Production And Pre-equilibrium Evolution Of Quark Gluon Plasma

Avijit K. Ganguly

Physical Research Laboratory
Navrangpura, Ahmedabad 380 009
India

A

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CERTIFICATE

I hereby declare that the work contained in this thesis was carried out at the Physical Research Laboratory, Ahmedabad. The results reported herein are original and have not formed the basis for the award of any degree or diploma by any University or Institution.

Author

(A. K. Ganguly)

Thesis Supervisor

(Prof. J. C. Parikh)

Co-Supervisor

Prof. M. S. Shah
Department of Physics
M. S. University Baroda
Vadodara
India.

Dedicated to my
parents

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Chapter 1

Physics Of Quark Gluon Plasma

1.1 Introduction

We very briefly review some basic aspects[1] of QCD and QGP in this chapter. Particle physicists today believe Quantum Chromodynamics (QCD) to be the candidate theory of strong interaction. It is the theory of particles that constitute the observed hadronic world. The evidence that the hadrons are made up of point like constituents, called quarks, came from deep inelastic scattering experiments of leptons on hadrons. These quarks come with different flavours, up(u), down (d), strange (s), charm (c), bottom (b) and another one (though not observed yet but have strong reasons to exist) top (t).

The deep inelastic scattering data also suggested that these quarks are Dirac particles and carry non-integer electric charge. For u, c, t it is $\frac{2}{3}$ and for d, s, b it is $\frac{-1}{3}$ in units of proton charge. Quarks of different flavours differ from each other through their flavour quantum number and mass. Current algebra techniques have suggested the typical values for the current quark masses to be

$$m_u = 5 \text{ Mev} \qquad m_d = 10 \text{ Mev} \qquad m_s = 150 \text{ Mev}$$

$$m_c = 1500 \text{ Mev} \qquad m_b = 5000 \text{ Mev} \qquad m_t \geq 140 Gev$$

The existence of resonances like Δ^{++}, Δ^{-} and Ω^{-}, to be consistent with

Pauli principle, suggested, within the quark model, the existence of another quantum number called color. From the ratio of cross sections of $e^+ e^- \rightarrow hadrons$ to $e^+ e^- \rightarrow \mu^+ \mu^-$ it has been established that the number of colors $N_c = 3$.

We believe that color degree of freedom is like electric charge, is exactly conserved and is a source of long range interaction. The theory of color interactions, QCD is derived from the principle of local gauge invariance in color space.

The Lagrangian that describes QCD is given by

$$L_{QCD} = \Sigma_{f=1}^{N_f} i \overline{\psi_f} \gamma^{\mu} \left(\partial_{\mu} - i g A_{\mu}^{a} T^{a} \right) \psi_f - m \overline{\psi_f} \psi_f - \frac{1}{4} F_{\mu\nu}^{a} F_{a}^{\mu\nu} \tag{1.1}$$

Here Lorentz index μ varies from 0 to 3, f is the flavour index with maximum value N_f and a varies from 1 to 8.

The color field tensor $F_{\mu\nu}$ is given by

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + ig \left[A_{\mu}, A_{\nu} \right] \tag{1.2}$$

It can be expressed as $F_{\mu\nu} = F_{\mu\nu}^{a} T^{a}$ with T^{a} as the generator of SU(3) color group. They obey the commutation relation

$$[T_a, T_b] = i\, f_{abc} T_c \tag{1.3}$$

where f_{abc} are the structure constants of the group.

Under a local gauge transformation U, the fields ψ and A_{μ} transform as

$$\psi'_f = U \psi_f \tag{1.4}$$

$$A_{\mu}\prime = U^{-1} A_{\mu} U - \frac{i}{g} U^{-1} \partial_{\mu} U \tag{1.5}$$

such that

$$F'_{\mu\nu} = U^{-1} F_{\mu\nu} U \tag{1.6}$$

From the structure of the Lagrangian, it is clear that though this theory looks like Quantum Electrodynamics(QED),but due to the presence of the non-abelian terms it differs significantly from QED. For instance, this difference can be seen from the expression of running coupling constant, at one loop level,

$$g^2_{QCD}(Q^2) = \frac{g^2(\mu^2)}{1 + \frac{g^2(\mu^2)}{12\pi}(33 - 2N_f)\, ln\left(\frac{Q^2}{\mu^2}\right)} \tag{1.7}$$

where μ^2 is a scale parameter to be deduced from experiments.

We can see that, for $N_f \leq 16$, as Q^2 the momentum transfer $\rightarrow \infty$, $g^2(Q^2)_{QCD} \rightarrow 0$. The running coupling constant in QED shows exactly opposite behaviour. i.e. $g^2_{QED} \rightarrow \infty$ as $Q^2 \rightarrow \infty$. This particular behaviour of the non-abelian coupling constant, termed asymptotic freedom, tells us that at high momentum transfer one has essentially free particles, and hence perturbation theory is applicable.

So far we have considered momentum scales Q^2 larger than μ^2. The important question is what happens to the running coupling constant at a scale $Q^2 < \mu^2$. It is worth mentioning that at this scale analytical studies are difficult and it is believed that g^2 increases with decrease in Q^2 and lattice results imply the same type of behaviour. Hence it is difficult for the quarks to separate themselves from each other beyond a distance of the order of one fermi. This particular phenomenon, called confinement, though not proved rigorously, confirms with our experience that no free quarks have been observed in experiments.

So these two properties, namely asymptotic freedom and confinement are the cornerstones of strong interactions and have profound consequences on the properties of hadronic matter subjected to high temperature or density or both.

1.2 QCD at High Density and Temparature

QCD predicts that in nuclear matter at densities ten to twenty times the nuclear density (≈ 0.15 GeV$/fm^3$) or at very high temperature (kT $\geq$ 200MeV), quarks and gluons will be liberated over a volume greater than a typical hadronic volume forming, a soup of quark gluon plasma.

QCD thermodynamics[2] is studied in the imaginary time formalism by compactifying the time direction, and putting a periodic boundary condition for the gauge fields in that direction. In this formalism one can define an order parameter,

$< L(x) >$, where

$$L(x) = N^{-1}\, tr\, P\, exp\left(i \int_0^{\frac{1}{T}} A_4(x,t)\, dt\right)$$

and P denotes path ordering.L(x) is called the polyakov loop. This quantity has been shown[3] to be related to the free energy of static quark in a gluonic bath at temperature T: $< L(x) >= e^{\frac{-F_q(x)}{T}}$. At low temperature one expects the free energy of the quark to be infinite and hence $< L(x) >= 0$ but at high temperature, if deconfinement of color occurs, then the free energy of the system will become finite and hence $< L(x) > \neq 0$. Numerical lattice studies of this quantity has suggested that the phase transition is of first order, and the critical temperature for such a transition to occur, is $T_c \geq 200 MeV$.

In view of these considerations, it appears that, the possible places in nature where such a phase may occur are (i) early universe[4], (ii) inside a neutron star, (iii) Relativistic Heavy Ion Collisions(RHIC).

In this thesis we will concentrate mostly on the RHIC. In RHIC, the plasma is expected to be produced when two heavy nuclei collide against each other at an energy of $\approx$ 200 GeV/nucleon. Once produced this plasma evolves in phase space to attain a state of thermal equilibrium following which it hadronises and particles stream out to the detectors. This whole process of formation to hadronisation of the plasma is supposed to take place within 5 to 10 fm/c.

The purpose of this thesis is to examine production and pre-equlibrium evolution of the plasma. Usually, as the plasma is produced it undergoes a simultaneous space time evolution, making it necessary to take these processes into account self-consistently. Since it is difficult to study these processes in entirety, we have studied some specific aspects of production and evolution of the plasma separately, with special emphasis on the non-abelian features of the underlying theory. We must mention that in the last few years extensive work has been done to study the production and equilibration of the plasma using parton cascade model[(5-7)], however we would not discuss it here.

The plan of this document is as follows. In chapter two we discuss the production mechanism of the plasma at zero temperature, in the color flux tube model[8] of Casher et.al. In contrast to the earlier studies we have done this analysis in the presence of a time varying external electric field and have tried to justify it, from the vacuum solutions of Yang-Mills equations. Results of our analysis show that, because of the presence of time varying chromo-electric field, the pair production rate instead of being exponentially suppressed (as in the constant field case of Schwinger[9]), follows a power law behaviour. Since the number of produced particles increases the probablity of producing a thermalised plasma also increases. Moreover the analysis[10] also shows that, a time varying field is capable of producing pairs even if the field strength is less than the critical field strength required to produce particles in the Schwinger model.

In chapter three we have examined the relevance of pair production at finite temperature in the presence of an external chromo electric field. Following which we have actually computed the pair production rate at finite temperature and have shown that because of screening the pair production rate becomes a space dependent quantity and it increases at high temperature[11].

In the following two chapters we have concentrated on the evolution of the plasma in phase space. In chapter five, we discuss the kinetic equation for gluons and suggest a simple model which shows that, color degrees of freedom can also give rise to new a mechanism for equilibration of the plasma. Chapter 6 contains a derivation of hydrodynamic equations for quarks and gluons starting from the kinetic equations. We also show that the non-abelian nonlinearities in the pre-equilibrium phase of the system lead to chaotic oscillations, that in turn tend to bring the system to thermal equilibrium. In the concluding chapter we have summarised our result and give a futuristic plan for further investigation along the same direction.

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Chapter 2

$Q\bar{Q}$Production In Presence Of Oscillating External Field

2.1 Introduction

In this chapter, we will discuss the process of formation of Quark Gluon Plasma in Relativistic Heavy Ion Collision (**RHIC**). In particular we will concentrate on the mechanism by which the initial beam energy in RHIC gets deposited in a small volume in the speculated form of quark gluon plasma through the production of quark anti-quark pairs. The process of quark anti-quark production in RHIC has attracted the attention of many workers for over a decade.

This complex process of pair production, inspite of being visited many a time by many workers taking into account different physical conditions, till today, stands as one of the most elegant model whose potential is far from being exhausted. The production of $q\bar{q}$ pairs from vacuum in the flux tube model [1], basically owes its existence to the classic paper of Schwinger[2], where in the context of Quantum Electro Dynamics (QED), it was shown that in the presence of very strong external electric field, QED vacuum becomes unstable and it starts emitting e^+e^- pairs at the expense of the electric field till the field strength falls bellow a critical value comparable to the square of the mass of the produced particles.

Along the same line, the $q\bar{q}$ pair production in RHIC is also assumed to take place by the decay of the flux tubes formed between the two receding nuclei due to the multiple exchange of soft gluons. This process continues till the energy stored in the chromo-electric field/unit length becomes less than the mass of the produced $q\bar{q}$ pairs. In addition to extending Schwinger's QED calculation to the QCD case, efforts have also been made to include effects such as, the screening of the external electric field[3], finite size of the nuclei[4], moving boundary conditions[5], radial confinement[6] etc. It is worth noting that in all these works the external chromo-electric field has always been considered to be constant in both space and time.

In this chapter we will contest the validity of this assumption and in fact argue that the basic nature of QCD lagrangian demands the electric field to be time dependent. The actual evaluation of $q\bar{q}$ pair production rate by us however has been carried out for an external field which is homogeneous in space but oscillating sinusoidally in time.

The organisation of this chapter is as follows. In section two we will review briefly the Schwinger mechanism followed by the physical picture of flux tube formation in relativistic heavy ion collisions. In section three we justify, from exact solutions of the classical SU(2) Yang-Mills equations, why the external chromo-electric field has to be time dependent rather than constant. This is followed by section four where we will try to give an order of magnitude estimate of the field strength and the frequency of oscillation attainable in relativistic heavy ion collision. In section five we compute the pair production rate in a time varying field with different values of field strength and frequency of oscillation. Lastly we conclude by stating the scope of further improvement of our results.

2.2 Schwinger Mechanism: A Brief Outline

The production of particle antiparticle pairs by a classical external field via Schwinger mechanism is a general phenomenon that reflects a much broader physical reality, i.e instability of vacuum under external perturbations. This idea has been used in a variety of theories in different contexts, ranging from QED, QCD, Transport theory[7], Relativistic Heavy Ion Collision, Gravitation[8], Early Universe[9] and even in String theory[10].In the following passage, we will elaborate on the physics[11] of this process for the simple case of QED.

Let us consider a system to consist of vacuum (including virtual particle antiparticle pairs) subjected to an external electric field. In order to create an on-shell particle antiparticle pair from the vacuum, the virtual particle antiparticle have to be moved away from each other over a distance d $\geq$ the compton wave length of the particles, with a corresponding energy loss (of the system) $\sim$ 2m. Now in the presence of an external electric field (with assumed strength $E \geq E_c \sim m^2$), because of vacuum polarisation,if the virtual particle-antiparticle are moved apart by a distance d the energy gained by the system, at the expense of the external field, will be gEd. If the distance d $\geq$ Compton wave length of the particles, the energy gained by the system in putting the pairs on shell becomes more than 2m. Since it is energetically always favourable for a system (i.e vacuum) to go to its lowest energy state, pairs will be emitted from vacuum till the field strength falls below the critical field strength E_c.

The production of $q\bar{q}$ pairs in RHIC has also been explained by the same principle via the flux tube model. This model was introduced independently by Low[1] and by Nussinov[1] to account for the observed scaling behaviour of scattering cross-sections in hadron hadron collisions.In nucleus nucleus collisions, it assumes that at high energy, when the two highly Lorentz contracted nuclei pass through each other, the partons of one nucleus interact with the partons of the other nucleus by the exchange of soft (color octet) gluons. If the fly by time of the nuclei is less than the time scale of interaction of the partons, the receding nuclei get randomly color charged by exchange of soft gluons. Since a colored object cannot exist free in nature the color octet partons in the receding nuclei get connected to each other by means of color flux tubes with color electric fields inside them. This color flux tube decays producing $q\bar{q}$ pairs in the same way as described previously for the QED case of Schwinger.

With this picture in mind, the dynamical evolution of the plasma produced in RHIC, including $q\bar{q}$ pair creation, has been studied[2-7] by many others. We however will be content to examine the effect of oscillating external chromo-electric field on the pair production rate, since this has not been investigated before.

2.3 Some Exact Solutions of Yang Mills Equations

In this section we will establish that, because of the presence of the nonlinear terms in the Lagrangian, the gluons produced in RHIC, polarise the medium between the two receding nuclei, generating an electric field that undergoes, characteristic non-linear, non-abelian oscillations in time. For this purpose we will make certain assumptions based on the geometry of the problem. These assumptions however do not change the qualitative nature of our observation.

The first assumption is that each of the color charged nucleus has a uniform distribution of color charge in the plane transverse to the direction of motion so that there exists no gradient of the fields in this direction. Our second assumption is that these color charges produce a chromo-electric field such that A_0 and A_z are the only nonzero potentials. Although in principle a magnetic field can also be present, we will not consider it here since it cannot transfer energy to the system to create pairs. Our third assumption is that the region between the two nuclei can be treated as vacuum and we will neglect the curvature effects near the boundaries. With these simplifying assumptions, the dynamics of the gluon fields can essentially be described in (1+1) dimensions rather than (3+1) dimensions. Therefore in order to get information about the nature of the classical gluon fields one needs to solve the classical Yang-Mills field equations in (1+1) dimensions.

2.3.1 Solution of Yang Mills equations in (1+1) dimensions.

We next show that in (1+1) dimensions the Yang-Mills equations have a solution with a sinusoidally time varying component whose frequency depends on the amplitude. We first write the sourceless Yang Mills equation in (1+1) dimensions

$$D_\mu F^{\mu\nu} = 0 \tag{2.1}$$

where the Greek indices μ and ν take values 0 and 1 only. The covariant derivative is defined as

$$D_\mu = \partial_\mu + ig\,[A_\mu,] \tag{2.2}$$

with g as the coupling constant and $g[A_\mu,]$ as the commutator bracket. Since we are working with an SU(2) color symmetry, A_μ is defined as $A_\mu = A_\mu{}^a \tau_a$ where τ_a are the generators obeying the commutation rules .

$$\left[\tau^a, \tau^b\right] = i\varepsilon_{abc}\tau^c \tag{2.3}$$

The indices a,b,c takes values from one to three. Further, the only non zero components of the vector field in this case are A_0 and A_z, and we have chosen the axial gauge $A_z = 0$. With this choice of the gauge we get from equation (2.1) for $\nu = 0$,

$$\partial^2{}_z A_a{}^0 = 0 \tag{2.4}$$

whose solution is

$$A_a{}^0(t) = \alpha_a(t)\, z + \beta_a \tag{2.5}$$

Here α_a and β_a are arbitrary integration constants. In order to find out an exact solution of this equation we take α_a to depend on time and β_a to be a constant. Equation (2.1) for $\nu = 1$ gives

$$\partial_0 \partial_z A_a{}^0 + g\epsilon_{abc} A_a{}^0 \partial_z A_c{}^0 = 0 \tag{2.6}$$

Substituting the solution (2.5) in equation (2.6) we arrive at

$$\dot{\alpha}_a(t) + g\epsilon_{abc}\alpha_c\beta_b = 0 \tag{2.7}$$

One can derive a conservation law from this equation namely

$$\alpha_a(t)\alpha_a(t) = constant \tag{2.8}$$

A summation over repeated indices is implied.

We solve this set of coupled first order linear differential equations by Euler's method; i.e we choose a solution of the form

$$\alpha_a(t) = a_a e^{pt} \tag{2.9}$$

Substituting equation (2.9) in equation (2.7), we obtain a set of coupled algebraic equations whose solution is of the form

$$\alpha_1 = \beta_1 + \beta_1\beta_3\,[\,e^{i\omega t} + e^{-i\omega t}] - i\omega\beta_2\,[\,e^{i\omega t} - e^{-i\omega t}] \tag{2.10}$$

$$\alpha_2 = \beta_2 + \beta_2\beta_3\,[\,e^{i\omega t} + e^{-i\omega t}] + i\omega\beta_1\,[\,e^{i\omega t} - e^{-i\omega t}] \tag{2.11}$$

$$\alpha_3 = \beta_3 + \beta_3{}^2\,[\,e^{i\omega t} + e^{-i\omega t}] - \omega^2\,[\,e^{i\omega t} + e^{-i\omega t}] \tag{2.12}$$

Here $\omega = [(\beta_1)^2 + (\beta_2)^2 + (\beta_3)^2]^{\frac{1}{2}}$.

Once the α's are known, one gets the solution for the A_0's by substituting equations (2.10), (2.11) and (2.12) in equation (2.5). Without giving the unnecessary mathematical details, the final expression is

$$A_0^1 = [\beta_1 + \beta_1\beta_3\,[\,e^{i\omega t} + e^{-i\omega t}] - i\omega\beta_2\,[\,e^{i\omega t} - e^{-i\omega t}]]\,z + \beta_1 \tag{2.13}$$

$$A_0^2 = [\beta_2 + \beta_2\beta_3\,[\,e^{i\omega t} + e^{-i\omega t}] + i\omega\beta_1\,[\,e^{i\omega t} - e^{-i\omega t}]]\,z + \beta_2 \tag{2.14}$$

$$A_0^3 = \left[\beta_3 + {\beta_3}^2\,[\,e^{i\omega t} + e^{-i\omega t}] - \omega^2\,[\,e^{i\omega t} + e^{-i\omega t}]\right]\,z + \beta_3\;. \tag{2.15}$$

Thus from the solution it is clear that the electric field inside a chromo-electric flux tube oscillates with frequency $\omega = [(\beta_1)^2 + (\beta_2)^2 + (\beta_3)^2]^{\frac{1}{2}}$, which depends on the amplitude of oscillation.

It may be pointed out that there also exists an exact time dependent vacuum solution of (SU(2)) Yang-Mills equations of the type[12]

$$A_\mu^\alpha = (0, H\delta_1^\alpha, H\delta_2^\alpha, H\delta_3^\alpha) \tag{2.16}$$

where

$$H = \frac{B}{\sqrt{g}} cn\left[\sqrt{2g}B\,(t - t_o)\right] \tag{2.17}$$

In eq.(2.16), μ (=0,1,2,3) is the Lorentz index, α (=1,2,3) is the color index and δ_i^α is the Kronecker delta. In eq. (2.17) cn represents the Jacobi elliptic function and B is a constant determining the amplitude of the oscillating field. Physically, this solution represents a non-linear collective oscillation of gluons with a characteristic amplitude dependent time period $\sim \left(\sqrt{2g}B\right)^{-1}$, which is a manifestation of the intrinsic nonlinearity present in the system.

As we will see this time varying nature of the field changes the pair production rate quite significantly over that due to the constant field i.e the Schwinger estimate.

2.4 Estimating The Parameters

Having established the fact, that the chromo-electric field inside the flux tube (because of the self-interaction of the fields) should be oscillating in time, we next explore its consequences on the rate of spontaneous pair production from vacuum.

For this purpose, one can in principle take either of the exact solutions and compute the rate of pair production. But considering the computational difficulties associated in working with such exact solutions, we will content ourselves with a spatially homogeneous chromo-electric field that oscillates sinusoidally in time. We take the vector potential to be

$$A^a_\mu(x) = (0,0,0,\ A_a(t))\ and\ A_a(t) = -\frac{E_a \cos\omega_o t}{\omega_o} \quad (2.18)$$

and for of SU(3) color symmetry a goes over (=1, 2,....8)

Here E_a's are constants and ω_0 is the characteristic collective frequency for the gauge fields. Now to determine the pair production rate one has to give an estimate of the frequency and amplitude of the external chromo-electric field produced in RHIC. For this purpose, we make use of the solution of Yang-Mills field equations given by equations(2.16)- (2.17).From this solution taking each $A^a \equiv A$ one can write the r.m.s chromo-electric field strength as

$$E \equiv \left(\sqrt{8}\right)\left(\sqrt{2}gB\right)\left(\frac{B}{\sqrt{g}}\right) = 4B^2 \quad (2.19)$$

apart from an uninteresting constant.
Since $\omega_0 = \left(\sqrt{2}gB\right)$, and replacing B in terms of E from the equations above, we get the expression for frequency

$$\omega_0 = \sqrt{\frac{gE}{2}} \quad (2.20)$$

in terms of a gauge invariant chromo-electric field defined as

$$E = \left[\sum_{a=1}{}^{8} E_a{}^2\right]^{\frac{1}{2}} \quad (2.21)$$

Once ω_0 is known in terms of the chromo-electric field strength, one is left with the determination of the strength of the external field attainable in RHIC. In order to estimate it, one has to first make an estimate of the color charge deposited on each of the receding nuclei after the collision. Following Kerman, Matsui and Svetitsky[13], it is usually assumed that at very high energy in a nucleon nucleus interaction multiple gluons are exchanged. In each interaction, with the exchange of each gluon there is an exchange of color charge t_a (where t_a is the matrix in the adjoint representation of the symmetry group). Thus after ν such exchanges of gluons, the total color charge that gets accumulated on the target nucleus is

$$\vec{T} = \sum_{j=1}^{\nu} \vec{t}_j \tag{2.22}$$

If the color orientations amongst these exchanged gluons are uncorrelated, one can assume, after ν such interactions, that the r.m.s color charge deposited on the target nucleus is

$$\langle T^2 \rangle^{\frac{1}{2}} = \sqrt{\nu} \langle t^2 \rangle \tag{2.23}$$

From this relation one can say that, after ν interactions, the amount of color charge deposited on the target nucleus is proportional to the square root of the number of interactions i.e

$$Q \propto \sqrt{\nu} \tag{2.24}$$

One can relate (see ref.13) the number of pairs produced to the number of interactions or the total color charge as

$$\frac{dN_{pair}}{dy} \propto \sqrt{\nu} \tag{2.25}$$

Here N_{pair} is the number of pairs produced and y is the rapidity. Moreover if one assumes the number of hadrons produced to be proportional to the number of pairs produced then

$$\left(\frac{dN}{dy}\right)_{pA} = \left(\frac{dN}{dy}\right)_{pp} \sqrt{\nu} \tag{2.26}$$

i.e the multiplicity for proton nucleus collision scales as the square root of the number of interactions times the multiplicity in proton proton collisions. So, from the multiplicities of the produced particles one can compute the number of collision that each nucleon undergoes in a p-A collision.

If σ_{p-p} and σ_{p-A} be the cross sections for proton proton and proton nucleus collisions then one can write phenomenologically that

$$\nu = A \frac{\sigma_{pp}}{\sigma_{pA}} \tag{2.27}$$

(where A is the mass number of the target nucleus).From simple geometrical considerations one can show that $\frac{\sigma_{pp}}{\sigma_{pA}}$ scales as $A^{-\frac{2}{3}}$ and hence the number of collisions from p-p to p-A should scale as $A^{\frac{1}{3}}$. This implies that the amount of color charge deposited in p-A collision on the target nucleus scales as $A^{\frac{1}{6}}$ times that in the p-p collision.

In high energy central collision of two heavy nuclei, the individual constituent nucleons of each nuclei can be thought of scattering through the other nuclei. So in the light of the foregoing discussion, total number of interactions, compared to p-p collision, should scale as $A_T^{\frac{1}{3}}\ A_P^{\frac{1}{3}}$, where A_T and A_P are the target and projectile mass numbers respectively. This implies, that the amount of color charge deposited should scale as $A_T^{\frac{1}{6}} A_P^{\frac{1}{6}}$ from p-p to A-A collision. The earlier relation implies that the chromo-electric field strength, should scale from p-p to A-A collision as $A_T^{\frac{1}{6}}\ A_P^{\frac{1}{6}}$.

After establishing the scaling behaviour of the chromo-electric field from p-p to A-A collision, the only task one is left with is to evaluate the strength of the chromo-electric field produced in p-p collisions. If the flux tube produced in p-p collision generates a string tension σ then the field energy stored per unit length of the tube is

$$E^2 = \frac{2\sigma}{area} \tag{2.28}$$

From Gauss law one can write[14]

$$E\ \text{area} = g, \qquad \text{where g is the coupling constant.} \tag{2.29}$$

On using the equations (2.28) and (2.29) one can derive that

$$gE = 2\sigma \tag{2.30}$$

The quantity σ is usually evaluated from the Regge slope parameter and its value has been estimated to be around $0.2GeV^2$[14]. Because of the final state interactions[15] (basically screening effect), the effective field strength generated initially in p-p collision, gets reduced to around .2 GeV^2. Once we know that the field strength produced in p-p collision is 0.2 GeV^2 one can compute the value of the field produced in A-A collision, from the scaling law

$$E_{A_T - A_P} \sim A_T^{\frac{1}{6}} A_P^{\frac{1}{6}} E_{p-p} \tag{2.31}$$

Following Pavel and Brink[6] the magnitude of the field strength produced in the collision of S^{32} on S^{32} has been estimated to be, $gE \leq 0.6GeV^2$ and for U - U collisions it is $gE \leq 1.2GeV^2$. These values of gE imply a variation of ω_0 between 0.32 GeV to 0.87 GeV,a number obviously not close to zero. This nonzero value of ω_0 certainly implies that caution should be exercised before estimating, the number of particles produced in RHIC, using Schwinger's expression[2].

2.5 Estimation Of Pair Production Rate

Having obtained estimates of the field strengths and the frequencies of oscillation of the fields produced in RHIC, we will concentrate next on the computation of the pair production rate of spin zero bosons with SU(2) color symmetry, in the presence of a sinusoidally oscillating background chromo-electric field. For fermions the final result will get modified by numerical factors only. In the discussion of our calculation we will not provide derivation of the standard field theory results, instead we will refer to the sources where they could be found.
The probability that the vacuum remains vacuum, in the presence of an external field, can be written in terms of the S matrix as

$$|\langle 0 \mid S \mid 0\rangle|^2 \equiv |S_o(A)|^2 = exp\left[-\int d^4x W(x)\right] \tag{2.32}$$

where $\langle 0 \mid S \mid 0\rangle$ is the vacuum expectation value of S-matrix in the presence of the color potential A^a_μ and $W(x)$ is the pair creation probability per unit volume per unit time. The quantity S_0 can be shown [16] to be equal to

$$S_0 = Det\left(G^{-1}G_0\right) = expTr\left[ln\left(G^{-1}G_0\right)\right] \tag{2.33}$$

where G_0 and G are the free propagator and the propagator in presence of the external field respectively, defined as

$$G_0 = \frac{1}{P^2 - m^2 + i\varepsilon} \quad \text{and} \quad G = \frac{1}{(P-gA)^2 - m^2 + i\varepsilon} \tag{2.34}$$

The trace in equation (2.33) is defined over spinor, color and coordinate spaces. In terms of scattering operators T and $\bar{T}$ defined as

$$T = V + V\frac{1}{P^2 - m^2 + i\varepsilon}T \quad and \quad \bar{T} = V + V\frac{1}{P^2 - m^2 - i\varepsilon}\bar{T} \tag{2.35}$$

(2.36)

with $\bar{T} = \gamma^0 T^\dagger \gamma^0$ and $V = G_o{}^{-1} - G^{-1}$, one can show that

$$|S_o(A)|^2 = exp\left[Trln\left(1 - T\rho_+ T^+ \rho_-\right)\right] \tag{2.37}$$

$$W(x) = -tr\langle x \mid ln\left(1 - T\rho_+ T^+ \rho_-\right) \mid x\rangle \tag{2.38}$$

Here $\rho_\pm$ are the projectors over positive and negative energy states defined as

$$\rho_\pm = 2\pi\theta_\pm\left(p^2\right)\delta\left(p^2 - m^2\right)$$

It should be noted that the operators $T\,(T^{+})$ as well as $\rho_{\pm}$ are matrices in color, spinor and coordinate space. In equation (2.37)the symbol Tr stands for integration over the continuous variables and trace over the color and spinor indices, whereas in equation (2.38) tr stands for trace over color and spinor indices only.
On expanding the logarithm in equation (2.38) and retaining the first term (i.e neglecting the production probability of 2 ,3 or more pairs) one gets

$$W = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{T} dt . \frac{1}{2\pi} \int \frac{d^3p}{(2\omega)^2} \mid \langle -\omega \mid T \mid \omega \rangle \mid^2 \tag{2.39}$$

Here $\omega = (p^2+m^2)^{1/2}$ and m is the mass of the spin zero colored particle. The backward "scattering" amplitude $\langle -\omega \mid T \mid \omega\rangle$ is then evaluated by solving the color coupled Klein Gordon equations in external color potential. For the color SU(2) group the equations to be solved are (τ_α , $\alpha = 1,2,3$ are Pauli matrices).

$$\left[\left(\partial_o^2 - \nabla^2\right) + 2igA_\alpha\tau_\alpha\partial_3 + g\,(A_\alpha)^2 + m^2\right] \begin{pmatrix} \varphi_+ \\ \varphi_- \end{pmatrix} = 0 \tag{2.40}$$

with appropriate asymptotic conditions in time.

More precisely, we look for solutions of eq. (2.40) having the form[17]

$$\begin{array}{llcl} t\to-\infty & \varphi_+(t) & = & e^{-i\omega t} + b_+ e^{i\omega t} \\ & \varphi_-(t) & = & e^{-i\omega t} + b_- e^{i\omega t} \\ t\to+\infty & \varphi_+(t) & = & a_+ e^{-i\omega t} \\ & \varphi_-(t) & = & a_- e^{-i\omega t} \end{array} \tag{2.41}$$

Since a negative energy particle at $t\to-\infty$ is equivalent to a positive energy antiparticle at $t\to+\infty$, the backward "scattering" amplitude $\langle -\omega \mid T \mid \omega\rangle$ and hence the pair creation probability, can be determined from the coefficients b_+ and b_-. Actually one has

$$W \propto \frac{\mid b_+ \mid^2 + \mid b_- \mid^2}{2} \tag{2.42}$$

In order to proceed with the solution of equation (2.40), it is easy to show that, the above equations can be decoupled by a unitary transformation in color space defined by,

$$U^+ = \begin{pmatrix} (E_3+E)\,/N_1, & (E_1-iE_2)\,/N_1 \\ (E_3-E)\,/N_2, & (E_1-iE_2)\,/N_2 \end{pmatrix} \tag{2.43}$$

where $E^2 = E_1^2+E_2^2+E_3^2$, $N_1^2 = 2E^2+2E_3E, N_2^2 = 2E^2-2E_3E$. The (column vector) wave function in turn transforms into

$$U^+ \begin{pmatrix} \varphi_+ \\ \varphi_- \end{pmatrix} = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} \tag{2.44}$$

For the spatially homogeneous system that we are considering (note that this ignores the confinement effect discussed by some earlier workers[6]), the decoupled equations are,

$$\left[\partial_o^2 + m^2 + p^2 \mp 2gp_3\left(\frac{E}{\omega_o}\right)\cos\omega_o t + g^2\left(\frac{E}{\omega_o}\right)^2\cos^2\omega_o t\right]\begin{pmatrix}\Psi_+\\ \Psi_-\end{pmatrix} = 0 \qquad (2.45)$$

with $p^2 \equiv p_1^2 + p_2^2 + p_3^2 \equiv p_\perp^2 + p_3^2$.
Following Brezin and Itzykson[17], these decoupled equations are solved using the boundary conditions

$$\begin{array}{lrcl} t \to -\infty & \Psi_+(t) & = & Ae^{-i\omega t} + Be^{i\omega t} \\ & \Psi_-(t) & = & Ce^{-i\omega t} + De^{i\omega t} \\ t \to +\infty & \Psi_+(t) & = & Ee^{-i\omega t} \\ & \Psi_-(t) & = & Fe^{-i\omega t} \end{array} \qquad (2.46)$$

After finding the coefficients A, B, C, D we finally express them in terms of b_+ and b_- respectively. We have solved for the coefficients A, B, C, D from equation (2.45) by W.K.B method, choosing a solution of the form

$$\begin{array}{rcl} \Psi_+(t) & = & \alpha_a(t)e^{-i\chi_a(t)} + \beta_a(t)e^{i\chi_a(t)} \\ \Psi_-(t) & = & \alpha_b(t)e^{-i\chi_b(t)} + \beta_b(t)e^{i\chi_b(t)} \end{array} \qquad (2.47)$$

where

$$\chi_a(t) = \int_0^t d\bar{t}\omega_a(\bar{t}) \quad \text{and} \quad \chi_b(t) = \int_0^t d\bar{t}\omega_b(\bar{t}) \qquad (2.48)$$

and assuming $\frac{gE}{m^2} \ll 1$ along with the conditions $\frac{\dot{\omega}_a(t)}{\omega_a{}^2(t)} \ll 1$ and $\frac{\dot{\omega}_b(t)}{\omega_b{}^2(t)} \ll 1$ where $\omega_a(t) = \left[m^2 + \left(p_3 - \frac{E}{\omega_0}t\right)^2\right]^{\frac{1}{2}}$ and $\omega_b(t) = \left[m^2 + \left(p_3 + \frac{E}{\omega_0}t\right)^2\right]^{\frac{1}{2}}$. One assumes here that the external field is switched on and off adiabatically.
From equation (2.46), we obtain an order of magnitude estimate of pair creation probability, in the case $\omega_o \ll m$,

$$W \simeq \frac{\alpha_s E^2}{2\pi}\frac{1}{g(\gamma) + \frac{1}{2}\gamma g'(\gamma)} exp\left[-\frac{\pi m^2}{gE}g(\gamma)\right] \qquad (2.49)$$

where

$$\begin{array}{rcl} g(z) & = & \frac{4}{\pi}\int_o^1 dy\left[\frac{1-y^2}{1+z^2y^2}\right]^{1/2} \\ and & & \\ \gamma & = & \frac{m\omega_o}{gE}, \quad \alpha_s = \frac{g^2}{4\pi} \end{array} \qquad (2.50)$$

As shown by Brezin and Itzykson[16], one can recover the static Schwinger limit from equations (2.49) and (2.50) by taking $\omega_o \to 0$ independently of gE in such a way that $\gamma = \frac{m\omega_o}{gE} \to 0$. In this case, one obtains the Schwinger result

$$W_s \simeq \frac{\alpha_s E^2}{2\pi} exp\left[-\frac{\pi m^2}{gE}\right] \tag{2.51}$$

To consider the case of oscillating non-abelian fields we must take ω_o to be dependent on E in the manner discussed after equation (2.20) i.e. that $\omega_o = \sqrt{gE/2}$. Equations (2.49) and (2.50) now show that $\gamma = \frac{m\omega_o}{gE} = \frac{m}{\sqrt{2gE}} = \frac{1}{2}\frac{m}{\omega_o} \gg 1$ and that the pair creation probability W takes the form of 'multigluon' production, viz.,

$$W_g \simeq \frac{\alpha_s E^2}{8}\left[\frac{g^2E^2}{4m^2\omega_o^2}\right]^{\frac{2m}{\omega_o}} \left[\omega_o = \sqrt{\frac{gE}{2}} \ll m\right] \tag{2.52}$$

where $\frac{2m}{\omega_o}$ is the minimum number of gluons required to produce a pair. Incidentally following Sakurai[17], one can also compute the pair production rate using ordinary perturbation theory, when $\omega_0 \gg m$. Here the transition amplitude is given by the S matrix element

$$S_{fi} = -g\langle q\bar{q} \mid \int d^4x \overline{\Psi}^{(-)}{}_\alpha (\gamma)_{\alpha\beta}\, \Psi^{(-)}{}_\beta A^a_\mu \tau^a \mid 0\rangle \tag{2.53}$$

Here Ψ are the quark fields operator, $A_\mu{}^a$ are the classical external fields and τ_a are the pauli matrices respectively. The square of this amplitude will give us the probability of transition from vacuum to $q\bar{q}$ pairs. An integration over the available phase space gives the total pair production probability. On taking the external field as sinusoidally oscillating in time and carrying out the integration one arrives at the pair production rate

$$W_p \simeq \frac{\alpha_s E^2}{6}\left(1 + \frac{2m^2}{\omega_0^2}\right)\sqrt{\left(1 - \frac{4m^2}{\omega_0^2}\right)} \tag{2.54}$$

If one considers the strong field limit i.e $gE \gg m^2$ then one can see that the perturbative formula for pair production reduces to

$$W_p \simeq \alpha_s E^2 \tag{2.55}$$

(ignoring the numerical factors). This result also follows from Schwinger's expression, since in the limit $m^2/gE \ll 1$ we can expand the exponential in powers of $\frac{m^2}{gE}$ and retain only the first term ignoring the others to arrive at the same expression. Next we consider the various limits, by defining $x = \frac{m^2}{gE}$ and $n = \frac{2m}{\omega_0}$. One can then see that the ratio between Schwinger and the multigluon production rate is

$$\frac{W_s}{W_g} \simeq (xn)^{2n} e^{-\pi x} \tag{2.56}$$

Since an exponential dominates over any finite order polynomial this expression shows that for $n \sim x \gg 1$ the multigluon ionisation process of vacuum dominates over the Schwinger process.

Before we obtain the numerical estimate of the pair production rate , we would like to comment on the numerical value of the particle mass to be used in the computation. In the literature the numerical estimate of the pair production rate has been carried out using constituent as well as current quark masses. In our view, since the flux tube model takes into account the localisation of color flux and the effect of confinement, it is more appropriate to consider constituent quark mass for numerical estimation. Moreover as has been discussed earlier, in order to produce an on shell $q\bar{q}$ pair from vacuum, the external field has to move them over a distance, of the order of compton wavelength ($\sim \frac{\hbar}{m_q c}$) of the particles. For current quark mass this distance is around $\sim 20fm$, which appears unreasonable for $A - A$ collisions. We therefore propose that for pair creation via flux tube model $\frac{\hbar}{m_q c} \leq 1fm$ i.e $m_q \geq 200MeV$.

In any case we have numerically evaluated the pair production rate using the expressions in the three limits i.e perturbative, multigluon ionisation and Schwinger, with different values of the chromo-electric field and mass. The results are shown in Table-I. They show the following features:

1.If m = 10 MeV, then for values of gE ranging from 0.05 GeV^2 to 1.5 GeV^2, the pair creation probability $W_s \approx W_g \approx W_p$. For $m_s = 150MeV$, W_p is larger in p-p collisions and W_g is significant in A - A collisions.

2.For the production of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, pairs with constituent quark masses and field strength $gE \leq 0.5GeV^2$ the pair creation probability W_p dominates in p-p and in A-A collisions. For $gE \geq 0.5GeV^2$ the multigluon ionisation of pairs from the vacuum is larger in A-A collisions.

Table Caption

Table 1. Pair creation probability W_s, W_g and W_p (in units $(\mathrm{fm})^{-4}$) for different values of mass m (GeV) and field strength gE ($(\mathrm{GeV})^2$).

Table 1

m	gE	W_s	W_g	W_p
0.01	0.05	0.021	0.015	0.022
	0.1	0.083	0.076	0.087
	0.2	0.334	0.331	0.350
	0.5	2.09	2.12	2.19
	1.0	8.35	8.35	8.75
	1.5	18.8	18.5	19.7
0.15	0.05	0.005	$\sim 2 \times 10^{-7}$	0.022
	0.1	0.041	$\sim 4 \times 10^{-4}$	0.087
	0.2	0.235	0.051	0.350
	0.5	1.81	3.03	2.19
	1.0	7.78	24.5	8.75
	1.5	18.0	65.9	19.7
0.300	0.05	7×10^{-5}	$\sim 2 \times 10^{-14}$	0.022
	0.1	5×10^{-3}	$\sim 6 \times 10^{-8}$	0.087
	0.2	0.081	7×10^{-4}	0.350
	0.5	1.19	1.06	2.19
	1.0	6.30	28.1	8.75
	1.5	15.6	112.5	19.7
0.500	0.05	3×10^{-9}	3×10^{-25}	0.022
	0.1	3×10^{-5}	6×10^{-14}	0.087
	0.2	7×10^{-3}	5×10^{-7}	0.350
	0.5	0.434	0.103	2.19
	1.0	3.81	17.5	8.75
	1.5	11.1	133.9	19.7

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Chapter 3

Pair Production at Finite Temperature

3.1 Introduction

Following the discussion in the last chapter on pair production from vacuum in the presence of an oscillating external chromo-electric field , in this chapter we will discuss the effect of heat bath on such a process.

Before going into the details of the calculation we will elaborate, on the physical situation relevant for this computation. In particular we will try to show that ,whether the time scale of reduction of the external field due to pair creation process is long enough for the system to come to thermal equilibrium.

It is worth emphasising here that, though in the earlier chapter , the vacuum chromo-electric field in the flux tube was taken to be oscillating in time, (since it followed from the solutions of the vacuum Yang-Mills (YM) equations), one need not assume the same, in the presence of a heat bath. To determine the nature of the chromo-electric field in the presence of the plasma, one has to solve the YM equations with a plasma source term. Although for a realistic study, one should compute the spontaneous pair production rate in the presence of such a field, as an approximation to the more realistic case we will restrict ourselves to a constant external chromo-electric field, as a proper investigation of this process has not been performed before.

Estimation Of The Characteristic Time Scales

In this section we first estimate the time scale for the production of pairs. If we recall, the expression for the spontaneous pair production rate from vacuum / unit time /unit volume is given by

$$W_p = \frac{\alpha_s E^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{\frac{-n\pi m^2}{gE}}$$

From this expression, one can crudely estimate the time scale of production of $q\bar{q}$ pairs in an unit volume, and it comes out as

$$t_p \sim \frac{\pi^2}{\alpha_s E^2} e^{\frac{\pi m^2}{gE}} \tag{3.1}$$

In the above expression we have assumed that just one pair is being produced , so we have neglected the sum over n in equation (3.1). After estimating the time scale of production of pairs, we will estimate next, the time scale of depletion of the external field. It is worth mentioning here that this time scale has been estimated by Gyulassy et.al[1] and Gatoff et.al[2] before. Gyulassy had estimated it assuming abelian dominance approximation for pair production rate and Gatoff et.al had estimated it using hydrodynamic equations. We will however estimate the same, from the principle of energy conservation , essentially following the argument of reference (3), assuming Schwinger picture for pair production to hold good.

Since in this model pairs are produced with zero longitudinal momentum but all possible values of the transverse momentum, $p_\perp$, the amount energy loss with the production of a pair, where each one of the produced particles is having average energy $\langle\sqrt{m^2 + p_\perp{}^2}\rangle$, is $2\langle\sqrt{m^2 + p_\perp{}^2}\rangle$, so after producing n such pairs, the total energy lost by the external field is $2n\langle\sqrt{m^2 + p_\perp{}^2}\rangle$. Since the production probability for a pair is given by $\frac{\alpha_s E^2}{\pi^2} e^{\frac{-\pi m^2}{gE}}$, the associated energy loss by the field can be written as

$$\frac{d\varepsilon(t)}{dt} = -2\langle\sqrt{m^2 + p_\perp{}^2}\rangle \frac{\alpha_s E^2}{\pi^2} e^{\frac{-\pi m^2}{gE}} \tag{3.2}$$

where $\varepsilon(t) = \frac{E^2}{2}$ is the field energy. From the solution of this differential equation, one arrives at the time required for the electric field to decay to $\frac{1}{e}$ th of its original value as

$$t_d = C\left[E_1\left(y_{max}\right) - E_1\left(y_{min}\right)\right] \tag{3.3}$$

Here $C = \frac{\pi^2}{2\alpha_s\langle\sqrt{p_\perp^2 + m^2}\rangle}$, $y_{max} = \frac{\pi m^2}{gE_{max}}$, $gE_{max} = gE(t = 0) = gE$, and $y_{min} = \frac{\pi m^2}{g(E/e)}$ and E_1 represents the exponential integral. So one needs to know the average value of $\sqrt{m^2 + p_\perp{}^2}$, for the proper estimation of the depletion time.The distribution of particles in the momentum interval $p_\perp$ to $p_\perp + dp_\perp$ can be computed from the solution of the Dirac equation in presence of the external field and it is (Ref Kerman[4], Nussinov[5]);

$$\frac{dN}{dp_\perp{}^2} \propto ln\left[1 - e^{\frac{-(m^2+p_\perp{}^2)}{gE}}\right]$$

From this equation one can compute the average energy of each produced particle to be

$$\langle\sqrt{m^2 + p_\perp{}^2}\rangle \simeq k\frac{\sqrt{gE}}{2}$$

where

$$k = O(1)$$

Hence

$$t_d = \frac{\pi^2}{k\alpha_s\sqrt{gE}}\left[E_1\left(y_{max}\right) - E_1\left(y_{min}\right)\right] \tag{3.4}$$

Finally from these relations one arrives at the ratio of depletion time to production time as

$$\frac{t_d}{t_p} = \left(\frac{1}{k(4\pi\alpha_s)}\right)(gE)^{\frac{3}{2}}e^{\frac{-\pi m^2}{gE}}\left[E_1\left(y_{max}\right) - E_1\left(y_{min}\right)\right] \tag{3.5}$$

With m = 0.2 GeV, $\alpha_s = 0.3$ and gE = 1 GeV^2 we get $t_d \simeq 5fm/c$ and $\frac{t_d}{t_p} \simeq 30$

One can see from these relationships that, as the ratio of field strength to mass square increases the time scale of reduction of the electric field also increases. Since the strength of the electric field is proportional to the mass number($A_P{}^{\frac{1}{6}}A_T{}^{\frac{1}{6}}$) of the colliding nuclei, for heavier nuclei, one can expect the external field to last for a time longer than the production time of the pairs.

Since these produced pairs come almost with a Boltzmann like distribution in momentum space[4] , (both in the case of constant as well as the time varying external field), they will come close to thermal equilibrium very fast through collision with each other. Moreover, other than the collisional processes, the joule heating of the plasma generated because of the conduction current produced by the external chromo-electric field will also contribute towards the thermalisation of the system. A quantitative estimate of momentum equlibration time, in a parton cascade model, has been obtained by Biro et. al.[15] who get a value of 0.31 fm/c. This value is essentially the same as the thermalization time ~ 0.3 fm/c for gluons at RHIC energies estimated by Shuryak[16]. For quarks the thermalization time ~ 1-2 fm/c. In brief, since the depletion time t_d is greater than the production time t_p, and the thermalisation time is smaller than the depletion time, one might be justified in assuming the existence of the external field in the thermally equilibrated plasma.

Though it is not very clear whether, initially the temperature of the system will be the same everywhere, but if the time scale of thermalisation is faster than the speed of separation of the two color charged, Lorentz contracted receding nuclei , one can expect the temperature to remain constant in the space, between them. Thus, in our view, it is pertinent to study the process of pair production at finite temperature, in **RHIC**.

As we go along we will see that because of the presence of heat bath, the rate of spontaneous creation of $q\overline{q}$ pairs in the presence of external field, is no more homogeneous in space ;rather it decreases towards the center.As a result of this differential rate of pair production, after all the field energy is exhausted in producing pairs, there will be an anisotropy in the temperature (global) distribution of the produced plasma.In our view, the following hydrodynamic evolution of the plasma will bear a signature of this anisotropic temperature distribution.

Having motivated the physical situation, we discuss the organisation of the chapter.In section 2 we will review the basics of finite temperature field theory .In section 3 we will be computing the finite temperature pair production rate in presence of external electric field following which we will conclude the chapter by discussing possible extension of our work to improve of our result.

3.2 Introduction To Thermal Effective Action:

In this section we will be introducing thermal field theory and the concept of thermal effective action.It is a well known fact that there are two different ways of introducing temperature in Quantum field theory.One of them being the imaginary time formalism[6] of Matsubara and the other is the real time[7] finite temperature field theory or thermo field dynamics.The real time formalism has distinct advantages over the former, in terms of computation of dynamical quantities.However as far the thermodynamic quantities are concerned the two formalisms give identical results.

The objective of the present work is to find out the rate of $q\bar{q}$ production at finite temperature in the presence of a static external chromo-electric field.One can compute this quantity either by evaluating the (reference(8)) thermal S matrix in presence of the external field or computing the imaginary part of the effective potential which is essentially the free energy[9] density of the system.In our work, we calculate the free energy density or the effective Lagrangian of the system.

3.3 Effective Action

Now let us recall that the expression for the partition function Z is given as

$$Z = Tre^{-\beta H} \equiv \sum_a \langle \phi_a \mid e^{-\beta H} \mid \phi_a \rangle \tag{3.6}$$

The first task in finite temperature studies of field theory is to write down the partition function in field theory as a functional integral involving Lagrangian density expressed in terms of the dynamical fields present in the theory. More precisely, given a theory, defined in Minkowski space, how does one compute the partition function Z, in relativistic quantum field theory. In order to illustrate the basic ideas , for the moment, we consider the case of a scalar quantum field theory with field operators (in Heisenberg picture), $\phi\,(t,x)$ with momenta $\pi\,(t,x)$ the Lagrangian density L and Hamiltonian density H.

If $\phi(\vec{x},0)$ is the Schrodinger-picture field operator having eigen states $\mid \phi_a\rangle$ and$\mid \phi_b\rangle$, with eigenvalues $\phi_a(x)$ and $\phi_b(x)$ then the transition amplitude for the system to go from the state $\phi_a(x)$ at t=0, to the state $\phi_b(x)$ at $t = t_f$ is

$$\langle \phi_b \mid e^{-iHt} \mid \phi_a \rangle = N' \int_{\phi_a}^{\phi_b} [d\phi] exp \left(-\int_0^{t_f} d\tau \int d^3x L\left(\phi, \dot{\phi}\right)\right) \tag{3.7}$$

Here N' as a normalisation constant, and the functional integral is defined over classical fields $\phi(t,x)$.

As it has been shown in number of places (see reference(6) and the references therein) one can use functional integral form of equation (3.7) to obtain a functional integral form for Z by a series of steps. (i) Choose the initial state and the final state to be the same,(ii) change the time coordinate t over to a variable defined as $\tau = it$ with the limits of integration varying between 0 to β and (iii) lastly as a consequence of the trace operation, perform the functional integration over the fields $\phi(\tau,x)$ with a periodic boundary conditions in τ i.e $\phi(0,x) = \phi(\beta,x)$.

For a system with no conserved charge one can equivalently show that,

$$Z = N' \int_{periodic} [d\phi] exp\left(\int_0^\beta d\tau \int d^3x L\left(\phi, \dot{\phi}\right)\right) \tag{3.8}$$

For a Lagrangian that is quadratic in the field variables, one can compute the partition function Z exactly by expanding the field variables $\phi(t,x)$ as

$$\phi(\beta, x) = \frac{1}{\beta}\sum_n \int \frac{d^3p}{(2\pi)^3} e^{-i(\omega_n\tau - p.x)} \tilde{\phi}(\omega_n, p) \tag{3.9}$$

and performing the Gaussian integration over the field variables. Here $\omega_n = \frac{2\pi n}{\beta}(n = -\infty$ to $\infty)$ are the (Matsubara) frequencies for bosons and have been defined to agree with the periodic boundary conditions of the field variables. The finite temperature Green functions defined as

$$G_\beta(\bar{x}_1, \bar{x}_2....\bar{x}_j) = \frac{Tre^{-\beta H}(T\phi(\bar{x}_1)....\phi(\bar{x}_j))}{Tre^{-\beta H}} \tag{3.10}$$

can be shown to be coming from

$$G_\beta(\bar{x}_1, \bar{x}_2....\bar{x}_j) = \frac{\delta^j Z(J)}{\delta J(\bar{x}_j).....\delta J(\bar{x}_1)}|_{J=0} \tag{3.11}$$

where

$$Z = \frac{\int_{periodic}[D\phi] exp\left(\int_0^\beta d\tau \int d^3x L(\phi, \partial_\mu\phi) + J\phi\right)}{\int_{periodic}[D\phi] exp\left(\int_0^\beta d\tau \int d^3x L(\phi, \partial_\mu\phi)\right)} \tag{3.12}$$

The generating functional for the connected Green function is defined through

$$W^\beta(J) = lnZ^\beta(J) \tag{3.13}$$

An effective action $\overline{\Gamma}(\phi_c)$ is defined in terms of the Legendre transform[10] of $W^\beta(J)$ as

$$\overline{\Gamma}(\phi_c) = W^\beta[J] - \int d\bar{x}\phi_c(\bar{x}) J(x) \tag{3.14}$$

where $\phi_c(\bar{x})$ is the classical field defined as

$$\phi_c(\bar{x}) = \frac{\delta W^\beta[J]}{\delta J[\bar{x}]} \tag{3.15}$$

and the source *J(x)* is given by

$$J(\bar{x}) = -\frac{\delta\Gamma[\phi_c]}{\delta\phi_c[\bar{x}]} \tag{3.16}$$

Here the vector $\bar{x} = (-i\tau, \vec{x})$.

The quantity $\overline{\Gamma}(\phi_c)$, evaluated semiclassically about some field configuration $\phi_c(\bar{x})$, gives the free energy of the system in that configuration. Usually $V_{eff}(\phi)$, the effective potential, the first term in a derivative expansion of $\overline{\Gamma}(\phi)$, is just the free energy density in a background constant field configuration. The quantity effective lagrangian L_{eff}, is defined[11] to be, L_{eff} = - V_{eff} This quantity is used for determining not only the thermal ground state energy of the system but also for determining the phase transition, symmetry breaking etc. In the case of a first order phase transition, a system can be trapped temporarily in a meta stable state leading to non-equilibrium phenomena. The rate of decay for such a system is determined from the imaginary part of its free energy (reference (9): Affleck, Langer). Though we have outlined the formalism for scalar bosons, it has been generalised for the case of fermions and gauge bosons too. For fermions one has to take the anti-periodic boundary condition because of anti commutation relation satisfied by the fermions. For vector bosons, other than periodic boundary conditions one also has to take care of the extra degrees of freedom carried by the gauge bosons. We are not going to elaborate on this point any further here. All the details can be found in reference (12).

3.4 Computation of Effective Lagrangian From The Fermionic Determinant

In this section we compute the effective action for a system of fermions with SU(2) color symmetry, in an external chromo-electric field.In our calculation we assume the plasma to consist of equal number of quarks and antiquarks, so the net baryon number as well as the chemical potential are zero. Further we do not include the dynamics of the gluon fields, though in a more realistic case one ought to do so.

It is also worth mentioning, that this problem has been studied earlier, in the context of quantum electrodynamics U(1) symmetry, by Loewe and Rojas[8] using real time thermal field theory and also by Cox, Helmann and Yildiz[13]. Unfortunately there is no agreement between the results obtained by them. Cox et al find no effect of temperature on pair production rate, whereas the authors of reference(8) do find a finite temperature contribution to pair production rate. In fact their result also shows that it increases dramatically with temperature. Our calculation, when performed with an U(1) symmetry, agrees qualitatively with the findings of reference(8) but it disagrees in other aspects.

For instance, in contrast to the result of Loewe and Rojas, we find that the finite temperature pair production rate has two distinct pieces in it, one being the vacuum contribution and the other the finite temperature contribution having a sign difference. The finite temperature contribution,unlike the vacuum contribution is a space dependent quantity implying that the pair production rate as well as all the thermodynamic quantities vary in space, in particular along the longitudinal direction. In fact, this striking result is due to shielding of the electric field by the polarised plasma in between. Consequently, as one moves away from the source, the field strength decreases, giving rise to a differential rate in pair production. Since the rate of pair production varies in space, the number density of produced particles will also vary in space leading to a similar behaviour of the thermodynamic quantities like pressure, entropy, temperature etc. Since we are interested in investigating the pair production rate, we will not discuss the thermodynamic quantities here.

3.4.1 Computation of Effective Lagrangian

We start from the "partition function" in Minkowski space defined by

$$Z[A] = \frac{\int D\bar{\psi} D\psi e^{i\int L d^4x}}{\int D\bar{\psi} D\psi e^{i\int L_o d^4x}} \tag{3.17}$$

where $L = \bar{\psi}\left(i\gamma_\mu \partial^\mu - g\gamma_\mu A_a{}^\mu \tau_a\right)\psi - m\bar{\psi}\psi$ is the fermionic Lagrangian in the presence of external vector field $A_\mu{}^a$, τ_a's are the Pauli matrices and $L_o = \bar{\psi}\left(i\gamma_\mu \partial^\mu - m\right)\psi$ is the free fermionic Lagrangian, such that $Z[0] = 1$.
Since we are interested in evaluating the effective action, in the presence of external chromo-electric field only, we choose $A_0{}^a = -E^a z$ and other components of A to be equal to zero. Following standard prescriptions (see reference(6) and (10)), we obtain the finite temperature partition function in terms of the Euclidean action S_β defined as,

$$S_\beta = \frac{1}{\beta^2} \sum_{n=-\infty}^{\infty} \int \bar{\psi}_n(x) \left[\left(\omega_n \gamma^0 + g A^a{}_o \tau^a \gamma^0\right) + i\gamma^j \partial_j - m\right] \psi_n(x) d^3x \tag{3.18}$$

and compactify the time direction by putting antiperiodic boundary condition to get

$$Z[A] = \frac{\prod_{n=-\infty}^{\infty} \int D\bar{\psi}_n D\psi_n e^{-S_\beta}}{\prod_{n=-\infty}^{\infty} \int D\bar{\psi}_n D\psi_n e^{-S_{o\beta}}} \tag{3.19}$$

Here ψ 's are the fermion fields in the fundamental representation of SU(2) defined as $\psi(x) = \frac{1}{\beta}\sum_n \int \frac{d^3x}{2\pi^3} e^{-i(\omega_n \tau - px)} \tilde{\psi}_n(\omega_n, p)$ and

$$S_{o\beta} = \frac{1}{\beta^2} \sum_{n=-\infty}^{\infty} \int \bar{\psi}_n(x) \left[\omega_n \gamma^0 + i\gamma^j \partial_j - m\right] \psi_n(x) d^3x$$

It should be noted that in Eq(3.19) γ^0 and $A_0{}^a$ are quantities in Euclidean space. On integrating over the fermion fields one arrives at

$$Z[A] = \prod_{n=-\infty}^{\infty} Det \left[\frac{(\gamma^0 \omega_n + g\gamma^0 A_o^a \tau^a) + i\gamma^j \partial_j - m}{(\omega_n \gamma^0 + i\gamma^j \partial_j - m)}\right] \tag{3.20}$$

This determinant is defined over color, spinor as well as the coordinate space.

Using well known techniques[14] one can further write it as

$$Z[A] = \prod_{n=-\infty}^{\infty} Det \left[\frac{(\omega_n + gA_o^a \tau^a)^2 - g \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix} E^a \tau_a - \partial_k^2 + m^2}{\omega_n^2 - \partial_k^2 + m^2}\right]^{1/2} \tag{3.21}$$

The determinant in Eq.(3.21) can further be diagonalised in color space using an unitary matrix of the form.

$$U^+ = \begin{pmatrix} (E_3 + E)/N_1 & (E_1 - iE_2)/N_1 \\ (E_3 + E)/N_2 & (E_1 - iE_2)/N_2 \end{pmatrix} \tag{3.22}$$

with $E = \sqrt{E_1^2 + E_2^2 + E_3^2}$; $N_1^2 = 2E^2 + 2E_3 E$, $N_2^2 = 2E^2 - 2E_3 E$.

After diagonalising Eq(3.21) in color space and using the identity $Det\hat{O} = e^{trln\hat{O}}$ and the integral representation $ln\hat{O} = \int_o^\infty \frac{ds}{s} e^{-s\hat{O}}$ one arrives at

$$\begin{aligned} -S_{eff} &= lnZ = -4tr \left[\sum_{n=-\infty}^{\infty} \int_0^\infty \frac{ds}{s} \left[cosh(gEs) e^{-s\left[(\omega_n + \bar{A}_o)^2 + p_j^2 + m^2\right]} \right.\right. \\ &- \left.\left. e^{-s\left[\omega_n^2 + p_j^2 + m^2\right]}\right]\right] \end{aligned} \tag{3.23}$$

with $\bar{A}_o = -Ez$. The trace is now defined only over coordinate space. We note that since A_μ is an external field, no Legendre transformation is required to go from the

connected vacuum functional to the effective action .
After doing some lengthy algebra one arrives at the expression for free energy density

$$-\mathcal{F} = -\frac{1}{4\pi^2}\left[\Sigma_{n=-\infty}^{\infty}(-1)^n\left[e^{in\beta gEz}\int_o^\infty \frac{ds}{s^2}(gE)coth(gEs)e^{-sm^2-n^2\frac{\beta^2 gE}{4}coth(gEs)}\right.\right.$$
$$\left.\left. - \int_0^\infty \frac{ds}{s^3}e^{-sm^2-\frac{n^2\beta^2}{4s}}\right]\right] \tag{3.24}$$

Expanding coth z in the asymptotic form $1 + 1/z$, we get after separating the $n = 0$ term from the other $n \neq 0$ terms

$$\mathcal{F} = \frac{1}{4\pi^2}\int_o^\infty \frac{ds}{s^3}(gEs)coth(gEs)e^{-sm^2} +$$
$$\frac{1}{2\pi^2}\ \Sigma_{n=1}^{\infty}(-1)^n\left[\cos\left(ng\beta\bar{A}_o\right)\int_o^\infty \frac{ds}{s^3}(gEs)coth(gEs)e^{-sm^2-\frac{n^2\beta^2}{4s}}e^{-n^2\beta^2 gE/4}\right.$$
$$\left. - \int_o^\infty \frac{ds}{s^3}e^{-sm^2-\frac{n^2\beta^2}{4s}}\right] \tag{3.25}$$

Since E in Eq.(3.25) is the Euclidean electric field we need to rotate the electric field back to Minkowski space i.e. $E \rightarrow -iE$ to obtain the expression for the thermal effective action

$$\mathcal{F} = \left[\frac{1}{4\pi^2}\int_o^\infty \frac{ds}{s^3}e^{-sm^2}\left[(gEs)cot(gEs) - 1\right] + \frac{1}{2\pi^2}\Sigma_{n=1}^{\infty}(-1)^n\int_o^\infty \frac{ds}{s^3}\right.$$
$$\left.\left[\cosh\left(ng\beta\bar{A}_o\right)(gEs)coth(gEs)e^{-in^2\beta^2 gE/4} - 1\right]e^{-sm^2-\frac{n^2\beta^2}{4s}}\right] \tag{3.26}$$

This is the main result of our work. We can clearly see that $n = 0$ provides the vacuum contribution to the effective action and $n \neq 0$ provides the finite temperature correction to it.

The spontaneous pair production rate is given by the imaginary part of the effective action given above.

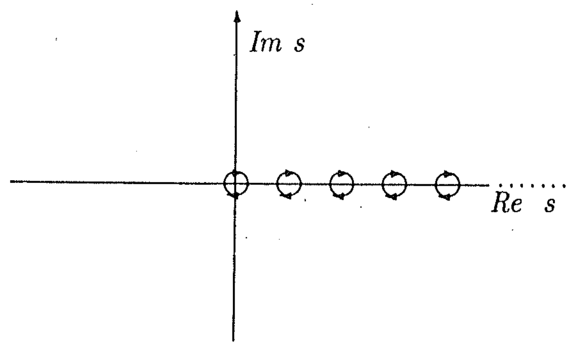

Fig-I

Contour Of Integration.

On carrying out the integration in Eq.(3.26), by choosing a contour as shown in figure-I with poles at $s = (l\pi/gE)$;we get for the imaginary part of the L_{eff}

$$Im\left[L_{eff}\right] = \frac{1}{4\pi^3}\sum_{n=1}^{\infty}\frac{(gE)^2}{n^2}e^{-\frac{m^2\pi n}{gE}} - \frac{1}{2\pi^2}\sum_{n=1}^{\infty}(-1)^n cosh(n\beta g A_o)\left[P.V.\int_0^{\infty}\frac{ds}{s^2}\, gEcot\, gEs \right.$$
$$\left. sin\frac{n^2\beta^2 gE}{4}e^{-sm^2-\frac{n^2\beta^2}{4s}} - \pi\sum_{l=1}^{\infty}\frac{(gE)^2}{(l\pi)^2}e^{\frac{-m^2\pi l}{gE}-\frac{n^2\beta^2 gE}{4\pi l}}\, cos\frac{n^2\beta^2 gE}{4}\right] \tag{3.27}$$

Here P.V means principal value. Although we have not evaluated the real part of the effective lagrangian, it will provide one with expressions for the thermodynamic quantities like pressure, entropy etc.

Analysis of Our Result

One can see from Eq. (3.27) that there is a sign difference between the zero temperature and the finite temperature part of the effective lagrangian. Depending on the temperature and field strength one or the other term will dominate. We have tried to evaluate the expression numerically for $z = 0$

$$Im\,[L_{eff}] = \frac{1}{4\pi^3}\sum_{n=1}^{\infty}\frac{(gE)^2}{n^2}e^{-\frac{m^2\pi n}{gE}}\left[1+\frac{\pi}{2}\sum_{l=1}^{\infty}e^{-\frac{n^2\beta^2 gE}{4\pi l}}\;cos\frac{n^2\beta^2 gE}{4}\right]$$
$$-\frac{1}{2\pi^2}\sum_{n=1}^{\infty}(-1)^n P.V.\int_0^{\infty}\frac{ds}{s^2}\,gE\;cot\;gEs\;sin\frac{n^2\beta^2 gE}{4}\;e^{-sm^2-\frac{n^2\beta^2}{4s}} \qquad (3.28).$$

For $z = 0$ (See eq.(3.28)) we find a dramatic increment in the pair production rate at high temperature over that of vacuum, though at some intermediate temperatures the rate decreases.

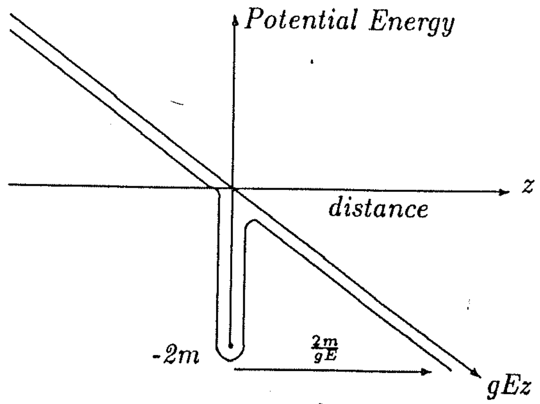

Fig-II

Potential well for quarks submitted to an external chromo-electric field gE .

It is possible to understand these phenomena, in terms of a simple potential well model, where the pair creation is viewed as tunnelling of pairs from vacuum through an energy barrier in the configuration space with maximum height $2m$ and width is $\frac{2m}{gE}$.In the presence of finite temperature the same picture still holds good. Due to thermal effects, the particles are lifted up from the bottom of the well, and as a consequence the effective barrier width, as seen by them becomes less, hence making it easier for them to tunnel out of the vacuum. This might be an explanation of the temperature corrected Schwinger expression i.e. the first term in Eq.(3.28).Moreover,

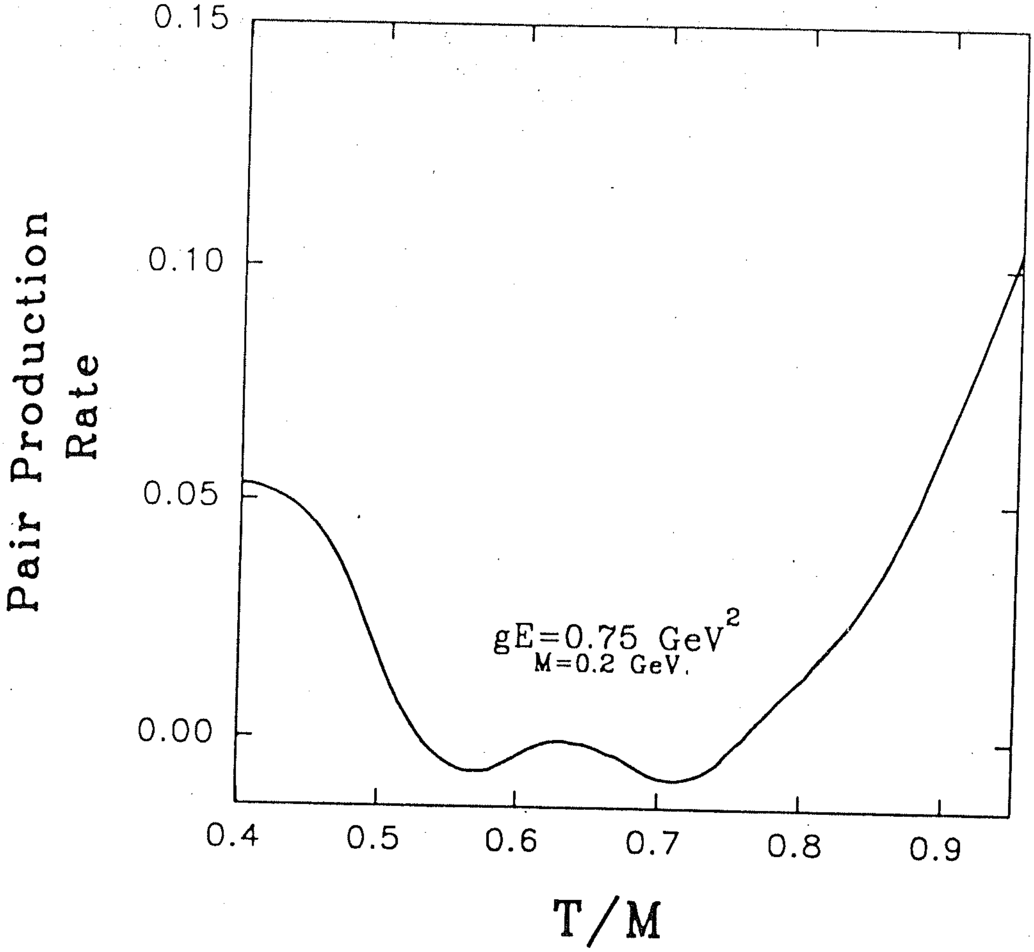

Figure 3.1: Pair Production Rate with Temperature/mass.

other than the temperature induced tunneling, at high temperature, the thermal excitations also push the particles over the barrier resulting in a significant increase in the pair production rate at high temperature. At low and intermediate temperatures, for some value of the external chromo-electric field, we find a decrease in pair production rate with respect to Schwinger's result. This effect probably reflects an increase in the width of the barrier due to thermal excitations.

From equation (3.26), we find that, at extremely high temperature the pair production rate goes as

$$Im\,[L_{eff}] \simeq \frac{gET^2}{6} \tag{3.29}$$

At $z \neq 0$ (eqn (3.27)), because of the presence of the cosine hyperbolic term, the pair production rate increases with increase in z [Figure].The reason behind this is that, in the presence of an electric field charged particles do not stay at rest. They move towards the source and try to cancel the electric field in the region in between. Thus, as one moves towards the source of the chromo-electric field, the field intensity increases, and hence one would expect a reasonable increment in the pair production rate as one approaches the color charged nuclear plates. In the context of heavy ion collision this would mean that if the flux tube model is correct then production rate of $q\bar{q}$ will be more as one moves away from the reaction plane. Considering the complexity of the underlying process and the successive phases that the plasma undergoes, it might be a difficult task at this stage to give a quantitative description about the signature of this phase but we believe, early signals like dilepton or direct photon might be an ideal candidate that might carry the information of this phase. From a simple minded approach to the problem, if one assumes the fluid to undergo Bjorken hydrodynamic expansion, in the following stage of its evolution, the effect of this phase may show up in the observed angular multiplicity distribution of the particles.

In summary, we have computed the pair production rate at finite temperature in the imaginary time formalism starting from the thermal partition function for a system of fermions with SU(2) color symmetry in the presence of a non-abelian external chromoelectric field.

Our results show the presence of two distinct pieces i.e. the vacuum contribution and the thermal correction to it. In the case of a $U(1)$ gauge symmetry it reduces to that of Loewe and Rojas but with a sign difference between the thermal and the vacuum contribution. It also clearly shows the spatial dependence of the temperature corrected part of the effective Lagrangian.

We have also tried to give a physical picture of the whole process in terms of particles in a simple potential well. We see that the pair production rate increases away from the plane at $z = 0$ and we expect that this effect might show up in future relativistic heavy ion collision experiments.

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Chapter 4

Evolution In Phase Space

4.1 Introduction

In the previous two chapters color flux tube model was studied to understand the process of plasma formation in A-A collision. In this chapter we examine how the plasma will evolve before it reaches color and thermal equilibrium.

The study of this phase is crucial because, it will give information about the dynamic processes that are important for reaching equilibrium and also the time it would take to reach the equilibrium. Furthermore the signals for detecting QGP might get modified depending on the pre-equilibrium evolution of the system. Since we are interested in the pre-equilibrium phase of the plasma , we will study the real time phase space evolution of the plasma through kinetic[1] theory followed by hydrodynamic equations.

As the number of degrees of freedom for gluons are more than the same for quarks and moreover since they are massless, the production rate of the gluons will be more than that for the quarks. This has already been evaluated in reference[2]. One can also get them, (approximately) apart from the numerical constants coming from color and spin degrees of freedom, from the rate expressions obtained by us for quarks by setting the, mass for the quarks equal to zero. Due to the color factors the g-g cross section is larger than gq and qq cross sections and as a result of this the gluons will equilibrate[3] faster than the quarks. So in this chapter we will concentrate on the the pre-equilibrium evolution of the gluons.

In the pre-equilibrium phase, right after the nuclear collision, the quarks and gluons will interact by means of binary (perhaps 3 body, 4 body) collisions and also through collective interactions to bring the system to a state of thermal equilibrium. The pre-equilibrium description of plasma has been studied by many authors[4] using kinetic description, by putting a collision term on the right hand side of Boltzman-Vlasov equation for the plasma.

For quarks a binary collision term is justified to some extent if one assumes the number density of quarks to be very small. For gluons this kind of assumption is not justified because of the presence of 3 body, 4 body interaction term in the Lagrangian. Therefore instead of using the Boltzman-Vlasov equation we will use the Vlasov kinetic equation, with the underlying assumption that collective effects arising out of mean fields are more important than the collision terms. This would be the case when a typical time scale for collective behavior $(1/\omega_p)$ is much shorter than the collision time $\frac{1}{\nu_c}$ i.e. $\omega_p >> \nu_c$. Further more there must be enough number of particles in a Debye sphere, i.e n ${\lambda_d}^3 \gg 1$, so that the collective effects dominate. We follow the phase space evolution of the gluonic plasma, starting from the gauge covariant operator valued quantum kinetic equations of gluons given by Elze,Gyulassy and Vasak and taking its classical limit. The classical description of the gluonic plasma is obtained as we take the ensemble average of this equation and then set terms proportional to $\hbar$ to zero. This is justified for studying those collective effects where the waves with wave length $\lambda > \frac{\hbar}{mc}$. It is also worth recalling that classical approaches reproduce many of the collective phenomena in quantal systems.

The organisation of this chapter is as follows. In section two we start with the gauge covariant distribution function[1] for gluons described by Elze, Gyulassy and Vasak and discuss how to obtain a classical kinetic description for gluons from there. In the following section we study a simple model to examine whether non-abelian color dynamics can provide a new equilibration mechanism. Finally we conclude by discussing the scope of further improvement of our result.

4.2 Kinetic Equations

In RHIC when the plasma is produced, the particles will have a characteristic momentum distribution. For the purpose of separating collective effects from the non-collective ones we assume that there are two types of gluons present in the system. The ones with very high four momentum (i.e short scale lengths) describe particle like properties , whereas those with low four momentum, i.e those generated by the interaction amongst the high frequency gluons, describe the collective i.e wave like properties. Therefore, as a result of this assumption the low four momentum gluons are described by the Yang Mills field equations with a source term (4-current) on the right hand side , generated by the high momenta gluons.

We are going to describe here the dynamics of these high momentum gluon fields which will interact among themselves to bring the system close to color and thermal equilibrium. Presently we take only the interaction of these high momentum gluons among themselves, which will be described by a Boltzman-Vlasov like equation for the gluons.

To describe the dynamics of these gluons, following Elze, Gyalassy and Vasak (EGV) (Ref. EGV[1], Elze[5]) one starts with the gauge covariant distribution function for the gluons defined as

$$G_{\mu\nu}(x,p) = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\mathrm{ip.y}/\hbar} \left[e^{-1/2\mathrm{y.D}(x)} \; \vec{F}^{\lambda}_{\mu}(x) \right] \left[e^{1/2\mathrm{y.D}(x)} \; \vec{F}_{\lambda\nu}(x) \right]^{\dagger} \quad (4.1)$$

which is an 3×3 matrix for $SU(2)$ case, expressed as a dyadic product of a 3 component vector (color) and its adjoint. In the component notation it can be written as

$$G^{ab}_{\mu\nu}(x,p) = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\mathrm{ip.y}/\hbar} \left[e^{-1/2\mathrm{y.D}(x)} \; \vec{F}^{\lambda}_{\mu}(x) \right]^a \left[e^{1/2\mathrm{y.D}(x)} \; \vec{F}_{\lambda\nu}(x) \right]^b \quad (4.2)$$

Here $y.D = y^0 D_0 + y^1 D_1 + y^2 D_2 + y^3 D_3$ and

$$D_\mu = \partial_\mu - ig\,[\,A_\mu,\,] \quad (4.3)$$

$$F_{\mu\nu} = \frac{[\,D_\mu, D_\nu\,]}{-\mathrm{ig}} \quad (4.4)$$

μ and ν go from 0 to 3 and a and b go from 1 to 3.

Now operating with $p^\mu.D_\mu$ term, on the distribution function one arrives at the kinetic equation of gluons (see ref. 1). Since we are interested in the classical description we set terms of the order of $\hbar$ equal to zero as in ref. (5) and from there arrive at the following expression.

$$p^\mu.D_\mu G_{\mu\nu} + g/2P^\sigma \partial_p^\tau \, [\, \mathcal{F}_{\sigma\tau}, G_{\mu\nu} \,]_+ \; = g\,(\, \mathcal{F}_{\mu\sigma} G^\sigma_\nu - \; G_{\mu\sigma}\mathcal{F}^\sigma_\nu \,) \tag{4.5}$$

Here $[,]_+$ means anti-commutator, and

$$D_\mu = \partial_\mu - ig\,[\,\mathcal{A}_\mu,] \tag{4.6}$$

where

$$\mathcal{A}^{ab}_\mu \equiv -i f_{\mathrm{abc}} A^c_\mu$$
$$\mathcal{F}^{ab}_{\mu\nu} \equiv -i f_{\mathrm{abc}} F^c_{\mu\nu}$$

f_{abc} is the antisymmetric structure constant for $SU(2)$

and g is the coupling constant.

In general, with regard to Lorentz indices, $G_{\mu\nu}$ has a symmetric part[4-5] as well as an antisymmetric part. We neglect the antisymmetric part by taking a spin equlibration ansatz, i.e.

$$G_{\mu\nu}(x,p) = p_\mu p_\nu G(x,p)$$

where $G(x,p)$ is a Lorentz scalar function.

So with this ansatz the r.h.s of equation(4.5) vanishes and, the gluon kinetic equation in color component notation takes the form

$$p^\mu \partial_\mu G^{mn} + g p^\mu . A^c_\mu \,[\, f_{\mathrm{cma}} G^{an} \; - \; G^{ma} f_{\mathrm{can}} \,] + i\frac{g}{2} p^\sigma \partial_p^\tau$$
$$[\; f_{\mathrm{ema}} G^{an} + f_{\mathrm{ean}} G^{ma} \,]\; F^e_{\sigma\tau} = 0 \tag{4.7}$$

All repeated indices are to be summed over.

The assumption of spin equilibration i.e $G_{\mu\nu}(x,p) = p_\mu p_\nu G(x,p)$ leads to the following expression for the gluon current ref(4,5)

$$J_c^{\mu} = ig \int G_{ab} f_{abc} p^{\mu} d^4 p \tag{4.8}$$

To study the collective behavior of the system, we solve the YM field equations with the current (equation(4.8)) on the right hand side. The basic idea here, as explained before, is that because of the self interaction the high momentum gluons generate a low momentum long wavelength mean field which in turn acts as a source term for a mean Yang-Mills field equations. For studying the collective properties one has to solve these equations self consistently,i.e

$$D_{\mu} F^{\mu\nu} = J^{\nu} \tag{4.9}$$

along with the gluon kinetic equations (equation(4.5)).

4.3 A New Mechanism For Equilibration

As mentioned earlier, in this section, we propose to analyse a simple model which exhibits mechanisms for equilibration arising entirely from the non-abelian nature of the color dynamics. In this model we assume that the equilibrium distribution function has the form

$$G_{ab}^{eq} = \frac{n_{ab}}{(e^{p_o \beta} - 1)} \tag{4.10}$$

Here n_{ab}'s are the elements of a matrix in color space p_o is the zeroth component of the four momentum and β is the temperature of the system and the important point is that, the off-diagonal elements of the distribution function are nonzero. In equilibrium, we have chosen the distribution to have a simple Bose-Einstein form, so as to avoid momentum space contribution to collective effects. The important point, that we would like to bring home, is the hitherto unconsidered role of the color degrees of freedom as a source of free energy. Further we take, the classical fields $F_{\mu\nu}$ and A in the kinetic equations (4.8) - (4.9) to be diagonal in color space(i.e abelian dominance[1,5] approximation) and the zeroth component of the vector field to be finite and other components are zero.

We then carry out a stability analysis of the resulting system of equations about the equilibrium distribution function G_{eq}^{ab}. On linearising the equations about the aforementioned equilibrium distribution, one arrives at

$$k_{\mu} p^{\mu} \delta G_{mk} = \frac{g}{2T} p_{\mu} F_e^{\mu o} \left[f_{ema} n_{ak} - f_{bek} n_{mb} \right] cosh^2 \left(\frac{p_o}{2T} \right) \tag{4.11}$$

Using equations (4.8) and (4.11) we have solved for the current produced by the fluctuations and it is

$$J_o^{\,n} = \frac{(gT)^2}{6}\left[\frac{\omega}{|k|} ln\left|\frac{\omega+k}{\omega-k}\right| - 2\right] A^n{}_o(k) \tag{4.12}$$

On using relation (4.9) we next get

$$k^2 A_o^{\,a}(k) = C(\omega,k)\left[2n_{kk}A_o^{\,a} - A_o^{\,b}(n_{ab}+n_{ba})\right] \tag{4.13}$$

Here the repeated indices are summed up and

$$C(\omega,k) = \frac{(gT)^2}{6}\left[\frac{\omega}{|k|} ln\left|\frac{\omega+k}{\omega-k}\right| - 2\right] \tag{4.14}$$

From equation (4.13) the matrix dispersion relation comes out to be

$$k^2 - C(\omega,k)\begin{bmatrix} 2(n_{22}+n_{33}) & -(n_{12}+n_{21}) & -(n_{13}+n_{31}) \\ -(n_{12}+n_{21}) & 2(n_{33}+n_{11}) & -(n_{32}+n_{23}) \\ -(n_{13}+n_{31}) & -(n_{32}+n_{23}) & 2(n_{22}+n_{11}) \end{bmatrix} = 0 \tag{4.15}$$

If we set $n_{11} = n_{22} = n_{33} = \frac{n}{2}$ and $n_{12} = n_{21} = n_{23} = n_{32} = n_{31} = n_{13} = s$ then ,in the long wavelength limit one gets the following dispersion relation (for the long wavelength gluons),

$$\omega^2 = \frac{3k^2(n-s)}{1+s-n} \tag{4.16}$$

From equation (4.16) we see that if $s > n$ there will be an instability in the system. Clearly the instability is related to the color degrees of freedom and would then drive the system towards a distribution which is diagonal in color space. This mechanism may provide us with some insight about the manner in which an arbitrary distribution function in color space becomes color diagonal and attains color equilibration.

4.4 Conclusion

In this chapter we have looked for the plasma oscillations in QGP through the semiclassical kinetic equations for gluons, derived by Elze, Gyulassy and Vasak. Though the dispersion relation has been derived under the approximations that the mean fields are basically abelian in nature and of them only A_o is finite, but these simplifying assumptions still carry some nontrivial nonabelian dynamical signatures in it. In particular the existence of the off-diagonal (in color space) components of the distribution function, is the signature of gluon gluon interactions , a purely non-abelian effect and is seen to be responsible for damping or instability. Incidentally on performing the same analysis with an equilibrium distribution function i.e diagonal in color space no such signature of instability or damping is found[5].

Usually, the damping, can originate from three different kinds of sources; for instance, it can be collisional relaxation damping, decay of plasmons into particle antiparticle pairs or gluon gluon pairs. Production of quark antiquark pairs from vacuum is similar to electron positron pair production through plasmon decay as encountered in high T QED plasma. On the the other hand gluon going to two gluons is a typical non-abelian effect, typical of QCD plasma. Since the physical situation we are considering here does not have any collisional relaxation process in it, and neither have we considered the presence of quarks and antiquarks here, so the existence of instability or damping corresponds to the last process. This damping signifies passage of energy from wave mode to particle mode. Conversely an instability would signify the passage of energy from particle mode to wave mode.

In our view, the non-abelian interactions amongst the gluons, try to take the system, with strong initial color fluctuations, to a stable equilibrium. To get a correct picture, of the physics of this process, one ought to solve these coupled partial non linear set of differential equations. Instead we will try to explore some special solutions numerically and some more of their collective properties under different approximation schemes.

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Chapter 5

Hydrodynamic Evolution Of The Plasma

In the last chapter we have tried to study the preequilibrium evolution of the plasma in phase space through kinetic description.But the kinetic equations are not always very convenient to the study the collective processes, because of their complicated structure. On the other hand, under appropriate conditions it is possible to study non-equilibrium plasma, with a little lesser difficulty , with the help of hydrodynamic equations. There have been efforts[1] to obtain such a description of pre-equilibrium phase from the Boltzman-Vlasov kinetic equations. It is worth noting here that though the usual hydrodynamic description is applicable when the mean free path is much less than the scale length of the system and as a consequence the system reaches local thermodynamic equilibrium. On the other hand, if one assumes that the plasma is cold[2] i.e $V_{ph} >> V_{thermal}$ (such that one can define a fluid element over which the particles can have a coherent velocity) and particle density inside the Debye sphere is $>> 1$, then one can factorize in the cold collisionless limit the distribution function as $f(x,p,t) = g(x,t)\delta(p-\overline{p})$ and obtain a closed hydrodynamic description even in the collisionless limit.

In view of this, we proceed to formulate a classical hydrodynamic description for gluons in the cold collisionless limit, starting from the gauge covariant kinetic equation given by Elze, Gyulassy and Vasak (EGV[3]) neglecting all the terms of the order of $\hbar$ and higher. This may be justified because the collective effects we consider have length scales much greater than the compton wavelength, so that quantal corrections may not be very important.

We take the moments of the EGV equation, as described in the last chapter to get a most general hydrodynamic description for the gluons. This involves a set of 48 coupled nonlinear partial differential equations and would be difficult to solve even numerically.

The organisation of this chapter is as follows. In section two we start from the gluon kinetic equation of chapter 4 and take the momentum moments of this equation to generate a set of hydrodynamic equations for gluons under certain approximations. In section three we start from the classical kinetic equation for quarks, take the momentum moment of those equations to generate a set of chromo hydrodynamical equations to describe the space-time evolution of pre-equilibrium quark matter. In section four we show the formal similarity between the gluon hydrodynamic equations and the quark hydrodynamic equations.

In section five we study the collective oscillations of the plasma. We show the existence of certain conservation laws following from the hydrodynamic equations and try to solve these equations numerically obeying these conservation laws. The numerical solutions x show the existence of chaotic oscillations.

5.1 Towards Hydrodynamics Of Quarks And Gluons

In this section starting from the gluon kinetic equations we derive the gluon hydrodynamic equations. To derive the gluon hydrodynamic equations one essentially starts from the gluon kinetic equations of chapter 4. The kinetic equation can be written in terms of the combinations of different components of the distribution function that get coupled to each other (as a consequence of the nonabelian nature of the fields) to describe the evolution of the plasma.

From equation (4.7) we define diagonal components,

$$\begin{aligned} G^{11} &= 2G^{1} \\ G^{22} &= 2G^{2} \\ G^{33} &= 2G^{3} \end{aligned} \tag{5.1}$$

the symmetric combinations,

$$\begin{aligned} 2S^1 &= G^{23} + G^{32} \\ 2S^2 &= G^{31} + G^{13} \\ 2S^1 &= G^{12} + G^{21} \end{aligned} \tag{5.2}$$

and lastly the antisymmetric combinations,

$$\begin{aligned} 2iQ^1 &= G^{23} - G^{32} \\ 2iQ^2 &= G^{31} - G^{13} \\ 2iQ^3 &= G^{12} - G^{21} \end{aligned} \tag{5.3}$$

Since gluons belong to the adjoint representation of the appropriate unitary group, the distribution function for the gluons, for the SU(2) case can have three distinct irreducible representations i.e scalar, vector and second rank symmetric tensor in color space. It is worth recalling that for the quarks the distribution function can have only the scalar and vector representations. In the equations above S corresponds to the symmetric rank two tensor , Q corresponds to the vector representation and $tr\,(G)$ corresponds to the scalar representation in color space. Hence the hydrodynamic equations for quarks and gluons in general would not be the same. It is difficult to make much progress with the most general distribution, because of lack of knowledge about the 'd' coefficients arising out of anticommutation relations of the generators in the adjoint representations of the unitary group. In order to make some progress, in the next step, we will make an assumption, that all the symmetric combinations of the distribution function are zero. With this assumption one can show that the equations (4.7) reduce to

$$\begin{aligned} p_\mu \partial^\mu Q^1 - g p^\mu \left[A_\mu^2 Q_3 - A_\mu^3 Q_2\right] + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^1 \left(G^2 + G^3\right)\right] &= 0 \\ p_\mu \partial^\mu Q^2 - g p^\mu \left[A_\mu^3 Q_1 - A_\mu^1 Q_3\right] + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^2 \left(G^1 + G^3\right)\right] &= 0 \\ p_\mu \partial^\mu Q^3 - g p^\mu \left[A_\mu^1 Q_2 - A_\mu^2 Q_1\right] + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^3 \left(G^1 + G^2\right)\right] &= 0 \end{aligned} \tag{5.4}$$

and

$$\begin{aligned} p_\mu \partial^\mu G^1 + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^3 Q^3 + F_{\mu\nu}{}^2 Q^2\right] &= 0 \\ p_\mu \partial^\mu G^2 + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^3 Q^3 + F_{\mu\nu}{}^1 Q^1\right] &= 0 \\ p_\mu \partial^\mu G^3 + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^1 Q^1 + F_{\mu\nu}{}^2 Q^2\right] &= 0 \end{aligned} \tag{5.5}$$

The space time and color evolution of the macroscopic observables will be described by quantities obtained after taking moments of the one particle distribution function. For instance, the 4-color current flow in space will be given by

$$\int p^{\mu} Q_a(x,p)\, d^4p = Q_a{}^{\mu}(x) \tag{5.6}$$

and the general four momentum flow is given by

$$\int p^{\mu} \sum_{a=1}^{3} G_a(x,p)\, d^4p = G^{\mu}(x) \tag{5.7}$$

This flow tensors can further be decomposed (following Stewart[4]) as

$$Q_a{}^{\mu} = n_a U^{\mu} E \qquad \text{and} \qquad G^{\mu} = n U^{\mu} E \tag{5.8}$$

Here n_a is the color triplet number density of gluons, E is their energy and $U^{\mu}(x)$ is the four velocity, with

$$EU^{\mu} = \frac{\int p^{\mu} Q_a(x,p)\, d^4p}{\int Q_a(x,p)\, d^4P} \qquad \text{and} \qquad U^{\mu}U_{\mu} = 0 \tag{5.9}$$

In fact following equation (5.8) one can decompose any higher order tensor. For example a second rank tensor can be written as,

$$Q_a{}^{\mu\nu} = n_a U^{\mu} U^{\nu} E^2 \tag{5.10}$$

To derive the hydrodynamic equation from the kinetic equations (5.4) and (5.5) we take the zeroth (and the first) momentum moments of those equations. In the next stage we assume $G_1 = G_2 = G_3 = G/3$ and add all the components of equations (5.5) to get

$$\begin{aligned} &\partial_{\mu} \quad G^{\mu} = 0 \\ &\partial_{\mu} \quad Q_a{}^{\mu} - g\epsilon_{abc} A_{\mu}{}^{b} Q^{\mu}{}_{c} = 0 \end{aligned} \tag{5.11}$$

In the following step, as was shown in equation (5.8) we decompose the 4-vectors in equation (5.11) to arrive at

$$\begin{aligned} &\partial_{\mu} \quad [n_a U^{\mu}] - g\epsilon_{abc} A_{\mu}{}^{b} n_c U^{\mu} = 0 \\ &\partial_{\mu} \quad [nU^{\mu}] = 0 \end{aligned} \tag{5.12}$$

From equation (5.12), one can show by defining a quantity called color charge, as $I_a = \frac{n_a}{n}$, that

$$\begin{aligned} &\partial_{\mu} \qquad [nU^{\mu}] = 0 \\ &\textit{and} \\ &U^{\mu}\partial_{\mu} \quad [I_a] - g\epsilon_{abc} A_{\mu}{}^{b} I_c U^{\mu} = 0 \end{aligned} \tag{5.13}$$

Similarly from the first momentum moment equation one can get, the force equation or conservation of energy momentum relation

$$U^\mu \partial_\mu U^\nu = \frac{g}{E} F_a{}^{\mu\nu} U_\mu I^a \tag{5.14}$$

For studying the collective behaviour of the system one needs to solve the equation

$$\begin{aligned} \partial_\mu \left[n U^\mu \right] = 0 \\ U^\mu \partial_\mu U^\nu = \tfrac{g}{E} F_a{}^{\mu\nu} U_\mu I^a \\ \partial_\mu \left[n_a U^\mu \right] - g \epsilon_{abc} A_\mu{}^b n_c U^\mu = 0 \end{aligned} \tag{5.15}$$

and

$$D_\mu F_a{}^{\mu\nu} = J_a{}^\mu = g n I_a U^\mu$$

self consistently. The first two equations above are the usual continuity and force balance equations respectively. The third equation, characteristic of non-abelian dynamics, is the color evolution equation. In the next section we will show that the quark hydrodynamic equations also take the same form but without any such approximation.

5.2 Towards Quark Hydrodynamics

In this section we try to arrive at the hydrodynamic equations for quarks as obtained by Kajantie and Montonen[5], starting from the kinetic equations of quarks as gotten by EGV. To derive the equations for classical quark matter, we proceed as follows. We start from the gauge covariant kinetic equations of Elze, Gyulassy and Vasak, setting the terms of the order of $\hbar = 0$.

$$p^\mu D_\mu W(x,p) + g/2 p^\mu \partial_p{}^\nu \left[F_{\mu\nu}, W(x,p) \right]_+ = 0 \tag{5.16}$$

Here $[,]_+$ means anticommutator. The distribution function W(x,p) apart from its spin structure, is a hermitian matrix in color space. It can be written in terms of a color singlet and triplet components for SU(2) as

$$W(x,p) = \frac{1}{2}\langle G \rangle 1 + \frac{1}{2} \sum_{a=1}^{3} \lambda_a \langle G^a \rangle \tag{5.17}$$

Here $\langle G \rangle = Tr W(x,p)$ and $\langle G_a \rangle = Tr \left[\lambda_a W(x,p) \right]$. Using equation (5.17) one can write equation(5.16) in terms of a set of coupled partial differential equations. The coupling is between the singlet and triplet distribution function for quarks. They are as follows,

$$\begin{aligned} p_\mu \partial^\mu \langle G \rangle + g p_\mu \partial_p{}^\nu \left[F_{\mu\nu}{}^a \langle G^a \rangle \right] &= 0 \\ p_\mu \partial^\mu \langle G^a \rangle + g \epsilon_{abc} p^\mu A^b{}_\mu \langle G^c \rangle + g p_\mu \partial_p{}^\nu \left[2 F_{\mu\nu}{}^a \langle G^a \rangle \right] &= 0 \end{aligned} \tag{5.18}$$

As it has been shown in the earlier section, one can take the momentum moments of these equations to generate the hydrodynamic equations. Before we go for generating the hydrodynamic equations it is worth noting that since the quarks are massive particles the 4-velocities for quarks obey

$$U^\mu U_\mu = 1 \qquad \text{and} \qquad mU^\mu = \frac{\int p^\mu G_a(x,p)\, d^4p}{\int G_a(x,p)\, d^4p} = \frac{\int p^\mu G(x,p)\, d^4p}{\int G(x,p)\, d^4p} \tag{5.19}$$

On taking the zeroth moment of equation (5.18) we arrive at

$$\begin{aligned} &\partial_\mu [G^\mu] = 0 \\ and \quad & \\ &\partial_\mu [G_a{}^\mu] + g\epsilon_{abc} A_\mu{}^b G_c{}^\mu = 0 \end{aligned} \tag{5.20}$$

Decomposing 4-vectors $G^\mu = mnU^\mu$ and $G_a{}^\mu = mn_a U^\mu$ we get,

$$\begin{aligned} &\partial_\mu [nU^\mu] = 0 \\ and \quad & \\ &\partial_\mu [U^\mu n_a] - g\epsilon_{abc} A_\mu{}^b n_c U^\mu = 0 \end{aligned} \tag{5.21}$$

Defining $I^a = \frac{n_a}{n}$ and using the two equations one can obtain the color evolution equation, namely

$$U^\mu \partial_\mu [I_a] - g\epsilon_{abc} A_\mu{}^b I_c U^\mu = 0 \tag{5.22}$$

Similarly on taking the first momentum moment of equation (5.18) and using similar decomposition as before we arrive at

$$U^\mu \partial_\mu U^\nu = \frac{g}{m} F_a{}^{\mu\nu} U_\mu I^a \tag{5.23}$$

Here repeated indices are summed up. If one multiplies equation (5.23) by U^ν one can show that the condition $U^\mu U_\mu = 1$ is satisfied. One can rewrite equation (5.23) as

$$U^\mu \partial_\mu U^\nu = \frac{g}{m} F_a{}^{\mu\nu} J_\mu{}^a \quad \text{with} \quad J^\mu{}_a = nU_\mu I^a \tag{5.24}$$

One can derive an identical set of hydrodynamic equations as Kajantie and Montonen, provided one considers a multicomponent distribution function, $G_A(x,p)$ where the label A stands for different species of quarks and one decomposes the color singlet and color triplet 4-vectors and Lorentz tensors as

$$\begin{aligned} G_A{}^\mu &= m_A n_A U_A{}^\mu \\ G_{aA}{}^\mu &= m_A n_{aA} U_A{}^\mu \\ G_a{}^{\mu\nu} &= m_A n_{aA} U_A{}^\mu U_A{}^\nu \\ G_{aA}{}^{\mu\nu} &= m_A n_{aA} U_A{}^\mu U_A{}^\nu \end{aligned} \tag{5.25}$$

The repeated indices A are not summed. Starting from gauge covariant kinetic equations of EGV, we have derived at the Kajantie Montonen hydrodynamic equations for both quarks and gluons. In the next section we will study their collective behaviour non perturbatively, by numerically solving the equations.

5.3 Study of Collective Oscillation of the Plasma

In the earlier section we have obtained the quark hydrodynamic equations. These equations for different species of quarks can be written as,

$$\begin{aligned}
\frac{\partial n_A}{\partial t} + \nabla\left(n_a V_A\right) &= 0 \\
\left(\frac{\partial}{\partial t} + V_A \nabla\right) V_A &= \frac{g}{m} I_A{}^a \left[E_A + V_A \times B_A\right] \\
\left[\frac{\partial}{\partial t} + V_A \nabla\right] I_A{}^a &= g\epsilon_{abc}\left[A_b{}^o - V_A A_b\right] I_A c
\end{aligned} \tag{5.26}$$

To study the collective oscillations of the system one has to solve these equations along with the Yang-Mills equations self-consistently. In order to extract the essential non-abelian physics, we have simplified the equations by removing most of the non-essential complications by assuming that in the x and y directions the plasma is homogeneous so that the fluid variables have zero gradient in x and y direction, i.e $\partial_x = \partial_y = 0$. Next we will look for special solutions (ref Coleman[6]) where the fluid as well as the field variables are functions of a single variable denoted by

$$\tau = t\beta + z \tag{5.27}$$

In this stationary frame ansatz, β is the frame velocity. We also assume that of the two species of quarks, one is much heavier than the other, such that the second species is relatively at rest compared to the first and it acts as a (neutralising) background. So with these assumptions one can solve the fluid equations analytically to get

$$n_A = \frac{n_o \beta}{V_A + \beta} \tag{5.28}$$

Here $n_{o1} = n_{o2} = n_o$ is the equilibrium density. Note that for $V_2 = 0$ we get $n_2 = n_o$. For the velocities we have assumed V_{1x} and V_{1z} to be nonzero, so that on solving the respective equations we get V_{1x} and V_{1z}

$$V_{1x} = V_x = \frac{(g I_a A^a{}_x)}{m} \tag{5.29}$$

$$V_{1z} = V_z = \frac{1}{2mn_0\beta}\left[(\beta^2-1)\left(\dot{A_a}^{x^2} - \dot{A}(0)_a^{x2}\right) + \beta^2\left(\dot{A_a}^{z^2} - A(\dot{0})_a^{z^2}\right)\right]$$
$$-g\left(\beta^2-1\right)\epsilon_{abc}\left(A^x{}_a A^z{}_b A^x{}_c\right) - \frac{g^2}{2}\left[\left(A_c^x A_b^z\right)^2 - \left(A_b^z A_c^z\right)\left(A_c^x A_b^x\right)\right]$$
$$-gI_{a0}A^z{}_a n_0\beta \tag{5.30}$$

$$I_a = -\frac{\epsilon_{abc}}{n_o\beta}\left[\left(\beta^2-1\right)\left(A_b^x\right)\left(\dot{A}^x{}_c\right) + \beta^2 A_b^z \dot{A}^z{}_c\right] - \frac{gA^x{}_b}{n_0\beta}\left(A_a^z A_b^x - A_a^x A_b^z\right) + I_{a0} \tag{5.31}$$

Thus we have expressed n_A, V_A and I_A in terms of the color potentials and their derivatives. The Yang Mills equations are

$$\left(\beta^2-1\right)\ddot{A_a}^x + g\epsilon_{abc}\left[2A^z{}_b\dot{A}^x{}_c + \dot{A}^x{}_c\dot{A}^z{}_b\right] + g^2A^z{}_b\left(A^x{}_aA^z{}_b - A^z{}_aA^x{}_b\right) = -\left(\frac{gn_o\beta I_a V_x}{V_z+\beta}\right) \tag{5.32}$$

$$\left(\beta^2\right)\ddot{A_a}^x - g\epsilon_{abc}\left[A^x{}_b\dot{A}^x{}_c\right] - g^2A^x{}_b\left(A^x{}_aA^z{}_b - A^z{}_aA^x{}_b\right) = -\left(\frac{gn_o\beta I_a V_z}{V_z+\beta}\right) \tag{5.33}$$

From the geometry that we have chosen, one can see that there is no force in the x direction. However in the x direction there exists a canonical momentum which is obviously conserved from equation (5.29). From these sets of equations (i.e (5.29) to (5.33)) one gets two conserved quantities

$$I_1{}^2 + I_2{}^2 + I_3{}^2 = constant \tag{5.34}$$

$$\begin{aligned}&\left(\beta^2-1\right)\left[\dot{A}_a^{x^2} - \dot{A}_a^{x^2}(0)\right]\\&+\beta^2\left[\dot{A}_a^{z^2} - \dot{A}_a^{z^2}(0)\right]\\&+g^2A^z{}_bA^x{}_c\left(A^z{}_bA^x{}_c - A^z{}_cA^x{}_b\right)\\&+n_om\left[V_x{}^2 + V_z{}^2\right] = \varepsilon\end{aligned} \tag{5.35}$$

where ε can be termed as energy. The first equation is the color conservation equation and the second equation is the energy conservation equation. These quantities depend on the initial value of the variables and the given parameters and they remain constant throughout the evolution of the system. We next scale the variables to bring the equations to dimensionless form. We choose

$$\begin{aligned}A_a{}^x &= a_oA_a{}^x\\ I_a{}^x &= I_oI_a{}^x\end{aligned} \tag{5.36}$$

Where, a_o and I_o have dimension of length and charge. Next we redefine our independent variable as

$$\zeta = \omega_p \tau \tag{5.37}$$

where τ is like time and ω_p is abelian plasma frequency parameter.

$$\omega_p{}^2 = \frac{\left(g^2 n_o{}^2 I_o{}^2\right)}{m} \tag{5.38}$$

We also introduce two more dimensionless parameter

$$\begin{aligned} t &= \left(\tfrac{g n_o I_o}{m}\right) \\ r &= \left(\tfrac{g a_o}{\omega_p}\right) \end{aligned} \tag{5.39}$$

One can check that $r * t = \epsilon$ is the non abelian parameter defined in (ref:7). The scaled equations then take the form

$$V_x = \left(I_a A^a{}_x\right) \tag{5.40}$$

$$\begin{aligned} V_z = \quad & \tfrac{t}{2\beta}\left[(\beta^2 - 1)\left(\dot{A_a}^{x^2} - \dot{A}(0)_a{}^{x2}\right) + \beta^2\left(\dot{A_a}^{z^2} - A(\dot{0})_a{}^{z^2}\right)\right] \\ & -\tfrac{rt}{\beta}\left(\beta^2 - 1\right)\epsilon_{abc}\left(A^x{}_a A^z{}_b A^x{}_c\right) - \tfrac{r^2 t}{2\beta}\left[\left(A^x_c A^z_b\right)^2 - \left(A^z_b A^z_c\right)\left(A^x_c A^x_b\right)\right] \\ & -A^z{}_a \end{aligned} \tag{5.41}$$

$$I_a = -\frac{rt\epsilon_{abc}}{\beta}\left[\left(\beta^2 - 1\right)\left(A^x_b\right)\left(\dot{A}^x{}_c\right) + \beta^2 A^z_b \dot{A}^z{}_c\right] - \frac{g r^2 t A^x{}_b}{\beta}\left(A^z_a A^x_b - A^x_a A^z_b\right) + 1 \tag{5.42}$$

The Yang Mills equations are

$$\left(\beta^2 - 1\right)\ddot{A_a}^x + r\epsilon_{abc}\left[2A^z{}_b\dot{A}^x{}_c + \dot{A}^x{}_c\dot{A}^z{}_b\right] + r^2 A^z{}_b\left(A^x{}_a A^z{}_b - A^z{}_a A^x{}_b\right) = -\left(\frac{\beta I_a V_x}{tV_z + \beta}\right) \tag{5.43}$$

$$\left(\beta^2\right)\ddot{A_a}^x - r\epsilon_{abc}\left[A^x{}_b\dot{A}^x{}_c\right] - r^2 A^x{}_b\left(A^x{}_a A^z{}_b - A^z{}_a A^x{}_b\right) = -\left(\frac{\beta I_a V_z}{tV_z + \beta}\right) \tag{5.44}$$

We will now solve these equation numerically with the help of 4th order Runge Kutta iteration scheme, as the analytical solution is beyond our reach. Before describing the numerical results we would like to show that in the absence of any nonlinearity parameter the equations for the potentials take the uncoupled form

$$\begin{aligned} (\beta^2 - 1)\, \ddot{A^x}_a + \textstyle\sum_{a=1}^{3} A_a{}^x &= 0 \\ (\beta^2)\, \ddot{A^z}_a + \textstyle\sum_{a=1}^{3} A_a{}^x &= 0 \end{aligned} \tag{5.45}$$

which indicates that in absence of all the nonlinearity the modes execute a simple harmonic oscillation with frequency $\frac{\sqrt{3}}{\beta}$ when β is large.

Next we choose the initial conditions as in reference (7), and Fig(5.1) shows the same earphone like oscillation obtained in ref:7 fig(2). Here the $A^x{}_a$ and $\dot{A}^x{}_a$ are all zero throughout the course of integration.

In our plots we show the momentum profiles. To see the effect of a small transverse perturbation on longitudinal oscillation we keep all $A^x = 0$, and $\dot{A}^x \simeq 10^{-5}$. One can see from the figures(5.2) and (5.3) that till t little beyond $600\omega_p$ the velocity profiles execute the same mode, when there is a catastrophic jump in the velocity profile in x direction and correspondingly the coherent oscillation in z direction breaks up into a chaotic one.

One interesting thing has been observed in all the oscillation that they try to execute a coherent mode for couple of periods, even if the profile is globally chaotic. We have carried out an extensive numerical analysis of the above hydrodynamic equations[7], namely taking different initial conditions for both the field variables and their first derivatives, varying the parameters of the equations such as non-abelian nonlinearity parameter, plasma nonlinearity parameter and the frame velocity β. The central observation of all the numerical experiment is that, except for a very few points in the parameter space, for most of the other points the system tends to go to a state of chaotic oscillation. On carrying out the FFT (fast fourier transform) analysis of the numerical solutions we have seen that the most dominant frequency for both V_x and V_z components are the same, implying energy equilibration.

5.4 Conclusion

The fundamental conclusion we reach, in addition to the conclusion of Bhatt et. al, is that, other than the non linearity parameters, even the presence of a small

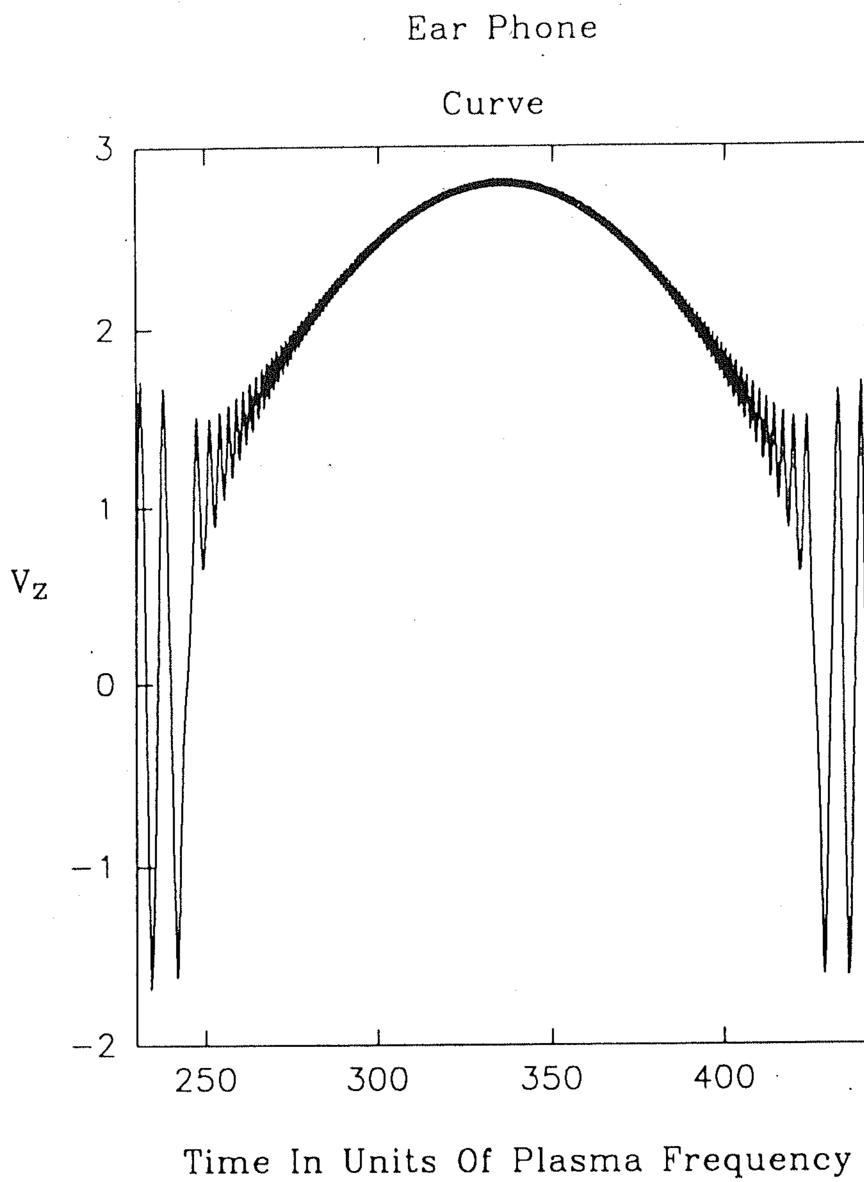

Figure 5.1: The Profile for V_z Oscillation (with out the transverse field).

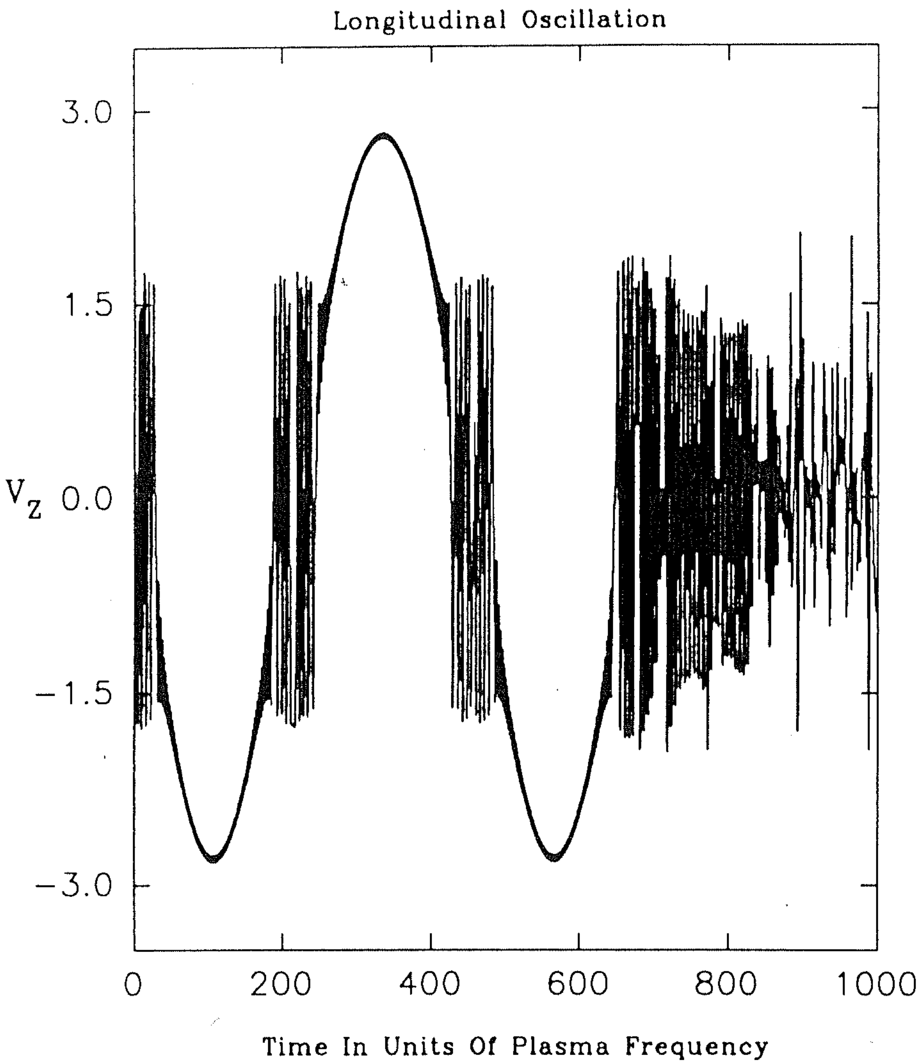

Figure 5.2: The Profile for V_z Oscillation (with the transverse field).

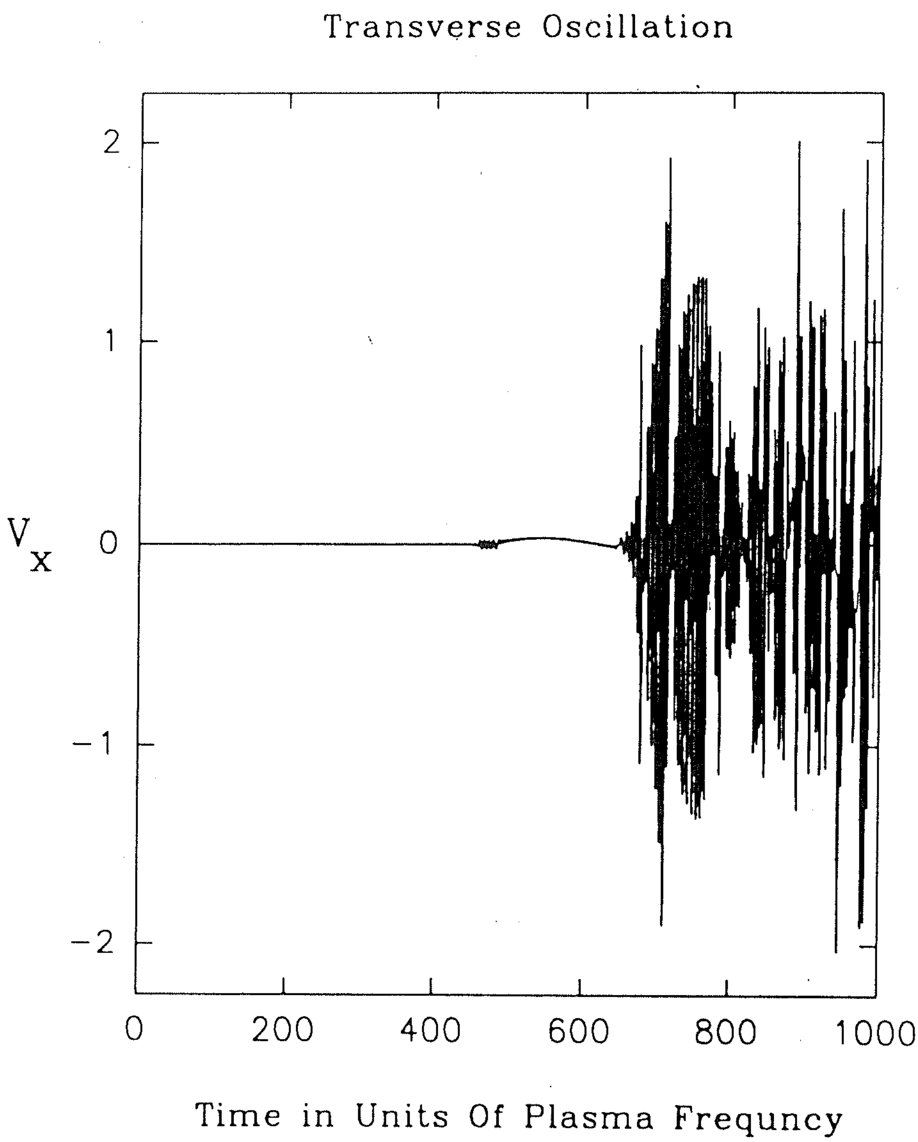

Figure 5.3: The Profile for V_x Oscillation

transverse field can also produce chaos in the otherwise regular longitudinal oscillation. Even though these transverse fields are orders of magnitude less,to begin with, but given sufficient enough time they become comparable to that of longitudinal oscillations. This onset of chaos is related to energy being equilibrated between the two components A_x and A_z . From the autocorrelation function one can find out the time taken by the system to reach a chaotic state of oscillation or the state of energy equilibrium. Secondly there exists some kind of a memory in the system that drives it to behave in the same way after a regular interval of time. Thirdly we have seen that the nonabelian nonlinearity is capable of setting the system into chaos. The factor on which the growth rate of oscillation in the x direction of the velocity depends, seems to be the quantity β, i.e the frame velocity. Totally chaotic regime shows that the energy transfer takes place around the most dominant frequency of oscillation and V_z component drives the V_x oscillation. Though We have not exhausted the space of all possible parameters and initial values for these system of equations, but we think we have been successful in showing a class of solutions where all the specialities as mentioned above, are all present.

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Chapter 6

Summary

In this chapter we will summarise our observation and achievements reported in the earlier chapters of the thesis. We started out with an objective to investigate the process of production and evolution of QGP, with a particular emphasis on the underlying 'nonperturbative' non-abelian dynamics. Studying non-abelian dynamics non perturbatively is by itself a tremendous task.

We started our investigation, 'the production of QGP' in the color flux tube model. It was observed that the chromo electric field inside the flux tube was taken to be constant and essentially abelian in all the previous investigations. Moreover most of the studies were done with an electric field existing in one particular direction in color space. We have argued in chapter two that due to non-abelian dynamics the chromo electric field is time dependent, and hence one has to take this fact into account in calculating the pair production rate. We have computed the pair production rate by taking the external field to be varying sinusoidally in time and have shown how this increases, relative to the constant field case, the pair production rate from vacuum, for A-A collisions. In chapter three, we have taken an unconsidered scenario of how the production rate gets modified in the presence of a heat bath and have shown a rise in the production rate in the presence of a bath.

Chapter four contains suggestions of a new mechanism for color equilibration of plasma. In chapter five we have considered the evolution of the plasma in phase space through kinetic and hydrodynamic equations. The thing that is worth mentioning here is that the non-perturbative studies reveal the existence of chaotic modes of oscillation, that can bring about thermalisation of the plasma.

As regards the future work it would be interesting and worthwhile to include some of the effects not considered by us. To name a couple of them, in our computation of pair production at zero and finite temperature we have (almost) ignored the effect of dynamical gluons. In some of the recent studies the effect of dynamical gluons has been taken into account but these are essentially abelian in nature and at zero temperature. In our view one ought to do the pair-production problem at zero and finite temperature, taking the non-abelian nature of the dynamical gluons into account.

So far as the evolution of the plasma is concerned we think one should incorporate the production and evolution of the plasma simultaneously using quantum kinetic theory.

List of Publications

1. Q $\bar{Q}$ pair production in an oscillating external field, A. K. Ganguly, P. K. Kaw and J. C. Parikh: Phys. Rev. D46 (1993), R2983

2. A. Collective non-abelian mechanism for color equilibration in quark gluon plasma, A. K. Ganguly, P. K. Kaw and J. C. Parikh: (paper in preparation)

3. A. Collective process for thermalisation in quark gluon plasma, A. K. Ganguly, P. K. Kaw and J. C. Parikh: (paper in preparation)

4. An exact nonlinear non-abelian oscillating solution in SU(2) Yang Mills theory and its consequences: A.K. Ganguly (paper in preparation)

5. Thermal tunneling of $Q\bar{Q}$ pairs in A-A collision PRL-Th-93/19: A.K. Ganguly, P.K.Kaw and J.C.Parikh

Papers accepted for presentation at Symposia/Conferences

1. $Q\bar{Q}$ Pair Production in non abelian gauge fields , A.K. Ganguly , P.K. Kaw and J.C. Parikh presented in ICPA-QGP '93 Calcutta.

2. Gluon Hydrodynamics, A.K. Ganguly , P. K. Kaw And J.C. Parikh Presented in Plasma '91 (Symphosium On Plasma Science and Technology, Indore)

3. Hydrodynamic Equations for Classical Gluonic Plasma A.K. Ganguly , P. K. Kaw And J.C. Parikh: Accepted in Quark Matter Conference '91,Gatlinberg,USA.