Theoretical Studies of Cosmological Models In The Light of Experimental Observations

A Thesis

submitted for the Award of Ph.D. degree of

MOHANLAL SUKHADIA UNIVERSITY

in the

Faculty of Science

By

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Year of submission: 2012

To

my parents

DECLARATION

I, **Mrs. Moumita Das**, D/o Dr. Chitta Ranjan Kar, resident of A-1, PRL residences, Navrangpura, Ahmedabad 380009, hereby declare that the research work incorporated in the present thesis entitled, **"Theoretical Studies of Cosmological Models In The Light of Experimental Observations"** is my own work and is original. This work (in part or in full) has not been submitted to any University for the award of a Degree or a Diploma. I have properly acknowledged the material collected from secondary sources wherever required. I solely own the responsibility for the originality of the entire content.

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CERTIFICATE

I feel great pleasure in certifying that the thesis entitled, **"Theoretical Studies of Cosmological Models In The Light of Experimental Observations"** by Mrs. Moumita Das under my guidance. She has completed the following requirements as per Ph.D regulations of the University.

(a) Course work as per the university rules.

(b) Residential requirements of the university.

(c) Regularly submitted six monthly progress reports.

(d) Presented her work in the departmental committee.

(e) Published/accepted minimum of one research paper in a referred research journal.

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Acknowledgements

First and foremost, I would like to express my sincere gratitude to my advisor Prof. Subhendra Mohanty for the continuous support during my Ph.D. study and research, with his patience, motivation, enthusiasm, and immense knowledge. His guidance helped me throughout my research period and preparing of this thesis. The courses, taken by him during PhD, help to understand the subject. The way, he explained the subject in a simple manner, enrich my knowledge to do the research. I could not have imagined having a better advisor and mentor for my PhD studies other than him.

I thank Prof. Saurabh Rindani, Dr. Srubabati Goswami, Dr. Raghavan Rangarajan, Dr. Dilip Angom, Dr. Namit Mahajan, Dr. Partha Konar, and Dr. D. K. Gosh for the courses they taught and many fruitful discussions. They were always very much encouraging and supportive and made me feel comfortable.

Dr. Raghavan Rangarajan took the initiative to organise 'cosmology journal club'. It is good platform to discuss the recent topics in our research. I want to acknowledge our cosmology group members namely, Dr. Raghavan Rangarajan, Prof. Subhendra Mohanty, Dr. Jitesh Bhatt, Dr. Namit Mahajan, Dr. Anjishnu Sarkar, Sasmita Mishra, Abhishek Basak, Tanushree Basak, Gaurav Tomar and Girish Chakravarty for valuable discussions.

I thank Prof. J. N. Goswami, the Director, Prof. Utpal Sarkar and Prof. A. K. Singhvi, former Deans, and Prof. Anjan Joshipura, Dean of PRL for providing necessary facilities and encouragements to carry out research work.

I thank all the staffs of PRL computer center for providing excellent computational and internet facilities.

I specially thank Jayati Chatterjee, Pravin Vaity, Bikkina Srinivas, Rajput Prasant, Tapas Baug, Suman Acharyya, Shivani Gupta, Ketan Patel, Paramita Deb, Bhaswar Chatterji, Amazad Hussain Laskar, MD. Rabiul Haque Biswas, Soumya Rao, Siddhartha Chattopadhyay, Joydeep Chakrabortty and Subrata Khan for there unconditional support and many memorable experiences with them.

I would like to thank my parents and my sister, Mim for their love, encouragement and support in all my pursuits, for being encouraging and patient with constant support during all the stages of this Ph.D. Finally, I would like to thank my husband, Saurabh Das. He is always there cheering me up and stood by me through the good times and bad.

The final words of my thesis are dedicated to those people whose name might be unsaid in this section but was always with me and helped me to choose the right direction during the last few years.

Moumita

ABSTRACT

Measurement of large angle correlations in the cosmic microwave background (CMB) anisotropy by COBE and WMAP experiments indicates that the universe went through a period of accelerated expansion in the past known as inflation. Inflation not only explains the long standing horizon and flatness problems of standard hot-big bang cosmology, it can also describe the structure formation very well in addition to the cosmic microwave anisotropy. In general, inflation is driven by the scalar field, which is known as inflaton. In this thesis we study the consequences of assumption that the Higgs field of the standard model can be the inflaton.

It is known that non-minimal coupling of the Higgs and gravity sector is needed to create a successful model of Higgs inflation. In this thesis we study magnetic field generation in the curvature coupled Higgs inflation model. It not only explains the magnitude of experimentally observed magnetic field at large scales, we also show that in this model there is no problem of back-reaction on the inflaton potential, which is normally seen in the generation of magnetic field studied in generic inflation model.

It is also known that in a potential with a large negative quartic coupling of a conformally coupled scalar field, one can generate scale invariant density perturbations to explain the structure formation of the universe and the CMB anisotropy. In this thesis we have implemented this idea in realistic inert doublet model. We show that we can generate the observed spectrum of the CMB anisotropy in this model by a suitable choice of the scalar Higgs couplings. With this choice of parameter one can tune the couplings to give a Higgs mass around 125.6 GeV along with light scalar dark matter candidate of mass 33.7 GeV which may be detected in the experiments.

In last part of this thesis we discuss the study of vacuum stability of the standard model Higgs potential which is the condition that the Higgs quartic coupling does not become negative under renormalization, all the way upto the Planck scale. In particular we study the phenomenological constraints on the heavy neutrino of Type-I seesaw models from the criterion of Higgs vacuum stability. We find that the Dirac mass of the neutrino is constrained to $m_D \leq 24.36$ GeV through the bound on the neutrino Yukawa coupling, $Y_\nu \leq 0.14$. This has application on the phenomenology of TeV scale heavy neutrinos, which can be tested in Large Hadron Collider. The three aspect of the heavy neutrino phenomenology, namely, Neutrino-less double beta decay ($0\nu\beta\beta$), Lepton flavor violating decays like $\mu \to e\gamma$ and Like-sign dilepton signals are studied in the light of the vacuum stability condition.

LIST OF PUBLICATIONS

1. **"Magnetic Field Generation in Higgs Inflation Model"**
   Moumita Das and Subhendra Mohanty
   Int. J. Mod. Phys. A **27**, 1250040 (2012)
   [arXiv:1004.1927 [astro-ph.CO]]

2. **"Constraints on background torsion from birefringence of CMB polarization"**
   Moumita Das, Subhendra Mohanty and A.R. Prasanna
   [arXiv:0908.0629 [astro-ph.CO]] (Communicated for publication)

3. **"Generation of scale invariant density perturbations in a conformally invariant Inert Higgs doublet model"**
   Moumita Das and Subhendra Mohanty
   [arXiv:1111.0799 [hep-ph]] (Communicated for publication)

4. **"Constraints on TeV scale Majorana neutrino phenomenology from the Vacuum Stability of the Higgs"**
   Joydeep Chakrabortty, Moumita Das and Subhendra Mohanty
   [arXiv:1207.2027 [hep-ph]] (Communicated for publication)

Contents

List of Figures

Chapter 1

Introduction

This chapter is the preface of this thesis. It helps to understand the motivation and importance of the thesis in current status of the particle and astroparticle physics.

The standard model (SM) is completely established with the discovery of the last remaining particle, the Higgs bosons [1]. The discovery of the Higgs particle opens up channels of model building where the Higgs particle may be useful in solving existing problems of cosmology and astroparticle physics. In this thesis we examine several issues where the Higgs plays a role in the standard model or in the models which extend the standard model. One primary area of cosmology is the theory of inflation, which is motivated by the observation of super-horizon correlations in the temperature anisotropy of cosmic microwave background observed by COBE [2, 3] and WMAP [4] satellite based experiments. While cosmological paradigm of inflation is generally accepted by cosmologists and particle physicists, however, there is no specific particle theory model of inflation which is universally accepted. The standard inflation model requires a scalar field which should have very small interactions with other particles and itself. This restricts severely the particle physics candidates which can serve as the 'inflaton', responsible for inflation.

In the standard model of electroweak interactions a scalar particle namely, the Higgs boson, is needed for making the theory renormalizable. The Higgs boson has finally been confirmed and focus now shifts to studying its couplings with other particles and to determine whether it is the Higgs boson of the standard model or its one of the many Higgs bosons of models beyond the SM like Supersymmetry and Inert-Higgs models. The main purpose of the models beyond the SM is to solve the problem of protecting the Higgs boson mass from getting quadratic divergence by loop corrections. If SM is an effective theory, which has a cutoff at some scale Λ, which may be the GUT scale 10^{16} GeV, then corrections to the Higgs mass m_h^2 are expected to be of the order Λ^2. This problem is solved by Supersymmetry and also by Inert-Higgs doublet model.

One more sector of particle physics, which directly involves the Higgs couplings, is the see-saw models of neutrino masses. In the simplest version, called Type-I see-saw, one introduces a number of right handed gauge singlet neutrinos, which can then form a gauge singlet Yukawa coupling term with the left-handed neutrinos of the standard model and the Higgs doublet. After electroweak symmetry breaking, when the neutral component of the Higgs gets a non-zero vacuum expectation value (vev), the neutrinos get Dirac mass. In addition the right-handed neutrinos can have a large Majorana mass. When the mass matrix of the light+heavy neutrinos is diagonalised, it is seen that the light neutrino masses is suppressed by the heavy neutrino mass scale. This is called the see-saw mechanism which accounts for the observed small neutrino masses.

In this thesis we have studied the consequences of the standard model Higgs and its variants in cosmology and particle physics phenomenology. The SM Higgs has been proposed as a candidate for being the inflaton by F. L. Bezrukov and M. E. Shaposhnikov [5]. There is a problem that the Higgs self coupling $\lambda \sim 0.1$, whereas to generate the observed CMB anisotropy amplitude $\Delta T/T \sim 10^{-5}$ and spectral index $n_s = 0.96 \pm 0.01$, the value of the quadratic coupling needed is $\lambda \sim 10^{-12}$. This problem is solved by introducing a large curvature coupling for the Higgs $\xi \phi^2 R$ with $\xi \sim 10^4$. This large coupling leads to a problem of non-unitary Higgs-graviton couplings and this aspect is being actively investigated [6, 7, 8, 9, 10, 11, 12]. We have investigated the idea of using the Higgs inflation model for generating the observed large scale magnetic fields in the universe. We start with the conformal symmetry breaking coupling of the Higgs with the photons which arises form loop corrections and add this interaction to the Higgs inflation model. We see that the Higgs photon coupling can generate a large scale magnetic field when the Higgs rolls down the inflaton potential. This mechanism has the advantages over similar inflaton-photon coupling terms, which can generate magnetic fields, that there is no problem of back-reaction due to the large curvature coupling. The Higgs-photon coupling term does not dominate the Higgs potential term as it is suppressed by the curvature coupling ξ.

The standard model of inflation involves a slow-roll potential where the inflaton potential term dominates the kinetic energy term. This gives rise to an exponential expansion of the universe which is responsible for the solving the horizon and flatness problems of cosmology but in addition it gives rise to scale-invariant density perturbations. One method of creating scale invariant density perturbations which does not need a slow roll potential was pointed out by Rubakov [13]. He showed that if a scalar is conformally coupled to gravity and it has a negative quartic potential where the radial part rolls down, then the phase of the scalar field has a scale invariant density perturbations. We apply this idea in the inert Higgs doublet model where we show that in the early universe the inert Higgs has a negative quartic potential and one can generate the observed scale invariant density perturbations without involving order 10^{-12} coupling constants. In addition the parameters of this model can be chosen to give correct Higgs mass of 125-126 GeV and viable dark matter candidate.

The mass of the Higgs boson lies in the range 125-126 GeV which is close to the vacuum stability limit [14, 15, 16, 17]. This Higgs mass corresponds to a Higgs self coupling of $\lambda \sim 0.13$ at the electroweak scale. The idea of vacuum stability is that the $\lambda(\mu)$ should remain positive for all μ upto the Planck scale otherwise the universe will roll down the negative potential making the vacuum unstable. This mass range of Higgs implies that any new physics which has interaction with the Higgs will influence the vacuum stability and therefore the vacuum stability condition can be used for constraining new physics. In this thesis we study the Type-I seesaw models where the neutrino Yukawa couplings change the renormalization group running of the Higgs coupling λ. So the vacuum stability condition implies that the neutrino Dirac mass cannot be larger than 20 GeV. This has important consequences for neutrino mass models and heavy neutrino phenomenology which we study in detail.

Chapter 2

Review of cosmological theory of Inflation

This chapter summarizes the previous work done in the proposed area of this thesis. It contains the standard model of cosmology and its success and drawbacks. This chapter describes how inflation can solves the problem of Big-bang model. Description of the inflationary dynamics and different kinds of inflation models are presented in this chapter. This chapter also includes the Higgs inflation model. The last section of this chapter describes the study of vacuum stability for the standard model Higgs.

2.1 Standard Model of Cosmology

Cosmological model of the universe predicts about the early stage of the universe. Big-bang theory is one of the successful cosmological models in explaining the early universe. This can be regarded as the starting point in our understanding of the evolution of the universe. According to this theory, the universe was extremely hot and dense about 12 to 14 billion years ago. Afterwards the universe started expansion and continues till date. The idea of expanding universe is mainly based on the observation of American astronomer, Edwin Hubble in 1929 and the observation was that the redshift of galaxies are proportional to their distances. The expansion caused the hot universe to cool down to the present stage and during this, it produced the matter. Hence the radiation decoupled from matter and continued travel through the space mostly without any hindrance. This radiation is known as Cosmic Microwave Background Radiation (CMBR). The discovery of CMBR by Arno Penzias and Robert Wilson in 1964 satisfied this theory from observational point of view. This theory rests on two theoretical pillars, one is "General Theory of Relativity" and the other one is the assumption, known as "Cosmological principle", that the universe is homogeneous and isotropic. Knowledge of General Relativity helps to understand the cosmology. The realization of the space is done by the Friedmann-Robertson-Walker metric (FRW) as follows,

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a(t)^2 & 0 & 0 \\ 0 & 0 & a(t)^2 & 0 \\ 0 & 0 & 0 & a(t)^2 \end{pmatrix} \tag{2.1}$$

where $a(t)$ denotes the scale factor of the universe. The Einstein equation relates the space with matter in universe [18],

$$G_{\mu\nu} \equiv \mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi G T_{\mu\nu} \tag{2.2}$$

where $G_{\mu\nu}$ and G are Einstein tensor and constant, $\mathcal{R}_{\mu\nu}$ and $\mathcal{R}$ are Ricci tensor and scalar, which depend on the metric $g_{\mu\nu}$, $T_{\mu\nu}$ is the energy momentum tensor. Using $T_{\mu\nu} = (p+\rho)u^{\mu}u^{\nu} + pg^{\mu\nu}$, we can solve the Einstein equation, which gives the following Friedmann equations,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2} \tag{2.3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{2.4}$$

where dot signifies the derivative with respect to time and $\frac{\dot{a}}{a}$ denotes the Hubble parameter and its present value is, 71.0 ± 2.5 km/s/Mpc [19]. Here κ indicates the curvature and $\kappa = (+1, 0, -1)$ respectively denotes closed, flat and open universe. The Friedmann equation alone is not sufficient to describe the universe and we need the fluid equation,

$$\frac{\dot{\rho}}{\rho} = -3(1+\omega)\frac{\dot{a}}{a} \tag{2.5}$$

where $\omega \equiv p/\rho$ signifies the equation of state. Different values of ω describes the different era of the universe as follows,

- Matter dominated era:
  $\omega = 0$, $\rho \propto \frac{1}{a^3}$ and $a \propto t^{2/3}$
- Radiation dominated era:
  $\omega = \frac{1}{3}$, $\rho \propto \frac{1}{a^4}$ and $a \propto t^{1/2}$
- Era dominated by cosmological constant: $\omega = -1$, $\rho =$ Constant and $a \propto e^{Ht}$

The density of the universe is defined by the density parameter Ω as follows,

$$\Omega \equiv \frac{\rho(t)}{\rho_c} \tag{2.6}$$

where $\rho_c = \frac{3H^2}{8\pi G}$ is the critical density. The values of $\Omega > 1, = 1$ and < 1 describe the closed, flat and open universe respectively. Eq. (2.3) can be rewritten in terms of the density parameter Ω as follows,

$$\Omega - 1 = \frac{\kappa}{a^2 H^2} \tag{2.7}$$

In spite of its success, Big-bang theory has unsolved problems such as Flatness problem and Horizon problem.

2.1.1 Flatness problem

Observations reveals that the density parameter at present $\Omega \simeq 1$ and hence the total energy density is almost equal to the critical energy density. If at present day, Ω is exactly equal to 1, then it has always been equal to one. However if $\Omega \simeq 1$ at present era, then it was extremely close to 1 at the time of the Hot Big Bang and extrapolating back in times gives $\Omega = 1 \pm 1 \times 10^{-60}$ at that time. In order to get the correct value of Ω at present, the value of Ω at early times has to be fine-tuned to the value amazingly close to zero, but without being exactly zero. Hence this flatness problem is also known as “fine-tuning problem”.

2.1.2 Horizon problem

Decoupling of the photons, from matter, produced Cosmic Microwave Background radiation at “Last scattering surface” with red shift of $z \simeq 1100$. The red shift, z can be defined in terms of scale factor as,

$$1 + z \equiv \frac{a(t_0)}{a(t)} \tag{2.8}$$

where $a(t_0)$ denotes the scale factor at present universe. Detection of CMBR reveals the snapshot of the universe of age around $300,000$ years old. This CMBR can be described by the a black body at temperature 2.73 K. If $R_H(t_0)$ denotes the present Hubble radius, then it was correspond to the length scale

$\lambda_H(t_{LS})$ at the time of last scattering surface and is given by,

$$\lambda_H(t_{LS}) = R_H(t_0)\,\frac{a_{LS}}{a_0} = R_H(t_0)\,\frac{T_0}{T_{LS}} \tag{2.9}$$

However the Hubble length was decreased with a different law during the matter-dominated period,

$$H^2 \propto a^{-3} \propto T^3 \tag{2.10}$$

Hence at last scattering surface horizon was,

$$H_{LS}^{-1} = R_H(t_0)\left(\frac{T_{LS}}{T_0}\right)^{-3/2} \tag{2.11}$$

and it is much smaller than $R_H(t_0)$. Comparing the volumes corresponding to these two era gives,

$$\frac{\lambda_H^3(t_{LS})}{H_{LS}^{-3}} = \left(\frac{T_0}{T_{LS}}\right)^{-3/2} \approx 10^6 \tag{2.12}$$

The volume, which corresponds to our horizon, contained $\sim 10^6$ number of casually disconnected regions in the past.

2.2 Historical developments of Inflation

These unsolved problems of Big-bang model motivated people to modify or extend this theory without losing its successful predictions. In 1980, an American physicist, Alan Guth was came up with the new idea, called *Inflation* [20]. This hypothesis is based on scalar field with first order phase transition.

The idea of inflation is a leading contender as a cosmological theory. In inflationary epoch, universe underwent an era of exponential expansion. Inflation can give the resolution to the flatness problem. During the inflationary period, space-time expanded to such an extent that its curvature have been smoothed out and driven the Universe to a very nearly spatially flat state, with almost exactly the critical density. This exponential expansion pushed the large regions of space well beyond our observable horizon and solved the Horizon problem. Inflation can also help to understand the problem of magnetic monopole. Due to the rapid expansion, inflation can dilute the abundance of these objects to such an extent that it can not be measured. This idea of inflation not only explains the long standing horizon and flatness problems of cosmology but also generates a scale invariant density perturbations needed to explain the galaxy formation.

2.3 Inflation solves the Flatness and Horizon problems

2.3.1 Flatness problem

Flatness, which is also known as fine-tuning problem can be solved in the inflation model. From Eq. (2.7), the relation of density parameter with Hubble parameter and scale factor is given by,

$$\Omega - 1 = \frac{\kappa}{a^2 H^2} \propto \frac{1}{a^2} \tag{2.13}$$

considering Hubble parameter, H remains constant in inflationary era. At present era, it is,

$$\Omega_0 - 1 \sim 1 \tag{2.14}$$

where 0 denotes the present epoch. Extrapolation of this present day value to the beginning of radiation-dominated era gives,

$$|\Omega - 1| \sim 10^{-60} \tag{2.15}$$

As the staring of the radiation era is considered as the end of inflation $t = t_f$, $|\Omega - 1|_{t=t_f} \sim 10^{-60}$. If we consider at the starting of inflation, $|\Omega - 1|_{t=t_i} \sim 1$, then it can be decreased upto $|\Omega - 1|_{t=t_f} \sim 10^{-60}$, by considering $N \approx 70$, number of e-foldings as follows,

$$\frac{|\Omega - 1|_{t=t_f}}{|\Omega - 1|_{t=t_i}} = \left(\frac{a_i}{a_f}\right)^2 = e^{-2N} \tag{2.16}$$

Hence we can avoid the fine tuning and solve the flatness problem in inflationary models.

2.3.2 Horizon problem

The physical scales which are larger than horizon scale during radiation-dominated or matter-dominated era, left the horizon. However the presence of sufficiently long inflation period can exponentially reduce these physical scales and these scales reentered the horizon afterwards. Microphysics can act on these scales as they are within the horizon and hence help to create approximately homogeneous universe. It can also resolve the issues relating to the homogeneity of CMB and the initial conditions for cosmological perturbations.

The largest scale, we observed today, is the present horizon H_0^{-1} and during inflation, the scale $\lambda_{H_0}(t_i)$, which corresponds to H_0^{-1} at present, should be smaller than the horizon length during inflation H_I^{-1}. This is the necessary condition to solve the horizon problem and now we can find out the number of e-foldings required for this condition as follows,

$$\lambda_{H_0}(t_i) = H_0^{-1}\left(\frac{a_{t_f}}{a_{t_0}}\right)\left(\frac{a_{t_i}}{a_{t_f}}\right) = H_0^{-1}\left(\frac{T_0}{T_f}\right)e^{-N} \leq H_I^{-1} \tag{2.17}$$

where T_f denotes the temperature at the end of inflation and also signifies the reheating temperature. The simplification of Eq. (2.17) gives,

$$N \geq \ln\left(\frac{T_0}{H_0}\right) - \ln\left(\frac{T_f}{H_I}\right) \approx 67 + \ln\left(\frac{T_f}{H_I}\right) \tag{2.18}$$

Hence number of e-foldings, $N \geq 70$ can solve the horizon problem.

To have the inflation in past, the scalar field with high energy density is required to be present at that time. This scalar field is known as *Inflaton*. In 1982, Linde, Albrecht and Steinhardt amongst other gave a new model for inflation [21, 22]. In subsequent years, the different idea for inflation came up in the literature such as Chaotic inflation, Hybrid inflation, Power-law inflation etc.

2.4 Basic of Inflation Model

In the next section, basics of the inflation models are provided for completeness of the theory.

2.4.1 Inflationary Dynamics

Idea of inflation is not the replacement to Big-bang model, rather accompanying with it, inflation can solve the cosmological problems. Inflation is defined as an epoch during which scale factor is accelerating,

$$\ddot{a} > 0 \tag{2.19}$$

In this era, the universe underwent exponential expansion. This relation can be reconstructed into the condition for equation of state ($\omega = p/\rho$). From 2nd Friedmann equation, (2.4), we find

$$(\rho + 3p) < 0 \tag{2.20}$$

$$p < -\frac{\rho}{3} \tag{2.21}$$

As the energy density is always a positive quantity, the pressure should be negative for the occurrence of the inflation.

The candidate responsible for inflation can be the simple scalar field, which can easily satisfy the criteria for negative pressure. The recent measurement of ATLAS and CMS [1, 23, 24] have confirmed the existence of a new boson which has mass in the range 126.5 GeV (ATLAS at 5.0σ) and 125.3 ± 0.6 GeV

(CMS at 4.9σ), and it is expected to be a Standard Model Higgs.

The Lagrangian which describes the inflaton field is as follows,

$$L = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \tag{2.22}$$

The stress energy momentum tensor will be,

$$T_{\mu\nu} = (p + \rho) u^\mu u^\nu + p g^{\mu\nu} \tag{2.23}$$

where u^μ is 4-velocity, ρ is density, p is pressure and T^{00} and T_{ii} are defined as ρ and $p\,\delta_{ii}$, respectively. Using the Lagrangian in Eq. (2.22) and the stress energy momentum tensor in Eq. (2.23), we can derive the equations for energy density and pressure of the scalar field ϕ as follows,

$$\begin{aligned} \rho_\phi &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p_\phi &= \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{aligned} \tag{2.24}$$

Here if $V(\phi) \gg \dot{\phi}^2$, called the slow roll approximation, then the relation between energy density and pressure of the scalar field will be,

$$p_\phi \simeq -\rho_\phi \tag{2.25}$$

This simple calculation, reveals that a scalar field whose energy is dominant in the universe and whose potential energy dominates over the kinetic term, gives inflation.

The equation of motion for the field ϕ derived from the Lagrangian Eq. (2.22) will be,

$$\ddot{\phi} + 3H\,\dot{\phi} + V'(\phi) = 0 \tag{2.26}$$

where H is the Hubble parameter and $V'(\phi)$ is the derivative of the potential with respect to ϕ. The Hubble parameter is,

$$H^2 = \frac{8\pi G}{3}\left(V(\phi) + \frac{\dot{\phi}^2}{2}\right) \tag{2.27}$$

2.4.2 Slow roll approximation

To analyze the inflation, we need to assume the slow roll approximation. According to this approximation, inflaton field is slowly rolling down a flat potential and hence the slow roll conditions will be,

$$V(\phi) \gg \dot{\phi}^2 \tag{2.28}$$

$$3H\dot{\phi} \gg \ddot{\phi} \tag{2.29}$$

These conditions can be rewritten in terms potential and Hubble parameter as,

$$\frac{(V')^2}{V} \ll H^2 \tag{2.30}$$

$$V'' \ll H^2 \tag{2.31}$$

A. Liddle and D. Lyth introduced two parameters ϵ and η which express these conditions mathematically. These are known as slow roll parameter and defined as,

$$\epsilon \equiv \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2 \ll 1 \tag{2.32}$$

$$\eta \equiv \frac{1}{8\pi G}\frac{V''}{V} \ll 1 \tag{2.33}$$

These parameters are very useful to quantify the predictions of the inflation.

Using this slow roll approximation, we can simplify the Eq. (4.19) as follows,

$$3H\,\dot{\phi} + V' \simeq 0 \tag{2.34}$$

and the Hubble parameters becomes,

$$H^2 \simeq \frac{8\pi G}{3} V(\phi) \tag{2.35}$$

In general, slow-roll inflation is attained if $\epsilon \ll 1$ and $\eta \ll 1$ and as soon as this condition fails, inflation ends. Therefore $\epsilon = 1$ signifies the end of inflation.

2.4.3 Duration of inflationary era

Duration of inflation is defined by the number of e-foldings N. The amount of inflation is in general quantified by the ratio of the scale factor at final to its initial value. The number of e-foldings N can be defined as,

$$N \equiv \ln \frac{a(t_f)}{a(t_i)} = \int_{t_i}^{t_f} H dt \tag{2.36}$$

where t_i and t_f denote the starting and the end of the inflation. Using slow roll approximation, the number of e-foldings N can be simplified as,

$$\begin{aligned} N &= \int_{t_i}^{t_f} H dt \\ &\simeq H \int_{\phi_i}^{\phi_f} \frac{d\phi}{\dot{\phi}} \\ &\simeq -3H^2 \int_{\phi_i}^{\phi_f} \frac{d\phi}{V'} \\ &\simeq -8\pi G \int_{\phi_i}^{\phi_f} \frac{V\, d\phi}{V'} \end{aligned} \tag{2.37}$$

To solve the horizon and flatness problems, approximately, 60 e-foldings are required.

2.4.4 Generation of scale invariance density perturbation

The important aspect of inflation model is the generation of density perturbation and the formation of large scale structure. Quantum fluctuations of the inflaton field generate the density perturbation, which subsequently produce galaxies. In inflationary era, scale factor grows exponentially and the vacuum structure is much more complicated compared to ordinary Minkowski space. According to quantum field theory, an empty space is not absolutely empty, instead it is full of quantum fluctuations of different physical fields. These fluctuations contain the waves of different wavelengths of these physical fields and are moving in every directions. Hence the average over macroscopically large time creates vacuum and these fields appear to us as empty.

If the fluctuation of the inflaton field ϕ is denoted by $\delta\phi$, then the equation of motion of these fluctuations will be,

$$\delta\ddot{\phi}_{\mathrm{k}} + 3H\,\delta\dot{\phi}_{\mathrm{k}} + \frac{k^2}{a^2}\delta\phi_{\mathrm{k}} = 0 \tag{2.38}$$

The wavelength of the fluctuation ($\lambda = 2\pi a/k$) within the horizon $\lambda \ll H^{-1}$ implies,

$$k \gg aH \tag{2.39}$$

and we can neglect the friction term $3H\,\delta\dot{\phi}_{\mathrm{k}}$ due to the expansion, then the Eq. (2.38) becomes,

$$\delta\ddot{\phi}_{\mathrm{k}} + \frac{k^2}{a^2}\delta\phi_{\mathrm{k}} = 0 \tag{2.40}$$

which simply describes the equation of motion for harmonic oscillator. When wavelength remains within the horizon, these fluctuations of the field oscillate.

For superhorizon case, $\lambda \gg H^{-1}$ gives the relation as follows,

$$k \ll aH \tag{2.41}$$

Now we can neglect the last term of the left hand side of Eq. (2.38) and it simplifies to,

$$\delta\ddot{\phi}_{\mathrm{k}} + 3H\,\delta\dot{\phi}_{\mathrm{k}} = 0 \tag{2.42}$$

This equation clearly shows that at superhorizon scale, the solution for $\delta\phi_{\mathrm{k}}$ becomes constant in time and hence the amplitude became frozen. This phenomena occurred in inflationary era, when fluctuations left the horizon. At the time of inflation, the wavelengths of the quantum fluctuation of the inflaton field ϕ grows exponentially. When the wavelength becomes larger than horizon H^{-1}, the oscillations of the fluctuations cease and the propagation of those fluctuations also stop. At this stage, the amplitude of these fluctuations becomes frozen to some value $\delta\phi_{\mathrm{k}}$. This happened due to the friction term $3H\,\delta\dot{\phi}_{\mathrm{k}}$, coming from the expansion.

Quantum fluctuations of the inflaton field are also connected to the perturbations of the metric through Einstein equation, Eq. (2.2). Now we will discuss the generation of density perturbations in details and use the simplest mathematical gauge ‘Longitudinal gauge’ [25, 26, 27, 28]. We have considered the only scalar degrees of freedom for the perturbed metric and the line element will be as follows,

$$\begin{aligned} ds^2 = a^2(\tau)\big[-(1+2A)\,d\tau^2 + 2\partial_i B d\tau dx^i + ((1-2\Psi)\delta_{ij} \\ + (\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2)E)dx^i dx^j\Big] \end{aligned} \tag{2.43}$$

where A, ψ, B and E are the scalar quantities, depend on space and time. For longitudinal gauge, the condition is $B = E = 0$ and putting $A = \Phi$, the metric will be,

$$\begin{aligned} ds^2 &= a^2(\tau)\left[-(1+2\Phi)\,d\tau^2 + ((1-2\Psi)\delta_{ij}dx^i dx^j\right] \\ &= -(1+2\Phi)\,d\tau^2 + a^2(t)(1-2\Psi)\delta_{ij}dx^i dx^j \end{aligned} \tag{2.44}$$

The potential Φ and Ψ are the gauge invariant quantities which remain unchanged under the conformal transformation. For diagonal energy-momentum tensor, these two become alike. Without cosmological constant $\Lambda = 0$, the Einstein equation is,

$$\delta G^{\mu}_{\nu} = 8\pi G\delta T^{\mu}_{\nu} \tag{2.45}$$

and the perturbed Einstein equations are,

$$\begin{aligned}
\Delta^2\Phi - 3\mathcal{H}\Phi' - \left(\mathcal{H}' + 2\mathcal{H}^2\right)\Phi &= \frac{8\pi G}{2}\left(\phi_0'\delta\phi_k + V'(\phi)a^2\delta\phi_k\right) \\
\phi' + \mathcal{H}\Phi &= \frac{8\pi G}{2}\phi_0'\delta\phi_k \\
\Phi'' - 3\mathcal{H}\Phi' + \left(\mathcal{H}' + 2\mathcal{H}^2\right)\Phi &= \frac{8\pi G}{2}\left(\phi_0'\delta\phi_k - V'(\phi)a^2\delta\phi_k\right)
\end{aligned} \tag{2.46}$$

where $\mathcal{H} \equiv a'/a$ and ϕ_0 and $\delta\phi_k$ are the homogeneous background and the perturbation of the inflaton field. Simplification of these equations give the equation for the gauge-invariant scalar field,

$$\delta\phi_k'' + 2\mathcal{H}\delta\phi_k' - \nabla^2\delta\phi_k + V''(\phi)a^2\delta\phi_k = 4\phi_0'\Phi' - 2V'(\phi)a^2\Phi \tag{2.47}$$

The relation between the scalar potential and fluctuations is $\Phi \simeq \epsilon H\delta\phi_k/\dot{\phi}_0$ On superhorizon scale, $|4\phi_0'\Phi'| \ll |2V'(\phi)a^2\Phi|$ and using the slow roll condition $3H\dot{\phi} \simeq -V'(\phi)$

$$\delta\phi_k'' + 2\mathcal{H}\delta\phi_k' + \left[k^2 + V''(\phi)a^2 + 6a^2\epsilon H^2\right]\delta\phi_k = 0 \tag{2.48}$$

After redefinition of the field $\delta\phi_k = \delta\sigma_k/a$, Eq. (2.48) becomes,

$$\delta\sigma_k'' + \left[k^2 - \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\right]\delta\sigma_k = 0 \tag{2.49}$$

where $\nu^2 = \frac{9}{4} + 9\epsilon - 3\eta$ and in this case,

$$\begin{aligned}
\frac{a''}{a} &= a^2\left(2-\epsilon\right)H^2 \\
&\simeq \frac{2+3\epsilon}{\tau^2}
\end{aligned} \tag{2.50}$$

Solution of this equation Eq. (2.49) can be written in term of Hankel function as follows,

$$\delta\sigma_k = \sqrt{-\tau}\left[c_1(k)H_\nu^{(1)}(-k\tau) + c_2(k)H_\nu^{(2)}(-k\tau)\right] \tag{2.51}$$

where $H_\nu^{(1)}$ and $H_\nu^{(2)}$ are the first and second Hankel functions. For superhorizon case ($k \gg aH$ or $-\,\mathrm{k}\tau \gg 1$), quantum fluctuations are plane-wave form $e^{-ik\tau}/\sqrt{2k}$. This solves the constant c_1 and c_2 as follows,

$$c_1(k) = \frac{\sqrt{\pi}}{2} e^{i(\nu+\frac{1}{2})\frac{\pi}{2}} \tag{2.52}$$

$$c_2(k) = 0 \tag{2.53}$$

and the solution becomes,

$$\delta\sigma_k = \frac{\sqrt{\pi}}{2} e^{i(\nu+\frac{1}{2})\frac{\pi}{2}} \sqrt{-\tau} H_\nu^{(1)}(-k\tau) \tag{2.54}$$

When $-k\tau \ll 1$, the Hankel function becomes,

$$H_\nu(-k\tau \ll 1) \simeq \sqrt{\frac{2}{\pi}} e^{-i\frac{\pi}{2}} 2^{\nu-\frac{3}{2}} \left(\frac{\Gamma(\nu)}{\Gamma(\frac{3}{2})}\right) (-k\nu)^{-\nu} \tag{2.55}$$

Using the Hankel function (2.55) and $\delta\phi_k = \delta\sigma_k/a$, the amplitude of the fluctuations becomes,

$$|\delta\phi_k| \simeq \frac{H}{\sqrt{2k^3}} \left(\frac{k}{aH}\right)^{\frac{3}{2}-\nu} \tag{2.56}$$

The calculation of the curvature power spectrum is given in the next section

2.4.5 Curvature power spectrum

Power spectrum is an important quantity to analyze the perturbations. Any quantity $f(x,t)$ can be written in Fourier space as,

$$f(x,t) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} f_{\mathbf{k}}(t) \tag{2.57}$$

Now the power spectrum of the function $f(x,t)$ is defined as,

$$\langle 0|f^*_{\mathbf{k_1}} f_{\mathbf{k_2}}|0\rangle \equiv \delta^{(3)}(\mathbf{k_1} - \mathbf{k_2}) \frac{2\pi^2}{k^3} \mathcal{P}_f(k) \tag{2.58}$$

Hence the relation of the power spectrum will be

$$\langle 0|f^2(x,t)|0\rangle = \int \frac{dk}{k}\mathcal{P}_f(k) \tag{2.59}$$

The gauge invariant comoving curvature perturbation is,

$$\mathcal{R}_{\mathbf{k}} = \frac{H\delta\phi_k}{\dot{\phi}_0} \tag{2.60}$$

Therefore, the power spectrum of comoving curvature perturbation $\mathcal{R}_{\mathbf{k}}$ will be as follows,

$$\mathcal{P}_{\mathcal{R}} = \frac{k^3}{2\pi^2}\langle|\mathcal{R}|^2\rangle = \frac{k^3}{2\pi^2}\frac{H^2}{\dot{\phi}_0^2}|\delta\phi_k|^2 \tag{2.61}$$

Substituting $\delta\phi_k$ from Eq. (2.56), power spectrum will look like,

$$\mathcal{P}_{\mathcal{R}} = \frac{4\pi}{M_p^2\epsilon}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{3-2\nu} \equiv \mathcal{A}_{\mathcal{R}}^2\left(\frac{k}{aH}\right)^{n_s-1} \tag{2.62}$$

where n_s denotes the spectral index. It can calculated from the power spectrum as,

$$n_s - 1 = \frac{d\ln\mathcal{P}_{\mathcal{R}}}{d\ln k} = 3 - 2\nu \tag{2.63}$$

ν is defined in Eq. (2.49) and it can simplified to $(3+6\epsilon-2\eta)/2$. Hence the spectral index will be,

$$n_s = 1 - 6\epsilon + 2\eta \tag{2.64}$$

n_s defines the tilt of the spectrum. $n_s \sim 1$ corresponds to the scale invariant perturbation, which is consistent with the WMAP observations [19]. In inflationary models, the slow-roll parameters ϵ and η are small, hence it is easy to produce the scale invariant perturbations. $n_s < 1$ corresponds to red-tilted spectrum, occurred in chaotic inflation model. However $n_s > 1$ signifies blue-tilted spectrum and power spectrum of the hybrid inflation is an example of it.

2.4.6 Observable parameters

One of the important tools to study the origin and characteristics of the universe is the knowledge of CMBR. Primordial vacuum fluctuations resulting from the quantum nature of the scalar field relate the inflation model and CMBR. WMAP observations of CMBR puts the bound on the inflation parameters, such as spectral index n_s, amplitude of curvature fluctuation $\mathcal{P}_{\mathcal{R}}$ etc [19] as follows,

$$n_s = 0.963 \pm 0.014 \tag{2.65}$$

$$\mathcal{P}_{\mathcal{R}} = (2.43 \pm 0.11) \times 10^{-9} \tag{2.66}$$

Both notation $\mathcal{P}_{\mathcal{R}}$ and Δ_R^2 have been used in this thesis.

I have mentioned only those parameters, which have been used in my calculations. However there are more parameters of inflation model, which are bounded from the CMBR observations.

2.5 Reheating

The discrepancy of the Big-bang theory has been smoothed out due to the rapid expansion of the universe during inflationary epoch. With the acceleration, $\ddot{a} > 0$, the temperature of the universe falls exponentially and the universe becomes supercooled. However, this is not a favorable situation for the formation of atomic nuclei, which requires far greater temperature. To achieve this, we need the reheating era in most of the inflation models. Reheating can be thought as a graceful exit to the inflation and it can transformed supercooled universe to the hot radiation-dominated universe.

The details of reheating mechanism depends on the interactions between the inflaton field and the surrounding fields. In standard inflationary theory, the production of particles happens during this oscillatory reheating phase.

Reheating process, in general, consists of three following parts,

- **Oscillation of the inflaton :**

  At the end of inflation, the inflaton field comes down to the minimum of the potential and starts oscillation. In this oscillation phase, the energy density follows the equation of the density of non-relativistic matter and hence density varies as $1/a^3$.

- **Decay of inflaton particle :**

  Decay of inflaton particle happens when the Hubble parameter becomes equal to the decay time. In this case, the equation of motion of the inflaton field is modified by the decay term "$\Gamma\dot{\phi}$" and becomes,

  $$\ddot{\phi} + (3H + \Gamma)\ \dot{\phi} + V'(\phi) = 0 \tag{2.67}$$

  This kind of equation is valid in slow-decay case and in this decay mode, only fermionic decays are available. However rapid decay of inflaton particle can produce the bosonic particles also. Parametric resonance allows this decay. In rapid decay of inflaton, the oscillating phase ends nearly as soon as it begins and this decay mode sometime termed as "preheating".

- **Thermalization of the decay products :**

  The decay product reach thermalization through the interactions, which depends on the accepted field theory. And it also determines the temperature, at which the universe reenter the standard Hot Big-bang scenario.

2.6 Different models of Inflation

In 1983, Andrei Linde proposed the most popular single field inflation model, *Chaotic inflation*, based on a scalar field with no phase transition and the shape of the potential automatically lead to inflation [29].

2.6.1 Chaotic inflation

The name "Chaotic" is related to the arbitrary initial conditions for the scalar field. The only restriction on the scalar field is that it should be greater than Plank scale M_p.

In general, this kind of inflation can be derived from the potential of the form,

$$V(\phi) = \frac{m^2\phi^2}{2} \tag{2.68}$$

In chaotic inflation, the inflaton field ϕ is moving towards the origin of its potential.

The field ϕ satisfies the following equation of motion,

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi \tag{2.69}$$

The term $3H\dot{\phi}$ behaves as the friction term. In this case, the Hubble parameter will be,

$$H^2 = \frac{8\pi G}{3}\left(\frac{\dot{\phi}^2}{2} + \frac{m^2\phi^2}{2}\right) \tag{2.70}$$

considering the flat universe ($\kappa = 0$). As ϕ is very large ($\sim M_p$), the friction term $3H\dot{\phi}$ will be large and the field rolls down the potential very slowly. Hence the kinetic energy in Hubble parameter in Eq. (2.70) can be neglected and in this scenario,

$$H^2 = \frac{8\pi G}{3}\frac{m^2\phi^2}{2} = \frac{m^2\phi^2}{6M_p^2} \tag{2.71}$$

where $(8\pi G)^{-1} = M_p^2$.

Solving the Eq. (2.71), gives exponential scale factor $a \sim e^{Ht}$ with $H = m\phi/\sqrt{6}M_p$. Note that, in the chaotic inflation scenario, the inflationary expansion does not depend on the choices of initial conditions. After inflation, reheating process will start. During reheating, the inflaton field oscillates coherently around the minimum of its potential and produces particles via parametric resonance.

Later on more than one scalar field are also used to describe the inflation. This kind of models in the literature are known as *Hybrid inflation*[30].

2.6.2 Hybrid inflation

In the mid-1990s, there was challenge to build inflation model based on particle physics motivation such as supersymmetry, supergravity and superstring theory. This motivation reveals the idea of Hybrid inflation and it was introduced by Linde [30, 31, 32].

This models consist of at least two scalar fields. One field behaves as inflaton field, rolling down its flat potential, and the other field is a symmetry breaking field that is trapped in a false vacuum state. Symmetry breaking field remains zero in inflationary era and hence does not contribute to the perturbation spectrum.

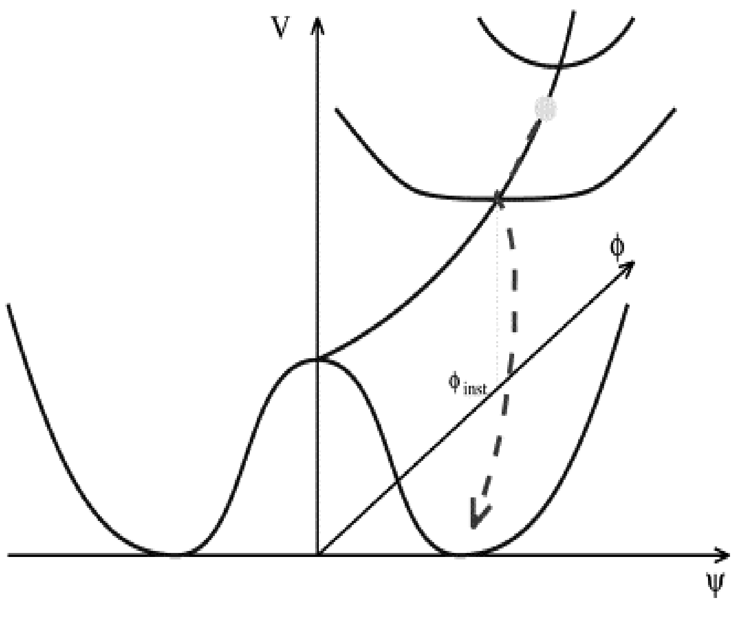

Figure 2.1: Schematic diagram of Hybrid inflation model

Hybrid inflation described by the two scalar fields can be generated from following potential shown in Fig. (2.1),

$$V(\psi, \phi) = \frac{1}{4\lambda} \left(M^2 - \lambda\psi^2\right)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\psi^2 \tag{2.72}$$

When $\phi > \phi_c = M/g$, the minimum of the potential will be at $\psi = 0$ and the universe is in inflationary era. Then the inflation ends at $\phi = \phi_c$ and subsequently the phase transition with the symmetry breaking occurs when $\phi < \phi_c$.

The most interesting feature of this model is that the inflation field only generates the inflationary expansion, while the symmetry breaking field helps in reheating the universe and particle production. There is no need to couple the inflaton field directly to other degrees of freedom, unlike in the simpler versions of chaotic inflation.

2.6.3 Power-law inflation

In this kind of inflation model, the universe underwent power-law expansion and because of this, it has named “Power-law inflation”. This kind of inflation can be generated from the exponential potential as follows,

$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{2}{p}}\frac{\phi}{M_p}\right) \tag{2.73}$$

The most interesting feature of this inflation model is that this model can be solved analytically and gives the scale factor as,

$$a(t) \propto t^p \tag{2.74}$$

where t denotes the time scale. To satisfy the conditions for the inflation p has to be greater than 1. In this case, the slow roll parameters do not depend on the inflaton field ϕ and becomes $\epsilon = \eta/2 = 1/p$. The WMAP constrain on the spectral index, $n_s = 1 + 2\eta - 6\epsilon$ in [19] is,

$$n_s = 0.963 \pm 0.012 \tag{2.75}$$

and this gives $p \sim 50$.

Inflation models discussed till now are basically "**Supercooled inflation**" model. In these kind of model the universe becomes supercooled during inflation and after that reheating is required to produce the light particles and to warm up the universe.

However, there are other kind of inflation, known as Warm inflation [33, 34, 35, 36, 37, 38]. In this case, the dissipative effects are important and the radiations are also produced simultaneously during inflation.

2.6.4 Warm inflation

In supercooled inflation models, as the universe expands, the temperature drops exponentially and immediately it becomes irrelevant for the particle production. If the interactions between the inflaton field with the other particles of the universe remain there, then the inflaton field transfers some of its energy to the thermal bath. In this way it prevents the temperature to fall to zero. This is the main idea for warm inflation.

Hence in warm inflation model, the dissipative effects become important during the inflationary epoch. In this case production of radiation occurs concurrently with the expansion, resulting in inflaton interactions.

In this case, the inflaton field satisfies the following equation of motion,

$$\ddot{\phi} + 3H\,(1+r)\,\dot{\phi} + V'(\phi) = 0 \tag{2.76}$$

where r is the ratio between the thermal damping factor Γ and the expansion damping, signifies by the Hubble parameter H. Small value of r suppresses the effect of the thermal damping and it becomes supercooled inflation model.

I have briefly discussed the basics of the inflation and the different kinds of inflation model. However depending upon the nature of the potential and type of the inflaton field, there are various models described in literature. Besides the standard inflation model, people came up with non-standard inflation model such as, natural inflation, inflation from f(r) gravity, SUSY F-term inflation, SUSY D-term inflation, tachyon inflation, DBI inflation etc, to describes the different aspect of the experimental results and phenomenology. Since my work is primarily dealt with Higgs-inflation, therefore, only this model is presented in next section.

2.7 Higgs inflation

The recent measurement of ATLAS and CMS [1, 23, 24] have confirmed the existence of a new boson which has mass in the range 126.5 GeV (ATLAS at 5.0σ) and 125.3 ± 0.6 GeV (CMS at 4.9σ), and it is expected to be the Standard Model Higgs. In particle physics, Higgs boson plays the fundamental role by generating the masses of all the known particles through electroweak symmetry breaking. Higgs is not only a favorable candidate for particle physics, it also has great importance in cosmology as most of the inflation models need scalar field to explain the early universe. Hence the study of the inflation model, in which standard model Higgs can be the inflaton field, has great importance in understanding the early universe.

2.7.1 Higgs minimally coupled with gravity

The subsequent attempt to make Higgs as inflaton is to consider the minimal coupling between the Higgs and the gravity. In this kind of model, action will be,

$$S = \int d^4x\sqrt{-g}\left[\frac{RM_p^2}{2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2\right] \tag{2.77}$$

First term denotes the Einstein-gravity term and second and third term of the action describes the kinetic and potential energy. In inflationary era, ϕ is large, we can neglect the vev of the Higgs, $v = 246$ GeV and the potential becomes,

$$\frac{\lambda}{4}\left(\phi^2 - v^2\right)^2 \simeq \frac{\lambda}{4}\phi^4 \tag{2.78}$$

In this case, equation of motion of the Higgs will be,

$$\ddot{\phi} + 3H\,\dot{\phi} + \lambda\phi^3 = 0 \tag{2.79}$$

and the Hubble parameter is,

$$H^2 = \frac{1}{6M_p^2}\left(\dot{\phi}^2 + \frac{\lambda}{2}\phi^4\right) \tag{2.80}$$

Slow-roll approximation gives,

$$\dot{\phi}^2 \ll \frac{\lambda}{2}\phi^4 \quad \text{and} \quad |\ddot{\phi}| \ll 3\text{H}\,|\dot{\phi}| \tag{2.81}$$

and Eq. (2.79) and (2.80) become,

$$H^2 \simeq \frac{\lambda\phi^4}{12M_p^2} \tag{2.82}$$

$$\text{and} \qquad \dot{\phi} \simeq -\frac{\lambda}{3H}\phi^3 \tag{2.83}$$

In inflationary era, slow roll parameter ϵ is,

$$\epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{8M_p^2}{\phi^2} \ll 1 \tag{2.84}$$

Form classical gravity, the Ricci scalar will be $R \simeq 12H^2$ and it should be much smaller than $M_p^2/2$ to avoid the quantum gravity. Using the expression for the Hubble parameter in (2.82), the relation will be,

$$12H^2 = \frac{\lambda\phi^4}{M_p^2} \ll M_p^2/2 \tag{2.85}$$

From Eq. (2.84) and (2.85), we can find the bound on the Higgs quartic coupling as follows,

$$\lambda \ll 0.008 \tag{2.86}$$

Again we can find values of λ from the observation of curvature power spectrum [29]. The curvature perturbation is as follows,

$$\mathcal{R} = \frac{H\delta\phi}{\dot{\phi}} \tag{2.87}$$

and in case of inflationary era $\delta\phi$ can simplified to $H/2\pi$. Hence, the curvature perturbation becomes,

$$\mathcal{R} = \frac{H^2}{2\pi\dot{\phi}} \tag{2.88}$$

Using the values of H and $\dot{\phi}$ from Eq. (2.82) and (2.83), we can find out,

$$\mathcal{R} \sim \lambda^{1/2} \left(\frac{\phi}{M_p}\right)^3 \tag{2.89}$$

Using the experimental observation on the power spectrum of curvature perturbation,

$$\mathcal{P}_{\mathcal{R}} \sim \lambda \left(\frac{\phi}{M_p}\right)^6 = 2.43 \times 10^{-9} \tag{2.90}$$

we can find the constrain on the Higgs quartic coupling λ as follows,

$$\lambda \sim 10^{-12} \tag{2.91}$$

However, according to the recent experimental results, Higgs mass is $\sim$ 125 GeV which gives Higgs quartic coupling $\lambda \sim 0.13$ [1, 23, 24] and it is inconsistent with the tiny value of λ coming from inflation. Hence standard model Higgs minimally coupled to gravity cannot be the slow-roll Inflaton.

2.7.2 Higgs non-minimally coupled with gravity

Finally F.L. Bezrukov and M. Shaposhnikov came up with a inflation model, in which standard model Higgs behaves as inflaton [5]. The main idea of this model is based on the non-minimal coupling ξ between gravity and scalar sector. However this couplings should not change the degrees of freedom. The number of degrees of freedom of gravity and the Higgs are 2 and 1 respectively. Inclusion of the coupling $\xi\phi^2 R$ will not alter the degrees of freedom and the Lagrangian will be,

$$S_J = \int d^4x\,\sqrt{-g}\left[-\frac{M_p^2+\xi\phi^2}{2}R+\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}-V\left(\phi\right)\right] \tag{2.92}$$

Here the Lagrangian is written in Jordan frame, denoted by the suffix J. The presence of the coupling between gravity and Higgs indicates the frame as Jordan frame. However it is easy to do the calculations in Einstein frame, in which this coupling can be removed by the following conformal transformation,

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad \text{where} \quad \Omega^2 = 1+\frac{\xi\phi^2}{\mathrm{M_p^2}} \tag{2.93}$$

This non-minimal coupling ξ makes the Higgs potential sufficiently flat in this frame,

$$\hat{V} = \frac{V}{\Omega^4} = \frac{\lambda M_p^4}{4\xi^2}\left(1+\frac{M_p^2}{\xi\phi^2}\right)^{-2} \tag{2.94}$$

where hat signifies the Einstein frame.

In this model, from the experimental results on temperature correlation gives,

$$\xi \simeq 10^4\,\sqrt{\lambda} = 10^4\frac{m}{\sqrt{2}v} \tag{2.95}$$

where m denotes the Higgs mass and $\xi \sim 10^4$ gives rise to the correct mass, predicted by Atlas and CMS experiment. This large value of ξ suffers the problem of unitarity. However there are various attempts reported in the literature to handle this problem. There are different approaches to consider Higgs as inflaton using this non-minimal coupling [9, 39, 40].

Complete discussion of this model and the study of the generation of magnetic field in this model are discussed in next chapter. This model matches the observation for the magnetic field very well with out facing backreaction problem, described in subsequent chapter.

Inflation helps to generate the density perturbations as well as explains the structure formation. There is another approach to get the density perturbation as discussed in [13, 41, 42]. It was shown that a conformally coupled field rolling down negative quartic potential can generate scale invariant density perturbation. These perturbations can become superhorizon in an inflationary era or in a ekpyrotic scenario. This kind of model can also accommodate Higgs. Thorough discussion of this type inflation model in realistic inert doublet model has been given in Chapter-4.

2.8 Vacuum stability

The last section of this chapter provides the study of vacuum stability for the potential of the standard model Higgs. This is motivated from the recent observation for Higgs mass from Atlas and CMS experiment [1, 23, 24]. The study of vacuum stability for Higgs and constraining Higgs mass from this study was started long way back as mentioned in [14, 15, 16, 17]. The Higgs potential called instable when it becomes negative and unbounded from below. The instability mainly comes from the loop correction to the potential.

To be consistent with the recent results of Higgs mass, this analysis has to be done again. Recently the behavior of the Higgs quartic coupling λ, staring from electroweak scale to Planck scale, is studied in [43]. In [44], Xing *et. al.* showed the implication of recent range of Higgs mass on the SM vacuum stability by using the two-loop renormalization-group equations (RGEs) and repeated the determination of the branching ratios of some important two-body Higgs decay modes. Chetyrkin *et. al.* [45] reproduced this analysis in more details by considering the three-loop β-functions for top-Yukawa and the Higgs self-interaction. One step ahead of the procedure ("one-loop matching–two-loop running") to calculate vacuum stability is presented in [46]. They considered the 3-loop RGEs of the couplings of the Standard Model. Subsequently the first complete next-to-next-to-leading order analysis of the Standard Model Higgs potential is presented in [47] and this detailed study reduced the theoretical uncertainties. The determination of vacuum stability is carried out beyond the standard model by including the right handed neutrino to the standard model [48, 49, 50, 51, 52] and adding extra singlet [53].

We also study the vacuum stability in seesaw model, going beyond the standard model and we have shown that this subject has important effect on the LHC signature. The details of this study are discussed in Chapter-5.

2.9 Conclusion

Inflationary paradigm has become widely accepted from its introduction by Alan Guth in 1980 to explain the early universe. In the literature there are various models depending on the nature of the inflaton field and different types of potential. Higgs can be the most favorable candidate for the inflation model. Hence the study of Higgs inflation has important prospect in cosmology as well as in particle physics.

Chapter 3

Magnetic field generation in Higgs Inflation

This chapter presents comprehensive study of the Higgs inflation model. Brief discussions of the conformal transformation are provided to elucidate the calculations of the inflationary observables in Einstein frame. This model successfully describes the generation of magnetic field. Complete derivation for the production of the magnetic field is also discussed in this chapter.

3.1 Introduction

The importance of the Higgs inflation model has already been established in Chapter-2. It is also pointed out that non-minimal coupling between Higgs and gravity sector is the main ingredient in this type of inflation model. Starting point of this model is the Lagrangian written in Jordan frame. The presence of the non-minimal coupling between Higgs and gravity defines the frame as a Jordan frame. However the subsequent calculations are easy to handle in Einstein frame as this frame is devoid of this coupling between Higgs and gravity. The change of the frame can be done by the conformal transformation, which is discussed next.

3.2 Conformal transformation

Conformal transformation is a mathematical tool in the field of General relativity [54, 55]. In current situation, the use of conformal transformation techniques has become widespread in the literature on cosmology, specifically on nonminimally coupled scalar fields. This transformation relates the Jordan frame and the Einstein frame. In this scenario, the metric transforms as field dependent way as follows,

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \tag{3.1}$$

where Ω is a field-dependent variable. Under this transformation, determinant transform as,

$$\det\left(\mathrm{g}_{\mu\nu}\right) = \mathrm{g} \to \hat{\mathrm{g}} = \Omega^4 \mathrm{g} \tag{3.2}$$

Quantities in the Einstein frame is denoted by a hat (e.g. $\hat{g}_{\mu\nu}$). The change of the metric is depicted in other quantities, such as Christoffel symbol $\Gamma^{\rho}{}_{\mu\nu}$, Ricci tensor $R_{\mu\nu\rho}{}^{\sigma}$ and Ricci scalar R derived from the metric. The definition of these quantities, deduced from the metric, are as follow,

$$\Gamma^{\rho}_{\ \mu\nu} \equiv \frac{1}{2}g^{\rho\sigma}\left(\frac{\partial g_{\nu\sigma}}{\partial x^{\mu}} + \frac{\partial g_{\mu\sigma}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}}\right) \tag{3.3}$$

$$R_{\mu\nu\rho}^{\ \ \ \ \sigma} \equiv \frac{\partial \Gamma^{\sigma}_{\ \mu\rho}}{\partial x^{\nu}} - \frac{\partial \Gamma^{\sigma}_{\ \nu\rho}}{\partial x^{\mu}} + \Gamma^{\alpha}_{\ \mu\rho}\Gamma^{\sigma}_{\ \alpha\nu} - \Gamma^{\alpha}_{\ \nu\rho}\Gamma^{\sigma}_{\ \alpha\mu} \tag{3.4}$$

$$R_{\mu\rho} \equiv R_{\mu\nu\rho}^{\ \ \ \ \nu} \tag{3.5}$$

$$R \equiv g^{\mu\rho}R_{\mu\rho} \tag{3.6}$$

Under the conformal transformation (3.1), these quantities will be transformed as,

$$\Gamma^{\rho}_{\ \mu\nu} \to \hat{\Gamma}^{\rho}_{\ \mu\nu} = \Gamma^{\rho}_{\ \mu\nu} + \Omega^{-1}\left(\delta^{\rho}_{\mu}\Omega_{;\nu} + \delta^{\rho}_{\nu}\Omega_{;\mu} - g_{\mu\nu}g^{\mu\alpha}\Omega_{;\alpha}\right) \tag{3.7}$$

$$\begin{aligned} R^{\nu}_{\mu} &\to \hat{R}^{\nu}_{\mu} = \Omega^{-2}R^{\nu}_{\mu} - (n-2)\Omega^{-1}\left(\Omega^{-1}\right)_{;\mu\rho} g^{\rho\nu} \\ &+ (n-2)^{-1}\Omega^{-\mu}\left(\Omega^{n-2}\right)_{;\rho\sigma} g^{\rho\sigma}\delta^{\nu}_{\mu} \end{aligned} \tag{3.8}$$

$$R \to \hat{R} = \Omega^{-2}R + 2(n-1)\Omega^{-3}\Omega_{;\mu\nu}g^{\mu\nu} + (n-2)(n-4)\Omega^{-4}\Omega_{;\mu}\Omega_{;\nu}g^{\mu\nu} \tag{3.9}$$

where $n\,(n \geq 2)$ signifies the dimension of the spacetime manifold. We will use this transformed quantities in dimension 4 and in this case Ricci scalar will be,

$$R \to \hat{R} = \Omega^{-2}R + 6\Omega^{-3}\Omega_{;\mu\nu}g^{\mu\nu} \tag{3.10}$$

and this will be used afterwards for Higgs inflation model.

3.3 Higgs inflation model

There has been much interest recently in Higgs-Inflation model [5], as already mentioned in chapter-2. In this case, Standard Model (SM) Higgs boson, non-minimally coupled to the Ricci scalar, can give rise to inflation without additional degrees of freedom to the SM. The cosmic microwave background anisotropy data estimates the very small value for the Higgs quadratic coupling λ and this tiny value for λ does not support the recent hint for Higgs mass from Atlas and CMS experiments [1, 23, 24]. However a large non-minimally coupling with gravity can resolve this issue. In this model, the resultant potential of Higgs inflaton in the inflationary domain is effectively flat. It can result in successful inflation for values of non-minimally coupling constant $\xi \sim 10^4$.

We will start with the action written in Jordan frame, in which Higgs non-minimally coupled to the Ricci scalar,

$$S_J = \int d^4x \, \sqrt{-g} \left[-\frac{M_p^2 + \xi\phi^2}{2} R + \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi) - \frac{1}{4} I^2(\phi) \, F^{\mu\nu} F_{\mu\nu} \right] \tag{3.11}$$

where suffix J signifies the Jordan frame, M_p denotes the Planck scale and Higgs scalar $\Phi = \frac{1}{\sqrt{2}}(0, v+\phi)^T$ written in unitary gauge and the Higgs potential is given by,

$$V(\phi) = \frac{\lambda}{4} \left(\phi^2 - v^2 \right)^2 \tag{3.12}$$

and we can ignore the Higgs vev $v = 246$ GeV in inflationary era, where ϕ takes Planck scale value. We also consider Higgs-photon interaction, denoted by the last term in the action. This term breaks the conformal symmetry and helps to generate the magnetic field. Here I is the inverse of the electromagnetic coupling and we assume that it has the explicit dependence on ϕ as,

$$I^2(\phi) = \frac{1}{e^2} + \frac{\phi^\dagger \phi}{M_p^2} \tag{3.13}$$

This Higgs photon interaction term is the lowest order term which is invariant under $SU(2)$ transformations of the Higgs. In the present epoch, when $\phi = v$, the effective photon kinetic term $(1/e^2 + v^2/M_P^2)F^2$ reduces to the standard photon kinetic term $(1/e^2)F^2$. When we deal with the perturbations during inflation, we drop the standard photon kinetic term and consider only the second term in $I(\phi)$.

3.3.1 Conformal transformation in Higgs inflation model

In Jordan frame, we consider the following metric,

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j \tag{3.14}$$

To transform the action (3.11) to the Einstein frame, we consider the following conformal transformation [56, 57],

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad \text{where} \quad \Omega^2 = 1 + \frac{\xi\phi^2}{\mathrm{M_p^2}} \tag{3.15}$$

and follow the prescription of conformal transformation as described in Section-3.2.

Let us look at each of these terms in turn. The first term of Eq. (3.11) becomes,

$$\begin{aligned} -\int d^4x\,\sqrt{-g}\,\frac{\left(M_p^2+\xi\phi^2\right)}{2}R &= -\int d^4x\,\frac{\sqrt{-\hat{g}}}{\Omega^4}\,\frac{\left(M_p^2+\xi\phi^2\right)}{2}\left[\Omega^2\hat{R}+\frac{6}{\Omega}\Box\Omega\right] \\ &= -\frac{M_p^2}{2}\int d^4x\,\frac{\sqrt{-\hat{g}}}{\Omega^2}\left[\Omega^2\hat{R}+\frac{6}{\Omega}\Box\Omega\right] \end{aligned} \tag{3.16}$$

Here we use the definition of Ω^2 from Eq. (3.15) and $\Box\Omega = g^{\mu\nu}\nabla_\mu\nabla_\nu\Omega = \frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}g^{\mu\nu}\partial_\nu\Omega\right]$. In this case, $\hat{\partial}_\mu = \partial_\mu$ as x^μ remains unaffected under this transformation. Because the covariant derivatives act only on scalar functions, we have,

$$\nabla_\mu\Omega = \partial_\mu\Omega \quad \text{and} \quad \hat{\nabla}_\mu\Omega = \nabla_\mu\Omega \tag{3.17}$$

The simplification of Eq. (3.16) gives,

$$-\int d^4x\,\sqrt{-g}\,\frac{\left(M_p^2+\xi\phi^2\right)}{2}R = \int d^4x\,\sqrt{-\hat{g}}\left[-\frac{M_p^2}{2}\hat{R}+\frac{3\xi^2}{M_p^2\Omega^4}(\phi\,\phi_{,\mu})^2\right] \tag{3.18}$$

Now we consider the transformation of the kinetic energy, potential energy and

Higgs-photon interaction term,

$$\begin{aligned}&\int d^4x\,\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}-V\left(\phi\right)-\frac{1}{4}I^2(\phi)\,F^{\mu\nu}F_{\mu\nu}\right]\\ &=\int d^4x\,\sqrt{-\hat{g}}\left[\frac{1}{2\Omega^2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}-\frac{V(\phi)}{\Omega^4}-\frac{1}{4}I^2(\phi)\,F^{\mu\nu}F_{\mu\nu}\right]\end{aligned} \tag{3.19}$$

Here it is important to notice that $\sqrt{-g}F^{\mu\nu}F_{\mu\nu}$ is a conformally invariant term. However the conformal invariance is broken by the presence of I^2, as it is explicitly depend on the Higgs field ϕ. In Einstein frame, the action will look like,

$$\begin{aligned}S_E=\int d^4x\,\sqrt{-\hat{g}}&\left[-\frac{M_p^2}{2}\hat{R}+\frac{1}{2\Omega^2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}+\frac{3\xi^2}{M_p^2\Omega^4}(\phi\,\phi_{,\mu})^2-\frac{V(\phi)}{\Omega^4}\right.\\ &\left.-\frac{1}{4}I^2(\phi)\,F^{\mu\nu}F_{\mu\nu}\right]\end{aligned} \tag{3.20}$$

This conformal transformation produces non-canonical kinetic terms for the scalar field ϕ. To make the kinetic term of the ϕ field canonical, we have to redefine the ϕ field in terms of new scalar $\hat{\phi}$,

$$\frac{d\hat{\phi}}{d\phi}=\sqrt{\frac{\Omega^2+\frac{6\xi^2\phi^2}{M_p^2}}{\Omega^4}} \tag{3.21}$$

When $\phi \ll M_P/\sqrt{\xi}$, $\Omega \simeq 1$ and $\hat{\phi}=\phi$. This corresponds to the situation in the present era where $\phi = v$. Inflation takes place when $\phi \gg M_P/\sqrt{\xi}$ and in this regime $\Omega \simeq \sqrt{\xi}\phi/M_P$ and the relation between ϕ and $\hat{\phi}$ obtained from (3.21) is,

$$\phi=\frac{M_p}{\sqrt{\xi}}\,\exp\left(\frac{\hat{\phi}}{\sqrt{6}M_p}\right) \tag{3.22}$$

The action (3.20), in terms of $\hat{\phi}$ is ,

$$S_E=\int d^4x\,\sqrt{-\hat{g}}\left[-\frac{M_p^2}{2}\hat{R}+\frac{1}{2}\hat{g}^{\mu\nu}\hat{\phi}_{,\mu}\hat{\phi}_{,\nu}-\frac{V(\hat{\phi})}{\Omega^4}-\frac{1}{4}I^2(\hat{\phi})\,F^{\mu\nu}F_{\mu\nu}\right] \tag{3.23}$$

where ϕ is an implicit function of $\hat{\phi}$.

In terms of new scalar field $\hat{\phi}$, Higgs potential will be,

$$\begin{aligned}\hat{V} &= \frac{V}{\Omega^4} \simeq \frac{\lambda\phi^4}{4\Omega^4} \\ &= \frac{\lambda M_p^4}{4\xi^2}\left(1+\frac{M_p^2}{\xi\phi^2}\right)^{-2} \\ &= \frac{\lambda M_p^4}{4\xi^2}\frac{\exp\left(\frac{4\hat{\phi}}{\sqrt{6}M_p}\right)}{\left(1+\exp\left(\frac{2\hat{\phi}}{\sqrt{6}M_p}\right)\right)^2}\end{aligned} \tag{3.24}$$

In inflationary era, $\phi \gg M_p/\sqrt{\xi}$ implies $\hat{\phi} \gg \sqrt{6}M_p$ and potential in this limit will be.

$$\hat{V} = \frac{\lambda M_p^4}{4\xi^2}\left(1+\exp\left(-\frac{2\hat{\phi}}{\sqrt{6}M_p}\right)\right)^{-2} \tag{3.25}$$

Since, $\phi >> \frac{M_p}{\sqrt{\xi}}$ $\left(\text{or } \hat{\phi} \gg \sqrt{6}\mathrm{M_p}\right)$, the Higgs-inflaton has the exponentially flat potential in the Einstein frame.

3.3.2 Renormalization of tree level potential

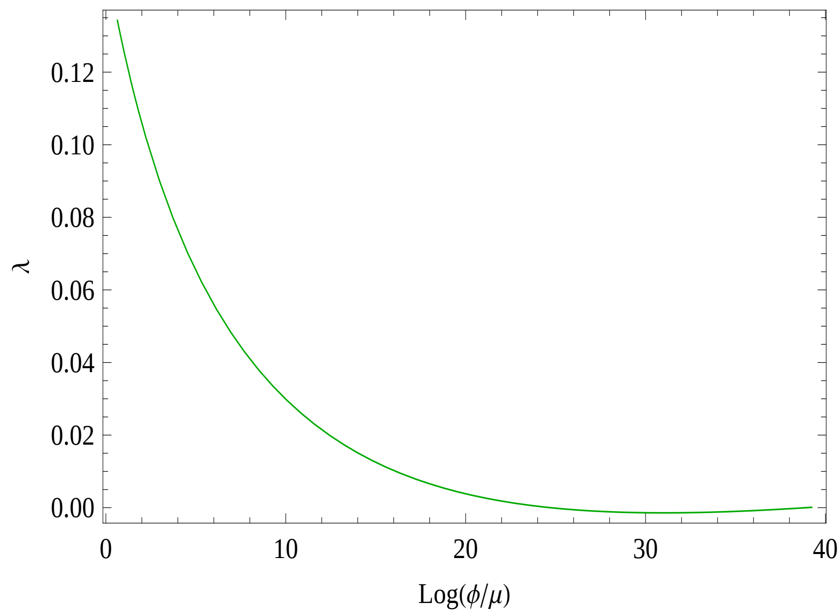

Figure 3.1: Running of λ as with the renormalization scale μ.

We consider the one loop correction to the tree level potential following the Coleman-Weinberg formalism in [58]. Loop correction for this kind of model is already discussed in [59, 60]. The effective potential is of the form,

$$\hat{V}_{eff.} = \hat{V} + \delta\hat{V} \tag{3.26}$$

where the loop correction is as follows,

$$\delta\hat{V} = \frac{m_\phi^4}{64\pi^2}\log\frac{m_\phi^2}{\mu^2} + \frac{6m_w^4}{64\pi^2}\log\frac{m_w^2}{\mu^2} + \frac{3m_z^4}{64\pi^2}\log\frac{m_z^2}{\mu^2} - \frac{3m_t^4}{16\pi^2}\log\frac{m_t^2}{\mu^2} \tag{3.27}$$

As was discussed in [5, 61, 62], loop corrections also change the values of λ and ξ at Planck scale compared to the values at electroweak scale. The main contribution comes from the following RG equation for the running of λ and ξ.

$$\begin{aligned} 16\pi^2\frac{d\lambda}{dt} &= 24\lambda^2 + 12\lambda y_t^2 - 9\lambda(g^2 + \frac{1}{3}g'^2) - 6y_t^4 + \frac{9}{8}g^4 + \frac{3}{8}g'^4 + \frac{3}{4}g^2g'^2 && (3.28)\\ 16\pi^2\frac{d\xi}{dt} &= \left(\xi + \frac{1}{6}\right)\left(12\lambda + 6y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g'^2\right) && (3.29) \end{aligned}$$

where $t = \log\left(\frac{\mu}{m_z}\right)$, y_t is the Yukawa coupling and g and g' are the gauge couplings. Neglecting the two loop contributions to the beta functions, from these two Eq. (3.28) and (3.29), we can see that,

$$16\pi^2\frac{d}{dt}\left(\frac{\lambda}{\xi^2}\right) = 0 \tag{3.30}$$

The effective potential (3.26) should be independent of the renormalization scale μ ,

$$\begin{aligned} \frac{d\hat{V}_{eff.}}{d\mu} = \frac{d}{d\mu}\left(\hat{V} + \delta\hat{V}\right) &= 0 \\ \frac{d}{d\mu}\left(\frac{\lambda(\mu)}{\xi(\mu)^2}\frac{M_p^4}{4} - \frac{3m_t^4}{16\pi^2}\log\frac{m_t^2}{\mu^2}\right) &= 0 && (3.31) \end{aligned}$$

Here $\hat{V} \sim \frac{\lambda}{\xi^2}\frac{M_p^4}{4}$ at inflationary era ($\hat{\phi} \gg \sqrt{6}M_p$) and we consider only top quark contribution as it is the dominant one. From Eq. (3.30), we see that this condition (3.31) can be satisfied if the logarithms is zero. This can be achieved by suitably choosing the value of μ as follows,

$$\mu^2 \simeq \frac{y_t(\mu)^2}{2}\frac{M_P^2}{\xi(\mu)}\left(1 - e^{-\frac{2\hat{\phi}}{\sqrt{6}M_P}}\right) \tag{3.32}$$

We shall use this value of μ in the renormalized Higgs potential. We will see that by choosing $\lambda(M_Z) = 0.132$ (which corresponds to $m_h = 126$ GeV, consistent with the recent measurements by Atlas and CMS [1, 23, 24]) and $\xi(M_Z) = 10^3$ we get the requisite value of the perturbation Δ_R and spectral index n_s at the time of inflation and in addition we can generate the comoving magnetic field of 10^{-7} Gauss.

3.3.3 Dynamics of Higgs inflation

The time derivative of the redefined field $\hat{\phi}$ in Einstein frame is follows,

$$\dot{\hat{\phi}} \equiv \frac{d\hat{\phi}}{d\hat{t}} \tag{3.33}$$

The inflaton field satisfies the equation,

$$\ddot{\hat{\phi}} + 3\dot{\hat{\phi}}\hat{H} + \hat{V}' = 0 \tag{3.34}$$

where prime denote the derivative with respect to $\hat{\phi}$ and $\hat{H}^2 = \hat{V}/\left(3M_p^2\right)$. Using slow-roll approximation, the field equation becomes,

$$\frac{d\hat{\phi}}{d\hat{t}} = -\frac{\hat{V}'}{3\hat{H}} \tag{3.35}$$

We can find the relation between ϕ and the number of e-foldings $N = \int_{\hat{\phi}_{end}}^{\hat{\phi}}(\hat{H}/\dot{\hat{\phi}})d\hat{\phi}$ using equation (3.35) for $\dot{\hat{\phi}}$,

$$\phi^2 = \phi_{end}^2 + \frac{4N}{3}\frac{M_p^2}{\xi} \tag{3.36}$$

The field value at the end of inflation, ϕ_{end}, corresponds to the slow-roll parameter $\epsilon = \frac{M_p^2}{2}\left(\frac{\hat{V}'}{\hat{V}}\right)^2 \simeq 4M_p^4/\left(3\xi^2\phi^4\right) = 1$ and it gives $\phi_{end} = 1.2\frac{M_p}{\sqrt{\xi}}$. Taking the horizon crossing of the large scale perturbations to be at N_k=60, the value of ϕ_k turns out to be $\phi_k = \frac{9M_p}{\sqrt{\xi}}$. Therefore, the relation between the scale factor and field can be written as,

$$\phi^2 = \left[1.2 + 1.3\log\left(\frac{\hat{a}_{end}}{\hat{a}}\right)\right]\frac{M_p^2}{\xi} \tag{3.37}$$

Using the equation (3.13) , we can write $I(\phi)$ as a function of $\hat{a}$ as follows,

$$I^2(\phi) = \frac{1}{\xi}\left[1.2 + 1.3\log\left(\frac{\hat{a}_{end}}{\hat{a}}\right)\right] \tag{3.38}$$

For the calculation of δ_B, we have to know the value of ξ, which can be calculated from the curvature perturbation. The amplitude of the curvature perturbation can be written as,

$$\Delta_R^2 = \frac{1}{4\pi^2}\left(\frac{\hat{H}^2}{d\hat{\phi}/d\hat{t}}\right)^2 = \frac{1}{8\pi^2 M_p^2}\frac{\hat{H}^2}{\epsilon} \tag{3.39}$$

Therefore, the amplitude of the curvature perturbation for the large scale perturbation is,

$$\Delta_R^2 = 5.19\frac{\lambda}{\xi^2} \tag{3.40}$$

WMAP measures $\Delta_R^2 = 2.43 \times 10^{-9}$ in [19] for the length scale of 3000 Mpc. From this we can calculate the amplitude of the curvature perturbation at length scale 100 Kpc,

$$\Delta_R(k = 2\pi/100\ Kpc^{-1}) = \Delta_R(k_0 = 2\pi/3000\ Mpc^{-1})\left(\frac{k_0}{k}\right)^{n_s-1} \tag{3.41}$$

Using the WMAP measured value of $n_s = 0.963$ in [19], we have $\Delta_R^2 = 5.2 \times 10^{-9}$ for the length scale of 100 Kpc, at which scale the magnetic field is measured. From RG equations (3.28), we can find out the values of λ and

ξ at inflationary era with their values at the electroweak scale as shown in Fig. (3.1). The recent measurement of Higgs mass 126 GeV measured by LHC, gives $\lambda(M_Z) \sim 0.132$. Therefore at inflationary era, $\lambda(M_P) = 10^{-4}$. Considering $\Delta_R^2 = 5.2 \times 10^{-9}$, we find out $\xi(M_P) = 3.16 \times 10^2$. In electroweak scale the values of ξ is $\sim 10^3$, (this is in the allowed range of parameter space which was studied in [61, 62]). We see that value of ξ which gives the correct amplitude of curvature perturbation predicts that the spectral index n_s is,

$$n_s = 1 + 2\eta - 6\epsilon = 0.966 \tag{3.42}$$

where $\eta = \left(M_p^2 \hat{V}''\right) / \hat{V} \simeq -4M_p^2 / \left(3\xi\phi^2\right)$. This value of n_s is consistent with the WMAP result $n_s = 0.963 \pm 0.012$ in [19]. However this large coupling $\xi \sim 10^4$ leads to problem with unitarity [6, 7, 8] of graviton-scalar scattering. Some ideas to solve the unitarity problem associated with Higgs inflation models are discussed in [9, 10, 11, 12]

3.4 Cosmological magnetic field

In 1908, the first extraterrestrial magnetic field was observed in sunspots [63]. Since then magnetic fields have been observed in galaxies (1949) [64], galaxy clusters and potentially in superclusters [63]. Afterward, the generation and the evolution of the magnetic field become an interesting subject for physicist. Magnetic fields are the only large-scale matter source, known in the universe today. Understanding the history of magnetic fields will help to understand the dynamics of the early universe and the formation of the first stars and galaxies. Primordial magnetic fields, which are present since before matter-radiation equality, affect the spectrum of temperature anisotropies and the polarization of the cosmic microwave background (CMB). Furthermore, a primordial magnetic field causes Faraday rotation of the orientation of the CMB polarization which leads to the creation of a B-mode.

Observations [65, 66] of magnetic field associated with high red-shift ($z > 1$)

galaxies suggest that the large scale magnetic fields have a cosmological rather than an astrophysical origin. For a long time, dynamo mechanism was the the preferred mechanism to explain the aforementioned observations. A dynamo is a mechanism in which kinetic energy of an electrically conducting fluid convert into the magnetic energy. Mean field dynamo is the most common approach in this kind of mechanisms. The assumption for mean field dynamo model is that the fluctuations in the magnetic and velocity fields are much smaller than the mean slowly varying components of the corresponding quantities. This mechanism depends on the idea that is the amplification of a tiny field created early enough by differential rotation of the galaxies and the subsequent generation of the galactic and cluster fields. This model is applicable to explain the amplification of large scale magnetic structures starting from small scale seed fields in the presence of a turbulent fluid.

In the dynamo theory of magnetic field amplification in galaxies [67, 63], the initial seed B-field of strength 10^{-20} Gauss can be amplified to the observed 10^{-7} Gauss by the magnetohydrodynamics of galactic rotation. However observations [65, 66] show that the magnetic fields associated with galaxies have a very narrow spread around a micro-Gauss and therefore independent of the number of rotations of the galaxies. Hence several experts of this field questioned about the effectiveness of this mechanism.

3.4.1 Magnetic field generation in inflation model

In the standard hot Big bang model of cosmology, the generation of magnetic fields of large coherence scales (1kpc-1Mpc) runs into the problems with causality. For example, if B-fields are generated in the electro-weak era then the present coherence length of the 100 kpc would correspond to a length scale $\lambda_{EW} = 100\text{kpc}(\text{T}_0/100\text{GeV}) = 10^7\text{cm}$. This length scale is much larger than the distance scale of causal Horizon at the electro-weak era $H_{EW}^{-1} = 10^{-2}\text{cm}$. This suggests that if B-fields have an origin in the fundamental interaction then the perturbations must be super-horizon and this can happen during inflation. The magnetic field generated with the coherence scale H_I during the time of inflation can be as large as the present horizon H_0. Generation of magnetic field during inflation has been studied extensively starting with Turner [63, 68, 69] and Widrow [70]. This kind of models are based on the coupling between electromagnetic fields and curvature. The coupled electromagnetic fields with the axion-inflaton was considered in [71]. Subsequently Bamba and Yokoyama studied the coupled electromagnetic field with the dilaton-inflaton in [72]. However in recent studies [73, 74, 75, 76], it has been observed that, in theories where B-field is generated during the inflation the fluctuations of the electromagnetic field are as large as the perturbations of the inflaton and spoil the prediction of near-scale invariant primordial density perturbation of inflation.

Magnetic field generation during inflation requires the breaking of conformal invariance of the electromagnetic action. We outline one particular scenario for such generation. We study the generation of magnetic field during Higgs inflation. We introduce a non-renormalisable coupling of the Higgs with the electromagnetic fields of the form $\frac{\phi^\dagger\phi}{M_P^2}F^2$ in the action (3.11). This term breaks the conformal symmetry and generates a magnetic field at the time of inflation when $\phi \sim M_P/\sqrt{\xi}$.

3.4.2 Generation of magnetic field during Higgs inflation

We consider separately the term containing massless vector field from the Einstein action (3.23),

$$S_E = \int d^4x\ \sqrt{-\hat{g}}\left[-\frac{1}{4}I^2(\hat{\phi})\,F^{\mu\nu}F_{\mu\nu}\right] \tag{3.43}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $A_\mu = (A_0, A_i)$. Decomposing the spatial part A_i in terms of its transverse and longitudinal components $A_i = A_i^T + \partial_i\chi$ and considering $\partial_i A_i^T = 0$ and $A_0 = \chi\prime$, we get the action as follows,

$$S_E = \int d^4x\ I^2\left(A_i^{T\prime}A_i^{T\prime} + A_i^T\Delta A_i^T\right) \tag{3.44}$$

where primes denotes the derivative with respect to the conformal time τ. The transverse component of A_i can be written in Fourier space as follows,

$$A_i^T(x,\hat{\tau}) = \sum_{\sigma=1,2} \int \frac{d^3k}{(2\pi)^{3/2}}\, A_{\mathbf{k}}^{\sigma}(\hat{\tau})\, \varepsilon_i^{\sigma}(\mathbf{k}) e^{i\mathbf{k}\cdot x} \tag{3.45}$$

where $\varepsilon_i^{\sigma}(\mathbf{k}), \sigma = 1,2$ are two orthogonal polarization vectors and they satisfy the relations $k_i\varepsilon_i^{\sigma}(\mathbf{k}) = 0$ and $\varepsilon_i^{\sigma}(-\mathbf{k})\varepsilon_i^{\rho}(\mathbf{k}) = \delta^{\sigma\rho}$, the action will be,

$$S_E = \frac{1}{2}\sum_{\sigma=1,2}\int I^2 \left(A_{\mathbf{k}}^{\sigma\prime} A_{-\mathbf{k}}^{\sigma\prime} - k^2 A_{\mathbf{k}}^{\sigma} A_{-\mathbf{k}}^{\sigma}\right) d\hat{\tau} d^3k \tag{3.46}$$

Defining $A_{\mathbf{k}}^{\sigma} = \tilde{A}_{\mathbf{k}}^{\sigma}/I$, the action becomes,

$$S_E = \frac{1}{2}\sum_{\sigma=1,2}\int \left[\tilde{A}_{\mathbf{k}}^{\sigma\prime}\tilde{A}_{-\mathbf{k}}^{\sigma\prime} - \left(k^2 - \frac{I''}{I}\right)\tilde{A}_{\mathbf{k}}^{\sigma}\tilde{A}_{-\mathbf{k}}^{\sigma}\right] d\hat{\tau} d^3k \tag{3.47}$$

We can expand $\tilde{A}_{\mathbf{k}}^{\sigma}$ in terms of the creation and annihilation operators as follows,

$$\tilde{A}_{\mathbf{k}}^{\sigma} = \frac{1}{\sqrt{2}}\left(u_{\mathbf{k}} a_{\mathbf{k}}^{\sigma} + u_{\mathbf{k}}^{\star} a_{\mathbf{k}}^{\sigma\dagger}\right) \tag{3.48}$$

where the creation and annihilation operators satisfy the relation $\left[a_{\mathbf{k}}^{\sigma}, a_{\mathbf{k}'}^{\rho\dagger}\right] = \delta^{\sigma\rho}\delta(\mathbf{k}-\mathbf{k}')$.

Therefore, the equation of motion of the mode function $u_{\mathbf{k}}$, derived from the action (3.47), is as follows,

$$u_{\mathbf{k}}'' + \left(k^2 - \frac{I''}{I}\right) u_{\mathbf{k}} = 0 \tag{3.49}$$

For large value of k, equation (3.49) reduces to

$$u_{\mathbf{k}}'' + k^2 u_{\mathbf{k}} = 0 \tag{3.50}$$

and the solution will be of the form,

$$u_{\mathbf{k}_>} = \frac{1}{\sqrt{2k}} e^{ik\hat{\tau}} \tag{3.51}$$

But for smaller value of k, the term $\frac{I''}{I}$ will dominate and the solution will be,

$$u_{\mathbf{k}_<} = cI + c'I \int \frac{d\hat{\tau}}{I^2} \simeq c\, I \tag{3.52}$$

where we have dropped the second term in equation (3.52) as it is suppressed by a factor $\frac{1}{\hat{a}}$ compared to the first term. By matching the equation (3.51) and equation (3.52) at $\hat{\tau} = \frac{1}{k}$, we determine the constant c,

$$c = \frac{e^i}{9}\sqrt{\frac{\xi}{2k}} \tag{3.53}$$

Therefore, the solution of the mode functions of the electromagnetic perturbations is of the form,

$$u_{\mathbf{k}} \simeq \frac{e^i}{9\sqrt{2k}} \left(1.2 + 1.3 \log\left(\frac{\hat{a}_{end}}{\hat{a}}\right)\right)^{1/2} \tag{3.54}$$

The correlation function will be,

$$\begin{aligned} < 0|\hat{A}_i^T(\hat{\tau}, x)\hat{A}^{Ti}(\hat{\tau}, y)|0 > &= \frac{1}{\hat{a}^2 I^2} \sum_{\sigma\sigma'} \int \frac{d^3k\, d^3k'}{(2\pi)^3} e^{i(\mathbf{k}\cdot\mathbf{x} + \mathbf{k}\cdot\mathbf{y})} < 0|u_{\mathbf{k}}^{\sigma} u_{\mathbf{k'}}^{\sigma'}|0 > \\ &= \frac{1}{4\pi^2 \hat{a}^2 I^2} \int \frac{dk}{k}\, |u_{\mathbf{k}}|^2 k^3\, \frac{\sin k(x-y)}{k(x-y)} \\ &\equiv \int \frac{dk}{k}\, \delta_A^2(k, \hat{\tau})\, \frac{\sin k(x-y)}{k(x-y)} \end{aligned} \tag{3.55}$$

The power spectrum of the vector field $\delta_A^2(k, \hat{\tau})$ can be identified with,

$$\delta_A^2(k, \hat{\tau}) = \frac{|u_{\mathbf{k}}|^2 k^3}{4\pi^2 \hat{a}^2 I^2} \tag{3.56}$$

Using the relation between magnetic field and vector field $B^2 = \frac{1}{2\hat{a}^4} F_{ik}F_{ik} = \frac{1}{\hat{a}^4}\left(\partial_i A_k \partial_i A_k - \partial_k A_i \partial_k A_i\right)$, we can calculate the power spectrum of the magnetic field $\delta_B^2(k,\hat{\tau})$ as follows,

$$\delta_B^2(k,\hat{\tau}) = \delta_A^2(k,\hat{\tau})\frac{k^2}{\hat{a}^2} = \frac{|u_{\mathbf{k}}|^2 k^5}{4\pi^2\hat{a}^4 I^2} \tag{3.57}$$

Using the expression for $u_{\mathbf{k}}$ from equation (3.54), we can calculate δ_B, at the time of horizon crossing as follows,

$$\delta_B^2(k) \simeq 1.5\times 10^{-4}\,\xi \hat{H}^4 \tag{3.58}$$

At the time of inflation, $\delta_{B_I}^2 = 1.04\times 10^{-6}\left((\lambda^2 M_p^4)/\xi^3\right) = 1.09\times 10^{52}\,\mathrm{GeV}^4$ and it is scale invariant. Now, we study the perturbation of the magnetic field in present era as follows,

$$\delta B_0 = \delta B_I \left(\frac{T_0}{T_{reh}}\right)^2 \tag{3.59}$$

As the experimental results for δB_0 is $\sim 10^{-7}$ Gauss at length scales of 100kp, we can calculate the reheat temperature $T_{reh}\sim\ 10^{13}$ GeV, which agrees with the results in [77]. We have to study the back-reaction of the generated electromagnetic field on the background. For this, we will calculate the energy density ρ_{em} which is defined as T_0^0 component of the energy-momentum tensor.

$$T_0^0 = I^2\left(\frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - F_{0\alpha}F^{0\alpha}\right) = \frac{I^2}{2\hat{a}^4}\left(A_i^{T\prime}A_i^{T\prime} + \partial_i A_k^T \partial_i A_k^T\right) \tag{3.60}$$

Using the relation (3.48) , the energy density ρ_{em} will be,

$$\rho_{em} =< 0|\hat{T}_0^0|0> \;=\; \frac{1}{8\pi^2\hat{a}^4}\int\frac{dk}{k}\;\left(|u'_{\mathbf{k}}|^2 + k^2|u_{\mathbf{k}}|^2\right)k^3 \tag{3.61}$$

neglecting the derivatives of I. Using the solution (3.54) for the mode function, in equation (3.61), the energy density perturbations of electromagnetic fields will be,

$$\begin{aligned}\rho_{em} &\simeq \frac{1.2\times 10^{-3}}{\pi^2}\,\hat{H}^4 &(3.62)\\ &\simeq 8.4\times 10^{-7}\frac{\lambda^2 M_p^4}{\xi^4} &(3.63)\\ &\simeq 2.81\times 10^{49}\ GeV^4 &(3.64)\end{aligned}$$

The energy density of the inflation will be

$$\begin{aligned}\rho_\phi &\simeq \hat{V}(\phi) = 0.25\,\frac{\lambda M_p^4}{\xi^2} &(3.65)\\ &\simeq 8.3\times 10^{63}\,GeV^4 &(3.66)\end{aligned}$$

As $\rho_\phi \gg \rho_{em}$, the back-reaction of the generated electromagnetic field can not spoil the inflation.

3.5 Conclusion

We have shown that the Higgs model of inflation [5, 77], in which a large Higgs-Ricci coupling gives rise to a flat Higgs potential in the Einstein frame in early universe, is also ideal for generation of magnetic field during inflation. Breaking the conformal invariance of electromagnetism by a non-renormalizable Higgs-photon coupling term in the Jordan frame enables us to generate large scale magnetic field during inflation while keeping the backreaction, as pointed out in [73]-[76], is under control. We find that by choosing $\lambda(M_Z) = 0.13$ (which corresponds to $m_h = 126$ GeV, in agreement with the recent measurements by ATLAS and CMS [1, 23, 24]) and $\xi(M_Z) = 10^3$ we get the correct values for the perturbation Δ_R and spectral index n_s. These results are consistent with WMAP measurements and we can also generate the comoving magnetic field of 10^{-7} Gauss in this model. The consequences of primordial magnetic field fluctuations on the CMB anisotropy have been studied in [78, 79]. The cosmological isotropy is broken by large scale magnetic fields which will show up in the CMB anisotropy and polarization spectrum. This points to the possibility that the magnetic field generation model studied in this work can be tested in the CMB anisotropy measurement experiments like PLANCK [80].

Chapter 4

Higgs inflation in Inert Doublet Model

This chapter includes the study of inert doublet model, which is the minimal extension to the standard model of particle physics. Complete discussion of the generation of density perturbation in this model is presented here. Moreover, the study of electroweak symmetry breaking reveals the scalar dark matter candidate in this model which is presented in the last section of this chapter.

4.1 Introduction

It is well known that to generate the density perturbation of the CMB as per the magnitude observed by COBE [2, 3] and WMAP [4], we need an inflationary period generated by the flat potential of a scalar field with coupling $\lambda \sim 10^{-10}$ in a $\lambda\phi^4$ theory. For standard model Higgs, λ is approximately ~ 0.1 and hence Higgs can not be used as inflaton. A way out was proposed by Bezrukov and Shaposhnikov [5] who coupled the standard model Higgs with the Ricci scalar with a large coupling constant $\xi \sim 10^4$. The exhaustive discussion of this kind of inflation model already presented in Chapter-2 of this thesis.

In this chapter we will follow a different approach for the generation of scale invariant density perturbations. It was shown by Rubakov and collaborators [13, 41, 42] that a conformally coupled field rolling down negative quartic potential can generate scale invariant density perturbation. These perturbations can become superhorizon in an inflationary era or in a ekpyrotic scenario [81]. We work with this idea in inert Higgs doublet (IDM) model [82, 83]. To start with, we present the brief discussion of inert doublet model in the next section.

4.2 Inert doublet model

In standard model, single Higgs doublet can successfully introduce the electroweak symmetry breaking and generate the masses for other particles. However considering more than one scalar doublet is not excluded instead it is an interesting extension of standard model. Inert doublet model is a special case of two Higgs doublets model. IDM was first proposed by Deshpande and Ma in 1970's [84]. However the importance of this model becomes prominent in [82] discussed by Barbieri *et.al.*

This model has been used in the literature to explain various physical processes. It has explained the 'LEP paradox' in [85, 82, 86]. The generation of light neutrino mass through one-loop radiative see-saw mechanism is discussed in IDM in [87]. This kind of models also help to generate the leptogenesis by the inclusion of TeV scale right handed neutrinos as pointed out in [88]. The detailed analysis of electroweak symmetry breaking in this model is discussed in [83].

In short, IDM, which is an economical extension of Standard Model, solves the problem of naturalness [82] and it can also explain the electroweak symmetry breaking [83]. The Lagrangian of this model respects the Z_2 symmetry, under which all the standard model particles including the SM Higgs doublet H_1 are even and an extra scalar doublet H_2 is odd. Due to this Z_2 symmetry, the cubic term and Yukawa term for H_2 doublet are forbidden. This makes the inert doublet stable and its neutral component can be a candidate for dark matter.

The two Higgs doublets H_1 and H_2 can be written in terms of their component fields as,

$$H_1 = \begin{pmatrix} h^+ \\ \frac{h+iG_0}{\sqrt{2}} \end{pmatrix} \qquad H_2 = \begin{pmatrix} H^+ \\ \frac{H_0+iA_0}{\sqrt{2}} \end{pmatrix}$$

The one of the attractive implication of this model is to provide the dark matter candidate. In literature there are various attempts to search dark matter candidates in IDM [89, 90, 91, 92]. However, here we able to use this model to generate the inflation in early universe and in this way, this model also has significant effect in explaining large scale structure of the universe.

4.2.1 Tree level potential of the model

We have already pointed out that the conformally coupled Higgs can produce the scale invariant density perturbations. We make use of this result in this realistic inert Higgs doublet model, where we have a pair of Higgs doublets conformally coupled to the gravity in the early universe. The main motivation of this work has been to show that a Higgs potential with not too small couplings can be a viable source of the observed scale invariant density perturbations. This is independent of the background metric as the scalar fields are assumed to be conformally coupled. The scale invariant density perturbations can occur in radiation or matter era also if the conditions of the potential are met. The scale invariant density perturbations become superhorizon during a phase of inflation at the electroweak scale. However other cosmological scenarios like a bounce models [81] of making the density perturbations superhorizon may be equally viable with our model.

We will start with tree level potential and again consider the loop-corrections. In this case, the most general renormalisable potential will be,

$$V_{tree} = V_c + \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_1^\dagger H_2|^2 + \frac{\lambda_5}{2}\left[\left(H_1^\dagger H_2\right)^2 + h.c.\right] \quad (4.1)$$

Here, we consider the conformal case where $\mu_1 = \mu_2 = 0$. V_c is the constant potential, which acts as cosmological constant and can be formed from the vev of different Higgs fields. We have chosen $V_c = 4 \times 10^7\,\mathrm{GeV}^4$ such that the minimum of the total potential becomes zero at present era. In the early universe the cosmological constant gave rise to an exponential expansion during which the scale invariant perturbations of the phase of the neutral component of H_2 became super-horizon. To achieve this we need that the potential is such that in the early universe, $V \sim -|\lambda_2||H_2|^4 \sim -|\lambda_2||H_0 + iA_0|^4$ and the neural component of H_2 rolls down this quartic potential while the minimum of H_1 is at $\langle H_1 \rangle = 0$. In the present era the potential should be such that the minima occurs at $\langle H_2 \rangle = 0$ and $\langle H_1 \rangle = v = 246$ GeV which gives rise to the electro-weak symmetry breaking. We show in the next section how this is achieved by radiative corrections starting from a scale invariant tree level potential.

4.2.2 Loop correction to the potential

We derive the one-loop correction to the potential (4.1) following Coleman-Weinberg formalism [58]. The generic one-loop correction to the potential can be written as [93],

$$\Delta V^1 = \frac{1}{2}\sum_i (-1)^{2J_i}\ (2J_i+1) \int \frac{d^3k}{(2\pi)^3}\sqrt{k^2+m_i^2} \tag{4.2}$$

where J_i are the spin of the fields and m_i are the tree level masses, function of the Higgs field. The double derivative of the tree level potential (4.1) with respect to the fields give the the tree level masses, which are,

$$\begin{aligned}
m_h^2 &= \lambda_1(G_0^2+3h^2+2h^+h^-)+\lambda_3 H^+H^- + \frac{\lambda_L}{2}H_0^2+\frac{\lambda_S}{2}A_0^2 \\
m_{G_0}^2 &= \lambda_1(h^2+3G_0^2+2h^+h^-)+\lambda_3 H^+H^- + \frac{\lambda_L}{2}A_0^2+\frac{\lambda_S}{2}H_0^2 \\
m_{h^\pm}^2 &= 2\lambda_1(G_0^2+h^2+6h^+h^-)+\lambda_3(H_0^2+A_0^2)+2\lambda_L H^+H^- \\
m_{H_0}^2 &= \lambda_2(A_0^2+3H_0^2+2H^+H^-)+\lambda_3 h^+h^- + \frac{\lambda_L}{2}h^2+\frac{\lambda_S}{2}G_0^2 \\
m_{A_0}^2 &= \lambda_2(H_0^2+3A_0^2+2H^+H^-)+\lambda_3 h^+h^- + \frac{\lambda_L}{2}G_0^2+\frac{\lambda_S}{2}h^2 \\
m_{H^\pm}^2 &= 2\lambda_2(A_0^2+H_0^2+6H^+H^-)+\lambda_3(h^2+G_0^2)+2\lambda_L h^+h^-
\end{aligned} \tag{4.3}$$

where $\lambda_{L,S} \equiv \lambda_3+\lambda_4\pm\lambda_5$. We regularize the divergent terms in Eq. (4.2) using the cut-off scale Λ and obtain

$$\Delta V^1 = \sum_i \left(\frac{m_i^2\Lambda^2}{32\pi^2}+\frac{m_i^4}{64\pi^2}\left(\ln\frac{m_i^4}{\Lambda^2}-\frac{1}{2}\right)\right) \tag{4.4}$$

The divergence in Eq. (4.4) can be removed by adding the counter terms in the potential of the form,

$$V_{ct}(\phi) = \delta\mu_\phi^2\phi^2+\delta\lambda_\phi\phi^4 \tag{4.5}$$

where ϕ denotes the scalar fields, considered in the model.

We impose the regularization condition on the effective potential, such that at early era (with high μ value), the potential is scale invariant form (4.1) by choosing the counter terms as follows,

$$\begin{aligned}
\delta\mu_\phi^2\phi^2 &= (6\lambda_1+2\lambda_3+\lambda_4+1/2)\ h^2+(6\lambda_2+2\lambda_3+\lambda_4)\ A^2 \\
&+ (6\lambda_1+2\lambda_3+\lambda_4)\ G^2+(6\lambda_2+2\lambda_3+\lambda_4)\ H_0^2 \\
&+ 2\,(8\lambda_1+2\lambda_3+\lambda_4+\lambda_5)\ h^+h^-+2\,(8\lambda_2+2\lambda_3+\lambda_4+\lambda_5)\ H^+H^-
\end{aligned} \tag{4.6}$$

$$\begin{aligned}
\delta\lambda_\phi\phi^4 &= h^4\left(9\lambda_1^2\,f(m_h^2)+\lambda_1^2\,f(m_{G_0}^2)+4\lambda_1^2\,f(m_{h^\pm}^2)+\frac{\lambda_L^2}{4}\,f(m_{H_0}^2)\right.\\
&+ \left.\frac{\lambda_S^2}{4}\,f(m_{A_0}^2)+\lambda_3^2\,f(m_{H^\pm}^2)\right)+H_0^4\left(\frac{\lambda_L^2}{4}\,f(m_h^2)+\frac{\lambda_S^2}{4}\,f(m_{G_0}^2)+\lambda_3^2\,f(m_{h^\pm}^2)\right.\\
&+ \left.9\lambda_2^2\,f(m_{H_0}^2)+\lambda_2^2\,f(m_{A_0}^2)+4\lambda_2^2\,f(m_{H^\pm}^2)\right)\\
&+ G_0^4\left(\lambda_1^2\,f(m_h^2)+9\lambda_1^2\,f(m_{G_0}^2)+4\lambda_1^2\,f(m_{h^\pm}^2)+\frac{\lambda_3^2}{4}\,f(m_{H_0}^2)+\frac{\lambda_L^2}{4}\,f(m_{A_0}^2)+\lambda_3^2\,f(m_{H^\pm}^2)\right)\\
&+ A_0\left(\frac{\lambda_S^2}{4}\,f(m_h^2)+\frac{\lambda_L^2}{4}\,f(m_{G_0}^2)+\lambda_3^2\,f(m_{h^\pm}^2)+\lambda_2^2\,f(m_{H_0}^2)+9\lambda_2^2\,f(m_{A_0}^2)+4\lambda_2^2\,f(m_{H^\pm}^2)\right)\\
&+ (h^+h^-)^2\left(4\lambda_1^2\,f(m_h^2)+4\lambda_1^2\,f(m_{G_0}^2)+144\lambda_1^2\,f(m_{h^\pm}^2)\right.\\
&+ \left.\lambda_3^2\,f(m_{H_0}^2)+\lambda_3^2\,f(m_{A_0}^2)+4\lambda_L^2\,f(m_{H^\pm}^2)\right)\\
&+ (H^+H^-)^2\left(\lambda_3^2\,f(m_h^2)+\lambda_3^2\,f(m_{G_0}^2)+4\lambda_L^2\,f(m_{h^\pm}^2)\right.\\
&+ \left.4\lambda_L^2\,f(m_{H_0}^2)+4\lambda_2^2\,f(m_{A_0}^2)+144\lambda_L^2\,f(m_{H^\pm}^2)\right)
\end{aligned} \tag{4.7}$$

where $f(m_i^2) = \log\left(\frac{\Lambda^2}{\mu^2}+\frac{\mu^2}{m_i^2}\right)$. With these counter terms, the form of the effective potential turns out to be,

$$V_{eff.}(H_0,h,\mu) = V_{tree}+\frac{1}{64\pi^2}\sum_i n_i m_i^4 \ln(\frac{m_i^2}{\mu^2}+1) \tag{4.8}$$

where n_i is degrees of freedom and m_i are tree level masses, shown in Eq. (4.3). In this case we have also considered the loop corrections from the top quark and gauge bosons. However the gauge boson loop has negligible effect on the results.

4.2.3 Renormalization group equations for IDM model

We have used the RG equations of the two Higgs doublet model to find out the values of the coupling constant for different renormalization scale. The one-loop renormalization group equations are as follows [94],

$$\frac{d\lambda_i}{d\log\mu} = \frac{1}{16\pi^2}\,\beta_i(\lambda) \tag{4.9}$$

where μ is the scale of renormalization and the β-functions are as follows,

$$\begin{aligned}
\beta_1 &= 12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + \frac{3}{4}\left[2g^4 + \left(g^2 + g'^2\right)^2\right] - 12\lambda_t^4 - 64\pi^2\lambda_1\gamma_1 \\
\beta_2 &= 12\lambda_2^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + \frac{3}{4}\left[2g^4 + \left(g^2 + g'^2\right)^2\right] - 64\pi^2\lambda_1\gamma_1 \\
\beta_3 &= 2\left(3\lambda_3 + \lambda_4\right)\left(\lambda_1 + \lambda_2\right) + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 + \frac{3}{4}\left[2g^4 + \left(g^2 + g'^2\right)^2\right] - 32\pi^2\lambda_3\left(\gamma_1 + \gamma_2\right) \\
\beta_4 &= 2\lambda_4\left(\lambda_1 + \lambda_2 + 4\lambda_3 + 2\lambda_4\right) + 8\lambda_5^2 + 3g^2g'^2 - 32\pi^2\lambda_4\left(\gamma_1 + \gamma_2\right) \\
\beta_5 &= 2\lambda_5\left(\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4\right) - 32\pi^2\lambda_5\left(\gamma_1 + \gamma_2\right)
\end{aligned} \tag{4.10}$$

where the anomalous dimensions of the two Higgs doublets are.

$$\gamma_1 = \frac{1}{64\pi^2}\left(9g^2 + 3g'^2 - 12\lambda_t^2\right) \tag{4.11}$$

$$\gamma_2 = \frac{1}{64\pi^2}\left(9g^2 + 3g'^2\right) \tag{4.12}$$

4.2.4 Values of coupling constants

To get the correct electro-weak symmetry breaking in the present era and the scale invariant density perturbation in the early era, we have chosen a set of λ values in present epoch as shown in Table-(4.1).

Now we have study the running of couplings λ_i, where $\{i = 1\,\text{to}\,5\}$ shown in Fig. (4.1) using the one-loop renormalization group equation mentioned in

λ_1	λ_2	λ_3	λ_4	λ_5
-0.14	-11	2.8	-1.52	-1.52

Table 4.1: The scalar couplings in the present era with $\mu = 165.6$ GeV

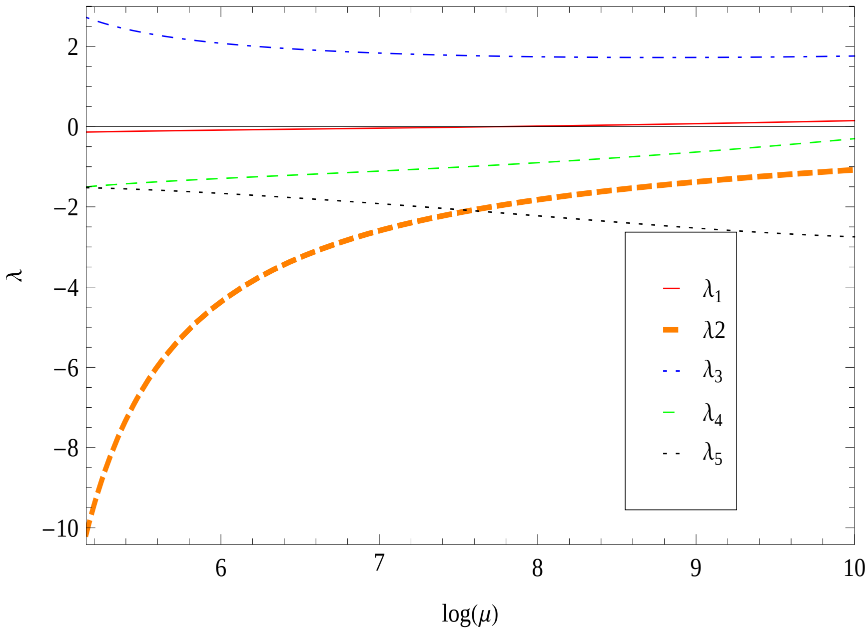

Figure 4.1: Running of scalar couplings from present to early era

the section 4.2.3 [82]. The variation of Yukawa coupling λ_t and the gauge couplings are shown in Fig. (4.2).

From Fig (4.1), we can find the λ values in the early era $\mu \simeq 10^5$ GeV and are given in Table-4.2. Only λ_2 at early universe is relevant for calculating

λ_1	λ_2	λ_3	λ_4	λ_5
0.49	-0.5	2.1	0.84	-2.2

Table 4.2: The scalar couplings in the early era with $\mu = 10^5$ GeV

the scale invariant density perturbation. We constrain the couplings of the scalar potential by comparing with the amplitude of the spectrum of CMB anisotropy measured by WMAP.

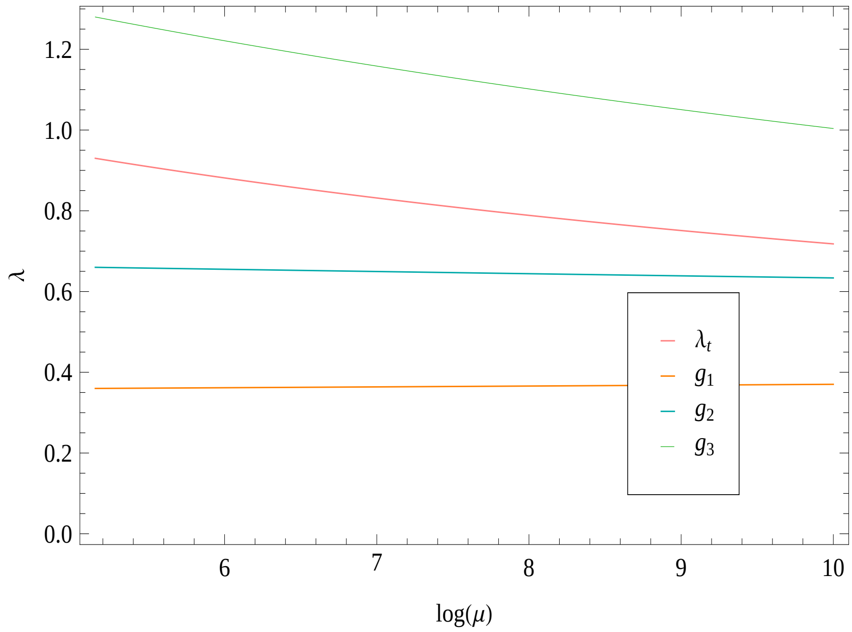

Figure 4.2: Running of Yukawa coupling λ_t and gauge couplings g_1, g_2 and g_3 from present to early era

4.2.5 Variation of potential in early era

The change in shape of the effective potential $V_{eff}(H_0, h, \mu)$ in Eq. (4.8) from the early universe where we take $\mu = 10^5$ to the present epoch where $\mu = 165.6$ GeV is shown in Fig (4.7). We see that in the early universe for a given value of H_0, the minima of $V_{eff}(h)$ (shown in Fig (4.3)) is at $h = 0$. We assume that in the early universe $h = 0$ and we see that $V_{eff}(H_0)$ is of the form as shown in Fig (4.4).

4.2.6 Variation of potential in present era

The one loop correction of the potential has significant contribution in present era. When we take $\mu = 165.6$ GeV then the potential (4.8) is of the form shown in Fig (4.5) and Fig (4.6). In this era, the $V_{eff}(H_0)$ has a minimum at $H_0 = 0$ as shown in Fig (4.6). With $H_0 = 0$, the potential as a function of the field h has a minimum at $h = v \sim 246GeV$ signifying the electroweak symmetry breaking.

It is important to note that we calculate the potential at zero temperature,

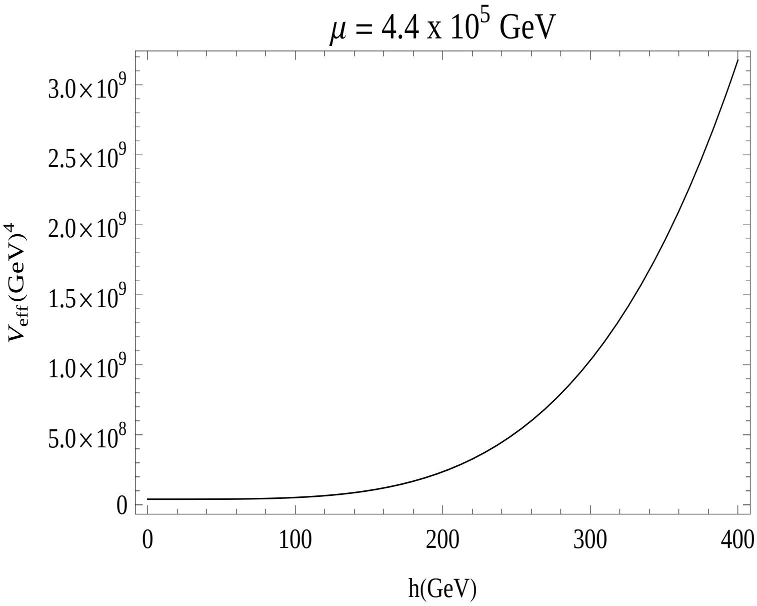

Figure 4.3: Effective potential in the early universe for SM Higgs doublet

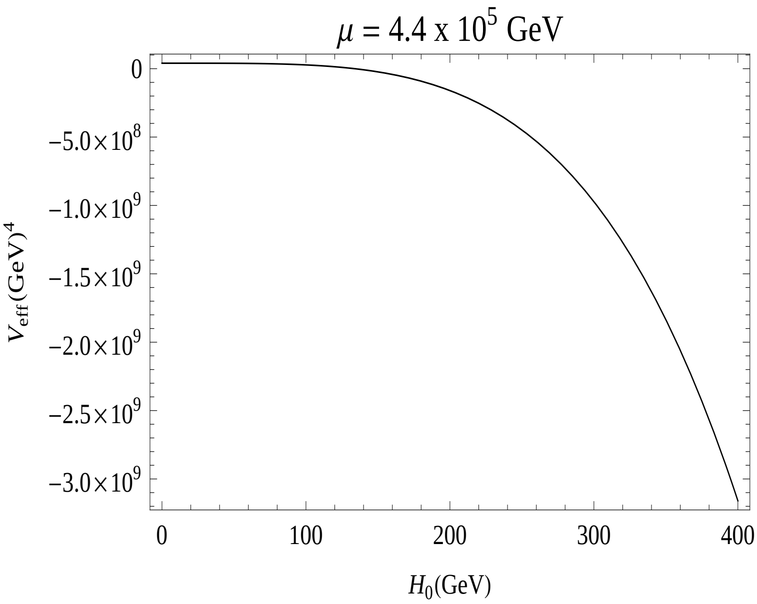

Figure 4.4: Effective potential in the early universe for inert Higgs doublet

which accurately describes the universe during inflation (when any prior temperature goes down exponentially in time) or in the present universe where the background temperature negligible compared to the electroweak scale. There is a radiation era after reheating at the end of inflation. The effective potential at high temperature has been computed for the inert Higgs doublet model in

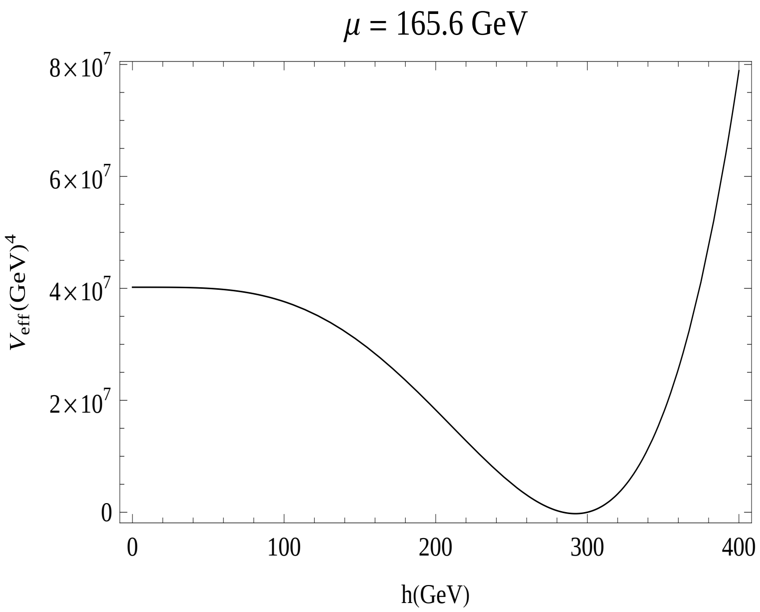

Figure 4.5: Effective potential the present universe for SM Higgs doublet

[95], where the thermal evolution of the effective potential has been shown. In this work we deal with the $T = 0$ case which is relevant during inflation and in the present universe.

4.3 Generation of the scale invariant density perturbation

We now turn to the question of the generation of density perturbations in the early era when V_{eff} (4.8) simplifies to the form,

$$V_{inf} \sim V_c + \frac{\lambda_2}{4} |H_2|^4, \tag{4.13}$$

where $V_c = 4 \times 10^7 \, \mathrm{GeV}^4$ and $\lambda_2 = -0.5$. The Hubble parameter H_u in this era can be calculated from Eq (4.13),

$$\begin{aligned} H_u|_{inf} &= \frac{1}{\sqrt{3}} \frac{V_{inf}^{1/2}}{M_p} \sim \frac{1}{\sqrt{3}} \frac{V_0^{1/2}}{M_p} & (4.14) \\ &= 3 \times 10^{-16} \mathrm{GeV} & (4.15) \end{aligned}$$

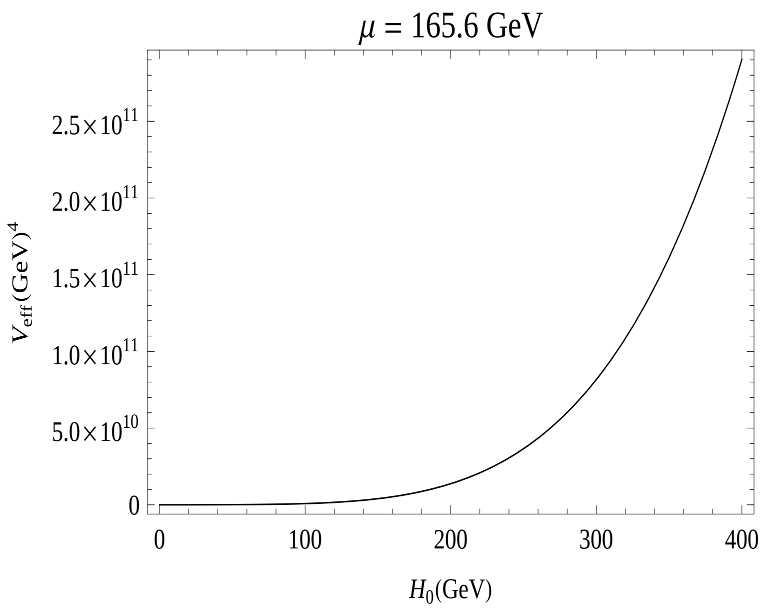

Figure 4.6: Effective potential the present universe for inert Higgs doublet

We take the inert Higgs doublet to be conformally coupled to gravity and the action for this field can be written as,

$$S = \int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu H_2^*\partial_\nu H_2 - \frac{R}{6}H_2^*H_2 - V_{inf}\right] \tag{4.16}$$

where H_2 contains the neutral part of the inert doublet i.e. $H_2 = \frac{H_0+i\,A_0}{\sqrt{2}}$ and R is the scalar curvature, which conformally coupled with the field H_2. The equation of the field H_2 will be,

$$\ddot{H}_2 + \left(\frac{k}{a}\right)^2 - 3H_u\,\dot{H}_2 + \frac{R}{6}H_2 + \frac{\partial V_{inf}}{\partial H_2} = 0 \tag{4.17}$$

where a and H_u are the scale factor and Hubble constant respectively. Now defining $H_2 = \chi_{H_2}/a$ and rewriting the Eq (4.17), we will get,

$$\chi^{''}_{H_2} + \left(k^2 - \frac{a^{''}}{a}\right)\chi_{H_2} + \frac{R}{6}a^2\chi_{H_2} + a^3\frac{\partial V_{inf}}{\partial H_2} = 0 \tag{4.18}$$

where $'$ denotes the derivative with respect to conformal time η. We note that both $a^{''}/a$ and $(R\,a^2)/6$ are equal to $2/\eta^2$ and hence these two terms in

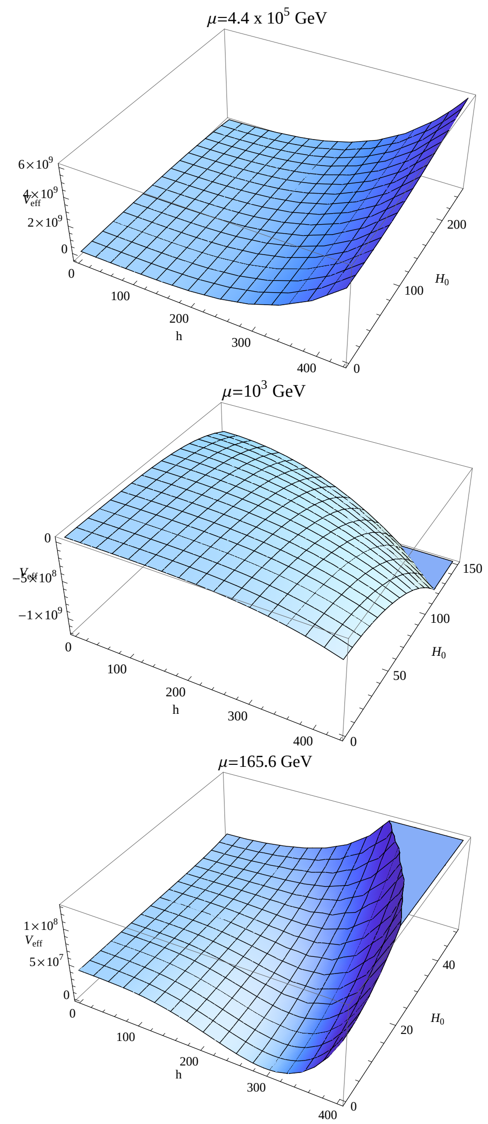

Figure 4.7: Variation of the potential at different era

Eq. (4.18) cancel each other. So the equation for H_2 becomes,

$$\chi''_{H_2} + k^2 \chi_{H_2} + a^3 \frac{\partial V_{inf}}{\partial H_2} = 0 \tag{4.19}$$

Expressing $\chi_{H_2} = \rho \exp(i\,\theta)$, the conserved current will be of the form,

$$\frac{d}{d\eta}(\rho^2 \theta') = 0 \tag{4.20}$$

Hence, the field rolls along the radial direction while the phase θ remains constant with the increase of ρ. Without loss of generality we can choose the fixed phase such that the field H_2 has only real neutral component χ_{H_0}. The perturbations of H_2 will be along the imaginary axis and we can denote the full H_2 with the perturbations as $\chi_{H_2} = \chi_{H_0} + i\,\delta\chi_{A_0}$. From Eq (4.19) the equation of motion of χ_{H_0} will be,

$$\chi''_{H_0} + k^2\chi_{H_0} - \frac{\lambda_2}{2}\chi^3_{H_0} = 0 \tag{4.21}$$

Considering $k \ll 1/\eta$ at late time, the solution will be,

$$\chi_{H_0} \approx \frac{1}{\sqrt{-\lambda_2}(\eta_* - \eta)} \tag{4.22}$$

where $\sqrt{-\lambda_2}$ is a real quantity as λ_2 is negative and η_* is a constant of integration. At the end of inflation when $\mu << 10^4$ the shape of the potential changes, and H_0 starts rolling back to zero shown in Fig. (4.6).

Starting from (16) we see that the equation of motion of the perturbation, $\delta\chi_{A_0}$ is given by,

$$\delta\chi''_{A_0} + k^2\delta\chi_{A_0} + \frac{\lambda_2}{2}\chi^2_{H_0}\delta\chi_{A_0} = 0 \tag{4.23}$$

Substituting χ_{H_0} from Eq (4.22), the equation becomes,

$$\delta\chi''_{A_0} + k^2\delta\chi_{A_0} - \frac{1}{2(\eta_* - \eta)^2}\delta\chi_{A_0} = 0 \tag{4.24}$$

This equation can be solved for early times and later times separately. At early time ($k(\eta_* - \eta) \gg 1$), third term can be neglected and the solution will be

$$\delta\chi_{A_0} = \frac{1}{(2\pi)^{3/2}\sqrt{2k}}\exp^{ik(\eta_* - \eta)} \tag{4.25}$$

At later times, when ($k(\eta_* - \eta) \ll 1$), third term will dominate and in this case solution will look like,

$$\delta\chi_{A_0} \sim \frac{1}{k^{3/2}(\eta_* - \eta)} \tag{4.26}$$

Hence, the super-horizon perturbations of the phase can be defined as,

$$\delta\theta \equiv \delta\chi_{A_0}/\chi_{H_0} \tag{4.27}$$

Therefore the perturbation of the phase $\delta\theta$ becomes,

$$\delta\theta = \frac{\sqrt{-\lambda_2}}{k^{3/2}} \tag{4.28}$$

The power spectrum of $\delta\theta$ is scale invariant,

$$\mathcal{P}_{\delta\theta} = \frac{k^3}{2\pi^2}|\delta\theta|^2 = \frac{-\lambda_2}{2\pi^2} \tag{4.29}$$

If one considers the k dependence of the equation of motion of H_0 as discussed in [42] there will be a deviation from the scale free power spectrum (4.29) which will give rise to a non-zero spectral index,

$$n_s - 1 = \frac{3\lambda_2}{4\pi^2} \tag{4.30}$$

From Table-(4.2) we see that in the early universe $\lambda_2 = -0.5$ which gives the spectral index $n_s - 1 = -0.04$. This value is consistent with the WMAP observation of $n_s = 0.967 \pm 0.014$ [96]. The perturbations of the phase $\delta\theta = \delta A_0/H_0$ can be converted to adiabatic perturbation by the decay of A_0 field into standard model fields as in the curvaton mechanism [97]. The amplitude of the adiabatic perturbation is related to the phase perturbation as

$$P_\zeta = r^2\frac{P_{\delta\theta}}{\theta_c^2} = r^2\frac{-\lambda_2}{2\pi^2\theta_c^2} \tag{4.31}$$

where r is the ratio of the energy density in the A_0 field oscillations to the total energy density. Taking the unperturbed phase to be $\theta_c \sim \pi/2$, and with

$\lambda_2 = -0.5$, we see that $r = 2 \times 10^{-4}$ can give the required $P_\zeta = 10^{-10}$.

4.4 Scalar mass spectrum

The identification of the dark matter particle is one of the most challenging problem in astro-particle physics today. As the field H_2 has a zero vev in the present universe the lightest neutral components of H_2 will be stable and can be the candidates for dark matter. We study the masses of the fields in present universe from the effective potential. Taking $< H_1 >= 246\,\text{GeV}$ and $< H_2 >= 0\,\text{GeV}$ and for λ_i as in Table-(4.1) we find the mass spectrum of scalars in the present universe and it is given in Table 3. We see that A_0 field can be a candidate for light dark matter. We also see that the Higgs mass is predicted to be $M_h = 125.6$ GeV which matches with the current observation of CMS and Atlas experiments.

M_h	M_{H^0}	M_{A^0}	$M_{H^\pm}$
125.6	273.6	33.7	433.5

Table 4.3: Scalar mass spectrum in GeV

4.5 Conclusions

The inert Higgs doublet model is a natural extension of the standard model and can be used for explaining the electroweak symmetry breaking by loop corrections [83] starting from a scale invariant tree level potential. We connect the scale invariance of the inert Higgs potential to the generation of scale invariant spectrum of a conformally coupled scalar as discussed by Rubakov and collaborators [13, 41, 42]. The requirement of scale invariance at high energy scale and electroweak symmetry breaking at low energies fix the coupling constants of the theory. Specifically we find that the quartic coupling of the inert doublet is $\lambda_2 = -0.5$ at $\mu = 10^5$ GeV and it predicts the spectral index of the power spectrum of the perturbations and it is consistent with WMAP observations. The amplitude of the power spectrum P_ζ can be tuned with the observations by choosing a suitable curvaton mechanism. We also make the predictions for masses of the Higgs bosons and the dark matter (which is the lightest neutral component of the inert doublet) which can be tested in forthcoming experiments.

Chapter 5

Phenomenology of vacuum stability of standard model Higgs

This chapter presents the discussions of the running of the Higgs quartic coupling in the standard model for the 125 GeV Higgs. We introduce the Yukawa couplings Y_ν between the Higgs and heavy neutrinos in the context of Type-1 see-saw models and study the effect of neutrino Yukawa's on the running of the Higgs quartic coupling. We establish the bound on Y_ν from the stability criterion. The three aspect of the heavy neutrino phenomenology, namely, Neutrino-less double beta decay ($0\nu\beta\beta$), Lepton flavor violating decays like $\mu \to e\,\gamma$ are studied in the light of the vacuum stability condition. We estimate the same-sign-dilepton signals at the LHC.

5.1 Introduction

The recent measurement of Atlas and CMS [1, 23, 24] have confirmed the existence of a new boson which has mass in the range 126.5 GeV (Atlas at 5.0σ) and 125.3 ± 0.6 GeV (CMS at 4.9σ), and it is expected to be a Standard Model Higgs. This mass range implies that quartic coupling λ of the Higgs has a value close to the vacuum stability limit [17, 43, 98, 44, 45, 46, 47]. The top-quark loops make a negative contribution to the β-function of λ_h while the gauge couplings give a positive contribution. If the quartic coupling $\lambda_h(\mu)$ becomes negative at large renormalization scale μ, it implies that in the early universe the Higgs potential would be unbounded from below and the vacuum would be unstable in that era. It has been pointed out that the Higgs mass $\sim$ 126 GeV range being close to the vacuum stability limit, one can put stringent constraint on new physics which affects the running of the Higgs quartic coupling.

One class model, which can be constrained from the stability criterion of the Higgs coupling, is the see-saw models of neutrino masses [48, 49, 50, 51, 99, 52]. In Type-I see-saw models [100] one introduces number of heavy gauge singlet Majorana neutrinos which have Yukawa couplings with the Higgs and lepton doublets. The electroweak symmetry breaking gives rise to the Dirac mass matrix $\mathcal{M}_D$,

$$-\mathcal{L} = \bar{N_R}\mathcal{M}_D\nu_L + \frac{1}{2}\bar{N_R}\mathcal{M}_R N_R^c + \text{h.c.} \tag{5.1}$$

If $\mathcal{M}_D << \mathcal{M}_R$ in pure Type-I models [100], the light neutrino masses are given by $\mathcal{M}_\nu = \mathcal{M}_D^T\mathcal{M}_R^{-1}\mathcal{M}_D$. It has been discussed earlier in many papers that light neutrino masses, which can explain the solar and atmospheric neutrino oscillations, are obtained by assuming the eigenvalues $M_D \sim 100$ GeV and $M_R > 10^{14}$ GeV. By a suitable choice of $\mathcal{M}_D$ and $\mathcal{M}_R$ one can set $\mathcal{M}_D^T\mathcal{M}_R^{-1}\mathcal{M}_D = 0$ and the light neutrino masses are given by higher order terms in $\mathcal{M}_D^T\mathcal{M}_R^{-1}$ [101, 102, 103]. In this way it is possible to generate viable light neutrino masses while reducing the scale M_R to less than a TeV.

In [99, 52], the constraints on various TeV-scale Type-I neutrino mass models from the vacuum stability criterion of Higgs coupling has been checked.

In this study we assume Yukawa couplings Y_ν of heavy Majorana neutrinos with the lepton doublets and the Standard Model Higgs and that the heavy neutrinos masses are in the 100 GeV-10 TeV range. We obtain the constraints on the Higgs-neutrino Yukawa couplings by calculating the renormalization group evolutions (RGEs) of $\lambda_h(\mu)$ (which is fixed at the electroweak scale by the Higgs mass). The vacuum stability condition is the requirement that $\lambda_h(M_W \leq \mu \leq M_P) \geq 0$. We find that this leads to the constraint $Y_\nu \leq 0.14$ on the elements of the Yukawa coupling matrix. We then apply this condition (which implies that the Dirac neutrino masses $M_D \leq 24.36$ GeV) on the phenomenology of TeV scale heavy neutrinos [104, 105, 106, 107, 108, 109].

First we will presents the Renormalization Group evolutions for standard model followed by the discussion of vacuum stability of the Standard Model Higgs potential.

5.2 Vacuum stability of the Standard Model Higgs potential

The Higgs mass measured by Atlas and CMS collaborations [1, 23, 24] is in the mass range 124.7 GeV-126.5 GeV and this range is close to the bound on Higgs mass from electro-weak vacuum stability condition [17]. In [43], it has been shown that in standard model, the Higgs boson quartic coupling λ_h can remain positive upto Planck scale with appropriate choice of top quark mass m_t, strong coupling constant α_s etc. The coupling $\lambda_h > 0$ ensure the stable vacuum and the bounded potential from below. Details of this study is done recently in [47].

The RG-improved Higgs potential can be written as,

$$V_{\text{eff}} = \frac{\lambda_h(t)}{4!}\left[\xi(t)\phi\right]^4 \tag{5.2}$$

where ξ signifies the wave function renormalization and $t \sim \log(\mu/M_Z)$, μ is the scale of renormalization. Here $\lambda_h(t)$ is the effective Higgs quartic coupling with loop corrections. Loop corrections can cause an instability of the potential if $\lambda(t)$ becomes negative at any scale $\mu < M_P$. The gauge boson loop makes a positive contribution whereas the top quark makes a negative contribution to the β function of λ_h. Hence the instability of the potential mainly comes from loop-correction of top quark.

We compute the RG running of $\lambda_h(\mu)$ using the two loop RG equations for the Standard Model [110, 111, 112, 113, 114, 43, 46].

5.3 Renormalization Group Equations

In general, the beta functions for the couplings λ_h can be written as,

$$\beta_\lambda = \mu \frac{d\lambda}{d\mu} = \sum_i \frac{\beta_\lambda^{(i)}}{(16\pi^2)^i} \tag{5.3}$$

where λ denotes the Higgs quartic coupling λ_h, top quark Yukawa coupling λ_t, gauge couplings g_i, $(i = 1, 2, 3)$ and neutrino coupling Y_ν for Type-I seesaw model respectively.

5.3.1 Higgs Quartic coupling λ_h

One-loop and two-loop corrections for λ_h are as follows,

$$\beta_{\lambda_h}^{(1)} = \lambda_h \left(-3g_1^2 - 9g_2^2 + 12\lambda_t^2\right) + \frac{3g_2^4}{4} + \frac{3}{8}\left(g_1^2 + g_2^2\right)^2 + 24\lambda_h^2 - 6\lambda_t^4 \tag{5.4}$$

$$\begin{aligned}\beta_{\lambda_h}^{(2)} &= \left(\frac{85g_1^2}{6} + \frac{45g_2^2}{2} + 80g_3^2\right)\lambda_h\lambda_t^2 + \frac{629}{24}g_1^4\lambda_h + \frac{39}{4}g_2^2g_1^2\lambda_h + 36\left(g_1^2 + 3g_2^2\right)\lambda_h^2 \\ &- \frac{73}{8}g_2^4\lambda_h - \frac{19}{4}g_1^4\lambda_t^2 - \frac{8}{3}g_1^2\lambda_t^4 + \frac{21}{2}g_2^2g_1^2\lambda_t^2 - 32g_3^2\lambda_t^4 - \frac{9}{4}g_2^4\lambda_t^2 - \frac{379g_1^6}{48} \\ &- \frac{559}{48}g_2^2g_1^4 - \frac{289}{48}g_2^4g_1^2 + \frac{305g_2^6}{16} - 3\lambda_h\lambda_t^4 - 144\lambda_h^2\lambda_t^2 - 312\lambda_h^3 + 30\lambda_t^6\end{aligned} \tag{5.5}$$

5.3.2 Yukawa coupling for Top quark λ_t

One and two loop β functions for λ_t are given by,

$$\beta^{(1)}_{\lambda_t} = \left(-\frac{17g_1^2}{12} - \frac{9g_2^2}{4} - 8g_3^2\right)\lambda_t + \frac{9\lambda_t^3}{2} \tag{5.6}$$

$$\begin{aligned}\beta^{(2)}_{\lambda_t} &= \lambda_t \left(\lambda_t^2\left(\frac{131g_1^2}{16} + \frac{225g_2^2}{16} + 36g_3^2 - 12\lambda_h\right) + \frac{1187g_1^4}{216} - \frac{3}{4}g_2^2g_1^2 + \frac{19}{9}g_3^2g_1^2 \right.\\ &- \left.\frac{23g_2^4}{4} + 9g_2^2g_3^2 - 108g_3^2 + 6\lambda_h^2 - 12\lambda_t^4\right)\end{aligned} \tag{5.7}$$

5.3.3 Gauge couplings g_1, g_2 and g_3

In standard model, gauge couplings have following one-loop and two-loop corrections [43],

$$\beta^{(1)}_{g_1} = \frac{41g_1^3}{6} \tag{5.8}$$

$$\beta^{(1)}_{g_2} = -\frac{19g_2^3}{6} \tag{5.9}$$

$$\beta^{(1)}_{g_3} = -7g_3^3 \tag{5.10}$$

$$\beta^{(2)}_{g_1} = g_1^3\left(\frac{199g_1^2}{18} + \frac{9g_2^2}{2} + \frac{44g_3^2}{3} - \frac{17\lambda_t^2}{6}\right) \tag{5.11}$$

$$\beta^{(2)}_{g_2} = g_2^3\left(\frac{3g_1^2}{2} + \frac{35g_2^2}{6} + 12g_3^2 - \frac{3\lambda_t^2}{2}\right) \tag{5.12}$$

$$\beta^{(2)}_{g_3} = g_3^3\left(\frac{11g_1^2}{6} + \frac{9g_2^2}{2} - 26g_3^2 - 2\lambda_t^2\right) \tag{5.13}$$

We have also included the proper matching conditions at top pole mass [115]. The Higgs boson pole mass M_H is determined by its relation to the running Higgs quartic coupling through the one-loop matching condition,

$$\lambda_h(M_t) = \frac{M_H^2}{v^2}\left(1 + \delta_h(M_t)\right) \tag{5.14}$$

and $\delta_h(M_t)$ is defined as,

$$\delta_h(M_t) = \frac{M_z^2}{32\pi^2 v^2}\left[\xi f_1(\xi) + f_0(\xi) + \xi^{-1} f_{-1}(\xi)\right] \tag{5.15}$$

where $\xi \equiv M_h^2/M_z^2$ and the functions f_i, $(i = 1, 0, -1)$ are as follows,

$$\begin{aligned}
f_1(\xi) &= -Z\left(\frac{c_w^2}{\xi}\right) - \log\left(c_w^2\right) + 6\log\left(\frac{M_t^2}{M_h^2}\right) - \frac{1}{2}Z\left(\frac{1}{\xi}\right) + \frac{3\log(\xi)}{2} \\
&+ \frac{9}{2}\left(\frac{25}{9} - \frac{\pi}{\sqrt{3}}\right) \qquad (5.16) \\
f_0(\xi) &= -6\log\left(\frac{M_t^2}{M_z^2}\right)\left(2c_w^2 - \frac{2M_t^2}{M_z^2} + 1\right) + \left(\frac{3c_w^2}{s_w^2} + 12c_w^2\right)\log\left(c_w^2\right) + 4c_w^2 Z\left(\frac{c_w^2}{\xi}\right) \\
&+ \frac{3\xi c_w^2 \log\left(\frac{\xi}{c_w^2}\right)}{\xi - c_w^2} - \frac{15}{2}\left(2c_w^2 + 1\right) - 3\frac{M_t^2}{M_z^2}\left(2Z\left(\frac{M_t^2}{\xi M_z^2}\right)\right. \\
&+ \left. 4\log\left(\frac{M_t^2}{M_z^2}\right) - 5\right) + 2Z\left(\frac{1}{\xi}\right) \qquad (5.17) \\
f_{-1}(\xi) &= 6\log\left(\frac{M_t^2}{M_z^2}\right)\left(2c_w^4 - \frac{4M_t^4}{M_z^4} + 1\right) - 12c_w^4 Z\left(\frac{c_w^2}{\xi}\right) + 8\left(2c_w^4 + 1\right) \\
&- 12c_w^4\log\left(c_w^2\right) + 24\frac{M_t^4}{M_z^4}\left(Z\left(\frac{M_t^2}{\xi M_z^2}\right) + \log\left(\frac{M_t^2}{M_z^2}\right) - 2\right) - 6Z\left(\frac{1}{\xi}\right) \qquad (5.18)
\end{aligned}$$

with $s_w^2 = \sin^2\theta_W = 0.23116(13)$ [116], $c_w^2 = \cos^2\theta_W$ (θ_W denotes the weak mixing angle) and

$$Z(z) = \begin{cases} 2A\arctan(1/A) & (z > 1/4) \\ A\ln\left[(1+A)/(1-A)\right] & (z < 1/4), \end{cases} \qquad (5.19)$$

with $A = \sqrt{|1-4z|}$ and we use the value $\alpha_s(M_z) = 0.1184$ [116].

Fig. (5.1) shows the variation of λ_h with different values top quark mass m_t. With increase of m_t, λ_h becomes negative even before the Planck scale. For subsequent calculations, we have chosen different sets of top mass keeping Higgs mass constant.

Now we move beyond the standard model by adding a Higgs-lepton doublet and heavy neutrino Yukawa coupling.

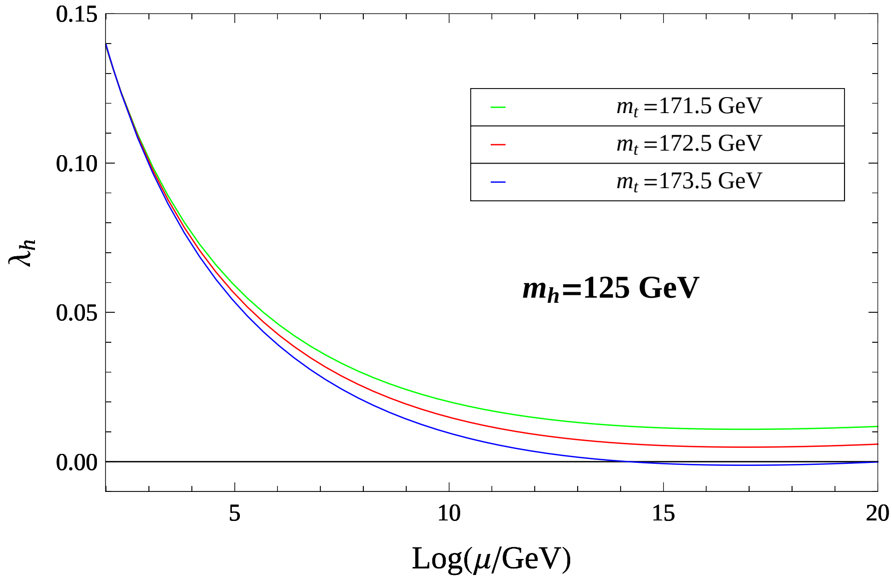

Figure 5.1: *Running of λ_h for different values of m_t in Standard Model* (m_h = 125 *GeV*, $\alpha_s = 0.1184$).

5.4 Higgs coupling with heavy neutrino

The Standard Model Higgs can couple to a singlet neutrino N_R via the gauge invariant interaction term,

$$-\mathcal{L}_Y = \mathcal{Y}_\nu L H N_R + \frac{1}{2}\bar{N_R}\mathcal{M}_R N_R^c + \text{h.c.}, \tag{5.20}$$

where $L = (\nu, l)^T$ is the lepton doublet, $H = (h^0, h^-)$ is the Higgs doublet and N_R is right-handed singlet neutrino. $\mathcal{M}_R$ are the Majorana masses for N_R. Once this neutral field h^0 acquires vacuum expectation value (vev) $v = 174$ GeV, the electroweak symmetry breaking occurs. This interaction generates Dirac mass term, $\mathcal{M}_D$, after electroweak symmetry breaking which reads as $\mathcal{M}_D = \mathcal{Y}_\nu v$ ($v = 174$ GeV).

In our further analysis we will not consider the flavour structures of both $\mathcal{Y}_\nu$ and $\mathcal{M}_R$, i.e., we will assume that $\mathcal{Y}_\nu = Y_\nu \text{diag}(1,1,1)$ and right-handed neutrinos are degenerate, i.e. $\mathcal{M}_R = M_R\,\text{diag}(1,1,1)$.

This new Yukawa coupling affects the RG evolutions of λ_h and thus gets constrained from vacuum stability. This Y_ν also plays an important role in the production and decays of N_R leading to same-sign-dilepton associated with

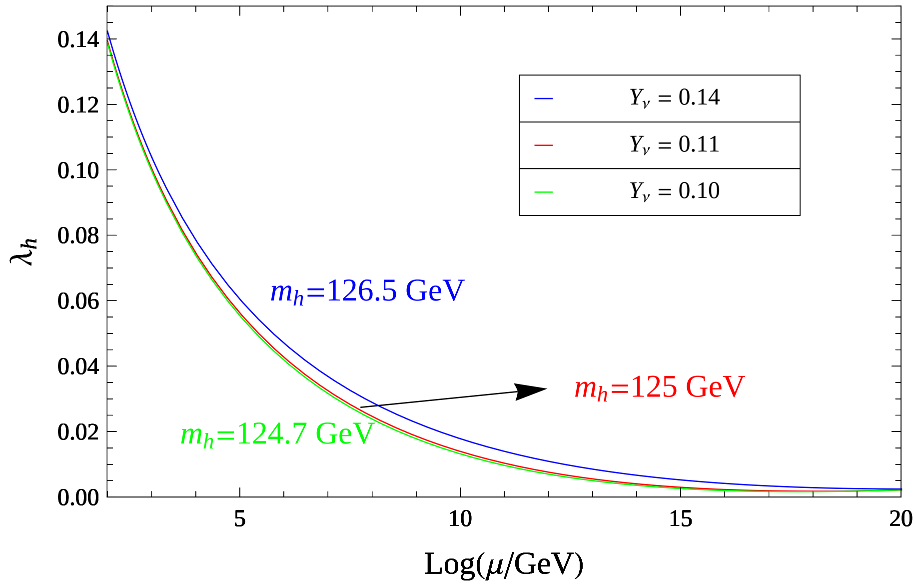

Figure 5.2: *Running of λ_h for different values of m_h in Standard Model* (m_t = 172.5 GeV, $\alpha_s = 0.1184$).

jets at the LHC.

The running of neutrino Yukawa coupling is as follows, [48, 50, 51]

$$\mu\frac{d}{d\mu}\left(\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\right) = \frac{1}{(4\pi)^2}\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\left[6\lambda_t^2 + 2\ \mathrm{Tr}\left(\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\right) - \left(\frac{9}{10}g_1^2 + \frac{9}{10}g_2^2\right) + 3\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\right] \tag{5.21}$$

The introduction of neutrino sector to Standard Model also modify the RG evolution of the Higgs quartic coupling λ_h and Yukawa coupling of top quark λ_t as follows,

The extra contribution for the singlet fermionic field to Higgs quartic coupling (λ_h) is

$$\hat{\beta}_{\lambda_h} = \frac{1}{(4\pi)^2}\left[-4\ \mathrm{Tr}(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger) + 4\lambda_h\ \mathrm{Tr}(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger)\right], \tag{5.22}$$

and to the top quark Yukawa coupling (λ_t) is

$$\hat{\beta}_{\lambda_t} = \frac{1}{(4\pi)^2}\left[\ \mathrm{Tr}\left(\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\right)\right]. \tag{5.23}$$

Keeping top quark mass m_t fixed, we can find different values of neutrino coupling for different Higgs masses. This variation of λ_h with different values

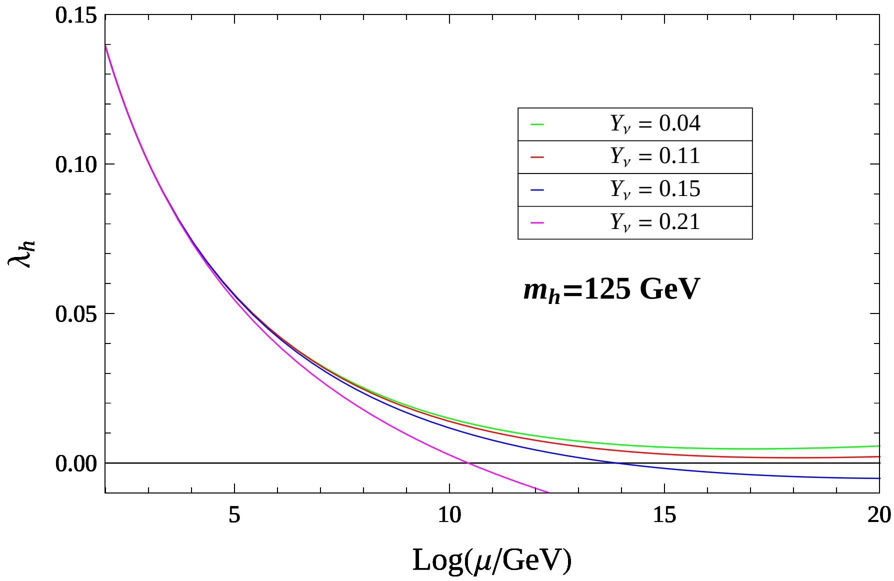

Figure 5.3: *Running of* λ_h *for different values of neutrino Yukawa coupling* Y_ν *with* $M_R =$ *0.1-1 TeV,* $(m_t = 172.5\ \text{GeV}, \alpha_s = 0.1184)$.

of Higgs mass is presented in Fig. (5.2). Using these RG equations, we can also study the effect of neutrino coupling on λ_h with fixed Higgs mass m_h and top quark mass m_t. This running of λ_h has been shown in Fig. 5.3. The impact of neutrino Yukawa coupling Y_ν on λ_h is in similar fashion as λ_t and λ_h becomes negative before Planck scale (M_P) with comparatively larger values of Y_ν. We know that there is an uncertainty in top mass measurement 173.2 ± 0.9 GeV [117] and 173.3 ± 2.8 GeV [118], and that feature has been grabbed in Fig. 5.1.

We outline the RGEs of λ_h for different sets of Y_ν for $m_h = 124.7 - 126.5$ GeV and $m_t = 172.5$ GeV. We check the stability condition, defined as $\lambda(\mu \leq M_P) > 0$ and reveal that, to avoid the instability of potential, the maximum value of the Yukawa coupling Y_ν at $\mu \sim$ TeV must be,

$$Y_\nu \quad \leq \quad 0.14. \tag{5.24}$$

This upper limit of Y_ν sets the tolerance of the vacuum in this model. It has been noted that the light-heavy mixing parameter can be encapsulated in terms of the Dirac mass, $M_D \sim Y_\nu v$, see [109, 119]. In other words, this mixing, which in turn also affects the production and decay of the heavy Majorana neutrino, gets constrained. Thus eventually this bound can be useful to adjudge the possibility of being probed or ruled out this TeV scale model at the LHC.

5.5 Gauge interactions of heavy neutrinos

We consider three generations of Standard Model $SU(2)_L$ lepton doublets $L_{lL} = (\nu_l, \ell_l)^T_L$, $(\ell = e, \mu, \tau)$ and three singlets N_R. The relation between the neutrino flavour and the mass eigenstates can be written as,

$$\begin{aligned} \nu_{lL} &= \sum_{i=1}^{3} U_{li}\nu_{iL} + \sum_{k=1}^{3} V_{lk}N^c_{kL} && (5.25) \\ U^\dagger U &+ V^\dagger V = I, && (5.26) \end{aligned}$$

where the mixing between the light and heavy neutrinos is $V^\dagger V \simeq (\mathcal{M}_D\mathcal{M}_R^{-1})^2 = (v\mathcal{Y}_\nu\mathcal{M}_R^{-1})^2$. In terms of the mass eigenstates the charged current interaction vertices can be written as

$$-\mathcal{L}^{cc}_{int} = \frac{g}{\sqrt{2}}W_\mu\left(\sum_{i=1}^{3} U^*_{li}\bar{\nu}_i\gamma^\mu P_L l + \sum_{k=1}^{3} V^*_{lk}\overline{N^c_k}\gamma^\mu P_L l + \text{h.c.}\right) \tag{5.27}$$

Our phenomenological studies involve $\Delta L = 2$ processes, like same-sign-dilepton (including $(0\nu\beta\beta)$) production at colliders where the source of the lepton number violation is the exchange of heavy Majorana neutrino. The coupling of the heavy neutrino to the charged leptons is parametrized by the mixing angles of V_{lk}. We use the upper bound of Y_ν from Eq. (5.24) to predict the parameter space where these processes may be observable. We also study lepton flavour violations like $\mu \to e\gamma$ whose upper limits are again restricted by Eq. (5.24).

5.5.1 Neutrinoless double beta decay

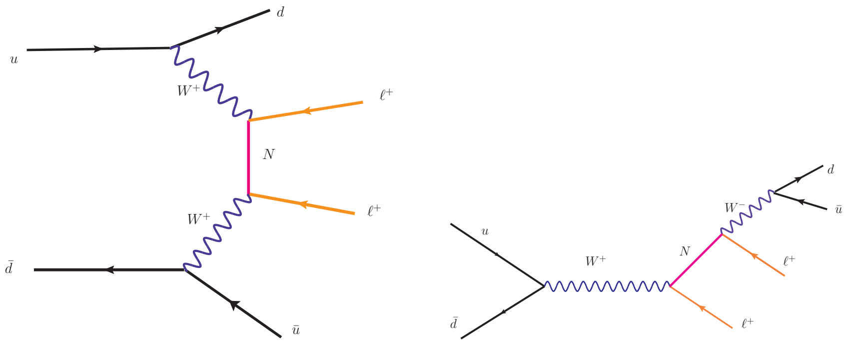

Figure 5.4: *Neutrinoless double beta diagrams involving heavy Majorana field*

Neutrinoless double beta decay is one of the important phenomena to probe the lepton number violation. In this process, the lepton number violation occurs by two units. The half-life time of this process is also ascribed by this mixing V_{li} as following,

$$T_{1/2}^{-1} = \kappa_{0\nu} \left| \frac{(M_\nu)_{ee}}{\langle p^2 \rangle} - \frac{|V_{ei}|^2}{M_R} \right|^2 , \tag{5.28}$$

where $\kappa_{0\nu} = \mathcal{G}_{0\nu} \left(\mathcal{M}_N m_p \right)^2$, nuclear matrix element (NME) for heavy neutrino, $\mathcal{M}_N = 363 \pm 44$, m_p is the proton mass, and $\mathcal{G}_{0\nu} = 7.93 \times 10^{-15}\ \mathrm{yr}^{-1}$. We assume that the second term, arising from the heavy neutrino mixing, dominates over the first term and the mixing parameter V_{ei} is explicitly related to the neutrino Yukawa coupling Y_ν via Dirac mass as,

$$|V_{ei}|^2 = \left| \left(M_D M_R^{-1} \right)_{ee} \right|^2 , \tag{5.29}$$

and the relation Eq. (5.28) for half-life time of neutrinoless double beta decay becomes,

$$T_{1/2}^{-1} \approx \frac{\kappa_{0\nu} \, |V_{ei}|^4}{M_R^2} = \frac{K_{0\nu}}{M_R^2} \left| \left(M_D M_R^{-1} \right)_{ee} \right|^4 . \tag{5.30}$$

The experimental bound on half-life time is $T_{1/2} = 2.23^{+0.44}_{-0.31} \times 10^{25}$ yr shown in [120]. The study of vacuum stability gives $M_D \leq 24.36$ GeV. Using the values for $T_{1/2}$ and M_D, we can put the limit on the mass of the heavy neutrino,

$$M_R \leq 4.5 \text{ TeV} \tag{5.31}$$

5.5.2 Lepton flavor violation

The mixing of active neutrinos with heavy neutrinos can give rise to lepton flavour violations (LFV) like $\mu \to e\gamma$ as shown in Fig. 5.5, if we generalize $\mathcal{M}_R$ matrix to contain off-diagonal terms. In this case, the structure of $\mathcal{M}_R$ matrix will be,

$$\mathcal{M}_R^{-1} = M_R^{-1} \begin{pmatrix} 1 & \epsilon_1 & \epsilon_2 \\ \epsilon_1 & 1 & \epsilon_3 \\ \epsilon_2 & \epsilon_3 & 1 \end{pmatrix}, \tag{5.32}$$

where ϵ_is can be chosen to satisfy the correct light neutrino maxing angles.

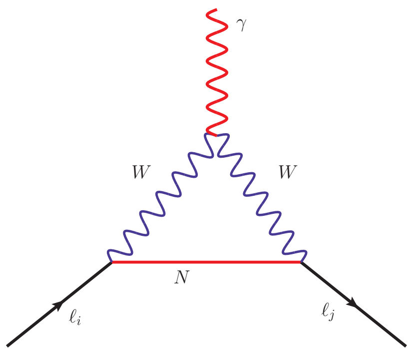

Figure 5.5: *Lepton Flavour Violating process $\ell_i \to \ell_j\gamma$.*

We know among the $\ell_i \to \ell_j\gamma$ type LFV decays, $\mu \to e\gamma$ holds the most stringent bound on its decay branching ratio (BR) which is 2.4×10^{-12} (Present) [121], and 1.0×10^{-13} (Future) [122].

We estimate the branching ratio of this process from vacuum stability and check its compatibility with the existing direct bounds. This branching ratio for $\mu \to e\gamma$ is accompanied by the mixing V_{li} $(l = e, \mu)$ between light to heavy neutrino [123, 124, 125, 104, 105, 106, 107, 108] as,

$$\mathrm{Br}\,(\mu \ \to \ e\ \gamma) = \frac{3\alpha}{8\pi}\left|\sum_i V_{ei}V^*_{\mu i}\,\hat{g}(r)\right|^2 , \tag{5.33}$$

where $\hat{g}(r) = r\,(1-6r+3r^2+2r^3-6r^2\ln(r))\,/(2\,(1-r)^4)$, and $r = M_R^2/M_W^2$.

Again taking the constraint from vacuum stability $M_D \simeq 24.36$ GeV, Eq. (5.33) is simplified to,

$$\mathrm{Br}\,(\mu \to e\gamma) = 2.82\times 10^{-10}\left(\frac{M_D}{24.36\ \mathrm{GeV}}\right)^4\left(\frac{\mathrm{TeV}}{M_R}\right)^4 . \tag{5.34}$$

Taking the experimental bound $\mathrm{Br}\,(\mu \to e\gamma) < 2.4\times 10^{-12}$ from [121] and if $M_D \simeq 24.36$ GeV (in order to give a sizable contribution to $(0\nu\beta\beta)$ and SSD signal at LHC), we see that $M_R \geq 3.3$ TeV. This implies that in order to observe $(0\nu\beta\beta)$ or like-sign-dilepton signals, we need M_R to be small and the texture of $\mathcal{Y}_\nu$ and $\mathcal{M}_R$ should be such that the $e-\mu$ flavour mixing is small.

5.6 Same-Sign-Dilepton signal at LHC

The processes for same-sign-dilepton (SSD) production are similar to the neutrinoless double beta decay, see Fig. 5.4. These processes have phenomenological importance as it involves both e and μ . The signal is identified as the same-sign-dileptons + N jets, $N > 2$. The interaction vertices of the heavy neutrino (N_R) are suppressed by the mixing parameters $\sim \mathcal{O}(Y_\nu v/M_R)$. Assuming again a flavour diagonal $\mathcal{Y}_\nu$ and degenerate N_R, we estimate the cross section for SSD at the LHC.

We have implemented this SM $\oplus$ Heavy Singlet neutrino model at Calchep [126] and estimated the cross-section for the process $pp \to e^\pm e^\pm$ + jets and $pp \to \mu^\pm\mu^\pm$ + jets. We have considered the range of M_R to be 0.1-1 TeV and no flavour structure for the simplification of study. It has been noted that in the Fig. 5.4 (left) the amplitude is suppressed more ($(M_D/M_R)^4$) than the other diagram Fig. 5.4 (right) (here the suppression is $\mathcal{O}(M_D/M_R)^2$). The choice of our M_R is such that the mixing is much smaller than 1 and that dictates us to work safely with the Fig. 5.4 (right).

In earlier section we have noted the maximum $M_D = Y_\nu v$ from the vacuum stability of the Standard Model Higgs field. In this section we have use that limit and estimate the largest possible maximum cross-section for the process Fig. 5.4 (right) with two different sets of center of mass energy at the LHC. These two cross-sections are calculated with center of mass energy ($\sqrt{s}$) 7 TeV and 14 TeV shown in Fig. 5.6 and Fig. 5.7 respectively.

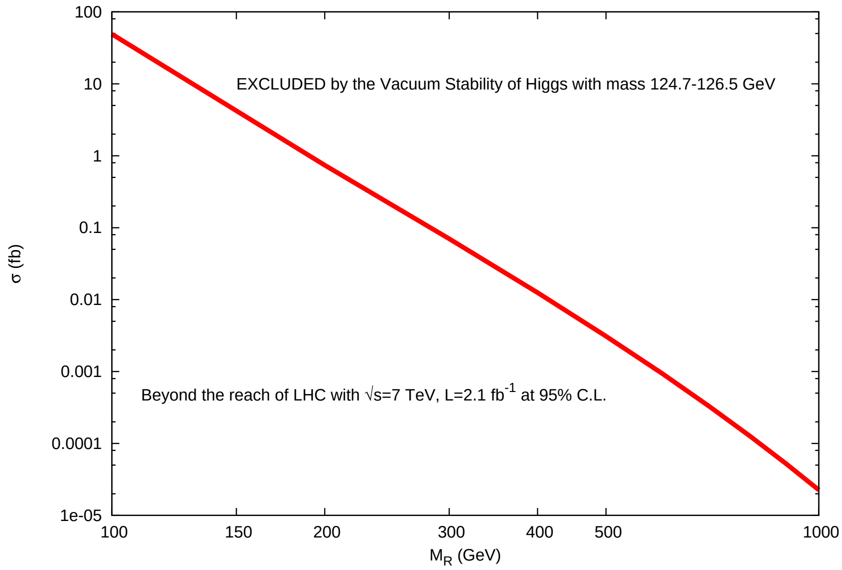

Figure 5.6: *Production cross-section Fig. 4 (right) with* $\sqrt{s}$*=7* TeV in the LHC.

In recent paper by Atlas [127] the Standard Model background has been estimated at 2.1 fb^{-1} luminosity. As shown in Fig. 5.6 and Fig. 5.7, the vacuum stability puts a stringent bound on the production cross-section, through the M_D and the maximum allowed cross-section is 49.02 fb at 7 TeV center of mass energy. This is the maximum cross-section that one attains using no cuts. But due to the stringent constraint from the demand of vacuum stability, the allowed cross-section is quite small. However LHC does not have enough data to see the process compared to the SM background [127]. Thus we have to wait for future data with 14 TeV center of mass energy and large integrated luminosity (L=$\int \mathcal{L}dt = \sim 100\ fb^{-1}$). The cross section for the SSD process at the LHC with $\sqrt{s}$=14 TeV is shown in Fig. 5.7 and the region above the ‘thick

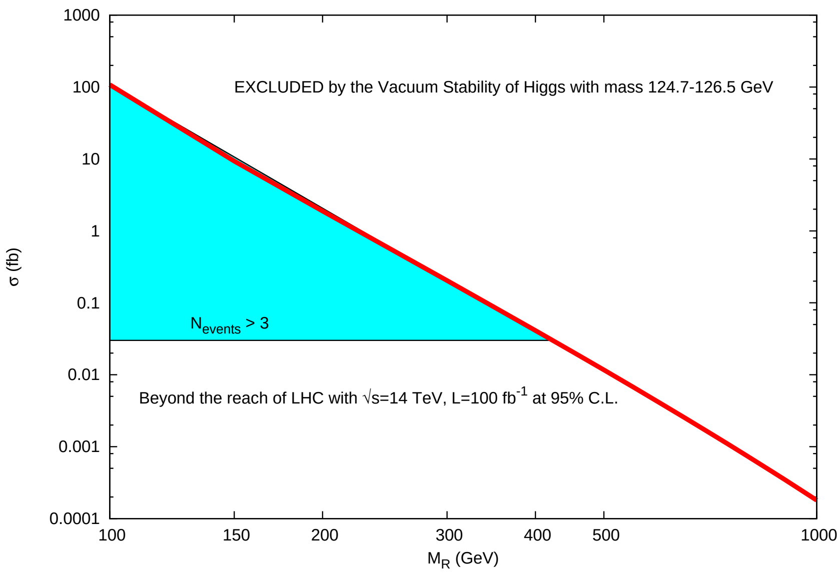

Figure 5.7: *Production cross-section of the process Fig. 4 (right) with* $\sqrt{s}$*=14* TeV in the LHC.

(red)' line is disallowed by the vacuum stability. In Fig. 5.7 we see that 'shaded (cyan)' area is the accessible region at the LHC with $\sqrt{s}$=14 TeV at L=100 fb^{-1} considering at least 3 events over the zero background, i.e, at 95% C.L. Hence taking into account the vacuum stability condition it may be possible to observe SSD signal at LHC if $M_R < 400$ GeV.

5.7 Conclusion

In this work we have focused on the vacuum stability of the Higgs field in a specific scenario where the Standard Model is extended by singlet Majorana fermions. We have studied the impact of such new field that couples to the light neutrinos via the SM Higgs doublet on the RG evolution of the Higgs quartic coupling (λ_h). We show that expectedly this new coupling (Y_ν) lowers the scale μ at which $\lambda_h(\mu)$ becomes negative. In this study the aim is to find the maximum value of Y_ν which is compatible with the vacuum stability with heavy neutrino field having mass $M_R \sim$ TeV.

We showed that the vacuum stability condition constrains the Dirac mass (which we have taken to be degenerate) to be $M_D \leq 24.36$ GeV. We studied

$\Delta L = 2$ processes like $(0\nu\beta\beta)$ and same-sign-dileptons at LHC and lepton flavour violating processes like $\mu \to e\gamma$ taking into account the vacuum stability bound on M_D. This bound restricts the mixing between the light and heavy neutrinos and the mixing varies as M_D/M_R.

We find that in order to observe $(0\nu\beta\beta)$ signal, which saturates the experimental bound $T_{1/2} = 2.23^{+0.44}_{-0.31} \times 10^{25}$ yr [120], the heavy neutrinos must have a mass $M_R < 4.5$ TeV.

For the LFV process $\mu \to e\gamma$ if we assume M_D at the largest possible value 24.36 GeV from vacuum stability (to maximize the chances for other signals), then we get the constraint $M_R > 3.3$ TeV. It may be possible to evade this bound on M_R by choosing the texture of $\mathcal{M}_D$ and $\mathcal{M}_R$ matrices such that $e - \mu$ mixing is suppressed.

Finally we estimate the maximal cross-section for the signal, like same-sign-dilepton associated with jets imposing the vacuum stability condition. We show that the data attained with 2.1 fb^{-1} integrated luminosity cannot rule out right-handed neutrinos as the vacuum stability criterion shows that the dilepton signal would be way below the SM background. It may be possible to observe the SSD at the LHC with $\sqrt{s}$=14 TeV and integrated luminosity of 100 fb^{-1} as long as $M_R <$ 400GeV. If a larger signal is seen at the LHC then it would be a sign of new physics beyond SM + sterile right-handed neutrinos.

Chapter 6

Summary and conclusions

This chapter summarizes the results of the thesis and we have drawn the conclusions for the work presented here.

This thesis covers the study of inflation model, which is an exponentially expanded era of early universe. Inflation can successfully discuss the early universe. Standard model of cosmology mainly depends on the Big-bang theory. Big-bang theory solely can not describe the early phenomena of the universe. Inflation is one of the important cosmological model, which along with Big-bang model can solve the horizon and flatness problem. The discussion of the drawback of Big-bang theory and how it can be solved by considering inflation is described in Chapter-2. The requirement of the inflationary model can be fulfilled by simple scalar field, named as 'Inflaton'. Dynamics of this inflaton field is presented here. For completeness of the basic of inflation, we also briefly discuss the different possible kinds of inflation model.

We have presented a comprehensive study of Higgs inflation model in Chapter-3. And it is the main focus of this thesis. The recent measurement of Higgs mass, around 125.3 ± 0.6 GeV at 4.9σ level from CMS experiment and 126.5 GeV at 5σ level from Atlas experiment [1], imply $\lambda \sim 0.14$. However inflation predicts $\lambda \sim 10^{-12}$ from the WMAP observation of curvature perturbation $\Delta_R^2 = 2.43 \pm 0.11 \times 10^{-9}$ [96]. Hence it seems impossible to consider standard model Higgs as inflaton to satisfy WMAP results as well as Atlas and CMS prediction for Higgs mass. However the large conformal coupling between Higgs and the gravity [5] helps to clarify both the observations and obtain the correct value for the Higgs quartic coupling λ. This large curvature coupling creates the problem of unitarity which also can be handled in this model. We couple the Higgs field to the Electromagnetic fields via a non-renormalizable dimension six operator suppressed by the Planck scale in the Jordan frame. We show that by choosing the Higgs coupling $\lambda(M_Z) = 0.132$ (which is related to $m_h = 126$GeV consistent with the recent results of Atlas and CMS experiments) and curvature coupling $\xi(M_Z) = 10^3$, we can generate comoving magnetic fields of 10^{-7} Gauss at present and at comoving coherence length of $100kpc$. We have calculated the magnetic field at inflationary era as $\delta_{B_I}^2 = 1.09 \times 10^{52}\,\mathrm{GeV}^4$. Along with this value and the observation for present magnetic field give the correct reheating temperature, $T_{reh} \sim 10^{13}$ GeV. This reheating temperature is matches with the results described in [77]. There is also a scope to check the back-reaction problem in this model. Energy density of the electromagnetic field is, $\rho_{em} = 2.81 \times 10^{49}$ GeV^4, whereas the energy density of the inflation is, $\rho_\phi = 8.3 \times 10^{63}$ GeV^4, which is much larger than ρ_{em}. Hence the back-reaction of the generated electromagnetic field can not spoil the inflation. The problem of large back-reaction which is generic in the usual inflation models of magneto-genesis is avoided in this Higgs inflation model as the back-reaction is suppressed by the large Higgs-curvature coupling.

In next chapter we have considered a distinct method to generate the density perturbation for explaining the structure formation of the universe. According to this method, conformally coupled field rolling down negative quartic potential can produce density perturbations. We have applied this idea in Inert doublet model, which is the minimal extension of the standard model. The predictions of this model for inflation are consistent with the WMAP observations. However, Inert doublet is an established and well-known model to produce dark matter candidate by imposing Z_2 symmetry. In this model, light neutral component of the inert Higgs doublet becomes the dark matter. Our study reveals light scalar dark matter of mass 33.7 GeV. Moreover the successful electroweak symmetry breaking makes possible to get correct Higgs mass around $\sim$ 126.5 GeV.

Subsequently we studied the vacuum stability of 125 GeV Higgs. This study have started long way back by G. Altereli [17]. The Higgs potential becomes instable if the coupling becomes negative in any scale within the scale, from electroweak to Planck scale. The unbounded potential from the below, can create problem to explain early universe, as energy density as well as Hubble parameter become negative. Keeping in mind the present observation of Higgs mass, we have done the full analysis. To study the phenomenology of TeV scale heavy neutrinos, we have added extra right handed neutrino to the standard model. We have constrained the neutrino yukawa coupling $Y_\nu < 0.14$ through the study of vacuum stability and this can reflected as the bound on Dirac mass as $m_D = Y_\nu v$, where Higgs vev, $v = 174$ GeV. This constraint effects the mixing parameter, $V^\dagger V = \left(m_D\, M_R^{-1}\right)^2$. Hence the processes such as, Neutrinoless double beta decay ($0\nu\beta\beta$), Lepton flavor violating decays like $\mu \rightarrow e\gamma$ and Like-sign dilepton signals at LHC have significant effects from vacuum stability.

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Publications attached with the thesis

1. **“Magnetic Field Generation in Higgs Inflation Model”**
   Moumita Das and Subhendra Mohanty
   Int. J. Mod. Phys. A **27**, 1250040 (2012)
   [arXiv:1004.1927 [astro-ph.CO]]