STUDY OF NONLINEAR STRUCTURES IN MAGNETISED PLASMAS

by

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CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis of research, for any degree or diploma, of any university or institution.

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Place: Ahmedabad
Date: June 1990

ABSTRACT

In recent years there has been an effort to model turbulence as the nonlinear interaction of self consistent coherent fluctuations of the system. Vortex theories in fluid turbulence and the clump theory for kinetic turbulence are the most prominent examples. This thesis reports the first detailed study of such coherent, stationary, nonlinear solutions for the case of a magnetised inhomogeneous plasma in both fluid and kinetic limits.

Following the basic theme of identification of nonlinear coherent exact solutions, the equations describing nonlinear fluid drift waves have been re-examined. It is shown that a new kind of two dimensional monopole vortex solution is possible if nonlinear parallel ion motion is retained. These solutions are different from the Hasegawa-Mima type dipole vortices or the ones due to scalar nonlinearities related to strong temperature gradients. The effect of weak temperature gradients on these solutions is also studied numerically.

One of the latest models for kinetic turbulence, is the theory of phase space holes. The primary direction was given by Dupree (1982) in which it was shown that the maximum entropy state of a self-trapped equilibrium is a BGK mode in the case of a homogeneous unmagnetised plasma. This most probable BGK mode was called a phase space hole.

In this thesis this concept has been extended to the

case of a magnetised inhomogeneous collisionless plasma. General steady state solutions of the drift kinetic equation are found and the distribution function in the mixing region is determined by entropy maximisation subject to appropriate constraints. Then an inverse BGK type problem is set up and the steady state equation for the electrostatic potential is solved numerically. We have looked for a general class of nonlinear oscillatory solutions.

Two main cases are dealt with. The first is the problem of the most probable nonlinear steady state of the one dimensional drift wave. The dominant nonlinear mechanism here is the phase space or parallel trapping of resonant electrons that drive the linear instability of the drift wave. Temperature gradient and ion trapping effects are also incorporated.

The second case is more general and includes a nonlinearity which, from recent gyrokinetic simulations, appears to be more dominant in two dimensions at lower amplitudes than parallel trapping. This is the physical space trapping of electrons due to the perpendicular ExB resonance. However, in the present calculation both effects are retained and the resulting two dimensional maximum entropy state is studied. The case of ExB resonant trapping of electrons is then combined with the ExB convection of fluid ions to set up some interesting almost maximum entropy monopole solutions of the system.

From the point of view of exploring the high frequency regime for the existence of nonlinear stationary states, the special case of an electrostatic wave traveling perpendicular to the ambient magnetic field is studied. A new mechanism is proposed to keep the particles trapped in the wave field without resorting to relativistic effect. It is demonstrated that using nonuniformities of the magnetic fields in the direction of wave propagation, the trapped particles can be accelerated indefinitely. The resultant damping of the wave is also estimated.

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CONTENTS

CHAPTER I

INTRODUCTION

A nonequilibrium plasma is a system that occurs frequently in both laboratory and astrophysical systems. The nonequilibrium nature of the system can be the result of the presence of several types of sources of free energy. These can be in the form of density and temperature gradients, nonuniform magnetic fields, presence of beams and flows, etc. The most prevalent tendency of such a system is to release this free energy in order to approach a state of equilibrium. This is done via the mechanism of generating a large spectrum of collective modes. These modes are characterised by the relevant driving mechanism and geometry of the system. Further, they are often unstable in the presence of the sources of free energy and grow beyond their linear limits. As a result the entire system of such interacting modes is highly nonlinear. The study of such nonlinear systems is of great importance in order to understand the transport of particles and energy in laboratory and space plasmas. A large body of very significant work exists today in the area of plasma turbulence.

Traditionally, turbulence theories take the view that turbulence can be described as a collection of nonlinearly interacting Fourier modes of the system. This is a natural consequence of the assumption that turbulence results from instabilities due to which waves grow from small amplitude linear perturbations to large amplitude fluctuations. The quasilinear theory and the renormalised perturbation theories have been developed to a great extent with these underlying principles (Kadomtsev 1965). These are the kinetic theories of turbulence. The basis of the majority of kinetic theories in plasma physics is the self-consistent Vlasov-Poisson system of equations. In a two dimensional (x,v) phase space these are:

$$\partial_t f + V \partial_x f + \frac{e}{m} E \partial_v f = 0 \ ,$$

$$\partial_x E = - 4 \pi e \int_{-\infty}^{\infty} dv\, f \ .$$

The quasilinear theory examines the effect of the turbulent fluctuations on the time evolution of certain spatially or ensemble averaged quantities. This approach is applied to 'weakly turbulent' situations. The nonlinearities are weak and can be written as perturbations to the linear behaviour. Then the distribution function can be separated as:

$$f(x, v, t) = f_0(v, t) + f_1(x, v, t),$$

where, $$f_0(v,t) = \langle f_0(x, v, t) \rangle_x \ .$$

f_o is a spatially averaged distribution and is a slow function of time. f_1 is the rapidly oscillating part and represents a system of oscillations with randomly distributed phases. In the quasilinear theory it is determined by the linearised form:

$$f_k = \frac{e}{m} \frac{i E_k}{\omega_k - kV} \partial_v f_0 (v,t) .$$

ω_k is the characteristic frequency of the k^{th} mode. This response forms the turbulent background. In the presence of such fields a typical plasma particle with a resonant velocity $V = \frac{\omega_k}{k}$, undergoes a random walk in velocity space, since the acceleration provided by these fields is random. The slow time evolution of $f_0(v,t)$ is then given by:

$$\partial_t f_0 = \partial_v (D_v \partial_v f_0) .$$

This is the quasilinear diffusion equation. The assumption used here is that the growth rate of the modes is smaller than the mode frequency; $\gamma_k << \omega_k$. So the interaction between individual modes can be neglected, and they can be considered to be a superposition of separate modes. This assumption breaks down even when there is weak interaction between modes. Then, the growth rates become functions of the fields due to these modes and the quasilinear theory cannot be applied.

The most representative result of this treatment is the case of Landau damping. The quasilinear theory predicts that the distribution in the resonant region will become progressively flatter, leading to a diminishing damping rate. In the long time limit it will become completely flat and the damping will stop.

The renormalised perturbation theories go one step beyond the quasilinear approximation. They take into account the interaction of the Fourier modes in the large amplitude limit. The response of the plasma is then described by an infinite set of kinetic equations derived from the Vlasov equation. They describe nonlinear phenomena such as wave-wave coupling, scattering of waves by particles, decay of a single mode into two waves, etc. It is impossible to handle the entire infinite set to study these effects, so truncation schemes have been developed to reduce the system. In the weak turbulence limit the 'random phase approximation' is used. This allows only the description of the modulus of the wave amplitudes, averaged over the phases of the waves. Correlations among the fluctuating quantities upto fourth order are retained, effectively truncating the set of coupled equations. In the strong turbulence limit the 'weak coupling approximation' is used. The turbulent motion is described by a system of nonlinear integral equations for the spectral density and the Green's function describing the response of the system to an external force. As the coupling between the modes decreases, these equations reduce to the kinetic wave equations.

All these theories assumed a coarse grained form of the distribution function as described by the Vlasov equation, where the fine scale was generally taken to be the Debye length. Correlations over a scale of the Debye length were neglected. The distribution function used in the Vlasov equation was essentially the one point distribution. Any correlations over scale lengths larger than the Debye length were self-consistently determined in terms of the individual local one point distributions. The two point distribution function could then be expressed as the product of two one point distributions. All perturbed quantities could be given in terms of the perturbed one point distribution to study phenomena whose scale lengths were at the least of the order of the Debye length.

Particle orbits in turbulent plasmas become random, since they see the combined field of a collection of modes with randomly distributed phases. This continued to be the basic assumption of the turbulence theories. If this happens in a region of phase space where the gradient of the average density is significant then these random orbits would bring about a considerable rearrangement of the local phase space density. This is called mixing and is the only mechanism through which a collisionless Vlasov system can relax to equilibrium and is inherent in all the previous theories.

What these theories neglected was the fact that, the time scale of this mixing process is finite and a function of the smallest cell size in phase space or the assumed fine scale. But this implies that for a given degree of coarse graining there would always remain a few elements of phase space which have not had time to phase mix down to fine scales. In fact, as was pointed out by Dupree (1972,1978), these could act as additional sources of turbulent fluctuations through their interaction with the background fields.

The theory of clumps introduced the existence of these entities into the renormalised perturbation theories. A clump is a fluctuation produced when the phase space density in a small region is moved randomly to a new location where its value $\tilde{f}$ differs from the local average f_o. It is clear, therefore, that the fluctuation cannot be described in terms of the local coarse grained distribution. It appears as a peak in the two-point correlation function. It was shown that a clump persists as a single coherent entity for a long enough time to interact with the background turbulence before velocity diffusion effects break it up. The typical clump life times are,

$$\tau_{cl} = -\tau_o \ln \left\{ \frac{1}{3} \left[k_o^2 \left(\Delta x^2 - 2 \Delta x \Delta v \tau_o + 2 \Delta v^2 \tau_o^2 \right) \right] \right\},$$

where, $\tau_o = (4 k_o^2 D_v)^{-1/3}$, k_o is the average wavenumber of the turbulence, D_v is the diffusion coefficient and Δx, Δv the clump scale lengths in phase space. τ_o is the mixing time and is finite. If a clump is preserved for a time τ_{cl}, then the mean square fluctuation produced due to its presence would be, by Dupree's estimates:

$$\langle \tilde{f}^2 \rangle \cong 2 \tau_{cl} D_v (\partial_v f_o)^2 .$$

It was recently pointed out by Dupree (1982) that the main deficiency of the clump theory was that it did not take into account electric fields that are generated self consistently by the clump. He proposed that, these self fields would be instrumental in bringing about the self trapping of these random fluctuations, effectively increasing their lifetime. As is clear from the expression for the fluctuation level due to a clump, this would also increase with τ_{cl}. This is also another reason for studying this phenomenon. However, it is not possible to include the self trapping effects in a traditional perturbative treatment. The orbits of the trapped particles in phase space are turned around and the bounce frequency of such a particle is typically proportional to the square root of the trapping potential. That is not to say that there is no wave-particle interaction of the trapping type included in these theories. A single particle, for example, would see the field of a fluctuation that is large enough to trap it but it will typically spend less than a single bounce period in the field of this wave before being taken out by the effect of the ambient turbulent field. It is not possible, therefore, to form coherent structures with the help of this interaction. Hence it is necessary to consider a complementary approach to the perturbation theories to include this effect.

The earliest observation of coherent structures in turbulent plasmas came from Morse and Nielson (1969). They analysed the simulation of the phase space dynamics of electron-electron two stream generated electrostatic turbulence and saw that it was dominated by large phase space structures having life times longer than the typical turbulent time scales. More specifically, the formation of coherent structures with particle trapping effects has also been studied by previous authors. Self-consistent saturated states of an electron plasma wave due to particle trapping were studied by Bernstein, Greene and Kruskal (1957). The BGK modes are nonlinear, coherent, exact solutions of the Vlasov-Poisson system with particle trapping effects. The particles making up the fluctuations are self-consistently trapped by the potentials they produce, giving rise to coherent structures in phase space. The time evolution of a large amplitude wave towards such structures due to nonlinear Landau damping was given by O'Neil (1965). He showed that the phase mixing of the particles trapped in the potential wells of the wave leads to its saturation at finite amplitudes. The concept of BGK modes was later extended to study trapped particle effects in ion acoustic waves (Schamel 1979 and references therein).

It was proposed by Dupree (1982) that, if clumps were to get trapped by their self fields, they would be like BGK modes and in the isolated state would have infinite lifetimes. This presents an entirely different picture of turbulence as against the traditional theories. Instead of a large spectrum of large amplitude interacting Fourier waves, there is now a collection of isolated nonlinear coherent structures in phase space. Dupree (1982), for a one dimensional unmagnetised case, proposed that phase space holes could be modeled as the most probable or maximum entropy BGK modes. The strength of their interaction would depend on their density in phase space. In the limiting case of sufficient hole density, the hole-hole interaction would overcome the self binding force of a single hole and the system would revert to the clump mode. However, in the case where holes are only weakly interacting, turbulence can be regarded as a collection of such almost-coherent structures. It is understood, of course, that turbulent fluctuations cannot be exact BGK modes since they are continually interacting with each other, but it is a good starting approximation. Coherent structures in the form of eddies or vortex forms are well known to be present in turbulent fluids. The phase space holes suggested by Dupree (1982) are the exact analogue of these vortex structures, representing self-organised motion in turbulent systems.

In this thesis, this concept of phase space holes as maximum entropy exact nonlinear coherent solutions of the relevant kinetic equations, is extended to the case of a magnetised, inhomogeneous, collisionless plasma. The typical Fourier spectrum of such a system consists of a large number of drift type modes, which have frequencies less than the ion cyclotron frequency. They are linearly unstable in most of the parameter space covered by systems like tokamaks and magnetospheric plasmas. Hence they are regarded as prime candidates for some important nonlinear phenomena, such as anomalous particle and energy transport, observed in these systems. It would be appropriate at this point to briefly review the physics of the linear drift wave and introduce some of the terminology used frequently henceforth.

Consider a plasma in slab geometry with a uniform magnetic field B in the z-direction and an equilibrium density gradient in the x-direction. The equilibrium fluid drifts are the electron and ion diamagnetic drifts, given by:

$$\underset{\sim}{V}_d = -\frac{c}{nq}\frac{\underset{\sim}{B} \times \underset{\sim}{\nabla} p}{B^2}$$

K_n is the scale of the density gradient, T_e the electron temperature, c the velocity of light and e the electronic charge. Typical drift waves have phase velocities in the range, $V_{thi} << \omega/k_{\parallel} << V_{the}$.

Then, typically electrons have the Boltzmann distribution and their linearised response can be written as:

$$\frac{n_{e1}}{n_o} = \frac{e\tilde{\phi}_1}{T_e}.$$

The ion continuity equation is:

$$\partial_t n_i + \underset{\sim}{\nabla} \cdot (n_i \underset{\sim}{V}) = 0,$$

$$\underset{\sim}{V}_\perp = \underset{\sim}{V}_E + \underset{\sim}{V}_p, \quad \underset{\sim}{V}_E = c\,\frac{\underset{\sim}{E} \times \underset{\sim}{B}}{B^2}, \quad \underset{\sim}{V}_p = -\frac{Mc^2}{eB^2} d_t \underset{\sim}{\nabla}_\perp \tilde{\phi}_1.$$

Neglecting parallel velocity, linearising and looking for plane wave solutions of the type, $\sim \exp(ik_y y - i\omega t)$, gives the perturbed ion density to be:

$$\frac{n_{i1}}{n_o} = \left(-\frac{k_y V_d}{\omega} + \rho_s^2 \nabla_\perp^2\right)\frac{e\tilde{\phi}_1}{T_e}.$$

It is assumed that for a low β plasma the perturbations are electrostatic and $k_x << k_y$. The first term on the RHS is due to the ExB drift coupling with the density gradient. The second term represents the charge separation coming from the ion polarisation drift. Drift oscillations are quasineutral, so setting $n_e \sim n_i$, we obtain the dispersion relation:

$$\omega = -\omega_* / (1 + k_y^2 \rho_s^2),$$

where, ρ_s is the ion Larmor radius at electron temperature and ω_* ($= k_y c T_e K_n / eB$), the drift frequency. When the linearised parallel ion motion is included for an oblique wave (having a finite $k_{\parallel}$ along B), there is another branch of the linear drift wave that represents the modified ion acoustic wave due to the density gradient. The dispersion relation then becomes:

$$\omega^2 - \omega\omega_* - k_{\parallel}^2 c_s^2 = 0.$$

In the kinetic limit the linear density gradient driven drift wave is unstable. The free energy residing in the density gradient is released via the mechanism of wave-particle interaction. It is convenient to describe low frequency phenomena like drift waves in the guiding centre or drift approximation. This means ignoring effects that have perpendicular scale lengths smaller than the ion Larmor radius. Frequencies larger than the ion Larmor frequency are averaged out and the gyromotion of particles around the field lines is ignored. Then the perpendicular velocities can be replaced by single particle drifts in those directions. In such a description the magnetic moment, $\underset{\sim}{\mu}$, is an invariant and in the guiding centre phase space the distribution function can be written as:

$$f(\underset{\sim}{x}, \underset{\sim}{v}, t) \equiv f(\underset{\sim}{x}, v_{\parallel}, \underset{\sim}{\mu}, t).$$

Conservation of the density of guiding centres then leads to the well known drift kinetic equation (DKE):

$$\partial_t f + \underset{\sim}{\nabla} \cdot (\underset{\sim}{V}_{\perp} f) + V_{\parallel} \nabla_{\parallel} f + \frac{q}{m} E_{\parallel} \partial_{V_{\parallel}} f = 0 .$$

The ordering of the DKE is typically taken to be:

$$\frac{e\tilde{\phi}}{T_e} \sim \frac{\omega}{\omega_{ce}} \sim \frac{\rho_s}{L_n} \sim \epsilon << 1 .$$

Electrons having velocities close to the parallel phase velocity of the wave undergo large ExB displacements in the direction of the density gradient. Electrons from the low density region are taken into high density regions enhancing the density fluctuation. This creates a phase difference between the potential and density perturbation giving a net growth. The growth rate is given by:

$$\gamma_l = \sqrt{\pi} \, \frac{\omega_r^{\,2}}{k_{\parallel} \sqrt{2} \, V_{the}} \left(k_y^2 \rho_s^2 \right) .$$

This mechanism of the drift instability relies entirely on the resonant interaction of the electrons with the wave. This wave-particle interaction is absent in the fluid theory. This lack of a path for the dissipation of the free energy makes the fluid drift modes stable.

Theories of drift turbulence in both fluid and kinetic limits are varied and well developed. As before, they are based on the principle of interacting nonlinear modes which are assumed to have grown due to the instability mechanisms described earlier. A large amount of work has been done to extend the quasilinear and perturbative theories for the understanding of drift turbulence. The concept of clumps has also been incorporated in the theory of strong drift turbulence (Dupree 1978). As pointed out earlier the major assumption of this thesis is that only completely coherent nonlinear steady state structures are constructed and studied. The time evolution of the system towards these states is not studied. However, the physical ideas used in this thesis have had considerable input from earlier attempts at understanding drift turbulence. Therefore, it will be useful to briefly go through the assumptions and results of these theories.

The Hasegawa-Mima equation (Hasegawa and Mima 1978) describes fluid drift turbulence in terms of the nonlinear ion dynamics with long parallel and short perpendicular wavelengths. The ExB and polarisation drift nonlinearities of ions are described and electrons are assumed to be adiabatic. Earlier authors have looked for exact coherent nonlinear solutions of this equation in different limits, in the form of dipole or monopole vortex solutions (Hasegawa et al. 1979, Miess and Horton 1983, Laedke and Spatschek 1988, Lakhin et al 1987). The phenomenon of self-organisation in fluids is well known and gives rise to eddies or vortex formation. These coherent solutions are the analogue of this concept of intermittancy in fluid turbulence. They establish the link between strong turbulence and self-organised motion. The stability of such structures to various perturbations is an area of much current interest. Attempts are being made to build theories for fluid drift turbulence on the basis of the interaction of a large number of such structures (Miess and Horton 1982, Horton 1988). In chapter II we have re-examined this equation for the presence of new monopole vortex solutions in the presence of nonlinear parallel ion motion.

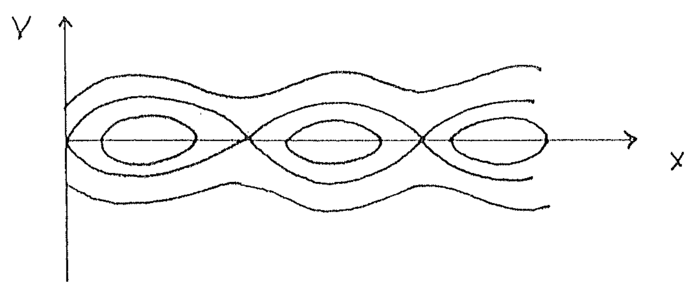
In the kinetic limit, the main focus of interest in this thesis is the nonlinear wave-particle interaction aspect. For a magnetised inhomogeneous plasma this has a special significance. We shall consider two cases of such an interaction. The first is the well known parallel or phase space trapping of particles having velocities close to the parallel phase velocity of the wave. This is the effect leading to the formation of BGK modes. For a simple one dimensional, homogeneous, unmagnetised plasma, the steady state conserved quantity is the total energy.

$$W = \frac{1}{2} m v^2 + q \tilde{\phi}$$

The parallel resonance, in a frame moving with the phase velocity ω/k, is given by:

$$\omega - kV = \left(\frac{e k^2 \tilde{\phi}}{m} \right)^{1/2}$$

Then the phase curves of the steady state distribution function have the following characteristic form:



The trapped particles have a bounce frequency given by,

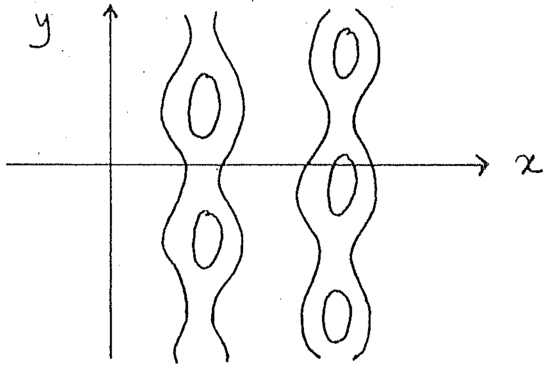
$$\omega_B^2 = e k^2 \tilde{\phi} / m$$

Henceforth, this effect will be referred to as, velocity space trapping, parallel trapping or phase space trapping. Dupree(1982) has formulated the maximum entropy states for a homogeneous unmagnetised plasma, with parallel trapping effects. We shall examine the analogous one dimensional case for a magnetised inhomogeneous plasma in chapter III.

The phenomenology considered in chapter IV of this thesis deals with the second kind of wave-particle interaction typical to the case of a drift wave; the physical space trapping of particles due to the resonance between the perpendicular phase velocity of the wave and the ExB drift velocity of the particles in that direction. This effect has been studied by Dupree(1967), Ching(1973) and Hirshman(1980) theoretically and by Lee et al.(1984), Smith et al.(1985), Federici et al.(1987), and Dimits(1988) through gyrokinetic particle simulations. As shown by Ching (1973), Hirshman (1980) and Smith (1985), the Hamiltonian for a single particle trapped in the perpendicular physical space for known form of the wave potential is,

$$H = \frac{c}{B} \tilde{\phi}(x, y) + \frac{\omega - k_{\parallel} V_{\parallel}}{k_{\perp}} x$$

It is clear that far away from the parallel resonance the (x,y) trajectories will be almost straight lines. But near the resonance the lines will close giving trapped trajectories. The following picture illustrates the situation:



Henceforth, this effect will be referred to as, perpendicular trapping, physical space trapping or ExB trapping. The bounce frequency of such a trapped particle is typically:

$$\omega_{E\times B} = \frac{c k_\perp^2 \tilde{\phi}}{B}$$

Note that the ExB trapping of both electrons and ions occurs at the same amplitude since the ExB drift is mass independent. Also, this effect becomes important at amplitudes lower than those required for parallel trapping to dominate. This can be seen by comparing the respective trapping frequencies:

$$\frac{\omega_B}{\omega_{E\times B}} : \frac{e\tilde{\phi}}{T_e} : \left(\frac{k_\parallel}{k_\perp}\right)^2 \left(\frac{M}{m}\right) << 1$$

The work of Dupree(1967) and Ching(1973) emphasised the role played by ions in the saturation mechanism of drift turbulence. Dupree(1967) showed that in the case of the current driven drift instability, the growth rate is suppressed by a nonlinearly generated 'broadening' of the parallel resonance. Basically, the point here was that, due to the presence of a large number of incoherent modes, there is a phase difference between the density and potential fluctuations, giving a net electron diffusion in the linear stage. As the instability grows, this flux becomes larger. If now the ions are treated linearly, the instability will never saturate, since in the linear limit there is no balancing ion diffusion. Also, typically, phase velocities of these waves are much larger than ion thermal velocities. Therefore ions cannot absorb energy until they get trapped by the wave. Therefore it becomes necessary to include the most dominant of the wave-particle interactions which is the perpendicular trapping. This led to a broadening of the parallel resonance and some parts of the ion distribution could then carry out ion Landau damping to saturate the spectrum.

The model presented in this thesis does not contain the effects studied by Dupree in the resonance broadening theory. The kinetic theory of the perpendicular trapping of resonant ions is not attempted. Also, the model studied here assumes a completely coherent response of the plasma and so that the system consists of a large number of phase space holes all propagating at the same phase velocity. So even though the relationship between the steady state density and potential fluctuations is complicated and contains effects of the order of the square root of potential, there is no net phase difference between the two. This in effect ignores any diffusion processes. It may become important when a statistical ensemble of such holes with a distribution in phase velocities is considered in the limit of weak hole-hole interaction. The presence of collisions can also introduce a certain incoherence and hence an enhanced diffusion. These effects are not studied in this thesis.

The work of Ching(1973) also emphasises ion effects in the saturation process. However, this treatment is completely coherent. It is shown that though the parallel resonance remains sharp, ion perpendicular trapping shifts it into the main ion distribution function, leading to the drift wave being ion Landau damped. There is no net flux of particles or diffusion in the treatment. The Hamiltonian for such a trapping for a single particle with an assumed form of the drift wave potential was studied. A similar treatment was carried out by Hirshman (1980) where the trajectory of a perpendicular physically trapped particle was studied by setting up the single particle equations of motion for an assumed potential and deriving a Hamiltonian of the system. In this thesis the effect of Ching(1973) where ions are the kinetic species with their resonant physical space trapping is not studied. Ion parallel trapping in one dimension is modeled in chapter III, but that is an essentially one dimensional case and does not have the perpendicular trapping effects in it.

A powerful tool for studying low frequency turbulence is the recently developed method of gyrokinetic particle simulation. The gyrokinetic equation is also a reduced Vlasov equation for studying phenomena with frequencies lower than the ion cyclotron frequency. It differs from the drift kinetic equation in the ordering of the scale lengths of the fluctuations; for the DKE, $k_\perp \rho_s << 1$, for the gyrokinetic equation, $k_\perp \rho_s \sim O(1)$. These simulations give an approximate solution of the gyrokinetic system of equations derived by Lee(1983) and Dubin et al.(1983). This system consists of the gyrokinetic Vlasov equation for the gyrophase averaged distribution function and the gyrokinetic Poisson equation which describes the electrostatic potential in terms of the particle gyrocentre densities. The code has a dimensionality of $2\frac{1}{2}$, with $f(x,y,v_\parallel,\mu)$. We shall concentrate here on the efforts begun by Lee et al.(1984) to examine, for the first time, the role played by electron nonlinearities in the saturation of drift wave turbulence and carried on later by Smith et al.(1985), Federici et al.(1987) and Dimits(1988).

Lee et al.(1984) in their study of the nonlinear effects leading to steady state fluctuations found that the primary cause of the saturation of the turbulent spectrum was the nonlinear behaviour of the electrons. It was found that modes with m=1 and n=+1 are dominant while most other modes were heavily damped. No ion heating was observed in the course of the simulation showing that nonlinear wave-particle interaction of the type studied by Dupree(1967) and Ching (1973) could not be the saturation mechanism. The parallel velocity space nonlinearity of electrons was seen to affect only the amplitude of the final saturated state but could not by itself lead to saturation. Similar conclusions were arrived at in the study by Smith et al.(1985). Both studies showed that the electron ExB advection of resonant electrons was the dominant saturation mechanism in the limit of weak nonlinearity. The saturation levels were seen to be:

$$\delta n / n \sim \left(\frac{k_\eta}{k_x} \right) \left(\frac{\gamma_l}{\omega_*} \right)$$

So, when $\gamma_l < \omega_*$, it was the electron ExB advection that could bring about saturation by itself, without taking account of ion nonlinear effects. However, in the limit of strong nonlinearity, $\gamma_l \sim \omega_*$, the electron flux in the nonlinear stage became too large for the instability to saturate by electron dynamics alone. But unlike in the resonance broadening theory, it was still not necessary to bring in the kinetic effects of ions (Dimits 1988). The fluid ExB convection of ions was sufficient to set up the balancing ion flux to bring about a steady state. So the two major nonlinear effects isolated by the simulation work are; the resonant ExB advection of electrons and their associated parallel velocity space nonlinearity and the ExB convection of fluid ions. These studies were more coherent than those of Dupree(1967) in that they considered a few modes to be the dominant ones in the system, while the rest were essentially damped out long before the nonlinear effect took over. Federici et al.(1987) and Dimits(1988) also essentially arrived at the same conclusions, but went further to study the effects of collisions on these saturation mechanisms.

In chapter IV we shall first consider the entire electron nonlinearity in two dimensions and set up the maximum entropy state treating the ions to be linear. Some limiting cases including those derived by Smith et al.(1985) will be recovered showing the importance of the electron ExB nonlinearity as compared to their parallel trapping one. Later in chapter IV we shall take up the problem of including the fluid ExB convection of ions and studying some interesting almost-maximum entropy monopole solutions of the system. Again there are no diffusion effects or net particle fluxes included in the problem.

As is inherent in the motivation behind studying such structures, their properties need not conform to what is predicted by any existing theories, since these are based on the assumption of interacting Fourier modes. Therefore, the parameter regimes set up by the previous studies can at best provide loose guidelines for the present approach.

The organisation of the thesis and its results in brief are as follows:

In the second chapter we deal with the possibility of the existence of stationary states of the fluid drift wave. A new 2-d monopole solution of the generalised Hasegawa-Mima equation with parallel ion nonlinearity is found. Unlike the earlier cases, this solution does not depend upon strong temperature gradients or higher order effects of the density gradient. A weak temperature gradient modifies its circular symmetry but monopole solutions continue to exist. It is also shown that the one dimensional analogous structure is unstable to two dimensional perturbations. Monopole solutions have been shown to have better stability properties than dipole vortices (Mikhailovskaya 1986, Su 1988). This solution is of significance since it lies in a physically realistic parameter regime.

In the limit of $k_x << k_y$, the response of a magnetised inhomogeneous plasma is essentially one dimensional. This simple case has been used in chapter III to formally set up the method of entropy maximisation in a magnetised, inhomogeneous plasma. The parallel velocity space or phase space trapping of particles is the nonlinear effect. This arises as a result of a resonance where particles with velocities close to the parallel phase velocity of the mode get trapped in its potential or when, $e k_\perp^2 \tilde{\phi}/m \omega^2 \sim 1$. The case of electron phase space trapping together with nonlinear fluid ion equations and later with ion phase space trapping effects is studied. An earlier calculation presented by Terry et al. (1987) had constructed isolated drift phase space holes in three dimensions with electron trapping effects and linear ion equations. However the treatment is a rather special case since it makes a specific choice for the form of the distribution function in the presence of the hole which cannot always be justified. As a result, it also ignores the important two dimensional effect of ExB trapping in physical space. No such assumptions are made here. A case demonstrating the inclusion of an electron temperature gradient is also studied.

In the limit of $k_x \sim k_y$, the two dimensional problem has been solved in chapter IV to study maximum entropy states. The additional nonlinearity here is due to the perpendicular ExB resonance. When the perpendicular phase velocity is close to the ExB drift speed of particles in that direction, they get trapped in physical space. The case of linear fluid ions with the ExB and parallel trapping nonlinearity of electrons is studied and some known results are recovered. The coupling between physical space and phase space trapping is shown analytically and numerically. The fluid ion ExB nonlinearity is then added to the problem to show that maximum entropy monopole vortex solutions can be formed through this approach. Thus the roles of both electron and ion nonlinearities in various parameter regimes, as indicated by earlier theoretical and computational results, are taken into account while studying these coherent structures. In most of these cases we have looked for general nonlinear oscillatory solutions of the final equations. These can be regarded as a periodic array of the isolated holes described by Dupree (1982) and Terry et al. (1987).

Finally, in order to extend the idea of the identification of coherent exact solutions to the high frequency regime, we have done a preliminary calculation with single particle effects. The wave-particle interaction in a high frdquency wave traveling perpendicular to an inhomogeneous magnetic field is studied in chapter V. Some interesting results regarding particle acceleration and associated wave damping are presented. It is possible that these can be applied and extended further to study some realistic cases for the formation of stationary states. For example, the beam-driven lower hybrid wave is an unstable mode which also propagates perpendicular to the magnetic field. It will be interesting to study these effects in such a system.

It would be appropriate to point out here that in this thesis only the exact, nonlinear, coherent states of a magnetised inhomogeneous plasma are formulated and studied. The time evolution of the drift instability to this steady state is not given. No attempts are made to go further and actually estimate the nature of interaction of a collection of such structures or its results. It is believed, however, that this study will be useful in constructing a turbulence model based on the principle of interacting coherent structures in phase space.

CHAPTER II

Nonlinear Fluid Drift Vortices

2.1 Introduction:

In this chapter we study nonlinear coherent fluid structures in magnetised inhomogeneous plasmas. This represents the first step towards the study of such structures in the more realistic kinetic picture. The major difference between these two limits is that there is no dissipation in the fluid limit and the expansion free energy cannot be released. As a result there is no instability of the normal modes. On the other hand, in the kinetic limit, the linear drift waves are unstable. The instability is driven by resonant wave particle interaction. This interaction or phase space effect is absent in the fluid theory. Therefore the study of coherent structures in this case, is simply the study of nonlinear fluid motion in a stable magnetised inhomogeneous plasma. These solutions, however, occupy an important place in the theories of fluid drift turbulence and transport studies, since they represent organised motion in the presence of nonlinear effects. As in ordinary fluids, these structures take the form of eddies or vortex forms. In this chapter we identify the parameter ranges of such solutions studied by earlier workers and pinpoint new effects that have not been examined so far. In the process we also learn which important nonlinear fluid motions can contribute to the overall theory of phase space structures in magnetised inhomogeneous plasmas.

The coherent nonlinear wave solutions of the fluid drift wave have been investigated by several authors in one and two dimensions. It was first suggested by Petviashvili (1967) that a new kind of one dimensional nonlinear solitary structure is possible if temperature gradient effects are retained in the limit of $k_{\parallel} << k_{\perp}$ and $k_{\perp}^2 \rho_s^2 << 1$, with $\nabla T_e / T_e \sim \rho_s$, where ρ_s is the ion Larmor radius at electron temperature. By retaining finite Larmor radius effects through the linear polarisation drift he arrived at a KdV type equation. This has both the solitary as well as the oscillatory cnoidal waves as its solutions. Oraevskii et al. (1969) also essentially arrived at the conclusion that the scalar nonlinearity due to ∇T_e was important in one dimension giving solitary and oscillatory solutions. However, they retained a higher order term of the nonlinear ion polarisation drift in the limit $k_{\perp}^2 \rho_s^2 \sim O(1)$. Later on Petviashvili (1977) showed that this solitary structure was unstable to two dimensional perturbations. He speculated that this instability would lead to the formation of two dimensional circularly symmetric localised potential structures or monopole vortices (Petviashvili 1981).

Hasegawa and Mima (1978) independently looked at nonlinear drift waves and pointed out the importance of the two dimensional nonlinearity of the polarisation drift in the absence of temperature gradients. This is a vector nonlinearity as against the scalar nonlinearity studied by Petviashvili (1977) and Oraevskii (1969). It was found that the presence of this nonlinearity led to dipole solutions or modons (Hasegawa et al. 1979, Miess and Horton 1983). While the monopole solution represents a net increase in the local charge density, the dipole vortex represents a local polarisation in the local charge density. The stability of these solutions to head-on or overtaking vortex-vortex collisions has been demonstrated (Makino et al. 1981, Swaters 1986). However, more recently, Mikhailovskaya (1986) and Su (1988) have shown that the presence of a scalar nonlinearity can destabilise dipole vortices. These vortices then separate into their constituent cyclone ,(ϕ< 0) and anticyclone (ϕ > 0) monopoles. These monopoles are long lived and very stable configurations. A more recent stability analysis undertaken by Marquardt et al. (1989) also seems to imply better stability properties of monopole vortices in the presence of scalar nonlinearities.

Both Petviashvili (1977, 1986) and Oraevskii et al. (1969) had ignored the vector nonlinearity of the Hasegawa-Mima (1978) type resulting from the nonlinear ion polarisation drift. Lakhin et al. (1987) pointed out that this was a serious flaw. It turns out that by keeping this nonlinearity in two dimensions, ∇T_e effects cancel identically. The implicit scaling used was,

$$\nabla T_e \sim \nabla^2 n_o .$$

This was weaker than that used by Petviashvili (1977)which had the strong gradient $\nabla \ln T_e \sim \rho_s$. The only contribution to the nonlinear term then came from the higher order expansions of ∇n_o. The reduced one dimensional form, therefore, had no contribution from ∇T_e. In two dimensions this treatment predicted monopole solutions. Recently Marquardt et al. (1989) have suggested that in fact the relative importance of the scalar nonlinearity against the vector nonlinearity is a matter of appropriate scaling of ∇T_e. We shall discuss this further in the next section.

The stability analysis has established the importance of the presence of a scalar nonlinearity. So far the only sources for this are the ∇T_e type given by Petviashvili (1977) and the $\nabla^2 n_o$ type given by Lakhin et al. (1987). In this chapter we show that retaining nonlinear parallel ion motion can lead to a scalar nonlinearity giving monopole solutions. These solutions also fit into a more realistic scaling of the equilibrium gradients since they require neither strong temperature gradients nor second order effects of density gradients. It is also shown that this additional parallel ion motion nonlinearity does not affect the stability of the one dimensional structure. It remains unstable to two dimensional perturbations. Effects of order $k_\perp^2 \rho_s^2 \sim O(1)$ have also been retained. Effects of ∇T_e on these monopole solutions will be examined and numerical solutions presented.

2.2 Basic equations:

We shall begin by setting up the nonlinear equations for the fluid drift waves retaining the ion parallel motion nonlinear effects. A few limiting cases and scalings used by previous authors will then be reviewed. Then the parallel motion nonlinearity will be included in the appropriate scaling to see if any new effects can be seen. Consider a uniformly magnetised, inhomogeneous, collisionless plasma, in a slab geometry with,

$$\underset{\sim}{B} = B\,\hat{z}, \qquad \frac{d_x T_e}{T_e} = K_T\,, \qquad \frac{d_x n_0}{n_0} = K_n\,.$$

Where T_e, n_o and B are the equilibrium electron temperature, density and magnetic field. K_n^{-1} and K_T^{-1} are the density and temperature gradient scale lengths. The waves are assumed to be electrostatic with $\underset{\sim}{E} = -\underset{\sim}{\nabla}\tilde{\phi}$. There is no equilibrium electric field.

As is standard in the calculation of fluid drift waves, we assume the parallel phase velocity of the waves to be much smaller than the electron thermal velocity. Since there is a small but finite $k_{\parallel}$, electrons can flow along the B field to to wash out the temperature fluctuations. Then in the parallel direction the velocity averaged response of the electrons is given by the Boltzmann relation:

$$n_e = n_0(x)\exp\left(e\tilde{\phi}/T_e\right).$$

We want to study the low frequency ($\omega \ll \omega_{ci}$) response of the ions. The parallel phase velocity is taken to be much larger than the ion thermal velocity. In this limit, when $T_e \gg T_i$, the continuity and parallel momentum equations for the ion fluid reduce to:

$$\partial_t n_i + \nabla \cdot (n_i \mathbf{V}) = 0 , \qquad (2.1)$$

$$\partial_t V_\parallel + (\mathbf{V} \cdot \nabla) V_\parallel = -\frac{e}{M} \nabla_\parallel \tilde{\phi} , \qquad (2.2)$$

where $E_\parallel = -\nabla_\parallel \tilde{\phi}$. In the low frequency drift approximation, the perpendicular ion velocity is given by

$$\mathbf{V}_\perp = \mathbf{V}_E + \mathbf{V}_p \; ; \; \mathbf{V}_E = c\frac{\mathbf{E} \times \mathbf{B}}{B^2} \; ; \; \mathbf{V}_p = -\frac{c}{B\omega_{ci}} d_t \nabla_\perp \tilde{\phi},$$

$$d_t \equiv \partial_t + V_\parallel \nabla_\parallel + \mathbf{V}_E \cdot \nabla . \qquad (2.3)$$

The oscillations are quasineutral and $n_e \cong n_i$. Using this and substituting equation (2.3) into (2.2),

$$(\partial_t + V_\parallel \nabla_\parallel - \partial_y \phi \partial_x + \partial_x \phi \partial_y)(\phi/q_0 + \ln n_0 - \nabla_\perp \phi)$$
$$+ \nabla_\parallel V_\parallel = 0 , \qquad (2.4)$$

$$(\partial_t + V_\parallel \nabla_\parallel - \partial_y \phi \partial_x + \partial_x \phi \partial_y) V_\parallel = -\nabla_\parallel \phi , \qquad (2.5)$$

where, all length scales have been normalised to ρ_s , all velocities to C_s, time to ω_{ci} and

$$\phi = \frac{e\tilde{\phi}}{T_e} , \quad q_0 = \exp(x K_T) .$$

The convective derivative in the polarisation drift has both, the parallel and ExB velocity. The nonlinear $V_{\parallel}$ terms are new. The linear dispersion relation can be obtained by linearising these equations and looking for plane wave solutions with frequency ω and wavenumber $\underset{\sim}{k}$. It is given by,

$$\omega^2(1+k_\perp^2 \rho_s^2) - \omega\omega_* - k_\parallel^2 c_s^2 = 0 .$$

Equations (2.4) and (2.5) are the complete equations that will be studied in this chapter. The effects studied by previous authors are all contained in these equations. The ExB terms give the vector nonlinearity of Hasegawa-Mima. The ordering used to recover the Hasegawa-Mima equation is:

$$\partial_t \sim \phi \sim K_n \sim \delta , \quad \nabla_\parallel \lesssim \delta^2 ,$$

$$\partial_x \sim \partial_y \sim O(1) , \quad K_T = 0 \text{ or } \delta .$$

The HM equation has no temperature gradients. Note that it also gives a reasonable description of the plasma when $K_T \sim K_n$. The solutions studied in the HM equation have perpendicular scales of the order of ρ_s. It is given by:

$$(\partial_t/q_0 - K_n\partial_y)\phi - \partial_t \nabla_\perp^2 \phi$$
$$+ (\partial_y\phi\,\partial_x - \partial_x\phi\,\partial_y)\nabla_\perp^2\phi = 0 . \qquad (2.6)$$

The last term gives the vector nonlinearity and the equation has standard modon or dipole solutions.

Petviashvili (1977) studied the one dimensional analogue of equations (2.4) and (2.5) in a slightly different parameter regime. He was looking for solutions where the temperature gradients scaled on the order of ρ_s. The ordering used was:

$$\partial_t \sim \phi \sim \kappa_n \sim \delta \ , \ \nabla_{\parallel} \sim \partial_x \sim \delta^2$$

$$\partial_y \sim O(1) \ , \ \kappa_T \sim O(1) \ .$$

So, for a strong temperature gradient, when $k_x \ll k_y$, the equations were:

$$\partial_t \left(1/q_0 - \nabla_{\perp}^2 \right) \phi - \kappa_n \partial_y \phi + \nabla_{\parallel} V_{\parallel} + \kappa_T/q_0 \, \phi \, \partial_y \phi = 0 \ , \tag{2.7}$$

$$\partial_t V_{\parallel} = - \nabla_{\parallel} \phi \ . \tag{2.8}$$

These were then reduced to the KdV form to obtain solitary wave solutions.

In the following sections we shall examine the effect of retaining the parallel ion nonlinearity on the various solutions studied so far. We shall also see if any qualitatively new solutions can be written down for these nonlinear equations.

2.3 One dimensional solitary structures:

In this section the effect of retaining parallel ion nonlinearity on the one dimensional solutions studied by Petviashvili (1977) will be examined. So, we no longer make the assumption of long parallel wavelengths and use the following ordering:

$$\partial_t \sim \phi \sim \kappa_n \sim \delta \ , \ \nabla_{\parallel} \sim V_{\parallel} \sim \delta \ .$$

$$\partial_y \sim O(1) \ , \ \partial_x \sim \delta^2 , \ \kappa_T \sim O(1) \ .$$

The equations (2.7) and (2.8) are modified as:

$$\partial_t (\phi/q_0 - \nabla_\perp^2 \phi) - K_n \partial_y \phi + \nabla_{\parallel} V_{\parallel} + {}^{K_T}/_{q_0}\, \phi \partial_y \phi + V_{\parallel} \nabla_{\parallel} \phi / q_0 = 0 , \quad (2.9)$$

$$\partial_t V_{\parallel} + V_{\parallel} \nabla_{\parallel} V_{\parallel} = - \nabla_{\parallel} \phi . \quad (2.10)$$

In order to study the solitary stationary states and their stability properties, we need to reduce this set of coupled equations. Since the effect of two dimensional perturbations is to be examined, the lowest order term in $k_x \rho_s$ has been retained. This comes from the linear polarisation drift. We now introduce the following change in variable to study the nonlinear stationary states:

$$\eta = y + \theta z - u t , \quad V = \theta V_{\parallel} ,$$

giving:

$$\partial_t (\phi/q_0 - \nabla_\perp^2 \phi) - u/q_0 \partial_\eta \phi - K_n \partial_\eta \phi + u \partial_\eta \nabla^2 \phi + \partial_\eta V + {}^{K_T}/_{q_0}\, \phi \partial_\eta \phi + V \partial_\eta \phi / q_0 = 0 , \quad (2.11)$$

$$\partial_t V - u \partial_\eta V + V \partial_\eta V = - \theta^2 \partial_\eta \phi . \quad (2.12)$$

In order to eliminate ϕ or V from equations (2.11) and (2.12) we employ the standard methods of studying solitary one dimensional structures. Introduce the stretched variables:

$$\bar{\eta} = A\eta , \quad \tau = A^3 u t , \quad \bar{x} = A x ,$$

where,

$$A^2 = (u^2/q_0 + u K_n - \theta^2)/u^2,$$

setting, $$\phi = u q_0 A^2 F(\eta)/K_T,$$

Substituting and retaining terms of the order A^2, the following equation is obtained:

$$\partial_\eta \left(\nabla^2 F + \left(1 + \frac{\theta^2}{K_T q_0 u}\right) \frac{F^2}{2} - F \right) = -\left(\frac{1}{q_0} + \frac{\theta^2}{u^2} \right) \partial_\tau F.$$

In the steady state, when $\partial_x^2 \ll \partial_y^2$, the potential is given by the one dimensional KdV type equation:

$$\partial_\eta^2 F + \left(1 + \frac{\theta^2}{K_T q_0 u}\right) \frac{F^2}{2} - F = 0 \qquad (2.13)$$

This is the new equation for the one dimensional steady state potential when ion parallel motion effects are retained. The coefficient of the nonlinear term now has an additional contribution due to the parallel nonlinear effects. In the limit $\theta^2 \ll 1$, and $K_x \ll k_y$, this equation was solved by Petviashvili (1977). It has both, solitary and periodic solutions. The stability of the solitary solution,

$$F_0(\eta) = 2\cosh^{-1}(\eta/2),$$

to two dimensional perturbations was estimated by Petviashvili (1977), and it was shown to be unstable. The presence of the extra nonlinear term does not make any qualitative difference in this result. It is to be remembered that the temperature gradient is sharp. T falls to the e-folding value on the scale of ρ_s. Also, the parallel wavelength is smaller than k_y^{-1}. So, the effect of the

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additional term is small and the solitary solution remains unstable to two dimensional perturbations.

The growth rate can be estimated to be,

$$\gamma = \frac{k_x}{(1/q_0 + \theta^2/u^2)^{1/2}} \partial_x^2 \left(\frac{\theta^2}{K_T q_0 u}\right).$$

This result is similar to that derived by Petviashvili (1977). Thus the inclusion of the parallel ion motion makes no qualitative difference to the stability of the one dimensional solution.

It is clear, therefore, that in the evolution of the fluid drift wave it is not possible to retain one dimensional coherent structures. They would be unstable and lead to the formation of two dimensional vortex structures.

When $\partial_x^2 \sim \partial_y^2$, Petviashvili (1977) suggested that these would take the form of two dimensional circularly symmetric monopole solutions described by:

$$\partial_\eta^2 F + \partial_x^2 F + \frac{F^2}{2} - F = 0 .$$

where the nonlinear term arises essentially due to ∇T_e effects. The equation has cylindrical symmetry and was shown to have circular solutions in the (r, θ) plane (Petviashvili 1981). We have given the modified form of this scalar nonlinearity in the presence of parallel ion motion in one dimension. It becomes interesting, therefore, to see if this additional scalar term can give some qualitatively new results in two dimensions.

In the next section we shall go on to the consideration of monopole structures in the presence of parallel nonlinear ion motion. We shall also consider the effects of weak temperature gradients on these solutions. Note that the stability of the one dimensional structure has been examined only for the strong temperature gradient case. As will be shown in the next section, a weak temperature gradient does not give a scalar nonlinearity in either one or two dimensions.

2.4 Two dimensional localised structures:

Lakhin et al. (1987) pointed out that it is incorrect to ignore the vector nonlinearity of the Hasegawa-Mima type coming through the nonlinear ion polarisation drift. They showed that in two dimensions this led to the exact cancellation of the scalar nonlinearity. However, the scaling used there was different from that of Petviashvili (1977). The temperature gradient was not so sharp. The equations (2.4) and (2.5) in the moving frame are:

$$\left(-u\partial_\eta + V\partial_\eta - \partial_\eta\phi\,\partial_x + \partial_x\phi\,\partial_\eta\right)\left(\phi/q_0 + \ln n_0 - \nabla_\perp^2\phi\right) + \partial_\eta V = 0\,, \qquad (2.14)$$

$$\left(-u\partial_\eta + V\partial_\eta - \partial_\eta\phi\,\partial_x + \partial_x\phi\,\partial_\eta\right) V = -\theta^2\partial_\eta\phi\,. \qquad (2.15)$$

Then the ordering used by Lakhin et al. (1987) was:

$$(u/q_0+K_n) \sim \delta^{3/2}, \quad \phi \sim K_n \sim u \sim \delta^{1/2}$$
$$\nabla_\perp \sim \delta^{1/2}, \quad k_\parallel = 0,$$
$$K_T \sim \partial_x K_n \sim \delta^{3/2}.$$

This shows that the scalar nonlinearity due to ∇T_e is of the same order as the vector nonlinearity and there is an exact cancellation. The structures studied by Lakhin et al.(1987) had scale lengths longer than the ion Larmor radius. Also the temperature gradient here has a more realistic scaling as compared to the sharp gradients of Oraevskii et al. (1969), Petviashvili (1977) and that of the previous section. In the treatment of Lakhin et al. (1987) the only source of a scalar nonlinearity then came from a second order term in the expansion of ∇n_0. In case K_n is a constant, the HM equation gives the correct description in this limit.

In this section we shall explore this region of parameter space for the existence of new monopole vortex solutions. As shown by recent workers (Mikhailovskaya 1986, Su 1988), dipole solutions are unstable to scalar nonlinearities of the type in equation (2.6). Monopole structures have been observed to have better stability properties. It is of interest, therefore, to identify any new sources of this nonlinearity.

Since we want to consider the role played by the nonlinear ion motion the new ordering used here has the effects of short parallel wavelengths:

$$V_{\parallel} \sim \delta^{1/2}, \quad \nabla_{\parallel} \sim \delta .$$

Following Lakhin et al. (1987), we now look for solutions of the type:

$$\nabla_{\perp}^2 \phi = f(x)\,\phi + g(x)\,\phi^2,$$

$$V = a(x)\,\phi + b(x)\,\phi^2 . \tag{2.16}$$

This particular choice of functional form is an attempt to look for monopole solutions. Later in this section we will justify this choice by obtaining the same result through a different method.

Using (2.16) in equation (2.14) and (2.15) we find,

$$a(x) = \theta^2/u , \qquad b(x) = \theta^4/2u^3$$

$$f(x) = \frac{1}{q_0} + \frac{K_n}{u} - \frac{\theta^2}{u^2} ,$$

$$g(x) = \frac{a(x)f(x)}{2u} - \frac{b(x)}{u} - \frac{a(x)}{2uq_0} + \partial_x(1/q_0)/2u .$$

Substituting in equation (2.16) for $\nabla^2\phi$ it can be seen that the ∇T_e term cancels identically, as shown by Lakhin et al. (1987). Then in normalised variables,

$$\nabla^2\phi = \left(\frac{1}{q_0} + \frac{K_n}{u} - \frac{\theta^2}{u^2}\right)\phi - \left(\frac{K_n'}{2u^2} + \frac{\theta^4}{u^4} - \frac{\theta^2 K_n}{2u^3}\right)\phi^2 . \tag{2.17}$$

The coefficient of gives the linear dispersion relation when $K_T = 0$. Both the drift and modified ion acoustic branches are included. In this limit, when $\theta^2 \ll 1$ and K_n = constant, this equation reduces to the one studied by previous authors for dipole solutions (Miess et al. 1983, Leadke et al. 1988). It is clear that the ion parallel motion has contributed to the nonlinear term. In fact, it is now not necessary to retain the K_n' terms to obtain a monopole solution and will be ignored henceforth in the limit of K_n = constant.

Equation (2.17), then may be obtained in another way. We look for generalised solutions of the type,

$$\nabla^2\phi = g(\phi) \quad , \quad V = f(\phi),$$

without any assumptions about the (x, η, ϕ) dependence of $\nabla^2\phi$. Substituting in (2.14) and (2.15),

$$f(\phi) = u - (u^2 - \theta^2\phi)^{1/2};$$

$$\nabla^2\phi = \phi - K_n u\left[\left(1 - \frac{2\theta^2}{u}\phi\right)^{1/2} - 1\right] + \ln\left(1 - \frac{2\theta^2}{u^2}\phi\right).$$

With the boundary conditions,

as, $x, y \to \infty$; $\phi \to 0$ and $V \to 0$,

we look for localised solutions. In the limit of small amplitudes, this reduces to,

$$\nabla^2\phi = \left(1 + \frac{K_n}{u} - \frac{\theta^2}{u^2}\right)\phi - \left(\frac{\theta^2}{u^2} - \frac{K_n}{2u}\right)\frac{\theta^2}{u^2}\phi^2. \tag{2.18}$$

which is the same as equation (2.17). So our assumed form of the functional dependence of $\nabla^2\phi$ was justified. It is to be noted that the coefficient of the nonlinear term must be positive for a monopole solution to exist. An equation of the type, $\nabla^2\phi = |\kappa|\,\phi + |\beta|\,\phi^2$, has no bounded solutions. Usually, the density gradient is taken to be in the negative x direction, so substituting $K_n = -K_n$ in equation (2.17) we obtain:

$$\nabla^2\phi = \left(\frac{1}{q_0} - \frac{K_n}{u} - \frac{\theta^2}{u^2}\right)\phi - \left(\frac{\theta^2}{u^2} + \frac{K_n}{2u}\right)\frac{\theta^2}{u^2}\,\phi^2 . \qquad (2.19)$$

This is the complete equation to be studied in order to see the difference between the work of Lakhin et al.(1987) and the more general approach given here. The coefficient of the nonlinear term is positive and the equation has monopole solutions.

The numerical solution of equation (2.19) is done using the DELSQPHI code by Hockney (1970) for solving a nonlinear Poisson equation in a two dimensional cartesian system. The grid is a rectangular 16x16 matrix and solutions are correct upto a relative error of 10^{-4}. The code uses a mixed Fourier transform - cyclic reduction method and has been widely used. The boundary conditions were Dirichlet with the potential zero on the boundary. So only bounded solutions were examined. A larger grid size gave the same results. The initial guess solution given was in the form of concentric circles of diminishing value commensurate with the zero boundary. Successive values of the coefficients of ϕ and

ϕ^2 were given the earlier solutions as initial guesses.

The numerical solutions of equation (2.19) for $K_T = 0$, give localised circularly symmetric potential contours of a monopole vortex as shown in fig. (2.1). These are similar to the ones predicted by Petviashvili (1981) and Lakhin et al. (1987). The potential is peaked at the centre and falls off rapidly, almost exponentially, but decreases smoothly to zero at the boundary. This fall off is consistent with the scale of the density gradient. The potential reaches its e-folding value on a scale similar to the density gradient, i.e. over a distance of about ten Larmor radii. This is reasonable since we are studying structures with long perpendicular scale lengths.

Introduction of the temperature gradient would mean solving (2.19) for a finite value of K_T. Fig.(2.2) shows the result for $K_T = 0.01$. The deviation from circular symmetry is small. As K_T increases, the deviation becomes larger, as shown by fig. (2.3) for $K_T = 0.1$. However, monopole solutions continue to exist. Therefore it is clear that for this scaling of the temperature gradient, the ion parallel motion alone is able to bring about the formation of monopole vortices. In parameter regimes where finite $k_{\parallel}$ effects are important, e.g. tokamak edge plasmas and magnetospheric plasmas, these solutions would be of interest.

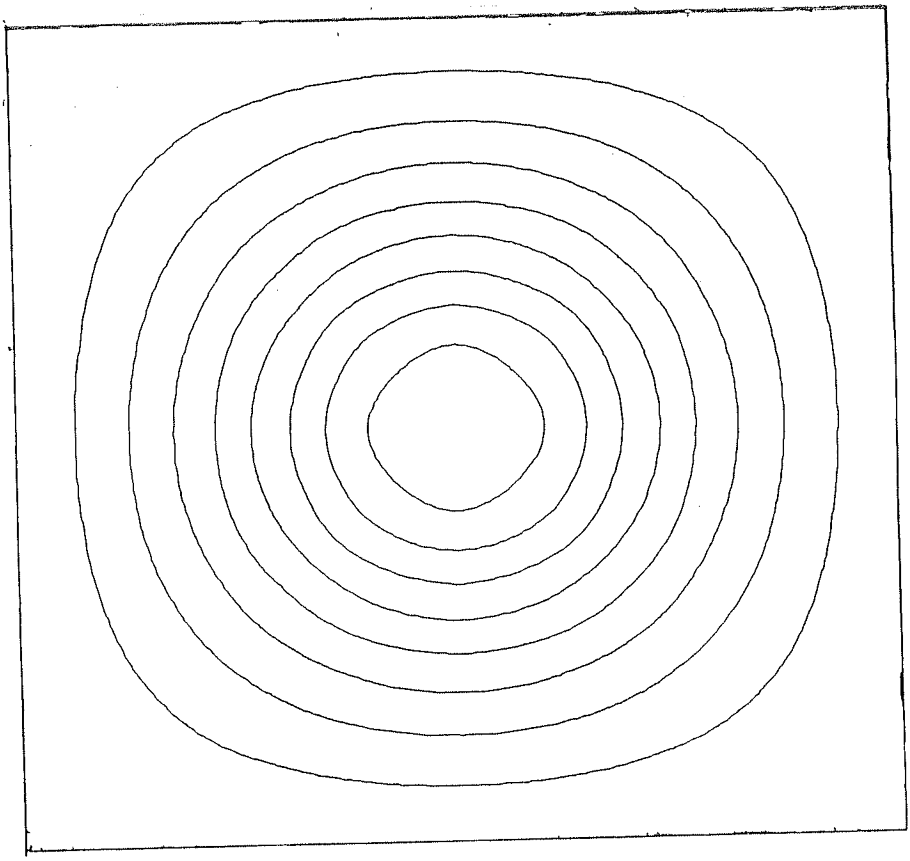


Fig. 2.1 : Circular potential contours for $K_T = 0$, $\theta = 0.02$, $K_n = 0.1$, $u/\theta = 1.2$ [Eq.(2.19)].

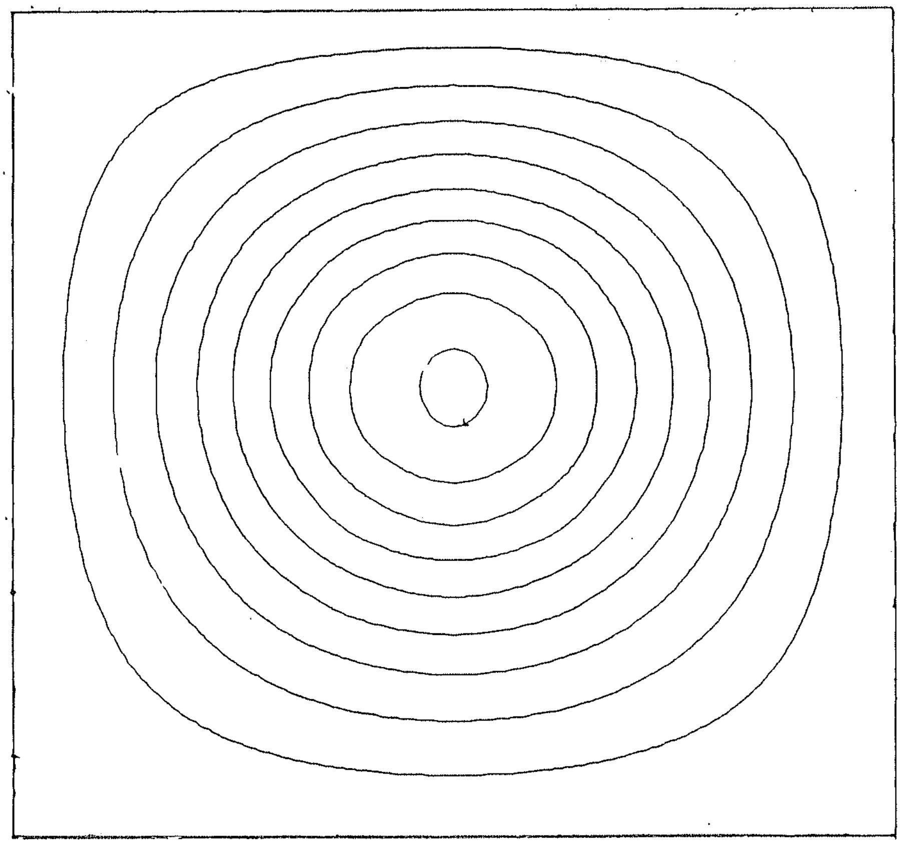


Fig. 2.2 : Potential contours for K_T = 0.01 [Eq.(2.19)].

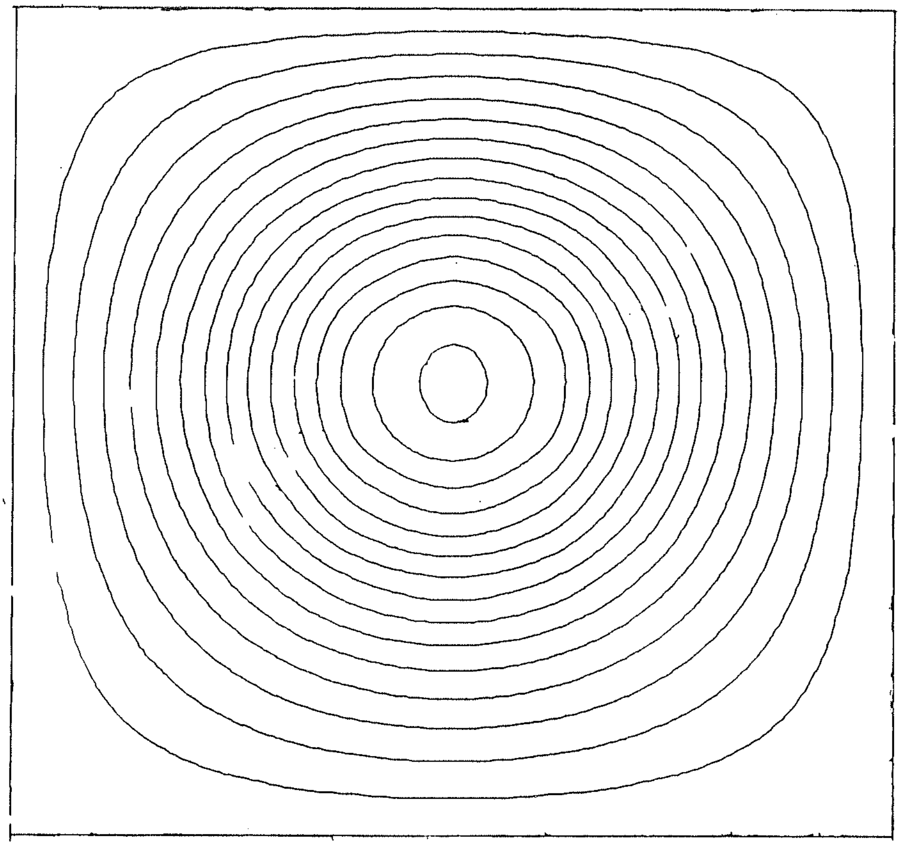


Fig. 2.3 : Potential contours for $K_T = 0.1$ [Eq.(2.19)].

2.5 Conclusion

In this chapter we studied the nonlinear coherent structures in a magnetised inhomogeneous fluid plasma. We looked for these solutions of the generalised HM equation, including the ion parallel motion effects, in both one and two dimensions. It was shown that one dimensional structures are unstable to two dimensional perturbations. This leads to the formation of two dimensional vortex solutions. The important new nonlinear effect has been identified to be due to the parallel ion motion. The effect of a weak temperature gradient on these solutions has been studied and it is shown that the vortex solutions become noncircular but continue to exist. As mentioned earlier, the monopole seems to have better stability properties than the dipole (Su 1988). This solution does not rely on strong temperature gradients or second order density gradients, unlike solutions obtained in earlier studies. Hence it would seem to be a more likely stable configuration in realistic parameter regimes. It has been shown, therefore, that finite $k_{\parallel}$ effects play an important role in the nonlinear dynamics of drift turbulence.

In the following chapter we shall turn to the formulation of one dimensional maximum entropy solutions of the drift kinetic equation.

CHAPTER III

One Dimensional Drift Phase Space Holes

3.1 Introduction:

In this chapter we shall study the maximum entropy solutions of the drift kinetic equation in one dimension. In the last chapter, coherent stationary states of the magnetised, inhomogeneous system were studied in the fluid limit. Thus the phase space effects were excluded and important fluid nonlinear terms identified and examined in various parameter regimes. New effects, leading to the formation of vortices in physical space, were studied. We now wish to take into account wave particle interaction to study the formation of phase space structures.

As a first step towards extending Dupree's formalism (1982) of phase space holes as maximum entropy states, we would like to consider the simplest system. The one dimensional DKE is an ideal point to begin the formulation of this concept for a magnetised, inhomogeneous plasma. In order to study some possible nonlinear steady state properties of this wave, it is important to note that a kinetic drift wave is unstable in the linear limit, as against the fluid case, which is always stable, since it does not take into account resonant wave-particle interactions in phase space. In the kinetic limit, the resonant electrons near the parallel phase velocity undergo large ExB displacements in the direction of the density gradient. As a result of this gradient there are more number of particles giving energy to the wave than taking energy from it, leading to wave growth. In order to achieve a steady state, there must exist some mechanism to counter this process. In general the nonlinear terms in the DKE arise due to the nonlinear ExB advection of particles and their motion in the parallel direction. However, in the limit when $k_x \ll k_y$, the ExB nonlinearity is ignored, effectively reducing the system to one dimension. The dominant nonlinear mechanism here is then due to parallel motion. This gives rise to phase space trapping of particles. In this chapter we shall set up one dimensional phase space holes due to this effect. There is a qualitative difference between electron and ion trapping in the case of a drift wave. Since electrons are responsible for the instability mechanism, their trapping should give a direct modification of the growth rate. The ion trapping effects are small even in the kinetic limit since their thermal velocities are generally smaller than the wave phase velocities.

Oraevskii, Sagdeev, Galeev and Rudakov (1969a) considered the quasilinear theory of stability and saturation of drift waves due to parallel trapping. It was shown that coherent parallel trapping in a single mode as well as quasilinear diffusion due to many waves can cause, a) steepening of the velocity distribution of electrons and b) a local flattening of the spatial gradient, near the parallel resonance. While the former is a stabilising effect, the latter leads to an effective reduction in the growth rate. This can be seen from the expression for the linear electron response , calculated from the drift kinetic equation.

$$\delta f_e = (\omega_r + i\gamma_L - k_{\parallel} v_{\parallel}) \frac{M}{m} k_y \tilde{\phi} \left(\frac{k_{\parallel}}{k_{\perp}} \partial v_{\parallel} + \frac{m}{M} \partial x \right) F_{oe}'$$

The first derivative describes standard Landau damping due to parallel acceleration and causes the net shift in the perturbed electron density to run behind the potential. The second derivative term is due to the ExB drift of electrons near resonance. This makes the net electron density run ahead of the potential. When $\omega_r > \omega_*$, as is usually the case, the latter effect dominates, giving a net growth. Therefore, a) enhances damping and b) makes the growth mechanism vanish.

The problem of maximum entropy steady states of the DKE with electron parallel trapping effects was examined by Terry et al. (1987) in the full three dimensional geometry. They made the assumption that, the electron distribution function could be taken to be a function of ϕ alone. So, $f(x,y,z,v_{\parallel}) = f(\phi)$. In effect they ignored any independent x-dependence of f in the form of a density gradient. However, this cannot be assumed to be the only configuration of the saturated state. In more realistic situations some modified form of the density gradient is maintained in the steady state phase. Also, in more than one dimensions the ExB nonlinearity leading to perpendicular physical space trapping of resonant particles becomes dominant at amplitudes lower than those required for parallel trapping to be effective. The assumption of $f(\phi)$ ignored this effect completely since $\underset{\sim}{V}_E \cdot \underset{\sim}{\nabla} f(\phi) = 0$, when, $f = f(\phi)$. This reduced their problem to an effective one dimensional case and may therefore be treated as a special case of the present formulation.

In the following sections we shall systematically reduce the system to a single dimension and formulate the most general maximum entropy state. Both electron and ion trapping effects are considered in various limiting cases of the hole velocity. Finally we shall apply this formalism to the case of a plasma with an equilibrium temperature gradient. It was pointed out in section (2.2) that the presence of a ∇T_e scalar nonlinearity gives interesting solutions in the one dimensional fluid limit. This case will be modified with the most probable phase space response with trapping effects. Numerical solutions of the resulting equations for the steady state potential will be presented.

3.2 Distribution function and entropy:

We begin by writing the drift kinetic equation (DKE) for a magnetised, inhomogeneous plasma with a density gradient. The equilibrium consists of a plasma in a slab geometry with a uniform B field in the z-direction. There is an equilibrium density gradient in the x-direction, $n_o(x)$.

The DKE is,

$$\partial_t f + \underset{\sim}{\nabla}_\perp \cdot (\underset{\sim}{V}_\perp f) + V_\parallel \nabla_\parallel f + \frac{q}{m} E_\parallel \partial_{V_\parallel} f = 0,$$

$$\underset{\sim}{V}_\perp = \underset{\sim}{V}_E + \underset{\sim}{V}_p \quad , \quad \underset{\sim}{E} = - \underset{\sim}{\nabla} \tilde{\phi} .$$

Consider the electron DKE in this geometry.

$$\partial_t f + \frac{c}{B} \partial_y \tilde{\phi} \partial_x f - \frac{c}{B} \partial_x \tilde{\phi} \partial_y f + V_\parallel \nabla_\parallel f + \frac{e}{m} \nabla_\parallel \tilde{\phi} \partial_{V_\parallel} f = 0 . \quad (3.1)$$

This equation was studied by Terry et al. (1987) for the three dimensional parallel trapping problem. However they assumed that the second and third terms vanished identically when f = f(ϕ). With this ansatz the only constant of motion in the steady state is the parallel kinetic energy,

$$W = \frac{1}{2} m V_\parallel^2 - e \tilde{\phi} .$$

Therefore the general solution of equation (3.1) in the steady state is,

$$f = f(W) .$$

This reduces the problem effectively to the one dimensional case. Let us consider the more general problem and not say anything about the dependence of f on $x, y, z, v_\parallel$. We want to look for stationary nonlinear solutions of the plane wave type in a moving frame. Then we can transform to a moving frame with velocity u,

$$\eta = y + \theta z - u t,$$

so that, $f(x,y,z,v_{\parallel},t) = f(x,\eta,v_{\parallel})$. u is the hole velocity in the perpendicular direction and, typically, is close to the drift velocity, V_d . The equation (3.1) has in general the following two conserved quantities.

$$E = \frac{1}{2} m v^2 - e\theta^2 \tilde{\phi},$$

$$K = V + \omega_{ce} \theta^2 x,$$

where $V = \theta v_{\parallel} - u$. Therefore now the general solutions of equation (3.1) is any arbitrary function of K and E,

$$f(x, \eta, V_{\parallel}) \equiv f(K, E). \tag{3.2}$$

E is the parallel kinetic energy and K the canonical momentum. This gives the complete description of the steady state. The parallel trapping of electrons is present through the dependence on E. In simple terms, the resonant particles at u/ will get trapped in a region of velocity space around it, $|V| \leqslant (2e\theta^2\tilde{\phi}/m)^{1/2}$. In the treatment of Terry et al. (1987) the dependence of K has been ignored, removing all gradients from the system. The equation (3.2) includes both the parallel as well as perpendicular trapping information. In order to study the one dimensional system we make the following assumptions.

It is specified that, in the unperturbed state, the density gradient has an exponential form,

$$\frac{\partial_x f}{f} = K_n .$$

Further, we make the approximation, $k_x << k_y$. Then, in one dimension, the equation (3.1) reduces to,

$$V \partial_\eta f + \frac{e\theta^2}{m} \partial_\eta \tilde{\phi} \partial_V f = \frac{c}{B} \partial_\eta \tilde{\phi} K_n f . \tag{3.3}$$

The RHS gives the correction due to the presence of the density gradient to the one dimensional unmagnetised case studied by Dupree (1982). The solutions of this equation are given by,

$$f(V, \eta) = g(E) \exp\left(\frac{K_n V}{\alpha_e}\right), \tag{3.4}$$

$\alpha_e = \omega_{ce}\theta^2$ and g(E) is any arbitrary function of E. This is the correct distribution function to be used to study the parallel trapping problem. $K_n = 0$ will reduce this to the case of Terry et al. (1987).

Particle trapping effects are now incorporated through the choice of g(E). In the original BGK approach (1957) the potential structure $\tilde{\phi}(\eta)$ and the untrapped particle distribution were specified. Then, in the simple case of a electron plasma wave, the Poisson equation was solved for the trapped distribution. However, this solution is artificial. Depending on the choice of $\tilde{\phi}(\eta)$ there are an infinity of such solutions. One could equally well solve the inverse problem. Specify all the distributions and solve for $\tilde{\phi}(\eta)$. This was attempted by BGK (1957) and later applied to the study of nonlinear ion acoustic solitary waves by Schamel

(1972). But the arbitrariness in the choice of these functions would persist.

In order to remove this ambiguity, the following approach, as suggested by Dupree (1982), is taken. We are not looking for any ad-hoc steady state. Only the most probable one. So the restriction on the choice of g(E) is that it should be commensurate with the maximum entropy state of the system. The boundaries of this structure will also be determined self-consistently. The resulting potential $\tilde{\phi}(\eta)$ would then describe a phase space hole in the sense of Dupree.

It is to be remembered, however, that it is only the form of g(E) in equation (3.4) that is being determined, not of $f(V, \eta)$ as a whole.

The form of the entropy to be used to describe a collisionless Boltzmann system is an area of much investigation. Lynden-Bell (1967) has proposed an alternative form of the entropy. However, the subject has not produced final answers yet. For lack of a better alternative and from the point of view of familiarity, we continue to use the Maxwell-Boltzmann form of entropy,

$$S = -\int f \ln f \, d\underset{\sim}{x} \, d\underset{\sim}{v} .$$

It is obvious from results of the statistical mechanics of the Boltzmann equation that the maximum entropy form of $f(\underset{\sim}{v}, \underset{\sim}{x})$ would be similar to the Maxwellian. However we shall carry out the process to see what the specifications are.

3.3 Entropy Maximisation

We shall continue to study the system where the kinetic species is the electrons. Ions will be described by fluid equations. This is a good approximation when the parallel phase velocity u/θ is close to or larger than the electron thermal velocity. Then the ions will not see the parallel resonance and can be treated adequately as a fluid. Let $F_0(V)$ be the equilibrium distribution of electrons. Let $u/\theta = u'$ be the parallel hole velocity. Then the amount of phase space density that is required to fill the area of the trapping region in order to create a phase space hole is, $F_0(u')$. This phase space density is lost by the untrapped region in a reversible fashion. So, the creation of the hole rearranges the phase space in the hole region irreversibly and contributes to the change in entropy. But the untrapped region remains unchanged except for losing some density and does not contribute to the change of entropy. Then the entropy of the entire system may be written as,

$$\sigma = n \iint_h \left[f_h \ln f_h - F_0(u') \ln F_0(u') \right] d\eta \, dV + \sigma_i , \tag{3.5}$$

where σ_i is the initial entropy.

$$f_h(V, \eta) = g(E) \exp\left(\frac{K n V}{\alpha e} \right) , \tag{3.6}$$

as given by equation (3.4). We now maximise the entropy subject to certain constraints. We want to keep the total mass, momentum and energy of the system constant during the maximisation process. Assuming that the region outside the hole may be treated as a linear dielectric, we can write the constraint equations to be:

$$\begin{bmatrix} M_o \\ P_o \\ T_o \end{bmatrix} = n \iint_h dv\, d\eta \left(f_h - F_o(u) \right) \begin{bmatrix} m \\ mV \\ E \end{bmatrix} \tag{3.7}$$

where M_o, P_o, T_o are the mass, momentum and energy of the hole. The definition for the case of general oscillatory solutions will be modified and M_o, P_o and T_o will be defined as quantities per unit wavelength. These quantities, in physical terms, refer to the hole density, hole velocity and hole temperature, respectively.

The potential $\tilde{\phi}(\eta)$ is given self consistently by the quasineutrality condition,

$$n_e \simeq n_i \tag{3.8}$$

We now wish to find $f_h(v, \eta)$ and the hole boundary in velocity space, $V(\eta)$ that will make maximum. Using Lagrange multipliers, a, b and $-\tau^{-1}$, for M_o, P_o and T_o respectively, we get two equations corresponding to independent variations in $f_h(v, \eta)$ and $V(\eta)$ on the boundary,

$$\delta\sigma = 0 = \iint_h d\eta\, dv \left(1 + \ln g(E) + \frac{K n V}{\alpha e} + a + bV - E/\tau \right) \delta f_h \tag{3.9}$$

$$\delta\sigma = 0 = \int_h d\eta\, \delta V(\eta) \Big[\left(f_h \ln f_h - F_o \ln F_o \right) + \left(a + bV - \frac{E}{\tau} \right) \left(f_h - F_o \right) \Big] \tag{3.10}$$

Since both δf_h and $\delta V(\eta)$ are arbitrary, from (3.9) we obtain,

$$g(E) = \exp\left[\frac{E}{\tau} - \left(b + \frac{K_\eta}{\alpha_e}\right)V - a - 1\right].$$

However, we know from equation (3.4) that g(E) is a function of E alone, fixing the value of 'b' to be,

$$b = -K_\eta / \alpha_e .$$

Therefore,

$$g(E) = \exp(E/\tau - a - 1).$$

Equation (3.10) gives the condition that, on the hole boundary,

$$F_0(u) = f_h(V, \eta),$$

giving

$$f_h(V, \eta) = F_0(u') \exp\left(\frac{E}{\tau} + \frac{K_\eta}{\alpha_e} V\right), \quad (3.10)$$

when,

$$\frac{E}{\tau} + \frac{K_\eta}{\alpha_e} V = 0; \quad \Rightarrow \quad V_{\pm} = -\frac{K_\eta \tau}{m \alpha_e} \pm \left[\frac{2\tau e \theta^2}{m}\phi + \frac{\tau^2 K_\eta^2}{m^2 \alpha_e^2}\right].$$

Thus the maximum entropy form of equation (3.5) is determined together with the relevant limits in velocity space where it is valid. Making the untrapped distribution continuous at the boundaries, we can write,

$$f_u(V, \eta) = \exp\left\{-\frac{1}{T_e}\left[\pm\left(E + \frac{\tau K_\eta}{\alpha_e} V\right)^{1/2} - u'\right]^2\right\},$$

$$\text{when, } V < V_-, \; V > V_+ \quad (3.11)$$

This represents a shifted Maxwellian. As $K_\eta \to 0$ this entire set reduces to the one studied by Schamel et al. (1972) for nonlinear ion acoustic waves. However, they had also taken account of ion trapping. When $K_n = 0$, the drift wave branch will be absent and these equations will describe the parallel propagating ion acoustic wave. In the limit of $\theta^2 < 1$, the trapping will be a very small effect and the Boltzmann response for electrons recovered. τ is the temperature of the trapped region and for a hole in phase density, must be positive, $-\tau < 0$.

The electron density is given by

$$n_e(\eta) = \int_u f_u(v,\eta)\, dv + \int_h f_h(v,\eta)\, dv\,. \tag{3.12}$$

The fluid limit is obtained when $\tau \to 0$, giving,

$$n_e(\eta) = \exp\left(\frac{e\tilde{\phi}}{T_e}\right)$$

Thus (3.12) together with (3.10) and (3.11) gives the most probable electron response. Note that by fixing the value of 'b', we are being restricted to a single value of hole momentum P_o. The choice of τ and u' will give the corresponding values of T_o and M_o. Thus all parameters have been determined without ambiguity.

As said earlier the ions will be treated as a cold fluid, in the limit, $\omega >> k_{\parallel} V_{thi}$.

We use the equations of continuity and parallel momentum for ions in the low frequency approximation, in the limit k_x

$\ll k_y$. We retain parallel motion nonlinearity and effects of the order of $k_\perp^2 \rho_s^2 \sim O(1)$ in the polarisation drift giving,

$$\underset{\sim}{V}_p = \frac{-c/_{B\omega_{ci}} \left(V_i \partial_\eta^2 \tilde{\phi} \right)}{1 + \frac{c}{B\omega_{ci}} \partial_\eta^2 \tilde{\phi}} \hat{y} \qquad (3.13)$$

where V_i is now the ion fluid velocity, $V_i = \theta v_\parallel - u$. The denominator in equation (3.13) arises due to finite Larmor radius effects. Note that, in chapter II this term had been ignored giving only the dominant numerator. Then the ion equations become,

$$\partial_\eta (V_i n) - \frac{cK_\eta}{B} n \partial_\eta \tilde{\phi} - \partial_\eta \left[\frac{n \frac{c}{B\omega_{ci}} V_i \partial_\eta^2 \tilde{\phi}}{1 + \frac{c}{B\omega_{ci}} \partial_\eta^2 \tilde{\phi}} \right] = 0$$

$$V_i \partial_\eta V_i = -\frac{e\theta^2}{M} \partial_\eta \tilde{\phi} \left(1 + \frac{c}{B\omega_{ci}} \partial_\eta^2 \tilde{\phi} \right)$$

Solving these, we get the following results,

$$n(\eta) = \frac{e^{-K_\eta V_i/\alpha_i} + \bar{K}}{V_i} \left(1 + \frac{c}{B\omega_{ci}} \partial_\eta^2 \tilde{\phi} \right) \qquad (3.13)$$

$$\overline{W} = \frac{M}{2} V_i^2 + e\theta^2 \tilde{\phi} + \frac{M^2 c^2 \theta^2}{eB^2} (\partial_\eta \tilde{\phi})^2 \qquad (3.14)$$

where $\bar{W}$ and $\bar{K}$ are arbitrary constants. $\bar{W}$ is the parallel kinetic energy of fluid ions, with contributions from the polarisation drift. Using (3.14) to eliminate V_i in (3.13) we use the following boundary conditions to determine $\bar{K}$ and $\bar{W}$.

at $\eta = 0$, $\tilde{\phi} = \phi_0$, $\partial_\eta \tilde{\phi} = \varepsilon$, $\partial_\eta^2 \tilde{\phi} = 0$

and $$V_i = -u \tag{3.15}$$

Then, using quasineutrality and equation (3.12) we can write in normalised variables,

$$\partial_\eta^2 \phi = \frac{n_e(\eta)\, V_i(\bar{W})}{\exp\left(-K_n V_i/\alpha_i\right) + \bar{K}} \tag{3.16}$$

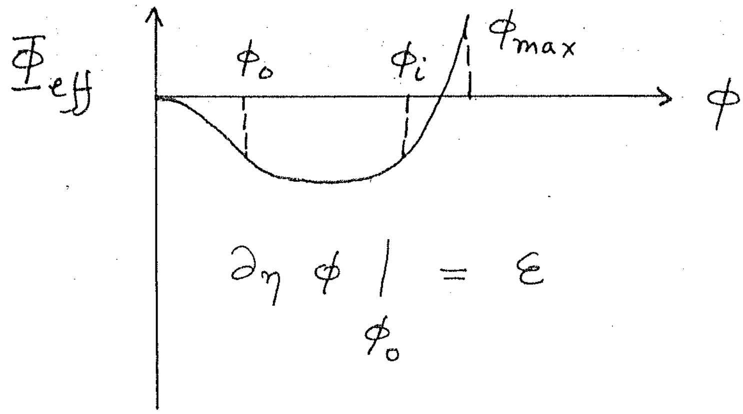
The electron density has been normalised to its value at, $\phi = \phi_0$.

Equation (3.16) describes the most probable steady state potential of a drift wave with electron trapping effects. When $\tau = 0$, the fluid limit for n (η) is recovered, since there is no trapping.

In chapter II we had studied the one dimensional fluid case in the presence of temperature gradients. Also, only the linear part of the polarisation drift was retained there in one dimension. So, both electrons and ions have been treated more exactly. It has been solved together with the conditions (3.15), numerically. Equation (3.16) can be written in terms of an effective potential,

$$\partial_\eta^2 \phi = -\, d_\phi \, \Phi_{eff}(\phi) \tag{3.18}$$

The concept of defining an effective potential has been used in the study of shocks in plasmas (Tidman and Krall 1971). For an ion acoustic shock, this is called the Sagdeev potential. It has the characteristic shape given below.



The equation (3.18) gives the description of a 'pseudoparticle' moving in the potential Φ_{eff}. The space variable becomes the effective time variable. The motion of such a pseudoparticle in Φ_{eff} gives the properties of $\phi(\eta)$. In our case the RHS. of (3.16) is not a function of ϕ, alone but also of $\partial_\eta \phi$. So it is not possible to write down an analytic form for Φ_{eff}. However, we have drawn an analogy with this description to explain the boundary conditions used. ϕ_0 is the turning point on the trajectory of such a pseudoparticle. At this point it is given an energy $\mathcal{E}$ and we look for oscillatory solutions of the system. This means specifying the value of $\partial_\eta \phi$ at ϕ_0. η acts as the time variable. Obviously, for larger values of $\mathcal{E}$, amplitudes will also be larger. In a single period the pseudoparticle will bounce upto some value ϕ_i and bounce back. The limit on $\mathcal{E}$ will come from,

$$u^2 + 2\theta^2 \phi_{max} + (\partial_\eta \phi)^2 \Big|_{\phi = \phi_{max}} - \varepsilon^2 = 0$$

for a given value of u. This is similar to the condition on the Mach number of an ion acoustic soliton. Since ϕ_0 is a turning point,

$$\partial_\eta \phi \Big|_{\phi_0} = \mathcal{E}, \quad \partial_\eta^2 \phi \Big|_{\phi_0} = 0$$

The effect of increasing $\mathcal{E}$ can be seen in Fig.(3.1). Curve (1) is the fluid electron limit and corresponds to the solution of the one dimensional problem in equation (2.13), when, $K_T = 0$. Note that the introduction of trapped electrons has lowered the amplitude for the same set of boundary conditions and values of K_n, u, $\mathcal{E}$. The minimum of the potential has been fixed at $\phi = 0$.

τ is the temperature of the particles in the hole region. The distribution function for $\tau > 0$ is shown in fig. (3.2). As τ becomes larger the dip becomes deeper. As a result a larger number of particles traveling close to u' are now available in the trapping region. So it can be expected that saturation should take place at larger amplitudes. This can be seen from the fig. (3.3). As τ becomes larger, the amplitude increases, together with the wavelength. The wavelengths of these periodic structures are typically of the order of a few ion Larmor radii (ρ_s), as expected from the results of the one dimensional treatment of chapter II.

3.4 A 1-D rectangular hole:

It would be interesting to see analytically how this maximum entropy state differs from that studied by Dupree (1982). Simple approximate solutions for an isolated hole may be obtained by using the rectangular hole approximation

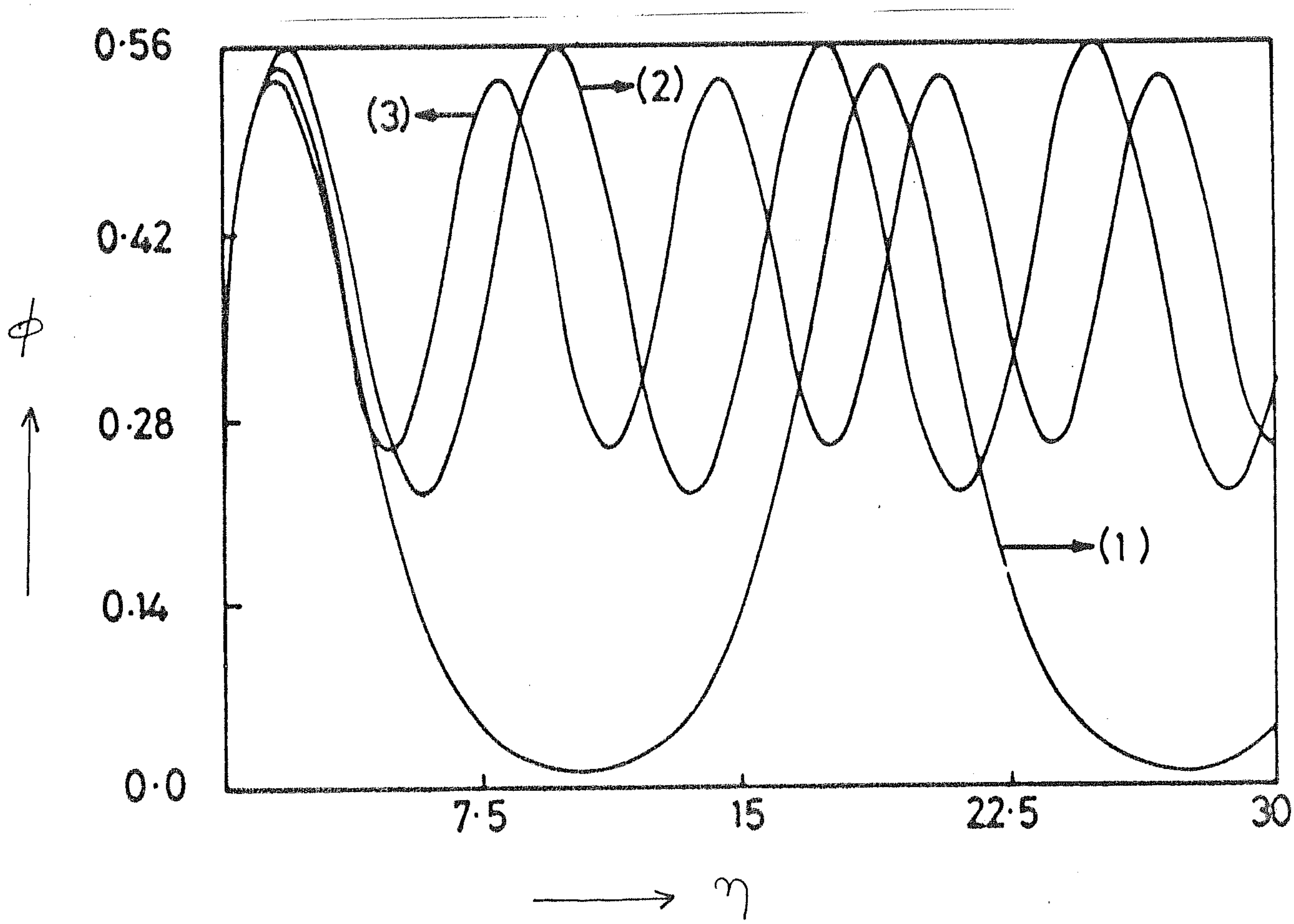


Fig. 3.1 : Potential showing effects of $\mathcal{E}$:

(1) The fluid limit of electrons

(2) $K_n = -0.1$, $u/\Theta = 1.2$, $\phi_0 = .2\ 5$, $\mathcal{E} = 0.3$

(3) $\mathcal{E} = 0.1$

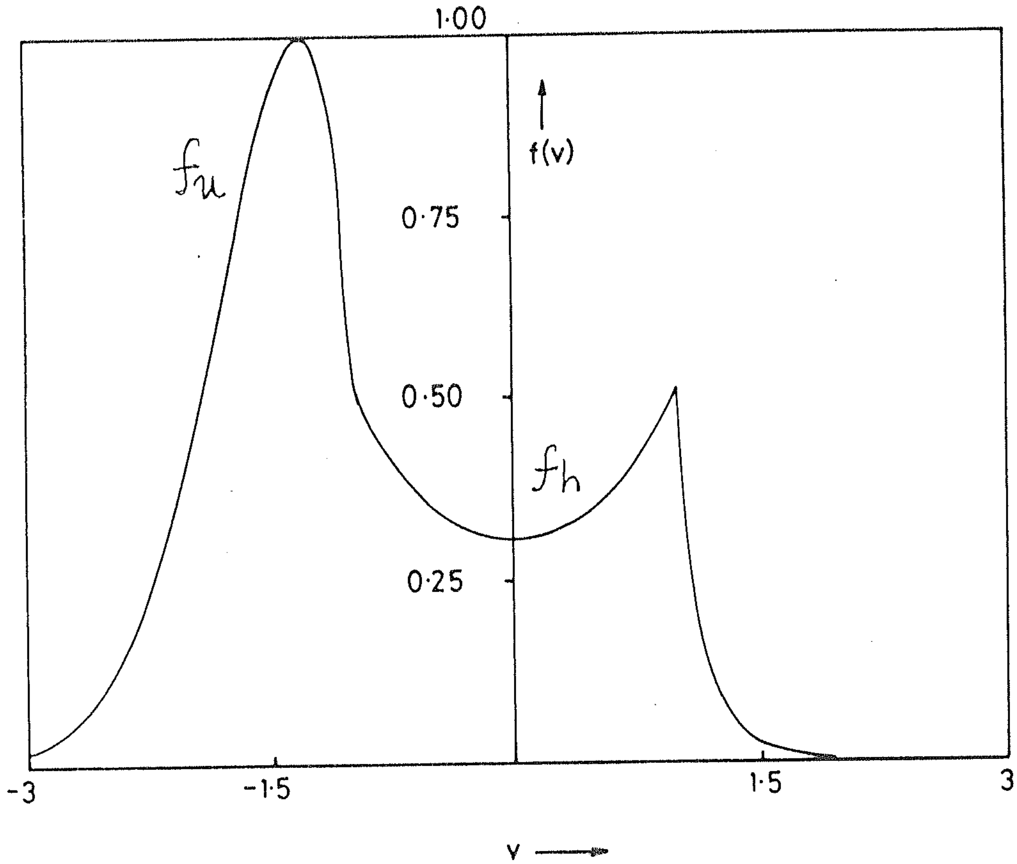


Fig. 3.2 : The complete distribution function for $\tau > 0$. The dip in the trapping region deepens with .

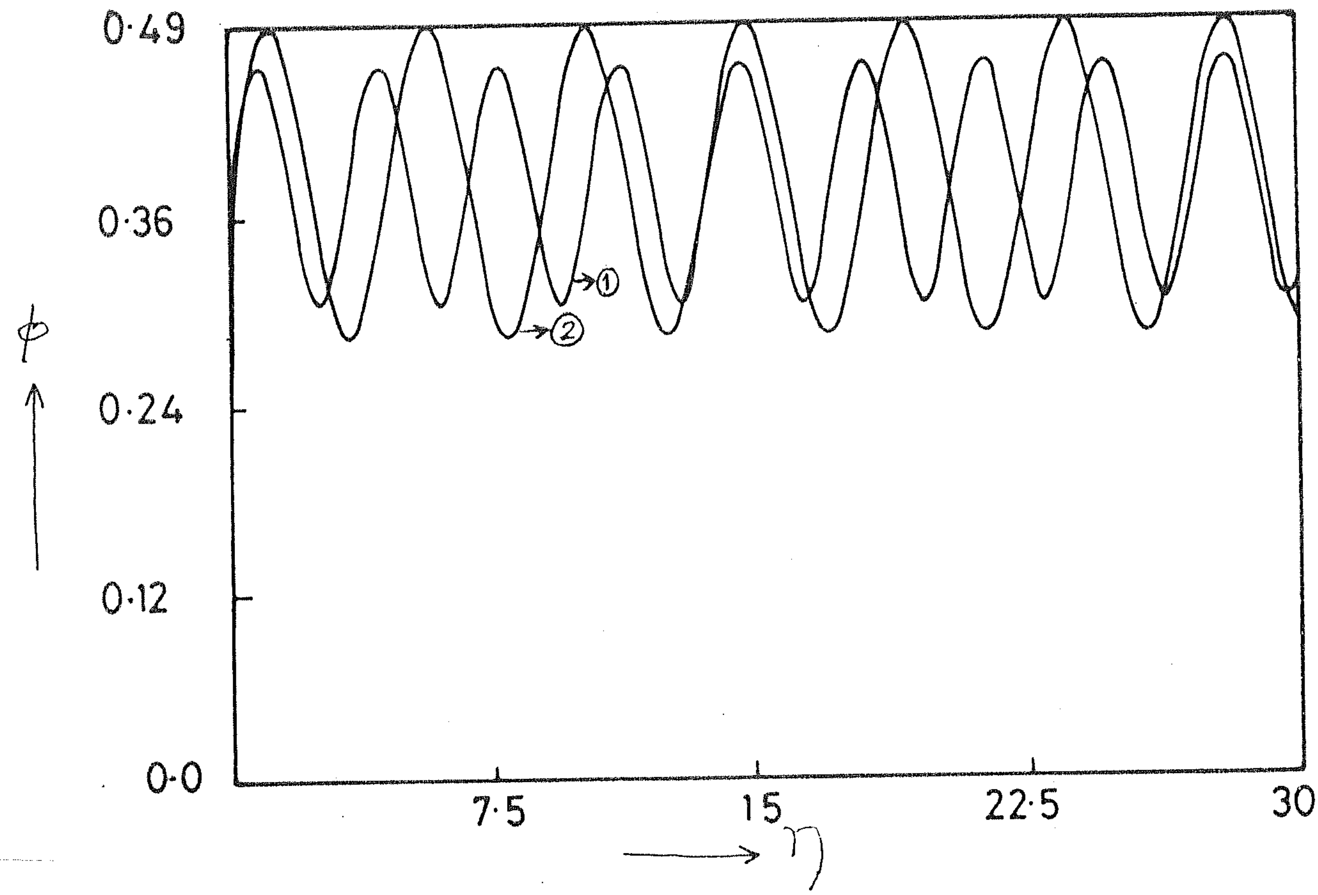


Fig. 3.3 : Potential showing effect of .

K_n = -0.1, u/Θ = 1.2, ϕ_o = 0.36, ε = 0.11:

(1) τ = 0.1, (2) τ = 0.23

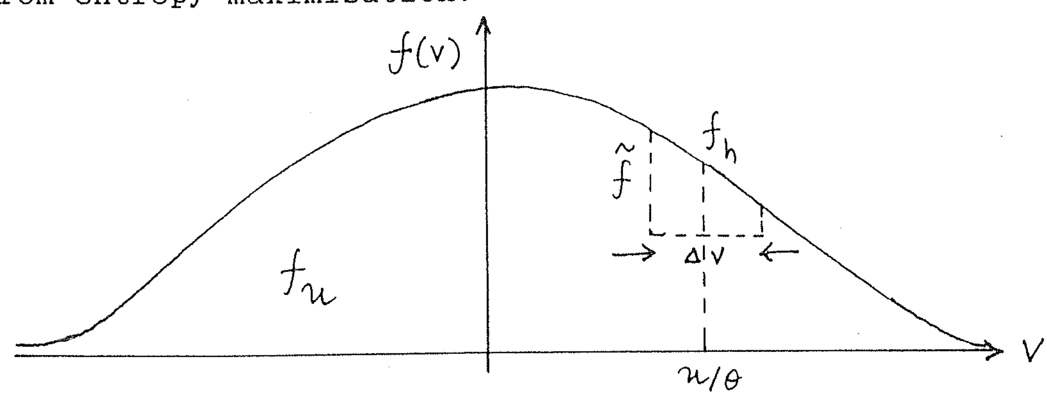
(Dupree 1982). The shape of the hole boundary is taken to be a rectangle instead of the one described by equation (3.10). When the hole is shallow, the hole depth can be approximated by a constant. The description of such a hole can be given by,

$$f(V,\eta) = \tilde{f}\, e^{K_\eta V/\alpha_e} + F_o(u)$$

$$\text{when,} \quad -\Delta\eta/2 \leq \eta \leq \Delta\eta/2$$
$$-\Delta V/2 \leq V \leq \Delta V/2$$

$$= F_o(u) \qquad \text{otherwise.} \tag{3.19}$$

for a hole moving with velocity u, and located at $\eta = 0$. The scale lengths in phase space, $\Delta\eta$, ΔV, will be determined now from entropy maximisation.



For the sake of simplicity we shall treat ions linear, giving,

$$\frac{n_i}{n_0} = \left(\rho_s^2 \nabla_\perp^2 - \frac{V_d}{u}\, \partial_\eta \right) \frac{e\tilde{\phi}}{T_e} \tag{3.20}$$

Equations (3.19) and (3.20) describe a special subset of hole solutions of the drift kinetic equation described in section (3.3). The parallel motion nonlinearity and FLR effects studied in chapter II and section (3.3) are neglected. Electron trapping effects are retained upto the $\sqrt{\phi}$ dependence of velocity limits of the trapping region. Higher order effects are ignored. The trapping parameter, τ, is replaced by $\tilde{f}$.

We now use the maximum entropy property of the hole to obtain the scale lengths $\Delta\eta$ and ΔV, subject to constraints that M_o, P_o and T_o as given by equation (3.7), are kept constant.

Using equation (3.17) and the definition of entropy given by equation (3.5) we get the expression,

$$\sigma = \frac{M_o \tilde{f}}{m F_o(u)} \tag{3.21}$$

for the entropy.

Using (3.19) and (3.20), we get the following equation for the potential $\tilde{\phi}(\eta)$,

$$(\partial_\eta^2 - \lambda^{-2})\,\tilde{\phi} = \frac{2T_e}{e}\,\tilde{f}\,\frac{\alpha_e}{K_n}\sinh\left(\frac{K_n \Delta V}{2\alpha_e}\right)$$

$$\text{where,} \quad \lambda^{-2} = (1 + V_d/u)\,/\,\rho_s^2 \tag{3.22}$$

Then using standard Greens function methods we can solve equation (3.22) to give the hole potential to be,

$$\tilde{\phi}(\eta) = -\frac{2 T_e \tilde{f} \alpha_e}{e L_\perp K_n} \sinh\left(\frac{K_n \Delta V}{2\alpha_e}\right) *$$

$$\times \frac{\lambda}{2}\left[1 - e^{-\Delta\eta/2}\cosh\left(\frac{\eta}{\lambda}\right)\right]. \qquad (3.23)$$

The hole mass, momentum and energy can be evaluated,

$$M_o = 2nm\tilde{f}\Delta\eta\,\frac{\alpha_e}{K_n}\sinh\left(\frac{K_n\Delta V}{2\alpha_e}\right)$$

$$P_o = \frac{M_o\alpha_e}{K_n}\left[\frac{K_n\Delta V}{2\alpha_e}\coth\left(\frac{K_n\Delta V}{2\alpha_e}\right) - 1\right]$$

$$T_o = \frac{K_n^2 M_o}{2\alpha_e}\left[\frac{K_n^2\Delta V^2}{4\alpha_e^2} + \frac{K_n P_o}{M_o\alpha_e}\right]$$

$$+ \frac{2nT_e}{L_\perp K_n^2}\tilde{f}^2\alpha_e^2\theta^2\sinh^2\left(\frac{K_n\Delta V}{2\alpha_e}\right)\left(\lambda\Delta\eta - \lambda^2 + \lambda^2 e^{-\Delta\eta/\lambda}\right)$$

In the limit of $K_n = 0$, these quantities revert to those calculated by Dupree (1982). Using these and maximising σ in (3.21), we get the following relation,

$$\frac{M_o}{2}(\Delta V)^2\sinh^2\left(\frac{K_n\Delta V}{2\alpha_e}\right) = Q_h\,\theta^2\tilde{\phi}(0)\,g\left(\frac{\Delta\eta}{\lambda}\right) \qquad (3.24)$$

where,

$$Q_h = qM_o/m$$

and $g(y) = \; dy\left[\frac{1}{y}\,\frac{(y + e^{-y} - 1)}{(1 - e^{-y/2})}\right]$

$$\tilde{\phi}(0) = \tilde{\phi}(\eta = 0)$$

In the limit of large scale length density gradients or small K_n,

$$(\Delta V)^2 \sim \tilde{\phi}(0)$$

recovering the parallel trapping limit of Dupree (1982).

There are corrections to this result due to the finite K_n. For a specified value of θ^2 and $\tilde{\phi}(0)$ equation (3.22) can be solved for ΔV. The potential $\tilde{\phi}(\eta)$ in the hole region is plotted in fig.(3.4). It has the right sign for trapping electrons and is an isolated hole solution unlike in the previous section, where we had looked for oscillatory solutions. $\Delta\eta$ typically varies from a few times ρ_s to some fraction of K_n^{-1}. Outside the hole region the $\tilde{\phi}(\eta)$ decays with the e-folding length λ. Note that, the solutions studied in section (3.3) can be regarded as a periodic array of such isolated holes.

3.5 Ion trapping effects:

In section (3.3) we have taken into consideration the nonlinear fluid ion response. This was a good description as long as u', the parallel hole velocity, was close to the electron thermal velocity. However, typically it lies anywhere between the ion and electron thermal velocities. If $T_i = T_e$, then as u' gets closer to the sound speed C_s, parts of the ion distribution will be affected by the resonance and get trapped by the wave field. Also as pointed out by Dupree(1982), the essential basis for this formalism is that, whatever happens in the nonlinear regime need not be predicted in a perturbative way only on the basis of the linear theory. So, it could very well happen that there are stationary nonlinear modes propagating close to the ion thermal velocity, even though linear theory does not predict them. In this parameter regime, we shall take into account

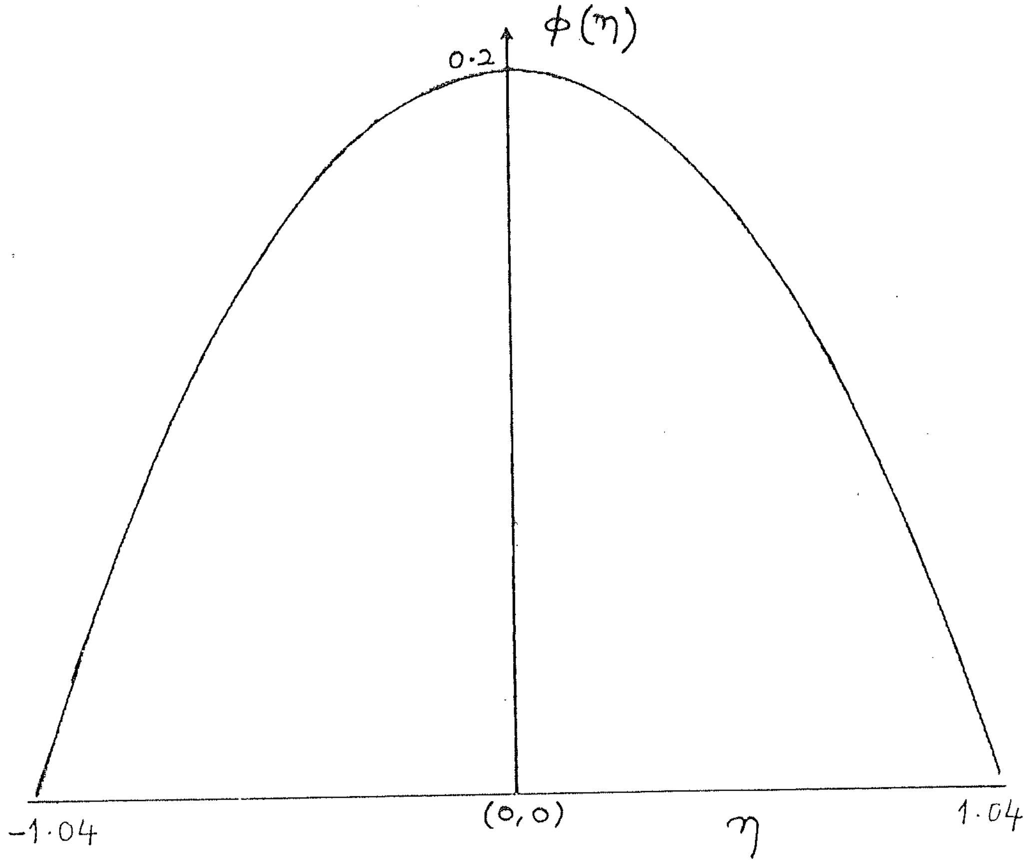


Fig. 3.4 : The one dimensional rectangular hole potential.

$\Delta\eta = 2.208$, $K_n = -0.1$, $u/\Theta = 1.2$, $\Theta = 0.02$

the ion trapping effects. In the ion DKE, perpendicular velocities would consist of the ExB and the polarisation drift as given by equation (3.13). V_1 is replaced by the phase space velocity $V = \theta v_{\parallel} - u$. Then the steady state ion DKE in one dimension becomes

$$\frac{V\partial_\eta f}{1+\frac{c}{B\omega_{ci}}\partial_\eta^2\tilde{\phi}} - \frac{e\theta^2}{M}\partial_\eta\tilde{\phi}\,\partial_V f = \frac{cK_n}{B}\partial_\eta\tilde{\phi} f + V\partial_\eta\left[\frac{\frac{c}{B\omega_{ci}}\partial_\eta^2\tilde{\phi}}{1+\frac{c}{B\omega_{ci}}\partial_\eta^2\tilde{\phi}}\right]f.$$

The general solution for this equation is,

$$f(V,\eta) = g(E_i)\exp\left(-\frac{K_n V}{\alpha_i}\right)\left(1+\frac{c}{B\omega_{ci}}\partial_\eta^2\tilde{\phi}\right),$$

where
$$\alpha_i = \omega_{ci}\theta^2,$$
$$E_i = \frac{M}{2}V^2 + e\theta^2\tilde{\phi} + \frac{M^2c^2\theta^2}{2B^2}(\partial_\eta\tilde{\phi})^2,$$

as in equation (3.14). The general distribution function to be determined, therefore, can be written as

$$f_{hs} = g_s(E_s)\exp\left(-\frac{K_n V}{\alpha_s}\right)\left(1+\gamma_s\partial_\eta^2\tilde{\phi}\right),$$

where 's' denotes the species.

$$E_e = \frac{m}{2}v^2 - e\theta^2\phi, \quad \alpha_e = -\omega_{ce}\theta^2, \quad \gamma_e = 0.$$

$$\alpha_i = \omega_{ci}\theta^2, \quad \gamma_i = \frac{c}{B\omega_{ci}}.$$

The total entropy of the system to be maximised is,

$$\sigma = \sum_s n_s \iint_h d\eta\, dv\,(f_h \ln f_h - F_{0s}\ln F_{0s}).$$

We now impose constraints for the maximisation. It turns out that it is not sufficient to say that the total mass, momentum and energy be constant. All the Lagrange multipliers do not get determined self consistently in that case. This is obvious since instead of a one species case as in the last section, we are now dealing with a two species system whose masses and charges are different. In order to take into account these we must introduce two additional constraints. Apart from saying that M_o, P_o, T_o as defined by,

$$\begin{bmatrix} M_0 \\ P_0 \\ T_0 \end{bmatrix} = \sum_s n_s \iint_h d\eta\, dv\, (f_{hs} - F_{0s}) \begin{bmatrix} m_s \\ m_s V \\ E_s \end{bmatrix}$$

be constant, we also say that the total current and charge of the system should remain unaltered, giving the following two additional constraints:

$$\begin{bmatrix} Q_0 \\ J_0 \end{bmatrix} = \sum_s n_s \iint_h d\eta\, dv\, (f_{hs} - F_{0s}) \begin{bmatrix} q_s \\ q_s V \end{bmatrix}$$

These additional constraints regarding constancy of total charge and current were not necessary in the last section. This is because, for a single species, constancy of total mass and charge and constancy of total momentum and current are degenerate constraints. Now introducing the Lagrange multipliers a, b, $-\tau^{-1}$, d and e for M_o, P_o, T_o, J_o and Q_o respectively and carrying out the maximisation processes, we obtain,

$$f_{hs} = \exp\left[-am_s - bv + \frac{E_s}{\tau} - dq_s v - eq_s - 1\right]$$

Or

$$g_s(E_s) = \exp\left[\frac{E_s}{\tau} + v\left(\frac{k\eta}{\alpha_s} - b - dq_s\right) - am_s - eq_s - \ln\left(1 + \gamma_s \partial_\eta^2 \tilde{\phi}\right)\right]$$

It is to be noted that the variations in the electron and ion trapped distributions are taken to be independent of each other, as are the variations in their respective velocity boundaries. Knowing that g_s is a function of E_s alone, at all times, we get the following algebraic equations for the Lagrange multipliers.

$$bm_s + dq_s = k\eta/\alpha_s$$

$$am_s + eq_s = -\ln\left(1 + \gamma_s \partial_\eta^2 \tilde{\phi}\right)$$

Thus, a, b, d and e are determined completely, fixing the values of the hole mass, momentum, current and charge. τ, the hole temperature remains a parameter. Then apart from the electron distributions in equations (3.10) and (3.11), we get the trapped and untrapped distributions of ions to be,

$$f_{iu}(v, \eta) = \exp\left\{-\frac{1}{T_i}\left[\pm\left(E_i + \frac{\tau k\eta v}{\alpha_i}\right)^{1/2} - u\right]^2\right\}$$

$$\text{when} \quad v > v_{i+}, \quad v < v_{i-}$$

and,

$$f_{hi}(v,\eta) = F_{oi}(u)\exp\left(\frac{E_i}{\tau} + \frac{K_\eta}{\alpha_i} v\right)$$

$$\text{when, } \quad V_{i-} \leq V \leq V_{i+}$$

where,

$$V_{i\pm} = -\frac{\tau K_\eta}{M^2 \alpha_i^2} \pm \left[\frac{2e\theta^2}{M}(\psi - \tilde{\phi}) + \frac{2Mc^2\theta^2}{eB^2}(\varepsilon^2 - \partial_\eta\phi)^2) + \frac{\tau^2 K_\eta^2}{M^2\alpha_i^2}\right]$$

ψ is related to the maximum of the potential.

In the limit of $\tau \rightarrow 0$, we recover the cold fluid limit of $n_i(\eta)$ given in the previous section.

Using these equations and quasineutrality we can write the following equation for the potential ϕ,

$$\partial_\eta^2 \phi = f(\phi) - 1$$

This equation was studied numerically in various parameter regimes. In case of the fluid ions it was seen that as u increases the amplitude of the potential decreases. This can be understood from the fact that as u increases the parallel resonance shifts further down the tail of the distribution. So lesser number of particles are available for trapping, even when τ is kept constant. However, for u′ close to C_s it was found that the amplitude of the potential in the trapped ion case was smaller than the fluid ion case for the same set of initial conditions. This is shown in fig. (3.5). The decrease is small. This can be understood again because the ion distribution function sampled by the resonance is very small, the effect of trapped ions is proportionately less. As u' increases beyond C_s this effect becomes negligible and the fluid description holds good.

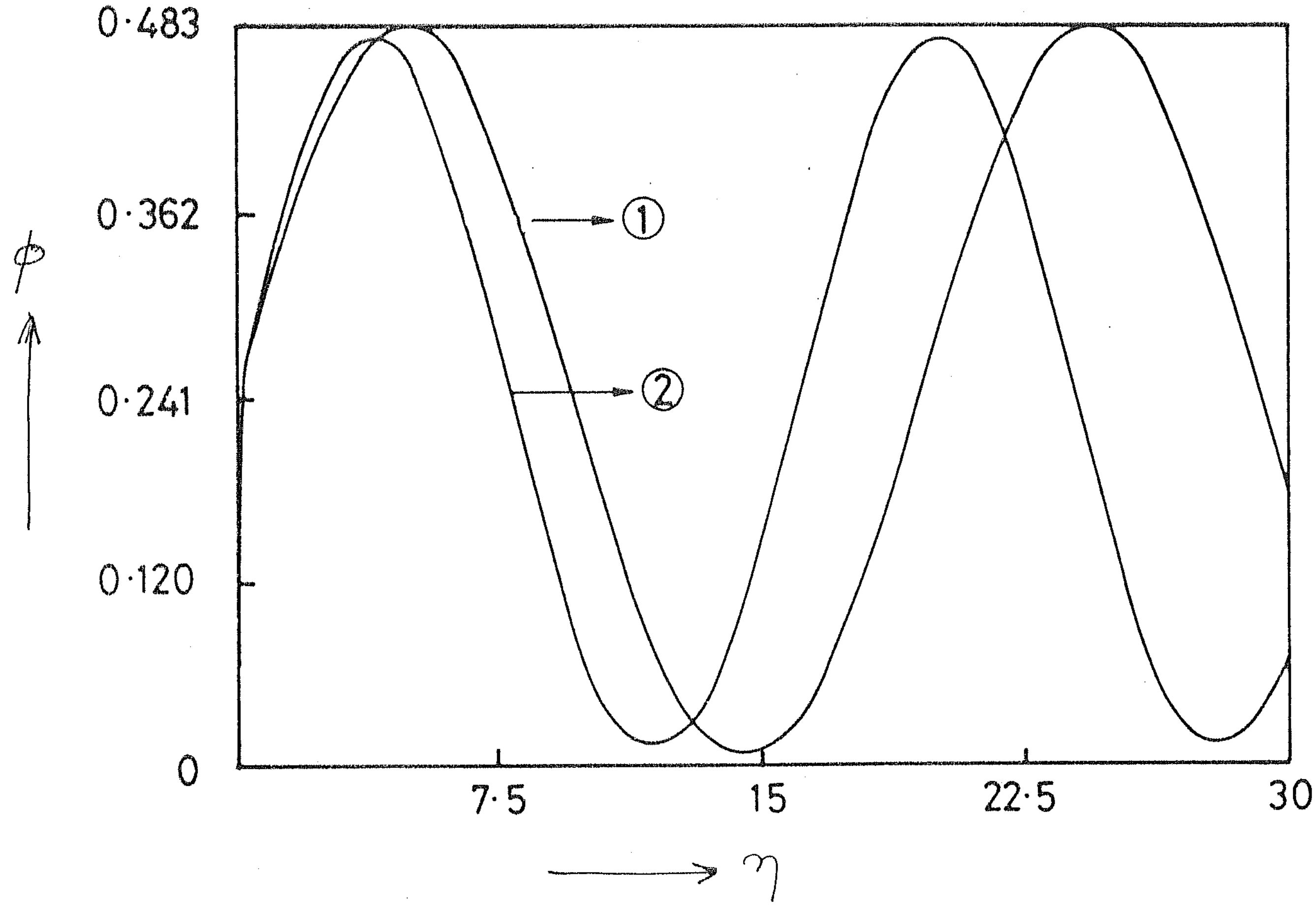


Fig. 3.5 : Trapped ion potential.

K_n = -0.1, u/Θ = 1.2, ϕ_o = 0.24,

ε = 0.11, τ = 0.1

(1) Fluid ions, (2) Kinetic ions

In the next section we shall consider a further modification in the form of an equilibrium temperature gradient. This is motivated by the interesting physics of the one dimensional fluid case studied in chapter II.

3.6 Temperature gradient effects:

In this section the maximum entropy state of a magnetised, inhomogeneous plasma with an equilibrium electron temperature gradient, due to phase space trapping in one dimension will be set up. In chapter II, the stationary solutions of the one dimensional fluid drift wave were studied in the presence of an electron temperature gradient (equation 2.13). It was shown that the scalar nonlinearity due to this term is important in certain parameter regimes and is, in fact, the most dominant nonlinear term. There, the electron response was taken to be adiabatic and given by the Boltzmann relation. However, as pointed out earlier, as the amplitude of the modes become large enough to trap particles, this is no longer a good description. In the following calculation, it will be modified to incorporate the parallel trapping effects. So, we are now applying the formalism for entropy maximisation, developed in the earlier sections, to a known case of interest, to see the difference in the properties of the final steady states.

In section (2.3) the parallel nonlinearity of fluid ions was also taken into account. In this section we shall continue to treat ions as a fluid and use the equation (3.14) to give their steady state response. So, their parallel fluid nonlinearity and the polarisation drift effects are also included.

As before, if we assume that there is an equilibrium temperature gradient in the x-direction, such that, when $k_x << k_y$, i.e. in one dimension,

$$\partial_x f / f = K_n + T K_T \partial_T f$$

Then in place of equation (3.3) we now get the following 1-dim. form for the DKE of electrons.

$$V \partial_\eta f + \frac{T c K_T}{B} \partial_\eta \tilde{\phi} \, \partial_T f + \frac{e\theta^2}{m} \partial_\eta \tilde{\phi} \, \partial_\eta f = \frac{c K_n}{B} \partial_\eta \tilde{\phi} f \qquad (3.25)$$

For $K_T = 0$, this reduces to equation (3.3). We assume that temperature T appears only as the normalisation of the energy E, therefore,

$$\partial_T f = \partial_{(E/T)} f \cdot \partial_T \left(\frac{E}{T}\right) = -\partial_E f \cdot \frac{E}{T}.$$

Substituting in (3.25) we obtain,

$$V \partial_\eta f - \frac{c K_T}{B} \partial_\eta \tilde{\phi} \, E \partial_E f + \frac{e\theta^2}{m} \partial_\eta \tilde{\phi} \, \partial_V f = \frac{c K_n}{B} \partial_\eta \tilde{\phi} f \qquad (3.26)$$

Apart from the parallel nonlinearity in the third term, there is an additional nonlinear term due to the temperature gradient. This term is similar to the scalar nonlinearity in the equation (2.9) and studied by Petviashvili (1977). In fact, if the parallel nonlinearity is neglected then, the trapping effect would vanish. The velocity average of equation (3.26) would then reduce to the Boltzmann relation for electrons with a temperature gradient. Then the equations (2.9), (2.10) would be recovered. It can be seen further that E is no longer a constant of motion. Changing the variables,

$$f(v,\eta) \equiv f(\eta, E),$$

equation (3.26) becomes,

$$\partial_\eta f - \frac{cK_T}{B}\frac{E}{V(E)}\partial_\eta \tilde{\phi}\, \partial_E f = \frac{cK_n}{B}\partial_\eta \tilde{\phi} f$$

Then, the new equation for the characteristics is,

$$d_\eta E = \frac{cK_T}{B}\partial_\eta \tilde{\phi}\frac{E}{V(E)},$$

where

$$V(E) = \left[\frac{2}{m}\left(E - e\theta^2\tilde{\phi}\right)\right]^{1/2}.$$

This equation can be solved perturbatively for the new constant of motion, $\bar{E}$,

$$\bar{E} = E\left(1 + \frac{K_T}{\alpha_e}V\right).$$

Then to lowest order in $(K_T \rho_s)$, the form of distribution function is,

$$f(v,\eta) = g(\bar{E})\exp\left(\frac{K_n V}{\alpha_e}\right).$$

Treating ions as a nonlinear fluid as in equation (3.14) and following the procedure for entropy maximisation given in section (3.3), we get an altered trapped electron distribution function:

$$f_h(V,\eta) = F_0(u)\exp\left[\frac{\bar{E}}{\tau} + \frac{K_\eta V}{\alpha_e}\right]$$

when, $\quad \dfrac{\bar{E}}{\tau} + \dfrac{K_\eta V}{\alpha_e} = 0$

$$\Rightarrow \quad V_\pm = -\frac{K_\eta \tau}{2\alpha_e V_{bo}} \pm \left(\frac{\tau^2 K_\eta^2}{4\alpha_e^2 V_{bo}^2} + \frac{2e\theta^2}{m}\tilde{\phi}\right)^{1/2}$$

where V_{bo} is the positive boundary value in the limit $K_T = 0$. Again, making the untrapped distributions continuous at the boundaries in velocity space, we can write,

$$f_u(V,\eta) = \exp\left[-\frac{1}{T_e}\left\{\pm\left(\bar{E} + \frac{\tau K_\eta}{\alpha_e}V\right)^{1/2} - u\right\}^2\right]$$

$$\text{when}, \quad V < V_-\ , \quad V > V_+$$

Using these distributions and the expression for $n_i(\eta)$ in equation (3.14) we can now solve for $\tilde{\phi}(\eta)$. These distributions reduce to the fluid limit when $\tau = 0$. The nonlinearity due the ∇T_e in equation (2.14) has now been modified by the presence of the trapped particles. Fig. (3.6) shows the results of the numerical calculations and the difference between the fluid electron case of section (2.3) and the effect trapping. As K_T increases both the amplitude and wavelength increase. However, the amplitude of the trapped electron case is lower than that of the solution of equation (2.14) for the same set of parameters. The difference between the amplitudes for successive values of K_T

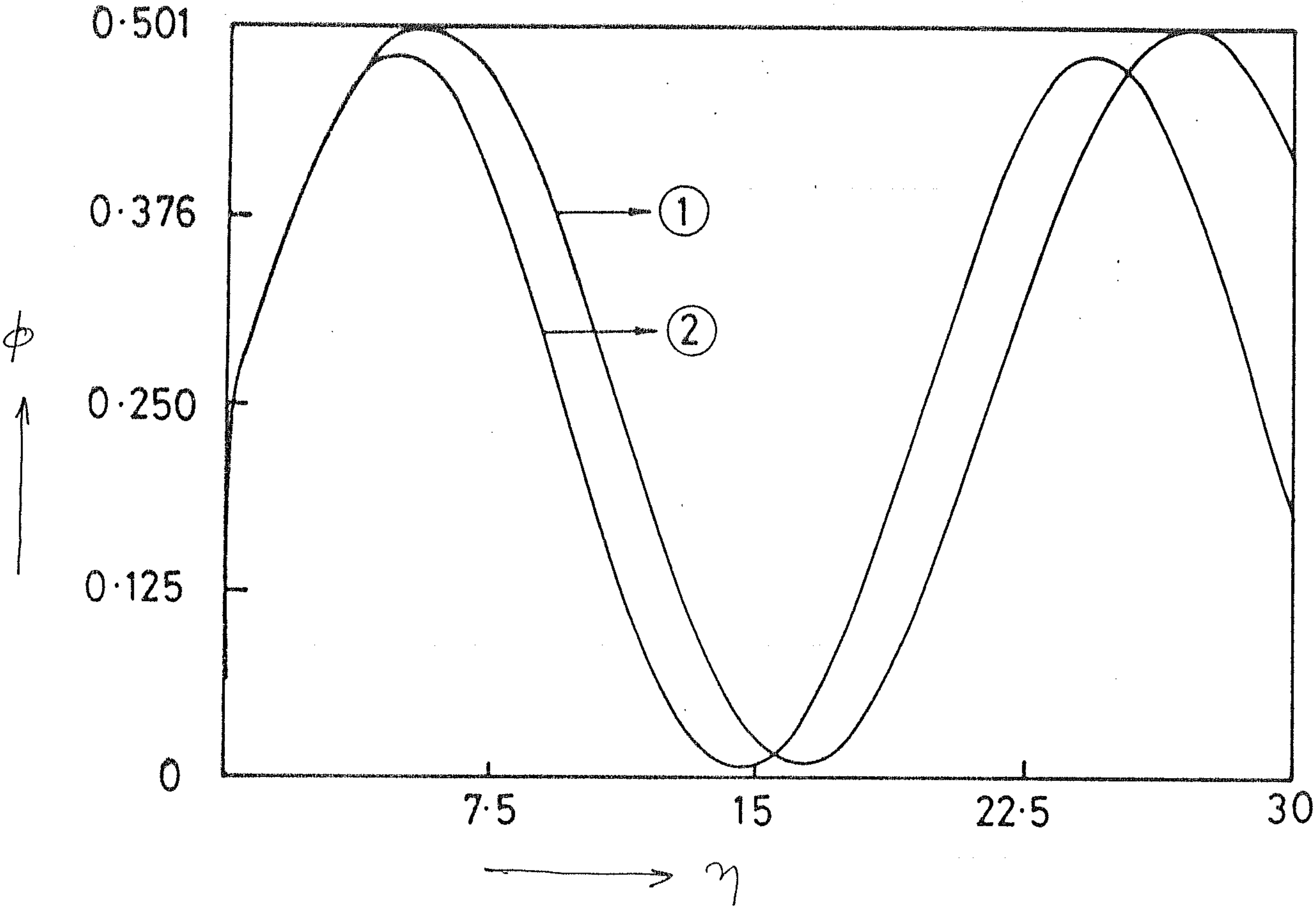


Fig. 3.6 : Temperature gradient case.

K_n = -0.1, u/Θ = 1.2, ϕ_0 = 0.25,

$\mathcal{E}$ = 0.11, τ = 0.1:

(1) K_T = 0.2, (2) K_T = 0.1.

is small since only first order terms in ($K_T \ell_s$) have been retained.

Thus we have successfully applied the formalism of maximum entropy stationary states to the interesting case of one dimensional structures with equilibrium temperature gradients. It should be remembered, however, that the fluid one dimensional solutions were unstable to two dimensional perturbations. This analysis is not attempted here for the trapped electron case.

3.7 Conclusion

In this chapter we extended the theory of phase space holes given by Dupree (1982) for an unmagnetised, homogeneous plasma to the case of a magnetised, inhomogeneous plasma. Electron and ion trapping effects were considered to build the most probable BGK modes in one dimension. In chapter II we had studied the stationary solutions of the fluid drift wave and found that they are vortices in two dimensional physical space. The solutions studied here are phase space structures in one dimension. The fluid solutions were essentially a result of the ExB and polarisation drift nonlinearities of ions. Therefore it would seem that in two dimensions there could be a coupling between these two effects in some parameter regime. The ExB nonlinearity would come into play when $k_x \sim k_y$ and then the system can no longer be reduced to a one dimensional form.

It would be appropriate at this point to explore the regime of existence of such states. These steady states may be regarded as saturated states of the drift instability. As the wave grows beyond the linear limit, the amplitude would become large enough for the resonant electrons near ω/k to be trapped in the potential of the wave. In the limit of sufficient density of trapped electrons the instability is quenched and a finite amplitude steady state established. In the limit $k_x \ll k_y$ used here the existence of such states is possible, even though the ExB nonlinearity has been ignored. However Ott et al. (1976) have examined the stability of drift waves with electrostatically trapped electrons in the presence of collisions. It appears that collisions can induce a detrapping of particles from the wave fields to give rise to a net growth. So the stability of these one dimensional phase space holes against collisions may restrict its stationarity to time periods smaller than the collisional detrapping time scales. Their stability to one and two dimensional perturbations will also have to be explored since the one dimensional fluid structures of chapter II were shown to be unstable to these.

In the next chapter we shall go on to the more complex problem of setting up two dimensional phase space holes with contributions from both the ExB and parallel phase space nonlinearity.

CHAPTER IV

Two dimensional drift phase space holes

4.1 Introduction:

In this chapter we take up the more complex problem of two dimensional maximum entropy states in magnetised, inhomogeneous plasmas.

In the preceding chapters we considered the exact solution of the nonlinear equations for such a system in two special cases. The first was, the fluid vortex solution of the generalised Hasegawa-Mima equation involving parallel ion nonlinearity and a temperature gradient. The next step was, to find maximum entropy solutions of the drift kinetic equation in one dimension. Parallel electron trapping, nonlinear fluid ions, parallel ion trapping and temperature gradient effects were examined. We set up the most probable steady states and studied their properties. The underlying assumption for the one dimensional nature was that, $k_x << k_y$. We now wish to study structures where $k_x \sim k_y$. Then the problem becomes two dimensional and the important ExB nonlinearity leading to physical space trapping of the particles comes into play. This trapping takes place when,

$$c k_\perp^2 \phi / B \omega \sim O(1)$$

As discussed in chapter I, there are several parameter regimes in which effects leading to the saturation of the drift turbulence have been examined. Earlier theories like the resonance broadening theory of Dupree(1967) and the ion Landau damping theory of Ching(1973) examined the importance of resonant ion nonlinearities. These models predicted the saturation amplitude of the turbulent spectrum to be:

$$\delta n / n \sim k_m / k_x$$

These theories directly took account of particle trapping due to the perpendicular ExB resonance effect in an obliquely propagating wave. The broadening of the parallel ion resonance due to the ExB trapping of ions predicted by Dupree(1967) brought a part of the ion distribution into the resonance leading to ion Landau damping. The associated trajectories of particles trapped in the physical space were first studied in detail by Ching(1973) in a completely coherent single mode picture. The process of saturation of a single drift mode was due to the shifting of the ion parallel resonance into the the bulk ion distribution as a result of the perpendicular trapping of ions. The drift wave then Landau damped heavily and saturated at a finite amplitude. However, unlike the resonance broadening theory, the resonance here remained sharp.

In this chapter we shall not attempt an analogous treatment taking account of resonant ion dynamics. Attention will be focused instead on the resonant electron nonlinearities, keeping the ions in the fluid limit. These were first studied by Lee et al.(1984) through gyrokinetic particle simulations. A more detailed study of the role played by the ExB trapping of resonant electrons in the saturation of drift turbulence was done by Smith et al.(1985). Their conclusions were reviewed in chapter I and will be repeated here for the sake of completeness.

The system studied in the gyrokinetic code was, to begin with, more coherent than that of Dupree(1967). A few modes were identified as being most dominant and contributing to the particle fluxes observed during the simulation. In the collisionless limit it was found that the dominant saturation mechanism at low levels of instability was the ExB advection of resonant electrons. No ion heating was observed so the Ching-type mechanism of ion parallel acceleration was absent and ions could be treated adequately in the fluid limit, with their nonlinear ExB convection and polarisation drift effects. Parallel electron acceleration was also seen to have only a higher order effect and could not carry out saturation by itself. In the weakly unstable limit (θ = 0.01) there was a small electron flux that could be effectively inhibited by the ExB advection of the resonant electrons. The saturation levels predicted by this mechanism were:

$$\delta n / n \sim (K_n / k_x)(\gamma_L / \omega_*)$$

So, when $\gamma_L < \omega_*$, the electrons were the saturating species. However, for θ = 0.002, the instability was stronger and the electron flux could not be effectively contained by their ExB advection. It was seen that when the instability is strong or $\gamma_L \sim \omega_*$ then the steady state reached also has a substantial contribution from the ExB ion nonlinearity. The energy exchange between the dominant modes leads to an enhanced electron velocity diffusion in the parallel direction. If then the ions are treated linear there is no saturation. The diffusion induces a nonlinearly generated phase shift between ion density modes and the potential, leading to growth. The only mechanism to counterbalance this phase shift is to bring in the ion ExB advection. They can still be treated as hydrodynamic, however, since $\omega/k >> V_{thi}$. This effect is seen to be important even in the absence of collisions for sufficiently small values of θ (Dimits 1988). However, due to the semi-coherent nature of the system under consideration, the resonance broadening mechanism was still not important. The only similarity was that, saturation was due to a balance between the electron and ion fluxes in the large amplitude limit.

As was pointed out in chapter I, the basic difference between all these efforts and the model developed in the present thesis is that, there are no particle fluxes in the saturated state. These arise primarily due to incoherent mode coupling effects which, by definition, are absent in this treatment. Therefore, the absence of any source of incoherence in the system (collisions could also contribute but are not included) is the major difference between the earlier theories and the present approach of studying maximum entropy coherent steady states. The incoherence would possibly enter when these structures are no longer isolated and start interacting at least weakly. Then particles could get detrapped and there could exist some finite particle fluxes. However, that interaction is out of the scope of this thesis.

Further, it is interesting to note that the electron ExB nonlinearity becomes important at lower amplitudes than that required for their parallel trapping to dominate. This can be seen most simply from the comparison of the two trapping frequencies.

$$\omega_{E\times B} = \frac{c k_\perp \tilde{\phi}}{B} \quad , \quad \omega_B{}^2 = \frac{e k_\parallel{}^2 \phi}{m} \quad .$$

Therefore, the estimate of the amplitude will be given by:

$$\frac{e\tilde{\phi}}{T_e} : \frac{\theta^2}{\delta^2} \quad , \qquad \text{where,} \quad \theta = \frac{k_\parallel}{k_\perp} \quad , \quad \delta^2 = \frac{M}{m}$$

and, $$k_\perp \rho_s \sim O(1) .$$

When $\theta^2 \sim \delta^2$, then both effects have to be accounted for, but when, $\theta^2 < \delta^2$, the dominant mechanism at low amplitude is the ExB trapping of resonant electrons with corrections due to phase space trapping.

In the following sections we shall treat two separate cases regarding the comparative weightage of the various nonlinear effects. We shall first take into account the entire nonlinear electron dynamics but treat the ions linear in the regime of $\gamma_i < \omega_*$. Some limiting cases will be recovered analytically and we shall also solve the problem of the complete maximum entropy state in such a case. So we have gone one step beyond the work in chapter III and included the effects of the perpendicular motion of resonant electrons.

We shall next present a model to include what seem to be the two most dominant saturation mechanisms at low amplitudes. Both ion and electron ExB trapping takes place at the same levels of amplitude. So the correct procedure would be to incorporate the perpendicular trapping of both species in the kinetic limit. However, we shall not attempt this complete problem here. Instead, we shall treat the two most dominant nonlinear effects leading to saturation isolated by the simulations: the ExB trapping of kinetic resonant electrons and the fluid ion ExB convection. Some methods learnt in the study of vortex solutions of fluid drift waves in the second chapter will be applied to study interesting monopole solutions in this case. The nonlinearity is entirely different from the fluid case. There, it was the parallel ion motion that led to the formation of these structures; here the ExB trapping of resonant electrons and the convection of fluid ions will play that role.

It would be appropriate to add a note about the processes leading to an increase in entropy in the situation considered here. In the earlier chapter the mixing in phase space due to parallel trapping and the resultant modification in the velocity distribution in the trapping region was the cause for the increase in entropy. However, the dominant mechanism here is due to the rearrangement of the physical space density gradient. This is reminiscent of the quasilinear theory where in the saturated state the density gradient in the resonant region is flattened out. However, it was seen in the simulations of Smith et al.(1985) that steady states with finite density gradients in the resonant region could be achieved. We shall look for such generalised solutions. The flat gradient would be only a special case.

4.2 <u>Two dimensional structures</u>:

In the last chapter we found that the electron DKE in the steady state has solutions of the form:

$$f(x,\eta,v) = f(K,E),$$

$$K = v + \omega_{ce}\theta^2 x,$$

$$E = \frac{1}{2}mv^2 - e\theta^2\tilde{\phi}.$$

The entropy of the system with a phase space hole is given by,

$$\Gamma = n\iiint_h dv\,d\eta\,dx\left(f_h \ln f_h - F_0(u,x)\ln F_0(u,x)\right).$$

In the most general two dimensional system the constraint equations would be,

$$\begin{bmatrix} M_0 \\ P_0 \\ T_0 \end{bmatrix} = n\iiint_h dv\,d\eta\,dx\,(f_h - F_0(u,x))\begin{bmatrix} m \\ mK \\ E \end{bmatrix},$$

where $F_0(u,x)$ is the untrapped or equilibrium distribution.

We wish to determine f_h and its boundary in the velocity space by maximising σ keeping M_o, P_o, T_o constant. This gives us two equations:

$$f_h = \exp\left(\frac{E}{\tau} - bK - a - 1\right) ,$$

and $F_o(u,x) = f_h$ on the boundary. $-1/\tau$,a,b are Lagrange multipliers as before. $F_o(u,x)$ is the equilibrium distribution and a function of x. We choose the equilibrium gradients to be of the form,

$$F_o(u,x) = F_o(u)\, e^{xK_n} .$$

F_o has an x-directed density gradient as before. Therefore, on the boundary,

$$F_o(u)\, e^{xK_n} = \exp\left(\frac{E}{\tau} - bK - a - 1\right) .$$

The equation of the boundary is given by,

$$\frac{E}{\tau} - bK - K_n x = \text{const.} ,$$

$$e^{-a-1} = F_o(u) .$$

The complete distribution function is,

$$f_h(x,\eta,v) = F_o(u) \exp(E/\tau - bK) \quad \text{in the hole region,}$$

$$= F_o(u) \quad \text{otherwise.} \tag{4.1}$$

The hole boundary in velocity space is given by,

$$V_{\pm} = \frac{\tau b}{m} \pm \left[\frac{2e\theta^2}{m}\tilde{\phi} + \frac{2\tau x}{m}(b\,\omega_{ce}\theta^2 + k_n) + \tau^2 b^2/m^2\right]^{1/2} \tag{4.2}$$

These equations give the complete description of the nonlinear electrons in the maximum entropy steady state. The combined dependence of the hole distribution on E and K takes account of both parallel as well as perpendicular trapping. This can be seen from, i) the $\sqrt{\phi}$ dependence typical of parallel trapping and ii) the x-dependence due to physical space trapping, in equation (4.2).

We shall treat ions linear and write the quasineutrality condition as in chapter I, to give,

$$\left(\rho_s^2 \nabla_\perp^2 - \frac{v_d}{u} - 1\right)\frac{e\tilde{\phi}}{T_e} = \int_h dv\,\left(f_h - F_0(u,x)\right) \tag{4.3}$$

Note that unlike in the case of the one dimensional problem 'b' does not get determined. For different values of b, P_0 will take different values. Also it will determine the form of the gradient in the mixing region. So we are looking for general solutions of the two dimensional problem.

Equation (4.3) will have to be solved numerically. We shall, however, do a simpler calculation first in order to see the essential physics of the problem and recover some known results. The solution of equation (4.3) will be presented later in the light of these limiting cases.

4.3 Rectangular hole approximation:

In the last section we maximised entropy to find the form of the distribution function in the mixing region and its boundaries in velocity space. The resulting equation (4.3) does not give any analytical insight into the problem. So we shall do the following reduced problem, following section 3.3.

We assume that the distribution function in the mixing region is known to us. Its boundaries in phase space, Δx, $\Delta\eta$ and ΔV, will be considered to be parameters. Then the entropy will be maximised to find these parameters, keeping M_o, P_o and T_o constant.

If the fluctuation level is small, then we can write the hole distribution to be:

$$f_h(x,\eta,v) = \tilde{f}\, e^{K_\eta K/\alpha_e} + F_o(u,x)$$

$$\text{when, } -\Delta x/2 \le x \le \Delta x/2$$
$$-\Delta\eta/2 \le \eta \le \Delta\eta/2$$
$$-\Delta V/2 \le v \le \Delta V/2 \quad (4.4)$$

$$= F_o(u,x) \quad \text{otherwise.}$$

Using this we can calculate the following quantities,

$$M_o = \frac{2\eta m \Delta x \Delta\eta\, \alpha_e}{K_\eta}\tilde{f}\sinh\left(\frac{K_\eta \Delta K}{2\alpha_e}\right), \quad (4.5)$$

$$P_0 = \frac{\alpha_e M_0}{K_n}\left[\frac{K_n \Delta K}{2\alpha_e}\coth\left(\frac{\Delta K\, K_n}{2\alpha_e}\right) - 1\right], \qquad (4.6)$$

$$T_{0KE} = \frac{M_0}{2}\left(\frac{\alpha_e}{K_n}\right)^2\left[\frac{K_n^2 \Delta K^2}{4\alpha_e^2} - \frac{2 P_0 K_n}{\alpha_e M_0}\right] + \frac{M_0^2 \alpha_e^2}{24}\Delta x^2 , \qquad (4.7)$$

where, $\Delta K = \Delta V + \alpha_e \Delta x$, and the entropy can be evaluated to be:

$$\sigma = M_0 \tilde{f} / m\, F_0(u, x) .$$

Equation (4.3) gives the equation for $\tilde{\phi}(x,\eta)$:

$$\left[\nabla_\perp^2 + (1 + V_d/u)/\rho_s^2\right]\frac{e\tilde{\phi}}{T_e} = \frac{2\alpha_e \tilde{f}}{\rho_s^2 K_n}\sinh\left(\frac{K_n \Delta K}{2\alpha_e}\right) . \qquad (4.8)$$

Define the Green's function,

$$G(x-x', \eta-\eta') = \frac{1}{L_\perp}\sum_m G_m(x-x')\, e^{ik_m(\eta-\eta')} ,$$

where $G_m(x,x')$ satisfies the equation,

$$(\partial_x^2 - \lambda^{-2})\, G_m(x-x') = \delta(x-x'),$$

and is given by,

$$G_m(x-x') = -\frac{\lambda}{2} e^{-|x-x'|/\lambda}$$

where, $\lambda^{-2} = (1 + k_m^2 \rho_s^2 + V_d/u)/\rho_s^2$

Therefore is given by:

$$\tilde{\phi}(x,\eta) = -\frac{4 T_e \alpha_e \lambda^2 \tilde{f}}{L_\perp e \rho_s^2 K_n k_m}\sinh\left(\frac{\Delta K\, K_n}{2\alpha_e}\right) *$$

$$* \; e^{ik_m\eta} \sin\left(\frac{k_m \Delta\eta}{2}\right)\left(1 - e^{-\Delta x/2\lambda}\cosh\left(\frac{x}{\lambda}\right)\right) . \tag{4.9}$$

The potential energy part of T_o can now be evaluated,

$$T_{OPE} = -e\theta^2 \iiint_h dx\,d\eta\,dK\; \tilde{\phi}\,\tilde{f}\, e^{K_n K/\alpha_e}$$

$$= \frac{E_o M_o}{m} \quad \frac{\sin(k_m \Delta\eta/2)}{k_m \Delta\eta} \cdot \frac{\lambda}{\Delta x}\, \frac{\left(\frac{\Delta x}{\lambda} + e^{-\Delta x/\lambda} - 1\right)}{\left(1 - e^{-\Delta x/2\lambda}\right)},$$

where, $E_o = -\,e\theta^2 \tilde{\phi}(x=0, \eta=0)$ and,

$$T_o = T_{OKE} + T_{OPE} . \tag{4.10}$$

We need to maximise σ in order to find Δx, $\Delta\eta$, ΔV . Differentiating (4.10) with respect to Δx keeping M_o, P_o constant and putting $\partial_{\Delta x} \tilde{f} = 0$, we get the following results,

$$\Delta\eta \simeq 2\pi / k_m \tag{4.11}$$

and

$$(\Delta K)^2 \operatorname{sech}^2\left(\frac{K_n \Delta K}{2\alpha_e}\right) = \frac{(\alpha_e \Delta x)^2}{3} + \frac{2E_o}{\pi m}\, g\left(\frac{\Delta x}{\lambda}\right) \tag{4.12}$$

where,

$$g(y) = d_y \left[\frac{1}{y}\, \frac{(y + e^{-y} - 1)}{(1 - e^{-y/2})} \right] .$$

Consider the following limits of equation (4.12).

1. As $K_n \to 0$ and $\theta^2 < 1$,

$$(\Delta K)^2 \sim \frac{2E_o}{\pi m}\, g\left(\frac{\Delta x}{\lambda}\right),$$

Or,

$$\frac{M_o}{2}(\Delta K)^2 \sim \frac{Q_h \theta^2}{\pi}\, \tilde{\phi}(0)\, g\left(\frac{\Delta x}{\lambda}\right). \qquad (4.13)$$

This recovers the parallel trapping limit with the dependence of square root of potential. The function g introduces corrections due to the two dimensionality of the problem.

2. However, for $\theta^2 << 1$, K_n small but finite and for small amplitudes $e\tilde{\phi}(0)/T_e < 1$, we recover the limit,

$$\Delta v \simeq \frac{Q_h}{M_o} \cdot \tilde{\phi}(0)\, g\left(\frac{\Delta x}{\lambda}\right) \Big/ \pi \omega_{ce} \Delta x \qquad (4.14)$$

This is the case studied by Lee et al.(1984) and Smith et al.(1985) in considering the pure perpendicular trapping of electrons. They get a Hamiltonian for the particle motion in a known potential $\tilde{\phi}(x,\eta)$ to be,

$$H = \frac{c}{B}\tilde{\phi}(x,\eta) + \frac{\omega - k_{\parallel} v_{\parallel}}{k_{\perp}} x$$

This is also the result derived by Hirshman(1980) for physical space trapping in an oblique wave and gives the closed trajectrories in (x,η) space discussed in chapter I. Far away from the resonance the trajectory of a single particle gives straight lines in the $x-\eta$ plane. Near the resonance they close giving the trapping effect. We have recovered this result in equation (4.14). It is therefore clear that for finite density gradients and small amplitudes the electron dynamics, especially their ExB advection effects, dominate. Equation (4.12) can be solved for values of Δx and ΔV in various parameter regimes.

Fig. (4.1) gives the behaviour of ϕ as a function of x and η from equation (4.9). It falls off smoothly and has the correct sign for trapping electrons.

As is shown in fig.(4.2), the fall of ϕ in x, becomes less sharp as θ^2 is increased. In the limiting case there would be a very shallow potential profile in the hole region and would essentially correspond to the parallel trapping limit.

These results are now used as inputs to the numerical solution of equation (4.3). We look for solutions that are periodic in η and bounded in x. The results show a surprising degree of matching. The value of b is varied from $-K_n/\alpha_e$ to 0. It is seen that the solution does not change character very much except for the value of the amplitude. But the value of θ^2 has the same effect as predicted by the results of the rectangular solution. Fig. (4.3) and (4.4) show that as θ^2 increases, the potential profile in the $x-\eta$ direction becomes shallower. Thus we can say that the rectangular hole is a good approximation to the actual complete problem. It gives the essential features of the structures and is easier to handle, without losing any significant information. Note that while the rectangular hole

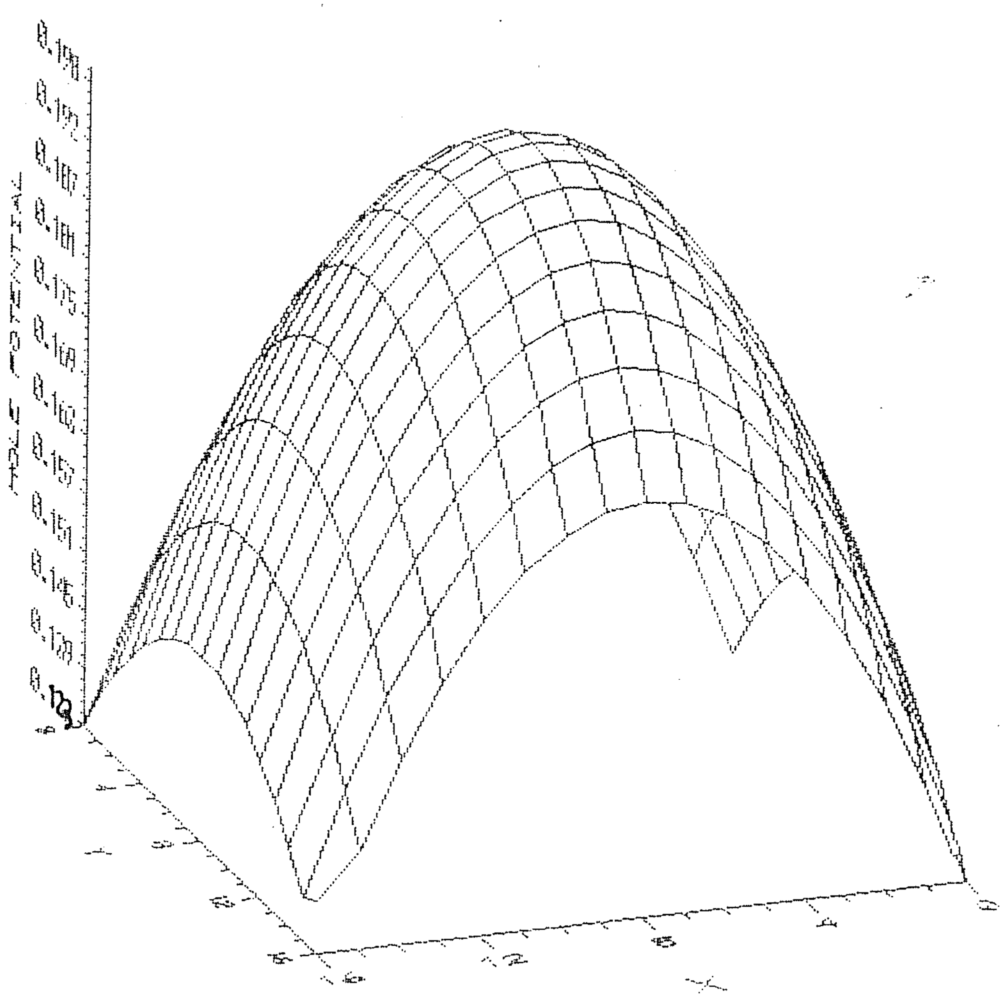


Fig. 4.1 : A 2-d box hole. $\Theta = 0.01$, $\phi_{min} = 0.123$.

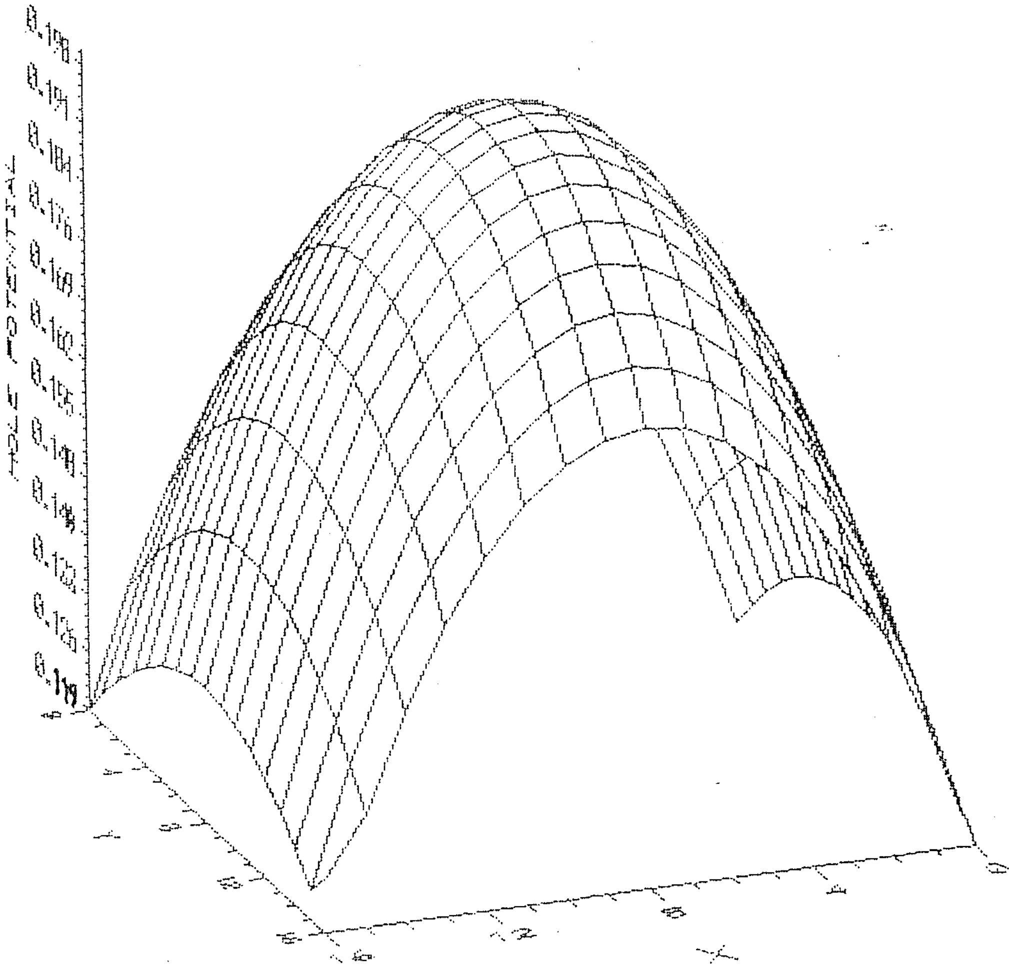


Fig. 4.2 : Box solution for $\Theta = 0.02$, $\phi_{min} = 0.119$.

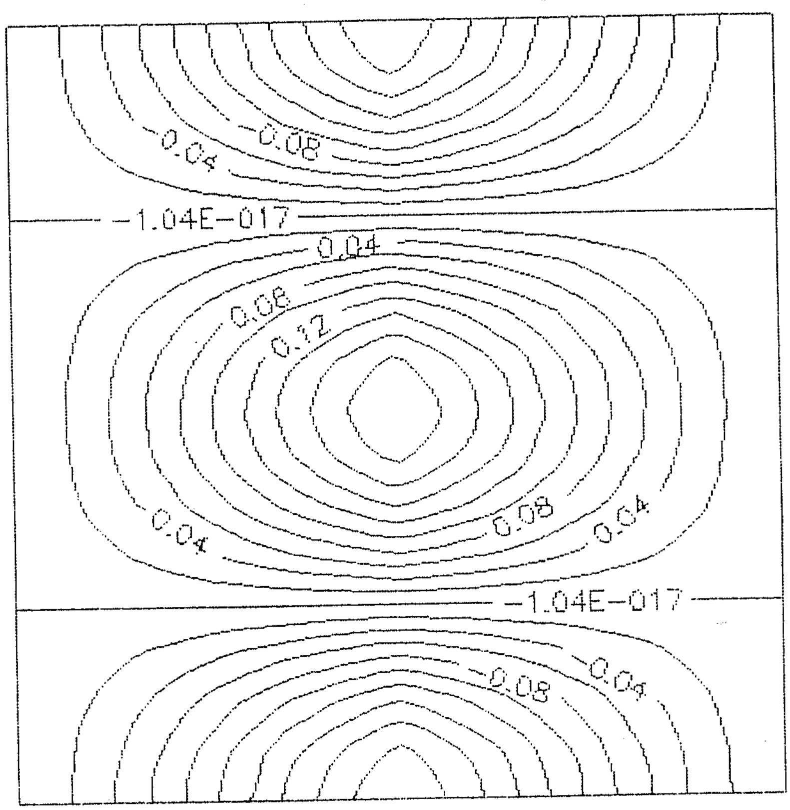


Fig. 4.3 : Solution of equation (4.3) for $\Theta = 0.01$.

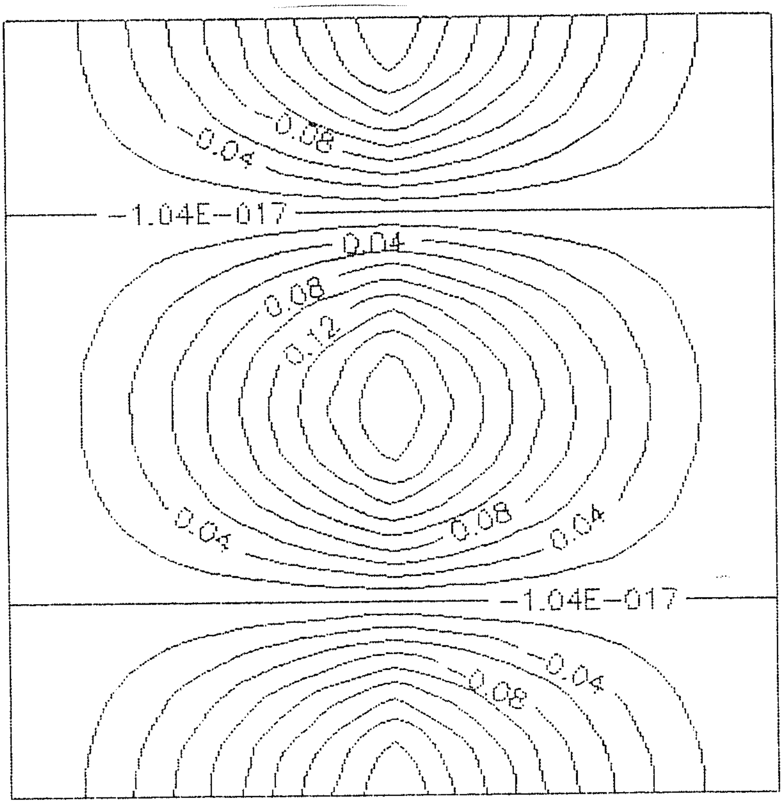


Fig. 4.4 : Solution of equation (4.3) for $\Theta = 0.02$.

has a finite potential on the boundary, the solutions of equation (4.3) have been set to have zero potetial on the boundary. We have also looked for solutions that are periodic in the η direction. This solution may be looked upon as a periodic array of the rectangular solutions.

As mentioned in chapter I, both electron and ion ExB trapping occur at the same amplitude levels since the ExB drift is mass independent. So in principle, both should be treated simultaneously in the kinetic limit for this nonlinearity. However, we shall not attempt this complex problem here. The simulation studies of Smith et al.(1985) and Dimits(1988) make the following observation in the range of strong instability. As the system evolves towards a steady state, the fluctuations in the electrostatic potential cause electron diffusion and an associated phase shift between the electron density modes and the respective potential modes. For the spectrum to saturate, there must also be an associated phase shift between the ion density modes and the potential. Therefore, if the ion dynamics are kept linear the modes keep growing. It seems that the only way to stop this growth is to include the fluid ion ExB convection.

In the next section we make use of this input in order to present a first attempt at including the fluid ion ExB convection, keeping electrons kinetic with the maximum entropy property to set up an equation for the steady state potential.

4.4 Nonlinear ion effects:

This section will concentrate on the task of combining the maximum entropy electron distribution retaining only the ExB nonlinear effect, with the fluid ion ExB nonlinearity. Some techniques learned in Chapter II will be used to set up an entirely new monopole vortex solution for the steady state potential. The issue of parallel trapping of ions will not be attempted here.

Again it is to be remembered that in the simulation studies the steady state has a balance of electron and ion fluxes generated by their ExB dynamics. In the present model there are no particle fluxes. So, this cannot be considered to be a direct analogue of the simulation results. However, we have incorporated what seem to be the most relevant nonlinear effects. In principle, what effects actually dominate the nonlinear stage of a turbulent system is still a matter of educated conjecture. So, to that extent, all parameter regimes should be examined without bias. This being an almost impossible task we have taken the cues from existing studies to explore the most prevalently interesting regime.

From the calculations of section 4.2 we know that maximising the entropy for the electron DKE gives the following results for the hole distribution,

$$f_h = \exp\left(\frac{E}{\tau} - bK - a - 1\right),$$

and, on the boundary,

$$F_0(x)\, e^{xK_n} = \exp\left(\frac{E}{\tau} - bK - a - 1\right),$$

giving,

$$e^{-a-1} = F_0(x),$$

and,

$$e^{E/\tau - bK - xK_n} = \text{constant}.$$

Since we do not wish to retain the parallel trapping effects, we now make the following assumption:

$$e^{E/\tau} \sim \tilde{f} / F_0(x),$$

where $\tilde{f}$ is a constant. This is an assumption similar to that made in the last section for rectangular holes. We are not interested in studying the fine structure of the trapping region in velocity space. So it is replaced by a flat rectangular dip with constant depth. However, in the last section the entropy was maximised after the assumption was made. The scale lengths in phase space were consistent with this assumption. Here, we are making this approximation after finding the maximum entropy form of both f_h and its boundary in velocity space. Therefore, this form of the hole distribution can no longer be considered strictly most probable. However, we feel justified in doing so since the last section has shown that for a large parameter regime the prediction of the rectangular hole calculation is an excellent approximation to the complete problem.

The equation for the velocity boundary now becomes,

$$\tilde{f}/F_0(x)\, \exp\left(-bK - xK_n\right) = \text{constant},$$

giving,

$$K + \frac{K_n}{b} x + \frac{\bar{\lambda}}{b} = 0 , \tag{4.15}$$

where,

$$e^{\bar{\lambda}} = \text{const.} \cdot F_0(u) / \tilde{f} \, .$$

Since the parallel resonance is not of interest here, we can expand K as,

$$K = \sqrt{E} \left(1 + \frac{\theta^2 \phi}{2\sqrt{E}} \right) + \theta^2 x \, .$$

Normalised quantities are used. To the lowest order in ϕ,

$$V = \sqrt{E} ,$$

and the equation (4.15) becomes,

$$V + \frac{\theta^2 \phi}{2V} + \left(\theta^2 x + \frac{K_n}{b} x + \frac{\bar{\lambda}}{b} \right) = 0 \, .$$

To first order in ϕ, this gives the boundary in V-space to be,

$$V_{\pm} = - \frac{(\chi x + \bar{\lambda}/b)}{2} \pm \left[\frac{(\chi x + \bar{\lambda}/b)}{2} - \frac{2\theta^2 \phi}{\chi x + \bar{\lambda}/b} \right] .$$

where,

$$\chi = \theta^2 + K_n / b$$

Note that this closely resembles the prediction of the single particle Hamiltonian of Smith et al.1985), which gives,

$$V \sim \phi / x$$

for the trapped region in physical space. The density of the trapped region is given by,

$$n_h = \int_h dv \, \left(f_h - F_0(u, x) \right)$$

knowing,

$$f_h = F_0(x)\,\tau\, e^{-bK} \; ; \quad V_- \leq V \leq V_+ ,$$

where,

$$\tau = \tilde{f} / F_0(x) .$$

Integrating and expanding for small ϕ, to order ϕ^2, we get n_h to be,

$$\frac{n_h}{n_0} = \frac{\tau}{b} e^{-b\theta^2 x}\Big[e^{b(\chi x+\lambda)} + \frac{2\theta^2\phi}{\chi x+\lambda}\left(2e^{K_n x} - e^{b(\chi x+\lambda)}\right)$$

$$+ \frac{2b^2\theta^4}{(\chi x+\lambda)}\,\phi^2 .\; e^{b(\chi x+\lambda)} \Big],$$

$$\lambda = \bar{\lambda}/b .$$

The untrapped density is approximated by the adiabatic electron response,

$$\frac{n_u}{n_0} = e^{xK_n}\left[1 + \theta^2\phi + \theta^4\phi^2/2\right].$$

The total density is,

$$\ln n = N = p(x) + q(x)\,\phi + r(x)\,\phi^2 ,$$

where,

$$p(x) = \ln\left[\frac{\tau}{b} e^{-b\theta^2 x + b(\chi x+\lambda)} . \frac{2b^2\theta^4}{(\chi x+\lambda)^2} + \frac{\theta^4}{2}\right]$$

$$q(x) = \frac{\dfrac{2\theta^2\phi}{\chi x+\lambda}\,\dfrac{\tau}{b}\, e^{-b\theta^2 x}\left(2e^{xK_n} - e^{b(\chi x+\lambda)}\right) + \theta^2 e^{xK_n}}{\tau/b\; e^{-b\theta^2 x + b(\chi x+\lambda)} + e^{xK_n}} .$$

$$r(x) = -q^2(x)/2 + \left[\frac{\tau}{b} e^{-b\theta^2 x + b(\chi x+\lambda)} . \frac{2b^2\theta^4}{(\chi x+\lambda)^2} + \frac{\theta^4}{4}\right] /$$

$$\left[\tau/b \, e^{-b\theta^2 x + b(\chi x + \lambda)} + e^{x \kappa_n} \right].$$

Let us calculate what the ion equation would look like. The ion continuity equation is,

$$\partial_t n + \underset{\sim}{\nabla} \cdot (n \underset{\sim}{v}) = 0 .$$

In the steady state we retain only the ExB nonlinear terms and the linear polarisation drift. Parallel motion and nonlinear polarisation effects are ignored. Then in normalised quantities,

$$\partial_\eta N + \frac{1}{u} \partial_\eta \phi \, \partial_x N - \frac{1}{u} \partial_x \phi \, \partial_\eta N - \partial_\eta \nabla_\perp^2 \phi = 0 , \tag{4.16}$$

where,

$$N = \ln n .$$

Using quasineutrality we know that,

$$N = p(x) + q(x)\phi + r(x)\phi^2. \tag{4.17}$$

Therefore we now know N completely as a function of x and η in terms of ϕ and parameters of the system.

We now look for monopole type vortex solutions as in chapter II. We want solutions of the form,

$$\nabla^2 \phi = m(x)\phi + n(x)\phi^2. \tag{4.18}$$

Substituting equations (4.17) and (4.18) in (4.16) and comparing coefficients, we can write,

$$m(x) = q(x) + \partial_x p(x)/u ,$$

$$n(x) = r(x) + \partial_x\, q(x)/2u,$$

giving,

$$\nabla^2\phi = \left[q(x) + \frac{\partial_x\, p(x)}{u}\right]\phi + \left[r(x) + \frac{\partial_x\, q(x)}{2u}\right]\phi^2 . \quad (4.19)$$

This gives the equation for the steady state potential with the resonant electron and fluid ion ExB effects included. The solutions of this equation would be vortex structures in (x, η) space. Remember that the phase space trapping effect has been neglected. So the modification of the phase space in the trapping region will arise solely due to the physical space trapping of electrons.

Fig. (4.5) describes the situation. For every constant-V surface, there is some ExB trapping that modifies the phase space in the (x, η) plane. The parallel trapping effect leads to mixing of two or more constant V surfaces leading to the modification of the distribution in velocity space.

In chapter II we had studied the monopole vortex solutions of the generalised Hasegawa-Mima equation by retaining the parallel ion motion. The treatment presented here is similar only to the extent that the LHS of equation (4.19) comes from the ion polarisation density. The coefficients of ϕ and ϕ^2 in the present case come from the perpendicular nonlinearity of electrons and ions. The adiabatic Boltzmann response used for electrons in chapter II is now severely modified with their resonant ExB dynamics.

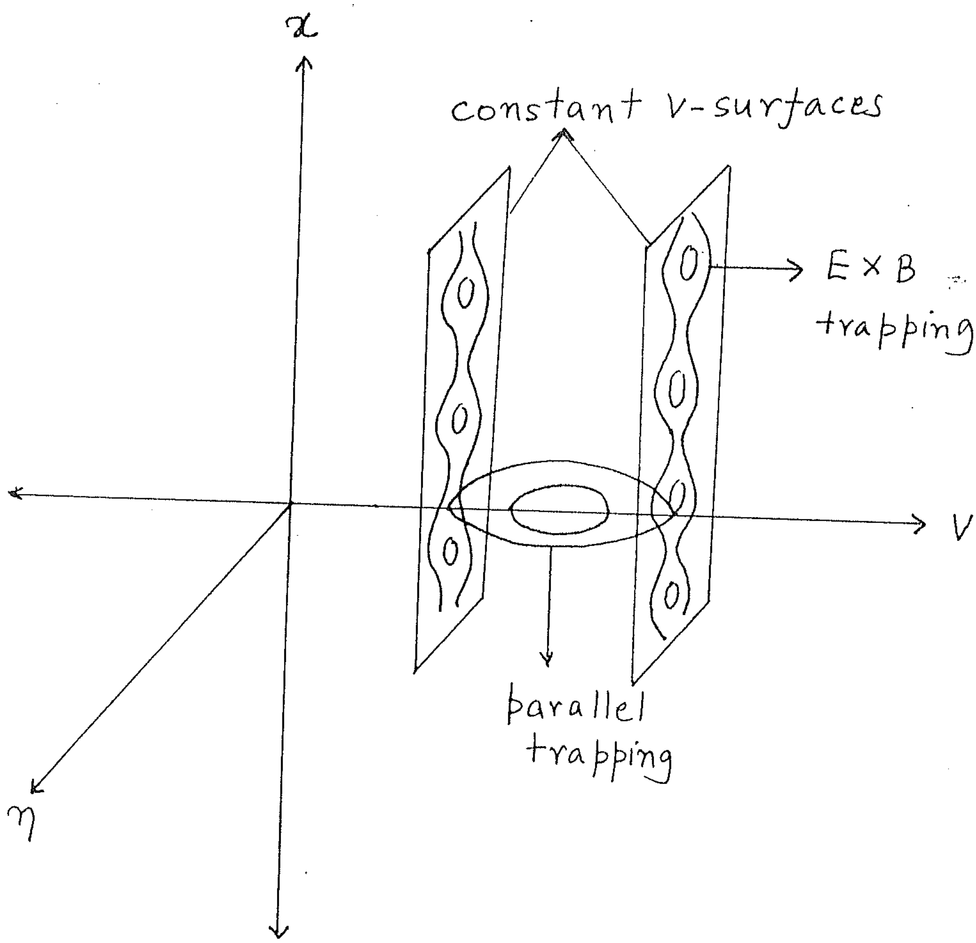


Fig. 4.5 : Trapped particle orbits for perpendicular and parallel trapping leading to mixing in phase space.

Numerical solutions of equation (4.19) were carried out for a few cases. It is seen that monopole solutions do exist for such a system. The parameter τ, which gives the effective temperature of the trapped region, is now substituted by the ratio, $\tilde{f} / F_0(x)$, giving the relative size of the hole in the distribution function. As seen in the earlier cases the amplitude of the potential increased with increasing τ, since more number of particles took part in the wave-particle interaction. We see a similar trend here also as τ is increased. Fig. (4.6) shows a typical monopole solution with $\tau = 0.01$. The potential on the boundaries is zero and peaks at an off-centre point in the x-η plane. Figs. (4.7) and (4.8) show the increasing amplitude for $\tau = 0.05$ and 0.1 respectively. Values higher than this would go against the shallow hole approximation.

It is quite clear, therefore, that a monopole solution exists for this particular combination of the electron and ion nonlinearities. The entire parameter regime for the existence of these solutions has not been worked out here. However, since the values of various parameters considered are realistic it is a physical solution that needs to be taken into account in any theory of drift turbulence that retains the existence of coherent structures.

4.5 Conclusion:

In this chapter we have effectively integrated all that was learnt in the earlier chapters in order to set up an

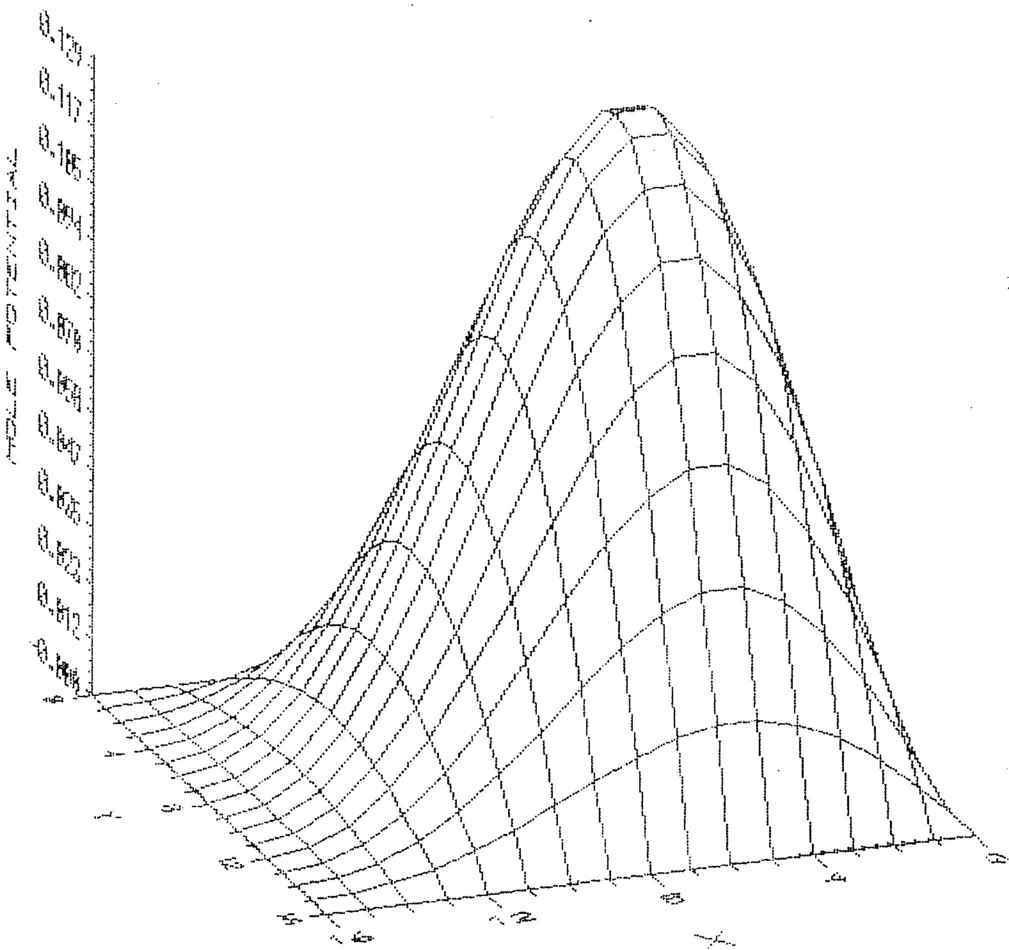


Fig. 4.6 : Monopole solution of eq. (4.19) for $\theta = 0.01$, $\phi_{max} = 0.129$.

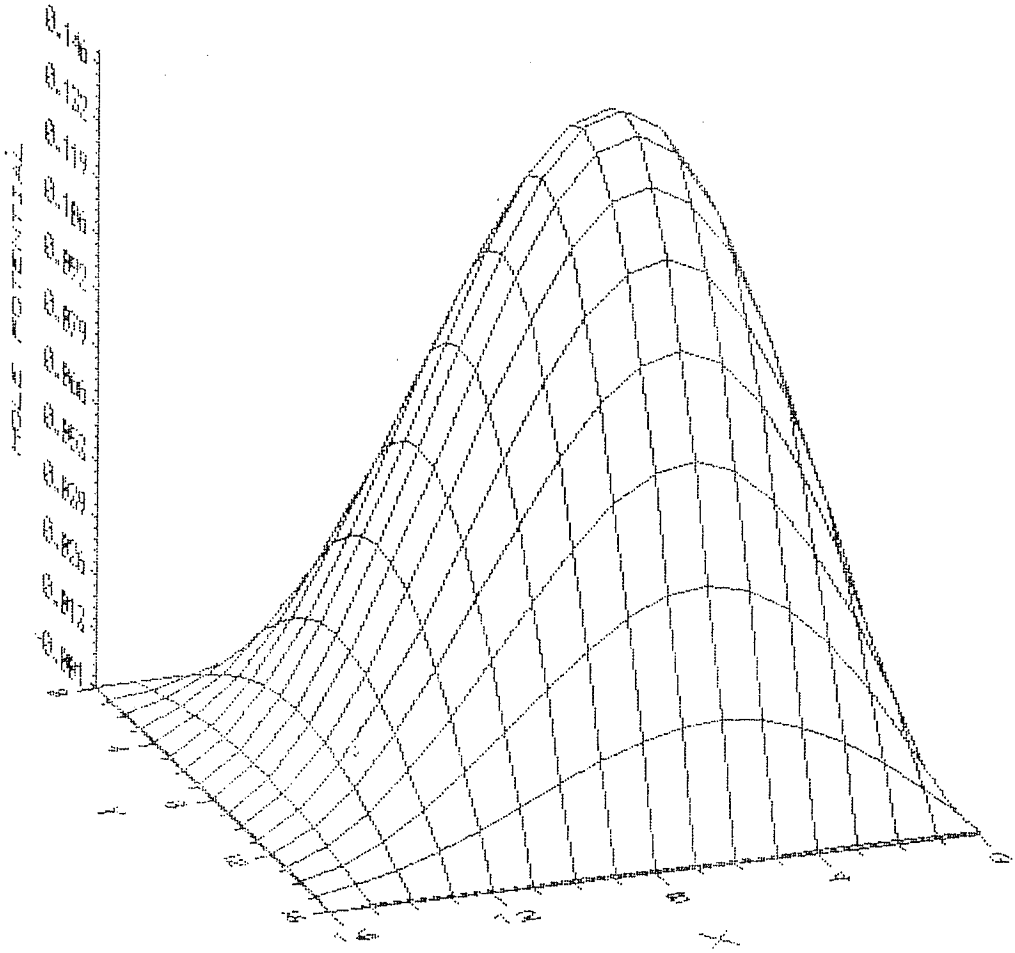


Fig. 4.7 : θ = 0.05, ϕ_{max} = 0.146.

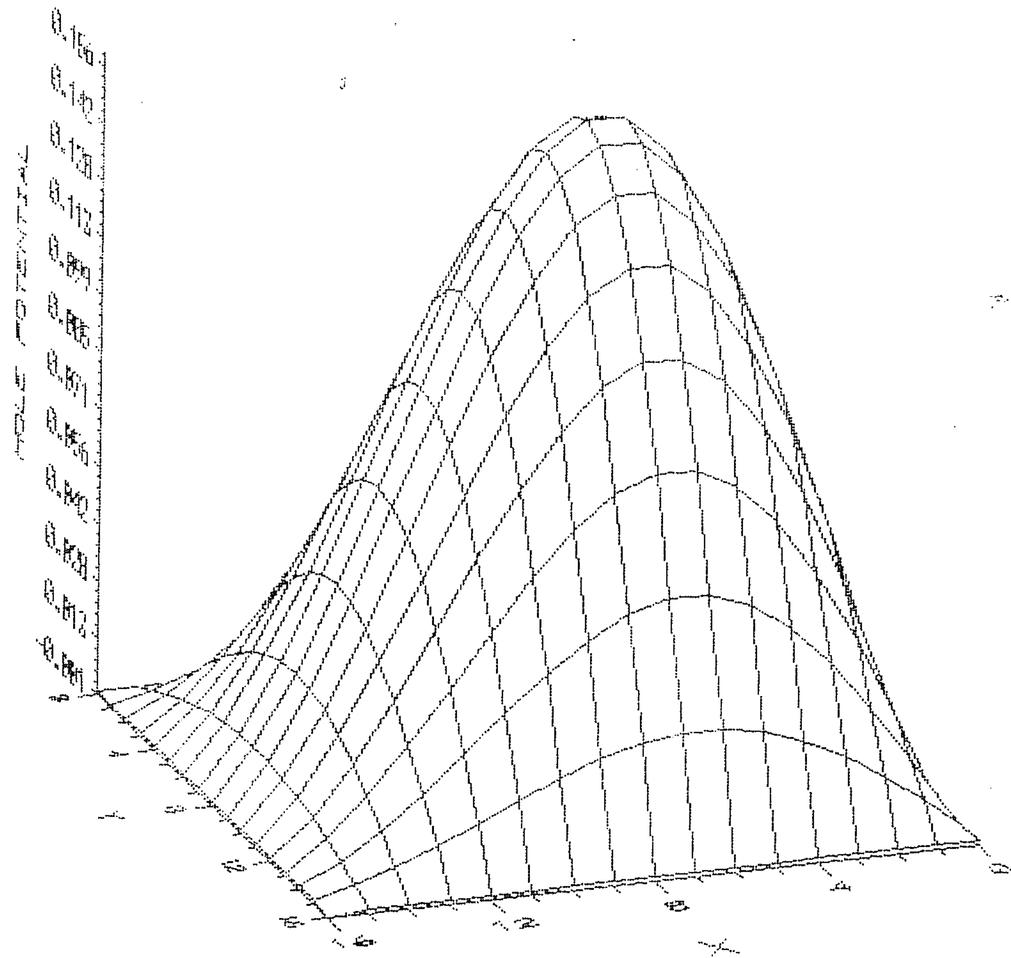


Fig. 4.8 : $\theta = 0.1$, $\phi_{max} = 0.156$.

almost-maximum entropy state for a combination of electron and ion nonlinearities that the simulations have shown to be of most importance. The resonant ExB trapping of electrons in perpendicular physical space was first added to the parallel trapping problem of chapter III keeping the ions linear. The limiting cases of pure phase space trapping and the Hamiltonian for physical space trapping (Ching 1973, Hirshman 1980, Smith et al. 1985) were recovered. It was shown that a reduced description in the form of the rectangular hole is accurate without losing any significant information. The resulting maximum entropy periodic structures in two dimensions were studied numerically.

Then we incorporated the ion fluid ExB convection into the problem neglecting the parallel electron nonlinearity to set up an almost maximum entropy monopole vortex solution. Unlike in the second chapter the nonlinear terms in the vortex equation came from the resonant ExB trapping of electrons. It was shown numerically that monopole solutions for realistic values of the parameters could indeed be found for the equation.

The results of this chapter are different from those of Dupree(1982) in that the dominant nonlinearity is not the phase space but the physical space trapping of particles which becomes important at a lower amplitude. This is the most interesting wave-particle interaction in this case.

It is obvious, however, that several important effects have been left out of the present treatment. The resonance broadening effects with resonant ion dynamics (Dupree 1967) and the ion Landau damping of Ching(1973) have not been included. In the last chapter we shall summarise the results and deficiencies of the work done so far in this thesis in brief and suggest some possible interesting paths along which it can be extended.

CHAPTER V

Particle acceleration in perpendicular waves

5.1 Introduction:

In the preceding chapters the low frequency response of a uniformly magnetised, inhomogeneous plasma was studied for the existence of nonlinear stationary states. This essentially involved the consideration of a reduced form of the Vlasov equation which takes into account phenomena in the frequency range $\omega \ll \omega_{ci}$. The perpendicular velocities in such a system are replaced by the particle drift velocities in those directions. The resultant maximum entropy steady states were assumed to be formed due to wave particle interaction. The two effects taken into consideration were: phase space trapping and perpendicular trapping in physical space. Their effects on the properties of the resultant stationary states were studied.

As a first step towards extending this formalism to the high frequency regime, the specific case of wave particle interaction in an electrostatic wave propagating perpendicular to an ambient magnetic field was taken up. This problem was studied by some earlier workers. It was found that for a negligibly small magnetic field, the wave particle interaction in similar to that in an unmagnetised plasma. The absorption of wave energy saturates due to particle trapping. For very large B fields the process of acceleration of particles is stochastic, as shown by Karney (1978). However, for intermediate B fields a novel non-stochastic acceleration process takes over. This was investigated by Sugihara and Midzuno (1979), Savdeev and Shapiro (1973) and Dawson et al. (1983). Basically, particle trapping occurs if the forces due to the electrostatic wave fields are larger than those resulting from the ambient B fields. Trapped particles are then accelerated to the ExB drift speeds in the wave frame. When this acquired velocity exceeds a critical value it leads to a detrapping ExB force that flings the particle out of the wave.

This process, however, is such that the maximum detrapping velocity that the particle can acquire is limited. Katsouleas and Dawson (1983a) have shown that relativistic effects can prevent detrapping and give unlimited acceleration across B. This requires the application of intense electric fields such that,

$$E_o > \gamma_{ph} B_o \qquad \left(\gamma_{ph} = \sqrt{1 - V_{ph}^2 / c^2} \right)$$

From the point of view of studying the possible formation of stationary states due to wave particle interaction, the interest was to keep the particles trapped in the wave field without resorting to relativistic effects. In this chapter such a mechanism is proposed and it's implications are worked out. Use is made of an inhomogeneity in the B field in the direction of wave propagation. As the trapped particle moves with the wave it experiences regions of lower B fields. As a result the VxB force never overcomes the electrostatic trapping force. This mechanism then predicts an indefinite particle acceleration, while keeping the particles trapped.

Indefinite acceleration would mean that ultimately the velocities would become large enough for relativistic effects to take over. Beyond this stage, the relativistic Katsouleas-Dawson mechanism (1983a) will have to be used for further acceleration. The method proposed here may be used in a preinjector that gives acceleration to near relativistic energies and injects them into a conventional or plasma base high energy accelerator.

However, the acceleration of particles implies wave damping. In order to study the formation of stationary states it might be interesting to consider putting in a source of instability, say, in the form of a beam of particles. Then the growth rate of this beam induced instability may be tailored to balance the wave damping self consistently to form a stationary situation. This extension is not attempted here.

5.2 Equations of motion:

Consider the motion of a charged particle of mass m and charge q, in an electrostatic wave field,

$$\underset{\sim}{E} = -E_0 \sin(ky - \omega t)\,\hat{y}$$

and a y-dependent magnetic field,

$$\underset{\sim}{B} = B(y)\,\hat{z} \equiv -\partial_y A_x(y).$$

Such an inhomogeneity may be generated externally or by using x-directed plasma currents in the region of particle acceleration.

The equations of motion for a particle in these fields are,

$$m\ddot{x} = \frac{q}{c} B(y)\,\dot{y} \tag{5.1}$$

$$m\ddot{y} = -qE_0 \sin(ky-\omega t) - \frac{q}{c} B(y)\,\dot{x}. \tag{5.2}$$

Equation (5.1) can be integrated once to give,

$$P_x = \dot{x} + \frac{q}{mc} A_x(y) = \text{constant}. \tag{5.3}$$

This shows the invariance of the x-component of canonical momentum that results from the translational symmetry of the fields in the x-direction.

Using (5.3), (5.2) becomes,

$$\ddot{y} + \frac{qE_0}{m} \sin(ky - \omega t) =$$

$$- \frac{q^2}{m^2c^2} \partial_y (A_x)^2 + \frac{q}{mc} P_x \partial_y A_x \quad . \tag{5.4}$$

This equation completely describes the motion of a particle in a large amplitude electrostatic wave and an arbitrary y-dependent magnetic field. In the wave frame, this motion can be seen to be that of a nonharmonic oscillator with time dependent potential and is, in general, very complex.

Consider the case when B is uniform. Then $A_x = -B_0 y$, and shifting to the wave frame, $\eta = y - \frac{\omega}{k} t$, we obtain

$$\dddot{\eta} + \left[\omega_c^2 + \frac{q E_0 k}{m} \cos(k\eta)\right] \dot{\eta} = - \omega_c^2 \frac{\omega}{k} \quad , \tag{5.5}$$

where, $\omega_c = {qB_0}/{mc}$. This is the equation solved by earlier authors (Dawson et al. 1983). The basic results may be qualitatively understood as follows.

When,
$$\omega_B^2 \left(= \frac{q E_0 k}{m}\right) \gg \omega_c^2 \, ,$$

then the particle velocity in the wave frame oscillates about the mean,

$$\overline{\dot{\eta}} = - \frac{\omega_c^2}{\omega_B^2} \left(\frac{\omega}{k}\right)$$

This mean motion results in a progressive dephasing between the trapped particle and the wave. Detrapping occurs when this phase lag is the equivalent of one wavelength, i.e., when

$$k \overline{\dot{\eta}} t_d \sim 1$$

giving

$$t_d \simeq \left(\frac{\omega_B^2}{\omega_c^2}\right)\frac{1}{\omega}.$$

The lab frame distance traveled by the particle would be,

$$y_d = t_d\left(\frac{\omega}{k}\right).$$

Then the $\dot{x}$ acquired in time t_d can be estimated from equation (5.3) to be:

$$\dot{x} \sim \omega_c y \sim \omega_c t_d \frac{\omega}{k} \sim \frac{cE_0}{B_0}.$$

So that the detrapping particle at escape has the ExB velocity. This was the result worked out by earlier authors (Dawson et al. 1983). Note that the detrapping particle gains its energy from the oscillating E-field and in turn must damp the wave. This damping effect is not taken into account self consistently here. If, as would be a realistic case, a large number of particles are under going acceleration then it would be necessary to introduce a distribution function of particle velocities and the trapped density would be a function of the wave amplitude. Also the damping rate would have to be found self-consistently.

5.3 Inhomogeneous Fields:

As mentioned earlier, the aim of this work was to find a way of keeping the particles trapped in the wave. This essentially meant ensuring that the detrapping VxB force never becomes larger than the trapping qE force. It was thought that this may be achieved by letting the trapped particle see smaller and smaller B-fields as the wave travels across it. This meant using an inhomogeneous magnetic field with a nonuniformity in the direction of propagation.

First consider the special case,

$$A_x^2 = A_0^2 (1 + y/L) \quad ; \quad P_x = 0 .$$

L is the scale length of nonuniformity in magnetic field and the initial conditions are set to be such that,

$$\dot{x}_0 = -\frac{q}{mc} A_x(y_0) \quad ; \quad B_0 = A_0/2L$$

$$B(y) = B_0 / (1 + y/L)^{1/2} .$$

With these specifications, equation (5.4) takes the simple form in the wave frame:

$$\ddot{\eta} + \frac{qE_0}{m} \sin(k\eta) = -2L\omega_c^2. \tag{5.6}$$

This is the equation of a nonlinear oscillator driven by a constant force. Let us consider the case of a well-trapped particle, such that,

$$\sin(k\eta) \simeq k\eta .$$

Then, the particle position oscillates about a mean displacement,

$$\bar{\eta} \simeq -2L \left(\frac{\omega_c^2}{\omega_B^2} \right) .$$

As long as $\bar{\eta}$ remains smaller than the wavelength, the particle cannot detrap, giving the condition,

$$\omega_B^2/\omega_c^2 \gg 2kL . \tag{5.7}$$

Unlike in the uniform B case there is no mean velocity of the particle in the wave frame and it will remain indefinitely trapped as long as this condition is satisfied.

The x-directed velocity of such a well trapped particle may be estimated from equation (5.3), giving,

$$\dot{x} = -\frac{q}{mc} A_0 \left(1 + \frac{\omega t}{kL} + \frac{\bar{c}}{kL} \sin(\omega t + \phi) - \frac{2\omega_c^2}{\omega_B^2}\right)^{1/2}.$$

Asymptotically, therefore, the $\dot{x}$ increases indefinitely as $t^{1/2}$. At near-relativistic velocities, this result would be modified. The condition for indefinite trapping may be derived using the uniform B field case. Following the Dawson mechanism (1983) of a mean drift in the wave frame, the laboratory frame distance traveled during the detrapping time is,

$$y_d \simeq \left(\frac{\omega_B^2}{\omega_c^2}\right)\frac{1}{\omega} .$$

Therefore, as long as,

$$(\omega_B^2/\omega_c^2)/k << L ,$$

the particle will detrap before it can feel the inhomogeneity in B, giving a condition for indefinite trapping consistent with equation (5.7).

Equations (5.1) and (5.2) were solved numerically for,

$$B = B_0 / (1 + y/L)^{1/2} ,$$

with the initial conditions,

$$\dot{x}_0 = -\frac{q}{mc} A_x(y_0) .$$

As shown in fig. (5.1), in the case of large $kL \sim 10^4$ the

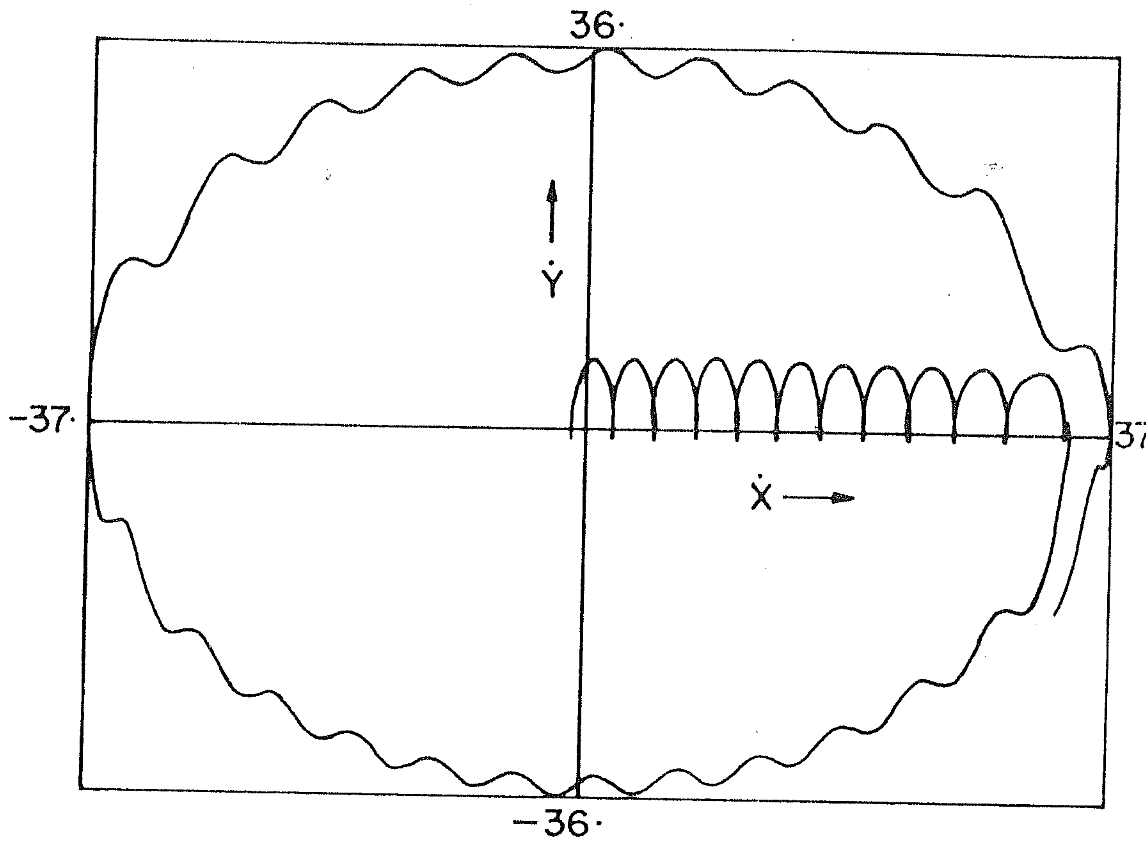


Fig. 5.1 : Particle trajectory in velocity space in the limit $\omega_B^2/\omega_c^2 << kL$ with $\omega_B^2/\omega_c^2 = 40$ and $kL=10^4$. The particle detraps.

uniform field case is recovered. The particle detraps and settles on a large Larmor orbit, since it does not see the inhomogeneity in the magnetic field before detrapping.

In the opposite limit, when kL = 1 and $\omega_B^2/\omega_c^2 = 40$, indefinite acceleration takes place, as shown by fig. (5.2). Fig. (5.3) verifies the $t^{1/2}$ dependence of $\dot{x}$ in this case, showing that not only does the particle stay trapped but is also accelerated. It should be pointed out that since B is being reduced continuously while the electric field remains unchanged, when $\omega_B^2/\omega_c^2 \sim O(1)$, the wave particle interaction will reduce to the unmagnetised problem of the type described by O'Neill (1965).

As stated earlier, once the particle acquires relativistic energies, it may be further accelerated by the Katsouleas-Dawson mechanism. For this method it is required that, $E_o > \gamma_{ph} B_o$. Thus if nonrelativistic acceleration can bring the particle to, say, a fraction f of the velocity of light c, then at that point the magnetic field would be given using equation (5.3),

$$B_{critical} = |2L\omega_c/fc|\,\gamma_{ph} B_o .$$

For the relativistic mechanism to take over,

$$E_o > \left|\frac{2L\omega_c}{fc}\right| \gamma_{ph} B_o .$$

This shows that the relativistic indefinite acceleration can be achieved at smaller values of E_o than stated by Katsouleas and Dawson (1983a). A few other variations of the B(y) were also considered.

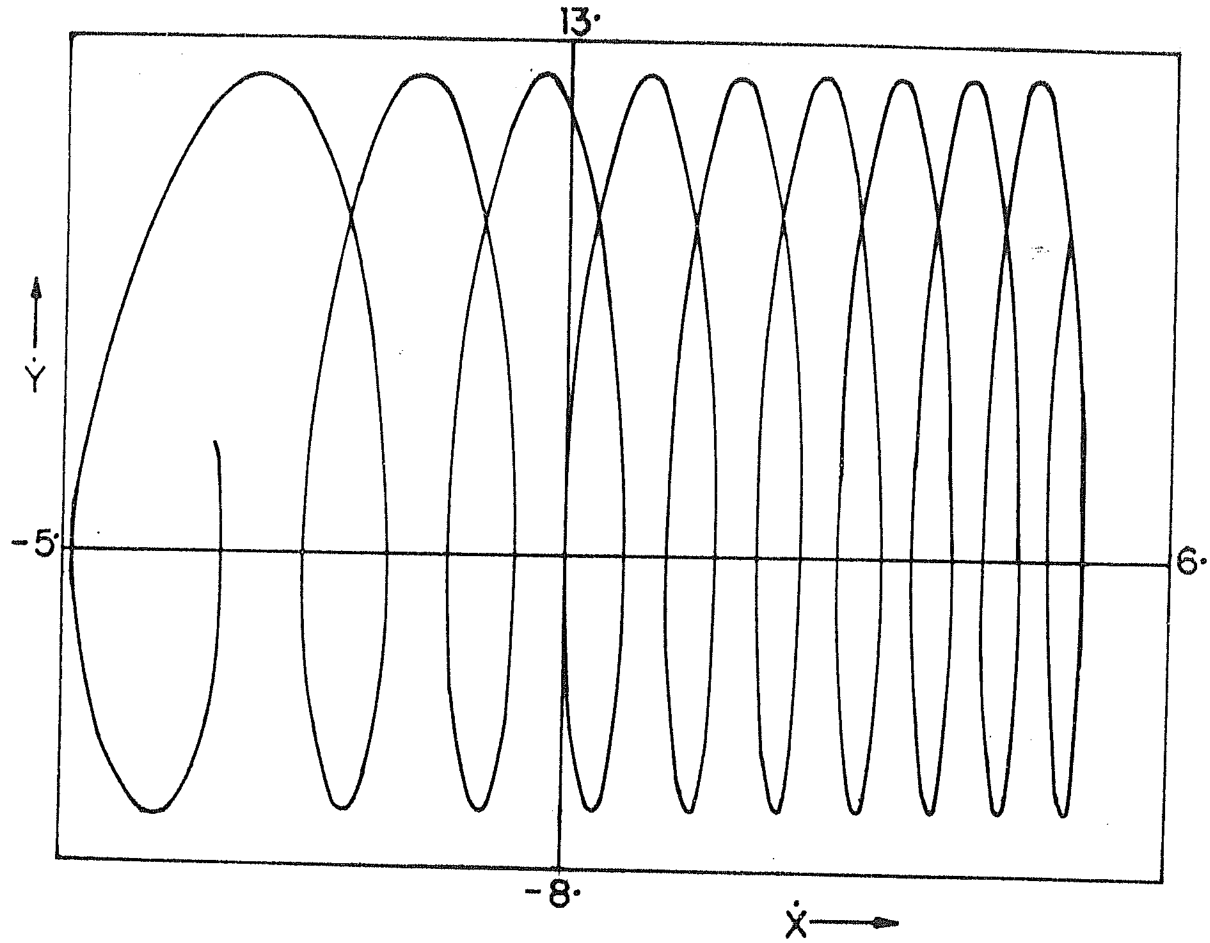


Fig. 5.2 : Particle trajectory in velocity space in the limit $\omega_B^2/\omega_c^2 >> kL$; $\omega_B^2/\omega_c^2 = 40$ and $kL = 1$. The particle remains trapped and accelerates.

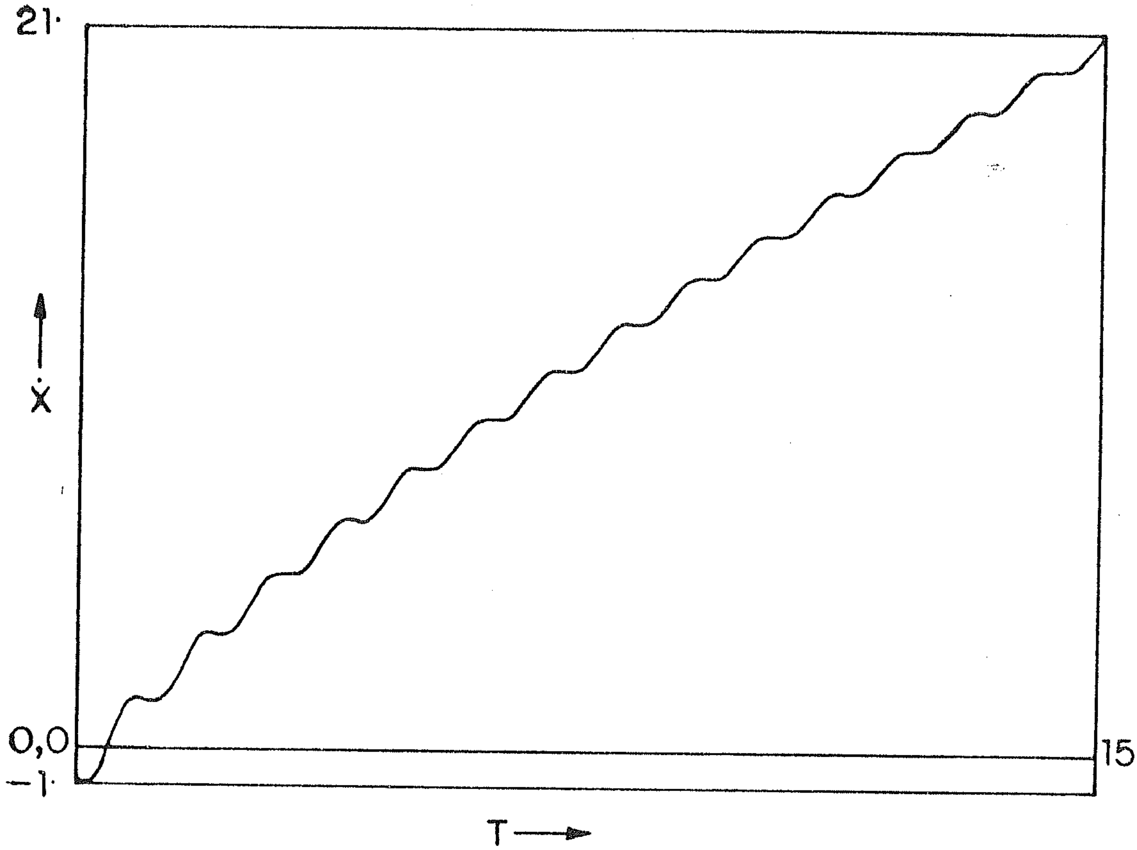


Fig. 5.3 : The x-velocity against time for $B = B_o/(1+y/L)^{1/2}$, $\omega_B^2/\omega_c^2 = 40$, $kL = 1$.

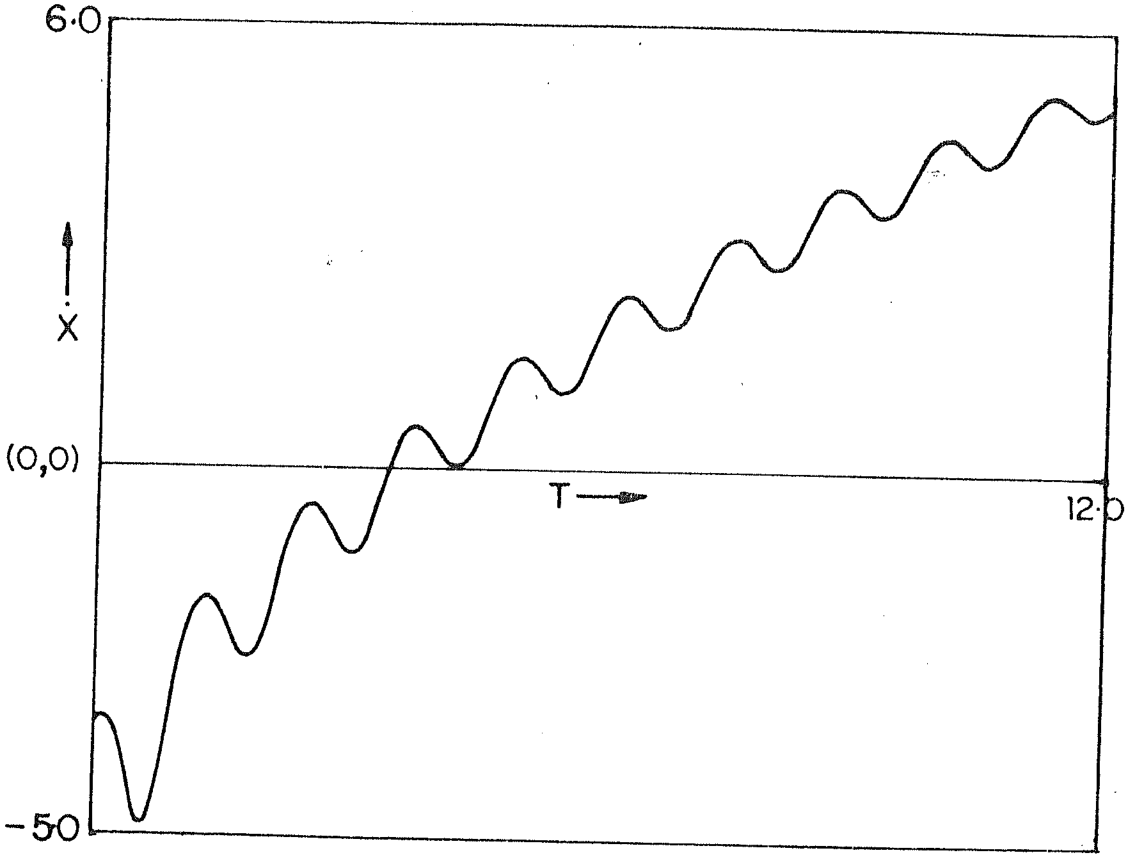


Fig. 5.4 : The x-velocity against time $B = B_o/(1+y/L)^{1/4}$, $\omega_B^2/\omega_c^2 = 40$, $kL = 1$.

For, $$B = B_0 / (1 + y/L)^{1/4},$$

we find that asymptotically, $\dot{x} \sim t^{1/4}$ as shown in fig. (5.4).

However, for variations faster than the square root, such as,

$$B = B_0 / (1 + y/L)^2,$$

$\dot{x}$ is seen to saturate and become asymptotically constant; fig. (5.5). In this case the effective acceleration stops after some time. The reason being, $A_x(y) \to 0$ asymptotically, and therefore,

$$\dot{x} \simeq \dot{x}_0 + \frac{q}{mc} A_x(y_0) .$$

Thus if the particle is injected in the wave at a finite vector potential and finally taken to a region where $A_x(y)$ vanishes, it is left with a finite energy. Similar arguments have been used by Kaw and Kulsrud (1973) for acceleration of particles by inhomogeneous electro-magnetic fields.

5.4 Damping effects:

As mentioned earlier, the calculations done so far do not self-consistently take into account the fact that, as particles are accelerated, the wave must damp. Though this is not attempted here, it would be interesting to estimate the damping rate of the wave, due to this single particle indefinite acceleration.

The case of $\dot{P}_x = 0$ and $B = B_0/(1+y/L)^{1/2}$ will be examined for the effect of particle acceleration on wave amplitude.

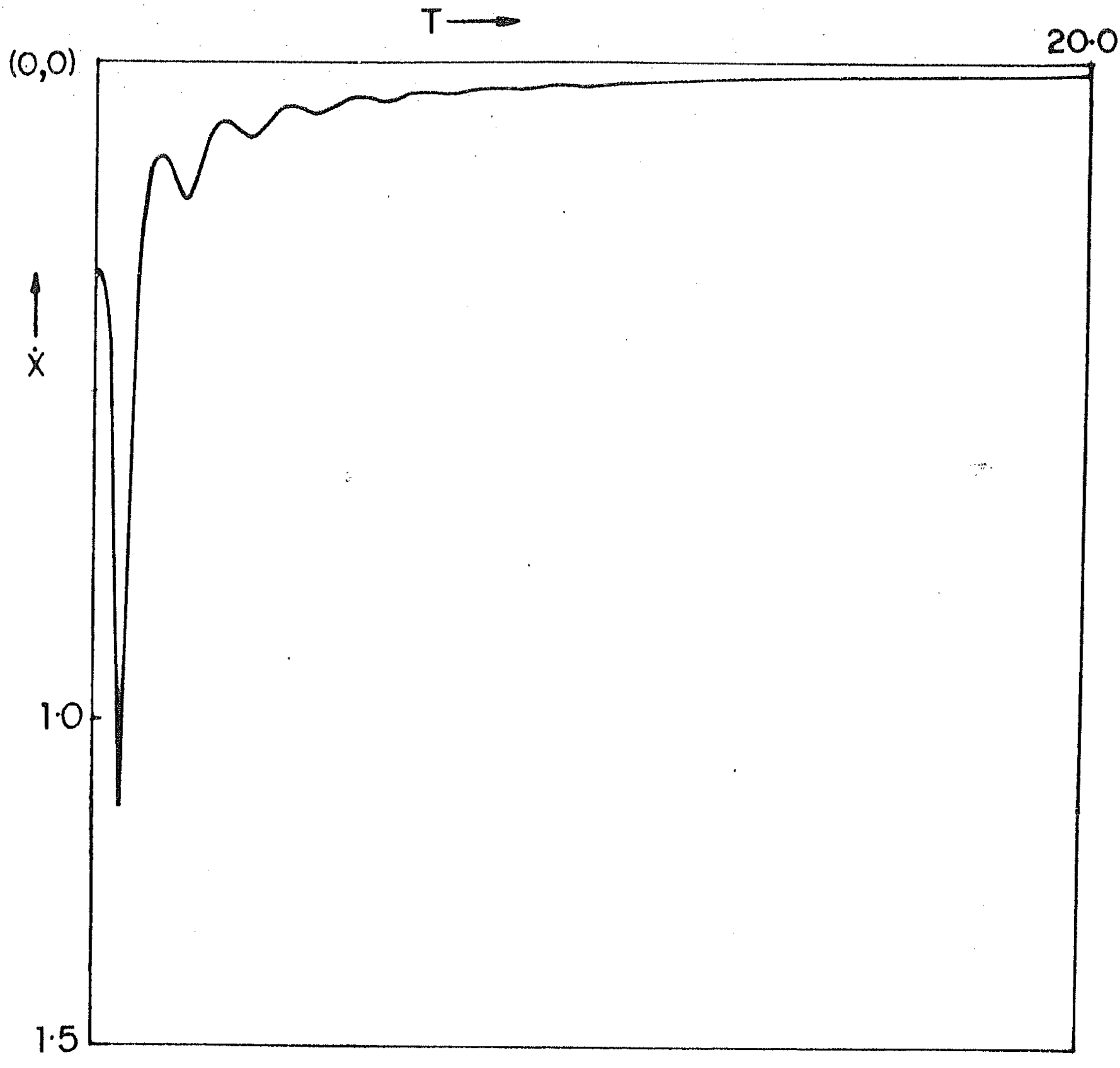


Fig. 5.5 : The x-velocity against time $B = B_o/(1+y/L)^2$, $\omega_B^2/\omega_c^2 = 40$, $kL = 1$.

In order to estimate this, it is necessary to calculate the particle energy. Consider equation (5.6). The corresponding effective potential that the particle sees is,

$$V(\eta) = -\frac{qE_0}{mk}\cos(k\eta) + 2L\omega_c^2\eta .$$

$V(\eta)$ must possess a minimum in order that trapping should take place.

$$-d_\eta V = -\frac{qE_0}{m}\sin(k\eta) - 2L\omega_c^2 = 0,$$

implying,

$$\sin(k\eta) = -2L\omega_c^2 m / qE_0 . \tag{5.8}$$

For a well trapped particle the necessary condition is,

$$|\sin(k\eta)| \leq 1,$$

or

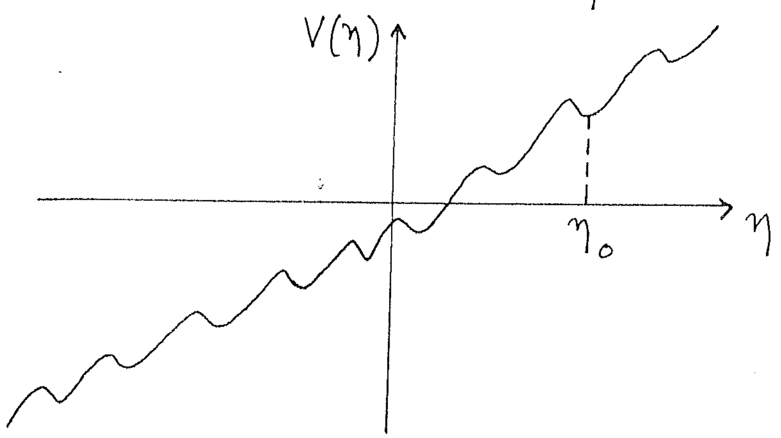
$$qE_0 > 2L\omega_c^2 m ,$$

recovering equation (5.7) and the condition for indefinite trapping.

Further for a minimum of $V(\eta)$,

$$d_\eta^2 V > 0,$$

giving the following picture of $V(\eta)$,



Consider the case of a well-trapped particle at a particular minimum η_0.

Then,

$$\sin(k\eta_0) = -2L\omega_c^2 m/qE_0 .$$

For such a particle, $y = \eta_0 + \frac{\omega}{k}t$ and $\dot{y} = \omega/k$. The particle is undergoing small oscillations about the bottom of the minimum of $V(\eta)$ at η_0. In the long time limits, however, these oscillations would average out and only the acceleration effect due to the mean displacement in the wave frame will contribute. So from equation (5.3),

$$\dot{x}^2 = 4L^2\omega_c^2(1 + y/L).$$

The total kinetic energy of the particle in the laboratory frame is,

$$T = \frac{m}{2}\dot{x}^2 + \frac{m}{2}\dot{y}^2 .$$

Then,

$$d_t T = 2mL^2\omega_c^2\dot{y}/L = 2L\omega_c^2 m\frac{\omega}{k} .$$

Since there are no other sources or sinks in the problem,

$$d_t W_{wave} = -n\, d_t T = -2nLm\omega_c^2\frac{\omega}{k}, \tag{5.9}$$

where n is the density of such trapped particles.

Thus the wave is damped due to particles accelerating along x. The same result may be obtained using the argument that,

$$d_t W_{wave} = \langle \underset{\sim}{J} \cdot \underset{\sim}{E} \rangle .$$

The RHS can be calculated to be,

$$\langle J \cdot \underset{\sim}{E} \rangle = - n q E_0 \langle \sin(k\eta) \dot{y} \rangle .$$

Since there is a mean displacement in the wave frame this does not average to zero, giving,

$$d_t W_{wave} = - n L \omega_c^2 m \, \omega/k \, ,$$

which is the same as equation (5.9). Thus the wave damps at the rate,

$$\gamma_d = 2 n L m \omega_c^2 \left(\omega/k \right)$$

In a more realistic calculation this effect would have to be taken into account self-consistently. In such a calculation, the density n would be determined by the amplitude of the wave field and the distribution function of particles.

5.5 Conclusion:

As a first step towards examining the possibility of the formation of stationary states of a high frequency electrostatic perpendicular wave, the nature of wave particle interaction was re-examined. A method was suggested to keep the particles trapped indefinitely in the wave. It was found that the associated effect of particle acceleration to large energies could be utilised in plasma accelerators.

It was found that particles trapped in the wave accelerate at the cost of the wave energy leading to wave damping. In order to achieve a stationary state, however, it would be necessary to be able to counteract this damping mechanism. This could be done by considering a mode with a linear instability mechanism. Then if the growth rate could be made to match this damping, a dynamic steady state may be achieved. This would require careful calculation of the trapped particle density, the effect of the B field inhomogeneity on wave propagation and the growth mechanism in a self-consistent treatment. Recent work has shown that oblique propagation in a nonuniform field gives higher acceleration rates and final velocities of particles. It would be interesting to include these effects also. One possible realistic case study can be the beam-driven lower hybrid wave.

Later work by Erokhin et al. (1989) has shown that from more general arguments regarding the conservation of the associated adiabatic invariants to keep the particles trapped, a similar form of nonuniformity of B fields emerges. In a self consistent treatment, therefore, it may be possible to optimise the form of the B fields to give rise to stationary states.

CHAPTER VI

CONCLUSIONS

In this thesis the nonlinear coherent exact stationary solutions of the equations governing a magnetised, inhomogeneous, collisionless plasma have been studied. This thesis contains the first detailed consideration of the modeling of such solutions in the kinetic limit as maximum entropy stationary states. This approach was motivated on one hand by the need to provide a complementary approach to the existing theories for drift wave turbulence. It is hoped that the phase space holes modeled in this thesis will prove to be useful building blocks for such a consideration. On the other hand, there is a large amount of ill understood physics in the nonlinear phase of low frequency turbulence. So from the point of view of the identification of nonlinear effects leading to a saturated spectrum of drift turbulence and their associated parameter regimes, this study has been instructive. The main results of this thesis are as follows.

It has been shown that when nonlinear ion parallel dynamics is included, the generalised HM equation has qualitatively new two dimensional monopole vortex solutions. Their existence does not depend upon strong temperature gradient effects or second order density gradients, as studied by earlier authors. The inclusion of a weak temperature gradient produces noncircular monopoles. However, the one dimensional analogue of these solutions remains unstable to two dimensional perturbations. Recent simulation studies of the stability of dipole and monopole vortices has indicated the better stability properties of the monopole. The monopole solutions studied here, therefore, might play an important role in the development of fluid drift turbulence in realistic parameter regimes.

The one dimensional maximum entropy state of the DKE has been formulated and studied in the limit $k_x \ll k_y$. The method of entropy maximisation outlined by Dupree(1982) has been formally extended to this case. Both ion and electron phase space trapping effects in relevant parameter spaces have been included. The case of a known temperature gradient is also attempted. The properties of the resulting steady state potential structures have been investigated numerically.

In the limit of $k_x \sim k_y$, the electron nonlinearities have been considered fully in a study of two dimensional phase space holes. Both their parallel phase space and perpendicular physical space trapping effects are taken into account. The rectangular hole approximation has been used to give analytical insight to the complex problem. From numerical solutions of the two dimension equation for the steady state potential it is shown that it is a good approximation to the complete problem for small amplitudes. The properties of the solutions for various values of the instability parameter $k_{\parallel} / k_{\perp}$ are studied.

In the limit of $k_x \sim k_y$, approximately maximal entropy solutions are built. The ExB nonlinearity of kinetic electrons and fluid ions is taken into consideration. It is shown that monopole type vortex solutions can be constructed. Numerical solutions of the resultant equation for the steady state potential are presented.

In determining the comparative weightage of the various nonlinear effects in their respective parameter regimes, the results of gyrokinetic simulations by earlier workers have been used extensively. These have provided valuable guidelines in the interpretation of the results obtained here, specially in the two dimensional calculations.

As a first step towards extending the concept of the identification of nonlinear, coherent, exact solutions to the high frequency regime, a preliminary attempt has been made. The specific case of wave particle interaction in a perpendicular wave in a magnetised plasma is studied. Important results regarding indefinite particle acceleration and associated wave damping are presented. Numerical results of single particle orbits supporting these are given. It is hoped that these results can be successfully extended to give self consistent stationary solutions for inherently unstable systems such as the beam-plasma interaction.

While studying the possible parameter spaces for the existence of such exact solutions several interesting questions arose, suggesting areas for further exploration. Some of them can be listed as follows:

* The stability analysis of the fluid monopole solutions in the presence and absence of temperature gradients can be carried out. The effects of a sheared magnetic field can be introduced as a realistic addition.

* The entire maximum entropy formalism must be studied in the presence of collisions. They would induce detrapping as pointed out by Ott et al.(1979) leading to a net growth. Collisions could also form a source of incoherence and lead to diffusion and further instability as suggested by the simulation results. Thus time scales of the validity of a possible turbulence model based on interacting holes may be determined by collisions.

* The main source of incoherence, in a collisionless system, will come from the interaction among phase space holes that have a random distribution in phase velocities. The model of turbulence built on this basis would necessarily have the associated diffusion effects and the resulting particle fluxes. A fully kinetic model that takes into account both, ion Landau damping as described by the resonance broadening theory and the electron nonlinearities studied in this thesis, would give the correct picture of the turbulent spectrum and its evolution towards a steady state. This will be the most interesting and possibly complete model for turbulence.

* It would be, of course, of great interest to include effects of shear and other inhomogeneities of the magnetic field in the maximum entropy formalism. A fully three dimensional treatment might prove useful in such a case. An interesting case for further study would be to include the effects of magnetic field perturbations in the parallel direction, in the parameter regime of Alfven-like modes. These effects might lead to the formation of maximum entropy island formation.

* As an immediate off shoot it would be of practical interest to make some heuristic predictions for the particle and energy transport in an interacting hole model for turbulence.

REFERENCES

Bernstein I.B., Greene J.M., Kruskal M.D., 1957 Phys. Review **108**, 546.

Ching H., 1973 Phys. Fluids, **16**, 130.

Dawson J.M., Decyk V.K., Huff R.W., Jechart I., Katsouleas T., Leboenf J.N., Lembege B., Martinez R.M., Ohsawa Y., Ratliff S.T., 1983 Phys. Rev. Lett. **50**, 1455.

Dimits A.M., 1988 Ph.D. Thesis, Princeton University.

Dubin D.H.E., Krommes J.A., Oberman C., Lee W.L., 1983 Phys. Fluids, **26**, 3524

Dupree T.H., 1967 Phys. Fluids **10**, 1049.

Dupree T.H. 1972 Phys. Fluids **15**, 334

Dupree T.H. 1978 Phys. Fluids **21**, 783

Dupree T.H. 1982 Phys. Fluids **25**, 277.

Erokhin N.S., 1989 Private Communication.

Federici J.F., Lee W.W., Tang W.M., 1987 Phys. Fluids, **30**, 425.

Hasegawa A., Mima R., 1978 Phys. Fluids, **21**, 87.

Hasegawa A., Macclennan C.G., Kodama Y., 1979 Phys. Fluids, **22**, 2122.

Hirshman S.P., 1980 Phys. Fluids, **23**, 562.

Hockney R.W., in Methods in Computational Physics, **9**, 135, Academic Press

Horton W., 1988 Phys. Fluids B, **1**, 524

Kadomtsev B.B. 1965 Plasma Turbulence, Academic Press

Karney C.F.F., 1978 Phys. Fluids **21**, 1584.

Katsouleas T. and Dawson J.M., 1983 Phys. Rev. Lett. **51**,392

Kaw P.K., Kulsrud R.M., 1973 Phys. Fluids, **16**, 321.

Laedke E.W., Spatschek K.H., 1988 Phys. Fluids, **31**, 1492.

Lakhin V.P., Mikhailovskii A.B., Onishckenko O.G., 1987 Phys. Lett. A., **119**, 348.

Lee W.W. 1983, Phys. Fluids, **26**, 556

Lee W.W., Krommes J.A., Oberman C.R., Smith R., 1984 Phys. Fluids, **27**, 2652.

Lynden-Bell D., 1967 Mon. Not. R. Astr. Soc. **136**, 101.

Makino M., Kamimura K., Taniuti T., 1981 J. Phys. Soc. Japan, **50**, 980.

Marquardt Chv, Wenk H., Spatschek K.H., Laedke E.W., Proceedings of the 1989 International Conference on Plasma Physics, New Delhi, 1989, Vol.II, 701.

Meiss J.D., Horton W., 1982 Phys. Fluids, **25**, 1838.

Meiss J.D., Horton W., 1983 Phys. Fluids, **26**, 990.

Mikhailovskaya, L.A., 1986 Sov. J. Plasma Phys. **12**, 507.

Morse R.L., Nielson C.W., 1968 Phys. Fluids, **12**, 2418

O'Neil T.M., 1965 Phys. Fluids, **8**, 2255.

Oraevskii V.N., Tasso H., Wobig H., 1969 Proceedings of the Third International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Novisibirsk, 1968, **1**, 671.

Oraevskii V.N., Sagdeev R.Z., Galeev A.A., Rudakov J.A., 1969a in Nonlinear Plasma Theory, ed. Sadeev R.Z. and Galeev A.A., W.A. Benjamin Inc.

Ott E., Manheimer W.M., 1976 Phys. Fluids, **19**, 1035.

Petviashvili V.I., 1967 Sov. Phys. Dokl., **15**, 456

Petviashvili V.I., 1977 Sov. J. Plasma Phys., **3**, 150.

Petviashvili V.I., 1981 in Plasma Physics, ed. Kadomtsev B.B., Mir Publishhers.

Petviashvili V.I., Pokhotelov O.A., 1986 Sov. J. Plasma Phys., **12**, 1986.

Sadgeev R.Z., Shapiro V.D., 1973 JETP Lett. **17**, 279.

Schamel H., 1972 J. Plasma Phys., **7**, 1

Schamel H., 1979 Physica Scripta, **20**, 306.

Smith R.A., Krommes J.A., Lee W.W., 1985 Phys. Fluids, **28**, 1089.

Su X, Horton W., Morrison P.J., Pavlenko V.P., 1988 Report EFSR # 328, The University of Texas at Austin, USA

Sugihara R., Midzuno Y., 1979 J. Phys. Soc. Japan, **47** 1290.

Swaters G.E., 1986 Phys. Fluids, **29**, 1419

Terry P.W., Diamond P.H., Hahm T.S., Presented at the International Conference on Plasma Physics, Kiev, USSR, 1987

Tidman D.A., Krall N.A., 1971 Shocks in Collisionless Plasmas, Krieger