

Characterization of Chaotic and Nonlinear Time Series

A THESIS

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BY

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CERTIFICATE

I feel great pleasure in certifying that the thesis entitled (**Characterization of Chaotic and Nonlinear Time Series**) embodies a record of the results of investigations carried out by Shri. **Balamurali Krishna Mayya K.** under my guidance. I am satisfied with the analysis of data, interpretation of results and conclusions drawn.

He has completed the residential requirement as per rules.

I recommend the submission of thesis.

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Contents

Acknowledgements	viii
Abstract	ix
1 Introduction	1
1.1 Time Series Analysis	2
1.1.1 Data analysis or description	5
1.1.2 Reconstruction of state space for a chaotic signal	8
1.1.3 Calculaton of invariants for the chaotic signal	12
1.1.4 Lyapunov exponents and spectrum	14
1.1.5 Modelling of data	17
1.2 Multivariate Analysis	20
1.2.1 Analysis techniques	21
1.3 Random matrix theory	26
1.3.1 Gaussian Orthogonal Ensemble	27
1.3.2 Spectral statistics for the Gaussian ensembles	28
1.3.3 Wishart Ensembles	30
1.4 Application of Random Matrix Theory concepts to Multivariate Analysis . .	31
1.4.1 Results for atmospheric data	32
1.4.2 Results for stock market data	33
1.4.3 Results for EEG data	35
1.5 Outline of the thesis	35
2 Delay covariance matrix	37
2.1 Delay covariance matrix	37
2.2 Wick's Theorem	39

2.3	Density distribution of eigenvalues:	39
2.3.1	Diagrammatic summation and the Dyson's self energy equation . . .	41
2.3.2	Self energy calculation for finite delay τ	45
2.3.3	Anti-symmetric case	49
2.3.4	Self energy calculation for $\tau = 0$	50
3	Analysis of observed data	51
3.1	Sea level pressure data for the NAO	51
3.1.1	North Atlantic Oscillation (NAO)	51
3.2	Monthly NAO sea level pressure data	53
3.2.1	Analysis of NAO data	54
3.2.2	Daily pressure data of NAO	64
3.3	Anti-symmetric delay correlation matrix analysis of NAO data	69
3.3.1	Results for the anti-symmetric delay correlation matrix	69
3.4	Stock Market data	83
3.4.1	Standard and Poor (S&P 500) US stock market data	83
3.4.2	Indian stock market data	83
3.5	Analysis of stock market data sets	84
3.5.1	Correlation matrix results	84
3.5.2	Symmetrized delay correlation matrix results	86
4	Complex data sets	100
4.1	Analytical results	100
4.2	Wind-stress-vector data analysis of the equatorial pacific region	101
4.2.1	Wind-stress-vector data analysis	101
4.2.2	Wind-stress-vector data for the equatorial pacific region	103
4.3	Correlation matrix analysis of equatorial pacific data	104
4.4	Results for Hermitian delay correlation matrix	104
4.4.1	Discussion	113
4.5	Results for the anti-hermitian delay correlation matrix	113
4.5.1	Discussion	118
5	Relative Entropy of Chaotic Dynamical Systems	120
5.1	Relative entropy	120
5.2	Relative Entropy method applied to sequence characterization	121
5.3	Numerical method and results	122
5.3.1	Sequence of symbols from a time series	122
5.3.2	Comparison of different dynamical systems	123

5.3.3 Logistic map	126
5.3.4 Lorenz attractor	127
5.4 Sunspot data	128
5.5 Frequencies of symbol sequences in chaotic data	130
5.5.1 Conclusions	131
Conclusions	134
References	135
List of Publications	144