Studies of Models of Fermion Masses and Mixing

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by

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To

my family

DECLARATION

I ***Mr. Kodrani Bhavikkumar Pravinkumar****, S/o Mr. Kodrani Pravinkumar, resident of E-203, PRL residences, Navrangpura, Ahmedabad 380009, hereby declare that the work incorporated in the present thesis entitled,* ***"Studies of Models of Fermion Masses and Mixing"*** *is my own and original. This work (in part or in full) has not been submitted to any University for the award of a Degree or a Diploma.*

Date :

(Kodrani Bhavikkumar Pravinkumar)

CERTIFICATE

I feel great pleasure in certifying that the thesis entitled, **"Studies of Models of Fermion Masses and Mixing"** embodies a record of the results of investigations carried out by Mr. Kodrani Bhavikkumar Pravinkumar under my guidance.

He has completed the following requirements as per Ph.D. regulations of the University.

(a) Course work as per the university rules.

(b) Residential requirements of the university.

(c) Presented his work in the departmental committee.

(d) Published minimum of two research papers in a referred research journal.

I am satisfied with the analysis of data, interpretation of results and conclusions drawn.

I recommend the submission of thesis.

Date :

Anjan S. Joshipura
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Countersigned by

Head of the Department

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Abstract

In this thesis we consider two Higgs doublet model (2HDM) as physics beyond standard model (SM) and study various aspects of CP and flavor violation in it with a view to explain the deviations in CP violating observables in B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing from SM predictions. 2HDM provide several new sources of CP and flavor violations which are not present in SM. These include flavor changing neutral current (FCNC) interactions, charged Higgs interactions and scalar-pseudoscalar mixing. We consider different variants of 2HDM which can be obtained by imposing some symmetry conditions or some other assumptions on the general 2HDM. These variants include

1. 2HDM with minimal flavor violation : In this model all the CP and flavor violations are described in terms of CKM matrix. Introduction of complex Higgs singlets can give rise to new phases to neutral mesons mixing. In this model new contribution to B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing gets correlated for the case of neutral Higgs dominance.

2. 2HDM with suppressed FCNC : In this model spontaneous CP violation along with the presence of FCNC leads to complex CKM matrix. FCNC in this model are suppressed using 23 symmetry which exchanges quarks of second and third generations.

3. 2HDM without FCNC: In this model there are no FCNC. Charged Higgs interactions contains new phases not present type-I or type-II 2HDM and can give new phases to neutral meson mixing. NP contribution to B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ in this model gets correlated in the limit when masses and couplings of first and second generation of quarks vanishes.

4. 2HDM with general FCNC.

All these variants have different pattern for CP and flavor violations and lead to interesting phenomenological consequences. We make phenomenological analysis of above models and obtain constraints on new physics parameters subjected to constraints from processes such as B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing, $\bar{B}_s \to \mu^+\mu^-$ and $\bar{B}_d \to \bar{K}\mu^+\mu^-$

Contents

Chapter 1

Introduction

High energy physics is a branch of physics which attempts to understand the world at its most fundamental level. The dream is to understand the basic building blocks of the world and their interactions. According to our current understanding, the basic building blocks of the world are the elementary particles known as quarks and leptons. Elementary means that, they don't have any substructure. The other ingredients of our world recipe are interactions experienced by these particles. There are four known interactions namely strong, weak, electromagnetic and gravitational all having different roles to play. Strong interaction is responsible for formation of nucleons such as protons and neutron from the quarks. It also keeps the nucleons together in the nucleus. Weak interaction gives rise to the processes such as β decay. Electromagnetic interaction is responsible for the formation of atoms from nucleus and electrons, and for the formation of molecules from atoms. Gravitational interaction is responsible for formation of stars, galaxies and so on. These interactions are mediated by force carriers which are called gauge bosons. Strong interaction is mediated by gluons, weak interaction is mediated by weak bosons, electromagnetic interaction is mediated by photon while gravitational interaction is believed to be mediated by graviton. The quest is on to see whether these interactions can be unified so that the four different interactions can be seen as different manifestations of one fundamental interaction. An important milestone in this journey is the development of Electro-weak standard model which unifies electromagnetic and weak interactions into a single electro-weak interaction [1]. It is based on the gauge group $SU(2)_L \times U(1)_Y$. Strong interaction is described by a theory based on $SU(3)_C$ gauge group [2].

1.1 Standard Model

Standard model (SM) as we know of today is based on gauge group $G_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$. The SM fields and their transformation properties under SM gauge group are given as

$$Q'_i \equiv \begin{pmatrix} u'_{iL} \\ d'_{iL} \end{pmatrix} \sim (3, 2, \frac{1}{6}), \; L'_i \equiv \begin{pmatrix} \nu'_{iL} \\ e'_{iL} \end{pmatrix} \sim (1, 2, -\frac{1}{2}) \tag{1.1}$$
$$u'_{iR} \sim (3, 1, \frac{2}{3}), \; d'_{iR} \sim (3, 1, -\frac{1}{3}), \; e'_{iR} \sim (1, 1, -1)$$

Here $i = 1, 2, 3$ represents generation or family index. Numbers in the parentheses represents transformation properties of the particles under $SU(3)_C, SU(2)_L$ and $U(1)_Y$ groups respectively. Q'_i, L'_i represents doublets of left handed quarks and leptons respectively in weak basis. $u'_{iR}, d'_{iR}, e'_{iR}$ represents right handed up type quarks, down type quarks and charged leptons respectively in weak basis. In SM there are no right handed neutrinos. SM also has a doublet of scalars known as Higgs doublet. It is defined as

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (1, 2, \frac{1}{2}) \tag{1.2}$$

Higgs doublet is required for the spontaneous breaking of $SU(2)_L \times U(1)_Y$ group to $U(1)_E$. SM Lagrangian can be written as

$$\mathcal{L}_{SM} = \mathcal{L}_F + \mathcal{L}_{YM} + \mathcal{L}_S + \mathcal{L}_{Yukawa} \tag{1.3}$$

The $\mathcal{L}_F$ part includes kinetic energy terms of fermions and their interaction with gauge bosons.

$$\mathcal{L}_F = i\bar{\psi}\gamma^\mu D_\mu \psi \tag{1.4}$$

Where $\psi = Q'_i, L'_i, u'_{iR}, d'_{iR}, e'_{iR}$. D_μ is covariant derivative given as

$$D_\mu = \partial_\mu - ig_s G^A_\mu \cdot \lambda^A - i\frac{g}{2} W^I_\mu \cdot T^I - ig' B_\mu Y \tag{1.5}$$

Here G_μ^A with $A = 1, 2, ...8$ represents $SU(3)_C$ gauge bosons, W^I with $I = 1, 2, 3$ represents $SU(2)_L$ weak bosons. B_μ is $U(1)_Y$ gauge field. $\mathcal{L}_{YM}$ represents self interactions of gauge fields. It is given as

$$\mathcal{L}_{YM} = -\frac{1}{4}G^{\mu\nu A}G^A_{\mu\nu} - \frac{1}{4}W^{\mu\nu I}W^I_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \tag{1.6}$$

with

$$\begin{aligned} G^A_{\mu\nu} &= \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_s\, i f^{ABC}\, G_{\mu B} G_{\nu C} \\ W^I_{\mu\nu} &= \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g\, i f^{IJK}\, W_{\mu J} W_{\nu K} \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \tag{1.7}$$

Where f^{ABC} and f^{IJK} represents the structure constants of the $SU(3)$ and $SU(2)$ groups respectively. If the Lagrangian has only two terms $\mathcal{L}_F$ and $\mathcal{L}_{YM}$ then the gauge bosons as well as fermions remain massless. Masses of weak gauge bosons and fermions are generated by a mechanism known as spontaneous symmetry breaking (SSB). Scalar part of Lagrangian is given as

$$\mathcal{L}_S = (D_\mu\phi)^\dagger(D_\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \tag{1.8}$$

For the scalar potential $V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$ to be bounded from below we require that $\lambda \geq 0$. For $\mu^2 < 0$ we can choose the vacuum expectation value (vev) of ϕ as

$$\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{1.9}$$

After this choice of the vacuum, gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ gets broken to $SU(3)_C \times U(1)_{EM}$. Hence the gauge bosons W^μ and B^μ combine to give massive weak bosons $W^{\pm\mu}$, Z^μ and a massless photon A^μ. The part of Lagrangian describing the Yukawa interaction of fermions in weak basis with scalar ϕ is given as

$$\mathcal{L}_{Yukawa} = \bar{Q}'_i\phi Y^d_{ij} d'_{jR} + \bar{Q}'_i\tilde{\phi} Y^u_{ij} u'_{jR} + \bar{L}'_i\phi Y^e_{ij} e'_{jR} + H.C. \tag{1.10}$$

Here $i, j = 1, 2, 3$ are generation indices and Y^d, Y^u, and Y^e are 3×3 non diagonal Yukawa matrices. $\tilde{\phi} = i\sigma_2\phi^*$, where σ_2 is Pauli matrix. Mass matrices of quarks and charged leptons are given as

$$M^u = vY^u,\ M^d = vY^d,\ M^e = vY^e \tag{1.11}$$

Mass matrices can be diagonalized by going to mass eigen basis $f_{L,R}$, where $f = u, d, e$, by applying the following basis transformations

$$f'_{L,R} = V^f_{L,R}\ f_{L,R} \tag{1.12}$$

In mass basis, the fermion mass matrix is given as

$$D^f = V_L^{f\dagger} M^f V_R^f \tag{1.13}$$

Here D^f is diagonal matrix and the diagonal entries of this matrix are the corresponding fermion masses.

1.1.1 Neutral current interaction

Neutral current interactions of quarks mediated by Z boson and photon in weak basis are given as

$$\begin{aligned}
\mathcal{L}^{NC,q} &= \frac{g}{2c_w} Z_\mu \left(\left(1 - \frac{4}{3}s_w^2\right) \bar{u}'_{iL}\gamma^\mu u'_{iL} - \left(1 - \frac{2}{3}s_w^2\right) \bar{d}'_{iL}\gamma^\mu d'_{iL} \right) \\
&\quad - \frac{g}{2c_w} Z_\mu s_w^2 \left(\frac{4}{3}\bar{u}'_{iR}\gamma^\mu u'_{iR} - \frac{2}{3}\bar{d}'_{iR}\gamma^\mu d'_{iR} \right) \\
&\quad - \frac{2}{3} e A_\mu \ \left(\bar{u}'_{iL}\gamma^\mu u'_{iL} + \bar{u}'_{iR}\gamma^\mu u'_{iR} \right) \\
&\quad + \frac{1}{3} e A_\mu \left(\bar{d}'_{iL}\gamma^\mu d'_{iL} + \bar{d}'_{iR}\gamma^\mu d'_{iR} \right)
\end{aligned} \tag{1.14}$$

Neutral Current interactions of leptons in weak basis are given as

$$\begin{aligned}
\mathcal{L}^{NC,l} &= \frac{g}{2c_w} Z_\mu \left(\bar{\nu}'_{iL}\gamma^\mu \nu'_{iL} - \left(1 - 2s_w^2\right) \bar{e}'_{iL}\gamma^\mu e'_{iL} + 2s_w^2 \bar{e}'_{iR}\gamma^\mu e'_{iR} \right) \\
&\quad + e A_\mu \left(\bar{e}'_{iL}\gamma^\mu e'_{iL} + \bar{e}'_{iR}\gamma^\mu e'_{iR} \right)
\end{aligned} \tag{1.15}$$

It can be seen from equations(1.14, 1.15) that neutral current interactions are flavor diagonal. This remains true even in the mass basis of fermions. Hence there are no flavor changing neutral currents at tree level in SM.

1.1.2 Charged current interaction

Charged current interaction of the quarks in weak basis is given as

$$\mathcal{L}^{CC,q} = \frac{g}{\sqrt{2}} \left(W^+_\mu \bar{u}'_L \gamma^\mu d'_L + W^-_\mu \bar{d}'_L \gamma^\mu u'_L \right) \tag{1.16}$$

The generation indices are not shown explicitly. In the mass basis above equation becomes

$$\begin{aligned}\mathcal{L}^{CC,q} &= \frac{g}{\sqrt{2}}\left(W_\mu^+ \bar{u}_L V_L^{u\dagger} V_L^d \gamma^\mu d_L + W_\mu^- \bar{d}_L V_L^{d\dagger} V_L^u \gamma^\mu u_L\right) \\ &= \frac{g}{\sqrt{2}}\left(W_\mu^+ \bar{u}_L V \gamma^\mu d_L + W_\mu^- \bar{d}_L V^\dagger \gamma^\mu u_L\right) \qquad (1.17)\end{aligned}$$

Where $V = V_L^{u\dagger} V_L^d$ is 3×3 mixing matrix known as Cabibbo Kobayashi Maskawa (CKM) matrix [3, 4]. Its elements are denoted as

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad (1.18)$$

1.2 Parameterization of CKM matrix

In SM Yukawa matrices are complex. Therefore V becomes complex and leads to CP violation in the charged current interactions of quarks. CKM matrix is a 3×3 unitary matrix. Hence it can be parameterized by nine parameters. Since quarks can be rephased without any physical consequence, five phases of V can be absorbed. Hence V can be parameterized by four parameters.

1.2.1 Chau-Keung Parameterization

A 3×3 orthogonal matrix can be parameterized by three angles. A unitary matrix is complex expansion of an orthogonal matrix. Hence, of the four parameters of V, three parameters can be identified as three angles [5]. The fourth parameter is identified as phase. This phase is a physical one and can not be rotated away. Particle Data Group (PDG) [6] advocates the use of Chau-Keung parametrization [7] which is given as

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \qquad (1.19)$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ with $i, j = 1, 2, 3$. The angles θ_{ij} may be chosen to lie in the range $[0, \frac{\pi}{2}]$. δ is the CP phase. This phase is responsible for all CP violating phenomena in flavor changing processes of quarks in SM.

1.2.2 Wolfenstein parameterization

It is known from the experimental results that $s_{13} << s_{23} << s_{12} << 1$ [6]. This becomes evident in the widely used Wolfenstein parameterization [8]. This parameterization can be obtained using following definitions [5, 6].

$$\begin{aligned} s_{12} &= \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}} \\ s_{23} &= A\lambda^2 = \lambda \left| \frac{V_{cb}}{V_{us}} \right| \\ s_{13}e^{i\delta} &= V_{ub}^* = A\lambda^3(\rho + i\eta) \\ &= \frac{A\lambda^3(\bar{\rho} + i\bar{\eta})\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]} \end{aligned} \tag{1.20}$$

When we put values from above equation in the exact parameterization given by eq.(1.19) we get exact parameterization in terms of λ, A, ρ and η [5]. PDG quotes the value $\lambda = 0.2253 \pm 0.0007$ [6]. Neglecting the terms of the $O(\lambda^4)$ in the exact parameterization we obtain the Wolfenstein parameterization

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4). \tag{1.21}$$

In this parameterization it is easy to see the hierarchical structure of CKM matrix. From eq.(1.19) and eq.(1.20), we can derive the following relation

$$\bar{\rho} + i\bar{\eta} = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*) \tag{1.22}$$

It can be shown that the quantity $\bar{\rho} + i\bar{\eta}$ is phase convention independent. Approximate formula for $\bar{\rho}$ and $\bar{\eta}$ are given as [6]

$$\begin{aligned} \bar{\rho} &= \rho(1 - \lambda^2/2 + \ldots) \\ \bar{\eta} &= \eta(1 - \lambda^2/2 + \ldots) \end{aligned} \tag{1.23}$$

From the eq.(1.20) we get the following relation for ρ and η

$$\begin{aligned} \rho &= \frac{s_{13}}{s_{12}s_{23}} \cos\delta \\ \eta &= \frac{s_{13}}{s_{12}s_{23}} \sin\delta. \end{aligned} \tag{1.24}$$

Once parameters of CKM matrix are determined from experiments, they can be used for predictions of different processes in SM.

1.3 Establishment of CKM picture of CP violation

From the eq.(1.18) and using the unitarity of CKM matrix we can write the following relations

$$\begin{aligned} V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* &= 0 \\ V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* &= 0 \\ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* &= 0 \end{aligned} \tag{1.25}$$

Since the CKM elements are complex numbers, these relations show that the sum of three complex numbers of each unitarity relation is zero. These relations can be represented by triangles in the complex plane. The triangle based on last relation is known as unitarity triangle(UT). Left panel in figure(1.1) shows the last relation of eq.(1.25) in the form of triangle in complex plane.

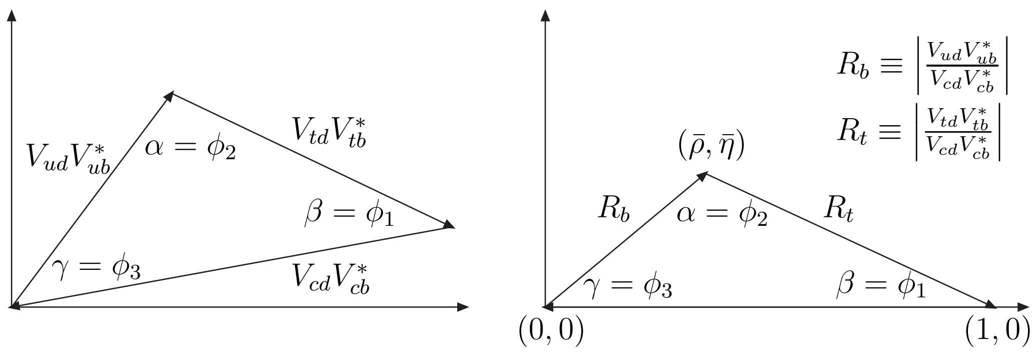

Figure 1.1: Left panel shows the triangle based on the last relation of the unitarity relations. Right panel shows the unitarity triangle obtained by dividing all the sides of triangle in the left panel by $V_{cd}V_{cb}^*$.

Dividing all the sides of the triangle in the left panel of figure(1.1) by $V_{cd}V_{cb}^*$, we obtain the triangle shown in the right panel which is referred as the unitarity triangle [6]. In this triangle one side is real and of unit magnitude. Also from eq.(1.22) we get

$$\begin{aligned} \bar{\rho} + i\bar{\eta} &= -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*) \\ \Longrightarrow \sqrt{\bar{\rho}^2 + \bar{\eta}^2} &= |(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)| \end{aligned} \tag{1.26}$$

Hence coordinates of the one of the vortex becomes $(\bar{\rho}, \bar{\eta})$ and lengths of the two complex sides are given as

$$\begin{aligned} R_b &= \sqrt{\bar{\rho}^2 + \bar{\eta}^2} \\ R_t &= \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} \end{aligned} \quad (1.27)$$

Two different notation α, β, γ and ϕ_1, ϕ_2, ϕ_3 are used to denote the angles of unitarity triangle which are given as [6]

$$\begin{aligned} \alpha &\equiv \phi_2 \equiv arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right] \\ \beta &\equiv \phi_1 \equiv arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right] \\ \gamma &\equiv \phi_3 \equiv arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right] \end{aligned} \quad (1.28)$$

If CKM matrix is real then the unitarity triangle collapses to a line and all the angles vanishes. Conversely, if any of the angle is non-zero then CKM matrix is complex. In absence of any new physics, CKM matrix is required to be complex as it is the only source of CP violation. However, any new physics (NP) that may be present can provide new sources of CP violation. In that case a real CKM matrix may be allowed. There are two ways to establish that CKM matrix is indeed complex : (A) From the determination of the observables in which NP contribution to CP violation is known to be small. (B) Allow for arbitrary new physics contribution to various CP violating observables and see if the fit to these observables require non-zero CKM phase. Presence of non-zero CP violating phase in CKM matrix is established using both of these methods. We discuss them in turn.

1.3.1 Measurement of angle γ of unitarity triangle

Many of the CP violating observables obtain their contributions at 1-loop level in the SM, e.g. from box diagrams or penguin diagrams. Even a small new physics contribution to these processes can be comparable to SM contribution. Hence any such process by itself can not be used to establish the existence of non-zero CKM phase. In contrast, if in some process CP violation arises from the interference between two tree level diagrams then the possible NP contribution can be neglected.

This happens in the decay chain $B^{\mp} \to \tilde{D}^{(*)0}K^{\mp}$ with $\tilde{D}^{(*)0} \to \tilde{D}^0\pi^0, \tilde{D}^0\gamma$ and $\tilde{D}^0 \to K_s^0\pi^+\pi^-$ which has been used to make relatively clean determination of angle γ of unitarity triangle.

The angle γ is measured through Dalitz plot analysis of decays of the neutral D mesons to $K_s^0\pi^+\pi^-$ observed by BABAR detector at the Stanford linear accelerator center PEP-II e^+e^- asymmetric-energy storage ring [9] and by Belle detector at KEK [10]. The center of mass energy of the e^+e^- collision was tuned to the $\Upsilon(4S)$ resonance peak, which is just above the threshold for decay into two B mesons. The method [11] based on $B^{\mp} \to \tilde{D}^{(*)0}K^{(*)\mp}$ where $\tilde{D}^{(*)0}$ represents $D^{(*)0}$ or $\bar{D}^{(*)0}$ and $D^{(*)0}$ represents D^0 or D^{0*} meson is theoretically clean because main contributions come from the tree level diagrams. Hence they are expected to remain unaffected by new physics. The diagrams contributing to this decay chain are shown on the next page.

The ratio of decay amplitudes for $B^- \to \bar{D}^{(*)0}K^{(*)-}$ to $B^- \to D^{(*)0}K^{(*)-}$ is defined as [9]

$$\frac{B^- \to \bar{D}^{(*)0}K^{(*)-}}{B^- \to D^{(*)0}K^{(*)-}} = r_B^{(*)}e^{i(\delta_B^{(*)}-\gamma)} \tag{1.29}$$

where $\delta_B^{(*)}$ is the relative strong phase. The neutral D meson is reconstructed in three body final states such as $K_S^0\pi^+\pi^-$. The final state $K_S^0\pi^+\pi^-$ can come from decay of D^0 or $\bar{D}^0$. The amplitude $A_{\mp}$ for decay chain $B^{\mp} \to \tilde{D}^{(*)0}K^{\mp}$ with $\tilde{D}^{(*)0} \to \tilde{D}^0\pi^0, \tilde{D}^0\gamma$ and $\tilde{D}^0 \to K_s^0\pi^+\pi^-$ is given as

$$A_{\mp}^{(*)} \propto A_{D\mp} + \lambda r_B^{(*)}e^{i(\delta_B^{(*)}\mp\gamma)}A_{D\pm} \tag{1.30}$$

where $A_{D-}(A_{D+})$ is amplitude for $D^0 \to K_s^0\pi^+\pi^-(\bar{D}^0 \to K_s^0\pi^-\pi^+)$. The factor λ is -1 for $B^{\mp} \to \tilde{D}^{(*)0}[\tilde{D}^0\gamma]K^{\mp}$ decay and 1 for the rest of the B decays.

The decay rate can be written as

$$\Gamma_{\mp}^{(*)} \propto |A_{D\mp}|^2 + r_B^{(*)2}|A_{D\pm}|^2 + 2\lambda[x_{\mp}^{(*)}Re(A_{D\mp}A_{D\pm}^*) + y_{\mp}^{(*)}Im(A_{D\mp}A_{D\pm}^*)] \tag{1.31}$$

where $x_{\mp}^{(*)} = r_B^{(*)}\cos(\delta_B^{(*)} \mp \gamma)$ and $y_{\mp}^{(*)} = r_B^{(*)}\sin(\delta_B^{(*)} \mp \gamma)$. For the decays $B^{\mp} \to \tilde{D}K^{(*)\mp}$ with $K^{(*)\mp} \to K^0\pi^{\mp}$, the decay rate is given as

$$\Gamma_{s\mp} \propto |A_{D\mp}|^2 + r_s|A_{D\pm}|^2 + 2\lambda[x_{s\mp}Re(A_{D\mp}A_{D\pm}^*) + y_{s\mp}Im(A_{D\mp}A_{D\pm}^*)] \tag{1.32}$$

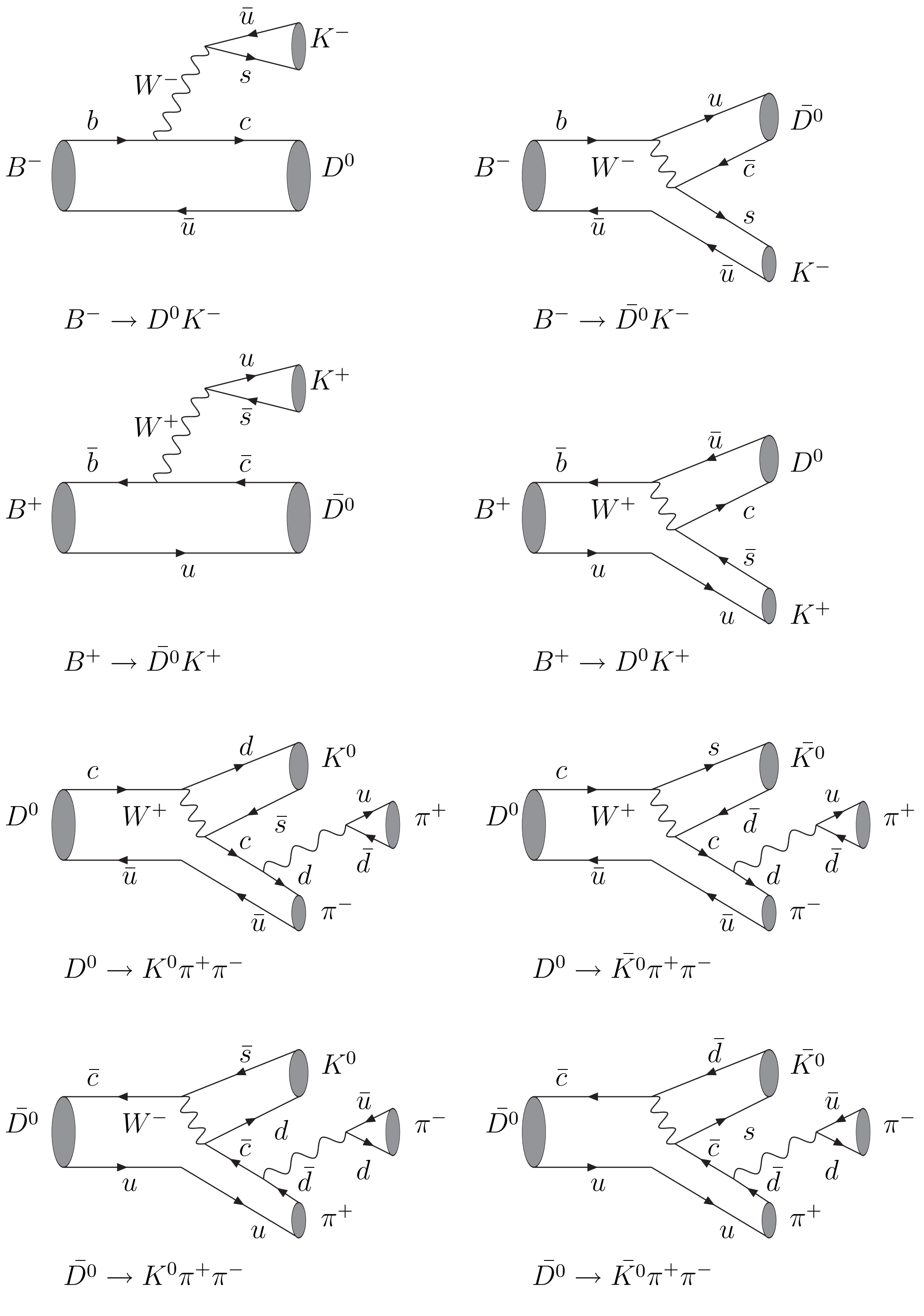

Figure 1.2: Tree level diagrams contributing to the decay chain $B^\mp \to \tilde{D}^{(*)0} K^\mp$ with $\tilde{D}^{(*)0} \to \tilde{D}^0\pi^0, \tilde{D}^0\gamma$ and $\tilde{D}^0 \to K_s^0\pi^+\pi^-$

where $x_{s\mp} = \kappa r_s \cos(\delta_s \mp \gamma)$ and $y_{s\mp} = \kappa r_s \sin(\delta_s \mp \gamma)$. Similarly for $B^{\mp} \to \tilde{D}K^{\mp}$ the decay rate can be obtained from expression of $\Gamma_{s\mp}$ by replacing r_s by $r_{B\mp}$ and $x_{s\mp}, y_{s\mp}$ by $x_{\mp}, y_{\mp}$.

The parameters $x_{\mp}^{(*)}$, $y_{\mp}^{(*)}$, $x_{s\mp}$, $y_{s\mp}$, $x_{\mp}$, $y_{\mp}$ are obtained from the analysis of the decay data and used to calculate the physically relevant quantities γ, r_B, $r_B^{(*)}$, κr_s, δ_B, $\delta_B^{(*)}$, δ_s. The value obtained by BABAR collaboration is $\gamma = 76° \pm 22°(\text{stat}) \pm 5°(\text{syst}) \pm 5°(\text{model})$ [9]. Belle collaboration obtained $\phi_3 \equiv \gamma = 53°^{+15°}_{-18°}(\text{stat}) \pm 3°(\text{syst}) \pm 9°(\text{model})$ [10]. Since the processes used in these analysis get main contributions from the tree level diagrams, they are expected to remain unaffected of the presence of new physics. Therefore this non-zero value of γ can be considered as an evidence for complex CKM matrix [12].

1.3.2 Determination of CKM parameters in presence of new physics

UTfit group has determined CKM parameters from the NP generalized fit to experimental data assuming the presence of arbitrary new physics contribution to K^0-$\bar{K}^0$, B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing. Their method is described in [13]. An outline of their method is given below.

Let $x_i(A, B_K, f_{B_d}, \ \ldots)$ be N parameters and $c_i(\Delta m_i, \epsilon_K, \ \ldots)$ be M constraints whose actual values depends upon parameters x_i and on CKM parameters $(\bar{\rho}, \bar{\eta})$. According to Bays theorem best determination of $(\bar{\rho}, \bar{\eta})$ is given by

$$\begin{aligned} f(\bar{\rho}, \bar{\eta}, x_1, x_2, \ \ldots, x_N | c_1, c_2, \ \ldots, c_M) \ &= \ \prod_{j=1,M} f_j(c_j | \bar{\rho}, \bar{\eta}, x_1, x_2, \ \ldots, x_N) \\ &\times \prod_{i=1,N} f_i(x_i) f_0(\bar{\rho}, \bar{\eta}) \end{aligned} \tag{1.33}$$

where f is probability distribution function(p.d.f) and f_0 is the a-priory probability for $(\bar{\rho}, \bar{\eta})$. The output p.d.f. for $(\bar{\rho}, \bar{\eta})$ is obtained by integrating over the parameters space:

$$f(\bar{\rho}, \bar{\eta}) \propto \int \prod_{j=1,M} f_j(c_j | \bar{\rho}, \bar{\eta}, x_1, x_2, \ \ldots, x_N) \prod_{i=1,N} f_i(x_i) f_0(\bar{\rho}, \bar{\eta}) \tag{1.34}$$

UTfit group has used constraints on following quantities [14]. UT angles : α, β, γ, CKM elements : $|V_{ub}/V_{cb}|$, mass difference in B_d and B_s systems : $\Delta m_d, \Delta m_s$, CP violating parameter in K meson system : ϵ_k, semi leptonic CP asymmetry A_{SL}, CP asymmetry in dimuon events : A_{CH}, and ratio of decay width difference to decay width in B_q^0-$\bar{B}_q^0$ mixing: $\Delta\Gamma_q/\Gamma_q$ where $q = d, s$. NP contribution to B_q^0-$\bar{B}_q^0$ mixing is parameterized as

$$C_{B_q} e^{2i\phi_{B_q}} = \frac{\langle B_q | H_{\rm eff}^{\rm full} | \bar{B}_q \rangle}{\langle B_q | H_{\rm eff}^{\rm SM} | \bar{B}_q \rangle} = 1 + \frac{A_q^{\rm NP}}{A_q^{\rm SM}} e^{2i\phi_q^{\rm NP}} \tag{1.35}$$

where $H_{\rm eff}^{\rm full} = H_{\rm eff}^{\rm SM} + H_{\rm eff}^{\rm NP}$. In SM expectation values of parameters C_{B_q} and ϕ_{B_q} are 1 and 0 respectively. New physics in K^0-$\bar{K}^0$ mixing is parameterized as

$$C_{\epsilon_K} = \frac{Im\langle K^0 | H_{\rm eff}^{\rm full} | \bar{K}^0 \rangle}{Im\langle K^0 | H_{\rm eff}^{\rm SM} | \bar{K}^0 \rangle} \tag{1.36}$$

Using the method and constraints described above, UTfit group have performed generalized fit in presence of NP and obtained following results for CKM parameters $\bar{\rho}, \bar{\eta}$ [14].

$$\begin{aligned} \bar{\rho} &= 0.20 \pm 0.06 \\ \bar{\eta} &= 0.36 \pm 0.04 \end{aligned} \tag{1.37}$$

These results shows that $\bar{\rho}, \bar{\eta}$ are non-zero while allowing for arbitrary NP contribution. Hence quantity $\bar{\rho} + i\bar{\eta} = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$ is complex. This is an evidence for complex CKM matrix.

From nonzero value of UT angle γ determined from the tree level decays of the type $B \to DK$ and nonzero values of CKM parameters $\bar{\rho}, \bar{\eta}$ determined in presence of arbitrary NP we can conclude that CKM matrix is complex even in presence of new physics. In addition, predictions based on the CKM matrix are in agreement with measurements of many CP violating observables [6]. Thus it can be said that CKM matrix provides the dominant source of CP violation.

1.4 Hints of physics beyond SM

Many of the SM predictions are in good agreement with the experimental results. However there are some experimental results which differ from the SM prediction and are considered as hints of physics beyond SM. We consider here two of them in the following processes.

1. B_d^0-$\bar{B}_d^0$ mixing

2. B_s^0-$\bar{B}_s^0$ mixing.

In the next section we describe formalism of the B_q^0-$\bar{B}_q^0$ mixing where $q = d, s$. Then we will describe hints of NP in the B_d^0-$\bar{B}_d^0$ mixing and B_s^0-$\bar{B}_s^0$ mixing.

1.4.1 Formalism for B_q^0-$\bar{B}_q^0$ mixing

B_q^0-$\bar{B}_q^0$ mixing refers to the transition from $B_q^0 \equiv \bar{b}q$ to $\bar{B}_q^0 \equiv b\bar{q}$; $q = d, s$. The time evolution of a beam of B_q^0 and $\bar{B_q}^0$ in Wigner-Weisskopf approximation is given as [5]

$$i\frac{d}{dt}\begin{pmatrix} |B_q^0(t)\rangle \\ |\bar{B}_q^0(t)\rangle \end{pmatrix} = \left(M_q - \frac{i}{2}\Gamma_q\right)\begin{pmatrix} |B_q^0(t)\rangle \\ |\bar{B}_q^0(t)\rangle \end{pmatrix}$$

where

$$\begin{aligned} M_q &= \begin{pmatrix} M_{q11} & M_{q12} \\ M_{q21} & M_{q22} \end{pmatrix} \\ \Gamma_q &= \begin{pmatrix} \Gamma_{q11} & \Gamma_{q12} \\ \Gamma_{q21} & \Gamma_{q22} \end{pmatrix} \end{aligned} \tag{1.38}$$

The physical eigen states $|B_{qH}^0\rangle$ and $|B_{qL}^0\rangle$ with the masses m_{qH}, m_{qL} and the decay widths Γ_{qH}, Γ_{qL} are obtained by diagonalizing $M_q - i\Gamma_q/2$. In the CPT invariant case mass eigen states are given as

$$|B_{qH}^0\rangle = p|B_q^0\rangle + q|\bar{B_q}^0\rangle \tag{1.39}$$

$$|B_{qL}^0\rangle = p|B_q^0\rangle - q|\bar{B_q}^0\rangle \tag{1.40}$$

Where $|p|^2 + |q|^2 = 1$. Time evolution of the mass eigen states is given as

$$|B_{qH,L}^0(t)\rangle = e^{-i(m_{qH,L} - i\Gamma_{qH,L}/2)t}\,|B_{qH,L}^0\rangle \tag{1.41}$$

Where $|B_{qH,L}^0\rangle$ denotes mass eigen state at time t = 0. Average mass and decay width are defined as

$$m_q = \frac{m_{qH} + m_{qL}}{2} = M_{q11} \tag{1.42}$$

$$\Gamma_q = \frac{\Gamma_{qH} + \Gamma_{qL}}{2} = \Gamma_{q11} \tag{1.43}$$

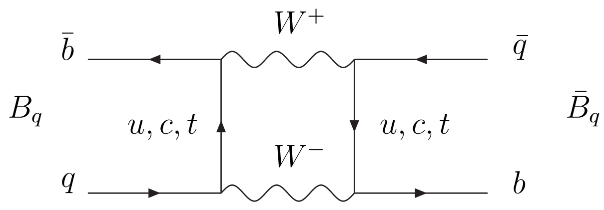

Figure 1.3: Box diagram for $B_q^0 - \bar{B}_q^0$ mixing in SM

Mass difference and decay width difference are defined as

$$\Delta m_q = m_{qH} - m_{qL} \tag{1.44}$$

$$\Delta\Gamma_q = \Gamma_{qH} - \Gamma_{qL} \tag{1.45}$$

Relative phase between M_{q12} and Γ_{q12} enters in many experimental observables. It is defined as

$$\phi_q = Arg\left(-\frac{M_{q12}}{\Gamma_{q12}}\right) \tag{1.46}$$

In SM B_q^0-$\bar{B}_q^0$ mixing occurs through the box diagram shown in the fig.(1.3).

SM matrix element for B_q^0-$\bar{B}_q^0$ mixing is given as [15, 16]

$$\Delta m_q = 2|M_{q12}^{SM}| = \frac{G_F^2 m_W^2 \eta_B m_{B_q} B_q f_{B_q}^2}{6\pi^2} S_0(m_t^2/m_w^2)|V_{tb}^* V_{tq}|^2 \,, \tag{1.47}$$

Where G_F is Fermi constant, m_W and m_{B_q} are the masses of W boson and B_q^0 meson. The Inami-Lim function[17] $S_0(m_t^2/m_W^2) \approx 2.35$ [18] for $m_t \sim 165$ GeV. $\eta_B \approx 0.55$ refers to the QCD correction to the Wilson operator in the SM. f_{B_q}, B_{B_q} are the decay constant and the bag parameter respectively. V_{tb} and V_{tq} are CKM matrix elements.

1.4.2 Hint of NP in B_d^0-$\bar{B}_d^0$ mixing

CP violating observable $\sin 2\beta$ of B_d^0-$\bar{B}_d^0$ mixing is measured in time dependent CP asymmetry of $B_d^0 \to J/\psi K_s$ by BABAR experiment at the Stanford linear accelerator center (SLAC) and by Belle experiment at KEK. BABAR experiment collected data at PEP-II asymmetric energy e^+e^- storage ring at SLAC for collision of 3.1 GeV positrons with 9.0 GeV electrons [19]. Belle experiment collected data at

KEK asymmetric energy collider for the collision of 3.5 GeV positron with 8.0 GeV electrons [20]. The e^+e^- collision results in the resonance $\Upsilon(4s)$ which decays in to two B mesons. Since the two mesons are produced together, they are correlated. The initial flavor of the reconstructed meson B_{rec} is determined by tagging the other meson B_{tag}. The decay rate $g_+(g_-)$ for $B^0_d(\bar{B}^0_d)$ meson, whose initial flavor is known from tagging, to decay into one of the CP eigen state is given as [19]

$$g_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}((1 \mp \Delta w) \pm (1-2w)[S_f \sin(\Delta m_d \Delta t) - C_f \cos(\Delta m_d \Delta t)]) \tag{1.48}$$

Where $S_f = \frac{2Im(\lambda_f)}{1+|\lambda_f|^2}$, and $C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}$. $\lambda_f = (q/p)(\bar{A}_f/A_f)$ where q, p are defined by the eq.(1.39). $\Delta t \equiv t_{rec} - t_{tag}$ is the difference between the proper decay times of B_{Rec} and B_{tag}. Δm_d is the mass difference between heavy and light mass eigen states of B_d meson. τ_B is the average lifetime. Δw is the difference between mistag probabilities for B^0 and $\bar{B}^0$. The time dependent CP violating asymmetry is given as

$$A_{CP}(\Delta t) = \frac{g_+(\Delta t) - g_-(\Delta t)}{g_+(\Delta t) + g_-(\Delta t)} = (1-2w)S_f \sin(\Delta M_d \Delta t) \tag{1.49}$$

Where $S_f = -\eta_f \sin 2\beta$ and η_f is +1 for CP even and -1 for CP odd final state. BABAR collaboration obtained following results [19].

$$\begin{aligned} -\eta_f S_f &= 0.687 \pm 0.028(\text{stat}) \pm 0.012(\text{syst}) \\ c_f &= 0.024 \pm 0.020(\text{stat}) \pm 0.016(\text{syst}) \end{aligned} \tag{1.50}$$

Since state $f = J/\psi K_s$ is CP odd state, $\eta_{J/\psi K_s} = -1$. Therefore BABAR results for $S_{J/\psi K_s}$ is

$$S_{J/\psi K_s} \equiv \sin 2\beta = 0.687 \pm 0.028(\text{stat}) \pm 0.012(\text{syst}) \tag{1.51}$$

Belle collaboration obtained value [20]

$$S_{J/\psi K_s} \equiv \sin 2\beta = 0.642 \pm 0.031(\text{stat}) \pm 0.017(\text{syst}) \tag{1.52}$$

SM prediction for $\sin 2\beta$ obtained using the constraints from $\frac{|V_{ub}|}{|V_{cb}|}$, ϵ_K, Δm_{B_s}, and Δm_{B_d} is [31]

$$\sin 2\beta = 0.78 \pm 0.04 \tag{1.53}$$

This value deviates from the value of $\sin 2\beta$ obtained by BABAR and Belle collaboration by about two standard deviation. The value of V_{ub} used to obtain the SM prediction in eq.(1.53) has considerable uncertainty as the value of V_{ub} determined from exclusive processes differs from the value determined from inclusive measurements. The discrepancy between the direct determination and SM prediction of $\sin 2\beta$ may be affected by the difference between the inclusive and exclusive determination of V_{ub}. SM prediction of $\sin 2\beta$ without using V_{ub} is given in [32]. The value of $\sin 2\beta$ obtained in this analysis is

$$\sin 2\beta = 0.87 \pm 0.09 \tag{1.54}$$

This value differs from the direct measurement of $\sin 2\beta$ by about two standard deviation. This deviation can be considered as hint of new physics.

1.4.3 Hint of NP in B_s^0-$\bar{B}_s^0$ mixing

CP violating phase ϕ_s of B_s^0-$\bar{B}_s^0$ mixing is obtained from analysis of time dependent angular distribution of decay products in flavor tagged decay $B_s^0 \to J/\psi\phi$ by CDF and D0 experiments at Fermilab Tevatron collider through decay chain $B_s^0 \to J/\psi\phi$, $J/\psi \to \mu^+\mu^-$, $\phi \to K^+K^-$[21, 22]. At Tevatron b quarks are produced in pairs from which B_s^0-$\bar{B}_s^0$ pair is produced. Flavor of B_s^0 ($\bar{B}_s{}^0$) is identified from the decay products of associated $\bar{B}_s{}^0$ (B_s^0) meson which is called opposite side tagging. Flavor of reconstructed B_s^0 or $\bar{B}_s{}^0$ can also be identified from the charge of associated K_s meson. This is referred as same side kaon tagging. Decay amplitude for decay of B_s^0 and $\bar{B}_s{}^0$ mesons can be decomposed into three independent components according to linear polarization states of the vector meson J/ψ and ϕ [23, 24]. These components are defined as follows.

1. J/ψ and ϕ are either longitudinal or transverse to the direction of motion : A_0

2. J/ψ and ϕ are parallel to each other : $A_{||}$

3. J/ψ and ϕ are perpendicular to each other $A_\perp$

These amplitudes depend upon the phase ϕ_s. The phase ϕ_s is determined from the fit to time-dependent angular distribution of the decay products which can be expressed in terms of A_0, $A_{||}$, $A_\perp$ and relative strong phases. CDF collaboration reports 68% confidence limit interval as $-2.82 < \phi_s < -0.32$ [21]. D0 collaboration obtained $\phi_s = -0.57^{+0.24}_{-0.30}(\text{stat})^{+0.08}_{-0.02}(\text{syst})$ [22] and 90% confidence limit interval is $-1.20 < \phi_s < 0.06$.

SM prediction for ϕ_s is [25, 26]

$$\phi_s = 2\,\arg[-(V_{tb}V_{ts}^*)/(V_{cb}V_{cs}^*)] = -0.038 \pm 0.002 \tag{1.55}$$

This does not agree with the value obtained by the CDF and D0 collaborations.

UTfit group has combined all the available constraints on B_s mixing and performed a model independent analysis of NP contribution to B_s^0-$\bar{B}_s^0$ mixing. Their method is described in previous section and in [13]. NP in B_s mixing is parameterized as

$$C_{B_s} e^{2i\phi_{B_s}} = \frac{\langle B_s|H_{\text{eff}}^{\text{full}}|\bar{B}_s\rangle}{\langle B_s|H_{\text{eff}}^{\text{SM}}|\bar{B}_s\rangle} = \frac{A_S^{\text{SM}} e^{-2i\beta_s} + A_S^{\text{NP}} e^{2i(\phi_s^{\text{NP}}-\beta_s)}}{A_S^{\text{SM}} e^{-2i\beta_s}} \tag{1.56}$$

where $H_{\text{eff}}^{\text{full}} = H_{\text{eff}}^{\text{SM}} + H_{\text{eff}}^{\text{NP}}$. In SM, $\beta_s = \arg\left(-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)\right) \approx 0.018 \pm 0.001$. Expectation values of parameters C_{B_s} and ϕ_{B_s} in SM are 1 and 0 respectively. In this analysis, following inputs are used in the B_s system [27].

Mass difference Δm_s, semi-leptonic asymmetry in B_s decays A_{SL}, the dimuon charge asymmetry $A_{SL}^{\mu\mu}$, measurement of B_s lifetime from flavor specific final states, two dimensional likelihood ratio $\Delta\Gamma_s$, ϕ_s and correlated constraints on Γ_s, $\Delta\Gamma_s$ and ϕ_s. All other experimental inputs are given in [28]. Following results for NP parameters in B_s^0-$\bar{B}_s^0$ mixing are obtained.

$$\begin{aligned}
\phi_{B_s} &= -19.9^\circ \pm 5.6^\circ \text{ (68\% Prob.) and } [-30.45^\circ, -9.29^\circ] \text{ (95\% Prob.)}, \\
&\quad -68.2^\circ \pm 4.9^\circ \text{ (68\% Prob.) and } [-78.45^\circ, -58.2^\circ] \text{ (95\% Prob.)} \\
C_{B_s} &= 1.07 \pm 0.29 \text{ (68\% Prob.) and } [0.61, 1.93] \text{ (95\% Prob.)}
\end{aligned} \tag{1.57}$$

This value of ϕ_{B_s} deviates from zero by about 3.7σ. From this analysis UTfit group has concluded that phase ϕ_s of B_s^0-$\bar{B}_s^0$ mixing deviates from the SM prediction by about 3σ [27, 29]. A similar analysis performed by CKMfitter group shows that the deviation from SM prediction is about 2.5σ [30].

Inconsistencies between SM prediction and the experimental determination in B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing discussed here can be resolved if there is some new physics beyond SM which can give extra contribution to above mentioned CP violating observables. Several extensions of SM have been suggested for this purpose which include (a) models with fourth generation, (b) model with extra vector-singlet quarks and (c) supersymmetric models. In the model with additional generation of leptons and quarks the CKM matrix becomes a 4×4, which can be parameterized in terms of six angles and three phases. In this model the box diagrams with fourth generation 2/3 quark t' can give additional contributions to B_q^0-$\bar{B}_q^0$ mixings [33, 34, 35]. In the model with extra $-1/3$ charge quark, there are FCNC mediated by Z bosons which can give extra contributions to B_q^0-$\bar{B}_q^0$ mixings [36, 37, 38]. In supersymmetric models [39, 40, 41], there are many new contributions to the above mentioned processes such as the charged Higgs and charge 2/3 quarks contributions, chargino and charge 2/3 squarks contributions, gluinos and charge $-1/3$ squarks contributions etc.

1.5 Motivation and Outline of the thesis

As discussed in the previous section, experimental determination of CP violating phases in B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing shows substantial deviation from their values predicted in SM. These are considered as hints of new physics beyond SM. Here we explore the possibility of explaining these deviations in two Higgs doublet model (2HDM). It is a simplest extension of the SM having several new sources of CP violation. In this thesis we study all sources of CP violation in the 2HDM with a view to explain deviations in the CP violating observables in the B_d and B_S system.

This thesis will be organized as follows. In chapter 2, a brief review of 2HDM will be presented. In chapter 3 study of 2HDM in which the FCNC display minimal flavor violation will be given. Chapter 4 will describe analysis of 2HDM with suppressed FCNC. Chapter 5 will present study of a 2HDM without FCNC in which new CP violating phases can come from charged Higgs contributions. In Chapter 6 analysis of 2HDM with general FCNC will be given. Chapter 7 will summarize this thesis.

Chapter 2

2HDM

We introduce two Higgs doublet model (2HDM) in this chapter and summarize salient features of the model. 2HDM is considered in most generality and two popular special cases are introduced. Subsequent chapters deal with phenomenological implications of other theoretically different special cases of the most general model developed here.

2.1 Scalar Lagrangian

2HDM is based on the same gauge group as of SM i.e $G_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$. Fermion sector is also same as that of SM but now we have two Higgs doublets $\phi_1 \equiv \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}$ and $\phi_2 \equiv \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$ with the hypercharge 1/2. Scalar Lagrangian is given as

$$\mathcal{L}_S = (D_\mu \phi_1)^\dagger (D^\mu \phi_1) + (D_\mu \phi_2)^\dagger (D^\mu \phi_2) - V(\phi) \tag{2.1}$$

Scalar potential becomes [5, 42]

$$\begin{aligned} V(\phi) \;=\; & m_1 \phi_1^\dagger \phi_1 + m_2 \phi_2^\dagger \phi_2 + m_3 (e^{i\delta_3} \phi_1^\dagger \phi_2 + e^{-i\delta_3} \phi_2^\dagger \phi_1) \\ & + a_1 (\phi_1^\dagger \phi_1)^2 + a_2 (\phi_2^\dagger \phi_2)^2 + a_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + a_4 (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \\ & + a_5 (e^{i\delta_5} (\phi_1^\dagger \phi_2)^2 + e^{-i\delta_5} (\phi_2^\dagger \phi_1)^2) + a_6 (\phi_1^\dagger \phi_1)(e^{i\delta_6} \phi_1^\dagger \phi_2 + e^{-i\delta_6} \phi_2^\dagger \phi_1) \\ & + a_7 (\phi_2^\dagger \phi_2)(e^{i\delta_7} \phi_1^\dagger \phi_2 + e^{-i\delta_7} \phi_2^\dagger \phi_1) \end{aligned} \tag{2.2}$$

The coupling constants $m_i (i = 1, 2, 3)$ and $a_j (j = 1, 2, ...7)$ are real and all the phases are shown explicitly. The vacuum expectation values (vevs) are given as

$\langle 0|\phi_1|0\rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$, $\langle 0|\phi_2|0\rangle = \begin{pmatrix} 0 \\ v_2 e^{i\theta} \end{pmatrix}$. We use following definitions.

$$\begin{aligned} v_1 &= v\cos\beta \\ v_2 &= v\sin\beta \end{aligned} \tag{2.3}$$

Here $v = 174$ GeV is the vev of the neutral component of Higgs doublet in the SM. Using above equation we get $\sqrt{v_1^2 + v_2^2} = v$ and $\tan\beta = \frac{v_2}{v_1}$.

2.2 Coupling of scalars to fermions

Yukawa Lagrangian for quarks in weak basis of quarks is given as

$$-\mathcal{L}_{Yukawa} = \bar{Q}'_i(\Gamma^d_{1ij}\phi_1 + \Gamma^d_{2ij}\phi_2)d'_{jR} + \bar{Q}'_i(\Gamma^u_{1ij}\tilde{\phi_1} + \Gamma^u_{2ij}\tilde{\phi_2})u'_{jR} + H.C. \tag{2.4}$$

with $\tilde{\phi_k} = i\sigma_2\phi_k^*(k = 1, 2)$. Here $i, j = 1, 2, 3$ are generation indices and $\Gamma^{u,d}_{1,2}$ are 3×3 non diagonal Yukawa matrices. It is useful to work in a special basis of scalars called Higgs basis. Advantage with this basis is that neutral component of only one Higgs doublet gets vev which is real and positive while vev of other Higgs doublet is zero. We will denote Higgs doublets in this basis by ϕ and ϕ_H. In this basis Higgs doublets are defined as

$$\begin{pmatrix} \phi \\ \phi_H \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 e^{-i\theta} \end{pmatrix} \tag{2.5}$$

Scalar doublets in this basis are written as $\phi = \begin{pmatrix} G^+ \\ v + \frac{h_1 + iG^0}{\sqrt{2}} \end{pmatrix}$ and $\phi_H = \begin{pmatrix} H^+ \\ \frac{h_2 + ih_3}{\sqrt{2}} \end{pmatrix}$. G^+ and G^0 are would be Goldstone bosons. H^+ is charged Higgs. Three neutral scalars h_1, h_2 and h_3 mix to give mass eigen states h, H and A as follows.

$$\begin{pmatrix} h \\ H \\ A \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} \tag{2.6}$$

In the Higgs basis the equation(2.4) becomes

$$
\begin{aligned}
-\mathcal{L}_{Yukawa} &= \bar{d'}_{iL}(\Gamma^d_{1ij}\cos\beta + \Gamma^d_{2ij}\sin\beta\ e^{i\theta})\phi^0 d'_{jR} \\
&+\bar{u'}_{iL}(\Gamma^u_{1ij}\cos\beta + \Gamma^u_{2ij}\sin\beta\ e^{-i\theta})\phi^{0*} u'_{jR} \\
&+\bar{u'}_{iL}(\Gamma^d_{1ij}\cos\beta + \Gamma^d_{2ij}\sin\beta\ e^{i\theta})\phi^{+} d'_{jR} \\
&-\bar{d'}_{i}(\Gamma^u_{1ij}\cos\beta + \Gamma^u_{2ij}\sin\beta\ e^{-i\theta})\phi^{-} u'_{jR} \\
&+\bar{d'}_{iL}(-\Gamma^d_{1ij}\sin\beta + \Gamma^d_{2ij}\cos\beta\ e^{i\theta})\phi^0_H d'_{jR} \\
&+\bar{u'}_{iL}(-\Gamma^u_{1ij}\sin\beta + \Gamma^u_{2ij}\cos\beta\ e^{-i\theta})\phi^{0*}_H u'_{jR} \\
&+\bar{u'}_{iL}(-\Gamma^d_{1ij}\sin\beta + \Gamma^d_{2ij}\cos\beta\ e^{-i\theta})H^{+} d'_{jR} \\
&-\bar{d'}_{iL}(-\Gamma^u_{1ij}\sin\beta + \Gamma^u_{2ij}\cos\beta\ e^{i\theta})H^{-} u'_{jR} + H.C. \\
&= \frac{1}{v}(\bar{d'}_L M^d \phi^0 d'_R + \bar{u'}_L M^u \phi^{0*} u'_R) \\
&+\frac{1}{v}(\bar{u'}_L M^d \phi^{+} d'_R - \bar{d'}_{iL} M^u \phi^{-} u'_R) \\
&+\frac{1}{v}(\bar{d'}_L F^{d'} \phi^0_H d'_R + \bar{u'}_L F^{u'} \phi^{0*}_H u'_R) \\
&+\frac{1}{v}(\bar{u'}_L F^{d'} H^{+} d'_R - \bar{d'}_L F^{u'} H^{-} u'_R) + H.C.
\end{aligned}
\tag{2.7}
$$

M^u, M^d are mass matrices of quarks. They are given as

$$
\begin{aligned}
M^u &= v\left[\Gamma^u_1\cos\beta + \Gamma^u_2\sin\beta\ e^{-i\theta}\right] \\
M^d &= v\left[\Gamma^d_1\cos\beta + \Gamma^d_2\sin\beta\ e^{i\theta}\right]
\end{aligned}
\tag{2.8}
$$

Matrices $F^{u'}, F^{d'}$ are 3×3 non-diagonal matrices given as

$$
\begin{aligned}
F^{u'} &= v\left[-\Gamma^u_1\sin\beta + \Gamma^u_2\cos\beta\ e^{-i\theta}\right] \\
F^{d'} &= v\left[-\Gamma^d_1\sin\beta + \Gamma^d_2\cos\beta\ e^{i\theta}\right]
\end{aligned}
\tag{2.9}
$$

Mass matrices can be diagonalized by going to mass eigen basis $f_{L,R}$ ($f = u, d$) by applying the following basis transformations.

$$f'_{L,R} = V^f_{L,R}\ f_{L,R} \tag{2.10}$$

In mass basis, fermion mass matrix is given as

$$D^f = V^{f\dagger}_L M^f V^f_R \tag{2.11}$$

Here D^f is diagonal matrix and the diagonal entries of this matrix are the corresponding fermion masses. An important feature of the 2HDM is flavor changing neutral currents (FCNCs). Part of Lagrangian describing FCNCs in mass basis is given as

$$-\mathcal{L}_{FCNC} = \frac{1}{v}(\bar{d}_L F^d \phi^0_H d_R + \bar{u}_L F^u \phi^{0*}_H u_R) \tag{2.12}$$

Matrices F^u, F^d are 3×3 non-diagonal matrices given as

$$\begin{aligned} F^u &= V_L^{u\dagger}\, v\left[-\Gamma_1^u \sin\beta + \Gamma_2^u \cos\beta\; e^{-i\theta}\right] V_R^u \\ F^d &= V_L^{d\dagger}\, v\left[-\Gamma_1^d \sin\beta + \Gamma_2^d \cos\beta\; e^{i\theta}\right] V_R^d \end{aligned} \tag{2.13}$$

Since F^d and F^u are 3×3 non-diagonal matrices, above equation contains terms which relates quarks of same charges through coupling to ϕ^0_H. Interactions represented by these terms are called flavor changing neutral current interactions.

2.3 2HDM with natural flavor conservation

Natural flavor conservation refers to absence of tree level FCNC. In general 2HDM, tree level FCNC interactions can give large contributions to the processes such as K^0-$\bar{K}^0$, B^0_d-$\bar{B}^0_d$ and B^0_s-$\bar{B}^0_s$ mixing. There are stringent constraints on the NP contributions to these processes. Hence FCNC are required to be suppressed or eliminated. To avoid FCNC a discrete symmetry is imposed in such a way that only one of the Higgs doublet gives mass to the quarks of given charge. [43, 44]. These models are called type-I or type-II 2HDM depending upon how the Higgs doublets couples to quarks.

2.3.1 Type - 1 2HDM

This model can be obtained by applying the discrete symmetry $\phi_2 \to -\phi_2$. In this case $\Gamma_2^{u,d} = 0$ and the Yukawa part of the total Lagrangian becomes

$$-\mathcal{L}_{Yukawa} = \bar{Q}'_i \Gamma^d_{1ij} \phi_1 d'_{jR} + \bar{Q}'_i \Gamma^u_{1ij} \tilde{\phi_1} u'_{jR} + H.C. \tag{2.14}$$

In this model quarks mass matrices are given as

$$\begin{aligned} M^u &= \Gamma^u_1 v_1 \\ M^d &= \Gamma^d_1 v_1 \end{aligned} \tag{2.15}$$

It can be easily seen that tree level FCNC are absent in this model.

2.3.2 Type - 2 2HDM

In this model action of symmetry is given as $\phi_2 \rightarrow -\phi_2, u_R \rightarrow -u_R$. In this case the Yukawa part of the total Lagrangian becomes

$$-\mathcal{L}_{Yukawa} = \bar{Q}'_i \Gamma^d_{1ij} \phi_1 d'_{jR} + \bar{Q}'_i \Gamma^u_{2ij} \tilde{\phi}_2 u'_{jR} + H.C. \tag{2.16}$$

In this model quarks mass matrices are given as

$$\begin{aligned} M^u &= \Gamma^u_2 v_2 \\ M^d &= \Gamma^d_1 v_1 \end{aligned} \tag{2.17}$$

From the above equation we can see that tree level FCNC are absent in this model.

2.4 Sources of CP violation in General 2HDM

CP violation can arise in the 2HDM in the following ways [45].

- Complex CKM matrix : CKM matrix can become complex in two ways (1) Explicit CP violation (2) Spontaneous CP violation. In the model with explicit CP violation Yukawa couplings are taken to be complex which leads to complex mass matrices and complex CKM matrix. In models with Spontaneous CP violation, the Yukawa couplings are real. Hence in this case Lagrangian is CP invariant. CP symmetry is broken by the vacuum of the theory i.e vev of the Higgs becomes complex. The phase of the complex vev enters mass matrices and makes them complex. Hence the CKM matrix becomes complex.

- Interaction involving exchange of the Higgs : The interactions in which exchange of Higgs takes places are proportional to the matrices F^u and F^d given by equation(2.13). F^u and F^d are in general complex and hence CP is violated in these interactions.

- Scalar-Pseudoscalar mixing: If CP is violated in the Higgs sector then the CP odd and CP even neutral Higgs mix with each other to give the mass eigen states. The neutral Higgs responsible for tree level FCNC can be written as

$$\phi^0_H = \frac{R + iI}{\sqrt{2}} \tag{2.18}$$

where $\frac{R}{\sqrt{2}}$ and $\frac{I}{\sqrt{2}}$ represents real and imaginary parts of ϕ_H^0 respectively. R and I can be written in terms of mass eigen states as

$$\begin{aligned} \frac{R}{\sqrt{2}} &= \sum_{\alpha=1}^{3} O_{R\alpha} H_\alpha \\ \frac{I}{\sqrt{2}} &= \sum_{\alpha=1}^{3} O_{I\alpha} H_\alpha \end{aligned} \tag{2.19}$$

Using above equation, ϕ_H^0 can be written as

$$\phi_H^0 = \sum_{\alpha=1}^{3} (O_{R\alpha} + iO_{I\alpha}) H_\alpha = \sum_{\alpha=1}^{3} C_\alpha H_\alpha \tag{2.20}$$

Here $C_\alpha; \alpha = 1, 2, 3$ are complex numbers. The phases of C_α can lead to interesting phenomenological consequences for CP violation.

Chapter 3

2HDM with minimal flavor violation

In SM all the flavor and CP violation arises in the charged current interactions. Neutral current interactions are flavor diagonal and do not give any contribution to CP violation. Hence all the flavor and CP violation are described by the CKM matrix. K and B meson decays and mixing have provided stringent tests of CKM mechanism and the SM predictions have been verified with some hints for possible new physics contributions [25, 46, 47, 48, 49, 50, 51, 52, 53]. Any new source of flavor violations resulting from the well-motivated extensions of the SM (*e. g.* supersymmetry) is now constrained to be small [14, 28, 54]. Several models of NP have the property that only source of CP and flavor violation are the SM Yukawa matrices. In other words all the flavor and CP violation in these models are described in terms of the CKM matrix even in presence of new interactions. This property is termed as minimal flavor violation (MFV)[55, 56, 57, 58, 59, 60, 61, 62].

First description of MFV in effective theory approach was given in [55]. We give essential points of this description here. Fermions in the SM are arranged in three families. Each family consists of two $SU(2)$ doublets: Q_L, L_L and three $SU(2)$ singlets: u_R, d_R, e_R. Largest group of unitary field transformation which commutes with the SM gauge group is $U(3)^5$. It can be decomposed as

$$G_F \equiv SU(3)_q^3 \otimes SU(3)_l^2 \otimes U(1)_B \otimes U(1)_L \otimes U(1)_Y \otimes U(1)_{PQ} \otimes U(1)_{E_R} \tag{3.1}$$

Where

$$\begin{aligned} SU(3)_q^3 &\equiv SU(3)_{Q_L}^3 \otimes SU(3)_{U_R}^3 \otimes SU(3)_{D_R}^3 \\ SU(3)_l^2 &\equiv SU(3)_{L_L} \otimes SU(3)_{E_R} \end{aligned} \tag{3.2}$$

The five $U(1)$ groups mentioned above can be related to baryon number (B), lepton number (L), hypercharge (Y), Peccei-Quinn symmetry of 2HDM and global rotation of $SU(2)$ singlet. Baryon number, lepton number, and hypercharge are respected by Yukawa interactions. In SM, the group $SU(3)_q^3 \otimes SU(3)_l^2 \otimes U(1)_{PQ} \otimes U(1)_{E_R}$ get broken by Yukawa matrices. Flavor invariance can be recovered by introducing three dimensionless auxiliary fields Y_U, Y_D, Y_L. Transformation properties of these fields under $SU(3)_q^3 \otimes SU(3)_l^2$ group are given as

$$Y_U \sim (3, \bar{3}, 1)_{SU(3)_q^3}, \quad Y_D \sim (3, 1, \bar{3})_{SU(3)_q^3}, \quad Y_L \sim (3, \bar{3})_{SU(3)_l^2} \tag{3.3}$$

For Y_U and Y_D numbers in the parentheses in above equation represents transformation properties under the groups $SU(3)_{Q_L}^3, SU(3)_{U_R}^3$ and $SU(3)_{D_R}^3$ respectively. For Y_L numbers represents transformation properties under the groups $SU(3)_{L_L}$ and $SU(3)_{E_R}$ respectively. Transformation properties of the fields given by above equation allows the appearance of Yukawa interactions which are invariant under the flavor symmetry transformation given by the group $SU(3)_q^3 \otimes SU(3)_l^2$ as

$$\mathcal{L} = \bar{Q}_L Y_D D_R \phi + \bar{Q}_L Y_U U_R \tilde{\phi} + \bar{L}_L Y_L E_R \phi + H.C. \tag{3.4}$$

Where $\tilde{\phi} = -i\sigma_2 \phi^*$. This equation represents the most general couplings of Y fields to renormalizable SM operators. Using the $SU(3)_q^3 \otimes SU(3)_l^2$ symmetry, background values of the Y fields can be rotated to give

$$Y_D = \lambda_d, \quad Y_L = \lambda_l, \quad Y_U = V^\dagger \lambda_u \tag{3.5}$$

Where λ are diagonal matrices and V is CKM matrix. If in an effective theory all higher dimensional operators constructed from SM fields and Y fields are invariant under CP and formally under Group G_F then the theory is said to satisfy the criteria of MFV. In other words all the flavor violation is completely determined by the structure of Yukawa couplings. Since in SM Yukawa couplings for all the quarks except top are small, only relevant non-diagonal structure is $Y_U Y_U^\dagger$. Hence all the flavor changing processes with external down type quarks are governed by

$$(\lambda_{FC})_{ij} \equiv (Y_U Y_U^\dagger)_{ij} \approx \lambda_t^2 V_{3i}^* V_{3j} \quad \text{for} \quad i \neq j \tag{3.6}$$

In some models satisfying the criteria of MFV the operators responsible for the flavor violations are also the same as in the SM. This class of model is referred to as the constrained MFV [63, 64]. In more general situations, MFV models contain more operators with coefficients determined in terms of the elements of CKM matrix. Some models also contain new phases not present in CKM matrix V. These are known as the next to minimal flavor violation (NMFV) models [65].

2HDM with natural flavor conservation (NFC) provides a simple example of MFV [43]. In this class of models a discrete symmetry is imposed in such a way that quarks get mass from only one Higgs doublet. Hence FCNC are absent in these models. Imposition of the discrete symmetry prevents any CP violation coming from the Higgs potential and the CKM matrix provides a unique source of CP and flavor violations. In this model, box diagram shown in figure(3.1) gives rise to the B_q^0-$\bar{B}_q^0$ $(q = d, s)$ transition amplitude M_{12}^q. The dominant top quark dependent

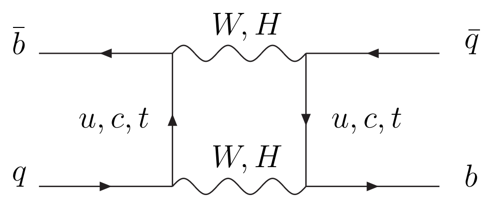

Figure 3.1: Box diagram for B_q-$\bar{B}_q$ mixing in 2HDM with NFC

part can be written [66, 67] as

$$M_{12}^q = \frac{G_F^2 M_W^2 M_{B_q} B_q f_{B_q}^2 \eta_B(x_t)(V_{tb}V_{tq}^*)^2}{12\pi^2}(1 + \kappa_H^+) \ , \tag{3.7}$$

where

$$\begin{aligned}
\kappa_H^+ &\equiv \frac{1}{4S_0(x_t)} \frac{\eta_B(x_t, y_t)}{\eta_B(x_t)} (\cot^4 \beta S_{HH}(y_t) + \cot^2 \beta S_{HW}(x_t, y_t)) \ , \\
&\approx \frac{\eta_B(x_t, y_t)}{\eta_B(x_t)} (0.12 \cot^4 \beta + 0.53 \cot^2 \beta) \ ,
\end{aligned} \tag{3.8}$$

where η_B are the QCD corrections [68, 69, 70], $\tan\beta$ is the ratio of the Higgs vacuum expectation values and $x_t = \frac{m_t^2}{M_W^2}, y_t = \frac{m_t^2}{M_H^{+2}}$. The functions appearing above can be found for example in [69, 70, 71] and the last line corresponds to the obtained numerical values in case of the charged Higgs mass $M_{H^+} = 200$ GeV. It can be seen from above equation that the transition amplitude is still described in terms of CKM elements. The only effect of the charged Higgs boson is an additional contribution to the function $S_0(x_t)$. The same happens in case of other observables.

2HDM with NFC lead to MFV but they do not represent the most generic possibilities. A general 2HDM contains additional sources of CP and flavor violation through the presence of FCNC. The principle of NFC now appears to conflict [72] with the idea of the spontaneous CP violation (SCPV) at low energy and both cannot coexist together. To have SCPV, Lagrangian is assumed to be CP invariant. Hence, in this case Yukawa couplings are real. Now if NFC is also imposed then each quark gets mass from only one Higgs doublet. Hence any phase in the Higgs vacuum expectation value can be absorbed by redefining the quark fields. Thus in this case quark mass matrices are real which leads to real CKM matrix [73]. This is inconsistent with the evidences which suggests that the CKM matrix is complex under very general assumptions as discussed in the first chapter and also in [12]. Thus attractive idea of low energy SCPV can only be realized by admitting the tree level FCNC [74]. Independent of this, the 2HDM without NFC become phenomenologically interesting if there is a natural mechanism to suppress FCNC. The phenomenology of such models has been studied in variety of context [31, 75, 76, 77, 78, 80, 170, 74]. One example of model with suppressed FCNC will also be discussed in chapter 4.

Our focus here is the discussion of a class of 2HDM with FCNC which satisfies the principle of MFV. In these models the FCNC couplings are determined completely in terms of the CKM matrix and the quark masses [81]. These models were presented long ago [79, 80, 81]. Here we update constraints on them in view of the substantial experimental information that has become available from the Tevatron and B factories. In the next section we will describe the model and present the structure of the FCNC couplings. Then we will present the analytic and numerical studies of the consequences in this model assuming that either the charged Higgs or a neutral Higgs dominates the P^0-$\bar{P}^0$ ($P = K, B_d, B_s$) mixing. The last section will summarizes the salient features.

3.1 Model and the structure of FCNC

Consider the SU(2) $\otimes$ U(1) model with two Higgs doublets ϕ_a, ($a = 1, 2$) and the following Yukawa couplings:

$$-\mathcal{L} = \bar{Q}'_L \Gamma^d_a \phi_a d'_R + \bar{Q}'_L \Gamma^u_a \tilde{\phi}_a u'_R + \text{H.c.} \; . \tag{3.9}$$

$Q^{'}_{iL}$ ($i = 1, 2, 3$) represent three families of weak doublets and $u^{'}_{iR}, d^{'}_{iR}$ are the corresponding singlets. Let us consider a class of models [79] represented by a specific choice of the matrices Γ^d_a and their permutations:

$$\Gamma^d_1 = \begin{pmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{pmatrix} \quad ; \; \Gamma^d_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{pmatrix} \; , \tag{3.10}$$

where x represents an entry which is allowed to be non-zero. We do not impose CP on eq.(3.9) allowing elements in $\Gamma^{u,d}_{1,2}$ to be complex. The above forms of $\Gamma^d_{1,2}$ are technically natural as they follow from imposition of discrete symmetries on eq.(3.9), the simplest being a Z_2 symmetry under which only $Q^{'}_{3L}$ and ϕ_2 change sign.

The down quark mass matrix M^d follows from eq.(3.10) when the Higgs fields obtain their vacuum expectation values (vev): $\langle\phi^0_1\rangle = v_1$ and $\langle\phi^0_2\rangle = v_2 e^{i\theta}$. Let $V^d_{L,R}$ be the unitary matrices connecting the mass (unprimed) and the weak (primed) basis $d^{'}_{L,R} = V^d_{L,R} d_{L,R}$. Then

$$V^{d\dagger}_L M^d V^d_R = D^d \; , \tag{3.11}$$

D^d being a diagonal matrix of the down quark masses m_i. M^d obtains contributions from two different Higgs fields leading to the Higgs induced FCNC in the down quark sector. Eqs. (3.9-3.11) are used to obtain:

$$-\mathcal{L}_{FCNC} = \frac{(2\sqrt{2}G_F)^{1/2}}{\sin\beta\cos\beta} F^d_{ij} \bar{d}_{iL} d_{jR} \phi^0 + \text{H.C.} \; , \tag{3.12}$$

where $\tan\beta = \frac{v_2}{v_1}$ and

$$\phi^0 \equiv \cos\beta\ \phi_2^0\ e^{-i\theta} - \sin\beta\ \phi_1^0 \tag{3.13}$$

is a specific combination of $\phi_{1,2}$ with zero vev. The orthogonal combination plays the role of the standard model Higgs. The strength of FCNC current is determined in the fermion mass basis by [79]:

$$\begin{aligned} F_{ij}^d &\equiv (V_L^{d\dagger}\Gamma_2^d v_2 e^{i\theta} V_R^d)_{ij} \\ &= (V_L^{d\dagger})_{i3}(\Gamma_2)_{3k}^d v_2 e^{i\theta}(V_R^d)_{kj} \end{aligned} \tag{3.14}$$

Also

$$\begin{aligned} & V_L^{d\dagger} M^d V_R^d = D^d \\ \Leftrightarrow\quad & M_{3k}^d (V_R^d)_{kj} = (V_L^d)_{3j} D_{jj}^d \end{aligned} \tag{3.15}$$

Using eq.(3.10) we get

$$(\Gamma_2^d)_{3k} v_2 e^{i\theta}(V_R^d)_{kj} = (V_L^d)_{3j} m_j \tag{3.16}$$

Substituting this in the eq.(3.14), we get

$$F_{ij}^d = (V_L^{d*})_{3i}(V_L^d)_{3j} m_j\ , \tag{3.17}$$

Thus F_{ij}^d depend on the left-handed mixing matrix V_L^d which is a priory unknown but would be correlated to the CKM matrix. One observes that

- independent of the values of elements of V_{dL}, the F_{ij}^d display hierarchy

$$|F_{12}^d| < |F_{13}^d|, |F_{23}^d| \tag{3.18}$$

  Due to this hierarchy the flavor violations in the K sector are suppressed relative to B mesons.

- all the FCNC couplings are suppressed if the off-diagonal elements of V_L^d are smaller than the diagonal ones. The model in this sense illustrates the principle of near flavor conservation [82, 83]. This is a generic possibility in view of the strong mass hierarchy among quarks unless there are some special symmetries.

- F^d_{ij} can be determined in terms of the CKM matrix elements for a specific structure of M^u [81] given as follows:

$$M^u = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{pmatrix} . \tag{3.19}$$

The above postulated structures of $M^{u,d}$ follow from discrete symmetries [81] rather than being ad-hoc. Particular example can be:

$$(Q^{'}_{1,2L}, \phi_1) \rightarrow \omega(Q^{'}_{1,2L}, \phi_1) \quad , \quad u^{'}_{1,2R} \rightarrow \omega^2 u^{'}_{1,2R} \ . \tag{3.20}$$

Here $\omega, \omega^2 \neq 1$ are complex numbers. The fields not shown above remain unchanged under the symmetry. Up quark mass matrix of the form given in eq.(3.19) can be diagonalized by unitary matrices $V^u_{L,R}$ of the following form.

$$V^u_{L,R} = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{3.21}$$

CKM matrix is given as

$$V = V^{u\dagger}_L V^d_L \tag{3.22}$$

The CKM elements relevant for B^0_q-$\bar{B}^0_q$ mixing are

$$V_{3i} = V^{u\dagger}_{L3j} V^d_{Lji} \tag{3.23}$$

From the eq.(3.21) $V^{u\dagger}_{L3i} = 0\ ; i = 1, 2$ and $V^{u\dagger}_{L33} = 1$. Hence the relevant CKM elements are given as

$$V_{3i} = V^d_{L3i} \tag{3.24}$$

Thus F^d_{ij} are completely determined in terms of the CKM matrix V.

$$F^d_{ij} = V^*_{3i} V_{3j} m_j \ . \tag{3.25}$$

As a consequence of eq.(3.20), M^u_{33} gets contribution from ϕ_2 while the first two generations from ϕ_1 with no mixing with the third one. As a result, there are no FCNC in the up quark sector while they are determined as in eq.(3.25) in the down quark sector.

The tree level couplings of the charged Higgs $H^+ \equiv \cos\beta\phi_2^+ - \sin\beta e^{i\theta}\phi_1^+$ can be read off from eq.(3.9) and are given by

$$(2\sqrt{2}G_F)^{1/2}H^+\left\{\bar{u}_R\hat{D}_u V d_L + \bar{u}_L(VD_d\tan\beta - \frac{1}{\sin\beta\cos\beta}VF^d)d_R\right\} + \text{H.C.}\ , \tag{3.26}$$

where $\hat{D}_u \equiv \text{diag.}(-m_u\tan\beta, -m_c\tan\beta, m_t\cot\beta)$.

It follows from eqs.(3.12, 3.25, 3.26) that all the Higgs fermion couplings are determined by the CKM matrix V giving rise to MFV. There can however be an additional source of CP violation in the model. This can arise if the scalar-pseudo scalar mixing contains a phase. As noted in [81], the discrete symmetry of eq.(3.20) prevents this mixing in the Higgs potential even if one allows for explicit CP violation and a bilinear soft symmetry breaking term $\mu(\phi_1^\dagger\phi_2) + \text{H.c.}$. Thus the minimal version of the model corresponds to the MFV scenario with no other CP violating phases present. CP violation in Higgs mixing can however be induced by adding a complex Higgs singlet field [81, 84]. In this case, there would be an additional phase which mixes the real and the imaginary parts of the Higgs ϕ^0 defined in eq.(3.13). An independent motivation for introducing the Higgs singlet is provided by the strong CP problem. It is known that the Peccei Quinn (PQ) solution [5] to this problem can be made phenomenologically viable by invoking a Higgs singlet. It would thus be natural to have singlet fields play a dual role of providing weak CP violation and solving the strong CP problem [84].

This can be done here by replacing the discrete symmetry in eq. (3.20) with a continuous symmetry defined by $w \to e^{i\beta}$. This symmetry can play the role of the PQ symmetry and would also enforce the desired structures of the Yukawa couplings $\Gamma_{1,2}^q$. But the Higgs potential gets further restricted. Now a simple Higgs potential with two doublets and a singlet and the above PQ symmetry does not admit CP violation, but this can be done by adding one more singlet. Consider the following PQ symmetric couplings between singlets and doublets in the Higgs potential:

$$(\phi_1^\dagger\phi_2)(\sigma_1\eta_1^2 + \sigma_2\eta_2^{*2} + \lambda_{12}(\eta_1\eta_2)^2 + \mu_{12}\eta_1\eta_2 + H.c. \tag{3.27}$$

Where $\sigma_{12}, \mu_{12}, \lambda_{12}$ are complex numbers. $\eta_{1,2}$ are two complex singlets such that $\eta_1 \to e^{(i/2)\beta}\eta_1, \eta_2 \to e^{-(i/2)\beta}\eta_2$ under PQ symmetry. Quark fields and ϕ_1 transform as in eq.(3.20) with $w \equiv e^{i\theta}$ while ϕ_2 and remaining fields are invariant. Minimization of the full potential including the above terms (but $\mu_{12} \equiv 0$) is carried out in [84] where it is shown that the desired mixing between the scalar and pseudoscalar components of ϕ^0 in eq.(3.12) indeed takes place.

Without committing to any of the above scenario, we will simply assume for phenomenological purpose that Higgs mixing contains an effective CP violating phase which could be generated through singlets as outlined above.

There is an important quantitative difference between the present scenario and the general MFV analysis following from the effective field theory approach [55]. There the effective dominant FCNC couplings between down quarks are given by

$$(\lambda_{FC})_{ij} \approx \lambda_t^2 V_{3i}^* V_{3j} \ ,$$

where λ_t denotes the top Yukawa coupling. The same factor controls the loop induced contributions here but the tree level flavor violations are given by eq.(3.25) which contains the same elements of V but involves the down quark masses linearly. Its contribution is still important or dominates over the top quark dependent terms because of its presence at the tree level.

One could consider variants of the above textures and symmetry obtained by permutations of flavor indices. These variants lead to different amount of FCNC. Labeling these variants by a, one has three models [81] with $F_{ij}^d(a) = V_{ai}^* V_{aj} m_j \ , (a = 1, 2, 3)$. Alternatively, one could also consider equivalent models in which FCNC in the down quarks are absent while in the up quark sector they would be related to the CKM matrix elements and the up quark masses. The case $a = 3$ is special. It leads to the maximum suppression of FCNC between first and second generation of quarks. We will mainly consider phenomenological implication of this case.

3.2 Experimental constraints and their implications

3.2.1 Basic Results

The strongest constraints on the model come from the $P^0 - \bar{P}^0$ $(P = K, B_d, B_s)$ mixing. In addition to the SM contribution, two other sources namely, the charged Higgs induced box diagrams and the neutral Higgs ϕ^0 induced tree diagram contribute to this mixing. Formalism for B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing is described in the first chapter. Here we describe formalism for K^0-$\bar{K}^0$ mixing.

$K^0 - \bar{K}^0$ mixing

$K^0 - \bar{K}^0$ mixing refers to the transition from $K^0 \equiv \bar{s}d$ to $\bar{K}^0 \equiv s\bar{d}$. The evolution of a beam of K^0 and $\bar{K}^0$ in Wigner-Weisskopf approximation is given as [5]

$$i\frac{d}{dt}\begin{pmatrix} |K^0(t)\rangle \\ |\bar{K}^0(t)\rangle \end{pmatrix} = \left(M_K - \frac{i}{2}\Gamma_K\right)\begin{pmatrix} |K^0(t)\rangle \\ |\bar{K}^0(t)\rangle \end{pmatrix}$$

Eigen states of above equations has definite mass and lifetime. These eigen states have large difference in lifetime. Hence they are denoted by $|K_L\rangle$ and $|K_S\rangle$ where subscripts L and S stands for long lived and short lived states. For these eigen states, masses M_L, M_S and the decay widths Γ_L, Γ_S are obtained by diagonalizing $M_K - i\Gamma_K/2$. In the CPT invariant case mass eigen states K_L and K_S are given as

$$|K_L\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle\right] \tag{3.28}$$

$$|K_S\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle\right] \tag{3.29}$$

Time evolution of the mass eigen state is given as

$$|K^0_{L,S}(t)\rangle = e^{-i(M_{L,S}-i\Gamma_{L,S}/2)t}\ |K^0_{L,S}\rangle \tag{3.30}$$

Where $|K^0_{L,S}\rangle$ denote mass eigen states at time t = 0. Average mass and decay width are defined as

$$m_K = \frac{M_L + M_S}{2} \tag{3.31}$$
$$\Gamma_K = \frac{\Gamma_L + \Gamma_S}{2} \tag{3.32}$$

Mass difference and decay width difference are defined as

$$\Delta m_K = M_L - M_S \tag{3.33}$$
$$\Delta\Gamma_K = \Gamma_L - \Gamma_S \tag{3.34}$$

The CP violating parameter ϵ can be related to measurable quantities as follows.

$$\epsilon = \frac{2\eta_{+-} + \eta_{00}}{3} \tag{3.35}$$

where η_{+-} and η_{00} are defined as follows.

$$\begin{aligned} \eta_{+-} &= \frac{\langle\pi^+\pi^-|T|K_L\rangle}{\langle\pi^+\pi^-|T|K_S\rangle} \\ \eta_{00} &= \frac{\langle\pi^0\pi^0|T|K_L\rangle}{\langle\pi^0\pi^0|T|K_S\rangle} \end{aligned} \tag{3.36}$$

In the 2HDM with FCNC, ϵ will have additional contributions from the charged Higgs and neutral Higgs. The charged Higgs leads to new box diagrams which follow from eq.(3.26). The last two terms of this equation are suppressed by the down quark masses (for modest $\tan\beta$) and the dominant contribution comes from the top quark. This term and hence the charged Higgs contributions remain the same as in 2HDM with NFC [66, 67]. Charged Higgs contribution to ϵ is given [71] by

$$\epsilon^{H^+} = \frac{G_F^2 M_W^2 f_K^2 m_K B_K A^2\lambda^6\bar\eta}{6\sqrt{2}\pi^2\Delta m_K}\left(f_1^H + f_2^H A^2\lambda^4(1-\bar\rho)\right)\ , \tag{3.37}$$

where functions $f^H_{1,2}$ can be read-off from expressions given in [71]. $\lambda, \bar\eta \equiv \eta(1-\frac{\lambda^2}{2}), \bar\rho \equiv \rho(1-\frac{\lambda^2}{2})$ and A are the Wolfenstein parameters. Contribution of f_1^H to ϵ is practically negligible while the f_2^H can compete with the corresponding term in the SM expression

$$\epsilon^{SM} = \frac{G_F^2 M_W^2 f_K^2 m_K B_K A^2\lambda^6\bar\eta}{6\sqrt{2}\pi^2\Delta m_K}(f_1(x_t) + f_2(x_t)A^2\lambda^4(1-\bar\rho)) \tag{3.38}$$

for moderate values of $\tan\beta$.

Neutral Higgs contribution

The neutral Higgs contributions to the above observables follow from eqs.(3.12) and (3.25). Define

$$\phi^0 \equiv \frac{R+iI}{\sqrt{2}} = \left(\frac{O_{R\alpha}+iO_{I\alpha}}{\sqrt{2}}\right) H^0_\alpha \equiv |C_\alpha| e^{i\eta_\alpha} H^0_\alpha \ ,$$

where H^0_α denote the mass eigen states with masses M_α. $\alpha = 1,2,3$ for the 2HDM while $\alpha = 1,...5$ in the presence of a complex singlet introduced to induce the scalar-pseudo scalar mixing leading to phases η_α in the Higgs mixing C_α. $O_{R\alpha,I\alpha}$ are elements of the mixing matrix. Using this definition and eq.(3.25) the neutral Higgs contribution to M^q_{12} can be written as

$$(M^q_{12})^{H^0} = \frac{5\sqrt{2}G_F m_b^2 m_{B_q} f^2_{B_q} B_{2q}}{12\sin^2 2\beta M^2_\alpha}\left(\frac{m_{B_q}}{m_b+m_q}\right)^2 C^2_\alpha (V^*_{3q}V_{33})^2 + \mathcal{O}\left(\frac{m_q}{m_b}\right) \ , \quad (3.39)$$

where we used the vacuum saturation approximation multiplied by the bag factor B_{2q}

$$\langle B^0_q|(\bar{q}_L b_R)^2|\bar{B}^0\rangle = -\frac{5}{24} m_{B_q} f^2_{B_q} B_{2q}\left(\frac{m_{B_q}}{m_b+m_q}\right)^2 \ .$$

The $\mathcal{O}\left(\frac{m_q}{m_b}\right)$ refer to contributions coming from the F^{d*}_{3q} terms in eq.(3.12). Using the vacuum saturation approximation and eq.(3.25), these terms are estimated to be only a few % of the first term in eq (3.39) for $q = s$ and much smaller for $q = d$. We do not display here the QCD corrections to $(M_{12})^{H^0}$. Such corrections can be significant and play important role in the precise determination of the SM parameters. In contrast, the above expressions contain several unknowns of the Higgs sector because of which we prefer to simplify the analysis and retain only the leading terms as far as the Higgs contributions to various observables are concerned. The SM contribution is given by

$$(M^q_{12})^{SM} = \frac{G_F^2 m_W^2 m_{B_q} f^2_{B_q} B_q \eta_B}{12\pi^2}(V^*_{3q}V_{33})^2 S_0(x_t) \ , \quad (3.40)$$

with $S_0(x_t) \approx 2.3$ for $m_t \approx 161$ GeV. Eqs.(3.39,3.40) together imply

$$\kappa^q \equiv \left|\frac{(M^q_{12})^{H^0}}{(M^q_{12})^{SM}}\right| = \left(\frac{5\sqrt{2}\pi^2|C_\alpha|^2}{G_F M_W^2 \sin^2 2\beta}\right)\left(\frac{m_b}{M_\alpha}\right)^2 \frac{B_{2q}}{B_{B_d}\eta_B}\left(\frac{m_{B_q}}{m_b+m_q}\right)^2 + \mathcal{O}\left(\frac{m_q}{m_b}\right) \ . \quad (3.41)$$

The neutral Higgs contribution to ϵ is given by

$$\epsilon^{H^0} = \frac{5G_F m_K f_K^2 B_{2K}}{12\sin^2 2\beta \Delta m_K M_\alpha^2}\left(\frac{m_K}{m_s+m_d}\right)^2 Im(F_{12}^d C_\alpha)^2 \ , \tag{3.42}$$

Using the expression of F_{12}^d from eq.(3.25) and the Wolfenstein parameterization, one can rewrite the above equation as

$$\epsilon^{H^0} \approx \frac{5G_F m_s^2 m_K f_K^2 B_{2K}}{12\sin^2 2\beta \Delta m_K M_\alpha^2}\left(\frac{m_K}{m_s+m_d}\right)^2 |C_\alpha|^2 A^4\lambda^{10}[(1-\bar\rho)^2+\bar\eta^2]^{1/2}\sin 2(\eta_\alpha-\beta_U), \tag{3.43}$$

where $\tan\beta_U = \frac{\bar\eta}{1-\bar\rho}$. $\beta_U \equiv arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$ is the angle of the unitarity triangle. The Higgs contribution to ϵ is suppressed here by the strange quark mass and ϵ^{H^0} is practically negligible compared to ϵ^{SM}:

$$\left|\frac{\epsilon^{H^0}}{\epsilon^{SM}}\right| \approx 3.810^{-4}\frac{B_{2K}}{B_K}\frac{|C_\alpha|^2}{\sin^2 2\beta}\left(\frac{100\text{GeV}}{M_\alpha}\right)^2 \frac{\sin 2(\eta_\alpha-\beta_U)}{\cos\beta_U + 0.1\sin\beta_U} \ . \tag{3.44}$$

The neutral Higgs contribution to the $K^0-\bar K^0$ mass difference is even more suppressed compared to its experimental value.

3.2.2 Experimental Inputs

Constraints on the present scheme come from several independent measurements. The complex amplitude M_{12}^d for B_d^0-$\bar B_d^0$ mixing is known quite well. The magnitude is given in terms of the $B_d^0-\bar B_d^0$ mass difference [28]:

$$\Delta M^d \equiv 2|M_{12}^d| = (0.507\pm 0.005)\ \text{ps}^{-1} \ . \tag{3.45}$$

The phase ϕ_d of B_d^0-$\bar B_d^0$ mixing is measured through the mixing induced CP asymmetry in the $B_d^0 \to J/\psi K_S$ decay:

$$\sin\phi_d = 0.668\pm 0.028 \ . \tag{3.46}$$

Likewise, the $B_s^0-\bar B_s^0$ mass difference is quite well determined:

$$\Delta M^s \equiv 2|M_{12}^s| = 17.77\pm 0.12\ \text{ps}^{-1} \ . \tag{3.47}$$

For the corresponding phase ϕ_s we have used value determined [85] by the $D0$ collaboration [86]

$$\phi_s = -0.70^{+0.47}_{-0.39} \ . \tag{3.48}$$

by combining their measurements of (1) the light and the heavy B_s^0 width difference (2) the time dependent angular distribution in the $B_s^0 \to J/\psi\phi$ decay and (3) the semileptonic charge asymmetries in the B^0 decays.

The SM predictions for the above quantities depend on the hadronic and the CKM matrix elements. The determination of Wolfenstein parameters $\bar\rho, \bar\eta$ is somewhat non-trivial when new physics is present. The conventional SM fits use the loop induced variables ϵ, M_{12}^d, ϕ_d for determining $\bar\rho, \bar\eta$. These variables are susceptible to new physics contributions. This makes extraction of $\bar\rho, \bar\eta$ model-dependent. It is still possible to determine these parameters and construct a universal unitarity triangle [87] for a unitary V by assuming that the tree level contributions in the SM are not significantly affected by new physics. In that case, one can use only the tree level measurements for determining $\bar\rho, \bar\eta$ [51, 52]. Alternatively one can allow for NP contributions [14, 28, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65] in the loop induced processes while determining elements of V. The tree level observables are the moduli of V and the unitarity angle γ [28].

$$\begin{aligned} \lambda = |V_{us}| = 0.2258 \pm 0.0014 \quad &, \quad A = \frac{|V_{cb}|}{\lambda^2} = 0.82 \pm 0.014 \ , \\ |V_{ub}|^{\text{excl.}} = 0.0034 \pm 0.0004 \quad &, \quad |V_{ub}|^{\text{incl.}} = 0.0045 \pm 0.0003 \ . \end{aligned} \tag{3.49}$$

γ is determined from purely tree level decay $B \to D^* K^*$. We will use the UTfit average value [28]:

$$\gamma = (83 \pm 19)^\circ \ . \tag{3.50}$$

In terms of the Wolfenstein parameters,

$$\begin{aligned} \bar\rho = R_b \cos\gamma \quad &, \quad \bar\eta = R_b \sin\gamma \ , \\ R_b \equiv (1 - \frac{\lambda^2}{2})\frac{1}{\lambda}|\frac{V_{ub}}{V_{cb}}| &= 0.46 \pm 0.03 \quad \text{inclusive determination} \ , \\ &= 0.35 \pm 0.04 \quad \text{exclusive determination} \ . \end{aligned} \tag{3.51}$$

Eqs.(3.50) and (3.51) provide a NP independent determination of $\bar\rho, \bar\eta$, *e.g.* with inclusive values in eq.(3.49),

$$\begin{aligned} \bar\rho &= 0.06 \pm 0.15 \\ \bar\eta &= 0.46 \pm 0.03 \end{aligned} \tag{3.52}$$

One could use the above values of $\bar{\rho}, \bar{\eta}$ to obtain predictions of ϵ and M_{12}^d in the SM. The errors involved are rather large but it has the advantage of being independent of any new physics contributing to these observables. This approach has been used for example in [14, 51, 52, 53] to argue that a non-trivial NP phase is required if $|V_{ub}|$ is close to its inclusive determination. We will use an alternative analysis which also leads to the same conclusion. The new physics contributions to the loop induced $\Delta F = 2$ observables is parameterized as follows

$$\begin{aligned} M_{12}^q &= (M_{12}^q)^{SM}(1+\kappa_q e^{i\sigma_q}) = \rho_q (M_{12}^q)^{SM} e^{i\phi_q^{NP}} \ , \\ \epsilon &= \rho_\epsilon \, \epsilon_{SM} \ . \end{aligned} \tag{3.53}$$

Model independent studies using the above or equivalent parameterization have been used to determine $\bar{\rho}, \bar{\eta}, \kappa_q, \sigma_q, C_\epsilon$ in number of different works [14, 28, 54, 51, 52]. We will use the results from UTfit group whenever appropriate.

In view of the several unknown Higgs parameters, we make a simplifying assumption that only one Higgs contributes dominantly. We distinguish two qualitatively different situations corresponding to the dominance of the charged Higgs H^+ or of a neutral Higgs.

3.3 Charged Higgs dominance

The effects of the charged Higgs on the P^0-$\bar{P}^0$ mixing as well as on $\Delta F = 1$ processes such as $b \to s\gamma$ have been discussed at length in the literature [66, 67, 69, 70, 71, 88, 89]. The present case remains unchanged compared to the standard two Higgs doublet model of type II as long as the down quark mass dependent terms are neglected in eq.(3.26). Just for illustrative purpose and completeness we discuss some of the restrictions on the charged Higgs couplings and masses in this subsection before turning to our new results on the neutral Higgs contributions to flavor violations.

The allowed values of $\bar{\rho}, \bar{\eta}$ in the presence of the charged Higgs follow from the detailed numerical fits in case of MFV scenario, e.g. fits in [14] give

$$\bar{\rho} = 0.154 \pm 0.032 \quad , \quad \bar{\eta} = 0.347 \pm 0.018 \ . \tag{3.54}$$

We can substitute these values in the SM expressions for ΔM^d and ϵ to obtain [28]

$$\begin{aligned} \rho_d \equiv & \quad \frac{\Delta M^d}{(\Delta M^d)^{SM}} & = 0.99 \pm 0.29 \ , \\ \rho_\epsilon \equiv & \quad \frac{\epsilon}{\epsilon^{SM}} & = 0.94 \pm 0.09 \ . \end{aligned} \tag{3.55}$$

This can be translated into bounds on M_{H^+} and $\tan\theta$ using eqs.(3.7, 3.37) and eq.(3.38). The 2σ bounds following from eq.(3.55) are shown in Fig.(3.2). The constraints from ϵ are stronger and allow the middle (dotted) strip in the $M_{H^+} - \tan\theta$ plane. These are illustrative bounds and we refer to literature [66, 67, 69, 70, 71, 88, 89] for more detailed results which include QCD corrections. Generally, there is sizable region in $\tan\theta$, M_{H^+} plane (e.g. $\tan\theta \gtrsim 1-2$ in Fig.(1)) for which the top induced charged Higgs contribution to $\rho_{d,\epsilon}$ is not important. But the neutral Higgs can contribute to these observables in these regions as we now discuss.

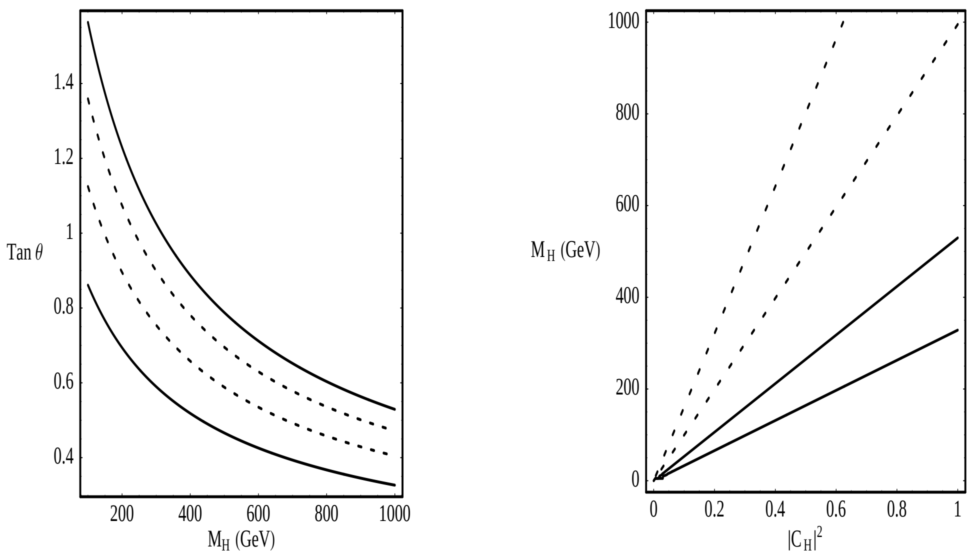

Figure 3.2: Left panel: The 2σ region in the $\tan\beta, M_{H^+}$ plane allowed by ρ_d (solid) and ρ_ϵ (dotted) given in eq.(3.55). Right panel: Allowed regions in $|C_H|^2, M_H$ plane following from the inclusive determination of $|V_{ub}|$ for $\tan\beta = 3$ (solid) and 10 (dotted). The left (right) panel is based on the assumption that the charged Higgs (neutral Higgs) alone accounts for the required new physics contribution to M_{12}^q.

3.4 Neutral Higgs dominance

We label the dominating neutral Higgs field by $\alpha = H$ and retain only one term in eq.(3.39). Unlike in the previous case, the neutral Higgs contribution to ϵ (and the $K^0 - \bar{K}^0$ mass difference) is very small. It can contribute significantly to $M_{12}^{d,s}$ but these contributions are strongly correlated. Using eq.(3.39,3.41) one finds that:

$$\begin{aligned} r = \frac{\kappa_s}{\kappa_d} &= \frac{B_{2s}}{B_{2d}}\frac{B_{B_d}}{B_{B_s}}\left(\frac{m_{B_s}}{m_s + m_b}\right)^2\left(\frac{m_d + m_b}{m_{B_d}}\right)^2 , \\ \sigma_d &= \sigma_s = 2\eta_H . \end{aligned} \quad (3.56)$$

This ratio does not involve most of the unknown parameters and is determined by masses and the bag parameters. The ratios of B parameter in eq.(3.56) and hence r is very close to 1. For example, the results in [90] for the bag parameters imply

$$r = 1.04 \pm 0.12 . \quad (3.57)$$

Assuming $r = 1$ leads to an important prediction:

$$\frac{\Delta M^s}{\Delta M^d} = \left(\frac{\Delta M^s}{\Delta M_d}\right)^{SM} .$$

This prediction holds good in various MFV scenario, e.g. SUSY MFV model at low $\tan\beta$ [56, 57, 58, 59, 60, 61, 62]. Here it remains true even in the presence of an extra phase η_H. The above prediction can be usefully exploited [87] for the determination of one of the sides of the unitarity triangle:

$$\begin{aligned} R_t &\equiv \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right| , \\ &= \frac{\xi}{\lambda}\sqrt{\frac{M_{B_s}}{M_{B_d}}}\sqrt{\left|\frac{\Delta M_{B_d}}{\Delta M_{B_s}}\right|} \approx\ 0.93 \pm 0.05, \end{aligned} \quad (3.58)$$

where $\xi = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}} = 1.23 \pm 0.06$ [28]. We used the SM expression, eq.(3.40) in the above equation and the approximation $|V_{ts}| = |V_{cb}|$.

The SM prediction for ΔM_s is independent of $\bar{\rho}, \bar{\eta}$. Using, $f_{B_s}\sqrt{B_s} = 0.262 \pm 0.035$ MeV [28] we obtain

$$\rho_s \equiv \left|\frac{\Delta M_s}{\Delta M_s^{SM}}\right| \approx\ 0.96 \pm 0.26 . \quad (3.59)$$

The existing fits to the $\Delta F = 2$ processes in the presence of NP are carried out in the context of the MFV [14, 28, 55, 56, 57, 58, 59, 60, 61, 62] or NMFV [65] scenario or in a model independent manner [14, 28]. Most of these assume that NP contributes significantly to $\Delta S = 2$ transition, particularly to ϵ. This is not the case here. On the other hand the model independent fits neglect correlations between $\Delta M_d, \Delta M_s$ as present here. In view of this, we performed our own but simplistic fits in the present case. We use $\phi_d, \gamma, R_b, R_t, \rho_s$ and ϵ in the fits assuming all errors to be Gaussian. The expressions and the experimental values for these quantities are already given in respective equations. We use the standard model expression for ϵ. We have used $r = 1$ in eq.(3.56) giving eq.(3.58) and $\rho_d = \rho_s \equiv \tilde{\rho}$ and $\sigma_d = \sigma_s \equiv \sigma$. The above six observables are fitted in terms of the four unknowns $\bar{\rho}, \bar{\eta}, \tilde{\rho}, \phi_d^{NP}$. The fitted values of the parameters are sensitive to $|V_{ub}|$. The table(3.1) contains values of the fitted parameters and 1σ errors obtained in three cases which use (a) inclusive (b) exclusive and (c) average value of $|V_{ub}|$ as quoted in [6]. The predictions based on the average values agree within 1σ with the corresponding detailed model independent fits by the UTfit group [28]: $\bar{\rho} = 0.167 \pm 0.051$, $\bar{\eta} = 0.386 \pm 0.035$. The values of $\bar{\rho}, \bar{\eta}$ in the fit directly determine the phase of $(M_{12}^d)^{SM}$:

$$\sin 2\beta_d = \frac{\bar{\eta}(1-\bar{\rho})}{\sqrt{\bar{\eta}^2 + (1-\bar{\rho})^2}} \; .$$

The phase ϕ_d as measured through $S(\psi K_S)$ is then given by

$$\phi_d = 2\beta_d + \phi_d^{NP} \; ,$$

where ϕ_d^{NP} is defined in eq.(3.53) and can also be written as

$$\tan \phi_q^{NP} = \frac{\kappa_q \sin \sigma_q}{1 + \kappa_q \cos \sigma_q} \; . \qquad (3.60)$$

Results in the table(3.1) imply that if $|V_{ub}|$ is close to the exclusive value then the present results are consistent with SM. If V_{ub} is large and close to the inclusive value then ϕ_d^{NP} is non-zero at 2σ level. This conclusion is similar to observations made [51, 52] on the basis of the use of R_b, γ alone but with somewhat different input values then used here. A non-zero ϕ_d^{NP} (and hence σ) has important qualitative implication for the model under consideration. Non-zero σ requires CP violating phase η_H from the scalar-pseudo scalar mixing. As already remarked the minimal 2HDM with symmetry as in (3.20) cannot lead to such a phase and more

	$\lvert V_{ub}^{\text{incl.}}\rvert$	$\lvert V_{ub}^{\text{excl.}}\rvert$	$\lvert V_{ub}^{\text{average}}\rvert$
$\bar{\rho}$	0.200 ± 0.039	0.121 ± 0.042	0.186 ± 0.039
$\bar{\eta}$	0.391 ± 0.028	0.320 ± 0.026	0.378 ± 0.027
$\rho_{d,s}$	0.96 ± 0.26	0.96 ± 0.26	0.96 ± 0.26
$\sin\phi_d^{NP}$	-0.18 ± 0.08	0.03 ± 0.08	$-0.14 \pm .08$

Table 3.1: Determination of NP parameters and $\bar{\rho}, \bar{\eta}$ from detailed fits to predictions of the neutral Higgs induced FCNC. See, text for more details

general model with an additional singlet field will be required. Also the charged Higgs contribution by itself cannot account for such a phase.

At the quantitative level, $\tilde{\rho} \neq 1$ implies restrictions on the Higgs parameters, $M_H, |C_H|, \beta$. These parameters are simply related to $\kappa \equiv |\tilde{\rho}e^{i\phi_d^{NP}} - 1|$ which is related to the said parameters through eq.(3.41). Results in table imply $\kappa = 0.18 \pm 0.08$ if $|V_{ub}| = |V_{ub}^{incl}|$. The values of M_H and $|C_H|^2$ which reproduce this κ within 1σ range is shown in Fig.(3.2) for two illustrative values of $\tan\beta = 3, 10$. Both these values of $\tan\beta$ are chosen to make the charged Higgs contribution to κ very small. Unlike general models with FCNC, relatively light Higgs is a possibility within the present scheme and there exist large ranges in β and C_H which allow this.

One major prediction of the model is equality of new physics contributions to CP violation in the B_d and B_s system. If the top induced charged Higgs contribution dominates then this CP violation is zero. In the case of the neutral Higgs dominance, the phases σ_d and σ_s induced by the Higgs mixing are equal see, eq.(3.56). Since the ratio r in this equation is nearly one, let us write $r = 1 + \delta_r$ with $\delta_r \approx \pm\mathcal{O}(0.1)$. Then ϕ_s^{NP} in eq.(3.60) can be approximated as

$$\begin{aligned} \tan\phi_s^{NP} &\approx \tan\phi_d^{NP}\left[1 + \delta_r(1 - \cot\sigma\tan\phi_d^{NP})\right] \;, \\ &\approx (1+\delta_r)\tan\phi_d^{NP} \;. \end{aligned} \tag{3.61}$$

This prediction is independent of the details of the Higgs parameters. Its important follows from the fact the standard CP phase in the B_s system is quite small, $\beta_s \sim -1.0^\circ$. Thus observation of a relatively large $\phi_s = 2\beta_s + \phi_s^{NP}$ will signal new physics. The predicted values of $\tan\phi_s$ based on eq.(3.61) and the numerical values given in table give

$$\begin{aligned} \tan\phi_s &\approx -0.18 \pm 0.08 \qquad \text{inclusive} \;, \\ &\approx \;\;\, 0.03 \pm 0.08 \qquad \text{exclusive} \;, \\ &\approx -0.14 \pm 0.08 \qquad \text{average} \;. \end{aligned} \tag{3.62}$$

Recent determination for ϕ_s of B_s^0-$\bar{B}_s^0$ mixing obtained from analysis of time dependent angular distribution of decay products in flavor tagged decay $B_s^0 \rightarrow J/\psi\phi$ decay by D0 collaboration is $\phi_s = -0.57^{+0.24}_{-0.30}(\text{stat})^{+0.08}_{-0.02}(\text{syst})$ [22] and 90% confidence limit interval is $-1.20 < \phi_s < 0.06$. Our predictions for $\tan\phi_s$ given in eq.(3.62) are consistent with the 90% interval of ϕ_s obtained by D0 collaboration. Significant improvements in the errors is foreseen in future at LHCb [91]. The above predictions show correlation with V_{ub} and also with the CP violating phase ϕ_d. So combined improved measurements of all three will significantly test the model. The predictions of ϕ_s in the present case are significantly different from several other new physics scenario allowing relatively large values for ϕ_s [35, 92].

3.5 Summary

The general two Higgs doublet models are theoretically disfavored because of the appearance of uncontrolled FCNC induced through Higgs exchanges at tree level. We have discussed here the phenomenological implications of a particular class of models in which FCNC are determined in terms of the elements of the CKM matrix. This feature makes these models predictive and we have worked out major predictions of the scheme. Salient aspects of the scheme discussed here are described below.

- Many of the predictions of the scheme are similar to various other models [56, 57, 58, 59, 60, 61, 62] which display MFV. The tree level FCNC couplings are governed by the CKM elements and the down quark masses while the dominant part of the charged Higgs couplings involve the same CKM factors but the top quark mass. Both contributions can be important and there exists regions of parameters ($\tan\theta \gtrsim 2-3$) in which the former contribution dominates. Unlike general FCNC models, the neutral Higgs mass as light as the current experimental bound on the SM Higgs is consistent with the restrictions from the P^0-$\bar{P}^0$ mixing, see Fig.(3.2).

- The neutral Higgs coupling to ϵ parameter is suppressed in the model by the strange quark mass. This prediction differs from the general MFV models where the top quark contributes equally to the B^0-$\bar{B}^0$ mixing and ϵ. Detailed fits to experimental data is carried out which determine the CKM parameters $\bar{\rho}, \bar{\eta}$ as displayed in the table(3.1).

- Noteworthy and verifiable prediction of the model is correlation (eq. (3.61)) between the CP violation in B_d-$\bar{B}_d$, B_s-$\bar{B}_s$ mixings and $|V_{ub}|$ as displayed in the table(3.1).

- Similar correlation between B_d-$\bar{B}_d$, B_s-$\bar{B}_s$ mixings can also occur in entirely different situation and will be discussed in chapter 5.

Chapter 4

2HDM with suppressed FCNC

CP violation has been one of the guiding principle for building models of elementary particles since its discovery in decays of K mesons [94]. Since then various experiments on mixing and decay of K, B and D mesons have given constraints on several CP violating observables [6]. Any model of elementary particles must satisfy these constraints. As discussed in the first chapter, the available data on CP violating observables can be utilized to obtain evidences for complex CKM matrix. One of the evidence comes from the determination of the angle $\gamma = -\mathrm{Arg}(V_{ud}V_{cb}V_{cd}^{*}V_{ub}^{*})$ of unitarity triangle. If CKM matrix is real then the unitarity triangle collapses to a line and all the angles become zero. Therefore a non-zero value for any of the angles of unitarity triangle will imply a complex CKM matrix. The angle γ is measured from the decays of the type $B \to D\ K$ and values reported by BABAR and Belle collaboration are [9, 10]

$$\begin{aligned}
\gamma &= 76^\circ \pm 22^\circ(\mathrm{stat}) \pm 5^\circ(\mathrm{syst}) \pm 5^\circ(\mathrm{model}) \quad \text{(BABAR collaboration)} \\
&= 53^{\circ}{}^{+15^\circ}_{-18^\circ}(\mathrm{stat}) \pm 3^\circ(\mathrm{syst}) \pm 9^\circ(\mathrm{model}) \quad \text{(Belle collaboration)} \qquad (4.1)
\end{aligned}$$

In SM, main contributions to the decay processes involved in determination of γ comes from the tree level processes. Hence under the assumption that NP does not contribute significantly to tree level processes, this determination of γ is considered to be free of any NP effects and provides evidence for complex CKM matrix [12]. Another evidence comes from the determination of Wolfenstein parameter $\bar{\eta}$. In case of real CKM matrix, $\bar{\eta}$ is zero. UTfit group has performed generalized fit to various observables in presence of arbitrary new physics and obtained $\bar{\eta}$ =

0.36 ± 0.04 [14]. This non-zero value of $\bar{\eta}$ implies that CKM matrix is complex in presence of arbitrary new physics.

Complex nature of CKM matrix does not provide any clue about exact source of CP violation. In SM CP is violated explicitly because the Yukawa couplings of fermions with the scalar doublet are complex. Therefore, as discussed in first chapter, mass matrices of the fermions becomes complex. This leads to a phase in the CKM matrix. This phase is responsible for all the CP violation in SM. The phase in CKM matrix can also be obtained through entirely different way of spontaneous CP violation (SCPV). Idea of SCPV was first suggested by T.D. Lee in the model with two Higgs doublets [95]. General theories of SCPV contain two or more Higgs doublets. For 2HDM, Yukawa terms for quarks in weak basis of quarks are given as

$$-\mathcal{L}_{Yukawa} = \bar{Q}'\Gamma_a^d \phi_a d'_R + \bar{Q}'\Gamma_a^u \tilde{\phi}_a u'_R + H.C. \tag{4.2}$$

Where Q' represents doublets of left handed quarks in weak basis. u'_R, d'_R represents right handed up type quarks and down type quarks respectively in weak basis. $\phi_a \equiv \begin{pmatrix} \phi_a^+ \\ \phi_a^0 \end{pmatrix}$ for $a = 1, 2$ and $\tilde{\phi}_a = i\sigma_2\phi_a^*$. σ_2 is the Pauli matrix. $\Gamma_a^{u,d}$ are 3×3 non diagonal Yukawa matrices. To obtain SCPV, Lagrangian should be invariant under CP transformations. Imposition of CP invariance on Lagrangian forces all the couplings to be real and all the phases vanish. CP symmetry is broken spontaneously by the vacuum of the theory. Vacuum expectation values of the Higgs doublets are given as

$$\begin{aligned} \langle 0|\phi_1|0\rangle &= \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \\ \langle 0|\phi_2|0\rangle &= \begin{pmatrix} 0 \\ v_2 e^{i\theta} \end{pmatrix} \end{aligned} \tag{4.3}$$

v_1, v_2 are real and positive and $\sqrt{v_1^2 + v_2^2} = v$. Here v is vev of the neutral Higgs in SM and $v = 174$ GeV. Defining $\tan\beta = v_2/v_1$, we get $v_1 = v\cos\beta$ and $v_2 = v\sin\beta$. Mass matrices of quarks are given as

$$\begin{aligned} M^u &= \Gamma_1^u v_1 + \Gamma_2^u v_2 e^{-i\theta} \qquad (4.4) \\ M^d &= \Gamma_1^d v_1 + \Gamma_2^d v_2 e^{i\theta} \end{aligned}$$

It can be seen from above equations that mass matrices becomes complex because of presence of phase θ. Mass matrices can be diagonalized by going from weak (primed) eigen basis $q'_{L,R};\ q' = u', d'$ to mass (unprimed) eigen basis $q_{L,R};\ q = u, d$ by applying following basis transformations

$$q'_{L,R} = V^q_{L,R}\ q_{L,R} \tag{4.5}$$

In mass basis, fermion mass matrix is given as

$$D^q = V^{q\dagger}_L M^q V^q_R \tag{4.6}$$

Here D^q is diagonal matrix and the diagonal entries of this matrix are the corresponding fermion masses. Part of Lagrangian describing FCNC for down type quarks is given as

$$-\mathcal{L}_{FCNC} = \frac{(2\sqrt{2}G_F)^{1/2}m_b}{\sin\beta\cos\beta} F^d_{ij}\bar{d}_{iL}d_{jR}\phi_H + \text{H.C.}\ , \tag{4.7}$$

Here ϕ_H is the Higgs responsible for all the tree level FCNC. $\phi_H \equiv \cos\beta e^{-i\theta}\phi^0_2 - \sin\beta\ \phi^0_1$ with $\langle 0|\phi^0_H|0\rangle = 0$. Similar expression describe FCNC in case of the up type quarks. In general 2HDM, tree level FCNC interactions give large contributions to K^0-$\bar{K}^0$, B^0_d-$\bar{B}^0_d$ and B^0_s-$\bar{B}^0_s$ mixing. Hence FCNC are required to be eliminated or suppressed. It is known that FCNC can be eliminated by imposing discrete symmetries. This goes under the name of natural flavor conservation (NFC). In the models with NFC all the quarks of a given charge get mass by couplings with only one of the Higgs doublets. Therefore phases in the mass matrices arising from Higgs vev can be absorbed by rephasing the right handed quark fields. Hence quark mass matrices become real. For the case of real mass matrix the matrices $V^q_{L,R}$ also become real. CKM matrix is given as

$$V = V^{u\dagger}_L V^d_L \tag{4.8}$$

Since $V^{u,d}_L$ are real, CKM matrix also becomes real. This is not consistent with the evidences for complex CKM matrix as discussed previously. To get the complex CKM matrix along with SPCV it is required that both the Higgs doublets couple to quarks of the both type. Hence one has to choose the option of suppressing the FCNC instead of eliminating them. FCNC couplings arising due to tree level exchange of ϕ_H are proportional to $1/M_H^2$, where M_H is mass of ϕ_H. Hence FCNC can be suppressed by choosing very large value for M_H. It is shown that CP phase in this model is proportional to M_W/M_H [72]. Here M_W is mass of the W boson. Therefore a large value for M_H leads to small value for CP phase and CKM matrix effectively becomes real. Thus suppressing the FCNC by making the Higgs heavy does not provide complex CKM matrix along with spontaneous CP violation.

The data on neutral meson mixing shows that the FCNC are required to be suppressed most between first two generations of quarks while some FCNC contribution between first and third and between second and third may be allowed. As discussed in the first chapter recent experimental data on CP violating observables in B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixings shows some deviation from SM predictions. One such observable is the angle β of the unitarity triangle. Value of β determined directly from the time dependent asymmetry in the $B \to J/\psi K_S$ decay does not quite agree with the indirect determination which uses $|V_{cb}|, \Delta M_s/\Delta M_d$ and ϵ [32] or only the tree level variables [51, 52]. Second observable is the CP violating phase ϕ_s in the B_s system. ϕ_s inferred [27] using the CDF [21] and D0 [22] measurements of the tagged $B_s \to J/\psi\phi$ decays differs from the SM prediction by about 3σ. These deviations are considered as hints of new physics beyond SM [25, 35, 46, 47, 48, 49, 50, 51, 52, 65]. FCNC may provide the new physics needed to explain these discrepancies. But any mechanism generating the required FCNC should also provide adequate suppression in FCNC between the first two generations.

It was pointed out long ago [79, 80, 81] that one could use a discrete symmetry or some assumptions on flavor structure of mass matrices [82, 83, 96] to obtain selective suppression of FCNC. Here we consider a 2HDM with a specific discrete symmetry (to be called 23 symmetry) which interchanges fermions of second and third generations. This discrete symmetry is a generalization of μ-τ symmetry studied extensively in lepton sector in order to explain the maximal atmospheric mixing [97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120] observed in neutrino oscillation experiments. Neutrino oscillation refers to the conversion of the neutrino of one flavor to other flavor during its propagation. To explain neutrino oscillation it assumed that flavor eigen states of neutrinos denoted by ν_e, ν_μ, ν_τ are different from neutrino mass eigen states denoted by ν_1, ν_2, ν_3. Mass eigen states of neutrinos have definite mass m_1, m_2, and m_3. The neutrino flavor eigen states are related to neutrino mass eigen states by the following relation.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V^{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{4.9}$$

Where V^{PMNS} is the Pontecorvo Maki Nakagawa Sakata (PMNS) matrix given as

$$V^{PMNS} = \begin{pmatrix} c^l_{12}c^l_{13} & s^l_{12}c^l_{13} & s^l_{13}e^{-i\delta^l} \\ -s^l_{12}c^l_{23} - c^l_{12}s^l_{23}s^l_{13}e^{i\delta^l} & c^l_{12}c^l_{23} - s^l_{12}s^l_{23}s^l_{13}e^{i\delta^l} & s^l_{23}c^l_{13} \\ s^l_{12}s^l_{23} - c^l_{12}c^l_{23}s^l_{13}e^{i\delta^l} & -c^l_{12}s^l_{23} - s^l_{12}c^l_{23}s^l_{13}e^{i\delta^l} & c^l_{23}c^l_{13} \end{pmatrix} \tag{4.10}$$

where superscript l stands for leptons. $c^l_{ij} = \cos\theta^l_{ij}$ and $s^l_{ij} = \sin\theta^l_{ij}$ with $i, j = 1, 2, 3$. The angles θ^l_{ij} may be chosen to lie in the range $[0, \frac{\pi}{2}]$. δ^l is the CP phase. Parameters θ^l_{ij} can be determined from neutrino oscillation experiments. No experimental bound on δ^l is available while bounds on angles are as follows [6].

$$\begin{aligned} \sin^2(2\theta^l_{23}) &> 0.92 \\ \sin^2(2\theta^l_{12}) &= 0.87 \pm 0.03 \\ \sin^2(2\theta^l_{13}) &< 0.15 \end{aligned} \tag{4.11}$$

Here the angle θ^l_{23} is referred as atmospheric mixing angle due to its importance in atmospheric neutrino oscillations. It can be seen that $\theta^l_{23} \sim \pi/4$. This is referred as maximal mixing. Various NP models have been suggested to explain maximal value of θ^l_{23} and other neutrino oscillation data. One way to achieve maximal mixing involves use of a discrete symmetry called as μ-τ symmetry. Action of this symmetry amounts to exchange of μ and τ fields.

Generalization of μ-τ symmetry to quark sector was considered in [121, 122, 123, 124, 125] and it was called 23 symmetry. In particular, it was shown [125] that one can obtain a natural understanding of the hierarchy $|V_{ub}| \ll |V_{cb}| \ll |V_{us}|$ among the elements of CKM matrix as an outcome of the mildly broken 23 symmetry. Here, we use the same symmetry to obtain a complex V in the context of SCPV.

In the next section we will describe the action of the symmetry and the form of mass matrices. Explanation about hierarchy of CKM elements in terms symmetry breaking parameters will also be given. In Section 4.2 approximate expressions for the FCNC in terms of symmetry breaking parameters and parameters of mass matrices will be shown. Section 4.3. will describe our numerical analysis. A summary will be given in last section.

4.1 Quark mass matrices and consequences of 23 symmetry

We impose 23 symmetry on the general 2HDM Lagrangian. This symmetry exchanges second and third generation fields : $f_2 \leftrightarrow f_3$. Higgs doublet ϕ_2 transforms as $\phi_2 \to -\phi_2$ under the action of this symmetry. Rest of the fields remain unchanged. The Yukawa couplings of ϕ_a^0 to quarks are given by

$$-\mathcal{L}_{\mathcal{Y}} = \bar{d}_L \Gamma_a^d \phi_a^0 d_R + \bar{u}_L \Gamma_a^u \phi_a^{0*} u_R + \text{H.C.} \ , \tag{4.12}$$

CP invariance makes $\Gamma_a^{u,d}$ real. Imposition of CP and the 23 invariance on the scalar potential results in a CP conserving minimum. Here we achieve SCPV by allowing soft breaking of 23 symmetry in the Higgs potential through a term $\mu_{12}\phi_1^\dagger\phi_2$ whose presence along with other 23 invariant terms violates CP spontaneously [126]. The assumption of $\langle\phi_1^0\rangle = v_1$ and $\langle\phi_2^0\rangle = v_2 e^{i\theta}$ leads to the quark mass matrices given as

$$M^q = \Gamma_1^q v_1 + \Gamma_2^q v_2 e^{i\theta^q} \ , \tag{4.13}$$

with $q = u, d$, $\theta^d = -\theta^u = \theta$ and

$$\Gamma_1^q v_1 \equiv \begin{pmatrix} X^q & A^q & A^q \\ A^q & B^q & C^q \\ A^q & C^q & B^q \end{pmatrix}, \Gamma_2^q v_2 \equiv \begin{pmatrix} 0 & -A^q\epsilon_1^q & A^q\epsilon_1^q \\ -A^q\epsilon_1^q & -B^q\epsilon_2^q & 0 \\ A^q\epsilon_1^q & 0 & B^q\epsilon_2^q \end{pmatrix} . \tag{4.14}$$

In addition to imposing the 23 symmetry, we have also assumed that M^q are symmetric as would be the case in $SO(10)$ with appropriate Higgs representations.

The phase θ in M^q cannot be rotated away and leads to a complex V. This can be seen by considering Jarlskog invariant $Im[Det[M^u M^{u\dagger}, M^d M^{d\dagger}]]$ which is found to be non-zero as long as even one of M^q is complex, i.e. $\epsilon^q_{1,2} \neq 0$ for ($q = u$ or d). Thus unlike earlier models [73], a complex CKM originates here from SCPV. We diagonalize eq.(4.13) using perturbation theory under the assumption that $|\epsilon^q_{1,2}| \leq 1$.

A general mass matrix can be diagonalized with following biunitary transformations as

$$V_L^{q\ \dagger} M^q V_R^q = D^q \tag{4.15}$$

In the present case mass matrices M^q are symmetric. It can be shown that for a symmetric mass matrix $V_R^q = V_L^{q*}$ and the mass matrix can be diagonalized by following transformation.

$$V_L^{q\ T} M^q V_L^q = D^q \ , \tag{4.16}$$

where D^q is the diagonal mass matrix with real and positive masses. V_L^q can be written as

$$V_L^q = V_{0L}^q (1 + i\Theta^q)$$

with $\Theta^{q\dagger} = \Theta^q$. V_{0L}^q diagonalizes the first term in eq.(4.14) and is given by

$$V_{0L}^q = R_{23}(\pi/4) R_{12}(\theta^q_{12}) \ , \tag{4.17}$$

where $R_{12}(\theta^q_{12})$ denotes rotation in the 12 plane by an angle θ^q_{12}. The Θ^q arise from perturbation given by the second term of (4.14). The fermion masses m_{iq} do not get any corrections to first order in $\epsilon^q_{1,2}$ and follow [125] from the first term in eq.(4.14). The mixing angles get corrected to first order in perturbation and are determined by

$$1 + i\Theta^q \equiv \begin{pmatrix} 1 & 0 & \kappa^q_{13} e^{i\theta^q} \\ 0 & 1 & \kappa^q_{23} e^{i(\phi^q + \theta^q)} \\ -\kappa^q_{13} e^{-i\theta^q} & -\kappa^q_{23} e^{-i(\phi^q + \theta^q)} & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2), \tag{4.18}$$

where

$$\begin{aligned} \phi^q &= Arg(1 + \frac{m_{2q}}{m_{3q}} e^{-2i\theta^q}) \ , \\ \kappa^q_{13} &\approx \frac{B^q \epsilon^q_2}{m_{3q}} \frac{\sin(\eta^q + \theta^q_{12})}{\cos \eta^q} \ , \\ \kappa^q_{23} &\approx \frac{B^q \epsilon^q_2}{m_{3q}} \frac{\cos(\eta^q + \theta^q_{12})}{\cos \eta^q} \ . \end{aligned} \tag{4.19}$$

θ^q_{12} is defined through eq.(4.17) while $\tan\eta^q \equiv \frac{\sqrt{2}A^q\epsilon_1^q}{B^q\epsilon_2^q}$. V_L^q along with the above expression for θ^q can be used to obtain the elements of the CKM matrix V:

$$\begin{aligned}
V_{us} &\equiv \sin\theta_c \approx -\sin(\theta^d_{12}-\theta^u_{12}) + \mathcal{O}(\epsilon^2) \\
V_{cb} &\approx \frac{B^d\epsilon^d_2}{m_b\cos\eta^d} e^{i\theta}(c_{\theta_c}\cos(\eta^d+\theta^d_{12})e^{i\phi^d} + s_{\theta_c}\sin(\eta^d+\theta^d_{12})) \\
&\quad -\frac{B^u\epsilon^u_2}{m_t\cos\eta^u} e^{-i(\theta-\phi^u)}\cos(\eta^u+\theta^u_{12}) \;, \\
V_{ub} &\approx \frac{B^d\epsilon^d_2}{m_b\cos\eta^d} e^{i\theta}(c_{\theta_c}\sin(\eta^d+\theta^d_{12}) - e^{i\phi^d}s_{\theta_c}\cos(\eta^d+\theta^d_{12})) \\
&\quad -\frac{B^u\epsilon^u_2}{m_t\cos\eta^u} e^{-i\theta}\sin(\eta^u+\theta^u_{12}) \;,
\end{aligned} \tag{4.20}$$

where $c_{\theta_c} \equiv \cos\theta_c$ and $s_{\theta_c} \equiv \sin\theta_c$. The angle θ_c is the Cabibbo angle. While these mixing angles depend on several parameters, one can get the correct pattern with the choice [125]

$$|X^q| \ll |\sqrt{2}|A^q| \ll |B^q| \sim |C^q| \approx \frac{m_{3q}}{2} \;. \tag{4.21}$$

In the approximation $\eta^q \ll 1$, eq.(4.20) reduces to a simple form

$$V_{cb} \approx \frac{1}{2}(\epsilon^d_2 - \epsilon^u_2)\cos\theta^u_{12} \;\; ; \;\; V_{ub} \approx \frac{1}{2}(\epsilon^d_2 - \epsilon^u_2)\sin\theta^u_{12} \;, \tag{4.22}$$

where we used the approximate relation $B^q \approx \frac{1}{2}m_{3q}$ valid to first order in perturbation theory and also put $\theta = 0$ for illustrative purpose. As can be seen, the choice of parameters as in eq.(4.21) lead to relative suppression of V_{ub} compared to V_{cb} since $\sin\theta^u_{12} \approx \sqrt{-m_u/m_c}$ in this case. The exact numerical diagonalization reveals that the above approximate expressions reproduce V_{us} and V_{cb} correctly while V_{ub} gets significant corrections from the terms neglected in the approximate treatment.

4.2 FCNC and neutral meson mixing

Like other 2HDMs, eq.(4.12) generates FCNC but they are linked here to 23 breaking which also generates V_{ub}, V_{cb}. Both remain small if 23 breaking is small. The flavor changing Higgs couplings in the down quark sector can be obtained by using eqs.(4.12,4.14,4.16). In case of down quarks, one gets

$$-\mathcal{L}_{FCNC} = \frac{(2\sqrt{2}G_F)^{1/2}m_b}{\sin\beta\cos\beta} F^d_{ij}\bar{d}_{iL}d_{jR}\phi_H + \text{H.C.} \;, \tag{4.23}$$

where $\phi_H \equiv \cos\beta e^{-i\theta}\phi_2^0 - \sin\beta\ \phi_1^0, \quad \tan\beta = \frac{v_2}{v_1}$ and

$$m_b F_{ij}^d \equiv (V_L^{d\ T}\Gamma_2^d v_2 e^{i\theta} V_L^d)_{ij}\ , \tag{4.24}$$

and we have introduced the physical third generation quark mass m_b as an overall normalization to make F_{ij}^d dimensionless. Analogous expressions hold in case of the up quarks. Since Γ_2^d is symmetric. F^d also become symmetric. Hence $|F_{ji}^d| = |F_{ij}^d|$. Eqs.(4.14,4.19) and (4.24) are used to show that

$$\begin{aligned}
F_{12}^d &\approx -\frac{B^d}{m_b} c_{12}^d s_{12}^d \epsilon_2^{d^2} + \mathcal{O}(\eta^d, \epsilon^3) \sim 6.8 \cdot 10^{-4}\ , \\
F_{13}^d &\approx \frac{B^d}{m_b} s_{12}^d \epsilon_2^d + \mathcal{O}(\eta^d, \epsilon^2) \sim 8.8 \cdot 10^{-3}\ , \\
F_{23}^d &\approx \frac{B^d}{m_b} c_{12}^d \epsilon_2^d + \mathcal{O}(\eta^d, \epsilon^2) \sim 3.8 \cdot 10^{-2}\ ,
\end{aligned} \tag{4.25}$$

Above expressions show that there is a clear hierarchy between the strengths of various FCNC couplings. The quoted numerical values are obtained using $B^d \approx \frac{1}{2}m_b$, $s_{12}^d \approx \sin\theta_c$ and $\epsilon_2^d \approx 2V_{cb}$, see eq.(4.20).

The strength and hierarchy of F_{ij}^q can be probed through flavor changing transitions, particularly through $P^0 - \bar{P}^0$, $(P = K, B_d, B_s, D)$ mixing. This mixing is generated in the SM at 1-loop level and thus can become comparable to the tree level FC effects in spite of the suppression in F_{ij}^q. $P^0 - \bar{P}^0$ mixing is induced by the element $M_{12}^P \equiv \langle P^0 | \mathcal{H}_{eff} | \bar{P}^0 \rangle$. The effective Hamiltonian here contains two terms $\mathcal{H}_{eff}^{SM} + \mathcal{H}_{eff}^H$ where the second term is induced from the Higgs exchanges. The charged Higgs contributes to $\mathcal{H}_{eff}^H$ through the box diagrams which have been studied in models with or without the tree level FCNC. In chapter 3 we had discussed the charged Higgs effects which were similar to the charged Higgs effects in the type - II 2HDM. In chapter 5 we will describe charged Higgs effects in a 2HDM in which FCNC can be eliminated without imposing any discrete symmetries. Here we will describe the FC effects due to neutral Higgs which contributes at tree level. We shall assume that this contribution dominates over the charged Higgs contribution and study its effects here. In this case the $\mathcal{H}_{eff}^H$ follows from eq.(4.23) in a straightforward manner:

$$\begin{aligned}\mathcal{H}^H_{eff}(ij) &= -\frac{2\sqrt{2}G_F m_b^2}{\sin^2 2\beta M_\alpha^2}(F^{d^2}_{ij}C^2_\alpha(\bar{d}_{iL}d_{jR})^2 + F^{d*^2}_{ji}C^{*2}_\alpha(\bar{d}_{iR}d_{jL})^2 \\ &+ 2F^d_{ij}F^{d*}_{ji}|C_\alpha|^2(\bar{d}_{iL}d_{jR})(\bar{d}_{iR}d_{jL}))\ , \end{aligned} \tag{4.26}$$

where $ij = 12, 13, 23$ respectively denote $\mathcal{H}^H_{eff}$ for the K, B_d, B_s mesons. The model contains three real Higgs fields H_α whose masses M_α appear above. The real and imaginary parts of $\sqrt{2}\phi_H \equiv R + iI$ in eq.(4.23) are related to H_α through a 3×3 orthogonal matrix O and one can write $\sqrt{2}\phi_H = (O_{R\alpha} + iO_{I\alpha})H_\alpha \equiv C_\alpha H_\alpha$ which defines the complex parameters C_α appearing in eq.(4.26).

The above effective Hamiltonian would receive QCD corrections. The relevant QCD corrections for the scalar operators are known [39, 127] to be large. But unlike in the standard model, the Higgs induced contributions involve several unknown parameters which are not well-determined. In view of this we do not consider the effects of QCD corrections in the Higgs contribution given in eq.(4.26).

Define $F^d_{ij} \equiv |F^d_{ij}|e^{is_{ij}}, C_\alpha \equiv |C_\alpha|e^{i\eta_\alpha}$. We then obtain the Higgs contribution to $M_{12}(P)$ from eq.(4.26) by using $|F^q_{ij}| = |F^q_{ji}|$ and the vacuum saturation approximation :

$$M^H_{12}(ij) = \frac{\sqrt{2}G_F m_b^2 f_P^2 M_P |C_\alpha|^2 |F^d_{ij}|^2}{6\sin^2 2\beta M_\alpha^2} Q_{ij} e^{i(s_{ij}-s_{ji})} \tag{4.27}$$

with

$$Q_{ij} = \left[A_P - 1 + 10A_P\sin^2(\frac{s_{ij}+s_{ji}}{2} + \eta_\alpha)\right]$$

and $A_P = \left(\frac{M_P}{m_a+m_b}\right)^2$, $(P^0 \equiv \bar{a}b)$ and as before $ij = 12, 13, 23$ refer to the $P = K, B_d, B_s$ mesons. $\Delta M^H(ij) \equiv 2|M^H_{12}(ij)|$ following from eq.(4.27) depends on several unknown parameters in the Higgs sector while its phase is determined by the phases of F^d_{ij} and η_α. The phases of F^d_{ij} depend on parameters in M^q while η_α depends on the parameters in the Higgs potential. For illustration, we retain the contribution of the lightest Higgs $\alpha \equiv H$ in eq.(4.27) and choose $M_H = 200$ GeV, $\sin^2 2\beta = 1, |C_H|^2 = 1/2, Q_{ij} = 1/2(Q_{ij})_{max}$. The numerical values of F^d_{ij} in eq.(4.25) then give

$$r^P \equiv |\frac{\Delta M^H(P)}{\Delta M^{exp}(P)}| \approx (0.22, 0.13, 0.07) \tag{4.28}$$

respectively for $P = B_d, B_s, K$. It follows that effect of the FCNC do get suppressed in the model even for relatively light Higgs and one does not need very heavy ($M_H \geq$ TeV) Higgs as in models with unsuppressed FCNC. But the Higgs induced effects are not negligible and would imply significant new physics contributions to the meson mixing. Such contributions to $M_{12}^{B_d,B_s} \equiv M_{12}^{d,s}$ have been parameterized in a model independent manner [25, 35, 46, 47, 48, 49, 50, 51, 52, 65] by

$$M_{12}^{d,s} = M_{12}^{d,s;SM}(1 + \kappa^{d,s} e^{i\sigma^{d,s}}).$$

The parameters $\kappa^{d,s}$ and $\sigma^{d,s}$ represent the effects of NP such as Higgs induced FCNC in this case. The SM contribution appearing above is given by

$$M_{12}^{q;SM} = \frac{G_F^2 m_W^2 m_{B_q} f_{B_q}^2 B_q \eta_B}{12\pi^2} (V_{3q}^* V_{33})^2 S_0(x_t) \ , \tag{4.29}$$

with $S_0(x_t) \approx 2.3$ for $m_t \approx 161$ GeV,B_q, f_{B_q} are standard parameters entering the expressions of the hadronic matrix elements and η_B represents the QCD corrections to the SM operator induced through the box diagram. Note that element V_{3q} entering above is not directly measured and its determination in the SM uses loop induced observables such as ϵ or B^0-$\bar{B}^0$ mixing to which new physics can contribute. $M_{12}^{q;SM}$ in this way indirectly depends on the NP. Its determination is done in two ways. In the general analysis of UTfit [14] and CKMfitter groups [128], the CKM phase (equivalently the Wolfenstein parameter $\bar{\eta}$) is also treated as unknown and is fitted along with the new physics parameters given above to determine V_{3q} and hence $M_{12}^{q;SM}$. Alternatively, one can use the tree level determination of γ to obtain $\bar{\eta}$ and then use it to determine $M_{12}^{d,s:SM}$. We will follow the latter option here. The hadronic matrix elements entering $M_{12}^{q;SM}$ are determined using lattice results and we will specifically use predictions based on [129]. The SM predictions along with the experimental determination of $\Delta M^{d,s} \equiv 2|M_{12}^{d,s}|$ are used in [51, 52] to obtain

$$\rho^d \equiv \left|\frac{\Delta M^d}{\Delta M_{SM}^d}\right| = 0.97 \pm 0.39, \rho^s \equiv \left|\frac{\Delta M^s}{\Delta M_{SM}^s}\right| = 1.08 \pm 0.19 \ . \tag{4.30}$$

It was argued in [51, 52] that the experimental value of $\gamma = (65 \pm 20)^\circ$ from the tree level measurement already provides a hint for new physics if $|V_{ub}|$ is close to its value $(4.4 \pm 0.7) \cdot 10^{-3}$ determined from the inclusive $b \to ul\nu_l$ transition. The phase ϕ_d of M_{12}^d is measured through the time-dependent CP asymmetry in the decay $B_d \rightarrow J/\psi K_S$, $\phi_d = 43.4^\circ \pm 2.5^\circ$. The corresponding phase in $M_{12}^{d;SM}$ gets determined by γ and the ratio $\frac{|V_{ub}|}{|V_{cb}|}$ and is given by [51, 52] $\phi_d^{SM} = 53.4^\circ \pm 3.8^\circ$ if one uses the inclusive $|V_{ub}|$. This implies a non-zero new physics contribution $\phi_d^{NP} = Arg(1+\kappa^d e^{i\sigma^d}) = -(10.1 \pm 4.6)^\circ$ which in the present case can come from the Higgs exchanges.

The values of ρ^d and ϕ_d have been used [51, 52] to determine allowed ranges in the parameters κ^d and σ^d. This is displayed in Fig.(4.1) in case of the B_d mesons. We can confront these observations now with the specific predictions in the present case.

4.3 Numerical analysis

Our strategy is to first determine parameters in M^q from the quark masses and mixing and then use them to determine F_{ij}^q which are used to obtain information on $M_{12}^H(P)$. As we discussed analytically, one needs breaking of 23 symmetry in order to obtain non-zero mixing angle. But once this is done, one has enough number of parameters to determine all quark masses and mixing angles. In fact, one has large class of solutions which reproduce the correct spectrum. We find them by minimizing the following χ^2:

$$\chi^2 = \sum_{i=1,10} \left(\frac{E_i(x) - \bar{E}_i}{\delta E_i} \right)^2 ,$$

where $E_i(x)$ represent predictions of six quark masses, three moduli $|V_{us}|, |V_{cb}|, |V_{ub}|$ and CP violation parameter Jarlskog invariant J calculated as functions of parameters of M^q. Jarlskog invariant is a CP violating quantity which remains invariant under rephasing of the quark fields. We have used following expression in our calculations.

$$J = Im(V_{ud} V_{us}^* V_{td}^* V_{ts}) \tag{4.31}$$

The quantities $\bar{E}_i \pm \delta E_i$ are values of the above mentioned parameters determined from experiments. We choose quark masses at M_Z given in [130] and all the CKM elements except $|V_{ub}|$ as in [6]. For the latter, we use the value $(4.4 \pm 0.3) \cdot 10^{-3}$ based on the determination [51, 52] from the inclusive b decays. We find many

	X (GeV)	A (GeV)	B (GeV)	C (GeV)	ϵ_1	ϵ_2
up	0.000807	0.0321	90.67	−90.05	0.299	0.0797
down	0.003	0.0188	1.54	−1.46	−0.219	0.0279

Table 4.1: An example of fit to the quark masses and mixing angles corresponding to $\chi^2 = 6.9 \cdot 10^{-7}$ and θ=-0.7789

solutions giving excellent fits with $\chi^2 \lesssim 10^{-7}$. One specific example is given in the Table (4.1). The parameters of the table lead to

$$\begin{aligned}
F^d_{12} &= \ \ (0.72 - 4.64\ i) \cdot 10^{-4}, & F^d_{21} &= \ \ (0.44 + 4.68\ i) \cdot 10^{-4};\\
F^d_{13} &= -(0.36 + 1.74\ i) \cdot 10^{-2}, & F^d_{31} &= -(1.73 + 0.41\ i) \cdot 10^{-2};\\
F^d_{23} &= \ \ (1.32 + 3.73\ i) \cdot 10^{-2}, & F^d_{32} &= -(3.73 + 1.34\ i) \cdot 10^{-2};\\
F^u_{12} &= \ \ (-3.14 + 2.05\ i) \cdot 10^{-2}, & F^u_{21} &= -(0.95 + 3.63\ i) \cdot 10^{-2}\ .
\end{aligned} \tag{4.32}$$

which are similar to the rough estimates in eq.(4.25). We note that the phases of the couplings are determined in the specific bases in which the CKM matrix assumes the standard form given in [6] corresponding to real $V_{ud}, V_{us}, V_{cb}, V_{tb}$.

The above fits strongly depend on some of the $\epsilon^q_{1,2}$ being non-zero since if they vanish then $|V_{ub}|, |V_{cb}|$ and CP violation also vanish. However, we could get excellent fits with $|\epsilon^q_{1,2}| < 0.2$ showing that an approximately broken 23 symmetry provides a very good description of the quark spectrum. In an alternative analysis, we fixed B^q, C^q from $|B^q - C^q| = m_{3q}, |B^q + C^q| = m_{2q}$ which correspond to the 23 symmetric limit in the two generation case. This limit is found to be quite good and gives good fits with $\chi^2 \lesssim 1$ when it is minimized with respect to the remaining nine parameters.

The predictability of the scheme comes from the fact that each set of parameters of M^q determined as above completely fix (complex) FCNC strengths $F^{u,d}_{ij}$ in all 18 independent real quantities. We use these predicted values to calculate Higgs contribution to M^P_{12} by randomly varying unknown parameters of eq.(4.27). We retain contribution of only one Higgs and vary its mass from 100-500 GeV. $|C_\alpha|$, $\sin^2 2\beta$ and the phase η_α are varied over their full range namely, $0-1$ and $0-2\pi$ respectively. We require that (i) $\rho^{d,s}$ and ϕ_d lie in the allowed 1σ range (ii) The Higgs contribution to the $D^0 - \bar{D}^0$ mixing amplitude satisfies the bound $|M^{DH}_{12}| < 2.2 \cdot 10^{-14}$ GeV derived in [131] from the BaBar and Belle measurements

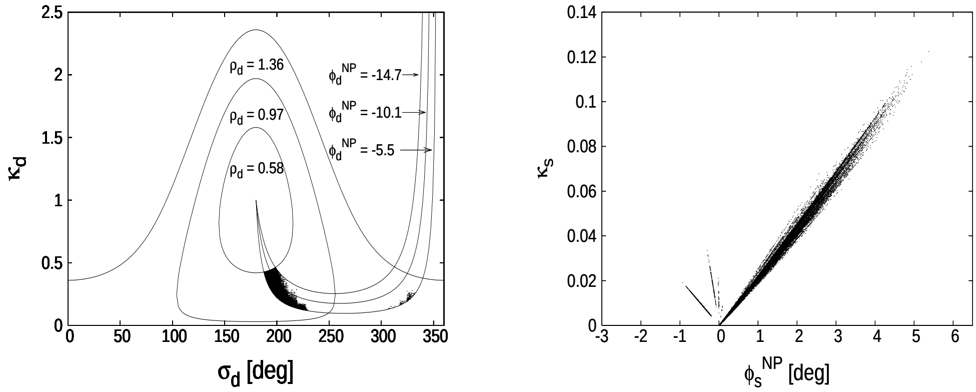

Figure 4.1: Allowed regions in the κ^d, σ^d (left panel) and κ^s, ϕ_s^{NP} (right panel) planes following from the inclusive determination of $|V_{ub}|$ and JLQCD result for the hadronic matrix element. Solid lines in the left panel corresponds to 1σ allowed values for ρ^d, ϕ_d^{NP} in model independent study. The scattered plots in both panels correspond to the predictions of the present model.

(iii) the Higgs contribution to the $K^0 - \bar{K}^0$ mass difference and to ϵ is an order of magnitude less than their central experimental values. Combined results of this analysis for several sets of allowed F_{ij}^q are shown as scattered plot in Fig.(4.1).

The solid curves describe restrictions on κ^d, σ^d following from eq.(4.30) and the measured value of ϕ_d in a model independent study. In the present case, the allowed values of κ^d, σ^d are indirectly affected by restrictions coming from mixing of other mesons as well since the same set of Higgs parameters determine these mixings. Thus simultaneous imposition of the above mentioned constraints considerably restrict the allowed ranges in parameter space shown as scattered plot in Fig.(4.1). σ^d is restricted in such a way that the Higgs contributes negatively to ρ^d (in most parameter space) and reduces the value of ρ^d compared to the SM. κ^d and σ^d are restricted in the range $0.2 < \kappa^d < 0.46$, $185^\circ \lesssim \sigma^d \lesssim 229^\circ$ which correspond to $0.58 \lesssim \rho^d \lesssim 0.9$ and $\phi_d^{NP} \approx -(5-15)^\circ$.

The right panel in Fig.(4.1) shows the predictions of κ^s and possible new physics phase $\phi_s^{NP} \equiv Arg(1 + \kappa^s e^{i\sigma^s})$ in M_{12}^s. The allowed values of κ^s after the combined constraints from all sources are relatively small ≤ 0.1. This also results in a small ϕ_s^{NP} although the Higgs induced CP phase σ^s could be large.

The phase ϕ_s has been measured by the $D0$ and the CDF [21, 22] group from their analysis of the time-dependent CP asymmetry in the tagged $B_s^0 \to J/\psi\phi$ decays. The relevant strong CP phases are extracted by the $D0$ group from the $B_d^0 \to J/\psi K^*$ decays using SU(3) symmetry. Under this assumption, they obtain $\phi_s \equiv -2\beta_s^{SM} - \phi_s^{NP} \equiv -0.57^{+0.24}_{-0.30}$, where $2\beta_s^{SM} \approx 0.04$. The predicted values of ϕ_s^{NP} in the present approach are consistent with this result. UTfit group has performed a detailed analysis of the above $D0$ results combining it with other CP violating observables without making specific assumption on the strong phases. They find [27] $\phi_s \equiv -0.34 \pm 0.09$ which deviates from the SM results and also from the ϕ_s^{NP} predicted here. Possible improvement in the value of ϕ_s at LHC would therefore provide a crucial test of the presently studied scenario. It should be stressed that our numerical analysis is aimed at reconciling possible 2σ deviations from the SM predictions in the CP phase ϕ_d in $B_d \to J/\psi K_s$ decay which arise if $|V_{ub}|$ is close to the inclusive value. The predictions on ϕ_s may change in a more general analysis using the average or exclusive value of $|V_{ub}|$.

It is found that the $D^0 - \bar{D}^0$ mixing plays an important role in ruling out some of the regions in parameter space and in most of the allowed regions $|M_{12}^{DH}|$ remains close to the limit $2.2 \cdot 10^{-14}$ GeV.

4.4 Summary

In this chapter we addressed the problem [78, 132] of obtaining a phenomenological consistent picture of SCPV. This is an important issue in view of the fact the earlier theories of SCPV led to a real CKM matrix while recent observations need it to be complex. The proposed picture is phenomenologically consistent and does not need very heavy Higgs to suppress FCNC present in general multi Higgs models. The hierarchy in FCNC eq.(4.25), obtained here through a discrete symmetry has observable consequences. The effect of Higgs is to reduce the $B_d^0 - \bar{B}_d^0$ mixing amplitude compared to the standard model prediction. The $D^0 - \bar{D}^0$ mixing can be close to the bound derived from observation [131]. The new contribution to the magnitude of $B_s^0 - \bar{B}_s^0$ mixing is relatively small. The Higgs induced phase in this mixing is found to be relatively low but much larger than in the SM.

Noteworthy feature of the proposal is universality of the discrete symmetry used here. The generalized $\mu - \tau$ symmetry used here can explain large atmospheric mixing angle in the manner proposed in [125] on one hand and can also account for the desirable features of the quark mixing and CP violation as discussed here.

Chapter 5

2HDM without FCNC

In this chapter we present study of a 2HDM in which tree level FCNC are absent while charged Higgs interactions contain phases which can give new CP phases to neutral meson mixing. Tree level FCNC also are absent in the 2HDM with NFC. But the charged Higgs interaction in 2HDM with NFC does not provide any new phases which are not present in SM. As it was discussed in first chapter, new phases not present in SM may be required to explain deviation of experimentally determined value of CP violating observables in B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing from their values predicted in SM. To obtain 2HDM in which tree level FCNC are absent and charged Higgs interaction have new phases not present in SM we make use of flavor symmetries.

Flavor symmetries are often invoked in SM and beyond to restrict the structure of Yukawa couplings all of which cannot be directly determined from experiments. Either one can impose some symmetry and study its consequences for fermion flavor structure or one can use experimental information to guess possible flavor symmetries under which fermion mass matrices remain invariant. Advantage of this approach is that it directly relates the experimental observations to some symmetries of mass matrices. But relating symmetries of mass matrices to symmetry of Lagrangian is not straightforward in this bottom up approach. Assumption that symmetries of mass matrices form a sub-group of the full symmetry at the Lagrangian level can lead to identification of possible interesting flavor symmetries and this approach has been pursued in [134, 135, 136]. General study of this approach, particularly, the relation between the structures of mass matrices and symmetries enjoyed by them was recently made in [135, 136, 137]. Lam in his study of the neutrino mass matrix found that an arbitrary neutrino mass matrix M^ν can always be linked to a symmetry S which leaves M_ν invariant, $S^T M^\nu S = M^\nu$. It was then pointed out by Grimus, Lavoura and Ludl [137] that any Hermitian mass matrix $M_f M_f^\dagger$ obtained from a fermion mass matrix M^f always possesses a symmetry $G^f = U(1) \times U(1) \times U(1)$ and the corresponding G for the mass matrix of the Majorana neutrinos is $Z_2 \times Z_2 \times Z_2$. This is easy to prove. If V is a unitary matrix which diagonalizes a Hermitian mass matrix $M^f M^{f^\dagger}$, i.e.

$$M^f M^{f\dagger} = V D^2 V^\dagger$$

with a diagonal D then one can always construct an $S = V P(\alpha_i) V^\dagger$ which leaves $M^f M^{f\dagger}$ invariant, i.e. $S^\dagger M^f M^{f\dagger} S = M^f M^{f\dagger}$. $P(\alpha_i)$ refers to a diagonal phase matrix with phases α_i and S therefore generates a $U(1) \times U(1) \times U(1)$ symmetry.

The above reasoning can easily be generalized to non Hermitian mass matrices. Define

$$S_{L,R} = V_{L,R} P(\alpha_i) V_{L,R}^\dagger \ , \tag{5.1}$$

where $V_{L,R}$ are unitary matrices diagonalizing a general non-Hermitian M

$$M = V_L D V_R^\dagger \ . \tag{5.2}$$

It then follows that $S_{L,R}$ define a symmetry of M:

$$S_L^\dagger M S_R = M \ . \tag{5.3}$$

The symmetries $S_{L,R}$ and the resulting form of M may look complicated depending on the choice of $V_{L,R}$ but eq.(5.3) is equivalent to the statement of the fermion number conservation of each generation by its mass term. This is trivial to see. Let $f_{L,R}$ denote the fermion fields in their mass basis corresponding to a diagonal mass matrix D. In this basis, individual fermion number is conserved by the mass term:

$$P^\dagger(\alpha_i) D P(\alpha_i) = D \ . \tag{5.4}$$

Arbitrary weak basis would be defined as $f'_{L,R} = V_{L,R} f_{L,R}$. The phase invariance of the mass term shown in eq.(5.4) then manifests itself in the weak basis as invariance under $S_{L,R}$ given in eq.(5.3). This can be seen by multiplying eq.(5.4) by $V_L(V_R^\dagger)$ from left(right) and using eq.(5.2). The Majorana mass terms for neutrinos do not conserve fermion number but mass of each neutrino is Z_2 invariant which reflects as $Z_2 \times Z_2 \times Z_2$ symmetry discussed in [137].

Eq.(5.3) remains true for any choice of $V_{L,R}$. This invariance thus holds for any choice of mass matrix as emphasized in [137]. However, if one wishes to understand symmetries of mass matrices as arising from some flavor symmetries at the Lagrangian level then only specific class of symmetry transformations $S_{L,R}$ would be admissible. It is desirable to specify $S_{L,R}$ a priory in this case and put some reasonable requirement on them. In this case mass matrix symmetries may have non-trivial content and can restrict the structure of the allowed theories. This is made explicit below within 2HDM.

5.1 Mass matrix symmetries and 2HDM

Two Higgs doublet models contain the following Yukawa couplings which provide sources of mass and additional flavor violations:

$$-\mathcal{L}_Y = \bar{Q}'_L(\Gamma_1^d\phi_1 + \Gamma_2^d\phi_2)d'_R + \bar{Q}'_L(\Gamma_1^u\tilde{\phi}_1 + \Gamma_2^u\tilde{\phi}_2)u'_R + \text{H.c.} \;, \tag{5.5}$$

where Γ_a^q $(a = 1, 2; q = u, d)$ are matrices in the generation space. $\phi_{1,2}$ denote Higgs doublets and $\tilde{\phi}_a = i\sigma_2\phi_a^*$. Q'_{iL} refer to three generations of doublet quarks and primed fields in the above equation refer to various quark fields in the weak basis. The neutral component of a specific linear combination of the Higgs fields namely,

$$\phi \equiv \cos\beta\phi_1 + \sin\beta e^{-i\theta}\phi_2 \tag{5.6}$$

is responsible for the mass generation

$$M^q = v(\cos\beta\Gamma_1^q + \sin\beta\Gamma_2^q e^{i\theta_q}) = V_L^q D^q V_R^{q\dagger} \;, \tag{5.7}$$

where

$$\left\langle\phi_1^0\right\rangle = v\cos\beta \;;\; \left\langle\phi_2^0\right\rangle = v\sin\beta e^{i\theta}$$

$v \sim 174$ GeV and $\theta^d = -\theta^u = \theta$. The matrices $V_{L,R}^q$ diagonalize M^q and also determine its symmetry

$$S_{L,R}^q = V_{L,R}^q P^q V_{L,R}^{q\dagger} \;, \tag{5.8}$$

$$S_L^{q\dagger} M^q S_R^q = M^q \ . \tag{5.9}$$

P^q is a diagonal phase matrix

$$P^q = \text{diag.}(e^{i\alpha_1^q}, e^{i\alpha_2^q}, e^{i\alpha_3^q})$$

In the most general situation, the matrices $S_{L,R}^q$ generate two independent $U(1) \times U(1) \times U(1)$ symmetries G^u and G^d for the up and the down quarks mass matrices respectively. $G^u \times G^d$ invariance holds for arbitrary M^u, M^d and specific $S_{L,R}^q$ determined from them. We put a mild requirement on possible $S_{L,R}^q$ namely that the form of $S_{L,R}^q$ be independent of the parameters $\tan\beta$ and θ which are determined entirely in the Higgs sector. This innocuous requirement has important consequences. Using the definition, eq.(5.7) of mass matrices, it immediately leads to

$$S_L^{q\dagger} \Gamma_i^q S_R^q = \Gamma_i^q \qquad i = 1, 2 \ . \tag{5.10}$$

This shows that not only the total mass matrix but the individual Yukawa couplings should also respect the symmetry. Let us parameterize Γ_i^q as

$$\Gamma_i^q \equiv V_L^q \tilde{\Gamma}_i^q V_R^{q\dagger} \ . \tag{5.11}$$

Eqs. (5.8,5.10) then imply

$$P^{q\dagger} \tilde{\Gamma}_i^q P^q = \tilde{\Gamma}_i^q \ . \tag{5.12}$$

If G^u, G^d refer to the full $U(1) \times U(1) \times U(1)$ symmetry with totally independent α_i^q then the only non-trivial solution of eq.(5.12) is a diagonal $\tilde{\Gamma}_i^q$ for every i and q. Yukawa couplings are then given as

$$\Gamma_i^q = V_L^q \gamma_i^q V_R^{q\dagger} \ , \tag{5.13}$$

where γ_i^q are diagonal matrices with complex entries. More general forms for $\tilde{\Gamma}_i^q$ are allowed if one demands invariance with respect to subgroups of $G^u \times G^d$ and we will discuss this case in the next section.

Eq.(5.13) has powerful phenomenological implications. To see these, let us note that the Higgs combination orthogonal to one in eq.(5.6) namely,

$$\phi_H \equiv -\sin\beta\phi_1 + \cos\beta\phi_2 e^{-i\theta} \tag{5.14}$$

generates all the Higgs induced flavor violations. The couplings of the neutral component ϕ_H^0 are given as

$$-\mathcal{L}_Y^0 = \frac{1}{v}\bar{q}_L F^q q_R \phi_H^0 + \text{H.c.} \tag{5.15}$$

with

$$\begin{aligned} F^q &\equiv V_L^{q\dagger} v(-\sin\beta\Gamma_1^q + \cos\beta\Gamma_2^q e^{i\theta_q})V_R^q \ , \\ &= v(-\sin\beta\gamma_1^q + \cos\beta\gamma_2^q e^{i\theta_q}) \ , \end{aligned} \tag{5.16}$$

where we have used eq.(5.13) to obtain the second line. It is seen that the FCNC matrix F^q becomes diagonal along with the mass matrices and the tree level FCNC are absent. But the phases of F_{ii}^q cannot be removed in the process of making the quark masses real and remain as physical parameters. The charged component of ϕ_H correspond to the physical charged Higgs field and its couplings are given in our case by

$$-\mathcal{L}_{H^+} = \frac{H^+}{v}\left(\bar{u}_{iL}V_{ij}F_{jj}^d d_{jR} - \bar{u}_{iR}V_{ij}F_{ii}^{u*} d_{jL}\right) + \text{H.c.} \tag{5.17}$$

The above couplings are similar to the charged Higgs couplings in 2HDM of type-I and II. In those models, F_{ii}^q are proportional to the corresponding quark masses m_i^q and are real. Here F_{ii}^q are general complex numbers which can provide new phases in the $B_{d,s}$-$\bar{B}_{d,s}$ mixing.

Let us make several important remarks:

(1) An interesting class of 2HDM without the tree level FCNC have been recently discussed in [138]. These models are obtained from general 2HDM by assuming that two Yukawa couplings Γ_1^q and Γ_2^q are proportional to each other. In the present case, the two Yukawa coupling matrices are not proportional to each other but the tree level FCNC are still absent. The phases of F_{ii}^q in the charged Higgs couplings are dependent on the flavor index i unlike in models of [138] which are characterized by two universal phases: one for the up type quarks and the other for the down type quarks. If the diagonal matrices γ_1^q and γ_2^q in eq.(5.13) are proportional then the present class of models reduce to the one in [138].

(2) Reference [139] proposed an idea of shared flavor symmetry and identifies

2HDM models without the tree level FCNC. According to this, the Yukawa coupling F^q and the mass matrix M^q share a flavor symmetry which results in the absence of FCNC as long as this symmetry is unbroken. Our eq.(5.8) explicitly defines this shared symmetry. Reference [139] assumes that the elements of the transformation matrix $V_L^q V_R^{q\dagger}$ are pure numbers in the symmetric limit while our definition of the symmetry and consequent absence of FCNC is more general and holds for arbitrary form of M^q. We are assuming in general that $S_L^u \neq S_L^d$. Such inequality can arise from some more fundamental flavor symmetry once $SU(2)_L \times U(1)$ is broken. There are well-known specific examples. For example D_4 symmetry [140] broken in a specific way leads to trivial phase symmetry for the (diagonal) charged lepton matrix and μ-τ symmetry for the neutrino mass matrix. Similarly, D_4 [135, 136] and A_4 [141] symmetries in the quark sectors also are shown to lead to effectively different S_L^u and S_L^d for the quark mass matrices.
(3) If $S_L^u \neq S_L^d$ then neither the Yukawa interactions (5.5) nor the charged current weak interactions remain invariant under symmetries of the mass matrices. This means that radiative corrections will not preserve [142] the structure implied by eq.(5.13). This equation thus should be regarded as a means of identifying class of models without the tree level FCNC. Just as in case of the 2HDM of type-I and type-II as well as the aligned models of [138], the full Lagrangian of the present model is formally invariant under the fermion number transformation $q_{iL,R} \to e^{i\alpha_i^q} q_{iL,R}$ accompanied by the change in the CKM matrix elements $V_{ij} \to e^{i\alpha_i^u} V_{ij} e^{-i\alpha_j^d}$. As a consequence of this all the radiative corrections in the model would display structure similar to the one obtained in the Minimal Flavor Violating [55] models.

5.2 Modeling the symmetries:

We have used the $G^u \times G^d$ symmetry of the quark mass matrices to identify models without the tree level FCNC. As already stated these symmetries do not commute with the $SU(2)_L$ group and are effective symmetries of the quark mass matrices in general. We wish to discuss here two examples. In the first example, the symmetries S_L^u and S_L^d are identified and thus can be imposed at the Lagrangian level. The other model provides a specific realization of the Yukawa alignment discussed in [138] and is a special case of the general 2HDM identified here.

Let us assume that

$$S_L^u = S_L^d \equiv S_L$$

It then follows from the defining equation (5.9) that

$$P^u V = V P^d \ . \tag{5.18}$$

where $V = V_L^{u\dagger} V_L^q$ defines the CKM matrix. P^u, P^d are a priori independent phase matrices generating $G^u \times G^d$. From the fact that the diagonal elements of V are non-zero and O(1), one immediately concludes that $P^u = P^d$. Moreover, if all entries in V are taken to be non-zero then one is further led to $P^u = P^d = I$ and the symmetry $S_L^u = S_L^d \equiv S_L$ becomes trivial. But since, the elements of V are known to be hierarchical one may assume as a first approximation

$$V \approx \begin{pmatrix} 1 & \lambda & 0 \\ -\lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{5.19}$$

This form is consistent with eq.(5.18) provided

$$\begin{aligned} \alpha_1^d &= \alpha_2^d = \alpha_1^u = \alpha_2^u \equiv \alpha \ , \\ \alpha_3^u &= \alpha_3^d \equiv \eta \ . \end{aligned} \tag{5.20}$$

This relation defines a $U(1) \times U(1)$ symmetry with a non-trivial S_L:

$$(S_L)_{ij} = \delta_{ij} e^{i\alpha} + (e^{i\eta} - e^{i\alpha})(V_L)_{i3}(V_L)^*_{j3} \ . \tag{5.21}$$

Imposition of this symmetry would thus lead to approximately correct description of the quark mixing. Specifically, let us impose

$$q'_L \to S_L q'_L \ ; \ q'_R \to S_R^q q'_R \ ,$$

as symmetries on the full Lagrangian. Here S_L is defined in eq.(5.21) and S_R^q is obtained from it by the replacement $V_L \to V_{qR}$.

The structure of the Yukawa couplings invariant under the above symmetry is given by:

$$\Gamma_1^u = V_L V_1^u \gamma_1^u V_R^{u\dagger} \quad ; \quad \Gamma_2^u = V_L V_2^u \gamma_2^u V_R^{u\dagger} \ ,$$
$$\Gamma_1^d = V_L V_1^d \gamma_1^d V_R^{d\dagger} \quad ; \quad \Gamma_2^d = V_L V_2^d \gamma_2^d V_R^{d\dagger} \ , \tag{5.22}$$

where $\gamma_{1,2}^q$ are diagonal matrices as discussed earlier and $V_1^{u,d}$ and $V_2^{u,d}$ are independent matrices with a block diagonal structure having a non-trivial 12 block. The mass matrices M^q have simple structure in the basis defined by $q'_L \to \tilde{q}_L = V_L q'_L$ and $q'_R \to \tilde{q}_R = V_R^q q'_R$:

$$\tilde{M}^q = \begin{pmatrix} X^q & A^q & 0 \\ B^q & Y^q & 0 \\ 0 & 0 & m_3^q \end{pmatrix} , \tag{5.23}$$

where $m_3^u = m_t$; $m_3^d = m_b$. Γ_i^q also have similar structure in the same basis. As expected, $\tilde{M}^q$ defined above leads to the CKM matrix as given in eq.(5.19).

The above considerations do not dictate any specific choice of V_L and remain true for arbitrary V_L. This may come from other independent considerations such as quark lepton unification. As an interesting example, let us assume that the S_L defined as above also defines the symmetry of the neutrino mass matrix in the flavor basis. Then V_L can be related to the leptonic mixing matrix in which case one can choose to a good approximation $(V_L)_{13} = 0$ and $(V_L)_{23} = -(V_L)_{33} = -\frac{1}{\sqrt{2}}$. Then

$$S_L = \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & \frac{1}{2}(e^{i\eta} + e^{i\alpha}) & -\frac{1}{2}(e^{i\eta} - e^{i\alpha}) \\ 0 & -\frac{1}{2}(e^{i\eta} - e^{i\alpha}) & \frac{1}{2}(e^{i\eta} + e^{i\alpha}) \end{pmatrix} \tag{5.24}$$

This corresponds to the generalized μ-τ symmetry which exchanges the second and third generation fermions if $\alpha = 0, \eta = \pi$. This symmetry was earlier discussed in the context of quarks in chapter 4 and also in [74, 121, 122, 124, 125, 143, 144, 145, 146]. Ii was shown in [125] that one obtains the CKM matrix of the form given in eq.(5.19) by imposing the generalized μ-τ symmetry. The discussion presented here shows that this result is not specific to the μ-τ symmetry but would follow for any $U(1) \times U(1)$ symmetry as given in eq.(5.20) with an arbitrary V_L. In this approach, V_{ub}, V_{cb} can arise from the small breaking of the μ-τ symmetry as discussed in details in [125].

Above model provides a good example of the "bottom up approach" in which starting with symmetries of mass matrices we were led to a symmetry of Lagrangian which leads to the approximately correct CKM matrix at zeroth order. But the imposed $U(1)\times U(1)$ sub-group lacks the power of the full $G^u\times G^d$ symmetry of the mass matrix. This follows from eq.(5.22). If $V_1^q=V_2^q$ in this equation then Γ_1^q,Γ_2^q and M^q all get diagonalized by $V_L^q=V_LV_1^q$ with the result that there are no FCNC as can be verified by substituting this V_L^q in the first line of eq.(5.16) and using eq.(5.22). Thus unlike $G^u\times G^d$ symmetry, one needs additional alignment condition $V_1^q=V_2^q$ in order to eliminate the FCNC. We now discuss alternative model where such alignment is in-built.

The model is based on an additional softly broken $SU(2)_H$ symmetry acting on the Higgs fields. (ϕ_1,ϕ_2) are taken as doublets under $SU(2)_H$. We also introduce two SM singlets $\chi_q\equiv(\chi_1^q,\chi_2^q)$; $q=u,d$ each transforming as doublet under $SU(2)_H$. Finally we impose a Z_2 symmetry under which χ^d and d'_R change sign. This ensures that only χ^d couples to d quarks and χ^u to the up quarks. Yukawa couplings are allowed as dimension five operators below some high scale Λ as in the Froggatt Nielsen approach [147]:

$$-\mathcal{L}_Y=\frac{1}{\Lambda}\left[\bar{q}'_L\Gamma^d(\chi_1^{d\dagger}\phi_1+\chi_2^{d\dagger}\phi_2)d'_R+\bar{q}'_L\Gamma^u(\chi_1^{u\dagger}\tilde{\phi}_1+\chi_2^{u\dagger}\tilde{\phi}_2)u'_R\right]\ . \tag{5.25}$$

Vacuum expectation values of χ^d,χ^u at a scale $\lesssim\Lambda$ leads to 2HDM Yukawa couplings as in eq.(5.5) with the property

$$\Gamma_1^q=\frac{\left\langle\chi_1^{q\dagger}\right\rangle}{\left\langle\chi_2^{q\dagger}\right\rangle}\Gamma_2^q=\frac{\left\langle\chi_1^{q\dagger}\right\rangle}{\Lambda}\Gamma^q$$

This relation realizes the alignments hypothesis in [138] and leads to models without the tree level FCNC. Since this is a subset of more general solution, eq.(5.13) implied by the $G^u\times G^d$ symmetry, the Yukawa couplings and the mass matrix M^q remain invariant under this symmetry. The $SU(2)_H$ symmetry needs to be broken softly by mass terms in the Higgs sector to obtain the general vacuum structure.

Top quark Yukawa couplings in the above example also get suppressed by the the Froggatt Nielsen factor $\frac{\langle\chi^{u\dagger}\rangle}{\Lambda}$. This may not be desirable. This is avoided by adding a third Higgs doublet ϕ_3 instead of $\chi_{1,2}^u$. The ϕ_3 is taken as singlet under

$SU(2)_H$ and one imposes $\phi_3 \to -\phi_3, u'_{iR} \to u'_{iR}$. In this case the up quarks get their masses only from ϕ_3 and the down quark Yukawa couplings remain the same as in the eq.(5.25). One gets Yukawa alignment in the down quark sector as before. Now the model has one more charged Higgs field which will mix with H^+ entering eq.(5.17). If one now denotes the lighter charged Higgs as H^+, then its couplings are obtained by the replacement $F^d_{ii} \to \eta_F F^d_{ii}$, $F^u_{ii} \to m_{iu}\eta_3$ in eq.(5.17). Here m_{iu} denote the up quark masses and $\eta_F(\eta_3)$ denotes the mixing of H^+ with $\phi^+_F(\phi^+_3)$.

5.3 Neutral meson mixing

As an example of the phenomenological application of the model, we discuss the neutral meson mixing induced by the charged Higgs couplings in eq.(5.17). Some other application of this scheme are discussed in [138, 148]. The F^d_{ii} and F^u_{ii} entering the H^+ couplings are determined by the diagonal Yukawa couplings γ^q_i which also determine corresponding quark masses, see eq.(5.13). Let us make a simplifying assumption that the first two generation quark masses and the corresponding F^q_{ii} are small compared to the third generation masses and F^q_{33}. Retaining only the latter, eq.(5.17) reduces to

$$-\mathcal{L}_{H^+} \approx \frac{H^+}{v}\left(\bar{u}_{iL}V_{i3}F^d_{33}b_R - \bar{t}_R V_{3j}F^{u^*}_{33}d_{jL}\right) + \text{H.C.} \tag{5.26}$$

The charged Higgs contribution to the K^0-$\bar{K}^0$ mixing arise only from the second term. The phase in F^u_{33} and can be absorbed in the definition of H^+. As a result the above Lagrangian does not generate any new CP violating phases in the K meson mixing as long as F^q_{jj} are neglected for $j = 1, 2$. In this limit, the additional H^+ contribution to the K^0-$\bar{K}^0$ mixing has the same structure as in the MFV scenario [55]. The same limit can however lead to non-trivial phases in the B^0_q-$\bar{B}^0_q$ mixing since the charged Higgs exchanges in this case involve both F^d_{33} and F^u_{33} and their phases cannot be simultaneously removed. More interestingly, the above interaction (5.26) distinguishes between the d and s quarks only through the CKM factor and not through additional phases. This results in strong correlations among the CP violations in the B_s and B_d system.

Now we make the above remarks more quantitative. The most general effective

Hamiltonian for the $\Delta B = 2$ processes beyond the SM has the following form [39].

$$\mathcal{H}_{eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i \tag{5.27}$$

The operators Q_i are defined as

$$\begin{aligned} Q_1 &= \bar{b}_L^\alpha \gamma^\mu q_L^\alpha \, \bar{b}_L^\beta \gamma_\mu q_L^\beta \\ Q_2 &= \bar{b}_R^\alpha q_L^\alpha \, \bar{b}_R^\beta q_L^\beta \\ Q_3 &= \bar{b}_R^\alpha q_L^\beta \, \bar{b}_R^\beta q_L^\alpha \\ Q_4 &= \bar{b}_R^\alpha q_L^\alpha \, \bar{b}_L^\beta q_R^\beta \\ Q_5 &= \bar{b}_R^\alpha q_L^\beta \, \bar{b}_L^\beta q_R^\alpha \end{aligned} \tag{5.28}$$

Here $q = d, s$ for B_d and B_s system. Operators $\tilde{Q}_i$ $(i = 1, 2, 3)$ can be obtained from Q_i by exchanging $L \leftrightarrow R$. Matrix elements of operators Q_i between neutral B_q mesons are given as [39]

$$\begin{aligned} \langle \bar{B}_q | Q_1(\mu) | B_q \rangle &= \frac{1}{3} m_{B_q} f_{B_q}^2 B_{1q}(\mu), \\ \langle \bar{B}_q | Q_2(\mu) | B_q \rangle &= -\frac{5}{24} \left(\frac{m_{B_q}}{m_b(\mu) + m_q(\mu)} \right)^2 m_{B_q} f_{B_q}^2 B_{2q}(\mu) \\ \langle \bar{B}_q | Q_3(\mu) | B_q \rangle &= \frac{1}{24} \left(\frac{m_{B_q}}{m_b(\mu) + m_q(\mu)} \right)^2 m_{B_q} f_{B_q}^2 B_{3q}(\mu) \\ \langle \bar{B}_q | Q_4(\mu) | B_q \rangle &= \frac{1}{4} \left(\frac{m_{B_q}}{m_b(\mu) + m_q(\mu)} \right)^2 m_{B_q} f_{B_q}^2 B_{4q}(\mu) \\ \langle \bar{B}_q | Q_5(\mu) | B_q \rangle &= \frac{1}{12} \left(\frac{m_{B_q}}{m_b(\mu) + m_q(\mu)} \right)^2 m_{B_q} f_{B_q}^2 B_{5q}(\mu). \end{aligned} \tag{5.29}$$

Here m_{B_q} and f_{B_q} are mass and decay constant of B_q meson. Matrix element of $\tilde{Q}_i(\mu)$ $(i = 1, 2, 3)$ are same as that of $Q_i(\mu)$ $(i = 1, 2, 3)$. $B_{iq}(\mu)$ $(i = 1, 2, \ldots 5)$ are Bag parameters. The matrix element for B_q^0-$\bar{B}_q^0$ mixing is

$$M_{12} = \sum_{r=1}^{5} C_r(\mu) \langle \bar{B}_q | Q_r(\mu) | B_q \rangle + \sum_{r=1}^{3} \tilde{C}_r(\mu) \langle \bar{B}_q | \tilde{Q}_r(\mu) | B_q \rangle \tag{5.30}$$

The detailed calculation of effective Hamiltonian in the present model is given in the appendix A. Our calculation shows that the effective Hamiltonian describing new contribution to $\Delta B = 2$ processes in the present model at the weak scale is given as:

$$\begin{aligned} \mathcal{H}^{NP} &\approx C_1 Q_1 + C_2 Q_2 \\ &\approx C_1 \bar{b}_L \gamma^\mu q_L \, \bar{b}_L \gamma_\mu q_L + C_2 \bar{b}_R q_L \, \bar{b}_R q_L. \end{aligned} \tag{5.31}$$

Wilson coefficients $C_{1,2}$ are given as

$$\begin{aligned}
C_1 &= -\frac{G_F^2}{4\pi^2}(V_{tb}^* V_{tq})^2 \\
&\Big(|F_{tt}|^4 D_{00}(m_t^2, m_t^2, m_H^2, m_H^2) \\
&- 2m_t^2 |F_{tt}|^2 m_W^2 D_0(m_t^2, m_t^2, m_H^2, m_W^2) \\
&+ 2m_t^2 |F_{tt}|^2 (D_{00}(m_t^2, m_t^2, m_H^2, m_W^2) + D_{00}(m_t^2, m_t^2, m_H^2, m_G^2)) \\
&+ 2m_t^2 |F_{tt}|^2 D_{00}(m_t^2, m_t^2, m_H^2, m_G^2)\Big) \\
C_2 &= -\frac{G_F^2}{4\pi^2}(V_{tb}^* V_{tq})^2 \\
&\Big(m_t^2 F_{tt}^{*2} F_{bb}^{*2}\ D_0(m_t^2, m_t^2, m_H^2, m_H^2) \\
&+2m_t^3 m_b F_{tt}^* F_{bb}^*\ D_0(m_t^2, m_t^2, m_H^2, m_G^2)\Big) \qquad (5.32)
\end{aligned}$$

Expressions for Passarino-Veltman one-loop four-point functions with zero external momenta $D_0(m_0^2, m_1^2, m_2^2, m_3^2)$ and $D_{00}(m_0^2, m_1^2, m_2^2, m_3^2)$ are given in the appendix A. m_t, m_W, m_H are the masses of top quark, W boson and charged Higgs respectively. $m_G = \xi m_W$. ξ is the gauge fixing parameter. We perform our calculations in Feynman gauge for which $\xi = 1$.

In our model new contribution to matrix element for B_q^0-$\bar{B}_q^0$ mixing can be obtained as

$$\begin{aligned}
M_{12q}^{NP} &= \langle \bar{B}_q | \mathcal{H}^{NP} | B_q \rangle \\
&= C_1 \langle \bar{B}_q | \bar{b}_L \gamma^\mu q_L\ \bar{b}_L \gamma_\mu q_L | B_q \rangle + C_2 \langle \bar{B}_q | \bar{b}_R q_L\ \bar{b}_R q_L | B_q \rangle \\
&= C_1 \langle \bar{B}_q | Q_{1q} | B_q \rangle + C_2 \langle \bar{B}_q | Q_{2q} | B_q \rangle \qquad (5.33)
\end{aligned}$$

Using matrix elements given in eq.(5.29) we get

$$M_{12q}^{NP} = m_{B_q} f_{B_q}^2 \left(C_1 \frac{1}{3} B_{1q}(\mu) - C_2 \frac{5}{24} \left(\frac{m_{B_q}}{m_b(\mu) + m_q(\mu)} \right)^2 B_{2q}(\mu) \right) \qquad (5.34)$$

The B_q^0-$\bar{B}_q^0$ $(q = d, s)$ mixing amplitude can be parameterized in the presence of new physics contribution as follows:

$$\begin{aligned}
\left\langle \bar{B}_q | \mathcal{H}^{SM} + \mathcal{H}^{NP} | B_q \right\rangle &\equiv \left\langle \bar{B}_q | \mathcal{H}^{SM} | B_q \right\rangle (1 + \kappa_q e^{i\sigma_q}) \qquad (5.35) \\
&\equiv |\left\langle \bar{B}_q | \mathcal{H}^{SM} | B_q \right\rangle|\ \rho_q e^{2i(\beta_q + \phi_q)}
\end{aligned}$$

where β_q represent the relevant phase in case of the SM and ϕ_q are the charged Higgs induced phases. The SM expression for B_q^0-$\bar{B}_q^0$ mixing is [15, 16]

$$M_{12q}^{SM} = \frac{G_F^2 m_W^2}{12\pi^2} m_{B_q} B_{1q} f_{B_q}^2 \eta_B S_0(m_t^2/m_W^2)(V_{tb}^* V_{tq})^2 \tag{5.36}$$

where G_F, m_W respectively denote the Fermi coupling constant and the W boson mass. The Inami-Lim function [17] $S_0(m_t^2/m_W^2) \approx 2.35$ [18] for $m_t \sim 165$ GeV. $\eta_B \approx 0.55$ refers to the QCD correction to the Wilson operator in the SM [51]. B_{1q} is the Bag parameter. Using the eqs.(5.34,5.35) and eq.(5.36) we get

$$\begin{aligned} \kappa_q e^{i\sigma_q} &= \frac{M_{12q}^{NP}}{M_{12q}^{SM}} \\ &= \frac{4\pi^2}{G_F^2 M_W^2 \eta_B S_0(x_t)} \left[C_1' - 5/8 C_2' \left(\frac{M_{B_q}}{m_b + m_q} \right)^2 \frac{B_{2q}(\mu)}{B_{1q}(\mu)} \right]. \end{aligned} \tag{5.37}$$

Here $C_i' = C_i/(V_{tb}^* V_{tq})^2$. It can be seen from eq.(5.32) that $C_{1,2}'$ are independent of the flavor $q = d, s$ of the light quark in B_q. Mild dependence of κ_q on q arise from the operator matrix element multiplying C_2' in eq.(5.37). This leads to two predictions: To a good approximation, (i) $\kappa_d \approx \kappa_s$ and (ii) $\sigma_d \approx \sigma_s$. This implies from eq.(5.35) that

$$\frac{\Delta M_d}{\Delta M_s} \approx \frac{\Delta M_d^{SM}}{\Delta M_s^{SM}} , \tag{5.38}$$

where ΔM_q denote the values of the $B_q^0 - \bar{B}_q^0$ mass difference in the presence of new physics. Equality of κ_d and κ_s as well as σ_d and σ_s also imply through eq.(5.35)

$$\phi_d \approx \phi_s \tag{5.39}$$

The detailed phenomenological consequences of this prediction are already discussed in [149] in a model independent manner. It appears to be in the right direction for explaining the CP violating anomalies. In case of B_d system the value

$$\sin 2\beta = 0.885 \pm 0.082$$

as determined [133] using the information from V_{cb}, ϵ_K and $\Delta M_d, \Delta M_s$ is found to be higher than the value

$$\sin 2\beta = 0.657 \pm 0.036$$

obtained from the mixing induced asymmetry in $B \to J/\psi K_S$ decay. Since the latter measures $\sin 2(\beta + \phi_d)$, the above information can be reconciled with a negative $\phi_d \approx -10°$. $\phi_d \approx \phi_s$ then implies a sizable asymmetry $S_{J\psi\phi} = \sin 2(\beta_s + \phi_s) \approx$

$\sin 2(-0.1^\circ - 10^\circ) \approx -0.34$ in B_s decay. Hence the total phase in $B \to J/\psi K_S$ decay will be $2(\beta + \phi_s) \approx -0.35$. This value is in agreement with determination by D0 collaboration [22].

Alternative and more detailed study of relations (5.38,5.39) is made by Lenz *et al* [18]. This analysis includes the most recent results on the measurement of the dimuon charge asymmetry by the D0 collaboration [151] and attributes it to a new physics phase in the B_s^0-$\bar{B}_s^0$ mixing. The new physics effects are parameterized in the standard way as in eq.(5.35). They carry out a global fit to various observables in different scenarios one of these being the assumptions $\kappa_d = \kappa_s$ and $\sigma_d = \sigma_s$ realized in the present scenario. The resulting fits are found to be better than the SM fits corresponding to $\kappa_d = \kappa_s = 0$ and imply [18] a common new physics phase

$$2\phi_s = 2\phi_d = -14.4^{+6.7}_{-4.2}$$

and

$$\sin 2\beta = 0.83 \pm 0.05$$

at 2σ. In particular, the scenario with no new physics phase is disfavored at 3.1σ in this analysis [18]. At the qualitative level, a negative new physics phase reduces the tension between the determination of β using different inputs on one hand and accounts for the new larger than SM phase in the Bs mixing. More constrained determination of either will provide a crucial test of the proposed scenario. It is worth emphasizing that the predictions eq.(5.38,5.39) also arise in other scenarios based on the Minimal Flavor Violation hypothesis. A specific example is a model with tree level FCNC and neutral Higgs exchange providing flavor blind CP violating phase through the scalar-pseudoscalar mixing [170]. We have discussed this model in chapter 3.

5.4 Numerical analysis

In this section we discuss our numerical analysis in which we obtain constraints on model parameters from study of B_s^0-$\bar{B}_s^0$ mixing and like-sign dimuon charge asymmetry A_{sl}^b of semileptonic b-hadron decays. First we concentrate on B_s^0-$\bar{B}_s^0$ mixing.

5.4.1 B_s^0-$\bar{B}_s^0$ mixing

Effective Hamiltonian and the relevant hadronic matrix elements for B_s^0-$\bar{B}_s^0$ mixing can be obtained from eqs.(5.27,5.29) with the substitution $q = s$. Non-zero Wilson coefficients in the present model at weak scale are given in the eq.(5.32) with $q = s$. Wilson coefficients at $\mu = m_b$ scale can be obtained as

$$C_r(m_b^{pole}) = \sum_i \sum_s (b_i^{(r,s)} + \eta c_i^{(r,s)})\eta^{\alpha_i} C_s(M_S) \tag{5.40}$$

where $\eta = \alpha_s(M_s)/\alpha_s(m_t)$. $C_s(M_S)$ are the Wilson coefficients at weak scale. a_i, b_i and c_i are the magic number used in the evolution from weak scale to scale $\mu = m_b$. In our calculation we use values of these magic numbers given in [39]. We produce these number below for the sake of completeness.

$$
\begin{aligned}
a_i &= (0.286, -0.692, 0.787, -1.143, 0.143), & & \\
b_i^{11} &= (0.865, 0, 0, 0, 0), & c_i^{11} &= (-0.017, 0, 0, 0, 0) \\
b_i^{22} &= (0, 1.879, 0.012, 0, 0), & c_i^{22} &= (0, -0.18, -0.003, 0, 0) \\
b_i^{23} &= (0, -0.493, 0.18, 0, 0), & c_i^{23} &= (0, -0.014, 0.008, 0, 0) \\
b_i^{32} &= (0, -0.044, 0.035, 0, 0), & c_i^{32} &= (0, 0.005, -0.012, 0, 0) \\
b_i^{33} &= (0, 0.011, 0.54, 0, 0), & c_i^{33} &= (0, 0, 0.028, 0, 0) \\
b_i^{44} &= (0, 0, 0, 2.87, 0), & c_i^{44} &= (0, 0, 0, -0.48, 0.005) \\
b_i^{45} &= (0, 0, 0, 0.961, -0.22), & c_i^{45} &= (0, 0, 0, -0.25, -0.006) \\
b_i^{54} &= (0, 0, 0, 0.09, 0), & c_i^{54} &= (0, 0, 0, -0.013, -0.016) \\
b_i^{55} &= (0, 0, 0, 0.029, 0.863), & c_i^{55} &= (0, 0, 0, -0.007, 0.019)
\end{aligned}
$$

Here $i = 1, 2, \ldots, 5$. Only non-vanishing entries are shown. The magic numbers for evolution of $\tilde{C}_i$ $(i = 1, 2, 3)$ are same as that of C_i $(i = 1, 2, 3)$. Bag parameters for B_s^0-$\bar{B}_s^0$ mixing at scale $\mu = m_b$ in RI/MOM scheme are given as [150]

$$B_{1s}(m_b) = 0.86 \tag{5.41}$$

$$B_{2s}(m_b) = 0.83 \tag{5.42}$$

$$B_{3s}(m_b) = 1.03 \tag{5.43}$$

$$B_{4s}(m_b) = 1.17 \tag{5.44}$$

$$B_{5s}(m_b) = 1.94 \tag{5.45}$$

We have not shown the uncertainty in the magic numbers and Bag parameters because we use their central values in our calculations. Matrix element for B_s^0-$\bar{B}_s^0$ mixing at the scale $\mu = m_b$ can be calculated using the expression and the values given here. New physics contribution to B_s^0-$\bar{B}_s^0$ mixing is parameterized by UTfit group as [27]

$$C_{B_S} e^{-2i\phi_{B_s}} = \left(1 + \frac{\Delta M_s^{NP}}{\Delta M_s^{SM}}\right) \tag{5.46}$$

Using eq.(5.35) we get,

$$\begin{aligned} C_{B_s} &= |1 + \kappa_s e^{i\sigma_s}| \\ \phi_{B_S} &= -\frac{1}{2}\mathrm{Arg}[1 + \kappa_s e^{i\sigma_s}] \end{aligned} \tag{5.47}$$

UTfit collaboration has obtained the allowed range of the parameters C_{B_S} and ϕ_{B_s} as [27]

$$\begin{aligned} C_{B_s} &= [0.68, 1.51]\ , \\ \phi_{B_s} &= [-30.5, -9.9] \cup [-77.8, -58.2] \end{aligned} \tag{5.48}$$

We calculate C_{B_s} and ϕ_{B_s} in the present model with $m_d = m_s = 0$ and $F_{11}^{u,d} = F_{22}^{u,d} = 0$ subjected to the bounds given by eq.(5.48). Matrix element for B_s^0-$\bar{B}_s^0$ mixing depends upon unknown parameters such as charged Higgs mass M_H and couplings F_{tt}, F_{bb}. From eq.(5.16) we get

$$\begin{aligned} F_{tt} &= v(-\sin\beta(\gamma_1^u)_{33} + \cos\beta(\gamma_2^u)_{33} e^{i\theta^u}) \\ F_{bb} &= v(-\sin\beta(\gamma_1^d)_{33} + \cos\beta(\gamma_2^d)_{33} e^{i\theta^d}). \end{aligned} \tag{5.49}$$

Here $v = 146\ GeV$ is the vev of the SM Higgs. β is defined as $\tan\beta = \frac{v_2}{v_1}$. $\theta^d = -\theta^u = \theta$, where θ phase of the vev of the Higgs doublet ϕ_2. In our calculation charged Higgs mass M_H is allowed to vary in the range $100 - 500$ GeV, while β and couplings $(\gamma_1^{u,d})_{33}, (\gamma_2^{u,d})_{33}$ are varied in a manner that they satisfy

$$\begin{aligned} v(\cos\beta(\gamma_1^u)_{33} + \sin\beta(\gamma_2^u)_{33} e^{i\theta^u}) &= m_t \\ v(\cos\beta(\gamma_1^d)_{33} + \sin\beta(\gamma_2^d)_{33} e^{i\theta^d}) &= m_b. \end{aligned} \tag{5.50}$$

Values of C_{B_s} and ϕ_{B_s} which can be obtained in the present model is shown in figure(5.1). It can be seen from the figure that the model presented here can give new contribution to B_s^0-$\bar{B}_s^0$ mixing as allowed by the analysis of UTfit group.

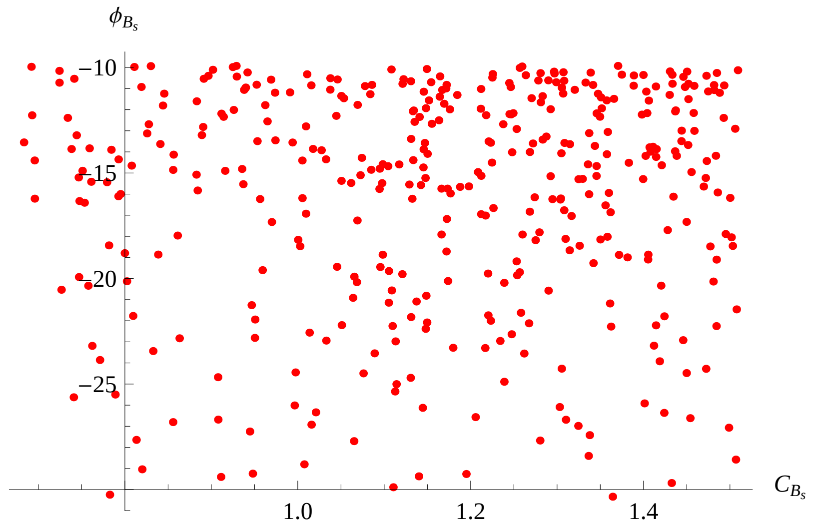

Figure 5.1: Values of NP parameter C_{B_s} and ϕ_{B_s} which can be obtained in the present model with $m_d = m_s = 0$ and and $F_{11}^{u,d} = F_{22}^{u,d} = 0$

5.4.2 Like-sign dimuon charge asymmetry of semileptonic b-hadron decays

In this section we calculate like-sign dimuon charge asymmetry A_{sl}^b of semileptonic b-hadron decays in the present model for model parameters which satisfies the bound on NP parameters for B_s^0-$\bar{B}_s^0$ mixing as described in the previous section. Like-sign dimuon charge asymmetry A_{sl}^b of semileptonic b-hadron decays is defined as [85, 151]

$$A_{sl}^b \equiv \frac{N_b^{++} - N_b^{--}}{N_b^{++} + N_b^{--}} \equiv \frac{f_d Z_d a_{sl}^d + f_s Z_s a_{sl}^s}{f_d z_d + f_s z_s} \tag{5.51}$$

Where $z_q \equiv \frac{1}{1-y_q^2} - \frac{1}{1+x_q^2}$, $y_q \equiv \frac{\Delta\Gamma_q}{2\Gamma_q}$, $x_q \equiv \frac{\Delta M_q}{\Gamma_q}$ and $q = d, s$. Values of x_d, y_d, x_s, y_s are given in [6].

$$\begin{aligned} x_d &= 0.774 \pm 0.008 \\ y_d &= 0 \\ x_s &= 26.2 \pm 0.5 \\ y_s &= 0.046 \pm 0.027 \end{aligned} \tag{5.52}$$

f_d and f_s are production cross section of $b \to B_d^0$ and $b \to B_s^0$ respectively. Their values are given in [6].

$$\begin{aligned} f_d &= 0.323 \pm 0.037 \\ f_s &= 0.118 \pm 0.015 \end{aligned} \tag{5.53}$$

SM prediction for A_{sl}^b is given as [25]

$$A_{sl}^b = (-2.3^{+0.5}_{-0.6}) \times 10^{-4} \tag{5.54}$$

A_{sl}^b has been recently obtained from the measurement of charge asymmetry of like-sign muon events by D0 experiment at Fermilab Tevatron proton-antiproton collider. D0 collaboration obtained [151]

$$A_{sl}^b = -0.00957 \pm 0.00251(\text{stat}) \pm 0.00146(\text{syst}) \tag{5.55}$$

This values deviates from SM prediction by 3.2σ. Using the values of x_d, y_d, x_s, y_s, f_d, f_s following expression for A_{sl}^b can be obtained

$$A_{sl}^b = (0.0506 \pm 0.043)a_{sl}^d + (0.0494 \pm 0.043)a_{sl}^s \tag{5.56}$$

Using current experimental value of a_{sl}^d, required value of a_{sl}^s is obtained as [152]

$$a_{sl}^s = -0.0146 \pm 0.0075 \tag{5.57}$$

Combining the CDF and D0 measurements of A_{sl}^b with measured value of a_{sl}^s, average value of a_{sl}^s has been obtained as [152]

$$(a_{sl}^s)_{ave} = -(12.5 \pm 5.0) \times 10^{-3} \tag{5.58}$$

This value is 2.5σ away from the SM prediction $(a_{sl}^s)_{SM} = (2.1 \pm 0.6) \times 10^{-5}$. If confirmed, this can be an indication for physics beyond SM. Theoretical expression for a_{sl}^s is [152]

$$a_{sl}^s = \frac{\sin\phi_s}{\sqrt{1 + \frac{(1-(\frac{\Delta\Gamma_s}{2\Delta M_S})^2)^2}{4(\frac{\Delta\Gamma_s}{2\Delta M_S})^2}\cos^2\phi_s}}. \tag{5.59}$$

Here $\Delta M_S = 2|M_{12s}|$ and $\Delta\Gamma_s = 2|\Gamma_{12s}|\cos\phi_s$. We use $\Gamma_{12s} = 0.048 \times 10^{12}\ sec^{-1}$ [25] in our calculation. We have calculated a_{sl}^s in our model and found that it is possible to obtain values allowed by the eq.(5.58). Table(5.1) shows values of a_{sl}^s which could be obtained in the present model along with the required values of model parameters. It can be seen that it possible to obtain the required value of a_{sl}^s with moderate Higgs mass.

β	θ	M_H	a^s_{sl}
0.022	3.41	228.55	-0.0088
0.015	4.10	125.14	-0.0091
0.016	3.89	382.85	-0.0099
0.017	4.48	277.56	-0.01

Table 5.1: Model parameters and values of a^s_{sl} obtained in the limit of vanishing first and second generation masses and $F^{u,d}_{ii}; (i = 1, 2)$

5.5 Summary

Using the flavor symmetries which does not depend on the Higgs parameter $\tan\beta$ and θ, we obtained a class of 2HDM in which FCNCs can be eliminated without imposing discrete symmetries. Unlike type - I and type-II 2HDM, charged higgs interaction in this model contains new phases which are flavor dependent. In the limit of vanishing first and second generation masses and corresponding couplings $F^{u,d}_{ii}; (i = 1, 2)$, K^0-$\bar{K}^0$ mixing does not get any new CP phase while the B^0_d-$\bar{B}^0_d$ and B^0_s-$\bar{B}^0_s$ mixing gets new phases. New contribution to CP violation in this model in B^0_d-$\bar{B}^0_d$ and B^0_s-$\bar{B}^0_s$ mixing are correlated and it is possible to explain CP violating anomalies in B_d and B_s systems. We also calculated the B^0_s-$\bar{B}^0_s$ mixing in the present model subjected to the bounds on NP parameters by UTfit group. It was also shown that the model parameters allowed by the UTfit bounds can also generate like sign dimuon charge asymmetry as required by the experimental data.

Chapter 6

2HDM with General FCNC

In this chapter we study implications of a 2HDM with general tree level FCNC in processes arising due to $b \leftrightarrow s$ transitions. We consider three processes originating from $b \leftrightarrow s$ transitions. (1) $\Delta B = 2$, B_s^0-$\bar{B}_s^0$ mixing (2) the leptonic decays $\bar{B}_s \to \mu^+\mu^-$ (3) The semi leptonic decays $\bar{B}_d \to (\bar{K}, \bar{K}^*)\mu^+\mu^-$. These processes contains observables which are predicted to be small in SM. If experimental determination of any of these observable gives large value then it must be due new physics beyond SM. Hence these observables are useful in search for NP. As discussed in the first chapter, the CP violating phase

$$\phi_s = Arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

where M_{12} and Γ_{12} respectively denote the real and absorptive parts of the B_s^0-$\bar{B}_s^0$ transition amplitude is predicted to be quite small $\sim 2^\circ$ in the SM. In contrast, the experimental determination of ϕ_s from the time-dependent CP asymmetry in $B_s \to J/\psi\phi$ decays by the CDF [21] and D0 [22] groups allow much larger phase: the 90% CL average reported by HFAG [153] requires ϕ_s to be in the range $[-1.47; -0.29] \cup [-2.85; -1.65]$. By including the D0 and the CDF results in their global analysis, UTfit group find around 3σ departure from the SM prediction on ϕ_s [27, 29]. Similar analysis by the CKMfitter group [30] also reports deviation from the SM result but at around 2.5σ. This may be a hint of the presence of new physics in the $b \leftrightarrow s$ transitions. Future measurement would provide a crucial test of this possibility.

The decay rate for $\bar{B}_s \to \mu^+\mu^-$ is also predicted [64] to be small in SM

$$Br(\bar{B}_s \to \mu^+\mu^-) = (3.51 \pm 0.50) \times 10^{-9} \ . \tag{6.1}$$

compared to an order of magnitude larger experimental limit [154]

$$Br(\bar{B}_s \to \mu^+\mu^-) < 5.8 \times 10^{-8} \quad (95\%\mathrm{CL}) \ . \tag{6.2}$$

This rate therefore can be an important observable in search of new physics. In contrast, the branching ratios for the exclusive processes $\bar{B}_d \to (\bar{K}, \bar{K}^*)\mu^+\mu^-$ are close to the SM predictions. But they still provide valuable constraints on any new physics that may be present. Moreover, the di-lepton spectrum and the angular distribution of leptons in these exclusive processes provide very sensitive test of the SM and possible indication of new physics [155, 156]. The LHCb [157] and the super-B factory will allow more sensitive determination of these observables and will strongly constrain or uncover any new physics that may be present.

The $b \leftrightarrow s$ transition is also interesting from the theoretical point of view since several extensions of SM predict relatively large effects in this transition. The most popular extensions studied are the two Higgs doublet models (2HDM) in which some symmetry (discrete or super) prevents FCNC at the tree level. In these models, the Higgs (like the W boson) contribute to the FCNC at the loop level. The supersymmetric standard model is one such example within which the Higgs and sparticle mediated flavor changing effects have been extensively studied [158]. In the Minimal Supersymmetric Standard Model (MSSM), the $d_i \leftrightarrow d_j$ transitions between the charged $-1/3$ quarks in large $\tan\beta$ limit are governed by the CKM factor $V_{3i}V_{3j}^*$ [159, 160, 161]. As a result, the effect becomes more prominent for the $b \leftrightarrow s$ transitions compared to others. The same thing also happens in the charged Higgs induced flavor transitions in certain class of two Higgs doublet models. In a general 2HDM quarks and leptons would couple and obtain their masses from both the Higgs doublets as discussed in the chapter 2. This however leads to the Higgs induced FCNC at the tree level. This is generally avoided [42] by imposing a discrete symmetry which ensures that all the fermions of a given charge obtain their masses from coupling to only one Higgs [44] doublet. This way of suppressing FCNC is technically natural since the loop induced FCNC couplings after spontaneous breaking of the discrete symmetry are calculable and finite. This way of suppressing FCNC is termed as natural flavor conservation (NFC) in the literature [42]. The charged Higgs induced $d_i \leftrightarrow d_j$ transitions in these models also involve the factor $V_{3i}V_{3j}^*$ as in the MSSM.

2HDM with NFC and the MSSM realize the Minimal Flavor Violation (MFV) [55] scenario and do not have any additional CP violating phase other than the CKM phase. In the context of the MSSM, one can consider scenarios which go beyond the MFV to accommodate a large ϕ_s [158, 162, 163]. This cannot easily be done for two Higgs doublet model with NFC. Large CP violating phases are possible in more general two Higgs doublet models (called type - III 2HDM) which allow the tree level FCNC. Most general model of this type can lead to large flavor violation in the $d \leftrightarrow s$ transitions and would imply a very heavy Higgs mass suppressing all other flavor violations. It is possible to imagine scenarios where the tree level FCNC couplings also show hierarchy as in the quark masses [82, 83]. This class of models would imply relatively large flavor violations in B transitions. The standard example of this is the so called Cheng- Sher ansatz [96] which postulates a relation between the down quark masses m_i and the FCNC couplings:

$$F_{ij} = \lambda_{ij}\frac{\sqrt{m_i m_j}}{v} \ , \tag{6.3}$$

with $\lambda_{ij} \sim \mathcal{O}(1)$ and $v \sim O(174\text{GeV})$.

There exist explicit models [31, 75, 76, 77, 79, 81] which lead to hierarchy in FCNC. Such models which are theoretically as natural as the two Higgs doublets with NFC can lead to interesting patterns of flavor violations. In this chapter we analyze the constrains and prediction of the Higgs induced tree level FCNC in the $b \leftrightarrow s$ transitions. Rather than looking at any specific model in this category we consider several classes of models which imply interesting patterns of flavor violation. We find that the predictions of some of these models for the leptonic and semi leptonic transitions mentioned above are distinctively different compared to the two Higgs doublet models with NFC and the MSSM. Moreover, it is possible within them to simultaneously look at the constraints from all three processes listed above and we find that the B_s^0-$\bar{B}_s^0$ mixing provides very stringent restrictions on the other two processes.

There have been earlier phenomenological studies of models with tree level FCNC [69, 71, 164]. Most of these are model specific and mainly use the Cheng-Sher ansatz and try to constrain parameters λ_{ij}. As we discuss, there are models which are distinctively different from this ansatz. So rather than specifying any specific model, we perform a model-independent analysis of the Higgs induced FCNC couplings. Unlike the Cheng-Sher ansatz, these couplings in general can have phases which are not included in the earlier analysis. As we show, the FCNC couplings may provide the source of a large ϕ_s and we identify models which explain large ϕ_s and those which can not do so.

We present the general structure of the Higgs induced FCNC in the next section where we also discuss various classes of models which lead to hierarchical FCNC couplings. In section (6.2), we give the details of the effective Hamiltonian for the $\Delta B = 1$ and 2 transitions. In the next section, we derive an important relation between the Higgs contributions to the B_s^0-$\bar{B}_s^0$ mass difference and the branching ratio for $\bar{B}_s \to \mu^+\mu^-$. This relation is independent of the FCNC couplings F_{23}^*, F_{32} under specific assumptions. In the same section, we study numerical implications of various classes of models and conclude in the last section.

6.1 FCNC: Structure and examples

This section is devoted to a discussion of classes of the 2HDM which we use as a guide to carry out a fairly model-independent analysis of the $b \to s$ transitions subsequently.

The general two Higgs doublet models [42] have the following Yukawa couplings in the down quark sector:

$$-\mathcal{L}_Y^d = \bar{d}'_L(\Gamma_1^d \phi_1^0 + \Gamma_2^d \phi_2^0) d'_R + \text{H.c.} \ . \tag{6.4}$$

Here, $d'_{L,R}$ denote (the column of) the weak eigenstates of down quarks. The models with NFC impose an additional discrete symmetry, *e.g.* $(d'_R, \phi_1) \to -(d'_R, \phi_1)$ which forbids the couplings Γ_2^d. As a result, the down quark couplings to ϕ_1 become diagonal in the mass basis and there are no tree level FCNC.

More general 2HDM allow both Γ_1^d and Γ_2^d in eq.(6.4) and contain the tree level

FCNC. Consider two orthogonal combinations of the Higgs fields ϕ_1, ϕ_2:

$$\begin{aligned} \phi^0 &\equiv \cos\beta\phi_1^0 + \sin\beta\phi_2^0 \ , \\ \phi_H^0 &\equiv -\sin\beta\phi_1^0 + \cos\beta\phi_2^0 \end{aligned} \tag{6.5}$$

with $\langle\phi_1\rangle = v\cos\beta$; $\langle\phi_2\rangle = v\sin\beta$ and $v \sim 174$ GeV. Here we assume that CP is conserved in the Higgs sector. ϕ^0 acquires a non-zero vacuum expectation value (vev) and leads to the quark mass matrix

$$M^d = v(\Gamma_1^d \cos\beta + \Gamma_2^d \sin\beta) \ . \tag{6.6}$$

ϕ is like the SM Higgs field with flavor conserving couplings to quarks. The ϕ_H^0 violates flavor and one can write using, eqs.(6.4,6.6)

$$-\mathcal{L}_{FCNC} = \sum_{i\neq j} F_{ij}\bar{d}_{iL}d_{jR}\phi_H^0 + \text{H.c.} \ . \tag{6.7}$$

$d_{L,R}$ denote the mass eigenstates. FCNC couplings are given as

$$F_{ij} \equiv (V_L^\dagger \Gamma_2^d V_R)_{ij}\frac{1}{\cos\beta} \ . \tag{6.8}$$

where $V_{L,R}$ are defined by

$$V_L^\dagger M^d V_R = D^d \ . \tag{6.9}$$

Here D^d is the diagonal mass matrix for the down quarks. The structure as in (6.7) can arise as an effective interactions from the loop diagrams as in MSSM [159] or the 2HDM with NFC [165, 166]. Phenomenology based on this structure therefore would include such cases also.

The leptonic and semi-leptonic FCNC transitions also depend on how the charged leptons couple to the fields $\phi_{1,2}$. For definiteness, we will assume that the charged lepton Yukawa couplings are given as in the MSSM. We thus assume

$$\begin{aligned} -\mathcal{L}_Y^l &= \bar{l}'_L\Gamma_1^l l'_R\phi_1^0 + \text{H.c.} \ , \\ &= \frac{1}{v\cos\beta}\bar{l}_L D_l l_R \phi_1^0 + \text{H.c.} \ . \end{aligned} \tag{6.10}$$

If coupling to ϕ_2 is also present then one would get flavor violations in the leptonic sector also.

General properties of F follow from its definition, eq.(6.8). We shall consider three specific class of FCNC and show that each of these imply different and interesting physics.

(A) Hermitian structures: Assume that quark mass matrices and $\Gamma^d_{1,2}$ are Hermitian. In this case, eq.(6.8) trivially implies

$$F_{ij} = F^*_{ji} \; . \tag{6.11}$$

(B) Symmetric structures: Assume that M^d and $\Gamma^d_{1,2}$ are symmetric. This trivially leads to symmetric FCNC couplings:

$$F_{ij} = F_{ji} \; . \tag{6.12}$$

(C) MSSM like structures: The FCNC in MSSM in large $\tan\beta$ limit [159, 161] can be described by an effective tree level Lagrangian similar to eq.(6.8) with the specific relation

$$F_{ij} = \frac{m_j}{m_i} F^*_{ji} \tag{6.13}$$

between the FCNC couplings. The same relation also holds in 2HDM with NFC where F_{ij} are induced by the charged Higgs at 1-loop [165, 166]. More interestingly, even the tree level FCNC can satisfy the same relation in some specific models [79, 81]. We have discussed one example of this class in chapter 3.

While the phenomenological analysis that we present in the above three cases would be model independent, we give below several examples of textures/models which can realize above scenarios and simultaneously explain the quark masses.

Yukawa textures and FCNC

The strongest constraints on FCNC come from the K^0-$\bar{K}^0$ mixing and the ϵ parameter. One needs very heavy Higgs$\sim$ O(TeV) to suppress this effect if $F_{12} \sim$ O(gauge coupling). Heavy Higgs would then suppress other flavor violations as well without leaving any signature at low energy. Interesting class of models would be the ones in which the coupling $|F_{12}|$ would be suppressed compared to the other couplings. As already discussed in the introduction, widely studied example of this is the Cheng-Sher ansatz, eq.(6.3). Here the suppression in F_{ij} comes from the suppression in the quark masses compared to the weak scale. F_{ij} may also be suppressed by mixing angles. This can come about naturally in large classes of 2HDM. Assume that the Higgs ϕ_2 in eq.(6.4) is responsible for only the third generation mass while the Higgs ϕ_1 accounts for the first two generation masses and the inter-generation mixing. Only the (33) element of Γ_2^d is assumed non-zero in this case and eq.(6.8) automatically implies

$$F_{ij} = \frac{m_b}{v\cos\beta\sin\beta} V^*_{L3i} V_{R3j} \ . \tag{6.14}$$

If M^d is Hermitian or symmetric one automatically obtains eq.(6.11) or (6.12). If the off-diagonal elements of $V_{L,R}$ are suppressed compared to the diagonal elements, then F_{12} will be more suppressed compared to others. In particular, $(V_{L,R})_{3i} \sim c_{L,R}\sqrt{\frac{m_i}{m_b}}$ reproduces the Cheng-Sher ansatz with $\lambda_{ij} \sim \frac{c_L c_R}{\cos\beta\sin\beta}$. Thus this class of models may be regarded as a generalization of the Cheng-Sher ansatz.

Let us take two concrete examples which are among the specific textures studied in the literature with a view to understand the fermion masses and mixings.

Consider

- $$\Gamma_1^d = y_{33}\begin{pmatrix} d\epsilon^4 & b\epsilon^3 & c\epsilon^3 \\ b\epsilon^3 & f\epsilon^2 & a\epsilon^2 \\ c\epsilon^3 & a\epsilon^2 & 0 \end{pmatrix} \ ; \ \Gamma_2^d = y_{33}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \ . \tag{6.15}$$

  These couplings together imply the down quark mass matrix studied long ago by Roberts, Romanino, Ross and Velesco-Sevilla [167] and recently in [168]. ϵ here is a small parameter which can be determined from the quark masses. $\epsilon \sim 0.1$ is determined in [168] assuming the above structure to be valid at the GUT scale. Above matrices imply in a straight forward way

  $$|V_{L32}| = |V^*_{R32}| \sim a\epsilon^2 \ ; |V_{L31}| = |V^*_{R31}| \sim |c|\epsilon^3 \ ; |V_{L12}| = |V^*_{R12}| \sim \frac{b}{f}\epsilon \ . \tag{6.16}$$

  This in turn implies

  $$\begin{aligned} |F_{12}| &\approx \frac{m_b}{v\cos\beta\sin\beta} a|c|\epsilon^5 \\ |F_{13}| &\approx \frac{m_b}{v\cos\beta\sin\beta} |c|\epsilon^3 \\ |F_{23}| &\approx \frac{m_b}{v\cos\beta\sin\beta} a\epsilon^2 . \end{aligned} \tag{6.17}$$

  Thus one obtains the desired hierarchical FCNC couplings with this ansatz.

- As an other example we consider the texture suggested in [169]:

$$\Gamma_1^d = y_{33} \begin{pmatrix} d\epsilon^6 & b\epsilon^4 & c\epsilon^3 \\ b\epsilon^4 & f\epsilon^2 & a\epsilon \\ c\epsilon^3 & a\epsilon & 0 \end{pmatrix} \; ; \; \Gamma_2^d = y_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{6.18}$$

where ϵ is a small expansion parameter (assumed to be ~ 0.2 in [169]) and other parameters are O(1). The quark mass matrix is of rank 1 if these parameters are exactly 1. Because of this feature, it is possible to simultaneously understand the large mixing in the neutrinos and small mixing in the quark sector. The above form of the quark matrix also implies the relation

$$(V_L)_{ij} \approx (V_L)_{ji} \approx \sqrt{\frac{m_i}{m_j}} \; , (i < j).$$

As a result, the FCNC couplings satisfy the Cheng-Sher ansatz given in eq.(6.3) with $\lambda_{ij} \sim \frac{1}{\cos\beta \sin\beta}$. M^d and Yukawa couplings are symmetric in both the above examples. One could consider instead similar Hermitian textures as well.

- Somewhat different illustration of the suppressed FCNC couplings is provided by the following textures of the Yukawa couplings:

$$\Gamma_1^d = \begin{pmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{pmatrix} \; ; \; \Gamma_2^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{pmatrix} , \tag{6.19}$$

where x denotes an entry which is not required to be zero. It is straightforward to show that the above Yukawa couplings imply

$$F_{ij} = \frac{1}{v \cos\beta \sin\beta} V^*_{L3i} V_{L3j} m_j \tag{6.20}$$

and therefore satisfy relation (6.13). Note that F_{ij} depend only on the left-handed mixing matrix and they remain suppressed and hierarchical if the mixing elements show hierarchy. The structure of FCNC in this example is different compared to the Cheng-Sher ansatz and earlier two examples. The earlier two examples reduce to the Cheng-Sher ansatz if $V_{Lij} \approx V_{Lji} \approx$

$\sqrt{\frac{m_i}{m_j}}, (i < j)$ while eq.(6.20) has an additional suppression by $\frac{m_j}{m_b}$ compared to them in this case when $j \neq 3$.

This particular example of the suppressed FCNC couplings was proposed in [79]. The hierarchy among F_{ij} is determined in the MSSM by the CKM matrix elements while here it is determined by the elements of the down quark mixing matrix. In particular, the F_{ij} can have new phases not present in the MSSM case. It is possible to construct models [81] in which V_L in eq.(6.20) gets replaced by the CKM matrix making the F_{ij} very similar to the MSSM model. We have presented phenomenological analysis of this model in chapter 3.

The examples given here are representative rather than exhaustive. One could consider several similar structures, e.g. one based on the Fritzsch ansatz [171] or on some different textures , e.g. based on the μ-τ interchange symmetry [74] as discussed in the chapter 4. All these model have the property of the suppressed and hierarchical FCNC. Without subscribing to any specific model we shall now consider the general implications for the $b \leftrightarrow s$ transitions.

6.2 Effective Hamiltonian for the $b \leftrightarrow s$ transitions

The basic interaction in eq.(6.7) leads to both $\Delta B = 1$ and 2 transitions. We give below the corresponding effective Hamiltonian.

6.2.1 B_s^0-$\bar{B}_s^0$ mixing

B_s^0-$\bar{B}_s^0$ mixing is governed by the transition amplitude [172]

$$M_{12}^{*s} \equiv \langle \bar{B}_s^0 | \mathcal{H}_{eff} | B_s^0 \rangle \ .$$

Here,

$$\mathcal{H}_{eff} \equiv \mathcal{H}_{eff}^{SM} + \mathcal{H}_{eff}^{NP}$$

includes the SM and the new physics contribution to the B_s^0-$\bar{B}_s^0$ transition. $\mathcal{H}_{eff}^{NP}$ arises in the present case from the tree level exchange of the ϕ_H^0 field and its complex conjugate. We use the parametrization and treatment given in [161] to evaluate the effective Hamiltonian.

$$\mathcal{H}_{eff}^{NP} = \frac{G_F^2 M_W^2}{16\pi^2}(V_{tb}^* V_{ts})\left[Q_2^{LR}C_2^{LR} + Q_1^{LL}C_1^{LL} + Q_1^{RR}C_1^{RR}\right] \ . \tag{6.21}$$

Here, G_F is the Fermi coupling constant, M_W is the mass of the W-boson and V denotes the CKM matrix. The operators Q induced by the FCNC couplings in this case are defined by

$$Q_2^{LR} = (\bar{b}_L s_R)(\bar{b}_R s_L) \ \ ; Q_1^{LL} = (\bar{b}_R s_L)(\bar{b}_R s_L) \ \ ; Q_1^{RR} = (\bar{b}_L s_R)(\bar{b}_L s_R)$$

The coefficients $C_{1,2}$ are the Wilson coefficients of these operators evaluated at the Higgs mass scale. They follow from the tree level Feynman diagrams involving the neutral Higgs field ϕ_H^0 and can be evaluated in a straightforward manner:

$$\begin{aligned} C_2^{LR} &= -\frac{16\pi^2}{G_F^2 M_W^2 (V_{tb}^* V_{ts})^2} F_{32} F_{23}^* \langle \phi_H | \phi_H^* \rangle \ , \\ C_1^{LL} &= -\frac{1}{2}\frac{16\pi^2}{G_F^2 M_W^2 (V_{tb}^* V_{ts})^2} {F_{23}^*}^2 \langle \phi_H^* | \phi_H^* \rangle, \\ C_1^{RR} &= -\frac{1}{2}\frac{16\pi^2}{G_F^2 M_W^2 (V_{tb}^* V_{ts})^2} F_{32}^2 \langle \phi_H | \phi_H \rangle \ . \end{aligned} \tag{6.22}$$

Here $\langle \phi_H | \phi_H \rangle$ is i times the propagator of ϕ_H^0 field evaluated at the zero momentum transfer and $\langle \phi_H | \phi_H^* \rangle$ and $\langle \phi_H^* | \phi_H^* \rangle$ are defined analogously. These propagators are evaluated by decomposing the ϕ_H^0 field in terms of the Higgs mass eigenstates denoted as h, H (scalars) and A (pseudo scalar). We shall assume that CP is conserved in the Higgs sector in which case decomposition of ϕ_H^0 is given by

$$\begin{aligned} Re(\phi_H^0) &= \frac{1}{\sqrt{2}}(\cos(\alpha-\beta)h + \sin(\alpha-\beta)H) \ , \\ Im(\phi_H^0) &= \frac{1}{\sqrt{2}}A \ , \end{aligned} \tag{6.23}$$

where $M_{h,H,A}$ are the masses of the fields h, H, A respectively. Using these expressions, one arrives at

$$\begin{aligned} \langle \phi_H | \phi_H^* \rangle &= \frac{\sin^2(\alpha-\beta)}{2M_H^2} + \frac{\cos^2(\alpha-\beta)}{2M_h^2} + \frac{1}{2M_A^2} \ , \\ \langle \phi_H | \phi_H \rangle &= \langle \phi_H^* | \phi_H^* \rangle = \frac{\sin^2(\alpha-\beta)}{2M_H^2} + \frac{\cos^2(\alpha-\beta)}{2M_h^2} - \frac{1}{2M_A^2} \ . \end{aligned} \tag{6.24}$$

Taking the matrix elements of eq.(6.21) between the $\bar{B}_s^0$ and B_s^0 mesons one arrives at [161]

$$(M_{12}^{s*})^{NP} = \frac{G_F^2 M_W^2}{48\pi^2} M_{B_s} f_{B_s}^2 (V_{tb}^* V_{ts})^2 \left[P_2 C_2^{LR} + P_1 C_1^{LL} + P_1 C_1^{RR}\right] \ . \tag{6.25}$$

and M_{B_S}, f_{B_s} are the mass and the decay constant of the B_s meson. $P_{1,2}$ summarize the effect of the evolution to the low scale and of the Bag factors. When Higgs scale is identified with the top mass one gets, $P_2 \approx 2.56$ and $P_1 \approx -1.06$ [161]. For definiteness, we will use these values in the numerical analysis. The total mixing amplitude is given by

$$M_{12}^{s*} = (M_{12}^{s*})^{SM} + (M_{12}^{s*})^{NP} \equiv (M_{12}^{s*})^{SM}(1 + \kappa_s^H e^{2i(\phi_s^H + \beta_s)}) \ , \tag{6.26}$$

The $(M_{12}^{*s})^{SM}$ is given [172] by

$$(M_{12}^{*s})^{SM} = \frac{G_F^2 M_W^2 M_{B_s} f_{B_s}^2 B_{B_s} \eta_B}{12\pi^2} (V_{tb}^* V_{ts})^2 S_0(x_t) \ , \tag{6.27}$$

with $S_0(x_t) \approx 2.3$ for $m_t \approx 161$ GeV and $\eta_B \approx 0.55$ represents the QCD corrections. Using eq.(6.22) we find

$$\begin{aligned} \kappa_s^H e^{2i\phi_s^H} \quad &= \quad -\frac{4\pi^2}{B_{B_s}\eta_B S_0(x_t) G_F^2 M_W^2 |V_{tb}^* V_{ts}|^2} \\ &\times \left[P_2 F_{32} F_{23}^* \langle \phi_H | \phi_H^* \rangle + \frac{1}{2} P_1 (F_{32}^2 \langle \phi_H | \phi_H \rangle + F_{23}^{*2} \langle \phi_H | \phi_H \rangle^*)\right] . \end{aligned} \tag{6.28}$$

The new physics induced phase in the above expression is determined by the phases of the FCNC couplings and the complex Higgs propagators. We assume throughout that the Higgs sector is CP conserving. In this case, the only source of the non-standard CP violation resides in the phases of F_{23}, F_{32}.

6.2.2 $\Delta B = 1$ transitions

The transition $b \to s$ occurs in SM at the 1-loop level. The corresponding effective Hamiltonian is described in terms of 10 different operators and associated Wilson coefficients. The complete list can be found for example in [155]. The Wilson coefficients are calculated at the electroweak scale and are then evaluated in the low energy theory in a standard way. If some new physics is present at or above the electroweak scale then (1) it can give additional contributions to some of the Wilson coefficients and/or (2) can lead to new sets of operators not present in the SM. We will mainly be concerned here with effects due to (2) induced by the presence of the non-standard Higgs field(s) but the effect (1) may also be simultaneously present.

The Higgs induced operators for the transition $b \to s\mu^+\mu^-$ may be parameterized as:

$$\mathcal{H}^H_{eff} \equiv -\frac{4G_F}{\sqrt{2}} V_{tb}V^*_{ts} \sum_{i=S,S',P,P'} C_i(\mu) O_i(\mu) \; . \tag{6.29}$$

where μ denotes the renormalization scale at which the operators and the Wilson coefficients appearing above are defined. The operators are defined as

$$\begin{aligned} O_S &= \frac{e^2}{16\pi^2} \bar{s}_L b_R \bar{\mu}\mu \quad ; \quad O_P = \frac{e^2}{16\pi^2} \bar{s}_L b_R \bar{\mu}\gamma_5\mu \\ O'_S &= \frac{e^2}{16\pi^2} \bar{s}_R b_L \bar{\mu}\mu \quad ; \quad O'_P = \frac{e^2}{16\pi^2} \bar{s}_R b_L \bar{\mu}\gamma_5\mu \; , \end{aligned} \tag{6.30}$$

The tree level Higgs exchange through eq.(6.7) induce the above operators with the Wilson coefficients given by

$$\begin{aligned} C_S &= -\frac{\sqrt{2}\pi}{\alpha G_F V_{tb} V^*_{ts}} \frac{F_{23} m_\mu}{2v\cos\beta} \Big(\frac{\sin(\alpha-\beta)\cos\alpha}{M_H^2} - \frac{\cos(\alpha-\beta)\sin\alpha}{M_h^2} \Big) \; , \\ C'_S &= -\frac{\sqrt{2}\pi}{\alpha G_F V_{tb} V^*_{ts}} \frac{F^*_{32} m_\mu}{2v\cos\beta} \Big(\frac{\sin(\alpha-\beta)\cos\alpha}{M_H^2} - \frac{\cos(\alpha-\beta)\sin\alpha}{M_h^2} \Big) \; , \\ C_P &= -\frac{\sqrt{2}\pi}{\alpha G_F V_{tb} V^*_{ts}} \frac{F_{23} m_\mu}{2v\cos\beta} \frac{\sin\beta}{M_A^2} \; , \\ C'_P &= -\frac{\sqrt{2}\pi}{\alpha G_F V_{tb} V^*_{ts}} \frac{F^*_{32} m_\mu}{2v\cos\beta} \Big(-\frac{\sin\beta}{M_A^2} \Big) \; . \end{aligned} \tag{6.31}$$

Eq.(6.29) contributes both to the $\bar{B}_s \to \mu^+\mu^-$ and the $\bar{B}_d \to \bar{K}(\bar{K}^*)\mu^+\mu^-$ processes. The Higgs contribution to the branching ratio for the former process follows [155, 165, 166] in a straightforward way from eq.(6.29):

$$\begin{aligned} Br(\bar{B}_s \to \mu^+\mu^-) &= \left(\frac{\alpha G_F |V_{tb} V^*_{ts}|}{\sqrt{2}\pi} \right)^2 \frac{f^2_{B_s} M^5_{B_s} \tau_{B_s}}{32\pi(m_b+m_s)^2} \left(1 - \frac{4m_\mu^2}{M^2_{B_s}} \right)^{1/2} \\ &\times \left(\Big(1 - \frac{4m_\mu^2}{M^2_{B_s}}\Big) |C_S - C'_S|^2 + |C_P - C'_P + 2\frac{m_\mu}{M^2_{B_s}} C_{10}|^2 \right) \; . \end{aligned} \tag{6.32}$$

The explicit expression for C_{10} in SM can be found for example in [175]. In view of the smallness of this contribution, we would be interested in exploring the region of parameter space where the Higgs contribution significantly dominates over the contribution from C_{10}. It is thus useful to separate out the Higgs contribution B_H alone to the above branching ratio and we define:

$$\begin{aligned}
B_H \;\equiv\; & \left(\frac{\alpha G_F |V_{tb}V_{ts}^*|}{\sqrt{2}\pi}\right)^2 \frac{f_{B_s}^2 M_{B_s}^5 \tau_{B_s}}{32\pi(m_b+m_s)^2}\left(1-\frac{4m_\mu^2}{M_{B_s}^2}\right)^{1/2} \\
& \times\left(\left(1-\frac{4m_\mu^2}{M_{B_s}^2}\right)|C_S-C_S'|^2+|C_P-C_P'|^2\right) \qquad (6.33)
\end{aligned}$$

We however use the full equation, (6.32) in our numerical study.

The process $\bar{B}_d \to \bar{K}\mu^+\mu^-$ is studied in detail in [155, 156] using the QCD factorization approach which works for the low q^2 region. The amplitude for this process depends on two hadronic form factors defined as [173, 174]

$$\langle K|\bar{s}\gamma_\mu b|\bar{B}\rangle = (2p_B-q)_\mu f_+(q^2)+\frac{M_B^2-M_K^2}{q^2}q_\mu[f_0(q^2)-f_+(q^2)]\ ,$$

where p_B and q respectively refer to four momenta of $\bar{B}$ meson and the dilepton pair respectively. Bobeth *et al* [156] evaluated the form factors using the QCD factorization and results based on the light cone sum rules to obtain predictions (details can be found in the appendix A,B of the reference [156]) for the angular distribution of the dilepton pair and the branching ratio for $\bar{B}_d \to \bar{K}\mu^+\mu^-$. Restricting the dilepton invariant (mass)^2 between the range $1\text{GeV}^2 < q^2 < 7\text{GeV}^2$, they derive [156]

$$\begin{aligned}
Br(\bar{B}_d\to\bar{K}\mu^+\mu^-) \;=\; & \left(\frac{\tau_B^+}{1.64\text{ps}}\right)\Big(1.91+0.02(|\tilde{C}_S|^2+|\tilde{C}_P|^2)-\frac{m_\mu}{GeV}\frac{Re(\tilde{C}_P)}{2.92} \\
& -\frac{m_\mu^2}{GeV^2}\left(\frac{|\tilde{C}_S|^2}{5.98^2}+\frac{|\tilde{C}_P|^2}{10.36^2}\right)+O(m_\mu^3)\Big)\times 10^{-7} \qquad (6.34)
\end{aligned}$$

where $\tilde{C}_S, \tilde{C}_P$ are given in terms of $C_{S,P}, C'_{S,P}$ in eq.(6.31) by

$$\begin{aligned}
\tilde{C}_S &= C_S + C_S'\ , \\
\tilde{C}_P &= C_P + C_P'\ . \qquad (6.35)
\end{aligned}$$

6.3 Constraining the FCNC couplings

Among the processes mentioned above, the B_s^0-$\bar{B}_s^0$ transition is the most accurately measured and provide sensitive test of the FCNC couplings. In particular, the presence of these couplings in some cases can explain the additional CP violating phase in the $B_s \to J/\psi\phi$ decay.

The new physics contribution to B_s^0-$\bar{B}_s^0$ mixing is parameterized in terms of

$$\begin{aligned} C_{B_s} &= |1+\kappa_s^H e^{2i(\phi_s^H+\beta_s)}| \;, \\ \phi_{B_S} &= -\frac{1}{2}\mathrm{Arg}(1+\kappa_s^H e^{2i(\phi_s^H+\beta_s)}) \;, \end{aligned} \tag{6.36}$$

where κ_s^H is given in our case by eq.(6.28). The 95% allowed ranges of C_{B_s} and ϕ_{B_s} given by UTfit collaboration are [29]

$$\begin{aligned} C_{B_s} &= [0.68, 1.51] \;, \\ \phi_{B_s} &= [-30.5, -9.9] \cup [-77.8, -58.2] \end{aligned} \tag{6.37}$$

We shall derive constraints on F_{23}, F_{32} based on the above values and look at its observable consequences for the processes $\bar{B}_s \to \mu^+\mu^-, \bar{B}_d \to \bar{K}\mu^+\mu^-$. The derived constraints depend on the Higgs masses and mixing angles. But a simple and F_{ij}-independent correlations between κ_s^H and the Higgs contribution B_H to the branching ratio for the process $\bar{B}_s \to \mu^+\mu^-$ follows in the decoupling limit if it is assumed that the Higgs potential is the same as in the case of MSSM. We first derive this relation. Then we give up these simplifying assumptions in the Higgs sector and explore the Higgs parameter space numerically and study the correlation between κ_s^H and the $\bar{B}_s \to \mu^+\mu^-$ branching ratio.

The Higgs masses and mixing angle satisfy the following two relations [42, 159] if the scalar potential coincide with the MSSM.

$$\begin{aligned} \langle\phi_H|\phi_H\rangle &= 0 \\ \cos^2(\alpha-\beta) &= \frac{M_h^2(M_Z^2-M_h^2)}{M_A^2(M_H^2-M_h^2)} \;. \end{aligned} \tag{6.38}$$

The first relation leads to the following simple expression for κ_s:

$$\kappa_s^H e^{2i\phi_s^H} = -\frac{4\pi^2 P_2 F_{32} F_{23}^*}{B_{B_s}\eta_B S_0(x_t) G_F^2 M_W^2 |V_{tb}^* V_{ts}|^2 M_A^2} \;. \tag{6.39}$$

Note that

$$e^{2i\phi_s^H} = -\frac{F_{32}F_{23}^*}{|F_{32}F_{23}^*|}$$

directly probes the CP violating phase in the FCNC couplings and would depend on the model for quark masses under consideration.

- In models with Hermitian mass matrices, $\phi_s^H = Arg(F_{32}) \pm \pi$. This class of models can account for possible large CP violating phase ϕ_s.

- In contrast, the models with symmetric mass matrices, automatically imply $\phi_s^H = \pm\pi$. Thus even the presence of FCNC in these models does not lead to large CP violation. Alternative source of CP violation can arise in these models if the Higgs sector violate CP. In this case, mixing between the scalar and pseudo-scalar generate additional phase which can contribute to ϕ_s^H. This scenario was studied in [170] in a specific model with symmetric quark mass matrices.

- ϕ_s^H is again given by the phase of F_{32} in class (C) models satisfying $F_{32} = \frac{m_s}{m_b} F_{23}^*$. In particular, MSSM with MFV as well as the 2HDM of ref([81]) predict $F_{32} \sim V_{tb}^* V_{ts}$. As a consequence, the Higgs generated phase ϕ_s^H coincide with the SM phase β_s which is known to be small. Thus, these type of models will also need additional source, e.g. scalar-pseudo scalar mixing if large ϕ_s is established.

The magnitude κ_s^H of the Higgs contribution to the $B_s^0 - \bar{B}_s^0$ mass difference relative to the SM contribution can be quite large for reasonable values of the unknown parameters. Eq.(6.39) implies that

$$\kappa_s^H \approx 0.6 \left(\frac{F_{23}^* F_{32}}{10^{-6}}\right) \left(\frac{300\mathrm{Ge}V}{M_A}\right)^2 \quad . \tag{6.40}$$

Consider various model expectations:

- If one uses the Cheng-Sher ansatz eq.(6.3) then $|F_{23}F_{32}| \approx \mathcal{O}(1)\frac{m_s m_b}{v^2} \approx 10^{-5}$. Eq.(6.40) then gives large contribution to κ_s^H.

- Eq.(6.14) gives the typical magnitude of FCNC in class of models discussed in section (2A). In case of Hermitian textures with $|F_{32}| = |F_{23}| \sim \frac{m_b}{v\cos\beta\sin\beta}|V_{L33}^* V_{L32}|$ we obtain $|F_{23}^* F_{32}| \sim 10^{-6}$ if $V_L \sim V$ leading to a sizable value of κ_s^H in this case also.

- Models with $F_{32}^* = \frac{m_s}{m_b} F_{23}$ have additional suppression by $\frac{m_s}{m_b}$ compared to the previous estimates and one would need a light A to obtain significant κ_s^H.

There is also an additional suppression by loop factors in MSSM but the F_{ij} can get enhanced by $\tan\beta$. Typical magnitude of F_{23} in MSSM is given by [161]

$$F_{23} \approx \frac{g|V_{tb}^* V_{ts}| m_b \epsilon_Y}{\sqrt{2} M_W} \tan^2\beta \ ,$$

where ϵ_Y depends on the squark masses, the trilinear coupling A_t and μ. Taking the former two at TeV and $\mu \sim 300$ GeV, $\epsilon_Y \sim 0.002$ leading to $F_{23} \sim 2\ 10^{-6} \tan^2\beta$. Thus one can get significant effect only for very large $\tan\beta$

The expression for B_H gets simplified in the decoupling limit corresponding to $M_A^2 \sim M_H^2 \gg M_Z^2, M_h^2$. In this limit, $\alpha - \beta \to \frac{\pi}{2}$ from eq.(6.38) and the couplings $C_{S,S',P,P'}$ satisfy

$$\frac{C_S}{F_{23}} \approx \frac{C_S'}{F_{32}^*} \approx -\frac{C_P}{F_{23}} \approx \frac{C_P'}{F_{32}^*} \approx \frac{\sqrt{2}\pi m_\mu}{\alpha G_F V_{tb} V_{ts}^*} \frac{\sin\beta}{2v\cos\beta M_A^2} \ . \tag{6.41}$$

Because of this, the B_H in eq.(6.33) reduces to

$$\begin{aligned} B_H &= \frac{f_{B_s}^2 M_{B_s}^5 \tau_{Bs}}{128\pi(m_b+m_s)^2} \left(\frac{m_\mu^2}{v^2}\right) \frac{\tan^2\beta}{M_A^4} \left(1 - \frac{4m_\mu^2}{M_{B_s}^2}\right)^{1/2} \\ &\quad \times \left(\left(1 - \frac{4m_\mu^2}{M_{B_s}^2}\right) |F_{23} - F_{32}^*|^2 + |F_{23} + F_{32}^*|^2 \right) \ . \end{aligned} \tag{6.42}$$

The above equation allows us to derive simple correlation between κ_s^H and B_H. Combining eqs.(6.39) and (6.42) we find

$$\begin{aligned} B_H &\approx \frac{4b\kappa_s^H \tan^2\beta}{\kappa M_A^2} \approx 2.2\ 10^{-8} \kappa_s^H \left(\frac{\tan\beta}{50}\right)^2 \left(\frac{300\text{GeV}}{M_A}\right)^2 \ \text{(Models(A)\&(B))} \ , \\ &\approx \frac{2b\kappa_s^H \tan^2\beta}{\kappa M_A^2} \frac{m_b}{m_s} \approx 1.7\ \ 10^{-8} \kappa_s^H \left(\frac{\tan\beta}{10}\right)^2 \left(\frac{300\text{GeV}}{M_A}\right)^2 \ \text{(Models(C))} \ , \end{aligned} \tag{6.43}$$

where

$$b \equiv \frac{f_{B_s}^2 M_{B_s}^5 \tau_{B_s}}{128\pi(m_b+m_s)^2} \left(\frac{m_\mu}{v}\right)^2 \approx 1.1\ 10^4 \text{GeV}^4 \ ,$$

$$\kappa \equiv \frac{4\pi^2}{B_{B_s} \eta_B S_0(x_t) G_F^2 M_W^2 |V_{tb}^* V_{ts}|^2} \approx 2.2\ 10^{10} \text{GeV}^2 \ .$$

These correlations are independent of the magnitude and phases of the FCNC couplings and therefore test the assumption of (1) the presence of FCNC and (2)

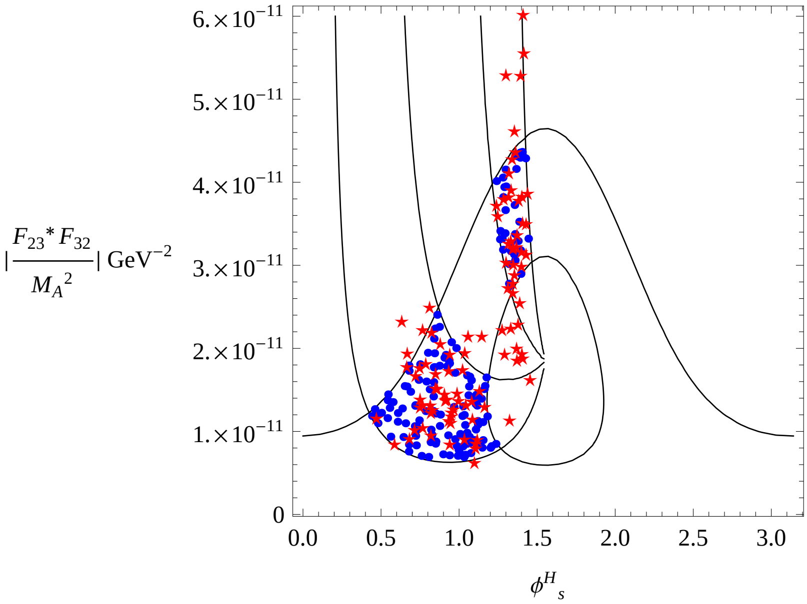

Figure 6.1: The region in $|\frac{F_{32}F_{23}^*}{M_A^2}| - \phi_s^H$ allowed by the UTfit constraints on B_s^0-$\bar{B}_s^0$ mixing. The solid lines and dots describe the region allowed under the assumption of the same Higgs potential as in MSSM. The stars correspond to assuming general Higgs sector and varying parameters as explained in the text.

the MSSM structure in the Higgs potential independent of the detailed structures of the quark mass matrices. These correlations also show that the FCNC would lead to sizable B_H provided it gives significant correction to κ_s^H also.

Let us now turn to the numerical analysis. If we assume the MSSM like Higgs structure then the allowed ranges of ϕ_{B_s} and C_{B_s} given in (6.37) determines the magnitude and phase of $F_{32}F_{23}^*$, see. eq.(6.39). The allowed region in $|\frac{F_{32}F_{23}^*}{M_A^2}|$-$\phi_s^H$ plane is shown in fig.(6.1). No specific assumption is made on the nature of the FCNC couplings. Therefore fig.(6.1) represents generic constraints on these couplings in all the 2HDM with tree level FCNC. The allowed values of $|F_{32}F_{23}^*|$ typically lie in the region $(1-5)\times 10^{-11} M_A^2$ GeV^{-2} with a strong correlation between its magnitude and phase. A generic 2HDM need not follow the MSSM structure and the decoupling would also correspond to only a part of the available parameter space. We study departures from these assumptions numerically as follows. We randomly vary the Higgs masses M_h, M_H, M_A between the range $100 - 500$ GeV keeping $M_h \leq M_H$. The mixing angles α, β are varied in their full range. From every set of these input parameters we allow those which give C_{B_s}, ϕ_{B_s} in the range

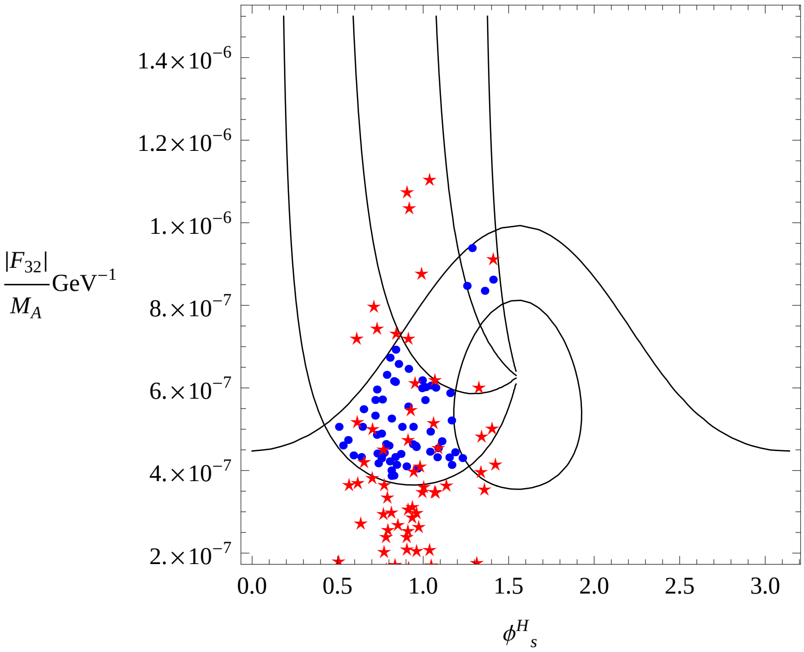

Figure 6.2: The region in $|\frac{F_{32}}{M_A}| - \phi_s^H$ allowed by the B_s^0-$\bar{B}_s^0$ mixing constraints in eq.(6.37) in class of models satisfying $F_{32} = \frac{m_s}{m_b} F_{23}^*$. Other details are as in Fig.(6.1)

in eq.(6.37) and the $Br(\bar{B}_s \to \mu^+\mu^-)$ below the limit in eq.(6.2). In this random analysis we distinguish two cases. One in which the MSSM relation eq.(6.38) remains true. These cases are shown as dots in our figure while the more general case without that assumption is shown as $\star$.

Fig.(6.2) shows the allowed region in the $\frac{|F_{32}|}{M_A}$-ϕ_s^H plane in classes of models which satisfy the constraints $F_{32} = \frac{m_s}{m_b} F_{23}^*$. One obtains the constraint $|F_{32}| \lesssim 1.2 \times 10^{-6} \frac{M_A}{\text{GeV}}$. This is to be compared with typical MSSM value $1.6 \;\; 10^{-6} \tan^2\beta$. Thus one would need $\tan^2\beta \approx \frac{M_A}{\text{GeV}}$ to account for the magnitude C_{B_s}. If F_{23} is given by eq.(6.14) then $F_{32} \approx 3 \times 10^{-5} \frac{1}{\sin\beta\cos\beta} \frac{.05}{|V_{L23}V_{L33}|}$. Thus, in this class of models one would need $|V_{L23}|$ somewhat smaller than $|V_{cb}| \sim 0.05$. In contrast to MSSM, large values of $\tan\beta$ are disfavored by the B_s^0-$\bar{B}_s^0$ mixing constraint in this class of models.

Fig.(6.3) shows the allowed values of the branching ratio for $\bar{B}_s \to \mu^+\mu^-$ obtained under the assumption $F_{32} = \frac{m_s}{m_b} F_{23}^*$ after imposing the UTfit constraints. It is possible to obtain relatively large branching ratios even for moderate values of $\tan\beta$ if M_A is light ~ 100 GeV.

Fig.(6.4) represents the corresponding constraints in class of models with Her-

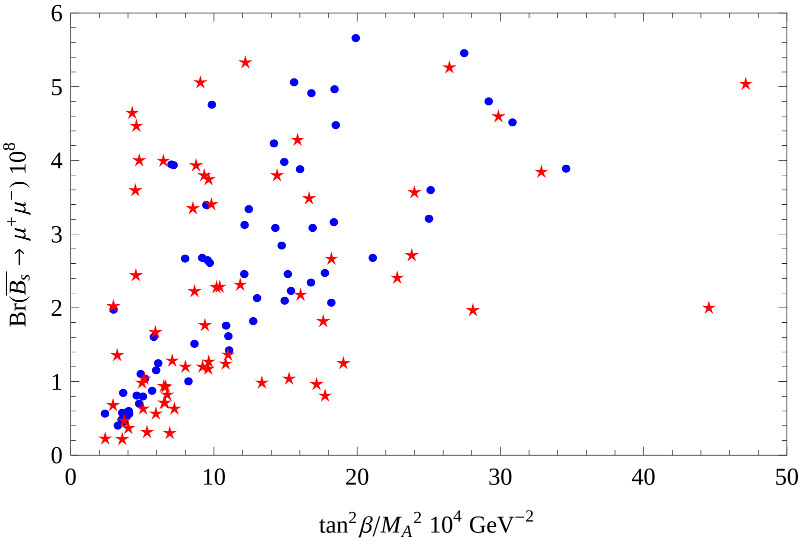

Figure 6.3: Variations for the branching ratio of the process $\bar{B}_s \longrightarrow \mu^+\mu^-$ with respect to $\tan^2\beta/M_A^2$ after incorporating the B_s^0-$\bar{B}_s^0$ constraints in model with $F_{32} = \frac{m_s}{m_b} F_{23}^*$. The dots and stars are defined as in Fig.(6.1)

mitian structure $F_{23} = F_{32}^*$. The required values for F_{32} are now $(2-6)\times 10^{-6} M_A$. But once again, one could obtain measurable rate for the dimuonic B_s decay even with moderate value of $\tan\beta$ as shown in fig.(6.5).

Fig.(6.6) displays the allowed values of $\bar{B}_s \longrightarrow \mu^+\mu^-$ in the case $F_{23} = F_{32}$. It is seen that one needs relatively large $\tan\beta$ typically $\tan^2\beta/M_A^2 \approx 10^{-2} GeV^{-2}$ in order to obtain a branching ratio larger than 10^{-8}. As already mentioned, this case also predicts vanishing Higgs induced phase if the Higgs sector is CP conserving.

While $\bar{B}_s \longrightarrow \mu^+\mu^-$ can receive significant contribution from the FCNC, the same is not the case with the semi leptonic process $\bar{B}_d \longrightarrow \bar{K}\mu^+\mu^-$. The FCNC induced contribution to this process can be qualitatively different than the 2HDM model based on the NFC. For example, if $F_{23} = F_{32}^*$ then eq.(6.33) and eq.(6.34) together imply that only the scalar Higgses contribute to $\bar{B}_d \longrightarrow \bar{K}\mu^+\mu^-$ while $\bar{B}_s \longrightarrow \mu^+\mu^-$ gets contribution from the pseudo scalar Higgs. Thus these processes are uncorrelated if the corresponding Higgs masses are not correlated. This is to be compared with the standard 2HDM or the MSSM where definite correlations between these processes have been pointed out [176, 177, 178]. At the quantitative level, we find numerically that after imposition of the $B_s^0 - \bar{B}_s^0$ mixing, the allowed numerical values of the couplings $\tilde{C}_{S,P}$ in all cases are such that the Higgs con-

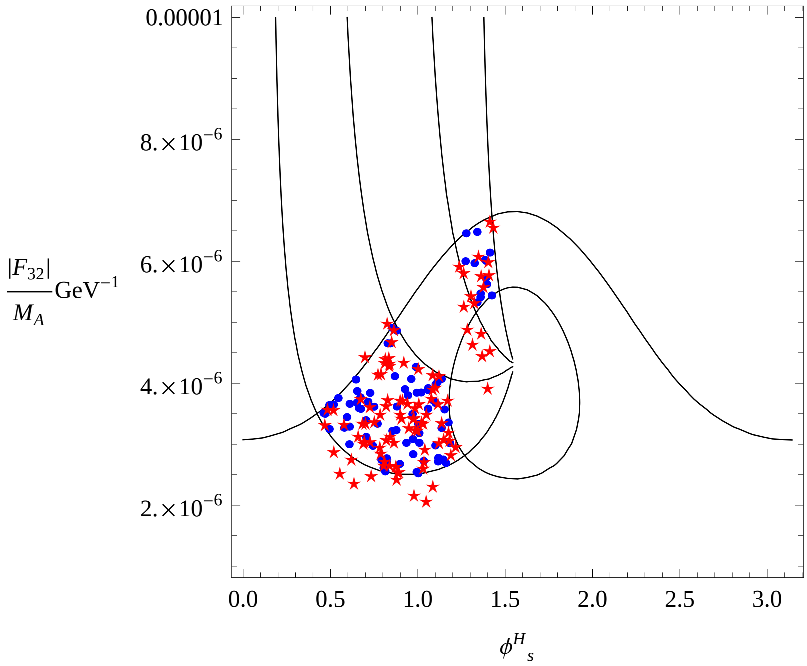

Figure 6.4: The region in $|\frac{F_{32}}{M_A}| - \phi_s^H$ allowed by the B_s^0-$\bar{B}_s^0$ mixing constraints in eq.(6.37) in class of models satisfying $F_{32} = F_{23}^*$. Other details are as in Fig.(6.1)

tribution to the branching ratio of $\bar{B}_d \to \bar{K}\mu^+\mu^-$amounts to at most few percent of the SM contribution . This is much smaller than the theoretical uncertainties. Therefore detecting Higgs effects in this branching ratio would need considerable reduction in theoretical errors. However one can conclude that if a significant new physics contribution to the branching ratio of this process is detected, it cannot be due to the presence of the Higgs induced FCNC.

6.4 Conclusion

$b \to s$ transition is known to be a good probe of physics beyond standard model. We have looked at the possibility of using this transition to test the Higgs induced FCNC assuming that the neutral Higgs provides the dominant contribution. In this case, several processes get described in terms of two complex parameters F_{23} and F_{32} and the Higgs mass parameters through equation(6.7). Phenomenological analysis in many of the earlier works [69, 71, 164] used the specific form for F_{23} and F_{32} motivated by the Cheng-Sher ansatz and often considered them to be real. We have tried to develop model-independent constraints on these parameters. In

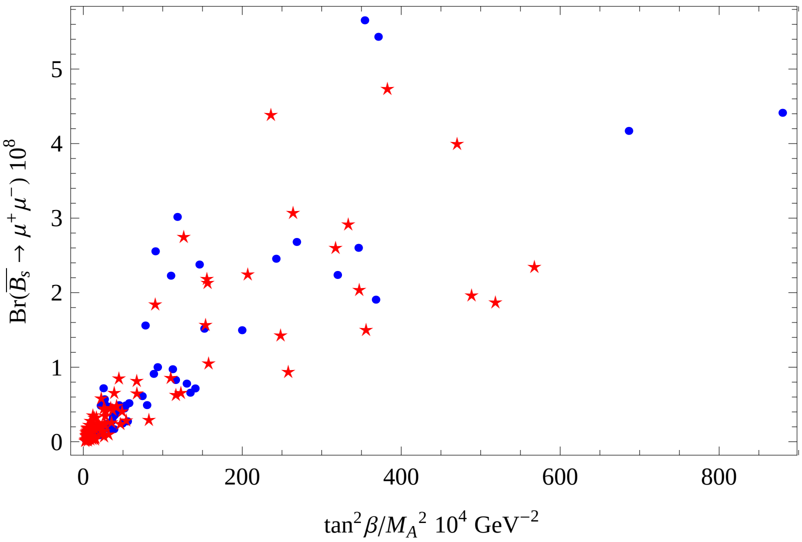

Figure 6.5: Variations for the branching ratio of the process $\bar{B}_s \to \mu^+\mu^-$ with respect to $\tan^2\beta/M_A^2$ after incorporating the B_s^0-$\bar{B}_s^0$ constraints in model with $F_{32} = F_{23}^*$. The dots and stars are defined as in Fig.(6.1)

particular, as shown here, the phases of the FCNC couplings can play an important role and may provide the large CP violating phase that may be needed to explain the CDF and $D0$ results on CP violation.

We discussed phenomenology of three broad classes of theories with FCNC satisfying the relations (1) $F_{23} = F_{32}^*$ (2) $F_{23} = F_{32}$ and (3) $F_{32} = \frac{m_s}{m_b} F_{23}^*$. We discussed several textures of the Yukawa couplings giving rise to these relations. In particular, MSSM and 2HDM with NFC provide examples of (3). We showed that the case (2) cannot account for large CP violating phase if the Higgs sector is CP conserving. The same applies to MSSM and the particularly predictive model of [81]. Our numerical analysis shows that one typically needs $F_{32} \sim (10^{-6} - 10^{-7}) M_A$ GeV^{-1}. As discussed here such values can arise within the textures discussed in section (6.1).

Using the available information on the B_s^0-$\bar{B}_s^0$ mixing we have worked out expectations for the leptonic branching ratio $\bar{B}_s \to \mu^+\mu^-$. It is found that the former constraints do allow measurable values for this branching ratio but the range for $\frac{\tan^2\beta}{M_A^2}$ required in these cases are different as seen from Figs. (6.3,6.5,6.6). In contrast, the Higgs contribution to the branching ratio of process $\bar{B}_d \to \bar{K}\mu^+\mu^-$ is constrained to be close to or smaller than the SM value in all these models. Thus

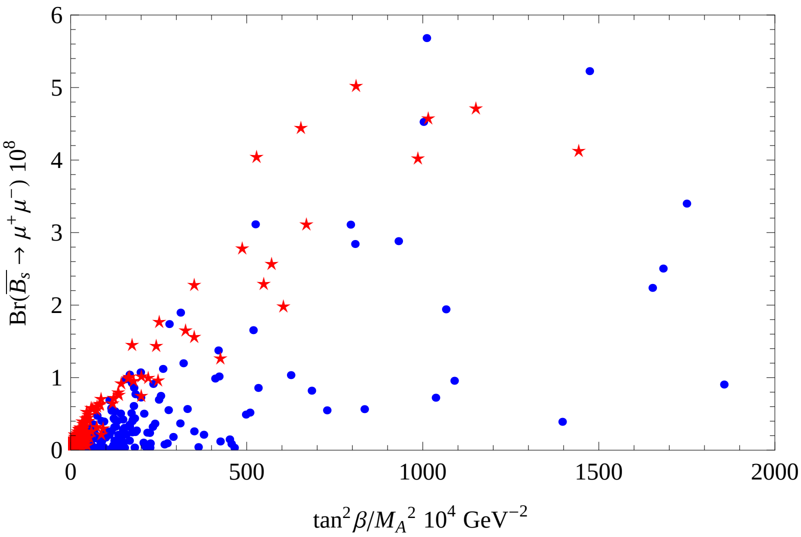

Figure 6.6: Variations for the branching ratio of the process $\bar{B}_s \to \mu^+\mu^-$ with respect to $\tan^2\beta/M_A^2$ after incorporating the B_s^0-$\bar{B}_s^0$ constraints in model with $F_{32} = F_{23}$. The dots and stars are defined as in Fig.(6.1)

any significant deviation in this branching ratio compared to the SM prediction will rule out all the models with FCNC in one shot under the assumption that these models are the only source of new physics in the B_s^0-$\bar{B}_s^0$ mixing.

Chapter 7

Summary

Standard model (SM) of elementary particles has been very successful and its predictions agrees to experimental results very well. In SM all the flavor and CP violations are described by CKM matrix. CP violation is explicit in SM because Yukawa couplings of the fermions with scalar doublet are complex. This leads to complex mass matrices for quarks and complex CKM matrix. Present experimental data provide evidences for complex CKM matrix. First evidence comes from the measurement of the angle γ of unitarity triangle. γ is measured through Dalitz plot analysis of decays of the neutral D mesons to $K_s^0\pi^+\pi^-$ observed by BABAR detector at the Stanford linear accelerator center [9] and by Belle detector at KEK [10]. The method [11] is based on decay chain $B^\mp \to \tilde{D}^{(*)0}K^{(*)\mp}$ where $\tilde{D}^{(*)0}$ represents $D^{(*)0}$ or $\bar{D}^{(*)0}$ and $D^{(*)0}$ represents D^0 or D^{*0} meson. BABAR collaboration obtained [9]

$$\gamma = 76^\circ \pm 22^\circ(\text{stat}) \pm 5^\circ(\text{syst}) \pm 5^\circ(\text{model})$$

Belle collaboration obtained [10]

$$\gamma = 53^{\circ}{}^{+15^\circ}_{-18^\circ}(\text{stat}) \pm 3^\circ(\text{syst}) \pm 9^\circ(\text{model})$$

Main contributions to the processes in this decay chain come from the tree level diagrams in SM. Hence under the assumption that new physics does not contribute significantly at tree level, the nonzero value of γ can be considered as evidence for complex CKM matrix. Another evidence for complex CKM matrix comes from determination of Wolfenstein parameter $\bar{\eta}$. In case of real CKM matrix $\bar{\eta} = 0$. UTfit group has determined CKM parameters from the NP generalized fit to several observables assuming the presence of arbitrary new physics contribution to K^0-$\bar{K}^0$, B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing. CKM parameters obtained from this fit are [14]

$$\begin{aligned}\bar{\rho} &= 0.20 \pm 0.06 \\ \bar{\eta} &= 0.36 \pm 0.04\end{aligned}$$

The nonzero value of $\bar{\eta}$ implies that CKM matrix is complex in presence of arbitrary new physics. In addition, prediction based on CKM mechanism are in good agreement with several CP violating observables. Hence CKM matrix is established as dominant source of CP violation. However the exact origin of CP violation is still unknown. A complex CKM matrix can arise from complex Yukawa couplings in case of explicit CP violation or from phase in the Higgs vacuum expectation value in case of spontaneous CP violation in models of new physics beyond SM having two or more Higgs doublets.

Experiments at B-factories at SLAC and Belle along with Tevatron have provided some hints of new physics beyond SM. One such hint comes from CP violating observable $S_{J/\psi K_S}$ of B_d^0-$\bar{B}_d^0$ mixing. Direct determination of $S_{J/\psi K_S}$ from the time dependent CP asymmetry of $B_d^0 \to J/\psi K_s$ by BABAR experiment at the Stanford linear accelerator center (SLAC) [19] and by Belle experiment at KEK does [20] not agree with the value determined from $\frac{|V_{ub}|}{|V_{cb}|}, \epsilon_K$, Δm_{B_s}, and Δm_{B_d} [31]. This deviation can be considered as hint of new physics. Another hint comes from CP violating phase ϕ_s of B_s^0-$\bar{B}_s^0$ mixing. Value of ϕ_s obtained from analysis of time dependent angular distribution of decay products in flavor tagged decay $B_s^0 \to J/\psi\phi$ decay by CDF and D0 experiments at Fermilab Tevatron collider through decay chain $B_s^0 \to J/\psi\phi$, $J/\psi \to \mu^+\mu^-$, $\phi \to K^+K^-$ [21, 22] does not agree with SM prediction for ϕ_s. UTfit group has combined all the available constraints on B_s mixing and performed a model independent analysis of NP contribution to B_s^0-$\bar{B}_s^0$ mixing and concluded that phase ϕ_s of B_s^0-$\bar{B}_s^0$ mixing deviates from the SM prediction by about 3σ [27, 29]. A similar analysis performed by CKMfitter group shows that the deviation from SM prediction is about 2.5σ [30].

Many new models have been suggested to explain the above mentioned deviations. Two Higgs doublet model (2HDM) is the simplest extension of SM having one extra Higgs doublet. 2HDM has two additional sources of flavor violation. (1)

Tree level flavor changing neutral currents (FCNC) and (2) Charged Higgs interactions. General tree level FCNC gives large contributions to K^0-$\bar{K}^0$ mixing on which very stringent constraints exists. Hence contribution to K^0-$\bar{K}^0$ mixing from tree level FCNC is required to be eliminated or suppressed. Tree level FCNC can be eliminated by imposing some discrete symmetries in such a way that quarks get their masses by coupling to only one of the scalar doublet [43, 44]. This goes under the name of natural flavor conservation (NFC). Examples of this type of model are type-I and type-II 2HDM. In these models charged Higgs couplings do not provide any new source of CP violation and hence they can not explain the large CP phase of B_s^0-$\bar{B}_s^0$ mixing if confirmed in the future. Hence we need to go beyond type-I and type-II 2HDM and allow tree level FCNC. 2HDM with general FCNC provides several sources of CP violation which includes (1) Complex CKM matrix (2) Tree level FCNC. (3) Charged Higgs interactions (4) scalar - pseudo scalar mixing [45]. In this thesis we have studied implications of these sources in several different variants of 2HDM having different structure for flavor and CP violation with a view to explain the deviations from SM predictions observed in B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixings.

In chapter 3 we have presented study of a 2HDM which satisfies the principle of minimal flavor violation (MFV). In a model with MFV all the flavor and CP violation are described by the CKM matrix only. In 2HDM, charged Higgs interactions are described in terms of CKM matrix elements. Tree level FCNC present in general 2HDM are not related to CKM matrix. 2HDM with natural flavor conservation satisfy the criteria of MFV but these models do not provide any new source of CP violation which may be required to explain deviation of phase ϕ_s of B_s^0-$\bar{B}_s^0$ mixing from SM prediction. Using some discrete symmetries it is possible to obtain a 2HDM in which tree level FCNC are also described in terms of CKM elements. Hence this model satisfy the criteria of MFV despite the presence of tree level FCNC. This model can be obtained from general 2HDM by imposition of following discrete symmetry

$$(Q'_{1,2L}, \phi_1) \rightarrow w(Q'_{1,2L}, \phi_1),\ u'_{1,2R} \rightarrow w^2 u'_{1,2R} \tag{7.1}$$

Here $Q'_{1,2L}$ represents doublet of left handed quarks for first two generations.

$w, w^2 \neq 1$ are complex numbers. ϕ_1 is Higgs doublet. $u'_{1,2R}$ are right handed up type quarks of first two generations. All other fields remain unchanged under the symmetry. In this model FCNC couplings in down quarks are given as,

$$F^d_{ij} = V_{3i} V^*_{3j} m_j \tag{7.2}$$

Where $i, j = 1, 2, 3$ and $i \neq j$. V_{ij} are CKM elements and m_j is the mass of down quark of j^{th} generation. In this model there are no FCNC in the up type quarks. It can be seen that FCNC couplings are determined in terms CKM elements and mass of down type quark. FCNC in this model have the following hierarchy

$$F_{12} < F_{13}, F_{23} \tag{7.3}$$

For this type of hierarchy, tree level FCNC contribution to K^0-$\bar{K}^0$ mixing is suppressed compared to B^0_d-$\bar{B}^0_d$ and B^0_s-$\bar{B}^0_s$ mixing. If this model contain only two Higgs doublet then it can not provide large CP phase required in B^0_s-$\bar{B}^0_s$ mixing. Large phase in B^0_s-$\bar{B}^0_s$ mixing can be obtained by introducing complex Higgs singlets. Introduction of Complex Higgs singlets will also allow for the Peccei-Quinn solution of strong CP problem. We assume that Higgs mixing contains an effective CP violating phase which can be generated through complex Higgs singlets and make a phenomenological study of this model in two separate cases of Charged Higgs dominance and neutral Higgs dominance. In the case of charged Higgs dominance, predictions of this model is similar to that of type-II 2HDM. We have shown that in case of neutral Higgs dominance this model has a special property that new contributions to B^0_d-$\bar{B}^0_d$ and B^0_s-$\bar{B}^0_s$ mixing are correlated. We have used this property in a fit to various observables and obtained Wolfenstein parameter $\bar{\rho}, \bar{\eta}$ in this model.

In chapter 4 we have considered a 2HDM with FCNC in which spontaneous CP violation gives rise to complex CKM matrix. The FCNC present in this model can give large contribution to the processes such as K^0-$\bar{K}^0$, B^0_d-$\bar{B}^0_d$, B^0_s-$\bar{B}^0_s$ mixing. There are very stringent constraints on the new contribution to K^0-$\bar{K}^0$ mixing, while some new contribution may be allowed in B^0_d-$\bar{B}^0_d$ and B^0_s-$\bar{B}^0_s$ mixing. Hence a selective suppression of FCNC is required. To obtain such a selective suppression of FCNC we have used 23 symmetry which is a generalization of μ-τ symmetry used extensively in lepton sector to explain maximal atmospheric mixing. Action of 23 symmetry is given as

$$f_2 \leftrightarrow f_3,\ \phi_2 \rightarrow -\phi_2 \tag{7.4}$$

Here f_2, f_3 are quarks of second and third generations. ϕ_2 is Higgs doublet. If 23 symmetry remains unbroken then the model is CP conserving. Spontaneous CP violation is achieved by soft breaking of 23 symmetry by the term $\mu_{12}(\phi_1^\dagger\phi_2)$ in the Higgs potential. Hence vacuum expectation value (vev) of one of the neutral Higgs becomes complex. Since both the Higgs doublet couple to quarks of the both the type, the phase of Higgs vev can not be removed from the quark mass matrices. Therefore quark mass matrices become complex and gives rise to complex CKM matrix. Thus in this model complex CKM matrix arises from spontaneous CP violation. FCNC couplings in this model satisfy the hierarchy $F_{12} < F_{13} < F_{23}$. Hence it is possible to obtain large neutral Higgs contribution to B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing while suppressing the new contribution to K^0-$\bar{K}^0$ mixing. In our numerical analysis we have determined parameters of quark mass matrices from a fit to observables which include six quark masses, CKM elements moduli $|V_{us}|, |V_{ub}|, |V_{cs}|$ and Jarlskog invariant. We could get very good fits for parameters of quark mass matrices corresponding to small breaking of 23 symmetry which shows that 23 symmetry gives good description of masses and mixing of quarks. Parameters of quark mass matrices obtained from these fit are used to calculate FCNC couplings. From these couplings we have calculated transition amplitudes and CP violating observables for K^0-$\bar{K}^0$, B_d^0-$\bar{B}_d^0$, and D^0-$\bar{D}^0$ mixing and compared them with the experimental data. From the set of fitted quark mass matrix parameters, for which the calculated mixing matrix elements and CP violating observables for K^0-$\bar{K}^0$, B_d^0-$\bar{B}_d^0$, and D^0-$\bar{D}^0$ lies in the ranges allowed by experimental data, we calculate NP contribution to matrix element and CP violating phase ϕ_s of B_s^0-$\bar{B}_s^0$ mixing. We find that new physics contribution to magnitude of B_s^0-$\bar{B}_s^0$ mixing is small while the NP phase is relatively small but larger than the SM phase.

In chapter 5 we have shown that it is possible to eliminate tree level FCNC in a 2HDM using flavor symmetries. It was shown that any non-Hermitian mass matrix M_q has an invariance defined as

$$S_L^{q\dagger} M^q S_R^q = M^q \tag{7.5}$$

Where $S^q_{L,R} = V^q_{L,R} P(\alpha_i) V^{q\dagger}_{L,R}$ and $P(\alpha_i) = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$. This statement is equivalent to statement of conservation of fermion number of each generation by fermion mass term. In most general situation matrices $S^q_{L,R}$ define two independent $U(1) \times U(1) \times U(1)$ symmetries G^u and G^d for up and down quark mass matrices respectively. Under the assumption that the forms of $S^q_{L,R}$ are independent of Higgs parameters such as $\tan\beta$ and phase θ of vev of the neutral Higgs, the Yukawa coupling matrices of quarks with Higgs doublets also remain invariant under the same symmetry defined by $S^q_{L,R}$. If G^u and G^d refer to full $U(1) \times U(1) \times U(1)$ symmetry with independent α_i then Yukawa coupling matrices of quarks to both the Higgs doublets can be simultaneously diagonalized. Hence there are no tree level FCNC in this case. In this model charged Higgs contribution to neutral meson mixing contains new phases not present in type-I or type-II 2HDM. We have also presented concrete model examples of above mentioned scenario. We have calculated charged Higgs contributions to neutral meson mixing in the present scenario under the simplifying condition that first two generation quark masses as well as flavor changing couplings entering in charged Higgs interaction can be neglected compared to third generation mass and flavor changing couplings. In this case, charged Higgs contribution does not provide any new CP phases to K^0-$\bar{K}^0$ mixings, while new contribution to B^0_d-$\bar{B}^0_d$ and B^0_s-$\bar{B}^0_s$ mixing are correlated. This is the clear prediction of present case. Correlation among charged Higgs contribution to B^0_d-$\bar{B}^0_d$ and B^0_s-$\bar{B}^0_s$ mixing are consistent with current data and other recent analysis. We have also obtained constraints on the parameters of this model from the available constraints on B^0_s-$\bar{B}^0_s$ mixing and like-sign dimuon charge asymmetry of semileptonic b-hadron decay.

In chapter 6 we have considered 2HDM with general tree level FCNC and studied its implications for the processes (1) B^0_s-$\bar{B}^0_s$ mixing (2) leptonic decays $\bar{B}_s \to \mu^+\mu^-$ (3) semileptonic decays $\bar{B}_d \to (\bar{K}, \bar{K}^*)\mu^+\mu^-$. These processes arises from the quark level interaction $b \leftrightarrow s$. These processes provide observables which are small in SM. NP models in general may give large contributions to these observables. Hence these observables can be used to search for new physics. One of the observable is the CP violating phase ϕ_s of B_s^0-$\bar{B}_s^0$ mixing which is predicted to be very small in SM while the experimental data from CDF and D0 groups allow larger value for ϕ_s. Second observables is the branching ratio of $\bar{B}_s \to \mu^+\mu^-$. SM prediction for this branching ratio is order of magnitude smaller then the current experimental limit. Branching ratio for $\bar{B}_d \to (\bar{K}, \bar{K}^*)\mu^+\mu^-$ are close to the SM prediction. But still these processes provide useful constraints on NP that may be present. We have analyzed constraints and predictions of tree level FCNC present in general 2HDM in the above mentioned processes. Most of the earlier phenomenological studies of 2HDM with the tree level FCNC used the Cheng-Sher ansatz. We have shown that there exists phenomenologically interesting models in which tree level FCNC are different from Cheng-Sher ansatz. In particular we have studied (1) Model with Hermitian structure for FCNC : $F_{ij} = F_{ji}^*$ (2) Model with symmetric structure for FCNC : $F_{ij} = F_{ji}$ (3) Model with minimal supersymmetric standard model (MSSM) like structure for FCNC : $F_{ij} = \frac{m_j}{m_i} F_{ji}^*$. Here m_i, m_j $(i, j = d, s, b)$ represent masses of the corresponding down type quarks. We have obtained expressions for the B_s^0-$\bar{B}_s^0$ mixing, $\bar{B}_s \to \mu^+\mu^-$ and $\bar{B}_d \to \bar{K}\mu^+\mu^-$ for 2HDM with general FCNC and also for 2HDM with special structures of FCNC mentioned above. It is seen that the 2HDM with symmetric structure for FCNC can not explain the large CP violating phase in B_s^0-$\bar{B}_s^0$ mixing if Higgs sector is also CP conserving. Using the experimental information and SM prediction for B_s^0-$\bar{B}_s^0$ mixing, we obtain predictions for branching ratios for $\bar{B}_s \to \mu^+\mu^-$ and $\bar{B}_d \to \bar{K}\mu^+\mu^-$ in case of the models with three specific structures for FCNC mentioned above. It is found that the branching ratio for $\bar{B}_s \to \mu^+\mu^-$ can go as high as current experimental bounds in this models while the branching ratio for $\bar{B}_d \to \bar{K}\mu^+\mu^-$ remains close to or smaller then the SM prediction.

All the variants of 2HDM considered in the different chapters of this thesis have different sources of new CP phases. In the 2HDM with MFV considered in chapter 3 the new phases arises due to scalar-pseudoscalar mixing due to presence of complex Higgs singlets. For the variant of 2HDM with spontaneous CP violation considered in chapter 4, phase present in the Higgs vacuum expectation value results in new phases in the FCNC couplings. In the variant considered in chapter

5, charged Higgs interaction contains new phases which can come either due to complex Yukawa couplings for the case of explicit CP violation or from phase in the Higgs vacuum expectation value for the case of spontaneous CP violation. For the 2HDM with general tree level FCNC considered in chapter 6 new phases arises in the FCNC couplings which are complex due to complex Yukawa matrices. All these models can give new phases to the B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing. Accurate determination of the CP violating observables along with further analysis of these models for different processes will enable us to test these models in the future.

Appendix A

Calculation of Wilson coefficients for B_q^0-$\bar{B}_q^0$ mixing in 2HDM without FCNC

Here we calculate contributions to B_q^0-$\bar{B}_q^0$ mixing ($q = d, s$) due to box diagrams arising from presence of charged Higgs and charged Goldstone bosons in a variant of 2HDM described in chapter 5. In this model FCNC are absent. Interactions of charged Higgs and charged Goldstone boson in this model are described by following Lagrangian.

$$\begin{aligned}
\mathcal{L}_{int} &= \frac{g}{\sqrt{2}m_w} H^+ \bar{u}_i V_{ij} \left(\gamma_L F_{ii}^{u*} - \gamma_R F_{jj}^d\right) d_j \\
&+ \frac{g}{\sqrt{2}m_w} H^- \bar{d}_i V_{ji}^* \left(\gamma_R F_{jj}^u - \gamma_L F_{ii}^{d*}\right) u_j \\
&+ \frac{g}{\sqrt{2}m_w} G^+ \bar{u}_i V_{ij} \left(\gamma_L D_{ii}^u - \gamma_R D_{ij}^d\right) d_j \\
&+ \frac{g}{\sqrt{2}m_w} G^- \bar{d}_i V_{ji}^* \left(\gamma_R D_{jj}^u - \gamma_L D_{ii}^d\right) u_j
\end{aligned} \tag{A.1}$$

Here $\gamma_L = \frac{1-\gamma_5}{2}$, $\gamma_R = \frac{1+\gamma_5}{2}$. D^u, D^d diagonal mass matrices for up and down type quarks respectively. D^u, D^d is obtained by diagonalizing the mass matrices $M^{u,d}$ as described in the following equation.

$$D^{u,d} = V_L^{u,d\dagger} M^{u,d} V_R^{u,d} \tag{A.2}$$

$V = V_L^{u\dagger} V_L^d$ is CKM matrix. Coupling matrices F^u, F^d are given as

$$F^u = V_L^{u\dagger}[v(-\gamma_1^u \sin\beta + \gamma_2^u \cos\beta e^{-i\theta})]V_R^u \quad (A.3)$$

$$F^d = V_L^{d\dagger}[v(-\gamma_1^d \sin\beta + \gamma_2^d \cos\beta e^{i\theta})]V_R^d \quad (A.4)$$

Here $v = 174$ GeV is the vacuum expectation value of SM Higgs. β is defined as $\tan\beta = v_2/v_1$ with $\langle\phi_1\rangle = v_1$, $\langle\phi_2\rangle = v_2 e^{i\theta}$. $\gamma_{1,2}^{u,d}$ are diagonal matrices with complex entries. We will denote the elements of F^u by $F_{\alpha\alpha}$ or $F_{\beta\beta}$ with $\alpha, \beta = u, c, t$ and elements of F^d by F_{qq} for $q = d, s$ or with F_{bb} for b quark. Charged current interaction of quarks is given as

$$\mathcal{L}^{CC,q} = \frac{g}{\sqrt{2}}\left(W_\mu^+ \bar{u}_L V \gamma^\mu d_L + W_\mu^- \bar{d}_L V^\dagger \gamma^\mu u_L\right) \quad (A.5)$$

A.1 Calculation of new contribution to B_q^0-$\bar{B}_q^0$ mixing

B_q^0-$\bar{B}_q^0$ mixing ($q = d, s$) matrix element arising due to new box diagrams is given as,

$$M_{12q}^{NP} = M_{12q}^{W^+H^-} + M_{12q}^{H^+W^-} + M_{12q}^{H^+H^-} + M_{12q}^{H^+G^-} + M_{12q}^{G^+H^-} \quad (A.6)$$

Following sections describe calculation of each of these box diagrams.

A.1.1 Box diagram with W^+, H^- in the loop

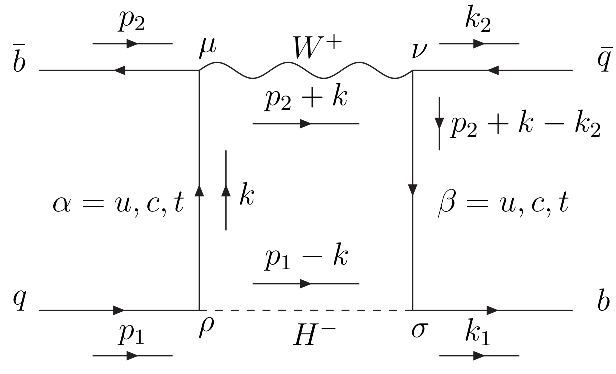

Figure A.1: Box diagram for B_q^0-$\bar{B}_q^0$ mixing in 2HDM with W^+, H^- in the loop

Box diagram contributing to B_q^0-$\bar{B}_q^0$ mixing with W^+ and H^- in the loop is shown in fig.(A.1). p_1, p_2 are initial momenta of $\bar{b}$ and q respectively. $q = d$ for B_d^0-$\bar{B}_d^0$ mixing and $q = s$ for B_s^0-$\bar{B}_s^0$ mixing. k_1, k_2 are final momenta of b and $\bar{q}$ respectively. α, β represent up type internal quarks. Matrix element due to above diagram is given as

$$\begin{aligned}
iM_{12q}^{W^+H^-} &= \sum_{\alpha,\beta=u,c,t} \int \frac{d^4k}{(2\pi)^4}\, \bar{v}_b\, \frac{ig}{\sqrt{2}}\gamma_\mu\gamma_L V_{\alpha b}^* \frac{i(\not{k}+m_\alpha)}{k^2-m_\alpha^2} \\
&\times \frac{ig}{\sqrt{2}m_W} V_{\alpha q}(\gamma_L F_{\alpha\alpha}^* - \gamma_R F_{qq}) u_q \\
&\times \left(\frac{-ig^{\mu\nu}}{(p_2+k)^2 - m_W^2} \right. \\
&\qquad \left. + \frac{i(p_2+k)^\mu (p_2+k)^\nu}{m_W^2} \left(\frac{1}{(p_2+k)^2 - m_W^2} - \frac{1}{(p_2+k)^2 - \xi m_W^2} \right) \right) \\
&\times \frac{i}{(p_1-k)^2 - m_H^2}\, \bar{u}_b \frac{ig}{\sqrt{2}m_W} V_{\beta b}^* (\gamma_R F_{\beta\beta} - \gamma_L F_{bb}^*) \\
&\left(\frac{i(\not{p}_2 + \not{k} - \not{k}_2 + m_\beta)}{(p_2+k-k_2)^2 - m_\beta^2} \right) \frac{ig}{\sqrt{2}} \gamma_\nu \gamma_L V_{\beta q} v_q
\end{aligned} \tag{A.7}$$

Here $u_{q,b}, v_{q,b}$ are the spinors of the q and b quark fields. ξ is gauge fixing parameter. We take the limiting case in which all external momenta and down and strange quark masses are zero. In this case above equation becomes

$$\begin{aligned}
iM_{12q}^{H^+H^-} &\approx \frac{g^4}{4m_W^2} \sum_{\alpha,\beta=u,c,t} V_{\alpha b}^* V_{\alpha q} V_{\beta b}^* V_{\beta q} \\
&\times \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2-m_\alpha^2} \frac{1}{k^2-m_\beta^2} \frac{1}{k^2-m_H^2} \\
&\times \bar{v}_b\, \gamma_\mu\gamma_L\, (\not{k}+m_\alpha)\, (\gamma_L F_{\alpha\alpha}^* - \gamma_R F_{qq}) u_q \\
&\times \left(\frac{-g^{\mu\nu}}{k^2-m_W^2} + \frac{k^\mu k^\nu}{m_W^2} \left(\frac{1}{k^2-m_W^2} - \frac{1}{k^2-\xi m_W^2} \right) \right) \\
&\times\ \bar{u}_b (\gamma_R F_{\beta\beta} - \gamma_L F_{bb}^*)(\not{k}+m_\beta)\gamma_\nu\gamma_L v_q
\end{aligned} \tag{A.8}$$

We use the following relation to simplify the above equation.

$$\begin{aligned}
\gamma_{L,R}\gamma_\mu &= \gamma_\mu\gamma_{R,L} \\
\not{k}\not{k} &= k^2 \\
\int \frac{d^4k}{(2\pi)^4} \frac{k^\mu k^\nu}{f(k^2)} &= \frac{g^{\mu\nu}}{4} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{f(k^2)}
\end{aligned} \tag{A.9}$$

It can be seen from the above expression of matrix element that denominator of the integral depends on k^2. Since $\int_{-\infty}^{\infty} f(x) = 0$ if $f(-x) = -f(x)$, hence terms with odd power of k in the numerator will vanish. Using all these results we get

$$\begin{aligned}
iM_{12q}^{W^+H^-} \approx & \frac{g^4}{4m_W^2} \sum_{\alpha,\beta=u,c,t} V_{\alpha b}^* V_{\alpha q} V_{\beta b}^* V_{\beta q} \\
& \times \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2-m_\alpha^2} \frac{1}{k^2-m_\beta^2} \frac{1}{k^2-m_H^2} \\
& \times \Bigg(-\frac{F_{qq}F_{bb}^*}{4(k^2-m_W^2)} \bar{v}_b\gamma_\mu\gamma_R\gamma_\rho u_q \; \bar{u}_b\gamma_L\gamma^\rho\gamma^\mu v_q \; k^2 \\
& + \frac{F_{qq}F_{bb}^*}{m_W^2} \bar{v}_b\gamma_R u_q \; \bar{u}_b\gamma_L v_q \; k^4 \left(\frac{1}{k^2-m_W^2} - \frac{1}{k^2-\xi m_W^2}\right) \\
& - \frac{F_{\alpha\alpha}^* F_{\beta\beta} m_\alpha m_\beta}{k^2-m_W^2} \bar{v}_b\gamma_\mu\gamma_L u_q \; \bar{u}_b\gamma_R\gamma^\mu v_q \\
& + \frac{F_{\alpha\alpha}^* F_{\beta\beta} m_\alpha m_\beta}{4m_W^2} \bar{v}_b\gamma_\mu\gamma_L u_q \; \bar{u}_b\gamma_R\gamma^\mu v_q \\
& \times k^2 \left(\frac{1}{k^2-m_W^2} - \frac{1}{k^2-\xi m_W^2}\right)\Bigg)
\end{aligned} \tag{A.10}$$

Now we take simplifying assumption that $F_{qq} = 0$; $q = d, s$. By taking $\alpha = \beta = t$ we get the dominant top quark contribution as follows.

$$\begin{aligned}
iM_{12q}^{W^+H^-} \approx & \frac{ig^4}{64\pi^2 m_W^4} (V_{tb}^* V_{tq})^2 \\
& \times \Bigg(-|F_{tt}|^2 m_W^2 m_t^2 D_0(m_t^2, m_t^2, m_H^2, m_W^2) \\
& + |F_{tt}|^2 m_t^2 (D_{00}(m_t^2, m_t^2, m_H^2, m_W^2) \\
& + D_{00}(m_t^2, m_t^2, m_H^2, m_G^2))\Bigg) \\
& \times (\bar{v}_b\gamma_\mu\gamma_L u_q \; \bar{u}_b\gamma^\mu\gamma_L v_q)
\end{aligned} \tag{A.11}$$

Here $m_G^2 = \xi m_W^2$. To obtain last expression we have used the Passarino-Veltman one-loop four-point functions with zero external momenta which are defined as

$$\begin{aligned}
\int \frac{d^4k}{i\pi^2} \frac{1}{(k^2-m_0^2)\,(k^2-m_1^2)\,(k^2-m_2^2)\,(k^2-m_3^2)} &= D_0(m_0^2, m_1^2, m_2^2, m_3^2) \\
\int \frac{d^4k}{i\pi^2} \frac{k^\mu k^\nu}{(k^2-m_0^2)\,(k^2-m_1^2)\,(k^2-m_2^2)\,(k^2-m_3^2)} &= g^{\mu\nu} D_{00}(m_0^2, m_1^2, m_2^2, m_3^2)
\end{aligned} \tag{A.12}$$

Where

$$
\begin{aligned}
D_0(m_0^2, m_1^2, m_2^2, m_3^2) &= \frac{1}{m_3^4}\Bigg(\frac{x\ln x}{(1-x)(x-y)(x-z)} + \frac{y\ln y}{(1-y)(y-x)(y-z)} \\
&\quad + \frac{z\ln z}{(1-z)(z-y)(z-x)}\Bigg) \\
D_{00}(m_0^2, m_1^2, m_2^2, m_3^2) &= \frac{1}{4m_3^2}\Bigg(\frac{x^2\ln x}{(1-x)(x-y)(x-z)} + \frac{y^2\ln y}{(1-y)(y-x)(y-z)} \\
&\quad + \frac{z^2\ln z}{(1-z)(z-y)(z-x)}\Bigg)
\end{aligned}
\tag{A.13}
$$

and

$$
x = \frac{m_0^2}{m_3^2},\ y = \frac{m_1^2}{m_3^2},\ z = \frac{m_2^2}{m_3^2}
$$

Box diagram with H^+ and W^- in the loop gives the same result as $M_{12q}^{W^+H^-}$ i.e

$$
M_{12q}^{H^+W^-} = M_{12q}^{W^+H^-}. \tag{A.14}
$$

A.1.2 Box diagram with H^+, H^- in the loop

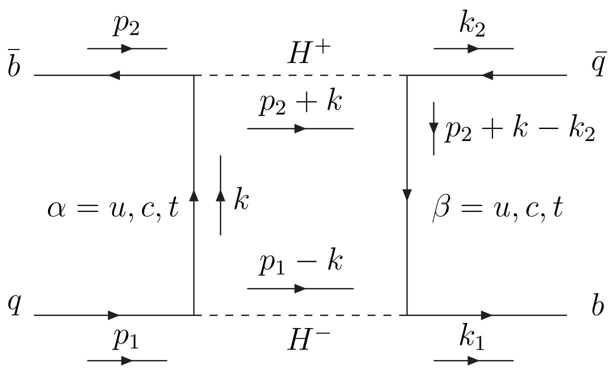

Figure A.2: Box diagram for B_q^0-$\bar{B}_q^0$ mixing in 2HDM with H^+, H^- in the loop

Fig.(A.2) shows box diagram contributing to B_q^0-$\bar{B}_q^0$ mixing with H^+ and H^-

in the loop. Matrix element due to above diagram is given as

$$
\begin{aligned}
iM_{12q}^{H^+H^-} &= \sum_{\alpha,\beta=u,c,t} \int \frac{d^4k}{(2\pi)^4}\; \bar{v}_b\; \frac{ig}{\sqrt{2}m_W} V_{\alpha b}^*(\gamma_R F_{\alpha\alpha} - \gamma_L F_{bb}^*)\; \frac{i(\not{k} + m_\alpha)}{k^2 - m_\alpha^2} \\
&\quad \times \frac{ig}{\sqrt{2}m_W} V_{\alpha q}(\gamma_L F_{\alpha\alpha}^* - \gamma_R F_{qq}) u_q \\
&\quad \times \frac{i}{(p_2+k)^2 - m_H^2} \times \frac{i}{(p_1-k)^2 - m_H^2} \\
&\quad \times \bar{u}_b \frac{ig}{\sqrt{2}m_W} V_{\beta b}^*(\gamma_R F_{\beta\beta} - \gamma_L F_{bb}^*) \\
&\quad \left(\frac{i(\not{p}_2 + \not{k} - \not{k}_2 + m_\beta)}{(p_2 + k - k_2)^2 - m_\beta^2}\right) \frac{ig}{\sqrt{2}m_W} V_{\beta q}(\gamma_L F_{\beta\beta}^* - \gamma_R F_{qq}) v_q \qquad \text{(A.15)}
\end{aligned}
$$

We take the limiting case in which all external momenta and down and strange quark masses are zero. In this case above equation becomes

$$
\begin{aligned}
iM_{12q}^{H^+H^-} &\approx \frac{g^4}{4m_W^4} \sum_{\alpha,\beta=u,c,t} V_{\alpha b}^* V_{\alpha q} V_{\beta b}^* V_{\beta q} \\
&\quad \times \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_\alpha^2} \frac{1}{k^2 - m_\beta^2} \frac{1}{(k^2 - m_H^2)^2} \\
&\quad \times \left(\bar{v}_b(\gamma_R F^{\alpha\alpha} - \gamma_L F_{bb}^*)(\not{k} + m_\alpha)(\gamma_L F_{\alpha\alpha}^* - \gamma_R F_{qq}) u_q\right) \\
&\quad \times \left(\bar{u}_b(\gamma_R F_{\beta\beta} - \gamma_L F_{bb}^*)(\not{k} + m_\beta)(\gamma_L F_{\beta\beta}^* - \gamma_R F_{qq}) v_q\right) \qquad \text{(A.16)}
\end{aligned}
$$

We use the relations given in eq.(A.9) to simplify the above expression. In the above equation denominator of the integral depends on k^2. Since $\int_{-\infty}^{\infty} f(x) = 0$ if $f(-x) = -f(x)$ hence terms with odd power of k in the numerator will vanish.

Using all these results we get

$$
\begin{aligned}
iM_{12q}^{H^+H^-} \approx \; & \frac{g^4}{4m_W^4} \sum_{\alpha,\beta=u,c,t} V_{\alpha b}^* V_{\alpha q} V_{\beta b}^* V_{\beta q} \\
& \times \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_\alpha^2} \frac{1}{k^2 - m_\beta^2} \frac{1}{(k^2 - m_H^2)^2} \\
& \times \Bigg(\frac{|F_{\alpha\alpha}|^2 |F_{\beta\beta}|^2}{4} \bar{v}_b \gamma_\mu \gamma_L u_q \; \bar{u}_b \gamma^\mu \gamma_L v_q \; k^2 \\
& + \frac{|F_{\alpha\alpha}|^2 F_{\beta\beta}^* F_{qq}}{4} \bar{v}_b \gamma_\mu \gamma_L u_q \; \bar{u}_b \gamma^\mu \gamma_R v_q \; k^2 \\
& + \frac{F_{bb}^* F_{qq} |F_{\beta\beta}|^2}{4} \bar{v}_b \gamma_\mu \gamma_R u_q \; \bar{u}_b \gamma^\mu \gamma_L v_q \; k^2 \\
& + \frac{(F_{bb}^* F_{qq})^2}{4} \bar{v}_b \gamma_\mu \gamma_R u_q \; \bar{u}_b \gamma^\mu \gamma_R v_q \; k^2 \\
& + F_{\alpha\alpha} F_{qq}^2 F_{\beta\beta} \; m_\alpha m_\beta \; \bar{v}_b \gamma_R u_q \; \bar{u}_b \gamma_R v_q \\
& + F_{\alpha\alpha} F_{qq} F_{bb}^* F_{\beta\beta}^* \; m_\alpha m_\beta \; \bar{v}_b \gamma_R u_q \; \bar{u}_b \gamma_L v_q \\
& + F_{bb}^{*2} F_{\alpha\alpha}^* F_{\beta\beta}^* \; m_\alpha m_\beta \; \bar{v}_b \gamma_L u_q \; \bar{u}_b \gamma_L v_q \\
& + F_{bb}^* F_{\alpha\alpha}^* F_{\beta\beta} F_{qq} \; m_\alpha m_\beta \; \bar{v}_b \gamma_L u_q \; \bar{u}_b \gamma_R v_q \Bigg)
\end{aligned}
\tag{A.17}
$$

Now we take simplifying assumption that $F_{qq} = 0$; $q = d, s$. By taking $\alpha = \beta = t$ we get the dominant top quark contribution as follows.

$$
\begin{aligned}
iM_{12q}^{H^+H^-} \approx \; & \frac{ig^4}{64\pi^2 m_W^4} (V_{tb}^* V_{tq})^2 \\
& \times \Big(|F_{tt}|^4 D_{00}(m_t^2, m_t^2, m_H^2, m_H^2) \; (\bar{v}_b \gamma_\mu \gamma_L u_q \; \bar{u}_b \gamma^\mu \gamma_L v_q) \\
& + m_t^2 F_{tt}^{*2} F_{bb}^{*2} D_0(m_t^2, m_t^2, m_H^2, m_H^2) \; (\bar{v}_b \gamma_L u_q \; \bar{u}_b \gamma_L v_q) \Big)
\end{aligned}
\tag{A.18}
$$

Here we have used Passarino-Veltman one-loop four-point functions with zero external momenta defined in eq.(A.12) and eq.(A.13).

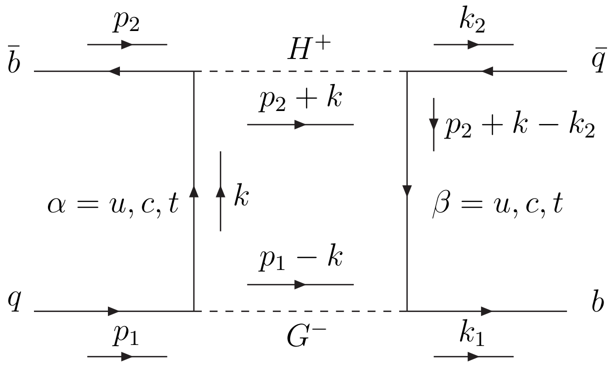

Figure A.3: Box diagram for B_q^0-$\bar{B}_q^0$ mixing in 2HDM with H^+, G^- in the loop

A.1.3 Box diagram with H^+, G^- in the loop

Fig.(A.3) shows box diagram contributing to B_q^0-$\bar{B}_q^0$ mixing with H^+ and G^- in the loop. Matrix element due to above diagram is given as

$$\begin{aligned}
iM_{12q}^{H^+G^-} &= \sum_{\alpha,\beta=u,c,t} \int \frac{d^4k}{(2\pi)^4}\, \bar{v}_b\, \frac{ig}{\sqrt{2}m_W} V_{\alpha b}^*(\gamma_R F_{\alpha\alpha} - \gamma_L F_{bb}^*)\, \frac{i(\not{k} + m_\alpha)}{k^2 - m_\alpha^2} \\
&\times \frac{ig}{\sqrt{2}m_W} V_{\alpha q}(\gamma_L m_\alpha - \gamma_R m_q) u_q \\
&\times \frac{i}{(p_2+k)^2 - m_H^2} \times \frac{i}{(p_1-k)^2 - \xi m_W^2} \\
&\times \bar{u}_b \frac{ig}{\sqrt{2}m_W} V_{\beta b}^*(\gamma_R m_\beta - \gamma_L m_b) \\
&\left(\frac{i(\not{p}_2 + \not{k} - \not{k}_2 + m_\beta)}{(p_2+k-k_2)^2 - m_\beta^2}\right) \frac{ig}{\sqrt{2}m_W} V_{\beta q}(\gamma_L F_{\beta\beta}^* - \gamma_R F_{qq}) v_q
\end{aligned} \tag{A.19}$$

We take the limiting case in which all external momenta and down and strange quark masses are zero. In this case above equation becomes

$$\begin{aligned}
iM_{12q}^{H^+G^-} &\approx \frac{g^4}{4m_W^4} \sum_{\alpha,\beta=u,c,t} V_{\alpha b}^* V_{\alpha q} V_{\beta b}^* V_{\beta q} \\
&\times \int \frac{d^4k}{(2\pi)^4}\, \frac{1}{k^2 - m_\alpha^2}\, \frac{1}{k^2 - m_\beta^2}\, \frac{1}{k^2 - m_H^2}\, \frac{1}{k^2 - \xi m_W^2} \\
&\times \left(\bar{v}_b(\gamma_R F_{\alpha\alpha} - \gamma_L F_{bb}^*)(\not{k} + m_\alpha)(\gamma_L m_\alpha - \gamma_R m_q) u_q \right) \\
&\times \left(\bar{u}_b(\gamma_R m_\beta - \gamma_L m_b)(\not{k} + m_\beta)(\gamma_L F_{\beta\beta}^* - \gamma_R F_{qq}) v_q \right)
\end{aligned} \tag{A.20}$$

We use the relations given in eq.(A.9) to simplify the above equation. As discussed earlier terms with odd power of k in the numerator will vanish. Using all these results we get

$$
\begin{aligned}
iM_{12q}^{H^+G^-} \approx\; & \frac{g^4}{4m_W^4}\sum_{\alpha,\beta=u,c,t} V_{\alpha b}^* V_{\alpha q} V_{\beta b}^* V_{\beta q} \\
& \times \int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-m_\alpha^2}\,\frac{1}{k^2-m_\beta^2}\,\frac{1}{k^2-m_H^2}\frac{1}{k^2-\xi m_W^2} \\
& \times \Bigg(\frac{F_{\alpha\alpha}F_{\beta\beta}^* m_\alpha m_\beta}{4}\;\bar{v}_b\gamma_\mu\gamma_L u_q\;\bar{u}_b\gamma^\mu\gamma_L v_q\; k^2 \\
& + \frac{F_{\alpha\alpha}F_{qq} m_\alpha m_\beta}{4}\;\bar{v}_b\gamma_\mu\gamma_L u_q\;\bar{u}_b\gamma^\mu\gamma_R v_q \\
& + F_{bb}^* F_{qq} m_\alpha^2 m_\beta^2\;\bar{v}_b\gamma_L u_q\;\bar{u}_b\gamma_R v_q \\
& + F_{bb}^* F_{\beta\beta}^* m_\alpha^2 m_\beta m_b\;\bar{v}_b\gamma_L u_q\;\bar{u}_b\gamma_L v_q\Bigg)
\end{aligned}
\tag{A.21}
$$

Now we take simplifying assumption that $F_{qq} = 0$; $q = d, s$. By taking $\alpha = \beta = t$ we get the dominant top quark contribution as follows.

$$
\begin{aligned}
iM_{12q}^{H^+G^-} \approx\; & \frac{ig^4}{64\pi^2 m_W^4}(V_{tb}^* V_{tq})^2 \\
& \times \Bigg(|F_{tt}|^2 m_t^2 D_{00}(m_t^2, m_t^2, m_H^2, m_G^2)\;(\bar{v}_b\gamma_\mu\gamma_L u_q\;\bar{u}_b\gamma^\mu\gamma_L v_q) \\
& + m_t^3 m_b F_{tt}^* F_{bb}^* D_0(m_t^2, m_t^2, m_H^2, m_G^2)\;(\bar{v}_b\gamma_L u_q\;\bar{u}_b\gamma_L v_q)\Bigg)
\end{aligned}
\tag{A.22}
$$

Here we have used Passarino-Veltman one-loop four-point functions with zero external momenta defined in eq.(A.12) and eq.(A.13). Box diagram with G^+ and H^- in the loop gives the same result as $M_{12q}^{H^+G^-}$ i.e

$$
M_{12q}^{G^+H^-} = M_{12q}^{H^+G^-}. \tag{A.23}
$$

A.1.4 Total new contribution

Total new contribution to B_q^0-$\bar{B}_q^0$ mixing due to all the diagrams described in earlier sections is given as

$$\begin{aligned}
iM_{12q}^{NP} \approx & \frac{ig^4}{64\pi^2 m_W^4}(V_{tb}^* V_{tq})^2 \\
& \Bigg[\Big(|F_{tt}|^4 D_{00}(m_t^2, m_t^2, m_H^2, m_H^2) \\
& - 2m_t^2|F_{tt}|^2 m_W^2 D_0(m_t^2, m_t^2, m_H^2, m_W^2) \\
& + 2m_t^2|F_{tt}|^2(D_{00}(m_t^2, m_t^2, m_H^2, m_W^2) + D_{00}(m_t^2, m_t^2, m_H^2, m_G^2)) \\
& + 2m_t^2|F_{tt}|^2 D_{00}(m_t^2, m_t^2, m_H^2, m_G^2)\Big) \\
& \times(\bar{v}_b\gamma^\mu\gamma_L u_q\ \bar{u}_b\gamma_\mu\gamma_L v_q) \\
& +\Big(m_t^2 F_{tt}^{*2} F_{bb}^{*2}\ D_0(m_t^2, m_t^2, m_H^2, m_H^2) \\
& +2m_t^3 m_b F_{tt}^* F_{bb}^*\ D_0(m_t^2, m_t^2, m_H^2, m_G^2)\Big) \\
& \times(\bar{v}_b\gamma_L u_q\ \bar{u}_b\gamma_L v_q)\Bigg]
\end{aligned} \tag{A.24}$$

To write the the total contribution from all the diagrams in terms of effective Hamiltonian we replace the spinors $v_{b,q}, u_{b,q}$ by corresponding fields as follows.

$$\begin{aligned}
(\bar{v}_b\gamma^\mu\gamma_L u_q\ \bar{u}_b\gamma_\mu\gamma_L v_q) &\rightarrow (\bar{b}\gamma^\mu\gamma_L q\ \bar{b}\gamma_\mu\gamma_L q) \\
(\bar{v}_b\gamma_L u_q\ \bar{u}_b\gamma_L v_q) &\rightarrow (\bar{b}\gamma_L q\ \bar{b}\gamma_L q)
\end{aligned}$$

Quark fields q,b are given as

$$\begin{aligned}
q(x) &= \int \frac{d^3k}{2\pi^3\sqrt{2k_0}} \sum_s((u_q^s(k) b_q^s(k) e^{-ik\cdot x} + (v_q^s(k) d_q^{s\dagger}(k) e^{ik\cdot x}) \\
b(x) &= \int \frac{d^3k}{2\pi^3\sqrt{2k_0}} \sum_s((u_b^s(k) b_b^s(k) e^{-ik\cdot x} + (v_b^s(k) d_b^{s\dagger}(k) e^{ik\cdot x})
\end{aligned} \tag{A.25}$$

Here subscripts q, b denotes the flavor of the quark. Superscript s denote the spin. Effective interaction Lagrangian is obtained as

$$\mathcal{L}_{eff}^{NP} = \frac{iM_{12q}^{NP}}{2i} \tag{A.26}$$

Since the operator $\bar{b}\gamma^\mu\gamma_L q\ \bar{b}\gamma_\mu\gamma_L q$ and $\bar{b}\gamma_L q\ \bar{b}\gamma_L q$ are product of two identical operators, we have divided M_{12q}^{NP} by 2 to get $\mathcal{L}_{eff}^{NP}$. Effective Hamiltonian is given as

$$\mathcal{H}_{NP} = -\mathcal{L}_{eff}^{NP} \tag{A.27}$$

We also use the relation $G_F^2 = \frac{g^4}{32 m_W^4}$ where G_F is Fermi constant. Hence the effective Hamiltonian for new contribution to the B_q^0-$\bar{B}_q^0$ mixing in variant of 2HDM presented in chapter 5 is given as

$$\begin{aligned}
\mathcal{H}^{NP} &\approx C_1(\bar{b}\gamma^\mu \gamma_L q\ \bar{b}\gamma_\mu \gamma_L q) + C_2(\bar{b}\gamma_L q\ \bar{b}\gamma_L q) \\
&\approx C_1(\bar{b}_L \gamma^\mu q_L\ \bar{b}_L \gamma_\mu q_L) + C_2(\bar{b}_R q_L\ \bar{b}_R q_L)
\end{aligned} \tag{A.28}$$

Where Wilson coefficients C_1, C_2 are as follows.

$$\begin{aligned}
C_1 &= -\frac{G_F^2}{4\pi^2}(V_{tb}^* V_{tq})^2 \\
&\quad \Big(|F_{tt}|^4 D_{00}(m_t^2, m_t^2, m_H^2, m_H^2) \\
&\quad - 2m_t^2 |F_{tt}|^2 m_W^2 D_0(m_t^2, m_t^2, m_H^2, m_W^2) \\
&\quad + 2m_t^2 |F_{tt}|^2 (D_{00}(m_t^2, m_t^2, m_H^2, m_W^2) + D_{00}(m_t^2, m_t^2, m_H^2, m_G^2)) \\
&\quad + 2m_t^2 |F_{tt}|^2 D_{00}(m_t^2, m_t^2, m_H^2, m_G^2)\Big) \\
C_2 &= -\frac{G_F^2}{4\pi^2}(V_{tb}^* V_{tq})^2 \\
&\quad \Big(m_t^2 F_{tt}^{*2} F_{bb}^{*2}\ D_0(m_t^2, m_t^2, m_H^2, m_H^2) \\
&\quad + 2m_t^3 m_b F_{tt}^* F_{bb}^*\ D_0(m_t^2, m_t^2, m_H^2, m_G^2)\Big)
\end{aligned} \tag{A.29}$$

Our results are in agreement with the results given in Ref. [164] in the case when $m_q = 0$ and $F_{qq} = 0$ for $q = d, s$.

Appendix B

List of publications

1. Minimal flavour violations and tree level FCNC, A. S. Joshipura, B. P. Kodrani, Phys. Rev. **D77**, 096003 (2008). [arXiv:0710.3020 [hep-ph]].

2. Complex CKM matrix, spontaneous CP violation and generalized $\mu - \tau$ symmetry, A. S. Joshipura, B. P. Kodrani, Phys. Lett. **B670**, 369-373 (2009). [arXiv:0706.0953 [hep-ph]].

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