PARAMETRICALLY INDUCED STABILITY AND INSTABILITY IN PLASMAS

BY

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Śreyo hi jñānam abhyāsāj
jñānād dhyānaṁ visiśyate
dhyānāt karmaphalatyāgas
tyāgāc chāntir anantaram

Better indeed is knowledge than the practice (of concentration); better than knowledge is meditation, better than meditation is the renunciation of the fruit of action; on renunciation (follows) immediately peace.

- Bhagavadgita
Chapter XII. 12

DEDICATED

TO MY PARENTS

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CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

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ABSTRACT

In this thesis we study some aspects of parametric processes in a plasma with a view to application in thermonuclear fusion devices. The possibility of stabilization of the kink instability, a dangerous mode in the toroidal confinement schemes, by parametric coupling to damped ion acoustic waves has been investigated. We have also investigated the linear saturation of some well known scattering instabilities, namely stimulated Raman and Brillouin scattering due to the presence of background turbulence, multiple ion species and nonresonant contributions in an inhomogeneous plasma. These factors would normally be present in actual experimental conditions for laser fusion. Finally we have studied the evolution of the modulational instability, which is known to significantly modify the corona of an expanding pellet plasma, in the presence of Langmuir Turbulence. The relevance of these studies to practical schemes has been discussed.

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CHAPTER I

INTRODUCTION

1.1 Brief History:

In recent years the study of parametric interactions in a plasma has been an active field of research, primarily because of its diverse applications in such areas like ionospheric modification schemes,laboratory heating devices and laser fusion. The concept of parametric amplification is an old one and has been known since the time of Lord Rayleigh[1]. To electrical engineers it is known for a long time from their experiments on parametric amplification in travelling wave tubes.

Parametric excitations can be simply defined as the amplification of an oscillation by a periodic variation of some parameter which is characteristic of the system. The amplification however occurs for some specific relationship between the modulating frequency and the natural frequency of the oscillator. In general an oscillator whose natural frequency is Ω, when modulated weakly at a frequency $2\pi/T$ becomes unstable if $\Omega T = n\pi$ (n being an integer).

In a more complicated system like a plasma, which has many elementary states of excitations, the above definition can be easily extended. In this case the modulation wave (very often called the pump wave) acts in conjunction with a normal mode, to excite another elementary state. The excited collective state then co-operates with the pump to enhance the amplitude of the first mode. Such a process is most efficient when the following conditions are met.

$$\omega_0 = \omega_1 + \omega_2 \quad , \quad k_0 = k_1 + k_2 \tag{1}$$

where the ω_i and k_i refer to the frequencies and wave numbers of the three modes involved. The subscript zero refers to the pump wave. Alternatively if one speaks in terms of quantised states associated with the elementary excitations, conditions (1) express the law of conservation of energy and momentum for the process. We can in passing mention that both the collective states which are excited need not be normal modes of the system under consideration.

Parametric interactions in plasmas, magnetized or unmagnetized, can be broadly categorised into two distinct classes. In the first class of interactions the electromagnetic pump decays into two electrostatic modes. Thus the energy of the pump wave gets transferred to these electrostatic modes sustained by the plasma. Subsequently this wave energy can get converted into thermal energy of the particles due to the damping of the waves.

The decay process therefore provides an efficient channel (since it is a resonant process) of energy transfer to the plasma and leads to anomalous heating. On the other hand there are those processes for which one of the decay products is electrostatic and the other electromagnetic. These processes act as frequency conversion mechanisms for the electromagnetic mode with a minimal of energy deposition in the plasma. These are termed as the stimulated scattering processes. The linear theory of all these processes and their interrelationships is very well developed and has been reviewed by several authors[2, 3].

Some of the earliest applications of parametric interactions were in the field of ionospheric research. The fabrication of high power, high frequency transmitting antennae, made the terrestrial plasma susceptible to parametric instabilities and also provided a testing ground for the linear theory developed by Silin[4] and Dubois and Goldman[5]. It further provoked theoretical investigations into the nonlinear saturated state of these instabilities. A review of the linear and nonlinear parametric instabilities and their relevance to ionospheric modification by Perkins et. al[6], very neatly summarises the theoretical status and its agreement with experimental findings. The essential emphasis has been on the anomalous absorption effects.

Following the advancement of laser technology and the installation of high power rf generators, laboratory experiments for the study of parametric instabilities have also been carried out. The experimental studies[7] involve devices ranging from S band waveguides containing one centimetre diameter plasma columns, to Q machines which produce plasmas of 5 cm. diameter, DP devices with source of 30 cm. diameter, to plasma columns of 2 metre diameter and four metres length. The larger devices are fabricated with a view to accommodating a large number of wave-lengths of the incident electromagnetic wave so as to reduce boundary effects. Hence they can be more favourable for comparison with the theories derived for uniform and homogeneous plasmas. Again the basic motivation for these experiments has been to study the anomalous absorption of the electromagnetic as well as electrostatic pumps.

In the field of laser fusion[8], the part played by parametric processes cannot be overemphasised. It is well recognised that the classical heating of plasmas by the method of inverse bremsstrahlung or for that matter by multiphoton inverse bremsstrahlung, becomes more and more ineffective at higher temperatures and higher incident powers of the laser radiation. As a consequence an alternative method had to be envisaged which could lead to efficient coupling of the laser radiation to the plasma. The anomalous absorption method discussed above appeared to be a very promising candidate. This stimulated concentrated effort on the determination of the nonlinear saturation effect in the regime of strong turbulence due to direct trapping of particles or by transfer of energy to lower phase velocity modes which are Landau damped[3].

On the other hand there was a growing apprehension, due to early theoretical investigations[9] aided by computer simulations[10] that the scattering parametric instabilities would inhibit anomalous absorption. Since the laser plasma was expected to be inhomogeneous these scattering instabilities in the underdense region of the plasma would thereby increase the opacity and prevent penetration of the laser light up to the critical layer where anomalous absorption could be effective. Most experimental evidence[11] however failed to show any significant scattering. This discrepancy has led to various speculations about the possible mechanisms of inhibition of scattering. Some of the effects studied are those arising from density gradients and their modifications, velocity gradients, finite bandwidth and background turbulence.

Besides the effect of anomalous absorption, parametric processes have been studied for other applications. One of the more interesting applications is to exploit it for dynamic stabilization of plasmas[12, 13]. The idea is to provide a parametric coupling between the growing mode and a naturally damped mode of the system which thereby prevents the instability from developing. The advantage such a method would have over linear dynamic (or feedback) stabilization techniques[14] lies in the fact that the parametric process is a resonant interaction and would therefore require far less energy. It would therefore provide an active means of suppressing instabilities in thermonuclear devices.

1.2 Scope of the thesis:

In the present thesis we have studied a few parametric processes which are relevant to the problem of thermonuclear fusion. At present the two schemes which seem most promising for achieving this goal and therefore encourage much activity are (i) the toroidal confinement scheme for plasmas (e.g. the tokamak) and (ii) laser fusion. Some aspects of both these schemes have been investigated and it is convenient to broadly divide the thesis into two sections. In the first part, which is essentially the second chapter, the possibility of inducing stability to a cylindrical plasma column by exploiting the phenomenon of parametric coupling between three linear modes of the system has been investigated.

Most of the conventional dynamic stabilization schemes consider oscillating fields with frequencies much larger than the ion acoustic frequency. However, as shown in this work, if the oscillating frequency is adjusted closer to that of the ion acoustic wave the efficiency of stabilization increases because of the resonant enhancement of the mode coupling coefficients. In a plasma with nearly equal electron and ion temperatures, ion acoustic waves are heavily damped and coupling to such damped modes can be usefully employed to reduce the growth rate of kink modes. We have investigated such a coupling process. Because the kink mode in the presence of the pump wave is no longer purely growing, we have considered the possibility of Landau damping of this mode by incorporating a phenomenological damping term in the equation of motion. We demonstrate the possibility of existence of low frequency stable modes and also reduction in the growth rate of the kink modes.

In the second part of the thesis a number of problems relating to laser fusion have been studied. It is well recognized that for strong heating the process of anomalous absorption which occurs when the frequency of the incoming light matches the local plasma frequency is highly effective. Thus in an inhomogeneous plasma the incident radiation should penetrate the plasma up to this critical layer. However there are well known nonlinear scattering processes that can occur in the underdense region of the plasma and work against the radiation penetration or greatly modify it. Stimulated Raman and Brillouin Scattering (i.e. scattering of electromagnetic waves from electron plasma waves and ion acoustic waves respectively) are the three wave parametric processes which can work against efficient coupling between the laser light and the inhomogeneous plasma, by preventing the required light penetration, whilst modulational instability (which can be looked upon as a parametric process involving four waves) can channelize the radiation into filaments and thereby significantly modify the laser light and plasma coupling. However in a laser pellet experiment there can be various extraneous factors which can significantly influence the evolution of these instabilities. The problems investigated in this section delineate such effects.

In chapter III we have investigated the effect of random density fluctuations on the process of stimulated Raman Scattering. In view of the fact that the plasma production mechanism is violent and also because of the existence of various instabilities, it is natural to expect the laser produced plasma to be in a turbulent state, and hence it is important to investigate the influence of random density fluctuations. We consider the turbulence to be characterised by long wavelength, slow frequency waves (compared to the scale length and time scale associated with the interacting waves) so as to justify the use of the Eikonal or W.K.B. approximation. A general integro differential equation for the ensemble average of the amplitude of one of the modes is derived. The effect of turbulence upto the order $|\epsilon^2|$ is retained where $|\epsilon| = |n(x,t) - n_0|/n_0 \ll 1$ We then consider the case of a homogeneous plasma with a statistically homogeneous turbulence, and evaluate the modified thresholds and growth lengths. We compare our results with those of Thomson[15] who has considered the effect of finite bandwidth and show the similarity between these two effects. Next we consider the convective amplification of the backscatter in the presence of quasistatic turbulence and show that if the homogeneous threshold is exceeded, the convective threshold is very nearly the same as the inhomogeneous quiescent threshold, so that the presence of turbulence does not greatly influence the inhomogeneity amplification.

Chapter IV deals with certain problems associated with stimulated Brillouin scattering. It consists of three small sections in which some new processes which can affect the threshold for the instability have been discussed.

In the first section we study the effect of two ion species on SBS in an inhomogeneous plasma. Since in the majority of the experiments the pellets have two species of ions, we investigate the effect they would have on the inhomogeneity threshold if the scale length for the two species is different. Such a situation is to be expected because of the different velocity of expansion for the two species brought about by the ambipolar field. The new threshold is found to be significantly modified depending on the ratio of the masses of the two species.

In the second section we look into the problem of threshold modifications for SBS brought about by Langmuir Turbulence. In contrast to the work presented in chapter III, we are here considering the effect of a high-frequency short-wavelength turbulence on the three wave process. We assess the modification brought about in the dispersion relation of the ion acoustic wave due to the Langmuir turbulence in an inhomogeneous medium. The adiabatic approximation is used so that the resonant wave-wave and wave-particle interactions can be neglected. We find that the level of backscatter can be significantly reduced even in the case of weak turbulence. The results obtained may explain the low level of backscattered radiation observed experimentally.

In the final section of this chapter we consider the problem of side scatter. Present experimental evidence does not show appreciable side scatter belying theoretical predictions to the contrary. A calculation is carried out by incorporating the nonresonant effects of the pump wave on the ion acoustic mode and it is found that the side scatter threshold is significantly increased and is higher than the corresponding threshold for backscatter.

In the fifth chapter the modulational instability of a large amplitude electromagnetic wave in the presence of Langmuir Turbulence is investigated. It is well known that an electromagnetic wave of sufficiently large amplitude, when perturbed along the transverse direction (i.e. perpendicular to the propagation vector) tends to break up the plasma into filaments. It is found that short wavelength, high frequency turbulence propagating essentially in the transverse direction tends to reduce the threshold for filamentation provided $\mathcal{E}_{fluc} / \mathcal{E}_{kin} < \Delta^2 \lambda_D^2$ where $\mathcal{E}_{fluc}$ is the energy density of the turbulent waves, $\mathcal{E}_{kin}$ the energy density of the particles, λ_D the electron Debye wave length and Δ the width in k-space of the turbulent waves. However under strong turbulence conditions, i.e. $\mathcal{E}_{fluc} / \mathcal{E}_{kin} > \Delta^2 \lambda_D^2$ the modulational instability gets totally quenched. We should however remark that the amplitude of the pump wave in our analysis is restricted by the condition $v_0 / c \ll 1$ where v_0 is the velocity of the electron in the pump field. In another case we have examined the dispersion relation when the turbulent waves propagate parallel to the electromagnetic mode and have discovered a new mode which is purely growing, the growth rate being determined by the level of turbulence.

Summarizing, the work presented aims at providing a stabilizing mechanism for a well known instability (the kink instability) and explaining some experimental data in laser plasma experiments which are of primary importance to the scheme of laser fusion. The basic motivation has been to study problems that have a bearing on the present experimental status in thermonuclear fusion research and to provide a better understanding of various effects (such as background turbulence, multiion composition, nonresonant contributions) on some three wave and four wave interactions in an unmagnetized plasma.

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CHAPTER II

PARAMETRIC EXCITATION AND SUPPRESSION OF THE KINK MODE

2.1 Introduction:

The problem of magnetic confinement of a plasma is an old one and has been extensively investigated for controlled fusion devices. Some of the currently active areas of research in this direction involve such well known configurations as the mirror machine, the theta-pinch and the Tokamak. Of these, the Tokomak holds much promise and the stability of the plasma column in such a device is an important problem.

One of the earliest works in this regard is due to Krushkal and Schwarzchild[1] who considered the stability of a uniform plasma column confined by the self magnetic field arising from a surface current. They found that for the m=1 perturbation, (where m/r_0 is the azimuthal wavenumber, r_0 is the radius of the column) the initial equilibrium is unstable. The physical mechanism leading to such an instability is evident from the nature of the perturbation. When the plasma column is distorted into a wavy structure, the azimuthal magnetic lines of force are separated from each other at the convex side of the cylinder and pressed together at the concave side. Due to this imbalance in the local magnetic pressure, the perturbation gets enhanced and thus leads to a purely growing instability.

Later Shafranov[(2)] examined the stability of a plasma column in the presence of a longitudinal field and surrounded by a coaxial conductor. He considered the case for all m and established that in the presence of a longitudinal field and conducting walls all perturbations with wave number $k > k_m$ are unstable. For m=0 and m=1, $k_m = 0$ so that all the modes are unstable. This agrees with Krushkal and Schwarzchild's result. Secondly in the case of the m=1 mode and a frozen-in longitudinal magnetic field (and no external magnetic field or conductor) the stability criterion is of the form

$$\frac{B_z^2}{B_\theta^2} > \ln \frac{2}{k r_0}$$

On the other hand if both external and internal longitudinal fields are present the stability condition deteriorates owing to the fact that the column can now exhibit perturbations which are parallel to the spiralling lines of force and these can grow readily because they do not have to distort the lines of force. The stability criterion assumes the form

$$\frac{B_z}{B_\theta} > \frac{mL}{2\pi r_0}$$

(where L is the length of the column and hence the upper limit on the scale length of the perturbation) provided the internal and external fields are equal. It is convenient to write this stability criterion in terms of a dimensionless quantity $q = \frac{2\pi r_0 B_z}{L B_\theta}$ so that now the criterion becomes

$$q > m$$

Physically this tells us that when the perturbation, for the maximum scale length L, becomes more tightly spiralling (this is measured by the pitch $2\pi r_0 / mL$) than the lines of force (B_θ / B_z), stability is ensured. For a toroidal geometry, the scale length along the toroidal axis will be quantized i.e. $k_z = 2\pi n / L$ where n is an integer. Hence for any general mode n, the stability criterion takes the form

$$nq > m$$

This further tells us that if the longest wavelength perturbation is stable, then the shorter wavelengths are necessarily stable.

Therefore the presence of the magnetic field introduces stability to the plasma column and one can find some wavelengths for which the column is stable. The next question that arises concerns the part played by the conducting wall. Shafranov(2) has shown that the region of instability given by $nq < m$ gets reduced because of the presence of the wall. The instability condition becomes

$$m > nq > 1 - \left(\frac{r_0}{R_0}\right)^{2m}$$

where R_0 is the radius of the conducting cylinder. This result is valid provided the scale length of the perturbations exceeds the radii r_0 and R_0 . Physically the stabilization is brought about because the conductor plays the role of a passive feedback element. It gives rise to a stabilizing field due to the currents induced on it because of the displacement of the plasma column. The long wavelength perturbations (i.e. small m) are more affected than the short wavelength. Hence the region of unstable k (wave vectors) is reduced.

The discussion so far considered only a surface current. However it has been shown by Shafranov[3,4] that for an incompressible plasma column with a uniform volume current the stability criterion remains the same. The parameter which characterises the current distribution along the cross-section of this column does not enter the stability criterion, thereby showing that the instability is brought about by the surface current and the field configuration at the surface and its immediate neighbourhood. This has also been shown by Lowder and Thomassen[5] in their calculation from first principles, where they derive the general stability criterion in the presence of feedback.

It is worth mentioning that the analysis for the cylindrical column for the general kink instability with a free boundary can be readily applied to the toroidal geometry, because the corrections due to the toroidal effects are of the order B_θ / B_z compared to the helical instability. Hence as far as the stability of the kink mode with a free boundary is concerned, the discussions for the cylindrical geometry are readily applicable to toroidal configurations.

The important aspect of the stability restriction $q > 1$ (for m=1) is that it imposes a limit on the operating current of the Tokamak. This in turn limits the heating that can be achieved by classical ohmic processes. Hence much effort has been directed towards relaxing the stability criterion by means of external stabilizing agencies. Berge[7] has given an excellent review of such dynamic stabilization schemes involving the use of oscillating magnetic fields in the axial (or toroidal) and or poloidal directions. However the papers reviewed by Berge consider oscillating fields with frequencies much larger than the ion acoustic frequency. In this case there is hardly any coupling to the ion acoustic waves in the plasma. If the oscillation frequency is adjusted close to the ion acoustic frequency, one can effectively increase the efficiency of dynamic stabilization by resonant enhancement of the mode coupling coefficients. This frequency range is, however avoided in general, because of the possibility of parametric excitation of undesirable modes. In a plasma with nearly equal electron and ion temperatures, ion acoustic waves are strongly damped. In tokamak-like plasmas, such damped waves exist even in the presence of strong longitudinal currents because of trapped particle effects[8]. Coupling to such damped modes can therefore be usefully employed to reduce the growth rate of kink modes, with the additional advantage that there is an accompanying heating of the ions. It is therefore of considerable interest to examine the effects of coupling kink modes to ion acoustic waves by external oscillating fields.

In this chapter, we will discuss the effect of an oscillating azimuthal magnetic field with a frequency close to the typical frequency of the ion acoustic wave on the dispersion relation for kink modes in a cylindrical plasmas column with surface current.

2.2 Derivation of Dispersion Relation:

2.2.1 Basic Equations and Equilibrium

We consider a linear plasma column in the pinch configuration with a longitudinal magnetic field inside and outside the column, together with an azimuthal field on the outside which is produced by a surface current. For simplicity we shall assume infinite conductivity and choose a scalar pressure. The use of infinite conductivity can be justified because we wish to study small wave number (or long wavelength) perturbations, which are the unstable ones. Hence the use of inifinite conductivity is not a bad one in the regime considered. The use of a scalar pressure can be justified by recalling[9] that from an energy principle analysis of the magnetohydrodynamic stability it has been shown that the potential energy associated with an MHD fluid column is less than that of a C.G.L. fluid (i.e. a plasma represented by the Chew, Goldberger, Low equations or the double adiabatic equations which use a pressure tensor) so that if stability is established for an MHD fluid then the C.G.L fluid is invariably stable.

Our basic MHD equations are

$$\rho \frac{d\vec{v}}{dt} = -\nabla p + \frac{\vec{j} \times \vec{B}}{c} \tag{2.1}$$

$$\frac{\partial \rho}{\partial t} + \nabla (\rho \vec{v}) = 0 \tag{2.2}$$

$$\vec{E} + \frac{\vec{v} \times \vec{B}}{c} = 0 \tag{2.3}$$

$$\frac{d}{dt}\left(\frac{p}{\rho^{\gamma}}\right) = 0 \tag{2.4}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} \tag{2.5}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{2.6}$$

In equations (1) - (6) we use obvious symbols for the fluids and field quantities. We next write down the necessary boundary conditions to supplement the above equations.

$$\left[p + \frac{B^2}{8\pi} \right] = 0 \tag{2.7}$$

$$\left[B \right]_n = 0 \tag{2.8}$$

The square bracket implies the difference in the enclosed quantity across the sharp plasma-vacuum boundary and the suffix n refers to the normal component (i.e. normal to the surface).

In equilibrium the plasma has a homogeneous density ρ_0, pressure p_0 and an axial magnetic field $\vec{B}_0^{(0)} = (0, 0, B_0)$ In the vacuum the field configuration is given by $\vec{B}_0^{(0)} = (0, B_\theta, B_e)$. The discontinuity in the magnetic field inside and outside the plasma is maintained by a surface current $\vec{j}_0^{(0)} = (0, j_{s\theta}, j_{sz})$. Hence the zero order equilibrium conditions can be written as

$$j_{s\theta} = \frac{c}{4\pi}\left[B_0 - B_e \right] \tag{2.9}$$

$$j_{sz} = \frac{c}{4\pi} B_\theta \tag{2.10}$$

$$P_0 = \frac{1}{8\pi}\left[B_\theta^2 + B_c^2 - B_o^2 \right] \tag{2.11}$$

The next step is to calculate the perturbed quantities associated with a normal mode existing in the plasma column which gives the 'dynamic equilibrium' state of the column. This is done by linearizing the basic equations and solving for $\vec{B}_1^{(o)}$, the amplitude of the perturbing field. The smallness parameter used in this expansion is $\epsilon = B_1^{(o)}/B_o$, the relative strength of the pump field to the ambient field. From equations (2.1), (2.3) and (2.6) we obtain

$$\rho_0 \frac{\partial \vec{v}_1^{(o)}}{\partial t} = \frac{\vec{J}_1^{(o)} \times \vec{B}_o^{(o)}}{c} \tag{2.12}$$

$$\frac{\partial \vec{B}_1^{(o)}}{\partial t} = \nabla \times \left(\vec{v}_1^{(o)} \times \vec{B}_o^{(o)} \right) \tag{2.13}$$

where the subscript (1) denotes the perturbed quantities.

In equation (12) we have not retained the pressure gradient term which vanishes at a later stage because we identify this mode as the shear Alfven mode. Differentiating Eq. (2.13) with respect to time and taking the θ component, we obtain

$$\rho_0 \frac{\partial^2 B_{\theta 1}^{(o)}}{\partial t^2} = -\frac{B_o^2}{c} \frac{\partial j_{r1}^{(o)}}{\partial z} \tag{2.14}$$

$\frac{\partial j_{r1}}{\partial z}$ is evaluated from the r component of Eq. (2.4).

$$\frac{\partial^2 B_{\theta 1}^{(o)}}{\partial t^2} = C_A^2 \frac{\partial^2 B_{\theta 1}^{(o)}}{\partial z^2} \tag{2.15}$$

where $C_A = (B_o^2/4\pi\rho_o)^{1/2}$ is the Alfven speed. Eqn. (15) gives a plane wave solution

$$B_{\theta 1}^{(0)} = A_1 e^{ik_0 z + i\omega_0 t} \tag{2.16}$$

The vacuum field satisfies the Laplace equation and can be solved to yield

$$B_{\theta 1}^{(0)\,ext} = A_2 k_0 I_0'(k_0 r) \tag{2.17}$$

where $I_0(k_0 r)$ is the modified Bessel function of zeroeth order. We have not considered any external conducting wall enclosing the plasma, and hence the vacuum fields vanish at infinity. The linearized forms of the boundary conditions can next be written as

$$\begin{aligned} (\vec{B}_1^{(0)} \cdot \vec{B}_0^{(0)})_{inside} &= (\vec{B}_1^{(0)} \cdot \vec{B}_0^{(0)})_{outside} \\ (\hat{e}_r \cdot \vec{B}_1^{(0)})_{inside} &= (\hat{e}_r \cdot \vec{B}_1^{(0)})_{outside} \end{aligned} \tag{2.18}$$

where $\hat{e}_r$ is a unit vector in the radial direction. Use of condition (18) yields $A_2 = 0$ and hence absence of an external field. This mode can be identified as the torsional Alfven mode whose azimuthal field $B_{\theta 1}^{(0)}$ has an associated velocity $V_{\theta 1}^{(0)}$ leading to relative shear between adjacent (along the z axis) layers. The motion due to $V_{\theta 1}^{(0)}$ across the magnetic field B_0 also leads to a polarization current $j_{r1}^{(0)}$. As a result we have a surface charge on which the radial electric field $E_{r1}^{(0)}$ terminates. We refer to this as the state of dynamic equilibrium.

2.2.2 Coupled Equations for the kink and Ion Acoustic Modes

We now study the behaviour of the kink mode and the ion acoustic modes arising out of the perturbation imposed on the equilibrium derived in the previous section. The presence of the Alfven wave causes a coupling between the kink mode and the ion acoustic modes and our aim is to derive such a coupled set of equations. This can be achieved by carrying out a further expansion in the parameter η, where η is a measure of the strength of the perturbed field compared to the ambient field. It is assumed that $\epsilon^2 < \eta < \epsilon$ and terms of order $\epsilon\eta$ are retained. The nonlinear coupling terms appear through the terms of order $\epsilon\eta$ [10].

Our equations now become

$$\rho^{(1)}\frac{\partial \vec{v}_1^{(0)}}{\partial t} + \rho_0 \frac{\partial \vec{v}^{(1)}}{\partial t} + \rho_0 (\vec{v}_1^{(0)}\cdot\nabla)\vec{v}^{(1)} + \rho_0 (\vec{v}^{(1)}\cdot\nabla)\vec{v}_1^{(0)}$$
$$= -\nabla p^{(1)} + \frac{1}{4\pi}\left\{-\nabla(\vec{B}_1^{(0)}\cdot\vec{B}^{(1)}) + (\vec{B}_1^{(0)}\cdot\nabla)\vec{B}^{(1)} + (\vec{B}^{(1)}\cdot\nabla)\vec{B}_1^{(0)} + (\nabla\times\vec{B}^{(1)})\times\vec{B}_0^{(0)}\right\} \tag{2.19}$$

$$\frac{\partial \rho^{(1)}}{\partial t} + \nabla\cdot\left[\rho_0 \vec{v}^{(1)} + \rho^{(1)}\vec{v}_1^{(0)}\right] = 0 \tag{2.20}$$

$$\frac{d}{dt}\left[p^{(1)} - C_s^2 \rho^{(1)}\right] = 0 \,, \text{ where } C_s^2 = \frac{\gamma p_0}{\rho_0} \tag{2.21}$$

$$\frac{\partial B^{(1)}}{\partial t} = \nabla\times\left[\vec{v}^{(1)}\times(\vec{B}_0^{(0)} + \vec{B}_1^{(0)})\right] \tag{2.22}$$

We Fourier analyse the perturbed quantities in the θ and z directions and write the perturbed quantities as

$$\xi^{(1)} = \xi_1^{\pm}\, e^{i(\omega_\pm t + m\theta + k_\pm z)} + \xi_1\, e^{i(\omega t + m\theta + kz)} \tag{2.23}$$

where $\omega_\pm = \omega \pm \omega_0$ and $k_\pm = k \pm k_0$. The quantity $\xi^{(1)}$ denotes any one of the perturbed quantities ($\vec{v}^{(1)}$, $p^{(1)}$, $\vec{B}^{(1)}$, $\rho^{(1)}$). The terms $\xi_1^{\pm}$ are the ion acoustic modes arising as side bands due to the coupling of the Alfven wave with the kink mode. We choose the pump wave such that

$$v_1^{(0)} = 2 v_0 \cos(\omega_0 t + k_0 z)$$

$$B_1^{(0)} = \frac{2 v_0}{c_A} B_0 \cos(\omega_0 t + k_0 z) \equiv B_{\theta 1}^{(0)} \tag{2.24}$$

Using equations (23) and (24), we can write the equations for the kink mode as

$$i\omega \rho_0 v_{1r} = -\frac{\partial p_1}{\partial r} - \frac{\partial}{\partial r}\left(\frac{B_{1z}B_0}{4\pi}\right) + ik\frac{B_{1r}B_0}{4\pi} \tag{2.25}$$

$$i\omega \rho_0 v_{1\theta} = -\frac{im}{r}p_1 - \frac{im}{r}\frac{B_{1z}B_0}{4\pi} + ik\frac{B_{1\theta}B_0}{4\pi} + ik_0\rho_0(v_{1z}^{+} - v_{1z}^{-})v_0 + i\omega_0 v_0(\rho_1^{+} - \rho_1^{-}) \tag{2.26}$$

$$i(\omega - i\nu')\rho_0 v_{1z} = -ikp_1 - \frac{im}{r}\rho_0(v_{1z}^{+} + v_{1z}^{-})v_0 \tag{2.27}$$

$$i\omega \rho_1 + \rho_0(\nabla \cdot \vec{v}_1) = -\frac{im}{r}v_0(\rho_1^{+} + \rho_1^{-}) \tag{2.28}$$

$$i\omega B_{1z} = ikB_0 v_{1z} - B_0(\nabla \cdot \vec{v}_1) + \frac{im}{r}B_{\theta 1}^{(0)}(v_{1z}^{+} + v_{1z}^{-}) \tag{2.29}$$

$$i\omega B_{1\theta} = ikB_0 v_{1\theta} - ikB_{\theta 1}^{(0)}(v_{1z}^{+} + v_{1z}^{-}) \tag{2.30}$$

$$i\omega B_{1r} = ikB_0 v_{1r} \tag{2.31}$$

In equation (27) we have included a phenomenological damping term ν' to account for possible collisionless damping of the kink mode. Such a situation is possible because the kink mode acquires a small real frequency from the coupling of ion acoustic waves and hence can suffer Landau damping on the ions[11].

The equations for the ion acoustic modes can similarly be written down as

$$i\rho_0\omega_\pm v_{1z}^\pm = ik_\pm p_1^\pm - ik_\pm \frac{B_{\theta 1}^{(0)} B_{1\theta}}{4\pi} + \frac{im}{r}\frac{B_{\theta 1}^{(0)} B_{1z}}{4\pi} - \frac{im}{r}\rho_0 V_0 v_{1z} \tag{2.32}$$

$$i\omega_\pm \rho_1^\pm + ik_\pm \rho_0 v_{1z}^\pm = -\frac{im}{r}\rho_1 V_0 \tag{2.33}$$

$$p_1^\pm = c_s^2 \rho_1^\pm \tag{2.34}$$

It is possible now to reduce the set of equations (25) to (34) to just three coupled equations for the ion acoustic variable $v_{1z}^\pm$ and a variable $\tilde{p}_1$ for the kink mode. The quantity $\tilde{p}_1 = c_s^2\rho_1/\rho_0 + c_A^2 B_{1z}/B_0$ is a measure of the perturbed pressure (both particle and magnetic) and enters the boundary conditions in an intrinsic manner. Carrying out the elimination in the equation (25) to (34), we write down the three final equations as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\tilde{p}_1}{dr}\right) - \left(\frac{m^2}{r^2}+\beta^2\right)\tilde{p}_1 = -\frac{mV_0}{r}\left(\Gamma^+ v_{1z}^+ + \Gamma^- v_{1z}^-\right)$$

where

$$\Gamma^\pm = \left(\pm k_0 \mp \frac{\omega_\pm\omega_0}{k_\pm c_s^2} - \frac{\omega}{c_A}\right) + \frac{\left(k^2-\frac{\omega^2}{c_A^2}\right)\left(\frac{\tilde{\omega}}{kc_A} - 1 - \frac{\omega_\pm\tilde{\omega}}{k_\pm c_s}\right)k}{\left(k^2 - \frac{\bar{\omega}^2}{c_s^2} - \frac{\bar{\omega}^2}{c_A^2}\right)} \tag{2.35}$$

and $\bar{\omega}^2 = \omega(\omega - i\nu')$, $\tilde{\omega} = \omega - i\nu'$

$$\left[\omega_\pm(\omega_\pm - i\nu) - k_\pm^2 c_s^2\right] v_{1z}^\pm = \frac{mV_0}{r}\frac{(k^2-\bar{\omega}^2/c_s^2)}{(k^2-\bar{\omega}^2/c_s^2-\bar{\omega}^2/c_A^2)}\left[c_A\left(1-\frac{k_\pm k}{\beta^2}\right) + \frac{\omega}{k^2c_s}(kc_s+\tilde{\omega})/(1-\bar{\omega}^2/k^2c_s^2)\right] \tag{2.36}$$

with $\beta^2 = (k^2-\bar{\omega}^2/c_s^2)(k^2-\omega^2/c_A^2)/(k^2-\bar{\omega}^2/c_s^2-\bar{\omega}^2/c_A^2)$

where ν is the measure of the Landau damping of ion acoustic waves. Equations (35) and (36) are the set of coupled equations representing the interaction of the kink mode and the ion acoustic mode through the pump wave. The terms on the right hand side are the driving terms which can in general modify the spring constant of the coupled modes. We ignore the effect of the pump depletion, assuming a constant external energy input to maintain the p ump amplitude. In equation (36) it will be noticed that in the absence of the pump, the dispersion relation of the ion acoustic wave corresponds to $\omega_\pm = k_\pm c_s$ (if we neglect the damping). This expression is valid for a strong mangnetic field so that $\Omega_i \gg k_\pm c_s$ and $k_\pm \lambda_D \ll 1$ where the quantity Ω_i is the ion gyrofrequency and λ_D is the Debye wavelength. Combining equations (35) and (36) we obtain a modified equation for the kink mode.

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\tilde{p}_1}{dr}\right) - \left\{\frac{m^2}{r^2}\left[1 - \frac{V_0^2}{C_A^2}(\alpha_+ + \alpha_-)\right] + \beta^2\right\}\tilde{p}_1 = 0 \tag{2.37}$$

$$\alpha_\pm = \omega_\pm \left[\left(\pm k_0 \mp \frac{\omega_\pm \omega_0}{k_\pm c_s^2} - \frac{\omega}{C_A}\right) + \frac{k\beta^2}{(k^2 - \bar{\omega}^2/C_A^2)}\left(\frac{\hat{\omega}}{kC_A} - 1 - \frac{\tilde{\omega}\omega_\pm}{k_\pm c_s^2}\right)\right]$$
$$\times \left[C_A(1 - k_\pm k/\beta^2) + \frac{\omega}{k^2 c_s}(\tilde{\omega} + kc_s)/(1 - \bar{\omega}^2/k^2 c_s^2)\right] \Big/ \left[\omega_\pm(\omega_\pm - i\nu) - k_\pm^2 c_s^2\right]$$

This can be readily solved to give

$$\tilde{p}_1 = R I_p(\beta r) \qquad \text{where} \quad p^2 = m^2\left[1 - \frac{V_0^2}{C_A^2}(\alpha_+ + \alpha_-)\right]$$

This solution is valid for $\mathrm{Re}\, p > 0$. The second solution is rejected by imposing boundedness on $\tilde{p}_1$ for $r = 0$. We note that the effect of nonlinearity enters through the order of the Bessel function and one must retain terms of order V_0^2/C_A^2 to account for any modifications in the linear eigenvalues of the mode. It is easily seen that because the equations (24) and (31) are not directly affected by the nonlinearity and because of the azimuthal character of the pump wave, the nonlinearity has manifested itself through the m^2/r^2 terms. The second point to be noticed is that for the case m = 0, the pump does not in any way lead to a coupling with the ion acoustic wave. This is easily understood by examining the equations (24) to (31) for m=0. For the pinch m=0, $V_{1\theta} = B_{1\theta} = 0$, and the effect of the pump only appears in the $V_{1\theta}$, $B_{1\theta}$ equation so that it does not in any way modify the dispersion relation for the pinch.

2.2.3 Boundary Conditions and Dispersion Relation

To apply the boundary conditions we have to first ascertain the perturbation brought about to the normal unit vector to the boundary. The equation for the imperturbed boundary is

$$\psi_0(r) = r - r_0 = 0 \tag{2.40}$$

where r_0 is the radius of the plasma column.

In the presence of the kink and the ion acoustic modes, the equation for the perturbed boundary can be written as

$$\frac{\partial \psi^{(1)}}{\partial t} + (V^{(0)}\cdot\nabla)\psi^{(1)} + (V^{(1)}\cdot\nabla)\psi_0 = 0 \tag{2.41}$$

where $\psi^{(1)} = \psi_1 + \psi_1^+ + \psi_1^-$, $V^{(1)} = V_1 + V_1^+ + V_1^-$

the quantities (ψ_1 , V_1) pertain to the kink mode and ($\psi_1^{\pm}$, $V_1^{\pm}$) to the ion acoustic modes.

Separating the above equation into three coupled equations for the three modes we determine ψ_1 as

$$\psi_1 = \frac{iV_{1r}}{\omega(1 - V_0^2/r_0^2\omega_0^2)} \simeq \frac{iV_{1r}}{\omega} \tag{2.42}$$

we choose to neglect the small nonresonant effect arising from the term $V_0^2/r_0^2\omega_a^2$

The normal to the perturbed boundary is now

$$\hat{n}_0 + \hat{n}_1 = \nabla\psi = (-1,0,0) + (0, -imr_1, -ikr_1), \quad r_1 = \frac{iV_{1r}}{\omega}$$

The perturbed magnetic field in the vacuum region outside the plasma column is given by

$$B_{1r}^{v} = kC_1 K_m'(kr) + kC_2 I_m'(kr) \qquad (2.44)(a)$$

$$B_{1\theta}^{v} = \frac{im}{r} K_m(kr)C_1 + \frac{im}{r} C_2 I_m(kr) \qquad (2.44)(b)$$

$$B_{1z}^{v} = ikC_1 K_m(kr) + ikC_2 I_m(kr) \qquad (2.44)(c)$$

The boundary conditions now give

$$-i\left(\frac{m}{r_0} + kb_e\right) r_1 B_\theta = B_{1r}^{v} \qquad (2.45)$$

$$\left(\tilde{p}_1 - r_1 \frac{B_\theta^2}{4\pi\rho_0}\right) = iB_\theta\left(B_{1\theta}^{v} + B_{1z}^{v} b_e\right) \qquad (2.46)$$

Using the expressions derived for $B^{v}_{1r,\theta,z}$ and $\tilde{p}_1$ and eliminating C, C_1 and C_2 we get the dispersion relation

$$\beta k r_0^2\left[b_i^2 - \frac{\omega^2}{2C_s^2}\frac{(1+b_e^2-b_i^2)}{(k^2-\bar{\omega}^2/C_s^2)}\right]\frac{I_p(\beta r_0)}{I_p'(\beta r_0)} = kr_0 + (kr_0 b_e + m)^2 \frac{K_m(kr_0)}{K_m'(kr_0)}$$

where $b_i = B_0/B_\theta$, $b_e = B_e/B_\theta$ and $C_2 = 0$ in the absence of a conducting wall.

2.3 Stability of the Kink Mode

The stability of the kink mode may be examined by solving equation (47) for m=1. We shall examine the stability in the long wavelength limit i.e. $k r_0 \ll 1$ so as to justify a small argument expansion of the Bessel functions. Such an analysis is required because the unstable modes correspond to $k r_0 < 1$ and for the maximally growing modes $\beta^2 = k^2 - \omega^2/c_A^2$ (provided $4\pi n kT/B_0^2 < 1$), $\beta^2 \simeq k^2 - \frac{c_{A\theta}^2}{r_0^2 c_A^2}$, $\beta^2 r_0^2 < 1$ where $c_{A\theta}^2 = B_\theta^2/4\pi\rho_0$

The dispersion relation then simplifies to

$$\frac{b_i^2 \beta^2 r_0^2}{p} = -k r_0 b_e \left(2 + k r_0 b_e\right) \tag{2.48}$$

In the absence of an external pump $p^2 = 1$ and for $c_A > c_s$ equation (48) can be solved to give[12]

$$\omega^2 = \frac{k^2 c_A^2}{b_i^2}\left(b_e^2 + b_i^2 + \frac{2 b_e}{k r_0}\right) \tag{2.49}$$

Instability results if $2 b_e / k r_0 > b_e^2 + b_i^2$. This readily reduces for $b_e = b_i$ to the Krushkal - Shafranov condition discussed earlier. The destabilizing term $2 b_e / k r_0$ arises because of the negative gradient of the azimuthal field in the radial direction.

The presence of the pump wave no longer permits a purely growing solution. Hence in this case the instabilities which will develop will be characteristically overstable[13]. A similar feature appears for the case of a rotating pinch[14]. The pump wave in our case therefore leads to overstability by providing a state of motion of lower order than the perturbed quantities. This therefore raises the interesting possibility of Landau damping of the modified kink modes when their parallel phase velocity is comparable to the ion thermal speed. We have incorporated such an effect by a phenomenological damping parameter ν' (see equation 27). We now examine equation (48) in two limits.

(i) $\frac{\omega}{kc_s}, \frac{\omega}{kc_A} < 1$

For this case, equation (48) can be simplfied to

$$\omega = \frac{i k^2 c_A^2}{b_i^2 \nu'} \left(b_i^2 + b_e^2 p + \frac{2 b_e p}{k r_o} \right) \tag{2.50}$$

p is in general a complex quantity whose real part would contribute to the instability. As is clear from the above equation, the destabilizing term is $2 b_e \mathrm{Re}\, p / k r_o$ (for $k < 0$) and it is possible to effect stabilization by choosing Re $p \to 0$. This can be achieved by introducing a proper frequency mismatch between the pump and the ionacoustic wave. We retain one of the resonances ($\omega_o \simeq k_+ c_s$). The expression for p^2 can be written as

$$p^2 = 1 - \frac{V_o^2 k_o}{c_A (\omega + \delta - i\nu/2)} \tag{2.51}$$

where $\delta = \omega_o - k_+ c_s$ is the frequency mismatch. Introducing this in equation (50) and after proper rearrangement, we get

$$(\omega - i\omega_k)(\omega + \delta - i\nu/2) = -i\lambda \tag{2.52}$$

where $$\omega_k = \frac{k^2 c_A^2}{b_i^2 \nu'} \left(b_i^2 + b_e^2 + \frac{2 b_e}{k r_o} \right) \tag{2.53}$$

and $$\lambda = \frac{k^2 k_o c_A V_o^2 b_e}{2 b_i^2 \nu'} \left(\frac{2}{k r_o} + b_e \right) \tag{2.54}$$

Equation (52) gives the roots

$$\omega_{1,2} = -\frac{1}{2}\left(\delta - i\omega_k - i\nu/_2\right) \pm \left[\left(\delta + i\omega_k - i\nu/_2\right)^2 - 4i\lambda\right]^{1/2} \tag{2.55}$$

Here

$$\omega_1 = i\omega_k - \frac{i\lambda\delta}{\delta^2 + (\omega_k - \nu/_2)^2} - \frac{(\omega_k - \nu/_2)\lambda}{\delta^2 + (\omega_k - \nu/_2)^2} \tag{2.56}$$

where we have expanded the term within the radical. Since we wish to study the effect of the coupling on unstable kink mode $\omega_k < 0,\ \lambda < 0$. We readily see that there is a stabilizing effect for $\delta > 0$. The maximum stabilizing effect can be obtained for that δ for which the quantity $(\lambda\delta)/\delta^2 + (\omega_k - \nu/_2)^2$ when differentiated with respect to δ gives zero and the second derivative becomes negative, i.e. $\delta^2 = (\omega_k - \nu/_2)^2$

The second root is

$$\omega_2 = \frac{i\nu}{2} + \frac{i\lambda\delta}{\delta^2 + (\omega_k - \nu/_2)^2} - \delta + \frac{\lambda(\omega_k - \nu/_2)}{\delta^2 + (\omega_k - \nu/_2)^2} \tag{2.57}$$

In this case for $\delta < 0$ we get a stabilizing effect to add to the damping of the ion acoustic mode. This mode does not exist in the absence of the coupling.

(iii) $\frac{\omega}{kc_s} > 1$, $\frac{\omega}{kc_A} < 1$

The dispersion relation in this case becomes

$$b_i^2\left(k^2 - \frac{\omega^2}{c_A^2}\right)r_0^2 = -kr_0b_e(2 + kr_0b_e)\left(1 - \frac{\alpha}{\omega + \delta - \frac{i\nu}{2}}\right)^{1/2} \tag{2.58}$$

where $\alpha = V_0^2 k_0 / 2c_A$

Again for the small pump wave amplitude we can expand the last term and rewrite (58) as

$$(\omega^2 + \omega_k^2)(\omega + \delta - i\nu/_2) = \bar{\alpha} \tag{2.59}$$

where $\omega_K^2 = \frac{k^2 c_A^2}{b_i^2}\left(b_e^2 + b_i^2 \quad \frac{2b_e}{kr_0}\right)$, $\bar{\alpha} = \frac{k^2 \alpha c_A^2 b_e}{2 b_i^2}\left(\frac{2+b_e}{kr_0}\right)$ (2.60)

Since we are studying unstable modes we have written the term with a negative sign.

Writing $\omega = -i\omega_K + \gamma$ where $|\gamma| \ll \omega_K$

we get

$$\gamma = \frac{i\bar{\alpha}\delta}{2\omega_K\left[\delta^2 + \left(\omega_K + \frac{\nu}{2}\right)^2\right]} - \frac{\bar{\alpha}\left(\omega_K + \nu/2\right)}{2\omega_K\left[\delta^2 + \left(\omega_K + \nu/2\right)^2\right]}$$

Here once again for $\delta > 0$ we get a stabilizing effect on the original kink mode due to coupling. Hence we see that there is no complete stabilization of the mode.

2.4 Discussion

We have investigated the possibility of applying dynamic stabilization methods at frequencies close to the ion acoustic frequency for reducing the growth rate of hydrodynamic kink modes. The principle advantage we wanted to exploit was the parametric coupling of kink modes to damped ion acoustic waves, thereby increasing the efficiency of the stabilization scheme. Our calculations show that ω/kc_s, $\omega/kc_A < 1$ there exist low-frequency damped modes and that the original kink mode gets dramatically modified if we take into account the possibility of Landau damping of the mode because of the small real part to the frequency induced by the pump. This overstability present in the system is to be expected because of the lower order dynamic equilibrium state existing in the column prior to the perturbations corresponding to the kink and ion acoustic waves. In general for low pump amplitudes we obtain a cubic equation in ω which on numerical solution supports the analytic expressions derived by us, showing the existence of a low frequency stable mode together with a weak stabilizing effect on the original kink mode.

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CHAPTER III

STIMULATED RAMAN SCATTERING

3.1 Introduction:

When an electromagnetic radiation travels through matter, various scattering processes can occur. In each case the light is scattered by fluctuations in the refractive index which are caused by well defined elementary excitations of the medium. If the amplitude level of the incident beam is not very large so as not to disturb the scattering medium, the scattered radiation is referred to as the spotaneous radiation. However, when the incident radiation exceeds a certain threshold, then the scattered radiation is no longer maintained at a negligible level and in fact exponentiates to amplitudes comparable to the incident wave. At the same time the elementary excitations, or collective states of the medium also suffer drastic modification. Such a process is called stimulated scattering. In crystals, the process of scattering from phonons (optical), electronic states, spin waves and plasmons is referred to as Raman Scattering. In unmagnetized plasmas the collective state involved in the process of Raman scattering is the electron plasma wave. Bloembergen has reviewed stimulated Raman scattering in certain solids and liquids[(1)(2)].

In plasmas the study of stimulated scattering of electromagnetic waves from collective states received attention with the advent of the laser. Goldman and Dubois[(3)] calculated the gain or spatial growth rate for the scattered radiation from a classical plasma by utilizing the differential cross section for thermally excited electron density fluctuations. Comisar[(4)] used the coupled mode analysis described by Bloembergen[(5)] to evaluate the gain and obtained an agreement with the work of Goldman and Dubois in the case of small damping for the plasma wave. At about the same time Bloembergen and Shen[(6)] investigated the optical nonlinearities of an unmagnetized plasma. They studied the problem of stimulated Raman Scattering by using the screened Coulomb potential, the screening being produced by the self consistent field and appropriately introduced through the self-consistent linear dielectric function. They also demonstrated, that the quantum mechanical treatment used by them is closely connected to the classical coupled mode approach so that the Raman effect can be viewed as a parametric interaction between two light waves and an electron plasma wave.

It is worth mentioning that the process of beat heating of a plasma, by two oppositely propagating laser beams (ω_0, $\vec{k}_0$) and (ω_1, $\vec{k}_1$) which resonantly excite a plasma wave (ω_2, $\vec{k}_2$) (provided $\omega_0 = \omega_1 + \omega_2$, $\vec{k}_0 = \vec{k}_1 + \vec{k}_2$) involves the same physical coupling between the three waves as SRS. The former process had been first examined by Kroll, Ron and Rostoker[7]. Subsequent work by Rosenbluth and Liu[8] and Schmidt[9] established conclusively the inefficacy of this method as a heating mechanism. On the contrary there was a growing apprehension that SRS may prove very undesirable for laser fusion and laser heating of magnetically confined plasmas.

The interest in scattering instabilities was revived with a view to studying the saturated level of the backscatter radiation under conditions which were less idealized. Since in the case of laser fusion (or for that matter also in ionospheric problems) the plasma would be far from homogeneous, Rosenbluth[10] investigated the development of a three wave interaction, in an inhomogeneous plasma (by using model coupled equations). The inhomogeneity was assumed to be either linear or quadratic. He showed that for a linear density profile, the three-wave interaction becomes a convective instability provided $K'(x) = d(k_0 - k_1 - k_2)/dx \neq 0$ where $K'(x)$ is the measure of the k vector mismatch introduced by the inhomogeneity. For quadratic density profile the absolute nature of the instability was restored with a modified inhomogeneity threshold. A subsequent detailed calculation for scattering instabilities in inhomogeneous quiescent plasmas was given by Liu, Rosenbluth and White[11].

Another effect which can be of importance to the three wave interaction is the finite band width[12], [13] of the incident radiation. Valeo and Oberman considered such a problem where they treated the pump to be monochromatic but with a random phase. They were able to obtain analytic results which showed that the growth rate can be drastically reduced for $D/\gamma_0 > 1$

where γ_0 is the growth rate in the case of a monochromatic pump and D is the diffusion coefficient for the random variation of the phase. J.J. Thomson has recently made a direct calculation of the effect of finite bandwidth $\Delta\omega$. He finds that the instability threshold increases and the growthrates get lowered. Physically this is because the power of the pump is dispersed over a frequency range $\Delta\omega$, while only that within the resonance range is available to drive the instability. Thus the effective power is reduced by a factor $\gamma/\Delta\omega$ (assumed $\ll 1$).

Finally attention has been focussed on the effect of random background turbulence in plasmas on the three-wave interaction(14)(15)(16). Since in many practical situations the plasma is likely to be turbulent (e.g. due to the violent mechanism of production, presence of instabilities etc.) it is important to investigate the influence of turbulent fluctuations on parametric coupling processes. For stimulated Raman Scattering the most significant influence would arise from density fluctuations since they modify the propagation characteristics of the plasma wave.

In the present chapter we shall carry out a detailed investigation of the influence of background random density fluctuation on stimulated Raman Scattering in a plasma. We investigate a one dimensional problem of stimulated scattering in a plasma in which the background density is an irregular function of the position variable as well as fluctuates in time. We assume a long wavelength, low frequency turbulence (compared to the wavelengths and time scales associated with the interacting waves). The assumption allows us to use the W.K.B. or eikonal approximation whereby the linear dispersion relation of each of the interacting modes is valid locally. Under these circumstances the coupled set of second order partial differential[15] equations reduce to first order equations which can be readily reduced to a second order stochastic integro-differential equation. The solution of this equation is attempted by using the method developed by Keller and others[16] for the wave propagation in random media. Assuming the random inhomogeneity to be small we retain terms of order $|\epsilon^2|$ where $|\epsilon| = |(n(x,t)-n_0)/n_0| \ll 1$ and obtain an equation for the ensemble average of the unknown variable. Assuming a Gaussian correlation function for the turbulent waves we evaluate the modified growth rates in the case of homogeneous plasma and also find the inhomogeneous threshold.

3.2 Basic Equations:

We consider a plasma in which the zero order plasma density is given by

$$n_0(x,t) = n_0\left(1 + \frac{x}{L} + \epsilon(x,t)\right) \tag{3.1}$$

where n_0 represents the density at $x = 0$ in the quiescent plasma L the scalelength associated with the inhomogeneity and $\epsilon(x,t)$ the random density fluctuations over the background inhomogeneity. The fundamental set of equations are

$$\frac{\partial n_\alpha}{\partial t} + \nabla\cdot\left(n_\alpha \vec{V_\alpha}\right) = 0 \tag{3.2}$$

$$\frac{\partial \vec{V_\alpha}}{\partial t} + \vec{V_\alpha}\cdot\nabla\vec{V_\alpha} = -\frac{\nabla p_\alpha}{m_\alpha n_\alpha} + \frac{e_\alpha}{m_\alpha}\left(\vec{E} + \frac{\vec{V_\alpha}\times\vec{B}}{c}\right) \tag{3.3}$$

$$\nabla\cdot\vec{E} = 4\pi\sum_\alpha e_\alpha n_\alpha \tag{3.4}$$

$$\nabla\times\vec{B} = +\frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\sum_\alpha n_\alpha e_\alpha \vec{V_\alpha} \tag{3.5}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{3.6}$$

The subscripts α denotes the species.

We will assume from the outset, that the turbulence does not affect the electromagnetic waves (the pump as well as the scattered mode). This is well justified if ω_0 and $\omega_1 > \omega_{pe}$. The nonlinear terms in equation (3.3) arise from the electromagnetic waves $\vec{E}_j$, $(j = 0, 1)$. We take the perturbed quantities associated with the e.m. waves to go as

$$\xi_j = \tilde{\xi}_j e^{i\epsilon_j k_j x + i\omega t} + \text{c.c.} \tag{3.7}$$

where
$$\epsilon_j = \begin{cases} 1 & j = 0 \\ -1 & j = 1 \end{cases}$$

Since we are dealing with the one dimensional case, we will treat the problem of backscatter specifically. For these e.m. waves, $n_j = 0$, $v_{je} = -eE_j/im\omega_j$; $\nabla \times v_{je} = eB_j/mc$ where v_{je} is the electron fluid velocity, the ion response being neglected at these high frequencies.

As a consequence the equation of motion for the electrostatic mode ($j = 2$) may be written as

$$\frac{\partial \vec{v}_2}{\partial t} + \frac{\nabla p_2}{m n_0} + \frac{e E_2}{m} = -\nabla(\vec{v}_0 \cdot \vec{v}_1) \tag{3.8}$$

Taking the divergence of (3.8) and using equations (3.2) and (3.4) we get

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2(x,t) - v_e^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2}{\partial t^2} + \omega_{pi}^2(x,t)\right)n_e - \omega_{pe}^2(x,t)\,\omega_{pi}^2 n_e$$
$$= \left(\frac{\partial^2}{\partial t^2} + \omega_{pi}^2(x,t)\right)n_0(x,t)\,\nabla^2(\vec{v}_0 \cdot \vec{v}_1) \tag{3.9}$$

We have used $T_i = 0$ since we are interested in a nonisothermal plasma with $T_e \gg T_i$. By virtue of the assumption of long wavelength and slow time dependence of the turbulence, together with the weak spatial background inhomogeneity, equation (3.9) is the same as that for a homogeneous plasma except for the fact that the coefficients now have a slow space and time dependence.

Taking the perturbations of the electrostatic mode to go as

$$\xi_2 = \tilde{\xi}_2 e^{-ik_2x - i\omega t} + c.c. \tag{3.10}$$

the equation, (3.9) reduces to

$$\left(\frac{\partial}{\partial t} - V_2\frac{\partial}{\partial x}\right)\tilde{n}_e = \frac{k_2^2 n_0 V_0^* \tilde{V}_1}{2i\omega_2} e^{-i\left[\int K dx + \int \Omega dt\right]} \tag{3.11}$$

where we have used the local resonance condition $\omega_2^2 = \omega_p^2(x,t) + k_2^2(x,t)V_e^2$ to be valid. Also $K = k_0 + k_1 - k_2$, $\Omega = \omega_0 - \omega_1 - \omega_2$

Similarly the scattered electromagnetic wave is governed by the equation

$$\nabla^2\vec{E}_1 - \frac{1}{c^2}\frac{\partial^2\vec{E}_1}{\partial t^2} - \frac{\omega_p^2(x,t)}{c^2}\vec{E}_1 = -\frac{4\pi}{c^2}\frac{\partial}{\partial t}(ne\vec{V}_0) \tag{3.12}$$

which subsequently reduces to

$$\left(\frac{\partial}{\partial t} + V_1\frac{\partial}{\partial x}\right)\tilde{E}_1 = \frac{4\pi e}{2}\tilde{n}_e V_0 e^{i\left[\int k dx + \int \Omega dt\right]} \tag{3.13}$$

Equations (3.11) and (3.13) are the coupled equations which represent stimulated Raman Scattering in a random medium.

In a homogeneous medium, the exponential factor would be zero for perfect matching conditions or some constant value if perfect resonance was not feasible. In this case the turbulent fluctuations in the density give rise to fluctuations in the k vector of the electrostatic mode and thereby affects the strength of the coupling in a stochastic manner.

Although the coupled equations readily depict the parametric process, the interaction matrix element i.e. the coefficient of the term on the R.H.S. is not the same for both the waves and this apparent asymmetry is due to the choice of the dependent variables. Using the action amplitude a_j where $|a_j|^2 = N_j = \epsilon_j/\omega_j$ ϵ_j, being the energy density of the j^{th} mode the equations (3.11) and (3.13) can be recast into a more symmetric form by putting $\tilde{n}_e = (n_0 k_2^2 / 2\omega_p m)^{1/2} a_2$ and $\tilde{V}_1 = (2\pi)^{1/2} e a_1 / m\omega_1^{1/2}$, $\gamma_0 = \frac{i}{2}\left(\frac{\omega_p}{\omega_1}\right)^{1/2} k_2 V_0$

$$\left(\frac{\partial}{\partial t} + V_1 \frac{\partial}{\partial x}\right) a_1 = \gamma_0 a_2 e^{i\left[\int k\,dx + \int \Omega\,dt\right]} \tag{3.14}$$

$$\left(\frac{\partial}{\partial t} - V_2 \frac{\partial}{\partial x}\right) a_2 = \gamma_0 a_1 e^{-i\left[\int k\,dx + \int \Omega\,dt\right]} \tag{3.15}$$

One readily sees the symmetry of the coupling coefficients which represent the growth rate γ_0 of the three wave process in a homogeneous medium. These two sets of stochastic coupled equations in general represent any parametric process in an unmagnetized turbulent plasma under the restricted conditions of long-wavelength, low-frequency turbulence. We have ignored the slow space and time dependence of the coupling coefficient.

Till now we have not determined the explicit manner in which the turbulence or inhomogeneity affects the propagation vector and frequency of the electrostatic mode. Since the eikonal S equals $-ik_2x - i\omega_2 t$, where $\omega_2 = W_2(k_2, x, t)$ is the dispersion relation of the mode[17], we have

$$\frac{dk_2}{dt} = \frac{\partial W_2}{\partial x} \tag{3.16}$$

$$\frac{d\omega_2}{dt} = \frac{\partial W_2}{\partial t} \tag{3.17}$$

where the total derivative is $\partial/\partial t - V_2\,\partial/\partial x$ i.e. the equation to the characteristics in a homogeneous quiescent medium is given by $dx/dt = -V_2$. These Hamiltons equations determine the variation of k_2 and ω_2 by the explicit spatial and temporal dependence of the properties of the medium brought in through the dispersion relation associated with the mode. Using the plasma wave dispersion relation $\omega_2^2 = \omega_{po}^2\left[1 + x/L + \epsilon(x,t) + k_2^2\lambda_D^2\right]$ where λ_D is the electron Debye length. Equations (3.16) and (3.17) can be formally solved to give

$$k_2(x,t) - k_2(0,0) = \frac{\omega_{po}}{2}\int_0^t \frac{\partial\epsilon(x',t')}{\partial x'}dt' - \frac{\omega_{po}x}{2LV_2} \tag{3.18 a}$$

$$\omega_2(x,t) - \omega_2(0,0) = \frac{\omega_{po}}{2}\int_0^t \frac{\partial\epsilon(x',t')}{\partial t'}dt' \tag{3.18 b}$$

where the integrations are carried out over the trajectories given by

$$\frac{dx'}{dt'} = -V_2$$

We adopt the convention that the phase mismatch is zero at the origin and at t=0. However to eliminate the initial value of the turbulence level we put the lower limit of the integral as $t = -\infty$

Eliminating a_2 from equations (3.14) and (3.15) we get

$$\left[\left(\frac{\partial}{\partial t} + V_1 \frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t} - V_2 \frac{\partial}{\partial x}\right) + (O_1 + O_2)\left(\frac{\partial}{\partial t} + V_1 \frac{\partial}{\partial x}\right)\right] a_1 = \gamma_0^2 a_1 \tag{3.19}$$

with

$$O_1 = -\frac{i\omega_{p0} V_2}{2} \int_0^t \frac{\partial \epsilon(x', t')}{\partial x'} dt' + \frac{\omega_{p0} x}{2L} \tag{3.20 a}$$

$$O_2 = \frac{i\omega_{p0}}{2} \int_0^t \frac{\partial \epsilon(x', t')}{\partial t'} dt' \tag{3.20 b}$$

Equation (3.19) is our basic differential equation with coefficients which are random functions of x and t. In the following sections we shall solve this equations in various limits by applying Keller's method for the solution of stochastic differential equations.

3.3 Keller's Method:

Let us briefly outline Keller's method for solving a linear stochastic differential equation of the type

$$(M + V)\, a = 0 \quad \text{with} \quad \langle V \rangle = 0 \tag{3.21}$$

where M is the nonrandom operator. We define the Green's function $G(r, r')$ and $G_0(r, r')$ by

$$[M(r) + V(r)]\, G(r, r') = \delta(r - r') \tag{3.22}$$

and

$$M(r)\, G_0(r, r') = \delta(r - r') \tag{3.23}$$

We can readily convert (3.22) into an integral equation

$$G(r,r') = G_0(r,r') - \int G_0(r,r_1)\,V(r_1)\,G(r_1,r')\,dr_1 \qquad (3.24)$$

This is the Dyson equation

Iterating once we get

$$G(r,r') = G_0(r,r') - \int G_0(r,r_1)V(r_1)G_0(r_1,r')\,dr_1$$
$$+ \int G_0(r,r_1)\,V(r_1)\int G_0(r_1,r_2)\,V(r_2)\,G(r_2,r')\,dr_1\,dr_2 \qquad (3.25)$$

Taking the ensemble average of this equation we obtain

$$\langle G(r,r')\rangle = G_0(r,r')$$
$$+ \iint G_0(r,r_1)\langle V(r_1)\,G_0(r_1,r_2)\,V(r_2)\,G(r_2,r')\rangle\,dr_1\,dr_2 \qquad (3.26)$$

where the angular brackets denote the process of averaging. Operating by $M(r)$ on equation (3.25) we arrive at the equation

$$M(r)\langle G(r,r')\rangle - \int\langle V(r)\,G_0(r,r_1)\,V(r_1)\,G(r_1,r')\rangle\,dr_1 = \delta(r-r') \qquad (3.27)$$

Till now there has been no approximation involved. If we now invoke the assumption of local statistical independence[18] of Bourret

$$\langle V(r)\,G_0(r,r_1)\,V(r_1)\,G(r_1,r')\rangle = \langle V(r)\,G_0(r,r_1)\,V(r_1)\rangle\langle G(r_1,r')\rangle$$

then equation (3.27) becomes an integro-differential equation for $\langle G(r,r')\rangle$ This result was derived by Keller[16] in a different way. The equation is also referred to as the bilocal approximation to the Dyson equation. The term bilocal refers to the fact that $G(r, r')$ is dependent on the two point correlation of the random field.

Dence and Spence[19] have shown that if the infinite Neumann expansion is made from the Dyson equation (3.25), then by assuming the random function to represent a centered multivariate normal process (so that the odd order correlation functions are zero and all even correlation functions can be expressed in terms of two point correlations) then a rather simple diagramatic scheme can be evolved and that the bilocal approximation then corresponds to summing an infinite subset of the Neumann expansion in a closed form. Physically this approximation represents the singly scattered mean wave.

Hence the equation we shall use for our problem is

$$M(r)\langle G(r,r')\rangle - \int \langle V(r)\, G_0(r,r_1)\, V(r_1)\rangle \langle G(r_1,r')\rangle\, dr_1 = \delta(r-r') \tag{3.28}$$

3.4 Homogeneous Time Dependent Turbulence:

For the case of a statistically homogeneous plasma, from equation (3.19) we have

$$M = \left(\frac{\partial}{\partial t} + V_1\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t} - V_2\frac{\partial}{\partial x}\right) \tag{3.29}$$

$$V = -i\frac{\omega_{p0}}{2}\left[V_2\int_0^t \frac{\partial\epsilon(x',t')}{\partial x'}\,dt' - \int_0^t \frac{\partial\epsilon(x',t')}{\partial t'}\,dt'\right]\left(\frac{\partial}{\partial t} + V_1\frac{\partial}{\partial x}\right)$$

$$= (O_1 + O_2)\left(\frac{\partial}{\partial t} + V_1\frac{\partial}{\partial x}\right) \tag{3.39}$$

Under these circumstances the renormalized equation for the average amplitude of mode a_1 is

$$\left(\frac{\partial}{\partial t}+V_1\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t}-V_2\frac{\partial}{\partial x}\right)\langle a_1\rangle - \int \left(O_1(x,t)+O_2(x,t)\right)\left(\frac{\partial}{\partial t}+V_1\frac{\partial}{\partial x}\right)$$

$$\times\, G_0(x,x',t,t')\left(O_1(x',t')+O_2(x',t')\right)\left(\frac{\partial}{\partial t'}+V_1\frac{\partial}{\partial x'}\right)\langle a_1\rangle dx' \tag{3.31}$$

where $G_0(x,x',t,t')$ is the Green's function for the deterministic operator M. We shall resort to Fourier transforms and thereby convert the integro-differential equation into an algebraic one. This is possible because of the assumption of statistical homogeneity of the medium. Assuming $\langle a_1(x',t')\rangle \sim \exp(ipx'-iqt')$ the equation now becomes

$$M(p,q)\langle a_1(p,q)\rangle + \frac{\omega_{po}^2}{4}|\epsilon^2|\langle a_1(p,q)\rangle(-iq+iV_1p)\times$$

$$(2\pi)^2\iint dk\,d\omega\, P(p-k,q-\omega)\, G_0(p-k,q-\omega)\, B(k,\omega) = 0 \tag{3.32}$$

where

$$P(p-k,q-\omega) = -i(q-\omega)+iV_1(p-k) \tag{3.33}$$

$$G_0(p,q) = -(2\pi)^{-2}\left[(q-V_1p)(q+V_2p)+\gamma_0^2\right]^{-1} \tag{3.34}$$

and

$$|\epsilon^2|B(k,\omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dx\,dt\, e^{-ikx+i\omega t}\langle\epsilon(x,t)\epsilon(x',t')\rangle$$

$$= \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} e^{-ikx+i\omega t} B(x-x',t-t')\,dx\,dt \tag{3.35}$$

What we have essentially done is to Fourier analysis the quantities $G_0(x,x',t,t')$, $B(x-x',t-t')$ so that the integration for $\langle O_i O_j\rangle$

can be readily carried out over the specified trajectory. The four terms corresponding to the various combinations of $\langle o_i(x,t)\, o_j(x',t')\rangle \quad i,j = 1,2$ readily combine to give the expression in (3.32) (See Appendix).

So far we have not specified the relationship that exists between the space and time part of the fluctuating field, i.e. how the coherence length is related to the coherence time. We assume that the turbulence is due to a set of modes satisfying a linear dispersion relation of the type $\omega(k) = kV$. Under such a choice the Fourier transform of the correlation function can be written as

$$B(k,\omega) = 2\pi\, B(k)\, \delta(\omega + \omega(k)) \tag{3.36}$$

The dispersion relation in this case therefore becomes

$$(q + pV_2)(q - pV_1) + \gamma_0^2 + i\,\frac{\pi^{1/2}\omega_{p0}^2\, |\epsilon^2|\, L_T\,(q - V_1 p)}{8^{1/2}\,(V_2 - V)} = 0 \tag{3.37}$$

where we have used a Gaussian spectral density

$$B(k) = \frac{1}{(2\pi)^{1/2}}\, |\epsilon^2|\, L_T \exp\left(-\frac{k^2 L_T^2}{2}\right)$$

and have made the assumption $\gamma_0 L_T/(V_1 V_2)^{1/2} < 1$ i.e. the scale length of the turbulence is shorter than the homogeneous growth length.

We can rewrite the dispersion relation as

$$\left(p + \frac{q}{V_2}\right)\left(p - \frac{q}{V_1}\right) - i\,\frac{\nu_{eff1}}{V_1 V_2}\,(q - V_1 p) - \frac{\gamma_0^2}{V_1 V_2} = 0 \tag{3.38}$$

where

$$\nu_{eff1} = \pi^{1/2}\,\omega_{p0}^2\,|\epsilon^2|\,L_T/[8^{1/2}(V_2 - V)]$$

Here ν_{eff1} is the effective damping brought about on the three wave process due to the random walk that the phase suffers due to the turbulent fluctuations. It is to be borne in mind that this effective collisional frequency does not in any way lead to the heating of the plasma. On the scale sizes and frequencies we are interested in, the effect of the turbulence is to merely weaken the strength of the coupling between the waves.

If we compare our equation to that derived by Thomson[8] we can readily appreciate the similarity between the finite band-width and turbulent effects.

Solving for p we get

$$p = -\left[\left(\frac{q}{V_2} - \frac{q}{V_1}\right) + i\frac{\nu_{eff1}}{V_2}\right] \pm \left[\left(\frac{q}{V_2} - \frac{q}{V_1} + i\frac{\nu_{eff1}}{V_2}\right)^2 + 4\left(\frac{q^2}{V_1V_2} + i\frac{\nu_{eff1}\,q}{V_1V_2} + \frac{\gamma_0^2}{V_1V_2}\right)\right]^{1/2} \tag{3.39}$$

The merging of the two roots for some imaginary positive value of q ensures the existence of an absolute instability.

For this we require

$$\left[\left(\frac{q}{V_2} + \frac{q}{V_1}\right) + i\frac{\nu_{eff1}}{V_2}\right]^2 = -\frac{4\gamma_0^2}{V_1V_2} \tag{3.40}$$

Hence for $V_1V_2 > 0$ we have an absolute instability. We may recall that as we had started out with the decay waves propagating in opposite directions the condition we get is the reverse of what Thomson has derived. The threshold is

$$\frac{\gamma_0}{\nu_{eff1}} = \frac{1}{2}\left(\frac{V_1}{V_2}\right)^{1/2} \tag{3.41}$$

If we had derived an equation for $\langle a_2 \rangle$ then our dispersion relation would have been the same as (3.37) with $V_1 \longleftrightarrow V_2$ and ν_{eff1} replaced by ν_{eff2}

where $\left(\frac{\pi}{8}\right)^{1/2} \frac{|\epsilon^2| L_T (V_1 + V)^2}{(V_2 - V)^3} = \nu_{eff2}$

Hence the effective damping is greater for the electrostatic plasma wave and the threshold would become

$$\frac{\gamma_0}{\nu_{eff2}} = \frac{1}{2} \left(\frac{V_2}{V_1}\right)^{1/2} \tag{3.42}$$

The important parameter for determining the threshold is the intensity and not the amplitude so that the required threshold as discussed by Thomson is,

$$\gamma_0 = \frac{1}{2} \min \left[\nu_{eff1} \left(\frac{V_1}{V_2}\right)^{1/2}, \; \nu_{eff2} \left(\frac{V_2}{V_1}\right)^{1/2} \right] \tag{3.43}$$

so that in our case it is the electromagnetic backscattered mode which will determine the necessary intensity requirements.

The effect of the time dependence is a slight reduction in the velocity of the electrostatic mode, so that the basic feature for the modification in the dispersion relation is the same as that for quasi-state turbulence. It is easily seen that for the convective case (i.e. $q = 0$) the result reduces to that derived by us[15] for the quasistatic turbulence using the more general fourth order equations and then taking the limit for long wavelength turbulence. The modified inverse growth length becomes

$$p = -i \left(\frac{8}{\pi}\right)^{1/2} \frac{\gamma_0^2}{\omega_{p0}^2} \frac{V_2}{V_1} \frac{1}{|\epsilon^2| L_T} \tag{3.44}$$

In this case the turbulence acts like a series of weak inhomogeneities which introduce random phase mismatch in the three interacting waves. On encountering a typical density fluctuation the characteristic diffusion coefficient for the phase change can be written as

$$\Delta = \left| \langle (\delta k)^2 \rangle L_T \right| \qquad \text{where} \qquad \delta k = k_0 + k_1 - k_2$$

Hence the modified inverse growth length is given by[7]

$$p = -i\,\gamma_0^2 / V_1 V_2 \Delta$$

Using the dispersion relation for plasma waves we can retrieve equation (3.44) from the above expression.

3.5 Inhomogeneous Plasma with Quasistatic Turbulence:

In this section we examine the role played by inhomogeneity on the three wave interaction. We shall only consider quasistatic turbulence because as we have seen in the previous section, the time dependent and quasistatic turbulence effects are identical except for the relative shift in the group velocities of the decay waves.

In this case

$$O_1 = \frac{i\omega_{po}}{2L}\left[x + \epsilon(x) L \right] , \qquad O_2 = 0$$

Laplace transforming the time variable we get

$$\frac{d^2 a_1}{dx^2} - \left[ik(x) - \left(\frac{s}{V_1} - \frac{s}{V_2} \right) \right] \frac{da_1}{dx} - \left[ik(x)\frac{s}{V_1} - \delta^2 + \frac{s^2}{V_1 V_2} \right] a_1 = 0 \tag{3.45}$$

We write $\quad k(x) = k_1(x) + k_2(x)$

where $\quad k_1(x) = k'x , \quad k_2(x) = k' L \epsilon(x) \quad$ and $\quad k' = \dfrac{\omega_{po}}{2 V_2 L}$

Putting $a_1 = A e^{\frac{1}{2}\int k_1(x)\,dx + \left(\frac{s}{V_2} - \frac{s}{V_1}\right)\frac{x}{2}}$

Equation (3.45) becomes

$$\Big[\frac{d^2}{dx^2} + \frac{1}{4}\left[k_1(x) - is\left(\frac{1}{V_1} + \frac{1}{V_2}\right)\right]^2 + \frac{1}{2} i k_1'(x) + \delta^2 - i k_2(x)\frac{d}{dx}$$

$$- \frac{i}{2} k_2(x) s \left(\frac{1}{V_1} + \frac{1}{V_2}\right) + \frac{1}{2} k_2(x) k_1(x) \Big] A = 0 \tag{3.46}$$

We can identify the deterministic and stochastic operators as

$$M = \frac{d^2}{dx^2} + \frac{1}{4}\left[k_1(x) - is\left(\frac{1}{V_1} + \frac{1}{V_2}\right)\right]^2 + \frac{i}{2} k_1'(x) + \delta^2 \tag{3.47}$$

and

$$V = -i k_2(x) \left[\frac{d}{dx} + \frac{s}{2}\left(\frac{1}{V_1} + \frac{1}{V_2}\right) + \frac{i}{2} k_1(x) \right] \tag{3.48}$$

We shall use the Green's function for the homogeneous case in evaluating the terms $\langle V G_0 V \rangle$. The assumption that goes into the approximation is that the scale length of the turbulence is shorter than the inhomogeneity scale height so that on the spatial scale sizes of the order of L_T (the coherence length of the turbulent waves) the inhomogeneity effects will not be discernible. Such a prescription facilitates the evaluation of the integrals in a closed form. We further demand that $\delta L_T \ll 1$, that is the growth length be larger than the coherence length of the waves so that in the distance over which the coupled waves exponentiate they suffer a large number of phase random changes. Since the equation is an integro-differential one, we Fourier transform it to get a second order ordinary differential equation

$$\Big\{ \frac{d^2}{dp^2} - \left[\frac{2s}{k'}\left(\frac{1}{V_1} + \frac{1}{V_2}\right) - \frac{2\bar{\alpha}}{k'} \right] \frac{d}{dp} + \frac{s^2}{k'^2}\left(\frac{1}{V_1} + \frac{1}{V_2}\right)^2 - \frac{4\delta^2}{k'^2}$$

$$- \frac{2i}{k'} - \frac{4i\bar{\alpha}}{k'^2}\left[\left(p - \frac{is}{2}\left(\frac{1}{V_1} + \frac{1}{V_2}\right)\right)\right] \Big\} \langle A(p) \rangle = 0 \tag{3.49}$$

where $\tilde{\alpha} = \alpha\left(1 + i\dfrac{k' L_T^2}{2}\right)$, $\alpha = (2\pi)^{1/2} k'^2 |\epsilon^2| L^2 L_T$

We can now transform away the 1st order term by writing

$$\tilde{a}(p) = \langle A(p)\rangle \exp \int \left[\frac{S}{k'}\left(\frac{1}{V_1} + \frac{1}{V_2}\right) - \frac{\tilde{\alpha}}{k'}\right] dp \tag{3.50}$$

We get

$$\frac{d^2 \tilde{a}(p)}{dp^2} + \frac{4}{k'^2}\left[p^2 - i\tilde{\alpha} p - \delta^2\right] \tilde{a}(p) = 0 \tag{3.51}$$

We observe that this equation does not contain terms containing S, the Laplace transform variable and hence we can only get convective modes.

The W.K.B. solution for this equation is

$$\tilde{a}(p) = \frac{1}{[V(p)]^{1/4}}\, e^{\pm \int^{p} [V(p')]^{1/2} dp'} \tag{3.52}$$

where $V(p) = \dfrac{4}{k'^2}\left[\delta^2 - p^2 + i\tilde{\alpha} p\right]$

The amplitude is therefore written as

$$\tilde{a}(x) = \int e^{\pm \int^{p} [V(p')]^{1/2} dp' + ipx}\, dp \tag{3.53}$$

where we have neglected the term $[V(p)]^{1/4}$ in the denominator as it has a small contribution (except at the turning points, where of course the solution itself is not valid).

Evaluating the integral by the saddle point method, (where $p_s = \pm \delta + \frac{1}{2} i \bar{\alpha}$, are the saddle points if terms of order $\bar{\alpha}^2$ are neglected) the integral becomes

$$\tilde{a}(x) \sim e^{\pi \delta^2 / K' - \frac{1}{2} \bar{\alpha} x_{max}} \tag{3.54}$$

where we have used the requirement $\delta > \alpha / 2$ (see Thomson). This convective threshold due to the damping effects is the same as the absolute instability threshold (3.41) as has been discussed by Thomson. Also

$$x_{max} = 2 \gamma_0 / K' (V_1 V_2)^{1/2}$$

Hence the inhomogeneity threshold becomes

$$\frac{\pi \gamma_0^2}{K' V_1 V_2} - \frac{\alpha \gamma_0}{(V_1 V_2)^{1/2}} > 1 \tag{3.55}$$

Since we have already demanded that $\delta > \alpha / 2$ the second term on the L.H.S. is a very small contribution near the threshold.

$$\frac{\pi \gamma_0^2}{K' V_1 V_2} > 1 \tag{3.56}$$

Hence if we consider $\delta \simeq \alpha / 2$, to the lowest order $|\epsilon^2|$ in the turbulence, there would be no modification in the inhomogeneity threshold.

3.6 Discussion:

We have investigated the effect of background low frequency, long wavelength turbulence on the process of Raman Scattering. Though we have specifically considered the K' to be for the Raman process, the set of equations are general and can be considered for any three wave process.

We find that in the homogeneous case, the turbulence introduces a threshold for the absolute instability. This threshold is given by

$$\frac{V_0}{V_e} = \left(\frac{\pi}{8}\frac{\omega_1}{\omega_p}\frac{c}{c_s}\right)^{1/2}\frac{|\epsilon^2| L_T \omega_{pe}^2}{k_2 c_s V_e}$$

For stimulated Raman Scattering with $\omega_0 \simeq 2\omega_{pe}$, the growth rate is maximum. Hence for $T_e = 1\,\mathrm{keV}$, $k_0 L_T \simeq 10$ $k_0 = 6\times10^4$ we find that $V_0/V_e \sim 1$ $\sqrt{|\epsilon^2|} \sim 10^{-3}$ which is indeed a very low level of turbulence. Therefore the Raman scattering in a homogeneous plasma can be effectively suppressed by the presence of background density fluctuations.

In the case of an inhomogeneous plasma the convective amplification factor virtually remain unaltered. This is of course derived on the assumption that the damping threshold which is identical to the absolute instability threshold is overcome. Therefore it is apparent that the damping threshold in the case of the turbulent plasma is the more important one. A similar result was obtained by Thomson for the case of the finite bandwidth effects so that we see that finite bandwidth and turbulent effects manifest themselves in a similar manner.

APPENDIX

Evaluation of $\int \langle V G_0 V \rangle \langle a_1 \rangle \, dx' dt'$

$$\langle V G_0 V \rangle \langle a_1 \rangle = \sum_{i,j=1}^{2} \langle V G_0 V \rangle_{ij} \langle a_1 \rangle$$

where

$$\iint \langle V G_0 V \rangle_{ij} \langle a_1 \rangle \, dx' dt' = \iint dx' dt' \langle O_i(x,t) \, P\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) \times$$

$$G_0(x,x',t,t') \, O_j(x',t') \, P\left(\frac{\partial}{\partial t'}, \frac{\partial}{\partial x'}\right) \langle a_1(x',t') \rangle \rangle$$

Here $P\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) = \frac{\partial}{\partial t} + V_1 \frac{\partial}{\partial x}$

$$\iint \langle V G_0 V \rangle_{11} \langle a_1 \rangle \, dx' dt' = -\frac{\omega_{p_0}^2 V_2^2}{4} |\epsilon^2| \iint dx' dt' \int_0^t \langle \frac{\partial \epsilon(x'',t'')}{\partial x''} dt'' \times$$

$$P\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) G_0(x,x',t,t') \int_0^{t'} \frac{\partial \epsilon(x''',t''')}{\partial x'''} dt''' \, P\left(\frac{\partial}{\partial t'}, \frac{\partial}{\partial x'}\right) \langle a_1(x',t') \rangle \rangle$$

$$= -\frac{\omega_{p_0}^2 V_2^2}{4} |\epsilon^2|^2 \iint dx' dt' \int_0^t \int_0^{t'} dt'' dt''' \frac{\partial^2}{\partial x'' \partial x'''} B(x''-x''', t''-t''') \times$$

$$P\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) G_0(x,x',t,t') \, P\left(\frac{\partial}{\partial t'}, \frac{\partial}{\partial x'}\right) \langle a_1(x',t') \rangle \rangle$$

where $|\epsilon^2| \, B(x''-x''', t''-t''') = \langle \epsilon(x'',t'') \, \epsilon(x''',t''') \rangle$

$$\iint \langle V G_0 V \rangle_{11} \langle a_1 \rangle \, dx' dt' = -\frac{\omega_{p_0}^2 V_2^2}{4} |\epsilon^2| \iiiint dx' dt' dk \, d\omega \int_0^t \int_0^{t'} k^2 \times$$

$$B(k,\omega) e^{ik(x''-x''') - i\omega(t''-t''')} dt'' dt''' \iiiint P(p',q') G_0(p',q') e^{ip'(x-x')} \times$$

$$e^{-ip(t-t')} P(p,q) \, e^{ipx' - iqt'} \langle a_1(p,q) \rangle \, dp \, dq \, dp' \, dq'$$

where $P(p,q) = -iq + iV_1 p$

Now we integrate over the trajectories

$$x'' - x = -V_2\,(t'' - t)$$
$$x''' - x' = -V_2\,(t''' - t')$$

and demand that at t=0 there is no turbulence so that the initial value does not enter the discussion. After that we can perform the integrations over p', q', x', t' remembering that

$$\delta(x - x') = \frac{1}{2\pi}\int d\alpha\, e^{i\alpha(x - x')}$$

Performing these straight forward manipulations we get

$$\iint \langle V G_0 V\rangle_{11} \langle a_1\rangle\, dx'\, dt' = \frac{i\omega_{p_0}^2 |\epsilon^2| V_2^2 (2\pi)^2}{4} \iiiint dp\, dq\, dk\, d\omega \langle a_1(p,q)\rangle$$

$$(q - V_1 p)\, e^{ipx - iqt}\, P[(p-k),(q-\omega)]\, \frac{G_0(p-k,\, q-\omega)\, B(k,\omega)}{(\omega + kV_2)^2}$$

Similarly

$$\iint [\langle V G_0 V\rangle_{12} + \langle V G_0 V\rangle_{21}]\, dx'\, dt' = \frac{2 i\omega_{p_0}^2 |\epsilon^2| V_2}{4} \iiiint dp\, dq\, dk\, d\omega\, \omega$$

$$(2\pi)^2 (q - V_1 p)\, e^{ipx - iqt}\, P[p-k,\, q-\omega]\, \frac{G_0(p-k,\, q-\omega)\, B(k,\omega)\, \langle a_1(p,q)\rangle}{(\omega + kV_2)^2}$$

and finally

$$\iint \langle V G_0 V\rangle_{22} \langle a_1\rangle\, dx'\, dt' = \frac{i\omega_{p_0}^2 |\epsilon^2| (2\pi)^2}{4} \iiiint dp\, dq\, dk\, d\omega\, \omega^2 \langle a_1(p,q)\rangle$$

$$(q - V_1 p)\, e^{ipx - iqt}\, P[p-k,\, q-\omega]\, \frac{G_0(p-k,\, q-\omega)\, B(k,\omega)}{(kV_2 + \omega)^2}$$

Hence $$\iint \langle V G_0 V\rangle\, dx'\, dt' = \frac{i\omega_{p_0}^2}{4} \epsilon^2 (2\pi)^2 \iiiint (q - V_1 p) \langle a_1(p,q)\rangle\, e^{ipx - iqt}\, x$$
$$P(p-k,\, q-\omega)\, G_0(p-k,\, q-\omega)\, B(k,\omega)\, dp\, dq\, dk\, d\omega$$

This is then used in equation (3. 32)

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CHAPTER IV

STIMULATED BRILLOUIN SCATTERING

4.1 Introduction:

In the last chapter, we have discussed the stimulated scattering of electromagnetic waves from a high frequency electrostatic mode of the electron gas with a neutralizing ion background. In this chapter we will study the scattering of the electromagnetic wave from the low frequency ion acoustic mode which involves density perturbations maintaining approximate charge neutrality so that the ions participate in the process. Hence the fundamental difference in Raman and Brillouin scattering depends on the collective mode involved. In solids and liquids the Raman process involves scattering from internal degrees of freedom and therefore reflects the internal structure of the constituents whilst Brillouin processes involve density perturbations of the media in which the basic constituents collectively participate. For a mechanical system, an alternative way of stating the difference is saying that Raman processes involve optical branches and Brillouin scattering the acoustical branch of the mechanical dispersion relation.

In a plasma the linear theory for stimulated Brillouin scattering has been dealt with quite exhaustively by Gorbunov[1][2] and Drake[3] et al. Being essentially a three-wave parametric coupling between an ion acoustic wave and two electromagnetic waves, the coupled mode analysis given byBloembergen[4] and used by Comisar[5] for SRS, can be readily applied to SBS.

With the advent of high powered lasers and the subsequent experiments in laser pellet interactions[6][7], the studies in stimulated scattering received an impetus more so because of the large discrepancy between the calculated levels of backscatter and the levels obtained experimentally.

Theoretical findings and computer simulations by Forslund et. al.[8][9][10] have shown that exceedingly large levels of backscatter (as high as ninety per cent) can take place whilst experimentally,[11][12] level not exceeding twenty per cent of the incident pump has been observed.

The theoretical investigations were then oriented to the study of parametric processes in less idealized conditions and the emphasis was on finding both linear as well as nonlinear stabilizing mechanisms for the backscatter instabilities[13]. Although Raman scattering has a higher growth rate it is not considered to be as dangerous as Brillouin scattering because of the strong saturation mechanism[14] brought about by electron heating by nonlinear processes and by the modulational instability of plasmons. Hence it is conjectured that SBS is the more important nonlinear process for preventing the absorption of laser light by a plasma. We shall therefore concentrate on some linear and nonlinear processes which have been proposed for saturating stimulated Brillouin Scattering.

In an inhomogeneous, quiescent plasma of infinite extent the exponential amplification of a parametric instability has been calculated by Rosenbluth[15] to be $\gamma_0^2 / V_1 V_2 k'$ where γ_0 is the growth rate in the case of a homogeneous plasma, V_1, V_2 are the group velocities of the scattered electromagnetic wave and ion acoustic wave (for SBS) respectively and $k' = d(k_0 - k_1 - k_2)/dx$ is a measure of the deviation of the phase matching condition, brought about by the density gradient. This does not turn out to be a very stringent requirement because in the under-dense region of the plasma $\omega_0, \omega_1 > \omega_p$ so that the e.m. waves are weakly affected by the density gradient and the ion acoustic wave for which $k_2 \lambda_D << 1$ does not suffer any change due to the weak density variations. A velocity gradient however increases the threshold by a factor of $L_\omega / L_n \; (\omega_p^2 / \omega_0^2)$[16][17]. In addition, if a steady state supersonic expansion of the underdense plasma occurs, the threshold can be increased in proportion to the Mach number if it is greater than 2.

Another linear stabilization mechanism which has been intensively studied is the finite bandwidth effect[18][19][20]. Thomson[20] has shown that the reflectivity drops off exponentially with the increase in the band width which is in agreement with the computer simulation of Kruer[19] et. al. Furthermore Thomson et. al[21] have also established by comparing their result to that of Bodner[22] that the basic effect is insensitive to the laser bandwidth mechanism, be it phase or amplitude modulation. However the necessary bandwidths required for substantial increase in the damping factor and hence the threshold is quite large.

Another means of stabilization involves the presence of either short or long wave length ion wave turbulence[13]. For the long wave length case, the level of turbulence required is very large. Short wave length turbulence which can be generated by the ambipolar electric field in the expanding corona can effectively enhance the damping ν_{eff}^{s} on the stimulated ion waves as well as the damping ν_{eff}^{t} on the electromagnetic waves. Hence the spatial gain length can be increased and the instability brought below the threshold for sufficiently large turbulence levels given by $\gamma_c^2 = \nu_{eff}^{s} \nu_{eff}^{t}$. By calculating ν_{eff}^{t}, the nonlinear Landau damping of light waves on ion waves from the weak turbulence theory and ν_{eff}^{s}, the Landau damping of ion waves on the tail of the ions due to the background ion wave turbulence, it is found that the energy density of the turbulence $\mathcal{E}_{fl}$ has to exceed $E_0^2 M^{1/2} / 8\pi m^{1/2}$ which is absurdly high.

The nonlinear saturation mechanisms, due to effective damping of the linearly excited waves by direct formation of energetic ion and electron tails or by the nonlinear wave-wave interaction have also been considered. The tail formation requires small scale fluctuation. However the ion sound wave steepening mechanism cannot create scales smaller than $k^2 \lambda_d^2 = \mathcal{E}_{fl} / n_0 T_e$ because of the balance on the steepening brought about by the dispersion effects. Hence the only acceptable nonlinear mechanism is the outflow of ion sound wave energy into shorter scales.

with the given restriction. Gorbunov[23] has considered the specific problem of generation of the second harmonic of the ion acoustic wave. However he finds that SBS can be considerable even for small increment over the instability threshold.

In general the influence of ion sound wave steepening does not reduce the amplification. Also the multiple scattering model by which the side scattered light degrades into smaller frequencies does not prove to be very effective in increasing the absorption and thereby reducing the backscatter[13].

So far we have been focussing our attention on backscatter instability. Since the velocity gradient is considered to be perhaps the most effective mechanism the problem of side scatter in an inhomogeneous expanding plasma has also been considered exhaustively by Liu[4] is his review article. Although the side scatter has a lower growth rate, it is expected to amplify to higher levels in an inhomogeneous plasma, because of the fact that it stays in the interaction region for longer times. Klein et. al[25] have investigated the problem by simulations and have shown that for steep density gradients for which backscatter can be suppressed side scatter is very efficient and that only onefifth of the incident wave can penetrate the plasma. Furthermore Liu et. al[17] have shown the existence of temporally growing instabilities at various angles for which $K' = 0$. However there has yet been no observation[6][7] of significant side scatter.

In this chapter we have made an attempt to examine some new processes which can enhance the inhomogeneity threshold for the backscatter and side scatter for stimulated Brillouin Scattering.

4.2 Effect of Two Ion Species on Stimulated Brillouin Scattering

We wish to discuss in this section the effect of the existence of two ion species in a laser-pellet interaction. When a plasma is produced by a nanosecond pulse and if the plasma consists of ions of two or more types with different Z_j / m_j (as e.g. the thermonuclear fuel consisting of deuterium and tritium or other combinations like LiD, CH_2, $C_{36}D_{74}$ [9]) then the ambipolar electric field $\vec{E} = -(T_e/e)\nabla \ln n_e$ will accelerate the ions unevenly[26]. In the regions of the rarefied corona where the collisional friction between the two species cannot equalize the velocities, the lighter ions will run ahead of the heavier one. Hence the two ions can set up different scale lengths in the inhomogeneous plasmas. In this model, the plasma corona consists of an inhomogeneous medium in which the two ion species m_j $(j=1,2)$ have two different scale lengths L_j. We consider the density profiles to be linear so that $n_{oj}(x) = n_{oj}(0)\,(1 + x/L_j)$. The electron density also assumes a linear inhomogeneity given by $n_{oe}(x) = n_{oe}(0)\,(1+x/L_n)$. We further demand that the inhomogeneity scale factors L_j, L_n are much larger than the scale length of the interacting waves so that we can resort to the W.K.B. approximation discussed in the previous chapter.

Using the material equations

$$m_j\left(\frac{\partial \vec{v}_j}{\partial t} + \vec{v}_j\cdot\nabla\vec{v}_j\right) = eZ_j\left(\vec{E} + \frac{v_j \times \vec{B}}{c}\right) \tag{4.1}$$

$$\frac{\partial n_j}{\partial t} + \nabla\cdot(n_j\vec{v}_j) = 0 \tag{4.2}$$

together with the quasineutrality condition $\sum_{j=1,2} n_j = n_e$

the electron equation of motion gives

$$0 = -e\vec{E}_2 - \frac{T_e}{n_0}\nabla n_{e2} - m_e\nabla(\vec{V_0}\cdot\vec{V_1}) \qquad (4.3)$$

where the inertia term has been neglected. We follow the same notations as used in the previous chapter so that the nonlinear terms are due to the two electromagnetic modes.

The ion equations are written as

$$\frac{\partial n_j}{\partial t} + n_{0j}\nabla V_j = 0 \qquad (4.4)$$

$$m_j\frac{\partial v_j}{\partial t} = -eZ_j\nabla\phi_2 \quad \text{where} \quad \vec{E}_2 = -\nabla\phi_2 \qquad (4.5)$$

Combining these equations we get

$$\frac{\partial^2\bar{n}_2}{\partial t^2} - e\sum_j \frac{Z_j n_{0j}}{m_j}\nabla^2\phi_2 = 0 \qquad (4.6)$$

with $\bar{n}_2 = \sum n_j$

Now using the quasineutrality condition together with equation (4.3) we obtain the equation

$$\left(\frac{\partial^2}{\partial t^2} - T_e\sum_j \frac{Z_j n_{0j}}{m_j n_0}\nabla^2\right)\bar{n}_2 = \sum \frac{m_e Z_j n_{0j}}{m_j n_0}\nabla^2(\vec{V_0}\cdot\vec{V_1}) \qquad (4.7)$$

we have used the condition $k_2 L > 1$ in deriving the equation

Writing $\bar{n}_2 = \tilde{n}_2(x)\exp(-i\int k_2 dx - i\omega t)$ we get

$$\left(\frac{\partial}{\partial t} - \bar{C}_s\nabla\right)\tilde{n}_2 = \frac{m_e\bar{C}_s^2 k_2^2}{2i\omega_2 T_e}(\vec{V_0}^*\cdot\vec{V_1})e^{-i\int K dx} \qquad (4.7)$$

where the dispersion relation for ion acoustic waves become

$$\omega_2^2 = k_2^2 \sum \frac{Z_j n_{oj} T_e}{m_j n_o} \qquad \bar{c}_s^2 = \sum \frac{Z_j n_{oj} T_e}{m_j n_o}$$

We remark here that in view of the fact that $n_{oj}(x)$ are linear functions of the space variable, the wave vector k_2 is also a function of x . Again using equation (3.13) for the electromagnetic wave

$$\left(\frac{\partial}{\partial t} + v_1 \frac{\partial}{\partial x}\right) \tilde{E}_1 = \frac{4\pi e}{\bar{z}} \tilde{n}_2 v_0 \, e^{i\int k\,dx} \tag{4.8}$$

Hence we see that the equation (4.7) and (4.8) are the coupled set of equations describing stimulated Brillouin scattering. Compared to the equations for the one ion species, (which can be readily obtained) we see that the subtle difference lies in the k vector of the ion acoustic wave which has developed a spatial dependence even in the case of long scale inhomogeneity.

The homogeneous growth rate in this case becomes

$$\gamma_{ON}^2 = \frac{\bar{\omega}_{pi}^2 k_2^2 v_0^2}{4 \omega_2 \omega_1}$$

$$\bar{\omega}_{pi}^2 = 4\pi e^2 \sum Z_j n_{oj} / m_j \tag{4.9}$$

Using the expression for the amplification factor given by Rosenbluth[15] which is readily obtained by solving the coupled equations, in the W.K.B. approximation, the new inhomogeneity threshold becomes

$$\gamma_{ON}^2 / c\bar{c}_s k' > 1 \tag{4.10}$$

Here we will essentially consider the contribution to K' from the ion acoustic wave and compare it with the K' due to the electromagnetic waves.

$$\frac{dk_2}{dx} = -\frac{1}{2} k_2 \left[\frac{\sum \frac{Z_j n_{oj}}{m_j L_j}}{\sum \frac{Z_j n_{oj}}{m_j}} - \frac{1}{L} \right] \tag{4.11}$$

so that the threshold condition can be readily evaluated as

$$W_1 = \left(\frac{V_0}{V_e}\right)^2 = \frac{2 k_0 c^2}{\omega_p^2} \left[\frac{\sum \frac{Z_j n_{oj}}{m_j L_j}}{\sum \frac{Z_j n_{oj}}{m_j}} - \frac{1}{L} \right] \tag{4.12}$$

Comparing this with the density inhomogeneity threshold for back-scatter $W_2 = (V_0/V_e)^2 = 2/k_0 L$ we get

$$\frac{W_1}{W_2} = \frac{\omega_0^2}{\omega_p^2} \left[\frac{\sum \frac{Z_j n_{oj}}{m_j L_j}}{\sum \frac{Z_j n_{oj}}{m_j L_n}} - 1 \right] \tag{4.13}$$

assuming the existence of macroscopic charge neutrality, which means that the ambipolar field is not very large we can obtain a relation between the scale lengths for the three species by differentiating the neutrality condition $\sum n_{oj}/L_j = n_0/L_n$

For the case in which the two ion species have the same concentration, which is often realized in practice, we can write $n_{01} = n_{02} = n_0/2$ so that

$$\frac{W_1}{W_2} = \frac{\omega_0^2}{\omega_p^2}\left[\frac{\frac{1}{m_1L_1}+\frac{1}{m_2L_2}}{\frac{1}{2}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\left(\frac{1}{L_1}+\frac{1}{L_2}\right)} - 1\right] \tag{4.14}$$

As has been pointed out earlier, the difference in the scale lengths arises from the uneven acceleration of the ions, by the ambipolar electric field. Therefore we shall assume that the scale length L_1 and L_2 are inversely proportional to the respective masses m_1 and m_2 of the two ions. With this assumption, we get

$$\frac{W_1}{W_2} = \frac{\omega_0^2}{\omega_p^2}\left[\frac{4m_1m_2}{(m_1+m_2)^2} - 1\right] \tag{4.15}$$

We see that for $m_1 = m_2$, the ratio becomes zero which is to be physically expected. When there is only one species the wave vector of the ion acoustic wave is independent of x and hence $k_2' = 0$. For $m_1 = 2m_2/3$ (i.e. deuterium and tritium) we get

$$\frac{W_1}{W_2} = 2\,\frac{\omega_0^2}{\omega_p^2} \tag{4.16}$$

Hence we see that in the under dense region of the plasma, the thresholds can be much higher than the usual density inhomogeneity threshold for SBS. However a more appropriate comparison should be with the velocity inhomogeneity threshold which is more dominant. We recall that the exponential amplification factor for SBS with a blow off velocity is given by(13)

$$\frac{\gamma_0^2}{V_1V_2K'} = \frac{1}{4}\left(\frac{V_0}{V_e}\right)^2\left(\frac{\omega_p}{\omega_0}\right)^2 k_0L_u \,\sin\theta/2 \Big/ \left(\sin^2\theta/2\,\cos\theta + \frac{\omega_p^2}{\omega_0^2}\frac{L_u}{L_n}(\cos\theta-1)\right) \tag{4.17}$$

For backscatter $\theta = \pi$, the threshold condition becomes

$$W_2 = \left(\frac{V_0}{V_e}\right)^2 = \frac{4k_0c^2}{\omega_p^2 L_u} \tag{4.18}$$

Comparing this with (4.12) after using the same assumptions we get

$$\frac{W_1}{W_2} = \frac{1}{2}\frac{L_u}{L_n}\left(\frac{4m_1m_2}{(m_1+m_2)^2} - 1\right) \tag{4.19}$$

Since $L_u > L_n$ [27] again we see that the two ion species threshold exceeds the blow-off velocity gradient threshold.

In the case of side scatter it is readily seen that the two ion species effect is absent. This is of course based on the assumption that the pellet expansion occurs only along one preferred direction. In the more general spherically symmetric case for which there is a radial expansion, the side scatter effects will be identical to backscatter.

Hence to conclude this section we can summarize our results. We have seen that if the two ion species can establish different scale lengths by virtue of the differential acceleration in the ambipolar field, then the density inhomogeneity threshold in the presence of the two ions can be significantly different from that for a single ion because of the spatial dependence of the k vector brought about by the different scale lengths. It is quite plausible that in the underdense region of the plasma, such a state may prevail and thereby suppress stimulated Brillouin scattering.

4.3 Effect of Langmuir Turbulence on Stimulated Brillouin Backscattering

In chapter three we had investigated the effect of long-wavelength, low-frequency turbulence on SRS. The method could have been utilized for study of SBS also where the effect of turbulence would enter through the wave vectors of the electromagnetic modes. In this section we will investigate the effect of high frequency short wavelength Langmuir Turbulence on SBS. Before we delve into the mathematical aspect of the problem let us get a physical picture of how the short wavelength turbulence will affect the propagation characteristics of an ion acoustic wave. The presence of a high frequency field will give rise to rapid oscillations of the electrons. The ions on the other hand do not experience the high frequency electric field directly, but are affected by the time averaged pressure of the high frequency mode so that, besides the thermal pressure of the electrons, the ions 'feel' an additional pressure $m\, v_{\sim}^2 / M$ where $v_{\sim} = i e E_{\sim} / m \omega_0$ $E_{\sim}$ being the high frequency oscillating field. In the case of turbulence which essentially constitutes a conglomeration of waves with a certain bandwidth in k space, this ponderomotive force as it is called, becomes additive and each wave contributes to the pressure. If we look at the expression for the pressure, it depends on the square of the fluctuating velocity so that for a whole set of waves it can be written as $m \sum_{k,k'} \langle v_k v_{k'} \rangle / M$. Now each wave contributes its own pressure if $k = -k'$ and this essentially the random phase approximation(28) so that now we can characterise the turbulence in terms of its energy density or action.

The basic equation describing the stationary, high-frequency, short-wavelength, electron-plasma-wave turbulence is wave kinetic equation in the adiabatic approximation.

$$\frac{\partial N_k}{\partial t} + v_g \cdot \frac{\partial N_k}{\partial \vec{x}} - \frac{\partial \omega_k}{\partial \vec{x}} \cdot \frac{\partial N_k}{\partial k} = 0 \qquad (4.20)$$

where $N_k = |E_k|^2 / 4\pi\omega_k$ is the plasmon distribution function or action and $V_g = \partial\omega_k / \partial k$ is the group velocity of the plasma waves. We wish to ascertain the effect a long wavelength slow-frequency ion acoustic wave has on the plasmon distribution and the reaction of the plasmons on this large scale perturbation. Hence we can readily adopt the 'adiabatic' formalism developed by Vedenov et. al[29] to study the required effect. The last term in this equation (which very closely resembles a Vlasov equation) is the effective force term. Due to the modulation of the plasmon distribution by the ion acoustic wave, the local frequency of the high frequency electrostatic mode suffers a change in its value and hence in the group velocity. This can be looked upon as an effective force which retards or accelerates the plasmon propagation.

The electron equation of motion gives

$$0 = -\frac{T_e}{n_0}\nabla n_{e2} + e\nabla\phi_2 - m_e\langle V_\sim \cdot \nabla V_\sim \rangle - m_e \nabla(V_0 \cdot V_1) \quad (4.21)$$

where once again we have utilized the same notations. The only new term is that for the electron plasma wave turbulence for which $V_\sim = \sum_k ieE_k / m\omega_k$. Using the ion equation of motion, continuity equation and the quasineutrality condition, we get

$$\left(\frac{\partial^2}{\partial t^2} - C_s^2 \frac{\partial^2}{\partial x^2}\right) n_2 = \frac{n_0 e^2}{2Mm} \sum_k \frac{\partial^2}{\partial x^2} \frac{E_k^2}{\omega_k^2} + \frac{m n_0}{M} \frac{\partial^2}{\partial x^2}(V_0 \cdot V_1)$$

$$= \frac{\omega_p}{2M} \sum_k' \frac{\partial^2}{\partial x^2} N_k + \frac{m n_0}{M}\frac{\partial^2}{\partial x^2}(V_0 V_1) \quad (4.22)$$

Equations (4.20) and (4.22) completely determine the mutual interaction between the ion acoustic wave and the turbulent, inhomogeneous plasma. We first determine the steady state solution for (4.20) and treat the ion acoustic modulation as a perturbation.

In a weakly inhomogeneous plasma $n_{0x} = n_0(1 + x/L_n)$ a steady state solution of equation (4.20) can be written as

$$N_k^{(0)} = \frac{N^{(0)}}{(2\pi)^{1/2}\Delta} \exp\left(-\frac{k^2(0)}{2\Delta^2}\right) \tag{4.23}$$

where $k^2(0) = k^2 + x / L_n \lambda_D^2$ is the constant of integration obtained by solving the characteristic of the time independent equation (4.20). Here k represents the local wave vector. This equation essentially tells us that the plasmons have a tendency to bunch up in regions of lower density because of their lower group velocity in weak density regions compared to high density regions, so that in an inhomogeneous plasma, the plasmons will be distributed nonuniformly with a scale length given by $L_n \Delta^2 \lambda_D^2$. We calculate the perturbation in the plasmon density $\tilde{N}_k$ due to the long wavelength ion acoustic wave propagating as $\tilde{n}_2(x) \exp - i\omega_2 t - i\int k_2 dx$

Since $$\frac{\partial \omega_k}{\partial x} = \frac{\partial}{\partial x}\omega_p\left(1 + \frac{n_2}{2n_0}\right) = -\frac{ik_2}{2}\omega_p \frac{\tilde{n}_2}{n_0} \tag{4.24}$$

$$\tilde{N}_k = \frac{\omega_p k_2 \tilde{n}_2}{2 \; n_0} \frac{1}{\omega_2 + k_2 v_g} \frac{\partial N_k^{(0)}}{\partial k} \tag{4.25}$$

Inserting (4.25) in (4.22) we get

$$\left(\frac{\partial}{\partial t} - c_s \frac{\partial}{\partial x}\right)\tilde{n}_2 = \frac{k_2^2 m}{2Mi\omega_2}(V_0^* V_1)\, e^{-\int ik\,dx} \tag{4.26}$$

where

$$\omega_2^2 - k_2^2 c_s^2 + \frac{k_2^3 \omega_p^2}{4 M n_0} \int \frac{\frac{\partial N_k^{(0)}}{\partial k} dk}{\omega_2 + k_2 v_g} = 0 \tag{4.27}$$

The modified dispersion relation for the ion acoustic wave is valid for $k \gg k_2 \gg (\Delta \lambda_D)^{-2} L_n^{-1}$

Again equation (4.26) together with equation (4.8) constitute the coupled equations for SBS with the modification due to the turbulence introduced through the dispersion relation for the ion acoustic wave.

Solving (4.27) for real k_2 one gets

$$k_2 = \omega_2 \left(c_s^2 + \frac{\omega_2 \omega_p^2}{4 c_s M n_0} P \int \frac{\partial N_k^{(0)}(x)/dk}{\omega_2 + k_2 v_g} dk \right)^{-1/2} \tag{4.28}$$

where P denotes the principal part. In general we should also consider the resonance $\omega_2 + k_2 v_g = 0$ whose pole contribution would give an imaginary contribution. The physical process involved is the absorption of the acoustic waves by the plasmons and this would be the damping due to the turbulent spectrum. We however wish to investigate how the modification in the real part of the frequency can contribute to the convective saturation of SBS. Also we are justified in neglecting the resonance because a very simple calculation shows that for resonance to occur, the k vector should either be very small in which case the gradient $\partial N_k^{(0)}/dk \simeq 0$ (provided the adiabatic approximation is still valid) or the density should be very large so that the resonance condition would be difficult to satisfy in the underdense region of the plasma. To gauge the effectiveness of the turbulence we shall compare $k_2' = dk_2/dx$ with k' due to the velocity blowoff.

Introducing a dimensionless parameter $\beta = (m/M)^{1/2}/\Delta\lambda_D$ one finds

$$k_2' = -\frac{1}{4}\,\frac{\mathcal{E}_{fluc}\,F(\beta)\,k_2}{\mathcal{E}_{kin}\,(\Delta\lambda_D)^4\,L_n} \tag{4.29}$$

with $$F(\beta) = (2\pi)^{-1/2}\int y\,dy\,e^{-y^2/2}/(y-\beta) \tag{4.30}$$

and $\mathcal{E}_{fluc}/\mathcal{E}_{kin} = N^{(0)}\omega_p/4n_0T$

One can get an order of magnitude estimate of dk_2/dx by evaluating the integral approximately for $\beta \ll 1$

$$k_2' = -\frac{1}{4}\,\frac{\mathcal{E}_{fluc}\,k_2}{\mathcal{E}_{kin}\,(\Delta\lambda_D)^4\,L_n} \tag{4.31}$$

Consequently

$$\frac{k_2'}{k'} = \frac{1}{4}\,\frac{\mathcal{E}_{fluc}\,L_w}{\mathcal{E}_{kin}\,(\Delta\lambda_D)^4\,L_n} \tag{4.32}$$

If we consider a spectrum for which $(\Delta\lambda_D)^2 \sim 3 \times 10^{-2}$ and take $L_w \sim 10L_n$ we find that for $\mathcal{E}_{fluc}/\mathcal{E}_{kin} \sim 10^{-3}$, $k_2'/k = 3$. For a broader spectrum the level of fluctuations has to be higher but can still be within the realm of weak Turbulence Theory(29)(30).

Therefore we have shown that by incorporating the effect of high-frequency, shortwavelength turbulence in a weakly inhomogeneous plasma, the threshold for SBS can be quite significantly modified and therefore may be responsible for the low level of backscatter observed in laser-pellet experiments.

4.4 Nonresonant Pump Modification on the Side Scatter Threshold:

In the previous sections we have investigated two new physical processes which may affect the backscattering. In the single beam[(6)(7)] laser experiments, the plasma is seen to expand in the anti beam direction so that the inhomogeneity effect may necessarily be confined to that direction. Under those circumstances the direction transverse to that of the beam may be uniform and therefore side scatter should take place quite efficiently as has been predicted[(17)(25)]. However experiments have failed to register such an effect.

In this section we calculate the modification in the side scatter threshold brought about by nonresonant effects due to the pumpfield. One can readily see that for the process of side scatter, the electric vector of the pump field has a component in the direction of the ion acoustic wave which is not so in the case of backscatter. It is the contribution of this effect in an inhomogeneous plasma that we wish to investigate.

The basic equations are

$$\nabla(\nabla\cdot\vec{E}_1) - \nabla^2\vec{E}_1 + \frac{1}{c^2}\frac{\partial^2\vec{E}_1}{\partial t^2} + \frac{4\pi\sigma}{c^2}\frac{\partial\bar{E}_1}{\partial t} = -\frac{4\pi}{c^2}\frac{\partial J_{N.L}}{\partial t} \tag{4.33}$$

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2\nabla\right)n_2 = \frac{n_0 m}{M}\nabla^2(\bar{V}_0\cdot\bar{V}_1) \tag{4.34}$$

where σ is the linear conductivity and $J_{N.L} = -n_2 e V_0$

Taking the curl and div. of (4.33) we get

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{4\pi\sigma}{c^2}\frac{\partial}{\partial t}\right)\nabla\times\bar{E}_1 = -\frac{4\pi}{c^2}\frac{\partial}{\partial t}\nabla\times J_{N.L.} \tag{4.35}$$

and

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{4\pi\sigma}{c^2}\frac{\partial}{\partial t}\right)\nabla.\bar{E}_1 = -\frac{4\pi}{c^2}\frac{\partial}{\partial t}\,\bar{\nabla}.\bar{J}_{N.L.} \tag{4.36}$$

Hence we have separated the longitudinal and transverse parts of the h.f. field.

Assuming the perturbations to go as

$$E_1 = \tilde{E}_1\, \exp\left(-i\int \bar{k}_-dx + i\omega_-t\right) \quad , \quad k_- = k - k_0 \;,\; \omega_- = \omega - \omega_0$$

$$n_2 = \tilde{n}_2(x)\,\exp\left(-i\int \bar{k}.dx - i\omega t\right)$$

$$V_0 = \tilde{V}_0\,\exp\left(i\bar{k}_0.x + i\omega_0 t\right) + c.c \text{ with } \tilde{V}_0 = ieE_0/m\omega_0$$

we define $\vec{E}_{T1} = \bar{k}_-\times\vec{E}_1/|k_-|$ and $E_{L1} = (\bar{k}_-.\bar{E}_1)/|k_-|$

Under these conditions equations (4.35) and (4.36) become

$$\left(\frac{\partial}{\partial t} + \frac{c^2\vec{k}_-}{\omega_-}\cdot\frac{\partial}{\partial \vec{x}}\right)\vec{E}_{T1} = \frac{4\pi e}{2|k_-|}\,(\bar{k}_-\times\bar{V}_0)\,\tilde{n}_2\, e^{i\int k dx} \tag{4.37}$$

and

$$E_{L1} = \frac{4\pi i e\omega_-(\vec{k}_-\cdot\tilde{V}_0)\tilde{n}_2}{k_-^3} \tag{4.38}$$

where we have used the resonance condition

$$k_-^2c^2 - \omega_-^2\varepsilon_- = 0$$

$$\text{with } \varepsilon(k,\omega) = \left(1 + \frac{4\pi i\sigma}{\omega}\right) \tag{4.39}$$

The electrostatic field E_{L1} is a nonresonant correction due to the component of the pump wave in the direction of the acoustic mode. For backscatter $\bar{k}_-\cdot\vec{V}_0 = 0$ so that such an effect is absent.

The ion acoustic wave equation (4.34) can be written as

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2 \nabla^2\right) n_2 = \frac{i n_0 e}{M\omega_-}\left[\vec{V_0}\cdot \vec{E}_{T1} + \vec{V_0}\cdot \vec{E}_{L1}\right]$$

$$\left[\frac{\partial^2}{\partial t^2} - c_s^2 \nabla^2 - \frac{k^2 \omega_{pi}^2(x)}{k_-^2 \omega_0^2}\left(\vec{k}_-\cdot \vec{V_0}\right)^2\right] n_2 = \frac{i n_0 e}{M\omega_-}\left(\vec{V_0}\cdot \vec{E}_{T1}\right) \quad (4.40)$$

and hence

$$\left(\frac{\partial}{\partial t} - \frac{c_s^2 \vec{k}\cdot\nabla}{\omega}\right)\tilde{n}_2 = \frac{k^2 n_0 e}{2M\omega_-\omega}\left(V_0^* \cdot E_{T1}\right) e^{-i\int k\,dx} \quad (4.41)$$

with

$$\omega^2 - k^2 c_s^2 + \frac{k^2 \omega_{pi}^2(x)}{k_-^2 \omega_0^2}\left(\vec{k}_-\cdot \vec{V_0}\right)^2 = 0$$

as the new dispersion relation for the ion acoustic waves.

Once again equations (4.37) and (4.41) are the coupled equations for the process of SBS. The modification is now a function of the inhomogeneity of the system. Let us consider the basic inhomogeneity to be along the x direction so that

$$n_0(x) = n_0(0)\left[1 + x/L_n\right]$$

To get a feel for the change in the threshold we compare the value of k_2' with k' due to the velocity blow off for large mach number (The Mach number M is defined as the ratio of the blow-off velocity to the ion acoustic speed). If the incident wave is propagating along the density gradient and the scattering is at an angle θ, then

$$k_2' = -\frac{\omega_{pe}^2}{\omega_0^2 L_n}\sin^2\theta\; k_0 \left(\frac{V_0}{V_e}\right)^2$$

Comparing this with

$$k' = \frac{-2k_0 \sin^2\theta/2}{L_u}$$

we see that

$$\frac{k_2'}{k'} = \frac{1}{2}\frac{\omega_{pe}^2}{\omega_0^2}\frac{L_u}{L_n}\left(\frac{V_0}{V_e}\right)^2$$

Hence for $\omega_{pe}^2 L_u > \omega_0^2 L_n$ we have a strong reduction in the side scatter even if $V_0/V_e \sim 1$. For higher powers, side scatter gets suppressed even at lower densities. Therefore we see that such nonresonant effects, which in homogeneous cases can be neglected, do play an important role in inhomogeneous plasmas and can contribute to the overall development of the instability and its linear saturation.

4.5 Discussion:

In this chapter we have discussed three new processes which can affect stimulated Brillouin scattering and modify the threshold requirements.

In the first case we have considered the possibility of enhancement of the threshold power requirement by considering a two ion species in-homogeneous plasma for which the scale length associated with the inhomogeneity is different for the two species. Such a situation is plausible because of the differential acceleration of the two ion species by the ambipolar electric field. We find that in the underdense region of the plasma, the threshold can be quite drastically enhanced.

In the next section the effect of high-frequency short-wave length turbulence on the inhomogeneity threshold for SBS is investigated. The presence of such a turbulence introduces a spatial dependence into the wave vector of the modified dispersion relation of the ion acoustic wave, which, when utilized for evaluating the inhomogeneity threshold, gives rise to an interesting enhancement even in the regime of weak turbulence.

In the last section, we have looked into the problem of side scatter. If one examines the orientation of the pump wave electric field in relation to that of the ion acoustic wave, one sees that there is an overlap between the two which is absent in the case of backscatter. Using this as our starting point we investigate the non resonant modifications brought about by the above mentioned effect and have shown that for $v_0 / v_e \gtrsim 1$, the side scatter will be very small compared to backscatter.

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CHAPTER V

MODULATIONAL INSTABILITY IN THE PRESENCE OF LANGMUIR TURBULENCE

5.1 Introduction:

The study of some exact nonlinear forms of waves and their envelopes, brought about by the balance of nonlinearity and dispersive effects (in the collisionless plasma) is a topic of much interest. A knowledge of these nonlinear states can provide an appropriate discription for the plasma in the turbulent state. The underlying philosophy of such a representation is to look upon these localized entities as forming the 'basis' states of the system with a weak residual interaction between them.

It has been shown by Vedenov, Gordeev and Rudakov[1] that a 'cold' plasmon gas (i.e. a plasmon gas for which the mean square of the spread in the group velocities is zero) tends to break up into blobs, provided the wave number q of the perturbation exceeds the wave number of the plasmon. In the case of a 'warm' plasmon distribution of width Δ in K-space, there is a threshold for this break up given by $\mathcal{E}_{fl}/\mathcal{E}_{kin} > \Delta^2\lambda_D^2$ where $\mathcal{E}_{fl}$ is the energy density of the turbulent waves, $\mathcal{E}_{kin}$ is the kinetic energy density of the particles and λ_D is the electron Debye length. The applicability of weak turbulence theory breaks down when $\mathcal{E}_{fl}/\mathcal{E}_{kin} > (k\lambda_D)^2$ This is because the characteristic rate of nonlinear interactions $\delta\omega \sim \omega_p\, \mathcal{E}_{fl}/\mathcal{E}_{kin}$ becomes greater than the frequency spread due to thermal effects $\delta\omega_k \sim \omega_p (k\lambda_D)^2$ (2). For the case $\Delta/k \ll 1$, this instability is identified with the decay instability or at higher amplitudes with the oscillating - two stream instability. In the opposite case $\Delta/k \sim 1$ since the resonant condition cannot be satisfied for the entire set of k, only the modulational instability can exist. We shall be interested more in the latter case.

Another modulational instability that has attracted much attention is that of an electromagnetic mode due to transverse perturbations. Kaw, Schmidt and Wilcox[3] have investigated the stability of a large amplitude electromagnetic mode in an unmagnetized plasma to transverse perturbations. The nonlinearity responsible for the existence of this instability is provided by the ponderomotive force exerted on the plasma by the electromagnetic wave. It is shown that a plane electromagnetic wave is unstable against modulation in a direction perpendicular to the direction of propagation. Furthermore due to the saturating nature of the nonlinearity the final steady state consists of light filaments from which the plasma has been expelled, in equilibrium with the surrounding plasma pressure.

An alternative way of looking at the filamentation instability is as a coherent four-wave interaction. Drake[4], et. al have synthesised the electromagnetic instabilities (Raman scattering and filamentation, Brillouin scattering, Compton scattering and modulational instability) by deriving a general dispersion relation in terms of the susceptibility functions of the unmagnetized plasma and studying it in various limits. We shall approach the problem along the same lines.

It is recognized that the modulational and filamentation instabilities may play an important role in laser plasmas. These instabilities may drastically modify the backscattering instabilities in the underdense region of the plasma by modulating the plasma density. It may also facilitate the decay of the electromagnetic mode to electrostatic modes. It has been shown by Langdon and Lasinski[5] that as a result of self focusing or filamentation of the light beam, strong plasma heating can occur in a wider range of densities than is usually expected. Previously the anomalous heating mechanism was considered to occur near the critical density surface where the local electron plasma frequency matches the laser frequency ω_c. However in a plasma which has undergone filamentation, the density changes are significant and this leads to an extension of the region in which frequency matching for parametric processes can occur. The authors[5] have specifically considered the $2\omega_{pe}$ instability, a decay of the laser-wave into two Langmuir plasmons at the quarter critical density. When a filament forms in a higher density region, the density depression establishes the frequency matching conditions necessary for the $2\omega_{pe}$ instability. These localized conversions lead to strong plasma heating. Therefore it is expected that filamentation may introduce such modifications in the plasma density so as to facilitate heating by parametric processes.

The problem investigated by us involves both these instabilities. We wish to study how the presence of Langmuir plasmons affects the filamentation instability of the electromagnetic wave and vice-versa. The plasmons are assumed to be governed by the wave kinetic equation in the adiabatic approximation i.e. in the absence of resonant wave-wave and wave-particle interactions. The wave vector of the perturbation has to be less than that of the plasmons to justify the use of the adiabatic behaviour. The background plasma in general can be inhomogeneous (as we have discussed for the cases of Raman and Brillouin scattering). However, it is known that the existence of an inhomogeneity does not affect the absolute nature of the four-wave interaction(6). Therefore we will restrict our analysis to the case of a homogeneous turbulent plasma.

5.2 Basic Equations and General Dispersion Relation:

The basic set of equations representing the turbulent plasma are given by

$$\frac{\partial n_\sigma}{\partial t} + \nabla\cdot(n_\sigma \vec{V}_\sigma) = 0 \tag{4.1}$$

$$m_\sigma\left(\frac{\partial \vec{V}_\sigma}{\partial t} + \vec{V}_\sigma\cdot\nabla\vec{V}_\sigma\right) = \epsilon_\sigma q\vec{E} + \epsilon_\sigma \frac{q}{c}\vec{V}_\sigma\times\vec{B} \tag{4.2}$$

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \tag{4.3}$$

$$\nabla\times\vec{B} = +\frac{1}{c}\frac{\partial E}{\partial t} + \frac{4\pi J}{c} \tag{4.4}$$

$$\nabla\cdot\vec{E} = 4\pi q\sum_\sigma n_\sigma\epsilon_\sigma \tag{4.5}$$

where $J = \sum_\sigma n_\sigma q v_\sigma \epsilon_\sigma$, $\epsilon_i = -\epsilon_e = 1$

Here σ denotes the species (electrons and ions) and the rest of the notations are standard.

In the equilibrium state we assume the existence of a large amplitude electromagnetic wave (plane polarized)

$$\vec{E}_0 = 2E_0\hat{e}_0 \cos(\vec{k}_0 \cdot \vec{x} - \omega_0 t) = \bar{E}_{0+} + \bar{E}_{0-} \tag{4.6}$$

propagating in a homogeneous turbulent medium. We assume (ω_0, k_0) satisfy the usual linear dispersion relation

$$\omega_0^2 = \omega_p^2 + k_0^2 c^2 \tag{4.7}$$

Physically one would expect the shortwavelength turbulence to affect the dispersion relation of the electromagnetic wave. However it is obvious that these enter through the thermal correction which is indeed very small. The validity of the use of the linear dispersion relation is $\alpha = eE_0/m\omega_0 c \ll 1$ so that the relativistic mass corrections can be neglected.

The equilibrium is now perturbed by considering a density perturbation going as $\exp(i\bar{k}.\bar{x} - i\omega t)$ and this perturbation may be due to some normal electrostatic mode of the system. Due to the time and space dependent equilibrium state, currents at $\omega \pm l\omega_0$ and $k \pm lk_0$ will be induced in the system (here l is an integer). These side band modes (which may be mixed e.s. and e.m modes in general) interact with the pump wave field and shape out an effective potential through the ponderomotive force which leads to the amplification of the initial perturbation. This in turn enhances the side band amplitude and this bootstrap effect leads to the simultaneous amplification of the side bands and the initial perturbation.

For the case $\alpha \ll 1$ it is only necessary to consider the lowest order coupling. The Fourier transformed wave equation for the side band modes $\omega_\pm = \omega \pm \omega_0$, $k_\pm = k \pm k_0$ may be written as

$$\left[\left(k_\pm^2 - \frac{\omega_\pm^2}{c^2} \right) \mathrm{II} - \frac{\vec{k}_\pm \vec{k}_\pm}{k_\pm^2} \right] \cdot \vec{E}_\pm = \frac{4\pi i \omega_\pm \vec{J}_\pm}{c^2} \tag{4.8}$$

where II denotes the unit dyadic and $\vec{E}_\pm$ represents $E(\omega_\pm, k_\pm)$. The total current can be writtên as

$$J_\pm = J_{linear} + J_{nonlinear} = \sigma_\pm \vec{E}_\pm + n_e(k,\omega) e \vec{V}_{0\pm} \tag{4.9}$$

where $\sigma_\pm = i\omega_\pm (\epsilon_\pm - 1)/4\pi$ is the linear conductivity and $\epsilon_\pm$ is the linear dielectric function, $V_{0\pm} = \pm ie E_{0\pm}/m\omega_0$ and $n_e(k,\omega)$ is the perturbed electron density. In view of the fact that the side bands are high frequency modes ($\omega_\pm > \omega_{pe}$) the contribution to the current comes only from the electrons because the ions fail to respond to high frequency fields due to their inertia. Inverting the expression for the side bands $\vec{E}_\pm$ after plugging in the expression for the current $\vec{J}_\pm$ we get

$$\vec{E}_\pm = -\omega_p^2 \frac{n_e(k,\omega)}{n_0} \left[\left(\mathrm{II} - \frac{\vec{k}_\pm \vec{k}_\pm}{k_\pm^2} \right) \Big/ D_\pm - \frac{\vec{k}_\pm \vec{k}_\pm}{\omega_\pm^2 k_\pm \epsilon_\pm} \right] \cdot \vec{E}_{0\pm} \tag{4.10}$$

where

$$D_\pm = k_\pm^2 c^2 - \omega_\pm^2 \epsilon_\pm = c^2 k^2 \pm 2\vec{k} k_0 c^2 \mp 2\omega\omega_0 - \omega^2 \tag{4.11}$$

The equation of motion for the electrons in the presence of Langmuir Turbulence becomes

$$\frac{\partial \vec{V}_e}{\partial t} + \frac{\nabla p_e}{m n_0} + \frac{e\vec{E}}{m} = -\nabla(\vec{V}_0 \cdot \vec{V}_1) - \frac{\omega_p}{2 m n_0} \nabla \sum_q N_q \tag{4.12}$$

where, as discussed in the last chapter, the plasmon density Nq is conserved in phase space, so that

$$\frac{dN_q}{dt} = \frac{\partial N_q}{\partial t} + \frac{d\vec{x}}{dt}\cdot\frac{\partial N_q}{\partial \vec{x}} + \frac{d\vec{q}}{dt}\cdot\frac{\partial N_q}{\partial \vec{q}} = 0 \tag{4.13}$$

$$\frac{d\vec{x}}{dt} = \vec{V}_q \quad , \quad \frac{d\vec{q}}{dt} = -\frac{\partial \omega_q}{\partial \vec{x}} \tag{4.14}$$

Since the plasmon density is modulated by a long scale length density perturbation, the perturbed distribution is given by

$$\hat{N}_q = -\frac{\dfrac{\omega_p n_e}{2 n_0}\,\vec{k}\cdot\dfrac{\partial N_q^{(0)}}{\partial \vec{q}}}{(\omega - k.V_q)} \quad , \quad \vec{V}_q = \frac{q V_e^2}{\omega_p} \tag{4.15}$$

Hence

$$\frac{\partial \vec{V}_e}{\partial t} + \left[T_e - \frac{\omega_p^2}{4 n_0}\int \frac{\vec{k}\cdot\dfrac{\partial N_q^{(0)}}{\partial \vec{q}}\,d\vec{q}}{(\omega - k.\vec{V}_q)}\right]\frac{\nabla n_e}{m n_0} + \frac{e\vec{E}}{m} = -\nabla(\vec{V}_0.\vec{V}_1) \tag{4.16}$$

Therefore we see that the turbulence provides on effective pressure which modifies the thermal pressure exerted by the electrons on the ions. In fact under the present circumstances we can have a negative temperature system when the plasmon pressure exceeds the thermal pressure.

Hence the equation of motion becomes

$$\frac{\partial \vec{V}_e}{\partial t} + T_e' \frac{\nabla n_e}{m n_0} + \frac{eE}{m} = -\nabla(\vec{V}_0.\vec{V}_1) \tag{4.17}$$

where

$$T_e' = T_e\left(1 - \frac{\mathcal{E}_{fluc}}{(2\pi)^{1/2}\,\mathcal{E}_{kin}\,\Delta^3 \lambda_D^2}\int \frac{q\, e^{-q^2/2\Delta^2}\,dq}{q - \alpha}\right) \tag{4.18}$$

where $\mathcal{E}_{fluc}/\mathcal{E}_{kin} = \Sigma |E_q|^2/16 n_0 \pi T$ defines the ratio of the plasmon energy density to the particle energy density and

$\alpha = \omega\omega_p/kV_e^2 \cos\theta$, $\quad \cos\theta = \vec{q}.\vec{k}/|\vec{q}.\vec{k}|$, q

denotes the direction and Δ denotes the spectral bandwidth of the turbulence.

Using Equations (4.1), (4.2) and (4.5) we get

$$\left(\frac{\partial^2}{\partial t^2} - \frac{T_e'}{m}\nabla^2 + \omega_{pe}^2 \right) n_e - \omega_{pe}^2 n_i = n_0 \nabla^2 (\vec{V}_0 \cdot \vec{V}_1) \tag{4.19}$$

and

$$\left(\frac{\partial^2}{\partial t^2} - \frac{T_i}{M} + \omega_{pi}^2 \right) n_i - \omega_{pi}^2 n_e = 0 \tag{4.20}$$

Eliminating n_i between equations (4.19) and (4.20) we get

$$\left[\left(\frac{\partial^2}{\partial t^2} - \frac{T_e'}{m}\nabla^2 + \omega_{pe}^2\right)\left(\frac{\partial^2}{\partial t^2} - \frac{T_i}{M}\nabla^2 + \omega_{pi}^2\right) - \omega_{pi}^2\omega_{pe}^2\right] n_e$$

$$= n_0\left(\frac{\partial^2}{\partial t^2} - \frac{T_i}{M}\nabla^2 + \omega_{pi}^2\right)\nabla^2(\vec{V}_0 \cdot \vec{V}_1) \tag{4.21}$$

Taking

$$n_e = \tilde{n}_e \, e^{i\vec{k}\cdot\vec{x} - i\omega t}$$

$$\vec{V}_1 = \vec{V}_1^{+} e^{i\vec{k}_+\cdot\vec{x} - i\omega_+ t} + \vec{V}_1^{-} e^{i\vec{k}_-\cdot\vec{x} - i\omega_- t}$$

$$\vec{V}_0 = \vec{V}_0^{+} e^{i\vec{k}_0\cdot\vec{x} - i\omega_0 t} + \vec{V}_0^{-} e^{-i\vec{k}_0\cdot\vec{x} + i\omega_0 t}$$

Hence equation (4.21) becomes

$$\left[(\omega^2 - k^2 v_e^2 - \omega_{pe}^2)(\omega^2 - k^2 v_i^2 - \omega_{pi}^2) - \omega_{pe}^2\omega_{pi}^2\right]\frac{\tilde{n}_e}{n_0} =$$

$$\frac{e^2 k^2}{m^2\omega_0^2}\left(\omega^2 - k^2 v_i^2 - \omega_{pi}^2\right)\left(\vec{E}_0\cdot\vec{E}_+ + \vec{E}_0\cdot\vec{E}_-\right) \tag{4.22}$$

where we have used $\vec{V}_0^{\pm} = \pm\, e\vec{E}_0 / i m\omega_0$ and $\omega \ll \omega_0$

Substituting the expressions $\vec{E}_+$ and $\vec{E}_-$ we get the required dispersion relation

$$\frac{1}{\chi_e} + \frac{1}{\chi_i + 1} = k^2 \left[\frac{|\vec{k}_+ \times \vec{v}_0|^2}{k_+^2 D_+} - \frac{|\vec{k}_+ \cdot \vec{v}_0|^2}{k_+^2 \varepsilon_+ \omega_+^2} + \frac{|\vec{k}_- \times \vec{v}_0|^2}{k_-^2 D_-} - \frac{|\vec{k}_- \cdot \vec{v}_0|^2}{k_-^2 \varepsilon_- \omega_-^2} \right] \quad (4.23)$$

where

$$\chi_j = -\omega_{pj}^2 / (\omega^2 - k^2 v_j^2) \quad (4.24)$$

This is the general dispersion relation derived by Drake et al. using the Vlasov equation. This dispersion relation describes the parametric coupling of a low frequency electrostatic wave at (ω, k) and two high frequency mixed electromagnetic - electrostatic side bands at ($\omega \pm \omega_0$, $k \pm k_0$). The effect of turbulence enters through the electron susceptibility function and it is the effect of this turbulence that we wish to investigate.

5.3 Dispersion Relation for Modulational Instability in Presence of Langmuir Turbulence:

For the case of modulational instability the coupling terms involving D_+, D_- will be considered and the electrostatic contributions from ε_+ and ε_- will be neglected. We investigate the excitation of long wavelength instabilities with $k < 2k_0 \cos\theta$. Since $\omega \ll \omega_{pi}$ and $\omega \ll kc$ equation (4.23) can be written as

$$\frac{1}{\chi_e} + \frac{1}{\chi_{i+1}} = -\frac{2 v_0^2 \delta^2}{c^2} \left[(\omega - \vec{k} \cdot \vec{v}_{gc})^2 - \delta^2 \right]^{-1} \quad (4.25)$$

where $\vec{v}_{gc} = c^2 k_0 / \omega_0$, $\delta = c^2 k^2 / 2\omega_0$

Hence

$$\omega = \vec{k} \cdot \vec{v}_{gc} + \delta^2 \left[1 - \frac{2 v_0^2 k_{De}^2}{k^2 c^2 \left(1 - \frac{\varepsilon_{fluc}}{\varepsilon_{kin} \Delta^2 \lambda_D^2} \right)} \right] \quad (4.26)$$

(with, k_{De} = Debye wave-number)

where we have assumed $\cos\theta \simeq 1$ i.e. the turbulence is essentially perpendicular to the direction of the incoming wave and therefore almost parallel to the modulation wave vector.

We have used an equilibrium plasmon distribution given by

$$N_q^{(0)} = \frac{N^0}{(2\pi)^{1/2}\Delta} e^{-\frac{q^2}{2\Delta^2}} \tag{4.27}$$

In this case the condition for the modulational instability becomes

$$\frac{2V_0^2 k_{De}^2}{k^2c^2} > \left(1 - \frac{\mathcal{E}_{fluc}}{\mathcal{E}_{kin}\Delta^2\lambda_D^2}\right) \tag{4.28}$$

Therefore, for $\mathcal{E}_{fluc} / \mathcal{E}_{kin} < \Delta^2\lambda_D^2$ we see that the effect of the turbulence is to reduce the threshold. Physically this can be understood as follows. If the damping of the electromagnetic wave can be neglected, (as we have done) then it is the particle pressure of the plasma which tends to inhibit modulational instability. This therefore determines the threshold for the instability. In the presence of turbulence the effective temperature of the electrons is reduced and this facilitates the instability.

In evaluating the integral for the effective temperature we have neglected the pole contribution. For the case of $\vec{k} \cdot V_{gc} = 0$, this is justified because there is no real part to the frequency. For $\vec{k} \cdot \vec{V_{gc}} \neq 0$ the approximation is valid as long as $\vec{k} \cdot \vec{V_{gc}} / k\lambda_D V_e \cos\theta \ll \Delta$

The other limit, i.e. $\mathcal{E}_{fluc} / \mathcal{E}_{kin} > \Delta^2\lambda_D^2$ is the condition for strong turbulence[2]. Such a high level of turbulence[7] can build up in the absence of any dissipative mechanism. We see that the modulational instability gets totally quenched. We know that under these circumstances the plasma breaks up into 'blobs' localized in space and having random position. One such localised state is the envelope soliton. Therefore the 'blobs' by interacting amongst themselves can give rise to a steady state turbulent spectrum as has been shown by Kinsep, Rudakov and Sudan[2]. It is the presence of the randomly distributed blobs which prevents the electromagnetic wave from further modulating the plasma. We must however remark that in our case the strength of the electromagnetic wave is restricted by the condition $eE_0/m\omega_0 c \ll 1$. For stronger pump waves the above result is not valid.

We next solve equation in the limit $\cos\theta \ll 1$ for which $\alpha > \Delta$ and also assume that $\vec{k}\cdot\vec{V}_{gc} = 0$. The dispersion relation in this case simplifies to

$$(\omega^2+\eta)(\omega^2-\delta^2) = -\delta^2\beta\omega^2 \tag{4.29}$$

where $\eta = \dfrac{\mathcal{E}_{fluc}}{\mathcal{E}_{kin}} k^2 V_e^2 \cos^2\theta$, $\beta = \dfrac{2 V_0^2 k_{Dc}^2}{k^2 c^2}$

Hence

$$\omega^2 = -\frac{1}{2}\left(\eta-\delta^2(1-\beta)\right) \pm \frac{1}{2}\left[\left(\eta-\delta^2(1-\beta)\right)^2+4\eta\delta^2\right]^{1/2} \tag{4.30}$$

Considering $\eta < \delta^2(1-\beta)$ we get

$$\omega_1^2 = -\left(\eta-\delta^2(1-\beta)\right) - \frac{\eta\delta^2}{\left(\eta-\delta^2(1-\beta)\right)} \tag{4.31}$$

$$\omega_2^2 = -\eta/(1-\beta) \tag{4.32}$$

The first pair of roots is that for the usual modulational instability if η is neglected and requires the usual threshold $\beta > 1$. The second gives new roots for $\beta < 1$, i.e. below the usual threshold for the electromagnetic modulational instability. We may recall that Vedenov et.al.[1] had obtained a condition for the modulational instability of plasmons and the growth rate above the threshold would depend on $(\mathcal{E}_{fluc})^{\frac{1}{2}}$. Similarly in our case although this is a new root the growth rate goes as $\eta^{1/2}$. The validity of this root requires $\alpha > \Delta$ which gives

$$\frac{\mathcal{E}_{fluc}}{\mathcal{E}_{kin}} \frac{1}{\Delta^2 \lambda_D^2 (1-\beta)} > 1 \tag{4.33}$$

Hence even if $\mathcal{E}_{fluc}/\mathcal{E}_{kin} < \Delta^2 \lambda_D^2$ we can now satisfy this requirement, so that even in the case of weak turbulence a condensation of the plasmons can take place with the help of the electromagnetic field.

.4 Discussion:

In this chapter we have investigated the effect of high-frequency short-wavelength turbulence on the modulational instability. The turbulent spectra we have chosen are highly anisotropic. Such situations may arise in laser fusion experiments where, if the prepulse produces the turbulence, the plasma wave turbulence due to the decay instability will be almost perpendicular to the wave vector of the incident wave and if due to the linear conversion at the critical density, followed by the decay into electrostatic modes, the turbulent waves will propagate essentially along the direction of the wave vector of the incoming light.

We have therefore investigated the effect of such anisotropic turbulence on the modulational instability and have shown that if the direction of the turbulent waves is perpendicular to the incoming light, under the case of weak turbulence, the threshold for the electromagnetic wave modulational instability gets reduced, whilst for the case of strong turbulence the instability is quenched. On the other hand if the turbulent spectrum is confined to a small angle in the direction of the incident pump, then a new mode exists whose growth rate depends on the square root of the energy density of the turbulent modes. This mode is in a way reminiscent of the modulational instability of plasmons, however, with the difference that it may occur even in the case of weak turbulence because of the presence of the pump.

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