A STUDY OF SOME NONLINEAR INTERACTIONS OF KINETIC ALFVEN WAVES

BY

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CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

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Abstract

The present thesis is devoted to the study of some nonlinear aspects of kinetic Alfven waves. It is motivated by the importance of kinetic Alfven waves in supplementary heating of tokamak plasmas. In particular, near the mode conversion layer, the wave has an enhanced amplitude and can thus interact nonlinearly with other normal modes of the plasma. Two such interactions have been chosen for detailed investigation in this thesis - namely, the nonlinear excitation of tearing modes and that of drift modes.

The basic non-linear process is the parametric decay of the kinetic Alfven wave into another Alfven wave and a low frequency wave (the tearing or the drift mode). Several aspects of this interaction are studied - contributions from resonant (side band coupling) terms, non-resonant (ponderomotive) terms, nonlinear equilibrium drifts as well as phase mixing effects. The low frequency modes considered include resistive m=1 and m=2 tearing modes, collisionless tearing modes, kinetic drift modes and drift temperature modes. For the drift modes the effect of background inhomogeneity is also taken into account.

The calculations are based on both fluid and kinetic descriptions of the plasma. The method of solution is mainly analytical - relying on variational and matched asymptotic techniques. Some numerical support to the analytical results is also provided. It is found that the growth rates of the nonlinearly excited low frequency modes are quite large for realistic tokamak parameters. They can be comparable or even exceed growth rates of other nonlinear processes proposed earlier [1] for heating purposes. Since drift waves play an important role in plasma transport and can significantly influence plasma confinement, their nonlinear excitation can have serious implications for the Alfven wave heating schemes. A brief discussion on this aspect is made in light of some of the preliminary experimental evidence of such low frequency activity in tokamak experiments.

1. Hasegawa, A. and Chen, L., Phys. Fluids 19 1924 (1976)

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CHAPTER I

INTRODUCTION

1.1 Background :

Alfven waves are well known low frequency oscillations of a uniform magnetised plasma and were first treated in a classic work by H. Alfven in 1942 [1] . Early investigations of these waves were in connection with astrophysical plasmas. For example, in the solar atmosphere they were considered to be responsible for the heating of the corona.

A simple physical analogy for understanding the Alfven wave propagation, is the 'violin string model' as proposed by Alfven. The magnetic field lines immersed in a plasma can be viewed as plasma loaded strings held under tension. This analogy is useful since under certain conditions, (e.g. for high plasma conductivity) the field lines are 'frozen' in the plasma and are thereby constrained to move with the plasma. When the field lines are 'plucked transversely', the tension in the field lines tends to straighten the distortion. The field lines are pulled back, but due to the inertia of the plasma particles, overshoot the equilibrium. The resulting oscillations are the Alfven waves, which are analogous to the transverse oscillations of stretched strings. In the latter the waves propagate with a velocity given by $V = \left(\frac{T}{\rho}\right)^{\frac{1}{2}}$, where 'T' is the tension and 'ρ' the mass density. By replacing 'T' by the magnetic field tension $\frac{B_o^2}{\mu_o}$ and 'ρ' by the mass density of the plasma which is $m_i n_i$ (where m_i and n_i are the mass and density of the ions respectively) the velocity of the Alfven wave (V_A) can be obtained as $\left(\frac{B_o^2}{\mu_o m_i n_i}\right)^{1/2}$. A simple dispersion relation for the Alfven wave propagation is then $\omega = k V_A$, where k is the wave vector along the field lines.

The actual physical mechanism for Alfven wave propagation can be understood in terms of the interaction between fields and particles in a plasma. For instance, when the magnetic field lines are perturbed by adding a small transverse component there arise induced electric fields in the system. These electric perturbations combine with the equilibrium magnetic field and set the plasma in oscillation. On account of the finite ion mass the ions lag behind the electrons. This gives rise to currents, which generate electromagnetic forces ($\vec{J} \times \vec{B}$). These forces and the ion inertia sustain the oscillations and generate the Alfven wave propagation along the field lines.

When compression effects are taken into consideration, there is another low frequency mode of oscillation possible in the plasma. This is a mode which propagates perpendicular to the magnetic lines of force, and whose restoring force is provided by the compression in the field lines. Such a mode is called a compressional Alfven wave.

Early investigations [2, 3] of Alfven waves were concerned with the study of propagation and dissipation of Alfven waves in a solar atmosphere. The observations of Belcher and Davis [4] of large amplitude Alfven waves in the solar wind stimulated extensive research on these waves. Holleweg [5] recognised the importance of such observations and was the first to develop a model of Alfven wave propagation in the solar atmosphere.

In laboratory plasmas, several investigations both theoretical and experimental have been performed in understanding the properties of Alfven waves [6], namely reflection, refraction properties, effect of finite conductivity and Hall current. In order to understand the Alfven wave propagation in bounded cylindrical plasmas several theoretical models were developed [7, 8] . When a cylindrical plasma is perturbed axisymmetrically, two modes of decoupled waves propagate in the system : the torsional or the shear Alfven wave and the compressional wave. The former is the plane transverse wave modified by geometric effects. This wave motion causes the magnetic surfaces to shear past each other, without however any mutual coupling in their motion. When pressure perturbations are taken into account the torsional Alfven mode is not affected. The properties of compressional wave however changes and the mode couples to the acoustic wave.

In a non-uniform plasma, on the other hand, the torsional and the compressional modes undergo several striking important changes. The density gradient introduces a frequency dependent coupling between the torsional and compressional waves of the uniform plasma. As a result an axial component of the magnetic perturbation which is associated only with the compressional wave is now carried by the torsional wave as well. Such an effect was detected by Pridmore-Brown [9] . In a non-uniform plasma, the Alfven wave speed (V_A) is a function of position in the direction of the inhomogeniety. This local variation of Alfven wave velocity gives rise to a singularity in the wave equation at the point where the wave phase velocity equals local Alfven velocity.

This in turn leads to a continuum spectrum of Alfven waves in the plasma.

Physically the continuum indicates that the plasma structure hosts a continuous spectrum of internal oscillators, each representing the oscillatory characteristics of an infinitesimally small piece of plasma. The study of Alfven waves in a non-uniform plasma was first initiated by Gajewski [10] . Actually the equation arrived at by Gajewski did not show any singular behaviour because the authors neglected the pressure perturbations. Pridmore-Brown [11] discussed various features of the singular behaviour of the wave equations, but did not arrive at the continuum spectrum of frequencies. Hasegawa-Chen [12] later showed the existence of the continuum and demonstrated that the singularity leads to the damping of surface mode of a discontinuous plasma. They showed that the damping arises because the normal modes which make up the continuum phase mix in time leading to the decay of macroscopic variables. This damping of surface waves has been observed both experimentally [13] and in MHD computer calculations [14] . The continuum spectrum for Alfven waves plays an important role in practical problems both in laboratory and space plasmas.

In the latter, this has led to an understanding of the phenomena of magnetic pulsations in the ULF range [15] .

The singularity in the solution of the Alfven wave equation originates from neglecting the non-ideal effects. The singularity is seen to be removed when finite resistivity or finite Larmor radius effects are introduced in the fluid equations. The effect of resistivity on Alfven surface waves has been considered by Kapparoff [16] and Uberoi [17]. This treatment is valid only when the plasma skin depth is shorter than the ion gyroradius. When finite temperature effects are taken into consideration the Larmor radius effects become significant and an important change in the Alfven wave propagation occurs. The ions are no longer closely tied to the magnetic field lines, whereas the electrons still follow the field lines because of their small Larmor radius. The ions can thus freely move across the magnetic lines of force and enable a perpendicular propagation as well. Hasegawa-Chen [18] using kinetic theory to study the wave propagation, showed that the singularity of the fluid equations is replaced by a resonant layer, where a mode conversion occurs generating a 'kinetic Alfven wave'. In a low β plasma (where β is the ratio of the plasma pressure to the magnetic field pressure, $(8\pi n_o T/B_o^2)$ this wave is represented by the dispersion relation $\omega^2/K_{\parallel}^2 V_A^2 = (1 + K_{\perp}^2 \rho_i^2)$ (where $K_{\parallel}$ and $K_{\perp}$ are the wave vectors perpendicular and parallel to the magnetic field). The kinetic Alfven wave can propagate across the magnetic field and experiences both electron and ion Landau damping.

The resonant mode conversion to kinetic Alfven waves in laboratory plasmas has received a great deal of attention recently, due to some important applications. One such important application is in using them for supplementary heating of tokamak plasma. In tokamaks, Ohmic heating provides the initial heating. However as the temperature rises, the resistivity decreases as $T^{-\frac{3}{2}}$ and radiation losses increase as $T^{\frac{1}{2}}$. Hence at some point the radiation losses overcome the Ohmic heating and puts an upper limit on the achievable temperature. This temperature is around 1 to 2 KeV for ions and thus far from the ignition regime. Therefore in most of the devices some form of supplementary heating will be necessary to reach ignition temperature. In this context both Hasegawa-Chen [18] , Grossman-Totaronis [19] proposed the use of Alfven waves as candidates for supplementary heating because of their high rate of resonant absorption around the mode conversion region. A few of the other attractive features of this scheme are 1) predictions that it may provide a relatively high efficiency for heating the plasma, 2) availability of low cost and reliable power supplies in this frequency range, 3) the fact that the principle is based on simple well established theory, 4) the heating is localised near the resonant layers, the location of which can be controlled by the frequency and the launching structures.

For typical tokamak parameters it was shown by Hasegawa-Chen [18] that the heating of the particles by linear processes, occurs through ion viscosity and electron Ohmic dissipations. For high temperatures the linear heating is dominated by electron Landau damping. Since the initial proposal, other authors namely Perkins-Karney [20] Appert et al [21] , Stix [22] and Puri [23] have elucidated the absorption processes of the kinetic Alfven wave. Recently Ross et al [24] and others [25] have studied the problem in cylindrical geometry with sheared magnetic fields, taking into consideration electron inertia, Landau damping, finite ion gyroradius and equilibrium current. They have obtained numerical solutions of the eigenmode equations and have observed the mode conversion to the kinetic Alfven wave. One important conclusion of the linear study was that the amplitude of the excited kinetic waves was strongly enhanced near the mode conversion region, due to spatial resonance. Consequently several non-linear processes were expected to take place. In fact one such process namely parametric decay of a kinetic Alfven wave into an acoustic wave was proposed as a means for heating ions and found to be quite feasible for existing tokamak parameters. It was shown that if the applied oscillating magnetic field had an intensity larger than a few tens of Gauss, non-linear dissipation of the kinetic Alfven waves was expected.

Although the theoretical proposal to use Alfven waves in r.f. heating schemes was made more than ten years ago, experiments have begun only recently at several places. The most notable of these results are from the work of the experimental group on TCA tokamak in Switzerland [26] . Additional experiments have been reported by the Texas group [27] , the Wisconsin group [28] and Australian torus tokamak group [29] . Initial experiments were conducted at low power levels ($P < 3$ KW). In all these experiments efficient absorption and heating have been observed and most of the results show quantitative agreement with the theoretical predictions. However, most results also show the existence of enhanced loss of plasma and occasional major disruptions of the plasma column. In stellarators nearly all the experiments involving Alfven wave heating report enhanced transport of particles [30] in addition to heating. The anomalous transport varies approximately linearly with respect to the amplitude of the r.f.field and is believed to be caused by magnetic island formation. At Lausanne, experiments have reported saw-tooth modulations of the resonance peaks (of antenna loading) indicating the coupling of Alfven waves to MHD activity [31] . Alfven wave experiments at Suhumi in addition to efficient heating showed clearly the non-linear aspect of heating as a function of the r.f. magnetic field intensity [32] . On the TCA tokamak at Lausanne recent advances have enabled an increase of power to over 500 KW. Appert et al [33] have already reported an increased probability of disruptions.

These several experimental results and the theoretical predictions, suggest that the excitation of the kinetic Alfven wave is accompanied by non-linear processes in the plasma, which could lead to enhanced loss of plasma and disruptions. In order to understand some of these features, it is important to study the non-linear aspects.

1.2 Motivation :

The present work is concerned with the study of some non-linear processes among kinetic Alfven waves.

The principal motivation for the study arose from the recent interest in Alfven waves which are considered as excellent candidates for supplementary heating schemes. It has been pointed out that the amplitude of the excited kinetic Alfven waves is strongly enhanced at the resonance region. Therefore it is highly probable that several non-linear processes would take place around this region. One such mechanism, i.e. parametric decay into acoustic waves has been considered [18] . Besides the non-linear interaction with acoustic waves, there could in principle exist other modes of parametric decay which could channel energy into deleterious modes (namely drift modes) and thereby lead to loss of confinement. In fact several results of experiments conducted at Lausanne, Wisconsin and other places support the idea that the kinetic Alfven waves could undergo non-linear processes. In order to understand these several experimental results and for proper design of the antenna system, a knowledge of the non-linear evolution of the kinetic Alfven waves is therefore essential. A systematic study of the non-linear properties of the mode converted kinetic Alfven wave is in order and so far very limited work in this field has been done. This thesis is devoted to the non-linear study of kinetic Alfven waves and in particular non-linear interaction of kinetic Alfven waves with drift and tearing modes.

In addition to their important applications to the Alfven heating scheme, non-linear processes among Alfven waves have importance in space plasmas and are therefore of quite general interest. The early pioneering works in this context were carried out by Sagdeev-Galeev [34] , Galeev-Oraeskii [35] who studied the decay instability of a large amplitude Alfven wave into an acoustic wave. Later Cohen examining this instability by assuming a broad band of Alfven waves, found the incoherent spectrum to be statistically stable [36] . Since then several analytical studies have been undertaken to understand other non-linear properties like modulational instabilities [37], existence of solitons [38] etc. In recent times it has been of interest to consider various parametric processes [39] associated with the Alfven waves, since in many schemes large or fairly large amplitude Alfven waves are produced. It has also been shown that Alfven waves may excite zero frequency vortex motion and magnetostatic modes [40] which cause cross field plasma diffusion having Bohm scaling. In addition, recently it has been shown that magnetosonic modes [41] could also be excited by kinetic Alfven waves. Our study of the non-linear interaction of kinetic Alfven waves with drift and tearing modes could contribute to the general understanding of the non-linear evolution of kinetic Alfven waves which besides its potential applications to Alfven heating schemes is also of basic interest.

We have chosen to investigate the non-linear interaction of kinetic Alfven waves with drift and tearing modes on account of the fact that both these modes play significant roles in the context of laboratory [42] and space plasmas. In the latter, reconnection through tearing instability is considered an excellent candidate for coronal heating and is the most popular flare mechanism proposed [43]. Much of the recent revived interest in the tearing modes has been due to the fact that these modes have been observed in laboratory plasma discharges [44]. The disruptive instability is an unexplained phenomenan occuring in tokamak plasmas which often results in the termination of the discharge on a very short time scale (1 millisec) and the long wave length tearing modes are believed to play an important part. The disruption is usually preceeded by the m = 2, n = 1 oscillation in the soft x-ray signal (where 'm' and 'n' are the poloidal and toroidal mode numbers respectively) and recently a causal relationship between the growth of these large amplitude tearing modes and disruptions has been established [44] .

Drift waves were until recently considered universal instabilities because their existence required only a density gradient which was a common feature of both laboratory and astrophysical plasmas. These collisionless drift waves in sheared magnetic fields, which are driven by wave particle interactions, have been the subject of numerous investigations [45] . They are hazardous for plasma confinement and are known to cause anomalous transport of particles across the field lines. The results of our analysis of non-linear interaction of kinetic Alfven waves which have been obtained using variational and asymptotic methods indicate that both drift and tearing modes can be resonantly excited by Alfven waves with large growth rates. In the context of Alfven wave heating schemes, the excitation of these modes may be responsible for enhanced transport and plasma disruptions.

1.3 Scope of the thesis :

We have organised the present thesis in the following manner. In Chapter II, we have carried out an investigation of the non-linear interaction of the resistive tearing modes with kinetic Alfven waves. We have studied the parametric decay of a pump kinetic Alfven wave into a lower (upper) side band Alfven wave and the resistive tearing mode using the fluid formalism. The quasineutrality condition and Ampere's law are used to obtain the expression for the Alfven wave potentials with contributions from the non-linear interactions. This contribution is proportional to the gradient of the tearing mode perturbed current '$J_{\parallel t}$' and is responsible for several interesting effects namely anomalous resistive and viscous effects. The dynamics of the tearing mode is described by Ohm's law and the momentum equation, in the incompressible hydromagnetic approximation. The ponderomotive force (P.F.) generated by the non-linear interaction simulates anomalous viscous and resistive forces in Ohm's law and additional convective forces in the momentum equation. For large enough fluctuation levels of the kinetic Alfven waves, the non-linear forces dominate and in the equation of motion the linear shear flow is driven against fluid inertia by the torque produced by the non-linear ponderomotive force. Similarly in Ohm's law the parallel electric field is balanced by the anomalous viscous effects rather than the collisional drag. Our calculations predict that the m = 1 and m = 2 tearing instabilities can indeed be resonantly excited with growth rates proportional to fractional powers of the Alfven pump amplitude.

We have also investigated the non-resonant interaction between kinetic Alfven waves and tearing modes, in which equilibrium flows generated by the kinetic Alfven waves couple non-linearly to the tearing mode perturbations (Chapter III). These drifts arise when quiver velocities of the particles in the Alfven wave field are averaged over the fast Alfvenic motions. This effect had been omitted in the study of parameteric interaction between the tearing and Alfven modes owing to the fact that the resonant terms were much larger than the non-resonant ones. Although the dominant drift is in the axial direction it is however the radial drift which plays a significant role in the dynamics of the tearing mode. Modelling the spatial variations of the kinetic Alfven waves by a simple cosine profile, we find that the azimuthal and axial drifts Doppler shift the mode frequency while the radial drift couples to a cubic derivative in the momentum equation.

The growth rate of the tearing instabilities is found to be proportional to the radial drift. The results of our analysis in agreementwith the scaling for the weakly unstable modes are obtained by Pollard-Taylor [46] , Bondeson [47] . In their assumption however the radial drift was of an arbitrary nature, whereas in the present work the radial drift is proportional to the amplitude of the kinetic Alfven waves.

Chapter IV contains an investigation of non-linear wave mixing phenomenon between kinetic Alfven waves and the resistive tearing modes. We have considered two large amplitude kinetic Alfven waves interacting to produce beat waves of the resistive tearing mode frequency and resonantly exciting it. Such a phenomenon is quite possible in the Alfven wave heating scheme, where the antenna are phased to excite several waves simultaneously. These kinetic Alfven waves excited at different resonant surfaces could interact and excite beat waves of the tearing mode type. This phenomenon differs from the earlier study of parametric interaction between kinetic Alfven waves and resistive tearing modes. In the present problem, the kinetic Alfven waves act as external driver waves which act on the low frequency tearing mode. The system acts like a harmonic oscillator, driven at the natural tearing frequency by the wave mixing phenomena between the kinetic Alfven waves. To describe the evolution of the resistive tearing modes, the single fluid equations are used. The non-linear interaction generates ponderomotive forces in the momentum equation and Ohm's law which are the two coupled equations describing the dynamics of the tearing mode. From the coupled set of equations, a third order inhomogeneous differential equation describing the evolution of the tearing mode is obtained. The homogeneous part of the differential equation has been the subject of several investigations. We present an alternative method of obtaining symmetric solutions of the differential equation in terms of certain convenient set of orthonormal basis functions, namely, Hermite polynomials. We find that the solutions are very sensitive to the parity of the driven Alfven waves and separate out into an odd and even series. For arbitrary wave lengths for the symmetric tearing mode the log derivative Δ' across the boundary layer is matched to the outer infinite conduction regions. It is shown that in the limit of vanishing pump amplitude the earlier results of Paris [48] are recovered. In the presence of the external driving forces in the limit of large Reynolds number the classical growth rates of the symmetric tearing mode with positive mode numbers are enhanced. For modes with negative 'm' values, the effect is stabilising. These driven tearing modes however grow more slowly than the non-linearly tearing instabilities due to parametric interaction. In laboratory plasmas both beat wave excitation and parametric excitation of tearing modes by kinetic Alfven waves are equally possible phenomena. However on account of their large growth rates the parametrically excited tearing modes are of greater importance than the driven tearing modes.

Drift waves play an important role both in astrophysical and laboratory plasmas. In the context of magnetospheric plasmas, the importance of drift waves in understanding the micropulsations in the earth's magnetosphere has been stressed by several authors. In laboratory plasmas, they are considered to cause anomalous transport of particles across the magnetic field lines. In Chapter V we have studied a non-linear interaction between collisionless drift waves and kinetic Alfven waves. We have investigated a parametric interaction wherein a pump kinetic Alfven wave decays into a side band kinetic Alfven wave and a drift mode.

Since the dynamics of the drift wave are sensitively dependant on the shear and finite Larmor radius effects the kinetic equations are used to describe the motion of the ions and electrons. The quasineutrality condition and Ampere's law are used to derive the coupled equations (which are of quite high order) describing the decay process. We have used the local approximation to simplify the differential operators. The calculated growth rates and thresholds for the drift wave decay process are found to be comparable to other modes of decay calculated by earlier workers [18] . We find that the temperature gradient drift waves could also be parametrically excited with large growth rates. In addition we have investigated the effects of density inhomogeneity on the decay process which entails retention of the full differential operators in the coupled equations. We have examined the equations in Fourier space, using WKB techniques, and perturbative methods. We have established the condition under which an absolute instability, which is a well behaved solution, could occur near the mode conversion region.

In Chapter VI we have discussed another important non-linear mechanism that of the effect of ponderomotive force generated by two interacting kinetic Alfven waves on the collisionless tearing and drift modes. As before, the spatial variations of the interacting kinetic Alfven waves are modelled by a simple cosine profile. In order to describe the collisionless tearing modes, a generalised Ohm's law is obtained from the electron dynamics. The kinetic equations with the Krook collision operator are used for this purpose. The electron orbit equations are significantly modified by the equilibrium P.F. The perpendicular P.F. ($F_{\perp o}$) Doppler shifts the mode frequency while the parallel P.F. ($F_{\parallel o}$) alters the resonant wave particle phenomena. This leads to a replacement of $\frac{\omega}{K_{\parallel t} V_e}$ by $\frac{\omega}{(K_{\parallel t}^2 V_e^2 - a)^{1/2}}$ (where $a = \frac{2i F_{\parallel o} K_{\parallel t}}{m_e}$). The other piece of information needed to study the mode characteristics describe the ion dynamics, which is given by the momentum conservation law. The equilibrium ponderomotive force has no effect on the ion motion. Using standard transformation the eigenmode equation in the slab model describing the evolution of the tearing instability is then derived. The eigen values of the coupled equations are obtained using variational methods prescribed by Hazeltine et al [49] . We find that the effect of the parallel P.F. is to enhance the growth rates of the tearing modes in the collisional and collisionless regimes. In the collisional regime however the enhancement factor produced by the parallel force is of 2nd order and hence quite feeble. For laboratory plasmas, for given tokamak parameters, the enhancement factor in the collisionless regime is however quite large.

We have in addition investigated the effect produced by the P.F. on the collisionless drift waves. It has been shown that effects which modify the electron orbits in the resonance region (namely turbulent diffusion) serves to alter the stability effects of shear and hence change the mode characteristics [50] . It is therefore of importance to investigate the effect of the P.F. on drift waves. In order to retain effects of shear, the electron response is modelled by the kinetic equations. As in the case of the collisionless tearing modes the parallel P.F. accelerates the particles along the field lines resulting in resonance broadening of electron response. For simplicity the ions are treated by the hydrodynamic approximation. From the quasineutrality condition, the radial eigen mode equation describing the drift wave dynamics is obtained. To obtain the eign values and to investigate the effect of the P.F., the variational principle analogous to the one prescribed by Ross et al [51] is employed. Our calculations show that the contribution from the equilibrium parallel force has a destablising effect on the drift mode and competes significantly with the shear stabilising effect.

The results of our analysis indicate that both drift and tearing modes could be excited by several non-linear effects produced by the kinetic Alfven waves. In laboratory plasmas these instabilities whose growth rates are proportional to the amplitude of the kinetic Alfven waves could account for the observed enhanced transport and power limitations in Alfven wave heating schemes [25].

Chapter VII contains a summary of the main conclusions of our analysis and applications of our results to laboratory plasmas. We have also discussed the limitations of our theoretical model and pointed out future direction of work in this connection.

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CHAPTER II

RESONANT EXCITATION OF TEARING INSTABILITIES BY KINETIC ALFVEN WAVES

2.1 Introduction :

In this chapter we have investigated a resonant interaction between kinetic Alfven waves and resistive tearing modes. It can be viewed as a parametric decay process wherein a pump kinetic Alfven wave decays into a side-band Alfven wave and a tearing mode. The process has particular relevance to the Alfven wave heating scheme in tokamak plasmas. Alfven waves are considered excellent candidates for laboratory supplementary heating schemes [1,2,3] . They are excited at the resonant surfaces, ($\omega = k_{\parallel} V_A$) by the mode conversion of an external source. Heating in the linear regime is mostly that of electrons and occurs through the Landau damping of the kinetic Alfven wave. Near the mode conversion these modes have large amplitudes and therefore several non-linear processes were expected to take place. One such process, that of the parametric decay of the kinetic Alfven wave into an acoustic wave [1] has been investigated. The excited ion acoustic wave could then damp and heat the ions. An important feature to examine at this juncture is whether there exist other channels of non-linear decay which compete with the ion acoustic decay process and thereby influence the heating scheme.

In this context we have investigated another such non-linear interaction wherein kinetic Alfven waves resonantly excite the tearing modes.

Tearing modes feature in a wide variety of important phenomena both in laboratory [4-8] and astrophysical plasmas [9] . They occur wherever there is a reversal in the magnetic field (i.e. $\vec{B}_o$ or a component of $\vec{B}_o$, goes through a zero). The simplest of such configurations where tearing modes feature is that produced by a sheet current, where the magnetic field changes sign at $x=0$ (as depicted in Fig.(2.1)). Suppose now the field lines are perturbed (as shown in Fig(2.2)) with the wave vector along $\vec{B}_o$, such that $k_{\parallel}(0)$ vanishes. Such a perturbation in an ideal infinitely conducting plasma gives rise to Alfvenic motion everywhere except in the small region where $k_{\parallel}$ vanishes. Within this region (on account of the fact that $k_{\parallel}$ is zero) non-ideal effects namely resistivity come into play and the perturbed field can be dissipated only by these non-ideal effects. This effect generally takes place on a very slow time scale compared to the Alfvenic motion. However around this singular region, on account of the small scale lengths, the resistive forces give rise to gradients which are very steep. These give rise to rapid localised diffusion of the perturbed field which is the tearing instability. The sheet current as a consequence of this instability breaks along the current flow lines to form parallel filaments. Basically it is the free energy associated with the magnetic field gradient which drives the instability. In a tokamak, there exist axial and azimuthal magnetic fields and confinement of plasma depends on the existence of nested magnetic surfaces. The perturbation aligned with the magnetic field $K_{\parallel}$ is related to the torodial and polodial mode numbers n, m by, $K_{\parallel} = \frac{m}{r} B_p + \frac{n}{R} B_t$ (where B_p, B_t are the polodial and toroidal magnetic fields respectively). This vanishes on surfaces where $q(r)$ is a rational number, namely 1, 2 ... etc.($q(r)$ is the safety factor given by rB_t/RB_p). So on these surfaces $(\vec{K}_{\parallel} \cdot \vec{B}) = 0$. The tearing modes develop around these rational surfaces leading to formation of magnetic islands. Since particles are tied to the field lines this eventually results in plasma transport through destruction of magnetic surfaces.

In laboratory plasmas they were first observed in pinches [10] and stellarators [11] in the limit of high electrical conductivity. Much of the recent interest in tearing modes is due to the fact that they have been observed in tokamak discharges [6] . It is believed that the disruptive instability, which often results in the abrupt termination of the discharge on a very short time scale is due to these long wave-length tearing modes [12] . The first systematic study of the tearing instability in a plane resistive current layer was done by Furth et al [4] . Since then other authors [13] , recognising that fusion plasmas are collision free extended the calculation to the collisionless regime. In recent times, non-linear excitation of tearing modes through mode-coupling has also been of great interest. Various numerical and theoretical analysis indicate that effects resulting from these interactions could greatly enhance the growth rate of these tearing instabilities [14] .

A study of the effect of MHD turbulence [15] and stochastic magnetic fields [16] on low 'm' linear tearing modes reveals that the non-linear interaction generates anomalous viscous and resistive forces which give rise to tearing instabilities. The growth rates of these instabilities, found to be proportional to the amplitude of the fluctuation levels are much larger than the classical growth rates.

The dynamics of the tearing modes could be significantly modified by non-linear interactions with electrostatic waves [17] . The effect of a turbulent spectrum of lower hybrid waves [18] on the tearing modes is to simulate resistive forces which drive tearing instabilities which evolve on a time scale much shorter than the classical ones.

We have in the present problem used the fluid picture to describe the non-linear interactions between kinetic Alfven waves and tearing modes. The special feature of the kinetic Alfven waves, the finite Larmor radius effects can be simulated through this formalism. Using quasineutrality condition and Ampere's law the expressions for the side-band potentials with contributions from the non-linear interaction are obtained. The dominant contribution arises through the non-linear side-band current, ($J_{\parallel s}^{\pm}$) which is proportional to the gradient of $J_{\parallel t}$ (the tearing mode current density). This gives rise to interesting anomalous viscous effects in the tearing mode equations.

The dynamics of the tearing mode are described by the momentum transfer equation and Ohm's law. In the former the fluid is driven against inertia by 1) the torque produced by the ponderomotive force generated by the non-linear interaction and 2) the linear $\vec{J}_t \times \vec{B}_o$ forces. In Ohm's law, the parallel electric field is balanced by the linear collisional drag and the anomalous viscous forces generated by the non-linear interaction. For large levels of fluctuation of the pump amplitude, the non-linear forces dominate and excite the tearing instabilities. Using variational [18] and asymptotic methods [19] we find that the growth rates of these m = 1 and m = 2 tearing modes are proportional to fractional powers of the Alfven pump amplitude.

The calculated growth rates of the non-linear tearing modes range from 10^4 - 10^6 sec^{-1} for typical tokamak parameters. Since these values are significantly large, these new resistive tearing modes could possibly contribute to the destruction of good magnetic surfaces and subsequent enhanced plasma transport.

The plan of the chapter is as follows. In section 2.2, we derive the basic equations for the non-linear decay process. The coupled differential equations are analytically studied in section 2.3 invoking the variational principle. In section 2.4, the asymptotic matching technique is applied to obtain approximate solutions and eigen values of the differential equations. The results are compared with those derived using variational formalism. Finally the last section summarises the main results and discusses their application for auxilary heating with Alfven waves.

2.2 Basic equations :

In this section we derive the basic equations for the decay process. We consider a mode converted kinetic Alfven wave $\phi_0(\vec{r},t)$, propagating in a cylindrical magnetised plasma with $\phi_0(\vec{r},t)$ given by a plane wave,

$$\phi_0(\vec{r},t) = \phi_0 \exp i\,[\vec{k}\cdot\vec{r} - \omega t]$$

Such a model is quite justified as long as we are away from the mode conversion region ($\omega = k_{\parallel} V_A$), where the kinetic Alfven wave can have a complicated radial structure (Airy's function) [1] . Our primary interest is in the mode rational surface which can be well separated from the mode conversion region.

The equilibrium magnetic field is assumed to have the form $\vec{B}_0 = \hat{e}_\theta B_\theta(r) + \hat{e}_z B_z$ where $\hat{e}_\theta$ and $\hat{e}_z$ are the unit vectors in the $\hat{\theta}$ and $\hat{z}$ directions respectively. The kinetic Alfven wave satisfies the dispersion relation

$$\omega^2 = k_{\parallel}^2 V_A^2 (1 + k_{\perp}^2 \rho_i^2) \qquad \ldots 2.1$$

where ρ_i is the ion geyro radius, V_A is the Alfven speed $k_{\parallel}$ and $k_{\perp}$ are the parallel and perpendicular wave vectors. The effect of the finite Larmor radius which is the special feature of the kinetic Alfven wave can be simulated by the two fluid equations. For a low β plasma the perturbations associated with the magnetic compressions $\tilde{b}_z$ can be neglected. The electric fields can then be represented in terms of parallel and perpendicular potentials, ψ and ϕ as follows [20]

$$E_z = -\nabla_z \psi \quad , \quad E_{\perp} = -\nabla_{\perp} \phi$$

From the electron equation of motion, for massless electrons, the balance between the pressure gradient and the electrostatic forces along the field lines leads to the Boltzmann relation.

$$\frac{n_e^{(1)}}{n_0} = \frac{e\psi}{T_e} \qquad \ldots 2.2$$

For the heavier ions, the contributions to the density perturbations, (which can be obtained from the equation of continuity for ions) arises mainly from the perpendicular motion;

$$\frac{n_i^{(1)}}{n_o} = -k_\perp^2 \rho_s^2 \frac{e\phi}{T_e} \qquad \ldots\ 2.2a$$

Here T_e is the electron temperature n_e and n_i are the density perturbations of electrons and ions respectively. $\rho_s = \frac{c_s}{\omega_{ci}}$ where c_s is the ion sound speed, ω_{ci} the ion gyrofrequency, and n_o is the equilibrium density. The quasineutrality condition

$$n_i = n_e \qquad \ldots\ 2.2b$$

provides one relation between ψ and ϕ

$$\psi = -k_\perp^2 \rho_s^2 \phi \qquad \ldots\ 2.2c$$

The other relation required to eliminate the potentials is provided by the parallel component of Ampere's law.

$$\frac{\partial}{\partial z}\left[\nabla_\perp^2(\phi - \psi)\right] = \frac{4\pi}{c} J_z \qquad \ldots\ 2.3$$

where J_z is obtained from the equation $\nabla . J = 0$. Solving (2.2c) and (2.3), the dispersion relation (2.1) can be obtained.

We now proceed to study the interaction of the pump Alfven wave with a low frequency tearing mode characterised by $(\omega_t, \vec{k}_t)$ giving rise to side-band modes $(\omega_t \pm \omega, \vec{k}_t \pm \vec{k})$. The dynamics of the side-band kinetic Alfven waves can be adequately described by the 2 fluid equations, as shown earlier. The interaction generates non-linear driving terms in the form of $(\vec{V} \times \vec{B}_t)$, $(\vec{V} \cdot \nabla)\vec{V}_t$ etc. as shown below,

$$\frac{\partial \vec{V}_{sj}^{\pm}}{\partial t} + (\vec{V}_{t_j} \cdot \nabla)\vec{V}_j^{\pm} + (\vec{V}_j^{\pm} \cdot \nabla)\vec{V}_{t_j} = \frac{e_j}{m_j}\left[\vec{E}_s^{\pm} + \frac{\vec{V}_{sj}^{\pm} \times \vec{B}_o}{c} + \frac{\vec{V}_{t_j} \times \vec{B}^{\pm}}{c} + \frac{\vec{V}_j^{\pm} \times \vec{B}_t}{c}\right] - \frac{T_j}{m_j n_o}\left[\nabla n_{sj}^{\pm} - \frac{n_j^{\pm}}{n_o}\nabla n_{t_j} - \frac{n_{t_j}}{n_o}\nabla n_j^{\pm}\right], \qquad \ldots\ 2.4$$

$$\frac{\partial n_{sj}^{\pm}}{\partial t} + n_o \nabla \cdot V_{sj}^{\pm} + \nabla \cdot \left(n_j^{\pm}\vec{V}_{t_j} + n_{t_j}\vec{V}_j^{\pm}\right) = 0 \qquad \ldots\ 2.5$$

$$\frac{4\pi}{c^2}\frac{\partial J_{\parallel s}}{\partial t} = \nabla_\perp^2 E_{\parallel s} - \nabla_\parallel(\nabla_\perp \cdot E_s), \qquad \ldots\ 2.6$$

$$\nabla_\parallel \cdot \vec{J}_{\parallel s}^{\pm} + \nabla \cdot \vec{J}_{\perp s}^{\pm} + \frac{(\vec{B}^{\pm} \cdot \nabla)}{|B_o|} J_{\parallel t} + \frac{(\vec{B}_t \cdot \nabla)}{|B_o|} J_\parallel^{\pm} = 0 \qquad \ldots\ 2.7$$

The subscripts $\pm, j, t$ denote the pump, the type of species and the tearing mode perturbations respectively. $\vec{V}_{t_j}$, n_{t_j} and $J_{\parallel t}$ are the low freuquency tearing mode velocity, density perturbation and parallel current perturbation respectively, while $\vec{V}_{js}^{\pm}$ $n_{js}^{\pm}$ and $\vec{E}_{s}^{\pm}$ are respectively the velocity, density and electric fields of the side-band modes at $(\omega_t \pm \omega, \vec{k}_t \pm \vec{k})$. The velocity and density perturbations $(\vec{V}_j^{\pm}, n^{\pm})$ of the pump wave are obtained by solving the linearised equations of motion and continuity for ions and electrons.

Eliminating $\vec{J}_j^{\pm}$ $\vec{V}_j^{\pm}$ and $n_j^{\pm}$ in equations (2.4) to (2.7) and following the procedure outlined before (i.e. using equations (2.3) and (2.2a)) in deriving the linear dispersion relation of the kinetic Alfven wave, the side-band potentials are obtained as

$$\phi_{\pm} = \Big[\overset{(1)}{3\frac{\vec{k}_\perp \cdot \vec{B}_t}{k_\parallel |\vec{B}_0|}} + \overset{(2)}{2\frac{\vec{k}_\perp \cdot \vec{V}_t}{k_\parallel V_A}} + \overset{(4)}{\frac{4\pi}{c}\frac{(\vec{k}\times\hat{b}\cdot\nabla) J_{\parallel t}}{k_\parallel k_\perp^2 |\vec{B}_0|}} \underset{(3)}{-} \frac{i(\vec{k}\times\hat{b}\cdot\nabla)(\vec{k}\times\hat{b}\cdot\vec{V})}{k_\perp^2 k_\parallel V_A} \Big] \begin{bmatrix} \phi_0/\epsilon_A^{+} \\ \phi_0^{*}/\epsilon_A^{-} \end{bmatrix} \quad \ldots 2.8$$

Where

$$\epsilon_A^{\pm} = (\omega_t \pm \omega)^2 \Big/ (k_{\parallel}^{\pm} v_A)^2 - 1 - k_{\perp\pm}^2 \rho_s^2$$

$$\vec{k}_s^{\pm} = \vec{k}_t \pm \vec{k}. \qquad \text{.. 2.8a}$$

ϕ^* is the complex conjugate of ϕ.

In arriving at equation (2.8) , the following approximations have been made. We have made use of the adiabatic relation namely $\vec{k}_t << \vec{k}$ and $\omega_t << \omega$ In the radial direction this implies that the width of the tearing layer is much larger than the wave length of the Alfven wave.

Typically the radial variations of the kinetic Alfven wave in the absence of the non-linear coupling with tearing modes (T.M.) are of the order of the Larmor radius, while the variations of the tearing modes in the fluid limit are much larger than ρ_i . This enables us to write the side-band potential as a product of two functions, $\phi_s^{\pm} \exp i(k_{\pm} r)$ where $\phi_s^{\pm}$ is a slowly varying component indicating the contribution from the non-linear coupling to the tearing mode and the second part, $\exp i(k_{\pm} r)$, represents the fast fluctuations of kinetic Alfven wave (with $k_{\pm} \sim \frac{1}{\rho_i}$); with this formalism, using the fact $\left|\frac{d\phi^{\pm}}{dr}\right| \ll e^{i(k_{\pm} r)}$

equation (2.8) is readily obtained.

Further in equation (2.8), we have retained the dominant non-linear contributions arising through the terms $(\vec{V}^{\pm} \times \vec{B}_t)$, $(\vec{V}_t \cdot \nabla)\vec{V}^{\pm}$ in equation (2.4) and the term $(\vec{B}^{\pm} \cdot \nabla) J_{\parallel t}$ in equation (2.7). The pump amplitude $|\phi_0|$ is assumed to be sufficiently weak so that only interactions upto the order of $|\phi_0|^2$ need be kept.

A short explanation of the origin of the terms in equation (2.8) are in order. Terms (1), (2) and (3) arise from the coupling terms $(\vec{V}^{\pm} \times \vec{B}_t)$, $(\vec{V}_t \times \vec{B}^{\pm})$ and $(\vec{V}^{\pm} \cdot \nabla)\vec{V}_t$ respectively. Term(4) which plays an important role in governing the dynamics of the tearing mode comes from the non-linear contribution to the current density $J_{\parallel \pm}$, entering through the coupling term $(\vec{B}_{\pm} \cdot \nabla) J_{\parallel t}$ in the equation $\nabla . J = 0$.

In order to describe the dynamics of the resistive tearing modes, we consider again the two component fluid equations with the inclusion of non-linear effects arising through the interaction of the side band mode with the kinetic Alfven wave.

These effects manifest via the terms $(\vec{V}_{ds}^{\pm} \cdot \nabla)\vec{V}_j^{\mp}$ $(\vec{V}_{ds}^{\pm} \times \vec{B}^{\mp})$ etc and provide additional sources for the tearing mode evolution. Taking into account these contributions, the two fluid equations can be combined to yield the MHD equations

$$\rho_o \frac{\partial \vec{V}_t}{\partial t} + \nabla p = \frac{\vec{J}_t \times \vec{B}_o}{c} + \frac{\vec{J}_o \times \vec{B}_t}{c} + F_{NL}^{(1)} \qquad \ldots\ 2.9$$

$$\vec{E}_t + \frac{\vec{V}_t \times \vec{B}_o}{c} = \eta \vec{J}_t + F_{NL}^{(2)} \qquad \ldots\ 2.10$$

$$\frac{\partial \rho_t}{\partial t} + \nabla \cdot (\rho_o \vec{V}_t) + F_{NL}^{(3)} = 0 \qquad \ldots\ 2.11$$

$$\nabla \times \vec{E}_t = -\frac{1}{c} \frac{\partial \vec{B}_t}{\partial t}, \quad \nabla \times \vec{B}_t = \frac{4\pi}{c} \vec{J}_t, \quad \nabla \cdot \vec{B}_t = 0 \qquad \ldots\ 2.12$$

Where ρ_o is the mass density η the classical resistivity and J_o the equilibrium current density. In the equations (2.9) to (2.11), the non-linear terms, F_{NL} are defined by the expressions

$$F_{NL}^{(1)} = -\sum_{j=i,e} m_j n_o \left[(\vec{V}_j^{+} \cdot \nabla) \vec{V}_{s_j}^{-} + (\vec{V}_{s_j}^{-} \cdot \nabla) \vec{V}_j^{+} \right] + \frac{\vec{J}^{+} \times \vec{B}_s^{-}}{c} + \frac{\vec{J}_s^{-} \times \vec{B}^{+}}{c} + c.\ c. \qquad \ldots\ 2.13$$

$$\overset{(2)}{F}_{NL} = \frac{m_e m_i}{e(m_e+m_i)}\left[(\vec{V}_i^{\,+}\cdot\nabla)\vec{V}_{is}^{\,-} + (\vec{V}_{is}^{\,-}\cdot\nabla)\vec{V}_i^{\,+}\right] -$$

$$\left[(\vec{V}_e^{\,+}\cdot\nabla)\vec{V}_{es}^{\,-} + (\vec{V}_{es}^{\,-}\cdot\nabla)\vec{V}_e^{\,+}\right] -$$

$$\frac{1}{(m_e+m_i)}\sum_j m_j\left[\frac{\vec{V}_j^{\,+}\times\vec{B}_s^{\,-}}{c} + \frac{\vec{V}_{s_j}^{\,-}\times\vec{B}^{\,+}}{c}\right] + c.c. \quad \ldots 2.14$$

$$\overset{(3)}{F}_{NL} = \nabla\cdot\left[m_j\,(n_j^{+}\vec{V}_{s_j}^{\,-}) + m_j\,(n_{s_j}^{+}\vec{V}_j^{\,-})\right] + c.c. \quad \ldots 2.15$$

In the above system of equations, the values of $\vec{V}_j^{\pm}$, $n_j^{\pm}$, $\vec{V}_{js}^{\pm}$, $n_{js}^{\pm}$ expressed in terms of $\phi_{\pm}$(obtained from equations(2.4)- (2.7)) are to be substituted.

The density perturbations associated with the tearing mode is due to the plasma inhomogeneity (ω_* effects) and due to the source term $\overset{(3)}{F}_{NL}$. Substituting the values of $\vec{V}_j^{\pm}$, $n_j^{\pm}$, $\vec{V}_{js}^{\pm}$, $n_{js}^{\pm}$ from(2.4)to (2.7), in $\overset{(3)}{F}_{NL}$, it can be shown that these terms are proportional to $\left(\frac{1}{\epsilon^+} - \frac{1}{\epsilon^-}\right)$ where $\epsilon^{\pm}$ is defined in equation(2.8a).

Taylor expanding ϵ^{+} and ϵ^{-} around $(\omega_A k_A)$ the expressions for $\left(\frac{1}{\epsilon^+} \pm \frac{1}{\epsilon^-}\right)$ can be obtained as :

$$\left(\frac{1}{\epsilon^+} + \frac{1}{\epsilon^-}\right) = - \frac{\hat{\delta}\,(k_{\parallel} v_A)}{(1 + k_{\perp}^2 \rho_s^2)(\omega_t^2 - \hat{\delta}^2)} \quad \ldots\ 2.15a$$

$$(\hat{\delta} = \omega - \omega_A)$$

while

$$\left(\frac{1}{\epsilon^+} - \frac{1}{\epsilon^-}\right) \sim 0$$

Hence the contributions $F_{NL}^{(3)}$ to the density perturbations of the tearing mode are negligible and can be ignored.

Further, earlier investigations have demonstrated that the effect of compressiblity is generally negligible for tearing modes [3] . It was shown that the allowance for deviation from incompressiblity affects does not directly affect the fluid motion involved in the tearing modes, which are then best described by the condition $\nabla \cdot \vec{V}_t = 0$. This assumption is justified for modes which evolve on time scales much slower than the Alfvenic time.

To examine the tearing mode characteristics, we express the system of equations (2.9) -(2.12) in terms of the variables $\vec{B}_t$ and $\vec{V}_t$. To bring the equations to the standard form the following procedure is adopted;

Equations(2.10) and (2.12) are combined to give

$$-\frac{\partial \vec{B}_t}{\partial t} = \nabla \times \left[-(\vec{V}_t \times \vec{B}_0) + \eta \frac{c^2}{4\pi} (\nabla \times \vec{B}_t) + F_{NL}^{(2)} \right] \qquad \ldots\ 2.16$$

The $\hat{z}$ component of the curl of equation(2.9) is taken to give

$$\rho_0 \frac{\partial}{\partial t}(\nabla \times \vec{V}_t) \cdot \hat{e}_z = \nabla \times \left[\frac{(\vec{J}_t \times \vec{B}_0)}{c} + \frac{(\vec{J}_0 \times \vec{B}_t)}{c} + F_{NL}^{(1)} \right] \cdot \hat{e}_z \qquad \ldots\ 2.17$$

The velocity and magnetic field perturbations can in turn be expressed in terms of scalar and vector potentials respectively as $\vec{V}_t = c\frac{(\hat{b} \times \nabla \phi)}{|B_0|}$ and $\vec{B}_t = \nabla \times A_{\parallel} \hat{b}$ [Since $\nabla \cdot \vec{V}_t = 0$, $\nabla \cdot \vec{B}_t = 0$.] We look for perturbations of the form

$$Q(r, \theta, z, t) \sim Q(r) \exp i [m\theta + k_{z_t} z - \omega_t t]$$

Taking the scalar product of equations(2.16) with $\hat{b}$ (where $\hat{b}$ is the unit vector in the direction of the equilibrium magnetic field), we obtain

$$\omega_t A_{\parallel} - k_{\parallel t} c\phi = i\eta \frac{c^2}{4\pi} \nabla^2 A_{\parallel} + G \qquad \ldots\ 2.18$$

$$\frac{4\pi \rho_o c \omega_t \nabla^2 \phi}{|B_o|} = -\frac{4\pi}{c} k_{\parallel t} |B_o| J_{\parallel t} + \frac{4\pi}{c} \frac{m}{r} \frac{dJ_{o\parallel}}{dr} A_{\parallel} + F \qquad \ldots\ 2.19$$

with $k_{\parallel t} = \frac{(\vec{B_o} \cdot \nabla)}{|\vec{B_o}|}$ and 'r' the radial co-ordinate of the cylindrical co-ordinate system. The non-linear terms 'F' and 'G' in equations (2.19) and (2.18) respectively are given by

$$F = \frac{3 i n_o T_e}{|B_o|} \left|\frac{e\phi_o}{T_e}\right|^2 \left(\frac{k_r k_\theta}{k_\perp^4}\right) (k_\perp^2 \rho_i^2) \left(\frac{T_e}{T_i}\right)$$

$$\times \left(\frac{1}{\epsilon_A^+} + \frac{1}{\epsilon_A^-}\right) \frac{d^3}{dr^3} \left[-\frac{i k_r c}{\omega} \frac{d\phi}{dr} + \frac{1}{k_\parallel} \frac{d^2 A_\parallel}{dr^2} \right] \qquad \ldots\ 2.20$$

in which it is assumed that $\frac{d}{dr} \gg k_t$

Consequently only terms that give rise to the highest derivatives are retained for studying the tearing mode dyamics.

Similarly $\sigma_1 =$

$$i\,\omega_{ci}\left(\frac{T_e}{T_i}\right)^2 k_\theta^2 \rho_i^4 \left|\frac{e\phi_0}{T_e}\right|^2 \left(\frac{1}{\epsilon^+} + \frac{1}{\epsilon^-}\right)\Big[3\Big(\frac{m_e}{m_i}\frac{k_r}{k_\theta} +$$

$$i\,\frac{k_{\parallel t}}{k_\parallel}\,\frac{\omega_{ci}}{\omega}\Big)\frac{d^2A_\parallel}{dr^2} + \left(\frac{ck_r}{\omega}\right)\left(\frac{\omega_{ci}}{\omega}\right)\frac{k_{\parallel t}}{k_\perp^2}\,\frac{d^3\phi}{dr^3} +$$

$$\frac{1}{k_\perp^2}\left(\frac{m_e}{m_i}\frac{k_r}{k_\theta} + i\,\frac{k_{\parallel t}}{k_\parallel}\,\frac{\omega_{ci}}{\omega}\right)\frac{d^4A_\parallel}{dr^4} \qquad \ldots\ 2.21$$

The stability properties of the resistive tearing modes are studied by solving the simultaneous differential equations (2.18) and (2.19) with appropriate boundary conditions.

We solve the equations in a small boundary layer (inner layer) where $k_{\parallel t} \sim 0$ and in an outer region, (where resistive and inertial effects are unimportant) separately, and match the solutions in the two different regions to obtain an expression for the growth rate.

For the inner layer, we define a local region around the mode rational surface where $k_{\parallel t} = 0$; in this region, since the width of the layer is very small, derivatives of equilibrium quantities are negligible and only the highest derivatives of perturbed quantities play an important role. We expand $k_{\parallel t}(r)$ about the mode rational surface as $k_{\parallel}(r) \simeq k_{\parallel}'(r - r_s)$ (r_s is the radial co-ordinate of the rational surface and the prime denotes the derivative with respect to the radial co-ordinate). We define $x = \frac{r - r_s}{r_s}$,the local radial co-ordinate and a new variable $\psi = \frac{\omega_t}{c k'_{\parallel t} r_s} A_{\parallel}$ The inner layer equations (2.18 and 2.19) acquire the form

$$\overset{(1)a}{\frac{d^2\phi}{dx^2}} = \overset{(2)a}{\frac{x}{\Lambda^2} \frac{d^2\psi}{dx^2}} + \overset{(3)a}{\frac{\epsilon_1}{\Lambda^2} \frac{d^5\psi}{dx^5}} + \overset{(4)a}{\frac{\epsilon_2}{\Lambda} \frac{d^4\phi}{dx^4}} , \quad \ldots 2.22$$

$$x\phi - \psi = \overset{(1)b}{\frac{\bar{\eta}}{\Lambda} \frac{d^2\psi}{dx^2}} + \overset{(2)b}{\left(\frac{\epsilon_R + \epsilon_3 x}{\Lambda} \right) \frac{d^2\psi}{dx^2}} +$$

$$\overset{(3)b}{\left(\frac{\epsilon_V + \epsilon_4 x}{\Lambda} \right) \frac{d^4\psi}{dx^4}} + \epsilon_5 x \frac{d^3\phi}{dx^3} \quad \ldots 2.23$$

where $\Lambda\ (= \omega_t \tau_H = \omega_t / k'_{\parallel t} V_A r_s)$ is the characteristic time scale of the tearing mode evolution, being measured in units of poloidal Alfven time, τ_H,

$$\Big[\ \tau_H = q(r)(4\pi n_o m_i)^{1/2} / q'(r_s)\, B_o(r_s)\ ,$$

$$q(r) = r\, B_z / R\, B_\theta(r)\ ,$$

R is the major radius and $\bar{\eta} = \eta c^2 / (4\pi i |k'_{\parallel t}| V_A r_s^3)$. The non-linear coupling co-efficients $\epsilon_1, \epsilon_2, \epsilon_R$ etc., are defined as follows.

$$\epsilon_1 = -\frac{3i}{4\pi}\left(\frac{k_\theta c}{k_\perp V_A}\right)^2\left(\frac{k_r}{k_\parallel}\right)\frac{|\phi_o|^2}{k'_{\parallel t}\, r_s^4\, B_o^2}\left(\frac{1}{\epsilon_A^+} + \frac{1}{\epsilon_A^-}\right),$$

$$\epsilon_2 = \frac{3}{4\pi}\left(\frac{k_\theta c}{k_\perp V_A}\right)^2\left(\frac{k_r}{k_\parallel}\right)\frac{k_r\, |\phi_o|^2}{k'_{\parallel t}\, r_s^3\, B_o^2}\left(\frac{1}{\epsilon_A^+} + \frac{1}{\epsilon_A^-}\right),$$

$$\epsilon_R = -3i\left(\frac{k_\theta c}{\omega_{pe}}\right)\left(\frac{c}{V_A}\right)\frac{k_r\, |\phi_o|^2}{k'_{\parallel t}\, r_s^3\, B_o^2}\left(\frac{1}{\epsilon_A^+} + \frac{1}{\epsilon_A^-}\right),$$

$$\epsilon_3 = 3\left(\frac{k_\theta c}{k_\parallel V_A}\right)^2\frac{|\phi_o|^2}{r_s^2\, B_o^2}\left(\frac{1}{\epsilon_A^+} + \frac{1}{\epsilon_A^-}\right),$$

$$\epsilon_3 = -i\left(\frac{k_\theta c}{k_\perp v_A}\right)\left(\frac{k_r c}{\omega_{ce}}\right)\frac{|\phi_0|^2}{k'_{\parallel t} k_\perp r_s^5 B_0^2}\left(\frac{1}{\epsilon_A^+} + \frac{1}{\epsilon_A^-}\right),$$

$$\epsilon_4 = \left(\frac{k_\theta c}{k_\perp v_A}\right)^2 \frac{|\phi_0|^2}{(k_\parallel r_s)^2 r_s^2 B_0^2}\left(\frac{1}{\epsilon_A^+} + \frac{1}{\epsilon_A^-}\right)$$

$$\epsilon_5 = -i\left(\frac{k_\theta c}{k_\perp v_A}\right)^2 \frac{k_r |\phi_0|^2}{(k_\parallel r_s)^2 r_s B_0^2}\left(\frac{1}{\epsilon_A^+} + \frac{1}{\epsilon_A^-}\right) \qquad \ldots\ 2.23a$$

(c is the velocity of light)

In the above expressions, all symbols stand for those variables previously defined; ω_{pe} is the plasma frequency, and $\epsilon_\pm$ are the linear dielectric functions defined by

$$\epsilon_A^{\pm} = \left(\frac{\omega_\pm}{k_\parallel^\pm v_A}\right)^2 - \left(1 + k_{\perp\pm}^2 \rho_i^2\right)$$

Within the tearing layer, where small scale fluctuations exist, the derivatives play an important role. The dominant coupling terms coming through $J_{\parallel \pm s}^{(N.L)}$ in equation (2.7) (N.L. denotes the non-linear component) give rise to a 4th order derivative in ϕ' , 5th order in ψ in equation (2.22) and to 4th order terms in ψ in equation (2.23).

Some of these non-linear terms in equations (2.22) and (2.23) can be readily associated with the well known neo-classical dissipative effects studied in literature.

In particular ϵ_R (in (2.23)) could identified with the anomalous resistivity and ϵ_V (in (2.23)) with the anomalous viscosity [23] .

A brief explanation of the origin of non-linear terms in equations (2.22) and (2.23) are in order. In equation (2.22) the non-linear interaction generates ponderomotive forces $(\vec{V}^+ \cdot \nabla)\,\vec{V}_s^- + c.c.$, which when expressed in terms of electrostatic potentials give rise to terms proportional to $\nabla_r \phi_+ + \nabla_r \phi_-$. As pointed out earlier, the side-band potentials ϕ_+ and ϕ_- (given by equation (2.8)) contain terms proportional to $\nabla_r J_{\parallel t}$ and $\nabla_r V_t$

Hence the ponderomotive force is proportional to $\nabla_r^2 J_{\parallel t}$, $\nabla_r^2 V_t$. The torque produced by this force along the field lines, which is given by the parallel component of the curl of the momentum equation, is proportional to $\nabla_r^3 J_{\parallel t}$ and $\nabla_r^3 V_t$.

The former, by virtue of the relation between $J_{\parallel t}$ and ψ, $(J_{\parallel t} = \frac{d^2\psi}{dr^2})$ and the latter through the relation between $\vec{V}_t$ and ϕ give rise to terms (3a) and (4a) respectively in equation (2.22).

In the equation (2.23), however the dominant non-linear contribution arises through the $\vec{V}^{\pm} \times \vec{B}_S^{\mp}$ forces, which in terms of the side-band potentials, are proportional to $\nabla_r\phi_+ + \nabla_r\phi_-$. Using the relation between the $\vec{k}$ vectors, $k_{\parallel}{}^{\pm}_{-} = k_{\parallel t} \pm k_{\parallel}$ and the expression for ϕ_+, ϕ_- in equation (2.8), it can be seen that the non-linear mechanism generates terms proportional to $\frac{d^2}{dr^2}J_{\parallel t}$ and $J_{\parallel t}$. The former gives rise to the anomalous viscous force, and the later anomalous resistive effects in equation (2.23).

Equation (2.22) the momentum transfer equation, represents a balance between the inertial flow of the fluid $(i.e.\ \frac{d^2\phi}{dr^2})$ and the torque produced by
1) ponderomotive forces of the non-linear interaction
2) the linear $\vec{J}_t \times \vec{B}_O$ forces.

For large enough fluctuation levels of the amplitude of the pump kinetic Alfven wave, the ponderomotive force $(\vec{V}\cdot\nabla)\vec{V}$ produced by the non-linear interaction (term (3a)) dominates the linear $\vec{J}_t \times \vec{B}_O$ forces (term (2a)). The linear shear flow (term (1a)) is driven against fluid inertia by the torque produced by the ponderomotive force.

In equation (2.23) the force due to the parallel electric field on the left hand side of the equation is balanced by a combination of the classical resistive forces ((1) b), the anomalous resistive ((2) b) and viscous forces ((3) b) produced by the non-linear coupling mechanism. When the amplitude $|\phi_0|$ of the pump kinetic Alfven wave is above a certain threshold value, the anomalous viscous and diffusive effects are much larger than the classical resistive effects. The parallel electric field is then balanced by these anomalous effects rather than the collisional drag.

In the outer region where $k_{\parallel} \neq 0$ the nominal ordering $\nabla \sim a^{-1} \sim k_{\parallel t}$, $V_t \sim V_A$ (where a, is the minor radius) can be used.

The classical and anomalous dissipative effects in Ohm's law have much larger time scales (using the above ordering) than the convective term $(\vec{V}_t \times \vec{B}_0)$. So rapid field annihilation is due to convective processes rather than the dissipative processes.

For the equation of motion, using the ordering we have mentioned, the contribution from $\frac{d^5\psi}{dx^5}$, $\frac{d^4\phi}{dx^4}$ which scale as $\frac{\psi}{a^5}$, $\frac{\phi}{a^4}$ are very small compared to the equilibrium current source, and can be neglected.

Hence the outer layer equations can be approximated as [5]

$$k_{\parallel t} |B_0| \nabla^2 A_\parallel + \frac{4\pi}{c} \frac{m}{r} \frac{dJ_{0\parallel}}{dr} A_\parallel = 0 \qquad \text{... 2.23b}$$

$$\omega A_\parallel = k_{\parallel t}\, c\, \phi \qquad \text{... 2.23c}$$

A brief examination of equations 2.22 and 2.23 reveals that they can be simplified further. On comparing the ϕ and ψ dependent terms in equation (2.22), the ϕ dependent term turns out to be small compared to the ψ [18] dependent term for the range of mode with $X_\omega \gg k_{r0} r_s \Lambda$ $(\Lambda \ll 1)$. The widths of the mode (as later investigations show) satisfy this relation. Further the terms proportional to ϵ_R, ϵ_3 in equation (2.23) (whose effects on the mode growth rate has been extensively investigated [18] play negligible roles in the present problem, since the higher derivatives control the temporal evolution of the resistive tearing modes. With these features taken into consideration, equations (2.22) and (2.23), take the form

$$\frac{d^2\phi}{dx^2} = \frac{x}{\Lambda^2} \frac{d^2\psi}{dx^2} + \frac{\epsilon_1}{\Lambda^2} \frac{d^5\psi}{dx^5} \qquad \text{... 2.24a}$$

$$x\phi - \psi = \frac{(\epsilon_v + \epsilon_4 x)}{\Lambda} \frac{d^4\psi}{dx^4} \qquad \text{... 2.24b}$$

The solutions of equation (2.24) will be discussed in the following sections by employing the variational 21 and asymptotic matching techniques [19].

2.3 Solutions by the variational method :

The variational principle is a powerful tool for solving systems of equations such as (2.24). The details of this technique are elaborated in reference [21]. Here we simply state the prescription to be followed. We note first that though the order of the equation in 'x' space could be quite high, in Fourier space it would be a 2nd order differential equation. We define

$$\psi_p = \int_{-\infty}^{\infty} \psi(x)\, e^{ipx}\, dx \; , \qquad \phi_p = \int_{-\infty}^{\infty} \phi(x)\, e^{ipx}\, dx$$

the Fourier transforms of $\psi(x)$ and $\phi(x)$. Replacing 'x' by the operator $\frac{d}{dp}$ and performing partial integration, the variable ϕ can be eliminated from the coupled equations (2.24), to yield a 2nd order equation in J_p where $J_p = p^2 \psi_p$ (the Fourier transform of the perturbed current).

$$\frac{d}{dp}\left[\frac{dJ_p}{dp}\,\frac{1}{(p^2\Lambda^2)}\right] + \left(\frac{i\epsilon_4 p^2}{\Lambda} - \frac{\epsilon_1 p}{\Lambda^2}\right)\frac{dJ_p}{dp} + J_p\left(\frac{1}{p^2} - \frac{\epsilon_1}{\Lambda^2} + \frac{2i\epsilon_4 p}{\Lambda} + \frac{\epsilon_v p^2}{\Lambda}\right) = 0 \qquad \ldots\; 2.25$$

Equation (2.25) has to be cast into the self adjoint form to make it amenable for variational treatment. For this purpose, we define

$$J_P = J_1(P)\exp\left[\left(\frac{\epsilon_1 P^4}{8}\right) - i\left(\frac{\epsilon_4 \Lambda P^5}{10}\right)\right]$$

The equation for $J_1(P)$ becomes

$$\frac{d}{dP}\left[\frac{1}{P^2\Lambda^2}\frac{dJ_1}{dP}\right] + J_1\left[\frac{1}{P^2} - \frac{\epsilon_1}{2\Lambda^2} + \frac{i\epsilon_4 P}{\Lambda} + \frac{\epsilon_Y P^2}{\Lambda} - \frac{P^4}{4}\left(i\epsilon_4 P - \frac{\epsilon_1}{\Lambda}\right)^2\right] = 0 \qquad \ldots\ 2.26$$

This equation admits a variational treatment. For this purpose, a functional S is constructed in the following manner. Equation (2.26) is multiplied by J_1 and integrated from $-\infty$ to $+\infty$. On performing the integrations the equation reduces to

$$S = \int_{-\infty}^{\infty}\left[-\frac{1}{P^2\Lambda^2}\left(\frac{dJ_1}{dP}\right)^2 + J_1^2\left[\frac{1}{P^2} - \frac{\epsilon_1}{2\Lambda^2} + \frac{i\epsilon_4 P}{\Lambda} + \frac{\epsilon_Y P^2}{\Lambda} - \frac{P^4}{4}\left(i\epsilon_4 P - \frac{\epsilon_1}{\Lambda}\right)^2\right]\right] dP \qquad \ldots\ 2.27$$

We note that the variation of equation (2.27) i.e. $\delta S=0$, leads back to equation (2.26). In order to determine the eigen values Λ we choose a simple trial function of the form, $\exp(-\frac{\alpha \rho^2}{2})$ with $Re(\alpha)>0$. We then solve $S = 0$, $\frac{dS}{d\alpha} = 0$ for a self consistent solution. The tearing modes fall into two categories. The m = 1 mode, (m being the azimuthal mode number and the m $\geqslant$ 2 modes. Of these the former (m = 1) is a highly localised mode. It is confined to the inner layer and does not manifest itself in the outer regions. The m = 2 mode has more of a global structure, and extends well into the outer region. For this mode, the solutions are obtained independently in the inner and outer regions. In order to connect the solutions between the inner and outer regions, a quantity Δ' defined as [3]

$$\Delta' = \left. \frac{\psi'}{\psi(0)} \right|_{\epsilon^-}^{\epsilon^+}$$

the logrithmic derivative of ψ across the tearing layer is matched to that of the outer layer [5]. Hazeltine et al [22] have shown that the effect of Δ' for the m = 2 mode can be incorporated into the equation in the following manner. The contribution from the equilibrium current source gives an additional term in the solution of Ampere's law.

This term can be accounted for by replacing $\frac{1}{P^2}$ in equation (2.26) by $\left(\frac{1}{P^2} - \frac{\delta(P)}{\Delta'}\right)$, where $\delta(P)$ is the Dirac delta function and Δ' is the stability parameter. Taking into account the above considerations, the expression for the functional can be written as,

$$- \frac{S(\alpha)}{\sqrt{\pi}} = \frac{\alpha^{3/2}}{\Lambda^2} + 2\alpha^{1/2} + \frac{\epsilon_1}{2\alpha^{1/2}\Lambda^2} + \frac{3\epsilon_1^2}{16\Lambda^2\alpha^{5/2}}$$

$$+ \frac{1}{\Delta'\sqrt{\pi}} - \left(\frac{\epsilon_v}{2\Lambda\alpha^{3/2}} + \frac{15\,\epsilon_4^2}{32\,\alpha^{7/2}}\right) \qquad \ldots\ 2.28$$

The dispersion relation can now be derived by eliminating α between $S(\alpha) = 0$, $\frac{ds}{d\alpha} = 0$. This relation in general is an algebraic polynomial in Λ and is therefore difficult to solve for the roots using analytical methods. We consider several limits of equation (2.27) in the parameter space to that it enables us to find the approximate roots of the polynomial. First we examine the case where 'ϵ_v' alone is present. In this case we find that the m = 1 and m = 2 resistive tearing modes are excited with their growth rates varying as $(\epsilon_v)^{1/5}$ and $(\epsilon_v)^{1/3}(\Delta')^{2/3}$ respectively. These results are analogous to an investigation on magnetic braiding effects reported elsewhere [23] . This non-linear coefficient however turns out to be much smaller than the parameters ϵ_1 and ϵ_4 and hence its effect will be ignored throughout further discussions. The resultant dispersion relation obtained by solving $S=0$ and $\frac{ds}{d\alpha}=0$ with finite contributions from ϵ_1 and ϵ_4 is discussed in two limits, namely $|\alpha| > |\epsilon_4 \Lambda|^{2/5}$ and $|\alpha| < |\epsilon_4 \Lambda|^{2/5}$

m = 1 Resistive tearing mode $(\sqrt{\alpha}\,\Delta' \gg 1)$

In this limit the ϵ_4 term can be dropped from the expression for the functional in equation(2.28). Hence retaining all terms except ϵ_4 and eliminating α between the two simultaneous equations a quadratic equation for Λ is obtained,

$$\Lambda^2 = \mp \frac{\epsilon_1^{1/2}}{4} \left[\frac{2 + (2 \pm \sqrt{10})^2}{(1 \pm \sqrt{10})^{3/2}} \right] \qquad \ldots\ 2.29$$

while the value of α is found to be

$$\alpha = \pm \frac{\epsilon_1^{1/2}}{2} (1 \pm \sqrt{10})^{1/2} \qquad \ldots\ 2.30$$

In order to ensure that the trial function ($e^{-\alpha p^2}$) be well behaved, we must choose that , value of Λ for which $\mathrm{Re}(\alpha)$ is $>$ 0. $\epsilon_1(\Lambda)$ is a function of Λ (refer equation 2.23d) by virtue of its dependence on the term $\left(\frac{1}{\epsilon_A^+} + \frac{1}{\epsilon_A^-}\right)$ (equation(2.15a)).Using the approximations, $\omega_\pm \sim \omega_o$ (the pump frequency) and noting that $\omega_+ = \omega_t + \omega_0$ and $\omega_- = \omega_t - \omega$ the expression for ϵ_1 simplifies to

$$\epsilon_1 = \frac{-i\,\hat{\epsilon}_1}{\Lambda^2 - \delta^2} \qquad \ldots\ 2.31$$

where $\delta = \dfrac{\omega - \omega_A}{k_{\parallel t}' V_A r_s}$ is the frequency mismatch in the dimensionless form and $\hat{\epsilon}_1$ is given by

$$\hat{\epsilon}_1 = \frac{3}{4\pi}\left(\frac{k_\theta c}{k_\perp V_A}\right)^2 \frac{k_r \delta\, |\phi_0|^2}{|k_{\parallel t}'|^2 r_s^5 B_0^2 (1 + k_\perp^2 \rho_s^2)} \qquad \ldots\ 2.32$$

In deciding the nature of the complex term, ϵ_1 we have made use of the fact that $k_{\parallel t}' = \dfrac{-q'(r)}{R\, q(r)^2}$ and assumed that $\omega > \omega_A$ (or $\delta > 0$). Thus the dispersion relation (2.29), in conjunction with equations (2.31) and (2.32), gives the mode characteristics for the m = 1 resistive tearing mode.

We have solved the dispersion relation (2.29) numerically. In general there are six complex roots of equation (2.29) in Λ for a given value of δ and $\hat{\epsilon}_1$ The correct root is the one that satisfies the consistency condition of Re $\alpha(\Lambda) > 0$.

Typical plots of Λ_I versus δ , (for two different values of $C_1 \sim \dfrac{\hat{\epsilon}_1}{\delta}$) are shown in Figs.(2.3), (2.4) and (2.5). It is seen that close to $\Lambda \sim \delta$ there is a resonant behaviour and Λ_I has a distinct peak.

In the regions $\Lambda \ll \delta$ and $\Lambda \gg \delta$, equation (2.29) can be solved approximately and analytical solutions are given by

$$\Lambda \simeq \left[\frac{2 + (2 \pm \sqrt{10})^2}{(1 \pm \sqrt{10})^{1/2}}\right]^{1/3} (\hat{\epsilon}_1)^{1/6} e^{i7\pi/12} \qquad (\Lambda > \delta)$$

... 2.33

$$\Lambda \simeq \left[\frac{2 + (2 \pm \sqrt{10})^2}{(1 \pm \sqrt{10})^{3/4}}\right]^{1/2} \frac{(\hat{\epsilon}_1)^{1/4}}{(\delta)^{1/2}} e^{i5\pi/8} \qquad (\Lambda < \delta)$$

... 2.34

For parameters of interest in tokamak plasmas and for moderate values of pump amplitude, $|\phi_0|$, the above results demonstrate the existence of unstable modes with growth rates varying as fractional powers of $|\phi_0|$.

Further in each case the criterion for neglecting the ϵ_4 term i.e. $|\alpha| > |(\epsilon_4 \Lambda)|^{2/5}$ is checked using the value of α derived in equation (2.30) and the assumption found to be consistent. On substituting the values of ϵ_1, ϵ_4 in equation (2.23 a) the inequality turns out to be proportional to $\left(\frac{k_r}{k_{\perp t}}\right)(k_{\parallel} L_s)$ where L_s is the shear length. This ratio for typical parameters is $\gg 1$.

Next we consider the case when $|\alpha| \lesssim |\epsilon_4 \Lambda|^{2/5}$ For this limit, the contributions from the ϵ_4 term (neglected before) in equation(2.26) becomes important. We retain the dominant contributions in equation (2.28) namely $\alpha^{3/2}$, ϵ_1^2, ϵ_4^2 . The dispersion relation can be obtained as

$$\Lambda^8 + \left(\frac{3}{5}\right)\left(\frac{2\epsilon_1}{5\epsilon_4}\right)^8 \epsilon_1^2 = 0 \qquad \ldots\ 2.35$$

This is similar to the result (2.29), except for the additional multiplicative factor $\left(\frac{\epsilon_1}{\epsilon_4}\right)^8$. As before, the solutions also give rise to unstable tearing modes corresponding to the frequency domains $\Lambda > \delta$ and $\Lambda < \delta$ The new feature is the fact that the growth rates are now enhanced by a large factor proportional to $\left|\frac{\epsilon_1}{\epsilon_4}\right| (\gg 1)$. For typical spectrum of wave lengths of both kinetic Alfven waves and tearing mode, this factor turns out to be large

$$\left(|\epsilon_1/\epsilon_4| \simeq k_r k_{\parallel}/k'_{\parallel t} \gg 1\right)$$

It may be remarked that a similar enhancement in growth rate characteristics occurs for a non-linear coupling between tearing modes and lower hybrid waves [18].

m = 2 Resistive tearing mode $(\alpha^{1/2}\Delta' \ll 1)$

For the Δ' driven modes, the widths are such that $\alpha \gg \Lambda^2$. The dispersion relation is derived from equation (2.28), reglecting the term $2\sqrt{\alpha}$ and following the same procedure as outlined for the m = 1 mode. When $|\alpha| > |\epsilon_4 \Lambda|^{2/5}$, the dispersion relation reduces to

$$\Lambda = \pm i\, \epsilon_1^{3/8} (\Delta' \sqrt{\pi})^{1/2} C_1 \qquad \ldots\ 2.36$$

where

$$C_1 = \left[\frac{(1+\sqrt{46})^2 + 6(1+\sqrt{46}) + 27}{(12)^{3/8}\,(1+\sqrt{46})^{5/8}}\right]^{1/2}$$

and ϵ_1 is defined by equation (2.31). Once again out of the roots of equation(2.36) only those which satisfy the consistency criterion $\mathrm{Re}\,\alpha(\Lambda) > 0$ will constitute admissible roots. Using (2.31), we obtain the dispersion relation near the resonance region $(\Lambda \sim \delta)$ to be

$$\Lambda \simeq -\delta + i\,\delta^{-11/3} (\Delta' \sqrt{\pi})^{4/3} C_1^{4/3} \hat{\epsilon}_1 \qquad \ldots\ 2.37$$

This root yields a growing mode with its growth rate directly proportional to $|\Phi_0|^2$, $\delta^{-8/3}$. This constitutes a mode which is resonantly excited by the kinetic Alfven waves.

Away from the resonant region $\Lambda > \delta$, $\Lambda < \delta$, the dispersion relation has the form

$$\Lambda = (\Delta'\sqrt{\pi})^{2/7}\,\epsilon_1^{3/14}\,c_1^{4/7}\,e^{\left(i\pi/56 + \frac{4\pi i n}{7}\right)} \qquad .. \; 2.38$$

Where n takes the values 0, 1, 2, 3, 4, 5, 6 .

Growing roots exist for values of $n = 0, 4.$

For $\Lambda < \delta$

$$\Lambda = + \frac{\epsilon_1^{3/8}}{\delta^{3/4}}\, c_1 \exp\left[\frac{11\pi i}{16}\right] \qquad .. \; 2.39$$

This again is a growing root.

It may be concluded that the unstable nature still persists away from the region where resonance occurs ($\Lambda \sim \delta$) with growth rates proportional to fractional powers of the pump amplitude.

In the limit $|\alpha| << |\epsilon_4 \Lambda|^{2/5}$, there arises an additional contribution from the ϵ_4 term besides the ϵ_1^2 and Δ' terms in equation (2.27). Keeping these terms, the eigen values, Λ are found to satisfy the relation

$$(7\Lambda^2)^7 = \frac{9\pi}{2}\left(\frac{\epsilon_1}{\epsilon_4}\right)^{14}(\epsilon_4)^4(\Delta')^2 \qquad .. \; 2.40$$

Where ϵ_4 is a function of Λ as defined in equation (2.23a). In this limit also, we conclude that the non-resonant tearing instabilities consistent with the condition Re $\alpha(\Lambda) > 0$ are excited by the Alfven pump. The enhancement factor $\left(\frac{\epsilon_1}{\epsilon_4}\right)$ again features in this case.

For typical tokamak parameters, i.e. a (minor radius) $\sim$ 45 cms, R(major radius) $\sim$ 130 cms. B_T (toroidal magnetic field) $\sim$ 50 KG, n_e(equilibrium density) $\sim 7 \times 10^{13}$, T_e (electron temperature in electronvolts) $\sim$ 2KeV, ρ_i (ion gyroradius) $\sim$ .1cms, r_s(co-ordinate of mode radional surface) $\sim \frac{a}{2}$ $\Delta' a$(stability parameter) ~ 4, L_s (shear length) $\sim$ 150 cms, $k_r^2 |\phi_0|^2 / |B_0|^2 \sim 10^{-6} - 10^{-8}$, the computed growth rates fall in the range $\sim 10^4 - 10^6$ sec^{-1}. In particular the modes with growth rates proportional to $\left(\frac{\epsilon_1}{\epsilon_4}\right)$ have larger growth rates. In each case it is clear that the gradients introduced by the non-linear interaction drive new m = 1, and m = 2 resistive tearing modes unstable with significantly large growth rates compared to their growth through classical dissipative processes.

2.4 Matched asymptotic solutions :

In the previous section, we employed the variational technique in solving equation (2.25) and computed the eigen values, Λ by demanding that the trial function be well behaved at infinity. Although this technique enables us to predict the instability characteristics of resistive tearing modes accurately, it gives little knowledge about the global structure of the eigen function or its asymptotic behaviour. Thus it becomes important to obtain analytical solution of equation (2.25) valid in the entire region of P-space.

The aymptotic matching [19] technique is an important method, which can be used to find approximate analytical solutions to differential equations and to obtain global properties like eigen values. The principle of this technique is as follows. The interval on which the eigen value problem is posed is devided into two or more overlapping regions. The equation is solved in each of the subintervals and matched in the overlap region. In this section, we demonstrate that the dispersion relation derived in the previous section, can be recovered using this technique.

In order to connect the results of the earlier section with the direct method used here, we retain the dominant terms in equation (2.26) and rewrite it as follows.

$$\frac{d}{dk}\left[\frac{1}{k^2}\frac{dJ_1}{dk}\right] + \frac{J_1}{\Lambda^2}\left[\frac{\Lambda^2}{k^2} - \frac{k^4}{4\Lambda^2}\left(\frac{i\epsilon_4 k}{\Lambda} - \frac{\epsilon_1}{\Lambda}\right)^2\right] = 0 \quad \ldots\ 2.41$$

Where $k = P\Lambda$ and the terms such as ϵ_V, $\frac{\epsilon_1}{\Lambda^2}$ etc. are neglected consistent with the approximation made in section (2.2).

The basic approach revolves around the fact that we devide the k-space into sub-regions wherein a few terms depicting the physical effect provide dominant balance. We then match the solutions in the overlap region in such a manner so as to obtain a uniformly valid solution in the entire 'k' space.

We devide the region into two parts, $|k| < |\Lambda^4/\epsilon_1|^{1/3}$ and $|k| > |\Lambda^4/\epsilon_1|^{1/3}$ in which the governing equations are

$$\frac{d}{dk}\left[\frac{1}{k^2}\frac{dJ_1}{dk}\right] + \frac{J_1}{k^2} = 0 \qquad \ldots 2.42$$

and

$$\frac{d}{dk}\left[\frac{1}{k^2}\frac{dJ_1}{dk}\right] - \frac{k^4}{4\Lambda^8}(i\epsilon_4 k - \epsilon_1)^2 J_1 = 0 \qquad \ldots 2.43$$

respectively.

These equations (2.42) and (2.43) are the Fourier transform of the inner region equations, valid in the dissipative layer. ($|k| \gg 1$). In order to obtain a global continuous solution, the solutions of

(2.42), and (2.43) have to be joined to those of the outer region, (equations (2.23b), (2.23c)) where the equilibrium current plays a significant role. For this purpose, $J_{\parallel o}$ (in equation (2.23b)) is expressed in terms of $k_{\parallel t}$ and solutions in the neighbourhood of mode-rational surface, ($k_{\parallel t} \simeq 0$) are sought.

The solutions of the resulting equations, for standard current profile need to be obtained. It has been shown that electrostatic potential ϕ and perturbed current $J_1 = \rho^2 \psi$, have the asymptotic forms [25]

$$\phi \sim \phi_0 + \frac{\phi_1}{k} \quad , \quad \frac{dJ_1}{dk} \sim k^2 \phi_0 + k \phi_1 \qquad \ldots 2.44$$

Where ϕ_o and ϕ_1 are constants. Δ' is related to these constants through the relation $\Delta' = \frac{\phi_1}{\phi_o}$. Equations (2.42)-(2.44) constitute the complete set for studying the evolution of m = 1 and m = 2 modes.

The solution of equation (2.42) can be easily obtained as

$$\overset{(1)}{J} = \overset{(1)}{A} (\sin k - k \cos k) - \overset{(1)}{B} (k \sin k + \cos k) \qquad \ldots 2.45$$

While the 2nd region, is further broken into two sub-regions, namely $1 << |k| << |\epsilon_1/\epsilon_4|$ and $|k| >> |\epsilon_1/\epsilon_4|$

In the former the equation has the form

$$\frac{d}{dk}\left[\frac{1}{k^2} \frac{dJ_1}{dk} \right] - \frac{k^4}{4\Lambda^8} (\epsilon_1^2) J_1 = 0 \qquad \ldots 2.46$$

the solutions of which are

$$\overset{(2)}{J_1} = k^{3/2}\left[\overset{(2)}{A}\, J_{3/8}\left(\frac{i\epsilon_1}{8\Lambda^4}k^4\right) + \overset{(2)}{B}\, N_{3/8}\left(\frac{i\epsilon_1}{8\Lambda^4}k^4\right)\right]$$

$$\left(1 \ll |k| \ll |\epsilon_1/\epsilon_4|\right) \qquad \ldots\ 2.47$$

While in the region $|k| \gg \left|\frac{\epsilon_1}{\epsilon_4}\right|$ the equation takes the form

$$\frac{d}{dk}\left(\frac{1}{k^2}\frac{dJ_1}{dk}\right) - \frac{k^4}{4\Lambda^8}(i\epsilon_4 k)^2 J_1 = 0 \qquad \ldots\ 2.48$$

whose solutions are

$$\overset{(3)}{J_1} = k^{3/2}\left[\overset{(3)}{A}\, J_{3/10}\left(\frac{i\epsilon_4}{10\Lambda^4}k^5\right) + \overset{(3)}{B}\, N_{3/10}\left(\frac{i\epsilon_4}{10\Lambda^4}k^5\right)\right.$$

$$\left(|k| \gg |\epsilon_1/\epsilon_4|\right) \qquad \ldots 2.49$$

In equations (2.47) and (2.49), the symbols J and N are the Bessel functions of the 1st and 2nd kind respectively and the subcripts on $\overset{(1,2,3)}{J_1}$ denote the sub-regions in 'k' space. A and B denote the arbitrary constants in the various sub-regions.

We now investigate the solutions corresponding to the m = 1 mode. For this mode, the stability parameter Δ' is very large and hence the constant Φ_0

(through the relation in equation (2.44)) is set to zero. The solutions are highly localised in the dissipative layer $(|k| >> 1)$, while in the outer region $(|k| << 1)$ the solutions asymtotically approach the form $\phi \sim \phi_1/k$, $dJ_1/dk \sim k\phi_1$ (as ϕ_0 is zero). These asymptotic forms of the solutions in the outer region must match with the solutions of region (1) in the small argument limit. In the small argument limit, the solutions in region (1) have the form

$$\frac{dJ_1}{dk} \sim \overset{(1)}{B} k + \overset{(1)}{A} k^2 \qquad \ldots\ 2.50$$

To ensure matching between the solutions of region (1) and the outer region, the coefficient $\overset{(1)}{A}$ must be chosen to be zero. The other boundary condition arises from the fact that the solutions must be well-behaved in the limit $|k| \to \infty$. For the situation when $|\alpha| > |\epsilon_4 \Lambda|^{2/5}$, where $\sqrt{\alpha}$ is to be identified as the width of the mode in 'x' space, (or $\frac{1}{\sqrt{\alpha}}$ the width of the mode in 'k' space) the solutions of equation (2.43) do not play an important role and the term proportional to ϵ_4 can be omitted from equation (2.43). This is the limit discussed in section (2.2) .

The matching of solutions need be done only between region (1) and region (2) to arrive at the dispersion relation. Choosing $B^{(2)}$ (the 2nd co-efficient of the solution in region (2)) to be iA_2, converts the Bessel solution into a Hankel solution and ensures convergence.

To connect the solutions in regions (1) and (2), we equate the coefficients of $k^{(0)}$ and $k^{(8)}$ (these are the dominant terms) in the overlap region. The ratio of the coefficients are as follows:

$$\overset{(1)}{B} = \frac{\overset{(2)}{B}}{(16)^{-3/8}} \frac{(i\theta)^{-3/8}}{\sin\left(\frac{3\pi}{8}\right)} \frac{1}{\Gamma 5/8} \qquad \ldots\ 2.51$$

$$-\overset{(1)}{B}\left[-\frac{1}{7!} + \frac{1}{8!}\right] = \frac{\overset{(2)}{B}}{\sin\left(\frac{3\pi}{8}\right)} \left(\frac{i\theta}{16}\right)^{13/8} \frac{1}{\Gamma 13/8} \qquad \ldots\ 2.52$$

where $\theta = \epsilon_1 / \Lambda^4$ ... 2.53

Deviding equation (2.52) by the equation (2.53) the dispersion relation relation is obtained as :

$$\Lambda^8 = -36\,\epsilon_1^2 \qquad \ldots\ 2.54$$

The scaling for Λ obtained from (2.54) agrees in its functional form with equation (2.29). Out of all the roots of (2.54), only those which ensure convergence of Hankel solution of region (2) (which asymptotically has the form $\sim e^{i\theta}$) are acceptable.

For the limit $|\alpha| > |\epsilon_4 \Lambda|^{2/5}$ discussed in section (2.2), the term proportional to ϵ_4 plays a significant role. The solutions in region (3) i.e. equation (2.49) have to be retained. The convergence property now depends on the solutions in region (3). For this purpose, we impose the relation $\overset{(3)}{B} = i \overset{(3)}{A}$. The solution in region (2) have now to be connected to those of region (3). Equating the coefficients of $k^{(0)}$ and $k^{(4)}$, the connection can be brought about.

$$-\frac{\overset{(3)}{B}}{\sin\left(\frac{3\pi}{10}\right)}\left[\frac{\theta^{1/2}\alpha_1^{1/2}}{20}\right]^{-3/10}\frac{1}{\Gamma 7/10} = -\frac{\overset{(2)}{B}}{\sin\frac{3\pi}{8}}\left(\frac{i\theta^{1/2}}{16}\right)^{-3/8}\frac{1}{\Gamma 5/8} \quad \ldots 2.55$$

$$\left(\overset{(3)}{A} + \overset{(3)}{B}\cot\frac{3\pi}{10}\right)\left(\frac{\theta^{1/2}\alpha_1^{1/2}}{20}\right)^{3/2} \times \frac{1}{\Gamma 13/10} =$$

$$\left(\overset{(2)}{A} + \overset{(2)}{B}\cot\frac{3\pi}{8}\right)\left(\frac{i\theta^{1/2}}{16}\right)^{3/8}\frac{1}{\Gamma 11/8}$$

$$\left(\alpha_1 = \frac{\epsilon_4^2}{\epsilon_1^2}\right) \quad \ldots 2.56$$

As before deviding (2.55) by (2.56), the dispersion relation is obtained as :

$$\Lambda^8 = \frac{(1\cdot 2)}{4^5} \left(\frac{\epsilon_1}{\epsilon_4}\right)^8 \epsilon_1^2 \qquad .. \ 2.57$$

The above relation is identical in its functional dependence with the solution derived in equation (2.35). As before, among the roots of (2.57), only those which ensure the convergence of the solutions are accepted.

To compute the eigen values for the m = 2 mode by the asymptotic method, we need to consider the solutions in regions $|k| \ll 1$ (outer regions) besides those in the dissipative regions. The stability parameter, Δ' in Fourier space is defined as [25]

$$\Delta' = \underset{k \to \pm\infty}{Lt} \mp \frac{k^2}{\phi} \frac{d\phi}{dk} = \frac{\phi_1}{\phi_0} \qquad .. \ 2.58$$

Using equation (2.50), in conjuction with (2.51) and (2.58), (for the limit $|\alpha| > |\epsilon_4 \Lambda|^{2/5}$) the ratio of the coefficients $\overset{(1)}{A}$ & $\overset{(1)}{B}$ is given by

$$-\frac{\overset{(1)}{B}}{\overset{(1)}{A}} = \Lambda \Delta' \qquad .. \ 2.59$$

While the connection formula between the regions (1) and (2) leads to the ratio of coefficients as

$$\Lambda \overset{(2)}{B} = \overset{(1)}{B} \Gamma_{5/8} \sin\left(\frac{3\pi}{8}\right)\left(\frac{i\epsilon_1}{16}\right)^{3/8} \qquad \ldots 2.59a$$

$$3\left[\overset{(2)}{A} + \overset{(2)}{B} \cot\left(\frac{3\pi}{8}\right)\right]\left(\frac{i\epsilon_1}{16}\right)^{3/8} = \Lambda^2 \Gamma_{11/8} \overset{(1)}{A} \qquad \ldots 2.59b$$

with the boundary condition $\overset{(2)}{B} = i\overset{(2)}{A}$. Using (2.59-2.59b), the dispersion relation for the m = 2 mode is obtained as

$$\Lambda^2 = \left[\exp i\frac{5\pi}{8}\right]\left[\frac{i\epsilon_1}{16}\right]^{3/4}\left[\frac{\Gamma_{5/8}}{\Gamma_{11/8}}\right]\Delta' \qquad \ldots 2.60$$

Where $\Gamma(x)$ is the gamma function. This result is similar to that given in equation (2.36).

Thus we conclude that the scaling for m = 1 and m = 2 resistive tearing modes can be obtained rigorously by the asymptotic matching technique. The results of this section quantitatively agree with the variational solutions of the previous section.

2.5 Summary

To summarise, we have studied an important mode coupling mechanism, in which the kinetic Alfven waves couple non-linearly among themselves to parametrically excite tearing modes.

We have shown that the ponderomotive force produced by the interaction, generates convective forces in the equation of motion and simulates anomalous viscous and resistive effects in Ohm's law. Within the tearing layer, these effects manifest themselves as steep radial gradients of the perturbed magnetic field and provide additional sources of free energy. Outside the boundary layer, the Alfvenic terms dominate and the solutions are basically that of the ideal kink mode. We have preserved the basic linear characteristic of the tearing mode by retaining the non-linear terms only in the inner region equations.

The momentum equation describes the motion of the fluid driven against inertia by the torque produced by the linear $\vec{J}_{r} \times \vec{B}_{0}$ forces and the ponderomotive forces of the non-linear interaction. For large enough fluctuation levels of the amplitude of the pump Alfven wave the ponderomotive force emerges as the dominant destabilising force.

The Ohm's law, represents a balance between the parallel electric field and the dissipative forces of the classical resistive type, the anomalous resistive and viscous types generated by the non-linear interaction. In the inner layer, where steep gradients dominate, the perturbed magnetic field is dissipated by the viscous forces of the non-linear interaction rather than the collisional drag.

The tearing instability evolves under the combined effect of the non-linear ponderomotive force and the viscous drag.

Using both variational and asymptotic matching methods, we have demonstrated the existence of m = 1 and m = 2 tearing instabilities with their growth rates varying as fractional powers of the pump amplitude, namely $|\phi_0|^{1/3}$ (equations (2.23) and (2.34)) and $|\phi_0|^{3/7}(\Delta')^{2/7}$ (equation (2.36)).

For rather moderate Alfven wave intensity $k_\gamma^2|\phi_0|^2/|B_0|^2 \sim 10^{-6}$ to 10^{-8} and typical choice of tokamak parameters namely a $\sim$ 45 cms, (the minor radius) R $\sim$ 130 cms (the major radius), $B_0 \sim 150$ KG, $n_e \sim 7\times10^{13}\,cm^{-3}$, $T_e \sim 2$ keV, $\rho_i \sim 0{\cdot}1$ cm, $r_s \sim a/2$, $\Delta' a \sim 4$ we find that the calculated growth rates fall in the range 10^4 to 10^6 sec^{-1}.

Alfven wave experiments are currently being carried out at several places. The most notable results are from the TCA tokamak at Lausamne [3] .

In most of the experiments in addition to efficient electron heating enhanced transport of particles have been observed to take place. It is believed that this is due to the onset of tearing activity which destroys good magnetic surfaces. It is however unclear as to whether the tearing activity is triggered by the kinetic Alfven waves. These experiments also report disruptions for r.f. power above a certain threshold. There is indirect evidence that these power limitations may be due to the excitation of tearing modes. However to make a direct correlation, data on the MHD activity of the plasma and the mode dependences of the power limitations are required. We believe that work is in progress in this direction.

It may be remarked in conclusion that the growth rate of these tearing instabilities triggered by the kinetic Alfven waves will eventually be controlled by non-linear saturation mechanisms which have not been studied here. In order to consider these non-linear effects, we need to retain second order terms of the tearing mode perturbation such as etc. Presumably the rapid growth of these new modes discussed here would be slowed down by considerations similar to earlier works on non-linear tearing modes [23, 24] .

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Figure captions for Chapter II

Fig (2.1) The equilibrium sheet current distribution is shown in slab geometry. The current J_z runs along the field B_z, generating a self-consistent $B_y(x)$. The field B_y changes sign at $x = 0$

Fig (2.2) The current slab of Fig (2.1) has been subjected to a tearing instability with $k_z = 0$, $k.B_o = k_y B_{oy} = 0$ at $x = 0$.

Fig (2.3) Plot of growth rate Λ_I versus the mismatch parameter δ for two different values of $C \sim \frac{\hat{\epsilon}_1}{\delta}$

Fig (2.4) Plot of growth rate Λ_I versus the mismatch parameter δ in the $\Lambda_I \ll \delta$ regime. The dotted curve is the analytical result, equation (2.34)

Fig (2.5) Plot of growth rate Λ_I versus δ the mismatch parameter in the $\Lambda_I \gg \delta$ regime. The dotted curve is the analytical result, equation (2.33)

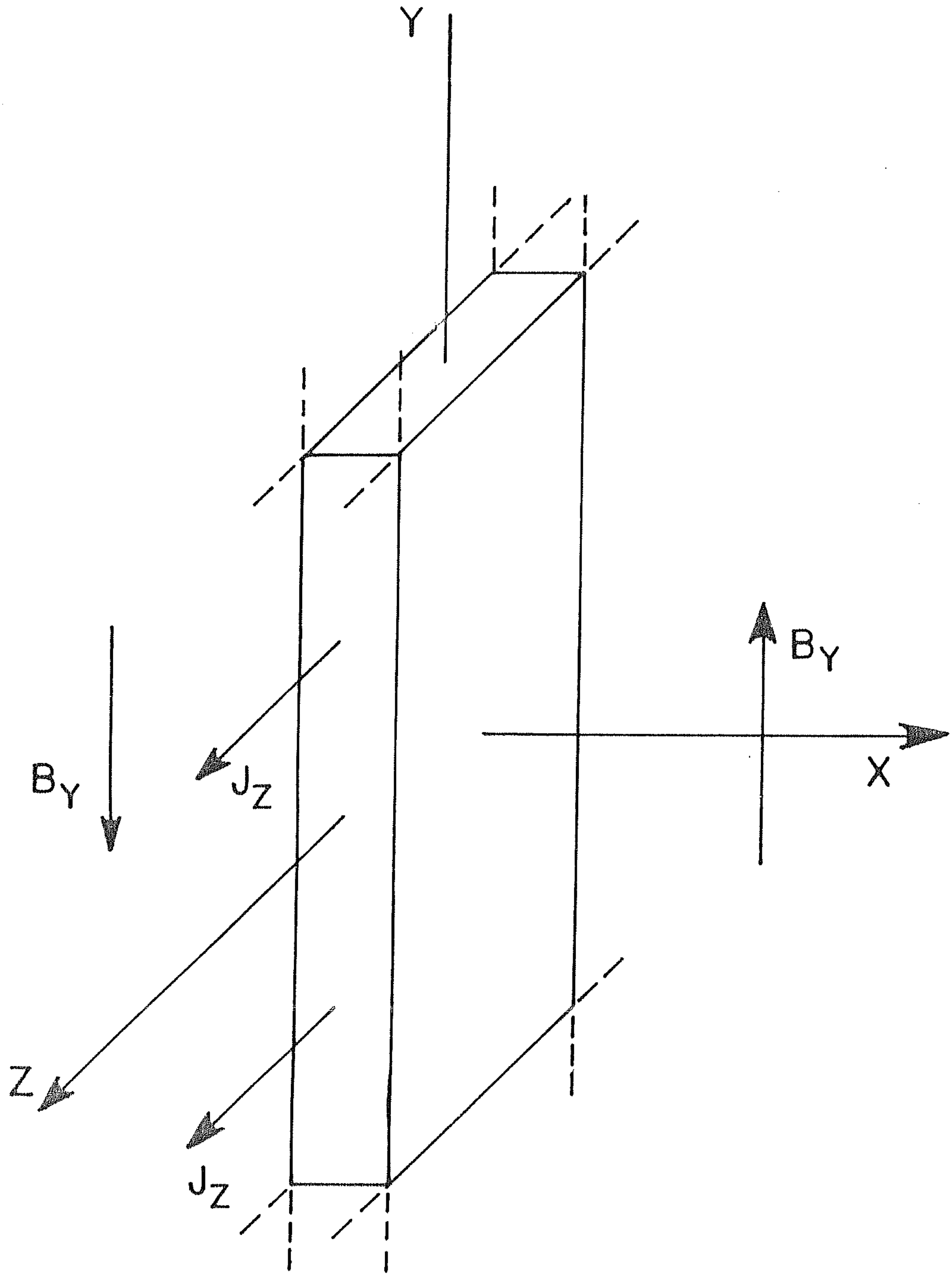

Fig. 2·1

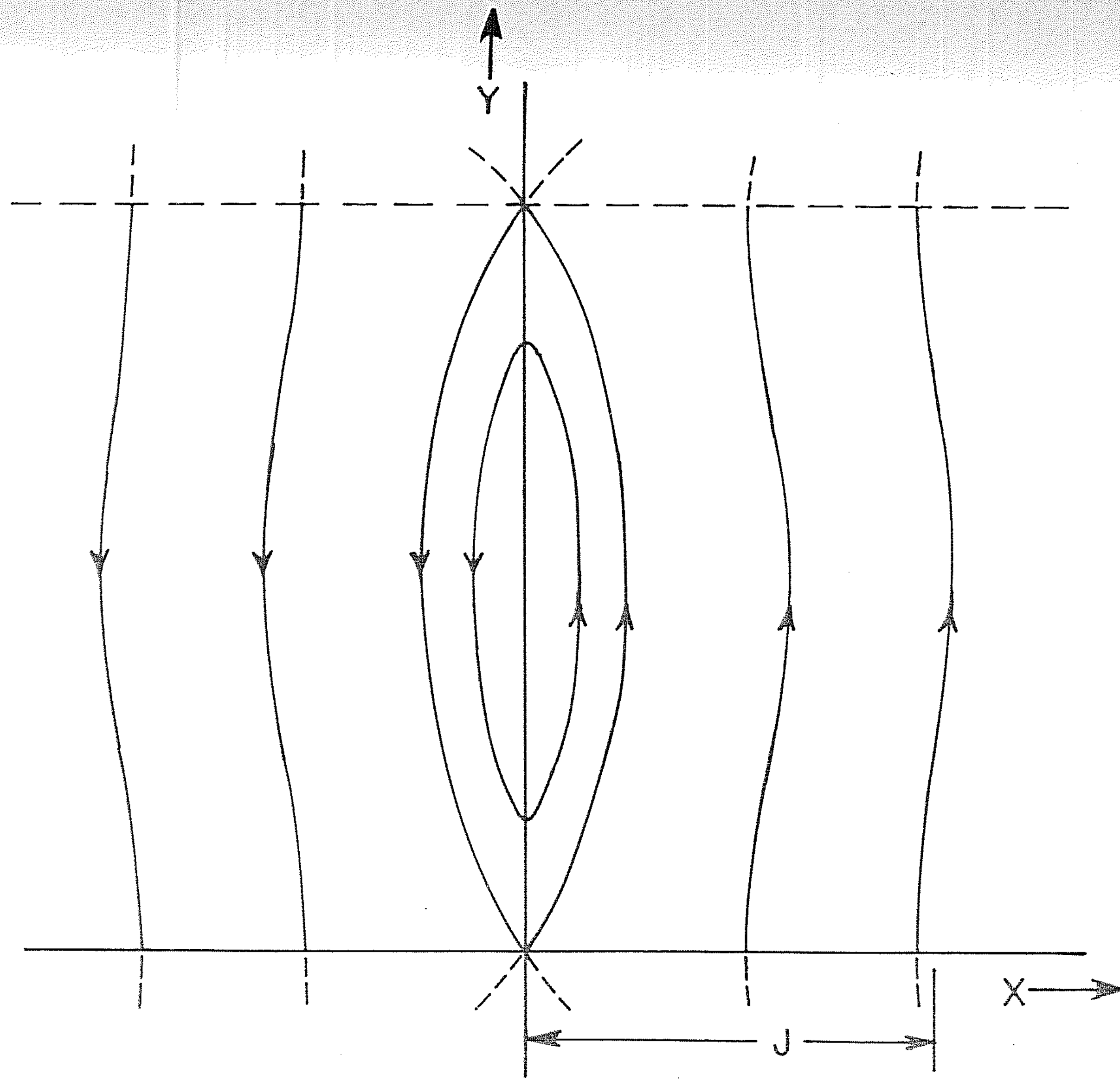

Fig. 2·2

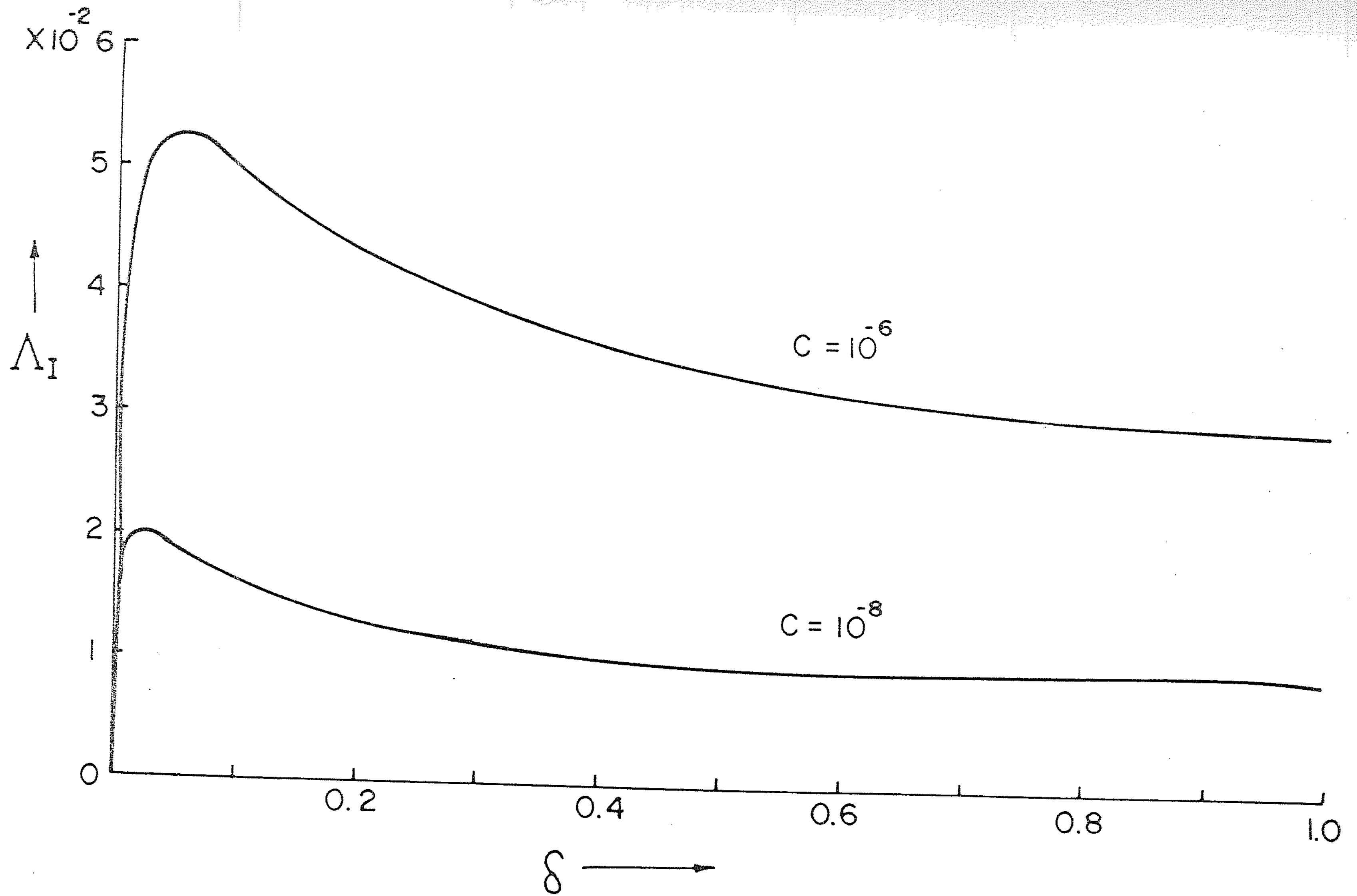

Fig. 2.3

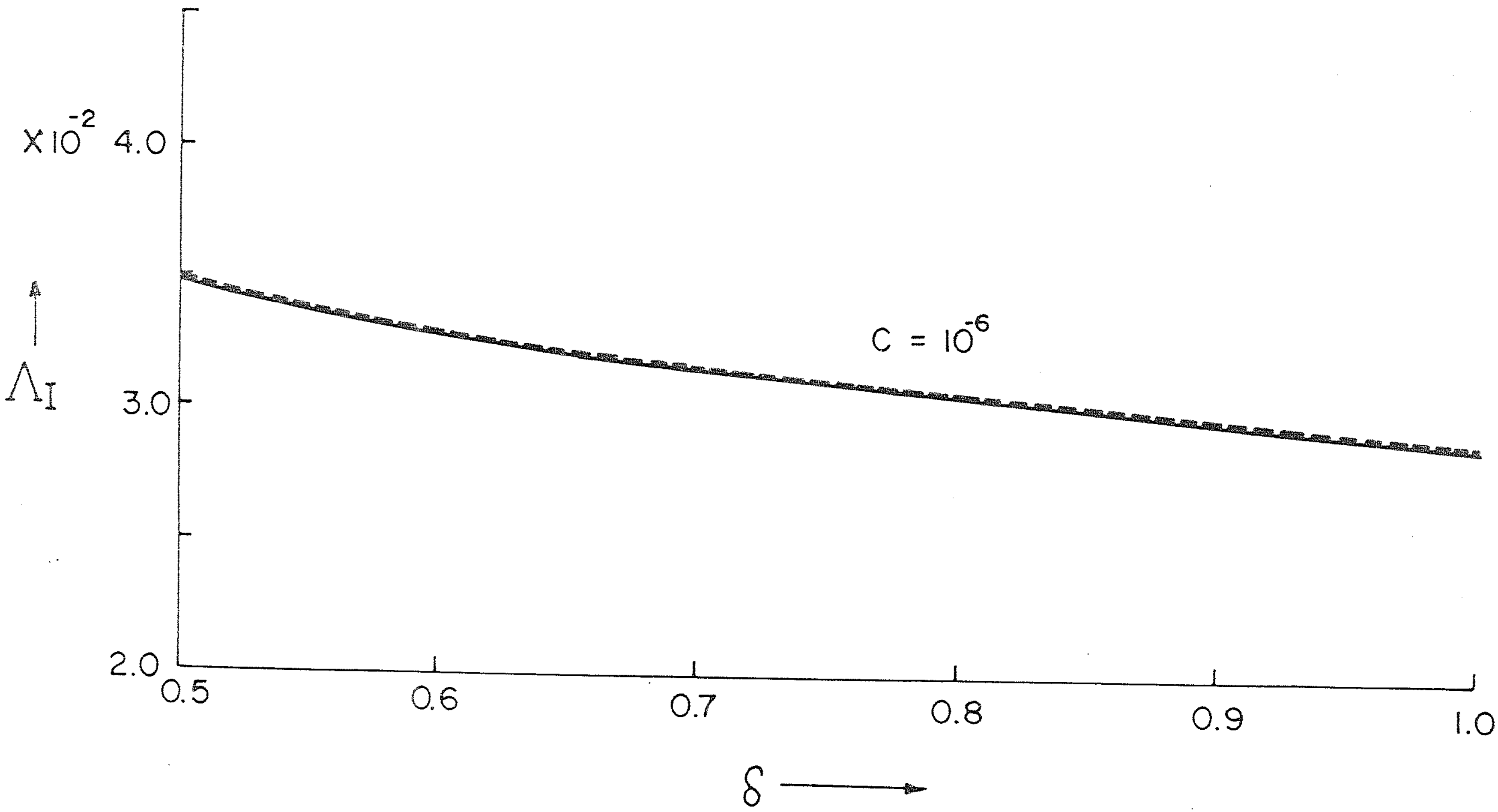

Fig. 2.4

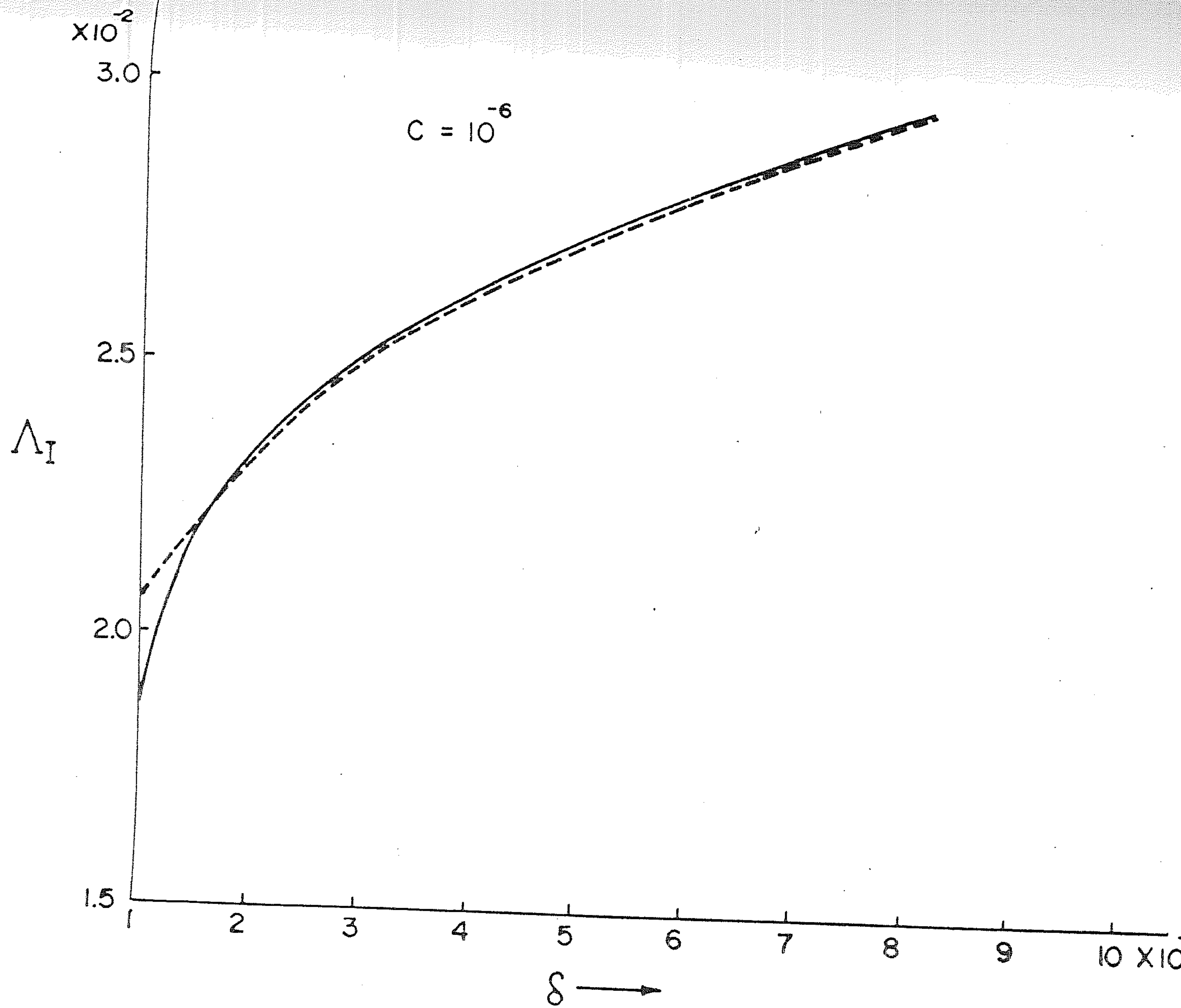

Fig. 2.5

CHAPTER III

PONDEROMOTIVE EFFECTS ON RESISTIVE TEARING MODES

3.1 Introduction :

In the last chapter, we had discussed a resonant interaction process by which an externally applied kinetic Alfven wave field could excite tearing modes with large growth rates in a tokamak plasma. This parametric decay process takes place when the frequency and wave vector matching conditions are satisfied between the kinetic Alfven waves and the tearing modes. However there are other non-linear effects which we have not taken into account in our calculation. These arise from the self interaction of the Alfven wave giving rise to a d.c. force, the so called ponderomotive force. The term responsible for the parametric interaction is usually referred to as the side-band or mode coupling term.

The ponderomotive force can give rise to two effects a) it can couple to the low frequency tearing mode perturbation and provide an additional driving term in the momentum equation, in the same manner as the mode coupling term. 2) it can induce equilibrium drifts. In this chapter we examine the stability of the tearing mode in the presence of these effects.

The equilibrium drifts have components in the axial, azimuthal and radial directions. Although the dominant drift is in the axial direction, we find that it is the radial drift which plays a significant role in the dynamics of the tearing mode. The radial drift couples to the gradient of the perturbed velocity of the tearing mode and generates large gradients in the momentum equation.

Our calculations show that the equilibrium flows play a more significant role in controlling the dynamics of the tearing mode than the convective driving force due to the ponderomotive term. Using a variational approach [1, 2] we find that a) the axial and azimuthal drifts Doppler shift the mode frequency. 2) the radial flow leads to steep gradients and drive the tearing mode unstable.

The present chapter has been organised as follows. In the next section, we derive the equilibrium flows experienced by the particles in the kinetic Alfven wave field. In section 3.3 the basic equations describing the tearing mode in the presence of the equilibrium drifts are obtained. Section 3.4 contains an analysis of the equations using a variational approach. The last section summarises and discusses the main results of the present chapter and compares them to earlier work [4, 5] done on the effect of d.c. flows on tearing modes.

3.2 Derivation of equilibrium flows :

In this section, we derive the equilibrium flows experienced by the particles in the fields of the kinetic Alfven waves. The drifts have components in the axial, azimuthal and radial directions. The salient features of the kinetic Alfven waves (namely longitudinal propagation and finite Larmor radius effects) can be simulated using the system of two fluid-Maxwell equations [6]

$$m_j n_0 \frac{d\vec{v}_j}{dt} = n_0 e_j \left[\vec{E} + \frac{\vec{v}_j \times \vec{B}_0}{c} \right] - \nabla p_j \qquad .. \ 3.1$$

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n \vec{v}_j) = 0 \qquad .. \ 3.2$$

$$\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{E}, \quad \nabla \times \vec{B} = \frac{4\pi \vec{J}}{c},$$

$$\nabla \cdot \vec{B} = 0 \qquad .. \ 3.3$$

Where the variables in equations (3.1) to (3.3) are as defined in chapter II. The quiver velocities of the particles (electrons and ions) obtained from the equation of motion (equation (3.1)), in the fields of the kinetic Alfven wave are given by the equation.

$$V_j = \frac{c(\vec{E} \times \vec{B})}{B_0^2} + \frac{c^2 m_j}{e_j B_0^2} \frac{d\vec{E}_\perp}{dt} \qquad \ldots 3.4$$

The first term is the crossed electric field drift common to both electrons and ions. The second term is the polarisation drift arising through the time dependent electric field. This term is typically proportional to $\frac{\omega_A}{\omega_{cj}}$ where ω_A and ω_{cj} are the Alfven and Larmor frequencies respectively. The ratio ω_A/ω_{ce} (for electrons) is one order of magnitude less than ω_A/ω_{ci} (for ions) and can be neglected.

The electric field drift in equation (3.4) has a non-linear contribution arising from the electric and magnetic fields of the wave itself i.e. $[\vec{E}_A(\omega_A) \times \vec{B}_A(\omega_A)]/B_0^2$. This second order drift has two components, one a d.c. component and the other at the higher harmonic of the Alfven wave fluctuation. In studying the problem of resonant interactions, we have retained only those contributions to the quiver velocities which were at the fundamental

Alfven frequency. The basic reason was that only these terms (at the fundamental Alfven frequency) couple with pump Alfven wave to produce beat waves of the tearing mode frequency and resonantly excite it. Here we study the second order contribution $(\vec{E}_A \times \vec{B}_A)/B_0^2$ to the quiver velocities and the implication of their effect on the tearing modes.

The tearing modes [7] evolve on the resistive time scale, which is many orders of magnetitude longer than the fast Alfvenic motions. Since we are interested in studying the evolution of the linear tearing instability in the presence of Alfven fluctuations, the fast Alfvenic wiggles in time can be smeared out. To achieve that equation(3.4) is averaged over the Alfvenic motion. The only finite contribution arises through the second order drift $(\vec{E}_A \times \vec{B}_A)/B_0^2$; so we notice that on a time scale longer than the Alfvenic time scale the particles execute certain average drifts. The electric fields are expressed in terms of perpendicular and parallel potentials as follows, $E_\perp = -\nabla_\perp \phi_A$, $E_\parallel = -\nabla_\parallel \psi_A$ (refer chapter II) . The magnetic field perturbations are related to the potentials ϕ_A and ψ_A (the subscript 'A' refers to the Alfven fluctuation) through the Maxwell's equations

$$\frac{\partial^2}{\partial r \partial z}(\phi_A - \psi_A) = \frac{1}{c}\frac{\partial B_\theta}{\partial t} \qquad \ldots 3.5$$

$$\frac{\partial^2}{\partial \theta \partial z}(\phi_A - \psi_A) = \frac{1}{c}\frac{\partial B_r}{\partial t} \qquad \ldots 3.6$$

We choose a simple cosine profile for the oscillating profiles as $\phi_A = \Phi_0 \cos(\vec{k}_A \cdot \vec{r} - \omega_A t)$ Substituting the values for electric and magnetic fields in the term $(\vec{E}_A \times \vec{B}_A)/B_0^2$ the axial drift V_a^{NL} has the form

$$\frac{c(E_{rA} B_{\theta A} - E_{\theta A} B_{rA})}{B_0^2} = \frac{\Phi_0^2}{B_0^2}\left(\frac{c k_z}{\omega}\right)(k_{rA}^2 + k_{\theta A}^2) \times$$

$$\sin^2(\vec{k}_A \cdot \vec{r} - \omega_A t) \qquad \ldots 3.7$$

the azimuthal drift V_θ^{NL}

$$\frac{(E_{zA} B_{rA})c}{B_0^2} = \frac{k_{zA}^2 k_{\theta A} c}{B_0^2 \omega_A} \psi_0 (\phi_0 - \psi_0) \sin^2(\vec{k}_A \cdot \vec{r} - \omega_A t) \qquad \ldots 3.8$$

the radial drift

$$-\frac{(E_{zA} B_{\theta A})c}{B_0^2} = \frac{k_{rA} k_{zA}^2 c}{\omega_A B_0^2} \psi_0 \Phi_0 \sin^2(k_A \cdot r - \omega_A t) \qquad \ldots 3.9$$

The relation between the parallel potential Ψ_A and perpendicular potential ϕ_A is given by the quasi-neutrality condition, $n_e^{(1)} = n_i^{(1)}$ (where $n_e^{(1)}$, & $n_i^{(1)}$ are the density perturbations of the electrons and ions respectively)

$$\Psi_A = -\bar{\lambda}_A \phi_A \qquad \ldots 3.10$$

where $\bar{\lambda}_A = k_{\perp A}^2 \rho_s^2$

The value of Ψ_A from equation (3.10) substituted in equations ((3.7) - (3.9)) and the averaging procedure is carried out over the fast Alfvenic motions;

$$\left(\frac{1}{T} \int_0^{2\pi/\omega_A} \alpha(t)\, dt \right)$$

The axial, azimuthal and radial drift respectively have the form

$$V_a^{NL} = \frac{\bar{\Phi}_0^2}{2} \bar{\lambda}_A \left(\frac{C_s}{V_A}\right) C_s \qquad \ldots 3.11$$

$$V_\theta^{NL} = \frac{\bar{\Phi}_0^2}{2} \bar{\lambda}_A (k_{\theta A}^2 \rho_s^2)\left(\frac{k_{zA}}{k_{\theta A}}\right)\left(\frac{C_s}{V_A}\right) C_s \qquad \ldots 3.12$$

$$V_r^{NL} = \frac{\bar{\Phi}_0^2}{2} \bar{\lambda}_A (k_{rA}^2 \rho_s^2)\left(\frac{k_{zA}}{k_{\theta A}}\right)\left(\frac{C_s}{V_A}\right) C_s \qquad \ldots 3.13$$

$$\left(\bar{\Phi}_0 = \frac{e\phi_0}{T_e} \right)$$

The factor $\frac{1}{2}$ in equations(3.11) 13 appears as a consequence of the averaging procedure. A brief examination of equations(3.11 - 3.13) reveals that the azimuthal and radial drifts are smaller than the axial drift by a factor$\left[(k_{\perp A}^{2}\rho_{s}^{2})\, k_{zA}/k_{\theta A}\right]$(For Alfven waves this factor is of the order of 0·1) Nevertheless it will be shown that the radial drift plays the most significant role in the evolution of the linear tearing mode.

The other important term due to the ponderomotive force is the non-linear convective derivative $\rho_t(\vec{V}\cdot\nabla)\vec{V}$ in the momentum equation for the tearning mode. The '$\vec{V}$' appearing in the expression is the quiver velocity of the particles having both perpendicular and parallel components and ρ_t the tearing mode density perturbation. The perpendicular component of '$\vec{V}$' consists of the electric field drift and the polarisation drift (equation (3.4)).

The parallel component can be obtained from the two fluid equations as follows:

For the lighter electrons, the balance between the electrostatic and pressure gradient forces in the equation of motion, gives rise to the density perturbation n_e^{ω} while the equation of continuity relates the parallel motion to n_e^{ω}. (the polarisation drift of the electrons is negligible).

$$\frac{\partial n_e^{(1)}}{\partial t} + n_0 \nabla_{\parallel} V_{\parallel e} = 0 \qquad \ldots 3.14$$

$$V_{\parallel e} = \frac{\omega}{k_{\parallel}} \frac{e\psi}{T_e} \qquad \ldots 3.15$$

For the heavier cold ions, the equation of motion provides the expression for $V_{\parallel i}$

$$m_i \frac{dv_{\parallel i}}{dt} = e E_{\parallel} \qquad \ldots 3.16$$

with $$V_{\parallel i} = \left(\frac{C_s}{V_A}\right) C_s \frac{e\psi}{T_e} \qquad \ldots 3.17$$

$C_s \ll V_A$ (the Alfven speed)

A comparison of (3.15) and (3.17) reveals that the parallel motion of electrons in much larger than that of ions by a factor $\left(\frac{V_A}{C_s}\right)^2$.

The perpendicular ponderomotive force for the ions and electrons is given by $\left\langle (\vec{V}_E + \vec{V}_{pj}) \cdot \nabla (\vec{V}_E - \vec{V}_{pj}) \right\rangle$. (Here the term $\vec{V}_p$ refers to the polarisation drift.) The averaging (as before) is performed over the fast Alfvenic motions. The phases of the electric field drift V_E and the polarisation drift are 90° apart. Hence $\langle (\vec{V} \cdot \nabla) V_{\perp} \rangle$ the $\perp^r$ component after the averaging process is done non-zero, while the parallel component of the ponderomotive force vasnishes.

Using the cosine profile for the pump Alfven wave, the perpendicular quiver velocities can be written as

$$V_{\perp A} = \frac{c(\hat{z} \times k_{\perp A}\phi_0)}{B_0} \sin(\vec{k}_A \cdot \vec{r} - \omega_A t)$$
$$- \frac{c m_j}{e_j B_0^2} \phi_0 \vec{k}_{\perp A} \cos(\vec{k}_A \cdot \vec{r} - \omega_A t) \qquad \ldots 3.18$$

where $\vec{k}_{\perp A} = \hat{r} k_{rA} + \hat{\theta} k_{\theta A}$

From equation(3.18) it is a short step to the expression $(\vec{V}.\nabla)V_{\perp j}$, $(F_{\perp j})$.

$$F_{\perp j} = \frac{m_j \omega_A}{e_j B_0^2} k_{\perp A}^2 \phi_0 \frac{(\hat{z} \times \vec{k}_{\perp A}\phi_0)}{B_0} \langle \cos^2(\vec{k}_A \cdot \vec{r} - \omega_A t) \rangle \quad \ldots 3.19$$

which can be further simplified to

$$F_{\perp j} = -\frac{\bar{\Phi}_0^2}{2} \bar{\lambda}_A \frac{\omega_A}{\omega_{cj}} c_s^2 (\hat{z} \times \vec{k}_{\perp A})$$

where $\left(\bar{\Phi}_0 = \frac{e\phi_0}{T_e}\right) \qquad \ldots 3.20$

Though the parallel component of $\langle(\vec{V}\cdot\nabla)V_{\parallel}\rangle$ produced by a single Alfven wave vanishes, the parallel ponderomotive force arising due to the interaction of two kinetic Alfven waves at slightly different wave numbers has a finite value. In chapter VI we we shall investigate the effect of this parallel force on the tearing modes.

3.3 Derivation of tearing mode equations with equilibrium drifts

We now go over to the study of the evolution of the linear tearing mode in the presence of equilibrium drifts and equilibrium ponderomotive force discussed in the earlier section.

The resistive tearing modes are modelled by a system of fluid - Maxwell equations,

$$\rho_0 \frac{\partial \vec{V}_t}{\partial t} + \rho_0 (\overset{NL}{\vec{V}} \cdot \nabla) \vec{V}_t + \rho_t \langle (\vec{V} \cdot \nabla) \vec{V} \rangle = \frac{\vec{J}_t \times \vec{B}_0}{c} - \nabla p \qquad \ldots 3.21$$

$$\frac{\partial \rho_t}{\partial t} + (\overset{NL}{V} \cdot \nabla) \rho_t + (\vec{V}_t \cdot \nabla) \rho_0 = 0 \qquad \ldots 3.22$$

$$E_t + \frac{(\overset{NL}{V} \times \vec{B}_t)}{c} + \frac{(\vec{V}_t \times \vec{B}_0)}{c} = \eta \vec{J}_t - \frac{\nabla p_e}{e n_0} \qquad \ldots 3.23$$

$$\frac{1}{c} \frac{\partial \vec{B}_t}{\partial t} = - \nabla \times \vec{E}_t \, , \; \nabla \times \vec{B}_t = \frac{4\pi \vec{J}_t}{c}$$

$$\nabla \cdot \vec{B} = 0 \, , \; \nabla \cdot \vec{V}_t = 0 \qquad \ldots 3.24$$

The term $\overset{NL}{V}$ and $\rho_t\langle(\vec{v}\cdot\nabla)\vec{v}\rangle$ refer to the average d.c. drifts and ponderomotive force. In the equation (3.21) the axial and azimuthal drifts give rise to terms proportional to V_t, thus Doppler shifting the mode frequency to $(\omega_t - k_\theta \overset{NL}{V_\theta} - k_z \overset{NL}{V_a})$ The radial drift $\overset{NL}{V_r}$ couples to the derivative $\frac{dV_t}{dr}$, while the ponderomotive force couples to the density perturbation. As in chapter II the perturbed magnetic field and the velocity are expressed in terms of potentials $A_{\parallel}$ and ϕ, as $B_t = \nabla \times \hat{b} A_{\parallel}$, $V_t = \frac{c(\hat{b}\times\nabla\phi)}{B_0}$ respectively. To study the tearing mode dynamics, we take the $\hat{b}$ component of the curl of equation (3.21), the equation of continuity and the parallel component of Ohm's law (3.23). The electric field and current perturbations are related to the vector $A_{\parallel}$ and scalar ϕ potentials of Maxwell's equations. The system of equations (3.21) to (3.23) are finally expressed in terms of the three variables, $A_{\parallel}$, ϕ and ρ_t. Expanding $k_{\parallel}$ about r_s as $k_{\parallel} = k_{\parallel}'(r - r_s)$, where r_s is the co-ordinate of the mode rational surface, and defining $x = \frac{(r - r_s)}{r_s}$ (a dimensionless quantity) the three coupled equations can be reduced to

$$\frac{\Omega}{r_s^2}\frac{d^2\phi}{dx^2} - \frac{\overset{NL}{V_r}}{i r_s^3}\frac{d^3\phi}{dx^3} + \frac{iB_0 F_\theta}{\rho_0 c r_s}\frac{d\rho_t}{dx}$$

$$= \frac{x}{r_s} c k_{\parallel}' V_A^2 \frac{d^2 A_{\parallel}}{dx^2} \qquad \ldots 3.25$$

$$-i k_{\parallel}' x c r_s \phi + i A_{\parallel}(\Omega - \omega_{*e}) - \frac{\overset{NL}{V_r}}{r_s}\frac{dA_{\parallel}}{dx}$$

$$= \frac{\eta c}{4\pi r_s^2}\frac{d^2 A_{\parallel}}{dx^2} \qquad \ldots 3.26$$

$$i\Omega\rho_t + \frac{\overset{NL}{V_r}}{r_s}\frac{d\rho_t}{dx} + \frac{ie\phi\rho_0\omega_{*e}}{T_e} = 0 \qquad \ldots 3.27$$

Where $\Omega = (\omega - k_z \overset{NL}{V_a} - k_\theta \overset{NL}{V_\theta})$ the Doppler shifted frequency. F_θ is the azimuthal component of the ponderomotive force (equation (3.20)) and ω_{*e} the diamagnetic drift frequency $(= -\frac{k_{\theta t} T_e}{eB_0^2}\frac{c}{L_n}$, where L_n = scale length of the inhomogeneity).

An important feature to note is the presence of a third derivative in the equation of motion due to the radial drift. Since the tearing modes are driven by steep gradients near the mode rational surface [8] we anticipate that the cubic derivative could significantly govern the dynamics of the tearing modes. A qualitative study of the equations at this juncture yields interesting results. The density perturbations are unimportant for the tearing mode dynamics and for the present qualitative analysis can be ignored. The coupled equations can be cast into dimensionless form as follows:

$$\overset{(1)}{\frac{d^2\phi}{dx^2}} - \overset{(2)}{\frac{V_R^{NL}}{\Lambda}\frac{d^3\phi}{dx^3}} = \overset{(3)}{\frac{x}{\Lambda^2}\frac{d^2\psi}{dx^2}} \qquad \ldots\ 3.28$$

$$\overset{(4)}{x\phi} - \overset{(5)}{\psi} = \overset{(6)}{\bar{\eta}/\Lambda\,\frac{d^2\psi}{dx^2}} \qquad \ldots\ 3.29$$

where $\Lambda = \dfrac{\omega_t}{k_{\parallel}' V_A r_s}$, $V_R^{NL} = \dfrac{V_r^{NL}}{i k_{\parallel}' V_A r_s^2}$, $\bar{\eta} = \dfrac{\eta c^2}{(4\pi i k_{\parallel}' V_A r_s^3)}$

$$\psi = \frac{\omega_t A_{\parallel}}{k_{\parallel}' r_s c}, \quad x = \frac{(r - r_s)}{r_s}$$

For the m = 2 mode one uses the nominal scaling $\frac{d^2\phi}{dx^2} \sim \frac{\phi}{x^2}$, $\frac{d\psi}{dx^2} = \frac{\Delta'\psi}{x}$ where 'x' is to be identified with the width of the tearing layer. By identifying the terms which contribute to the various physical processes the scaling for Λ can be obtained. The classical growth rates can be obtained in the following manner. For the m = 1 mode, a balance between (1) and (3) (with $\frac{d^2\psi}{dx^2} \sim \frac{\psi}{x^2}$) leads to an expression for 'x' as $x \sim \Lambda$. Physically Term (3) arises from the $\vec{J}_t \times \vec{B}_0$ force, while (1) arises from the fluid inertia. A balance between them can be interpreted as the $\vec{J}_t \times \vec{B}_0$ forces working against the inertial forces and causing a vortex flow. Balancing (4) versus (5) and (5) versus (6), gives a second relation between x and Λ $\left(x^2 = \frac{\eta}{\Lambda}\right)$. Eliminating x between the two relations leads to the scaling for Λ as $\Lambda \sim (\bar{\eta})^{1/3}$. The scaling for m = 2 mode, i.e. $\Lambda \sim (\Delta')^{4/5} (\bar{\eta})^{3/5}$ can be obtained using similar dimensional arugments.

In the presence of equilibrium flows, the equation of motion has an additional third derivative (term (2)). For the m = 2 mode balancing (1), (2) versus (3), we obtain,

$$\frac{1}{x^3}\left(1 - \frac{\overset{NL}{V_R}}{\Lambda x}\right) \sim \frac{\Delta'}{\Lambda^2} \qquad \ldots 3.30$$

The other relation is obtained from the second equation

$$x \sim \bar{\eta}\frac{\Delta'}{\Lambda} \qquad \ldots 3.31$$

elimnating 'x' between (3.30) and (3.31) we get

$$\Lambda^5 \left[1 - \frac{\overset{NL}{V_r}}{i V_A (k_{\parallel}' r_s^2) \bar{\eta}\, \Delta'}\right] = (\bar{\eta})^3 \Delta'^{4/5} \qquad \ldots 3.32$$

In the event of $\overset{NL}{V_r} \to 0$, the classical growth rate for the m = 2 is recovered. Note that $\overset{NL}{V_r}$ is a positive quantity.

Substituting the expression for $\bar{\eta}$,

$$\bar{\eta} = \eta c^2 / 4\pi i k_{\parallel}' v_A r_s^3 \qquad \ldots\ 3.33$$

for values of $\overset{NL}{V_r}$ less than $\frac{\Delta' \eta c^2}{4\pi r_s}$, the classical growth rate is either enhanced or reduced depending on the direction of the radial drift. In the opposite limit, equation (3.33) suggests the existence of a new class of instabilities with typical growth rates scaling as

$$\Lambda_i \sim \Delta' |(\bar{\eta})|^{4/5} |\overset{NL}{V_R}|^{-1/5} \qquad \ldots\ 3.34$$

Physically the instability arises because the torque produced by the linear $\vec{J}_t \times \vec{B}_0$ forces (term (3) in equation (3.28)) along the direction of the equilibrium magnetic field drives the convective flow of the fluid, represented by term (2) in equation (3.28), against the fluid inertia. In the next section using variational solutions, we obtain the scaling given in equation (3.34) in a more rigourous fashion.

3.4 Variational solutions

Having obtained the scaling for Λ on purely dimensional arguments, we proceed to verify it and solve the coupled equations in a more detailed and systematic manner. To achieve this, we take the

Fourier transform of the system of equations (3.25 - 3.27)

$$-\frac{\Omega}{r_s^2} k^2 \phi_k + \frac{\overset{NL}{V_Y}}{r_s^3}(k^3 \phi_k) + \frac{i B_0}{\rho_0} \frac{F_\theta}{c r_s}(i k \rho_k)$$

$$= \frac{i k_{\parallel}' v_A c}{r_s} \frac{d}{dk}(k^2 A_k) \qquad \ldots \; 3.35$$

$$-i \frac{d\phi_k}{dk} - (\Omega - \omega_{*e}) \frac{A_k}{c k_{\parallel}' r_s} + \frac{\overset{NL}{V_Y} k A_k}{k_{\parallel}' r_s^2}$$

$$= - \frac{\eta_c (k^2 A_k)}{4\pi r_s^2} \qquad \ldots \; 3.36$$

$$\Omega \rho_k + \frac{\overset{NL}{V_Y}}{r_s}(k \rho_k) - \frac{e\phi_k}{T_e} \rho_0 \omega_{*e} \qquad \ldots \; 3.37$$

Where the Fourier transformed variable is defined as

$$\phi_k = \int_{-\infty}^{\infty} \phi(x) e^{ikx} dx$$

Eliminating ϕ_k, ρ_k between the equations (3.35 - 3.37) and defining a variable, $J_k = k^2 \psi_k$ a second order differential equation in 'k' space is obtained.

$$\frac{d}{dk}\left[\frac{dJ_k}{dk} \frac{1}{(\Lambda k^2)} \frac{(\Lambda - k \overset{NL}{V_R})}{[(\Lambda - k \overset{NL}{V_R})^2 + \epsilon_p \Lambda_* k]}\right] - J_k \left[\frac{1}{k^2} - \frac{\overset{NL}{V_R}}{\Lambda k} - \frac{\eta}{\Lambda}\right] = 0$$

$$\ldots \; 3.38$$

We use the variational technique to solve the above equation and obtain the expression for the growth rate. In equation (3.38) ϵ_p is given by $\frac{F_\theta \omega_{ci}}{k_\parallel' C_s^2 V_A}$ and Λ_* is $\omega_{*e}/(k_\parallel' V_A \tau_s)$. For typical tokamak parameters mentioned in chapter II, ϵ_p is estimated to be much less than unity. On comparing the terms in the denominator of the expression in the parenthesis (of equation (3.38)), we find that the term proportional to ϵ_p becomes important only in the region of 'k' space such that $k \ll \epsilon_p \frac{\Lambda_*}{\Lambda}$ (i.e. $k \ll 1$, as $\Lambda_* \sim \Lambda$)

This region corresponds to the outer layer, where the tearing mode exhibits an ideal kink behaviour. The dynamics of the mode in this region are dominated by the equilibrium current profiles and dissipative effects are negligible.

It has been shown that field annihilation occurs only through convective effects, hence both non-linear source terms and classical resistive terms can be ignored.

Hence we conclude that the perpendicular ponderomotive force, does not play a significant role in the evolution of the tearing mode, and we shall therefore henceforth exclude this term from our analysis.

The prescription for obtaining eigen values of differential equation of type given in equation(3.38), has been outlined in the earlier chapter. Following the method, we choose a simple Gaussian trial function ($e^{-\alpha k^2}$) and obtain the expression for the functional 'S'.

$$S = \frac{\Lambda}{V_R^{NL}} \frac{\alpha^2}{\sqrt{\pi}} Z\left(\frac{\Lambda\sqrt{\alpha}}{V_R^{NL}}\right) - 2\Lambda^2\sqrt{\alpha}$$

$$- \bar{\eta}\Lambda/\sqrt{\alpha} - \frac{\Lambda^2}{\Delta'} \qquad \text{... 3.39}$$

where 'Z' is the plasma dispersion function. Making a small argument expansion of the Z function in the above equation the expression for the functional is obtained as

$$\frac{S}{\sqrt{\pi}} = \frac{i\Lambda\alpha^2}{V_R^{NL}} - 2\Lambda^2\sqrt{\alpha} - \bar{\eta}\Lambda\sqrt{\alpha}$$

$$- \frac{\Lambda^2}{\Delta'} \qquad \text{.. 3.40}$$

The condition for the small argument expansion to be valid is that $\left(\Lambda\sqrt{\alpha} / V_R^{NL}\right) << 1$ or $\Lambda << \frac{V_R^{NL}}{\sqrt{\alpha}}$ This condition imposes a constraint on the pump amplitude which is readily satisfied for typical tokamak parameters and realistic levels of Alfven wave intensities.

Solving S = O and $\frac{ds}{d\alpha} = 0$ simultaneously provides the dispersion relation. The equations are

$$\frac{i\Lambda\alpha^2}{\overset{NL}{V_R}} - \bar{\eta}\Lambda/\sqrt{\alpha} = \frac{\Lambda^2}{\Delta'} \qquad \ldots \; 3.41$$

$$-2\frac{\Lambda\alpha i}{\overset{NL}{V_R}} + \bar{\eta}\frac{\Lambda}{2}\alpha^{-3/2} = 0 \qquad \ldots \; 3.42$$

Solving equations (3.41) and (3.42) simultaneously, the scaling for the growth rate Λ_i is readily obtained as

$$\Lambda_i = \Delta' \frac{5}{4^{4/5}} |\bar{\eta}|^{4/5} |\overset{NL}{V_R}|^{-1/5} \qquad \ldots \; 3.43$$

$$\Lambda = i\,\Delta'\, 5/4^{4/5}\, |\bar{\eta}|^{4/5} (\overset{NL}{V_R})^{-1/5}$$

This is the same scaling as that obtained in equation (3.34) through physical arguments. This again is a purely driven mode. In obtaining the scaling we had made a small argument expansion of the 'Z' function in equation (3.39) and consequently the classical contribution had been neglected. This is equivalent to neglecting the $\frac{d^2\phi}{dx^2}$ term as compared with $\frac{\overset{NL}{V_R}}{\Lambda}\frac{d^3\phi}{dx^3}$ in the equation of motion. This instability is the consequence of the balance between the forces and the convective flow arising due to the

Alfven fluctuations. The growth rate is proportional only to the equilibrium radial drift, (given by equation (3.43)) while the real part of the frequency has contributions from the axial and azimuthal drifts. Substituting the expression for the radial drift into equation (3.43), it can be seen that the growth rate scales as fractional powers of the Alfven pump amplitude.

3.5 Discussion :

In this chapter we have investigated a non-resonant interaction of kinetic Alfven waves with tearing modes in which the d.c.ponderomotive force couples non-linearly to the tearing mode perturbations. This force gives rise to two effects. One is a convective term which couples to the density perturbation of the tearing mode in the momentum equation and the other is the equilibrium flow. Of the two, the dominant effect is that produced by the equilibrium flows. The axial and azimuthal components of the equilibrium drifts Doppler shift the mode frequency while the radial drift gives rise to a third derivative in the momentum equation. This fact is significant as the tearing modes are driven by steep gradients around the mode rational surface. The variational method was used to analyse the tearing mode equations. It was found that weakly growing tearing modes with their growth rates (given by equation (3.43)) proportional to fractional powers of the radial drift could be excited.

The effect of equilibrium flows on the stability of linear tearing modes has been investigated by several authors in other contexts. Dobrott et al [3] found that the effect of 'natural diffusion' on the tearing mode was to have a stabilising influence, while a velocity in the reverse direction could be destabilising. Pollard and Taylor [4] examined the effect of arbitrary equilibrium flow on the stability of the tearing mode. They found weakly growing modes with growth rates proportional to the radial velocity. Recently Bondeson et al have presented a numerical and analytical study of the stability problem of tearing modes in the presence of equilibrium flows. In the weakly unstable regime they find an instability with growth rate proportional to fractional power of the perpendicular flow. This result agrees with earlier investigation of Pollard and Taylor [4] .

In the present work using a fluid model we have shown that the radial flows induce tearing instabilities with growth rates proportional to fractional powers of the radial flow. The results of our analysis are in agreement with the scaling obtained for weak tearing instabilities by Pollard-Taylor, Bondeson. In their investigations however the equilibrium flows are of an arbitrary nature. In the present analysis the d.c. drifts are generated by the kinetic Alfven wave fields, and the growth rates are therefore proportional to fractional powers of the Alfven pump amplitude. These driven instabilities induced by the radial flow however grow on a much larger time scale (for typical tokamak parameters given in section 2.5) than the resonantly excited tearing modes (of chapter II). Both of these non-linear processes are equally possible phenomena and could take place simultaneously in a tokamak plasma. The resonant parametric decay takes place under rather special conditions when the frequency matching conditions are satisfied. The second effect which is due to the ponderomotive force generated by the kinetic Alfven wave could take place under more general conditions and is a more likely feature. Tearing instabilities induced by these non-resonant effects are hazardous for plasma confinement and could eventually lead to transport of plasma particles.

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CHAPTER IV

BEAT WAVE EXCITATION OF TEARING MODES

4.1 Introduction :

The non-linear excitation of oscillations in a plasma at the beat frequency of two high frequency electromagnetic waves is of considerable interest in plasma physics on account of its various applications. It can be used as a density probe for plasma diagnostics [1] and for ionospheric sounding [2]. Recently it has been proposed as a basis for high energy particle accelerators [3, 4] and also for current drive in magnetically confined plasmas [5, 6]. Another important application of the beat wave mechanism is as a possibility of heating the plasma [7, 8, 9]. The method consists in the excitation of longitudinal plasma waves by resonance with the difference frequency of a set of transverse waves.

We have studied the beat wave interaction in the following context. In Alfven wave heating scheme in laboratory plasmas the antennae excite several waves at the same time. (e.g. the antennae on the TCA tokamak were phased to excite modes with $n = \pm 2, \pm 6 \ldots$ and $m = \pm 1, \pm 3 \ldots$ [10]). These waves have a single frequency, (or a small spread in frequencies) and a range of mode numbers (i.e. different poloidal and toroidal mode numbers.). This gives rise to the possibility of two such excited kinetic Alfven waves, with slightly different frequencies and mode numbers to interact with each other. Such an interaction or mixing as it is called, will result in the excitation of waves at the sum and difference frequencies, due to the presence of non-linear terms which are present in the governing equations. If one of the beat waves corresponds to a natural mode of the plasma, then such an oscillation will be strongly enhanced. We consider two kinetic Alfven waves with slightly different frequencies ω_1, ω_2 and wave vectors $\vec{k}_1$ and $\vec{k}_2$, interacting to produce beat waves ($\omega_1 - \omega_2 = \omega_t$ and $\vec{k}_1 - \vec{k}_2 = \vec{k}_t$) at the tearing mode frequency ($\omega_t \vec{k}_t$) and resonantly exciting it.

The beat wave interaction between kinetic Alfven waves differs from the earlier parametric interaction with resistive tearing modes (investigated in chapter II) in the following manner. For the parametric interaction the non-linear coupling terms were proportional to the tearing mode perturbation. In the beat wave mechanism, the beat waves produced by the interacting kinetic Alfven waves are independent of the tearing mode perturbations. Thus they act as external drivers, exciting the system at its natural tearing frequency.

To describe the evolution of the tearing mode, the single fluid equations are used. The basic equations are the Ohm's law and the momentum transfer equation. By appropriate manipulation of the two equations, an inhomogeneous third order differential equation in the magnetic vector potential is obtained. The equation has the form of a driven harmonic oscillator. The driving terms which arise due to the non-linear forces generated by the beat wave interaction are in resonance with the tearing mode frequency, but are external to the system. On account of the fact that the differential equation has an inhomogeneous form, the method we employ to obtain the solutions differs from the earlier methods adopted in chapters II and III.

The corresponding homogeneous cubic equation for the symmetric tearing mode has been studied earlier by Furth et al [11] and more recently by Paris [12] . Paris obtained the solutions of the cubic equation of the symmetric tearing mode in the long wave length limit. He obtained analytical solutions for the normalised magnetic field and velocity perturbations within the boundary layer in the form of rapidly convergent series involving hypergeometric functions.

Our motive is to study the effect of the driver waves, i.e. the kinetic Alfven waves on the evolution of the plane symmetric tearing mode represented by the inhomogeneous third order differential equation. We have presented an alternative method of obtaining solutions of the cubic equation, in terms of certain convenient set of basis functions namely Hermite polynomials. We find that the solutions are very sensitive to the parity of the driver Alfven waves.

In order to obtain a global solution, the logarithmic derivative Δ' across the boundary layer is calculated for the symmetric tearing mode and matched to the outer infinite conduction regions. It is found that for certain values of parameters, the classical growth rates for the symmetric tearing modes are enhanced due to the presence of the external non-linear forces.

The chapter is organised in the following manner. In the next section the principal equations describing the beat wave interaction are obtained. In section 4.3, the solutions to the basic inhomogeneous differential equation are derived and discussed. Section 4.4 contains a summary of the work and the principal results.

4.2 Basic equations of the non-linear process :

In this section, the principal equations which describe the excitation of tearing modes due to resonant interaction with beat waves produced by two kinetic Alfven waves are derived and discussed.

We consider a cylindrical plasma with the equilibrium magnetic field given by $\vec{B}_0 = [B_z\hat{e}_z + B_\theta(r)\hat{e}_\theta]$ Two pump kinetic Alfven waves with oscillating profiles having the form

$$\phi_{1,2} = \phi_0(r) \exp i[m_{1,2}\theta + k_{1,2}z - \omega_{1,2}t] \quad \ldots 4.1$$

are assumed to be propagating in this equilibrium. The frequencies and wave numbers of the kinetic Alfven waves are such that $\omega_1 - \omega_2 = \omega_t$, $k_1 - k_2 = k_t$ The subscripts 1, 2 and 't' refer to the interacting Alfven modes and tearing modes respectively. (ω_1, k_1) and $(\omega_2, \vec{k}_2)$ satisfy the kinetic Alfven wave dispersion relation.

$$\frac{\omega^2}{k_\parallel^2 V_A^2} = (1 + k_\perp^2 \rho_s^2) \quad \ldots 4.2$$

The kinetic Alfven waves at frequencies ω_1, ω_2, interact among themselves giving rise to beat waves. These arise due to the presence of non-linear terms governing the response of the plasma to a given perturbation. They are at the sum and difference frequencies and wave vectors of the interacting waves i.e. $\omega_1 \pm \omega_2$, $\vec{k}_1 \pm \vec{k}_2$.

The evolution of the tearing modes are described by the following system of fluid - Maxwell equations :

$$\rho\left[\frac{\partial}{\partial t} + \vec{V}\cdot\nabla\right]\vec{V} = \frac{\vec{J}\times\vec{B}}{c} - \nabla P , \qquad \ldots 4.3$$

$$\vec{E} + \frac{\vec{V}\times\vec{B}}{c} = \eta\vec{J} , \qquad \ldots 4.4$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0 , \qquad \ldots 4.5$$

$$-\frac{1}{c}\frac{\partial\vec{B}}{\partial t} = \nabla\times\vec{E} , \qquad \ldots 4.6$$

$$\nabla\times\vec{B} = \frac{4\pi\vec{J}}{c} , \qquad \ldots 4.7$$

$$\nabla\cdot\vec{B} = 0 \qquad \ldots 4.8$$

Linearising the above system of equations and eliminating the electric field perturbations from (3.4), (3.6), and (3.8) we obtain

$$\frac{\partial \vec{B}_t}{\partial t} = \nabla \times (\vec{V}_t \times \vec{B}_0) + \nabla \times (\vec{V}_A \times \vec{B}_A) + \frac{\eta c^2}{4\pi} \nabla^2 \vec{B}_t \qquad \ldots 4.9$$

While the current perturbation is eliminated between (3.3) to (3.8), to give

$$\rho_0 \left[\frac{\partial \vec{V}_t}{\partial t} + (\vec{V}_A \cdot \nabla) \vec{V}_A \right] = \frac{\vec{J}_A \times \vec{B}_A}{c} + \frac{\vec{J}_0 \times \vec{B}_t}{c} + \frac{\vec{J}_t \times \vec{B}_0}{c} - \nabla p . \qquad \ldots 4.10$$

The equation of continuity is replaced by the incompressibility condition

$$\rho_0 \nabla \cdot \vec{V}_t + \nabla \cdot (\rho_A \vec{V}_A) = 0 \qquad \ldots 4.11$$

Equations (4.9) to (4.11), form the basic set of equations describing the dynamics of the resistive tearing mode. In the above system of equations, the non-linear terms are $(\vec{V}_A \cdot \nabla)\vec{V}_A$, $\vec{J}_A \times \vec{B}_A$ in the momentum equation and $(\vec{V}_A \times \vec{B}_A)$ in Ohm's law. (The subscripts 'A' refer to the interacting Alfven waves.) These can be identified as ponderomotive and electromagnetic forces generated by the coupling between two kinetic Alfven waves, and are independent of the tearing mode perturbation. These non-linear forces are at the sum and difference of the Alfven wave frequencies. We have assumed the frequencies of the interacting waves to be such that the difference frequency $(\omega_1 - \omega_2)$ is equal to the tearing mode frequency while the sum of the frequencies is much larger than ω_t.

We retain only those non-linear terms which produce beat waves of the tearing mode frequency, since the tearing mode perturbation can be resonantly excited only when the oscillations of the external force is in phase with tearing frequency ω_t. We neglect the beat waves at the frequency $(\omega_1 + \omega_2)$, since they are considered as off-resonant.

To describe the kinetic Alfven waves, we use the two fluid equations. The Larmor radius effects and longitudinal propagation can be described in this formalism (chapter II).

The electrons in the field of the Alfven waves, on account of their massless nature move rapidly along the magnetic field lines and attain Boltzmann distribution, given by equation (2.2). For the same reason the dominant drift excuted by them is the electric field drift. The inertial drift which is proportional to the electron mass can be neglected.

The dynamics of the heavier ions are predominantly in the perpendicular plane, and the density perturbation which arises mainly due to the gyro motion, is given by equation (2.2a). Although the dominant motion for the ions is the electric field drift, the finite inertia makes an important contribution giving rise to the polarisation drift (equation (3.4)).

The variables $\vec{V}_A$, $\vec{B}_A$ and ρ_A in equations (4.9)to(4.11) refer to the fluid velocity, magnetic field and density perturbation respectively, of the kinetic Alfven waves. These fluid variables are perturbations defined in the center of mass frame and are given by

$$\vec{V}_A = \frac{\vec{V}_i m_i + \vec{V}_e m_e}{m_i + m_e} \qquad \ldots 4.12$$

$$\rho_A = \frac{m_e n_e + m_i n_i}{m_i + n_i} \qquad \ldots 4.13$$

Since the Alfven wave is a low frequency mode ($\omega_A < \omega_{ci}$), implying that quasineutrality condition is satisfied ($n_i = n_e$), the principal contribution to the fluid perturbations arise through the ion dynamics. The contributions from the electron motion, on account of their negligible mass are very small.

The electric and magnetic field perturbations are expressed in terms of potentials ϕ and ψ as

$$E_{\perp A} = -\nabla_\perp \phi_A \,, \quad B_A = (\hat{z} \times \nabla \psi_A) \qquad \ldots 4.14$$

Using equation (4.14) in equations (4.12) and (4.13), the expressions for fluid velocity and density perturbations, in the fields of the Alfven waves can be reduced to

$$\vec{V}_{\perp A} = \frac{c}{B_0}(\hat{z} \times \nabla\phi_A) + \frac{i\omega_A}{\omega_{ci}} \frac{c}{B_0} \nabla_\perp \phi_A \qquad \ldots 4.14a$$

$$\frac{\rho_A}{\rho_0} = \rho_s^2 \nabla_\perp^2 \bar{\phi}_A \qquad \ldots 4.15$$

The tearing mode has an ideal kink like behaviour every where except within a small boundary layer , where steep radial gradients dissipate the perturbed field. The non-linear ponderomotive and electromagnetic forces, generated by the wave mixing process between the two kinetic Alfven waves namely $(\vec{V}_A \cdot \nabla) \vec{V}_A$ in the momentum equation and $(\vec{V}_A \times \vec{B}_A)$ in Ohm's law, are in phase with the tearing frequency and resonantly drive it. These external driving forces are significant only inside the resistive layer.

In the outer infinite conduction regions as demonstrated in the previous chapter, both classical dissipation and non-linear effects can be neglected, owing to the fact that the dominant role is played by fluid convection and the equilibrium current profile [13] .

To study the dynamics of the tearing mode within the boundary layer, we expand $k_\parallel$ in the radial direction, around the singular surface r_s as $k_\parallel'(r - r_s)$ and define a dimensionless radial variable $x = \frac{r - r_s}{r_s}$. By virtue of equation (4.7), we note that the velocity and magnetic field perturbation can be expressed in terms of potentials ϕ_t and $A_\parallel$ such

$$\vec{V}_t + \frac{\rho_A \vec{V}_A}{\rho_0} = \frac{(\hat{b} \times \nabla\phi_t)c}{|B_0|} \qquad \ldots\ 4.16$$

$$\vec{B}_t = (\nabla \times A_\parallel \hat{b}) \qquad \ldots\ 4.17$$

The two equations which are then required to describe the tearing mode evolution are the parallel component of Ohm's law (equation(4.9)) and the parallel component of the curl of equation (4.10). In the latter equation, the contribution from the equilibrium current source is neglected as it plays an insignificant role in controlling the dynamics of the tearing mode within the boundary layer.

The inner layer equations in dimensionless radial variables are given by

$$\overset{(1)}{\frac{d^2\phi}{dx^2}} = \frac{x}{\Lambda^2}\overset{(2)}{\frac{d^2\psi}{dx^2}} + \overset{(3)}{F(x)} \qquad \ldots\ 4.18$$

$$\overset{(4)}{x\phi} - \psi = \bar{\eta}/\Lambda\ \overset{(5)}{\frac{d^2\psi}{dx^2}} + \overset{(6)}{G(x)} \qquad \ldots\ 4.19$$

Where $F(x) =$

$$\frac{1}{(k_{\parallel}' r_s^2 \Lambda^2)} \left[m_1 \bar{\psi}_1 \frac{d^3 \bar{\psi}_2^*}{dx^3} + m_2 \frac{d\bar{\psi}_1}{dx} \frac{d^2 \bar{\psi}_2^*}{dx^2} \right. \qquad \ldots 4.20$$

$$\left. - \left(\frac{c}{V_A}\right)^2 \left[m_1 \bar{\Phi}_1 \frac{d^3 \phi_2^*}{dx^3} + m_2 \frac{d\bar{\Phi}_1}{dx} \frac{d^2 \phi_2^*}{dx^2} \right] \right] + c.c.$$

$$\left(\bar{\Phi}_{1/2} = \frac{e\phi_{1/2}}{T_e} \right)$$

and

$$G(x) = \frac{c}{(k_{\parallel}' r_s^2) V_A \Lambda} \left[\frac{k_{\parallel_1} c}{\omega_{ci}} \bar{\Phi}_1 \frac{d^2 \bar{\phi}_2^*}{dx^2} \right.$$

$$\left. - \left[m_1 \bar{\Phi}_1 \frac{d\bar{\psi}_2^*}{dx} + m_2 \frac{d\bar{\Phi}_1}{dx} \bar{\psi}_2^* \right] \right] + c.c.$$

$$\ldots 4.21$$

The normalisation of the parameters in the equations (4.18) and (4.19) is the same as those in chapter II. F(x) and G(x) are non-linear terms which, arise out of the beat wave mechanism between kinetic Alfven waves.

Equation (4.18), the momentum transfer equation describes the vortex flow within the boundary layer, (represented by term (1)) acted on by the torque produced by the linear $\vec{J}_t \times \vec{B}_0$ forces, (given by term (2)) and the non-linear ponderomotive forces (term (3)) of the wave mixing mechanism. These non-linear forces are external to the system and act as additional destabilising mechanisms.

Similarly in equation (4.19), the Ohm's law, the parallel component of the electric field (given by term (4)) is balanced by the collisional drag (term (5)) and the non-linear ($\vec{V}_A \times \vec{B}_A$) forces. (term (6)).

The non-linear terms F(x) and G(x) in equations (4.18) and (4.19) are functions of the radial co-ordinate through their dependence on the kinetic Alfven wave potentials.

Our interest is principally around the mode rational surface, where $k_{\parallel t} \simeq 0$. The tearing modes evolve around these surfaces and our objective is to study the effect of the beat waves produced by the interaction between kinetic Alfven waves, and the tearing mode dynamics.

For this purpose, we Taylor expand the kinetic Alfven wave potential functions around the mode rational surface, (which is at x = 0, ($r = r_s$)), as

$$\phi_{1,2} = \phi_{1,2}(x=0) + x\,\phi'_{1,2}\Big|_{x=0} + x^2\,\phi''_{1,2}\Big|_{x=0} \qquad \ldots\ 4.22$$

Such an expansion is valid in a small region around the surface defined by x = 0. With this expansion the source terms F(x) and G(x) reduce to

$$F(x) = \frac{1}{k_{\parallel}' r_s^2 \Lambda^2} \Big[\overset{(1)}{2m_2 \phi_2'' (\phi_1' + 2x\phi_1'')} - \overset{(2)}{\left(\frac{c}{V_A}\right)^2 (\phi_1' + 2x\phi_1'') \times 2\phi_2'' m_2} \Big] \quad \ldots 4.23$$

$$G(x) = \frac{c}{\Lambda k_{\parallel}' r_s^2 V_A} \Big[\overset{(3)}{2k_{\perp 1}\rho_s \frac{k_{\parallel 1}}{k_{\perp 1}} \phi_2'' (\phi_1 + x\phi_1' + x^2\phi_1'')} - \overset{(4)}{m_1 (\psi_2' + 2x\psi_2'')(\phi_1 + x\phi_1' + x^2\phi_1'')} \Big]$$

$$\ldots 4.24$$

In the momentum equation, (equation(4.18)), the contribution to the non-linear forces F(x) arise through the ponderomotive force $(\vec{V}_A \cdot \nabla)\vec{V}_A$ and the electromagnetic force $(\vec{J}_A \times \vec{B}_A)$. Term (2) in equation (4.23), arises through the former effect and is larger by a factor $\left(\frac{C}{V_A}\right)^2$, than the electromagnetic forces given by term (1). Therefore only the contribution coming from the ponderomotive force produced by the interacting kinetic Alfven waves is retained in the equation(4.18).

The dominant contribution to the non-linear force G(x) in Ohm's law, comes from the $(\vec{V}_A \times \vec{B}_A)$ forces which is given by term (4) in equation(4.24). Term (3), represents the characteristic feature of the kinetic Alfven wave and comes from the density perturbations due to the longitudinal nature of the kinetic Alfven fluctuations. This term is proportional to Larmor radius correction and is smaller than term (4) by a factor $k_\perp \rho_s$

With these simplifications of the two non-linear terms F(x) and G(x), the coupled equations describing the evolution of the tearing mode in the resitive layer reduce to

$$\frac{d^2\phi}{dx^2} - \frac{x}{\Lambda^2}\frac{d^2\psi}{dx^2} = -\frac{C_1 x}{\Lambda^2} + \frac{C_2}{\Lambda^2} \qquad \ldots 4.25$$

$$x\phi - \psi = \bar{\eta}/\Lambda \, \frac{d^2\psi}{dx^2} + \frac{C_3}{\Lambda} + \frac{C_4 x}{\Lambda} + \frac{C_5 x^2}{\Lambda} \qquad \ldots 4.26$$

where the coefficients are given by :

$$C_1 = 2\left(\frac{c}{V_A}\right)^2 \frac{(m_1 - m_2)}{k_{\parallel}' r_s^2} \phi_0''^2$$

$$C_2 = 2\left(\frac{c}{V_A}\right) \frac{(m_1 - m_2)}{k_{\parallel}' r_s^2} \phi_0' \phi_0''$$

$$C_3 = -\left(\frac{c}{V_A}\right) \frac{(m_1 - m_2)}{k_{\parallel}' r_s^2} \phi_0 \phi_0'$$

$$C_4 = 2\frac{c}{(V_A)}\frac{1}{(k_\perp r_s)}(k_\perp \rho_s)\ \phi_0 \phi_0''$$

$$C_5 = -\left(\frac{c}{V_A}\right)\frac{1}{(k_\parallel' r_s^2)}(m_1 - m_2)\left[2\phi_0' \psi_0'' + \phi_0'' \psi_0'\right] \quad .. \ 4.26a$$

The form of these driving terms in equations (4.25) and (4.26), can be contrasted with non-linear terms produced by the parametric interaction, which were functions of large order derivatives of the tearing mode perturbations, and could be identified with anomalous viscous and resistive forces.

In the present work, the source terms in the momentum equation and Ohm's law, are linear functions only of the interacting Alfven waves. They are independent of the tearing mode perturbation and act as external sources, resonantly exciting it. It may be remarked that in the earlier studies of beat wave excitation of plasma waves by two electro-magnetic waves, by Tajima et al [3] Rosenbluth -Liu [7] and several authors, the beat waves give rise to similar non-linear terms, which are independent of the longitudinal mode, but act as external forces, driving the plasma at its natural frequency.

We now look for the solutions of equations (4.25) and (4.26) with appropriate boundary conditions. We study the evolution of the symmetric tearing mode and in particular look for modifications introduced by the external driving sources.

There are several methods of obtaining solutions of the coupled differential equations (4.25) and (4.26). One such method had been applied to study the resonant parametric interaction between Alfven and tearing modes. This involved converting the spatial variable 'x' to its Fourier transform variable 'k'. The resulting equation in Fourier space could be solved by variational and asymptotic techniques. For the present problem, such a method is not particularly convenient and leads to several difficulties. We propose to solve the coupled equations, using a well known method, which is particularly amenable for application to equations (4.25) and (4.26), but which has hitherto, not been applied, to solve this system of equations.

We note that the variable ϕ. can be eliminated between equations (4.25) and (4.26). From equation (4.25), the value of ϕ'' can be obtained as :

$$\phi'' = \frac{d^2}{dx^2}\left[\frac{\psi}{x} + \frac{\eta}{\Lambda x}\frac{d^2\psi}{dx^2} + \frac{C_3}{\Lambda x} + \frac{C_4}{\Lambda} + \frac{C_5 x^2}{\Lambda}\right] \quad \ldots 4.27$$

This value is substituted in equation (4.26) to obtain an inhomogeneous fourth order differential equation of the form

$$\frac{\bar{\eta}}{\Lambda}\psi^{IV} - \frac{2\bar{\eta}}{\Lambda}\frac{\psi'''}{x} + \frac{2\bar{\eta}}{\Lambda}\frac{\psi''}{x^2} + \psi'' - \frac{2\psi'}{x}$$

$$+ \frac{2\psi}{x^2} = + \frac{C_1 x^2}{\Lambda^2} + \frac{C_2 x}{\Lambda^2} - \frac{2C_3}{\Lambda x^2} \qquad \ldots\ 4.28$$

The homogeneous part of equation (4.28) is a familiar one and has been studied by various authors. A simpler equation to handle is a third order differential equation in $\frac{d^2\psi}{dx^2}$ which can be derived from equation (4.28). To obtain the desired equation, equation (4.38) is differentiated once to get a fifth order differential form, and ψ^{IV} is eliminated between the resulting equation and equation (4.28).

$$\psi^5 - \psi'''\left(\frac{\Lambda}{\bar{\eta}} - \frac{x^2}{\Lambda^2}\right) + 4\frac{\psi'' x}{\Lambda\bar{\eta}} =$$

$$+ 4\frac{C_1 x}{\Lambda\bar{\eta}} + \frac{3C_2}{\Lambda\bar{\eta}} \qquad \ldots\ 4.29$$

Defining $\psi'' = z$, and making the transformation

$$x = \alpha X, \quad \frac{\alpha^4}{\Lambda\bar{\eta}} = -1, \quad \frac{\alpha^2\Lambda}{\bar{\eta}} = \lambda$$

equation (4.29) reduces to the form

$$\frac{d^3 z}{dX^3} - \frac{dz}{dX}(\lambda + X^2) - 4zX = -4C_1 X$$

$$+ \frac{3C_2}{(-1)^{1/4}(\Lambda\bar{\eta})^{1/4}}$$

$$\ldots\ 4.30$$

Equation (4.30) has a form similar to that of a driven harmonic oscillator. The equation describes the dynamics of the linear resistive tearing mode represented by the homogeneous part of equation (4.30), acted on by external forces generated by the beat wave interaction between kinetic Alfven waves.

It can be seen from the form of equation (4.30), that the equation admits solutions of definite parity. For example the magnetic field perturbations for the symmetric tearing mode ($m = 2$ mode) which is a solution of equation (4.30), is an even function of the radial co-ordinate, while that of the $m = 1$ mode has an odd parity. As will be presently shown, in order that the non-linear forces be in resonance with the linear tearing oscillations the parity of the driving forces, the odd or even nature of the radial variations of the non-linear terms plays a significant role in the resonant interaction.

Several authors [11, 12] have obtained the solutions of the third order differential equation, given by equation (4.30). Paris [12] obtained exact analytical solutions for the perturbed magnetic field potential ψ and the velocity potential ϕ in the resistive boundary layer about the resonant surface described by equation (4.30). He also determined analytically the value of λ across the boundary layer for the symmetric tearing mode for arbitrary values of wave numbers. The analytical solutions for ψ and ϕ were obtained in the form of infinite series involving the Hypergeometric functions, thus avoiding the necessity of solving numerically the differential system of equations (4.25) and (4.26).

Here we present an alternate method for solving the inhomogeneous equation (4.30). The solutions 'ζ' are expanded in terms of certain suitable orthonormal basis functions. We choose Hermite functions for this purpose. The cubic equation (4.30) bears a close resemblance to a particular form of Hermite differential equation [14]; therefore Hermite functions are the most natural basis for the expansion procedure.

The expansion co-efficients are obtained using the orthonormal properties of the basis function. Using appropriate boundary conditions to define the radial structure of the plane symmetric tearing mode, valid within the boundary layer, we obtain a symmetric solution which is an even function of the radial co-ordinate as an infinite summation of Hermite polynomials. The solution consists of two parts, a complimentary function, which represents the natural linear tearing mode, and a particular integral which is the response of the plasma to the external forces.

The solutions obtained within the resistive layer are then to be linked to those of the infinite conduction region. For this purpose, the logarithmic derivative Δ', across the boundary layer is calculated and matched to that of the infinite conduction regions. In the limit of vanishing pump amplitude, in the absence of non-linear forces, the value of Δ', reduces to that obtained by earlier authors for arbitrary wavelengths. In the presence of the driving forces the response of the plasma, for a symmetric tearing mode is studied. In the next section, the details of the procedure are outlined, and the growth rates obtained.

4.3 Solutions in terms of Hermite polynomials

In this section we obtain the symmetric solutions of the inhomogeneous cubic equation (4.30); we derive a general dispersion relation by matching the logarithmic derivative in ψ to the outer solutions. We also show that in the event of the source terms going to zero, the solutions reduce to that obtained by earlier authors.

The solutions of equation (4.30) is represented as a summation of basis functions i.e.

$$\mathfrak{z} = \sum_{n\geq 0}^{\infty} A_n \, He_n(x)$$

Where He_n's are the Hermite functions [14].
The boundary conditions we employ are that the solutions must be well behaved at $x = \infty$ and have a finite value at the origin [11].

$$z(\infty) = 0 \;, \quad z(0) = 1 \;, \quad z'(0) = 0 \qquad \ldots 4.31$$

The first condition ensures the well behavedness of the solution. The second normalises the flux in the tearing layer and along with the third determines the symmetry of the solutions. The contribution to Δ' the stability parameter comes only from the even parity mode.

Substituting equation (4.31) into (4.30), we get

$$-\sum_{n=0}^{\infty} A_n \left[2n^2 + n(\lambda+3)\right] He_{n-1} =$$

$$+ \sum_{n=0}^{\infty} A_n (n+4) He_{n+1} - 4C_1 x + B$$

$$\left[B = \frac{+3C_2}{(-1)^{1/4}(\lambda\,\eta)^{1/4}} \right] \qquad \ldots 4.32$$

Multiplying equation (4.32) by $\int_{-\infty}^{\infty} He_m e^{-x^2/2} dx$ provides the recursion relation between the co-efficients A_n. In arriving at the recursion relation, we use the orthonormal conditions between the Hermite polynomials

$$\int_{-\infty}^{\infty} He_m He_n \, e^{-x^2/2} \, dx = n! \, (2\pi)^{1/2} \qquad (m = n)$$

$$= 0 \qquad (m \neq n) \qquad \ldots 4.33$$

$$A_n \left[2n^2 + n(\lambda+3)\right] \delta_{n-(m+1)} =$$

$$-A_n (n+4) \, \delta_{n-(m-1)} + 4C_1 \int_{-\infty}^{\infty} He_1 He_m \, e^{-x^2/2} \, dx$$

$$- B \int_{-\infty}^{\infty} He_0 He_m \, e^{-x^2/2} \, dx \qquad \ldots 4.34$$

where $\delta_{n,m}$ is the Dirac delta function, and 'm' takes values 0 to ∞. We note that in equation (4.34) the integral proportional to C_1 exists only for m = 1 , while that containing B exists only for m = 0. For even values of 'm' i.e. m = 0, 2, 4 etc. the recursion equation relates A_1 to B, A_3 to A_1 etc. giving rise to an odd series. For odd values of m, i.e. m = 1, 3, 5 etc. equation (4.34) connects A_2 to A_0 and C_1, A_4 to A_2 etc. In particular to demonstrate we write down the relation between the first few coefficients.

For m = 1 the recursion relation leads to

$$A_2 = -\frac{(A_0 - C_1)}{2\left(\frac{\lambda}{4} + \frac{7}{4}\right)} \qquad \ldots 4.35$$

For m = 3

$$A_4 = \frac{-A_2\ 3/2}{4\left(\frac{\lambda}{4} + \frac{11}{4}\right)} \qquad \ldots 4.36$$

By virtue of equation (4.35), A_4 can be expressed in terms of A_0 and C_1. Thus all the even order coefficients can be expressed in terms of A_0 and C_1, while the odd order coefficients can be expressed in terms of A_1. The solution of equation (4.30) separates into an odd part proportional to A_1 and an even part proportional to A_0 and C_1 .

We note that the term proportional to C_1 in equation (4.30), which has an odd parity contributes to the even component of the solution, while B which has an even parity contributes to the odd part. Our interest lies in studying the modifications introduced by the source terms C_1 and B. The odd solution proportional to does not contribute to Δ' . This can be readily verified.

The constant A_0 is determined from the boundary conditions given by equation (4.31) . Applying the normalising condition $z(0) = 1$, we get

$$A_0 = \frac{\Gamma(1/2)}{\Gamma\left(\frac{\lambda}{4}+\frac{7}{4}\right)} \left[\sum_{n=0}^{\infty} \frac{\Gamma(n+1/2)\,\Gamma(n+2)}{\Gamma\left(\frac{\lambda}{4}+\frac{7}{4}+n\right)} \times \frac{1}{n!}\,\frac{1}{2^n} \right]^{-1} \qquad \ldots\ 4.37$$

The complete solution can be written as

$$z = \left[\overset{(1)}{\left[\sum_{n=0}^{\infty} \frac{\Gamma(n+1/2)\,\Gamma(n+2)}{\Gamma\left(\frac{\lambda}{4}+\frac{7}{4}+n\right)}\,\frac{1}{n!}\,\frac{1}{2^n} \right]^{-1}} - \overset{(2)}{c_1 \frac{\Gamma\left(\frac{\lambda}{4}+\frac{7}{4}\right)}{\Gamma(1/2)}} \right]$$

$$\times \left[\sum_{n=0}^{\infty} \frac{(-1)^n\,\Gamma(n+1/2)\,\Gamma(n+2)}{(2n)!\,\Gamma\left(\frac{\lambda}{4}+\frac{7}{4}+n\right)}\, He_{2n}(x) \right] +$$

$$\overset{(3)}{\frac{A_1\,\Gamma\left(\frac{\lambda}{4}+\frac{9}{4}\right)}{\Gamma(5/2)}} \left[\sum_{n=0}^{\infty} (-1)^n \frac{\Gamma(5/2+n)\,\Gamma(n+1)}{(2n+1)!\,\Gamma\left(\frac{\lambda}{4}+\frac{9}{4}+n\right)}\, He_{2n+1}(x) \right]$$

where $\qquad \ldots\ 4.38$

$$A_1 = \frac{B}{(\lambda+5)} \int_{-\infty}^{\infty} He_0\, He_m\, e^{-x^2/2}\, dx \qquad \ldots\ 4.39$$

In equation(4.38) term (1) , which is independent of the source terms constitutes the complimentary function, while terms(2) and(3) form the particular integrals of the solution representing the response of the system to the external ponderomotive forces. The former is an even function of x representing the driven symmetric tearing mode, while the latter is an odd mode.

To obtain a global solution, the complete solutions given by equation(4.38) has to be matched with the outer kink solutions. For this purpose the logarithmic derivative Δ' evaluated across the boundary layer is equated to that obtained from the infinite conduction regions.

The contribution to Δ' from the odd part of the particular solution vanishes, since for a function $f(x)$ such that $f(x) = -f(-x)$

$$\int_{-\infty}^{+\infty} f(x)\,dx = 0 \qquad \ldots\ 4.40$$

For matching to the outer infinite conductivity solution, the quantity Δ' for the symmetric tearing mode is given by [11]

$$\Delta' = 2 \lim_{\mu \to \infty} \left[\frac{\psi'}{(\psi - \mu\psi')} \right] \qquad \ldots\ 4.41$$

where the variable μ is proportional to the radial co-ordinate. The denominator represents the intersection of the asymptote of ψ with the ψ-axis at $\mu = 0$

Equation (4.41) expressed in terms of 'x' reduces to

$$\Delta' = \frac{2^{7/2}\Omega \int_0^\infty \frac{d^2\psi}{dx^2}\,dx}{\left[\psi(0) - \lambda\int_0^\infty x\,\frac{d^2\psi}{dx^2}\,dx\right]} \qquad \ldots\ 4.42$$

$$\Omega = \frac{1}{2^{5/2}}\,\frac{\Lambda^{5/4}}{a^2}\left(\frac{\tau_R}{\tau_H}\right)^{3/4}\frac{1}{(k_{\parallel}'r_s)^2}$$

Using this formalism, we first demonstrate that the earlier results of Paris [12], the expression for the logarithmic derivative Δ' for arbitrary wavelengths, can be recovered. For this purpose, we take the limit of the Alfven pump amplitude tending to zero, and look for the linear symmetric tearing solutions. Substituting equation (4.38) into (4.42) the expression for Δ' becomes :

$$\Delta' = \frac{(-1)^{1/4}\,2\,(\Lambda)^{5/4}\,(\bar{\eta})^{-3/4}\displaystyle\int_0^\infty A_0\,\frac{\Gamma\left(\frac{\lambda}{4}+\frac{7}{4}\right)}{\Gamma\left(\frac{1}{2}\right)}\left[\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\,\frac{\Gamma\left(n+\frac{1}{2}\right)\Gamma\left(n+2\right)}{\Gamma\left(\frac{\Lambda}{4}+\frac{7}{4}+n\right)}\right]He_{2n}(x)\,dx}{\left[1 - \frac{\Lambda}{\bar{\eta}}\left(-\Lambda\bar{\eta}\right)^{1/2}\displaystyle\int_0^\infty x\left[A_0\,\frac{\Gamma\left(\frac{\Lambda}{4}+\frac{7}{4}\right)}{\Gamma\left(\frac{1}{2}\right)}\right]\sum_{n=0}^{\infty}(-1)^n\,\frac{\Gamma\left(n+\frac{1}{2}\right)\Gamma\left(n+2\right)}{(2n)!\,\Gamma\left(\frac{\Lambda}{4}+\frac{7}{4}+n\right)}\,He_{2n}(x)\,dx\right]}$$

$$\ldots\ 4.43$$

To evaluate the integrals in equation(4.43), we note that the constant can be expressed in terms of the Hypergeometric functions [15] as

$$\left[\sum_{n=0}^{\infty} \frac{\Gamma(n+\frac{1}{2})\,\Gamma(n+2)}{\Gamma(\frac{\lambda}{4}+\frac{7}{4}+n)} \frac{1}{n!} 2^n\right]^{-1} =$$

$$\frac{\Gamma(\frac{\lambda}{4}+\frac{7}{4})}{\Gamma(\frac{1}{2})} \cdot {}_2F_1\left[\frac{1}{2}, 2, \frac{\lambda}{4}+\frac{7}{4}, \frac{1}{2}\right]^{-1}$$

... 4.44

The latter has an integral representation [15] as

$$\frac{\Gamma(\frac{1}{2})}{\Gamma(\frac{\lambda}{4}+\frac{7}{4})} \; {}_2F_1 = \int_0^{\infty} y^{-1/2} W_{-\frac{\lambda}{4},\frac{3}{4}}(y)\, dy$$

... 4.45

where $W_{-\frac{\lambda}{4},3/4}(y)$ is the Whittaker function of the second kind.

We also note that $\left[\sum_{n=0}^{\infty} \frac{(-1)^n\,\Gamma(n+\frac{1}{2})\,\Gamma(n+2)}{[\Gamma(\frac{\lambda}{4}+\frac{7}{4}+n)]} \frac{He_{2n}(x)}{(2n)!}\right]$ can be written as

$$2\int_0^{\infty} \cos xy \; f(y)\, dy$$

where $\phi(y) =$

$$y^{1/2} W_{-\frac{\Delta}{4}, 3/4}(y^2) \qquad \ldots\ 4.46$$

Using equations(4.44),(4.45) and (4.46) in equation (4.43) the expression for Δ' can be readily obtained. The properties of Fourier transforms can be used to evaluate the numerator of equation(4.43), which reduces to

$$4\left(\frac{\Lambda}{\bar{\eta}}\right)\left(-\Lambda\bar{\eta}\right)^{1/4} \phi(0)$$

where

$$\phi(0) = \frac{\sqrt{\pi}}{2} \frac{1}{\Gamma\left(\frac{5}{4}+\frac{\Delta}{4}\right)} \qquad \ldots\ 4.47$$

The denominator(D_Y) of equation(4.43) is given by

$$D_Y = \int_0^\infty \left[y^{-1/4} W(y) \quad - 2\Lambda y^{-1/2} \frac{d}{dy}\left(y^{1/4} W(y)\right)\right] dy$$

$$\ldots\ 4.48$$

Performing partial integration on the second term of the above equation and using the contiguous relation between Hypergeometric functions [16] equation(4.48) reduces to

$$D_Y = \frac{2\sqrt{\pi}\,(1-\lambda)}{\Gamma\left(3/4+\frac{\Delta}{4}\right)} \qquad \ldots\ 4.49$$

Using (4.47) and (4.49) together, the dispersion relation obtained by earlier authors [11, 12] can be recovered,

$$\Delta' = \frac{2\pi\left(\frac{\Lambda}{\bar{\eta}}\right)\left(-\Lambda\bar{\eta}\right)^{1/4}}{\left(1-\lambda^{2}\right)} \; \frac{\Gamma\left(\frac{3}{4}+\frac{\lambda}{4}\right)}{\Gamma\left(\frac{\Lambda}{4}+\frac{1}{4}\right)} \qquad \text{.. 4.50}$$

The growth rate as a function of the wave length can be obtained by matching the value of Δ' in equation (4.50) to that in the outer regions, to reproduce the numerical curve obtained in Appendix D of Furth et al [11].

The classical growth rate for the m = 2 mode is obtained in the limit of large magnetic Reynolds number, $\tau_R/\tau_H \to \infty$, where τ_R and τ_H are the resistive and hydrodynamic time scales respectively. In the regime $\tau_R/\tau_H \to \infty$, which is equivalent to Λ tending to 0 the expression for Δ' reduces to the constant approximation result [11] .

We now investigate the modifications introduced in the dispersion relation 4.50, by the non-linear terms of the wave mixing phenomena. Substituting for ψ'' from equation (4.38) into the expression of Δ' given by equation (3.42) we find that the numerator (Nr) is modified to

$$N_r = 4\left(\frac{\Lambda}{\bar{\eta}}\right)\left(-\Lambda\bar{\eta}\right)^{1/4} f(0)\left[1 - \frac{C_1}{\Gamma\frac{1}{2}}\,\Gamma\left(\frac{\Lambda}{4}+\frac{7}{4}\right)\,{}_2F_1\right] \qquad \text{... 4.51}$$

while the denominator (Dr) becomes

$$D_r = \frac{2\sqrt{\pi}\left(1-\lambda\right)}{\Gamma\left(\frac{\Lambda}{4}+3/4\right)} - \frac{iC_1\lambda}{2}\,\frac{\Gamma\left(\frac{\Lambda}{4}+\frac{7}{4}\right)}{\Gamma\left(\frac{\Lambda}{4}+3/4\right)}\,{}_2F_1\left(-\frac{1}{2}, 1, \frac{3}{4}+\frac{\Lambda}{4}, -\frac{1}{2}\right)$$

$$\text{... 4.52}$$

Combining equations (4.51) and (4.52) the dispersion relation for arbitrary wave lengths can be readily obtained. The resulting equation is a transcendental equation in λ and as such cannot yield much information. Hence we look for modifications introduced in the classical m = 2, tearing mode, in the limit of large magnetic Reynolds number. Setting $\lambda \to 0$ (which is equivalent to making the constant ψ approximation) the dispersion relation, for the symmetric tearing mode becomes,

$$\Delta' = 2\pi\left(\frac{\Delta}{\bar{\eta}}\right)\left(-\Lambda\bar{\eta}\right)^{1/4}\frac{\Gamma 3/4}{\Gamma 1/4}\left[1 - c_1\frac{(\Gamma 7/4)^2}{(\Gamma 3/4)^2}\right] \quad \ldots 4.53$$

Where F_{21} in equation (4.51) has been expressed in terms of gamma functions [16]

$$F_{21} = \frac{\sqrt{\pi}\,(\Gamma 7/4)}{(\Gamma 3/4)^2} \quad \ldots 4.54$$

and c_1 is given by equation (4.26a).

Equation (4.53) is the growth rate for the symmetric tearing mode. The expression in the parenthesis represents the enhancement factor for the linear growth rate, generated by the non-linear forces of the wave mixing phenomena.

The resistive tearing modes with positive 'm' numbers are resonantly excited by the external ponderomotive force, while for modes with negative 'm' numbers, the effect of the driving force is to have a stabilising effect. For typical tokamak parameters given in chapter IV this enhancement factor is between 2 to 3. However, these non-linear instabilities excited by beat wave mechanism, grow on a longer time scale than the parametrically excited tearing modes investigated in the previous chapter.

4.4 Conclusions :

In this chapter we have investigated the excitation of tearing modes through beat wave interactions between two kinetic Alfven waves with frequencies and wave vectors given by (ω_1, k_1) and (ω_2, k_2). The interaction results in the excitation of waves at the sum and difference frequencies due to the presence of non-linear terms in the governing equations. We have considered a situation where the difference frequency and wave vector combinations $(\omega_1 - \omega_2, k_1 - k_2)$ is equal to that of the tearing mode (ω_t, k_t). These non-linear beat waves at as external forces driving the system at its natural tearing frequency.

To describe the dynamics of the kinetic Alfven wave we have used the two fluid equations. The characteristic features of the kinetic Alfven wave (finite Larmor radius effects and longitudinal propagation) can be represented in this formalism. The principal equations governing the evolution of the tearing mode, are the Ohm's law and the momentum equation. These were obtained using a one fluid model. From these two coupled equations, a third order inhomogeneous differential equation in $\frac{d^2\Psi}{dx^2}$ (where Ψ is proportional to magnetic vector potential and x is the radial co-ordinate) was derived. The equation has a structure of a driven harmonic oscillator, with the non-linear term $(V_A.\nabla)\, V_A$ in the momentum equation playing the role of the external driver.

We have obtained solutions of the cubic differential equation in terms of certain orthonormal basis functions i.e. Hermite polynomials. The logarithmic derivative was then calculated for arbitrary wavelengths and matched to the outer infinite conduction regions. We have demonstrated that in the limit of vanishing pump amplitude, the earlier results of Paris and Furth et al [11, 12] are recovered. In the presence of non-linear effects, it was shown that in the limit of large magnetic Reynold's number given by $\tau_R/\tau_H \rightarrow \infty$ (where τ_R is the resistive time scale and τ_H the hydrodynamic time scale) the external driving force enhances the growth rates of the symmetric tearing modes with positive 'm' numbers. For modes with negative mode numbers the external force was found to have a stabilising influence. The enhancement factor in the growth rate of the tearing mode, which is proportional to the amplitude of the kinetic Alfven wave is calculated for typical tokamak parameters (given in chapter II, section 2.6). It is found that the growth rate of these driven instabilities, (proportional to $(\eta)^{3/5}(\Delta')^{4/5}$) is increased by a small factor approximately by a factor of (2 or 3).

Our analysis has application in the Alfven wave heating schemes in tokamak plasmas. The antennae in these experiments excite several modes simultaneously which are resonant at different surfaces. When the resonant surfaces are widely spaced, the non-linear processes are that due to a single kinetic Alfven wave (namely parametric decay). This has been discussed in chapter II. When the resonant surfaces are closely spaced, it is possible for the excited kinetic Alfven waves to interact and give rise to beat waves. Our investigations show that when the beat wave frequency corresponds to that of the tearing mode, these modes are driven into enhanced oscillations. The time scales of these driven instabilities are only slightly shorter than the classical resistive instabilities and much larger than the parametrically excited tearing modes (chapter II). However the excitation of these driven modes are harmful for confinment of tokamak plasmas. They could help to explain the enhanced transport of particles in Alfven wave heating schemes.

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CHAPTER V

NON-LINEAR EXCITATION OF DRIFT WAVES BY KINETIC ALFVEN WAVES

5.1 Introduction :

Over the years drift waves have been extensively studied because of the wide range of conditions under which they are unstable both in laboratory [1-7] and space plasmas [8] . Their presence is believed to contribute significantly to anomalous diffusion which is a principal factor in the confinement of plasmas in many laboratory experiments. These waves arise due to the inhomogeneity in the plasma density. They are driven unstable by the free energy associated with the spatial gradients, $\nabla n(x)$. Since the presence of density gradients in a necessary feature of all magnetically confined plasmas, the drift waves were initially considered universal instabilities. In addition to containing spatial density gradients, a tokamak contains sheared magnetic fields. It is now established that the shear in the magnetic field lines plays an important role in the stability of the drift wave.

Since the pioneering work of Pearlstein and Berk [1] the instability of the collisionless drift waves in sheared magnetic fields has been the subject of numerous investigations. By recognising the existence of outgoing wave solutions, it was concluded on the basis of WKB analysis that there existed absolutely unstable collisionless drift waves. Subsequent work by Gladd-Horton [2], Liu et al [3] based on perturbative methods seemed to confirm the existence of the instability. In reference(4),the same differential equation representing the evolution of drift waves in sheared magnetic fields was solved by breaking up the spatial domain into inner and outer regions .Later Tsang et al [5] extended the work of reference [4] to obtain an improved eigen value equation for all even and odd radial eigen modes. As a result of their investigation there emerged the possibility that the perturbation theory form of the dispersion relation was inadequate because it could only be recovered in the limit in which small corrections could be important. In particular, the perturbative theory form was found to be more accurate for more strongly damped modes.

Recently Ross-Mahajan [6] and several others [7] retaining the full electron Z function showed that the drift waves in slab model were actually stable. They confirmed the stability of the collisionless drift waves by numerical and analytical solutions of the appropriate second order differential equation for parameter range of interest in tokamak plasmas. It was pointed out that near the rational surface, the electron Z function varies rapidly and is poorly represented by its residue. Further more, away from the rational surface, where the residue becomes an accurate approximation a comparison of perturbation theory with numerical and improved analytical results of Tsang et al [5] indicates that the wave-electron interaction is somewhat less destabilising than is believed.

Although most of the drift waves are stabilised by magnetic shear, there remain several destabilising mechanisms even in a sheared magnetic field. These are force-free currents, toroidal effects, trapped electrons, and non-linear [9] effects among others. Of these the non-linear coupling of drift-waves, namely parametric [10] effects has been extensively studied. The principal motivation for the studies have been the understanding of wave phenomena, occurring in r.f. heating schemes.

The initial investigation of parametric excitation and stabilisation of drift waves was done by Fainberg Shapiro [12] who studied the stabilisation of collisionless and collisional drift instabilities, by high frequency electric fields along a steady magnetic field. Later Amano et al [12] analysed the effects of r.f. electric field on the excitation and stabilisation of various collisional and collisionless drift waves. The effect of large amplitude spatially uniform dipole electric field, at the lower hybrid frequency on the drift waves in collisionless plasmas was investigated by Sundaram and Kaw [13] . It was shown that the lower hybrid waves could parametrically excite or suppress the drift waves. Subsequently Tripathi [14] included finite wave length effects and found that the drift wave spectrum was stabilised because of parametric coupling to lower hybrid waves. Antani-Kaup [15] have considered a three wave decay, involving the scattering of pump whistler from a drift wave. They found that the scattering would be mainly restricted to the forward direction and the drift wave has a large growth for parameters of interest.

In the context of Alfven wave heating scheme [16, 17] an important problem to investigate is the parametric interaction between kinetic Alfven waves and drift waves. In a tokamak plasma the kinetic Alfven waves have enhanced amplitude near the mode conversion surface and several non-linear processes are expected to take place. One such process, that of the parametric decay of kinetic Alfven wave into the acoustic wave has been investigated earlier [16] .

We have studied the problem of non-linear interaction of kinetic Alfven waves with tearing modes in chapters II, III and IV. It was found that the tearing modes could be resonantly excited with large growth rates. In this chapter we examine the question of an alternate channel of Alfven wave decay with particular reference to drift waves, since they are known to influence plasma confinement in tokamak devices.

We have studied the non-linear decay of the mode converted kinetic Alfven wave into another kinetic Alfven wave and a drift wave. We model the dynamics of the electrons and ions using kinetic equations, to retain the effects of shear and finite Larmor radius corrections.

Using quasineutrality condition and Ampere's law, we derive the coupled equations for the decay process. These equations turn out to be quite complicated and are not amenable to easy solutions.

Under a local approximation however, the differential operators simplify and reduce to algebraic expressions. The growth rates and thresholds for the decay process are calculated, and found to be comparable to that of the ion acoustic process obtained by Hasegawa-Chen [16]. The ratio of the growth rates for the two processes is proportional to $\frac{\omega_*}{k_\parallel c_s} \sim O(1)$ where ω_* is the diamagnetic drift frequency and C_s is the velocity of sound.We have demonstrated that the temperature gradient driven drift waves could also be parametrically excited. These long wave length modes are found to have larger growth rates.

These modes can jeopardise the heating efficiency by providing alternate channels of non-linear energy transfer as well as by their deleterious effect on plasma confinement.

We have in addition investigated the effects of the background inhomogeneity on the decay process. Near the mode conversion region $(\omega = k_{\parallel} v_A)$ where the wavelengths of the decay waves could become comparable to the background inhomogeneity scale lengths, the linear dielectrics of the decay waves are expanded linearly. Although the resulting equation in 'x' space is of a high order, in Fourier space it is only of second order and hence amenable to WKB analysis. Treating the inhomogeneity scale length as a perturbation on the homogeneous plasma, we establish the condition under which an absolute instability, which is a well behaved solution of the differential equation can occur near the mode conversion region. We find that for values of the pump amplitude above a threshold value, the drift waves can be parametrically excited with large growth rates.

We present the basic coupled equations for the parametric decay process in section (5.2) and obtain growth rates and threshold conditions from local approximation in section (5.3). Section (5.4) contains a discussion on the effects of background inhomogeneity and establishes the conditions for absolute instability. The concluding section summarises the results and discusses their relevance to present day tokamaks.

5.2 Bssic equations for the decay process :

In this section, we shall derive the basic coupled equations describing the parametric decay of a mode converted kinetic Alfven wave into another kinetic Alfven wave and a drift wave. We choose a simple slab geometry with an equilibrium field, $\vec{B}_0 = B_0(\hat{e}_z + \frac{x}{L_s}\hat{e}_y)$. where $\hat{e}_y$ and $\hat{e}_z$ are unit vectors in the y and z directions and L_s is the shear length. Background inhomogeneities in physical quantities such as density, temperature are assumed to be in the 'x' direction and have simple linear variations. On this equilibrium a self consistent pump wave $\phi_0(\vec{x},t)$ (the kinetic Alfven wave) of the form

$$\phi_0(\vec{x},t) = \phi_0 \exp i\left[\vec{k}_0\cdot\vec{x} - \omega_0 t\right] + c.c. \qquad \ldots 5.1$$

is imposed where $(\omega_0, \vec{k}_0)$ satisfies the linear dispersion relation for the kinetic Alfen wave.

$$\omega_0^2 = k_{\parallel 0}^2 V_A^2 (1 + k_{\perp 0}^2 \rho_s^2) \qquad \ldots 5.2$$

We consider the non-linear coupling between the pump wave, $(\omega_0, \vec{k}_0)$, the lower side-band $(\omega_-, \vec{k}_-) = (\omega - \omega_0, \vec{k} - \vec{k}_0)$ and the low frequency drift wave $(\omega, \vec{k})$. Interactions with the upper side band mode are neglected as they are off resonant for the decay process. The pump field ϕ_0 is also assumed to be sufficiently weak to that only interactions upto order $|\phi_0|^2$ need be kept.

For a low β plasma $(\beta \sim \sqrt{\frac{m_e}{m_i}})$, the compressional perturbation of the magnetic field $\tilde{b}_z$ is negligible. We adopt the classic two potential representation for the electric field [18].

$$E_\perp = -\nabla_\perp \phi \quad , \quad E_\parallel = -\nabla_\parallel \Psi \qquad \ldots 5.3$$

In adopting different potentials for the parallel and perpendicular electric perturbation, the shear in the magnetic field lines is taken into consideration but the compression of the field lines is neglected. From Maxwell's equations

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}_t}{\partial t} \qquad \ldots 5.4$$

'x' and 'y' components of equation (5.4) are given by

$$\frac{\partial^2}{\partial z \partial y}(\phi - \psi) = -\frac{1}{c}\frac{\partial B_x}{\partial t} \qquad \ldots 5.5$$

$$-\frac{\partial^2(\phi - \psi)}{\partial z \partial x} = \frac{1}{c}\frac{\partial B_y}{\partial t} \qquad \ldots 5.6$$

while the 'z' component vanishes.

The coupled equations can then be derived from the quasineutrality condition and the parallel component of Ampere's law

$$\overset{L}{n_i} + \overset{NL}{n_i} = \overset{L}{n_e} + \overset{NL}{n_e} \qquad \ldots 5.7$$

$$\frac{\partial}{\partial z} \nabla_\perp^2 (\phi - \psi) = \frac{4\pi}{c} (\overset{L}{J_{ze}} + \overset{NL}{J_{ze}} + \overset{L}{J_{zi}} + \overset{NL}{J_{zi}}) \qquad \ldots 5.8$$

where subscripts 'e' and 'i' stand for electrons ions. 'n' and 'J' are the number and current densities respectively. The superscripts (L) and (NL) stand for linear and non-linear contributions being proportional to the first and second order perturbations in wave amplitudes.

To calculate the above perturbed quantities, we model the dynamics of the plasma by kinetic equations. In the low frequency range, the finite Larmor radius of the electrons are unimportant and the electrons can be adequately described by the drift kinetic equation.

$$\frac{\partial f_e}{\partial t} + v_z \frac{\partial f_e}{\partial z} + \nabla \cdot (\vec{V}_\perp f_e) - \frac{e}{m_e} [E_z + (\vec{V}_\perp \times \vec{B}_\perp) \cdot \hat{e}_z] \frac{\partial f_e}{\partial v_z} = 0 \qquad \ldots 5.9$$

where

$$\vec{V}_\perp = \vec{V}_E + \vec{V}_p + \vec{V}_B \,, \quad V_E = c\,\frac{\vec{E}\times\vec{B}_0}{B_0^2}$$

$$V_p = -\frac{mc^2}{eB_0^2}\frac{d\vec{E}_\perp}{dt} \,, \quad V_B = \frac{V_z E_\perp}{B_0} \,, \quad \frac{d}{dt} = \frac{\partial}{\partial t} + (V\cdot\nabla)$$

and f_e is the drift distribution function.

For the ions, we use the Vlasov description, in order to retain effects like Larmor radius, magnetic shear etc. which are important for drift waves.

$$\frac{\partial f_i}{\partial t} + (\vec{V}\cdot\nabla)f_i + \frac{e}{m_i}\left[\vec{E} + \frac{\vec{V}\times\vec{B}}{c}\right]\cdot\frac{\partial f_i}{\partial V} = 0 \qquad \ldots\ 5.10$$

Writing $f_j = f_j^{(0)} + f_j^{(L)} + f_j^{(NL)}$ ... 5.11

(where superscript 'O' refers to equilibrium quantities, and j = e, i) using (5.9) and (5.10), we now calculate the first and second order perturbed number and current densities to be substituted in equations (5.7) and (5.8).

The perturbed quantities are assumed to be of the form $Q \sim Q(x)\exp i\left[k_y y + k_z z - \omega t\right]$ where 'Q' stands for any physical quantity. ... 5.12

The linear responses are straight forward to calculate. For the low frequency electrostatic drift waves, the first order density perturbation is then given by

$$n_e^{(L)} = n_0\left[1 + \frac{\omega - \omega_{*e}}{|k_\parallel V_e|}\cdot Z\left(\frac{\omega}{k_\parallel V_e}\right)\right]\hat{\phi} \qquad \ldots\ 5.13$$

where $\hat{\phi} = \frac{e\phi}{T_e}$, $k_{\parallel} = k_z + k_y \frac{x}{L_s}$, $\omega_{*e} = -\frac{cT_e}{eB_0}\frac{k_y}{L_n}$

$L_n^{-1} = \frac{d}{dx}(\ln n_0)$,

$V_j = \frac{2T_j}{m_j}$ $(j = i, e)$, 'n_0' is the equilibrium number density and $Z(x)$ is the plasma dispersion function.

The first order density perturbation for the ions is calculated from the linearised Vlasov equation(5.10). For this purpose, the expressions for the velocities of the particles are substituted from the standard orbit equations.

Carrying out the integration over the unperturbed orbits, the density perturbation , given by

$$n_i = \int_{-\infty}^{\infty} dv_{\parallel} \int_0^{\infty} v_{\perp} dv_{\perp} \tilde{f}_i$$

is obtained.

$$n_i = -n_0\left[\tau + \frac{\omega\tau + \omega_{*e}}{|k_{\parallel} V_i|} Z\left(\frac{\omega}{k_{\parallel} V_i}\right)\left\{1 + \rho_i^2 |\Gamma_0'(b_i)| \frac{d^2}{dx^2}\right\}\right]\hat{\phi} \quad \ldots 5.14$$

$\tau = T_e/T_i$, $b_i = k_y^2 \rho_i^2$.

Substituting equations (5.13) and (5.14) in equation (5.7), we get

$$\epsilon_D \hat{\phi}_D = 0 \quad \ldots 5.15$$

where subscript 'D' is used to denote drift waves and

$$\epsilon_D = \left[\frac{\omega\tau + \omega_{*e}}{k_{\parallel} V_i} Z\left(\frac{\omega}{k_{\parallel} V_i}\right) \rho_i^2 |\Gamma_0'(b_i)| \frac{d^2}{dx^2}\right] +$$

$$\left\{1+\left(+\frac{\omega-\omega_{*e}}{|k_{\parallel}v_e|}\, Z\left(\frac{\omega}{k_{\parallel}v_e}\right)+\frac{(\omega\tau+\omega_{*e})}{|k_{\parallel}v_i|}\Gamma_0(b_i)\, Z\left(\frac{\omega}{k_{\parallel}v_i}\right)\right\} \qquad \ldots 5.16$$

where $\Gamma_n = I_n(b_i)\, e^{-b_i}$, I_n is the modified Bessel function of order n, and

$$\Gamma_0'(b_i) = \frac{d\,\Gamma_0(b_i)}{d b_i}$$

Equation (5.15) the linear eigen mode equation for drift waves has been extensively discussed in literature. In order to obtain equation (5.15) we have used the quasineutrality condition, since for electrostatic modes, the parallel and perpendicular potentials ϕ, ψ are identical, $(\phi = \psi)$. For the lower side band (kinetic Alfven wave) electromagnetic mode, both equations (5.7) and (5.8) need to be used.

The linear density perturbations calculated from (5.9) and (5.10) for electrons and ions respectively are

$$n_e^{(1)}(\omega_{-,0}) = n_0\left\{\frac{\omega_*}{k_y}\frac{k_{y-,0}}{\omega_{-,0}}\hat{\Phi}_{-,0}+\left(1-\frac{\omega_*}{k_y}\frac{k_y^{-,0}}{\omega_{-,0}}\right)\hat{\psi}_{-,0}\right\}$$

$$\ldots 5.17$$

$$\overset{(1)}{n_i} = n_o \frac{\omega_{*e}}{\omega_{o,-}} \frac{k_{y_{o,-}}}{k_y} \hat{\phi}_{o,-} \qquad \ldots 5.18$$

In equations (5.17) and (5.18) the subscripts $(-,o)$ stand for the side band mode and pump mode respectively. Further in arriving at the equations we have made use of the fact that $k_{\parallel_{-,o}} v_i < \omega_{-,o} < k_{\parallel_{-,o}} v_e$.
The current density perturbations defined by

$$\overset{(1)}{J_{z\,i,e}} = \int_{-\infty}^{\infty} v_z \overset{(1)}{n_j} \, dv_z \qquad \ldots 5.19$$

is given by

$$\overset{(1)}{J_{z e}} (\omega_{-,o}) = - \frac{\omega_{-,o}}{k_{\parallel_{-,o}}} e n_o \left\{ \frac{\omega_*}{k_y} \frac{k_{y_{-,o}}}{\omega_{-,o}} \hat{\phi}_{-,o} \right.$$

$$\left. + \left(1 - \frac{\omega_*}{k_y} \frac{k_{y_{-,o}}}{\omega_{-,o}} \right) \psi_{-,o} \right\} \qquad \ldots 5.20$$

The parallel current is mainly carried by the massless electrons and hence the ion current density perturbation is neglected. Substituting equations (5.17) - (5.20) in equations (5.7) and (5.8), we obtain

$$\epsilon_A \rho_s^2 \frac{d^2 \hat{\phi}}{dx^2} = 0 \qquad \ldots 5.21$$

with ψ, ϕ related through the equation

$$\hat{\psi} = \rho_s^2 \frac{d^2 \hat{\phi}}{dx^2} \qquad \ldots 5.22$$

ϵ_A is given by

$$\epsilon_A = \rho_s^2 \frac{d^2}{dx^2} + \frac{\omega_-^2}{(k_{\parallel -} v_A)^2} - 1 \qquad \ldots 5.23$$

In the next order equations (5.15) and (5.21) get coupled through the pump potential ϕ_0 .

For the drift waves, the important electron non-linear terms in equation (5.9) are those that originate from

$$\left\{ \left[\frac{\vec{V}_E(\omega_0,k_0) \times \vec{B}_\perp(\omega_-,k_-)}{c} \right] \cdot \hat{e}_z + 0 \rightleftharpoons - \right\} \frac{\partial f_e^{(0)}}{\partial v_z} ,$$

$$\nabla \cdot \left\{ \left[\vec{V}_E(\omega_-,k_-) + \vec{V}_B(\omega_- k_-) \right] f_e^L(\omega_0, k_0) + 0 \rightleftharpoons - \right\} \qquad \ldots 5.24$$

For the ion response the leading contributions in equation (5.10) arise from

$$\left\{ \frac{e}{m_i} \left[\vec{E}(\omega_-,k_-) + \frac{\vec{V} \times \vec{B}(\omega_-,k_-)}{c} \right] \frac{\partial f_i^L}{\partial v}(\omega_0,k_0) + 0 \rightleftharpoons - \right\}$$

and

$$\left\{ \frac{e}{m_i c} \left(\vec{V}_E(\omega_0, k_0) \times \vec{B}_\perp(\omega_-, k_-) \right) \cdot \hat{e}_z \right\} \frac{\partial f_i^0}{\partial v_z} + 0 \rightleftharpoons - \qquad \ldots 5.25$$

The last term is due to the parallel ponderomotive force $\frac{e}{m_i c}(V_E \times B_\perp) \cdot \hat{e}_z$ acting on the ions. The expressions for n_j^{NL} , are given by

$$n_i^{NL} = \frac{n_0 \tau c_s^2 \Gamma_0}{\omega_{ci}|k_\parallel|} \left\{ \frac{\omega_{*i}}{k_y v_i} \left(\frac{k_{y-}}{\omega_-} - \frac{k_{y0}}{\omega_0} \right) Z\left(\frac{\omega}{|k_\parallel v_i|}\right) \right.$$

$$\left. + \left(\frac{k_{z-}}{\omega_-} - \frac{k_{z0}}{\omega_0} \right) \left[1 + \frac{\omega}{|k_\parallel v_i|} Z\left(\frac{\omega}{|k_\parallel v_i|} \right) \right] \right\} \times$$

$$\left[k_{y0} \frac{d}{dx} (\hat{\phi}_0 \hat{\phi}_-) - k_y \hat{\phi}_- \frac{d\hat{\phi}_0}{dx} \right], \qquad \ldots 5.26$$

$$n_e^{NL} = \frac{c_s^2}{\omega_{ci}|k_\parallel|} \left\{ 1 + \frac{\omega - \omega_{*e}}{|k_\parallel v_e|} Z\left(\frac{\omega}{|k_\parallel v_e|} \right) \right\} \times$$

$$\left(\frac{k_{z0}}{\omega_0} - \frac{k_{z-}}{\omega_-} \right) \left[k_{y0} \frac{d}{dx} \left\{ \left(1 - \rho_s^2 \frac{d^2}{dx^2}\right) \hat{\phi}_0 \left(1 - \rho_s^2 \frac{d^2}{dx^2}\right) \hat{\phi}_- \right\} \right.$$

$$\left. - k_y \left(1 - \rho_s^2 \frac{d^2}{dx^2}\right) \hat{\phi}_- \frac{d}{dx} \left(1 - \rho_s^2 \frac{d^2}{dx^2}\right) \hat{\phi}_0 \right] \qquad \ldots 5.27$$

where some algebric simplification has been effected through the usual approximations of $(k_\perp^{-,0}\rho_e)^2 << 1$ $k_\parallel^{0,-} v_i < \omega_{-,0} << k_\parallel^{-,0} v_e$ for the kinetic Alfven waves.

Substituting (5.26) and (5.27) in (5.7), the eigen mode equation for the drift wave that is driven by the mode coupling term proportional to the amplitude of the pump wave and the lower side-band is obtained.

$$\epsilon_D \hat{\Phi}_D = \frac{C_s^2}{\omega_{ci}|k_\parallel|}\left[\frac{k_z^-}{\omega^-} - \frac{k_z^o}{\omega^o}\right]\Big[\epsilon_D\Big\{k_y^o \frac{d}{dx}(\hat{\Phi}_o\hat{\Phi}_-) - k_y\hat{\Phi}_- \frac{d\hat{\Phi}_o}{dx}\Big\} + \Big\{1 + \frac{(\omega - \omega_*)}{|k_\parallel V_e|} Z\left(\frac{\omega}{k_\parallel V_e}\right)\Big\}\Big\{\Big[k_y\hat{\Phi}_- \times \frac{d\hat{\Phi}_o}{dx} - k_y^o \frac{d}{dx}(\hat{\Phi}_o\hat{\Phi}_-)\Big]$$

$$+ k_{yo}\frac{d}{dx}\left[\left(1 - \rho_s^2\frac{d^2}{dx^2}\right)\hat{\Phi}_-\left(1-\rho_s^2\frac{d^2}{dx^2}\right)\hat{\Phi}_o\right] - k_y\left(\hat{\Phi}_- - \rho_s^2\frac{d^2\hat{\Phi}_-}{dx^2}\right)\times$$

$$\left(\frac{d\hat{\Phi}_o}{dx} - \rho_s^2\frac{d^3\hat{\Phi}_o}{dx^3}\right)\Big\}\Big] \qquad \ldots 5.28$$

Similarly, one needs to derive an equation for the lower side band that is coupled to the drift wave through the pump potential. For this, we need to calculate $n_j^{NL}(\omega_-\ k_-)$ and $J_{zj}^{NL}(\omega_- k_-)$.

For the electrons from the drift kinetic equation, the dominant non-linear contributions once again fome from $\vec{V}_B(\omega_0, k_0)\, f_e^{L}(\omega, k)$ and $[\vec{V}_E(\omega,k) \times \vec{B}_\perp(\omega_0,k_0)]\cdot\hat{e}_z$ For ion density perturbations, $\overset{NL}{n_i}$ can be calculated from the equation of continuity,

$$\frac{\partial}{\partial t}(e\overset{NL}{n_i}) + \nabla_\perp \overset{NL}{\vec{J}_{\perp i}} = 0 \qquad \ldots 5.29$$

(The parallel ion current is negligible and can be ignored.) where $\overset{NL}{J_{\perp i}}$ is composed of

$e\,[\overset{(1)}{n_i}(\omega)\,\vec{V}_\perp^{*}(\omega_0) + \overset{(1)}{n_i}(\omega_0)\,\vec{V}_\perp(\omega)]$, ($\vec{V}_\perp(\omega_{0,D})$ are the fluid drifts).

The second order electron and ion density perturbation for the side band mode obtained from equations (5.9) and (5.29) are given by

$$\overset{NL}{n_i}(\omega_-) = \frac{n_0 c_s^2}{\omega_{ci}\omega_-}\left\{ -k_{y_0}\hat{\Phi}_0^{*}\frac{d}{dx}(\epsilon_{Di}\hat{\Phi}_D) + k_y \epsilon_{Di}\hat{\Phi}_D\frac{d\phi_0^{*}}{dx} - \frac{k_{y_0}^2}{k_y}\frac{\omega_{*e}}{\omega}\hat{\Phi}_0^{*}\frac{d\hat{\Phi}_D}{dx} - k_y\hat{\Phi}_D\frac{\omega_*}{\omega}\frac{k_{y_0}}{k_y}\frac{d\phi_0^{*}}{dx}\right\} \qquad \ldots 5.30$$

$$\overset{NL}{n_e}(\omega_-) = \frac{C_s^2}{\omega_{ci}} \frac{k_{z_0}}{k_{z_-}} \frac{n_o}{\omega_o} \frac{(\omega - \omega_{*e})}{|k_\parallel V_e|} Z\left(\frac{\omega_-}{k_\parallel V_e}\right) \times$$

$$\left[k_{y_0} (\hat{\phi}_o^* - \hat{\psi}_o^*) \frac{d\hat{\phi}_D}{dx} + k_y \hat{\phi}_D \frac{d}{dx} (\hat{\phi}_o^* - \hat{\psi}_o^*) \right] \quad \ldots 5.31$$

We note that $\overset{NL}{n_e}(\omega_-)$ defined in equation (5.31) is proportional to $(\omega - \omega_{*e})$. For $\omega \sim \omega_{*e}$ the drift frequency, $\overset{NL}{n_e}(\omega_-)$ is negligible compared to $\overset{NL}{n_i}(\omega_-)$. Previous analysis have revealed the same result [16]. One needs to now obtain the electron current contribution $\overset{NL}{J_{ze}}$. This can be derived from the continuity equation

$$\nabla \cdot \overset{NL}{\vec{J}_e} - e \frac{\partial \overset{NL}{n_e}}{\partial t} = 0 \quad \ldots 5.32$$

For ions J_{zi} can be neglected since the ion dynamics is mainly in the plane perpendicular to $\vec{B_o}$. Carrying out these calculations and substituting the resulting expressions in equations (5.7) and (5.8), the non-linear dispersion relation for the side band mode is obtained

$$\epsilon_A \rho_s^2 \frac{d^2\hat{\phi}_-}{dx^2} = \frac{-C_s^2}{\omega_{ci}\,\omega_-} \left\{ \left(\frac{\omega_-}{k_\parallel V_A}\right)^2 \left(k_{y_0} \rho_s^2 \frac{d^2\hat{\phi}_o^*}{dx^2} \frac{d\hat{\phi}_D}{dx} + k_y \hat{\phi}_D \rho_s^2 \frac{d^3\hat{\phi}_o^*}{dx^3} \right) + \rho_s^2 \frac{d^2}{dx^2} \left[k_{y_0} \hat{\phi}_o^* \frac{d\hat{\phi}_D}{dx} + k_y \hat{\phi}_D \frac{d\hat{\phi}_o^*}{dx} \right] + \rho_s^2 \frac{d^2 A}{dx^2} \right\} \quad \ldots 5.33$$

where $\hat{\phi}_o^*$ is the complex conjugate of $\hat{\phi}$ and 'A' is given by

$$A = \frac{(\omega - \omega_{*e})}{|k_{\parallel} V_e|} \; Z\left(\frac{\omega}{|k_{\parallel} V_e|}\right) \Big[k_{yo} \frac{d\hat{\phi}_D}{dx} \Big\{ \hat{\phi}_o^* \left(1 - \frac{k_{zo}\,\omega_-}{k_{z-}\,\omega_o}\right) + \rho_s^2 \frac{k_{zo}\,\omega_-}{k_{z-}\,\omega_o} \frac{d^2\hat{\phi}_o^*}{dx^2} \Big\} +$$

$$k_y \hat{\phi}_D \Big\{ \frac{d\hat{\phi}_o^*}{dx} \left(1 - \frac{k_{zo}\,\omega_-}{k_{z-}\,\omega_o}\right) + \rho_s^2 \frac{k_{zo}\,\omega_-}{k_{z-}\,\omega_o} \frac{d^3\hat{\phi}_o^*}{dx^3} \Big\} \Big] \qquad \ldots\ 5.34$$

Equations (5.28) and (5.33) constitute the general set of coupled equations between the kinetic Alfven mode and the drift mode.

5.3 Dispersion relation under local approximation :

In this section, we shall examine the simplified versions of coupled equations (5.28) and (5.33) under the local approximation.

The local approximation is a major simplification of the drift wave problem and will be employed here without further proof. Extensive literature on this subject is available. The physical basis for the local approximation is that the mode is localised in a distance much less than the scale length of the gradients.

$$\lambda = \frac{2\pi}{k} << \left(\frac{1}{n}\frac{dn}{dx}\right)^{-1} \qquad \ldots \; 5.35$$

Choosing a fourier wave field for the pump, the side band and the low frequency modes, equations (5.28) and (5.33) can be reduced to a set of algebraic equations.

$$\bar{\epsilon}_D \hat{\Phi}_D = \frac{i c_s^2}{\omega_{ci} k_{\parallel}} \left(\frac{k_{\parallel}^-}{\omega_-} - \frac{k_{\parallel}^o}{\omega_o}\right)\left[(\vec{k} \times \vec{k}_o)\cdot \hat{e}_z\right]\Big[\bar{\epsilon}_D +$$

$$\left\{1 + \frac{\omega - \omega_{*e}}{k_{\parallel} V_e} Z\left(\frac{\omega}{k_{\parallel} V_e}\right)\right\} d(b) \Big] \hat{\Phi}_o \hat{\Phi}_- , \qquad \ldots \; 5.36$$

$$\bar{\epsilon}_A \hat{\Phi}_- = \frac{-i c_s^2}{\omega_{ci} \omega_- b_-} (\vec{k} \times \vec{k}_o \cdot \hat{e}_z) \Big[b_o + b_- \left(\frac{k_{\parallel -} V_A}{\omega_-}\right)^2 \Big\{ 1$$

$$+ \frac{(\omega - \omega_{*e})}{|k_{\parallel} V_e|} Z\left(\frac{\omega}{k_{\parallel} V_e}\right)\left[1 - \frac{k_{\parallel}^o \omega_-}{k_{\parallel}^- \omega_o} (1 + b_o)\right]\Big\}\Big] \hat{\Phi}_o^* \hat{\Phi}_D$$

$$\ldots \; 5.37$$

where $b_{-,o} = (k_{x_{o,-}} \rho_s)^2$, $d(b) = b_- + b_o + b_- b_o$. $\bar{\epsilon}_A$ and $\bar{\epsilon}_D$ are given by

$$\bar{\epsilon}_A = 1 - (1 + b_-) \left(\frac{k_{\parallel}^- V_A}{\omega_-}\right)^2 \qquad \ldots \; 5.38$$

$$\bar{\epsilon}_D = 1 + \tau + \frac{(\omega\tau + \omega_{*e})}{|k_{\parallel} V_i|} Z\left(\frac{\omega}{|k_{\parallel} V_i|}\right) + \frac{(\omega - \omega_{*e})}{|k_{\parallel} V_e|} Z\left(\frac{\omega}{k_{\parallel} V_e}\right)$$

$$\ldots \; 5.39$$

Eliminating $\hat{\Phi}_-$ and $\hat{\Phi}_D$ from equations (5.36) and (5.37) and taking $|\omega_-| \simeq \omega_o$, the local dispersion relation can be written in the form

$$\bar{\epsilon}_A \bar{\epsilon}_D \simeq \left(\frac{C_s^2}{\omega_{ci}\omega_o}\right)^2 \frac{(\vec{k} \times \vec{k}_o \cdot \hat{e}_z)^2 \phi^2(b)}{b_-(1+b_-)} \times$$

$$\left[1 + \frac{\omega - \omega_{*e}}{|k_\parallel v_e|} Z\left(\frac{\omega}{|k_\parallel v_e|}\right)\right] \quad \ldots \; 5.40$$

In reducing equation (5.40) to its simplified form, we have used the fact that $\bar{\epsilon}_D$ and $\bar{\epsilon}_A$ are nearly zero on the r.h.s. of equations (5.36) and (5.37). We shall now analyse equation (5.40) to obtain some simple estimates for the resonant decay instability. For the drift waves, we make use of the approximation, $b_i \ll 1$, $k_\parallel v_i \ll |\omega| \ll k_\parallel v_e$ so that $\bar{\epsilon}_D$ simplifies to

$$\bar{\epsilon}_D = 1 + \tau b_i - \frac{\omega_{*e}}{\omega} + \frac{i\sqrt{\pi}}{k_\parallel v_e} (\omega - \omega_{*e}) \quad \ldots \; 5.41$$

It may be recalled that the last term (being the inverse electron damping effect in equation (5.42)) is the source of the universal drift instability. We shall omit this term in equation (5.41) for resonant decay instability and set $\omega = \omega_R + i\gamma$; $\omega_- = -\omega_A + i\gamma$ where $\omega_A = \omega_o - \omega_R$ is the kinetic Alfven wave frequency. Assuming $\gamma \ll \omega_R, \omega_A$ and Taylor expanding the dielectric functions $\bar{\epsilon}_A$ and $\bar{\epsilon}_D$ about ω_A, ω_R respectively as

$$\bar{\epsilon}_A = \epsilon_A(\omega_A) + (\omega_- - \omega_A)\, \partial\epsilon_A/\partial\omega_- \big|_{\omega_- = \omega_A} \quad \text{and}$$

$$\bar{\epsilon}_D = \epsilon_D(\omega_R) + (\omega - \omega_R)\, \partial\epsilon/\partial\omega \big|_{\omega = \omega_R}, \quad \text{equation (5.40)}$$

reduces to

$$(\gamma + \nu_A)(\gamma + \nu_D) = \frac{\omega_{*e}\,\omega_A}{2(1+\tau b_i)^2} \left(\frac{C_s^2}{\omega_{ci}\omega_o}\right)^2 \left[\frac{(\vec{k} \times \vec{k}_o \cdot \hat{e}_z)^2 \phi^2(b)\, |\hat{\phi}_o|^2}{b_-(1+b_-)}\right] \quad \ldots \; 5.42$$

Where ν_A and ν_D represent the linear damping rates corresponding to kinetic Alfven waves and drift waves respectively. The threshold amplitude for the potential $|\hat{\phi}_0|$ can be obtained by setting $\gamma = 0$ in equation (5.43). The growth rate, well above the pump threshold value turns out to be

$$\gamma_T \simeq \frac{\omega_{ci}}{\sqrt{2}} \left(\frac{\omega_*}{\omega_A}\right)^{1/2} \beta^{-1/2} \left|\frac{B_{\perp 0}}{B_0}\right| \frac{f(b) \sin\theta}{(1+\tau b_i)[(1+b_0)(1+b_-)]^{1/2}} \qquad \ldots 5.43$$

In deriving the expression for γ_T, we have used the relationship between the pump magnetic field, $B_{\perp 0}$ and $\hat{\phi}_0$ namely,

$$B_{\perp 0}^2 = \left(k_{\parallel 0} k_{\perp 0} c/\omega_0\right)^2 (1+b_0)^2 |\phi_0|^2 \qquad \ldots 5.44$$

'θ' is the angle between the vectors $k_\perp^-$ and $k_\perp^0$ and β is the ratio of plasma pressure to magnetic field pressure. The growth rate γ_T thus derived for resonant excitation of drift waves is found to be comparable in magnitude to the growth rate for excitation of ion acoustic waves as calculated by Hasegawa and Chen [16]. The ratio of the two growth rates can be readily calculated as

$$\frac{\omega_*}{k_\parallel c_s} \simeq O(1) \qquad \ldots 5.45$$

In fact it is possible to obtain larger growth rates if one couples to other branches of drift modes e.g. a temperature gradient driven drift wave. The effect of temperature gradients is appropriately taken into account by modifying the equilibrium distribution functions [13] .

For an equilibrium temperature gradient in the 'x' direction, ϵ_D is modified [13]

$$\bar{\bar{\epsilon}}_D = 1 + b_i - \frac{\omega_{*e}}{\omega} - \frac{k_{\parallel}^2 T}{m_i \omega^2}\left\{\frac{\omega_{*e}}{\omega}(1+\eta) + 1\right\} = 0 \qquad \ldots\ 5.46$$

where $\eta = \frac{d \ln T}{d \ln n_0}$. In equation (5.46) the electron Landau damping term has been neglected as its effect is unimportant for the macroscopic mode under consideration . For $\eta \gg 1$, $\omega \gg \omega_{*e}$, $k_{\parallel}\left(\frac{T}{m_i}\right)^{1/2}$, equation (5.46) reduces to

$$\bar{\bar{\epsilon}}_D = 1 - \frac{k_{\parallel}^2 T}{m_i \omega^3}\,\omega_{*e}\,\eta \qquad \ldots\ 5.47$$

Considering the non-linear decay of a kinetic Alfven wave into a stable branch of the mode given by equation (5.47), we find the growth rate of the decay instability to be given by

$$\bar{\gamma}_T \simeq \frac{\omega_{ci}}{\sqrt{6}}\left|\frac{B_{\perp 0}}{B_0}\right| \frac{\left(k_{\parallel}^2 T \omega_{*e}\eta/m_i\right)^{1/6}}{\omega_A^{1/2}} \frac{\beta^{-1/2} q(b) \sin\theta}{(1+b_-)^{1/2}(1+b_0)^{1/2}} \qquad \ldots\ 5.48$$

Since $\omega_{*T} (= \omega_{*e}\eta) \gg k_{\parallel} c_s$, this growth rate is much larger than that for ion acoustic waves or ordinary drift waves. The excitation of such macroscopic modes with large growth rates can pose a serious drawback to the efficacy of non-linear ion heating schemes using kinetic Alfven waves.

5.4 Decay instability in an inhomogenous medium :

The results of the previous section are based on the solution of a local dispersion relation where the effect of back ground inhomogeneities have not been taken into account (except for inclusion of the diamagnetic drift frequency). Further, near the mode conversion region ($\omega = k_{\parallel} V_A$) the wave lengths of the decay waves could become comparable to the background inhomogeneity scale lengths, hence the spatial operators have large values and have to be retained. We now study the coupled differential equations (5.29) and (5.34) and analyse the stability properties of the solutions. The problem of interest is that of determining the nature of unstable waves (if they exist) supported by the system. A wave is said to be unstable if a complex $\omega = \omega_r + i\omega_i$ with positive ω_i is obtained from the dispersion relation, signifying growth in time of the disturbance. In an infinite system [19] a pulse disturbance that is initially of finite spatial extent may grow in time, without limit at every point in space, or it may 'propagate along' the system so that its amplitude eventually decreases with time at any fixed point in space. The former is termed 'absolute instability' and the latter 'convective instability'. It is ofcourse the former which is more dangerous because the distinguishing characteristics of the absolute instability is that it grows everywhere in space as a function of time. The convective instability on the other hand 'propagates along' the system as it grows in time, so that the disturbance eventually disappears if one stands at a fixed point. We wish to ascertain whether a growing solution, an absolute instability can be supported by the system formed by coupled equations (5.33), (5.28). The coupled equations are quite complex in view of the complicated spatial structure of the interacting waves in the region. To simplify the analysis somewhat, we shall drop some unimportant terms, (e.g. the Landau damping term) neglect $\epsilon_D (\simeq 0)$ on the right hand side of equation (5.28) and set $\frac{\omega}{k_{\parallel} V_A} \simeq 1$ on the right hand side of equation (5.33). With these simplifications and setting $X = \frac{2\zeta}{\rho_i}$ equations (5.33) and (5.28) can be written in the form

$$\left\{\frac{d^2}{dx^2} + g(x)\right\} \hat{\Phi}_D = -\left(\frac{\omega_{ci}}{\omega_o}\right)\left[|k_{\parallel}(x)v_i| / (\omega + \omega_{*i})\, \Gamma_o'(b_i)\right.$$

$$\times Z\left(\frac{\omega}{|k_{\parallel} v_i|}\right)\left[k \hat{\Phi}_- \frac{d\hat{\Phi}_0}{dx} - k_0 \frac{d}{dx}(\hat{\Phi}_0 \hat{\Phi}_-) + k_0 \frac{d}{dx}\left\{\left(\hat{\Phi}_- - \frac{d^2\hat{\Phi}_-}{dx^2}\right)\right.\right.$$

$$\left.\left.\times\left(\hat{\Phi}_0 - \frac{d^2\hat{\Phi}_0}{dx^2}\right)\right\} - k\left(\hat{\Phi}_- - \frac{d^2\hat{\Phi}_-}{dx^2}\right)\left(\frac{d\hat{\Phi}_0}{dx} - \frac{d^3\hat{\Phi}_0}{dx^3}\right)\right], \quad \ldots 5.49$$

$$\left\{\frac{d^2}{dx^2} + h(x)\right\} \frac{d^2\hat{\Phi}_-}{dx^2} = -\left(\frac{\omega_{ci}}{\omega_-}\right)\left\{k_0 \frac{d^2\hat{\Phi}_0}{dx^2}\frac{d\hat{\Phi}_D}{dx}\right.$$

$$\left. + k \hat{\Phi}_D \frac{d^3\hat{\Phi}_0^*}{dx^3} + \frac{d^2}{dx^2}\left[k_0 \hat{\Phi}_0^* \frac{d\hat{\Phi}_0}{dx} + k \hat{\Phi}_D \frac{d\hat{\Phi}_0^*}{dx}\right]\right\} \quad \ldots 5.50$$

where

$$k = k_y \rho_s, \; k_0 = k_{y_0} \rho_s, \; h(x) = \left(\frac{\omega_-}{k_{\parallel}^- v_A}\right)^2 - 1,$$

$$g(x) = \left\{1 + \tau + \frac{\omega\tau + \omega_{*e}}{|k_{\parallel} v_i|} \Gamma_o Z\left(\frac{\omega}{|k_{\parallel} v_i|}\right)\left[\frac{|k_{\parallel} v_i|}{(\omega + \omega_i^*)} |\Gamma_o(b_i)|\right]\right.$$

$$\times Z\left(\omega / |k_{\parallel} v_i|\right) \quad \ldots 5.51$$

Near the mode conversion region of the pump wave, the drift and the side band kinetic Alfven waves are also close to their respective resonance points. The functions g(x) and h(x) can therefore be Taylor expanded around these points and expressed as

$$g(x) = K_D (x - x_D) \; ; \; h(x) = K_A (x - x_A) \quad \ldots 5.53$$

where (x_D)and (x_A) are the resonance points for the drift and kinetic Alfven waves respectively, K_D and K_A are typical inverse scale lengths of shear variation and density inhomogeneity respectively. Since at x_D and x_A, the dispersion relation for the drift and Alfven waves are satisfied, $g(x_D)$ and $h(x_A)$ are set equal to zero. The linear operators on the left band sides of equations (5.49) and (5.50) therefore indicate an Airy function kind of spatial behaviour for the daughter waves. A similar spatial structure also exists for the pump wave [16] , which couples (5.49) and (5.50). To solve this coupled set is still quite formidable. For analytical simplification we adopt a plane wave model for the pump wave and study the spatial evolution of the daughter waves. We follow the method of White et al [20] , for analysising the instability. We Fourier analyse the coupled equations (5.49) and (5.50) defining

$$\Psi_p = \int_{-\infty}^{\infty} e^{-ipx} \hat{\phi}_-(x)\, dx$$

$$\phi_p = \int_{-\infty}^{\infty} \hat{\phi}_D\, e^{-ipx} dx \qquad \ldots\ 5.54$$

$$\hat{\phi}_0 = \phi_0 \exp(i k_0 x)$$

Eliminating the variable ϕ_p we obtain a single second order equation in Ψ_p as

$$\frac{d^2\Psi_p}{dp^2} + F^2(p)\Psi_p = 0 \qquad \ldots\ 5.55$$

where ψ_p is related to $\hat{\phi}_-$ through the relation (5.57) and $F^2(P)$ is given by,

$$F^2(P) = \frac{1}{K_A^2}\left[i\left(PK_A - (P+k_0)\,a\,k_A\right) + \frac{(k_0^2-p^2)K_A^2}{(k_0^2+p^2)}\right.$$

$$- \frac{k_0^2 K_A^2}{2\left[k_0(P+k_0) - k k_0\right]^2} - \frac{1}{4}\left\{ i\left[p^2 - a(P+k_0)^2\right.\right.$$

$$\left.\left. + Q\right] + \frac{2pK_A}{p^2+k_0^2} + \frac{k_0 K_A}{(k_0(P+k_0) - k k_0)}\right\}^2$$

$$- \frac{\omega a}{(\omega + \omega_{*i})}\left(\frac{\omega_{ci}}{\omega_0}\right)^2 \frac{|\hat{\Phi}_0|^2}{p^2}(p^2+k_0^2)^2\left\{ k k_0\right.$$

$$\left.\left. - k_0(P+k_0)\right\}^2\right] \qquad \ldots\ 5.56$$

where

$$Q = \left(\frac{\omega}{k_{\parallel} V_A}\right)^2 - 1 - \left[\frac{a\omega}{(\omega+\omega_{*i})}\right]\left[1 + \tau b_i - \frac{\omega_{*e}}{\omega}\right] \qquad \ldots\ 5.57$$

and $a = K_A/K_D$ (treated as a constant in this analysis).

Solutions of equation (5.55) are given in terms the usual WKB expressions

$$\Psi_{\pm} = F^{-1/2} \exp\left[\pm i \int^{P} F \, dp'\right] \qquad \ldots \; 5.58$$

We require that the solution be localised in k-space with finite extent of localisation, which implies the localisation of the Fourier transform solution in x-space. Such a localisation exists if the Bohr-Sommerfield quantisation condition [20] is satisfied.

$$\int_{P_1}^{P_2} F \, dp = (n + \tfrac{1}{2}) \pi \qquad \ldots \; 5.59$$

where P_1 and P_2 are called 'turning points', which are the roots of $F^2(P) = 0$. To solve for the exact solutions for the turning points is quite complicated; we shall therefore look for approximate solutions and eigen value condition by using a perturbation procedure. We treat L_n^{-1} as a small parameter and seek corrections to the eigen values in successive orders of L_n^{-1} by perturbation.

In the limit of a homogeneous plasma, $L_n \to \infty \; (k_n \to 0)$ the Bohr-Sommerfeld condition requires that the integral

$$\int_{P_1}^{P_2} F \, dP \text{ be vanishingly small}$$

This implies that the two turning points must coalesce. In the limit of homogenous plasmas, the turning points obtained by setting $F^2_{L_{n\to\infty}} = 0$ are the solutions of

$$P^2 - (P+K_0)^2 a + Q - 2\frac{|\phi_0|}{P}\frac{\omega_{ci}}{\omega_0}\sqrt{a}\,(P^2+K_0^2)\times [k_0(k-k_0) - k_0 P] = 0 \quad \ldots 5.60$$

From the above equation, calculation of the coalescence condition is quite straight forward and is given by

$$K_0^2 a^2 = (1-a)\left[Q - Q_1 - a K_0^2\right] \quad \ldots 5.61$$

Where

$$Q_1 = \frac{2|\phi_0|\,K_0\sqrt{a}}{(a-1)^2}\left(\frac{\omega_{ci}}{\omega_0}\right)\left[k\left(3a+\frac{1}{a}-2a^2-2\right) + k_0\left(1-\frac{1}{a}-2a\right)\right] \quad \ldots 5.62$$

Solving for $Q = Q_c$ from equation (5.61) we get

$$Q_c = -\frac{k_0^2 a}{a-1} + 2|\phi_0|\frac{\omega_{ci}}{\omega_0}\frac{K_0^2\sqrt{a}}{(a-1)^2}\left\{k\left(3a+\frac{1}{a}-2a^2-2\right) + k_0\left(1-\frac{1}{a}-2a\right)\right\} \quad \ldots 5.63$$

and the value of the coalesced variable obtained from equation (5.60) is

$$P_c = \frac{k_0 a}{a-1} \quad \ldots 5.64$$

F^2_∞ can now written as

$$F^2_\infty = \frac{1}{4K_A^2}(P-P_c)^2(P-P_3)(P-P_4) \quad \ldots 5.65$$

Where P_c is given by equation (5.64), and $P_{3,4}$ are given by

$$P_{3,4} = \frac{k_0 a \pm \sqrt{k_0^2 a^2 - (1-a)(Q+Q_1 - a k_0^2}}{(1-a)} \qquad \ldots 5.66$$

The introduction of a large inhomogeneity scale length 'L_n' produces a splitting of the coincident turning points at $P = P_c$. This can be readily investigated by examining the behaviour of $F^2(x)$ near $Q = Q_c$. Thus we write

$$F^2 = F^2_\infty + \frac{dF^2}{dQ}\Delta Q + \frac{dF^2}{d\left(\frac{1}{L_n}\right)}\left(\frac{1}{L_n}\right) \qquad \ldots 5.67$$

If P_3 and P_4 in equation (5.65) are sufficiently far away from P_c, we can treat $(P-P_3)(P-P_4)$ $\simeq (P_c - P_3)\ (P_c - P_4)$ as a constant. The resultant equation for F^2 is then a quadratic in 'P' and can be solved for the two turning points $P_{1,2}$ in the neighbourhood of P_c. We hence conclude that in the vicinity of $P = P_c$, the equation for F^2 assumes the form of a simple harmonic potential. In the region of interest (i.e. around $P \simeq P_c$), we substitute $Q = Q_c$, $P = P_c$ everywhere in $F^2(P)$ in equation (5.65) except in the fast oscillating factor $(P-P_c)$. We then find that equation (5.67) reduces to

$$F^2 = \frac{Q_1}{2K_A^2(a-1)}\left[q_t^2 - q^2\right]$$

where

$$q_t = (a-1)\,\Delta Q + \frac{i}{k_A}\left[\frac{k_o a}{Q_1} + \frac{(a-1)^2}{2K_o}\left\{\frac{2a}{a^2+(a-1)^2} + \frac{k_o}{(k_o-k+ka)}\right\}\right], \quad q = (p-p_c) \qquad \ldots\ 5.69$$

Applying the quantisation condition between the split roots obtained by solving $F^2 = 0$ (equation (5.76)) the Bohr-Sommerfeld condition reduces to

$$\left[\frac{Q_1}{2k_A(a-1)}\right]^{1/2} q_t^2 = 2n+1 \qquad \ldots\ 5.70$$

The expression for real and imaginary parts of obtained from the above dispersion relation is given by

$$1 + \tau b_i - \frac{\omega_{*e}}{\omega_R} = \frac{\omega_R + \omega_{*i}}{\omega_R a}\left[\frac{k_o^2 a}{(a-1)} - Q_1\right] \qquad \ldots\ 5.71$$

where

$$\gamma = \frac{\omega_R(\omega_R+\omega_{*i})}{a\,\omega_{*e}K_A}\left[\frac{k_o a}{(a-1)\,Q_1} + \frac{(a-1)}{2k_o} \times \left\{\frac{2a}{[a^2+(a-1)^2]} + \frac{k_o}{(k_o-k+ka)}\right\}\right] + \frac{\sqrt{2}\,(\omega_R+\omega_{*e})(2n+1)\,K_A}{\omega_R a\,(a-1)^{1/2}\,|Q_1|^{1/2}} \qquad \ldots\ 5.72$$

In deriving equations (5.71) and (5.72) we have assumed that γ (the non-linear growth rate) is $<< \omega_R$, $\omega_A = \omega_o - \omega_R$ and real $\left(\frac{\omega_-}{k_{\parallel -} V_A}\right)^2 \simeq 1$ Typically $k_A = \frac{1}{L_n}$ and $k_D = \left|\frac{k_\parallel'}{k_\parallel}\right| \frac{k_\parallel^2 c_s^2}{\omega^2} \simeq \frac{1}{L_s^2} \frac{k_\parallel^2 c_s^2}{\omega^2}$. For $\omega \sim \omega_{*e}$, $\frac{k_\parallel c_s}{\omega} << 1$

Further L_s is much larger than L_n. It follows therefore that K_D is much smaller than K_A. ($a = \frac{k_A}{k_D} >> 1$). For this value of a, we find from equation(5.62) that Q_1 is much less than zero. For $K_A > 0$ we observe that for certain threshold value ($|Q_1| > K_o^2$) of the pump field, 'γ' can be positive and therefore an absolute instability can exist. It must be noted that the threshold value for temporal growth of excited drift mode predicted from (5.72) should be treated as approximate since we have used a perturbative scheme, i.e. treating L_n^{-1} as a perturbative parameter.

5.5 Summary

We have studied the parametric decay of a pump kinetic Alfven wave into a side band kinetic Alfven and a drift mode. The dynamics of the drift wave are sensitively dependent on the shear and finite Larmor radius effects. We have therefore used the kinetic equations to describe the motion of electrons and ions. Using quasineutrality condition and Ampere's law, the coupled equations for the decay process are derived. The coupled equations in 'x' space are quite complicated and are not amenable to easy solutions.

Under a local approximation, however, the differential operators reduce to algebraic expressions, and the dispersion relation could be readily obtained. We have calculated the threshold value for the decay process and the growth rate of the drift instability far above the threshold value. We find that the calculated growth rates are quite large and compete significantly with the growth rates of excited ion acoustic waves, calculated by Hasegawa-Chen [16] . The ratio of the growth rates in the two processes was shown to be $\frac{\omega_{*}}{k_{\parallel} c_{s}} \sim O(1)$. We have demonstrated that the kinetic Alfven waves could couple to temperature gradient drift waves which have larger growth rates and longer wave lengths. In addition we have investigated the effects of the background inhomogeneity on the decay process. Near the mode conversion region when the wave lengths of the decay waves become comparable to the inhomogeneity scale lengths, the differential operators play a significant role. Expanding the linear dielectrics linearly around their resonant surfaces, we have obtained a second order differential equation in Fourier space. The equation is amenable to perturbative WKB analysis. Treating the inhomogeneity scale length L_n as perturbation parameter, we establish the condition under which an absolute drift instability can exist in the plasma. We have shown that for values of the pump amplitude above a certain threshold value, the drift waves could be parametrically excited with large growth rates.

Our analysis has important application in Alfven wave heating schemes in laboratory plasmas. It has been shown by earlier investigations [16] that the excited kinetic Alfven wave has an enhanced amplitude at the mode conversion layer, which could lead to several non-linear processes.Hence a study of the non-linear properties of the kinetic Alfven wave is essential for better understanding of the propagation of the mode converted wave. In laboratory plasmas several experiments on Alfven wave heating have reported enhanced diffusion of particles [21] in addition to efficient heating. The diffusion of particles may have been caused by the excitation of drift waves. These modes could seriously jeopardise the heating efficiency by providing alternate channels of non-linear energy transfer, as well as by degrading plasma confinement.

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CHAPTER VI

EFFECT OF PONDEROMOTIVE FORCE ON THE COLLISIONLESS TEARING AND DRIFT MODES

6.1 Introduction :

In the earlier chapters, we had studied the non-linear interactions between the kinetic Alfven waves and the resistive tearing modes, on account of the important role played by the latter in tokamak discharges. Traditionally these modes are analysed by the use of resistive MHD theory which predicts instability for $\Delta' > 0$. Early studies of the linear theory were based on such simple models of the Ohm's law $\vec{E} + \frac{\vec{v} \times \vec{B}}{c} = \eta \vec{J}$, valid only in the collisional limit. However as the temperature of the plasma increases, the resistivity decreases as $T^{-3/2}$ and in the fusion regime, the plasma is virtually collisionless. In this regime the collisionless version of the tearing modes are important and play significant roles in both space [1] and laboratory plasmas. These modes are destabilised by the dissipative Landau resonance of electrons and ions which move parallel to the d.c. magnetic field. For comparable electron and ion temperatures, the electrons make a dominant contribution to the linear growth rate. Several authors [2, 3, 4] have studied the kinetic theoretical calculations including the full electron-electron, electron-ion collision operators. Drake-Lee [3] showed that the tearing mode, as a function of the collisionality of the plasma falls into three categories, referred to as the collisionless, semi-collisional and collisional. An important conclusion of their work was that the width of the current layer becomes smaller than ρ_i, the ion gyroradius, as the plasma approaches the collisionless regime; therefore previous theories based on hydrodynamical model were not valid. Recently studies of non-linear effects on the evolution of collisionless tearing modes have been of interest [5, 6, 7]. In these investigations non-linear effects enter from the perturbation of particle orbits near the singular layer. Coroniti [8] examined the non-linear evolution of a broad 'k' spectrum of tearing modes. For magnetised particles and low frequency modes the turbulence produces a spatial diffusion His calculations using ad-hoc models showed that the wave induced turbulent drifts lead to non-linearly enhanced growth rates. Esarey [9] investigating the effect of turbulent electron diffusion from stocastic electron orbits, found that for the tearing modes, stability is obtained for large values of the diffusion coefficient.

These several diverse investigations [10] make one fundamental point clear, which is that the collisionless tearing modes are destabilised by the dissipative Landau resonance of electrons and ions. Therefore any effect which modifies the particle motion within the singular layer strongly influences the growth rate of these collisionless tearing modes.

Another mode whose stability depends on the wave particle interaction around the resonant surface is the collisionless drift wave. Within the framework of linear theory absolutely unstable drift modes do not exist in slab geometry with a single rational surface. However recently Hrishman-Molvig [11] investigated the turbulent diffusion of electrons in the vicinity of a mode rational surface and found that the stabilising influence of the non-resonant electrons could be eliminated leading to absolute instability.

Later, extending the work numerically Beasely et al [12] showed that the unstable range could extend down to almost zero turbulence levels. Since then several authors have pointed out the sensitivity of the electron motion to external modifications at the singular layer.

We have in this context investigated one such non-linear phenomena, which modifies the particle orbits around the singular surfaces and alters the growth rates of the collisionless tearing and drift modes, We have considered the effect of the ponderomotive force generated by two interacting kinetic Alfven waves on the collisionless drift and tearing modes. The ponderomotive force generated by the kinetic Alfven waves is obtained from the two-fluid equations.

The basic equations required to study the tearing mode evolution are the Ohm's law $\sigma E_{\parallel} = J_{\parallel}$ (where σ is a generalised conductivity) and the momentum conservation equation. The former is obtained from the electron dynamics, described by the Vlasoo equation. In the presence of the equilibrium ponderomotive force, the electron orbits are strongly modified, the perpendicular ponderomotive force Doppler shifts the mode frequency. The effect of the parallel P.F. ($F_{\parallel o}$) is to alter the resonant wave particle phenomenon, leading to a replacement of $\frac{\omega_t}{k_{\parallel_t} v_e}$ by $\frac{\omega_t}{(k_{\parallel_t}^2 v_e^2 - a)^{1/2}}$, where $a = \frac{2i F_{\parallel 0} k_{\parallel_t}}{m_e}$. Consequently the conductivity profile 'σ' is altered significantly. This phenomenon is similar to resonance broadening effects studied by several authors [5-7].

The conductivity profile contained in the coupled equation is in general a complicated function of 'x' and as a result the analysis is rather involved. However recent simplifications based on variational methods by Hazeltine et al [4] have simplified the treatment. Following the earlier methods, we obtain the modifications introduced by the parallel force in the growth rates of the tearing modes in the collisional and collisionless regimes. We find that the effect of the ponderomotive force is to enhance the growth rates of Laval et al [13] and Drake and Lee [3] in the two regimes. In the collisional regime however, the enhancement factor produced by the parallel force is second order and hence quite feeble. For given tokamak parameters, the enhancement factor in the collisionless regime is however quite large.

The P.F. has a similar effect on the dynamics of the drift wave. To retain shear effects the electron motion is modelled by the kinetic equations. As in the case of the tearing mode, the electron orbits are greatly modified due to the presence of the equilibrium P.F. The electron wave particle response $\frac{\omega}{k_\parallel v_e}$ is replaced by $\frac{\omega}{(k_\parallel^2 v_e^2 - a)^{1/2}}$

For simplicity the ions are treated by the hydrodynamic approximation. Then from the quasineutrality condition, the radial eigen mode equation is obtained. To obtain the eigen values we employ a variational principle analogous to the one employed by Ross et al [4] . Our calculations show that the P.F. generated by the kinetic Alfven waves has a destabilising effect on the drift mode. For typical tokamak parameters the destabilising effect could be quite large and competes significantly with the shear stabilising effect.

6.2 Derivation of the Ponderomotive force :

In this section, we obtain the expression for the ponderomotive force produced by the interaction of two pump kinetic Alfven waves. We consider the non-linear coupling between two kinetic Alfven waves excited at the mode conversion regions by an external r.f. source. Close to their resonance regions, the excited waves have enhanced amplitudes with complicated radial structures and propagate into the plasma towards regions of increasing density [15] .

We use the two fluid equations to describe the propagation characteristics of the kinetic Alfven waves. As demonstrated earlier, the special features of the wave, namely finite Larmor radius effects can be adequately reproduced in this formalism. The non-linear coupling between the kinetic Alfven waves manifests through the terms $(\vec{V}_A \times \vec{B}_A)$ and $(\vec{V}_A . \nabla)\vec{V}_A$ in equation (3.1) , where the subscript 'A' stands for the interacting Alfven waves. In the first order, equation (3.1) can be readily solved to obtain the quiver velocities of the electrons and ions $V_{\perp j}$ and $V_{\parallel j}$ $(j = i, e)$ which are given by equations (3.4), (3.15) and (3.17).

The second order part of the equation (3.1), is then obtained as

$$m_j \frac{d\vec{V}_2}{dt} = \left[m_j (\vec{V}_A \cdot \nabla)\vec{V}_A + e_j \frac{(\vec{V}_A \times \vec{B}_A)}{c} \right] \qquad \ldots 6.1$$

Where V_2 is the second order fluid velocity, $\vec{V}_A$ the linear quiver velocity, and $\vec{B}_A$, the linear perturbed magnetic field of the Alfven wave. The ponderomotive force (P.F.), F_{NL}, acting on the electron and ion fluids is obtained by averaging equation (6.1) over the fast Alfvenic fluctuations. In order to obtain the P.F. generated by the kinetic Alfven waves, we require an explicit form for the spatial variations of the field potentials. For this purpose we choose oscillating cosine profiles for the pump Alfven wave amplitudes as

$$\phi_A = \phi_0 \cos(\vec{k}_{1,2} \cdot \vec{x} - \omega_A t) \qquad \ldots 6.2$$

Where the subscripts 1, 2, refer to the interacting waves.

Such a model for the spatial variation of the potential function is valid in the region away from the mode conversion region, which is our region of interest. In this region the kinetic Alfven waves have rapidly oscillating structures in the radial direction and can be approximately described by equation (6.2).

We assume that the interacting kinetic Alfven waves have the same frequencies ω_A, but different wave vectors $\vec{k}_1$ and $\vec{k}_2$. Our assumptions have particular relevance to the Alfven wave heating scheme, where the interacting waves arise, through mode conversion of an external r.f. source at their respective resonance surfaces. These waves excited by an external antenna have the same frequency, but different wave numbers and our analysis which has been done with a motive to understand the non-linear features of this scheme [16] uses this model for the kinetic Alfven waves.

Substituting the expression for the quiver velocities into equation (6.1) and using equation (6.2), the second order equation of motion for the particles in terms of the potential functions ϕ and ψ can be readily obtained. In order to obtain the non-linear ponderomotive force F_{NL}, acting on the electron and ion fluids, equation (6.1) is averaged over the fast Alfvenic motions. The averaging is defined in the following manner;

$$F_{NLj} = \frac{1}{T}\int_0^{2\pi/\omega} f(t)\, dt \qquad \ldots 6.3$$

where $f(t) = e_j \left(\frac{\vec{V}_A \times \vec{B}_A}{c}\right) + m_j (\vec{V}_A \cdot \nabla)\vec{V}_A$

Equation (6.3) describes the d.c. (P.F.) acting on the electron and ion fluids due to the wave mixing phenomena between the kinetic Alfven waves.

Of the two components of the ponderomotive force described in equation (5.3), the dominant part comes from the $(\vec{V}_A . \nabla)\vec{V}_A$ term and is given by

$$F_\perp = \left(\frac{\omega_A}{\omega_{cj}}\right)(k_\perp^2 \rho_j^2)\left(\frac{e_j \phi_0}{T_j}\right)^2 m_j V_j^2 \left[\hat{y}\vec{k}_x - \hat{x}\vec{k}_y\right] \qquad \ldots 6.4a$$

$$F_\parallel = \left(\frac{\omega_A}{\omega_{cj}}\right)\left(\frac{\vec{k}_1 \times \vec{k}_2}{k_{\parallel A}}\right)\cdot \hat{e}_z \, \underline{V_j}^2 m_j \bar{\lambda}_A \left(\frac{e_j \phi_0}{T_j}\right)^2 \left(\frac{\Delta k_\perp}{k_\perp}\right) \qquad \ldots 6.4b$$

where $\Delta k_\perp$ is $|(k_{\perp 1} - k_{\perp 2})|$ the difference in the perpendicular wave vectors of the interacting kinetic Alfven waves. On account of the short wave lengths [15] (of the interacting Alfven waves) in the '$\hat{x}$' direction, which is the direction of the density inhomogeneity, k_x for the Alfven wave is much larger than k_y. Therefore the principal value of $\vec{F}_0$ in equation (6.4a) is along the $\hat{y}$ direction.

It is to be noted that in equations (6.4a) and (6.4b), the perpendicular and parallel ponderomotive forces $F_{\perp o}$ and $F_{\parallel o}$, are proportional to the finite Larmor radius corrections, indicating that these forces arise because of the kinetic characteristics of the interacting Alfven waves.

We remark that for the given profiles for the field variations, the perpendicular ponderomotive force in equation (6.4a) is a positive definite quantity, while the sign of the parallel P.F. could be negative or positive depending upon the combination of the parameters. For $(\vec{k}_1 \times \vec{k}_2).\hat{e}_z$ greater than zero i.e. $\frac{k_{x_1}}{k_{x_2}} > \frac{k_{y_1}}{k_{y_2}}$, $F_{\parallel o}$ is a positive quantity, while for the reverse inequality, it has a negative value. We have, in the subsequent sections examined the effect of the P.F. on the low frequency drift and tearing modes. It was found that the sign of $F_{\parallel o}$, plays a significant role in the stability of the modes examined.

6.3 Interaction of Ponderomotive force with collision - less Tearing modes :

In this section, we investigate the interaction between the P.F.(derived in the earlier section, produced by the non-linear coupling between two kinetic Alfven waves) and the collisionless tearing modes. In a plane one-dimensional magnetic neutral sheet, the collisionless tearing modes are destabilised by the dissipative Landau resonance of electrons and ions which move parallel to the d.c. magnetic field. For comparable electron and ion temperatures, the dominant contribution to the linear growth rate comes from the electrons.

Earlier theories of the tearing mode were based on such resistive models as Ohm's law,

$$\vec{E} + \frac{\vec{V} \times \vec{B}}{c} = \eta \vec{J} \quad \dots\ 6.4c$$

All these calculations continued to assume at least implicitly equation (6.4c). The width Δ of the current layer produced by the induced parallel electric field is an important quantity which can be determined from the plasma dynamics in the layer. Previous authors have assumed $\Delta \gg \rho_i$, where ρ_i is the ion gyro-radius. However an important conclusion reached by later investigations [3] was that Δ becomes smaller than ρ_i as the plasma approaches collisionless regime. Using heuristic consideration it is in fact possible to show that $\Delta \ll \rho_i$. The correct basis is therefore to replace equation (6.4c) by a suitably ordered version of the kinetic equations, which employs the collision operator.

We calculate the electron response by means of Vlasovequation with Krook collision operator on the r.h.s. The perturbed current $\vec{J} = \vec{J}_i + \vec{J}_e$ which is produced by the induced electric field is primarily along the direction of the equilibrium magnetic field. The parallel response of the electrons to the induced field is much larger than that of the ions so that $J_{\parallel e} > J_{\parallel i} \ (\simeq 0)$.

We obtain the electron response to the equilibrium consisting of a sheared magnetic field and a ponderomotive force generated by the wave mixing phenomena. The equilibrium distribution function is to be constructed from the constants of motion obtained, in the presence of the equilibrium force, and is given by [17]

$$f_{eo} = \left(\frac{m_e}{2\pi T_e}\right)^{3/2} \exp\left[-\frac{m_e}{2T_e}(v_\perp^2 + v_\parallel^2) + \frac{(F_{x_0} x + F_{y_0} y)}{T_e} + \frac{F_{\parallel 0} z}{T_e}\right]\left[1 - \epsilon'\left(x + \frac{v_y}{\omega_{ce}}\right)\right] \quad \ldots 6.5$$

where $F_{\perp 0} = \hat{x} F_{x0} + \hat{y} F_{y0}$ and $F_{\parallel 0}$ are the perpendicular and parallel ponderomotive forces given by equations (6.4a) and (6.4b) respectively, and $\epsilon' = \frac{1}{L_n}$ the scale length of inhomogeneity. The perturbed distribution function of the electrons is obtained from the linearised Vlasov equation with Krook collision term,

$$\left[\frac{\partial}{\partial t} + v\cdot\nabla + \frac{e}{m_e}\frac{(v \times B_0)}{c} + \frac{F_0}{m_e}\cdot\nabla_v\right] f_1 = \frac{e}{m_e}\left[E_1 + \frac{v \times B_1}{c}\right]\nabla_v f_{eo} + \left(\frac{\partial f}{\partial t}\right)_c \quad \ldots 6.6$$

where

$$\left(\frac{\partial f}{\partial t}\right)_c = -\nu\left(f_1 - \frac{n_1}{n_0} f_{eo}\right)$$

(ν is the collision frequency)

We consider the response of this equilibrium to a general perturbation of the form

$$Q = Q(x)\, \exp i[k_{z_t} z + k_{y_t} y - \omega_t t]$$

Where Q is any physical quantity. The perturbed fields are represented by a vector potential $\vec{A}(\vec{x},t)$ and a scalar potential $\phi(\vec{x},t)$ as

$$\vec{B} = \nabla \times \vec{A} \quad , \quad \vec{E} = \frac{i\omega\vec{A}}{c} - \nabla\phi \qquad \ldots\ 6.7$$

'A' is driven by the perturbed plasma currents

$$\nabla^2 \vec{A} = -\frac{4\pi}{c}(\vec{J}_e + \vec{J}_i) \qquad \ldots\ 6.8$$

The current $\vec{J}$ which is produced by the induced electric field is primarily along the $\hat{z}$ directions, so the vector potential $\vec{A}$ defined in equation (6.7) is dominated by its $\hat{z}$ component and therefore the perpendicular component $(A_\perp)$ can be neglected. The parallel response of the electrons in the induced field is much larger than that of the ions, so $J_{ze} > J_{zi}$. Further the radial gradients are much larger than the spatial variations in the other two directions i.e. $\frac{\partial}{\partial x^2} >> k_{y_t}^2, k_{z_t}^2 (\sim a^{-2})$

Equation (6.8), therefore simplifies to

$$\frac{d^2 A_{\parallel}}{dx^2} = -\frac{4\pi}{c} J_{ze} \quad \text{... 6.9}$$

The orbit equations of the electrons in the presence of the ponderomotive force are obtained and substituted in equation (6.6) to derive the linearised electron response. The trajectory of the particles is obtained from the equations

$$\frac{d\vec{x}}{dt} = \vec{V}$$

$$\frac{d\vec{V}}{dt} = \omega_{ce}(\vec{V} \times \hat{z}) + \frac{\vec{F}_x x}{m_e} + \frac{\vec{F}_y y}{m_e} + \frac{\vec{F}_{\parallel} z}{m_e} \quad \text{...6.10}$$

Using the above equations in equation (6.6), the perturbed electron density is readily calculated to give

$$n_{e1} = \int d^3v\, f_{e1} \quad \text{... 6.11}$$

$$n_{e1} = \frac{ieE_{\parallel}}{k_{\parallel} T_e}\left[1 + \frac{\Omega}{(k_{\parallel}^2 V_e^2 - a)^{1/2}} Z\left[\frac{\overline{\Omega}}{(k_{\parallel}^2 V_e^2 - a)^{1/2}}\right]\right.$$
$$- \frac{E_y \epsilon' e}{m_e \omega_{ce}^2}\left[\frac{1}{(k_{\parallel}^2 V_e^2 - a)^{1/2}} Z\left[\frac{\overline{\Omega}}{(k_{\parallel}^2 V_e^2 - a)^{1/2}}\right]\right]$$
$$+ \frac{iB_x e\epsilon'}{k_{\parallel}\omega_{ce} m_e}\left[1 + \frac{\Omega}{(k_{\parallel}^2 V_e^2 - a)^{1/2}} Z\left[\frac{\overline{\Omega}}{(k_{\parallel}^2 V_e^2 - a)^{1/2}}\right]\right]$$
$$- \frac{i\nu n_{e1}}{(k_{\parallel}^2 V_e^2 - a)^{1/2}} Z\left[\frac{\overline{\Omega}}{(k_{\parallel}^2 V_e^2 - a)^{1/2}}\right] \quad \text{...6.12}$$

where $a = \frac{2i F_{\parallel o} k_{\parallel t}}{m_e}$, $\Omega = \omega_t + i\nu$ and $\bar{\Omega} = \Omega - k_{yt} v_{yo}$ is the Doppler shifted frequency due to the $(\vec{F}_\perp \times \vec{B}_o)$ drift.

We note that the equilibrium perpendicular force gives rise to a Doppler shift of the mode frequency from Ω to $\bar{\Omega}$ due to the $(F_\perp \times B_o)$ drift. However the frequency shift $\bar{\Omega} - \Omega = k_{yt} v_{yo}$ which is proportional to the electron Larmor radius is very small and plays no significant role in the mode dynamics. We shall therefore omit this effect henceforth in our analysis. The parallel ponderomotive force on the other hand alters the characteristics of the wave-particle response of the electrons. The resonant interaction term undergoes a shift from $\frac{\omega_t}{k_{\parallel t} v_e}$ to $\frac{\omega_t}{[k_{\parallel t}^2 v_e^2 - a]^{1/2}}$ with the Landau interaction occurring at $\omega_t \sim (k_\parallel^2 v_e^2 - a)^{1/2}$. On account of the fact the modification is spatially dependent, the resultant effect induces a spatial resonance broadening of the electron response.

The parallel perturbed electron current density is then calculated in a straight forward manner from the relation

$$J_{\parallel e} = e n_o \int_{-\infty}^{\infty} v_\parallel f_{e1} \, dv_\parallel \qquad \ldots 6.13$$

with

$$J_{\parallel e} = \frac{i e E_\parallel}{T_e} \frac{(\omega - \omega_{*e})}{\left(k_{\parallel t}^2 - \frac{a}{v_e^2}\right)} \frac{[1 + \zeta Z(\zeta)]}{\left[1 + \frac{i\nu}{(k_{\parallel t}^2 v_e^2 - a)^{1/2}} Z(\zeta)\right]} \qquad \ldots 6.14$$

where $E_{\parallel}$ expressed in terms of the potentials $\vec{A}$ and ϕ is given by $E_{\parallel} = \frac{i\omega}{c} A_{\parallel} - \nabla_{\parallel}\phi$ ..6.14a

Equation (6.14), can be written as

$$J_{\parallel} = \sigma E_{\parallel} \qquad ..6.15$$

where σ is the conductivity profile. Equation (6.15) is the generalised Ohm's law which describes the force balance equation in the parallel direction.

The conductivity profile is now a complicated function of the radial co-ordinate 'x' through its dependence not only on $k_{\parallel}(x) v_e$ but also on the factor which is a function of $F_{\parallel o}$. In chapter II, we pointed out that the parallel ponderomotive force did not manifest itself in the fluid model. In fact, the parallel component of Ohm's law does not contain the parallel force, while the conductivity profile given by equation (6.15), is a function of the parallel force. This apparent contradiction is resolved by the fluid limit of equation (6.15) and demonstrating that in this limit, the parallel force, cancells exactly. The hydrodynamic limit is taken by imposing the condition that $\left|\frac{(\omega + i\nu)}{(k_{\parallel t}^2 v_e^2 - a)^{1/2}}\right| \gg 1$. Making a large argument expansion of the plasma dispersion function the expression for the conductivity becomes

$$\sigma = \eta^{-1} \qquad ..6.16$$

where η is the classical Spitzer resistivity and the parallel component of Ohm's law reduces to the standard form $E_{\parallel} = \eta J_{\parallel}$

Expressing $J_{\parallel}$, $E_{\parallel}$, the perturbed current and parallel electric field variations in terms of the potentials ϕ, $A_{\parallel}$ using equations (6.9) and (6.14a) equation (6.15) becomes

$$\frac{c}{4\pi}\frac{d^2A_{\parallel}}{dx^2} = \sigma\left[\frac{i\omega_k A_{\parallel}}{c} - \nabla_{\parallel}\phi\right] \qquad \ldots\ 6.17$$

The other relation between ϕ and $A_{\parallel}$ is given by the equation which describes the ion dynamics. A heuristic method for investigating the ion response is provided by the momentum conservation law. The ponderomotive force generated by the kinetic Alfven waves couples to the fluid density perturbation. It had been shown earlier (chapter II) that this effect is negligible within the tearing layer. We shall therefore omit the coupling of the P.F. to the density perturbation. The equation which describes the ion dynamics then has the standard form [4] .

$$ck_{\parallel t}\frac{d^2A_{\parallel}}{dx^2} = \left(\frac{c}{V_A}\right)^2(\omega_k - \omega_{*i})\frac{d^2\phi}{dx^2} \qquad \ldots\ 6.18$$

Equations (6.17) and (6.18), provide the coupled system of equations in ϕ and $A_{\parallel}$ describing the dynamics of the tearing mode in the collisional and collisionless regimes. To solve the system of equations and obtain eigen values in the presence of the parallel force, we first cast the equations into a more convenient form. We expand $k_{\parallel t}$ as $k'_{\parallel t}x$ around the mode rational surface defined at $x = 0$. In terms of x, equations (6.17) and (6.18) reduce to respectively

$$\frac{d^2\psi}{dx^2} = \frac{\sigma}{x}\left[\frac{\psi}{x} - \phi\right] \qquad \ldots 6.19$$

$$\frac{d^2\phi}{dx^2} = \frac{\sigma}{x_A^2}\left[\frac{\psi}{x} - \phi\right] \qquad \ldots 6.20$$

In equations (6.19) and (6.20) σ is given by

$$\sigma = -\frac{x_A^2(\omega_t - \omega_{*e})}{(\omega_t + \omega_{*i})\left(1 - \frac{a}{k_{\parallel t}^2 V_e^2}\right)} \frac{[1 + \zeta Z(\zeta)]}{\left[1 + \frac{i\nu Z(\zeta)}{(k_{\parallel t}^2 V_e^2 - a)^{1/2}}\right]}$$

with

$$x_A^2 = \frac{\omega_t(\omega_t + \omega_{*i})}{k'^2_{\parallel t} V_A^2} \qquad \zeta = \frac{\omega_t + i\nu}{(k_{\parallel t}^2 V_e^2 - a)^{1/2}} \qquad \ldots 6.21$$

The other parameters in the equations (6.19) and (6.20) are the same as those defined in the earlier chapters.

Equations (6.19) and (6.20) are now in the standard form for which methods of solutions have been developed. Recently treatments based on variational techniques [4] have brought considerable simplification to the solution of the eigen mode problem. The method allows uniform analytical treatment in addition to enabling handling of more complicated models of plasma behaviour, readily. The coupled equations are first reduced to a simple second order differential equation in $y = (\frac{\psi}{x} - \phi)$. We observe that equation (6.20) can be written as,

$$\frac{d^2\phi}{dx^2} = \frac{\nabla}{x_A^2} y = \frac{x}{x_A^2} \frac{d^2\psi}{dx^2} \qquad \ldots(6.22)$$

and further $x\frac{d^2\psi}{dx^2}$ can be expressed as

$$x\frac{d^2\psi}{dx^2} = \frac{d}{dx}\left[x^2 \left(\frac{\psi}{x}\right)'\right] \qquad \ldots(6.23)$$

Substituting equation (6.23) into (6.22) and integrating once, we obtain

$$\frac{d\phi}{dx} = \frac{1}{x_A^2}\left[x^2 \left(\frac{\psi}{x}\right)'\right] + C \qquad \ldots(6.24)$$

where C is the constant of integration. Differentiating equation (6.21a) once and eliminating $\left(\frac{\psi}{x}\right)'$ between the resulting equation and (6.24) we obtain

$$\phi' = \frac{x^2 y'}{x_A^2 - x^2} + \frac{C}{x_A^2 - x^2} \qquad \ldots (6.25)$$

Differentiating equation (6.25) and eliminating ϕ'' between the resulting equation and (6.22) the required second order equation is y is obtained.

$$\frac{d}{dx}\left[\frac{x^2 y'}{x^2 - x_A^2}\right] + \frac{\sigma}{x_A^2} y = -\frac{2Cx}{(x^2 - x_A^2)^2}$$

$$\ldots (6.26)$$

where the constant C in equation (6.24) is related to Δ' of the kink tearing theory by

$$C = -(\Delta')^{-1} \int_{-\infty}^{\infty} \frac{\sigma y}{x} \, dx \qquad \ldots (6.27)$$

Equation (6.26) can then be cast into the self adjoint form [4],

$$\left[\frac{x_A^2 x^2 y'}{x^2 - x_A^2}\right]' + \sigma y = -\left(\Delta' + \frac{i\pi}{x_A}\right)^{-1} \frac{\left[4 x x_A^2\right]}{(x^2 - x_A^2)^2} \times \int_{-\infty}^{\infty} \frac{x y}{(x^2 - x_A^2)} \, dx$$

$$\ldots (6.28)$$

Equation (6.28) is an integro-differential equation, which is amenable to variational treatment. The variational method consists in constructing a functional 'S', by means of an appropriate trial function which is variational. The equations $S=0$, $\frac{dS}{d\alpha}=0$ (α is the variational parameter) are to be solved simultaneously to obtain the eigen values. In the next section, we solve for the eigen values of the equation (6.28) using variational methods.

6.4 Variational solutions for tearing modes :

In this section, we solve for the eigenvalues of equation (6.28) by setting up a variational principle for the variable 'y'. Multiplying equation (6.28) by y and integrating from $-\infty$ to $+\infty$, we obtain

$$S = \left(\Delta' + \frac{i\pi}{x_A}\right)\left(I_1 + I_2\right) + I_3^2 \qquad \ldots\ 6.29$$

where I_1, I_2 and I_3 are integrals given by

$$I_1 = \int_{-\infty}^{\infty} y \left[y' \frac{x^2 x_A^2}{(x^2 - x_A^2)} \right] dx \qquad \ldots\ 6.30$$

$$I_2 = \int_{-\infty}^{\infty} \tau y^2 \, dx \qquad \ldots\ 6.31$$

$$I_3 = 2x_A^2 \int_{-\infty}^{\infty} \frac{xy}{(x^2 - x_A^2)} \, dx \qquad \ldots\ 6.32$$

It can readily be shown that the functional defined by equation (6.29) is variational in that $\delta S = 0$ generates the equation (6.28) and hence equations (6.19) and (6.20). The next step is to choose an appropriate trial function for y, evaluate the integrals and solve the set of equations $\frac{dS}{d\alpha} = 0$ and $S = 0$ to obtain the dispersion relation as well as the variational parameter α. The choice of the trial function is dictated by the nature of the mode we need to study. The tearing modes are characterised by localised symmetric solutions of $E_{\parallel}$ which tear the magnetic surfaces. The trial function for $E_{\parallel}$ therefore must be even. We deal with modes of the tearing symmetry and choose the trial function for $y = \frac{E_{\parallel}}{x}$, as $e^{-\alpha x^2}$, with $Re\,\alpha > 0$. 'α' is the variational parameter and the condition $Re\,\alpha > 0$ is imposed to ensure the well behavedness of the solutions.

With this choice of y, I_1 and I_3 can be readily calculated.

$$I_1 = -\pi^{1/2} x_A^2 \left[\alpha^{3/2} + \frac{2\alpha^{1/2}}{x_A} + \left(\frac{1}{x_A^3} + \frac{2\alpha}{x_A} + \alpha^2 x_A \right) Z(\sqrt{\alpha}\, x_A) \right] \quad \ldots 6.33$$

$$I_3 = -\pi^{1/2} \left[(2\alpha)^{1/2} + \left(\alpha x_A + \frac{1}{x_A} \right) Z\left[\left(\frac{\alpha}{2}\right)^{1/2} x_A \right] \right] \quad \ldots 6.34$$

here Z is the plasma dispersion function. To complete the evaluation of S, we need to determine $I_2 = \int \frac{\sigma}{x^2} e^{-\alpha x^2}$

Inspite of the complicated x dependence of σ , I_2 can be expressed in terms of known functions for many cases of interest. In the presence of the non-linear interaction, however, the modification introduced by the parallel force in the conductivity profile makes the integral complicated. We first study the modifications introduced by the equilibrium parallel force in the limit of collisionless plasmas.

The electron parallel conductivity can be described by a collisionless model when ever the electron collision frequency ν is much smaller than the mode frequency ω_t. The latter is typically of the order of the electron diamagnetic drift frequency ω_{*e}. For this case σ is given by

$$\sigma = - x_A^2 \frac{(\omega_t - \omega_{*e}) \left[1 + \zeta Z(\zeta) \right]}{(\omega_t + \omega_{*i}) \left[1 - \frac{a}{k_{\parallel t}^2 V_e} \right]} \qquad \ldots 6.35$$

where

$$\zeta = \frac{\omega_t}{(k_{\parallel t}^2 V_e^2 - a)^{1/2}}$$

Despite the simplified form for the conductivity profile, the exact evaluation of the integral I_2 is not possible. We therefore obtain approximate values of the integral by treating the term proportional to the parallel force i.e. $\frac{a}{k_{\parallel t}^2 V_e^2}$ in equation (6.33) as a small parameter, $(\epsilon \ll 1)$ and accordingly make Taylor expansion of the plasma dispersion function. The assumption

$\frac{a}{k_{\parallel t}^2 V_e^2} \ll 1$ implies that $\frac{F_{\parallel o}}{m_e V_e^2} \ll k_{\parallel t}$ (with $F_{\parallel o}$ defined by equation (6.46)) and puts an upper bound on the pump amplitude. However for nominal levels of Alfven fluctuations, this condition is readily fulfilled.

With this approximation the expression for the conductivity reduces to

$$\sigma = -x_A^2 \frac{(\omega_t - \omega_{*e})}{(\omega + \omega_{*i})} \left[1 + \frac{a}{k_{\parallel t}^2 V_e^2}\right] \left\{ \left[1 + \frac{\omega_t}{k_{\parallel t} V_e} Z\left(\frac{\omega}{k_{\parallel t} V_e}\right)\right] \right.$$

$$+ \frac{1}{2} \frac{a}{k_{\parallel t}^2 V_e^2} \left(\frac{\omega_t}{k_{\parallel t} V_e}\right) Z\left(\frac{\omega_t}{k_{\parallel t} V_e}\right) +$$

$$\frac{1}{2} \left(\frac{a}{k_{\parallel t}^2 V_e^2}\right)^2 Z'\left(\frac{\omega_t}{k_{\parallel t} V_e}\right)$$

The integral I_2 with the conductivity profile defined by equation (6.35), can now be readily evaluated.

$$I_2 = -x_A^2 \frac{(\omega_t - \omega_{*e})}{(\omega_t + \omega_{*i})} \int_{-\infty}^{\infty} \frac{e^{-\alpha x^2}}{x^2} \left[\left\{ 1 + \frac{x_e}{x} Z\left(\frac{x_e}{x}\right)\right\} + \right.$$

$$\left. \frac{A}{x^2} Z\left(\frac{x_e}{x}\right) + \frac{B}{x^3} \left[1 + \frac{x_e}{x} Z\left(\frac{x_e}{x}\right)\right] \right] dx$$

$$\left\{ A = \frac{3 i F_{\parallel o}}{m_e V_e^2} \left(\frac{\omega_t}{k_{\parallel t}'^2 V_e}\right), \quad B = -\frac{2 i F_{\parallel o}}{m_e k_{\parallel t}' V_e^2} \left(\frac{\omega_t}{k_{\parallel t}' V_e}\right)^2 \right.$$

$$\left. x_e = \left(\frac{\omega_t}{k_{\parallel t}' V_e}\right) \right\}$$

The expression for I_1, I_2, I_3 are then substituted in equation (6.29), to obtain the expression for the functional S. The resulting equation is an extremely complicated function of α and to obtain analytical solutions retaining the exact form for S is a forbidding task. It is possible however to solve for the dispersion relation in various interesting limits. One such class of modes which have been extensively studied are the current channel modes [18] .

It hasbeen pointed out by earlier investigation [18] that in the case of kinetic drift tearing modes, there exist two singular layers. The outer singular layer is the region within which the parallel electric field is significant and the typical variations of this region is $|x| \gg \frac{\omega}{k_{\parallel t}' v_e}$. There is also, an inner singular layer the variations of which typically are $|x| \sim \frac{\omega}{k_{\parallel t} v_e}$. Physically the electrons can be readily accelerated by the parallel electric field only if $\omega \gg k_{\parallel t} v_e$, so that most of the perturbed current lies within this inner region. The width of the current channel λ_σ is the typical scale length over which the conductivity varies [4] . Whenever the extent of the field variable '$E_\parallel$' is greater than λ_σ , the modes have a current channel or alternately they are called current channel modes. The interesting class of modes are likely to occur in the shear and 'β' range of present day tokamaks. In fact it has been shown that the condition that the width of the outer layer, i.e. variations of $E_{\parallel}$ be larger than the inner layer, (the variations of conductivity profile) becomes $\frac{r_n}{L_s} > \left(\frac{m_e}{m_i}\right)^{1/2}$, a condition that is usually satisfied in tokamaks.

We study the modifications introduced in these current channel modes by the parallel force. These modes have widths such that X_ω (the mode width) is much larger than x_e $\left(= \frac{\omega}{k_{\parallel}' v_e}\right)$. This leads to the condition that $|\alpha^{1/2} x_e| \ll 1$, where $\frac{1}{\sqrt{\alpha}}$ is the mode width.

Expanding I_2 for $|\sqrt{\alpha}\, x_e| \ll 1$, we write the functional in the form,

$$S = S_0 + \sqrt{\alpha}\, S_1 + \alpha\, S_2 \qquad \ldots\ 6.38$$

where $S_0 =$

$$\Delta' \left[-\frac{i\pi}{x_A} - \frac{i\pi x_A^2}{x_e} \frac{(\omega_t - \omega_{*e})}{(\omega_t + \omega_{*i})} \right] +$$

$$\pi^{3/2} \frac{x_A}{x_e} \frac{(\omega_t - \omega_{*e})}{(\omega_t + \omega_{*i})} + \left[\Delta' + \frac{i\pi}{x_A}\right] \left\{ x_A^2 \frac{(\omega_t - \omega_{*e})}{(\omega_t + \omega_{*i})} \times \right.$$

$$\left. \frac{Ai\pi}{2x_e^3} - \frac{3}{4} x_A^2 \frac{(\omega_t - \omega_{*e})}{(\omega_t + \omega_{*i})} \frac{i\pi B}{x_e^4} \right\}$$

$$S_1 = 2\left(\Delta' + \frac{i\pi}{x_A}\right)\sqrt{\pi}\; x_A^2 \times$$

$$\frac{(\omega_t - \omega_{*e})}{(\omega_t + \omega_{*i})}$$

$$S_2 = i\pi\Delta' x_A - \left[\Delta' + \frac{i\pi}{x_A}\right]\left\{\frac{2}{x_e}\, x_A^2 A \times\right.$$

$$i\pi\frac{(\omega_t - \omega_{*e})}{(\omega_t + \omega_{*i})} + \frac{6B\, i\pi\, x_A^2}{x_e^2}\frac{(\omega_t - \omega_{*e})}{(\omega_t + \omega_{*i})}$$

$$- \frac{i\pi B\, x_A^2}{x_e^2}\frac{(\omega_t - \omega_{*e})}{(\omega_t + \omega_{*i})}$$

A simultaneous solution of $\frac{ds}{d\alpha} = 0$ and $S = 0$ yields

$\sqrt{\alpha} = -\frac{S_1}{2S_2}$ and the dispersion relation is

$$S_0 = \alpha S_2 = \frac{S_1^2}{4S_2} \qquad \text{... 6.38a}$$

We determine the eigenvalues ω by solving equation (6.38a), keeping in mind the consistency criteria $Re\,\alpha > 0$, which ensures that the solutions are well behaved at $x = \pm\infty$. We obtain

$$(\omega - \omega_{*e})\left\{1 - \left[\Delta' + \frac{i\pi\, k_{\parallel}' V_A}{\omega}\right]\left[\frac{3 F_{\parallel o}\sqrt{\pi}}{k_{\parallel}' V_e^2 m_e}\,\frac{V_A}{V_e}\right]\right\}$$

$$= i\gamma_L \qquad \text{...6.39}$$

where $$\gamma_L = \frac{\Delta' k_{\parallel}' V_A^2}{\sqrt{\pi}\, V_e} \qquad \text{... 6.40}$$

is the growth rate of the collisionless tearing mode discussed by Laval et al [13] for the m $\geqslant$ 2 modes.

When $(\Delta')^{-1}$ is large that is $\Delta' << \left(\frac{\pi}{k_{\parallel}' x_A}\right)$ and positive, the solutions for ω_t approaches the results of Laval et al, with modifications from the parallel force. In this limit the dispersion relation becomes

$$(\omega_t - \omega_{*e}) = i\gamma_L \left[1 - \frac{3F_{\parallel o}\Delta'}{m_e k_{\parallel}' V_e^2}\left(\frac{V_A}{V_e}\right)\right] \qquad \ldots\ 6.41$$

where the expression in the parenthesis is the effect produced by the parallel ponderomotive force generated by the wave mixing phenomenon. It is to be noted that the growth rate of the collisionless tearing mode is a linear function $F_{\parallel o}$, and therefore the sign of $F_{\parallel o}$, plays a crucial role in deciding the stability of the mode. For certain combination of parameters i.e. for $\frac{k_{x_1}}{k_{x_2}} > \frac{k_{y_1}}{k_{y_2}}$ the parallel ponderomotive force $F_{\parallel o}$, has a positive sign and therefore by equation (6.41) has a destabilising effect on the mode. For the reverse inequality, $F_{\parallel o}$ has a stabilising effect on the collisionless tearing modes. For typical tokamak parameters given in chapter II, the enhancement factor although not large is $\lesssim O(1)$.

Collisional limit

In this limit, the full parallel conductivity profile σ, defined by equation (6.21) including the electron-ion collisions ν has to be retained in the integral I_2. The width of the electron layer x_c in this regime is now given by $x_c = \left| \frac{\omega + i\nu}{k'_{\parallel t} v_e} \right|$. This value could much larger than its collisionless counter part $x_e = \left| \frac{\omega}{k'_{\parallel t} v_e} \right|$. To solve for the eigenvalues in this regime, the integral with full conductivity profile has to be evaluated. As pointed out earlier, this cannot be done exactly. We therefore make some simplifying approximations.

In the large ν limit, the electron layer becomes large and tearing modes with widths much less than the electron layer become important. For these modes the conductivity can be brought to a standard form with appropriate contributions from the parallel force, by making use of the assumption that x_c, the width of the electron layer is much larger than the width of the tearing mode under consideration. Making a large argument expansion of the plasma dispersion function we obtain [4],

$$\sigma = \frac{\bar{x}^2 (x^2 - x\bar{b})}{\left[x_R^2 - (x^2 - x\bar{b})\right]} \qquad \ldots 6.42$$

where

$$\bar{x}^2 = \frac{(\omega_t - \omega_*)(\omega_t + i\nu)}{i\nu k'_{\parallel} v_A}, \quad x_R^2 = \frac{2\omega_t}{i\nu} \frac{(\omega_t + i\nu)}{k'_{\parallel} v_A^2}, \qquad \ldots 6.43$$

$$\bar{b} = \frac{2iF_{\parallel 0}}{m_e} \times \frac{1}{k'_{\parallel t} v_e^2}.$$

X_R defined in the above equation defines another scale length which lies between x_c and X_e . If the collision frequency is increased still further, such that X_R exceeds the spatial width of the mode, σ takes on its classical collision dominated form

$$\sigma = x^2 \left(\frac{x}{x_R}\right)^2 \qquad \ldots\ 6.44$$

for which solutions are known.

The parallel force in this regime identically cancells which is consistent with our analysis of the resistive tearing modes in chapter II, .

The conductivity in the regime defined by equation (6.42) varies on a scale length $|X_R|$, and therefore the width of the current channel is determined by X_R .

To obtain the functional, we now have to evaluate the integral $I_2 = \int \frac{\sigma}{x^2} e^{-\alpha x^2} dx$ with given by equation (6.42). As before we treat '$\bar{b}$' as a perturbation parameter and solve the coupled equations $S=0$ and $\frac{d\delta}{d\alpha}=0$ simultaneously, to obtain the eigenvalues.

We obtain modifications to the dispersion relation obtained by Drake and Lee [3] , due to the presence of the external ponderomotive force in this regime.

$$\omega^{1/2}(\omega - \omega_{*e}) = i^{3/2}\left[(2\pi\nu)^{1/2}\frac{k_{\parallel}' V_A}{\pi}\left(\frac{V_A}{V_e}\right)\right]\left[1 + i^{3/8}\left[\frac{\bar{b}^2\sqrt{\pi}}{\nu x_R} V_e k_{\parallel}'\left(\frac{V_e}{V_A}\right)\right]\right] \qquad \ldots\ 6.45$$

Solving for the growth rates, we find that the linear growth rates predicted by Drake and Lee [3] contained in the first paranthesis of r.h.s.) are modified by the parallel P.F. produced by the Alfvenic fluctuations. Equation (6.45) demonstrates that the contribution from the parallel force contained in the second paranthesis is proportional to $\bar{b}^2$ and therefore to $F_{\parallel o}^{\,2}$. Hence unlike the results of the collisionless regime, this implies that the modifications introduced by $F_{\parallel o}$, are independent of the sign of the parallel force. However we note that these corrections in equation (6.45), are only of the second order and therefore not very significant. In fact our calculations of these modifications due to the parallel force in the collisional regime indicate, that these corrections are of no importance.

6.5 Interaction of the ponderomotive force with collisionless drift waves :

In this section, we study the effect of the ponderomotive force generated by the Alfven waves on the collisionless drift waves. These waves have been extensively studied in the last few years on account of their wide application in both laboratory and space plasmas. They are endemic to tokamak plasmas and are considered to be the cause for the observed anomalous diffusion. Although within the framework of linear theory the drift waves have been shown to be stable in a sheared magnetic field, in slab geometry, they could exhibit highly non-linear behaviour. Several investigations have to be devoted to the understanding of this feature.

In this context in the previous chapter, we had investigated the parametric excitation of drift waves through decay of a mode converted kinetic Alfven wave in an inhomogeneous plasma and established the existence of various drift instabilities. In this section, we consider a non-linear interaction between two pump kinetic Alfven waves and a drift wave. We investigate the effect of ponderomotive force (generated by two interacting kinetic Alfven waves) (derived in section 6.1) on the drift mode.

We consider the plasma in slab geometry with gradient in density along the 'x' direction and with a sheared magnetic field given by

$$\vec{B} = B_0 \left[\hat{e}_z + \frac{x}{L_s} \hat{e} \right] \qquad \ldots 6.46$$

Where L_s is the shear length.

The gradient in density is assumed to be weak in the sense that

$$\rho_i \left| n^{-1} \frac{dn}{dx} \right| << 1 \qquad \ldots 6.47$$

where ρ_i is the ion Larmor radius.

Using simple fluid model, we first describe the propagation characteristics and the linear stability properties of drift waves. In the equilibrium described by equations (6.46) and (6.47), the electrons drift with a velocity, $v_o = -cT_e/e\omega_{ce} \frac{1}{n_o}\frac{dn}{dx}$. For simplicity we take the ions to be cold. For longitudinal oscillations, assuming the electric field to be curl free, and writing $E = -\nabla\phi$, the perturbation of the electron density 'n_e' which follows the Boltzman distribution, can be expressed in terms of ϕ

$$\frac{n_{e1}}{n_o} = \frac{e\phi}{T_e} \qquad \ldots 6.48$$

Physically the above relation implies that the massless electrons attain thermal equilibrium very fast along the field lines. The ion density perturbation can be readily obtained from the continuity equation,

$$\frac{\partial n_{i1}}{\partial t} + \nabla\cdot(n_o\vec{v}_{i1}) = 0 \qquad \ldots 6.49$$

where $\vec{V}_i$, is the macroscopic velocity and can be determined from the linearised equation of motion

$$\frac{\partial\vec{v}_i}{\partial t} = -\frac{e}{m_i}\nabla\phi + \frac{e}{m_i c}(\vec{v}_i\times\vec{B}_o) \qquad \ldots 6.50$$

For $\omega \ll \omega_{ci}$, the second term can be neglected and then the velocity of the ions is determined by the drift in the electric field. Substituting the value of $\vec{V}_i$ obtained into the equation of continuity, we obtain

$$\frac{n_{i1}}{n_o} = \frac{\omega_{*e}}{\omega} \frac{e\phi}{T_e} \qquad \ldots\ 6.51$$

where ω_{*e} is the diamagnetic drift frequency of the electrons given by $- k_y T_e/eB_o \frac{1}{L_n}$.

The effect of the inhomogeneity on the oscillation frequency arises from the transverse drift of the ions. By itself the motion is incompressible i.e. $\nabla \cdot \vec{V}_{\perp i} = 0$ and in a homogeneous plasma does not lead to a change in density. However in an inhomogeneous plasma, even an incompressible displacement leads to a perturbation of the density n_i.

Using quasineutrality condition, solving equations (6.48) and (6.51), the dispersion relation is obtained

$$\omega = \omega_* = k_y v_{yo} \qquad \ldots\ 6.52$$

To understand why the drift waves are unstable, one must realise that $\vec{V}_i$ is not quite $\frac{\vec{E}_y}{B_o}$ (the electric field drift) for the ions. There are corrections due to polarisation drift and non-uniform 'E' drift. The result of these drifts is always to make the potential ϕ lag behind the density perturbation.

As long as the electrons are free to move along $\vec{B}_0$ to cancel space charge, the Boltzmann relation is fulfilled and the drift wave is stable. There are however several effects that may limit the mobility of the electrons. These effects are generally more important for small $k_{\parallel}$ and could be electron ion collision, Landau damping. If the electrons are not able to move completely freely there will appear a phase shift corresponding to a time lag between density and potential. We then write

$$\frac{n_{e1}}{n_0} = \frac{e\phi}{T_e}\left[1 - i\delta\right] \qquad \ldots 6.53$$

where 'δ' signifies the phase lag. Subsequently the dispersion relation gets modified to

$$\omega = \omega_* \left(1 + i\delta\right) \qquad \ldots 6.54$$

(if we assume $|\delta| \ll 1$)

We note that a time variation $\exp(-i\omega t)$, $\delta > 0$, means that the potential lags behind the density. This situation corresponds to an instability, the energy for which arises due to the spatial gradient.

In a collisionless plasma, the instability arises from the interaction between drift waves and resonant particles, which must be represented on the basis of kinetic considerations. It is straight forward to show that the growth rate is proportional to $(\omega - \omega_*)$ where ω is the frequency of the oscillating perturbations. Any effect which shifts the oscillation frequency from the value ω_* leads to growth (or damping) of the drift waves.

In order to describe drift waves in an inhomogeneous system with magnetic shear, a differential equation for the field variation in 'x' has to be solved, and the solution for the mode frequency becomes an eigenvalue problem. The effect of the magnetic shear will be to twist the magnetic field. In a tokamak for example a toroidal eigen mode will also be twisted according to its toroidal and pooidal numbers. At a certain value of 'r', where 'r' is the radial co-ordinate, the drift eigen mode has the same degree of twisting as the magnetic field and the wave number parallel to the field, i.e. $k_{\parallel} = 0$. The drift waves have the strongest tendency of instability, for small $k_{\parallel}$ where the electron shielding is inefficient. They are generated near $k_{\parallel} = 0$ and propagate towards larger 'r'. When $k_{\parallel}$ has grown so that $k_{\parallel} V_{i_{th}} = \omega$ (where $V_{i_{th}}$ is the ion thermal velocity) the ion Landau damping sets in and absorbs the wave.

In order to have an absolute instability the growth rate of the drift instability must exceed the damping due to ions. It has been shown both analytically and numerically that both the collisional and universal drift waves are stable. In toroidal systems with poloidal variations of $\vec{B}_o$, however, toroidal couplings may introduce absolute instabilities. It is clear that since the shear in the magnetic field plays an important role in the stability properties of the drift wave, any modification in the shear effect alters the mode characteristics. In this context, a study of the effect of P.F. on the drift mode, is an important problem, for as demonstrated in the previous section, the equilibrium P.F. significantly modifies the electron response and alters the mode characteristics.

In the equilibrium described by equations (6.46) and (6.47), we consider two mode converted kinetic Alfven waves interacting to generate the ponderomotive force (F). We consider a general perturbation of the equilibrium of the form

$$Q = Q(x) \exp i(k_y y + k_z z - \omega t) \qquad \ldots 6.55$$

Where Q is any physical quantity.

To retain the effects of the shear, we model the electron response by the Vlasov equation.

As demonstrated in the previous analysis the electron response to the equilibrium constructed from the constants of motion is given by

$$f_{eo} = \frac{m_e}{(2\pi T_e)^{3/2}} \exp\left[-\frac{m}{2T_e}(v_\perp^2 + v_\parallel^2) + \frac{F_{x_0}x + F_{y_0}y}{T_e} + \frac{F_{\parallel 0}z}{T_e}\right]\left[1 - \epsilon'\left(x + \frac{v_y}{\omega_{ce}}\right)\right] \qquad \ldots\ 6.56$$

The perturbed electron distribution function is obtained from the linearised form of Vlasov equation. As shown earlier, the perpendicular ponderomotive force given by equation (6.4a), Doppler shifts the mode frequency, while the parallel P.F. gives rise to acceleration of particles along the field lines, leading to modification of the wave-particle response.

The perturbed electron density given by

$$n_{e1} = \int d^3v\, f_{e1} \qquad \ldots\ 6.57$$

is obtained as

$$\frac{n_{e1}}{n_0} = \frac{e\phi}{T_e}\left[1 + \frac{\omega - \omega_{*e}}{(k_\parallel^2 v_e^2 - a)^{1/2}} Z(\zeta)\right] \qquad \ldots\ 6.58$$

with $a = \dfrac{2i F_{\parallel 0} k_\parallel}{m_e}$, , and $\Omega = \omega - k\cdot V_0$

'Ω' represents the Doppler shift of the mode frequency corresponding to the $(\vec{F}_\perp \times \vec{B}_0)$ drift. We note however that this shift is proportional to the electron Larmor radius and therefore very small. As in the previous case, this effect shall be neglected, as it plays an insignificant role in the mode dynamics. The parallel ponderomotive force serves to modify the electron orbits around the resonance region. The electron wave-particle response is changed from $\left(\dfrac{\omega}{k_\parallel v_e}\right)$ to $\dfrac{\omega}{[k_\parallel^2 v_e^2 - a]^{1/2}}$.

This spatial broadening of the electron response due to the parallel force alters the mode characteristics considerably.

For the ions, however, we make the hydrodynamic approximation. The dynamics of the ions therefore are governed by the fluid equations. For simplicity we treat the ions as cold. For the electrostatic drift waves, since the electric field is curl free, it can be represented as the gradient of a scalar potential i.e.

$$E = -\nabla \phi \qquad \ldots\ 6.59$$

such that $$E_\perp = -i k_y \phi - \frac{d\phi}{dx} \qquad \ldots\ 6.60$$

Substituting the value of the perturbed velocity from equation (6.50) into equation (6.49), the expression for the perturbed ion density is obtained as

$$\frac{n_i}{n_0} = \frac{d^2\phi}{dx^2}\frac{e}{T_e}\frac{C_s^2}{\omega_{ci}^2} - k_y^2 \frac{e\phi}{T_e}\frac{C_s^2}{\omega_{ci}^2} + \frac{e\phi}{T_e}\omega_{*e} + \frac{k_{\parallel}^2 c_s^2}{\omega^2}\frac{e\phi}{T_e} \qquad \ldots\ 6.61$$

where $$k_{\parallel} = \frac{k \cdot B_0}{|B_0|} = k_z + \frac{x}{L_s} k_y \qquad \ldots\ 6.62$$

Using quasineutrality condition we now obtain the eigen mode equation with appropriate contributions from the parallel force.

$$\rho_s^2 \frac{d^2\phi}{dx^2} - k_y^2 \rho_s^2 \phi + \frac{\omega_{*e}}{\omega} + \frac{k_{\parallel}^2 c_s^2}{\omega^2}\phi$$

$$- \left[1 + \frac{\omega - \omega_{*e}}{[k_{\parallel}^2 v_e^2 - a]^{1/2}} Z(\zeta) \right] \qquad \ldots 6.63$$

...ere $a = \dfrac{2iF_{\parallel_0} k_{\parallel}}{m_e}$ and $\zeta = \dfrac{\omega}{[k_{\parallel}^2 v_e^2 - a]^{1/2}}$ ... 6.64

Since drift waves have the largest tendency for instability around $k_{\parallel} = 0$, we expand $k_{\parallel}$ in Taylor expansion as $k_{\parallel} = k_{\parallel}' x = k_y \frac{x}{L_s}$. The shear length L_s is the characteristic length over which the magnetic field changes direction.

Defining the dimensionless variable

$X = \dfrac{x}{\rho_s}$ equation (6.63) reduces to

$$\frac{d^2\phi}{dX^2} + \phi \left[\frac{\omega_{*e}}{\omega} - k_y^2 \rho_s^2 + \frac{k_{\parallel}'^2 \rho_s^2 c_s^2}{\omega^2} - \right.$$

$$\left. \left\{ 1 + \frac{\omega - \omega_{*e}}{[k_{\parallel}^2 v_e^2 - a]^{1/2}} Z(\zeta) \right\} \right] \qquad \ldots 6.65$$

In the absence of the electron resonant term, equation (6.65) is a Weber differential equation having solutions in terms of Hermite polynomials with discrete eigenvalues. Previous analysis [19] of the problem centred on the treatment of a Weber-like differential equation (including the electron resonant terms) using perturbation methods. In particular the destabilising effect of the inverse Landau damping as a perturbation on the eigen solutions of the Weber equation.

6.6 Variational solutions for the drift wave :

We now preceed to apply the variational method to equation (6.65). We construct a functional 'S' which is variational, in that $\delta S = 0$ reproduces equation (6.65). The functional so constructed is given by

$$\frac{S}{\langle\phi^2\rangle^{-1}} = -\int_{-\infty}^{\infty}\left(\frac{d\phi}{dx}\right)^2 dx + \left(\frac{\omega_*}{\omega} - k_y^2\rho_s^2 - 1\right)\int_{-\infty}^{\infty}\phi^2 dx + \frac{k_{\parallel}'^2\rho_s^2 c_s^2}{\omega^2}\int_{-\infty}^{\infty} x^2\phi^2\, dx - (\omega-\omega_*) I \qquad \ldots 6.66$$

In equation (6.66), $I = \int_{-\infty}^{\infty} \frac{1}{[k_{\parallel}^2 v_e^2 - a]^{1/2}} Z(\zeta)$ ... 6.67

We restrict ourselves to the lowest Pearlstein-Berk [20] mode and choose as a trial function the $n=0$ Hermite function [14] .

$$\phi = \exp\left(-\frac{i\alpha x^2}{2}\right) \qquad \ldots 6.68$$

where α is the variational parameter. The equations $S=0, \frac{dS}{d\alpha}=0$ have now to be solved simultaneously to obtain the dispersion relation. The problem of solving the differential equation (6.65) is now converted to that of evaluating the integral I in equation (6.67). In the absence of the parallel force the integral I can be evaluated exactly, however the presence of the ponderomotive force adds several complications, and renders the integral I difficult to solve exactly.

We therefore resort to approximate methods such as by treating the modiciations due to the parallel force as a perturbation on the shear terms. We assume that $\left| \frac{a}{k_{\parallel}^2 v_e^2} \right| << 1$. This implies that $\left| \frac{2i\ F_{\parallel o}}{m_e k_{\parallel} v_e^2} \right| << 1$ which puts an upper bound for the amplitude $|\phi_o|$ of the interacting Alfven waves. For nominal values of Alfven wave intensities and tokamak parameters, the above inequality is readily satisfied.

With this condition on the pump amplitude the integral I is expanded in terms of the small parameter and the functional equation (6.66) reduces to

$$\frac{S}{\langle \phi^2 \rangle^{-1}} = \int_{-\infty}^{\infty} \phi'^2 dx - \int_{-\infty}^{\infty} \phi^2 \left[\epsilon_* + \frac{k_{\parallel}'^2 \rho_s^2 c_s^2 x^2}{\omega^2} \right] dx -$$

$$a I_1 - c I_2 + d I_3 \qquad \ldots 6.69$$

where

$$\epsilon_* = \frac{\omega_*}{\omega} - (1 + k_y^2 \rho_s^2)$$

$$a = \frac{(\omega - \omega_{*e})}{k_{\parallel}' v_e \rho_s}, \quad c = \frac{i F_{\parallel o}}{m_e} \frac{(\omega - \omega_*)}{(k_{\parallel}' \rho_s)^2 v_e^3},$$

$$d = \frac{i F_{\parallel o}}{m_e} \frac{(\omega - \omega_{*e})}{(k_{\parallel}' \rho_s)^3 v_e^4}$$

$$I_1 = \int_{-\infty}^{\infty} \frac{\phi^2}{x} Z\left(\frac{b}{x}\right) dx, \quad I_2 = \int_{-\infty}^{\infty} \frac{\phi^2}{x^2} Z\left(\frac{b}{x}\right) dx$$

$$I_3 = \int_{-\infty}^{\infty} \frac{\phi^2}{x^3} Z'\left(\frac{b}{x}\right) dx, \quad b = \frac{\omega}{(k_{\parallel}' \rho_s v_e)} .$$

$$\ldots 6.70$$

The coefficients f, c, d are proportional to $(\omega-\omega_{*e})$, the deviation of the mode frequency from the diamagnetic drift frequency and are of the order of $k_y^2\rho_s^2$. In order to now evaluate the integrals and obtain the dispersion relation in the presence of F_{11o} we first express the plasma dispersion function in terms of error function as

$$Z(\varsigma) = i\sqrt{\pi}\, \bar{e} \, (1 + \Phi(i\varsigma)) \qquad \ldots\ 6.71$$

where Φ is the error function.

The integral I_2 expressed in terms of error function is now given by

$$I_2 = i\sqrt{\pi}\int_{-\infty}^{\infty} \frac{1}{x^2}\, e^{-i\alpha x^2 - \frac{b^2}{x^2}} \left(1 + \Phi\left(\frac{ib}{x}\right)\right) dx \qquad \ldots\ 6.72$$

Using the fact that $\Phi(\zeta) = -\Phi(-\zeta)$ the above integral reduces to

$$I_2 = 2i\sqrt{\pi}\int_0^{\infty} \frac{1}{x^2}\, e^{-i\alpha x^2 - b^2/x^2}\, dx \qquad \ldots\ 6.73$$

which can be straight away integrated to give

$$I_2 = \frac{i\sqrt{\pi}}{\sqrt{b^2}}\,(\pi)\, e^{-2\sqrt{i\alpha b^2}} \qquad \ldots\ 6.74$$

Similarly I_3 can be readily evaluated as

$$I_3 = -4\pi\alpha + i\pi b\left[\frac{-e^{-2\sqrt{i\alpha b^2}}}{b^3}\left(1 + 2\sqrt{i\alpha b^2}\right)\right] \qquad \ldots\ 6.75$$

I_1 expressed in terms of the error function can also be readily integrated to give

$$I_1 = 2i\sqrt{\pi}\left[K_0\left(2b(i\alpha)^{1/2}\right) + \frac{i\pi}{2} J_0\left(2ib\sqrt{i\alpha}\right) - \frac{\pi}{2} H_0\left(2ib\sqrt{i\alpha}\right)\right]$$

In equation (6.76), J_o and K_o are the modified Bessel functions of the first and second kinds respectively and H_o is the Strauve function.

Substituting the values of I_1, I_2 and I_3 into the expression for the functional, we observe that the expression is a transcendental equation in α. Fortunately a considerable amount of difficulty can be circumvented by noting that the argument of the function $b\sqrt{i\alpha}$ is very small, i.e. for $\left(\frac{m_e}{2m_i}\right) \ll \frac{L_n}{L_s}$ the argument is $\ll 1$. Hence making a small argument expansion, for the special functions, the expression for the functional reduces to

$$S = i\alpha^{3/2} - \epsilon_* + \frac{i}{2\alpha b^2} + \frac{(\omega-\omega_*)}{(k_{\parallel}' v_e \rho_s)}\Big\{ 2i^{3/2}\sqrt{\alpha}\Big[-\ln(b\sqrt{\alpha}) - \left(-\frac{i\pi}{2} + \frac{i\pi}{4}\right) - C_1 - 2i\,b\sqrt{i\alpha}\Big] + \frac{F_{110}}{m_e}\Big[-\frac{i\sqrt{\alpha}}{b}\,\frac{(1-2b\sqrt{i\alpha})}{k_{\parallel}' v_e \rho_s^2} - \frac{i^{3/2}4b\sqrt{\pi}\,\alpha^{3/2}}{k_{\parallel}' v_e \rho_s} + \frac{i\sqrt{\alpha}}{k_{\parallel}' v_e \rho_s}(1-4b^2 i\alpha)\Big]\Big\} \qquad \ldots 6.77$$

Where C_1 is the Euler's constant.

A significant detail to note is that the contributions from the principal part of the plasma dispersion function in the first parenthesis of the expression in equation (6.77) $\left(-\frac{i\pi}{2}\right)$, overcomes the contribution arising from the resonant particles to provide stability to the mode. We solve the simultaneous equation $S = 0$, $\frac{dS}{d\alpha} = 0$, using the prescription of the previous analysis [14]. The coefficient f, c, d in equation as pointed out earlier are proportional to $(\omega - \omega_{*e})$ which is of the order of $k_y^2 \rho_s^2$ and much less than unity. Hence in solving for α, the effect of the Landau term is neglected. The unperturbed value of $\alpha = \frac{k_{\parallel}' v_e \rho_s}{\omega}$ is now substituted in the equation $S = 0$, to obtain the dispersion relation.

In the presence of $F_{\parallel 0}$ the expression for the growth rate is given by

$$i\left(\frac{\omega_i}{\omega_*}\right)\left(\frac{k_{\parallel}' v_e \rho_s}{\omega_*}\right)^{1/2} = -i\, k_y^2 \rho_s^2 \left(\frac{\omega_*}{(k_{\parallel}' v_e \rho_s)}\right)\frac{\pi}{4}$$

$$+ i\, 4\sqrt{2\pi}\left(\frac{k_{\parallel}' v_e \rho_s}{\omega_*}\right)^{1/2}\left[\frac{F_{\parallel 0}}{m_e}\frac{L_s^2}{v_e^3}\right] b \qquad \ldots\ 6.78$$

Equation (6.78) gives the growth rate of the drift instability in the presence of the equilibrium force. The first term which is proportional to the finite Larmor radius corrections, $k_y^2 \rho_s^2$ describes the linear shear damping effect, while the second term represents the contribution from the parallel force. This contribution has a destabilising effect on the drift instability for values of parameters such that $F_{\parallel 0} > 0$. In parameter space this leads to the condition $\frac{k_{x_1}}{k_{y_1}} > \frac{k_{x_2}}{k_{y_2}}$. For the reverse inequality, $F_{\parallel 0}$ is < 0 and therefore has a stabilising effect. This aspect could be of interest in Alfven wave heating schemes. Comparing the two terms in equation (6.78), the linear shear damping term and the parallel force effect, we find that for $\frac{F_{\parallel}}{m_e}\left(\frac{L_s}{v_e^2}\right)\frac{1}{k_y \rho_s} \gg 1$

the destabilising force overcomes the shear damping effect. For laboratory plasma parameters and typical Alfven functions $(|\phi_0|^2 \ll 1)$, we find that the combination of parameters are such that the inequality is not satisfied. This indicates that the shear effects cannot be overcome by the ponderomotive force effects. However the latter competes significantly with the damping effect, leading to stabilisation or destabilisation depending on the combination of the parameters.

6.7 Conclusions :

In this chapter we have investigated the non-linear interaction between the kinetic Alfven waves and 1) the collisionless tearing modes, 2) the collisionless drift modes. We have studied the influence of the ponderomotive force (P.F.) generated by two kinetic Alfven waves on these modes. The kinetic Alfven waves are described by the two fluid model with simple cosine profile to represent the spatial variations. The collisionless tearing modes are described by a generalised Ohm's law and the momentum transfer equations. In the presence of the equilibrium P.F. generated by the Alfven waves, the electron orbit equations are modified. The perpendicular $(\perp^r)$ P.F. Doppler shifts the mode frequency while the parallel P.F. $(F_{\parallel 0})$ leads to a broadening of electron wave particle response $\left(\frac{\omega}{(k_\parallel^2 v_e^2 - a)^{1/2}},\ a = \frac{2 i F_{\parallel 0} k_\parallel}{m_e}\right)$

This phenomenon is similar to resonance broadening due to stochastic electron orbits studied by other authors [10].

The coupled equations describing the evolution of the collisionless tearing modes are solved using variational methods developed by Hazetline et al [4]. It is found that on account of the complications involved in the evaluation of the integrals containing the conductivity profile certain approximations have to be made. The parallel force has been treated as a perturbation parameter and suitable expansions of the plasma dispersion function made. With this constraint on the amplitude of the kinetic Alfven waves, the modification introduced by the equilibrium force in the growth rate of the collisional and collisionless tearing modes are obtained. In the collisionless regime, the parallel P.F. produces a modification of the growth rate of Laval et al [13]. It is found that for $F_{\parallel_0} > 0$, the growth rates are enhanced, while for $F_{\parallel_0} < 0$, the effect is stabilising. In parameter space, this translates into a relation between the wave vectors of interacting kinetic Alfven waves. The parallel force, $F_{\parallel_0}$, is < 0 for $\frac{k_{x_1}}{k_{y_1}} < \frac{k_{x_2}}{k_{y_2}}$ and for the reverse inequality $F_{\parallel_0}$ is > 0. For typical tokamak parameters and Alfven fluctuations ($|\Phi_0|^2 << 1$), the enhancement factor due to the parallel force is < 1. Therefore the destabilising effect in equation (6.41) is quite small. However the stabilising effect could of significant interest in tokamak plasmas which are plagued by the tearing instabilities. In the collisional regime, the modifications to the growth rates are found to be minimal. The enhancement factor is of second order in $F_{\parallel o}$ and therefore proportional to $[|\phi_o|^2]^2$. This factor consequently is too small to be of significance.

The ponderomotive force produces a similar effect on the collisionless drift waves. To retain shear effects, the electron response has been modelled by the kinetic equations. As in the case of the tearing modes, the particle orbits are considerably altered. The parallel P.F. accelerates the electrons along the field lines resulting in the broadening of the wave particle resonance. Treating the ions by the hydrodynamic approximation and using the quasineutrality condition, the drift eigen mode equation is obtained. In order to obtain the eigenvalues and investigate the effects of P.F., a variational principle analogous to the one used by Ross et al [14] is employed. Solving for the eigenvalues it is found that the growth rate of the drift waves is modified. For $F_{\parallel o} < 0$, the effect of the parallel force is to have a stabilising influence. For the reverse inequality the effect is destabilising. For laboratory plasma parameters and typical Alfven fluctuation $(|\bar{\Phi}_o|^2 \ll 1)$ it is found that the P.F. effect although competes significantly does not overcome the linear shear damping effect. However the former stabilising effect of the P.F. could be of interest as a means of stabilisation of drift waves in laboratory plasmas.

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CHAPTER VII
CONCLUSION

7.1 Summary :

In this thesis we have studied some non-linear interactions involving kinetic Alfven waves. In particular we have investigated the nonlinear interactions of the kinetic Alfven waves with drift modes and tearing modes. Such interactions have important applications in laboratory and astroplasmas. The motivation for the present work arose from the particular relevance of these interactions in Alfven wave heating schemes in tokamak plasmas. Alfven waves are considered excellent candidates for supplementary r.f heating schemes. Theoretical considerations show that near the resonance region these waves have enhanced amplitudes and that several non-linear processes can take place [1]. Therefore, a systematic study of the non-linear properties is necessary. In this context, we have investigated the non-linear interactions of kinetic Alfven waves with two important modes, namely the drift and tearing modes.

In Chapters II and III, we have examined the non-linear interactions of the kinetic Alfven waves with the resistive tearing mode. In Chapter II, we have discussed the resonant excitation of tearing modes through parametric interaction with the kinetic Alfven waves, using a fluid model. The momentum equation and Ohm's law are the basic equations which describe the evolution of the T.M. The non-linear interaction generates additional convective forces in the former and anomalous viscous and resistive effects in Ohm's law. Using variational and asymptotic matching methods, we find that the m = 1 and m = 2 tearing instabilities are excited by the kinetic Alfven waves with their typical growth rates scaling as $|\phi_0|^{1/3}$, $|\phi_0|^{3/7}\Delta'^{2/7}$ respectively These excited growth rates fall in the range $10^6 - 10^4$ sec^{-1} for typical tokamak parameters (given in Chapter II) [2] . Several experiments which have been conducted in Alfven wave heating have reported enhanced transport of particles and plasma disruptions [3]. It may be possible that these disruptions are caused by excitation of tearing modes. However, so far, no direct evidence has been obtained.

In Chapter III, we have investigated the non-resonant interaction between kinetic Alfven waves and resistive tearing modes, in which equilibrium flows generated by the Alfven waves couple non-linearly to the tearing mode perturbations. These non-linear drifts arise due to the interaction between the electric and the magnetic fields of the wave $(\vec{E}_A \times \vec{B}_A)/B_0^2$. The drifts have components in the axial, azimuthal and radial directions. The former Doppler shifts the mode frequency, while the latter components give rise to large gradients in the momentum equation. We find weakly growing tearing instabilities with growth rates proportional to the radial drift. The results of our analysis are in agreement with the weakly unstable modes obtained by Pollard-Taylor, Bondeson [4] . For tokamak parameters, these instabilities are found to grow on a longer time scale than the parametrically excited tearing modes. In a tokamak plasma, both the resonant and non-resonant processes could occur simultaneously; the former occurs when the resonant wave matching conditions are satisfied, while the latter is a more general phenomena.

In Alfven wave experiments, the antenna excites several modes which have a single frequency (or a small spread in frequency) simultaneously [3] . These kinetic Alfven waves could undergo non-linear interactions among themselves. Such a mechanism results in the excitation of waves at the sum and difference frequencies due to the presence of non-linear terms in the fluid equations. We have considered a situation where one of the resulting frequency, wave vector combination corresponds to that of the resistive T.M. and resonantly excites it (Chapter IV). The system responds like a driven harmonic oscillator wherein the non-linear interaction between the kinetic Alfven waves act as external forces driving the system at its natural tearing frequency. Using a fluid formalism, we obtain an inhomogeneous third order differential equation describing the evolution of the T.M. This problem differs from earlier investigations (ref. Chapter II) in that the non-linear terms are independent of the tearing mode perturbations. We have obtained solutions in terms of orthonormal basis functions, namely Hermite polynomials. It is found that the solutions are very sensitive to the parity of the driven Alfven waves. For arbitrary wavelengths, in the limit of vanishing pump amplitudes, the earlier results of Paris [5] are recovered. In the presence of the non-linear external forces, the growth rate for the symmetric tearing modes with positive 'm' numbers are enhanced, while for modes with negative 'm' numbers the effect is stablising. These driven tearing modes (for typical tokamak parameters) are found to grow more slowly than the parametrically excited modes. Excitation of these tearing modes could lead to enhanced transport through destruction of good magnetic surfaces.

Drift waves in sheared magnetic fields have been extensively studied. In laboratory plasmas, they are considered responsible for anomalous transport of particles. In Chapter V, we have investigated the problem of parametric excitation of drift waves by kinetic Alfven waves. The kinetic equations are used to describe the decay of a pump kinetic Alfven wave into a side-band Alfven wave and a drift wave. The quasi-neutrality condition and Ampere's law are used to obtain the coupled equations for the decay process. The dispersion relation was obtained under a local approximation. We find that the calculated growth rate of the excited drift wave is quite large and competes significantly with the growth rate of the ion acoustic wave calculated by Hasegawa-Chen [1] . The ratio of the growth rates of the two processes is found to be $\frac{\omega_*}{k_\parallel c_s} \sim O(1)$. We have also demonstrated that the kinetic Alfven waves could excite temperature gradient drift waves which have larger growth rates. In addition, we have investigated the effects of the back-ground inhomogeneity on the decay process, which calls for the retention of the full differential operators in the coupled equations. Treating the inhomogeneity scale length as a perturbation parameter, and using WKB methods, we have established the conditions under which an absolute instability can occur.

In the high temperature regime, the plasma is basically collisionless and the collisionless version of the tearing and drift modes are believed to play a very significant role in the reconnection mechanism. In Chapter VI, we have investigated two non-linear coupling processes -(1) between the kinetic Alfven waves and collisionless tearing modes 2) between the kinetic Alfven waves and the collisionless drift waves. We have investigated the effect of the P.F. generated by two kinetic Alfven waves on the two modes. The two fluid equations are used to describe the salient features of the kinetic Alfven waves. A generalised Ohm's law and the momentum equation describe the dynamics of the collisionless T.M. The former describes the electron response, while the latter describes the ion motion. The equilibrium P.F. (generated by the kinetic Alfven waves) broadens the electron wave particle response and modifies the conductivity profile in Ohm's law. The eigenvalues of the coupled equations have been obtained using variational methods prescribed by Hazeltine et al [6] . The modifications produced by the P.F. in the collisional and collisionless regimes have been obtained. In the latter, the parallel P.F. ($F_{\parallel o}$) modifies the growth rate of Laval et al [7]. For $F_{\parallel o} > 0$, the growth rates are strongly enhanced while for $F_{\parallel o} < 0$, the effect is stabilising. In parameter space the condition leads to a relation between the wave vectors of the kinetic Alfven waves, namely $F_{\|o} < 0$ for $\frac{k_{x_1}}{k_{x_2}} < \frac{k_{y_1}}{k_{y_2}}$ and for the reverse condition $F_{\|o}$ is positive. For typical tokamak parameters and Alfven fluctuations $(|\phi_o|^2 << 1)$ the enhancement factor due to the parallel force is however small. Although the destabilising effects are small, the stabilising effect of the P.F. could be of interest in tokamak plasmas.

In the collisional regime, consistent with expectations, the modifications to the growth rate of Drake and Lee [8] were found to be minimum. The enhancement factor in the growth rate is however of second order in $F_{\|o}$ and this factor is too small to be of any significance.

The dynamics of the collisionless drift waves are delicately controlled by the inverse Landau damping of electrons and shear effects. The electron response is therefore modelled by kinetic equations. The parallel P.F., as in the case of T.M., broadens the electron wave particle response, while the perpendicular P.F. Doppler shifts the mode frequency. The motion of the ions is described by the hydrodynamic approximation. The radial eigenmode equation is obtained from the quasineutrality condition and the eigenvalues are derived using the variational principle [9]. It is found that the parallel P.F. has a significant effect on the linear growth rate of the drift wave and competes with the shear stabilishing effect. For $F_{\|_0} > 0$, $\left(\frac{k_{x_1}}{k_{x_2}} > \frac{k_{y_1}}{k_{y_2}}\right)$ the parallel force contributes to the shear effect and stabilises the mode. For the reverse inequality, $F_{\|_0} < 0$, and the effect is destabilising. It is found that for typical tokamak parameters, the shear damping is not overcome by the destabilising contribution from the parallel force. However, the stabilisation of the drift modes by external P.F. generated by the interacting kinetic Alfven waves could be of interest in laboratory plasmas, where these modes are known to have deleterious effect on the plásma confinement.

7.2 Discussion :

It is appropriate at this point to discuss some of the simplifying assumptions that have been made in our calculations and view the results in that context. In the investigation of the non-linear interaction of the kinetic Alfven waves with the tearing modes, we have preserved the basic linear characteristics of the mode by retaining the non-linear terms only in the 'inner' region equations i.e. around $k_{\parallel t}(r_s) \sim 0$. Within this region, the non-linear terms are comparable to the linear resistive and inertial terms and play a significant role in governing the growth rate of the linear tearing mode. Outside this region, the linear Alfvenic terms dominate and the non-linear terms have been ignored. This approximation has been adopted by several earlier investigators [10] . One further simplification is the assumption that the radial variation of the T.M. is much larger than the wavelength which is implicit in the derivation of equation(2.8)(Chapter II). In the collisional regime where the fluid módel description of the T.M. is valid, this assumption is justified. In the collisionless regime the scale lengths would be comparable and the coupled differential equations for the decay process would have to be solved. However, as shown in ref. [10] of Chapter II, this does not change the qualitative aspects of the results.

In solving the differential equations describing the evolution of the tearing and drift modes, we have mainly used analytical methods. We have employed the variational and asymptotic matching techniques to obtain the eigenvalues of the differential equations. Both approaches provide approximate solutions. The variational method provides more accurate eigenvalues than eigenfunctions. However, the results obtained by using the two methods agree quite well except for the numerical factors.

In our analysis, the kinetic Alfven waves have been modelled using a plane wave approximation. Realistically the mode converted Alfven waves near their resonance regions have enhanced amplitudes and complicated radial structures (Airy functions) [1]. Therefore the equations describing the non-linear processes have to be solved with the Airy function profile for the pump Alfven waves. However solving the complicated differential equation with the exact Airy profiles calls for an extensive amount of computation.

7.3 Future Directions of Work :

The present work can be extended in several directions. Within the constraints of the plane wave approximation for the pump kinetic Alfven wave, the eigenmode equations for the tearing modes need to be solved numerically and the results compared with those obtained by analytical methods. In investigating the problem of the parametric decay of drift waves (Chapter IV), the spatial inhomogeneity needs to be appropriately considered. This requires a numerical solution of the coupled fourth and second order differential equations. At present, work is in progress in this direction. In addition, modelling the spatial variations of the kinetic Alfven waves by the Airy function profile renders the differential equation quite complicated and again calls for an extensive amount of numerical work. This work will be undertaken in the future.

In the context of astroplasmas, both tearing [11] and drift modes [12] play significant roles. Tearing modes are considered excellent candidates for reconnection processes in the magnetospheric plasmas, while the drift waves are considered responsible for the micropulsations in the solar wind. The non-linear excitation of these modes through kinetic Alfven waves could have important applications in this context and this feature must be explored.

In laboratory plasmas, in Alfven wave experiments, the antenna generally excites a spectrum of waves which are resonant at different surfaces (defined by $\omega = k_{\parallel}(r)\, V_A(r)$). The effect of this kinetic Alfven wave turbulence on the low frequency tearing and drift modes has not been investigated hitherto and is an important problem to examine.

Finally, the drift and tearing instabilities induced by the kinetic Alfven waves studied in the present work, will be controlled by non-linear mechanisms such as those discussed by Rutherford [13]. In order to study the non-linear evolution of these instabilities, we need to retain terms in the governing equations, which are of second order in the tearing or drift perturbations. Presumably, the rapid growth of these modes will be slowed down by the non-linear saturation mechanisms.

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