SOME PROBLEMS IN TERRESTRIAL MAGNETISM

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TO

MY FATHER

SHRI HIRALAL CHHOTALAL GANDHI

AND

MY MOTHER

SMT. HASUMATIBEN HIRALAL GANDHI

CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

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C O N T E N T S

STATEMENT

This thesis is an outcome of our study of the interaction between the ionosphere and terrestrial magnetic field generating a three dimensional current system responsible for the observed magnetic field variations on the surface of the Earth.

We have generalized the tensor conductivity which could exist at the ionospheric heights in terms of a diagonal symmetric and an antisymmetric tensor (Pratap and Gandhi (1975)). We have also shown how such a tensor could be obtained by taking nonlinear interaction in a dynamical magneto-active plasma.

We have used the above conductivity expression in evolving three dimensional current system at the ionospheric heights which could be contiguous with the dynamo theory developed for the magnetosphere. We have shown that the dynamo action does not exist for the symmetric part of the conductivity tensor when this depends only on magnetic field.

We have developed here a new method of studying the dynamics of S_q focus from a single station data. This method can give both day-to-day and seasonal variation of S_q focus as studied by earlier workers. Furthermore, this method is versatile and can also give the motion of S_q focus as a function of local time i.e. the daily variation of S_q focus. We have made the calculations of the motion of S_q focus for two different stations for the same day and have shown that the results are consistent and thereby justified the validity of above theory.

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CHAPTER I

INTRODUCTION

1.1. History of the Subject

The variations of the Earth's magnetic field had been the subject of study ever since Gauss. He suggested that spherical harmonic analysis could be applied successfully to the study of these variations. The harmonic analysis of magnetic records showed different types of periodic motions with periods ranging from a year to a few seconds. The study of the long period variations was greatly hampered due to lack of data and inaccurate recording. Nevertheless the variations having a not too long period such as diurnal, semidiurnal and seasonal, were studied extensively and various theories have been suggested to explain these.

Schuster (1889) made an extensive analysis and showed that the magnetic variations are essentially due to two sources - one existing outside the Earth (external) and the other inside (internal). He made an estimate of the ratio of the internal to the external as 1:4. The inadequacy of his data was rectified by Chapman (1919) when he used the data obtained from twenty one observatories distributed widely over the globe. Chapman obtained the above-said ratio as 1:2·5. Chapman therefore suggested the possibility of the internal part as being due to the induction effect given by external current system. Solar quiet (S_q) variation is a global phenomenon. One can reproduce this variation by means of an overhead current system. It was observed by McNish (1937) that the station in the neighbourhood of the magnetic equator had abnormally high S_q variation compared to the variation at higher latitudes. Chapman (1951) named this as the 'electrojet' and it was found that this region is centered around the magnetic equator with $\pm 3^{\circ}$ latitude spread. Efforts were made then to postulate ring currents in this region. Baker and Martyn (1953) tried to explain this enhanced variation in the magnetic field as due to an increase in the conductivity when one considers an anisotropic conductivity tensor. The Hall current becomes more dominant in the electrojet region. Lucid accounts of various theories as well as methods of analysis of the observed data are given by Chapman and Bartels (1940), Fleming (1939), Matsushita and Campbell (1967) and Akasofu and Chapman (1972). Recently Price (1969) and Kane (1976) have given extensive reviews of this phenomenon.

Baker and Martyn (1953)'s anisotropic conductivity model has a shortcoming, in that, the elements of the tensor were not considered as functions of space and time. Hence at present there does not exist a single theory to explain the diurnal geomagnetic field variations with a superimposed electrojet, which corresponds to observed amplitude and phase. This thesis is an attempt to formulate such a theory and connect it to the current system in the upper region of the atmosphere such as magnetosphere given by Pratap et al. (1973) and present coherent and consistent picture of the physical phenomena.

1.2. Theories of Sq Current System

Three main theories put forward to explain S_q variation, are

1. Diamagnetic Theory,
2. Drift Current Theory,
3. Dynamo Theory.

Though the dynamo theory is the earliest one, it is still the most satisfactory theory and it will be taken up last in the following discussion.

1. Diamagnetic theory

In this theory Ros Gunn (1928) considers the motion of a charged particle in a magnetic field. A free charged particle in the presence of a magnetic field executes a spiral motion, the projection of which on a plane normal to the line of force will be a circle of radius r known as the spiral radius or radius of gyration. This rotating particle will be equivalent to a magnetic shell, the axis of magnetisation, being in a direction opposite to the initial field therefore giving rise to a diamagnetic field. The effect stated is very much dependent on the density of the plasma. In a low density plasma the mean free path λ is greater than the radius of gyration r i.e. $\lambda >> r$. Hence the effect of gyration is pronounced. The expressions for λ and r are

$$\lambda = \frac{1}{\sqrt{2}} \frac{1}{\pi n d^2} \tag{1.1}$$

and
$$r = \frac{m v_\perp}{H q} \tag{1.2}$$

where m = mass of the particle.

$v_\perp$ = the component of the velocity of the particle, perpendicular to the direction of initial magnetic field H.

H = initial magnetic field in which particle is gyrating.

q = charge of the particle.

n = number density.

d = diameter of the particle.

When a charged particle moves in a magnetic field, two forces acting on it are the centrifugal force and the Lorentz force. Under the balance of these forces, the particle will gyrate with radius r and frequency Ω , given by

$$\frac{m v_\perp^2}{r} = H q v_\perp \tag{1.3}$$

$$\therefore \quad r = \frac{m v_\perp}{H q} \tag{1.4}$$

and
$$v_\perp = r \Omega \tag{1.5}$$

So that $$\Omega = \frac{Hq}{m} \tag{1.6}$$

In the plasma, V, the mean molecular velocity, is given by

$$V = \left\{ 8KT / \pi m \right\}^{1/2} \tag{1.7}$$

where K is Boltzmann's constant, T is temperature in degrees Kelvin.

The collision frequency in the plasma is given by

$$\nu = V / \tau \tag{1.8}$$

Substituting (1.1) and (1.7) in (1.8), we get

$$\nu = 4nd^2 \left(\pi KT / m \right)^{1/2} \tag{1.9}$$

It is known that the diamagnetic effect is independent of the sign of the charge since particles of opposite charge will gyrate in opposite directions, thus giving rise to a field in the same direction. This condition is satisfied in the ionosphere because both types of charged particles exist. For the condition $\lambda \gg r$ we see the following values at the ionospheric E-region heights (viz. ~ 100 km.).

$\Omega_{ce} = 8.4 \times 10^6$ Sec.$^{-1}$ $\qquad$ $\nu_{en} = 9.4 \times 10^4$ Sec.$^{-1}$

$\Omega_{ci} = 160$ Sec.$^{-1}$ $\qquad$ $\nu_{in} = 5.8 \times 10^3$ Sec.$^{-1}$

$\nu_{en} \ll \Omega_{ce}$ and $\nu_{in} > \Omega_{ci}$ (1.10)

So that $$\Omega = \frac{Hq}{m} \qquad (1.6)$$

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In the ionosphere therefore the motion of charged particles can be described as follows. In the E-region, only electrons are magnetized and follow the spiral motion. However, in the F-region (heights $\geq$ 150 km.) both the species (ions and electrons) are magnetized, since $\nu_{en} \ll \Omega_{ce}$ and $\nu_{in} \ll \Omega_{ci}$.

The magnetic moment (μ) due to a gyrating charged particle of charge q (in e.m.u.) describing an orbit of area A in time τ is given by

$$\mu = qA/\tau \tag{1.11}$$

Now $$\frac{A}{\tau} = \frac{\pi r^2}{\frac{2\pi r}{v_\perp}} = \frac{1}{2} v_\perp r = \frac{1}{2}\frac{m v_\perp^2}{Hq} \tag{1.12}$$

where r is given by equation (1.2).

Hence the magnetic moment

$$\mu = \frac{1}{2}\frac{m v_\perp^2}{H} \tag{1.13}$$

If there are N charged particles, then the diamagnetic intensity I will be given by

$$I = -\frac{1}{2}\frac{N m v_\perp^2}{H} \tag{1.14}$$

Taking the absolute temperature of the ion gas as T, the above relation can be written as

$$I = -2/3\,\frac{NkT}{H} \tag{1.15}$$

In evaluating the variation of I, Ros Gunn has made the following assumptions viz.

1. the ionized gas is at a height of 150 - 180 km.,
2. the geographic and the magnetic axes are in the same direction and
3. the temperature is uniform throughout the ionized gas.

Assuring that the atmospheric ionization is entirely due to solar radiation, the number of ions N is proportional to the intensity of the incident radiation i.e.

$$N = N_0 \cos\phi + \delta \tag{1.16}$$

where ϕ is the latitude.

N_0 is the number of ions at the equator at noon.

δ is the average number of residual ions.

δ is often neglected in the calculation of I. In the expression for I, H is the dipole field at any given place and is given as

$$H = H_0 (1 + 3\sin^2\phi)^{1/2} \tag{1.17}$$

then

$$I_\phi = -\frac{2}{3} \frac{N_0 K T \cos\phi}{H_0 (1 + 3\sin^2\phi)^{1/2}} \tag{1.18}$$

The above calculations gave only qualitative agreement with the observed data.

2. Drift current theory

Chapman showed that an electric field exists due to charge separation because of the gravitational effect on the different masses of electrons and ions with predominantly lower layer of ions. Thus there will exist an electrostatic field downwards which is known as a Pennekock - Rosseland field. Let this field have components $F_{\perp}$ and $F_{\parallel}$ along and transverse to the magnetic field respectively.

Let us consider the motion of a charged article within the layer. Besides the Lorentz force $Hq\nu$ and the centrifugal force $\frac{m\nu^2}{r}$ acting on the particle, other forces $F_{\perp}$ and $F_{\parallel}$, mentioned above also act. In this case the velocity component along the magnetic field experiences an acceleration $F_{\parallel}/m$. The component $F_{\perp}$ on the other hand produces no mean velocity or acceleration in its own direction, but imparts a velocity or drift to the charged particle at right angles to itself as well as to the magnetic field by an amount $F_{\perp}/Hq$, independent of mass of the particle. This may be seen from the following.

If the z-axis is taken along the direction of H and X-axis along that of $F_{\perp}$ and if the y-axis is taken such that it forms a right handed system, then we can write the equation of motion componentwise as

$$m\ddot{x} = F_{\perp} + Hq\dot{y} \qquad (1.19)$$

$$m\ddot{y} = -Hq\dot{x} \tag{1.20}$$

$$m\ddot{z} = F_{\parallel} \tag{1.21}$$

The solutions of the above equations are

$$\dot{x} = \dot{x}_0 \cos\omega t + \dot{y}_0 \sin\omega t + \frac{F_{\perp}}{Hq}\sin\omega t \tag{1.22}$$

$$\dot{y} = -\dot{x}_0 \sin\omega t + \dot{y}_0 \cos\omega t - \frac{F_{\perp}}{Hq}(1-\cos\omega t) \tag{1.23}$$

$$\dot{z} = \dot{z}_0 + \frac{F_{\parallel}}{m}t \tag{1.24}$$

where the suffix o refers to the value at t = o. The average value of $\dot{x}$ over a cycle ($\omega\tau = 2\pi$) is zero and that of $\dot{y}$ is $-F_{\perp}/Hq$. The average value of $\dot{z}$ increases at the rate of $F_{\parallel}/m$. This average value of $\dot{y}$ is known as the drift velocity.

Chapman has shown that the above drift currents will flow in closed circuits and have the same shape as the ring currents obtained by Bartels (1928) to explain S_q variation. Chapman has shown on the basis of drift current theory that the necessary ionic density to produce the observed maximum effect is about 2 x 10^{14} per square cm. column of the atmosphere while that required by the diamagnetic theory is about 5 x 10^{16} per sq. cm. column of the atmosphere. Experimentally observed ionic density is $\sim 10^{12}$ per square centimeter column of the atmosphere which is too less than those required by both the theories.

3. Dynamo Theory

The rarefied air which has been rendered highly conducting by the ionization effected by ultra violet radiation from the Sun gives rise to a system of Foucault's currents when it moves across the Earth's permanent magnetic field. This action is compared to that of a dynamo wherein the Earth acts as a magnet, the moving current of ionized gas represents the winding of the armature. The three factors here are the Earth's magnetic field, the motion of the air existing in the upper atmosphere and the conductivity of ionized gas.

The most dominant part of the Earth's magnetic field is the dipole component with its axis situated eccentrically from the centre of the Earth and inclined at an angle of 11° to the axis of rotation. The motion of the air in the upper atmosphere manifests itself as the pressure oscillation as recorded by a barometer at the surface. The theoretical investigations of Lamb (1910), Taylor (1929, 1930, 1932, 1936), Pekeris (1937, 1939), Wilkes (1949), Sibert (1961), Lindzen (1967) and Chapman and Lindzen (1970) have shown that the oscillation is very critically dependent on the temperature distribution. The existence of the ionized layers is evident from the radio wave propagation but the presence of the dipole magnetic field makes the conductivity anisotropic.

1.3. Merits and Demerits of the above theories.

All the three theories given above have their own merits and demerits. The diamagnetic and drift current theories are simple and elegant in as much as they depend solely on the ionic and electronic density existing in the ionosphere. Density becomes maximum in the afternoon at about 1400 hours local time (Chapman and Bartels, 1940). According to these two theories, magnetic variation must be maximum at this hour. Observations have however shown that the maximum is at 1100 hours local time. This discrepancy in phase can not be explained on the basis of these two theories.

Chapman concluded that if the drift currents in the Earth's atmosphere are to have the same intensity as the actual S_q, the number N of the 'drift effective' charges per square centimeter column of atmosphere above the equator at noon must be of the order 10^{14}; by 'drift effective' he means that each charge is not counted as one but as $\Omega_{ci}^2/(\Omega_{ci}^2+\nu^2)$, where Ω_{ci} is ion cyclotron frequency and ν is the collision frequency. Thus he accounts for the reduction of the drift currents by collisions. He estimated the actual number of ions per square centimeter column of the atmosphere as 10^{12} per sq. cm. He has thus found that drift currents are inadequate by a factor between 3 and 10 to account for the intensity of the S_q field. If we consider the upward and downward motions of the charged (which must exist in addition to the drift considered so far), this will again go counter to the theory, and hence he concluded that the magnetic field due to the drift currents will not account for the S_q current intensity.

The current intensity due to diamagnetic theory is also small to account for the S_q field. As it is shown by Chapman (1929) its contribution is only one part in two hundred and fifty towards the S_q field.

Dynamo theory therefore seems to be the only plausible one to explain the S_q phenomena. For developing a dynamo theory we need to know besides the magnetic field, conductivity and the velocity field also. The harmonic analysis of the barometric variation as observed on the surface of the Earth reveals a predominant semidiurnal mode and a small diurnal variation. This diurnal variation however will not account for S_q field. In a recent study by Lindzen (1967) on the theory of tides on a rotating Earth, it has been shown that the amplitude of the various components depends critically on vertical temperature distribution. He has also shown the possible existence of a strong diurnal component of ionospheric heights by taking into account the known temperature distribution with heights. Recently Mathews (1976) has observed experimentally that it is indeed so.

A satisfactory dynamo theory requires a knowledge of a realistic conductivity profile. Schuster (1908) calculated the required conductivity to account for the observed S_q variation, as 3×10^{-6} emu, Chapman's (1929) calculations gave the value 20×10^{-6} emu; while taking self-induction into account, it came to 25×10^{-6} emu.

Chakrabarty and Pratap (1954) developed this theory in an entirely different frame-work using orthogonality property of tesseral harmonics as against the method of successive approximation used by Chapman and Schuster. They could reproduce the current system in the E-layer which agreed well with the one given by Bartels (1928). They did not however calculate the conductivity explicitly.

1.4. Present Status of the Problem.

It is traditional to treat the global properties of the ionospheric dynamo on a two-dimentional basis in which vertical currents are neglected or not treated in a fully self-consistent manner.

Fukushima and Maeda (1959) developed a three-dimensional dynamo theory in a concentric spherical sheet of finite thickness. They took into account both toroidal and poloidal magnetic fields and showed that vertical component has comparatively small effect in the generation of polar elementary storm. They took a constant isotropic conductivity within the shell.

Nishida and Fukushima (1959) also considered a simple three-dimensional model in which they derived velocity from a potential and obtained the stream function, current contours and magnetic lines of force.

When one considers three-dimensional current system one cannot assume the existence of a current function (stream function) and this was pointed out by (Price (1968)).

Price (1968) tried to develope a three - dimensional current system but assuming a velocity field which is derived from a velocity potential. This velocity field however is not irrotational. He then went over to a slab geometry and chose the boundary condition in such a way that $J_z = 0$ at the lower boundary. He also took J_z profile to be parabolic and solved for vector $\vec{J}$ which is divergence - free in the slab. This assumption of vertical distribution, however, is arbitrary.

Cocks and Price (1969) continued the work of Price (1968) with a tensor conductivity, the components of which are functions of x only and tried to obtain the e.t.f. in the domain $x = \pm \frac{\pi a}{2}$ and $y = \pm \frac{\pi a}{2}$, where 'a' is radius of Earth. They claimed to have obtained the result that the currents are confined to this horizontal slab. It is obvious that this result is due to assumption that the conductivity components are functions of x only.

Mishin (1971) and his coworkers were the first to realize the importance of the height integrated Cowling conductivity (Σ_3) being a function of space and time in the analysis and also they have shown that the integrated Pederson (Σ_1) and Hall (Σ_2) conductivities are comparable with that of the parallel conductivity (Σ_0).

Forbes and Lindzen (1975) tried to study the electrodynamic effects of atmospheric tides obtained by Lindzen earlier by taking both diurnal and semidiurnal components. For developing this global model they made the two following important assumptions.

1. Vertical and latitudinal structures of these modes were assumed to be seperable in the dynamo region.

2. This seperability is valid more for semidiurnal component than the diurnal one.

They, however, were not able to account for discrepancies of the order of 20% in amplitude and the phase of 1 to 2 hours between the observed and calculated data.

In the present thesis we have developed a three dimensional dynamo theory effective in the ionospheric heights. The motivation has been to connect it with the dynamo theory effective in the magnetosphere as developed by Pratap et al. (1973). In the magnetospheric heights, though the conductivity is anisotropic in nature, it has been shown by Spitzer (1962) that the Hall component is very small while the Pederson and direct components are of the same order. Thus the deviation from isotropic conductivity in the magnetospheric heights is negligible while that in the ionospheric heights will be dominant. Hence the ideal conductivity distribution would be to have tensorial conductivity in the ionosphere in which only the Pederson and direct will exist as we increase the height. We have taken such a conductivity distribution in to account. Secondly while the conductivity is a function of magnetic field, it could, in general be a function of velocity field $\vec{V}$, and the electric field $\vec{E}$, besides being an explicit function of Υ. It is obvious that the conductivity used by Baker and Martyn (1953) could be a special case of the theory developed here. The wind velocity potential adopted here is based on the result developed by Lindzen (1967) and we have adopted the diurnal mode in the ionospheric heights and this could match well with that existing in the magnetosphere.

The second problem is an attempt to determine the focus of the solar quiet day variation (S_q) current system from the data of a single station. The earlier attempts in the determination of the S_q focus are from a chain of stations distributed over a longitude circle and lying between $45^{\circ} \pm 10^{\circ}$ in latitude and the limitations of these have been discussed at length by

Hasegawa (1960). The present method is devoid of these difficulties and further-more this also gives the motion of the S_q focus during a day - a result which could not have been derived from the earlier methods. This method therefore gives the existence of a global perturbation of the wind system and shows that any harmonic of the velocity potential will now have a phase which has a latitudinal and longitudinal dependence (or space time dependence).

1.5. Brief Description of the Chapters.

In this section a brief description of the chapters of the present thesis are given.

The Second Chapter deals with the wind systems. The work initiated by Lamb (1910) and carried through by Taylor (1929, 1930, 1932, 1936), Perkeris (1937, 1939), Wilkes (1949) etc. has recently been given a fresh consideration by Chapman and Lindzen (1970). It has been shown by Lindzen (1967) that the theory predicts a dominant diurnal component of the wind system at the ionospheric heights even though this component has a negligible amplitude at the surface of the earth. We have reviewed this theory in this chapter and have shown the critical role played by the temperature distribution with height.

The third chapter discusses the new conductivity tensor we have developed recently (Pratap and Gandhi, 1975). It has been shown that just as the conductivity depends on the magnetic field giving the direct, Hall and Pederson conductivity, it could also depend on the other vector fields as well as fields generated by the combination of these fields. We have shown from the general plasma theory point of view that this could happen if one includes nonlinear processes that could possibly exist in the ionospheric heights. We have also discussed and justified our taking the dipole field instead of a field of 48 components as considered by Sugiura and Poros (1969).

Chapter IV gives a general review of the currents existing at various heights responsible for the magnetic field variation as observed at the surface of the Earth. We have discussed the magnetospheric current systems, ionospheric current systems, ring currents and polar current systems. We have also shown the necessity of a general global three-dimensional dynamo theory which could explain these separate current systems in a unified manner.

Chapter V is devoted to the development of the three-dimensional global dynamo theory. This incorporates the essential features of the theories developed by the earlier workers and we have obtained the current density variations as a function of latitude, longitude and height. We have also derived the corresponding magnetic field variations.

In the sixth chapter we have developed a new theory to study the dynamics of S_q focus from a single station data. This theory is different from the existing one and as mentioned in (1.4) would enable us to study the movement of the S_q focus during a day. We have applied this theory to data from Alibag and Tashkent and have discussed the results in detail.

The last chapter gives the summary of the thesis and we have also discussed some of the consequences of the theories developed in the earlier chapters.

CHAPTER II

WIND SYSTEM

2.1. Introduction

Tides are raised by the Sun and the Moon in the neutral atmosphere. Those raised by the Sun are both thermal and gravitational in origin and therefore will have the same period as that of the rotation of the Earth. Lunar tides are, on the other hand raised by the gravitational force due to Moon, and hence its period would depend on the period of rotation of the Moon around the Earth.

Lord Kelvin (1882) propounded the resonance theory to explain the semi-diurnal barometric pressure variation observed on the ground. Lamb (1910) made a serious attempt to develop a mathematical theory by taking into consideration the temperature distribution and showed that the solar component is more dominant than the Lunar, the ratio being 11:5. Taylor (1936) worked on this tidal theory again and showed that the atmosphere can have more than one free mode of oscillation. Perkeris (1937) undertook a reexamination of this problem and showed that there can exist a freeperiod of 10.5 hours besides 12 hours. He also showed that the semidiurnal vibration can have a nodal plane at 30 km. altitude. He pointed out the possibility of the semidiurnal mode having a very high amplitude at a height of 100 km. (of the order of 200 times) which was demanded by Chapman (1919) in the development of the dynamo theory. An exhaustive review of this till 1949 has been given by Wilkes (1949), in which he has shown that the thermal energy available for exciting the atmospheric tides should have a larger diurnal component than the semi-diurnal one, and that the atmosphere has a vertical temperature distribution favourable for trapping the diurnal wave between the ground and mesosphere. This apparently attractive theory had to be revised when it was realized through rocket experiments that the peak value of mesospheric temperature is lower than the expected one, and unsuitable for this resonance (Haurwitz 1964).

Pratap (1954b) has made an attempt to obtain theoretically from the dynamo theory the nature of the atmospheric oscillation both in phase as well as in amplitude, at the ionospheric level which is supposed to be the seat of the dynamo current. He has also discussed how critically the geomagnetic field variations depend on the nature of the atmospheric oscillations. It was found by Kato (1956, 1957) and Maeda (1955 , 1957) that the ionospheric periodic wind system, causing an electric current system which in turn is responsible for the geomagnetic variation under quiet condition (S_q), contains a larger diurnal component than the semidiurnal one. Their results suggested that the tidal oscillation in the upper atmosphere is different fron that near the ground. It is remarkable that Kato (1966a, b, c) and

Lindzen (1966 .) independently and almost simultaneously theoretically predicted the existence of the diurnal negative mode which is evanescent in the vertical direction. Further, Kato found that the first negative mode can explain well the wind pattern causing the geomagnetic S_q variation. This finding gave evidence for the existence of the negative mode in the atmosphere. Based on rocket observation in the mesosphere Lindzen (1967) discussed the thermal excitation of the diurnal and semidiurnal tides by ozone and water vapour absorbing solar radiation. He explained fairly successfully the observation near the ground as well as in the mesosphere but not in the dynamo region, which the negative mode excited in the mesosphere cannot reach.

In Section (2.2) we have discussed the approximations made by Chapman and Lindzen (1970) in developing the tidal wave theory. In Section (2.3) we have given complete theory as developed by Chapman and Lindzen. In Section (2.4) we have discussed the two modes viz., semidiurnal and diurnal with temperature profiles with respect to height. In section (2.5) we have discussed the plausible modes which could explain the geomagnetic variation observed on the surface of the earth.

2.2. Approximations.

We shall present here the theory as developed by Chapman and Lindzen (1970). The following approximations have been made by them in solving the problem:

(1). The atmosphere is taken as a compressible fluid obeying Navier - Stokes equations.

(2). The heating of the atmosphere takes place through a series of continuous equilibrium states and hence the equation of state is same as that for a perfect gas.

(3) The atmosphere is regarded as a thin fluid layer; the thickness being small, compared to the radius of Earth. This enables one to take the acceleration of gravity 'g' as a constant throughout the layer.

(4) The atmosphere is in hydrostatic equilibrium.

(5) The Earth's ellipticity and surface topography are ignored. The dissipative processes such as turbulence, viscosity, etc. have also been neglected.

(6) They have linearized the equations and adopted a perturbation method.

(7) They also assumed that the basic flow is zero and that the unperturbed values of temperature and pressure are uniform.

2.3. Theory.

Approximations 2, 3, 4, 7 will give the following equations respectively.

$$P_0 = \rho_0 R T_0 \tag{2.1}$$

$$r = a + z \tag{2.2}$$

$$\frac{1}{\rho}\frac{\partial P_0}{\partial z} = -g \tag{2.3}$$

Substituting (2.1) in (2.3) and realizing the fact that it has a distribution with height we can obtain

$$P_0 = P_0(0)\, e^{-x} \tag{2.4}$$

where $P_0(0)$ is the pressure at the ground level and

$$x = \int_0^z dz/H \tag{2.5}$$

with
$$H = RT_0/g \tag{2.6}$$

The equation of motion of the fluid after neglecting inertial term, and viscosity, is given by

$$\frac{\partial \vec{V}}{\partial t} + 2\vec{\omega}\times\vec{V} = -\frac{1}{\rho}\nabla P + \vec{g} \tag{2.7}$$

where $\vec{V}$ velocity of a fluid element

$\vec{\omega}$ = angular velocity of the Earth

$$\vec{g} = -\nabla \Omega \tag{2.8}$$

Ω being gravitational tidal potential.

In this equation the coriolis term $\vec{\omega} \times \vec{v}$ describes the advection of Earth's momentum due to its rotation.

We can assume $p = p_0 + \delta p$, $\rho = \rho_0 + \delta\rho$, with p_0 and ρ_0 the values of pressure and density in the static case. This equation in θ (increasing southward), ϕ (increasing eastward) and z (radially outward) coordinate system can be written as

$$\frac{\partial u}{\partial t} - 2\omega v \cos\theta = -\frac{1}{a}\frac{\partial}{\partial\theta}\left(\frac{\delta p}{\rho_0} + \Omega\right) \tag{2.9}$$

$$\frac{\partial v}{\partial t} + 2\omega u \cos\theta = -\frac{1}{a\sin\theta}\frac{\partial}{\partial\phi}\left(\frac{\delta p}{\rho_0} + \Omega\right) \tag{2.10}$$

$$\frac{\partial \delta p}{\partial t} = -g\,\delta\rho - \rho_0\frac{\partial\Omega}{\partial z} \tag{2.11}$$

In writing (2.11) we have assumed w to be independent of time.

In the above equations

u = northerly velocity

v = westerly velocity

w = upward velocity

a = radius of Earth.

The equation of continuity is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \tag{2.12}$$

In the linearized form, this equation is written as

$$\frac{\partial \delta\rho}{\partial t} + w \frac{d\rho_0}{dz} + \rho_0 (\nabla \cdot \vec{V}) = 0 \tag{2.13}$$

We can define an operator $\frac{D}{Dt}$ such that

$$\frac{D\rho}{Dt} = \frac{\partial \delta\rho}{\partial t} + w \frac{d\rho_0}{dz}$$

and (2.12) then takes the form

$$\frac{D\rho}{Dt} = \frac{\partial \delta\rho}{\partial t} + w \frac{d\rho_0}{dz} = -\rho_0 \chi \tag{2.14}$$

where

$$\chi \equiv \nabla \cdot \vec{V} = \frac{1}{a \sin\theta} \frac{\partial}{\partial \theta} (u \sin\theta) + \frac{1}{a \sin\theta} \frac{\partial v}{\partial \phi} + \frac{\partial w}{\partial z} \tag{2.15}$$

Further, the equation for energy transfer in the linearized form is given by

$$\frac{R}{\gamma - 1} \frac{DT}{Dt} = \frac{R}{\gamma - 1} \left(\frac{\partial \delta T}{\partial t} + w \frac{dT_0}{dz} \right) = \frac{gH}{\rho_0} \frac{D\rho}{Dt} + J \tag{2.16}$$

In writing equation (2.16) we have taken into account the heat generated by adiabatically compressing the atmosphere. The solar heat received by the atmosphere is the second source of energy which is included in the equation through J - the source term and $\gamma = c_p/c_v$.

From (2.1) by taking the variation, we have

$$\frac{\delta p}{p_0} = \frac{\delta T}{T_0} + \frac{\delta \rho}{\rho_0} \tag{2.17}$$

Substituting δT from (2.17) in (2.16) we get

$$\frac{Dp}{Dt} = \gamma g H \frac{D\rho}{Dt} + (\gamma - 1)\rho_0 J \tag{2.18}$$

We will define a new variable G which is related to $\frac{Dp}{Dt}$ in the following manner

$$G = -\frac{1}{\gamma p_0}\frac{Dp}{Dt} \tag{2.19}$$

Fields in tidal theory are complex and are generally functions of time and longitude. Any general field component

$$f = f^{\sigma,s}(\theta, z)\, e^{i(\sigma t + s\phi)} \tag{2.20}$$

where $2\pi/\sigma$ represents either a solar or lunar day or some suitable fraction thereof

$$s = 0, \pm 1, \pm 2, ----$$

If we take the appropriate terms in u and v as given in (2.20), and substituting in (2.9) and (2.10) and solving for u and v, we get

$$u^{\sigma,s} = \frac{i\sigma}{4a\omega^2(f^2-\cos^2\theta)}\left(\frac{\partial}{\partial\theta} + \frac{s\cot\theta}{f}\right)\left(\frac{\delta p^{\sigma,s}}{\rho_0} + \Omega^{\sigma,s}\right) \tag{2.21}$$

and

$$v^{\sigma,s} = \frac{-\sigma}{4a\omega^2(f^2-\cos^2\theta)}\left(\frac{\cos\theta}{f}\frac{\partial}{\partial\theta} + \frac{s}{\sin\theta}\right)\left(\frac{\delta p^{\sigma,s}}{\rho_0} + \Omega^{\sigma,s}\right) \tag{2.22}$$

where $f = \sigma/2\omega$.

Here δp is a complex function, and $\left(\frac{\partial}{\partial\theta} + \frac{s\cot\theta}{f}\right)$ and $\left(\frac{\cos\theta}{f}\frac{\partial}{\partial\theta} + \frac{s}{\sin\theta}\right)$ are different operators; hence (2.21) and (2.22) do not imply that u lags 90° in phase behind v in the northern hemisphere. In addition, u and v may change sign at different latitudes; thus there may exist a latitude band where u leads v. (Blamont and Teitelbaum 1968). It also appears that for a certain value of θ, u and v may become singular when $f = \pm\cos\theta$.

Substituting (2.21) and (2.22) in (2.15) we get

$$\chi - \frac{\partial W}{\partial z} = \frac{i\sigma}{4a^2\omega^2} F\left(\frac{\delta p}{\rho_0} + \Omega\right) \tag{2.23}$$

where F is an operator given by

$$F \equiv \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\sin\theta}{f^2-\cos^2\theta}\frac{\partial}{\partial\theta}\right)$$

$$-\frac{1}{f^2-\cos^2\theta}\left(\frac{s}{f}\frac{f^2+\cos^2\theta}{f^2-\cos^2\theta}+\frac{s^2}{\sin^2\theta}\right) \qquad (2.24)$$

Using the equations (2.11), (2.14), (2.18), (2.19) and (2.23) and eleminating $\mathcal{G}$, δP, $\delta\rho$, W and χ successively and using the fact that the variables depend only through harmonic terms, we can write the final equation for G as

$$H\frac{\partial^2 G^{\sigma,s}}{\partial z^2}+\left(\frac{dH}{dz}-1\right)\frac{\partial G^{\sigma,s}}{\partial z}-\frac{i\sigma}{g}\frac{\partial^2\Omega^{\sigma,s}}{\partial z^2}$$

$$=\frac{g}{4a^2\omega^2}F\left[\left(\frac{dH}{dz}+K\right)G^{\sigma,s}-\frac{KJ^{\sigma,s}}{\gamma g H}\right] \qquad (2.25)$$

where
$$K=\frac{\gamma-1}{\gamma}$$

Since we have assumed a shallow atmosphere the variation of Ω through the atmospheric thickness will be small; hence the second derivative of Ω with respect to z could be neglected. This gives,

$$H\frac{\partial^2 G^{\sigma,s}}{\partial z^2}+\left(\frac{dH}{dz}-1\right)\frac{\partial G^{\sigma,s}}{\partial z}=\frac{g}{4a^2\omega^2}F\left[\left(\frac{dH}{dz}+K\right)G^{\sigma,s}-\frac{KJ^{\sigma,s}}{\gamma g H}\right] \qquad (2.26)$$

This is solved by the method of separation of variables. $G^{\sigma,s}$ may be written as

$$G^{\sigma,s} = \sum L_n^{\sigma,s}(z)\,\Theta_n^{\sigma,s}(\theta) \tag{2.27}$$

The set $\{\Theta_n^{\sigma,s}(\theta)\}$ for all n is a complete set of functions defined on a unit sphere. J can also be expanded in a similar manner viz.

$$J^{\sigma,s} = \sum J_n^{\sigma,s}(z)\,\Theta_n^{\sigma,s}(\theta) \tag{2.28}$$

Substitution of (2.27) and (2.28) in (2.26) yields the following set of equations for $L_n^{\sigma,s}$ and $\Theta_n^{\sigma,s}$, viz.,

$$F[\Theta_n^{\sigma,s}(\theta)] = -\frac{4a^2\omega^2}{g h_n^{\sigma,s}}\,\Theta_n^{\sigma,s} \tag{2.29}$$

and $$H\frac{d^2L_n^{\sigma,s}}{dz^2} + \left(\frac{dH}{dz}-1\right)\frac{dL_n^{\sigma,s}}{dz} + \frac{1}{h_n^{\sigma,s}}\left(\frac{dH}{dz}+\kappa\right)L_n^{\sigma,s}$$
$$= \frac{\kappa}{\gamma g H h_n^{\sigma,s}}\,J_n^{\sigma,s} \tag{2.30}$$

where $h_n^{\sigma,s}$ is the constant of separation.

The boundary conditions on $\Theta_n^{\sigma,s}(\theta)$ are that they are bounded at the poles (i.e. at $\theta = 0, \pi$). With these conditions, (2.29) defines an eigenvalue problem with h_n as the eigenvalues. Laplace first derived (2.29) for the free surface oscillation of a spherical ocean envelope. By historical analogy with this problem, $\{h_n\}$ is often called the set of equivalent depths (Taylor 1936). The eigenfunction $\{\Theta_n\}$ is called the Hough function after Hough (1897, 1898) who pioneered the solution of (2.29).

Equation (2.30) is an inhomogeneous equation which, given two boundary conditions, has a unique solution for the vertical structure. Equation (2.30) is called the vertical structure equation.

We shall now change the variable z to x by using equation (2.5) or (2.4) as

$$x = -\log\left(\frac{p_0(z)}{p_0(0)}\right) \tag{2.31}$$

and shall try a solution of the form

$$L_n = e^{x/2} y_n \tag{2.32}$$

This transforms the equation (2.30) to

$$\frac{d^2 y_n}{dx^2} - \frac{1}{4}\left[1 - \frac{4}{h_n}\left(kH + \frac{dH}{dx}\right)\right] y_n = \frac{kJ_n}{\gamma g h_n} e^{-x/2} \tag{2.33}$$

Equations (2.11), (2.14), (2.17), (2.18), (2.19) and (2.23) together with equation (2.27) imply that

$$\delta P = \Sigma \, \delta P_n(x) \, \Theta_n \tag{2.34}$$

$$\delta \rho = \Sigma \, \delta \rho_n(x) \, \Theta_n \tag{2.35}$$

$$\delta T = \Sigma \, \delta T_n(x) \, \Theta_n \tag{2.36}$$

$$W = \Sigma \, W_n(x) \, \Theta_n \tag{2.37}$$

Since the above equations are linear, we can obtain the coefficient functions by comparing the harmonics as

$$\delta P_n(x) = \frac{P_0(0)}{H(x)} \left[-\frac{\Omega_n(x)}{g} e^{-x} + \frac{\gamma h_n}{i\sigma} e^{-x/2} \left(\frac{dy_n}{dx} - \frac{y_n}{2} \right) \right] \tag{2.38}$$

$$\begin{aligned} \delta \rho_n(x) = \frac{P_0(0)}{(gH)^2} \Bigg[& -\Omega_n e^{-x} \left(1 + \frac{1}{H}\frac{dH}{dx} \right) \\ & + \frac{\gamma g h_n}{i\sigma} e^{-x/2} \left\{ \left(1 + \frac{1}{H}\frac{dH}{dx} \right) \left(\frac{dy_n}{dx} - \frac{1}{2} \right) \right. \\ & \left. + \frac{H}{h_n} \left(K + \frac{1}{H}\frac{dH}{dx} \right) y_n \right\} \\ & - \frac{K J_n}{i\sigma} \Bigg] \end{aligned} \tag{2.39}$$

$$\begin{aligned} \delta T_n(x) = \frac{1}{R} \Bigg\{ & \frac{\Omega_n(x)}{H} \frac{dH}{dx} - \frac{\gamma g h_n}{i\sigma} e^{x/2} \left[\frac{KH}{h_n} + \frac{1}{H} \left(\frac{dy_n}{dx} + \frac{H}{h_n} - \frac{1}{2} \right) \right] \\ & + \frac{K J_n}{i\sigma} \Bigg\} \end{aligned} \tag{2.40}$$

$$W_n(x) = -\frac{i\sigma}{g}\Omega_n + \gamma h_n e^{x/2}\left[\frac{dy_n}{dx} + \left(\frac{H}{h_n} - \frac{1}{2}\right)y_n\right] \tag{2.41}$$

From (2.21), (2.22) and (2.38) it also follows that

$$u = \Sigma\, u_n(x)\, U_n(\theta) \tag{2.42}$$

$$v = \Sigma\, v_n(x)\, V_n(\theta) \tag{2.43}$$

where

$$U_n = \frac{1}{f^2 - \cos^2\theta}\left(\frac{d}{d\theta} + \frac{s\cot\theta}{f}\right)\Theta_n \tag{2.44}$$

$$V_n = \frac{1}{f^2 - \cos^2\theta}\left(\frac{\cos\theta}{f}\frac{d}{d\theta} + \frac{s}{\sin\theta}\right)\Theta_n \tag{2.45}$$

$$u_n(x) = \frac{\gamma g h_n}{4a\omega^2}\left(\frac{dy_n}{dx} - \frac{y_n}{2}\right)e^{x/2} \tag{2.46}$$

$$v_n(x) = \frac{i\gamma g h_n}{4a\omega^2}\left(\frac{dy_n}{dx} - \frac{y_n}{2}\right)e^{x/2} \tag{2.47}$$

In writing the above, we have already expanded Ω as

$$\Omega = \Sigma\, \Omega_n(x)\, \Theta_n \tag{2.48}$$

We shall now obtain an estimate of λ the wavelength of the tidal modes. At the lower boundary if W = 0, then equation (2.41) would give a condition on y_n viz., y_n should be a solution of the differential equation

$$\frac{dy_n}{dx} + \left(\frac{H}{h_n} - \frac{1}{2}\right) y_n = \frac{i\sigma}{\gamma g h_n} \Omega_n \tag{2.49}$$

At the upper boundary the outward flow of energy should be zero i.e. $\frac{1}{2}\rho V^2 \rightarrow 0$ as $z \longrightarrow \infty$. This implies that y_n should remain bounded as $z \longrightarrow \infty$. If we consider an isothermal atmosphere with constant H and solar radiation input to be zero i.e. $J_n = 0$, we can simplify the equation (2.33) in the following form

$$\frac{d^2 y_n}{dx^2} - \frac{1}{4}\left(1 - \frac{4KH}{h_n}\right) y_n = 0 \tag{2.50}$$

Equation (2.49) gives harmonic solution for $h_n < 4KH$. For $h_n > 4KH$ the equation (2.49) yields an hyperbolic solution; therefore y_n ceases to be bounded as $x \longrightarrow \infty$. Hence boundary condition $h_n < 4KH$ gives a real number λ as

$$\lambda = \left\{\frac{KH}{h_n} - \frac{1}{4}\right\}^{1/2} \tag{2.51}$$

with this λ, y_n can be written down as

$$y_n = A e^{i\lambda x} + B e^{-i\lambda x} \tag{2.52}$$

2.4. Excitation of Semidiurnal and Diurnal Modes

General methods for solving the vertical structure equation (2.33) and Hough function equation (2.29) have been given by Chapman and Lindzen (1970). In considering the heat input into the atmosphere they have considered the direct reception of heat from the sun as well as the heat input from the ground. They have also discussed the mode excited by gravitation.

(a) The propagating semidiurnal thermal tide:

The heat input function $J^{\sigma,s}$ which was used for a thermal absorption mode can be represented by

$$J_G = Re \sum_{n=0}^{\infty} J_G^{n\sigma,s}(z,\theta)\, e^{in(\sigma t + \varphi)} \tag{2.54}$$

$J_G^{n\sigma,s}(z,\theta)$ was expanded in terms of Hough Functions and the equation (2.26) was solved for a propagating semidiurnal mode $s = 2$ and $f = \sigma/2\omega = 1$. The solution was separated into symmetric and antisymmetric parts with respect to equator and then coefficient $C_{n,m}$ was obtained. The specific advantage of this method is that one can obtain the velocity profiles for different temperature profiles obtained from the absorption data of Ozone and water vapour. One infers from this analysis that $\Theta_2^{\sigma,s}$ is dominant and that the equivalent depths associated with this are of the order of 8 km. The quantity λ in equation (2.51) was then found to be almost zero showing thereby that this mode has a very long wavelength.

(b) The propagating diurnal thermal tide:

For diurnal thermal tide we take $s = 1$ and $f = \sigma/2\omega = \frac{1}{2}$ and then we set up the series for the temperature distribution (T) we get Hough function with negative indices showing the existence of negative equivalent depths. This implies the existence of internal gravity waves and one may consider the atmospheric tides as one particular excitation of the internal gravity wave. Using a similar expansion, one can also obtain the variation of the pressure. It is found that the diurnal pressure oscillation due to the solar heat input is very weak and irregular.

(c) Relation between Hough Functions and Legendre Functions.

The Hough function can be expanded in terms of the associated Legendre function. Both Hough functions and Legendre functions are orthonormal functions. We can therefore write

$$\Theta_n^m(\theta) = \sum_{n=m}^{\infty} C_n^m P_n^m(\cos\theta) \qquad (2.55)$$

where one can determine C_n^m by using the usual orthogonality condition. It may be observed that the Hough function and Legendre function have the same degree but different orders. One can see that Θ_1^1 and Θ_1^{-1} have the leading term $P_1^1(\cos\theta)\sin\phi$ and hence we have used this to develope the three-dimensional dynamo theory instead of the entire Hough function.

2.5. Discussion and Conclusion.

The general tidal theory suggests that the diurnal thermotidal fields dominate the semidiurnal fields in the upper atmosphere in the belt of $\pm 40^\circ$ latitude. It also shows that the solar diurnal tidal winds constitute the major component above 80 km. in height in the belt of $\pm 30^\circ$ latitude. Nonpropagating components also contribute to northerly component of the diurnal tidal wind below 15 mb.

In conclusion one can generally infer that the agreement between theory and observation is fairly good upto 105 km. while the theory fails to predict the decay of amplitude above 105 km. The theory again is to some extent unsatisfactory since it neglects the transport features such as viscosity and conductivity and the nonlinear aspects, thus we have neglected the nonlinear inertial term ($\vec{V}\cdot\nabla\vec{V}$). Inclusion of this term would not have been possible since we could hot have expanded the relevant

CHAPTER III

MAIN GEOMAGNETIC FIELD AND CONDUCTIVITY TENSOR

3.1. Main geomagnetic field

The main geomagnetic field observed in a magnetic survey as conducted at various points on the surface of the Earth could be written as the one generated by a potential

$$V = a\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left[\left\{C_n^m\left(\frac{r}{a}\right)^n+(1-C_n^m)\left(\frac{a}{r}\right)^{n+1}\right\}g_n^m\cos m\phi + \left\{S_n^m\left(\frac{r}{a}\right)^n+(1-S_n^m)\left(\frac{a}{r}\right)^{n+1}\right\}h_n^m\sin m\phi\right]\left[P_n^m(\cos\theta)\right] \quad (3.1)$$

wherein 'a' is the radius of the Earth. $P_n^m\cos m\phi$ and $P_n^m\sin m\phi$ are tesseral harmonics.

C_n^m and S_n^m are positive numbers representing fractions of the harmonic terms of external origin.

g_n^m and h_n^m are Gauss coefficients.

As can be seen from the above equation the potential is written in two parts, one with a source inside the Earth and the other with a source situated outside. It has been shown by Vestine et al. (1947) that the part due to external source is very small i.e. of the order of 100 γ and that due to internal source is of the order of 30000 γ. Having removed the external part if we express the potential of internal origin as,

$$V^{i} = a \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \left(\frac{a}{r}\right)^{n+1} \{ g_n^m \cos m\phi + h_n^m \sin m\phi \} P_n^m (\cos\theta)$$

(3.2)

then according to geomagnetic reference field $g_1^0 = -30339\gamma$, $g_1^1 = -2123\gamma$ and $h_1^1 = 5738\gamma$ with an equatorial value of H as $H_0 = 30950\gamma$.

This analysis clearly shows that the dipole term gives most of the field while the other harmonics constitute only 10% of the total field. Sugiura and Poros (1969) have taken 48 components of the series (3.2) in their theory of electrojet. As can be seen the change in the dynamo field will mainly be from the dipole component and hence in our analysis, we retain only the dipole component.

3.2. Varying Elements.

The part of the geomagnetic field of the external source has a very complicated space-time structure because of the different sources such as the Sun, the Moon, etc. This varying component is denoted by x (northwards), y (eastwards) and z (vertically downwards) or by a set of elements H the horizontal component, D the declination and I the inclination or dip.

The variations in these elements can be considered as due to the current system existing outside the Earth and also due to the induction effect within the Earth by the outside current system. These components are written as

$$\Delta X = \Delta H \cos D - H \Delta D \sin D \tag{3.3}$$

$$\Delta Y = \Delta H \sin D + H \Delta D \cos D \tag{3.4}$$

$$\Delta Z = \Delta F \sin I + F \Delta I \cos I \tag{3.5}$$

$$\Delta H = \Delta F \cos I - F \Delta I \sin I \tag{3.6}$$

3.3. Analysis of Various Formulations.

Stewart (1882) postulated a large movement of ionized air in the dipole field of the Earth generating current system as in the case of a dynamo, at a time when the existence of the ionosphere was not known. Schuster (1889, 1908) gave a mathematical formulation of the dynamo problem where he used the conductivity as a scalar quantity which is a function of the zenith distance of the Sun thereby taking the Sun as the source of ionization. He wrote down the general relation for the integrated conductivity as

$$\rho_1 = (\rho e) = \sum_{s=0}^{\infty} a_s \cos^s \chi \tag{3.7}$$

where ρ is the conductivity of the shell,

e is the thickness of the shell,

and χ is the zenith distance of the Sun given by

$$\cos\chi = \sin\delta\cos\theta + \cos\delta\sin\theta\cos(\phi-\phi_0) \tag{3.8}$$

with δ as the declination of the sun,

θ the colatitude of the observer,

and $(\phi-\phi_0)$ is the local time of the observer reckoned from the local mid-day meridian.

a_s are constants to be fitted with observations.

The dynamo equation with this conductivity ρ_1 in an ionospheric sheet then can be written as

$$a(\rho_1)^2\left[\frac{\partial}{\partial\phi}(vH_z) + \frac{\partial}{\partial\theta}(uH_z\sin\theta)\right]$$

$$= \rho_1\left[\frac{1}{\sin\theta}\frac{\partial^2 R}{\partial\phi^2} + \frac{\partial}{\partial\theta}\sin\theta\frac{\partial R}{\partial\theta}\right]$$

$$- \left[\frac{1}{\sin\theta}\frac{\partial R}{\partial\phi}\frac{\partial\rho_1}{\partial\phi} + \sin\theta\frac{\partial R}{\partial\theta}\frac{\partial\rho_1}{\partial\theta}\right] \tag{3.9}$$

where u is the northerly velocity,

v the easterly velocity,

R a current function,

a the radius of earth,

and H_z the vertical component of total field H.

Schuster solved this equation using a method of successive approximation and in this he truncated the series for ρ_1 by retaining only the first two terms. Chapman (1919) used the same method but extended the Schuster's analysis by retaining the first three terms in the series viz. $s = 0, 1, 2$. He chose $a_1 = 3a_0$ and $a_2 = \frac{9}{4} a_0$ to make the conductivity positive definite. The conductivity then becomes

$$\rho_1 = (\rho e) = a_0 \left[1 + \tfrac{3}{2} \cos \chi \right]^2 \tag{3.10}$$

Later attempts made by Chakrabarty and Pratap (1954) and Pratap (1954a, 1954b, 1955, 1957) in solving the dynamo equation were based on a more exact method using the orthogonal property of the Legendre harmonics. A new condition was then obtained between a_0, a_1, a_2 viz.

$$a_1^2 = \frac{9}{4} a_0 a_2 \tag{3.11}$$

which could make the solution exact.

Baker and Martyn (1953) for the first time used a tensor conductivity in the dynamo problem for the ionospheric medium. They used a coordinate system in which the magnetic field formed one of the axes, and wrote the direct, Pedersen and Hall conductivities based on the theory of mean free - path developed by Chapman and Cowling (1960). Tensor form of conductivity used by Baker and Martyn (1953) was written in a generalised coordinate system by Möhlmann (1972) as

$$\overline{\overline{\sigma}} = \sigma_1 \delta_{ij} + (\sigma_0 - \sigma_1) H_i H_j + \sigma_2 \epsilon_{ijk} H_k \tag{3.12}$$

where direct conductivity

$$\sigma_0 = n_e q_e^2 \left[\frac{1}{m_e \nu_{en}} + \frac{1}{m_i \nu_{in}} \right] \tag{3.13}$$

Pederson conductivity

$$\sigma_1 = n_e q_e^2 \left[\frac{\nu_{en}}{m_e(\nu_{en}^2 + \Omega_{ce}^2)} + \frac{\nu_{in}}{m_i(\nu_{in}^2 + \Omega_{ci}^2)} \right] \tag{3.14}$$

and Hall conductivity

$$\sigma_2 = n_e q_e^2 \left[\frac{\Omega_{ce}}{m_e(\nu_{en}^2 + \Omega_{ce}^2)} - \frac{\Omega_{ci}}{m_i(\nu_{in}^2 + \Omega_{ci}^2)} \right] \tag{3.15}$$

with Ω_{ce} = cyclotron frequency of electron,

Ω_{ci} = cyclotron frequency of ion,

ν_{in} = collision frequency of ion with neutral,

ν_{en} = collision frequency of electron with neutral,

n_e = density of electron,

q_e = charge of an electron in e.m.u.

The derivation of σ_0, σ_1, σ_2 is given in Section (3.5).

These forms have been used very extensively in recent years in constructing the models of electrojet. (Sugiura and Poros (1969)) as well as S_q current systems (Mishin, (1971)).

The above discussion brings out the following features.

1. The scalar conductivity taken as a series in equation (3.7) explains the general pattern by a dynamo theory based on a y_2^2 wind system (Pratap, 1954b). The fact that a semidiurnal wind system generates a diurnal magnetic field variation of the form y_2^1 shows the importance of the space dependence of the conductivity function, since it is this that converts the second harmonic in wind to a first harmonic in the current by the method of beats.

2. Tensor conductivity given by equation (3.12), however depends on the magnetic field, the number density as well as collision frequency appearing through the functions σ_0, σ_1

and σ_2. Baker and Martyn (1953) could explain the enhancement of the S_q in the equatorial region. The theory however was a poor model in explaining the global feature.

3.4. Generalized Conductivity Tensor

We, therefore, come to the conclusion that

(1) the conductivity should be tensorial in form, and

(2) its elements must be space-time dependent as well as dependent on the field quantities. One, therefore is faced with the problem : what is the general form of a tensor conductivity whose elements are functions of space-time and fields, and secondly, can we derive this consistantly from plasma theory?

Sen Gupta (1965) answered the first part of the question, viz., that the most general form of a tensor whose elements are functions of a vector field $\vec{A}$ and which obeys the usual transformation laws of a tensor is given by

$$\overline{\overline{T}}_{ij} = \Phi_0 \delta_{ij} + \Phi_1 A_i A_j + \Phi_2 \epsilon_{ijk} A_k \quad (3.16)$$

The above tensor consists of three parts, the first being a diagonal tensor, the second, the symmetric part and the third, the antisymmetric one. The functions Φ_0, Φ_1, and Φ_2 are arbitrary functions constructed out of the scalars formed in turn from the vector field $\vec{A}$. If there are two vector fields $\vec{A}$ and $\vec{H}$, then the tensor $\bar{\bar{T}}$ is given by

$$\begin{aligned}\bar{\bar{T}}_{ij} = \Phi_0 \delta_{ij} &+ \Phi_1 A_i A_j + \Phi_2 \epsilon_{ijk} A_k \\ &+ \Phi_3 H_i H_j + \Phi_4 \epsilon_{ijk} H_k \\ &+ \frac{\Phi_5}{2} (A_i H_j + H_i A_j) \\ &+ \frac{\bar{\Phi}_5}{2} (A_i H_j - H_i A_j)\end{aligned} \tag{3.17}$$

One can easily see that equation (3.12) is a special case of equation (3.17) when the vector field $\vec{H}$ is a magnetic field and $\vec{A} = 0$. But in general, in addition to the magnetic field, we have dynamo electric field $\vec{E}$ as well as the velocity field $\vec{V}$ besides the intrinsic field $\vec{\gamma}$. Thus the general conductivity tensor should be a function of these fields and hence be of the form

$$\begin{aligned}\bar{\bar{T}}_{ij} = \Phi_0 \delta_{ij} &+ \Phi_1 \gamma_i \gamma_j + \Phi_2 \epsilon_{ijk} \gamma_k \\ &+ \Omega_1 H_i H_j + \Omega_2 \epsilon_{ijk} H_k \\ &+ \epsilon_1 E_i E_j + \epsilon_2 \epsilon_{ijk} E_k \\ &+ \mu_1 V_i V_j + \mu_2 \epsilon_{ijk} V_k \\ &+ \frac{\alpha_1}{2} (\gamma_i H_j + H_i \gamma_j) + \frac{\alpha_2}{2} (\gamma_i H_j - H_i \gamma_j) \\ &+ \frac{\alpha_3}{2} (\gamma_i E_j + E_i \gamma_j) + \frac{\alpha_4}{2} (\gamma_i E_j - E_i \gamma_j)\end{aligned}$$

$$+\frac{\alpha_5}{2}(r_iV_j+V_ir_j)+\frac{\alpha_6}{2}(r_iV_j-V_ir_j)$$
$$+\frac{\beta_1}{2}(H_iE_j+E_iH_j)+\frac{\beta_2}{2}(H_iE_j-E_iH_j)$$
$$+\frac{\beta_3}{2}(H_iV_j+V_iH_j)+\frac{\beta_4}{2}(H_iV_j-V_iH_j)$$
$$+\frac{\gamma_1}{2}(E_iV_j+V_iE_j)+\frac{\gamma_2}{2}(E_iV_j-V_iE_j)$$

(3.18)

where, as has already been mentioned, $\Phi_i, \Omega_i, \epsilon_i, \mathcal{H}_i, \alpha_i, \beta_i$ and γ_i are all arbitrary scalar functions of the scalars constructed out of different vector fields.

3.5. Derivation of Conductivity Tensor.

The fluid equation of motion for particles of charge q, mass m, and particle number density n is given as

$$nm\left(\frac{\partial\vec{V}}{\partial t}+\vec{V}\cdot\nabla\vec{V}\right)=nq\left(\vec{E}+\vec{V}\times\vec{H}\right)-\nabla\cdot\overline{\overline{\Psi}}-nm\nabla\phi+\overline{P}$$

(3.19)

where $\vec{V}$ is the velocity of the particle,

$\vec{E}$ the D.C. electric field,

$\overline{\overline{\Psi}}$ the stress tensor,

ϕ the gravitational potential,

and $\vec{P}$ the net momentum gain.

We shall make the following approximations in the equation (3.19):

1. The charged particles experience a very negligible force due to gravity as compared to the force due to electromagnetic fields.

2. We shall assume that the pressure is isotropic and constant, so that, $\overline{\overline{\psi}}$ becomes $\vec{p}\,\delta_{ij}$ and $\nabla p = 0$.

3. We shall neglect nonlinear inertial term $(\vec{V}\cdot\nabla\vec{V})$.

4. Density of neutral particles is very high as compared to that of electrons and ions in the ionosphere; therefore we have neglected electron-electron, electron-ion, ion-ion and ion-electron collisions.

5. We shall consider the system in a steady state.

6. We shall assume that ions are singly charged i.e. z = 1, and that there exists charge neutrality, i.e., $n_i = n_e$.

Let us choose the electric field in the $\hat{xz}$ plane and magnetic field in the $\hat{z}$ direction.

In the light of the above assumptions, the equations of motion take the form

$$0 = n_i q_i [\vec{E} + \vec{V}_i \times \vec{H}] + \vec{P}_{in} \quad (3.20)$$

$$0 = -n_e q_e [\vec{E} + \vec{V}_e \times \vec{H}] + \vec{P}_{en} \quad (3.21)$$

where $\vec{P}_{in} = -m_i n_i \nu_{in} \vec{V}_i$

$\vec{P}_{en} = -m_e n_e \nu_{en} \vec{V}_e$

Here we have assumed that the neutrals are stationary.

Substituting $\vec{P}_{in}$ in equation (3.20) and $\vec{P}_{en}$ in equation (3.21), we get

$$m_i \nu_{in} \vec{V}_i = q_i [\vec{E} + \vec{V}_i \times \vec{H}] \quad (3.22)$$

$$m_e \nu_{en} \vec{V}_e = -q_e [\vec{E} + \vec{V}_e \times \vec{H}] \quad (3.23)$$

Writing equations (3.22) and (3.23) component-wise, we get

$$m_i \nu_{in} V_{ix} = q_i E_x + q_i V_{iy} H \quad (3.24)$$

$$m_i \nu_{in} V_{iy} = -q_i H V_{ix} \quad (3.25)$$

$$m_i \nu_{in} V_{iz} = q_i E_z \quad (3.26)$$

$$m_e \nu_{en} V_{ex} = -q_e E_x - q_e H V_{ey} \quad (3.27)$$

$$m_e \nu_{en} V_{ey} = q_e V_{ex} H \tag{3.28}$$

$$m_e \nu_{en} V_{ez} = -q_e E_z \tag{3.29}$$

Solving (3.24) to (3.26) we get V_{ix}, V_{iy}, V_{iz} and solving (3.27) to (3.29) we get V_{ex}, V_{ey}, V_{ez}. Substituting these values in the following equations,

$$J_x = n_e q_e (V_{ix} - V_{ex}) \tag{3.30}$$

$$J_y = n_e q_e (V_{iy} - V_{ey}) \tag{3.31}$$

$$J_z = n_e q_e (V_{iz} - V_{ez}) \tag{3.32}$$

we get

$$J_x = n_e q_e^2 \left[\frac{\nu_{en}}{m_e(\nu_{en}^2 + \Omega_{ce}^2)} + \frac{\nu_{in}}{m_i(\nu_{in}^2 + \Omega_{ci}^2)} \right] E_x \tag{3.33}$$

$$J_y = n_e q_e^2 \left[\frac{\Omega_{ce}}{m_e(\nu_{en}^2 + \Omega_{ce}^2)} - \frac{\Omega_{ci}}{m_i(\nu_{in}^2 + \Omega_{ci}^2)} \right] E_y \tag{3.34}$$

$$J_z = n_e q_e^2 \left[\frac{1}{m_e \nu_{en}} + \frac{1}{m_i \nu_{in}} \right] E_z \tag{3.35}$$

Making use of the notations for σ_0, σ_1 and σ_2 we can rewrite the equations (3.33) to (3.35) as,

$$J_x = \sigma_1 E_x \; , \; J_y = \sigma_2 E_y \; , \; J_z = \sigma_0 E_z \quad (3.36)$$

From the above discussion it is obvious that a series of untenable assumptions has gone into the derivations of the simple formulae for σ_0, σ_1 and σ_2. We shall consider in the following, a more exact formulation and show the possibility of the field components entering into various quantities. Our starting point would be the equations of motion for a three component plasma consisting of electrons, ions and neutrals. We shall also include the electron-ion and ion-electron collisions in this formulation so that we can consistently go over to 'F'-region and beyond where electron-ion and ion-electron collisions are more important than electron-neutral and ion-neutral.

$$n_e m_e \left[\frac{\partial \vec{V}_e}{\partial t} + \vec{V}_e \cdot \nabla \vec{V}_e \right] = -n_e q_e \left[\vec{E} + \vec{V}_e \times \vec{H} \right] - \nabla P_e - n_e m_e \nabla \phi + \vec{P}_{en} + \vec{P}_{ei} \quad (3.37)$$

$$n_i m_i \left[\frac{\partial \vec{V}_i}{\partial t} + \vec{V}_i \cdot \nabla \vec{V}_i \right] = n_i q_i \left[\vec{E} + \vec{V}_i \times \vec{H} \right] - \nabla P_i - n_i m_i \nabla \phi + \vec{P}_{in} + \vec{P}_{ie} \quad (3.38)$$

$$n_n m_n \left[\frac{\partial \vec{V}_n}{\partial t} + \vec{V}_n \cdot \nabla \vec{V}_n \right] = -n_n m_n \nabla \phi - \nabla P_n + \vec{P}_{ni} + \vec{P}_{ne} . \quad (3.39)$$

The momentum transfer term $\vec{P}_{kj}$ could in general be a highly nonlinear one and it is this term that introduces the nonlinearity into the problem besides the inertial term. If, however following Spitzer (1962) we write

$$\vec{P}_{kj} = -n_k \nu_{kj} m_k (\vec{V}_k - \vec{V}_j) \tag{3.40}$$

Then substituting this term in (3.37) to (3.39) and using the usual definitions for the mean velocity and the bulk current we can eliminate electron velocity and ion velocity and obtain equation of motion and generalized Ohm's law as,

$$\rho \frac{\partial \vec{V}}{\partial t} = \vec{J} \times \vec{H} - \nabla (p_i + p_e + p_n) - \rho \nabla \phi$$

$$+ \frac{\vec{J}}{e n_e} (n_i m_i \nu_{ie} - n_e m_e \nu_{ei})$$

$$+ \frac{\vec{J}}{e(n_i m_i + n_e m_e)} \left\{ m_e (n_i m_i \nu_{in} - n_n m_n \nu_{ni}) - \frac{m_i}{Z_i} (m_e n_e \nu_{en} - m_n n_n \nu_{ne}) \right\}$$

$$+ \frac{\rho (\vec{V} - \vec{V}_n)}{n_i m_i + n_e m_e} \left\{ (n_i m_i \nu_{in} - n_n m_n \nu_{ni}) + (n_e m_e \nu_{en} - n_n m_n \nu_{ne}) \right\}$$

$$\frac{\partial \vec{J}}{\partial t} = \frac{e^2}{m_i m_e} Z_i (n_i m_i + n_e m_e) (\vec{E} + \vec{V} \times \vec{H}) \tag{3.41}$$

$$+ e \vec{J} \times \vec{H} \left\{ \frac{m_e Z_i - m_i}{m_e m_i} + \frac{n_n m_n Z_i}{\rho m_i m_e} \frac{\left(\nu_{ne} \frac{m_i}{Z_i} - m_e \nu_{ni}\right)}{(\nu_{ne} + \nu_{ni})} \right\}$$

$$+ e^2 \vec{H} \times \left[\frac{z_i(n_i m_i + n_e m_e)}{\rho m_i m_e} (\nabla p_n + n_n m_n \nabla \phi) \right]$$

$$+ \vec{J} \left[\nu_{ie} + \nu_{ei} + \frac{\nu_{in} m_e n_e + \nu_{en} n_i m_i}{n_e m_e + n_i m_i} \right.$$

$$\left. + \frac{n_e(\nu_{in} - \nu_{en})}{(n_e m_e + n_i m_i)} \frac{(\nu_{ne} \frac{m_i}{z_i} - \nu_{ni} m_e)}{(\nu_{ni} + \nu_{ne})} \right]$$

$$- \frac{e\, n_e}{n_n m_n} (\nu_{in} - \nu_{en})(\nabla p_n + n_n m_n \nabla \phi)$$

(3.42)

where

$$\rho = n_i m_i + n_e m_e + n_n m_n ,$$

$$\vec{V} = (n_i m_i \vec{v}_i + n_e m_e \vec{v}_e + n_n m_n \vec{v}_n)/\rho$$

$$q_e = -e \quad , \quad q_i = z_i e$$

As one can see from the equation (3.42), in a steady state system we have currents due to the gravitational term and pressure term and this equation in general can be written as

$$A_1 \vec{E}' = A_2 \vec{J} \times \vec{H} + A_3 \vec{J} \qquad (3.43)$$

By inverting equation (3.43) and writing $\vec{J}$ in terms electric field as

$$\vec{J} = \bar{\bar{\sigma}} \cdot \vec{E}' \tag{3.44}$$

It becomes very obvious that $\bar{\bar{\sigma}}$ can be a function of the velocity and it can also be a function of the electric field $\vec{E}$, implicitly through P_n, ϕ . To avoid this complication we wrote the general form of the conductivity as given by equation (3.18).

It may now be observed that if the limit of neutral density goes over to zero (i.e. $n_n \rightarrow 0$) we do not end up with $\sigma_0 \rightarrow \infty$ and therefore when one includes ion-electron and electron-ion collisions and goes over to a region where neutral density is negligible, we do not get infinite parallel conductivity and hence the magnetic line does not become an equipotential line.

3.6. Discussion and Conclusion

The above theory has been developed by linear momentum transfer in the collision process. One can generalize this theory by taking nonlinear momentum transfer processes. Recently Jayram et al. (1973) made an effort to obtain the transport coefficients by taking into account the nonlinear processes. It is in the light of this analysis that we have written the general form of the conductivity tensor.

CHAPTER IV

IONOSPHERIC AND NON-IONOSPHERIC CURRENT SYSTEMS

4.1 Introduction

It was always thought during the first half of the century that the geomagnetic variations could be completely explained in terms of the two dimensional dynamo theory. With the advent of the satellite and the discovery of the solar wind it was found that besides the ionospheric current system there could exist other sets of currents such as magnetospheric currents and magnetopausic currents etc. These currents could also give geomagnetic field variations similar to S_q. At times it was even thought the major S_q system could be the magnetospheric one and that the ionospheric contribution could be negligible (Sarabhai and Nair (1969a,b 1971)). It is at the moment however realized that while a significant part does come from the ionosphere, the magnetospheric part is not negligible. Kane (1970) has tried to study the magnetospheric contribution and the ionospheric contribution and he showed that during day time the ionospheric contribution is the more dominant one. Olson (1970b) made a model calculation and came to the conclusion that the magnetospheric current systems contribute about 10 γ variation to the S_q field. He has also concluded that the large fluctuations in the solar wind parameters could make significant contribution towards the variability in S_q.

Kane (1971) also came to similar conclusions.

In the next section we have discussed the ionospheric current systems. In the section (4.3) we have considered the non-ionospheric current systems and we have later posed the problem as to whether we can connect these two and obtain a coherent picture.

4.2. Ionospheric current systems

In fig. (4-1), we have plotted the ΔX variation of Agincourt (geographic latitude 50.5° N, longitude 2° E and geomagnetic latitude 55.0° N) and Alibag (geographic latitude 18.5° N, longitude 72.8° E and geomagnetic latitude 9.5° N). It could be seen that Alibag has maximum at 1100 hours while Agincourt has a minimum at this time, which implies that the two stations travel under opposite current elements and we therefore have a system of closed currents in the northern hemisphere. The two stations we have selected are on the either side of the centre of the system. This point is called the focus of the system.

The situation is however different if we plot the ΔH at Alibag and that at Trivandrum (geographic latitude 8.5° N, longitude 77° E and geomagnetic latitude 1 .1° S). We observe in fig. (4-2) that while Alibag gives an amplitude of about 40 γ, that of Trivandrum is about 100 γ which is far greater than the

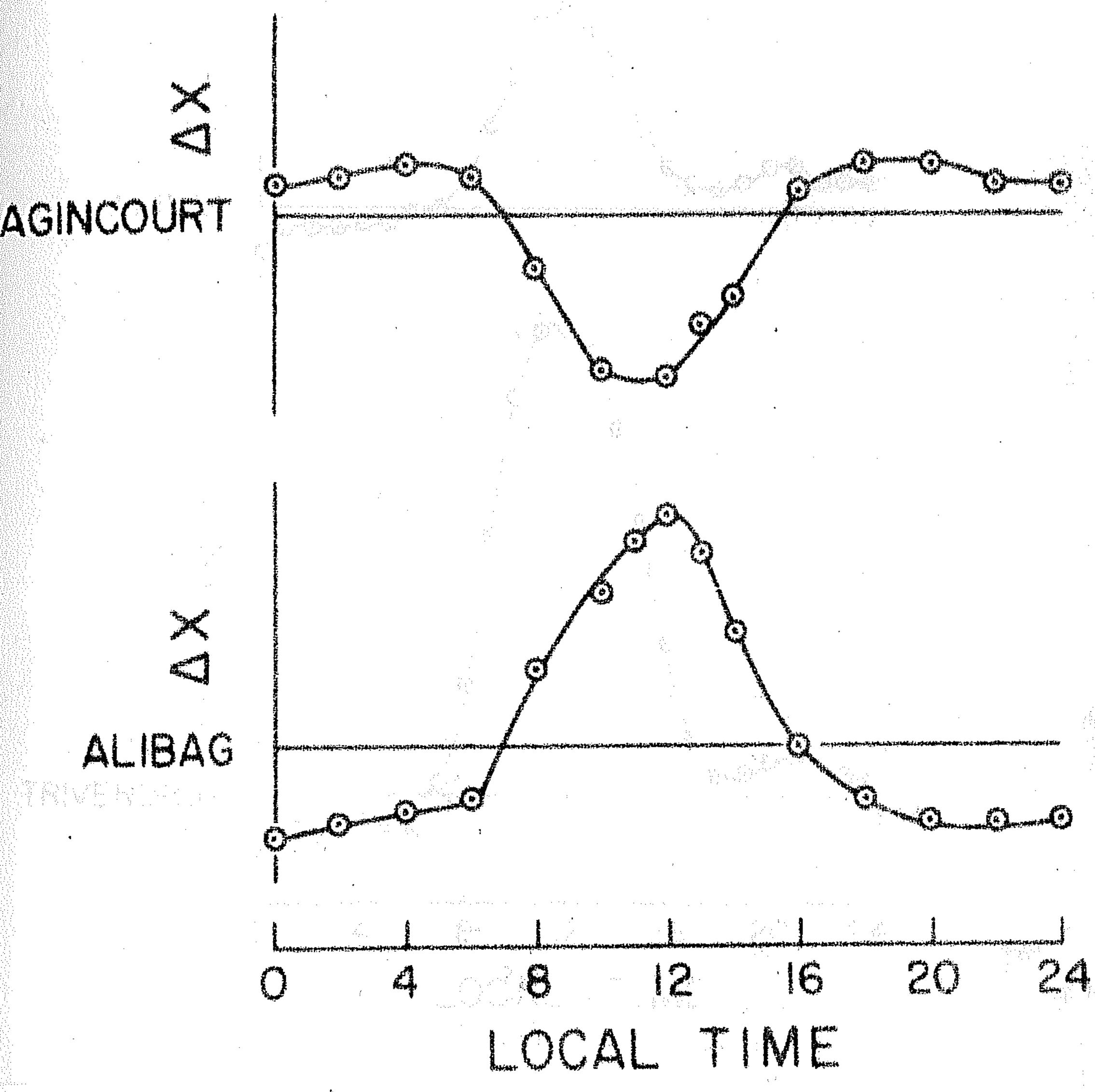

Figure (4-1) : The plot of ΔX with local time for Alibag and Agincourt. (From Vestine et al. (1947)).

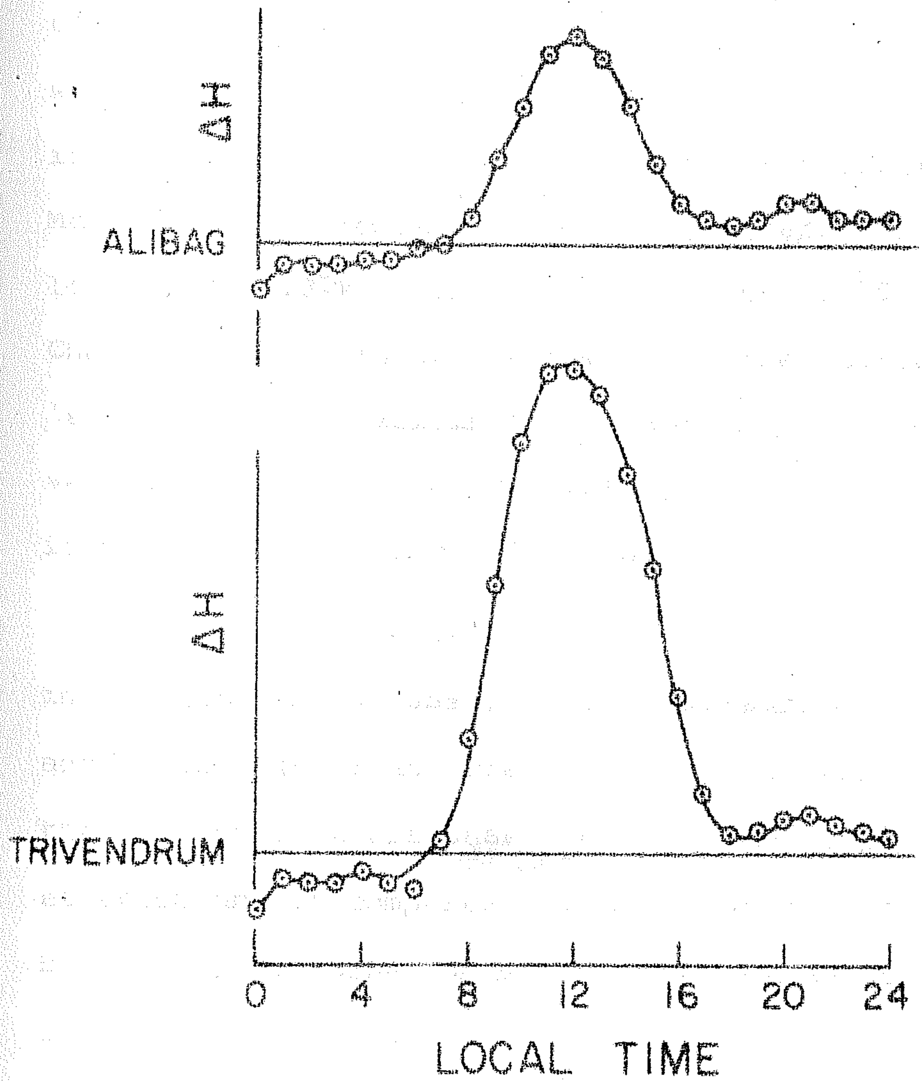

Figure (4-2) : The plot of ΔH with local time for Trivandrum and Alibag records.

S_q contribution at Trivandrum as calculated on the basic of Alibag variation. Trivandrum may have a variation of 60γ when extrapolated from the Alibag data but the actual variation is 40γ more. This phenomenon was first observed by McNish (1937) from the Huancayo (geographic latitude 12.1°S, longitude 75.3°W; geomagnetic latitude 0.6°S) data and Chapman (1951) proposed a new equatorial current system called 'electrojet' to explain this abnormality. Thus while Alibag variation is due to S_q current system, the one at Trivandrum is the combined effect of S_q and electrojet.

The global phenomenon of S_q variation was theoretically investigated by Schuster (1889), Chapman (1919), Chakrabarty and Pratap (1954) and Pratap (1957). Pratap (1955) has plotted the current contours based on his solution of the Dynamo equation and it compares favourably with that drawn by Bartles (1928) based on world wide observations. The electrojet phenomenon was however investigated in steady state by Chapman (1951), Baker and Martyn (1953), Untiedt (1967), Sugiura and Poros (1969), Forbes and Lindzen (1975). The above authors have however not taken into consideration the local irregularities existing in the ionosphere. Recently the role of local irregularities were investigated by Fejer (1959), Farley (1963), Kato (1965, 1972), Rogister (1971), 1972), Sato (1971), Ricmond (1973), Kaw et al. (1974)

by invoking cross-field and two-stream instabilities.

The S_q current system was observed by an in-situ measurement by means of rocket-borne magnetometer (Singer et al. (1961) Similar measurements were made by Sastry (1968) at Thumba and it was found that the magnetic field experiences a discontinuity when the rocket crosses the 105 Km (E-region, electrojet layer). These measurements conclusively showed that the E-region is indeed a sheet of electric current with a thickness of about 10 km. Prakash et al. (1970, 1970a, 1971b) have also studied the electron distribution at night at equatorial region using Langmuir probe and plasma noise probe and the detected irregularities in this region.

4.3. Non-Ionospheric Current Systems

(a). Magnetopause current systems

The solar wind from the Sun at supersonic speed impinging on the Earth's dipole field, creats a cavity in which the Earth's dipole field is compressed on the daylit hemisphere and blown out in the night hemisphere. This cavity is called the magnetosphere and the surface of separation is called the magnetopause. Olson (1970a) has shown that the positive and negative charged particles coming from the Sun interact with the Earth's main field and drift oppositely. These charges then form closed currents on the magnetopause as shown in the fig. (4-3). Viewing from the Sun the currents in the northern hemisphere will go counter clock-wise while those in the southern hemisphere are clock-wise. These current systems are more or less parallel to S_q current systems and therefore could enhance the field due to ionospheric S_q current system. Olson (1969,1970a) has estimated the contribution of this current system to be 3 - 4 γ (i.e. on the average $\sim$ 10 γ). This current system is identical to the one propounded by Chapman and Ferraro (1930, 1931) when they developed a theory for magnetic storm.

(b) Ring Current

Olson (1970b) has also suggested that there could exist a ring cufrent, the ring being confined to the ecliptic plane. Siscoe (1966) has argued that such a ring current should necessarily be antisymmetric with its centre displaced towards the Sun i.e. the ring will be closer to the Earth in the night hemisphere than in the daylit hemisphere. The particles in the ring currents move faster in the night side as they see greater inhomogeneity in the field. Olson (1970b,1974) made an estimate of the contribution of this ring current which turned out to be of the order of 2 γ . This indeed is a feable contribution to the equatorial station.

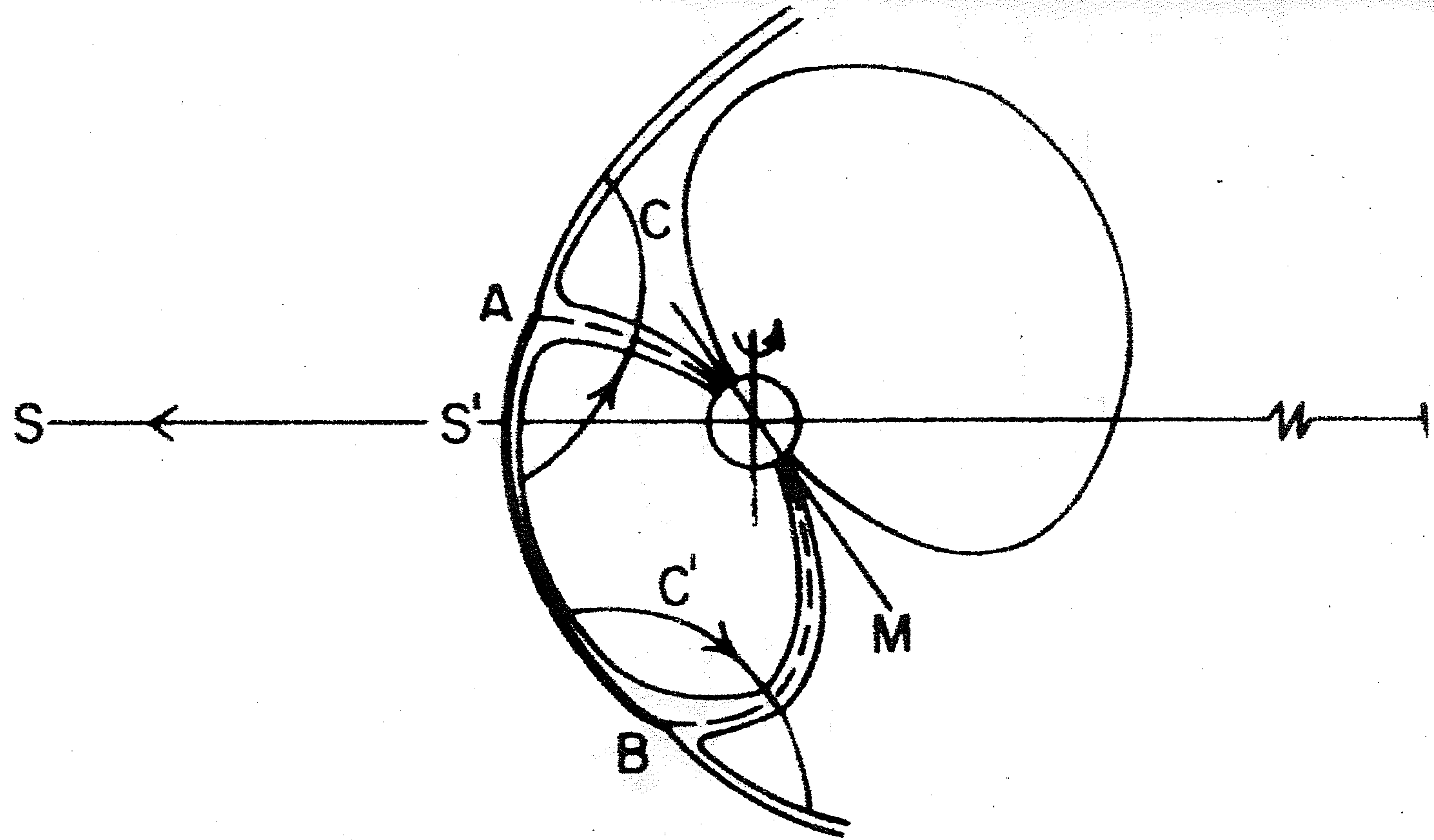

Figure (4-3) : Magnetopause current system in daylit hemisphere. M is magnetic axis, C and C' are current contours of magnetopause current system. S' is subsolar point. A and B are foci of the current system.

(c) Neutral Sheet Current

Olson (1970b) has developed a model in which he has shown the set of currents antiparallel to each other on either side of the neutral sheet to join together in the magnetopause. These currents again were estimated to contribute about 2 γ at the Earth surface; the ring field and the neutral sheet field being in the same direction. Olson and Cumming (1970) have shown from a model calculation that the magnetopause, the neutral sheet as well as the ring currents can describe most of the variation observed by A-T-S-1. It may be concluded then that the effect of these currents at A.T.S. height viz. 6.6 R_E may be significant but at the ground level the contributions are small and hence justifies Kane's (1973) finding.

4.4. Conclusion

We now come to the important problem, viz., whether one can have a three dimensional current system in the ionospheric layers which could consistently be joined to the current system in the magnetosphere and magnetopause. Pratap et al. (1973) developed a dynamo theory for the magnetosphere and we proposed to work a three dimensional dynamo theory in the ionosphere which will then be matched with their model in the magnetosphere. This theory is presented in the next Chapter.

CHAPTER V

THREE DIMENSIONAL IONOSPHERIC DYNAMO THEORY

5.1. Introduction

We have discussed in Chapter III the conductivity tensor as an explicit function of position vector $\vec{r}$. In this chapter we shall discuss the three dimensional dynamo theory with the conductivity tensor given in Chapter III. We shall represent the magnetic field by the dipole component since that represents the major part (about 90%) as discussed in Chapter III. We shall also take the diurnal mode in the velocity potential following the discussions in Chapter II.

In the next section, we shall give the three dimensional dynamo theory in which we have formulated the relevant equations. We get a system of coupled differential equations for the radial part of the potential function by imposing the condition that the current vector is divergence - free. Secondly we solved this equation by taking separately the different components of the conductivity tensor and this is possible since the equation is linear in conductivity. In Section (5.3) we have shown the method of obtaining the magnetic field induced by this current system. In the last section we have high-lighted the results and given the discussion. In solving the differential equation in r we resorted to the numerical method and this is shown in appendix I. For the sake of completeness we have added another appendix giving the recurrence relations for the Legendre functions which we have used here.

5.2 Three Dimensional Dynamo Theory

Our starting point of the theory would be the Ohm's law given by

$$\vec{J} = \bar{\bar{\sigma}} \cdot \left[\vec{E} + \vec{V} \times \vec{H} \right] \tag{5.1}$$

where $\bar{\bar{\sigma}}$ is as discussed in Chapter III, $\vec{V}$ is the velocity field, $\vec{H}$ is the magnetic field of the Earth, $\vec{E}$ is the electrostatic field and $\vec{J}$ is the current density vector.

We assume that there is no accumulation of charged particles and this necessitates a divergence - free condition, viz.,

$$\nabla \cdot \vec{J} = 0 \tag{5.2}$$

Substituting (5.1) in (5.2) we get

$$\nabla \cdot \left[\bar{\bar{\sigma}} \cdot \{ \vec{E} \} \right] = - \nabla \cdot \left[\bar{\bar{\sigma}} \cdot \{ \vec{V} \times \vec{H} \} \right] \tag{5.3}$$

The Lorentz force appearing in equation (5.3) was taken by

Möhlmann (1974) following the Helmoltz theorem as

$$\vec{\nabla} \times \vec{H} = -\nabla U + \nabla \times \vec{W} \tag{5.4}$$

where U is a scalar field and $\vec{W}$ is a vector field. Electric field $\vec{E}$ is given as

$$\vec{E} = -\nabla \Omega \tag{5.5}$$

where Ω is the electrostatic potential. With these equations (5.4) and (5.5) the equation (5.3) would be read as

$$\nabla \cdot \left[\bar{\bar{\sigma}} \cdot \{ -\nabla \Omega \} \right] = -\nabla \cdot \left[\bar{\bar{\sigma}} \cdot \{ -\nabla U + \nabla \times \vec{W} \} \right] \tag{5.6}$$

Möhlmann wrote the equation (5.6) in three parts; one being homogeneous and the other two, inhomogeneous. It may be pointed out here that this separation is really arbitrary. By this separation he achieved the following :

Ω_0 is the solution of homogeneous equation. The second equation is satisfied by

$$\Omega_1 = -U \tag{5.7}$$

So he was left with the third inhomogeneous equation. By this method he determined part of the electrostatic field. Nevertheless, when one takes the divergence - free condition of

$\vec{J}$ one still is left with Ω_2 to be determined. Therefore, one has to solve the third equation to determine Ω_2. Thus, mathematically we do not have any special advantage by this separation.

In the development of the theory here we have taken

$$\vec{V} = -\nabla\psi \tag{5.8}$$

where ψ is the velocity potential and

$$\vec{H} = -\nabla\Phi \tag{5.9}$$

with Φ as the magnetic potential so that

$$\vec{V} \times \vec{H} = \nabla\psi \times \nabla\Phi = \nabla \times (\psi\nabla\Phi) \tag{5.10}$$

We therefore have the equation for current as

$$\vec{J} = \bar{\bar{\sigma}} \cdot \left[-\nabla\Omega + \nabla \times (\psi\nabla\Phi)\right] \tag{5.11}$$

With $\Phi = \dfrac{M\cos\theta}{r^2}$, the explicit form of the equation (5.3) is given by

$$\frac{\partial}{\partial r}\left(\phi_0 \frac{\partial \Omega}{\partial r}\right) + \frac{2\phi_0}{r}\frac{\partial \Omega}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\, \phi_0 \frac{\partial \Omega}{\partial \theta}\right)$$

$$+ \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(\phi_0 \frac{\partial \Omega}{\partial \phi}\right) + \frac{\partial}{\partial r}\left(r^2\phi_1 \frac{\partial \Omega}{\partial r}\right) + 2r\phi_1\frac{\partial \Omega}{\partial r}$$

$$+\frac{1}{r\sin\theta}\left[\frac{\partial\Phi_2}{\partial\theta}\frac{\partial\Omega}{\partial\phi}-\frac{\partial\Phi_2}{\partial\phi}\frac{\partial\Omega}{\partial\theta}\right]+4\cos^2\theta\frac{\partial}{\partial r}\left(\frac{\Phi_3}{r^6}\frac{\partial\Omega}{\partial r}\right)$$

$$+2\sin\theta\cos\theta\left[\frac{\partial}{\partial r}\left(\frac{\Phi_3}{r^7}\frac{\partial\Omega}{\partial\theta}\right)+\frac{\partial}{\partial\theta}\left(\frac{\Phi_3}{r^7}\frac{\partial\Omega}{\partial r}\right)\right]$$

$$+(12\cos^2\theta-2\sin^2\theta)\frac{\Phi_3}{r^4}\frac{\partial\Omega}{\partial r}+\frac{6\sin\theta\cos\theta}{r^8}\Phi_3\frac{\partial\Omega}{\partial\theta}$$

$$+\sin\theta\frac{\partial}{\partial\theta}\left(\frac{\sin\theta\,\Phi_3}{r^8}\frac{\partial\Omega}{\partial\theta}\right)+\frac{1}{r^4}\left(\frac{\partial\Phi_4}{\partial\phi}\frac{\partial\Omega}{\partial r}-\frac{\partial\Phi_4}{\partial r}\frac{\partial\Omega}{\partial\phi}\right)$$

$$+\frac{2\cot\theta}{r^5}\left(\frac{\partial\Phi_4}{\partial\theta}\frac{\partial\Omega}{\partial\phi}-\frac{\partial\Phi_4}{\partial\phi}\frac{\partial\Omega}{\partial\theta}\right)+\cos\theta\left[2\frac{\partial}{\partial r}\left(\frac{\Phi_5}{r^5}\frac{\partial\Omega}{\partial r}\right)+\frac{5\Phi_5}{r^3}\frac{\partial\Omega}{\partial r}\right]$$

$$+\sin\theta\left[\frac{\partial}{\partial r}\left(\frac{\Phi_5}{2r^3}\frac{\partial\Omega}{\partial\theta}\right)+\frac{\partial}{\partial\theta}\left(\frac{\Phi_5}{2r^3}\frac{\partial\Omega}{\partial r}\right)+\frac{\Phi_5}{r^4}\frac{\partial\Omega}{\partial\theta}\right]$$

$$+\sin\theta\left[\frac{\partial}{\partial r}\left(\frac{\bar{\Phi}_5}{2r^3}\frac{\partial\Omega}{\partial\theta}\right)-\frac{\partial}{\partial\theta}\left(\frac{\bar{\Phi}_5}{2r^3}\frac{\partial\Omega}{\partial r}\right)+\frac{\bar{\Phi}_5}{r^4}\frac{\partial\Omega}{\partial\theta}\right]$$

$$-\frac{\cos\theta}{r^3}\bar{\Phi}_5\frac{\partial\Omega}{\partial r}$$

$$=M\left[\frac{\partial}{\partial\phi}\left(\frac{\Phi_0}{r^4}\frac{\partial\psi}{\partial r}\right)-\frac{\partial}{\partial r}\left(\frac{\Phi_0}{r^4}\frac{\partial\psi}{\partial\phi}\right)-\frac{4\Phi_0}{r^5}\frac{\partial\psi}{\partial\phi}\right.$$

$$+2\cot\theta\left\{\frac{\partial}{\partial\theta}\left(\frac{\Phi_0}{r^5}\frac{\partial\psi}{\partial\phi}\right)-\frac{\partial}{\partial\phi}\left(\frac{\Phi_0}{r^5}\frac{\partial\psi}{\partial\theta}\right)\right\}$$

$$-\frac{\partial}{\partial r}\left(\frac{\Phi_1}{r^2}\frac{\partial\psi}{\partial\phi}\right)-\frac{2\Phi_1}{r^3}\frac{\partial\psi}{\partial\phi}+\frac{\partial}{\partial\theta}\left(\frac{\sin\theta\,\Phi_2}{r^3}\frac{\partial\psi}{\partial r}\right)$$

$$-2\frac{\partial}{\partial\theta}\left(\frac{\cos\theta\,\Phi_2}{r^4}\frac{\partial\psi}{\partial\theta}\right)+\frac{\cos\theta}{r^3}\Phi_2\frac{\partial\psi}{\partial r}$$

$$-2\cot\theta\left\{\frac{\cos\theta\,\Phi_2}{r^4}\frac{\partial\psi}{\partial\theta}+\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\left(\frac{\Phi_2}{r^4}\frac{\partial\psi}{\partial\phi}\right)\right\}$$

$$- \sin^2\theta \frac{\partial}{\partial r}\left(\frac{\Phi_4}{r^6}\frac{\partial\psi}{\partial r}\right) + 4(2\cos^2\theta - 1)\frac{\Phi_4}{r^7}\frac{\partial\psi}{\partial r}$$

$$+ 2\sin\theta\cos\theta\left\{\frac{\partial}{\partial r}\left(\frac{\Phi_4}{r^7}\frac{\partial\psi}{\partial\theta}\right) + \frac{\partial}{\partial\theta}\left(\frac{\Phi_4}{r^7}\frac{\partial\psi}{\partial r}\right)\right\}$$

$$+ \frac{2\cos\theta\,(6 - 8\cos^2\theta)}{\sin\theta}\frac{\Phi_4}{r^8}\frac{\partial\psi}{\partial\theta}$$

$$- 4\cos^2\theta \frac{\partial}{\partial\theta}\left(\frac{\Phi_4}{r^8}\frac{\partial\psi}{\partial\theta}\right)$$

$$- \frac{(3\cos^2\theta + 1)}{\sin^2\theta}\frac{\partial}{\partial\phi}\left(\frac{\Phi_4}{r^8}\frac{\partial\psi}{\partial\phi}\right)$$

$$- 2\cos\theta \frac{\partial}{\partial r}\left(\frac{\Phi_5 - \bar{\Phi}_5}{2r^6}\frac{\partial\psi}{\partial\phi}\right)$$

$$- \sin\theta \frac{\partial}{\partial\theta}\left(\frac{\Phi_5 - \bar{\Phi}_5}{2r^7}\frac{\partial\psi}{\partial\phi}\right)$$

$$- \frac{3\cos\theta}{r^7}(\Phi_5 - \bar{\Phi}_5)\frac{\partial\psi}{\partial\phi}\Bigg]$$

(5.12)

where M is the dipole moment of the Earth. Φ_0, Φ_1, Φ_2, Φ_3, Φ_4, Φ_5, $\bar{\Phi}_5$ are as discussed in Chapter III. ψ is the velocity potential given by $f(r)\,P_1^1 \sin\phi$.

We shall obtain the solution of the above equation by taking the diagonal symmetric and antisymmetric parts of the conductivity tensor one at a time. In this method while we do separate the diagonal term as δ_{ij} part we have a diagonal term also appearing in the symmetric part. In principle we could have absorbed that part in δ_{ij} tensor. However, by taking it separately, we shall have no loss of generality. We shall expand electrostatic potential in a series of harmonic terms as

$$\Omega(r,\theta,\phi) = \sum_{n,m} \left\{ C_n^m(r)\cos m\phi + S_n^m(r)\sin m\phi \right\} P_n^m(\theta)$$

(5.13)

where C_n^m and S_n^m are functions of r, the radial distance.

We shall write the function $\Phi_0(r,\theta,\phi)$ appearing in the conductivity tensor as

$$\Phi_0(r,\theta,\phi) = \left\{ \Phi_{0,0}^{0}(r) + \Phi_{0,1}^{0}(r)\cos\theta + \Phi_{0,1}^{1}(r)\sin\theta\cos\phi \right\}$$

(5.14)

where $\Phi_{0,0}^{0}(r)$, $\Phi_{0,1}^{0}(r)$ and $\Phi_{0,1}^{1}(r)$ are functions of r only. It may be remembered that from this equation, one can obtain the $\cos\chi$ dependence of Φ_0 by defining suitably the functions $\Phi_{0,q}^{p}$ since we also know the dependence of conductivity on height through the electron density profile. One can choose these functions with this in mind : we get the final equation by using the orthogonality property of Y_n^m. In this we have collected the terms in the L.H.S. as coefficients of $Y_{n,c}^m$ and $Y_{n,s}^m$ as

$$\Phi_{0,0}^{0}(r)\, U_n^m\, C_n^m(r)$$

$$+\ \Phi_{0,1}^{0}(r)\, \frac{n-m}{(2n-1)}\, V_n^m\, C_{n-1}^m(r)$$

$$+\ \Phi_{0,1}^{0}(r)\, \frac{n+m+1}{2n+3}\, W_n^m\, C_{n+1}^m(r)$$

$$+\ \Phi_{0,1}^{1}(r)\, \frac{1}{2(2n-1)}\, V_n^m\, C_{n-1}^{m-1}(r)$$

$$-\ \Phi_{0,1}^{1}(r)\, \frac{1}{2(2n+3)}\, W_n^m\, C_{n+1}^{m-1}(r)$$

$$+\ \Phi_{0,1}^{1}(r)\, \frac{(n+m+1)(n+m+2)}{2(2n+3)}\, W_n^m\, C_{n+1}^{m+1}(r)$$

$$-\ \Phi_{0,1}^{1}(r)\, \frac{(n-m)(n-m-1)}{2(2n-1)}\, V_n^m\, C_{n-1}^{m+1}(r)$$

$$+\Big[\varphi_{0,0}^{0}(r)\,U_n^m\,S_n^m(r)$$

$$+\varphi_{0,1}^{0}(r)\,\frac{(n-m)}{(2n-1)}\,V_n^m\,S_{n-1}^m(r)$$

$$+\varphi_{0,1}^{0}(r)\,\frac{(n+m+1)}{(2n+3)}\,W_n^m\,S_{n+1}^m(r)$$

$$+\varphi_{0,1}^{1}(r)\,\frac{1}{2(2n-1)}\,V_n^m\,S_{n-1}^{m-1}(r)$$

$$-\varphi_{0,1}^{1}(r)\,\frac{1}{2(2n+3)}\,W_n^m\,S_{n+1}^{m-1}(r)$$

$$+\varphi_{0,1}^{1}(r)\,\frac{(n+m+1)(n+m+2)}{2(2n+3)}\,W_n^m\,S_{n+1}^{m+1}(r)$$

$$-\varphi_{0,1}^{1}(r)\,\frac{(n-m)(n-m-1)}{2(2n-1)}\,V_n^m\,S_{n-1}^{m+1}(r)\Big]$$

$$= M\Big[P_2^2\cos 2\phi\left\{\frac{\varphi_{0,1}^{0}(r)}{6r^4}\,f'(r)-\frac{f(r)}{6}\,\frac{1}{r^4}\,\frac{\partial\varphi_{0,1}^{0}(r)}{\partial r}\right\}$$

$$+P_2^1\cos\phi\left\{-\frac{f(r)}{3r^4}\,\frac{\partial\varphi_{0,1}^{0}(r)}{\partial r}-\frac{2f(r)}{3r^5}\,\varphi_{0,1}^{0}(r)\right\}$$

$$+P_1^1\cos\phi\left\{-\frac{f(r)}{r^4}\,\frac{\partial\varphi_{0,0}^{0}(r)}{\partial r}\right\}\Big]$$

$$+ P_2^0 \left\{ \frac{f'(r)}{3r^4} \varphi_{0,1}^1(r) + \frac{f(r)}{3r^4} \frac{\partial \varphi_{0,1}^1(r)}{\partial r} + \frac{4 f(r)}{3r^5} \varphi_{0,1}^1(r) \right\}$$

$$+ P_\phi^0 \left\{ - \frac{f'(r)}{3r^4} \varphi_{0,1}^1(r) - \frac{f(r)}{3r^4} \frac{\partial \varphi_{0,1}^1}{\partial r} + \frac{2}{3} \frac{f(r)}{r^5} \varphi_{0,1}^1(r) \right\} \Bigg] \tag{5.15}$$

where the operators U_n^m, V_n^m and W_n^m are defined as

$$U_n^m = \frac{1}{r^2} \frac{d}{dr}\left(r^2 \frac{d}{dr}\right) + \frac{d}{dr}\left(\log \varphi_{0,q}^p(r)\right) \frac{d}{dr} - \frac{n(n+1)}{r^2} \tag{5.16}$$

$$V_n^m = \frac{1}{r^2} \frac{d}{dr}\left(r^2 \frac{d}{dr}\right) + \frac{d}{dr}\left(\log \varphi_{0,q}^p(r)\right) \frac{d}{dr} - \frac{(n^2-1)}{r^2} \tag{5.17}$$

$$W_n^m = \frac{1}{r^2} \frac{d}{dr}\left(r^2 \frac{d}{dr}\right) + \frac{d}{dr}\left(\log \varphi_{0,q}^p(r)\right) \frac{d}{dr} - \frac{n(n+2)}{r^2} \tag{5.18}$$

with p, q taking values 0 or 1 and $f'(r) = \dfrac{df(r)}{dr}$

In solving the equation (5.15) we multiply it by $Y_{n'c}^{m'}$ $Y_{n's}^{m'}$ and integrate over θ and ϕ. We realize that the only surviving terms would be those harmonics that exist in R.H.S. of the equation. Since $Y_{n,c}^{m}$ and $Y_{n,s}^{m}$ are independent functions. We can write the above equation as two sets and then we find that on R.H.S. we do not have any term of Y_{ns}^{m} therefore we get a set of equations for Y_{nc}^{m}. Again we get a series of equations by setting m = n = 2, m = 1 and n = 2, n = m = 1, and n = 2, 1, 0 when m = 0.

For m = n = 2 we get the differential equation as

$$\Phi_{0,0}^{0}(r)\,U_2^2 C_2^2 + \frac{5}{7}\Phi_{0,1}^{0}(r)\,W_2^2 C_3^2 + \frac{\Phi_{0,1}^{1}(r)}{6}V_2^2 C_1^1 - \frac{\Phi_{0,1}^{1}(r)}{14}W_2^2 C_3^1 + \frac{15}{7}\Phi_{0,1}^{1}(r)\,W_2^2 C_3^3$$
$$= M\left[\frac{\Phi_{0,1}^{1}(r)\,f'(r)}{6r^4} - \frac{f(r)}{6r^4}\frac{\partial \Phi_{0,1}^{1}(r)}{\partial r}\right] \qquad (5.19)$$

$$\Phi_{0,0}^{0}(r)\,U_2^2 S_2^2 + \frac{5}{7}\Phi_{0,1}^{0}(r)\,W_2^2 S_3^2 + \frac{\Phi_{0,1}^{1}(r)}{6}V_2^2 S_1^1 - \frac{\Phi_{0,1}^{1}(r)}{14}W_2^2 S_3^1 + \frac{15}{7}\Phi_{0,1}^{1}(r)\,W_2^2 S_3^3 = 0$$

(5.20)

As one can realize that this set of equations forms a hierarchy of equations i.e. to determine C_1^1 we should know C_3^1, C_2^2, C_3^2 and C_3^3. Similarly in determining C_3^1 we have to know higher terms. Therefore we shall be getting an open chain with m and n going to infinity. However all the higher equations would be homogeneous with the R.H.S. being zero. Therefore one can break this hierarchy by defining $C_3^1 = C_2^2 = C_3^2 = C_3^3 = 0$. A similar definition can be imposed on S_n^m as well. Under this assumption the equations reduce to

$$\frac{\varphi_{0,1}^{1}(r)}{6}\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)+\frac{d}{dr}\left(\log\varphi_{0,1}^{1}(r)\right)\frac{d}{dr}-3/r^2\right]C_1^1(r)$$

$$= M\left[\frac{\varphi_{0,1}^{1}(r)}{6r^4}f(r)-\frac{f(r)}{6r^4}\frac{d\varphi_{0,1}^{1}(r)}{dr}\right] \tag{5.21}$$

and

$$\frac{\varphi_{0,1}^{1}(r)}{6}\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)+\frac{d}{dr}\left(\log\varphi_{0,1}^{1}(r)\right)\frac{d}{dr}-\frac{3}{r^2}\right]S_1^1(r)=0$$

$$\tag{5.22}$$

In solving these equation (5.21) and (5.22) we have to choose the function $\varphi_{0,q}^{p}(r)$ and $f(r)$. Since the electron density profile is a combination of a Gaussian and exponentially decaying function of height, we choose here the radial part of conductivity

$$\Phi_{0,q}^{p}(r) \quad \text{as} \quad r^2 \exp\{-\alpha(r-h)^2\} \quad .$$

This Gaussian distribution has a maximum at $r \doteq h$ and then decays with neight. We shall leave h and α as two parameters. The function $f(r)$ appearing in the velocity potential may be taken as proportional to r i.e. $f(r) \simeq r$ and hence feeding these forms into the above equations we get the final equation for C_1^1 as

$$\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr} - 3/r^2\right] C_1^1(r)$$

$$= -\frac{M}{r^4}\left[1 - 2\alpha r(r-h)\right] \qquad (5.23)$$

and similar equation for S_1^1

$$\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr} - 3/r^2\right] S_1^1(r) = 0$$

$$(5.24)$$

when we take $m = 1, n = 2$ we get the defining equations for C_1^0 and S_1^0 as follows.

$$\frac{1}{2}\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr} - 3/r^2\right] C_1^0(r)$$

$$+ \left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr} - 3/r^2\right] C_1^1(r)$$

$$= M\left[-\frac{4}{r^4} + \frac{2\alpha(r-h)}{r^3}\right] \qquad (5.25)$$

$$\frac{1}{2}\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\frac{d}{dr}\right\} - 3/r^2\right]S_1^0(r)$$

$$+\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\frac{d}{dr}\right\} - 3/r^2\right]S_1^1(r)$$

$$= 0$$

(5.26)

In the equation (5.25) C_1^0 can be determined once we know C_1^1 from equation (5.23). Similarly S_1^0 can be determined when once we know S_1^1 from equation (5.24).

In this way we get a set of coupled equations viz., C_1^0 being coupled to C_1^1 and S_1^0 to S_1^1 .

The choice m = 0 and n = 2 gives the defining equations for C_1^{-1} and S_1^{-1} as

$$\frac{1}{2}\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr} - 3/r^2\right]C_1^{-1}(r)$$

$$+2\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr} - 3/r^2\right]C_1^0(r)$$

$$-\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr} - 3/r^2\right]C_1^1(r)$$

$$= \frac{M}{r^4}\left[7 - 2\alpha r(r-h)\right]$$

(5.27)

$$\frac{1}{2}\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}-3/r^2\right]S_1^{-1}(r)$$

$$+2\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}-3/r^2\right]S_1^{0}(r)$$

$$-\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}-3/r^2\right]S_1^{1}(r)=0 \tag{5.28}$$

We shall take now the set n = 1. In this m can have either 0 or 1. The set m = 1, n = 1 gives the defining equations for C_0^0 and S_0^0. The explicit form of these equations is

$$\frac{1}{2}\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}\right]C_0^0(r)$$

$$+\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}-\frac{2}{r^2}\right]C_1^1(r)$$

$$=-\frac{2M}{r^4}\left[1-\alpha r(r-h)\right] \tag{5.29}$$

$$\frac{1}{2}\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}\right]S_0^0(r)$$

$$+\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}-2/r^2\right]S_1^1(r)=0 \tag{5.30}$$

The remaining member in this set is m = 0, n = 1 and this gives defining equations for C_2^{-1} and S_2^{-1}. These equations are both homogeneous, i.e. they are explicitly independent of velocity field but they are however implicitly dependent on the velocity field through C_0^0 and C_1^0 or S_0^0 and S_1^0

$$-\frac{1}{10}\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}-3/r^2\right]C_2^{-1}(r)$$
$$+\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}\right]C_0^0(r)$$
$$+\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}-2/r^2\right]C_1^0(r)=0 \tag{5.31}$$

$$-\frac{1}{10}\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}-3/r^2\right]S_2^{-1}(r)$$
$$+\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}\right]S_0^0(r)$$
$$+\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}-2/r^2\right]S_1^0(r)=0 \tag{5.32}$$

The final set, viz., m = 0 and n = 0 gives the compatibility condition between these determined coefficients and this condition turns out to be

$$\frac{1}{3}\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}\right]c_1^1(r)$$

$$+\frac{1}{3}\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}\right]c_1^0(r)$$

$$+\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}\right]c_0^0(r)$$

$$+\frac{1}{6}\left[\frac{d^2}{dr^2}+\left\{\frac{4}{r}-2\alpha(r-h)\right\}\frac{d}{dr}\right]c_1^{-1}(r)$$

$$=-\frac{M}{3r^3}\left[\frac{1}{r}-2\alpha(r-h)\right] \tag{5.33}$$

Equation (5.33) can be written in algebraic form by using equations (5.23), (5.25), (5.27) and (5.29) as

$$\frac{1}{3}\left[3/_{r^2}\, c_1^1(r)-\frac{M}{r^4}\left\{1-2\alpha r(r-h)\right\}\right]$$

$$+\frac{1}{3}\Big[2\Big\{-\frac{2M}{r^4}\big(2-\alpha r(r-h)\big)-\frac{M}{r^4}\big(1-2r\alpha(r-h)\big)\Big\}$$

$$+\ {}^{3}/_{r^2}\ C_1^0(r)\ \Big]$$

$$+\Big[2\Big\{-\frac{2M}{r^4}\big(1-\alpha r(r-h)\big)+\frac{M}{r^4}\big(1-2r\alpha(r-h)\big)\Big\}$$

$$-\ {}^{2}/_{r^2}\ C_1^1(r)\ \Big]$$

$$+\frac{1}{6}\Big[2\Big\{\frac{M}{r^4}\big(7-2\alpha r(r-h)\big)+\frac{M}{r^4}\big(1-2\alpha r(r-h)\big)$$

$$+2\Big(-\frac{2M}{r^4}\big(2-\alpha r(r-h)\big)-\frac{M}{r^4}\big(1-2\alpha r(r-h)\big)\Big)\Big\}$$

$$+\ {}^{3}/_{r^2}\ C_1^{-1}(r)\ \Big]=-\frac{M}{3r^4}\big[1-2\alpha r(r-h)\big]$$

(5.34)

With the above set of coefficients we can write the electrostatic potential function Ω as

$$\Omega(r,\theta,\phi)=P_1^1\big[C_1^1(r)\cos\phi+S_1^1(r)\sin\phi\big]$$

$$+\ P_1^0\,C_1^0(r)\ +\ C_0^0(r)$$

$$+\ P_1^{-1}\big[C_1^{-1}(r)\cos\phi-S_1^{-1}(r)\sin\phi\big]$$

$$+\ P_2^{-1}\big[C_2^{-1}(r)\cos\phi-S_2^{-1}(r)\sin\phi\big]$$

(5.35)

We use

$$P_n^{-m} = (-1)^n \frac{\Gamma(n-m+1)}{\Gamma(n+m+1)} P_n^m \qquad (5.36)$$

in (5.35)

This gives the final form of the current vector as

$$\vec{J} = \varphi_0 \delta_{ij} \left[-\nabla \Omega + \vec{V} \times \vec{H} \right] \qquad (5.38)$$

where δ_{ij} is the diagonal tensor. Writing the r, θ, ϕ components of equation (5.38), we have,

$$J_r = r^2 \exp\{-\alpha(r-h)^2\}\{1 + \cos\theta + \sin\theta\cos\phi\} \cdot \left\{ -\frac{\partial \Omega}{\partial r} + \frac{M P_1^1 \cos\phi}{r^3} \right\} \qquad (5.39)$$

$$J_\theta = r^2 \exp\{-\alpha(r-h)^2\}\{1 + \cos\theta + \sin\theta\cos\phi\} \cdot \left\{ -\frac{1}{r}\frac{\partial \Omega}{\partial \theta} - 2\frac{M P_1^0 \cos\phi}{r^3} \right\} \qquad (5.40)$$

$$J_\phi = r^2 \exp\{-\alpha(r-h)^2\}\{1+\cos\theta+\sin\theta\cos\phi\}$$

$$\cdot\left\{-\frac{1}{r\sin\theta}\frac{\partial\Omega}{\partial\phi}+\frac{M\left(2(P_1^0)^2-(P_1^1)^2\right)\sin\phi}{r^3}\right\}$$

$$\text{where} \quad P_1^1 = \sin\theta \; , \quad P_1^0 = \cos\theta \tag{5.41}$$

We shall now take the second part of the conductivity tensor viz. $\Phi_1 r_i r_j$. If we again expand Ω and Φ_1 we get the dynamo equation as

$$\Phi_{1,0}^{0}(r)\; O_{1,0}^{0}\;\{C_n^m(r)+S_n^m(r)\}$$

$$+\frac{(n-m)}{(2n-3)}\;\Phi_{1,1}^{0}(r)\;O_{1,1}^{0}\;\{C_{n-1}^m(r)+S_{n-1}^m(r)\}$$

$$+\frac{(n+m+1)}{(2n+1)}\;\Phi_{1,1}^{0}(r)\;O_{1,1}^{0}\;\{C_{n+1}^m(r)+S_{n+1}^m(r)\}$$

$$+\frac{1}{2(2n-3)}\;\Phi_{1,1}^{1}(r)\;O_{1,1}^{1}\;\{C_{n-1}^{m-1}(r)+S_{n-1}^{m-1}(r)\}$$

$$-\frac{1}{(2n+1)}\;\Phi_{1,1}^{1}(r)\;O_{1,1}^{1}\;\{C_{n+1}^{m-1}+S_{n+1}^{m-1}\}$$

$$+\Phi_{1,1}^{1}(r)\,\frac{(n+m+1)(n+m+2)}{2(2n+1)}\,O_{1,1}^{1}\left\{C_{n+1}^{m+1}(r)+S_{n+1}^{m+1}(r)\right\}$$

$$-\Phi_{1,1}^{1}(r)\,\frac{(n-m)(n-m-1)}{2(2n-3)}\,O_{1,1}^{1}\left\{C_{n-1}^{m+1}(r)+S_{n-1}^{m+1}(r)\right\}$$

$$=M\left[P_2^2\cos 2\phi\left\{-\frac{\Phi_{1,1}^{1}(r)}{6r^2}f'(r)-\frac{f(r)}{6r^2}\frac{\partial\Phi_{1,1}^{1}(r)}{\partial r}\right\}\right.$$

$$+P_2^1\cos\phi\left\{-\frac{\Phi_{1,1}^{0}(r)}{3r^2}f'(r)-\frac{f(r)}{3r^2}\frac{d\Phi_{1,1}^{0}(r)}{dr}\right\}$$

$$+P_1^1\cos\phi\left\{\frac{\Phi_{1,0}^{0}(r)}{r^2}f'(r)+\frac{f(r)}{r^2}\frac{d\Phi_{1,0}^{0}(r)}{dr}\right\}$$

$$+P_2^0\left\{\frac{\Phi_{1,1}^{1}(r)}{3r^2}f'(r)+\frac{f(r)}{3r^2}\frac{d\Phi_{1,1}^{1}(r)}{dr}\right\}$$

$$\left.+P_0^0\left\{-\frac{\Phi_{1,1}^{1}(r)}{3r^2}f'(r)-\frac{f(r)}{3r^2}\frac{d\Phi_{1,1}^{1}(r)}{dr}\right\}\right]$$

(5.42)

where

$$O_{1,q}^{p} = r^2 \frac{d^2}{dr^2} + 4r \frac{d}{dr} + r^2 \frac{d}{dr}\left\{\log\left(\varphi_{1,q}^{p}(r)\right)\right\}\frac{d}{dr}$$

(5.43)

p and q take the values 0 or 1.

Here again we do not get odd harmonics in the R.H.S. but only the even harmonics. For the 1st term $P_2^2 \cos 2\phi$ we can take m = 2, n = 2 and we get the equation for $C_1^1(r)$ as

$$\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr}\right] C_1^1(r)$$
$$= -M/3r^4 \left[1 - 2\alpha r(r-h)\right] \quad (5.44)$$

The corresponding equation for $S_1^1(r)$ is homogeneous viz.,

$$\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr}\right] S_1^1(r) = 0 \quad (5.45)$$

where we have chosen this time, the r dependence of $\varphi_{1,0}^{0}(r)$, $\varphi_{1,1}^{0}(r)$ and $\varphi_{1,1}^{1}(r)$ as $\exp\{-\alpha(r-h)^2\}$.

For m = 1, n = 2 we get the defining equations for C_1^0

and S_1^0 as

$$\left[\frac{d^2}{dr^2} + \left\{ \frac{4}{r} - 2\alpha(r-h) \right\} \frac{d}{dr} \right] C_1^0(r)$$

$$+ 2 \left[\frac{d^2}{dr^2} + \left\{ \frac{4}{r} - 2\alpha(r-h) \right\} \frac{d}{dr} \right] C_1^1(r)$$

$$= - \frac{2M}{3r^4} \left[1 - 2\alpha r(r-h) \right] \qquad (5.46)$$

$$\left[\frac{d^2}{dr^2} + \left\{ \frac{4}{r} - 2\alpha(r-h) \right\} \frac{d}{dr} \right] S_1^0(r)$$

$$+ 2 \left[\frac{d^2}{dr^2} + \left\{ \frac{4}{r} - 2\alpha(r-h) \right\} \frac{d}{dr} \right] S_1^1(r) = 0 \qquad (5.47)$$

As we have seen before the equation for S_1^0 depends only on S_1^1 otherwise we have no term on the R.H.S.

Similarly for the set m = 0, n = 2 we get the equations for C_1^{-1} and S_1^{-1}

$$\left[\frac{d^2}{dr^2} + \left\{ \frac{4}{r} - 2\alpha(r-h) \right\} \frac{d}{dr} \right] C_1^{-1}(r)$$

$$+ 4 \left[\frac{d^2}{dr^2} + \left\{ \frac{4}{r} - 2\alpha(r-h) \right\} \frac{d}{dr} \right] C_1^0(r)$$

$$- 2 \left[\frac{d^2}{dr^2} + \left\{ \frac{4}{r} - 2\alpha(r-h) \right\} \frac{d}{dr} \right] C_1^1(r)$$

$$= \frac{2M}{3r^4} \left[1 - 2\alpha r(r-h) \right] \qquad (5.48)$$

$$\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr}\right] S_1^{-1}(r)$$

$$+4\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr}\right] S_1^{0}(r)$$

$$-2\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr}\right] S_1^{1}(r) = 0 \qquad (5.49)$$

When we consider next set of value m = 1, n = 1 we get the defining equations for C_0^0 and S_0^0 as

$$\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr}\right] C_0^0(r)$$

$$+2\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr}\right] C_1^1(r) \qquad (5.50)$$

$$= -\frac{M}{2r^4}\left[1 - 2\alpha r(r-h)\right]$$

$$\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr}\right] S_0^0(r)$$

$$+2\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr}\right] S_1^1(r) = 0$$

(5.51)

As we have seen in previous case, for $\bar{\bar{\sigma}} = \varphi_0 \delta_{ij}$ we got the compatibility condition for m = 0, n = 0. Here also we get a compatibility condition for m = 0 and n = 0 as

$$
\begin{aligned}
&2\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr}\right] c_1^1(r) \\
&+2\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr}\right] e_1^0(r) \\
&+2\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr}\right] c_0^0(r) \\
&+\left[\frac{d^2}{dr^2} + \left\{\frac{4}{r} - 2\alpha(r-h)\right\}\frac{d}{dr}\right] c_1^{-1}(r) \\
&= -\frac{2M}{3r^4}\left[1 - 2\alpha r(r-h)\right]
\end{aligned}
\tag{5.52}
$$

Equation (5.52) can be written in algebraic form by using equations (5.44), (5.46), (5.48) and (5.50) as

$$
\begin{aligned}
&2\left[-\frac{M}{3r^4}\left\{1 - 2\alpha r(r-h)\right\}\right] \\
&+2\left[\frac{2M}{3r^4}\left\{1 - 2\alpha r(r-h)\right\} - \frac{2M}{3r^4}\left\{1 - 2\alpha r(r-h)\right\}\right]
\end{aligned}
$$

$$+2\left[\frac{2M}{3r^4}\{1-2\alpha r(r-h)\}-\frac{M}{2r^4}\{1-2\alpha(r-h)r\}\right]$$

$$+\left[-\frac{2M}{3r^4}\{1-2\alpha(r-h)r\}+\frac{2M}{3r^4}\{1-2\alpha r(r-h)\}\right.$$

$$\left.-4\left\{-\frac{2M}{3r^4}\left(1-2\alpha r(r-h)\right)-\frac{2M}{3r^4}\left(1-2\alpha r(r-h)\right)\right)\right]$$

$$=-\frac{2M}{3r^4}\left[1-2\alpha r(r-h)\right] \qquad (5.53)$$

With the above set of coefficients we can write the electric potential function Ω as

$$\Omega(r,\theta,\varphi)=P_1^1\left\{C_1^1(r)\cos\varphi+S_1^1(r)\sin\varphi\right\}$$
$$+P_1^0C_1^0(r)+R_0^0(r)$$
$$+P_1^{-1}\left\{C_1^{-1}(r)\cos\varphi-S_1^{-1}\sin\varphi\right\} \qquad (5.54)$$

We use equation (5.36). This gives the final form of the current vector as

$$\vec{J} = \varphi_1 r_i r_j \left[-\nabla\Omega + \vec{V} \times \vec{H} \right] \tag{5.55}$$

where $r_i r_j$ is the symmetric tensor. Writing the components of equation (5.55), we have,

$$\begin{aligned} J_r = {} & \exp\{-\alpha(r-h)^2\}\left[1+\cos\theta+\sin\theta\cos\varphi\right] \\ & r^2 \sin\theta\cos\varphi\left[\sin\theta\cos\varphi\left\{-\frac{\partial\Omega}{\partial r}+\frac{M\rho_1^1\cos\varphi}{r^3}\right\}\right. \\ & +\sin\theta\sin\varphi\left\{-\frac{1}{r}\frac{\partial\Omega}{\partial\theta}-\frac{2M\rho_1^0\cos\varphi}{r^3}\right\} \\ & \left.+\cos\theta\left\{-\frac{1}{r\sin\theta}\frac{\partial\Omega}{\partial\varphi}+\frac{M}{r^3}\left(2(\rho_1^0)^2-(\rho_1^1)^2\right)\sin\varphi\right\}\right] \end{aligned} \tag{5.56}$$

$$\begin{aligned} J_\theta = {} & \exp\{-\alpha(r-h)^2\}\left[1+\cos\theta+\sin\theta\cos\varphi\right] \\ & r^2 \sin\theta\sin\varphi\left[\sin\theta\cos\varphi\left\{-\frac{\partial\Omega}{\partial r}+\frac{M\rho_1^1\cos\varphi}{r^3}\right\}\right. \\ & +\sin\theta\sin\varphi\left\{-\frac{1}{r}\frac{\partial\Omega}{\partial\theta}-\frac{2M\rho_1^0\cos\varphi}{r^3}\right\} \\ & +\cos\theta\left\{-\frac{1}{r\sin\theta}\frac{\partial\Omega}{\partial\varphi}+\frac{M}{r^3}\left(2(\rho_1^0)^2-(\rho_1^1)^2\right)\right. \\ & \left.\left.\sin\varphi\right\}\right] \end{aligned} \tag{5.57}$$

$$J_\varphi = \exp\{-\alpha(r-h)^2\}\left[1+\cos\theta+\sin\theta\cos\varphi\right]$$

$$r^2\cos\theta\left[\sin\theta\cos\varphi\left\{-\frac{\partial\Omega}{\partial r}+m\frac{P_1^1\cos\varphi}{r^3}\right\}\right.$$

$$+\sin\theta\sin\varphi\left\{-\frac{1}{r}\frac{\partial\Omega}{\partial\theta}-\frac{2m\,P_1^0\cos\varphi}{r^3}\right\}$$

$$\left.+\cos\theta\left\{-\frac{1}{r\sin\theta}\frac{\partial\Omega}{\partial\varphi}+m\left(\frac{2(P_1^0)^2-(P_1^1)^2}{r^3}\right)\sin\varphi\right\}\right]$$

(5.58)

where P_1^1 and P_1^0 as defined earlier.

The next symmetric term in the conductivity function is the one containing the magnetic field i.e. $\bar{\bar{\sigma}} = \Phi_3 H_i H_j$. Since on the R.H.S. we have the term $\vec{\nabla}\times\vec{H}$, the scalar product of the tensor $\bar{\bar{\sigma}}$ with the vector $\vec{\nabla}\times\vec{H}$ becomes zero. Thus we get a homogeneous equation.

The fourth symmetric part is the one containing $\frac{\Phi_5}{2}(r_i H_j + H_i r_j)$. The general equation corresponding to this is given as

$$\frac{(n+m+3)(n+m+2)(n+m+1)}{2(2n+5)(2n+3)}\left[\frac{2\varphi_{5,1}^1(r)}{r^2}\frac{d^2}{dr^2}+\frac{\varphi_{5,1}^1(r)}{2r^3}\frac{d}{dr}\right.$$

$$\left.+\left(\frac{2}{r^2}\frac{d\varphi_{5,1}^1(r)}{dr}+\frac{\varphi_{5,1}^1(r)}{r^3}\right)\frac{d}{dr}-(n+3)\left\{\frac{\varphi_{5,1}^1}{r^3}\frac{d}{dr}+\left(\frac{1}{2r^3}\frac{d\varphi_{5,1}^1}{dr}-\frac{\varphi_{5,1}^1}{2r^4}\right)\right\}\right]$$

$$\times \left[C_{n+2}^{m+1} + S_{n+2}^{m+1} \right]$$

$$+ \frac{(n-m)}{2(2n+1)} \Bigg[\left[\left\{ \frac{(n+m+2)(n+m+1)}{(2n+3)} - \frac{(n-m-1)}{(2n-1)} \right\} \right.$$

$$\left\{ \frac{2\varphi_{s,1}^{1}(r)}{r^2} \frac{d^2}{dr^2} + \left(\frac{2}{r^2} \frac{d\varphi_{s,1}^{1}(r)}{dr} + \frac{\varphi_{s,1}^{1}(r)}{r^3} \right) \frac{d}{dr} \right\}$$

$$+ \left\{ (n+m+1) \left(\frac{n(n+m+2)}{(2n+3)} + \frac{(n+1)(n-m-1)}{(2n-1)} \right) \right\}$$

$$\left\{ \frac{\varphi_{s,1}^{1}(r)}{r^3} \frac{d}{dr} + \left(\frac{1}{2r^3} \frac{d\varphi_{s,1}^{1}(r)}{dr} - \frac{\varphi_{s,1}^{1}(r)}{2r^4} \right) \right\}$$

$$+ \left\{ \frac{(n+m+1)(n+m)}{(2n-1)} - \frac{(n-m+1)(n+m-1)}{(2n+3)} \right\}$$

$$\left. \left\{ \frac{\varphi_{s,1}^{1}(r)}{2r^3} \frac{d}{dr} \right\} \right] \left[C_n^{m+1} + S_n^{m+1} \right]$$

$$- \frac{(n-m-1)(n-m)(n-m-2)}{2(2n-3)(2n-1)} \Bigg[\frac{2\varphi_{s,1}^{1}(r)}{r^2} \frac{d^2}{dr^2} + \frac{\varphi_{s,1}^{1}(r)}{2r^3} \frac{d}{dr}$$

$$+ \left(\frac{2}{r^2} \frac{d\varphi_{s,1}^{1}(r)}{dr} + \frac{\varphi_{s,1}^{1}(r)}{r^3} \right) \frac{d}{dr}$$

$$+ (n-2) \left\{ \frac{\varphi_{s,1}^{1}(r)}{r^3} \frac{d}{dr} + \left(\frac{1}{2r^3} \frac{d\varphi_{s,1}^{1}(r)}{dr} - \frac{\varphi_{s,1}^{1}(r)}{2r^4} \right) \right\} \Bigg]$$

$$\times \left[C_{n-2}^{m+1} + S_{n-2}^{m+1} \right]$$

$$+\frac{(n+m+2)(n+m+1)}{(2n+5)(2n+3)}\left[2\frac{\varphi_{s,1}^{0}(r)}{r^2}\frac{d^2}{dr^2}+\frac{\varphi_{s,1}^{0}(r)}{2r^3}\frac{d}{dr}\right.$$

$$+\left\{\frac{2}{r^2}\frac{d\varphi_{s,1}^{0}(r)}{dr}+\frac{\varphi_{s,1}^{0}(r)}{r^3}\right\}\frac{d}{dr}$$

$$\left.-(n+3)\left\{\frac{\varphi_{s,1}^{0}(r)}{r^3}+\left(\frac{1}{2r^3}\frac{d\varphi_{s,1}^{0}(r)}{dr}-\frac{\varphi_{s,1}^{0}(r)}{2r^4}\right)\right\}\right]$$

$$\left[C_{n+2}^{m}+S_{n+2}^{m}\right]$$

$$+\frac{(n+m+1)}{(2n+1)}\left[\frac{2\varphi_{s,0}^{0}(r)}{r^2}\frac{d^2}{dr^2}+\left\{\frac{2}{r^2}\frac{d\varphi_{s,0}^{0}(r)}{dr}+\frac{\varphi_{s,0}^{0}(r)}{r^3}\right\}\frac{d}{dr}\right.$$

$$\left.-(n+2)\left\{\frac{\varphi_{s,0}^{0}(r)}{r^3}\frac{d}{dr}+\left(\frac{1}{2r^3}\frac{d\varphi_{s,0}^{0}(r)}{dr}-\frac{\varphi_{s,0}^{0}(r)}{2r^4}\right)\right\}\right]$$

$$\left[C_{n+1}^{m}+S_{n+1}^{m}\right]$$

$$+\frac{1}{(2n+1)}\left[\left\{\frac{(n-m+1)(n+m+1)}{(2n+3)}+\frac{(n+m)(n-m)}{(2n-1)}\right\}\right.$$

$$\left\{\frac{2\varphi_{s,1}^{0}(r)}{r^2}\frac{d^2}{dr^2}+\left(\frac{2}{r^2}\frac{d\varphi_{s,1}^{0}(r)}{dr}+\frac{\varphi_{s,1}^{0}(r)}{r^3}\right)\frac{d}{dr}\right\}$$

$$+\left\{\frac{n(n-m+1)(n+m+1)}{(2n+3)}-\frac{(n+1)(n+m)(n-m)}{(2n-1)}\right\}$$

$$\times\left\{\frac{\varphi_{s,1}^{0}(r)}{r^3}\frac{d}{dr}+\left(\frac{1}{2r^3}\frac{d\varphi_{s,1}^{0}(r)}{dr}-\frac{\varphi_{s,1}^{0}(r)}{2r^4}\right)\right\}$$

$$+\left\{\frac{(n+m+2)(n+m+1)}{(2n+3)}+\frac{(n-m+1)(n-m)}{(2n-1)}\right\}$$

$$\left\{-\frac{\varphi_{s,1}^{0}(r)}{2r^3}\frac{d}{dr}\right\}\Bigg]\left[C_n^m+S_n^m\right]$$

$$+\frac{(n-m)}{(2n-1)}\left[\frac{2\varphi_{s,0}^{0}(r)}{r^2}\frac{d^2}{dr^2}+\left(\frac{2}{r^2}\frac{d\varphi_{s,0}^{0}(r)}{dr}+\frac{\varphi_{s,1}^{0}(r)}{r^3}\right)\frac{d}{dr}\right.$$

$$\left.+(n-1)\left\{\frac{\varphi_{s,0}^{0}(r)}{r^3}\frac{d}{dr}+\left(\frac{1}{2r^3}\frac{d\varphi_{s,0}^{0}(r)}{dr}-\frac{\varphi_{s,0}^{0}(r)}{2r^4}\right)\right\}\right]$$

$$\left[C_{n-1}^m+S_{n-1}^m\right]$$

$$+\frac{(n-m+1)(n-m)}{(2n-3)(2n-1)}\left[\frac{2\varphi_{s,1}^{0}(r)}{r^2}\frac{d^2}{dr^2}+\frac{\varphi_{s,1}^{0}(r)}{2r^3}\frac{d}{dr}\right.$$

$$+(n-2)\left\{\frac{\varphi_{s,1}^{0}(r)}{r^3}+\left(\frac{1}{2r^3}\frac{d\varphi_{s,1}^{0}(r)}{dr}-\frac{\varphi_{s,1}^{0}(r)}{2r^4}\right)\right\}$$

$$\left.+\left(\frac{2}{r^2}\frac{d\varphi_{s,1}^{0}(r)}{dr}+\frac{\varphi_{s,1}^{0}(r)}{r^3}\right)\frac{d}{dr}\right]$$

$$\left[C_{n-2}^m+S_{n-2}^m\right]$$

$$-\frac{(n+m+2)}{2(2n+5)(2n+3)}\left[\frac{2\varphi_{5,1}^{1}(r)}{r^2}\frac{d^2}{dr^2}+\frac{\varphi_{5,1}^{1}(r)}{2r^3}\frac{d}{dr}\right.$$

$$+\left(\frac{2}{r^2}\frac{d\varphi_{5,1}^{0}(r)}{dr}+\frac{\varphi_{5,1}^{0}(r)}{r^3}\right)\frac{d}{dr}$$

$$\left.-(n+3)\left\{\frac{\varphi_{5,1}^{1}(r)}{r^3}\frac{d}{dr}+\left(\frac{1}{2r^3}\frac{d\varphi_{5,1}^{1}(r)}{dr}-\frac{\varphi_{5,1}^{1}(r)}{2r^4}\right)\right\}\right]$$

$$\left[C_{n+2}^{m-1}+S_{n+2}^{m-1}\right]$$

$$+\frac{1}{2(2n+1)}\left[\left\{\frac{n+m-1}{2n-1}-\frac{n-m+2}{2n+3}\right\}\right.$$

$$\left\{\frac{2\varphi_{5,1}^{1}(r)}{r^2}\frac{d^2}{dr^2}+\left(\frac{2}{r^3}\frac{d\varphi_{5,1}^{1}(r)}{dr}+\frac{\varphi_{5,1}^{1}(r)}{r^3}\right)\frac{d}{dr}\right\}$$

$$-\left\{\frac{n(n-m+2)}{(2n+3)}+\frac{(n+1)(n+m-1)}{(2n-1)}\right\}$$

$$\left\{\frac{\varphi_{5,1}^{1}(r)}{r^3}\frac{d}{dr}+\left(\frac{1}{2r^3}\frac{d\varphi_{5,1}^{1}(r)}{dr}-\frac{\varphi_{5,1}^{1}(r)}{2r^4}\right)\right\}$$

$$\left.+\left\{\frac{n+m+1}{2n+3}-\frac{n-m}{2n-1}\right\}\left\{\frac{\varphi_{5,1}^{1}(r)}{2r^3}\frac{d}{dr}\right\}\right]$$

$$\left[C_{n}^{m-1}+S_{n}^{m-1}\right]$$

$$+\frac{(n-m)}{2(2n-3)(2n-1)}\left[\frac{2\varphi_{s,1}^{1}(r)}{r^2}\frac{d^2}{dr^2}+\frac{\varphi_{s,1}^{1}(r)}{2r^3}\frac{d}{dr}\right.$$

$$+\left(\frac{2}{r^2}\frac{d\varphi_{s,1}^{1}(r)}{dr}+\frac{\varphi_{s,1}^{1}(r)}{r^3}\right)\frac{d}{dr}$$

$$\left.+(n-2)\left\{\frac{\varphi_{s,1}^{1}(r)}{r^3}\frac{d}{dr}+\left(\frac{1}{2r^3}\frac{d\varphi_{s,1}^{1}(r)}{dr}-\frac{\varphi_{s,1}^{1}(r)}{r^4}\right)\right\}\right]$$

$$\left[C_{n-2}^{m-1}+S_{n-2}^{m-1}\right]$$

$$=M\left[P_3^2\cos 2\varphi\left\{\frac{1}{10}\left(-\frac{f'(r)}{r^6}\frac{\varphi_{s,1}^{1}(r)}{3}-\frac{1}{3}\frac{f(r)}{r^6}\frac{d\varphi_{s,1}^{1}(r)}{dr}\right.\right.\right.$$

$$\left.+\frac{5}{6}f(r)\frac{\varphi_{s,1}^{1}(r)}{r^7}\right)$$

$$\left.\left.-\frac{f(r)}{2r^7}\frac{\varphi_{s,1}^{1}(r)}{30}\right\}\right]$$

$$+P_3^1\cos\varphi\left\{\frac{2}{5}\left(-\frac{f'(r)}{r^6}\frac{\varphi_{s,1}^{0}(r)}{3}-\frac{f(r)}{3r^6}\frac{d\varphi_{s,1}^{0}(r)}{dr}\right.\right.$$

$$\left.+\frac{5}{6}\frac{\varphi_{s,1}^{0}(r)}{r^6}f(r)\right)$$

$$\left.-\frac{\varphi_{s,1}^{0}(r)}{15r^7}f(r)\right\}$$

$$+ P_3^0 \left\{ -\frac{3}{5}\left(-\frac{f'(r)}{r^6} \frac{\varphi_{s,1}^1(r)}{3} - \frac{f(r)}{3r^6} \frac{d\varphi_{s,1}^1(r)}{dr} + \frac{5}{6r^7}\varphi_{s,1}^1(r) f(r) \right) \right.$$

$$\left. + \frac{1}{10} f(r) \frac{\varphi_{s,1}^1(r)}{r^7} \right\}$$

$$+ P_2^1 \cos\varphi \left\{ -\frac{f'(r)}{r^6} \frac{\varphi_{s,0}^0(r)}{3} - \frac{f(r)}{3r^6} \frac{d\varphi_{s,0}^0(r)}{dr} \right.$$

$$\left. + \frac{5}{6r^7} \varphi_{s,0}^0(r) f(r) \right\}$$

$$+ P_1^1 \cos\varphi \left\{ \frac{3}{5}\left(-\frac{\varphi_{s,1}^0(r)}{3r^6} f'(r) - \frac{f(r)}{3r^6} \frac{d\varphi_{s,1}^0(r)}{dr} \right. \right.$$

$$\left. + \frac{5}{6r^7} \varphi_{s,1}^0(r) f(r) \right)$$

$$\left. + \frac{2}{5} \varphi_{s,1}^0(r) \frac{f(r)}{r^7} \right\}$$

$$+ P_1^0 \left\{ 3/5 \left(-\frac{\varphi_{s,1}^1(r)}{3r^6} f'(r) - \frac{f(r)}{3r^6} \frac{d\varphi_{s,1}^1(r)}{dr} \right. \right.$$

$$\left. \left. + \frac{5}{6} \frac{f(r)}{r^7} \varphi_{s,1}^1(r) \right) - \frac{f(r)}{10r^7} \varphi_{s,1}^1(r) \right\} \Bigg] \qquad (5.59)$$

As in the previous case we get the defining equations for $C_1^1, C^0, C_0^0, C_1^{-1}, C_2^{-1}$ and S_1^1, S_1^0

S_0^0, S_1^{-1} and S_2^{-1} as follows.

For m = 2, n = 3,

$$\left[\frac{d^2}{dr^2} + \left\{\frac{21}{4r} - 2\alpha(r-h)\right\}\frac{d}{dr} + \frac{1}{2r^2}\{1-\alpha r(r-h)\}\right]c_1^1(r)$$

$$= -\frac{3M}{2r^4}\left[1 - {}^2\!/_3\, r\alpha(r-h)\right]$$

(5.60)

$$\left[\frac{d^2}{dr^2} + \left\{\frac{21}{4r} - 2\alpha(r-h)\right\}\frac{d}{dr} + \frac{1}{2r^2}\{1-\alpha r(r-h)\}\right]s_1^1(r)$$

$$= 0$$

(5.61)

For m = 1, n = 3

$$\frac{1}{2}\left[\frac{d^2}{dr^2} + \left\{\frac{21}{4r} - 2\alpha(r-h)\right\}\frac{d}{dr} + \left\{\frac{3}{4r^2} - \frac{\alpha(r-h)}{2r}\right\}\right]c_1^0(r)$$

$$+ \left[\frac{d^2}{dr^2} + \left\{\frac{21}{4r} - 2\alpha(r-h)\right\}\frac{d}{dr} + \left\{\frac{3}{4r^2} - \frac{\alpha(r-h)}{2r}\right\}\right]c_1^1(r)$$

$$= -\frac{3M}{2r^4}\left[1 - {}^2\!/_3\,\alpha(r-h)r\right] \quad (5.62)$$

$$\frac{1}{2}\left[\frac{d^2}{dr^2} + \left\{\frac{21}{4r} - 2\alpha(r-h)\right\}\frac{d}{dr} + \left\{\frac{3}{4r^2} - \frac{\alpha(r-h)}{2r}\right\}\right]s_1^0(r)$$

$$+ \left[\frac{d^2}{dr^2} + \left\{\frac{21}{4r} - 2\alpha(r-h)\right\}\frac{d}{dr} + \left\{\frac{3}{4r^2} - \frac{\alpha(r-h)}{2r}\right\}\right]s_1^1(r) \quad (5.63)$$

$$= 0$$

For m = 0, n = 3

$$\frac{1}{4}\left[\frac{d^2}{dr^2}+\left\{\frac{21}{4r}-4\alpha(r-h)\right\}\frac{d}{dr}+\frac{1}{2r^2}\{1-\alpha r(r-h)\}\right]c_1^{-1}$$

$$-\frac{1}{2}\left[\frac{d^2}{dr^2}+\left\{\frac{21}{4r}-4\alpha(r-h)\right\}\frac{d}{dr}+\frac{1}{2r^2}\{3/2-\alpha r(r-h)\}\right]c_1^{0}$$

$$+\left[\frac{d^2}{dr^2}+\left\{\frac{21}{4r}-2\alpha(r-h)\right\}\frac{d}{dr}+\frac{1}{2r^2}\left\{\frac{3}{2}-\alpha r(r-h)\right\}\right]c_1^{0}$$

$$=\frac{M}{4r^4}\left[3-2\alpha r(r-h)\right] \tag{5.64}$$

$$\frac{1}{4}\left[\frac{d^2}{dr^2}+\left\{\frac{21}{4r}-4\alpha(r-h)\right\}\frac{d}{dr}+\frac{1}{2r^2}\{1-\alpha r(r-h)\}\right]s_1^{-1}$$

$$-\frac{1}{2}\left[\frac{d^2}{dr^2}+\left\{\frac{21}{4r}-4\alpha(r-h)\right\}\frac{d}{dr}+\frac{1}{2r^2}\left\{\frac{3}{2}-\alpha r(r-h)\right\}\right]s_1^{0}$$

$$+\left[\frac{d^2}{dr^2}+\left\{\frac{21}{4r}-2\alpha(r-h)\right\}\frac{d}{dr}+\frac{1}{2r^2}\left\{\frac{3}{2}-\alpha r(r-h)\right\}\right]s_1^{0} \tag{5.65}$$

$$=0$$

For m = 1, n = 2

$$\left[\frac{1}{2}\frac{d^2}{dr^2}+\left\{\frac{17}{2r}-\alpha(r-h)\right\}\frac{d}{dr}\right]c_0^{0}$$

$$+\left[\frac{d^2}{dr^2}+\left\{\frac{5}{r}-2\alpha(r-h)\right\}\frac{d}{dr} \tag{5.66}$$

$$+\left\{\frac{3}{4r^2}-\frac{\alpha(r-h)}{r}\right\}\right]c_1^{1}$$

$$=-\frac{M}{r^4}\left[\frac{5}{4}-\alpha r(r-h)\right]$$

$$\left[\frac{1}{2}\frac{d^2}{dr^2}+\left\{\frac{17}{2r}-\alpha(r-h)\right\}\frac{d}{dr}\right]S_0^0(r)$$

$$+\left[\frac{d^2}{dr^2}+\left\{\frac{5}{r}-2\alpha(r-h)\right\}\frac{d}{dr}+\left\{\frac{3}{4r^2}-\frac{\alpha(r-h)}{r}\right\}\right]S_1^1(r)$$

$$=0 \tag{5.67}$$

For m = 0, n = 2,

$$-\frac{1}{28}\left[\frac{d^2}{dr^2}+\left\{\frac{37}{4r}-2\alpha(r-h)\right\}\frac{d}{dr}+\left\{\frac{3/2-\alpha r(r-h)}{2r^2}\right\}\right]C_2^{-1}$$

$$+\left[\frac{d^2}{dr^2}+\frac{1}{2}\left\{\frac{10}{r}-4\alpha(r-h)\right\}\frac{d}{dr}+\left\{\frac{\frac{3}{2}-\alpha r(r-h)}{2r^2}\right\}\right]C_1^0$$

$$+\left[\frac{d^2}{dr^2}+\frac{1}{2}\left\{\frac{19}{2r}-4\alpha(r-h)\right\}\frac{d}{dr}\right]C_0^0=0 \tag{5.68}$$

$$-\frac{1}{28}\left[\frac{d^2}{dr^2}+\left\{\frac{37}{4r}-2\alpha(r-h)\right\}\frac{d}{dr}\right.$$

$$\left.+\left\{\frac{\frac{3}{2}-\alpha r(r-h)}{2r^2}\right\}\right]S_2^{-1}$$

$$+\left[\frac{d^2}{dr^2}+\frac{1}{2}\left\{\frac{10}{r}-4\alpha(r-h)\right\}\frac{d}{dr}\right.$$

$$\left.+\frac{1}{2r^2}\left\{\frac{3}{2}-\alpha r(r-h)\right\}\right]S_1^0 \tag{5.69}$$

$$+\left[\frac{d^2}{dr^2}+\frac{1}{2}\left\{\frac{19}{2r}-4\alpha(r-h)\right\}\frac{d}{dr}\right]S_0^0=0$$

In this set we get two compatibility conditions for $m = 1$, $n = 1$ and $m = 0$, $n = 1$. However, we write here only one compatibility condition for $m = 1$ and $n = 1$.

$$2\left[\frac{d^2}{dr^2} + \left(\frac{21}{4r} - 4\alpha(r-h)\right)\frac{d}{dr} + \frac{3}{2r^2}\left\{\frac{3}{2} - \alpha r(r-h)\right\}\right]c_1^1$$

$$+3\left[2\frac{d^2}{dr^2} + \left(\frac{8}{r} - 4\alpha(r-h)\right)\frac{d}{dr} - \frac{1}{3r^2}\left\{\frac{3}{2} - \alpha r(r-h)\right\}\right]c_1^0$$

$$+\left[\frac{d^2}{dr^2} + \left(\frac{21}{4r} - 4\alpha(r-h)\right)\frac{d}{dr} + \frac{1}{2r^2}\left\{3/2 - \alpha r(r-h)\right\}\right]c_1^{-1}$$

$$+\left[2\frac{d^2}{dr^2} + \left(\frac{9}{r} - 4\alpha(r-h)\right)\frac{d}{dr}\right]c_0^0$$

$$= M\left[-\frac{3}{r^4} + \frac{2\alpha(r-h)}{r^3}\right]. \qquad (5.70)$$

Using equations (5.60), (5.62), (5.64), (5.66), one can write (5.70) as a first order differential equation. Here the compatibility condition cannot be written in algebraic form, as has been done in the previous case.

With the above set of coefficients we can write the electrostatic potential function as

$$\Omega(r,\theta,\varphi) = P_1^1\left[c_1^1 \cos\varphi + s_1^1 \sin\varphi\right]$$

$$+ P_1^0\, c_1^0(r) + c_0^0(r)$$

$$+ P_1^{-1}\left[C_1^{-1}(r)\cos\varphi - S_1^{-1}(r)\sin\varphi\right]$$

$$+ P_2^{-1}\left[C_2^{-1}(r)\cos\varphi - S_2^{-1}(r)\sin\varphi\right] \tag{5.71}$$

We use eqn. (5.36) in above equation (5.71).

This gives the final form of the current vector as

$$\vec{J} = \frac{\varphi_S}{2}(r_i H_j + H_i r_j)\left[-\nabla\Omega + \vec{V}\times\vec{H}\right] \tag{5.72}$$

where $(r_i H_j + H_i r_j)$ is the symmetric tensor. Writing the components of eqn. (5.72),

$$J_r = r^4 \exp\{-\alpha(r-h)^2\}\left[1+\cos\theta+\sin\theta\cos\varphi\right]$$

$$\frac{\sin\theta\cos\varphi}{r^2}\left[6\sin\theta\cos\theta\cos\varphi\left\{-\frac{\partial\Omega}{\partial r}+\frac{m P_1^1\cos\varphi}{r^3}\right\}\right.$$

$$+6\sin\theta\cos\theta\sin\varphi\left\{-\frac{1}{r}\frac{\partial\Omega}{\partial\theta}-\frac{2mP_1^0\cos\varphi}{r^3}\right\}$$

$$\left.+(6\cos^2\theta-1)\left\{-\frac{1}{r\sin\theta}\frac{\partial\Omega}{\partial\varphi}+\frac{m(2(P_1^0)^2-(P_1^1)^2)\sin\varphi}{r^3}\right\}\right] \tag{5.73}$$

$$J_\theta = r^4 \exp\{-\alpha(r-h)^2\}\left[1+\cos\theta+\sin\theta\cos\varphi\right]$$

$$\frac{\sin\theta\sin\varphi}{r^2}\left[6\sin\theta\cos\theta\cos\varphi\left\{-\frac{\partial\Omega}{\partial r}+\frac{m\,p_1^1\cos\varphi}{r^3}\right\}\right.$$

$$+6\sin\theta\cos\theta\sin\varphi\left\{-\frac{1}{r}\frac{\partial\Omega}{\partial\theta}-\frac{2m\,p_1^0\cos\varphi}{r^3}\right\}$$

$$+(6\cos^2\theta-1)\left\{-\frac{1}{r\sin\theta}\frac{\partial\Omega}{\partial\theta}\right.$$

$$\left.\left.+\frac{m}{r^3}\left(2(p_1^0)^2-(p_1^1)^2\right)\sin\varphi\right\}\right] \qquad (5.74)$$

$$J_\varphi = r^4 \exp\{-\alpha(r-h)^2\}\left[1+\cos\theta+\sin\theta\cos\varphi\right]$$

$$\frac{1}{r^2}\left\{(6\cos^2\theta-1)\sin\theta\left[\cos\varphi\left(-\frac{\partial\Omega}{\partial r}+\frac{m\,p_1^1\cos\varphi}{r^3}\right)\right.\right.$$

$$\left.+\sin\varphi\left(-\frac{1}{r}\frac{\partial\Omega}{\partial\theta}-\frac{2m\,p_1^0\cos\varphi}{r^3}\right)\right]$$

$$+2\cos\theta(3\cos^2\theta-1)\left[-\frac{1}{r\sin\theta}\frac{\partial\Omega}{\partial\varphi}+\frac{m}{r^3}\left((p_1^0)^2\cdot 2-(p_1^1)^2\right)\right.$$

$$\left.\left.\times\sin\varphi\right]\right\} \qquad (5.75)$$

where p_1^1 and p_1^0 are defined earlier.

This completes our analysis on the symmetric part of the tensor $\bar{\bar{\sigma}}$.

The antisymmetric part of the tensor consists of $\varphi_2 \epsilon_{ijk} r_k$, $\varphi_4 \epsilon_{ijk} H_k$ and $\frac{\Phi_5}{2}(r_i H_j - H_i r_j)$. When we substitute this in the dynamo equation, the order of equation reduces by one i.e. either we get 1st order equation or algebraic equation. The solution depends on the boundary condition. We have chosen a boundary condition for the coefficients of Ω in such a way that they vanish at the lower boundary and are contiguous at the upper boundary with the values obtained by Pratap et al. (1973). These coefficients vanish because of the lower boundary conditions.

We therefore derive an important conclusion viz. the part depending on the magnetic field in the conductivity reduces to a set of homogeneous equations when we take symmetric part of the tensor and depends very critically on the boundary condition for the antisymmetric part.

When one considers the conductivity such as direct, Hall and Pederson this effect will show itself up. <u>This is a result which is not recognized before</u>.

The significance of this is that the Lorentz - force or the dynamo force does not play any vital role as far as symmetric part of the field is concerned.

We have numerically integrated the above equations and boundary conditions have been such that the coefficients are zero at $r = 1.01\ R_E$. This is the base of the D layer. At the upper boundary ($r = 1.50\ R_E$) the condition is contiguous with Pratap et al. (1973) i.e. the coefficients are zero at the upper boundary. The integration has been carried out from $r = 1.01\ R_E$ to $r = 1.50\ R_E$ in steps of $0.01\ R_E$. i.e. 60 to 3000 Km. The discussion of the results is given in the Section (5.4).

5.3. Induced Magnetic Field

In this section we shall discuss the magnetic field induced by the current system. Since the magnetic field is divergence - free one can write the same as a combination of toroidal field $\vec{T}$, and a poloidal field $\vec{P}$. Thus

$$\vec{H} = \vec{P} + \vec{T} \tag{5.76}$$

where

$$\vec{T} = \nabla \times (\vec{r} u) \tag{5.77}$$

and

$$\vec{P} = \nabla \times \nabla \times (\vec{r} v) \tag{5.78}$$

where u and v are scalar functions of (r, θ, φ).

Maxwell's equation for induction is given by

$$\nabla \times \vec{H} = 4\pi \vec{J} \tag{5.79}$$

Substituting equation (5.76) in equation (5.79) and writing the components, we get

$$\frac{1}{r^2} L^2 u = 4\pi J_r \tag{5.80}$$

$$\frac{1}{r}\frac{\partial^2 u}{\partial r \partial \theta} + \frac{1}{r \sin\theta}\frac{\partial}{\partial \varphi}\left(\nabla^2\left(\frac{v}{r}\right)\right) = 4\pi J_\theta \tag{5.81}$$

$$\frac{1}{r\sin\theta}\frac{\partial^2 u}{\partial r \partial \theta} - \frac{1}{r}\frac{\partial}{\partial \theta}\left(\nabla^2\left(\frac{v}{r}\right)\right) = 4\pi J_\varphi \tag{5.82}$$

Expanding u and v in terms of spherical harmonics,

$$u = \sum_{n,m}\left[U_{nc}^{m}(r)\cos m\varphi + U_{ns}^{m}(r)\sin m\varphi\right] P_n^m \tag{5.83}$$

$$v = \sum_{n,m}\left[V_{nc}^{m}(r)\cos m\varphi + V_{ns}^{m}(r)\sin m\varphi\right] P_n^m \tag{5.84}$$

We can evaluate u from equation (5.80) as

$$\frac{1}{r^2}\left[U_{nc}^{m} Y_{nc}^{m} + U_{ns}^{m} Y_{ns}^{m} \right] n(n+1) = 4\pi J_r \tag{5.85}$$

A simple comparison of coefficient of harmonics gives the function $U_{n\,c,s}^{m}$.

We can substitute this in the equation (5.81) and we get a defining equation for $V_{n\,c,s}^{m}$ in terms of J_θ and $U_{n,c,s}^{m}$. The equation (5.82) will trivially be satisfied since J is also divergence - free. We can get, thus the magnetic field induced by this current system.

5.4. Discussion

In this section we shall discuss the nature of variation of the current components J_r, J_θ , J_φ for different values of $\bar{\bar{\sigma}}$. We would again like to mention the fact that our solution does not contain the homogeneous part because , as we have already mentioned, this part would not contain the dynamo terms.

In figs. (5-1a, b, c), we have plotted J_φ, J_θ and J_r as functions of θ and L_T at $r = 1.02\ R_E$ with $\bar{\bar{\sigma}} = \varphi_0 \delta_{ij}$ and fig. (5-1d) gives J_r as a function of r at $\varphi = 0°$ (mid-day meridian).

One could observe here in fig. (5-1a) which gives the

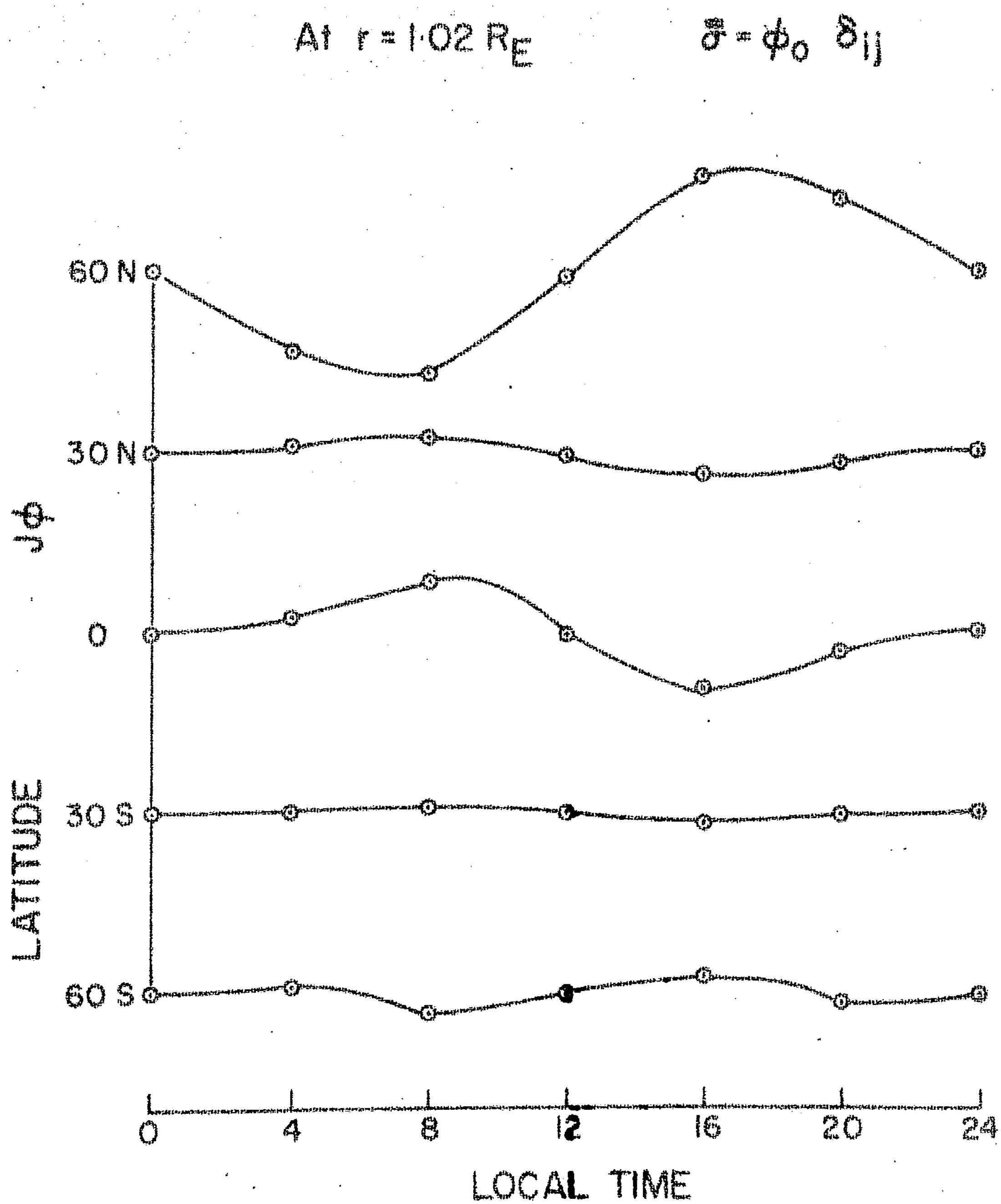

Figure (5-1a) : The plot of J_ϕ (or ΔX) with local time at $r = 1.02R_E$ when $\bar{\bar{\sigma}} = \phi_o \delta_{ij}$.

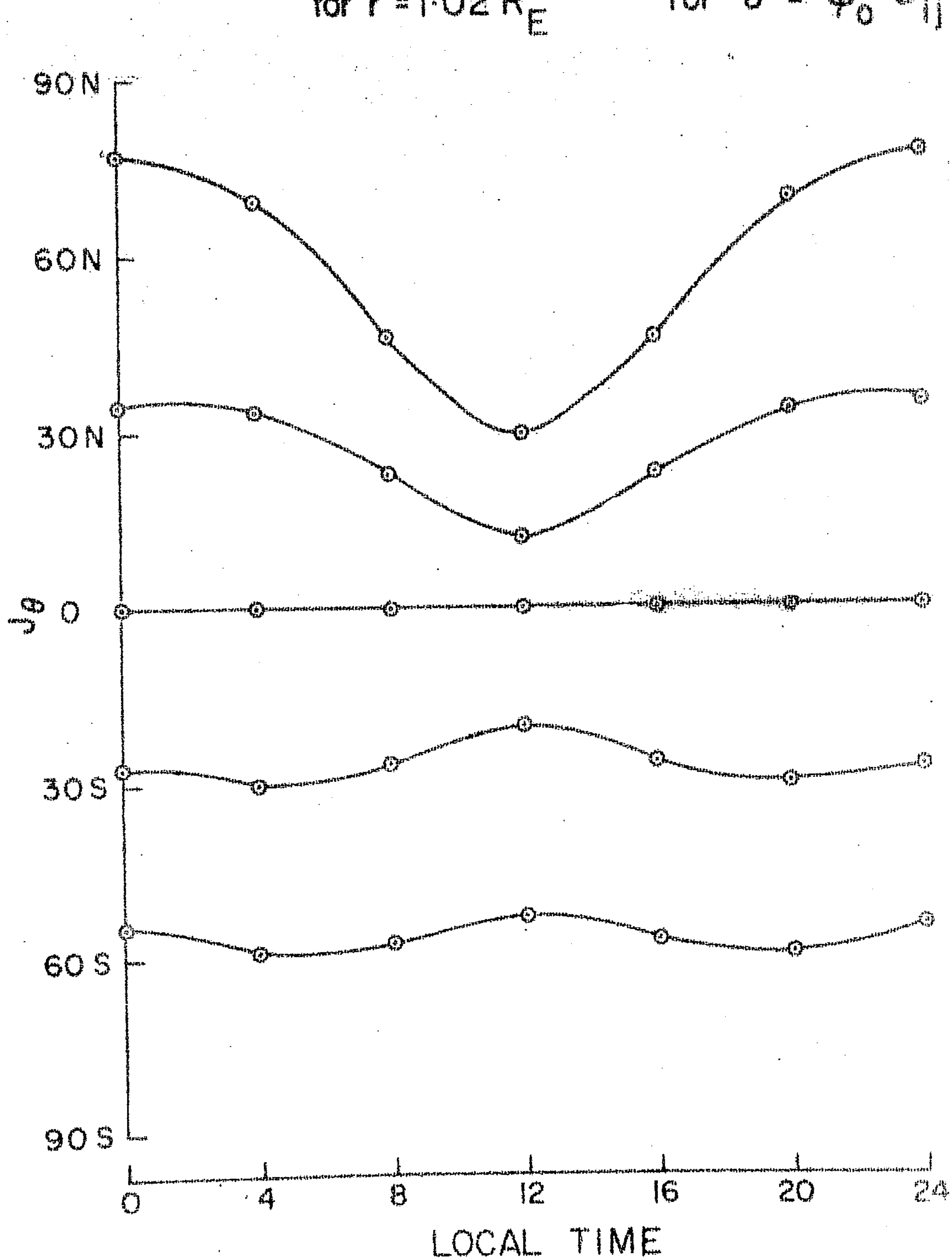

Figure (5-1b) : The plot of J_θ (or ΔY) with local time at $r = 1.02R_E$ when $\bar{\bar{\sigma}} = \phi_0 \delta_{ij}$.

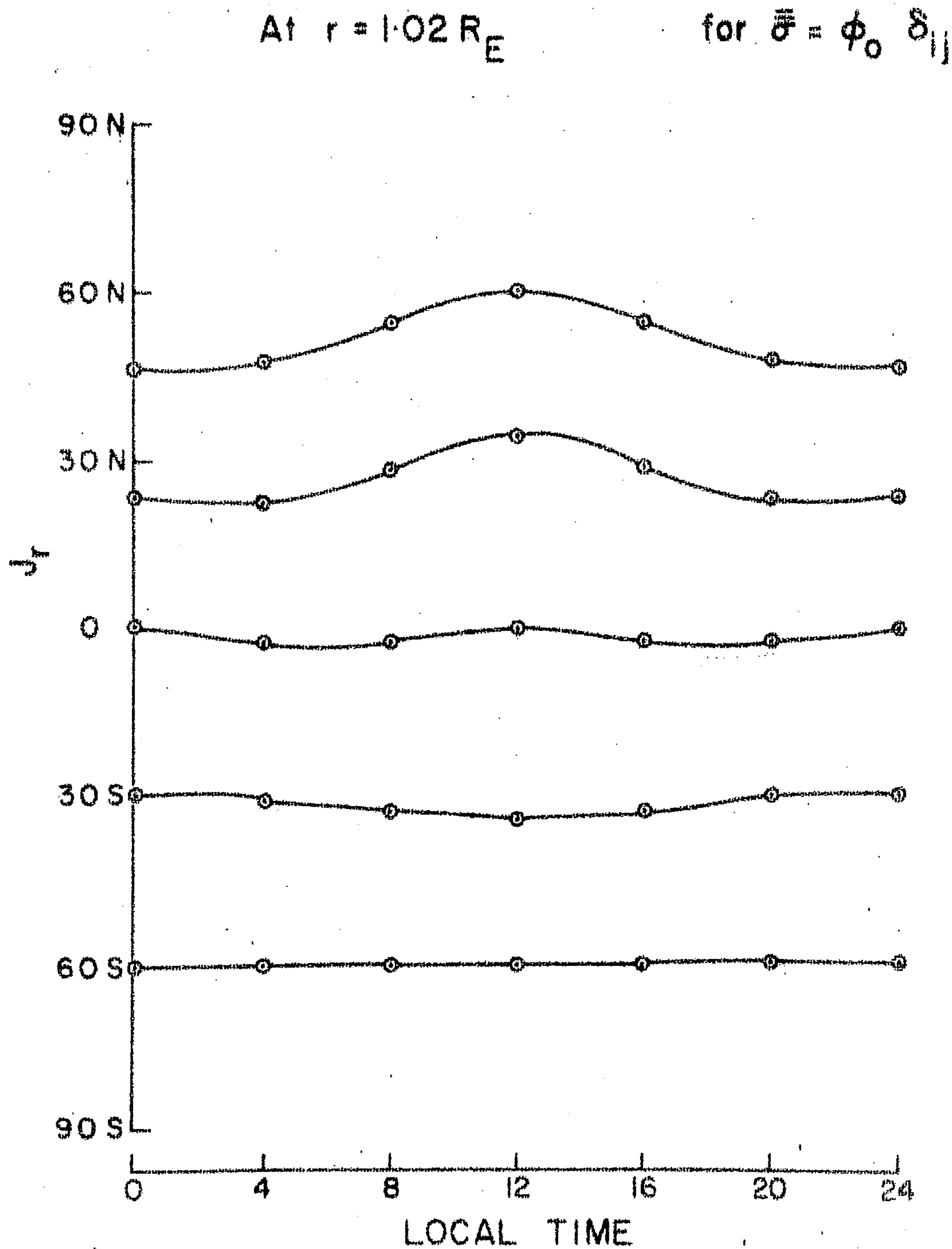

Figure (5-1c) : The plot of J_r with local time at $r = 1.02R_E$ when $\bar{\bar{\sigma}} = \phi_o \delta_{ij}$.

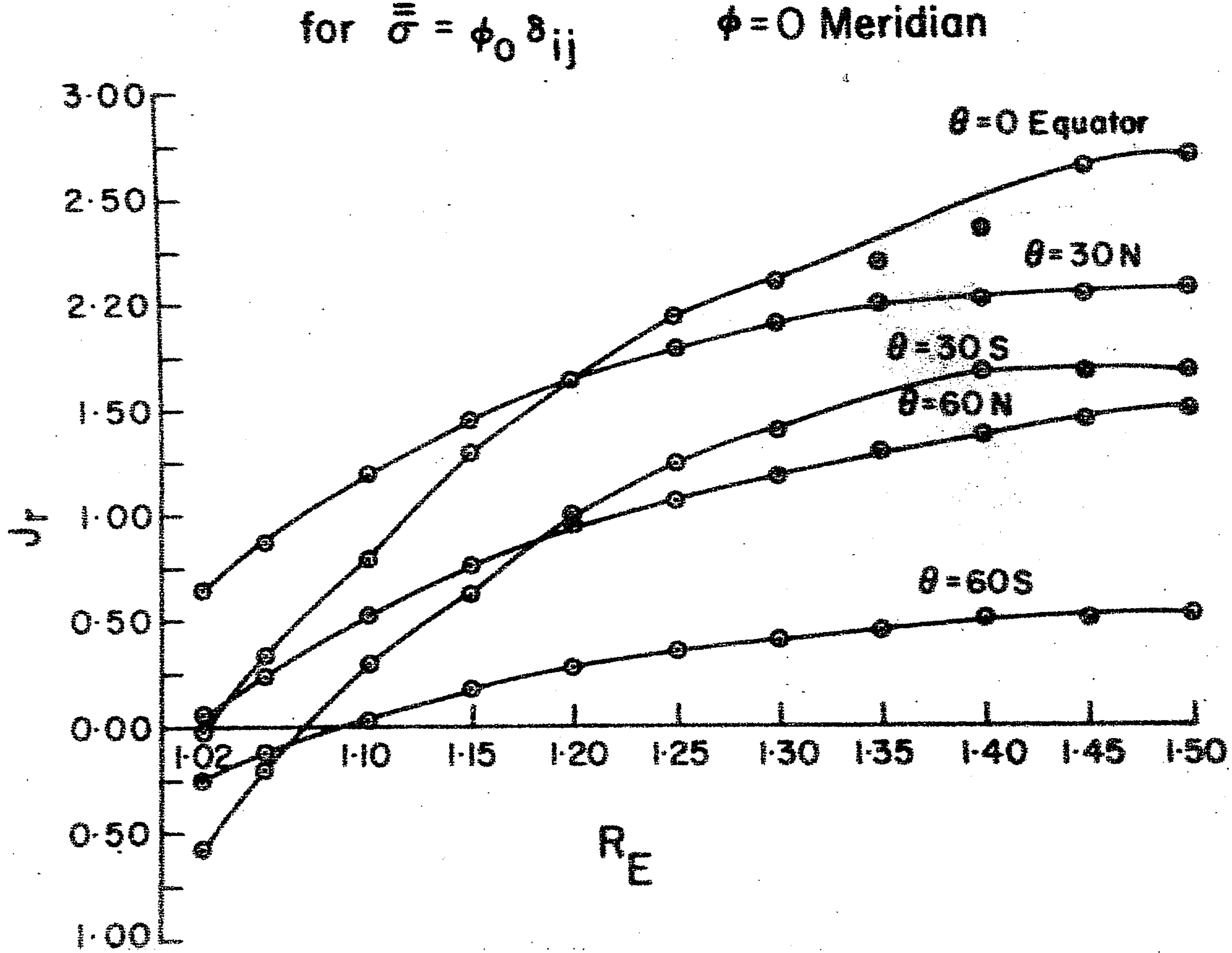

Figure (5-1d) : The plots of $\overline{J_r}$ with radial distance from $r = 1.02R_E$ to $r = 1.50R_E$ for $\theta = 0$ (equator), $\theta = 30N$, $30S$ and $\theta = 60N$, $60S$ when $\bar{\bar{\sigma}} = \phi_o \delta_{ij}$.

variation or in other words the ΔX variation, that there is a change of phase at about $\pm 40^{\circ}$ in latitude. We can also find the maximum in equatorial latitudes occurs at about 0900 hours local time, while the observed variation has a maximum at 1100 hours. This is primarily because we have not included a phase in the velocity potential function. The fact, that the phase in velocity potential is very crucial, was discussed at length by Pratap (1964). If we include the phase of 270° as discussed by Pratap, we would get maximum at 1100 hrs. local time.

The fig. (5-1b) is the plot of J_{θ} which corresponds to ΔY showing that it is asymmetric about the equator. Here again a phase shift will bring this curve in line with the observed one.

The fig. (5-1c) for J_{γ} shows that there exist maxima in the northern hemisphere while feeble minima exist in the southern hemisphere. Here again a shift of 270° will lead to a minimum at noon consistent with the observed variations.

In fig. (5-1d) we have plotted the value of J_{γ} as a function of γ when $\varphi = 0^{\circ}$ i.e. mid-day meridian and $\theta = 0^{\circ}$, $\theta = \pm 30^{\circ}$ and $\pm 60^{\circ}$. In this calculation J_{γ} increases as we go from 1.02 R_E and then stabilizes and takes a constant value as increases further. Again it is observed that the rate of increase of J_{γ} is more at the equator than at higher latitudes.

For 30°N and 30° S the curves are more or less parallel but the magnitude at the northern latitudes is higher than that at the Southern latitudes. This implies that while the rate of change is more or less the same, the intrinsic value of J_γ is greater in northern latitudes than the southern latitudes. The same feature is obtained for 60° N and 60° S as well.

This asymmetry is mainly due to the asymmetry in the conductivity function. This asymmetry has been introduced in the choice of the function φ_0. If we had chosen φ_0 as

$$\varphi_0 = 1 + \cos\chi$$

where $\cos\chi$ is as defined by equation (3.8), then this function would be asymmetric for the solstice when δ is 23½$^\circ$. On the other hand, the function becomes symmetric when δ is zero. Hence our calculation would be akin to the solstice part.

In the second set of the curves (see fig. (5-2a, b, c)) corresponding to $J_\varphi, J_\theta, J_\gamma$, the variations are not very prominent, expect those of J_γ. We observe that the maximum value at local noon decreases as height increases. It continues to decrease till it saturates finally. This can be seen in fig. (5.2d) where we have plotted J_γ as a function of γ for the various sets of latitudes. Here again asymmetry creeps in, for the reason given earlier for the case $\bar{\bar{\sigma}} = \varphi_0 \delta_{ij}$.

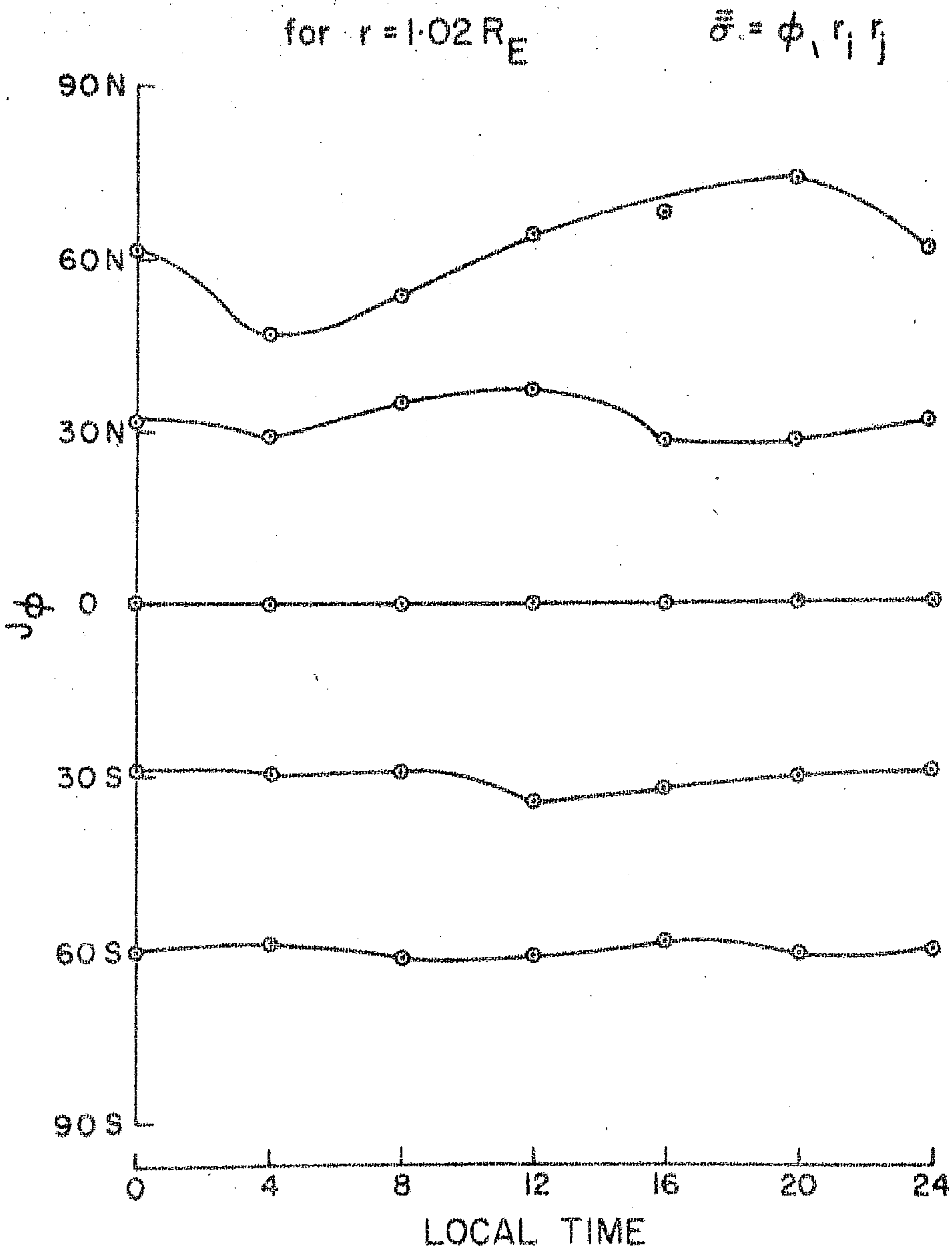

Figure (5-2a) : The plot of J_ϕ (or Δx) with local time at $r = 1.02R_E$ when $\bar{\bar{\sigma}} = \phi_1 \gamma_i \gamma_j$

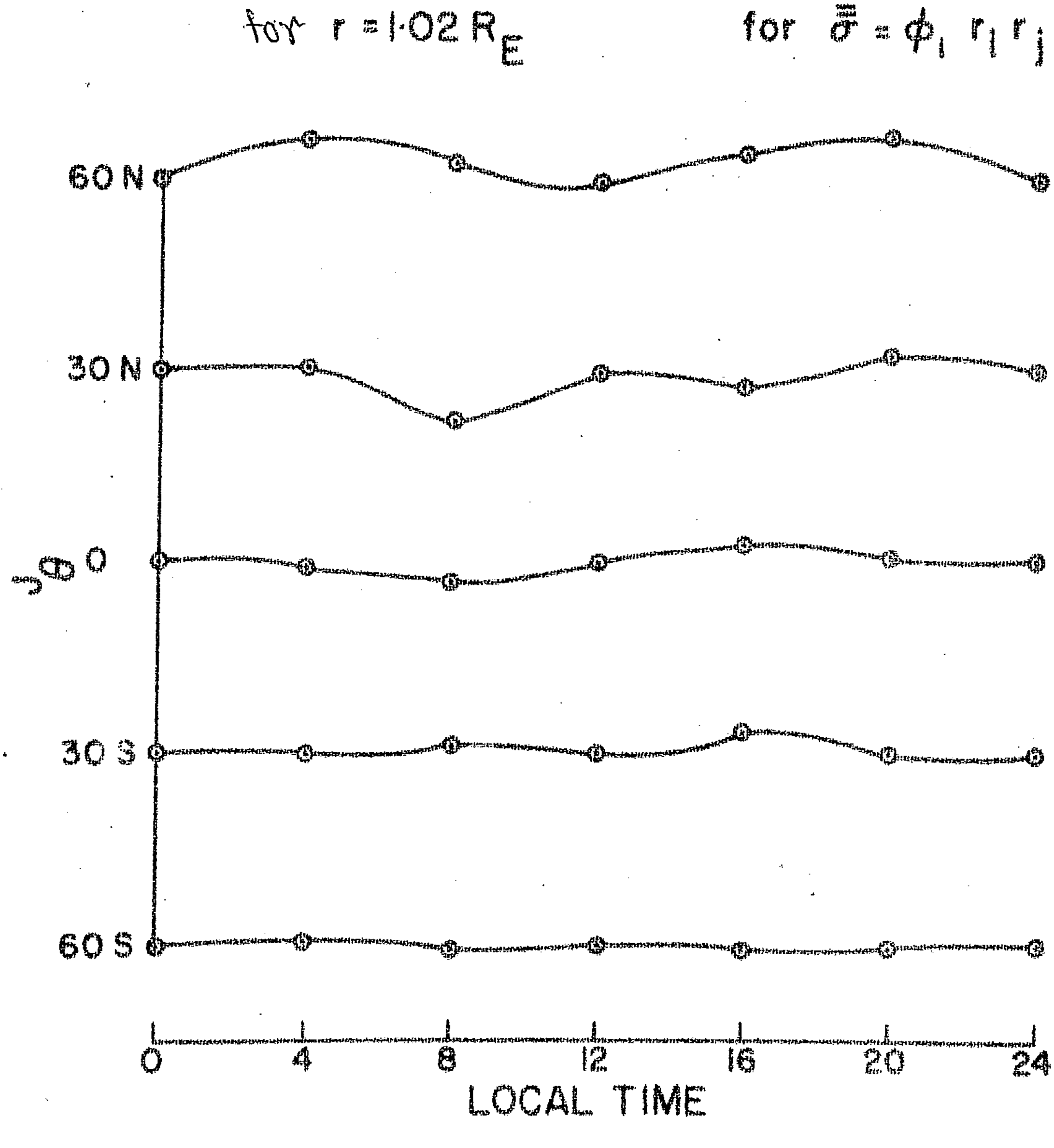

Figure (5-2b) : The plot of J_θ (or ΔY) with local time at $r = 1.02R_E$ when $\bar{\bar{\sigma}} = \phi_1 r_1 r_j$

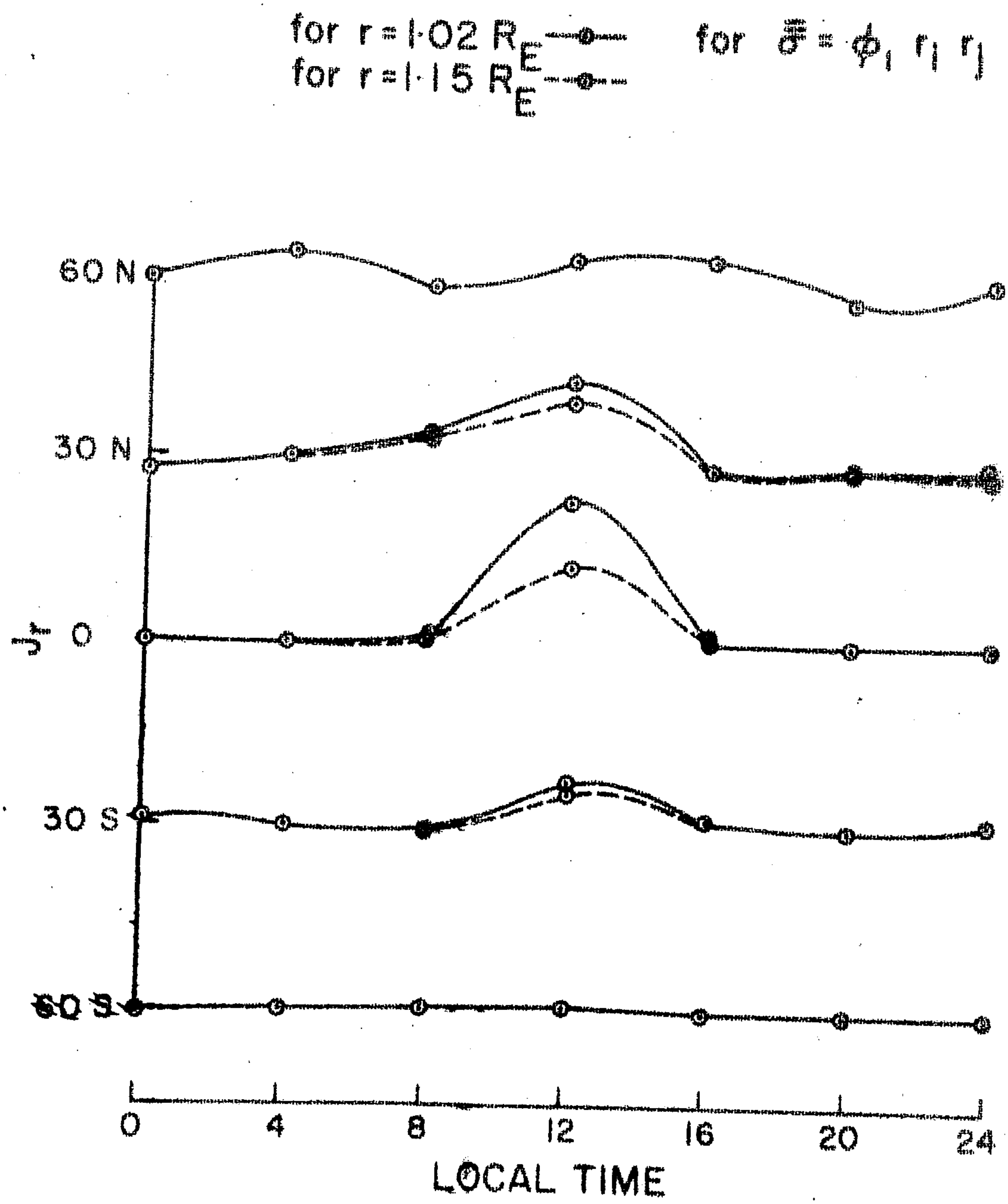

Figure (5-2c) : The plots of J_r with local time at $r = 1.02R_E$ and $r = 1.15R_E$ when $\bar{\bar{\sigma}} = \phi_1 \gamma_i \gamma_j$.

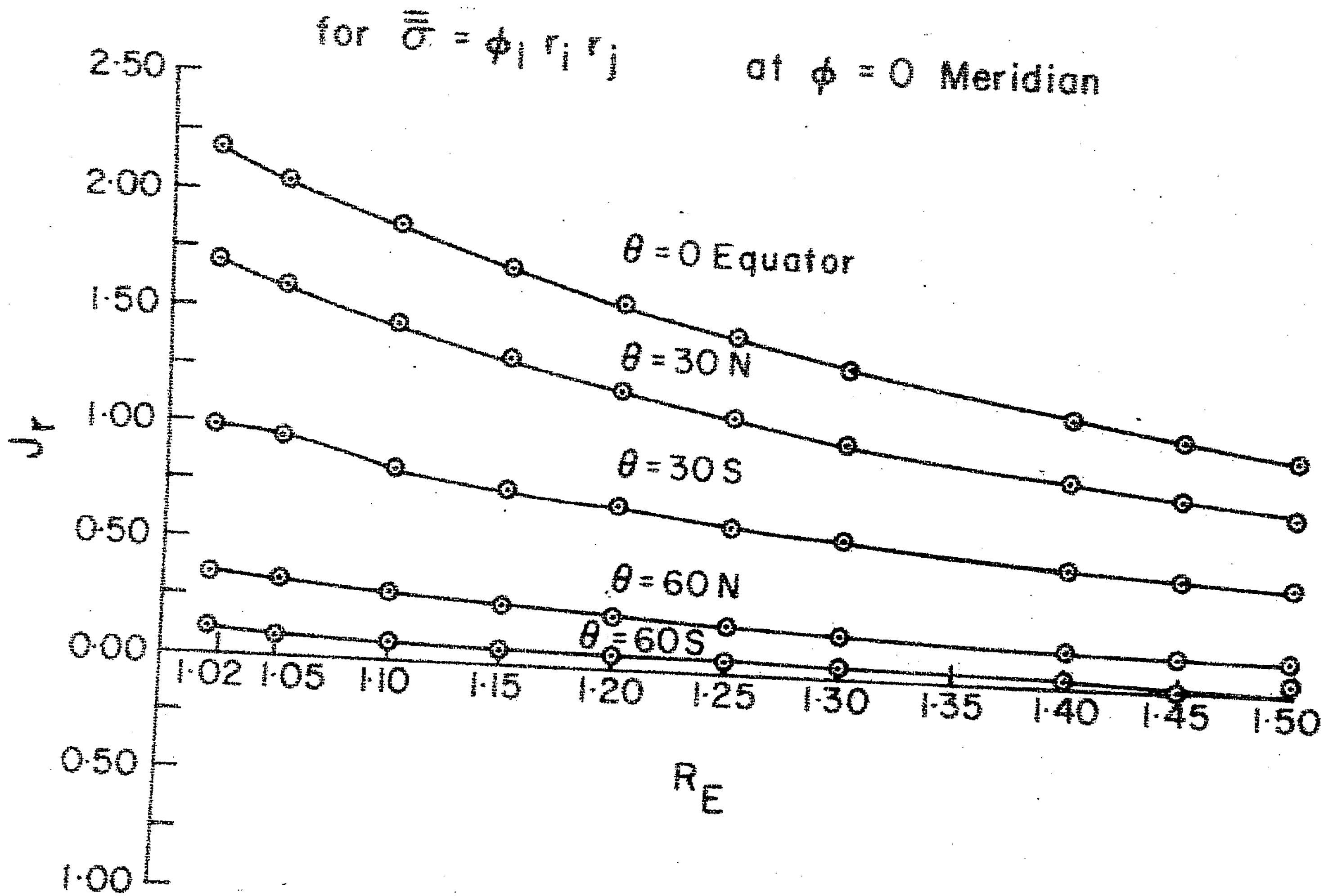

Figure (5-2d) : The plots of J_r with radial distance from $r = 1.02R_E$ to $r = 1.15R_E$ for $\theta = 0$ (equator), θ = 30N, 30S and θ = 60N, 60S when $\bar{\bar{\sigma}} = \Phi_1 \gamma_i \gamma_j$

For the third set of calculations where the symmetric form of tensor formed by γ and H, viz., $\bar{\bar{\sigma}} = \frac{\phi_s}{2}(\gamma_i H_j + H_i \gamma_j)$ the contribution for J_φ, J_θ, J_r at the equator is negligible while at higher latitudes, the current components obtained from this part of conductivity tensor exhibit a semi-diurnal feature, since S_q field is essentially diurnal. We remark that the contribution from this part of conductivity does not conform to the observations. This completes our discussion of the results.

CHAPTER VI

DYNAMICS OF S_q – FOCUS FROM A SINGLE STATION

6.1. Introduction

Hasegawa was the first to suggest that the centre of the S_q current system (S_q focus) need not be stationary but can have motion both latitude and longitude-wise. A first attempt to discover this phenomena was made by Ota (1950) and was followed by Hasegawa (1960) himself. The method adopted by these people has been to find the centre of a potential contour system which generates the S_q current system and sets it identically equal to the focus of the S_q current system. Hasegawa (1960) however tried to get S_q focus from a chain of stations distributed in the same longitude circle and between 25° to 40° on the latitude. The basic principle involved in this method is to plot the maximum value of ΔH or ΔX as a function of latitude as shown in fig. (6-1). The latitude point at which ΔH changes its sign is taken as the focus of the current system. While they could obtain day-to-day variation of the S_q focus, they have not been able to get the daily variation of the S_q focus. Ota has taken three longitude circles far-east (110° to 150°), American (270° to 300°) and European (0° to 20°) and also

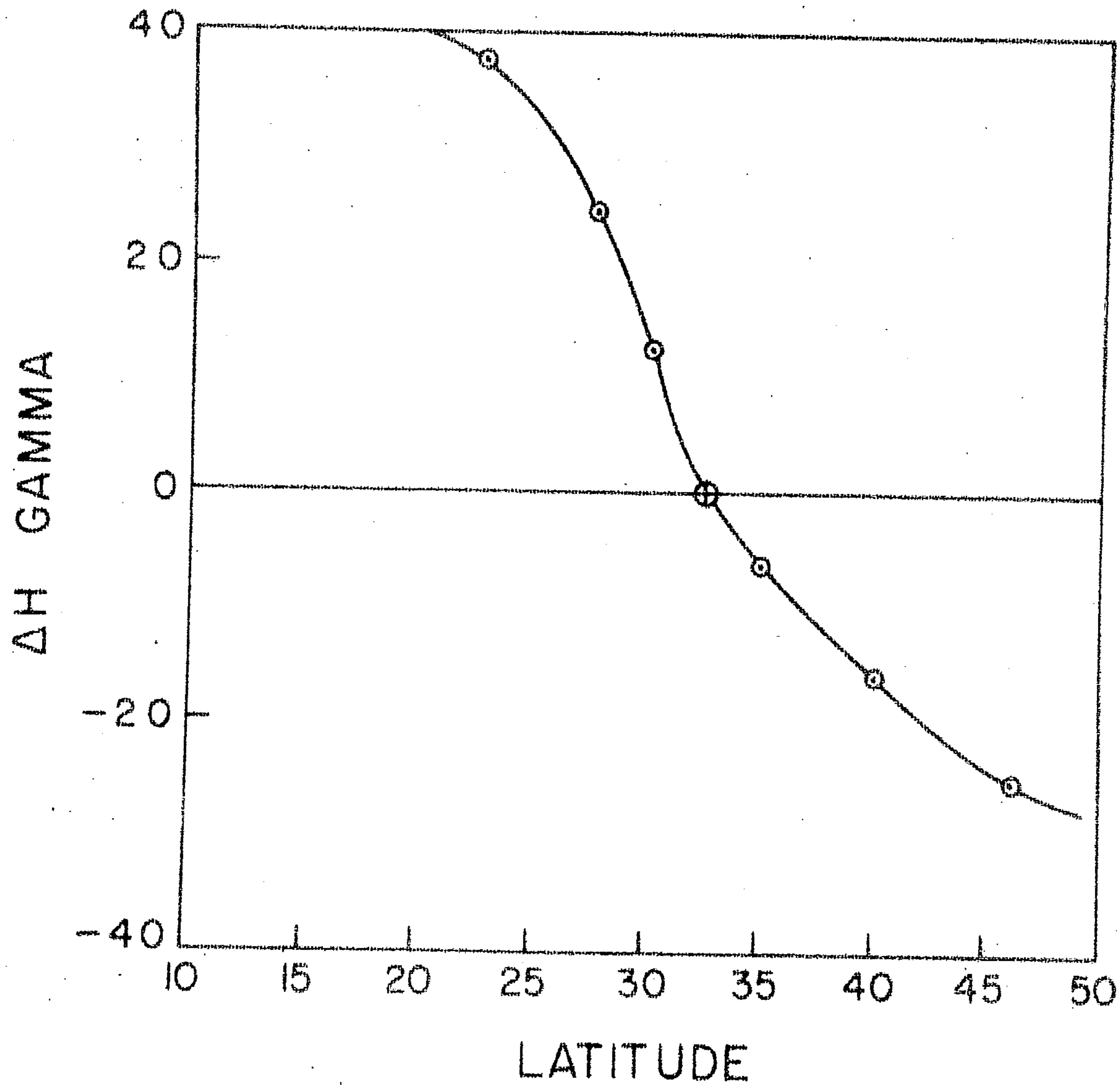

Figure (6-1) : A typical plot of ΔH with latitude. The latitude at which ΔH changes its sign i.e. $\Delta H = 0$ is the focus latitude. In the diagram focus latitude is represented by the symbol $\oplus$.

tried to obtain the seasonal variation.

There are inherent difficulties in the determination of S_q focus from different stations on the same longitude circle as follows :

(1) The effect of the internal current system induced by the external source would very critically depend on the conductivity of the crust and the upper mantle just below the station. This could introduce phase lag in the magnetic field recordings and therefore one ends up in an erroneous determination of the S_q focus.

(2) Again the distribution of land and ocean also has an effect on the recordings of the magnetic variometers and this will again give inaccurate determination S_q focus.

(3) It is very hard indeed to determine the contribution of the disturbance field towards the quiet day variation and this can again cause errors in the determination of S_q focus.

The method originally suggested by Hasegawa was followed by Matsushita (1960) by taking three American stations Fredericksburg (geographic latitude 38° 12' N, longitude 77° 22' W; geomagnetic latitude 49.6° N), Tucson (geographic latitude 32° 14' N, longitude 110° 57' W; geomagnetic latitude 40.4° N) and San Juan (geographic latitude 18° 23' N, longitude 66° 07' W; geomagnetic latitude 29.9° N). He plotted the deviation of the determined value of the focus from its central position for the three stations and thereby he obtained the seasonal variation of the central position of S_q current system. While Ota has taken the data for the entire month, Matsushita confined to ten internationally quiet days every month and observed that the focus is located at a higher latitude in local summer than in local winter. One can easily see that while the internationally quiet days are quiet with respect to the other days in the month, it is not certain that these days are really quiet. To ascertain this one has to really take into consideration the other parameters such as solar activity indices like A_p or K_p. Hence the basic difficulty which Hasegawa has pointed out still remained in Matsushita's work.

Tarpley (1973) followed Matsushita's line of thought and made an extensive analysis with four stations in the northern hemisphere and two in the southern hemisphere. Of the four in the northern hemisphere the three, Membetsu (Geographic latitude 43° 54', longitude 144° 12'E; geomagnetic latitude 34.0°N), Kakioka (geographic latitude 36° 14'N, longitude 140° 11'E; geomagnetic latitude 26.0°N), Kanoya (geographic latitude 31° 25'N, longitude 130° 53'E, geomagnetic latitude 20.5°N) are on Japanese island and one, Lunping (geographic latitude 25°N, longitude 121° 10'E; geomagnetic latitude 13.7°N) is at Formosa. The southern stations he has choosen are Tsumeb (geographic latitude 19° 13' S, longitude 17° 42' E; geomagnetic latitude 18.2° S) in south-west Africa deep inside the land mass while Hermanus (geographic latitude 34° 25' S, longitude 19° 14' E; geomagnetic latitude 33.3° S) near the Capetown on the coast. The difficulties pointed out by Hasegawa are very much inherent in this choice of stations as well. The main results of Tarpley again are the seasonal and day-to-day variation of the focus and he has inferred that the existence of this variation during very low magnetic activity suggests that the perturbation is ionospheric rather than magnetospheric.

Gupta (1973) repeated the same analysis by taking three different sectors viz. American sector, European - African sector and Asian - Australian sector. He has inferred that the foci in the two hemispheres move together in the same direction. He has also inferred that in general the foci are closer to the equator in local winter than in local summer. This could be due to a geomagnetic control. Thirdly he found that the focus variations are larger in the north American sector than in the Asian - Australian sector. If one assumes the S_q focus system to be global one, then this variation could partly be due to the structure of the continent. The north American stations are distributed mainly on the land mass while the Asian - Australian stations are approximately closer to sea-shore.

In a recent discussion Kane (1974) has pointed out that the non-ionospheric contribution towards the observed daily H variation should not be taken negligible. We have also discussed this point in Chapter IV. Kane has argued that the non-ionospheric contribution may be small but could effect the dynamics of the S_q focus more so because we are looking for that latitude for which ΔH is zero.

6.2. An alternative Method

In this section we propose a new method of determining the S_q focus from the data of a single station. We do believe that from the single station data the focus determination would be free from the main objections levelled by Hasegawa for a single station.

(1) the crust-mantle structure will be the same and therefore any time lag introduced by the induction process would be uniform through out the period.

(2) the effect of sea or land will also be the same and hence the S_q determination would be more reliable.

We propose to develope the main theory in this section and application of this theory in determining S_q focus dynamics is given in the next section. The final section consists of a detailed discussion of results obtained from our calculation of the S_q dynamics from two stations Alibag and Tashkent (geographic latitude

41.25°, longitude 69.0° E; geomagnetic latitude 32.5° N).

(a) Theory

We have made the following basic assumptions:

(1) That the current loops forming the S_q current systems are concentric ellipses.

(2) The induced magnetic field due to this current sheet is taken as that due to an infinite sheet and hence the magnetic field induced is directly proportional to the over-head current vector.

The first assumption that the current contours form ellipses needs some justification. If one goes through the diagrams given by Hasegawa and Ota (1950) from the observed data at the interval of every two hours, one can see clearly that while the contours close to the focus are indeed realistic ellipses (or circles which are degenerated ellipses), this approximation breaks down as we go to morning or evening hours. Hence this approximation is more and more realistic as we approach the focus. Secondly if the contour is not a regular geometric figure viz. circle or ellipse the focus looses most of its meaning. However, one can see these ellipses have their principal axes not necessarily parallel to the equator or to the longitude circle. We give a typical contour in fig. (6-2). In this figure we

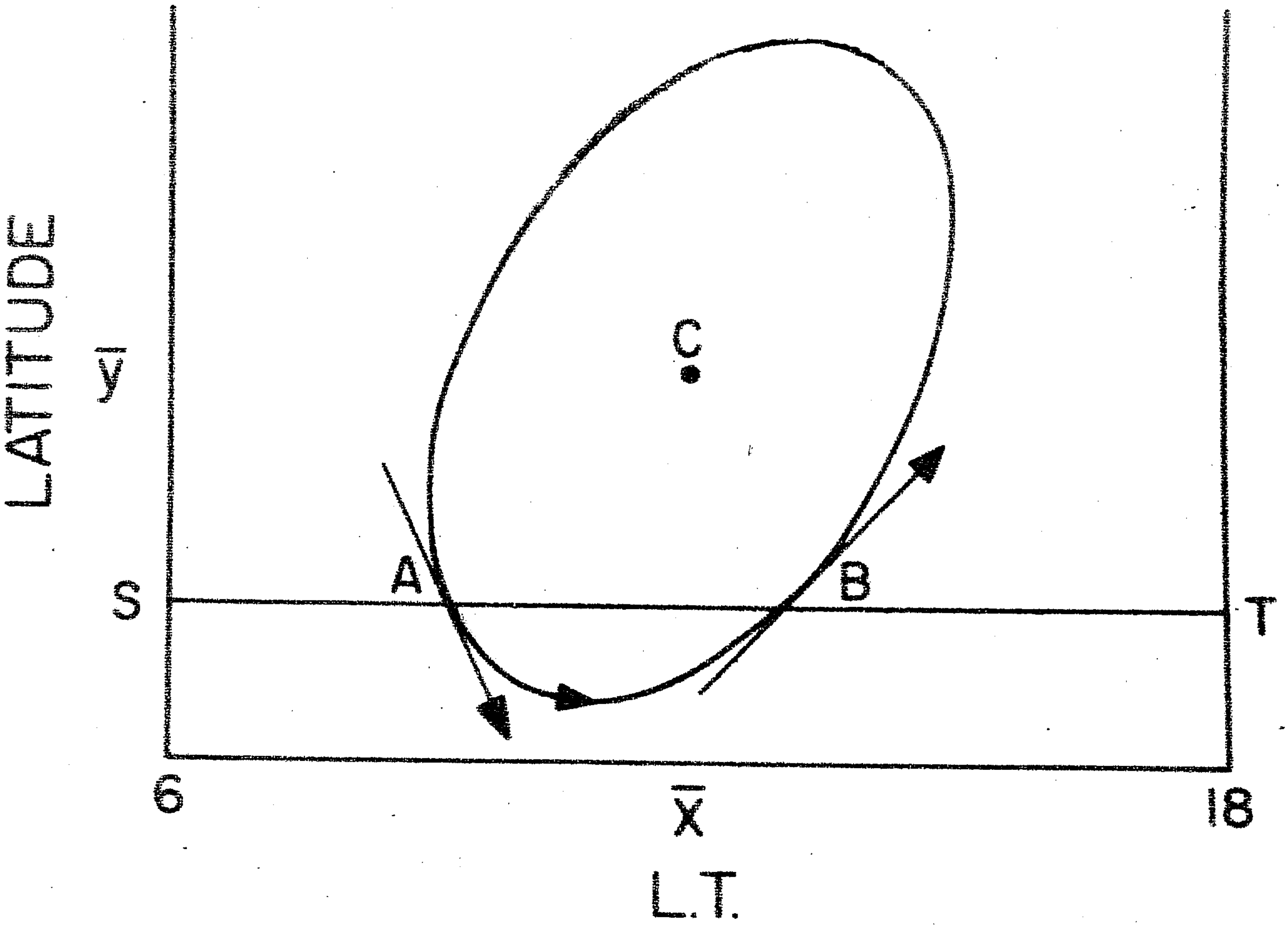

Figure (6-2) : A typical current contour of the S_q current system. ST is the latitude circle described by the station. A and B are points of intersection of this latitude circle with the contour. The station observes the same overhead current density at the local times of A, B. C is the centre of ellipse (focus of the current system) and the X coordinate of C is $\bar{X}$.

have drawn a current contour with C as its centre, whose coordinates are ($\bar{x}$, $\bar{y}$). These current contours are fixed with respect to the Earth - Sun line and is contained in the surface which is normal to the plane containing the Sun - Earth line and normal to ecliptic. In this configuration, the Earth rotates below the sheet containing this contour : The station is taken along the line ST and will be under the contour at two points A and B at two different local times. The station will observe a current vector at A pointing downwards while at B pointing upwards. Hence the induced magnetic field for the station will have an upward trend at A and downward trend at B. Opposite will be the case above C. If therefore we know the magnetic field variation at A and B we shall then know the current vectors at A and B which are tangents to ellipse. We shall then be able to determine the centre C.

The magnetic field inducedby a current element of strength I and flowing through a line element $\vec{dl}$ at a distance $\vec{r}$ is given by

$$d\vec{H} = \frac{I}{r^3} (d\vec{l} \times \vec{r}) \qquad (6.1)$$

Since the element is over-head the vector $\vec{r}$ will be normal to the surface of the Earth. Therefore $d\vec{H}$ will be parallel to the surface of the Earth. $d\vec{H}$ will be changing its direction and magnitude as station proceeds from S to T. If $\overrightarrow{\Delta H}$ has components ΔX and ΔY parallel to latitude and longitude circles respectively, we then have as pointed out in Chapter III

$$\overline{\Delta H} = \left\{(\Delta X)^2 + (\Delta Y)^2\right\}^{1/2} = \left\{(\Delta H)^2 + (H\Delta D)^2\right\}^{1/2} \tag{6.2}$$

and the slope

$$m_H = \frac{\Delta Y}{\Delta X} \tag{6.3}$$

It may be remarked that to determine the slopes of the contour at any point we should consider $\overline{\Delta H}$ defined as equation (6.2) rather than ΔH measured in the observatory. As one can see from this analysis $H\Delta D$ is often comparable with ΔH. This is a fact which is usually ignored by analysts.

The slope of the current vector can be written as

$$m_J = -(m_H)^{-1} \tag{6.4}$$

Since these two are mutually orthogonal, thus as we go from S to T we find the local times at which $\overline{\Delta H}$ takes the same value and at these local times we then determine m_{H1} and m_{H2} and hence m_{J1} and m_{J2}. The problem reduces to one of coordinate geometry in which one has to determine the equation for a conic with two tangents at A and B whose equations are written as $l_1 = 0$ and $l_2 = 0$ respectively and the chord of contact AB with the equation given by $l_3 = 0$. The general equation for a set of concentric ellipses is then given by

$$l_1 l_2 - K l_3^2 = 0 \tag{6.5}$$

where

$$l_1 \equiv y - y_1 - m_1 (x - x_1) = 0 \tag{6.6}$$

$$l_2 \equiv y - y_1 - m_2 (x - x_2) = 0 \tag{6.7}$$

$$l_3 \equiv y - y_1 = 0 \tag{6.8}$$

K forms a parameter which characterizes any particular ellipse. Substituting (6.6) to (6.8) in (6.5) we get after some reduction

$$\begin{aligned} & m_1 m_2 x^2 - (m_1 + m_2) xy + (1-K) y^2 \\ & + \{ y_1 (m_1 + m_2) - m_1 m_2 (x_1 + x_2) \} x \\ & + \{ m_1 x_1 + m_2 x_2 - 2 y_1 (1-K) \} y \\ & + \{ y_1^2 (1-K) - y_1 (m_1 x_1 + m_2 x_2) + m_1 m_2 x_1 x_2 \} = 0 \end{aligned} \tag{6.9}$$

The condition that equation (6.8) should represent the system of ellipse is

$$-K > (m_1 - m_2)^2 / 4m_1 m_2 \qquad (6.10)$$

This implies that one of the slopes m_1 or m_2 must be negative definite. For these system of ellipses the coordinates of the centre $(\bar{x}, \bar{y})$ is given by

$$\bar{x} = \frac{2(1-K)\,m_1 m_2 (x_1 + x_2) - (m_1 + m_2)(m_1 x_1 + m_2 x_2)}{4 m_1 m_2 (1-K) - (m_1 + m_2)^2} \qquad (6.11)$$

and

$$\bar{y} = y_1 + \frac{m_1 m_2 (m_2 - m_1)(x_1 - x_2)}{4 m_1 m_2 (1-K) - (m_1 + m_2)^2} \qquad (6.12)$$

We can determine $\bar{y}$ by knowing K which in turn can be determined if we know $\bar{x}$. On the other hand if we eliminate K between (6.11) and (6.12) we get the locus of the point $(\bar{x}, \bar{y})$. In the first case we shall determine the focus knowing the local time at which $\overline{\Delta H}$ is maximum or minimum while in the second we shall be determining the movement of the S_q focus in a day.

(b) Focus Determination

We have chosen 28th March, 1964 which satisfies both the equinoxial and the solar manimum conditions, to test the theory. In fig. (6-3), we have plotted $\overline{\Delta H}$ with local time for Alibag station. We have taken readings on the magnetograms at a very close interval (10 to 15 minutes) since the time of minimum occurrence is very crucial on this day $K_p = 1-$ and hence a real quiet day. In Table (6-1) we have given ΔH, ΔD and $H\Delta D$ from which we have calculated ΔX and ΔY. One may note ΔX and Δy here are comparable. We have same value of $\overline{\Delta H}$ at 1030/1215 and 1015/1230 etc. We also have maximum as $\overline{\Delta H}$ at 1145 local time. Taking 1145 as $\overline{x}$ (converting in to degrees) we determine (1-K) from equation (6.11) and substituting this in equation (6.12) we determine $\overline{y}$. As can be seen, the deviation of consecutive value of $\overline{y}$ goes on increasing as we go down the list in the table (6-1). This clearly indicates the deviation from the ellipse is more dominant in the 0936/1312 pair and 0930/1324 as compared to 1030/1215 and 1015/1230. As it has already been pointed out the theory would be more accurate when we take the neighbourhood close to 1145.

We repeated this calculation for Tashkent (geographic latitude 41.25° N, longitude 69.0° E, geomagnetic latitude 32.6° N) for the same day. The minimum occurred in $\overline{\Delta H}$ at

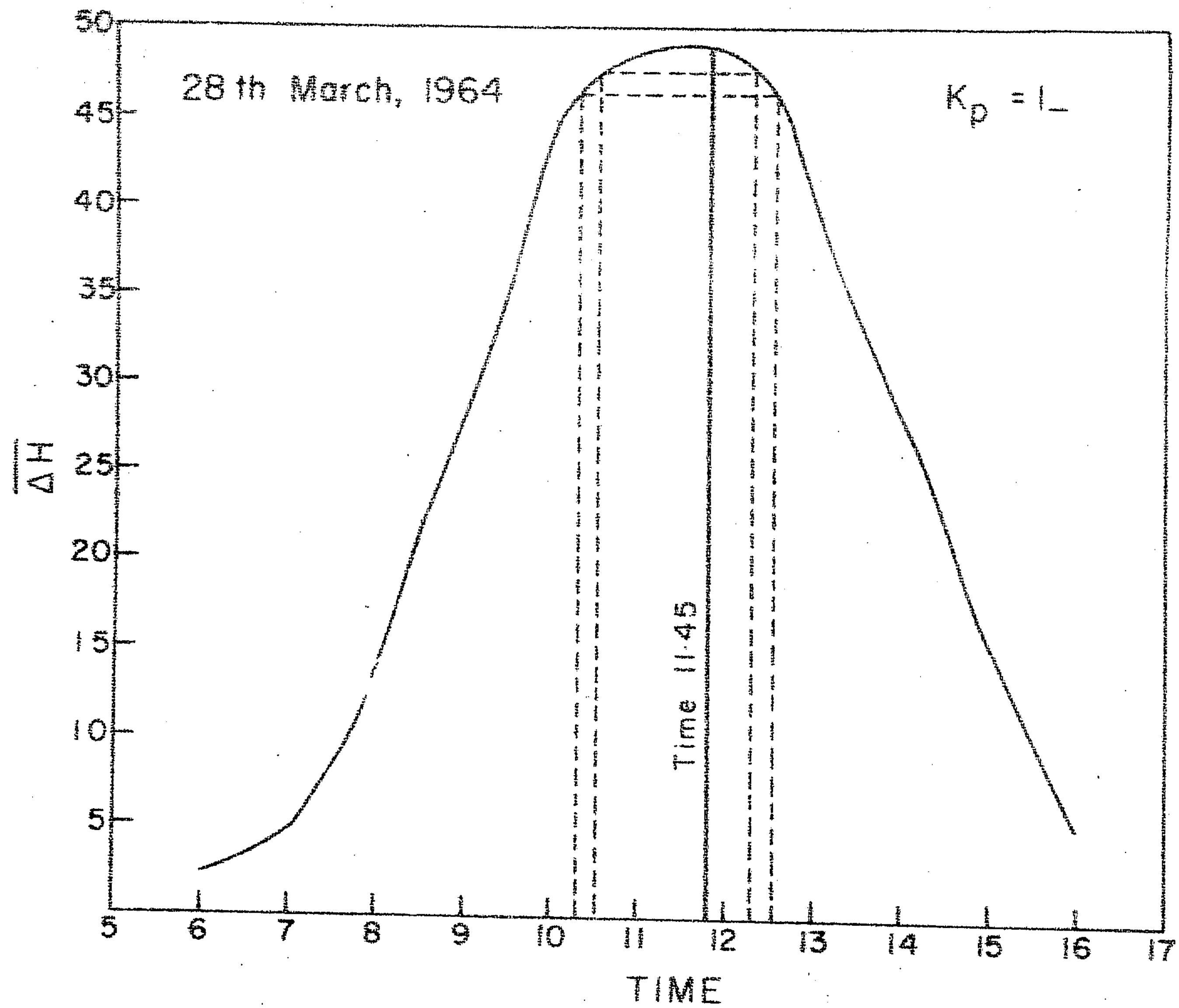

Figure (6-3) : The plot of $\overline{\Delta H}$ (as defined by equation (6.2)) for March 28, 1964 when $K_p = 1_-$ for Alibag. In this $\overline{X}$ is the LT (1145 hrs.) at which $\overline{\Delta H}$ is maximum.

Table (6-1) showing the magnetic data for March 28, 1964 at Alibag and the details regarding the determination of S_q Focus. Here $H = 38775.66\gamma$, $D = 1^{\circ}\ 1.5068'$, $\bar{X} = 1145$ hrs.

Sr. No. of Pairs	LT	ΔH	ΔD	$H \cdot \Delta D$	ΔX	ΔY	m_H	m_J	$\bar{Y}$
1	1030	46.34	1.0025'	11.312	46.13	12.15	0.2633	-3.797	39.62°
	1215	40.34	-1.041'	-11.746	40.54	-11.02	-0.2718	3.679	
2	1015	44.34	1.126'	12.705	44.10	13.50	0.3061	-3.267	37.55°
	1230	44.34	-1.207'	-13.618	44.58	-12.82	-0.2876	3.477	
3	1003	42.34	1.293'	14.590	42.07	15.35	0.365	-2.740	36.92°
	1239	42.34	-1.290'	-14.556	42.59	-13.79	-0.3238	3.088	
4	0954	40.34	1.293'	14.59	10.07	15.32	0.3823	-2.616	34.99°
	1248	39.34	-1.290'	-14.556	39.59	-13.85	-0.3498	2.859	
5	0945	38.34	1.2093'	13.651	38.08	14.34	0.3765	-2.656	34.94°
	1257	38.34	-1.285'	-14.501	38.59	-13.81	-0.3579	2.795	
6	0936	36.34	1.293'	14.590	46.07	15.24	0.4235	-2.352	32.97°
	1312	36.34	-1.207'	-13.618	36.58	-12.96	-0.3543	2.822	
7	0930	32.34	1.3765'	15.532	32.05	16.11	0.5027	-1.989	28.73°
	1324	32.34	-1.165'	-13.148	32.57	-12.57	-0.3859	2.591	

1100 hrs. local time is seen in fig. (6-4). The difference between the occurrence of maxima and minima at Alibag (11. and Tashkent (1100) could be due to the effect of induced field and may be due to the fact that Alibag is on the sea-shore while Tashkent is land-locked. Further-more Tashkent is very close to focus and hence the magnetic field variation at times is like that of Alibag and at other times is like that of Agincourt. Nevertheless the $\bar{y}$ determined from Tashkent agrees remarkably well with that determined from Alibag data. This clearly vindicates the correctness of the assumption. Table (6.2) gives the ΔH, ΔD and $H\Delta D$ values for Tashkent station.

6.3. Dynamics of S_q focus

As has been pointed out, if we eliminate (1-K) from equations (6.11) and (6.12) we get an equation connecting $\bar{y}$ and $\bar{x}$ and can be written as

$$\frac{1}{m_1}(\bar{y}-y_1)-(\bar{x}-x_1)+\frac{1}{m_2}(\bar{y}-y_1)-(\bar{x}-x_2)=0 \qquad (6.13)$$

We can consider the above equation as an equation connecting $\bar{x}$ and $\bar{y}$ with m_1 and m_2 changing in time. Since m_1 and m_2 are determined at (x_1, y_1) and (x_2, y_1), we can

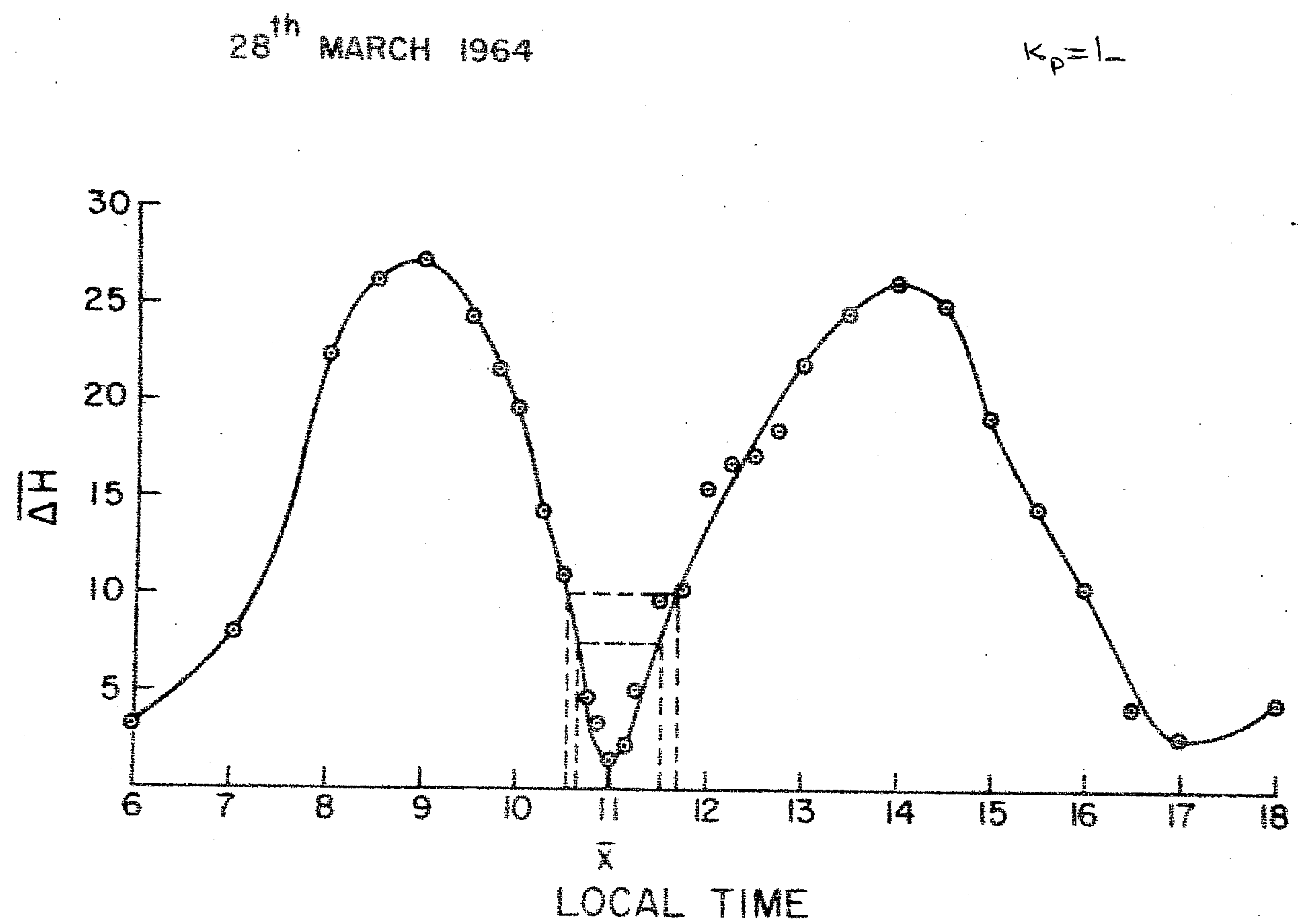

Figure (6-4) : The plot of $\overline{\Delta H}$ for March 28, 1964 when $K_p = 1_-$ for Tashkent. In this $\overline{X}$ is the LT (1100 hrs.) at which $\overline{\Delta H}$ is minimum.

Table (6-2) showing the magnetic data for March 28, 1964 at Tashkent and the details regarding the determination of S_q Focus. Here $H = 25768.69\gamma$, $D = 4^{\circ}\ 53.01'$, $\bar{X} = 1100$ hrs.

Sr. No. of Pairs	LT	ΔH	ΔD	$H \cdot \Delta D$	ΔX	ΔY	m_H	m_J	$\bar{Y}$
1	1039	-5.67	0.774'	5.823	-6.1462	5.3207	-0.8657	1.1551	40.7696°
	1132	1.14	-1.28'	-9.629	1.9554	-9.4993	-4.859	0.2058	
2	1033	-5.74	0.953'	7.162	-6.3290	6.6491	-1.0506	0.9519	40.6354°
	1142	1.14	-1.37'	-10.291	2.0119	-10.159	-5.0494	0.1980	
3	1028	-7.38	1.131'	8.507	-8.0788	7.8648	-0.9735	1.0272	39.2925°
	1154	5.12	-1.55'	-11.662	6.095	-11.186	-1.8354	0.5448	
4	1007	-10.22	2.014'	15.152	-11.4746	14.2308	-1.240	0.8063	38.906°
	1225	5.69	-2.08'	-15.64	7.0016	-15.1066	-2.1575	0.4635	
5	0942	-10.22	2.559'	19.1719	-11.8167	18.3370	-1.5518	0.6444	37.727°
	1306	7.96	-2.449'	-18.345	9.4941	-17.6052	-1.8542	0.5393	

write this equation separately

$$\frac{1}{m_1}(\bar{y} - y_1) - (\bar{x} - x_1) = 0$$

$$\frac{1}{m_2}(\bar{y} - y_1) - (\bar{x} - x_2) = 0 \qquad (6.14)$$

If we draw these lines at various local times (x_i) and on the station line y_1 keeping in mind x_1 and x_2 are on either side of the centre C, we can draw a curve which is an envelope generated by these lines. This curve will then be the locus of ($\bar{x}$, $\bar{y}$) to which (6.14) gives the tangent at various points. This curve therefore represents the motion of S_q focus as a function of local time during the day. In fig. (6.5) and (6.6) we have generated this curve for Alibag and Tashkent. The y axis in both these cases are drawn in arbitrary scale. They are of the order 3° to 4° latitude

6.4. Discussion

We have discussed in this chapter a method for obtaining S_q focus from a single station data and we have shown this method is much superior to the methods already known. We would however like to point out the following facts as regards the choice of data as well as plotting.

1. We have taken in this case a very quiet day during an equinoxial period of solar minimum. If we take other days the

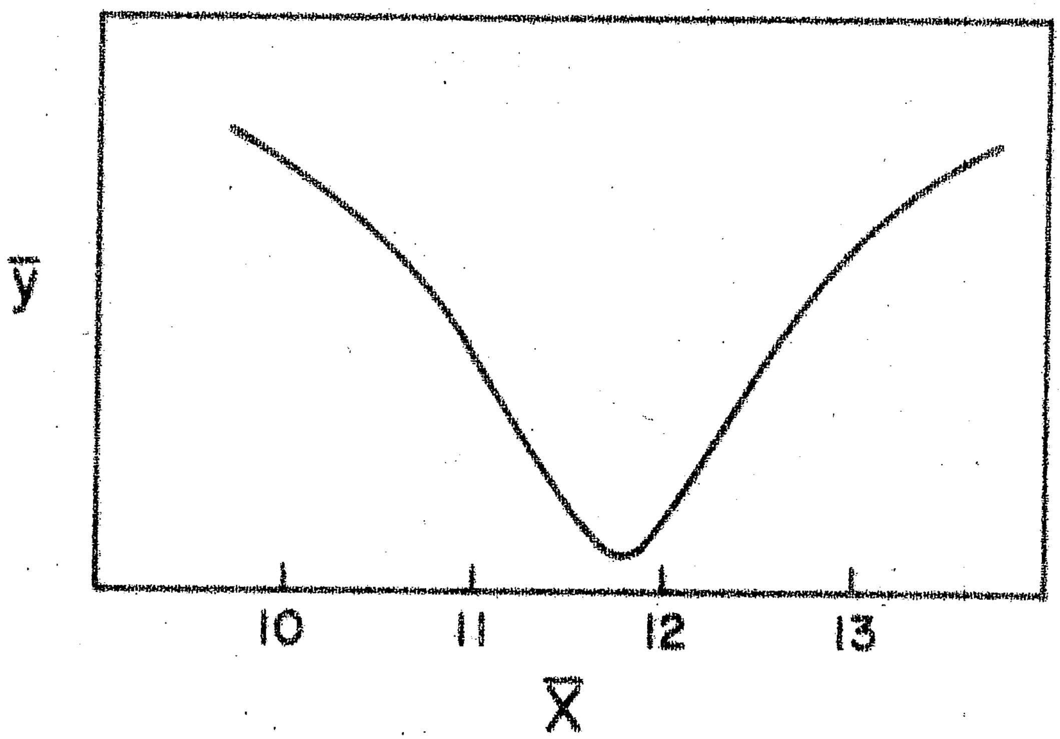

Figure (6-5) : Envelope curve for Alibag for which straight lines (as given by ecuation (6.14)) are tangents. The curve depicts the movement of the S_q focus C ($\overline{X}$, $\overline{Y}$) during the time of the day.

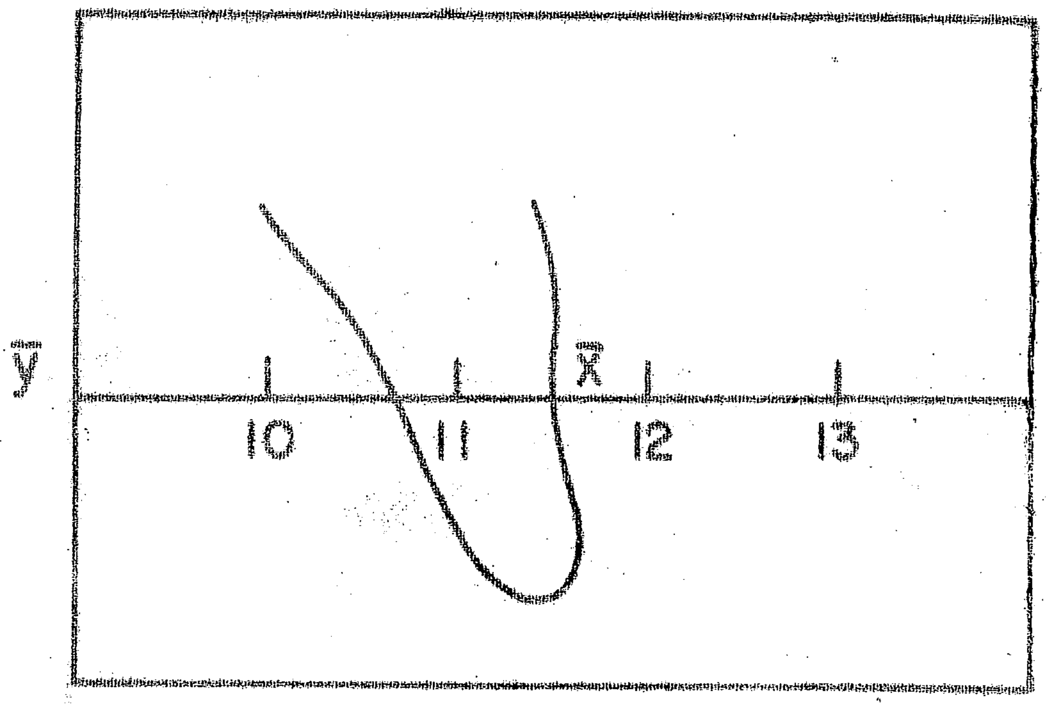

Figure (6-6) : Envelope curve for Tashkent for which straight lines are tangents. The curve depicts the movement of the S_q focus $C(\bar{X}, \bar{Y})$ during the time of the day.

disturbance field must be completely removed to obtain the pure S_q field. This is discussed by Kane (1973) at length.

2. As has already been pointed out, we should take $\overline{\Delta H}$ as defined in equation (6.2) instead of ΔH since $H/\Delta Q$ can effect $\overline{\Delta H}$ considerably.

3. The time of occurrence of maxima or minima depending on as to whether the station is south or north of the focus is very crucial. Hence in digitising the magnetograms one should take very close interval in this neighbourhood.

4. In taking the station like Tashkent whose latitude is very close to the focus, extreme care has to be taken in determining the slopes as well as the envelope curve since we do not get a change in sign of slope as was observed in Alibag.

5. In calculating $\overline{\gamma}$ we have considered the magnitude of slopes and not its sign since sign has already appeared in eliminating (1-K).

The oscillation of the S_q as determined from this could be either due to rotation of the dipole field since the dipole axis and rotation axis do not coincide or it could also be due to a time dependent phase factor in the velocity potential. Pratap (1955) has shown that the inclination of the dipole axis to the rotation axis introduces only a very insignificant contribution towards the S_q field. Therefore one should really seek for the reason for this movement in the velocity field.

Recently it was found by Spizzichino (1969, 1970) that the velocity potential has a time dependent phase factor and this in turn would imply nonlinear processes at these heights.

There could be a non-ionospheric contribution towards the oscillation of S_q focus during a day as well as day-to-day and season-to-season. It has been shown by Olson (1969, 1974) that there could exist currents on the magnetopause which can have its focus in the neighbourhood of the cleft.

In the figure (6-7a) we have shown the Earth as a rotating sphere. The inclined axis represents the magnetic axis and the plane of the paper is the plane containing the geomagnetic axis, the rotation axis and the Sun.

In the figure (6-7a) the magnetic north pole is in the daylit hemisphere while in figure (6-7b) it is in the neight hemisphere. Points A and B are the singular points at the surface as the magnetosphere and point S is the subsolar point.

The figure has been largely exaggerated to make the points mentioned below clear.

C and C' are current contours. It is not clear that the points A and B are the foci of C and C' respectively. Nevertheless if we rotate the Earth from mid-day to mid-night i.e. a to b we find points A and B shifting north-ward while completing the rotation in a day. These points come back to their original positions. This implies that the segment AB slips up and down

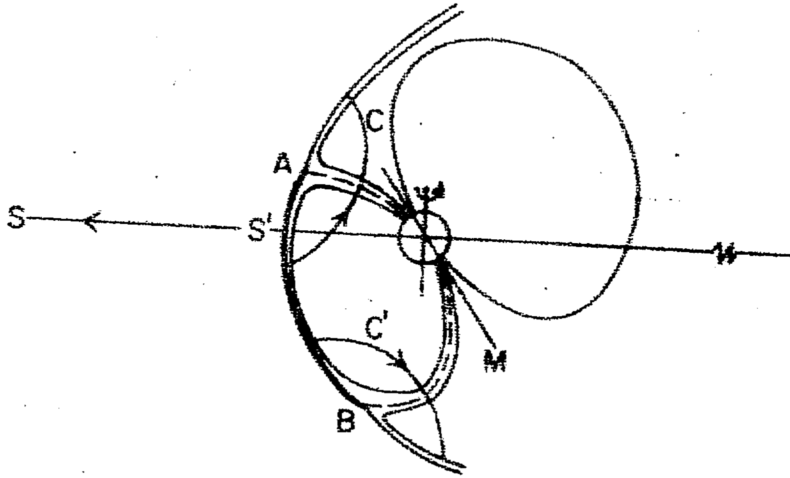

Figure (6-7a)

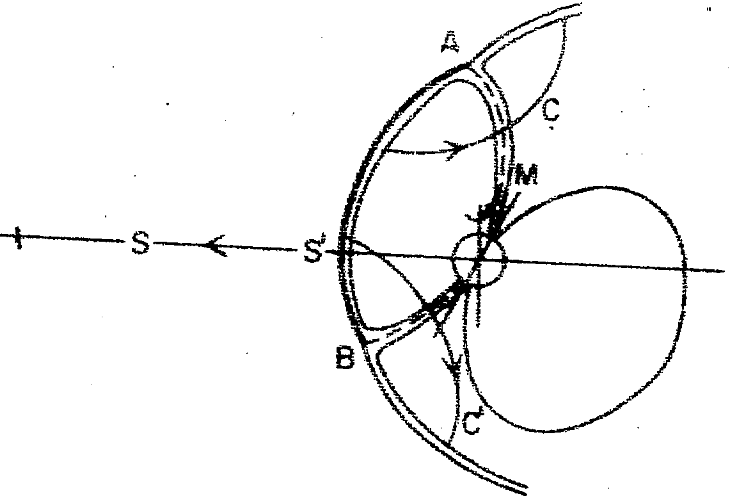

Figure (6-7b)

Figure (6-7a, 6-7b) : The movement of the Magnetopause current system in a day. Figure (6-7a) shows the magnetic N pole in the daylit hemisphere and Figure (6-7b) shows the N pole in the night hemisphere. The line segment AB moves up and down during a day. M is the magnetic axis. C and C' are current contours in both hemispheres. S' is subsolar point.

on the magnetopause as the Earth rotates and therefore executes an oscillation. It can be seen that as the Earth rotates from dawn to dusk, the points come from above to below and then again go up.

This shows that the focus comes towards the equator and attains the nearest point to the equator at local noon and then goes up. This is precisely what we have obtained in the analysis.

Point A is on the axis of the cone which rotates around the north pole since A and B are always on the mid-day meridian, the magnetic axis comes in the dawn-dusk meridian plane. The cone will be twisted when the magnetic axis goes back into night meridian i.e. the apex of the cone goes into midnight meridian. The axis of the cone goes from this point across the pole of Earth to the daylit hemisphere as shown in figure (6-7b).

This cone therefore will be wobbling around the rotation axis. Thus while the base of the cone makes a small retrace on the magnetosphere the apex of the cone is fixed at the pole. This could thus enhance the oscillation of the S_q focus. Between the equinox and solstice the magnetopause will have changed in its orientation by $23\frac{1}{2}^{\circ}$ and this can again contribute to the seasonal oscillation of the S_q focus as observed by Tarpley (1973) and Gupta (1973).

The implication of this method is that from a station like Alibag, having a long record of magnetic data (for the past 120 years), we can reconstruct the solar activity and wind patterns.

CHAPTER VII

SUMMARY

Recent advances in the study of the upper atmospheric physics have shown that the usual assumption of the linear tidal theory is a very crude approximation and that the actual observations reveal a high degree of nonlinearity. In developing a three dimensional dynamo theory we have considered this aspect when we generalized the conductivity tensor and writing it as a function of various field quantities.

The velocity potential that one observes on the surface of the Earth is essentially semidiurnal in nature, while Lindzen has shown that the nonlinear effects could attenuate the semi-diurnal and amplify the diurnal mode in the presence of temperature distribution. In view of this we have taken the diurnal mode of the atmospheric oscillation in the development of the three dimensional dynamo theory. We have shown that the symmetric part of the conductivity tensor containing the magnetic field only does not contribute towards the dynamo field. This is a result which has not been realized by Baker and Martyn and later authors. The results we have obtained conform the observations if we introduce a phase of about 270° for the velocity potential. The current intensity decreases with increasing Υ when we take the conductivity tensor to be dependent on the intrinsic vector field $\vec{\gamma}$.

We have developed a new theory for obtaining the dynamics of S_q focus from a single station data and we have shown that this method gives a new feature viz. the daily variation of the S_q focus in addition to the already known day-to-day and seasonal variations. Using this formula we calculated the motion of S_q focus for two different stations Alibag and Tashkent for March 28, 1964. We interpreted this motion as due to the presence of a phase in the velocity potential which could be time dependent. This is a clear and direct manifestation of the nonlinear effects present at the seat of the current system. Such a motion has not been discovered before and we propose to continue the study by taking the data for the past century at Alibag and thereby obtain the history of the solar activity in the past. We would like to interpret the small differences in the variation between Tashkent and Alibag as due to the fact that Tashkent is landlocked while Alibag is on the sea-cost. This justifies the conjectures of Hasegawa.

Appendix I

NUMERICAL METHOD FOR SOLVING THE BOUNDARY VALUE PROBLEM

In obtaining the solution of the set of equations we required the numerical method. It is to be noted here that the ratio of the dipole moment M and radius of the Earth R_E i.e. $\frac{M}{R_E^3}$ is taken as one. Parameters $\alpha = 1$ and $h = 1.01R_E$.

Now we discuss the method of solving the linear inhomogeneous differential equation of the type

$$DU \equiv \frac{d^2u}{dr^2} + f_1(r)\frac{du}{dr} + f_2(r)u = f_3(r) \quad (1)$$

With the boundary conditions

$$u(r_{min}) = 0 \quad , \quad u(r_{max}) = 0$$

We first choose an arbitrary value for $u'(r_{min}) = E_1$ and solve the equation as an initial value problem by Runge - Kutta method of fourth order followed by Adam's predictor - corrector algorithm of the same order upto r_{max}. Let this solution be $u_1(r)$. Similarly write $u'(r_{min}) = E_2 (\neq E_1)$. We obtain another solution $u_2(r)$. For any scalars α_1 and β_1,

$u = \alpha_1 u_1 + \beta_1 u_2$ satisfies the differential equation (1) if $\alpha_1 + \beta_1 = 1$ since

$$DU = \alpha_1 DU_1 + \beta_1 DU_2 = (\alpha_1 + \beta_1) f_3(r)$$

The condition at the lower boundary is automatically satisfied. Another condition is obtained by imposing the only remaining condition at the upper boundary

$$U(r_{max}) = \alpha_1 U_1(r_{max}) + \alpha_2 U_2(r_{max})$$

Solving for α_1 and β_1, from these two equation, we have

$$\alpha_1 = U_2(r_{max}) / \{ U_2(r_{max}) - U_1(r_{max}) \}$$

$$\beta_1 = 1 - \alpha_1$$

With these values for α_1 and β_1, the two solutions are combined to get the solution of the boundary value problem.

Very often it is necessary to use this solution $U(r)$ and its derivatives as known functions in the successive equation.

In some cases, the equations are of the form

$$g_{11}(r) U_1'' + g_{12}(r) U_2'' + g_{13}(r) U_3''$$

$\doteq$ terms involving lower order derivative

$$g_{21}(r) U_1'' + g_{22}(r) U_2'' + g_{23}(r) U_3''$$

$\doteq$ terms involving lower order derivative

$$g_{31}(r)\,u_1'' + g_{32}(r)\,u_2'' + g_{33}(r)\,u_3''$$

$$= \text{terms involving lower order derivative}$$

While solving these equations, simultaneously as an initial value problem, the second derivative, necessary for the Runge-Kutta-Adam's method, were calculated by inverting the matrix $G = [g_{ij}]$

Appendix II

The recurrence relations for the Legendre functions which have been used in this analysis are given here for the sake of completeness.

$$(2n+1)\cos\theta\, P_n^m = (n-m+1)\,P_{n+1}^m + (n+m)\,P_{n-1}^m$$

$$\begin{aligned}(2n+1)\sin\theta\, P_n^m &= P_{n+1}^{m+1} - P_{n-1}^{m+1}\\ &= (n+m)(n+m-1)\,P_{n-1}^{m-1}\\ &\quad - (n-m+1)(n-m+2)\,P_{n+1}^{m-1}\end{aligned}$$

$$2\,\frac{dP_n^m}{d\theta} = (n+m)(n-m+1)\,P_n^{m-1} - P_n^{m+1}$$

$$(2n+1)\sin\theta\,\frac{dP_n^m}{d\theta} = n(n-m+1)\,P_{n+1}^m - (n+1)(n+m)\,P_{n-1}^m$$

$$\begin{aligned}\frac{2m\,P_n^m}{\sin\theta} &= (n+m)(n+m-1)\,P_{n-1}^{m-1} + P_{n-1}^{m+1}\\ &= (n-m+1)(n-m+2)\,P_{n+1}^{m-1} + P_{n+1}^{m+1}\end{aligned}$$

$$2m\cos\theta\,P_n^m = \sin\theta\,P_n^{m+1} + (n+m)(n-m+1)\sin\theta\,P_n^{m-1}$$

$$(2n+1)\left(\cos\theta \frac{dP_n^m}{d\theta} + \frac{m P_n^m}{\sin\theta}\right) = (n+1)(n+m)(n+m-1)P_{n-1}^{m-1}$$

$$+ n(n-m+1)(n-m+2)P_{n+1}^{m-1}$$

$$(2n+1)\left(\cos\theta \frac{dP_n^m}{d\theta} - \frac{m P_n^m}{\sin\theta}\right) = -\left\{n P_{n+1}^{m+1} + (n+1) P_{n-1}^{m+1}\right\}$$

$$2m \cot\theta \, P_n^m = P_n^{m+1} + (n+m)(n-m+1) P_n^{m-1}$$

$$\frac{dP_n^m}{d\theta} - m \cot\theta \, P_n^m = - P_n^{m+1}$$

$$\frac{dP_n^m}{d\theta} + m \cot\theta \, P_n^m = (n+m)(n-m+1) P_n^{m-1}$$

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