

Origin and dynamics of the Primordial Magnetic field in a parity violating plasma

A thesis submitted in partial fulfilment of
the requirements for the degree of

Doctor of Philosophy

by

Arun Kumar Pandey

(Roll No. 11330014)

Under the guidance of

Prof. Jitesh R. Bhatt

Theoretical Physics Division

Physical Research Laboratory, Ahmedabad, India.



DISCIPLINE OF PHYSICS

INDIAN INSTITUTE OF TECHNOLOGY GANDHINAGAR, GUJARAT, INDIA

2017

To
My Mummy Papa



Declaration

I declare here that this thesis report represents my own ideas in my own words and I have included others ideas with appropriate citations from original sources. I also declare that I have followed all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/fact/source/data in my submission. I understand that any violation of the above can cause disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

Arun Kumar Pandey

(Roll No: 11330014)

Date: December 5, 2017

CERTIFICATE

It is certified that the work contained in the thesis titled “**Origin and dynamics of the Primordial Magnetic field in a parity violating plasma**” by **Mr. Arun Kumar Pandey** (Roll No. 11330014), has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

Prof. Jitesh R. Bhatt

(Thesis Supervisor)

Professor,

Theoretical Physics Division

Physical Research Laboratory,

Ahmedabad, India.

Date:

Acknowledgements

It has been a great prerogative to stay in Physical Research Laboratory for several years, where I got a research as well as homely atmosphere throughout my PhD tenure. It gives me a great pleasure to acknowledge all those who supported me directly or indirectly and made this thesis possible.

First and foremost, I wish to express my sincere gratitude to my supervisor, Prof. Jitesh R. Bhatt, for giving me guidance, encouragement and support throughout my PhD tenure that lead up to this thesis. I am very thankful to him for keeping faith in me and giving me encouragement time to time, which was invaluable source of confidence. I am also grateful to him for introducing me to the interesting and challenging problems of the field, which will play an important role in my future research. Special thanks to my collaborator Sampurn Anand for his invaluable support and for careful reading of my thesis.

I am also thankful to Dr. Ramitendranath Bhattacharyya, Dr. Sanjay Saini, Alok Tiwari and Rahul for providing me a very open and friendly atmosphere whenever I visited Udaipur Solar Observatory (USO) for learning the EULAG code. I would also like to thank Prof. Nandita Srivastava for her wonderful wisdom and insight during my second coursework project on the topic of “CME arrival and predictions on Earth and Mars”. I take the opportunity to thank Prof. R. Rangarajan, Dr. Namit Mahajan, Prof. Dilip Angom, Prof. Subhendra Mohanty, Prof. S. Goswami, Dr. B.K.Sahoo and Dr. Partha Konar for teaching several useful courses at various times during my Ph.D. I also thank the other faculty members of theory group: Prof. Rindani, Prof. Hiranmaya Mishra, and Dr. Navinder Singh for creating a friendly, interactive atmosphere to work. I am especially thankful to Dr. Namit Mahajan and Prof. R. Rangarajan for useful discussions from which I learned a lot.

I would like to thank my seniors Gulab Bhai, Avdheshji, Naveen Negiji, Yashpal, Dilip Nandi, Girish C., Upendra, Whagesh, Gaurav Tomar, Manojit, Arkoy, Tanmoy C., Bhavya, Anjali, Priyanka, Lata Thakur, Abhishek Attreya, and Shweta Di for their presence at several times. I would also like to give a sincere thank to my batch mate friends Abhaya, Alok, Anirban, Chitra, Gaurava, Girish, Guru, Ikhsu, Kuldeep, Manu, Sanjay, Shraddha and Tanmoy for making a cheerful environment during my Ph.D stay in PRL. Without them my life here in Ahmedabad would have been a hell. They were present at each and every moment of my life. I would also like to thank my Juniors with whom, I interacted most, Rahul Yadav, Bivin, Diptiranjan, Lalit, Pankaj, Newton, Chandan Hati, Rukmani Bai, Kuldeep Pandey, Navpreet, Rupa, Prahlad, Jabir, Ali, Chandan Gupta, Soumik, Aman,

Vishnudath, Arvind Mishraji, Nidhi, Shefali, Rahul K., Varun, Subir, Kaustav, Akanksha, Archita, Balbeer, Ashish, Richa, Ayan, Bhavesh, Bharti and Naveen. Especially, I would like to thank Nidhi, Ayan and Shefali for their mental support and presence during the stressful days of thesis writing. Their fun, loving and friendly environment made my days easy while I was staying in PRL thaltez hostel.

I am also grateful to Bhagyashree and Sujata ben for their sincere support and help whenever I asked. Last, but not the least, I would be nowhere around here without the love and support of my family. Finally, I must also acknowledge the love, support and blessings of friends, teachers, relatives and all known and unknown persons whom I met with or interacted with in life.

Arun Kumar Pandey

Abstract

The Universe is magnetized on all scales that we have observed so far: stars, galaxies, cluster of galaxies etc. Recent observations indicates that typical magnetic field strength in a galaxy or a galaxy-cluster can have is about a few μG and its coherent length is around ten kpc. Recent observations suggest that the intergalactic medium (IGM) have magnetic field in the range of $(10^{-16} - 10^{-9})$ G with the coherent length scales around Mpc. Though it might be possible to explain the observed magnetic field of the galaxies and stars by some kind of astrophysical process, it is hard to explain the observed magnetic field of an IGM void. There exists an intriguing possibility of relating the origin of these large scale magnetic fields with some high-energy process in the early Universe. Thus it is of great interests to study the origin, dynamics and constraints on the magnetic field generated by such mechanism. This forms the prime focus of this thesis which considers the above problem in the context of a high-energy parity violating plasma.

In recent times there has been considerable interest in studying the magnetic field evolution in high-energy parity violating plasma. It is argued that there can be more right-handed particles over left-handed particles due to some process in the early Universe at temperatures T very much higher than the electroweak phase transition (EWPT) scale ($T \sim 100\text{GeV}$). Their number is effectively conserved at the energy scales much above the electroweak phase transitions and this allows one to introduce the chiral chemical potentials $\mu_R(\mu_L)$. However, at temperature lower than $T \sim 80$ TeV processes related with the electron chirality flipping may dominate over the Hubble expansion rate and the chiral chemical potentials can not be defined. Further the right handed current is not conserved due to the Abelian anomaly in the standard model (SM) and it is the number density of the right handed particles are related with the helicity of the fields as $\partial_\tau(\Delta\mu + \frac{\alpha_2}{\pi} \mathcal{H}_B) = 0$. (Here $\Delta\mu$, α_2 and $\mathcal{H}_B$ represents the chiral chemical imbalance, fine structure constant and magnetic helicity respectively). Therefore even if initially when there is no magnetic field, a magnetic field can be generated at the cost of the asymmetry in the number density of the left and right handed

particles in the plasma. Recently, it has been interesting development in incorporating the parity -violating effects into a kinetic theory formalism by including the effect of the Berry curvature. Berry curvature term takes into account chirality of the particles. By using this modified kinetic equation, we show that chiral-imbalance leads to generation of hypercharge magnetic field in the plasma in both the collision dominated and collisionless regimes. We show that in the collision dominated regime, chiral vortical effects can generate a chiral vorticity and magnetic field. Typical strength of the magnetic field in the collision and collisionless regime are 10^{27} G at $10^5/T$ length scale and a magnetic field of strength 10^{31} G at $10/T$ length scale at a temperature $T \sim 80$ TeV. We also show that the estimated values are consistent with the present observations.

We also show that in the presence of chiral imbalance and gravitational anomaly, a magnetic field of the strength 10^{30} G can be generated at a scale of 10^{-18} cm (much smaller than the Hubble length scale i.e. 10^{-8} cm). The idea is that in the presence of the gravitational anomaly, the current expression for the chiral plasma consists of a vortical current, proportional to square of temperature i.e. T^2 . The silent feature of this seed magnetic field is that it can be generated even in absence of chiral charge. In this work we have considered the scaling symmetry of the chiral plasma to obtain the velocity spectrum. Under this scaling symmetry in presence of gravitational anomaly, amount of energy at any given length scale is much larger than the case where only chiral asymmetry is considered. We also show that under such scenario energy is transferred from large to small length scale, which is commonly known as inverse cascade.

In the context of chiral plasma at high energies, there exist new kind of collective modes [*e.g.*, the chiral Alfvén waves] in the MHD limit and these modes exist in addition to the usual modes in the standard parity even plasma. Moreover, it has been shown that chiral plasmas exhibit a new type of density waves in presence of either of an external magnetic field or vorticity and they are respectively known as Chiral Magnetic Wave (CMW) and the Chiral Vortical Wave (CVW). In this regard we have investigated the collective modes in a magnetized chiral plasma and the damping mechanisms of these modes using first order and second

hydrodynamics. Using first order conformal hydro, we obtained previously derived modes in the chiral plasma. However we show in addition that these modes get split into two modes in presence of the first order viscous term. By using second order conformal magnetohydrodynamics, we show that there are a series of terms in the dispersion relation and these terms are in accordance with the results obtained using AdS/CFT correspondence. We also calculated one of the transport coefficients related with the second order magnetohydrodynamics.

Keywords: Cosmology, Primordial Magnetic Field, EW phase transition, Left Right symmetric model, Turbulence theory, Kinetic theory, CMB, BBN

Contents

Acknowledgements	i
Abstract	iii
Contents	vii
List of Figures	1
1 Introduction	5
1.1 Magnetogenesis and magnetic fields	5
1.1.1 Astrophysical models	6
1.1.2 Models based on early Universe processes	12
1.2 Evolution of the Primordial magnetic fields	23
1.3 Observations and constraints	26
1.3.1 Observational bounds	27
1.4 Objectives of the study	29
1.5 Overview of the chapters	31
2 Theoretical Foundation	33
2.1 Kinetic theory of a classical plasma system	36
2.1.1 Newtonian Kinetic theory	37
2.1.2 Relativistic Boltzmann theory	39
2.2 Kinetic theory with Berry Curvature	41
2.2.1 Basics of Berry curvature	41
2.2.2 Relativistic description with anomaly	49

3	Generation of magnetic fields	51
3.1	Introduction	52
3.2	Generation of magnetic fields above EW scale	54
3.2.1	Current and polarization tensor for chiral plasma	57
3.2.2	Vorticity generated from chiral imbalance in the plasma	62
3.3	Discussion	64
4	Chiral Battery, scaling laws and magnetic fields	69
4.1	Introduction	69
4.2	Chiral battery	72
4.2.1	Mode decomposition	73
4.2.2	Scaling properties in chiral plasma	74
4.2.3	Evolution of Magnetic field	76
4.3	Results and discussion	77
4.4	Conclusion	79
5	Damping of the generated magnetic field	81
5.1	Introduction	82
5.2	Waves in the non dissipative chiral plasma	85
5.2.1	Chiral Magnetic Waves (CMW)	85
5.2.2	Chiral Vortical Waves (CVW)	87
5.2.3	Chiral Alfvén Wave (CAW)	88
5.3	Conformal viscous chiral hydrodynamics	90
5.3.1	Chiral magnetohydrodynamics with first order viscous terms	91
5.3.2	Chiral magnetohydrodynamics with second order viscous terms	95
5.4	Results and Discussions	97
6	Summary and Future Directions	99
6.1	Summary	99
6.2	Future Direction	101
7	Appendix	103

Bibliography	105
List of publications	125

List of Figures

1.1	(Bounds on Extra Galactic Magnetic fields B (EGMFs), from several observations and experiments): Light, medium and dark gray color shows known bounds on the strength of EGMFs. The bound from the Big Bang Nucleosynthesis is taken from [10]. The black hatched region in the lower side of the figure, shows the lower bound on the EGMF derived from observations of 1ES 0347-121(cross-hatching) and 1ES 0229+200 (single diagonal hatching) [102]. Orange hatched regions show the allowed ranges of magnetic field and it's correlation scale generated during inflation models (horizontal orange hatching), during EW phase transition (dense vertical hatching), QCD phase transition (medium vertical hatching), and epoch of recombination (light vertical hatching) [99]. White ellipses show the range of measured magnetic field strengths and correlation lengths in galaxies and galaxy clusters. Reprinted figure from [102], Copyright ©The American Association for the Advancement of Science	28
1.2	(a).Time evolution of kinetic (dashed lines) and magnetic (solid lines) in the turbulent regime($R_e \gg 1$) without initial helicity. (b).Evolution of magnetic-energy spectra in the turbulent regime ($R_e \gg 1$) with no initial helicity. Reprinted figure from [39], Copyright ©APS	29

- 4.1 Magnetic energy spectrum as a function of k is shown: (a) when $n = 0$, magnetic energy spectrum E_B goes as k^7 at large length scale. However, the power is more than what authors of ref. [107] has found for a plasma with chiral asymmetry for a given k value. (b) when $n \neq 0$, E_B goes as k^5 at large length scale. In both the figures we have set $T = 10^9$ GeV and $v = 10^{-4}$ 78
- 4.2 (a) Evolution of magnetic energy density E_B as a function of k and (b) Evolution of magnetic helicity density $\mathcal{H}_B$ as a function of k for different temperatures. As temperature ($T \sim 1/\tau$) decreases, peak shifts from higher to lower k values which shows inverse cascading *i.e.* transfer of magnetic energy from small to large length scale. In both the figures we have set $v = 10^{-4}$ and $n = 10^{-4}$ 78

List of terms used in the thesis

Short name/ Symbol	Definition / meaning
IGM	Inter Galactic Medium
ICM	Inter Cluster Medium
EGMF	enter galactic magnetic field
MHD	Magnetohydrodynamics
CMB	Cosmic Microwave Background
BBM	Biermann Battery Mechanism
BBN	Big Bang Baryogenesis
CME	Chiral Magnetic Effect
ChMHD	Chiral Magnetohydrodynamics
CVE	Chiral Vortical Effect
CVW	Chiral Vortical Waves
CAW	Chiral Álfven Waves
CMW	Chiral Magnetic Waves
CSE	Chiral separation effect
EM	Electromagnetic
SM	Standard Model
EW	Electroweak
GUT	Grand Unified Theory
QCD	Quantum Chromodynamics
BAU	Baryon Asymmetry of the Universe

Short name/ Symbol	Definition / meaning
n D	n dimensional space
kpc/Mpc	kiloparsec/megaparsec
eV	electron volt
keV	kilo electron volt
MeV	mega electron volt
GeV	giga electron bolt
H	Hubble constant
ρ	Matter energy density
ρ_γ	Radiation energy density
$\boldsymbol{\omega}$	vorticity vector
m_p	mass of the ions
k_B	Boltzmann's coefficient
c	speed of light
σ	electrical conductivity
α	fine structure constant
m_e	electron mass
μ	micro
$\hbar$	reduced Planck constant ($\hbar = h/2\pi$)
M_{pl}	Planck Mass
G	Gauss
$\Re$	set of Real number

Chapter 1

Introduction

Magnetic fields are present on almost all scales, from stars, galaxies to intergalactic medium. A field strength of few μ G has been observed in galaxies at kpc (Kilo Parsec) length scale. However in the intra-cluster medium (ICM) a magnetic field of strength of 10^{-16} G at a coherence length scale of the order of $(10 - 100)$ kpc can be present [1]. One of the unsettled issues of the modern cosmology and astrophysics is the origin of these magnetic fields. One understanding is that these fields could have been originated during structure formation or in the early stages of Universe. However, one of the intriguing possibility is that they are relic from the early Universe and subsequently amplified and maintained by a dynamo during structure formation. During structure formation, it is necessary to have a seed magnetic field of $B \sim 10^{-20}$ G to explain currently observed magnetic field [2, 3]. However a magnetic field of the order of $B \sim 10^{-9}$ G can explain the observed galactic magnetic fields [4]. In the former scenario, seed magnetic fields get amplified via dynamo mechanism [2, 5–7], while in the latter scenario it is not necessary to have a dynamo mechanism for amplification. This chapter provides the overview of current understanding of the astrophysical and cosmological magnetic fields.

1.1 Magnetogenesis and magnetic fields

Models for generation of the magnetic field is broadly classified into two classes:

- Astrophysical models
- Models based on early universe processes

We can separate creation and amplification mechanism based on the early universe model into three process namely

1. Before recombination
2. During recombination
3. After recombination

Astrophysical models mainly comes into the category of processes after recombination. However few authors have discussed generation of primordial magnetic field due to some astrophysical phenomenon during recombination era [8] (for review see the reference [9]).

1.1.1 Astrophysical models

Post recombination processes include explanation for the amplification of the seed fields in galaxies by the dynamo [10–12] (Review Ref. [13]). Models based on astrophysical process can be divided into two categories [9]: a bottom-up (astrophysical processes) scenario, where the seed field is typically very weak and the observed large-scale magnetic field is transported from local sources within galaxies to larger scales [14], and a top-down (cosmological processes) scenario where a significant seed field is generated prior to galaxy formation in the early universe on a scale of a cosmological interest [15]. The “*Biermann Battery process*” (BBM) [16] is one process, which discuss generation of magnetic field in the prototype galaxies, clusters or on shock fronts [11, 17–19]). Harrison in [20, 21], discussed another generation mechanism. The idea was that, in presence of plasma vortical motion during radiation dominated era, can produce a sufficient large magnetic field.

Generation from vortical motion of the plasma during radiation dominated era

Harrison mechanism [20–23] for generation of magnetic fields depend on the transfer of angular momentum between the ion and electron-photon gases in the expanding radiation dominated plasma. This can be achieved by considering that rotational velocities of the ion and the electrons decreases differently in the expanding background before recombination takes place. The generating mechanism depends on the transfer of angular momentum between the ion and electron-photon gases in the expanding radiation dominated plasma. To understand this behaviour, let us take a simple example of a spherically symmetric region of radius r , uniformly rotating, and containing radiation of uniform density ρ_γ , and matter of uniform density ρ consisting of ions and non-relativistic electrons. As the eddy expands, both energy in the matter and the radiation in the regime of radius r , *i.e.* ρr^3 and $\rho_\gamma r^4$ remains constant. Assume that the angular velocities of matter and radiation are ω and ω_γ ; then in the absence of interactions the angular momentum proportional to $\rho\omega r^5$ and $\rho_\gamma\omega_\gamma r^5$ are separately conserved. Hence, $\omega \propto a^{-2}$ and $\omega_\gamma \propto a^{-1}$. So as eddy expands, radiation spins down more slowly than matter. Here a is scale factor for the expanding Universe. Above assumption was based on the difference in the effectiveness of Thomson scattering for electrons than ions. The Thomson cross-section is much larger for the electron than for ions and the electrons tend therefore to be dragged along by the photon gas. In the radiation dominated era when $\rho_\gamma > \rho$, the electron-photon coupling is relatively tight. Therefore there are only two fluids: one positively charged ion gas of density ρ and relatively charged photon gas of density ρ_γ . The differences in the angular velocities of the ion and electron-photon gases in an expanding eddy generated a magnetic field, which is follows following equation [20]

$$\frac{\partial}{\partial \tau} \left(a^2 \boldsymbol{\omega} + \frac{e}{m_p} \mathbf{B} \right) = \frac{e}{4\pi\sigma m_p} \nabla^2 \mathbf{B}. \quad (1.1)$$

where m_p and τ are mass of the ions and conformal time respectively. If we assume that there exist some initial vorticity prior to the decoupling, then above equation can be solved and the magnetic field can be related to the initial vorticity

as

$$\mathbf{B} \approx -\frac{m_p}{e} \left(\frac{a_i}{a} \right)^2 \boldsymbol{\omega}_i \quad (1.2)$$

So, it is clear that if an appropriate value of initial vorticity is assumed, current observed bounds of magnetic field strength can be obtained. In this work, Harrison showed that in an expanding eddy, a magnetic field of strength $\mathbf{B} = -2.1 \times 10^{-4} \boldsymbol{\omega}$ G can be generated in the case of a hydrogen plasma. Primordial turbulence, if present at the epoch of recombination, can lead to a magnetic field of strength 10^{-8} G on a length scale of 1 Mpc at current time. In the context of topological defects, it has been argued that the velocity gradients induced as matter flows in the baryonic wakes, can generate vorticity required for the generation of magnetic fields [24]. Vilenkin [25] has also shown, in a different scenario, that the vorticity can be induced in a parity violating processes in the Weinberg-Salam model of the electroweak interactions, as macroscopic parity violating currents may develop a vortical motion in the thermal background. Even though this process can generate magnetic field at a length scale of cosmological interest, it was realized (see ref. [25]) that currently observed magnetic field in the proto-galaxies can not be obtained by the processes discussed above. As a consequence of the Helmholtz-Kelvin circulation theorem, small deviation from homogeneity and isotropy can not generate large rotational perturbations near the initial singularity as compared to the scalar and tensor perturbations. Rebhan et. al. [26] showed that the existence of collisionless matter exist in the very beginning of the Universe, can lead to growing modes of the vorticity on super-horizon scale. The lack of too large anisotropies in the Cosmic Microwave Background (CMB) spectrum constrains the amount of primordial vorticity. The upper bound on the strength of magnetic field at a length scale of L at present time, consistent with the CMB observations, is [26]

$$B_0(L) < 3 \times 10^{-18} h^{-2} L_{Mpc}^{-3} G, \quad (1.3)$$

and this field acts as a seed for galactic dynamo.

Another mechanism that can explain the generation of seed magnetic fields is Biermann battery Mechanism [27], which we are going to discuss in the next section.

Biermann Battery process

The Biermann mechanism, which was first discussed in the context of stellar, exploits the difference between the accelerations of electrons and ions which is triggered by their pressure gradients. This would in return generate electric current and hence the magnetic field. Force balance on a single electron in the fluid can be given as

$$n_e m_e \left(\frac{d\mathbf{v}}{dt} \right) = -n_e e \mathbf{E} - \nabla p_e - \frac{en_e}{c} (\mathbf{v} \times \mathbf{B}) \quad (1.4)$$

where e , n_e , m_e , and p_e respectively denote charge, number density, mass of the electrons and electron pressure in the fluid. Electron pressure in the fluid is defined as $n_e k_B T$. We can drop any term with m_e , as it is very small compared to the others terms. In this case, right hand side of the eq. (1.4) will be non-zero, while left hand side can be set to zero. Taking curl of equation (1.4), under above assumption and using Maxwell equation, one can obtain

$$\frac{d\mathbf{B}}{dt} = \nabla \times (\mathbf{v} \times \mathbf{B}) - c \frac{\nabla n_e \times \nabla p_e}{en_e^2} \quad (1.5)$$

Second term on the right hand side of eq. (1.5) is known as Biermann Battery term and has been well exploited to explain the generation of magnetic field [16]. Note that this term will work only when ∇p_e and ∇n_e are not parallel to each other. Davies and Widrow in [11], showed that in presence of vorticity in the plasma, Biermann Battery term gives a non zero value. Consider the evolution of a fluid described by Euler equation

$$\frac{d\mathbf{v}}{dt} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p, \quad (1.6)$$

where ρ is the energy and p is the pressure density of the fluid. Let's take the curl of this equation, and substitute $\boldsymbol{\omega} = \nabla \times \mathbf{v}$

$$\frac{d\boldsymbol{\omega}}{dt} - \nabla \times (\mathbf{v} \times \boldsymbol{\omega}) = \frac{\nabla \rho \times \nabla p}{\rho^2}. \quad (1.7)$$

In the limit of very small fluid velocity, the above equation takes the following form

$$\frac{d\boldsymbol{\omega}}{dt} = \frac{\nabla \rho \times \nabla p}{\rho^2}. \quad (1.8)$$

Going back to magnetic field evolution equation (1.5), in the limit of no background field and very small v

$$\frac{d\mathbf{B}}{dt} = -c \frac{\nabla n_e \times \nabla p_e}{en_e^2} \quad (1.9)$$

Assuming local charge neutrality and thermal equilibrium, we find

$$\frac{d\mathbf{B}}{dt} = -\alpha \frac{\nabla \rho \times \nabla p}{\rho^2}, \quad (1.10)$$

where $\alpha = m_p c / e(1 + \xi)$ with m_p being the proton mass and ξ being the ionization fraction. Therefore, from above two equations, one can write $\mathbf{B} = \alpha \boldsymbol{\omega}$. Under these conditions, the growth of the magnetic fields mimics the growth of vorticity. By this mechanism, magnetic field can be generated with initial conditions $t = 0$ and $B = 0$. One should note that, Biermann Battery term works only when $\nabla \rho \times \nabla p \neq 0$. The power of the Biermann Battery is realized in the early universe through shocks and subsequent vorticity [11]. This idea has been well exploited to generate the magnetic field on astrophysical scale (Kpc scale).

Dynamo Mechanism

There are several alternative scenarios available in the literature, that discuss generation of magnetic seed fields using battery-type mechanism during the post-recombination era. Basic understanding about these theories are that the magnetic fields are generated at the cost of the kinetic energy of the turbulent motions of the conductive interstellar medium [14]. Once seed fields of sufficient strength are generated, they grow due to dynamos for example in the galaxies. For galactic dynamo to work, following ingredients are required

1. Small but non vanishing, conductivity of the interstellar medium,
2. Turbulent motion of the fluid,
3. The differential rotation of the galactic system.

Qualitatively, the dynamo action can be understood by a reasonable differential equation. Steenbeck *et. al.* in 1966 [28] first time derived a differential equation describing dynamo mechanism. Later many authors explored dynamos in different

astrophysical objects [29,30]. To bring out the physics, a simple discussion is given below.

The generated magnetic field follows induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}. \quad (1.11)$$

The velocity and magnetic fields can be written as $\mathbf{v}$ and $\mathbf{B}$ respectively as

$$\begin{aligned} \mathbf{u} &= \bar{\mathbf{u}} + \mathbf{v} \\ \mathbf{B} &= \bar{\mathbf{B}} + \mathbf{b} \end{aligned}$$

where $|\bar{\mathbf{B}}| \gg |\mathbf{b}|$ and $|\bar{\mathbf{u}}| \gg |\mathbf{v}|$ represents ensemble average and η is the magnetic diffusivity. Averaging of the induction equation (1.11) over the small scale fluctuations, one can obtain the evolution of mean field as

$$\frac{\partial \bar{\mathbf{B}}}{\partial t} = \nabla \times [\bar{\mathbf{u}} \times \bar{\mathbf{B}}] + \nabla \times [\overline{\mathbf{v} \times \mathbf{b}}] + \eta \nabla^2 \bar{\mathbf{B}}. \quad (1.12)$$

We emphasize that for dynamo mechanism to work, the second term on the right hand side is crucial. Evolution of the time-varying component is

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times [\mathbf{v} \times \bar{\mathbf{B}}] + \nabla \times [\bar{\mathbf{u}} \times \mathbf{b}] + \nabla \times [\mathbf{v} \times \mathbf{b} - \overline{\mathbf{v} \times \mathbf{b}}] + \eta \nabla^2 \mathbf{b}. \quad (1.13)$$

We can drop term with $\bar{\mathbf{u}} \times \mathbf{b}$, as it can be removed by a Galilean transformation. This equation can be solved for each realization of the turbulent motions, and the ensemble average of $\mathbf{v} \times \mathbf{b}$ is to be calculated at each point. Assuming isotropic helical turbulence, one can write $\overline{\mathbf{v} \times \mathbf{b}} = -c_1 \bar{\mathbf{B}} + c_2 \nabla \times \bar{\mathbf{B}}$, where $c_1 = (\overline{\mathbf{v} \cdot \nabla \times \mathbf{v}}) \tau_v / 3$ and $c_2 = (\overline{\mathbf{v} \cdot \mathbf{v}}) \tau_v / 3$ [31]. Here τ_v is the velocity correlation time of the turbulence. The dynamo equation then becomes

$$\frac{\partial \bar{\mathbf{B}}}{\partial t} = \nabla \times [\bar{\mathbf{u}} \times \bar{\mathbf{B}}] + \nabla \times (\alpha \bar{\mathbf{B}}) + (\beta + \eta) \nabla^2 \bar{\mathbf{B}}. \quad (1.14)$$

c_2 represents turbulent diffusivity and acts on the mean field. For a fast dynamo, $c_2 \gg \eta$. The effect of first term is known as ω -effect and it vanishes if there is no differential rotation. The term with c_1 , represents the effect of the cyclonic motions on the mean field. c_1 effect corresponds to a mean turbulent electromotive force parallel to magnetic field. This effect deforms a straight magnetic field

into a helix. This effect survive only when there is finite fluid helicity. It has been shown in Ref. [31], that the few modes of the magnetic field grow exponentially. Several authors have questioned the operational efficiency of the galactic dynamo, as dynamo mechanism itself could be shut down before the generation of large scale magnetic fields. Such a termination of the dynamo mechanism could happen when small scale magnetic fields are strongly amplified and come to equipartition with the turbulent motions which in turn leads to the no net transfer of kinetic energy into magnetic energy. This early termination can be due to the strong amplification of small scale magnetic fields which may come to equipartition with the turbulent motions resulting in the end of transfer of kinetic energy into magnetic energy. Furthermore, the origin of the seed fields which are needed by the dynamo amplification, are not explained by the dynamo theory itself, but must be assumed to preexist. In a recent work, S. Naoz and R. Narayan [8], showed that the vorticity induced by temperature fluctuations, which are scale dependent, or spatially varying sound speed in the gas on linear scale can lead to the generation of primordial fields. This process operates at the time of recombination and works even when there is relative velocity between dark matter and baryonic matter. This modifies the predicted spatial power spectrum of the magnetic field. They estimated the strength of the magnetic field to be $(10^{-25} - 10^{-24})$ G on a comoving scales ~ 10 kpc.

1.1.2 Models based on early Universe processes

Models based on the astrophysical processes can satisfactorily explain the magnetic field in astrophysical objects but fails to explain large scale magnetic fields due to its large correlation length scale. Therefore we expect that the cosmic magnetic are generated in the early Universe. In literature, there are many mechanism available, but one can broadly classify them in following three categories

1. Inflationary scenario: when conformal invariance of the electromagnetic interaction are broken [9, 15, 32–39].
2. Phase transition: Bubble collision during the first order phase transitions

in the early universe [40]. Topological defects, cosmic strings, domain wall etc are also related with phase transitions [24].

3. Asymmetries in the early Universe [41, 42]

Based on this categorization, there are many models available, for example during reheating period after inflation [36, 43], cosmological defects [24, 44, 45] phase transitions [46–49], electroweak Abelian anomaly [42, 50], temporary electric charge non conservation [32], trace anomaly [32] or breaking gauge invariance [33]. Below we discussed some of the important processes, by which cosmic magnetic fields can be generated.

Generation during Inflation

Problem with the magnetic fields generated after inflation and before recombination era was that fields had very small coherence length scale, and so may not have importance on cosmological length scale ($\sim$ Mpc). One of the fundamental reason behind this is that the coherence scale of the fields are within the horizon at the time of magnetogenesis. One possibility is that these fields are generated during inflation. As inflation can inflate the scale of the generated magnetic fields to the horizon size, they provide an ideal set up for the generation of primordial magnetic fields [33]. Although model based on inflation explains the existing large coherence length of the magnetic field, yet it fails to generate the enough strength of the magnetic field to seed the galactic dynamos. In particular, B-fields that has survived a period of standard de-Sitter inflation are typically too weak to sustain the galactic dynamo. Turner and Widrow in [33] showed that magnetic field of astronomical interest (of large scale $\sim$ Mpc) could be produced during this period of accelerated expansion provided the conformal symmetry of the electromagnetic theory is broken. Inflation provides following important ingredients for the production of primeval magnetic fields [33]

1. Inflation provides the kinematic means of producing very-long wavelength effects at very early times through micro-physical processes operating on scales less than the Hubble scale. An electromagnetic wave with wavelength

larger than Hubble length scale i.e., $\lambda \gtrsim H^{-1}$, appears as static fields. This will leads to large-scale

2. If there is mechanism to break the conformal invariance of the electromagnetic action then magnetic field could be excited during the expansion phase of the Universe. Modes with length scale $\lambda \lesssim H^{-1}$ could be excited from the quantum fluctuations. However due to expansion of the Universe length scale also get stretched and the energy density of the modes with length scale $\lambda \simeq H^{-1}$ follows $d\rho/dk \sim H^3$.
3. During the inflationary period conductivity of the Universe was negligible due to absence of charge particles. However during reheating period huge number of charged particles were produced. Therefore, magnetic fields produced during inflationary period will be frozen.

Turner and Widrow also [33] proposed three methods of breaking the conformal invariance, which are as follows:

- *Coupling of the photon to the gravitational field* : A term of following form $RA_\mu A^\mu$, $R_{\mu\nu}A^\mu A^\nu$, $R_{\mu\nu\lambda\kappa}F^{\mu\nu}F^{\lambda\kappa}/m^2$, $RF^{\mu\nu}F_{\mu\nu}$, (where A_μ is the electromagnetic field, $F^{\mu\nu}$ is field strength tensor, $R_{\mu\nu\lambda\kappa}$ is Ricci tensor of fourth order, $R_{\mu\nu}$ is the Ricci tensor, R is the curvature scalar and m denotes the mass scale required by dimensional considerations) leads to breaking of conformal invariance. As a consequence of the conformal symmetry breaking the photons gain an effective time-dependent mass.
- *Coupling of the photon to a charged scalar field* [51–53],
- *Anomalous coupling of the axions and photons* [54].

During inflationary period, quantum vacuum fluctuations of the electromagnetic fields can be excited within the horizon and later when they leave the horizon, they transformed to the classical fluctuations. Due to rapid expansion of the Universe, the existing charged particles, if any, diluted drastically which leads to the negligible conductivity of the Universe. However, during reheating inflaton

decays to produce huge amount of charged particles and hence the plasma becomes almost perfect conductor. As a consequence the magnetic flux gets frozen in the plasma.

One of the most general form of the action may include coupling of the EM fields with inflatons, scalar fields or with charged fields. Moreover if there exist extra-spatial dimensions with scale factor $b(t)$ then one of the possible action in $(1 + 3)$ dimension can be written as

$$S = \int \sqrt{-g} d^4x b(t) \left[-\frac{f^2(\phi, R)}{16\pi} F_{\mu\nu} \tilde{F}^{\mu\nu} - g_1 R A^2 + g_2 \theta F_{\mu\nu} \tilde{F}^{\mu\nu} - (D_\mu \psi)(D^\mu \psi)^* \right] \quad (1.15)$$

where g_1 and g_2 are some constant related with the coupling constants. This is a qualitative picture of magnetic field generation during the era of inflation and reheating. In the recent past, it has been shown that, the magnetic field generated in this fashion are model dependent and produces very weak seed fields [36].

Example: To understand the generation of magnetic fields from inflationary process, let us assume that a scalar field ϕ is inflaton field and is responsible for the breaking of conformal invariance in equation (1.15) (first term in above action is just $\phi(\tau) F_{\mu\nu} \tilde{F}^{\mu\nu}$). Further assume that the electromagnetic field is weak enough that it does not perturb inflaton field and flat FRW expanding background space (a being scale factor). However, in references [55, 56] authors have discussed back reaction of these generated magnetic fields. In reference, [55] authors have given a upper limit on the amplitude of the these field so that it could not spoil the inflation. In the present example, we are interested in a case where these seed fields are very weak and do not spoil inflation. For such a case, one can obtain the generalised Maxwell's equation, by using equation (1.15) with the assumptions made in the current example. The space component gives

$$A_i'' + 2\frac{f'}{f} A_i' - a^2 \partial_j \partial^j A_i = 0 \quad (1.16)$$

where prime denotes derivative with respect to conformal time τ and $\partial^i = g^{jk} \partial_k = a^{-2} \delta^{jk} \partial_k$. In deriving equation (1.16), we have considered Coulomb gauge for which electromagnetic four vector gauge field A^μ follows $\partial_i A^i(t, x) = 0$ and $A_0(t, x) = 0$. Electric and magnetic fields can be written in terms of three space

component of the gauge field A_i as: $E_i = \frac{1}{a}A'_i$ and $B_i = \bar{\epsilon}_{ijk}\delta^{jl}\delta^{km}\partial_l A_m$. One can obtain the Maxwell's equation by varying action with respect to A_μ , which is given as

$$\nabla_\nu F^{\mu\nu} = \frac{4\pi J^\mu}{f_0^2}. \quad (1.17)$$

In this example, value of f_0 (here f_0 is background value of f) normalises the electric charge e as e/f_0^2 . For a redefined variable $\mathcal{A} = a(\tau)f(\tau)A(\tau, k)$, above equation (1.16) can be reduced to following form

$$\mathcal{A}''(\tau, k) + \left(k^2 - \frac{f''}{f}\right)\mathcal{A}(\tau, k) = 0, \quad (1.18)$$

which is similar to the equation of evolution of harmonic oscillator *i.e.*

$$\mathcal{A}''(\tau, k) + \omega(\tau)^2\mathcal{A}(\tau, k) = 0 \quad (1.19)$$

For $f = \text{const.}$, above equation reduces to the standard EM action and $\mathcal{A}$ oscillates in time like a standing wave.

The energy density T_0^0 is the sum of magnetic contribution, $T_0^{0B} = -f^2 B^2/8\pi$ and electric contribution $T_0^{0E} = -f^2 E^2/8\pi$. Magnetic and electric energy densities can be defined as: $\rho_B = \langle 0|T_0^{0B}|0 \rangle$ and $\rho_E = \langle 0|T_0^{0E}|0 \rangle$. The spectral distribution of energy in the electric and magnetic field is

$$\frac{d\rho_E}{d\ln k} = \frac{f^2 k^3}{2\pi^2 a^4} \left| \frac{\mathcal{A}(\tau, k)}{f} \right|^2 \quad (1.20)$$

$$\frac{d\rho_B}{d\ln k} = \frac{1}{2\pi^2} \left(\frac{k}{a} \right)^4 k |\mathcal{A}(\tau, k)|^2, \quad (1.21)$$

Thus, once we know the mode function $\mathcal{A}(k, \tau)$, the evolution of energy densities ρ_E and ρ_B can be calculated. During reheating the inflaton fields decays and convert the energy in the inflaton field to radiation. In the present scenario, the strength of the magnetic fields as observed today can be given as

$$B_0 \sim 0.7 \times 10^{-10} G \left(\frac{H}{10^{-5} M_{pl}} \right). \quad (1.22)$$

Magnetic field with this strength can, in principle, be generated by appropriately choosing the coupling function.

Magnetic fields from phase transitions

There are some other mechanism of magnetogenesis based on the phase transitions in the early Universe, motivated from particle physics. The first pioneer work based on phase transition was done by Hogan [40]. This was based on the idea that phase transition proceeds via bubble nucleation. During this course the bubbles of new phase collides violently leading to turbulence in the plasma. Similar to the “*Biermann Battery Mechanism*”, a charge separation of the positive and negative occurs in the plasma during this process [57] which leads to the generation of electric currents, resulting in the generation of magnetic fields (actually hypermagnetic magnetic fields). In a ref. [58], authors have shown that a primordial magnetic fields, generated due to some causal process in the early Universe, can be suppressed than usually assumed. However in ref. [59] it is discussed that the helical fields will suppressed even more. In ref. [42], it was shown that in presence of asymmetry in the left handed and right particles in the early Universe, some modes of the magnetic fields grow exponentially and leads to the generation of helical magnetic fields. This happens in presence of external gauge fields. In the presence of external gauge fields, a current flows in a particular direction. This is known as Chiral magnetic effect [60].

Another approach to discuss the magnetogenesis during phase transitions is that the phases of the complex order parameter of the nucleated bubbles are not correlated and so during collision there exists a phase gradient. This gradient leads to the source term for the evolution equation of the gauge field [48]. This idea has been investigated in the context of the Abelian-Higgs model [61–63]. In the Abelian Higgs model, the collision of two spherical bubbles leads to a magnetic field which is localized in the interaction region of two bubbles. The strength of the generated magnetic field in this model depends on the velocity of the bubble wall. In the context of Electroweak phase transition, Vachaspati [46] showed that the magnetic fields can be generated even if the transition is second order. However, the model parameters were gauge dependent quantities. So it is necessary to give a gauge-invariant definition of the photon in terms of the standard model fields. In an interesting approach [42], authors discussed

generation of primordial magnetic fields at the electroweak scale using anomalous magneto-hydrodynamics [64]. The basic assumption in this scenario was that the chemical potential of the pre-existing excess of right-electron converted to the hypercharge fields. The magnetic fields obtained in this case are sufficiently strong i.e., $\sim 10^{22}$ G at the electroweak epoch. However the correlation scale is very small and of the order of $10^{-6}H_{ew}^{-1}$. There are ideas related with the generation of magnetic fields at QCD phase transition. As the charge of the strange quarks and down quarks are different, it may be possible that the quarks develop a net charge. Therefore the first order phase transition at QCD scale may develop the different currents for leptons and quarks as the bubble wall moves in the quark-gluon plasma. In ref. [49] the magnetic field has been estimated to be of the order of $B \sim \text{G}$ at the time of the QCD phase transition with a typical scale of the order of the meter at the same epoch.

Generation of magnetic field at electroweak phase transition were studied (at temperature $T_c \sim 100 \text{ GeV}$) in [9, 10, 46, 47, 65] and at QCD transitions (at temperature $T_c \sim 150 \text{ MeV}$) in [66–69]. Another aspect of the consideration magnetogenesis at phase transitions is that they provides non-equilibrium conditions for processes like baryogenesis [70, 71] and leptogenesis [72–74] in the early universe.

Helicity of fields and its production:

It was shown in ref. [75], that the generated turbulence during the phase transition can amplify the seed magnetic field further by dynamo action. Model based on phase transitions explain the current observed strength of magnetic fields but they fails to explain the existence of the magnetic fields at large scale. Later it was shown that if generated fields have helicity, then the coherence length can increase because of the “*inverse cascade*” in presence of MHD turbulence [4, 76, 77]. In 3-D MHD, the total (kinetic plus magnetic) energy cascades toward smaller scales, where it is dissipated by viscosity and resistivity. This is known as direct cascading of the total energy. However in the presence of magnetic helicity, energy cascade towards large length scale. This is known as “*inverse cascades process*”.

There are some other work [78], in which authors have shown that energy transfer can occur even in the absence of helical field. This work was later confirmed by other groups [79–81]. The magnetic helicity density is defined as

$$\mathcal{H}_M(t) = \frac{1}{V} \int_V d^3x \mathbf{Y} \cdot \mathbf{B}, \quad (1.23)$$

where $\mathbf{Y}$ is the (hyper)-electromagnetic vector potential and $\mathbf{B} = \nabla \times \mathbf{Y}$, the corresponding (hyper-)magnetic field. Helicity given above is equivalent to the Chern-Simon number of the particle physics [50]. The time evolution of $\mathcal{H}_M$ is given by

$$\frac{d\mathcal{H}_M}{dt} = -\frac{1}{\sigma} \int d^3x \mathbf{B} \cdot \nabla \times \mathbf{B}. \quad (1.24)$$

For perfectly conducting medium, magnetic helicity remain conserved. Helicity is an ideal invariant and asymptotically conserved within the resistive MHD approximation [82]. Apart from being a conserved quantity, helicity is a interesting for some reasons, given below

- Helicity in the MHD and Chern-Simon number, which is related with the topological property of the gauge fields in the field theory, coincides with each other.
- As $\mathcal{H}_M$ is a parity P (parity) and CP-odd variable, the presence of a non-vanishing value of $\mathcal{H}_M$ would be a direct demonstration of breaking of these two symmetries.
- The presence of $\mathcal{H}_M$ implies the amplification and reorganization of field configurations towards large scale (which is known as inverse cascading) [83, 84].

So physically $\mathcal{H}_M$ describes the topology of the magnetic fields lines. Also the inverse cascade allows the magnetic energy to shift from smaller length scale to larger length scales, as system tries to minimize its energy while conserving magnetic helicity [76, 85–87]. In the last few years, there are several proposed mechanism to produce a helical magnetic fields in the early Universe. One of the important mechanism was discussed by Cornwall [50]. In the model, discussed in

ref. [50], it was suggested that a finite baryon number (B) and lepton number (L) stores the information about the initial helicity $\mathcal{H}_M$. These finite baryons (B) and leptons (L) were generated by some GUT scale baryogenesis mechanism. Assumptions was that a small amount of the total classically conserved $B + L$ charge, dissipated because of the anomalous processes in the early Universe. In this work, it was shown that $n_{B+L}T^{-3}$ amount of dissipated charged may have converted in to the helicity. The order of the produced helicity from the dissipated charge in natural units is

$$\mathcal{H}_M \sim \frac{1}{\alpha}(N_B + N_L) \simeq 10^{71}. \quad (1.25)$$

Above, we have used B and L for baryon number and lepton numbers respectively. However, in the later part of the thesis, we have used B only for magnetic field amplitude.

Helicity and Chern-Simon number (CS)

Cornwall [50] and Vachaspati [84], showed in their work that baryogenesis and helicity of the magnetic field are actually connected with each other during EW phase transition. Classically B remain conserved in the EW theory, however it is broken in the quantum theory in the presence of classical Gauge field configuration. This is because of anomalous behaviour of EW plasma. In the standard model, the global baryon current $j_B^\mu = \Sigma_q \frac{1}{3} \bar{q} \gamma^\mu q$ and lepton number currents $j_L^\mu = \Sigma_l (\bar{l} \gamma^\mu l + \bar{\nu}_l \gamma^\mu \nu_l)$ are exactly conserved at the classical level. However, the anomaly equation give

$$\partial_\mu j_B^\mu = \frac{iN_F}{32\pi^2} \left(-g_2^2 W^{\mu\nu a} \tilde{W}_{\mu\nu}^a + g_1^2 Y^{\mu\nu} \tilde{Y}_{\mu\nu} \right), \quad (1.26)$$

here $W^{\mu\nu a}$ and $\tilde{W}_{\mu\nu}^a$ are the field strength corresponding to the $SU(2)_L$ gauge potential W_μ^a and it's dual respectively. Dual is defined as $\tilde{W}_{\mu\nu}^a = \epsilon_{\mu\nu\alpha\beta} W^{\alpha\beta a} / 2$. Similarly $Y^{\mu\nu}$ and $\tilde{Y}_{\mu\nu}$ are the field strength of $U(1)_Y$ group with coupling constant g_1 . $N_F = 3$ is the number of families and g_2 is the coupling constant related to $SU(2)_L$ group. One can also obtain corresponding current expression for each Lepton species as

$$\partial_\mu j_l^\mu = i \frac{1}{32\pi^2} \left(-g_2^2 W^{\mu\nu a} \tilde{W}_{\mu\nu}^a + g_1^2 Y^{\mu\nu} \tilde{Y}_{\mu\nu} \right), \quad (1.27)$$

here j_l^μ is defined for each lepton species. Even B and L are separately not conserved, $B - L$ remain safe from the anomalous processes. Integrating equation (1.26) over 4-D space, baryon number changes by $\Delta B = N_F(N_{CS} - n_{CS})$. N_{CS} and n_{CS} are Chern-Simon number defined as

$$N_{CS} = -\frac{g_2^2}{16\pi^2} \epsilon^{ijk} \int d^3x \text{Tr} \left[W_k \partial_i W_j + i \frac{2}{3} g_2 W_i W_j W_k \right] \quad (1.28)$$

$$n_{CS} = -\frac{g_1^2}{16\pi^2} \int d^3x \epsilon^{ijk} Y_k \partial_i Y_j \quad (1.29)$$

One can write equation (1.26) as follows

$$\partial_\mu j_B^\mu = i \frac{N_F}{32\pi^2} (-g_2^2 \partial_\mu K^\mu + g_1^2 \partial_\mu k^\mu) \quad (1.30)$$

where

$$K^\mu = 2\epsilon^{\mu\nu\alpha\beta} (W_\beta^a \partial_\nu W_\alpha^a - \frac{1}{3} g_2 \epsilon_{abc} W_\nu^a W_\alpha^b W_\beta^c) \quad (1.31)$$

$$k^\mu = 2\epsilon^{\mu\nu\alpha\beta} Y_\beta \partial_\nu Y_\alpha \quad (1.32)$$

One can also write baryon current as

$$j_B^\mu = \frac{N_F}{32\pi^2} (-g_2^2 K^\mu + g_1^2 k^\mu) \quad (1.33)$$

Although N_{CS} and n_{CS} are gauge non-invariant objects though $(N_{CS} - n_{CS})$ at different times remain gauge invariant. Chern-Simon number for the electromagnetic field is usual magnetic helicity. Each Chern-Simon number of the non-Abelian fields defines pure gauge configurations with zero energy and they are separated by barrier of height $E_{sph} \sim M_W/g_2^2$ in field configuration space. Transmission through the barrier happens only via quantum tunnelling at zero temperature and fermion density. This tunnelling is known as instanton [88]. The tunnelling at zero temperature and zero fermion density is suppressed exponentially as $\Gamma \propto e^{-4\pi/\alpha_w}$ [89, 90]. However at finite temperature, this barrier can be overcome by thermal jumps on the top of the barrier. The field configuration with maximum energy is referred as Sphaleron. If the field configuration with energy E_{sph} is smaller than the temperature of the thermal bath, *i.e.* $E_{sph} < T$, the jump on the barrier are suppressed by exponential factor $\Gamma_{sph} \propto e^{-E_{sph}/T}$. With the change of CS-number or decay of sphaleron configurations, baryons are

produced or annihilated. The range of temperature in which electroweak number violating processes are in thermal equilibrium in the early universe, can be calculated by comparing rate of sphaleron processes with Hubble expansion rate; *i.e.* $\Gamma_{sph} \geq nH$. Here $n \sim T^3$ is the number density of particles of a given type. Therefore sphaleron remain in the equilibrium for

$$\frac{\Gamma_{sph}}{T^3} \geq H(T) = \frac{T^2}{M_{pl}}. \quad (1.34)$$

Using this inequality, we can get temperature bound over which baryon number violating processes are in thermal equilibrium. Bounds are $100 \text{ GeV} \leq T \leq 10^{12} \text{ GeV}$. So for $T \leq 10^{12} \text{ GeV}$, any preexisting $(B + L)$ is erased exponentially with a typical time scale $\tau_c \sim 2N_F T^3 / 13\Gamma_{sph}$. This can be understood from the following master equation for the evolution of the number density n_{B+L} as [91]

$$\frac{dn_{B+L}}{dt} = -\frac{13}{2} N_F \frac{\Gamma_{sph}}{T^3} n_{B+L}. \quad (1.35)$$

But the baryon number violation within the SM and GUT baryogenesis can occur, if there is preexisting $B - L$ asymmetry at the GUT scale, because the sphaleron mechanism decays only $(B + L)$ asymmetry and does not touch the $(B - L)$ asymmetry [92]. One can see from the following combination for B

$$B = \frac{B + L}{2} + \frac{B - L}{2}. \quad (1.36)$$

The fact that the combination $B - L$ is left unchanged by the sphaleron transitions opens up the possibility of generating the baryon asymmetry from the lepton asymmetry [93]. It is known that, to generate finite baryon number density at GUT scale, *Sakharov conditions* must be satisfied. However in EW theory, since C and CP are known to be violated by electroweak interactions, it is possible to satisfy all Sakharov's conditions within the SM if the EW phase transition leading to the breaking of $SU(2)_L \otimes U(1)_Y$ is of the first order [94]. At temperature T less than the temperature at which EW phase transition occur (*i.e.* $T < T_c \sim 100 \text{ GeV}$) and small densities of the different fermionic charges, the EW symmetry is broken. And the $U(1)_{em}$ fields are only long range fields [46, 65]. At the electroweak epoch the typical size of the Hubble radius is of the order of 3 cm. The typical diffusion scale (defined as $\sqrt{t/\sigma}$, here t is time scale of diffusion

and σ is the conductivity) of the plasma is of the order of 10^9 cm. Therefore, over roughly eight orders of magnitude hypermagnetic fields can be present in the plasma without being dissipated [95].

In the upcoming section, our main focus will to discuss the evolution of the generated magnetic fields and their correlation length scale from the epoch of magnetogenesis to present time.

1.2 Evolution of the Primordial magnetic fields

To understand the existence of the magnetic field at the galactic scale and in the intergalactic scale, it is crucial to understand the evolution of the magnetic field from the time of generation to the present time. At the time of magnetogenesis, nonlinear interactions of the magnetic field and the plasma fluid can govern the evolution of the magnetic fields. In this case the growth of the peak value of the magnetic field B_p and physical correlation length scale λ_p may not evolve adiabatically. Which means that $B_p \not\propto a^{-2}$ or $\lambda_p \not\propto a$. In absence of the free charges and small diffusion time of the fields in comparison to the age of the Universe, the stable modes of the fields sustain for a long time. However at high energy for example in the case of QCD plasma, presence of the strong magnetic fields produces a non-trivial instability in the plasma [96]. There are three effects that play major role in the evolution of magnetic fields in the early Universe [4]

- The effect of viscous diffusion
- The interaction of the fields with the turbulent fluid
- The free streaming of photon and neutrino

When a typical scale of the turbulence λ_T at a certain time t becomes of the order of $\lambda_T \simeq v_T t$ (here v_T is the velocity of the fluid), the coupling of the magnetic fields and turbulence may become important. However in the case when the magnetic field correlation scale is much larger than the turbulence, then magnetic fields grows adiabatically. For the case of $\lambda_T \simeq \lambda_B$, inverse cascade starts. Under following scenarios, magnetic fields grows

1. **Purely inverse cascade case:** the magnetic fields undergo the inverse cascade just after their generation.
2. **The period when adiabatic and inverse cascade are simultaneously important *i.e.* transition case:** First magnetic fields adiabatically evolve, and subsequently the inverse cascade starts at a temperature $T = T_{TS}$.
3. **And the purely adiabatic case:** The magnetic fields always evolve adiabatically and never experience the inverse cascade process.

To understand these three scenarios, let us first study evolution of a conducting plasma. The time evolution law of a magnetic field in a standard conducting plasma is given as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{4\pi\sigma} \nabla^2 \mathbf{B} \quad (1.37)$$

This equation is known as diffusion equation. In a time $\tau_{diff}(L) = 4\pi\sigma L^2$, initial magnetic field configuration will vanished. Here L is the characteristic length scale of the spatial variation of $\mathbf{B}$. A magnetic field generated at some time t in the early Universe with coherence length scale L_0 will survive till today t_0 when condition $\tau_{diff}(L_0) > t_0$ is satisfied. Length scale L_0 corresponds to present time determined by Hubble law.

$$L_0 = L(t_i) \frac{a(t_0)}{a(t_i)}, \quad (1.38)$$

where $L(t_i)$ is the length scale at the time of formation of the magnetic configuration. Magnetic fields evolved mostly in the matter dominated Universe and therefore when $\sigma \rightarrow \infty$, the magnetic flux moving with fluid remain frozen in. Magnetic flux can be calculated using

$$\frac{d\Phi_S(B)}{dt} = -\frac{1}{\sigma} \int_S \nabla \times (\nabla \times \mathbf{B}) \cdot d\mathbf{S} \quad (1.39)$$

where S stands for surface integral. On scale where diffusion can be neglected, the field is said to be *frozen-in*. For the case of isotropic expanding Universe, flux conservation implies

$$B(t) = B(t_i) \left(\frac{a(t_i)}{a(t)} \right)^2. \quad (1.40)$$

Similar to the flux, another quantity which remain conserved is magnetic helicity (defined in eq. (1.23)). In the presence of turbulence, the characteristics of the initially created magnetic field are vastly modified during cosmic evolution between the epoch of magnetogenesis and the present [97, 98].

In the remaining part of this section, we will focus on general features of the evolution of the magnetized fluids in the turbulent regime. Due to the finite conductivity in the early Universe, one can neglect dissipative effects. It is generally believed that the generated magnetic field freely decays without any further input of kinetic or magnetic energy, i.e., as freely decaying MHD (due to viscosity). Lets us first write general equations that describe incompressible MHD with dissipative contributions [76]

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} - (\mathbf{v}_A \cdot \nabla) \mathbf{v}_A = \mathbf{F} \quad (1.41)$$

$$\frac{\partial \mathbf{v}_A}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v}_A - (\mathbf{v}_A \cdot \nabla) \mathbf{v} = \frac{1}{\sigma} \nabla^2 \mathbf{v}_A. \quad (1.42)$$

In above $\mathbf{v}_A(x)$ is known as local Alfvén velocity, which is given by $\mathbf{v}_A(x) = \mathbf{B}(x)/\sqrt{4\pi(\epsilon + p)}$. Other variables are $\mathbf{v}$, $\mathbf{B}$, ϵ and p known as the fluid velocity, magnetic field, mass energy density, and pressure respectively. Equation (1.41) and (1.42) are known as Euler and diffusivity equations respectively. The function $\mathbf{F}$ in (1.41) is dissipative term and depends on the scale.

$$\mathbf{F} = \begin{cases} \eta \nabla^2 \mathbf{v} & \lambda_{mfp} \ll l \implies \text{dissipation due to diffusing particles} \\ -\alpha \mathbf{v} & \lambda_{mfp} \gg l \implies \text{dissipation due to free streaming} \end{cases} \quad (1.43)$$

here l is some length scale, α and η are parameters. In the early Universe, both of these two regime are important. The kinetic Reynolds number define the characteristic of the fluid flow and can be given as

$$R_e(l) = \frac{v^2/l}{|\mathbf{F}|} = \begin{cases} \frac{vl}{\eta} & \lambda_{mfp} \ll l \\ \frac{v}{\alpha l} & \lambda_{mfp} \gg l \end{cases} \quad (1.44)$$

Reynolds number represents the relative importance of fluid advective terms and dissipative terms in the Euler equation. Some times it is given by the ratio of a typical dissipative time scale $\tau_d = (l^2/\eta, 1/\alpha)$ to the eddy-turnover time scale $\tau_{eddy} = l/v$. For the turbulent flow *i.e.* $R_e(L) \gg 1$ (here L is the coherence scale

of the magnetic field), the decay rate of the total energy is independent of dissipative effects and it solely depends on the flow property of the coherence scale and flux lines of the magnetic field will be frozen into the plasma element. However in the regime where $R_e(L) \ll 1$, total decay rate depends on the magnitude of the viscosity. Therefore the flux lines of the magnetic field will diffuse through the plasma. In equation (1.41), third term on the left hand side will establish fluid motions of the order of $v \approx v_A$ within an Alfvén crossing time $\tau_A \simeq l/v_A$, at which point back reaction of the fluid flow on the magnetic fields will prevent further conversion of magnetic field into kinetic energy. The resultant fully turbulent state is characterized by close-to-perfect equipartition (in the absence of net helicity) between magnetic and kinetic energy which can be represented by $\langle \mathbf{v}^2 \rangle \approx \langle \mathbf{v}_A^2 \rangle$. The transport of the fluid energy from the integral scale L to the dissipation scale l_{diss} occurs via a cascading of energy from large scales to small scales, referred *direct cascade*. It was shown that at the smallest length scale, magnetic and kinetic energy follows Kolmogorov spectrum ($E_k \propto k^n$).

1.3 Observations and constraints

One can divide the observed magnetic fields into two components: first one is the uniform component) and another is the non-uniform component.

$$B_{tot} = \bar{B} + \delta B,$$

here B_0 is the homogeneous part of the total magnetic field (B_{tot}) and (δB) is the non-homogeneous part (where $\delta B \ll B_0$). Depending on the experimental techniques, the total magnetic field or its homogeneous part can be measured. Some measurements do measure two important components $B_\perp$ and $B_\parallel$. Component $B_\perp$ and $B_\parallel$ are defined as the component that are perpendicular and parallel to the line of sight respectively. Broadly measurement methods can be classified: Faraday rotation measurements, Zeeman effect and Synchrotron emission. However, recently Neronov and Semikoz [99] have proposed a new methodology based on Blazar's observations to measure the magnetic fields in the inter galactic medium. When a polarized radio signal passes through a region of space of size

δl containing a magnetized plasma, the plane of polarization of the wave gets rotated by a amount $\delta\phi \propto \omega_B \frac{\omega_p}{\omega} \delta l$ [100]. Here ω_p is plasma frequency and ω_B is Larmour frequency ($\omega_p \propto B_0$). By measuring rotation angle, we can measure the strength of background magnetic fields. In the case Zeeman effect measurement, the measurement of the splitting of spectral lines (which is characterized by the deviation of the spectral lines $\Delta\nu = \frac{eB_0^{\parallel}}{2\pi m_e}$ (here m_e is the electron mass) [101] gives information about the background magnetic field along the line of sight. The synchrotron emission is sensitive to measure the transverse component $B_{\perp}$, however Faraday Rotation measurement and the Zeeman splitting of spectral lines are sensitive to the longitudinal component of the magnetic field along the line of sight. The key to detection of the magnetic field is the polarization of the emitted light from different sources in the optical, in the infrared, in sub-millimeter and in the radio wavelengths. The knowledge about the galactic and intergalactic magnetic fields comes from the measurements of the radio observations. The optical polarization occur from the extinction of the magnetic fields while passing through the dust grains in the interstellar medium along the line of sight.

1.3.1 Observational bounds

Nucleosynthesis provides the earliest probe to measure the magnetic fields as in the presence magnetic field can alters the rate of nucleosynthesis. One of the pioneer work related with the estimate of the magnetic field strength was done by Greenstein in ref. [103] and later it was done in more details by several others in ref. [10, 104]. The basic idea was that, the rate of β -decay and rate of expansion of the Universe can be affect by the primordial magnetic fields. In ref. [105], authors have given bounds on the amplitude of the magnetic fields from large scale structure data. Another source to know about the presence of the primordial magnetic fields is the CMB measurements. In a recent observation of PLANCK, it is shown that upper and lower bounds on the peak value of the magnetic field is $10^{-17} \text{ G} \lesssim B_0 \lesssim 10^{-9} \text{ G}$ [106]. A summary of the some observations and theoretical predictions has been shown in figure- (1.3.1). The

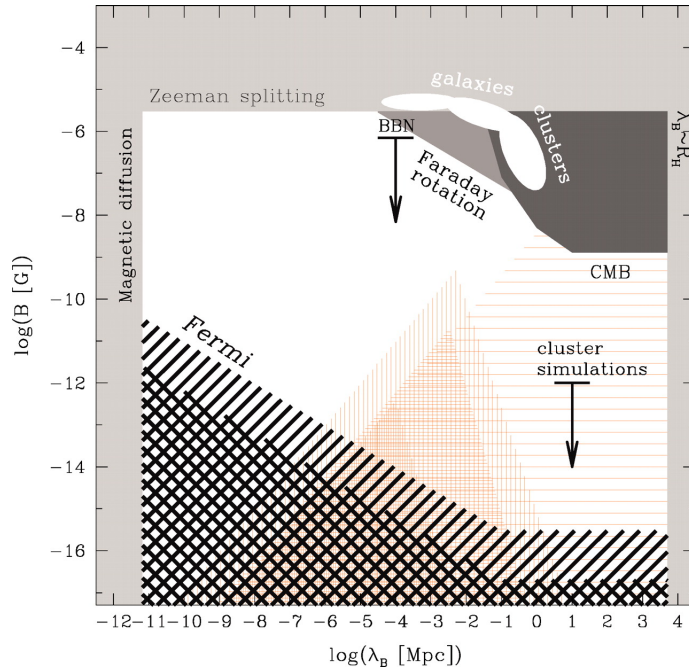


Figure 1.1: **(Bounds on Extra Galactic Magnetic fields B (EGMFs), from several observations and experiments):** Light, medium and dark gray color shows known bounds on the strength of EGMFs. The bound from the Big Bang Nucleosynthesis is taken from [10]. The black hatched region in the lower side of the figure, shows the lower bound on the EGMF derived from observations of 1ES 0347-121 (cross-hatching) and 1ES 0229+200 (single diagonal hatching) [102]. Orange hatched regions show the allowed ranges of magnetic field and its correlation scale generated during inflation models (horizontal orange hatching), during EW phase transition (dense vertical hatching), QCD phase transition (medium vertical hatching), and epoch of recombination (light vertical hatching) [99]. White ellipses show the range of measured magnetic field strengths and correlation lengths in galaxies and galaxy clusters. Reprinted figure from [102], Copyright ©The American Association for the Advancement of Science

strength of the magnetic field varies with different length scale which is clearly depicted in the figure-(1.3.1). Another work which catches much attentions of ours is a work by the authors of ref. [39]. In this work authors have calculated a much tight bound on the strength of the magnetic fields which comes from the early Universe baryon asymmetry. A magnetic fields with present strength of

$10^{-14} \text{ G} \lesssim B_0 \lesssim 10^{-12} \text{ G}$ can be obtained if the magnetic fields were undergone a inverse cascading before the electroweak phase transition. They predicted that beyond this bound any magnetic field can be favoured because of the limited observed baryon asymmetry.

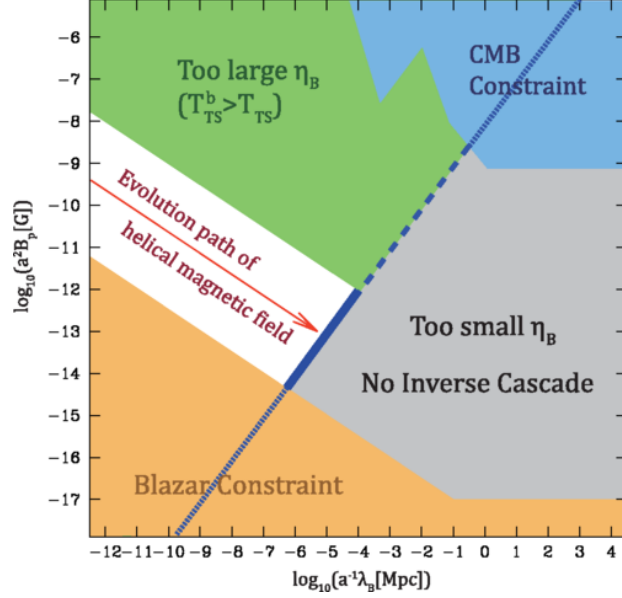


Figure 1.2: **(a)**.Time evolution of kinetic (dashed lines) and magnetic (solid lines) in the turbulent regime($R_e \gg 1$) without initial helicity. **(b)**.Evolution of magnetic-energy spectra in the turbulent regime ($R_e \gg 1$) with no initial helicity. Reprinted figure from [39], Copyright ©APS

1.4 Objectives of the study

The universe is magnetized on all scales probed so far. A variety of observations imply that stars, planets, galaxies, clusters of galaxies are all magnetized. The typical strength of the magnetic field ranges from few $\mu \text{ G}$ (in the galaxies and galaxy clusters) to few G (in planets, like earth) and up to 10^{12} G (in the neutron stars). There are several astrophysical and particle models available but no model alone is able to explain the presence of homogeneous magnetic field at all scales. An intriguing possibility is that these observed magnetic fields are a relic from the early Universe, albeit one which has been subsequently amplified and maintained by a dynamo in collapsed objects. So it is quite important to study and construct

a model which can explain the generation of magnetic field and its evolution throughout the Universe evolution.

In our work, we study the generation of magnetic fields above the EW scale due to an anomaly in the primordial plasma consisting of the standard model particles. In our work we also look at the subsequent evolution of the generated magnetic fields in the early Universe. To study the generation of magnetic field, we use kinetic equations modified with Berry curvature. The new kinetic theory framework explains some of the very important features of the plasma with anomaly, like Chiral Magnetic Effects (CME) and Chiral Vortical Effects (CVE). We derive the expression of the Chiral Magnetic and Chiral Vortical currents. We also calculate chiral magnetic and chiral vortical conductivities by using kinetic theory. One of the main feature of our study is that the derived expressions for current and conductivities match with the studies done earlier, in a different context [42, 107, 108]. We also calculate the strength of magnetic field at the EW scale, which comes into the bounds obtained observationally. In another work, we study the generation and evolution of magnetic field in the presence of chiral imbalance and gravitational anomaly which gives an additional contribution to the vortical current. The contribution due to gravitational anomaly is proportional to T^2 . This contribution to the current can generate seed magnetic field of the order of 10^{30} G at $T \sim 10^9$ GeV, with a typical length scale of the order of $10^6/T$, even in absence of chiral charges (when chiral chemical potential is zero). Moreover, such a system possess scaling symmetry. We show that the T^2 term in the vorticity current along with scaling symmetry leads to more power transfer from lower to higher length scale as compared to only chiral anomaly without scaling symmetry. Next we study the evolution of the hydrodynamic excitation in the chiral plasma in the early universe. In this work, we have included the first and second order viscous terms in the hydrodynamic equation to study the effect of these first and second order viscous term on these hydrodynamic excitations. We have calculated few of the second order transport coefficients, and have found that the values of these coefficients fall under current bounds.

1.5 Overview of the chapters

The thesis is organized as follows: Chapter (2) contains the theoretical foundation for our work. This chapter contains an introduction to the kinetic theory of plasmas. We also discuss a modification of the kinetic equations in the presence of external magnetic fields. In Chapter (3), we discuss the generation of the magnetic field before EW phase transition using kinetic theory. In chapter (4), we have discussed the generation of primordial magnetic in presence of the chiral imbalance and gravitational anomaly. In this chapter we have also studied evolution in details. In Chapter (5), we discuss anomalous magnetohydrodynamics. The final Chapter (6), contains a summary of our work and the future scope of our work.

Chapter 2

Theoretical Foundation

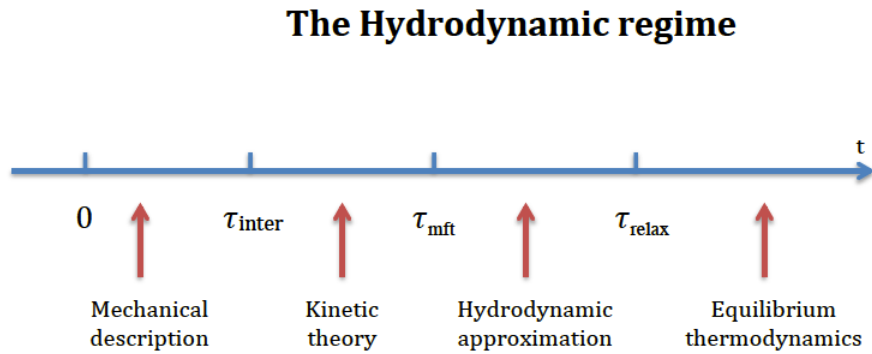
Plasma is a many body system of charged and neutral particles, whose behaviour is dominated by collective effects mediated by the electromagnetic force. Here, “collective” designates phenomena determined by the whole ensemble of particles in the system. The long range behaviour of the electromagnetic force determine the collective aspects of the plasma physics. The collective aspects of plasma physics are due to the long-range behaviour of the electromagnetic force. Temperature and number density of the charge particles are the one of the basic plasma parameters. Standard Big Bang cosmology tells us that in the early Universe, temperature was so high that no atoms or molecules could exist. Hence the ionized gas was in the plasma state. It is currently believed that almost 99% of the matter in the Universe is made up of plasma, but such estimates are obviously hard to verify [109]. As temperature of the Universe decreases, some fraction of the ionized gas combine to form atoms and we could have partially ionized plasma. As Universe cools down even more, system can be considered as a neutral gas of weakly interacting atoms and molecules. A system with density comparable to the 10^{36} meter⁻³, is believed to be strongly coupled system. Whenever thermal energies are larger than interaction energies or of the order of Fermi energy, it is said that system of plasma is strongly coupled. These strongly coupled plasma system have more common property with a liquid than with a standard weakly coupled plasma [110]. Examples of strongly coupled plasmas, are the interior of giant planets like Jupiter, solid-laser ablation plasmas, plasmas

in high pressure arc discharges where the thermal and ionization energies are similar, and cold dusty plasmas. On the other hand, the systems found in space plasma physics, astrophysics, controlled nuclear fusion, and ionospheric physics are all weakly coupled.

A complete description of a non-relativistic plasma can be given by tracking individual particles, using the laws of classical mechanics and Maxwell's equations. Exact formulations of a system of very large number of charged particles (for example plasma systems) are exceedingly complicated, because tracking individual particles is almost impossible. Thus it is customary to formulate approximate description of such systems, that describe macroscopic properties of plasma. A hierarchy of approximations leads to the three leading plasma theories

- Kinetic theory
- Multifield theory
- Magnetohydrodynamics (MHD) descriptions

One can also divide different regime based on the time scale of interaction of individual particles. This is shown in the following figure (2) Here τ_{inter} , τ_{mft} and



τ_{relax} are time scale known as interaction time, mean free time and relaxation time of the plasma. Whenever time scale is less than interaction time of the individual particles, then mechanical description is enough. However when time scale is larger than τ_{inter} and less than τ_{mft} , then kinetic theory must be used to describe the dynamics of the plasma system. Hydrodynamics approximations

work for the times $\tau_{mft} \leq t \leq \tau_{relax}$ and for time t larger than τ_{relax} , system is in equilibrium state.

Fluid approximations

Depending on the number of free particles interacting through some coupling, there are number of different ways to describe the dynamics of the system. These approaches can be distinguished in terms of the dimensionless quantity: $\mathcal{R} = \lambda_{DB}/l$, where λ_{DB} is the de Broglie wave length associated to each particle and l is the typical inter-particle separation.

- $\mathcal{R} \gtrsim 1 \implies$ the waveforms of the various particles overlap and a quantum-mechanical description is necessary. In this case, system is described by the N-particle wave function evolving in time following the Schroedinger equation.
- $\mathcal{R} < 1 \implies$ then the wave-functions of the different particles are widely separated, the quantum interference is not important, and the individual wave packets evolve according to the Schrödinger equation in an isolated fashion, moving like classical particles.

Above result is known as Ehrenfest theorem. If the system extends over a length scale L so much larger than the typical inter particle separation l (of-course much larger than λ_{DB}), that the dynamics of the individual particle can not be followed, not even in statistical terms, the collective dynamics of the system can be approximated by a continuous description in terms of a so called “*fluid*”. The fluid description is valid only upto the limit of $l/L := K_n$ (this is known as *Knudsen number* and fluid description is valid only for $K_n \ll 1$). When a fluid description is possible, the dynamics can be described in terms quantities averaged over representative “elements”, which are large enough to contain a high number of particles, and small enough to guarantee homogeneity over the element. So the velocity of the particle in each element is same and are in thermal equilibrium. This description is given by classical plasma physics. However sometimes quantum mechanics is necessary in limited instances of the usual plasma

theory, as for the calculation of nuclear fusion cross sections [111]. A system with very high collision rate, has larger associated coupling constant. Due to this a system of plasma with sufficiently large collisional frequency behaves quantum mechanically. So it is relevant to ask, when is quantum and when is classical description of the plasma system are necessary? As mentioned above, extremely dense plasma behave like a quantum ideal gas, due to the exclusion principle. It is said that, if the typical length scale L of the system is comparable to the de Broglie wavelength $\lambda_{DB} = \hbar/mv_T$ (where v_T is thermal velocity and is defined as $v_T = (k_B T/m)^{1/2}$, k_B is Boltzmann constant, $\hbar = h/2\pi$ is reduced Planck's constant, m is the mass of the charge carriers), *i.e.* $\lambda_{DB} \sim L_0$, then the quantum effects should be taken into account. This can be the case, for instance, in charged particle systems like semiconductor quantum wells, thin metal films, and nanoscale electronic devices in general [111–113]. In the later part of this chapter, we will mainly focus on classical plasma. To discuss the system, we need to use kinetic theory, as the length scale of the system under consideration is equivalent to the typical inter particle distance. In the later part of this chapter we have addressed the relativistic kinetic theory for our system (chiral plasma), for which relativistic theory is needed.

2.1 Kinetic theory of a classical plasma system

Rather than to describe position and velocity of every individual particle in a plasma, kinetic theory divides the plasma into different classes of particles (or species) and describe the evolution of the probability distribution of each species of particles. The classification based on species are done using same mass or same charge. Most of the modern kinetic theory was first introduced by Landau [114]. A theory that self-consistently accounts for long range force in plasma is the Lenard-Balesescu-equation [115, 116]. An important physical effect that kinetic theory captures, but conventional fluid and MHD approximations do not, is Landau damping [117]. Landau damping is a process by which waves can either damp, or grow. Waves can damp or grow by different physical mechanisms in

fluid descriptions as well, which are also captured by kinetic theory, but Landau damping is fundamentally a kinetic process. In stable plasma all fluctuations damp, often by Landau damping, and scattering is dominated by conventional Coulomb interactions between individual particles. Landau's and Lenard-Balescu kinetic equations assume the plasma is stable. But plasmas are not always stable. In the presence of a free energy source, fluctuations may grow. The growth of these fluctuations in the system is known as instability. Theories that describe the scattering of particles from collective wave motion, typically assume that the instability amplitude is so large that conventional Coulomb interactions are negligible compared to the wave-particle interactions. However the theory of stable plasma assume that Coulomb interactions dominate. It is interesting to study dynamics of the plasma system in the intermediate regime, where plasma is a weakly interacting and collective fluctuation amplitude is sufficiently weak and the collective fluctuations may be, but not necessarily dominant scattering mechanism. In this case the non-linear wave-wave interactions can be regarded to be sub-dominant. These instabilities can modify the particle distribution functions and hence the amplitude of the fluctuations in the plasma.

For a very large number of particle, it is not realistic to solve the equations of motion for all of the particles. It is more useful to switch to a statistical approach in terms of a distribution function $f(t, \mathbf{x}, \mathbf{u})$. Here $\mathbf{x}$ and $\mathbf{u}$ are three dimensional position and velocity vector. In the six dimensional phase space $(\mathbf{x}, \mathbf{u})$, the quantity $f(t, \mathbf{x}, \mathbf{u})d\mathbf{x}^3d\mathbf{u}^3$ represents particle density in the range of $\mathbf{x} + d\mathbf{x}$ and $\mathbf{u} + d\mathbf{u}$. The dynamics of the system is then described by the evolution of the distribution function $f(t, \mathbf{x}, \mathbf{u})$. This distribution function can change in time either through simple advection in phase space in absence of collisions among the particles or in a more complex way when collisions are present and strongly influence the evolution.

2.1.1 Newtonian Kinetic theory

Evolution of $f(t, \mathbf{x}, \mathbf{u})$ can be described by the *Boltzmann Equation*, which represents the foundation of the kinetic theory. There are two important extensions of

this equation in which long range forces can play a very important role. In plasma physics, in presence of Coulomb forces, the Boltzmann equation is replaced by the *Vlasov-Maxwell equation*, and in presence of long-range gravitational forces, this equation becomes *Einstein-Vlasov-Maxwell equation* or *Einstein-Vlasov equation*. The Newtonian (i.e., non-relativistic) kinetic theory provide the simplest framework to study a system of interacting particles. Total number of particles in the system can be written as

$$N = \int_{-\infty}^{\infty} d^3x \int_{-\infty}^{\infty} d^3u f(t, \mathbf{x}, \mathbf{u}) = \int f(t, \mathbf{x}, \mathbf{u}) d^3x d^3u \quad (2.1)$$

It is necessary that the volume element d^3x , contain a large number of particles ensuring a small statistical variance and yet small enough with respect to the size of the system so that they can be considered as “points” in phase space and the continuum approach is justified. In a finite time element, dt , the particle coordinates change to

$$\hat{\mathbf{x}} = \mathbf{x} + \mathbf{u} dt \quad \text{and} \quad \hat{\mathbf{u}} = \mathbf{u} + \frac{\mathbf{F}}{m_g} dt \quad (2.2)$$

Here $\mathbf{F}$ is an external force and m_g is the mass of the particle. In the absence of collisions, we would have number densities at two different phase space points equal i.e.,

$$f(t, \mathbf{x}, \mathbf{u}) d^3x d^3u = f(t + dt, \mathbf{x} + \mathbf{u} dt, \mathbf{u} + \frac{\mathbf{F}}{m_g} dt) d^3x' d^3u'. \quad (2.3)$$

Here $d^3x' d^3u'$ represents the volume element at a later time. With collisions, the distribution can change over time dt as

$$f(t + dt, \mathbf{x} + \mathbf{u} dt, \mathbf{u} + \frac{\mathbf{F}}{m_g} dt) d^3x' d^3u' - f(t, \mathbf{x}, \mathbf{u}) d^3x d^3u = \left(\frac{\partial f(t, \mathbf{x}, \mathbf{u})}{\partial t} \right)_{coll} d^3x d^3u. \quad (2.4)$$

The Boltzmann equation for the collisional plasma can be obtained from above equation as

$$\frac{\partial f}{\partial t} + \mathbf{u} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{\mathbf{F}}{m_g} \cdot \frac{\partial f}{\partial \mathbf{u}} = \left(\frac{\partial f}{\partial t} \right)_{coll} \equiv C_B(f), \quad (2.5)$$

here $C_B(f)$ is the collision operator (or some times referred as collision integral) and depends on the nature of the interaction between particles. In the simplest case in which binary collision occurs with velocity of two colliding particles $\mathbf{u}_1$ and $\mathbf{u}_2$, in absence of the external forces, is defined as

$$C_B(f) = \int d^3u_2 \int d\Omega \frac{d\sigma}{d\Omega} |\mathbf{u}_1 - \mathbf{u}_2| (f'_2 f'_1 - f_2 f_1), \quad (2.6)$$

where $f_{1,2} = f(t, \mathbf{x}, \mathbf{u}_{1,2})$ and $f'_{1,2} = f(t, \mathbf{x}, \mathbf{u}'_{1,2})$ are the distribution functions before and after a collision at time t and position $\mathbf{x}$, while $d\sigma/d\Omega$ is the differential cross-section over solid angle $d\Omega$ of the short range interaction responsible for the collisions. Thus Boltzmann equation with the collision integral is a non-linear integro-differential equation. So finding out the collision integral is one of the important problem in the kinetic theory. From the distribution function obtained from solving equation (2.5), it is possible to define the averaged value of the quantity Ψ with respect to the distribution function f as

$$\langle \Psi \rangle = \frac{1}{n} \int \Psi f d^3u, \quad (2.7)$$

where n is the number density, i.e., the number of particles per unit volume, which is defined as

$$n = \int f d^3u \quad (2.8)$$

$$N = \int n d^3x \quad (2.9)$$

. Using above definition of the average quantity (2.7), one can write expression for the mean macroscopic velocity $\mathbf{v}$ as

$$\mathbf{v} = \langle \mathbf{u} \rangle = \frac{1}{n} \int \mathbf{u} f d^3u.$$

Here $\mathbf{v}$ is referred as fluid velocity.

2.1.2 Relativistic Boltzmann theory

The origin of relativistic kinetic theory dates back to 1911, when Juttner derived the relativistic Maxwell-Boltzmann equilibrium distribution for a relativistic fluid [118]. The relativistic description is valid in the case of high energy plasma

system. In the following section, we will discuss basic concepts of the relativistic kinetic theory in the flat space time. The four space time coordinate point and four-momentum of a particle of rest mass m , can be indicated by x^μ and $p^\mu = mcu^\mu = (p^0, p^i)$. The four-momentum satisfies $p_\mu p^\mu = m^2 c^2$. As in the Newtonian description, a distribution function f can be defined such that the quantity

$$f d^3x d^3p = f dx^1 dx^2 dx^3 dp^1 dp^2 dp^3 , \quad (2.10)$$

gives the number of particles in a given volume elements in the six dimensional space. Consider an observer in a frame $\mathcal{O}'$ comoving with the particle, and a second observer $\mathcal{O}$ moving with a speed $\mathbf{v}$ with respect to observer $\mathcal{O}'$. Lets assume that $\mathbf{v}$ is aligned to the x-axis. So the proper volume measured by observer in frame $\mathcal{O}'$ is given by

$$d^3x' = W d^3x \quad (2.11)$$

where W is the Lorentz contraction factor between the two frame. Similarly

$$\frac{d^3p'}{p'_0} = \frac{d^3p}{p_0} . \quad (2.12)$$

Above equations shows that the ratio d^3p'/p'_0 is a Lorentz invariant (here $p'_0 = p_0/W$). So using above relations, one can get $d^3x' d^3p' = d^3x d^3p$. So even d^3x is not Lorentz invariant, the product $d^3x d^3p$ is Lorentz invariant. The number of particles measured in two frames must not changed. So

$$f'(x', u') d^3x' d^3p' = f(x, u) d^3x d^3p . \quad (2.13)$$

Therefore distribution function f itself must also be Lorentz invariant, i.e., $f'(x', u') = f(x, u)$. One can write relativistic Boltzmann equation as

$$p^\mu \frac{\partial f}{\partial x^\mu} + m \frac{\partial(f F^\mu)}{\partial p^\mu} = \Pi(f) , \quad (2.14)$$

here F^μ is the four-force acting on a particle, that may or may not be dependent on the four momentum p^μ . However scalar quantity on the right side of the above equation is the relativistic generalization of the collision integral and can be written as [119]

$$\Pi(f) := \left(\frac{\partial f}{\partial t} \right)_{coll} = \int \frac{d^3p_2}{(p_2)^0} \int d\Omega \sigma(\Omega) \sqrt{(p_1)^\alpha (p_2)_\alpha - m^4 c^4} (f'_2 f'_1 - f_2 f_1) . \quad (2.15)$$

Here $\sigma(\Omega)$ represents differential cross section. The first moment of the distribution function defines the number density current

$$N^\mu := c \int p^\mu f \frac{d^3p}{p^0} . \quad (2.16)$$

Zeroth order of the above 4-vector current density is referred as number density

$$N^0 = c \int f d^3p = c n , \quad (2.17)$$

and the spatial (contravariant) components of the 4-vector current density can be given as

$$N^i = c \int p^i f \frac{d^3p}{p^0} = c^2 \int W m v^i f \frac{d^3p}{E} = \int v^i f d^3p . \quad (2.18)$$

The second moment of the distribution function is stress-momentum tensor.

$$T^{\mu\nu} = c \int p^\mu p^\nu f \frac{d^3p}{p^0} . \quad (2.19)$$

This quantity measures the flux of μ -momentum across a surface at $x^\nu = \text{const.}$, i.e., in the ν -direction. This is relativistic generalization of the classical energy momentum tensor.

2.2 Kinetic theory with Berry Curvature

The relativistic kinetic theory framework discussed above misses the effects of triangle anomalies, responsible for parity and CP violations [120, 121]. To discuss such kind of effects, a modified kinetic theory formalism from the underlying quantum field theory [122] has been developed by taking into account the Berry curvature [123]. In the following section, we shall discuss in brief about the Berry curvature and derivation of the Kinetic equations, modified with Berry curvature.

2.2.1 Basics of Berry curvature

We know that a quantal system can be described by its Hamiltonian $\hat{H}$. If the Hamiltonian is not a function of time then we can describe the system by its stationary state. If the system and hence $\hat{H}$, is slowly altered then from

the adiabatic theorem, at any instant the system will be in an eigenstate of the instantaneous Hamiltonian. If Hamiltonian is returned to its original form, the system will return to its original state, apart from a small phase factor may be large phase factor in some cases. This phase factor is observable by interference experiment.

Let us suppose a cubic box of size L , whose ground state is represented by 'ket' state $|n(L(0))\rangle$. Suppose box slowly expands with time, which implies length is a function of time *i.e.* $L(t)$. The adiabatic principle states that if the expansion is slow enough, the particle will be in the instantaneous ground state $|n(L(t_1))\rangle$ of the box of size $L(t_1)$. More generally, if particle Hamiltonian is given by $H(R(t))$, where R is some external coordinate which changes slowly in Hamiltonian. So Hamiltonian equation and its solution can be given as

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar} \int_0^t E_n(t') dt'\right) |n(t)\rangle \quad (2.20)$$

$$H(t) |n(t)\rangle = E_n(t) |n(t)\rangle. \quad (2.21)$$

It is to be noted here that if the Hamiltonian H is independent of the time, the above description is true, with the phase factor appropriate to energy E_n . But one can see that the instantaneous energy varies with time and gives the accumulated phase factor shift. To see what is missing in the above ansatz, let us modify it as follows:

$$|\psi(t)\rangle = c(t) \exp\left(-\frac{i}{\hbar} \int_0^t E_n(t') dt'\right) |n(t)\rangle, \quad (2.22)$$

here the extra factor $c(t)$ must equal to unity if the old ansatz is right. Now applying Schrodinger equation to this state

$$\left(i\hbar \frac{\partial}{\partial t} - H(t)\right) |\psi(t)\rangle = 0. \quad (2.23)$$

From this equation one can get

$$\dot{c}(t) = c(t) \langle n(t) | \frac{d}{dt} | n(t) \rangle. \quad (2.24)$$

Solution of the above equation for $c(t)$ gives

$$c(t) = c(0) \exp\left(-\int_0^t \langle n(t') | \frac{d}{dt'} | n(t') \rangle dt'\right) = c(0) e^{i\gamma}, \quad (2.25)$$

where γ is given as

$$\gamma = i \int_0^t \langle n(t') | \frac{d}{dt'} | n(t') \rangle dt' \quad (2.26)$$

This extra phase factor is known as *Berry phase* or the *geometric phase*. It is not recognize here that it has a measurable consequences. Since instantaneous kets are defined only up to a phase factor, and we can choose a new set and modify the extra phase. If we choose new phase factor $\xi(t)$, then state can be written as $|n'(t)\rangle = e^{i\gamma(t)} |n(t)\rangle$. So one can get

$$i \langle n'(t) | \frac{d}{dt} | n'(t) \rangle = i \langle n(t) | \frac{d}{dt} | n(t) \rangle - \frac{d\xi(t)}{dt}. \quad (2.27)$$

which suggests that, one can choose this new phase so that they cancel. Let us suppose a case where Hamiltonian return back to its starting value after time t_1 so that $H(t_1) = H(0)$. So in this case, it is obvious that one can not get rid of the extra phase factor.

$$i \oint \langle n'(t) | \frac{d}{dt} | n'(t) \rangle dt = i \oint \langle n(t) | \frac{d}{dt} | n(t) \rangle - (\xi(t_1) - \xi(0)). \quad (2.28)$$

So the choice of phase factor is quite arbitrary, but it must satisfies the requirement of single value in the region containing the closed loop. So it is necessary that choosing different set of basis do not altered the phase factor. The phase factor should depends only on the path in the parameter space, which explains the name “geodesic phase”. To see meaning of $\gamma \neq 0$, let us consider a system of some nucleus of some heavy object. Let $\mathbf{R}$ and $\mathbf{r}$ be the coordinate of some nucleus and electron respectively (electron is orbiting the nucleus). The electron moves under the influence of Coulomb potential created by nucleus. In this case the phase factor can be written as-

$$\begin{aligned} & \exp \left(- \int_0^t \langle n(t') | \frac{d}{dt'} | n(t') \rangle dt' \right) \\ &= \exp \left(\frac{i}{\hbar} \int_0^t \langle n(t') | \frac{d}{dt'} | n(t') \rangle dt' \right) \\ &= \exp \left(\frac{i}{\hbar} \int_0^t i \hbar \langle n(\mathbf{R}(t')) | \frac{d}{d\mathbf{R}} | n(\mathbf{R}(t')) \rangle \frac{d\mathbf{R}}{dt'} dt' \right) \\ &= \exp \left(\frac{i}{\hbar} \int_0^t \mathcal{A}^n(\mathbf{R}) \frac{d\mathbf{R}}{dt'} dt' \right), \end{aligned} \quad (2.29)$$

where

$$\mathcal{A}^n(\mathbf{R}) = i\hbar \langle n(\mathbf{R}) | \frac{d}{d\mathbf{R}} | n(\mathbf{R}) \rangle. \quad (2.30)$$

The quantity $\mathcal{A}^n(\mathbf{R})$, is known as Berry potential or Berry connection. Also normalisation condition satisfies i.e., $\langle n(\mathbf{R}(t)) | n(\mathbf{R}(t)) \rangle = 1$. So Berry phase can be written also in the following form

$$\gamma(t) = i \int_{\mathbf{R}_0}^{\mathbf{R}(t)} \mathcal{A}^n(\mathbf{R}) \cdot d\mathbf{R}. \quad (2.31)$$

Berry connections depends on the state, in which electronic degree of freedom is in. C in the integral represent integral over a path over which adiabatic process take place and $\mathbf{R}$ changes from $\mathbf{R}_0$ to $\mathbf{R}(t)$. As $\mathcal{A}^n(\mathbf{R})$ is purely imaginary, the Berry phase defined in equation (2.31) is real. For a closed path (for which $\mathbf{R}_0 = \mathbf{R}(t_1)$) of integration, above problem becomes much simpler. For this closed loop, using Stoke's theorem one can write

$$\gamma(C) = i \oint_C \langle n(\mathbf{R}) | \nabla_{\mathbf{R}} n(\mathbf{R}) \rangle d\mathbf{R} = - \int \int_C \Omega^n(\mathbf{R}) \cdot d\mathbf{S}, \quad (2.32)$$

here we changed notation of derivative from $d/d\mathbf{R}$ to $\nabla_{\mathbf{R}}$. Also Ω^n is known as Berry curvature. The right hand side of equation (2.32) is the flux of the Berry curvature on the surface S ; such flux remains meaningful even on a closed surface (e.g. a sphere or a torus). The key result is that such an integral is quantized [124].

Example: Let us consider a single chiral fermion described by Hamiltonian $H = \sigma \cdot \hat{\mathbf{p}}$. So energy eigen equation can be written as (from now on we will use natural units)

$$H u_{\mathbf{p}}(e) = \pm e |p| u_{\mathbf{p}},$$

where

$$H = \begin{bmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta \end{bmatrix}$$

and $\pm$ signs are for two component spinor. $+/-$ signs are for right/left handed fermions respectively. We can construct following form for the spinors

$$u_p(+) = \begin{bmatrix} e^{-i\phi}\cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{bmatrix}, \quad u_p(-) = \begin{bmatrix} -e^{-i\phi}\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{bmatrix},$$

with this definition one can get Berry correction and Berry curvature for chiral fermions as

$$\begin{aligned}\mathcal{A}(\mathbf{p}) &= -i u_p^\dagger(+) \nabla_p u_p(+) \\ &= -\hat{\mathbf{e}}_\phi \frac{1}{2|\mathbf{p}|} \frac{\cos(\theta/2)}{\sin(\theta/2)} \\ \Omega_p &= \nabla_p \times \mathcal{A}(\mathbf{p}) = \pm \frac{\mathbf{p}}{2|\mathbf{p}|^3},\end{aligned}$$

where

$$\nabla_p = \hat{\mathbf{e}}_p \frac{\partial}{\partial |\mathbf{p}|} + \hat{\mathbf{e}}_\theta \frac{1}{|\mathbf{p}|} \frac{\partial}{\partial \theta} + \hat{\mathbf{e}}_\phi \frac{1}{|\mathbf{p}| \sin \theta} \frac{\partial}{\partial \phi}.$$

One can see for $\mathbf{p} \neq 0$, the divergence is vanishing i.e., $\nabla \cdot \Omega_p = 0$ but is non-vanishing when calculating the total flux on a sphere $\int d^3p \nabla_p \cdot \Omega_p = 4\pi \Rightarrow \nabla_p \cdot \Omega_p = 2\pi \delta^{(3)}(\mathbf{p})$.

Anology to magnetic field:

Berry part	Magnetic field part
Berry curvature $\Omega(\mathbf{R})$	Magnetic field $\mathbf{B}(\mathbf{r})$
Berry connection $\mathcal{A}(\mathbf{R})$	Vector potential $\mathbf{A}(\mathbf{r})$
Geometric Phase	Ahanonrov-Bohm phase
Chern-Simons number	Dirac monopole

where Geometric phase, Ahanorov-Bohm phase, Chern-Simons number and Dirac monopole are defined as

$$\begin{aligned}\text{Geometric phase} &= \oint_C d\mathbf{R} \cdot \mathcal{A}(\mathbf{R}) = \int \int d\mathbf{S} \cdot \Omega \mathbf{R}, \\ \text{Ahanorov-Bohm phase} &= \oint_C d\mathbf{r} \cdot \mathcal{A}(\mathbf{r}) = \int \int d\mathbf{S} \cdot \mathbf{B}(\mathbf{R}), \\ \text{Chern-Simons number} &= \oint d\mathbf{S} \cdot \Omega(\mathbf{R}) = \text{integer} \\ \text{Dirac monopole} &= \oint d\mathbf{S} \cdot \mathbf{B}(\mathbf{r}) = \text{integer} \times \frac{h}{e}.\end{aligned}$$

Therefore, a non-zero Berry connection and curvature can be treated as the fictitious vector potential and magnetic field in the momentum space. Therefore, Berry curvature can affect the motion of chiral fermion in the momentum space and one can write the corresponding action as [125–128],

$$S(x, p) = \int dt [(p^i + eA^i(x))\dot{x}^i - \mathcal{A}^i(\mathbf{p})\dot{\mathbf{p}}^i - \epsilon_{\mathbf{p}}(\mathbf{x}) - \mathbf{A}^0(\mathbf{x})], \quad (2.33)$$

here $\varepsilon_p(p)$ is energy of the particles and A_0 and A^i are the scalar and magnetic potentials. This equation can also be written as

$$S(\xi) = \int dt [\Sigma_a(\xi) \dot{\xi}^a - H(\xi)], \quad (2.34)$$

Here ξ^α represents the phase space coordinates $\mathbf{r}$ and $\mathbf{k}$ collectively. Also $\Sigma_a(\xi) = (p^i + eA_i(x), -\mathbf{Q}^i(\mathbf{p}))$, $\xi^a = (x^i, p^i)$ and $H(\xi) = \varepsilon_p(p) + A^0(x)$. $\mathcal{A}^i(\mathbf{p}) = -i \mathbf{u}_{\mathbf{p}}^\dagger \nabla_{\mathbf{p}} \mathbf{u}_{\mathbf{p}}$ is the Berry connection for chiral fermion. Now the equations of motion of the action read as,

$$\Sigma_{ab} \dot{\xi}^b = -\frac{\partial H(\xi)}{\partial \xi^a}, \quad (2.35)$$

Hamilton's equation of motion is,

$$\dot{\xi}^a = -\{\xi^a, H(\xi)\} = -\{\xi^a, \xi^b\} \frac{\partial H(\xi)}{\partial \xi^b}. \quad (2.36)$$

One can find easily following relation $\{\xi^a, \xi^b\} = (\Sigma^{-1})^{ab}$. Using the above equation, we can write down the explicit form of Poisson brackets for variables x^i, p^i with berry curvature as follows,

$$\{x^i, x^j\} = \frac{\epsilon_{ijk} \Omega_k}{1 + e\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}}, \quad (2.37)$$

$$\{x^i, p^j\} = -\frac{\delta_{ij} + e\Omega_i B_j}{1 + e\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}}, \quad (2.38)$$

$$\{p^i, p^j\} = -\frac{e\epsilon_{ijk} B_k}{1 + e\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}}, \quad (2.39)$$

where, $B^i = \epsilon^{ijk} \frac{\partial A_k}{\partial x^j}$ and $\mathbf{\Omega}_{\mathbf{p}} = \nabla_{\mathbf{p}} \times \mathcal{A}_{\mathbf{p}}$. As a result of the modification of the Poisson Bracket, the invariant phase space gets modified [126, 127],

$$d\Gamma = \sqrt{\det \Sigma_{ab}} d\xi = (1 + e\mathbf{B} \cdot \mathbf{\Omega}) \frac{dp dx}{(2\pi)^3}. \quad (2.40)$$

Equivalent Liouville's theorem will give,

$$\dot{f}_{\mathbf{p}} - (\Sigma)_{ab}^{-1} \frac{\partial H(\xi)}{\partial \xi^b} \frac{\partial f_{\mathbf{p}}}{\partial \xi^a} = 0,$$

where, f_p is the distribution function of chiral fermion. Taking $H = \varepsilon_p + A_0$, one can get the following equation,

$$\dot{f}_{\mathbf{p}} + \dot{\mathbf{x}} \cdot \frac{\partial f_{\mathbf{p}}}{\partial \mathbf{x}} + \dot{\mathbf{p}} \cdot \frac{\partial f_{\mathbf{p}}}{\partial \mathbf{p}} = 0, \quad (2.41)$$

where,

$$\begin{aligned}\dot{\mathbf{x}} &= \frac{1}{1 + e\mathbf{B} \cdot \boldsymbol{\Omega}_{\mathbf{p}}} \left(\tilde{\mathbf{v}} + e\tilde{\mathbf{E}} \times \boldsymbol{\Omega}_{\mathbf{p}} + e(\tilde{\mathbf{v}} \cdot \boldsymbol{\Omega}_{\mathbf{p}})\mathbf{B} \right), \\ \dot{\mathbf{p}} &= \frac{1}{1 + e\mathbf{B} \cdot \boldsymbol{\Omega}_{\mathbf{p}}} \left[\left(e\tilde{\mathbf{E}} + e\tilde{\mathbf{v}} \times \mathbf{B} + e^2(\tilde{\mathbf{E}} \cdot \mathbf{B})\pm\boldsymbol{\Omega}_{\mathbf{p}} \right) \right].\end{aligned}\quad (2.42)$$

Note that here, $\tilde{\mathbf{v}} = \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}}$, $e\tilde{\mathbf{E}} = e\mathbf{E} - \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{x}}$, $\epsilon_{\mathbf{p}} = p(1 - e\mathbf{B} \cdot \boldsymbol{\Omega}_{\mathbf{p}})$ and $\boldsymbol{\Omega}_{\mathbf{p}} = \pm \mathbf{p}/2p^3$. Above $\pm$ sign respectively corresponds to the right and left handed particles. If $\boldsymbol{\Omega}_{\mathbf{p}} = 0$, the above equation reduces to Vlasov equation. It is easy to check that Eq.(2.41) gives the anomaly equation as follows,

$$\partial_t n + \nabla \cdot \mathbf{j} = e^2 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left(\boldsymbol{\Omega}_{\mathbf{p}} \cdot \frac{\partial \mathbf{f}_{\mathbf{p}}}{\partial \mathbf{p}} \right) \mathbf{E} \cdot \mathbf{B}, \quad (2.43)$$

where n and $\mathbf{j}$ are defined as

$$n = \int \frac{d^3 p}{(2\pi)^3} (1 + e\mathbf{B} \cdot \boldsymbol{\Omega}_{\mathbf{p}}) f_{\mathbf{p}}, \quad (2.44)$$

and

$$\mathbf{j} = -e \int \frac{d^3 p}{(2\pi)^3} \left[\epsilon_{\mathbf{p}} \frac{\partial f_{\mathbf{p}}}{\partial p} + e \left(\boldsymbol{\Omega}_{\mathbf{p}} \cdot \frac{\partial \mathbf{f}_{\mathbf{p}}}{\partial \mathbf{p}} \right) \epsilon_{\mathbf{p}} \mathbf{B} + \epsilon_{\mathbf{p}} \boldsymbol{\Omega}_{\mathbf{p}} \times \frac{\partial f_{\mathbf{p}}}{\partial x} \right] + \mathbf{E} \times \sigma. \quad (2.45)$$

here σ is given by,

$$\sigma = \int \frac{d^3 p}{(2\pi)^3} \boldsymbol{\Omega}_{\mathbf{p}} f_{\mathbf{p}}. \quad (2.46)$$

The integration on the right hand side of Eq.(2.43) is not simple because at $p = 0$ there is singularity. At this point motion of particles can be described by quantum mechanics [129]. To integrate this integral in the regime $|p| < R$, we have exclude the the point surrounding $p = 0$ and integrated only in the classical regime $|p| > R$. In this regime, particles can not be destroyed, they can only enter or exit through the surface at R . Therefore, for the region $|p| > R$ we can write Eq.(2.43),

$$\partial_t n + \nabla \cdot \mathbf{j} = e^2 \int_{S^2(\mathbf{R})} \frac{d\mathbf{S}}{(2\pi)^3} \cdot \boldsymbol{\Omega}_{\mathbf{p}} (\mathbf{E} \cdot \mathbf{B}) f_{\mathbf{p}}, \quad (2.47)$$

where, dS is the surface element of the sphere. One can see that in the limits of $R \rightarrow 0$, which implies that $p = 0$ and carry out surface integral we get,

$$\partial_t n + \nabla \cdot \mathbf{j} = \frac{e^2}{4\pi^2} \mathbf{f}_{\mathbf{p}=0} \mathbf{E} \cdot \mathbf{B}. \quad (2.48)$$

Thus total flux remains finite even at $p = 0$ due to anomaly ($\mathbf{E} \cdot \mathbf{B}$ term) [120,130]. The presence of $f_{\mathbf{p}=0}$ in the above equation shows that there must be some thermal correction. However, it is important to note that at finite temperature one must also consider anti-fermions. Therefore, if we consider both right-handed and left-handed particles/antiparticles and write the same sort of transport equation as above, we can arrive to the following equation,

$$\partial_\mu J^\mu = \frac{e^2}{4\pi^2} (f_{\mathbf{p}=0}^R + f_{\mathbf{p}=0}^{\bar{R}} + f_{\mathbf{p}=0}^L + f_{\mathbf{p}=0}^{\bar{L}}) (\mathbf{E} \cdot \mathbf{B}), \quad (2.49)$$

which can be simplified to the following form,

$$\partial_\mu J^\mu = \frac{e^2}{4\pi^2} \mathbf{E} \cdot \mathbf{B}. \quad (2.50)$$

Thus, chiral anomaly does not receive any thermal corrections, which is well known result in the literatures of quantum field theory [131,132]. One can also define modified energy density and momentum density of the particle.

$$\epsilon = \int \frac{d^3p}{(2\pi)^3} (1 + \mathbf{B} \cdot \boldsymbol{\Omega}_{\mathbf{p}}) \epsilon_{\mathbf{p}} f_{\mathbf{p}}, \quad \pi^i = \int \frac{d^3p}{(2\pi)^3} (1 + \mathbf{B} \cdot \boldsymbol{\Omega}_{\mathbf{p}}) p^i f_{\mathbf{p}}, \quad (2.51)$$

by multiplying Eq.(2.41) by $\epsilon_{\mathbf{p}} \sqrt{\Sigma_{ab}}$ and $p^i \sqrt{\Sigma_{ab}}$ and performing the integral over momentum $\mathbf{p}$, we can get energy and momentum conservation laws as follows,

$$\partial_\mu T^{0\mu} = E^i j^i, \quad \partial_\mu T^{i\mu} = n E^i + \epsilon^{ijk} j^j B^k, \quad (2.52)$$

where,

$$\begin{aligned} T^{0i} &= - \int \frac{d^3p}{(2\pi)^3} \left[(\delta^{ij} + B^i \Omega^j) \frac{\epsilon_{\mathbf{p}}^2}{2} \frac{\partial f_{\mathbf{p}}}{\partial p^j} + \epsilon^{ijk} \frac{\epsilon_{\mathbf{p}}^2}{2} \Omega^j \frac{\partial f_{\mathbf{p}}}{\partial x^k} \right], \\ T^{ij} &= - \int \frac{d^3p}{(2\pi)^3} p^i \left[\epsilon_{\mathbf{p}} (\delta^{jk} + B^j \Omega^k) \frac{\partial f_{\mathbf{p}}}{\partial p^k} + \epsilon^{jkl} \Omega^k \left(E^l f_{\mathbf{p}} + \epsilon_{\mathbf{p}} \frac{\partial f_{\mathbf{p}}}{\partial x^l} \right) \right] \\ &\quad - \delta^{ij} \epsilon. \end{aligned} \quad (2.53)$$

In the above equations, expression of $\epsilon_{\mathbf{p}}$ is still not known. It can be determined using the constraint due to Lorentz invariance, which demands that the energy flux is equal to the momentum density i.e.

$$T^{0i} = \pi^i. \quad (2.54)$$

According to Lorentz invariance above equation is valid at any order of perturbation. Using the expression of for T^{0i} and π^i from Eqs.(2.53) and (2.51), writing

down the final equation to the first order in perturbations in the quantities $n_{\mathbf{p}}$ and $\epsilon_{\mathbf{p}}$, from Eq.(2.54) one can obtain,

$$\epsilon_{\mathbf{p}}^0 = p - \frac{\mathbf{B} \cdot \hat{\mathbf{p}}}{2p}. \quad (2.55)$$

This is the dispersion relation of chiral fermions near Fermi surface in the presence of magnetic field [122, 133]. We will discuss application of the modified kinetic theory in detail in next chapter.

2.2.2 Relativistic description with anomaly

The relativistic Boltzmann equation in the presence of external background magnetic field can be written as [134]

$$p^\mu \left(\frac{\partial}{\partial x^\mu} - e F_{\mu\nu} \frac{\partial}{\partial p_\nu} \right) f_p = \Pi[f_p] \quad (2.56)$$

where the charge of the particle is $e = \pm 1$. Here p denotes the on shell momentum satisfying $p^2 = m^2$ (here m is mass of the particles). Also $\Pi[f_p]$ contains collision terms. In the presence of external electromagnetic field conservation equation becomes

$$\partial_\mu j^\mu = -C E^\mu B_\mu \quad (2.57)$$

Here j^μ is the current and $E^\mu = u_\nu F^{\mu\nu}$ and $B^\mu = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} u^\nu F^{\alpha\beta}$ are electric and magnetic field vectors, respectively, where u_μ is the fluid velocity. In this case, one can verify that the equilibrium solution of the distribution function,

$$f_p(x) = \frac{1}{\exp[\beta u_\mu (p^\mu - e F^{\mu\nu} x_\nu) - e \beta \mu_0] \pm 1} \quad (2.58)$$

where $\beta = 1/T$ (T is the local temperature), μ_0 is the local chemical potential without electromagnetic field. The plus/minus signs are for Bose or Fermi distributions. One can write above distribution function as follows

$$f_p(x) = f_{p_0}(x) + \delta f_p(x) \quad (2.59)$$

where the two parts are given by-

$$f_{p_0}(x) = \frac{1}{e^{(\mathbf{u} \cdot \mathbf{p} - e \mu_0)/T} \pm 1}$$

$$\delta f_p(x) = -f_{p_0}(x)[1 \pm f_{p_0}(x)]\chi_p(x)$$

here $\chi_p(x) = \xi_p(\mu_0, T)(\mathbf{p} \cdot \boldsymbol{\omega}) + \xi_B(\mu_0, T)(\mathbf{p} \cdot \mathbf{B})$. Here ξ_p are coefficients and can be obtained from thermodynamic constraints [108].

Chapter 3

Generation of magnetic fields

In this chapter, our aim is to discuss generation of the magnetic field above EW scale in the presence of Abelian anomaly, using kinetic theory modified with Berry curvature. It has been shown in previous chapter (chapter-(2)) that the Abelian anomaly can be incorporated in the kinetic theory by including Berry phase effect. It is known that in the presence of chiral asymmetry, there can be instability in the chiral plasma, which leads to the creation of turbulence in the plasma and because of this magnetic fields can be generated in the early Universe. Typical strength of the generated magnetic field in a regime where collision dominates is 10^{27} G at a length scale $10^5/T$. Furthermore, the instability can also generate a magnetic field in the collisionless regime, of the order 10^{31} G at a typical length scale $10/T$. We show that the estimated values of the magnetic field are consistent with the bounds obtained from current observations.

This chapter is organized as follows. In section-(3.1), we give a brief introduction to this chapter and in section-(3.2), we apply modified kinetic theory to study the primordial magnetic field generation in presence of the chiral asymmetry. We will also calculate the vorticity generation within the plasma due to the chiral imbalance. Section-(3.3) is dedicated for result and discussion.

3.1 Introduction

The dominance of matter over antimatter in the observable Universe is one of the vexing issues in the standard model of cosmology. This is due to the fact that inflation, a phase of accelerated expansion of the Universe in early time, would wash out any initial asymmetry if present. However, there are certain conditions namely, non-conservation of baryon number (B), violation of Charge (C) and Parity (P) symmetries, and occurrence of out of equilibrium processes in the early Universe would lead to the generation of baryon asymmetry in the universe [135]. In the simplest GUT models, the generated asymmetry in the baryon number suggests same amount of asymmetry in the lepton number, so that there is no net (B-L) asymmetry [135–137]. If the Electroweak (EW) phase transition is not of the first order, then any primordial (B+L) asymmetry generated at the GUT scale, will be washed out [93]. It may be possible that the Baryon Asymmetry of the Universe (BAU) could actually be reproduced from a lepton flavor asymmetries if one or more generations of leptons were out of equilibrium [70, 89, 138]. Authors of the Ref. [138] showed that the BAU could be encoded in a primordial symmetry for the right-handed electron field e_R (and its antiparticles). This can be understood as follows. Out of the 45 chiral states composing the three standard model fermion generations, only right-handed electrons are the most weakly coupled to the others. At high temperature electron chiral number densities, in the absence of the reactions that flip the chirality of the interacting particles, remain conserved. At such a high temperature the chirality flipping rates are out of equilibrium, which means that they are smaller than the expansion rate of the Universe (*i.e.* $\Gamma_R \sim T < H \sim T^2$) [42]. However at lower temperature, the processes that flip the chirality of the electrons need to be considered [50]. At temperature $T \geq 80$ TeV, right electron number density remains in the thermal equilibrium via its coupling with the hypercharge gauge bosons. It means that any primordially generated lepton number that occurred as e_R would be decoupled from non-perturbative electroweak effects until $T = 80$ TeV is reached. The only interaction by which it connects to the other standard model states is its Yukawa coupling to the Higgs ($H \rightarrow e_R \bar{e}_L$). This interaction with the Higgs

particle can change its chirality. However a very small electron mass tends to very small Yukawa coupling $h_e \simeq \sqrt{2}m_e/v \simeq 2.1 \times 10^{-6}$ [42]. At temperature below 80 TeV, left and right electron asymmetry evolve parallel through the corresponding Abelian anomalies in the standard model in the presence of a seed hypermagnetic field. In this introductory part of the chapter, we have considered only the right handed particles to understand generation of magnetic field qualitatively (assumption is that there is an excess of right handed particles). However for completeness, one needs to consider right as well as left handed particles (also their antiparticles). From section- (3.2) and onward of this chapter, we have considered left and right handed particles both (and their antiparticles).

Current density for the right handed electron satisfies following anomaly equation [42]

$$\partial_\mu j_R^\mu = -\frac{g_1^2 y_R^2}{64\pi^2} Y^{\mu\nu} \tilde{Y}_{\mu\nu}, \quad (3.1)$$

and therefore it is coupled to the hypermagnetic field. Here, $Y^{\mu\nu} = \partial^\mu Y^\nu - \partial^\nu Y^\mu$ is the field tensor associated with the hyper-charge gauge field Y^μ and $\tilde{Y}^{\mu\nu} = 1/2\epsilon^{\mu\nu\rho\lambda}Y_{\rho\lambda}$. The coefficient, g_1 is a associated gauge coupling, and $y_R = -2$ represents the hypercharge of the right-handed electrons. The conservation of the axial current is spoiled due to quantum effects. It has been shown that, the term after equality sign on the right hand side of the Eq. (3.1) are related with the Chern-Simon (CS) number N_{CS} (some times referred as helicity). The anomaly Eq. (3.1) relates variation of the number density of the right handed electrons with the Chern-Simon number [139]. CS number for hypercharge gauge fields, is defined in Eq. (1.29). The magnetic fields generated in the presence of chiral asymmetry can be seen through following equation

$$\partial_t \left(n_R + \frac{\alpha_1}{\pi} \mathcal{H}_M \right) = 0, \quad (3.2)$$

which can be obtained easily by using Eq. (3.1). The finite helicity state can be created even if the initial state has no helicity ($\mathcal{H}_M = 0$) but $n_R \neq 0$. Thus, the magnetogenesis by net non-zero chiral-charges may not require any preexisting seed field. It was shown that the chiral imbalance in the early Universe can give rise to a magnetic field of the order of $B \sim 10^{22}$ G at temperature $T \sim 100$ GeV with a typical inhomogeneity scale $\sim 10^6/T$ [42]. If these hyper-magnetic fields

survives at the time of EWPT, they will give ordinary magnetic fields due to the electroweak mixing $A_\mu = \cos \theta_w Y_\mu$.

Recently there has been an interesting development in incorporating the parity-violating effects into a kinetic theory formalism [128, 129, 133, 140]. In this approach the kinetic (Vlasov) equation is modified with the Berry curvature term, which takes into account chirality of the particles. Incorporation of the parity odd physics in kinetic theory leads to a redefinition of the Poisson brackets which includes contribution from the Berry connection. The modified kinetic equation can also be derived from the Dirac Hamiltonian by performing semi-classical Foldy-Wouthuysen diagonalization [141, 142]. The confidence that the new kinetic equation captures the proper physics stems from the fact that the equation is consistent with the anomaly Eq. (3.1) and it also reproduces some of the known results obtained using the quantum field theory with the parity odd interaction [96]. Keeping the above discussion in mind, we believe that it would be highly useful to consider the problem of generation of the primordial magnetic field in presence of the Abelian anomaly by using the Berry curvature modified kinetic theory. In this chapter we incorporate the effect of collisions in the modified kinetic theory and derive expressions for the electric and magnetic resistivities. The new kinetic framework also allows us to calculate the generation of the primordial magnetic field and vorticity. We show that our estimated value of the peak magnetic field actually falls within the constraints obtained from current observations.

3.2 Generation of magnetic fields above EW scale

In order to know about the generation of magnetic field, we intend to solve the coupled system of the modified kinetic and the Maxwell's equations in an expanding background. In this chapter, we have ignored the fluctuations in the metric due to the matter perturbation. We write the underlying equations in a general covariant form, developed in Refs. [143–146]. This allows one to write the system of kinetic and Maxwell's equations in the expanding background in the

form that look similar to their flat space-time form. In this formalism the well developed intuition and techniques of the flat space-time plasma physics can be exploited to study the problem at hand. This can be accomplished by choosing a particular set of fiducial observers (FIDO's) [143] at each point of space-time with respect to which all the physical quantities including hyper-electric and magnetic fields are measured. The line element in an expanding background can be written using the Friedmann-Lemaître-Robertson-Walker metric as

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \quad (3.3)$$

where x , y and z represent comoving coordinates and t denotes the proper time seen by observers at the fixed x , y & z . The quantity $a(t)$ is scale factor. One can introduce the conformal time τ using the definition $\tau = \int dt/a(t)$ to write this metric as:

$$ds^2 = a^2(\tau)(-d\tau^2 + dx^2 + dy^2 + dz^2) \quad (3.4)$$

The physical (hyper)-electric field $\mathcal{E}$, (hyper)-magnetic field $\mathcal{B}$ and the current density $\mathcal{J}$ are related to the corresponding fiducial quantities by transformations: $\mathbf{E}^* = a^2\mathcal{E}$, $\mathbf{B}^* = a^2\mathcal{B}$, $\mathbf{J}^* = a^3\mathcal{J}$. One can now write the Maxwell's equations in the fiducial frame as [76, 143]:

$$\frac{\partial \mathbf{B}^*}{\partial \tau} + \nabla \times \mathbf{E}^* = 0 \quad (3.5)$$

$$\nabla \cdot \mathbf{E}^* = 4\pi\rho_e^* \quad (3.6)$$

$$\nabla \cdot \mathbf{B}^* = 0 \quad (3.7)$$

$$\nabla \times \mathbf{B}^* = 4\pi\mathbf{J}^* + \frac{\partial \mathbf{E}^*}{\partial \tau} \quad (3.8)$$

where $\mathbf{B}^*$, $\mathbf{E}^*$, ρ_e and $\mathbf{J}^*$ are respectively magnetic field, electric field, charge density and current density seen by the fiducial observer. To make our notations simpler, we will not use star on the variables in the discussions below. The charge ρ_e and the current $\mathbf{J}$ in the Maxwell's eqs. (3.5-3.8) can be calculated using the Berry curvature modified kinetic equation which is also consistent with

the quantum anomaly equation. The modified kinetic equation is given by

$$\begin{aligned} \frac{\partial f}{\partial \tau} + \frac{1}{1 + e\mathbf{\Omega}_p \cdot \mathbf{B}} \left[(e\tilde{\mathbf{E}} + e\tilde{\mathbf{v}} \times \mathbf{B} + e^2(\tilde{\mathbf{E}} \cdot \mathbf{B})\mathbf{\Omega}_p) \cdot \frac{\partial f}{\partial \mathbf{p}} \right. \\ \left. + (\tilde{\mathbf{v}} + e\tilde{\mathbf{E}} \times \mathbf{\Omega}_p + e(\tilde{\mathbf{v}} \cdot \mathbf{\Omega}_p)\mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{r}} \right] = \left(\frac{\partial f}{\partial \tau} \right)_{coll} \end{aligned} \quad (3.9)$$

Here $\tilde{\mathbf{v}} = \partial \epsilon_p / \partial \mathbf{p} = \mathbf{v}$ and $e\tilde{\mathbf{E}} = e\mathbf{E} - \partial \epsilon_p / \partial \mathbf{r}$. $\mathbf{\Omega}_p = \pm \mathbf{p} / (2p^3)$ is Berry curvature. ϵ_p is defined as $\epsilon_p = p(1 - e\mathbf{B} \cdot \mathbf{\Omega}_p)$ with $p = |\mathbf{p}|$. The positive sign corresponds to right-handed fermions where as the negative sign is for left-handed ones. In the absence of Berry correction i.e. $\mathbf{\Omega}_p = 0$, Eq. (3.9) reduces to the Vlasov equation when the collision term on the right hand side is absent.

As we have already stated that we are interested in temperature regime $T > T_f (\sim 80 \text{ TeV}) \gg T_{EW} (\sim 100 \text{ GeV})$, where electrons are ultra-relativistic and the only process that can change electron chirality is its Yukawa interaction with Higgs boson. However, at this temperature this interaction is not strong enough to alter the electron chirality. It is important to note that for temperature smaller than T_R , electron mass may play a major role in left-right asymmetry. In Ref. [147], authors have shown that the mass of the electron plays an important role in determining the magnetic properties of the proto-neutron star by suppressing the chiral charge density during the core collapse of supernova at temperature of the order of MeV. However for the present case we ignore the electron mass by considering only $T > T_f$ regime.

One can use Eq. (2.45 given in chapter-(2)), for writing the expression of the current for each species. Total current can be written as the sum of current contribution from each species i.e., $\mathbf{j} = \sum_s \mathbf{j}_s$. Here index "s" denotes current contribution from different species of the fermion e.g. right and left handed particles and their antiparticles. $\mathbf{j}_s$ is defined as:

$$\mathbf{j}_s = -e^s \int \frac{d^3 p}{(2\pi)^3} \left[\epsilon_p^s \frac{\partial f_s}{\partial \mathbf{p}} + e^s (\mathbf{\Omega}_p^s \cdot \frac{\partial f_s}{\partial \mathbf{p}}) \epsilon_p^s \mathbf{B} + \epsilon_p^s \mathbf{\Omega}_p^s \times \frac{\partial f_s}{\partial \mathbf{r}} \right] \quad (3.10)$$

where $\epsilon_p^s = p(1 - e^s \mathbf{B} \cdot \mathbf{\Omega}_p^s)$ with $p = |\mathbf{p}|$. Depending on the species, charge e , energy of the particles ϵ_p and Berry curvature $\mathbf{\Omega}_p$, form of the distribution function f changes.

For the right-handed particle, $s = R$, hyper-charge is e while for right-handed anti-particle $s = \bar{R}$ and charge is $-e$ and similarly for other particles. The first

term in Eq. (3.10) is the usual current equivalent to the kinetic theory and remaining second and third terms are the current contribution due to the Berry correction. If we follow the power counting scheme as in [133] i.e. $\mathbf{Y}_\mu = \mathcal{O}(\epsilon)$, $\partial_r = \mathcal{O}(\delta)$ and considering only terms of the order of $\mathcal{O}(\epsilon\delta)$ in Eq. (3.9), we will have:

$$\left(\frac{\partial}{\partial\tau} + \mathbf{v} \cdot \frac{\partial}{\partial\mathbf{r}}\right)f_s + \left(e^s \mathbf{E} + e^s(\mathbf{v} \times \mathbf{B}) - \frac{\partial\epsilon_p^s}{\partial\mathbf{r}}\right) \cdot \frac{\partial f_s}{\partial\mathbf{p}} = \left(\frac{\partial f_s}{\partial\tau}\right)_{\text{coll.}} \quad (3.11)$$

Above equation is similar to the Boltzmann equation with one additional term $\frac{\partial\epsilon_p^s}{\partial\mathbf{r}} \cdot \frac{\partial f_s}{\partial\mathbf{p}}$. This term comes only because of inclusion of Berry curvature.

3.2.1 Current and polarization tensor for chiral plasma

Here we assume that the plasma of the standard model particles is in a state of “thermal-equilibrium” at temperature $T > T_R$, where masses of the plasma particles can be ignored. We also assume that there exist a left-right asymmetry and there is no large-scale electromagnetic field. Thus the equilibrium plasma is considered to be in a homogeneous and isotropic state which is similar to the assumptions made in Ref. [42, 148]. For a homogeneous and isotropic conducting plasma in thermal equilibrium, distribution function for different species are:

$$f_{0a}(p) = \frac{1}{\exp(\frac{\epsilon_p^0 - \mu_a}{T}) + 1} \quad (3.12)$$

If δf_R and $\delta f_{\bar{R}}$ are fluctuations in the distribution functions of the right electron and right-antiparticles around their equilibrium distribution, we can write perturbed distribution functions as

$$f_R(\mathbf{r}, \mathbf{p}, \tau) = f_{0R}(p) + \delta f_R(\mathbf{r}, \mathbf{p}, \tau) \quad (3.13)$$

$$f_{\bar{R}}(\mathbf{r}, \mathbf{p}, \tau) = f_{0\bar{R}}(p) + \delta f_{\bar{R}}(\mathbf{r}, \mathbf{p}, \tau) \quad (3.14)$$

Subtracting equation for $a = \bar{R}$ from $a = R$ and using Eq. (3.11) one can write:

$$\begin{aligned} \left(\frac{\partial}{\partial\tau} + \mathbf{v} \cdot \frac{\partial}{\partial\mathbf{r}}\right)f(\mathbf{r}, \mathbf{p}, \tau) + e p \frac{\partial(\mathbf{B} \cdot \boldsymbol{\Omega}_p)}{\partial\mathbf{r}} \cdot \frac{\partial f_0}{\partial\mathbf{p}} + (\mathbf{E} \cdot \mathbf{v}) \frac{df_0}{dp} \\ = \left(\frac{\partial f(\mathbf{r}, \mathbf{p}, \tau)}{\partial\tau}\right)_{\text{coll.}} \end{aligned} \quad (3.15)$$

where, $f(\mathbf{r}, \mathbf{p}, \tau) = (f_R - f_{\bar{R}})$ and $f_0 = f_{0R} + f_{0\bar{R}}$. Here we have used $\frac{\partial f_0}{\partial \mathbf{p}} = \mathbf{v} \frac{df_0}{dp}$. This equation relates the fluctuations of the distribution functions of the charged particles with the induced gauge field fluctuations. The gauge field fluctuations can be seen from the Maxwell's electromagnetic eqs. (3.5-3.8). Under the relaxation time approximation, the collision term can be written as $(\partial f_a / \partial \tau)_{coll.} \approx -\nu_c(f_a - f_{0a})$ (one can also look at some studies in chiral kinetic theory with collision in Ref. [149, 150]). As above differential Eq. (3.15), is linear in the distribution function, one can use Fourier transform to reduce this differential equation into a algebraic form. For this, we have taken Fourier transform of the perturbed quantities namely $\mathbf{E}$, $\mathbf{B}$ and $f(\mathbf{r}, \mathbf{p}, \tau)$ by considering the spatio-temporal variation of these quantities as $\exp[-i(\omega\tau - \mathbf{k} \cdot \mathbf{r})]$. Then using Eq. (3.15) one can get

$$f_{\mathbf{k}, \omega} = \frac{-e[(\mathbf{v} \cdot \mathbf{E}_{\mathbf{k}}) + \frac{i}{2p}(\mathbf{v} \cdot \mathbf{B}_{\mathbf{k}})(\mathbf{k} \cdot \mathbf{v})] \frac{df_0}{dp}}{i(\mathbf{k} \cdot \mathbf{v} - \omega - i\nu_c)}, \quad (3.16)$$

here subscript k with the variables represent their Fourier transformed values. One can calculate, current for the right handed particle and right handed antiparticles in terms of the mode functions using Eq.(3.10) as:

$$\begin{aligned} \mathbf{J}_{\mathbf{k}R} = & e \int \frac{d^3p}{(2\pi)^3} \left[\left\{ \mathbf{v} - \frac{i}{2p}(\mathbf{v} \times \mathbf{k}) \right\} f_{\mathbf{k}\omega R} \right. \\ & \left. - \frac{e}{2p^2} \{ \mathbf{B}_{\mathbf{k}} - \mathbf{v}(\mathbf{v} \cdot \mathbf{B}_{\mathbf{k}}) \} f_0 + \frac{e}{2p} \mathbf{B}_{\mathbf{k}} \frac{df_0}{dp} \right] \end{aligned} \quad (3.17)$$

In the similar way we can get currents due to left handed particles and left handed antiparticles. So the total current can be obtained by adding the contributions from both left and right handed particles and antiparticles by putting perturbations $f_{\mathbf{k}\omega}$ for all species in Eq. (3.17) and adding as:

$$\begin{aligned} \mathbf{J}_{\mathbf{k}} = & -m_D^2 \int \frac{d\Omega}{4\pi} \frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{E}_{\mathbf{k}})}{i(\mathbf{k} \cdot \mathbf{v} - \omega - i\nu_c)} \\ & - \frac{h_D^2}{2} \int \frac{d\Omega}{4\pi} \{ \mathbf{B}_{\mathbf{k}} - \mathbf{v}(\mathbf{v} \cdot \mathbf{B}_{\mathbf{k}}) \} \\ & - \frac{ig_D^2}{4} \int \frac{d\Omega}{4\pi} \frac{(\mathbf{v} \times \mathbf{k})(\mathbf{v} \cdot \mathbf{B}_{\mathbf{k}})(\mathbf{k} \cdot \mathbf{v})}{(\mathbf{k} \cdot \mathbf{v} - \omega - i\nu_c)} \\ & - \frac{c_D^2}{2} \int \frac{d\Omega}{4\pi} \left\{ \frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{B}_{\mathbf{k}})(\mathbf{k} \cdot \mathbf{v}) - (\mathbf{v} \times \mathbf{k})(\mathbf{v} \cdot \mathbf{E}_{\mathbf{k}})}{(\mathbf{k} \cdot \mathbf{v} - \omega - i\nu_c)} + \mathbf{B}_{\mathbf{k}} \right\}, \end{aligned} \quad (3.18)$$

where Ω represent angular integrals. In Eq. (3.18), we have defined $m_D^2 = e^2 \int \frac{p^2 dp}{2\pi^2} \frac{d}{dp} (f_{0R} + f_{0\bar{R}} + f_{0L} + f_{0\bar{L}})$, $c_D^2 = e^2 \int \frac{p dp}{2\pi^2} \frac{d}{dp} (f_{0R} - f_{0\bar{R}} - f_{0L} + f_{0\bar{L}})$, $g_D^2 = e^2 \int \frac{dp}{2\pi^2} \frac{d}{dp} (f_{0R} + f_{0\bar{R}} + f_{0L} + f_{0\bar{L}})$, $h_D^2 = e^2 \int \frac{dp}{2\pi^2} (f_{0R} - f_{0\bar{R}} - f_{0L} + f_{0\bar{L}})$.

Expression for the polarization tensor Π^{ij} can be obtained from Eq. (3.18) by writing the total current in the following form $J_k^i = \Pi^{ij}(k)Y_j(k)$ using $\mathbf{E}_k = -i\omega\mathbf{Y}_k$ and $\mathbf{B}_k = i(\mathbf{k} \times \mathbf{Y}_k)$. One can express Π^{ij} in terms of longitudinal $P_L^{ij} = k^i k^j / k^2$, transverse $P_T^{ij} = (\delta^{ij} - k^i k^j / k^2)$ and the axial $P_A^{ij} = i\epsilon^{ijk} k^k$ projection operators as $\Pi^{ik} = \Pi_L P_L^{ik} + \Pi_T P_T^{ik} + \Pi_A P_A^{ik}$. After performing the angular integrations in Eq. (3.18) one obtains Π_L , Π_T and Π_A as given below

$$\Pi_L = -m_D^2 \frac{\omega\omega'}{k^2} [1 - \omega' L(k)], \quad (3.19)$$

$$\Pi_T = m_D^2 \frac{\omega\omega'}{k^2} \left[1 + \frac{k^2 - \omega'^2}{\omega'} L(k) \right], \quad (3.20)$$

$$\Pi_A = -\frac{h_D^2}{2} \left[1 - \omega \left(1 - \frac{\omega'^2}{k^2} \right) L(k) - \frac{\omega'\omega}{k^2} \right] \quad (3.21)$$

where, $L(k) = \frac{1}{2k} \ln \left(\frac{\omega' + k}{\omega' - k} \right)$, $\omega' = \omega + i\nu_c$, $m_D^2 = 4\pi\alpha_1 \left(\frac{T^2}{3} + \frac{\mu_R^2 + \mu_L^2}{2\pi^2} \right)$ and $h_D^2 = \frac{2\alpha_1 \Delta\mu}{\pi}$ (with $\Delta\mu = (\mu_R - \mu_L)$). In the above integrals, we have replaced, e by α_1 using relation $e^2 = 4\pi\alpha_1$.

One can consider two cases: collision dominated regime (where $\nu_c \neq 0$) and collisionless regime ($\nu_c = 0$). First, consider case when $\nu_c \neq 0$. In the limit $\omega \rightarrow 0$, Π_L and Π_T vanish and the parity odd part of the polarization tensor $\Pi_A = h_D^2/2 \approx \alpha_1 \Delta\mu/\pi$. Note that Π_A does not get thermal correction. This could be due to the fact that origin of Π_A term is related with the axial anomaly and it is well known that anomaly does not receive any thermal correction [131, 132, 151]. This form of Π_A is similar to the result obtained in [152] using quantum field theoretic arguments at $T \leq 40$ GeV. In this work, authors discussed that the ground state of the plasma of standard model particles can be determined by the temperature and exactly conserved combination of baryon and lepton numbers in the thermal equilibrium. They show that at non-zero values of the global charge, states that are translationally invariant and homogeneous becomes unstable. In such a situation, in the thermal equilibrium state a large scale magnetic field exists. However, in the kinetic theory approach presented in our work, we have not made any such assumption. Normal modes for the plasma can be obtained

by using expressions for Π_L , Π_T and Π_A . Using the equation $\partial_\nu F^{\mu\nu} = -4\pi J^\mu$, we can write the following relation

$$[M^{-1}]^{ij} Y_j(k) = -4\pi J_k^i, \quad (3.22)$$

where, $[M^{-1}]^{ik} = [(k^2 - \omega^2)\delta^{ik} - k^i k^k + \Pi^{ik}]$. Dispersion relations can be obtained from the poles of $[M^{-1}]^{ik}$, which are given below

$$\begin{aligned} \omega^2 &= \Pi_L, \\ \omega^2 &= k^2 + \Pi_T(k) \pm k\Pi_A. \end{aligned}$$

Once dispersion relations are known, one can study dynamical behavior of the magnetic fields. Expression for the total current described by Eq.(3.18) can be written as $J_k^i = \sigma_E^{ij} E_k^j + \sigma_B^{ij} B_k^j$ where σ_E^{ij} and σ_B^{ij} are electrical and magnetic conductivities. The integrals involved in Eq. (3.18) are rather easy to evaluate in the limit $k, \omega \ll \nu_c$ and one can write the expression for σ_E^{ij} and σ_B^{ij} as:

$$\sigma_E^{ij} \approx \left(\frac{m_D^2}{3\nu_c} \delta^{ij} + \frac{i}{3\nu_c} \frac{\alpha_1 \Delta\mu}{\pi} \epsilon^{ijl} k^l \right) \quad (3.23)$$

$$\sigma_B^{ij} \approx -\frac{4}{3} \frac{\alpha_1 \Delta\mu}{\pi} \delta^{ij} \quad (3.24)$$

Here, we would like to highlight that the Berry curvature correction in the kinetic equation gives us an additional contribution in the expression for σ_E^{ij} which was not incorporated in Ref. [42]. Note that, first term is the usual dissipative part of the electric current and it contributes to the Joules dissipation. The second term is due to the chiral imbalance and it does not give any contribution to the Joules heating. As we shall demonstrate later, this term is responsible for the vorticity current [153]. One can write the total current as $J_{\mathbf{k}}^i = \sigma_E^{ij} E_{\mathbf{k}}^j + \sigma_B^{ij} B_{\mathbf{k}}^j$ and the Maxwell's equation: $i(\mathbf{k} \times \mathbf{B}_{\mathbf{k}})^i = 4\pi J_{\mathbf{k}}^i$. Here we have dropped the displacement current term (this is valid when $\frac{\omega}{4\pi\sigma} \ll 1$). Next by taking vector product of $\mathbf{k}$ with the above Maxwell equation one obtains

$$\begin{aligned} \frac{\partial \mathbf{B}_{\mathbf{k}}}{\partial \tau} + \left(\frac{3\nu_c}{4\pi m_d^2} \right) k^2 \mathbf{B}_{\mathbf{k}} - \left(\frac{\alpha_1 \Delta\mu}{\pi m_D^2} \right) (\mathbf{k} \times (\mathbf{k} \times \mathbf{E}_{\mathbf{k}})) \\ + i \frac{4\alpha_1 \nu_c \Delta\mu}{\pi m_D^2} (\mathbf{k} \times \mathbf{B}_{\mathbf{k}}) = 0. \end{aligned} \quad (3.25)$$

This is the magnetic diffusivity equation for the chiral plasma. By replacing $(\mathbf{k} \times \mathbf{E}_k)$ by $-\frac{1}{i} \frac{\partial \mathbf{B}_k}{\partial \tau}$ in Eq. (3.25), we can solve this equation. Without a loss of generality, consider the propagation vector $\mathbf{k}$ in the z -direction and the magnetic field having components perpendicular to the z -axis. After defining two new variables: $\tilde{B}_k = (B_k^1 + iB_k^2)$ and $\tilde{B}'_k = (B_k^1 - iB_k^2)$ one can rewrite Eq. (3.25) as,

$$\frac{\partial \tilde{B}_k}{\partial \tau} + \left[\frac{\left(\frac{3\nu_c}{4\pi m_d^2} \right) k^2 - \left(\frac{4\alpha_1 \nu_c \Delta \mu}{\pi m_D^2} \right) k}{\left(1 + \frac{\alpha_1 \Delta \mu k}{\pi m_D^2} \right)} \right] \tilde{B}_k = 0, \quad (3.26)$$

$$\frac{\partial \tilde{B}'_k}{\partial \tau} + \left[\frac{\left(\frac{3\nu_c}{4\pi m_d^2} \right) k^2 + \left(\frac{4\alpha_1 \nu_c \Delta \mu}{\pi m_D^2} \right) k}{\left(1 - \frac{\alpha_1 \Delta \mu k}{\pi m_D^2} \right)} \right] \tilde{B}'_k = 0. \quad (3.27)$$

Thus, the magnetic field vector $\mathbf{B}$ can be decomposed into these new variables $\tilde{\mathbf{B}}_k$ & $\tilde{\mathbf{B}}'_k$ having definite helicity (or circular polarization). In the above equation, the effect of Ohmic decay is already taken into account by including the collisions. It should be noted here that if $\alpha_1 \Delta \mu k / \pi m_D^2 \ll 1$ Eq. (3.26) is similar to the magnetic field evolution equation considered in Ref. [42]. In this limit Eq. (3.27) will give a purely damping mode and the dispersion relation reduces to

$$i\omega = \frac{3\nu_c}{4\pi m_d^2} k^2 - \frac{4\alpha_1 \nu_c \Delta \mu}{\pi m_D^2} k \quad (3.28)$$

The dispersion relation obtained here using kinetic theory matches with the dispersion relation obtained in [96].

The instability can also occur in the collisionless regime ($\nu_c = 0$) [96]. In the quasi-static limit i.e. $\omega \ll k$, one can define the electric conductivity as $\sigma_E^{ij} \approx \pi(m_D^2/2k)\delta^{ij}$ and magnetic conductivity $\sigma_B^{ij} \approx (h_D^2/2)\delta^{ij}$. Note that the above conductivities are independent of collision frequency. Similar to the previous case one can take the propagation vector in the z -direction and consider components of the magnetic field in the direction perpendicular to the z -axis. One can write a set of decoupled equations describing the evolution of magnetic field using the variables $\tilde{\mathbf{B}}_k$ and $\tilde{\mathbf{B}}'_k$ as:

$$\frac{\partial \tilde{\mathbf{B}}_k}{\partial \tau} + \left[\frac{k^2 - \frac{4\alpha_1 \Delta \mu k}{3}}{\frac{\pi m_D^2}{2k}} \right] \tilde{\mathbf{B}}_k = 0, \quad (3.29)$$

$$\frac{\partial \tilde{\mathbf{B}}'_k}{\partial \tau} + \left[\frac{k^2 + \frac{4\alpha_1 \Delta \mu k}{3}}{\frac{\pi m_D^2}{2k}} \right] \tilde{\mathbf{B}}'_k = 0. \quad (3.30)$$

If one replaces $\partial/\partial\tau$ by $-i\omega$ Eqs. (3.29) and (3.30) gives the same dispersion relation for the instability as discussed in Ref. [96].

3.2.2 Vorticity generated from chiral imbalance in the plasma

It would be interesting to see if the instabilities arising due to chiral-imbalance can lead to vorticity generation in the plasma. In order to study vorticity of the plasma, we define the average velocity as:

$$\langle \mathbf{v} \rangle = \frac{1}{\bar{n}} \int \frac{d^3p}{(2\pi)^3} \mathbf{v} (\delta f_R - \delta f_{\bar{R}} + \delta f_L - \delta f_{\bar{L}}) \quad (3.31)$$

Here we have used the perturbed distribution function in the numerator of the above equation which is due to the fact that the equilibrium distribution function are assumed to homogeneous and isotropic and therefore will not contribute to vorticity dynamics. The denominator is the total number density and is defined as in Ref. [136]:

$$\begin{aligned} \bar{n} &= n_{particle} - n_{antiparticle} \\ &= d_f \int_0^\infty \frac{d^3p}{(2\pi)^3} \left(\frac{1}{1 + \exp(\frac{p-\mu}{T})} - \frac{1}{1 + \exp(\frac{p+\mu}{T})} \right) \end{aligned} \quad (3.32)$$

which in the case of chiral plasma reduces to $\bar{n} = \frac{2}{3}T^2(\mu_R + \mu_L)$. d_f is degree of freedom of the particles and μ is the chemical potential. We consider $k, \omega \ll \nu_c$ regime. In this case the perturbed distribution function, say for the right-handed particles, can be written as:

$$\delta f_{\mathbf{k}, \omega R} = -\frac{e}{\nu_c} [(\mathbf{v} \cdot \mathbf{E}_{\mathbf{k}}) + \frac{i}{2p} (\mathbf{v} \cdot \mathbf{B}_{\mathbf{k}})(\mathbf{k} \cdot \mathbf{v})] \frac{df_{0R}}{dp}. \quad (3.33)$$

If we add the contribution for all the particles species and their anti-particles we can write the numerator in equation (3.31) as

$$\sqrt{\frac{\alpha_1}{\pi^3}} \frac{1}{\nu_c} \left(\frac{T^2}{3} + \frac{\mu_R^2 + \mu_L^2}{2\pi^2} \right) \mathbf{E}_{\mathbf{k}} \approx \sqrt{\frac{\alpha_1}{\pi^3}} \frac{T^2}{3\nu_c} \mathbf{E}_{\mathbf{k}}. \quad (3.34)$$

Above we have neglected $\frac{3}{2\pi^2} \frac{\mu_R^2 + \mu_L^2}{T^2}$ in comparison to one, as we have considered $\mu_R/T \ll 1$ and $\mu_L/T \ll 1$. One can write the average velocity as follows

$$\langle \mathbf{v}_k \rangle = \sqrt{\frac{\alpha'}{\pi^3}} \frac{1}{2\nu_c} \frac{1}{\mu_R + \mu_L} \mathbf{E}_{\mathbf{k}} \quad (3.35)$$

Assuming that the chiral chemical potential and temperature are independent of space and time, we can obtain vorticity by taking curl of the Eq. (3.35), which takes the following form

$$\langle \boldsymbol{\omega}_k \rangle = i \sqrt{\frac{\alpha'}{\pi^3}} \frac{1}{2\nu_c \mu_R + \mu_L} (\mathbf{k} \times \mathbf{E}_k) \quad (3.36)$$

One can find contribution of the vorticity to the total current from eqs. (3.23) and $J_{k\omega}^i = \sigma_E^{ij} E_k^j + \sigma_B^{ij} B_k^j$. By using Eq. (3.36) the vorticity current can be written as

$$\mathbf{J}_\omega \approx -\sqrt{\frac{4\pi\alpha_1}{9}} (\mu_R^2 - \mu_L^2) \boldsymbol{\omega} = \xi \boldsymbol{\omega} \quad (3.37)$$

It is clear from the Eq. (3.37) that in the absence of any chiral-imbalance there is no vorticity current. Here we note that our definition agrees with the Ref. [108]. Further using Eq. (3.36), one can eliminate $(\mathbf{k} \times \mathbf{E}_k)$ in Eq. (3.25) and obtain:

$$\begin{aligned} \frac{\partial \mathbf{B}_k}{\partial \eta} + \frac{3\nu_c}{4\pi m_d^2} k^2 \mathbf{B}_k + i \frac{\sqrt{4\pi\alpha_1}\nu_c}{m_D^2} (\mu_R^2 - \mu_L^2) (\mathbf{k} \times \boldsymbol{\omega}_k) \\ + i \left(\frac{4\alpha_1\nu_c}{\pi m_D^2} \right) (\mu_R - \mu_L) (\mathbf{k} \times \mathbf{B}_k) = 0. \end{aligned} \quad (3.38)$$

In this equation, second term is the usual diffusivity term, however third and fourth terms are additional terms, which respectively represent vorticity and chiral magnetic effects in the chiral plasma. Therefore Eq. (3.25) actually contains terms due to vorticity and magnetic effect. The saturated state of the instability can be studied by setting $\partial_\tau \mathbf{B}_k = 0$ in Eq. (3.38). After taking dot product of Eq. (3.38) with fluid velocity $\mathbf{v}_k$ in the saturated state, $\partial_\tau \mathbf{B}_k = 0$, leads to,

$$\left(\boldsymbol{\omega}_k - i \frac{16T\delta}{3} \mathbf{v}_k \right) \cdot \mathbf{B}_k = 0. \quad (3.39)$$

where we have defined $\delta = \alpha_1(\mu_R - \mu_L)/T$. We can write expression for magnetic field, which satisfies above Eq. (3.39) as:

$$\mathbf{B}_k = g(\mathbf{k}) \mathbf{k} \times \left[\boldsymbol{\omega}_k - i \frac{16T\delta}{3} \mathbf{v}_k \right] \quad (3.40)$$

Where $g(k)$ is any general function, which can be determined by substituting the above expression for the magnetic field into Eq. (3.38) in the case of steady state. On very large length scales, i.e. $\mathbf{k} \rightarrow 0$:

$$g(\mathbf{k}) = -\frac{3}{32} \sqrt{\frac{\pi^3}{\alpha'^3}} \frac{\mu_R^2 - \mu_L^2}{(\mu_R - \mu_L)^2} \quad (3.41)$$

Therefore, in this limit, for a very large length scale, $\mathbf{k} \rightarrow 0$, magnetic field in the steady state has the form:

$$\mathbf{B}_{\mathbf{k}} = -i \sqrt{\frac{\pi^3}{4\alpha_1} \frac{\mu_R^2 - \mu_L^2}{(\mu_R - \mu_L)}} \boldsymbol{\omega}_{\mathbf{k}} \quad (3.42)$$

This equation relates the vorticity generated during the instability with the magnetic field in the steady state.

However in the collisionless regime ($\omega \ll k$ and $\nu_c = 0$), one can have an instability described by Eq. (3.29) with typical scales $k \sim \alpha_1 \Delta\mu$ and $|\omega| \sim \alpha'^2 T \delta$ [96]. Using the expression for electric and magnetic conductivities for modes in this regime one can write the magnetic diffusivity equation as:

$$\frac{\partial \mathbf{B}_{\mathbf{k}}}{\partial \tau} + \frac{k^2}{4\pi\sigma_1} \mathbf{B}_{\mathbf{k}} - i \frac{T\delta}{\pi\sigma_1} (\mathbf{k} \times \mathbf{B}_{\mathbf{k}}) = 0 \quad (3.43)$$

where $\sigma_1 = \pi m_D^2/2k$. Here it should be noted that unlike Eq. (3.38), the above equation does not have a vorticity term. The last term on the left-hand side arises due to the chiral-magnetic effect. In the steady state, one get $\nabla \times \mathbf{B} = (4T\delta)\mathbf{B}$. This equation resembles the case of magnetic field in a force free configuration of the conventional plasma where the plasma pressure is assumed to be negligible in comparison with the magnetic pressure [154]. But for our case no such assumption about the plasma pressure is required.

3.3 Discussion

In this chapter, we have applied the modified kinetic theory in the presence of chiral imbalance and obtained equations for the magnetic field generation for both the collision dominated and the collisionless regimes. The instability can lead to generation of the magnetic field at the cost of the chiral imbalance. One can estimate the strength of the generated magnetic field as follows. From Eqs. (3.1) and (1.29), one can notice that right-handed electron number density n_R changes with the Chern-Simon number n_{CS} of the hyper-charge field configuration as

$$\Delta n_R = \frac{1}{2} y_R^2 n_{cs} \quad (3.44)$$

Here $n_{CS} \approx \frac{g_1^2}{16\pi^2} kY^2$ and $\Delta n_R = \mu_R T^2 = \frac{88}{783} \delta T^3$ [136]. From this, one can estimate the magnitude of the generated physical magnetic field to be

$$B_Y^{phy} \approx 10^{-3} \left[\frac{\pi^2 k}{g_1^2 \alpha_1 T} \right]^{\frac{1}{2}} \left(\frac{\delta}{10^{-6}} \right)^{1/2} T^2, \quad (3.45)$$

where we have used $\mathcal{B}_Y \sim kY$ and k^{-1} is physical length scale, which is related with the comoving length by $k_{phy}^{-1} = (a/k_c)^{-1}$.

Now consider the regime $\omega, k \ll \nu_c$ where dynamics for the magnetic field is described by Eqs.(3.26-3.27). Eq.(3.26) clearly gives unstable modes if $\left(\frac{T\delta}{\pi m_D^2} \right) k < 1$ is satisfied. However, Eq.(3.27) gives a purely damping mode if the condition $\left(\frac{T\delta}{\pi m_D^2} \right) k \ll 1$ is satisfied. One can rewrite this condition as $\left(\frac{T\delta}{3\pi\sigma\nu_c} \right) k \sim \left(\frac{10^{-2}}{3\pi} \frac{\delta}{\nu_c} \right) k \ll 1$. Here we have used $m_D^2 = 3\nu_c\sigma$ with $\nu_c \sim \alpha'^2 \ln(\frac{1}{\alpha_1})T$ [98] and $\sigma = 100T$. Thus for $k \ll \nu_c$ and $\delta \ll 1$, Eq. (3.27) can only give purely damped modes. For these values of k and δ , Eq. (3.26) assumes similar form as the equation for the magnetic field dynamics considered in Ref. [42]. If one replaces $\frac{\partial}{\partial \tau}$ by $-i\omega$ in Eq.(3.26), the dispersion relation for the unstable modes can be obtained. The fastest growth of the perturbation occurs for $k_{max1} \sim \frac{8T\delta}{3}$ and the maximum growth rate can be found to be $\Gamma_1 \sim \frac{16}{3\pi} \frac{T^2\delta^2}{m_D^2} \nu_c$. Here we note that our k_{max1} differs by a numerical factor from the value of k where there is a peak in the magnetic energy calculated using chiral magnetohydrodynamics [107]. For $\delta \sim 10^{-6}$ and $\alpha_1 \sim 10^{-2}$ one can show that $\frac{k_{max1}}{\nu_c} \ll 1$ and $\frac{\Gamma_1}{\nu_c} \ll 1$ is satisfied. For these values of k_{max1} , α_1 and δ one can estimate the magnitude of the generated magnetic field using Eq. (3.45). We find $\mathcal{B} \sim 10^{26}$ Gauss for $\alpha_1 \sim 10^{-2}$ and the typical length scale $\lambda \sim 10^5/T$. Here we would like to note that the typical Hubble length scale $\sim 10^{13}/T$, which is much larger than the typical length scale of instability. Our estimate of the magnetic field strength $\mathcal{B}$ in the collision dominated regime broadly agree with Ref. [42]. Here we note again that Eq. (3.26) includes effect of the Ohmic decay due to presence of the collision term. Our analysis shows that Ohmic decay is not important for the instability. Further we have shown that the chiral instability can also lead to generation of vorticity in the collision dominated regime. Typical length scale for vorticity is similar to that of the magnetic field. From Eq.(3.42) magnitude of the vorticity is found to be $\omega_v \sim 10^{-4}B/T$. Next, we analyse the chiral instability

in collisionless regime $\nu_c \ll \omega \ll k$, considering eqs. (3.29-3.30). Here one finds the wave number $k_{max2} = \frac{8\delta T}{9}$ at which the maximum growth rate $\Gamma_2 = \frac{1}{2\pi} \frac{T^3 \delta^3}{m_D^2}$ occurs. Now $\frac{k_{max2}}{\nu_c} = \frac{8}{9} \frac{\delta}{\alpha_1^2} \ll 1$ and $\frac{\Gamma_2}{\nu_c} \sim \frac{3\delta^3}{8\pi^2 \alpha_1^3 \ln(1/\alpha_1)} \ll 1$ this puts constraints on the allowed values of δ . For $\delta \sim 10^{-1}$, $\alpha_1 \sim 10^{-2}$ and $T \sim T_f$, one can estimate magnitude of the magnetic field to be 10^{31} Gauss. Typical length scale for the magnetic field $\lambda_2 \sim 10/T$ and which is much smaller than the length scale in the collision dominated case. This is expected as the typical length scale associated with kinetic theory are smaller than the hydrodynamical case.

The upper and lower bounds on the present observed magnetic field strength from PLANCK 2015 results [106] and blazars [102] are between $10^{-17}G - 10^{-9}G$. However, recently in Ref. [39] it has been shown that if the magnetic field is helical and created before the electroweak phase transition then it can produce some baryon asymmetry. This can put more stringent bounds on the magnetic field ($10^{-14}G - 10^{-12}G$). Since the magnetic fields and the plasma evolution are coupled, the produced magnetic field may not evolve adiabatically i.e. like $a(\tau)^{-2}$ due to the plasma processes like turbulence. Similarly the magnetic correlation length $\lambda_B \propto k_{max}^{-1}$ may not be proportional to $a(\tau)$. Typical values of λ_B for the collision dominated and collisionless regimes in our case are $10^5/T$ and $10/T$ respectively. At EW phase transition, this length is of the order of few centimetres. One of the most important length scale is the length scale of turbulence. Which can be written in the following form

$$\lambda_T \approx \frac{\mathcal{B}_p}{\sqrt{\varepsilon^{ch} + p^{ch}}} \tau \sim \frac{\mathcal{B}_p}{\sqrt{\varepsilon^{ch} + p^{ch}}} H^{-1}, \quad (3.46)$$

where $\mathcal{B}_p$ is the physical value of magnetic field and ε^{ch} and p^{ch} are respectively energy and pressure densities of the charged particles. For $\lambda_B \gg \lambda_T$ the effect of turbulence can be neglected. However, the maximum value of the magnetic field (for $\nu_c = 0$) is about $10^{31} G$ in our case and this gives $\lambda_T \approx 10^6/T$. Thus we have $\lambda_B \ll \lambda_T$ and following Ref. [39] we assume that the generated magnetic fields will undergo inverse cascade soon after their generation. One can relate $\mathcal{B}_p$ and λ_B that are undergoing the process of inverse cascade with the present day values

of magnetic field $\mathcal{B}_0$ and the correlation length λ_0 using the following eqs. [39]:

$$\mathcal{B}_p^{IC}(T) \simeq 9.3 \times 10^9 \text{ G} \left(\frac{T}{10^2 \text{ GeV}} \right)^{7/3} \left(\frac{\mathcal{B}_0}{10^{-14} \text{ G}} \right)^{2/3} \times \left(\frac{\lambda_0}{10^2 \text{ pc}} \right)^{1/3} \mathcal{G}_B(T) \quad (3.47)$$

$$\lambda_B^{IC}(T) \simeq 2.4 \times 10^{-29} \text{ Mpc} \left(\frac{T}{10^2 \text{ GeV}} \right)^{-5/3} \left(\frac{\mathcal{B}_0}{10^{-14} \text{ G}} \right)^{2/3} \times \left(\frac{\lambda_0}{1 \text{ pc}} \right) \mathcal{G}_\lambda(T) \quad (3.48)$$

Where

$$\mathcal{G}_B(T) = (g_*^{total}(T)/106.75)^{1/6} (g_*^{ch}(T)/82.75)^{1/6} (g_{*s}(T)/106.75)^{1/3} \quad (3.49)$$

$$\mathcal{G}_\lambda(T) = (g_*^{total}(T)/106.75)^{-1/3} (g_*^{ch}(T)/82.75)^{-1/3} (g_{*s}(T)/106.75)^{1/3}. \quad (3.50)$$

Here $g_*^{ch}(T)$ and $g_*^{total}(T)$ are the number of degree of freedom of the $U(1)$ charged particles in the thermal bath. From these equations one can see that for collision dominated case $\mathcal{B}_p \simeq 10^{26} \text{ G}$, can be achieved when $\mathcal{B}_0 \simeq 10^{-12} \text{ G}$ and $\lambda_0 \simeq 100 \text{ Kpc}$. However in collisionless regime a value of $\mathcal{B}_0 \simeq 10^{-11} \text{ G}$ and $\lambda_0 \simeq 1 \text{ Mpc}$ at temperature $T = 80 \text{ TeV}$ gives the values that we have found in our estimates for the peak value of the magnetic field. Thus the values of the magnetic field and the correlation length scale estimated by us can be consistent with the current bounds obtained from CMB observation and necessary for current observed baryon asymmetry. Since the values of $\mathcal{B}_p$ and λ_B for the collision dominated case are similar to those given in the Ref. [95], we believe that they are also consistent with BBN constraints.

In conclusion we have studied the generation of magnetic fields due to the anomaly in primordial plasma consisting of the standard model particles. We have applied the Berry curvature modified kinetic theory to study this problem. The effect of collisions in the kinetic equation was incorporated using the relaxation time approximation. We find that the chiral instability can occur in presence of the dissipation in both collision dominated and collisionless regimes. We find that in the collision dominated case the chiral instability can produce a magnetic field of the order of 10^{27} Gauss with the typical length scale $10^5/T$. These results are in broad agreement with Ref. [42]. However in this work authors have used heuristic kinetic equation and the collision term was not explicitly written in the

kinetic equation. While the expression for the total current included the Ohm's law. We have obtained expressions for electric and magnetic conductivities using the modified kinetic theory. We find that expression for electric conductivity in chiral plasma has a non-dissipative term in addition to the standard Ohmic term. It is shown that this new term is related to the vorticity current term found in the chiral magnetohydrodynamics [64]. Further we have also studied the chiral instability in the collisionless regime. It is shown that in this regime magnetic field of strength 10^{31} Gauss can be generated at length scale $10/T$. These length scales are much smaller than the length scale of the magnetic field in the collision dominated regime. Further the obtained values of magnetic-field strength and the length scale are shown to be consistent with the recent constraints from CMB data. We have also shown that in the collision dominated regime results of kinetic theory agree with the hydrodynamic treatment.

Chapter 4

Chiral Battery, scaling laws and magnetic fields

In this chapter, we study the generation and evolution of magnetic field in the presence of chiral imbalance and gravitational anomaly which gives an additional contribution to the vortical current. The contribution due to gravitational anomaly is proportional to T^2 . This contribution to the current can generate seed magnetic field of the order of 10^{30} G at $T \sim 10^9$ GeV, with a typical length scale of the order of $10^6/T$, even in the absence of chiral charges (when chiral chemical potential is zero). Moreover, such a system possess scaling symmetry. We show that the T^2 term in the vorticity current along with scaling symmetry leads to more power transfer from lower to higher length scale as compared to only chiral anomaly without scaling symmetry.

4.1 Introduction

Chiral fluid carries both electric charges j_{elec}^μ (which can be written in terms of vector current $j_V^\mu = \bar{\psi}\gamma^\mu\psi$ as $j_{elec}^\mu = ej_V^\mu$, where e is electric charge) and chiral charges $j_A^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$. Above ψ represents the fermions, γ^μ are the Dirac matrices and γ_5 is defined as $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. Sometimes it is convenient to work in the basis of the left handed and right handed currents, which are defined as $j_R^\mu = \frac{1}{2}(j_V^\mu + j_A^\mu)$ and $j_L^\mu = \frac{1}{2}(j_V^\mu - j_A^\mu)$.

The dynamics of a plasma consisting of chiral fermions in presence of background fields is governed by the following hydrodynamic equations [155]

$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda, \quad (4.1)$$

$$\nabla_\mu j^\mu = 0, \quad (4.2)$$

$$\nabla_\mu j^\mu = C E_\mu B^\mu \quad (4.3)$$

where the vector current $j^\mu = j_R^\mu + j_L^\mu$ and chiral current $j_5^\mu = j_R^\mu - j_L^\mu$ with $j_{R(L)}$ being the current density for right (left) handed particles. Here ∇_μ represent covariant derivative with respect to the curved space. Greek index runs from 0 to 3. In the present work, we have used Latin index for the three-space coordinate points (1–3). Here $F^{\mu\nu}$ is the field strength and C is the anomaly coefficient. E^μ and B^μ are electric and magnetic four vectors, respectively, and they are measured with respect to the comoving observer. These are purely spatial vectors and are defined as: $E^\mu = F^{\mu\nu} u_\nu$, $B^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}$ and $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$. The Maxwell equations read

$$\nabla_\nu F^{\mu\nu} = j^\mu \quad \text{and} \quad F_{[\mu\nu,\lambda]} = 0 \quad (4.4)$$

In the state of local equilibrium, the vector current j_v^μ and the chiral current j_5^μ can be expressed in terms of the four velocity of the fluid u^μ , energy density ρ , vector charge density n_v and axial charge density n_5 . These quantities respectively takes the following form

$$j_v^\mu = n_v u^\mu + \sigma E^\mu + \xi_v \omega^\mu + \xi_v^B B^\mu, \quad (4.5)$$

$$j_5^\mu = n_5 u^\mu + \xi_5 \omega^\mu + \xi_5^B B^\mu, \quad (4.6)$$

where $n_{v,5} = n_R \pm n_L$, $\xi_{v,5} = \xi_R \pm \xi_L$, $\xi_{v,5}^B = \xi_R^B \pm \xi_L^B$, $\omega^\mu = \epsilon^{\mu\nu\sigma\delta} u_\nu \partial_\sigma u_\delta$ is the vorticity four vector, $E^\mu = F^{\mu\nu} u_\nu$, and $B^\mu = 1/2 \epsilon^{\mu\nu\sigma\delta} u_\nu F_{\sigma\delta}$. In presence of chiral imbalance and gravitational anomaly, consistency with second law of thermodynamics ($\partial_\mu s^\mu \geq 0$, with s^μ being the entropy density) demand that the coefficients, for each right and left particle, have following form [108, 156, 157]

$$\xi_i = C \mu_i^2 \left(1 - \frac{2 n_i \mu_i}{3(\rho + p)} \right) + \frac{D T^2}{2} \left(1 - \frac{2 n_i \mu_i}{(\rho + p)} \right) \quad (4.7)$$

$$\xi_i^B = C \mu_i \left(1 - \frac{n_i \mu_i}{2(\rho + p)} \right) - \frac{D}{2} \left(\frac{n_i T^2}{(\rho + p)} \right), \quad (4.8)$$

The constants C and D are related to those of the chiral anomaly and mixed gauge-gravitational anomaly as $C = \pm 1/4\pi^2$ and $D = \pm 1/12$ for right and left handed chiral particles respectively. It is now easy to see from eq.(4.7) and eq.(4.8) that for $\mu_i/T \ll 1$, which is the scenario in the early universe, $\xi_i \sim \frac{D}{2} T^2$ and $\xi_i^B \sim C \mu_i$. Note that, $\xi_v = 0 = \xi_v^B$. However, $\xi_5 \sim DT^2$ and $\xi_5^B = C(\mu_R - \mu_L) = C \mu_5$. We would like to emphasize here that the ξ_5 is independent of chiral imbalance and depends only on the temperature. Thus, we expect such a term to dominate at high temperature. We exploit this dominance to generate the magnetic field in the early Universe.

In an expanding background, the line element can be described by Friedmann-Robertson-Walker metric, given in the chapter (3) in equation (3.4),

$$ds^2 = a^2(\tau) (-d\tau^2 + \delta_{ij} dx^i dx^j) , \quad (4.9)$$

where a is the scale factor and τ is the conformal time. We choose $a(\tau)$ to have dimensions of length, and τ , x^i to be dimensionless. Using the fact that the scale factor $a = 1/T$, we can define the conformal time $\tau = M_*/T$, where $M_* = \sqrt{90/8 \pi^3 g_{\text{eff}}} M_P$ with g_{eff} being the effective relativistic degrees of freedom that contributes to the energy density and $M_P = 1/\sqrt{G}$ being the Planck mass. We also define the following comoving variables

$$\begin{aligned} \mathbf{B}_c &= a^2(\tau) \mathbf{B}(\tau), \quad \mu_c = a(\tau) \mu, \quad \sigma_c = a(\tau) \sigma \\ , \quad T_c &= a(\tau) T, \quad x_c = x/a(\tau). \end{aligned} \quad (4.10)$$

In terms of comoving variables, the evolution equations of fluid and electromagnetic fields are form invariant [143, 144, 146]. Thus, in the discussion below, we will work with the above defined comoving quantities and omit the subscript c .

Using the effective Lagrangian for the standard model, one can derive the generalized Maxwell's equation [158],

$$\nabla \times \mathbf{B} = \mathbf{j}, \quad (4.11)$$

with $\mathbf{j} = \mathbf{j}_v + \mathbf{j}_5$ being the total current. In the above equation, we have ignored the displacement current. Taking $u^\mu = (1, \mathbf{v})$ and using eq.(4.5) and eq.(4.6), one

can show that

$$\begin{aligned} j^0 &= n = n_v + n_5 \\ \mathbf{j} &= n\mathbf{v} + \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \xi \boldsymbol{\omega} + \xi^B \mathbf{B}, \end{aligned} \quad (4.12)$$

with $\xi = \xi_v + \xi_5$ and $\xi^B = \xi_v^B + \xi_5^B$. For our analysis, we assume that the velocity field is divergence free, *i.e.* $\nabla \cdot \mathbf{v} = 0$. Taking curl of eq.(4.11) and using the expression for current from eq.(4.12), we get

$$\frac{\partial \mathbf{B}}{\partial \tau} = \frac{n}{\sigma} \nabla \times \mathbf{v} + \frac{1}{\sigma} \nabla^2 \mathbf{B} + \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\xi}{\sigma} \nabla \times \boldsymbol{\omega} + \frac{\xi^B}{\sigma} \nabla \times \mathbf{B}. \quad (4.13)$$

In obtaining the above equation, we have also assumed that chemical potential and the temperature are homogeneous. Note that, in the limit $\sigma \rightarrow \infty$, we will obtain magnetic fields which are flux frozen. There are other conditions in the cosmological and astrophysical settings in which the advection term $\nabla \times (\mathbf{v} \times \mathbf{B})$ can be ignored compared to other terms and reduces the above equation to linear equation in v and B . We ignore advection term in our analysis.

4.2 Chiral battery

In absence of any background magnetic fields $\mathbf{B} = 0$, eq.(4.13) reduces to

$$\frac{\partial \mathbf{B}}{\partial \tau} = \frac{1}{\sigma} n \nabla \times \mathbf{v} + \frac{1}{\sigma} \xi \nabla \times \boldsymbol{\omega}. \quad (4.14)$$

Note that the two terms on the right hand side of eq.(4.14) no longer depend on B . This situation is similar to that of the Biermann battery mechanism where $\nabla \rho \times \nabla p$ term is independent of B [27]. For Biermann mechanism to work, $\nabla \rho$ and ∇p have to be in different directions which can be achieved if the system has vorticity. In our scenario, it is interesting to note that even if $n = 0$, T^2 term in ξ will act as a source to generate the seed magnetic field at high temperature. On the other hand, in presence of finite chiral imbalance (such that $\mu/T \ll 1$) in the early universe, T^2 term in ξ will still be the source to generate the seed magnetic field, but non-zero $\xi^{(B)}$ will lead to an instability in the system. An order of magnitude estimate gives the first term of eq.(4.14) to be $\frac{\alpha}{L} \left(\frac{\mu}{T}\right) T^2 v$ while the second term is of the order $\frac{\alpha}{L^2} D T v$. On comparing the two terms we get a critical length $\lambda_c \sim (D/T)(\mu/T)^{-1}$ below which first term will be sub-dominant.

4.3 Results and discussion

In the early Universe, much before electroweak symmetry breaking, collision of bubble walls during first order phase transition at GUT scale, can generate initial vorticity in the plasma. With these assumptions, we have shown that magnetic field can be generated, in presence of either of the two terms on the right hand side of eq.(4.14). It is also important to note that, second term has purely temperature dependence ($\propto D T^2 \omega$) which comes due to gravitational anomaly. In absence of any background field, the two term $n k$ and ξk^2 in the right hand side of eq.(4.41) can be source of the seed field. Further, when $n = 0$, only ξk^2 term acts as a source to generate the seed magnetic field. However, there may not be any instability in the system if $\xi^B = 0$ (Fig(4.3)). It is also evident from eq.(4.41) and eq.(4.23) that the magnetic energy spectrum will go as $\sim k^7$ at large length scale (Fig(4.1 a)). This feature is similar to [107]. However, for a given k the power transferred to larger length scale is more with the scaling symmetry (see Fig(4.1 a)). On the other hand, when $n \neq 0$, then $n k$ term dominates over ξk^2 term at larger length scale (see figure 4.2 b). In the intermediate range, there is a competition between the two terms on the right hand side of eq.(4.14). Consequently, the energy spectrum goes as $\sim k^5$ at large length scales (see Fig(4.1 b)). We estimate the strength of the magnetic field produced to be: $\frac{B}{T^2} \approx \frac{g}{D} \left(\frac{\mu}{T}\right)^2 T L(\tau)$, where $L(\tau) = v \tau \approx \frac{D}{T} \left(\frac{\mu}{T}\right)^{-1}$. For temperature $T = 10^9$ GeV and $\mu/T \approx 10^{-6}$, the strength of the magnetic field is of the order of 10^{30} G at a length scale of the order of 10^{-18} cm, which is much smaller than the Hubble length (10^{-9} cm) at that temperature.

It is evident from eq.(4.41) that once the magnetic field of sufficiently large strength is produced, it starts influencing the subsequent evolution of the system and the magnetic energy grows rapidly. This phenomena is known as the chiral plasma instability [96, 161]. In this regime, we solve eq.(4.41) and obtain mode expression in (4.42), which clearly shows that the modes of the magnetic fields grow exponentially for the wave number $k \simeq k_{ins}/2 \approx C\mu_5/2$. As mentioned earlier, for $T \geq 80$ TeV chiral flipping rate is slower than the expansion rate of the Universe, one can safely ignore $\Gamma_f \mu_5$ term in eq.(4.44). However, the magnetic

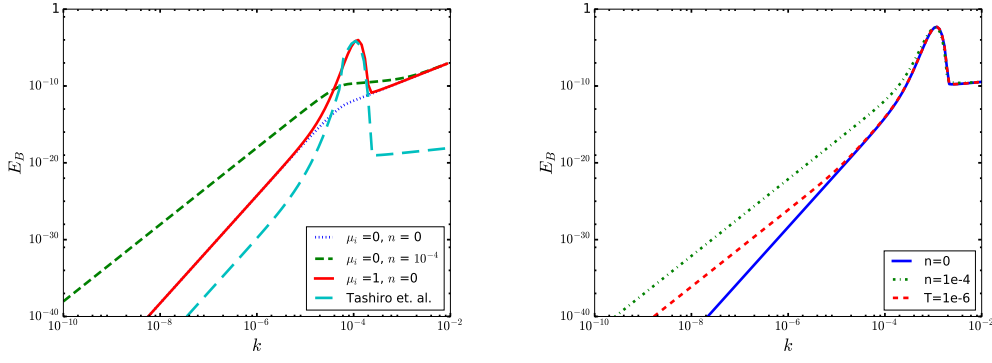


Figure 4.1: Magnetic energy spectrum as a function of k is shown: (a) when $n = 0$, magnetic energy spectrum E_B goes as k^7 at large length scale. However, the power is more than what authors of ref. [107] has found for a plasma with chiral asymmetry for a given k value. (b) when $n \neq 0$, E_B goes as k^5 at large length scale. In both the figures we have set $T = 10^9$ GeV and $v = 10^{-4}$.

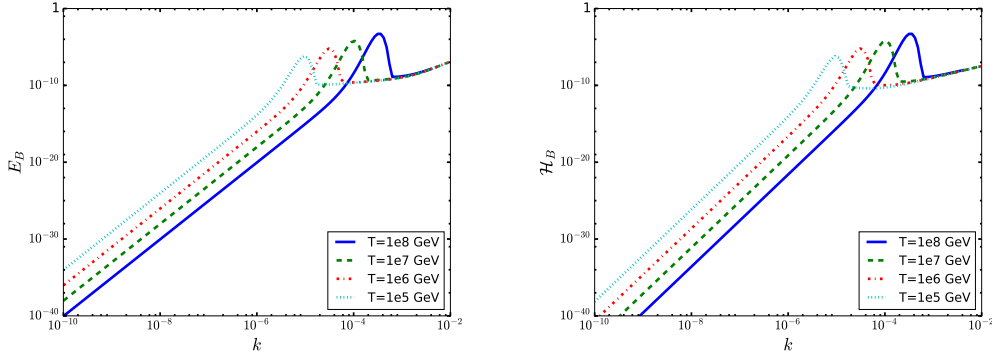


Figure 4.2: (a) Evolution of magnetic energy density E_B as a function of k and (b) Evolution of magnetic helicity density $\mathcal{H}_B$ as a function of k for different temperatures. As temperature ($T \sim 1/\tau$) decreases, peak shifts from higher to lower k values which shows inverse cascading *i.e.* transfer of magnetic energy from small to large length scale. In both the figures we have set $v = 10^{-4}$ and $n = 10^{-4}$.

helicity generated due to DT^2 will drive the evolution of chemical potential. Below 80 TeV, flipping rate becomes comparable to the expansion rate of the Universe and the chirality of the right particles changes to the left handed. So to know the complete dynamics of the magnetic field energy, we have solved

the coupled equations (4.41) and (4.44) simultaneously. In figure (4.2 a) we have shown the variation of magnetic energy spectrum E_B with k for different temperature. Since, the chemical potential is a decreasing function of time, so it is expected that with decrease in temperature the instability peak will shift towards the smaller k which implies the transfer of energy from small to large length scale. Similarly, we have shown the magnetic helicity spectrum in figure (4.2 b).

4.4 Conclusion

In the present chapter, we have discussed the generation of magnetic field due to gravitational anomaly which induces a term $\propto T^2$ in the vorticity current. The salient feature of this seed magnetic field is that it will be produced at high temperature irrespective of the whether fluid is charged or neutral. In ref. [159], authors have shown that the equations of chiral magnetohydrodynamics (ChMHD), in absence of other effects like charge separation effect and cross helicity, follow unique scaling property and transfer energy from small to large length scales known as inverse cascade.

Chapter 5

Damping of the generated magnetic field

In chapters (3) and (4), we have shown that the spontaneous magnetization can occur in a high energy plasma at temperature $T \geq 80$ TeV in the presence of an excess amount of right handed electrons. Therefore it is natural to study the collective modes in a magnetized chiral plasma and the damping mechanisms of these modes. Similar problem has been studied in the literature of cosmology albeit for a low energy plasma [76, 162, 163]. In ref. [162] Jedamzik et. al. have examined evolution of the collective modes in a magnetized plasma in the FRW background. In this work the authors study the effects of finite viscosity, resistivity and heat conductivity on the collective modes. It has been shown that the Alfven and slow magnetosonic waves survive below the Silk mass scale.

In the context of chiral plasma at high energies, there exist new kind of collective modes [*e.g.*, the chiral Alfven waves in ref. [155]] in the MHD limit and these modes exist in addition to the usual modes in the standard parity even plasma [164]. Moreover, it has been shown that chiral plasmas exhibit a new type of density wave in presence of either an external magnetic field or vorticity and they are respectively known as Chiral Magnetic Wave (CMW) [165, 166] and the Chiral Vortical Wave (CVW) [167]. Here we would like to note that there are some attempts in studying the damping of MHD waves of these new modes [168, 169] in the context of quark-gluon plasma. In ref. [170], the authors

have studied damping of thermal chiral vortical and magnetic waves. In these work the first order dissipative MHD dynamics was used. However it should be emphasized here that the first order dissipative hydrodynamics is known for its difficulties in dealing with the acausal behaviour and unphysical instability. These issues with the first order hydrodynamics can be resolved by using the causal hydrodynamics [171–173]. The causal hydrodynamics for the parity violating MHD has been developed in refs. [174–177]. In this chapter we study normal modes in the chiral plasma using the causal MHD equations [175]. We believe this kind of study was not systematically done earlier.

This chapter is divided in the following sections and subsections: in section-(5.1) we have given a brief introduction and motivation for this chapter; in section-(5.2) we have discussed waves in the non-dissipative chiral plasma. Section-(5.3) and its subsections contains a very brief introduction to the conformal viscous hydrodynamics. Last section-(5.4) contains summary and discussion of this chapter.

5.1 Introduction

The motion of a relativistic fluid can be described by relativistic hydrodynamics for the fluid near equilibrium. Relativistic hydrodynamics play important role in exploring various areas of cosmology, astrophysics and nuclear physics. The fundamental variables are local fluid velocities $u^\mu(x)$, temperature $T(x)$, all conserved charges or their chemical potentials $\mu_s(x)$ (here 's' stands for different species of the fluid and x represents space-time dependence of the variables). The conservation equation for stress tensor $T^{\mu\nu}$ and the currents J_s^μ determine evolution of the fluid. The energy momentum tensor and the currents can be expanded in terms of the above mentioned variables, as a series of the *Kundsen number* [172, 178, 179]. *Kundsen number* is defined as a ratio of mean free path λ_{mfp} to the characteristic length scale l of the system under consideration, i.e., $K = (\lambda_{\text{mfp}}/l)$. Each term in the series expansion of K is multiplied by a coefficient, also known as a transport coefficient [180]. These transport coefficients

are functions of temperature and chemical potentials and are generally obtained from experiments.

The zeroth order of this expansion represents contribution for the ideal fluid. From the first order, the Navier-Stokes (NS) equations are obtained. There are several representations for the second order terms. For example, the theory of the conformal fluid [181], Israel-Stewart (IS) theory [171, 172] and others [173]. It has been shown that the first order theory does not satisfy causality and it do not explain the presence of the unphysical behaviour in the plasma [182–185]. To demonstrate, we note that the dispersion relation obtained from the velocity diffusion equation is $\omega = -i\frac{\eta_0}{\varepsilon_0 + p_0}k^2$ and the speed of diffusion is $v_T = \left|\frac{d\omega}{dk}\right| = 2\frac{\eta_0}{\varepsilon_0 + p_0}k$ (here η_0 , ε_0 and p_0 are background shear viscosity, energy density and pressure of the fluid). For $k \rightarrow \infty$, the diffusion speed goes to infinity and therefore violates causality [185]. Therefore the relativistic Navier-Stokes equation is only valid for the short wavenumber. Muller, Stewart and Israel in their series of papers [171, 172] addressed the causality problem. It was shown that one can avoid problems related to the causality, if we include second and higher order terms of thermodynamic variables. With the second order hydrodynamic terms, for $k \rightarrow \infty$, longitudinal and transverse modes have group velocity, given by

$$v_L^{max} = \sqrt{v_s^2 + \frac{4}{3}\frac{\eta_0}{(\varepsilon_0 + p_0)\tau_\pi} + \frac{\zeta_0}{(\varepsilon_0 + p_0)\tau_\Pi}}, \quad (5.1)$$

$$v_T^{max} = \sqrt{\frac{\eta_0}{(\varepsilon_0 + p_0)\tau_\pi}}. \quad (5.2)$$

From the above equations, it is clear that there is no causality problem for sufficiently large values of relaxation times τ_π and τ_Π [174]. However problem with this model is that, we still don't know the transport coefficients τ_π and τ_Π . Later, it was realized that the theory of Muller-Israel-Stewart does not give a complete picture of evolution of the relativistic fluid under viscous hydrodynamics and it does not address second order terms properly. The generalization to the second and higher order terms has been addressed by many authors (see following Ref. [174–177] and references there in). These works are focused on constructing all possible second order derivative terms of the hydrodynamical degrees of free-

dom and then evaluating transport coefficients. In literature, few coefficients have been determined from underlying microscopic description. Transport coefficients from microscopic theory are calculated using kinetic theory [186, 187], Kubo formalism [156], fluid/gravity correspondence [188] and diagrammatic methods [189]. In Ref. [175], the authors have shown that out of a total 24 terms of second order derivative expansions, only 18 terms are possible for the parity odd fluid in the presence of external electric/magnetic fields and vorticity in the fluid. It was shown that the number of possible terms can be limited by using some symmetry property namely, conformal symmetry and discrete symmetries (charge conjugation and parity). Earlier in refs. [176, 177], the number of possible terms were constrained for the second order conformal viscous fluid in the absence of external electric/magnetic fields in the 4-D space. Few terms of the second order viscous chiral hydro are odd under the parity transformation and are related with the quantum anomalies of the microscopic theory [108] and can be explained by quantum field theory [122, 141]. The phenomenon of quantum anomaly was first studied in holographic waves of strongly coupled Supersymmetric Yang-Mills (SYM) plasma in the presence of non-zero chiral chemical potential in the chiral fluid [190, 191]. It was shown that the vorticity as well magnetic effect term is allowed not only from symmetry, but also from the hydrodynamic constraint ($\partial_\mu s^\mu \geq 0$, where s^μ is an entropy density tensor). It has been shown that the coefficients of the CME [192] and CVE [193] terms in the current are related with the anomaly coefficient at non zero chiral chemical potential.

In the above backdrop, in this chapter we discuss the first and second order conformal hydrodynamics for chiral plasma to study the hydrodynamic excitations. By linearization of the first and second order ChMHD equations, we have discussed some of the interesting aspect of the chiral plasma.

5.2 Waves in the non dissipative chiral plasma

5.2.1 Chiral Magnetic Waves (CMW)

In this subsection, we will discuss the presence of a new type of collective excitation in the plasma, due to the coupling of the electric and chiral charge. These waves are generated in the presence of the external magnetic field, when CME [192] and *Chiral Charge Separation* effects (CSE) interplay [165]. Due to this, they carry several important new transport properties of the hot QCD plasma. For simplicity let us consider single flavor massless QCD with chiral symmetry. The axial symmetry is violated via triangle anomaly of the global chiral symmetry. Let us assume that because of the presence of the axial and vector chemical potentials ($\mu_{A/V}$), there are two currents in the chiral plasma, namely vector and axial current

$$\mathbf{j}_V = \frac{N_c e}{2\pi^2} \mu_5 \mathbf{B}, \quad (5.3)$$

$$\mathbf{j}_A = \frac{N_c e}{2\pi^2} \mu_V \mathbf{B}. \quad (5.4)$$

Here N_c is the number of colour charges and $\mathbf{B}$ is the external magnetic field. One can establish a close connection between CME and CSE by the method of dimensional reduction appropriate in the case of strong magnetic field [194]. The simple relations $j_V^0 = j_A^1$ and $j_A^0 = j_V^1$ between the vector and axial currents in the dimensionally reduced (1+1) theory imply that the density of the baryon charge (which is proportional to $\mu_R + \mu_L$) must induce the axial current and the density of axial charges must induce the current of electric charge (CME). This can be seen as follows: Consider a local perturbation in electric charge density, which implies from equation (5.4), perturbation in the axial current. In turn the perturbation in the axial current, would induce a local perturbation in the μ_5 . Thus according to equation (5.3), perturbations in the electric current. The resulting fluctuations in the electric charge density complete the cycle and lead to the excitations that combine the density waves of electric and chiral charges. This density wave is known as “*Chiral Magnetic Waves*”. In the matrix form the

$\chi_\mu = \partial n_{L,R} / \partial \mu_{L,R}$. Let us suppose a case, where background fluid is not neutral i.e. $\mu_0 \neq 0$. In this case, one can linearise above equation using $\delta n = \chi_0 \delta \mu$. So

$$\partial_t(\delta n) + \frac{\mu_0 \omega}{2\pi^2 \chi_{\mu_0}} \partial_x(\delta n) = 0. \quad (5.18)$$

This is just a wave equation describing a propagating mode with a gapless dispersion relation

$$V_\omega = \frac{\mu_0 \omega}{2\pi^2 \chi_{\mu_0}} \quad (5.19)$$

This is the chiral vortical wave (CVW) corresponding to the right handed wave with wave speed V_ω [167]. One can get left handed wave modes in the similar way but propagation velocity in the negative direction of ω . The CVW obtained above is essentially a hydrodynamic density wave arising from slowly varying vector and axial density fluctuations that are coupled together through vortical effects.

5.2.3 Chiral Alfven Wave (CAW)

At high temperature authors of the ref. [155] have shown that a new type of gapless collective excitation can be induced in the presence of the external magnetic fields. They showed that these waves are transverse in nature and they can sustain even in the fluids, where $\nabla \cdot \mathbf{v}$ i.e. incompressible fluids. Physical argument for this is: let us assume that an external magnetic field is present in the fluid along the $\hat{z}$ -direction, and the perturbation in the velocity fields is in the $\hat{y}$ -direction, such that $\mathbf{v} = v(z)\hat{y}$ with $\partial_z v(z) < 0$. In this case vorticity in the plasma will be in the positive $\mathbf{x}$ -direction. Therefore the local current due to vorticity will be $\mathbf{j} \propto T^2 \omega$ [156]. As a result, there will be a Lorentz force ($\mathbf{j} \times \mathbf{B}$) in the $\mathbf{y}$ -direction. This is in the opposite direction of fluid velocity and so it will act as a restoring force and makes the perturbed velocity $\mathbf{v}$ to oscillate. This is the basic understanding of the waves in the chiral plasma. We have seen that stress-energy tensor and the axial currents follow following conservation equations

5.4 Results and Discussions

In this chapter we have discussed the transport phenomenon of viscous parity odd chiral plasma. We have shown that modes with wave number $k < \xi^B$ grow rapidly. This is known as Chiral Plasma Instability [96]. Modes with $k \sim \xi^B/2$ grows maximally and the growth rate is proportional to the $|\omega|_{max} \approx \pi \xi^{B^2}/2\sigma_0$. In the case of turbulent plasma (for example at the time of EW phase transition, turbulence can be generated), the evolution of the fluid becomes more complicated and one can consider that the system is in quasi-equilibrium. We did linear analysis of the viscous hydro with first and second order terms. We analyse two cases where perturbation in the magnetic field is either normal to the background field or it is parallel to the background field. In the first case, when $\delta\tilde{B} \parallel \hat{n}$, modes damp because of two dissipative phenomenon: finite conductivity and viscosity. However in the same case, when $\delta\tilde{v} \parallel \tilde{k}$ in the absence of any viscous force, modes evolve with the sound speed. In the second case when $\delta\tilde{B} \perp \hat{n}$, and there is no viscosity, modes move either with the speed of sound or with Alfvén speed [155] depending on the fluid flow direction with respect to the propagation vector. One case is especially very interesting, when $\delta\tilde{B} \perp \hat{n}$, and $\tilde{k} \parallel \hat{n}$, $\delta\tilde{v} \parallel \tilde{k}$. For the case of $\tilde{k} < 1 \Rightarrow k < \xi^B$, there will be instability in the plasma (see eq.(5.52) for '+' sign). This kind of phenomenon occurs only in the chiral plasma.

So far we have discussed magnetohydrodynamics of chiral plasma considering only first order derivative expansion terms in the energy momentum tensor. And we have shown that our results are consistent with some other works for chiral plasma [96, 108]. Next we did the linear analysis of the second order hydrodynamics for the transverse modes. In eq.(5.65), $\pm$ sign depends on the helicity of the fields. The third term is similar to the form obtained in AdS/CFT computations for N=4 super Yang-Mills (SYM) with $U(1)_R$ symmetry [190]. First term is the oscillatory term, however higher order terms are clearly damping terms. In comparison to the first order hydro for the transverse motion, we have an additional term proportional to k^3 . The chiral shear wave was first studied in refs. [181, 190] for strongly interacting plasma using AdS/CFT correspondence. In Ref. [181], authors have discussed a sound mode in a strongly interacting N=4 SYM theory,

with $AdS_5 \times S_5$ background, and the dispersion relations are

$$\omega = \pm \frac{k}{\sqrt{3}} + \frac{ik^2}{6\pi T} \pm \frac{3 - 2\ln 2}{6\sqrt{3}(2\pi T)^2} k^3 + \mathcal{O}(k^4) \quad (5.66)$$

As best values of the transport coefficients were calculated using the AdS/CFT correspondence, it is a good idea to calculate some of the transport coefficients by comparing our dispersion relation with the dispersion relations obtained by AdS/CFT correspondence. So comparing eq.(5.65), one can see that for a neutral plasma

$$\lambda_{1(0)} \propto \frac{3 - 2\ln 2}{3 \times 2^7 \pi^4} \left(\frac{\eta_{(0)}}{T} \right) \quad (5.67)$$

We have obtained this proportionality relation, using the condition that shear viscosity to entropy must be larger than or equals to $1/4\pi$. This is the estimates of one of the transport coefficients of the second order hydro. Clearly higher order contributions have very little impact on the shear motion of the fluid. We have seen in our calculations that it is very hard to calculate other transport coefficients simply by comparing two dispersion relations. So we have left it for our future work.

To conclude the chapter, we have shown using first and second order conformal hydrodynamics that our results are consistent with the previous results. In the case of second order conformal hydrodynamics, for the chiral plasma, we have shown that the transport coefficients related with the second order conformal hydrodynamics actually contribute to the dispersion relations. We also calculated one of the transport coefficients related with the second order conformal hydrodynamics by comparing our dispersion relation with the result obtained from AdS/CFT correspondence. The work discussed in this chapter is done using conformal hydro. It is interesting to look at non-conformal theories for the chiral plasma using first and second order hydrodynamics for a parity odd plasma.

Chapter 6

Summary and Future Directions

6.1 Summary

The main goal of this thesis is to study the generation and evolution of the Primordial Magnetic fields in presence of Abelian anomaly in the early Universe. In particular we have focused on the generation and dynamics of magnetic field in a parity violating chiral plasma, where there is finite chiral asymmetry in the number density of the left and right handed particles. Chapter wise summary of the thesis is given below

Chapter 1 : In this chapter, we have briefly reviewed the previous work done on the generation and evolution of the magnetic fields. We have also discussed in brief about observations and current bounds. In the end of this chapter, we discussed our motivation behind this study.

Chapter 2 : This chapter contains the theoretical foundation of our work. In the first half part of this chapter, we gave a very brief theory of the Kinetic theory of the classical plasma as well as relativistic plasma. In the later part of the chapter, kinetic theory modified with the Berry curvature has been given.

Chapter 3: In this chapter, our aim is to discuss the generation of magnetic field above EW scale in presence of Abelian anomaly, using kinetic theory, modified with Berry curvature. We have shown that the Abelian anomaly can be incorporated in the kinetic theory by considering a modified form of the kinetic equations with Berry curvature. We studied with this modification that, in presence of the

chiral asymmetry there can be instability in the chiral plasma and which leads to the generation of turbulence and magnetic fields above a typical temperature $T \sim 80$ TeV in both collision dominated regime and collisionless regime.

Chapter 4: In this chapter, we have discussed the generation of magnetic field due to gravitational anomaly which induces a term $\propto T^2$ in the vorticity current. The salient feature of this seed magnetic field is that it will be produced at high temperature irrespective of the whether fluid is charge or neutral. Once seed magnetic field is generated, we have studied its subsequent evolution in a plasma which have chiral asymmetry.

Chapter 5 : In this chapter we examine the evolution of the magnetic fields in an expanding fluid of matter and radiation with particular interest in the evolution of cosmic magnetic fields. In the context of chiral plasma at high energies, there exist new kind of collective modes [*e.g.*, the chiral Alfvén waves] in the MHD limit and these modes exist in addition to the usual modes in the standard parity even plasma. Moreover, it has been shown that chiral plasmas exhibit a new type of density waves in presence of either of an external magnetic field or vorticity and they are respectively known as Chiral Magnetic Wave (CMW) and the Chiral Vortical Wave (CVW). In this regard we have investigated the collective modes in a magnetized chiral plasma and the damping mechanisms of these modes using first order and second hydrodynamics. Using first order conformal hydro, we obtained previously derived modes in the chiral plasma. However we show in addition that these modes get split into two modes in presence of the first order viscous term. By using second order conformal magnetohydrodynamics, we show that there are a series of terms in the dispersion relation and these terms are in accordance with the results obtained using AdS/CFT correspondence. We also calculated one of the transport coefficients related with the second order magnetohydrodynamics.

6.2 Future Direction

In future, we would like to work on another possible aspect of the chiral plasma related with the chiral instability.

- It is known that, bubble nucleation, expansion and collision during the first order phase transition, leads to the generation of turbulence in the plasma. In presence of the magnetic fields and turbulence authors of the ref. [197–205] showed that magnetic field and turbulence can act as a source of gravitational waves. However in the case of chiral plasma with finite chiral imbalance, for certain wave vector k , modes of the magnetic fields grows exponentially and leads to the instability in the chiral plasma and hence generate magnetic field together with turbulence. In references [204, 205], authors have shown that, during EW and QCD phase transitions a circularly polarized GWs can be generated.

It is known that cosmic magnetic field, if present, can generate anisotropic stress in the plasma and hence, can act as a source of gravitational waves. On the other hand, cosmic magnetic can be generated even at very temperature, much above electroweak scale, due to gravitational anomaly. Such magnetic field, in its due course of evolution, generates instability in the chiral plasma. So, our proposed work is that, we will discuss the generation of gravitational waves due to turbulence in the chiral plasma sourced by the magnetic field generated due to gravitational anomaly. This kind of gravitational waves will have unique spectrum. We estimate the amplitude and frequency of such gravitational waves.

Chapter 7

Appendix

Chiral Magnetic effects

In presence of chiral imbalance of the chiral particles, the separation of electrical charge along the external magnetic fields is known as chiral magnetic effects [60, 206]. It is a quantum effect and is a manifestation of chiral anomaly. The anomaly refers to the case when a classical symmetry of the theory is broken by quantum effects. For example, in the chiral limit in QCD, the massless quarks possess the chiral and scale invariances, which leads to the conservation of axial and vector currents by Noether's theorem. However this current conservation breaks when a gauge field is present. This leads to the breaking of chiral symmetry and the scale invariance gives a spectacular consequence for the hadron spectrum. The fermions with different chiralities have opposite contribution to the triangle diagram and hence in total vector current ($J_v = J_R + J_L$) their effects are absent. However, for the axial current ($J_A = J_R - J_L$), their interaction with the gauge fields, leads to the non-conservation as: $\partial_\mu \mu J_A^\mu \propto \mathbf{E} \cdot \mathbf{B}$ (where $\mathbf{E}$ and $\mathbf{B}$ are the external electric and magnetic fields). It is to be noted here that the key to the CME is the presence of chiral chemical potential $\mu_A \propto (\mu_R - \mu_L)$ and even though the global electric charge Q remains conserved. This can be seen through the following equation,

$$\frac{dQ}{dt} = - \int_S j_v d\mathbf{S} = 0. \quad (7.1)$$

Here S represents the boundary of a closed domain, where $\mu \neq 0$.

Chiral vortical effects

A phenomenon similar to the CME arises, when there is an angular momentum (vorticity) is present instead of external magnetic fields. This phenomenon is known as Chiral Vortical Effect (CVE) [[156](#), [175](#), [177](#), [207](#), [208](#)].

Bibliography

- [1] K.-T. Kim, P. C. Tribble, and P. P. Kronberg, *Detection of excess rotation measure due to intracluster magnetic fields in clusters of galaxies*, *Astrophys. J.* **379** (sep, 1991) 80–88.
- [2] M. L. Anne-Christine Davis and O. Tornkvist, *Relaxing the bounds on primordial magnetic seed fields*, *Phys. Rev. D* **60** (Jun, 1999) 021301.
- [3] A. Ruzmaikin, D. Sokoloff, and A. Shukurov, *Magnetic Fields of Galaxies*. Springer Netherlands, Astrophysics and Space Science Library, 2013.
- [4] R. Banerjee and K. Jedamzik, *The evolution of cosmic magnetic fields: From the very early universe, to recombination, to the present*, *Phys. Rev. D* **70** (2004) 123003, [[astro-ph/0410032](#)].
- [5] K. Subramanian, D. Narasimha, and S. M. Chitre, *Thermal generation of cosmological seed magnetic fields in ionization fronts*, *mnras* **271** (Nov., 1994).
- [6] R. M. Kulsrud, R. Cen, J. P. Ostriker, and D. Ryu, *The protogalactic origin for cosmic magnetic fields*, *Astrophys.J.* **480** (1997) 481.
- [7] A. M. Howard and R. M. Kulsrud, *The evolution of a primordial galactic magnetic field*, *The Astrophysical Journal* **483** (1997), no. 2 648.
- [8] S. Naoz and R. Narayan, *Generation of primordial magnetic fields on linear over-density scales*, *Phys.Rev.Lett.* **111** (2013) 051303, [[arXiv:1304.5792](#)].

- [9] R. Durrer and A. Neronov, *Cosmological magnetic fields: Their generation, evolution and observation*, *Astron. Astrophys. Rev.* **21** (2013) 62, [[arXiv:1303.7121](#)].
- [10] D. Grasso and H. R. Rubinstein, *Magnetic fields in the early universe*, *Phys. Rept.* **348** (2001) 163–266, [[astro-ph/0009061](#)].
- [11] G. Davies and L. M. Widrow, *A possible mechanism for generating galactic magnetic fields*, *The Astrophysical Journal* **540** (2000), no. 2 755.
- [12] A. A. R. Ya. B. Zeldovich and D. D. Sokoloff, *Magnetic fields in astrophysics*. Astrophysics and Space Science Library. Gordon & Breach Science Pub, 1990.
- [13] L. M. Widrow, *Origin of galactic and extragalactic magnetic fields*, *Rev. Mod. Phys.* **74** (2002) 775–823.
- [14] R. M. Kulsrud and E. G. Zweibel, *The origin of astrophysical magnetic fields*, *Rept. Prog. Phys.* **71** (2008) 0046091, [[arXiv:0707.2783](#)].
- [15] A. Kandus, K. E. Kunze, and C. G. Tsagas, *Primordial magnetogenesis*, *Physics Reports* **505** (2011), no. 1 1 – 58.
- [16] L. Biermann, *Über den ursprung der magnetfelder auf sternem und im interstellaren raum (miteinem anhang von a. schluter) (on the origin of the magnetic fields on stars and in interstellar space)*, *Z. Naturforschg* (1950), no. 5a 65–71.
- [17] M. Ando, K. Doi, and H. Susa, *Generation of seed magnetic field around first stars: Effects of radiation force*, *The Astrophysical Journal* **716** (2010), no. 2 1566.
- [18] N. Y. Gnedin, A. Ferrara, and E. G. Zweibel, *Generation of the primordial magnetic fields during cosmological reionization*, *Astrophys. J.* **539** (2000) 505–516, [[astro-ph/0001066](#)].
- [19] J. Cho, *Origin of magnetic field in the intrachuster medium: Primordial or astrophysical?*, *Astrophys. J.* **797** (2014), no. 2 133, [[arXiv:1410.1893](#)].

- [20] E. R. Harrison, *Generation of magnetic fields in the radiation era*, *mnras* **147** (1970) 279.
- [21] E. R. Harrison, *Origin of magnetic fields in the early universe*, *Phys. Rev. Lett.* **30** (Jan, 1973) 188–190.
- [22] E. R. Harrison, *Baryon inhomogeneity in the early universe*, *Phys. Rev.* **167** (Mar, 1968) 1170–1175.
- [23] E. R. Harrison, *Matter, antimatter, and the origin of galaxies*, *Phys. Rev. Lett.* **18** (Jun, 1967) 1011–1013.
- [24] T. Vachaspati and A. Vilenkin, *Large-scale structure from wiggly cosmic strings*, *Phys. Rev. Lett.* **67** (Aug, 1991) 1057–1061.
- [25] A. Vilenkin, *Parity nonconservation and rotating black holes*, *Phys. Rev. Lett.* **41** (Nov, 1978) 1575–1577.
- [26] A. Rebhan, *Large-scale rotational perturbations of a Friedmann universe with collisionless matter and primordial magnetic fields*, *Astrophys. J.* **392** (June, 1992) 385–393.
- [27] V. L. BIERMANN, *On the origin of the magnetic fields on stars and in interstellar space*, *Z. Naturforschg* **5a** (Jul, 1950) 65–67.
- [28] M. Steenbeck, F. Krause, and K.-H. Radler, *Berechnung der mittleren LORENTZ-Feldstarke fur ein elektrisch leitendes Medium in turbulenter, durch CORIOLIS-Krafte beeinfluter Bewegung*, *Zeitschrift Naturforschung Teil A* **21** (Apr., 1966) 369.
- [29] E. N. Parker, *The origin of magnetic fields*, *Astrophys. J.* **160** (May, 1970) 383.
- [30] S. Vainshtein and F. Cattaneo, *Nonlinear restrictions on dynamo action*, *Astrophys. J.* **393** (July, 1992) 165–171.

- [31] R. M. Kulsrud, *A critical review of galactic dynamos*, *Annual Review of Astronomy and Astrophysics* **37** (1999), no. 1 37–64, [<http://dx.doi.org/10.1146/annurev.astro.37.1.37>].
- [32] A. Dolgov, *Breaking of conformal invariance and electromagnetic field generation in the universe*, *Phys. Rev.* **D48** (1993) 2499–2501, [[hep-ph/9301280](http://arxiv.org/abs/hep-ph/9301280)].
- [33] M. S. Turner and L. M. Widrow, *Inflation-produced, large-scale magnetic fields*, *Phys. Rev. D* **37** (1988) 2743–2754.
- [34] B. Ratra, *Cosmological 'seed' magnetic field from inflation*, *Astrophys. J.* **391** (1992) L1–L4.
- [35] M. Gasperini, M. Giovannini, and G. Veneziano, *Primordial magnetic fields from string cosmology*, *Phys. Rev. Lett.* **75** (1995) 3796–3799.
- [36] M. Giovannini and M. E. Shaposhnikov, *Primordial magnetic fields from inflation?*, *Phys. Rev.* **D62** (2000) 103512, [[hep-ph/0004269](http://arxiv.org/abs/hep-ph/0004269)].
- [37] J. Martin and J. Yokoyama, *Generation of large scale magnetic fields in single-field inflation*, *jcap* **1** (Jan., 2008) 025, [[arXiv:0711.4307](http://arxiv.org/abs/0711.4307)].
- [38] K. Subramanian, *Magnetic fields in the early universe*, *Astron. Nachr.* **331** (2010) 110–120, [[arXiv:0911.4771](http://arxiv.org/abs/0911.4771)].
- [39] T. Fujita and K. Kamada, *Large-scale magnetic fields can explain the baryon asymmetry of the universe*, *Phys. Rev. D* **93** (Apr, 2016) 083520.
- [40] C. J. Hogan, *Magnetohydrodynamic effects of a first-order cosmological phase transition*, *Phys. Rev. Lett.* **51** (Oct, 1983) 1488–1491.
- [41] A. Dolgov and J. Silk, *Electric charge asymmetry of the universe and magnetic field generation*, *Phys. Rev.* **D47** (1993) 3144–3150.
- [42] M. Joyce and M. E. Shaposhnikov, *Primordial magnetic fields, right-handed electrons, and the abelian anomaly*, *Phys. Rev. Lett.* **79** (1997) 1193–1196, [[astro-ph/9703005](http://arxiv.org/abs/astro-ph/9703005)].

- [43] S. M. Carroll, G. B. Field, and R. Jackiw, *Limits on a lorentz and parity violating modification of electrodynamics*, *Phys. Rev.* **D41** (1990) 1231.
- [44] A. Vilenkin, *Cosmic string dynamics with friction*, *Phys. Rev.* **D43** (1991) 1060–1062.
- [45] A. Vilenkin, *Topological defects and open inflation*, *Phys. Rev.* **D56** (1997) 3238–3241, [[astro-ph/9703201](#)].
- [46] T. Vachaspati, *Magnetic fields from cosmological phase transitions*, *Phys.Lett.* **B265** (1991) 258261.
- [47] K. Enqvist and P. Olesen, *On primordial magnetic fields of electroweak origin*, *Phys. Lett.* **B319** (1993) 178–185, [[hep-ph/9308270](#)].
- [48] T. W. B. Kibble and A. Vilenkin, *Phase equilibration in bubble collisions*, *Phys. Rev. D* **52** (1995) 679–688.
- [49] J. M. Q. A. Loeb and D. N. Spergel, *Field generation during the cosmological qcd phase transition*, *ApJ* **344** (sep 1989) L49–L51.
- [50] J. M. Cornwall, *Speculations on primordial magnetic helicity*, *Phys. Rev. D* **56** (Nov, 1997) 6146–6154.
- [51] G. Denardo and E. Spallucci, *GAUGE FIELDS IN $S(1) \times R(3)$* , *Nuovo Cim.* **A58** (1980) 243–253.
- [52] G. M. Shore, *Radiatively Induced Spontaneous Symmetry Breaking and Phase Transitions in Curved Space-Time*, *Annals Phys.* **128** (1980) 376.
- [53] D. J. Toms, *Interacting Twisted and Untwisted Scalar Fields in a Nonsimply Connected Space-time*, *Annals Phys.* **129** (1980) 334.
- [54] S. L. Cheng, C. Q. Geng, and W. T. Ni, *Axion - photon couplings in invisible axion models*, *Phys. Rev.* **D52** (1995) 3132–3135, [[hep-ph/9506295](#)].
- [55] V. Demozzi, V. Mukhanov, and H. Rubinstein, *Magnetic fields from inflation?*, *JCAP* **0908** (2009) 025, [[arXiv:0907.1030](#)].

- [56] L. Parker, *Particle creation in expanding universes*, *Phys. Rev. Lett.* **21** (Aug, 1968) 562–564.
- [57] G. Baym, D. Bodeker, and L. McLerran, *Magnetic fields produced by phase transition bubbles in the electroweak phase transition*, *Phys. Rev. D* **53** (Jan, 1996) 662–667.
- [58] R. Durrer and C. Caprini, *Primordial magnetic fields and causality*, *JCAP* **0311** (2003) 010, [[astro-ph/0305059](#)].
- [59] K. E. Kunze, *Effects of helical magnetic fields on the cosmic microwave background*, *Phys. Rev. D* **85** (Apr, 2012) 083004.
- [60] D. E. Kharzeev, *The chiral magnetic effect and anomaly-induced transport*, *Prog. Part. Nucl. Phys.* **75** (2014) 133–151, [[arXiv:1312.3348](#)].
- [61] J. Ahonen and K. Enqvist, *Magnetic field generation in first order phase transition bubble collisions*, *prd* **57** (Jan., 1998) 664–673, [[hep-ph/9704334](#)].
- [62] E. Copeland, P. Saffin, and O. Tornkvist, *Phase equilibration and magnetic field generation in $U(1)$ bubble collisions*, *prd* **61** (May, 2000) 105005, [[hep-ph/9907437](#)].
- [63] P. Saffin and E. Copeland, *Bubble collisions in $SU(2) \times U(1)$ gauge theory and the production of nontopological strings*, *prd* **56** (July, 1997) 1215–1219, [[hep-th/9702034](#)].
- [64] M. Giovannini, *Anomalous magnetohydrodynamics*, *Phys. Rev.* **D88** (2013) 063536, [[arXiv:1307.2454](#)].
- [65] G. Sigl, A. V. Olinto, and K. Jedamzik, *Primordial magnetic fields from cosmological first order phase transitions*, *Phys. Rev.* **D55** (1997) 4582–4590, [[astro-ph/9610201](#)].
- [66] M. M. Forbes and A. Zhitnitsky, *Primordial galactic magnetic fields from domain walls at the qcd phase transition*, *Phys. Rev. Lett.* **85** (Dec, 2000) 5268–5271.

- [67] J. M. Quashnock, A. Loeb, and D. N. Spergel, *Magnetic field generation during the cosmological qcd phase transition*, *apjl* **344** (Sept., 1989) L49–L51.
- [68] A. G. Tevzadze, L. Kisslinger, A. Brandenburg, and T. Kahniashvili, *Magnetic fields from qcd phase transitions*, *The Astrophysical Journal* **759** (2012), no. 1 54.
- [69] B.-l. Cheng and A. V. Olinto, *Primordial magnetic fields generated in the quark-hadron transition*, *Phys. Rev.* **D50** (1994) 2421–2424.
- [70] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, *Anomalous electroweak baryon number nonconservation and gut mechanism for baryogenesis*, *Phys. Lett.* **B191** (1987) 171.
- [71] N. Turok and J. Zadrozny, *Dynamical generation of baryons at the electroweak transition*, *Phys. Rev. Lett.* **65** (Nov, 1990) 2331–2334.
- [72] E. Nardi, Y. Nir, J. Racker, and E. Roulet, *On higgs and sphaleron effects during the leptogenesis era*, *Journal of High Energy Physics* **2006** (2006), no. 01 068.
- [73] A. J. Long, E. Sabancilar, and T. Vachaspati, *Leptogenesis and primordial magnetic fields*, *JCAP* **2014** (2014), no. 02 036.
- [74] L. M. Krauss and M. Trodden, *Baryogenesis below the electroweak scale*, *Phys. Rev. Lett.* **83** (Aug, 1999) 1502–1505.
- [75] A. Brandenburg and K. Subramanian, *Astrophysical magnetic fields and nonlinear dynamo theory*, *Phys. Rept.* **417** (2005) 1–209, [[astro-ph/0405052](#)].
- [76] A. Brandenburg, K. Enqvist, and P. Olesen, *Large scale magnetic fields from hydromagnetic turbulence in the very early universe*, *Phys. Rev.* **D54** (1996) 1291–1300, [[astro-ph/9602031](#)].

- [77] U. Frisch, A. Pouquet, J. Leorat, and A. Mazure, *Possibility of an inverse cascade of magnetic helicity in magnetohydrodynamic turbulence*, *Journal of Fluid Mechanics* **68** (Apr., 1975) 769–778.
- [78] A. Brandenburg, T. Kahniashvili, and A. G. Tevzadze, *Nonhelical inverse transfer of a decaying turbulent magnetic field*, *Phys. Rev. Lett.* **114** (2015), no. 7 075001, [[arXiv:1404.2238](#)].
- [79] J. Zrake and W. E. East, *Freely decaying turbulence in force-free electrodynamics*, *Astrophys. J.* **817** (2016), no. 2 89, [[arXiv:1509.00461](#)].
- [80] M. F. Linkmann, A. Berera, M. E. McKay, and J. Jger, *Helical mode interactions and spectral transfer processes in magnetohydrodynamic turbulence*, *J. Fluid Mech.* **791** (2016) 61–96, [[arXiv:1508.05528](#)].
- [81] J. Reppin and R. Banerjee, *Nonhelical turbulence and the inverse transfer of energy: A parameter study*, [arXiv:1708.07717](#).
- [82] R. Beck and R. Wielebinski, *Magnetic fields in galaxies*, in *Planets, Stars and Stellar Systems: Volume 5: Galactic Structure and Stellar Populations* (T. D. Oswalt and G. Gilmore, eds.), pp. 641–723. Springer Netherlands, Dordrecht, 2013.
- [83] G. Sigl, *Cosmological magnetic fields from primordial helical seeds*, *Phys. Rev. D* **66** (Dec, 2002) 123002.
- [84] T. Vachaspati, *Estimate of the primordial magnetic field helicity*, *Phys. Rev. Lett.* **87** (2001) 251302, [[astro-ph/0101261](#)].
- [85] T. Kahniashvili, A. G. Tevzadze, A. Brandenburg, and A. Neronov, *Evolution of primordial magnetic fields from phase transitions*, *Phys. Rev. D* **87** (2013), no. 8 083007, [[arXiv:1212.0596](#)].
- [86] T. Kahniashvili, A. Brandenburg, and A. G. Tevzadze, *Evolution of primordial magnetic fields: From generation till today*, in *ICTP Summer School on Cosmology 2014 Trieste, Italy, August 4-21, 2014*, 2015. [arXiv:1507.00510](#).

- [87] M. Christensson, M. Hindmarsh, and A. Brandenburg, *Inverse cascade in decaying 3-d magnetohydrodynamic turbulence*, *Phys. Rev.* **E64** (2001) 056405, [[astro-ph/0011321](#)].
- [88] C. G. Callan, R. Dashen, and D. J. Gross, *Toward a theory of the strong interactions*, *Phys. Rev. D* **17** (May, 1978) 2717–2763.
- [89] G. 't Hooft, *Symmetry breaking through bell-jackiw anomalies*, *Phys. Rev. Lett.* **37** (1976) 8–11.
- [90] G. 't Hooft, *Computation of the quantum effects due to a four-dimensional pseudoparticle*, *Phys. Rev. D* **14** (Dec, 1976) 3432–3450.
- [91] A. I. Bochkaev and M. E. Shaposhnikov, *Electroweak production of baryon asymmetry and upper bounds on the higgs and top masses*, *Mod. Phys. Lett.* **A2** (1987) 417.
- [92] J. A. Harvey and M. S. Turner, *Cosmological baryon and lepton number in the presence of electroweak fermion-number violation*, *Phys. Rev. D* **42** (Nov, 1990) 3344–3349.
- [93] M. Fukugita and T. Yanagida, *Baryogenesis without grand unification*, *Physics Letters B* **174** (1986), no. 1 45 – 47.
- [94] S. Yu. Khlebnikov and M. E. Shaposhnikov, *The Statistical Theory of Anomalous Fermion Number Nonconservation*, *Nucl. Phys.* **B308** (1988) 885–912.
- [95] M. Giovannini and M. E. Shaposhnikov, *Primordial hypermagnetic fields and triangle anomaly*, *Phys. Rev.* **D57** (1998) 2186–2206, [[hep-ph/9710234](#)].
- [96] Y. Akamatsu and N. Yamamoto, *Chiral plasma instabilities*, *Phys.Rev.Lett.* **111** (2013) 052002, [[arXiv:1302.2125](#)].
- [97] K. Brecher and G. R. Blumenthal, *On the Origin of Cosmic Magnetic Fields*, *aplett* **6** (July, 1970) 169.

- [98] G. Baym and H. Heiselberg, *Electrical conductivity in the early universe*, *Phys. Rev. D* **56** (Oct, 1997) 5254–5259.
- [99] A. Neronov and D. V. Semikoz, *Sensitivity of γ -ray telescopes for detection of magnetic fields in the intergalactic medium*, *Phys. Rev. D* **80** (Dec, 2009) 123012. bound from IGM.
- [100] P. Blasi, S. Burles, and A. V. Olinto, *Cosmological magnetic fields limits in an inhomogeneous universe*, *Astrophys. J.* **514** (1999) L79–L82, [[astro-ph/9812487](#)].
- [101] M. Giovannini, *Primordial magnetic fields*, in *7th Paris Cosmology Colloquium on High Energy Astrophysics for and from Space Paris, France, June 11-15, 2002*, 2002. [hep-ph/0208152](#).
- [102] A. Neronov and I. Vovk, *Evidence for strong extragalactic magnetic fields from fermi observations of tev blazars*, *Science* **328** (Apr., 2010) 73, [[arXiv:1006.3504](#)].
- [103] Greenstein, *Primordial helium production in magnetic cosmologies*, *Nature* **233** (1969).
- [104] J. Matese and R. F. O’Connell, *Production of Helium in the Big-Bang Expansion of a Magnetic Universe*, *The Astrophysical Journal* **160** (May, 1970) 451.
- [105] T. Kahniashvili, Y. Maravin, A. Natarajan, N. Battaglia, and A. G. Tevzadze, *Constraining primordial magnetic fields through large scale structure*, *Astrophys. J.* **770** (2013) 47, [[arXiv:1211.2769](#)].
- [106] **Planck** Collaboration, P. A. R. Ade et al., *Planck 2015 results. XIX. Constraints on primordial magnetic fields*, *Astron. Astrophys.* **594** (2016) A19, [[arXiv:1502.01594](#)].
- [107] H. Tashiro, T. Vachaspati, and A. Vilenkin, *Chiral effects and cosmic magnetic fields*, *Phys. Rev.* **D86** (2012) 105033, [[arXiv:1206.5549](#)].

- [108] D. T. Son and P. Surówka, *Hydrodynamics with triangle anomalies*, *Phys. Rev. Lett.* **103** (Nov, 2009) 191601.
- [109] P. M. Julio Puerta, *Plasma Physics: Proceedings of the 1997 Latin American Workshop : VII LAWPP 97*. Springer, Springer, ISBN 079235527X, Page 57, 1999.
- [110] R. Fitzpatrick, *The Physics of Plasmas*. Lulu Inc., Raleigh, 2011.
- [111] G. Manfredi, *How to model quantum plasmas*. Fields Inst. Commun. 46, 263287, 2005.
- [112] A. Jungel, *Transport Equations for Semiconductor Devices*. Springer, Berlin-Heidelberg, 2009.
- [113] M. Di Ventra, *Electrical Transport in Nanoscale Systems*. Cambridge, New York, 2008.
- [114] H. D. TER, **Die Kinetische Gleichung fur den fall Coulombscher physik. z. sowjetunion 10, 154 (1936).** [english translation **the transport equation in the case of coulomb interactions**], in *Collected Papers of L.D. Landau* (D. T. HAAR, ed.). Pergamon, 1965.
- [115] A. Lenard, *On bogoliubov's kinetic equation for a spatially homogeneous plasma*, *Annals of Physics* **10** (1960), no. 3 390 – 400.
- [116] R. Balescu, *Irreversible processes in ionized gases*, *Physics of Fluids* **3** (1960), no. 1 52–63.
- [117] L. D. Landau, *On the vibrations of the electronic plasma*, *J. Phys. (USSR)* **10** (1946) 25–34. [Zh. Eksp. Teor. Fiz.16,574(1946)].
- [118] F. Jttner, *Das marxwellsche gesetz der geschwindigkeitsverteilung in der relativtheorie*, *Annalen der Physik* **339** (1911), no. 5 856–882.
- [119] G. M. K. Carlo Cercignani, *The Relativistic Boltzmann Equation: Theory and Applications (Birkhauser Basel 2002)*. Birkhauser Basel, 2002.

- [120] S. L. Adler, *Axial-vector vertex in spinor electrodynamics*, *Phys. Rev.* **177** (Jan, 1969) 2426–2438.
- [121] J. S. Bell and R. Jackiw, *A pcac puzzle: $\pi^0 \rightarrow \gamma\gamma$ in the σ model*, *Il Nuovo Cimento A (1965-1970)* **60** (1969), no. 1 47–61.
- [122] D. T. Son and N. Yamamoto, *Kinetic theory with berry curvature from quantum field theories*, *Phys. Rev. D* **87** (Apr, 2013) 085016.
- [123] M. V. Berry, *Quantal phase factors accompanying adiabatic changes*, *Proc. Roy. Soc. Lond.* **A392** (1984) 45–57.
- [124] D. J. Thouless, *Topological Quantum Numbers in Nonrelativistic Physics* (World Scientific, Singapore, 1998). World Scientific, 1998.
- [125] G. Sundaram and Q. Niu, *Wave-packet dynamics in slowly perturbed crystals: Gradient corrections and berry-phase effects*, *Phys. Rev.* **B59** (1999) 14915–14925.
- [126] C. Duval, Z. Horváth, P. A. Horváthy, L. Martina, and P. C. Stichel, *Berry phase correction to electron density in solids and "exotic" dynamics*, *Modern Physics Letters B* **20** (2006) 373–378, [[cond-mat/0506051](#)].
- [127] D. Xiao, J. Shi, and Q. Niu, *Berry phase correction to electron density of states in solids*, *Phys. Rev. Lett.* **95** (Sep, 2005) 137204.
- [128] J.-W. Chen, S. Pu, Q. Wang, and X.-N. Wang, *Berry curvature and four-dimensional monopoles in the relativistic chiral kinetic equation*, *Phys. Rev. Lett.* **110** (Jun, 2013) 262301.
- [129] M. A. Stephanov and Y. Yin, *Chiral kinetic theory*, *Phys. Rev. Lett.* **109** (2012) 162001, [[arXiv:1207.0747](#)].
- [130] S. L. Adler, C. G. Callan, D. J. Gross, and R. Jackiw, *Constraints on anomalies*, *Phys. Rev. D* **6** (Nov, 1972) 2982–2988.
- [131] H. Itoyama and A. H. Mueller, *The axial anomaly at finite temperature*, *Nucl.Phys.* **B218** (1983) 349.

- [132] Y.-l. Liu and G.-j. Ni, *Chiral anomaly at finite temperature*, *Phys. Rev. D* **38** (Dec, 1988) 3840–3842.
- [133] D. T. Son and N. Yamamoto, *Berry curvature, triangle anomalies, and the chiral magnetic effect in fermi liquids*, *Phys. Rev. Lett.* **109** (2012) 181602, [[arXiv:1203.2697](#)].
- [134] S. Pu, J.-h. Gao, and Q. Wang, *Consistent description of kinetic equation with triangle anomaly*, *Phys. Rev. D* **83** (May, 2011) 094017.
- [135] A. D. Sakharov, *Violation of CP Invariance, c Asymmetry, and Baryon Asymmetry of the Universe*, *Pisma Zh. Eksp. Teor. Fiz.* **5** (1967) 32–35. [*Usp. Fiz. Nauk*161,61(1991)].
- [136] E. W. Kolb and M. S. Turner, *The Early Universe*. Perseus Books Group, 1994.
- [137] M. Yoshimura, *Unified gauge theories and the baryon number of the universe*, *Phys. Rev. Lett.* **41** (Jul, 1978) 281–284.
- [138] B. A. Campbell, S. Davidson, J. R. Ellis, and K. A. Olive, *On the baryon, lepton flavor and right-handed electron asymmetries of the universe*, *Phys.Lett.* **B297** (1992) 118–124.
- [139] V. B. Semikoz and D. D. Sokoloff, *Magnetic helicity and cosmological magnetic field*, *Astron. Astrophys.* **433** (2005) L53, [[astro-ph/0411496](#)].
- [140] D. T. Son and B. Z. Spivak, *Chiral anomaly and classical negative magnetoresistance of weyl metals*, *Phys. Rev.* **B88** (2013) 104412, [[arXiv:1206.1627](#)].
- [141] C. Manuel and J. M. Torres-Rincon, *Kinetic theory of chiral relativistic plasmas and energy density of their gauge collective excitations*, *Phys. Rev. D* **89** (May, 2014) 096002.
- [142] A. J. Silenko, *Foldy-wouthyusen transformation and semiclassical limit for relativistic particles in strong external fields*, *Phys.Rev.* **A77** (2008) 012116.

- [143] C. P. Dettmann, N. E. Frankel, and V. Kowalenko, *Plasma electrodynamics in the expanding universe*, *Phys. Rev. D* **48** (1993) 5655–5667.
- [144] K. A. Holcomb and T. Tajima, *General-relativistic plasma physics in the early universe*, *Phys. Rev. D* **40** (Dec, 1989) 3809–3818.
- [145] R. M. Gailis, C. P. Dettmann, N. E. Frankel, and V. Kowalenko, *Magnetized plasmas in the early universe*, *Phys. Rev. D* **50** (Sep, 1994) 3847–3858.
- [146] R. M. Gailis, N. E. Frankel, and C. P. Dettmann, *Magnetohydrodynamics in the expanding universe*, *Phys. Rev. D* **52** (Dec, 1995) 6901–6917.
- [147] D. Grabowska, D. B. Kaplan, and S. Reddy, *Role of the electron mass in damping chiral plasma instability in supernovae and neutron stars*, *Phys. Rev. D* **91** (Apr, 2015) 085035.
- [148] A. Boyarsky, J. Frohlich, and O. Ruchayskiy, *Self-consistent evolution of magnetic fields and chiral asymmetry in the early universe*, *Phys. Rev. Lett.* **108** (2012) 031301, [[arXiv:1109.3350](#)].
- [149] J.-Y. Chen, D. T. Son, M. A. Stephanov, H.-U. Yee, and Y. Yin, *Lorentz invariance in chiral kinetic theory*, *Phys.Rev.Lett.* **113** (2014), no. 18 182302, [[arXiv:1404.5963](#)].
- [150] J.-Y. Chen, D. T. Son, and M. A. Stephanov, *Collisions in chiral kinetic theory*, *Phys. Rev. Lett.* **115** (Jul, 2015) 021601.
- [151] A. Gomez Nicola and R. Alvarez-Estrada, *Anomalies at finite temperature and density*, *Int.J.Mod.Phys.* **A9** (1994) 1423–1442.
- [152] A. Boyarsky, O. Ruchayskiy, and M. Shaposhnikov, *Long-range magnetic fields in the ground state of the standard model plasma*, *Phys. Rev. Lett.* **109** (2012) 111602, [[arXiv:1204.3604](#)].
- [153] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, *Chiral magnetic effect*, *Phys. Rev. D* **78** (Oct, 2008) 074033.

- [154] S. Chandrasekhar and L. Woltjer, *On force free magnetic fields*, *Proceedings of the National Academy of Sciences* **44** (1958), no. 4 285–289.
- [155] N. Yamamoto, *Chiral alfvén wave in anomalous hydrodynamics*, *Phys. Rev. Lett.* **115** (Sep, 2015) 141601.
- [156] K. Landsteiner, E. Megias, and F. Pena-Benitez, *Gravitational anomaly and transport phenomena*, *Phys. Rev. Lett.* **107** (Jul, 2011) 021601.
- [157] Y. Neiman and Y. Oz, *Relativistic hydrodynamics with general anomalous charges*, *JHEP* **2011** (2011), no. 3 23.
- [158] V. Semikoz and J. Valle, *Chern-simons anomaly as polarization effect*, *JCAP* **1111** (2011) 048, [[arXiv:1104.3106](#)].
- [159] N. Yamamoto, *Scaling laws in chiral hydrodynamic turbulence*, *Phys. Rev.* **D93** (2016), no. 12 125016.
- [160] P. Olesen, *On inverse cascades in astrophysics*, *Phys. Lett.* **B398** (1997) 321–325.
- [161] Y. Akamatsu and N. Yamamoto, *Chiral langevin theory for non-abelian plasmas*, *Phys. Rev. D* **90** (Dec, 2014) 125031.
- [162] K. Jedamzik, V. Katalinic, and A. V. Olinto, *Damping of cosmic magnetic fields*, *Phys. Rev.* **D57** (1998) 3264–3284, [[astro-ph/9606080](#)].
- [163] K. Subramanian and J. D. Barrow, *Magnetohydrodynamics in the early universe and the damping of nonlinear alfvén waves*, *Phys. Rev.* **D58** (1998) 083502, [[astro-ph/9712083](#)].
- [164] L. Landau and E. Lifshitz, *Fluid Mechanics (Second Editions)*. Pergamon Press, 1987.
- [165] D. E. Kharzeev and H.-U. Yee, *Chiral magnetic wave*, *Phys. Rev.* **D83** (2011) 085007, [[arXiv:1012.6026](#)].
- [166] G. M. Newman, *Anomalous hydrodynamics*, *JHEP* **01** (2006) 158, [[hep-ph/0511236](#)].

- [167] Y. Jiang, X.-G. Huang, and J. Liao, *Chiral vortical wave and induced flavor charge transport in a rotating quark-gluon plasma*, *Phys. Rev. D* **92** (Oct, 2015) 071501.
- [168] N. Abbasi and A. Davody, *Dissipative Charged Fluid in a Magnetic Field*, *Phys. Lett.* **B756** (2016) 161–165, [[arXiv:1508.06879](#)].
- [169] N. Abbasi, A. Davody, and Z. Rezaei, *Hydrodynamic waves in an anomalous charged fluid*, [arXiv:1509.08878](#).
- [170] T. Kalaydzhyan and E. Murchikova, *Thermal chiral vortical and magnetic waves: New excitation modes in chiral fluids*, *Nuclear Physics B* **919** (2017) 173 – 181.
- [171] J. M. Stewart, *On transient relativistic thermodynamics and kinetic theory*, *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences* **357** (1977), no. 1688 59–75.
- [172] W. Israel and J. M. Stewart, *Transient relativistic thermodynamics and kinetic theory*, *Annals Phys.* **118** (1979) 341–372.
- [173] G. S. Denicol, T. Kodama, T. Koide, and P. Mota, *Causal theory of relativistic dissipative hydrodynamics*, *Brazilian Journal of Physics* **37** (09, 2007) 1047 – 1054.
- [174] P. Romatschke, *New developments in relativistic viscous hydrodynamics*, *Int. J. Mod. Phys.* **E19** (2010) 1–53, [[arXiv:0902.3663](#)].
- [175] D. E. Kharzeev and H.-U. Yee, *Anomalies and time reversal invariance in relativistic hydrodynamics: The second order and higher dimensional formulations*, *Phys. Rev. D* **84** (Aug, 2011) 045025.
- [176] J. Erdmenger, M. Haack, M. Kaminski, and A. Yarom, *Fluid dynamics of r -charged black holes*, *Journal of High Energy Physics* **2009** (2009), no. 01 055.

- [177] N. Banerjee, S. Bhattacharya, Jyotirmoy and Bhattacharyya, S. Dutta, R. Loganayagam, and P. Surowka, *Hydrodynamics from charged black branes*, *JHEP* **01** (2011) 094, [[arXiv:0809.2596](#)].
- [178] B. Betz, D. Henkel, and D. H. Rischke, *From kinetic theory to dissipative fluid dynamics*, *Prog. Part. Nucl. Phys.* **62** (2009) 556–561, [[arXiv:0812.1440](#)].
- [179] B. Betz, D. Henkel, and D. H. Rischke, *Complete second-order dissipative fluid dynamics*, *J. Phys.* **G36** (2009) 064029.
- [180] P. Kovtun, *Lectures on hydrodynamic fluctuations in relativistic theories*, *J. Phys.* **A45** (2012) 473001, [[arXiv:1205.5040](#)].
- [181] R. Baier, P. Romatschke, D. T. Son, A. O. Starinets, and M. A. Stephanov, *Relativistic viscous hydrodynamics, conformal invariance, and holography*, *Journal of High Energy Physics* **2008** (2008), no. 04 100.
- [182] T. S. Olson and W. A. Hiscock, *Stability, causality, and hyperbolicity in carter’s “regular” theory of relativistic heat-conducting fluids*, *Phys. Rev. D* **41** (Jun, 1990) 3687–3695.
- [183] S. Pu, T. Koide, and Q. Wang, *Causality and stability of dissipative fluid dynamics with diffusion currents*, in *American Institute of Physics Conference Series*, vol. 1235 of *American Institute of Physics Conference Series*, pp. 186–192, May, 2010.
- [184] T. S. Olson and W. A. Hiscock, *Relativistic dissipative hydrodynamics and the nuclear equation of state*, *Phys. Rev.* **C39** (1989) 1818–1826.
- [185] T. Koide, G. S. Denicol, P. Mota, and T. Kodama, *Relativistic dissipative hydrodynamics: A Minimal causal theory*, *Phys. Rev.* **C75** (2007) 034909, [[hep-ph/0609117](#)].
- [186] M. A. York and G. D. Moore, *Second order hydrodynamic coefficients from kinetic theory*, *Phys. Rev. D* **79** (Mar, 2009) 054011.

- [187] P. B. Arnold, G. D. Moore, and L. G. Yaffe, *Transport coefficients in high temperature gauge theories. 1. Leading log results*, *JHEP* **11** (2000) 001, [[hep-ph/0010177](#)].
- [188] S. Bhattacharyya, V. E. Hubeny, S. Minwalla, and M. Rangamani, *Nonlinear Fluid Dynamics from Gravity*, *JHEP* **02** (2008) 045, [[arXiv:0712.2456](#)].
- [189] J. L. Mañes and M. Valle, *Parity violating gravitational response and anomalous constitutive relations*, *Journal of High Energy Physics* **2013** (2013), no. 1 8.
- [190] B. Sahoo and H.-U. Yee, *Holographic chiral shear waves from anomaly*, *Phys. Lett.* **B689** (2010) 206–212, [[arXiv:0910.5915](#)].
- [191] G. Policastro, D. T. Son, and A. O. Starinets, *Shear viscosity of strongly coupled $n = 4$ supersymmetric yang-mills plasma*, *Phys. Rev. Lett.* **87** (Aug, 2001) 081601.
- [192] A. Vilenkin, *Equilibrium parity violating current in a magnetic field*, *Phys. Rev.* **D22** (1980) 3080–3084.
- [193] A. Vilenkin, *Macroscopic parity violating effects: Neutrino fluxes from rotating black holes and in rotating thermal radiation*, *Phys. Rev.* **D20** (1979) 1807–1812.
- [194] G. Basar, G. V. Dunne, and D. E. Kharzeev, *Chiral magnetic spirals*, *Phys. Rev. Lett.* **104** (Jun, 2010) 232301.
- [195] J. R. Bhatt and A. K. Pandey, *Primordial generation of magnetic fields*, *Springer Proc. Phys.* **174** (2016) 409–413, [[arXiv:1507.01795](#)].
- [196] J. R. Bhatt and A. K. Pandey, *Primordial magnetic field and kinetic theory with berry curvature*, [arXiv:1503.01878](#).
- [197] C. Caprini, R. Durrer, and G. Servant, *The stochastic gravitational wave background from turbulence and magnetic fields generated by a first-order phase transition*, *JCAP* **0912** (2009) 024, [[arXiv:0909.0622](#)].

- [198] A. Kosowsky, A. Mack, and T. Kahniashvili, *Gravitational radiation from cosmological turbulence*, *Phys. Rev. D* **66** (Jul, 2002) 024030.
- [199] C. Caprini and R. Durrer, *Gravitational wave production: A strong constraint on primordial magnetic fields*, *Phys. Rev. D* **65** (Dec, 2001) 023517.
- [200] C. Caprini, R. Durrer, and T. Kahniashvili, *Cosmic microwave background and helical magnetic fields: The tensor mode*, *Phys. Rev. D* **69** (Mar, 2004) 063006.
- [201] D. V. Deriagin, D. Y. Grigoriev, V. A. Rubakov, and M. V. Sazhin, *Generation of gravitational waves by the anisotropic phases in the early universe*, *Monthly Notices of the Royal Astronomical Society* **229** (1987), no. 3 357–370.
- [202] A. D. Dolgov, D. Grasso, and A. Nicolis, *Relic backgrounds of gravitational waves from cosmic turbulence*, *Phys. Rev. D* **66** (Nov, 2002) 103505.
- [203] A. Nicolis, *Relic gravitational waves from colliding bubbles and cosmic turbulence*, *Classical and Quantum Gravity* **21** (2004), no. 4 L27.
- [204] T. Kahniashvili, A. Kosowsky, G. Gogoberidze, and Y. Maravin, *Detectability of Gravitational Waves from Phase Transitions*, *Phys. Rev. D* **78** (2008) 043003, [[arXiv:0806.0293](#)].
- [205] L. Kisslinger and T. Kahniashvili, *Polarized Gravitational Waves from Cosmological Phase Transitions*, *Phys. Rev. D* **92** (2015), no. 4 043006, [[arXiv:1505.03680](#)].
- [206] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, *The Effects of topological charge change in heavy ion collisions: 'Event by event P and CP violation'*, *Nucl. Phys. A* **803** (2008) 227–253, [[arXiv:0711.0950](#)].
- [207] D. E. Kharzeev, *Topologically induced local P and CP violation in QCD x QED*, *Annals Phys.* **325** (2010) 205–218, [[arXiv:0911.3715](#)].

- [208] J. Erdmenger, M. Haack, M. Kaminski, and A. Yarom, *Fluid dynamics of R -charged black holes*, *JHEP* **01** (2009) 055, [[arXiv:0809.2488](#)].

List of Publications

1. **Arun Kumar Pandey**, Jitesh R. Bhatt, *Primordial magnetic field and kinetic theory with Berry curvature*, Phys. Rev. D, 94, 043536, doi: [10.1103/PhysRevD.94.043536](https://doi.org/10.1103/PhysRevD.94.043536).
2. **Arun Kumar Pandey**, Jitesh R. Bhatt, *Primordial Generation of Magnetic Fields*, XXI DAE-BRNS High Energy Physics Symposium: Proceedings, Guwahati, India, December 8-12, 2014, Springer Proceedings in Physics, doi: [10.1007/978-3-319-25619-1_62](https://doi.org/10.1007/978-3-319-25619-1_62).
3. **Arun Kumar Pandey**, Jitesh R. Bhatt, Sampurnand ‘*Chiral Battery, scaling laws and magnetic fields*, JCAP **1707**, no. 07, 051 (2017) doi:[10.1088/1475-7516/2017/07/051](https://doi.org/10.1088/1475-7516/2017/07/051)

Publications attached with the thesis

1. **Arun Kumar Pandey**, Jitesh R. Bhatt, *Primordial magnetic field and kinetic theory with Berry curvature*, Phys. Rev. D, 94, 043536, doi: [10.1103/PhysRevD.94.043536](https://doi.org/10.1103/PhysRevD.94.043536).
2. **Arun Kumar Pandey**, Jitesh R. Bhatt, *Primordial Generation of Magnetic Fields*, XXI DAE-BRNS High Energy Physics Symposium: Proceedings, Guwahati, India, December 8-12, 2014, Springer Proceedings in Physics, doi: [10.1007/978-3-319-25619-1_62](https://doi.org/10.1007/978-3-319-25619-1_62).
3. **Arun Kumar Pandey**, Jitesh R. Bhatt, Sampurnand ‘*Chiral Battery, scaling laws and magnetic fields*, JCAP **1707**, no. 07, 051 (2017) doi:[10.1088/1475-7516/2017/07/051](https://doi.org/10.1088/1475-7516/2017/07/051)

Primordial magnetic field and kinetic theory with Berry curvatureJitesh R. Bhatt^{1,*} and Arun Kumar Pandey^{1,2,†}¹*Theory Division, Physical Research Laboratory, Ahmedabad 380 009, India*²*Department of Physics, Indian Institute of Technology, Gandhinagar, Ahmedabad 382 424, India*
(Received 16 March 2015; revised manuscript received 6 July 2016; published 30 August 2016)

We study the generation of a magnetic field in primordial plasma of standard model particles at a temperature $T > 80$ TeV—much higher than the electroweak scale. It is assumed that there is an excess number of right-handed electrons compared to left-handed positrons in the plasma. Using the Berry-curvature modified kinetic theory to incorporate the effect of the Abelian anomaly, we show that this chiral imbalance leads to the generation of a hypermagnetic field in the plasma in both the collision dominated and collisionless regimes. It is shown that, in the collision dominated regime, the chiral-vorticity effect can generate finite vorticity in the plasma together with the magnetic field. Typical strength of the generated magnetic field is 10^{27} G at $T \sim 80$ TeV with the length scale $10^5/T$, whereas the Hubble length scale is $10^{13}/T$. Furthermore, the instability can also generate a magnetic field of the order 10^{31} G at a typical length scale $10/T$. But there may not be any vorticity generation in this regime. We show that the estimated values of the magnetic field are consistent with the bounds obtained from current observations.

DOI: [10.1103/PhysRevD.94.043536](https://doi.org/10.1103/PhysRevD.94.043536)**I. INTRODUCTION**

There is a strong possibility that the observed magnetic fields in galaxies and in the intergalactic medium could be due to a process in the very early Universe. Understanding the origin and dynamics of the primordial magnetic field is one of the most intriguing problems of the cosmology (see the recent reviews [1–3]). It should be noted here that there still exists a possibility that the fields may not be of primordial origin but might be created during the gravitational collapse of galaxies [4,5]. In this work we are interested in the primordial origin of the magnetic fields. There exist several models describing the generation of primordial magnetic fields in terms of cosmological defects [6–8]: phase transitions [9–12], inflation [13,14], the electroweak Abelian anomaly [15,16], string cosmology [17,18], temporary electric charge nonconservation [19], the trace anomaly [20], and breaking gauge invariance [21]. In a recent work [22], it was shown that a process like Biermann battery can play a role in generating the primordial magnetic field just after the recombination era.

In recent times there has been considerable interest in studying the role of the quantum chiral anomaly in generation of the primordial magnetic field [23–25]. In Ref. [15] (see also [16]), it was argued that there can be more right-handed electrons over left-handed positrons due to a process in the early Universe at temperatures T very much higher than the electroweak phase transition (EWPT) scale (~ 100 GeV). Their number is effectively conserved at energy scales significantly above the electroweak phase transitions and this allows one to introduce the chiral

chemical potentials $\mu_R(\mu_L)$. At temperatures lower than $T_R \sim 80$ TeV, processes related to electron chirality flipping may dominate over the Hubble expansion rate and the chiral chemical potentials are not defined [26–28]. Furthermore, the right-handed current is not conserved due to the Abelian anomaly in the standard model (SM), and it satisfies the following equation:

$$\partial_\mu \mathcal{J}_R^\mu = -\frac{g'^2 y_R^2}{64\pi^2} \mathcal{Y}^{\mu\nu} \tilde{\mathcal{Y}}^{\mu\nu} = -\frac{g'^2}{4\pi^2} \mathcal{E}_Y \cdot \mathcal{B}_Y. \quad (1)$$

Here, $\mathcal{Y}^{\mu\nu} = \partial^\mu Y^\nu - \partial^\nu Y^\mu$ is the field tensor associated with the hypercharge gauge field Y^μ , and $\tilde{\mathcal{Y}}^{\mu\nu} = 1/2\epsilon^{\mu\nu\rho\lambda} \mathcal{Y}_{\rho\lambda}$. $\mathcal{E}_Y$ and $\mathcal{B}_Y$ denote hyperelectric and hypermagnetic fields, respectively. Furthermore, g' indicates associate gauge coupling, and $y_R = -2$ represents the hypercharge of the right electrons. The right-hand sides of the first and second equality signs are related with the Chern-Simons number n_{CS} :

$$n_{CS} = -\frac{g'^2}{32\pi^2} \int d^3x \mathcal{B}_Y \cdot \mathcal{Y}. \quad (2)$$

The anomaly equation (1) relates the change in the right-handed electron density to the variation of the topological (Chern-Simons or helicity) charge of the gauge fields. It was shown in Ref. [29] that the Chern-Simons term contributes, in the effective standard model Lagrangian of the field Y_μ , by the polarization effect through the nonzero mean pseudovector current $\mathcal{J}_{j5} = g'^2 y_R^2 / 2 \langle \bar{e}_R \gamma_j \gamma_5 e_R \rangle = -g'^2 y_R^2 \mu_{eR} \mathcal{B}_j / 4\pi^2$, and the effective Lagrangian for gauge field Y_μ in the SM is [29,30]

*jeet@prl.res.in

†arunp@prl.res.in, arun_pandey@iitgn.ac.in

$$L_Y = -\frac{1}{4}\mathcal{Y}_{\mu\nu}\mathcal{Y}^{\mu\nu} - \mathcal{J}_\mu Y^\mu - \frac{g'^2 Y_R^2 \mu_{eR}}{4\pi^2} \mathcal{B}_Y \cdot Y. \quad (3)$$

If these hypermagnetic fields survive at the time of the EWPT, they will produce ordinary magnetic fields due to the electroweak mixing $\mathcal{A}_\mu = \cos\theta_w Y_\mu$. Where Y_μ is the massless $U(1)_Y$ Abelian gauge hypercharge field. It was shown that the chiral imbalance in the early Universe could give rise to a magnetic field $B \sim 10^{22}$ G at the temperature $T \sim 100$ GeV with a typical inhomogeneity scale $\sim 10^6/T$ [15]. In this work the authors studied the Maxwell equations with the Chern-Simons term and a kinetic equation consistent with Eq. (1). It was found that the transverse modes could become unstable and give rise to the hypercharge magnetic field [31,32]. In Ref. [23] the authors used magnetohydrodynamics in the presence of chiral asymmetry to study the evolution of the magnetic field. They showed that the chiral-magnetic [31,33,34] and chiral-vorticity effects [35] can play a significant role in the generation and dynamics of the primordial magnetic field. Furthermore, it was demonstrated in Ref. [25] that evolution of the primordial magnetic field is strongly influenced by the chiral anomaly, even at a temperature as low as 10 MeV. It was shown that an isotropic and translationally invariant initial state of the standard model plasma in thermal equilibrium can become unstable in the presence of global charges [24]. The most general form of the polarization operator Π^{ij} can be written

$$\Pi^{ij}(\mathbf{k}) = (k^2 \delta^{ij} - k^i k^j) \Pi_1(k^2) + i \epsilon^{ijk} k^k \Pi_2(k^2), \quad (4)$$

where $\mathbf{k}$ is a wave vector and $k^2 = |\mathbf{k}|^2$. This equation satisfies the transversality condition $k_i \Pi^{ij} = 0$. It should be noted here that the Chern-Simons term is $\propto \mathbf{Y} \cdot \partial \mathbf{Y}$, whereas the kinetic term is $\propto (\partial \mathbf{Y})^2$, and therefore the Chern-Simons term can dominate over large length scales. Thus, a nonzero value of Π_2 when $k \rightarrow 0$, implying the presence of a Chern-Simons term in the expression for free energy. Using the field theoretic framework in [24], it was shown that, for a sufficiently small $k < \Pi_2(k^2)/\Pi_1(k^2)$, the polarization tensor Π^{ij} has a negative eigenvalue, and the corresponding eigenmode provides instability.

Recently, there has been an interesting development in incorporating the parity-violating effects into a kinetic theory formalism (see Refs. [31,36–38]). In this approach the kinetic (Vlasov) equation is modified by including the Berry-curvature term which takes into account chirality of the particles. The modified kinetic equation is consistent with the anomaly equation (1). Incorporation of the parity-odd physics in kinetic theory leads to a redefinition of the Poisson brackets which includes a contribution from the Berry connection. Confidence that the new kinetic equation captures the proper physics stems from the fact that the equation is consistent with the anomaly equation (1) and that it also reproduces some of the known results obtained

using the quantum field theory with the parity-odd interaction [39]. In fact, the “classical” kinetic equation can reproduce—in the leading order in the hard dense loop approximation—the parity-odd correlation of the underlying quantum field theory [30,40]. The modified kinetic equation can also be derived from the Dirac Hamiltonian by performing a semiclassical Foldy-Wouthuysen diagonalization [41,42]. The modified kinetic equation can be applied to both the high density and the high temperature regime [41]. Furthermore, in Ref. [39] normal modes of chiral plasma were analyzed using the modified kinetic theory in the context of heavy-ion collisions. In that work the authors found that, in the collisionless limit, the transverse branch of the dispersion relation could become unstable with a typical wave number $k \sim \alpha' \mu / \pi$, where α' is the coupling constant and μ refers to the chiral chemical potential.

Here, we would like to note that the authors in Ref. [15] have used a heuristically written kinetic equation which is consistent with Eq. (1) to study generation of the primordial magnetic field. In addition, authors have used an expression for the current by incorporating standard electric resistivity and the chiral-magnetic effect. The chiral-vorticity effect was not considered. It should be emphasized that the forms of the kinetic equation used in Ref. [15] and in the Berry-curvature modified theory [37] are very different. As both the approaches describe the same physics, it would be interesting to see under what conditions they give similar predictions. Keeping the above discussion in mind, we believe that it would be highly useful to consider the problem of generating a primordial magnetic field in the presence of the Abelian anomaly by using the Berry-curvature modified kinetic theory. In this work we incorporate the effect of collisions in the modified kinetic theory and derive expressions for the electric and magnetic resistivities. The new kinetic framework also allows us to calculate the generation of the primordial magnetic field and the vorticity. Further on in our calculation, we considered an isotropic and homogeneous initial state of the particle distribution function. The magnetic field is generated by the unstable transverse modes in the presence of chiral charges (Q_5). This can be seen by integrating Eq. (1) over space. One can write $\partial_0(Q_5 + \frac{g'}{\pi} \mathcal{H}) = 0$, where $Q_5 = \int j^0 d^3x$ and the helicity $\mathcal{H} = \frac{1}{V} \int d^3x (\mathbf{Y} \cdot \mathcal{B}_Y)$. The finite helicity state can be created even if the initial state has $\mathcal{H} = 0$, but $Q_5 \neq 0$. Thus, the magnetogenesis by net nonzero chiral charges may not require any preexisting seed field.

This manuscript is organized into three forthcoming sections. In Sec. II we briefly state the (3 + 1) formalism of MacDonald and Thorne [43] and the kinetic theory with the Berry curvature. In Sec. III we apply this formalism to a study of the primordial magnetic field generation in the presence of chiral asymmetry. We also calculate the vorticity generation in the plasma due to the chiral

imbalance. Section IV contains our results and a brief discussion. We show that our estimated value of the peak magnetic field actually falls within the constraints obtained from current observations.

II. BASIC FORMALISM

A. Maxwell's equations in the expanding Universe

In this work we shall study the generation of a primordial magnetic field at the time when the temperature of the Universe was much higher than $T_R \sim 80$ TeV (much higher than the EWPT temperature). We intend to solve the coupled system of the modified kinetic and the Maxwell's equations in the expanding Universe background. Here, we note that we ignore the fluctuations in the metric due to the matter perturbation. For this, one needs to write the underlying equations in a general covariant form. Interestingly, the techniques developed in Refs. [44–47] allow one to write the system of kinetic and Maxwell's equations in the expanding background in a form that looks similar to their flat space-time form. In this formalism the well developed intuition and techniques of the flat space-time plasma physics can be exploited to study the problem at hand. This can be accomplished by choosing a particular set of fiducial observers [44] at each point of space-time at which all of the physical quantities, including hyperelectric and magnetic fields, are measured. A line element for the expanding background can be written using the Friedmann-Lemaître-Robertson-Walker metric as

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \quad (5)$$

where x , y , and z represent comoving coordinates. Here, t is the proper time seen by observers at a fixed x , y , and z and $a(t)$ is the scale factor. One can introduce the conformal time η using the definition $\eta = \int dt/a^2(t)$ to write this metric as

$$ds^2 = a^2(\eta)(-d\eta^2 + dx^2 + dy^2 + dz^2). \quad (6)$$

The (hyper)electric $\mathbf{E}_{\text{phy}}$, the (hyper)magnetic $\mathbf{B}_{\text{phy}}$, and the current density $\mathbf{J}_{\text{phy}}$ are related to the corresponding fiducial quantities by transformations: $\mathcal{E} = a^2 \mathbf{E}_{\text{phy}}$, $\mathcal{B} = a^2 \mathbf{B}_{\text{phy}}$, $\mathcal{J} = a^3 \mathbf{J}_{\text{phy}}$. One can now write the Maxwell's equations in the fiducial frame as

$$\frac{\partial \mathcal{B}}{\partial \eta} + \nabla \times \mathcal{E} = 0, \quad (7)$$

$$\nabla \cdot \mathcal{E} = 4\pi \rho_e, \quad (8)$$

$$\nabla \cdot \mathcal{B} = 0, \quad (9)$$

$$\nabla \times \mathcal{B} = 4\pi \mathcal{J} + \frac{\partial \mathcal{E}}{\partial \eta}, \quad (10)$$

where $\mathcal{B}$, $\mathcal{E}$, ρ_e , and $\mathcal{J}$ are, respectively, the magnetic field, the electric field, the charge density, and the current density seen by the fiducial observer.

B. Kinetic theory with Berry curvature

The charge ρ_e and current $\mathcal{J}$ in the Maxwell's equations (7)–(10) can be calculated using the Berry-curvature modified kinetic equation, which is also consistent with the quantum anomaly equation. The modified kinetic equation is given by

$$\begin{aligned} \frac{\partial f}{\partial \eta} + \frac{1}{1 + e\Omega_p \cdot \mathcal{B}} \left[(e\tilde{\mathcal{E}} + e\tilde{\mathbf{v}} \times \mathcal{B} + e^2(\tilde{\mathcal{E}} \cdot \mathcal{B})\Omega_p) \cdot \frac{\partial f}{\partial \mathbf{p}} \right. \\ \left. + (\tilde{\mathbf{v}} + e\tilde{\mathcal{E}} \times \Omega_p + e(\tilde{\mathbf{v}} \cdot \Omega_p)\mathcal{B}) \cdot \frac{\partial f}{\partial \mathbf{r}} \right] = \left(\frac{\partial f}{\partial \eta} \right)_{\text{coll}}, \end{aligned} \quad (11)$$

where e is the charge of the particles and has relation with the electroweak (EW) mixing angle θ_w : $e = g' \cos \theta_w$. Also, the g' is related with the $U(1)_Y$ gauge coupling constant α' as $\alpha' = g'^2 \cos^2 \theta_w / 4\pi$. $\tilde{\mathbf{v}} = \partial \epsilon_p / \partial \mathbf{p} = \mathbf{v}$, and $e\tilde{\mathcal{E}} = e\mathcal{E} - \partial \epsilon_p / \partial \mathbf{r}$. $\Omega_p = \pm \mathbf{p} / (2p^3)$ is the Berry curvature. ϵ_p is defined as $\epsilon_p = p(1 - e\mathcal{B} \cdot \Omega_p)$, with $p = |\mathbf{p}|$. The positive sign corresponds to right-handed fermions, whereas the negative sign is for left-handed ones. In the absence of a Berry correction, i.e., $\Omega_p = 0$, the above equation reduces to the Vlasov equation when the collision term on the right-hand side of Eq. (11) is absent.

As we have already stated, we are interested in the temperature regime $T > T_R \gg T_{\text{EW}}$. At these temperatures, electrons are massless. The only process that can change electron chirality is its Yukawa interaction with the Higgs boson. However, at this temperature this interaction is not strong enough to alter electron chirality. It is important here to note that, for a temperature smaller than T_R , electron mass plays a major role in left-right asymmetry. Recently, in Ref. [48], it was shown that at a temperature of the order of MeV, the mass of the electron plays an important role in determining the magnetic properties of the protoneutron star by suppressing the chiral charge density during the core collapse of the supernova. However, for the present case we ignore the electron mass by considering only the $T > T_R$ regime. Thus, we write the particle number density modified by the Berry term as

$$N = \int \frac{d^3 p}{(2\pi)^3} (1 + e\mathcal{B} \cdot \Omega_p) f. \quad (12)$$

The above equation (11) can be converted to the following form by multiplying it by $(1 + e\mathcal{B} \cdot \Omega_p)$ and integrating over p :

$$\frac{\partial N}{\partial \eta} + \nabla \cdot \mathcal{J} = -e^2 \int \frac{d^3 p}{(2\pi)^3} \left(\Omega_p \cdot \frac{\partial f}{\partial \mathbf{p}} \right) (\mathcal{E} \cdot \mathcal{B}). \quad (13)$$

In Eq. (13) $\mathcal{J}$ is the total current and is defined as $\mathcal{J} = \sum_a \mathcal{J}_a$. Here, index a denotes the current contribution from different species of the fermion, e.g., right-left particles and their antiparticles. $\mathcal{J}_a$ is defined as

$$\begin{aligned} \mathcal{J}_a = & -e^a \int \frac{d^3 p}{(2\pi)^3} \left[\epsilon_p^a \frac{\partial f_a}{\partial \mathbf{p}} + e^a \left(\Omega_p^a \cdot \frac{\partial f_a}{\partial \mathbf{p}} \right) \epsilon_p^a \mathcal{B} \right. \\ & \left. + \epsilon_p \Omega_p^a \times \frac{\partial f_a}{\partial \mathbf{r}} \right] + e^a (\mathcal{E} \times \sigma^a), \end{aligned} \quad (14)$$

where $\sigma^a = \int d^3 p / (2\pi)^3 \Omega_p^a f_a$ and $\epsilon_p^a = p(1 - e^a \mathcal{B} \cdot \Omega_p^a)$, with $p = |\mathbf{p}|$. Depending on the species, the charge e , the energy of the particles ϵ_p , the Berry curvature Ω_p , and the form of the distribution function f changes. For the right-handed particle, $a = R$ with a hypercharge e , for the right-handed antiparticle, $a = \bar{R}$ with a charge of $-e$, etc. It is clear from Eq. (13) that, in the presence of external electric and magnetic fields, the chiral current is no longer conserved. The first term in Eq. (14) is the usual current equivalent to the kinetic theory and the remaining second and third terms are the current contribution by the Berry correction. The last term results from the anomalous Hall effect and it vanishes for a spherically symmetric distribution function. If we follow the power counting scheme used in [31], i.e., $Y_\mu = \mathcal{O}(\epsilon)$, $\partial_r = \mathcal{O}(\delta)$ [where Y_μ represents the $U(1)_Y$ gauge field] and considering only those terms of the order of $\mathcal{O}(\epsilon\delta)$ in Eq. (11), we have

$$\begin{aligned} \left(\frac{\partial}{\partial \eta} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} \right) f_a + \left(e^a \mathcal{E} + e^a (\mathbf{v} \times \mathcal{B}) - \frac{\partial \epsilon_p^a}{\partial \mathbf{r}} \right) \cdot \frac{\partial f_a}{\partial \mathbf{p}} \\ = \left(\frac{\partial f_a}{\partial \eta} \right)_{\text{coll}}, \end{aligned} \quad (15)$$

where we have taken $\mathbf{v} = \mathbf{p}/p$. In the subsequent discussion, we shall apply this equation to a study of the evolution of the primordial magnetic field.

C. Current and polarization tensor for chiral plasma

Here, we assume that the plasma of the standard particles is in a state of “thermal equilibrium” at a temperature $T > T_R$, and at these temperatures the masses of the plasma particles can be ignored. We also assume that a left-right asymmetry exists and that there is no large-scale electromagnetic field. Thus, the equilibrium plasma is considered to be in a homogeneous and isotropic state, which is similar to the assumptions made in Ref. [15,24]. For a homogeneous and isotropic conducting plasma in thermal equilibrium, the distribution function for different species is

$$f_{0a}(p) = \frac{1}{\exp(\frac{\epsilon_p^0 - \mu_a}{T}) + 1}. \quad (16)$$

If δf_R and $\delta f_{\bar{R}}$ are fluctuations in the distribution functions of the right electron and the right antiparticles around their equilibrium distribution. Then we can write the perturbed distribution functions as

$$f_R(\mathbf{r}, \mathbf{p}, \eta) = f_{0R}(p) + \delta f_R(\mathbf{r}, \mathbf{p}, \eta), \quad (17)$$

$$f_{\bar{R}}(\mathbf{r}, \mathbf{p}, \eta) = f_{0\bar{R}}(p) + \delta f_{\bar{R}}(\mathbf{r}, \mathbf{p}, \eta). \quad (18)$$

Subtracting the equation for $a = \bar{R}$ from $a = R$ using Eq. (15), one can write

$$\begin{aligned} \left(\frac{\partial}{\partial \eta} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} \right) f(\mathbf{r}, \mathbf{p}, \eta) + e p \frac{\partial (\mathcal{B} \cdot \Omega_p)}{\partial \mathbf{r}} \cdot \frac{\partial f_0}{\partial \mathbf{p}} + (\mathcal{E} \cdot \mathbf{v}) \frac{df_0}{dp} \\ = \left(\frac{\partial f(\mathbf{r}, \mathbf{p}, \eta)}{\partial \eta} \right)_{\text{coll}}, \end{aligned} \quad (19)$$

where $f(\mathbf{r}, \mathbf{p}, \eta) = (f_R - f_{\bar{R}})$ and $f_0 = f_{0R} + f_{0\bar{R}}$. Here, we have used $\frac{\partial f_0}{\partial \mathbf{p}} = \mathbf{v} \frac{df_0}{dp}$. This equation relates the fluctuations of the distribution functions of the charged particles to the induced gauge field fluctuations. The gauge field fluctuations can be seen from the Maxwell's electromagnetic equations (7)–(10). Under the relaxation time approximation, the collision term can be written as $(\partial f_a / \partial \eta)_{\text{coll}} \approx -\nu_c (f_a - f_{0a})$ (one can also look at some studies of chiral kinetic theory with collisions in Refs. [49,50]). Next, we take the Fourier transform of all of the perturbed quantities, namely, $\mathcal{E}$, $\mathcal{B}$, and $f(\mathbf{r}, \mathbf{p}, \eta)$, by considering the spatiotemporal variation of these quantities as $\exp[-i(\omega\eta - \mathbf{k} \cdot \mathbf{r})]$. Then, using Eq. (19), one can get

$$f_{\mathbf{k}, \omega} = \frac{-e[(\mathbf{v} \cdot \mathcal{E}_{\mathbf{k}}) + \frac{i}{2p}(\mathbf{v} \cdot \mathcal{B}_{\mathbf{k}})(\mathbf{k} \cdot \mathbf{v})] \frac{df_0}{dp}}{i(\mathbf{k} \cdot \mathbf{v} - \omega - i\nu_c)}. \quad (20)$$

Thus, the current contribution for the right-handed particle and the right-handed antiparticles in terms of a mode function can be written using Eq. (14) as (ignoring the anomalous Hall current part)

$$\begin{aligned} \mathcal{J}_{kR} = & e \int \frac{d^3 p}{(2\pi)^3} \left[\left\{ \mathbf{v} - \frac{i}{2p}(\mathbf{v} \times \mathbf{k}) \right\} f_{k\omega R} \right. \\ & \left. - \frac{e}{2p^2} \{ \mathcal{B}_{\mathbf{k}} - \mathbf{v}(\mathbf{v} \cdot \mathcal{B}_{\mathbf{k}}) \} f_0 + \frac{e}{2p} \mathcal{B}_{\mathbf{k}} \frac{df_0}{dp} \right]. \end{aligned} \quad (21)$$

In a similar way, we can get current contributions from the left-handed particle and the left-handed antiparticle. Thus, we can obtain the total current by adding the contributions from both the left- and right-handed particles

and antiparticles by putting perturbations $f_{k\omega}$ for all species into Eq. (21) and adding them:

$$\begin{aligned} \mathcal{J}_k = & -m_D^2 \int \frac{d\Omega}{4\pi} \frac{\mathbf{v}(\mathbf{v} \cdot \mathcal{E}_k)}{i(\mathbf{k} \cdot \mathbf{v} - \omega - i\nu_c)} \\ & - \frac{h_D^2}{2} \int \frac{d\Omega}{4\pi} \{ \mathcal{B}_k - \mathbf{v}(\mathbf{v} \cdot \mathcal{B}_k) \} \\ & - \frac{ig_D^2}{4} \int \frac{d\Omega}{4\pi} \frac{(\mathbf{v} \times \mathbf{k})(\mathbf{v} \cdot \mathcal{B}_k)(\mathbf{k} \cdot \mathbf{v})}{(\mathbf{k} \cdot \mathbf{v} - \omega - i\nu_c)} \\ & - \frac{c_D^2}{2} \int \frac{d\Omega}{4\pi} \left\{ \frac{\mathbf{v}(\mathbf{v} \cdot \mathcal{B}_k)(\mathbf{k} \cdot \mathbf{v}) - (\mathbf{v} \times \mathbf{k})(\mathbf{v} \cdot \mathcal{E}_k)}{(\mathbf{k} \cdot \mathbf{v} - \omega - i\nu_c)} + \mathcal{B}_k \right\}. \end{aligned} \quad (22)$$

Here, Ω represents the angular integrals. In Eq. (22), we have defined $m_D^2 = e^2 \int \frac{p^2 dp}{2\pi^2} \frac{d}{dp} (f_{0R} + f_{0\bar{R}} + f_{0L} + f_{0\bar{L}})$, $c_D^2 = e^2 \int \frac{p^2 dp}{2\pi^2} \frac{d}{dp} (f_{0R} - f_{0\bar{R}} - f_{0L} + f_{0\bar{L}})$, $g_D^2 = e^2 \int \frac{p^2 dp}{2\pi^2} \frac{d}{dp} \times (f_{0R} + f_{0\bar{R}} + f_{0L} + f_{0\bar{L}})$, and $h_D^2 = e^2 \int \frac{p^2 dp}{2\pi^2} (f_{0R} - f_{0\bar{R}} - f_{0L} + f_{0\bar{L}})$.

Expression for the polarization tensor Π^{ij} can be obtained from Eq. (22) by writing the total current in the following form: $\mathcal{J}_k^i = \Pi^{ij}(k)Y_j(k)$, using $\mathcal{E}_k = -i\omega\mathbf{Y}_k$ and $\mathcal{B}_k = i(\mathbf{k} \times \mathbf{Y}_k)$. One can express Π^{ij} in terms of the longitudinal $P_L^{ij} = k^i k^j / k^2$, transverse $P_T^{ij} = (\delta^{ij} - k^i k^j / k^2)$, and axial $P_A^{ij} = i\epsilon^{ijk} k^k$ projection operators as $\Pi^{ik} = \Pi_L P_L^{ik} + \Pi_T P_T^{ik} + \Pi_A P_A^{ik}$. After performing the angular integrations in Eq. (22), one obtains Π_L , Π_T , and Π_A as given below:

$$\Pi_L = -m_D^2 \frac{\omega\omega'}{k^2} [1 - \omega' L(k)], \quad (23)$$

$$\Pi_T = m_D^2 \frac{\omega\omega'}{k^2} \left[1 + \frac{k^2 - \omega'^2}{\omega'} L(k) \right], \quad (24)$$

$$\Pi_A = -\frac{h_D^2}{2} \left[1 - \omega \left(1 - \frac{\omega'^2}{k^2} \right) L(k) - \frac{\omega'\omega}{k^2} \right], \quad (25)$$

where $L(k) = \frac{1}{2k} \ln(\frac{\omega'+k}{\omega'-k})$ and $\omega' = \omega + i\nu_c$. Also, $m_D^2 = 4\pi\alpha'(\frac{T^2}{3} + \frac{\mu_R^2 + \mu_L^2}{2\pi^2})$ and $h_D^2 = \frac{2\alpha'\Delta\mu}{\pi}$. We have defined $\Delta\mu = (\mu_R - \mu_L)$. In the above integrals, we have replaced e by α' using the relation $e^2 = 4\pi\alpha'$. First, consider the case where $\nu_c = 0$. In the limit $\omega \rightarrow 0$, Π_L and Π_T vanish and the parity-odd part of the polarization tensor $\Pi_A = h_D^2/2 \approx \alpha'\Delta\mu/\pi$. Here, it should be noted that Π_A does not get thermal correction. This could be due to the fact that the origin of the Π_A term is related to the axial anomaly, and it is well known that the anomaly does not receive any thermal correction [51–53]. This form of Π_A is similar to the result obtained in [24] using quantum field theoretic arguments at $T \leq 40$ GeV. However, in the

kinetic theory approach presented here, no such assumption is made. Normal modes for the plasma can be obtained by using expressions for Π_L , Π_T , and Π_A . Using the equation $\partial_\nu F^{\mu\nu} = -4\pi\mathcal{J}^\mu$, we can write the following relation:

$$[M^{-1}]^{ij} Y_j(k) = -4\pi\mathcal{J}_k^i, \quad (26)$$

where $[M^{-1}]^{ik} = [(k^2 - \omega^2)\delta^{ik} - k^i k^k + \Pi^{ik}]$. Dispersion relations can be obtained from the poles of $[M^{-1}]^{ik}$, which are as given below:

$$\omega^2 = \Pi_L,$$

$$\omega^2 = k^2 + \Pi_T(k) \pm k\Pi_A.$$

One can study the normal modes of the chiral plasma and instabilities using these dispersion relations. However, it is more instructive to study the dynamical evolution of the magnetic field by explicitly writing time dependent Maxwell equations.

III. GENERATION OF THE PRIMORDIAL MAGNETIC FIELD AND VORTICITY

Plasmas with chirality imbalance are known to have instabilities that can generate magnetic fields in two different regimes: (i) collision dominated, $k, \omega \ll \nu_c$ [15], and (ii) the collisionless case, i.e., $\nu_c = 0$ [39]. In this section we analyze how the magnetic fields evolve in the plasma due to these instabilities, within the modified kinetic theory framework. Expression for the total current described by Eq. (22) can be written as $\mathcal{J}_k^i = \sigma_E^{ij} \mathcal{E}_k^j + \sigma_B^{ij} \mathcal{B}_k^j$, where σ_E^{ij} and σ_B^{ij} are electrical and magnetic conductivities. The integrals involved in Eq. (22) are rather easy to evaluate in the limit $k, \omega \ll \nu_c$, and one can write the expression for σ_E^{ij} and σ_B^{ij} as

$$\sigma_E^{ij} \approx \left(\frac{m_D^2}{3\nu_c} \delta^{ij} + \frac{i}{3\nu_c} \frac{\alpha' \Delta\mu}{\pi} \epsilon^{ijl} k^l \right), \quad (27)$$

$$\sigma_B^{ij} \approx -\frac{4}{3} \frac{\alpha' \Delta\mu}{\pi} \delta^{ij}, \quad (28)$$

Here, we would like to note that the Berry-curvature correction in the kinetic equation gives us an additional contribution in the expression for σ_E^{ij} which was not incorporated in Ref. [15]. The first term is the usual dissipative part of the electric current and it contributes to the Joules dissipation. The second term results from the chiral imbalance and it does not give any contribution to the Joules heating. As we shall demonstrate later, this term is responsible for the vorticity current [33]. One can write the total current as $\mathcal{J}_k^i = \sigma_E^{ij} \mathcal{E}_k^j + \sigma_B^{ij} \mathcal{B}_k^j$ and the Maxwell's equation as $i(\mathbf{k} \times \mathcal{B}_k)^i = 4\pi\mathcal{J}_k^i$. Here, we have dropped the displacement current term (this is valid when $\frac{\omega}{4\pi\sigma} \ll 1$).

Next, by taking a vector product of $\mathbf{k}$ with the above Maxwell equation, one obtains (using Maxwell's equations and after some simplification)

$$\frac{\partial \tilde{\mathcal{B}}_k}{\partial \eta} + \left(\frac{3\nu_c}{4\pi m_d^2} \right) k^2 \tilde{\mathcal{B}}_k - \left(\frac{\alpha' \Delta \mu}{\pi m_D^2} \right) (\mathbf{k} \times (\mathbf{k} \times \mathcal{E}_k)) + i \frac{4\alpha' \nu_c \Delta \mu}{\pi m_D^2} (\mathbf{k} \times \tilde{\mathcal{B}}_k) = 0. \quad (29)$$

This is the magnetic diffusivity equation for the chiral plasma. By replacing $(\mathbf{k} \times \mathcal{E}_k)$ by $-\frac{1}{i} \frac{\partial \tilde{\mathcal{B}}_k}{\partial \eta}$ in Eq. (29), we can solve this equation without a loss of generality by considering the propagation vector $\mathbf{k}$ in the z direction and the magnetic field having components perpendicular to the z axis. After defining two new variables, $\tilde{\mathcal{B}}_k = (\mathcal{B}_k^1 + i\mathcal{B}_k^2)$ and $\tilde{\mathcal{B}}'_k = (\mathcal{B}_k^1 - i\mathcal{B}_k^2)$, one can rewrite Eq. (29) as

$$\frac{\partial \tilde{\mathcal{B}}_k}{\partial \eta} + \left[\frac{\left(\frac{3\nu_c}{4\pi m_d^2} \right) k^2 - \left(\frac{4\alpha' \nu_c \Delta \mu}{\pi m_D^2} \right) k}{\left(1 + \frac{\alpha' \Delta \mu k}{\pi m_D^2} \right)} \right] \tilde{\mathcal{B}}_k = 0, \quad (30)$$

$$\frac{\partial \tilde{\mathcal{B}}'_k}{\partial \eta} + \left[\frac{\left(\frac{3\nu_c}{4\pi m_d^2} \right) k^2 + \left(\frac{4\alpha' \nu_c \Delta \mu}{\pi m_D^2} \right) k}{\left(1 - \frac{\alpha' \Delta \mu k}{\pi m_D^2} \right)} \right] \tilde{\mathcal{B}}'_k = 0. \quad (31)$$

Thus, the magnetic field vector $\mathcal{B}$ can be decomposed into these new variables, $\tilde{\mathcal{B}}_k$ and $\tilde{\mathcal{B}}'_k$, having definite helicity (or circular polarization). The effect of Ohmic decay is already there in the above equations due to the inclusion of collision frequency ν_c . It should be noted here that, if $\alpha' \Delta \mu k / \pi m_D^2 \ll 1$, Eq. (30) is similar to the magnetic field evolution equation considered in Ref. [15]. In this limit, Eq. (31) will give a purely damping mode. In this case, the dispersion relation will be

$$i\omega = \frac{3\nu_c}{4\pi m_d^2} k^2 - \frac{4\alpha' \nu_c \Delta \mu}{\pi m_D^2} k. \quad (32)$$

In the Appendix we show that the dispersion relation we have found here using kinetic theory matches the dispersion relation obtained in [39].

The instability can also occur in the collisionless regime ($\nu_c = 0$) [39]. In the quasistatic limit, i.e., $\omega \ll k$, one can define the electric conductivity as $\sigma_E^{ij} \approx \pi(m_D^2/2k)\delta^{ij}$ and the magnetic conductivity as $\sigma_B^{ij} \approx (h_D^2/2)\delta^{ij}$. Here, it should be noted that the above conductivities do not depend upon the collision frequency. Similar to the previous case, one can take the propagation vector in the z direction and then consider the components of the magnetic field in the direction perpendicular to the z axis. One can write a set of decoupled equations describing the evolution of the magnetic field using the variables $\tilde{\mathcal{B}}_k$ and $\tilde{\mathcal{B}}'_k$ as

$$\frac{\partial \tilde{\mathcal{B}}_k}{\partial \eta} + \left[\frac{k^2 - \frac{4\alpha' \Delta \mu k}{3}}{\frac{\pi m_D^2}{2k}} \right] \tilde{\mathcal{B}}_k = 0, \quad (33)$$

$$\frac{\partial \tilde{\mathcal{B}}'_k}{\partial \eta} + \left[\frac{k^2 + \frac{4\alpha' \Delta \mu k}{3}}{\frac{\pi m_D^2}{2k}} \right] \tilde{\mathcal{B}}'_k = 0. \quad (34)$$

Here, we note that, if one replaces $\partial/\partial \eta$ by $-i\omega$, Eqs. (33) and (34) give the same dispersion relation for the instability as discussed in Ref. [39].

A. Vorticity generated from chiral imbalance in the plasma

It would be interesting to see whether the instabilities arising due to chiral imbalance can lead to vorticity generation in the plasma. In order to study the vorticity of the plasma, we define the average velocity as

$$\langle \mathbf{v} \rangle = \frac{1}{\bar{n}} \int \frac{d^3 p}{(2\pi)^3} \mathbf{v} (\delta f_R - \delta f_{\bar{R}} + \delta f_L - \delta f_{\bar{L}}). \quad (35)$$

Here, we have used the perturbed distribution function in the numerator of the above equation, which is due to the fact that the equilibrium distribution function is assumed to be homogeneous and isotropic and therefore will not contribute to vorticity dynamics. The denominator is the total number density and is defined as (in Ref. [54], p. 63)

$$\bar{n} = n_{\text{particle}} - n_{\text{antiparticle}} = d_f \int_0^\infty \frac{d^3 p}{(2\pi)^3} \left(\frac{1}{1 + \exp(\frac{p-\mu}{T})} - \frac{1}{1 + \exp(\frac{p+\mu}{T})} \right), \quad (36)$$

which, in the case of chiral plasma, gives $\bar{n} = \frac{2}{3} T^2 (\mu_R + \mu_L)$. We consider the $k, \omega \ll \nu_c$ regime; in this case, the perturbed distribution function for, say, the right-handed particles can be written as

$$\delta f_{\mathbf{k}, \omega R} = -\frac{e}{\nu_c} \left[(\mathbf{v} \cdot \mathcal{E}_k) + \frac{i}{2p} (\mathbf{v} \cdot \mathcal{B}_k) (\mathbf{k} \cdot \mathbf{v}) \right] \frac{df_{0R}}{dp}. \quad (37)$$

If we add the contribution for all of the particles species and their antiparticles, one can write the numerator in Eq. (35) as

$$\sqrt{\frac{\alpha'}{\pi^3}} \frac{1}{\nu_c} \left(\frac{T^2}{3} + \frac{\mu_R^2 + \mu_L^2}{2\pi^2} \right) \mathcal{E}_k \approx \sqrt{\frac{\alpha'}{\pi^3}} \frac{T^2}{3\nu_c} \mathcal{E}_k. \quad (38)$$

Above, we have neglected $\frac{3}{2\pi^2} \frac{\mu_R^2 + \mu_L^2}{T^2}$ in comparison to one, as the values of μ_R/T and μ_L/T are very small [$O(10^{-4})$]. One can write the average velocity as follows:

$$\langle \mathbf{v}_k \rangle = \sqrt{\frac{\alpha'}{\pi^3}} \frac{1}{2\nu_c \mu_R + \mu_L} \mathbf{E}_k. \quad (39)$$

The vorticity can now be obtained by taking the curl of Eq. (39) and assuming that the chemical potentials and the temperature are constant in space and time:

$$\langle \boldsymbol{\omega}_k \rangle = i \sqrt{\frac{\alpha'}{\pi^3}} \frac{1}{2\nu_c \mu_R + \mu_L} (\mathbf{k} \times \mathbf{E}_k). \quad (40)$$

One can find the contribution of the vorticity to the total current from Eq. (27) and $\mathcal{J}_{kw}^i = \sigma_E^{ij} \mathcal{E}_k^j + \sigma_B^{ij} \mathcal{B}_k^j$. By using Eq. (40), the vorticity current can be written as

$$\mathcal{J}_\omega \approx -\sqrt{\frac{4\pi\alpha'}{9}} (\mu_R^2 - \mu_L^2) \boldsymbol{\omega} = \xi \boldsymbol{\omega}. \quad (41)$$

Thus, in the absence of any chiral imbalance, there is no vorticity current. Here, we note that our definition agrees with Ref. [55]. In the Appendix we demonstrate that our kinetic theory is also consistent with the second law of thermodynamics. Furthermore, using Eq. (40), one can eliminate $(\mathbf{k} \times \mathbf{E}_k)$ in Eq. (29) and obtain

$$\begin{aligned} \frac{\partial \mathcal{B}_k}{\partial \eta} + \frac{3\nu_c}{4\pi m_d^2} k^2 \mathcal{B}_k + i \frac{\sqrt{4\pi\alpha'} \nu_c}{m_D^2} (\mu_R^2 - \mu_L^2) (\mathbf{k} \times \boldsymbol{\omega}_k) \\ + i \left(\frac{4\alpha' \nu_c}{\pi m_D^2} \right) (\mu_R - \mu_L) (\mathbf{k} \times \mathcal{B}_k) = 0. \end{aligned} \quad (42)$$

In this equation, the second term is the usual diffusivity term. However, the third and fourth terms are additional terms, which, respectively, represent the vorticity and chiral-magnetic effects on the chiral plasma. Therefore, Eq. (29) actually contains terms resulting from vorticity and the magnetic effect. The saturated state of the instability can be studied by setting $\partial_\eta \mathcal{B}_k = 0$ in Eq. (42). After taking a dot product of Eq. (42) with fluid velocity $\mathbf{v}_k$ after setting $\partial_\eta \mathcal{B}_k = 0$, one can obtain

$$\left(\boldsymbol{\omega}_k - i \frac{16T\delta}{3} \mathbf{v}_k \right) \cdot \mathcal{B}_k = 0. \quad (43)$$

Here, we have defined $\delta = \alpha'(\mu_R - \mu_L)/T$. We can write an expression for the magnetic field which satisfies the above equation (43) as

$$\mathcal{B}_k = g(k) \mathbf{k} \times \left[\boldsymbol{\omega}_k - i \frac{16T\delta}{3} \mathbf{v}_k \right]. \quad (44)$$

Where $g(k)$ is any general function, which can be determined by substituting the above expression for the magnetic field into Eq. (42) in a case of steady state. In a very large length scale, i.e., $\mathbf{k} \rightarrow 0$,

$$g(\mathbf{k}) = -\frac{3}{32} \sqrt{\frac{\pi^3}{\alpha'^3}} \frac{\mu_R^2 - \mu_L^2}{(\mu_R - \mu_L)^2}. \quad (45)$$

Thus, for a very large length scale $\mathbf{k} \rightarrow 0$, the magnetic field in the steady state is

$$\mathcal{B}_k = -i \sqrt{\frac{\pi^3}{4\alpha'}} \frac{\mu_R^2 - \mu_L^2}{(\mu_R - \mu_L)} \boldsymbol{\omega}_k. \quad (46)$$

This equation relates the vorticity generated during the instability to the magnetic field in the steady state.

However, in the collisionless regime ($\omega \ll k$ and $\nu_c = 0$), one can have an instability described by Eq. (33) with the typical scales $k \sim \alpha' \Delta\mu$ and $|\omega| \sim \alpha'^2 T \delta$ [39]. Using the expression for electric and magnetic conductivities for modes in this regime, one can write the magnetic diffusivity equation as

$$\frac{\partial \mathcal{B}_k}{\partial \eta} + \frac{k^2}{4\pi\sigma_1} \mathcal{B}_k - i \frac{T\delta}{\pi\sigma_1} (\mathbf{k} \times \mathcal{B}_k) = 0, \quad (47)$$

where $\sigma_1 = \pi m_D^2 / 2k$. Here, it should be noted that, unlike Eq. (42), the above equation does not have a vorticity term. The last term on the left-hand side arises from the chiral-magnetic effect. In the steady state ($\partial_\eta \mathcal{B}_k = 0$), one can get $\nabla \times \mathcal{B} = (4T\delta) \mathcal{B}$. This equation resembles the case of a magnetic field in a force free configuration of the conventional plasma where the plasma pressure is assumed to be negligible in comparison to the magnetic pressure [56]. However, for our case no such assumption about the plasma pressure is required.

IV. RESULTS AND DISCUSSION

In the previous sections, we applied the modified kinetic theory in the presence of chiral imbalance and obtained equations for the magnetic field generation for both the collision dominated and the collisionless regime. The instability can lead to generation of the magnetic field at the cost of the chiral imbalance. This can be seen from the anomaly equation $n_L - n_R + 2\alpha' \mathcal{H} = \text{constant}$ above $T > 80 \text{ TeV}$, where $n_{L,R} = \frac{\mu_{L,R} T^2}{6}$ and $\mathcal{H}$ is the magnetic helicity, defined as

$$\mathcal{H} = \frac{1}{V} \int d^3x (\mathbf{Y} \cdot \mathcal{B}_Y). \quad (48)$$

One can estimate the strength of the generated magnetic field as follows. From Eqs. (1) and (2), one can notice that the right-handed electron number density n_R changes with the Chern-Simons number n_{cs} of the hypercharge field configuration, as $\Delta n_R = \frac{1}{2} \mathcal{Y}_R^2 n_{cs}$. Here, $n_{cs} \approx \frac{q^2}{16\pi^2} k Y^2$ and $\Delta n_R = \mu_R T^2 = \frac{88}{783} \delta T^3$ [54]. From this, one can estimate the magnitude of the generated physical magnetic field as

$$B_Y^{\text{phy}} \approx \left[\frac{\pi^2 k \delta}{g'^2 \alpha' T} \right]^{\frac{1}{2}} T^2, \quad (49)$$

where we have used $B_Y^{\text{phy}} \sim kY$ and k^{-1} is the physical length scale, which is related to the comoving length by $k_{\text{phy}}^{-1} = (a/k_c)^{-1}$.

Now consider the regime $\omega, k \ll \nu_c$, where the dynamics for the magnetic field is described by Eqs. (30) and (31). Equation (30) clearly gives unstable modes for which $(\frac{T\delta}{\pi m_D^2})k < 1$ is satisfied. However, Eq. (31) gives a purely damping mode if the condition $(\frac{T\delta}{\pi m_D^2})k \ll 1$ is satisfied. One can rewrite this condition as $(\frac{T\delta}{3\pi\sigma\nu_c})k \sim (\frac{10^{-2}}{3\pi}\frac{\delta}{\nu_c})k \ll 1$. Here, we have used $m_D^2 = 3\nu_c\sigma$ with $\nu_c \sim \alpha'^2 \ln(\frac{1}{\alpha'})T$ [57] and $\sigma = 100T$. Thus, for $k \ll \nu_c$ and $\delta \ll 1$, Eq. (31) can only give purely damped modes. For these values of k and δ , Eq. (30) assumes a form similar to the equation for the magnetic field dynamics considered in Ref. [15]. If one replaces $\frac{\partial}{\partial \eta}$ by $-i\omega$ in Eq. (30), the dispersion relation for the unstable modes can be obtained. The fastest growth of the perturbation occurs for $k_{\text{max}1} \sim \frac{8T\delta}{3}$, and the maximum growth rate can be found to be $\Gamma_1 \sim \frac{16}{3\pi} \frac{T^2\delta^2}{m_D^2} \nu_c$. Here, we note that our $k_{\text{max}1}$ differs by a numerical factor from the value of k where the peak in the magnetic energy is calculated using chiral magnetohydrodynamics [23]. For $\delta \sim 10^{-6}$ and $\alpha' \sim 10^{-2}$, one can show that $\frac{k_{\text{max}1}}{\nu_c} \ll 1$ and $\frac{\Gamma_1}{\nu_c} \ll 1$ is satisfied. For these values of $k_{\text{max}1}$, α' , and δ , one can estimate the magnitude of the generated magnetic field using Eq. (49). We find $B \sim 10^{26}$ G for $\alpha' \sim 10^{-2}$ and the typical length scale $\lambda \sim 10^5/T$. Here, we would like to note that the typical Hubble length scale is $\sim 10^{13}/T$, which is much larger than the typical length scale of instability. Our estimate of magnetic field strength B in the collision dominated regime broadly agrees with Ref. [15]. Here, we note again that Eq. (30) includes the effect of Ohmic decay due to the presence of the collision term. Our analysis shows that Ohmic decay is not important for instability. Furthermore, we have shown that the chiral instability can also lead to the generation of vorticity in the collision dominated regime. The typical length scale for vorticity is similar to that of the magnetic field. From Eq. (46), we take the magnitude of the vorticity to be $\omega_v \sim 10^{-4}B/T$.

Next, we analyze the chiral instability in collisionless regime $\nu_c \ll \omega \ll k$, considering Eqs. (33) and (34). Here, one finds the wave number $k_{\text{max}2} = \frac{8\delta T}{9}$, at which the maximum growth rate $\Gamma_2 = \frac{1}{2\pi} \frac{T^3\delta^3}{m_D^2}$ can occur. Now, $\frac{k_{\text{max}2}}{\nu_c} = \frac{8}{9} \frac{\delta}{\alpha'^2} \ll 1$ and $\frac{\Gamma_2}{\nu_c} \sim \frac{3\delta^3}{8\pi^2\alpha'^3 \ln(1/\alpha')} \ll 1$, which puts constraints on the allowed values of δ . For $\delta \sim 10^{-1}$, $\alpha' \sim 10^{-2}$, and $T \sim T_R$, one can estimate the magnitude of the magnetic field to be 10^{31} G. The typical length scale for the magnetic field is $\lambda_2 \sim 10/T$, which is much smaller than the length scale in the collision dominated case. This is to be expected, as the typical length scale associated with kinetic theory is

smaller than the hydrodynamical case (related to the collision dominated regime).

The upper and lower bounds on the present observed magnetic field strength from the Planck 2015 results [58], and blazars [59,60] are between 10^{-17} G and 10^{-9} G. However, recently, in Ref. [61], it was shown that, if the magnetic field is helical and was created before the electroweak phase transition, then it can produce some baryon asymmetry. This can put more stringent bounds on the magnetic field (10^{-14} G– 10^{-12} G). Since the magnetic fields and the plasma evolutions are coupled, the produced magnetic field may not evolve adiabatically—i.e., like $a(\eta)^{-2}$ —due to plasma processes like turbulence. Similarly, the magnetic correlation length $\lambda_B \propto k_{\text{max}}^{-1}$ may not be proportional to $a(\eta)$. Typical values of λ_B for the collision dominated and collisionless cases in our case are $10^5/T$ and $10/T$, respectively. The length scale of turbulence can be written as $\lambda_T \approx \frac{B_p}{\sqrt{\epsilon^{ch} + p^{ch}}} \eta \sim \frac{B_p}{\sqrt{\epsilon^{ch} + p^{ch}}} H^{-1}$, where B_p is the physical value of the magnetic field and ϵ^{ch} and p^{ch} are, respectively, the energy and pressure densities of the charged particles. $g_*^{ch}(T)$ and $g_*^{\text{total}}(T)$ are the number of degrees of freedom of the $U(1)$ charged particles in the thermal bath. For $\lambda_B \gg \lambda_T$, the effect of turbulence can be negligible. However, the maximum value of the magnetic field (for $\nu_c = 0$) is about 10^{31} G in our case, and this gives $\lambda_T \approx 10^6/T$. Thus, we have $\lambda_B \ll \lambda_T$ and, following Ref. [61], we assume that the generated magnetic fields will undergo an inverse cascade soon after their generation. One can relate B_p and λ_B , which undergo the process of inverse cascading with the present day values of magnetic field B_0 and the correlation length λ_0 using the following equations [61]:

$$B_p^{\text{IC}}(T) \simeq 9.3 \times 10^9 \text{ G} \left(\frac{T}{10^2 \text{ GeV}} \right)^{7/3} \left(\frac{B_0}{10^{-14} \text{ G}} \right)^{2/3} \times \left(\frac{\lambda_0}{10^2 \text{ pc}} \right)^{1/3} \mathcal{G}_B(T), \quad (50)$$

$$\lambda_B^{\text{IC}}(T) \simeq 2.4 \times 10^{-29} \text{ Mpc} \left(\frac{T}{10^2 \text{ GeV}} \right)^{-5/3} \left(\frac{B_0}{10^{-14} \text{ G}} \right)^{2/3} \times \left(\frac{\lambda_0}{1 \text{ pc}} \right) \mathcal{G}_\lambda(T), \quad (51)$$

where $\mathcal{G}_B(T) = (g_*^{\text{total}}(T)/106.75)^{1/6} (g_*^{ch}(T)/82.75)^{1/6} \times (g_{*s}(T)/106.75)^{1/3}$ and $\mathcal{G}_\lambda(T) = (g_*^{\text{total}}(T)/106.75)^{-1/3} (g_*^{ch}(T)/82.75)^{-1/3} (g_{*s}(T)/106.75)^{1/3}$. From these equations, one can see that, for the collision dominated case, $B_p \simeq 10^{26}$ G can be achieved when $B_0 \simeq 10^{-12}$ G and $\lambda_0 \simeq 100$ Kpc. However, in the collisionless regime, a value of $B_0 \simeq 10^{-11}$ G and $\lambda_0 \simeq 1$ Mpc at temperature $T = 80$ TeV gives the values that we have found in our estimates for the peak value of the magnetic field. Thus, the

values of the magnetic field and the correlation length scale estimated by us can be consistent with the current bounds obtained from cosmic microwave background (CMB) observation and necessary for current observed baryon asymmetry. Since the value of B_p and λ_B for the collision dominated case are similar to that given in Ref. [62], we believe that they are also consistent with big bang nucleosynthesis constraints.

In conclusion, we have studied the generation of a magnetic field due to the anomaly in primordial plasma consisting of standard model particles. We have applied the Berry-curvature modified kinetic theory to the study of this problem. The effect of collision in the kinetic equation had been incorporated using the relaxation time approximation. We have found that chiral instability can occur in the presence of dissipation in both collision dominated and collisionless regimes. We have found in the collision dominated case that chiral instability can produce a magnetic field of the order of 10^{27} G, with the typical length scale being $10^5/T$. These results are in broad agreement with Ref. [15]. However, in that work the authors used a heuristic kinetic equation, and the collision term was not explicitly written in the kinetic equation. However, the expression for the total current included the Ohm's law. We have obtained expressions for electric and magnetic conductivities using the modified kinetic theory. We have found that the expression for electric conductivity in chiral plasma has a nondissipative term in addition to the standard Ohmic term. It has been shown that this new term is related to the vorticity current term found in chiral magnetohydrodynamics [63]. Furthermore, we have also studied chiral instability in the collisionless regime. It has been shown that, in this regime, a magnetic field of strength 10^{31} G can be generated at the length scale $10/T$. These length scales are much smaller than the length scale of the magnetic field in the collision dominated regime. Furthermore, the obtained values for the magnetic-field strength and the length scale have been shown to be consistent with the recent constraints from CMB data. We have also shown that in the collision dominated regime results of kinetic theory agree with the hydrodynamic treatment.

ACKNOWLEDGMENTS

We would like to thank our colleagues Manu George and Avadesh Kumar for their helpful comments and discussions. We also thank the anonymous referee for this work, whose comments helped us to improve our presentation.

APPENDIX: SECOND LAW OF THERMODYNAMICS AND DISPERSION RELATION FROM HYDRODYNAMICS FOR CHIRAL PLASMA

In Ref. [55] it was shown that parity-violating hydrodynamics violate the second law of thermodynamics,

$\partial_\alpha s^\alpha \geq 0$, where s^α is the entropy current density, unless certain constraints on the transport coefficients are imposed. Therefore, our results in the collision dominated regime should be in agreement with Ref. [55]. The most general equations for U(1) and entropy currents can be written as [55]

$$\nu^\alpha = \nu'^\alpha + \xi(\mu, T)\omega^\alpha + \xi_B(\mu, T)B^\alpha, \quad (\text{A1})$$

$$s^\alpha = s'^\alpha + D(\mu, T)\omega^\alpha + D_B(\mu, T)B^\alpha, \quad (\text{A2})$$

where $\nu'^\alpha = \rho U^\alpha + \sigma \mathcal{E}^\alpha$ and $s'^\alpha = sU^\alpha - \frac{\mu}{T}\nu^\alpha$, with ρ and U^α being the charge density and the hydrodynamic four-velocity, respectively. Entropy density can be found using the thermodynamic relation $sT + \mu\rho = (\varepsilon + p)$, where ε denotes the energy density and p denotes the pressure. In the collision dominated limit there is no charge separation in the plasma, and one can regard the total charge density as zero. Using the energy-momentum tensor $T^{\alpha\beta} = (\varepsilon + p)u^\alpha u^\beta + pg^{\alpha\beta}$, one can write the equation of motion as $\partial_\alpha T^{\alpha\beta} = F^{\beta\gamma}j_\gamma$ and the divergence of the entropy current as

$$\begin{aligned} \partial_\alpha (s^\alpha - D\omega^\alpha - D_B B^\alpha) = & -(\nu^\alpha - \xi\omega^\alpha - \xi_B B^\alpha) \\ & \times \left(\partial_\alpha \frac{\mu}{T} - \frac{E_\alpha}{T} \right) - C \frac{\mu}{T} E_\alpha B^\alpha. \end{aligned} \quad (\text{A3})$$

According to Ref. [55], the second law of thermodynamic is satisfied if the following four equations are satisfied:

$$\partial_\alpha D - \frac{2\partial_\alpha p}{\varepsilon + p} D - \xi \partial_\alpha \frac{\mu}{T} = 0, \quad (\text{A4})$$

$$\partial_\alpha D_B - \frac{\partial_\alpha p}{\varepsilon + p} D_B - \xi_B \partial_\alpha \frac{\mu}{T} = 0, \quad (\text{A5})$$

$$-2D_B + \frac{\xi}{T} = 0, \quad (\text{A6})$$

$$\frac{\xi_B}{T} - C \frac{\mu}{T} = 0. \quad (\text{A7})$$

In Ref. [55] these equations are solved and one can know the dependence of ξ , ξ_B , D , and D_B on μ and T up to an arbitrary constant. Next, we assume the perturbation scheme considered for the kinetic approach. With no background field tensor, one can write $F^{\alpha\beta} = \delta F^{\alpha\beta}$. Also, the energy density ε , the pressure p , and the flow velocity U^α can be written in this scheme as $\varepsilon = \varepsilon_0 + \delta\varepsilon$, $p = p_0 + \delta p$, and $U^\alpha = U_0^\alpha + \delta u^\alpha$, respectively, where all quantities with the subscript 0 are background values. The background and perturbed velocities are defined as $U_0^\alpha = (1, 0, 0, 0)$ and $\delta u^\alpha = (0, \delta \mathbf{u})$, respectively. Here, $\mathbf{u}$ is the

three flow velocity. We assume that the background is homogeneous and isotropic. The equation of motion for the background gives $\partial_0 \varepsilon_0 = 0$ & $\partial_0 p_0 = 0$. Since the ε_0 and p_0 are functions of temperature and chemical potential, we regard the background temperature as constant. In this scheme the zeroth and i th components of the equation of motion can be written as

$$\partial_0 \delta \varepsilon + (\varepsilon_0 + p_0) \partial_i \delta u^i = 0, \quad (\text{A8})$$

$$(\varepsilon_0 + p_0) \partial_0 \delta u^i + g^{ij} \partial_j \delta p = 0. \quad (\text{A9})$$

Ignoring the time derivative term in Maxwell's equation, one can write $\nabla \times \delta \mathbf{B} = 4\pi \delta \mathbf{j}$, and by using the expression for the perturbed current, one can obtain the following dispersion relation:

$$i\omega = \frac{k^2}{4\pi\sigma_0} \pm \frac{\xi_{0B}}{\sigma_0} k, \quad (\text{A10})$$

where we have used $\mathbf{k} = k\hat{z}$ and $\delta \mathbf{B}_k = \delta \mathbf{B}_{kx}\hat{x} + \delta \mathbf{B}_{ky}\hat{y}$. It should be noted that Eq. (A10) matches the dispersion relation obtained by the kinetic theory approach [Eq. (32)].

We first emphasize that there is no current in the background, and therefore the transport coefficients that appear in the expression for the perturbed current depend only on the background temperature and the chemical potentials. Now consider Eqs. (A4)–(A7), which, for the background quantities, can be described by the following two equations:

$$-2D_{B0} + \frac{\xi_0}{T_0} = 0, \quad (\text{A11})$$

$$\frac{\xi_{B0}}{T_0} - C \frac{\mu_0}{T_0} = 0. \quad (\text{A12})$$

These equations are satisfied by each species considered. Thus, one can write $\xi_0 = \xi_{R0} + \xi_{L0}$ and $\xi_{B0} = \xi_{BR0} + \xi_{BL0}$. Using the expression for ξ_0 from the kinetic equation, one can calculate D_{B0} from Eq. (A11). It agrees with the expression obtained in Ref. [55] and the expression obtained for ξ_B using kinetic theory is also in agreement with it. Thus, we have shown that the modified kinetic theory respects the constraint implied by the second law of thermodynamics.

-
- [1] A. Kandus, K. E. Kunze, and C. G. Tsagas, *Phys. Rep.* **505**, 1 (2011).
 - [2] L. M. Widrow, D. Ryu, D. R. G. Schleicher, K. Subramanian, C. G. Tsagas, and R. A. Treumann, *Space Sci. Rev.* **166**, 37 (2012).
 - [3] D. G. Yamazaki, T. Kajino, G. J. Mathew, and K. Ichiki, *Phys. Rep.* **517**, 141 (2012).
 - [4] R. M. Kulsrud, R. Cen, J. P. Ostriker, and D. Ryu, *Astrophys. J.* **480**, 481 (1997).
 - [5] D. R. Schleicher, S. Sur, R. Banerjee, R. S. Klessen, C. Federrath *et al.*, *Proc. Sci.*, CRF2010 (2010) 027 [arXiv:1102.3558].
 - [6] A. Vilenkin, *Phys. Rev. D* **43**, 1060 (1991).
 - [7] T. Vachaspati and A. Vilenkin, *Phys. Rev. Lett.* **67**, 1057 (1991).
 - [8] A. Vilenkin, *Phys. Rev. D* **56**, 3238 (1997).
 - [9] T. Vachaspati, *Phys. Lett. B* **265**, 258 (1991).
 - [10] K. Enqvist and P. Olesen, *Phys. Lett. B* **319**, 178 (1993).
 - [11] T. W. B. Kibble and A. Vilenkin, *Phys. Rev. D* **52**, 679 (1995).
 - [12] J. M. Q. A. Loeb and D. N. Spergel, *Astrophys. J.* **344**, L49 (1989).
 - [13] S. M. Carroll, G. B. Field, and R. Jackiw, *Phys. Rev. D* **41**, 1231 (1990).
 - [14] M. Giovannini and M. Shaposhnikov, *Phys. Rev. D* **62**, 103512 (2000).
 - [15] M. Joyce and M. Shaposhnikov, *Phys. Rev. Lett.* **79**, 1193 (1997).
 - [16] J. M. Cornwall, *Phys. Rev. D* **56**, 6146 (1997).
 - [17] M. Gasperini, M. Giovannini, and G. Veneziano, *Phys. Rev. Lett.* **75**, 3796 (1995).
 - [18] D. Lemoine and M. Lemoine, *Phys. Rev. D* **52**, 1955 (1995).
 - [19] A. Dolgov and J. Silk, *Phys. Rev. D* **47**, 3144 (1993).
 - [20] A. D. Dolgov, *Phys. Rev. D* **48**, 2499 (1993).
 - [21] M. S. Turner and L. M. Widrow, *Phys. Rev. D* **37**, 2743 (1988).
 - [22] S. Naoz and R. Narayan, *Phys. Rev. Lett.* **111**, 051303 (2013).
 - [23] H. Tashiro, T. Vachaspati, and A. Vilenkin, *Phys. Rev. D* **86**, 105033 (2012).
 - [24] A. Boyarsky, O. Ruchayskiy, and M. Shaposhnikov, *Phys. Rev. Lett.* **109**, 111602 (2012).
 - [25] A. Boyarsky, J. Fröhlich, and O. Ruchayskiy, *Phys. Rev. Lett.* **108**, 031301 (2012).
 - [26] B. A. Campbell, S. Davidson, J. R. Ellis, and K. A. Olive, *Phys. Lett. B* **297**, 118 (1992).
 - [27] J. M. Cline, K. Kainulainen, and K. A. Olive, *Phys. Rev. D* **49**, 6394 (1994).
 - [28] J. M. Cline, K. Kainulainen, and K. A. Olive, *Phys. Rev. Lett.* **71**, 2372 (1993).
 - [29] V. Semikoz and J. Valle, *J. Cosmol. Astropart. Phys.* **11** (2011) 048.
 - [30] M. Laine, *J. High Energy Phys.* **10** (2005) 056.
 - [31] D. T. Son and N. Yamamoto, *Phys. Rev. Lett.* **109**, 181602 (2012).

- [32] A. Avdoshkin, V.P. Kirilin, A.V. Sadofyev, and V.I. Zakharov, *Phys. Lett. B* **755**, 1 (2016).
- [33] K. Fukushima, D.E. Kharzeev, and H.J. Warringa, *Phys. Rev. D* **78**, 074033 (2008).
- [34] C. Manuel and J.M. Torres-Rincon, *Phys. Rev. D* **92**, 074018 (2015).
- [35] A. Vilenkin, *Phys. Rev. D* **20**, 1807 (1979).
- [36] M. A. Stephanov and Y. Yin, *Phys. Rev. Lett.* **109**, 162001 (2012).
- [37] D. T. Son and N. Yamamoto, *Phys. Rev. D* **87**, 085016 (2013).
- [38] J.-W. Chen, S. Pu, Q. Wang, and X.-N. Wang, *Phys. Rev. Lett.* **110**, 262301 (2013).
- [39] Y. Akamatsu and N. Yamamoto, *Phys. Rev. Lett.* **111**, 052002 (2013).
- [40] A. N. Redlich and L. C. R. Wijewardhana, *Phys. Rev. Lett.* **54**, 970 (1985).
- [41] C. Manuel and J.M. Torres-Rincon, *Phys. Rev. D* **89**, 096002 (2014).
- [42] A. J. Silenko, *Phys. Rev. A* **77**, 012116 (2008).
- [43] D. MacDonald and K. Thorne, *Mon. Not. R. Astron. Soc.* **198**, 345 (1982).
- [44] C. P. Dettmann, N. E. Frankel, and V. Kowalenko, *Phys. Rev. D* **48**, 5655 (1993).
- [45] K. A. Holcomb and T. Tajima, *Phys. Rev. D* **40**, 3809 (1989).
- [46] R. M. Gailis, C. P. Dettmann, N. E. Frankel, and V. Kowalenko, *Phys. Rev. D* **50**, 3847 (1994).
- [47] R. M. Gailis, N. E. Frankel, and C. P. Dettmann, *Phys. Rev. D* **52**, 6901 (1995).
- [48] D. Grabowska, D. B. Kaplan, and S. Reddy, *Phys. Rev. D* **91**, 085035 (2015).
- [49] J.-Y. Chen, D. T. Son, M. A. Stephanov, H.-U. Yee, and Y. Yin, *Phys. Rev. Lett.* **113**, 182302 (2014).
- [50] J.-Y. Chen, D. T. Son, and M. A. Stephanov, *Phys. Rev. Lett.* **115**, 021601 (2015).
- [51] Y.-l. Liu and G.-j. Ni, *Phys. Rev. D* **38**, 3840 (1988).
- [52] H. Itoyama and A. H. Mueller, *Nucl. Phys.* **B218**, 349 (1983).
- [53] A. Gomez Nicola and R. Alvarez-Estrada, *Int. J. Mod. Phys. A* **09**, 1423 (1994).
- [54] E. W. Kolb and M. S. Turner, *The Early Universe* (Perseus Books Group, New York, 1994).
- [55] D. T. Son and P. Surówka, *Phys. Rev. Lett.* **103**, 191601 (2009).
- [56] S. Chandrasekhar and L. Woltjer, *Proc. Natl. Acad. Sci. U.S.A.* **44**, 285 (1958).
- [57] G. Baym and H. Heiselberg, *Phys. Rev. D* **56**, 5254 (1997).
- [58] P. A. R. Ade *et al.* (Planck Collaboration), *arXiv*: 1502.01594.
- [59] A. Neronov and I. Vovk, *Science* **328**, 73 (2010).
- [60] F. Tavecchio, G. Ghisellini, L. Foschini, G. Bonnoli, G. Ghirlanda, and P. Coppi, *Mon. Not. R. Astron. Soc. Lett.* **406**, L70 (2010).
- [61] T. Fujita and K. Kamada, *Phys. Rev. D* **93**, 083520 (2016).
- [62] M. Giovannini and M. E. Shaposhnikov, *Phys. Rev. D* **57**, 2186 (1998).
- [63] M. Giovannini, *Phys. Rev. D* **88**, 063536 (2013).

Chapter 62

Primordial Generation of Magnetic Fields

Jitesh R. Bhatt and Arun Kumar Pandey

Abstract We reexamine generation of the primordial magnetic fields, at temperature $T > 80$ TeV, by applying a consistent kinetic theory framework which is suitably modified to take the quantum anomaly into account. The modified kinetic equation can reproduce the known quantum field theoretic results upto the leading orders. We show that our results qualitatively matches with the earlier results obtained using heuristic arguments. The modified kinetic theory can give the instabilities responsible for generation of the magnetic field due to chiral imbalance in two distinct regimes: (a) when the collisions play a dominant role and (b) when the primordial plasma can be regarded as collisionless. We argue that the instability developing in the collisional regime can dominate over the instability in the collisionless regime.

62.1 Introduction

Observations suggest that we live in a magnetized Universe: Magnetic fields are present from stellar scales to the intergalactic scales. However, it is yet not clear that how these magnetic field arises. One of the possible thought is that their origin is due to some process in the early universe like inflation [1] or phase transitions [2] etc. It may also possible that the magnetic fields may not have any primordial origin but they may be generated during a gravitational collapse [3]. Currently the generation and dynamics of the primordial magnetic field is one of the most intriguing problem in cosmology [4].

In recent time, there has been great interests in generation of primordial fields by quantum anomaly [5]. A chiral imbalance of the leptons can occur in the very early universe, due to some electroweak(EW) anomalous processes like parity violating decays of massive particles, just before EW scale [6]. It has been shown that, the

J.R. Bhatt (✉) · A.K. Pandey

Theoretical Physics Division, Physical Research Laboratory, Ahmedabad 380009, India
e-mail: jeet@prl.res.in

A.K. Pandey

Department of Physics, Indian Institute of Technology, Gandhinagar, Ahmedabad, India
e-mail: arunp@prl.res.in

© Springer International Publishing Switzerland 2016

B. Bhuyan (ed.), *XXI DAE-BRNS High Energy Physics Symposium*,
Springer Proceedings in Physics 174, DOI 10.1007/978-3-319-25619-1_62

409

presence of chiral imbalance creates instability in the the hot matter [7]. At temperatures $T > 80 \text{ TeV} (\sim T_R)$ and below Hubble radius, the chirality flipping processes are highly suppressed and hence asymmetry between the right-handed and left-handed particle is preserved. It has been shown for a temperature $T > 80 \text{ TeV}$ and for a time scale $t < \Gamma_R^{-1}$ the number density of the chiral particles do not change very much. So one can ignore space-time dependence of the chemical potential in that case. However for temperature $T < 80 \text{ TeV}$ one should consider space-time dependence. It has been found that, for a temperature above 80 TeV , the modes of the magnetic field has maximum growth rate and for a $k \sim \alpha \Delta\mu$ mode of the magnetic field evolve maximally. In the regime $T > T_R$, the EW symmetry is unbroken and the generated field will be $U(1)$ hypermagnetic(hypercharge) fields [8]. After EW phase transition, these hypermagnetic field will be converted into the electromagnetic field. Recently there has been growing interests in including the parity violating effects into a kinetic theory formalism. It was found that the chiral anomaly can be incorporated in a kinetic theory framework by including the Berry curvature correction [9]. The resulting theory can reproduce the triangle anomaly and provide the descriptions for the chiral magnetic and chiral vorticity effects [10]. In this work we apply the Berry curvature modified kinetic theory to the chiral plasma for the study of generation of primordial magnetic field. In this work, we discuss the generation and evolution of magnetic field in two regions: a) where collision play a dominant role i.e. $v_c \gg \omega \gg k$ where v_c is the collision frequency, ω and k respectively denote the typical frequency and wave-number for the perturbations. b) And in the collisionless regime $k \gg \omega \gg v_c$. The instabilities can grow in these two regimes at the expense of the chiral-imbalance and surprising the maximum growth rates for both regimes occur at the similar length scales. In this work, we analyze the characteristics of these instabilities within the framework of the modified kinetic theory and discuss the relationship between our results and the earlier results. The manuscript is organized into two sections. In first section we discussed the generation of primordial magnetic field incorporating Berry correction. Second section contains results and discussion.

62.2 Generation of the Primordial Magnetic Field

In this work we solve a coupled system of the Maxwell equations and the modified kinetic theory for relativistic particles in the spatially flat Friedmann-Lemaître-Robertson-Walker universe. Conformal flatness of this space-time insures that equations in the conformal space have same forms as those of the flat space-time [11]. A conformal metric can be written as:

$$ds^2 = a^2(\eta)(-d\eta^2 + dx^2 + dy^2 + dz^2). \quad (62.1)$$

where conformal time η is related with proper time t by $\eta = \int dt/a^2(t)$ The quantities like (hyper)-electric $\mathbf{E}$, (hyper)-magnetic $\mathbf{B}$ and the current density $\mathbf{J}$ measured

Chiral battery, scaling laws and magnetic fields

Sampurn Anand,^a Jitesh R. Bhatt^a and Arun Kumar Pandey^{a,b}

^aPhysical Research Laboratory,
Ahmedabad, 380009, India

^bIndian Institute of Technology,
Gandhinagar, 382424, India

E-mail: sampurn@prl.res.in, jeet@prl.res.in, arunp@prl.res.in

Received June 1, 2017

Accepted July 13, 2017

Published July 28, 2017

Abstract. We study the generation and evolution of magnetic field in the presence of chiral imbalance and gravitational anomaly which gives an additional contribution to the vortical current. The contribution due to gravitational anomaly is proportional to T^2 which can generate seed magnetic field irrespective of plasma being chirally charged or neutral. We estimate the order of magnitude of the magnetic field to be 10^{30} G at $T \sim 10^9$ GeV, with a typical length scale of the order of 10^{-18} cm, which is much smaller than the Hubble radius at that temperature (10^{-8} cm). Moreover, such a system possess scaling symmetry. We show that the T^2 term in the vorticity current along with scaling symmetry leads to more power transfer from lower to higher length scale as compared to only chiral anomaly without scaling symmetry.

Keywords: Magnetohydrodynamics, primordial magnetic fields, physics of the early universe, baryon asymmetry

ArXiv ePrint: [1705.03683](https://arxiv.org/abs/1705.03683)

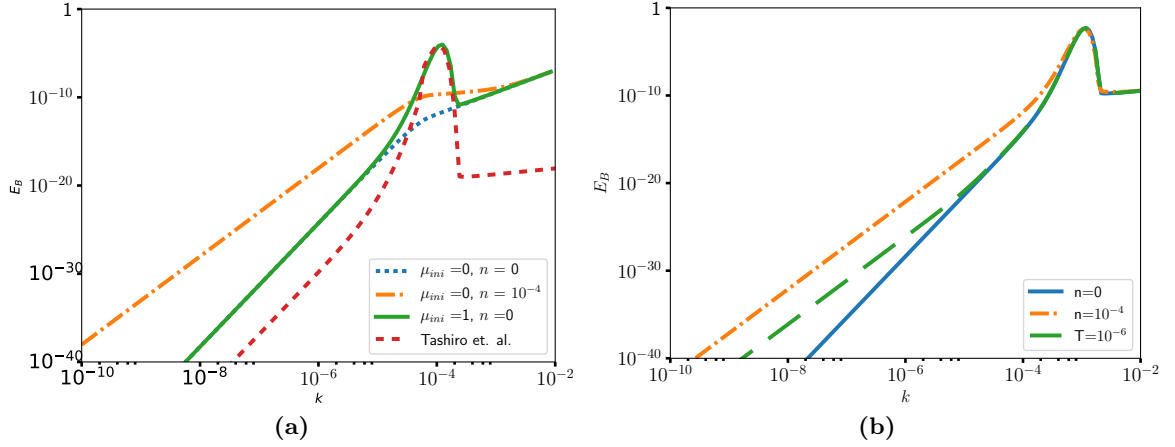


Figure 1. Magnetic energy spectrum as a function of k is shown: (a) when $n = 0$, E_B goes as k^7 at large length scale. However, the power is more than [29] at given k . (b) when $n \neq 0$, E_B goes as k^5 at large length scale. In both the figures we have set $T = 10^9$ GeV and $v = 10^{-4}$.

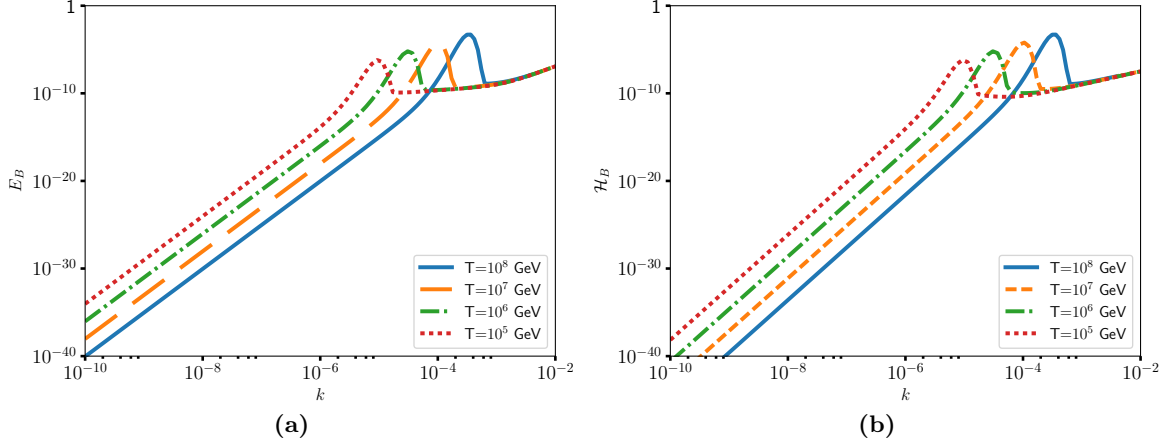


Figure 2. (a) Evolution of magnetic energy density E_B as a function of k and (b) Evolution of magnetic helicity density $\mathcal{H}_B$ as a function of k for different temperature. As temperature ($T \sim 1/\eta$) decreases, peak shifted from higher to lower k values which shows transfer of magnetic energy from small to large length scale. In both the figures we have set $v = 10^{-4}$ and $n = 10^{-4}$.

3.3 Evolution of magnetic field

Once seed field is generated it may start influencing the subsequent evolution of the system at certain mode. In that case, there will be a current flowing parallel to these generated magnetic fields. Therefore, we need to consider the effect of CME in the evolution eq. (2.13). We can decompose this eq. (2.13) in terms of polarization to obtain

$$\frac{\partial |\tilde{B}^\pm|^2}{\partial \eta} = \frac{2}{\sigma} \left(-k^2 \pm \xi^{(B)} k \right) |\tilde{B}^\pm|^2 + \frac{2}{\sigma^2} (\pm n k + \xi k^2)^2 |v^\pm|^2 f(\eta, k) \quad (3.26)$$

It is evident from the above equation that when the first term dominates over the second, then

$$|B^\pm|^2 = |B_0^\pm|^2 \exp \left(\frac{2\eta}{\sigma} k_{\text{ins}}^2 \right) \exp \left(\frac{2\eta}{\sigma} (k \mp k_{\text{ins}})^2 \right) \quad (3.27)$$

$k_{ins}/2 \approx C\mu_5/2$. As mentioned earlier, for $T \geq 80$ TeV is slower than the expansion rate of the Universe, one can safely ignore $\Gamma_f \mu_5$ term in eq. (3.29). However, the magnetic helicity generated due to DT^2 will drive the evolution of chemical potential. Below 80 TeV, flipping rate becomes comparable to the expansion rate of the Universe and the chirality of the right particles changes to the left handed. So to know the complete dynamics of the magnetic field energy, we have solved the coupled equations (3.26) and (3.29) simultaneously. In figure 2a we have shown the variation of magnetic energy spectrum E_B with k for different temperature. Since, the chemical potential is a decreasing function of time, so it is expected that with decrease in temperature the instability peak will shift towards the smaller k which implies the transfer of energy from small to large length scale. Similarly, we have shown the magnetic helicity spectrum in figure 2b.

5 Conclusion

In the present work, we have discussed the generation of magnetic field due to gravitational anomaly which induces a term $\propto T^2$ in the vorticity current. The salient feature of this seed magnetic field is that it will be produced at high temperature irrespective of whether fluid is charged or neutral. In ref. [32], authors have shown that the equations of chiral magneto-hydrodynamics (ChMHD), in absence of other effects like charge separation effect and cross helicity, follow unique scaling property and transfer energy from small to large length scales known as inverse cascade. Under this scaling symmetry more power is transferred from lower to higher length scale as compared to only chiral anomaly without scaling symmetry.

References

- [1] A. Vilenkin, *Cosmic string dynamics with friction*, *Phys. Rev. D* **43** (1991) 1060 [INSPIRE].
- [2] T. Vachaspati and A. Vilenkin, *Large scale structure from wiggly cosmic strings*, *Phys. Rev. Lett.* **67** (1991) 1057 [INSPIRE].
- [3] A. Vilenkin, *Topological defects and open inflation*, *Phys. Rev. D* **56** (1997) 3238 [astro-ph/9703201] [INSPIRE].
- [4] T. Vachaspati, *Magnetic fields from cosmological phase transitions*, *Phys. Lett. B* **265** (1991) 258 [INSPIRE].
- [5] K. Enqvist and P. Olesen, *Ferromagnetic vacuum and galactic magnetic fields*, *Phys. Lett. B* **329** (1994) 195 [hep-ph/9402295] [INSPIRE].
- [6] K. Enqvist and P. Olesen, *On primordial magnetic fields of electroweak origin*, *Phys. Lett. B* **319** (1993) 178 [hep-ph/9308270] [INSPIRE].
- [7] T.W.B. Kibble and A. Vilenkin, *Phase equilibration in bubble collisions*, *Phys. Rev. D* **52** (1995) 679 [hep-ph/9501266] [INSPIRE].
- [8] J.M. Quashnock, A. Loeb and D.N. Spergel, *Magnetic field generation during the cosmological QCD phase transition*, *Astrophys. J.* **344** (1989) L49 [INSPIRE].
- [9] S.M. Carroll, G.B. Field and R. Jackiw, *Limits on a Lorentz and parity violating modification of electrodynamics*, *Phys. Rev. D* **41** (1990) 1231 [INSPIRE].
- [10] M. Giovannini and M.E. Shaposhnikov, *Primordial magnetic fields from inflation?*, *Phys. Rev. D* **62** (2000) 103512 [hep-ph/0004269] [INSPIRE].
- [11] J.M. Cornwall, *Speculations on primordial magnetic helicity*, *Phys. Rev. D* **56** (1997) 6146 [hep-th/9704022] [INSPIRE].

- [12] M. Gasperini, M. Giovannini and G. Veneziano, *Primordial magnetic fields from string cosmology*, *Phys. Rev. Lett.* **75** (1995) 3796 [[hep-th/9504083](#)] [[INSPIRE](#)].
- [13] D. Lemoine and M. Lemoine, *Primordial magnetic fields in string cosmology*, *Phys. Rev. D* **52** (1995) 1955 [[INSPIRE](#)].
- [14] A. Dolgov and J. Silk, *Electric charge asymmetry of the universe and magnetic field generation*, *Phys. Rev. D* **47** (1993) 3144 [[INSPIRE](#)].
- [15] A. Dolgov, *Breaking of conformal invariance and electromagnetic field generation in the universe*, *Phys. Rev. D* **48** (1993) 2499 [[hep-ph/9301280](#)] [[INSPIRE](#)].
- [16] M.S. Turner and L.M. Widrow, *Inflation produced, large scale magnetic fields*, *Phys. Rev. D* **37** (1988) 2743 [[INSPIRE](#)].
- [17] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, *On the anomalous electroweak baryon number nonconservation in the early universe*, *Phys. Lett. B* **155** (1985) 36 [[INSPIRE](#)].
- [18] G. 't Hooft, *Symmetry breaking through Bell-Jackiw anomalies*, *Phys. Rev. Lett.* **37** (1976) 8 [[INSPIRE](#)].
- [19] K. Fukushima, D.E. Kharzeev and H.J. Warringa, *The chiral magnetic effect*, *Phys. Rev. D* **78** (2008) 074033 [[arXiv:0808.3382](#)] [[INSPIRE](#)].
- [20] A. Vilenkin, *Equilibrium parity violating current in a magnetic field*, *Phys. Rev. D* **22** (1980) 3080 [[INSPIRE](#)].
- [21] Y. Neiman and Y. Oz, *Relativistic hydrodynamics with general anomalous charges*, *JHEP* **03** (2011) 023 [[arXiv:1011.5107](#)] [[INSPIRE](#)].
- [22] H.B. Nielsen and M. Ninomiya, *Adler-Bell-Jackiw anomaly and Weyl fermions in crystal*, *Phys. Lett. B* **130** (1983) 389 [[INSPIRE](#)].
- [23] A. Yu. Alekseev, V.V. Cheianov and J. Fröhlich, *Universality of transport properties in equilibrium, Goldstone theorem and chiral anomaly*, *Phys. Rev. Lett.* **81** (1998) 3503 [[cond-mat/9803346](#)] [[INSPIRE](#)].
- [24] A. Vilenkin, *Macroscopic parity violating effects: neutrino fluxes from rotating black holes and in rotating thermal radiation*, *Phys. Rev. D* **20** (1979) 1807 [[INSPIRE](#)].
- [25] J. Erdmenger, M. Haack, M. Kaminski and A. Yarom, *Fluid dynamics of R-charged black holes*, *JHEP* **01** (2009) 055 [[arXiv:0809.2488](#)] [[INSPIRE](#)].
- [26] N. Banerjee, J. Bhattacharya, S. Bhattacharyya, S. Dutta, R. Loganayagam and P. Surówka, *Hydrodynamics from charged black branes*, *JHEP* **01** (2011) 094 [[arXiv:0809.2596](#)] [[INSPIRE](#)].
- [27] D.T. Son and P. Surówka, *Hydrodynamics with triangle anomalies*, *Phys. Rev. Lett.* **103** (2009) 191601 [[arXiv:0906.5044](#)] [[INSPIRE](#)].
- [28] K. Landsteiner, E. Megías and F. Pena-Benitez, *Gravitational anomaly and transport*, *Phys. Rev. Lett.* **107** (2011) 021601 [[arXiv:1103.5006](#)] [[INSPIRE](#)].
- [29] H. Tashiro, T. Vachaspati and A. Vilenkin, *Chiral effects and cosmic magnetic fields*, *Phys. Rev. D* **86** (2012) 105033 [[arXiv:1206.5549](#)] [[INSPIRE](#)].
- [30] J.R. Bhatt and A.K. Pandey, *Primordial magnetic field and kinetic theory with Berry curvature*, *Phys. Rev. D* **94** (2016) 043536 [[arXiv:1503.01878](#)] [[INSPIRE](#)].
- [31] L. Álvarez-Gaumé and E. Witten, *Gravitational anomalies*, *Nucl. Phys. B* **234** (1984) 269 [[INSPIRE](#)].
- [32] N. Yamamoto, *Scaling laws in chiral hydrodynamic turbulence*, *Phys. Rev. D* **93** (2016) 125016 [[arXiv:1603.08864](#)] [[INSPIRE](#)].
- [33] K.A. Holcomb and T. Tajima, *General relativistic plasma physics in the early universe*, *Phys. Rev. D* **40** (1989) 3809 [[INSPIRE](#)].

- [34] C.P. Dettmann, N.E. Frankel and V. Kowalenko, *Plasma electrodynamics in the expanding universe*, *Phys. Rev. D* **48** (1993) 5655 [[INSPIRE](#)].
- [35] R.M. Gailis, N.E. Frankel and C.P. Dettmann, *Magnetohydrodynamics in the expanding universe*, *Phys. Rev. D* **52** (1995) 6901 [[INSPIRE](#)].
- [36] V.B. Semikoz and J.W.F. Valle, *Chern-Simons anomaly as polarization effect*, *JCAP* **11** (2011) 048 [[arXiv:1104.3106](#)] [[INSPIRE](#)].
- [37] V.L. Biermann, *On the origin of the magnetic fields on stars and in interstellar space*, *Z. Naturforsch. A* **5** (1950) 65.
- [38] P. Olesen, *On inverse cascades in astrophysics*, *Phys. Lett. B* **398** (1997) 321 [[astro-ph/9610154](#)] [[INSPIRE](#)].
- [39] B.A. Campbell, S. Davidson, J.R. Ellis and K.A. Olive, *On the baryon, lepton flavor and right-handed electron asymmetries of the universe*, *Phys. Lett. B* **297** (1992) 118 [[hep-ph/9302221](#)] [[INSPIRE](#)].
- [40] Y. Akamatsu and N. Yamamoto, *Chiral plasma instabilities*, *Phys. Rev. Lett.* **111** (2013) 052002 [[arXiv:1302.2125](#)] [[INSPIRE](#)].
- [41] Y. Akamatsu and N. Yamamoto, *Chiral Langevin theory for non-Abelian plasmas*, *Phys. Rev. D* **90** (2014) 125031 [[arXiv:1402.4174](#)] [[INSPIRE](#)].