontaneous symmetry breaking of the gauge symmetry $SU(2)_L \otimes U(1)_Y$ by introducing a complex scalar field (higgs sector) ϕ, which couples gauge invariantly to the gauge bosons through the covariant derivative

$$
\partial_\mu\phi\partial^\mu\phi^\dagger = |\partial_\mu\phi|^2 \longrightarrow |(\partial_\mu + igT_iW_\mu^i + ig'\frac{Y}{2}B_\mu)\phi|^2, \tag{2.12}
$$

and to the fermions through so-called "Yukawa" couplings of the form

$$
-h_f[(\overline{\psi_L}\phi)\psi_R + \overline{\psi_R}(\phi^\dagger\psi_L)] \tag{2.13}
$$

Evidently, the field ϕ should transform as $(1, 2, 1)$ under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ to preserve the gauge invariance. Hence, we write

$$
\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \text{where} \quad \begin{pmatrix} \phi^+ \equiv (\phi_1 + i\phi_2)/\sqrt{2} \\ \phi^0 \equiv (\phi_3 + i\phi_4)/\sqrt{2} \end{pmatrix} \tag{2.14}
$$

with ϕ_i real, while the Hermitian conjugate doublet $\phi^\dagger$ describes the antiparticles ϕ^- and ϕ^{0*}.

Besides the above interactions, there can be a self-interaction between the higgs fields. The most general $SU(2)$ invariant and renormalizable form (with dimension ≤ 4) for such a self-interaction term in the Lagrangian is

$$V(\phi) = -\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2, \tag{2.15}$$

where λ must be positive to keep the potential bounded below. For $\mu^2 > 0$, $V(\phi)$ is at its minimum when $|(\phi^\dagger\phi)| = \mu^2/2\lambda$. The minima that has the vacuum expectation values (*vevs*)

$$< 0|\phi_i|0 >= 0, i = 1,2,4; \;\; < 0|\phi_3|0 >\equiv \frac{v}{\sqrt{2}} \equiv \sqrt{\mu^2/\lambda} \tag{2.16}$$

is chosen and then the field ϕ is expanded about this minimum such that the particle quanta of the theory (i.e. the physical higgs) correspond to quantum fluctuations of $\phi_3(x)$ about the value $\phi_3 = v$ rather than to $\phi_3(x)$ itself, that is, to

$$h(x) \equiv \phi_3(x) - v. \tag{2.17}$$

The choice of the non-zero *vev* for the neutral field ϕ_3 ensures that the vacuum is invariant under $U(1)_{em}$ of QED, and the photon remains massless.

When the relevant term in the Lagrangian is rewritten[6] in terms of $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$, the mass term for gauge bosons read

$$\begin{aligned} |(\partial_\mu + igT_iW^i_\mu + ig'\frac{Y}{2}B_\mu) < \phi > |^2 &= (\frac{1}{2}vg)^2W^+_\mu W^{-\mu} + \\ &\frac{1}{8}v^2(gW^3_\mu - g'B_\mu)^2 + 0(g'W^3_\mu - gB_\mu)^2, \end{aligned} \tag{2.18}$$

where

$$W^\pm \equiv (W^1 \pm iW^2)/\sqrt{2}. \tag{2.19}$$

The mass matrix of the neutral fields is off-diagonal in the (W^3, B) basis. As expected, one of the mass eigenvalues is zero and, thus the normalised neutral mass eigenstates are

$$\begin{aligned} Z_\mu &= \frac{(gW^3_\mu - g'B_\mu)}{\sqrt{g^2+g'^2}} \equiv W^3_\mu\cos\theta_W - B_\mu\sin\theta_W, \\ A_\mu &= \frac{(g'W^3_\mu + gB_\mu)}{\sqrt{g^2+g'^2}} \equiv W^3_\mu\sin\theta_W + B_\mu\cos\theta_W; \end{aligned} \tag{2.20}$$

where θ_W is the Weinberg angle defined by

$$\theta_W = \arctan\frac{g'}{g}. \tag{2.21}$$

Comparision of eqn.(2.18) with the mass terms in the Lagrangian of the physical $W^\pm_\mu, Z_\mu$ and photon A_μ fields, namely,

$$M^2_W W^+_\mu W^{-\mu} + \frac{1}{2}M^2_Z Z^2_\mu + \frac{1}{2}M^2_\gamma A^2_\mu \tag{2.22}$$

yields

$$M_W = \frac{1}{2}vg, \quad M_Z = \frac{1}{2}v\sqrt{(g^2 + g'^2)}, \quad M_\gamma = 0, \tag{2.23}$$

and so

$$\cos\theta_W = \frac{M_W}{M_Z}. \tag{2.24}$$

It is easy to see that the higgs mass comes out to be

$$M_H^2 = 2\lambda v^2 = 2\mu^2 \tag{2.25}$$

which cannot be predicted since neither μ^2 nor λ is determined, only their ratio v^2 is.

The $\rho - parameter$ that specifies the relative strength of the neutral and charged current weak interactions is defined as

$$\rho \equiv \frac{M_W^2}{M_Z^2 cos^2\theta_W}. \tag{2.26}$$

The GSW model with a single higgs doublet has $\rho = 1$, which is in excellent agreement with experiment. If the higgs sector is such that there are several representations (i=1,...,N) of higgs scalars whose neutral members acquire *vevs* v_i, then

$$\rho = \frac{\sum v_i^2[T_i(T_i+1) - \frac{1}{4}Y_i^2]}{\sum \frac{1}{2}v_i^2 Y_i^2}, \tag{2.27}$$

where T_i and Y_i are, respectively, the weak isospin and hypercharge of representation i.

To see how the charged leptons acquire mass consider the Yukawa coupling term for the electron doublet :

$$\mathcal{L}_Y^e = -h_e[(\overline{\nu_e}, e)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \overline{e_R}(\phi^-, \phi^{0*}) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L]. \tag{2.28}$$

After the symmetry breaking

$$\mathcal{L}_Y^e = -\frac{h_e}{\sqrt{2}}(v+h)(\overline{e_R}e_L + \overline{e_L}e_R) \tag{2.29}$$

from which we read out the electrons mass and couplings to be

$$m_e = \frac{h_e v}{\sqrt{2}}, \quad g(hee) = \frac{gm_e}{2M_W}. \tag{2.30}$$

Although the electron's coupling to higgs is well specified, the actual mass of the electron is not predicted as h_e is arbitrary. Similarly the most general $SU(2) \otimes U(1)$ invariant Yukawa terms for the quark doublet (u, d) are

$$\mathcal{L}_Y^q = -h_d(\overline{u}, \overline{d})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R + h_u(\overline{u}, \overline{d})_L \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} u_R + h.c. \tag{2.31}$$

here $\begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$ is $i\tau_2\phi^*$ that has a neutral upper member. Due to special properties of $SU(2)$ it has $Y = -1$. After the symmetry breaking

$$\mathcal{L}_Y^q = -(m_d\overline{d}d + m_u\overline{u}u)(1 + \frac{h}{v}) \tag{2.32}$$

where mass of the quarks are given by

$$m_q = \frac{h_q v}{\sqrt{2}}. \tag{2.33}$$

2.3 Current mass and Physical mass

Unlike the electroweak part of the SM, the renormalised masses in QCD cannot be naturally defined by on-shell renormalisation due to the confinement of quarks. The quark mass parameters of the Lagrangian can be simply considered as additional coupling constants. Hence, like the running coupling constants, their measurement first requires a careful mention of the conventions needed for the unique definition of a renormalized running quark mass of the theory. We discuss within the domain of the $\overline{MS}$ scheme which has the advantage that renormalization group equations are flavour diagonal. The evolution of the quark masses and the strong coupling constant with the renormalisation scale μ is governed by the RG equations

$$\mu\frac{dg}{d\mu} = \beta(g)$$
$$\mu\frac{dm_i}{d\mu} = -\gamma_{m_i}(g)m_i. \tag{2.34}$$

In the modified minimal subtraction ($\overline{MS}$) scheme, the beta function and the anomalous dimension are respectively given[7] by

$$\beta(g) = -\frac{\beta_0}{(4\pi)^2}g^3 - \frac{\beta_1}{(4\pi)^2}g^5 + O(g^7), \tag{2.35}$$

and

$$\gamma_m(g) = \frac{\gamma_0}{(4\pi)^2}g^2 - \frac{\gamma_1}{(4\pi)^2}g^4 + O(g^6), \tag{2.36}$$

with

$$\begin{aligned}
\beta_0 &= (11C_G - 4T_RN_f)/3, \\
\beta_1 &= [(34C_G^2 - 45C_G + 3C_F)T_RN_f]/3, \\
\gamma_0 &= 6C_F, \\
\gamma_1 &= C_F[9C_F + 97C_G - 20T_RN_f]/3,
\end{aligned} \tag{2.37}$$

where $N_f=$ number of quark flavours, T_R is given by the normalization of the generators $[Tr(T^aT^b) = N_fT_R]$ and C_G, C_F are the values of the quadratic Casimir operator on the gluons and quarks respectively. Following the convention for $SU(3)$ i.e. $T_R = 1/2, C_G = 3$, and $C_F = 4/3$ we have

$$\begin{aligned}
\beta_0 &= (11 - \frac{2}{3}N_f), \\
\beta_1 &= 102 - \frac{38}{3}N_f, \\
\gamma_0 &= 8, \\
\gamma_1 &= \frac{4}{3}(101 - \frac{10}{3}N_f).
\end{aligned} \tag{2.38}$$

The solution to the differential equations are

$$\alpha_s(\mu) \equiv \frac{g^2(\mu)}{4\pi} = \frac{4\pi}{\beta_0}L[1 - \frac{\beta_1}{\beta_0^2}\frac{lnL}{L} + O((\frac{lnL}{L})^2)], \tag{2.39}$$

and

$$m_i(\mu) = \overline{m_i}(\frac{L}{2})^{-\gamma_0/2\beta_0}[1 - \frac{\beta_1\gamma_0}{2\beta_0^2}\frac{1+lnL}{L} + \frac{\gamma_1}{2\beta_0^2 L} + O((\frac{lnL}{L})^2)], \quad (2.40)$$

where $L = ln(\mu^2/\Lambda^2)$. Here Λ and $\overline{m_i}$ are the RG invariant scale parameter and masses, respectively defined through

$$e^{-\beta_0 g^2(0)} = \frac{\lambda^2}{\Lambda^2}(ln\frac{\lambda^2}{\Lambda^2})^{\beta_1/\beta_0^2}, \quad (2.41)$$

and

$$m_i(0) = \overline{m_i}(ln\frac{\lambda^2}{\Lambda^2})^{\gamma_0/2\beta_0}, \quad (2.42)$$

λ being the momentum cut-off.

The physical mass of a quark is its value calculated at the same scale. Thus to one loop order, the physical mass of the ith quark is given by

$$m_i^{phy} = m_i(m_i)[1 + \frac{4}{3\pi}\alpha_s(m_i)]. \quad (2.43)$$

Although the determination of the light quark masses involves larger errors, still they are best estimated by the use of Chiral QCD perturbation theory as well as meson and baryon spectroscopy [7] :

$$\begin{aligned} m_u &= 5.1 \pm 1.5 \quad MeV \\ m_d &= 8.9 \pm 1.5 \quad MeV \\ m_s &= 175 \pm 55 \quad MeV \end{aligned} \quad (2.44)$$

Similarly the physical masses of the charm and bottom-quarks are obtained from e^+e^- data by using QCD sum rules for the vacuum polarisation amplitude. The running masses at $1GeV$ and $\Lambda_{QCD} = 100MeV$ [7] are

$$\begin{aligned} m_c(1GeV) &= 1.35 \pm 0.5 \quad GeV \\ m_b(1GeV) &= 5.3 \pm 0.1 \quad GeV \end{aligned} \quad (2.45)$$

While non-observation of the top-quark puts a lower limit to its mass

$$m_t^{phy} \geq 103 \quad GeV, \quad (2.46)$$

experimental consistency with the radiative corrections[9] in the SM requires

$$m_t \leq 180 \quad GeV. \quad (2.47)$$

2.4 The quark mixing matrix

In the early sixties, the LH quark states that take part in weak interactions were LH doublets, and a lonely strange singlet; and all the RH quarks were singlets :

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, s_L; u_R, d_R, s_R, \quad (2.48)$$

where d' is a mixed state of d and s states (Cabibbo's hypothesis)[10],

$$d' = V_{ud}d + V_{us}s, \tag{2.49}$$

when expressed in terms of the Cabibbo angle θ_C, $V_{ud} = \cos\theta_C, V_{us} = \sin\theta_C$. Cabibbo's hypothesis acccounted for relative coupling strength of stangness-conserving and strangness-violating baryon semi-leptonic decays, for the ratio of leptonic decay rates of pions and kaons, and for many other important features of the charged weak interactions. However, there is the strangness changing neutral current $(\overline{d}s + \overline{s}d)$ term arising from $\overline{d'}d' = \cos^2\theta_C\overline{d}d + \sin^2\theta_C\overline{s}s + \cos\theta_C\sin\theta_C(\overline{d}s + \overline{s}d)$. The too large rate of $K_L \to \mu^+\mu_-$ given by such a neutral strangeness changing current compared to the measured value of the branching ratio

$$Br(K_L \longrightarrow \mu^+\mu^-) = (9.1 \pm 1.8) \times 10^{-9}, \tag{2.50}$$

leads to the introduction of the 'charm' quark to form another left-handed doublet (GIM scheme[11]) (1970)

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L; u_R, d_R, c_R, s_R, \tag{2.51}$$

where $(d', s') = (d, s)V^T$, and V^T is the transposed matrix of

$$V = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}, \tag{2.52}$$

which represents a rotation by an angle θ_C in the two dimensional abstract space and all RH quarks are still singlets. The orthogonality of the mixing matrix guarrantees the absence of strangness-flavour changing neutral current terms. Kobayashi and Maskawa (1973) extended[5] the quark sector to

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L; u_R, d_R, c_R, s_R, t_R, b_R, \tag{2.53}$$

by introducing two more quarks, namely the top (t) and the bottom (b) on the basis of the theoretical observation that the 'reality' of the matrix V would not allow $\mathcal{CP}$ violation via the intermediate bosons $W^\pm$ coupling in the SM. The matrix V that express (d', s', b') in terms of (d, s, b) is

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \tag{2.54}$$

and is called the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The V_{ij}'s characterize the modification of the charged current vertices for the physical quark fields induced by quark mixing. The unitarity of the V matrix i.e. $VV^\dagger = \mathrm{I}$ ensures the absence of FCNC.

Ever since it was noted that the quark flavour states (states that take part in the weak interactions) are mixed states of the physical states (states that have well-defined mass), attempts have been made to comprehend the dynamical origins of the mixing angles. The observation that the Cabibbo angle θ_C is very close to the mass ratios,

$$\theta_C \sim m_\pi/m_K \sim m_d/m_s, \tag{2.55}$$

initiated many studies around expressing the elements of CKM matrix in terms of the quark masses.

To see how the CKM matrix is related to the procedure of quark mass matrix diagonalisation, consider the part of the Lagrangian containing the most general quark mass terms i.e.

$$\mathcal{L}_Y = h_{ij}^{(u)} \overline{u'}_{iL} u'_{jR} \phi^{0*} + h_{ij}^{(d)} \overline{d'}_{iL} d'_{jR} \phi^0 + h.c. \tag{2.56}$$

After symmetry breaking it reduces to

$$\mathcal{L}_Y = \frac{v}{\sqrt{2}} [\overline{u'}_{iL} h_{ij}^{(u)} u'_{jR} + \overline{d'}_{iL} h_{ij}^{(d)} d'_{jR}] + h.c. \tag{2.57}$$

with the generation index $i, j = 1, ..., N$ and $u'_{iL,iR} = \frac{1}{2}(1 \mp \gamma_5) u'_i$. The complex Yukawa couplings h_{ij} constitute $N \times N$ matrices which are generally neither Hermitian nor diagonal and $v(246\ GeV)$ is the vev of the neutral higgs field. Thus the quark mass matrices are given in the flavour basis as

$$M_{ij}^{(u)} = -(v/\sqrt{2}) h_{ij}^{(u)}, M_{ij}^{(d)} = -(v/\sqrt{2}) h_{ij}^{(d)}, \tag{2.58}$$

where the matrices $M^{(u)}$ and $M^{(d)}$ denote the quark mass matrices for charge $2/3$ (up-type) and $-1/3$ (down-type) quarks respectively. In order to find the physical fields, the quark mass matrices $M^{(u)}$ and $M^{(d)}$ must be diagonalised. As it is well-known from the theory of matrices, any square matrix (hermitian or not) can be diagonalised by a bi-unitary transformation. Since the mutually exclusive left and right-handed fields in the standard model can be rotated differently i.e.

$$\begin{aligned} u_{L,R} &= U_{L,R} u'_{L,R}, \\ d_{L,R} &= D_{L,R} d'_{L,R}; \end{aligned} \tag{2.59}$$

we can find four matrices such that for three generations

$$\begin{aligned} U_L M^{(u)} U_R{}^\dagger &= \mathcal{M}^{(u)} = diag(m_u, m_c, m_t, ...), \\ D_L M^{(d)} D_R{}^\dagger &= \mathcal{M}^{(d)} = diag(m_d, m_s, m_b, ...). \end{aligned} \tag{2.60}$$

This defines the basis of the physical quarks, and we have

$$\mathcal{L}_Y = -(\overline{u_{iL}} \mathcal{M}_{ij}^{(u)} u_{jR} + \overline{d_{iL}} \mathcal{M}_{ij}^{(d)} d_{jR}) + h.c. \tag{2.61}$$

It should be noted that the matrix $S^{(u)} = (M^{(u)} M^{(u)\dagger})$ and $S^{(d)} = (M^{(d)} M^{(d)\dagger})$ are hermitian and can thus be diagonalised by a single unitary transformation i.e.

$$\begin{aligned} U_L M^{(u)} M^{(u)\dagger} U_L^\dagger &= (\mathcal{M}^{(u)})^2 \equiv ({m_u}^2, {m_c}^2, {m_t}^2, ...) \\ D_L M^{(d)} M^{(d)\dagger} D_L^\dagger &= (\mathcal{M}^{(d)})^2 \equiv ({m_d}^2, {m_s}^2, {m_b}^2, ...). \end{aligned} \tag{2.62}$$

On the other hand, the transformations that relate the flavour basis to the physical basis introduces non-diagonal coupling into the charged currents, when they are expressed in terms of the physical quark basis,

$$j_\mu{}^\dagger = \overline{u_{iL}} \gamma_\mu V_{ij} d_{jL}, \tag{2.63}$$

where V is the unitary $N \times N$ flavour mixing matrix (CKM matrix) and is given by

$$V = U_L^{\dagger} D_L. \tag{2.64}$$

The unitarity of the $N \times N$ complex matrix V reduces the number of real parameters from $2N^2$ to N^2. An orthogonal matrix in N dimensions can be parametrized by $N(N-1)/2$ rotation angles. Thus, out of N^2 real parameters of V, $N(N-1)/2$ are rotation angles and $N(N+1)/2$ are phase angles. Since under rephasing of the up and down quark fields the non-physical individual phases γ_j and β_i of V_{ij} transform as:

$$V_{ij} \to (V'_{ij}) = V_{ij} exp(\gamma_j - \beta_i), \tag{2.65}$$

$(2N-1)$ of these phase angles can be absorbed into the definition of the quark field phases without loss of generality. So an $N \times N$ flavour mixing matrix V can be parametrised by $\mathrm{N(N-1)/2}$ rotation angles and $\mathrm{(N-1)(N-2)/2}$ phase angles. Since there are many ways to absorb the phases as relative phases between quark fields and to introduce the rotation angles in a particular way, there exists no unique parametrization of V. Physical quantities do not depend upon the particular choice of the parametrization. However, some non-measurable parameters (like the phase of a transition amplitude) are sensitive to the phase convention accepted for the quark fields. The chioce of a particular parametrization for the CKM matrix always implies the adoption of a definite phase choice. Now we consider three frequently used parametrization of V for three generations. For three generations, V can be parametrised in terms of 3 Euler angles and one phase (since five phases of the quark fields can be rotated away).

Kobayashi and Maskawa were the first to point out the matrix V for three generations cannot be transformed into a real form. Then, they suggested a parametrization where quark phases are so choosen that the first row and column of V are real,

$$V = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 c_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \tag{2.66}$$

where $c_i = \cos\theta_i$; $s_i = \sin\theta_i$ with $i = 1,2,3$. Without loss of generality θ_i can be chosen to lie in the first quadrant $i.e. 0 \le \theta_i \le \pi/2$ provided we allow the phase angle δ to take values in its full period, $i.e. -\pi \le \delta \le \pi$.

An alternative parametrization proposed by Maiani[12] and advocated by PDG to be the standard one is given as:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{2.67}$$

where the standard notation $c_{ij} = \cos\theta_{ij}$; $s_{ij} = \sin\theta_{ij}$ with $i,j = 1,2,3$ is used. It has the advantage that it makes it easy to incorporate the experimental results on B - meson deacy.

A third parametrization was introduced by Wolfenstein[14] in which he expanded the elements of the matrix V in terms of a small parameter $\lambda = \sin\theta_C$, exploiting the experimental information about the smallness of the mixing angles. The remaining structure is then determined by the unitarity constraint. This parametrization reads

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta + i\eta\frac{1}{2}\lambda^2) \\ -\lambda & 1-\frac{1}{2}\lambda^2 - i\eta A^2\lambda^4 & A\lambda^2(1+i\eta\lambda^2) \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{2.68}$$

This matrix is approximately unitary, the imaginary part of the unitarity relation is satisfied to order λ^5 and the real part to order λ^3. The coefficients A, ρ and η are of order one or even smaller.

Now we discuss the measured values of these mixing angles. While θ_{12} is very accurately determined from K_{e3} and hyperon decays[15]

$$s_{12} = .221 \pm 0.002, \tag{2.69}$$

θ_{23} and θ_{13} are rather poorly determined. The value of s_{23} may be extracted from a determination of V_{cb} (since $s_{23} \approx |V_{cb}|$ to a very good approximation) from the semileptonic B-meson partial width, under the assumption that it is given by the W-mediated process to be

$$\Gamma(b \to c\, l\, \overline{\nu}_l) = (\frac{G_F^2 m_b^5}{192\pi^3}) F(m_c^2/m_b^2)|V_{cb}|^2 \tag{2.70}$$

where $F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln(x)$ is a phase space factor. Thus

$$s_{23}^2 = (\frac{192\pi^3}{G_F^2}) \frac{\mathrm{Br}(b \to c\, l\, \overline{\nu}_l)}{\tau_b m_b^5 F(m_c^2/m_b^2)} \quad . \tag{2.71}$$

Using the experimental results for the branching ratio and the B-meson lifetime[16]

$$\mathrm{Br}(b \to c\, l\, \overline{\nu}_l) = .121 \pm 0.008 \qquad \tau_b = (1.16 \pm 0.16) 10^{-12} \mathrm{sec} \tag{2.72}$$

and the estimation for the quark masses :

$$m_c = 1.5 \pm 0.2\ GeV \qquad m_b = 5.0 \pm 0.3\ GeV \tag{2.73}$$

we get

$$s_{23} = 0.044 \pm .009 \tag{2.74}$$

The charmless B-meson decay width imposes the limit[17]

$$0.05 \leq s_{13}/s_{23} \leq 0.13 \tag{2.75}$$

The $\mathcal{CP}$-violating phase δ is allowed to adopt any value in the range $[0, \pi]$ by these current experimental results.

2.5 $\mathcal{CP}$ violation and quark mixing

Apart from the continuous symmetries that leads to conservation laws through the Noether currents the interactions that the particles undergo also respect certain discrete symmetries. Each discrete symmetry corresponds to a definite inversion and can be described in terms of a single operator.

Parity ($\mathcal{P}$) : Space-inversion. Invariance under $\mathcal{P}$ means that the LH frame obtained from RH frame by changing the signs of all spatial coordinates is an equally valid frame for expressing the laws of physics. In otherwords, the mirror image of an experiment would yield the same result in the reflected frame of reference as the original would do in the initial frame. Under $\mathcal{P}$ operation the 3-momenta are reversed. Interactions can be classified according to their transformations under the $\mathcal{P}$ operation. Since particles can be created or absorbed, intrinsic parity can also be assigned to particles. The over-all parity of a state is its parity under space inversion times the intrinsic parities of the particles in the state.

Charge-Conjugation ($\mathcal{C}$) : It transforms particles into anti-particles (i.e. reverses all additive quantum numbers) while spins and momenta are preserved. Invariance under $\mathcal{C}$ means that by turning all particles in a process into their anti-particles, we would get another process that would happen with equal probability. Under this operation, a spinor field transforms as :

$$\mathcal{C} : \psi \to \psi^c \equiv C\overline{\psi}^T , \tag{2.76}$$

where C is a matrix in the Dirac space satisfying

$$C\gamma_\mu^T C^{-1} = -\gamma_\mu, \quad C^\dagger C = 1, \quad C^T = -C. \tag{2.77}$$

A look at the Dirac equation tells the way the chiral components transform under $\mathcal{C}$:

$$\begin{aligned} \mathcal{C} : \quad \psi_L \to \quad (\psi_R)^c = C\overline{\psi_R}^T &= P_L\psi^c \\ \mathcal{C} : \quad \psi_R \to \quad (\psi_L)^c = C\overline{\psi_L}^T &= P_R\psi^c , \end{aligned} \tag{2.78}$$

where $P_{L,R}$ are the left– and right–projection operators respectively.

Time-Reversal ($\mathcal{T}$) : It refers to the reversal of the flow of time. Under this anti-unitary transformation, the initial and the final states are interchanged and spins and momenta are reversed.

Luders and Pauli had proved[18] that any Lorentz-invariant unitary local field theory is invariant under the combined transformation $\mathcal{CPT}$ (in any order). But, the invariance under any individual discrete symmetry is not assured by any deep-rooted theoretical motivations. Three out of four basic interactions i.e. gravitation, electromagnetism, and the strong nuclear interactions respect each of these discrete symmetries to quite a great extent whereas weak interactions violate both $\mathcal{C}$ and $\mathcal{P}$ invariance maximally.

Since the two terms (having same strength) present in the V-A structure of the Noether currents for the weak interactions can completely interfere, a system of quarks or leptons can change its parity and it is said that $\mathcal{P}$ is violated maximally. Similarly, SM also violates $\mathcal{C}$

invariance maximally since (for example) processes occur involving LH neutrinos, but never LH anti-neutrinos. However, under the combined operation $\mathcal{CP}$, the LH neutrino is transformed into a RH anti-neutrino and in SM, we do have electroweak interactions of LH neutrinos as well as the RH anti-neutrinos. Thus it was believed that (mid 1950's) that even though SM violates $\mathcal{C}$ and $\mathcal{P}$ separately it conserves $\mathcal{CP}$ in the sense that if a process occurs, so does the $\mathcal{CP}$ transformed process. But in the mid-1960's, it was found[19] out that although processes and their $\mathcal{CP}$ conjugate processes occur in SM, their probabilities to occur are not identical but differ by a small amount, about a one part in a thousand (see next section for detailed discussion). This small difference in the probabilities is called the $\mathcal{CP}$ violation.

$\mathcal{CP}$ violation was incorporated into SM by noticing that $\mathcal{CP}$ violation implies a violation of T or vice versa since $\mathcal{CPT}$ is a good symmetry for all quantum field theories. It is well known that if T is a good symmetry, then the quantum mechanical transformation gives $<\psi'|H|\psi> = < T\psi|THT^{-1}|T\psi'>$. Since the anti-unitary time-reversal operation T involves complex conjugation, the T (and $\mathcal{CP}$) is violated if the Hamiltonian H is not real, as the complex conjugation will, then, mean that $THT^{-1} \neq H$. From the structure of charged-current in SM, it is evident that $\mathcal{L}$ (hence H) is complex if the phase angle in the CKM matrix is non-zero. Thus the $\mathcal{CP}$ violation in SM is attributed to the non-zero CKM phase.

It should be noted that for less than three generations of quark flavours, there is no $\mathcal{CP}$ violation in SM as CKM matrix could, with full generality, be made real in such a case. However, for the $\mathcal{CP}$ violating character of V not only the phase δ is important. If any of the mixing angles is zero, the theory can be made $\mathcal{CP}$ invariant by reabsorbing the $\mathcal{CP}$ violation phase δ into a redefinition of the quark field phases. As for the leptonic sector, $\mathcal{CP}$ violation is identically zero in the minimal SM, but could arise if neutrinos are made massive.

Another concept that has been advocated is whether $\mathcal{CP}$ symmetry might be violated in a 'maximal' way as $\mathcal{P}$ and $\mathcal{C}$ are separately. But, it is not easy to find a resonable definition of "maximal $\mathcal{CP}$ violation" because the definition should be invariant under a change of phase convention or a parametriztion. The condition that the $\mathcal{CP}$ phase angle is equal to $\pi/2$ for maximal $\mathcal{CP}$ violation in some parametrization does not meet this standard. In fact, the rephasing invariant quantitative measure of $\mathcal{CP}$ violation was given in terms of elements of flavour mixing matrix elements as

$$J_{i\alpha} \equiv Im(V_{j\beta}V_{k\gamma}V^*_{j\gamma}V^*_{k\beta}) \tag{2.79}$$

where i, j, k and α, β, γ are cyclic. There are nine of these invariants for three generations case and they all are the same. this invariant $J_{i\alpha}$ is a small quantity bounded from above as

$$J_{i\alpha} < |V_{us}||V_{ub}||V_{cb}||V_{cs}| < 1.8 \times 10^{-4}, \tag{2.80}$$

and is given explicitly in various parametrizations as follows

$$\begin{aligned} KM : J_{i\alpha} &= c_1 c_2 c_3 s_1^2 s_2 s_3 \sin\delta \\ Maiani : J_{i\alpha} &= c_1 c_2 c_3 s_1^2 s_2 s_3 \sin\delta \\ Wolfenstein : J_{i\alpha} &= \lambda^6 A^2 \eta (1 - \frac{1}{2}\lambda^2) \end{aligned} \tag{2.81}$$

Then the "minimal $\mathcal{CP}$ violation" can be defined in terms of $J_{i\alpha}$. $J_{i\alpha}$ is maximal if

$$\cos\theta_1 = 1/\sqrt{3},\ \sin\theta_2 = \sin\theta_3 = 1/\sqrt{2},\ |\sin\delta| = 1. \tag{2.82}$$

Obviously, $\mathcal{CP}$ in nature is not violated maximally according to this definition.

Jarlskog[20] has defined a convention independent measure of $\mathcal{CP}$ violation in terms of quark mass matrices as

$$[M^{(u)}M^{(u)\dagger}, M^{(d)}M^{(d)\dagger}] = iC, \tag{2.83}$$

where C is a traceless hermitian matrix whose determinant is a convention independent measure of $\mathcal{CP}$ violation,

$$detC = -2FF'J_{11}, \tag{2.84}$$

with

$$\begin{aligned} F &= (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \\ F' &= (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \end{aligned} \tag{2.85}$$

and it vanishes if two of the quark masses with charge $(2/3)e$ or any two masses with charges $(-1/3)e$ are degenerate.

2.6 $\mathcal{CP}$ violation in neutral meson systems

As we have seen how SM incorporates $\mathcal{CP}$ violation, the next thing we would discuss is the extent to which SM can account for observed $\mathcal{CP}$ violating effects through the phase in CKM quark mixing matrix. The system of neutral Kaons is still the only experimentally established system having $\mathcal{CP}$ violation since its discovery by Cronin and Fitch (1964). In this section we introduce a general formalism that describes the $\mathcal{CP}$ properties of neutral mesons like $K^0 - \overline{K^0}, D^0 - \overline{D^0}, B^0 - \overline{B^0}$ etc. Although the $K^0 - \overline{K^0}$ system is taken as the prototype , the results hold for other mesons as well.

2.6.1 The Neutral Kaons system

Since both the strong and electromagnetic interactions conserve strangeness, the neutral Kaons K^0 and $\overline{K^0}$ (characterised by definite strangeness) form the basis for the Hamiltonian $H_S + H_{em}$. But they do not possess well defined masses or life-times as a mixing between K^0 and $\overline{K^0}$ is caused by strangeness violating weak interactions. Instead there exist two independent linear combinations of these states, namely K_L and K_S (having no definite strangeness but having definite mass and decay rates) that are characterized by the differences in the mass and life-times. The short-lived state K_S decays primarily through the 2π channel (with $\mathcal{CP}$ eigenvalue $+1$), the long-lived state K_L has many decay channels mostly going to final states with $\mathcal{CP}$ eigenvalue -1 i.e. 3π or $\pi^{\pm}l^{\mp}\overline{\nu}$ mode. If $\mathcal{CP}$ is respected in the above decays then it would follow that K_S and K_L are eigenstates of $\mathcal{CP}$ with eigenvalues $+1$ and -1 respectively.

With a convenient phase choice

$$\mathcal{CP}|K^0> = -|\overline{K^0}> \quad \text{and} \quad \mathcal{CP}|\overline{K^0}> = -|K^0> \tag{2.86}$$

we define two $\mathcal{CP}$ eigenstates as follows :

$$|K^0_{1,2}> \equiv \frac{1}{\sqrt{2}}[|K^0> \pm|\overline{K^0}>], \ \mathcal{CP} \equiv \mp 1 \tag{2.87}$$

Then $\mathcal{CP}$ invariance would imply that

$$|K_L> \equiv |K^0_1> \quad \text{and} \quad |K_S> \equiv |K^0_2> . \tag{2.88}$$

But it was observed by Cronin et. al that $|K_L>$ does decay into the $\pi^+\pi^-$ mode ($\mathcal{CP} \equiv +1$) with a small branching ratio 2×10^{-3}. Hence, the states $|K_L>$ and $|K_S>$ should be a more general superposition of K^0 and $\overline{K^0}$ as

$$|K_{L,S}> \equiv N_{L,S}[|K^0> \pm e^{i\xi_{L,S}}|\overline{K^0}>], \tag{2.89}$$

where $\xi_{L,S}$ are complex numbers and $N_{L,S}$ the wavefunction normalization constants.

The mixing and decay of $|K^0>$ and $|\overline{K^0}>$ are governed by an effective Hamiltonian (non-hermitian) $H = H_S + H_{em} + H_{wk} = M - i\Gamma$ where M and Γ are 2×2 hermitian matrices called the mass and decay matrices respectively. In order to study the time evolution of the states we write them as a two-component vector which satisfies the Schrodinger's equation,

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix} = H\begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix} \equiv (M - i\Gamma/2)\begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix}. \tag{2.90}$$

The eigenvalues of H are

$$E_{L,S} \equiv m_{L,S} - i\gamma_{L,S}/2 = \frac{1}{2}[H_{11} + H_{22} \pm \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}] \tag{2.91}$$

and the difference is given by

$$E_L - E_S = \Delta m - i\Delta\gamma/2 = \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}]. \tag{2.92}$$

For $K_{L,S}$ to be the eigenstate of H we must have

$$e^{i\xi_L} = \frac{E_L - H_{11}}{H_{12}} \quad \text{and} \quad e^{i\xi_S} = \frac{E_S - H_{22}}{H_{21}}. \tag{2.93}$$

Now we use both $\mathcal{CPT}$ and $\mathcal{CP}$ invariances to relate the elements of H as follows :

$$\begin{aligned}
\mathcal{CPT} : H_{11} &\equiv <K^0|H|K^0> = <K^0|(\mathcal{CPT})^{-1}H(\mathcal{CPT})|K^0> \\
&= <\overline{K^0}|H|\overline{K^0}> \equiv H_{22} \\
&= M_{11} = M_{22}, \ \Gamma_{11} = \Gamma_{22} \\
\mathcal{CP} : H_{12} &\equiv <K^0|H|\overline{K^0}> = <K^0|(\mathcal{CP})^{-1}H(\mathcal{CP})|\overline{K^0}> \\
&= <\overline{K^0}|H|K^0> \equiv H_{21} \\
&= M_{12} = M_{21}, \ \Gamma_{12} = \Gamma_{21}
\end{aligned} \tag{2.94}$$

Using the above relations, we get

$$\begin{aligned} \mathcal{CPT} : \xi_L &= \xi_S = \xi = -\frac{i}{2}\ln(\frac{H_{21}}{H_{12}}) \\ \mathcal{CP} : e^{i\xi} &= 1. \end{aligned} \quad (2.95)$$

However the relation $e^{i\xi} = 1$ depends on the phase convention. To see this let us define the phase rotation on Kaon wavefunctions as

$$\begin{aligned} |K^0> \longrightarrow |K^0>' &= e^{i\alpha}|K^0>, \\ |\overline{K^0}> \longrightarrow |\overline{K^0}>' &= e^{-i\alpha}|\overline{K^0}>. \end{aligned} \quad (2.96)$$

Under this rephasing of the Kaon fields the diagonal elements of any operator $\mathcal{O}$ remain unchanged whereas the off-diagonal elements pick up phases

$$\mathcal{O}_{12} \longrightarrow \mathcal{O'}_{12} = e^{-2i\alpha}\mathcal{O}_{12} \text{ and } \mathcal{O}_{21} \longrightarrow \mathcal{O'}_{21} = e^{2i\alpha}\mathcal{O}_{21}, \quad (2.97)$$

and, hence

$$\xi \longrightarrow \xi' = \xi + 2\alpha. \quad (2.98)$$

thus the basis independent condition for $\mathcal{CP}$ invariance is that ξ be real. There exists a phase convention dependant parameter ϵ that is often used as a measure of $\mathcal{CP}$ violation as follows :

$$\epsilon \equiv \frac{1 - e^{i\xi}}{1 + e^{i\xi}}. \quad (2.99)$$

Next we consider the decay of neutral Kaons to 2π mode. Bose-Einstein statistics demands that the 2π state be in either I=0 or I=2 where I denotes the total isospin. Parametrising the $K^0 \rightarrow 2\pi$ amplitudes as

$$< n|H_{wk}|\overline{K^0} >= \overline{a_n}e^{i\delta_n}; \; n = 0, 2, \quad (2.100)$$

where $|n >= |2\pi; I = n >$ and δ_n is the 2π s-wave phase shift in the I=n channel, we found out that $\mathcal{CPT}$ invariance implies $\overline{a_n} = -a_n^*$ whereas $\mathcal{CP}$ invariance demands that the ratio a_2/a_0 be real. Under the phase rotation

$$a_n \longrightarrow a'_n = a_n e^{i\alpha} \quad (2.101)$$

and hence the following combinations are independent of phase choice convention :

$$\begin{aligned} \epsilon_0 &\equiv \frac{< 0|H_{wk}|K_L >}{< 0|H_{wk}|K_S >} = \frac{a_0 - a_0^* e^{i\xi}}{a_0 + a_0^* e^{i\xi}} \\ \epsilon_2 &\equiv \frac{1}{\sqrt{2}}\frac{< 2|H_{wk}|K_L >}{< 0|H_{wk}|K_S >} = \frac{1}{\sqrt{2}}\frac{a_2 - a_2^* e^{i\xi}}{a_0 + a_0^* e^{i\xi}} e^{i(\delta_2 - \delta_0)} \\ \omega &\equiv \frac{< 2|H_{wk}|K_S >}{< 0|H_{wk}|K_S >} = \frac{a_2 - a_2^* e^{i\xi}}{a_0 + a_0^* e^{i\xi}} \end{aligned} \quad (2.102)$$

For the neutral Kaons system the $\mathcal{CP}$ violating quantities, which are directly related to physical observables, are

$$\begin{aligned} \eta^{+-} &\equiv \frac{<\pi^+\pi^-|H_{wk}|K_L>}{<\pi^+\pi^-|H_{wk}|K_S>} = \frac{\epsilon_0+\epsilon_2}{1+\omega/\sqrt{2}} = \epsilon_0 + \frac{\epsilon'}{1+\omega/\sqrt{2}} \\ \eta^{00} &\equiv \frac{<\pi^0\pi^0|H_{wk}|K_L>}{<\pi^0\pi^0|H_{wk}|K_S>} = \frac{\epsilon_0-2\epsilon_2}{1-\sqrt{2}\omega} = \epsilon_0 - \frac{2\epsilon'}{1-\sqrt{2}\omega}. \end{aligned} \tag{2.103}$$

where

$$\epsilon' \equiv \epsilon_2 - \frac{\omega\epsilon_0}{\sqrt{2}}. \tag{2.104}$$

In terms of the matrix elements of M and Γ then

$$\begin{aligned} \epsilon_0 &= i\frac{Im(M_{12}a_0^2) - iIm(\Gamma_{12}a_0^2)}{Re(a_0^2M_{12}) - \frac{i}{2}Re(a_0^2\Gamma_{12}) + \frac{|a_0^2|}{2}(\Delta m - \frac{i}{2}\Delta\gamma)} \\ \epsilon' &= \frac{i}{\sqrt{2}}\frac{Im(a_2a_0^*)(\Delta m - \frac{i}{2}\Delta\gamma)e^{i(\delta_2-\delta_0)}}{Re(a_0^2M_{12}) - \frac{i}{2}Re(a_0^2\Gamma_{12}) + \frac{|a_0^2|}{2}(\Delta m - \frac{i}{2}\Delta\gamma)}. \end{aligned} \tag{2.105}$$

Now we will simplify these general expessions to the special case of neutral Kaons and use some experimental results to obtain the approximate but easy to handle expressions. Experimentally we have

$$\begin{aligned} m_K &= 0.498\ GeV \\ \Delta m_K &= 3.5\times10^{-15}\ GeV \\ \Delta\gamma_K &\approx -\gamma_{K_S} = -7.3\times10^{-15}\ GeV \\ |\eta^{+-}| &= (2.275\pm0.021)\times10^{-3} \\ |\eta^{00}| &= (2.299\pm0.036)\times10^{-3}. \end{aligned} \tag{2.106}$$

The dominant contribution to Γ_{12} comes from the 2π states and more specifically the I=0 state. Thus

$$\Gamma_{12} \sim <K^0|H_{wk}^{\Delta S=1}|0><0|H_{wk}^{\Delta S=1}|\overline{K^0}>, \tag{2.107}$$

and hence

$$\frac{Im\Gamma_{12}}{Re\Gamma_{12}} \approx \frac{Im(a_0^*)^2}{Re(a_0^*)^2} \tag{2.108}$$

The $\Delta I = \frac{1}{2}$ rule for neutral Kaon decays manifests itself through a small suppression factor

$$\omega \approx 0.045 \tag{2.109}$$

Using the value of ω along with the experimental numbers for η^{+-} and η^{00} we get

$$|\epsilon_0| = 2.3\times10^{-3}, \tag{2.110}$$

and the phase of ϵ_0 is approximately $\pi/2$. this small value of $|\epsilon_0|$ gives the inequalities

$$\begin{aligned} ImM_{12}/ReM_{12} &\ll 1, \\ Im\Gamma_{12}/Re\Gamma_{12} &\ll 1. \end{aligned} \tag{2.111}$$

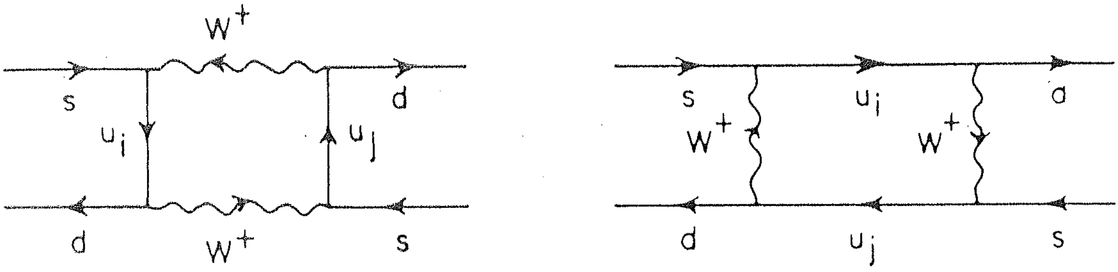

Figure 2.1: "Box"-diagram generating $K_0 - \overline{K^0}$ mixing and ϵ_K in the SM

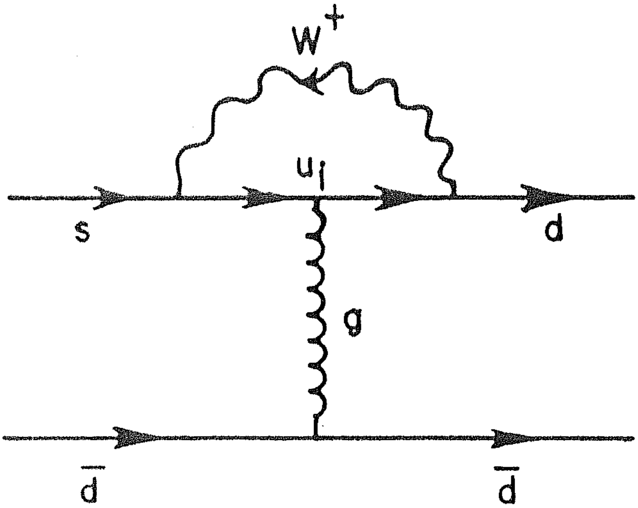

Figure 2.2: "Penguin"-diagram responsible for ϵ'_K in the SM

Consequently, the mass and width differences in the above approximations are given as

$$\Delta m_K \approx 2ReM_{12}, \quad \Delta\gamma_K \approx 2Re\Gamma_{12}, \tag{2.112}$$

and

$$|\epsilon_0| = \frac{1}{\sqrt{2}}\frac{ImM_{12}}{\Delta M_K} \approx \frac{1}{2\sqrt{2}}\frac{ImM_{12}}{ReM_{12}}. \tag{2.113}$$

In the 3-generation SM, which, for a complex CKM matrix, is a milliweak theory, $K_0 - \overline{K^0}$ mixing and $K_L \to 2\pi$ come about because of the 1-loop Feynman diagrams in Figures (2.1) and (2.2) respectively, giving rise to

$$ImM_{12} = \frac{G_F^2}{12\pi^2} f_K^2 m_K m_W^2 B_K \left[\lambda_c^2 \eta_1 S(y_c) + \lambda_t^2 \eta_2 S(y_t) + \lambda_c \lambda_t \eta_3 S(y_c, y_t)\right], \tag{2.114}$$

and

$$\tan\theta_0 = \frac{s_{13}s_{23}}{s_{12}} \sin\delta \left[\frac{150MeV}{m_s(1\ GeV)}\right]^2 \bar{H}, \tag{2.115}$$

where

$$\begin{array}{llllll} \lambda_i & \equiv & K_{id}^* K_{is} & y_i & \equiv & m_i^2/m_W^2 \\ f_K & = & 0.16\, GeV & m_W & = & 81.8\, GeV. \end{array} \tag{2.116}$$

Whereas f_K is the pion decay constant, the bag parameter B_K reflects our ignorance of the hadronic matrix elements. If vacuum saturation approximation were correct then one would have $B_K = 1$, but theoretical estimates only put the rather loose bound of $1/3 \leq B_K \leq 1$. The functions $S(x)$ and $S(x,y)$ arise from the loop integral and are given by

$$\begin{aligned} S(x) &= x\left[\frac{1}{4} + \frac{9}{4(1-x)} - \frac{3}{2(1-x)^2}\right] + \frac{3}{2}\left[\frac{x}{x-1}\right]^3 \ln x \\ S(x,y) &= xy\left[\left\{\frac{1}{4} + \frac{3}{4(1-x)} - \frac{3}{4(1-x)^2}\right\}\frac{\ln x}{x-y} - \frac{3}{8}\frac{1}{1-x}\frac{1}{1-y}\right] + (x \leftrightarrow y) \end{aligned}$$

The quantities η_i represent QCD corrections [25]. While η_1 does not depend on m_t and is evaluated to be 0.85, η_2 is essentially independent of m_t for $40\, GeV \leq m_t^{\rm phys} \leq 130\, GeV$ and $\eta_2 = 0.61$. η_3 and $\bar{H}$ are slowly varying functions of m_t and are approximately 0.25 and 0.37 respectively [26]. However we shall allow for their full variation in our calculations.

2.6.2 The Neutral Beauty meson system

Although $\mathcal{CP}$ violation has been observed so far only in neutral Kaon decays, one would expect to have non-zero effects in other processes onvolving heavy neutral mesons like $B^0 - \overline{B^0}$ and $D^0 - \overline{D^0}$ if one believes the KM mechanism for $\mathcal{CP}$ violation is correct. The phenomenology of the $B^0 - \overline{B^0}$ systems is quite similar to that of $K^0 - \overline{K^0}$. The physical situation, however, is very different since B^0 involves the bound states of a heavy quark a light quark and there are many intermediate states and the multi-particle final states dominate the decay as the case in deep inelastic scattering.

A new property which makes the B system very interesting is the recent observation of $B_d - \overline{B_d}$ mixing by the ARGUS collaboration. Their result fully justifies the expectation that the studies regarding neutral beauty mesons can reveal new phenomena and motivates the serious consideration of $\mathcal{CP}$ assymetries in this system. To study this particle-antiparticle mixing consider the time-integrated mixing parameters proposed by Pais and Treiman [6.4 p]

$$\begin{aligned} r_d &\equiv \frac{\int_0^\infty |<\overline{B_d}|B_d>|^2 dt}{\int_0^\infty |<B_d|B_d>|^2 dt} = |e^{i\xi_{B_d}}|^2 \frac{(\Delta m_B)^2 + (\Delta\Gamma_B)^2/4}{2\Gamma_B^2 + (\Delta m_B)^2 + (\Delta\Gamma_B)^2/4} \\ \overline{r_d} &\equiv \frac{\int_0^\infty |<B_d|\overline{B_d}>|^2 dt}{\int_0^\infty |<\overline{B_d}|\overline{B_d}>|^2 dt} = |e^{i\xi_{\overline{B_d}}}|^2 \frac{(\Delta m_{\overline{B_d}})^2 + (\Delta\Gamma_{\overline{B_d}})^2/4}{2\Gamma_{\overline{B_d}}^2 + (\Delta m_{\overline{B_d}})^2 + (\Delta\Gamma_{\overline{B_d}})^2/4}. \end{aligned} \tag{2.117}$$

If $\mathcal{CP}$ is violated we expect r to differ from $\overline{r}$ by a quantity proportional to

$$|e^{i\xi_{\overline{B_d}}}|^2 - |e^{i\xi_{B_d}}|^2 \approx 8 Re\,\epsilon_B, \quad |\epsilon_B| \ll 1, \tag{2.118}$$

otherwise $r = \overline{r}$. The above considerations are relevant for reactions where only one B^0 or $\overline{B_0}$ meson is produced. However, often in the actual experimental situation a pair of B^0 and $\overline{B_0}$ is produced instead of a single B^0 or $\overline{B_0}$. As the beam evolves in time, both of them oscillate in their B^0 and $\overline{B_0}$ content and one cannot directly measure either r_d or $\overline{r_d}$.

Okun, Zakharov and Pontecorvo proposed the observations of following two parameters in dilepton decay mode, characterising particle-antiparticle mixing

$$R_d = \frac{N^{++} + N^{--}}{N^{+-} + N^{-+}} \tag{2.119}$$

and the $\mathcal{CP}$ violating leptonic charge assymetry

$$A_d = \frac{N^{++} - N^{--}}{N^{++} + N^{+-} + N^{-+} + N^{--}} \tag{2.120}$$

where N's denote the number of dilepton pairs with the associated charges. For example, in the process

$$e^+e^- \longrightarrow \Upsilon(4S) \longrightarrow B_d^0\overline{B_d} \tag{2.121}$$

these relations reduce to

$$R_d = \frac{1}{2}(r_d + \overline{r_d}), \quad A_d = \frac{r_d - \overline{r_d}}{2 + r_d + \overline{r_d}} \tag{2.122}$$

Neglecting the possibility of large $\mathcal{CP}$ violation and introducing the approximation $\Delta\Gamma/\Delta m \approx 0$ we have

$$r_d = \frac{(\Delta m_d/\Gamma_d)^2}{2 + (\Delta m_d/\Gamma_d)^2} \tag{2.123}$$

If one assumes that for 3 generations SM with a relatively heavy top quark, the dominant contribution to r_d comes from the corresponding box-diagram with the top flowing in it then

$$x_d = (\Delta m_d/\Gamma_d) = \tau_b \frac{G_F^2}{6\pi^2}\eta M_B \left(B_B f_B^2\right) M_W^2 y_t f_2(y_t) |V_{tb}V_{td}^*|^2 \tag{2.124}$$

where τ_B is the B_d^0 lifetime, f_B the decay constant, B_B the bag factor and η a QCD correction factor.

The ARGUS result permits the range

$$\begin{aligned} \Delta m_B &= (4.2 \pm 0.9) \times 10^{-13}\text{GeV} \\ \text{or } x_d &= 0.73 \pm 0.18 \\ \text{and } m_B &= 5.28GeV, \quad B_B f_b^2 = (0.15 \pm 0.05GeV)^2 \\ \eta &= 0.85 \quad \tau_B = (1.16 \pm 0.16) \times 10^{-12}s \end{aligned} \tag{2.125}$$

Chapter 3

Symmetric Quark Mixing & Some Consequences

3.1 Symmetric Ansatz for quark mixing

3.1.1 CKM matrix with symmetric moduli

Apart from the masses, the other existing free parameters in the standard model are the three mixing angles and a CP-violating phase, which are incorporated into the quark sector of the standard model via the Cabibbo-Kobayashi-Maskawa (CKM) matrix V. All the presently available data [21] is consistent with having symmetric moduli for CKM matrix i.e.

$$|V_{ij}| = |V_{ji}|. \tag{3.1}$$

It should be noted that for three generations, the assumption that V has symmetric moduli implies a single constraint on the matrix V because the unitarity requirement alone yields

$$A = |V_{12}|^2 - |V_{21}|^2 = |V_{31}|^2 - |V_{13}|^2 = |V_{23}|^2 - |V_{32}|^2 \tag{3.2}$$

for three generations. The fact that experimentally the asymmetry parameter A is, in general, small i.e. $A < 10^{-4}$ and in particular $|V_{12}|$ and $|V_{21}|$ are quoted to be same modulo the errors and both of them lie between 0.217 and 0.223 prompted us to believe that V has symmetric moduli.

It is well known that the individual phases of V_{ij} is devoid of any physical meaning, since under rephasing of the up and down quark fields the non-physical individual phases γ_j and β_i of V_{ij} transform as:

$$V_{ij} \rightarrow (V')_{ij} = V_{ij} exp(\gamma_j - \beta_i). \tag{3.3}$$

Now, we consider the question whether starting with symmetric moduli one may use the rephasing freedom of the CKM matrix to obtain also symmetric phases. In other words, whether starting from an arbitrary V, it is possible to achieve $arg(V')_{ij} = arg(V')_{ji}$ by an appropriate choice of γ_j, β_i. It was shown by Branco and Parada that in general this is not possible for arbitrary V, but it is possible for a three-generation CKM matrix with symmetric moduli. In fact, to achieve $arg(V')_{ij} = arg(V')_{ji}$, the following relations has to be satisfied

$$arg(V')_{ij} - arg(V')_{ji} = \gamma_i - \gamma_j + \beta_i - \beta_j + 2n\pi. \tag{3.4}$$

In order for the above equation to yield a solution for γ_j, β_i, the imaginary part of a rephasing invariant sextet consisting of the off-diagonal matrix elements of V, namely

$$Im(V_{12}V_{23}V_{31}V_{21}{}^*V_{13}{}^*V_{32}{}^*) = 0. \tag{3.5}$$

The above equation is a necessary condition to have symmetric phase of V for any number of generations ($N \geq 3$). Obviously, for $N > 3$ there are other conditions, analogous to the above equation, which need also be satisfied in order to obtain symmetric phases. For three generations, symmetric moduli of the CKM matrix lead, through unitarity, to the above condition. To see this, consider the orthogonality conditions for the first two rows and first two columns of the CKM martix :

$$\begin{aligned} V_{11}V_{21}^* + V_{12}V_{22}^* + V_{13}V_{23}^* &= 0, \\ V_{11}V_{12}^* + V_{21}V_{22}^* + V_{31}V_{32}^* &= 0. \end{aligned} \tag{3.6}$$

If one multiplies the first equation by V_{21} and the second by V_{21}, and assumes $|V_{ij}| = |V_{ji}|$, then one obtains, by subtracting the resulting equations,

$$V_{13}V_{23}^*V_{21} - V_{31}V_{32}^*V_{12} = 0, \tag{3.7}$$

which in turn implies the vanishing of the imaginary part of a rephasing invariant sextet consisting of the off-diagonal matrix elements of V. It can be readily verified that for more than three generations, symmetric moduli of V do not imply symmetric phases through unitarity. For example, for four generations, even if one has exact knowledge of the moduli of V, with $|V_{ij}| = |V_{ji}|$, this would not imply a symmetric V.

3.1.2 Generalised two-angle parametrization

In general, four independent parameters are required to characterize the CKM matrix for three generations. But, assuming V to be symmetric implies a single constraint and, as a result, one needs three parameters to characterize the most general symmetric CKM matrix for three generations. A symmetric form for the CKM matrix, with two parameters was first proposed by Kielanowski and was generalised later by Blundell, Mann and Sarkar. It was also pointed out by Blundell et. al and Branco and Parada that Kielanowski had implicitly assumed a restriction on the free parameters of a symmetric CKM matrix.

Now we consider the generalised two-angle parametrization of the CKM matrix. The rephasing freedom of the quark fields implies that two CKM matrices V and V' are physically equivalent provided

$$V = U_1 U V' \dagger U_1, \tag{3.8}$$

where $U = diag(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ and $U_1 = diag(1, e^{i\psi_1}, e^{i\psi_2})$. Let λ_i and ω_i denote the eigenvalues and the eigenvectors of KM matrix V. The eigenvectors of V may be constructed in terms of three angles $(\beta_1, \beta_2, \beta_3)$ and one phase α. The eigenvalues of V satisfy its characteristic equation

$$\lambda^3 - k_1\lambda^2 + k_2\lambda - k_3 = 0 \tag{3.9}$$

where $k_1 = trV, k_2 = \frac{1}{2}[(trV)^2 - tr(V^2)]$, and $k_3 = detV$. The unitarity of V gives $k_2 = k_1^* k_3$. Let $trV = xe^{i\phi/3}$, a general complex number with real parameters and $detV = e^{i\phi}$, a phase. Then the characteristic equation becomes

$$\lambda^3 - xe^{i\phi/3}\lambda^2 + xe^{i2\phi/3}\lambda - e^{i\phi} = 0 \tag{3.10}$$

whose solutions are

$$\begin{aligned}
\lambda_1 &= \frac{1}{2}e^{i\phi/3}(x - 1 - i\sqrt{3 + 2x - x^2}) \\
\lambda_2 &= \frac{1}{2}e^{i\phi/3}(x - 1 + i\sqrt{3 + 2x - x^2}) \\
\lambda_3 &= e^{i\phi/3}
\end{aligned} \tag{3.11}$$

for $-1 \le x \le 3$. Unitarity of V implies that the only relevant range of x is $-1 \le x \le 3$. Note that the factor $e^{i\phi/3}$ will vanish in the magnitudes of KM matrix elements, and in the rephasing invariant plaquette J, so the observables are independent of $detV$. Since $J = 0$ for $x \le -1$ and $x \ge 3$ we will examine the range $-1 < x < 3$.

The CKM matrix has three orthonormal complex eigenvectors. The normalised eigenvectors are determined upto a phase. Thus we can choose one nonvanishing component of each vector to be real. The two remaining arbitrary phases can be chosen in such a way that one eigenvector is real. We use the following parametrization of the three eigenvectors of KM matrix with the above properties:

$$w_1 = \begin{pmatrix} c_1 \\ s_1c_2 \\ s_1s_2 \end{pmatrix}, \; w_2 = \begin{pmatrix} -s_1c_3 \\ c_1c_2c_3 - s_2s_3e^{i\alpha} \\ c_1s_2c_3 + c_2s_3e^{i\alpha} \end{pmatrix}, \; w_3 = \begin{pmatrix} s_1s_3 \\ -c_1c_2s_3 - s_2c_3e^{i\alpha} \\ -c_1s_2s_3 + c_2c_3e^{i\alpha} \end{pmatrix} \tag{3.12}$$

where $c_i \equiv \cos(\beta_i)$ and $s_i \equiv \sin(\beta_i)$ and the KM matrix V is written as

$$V = \sum_{i=1}^{3} \lambda_i \omega_i \otimes \omega_i^\dagger \tag{3.13}$$

In the case where $\beta_3 = 0$ and α drops out, we write the magnitudes of the elements of symmetric KM matrix in terms of three parameters *i.e.* x and two angles β_1, β_2 as:

$$\begin{aligned}
|V_{11}| &= \sqrt{1 - \frac{1}{4}sin^2(2\beta_1)(3 + 2x - x^2)} \\
|V_{12}| &= \frac{1}{2}sin(2\beta_1)cos(\beta_2)\sqrt{(3 + 2x - x^2)} \\
|V_{13}| &= \frac{1}{2}sin(2\beta_1)sin(\beta_2)\sqrt{(3 + 2x - x^2)}
\end{aligned}$$

$$
\begin{aligned}
|V_{22}| &= \sqrt{1-\frac{1}{4}sin^2(2\beta_2)[3-x]-\frac{1}{4}sin^2(2\beta_1)cos^4(\beta_2)[3+2x-x^2]} \\
|V_{23}| &= \frac{1}{2}sin(2\beta_2)\sqrt{[3-x]-\frac{1}{4}sin^2(2\beta_1)[3+2x-x^2]} \\
|V_{33}| &= \sqrt{1-\frac{1}{4}sin^2(2\beta_2)[3-x]-\frac{1}{4}sin^2(2\beta_1)sin^4(\beta_2)[3+2x-x^2]}
\end{aligned}
\tag{3.14}
$$

3.1.3 Restrictions on the eigenstates and eigenvalues

Since the CKM matrix is unitary, it can be diagonalised by a unitary transformation

$$V = WKW^{-1}; \quad K = diag(e^{i\sigma_1}, e^{i\sigma_2}, e^{i\sigma_3}) \tag{3.15}$$

where $exp(i\sigma_i)$ are the eigenvalues of the CKM matrix, corresponding to the eigenstates with components $w_{ij}(j = 1,2,3)$. The asymmetry parameter A can be expressed in terms of the eigenvalues of V and the combinations of the elements of the matrix W as follows:

$$A = -4I[\sin(\sigma_1-\sigma_2)+\sin(\sigma_3-\sigma_1)+\sin(\sigma_2-\sigma_3)], \tag{3.16}$$

where $I = Im(W_{11}W_{22}W_{12}^*W_{21}^*)$. From the above expression for V it is obvious that the reality of W is sufficient in order to have a symmetric V. It was shown[21] that the CKM matrix is symmetric if and only if the matrix W is real, apart from irrelevant overall phases for each one of its columns. We have also reached the same conclusion (see the subsection 3.2.2). One can easily verify that if two of the eigenvalues are degenerate, then $|V|$ is necessarily symmetric and the eigenvectors can be chosen to be real. Note that the asymmetry parameter A vanishes when two of the eigenvalues are degenerate and / or when the matrix W is effectively real (i.e. $I = 0$). The fact that experimentally A is small provides an indication that two of the eigenvalues of V are close to beiing degenerate and / or W is close to be 'effectively' real i.e. $I << 1$.

3.2 Consequences of Symmetric quark mixing

3.2.1 Top Quark Mass and a Symmetric CKM matrix

We pursued[1] the investigation of symmetric quark mixing (*ie* a symmetric CKM matrix) in conjunction with CP-violation in the neutral kaon-system and the extent of the B_d^0–$\overline{B}_d^0$mixings to find out what constraints it put on parameters like m_t etc. of SM. We used the standard parametrization[12, 13] for CKM matrix described in the introduction.

The relation $|V_{12}| = |V_{21}|$ obviously restricts one to a three dimensional hypersurface in the parameter space spanned by s_{12}, s_{23}, $q = |V_{13}|/|V_{23}|$ and δ. While $J = Im(V_{11}V_{22}V_{12}^*V_{21}^*)$, the rephasing invariant measure of CP-violation, does vary with s_{23}, q and δ do not show any such variations, as their dependence on θ_{23} is very weak. Taking s_{12} and s_{23} as phenomenological

[1]This section is based on the work reported in ref.[22]

inputs from (2.69) and (2.74) leaves us with a a curve in the q–δ plane for fixed values of s_{12} and s_{23}. For the situation described in ref. [23] the curve shrinks to a point. By determining whether this curve lies within the region in the q–δ plane allowed by the ϵ_K and B–$\overline{B}$ mixing data we are therefore able to obtain limits on the mass m_t of the t-quark as a consequence of the symmetric CKM ansatz since these latter quantities depend upon m_t.

The K^0–$\overline{K}^0$ system indirect CP-violating measure ϵ_K in the CKM picture is expressed as [24]

$$|\epsilon_K| = C \cdot B_K \cdot s_{23}^2 q \sin(\delta) \left[(\eta_3 f_3(y_t) - \eta_1)\, y_c s_{12} + \eta_2 y_t f_2(y_t) s_{23}^2 \,(s_{12} - q\cos(\delta)) \right] \qquad (3.17)$$

where

$$\begin{aligned} C &\equiv \frac{(G_F f_K M_W)^2 M_K}{6\pi^2 \sqrt{2}(\Delta M_K)} \\ f_2(y_t) &= 1 - \frac{3}{4}\frac{y_t(1+y_t)}{(1-y_t)^2}\left[1 + \frac{2y_t}{1-y_t^2}\ln(y_t)\right] \\ f_3(y_t) &= \ln\left(\frac{y_t}{y_c}\right) - \frac{3}{4}\frac{y_t}{1-y_t}\left[1 + \frac{y_t}{1-y_t}\ln(y_t)\right] \end{aligned} \qquad (3.18)$$

and $y_i \equiv m_i^2/M_W^2$ $(i = c, t)$. The parameters η_i are QCD corrections [25]

$$\eta_1 = 0.7 \ , \qquad \eta_2 = 0.6 \ , \qquad \eta_3 = 0.4 \ . \qquad (3.19)$$

The experimental result $|\epsilon_K| = 2.3 \times 10^{-3}$ gives a parabola in the q–δ plane for given B_K, s_{23} and m_t. The Bag factor B_K is very poorly determined and various theoretical estimates only find the bounds $1/3 \leq B_K \leq 1$. The expression for the B_d^0–$\overline{B}_d^0$ mixing parameters $x_d = \Delta M/\Gamma$ is on the other hand,

$$x_d = \tau_b \frac{G_F^2}{6\pi^2} \eta M_B \left(B_B f_B^2\right) M_W^2 y_t f_2(y_t) |V_{tb} V_{td}^*|^2 \qquad (3.20)$$

where $M_B = 5.28 GeV$, $B_B f_b^2 = (0.15 \pm 0.05 GeV)^2$ and the QCD correction $\eta = 0.85$. Experimentally $|V_{tb}| \approx 1$ to a high degree of accuracy and

$$|V_{td}|^2 = s_{23}^2 (s_{12}^2 + q^2 - 2 s_{12} q \cos\delta) \quad . \qquad (3.21)$$

The ARGUS result[27]

$$x_d = 0.73 \pm 0.18 \qquad (3.22)$$

thus gives another curve in the q–δ plane for given s_{23} and m_t.

It is straightforward to see that the symmetric ansatz implies a strong lower bound on m_t. Eq. (3.20) shows that $x_d \approx m_t^2 |V_{31}|^2$ which by the symmetric ansatz is $m_t^2 |V_{13}|^2$. However eqs. (2.74,2.75) impose a severe upper limit on $|V_{13}|$, in turn yielding a strong lower bound on m_t.

In our numerical analysis we hold B_K and m_t fixed and consider the total variation of all other parameters, taken in quadrature. Thus we get two interesting bands in the q–δ plane

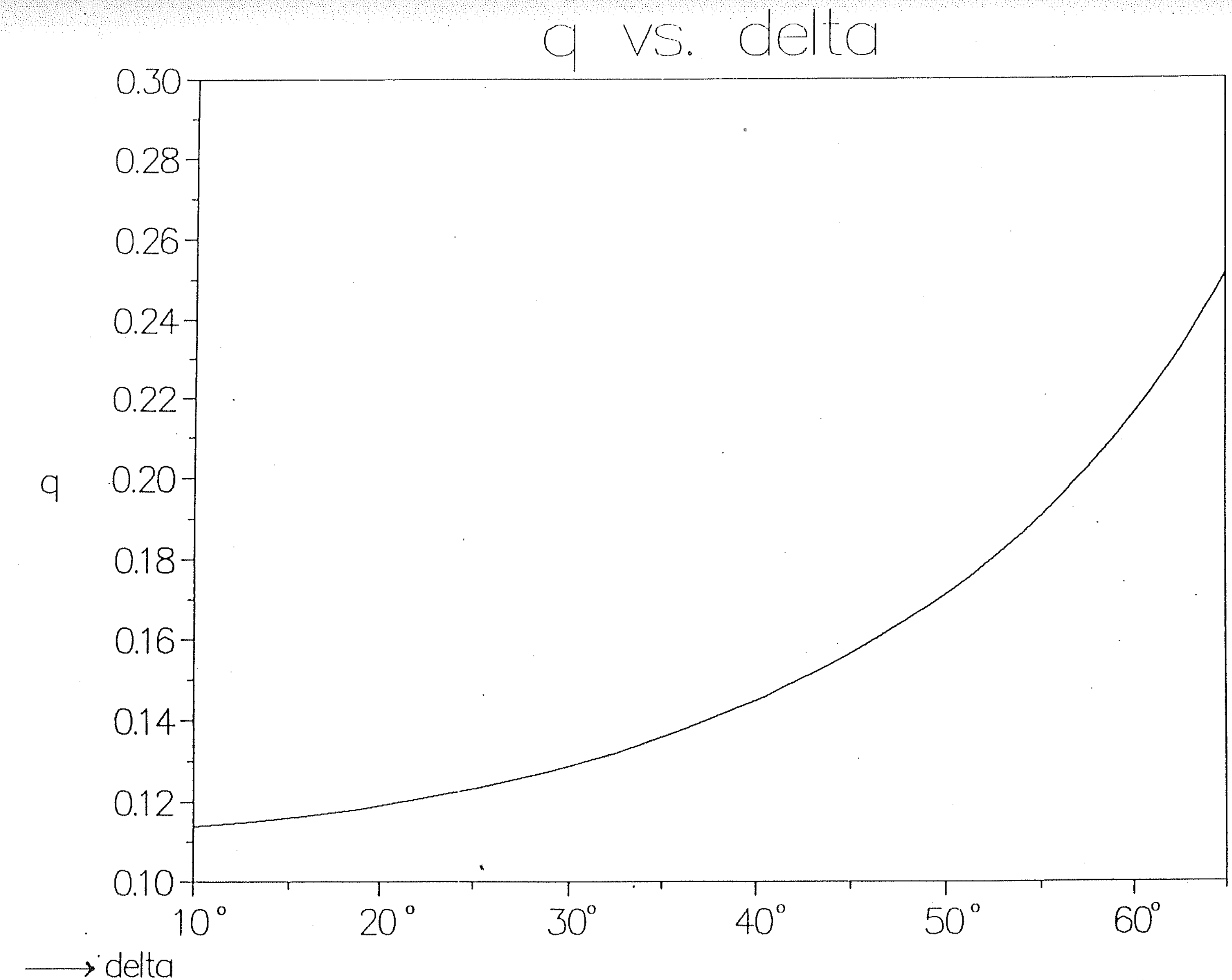

Figure 3.1: The symmetry curve for q vs. δ. Note that the existing data implies that $8.0° \leq \delta \leq 32.0°$

coming from ϵ_K and x_d. If this zone does contain the curve obtained from the symmetrical ansatz, then the assumptions are obviously valid for the given choice of m_t and B_K.

A plot of the curve in the q–δ plane for the symmetric ansatz (henceforth called the symmetric curve) is given in fig.(3.1)

We find a very narrow curve considering all the variations of s_{12} and s_{23}. We next superimpose on the symmetric curve in the q–δ plane curves parametrizing the regions allowed by the experiments with B–$\overline{B}$ mixing and the measurement of ϵ_K [28].

We find that when the top quark mass is lighter than 180 GeV, the symmetric curve does not intersect with the ARGUS measurement of x_d, implying that the top quark must be at least this heavy if the symmetric ansatz is correct. Imposing the K–$\overline{K}$ mixing result we find that for $B_K = 1/3$, the symmetric ansatz implies $m_t > 275$ GeV (although for $B_K = 2/3$ and 1 the lower limit of 180 GeV is unaltered). Alternatively, for given values of m_t, when the symmetric curve overlaps with the measurements of x_d and ϵ_K we find that the symmetric ansatz allows only a restricted range of values for q and δ, *i.e.*, the CP-violating phase is not completely arbitrary. The value of δ lies between 8° and 31°, while q is restricted to lie between .113 and .13. We have shown the allowed regions of q and δ for different m_t values in figs.(3.2, and 3.3) for two different values of B_K, namely, $B_K = 2/3$ and 1.

The experimental constraints imply that x must lie between -0.882 and .02. We also show the allowed regions of the parameter x for different values of m_t in fig.(3.4). From the allowed region of x for different m_t, we can immediately conclude that $x = 0$ is allowed for m_t about 185 GeV, in accord with an earlier result of Rosner [30].

We find that if the CKM matrix is symmetric then the top quark mass has to be heavier than 180 GeV, to be consistent with the experiments on B–$\overline{B}$ mixing and the measurement of ϵ_K; if the bag constant $B_K = 1/3$ then $m_t > 275$ GeV. The parameters q and δ are constrained to be in the range

$$.130 \geq q \geq .113 \qquad 8.0^\circ \leq \delta \leq 31.1^\circ \tag{3.23}$$

for the symmetric CKM matrix over the allowed range of the top quark mass.

3.2.2 Symmetric CKM matrix and Quark Mass matrices

The importance of studying the mass matrices lies in the fact that the structure of the quark and lepton mass matrices determines the flavour dynamics of the standard electroweak theory. However, the elements of these matrices cannot be predicted within the standard model as quark and lepton masses are the free parameters within the model. Furthermore, there exits an infinite number of mass matrices, related to each other by unitarity transformations, which yield the same physics. We have tried[2] to find out the constraints imposed on the form of the mass matrices due to the symmetric CKM matrix. In the basis, where the up-quark fields are mass

[2] This section is based on the work reported in ref.[29]

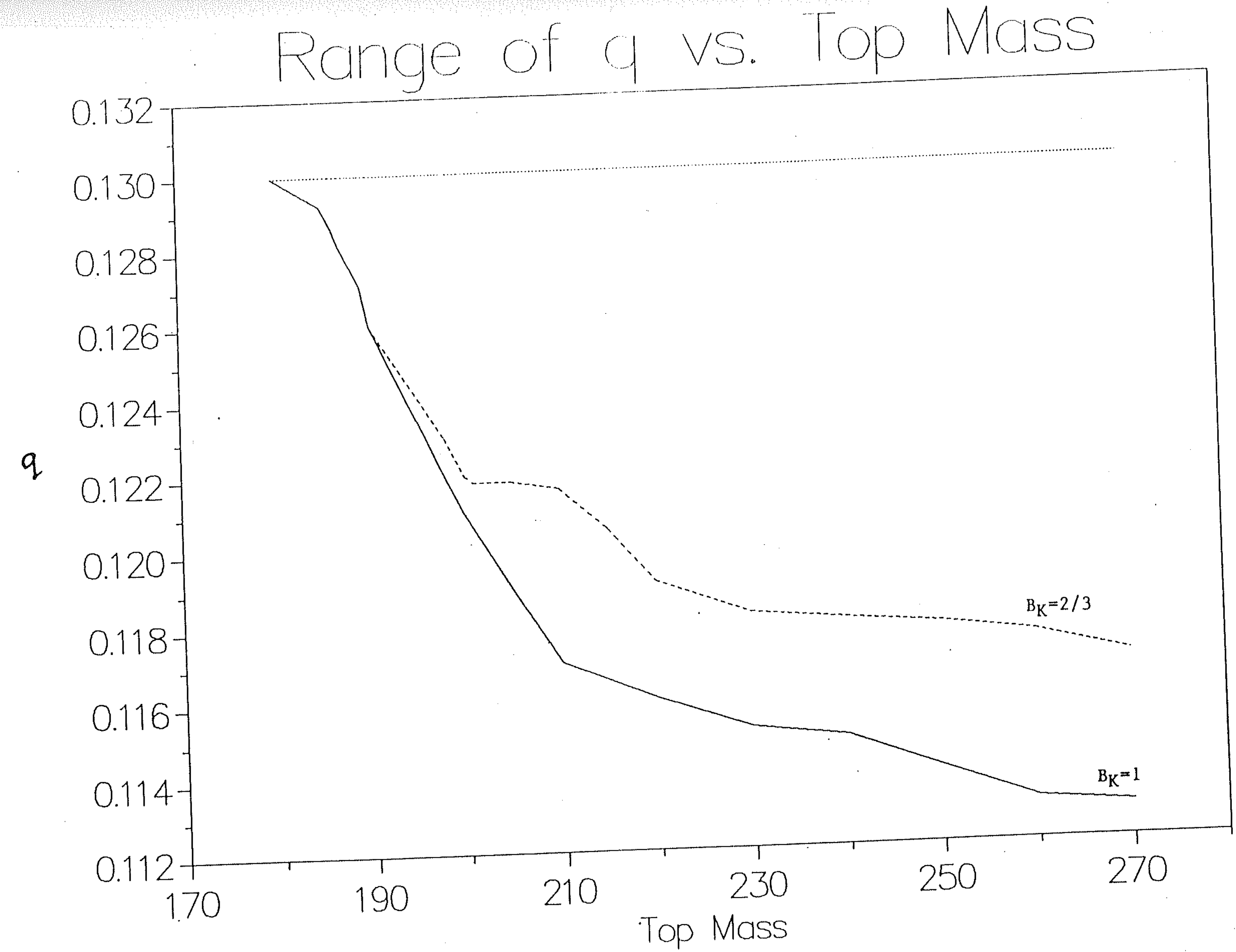

Figure 3.2: Allowed region of q as a function of m_t. The dotted line is the upper limit on q, valid for all B_K.

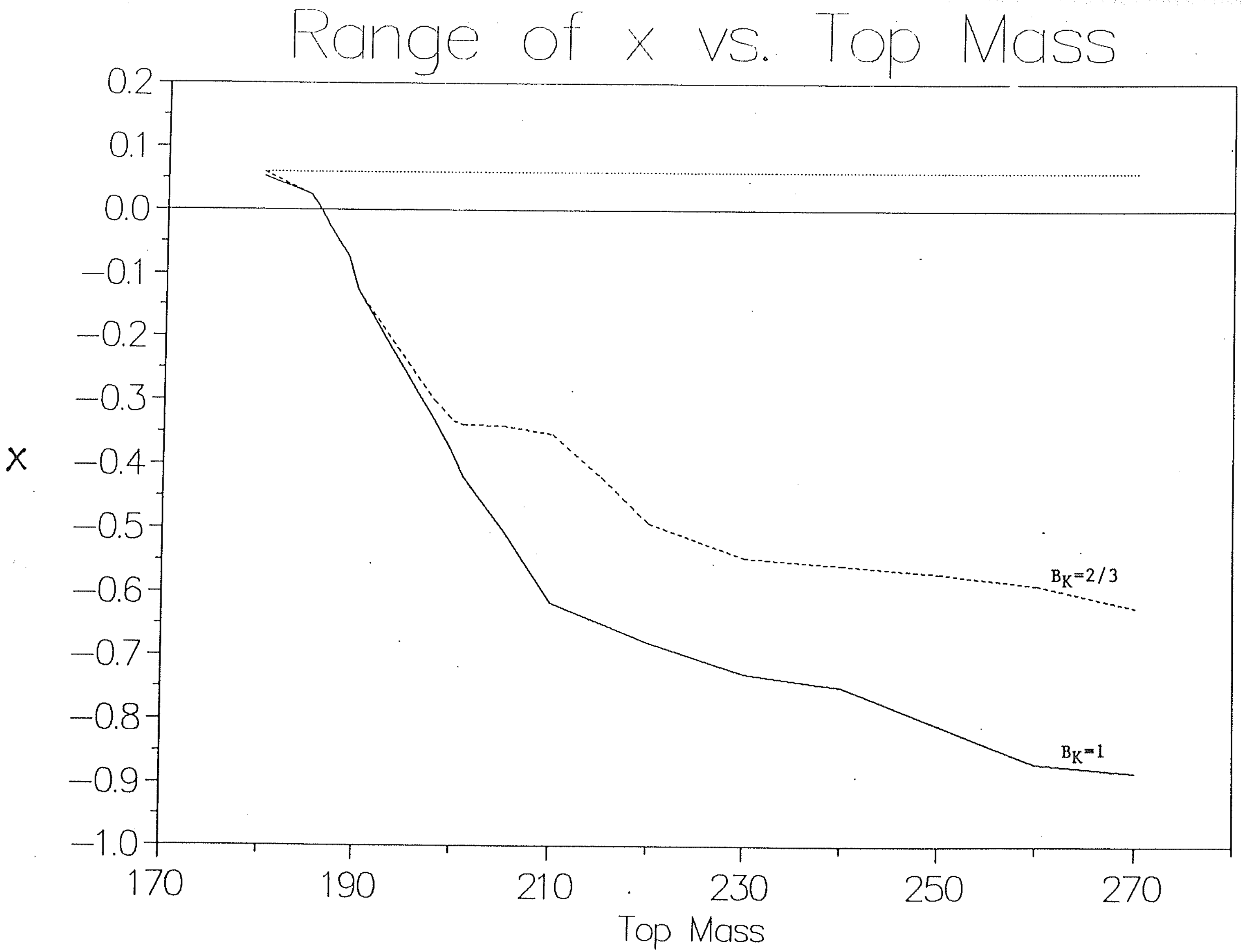

Figure 3.3: Allowed region of δ as a function of m_t. The dotted line is the upper limit on δ, valid for all B_K.

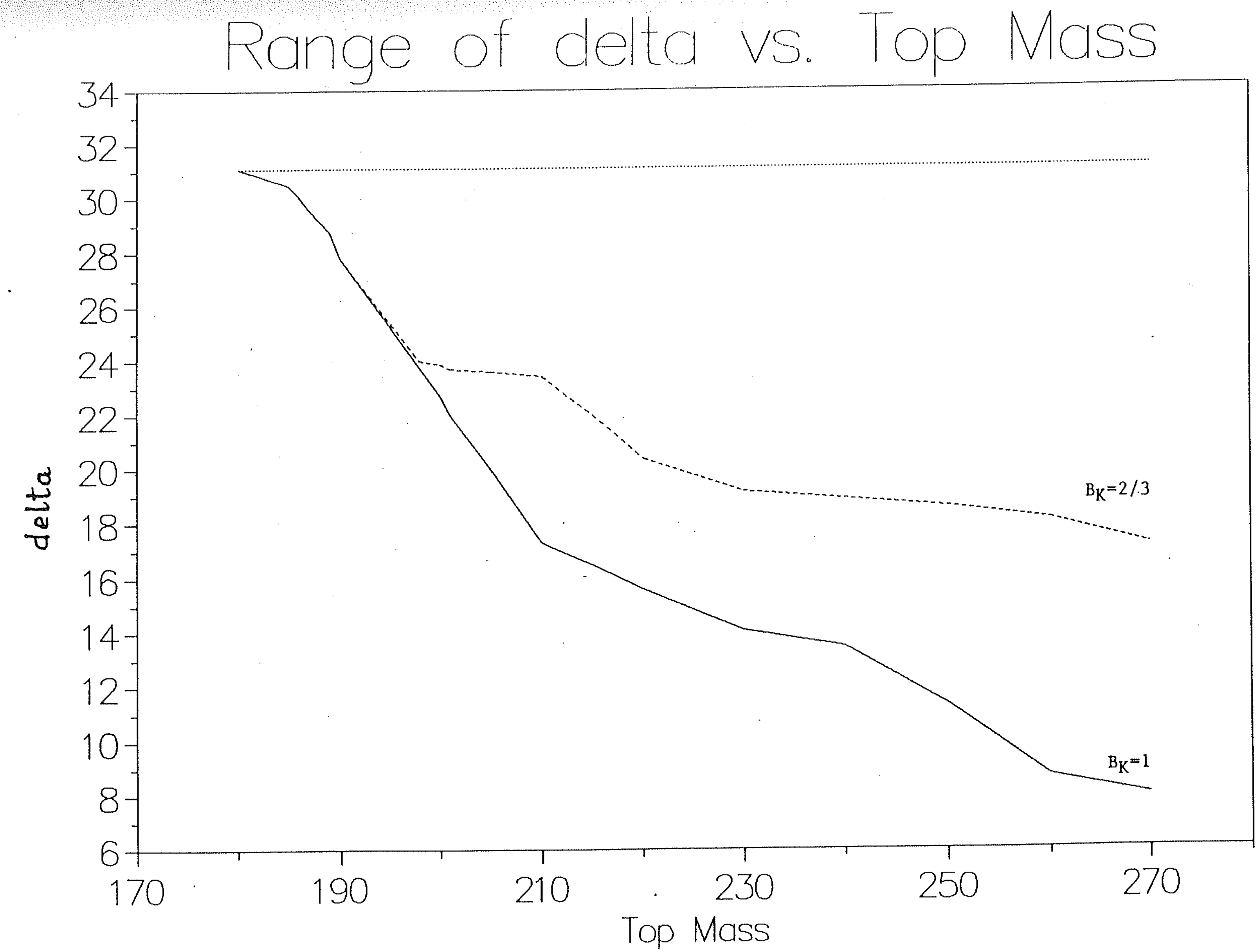

Figure 3.4: Allowed region of x as a function of m_t. The dotted line is the upper limit on x, valid for all B_K.

eigenstates, $M^{(u)}$ is diagonal i.e.

$$\mathcal{M}_u = diag(m_u, m_c, m_t) \tag{3.24}$$

In general the matrix $M^{(d)}$ is not hermitian, but we assume $M^{(d)}$ to be hermitian and write the most general hermitian $M^{(d)}$ is given by

$$M^{(d)} = h\mathcal{M}_u + A, \tag{3.25}$$

where

$$A = \begin{pmatrix} 0 & R_1 e^{i\rho_1} & R_2 e^{i\rho_2} \\ R_1 e^{-i\rho_1} & f & R_3 e^{i\rho_3} \\ R_2 e^{-i\rho_2} & R_3 e^{-i\rho_3} & d \end{pmatrix}. \tag{3.26}$$

Thus the mass matrices are a ten parameter family determined by m_u, m_c, $m_t, h, f, d, R_{1,2,3}$ and the invariant phase $(\rho_1 + \rho_3 - \rho_2)$ [32]. Taking the trace of both the sides of equation, we obtain the constant h in terms of parameters of mass matrices as

$$h = \frac{(m_d + m_s + m_b) - f - d}{(m_u + m_c + m_t)}. \tag{3.27}$$

Since the identity of the quarks is defined in the basis where the mass matrix is diagonal, the flavour projection operators[20], denoted by P_α and P'_j $(\alpha, j = 1, 2, ..., n)$ are introduced to keep track of the identity of quarks in any arbitrary basis, where the mass matrices are arbitrary, by projecting out the appropriate flavour. They are given by

$$\begin{aligned} P_\alpha(S) &= v_\alpha(S)/v, \\ P'_j(S') &= v'_j(S')/v', \end{aligned} \tag{3.28}$$

where the hermitian matrices $S(= M^{(u)}M^{(u)\dagger})$ and $S'(= M^{(d)}M^{(d)\dagger})$ has non-negative eigenvalues

$$\begin{aligned} (x_1, x_2, ..., x_n) &= ({m_u}^2, {m_c}^2, ...), \\ (x'_1, x'_2, ..., x'_n) &= ({m_d}^2, {m_s}^2, ...), \end{aligned} \tag{3.29}$$

respectively and v is a Vandermonde-type determinant given by

$$v = v(x_1, x_2, ..., x_n) = \Pi_{\beta,\alpha}(x_\beta - x_\alpha); \quad \beta > \alpha. \tag{3.30}$$

The quantity v' is the primed version of v, whereas the quantity v_α is obtained from the v by replacing x_α by the matrix S and all other $x_\beta, \beta \neq \alpha$ by $x_\beta I$ where I is the unit matrix. Thus v_α is a $n \times n$ matrix. For example, for $n = 3$ we have

$$v = v(x_1, x_2, x_3) = (x_3 - x_1)(x_3 - x_2)(x_2 - x_1), \tag{3.31}$$

and

$$v_1(S) = (x_3 - x_2)(x_3 - S)(x_2 - S). \tag{3.32}$$

These projection operators are hermitian and have unit traces.They can be used to express the measurable combinations of the CKM matrix elements in terms of invariant functions of the mass matrices.

To incorporate the constraint due to the symmetry of CKM,we use Jarlskog's flavour projection [20] operators to express the mod square elements of V in terms of the matrices S and S' as

$$|V_{\alpha j}|^2 = tr[P_\alpha(S)P'_j(S')], \tag{3.33}$$

where the first and second indices denote the up and down quark sectors respectively, and the flavour projection operator in S is given as

$$P_\alpha(S) = \frac{[(S-x_1)(S-x_2)...(S-x_n)]}{[(x_\alpha - x_1)(x_\alpha - x_2)...(x_\alpha - x_n)]'}, \tag{3.34}$$

with $[...]'$ to mean that the factor $(S - x_\alpha)$ in the numerator and the factor $(x_\alpha - x_\alpha)$ in the denominator must be left out. The expression for $P_a(S')$ is obtained by replacing α, S, x_n by j, S' and x'_n respectively. Then, the symmetry condition

$$|V_{\alpha j}|^2 = |V_{j\alpha}|^2 \tag{3.35}$$

is translated into a relation involving the matrices S and S' as

$$tr[P_\alpha(S)P'_j(S')] = tr[P_j(S)P'_\alpha(S')]. \tag{3.36}$$

Since the matrices $M^{(u)}$ and $M^{(d)}$ of our choice are hermitian, we have done all the calculations in terms of invariant functions of the matrices $M^{(u)}$ and $M^{(d)}$ instead of S and S'. Considering, in particular

$$|V_{12}|^2 = |V_{21}|^2, \tag{3.37}$$

we obtain the constraint condition due to symmetry of CKM matrix involving the parameters of the mass matrices as

$$\begin{array}{l} [R_1^2 + R_3^2 + (hm_c + f - m_s)(hm_c + f - m_b)] + \\ \frac{m_b - m_d}{m_b - m_s}[R_1^2 + R_2^2 + (hm_u - m_d)(hm_u - m_b)] \end{array} = 0. \tag{3.38}$$

In general, it was not possible to find out the form of $M^{(d)}$ based on the general consraint involving all the parameters. But, an interesting point was noticed when we calculated the CP violation mesuring plaqutte J in terms of S and S' using [33]

$$\pm J = Im \frac{tr[v_1(S)v'_2(S')v_3(S)v'_1(S')}{vv'}. \tag{3.39}$$

It was found that if any of the R_1, R_2, R_3 is chosen to be zero along with $M^{(u)}$ being diagonal, then J is zero impling such a choice is not allowed for three generations. Thus, we note that in the basis in which $M^{(u)}$ is diagonal, no off-diagonal elements of $M^{(d)}$ can be made zero consistent with the CP violation in the quark sector for three generations.

The numerical calculation was done to find out whether any of the off-diagonal elements of the mass matrix $M^{(d)}$ is consistent with zero. To find out numerically the allowed ranges for the elements of the mass matrix $M^{(d)}$ we note that $M^{(d)}$ can be written as

$$M^{(d)} = D^\dagger \mathcal{M}_d D = V\mathcal{M}_d V^\dagger, \tag{3.40}$$

because a diagonal form for $M^{(u)}$ implies $U = \mathbf{I}$ and $D = V^\dagger$. For a symmetric V it reduces to

$$M^{(d)} = V\mathcal{M}_d V^*. \tag{3.41}$$

Since any unitary matrix that diagonalises a hermitian matrix can be written as the product of an orthogonal matrix and a phase matrix, we write V , in this basis, as

$$V = O_v P_v, \tag{3.42}$$

where the phase matrix P_v carries all the informations regarding the CP violation in quark sector for three generations. Then, the ranges for the elements of the $M^{(d)}$ were calculated using the eigenvalues of $M^{(d)}$ and the mod of the elements of V. The allowed ranges for the elements of $M^{(d)}$ in GeV are found out to be

$$M^{(d)} = \begin{pmatrix} 0.0117-0.0052 & 0.0549-0.0207 & 0.0409-0.0059 \\ 0.0549-0.0207 & 0.2374-0.1186 & 0.3261-0.1591 \\ 0.0409-0.0059 & 0.3261-0.1591 & 5.3962-5.1824 \end{pmatrix}. \tag{3.43}$$

Similarly the allowed ranges for the elements of $M^{(u)}$ are found to be

$$M^{(u)} = \begin{pmatrix} 0.1137-0.0657 & 0.4209-0.2818 & 1.9774-0.1881 \\ 0.4209-0.2818 & 2.2736-1.3885 & 16.312-5.4287 \\ 1.9774-0.1881 & 16.312-5.4287 & 279.78-179.38 \end{pmatrix}, \tag{3.44}$$

in the basis where $M^{(d)}$ is diagonal.

Since the CKM matrix $V = UD^\dagger$, where U and D are unitary matrices that diagonalise the mass matrices $M^{(u)}$ and $M^{(d)}$ respectively, then the symmetry condition for V i.e. $V = V^T$ will be fulfilled by the necessary and sufficient condition involving the matrices U and D

$$U = D^* U^T D. \tag{3.45}$$

Consider the product of the matrix $P(= U^T D)$ with its complex conjugate P^*:

$$P^* P = (U^T D)^*(U^T D) = U^\dagger D^* U^T D. \tag{3.46}$$

Now, the use of symmetry condition and the unitarity of U yields

$$PP^* = U^\dagger U = \mathbf{I}. \tag{3.47}$$

Thus, we have seen that P is a unitary matrix which is also symmetric. Hence, the most general condition for V to be symmetric is

$$D = U^* P, \tag{3.48}$$

which helps us to write the symmetric V as

$$V = UD^\dagger = UP^* U^T. \tag{3.49}$$

Since the unitary matrix U can be written as the product of a phase matrix P_u and an orthogonal matrix O_u i.e.

$$\begin{aligned} U &= O_u P_u, \\ U^\dagger &= P_u^* O_u^T, \end{aligned} \tag{3.50}$$

we reduce the symmetric V to

$$V = U(U^*P)^\dagger = O_u P_u P^* P_u O_u^T. \tag{3.51}$$

The choice of P to be a phase matrix is a special but interesting case because for such a choice of P we can write either

$$M^{(d)} = f(M^{(u)^*}) \quad \text{or} \quad M^{(u)} = g(M^{(d)^*}). \tag{3.52}$$

For such a chioce of P, we write the CKM matrix as

$$V = O_u \widetilde{P} O_u^T, \tag{3.53}$$

where $\widetilde{P}$ is a phase matrix. Then one of the choices for the mass matrix $M^{(d)}$ is a function of $M^{(u)^*}$ as follows:

$$M^{(d)} = p(M^{(u)^*})^2 + qM^{(u)^*} + rI, \tag{3.54}$$

where the parameters p, q, r are introduced to retain the mass hierarchy for the down quark sector. Upon diagonalisation of both sides of the above equation,we obtain three equations involving six quark masses and three unknown parameters p, q, r which can be determined uniquely. These three parameters are given in terms of the quark masses as

$$\begin{aligned} p &= \frac{m_s}{m_c m_t}, \\ q &= \frac{m_s}{m_c}, \\ r &= m_d - m_u \frac{m_s}{m_c}. \end{aligned} \tag{3.55}$$

To get the ranges of the mod elements of the mass matrices for the case when P is a phase matrix we proceed with the numerical calculation using a convenient parametrization [34].

Comparing this general form with the form of symmetric V, we see that if Λ is recognised as $\widetilde{P}$ then the general form is reducible to symmetric form only if W is real. Thus we conclude that the reality of W is a necessary and sufficient condition for having a symmetric CKM matrix. Then it is evident that the choice $\alpha = 0$ will make V symmetric within the above parametrization. In this parametrization all the mod elements of V were written in terms of x as well as the angles. Consider the case when $\beta_3 = 0$ and α drops out. Then the mod elements of CKM matrix relevant to our discussion are:

$$\begin{aligned} |V_{11}| &= \sqrt{1 - (1/4)sin^2(2\beta_1)(3 + 2x - x^2)}, \\ |V_{12}| &= (1/2)sin^2(2\beta_1)cos(2\beta_2)\sqrt{(3 + 2x - x^2)}, \\ |V_{13}| &= (1/2)sin^2(2\beta_1)sin(2\beta_2)\sqrt{(3 + 2x - x^2)}. \end{aligned} \tag{3.56}$$

The experimental constraints i.e.the values of the magnitudes, $\rho = |V_{13}/V_{23}|$ and J imply[22] that x must lie between -0.882 and 0.02. We then solve for β_1 and β_2 by inverting the above equation and using the the magnitudes of the first row of V and found out the allowed ranges to be

$$\begin{aligned} \beta_1 &= 0.1265 \;\; to \;\; 0.3605, \\ \beta_2 &= 0.0040 \;\; to \;\; 0.0320. \end{aligned} \tag{3.57}$$

Consider the case of $\alpha = 0$. Then the elements of the matrix W are functions of 3 mixing angles β_1, β_2 and β_3 out of which two are independent and we recognise $O_u = W$. Then, the unitary matrix U is given as

$$\begin{aligned} U &= O_u P_u = W P_u; \\ P_u &= diag(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}). \end{aligned} \tag{3.58}$$

Using the matrix U and assuming the matrix $M^{(u)}$ to be hermitian, we can write the mass matrix $M^{(u)}$ as

$$M^{(u)} = U^\dagger \mathcal{M}_u U = P_u^* W^T \mathcal{M}_u W P_u, \tag{3.59}$$

and the mass matrix $M^{(d)}$ as

$$M^{(d)} = D^\dagger \mathcal{M}_d D = P^* P_u W^T \mathcal{M}_d W P_u^* P. \tag{3.60}$$

In our numerical calculation, we use above mentioned ranges of the angles β_1 and β_2 to calculate the ranges for the mod elements of the mass matrices using the above equations in this two-angle parametrization of CKM matrix. The allowed ranges in GeV for the mod elements of $M^{(u)}$ in GeV are:

$$M^{(u)} = \begin{pmatrix} 0.1799 - 0.0242 & 0.4609 - 0.1617 & 0.0147 - 0.0006 \\ 0.4609 - 0.1617 & 1.6636 - 1.1413 & 8.9172 - 0.7141 \\ 0.0147 - 0.0006 & 8.9172 - 0.7141 & 279.93 - 179.87 \end{pmatrix}. \tag{3.61}$$

and for the matrix $M^{(d)}$ are:

$$M^{(d)} = \begin{pmatrix} 0.0386 - 0.0081 & 0.0738 - 0.0135 & 0.0023 - 0.00005 \\ 0.0738 - 0.0135 & 0.2318 - 0.1059 & 0.1692 - 0.0198 \\ 0.0023 - 0.00005 & 0.1692 - 0.0198 & 5.3995 - 5.1947 \end{pmatrix}. \tag{3.62}$$

Basis independent symmetry constraint

In the previous sections, we have given the ranges of the elements of the mass matrices M_u and M_d allowed by the symmetric CKM in two different bases. In this section we give the symmetry constraint written in a basis independent form. As we have seen in the previous section, the condition $|V_{12}| = |V_{21}|$ implies

$$tr[P_1(M_u)P_2(M_d)] = tr[P_2(M_u)P_1(M_d)] \tag{3.63}$$

which can be rewritten as

$$tr[c_1 V^\dagger \widehat{P}_1(M_u) V \widehat{P}_2(M_d) - c_2 V^\dagger \widehat{P}_2(M_u) V \widehat{P}_1(M_d)] = 0 \tag{3.64}$$

where the constants c_1 and c_2 are functions of the mass eigenvalues and

$$\begin{aligned} \widehat{P}_{1,2}(M_u) &= UP_{1,2}(M_u)U^{\dagger}; \\ \widehat{P}_{1,2}(M_d) &= DP_{1,2}(M_d)D^{\dagger} \end{aligned} \tag{3.65}$$

Consider going from a unprimed basis to a primed basis by the following transformations:

$$U' = AU; \quad D' = BD \tag{3.66}$$

where A and B are unitary matrices. Then the CKM matrix in the primed basis is

$$V' = AVB^{\dagger} \tag{3.67}$$

Requiring $V' = V$ relates A and B through the matrix V as follows:

$$A = VBV^{\dagger} \tag{3.68}$$

Then use of symmetry of CKM matrix in the primed basis yields

$$A = VBV^{*}. \tag{3.69}$$

The mass matrices transform under this basis transformation as followes:

$$M_u' = AM_uA^{\dagger} \quad , M_d' = BM_dB^{\dagger} \tag{3.70}$$

But the difficulty in using these expressions to find out how the mod elements of the mass matrices transform under this basis transformation is that it is not possible to seperate out the phase from the the mass matrices in the primed basis for any general unitary matrix A and B after the transformation.

Firstly, we write [31] the symmetry constraint as an equation involving the parameters of the mass matrices using flavour projection operators of Jarlskog[20] in a basis where $M^{(u)}$ is diagonal. In general, it was not possible to find out the form of $M^{(d)}$ based on the general consraint involving all the parameters. Also we give the numerical ranges for the mod elements of $M^{(d)}$ in this basis. Then, we wrote the necessary condition for having a symmetric V in terms of the matrices U and D as

$$U = D^{*}U^{T}D \tag{3.71}$$

We chose a particularly interesting basis where $U = D^{*}P$;P being a phase matrix and gave the ranges for the mod elements of $M^{(u)}, M^{(d)}$ in that basis using a convenient parametrization for V. We noticed that non of the off-diagonal elements of $M^{(u)}$ and $M^{(d)}$ is consistent with zero for a symmetric V, which means such forms for mass matrices cannot be obtained from any symmetry. But, in principle there exists infinite number of other bases related to each other by similarity transformations. So it is apparent that the numbers we provided for the allowed ranges of the mod elements of mass matrices are not basis independent. Finally the symmetry constraint is written in a basis-independent form.

3.2.3 Rank One quark mass matrices and Phenomenological constraints

Recently in an interesting letter[35], the results of the studies on the approximately symmetric KM matrix based on eigenvalues of the KM matrix and the rank-one quark mass matrices were reported. In this new scheme, the up and down quark mass matrices are given as

$$M^{(u)} = \kappa_U M_0 + X_U; \quad M^{(d)} = \kappa_D M_0 + X_D \tag{3.72}$$

where κ_U and κ_D are numerical constants; M_0 is a $3X3$ rank-one matrix defined as

$$(M_0)_{ij} = h_i h_j; \quad \mathrm{h} = (g_1, g_2, g_3). \tag{3.73}$$

with $g_i(i = 1,2,3)$ being real and the matrices X_U and X_D are correction terms that have to be added to M_0 to obtain the non-zero masses of the light two generation quarks since the rank-one mass matrix M_0 has only one non-vanishing eigenvalue. These quark mass matrices are diagonalised by unitary matrices as follows

$$\begin{aligned} M^{(u)}(diag) &= U_U U_0(\kappa_U M_0 + X_U)(U_U U_0)^{-1}, \\ M^{(d)}(diag) &= U_D U_0(\kappa_D M_0 + X_D)(U_D U_0)^{-1}. \end{aligned} \tag{3.74}$$

where U_0 diagonalises the rank-one matrix M_0 and is given as

$$U_0 = \begin{pmatrix} \frac{g_2}{N_1} & \frac{-g_1}{N_1} & 0 \\ \frac{-g_1 g_3}{N_2} & \frac{-g_2 g_3}{N_2} & \frac{(g_1^2+g_2^2)}{N_2} \\ \frac{g_1}{N_3} & \frac{g_2}{N_3} & \frac{g_3}{N_3} \end{pmatrix}, \tag{3.75}$$

with

$$N_1 = \sqrt{g_1^2 + g_2^2}, \quad N_3 = \sqrt{g_1^2 + g_2^2 + g_3^2}, \quad N_2 = N_1 \times N_3. \tag{3.76}$$

Then the KM matrix V is written in terms of its eigenvalues and unitary matrices U_0, U_U, and U_D as follows

$$V = (U_U U_0)K(U_D U_0)^{-1}; \quad K = diag(e^{i\sigma_1}, e^{i\sigma_2}, e^{i\sigma_3}). \tag{3.77}$$

In this scheme KM matrix is symmetric if U_U and U_D are the unit matrix because then matrix U_0 is real. In this work[3], we mainly comment on the results given in this scheme related to perfectly symmetric KM matrix. We started with the most general parametrization of KM matrix for three generations in terms of three angles and a phase[12]

It is easy to see that the symmetry condition for KM matrix reduces the number of independent parameters from four to three. For example, taking

$$|V_{13}|^2 = |V_{31}|^2 \tag{3.78}$$

puts the constraint

$$s_{13}^2 = s_{23}^2(s_{12}^2 + R^2 - 2s_{12}R\cos\delta) \tag{3.79}$$

where $R = |V_{13}/V_{23}|$. Hence the four parameters $\sigma_1 - \sigma_3, \sigma_2 - \sigma_3, g_1/g_3, g_2/g_3$ used in Tanimato's paper[35] to express the matrix elements of the perfectly symmetric KM matrix cannot

[3]This section is based on the work reported in ref.[31]

be independent of each other as the above constraint can be translated into an equation relating them. To demonstrate this in a simpler way, consider the generalised two-angle parametrization of KM matrix[34]

To establish the link between this parametrization and the new scheme[35] consider

$$\lambda_i = exp(i\sigma_i); \quad i = 1,2,3. \tag{3.80}$$

Then we can write

$$\begin{aligned} e^{i(\sigma_1-\sigma_3)} &= \lambda_1/\lambda_3 \\ e^{i(\sigma_2-\sigma_3)} &= \lambda_2/\lambda_3 \end{aligned} \tag{3.81}$$

Denoting $\sigma_1 - \sigma_3$, $\sigma_2 - \sigma_3$ by δ_1, δ_2 respectively and using the expressions for the eigenvalues we obtain

$$e^{i(\delta_1+\delta_2)} = 1 \tag{3.82}$$

which implies

$$(\delta_1 + \delta_2) = 0 \tag{3.83}$$

Thus we see that the parameters $\sigma_1 - \sigma_3$ and $\sigma_2 - \sigma_3$ are not independent in general and we have to be careful while choosing their values.

Now we relate the angles β_1, β_2 to the parameters $g_1/g_3, g_2/g_3$. Since the eigenvectors of KM matrix are given by eqn.(3.12), we compare the elements of the matrix that diagonalises V for the case $\beta_3 = 0$ with that of the matrix U_0 and get

$$\begin{aligned} g_1/g_3 &= -s_1 s_2/c_2 \\ g_2/g_3 &= -c_1 s_2/c_2. \end{aligned} \tag{3.84}$$

Using the expressions for the KM matrix elements given in ref[35], it is easy to see that the CP violation measuring plaquette J can be written in terms of the parameters $G_1(= g_1/g_3)$, $G_2(= g_2/g_3)$, δ_1, δ_2 as follows

$$J = \frac{2[1 - cos(\delta_1 - \delta_2)](G_1^2 sin\delta_1 + G_2^2 sin\delta_2)G_1^2 G_2^2}{(G_1^2 + G_2^2)^2(1 + G_1^2 + G_2^2)^2}. \tag{3.85}$$

The ranges for the parameters $\sigma_1 - \sigma_3, \sigma_2 - \sigma_3, g_1/g_3, g_2/g_3$ can be found out using the allowed ranges of x, β_1, β_2. It has been shown[34] that the experimental constraints i.e. the values of the magnitudes of the KM matrix elements, $R = |V_{13}/V_{23}|$ and CP violation measuring plaquette J imply that x must lie between -0.882 and 0.02. Hence the allowed ranges for β_1 and β_2 was found to be[31]

$$\begin{aligned} \beta_1 &= 0.1265 \quad to \quad 0.3605, \\ \beta_2 &= 0.0040 \quad to \quad 0.0320; \end{aligned} \tag{3.86}$$

which in turn decide the allowed ranges for parameters $g_1/g_3, g_2/g_3$ to be

$$\begin{aligned} g_1/g_3 &= 0.0005 \quad to \quad 0.0112, \\ g_2/g_3 &= 0.003 \quad to \quad 0.031. \end{aligned} \tag{3.87}$$

We also found out that[31] the experimental constraint

$$0.05 \leq q(= \frac{|V_{13}|}{|V_{23}}|) \leq 0.13 \tag{3.88}$$

restricts the allowed range in δ to be

$$8^0 \leq \delta \leq 32^0. \tag{3.89}$$

Cosequently, considering the CP violation measuring rephasing invariant plaquette J to be given as

$$\begin{aligned} J &= s_{12}^2 s_{23} s_{13} c_{12} c_{23} c_{13} sin\delta, \\ &= \frac{4sin(\delta_2)^3 G_1^2 G_2^2 (G_1^2 - G_2^2)}{(G_1^2 + G_2^2)^2 (1 + G_1^2 + G_2^2)^2}, \end{aligned} \tag{3.90}$$

and using the experimental numbers

$$\begin{aligned} s_{12} &= 0.221 \pm 0.002, \\ s_{23} &= 0.044 \pm 0.009, \\ s_{13}/s_{23} &= 0.09 \pm 0.05, \end{aligned} \tag{3.91}$$

the allowed region for the parameter $(\sigma_2 - \sigma_3)$ can be found out. Now consider the R versus δ curve for symmetric KM matrix which is plotted using eqn.(3.79). Then recognising $-ArgV_0^{KM}(ub) = \delta$, it seems from the numbers provided in ref[35] that the solutions A and B correspond to two different points whereas the solution C corresponds to a spread in the allowed ranges of R versus δ curve. In the generalised two angle parametrization, δ is a function of x and consequently the solutions A, B, C seem to correspond to suitable choices of x in the generalised parametrization. For example, the Kielanowski's solution *i.e.* $\delta = 30^0$ corresponds to $x = 0$.

Now let us analyse the solutions provided in ref[35] from the viewpoint of the generalised two angle parametrization. The conclusions regarding the observables should be the same in both the schemes.

$$Case \quad A : g_1 << g_2 << g_3, \quad \sigma_1 = \sigma_3$$

This case corresponds to symmetric KM matrix by construction, since two of the eigenvalues are taken to be degenerate[21] The numerical values

$$g_1/g_3 = 0.0024 \quad and \quad g_2/g_3 = 0.021$$

lie well within the allowed ranges for the parameters g_1/g_3 and g_2/g_3. The choices $\sigma_2 - \sigma_3 \approx 180^0$ and $\sigma_1 = \sigma_3$ are also consistent with each other as the constraint $(\delta_1 + \delta_2) = 0$ is not applicable to this case. To see the allowed value of top quark mass (m_t) in this case, we consider the δ versus m_t curve [3] which is consistent with the experimental constraints from $B_0 - \overline{B}_0$ mixing and ϵ parameter in the neutral K meson system. We found out that this case requires $m_t \approx 255\ GeV$ provided the Bag factor $B_K = 1$; otherwise this solution is ruled out experimentally.

$$Case\ B : g_1 << g_2 << g_3, \quad \sigma_1 - \sigma_3 = -(\sigma_2 - \sigma_3) = 120^0$$

This is Kielanowski's solution[23] which has been discussed extensively in the literature. In terms of the parameters of the generalised two angle parametrization this case corresponds to

$$\beta_1 = 0.1285, \quad \beta_2 = 0.0300, \quad x = 0 \,. \tag{3.92}$$

$$Case \quad C : g_1 = g_2 << g_3$$

The condition $g_1 = g_2$ requires $sin\beta_1 = cos\beta_1$ implying $\beta_1 = 45^0$. Then the CP violation measuring rephasing invariant plaquette J vanishes for this case as we have

$$J = \frac{1}{32} cos(2\beta_1) sin^2(2\beta_1) sin^2(2\beta_2)[3 + 2x - x^2]^{3/2} \tag{3.93}$$

Secondly, the given numerical value of the parameter g_1/g_3 does not lie within its allowed region. Hence it is difficult to see the consistency as well as the physical significance of the solution C.

To summarize, we found out that the solutions A and B are special cases of the allowed solutions for symmetric KM matrix corresponding to different values of the parameter x in the generalised two angle parametrization. The solution A predicts $m_t \approx 255 GeV$ only if $B_K = 1$; otherwise it is ruled out experimentally. The solution C was found to be inconsistent with the experimental constraints.

Chapter 4

Studies related to massive neutrinos

The question of whether the neutrino has a non-zero mass is one of the important questions of particle physics today. Neutrino mass has also great significance for astrophysics and cosmology. In this chapter, we present a brief review of neutrino masses and mixing followed by a study related to the 17 keV neutrino mass eigenstate. At the end, we present an analysis of neutrino masses in left-right symmetric extensions of the SM with various choices of the higgs scalars. We minimise the scalar potentials in all these cases.

4.1 Review of neutrino masses and mixing

4.1.1 Neutrino Mass

In the minimal SM the neutrinos are strictly massless due to the absence of right handed neutrinos and lepton-number violating processes. This choice is made not by any deeper theoretical motivations (like gauge invariance, which keeps the photon and gluons massless or the spontaneous symmetry breaking of global gauge that renders Goldstone bosons massless) but rather by our limitations regarding the apparatus to conclusively detect any non-zero mass for the neutrinos. Current experiments provide only the upper limits for the neutrino masses which is consistent with any or all of the neutrinos having zero mass and these limits are not very restrictive.

Any discussion of massive neutrinos takes us beyond the minimal SM. Extensions of the SM that allows non-zero neutrino mass can be models involving new $SU(2)$ singlet (such that the anomaly cancellation is not affected) neutral fermions or extension of the higgs sector, or both. In this subsection, first, we will be discussing the types of neutrino masses and then some models for neutrino masses.

Types of neutrino mass

Since each LH (RH) particle is necessarily associated with a RH (LH) antiparticle, the RH antiparticle field ψ_R^c is not independent of ψ_L, but is closely related to $\psi_L^\dagger$ as $\psi_R^c \equiv C\overline{\psi_L}^T$. Similarly, for a RH Weyl spinor, $\psi_L^c \equiv \overline{\psi_R}^T$. In the special case that ψ_L is the chiral projection $P_L\psi$ of a Dirac field ψ, ψ_R^c is just the RH projection $P_R\psi^c$ of the antiparticle field $\psi^c = C\overline{\psi}^T$. Since the quarks and charged leptons carry conserved quantum numbers (like colour and electric charge etc.), they must be Dirac fields i.e. ψ_R and ψ_R^c are distinct and have the opposite values for all additive quantum numbers. But, the charge neutrality of the neutrinos (which take part only in weak interactions) leaves only one quantum number, namely lepton number to be associated with the neutrino. This allows the neutrino to have both lepton number conserving as well as lepton number violating mass terms.

In general a mass term for a fermion field consists of fields with opposite chirality. Keeping this in mind, we consider all such combinations of the fields ν_L, N_R, ν_R^c, $N_L^c(\equiv C\overline{N_R}^T)$, and their Hermitian conjugates as follows:

$$\begin{aligned}
&(1) \qquad \overline{\nu_L}N_R + \overline{N_R}\nu_L \\
&(2) \qquad \overline{N_L}^c\nu_R^c + \overline{\nu_R}^c N_L^c \\
&(3) \qquad \overline{\nu_R}^c\nu_R + \overline{N_R}\nu_R^c + \overline{N_L}^c\nu_L + \overline{\nu_L}N_L^c.
\end{aligned} \tag{4.1}$$

The first two combinations are invariant under the global gauge transformations, and consequently can be rewritten as a generalised lepton number conserving **Dirac** mass term

$$-\mathcal{L}_D = m_D\overline{\nu_L}N_R + H.c., \tag{4.2}$$

which connects N_R and ν_L. The fields ν_L, N_R, ν_R^c and ν_R^c form a 4-component Dirac particle i.e. we can define $\nu \equiv \nu_L + N_R, \nu^c \equiv N_L^c + \nu_R^c = C\overline{\nu}^T$, so that $-\mathcal{L}_D = m_D\overline{\nu}\nu$. Usually, the N_R is an $SU(2)\otimes U(1)$ singlet, with m_D generated by the SM doublet, and $L = L_e + L_\mu + L_\tau$ is conserved in the three family generalisation. For N generations

$$-\mathcal{L}_D = \overline{\nu_L'}m_D N_R' + h.c., \tag{4.3}$$

where m_D is an arbitrary $N\times N$ matrix, and ν_L' N_R' are N component vectors; thus $\nu_L' = (\nu_{1L}', \nu_{2L}', ..., \nu_{NL}')^T$, where ν_{iL}' are the weak eigenstate neutrinos.

The third combination violates lepton number by $\Delta L = 2$ and is generally known as the **Majorana** mass term

$$L_{m_D} = L_{m_1} + L_{m_2} = -m_M\overline{\psi}_D\psi_D \tag{4.4}$$

In fact a Majorana mass term can be written without introducing any new fermion field N_R. This is done by coupling the ν_L to its CP conjugate ν_R^c:

$$-\mathcal{L}_M = \frac{1}{2}m\overline{\nu_L}\nu_R^c + h.c. = \frac{1}{2}m\overline{\nu_L}C\overline{\nu_L}^T + h.c. \tag{4.5}$$

For N generations, the Majorana term is

$$-\mathcal{L}_M = \frac{1}{2}\overline{\nu_L'}m_M\nu_R'^c + h.c., \tag{4.6}$$

where m_M is a $N \times N$ Majorana mass matrix and ν'_L and ν'^c_R are N-component vectors i.e. $\nu'_L = (\nu'_{1L}, \nu'_{2L}, ..., \nu'_{NL})^T$, $\nu'c_L = (\nu'^c_{1L}, \nu'^c_{2L}, ..., \nu'^c_{NL})^T$ with weak eigenstate neutrinos ν'_{iL} and antineutrinos ν'^c_{NL}, related by

$$\nu'^c_{iR} = C\overline{\nu'^T_{iL}}, \tag{4.7}$$

from which it follows that $\overline{\nu'_{iL}}\nu'^c_{jR} = \overline{\nu'_{jL}}\nu'^c_{iR}$. This identity in turn implies that the Majorana mass matrix M must be symmetric i.e. $m_M = m_M^T$.

The most general mass term for any field having no Abelian charge consists of both Dirac and Majorana mass terms. For example, in a model having one doublet neutrino ν'_L (with $\nu'^c_R = C\overline{\nu'^T_L}$) and one singlet neutrino N'_R (with $N'^c_L \equiv C\overline{N_R}'^T$). One could have the general mass term

$$-\mathcal{L}_m = \frac{1}{2}(\overline{\nu_L}' \quad \overline{N_L}'^c)\begin{pmatrix} m_T & m_D \\ m_D^\dagger & m_S \end{pmatrix}\begin{pmatrix} \nu'^c_R \\ N'_R \end{pmatrix} + h.c. \tag{4.8}$$

where $m_D = m_D^T$ is a Dirac mass generated by a higgs doublet, m_T is a Majorana mass for ν'_L generated by a higgs triplet and m_S is a Majorana mass for N'_R, generated by a higgs singlet. The mass eigenstates are the mixed states

$$\begin{aligned} \nu_{1L} &= \cos\theta\nu_L - \sin\theta\nu^c_L \\ \nu_{2L} &= \sin\theta\nu_L + \cos\theta\nu^c_L, \end{aligned} \tag{4.9}$$

with the mixing angle

$$\theta = \frac{1}{2}arctan\frac{2m_D}{m_T - m_S}, \tag{4.10}$$

with eigenvalues

$$m_{1,2} = \frac{1}{2}\{m_T + m_S \pm [(m_T - m_S)^2 + 4{m_D}^2]^{\frac{1}{2}}\} \tag{4.11}$$

Interpreting ν'_L, N'_R, ν'^c_R, N'^c_L as N-component vectors, and m_T, m_D, m_S as $N \times N$ matrices (with $m_T = m_T^\dagger, m_S = m_S^\dagger$ we can generalise the above Lagrangian for N generations as

$$-\mathcal{L}_m = \frac{1}{2}\overline{n'_L}Mn'^c_R + h.c., \tag{4.12}$$

where $n'_L \equiv (\nu'_L \quad N'^c_L)^T$ and $n'^c_R \equiv (\nu'^c_R \quad N'_R)^T$ are $2N$ component vectors and M is the symmetric $2N \times 2N$ Majorana mass matrix.

To see how the **Dirac** case ($m_T = m_S = 0$) emerges as a limiting case of the general mass term we consider only a single family such that $M = m_D\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Since M is Hermitian (for m_D real) it can be diagonalised by a unitary transformation U as

$$U^\dagger MU = m_D\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{4.13}$$

with $U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, and then the mass eigenstates are

$$\begin{aligned} \nu_{1L} &= \frac{1}{\sqrt{2}}(\nu'_L + N'_L), \quad \nu^{0c}_{1R} = \frac{1}{\sqrt{2}}(\nu'^c_R + N'_R); \\ \nu_{2L} &= \frac{1}{\sqrt{2}}(\nu'_L - N'_L), \quad \nu^{0c}_{2R} = \frac{1}{\sqrt{2}}(\nu'^c_R - N'_R); \end{aligned} \tag{4.14}$$

The negative mass eigenvalue can be removed by redefining the RH fields $\nu_{1R}^c = \nu_{1R}^{0c}$ and $\nu_{2R}^c = -\nu_{2R}^{0c}$. Since the two Majorana states $\nu_1 = \nu_{1L} + \nu_{1R}^c$ and $\nu_2 = \nu_{2L} + \nu_{2R}^c$ and degenerate, we can rewrite the Lagrangian $\mathcal{L}_m$ in the new basis

$$\begin{aligned} \nu &\equiv \frac{1}{\sqrt{2}}(\nu_1 + \nu_2) = \nu_L' + N_R', \\ \nu^c &\equiv \frac{1}{\sqrt{2}}(\nu_1 - \nu_2) = N_L'^c + \nu_R'^c; \end{aligned} \tag{4.15}$$

yielding

$$\mathcal{L}_m = \frac{1}{2} m_D (\overline{\nu_{1L}} \nu_{1R}^c + \overline{\nu_{2L}} \nu_{2R}^c) + h.c. = m_D \overline{\nu} \nu, \tag{4.16}$$

which is just the standard Dirac mass term, with a conserved lepton number. Therefore, a Dirac neutrino is nothing but a pair of degenerate 2-component Majorana neutrinos (ν_1 and ν_2), combined to form a 4-component neutrino with a conserved lepton number.

A pseudo-Dirac neutrino is just a Dirac neutrino to which a small lepton number-violating perturbative term has been added. This can be seen by modifying the Dirac mass to

$$M = \begin{pmatrix} \epsilon & m_D \\ m_D & 0 \end{pmatrix}, \tag{4.17}$$

with $\epsilon \ll m_D$. Then we have two Majorana mass eigenstates $\nu_\pm$, with

$$\nu_{+L} = \nu_{1L} + \frac{\epsilon}{4}\nu_{2L}, \nu_{-L} = -\frac{\epsilon}{4}\nu_{1L} + \nu_{2L}, \tag{4.18}$$

with masses

$$m_\pm = m_0 \pm \frac{\epsilon}{2}. \tag{4.19}$$

Models of neutrino mass

There are many models for neutrino mass, all of which have good and bad features. But we will be discussing few models that involve either an enlarged fermion sector or an extended higgs sector.

As we have discussed earlier, an enlargement of the fermion sector through the inclusion of a RH neutrino field N_R that transforms as (1, 1, 0) under $SU(3)_C \otimes SU(2)_L \otimes U(1)$ leads to Dirac mass for neutrinos. This model treats neutrino mass exactly on the same footing as the masses of the other fermions since the mass is generated by the *vev* of the neutral component of a doublet higgs through the Yukawa couplings. However, the model has some drawbacks. First, it cannot predict the neutrino mass as it turns out to be proportional to arbitrary Yukawa coupling constants. Secondly, it fails to explain the smallness of neutrino mass. Since $v = 246GeV$, a ν_e mass in the 10 eV range would require an anomalously small Yukawa coupling $h_{\nu_e} \leq 10^{-10}$. Of course, if the coupling constants are fine-tuned such that h_νs are very small compared to the corresponding coupling constants which generates masses for charged leptons or quarks, the neutrinos can have lighter mass in commensurate with experimental bounds. But there is no good reson why h_νs must be small in this model.

If we donot enlarge the fermion sector of the minimal SM, we have, in each generation, only two degrees of freedom corresponding to neutrinos i.e. ν_L and $\overline{\nu_R}$. Then the mass of the neutrino must be of the Majorana type i.e. violate B-L symmetry irrespective of the mass-generating mechanism. Thus, we are motivated to extend the higgs sector through the inclusion of new higgs which can violate B-L symmetry. Also, the neutrino masses must somehow be induced by the Yukawa coupling. Majorana mass terms for the ordinary $SU(2)$ doublet neutrino involve a transition from $\nu_R^c(T_3 = -\frac{1}{2})$ into $\nu_L(T_3 = \frac{1}{2})$, and therefore must be generated by an operator transforming as a triplet under weak $SU(2)_L$. The simplest possibility is the Gelmini-Roncadelli[38] model, in which one introduces a triplet of higgs fields $\Delta \equiv (\Delta^0, \Delta^-, \Delta^{--})$ into the theory. The Yukawa coupling

$$\mathcal{L}_y = \frac{1}{2}\begin{pmatrix}\overline{\nu_L} & \overline{e_L}\end{pmatrix}\begin{pmatrix}\Delta^- & \sqrt{2}\Delta^0 \\ \sqrt{2}\Delta^{--} & -\Delta^-\end{pmatrix}\begin{pmatrix}e_R^c \\ \nu_R^c\end{pmatrix} \tag{4.20}$$

then generates a Majorana mass $m_T = h_\Delta v_\Delta$ for the ν, when the higgs triplet field Δ_L acquires a $v.e.v.$ $v_\Delta = \sqrt{2} < \Delta^0 >$ is the vev of the higgs triplet. Since both h_Δ and v_Δ are unknown, the neutrino mass is unrelated to that of the other fermions and can in principle be arbitrarily small, at least in the tree level. There will be massless Goldstone boson called Majoron in this model if the Lagrangian $\mathcal{L}$ conserves lepton number since the vev $< \Delta^0 > \neq 0$ violates lepton number conservation by two units.

There is a popular scheme called the **see-saw mechanism** to explain the smallness of the neutrino mass. The see-saw mechanism[40] for one generation is a special case of the general mass matrix, in which m_D is a typical Dirac mass (comparable to m_u or m_e for the first generation) connecting ν_L' to a new $SU(2)_L$ singlet N_R' and $m_S \gg m_D$ is a Majorana mass for N_R', presumably comparable to some new physics scale. It is usually assumed in this model that $m_T = 0$ i.e. there exists no higgs triplet. Then eqn.4.8 yields two Majorana mass eigenstates ν_1 and ν_2 with

$$\begin{aligned} \nu_L' &= (\nu_{1L}\cos\theta + \nu_{2L}\sin\theta), \quad \nu_R'^c = -(\nu_{1R}^c\cos\theta + \nu_{2R}^c\sin\theta), \\ N_L^c &= -(\nu_{1L}\sin\theta + \nu_{2L}\cos\theta), \quad N_R' = -(\nu_{1R}^c\sin\theta + \nu_{2R}^c\cos\theta). \end{aligned} \tag{4.21}$$

Then the physical masses (i.e. the eigenvalues) are

$$m_1 \approx \frac{m_D^2}{m_S} \ll m_D, \quad m_2 \approx m_S, \tag{4.22}$$

and the mixing angle is

$$\tan\theta = \frac{m_1}{m_2}^{\frac{1}{2}} \approx \frac{m_D}{m_S} \ll 1. \tag{4.23}$$

Since $m_S \gg m_D$, it follows that $m_1 \ll M$, which means that there is one very light neutrino compared to the charged fermions, which is mainly the $SU(2)$ doublet $(\nu_L', \nu_R'^c$, and there exists one heavy neutrino, that is mainly the singlet $(N_L'^c, N_R')$.

This mechanism of making one particle light at the expense of making another one heavy is called the see-saw mechanism. But, it should be noted that cosmological constraints restricts $m_S \geq 10^8 GeV$ which is much larger that the weak scale. In the case when $m_T \neq 0$ (but

$\ll m_S$) there exist two Majorana neutrinos with masses $|m_T - m_D^2/m_S|$ and m_S respectively, while $\theta \sim m_D/m_S \ll 1$ still holds. However, in such a case one does not have the natural explanation of why m_1 is so small, unless m_T is itself induced by the underlying physics and is of the order as m_D^2/m_S.

4.1.2 Lepton Mixing and Neutrino Oscillations

One immediate consequence of neutrinos being massive is the possibility of lepton mixing and neutrino oscillations. Thus if the neutrino oscillations are observed that will be an indication of non-zero mass for neutrinos and of physics beyond the Standard Model.

Lepton Mixing

In the previous section we have seen how neutrinos can be either Dirac or Majorana particles. Consequently, unlike the quarks there exists more than one mixing scheme for neutrinos. The mixing scheme for neutrinos are classified according to the types of mass terms, whose diagonalisation leads to the corresponding mixing. Defining the charged lepton mass basis by

$$l_L = L_L l'_L, \quad l_R = L_R l'_R, \tag{4.24}$$

where $L_{L,R}$ diagonalize the lepton mass matrix M^l through the biunitary transformation

$$L_L^\dagger M_l L_R = \widehat{M}_l \tag{4.25}$$

and assuming that all the LH neutrinos are part of $SU(2)_L$ doublets with hypercharge $Y = -\frac{1}{2}$, whereas all RH neutrinos are gauge singlets, we obtain the relevant charged current

$$J_\mu^+ = \sum_{i,j=1}^{n} \sum_{\alpha=1}^{n+m} \overline{l_{Li}} \gamma_\mu \chi_{L\alpha} (L_L^\dagger)_{ij} (U^*)_{j\alpha}. \tag{4.26}$$

This leads to an effective neutrino mixing matrix (analogous to the CKM matrix)

$$(K^\nu)_{i\alpha} = \sum_{j=1}^{n} (L_L^\dagger)_{ij} (U^*)_{j\alpha}. \tag{4.27}$$

which is, unlike in the hadronic case, a non–unitary and rectangular $[n \times (n+m)]$ matrix that satisfies:

$$(K^\nu K^{\nu\dagger})_{ik} = \delta_{ik} \quad \text{but} \quad (K^{\nu\dagger} K^\nu)_{\alpha\beta} = \sum_{k=1}^{n} U_{\alpha k}^T U_{k\beta}^*.$$

The non–orthogonality also manifests itself in the neutral current interactions, the relevant isotriplet part of which is given by

$$j_\mu^3 = \sum_{i=1}^{n} \overline{\nu_{iL}} \gamma_\mu \nu_{iL} = \sum_{\alpha,\beta=1}^{n+m} (K^{\nu\dagger} K^\nu)_{\alpha\beta} \overline{\chi_{\alpha L}} \chi_{\beta L}.$$

Parameter counting in this case is slightly different from that in the hadronic sector. K^ν is best recognized as being a rectangular part of a $(n+m) \times (n+m)$ unitary matrix and hence, in the most general case is given by ${}^{n+m}C_2$ angles and ${}^{n+m+1}C_2$ phases. However, we can't proceed as for the quarks and eliminate $2(n+m)-1$ phases by redefinition of wavefunctions, for the Majorana neutrinos obviously cannot absorb phase transformations. At most n phases can be eliminated by redefining only the charged lepton wavefunctions and thus we are left with ${}^nC_2 + \frac{m(2n+m+1)}{2}$ CP violating phases. Unlike the case of quarks, even for two generations we can have $\mathcal{CP}$ violation. It seems natural then that this difference can be exploited to distinguish a Majorana neutrino from a Dirac one, but Schechter and Valle [41] have shown that these extra CP violating effects are always suppressed by an additional factor of $(m_\nu/E_\nu)^2$, where m_ν and E_ν respectively are the mass and energy of the Majorana neutrino taking part in the process. The suppression is easily understood by appreciating that a process dependent on the Majorana mass must have an amplitude proportional to the latter and hence for dimensional reasons there has to be a suppression factor given by the relevant energy scale in the problem.

As in the case of the $K_0 - \overline{K^0}$ system, we have, in the general case, a number of neutrinos with possibly all different masses mixing with each other. While the interaction terms in the Lagrangian conserve the individual lepton numbers, the mass terms do not, and in the case of Majorana neutrinos even the total lepton number is not preserved. As a neutrino with definite interaction properties evolves in time, each of its massive modes propagates differently resulting in a periodic variation in their relative proportions in the generic neutrino 'beam'. Analogous to strangeness oscillations for the neutral kaons, we have then the possibility of lepton number oscillations [42].

Neutrino Oscillations

To explore the consequences of the mixing hypothesis for neutrinos consider first the mixing of only two species of neutrino, ν_e and ν_μ. In analogy to the quark sector, we express the weak eigenstates as linear combination of mass eigenstates ν_1 and ν_2 at time $t=0$ as

$$\begin{aligned} |\nu_e(0)> &= |\nu_1(0)> \cos\alpha + |\nu_2(0)> \sin\alpha \\ |\nu_\mu(0)> &= -|\nu_1(0)> \sin\alpha + |\nu_2(0)> \cos\alpha \end{aligned} \tag{4.28}$$

where α is the angle that parametrizes the mixing. A non-zero α implies that some neutrinos masses are non-zero and that the mass eigenstates are non-degenerate. Then in a production process, like $\pi^+ \longrightarrow e^+\mu_e$, for example, we start with the weak eigenstate $|\mu_e>$. But it is the mass eigenstates $|\nu_i>$ that have a definite time evolution of the form

$$|\nu_i(t)> = |\nu_i(0)> e^{-iE_it};\ i = 1,2, \tag{4.29}$$

where their energies are

$$E_i = (p^2 + m_i^2)^{1/2} \approx p + \frac{m_i^2}{2p} \tag{4.30}$$

if $p >> m_i$, since our concern is with spatially coherent states in which the neutrinos have essentially identical momenta p. After a time t has elapsed, the pure ν_e state therefore becomes

$$|\nu(t)> = |\nu_1(0)> \cos\alpha e^{-iE_1t} + |\nu_2(0)> \sin\alpha e^{-iE_2t}. \tag{4.31}$$

Substituting for $\nu_1(0)$ and $\nu_2(0)$ in terms of $\nu_e(0)$ and $\nu_\mu(0)$, we have

$$|\nu(t)> = |\nu_e(0)> [e^{-iE_1t}\cos^2\alpha + e^{-iE_2t}\sin^2\alpha] + |\nu_\mu(0)> \sin\alpha\cos\alpha[e^{-iE_2t} - e^{-iE_1t}]. \quad (4.32)$$

The probability that an initial beam of ν_e later contains some ν_μ is given by

$$\begin{aligned} P(\nu_e \to \nu_\mu, t) &= |<\nu_\mu|\nu(t)>|^2 \\ &= \frac{1}{2}\sin^2 2\alpha[1 - \cos(E_2 - E_1)t], \end{aligned} \quad (4.33)$$

while that of ν_e is

$$\begin{aligned} P(\nu_e \to \nu_e, t) &= |<\nu_e|\nu(t)>|^2 \\ &= 1 - \frac{1}{2}\sin^2 2\alpha[1 - \cos(E_2 - E_1)t]. \end{aligned} \quad (4.34)$$

Since the difference in energy

$$E_2 - E_1 = \frac{E_1^2 - E_2^2}{E_1 + E_2} \approx \frac{(m_2^2 - m_1^2)}{2p}$$

for $E_1 \approx E_2 \sim E >> m_i$, and since the distance travelled $r \approx ct$ is essentially the same for both ν_1 and ν_2 if the state is to remain coherent spatially, we rewrite the oscillation probalities as

$$\begin{aligned} P(\nu_e \to \nu_\mu, t) &= \sin^2 2\alpha \sin^2 \frac{\pi r}{L}, \\ P(\nu_e \to \nu_e, t) &= 1 - \sin^2 2\alpha \sin^2 \frac{\pi r}{L}, \end{aligned} \quad (4.35)$$

where the so-called "oscillation length" L is defined as

$$L \sim \frac{4\pi E}{\Delta m^2} \quad (4.36)$$

and it is an effective length that determines the distance over which one might expect to detect the neutrino oscillations effect. Note that the detectability depends on $(m_2^2 - m_1^2)$, not on m_2 or m_1 themselves. Thus the oscillation effect of the species of neutrino that is observed will change as a function of the distance r from the source, provided that (a) the mixing angle $\alpha \neq 0$, and (b) $m_1 \neq m_2$.

For the general case of more number of neutrino flavours $\nu_i(i = e, \mu, \tau, ...)$, we have

$$\begin{aligned} |\nu_i> &= \sum_i U_{li}|\nu_i>, \\ P(\nu_l \longrightarrow \nu_{l'}) &= \delta_{ll'} - \sum_{i>j} 4U_{li}U_{l'i}^*U_{lj}^*U_{l'j}\sin^2(\frac{\pi r}{L_{ij}}), \end{aligned} \quad (4.37)$$

where the U_{ij} are the lepton mixing matrix elements and

$$L_{ij} \sim \frac{4\pi E}{|m_i^2 - m_j^2|} = \frac{2.48E/(MeV)}{\Delta m_{ij}^2/(eV)^2} \quad meters \quad (4.38)$$

For a direct observation of such oscillations, we need to perform experiments such that

$$\frac{r}{E} \sim \frac{L}{E} \sim \frac{1}{\Delta m^2}, \quad (4.39)$$

though the effect of mixing will be significant for $r \geq L$. The null results obtained so far indicate either that $\Delta m^2 < E/r$ or that the relevant mixing matrix elements U_{ij} are very small.

4.1.3 Experimental Evidences

Accurate measurements of the charged particle momenta in the processes

$$^3He \longrightarrow ^3He^+ + e^- + \overline{\nu_e}, \quad \pi^- \longrightarrow \mu^- \overline{\nu_\mu}, \quad \tau^- \longrightarrow \pi^+\pi^-\pi^+\pi^-\pi^-\nu_\tau$$

yields the upper bounds on the masses (in GeV) of the neutrinos:

$$\begin{aligned} m_{\nu_e} &\leq 1.8 \times 10^{-8}, \\ m_{\nu_\mu} &\leq 2.5 \times 10^{-4}, \\ m_{\nu_\tau} &\leq 3.5 \times 10^{-2}. \end{aligned} \tag{4.40}$$

There exists no heavy neutrinos, that might be detected in processes such as $\pi^+ \longrightarrow \mu^+\nu_H$ or $\nu_H \longrightarrow e^+e^-\nu_e$, in the mass range 10 MeV- 10 GeV unless their coupling to e and μ are extraordinarily small ($< 10^{-5}G_F$). Besides these above mentioned kinematics, the process of neutrinoless double β-decays of the nuclei might provide us with a clue for finiteness of neutrino masses. Normally, the double β-decay of a nucleus of mass number A and charge Z is

$$(A, Z) \longrightarrow (A, Z+2) + e^- + e^- + \overline{\nu_e} + \overline{\nu_e},$$

but with a massive Majorana neutrino the decay

$$(A, Z) \longrightarrow (A, Z+2) + e^- + e^-$$

is possible through the $\Delta L = 2$ transition $\overline{\nu_R} \rightarrow \nu_L$. Searches for such decays in

$$^{76}Ge \longrightarrow ^{76}Se + e^- + e^-$$

give the lifetime bounds $\tau \geq 10^{23}$ years, from which a rather model-dependent limit to the Majorana mass $m_{\nu_e} < 1 - 2eV$ can be deduced[39] But such experiments say nothing about the masses of Dirac neutrinos for which these decays are forbidden.

Another possible source of evidence for finite masses for neutrinos would be the observation of neutrino oscillations. The oscillation length L_{ij} and the difference of squares of neutrino masses Δm^2 disscussed earlier decide the way in which such oscillations can be detected. Although various experiments in the context of solar neutrinos, cosmic rays, nuclear reactors and particle accelerators have been performed, there is as yet no convincing evidence that the neutrinos mix under the weak interaction like the quarks.

4.2 17 keV Nondegenerate Majorana Neutrino and neutrino mixing

In 1985, it was observed by Simpson[44] that there exists an anomalous kink in the Curie plot of the β-spectrum in Tritium decay. This was interpreted as a mixture (with a 3% mixing) of a $17keV$ neutrino with the ν_e i.e. $|U_{17e}|^2 = 0.03$. This was reobserved by others[46] in 1991.

As compared to the original claim, the mixing ($|U_{17e}|^2 = x$) of the 17 kev neutrino with ν_e had changed, the later value being close to 1% i.e.$x = 0.01$ [45, 46]. We have studied[1] the limits on the elements of the neutrino mixing matrix consistent with neutrinoless double beta decay and the neutrino oscillation experiments as a function of the mixing probability (x) of ν_e with the $17keV$ neutrino assuming only three generations of left-handed neutrinos and no sterile neutrinos.

We considered all x between 0.003 and 0.03, and found that $x > 0.015$ is not allowed. Stringent limits on m_{ν_μ} (when $m_{\nu_\mu} \gg m_{\nu_\tau}$) and the mass difference ($m_{\nu_\mu} - m_{\nu_\tau}$) (when ν_μ and ν_τ form a pseudo-Dirac particle) are found. Allowed values of the various mixing angles are obtained as function of x, when ν_e, ν_μ and ν_τ are nondegenerate Majorana neutrinos.

In our analysis we take x as a parameter and quote limits on other quantities as function of x. Since the present limit on the number of light neutrino species (as obtained from the Z width) is very close to 3, we shall study the constraints on the mixing matrix for three Majorana neutrinos. When the masses are non-degenerate at the tree level i.e.the mass differences are large,we parametrize the mixing matrix by three angles (assuming no CP violation in the leptonic sector).Then the limits on ($\nu_e \to \nu_\mu$) oscillaton, the neutrinoless double beta decay and the value of x can set limits on the three angles and hence on all the elements of the mixing matrix. For consistency we then calculate the ($\nu_\mu \to \nu_\tau$) and ($\nu_e \to \nu_\tau$) oscillation probabilities, and compare them with the experimental limits.We find extremely narrow allowed regions for the three mixing angles and hence the elements of the mixing matrix.

Next we analyse the situation when the $17keV$ neutrino is a pseudo-Dirac particle,that is, at tree level ν_μ and ν_τ combine together to form a Dirac particle,but a small mass difference is generated radiatively.In this case the strongest bound on the mass difference comes from the ν_μ disappearance experiment. If one starts with a ($L_e + L_\tau - L_\mu$) type of symmetry [47], which is broken at low energy,then at the tree level ν_e is massless and 17 kev neutrino is a Dirac neutrino. The symmetry breaking will induce new contributions to the zero elements of the mass matrix.From the limit on the allowed mass difference,we find that the limits on these non-zero elements and found them to be unnaturally small.

We shall first consider that three neutrinos (ν_i) have nondegenerate Majorana masses m_i [48]. The weak eigenstates of the neutrino $\nu_\alpha (\alpha = e, \mu, \tau)$ are related to the mass eigenstates $\nu_i (i = 1, 2, 3)$ through the relation

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i} \nu_i \tag{4.41}$$

where $U_{\alpha i}$ is the mixing matrix.If we assume that there is no CP violation in the leptonic sector, then $U_{\alpha i}$ is real and is an orthogonal matrix. We start with the most general 3x3 orthogonal matrix which has three independent parameters:

$$U = \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 \\ -s_1 c_2 - c_1 s_2 s_3 & c_1 c_2 - s_1 s_2 s_3 & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 & -c_1 s_2 - s_1 c_2 s_3 & c_2 c_3 \end{pmatrix} \tag{4.42}$$

[1] This section is based on the work reported in ref.[43]

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$. In the mass eigen-state basis, the charged-lepton-neutrino-W coupling is of the form

$$L_w = \bar{l}W^- U\nu + h.c. \tag{4.43}$$

The most general neutrino mass Lagrangian L_m has the form

$$L_m = -\frac{1}{2}\overline{(\nu_\alpha)^c}M\nu_\alpha + h.c. \tag{4.44}$$

Using $\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i}\nu_i$, in the expression for L_m, we obtain the mass matrix M in terms of the mixing matrix U and M^{diag} where $M^{diag} = diag(m_1, m_2, m_3)$; and m_1, m_2 and m_3 being the mass eigenvalues, as

$$M = UM^{diag}U^T \tag{4.45}$$

Recognising s_1, s_2, s_3 as the mixing angles we shall try to find out the allowed regions for each of them satisfying the following constraints:

a)The mixing of the $17keV$ neutrino with the ν_e should be commensurate with the latest experimental result. But for completeness we take it as a parameter in our analysis i.e.

$$|U_{17\,e}|^2 = x \tag{4.46}$$

and vary x between 0.003 to 0.030, which includes the present experimental value.

b)The 17 keV neutrino cannot be ν_μ, since $(\nu_e - \nu_\mu)$ oscillation will be too fast in that case. So, the third eigenvalue m_3 is dominantly the ν_μ mass. We shall thus use the present experimental limit [49] on ν_μ for m_3 i.e.,

$$m_3 < 250keV \tag{4.47}$$

and $m_2 = 17keV$ is mostly ν_τ mass. The limits for m_1 comes from the limit [17] on ν_e mass i.e.

$$m_1 < 17eV \tag{4.48}$$

The non-observation of neutrinoless double beta decay implies limit [17] on,

$$\left|\sum_{i=1}^{3} U_{ei}^2 m_i\right| < 1.8eV \tag{4.49}$$

c)In addition to these, we have, further constraints coming from neutrino oscillation. When

$$|m_i^2 - m_j^2| > 100ev^2 \tag{4.50}$$

the best limits [50] for various neutrino oscillations are

$$P_{\nu_e\to\nu_\mu} < 2\times10^{-3}, \quad P_{\nu_\mu\to\nu_\tau} < 3\times10^{-3}, \quad P_{\nu_\tau\to\nu_e} < 0.21 \tag{4.51}$$

The probability for a neutrino of flavour a to oscillate into a neutrino of flavour b is given by

$$P_{a\to b} = \left|\sum_{j=1}^{3} U_{aj}U_{bj}^{*}e^{(im_j^2 L/2E)}\right|^2 \tag{4.52}$$

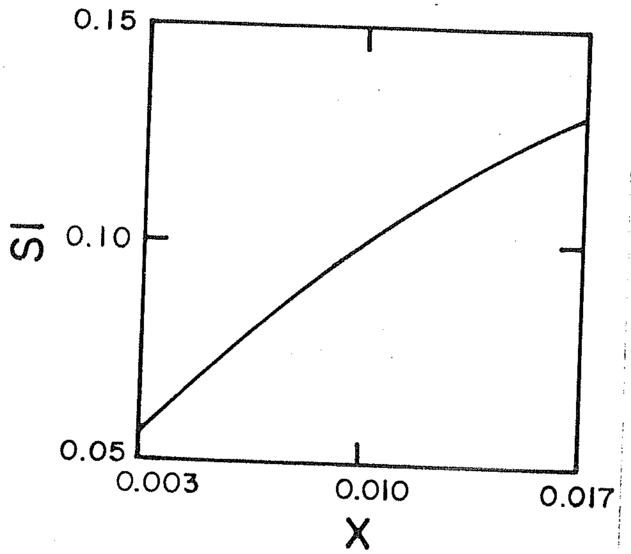

Figure 4.1: The allowed region for s_1 versus x. Note that the curves corresponding to s_{1max} and s_{1min} coincide.

Using the above mentioned parametrization for U [eqn(4.42)], we can express $P_{a\to b}$ as a function of the mixing angles s_1, s_2 and s_3. The $\nu_e \to \nu_\mu$ oscillation probabillity is expressed as

$$P_{\nu_e\to\nu_\mu} = 2[s_2^2\{s_3^2c_3^2(1 - c_1^2s_1^2) - (s_1c_1c_3)^2\} + s_2c_2s_1c_1s_3c_3^2(c_1^2 - s_1^2) + (s_1c_1c_3)^2], \quad (4.53)$$

since m_i are nondegenerate [they satisfy eqn(4.50)] and the interference terms average out to zero.

In our numerical analysis, we vary s_2 and s_3 between 0 and 1 and the parameter x in the range of 0.003 to 0.03 and calculate s_1 using

$$s_1 = (x/c_3^2)^{\frac{1}{2}}. \quad (4.54)$$

Taking $m_2 = 17keV$, m_3 was calculated using the constraint from the neutrinoless double beta decay as

$$m_3 = m_2\frac{|U_{e\,17}|^2}{|U_{e\,3}|^2} = m_2\frac{x}{s_3^2}. \quad (4.55)$$

Corresponding values of the $\nu_e - \nu_\mu$, $\nu_\mu - \nu_\tau$, and $\nu_\tau - \nu_e$ oscillation probabilities were calculated. It was found out that the upper limit of $P_{\nu_e-\nu_\mu}$ rules out most of the allowed regions of the various angles. The allowed region for each of the angles s_1, s_2, s_3 and m_3, are plotted versus the mixing parameter x (figs.(4.1,4.2,4.3,4.4).

The allowed region of the parameter space is extremely narrow. The fig.(4.1) shows the allowed region of s_1 as a function of x, in which both the curves coresponding to upper and

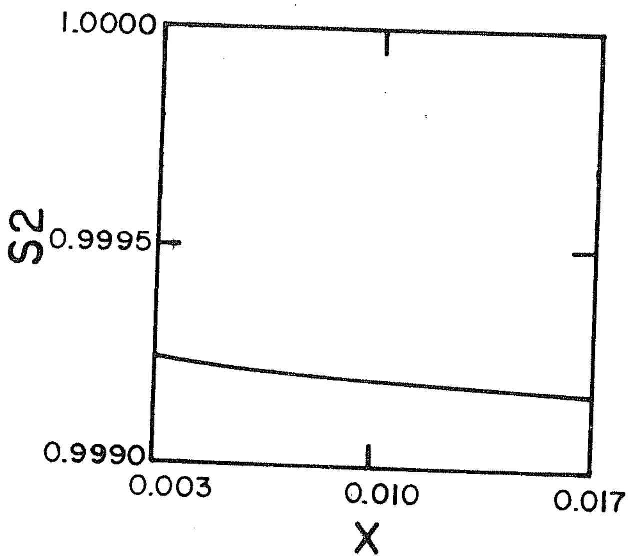

Figure 4.2: The lower limit for s_2 as a function of x. The upper limit is 1.

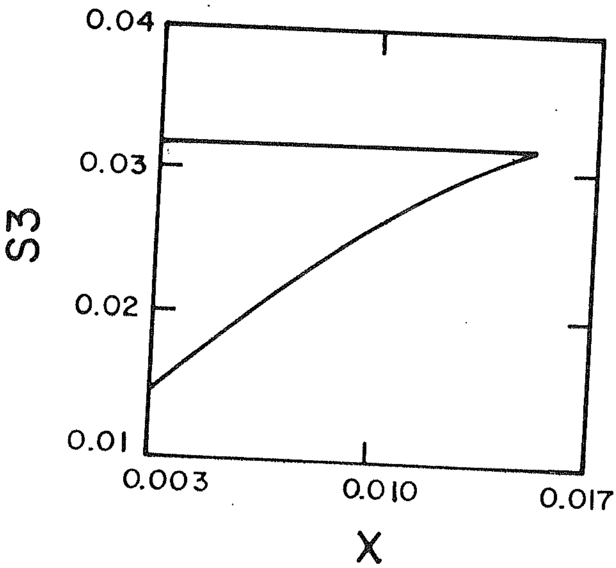

Figure 4.3: Allowed region of s_3 as a function of x.

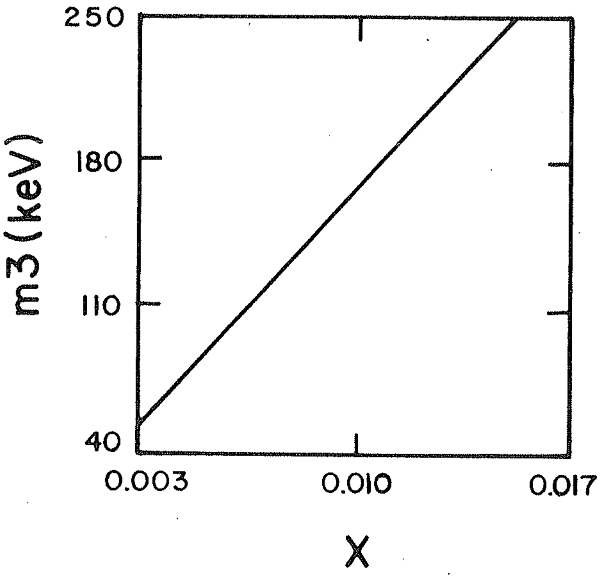

Figure 4.4: Allowed region of m_3 as a function of x. The lower limit is given by the curve, and the upper limit is the experimental upper limit of 250 keV.

lower limits merge. As it is shown in fig.(4.2), where we plot the lower limit for s_2 as a function of x, we find the allowed region to be extremely narrow. The upper limit of s_2 is always 1 and as a result, the upper limit of s_3 can be obtained from the expression

$$P_{\nu_e-\nu_\mu} = 2s_3^2c_3^2(1 - s_1^2c_1^2), \tag{4.56}$$

where s_1 and c_1 are also given in terms of s_3 [eq. 4.54]. The allowed region of s_3 and m_3 as a function of x are shown in figs.(4.3,4.4) and respectively. For $x \geq 0.015$ there is no allowed region in the parameter space. To get an idea of how much restriction is imposed on the various elements of the mixing matrix, we give the allowed ranges of the elements of the mixing matrix U for $x = 0.1$,

$$U = \begin{pmatrix} 0.9945 - 0.9946 & 0.0999 - 0.0999 & 0.0261 - 0.0318 \\ (-0.0259) - (-0.0356) & (-0.0026) - (+0.0365) & 0.9988 - 0.9995 \\ 0.0988 - 0.0999 & (-0.9942) - (-0.9951) & 0.0000 - 0.0398 \end{pmatrix} \tag{4.57}$$

We shall now consider the case, when ν_μ and ν_τ combine to form a pseudo-Dirac neutrino with the tree level mass of 17 keV and about 1 % mixing with ν_e. This scenario can be incorporated in a model with just three conventional neutrinoes in the low energy theory, which has $(L_e + L_\tau - L_\mu)$ as a good approximate symmetry. If the lepton mass matrices have a good approximate global symmetry $(L_e + L_\tau - L_\mu)$ then in the basis in which the first (second,third) row and column refers to the $e(\mu, \tau)$ weak eigenstates and in which the charged lepton mass matrix is diagonal, the most general mass matrix [47] is given by

$$M\begin{pmatrix} 0 & sin\theta & 0 \\ sin\theta & 0 & cos\theta \\ 0 & cos\theta & 0 \end{pmatrix} \quad (4.58)$$

where $M = 17keV$ to reproduce the massive neutrino states and $sin^2(\theta) = 0.03$ to reproduce 3% mixing. But the symmetry breaking will induce new contributions to the the zero elements of the mass matrix. Thus the limit on the mass difference ϵ will fix the bounds of the contributions to the zero elements of the mass matrix M after the symmetry breaking.

To consider the constraints on the lepton weak mixing matrix U for the case of

$$|m_3 - m_2| < 3 \times 10^{-03}eV \quad (4.59)$$

we write, in the flavour basis, the neutrino mass matrix as

$$M' = U^* M'^{diag} U^\dagger \quad (4.60)$$

where

$$M'^{diag} = diag(\delta, 17keV + \epsilon, 17keV - \epsilon) \quad (4.61)$$

Using the parametrization [48], which is ideal for the limit $\epsilon \longrightarrow 0$

$$U_M = \begin{pmatrix} c_\gamma e^{i\sigma} & s_\gamma B^* & s_\gamma A^* \\ s_\alpha s_\gamma e^{i\sigma} & e^{i\rho} c_\alpha A - s_\alpha c_\gamma B^* & -e^{i\rho} c_\alpha B - s_\alpha c_\gamma A^* \\ -s_\gamma c_\alpha e^{i\sigma} & e^{i\rho} s_\alpha A + c_\alpha c_\gamma B^* & -e^{i\rho} s_\alpha B + c_\alpha c_\gamma A^* \end{pmatrix} \quad (4.62)$$

where

$$\begin{pmatrix} A & -B \\ B^* & A^* \end{pmatrix} = \begin{pmatrix} c_\beta & is_\beta \\ is_\beta & c_\beta \end{pmatrix} \begin{pmatrix} c_\lambda & -s_\lambda \\ s_\lambda & c_\lambda \end{pmatrix} \quad (4.63)$$

and $m_1 \leq 13eV$, we have, for the mixing parameter x,

$$C_{2\beta} < \frac{13eV}{17keV}\frac{1}{x} \quad (4.64)$$

The $\nu_\mu \longrightarrow \nu_e$ oscillation gives

$$s_\alpha^2 < \frac{2 \times 10^{-3}}{x(1-x)} \quad (4.65)$$

The $\nu_\mu \longrightarrow \nu_\tau$ oscillations and ν_μ disappearance experiments fix the limit on the ν_τ and ν_μ mass difference as $4 \times 10^{-5}eV$ [51] and $9 \times 10^{-8}eV$ [52] respectively.

We found out that to satisfy such a strong limit the bounds on $ee, e\tau, \tau e$, $\mu\mu$ and $\tau\tau$ elements after the symmetry breaking is unnaturally small compared to the bounds suggested in Dugan et.al's paper. It was noted in their paper that for the lightest neutrino to be less than $40eV$, the bound on $e\tau$ and τe entries is about $250eV$ and limits from neutrinoless double beta decay require the ee element to be less than $1eV$. It was also noted that rapid oscillations $\nu_\mu \longrightarrow \nu_\tau$ caused by small non-degeneracy restrict the $\mu\mu$ and $\tau\tau$ elements to be less than $1eV$. But, we notice that $\epsilon < 9 \times 10^{-08}eV$ fixes the bounds for all the zero elements at $10^{-10}keV$. This conclusion is based on a numerical calculation in which $ee, e\tau, \tau e, \mu\mu, \tau\tau$ elements were varied slowly so far as $|m_3 - m_2| < 10^{-08}eV$.

To summarize, we studied constraints on the neutrino mixing matrix from the various oscillation data, neutrinoless double beta decay and the limit on the ν_e, ν_μ and ν_τ masses assuming only three generations of left-handed neutrinos and no sterile neutrinos. In the limit when all the three eigenvalues are nondegenerate and the mass differences are larger than $100eV^2$, we identify ν_τ with the 17 keV neutrino and vary the mixing probability between 3% and 0.3%. We find a very narrow allowed region for the various mixing angles. The allowed values of m_{ν_μ} lie between 145 keV and 205 keV for 1% mixing and between 135 keV and 240 keV for 3% mixing (our result differs from the earlier similar works with 3% mixing [48], where some approximations were made). The $\nu_e \to \nu_\mu$ oscillation probability is found to lie between .001 and .002 for consistency. We then considered the allowed amount of the symmetry breaking when ν_μ and ν_τ form a pseudo-Dirac particle. We found the mass difference to be less than $9 \times 10^{-08}eV$, which puts stringent limits on the symmetry breaking effect.

4.3 Potential Minimisation in Left-Right symmetric models and neutrino masses

4.3.1 Introduction

Although neutrinos are massless in the SM, they can be given masses in extensions of the SM based on either an extended fermion sector or an extended higgs sector. In the previous section we have seen how to accomodate the 17 keV massive neutrino without extending the fermion sector of the SM. The models incorporating an extension of the higgs sector have the advantage of explaining the smallness of neutrino mass through the see-saw mechanism. The see-saw mechanism can be very easily incorporated into the left-right symmetric extension[54] of the SM. In this section we present an analysis[2] of the minimization of the scalar potentials in left-right symmetric extensions of the SM with various choices of the higgs scalars and subsequently study their phenomenological implications regarding the neutrino masses.

4.3.2 Rudiments of Left-Right symmetric model

Left-Right symmetric models[55] are considered to be the most natural extensions of the standard model. Popularly one chooses the gauge group $G_{3221} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ or $G_{224} = SU(2)_L \times SU(2)_R \times SU(4)_c$ to describe the invariance properties of the model. When G_{3221} or G_{224} admits spontaneous symmetry breaking one recovers the standard model. Spontaneous symmetry breakdown takes place when the higgs fields transforming nontrivially under the higher symmetry group but not transforming under the lower symmetry group acquires a vacuum expectation value (vev). If one embeds the group G_{3221} or G_{224} in a grand unified theory or a partially unified theory then LEP constraints on $sin^2\theta_w$[56] can put strong bounds

[2] This section is based on the work reported in ref.[53]

on the breaking scale of the right handed $SU(2)_R$ group. On the other hand if one considers the left-right symmetric model with $g_L \neq g_R$ the right handed breaking scale can be lowered[57]. In this case the model becomes interesting as a rich set of phenomenological consequences can be directly tested in the next generation colliders. To achieve the inequality of the couplings a D-odd singlet higgs field η is introduced which on acquiring vev breaks the left-right parity (D-parity).

We are interested in the following symmetry breaking pattern:

$$\begin{aligned} SU(2)_L \times SU(2)_R \times SU(4)_c \ &\underset{M_X}{\longrightarrow} \ SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ &\underset{M_R}{\longrightarrow} \ SU(2)_L \times SU(3)_c \times U(1)_Y \\ &\underset{M_W}{\longrightarrow} \ SU(3)_c \times U(1)_Q \end{aligned} \tag{4.66}$$

If G_{224} is embedded in any higher symmetry group, then also most of the analysis will not change. In this sense our analysis is quite general. The advantage of starting with the group G_{224} instead of the group G_{3221} is that, we can discriminate between the fields which do and donot distinguish between quarks and leptons. This is important to understand the mass ratios of quarks and leptons.

We will also assume that $M_X = M_R$ which will imply that the scale of breaking of SU(4) color is the same as that of the breaking of the left right symmetry. This will not cause any loss of generality of our analysis. To specify the model further let us state the transformation properties of the fermions.

$$\begin{aligned} \psi_L &= \begin{pmatrix} \nu_L \\ e^-{}_L \end{pmatrix} : (2,1,4) \ ; \ \ \psi_R = \begin{pmatrix} \nu_R \\ e^-{}_R \end{pmatrix} : (1,2,4) \\ Q_L &= \begin{pmatrix} u_L \\ d_L \end{pmatrix} : (2,1,4) \ ; \ \ Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} : (1,2,4) \end{aligned} \tag{4.67}$$

The scalar fields which may acquire vev are stated below.

$$\begin{aligned} \phi_1 &\equiv (2,2,1) \ ; \ \ \phi_2 \equiv \tau_2 \phi_1^* \tau_2 \ ; \ \ \xi_1 \equiv (2,2,15) \ ; \ \ \xi_2 \equiv \tau_2 \xi_1^* \tau_2 \\ \Delta_L &\equiv (3,1,10) \ ; \ \ \Delta_R \equiv (1,3,10) \ , \ \ \eta \equiv (1,1,0) \end{aligned}$$

It has been shown in recent past that the LEP constraints on $sin^2\theta_w$[56] can put strong lower bound on the scale M_R. From renormalization group equations one can show that the right handed breaking scale has to be greater than 10^9 GeV. However one can show that when the D-Parity is broken the right handed breaking scale can be lowered. In that case a rich set of phenomenological predictions can be experimentally tested in high energy colliders. Here we consider the singlet field η which is odd under D-Parity. It breaks D-Parity when it acquires vev[57].

If we consider an underlying GUT, and start with the masses of the quarks and leptons to be the same at the unification scale, then, in the absence of ξ the low energy mass relations of fermions are not correct. This is because the field (2,2,1) contributes equally to the masses of the quarks and leptons. The situation can be corrected by the introduction of the field (2,2,15)[58]. This is the initial motivation to introduce the field ξ. Once it is there it allows new interesting baryon number violating decay modes which we discuss below.

Recently a lot of interest has been generated in the three lepton decay of the proton in SU(4) color gauge theory[59]. It can be shown that if the SU(3) triplet component of ξ remains sufficiently light it can mediate the three lepton decay mode of proton with a lifetime of 4×10^{31} years. In that case sufficient number of extra electron type neutrinos can be produced in the detector which can explain atmospheric neutrino anomaly. To keep the SU(3) triplet component of ξ sufficiently light, the following mechanism was proposed by Pati, Salam and Sarkar. If an extra (2,2,15) or (2,2,6) higgs field (henceforth called ξ' and χ) is introduced, its SU(3) triplet component will mix with the triplet component of ξ and hence there will be a light triplet in the model. These extra fields do not acquire *vev*. However the terms in the scalar potential which are linear in these extra fields can strongly constrain the other parameters of the model. In this paper we introduce such extra fields which do not acquire *vev* and study the terms in the scalar potential which are linear in these extra fields. The extra fields we consider here are,

$$\xi' = (2,2,15)\ ;\ \chi = (2,2,6)\ ;\ \delta = (3,3,0). \tag{4.68}$$

We shall see below that the linear term in the extra field δ will constrain the ratio of the D-parity breaking scale and the right handed symmetry breaking scale. We emphasise that in different models with extra scalars such study is necessary as it points out the extra scalar which is not favourable by the existing phenomenology.

4.3.3 Minimization of potential

Minimal choice of higgs scalars

The general procedure we adopt here is the following. First we write down the most general scalar potential which is allowed by renormalizability and gauge invariance. Next we substitute the vacuum expectation values in the potential and find out the minimization conditions. Here let us first write down the scalar potential with the scalar fields ϕ, Δ and η[60],

$$V(\phi_1, \phi_2, \Delta_L, \Delta_R, \eta) = V_\phi + V_\Delta + V_\eta + V_{\eta\Delta} + V_{\eta\Delta} + V_{\eta\phi}, \tag{4.69}$$

where the different terms in this expression are given by,

$$\begin{aligned}
V_\phi &= -\sum_{i,j} \mu_{ij}^2\ tr(\phi_i^\dagger \phi_j) + \sum_{i,j,k,l} \lambda_{ijkl}\ tr(\phi_i^\dagger \phi_j)\ tr(\phi_k^\dagger \phi_l) \\
&\quad + \sum_{i,j,k,l} \lambda_{ijkl}\ tr(\phi_i^\dagger \phi_j \phi_k^\dagger \phi_l) \\
V_\Delta &= -\mu^2\ (\Delta_L^\dagger \Delta_L + \Delta_R^\dagger \Delta_R) + \rho_1\ [tr(\Delta_L^\dagger \Delta_L)^2 + tr(\Delta_R^\dagger \Delta_R)^2] \\
&\quad + \rho_2\ [tr(\Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L) + tr(\Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R)] + \rho_3\ tr(\Delta_L^\dagger \Delta_L \Delta_R^\dagger \Delta_R)
\end{aligned}$$

$$V_\eta = -\mu_\eta^2\ \eta^2 + \beta_1\ \eta^4$$

$$\begin{aligned} V_{\Delta\phi} = & +\sum_{i,j}\alpha_{ij}\ (\Delta_L^\dagger\Delta_L + \Delta_R^\dagger\Delta_R)\ tr(\phi_i^\dagger\phi_j) + \sum_{i,j}\beta_{ij}\ [\ tr(\Delta_L^\dagger\Delta_L\phi_i\phi_j^\dagger) \\ & + tr(\Delta_R^\dagger\Delta_R\phi_i^\dagger\phi_j)] \\ & + \sum_{i,j}\gamma_{ij}\ tr(\Delta_L^\dagger\phi_i\Delta_R\phi_j^\dagger) \end{aligned}$$

$$V_{\eta\Delta} = M\ \eta\ (\Delta_L^\dagger\Delta_L - \Delta_R^\dagger\Delta_R) + \beta_2\ \eta^2\ (\Delta_L^\dagger\Delta_L + \Delta_R^\dagger\Delta_R)$$

$$V_{\eta\phi} = \sum_{i,j}\delta_{ij}\ \eta^2\ tr(\phi_i^\dagger\phi_j)$$

The vacuum expectation values of the fields have the following form:

$$\begin{aligned} <\phi> &= \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix}\ ;\ <\Delta_L>=\begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}\ ;\ <\eta>=\eta_0; \\ <\tilde{\phi}> &= \begin{pmatrix} k' & 0 \\ 0 & k \end{pmatrix}\ ;\ <\Delta_R>=\begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \end{aligned}$$

The phenomenological consistency requires the hierarchy $<\Delta_R>\ >>\ <\phi>\ >>\ <\Delta_L>$ and also that $k' << k$. Now the minimization conditions of the potential V are found out by differentiating it with respect to the parameters k, k', v_L, v_R and η_0 and separately equating them to zero. This will give us five equations for five parameters present. Solving the equations involving the derivatives with respect to v_L and v_R we get the relation between v_L and v_R :

$$v_L v_R = \frac{\beta k^2}{[(\rho-\rho') + \frac{4M\eta_0}{(v_L^2-v_R^2)}]},$$

where we have defined $\beta = 2\gamma_{12}$. The details of the derivation are presented in a subsequent section. We get in the M=0 limit,

$$v_L v_R \simeq \frac{\beta k^2}{[\rho-\rho']} \simeq \gamma k^2 \tag{4.70}$$

Here γ is a function of the couplings. However when the field η is present, v_L becomes differently related to v_R in the limit of large η_0.

$$v_L \simeq -(\frac{\beta k^2}{4M\eta_0})v_R \simeq (\frac{\beta k^2}{\eta_0^2})v_R \tag{4.71}$$

Here we see the important difference between the D-conserving and D-breaking scenarios.

This result was discussed in details in ref. [61]. In the D-parity conserving case, when the η field is absent one has to fine tune parameters to make γ arbitrarily snall so that the see-saw neutrino mass can be comparable to the Majorana mass of the left-handed neutrinos given by v_L. This fine tuning becomes redundant when the field η acquires *vev*.

In the presence ξ=(2,2,15)

When ξ is present, the most general scalar potential takes the following form:

$$V(\phi_1,\phi_2,\Delta_L,\Delta_R,\xi_1,\xi_2,\eta) = V_\phi + V_\Delta + V_\eta + V_\xi + V_{\phi\eta} + V_{\eta\Delta} + V_{\eta\phi} + V_{\phi\xi} + V_{\Delta\xi} + V_{\eta\xi} \quad (4.72)$$

The explicit forms of the terms involving ξ are listed below:

$$\begin{aligned}
V_\xi &= -\sum_{i,j} m_{ij}^2\, tr(\xi_i^\dagger \xi_j) + \sum_{i,j,k,l} n_{ijkl}\, tr(\xi_i^\dagger \xi_j \xi_k^\dagger \xi_l) + \sum_{i,j,k,l} p_{ijkl}\, tr(\xi_i^\dagger \xi_j)\, tr(\xi_k^\dagger \xi_l) \\
V_{\phi\xi} &= \sum_{i,j,k,l} u_{ijkl}\, tr(\phi_i^\dagger \phi_j \xi_k^\dagger \xi_l) + \sum_{i,j,k,l} v_{ijkl}\, tr(\phi_i^\dagger \phi_j)\, tr(\xi_k^\dagger \xi_l) \\
V_{\Delta\xi} &= +\sum_{i,j} a_{ij}\, [\, tr(\Delta_L^\dagger \Delta_L) + tr(\Delta_R^\dagger \Delta_R)]\, tr(\xi_i^\dagger \xi_j) \\
&\quad + \sum_{i,j} b_{ij}\, [\, tr(\Delta_L^\dagger \Delta_L \xi_i \xi_j^\dagger) + tr(\Delta_R^\dagger \Delta_R \xi_i^\dagger \xi_j)] \\
&\quad + \sum_{i,j} c_{ij}\, tr(\Delta_L^\dagger \xi_i \Delta_R \xi_j^\dagger) \\
V_{\eta\xi} &= \sum_{i,j} d_{ij}\, \eta^2\, tr(\xi_i^\dagger \xi_j)
\end{aligned}$$

The vacuum expectation value of ξ has the following form,

$$< \xi >= \begin{pmatrix} \tilde{k} & 0 \\ 0 & \tilde{k}' \end{pmatrix} \times (1,1,1,-3). \quad (4.73)$$

Here we may briefly mention the need to introduce the field ξ. The vacuum expectation value of the field ϕ is given by,

$$< \phi >= \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} \times (1,1,1,1). \quad (4.74)$$

Note that in the SU(4) color space the fourth entry is 1 for the *vev* of ϕ whereas it is -3 for the *vev* of ξ. Hence the *vev* of ϕ treats the quarks and the leptons on the same footing, whereas the *vev* of ξ differentiates between the quarks and the leptons. For example in the absence of ξ one gets $m_e^0 = m_d^0$, $m_\mu^0 = m_s^0$ and $m_\tau^0 = m_b^0$. Now including the QCD and electroweak renormalization effects in the symmetric limit it leads to the relation $\frac{m_e}{m_\mu} = \frac{m_d}{m_s}$. However when the field ξ is included in the masses in the symmetric limit they take the form $m_e^0 = m_e^\phi - 3m_e^\xi$ and $m_d^0 = m_d^\phi - m_d^\xi$.

The minimization conditions are again found by taking the derivatives of V with respect to the parameters $k, k', v_L, v_R, \tilde{k}^2$ and $\tilde{k'}^2$ and separately equating them to zero. Solving the equations involving the derivatives of v_L and v_R yields in the limit $\tilde{k}' << \tilde{k}$:

$$v_L v_R = \frac{[(w\tilde{k}^2 + \beta k^2)\, (v_L^2 - v_R^2)}{[(\rho - \rho')(v_L^2 - v_R^2) + 4M\eta_0]}. \quad (4.75)$$

Here we have defined $w = 2c_{12}$. Let us again check the special cases. Firstly the case without ξ can be recovered in the limit w=0, on the other hand the case with unbroken D-parity can be restored in the limit M=0; which is,

$$v_L v_R \simeq \frac{w\tilde{k}^2 + \beta k^2}{[(\rho - \rho')]}. \tag{4.76}$$

When D-parity is broken the v_L can be suppressed by η_0,

$$v_L = \frac{w\tilde{k}^2 + \beta k^2}{\eta_0^2} v_R. \tag{4.77}$$

We infer that the field ξ is allowed by the potential minimization and its introduction does not alter the general features of the see-saw condition between v_L and v_R.

4.3.4 Introduction of extra fields

Introduction of ξ'=(2,2,15)

We have already mentioned that there exist interesting models in the literature where the field ξ' is introduced to induce a sufficiantly large amplitude of the three lepton decay width of the proton. In these models the field ξ does not acquire *vev*. Hence after the minimization all terms other than the ones which are linear in ξ' drops out whereas the ones which are linear in ξ' puts constraints on the parameters of the model. Usually when any new fields are introduced in any model, which do not acquire *vevs*, it is assumed that it will not change the minimization conditions. As a result potential minimization with such fields were not done so far.

In this section we will first write down the linear couplings of the field ξ' *i.e.*

$$\begin{aligned}
V'_\xi &= -\sum_{i,j} \tilde{m}_{ij}^2 \, tr(\xi_i^\dagger \xi_j') + \sum_{i,j,k,l} n_{ijkl} \, tr(\xi_i^\dagger \xi_j \xi_k^\dagger \xi_l') \\
&\quad + \sum_{i,j,k,l} p_{ijkl} \, tr(\xi_i^\dagger \xi_j) \, tr(\xi_k^\dagger \xi_l') \\
&+ \sum_{i,j,k,l} u_{ijkl} \, tr(\phi_i^\dagger \phi_j \xi_k^\dagger \xi_l') + \sum_{i,j,k,l} v_{ijkl} \, tr(\phi_i^\dagger \phi_j) \, tr(\xi_k^\dagger \xi_l') \\
&+ \sum_{i,j} \tilde{a}_{ij} \, (\Delta_L^\dagger \Delta_L + \Delta_R^\dagger \Delta_R) \, tr(\xi_i^\dagger \xi_j') \\
&\quad + \sum_{i,j} \tilde{b}_{ij} \, [\, tr(\Delta_L^\dagger \Delta_L \xi_i' \xi_j^\dagger) + tr(\Delta_R^\dagger \Delta_R \xi_i^\dagger \xi_j')] \\
&+ \sum_{i,j} \tilde{c}_{ij} \, tr(\Delta_L^\dagger \xi_i' \Delta_R \xi_j^\dagger) \\
&+ \sum_{i,j} \tilde{d}_{ij} \, \eta^2 \, tr(\xi_i^\dagger \xi_j')
\end{aligned}$$

When this potential is minimised with respect to ξ' we get a relation between the couplings and the *vev* s. Obviously in this case due to large number of couplings of the field ξ' (which are independent parameters) this condition can be easily satisfied. A more stringent and interesting situation is the case where an extra field χ is introduced instead of ξ'.

Introduction of χ=(2,2,6)

It has been pointed out by Pati[62] that the field χ is a very economical choice for the mechanism that leads to appreciable three lepton decay of proton. The field χ is contained in the field 54-plet of SO(10) which has to be present for the breaking of SO(10). The terms linear in χ can be written as:

$$V_\chi = P\ \eta\xi\chi(\Delta_R - \Delta_L) + M\ \chi\xi(\Delta_R + \Delta_L). \tag{4.78}$$

These terms upon minimization give the condition

$$v_L = \frac{P\eta_0 - M}{P\eta_0 + M} v_R. \tag{4.79}$$

This means that to get $v_R >> v_L$ one has to fine tune $P\eta_0 - M << P\eta_0 + M$. This is interesting in the context of the three lepton decay of proton which will be discussed elsewhere [63].

Introduction of δ=(3,3,0)

In this case we first write down the linear couplings of the field δ:

$$V_\delta = M_1\delta(\Delta_L\Delta_R^\dagger + \Delta_R\Delta_L^\dagger) + M_2\delta\phi\phi^\dagger + C_1\eta\delta(\Delta_L\Delta_R^\dagger + \Delta_R\Delta_L^\dagger) + C_2\eta\delta\phi^\dagger\phi \tag{4.80}$$

These terms upon minimization gives the following conditions,

$$v_L v_R = -\frac{M_2 + C_2\eta_0}{2M_1 + C_1\eta_0} k^2 \tag{4.81}$$

In the limit of very large η_0 we can write,

$$v_L v_R \simeq k^2 \tag{4.82}$$

If we compare this relation with the see-saw relation of eqn(4.77) we get,

$$\frac{v_R^2}{\eta_0^2} = \frac{k^2}{w\tilde{k}^2 + \beta k^2} \simeq O(1). \tag{4.83}$$

Thus due to the introduction of δ the left-right parity and the left right symmetry gets broken almost at the same scale.

4.3.5 Details of potential minimization

When the spontaneous symmetry breakdown (SSB) occurs the scalar fiields acquire *vev*. Let us first consider the case when we include only the fields ϕ, Δ_L and Δ_R. After the SSB, the potential looks like :

$$\begin{aligned} V_1 &= -\mu^2\,(v_L^2+v_R^2)+\frac{\rho}{4}\,(v_L^4+v_R^4)+\frac{\rho'}{4}\,(v_L^2v_R^2)+2v_Lv_R[(\gamma_{11}\\ &+\gamma_{22})kk'+\gamma_{12}(k^2+k'^2)]+(v_L^2+v_R^2)\,[(\alpha_{11}+\alpha_{22}+\beta_{11})\,k^2\\ &+(\alpha_{11}+\alpha_{22}+\beta_{22})\,k'^2+(4\alpha_{12}+2\beta_{12})\,kk']\\ &+\text{terms containing k and k}'\text{ only} \end{aligned} \tag{4.84}$$

We have defined the new parameters as $\rho = 4(\rho_1+\rho_2)$ and $\rho' = 2\rho_3$. Minimisation with respect to v_L and v_R yields,

$$v_Lv_R = \frac{2\,[(\gamma_{11}+\gamma_{22})kk'+\gamma_{12}(k^2+k'^2)]}{\rho-\rho'} \tag{4.85}$$

This expression simplifies in the limit $k' << k$ to

$$v_Lv_R = \frac{2\,\gamma_{12}}{\rho-\rho'}k^2 \tag{4.86}$$

Here let us introduce the new scalar η which has a *vev* η_0. After the SSB, the scalar potential will be,

$$V_2 = V_1 - \mu_\eta^2\,\eta_0^2+\beta_1\,\eta_1\,\eta_0^4+M\,\eta_0\,(v_L^2-v_R^2)+\beta_2\,\eta_0^2\,(v_L^2+v_R^2)+\gamma\,\eta_0^2\,(k^2+k'^2) \tag{4.87}$$

Now the minimization with respect to v_L and v_R gives the following relation in the limit $k' << k$,

$$v_Lv_R = \frac{2\,\gamma_{12}k^2}{[\rho-\rho'+\frac{4M\eta_0}{(v_L^2-v_R^2)}]} = \frac{\beta k^2}{[\rho-\rho'+\frac{4M\eta_0}{(v_L^2-v_R^2)}]} \tag{4.88}$$

We have defined the new parameter $\beta = 2\gamma_{12}$. At this stage let us introduce the scalar field ξ. This will again introduce new terms in the scalar potential. The scalar potential after SSB now becomes,

$$\begin{aligned} V_3 &= V_2+(v_L^2+v_R^2)\,[(a_{11}+a_{22}+b_{11})\,\tilde{k}^2+(a_{11}+a_{22}+b_{22})\,\tilde{k}'^2\\ &+(4a_{12}+b_{12})\,\tilde{k}\tilde{k}']+2v_Lv_R[(c_{11}+c_{22})\tilde{k}\tilde{k}'+c_{12}(\tilde{k}^2+\tilde{k}'^2)]\\ &+\text{terms containing }\tilde{\text{k}}\text{ and }\tilde{\text{k}}'\text{ only} \end{aligned} \tag{4.89}$$

Now we minimise V_3 with respect to v_L and v_R. The see-saw relation becomes,

$$v_Lv_R = \frac{\beta k^2+2c_{12}\,(\tilde{k}^2+\tilde{k}'^2)}{[\rho-\rho'+\frac{4M\eta_0}{(v_L^2-v_R^2)}]} \tag{4.90}$$

This relation in the limit $\tilde{k}' << \tilde{k}$ becomes,

$$v_Lv_R = \frac{\beta k^2+w\,\tilde{k}^2}{[\rho-\rho'+\frac{4M\eta_0}{(v_L^2-v_R^2)}]} \tag{4.91}$$

Here we have defined $w = 2c_{12}$. This is the see-saw condition in the presence of ξ .

4.3.6 Neutrino mass matrix

The fermions acquire masses through the Yukawa terms in the lagrangian when the higgs fields acquire *vev*. The Yukawa part in the Lagrangian written in terms of fermionic and higgs fields is given by,

$$\begin{aligned} L_{Yukawa} &= y_1(\bar{f}_L f_R \phi_1) + y_2(\bar{f}_L f_R \phi_2) + y_3(\bar{f}_L^c f_L \Delta_L + \bar{f}_R^c f_R \Delta_R) \\ &\quad + y_4(\bar{f}_L f_R \xi_1) + y_5(\bar{f}_L f_R \xi_2) \end{aligned} \tag{4.92}$$

where y_i (i=1,5) are Yukawa couplings. With this notation neutrino mass matrix written in the basis (ν_L, ν_L^c) is

$$M = \begin{pmatrix} m_{M_L} & m_D \\ m_D & m_{M_R} \end{pmatrix} \tag{4.93}$$

where m_{M_L} (m_{M_R}) is the left (right) handed Majorana mass term whereas m_D is the Dirac mass term. These terms can be related to the Yukawa couplings and *vevs* through the following relation,

$$\begin{aligned} m_{M_L} &= y_3 v_L \\ m_D &= (y_1 + y_2)(k + k') + (y_4 + y_5)(\tilde{k} + \tilde{k}') \\ m_{M_R} &= y_3 v_R \end{aligned} \tag{4.94}$$

Upon diagonalization of the mass matrix we obtain the mass eigenvalues. Now let us consider the simplifying assumption that all the Yukawa couplings are of order h and the *vev* s k' and $\tilde{k}'$ are much smaller than the *vev* s k and $\tilde{k}$ respectively. Under this assumption the eigenvalues become,

$$\begin{aligned} m_1 &= y_3 v_R \\ m_2 &= m_{M_L} - \frac{M_D^2}{m_{M_R}} = y_3 v_L - \frac{h^2(k^2 + \tilde{k}^2)}{y_3 v_R} \end{aligned}$$

We substitute for v_L from the see-saw condition to get in the D-parity conserving $g_L = g_R$ case,

$$m_2 = y_3 \frac{(\beta k^2 + w\tilde{k}^2)}{v_R} - \frac{h^2(k^2 + \tilde{k}^2)}{y_3 v_R} \tag{4.95}$$

We notice that the second term in the right hand side is suppressed by the square of the Yukawa coupling. Due to this the first term dominates. If we want to make the first term small compared to the second we need to fine tune the parameters. Hence one has to fine tune such that $\beta k^2 + w\tilde{k}^2 \simeq 0$ to get acceptable value of the the light neutrino mass. However in the presence of the *vev* of η we get,

$$m_2 = y_3 \frac{w\tilde{k}^2 + \beta k^2}{\eta_0^2} v_R - \frac{h^2(k^2 + \tilde{k}^2)}{y_3 v_R}. \tag{4.96}$$

In the limit of very large η_0 the first term drops out of the expression and one gets rid of the fine tuning problem. However, if the field δ (which does not acquire any *vev*) is present, we cannot get away with the fine tuning problem, since it is difficult to maintain $v_R << \eta_0$.

4.3.7 Conclusion

We have incorporated the scalar field ξ=(2,2,15) in the scalar potential of the $SU(4)_{color}$ left-right symmetric extension of the standard model. This field is necessary to predict correct mass relationships of the quarks and the leptons. After including the field ξ in the scalar potential we have carried out the minimization of potential, and worked out the relationship between the *vevs* of the left-handed and the right-handed triplets (see-saw relationship). We have shown that the field ξ is allowed by potential minimization and its inclusion does not change the qualitative nature of the see-saw relationship existing in literature. Once the see-saw relationship between the v_L and v_R is known we have gone ahead to construct the neutrino mass matrix. We have shown that even after the inclusion of the field ξ one needs to fine tune the parameters in the $g_L = g_R$ case to predict small mass for the left handed neutrino, while in the $g_L \neq g_R$ case one naturally gets a large suppression for the left handed neitrino mass. This happens because even after the inclusion of the field ξ the light neutrino mass gets suppressed by the *vev* of the D-odd singlet η rather than the *vev* of Δ_R.

If there are new scalar fields which donot acquire any *vev*, then to check the consistency one has to write down their linear couplings with other fields and after minimizing the potential use the appropriate *vevs* of the various fields. In some cases the presence of such fields can give new interesting phenomenology. We studied some such cases for demonstration.

In recent past it has been shown that the three lepton decay of the proton can successfully explain the atmospheric neutrino anomaly by producing excess of electron type neutrino in the detector. To produce phenomenologically acceptable decay rate in the three lepton decay mode a mechanism was suggested by Pati, Salam and Sarkar, and later by Pati. In this mechanism one has to include extra scalars ξ'=(2,2,15) or χ=(2,2,6) which do not acquire *vevs*. We have calculated the linear couplings of such terms in the scalar potential and shown that these terms give relations that constrain the values of parameters and *vevs* of the model. In this work, we have given these constraints. We have also included, as a special case, the extra scalar δ=(3,3,0) and shown that its inclusion forces the right handed breaking scale and the D-parity breaking scale to become almost equal.

Chapter 5

Summary and Conclusions

In this concluding chapter, we summarize the work presented in this thesis. The objective of this study was to understand fermion mass hierarchy and the origin of quark mixing within the domain of the SM and the possible extensions of SM that are allowed phenomenologically. This thesis consists of two distinct but connected parts. The first part deals with quarks whereas the second part is devoted to neutrinos. The results are summarized below.

5.1 Studies related to quarks

We study the quark masses within the framework of the SM given the CKM quark mixing matrix to be symmetric. First it was shown that if the CKM matrix is symmetric then the top quark mass has to be heavier than 180 GeV, to be consistent with the experimental results of ϵ_K, the parameter describing the indirect CP violation in the interactions changing strangeness by two units ($\Delta S = 2$), and the measurement on B_d–$\overline{B_d}$ mixing parameter x_d (which gives the time-integrated probability of a $\overline{B_d}$ appearing in a B_d beam) for the Bag constant $B_K = 1, 2/3$; if the Bag constant $B_K = 1/3$ then $m_t > 275$ GeV. The parameters q and δ(CP violating phase) are constrained to be in the range

$$.113 \leq q \leq .130 \qquad 8.0^\circ \leq \delta \leq 31.1^\circ$$

for the symmetric CKM matrix over the allowed range of the top quark mass $80GeV \leq m_t \leq 270GeV$. To get a comparative idea it should be noted that accurate measurements, especially at LEP of the properties of Z^0, together with the collider and ν data yield an indirect value for m_t :

$$m_t = 164^{+16}_{-17}\ ^{+18}_{-21} GeV.$$

Secondly, we address the important question of how to derive the symmetric quark mixing starting from the Yukawa couplings in a natural way. In this regard we tried to find the constraints on the quark mass matrices for the symmetric CKM matrix. The symmetry constraint was written as an equation involving the parameters of the mass matrices using flavour projection operators in a basis where M_u is diagonal. The numerical ranges for the mod elements of

M_d were given in this basis. This procedure was repeated in the basis where M_d is diagonal. Then, the necessary condition for having a symmetric V in terms of the matrices U and D was derived. A particularly interesting basis was chosen where $U = D^*P$; P being a phase matrix and the ranges for the mod elements of M_u, M_d in that basis was found out using a convenient parametrization for V. It was noticed that none of the off-diagonal elements of M_u and M_d is consistent with zero for a symmetric V, which means such forms for mass matrices cannot be obtained from any symmetry. But, in principle there exists infinite number of other bases related to each other by similarity transformations. So it is apparent that the numbers provided for the allowed ranges of the mod elements of mass matrices are not basis independent. Finally the symmetry constraint was presented in a basis-independent form.

Then, we cheked the phenomenological validity of a new scheme, in which there was an attempt to obtain an approximately symmetric CKM matrix starting from mass matrices of the type $M_{U,D} = \kappa_{U,D}M_0 + X_{U,D}$ where $\kappa_{U,D}$ are numerical constants; M_0 is a real $3X3$, rank-one matrix and the matrices X_U and X_D are correction terms that have to be added to M_0 to obtain the non-zero masses of the light two generation quarks since the rank-one mass matrix M_0 has only one non-vanishing eigenvalue. We have shown that out of the three interesting solutions of the symmetric CKM matrix discussed in this scheme one is inconsistent with experiments, whereas another one requires a very heavy top quark mass ($m_t \approx 255GeV$) to be consistent.

5.2 Studies related to neutrinos

We studied the phenomenological consequences of massive neutrinoes. First, we have made an analysis of the spontaneous symmetry breaking for the Higgs sector taking various Higgs representations in the context of generalised ($g_L \neq g_R$) left-right symmetric model, including the higgs field $\xi = (2,2,15)$ that predicts the correct low energy ratio of $\frac{m_b}{m_\tau}$ and a singlet field η which breaks the left-right parity. As special cases we also include $\xi' = (2,2,15)$ and $\chi = (2,2,6)$ (which are interesting in the context of the three lepton decay mode of the proton) and field $\delta = (3,3,0)$ none of which acquire *vev*. We show that the linear couplings of these fields upon minimization put fine tuning conditions on the parameters of the model. We carry out the minimization of these potentials explicitly. In all the cases the relationship between the *vev* s of the left and right handed triplets v_L and v_R are given. The phenomenological consequences of this minimization regarding the neutrino masses are also studied.

Secondly, we studied constraints on the neutrino mixing matrix from the various oscillation data, neutrinoless double beta decay and the limit on the ν_e, ν_μ and ν_τ masses assuming only three generations of left-handed neutrinos and no sterile neutrinos to accomodate a 17 keV neutrino without extending the fermion sector. In the limit when all the three eigenvalues are nondegenerate and the mass differences are larger than $100eV^2$, we identify ν_τ with the 17 keV neutrino and vary the mixing probability between 3% and 0.3%. We found a very narrow allowed region for the various mixing angles. The allowed values of m_{ν_μ} lie between 145 keV and 205 keV for 1% mixing and between 135 keV and 240 keV for 3% mixing (our result differs from the earlier similar works with 3% mixing, where some approximations were made). The

$\nu_e \rightarrow \nu_\mu$ oscillation probability is found to lie between .001 and .002 for consistency. We then considered the allowed amount of the symmetry breaking when ν_μ and ν_τ form a pseudo-Dirac particle. We found the mass difference to be less than $9 \times 10^{-8} eV$, which puts stringent limits on the symmetry breaking effect on the neutrino mass matrix.

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