Physics Beyond Standard Model and Horizontal Symmetries

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Certificate

This is to cerify that the thesis entitled 'Beyond Standard Model and Horizontal Symmetries' contains the original research work of the candidate and that neither this thesis nor any part of it has been submitted for any degree or diploma before.

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Chapter 1

INTRODUCTION

The electroweak interaction of leptons and quarks are described by a unified model of the weak and the electromagnetic interactions developed by Glashow, Weinberg and Salam [1]. This model, together with Quantum chromodynamics (QCD), gives the correct description of all the fundamental interactions in nature except gravity and is reffered to as the Standard model. We restrict ourselves to the electroweak sector of this theory. The internal consistency of the standard model has been verified in various experiments to a high degree of precision. However certain aspects of this model are unsatisfactory. The fermions that take part in the electroweak interactions are classified into three distinct sets called families. These families of fermions form three identical representations of the standard model. These families are identical in all respects except for an hierarchy in their masses. The fermions from different families mix with each other to form physical mass states. The pure weak states and the mass states are connected by mixing matrices determined by mixing angles and the relative phases of the states. The masses of the fermions enter as parameters in the standard model. Along with the mixing angles and the phases of these mixing matrices, the number of parameters in the model becomes too large. Symmetries that connect the families can produce relationships between masses and mixing and reduce the arbitrariness in this sector. Such symmetries are known as horizontal symmetries. This thesis studies the simplest of such symmetries, namely the abelian one and investigates physics beyond the standard model implied by them.

In this chapter we give a brief account of the study of weak interactions, the considerations that lead to development of the standard model and the need to go beyond. Section 1.1 introduces the weak interaction phenomenon. Section 1.2 gives the essential ideas involved in the standard model. Section 1.3 discusses the problem of fermion masses within the Standard model. In section 1.4 we discuss the fermion mixing. Section 1.5 gives an account of the problems with Standard model and discusses possible ways to go beyond. Section 1.6 discusses phenomenological ansatz that restricts fermion masses and mixing. Section 1.7 introduces the idea of Horizontal symmetries that can generate the required structure for the mass matrices. Section 1.8 gives a brief summary of this chapter and a plan of the thesis.

1.1 The weak interaction Phenomena

The study of weak interaction begins with the discovery of beta decay. In this a neutron inside the nucleus of an atom decays to a proton and emits an electron. The energy of the emitted electron in such a scattering is expected to be unique. However it was found that they are emitted with continuous spectrum of energy. Also the spin of all the particles involved, the neutron, the proton and the emitted electron is half. This is impossible, since if two final state particles have spin half then the initial single particle state should be spin 0 or spin 1, whereas the neutron is a spin half particle. It was suggested by Pauli that the beta decay must be accompanied by a spin $\frac{1}{2}$ particle with a very small or zero mass. This particle called neutrino is chargeless and evades detection. Such a hypothesis by Pauli remedied the above mentioned problems in beta decay. In the line of Qauntum electrodynamics (QED), Fermi suggested the current-current form of weak interaction [2]. In Fermi's theory the interaction Lagrangian for beta decay is given by

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} j_{pn} j_{\bar{e}\nu_e}$$

Where the two interacting currents are

$$j_{pn} = \psi_p \gamma_\mu \psi_n$$

$$j_{\bar{e}\nu e} = \bar{\psi}_e \gamma_\mu \psi_{\nu e}$$

However there are certain essential differences in the case of beta decay. Firstly the two currents that interact have to be charged current. This is because the neutrino is chargeless while the electron is negatively charged. The proton is positively charged whereas the neutron is chargeless. Secondly this interaction must be a point interaction. This is because the weak interaction is a very short range interaction.

In Fermi theory weak interaction cross section are proportional to center of mass energy and blow up at high energies. It was felt that such a difficulty can be overcome if the interaction can be induced by a gauge symmetry in analogy with quantum electrodynamics. In such theories interaction between currents are mediated by bosons called gauge bosons. These gauge bosons appear automatically in the theory when one demands local symmetry. Such local symmetries are reffered to as gauge symmetries. We shall discuss this in section 2. However such a gauge symmetry as in the Yang Mills [3] theory would introduce massless gauge bosons that mediate the interaction. Weak interactions being extremely short range must be mediated by massive gauge bosons and not massless ones as necessitated by the gauge symmetries. A massive gauge boson being essential the intermediate vector boson hypothesis was introduced where mass terms for gauge bosons were explicitly written. This breaks the gauge symmetry explicitly. But such a theory being non-renormalizable ran into difficulties with infinite cross sections at higher orders. Thus one needs a theory where the gauge symmetries are broken at low energies but are restored at sufficiently high energies giving a renormalized theory. Such a phenomenon is called as spontaneous symmetry breaking. The mass terms of the gauge bosons are not explicitly introduced. They get masses by interaction with scalar fields at low energies. It was shown by G. t'Hooft [4] that spontaneously broken gauge theories are renormalizable.

As weak interactions involve charged current, the gauge group was taken to be $SU(2)$. It was found that weak interaction violates parity maximally [5, 6]. Hence the gauge group has to operate only on left handed fields. The particles that take part in weak interactions are called the leptons and quarks. Quarks form the constituents of hadrons like the protons and the neutrons. The left handed leptons and quarks behave as doublets under the gauge transformations of weak interaction. The lepton doublet consists of a charged lepton like the electron and a chargeless one, the neutrino. The quark doublet consists of an up quark and a down quark. The required gauge group is denoted as $SU(2)_L$. It was felt that QED which is an $U(1)$ gauge theory should be also incorporated in the same gauge theory. However QED conserves parity. Moreover, the electric charge of the members of a doublet are not same. So the $U(1)$ of electromagnetic theory cannot be directly incorporated as a gauge group of the so called electroweak interaction. So a new charge, namely hypercharge, Y, was introduced. The $U(1)$ symmetry generated by Y is denoted as $U(1)_Y$. Members of the multiplets of $SU(2)_L$ are assigned the same hypercharge. A combination of $U(1)_Y$ with the subgroup of $SU(2)_L$ generated by its diagonal generator is the $U(1)_Q$ of electromagnetic interaction. This combination is specified by the definition of the electromagnetic charge

$$Q = T_3 + \frac{Y}{2} \tag{1.1}$$

1.2 The Standard model

The electroweak sector of the standard model is a gauge theory based on the gauge group $SU(2)_L \times U(1)_Y$. The fermions that take part in electroweak interactions are classified as the leptons and the quarks. The leptons consists of the electron, the muon and the tau lepton which are negatively charged and their neutral counterparts, the neutrinos. There are six quarks arranged in the up sector and the down sector. The up sector consists of the up, the charm and the top quarks, while the down sector consists of the down, the strange and the bottom quarks. These fermions form three families of the representation of the standard model gauge group. These families of leptons and quarks are as follows:

$$\text{left handed leptons:} \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

$$\text{left handed quarks:} \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\text{right handed leptons:} \quad e^-_R \qquad \mu^-_R \qquad \tau^-_R$$

$$\text{right handed quarks:} \quad u_R,\ d_R \qquad c_R,\ s_R \qquad t_R,\ b_R$$

The left handed fermions transform as doublets under $SU(2)_L$ while the right handed fields transform as singlets. Hypercharge assignments are such that the left handed and the right handed components of a field has the same charge Q. Right handed neutrinos are not introduced in the model. This, as we will see, leads to massless neutrinos in the model. There are four gauge bosons in the model. Three corresponding to the gauge symmetry $SU(2)_L$, viz. W^+, W^- and W^3 and one corresponding to the $U(1)_Y$ of hypercharge,B. It also contains a complex scalar doublet Φ called the Higgs boson.

The interaction of the fermions with gauge bosons comes about automatically by demanding the local gauge invariance of the free fermionic Lagrangian.

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \tag{1.2}$$

Under the gauge transformation

$$\psi \rightarrow U\psi \quad \text{where} \quad U = e^{i\frac{\tau}{2}\cdot\alpha} \tag{1.3}$$

$$\begin{aligned} \partial_\mu\psi &\rightarrow \partial_\mu(U\psi) \\ &= U\partial_\mu\psi + (\partial_\mu U)\psi \end{aligned} \tag{1.4}$$

Here τ^a are the generators of the gauge group. In case of the Standard model, τ^a are the three Pauli matrices generating the $SU(2)_L$ which acts only on the left handed fields. τ^a is identity when it generates the $U(1)_Y$ which acts on both, the left and the right handed fields. The presence of the second term in eq.(1.4) above spoils the gauge invariance of the Lagrangian. In order to restore the gauge invariance, the ordinary differentials $\partial_\mu\psi$ are replaced by what is known as the covariant differentials $D_\mu\psi$ which transform in the same way as ψ under gauge transformations. The covariant differentials are given as follows:

$$D_\mu\psi = (\partial\mu - ig\frac{T}{2}\cdot W_\mu)\psi \tag{1.5}$$

W_μ are gauge fields which transform in such a way so as to make D_μ covariant. Under the gauge transformation $D_\mu\psi$ transforms as follows

$$\begin{aligned} D_\mu\psi &\rightarrow U\partial_\mu\psi + (\partial_\mu U)\psi - ig\frac{T}{2}\cdot W_\mu\psi \\ &= U\left(\partial_\mu + U^\dagger(\partial_\mu U) - igU^\dagger\frac{T}{2}\cdot W_\mu U\right)\psi \end{aligned} \tag{1.6}$$

Under infinitesimal transformation

$$U^\dagger(\partial_\mu U) \approx i\frac{T}{2}\cdot\partial_\mu\alpha$$

We have from (1.6) and (1.2)

$$D_\mu\psi \rightarrow U\left(\partial_\mu + i\frac{T}{2}\cdot\partial_\mu\alpha - igU^\dagger\frac{T}{2}\cdot W_\mu U\right)\psi \tag{1.7}$$

Covariance of $D_\mu\psi$ demands

$$D_\mu\psi \rightarrow UD'_\mu\psi$$

where

$$D'_\mu\psi = (\partial\mu - ig\frac{T}{2}\cdot W'_\mu)\psi \tag{1.8}$$

W'_μ is the transformation of the gauge field W_μ under the gauge transformation. From eq.(1.7) and (1.8) we have

$$\tau\cdot W'_\mu = -\frac{1}{g}\tau\cdot\partial_\mu\alpha + U^\dagger\tau\cdot W_\mu U \tag{1.9}$$

$$= -\frac{1}{g}\tau\cdot\partial_\mu\alpha + \tau\cdot W''_\mu \tag{1.10}$$

where $W''_\mu = e^{iT\cdot\alpha}W_\mu$. T^a are the generators of the adjoint representation of the gauge group. Thus

$$\begin{aligned} W'_\mu &= -\frac{1}{g}\partial_\mu\alpha + e^{iT\cdot\alpha}W_\mu \\ &= W_\mu - \frac{1}{g}\partial_\mu\alpha - \alpha\times W_\mu \end{aligned}$$

The gauge bosons W_μ are massive. But the explicit mass terms, namely, $\frac{1}{2}m^2 W_\mu W^\mu$ would break the gauge symmetry. Higgs mechanism provides a way to generate the gauge boson masses through the spontaneous gauge symmetry breaking. In this, a scalar field known as Higgs field Φ is introduced into the model. The potential of this field is such that its minimum has a non-zero vacuum expectation value. The minimal coupling of the gauge bosons to the Higgs field, as in eq.(1.5) above, generates the masses of the gauge bosons. The Higgs field chosen for the Standard model is a doublet of $SU(2)_L$ with a hypercharge $Y = 1$. The $SU(2)_L \times U(1)_Y$ invariant Lagrangian for Φ is

$$\mathcal{L} = \left|(\partial_\mu - ig_1\frac{\tau}{2}\cdot W_\mu - ig_2\frac{Y}{2}B_\mu)\Phi\right|^2 - V(\Phi)$$

The doublet Φ is conveniently represented with the electric charges as

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} \qquad \text{with} \qquad \begin{array}{l} \Phi^+ = (\Phi_1 + i\Phi_2)/\sqrt{2} \\ \Phi^0 = (\Phi_3 + i\Phi_4)/\sqrt{2} \end{array} \tag{1.11}$$

$V(\Phi)$ is the Higgs potential given by

$$V(\Phi) = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

$V(\Phi)$ has non-zero vacuum expectation value (vev) for $\mu^2 < 0$ and $\lambda > 0$. The minimum occurs when

$$\Phi^\dagger\Phi = \frac{v^2}{2} = \frac{-\mu^2}{2\lambda}$$

The vacuum expectation value of Φ which breaks the $SU(2)_L \times U(1)_Y$ symmetry but retain the $U(1)_Q$ is

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

The masses of the gauge bosons come from the following:

$$\left|\left(-ig_1\frac{\tau}{2}\cdot W_\mu - i\frac{g_2}{2}B_\mu\right)\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}\right|^2 \tag{1.12}$$

$$= (\frac{1}{2}vg_1)^2 W_\mu^+ W^{-\mu} + \frac{1}{8}v^2(W_\mu^3 \;\; B_\mu)\begin{pmatrix} g_1^2 & -g_1g_2 \\ -g_1g_2 & g_2^2 \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} \tag{1.13}$$

Thus $M_W = \frac{1}{2}vg_1$ is the mass of the charged gauge boson. The neutral gauge bosons, $W^{3\mu}$ and B^μ, don't have a well defined mass. The physical or mass states are obtained by a linear combination of these neutral fields as follows,

$$
\begin{aligned}
Z_\mu &= (g_1 W_\mu^3 - g_2 B_\mu)\frac{1}{\sqrt{g_1^2+g_2^2}} = \cos\theta_W \cdot W_\mu^3 - \sin\theta_W \cdot B_\mu \\
A_\mu &= (g_1 W_\mu^3 + g_2 B_\mu)\frac{1}{\sqrt{g_1^2+g_2^2}} = \sin\theta_W \cdot W_\mu^3 + \cos\theta_W \cdot B_\mu
\end{aligned}
$$

where $\frac{g_2}{g_1} = \tan\theta_W$.

$$M_Z = \frac{1}{2}v\sqrt{g_1^2+g_2^2}, \quad \text{and} \quad M_A = 0$$

Thus

$$\frac{M_W}{M_Z} = \frac{g_1}{\sqrt{g_1^2+g_2^2}} = \cos\theta_W.$$

θ_W, the mixing angle between W_μ^3 and B_μ is called the Weinberg angle.

The strong interaction of the quarks are described by a gauge group $SU(3)_C$ of color. The quarks are assigned color charges denoted as red(R), blue(B) and green(G). We do not discuss the strong interactions in this thesis.

1.3 Fermion Masses

The free fermions are described by the Dirac Lagrangian given in eq.(1.2)

$$\mathcal{L}_D = \overline{\psi}(\partial\!\!\!/ - m)\psi$$

The term $m\overline{\psi}\psi$ is called the Dirac mass term of the fermion ψ and can be written as follows:

$$m\overline{\psi}\psi = m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L)$$

This is because the terms $\overline{\psi_L}\psi_L$ and $\overline{\psi_R}\psi_R$ vanish.

In Standard model the left and the right handed fields transform differently under its gauge symmetry. The left handed fields transform as doublets while the right handed fields transform as singlets. Hence explicit fermion mass terms $m\overline{\psi}\psi$ are not invariant under the gauge group of the standard model. However the mass terms can be generated in a gauge invariant way through the interaction terms of the fermions with the Higgs scalar after spontaneous symmetry breaking. These interactions are generated through the Yukawa couplings given by the following Lagrangian

$$\mathcal{L} = -G_a \bar{\psi}_L^a \Phi \psi_R^a$$

Here ψ_L is a doublet while ψ_R is a singlet of $SU(2)$. a stands for leptons or quarks. These terms give mass only to the electrons and the down quarks. To give mass to the up quark one considers the Higgs field $\Phi^C = i\tau_2\Phi^*$.

Though all the charged fermions get mass through the vacuum expectation value of the same Higgs, their coupling with the Higgs is still arbitrary and hence there is no prediction of fermion masses in the standard model.

Dirac mass terms cannot be written for neutrinos in standard model as right handed neutrinos are not introduced. However for chargeless fermions like neutrinos the Dirac mass terms, as discussed above, is not the only way to generate mass. One can write the Majorana mass terms given by

$$M\bar{\psi}^C\psi + H.C.$$

Where ψ^C is the charge conjugate of ψ given by $\psi^C = i\gamma_2\psi^*$. Obviously such a mass term cannot be written for charged fermions as it would break the $U(1)_Q$ symmetry and lead to the violation of charge conservation. In terms of the left and the right handed fields the Majorana mass term can be written as

$$-\mathcal{L}_M = \frac{1}{2}M(\bar{\psi}_L^C\psi_L + \bar{\psi}_R^C\psi_R) + H.C$$

As right handed neutrinos are not introduced in the Standard model, the Majorana mass term for neutrinos may be written as

$$-\mathcal{L}_M = \frac{1}{2}M(\bar{\nu}_L^C\nu_L) + H.C$$

However it may be noted that left handed neutrinos are part of $SU(2)_L$ doublets. T_3 isospin of ν_L and ν_L^C are $+1/2$ and $-1/2$ respectively. The Majorana mass term for neutrinos thus violates the $SU(2)_L$ symmetry. To preserve the $SU(2)_L$ symmetry the Majorana mass term has to couple to a triplet Higgs. Thus neutrinos can neither have Dirac mass nor Majorana mass within the Standard model.

1.4 Mixing of fermions

As mentioned in section 1.2 the fermions form three identical families of representation of Standard model. Such a classification of the fermions into different families is possible because weak interactions do not generally transform them from one family to another. However decays of certain hadrons and mesons showed that such a strict distinction is not true. In order to understand these decays one had to consider charged transitions amongst members of different families of quark. In the case of two flavors, the strength of flavor conserving charged transitions to the strength of flavor violating ones was parameterized with an angle called the Cabbibo angle [7]. The mass states of down quarks is a linear combination of flavor states. In the two family case if we denote the flavor states with a prime viz. d' and s', then the mixing can be parameterized with an angle θ_C as follows [8]:

$$\begin{aligned} d' &= \cos\theta_C d + \sin\theta_C s \\ s' &= -\sin\theta_C d + \cos\theta_C s \end{aligned}$$

θ_C is called the Cabbibo mixing angle. The mixing can involve complex phases. The unitarity of the mixing restricts the number of phases to three in two dimensional case. All the three phases can be absorbed in the arbitrary phases of the individual quark states. However, as we will see below, if there are N families in general, all the phases in a $N \times N$ unitary matrix cannot be removed.

To get flavor changing currents amongst N families of quarks, one can consider mixing between the families of quarks. The up quark and the down quark can mix in different way but due to the fact that only flavor changing charged current are observed and not the neutral current, only a relative mixing between the up and the down quarks is observable. In such a case one can consider only an effective mixing in the down quarks and no mixing in the up quarks. Suppose the up sector of the quark mixing is given by a Unitary matrix U and the down sector mixing is given by D. Let an up and a down quark of i^{th} family be denoted as u'_i and d'_i respectively. Let the mass states be represented by u_i and d_i. Then we have

$$
\begin{aligned}
u'_i &= U_{ij}u_j \\
d'_i &= D_{ij}d_j
\end{aligned}
\tag{1.14}
$$

Charged weak current is possible only between pure flavor states. Thus a pure weak current is flavor conserving and would look like

$$
j'_{ii} = \bar{u}'_i \Gamma d'_i \tag{1.15}
$$

where Γ is an operator in the spinor space. Γ depends upon the nature of the current and is not important here. From (1.14) and (1.15) we get the charged current in terms of the mass states u and d

$$
\begin{aligned}
j'_{ii} &= \bar{u}_j U^{\dagger}_{ji} \Gamma D_{il} d_l \\
&= K_{jl} j_{jl}
\end{aligned}
$$

where $j_{jl} = \bar{u}_j \Gamma d_l$ are the flavor changing charged current for $j \neq l$ and $K = U^{\dagger}D$. The amplitude of the flavor changing charged current is given by the non diagonal elements of K. K is the relative mixing matrix between the up and the down sectors. It is called the Cabbibo Kobayashi Maskawa (CKM) matrix [9]. It is obvious from the above discussion that the amplitude for the flavor changing neutral current (FCNC) is zero. This is because in this case the matrix similar to the CKM matrix will be given by $U^{\dagger}U$ or $D^{\dagger}D$ which are identity.

The CKM matrix, being an unitary matrix has $\frac{N(N+1)}{2}$ independent phases. But as the $2N$ quarks can have $2N$ arbitrary phase, the mixing matrix has $2N$ redundant phases . An overall phase in the whole quark spectrum will not show up at all in the mixing matrix. So the redundant phases are $(2N - 1)$. The number of meaningful phases left in the CKM matrix is thus

$$
\frac{N(N+1)}{2} - (2N - 1) = \frac{1}{2}(N - 1)(N - 2)
$$

For three families the number of arbitrary phase is one.

Within standard model an equivalent of CKM matrix does not exist in the lepton sector. This is because the neutrinos are massless in Standard model. This makes them degenerate in mass. Thus the flavor states or any mixing of them are always simultaneous mass eigenstates. Hence one can consider the same mixing in the neutrino sector as in the charged lepton sector. There will be no relative mixing between the charged leptons and the neutrinos. However experimental observations do not rule out a small mass for neutrinos and mixing amongst them [10]. In models beyond Standard model one can consider massive neutrinos and mixing between them.

The mass term for a fermion ψ in the standard model Lagrangian is written in terms of ψ_L and ψ_R. Here one can have different rotation in the left and the right handed sectors. Thus

$$\bar{\psi}_L m_D \psi_R = \bar{\psi}'_L m \psi'_R$$

where

$$\psi'_L = U_L \psi_L, \quad \psi'_R = U_R \psi_R \text{ and } m_D = U_L^\dagger m U_R$$

The weak states ψ_L and ψ_R are rotated in such a way that the mass matrix m_D is diagonal. Such a diagonalisation of the mass matrix is known as biunitary diagonalisation. ψ_L and ψ_R are the left and the right handed mass states. As the weak charged current operates only in the left handed sector, the CKM matrix is defined as $U_L^\dagger D_L$. The complex phase in the CKM matrix is a measure of the CP violation observed in certain weak processes. We discuss this in chapter 3.

1.5 Problems with Standard Model

Standard model has been very successful in describing the strong and the electroweak interactions. It accounts for all the known physics below TeV scale [11]. The internal consistency of this model has been tested in various experiments. However there are a large number of arbitrary parameters in the model. These are three coupling constants for the three gauge groups, six masses of quarks, three mixing angles and one phase in the CKM matrix, the strong CP violating parameter θ_{QCD}, the three charged lepton masses, the Higgs vacuum expectation value v and the Higgs mass. These make for 19. If one allows for right handed neutrinos, they increase to 26 with the addition of three neutrino masses, three mixing angles and a phase in the lepton sector. Unlike other fermions, the neutrinos can have Majorana masses which increases the number of parameters to more than 26.

The gauge group of standard model is a direct product of three distinct gauge groups. Attempts have been made to embed the gauge symmetries of the standard model into a single larger gauge group with one coupling constant. The standard model would then be the low energy manifestation of such a gauge group. Such theories are called grand unified theories (GUT) [12]. The coupling strength of the strong, weak and electromagnetic interactions which are widely different at low scales may merge to form a single coupling constant at some high scale called the grand unification scale. In general one will have additional gauge bosons in such a theory, such as the X and the Y bosons in the $SU(5)$ GUT. These bosons couple to the leptoquark current and bring about proton decay. The present lower bound on the lifetime of proton is about 10^{30} years. This gives the lower bound on the mass of X bosons to be 10^{14}GeV which is taken as the typical grand unification scale. Alternatively grand unification scale is identified with unification of three couplings. This is about 10^{16}GeV.

Even the GUT fail to provide an understanding of some of the problems mentioned above. Standard model and GUT operate within a single family of fermions. The three families of fermions exist totally independent of each other in these theories. Though the present knowledge indicates that there are three families there is no reason why there should be only three. There is no relation between the masses and the mixing between the families. The mass matrices are completely arbitrary within standard model. CP is expected to be violated in Standard model due to the the presence of a complex phase in the CKM matrix. However, this phase being arbitrary, there is no relation of the CP violation to the fermion masses and mixing.

In order to get any relation between the masses and mixing angles, the arbitrariness of the fermion mass matrices need to be reduced. Some phenomenological ansatz have been proposed that restrict the structure of the fermion mass matrices in order to have successful relations between masses and mixing angles. We present here a brief discussion of the Fritzsch [13] ansatz.

1.6 The Fritzsch Structure

In this ansatz the mass matrices are taken to be hermitian. They are restricted to have non-zero entries corresponding to the nearest neighbor interaction between families. In an appropriate basis one could obtain the following Fritzsch structure for the up and the down quark mass matrices $M_{u,d}$:

$$M_u = \begin{pmatrix} 0 & A_u & 0 \\ A_u & 0 & B_u \\ 0 & B_u & C_u \end{pmatrix}, \qquad M_d = P \begin{pmatrix} 0 & A_d & 0 \\ A_d & 0 & B_d \\ 0 & B_d & C_d \end{pmatrix} P^\dagger \tag{1.16}$$

Here $A_{u,d}$, $B_{u,d}$, $C_{u,d}$ are real and P is a diagonal phase matrix,$P = \text{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$ The eigenvalues of M_u and M_d are diag$(m_u, -m_c, m_t)$ and diag$(m_d, -m_s, m_b)$.

With this one gets the following approximate relations

$$\begin{array}{lllllllll} C_u & \approx & m_t, & A_u & \approx & \sqrt{m_u m_c}, & B_u & \approx & \sqrt{m_c m_t} \\ C_d & \approx & m_b, & A_d & \approx & \sqrt{m_d m_s}, & B_d & \approx & \sqrt{m_s m_b} \end{array} \tag{1.17}$$

The orthogonal matrices which diagonalizes M_u and M_d are

$$U \approx \begin{pmatrix} 1 & \sqrt{\frac{m_u}{m_c}} & \sqrt{\frac{m_u}{m_t}} \\ -\sqrt{\frac{m_u}{m_c}} & 1 & \sqrt{\frac{m_c}{m_t}} \\ -\sqrt{\frac{m_u}{m_t}} & -\sqrt{\frac{m_c}{m_t}} & 1 \end{pmatrix} \quad \text{and} \quad D \approx P \begin{pmatrix} 1 & \sqrt{\frac{m_d}{m_s}} & \sqrt{\frac{m_d}{m_b}} \\ -\sqrt{\frac{m_d}{m_s}} & 1 & \sqrt{\frac{m_s}{m_b}} \\ -\sqrt{\frac{m_d}{m_b}} & -\sqrt{\frac{m_s}{m_b}} & 1 \end{pmatrix} \tag{1.18}$$

The CKM matrix $K = U^\dagger D$ gives a prediction of the Cabbibo angle θ_C. One gets

$$\theta_C = |K_{12}|$$

where

$$K_{12} = \left(\sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}}\right) e^{i\beta_1}$$

The experimental value of θ_C is consistent with this prediction. The other offdiagonal elements of the CKM matrix are

$$K_{23} = \left(\sqrt{\frac{m_s}{m_b}} - \sqrt{\frac{m_c}{m_t}}\right) e^{i\beta_2}, \quad K_{13} = \sqrt{\frac{m_d}{m_s}} K_{23} e^{i(\beta_1 - \beta_2)}$$

The experimental upper bound on K_{23} along with the bounds on m_s, m_b and m_c [14] gives the bound on top quark mass $m_t \leq 50$GeV. With the discovery of top [15] with $m_t \approx 170$GeV one obviously needs modification of Fritzsch structure.

1.7 Horizontal Symmetries

Phenomenological ansatz like the Fritzsch structure are reasonably successful in giving relations between quark masses and mixing. However, one would like to explain the reasons for having such structure rather than having them ad-hoc. It is natural to expect symmetries that operate on the family space of the fermions to restrict the structure of mass matrices. These symmetries are popularly known as horizontal symmetries. In the simplest case one can take the horizontal symmetry to be $U(1)_H$. As an example consider the fermions to transform as

$$\psi_i \to e^{i\alpha_i}\psi_i$$

The quark mass matrices now is no more arbitrary. In order to preserve the horizontal symmetry only diagonal terms like $g_{ii}\bar{\psi}_{iL}\psi_{iR}\Phi_0$ are allowed if $\alpha_i \neq \alpha_j$ for $i \neq j$. Here Φ_0 is the standard model Higgs doublet. However this does not allow any mixing. Consider an additional doublet Higgs Φ_1 with horizontal charge α such that $\alpha = \alpha_i - \alpha_j$. Now one can have non-diagonal entries in the mass matrix in the $(i,j)^{th}$ position through the Yukawa coupling $g_{ij}\bar{\psi}_{iL}\psi_{jR}\Phi_1$. Such non diagonal mass matrices leads to quark mixing. Suitable choice of horizontal charges and Higgs fields give Fritzsch like structures for mass matrices.

The horizontal symmetry $U(1)_H$ can be either global or local. Global $U(1)_H$ can be used to implement a multigenerational Peccei-Quinn [16] symmetry that rotate away the strong CP problem [17]. Such a global horizontal symmetry preserves the features of single generational Grand Unified theories (GUT)while producing the required structure of masses and mixing [17]. In these models CP violation only comes from Higgs boson exchange and not from gauge boson exchange. In a local $U(1)_H$ the horizontal charge assignments are restricted by anomaly cancellation. In such theories CP is violated through gauge boson exchange but its strength is not constrained to be small [18].

We will discuss a scheme in which horizontal symmetry plays a crucial role in obtaining a CP conserving theory and breaking of horizontal symmetry at high scale naturally leads to small CP violation in low energy theory.

Horizontal symmetries are also used to constrain neutrino masses [19]. It has been studied to obtain large mixing of neutrinos to solve the solar neutrino problem [10, 20]. We shall present a model in chapter 2 where horizontal symmetry gives degenerate neutrino masses in Seesaw scheme.

If $U(1)_H$ is local and is broken near the weak interaction scales, then its effect can be seen in known processes. It was shown [21] that a class of models are possible if one demands anomaly cancellations. The bounds on the extra gauge boson mass and coupling constant were obtained through the forward backward asymmetry and FCNC in certain leptonic processes. The extremely stringent limits on such processes in the charged leptonic sector are phenomenologically very useful in constraining the masses, the mixing and the coupling of the extra gauge boson and any extra Higgs that are introduced. In certain cases these bounds are more stringent than the precision tests. We discuss these in chapter four.

1.8 Outline of the thesis

The observed hierarchy in the fermion masses and its relation to the mixing angles indicates structures in the mass matrices. Such structure must also give an understanding of the smallness of the CP violation in the quark sector. The neutrino sector needs hierarchical mass scales. Large mixing is necessary for the solution of the atmospheric neutrino problem. In this thesis, horizontal symmetries are introduced to obtain the above desirable features. The thesis is arranged as follows:

Chapter 2: This discusses the problem of neutrino masses. The possibility of Dirac and Majorana masses are discussed. The smallness of neutrino masses through Seesaw mechanism is shown. The need to have Pseudo Dirac structure in neutrino masses is discussed. A model where the Majorana mass matrix takes the

Fritzsch structure by the imposition of horizontal symmetry is presented. Such a model generates a pair of Pseudo Dirac neutrinos while having the possibility of neutrinos as hot dark matter component.

Chapter 3: This discusses CP violation in the charged current sector. CKM matrix is introduced. The ϵ parameter that measures CP violation in neutral kaon system is introduced. The problem of small CP violation related to the spontaneous symmetry breaking is discussed. A specific model based on left-right symmetric theory with a horizontal symmetry is presented. The smallness of the CP violation here is shown to be connected with the scale at which the horizontal symmetry breaks.

Chapter 4: This discusses the case where the horizontal symmetry is broken at low scales. Physics beyond the standard model and leptonic flavor changing processes due to the presence of an extra Z is discussed. The constraints come from the precision tests of standard model, the leptonic and hadronic widths and the rare flavor changing neutral current processes. A specific example is presented where $U(1)_H$ charge is $L_e - L_\mu$.

Chapter 5: Presents the summary and conclusions.

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Chapter 2

Neutrino Masses

This chapter discusses neutrino masses. The possibility of both Dirac and Majorana types of mass terms beyond Standard model are considered. The seesaw mechanism is introduced which gives a very small mass to the neutrinos compared to other fermions. The need to have pseudo-Dirac structure in neutrino masses is emphasized. A model is presented where one gets such a structure by imposing a horizontal symmetry.

2.1 General mass terms

We have seen in section 1.3 that neutrinos can neither have Dirac mass nor Majorana mass within Standard model. Dirac and Majorana mass generation needs right handed neutrinos and a $SU(2)$ triplet Higgs respectively. These fields are not included in the Standard model. However in certain extensions of Standard model, where both of these fields are included like the left right symmetric model [1], both Dirac and Majorana type mass terms are possible for neutrinos. In the left right model right handed neutrinos are present and the Higgs sector contains an additional triplet Higgs which is necessary to generate Majorana mass for neutrinos. Let us consider a general mass Lagrangian for neutrinos which consists of a Dirac and a Majorana mass term.

$$-\mathcal{L}_{D-\mathcal{M}} = m_D(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L) + \frac{1}{2}M_L(\nu_L^C\nu_L + \bar{\nu}_L\nu_L^C) + \frac{1}{2}M_R(\nu_R^C\nu_R + \bar{\nu}_R\nu_R^C)$$

Consider the following basis for neutrinos

$$\nu = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix}.$$

In such a basis the above mass term can be written as

$$-\mathcal{L}_{D-M} = \frac{1}{2}\bar{\nu}^C\mathcal{M}\nu + H.C$$

where

$$\mathcal{M} = \begin{pmatrix} M_L & m_D \\ m_D & M_R \end{pmatrix} \tag{2.1}$$

2.2 Structure of neutrino masses

The upper bounds on neutrino masses in most experiments are very low. Hence even if neutrinos are massive, they have to be very light compared to their charged partners. The following are the upper bounds on the masses of ν_e, ν_μ and ν_τ [2]

$$m(\nu_e) < 15\ eV \tag{2.2}$$

$$m(\nu_\mu) < 170\ KeV \tag{2.3}$$

$$m(\nu_\tau) < 24\ MeV \tag{2.4}$$

Observations on neutrino mixing and oscillations suggest hierarchical scales in their mass square differences. The solution to the solar neutrino problem through the MSW mechanism demands oscillation between a pair of neutrinos with mass squared difference, Δm^2 of the order of $10^{-5}eV^2$. The solution to the atmospheric neutrino problem requires $\Delta m^2 \sim 10^{-2}eV^2$. If the mass of electron neutrino is of the order of eV then the solution to the solar neutrino problem demands a pair of neutrinos which are almost degenerate in mass.

Though neutrinos can be massive, the mass of the neutrinos are expected to be about 3 to 4 orders of magnitude smaller than their charged counterpart. The most natural explanation so far for the smallness of neutrino masses is the seesaw mechanism. We discuss it in the following section.

2.3 The Seesaw mechanism

Here one considers both the Majorana and the Dirac mass terms for the neutrinos as in eq. (2.1). In reality $\mathcal{M}$ of eq.(2.1) is a matrix of dimension 2n where n is the number of fermion generation in the model. As there are three generation of fermions $\mathcal{M}$ is 6×6. Each entry in $\mathcal{M}$ is a 3×3 matrix. In the basis $\nu^T = (\nu_L, \nu_R^C)$, we have

$$\mathcal{M} = \begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix} \tag{2.5}$$

M_R is generally much larger compared to m_D and M_L. This happens naturally in most theories beyond standard model like left-right symmetric model or the grand unified theories. For example, in left-right symmetric theory M_R is related to the scale of $SU(2)_R$ breaking which is of the order of $M_I \sim 10^{10} - 10^{12}$GeV. In $SO(10)$ grand unified theory, M_R is of the scale of M_X or M_I. m_D is taken to be at the scale of the charged lepton or quark masses. The scale of M_L is much smaller than m_D and M_R. Usually it is taken to be zero. In this approximation $\mathcal{M}$ becomes

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

It is possible to transform $\mathcal{M}$ to a block diagonal form by an appropriate unitary transformation [3]. The unitary matrix U which is used for this transformation is of the following form

$$U = \begin{pmatrix} 1 - \frac{1}{2}\rho\rho^T & \rho \\ -\rho & 1 - \frac{1}{2}\rho^T\rho \end{pmatrix}; \quad \text{where } \rho = m_D M_R^{-1}$$

After block diagonalisation we get

$$\mathcal{M}_{eff} \approx \begin{pmatrix} -m_D M_R^{-1} m_D^T & 0 \\ 0 & M_R \end{pmatrix} = \begin{pmatrix} m_{eff} & 0 \\ 0 & M_R \end{pmatrix} \tag{2.6}$$

The eigenvalues of m_{eff} are the observed masses of neutrinos. They are expected to be very small. If there is no mixing in m_D and M_R is proportional to identity then m_{eff} is diagonal and the neutrino masses can be read off directly as

$$m_{\nu i} = \frac{m_{fi}^2}{M_R}$$

where m_{fi} is the mass of the charged lepton or the quarks. We see that the hierarchy in the generations of the neutrino masses is proportional to the hierarchy in the square of the charged lepton or quark masses. In such a case the two mass squared differences are

$$\Delta_{32} = m_{\nu_\tau}^2 - m_{\nu_\mu}^2 = \frac{m_t^4 - m_c^4}{M_R^2} \approx \frac{m_t^4}{M_R^2}$$

and

$$\Delta_{21} = m_{\nu_\mu}^2 - m_{\nu_e}^2 = \frac{m_c^4 - m_u^4}{M_R^2} \approx \frac{m_c^4}{M_R^2}$$

These give [4]

$$\frac{\Delta_{32}}{\Delta_{21}} \approx \left(\frac{m_t}{m_c}\right)^4 \approx 10^8 \tag{2.7}$$

The above ratio of two mass square differences is incompatible with what is required for the simultaneous solution of the solar neutrino problem through MSW mechanism and the atmospheric neutrino problem as mentioned in section 2.2. The above result is model dependent and can change if one assumes some hierarchy in the elements of diagonal M_R. For example if one takes the hierarchy in M_R to be the same as m_{fi}, then

$$m_{\nu i} = m\frac{m_{fi}}{M_R}$$

where m is a proportionality constant with a mass dimension. Here we get the neutrinos with the same hierarchy as that of m_{fi}. The ratio $\frac{\Delta_{32}}{\Delta_{21}} \approx 10^4$. This is compatible with the required mass squared differences in solar and the atmospheric neutrino problems. However with such hierarchical patterns in neutrino masses the mass of the heaviest neutrino cannot be more than 0.1 eV, if the maximum mass squared difference has to be of the order of $10^{-2}eV^2$. Such a small mass will be unobservable in the present laboratory experiments. If one assumes the hierarchy in M_R as the square of m_{fi} then all the neutrino masses turn out to be of the same order. Such a structure would be compatible with the small mass square differences giving a set of almost degenerate neutrinos. The individual neutrino masses are now not restricted to be small. But such hierarchy in the structure of M_R is a very unnatural assumption. One can avoid such hierarchical entries in M_R if one takes a non-diagonal structure. Such structures in M_R can be obtained through horizontal symmetries. We present an example of such a model in section 2.6.

2.4 Pseudo Dirac neutrinos

In this section we discuss models that describe the possibilities of pseudo Dirac structure in neutrinos. Let us first introduce the Pseudo-Dirac structure with two flavors of left handed Majorana neutrinos [5, 6, 7]. The Majorana mass Lagrangian in the basis $\nu_L^T \equiv (\nu_{eL}\ \nu_{\mu L})$ is given as follows:

$$\begin{aligned} -\mathcal{L}_M &= \frac{1}{2}\{m_{ee}\bar{\nu}_{eL}^C\nu_{eL} + m_{\mu\mu}\bar{\nu}_{\mu L}^C\nu_{\mu L} + m_{e\mu}(\bar{\nu}_{eL}^C\nu_{\mu L} + \bar{\nu}_{\mu L}^C\nu_{eL})\} + H.C \\ &= \frac{1}{2}\bar{\nu}_L^C\mathcal{M}_L\nu_L + H.C \end{aligned} \tag{2.8}$$

where

$$\mathcal{M}_L = \begin{pmatrix} m_{ee} & m_{e\mu} \\ m_{e\mu} & m_{\mu\mu} \end{pmatrix} \tag{2.9}$$

The mass eigenvalues are given as

$$m_{1,2} = \frac{1}{2}\left(m_{ee} + m_{\mu\mu} \pm \sqrt{(m_{ee} + m_{\mu\mu})^2 - 4(m_{ee}m_{\mu\mu} - m_{e\mu}^2)}\right) \tag{2.10}$$

It can be seen that if $\mathcal{M}_L$ is traceless then we get a pair of equal but opposite mass eigenvalues given by

$$m_{1,2} = \pm\sqrt{m_{\mu\mu}^2 + m_{e\mu}^2} \tag{2.11}$$

The states which diagonalize $\mathcal{M}_L$ are a pair of degenerate Majorana mass eigenstate. Let us denote them by N_{1L} and N_{2L}. We have

$$\begin{pmatrix} N_{1L} \\ N_{2L} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \end{pmatrix} \quad \text{where} \quad \tan 2\theta = \frac{m_{e\mu}}{m_{\mu\mu}} \tag{2.12}$$

In this basis the Majorana mass Lagrangian takes the form

$$-\mathcal{L}_M = \frac{1}{2} m(\overline{N^C_{1L}} N_{1L} - \overline{N^C_{2L}} N_{2L}) \tag{2.13}$$

Here m is the absolute value of the equal and opposite mass eigenvalues $m_{1,2}$ in eq.(2.11) Let us define the states

$$\begin{aligned} \chi_1 &= N_{1L} + N^C_{1L} \\ \chi_2 &= N_{2L} - N^C_{2L} \end{aligned} \tag{2.14}$$

It is clear that $\chi_1^C = \chi_1$ and $\chi_2^C = -\chi_2$. Hence these are Majorana states with opposite C phases. In the basis χ_1 and χ_2 the mass term becomes

$$-\mathcal{L}_M = \frac{1}{2} m(\bar{\chi}_1 \chi_1 + \bar{\chi}_2 \chi_2) \tag{2.15}$$

If $\Psi = \frac{1}{\sqrt{2}}(\chi_1 + \chi_2)$ then the mass term becomes

$$-\mathcal{L}_M = m\bar{\Psi}\Psi \tag{2.16}$$

This along with the fact that $\Psi^C \neq \Psi$ suggests that the mass term describes a Dirac particle with mass m. Hence we see that two Majorana neutrinos with opposite CP phases and degenerate mass m behave as a Dirac particle of the same mass. The Dirac mass Lagrangian is invariant under a global gauge transformation $\Psi \to e^{i\alpha}\Psi$. Hence the Dirac neutrino given by Ψ has a definite lepton number. The flavor states ν_e and ν_μ can be written as a combination of Ψ and Ψ^C which have opposite lepton numbers. Thus the charged current given by $j_\mu = \bar{e}_L \gamma_\mu \nu_L$ will not conserve the lepton number symmetry satisfied by the Dirac mass Lagrangian in eq.(2.16). The lepton number is conserved only in the special case when the mixing between Ψ and Ψ^C is maximal. This non-conservation of the lepton number lead to a correction in the mass of the Dirac neutrinos Ψ and Ψ^C. A split in the degenerate mass is generated. Such pairs of neutrinos are known as Pseudo Dirac neutrinos [5]. Wolfenstein [5] showed that such neutrinos contribute to the rate of double beta decay with opposite signs and hence tend to cancel each other. If the mixing between the weak states and mass states is maximal, these contributions cancel completely and the neutrinos behave exactly as expected of a Dirac neutrino [5].

Zeldovich, Koponinsky and Mahmoud (ZKM) [7] showed that a Dirac mass term for a neutrino can be constructed if one combines the left handed neutrino of one flavor with the right handed antineutrino of another. The four component Dirac field for ZKM neutrinos are

$$\Psi = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L}^C \end{pmatrix} \qquad \Psi^C = \begin{pmatrix} \nu_{\mu L} \\ \nu_{eL}^C \end{pmatrix} \tag{2.17}$$

The Dirac mass terms for these neutrinos are

$$\begin{aligned} -\mathcal{L}_M &= \frac{1}{2} m(\bar{\Psi}\Psi + \bar{\Psi}^C \Psi^C) \\ &= \frac{1}{2} m(\bar{\nu}_{eL}\nu_{\mu L}^C + \bar{\nu}_{\mu L}^C \nu_{eL} + \bar{\nu}_{\mu L}\nu_{eL}^C + \bar{\nu}_{eL}^C \nu_{\mu L}) \\ &= \frac{1}{2} m(\bar{\nu}_{\mu L}^C \nu_{eL} + \bar{\nu}_{eL}^C \nu_{\mu L}) + H.C \end{aligned} \tag{2.18}$$

This is exactly the mass Lagrangian of eq.(2.8) with m_{ee} and $m_{\mu\mu}$ equal to zero. Thus the Majorana mass matrix $\mathcal{M}_L$ of eq.(2.9) takes the form

$$\mathcal{M}_L = \begin{pmatrix} 0 & m_{e\mu} \\ m_{e\mu} & 0 \end{pmatrix} \tag{2.19}$$

The mixing matrix that diagonalize this is the maximal mixing matrix given by $\theta = \frac{\pi}{4}$ in eq.(2.12). The lepton number conserved in the mass Lagrangian here is $L_e - L_\mu$. In this special case this lepton number is not broken in the charged current interaction of the neutrinos. This is as noted earlier due to the maximal rotation needed for diagonalizing $\mathcal{M}_L$

2.5 Horizontal Symmetries

To get pseudo Dirac structure in neutrino masses one has to get a suitable structure in neutrino mass matrices. Such structure may come from horizontal symmetries rather than being ad hoc. Horizontal $U(1)$ symmetries have been used long ago in order to constrain the quark [8] as well as neutrino [9] masses. The horizontal symmetries have also been studied [10] with a view of obtaining large mixing among neutrinos required in order to solve the atmospheric neutrino problem [11, 12].

While the horizontal symmetries can give the required pseudo Dirac structure to the neutrino masses, it would be interesting to see how it can be incorporated within the Seesaw mechanism introduced above in section 2.3. This is because in most models beyond standard model with right handed neutrinos, Seesaw mechanism is the most natural way to get a very small mass for the neutrinos. We study such a model in the next section.

2.6 Pseudo Dirac Neutrino in Seesaw model

2.6.1 The Model

The horizontal symmetries can give structure to M_L, m_D and M_R of the neutrino mass matrix of eq.(2.5). Let us consider an $SU(2)_L \times U(1)_Y$ model containing right handed neutrinos and a singlet Higgs η. A horizontal $U(1)$ is imposed in addition to $SU(2)_L \times U(1)_Y$ [13].

We shall require $U(1)$ to be vectorial and assume that the i^{th} generation of leptons carry the $U(1)$ charge X_i and that no two X_i are identical. The ordinary Higgs doublet Φ is assumed to be neutral under the $U(1)$ symmetry. As an immediate consequence, both the charged leptons as well as Dirac mass matrix m_D in eq.(2.5) are forced to be diagonal. The Majorana mass term can still have texture. We are interested in the Fritzsch type [14] of structures for M_R. The successful predictions of the Fritzsch type of structures for quark masses are discussed in chapter 1. Such structures can be easily obtained here by assigning a non-trivial $U(1)$ charge X to η. The M_R receives contributions from the following terms:

$$-\mathcal{L}_R = \frac{1}{2}\nu_{Ri}^T(M_{ij} + \Gamma_{ij}\eta + \Gamma'_{ij}\eta^*)C\nu_{Rj} + h.c \tag{2.20}$$

All the three terms are possible if total lepton number is not assumed to be conserved [15].

Now consider the specific assignment

$$\begin{aligned} X_2 &= -\tfrac{1}{3}X_3 = -\tfrac{1}{2}X \\ X_1 &= -\tfrac{5}{3}X_3 \end{aligned} \tag{2.21}$$

with $X_3 \neq 0$. Then M_{ij} in eq.(2.20) are zero for all i and j. Moreover only $\Gamma_{13} = \Gamma_{31}$, Γ_{22} and $\Gamma'_{23} = \Gamma'_{32}$ are allowed to be non-zero. This leads to the following texture for M_R:

$$M_R = \begin{bmatrix} 0 & 0 & M_1 \\ 0 & M_3 & M_2 \\ M_1 & M_2 & 0 \end{bmatrix} \tag{2.22}$$

$M_1 = \Gamma_{13}\langle\eta\rangle, M_3 = \Gamma_{22}\langle\eta\rangle$ and $M_2 = \Gamma'_{23}\langle\eta^*\rangle$. If we denote the elements of the diagonal matrix m_D by $m_i (i = 1, 2, 3)$ then the effective mass matrix for the light neutrinos is given by:

$$\begin{aligned} m_{eff.} &= -m_D M_R^{-1} m_D^T \\ &= -\begin{bmatrix} m_1^2 M_2^2 & -M_1 M_2 m_1 m_2 & M_1 M_3 m_1 m_3 \\ -M_1 M_2 m_1 m_2 & M_1^2 m_2^2 & 0 \\ M_1 M_3 m_1 m_3 & 0 & 0 \end{bmatrix} \frac{1}{M_1^2 M_3} \end{aligned} \tag{2.23}$$

We shall assume parameters $M_{1,2,3}$ to be similar in magnitude (often we will take them identical for some of the numerical estimates). In addition we will also assume hierarchy in the Dirac masses $m_1 << m_2 << m_3$. Both the above assumptions are natural assumptions made in the usual seesaw picture [12, 16]. But since M_R is different from identity, the resulting pattern of neutrino masses is completely different from the usual seesaw predictions. The eigenvalues of $m_{eff.}$ are given as follows:

$$\begin{aligned} m_{\nu_1} &\approx -\frac{m_1 m_3}{M_1}\left\{1 + \frac{1}{2}\frac{\epsilon}{\delta - 1}\right\} \\ m_{\nu_2} &\approx -\frac{m_2^2}{M_3}\left\{1 + \frac{\epsilon}{1 - \delta^2}\right\} \\ m_{\nu_3} &\approx \frac{m_1 m_3}{M_1}\left\{1 + \frac{1}{2}\frac{\epsilon}{\delta + 1}\right\} \end{aligned} \tag{2.24}$$

where

$$\begin{aligned} \epsilon &\equiv \left(\frac{m_1}{m_2}\right)^2 \left(\frac{M_2}{M_1}\right)^2 \\ \delta &\equiv \frac{m_1 m_3}{m_2^2}\left(\frac{M_3}{M_1}\right) \end{aligned} \tag{2.25}$$

The parameter $\epsilon << 1$ with the above stated assumptions while δ could be $O(1)$. In the $\epsilon \to 0$ limit, two of the neutrinos are exactly degenerate while the presence of the ϵ term introduces small splitting with the (mass)^2 difference

$$\Delta_{31} \approx 2\left(\frac{m_1 m_3}{M_1}\right)^2 \frac{\epsilon\delta}{\delta^2 - 1} \tag{2.26}$$

Hence for $\epsilon << 1$, these neutrinos form a pair of pseudo Dirac particles [5].

The conventional Seesaw mechanism gives a hierarchical spectrum in the neutrino masses as discussed in section 2.3. To remove such a hierarchical structure one had to assume hierarchical structure in the eigenvalues of the Majorana mass matrix M_R. Such an assumption is rather unnatural. In contrast one could obtain non-hierarchical structure here without postulating any hierarchy in elements of M_R.

The occurrence of a pseudo Dirac neutrino here is somewhat of a surprise. One would have expected it [6] if both m_D and M_R possessed some approximate global symmetry. For example, if M_2 is taken to be small, M_R is approximately invariant under $L_e - L_\tau$ and one could have expected a pseudo Dirac neutrino. But M_R in eq.(2.22) does not possess any approximate global symmetry as long as $M_1 \sim M_2 \sim M_3$. Despite this, the hierarchy in m_i (combined with specific texture for M_R) makes $m_{eff.}$ approximately invariant under $L_e - L_\tau$ symmetry resulting in a pseudo Dirac neutrino. In practice ϵ could be quite small, e.g. if $m_1(m_2)$ is identified with $m_u(m_c)$ then $\epsilon \sim 10^{-5}$ for $M_2 \sim M_1$. Thus degeneracy of two neutrinos is expected to be quite good.

The mixing among neutrinos implied by eq.(2.23) can be easily worked out for $\epsilon << 1$. In general, the eigen vectors of $m_{eff.}$ are given by

$$\Psi_i = N_i \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$$

with

$$\begin{aligned} x_i &= \frac{m^0_{\nu_2} - m_{\nu_i}}{B} y_i \\ z_i &= -\frac{m^0_{\nu_1}}{m_{\nu_i}} x_i \end{aligned} \tag{2.27}$$

where

$$m^0_{\nu_1} \equiv -\frac{m_1 m_3}{M_1} \equiv -m^0_{\nu_3}; \quad m^0_{\nu_2} \equiv \frac{m_2^2}{M_3} \text{ and } B \equiv \frac{m_1 m_2 M_2}{M_1 M_3}.$$

With m_{ν_i} given in eq.(2.24), one could determine Ψ_i and hence the mixing angles. The three wave functions are approximately given by

$$\Psi_1 \approx \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \Psi_2 \approx \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \Psi_3 \approx \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \tag{2.28}$$

Ψ_1 and Ψ_3 are maximally mixed to form a pseudo Dirac neutrino. Deviation of this mixing angle from 45^0 is very small. Using eqs.(2.27) and (2.24), we see that

$$\tan\theta_{13} \approx 1 - \frac{1}{2}\frac{\epsilon}{\delta - 1} \tag{2.29}$$

The angle θ_{13} represents mixing among ν_e and ν_τ states produced in association with e and τ respectively if the charged lepton mass matrix is diagonal as is the case here. This has important implications for the solar neutrino problem as we will see in the next section.

M_R of eq.(2.22) possesses a generalized Fritzsch type [14] structure with one pair of off-diagonal and two diagonal elements being zero. Other similar structures with this feature are as follows.

$$(M_R1) \equiv \begin{bmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{bmatrix}; \quad (M_R2) \equiv \begin{bmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{bmatrix}; \quad (M_R3) \equiv \begin{bmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{bmatrix};$$

$$(M_R4) \equiv \begin{bmatrix} 0 & \times & \times \\ \times & \times & 0 \\ \times & 0 & 0 \end{bmatrix}; \quad (M_R5) \equiv \begin{bmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & 0 \end{bmatrix}; \quad (M_R6) \equiv \begin{bmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & 0 \end{bmatrix}$$

Here $\times$ denotes a non-zero entry. Any of these structures can be obtained by imposing a $U(1)$ symmetry similar to the one considered in the text. If all the entries in a given M_R are assumed to be identical (and denoted by M) then eigenvalues $\lambda_i (i = 1, 2, 3)$ of $m_{eff.}$ satisfy the following characteristic equations:

$$\lambda_1\lambda_2\lambda_3 = \frac{m_i^2 m_j^2 m_k^2}{M^3} \tag{2.30}$$

$$\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 = \frac{m_j^2 m_k^2}{M^2} \tag{2.31}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = \frac{(-m_p^2 - m_i^2)}{M} \tag{2.32}$$

with $i \neq j \neq k$ and p either j or k. In the absence of $\frac{m_p^2}{M}$ in eq.(2.32), the above eqs. are solved by the eigenvalues, $\frac{m_j m_k}{M}$, $\frac{-m_j m_k}{M}$ and $\frac{-m_1^2}{M}$. Hence as long as m_p^2 term represents a small correction, one would get a Pseudo Dirac neutrino. This naturally depends upon the exact value of the masses $m_{1,2,3}$. If $m_{1,2,3}$ are identified with $m_{u,c,t}$ (or $m_{e,\mu,\tau}$), the m_p^2 term amounts to a small corrections and one would obtain Pseudo Dirac neutrino in all cases except (M_R4) and (M_R5). Note that the example studied here have the Majorana mass matrix M_R in eq.(2.22) like M_R3

2.6.2 Phenomenological Implications

We shall now investigate the implications of the specific structures for the neutrino masses and mixing derived in the previous subsection. These clearly depend upon the unknown values of the Dirac masses m_i. To be specific, we shall assume these masses to coincide with the up-quark masses. Moreover, we shall assume $M_1 = M_2 = M_3$ and denote them by a common scale M. The parameters ϵ, δ and M then determine neutrino masses and mixing. It follows from eq.(2.25) that

$$\delta \equiv \left(\frac{m_u m_t}{m_c^2}\right)\left(\frac{M_3}{M_1}\right) \approx \frac{m_{\nu_e}}{m_{\nu_\mu}} \approx 0.4\text{-}0.8$$
$$\epsilon \approx \left(\frac{m_u}{m_c}\right)^2 \approx 4 \times 10^{-5}$$

where we have chosen $m_u = 10MeV$, $m_t = 100\text{-}200GeV$ and $m_c = 1.5GeV$. Also from eq.(2.24), we have $|m_{\nu_e}| \sim |m_{\nu_\tau}|$. Hence independent of the numerical value of M, all three neutrino masses are expected to be of the same order. This has to be contrasted with the conventional seesaw prediction

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = m_u^2 : m_c^2 : m_t^2$$

obtained with similar assumptions on parameters but with M_R proportional to identity. The common mass of all three neutrinos would lie in the range (10^{-7}-$1eV$) for the Majorana mass scale $M \sim (10^{16}\text{-}10^9)GeV$. Hence for values of M in the intermediate range $\sim 10^9 GeV$, all three neutrinos would have masses in the eV range. These neutrinos can together provide the necessary hot component of the dark matter [17] which requires $\sum m_\nu = 7eV$. Moreover, such neutrino spectrum could have observable consequences for laboratory experiments as well. Note that two of the neutrinos are highly degenerate. Their (mass)^2 difference is given by eq.(2.26)

$$\Delta_{31} \approx 2(m_{\nu_e})^2 \frac{\epsilon\delta}{\delta^2 - 1} \tag{2.33}$$

It follows that if $m_{\nu_e} \sim m_{\nu_\tau} \sim O(eV)$ then their (mass)^2 difference is naturally expected to be around $\sim 10^{-5} eV^2$. This value falls in the range required to solve the solar neutrino problem through Mikheyev-Smirnov-Wolfenstein [18] mechanism. Thus one can solve the solar neutrino problem and at the same time obtain an electron neutrino with mass in the observable range unlike in the seesaw models considered so far in the literature [12, 16].

The detailed analysis of the four solar neutrino experiments constrain the mixing angle as well [19]. The mixing angle between ν_e-ν_τ is predicted to be large in our case. It turns out in fact to be too large to be consistent with observations if charged leptons do not mix among themselves. If the vacuum mixing angle is close to $\pi/4$ then the survival probability for ν_e is independent of energy. Such an energy independent survival probability is not favored when the results of all four experiments are combined. They do allow large angle solution but $\sin^2 2\theta$ (in our case $\theta \equiv \theta_{13}$) is required to be [19]

$$\sin^2 2\theta_{13} < 0.85$$

This constrain is not satisfied by the angle θ_{13} of eq.(2.29). θ_{13} represents the mixing between physical ν_e-ν_τ states if the charged lepton mass matrix is diagonal as is the case in subsection 2.6.1. The correction to $\theta_{e\tau} \equiv \theta_{13}$ is proportional to $\epsilon \sim 10^{-5}$ and is too small to cause significant deviation from $\pi/4$. Hence, one must have contribution from the charged lepton mixing in order to obtain MSW solution consistently. This needs enlargement of the model. For example, consider adding two more Higgs doublets $\Phi'_{1,2}$ with $U(1)_X$ charges 0 and $-\frac{8}{3}X_3$ respectively to the model presented in the last section. With a suitable discrete symmetry ($\Phi'_{1,2} \rightarrow -\Phi'_{1,2}$; $e_R \rightarrow -e_R$), $\Phi'_{1,2}$ can be made to contribute only to the charged lepton masses. These are now described by a mass matrix

$$M_l = \begin{bmatrix} m_{ee} & 0 & m_{e\tau} \\ 0 & m_{\mu} & 0 \\ 0 & 0 & m_{\tau\tau} \end{bmatrix} \tag{2.34}$$

The neutrino mass matrix m_D and hence $m_{eff.}$ remains the same as before. Because of the structure for M_l in eq.(2.34), the effective mixing angle describing ν_e-ν_τ mixing is now given by

$$\theta_{e\tau} \approx \theta_{13} - \phi$$

with

$$\tan 2\phi = \frac{2m_{e\tau}}{m_{ee}^2 + m_{e\tau}^2 - m_{\tau\tau}^2}$$

Due to contribution from ϕ, $\theta_{e\tau}$ need not be very close to $\pi/4$. The large angle MSW solution typically needs [18] $\sin^2 2\theta_{e\tau} \approx 0.65$-$0.85$. With $\theta_{13} \sim 45^0$, this translates to $\phi \sim 10^0$-20^0.

The present model makes a definite prediction for the neutrinoless $\beta\beta$ decay. The amplitude for this process is related to the (11) element of the neutrino mass matrix in the basis in which charged lepton mass matrix is diagonal [12]. It follows therefore from eq.(2.23) that in the model of the earlier section, neutrinoless $\beta\beta$ decay amplitude is proportional to

$$\frac{m_1^2}{M_3}\left(\frac{M_2}{M_1}\right)^2 \approx \epsilon m_{\nu_2}$$

Hence unless m_{ν_2} is very large $\sim 10^5 eV$, the ν-less $\beta\beta$ decay is not observable. If the charged lepton mass matrix is non-diagonal as is required here in order to obtain the right amount of mixing for the MSW solution to work then the ν-less $\beta\beta$ decay amplitude also changes and is now proportional to

$$\cos^2\phi\epsilon m_{\nu_2} - 2\sin\phi\cos\phi m_{\nu_1}$$

with m_{ν_2} and m_{ν_1} given by eqs.(2.24). For $\phi \sim 10^0$-20^0 and $m_{\nu_1} \sim m_{\nu_2}$ this corresponds to $\sim (0.3$-$0.6)m_{\nu_1}$. Hence, the ν_e mass $\sim 2eV$ could lead to an observable signal in the ν-less $\beta\beta$ decay [20]. With $m_{\nu_1} \sim m_{\nu_2} \sim m_{\nu_3} \sim 2eV$, one can also obtain the $\sim 7eV$ needed for obtaining hot component in the dark matter [17]. We have concentrated here on a specific structure among various possibilities (M_R1 to M_R6) that lead to pseudo Dirac neutrinos. The quantitative consequences of other structures could be considerably different. Moreover, within the specific texture, we have assumed $m_{1,2,3}$ to coincide with $m_{u,c,t}$. Such an identification need not hold [21]. But the qualitative conclusions, namely the occurrence of pseudo Dirac neutrino due to approximate $L_e - L_\tau$ symmetry of m_{eff}, is more general and holds as long as $m_1 << m_2 << m_3$ and $M_1 \sim M_2 \sim M_3$. This is a naturally expected pattern even if $m_{1,2,3}$ do not exactly coincide with $m_{u,c,t}$. Interesting predictions discussed above still remain true if $m_{1,2,3}$ are chosen to coincide with the corresponding charged lepton masses instead of the up-quark masses. Now $\epsilon \sim (m_e/m_\mu)^2$ and $\delta \sim m_e m_\mu/m_\tau^2$. Hence Δ_{31} given in eq.(2.33) now becomes

$$\Delta_{31} \sim \frac{1}{8}(m_{\nu_e})^2 \times 10^{-6}$$

Hence for $m_{\nu_e} \sim 1eV$, one can still solve the solar neutrino problem through MSW mechanism. $m_{\nu_e} \sim \frac{m_e m_\tau}{M}$ falls in the eV range if $M \sim 10^{12}GeV$.

2.7 Conclusions

Experiments indicate that neutrinos are possibly not massless. However the upper bounds on their masses are three to four orders of magnitude lower than the corresponding charged lepton masses. As right handed neutrinos are not introduced in the standard model, the Dirac mass term cannot be written for them. Majorana mass cannot be introduced as it needs a triplet Higgs. If one goes beyond standard model then both Dirac and Majorana mass terms can be written for neutrinos. In such models the most natural way to understand the smallness of neutrino mass is through the Seesaw mechanism.

The solution to the solar neutrino problem through MSW mechanism suggests a mass square difference between two flavors of neutrinos to be $10^{-5}eV^2$. The solution to the atmospheric neutrino problem require the presence of a pair of neutrinos with a mass square difference of $10^{-2}\ eV^2$. If neutrino masses are in the range of eV or above then the required mass squared differences restricts the neutrinos to be approximately degenerate in mass. Thus one would need a Pseudo Dirac structure in the neutrino masses.

Wolfenstein [5] showed the possibility that two Majorana neutrinos that are degenerate in mass but have opposite C phases can combine to form a Dirac particle and may cancel their contributions to double beta decay rates. The lepton number which is conserved in such a Dirac mass term in the Lagrangian is not in general conserved in the charged weak interaction terms. The degeneracy in the Majorana neutrino masses is thus lifted at higher orders giving a pair of Pseudo Dirac neutrinos.

Seesaw model as conventionally analyzed generally lead to hierarchical neutrino masses. In particular, if the Majorana masses of the right handed neutrinos are large ($> 10^9 GeV$) then at most ν_τ could have mass around eV range and ν_e is not expected to have mass near its laboratory limit. Such hierarchical pattern along with the aforementioned small mass squared differences would constrain all the neutrinos to have masses below 0.1 eV. However certain textures for neutrino masses lead to very different predictions. A particular model analyzed in detail has all three neutrino masses in the eV range. Two of the neutrinos are nearly degenerate constituting a pair of pseudo Dirac neutrinos. Their (mass)^2 difference could naturally be in the range required to solve the solar neutrino problem. Interesting aspect of the model worth reemphasizing here is the fact that (near) degeneracy of two of the neutrinos result here in spite of the fact that M_R does not possess any global symmetry. The hierarchy in eigenvalues of m_D and texture of M_R conspire to make $m_{eff.}$ approximately invariant under a global $U(1)$ symmetry resulting in almost degenerate pseudo Dirac neutrinos. This feature is shown to follow if m_D is diagonal and M_R has a generalized Fritzsch structure. Both these can be enforced by a global $U(1)$ symmetry. The study made here highlights the fact that the seesaw model can accommodate a completely different pattern of neutrino masses which is not thought to be among the conventional predictions of the scheme.

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Chapter 3

CP Violation

Weak Interactions violate parity maximally. It was not known whether charge conjugation and parity together is violated. The CPT theorem [1] states that every relativistically invariant local quantum field theory is automatically invariant under the combined operation of charge conjugation(C), parity(P), and time reversal(T). But there is no such restriction on CP. A violation of CP indicates a violation of T. In this chapter we discuss processes that give indications of CP violations though very small. We also discuss theoretical models to explain the small CP violation.

3.1 Neutral Kaons and CP violations

Consider the neutral kaons K^0 and $\bar{K}^0$ which are charge conjugates of each other. They are strange mesons with the strangeness quantum number +1 and -1. The states $|K^0\rangle$ and $|\bar{K}^0\rangle$ denotes the neutral kaon states at rest. Thus we will have

$$\begin{aligned} CP|K^0\rangle &= -|\bar{K}^0\rangle \\ CP|\bar{K}^0\rangle &= -|K^0\rangle \end{aligned} \tag{3.1}$$

The negative sign is taken as a convention. The CP eigen states are given by

$$|K^0_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \tag{3.2}$$

$$|K^{20}\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$

with CP eigenvalues -1 and +1 respectively.

The strong and the electromagnetic interactions conserve strangeness but the weak interactions do not. Thus K^0 and $\bar{K}^0$ do not have definite lifetimes for weak decays nor definite mass. The states with definite mass and lifetimes are denoted by the short lived $|K_S^0\rangle$ and the long lived $|K_L^0\rangle$. They are combinations of $|K^0\rangle$ and $|\bar{K}^0\rangle$. $|K_S^0\rangle$ decays into two significant modes $\pi^+\pi^-$ and $\pi^0\pi^0$ while $|K_L^0\rangle$ predominantly decays into $\pi^+\pi^-\pi^0$. The decay products of $|K_S^0\rangle$ have a CP of +1 whereas that of K_L^0 have CP of -1. If CP is a good symmetry of nature then one can identify K_L^0 with K^{10} and K_S^0 with K^{20}.

It was found [2] that K_L^0 decays into $\pi^+\pi^-$ though one part in thousand. Such a decay indicates that CP is violated in weak decays however small. K_L^0 and K_S^0 cannot be identified with the CP eigenstates K^{10} and K^{20}. However as the observed CP violation is small K_L^0 and K_S^0 can be written as

$$\begin{aligned} |K_L^0\rangle &= (1+|\epsilon_1|^2)^{-\frac{1}{2}}(|K^{10}\rangle + \epsilon_1|K^{20}\rangle) \\ |K_S^0\rangle &= (1+|\epsilon_2|^2)^{-\frac{1}{2}}(|K^{20}\rangle + \epsilon_2|K^{10}\rangle) \end{aligned}$$

where ϵ_1 and ϵ_2 are small parameters. In terms of strange states K^0 and $\bar{K}^0$ we have

$$\begin{aligned} |K_L^0\rangle &= \frac{(1+|\epsilon_1|^2)^{-\frac{1}{2}}}{\sqrt{2}}\left[(1+\epsilon_1)|K^0\rangle + (1-\epsilon_1)|\bar{K}^0\rangle\right] \\ |K_S^0\rangle &= \frac{(1+|\epsilon_2|^2)^{-\frac{1}{2}}}{\sqrt{2}}\left[(1+\epsilon_2)|K^0\rangle - (1-\epsilon_2)|\bar{K}^0\rangle)\right] \end{aligned} \quad (3.3)$$

Let us denote H_{WK} in the $K^0 - \bar{K}^0$ basis as

$$H_W = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \quad (3.4)$$

Under CPT

$$H_{11} = \langle K^0|H_W|K^0\rangle = \langle \bar{K}^0|H_W|\bar{K}^0\rangle = H_{22} \quad (3.5)$$

When H_W acts on $|K_L\rangle$ and $|K_S\rangle$ we get

$$\begin{aligned} H_W|K_L^0\rangle &= \lambda_L|K_L^0\rangle \\ H_W|K_S^0\rangle &= \lambda_S|K_S^0\rangle \end{aligned} \quad (3.6)$$

From eq.(3.3) and (3.6) we get

$$
\begin{aligned}
H_{11}(1+\epsilon_1) + H_{12}(1-\epsilon_1) &= \Lambda_L(1+\epsilon_1) \\
H_{21}(1+\epsilon_1) + H_{22}(1-\epsilon_1) &= \Lambda_L(1-\epsilon_1)
\end{aligned}
\tag{3.7}
$$

With the CPT condition eq(3.5) and eq(3.7) we get

$$
\frac{1-\epsilon_1}{1+\epsilon_1} = \sqrt{\frac{H_{21}}{H_{12}}} \tag{3.8}
$$

Similarly

$$
\frac{1-\epsilon_2}{1+\epsilon_2} = \sqrt{\frac{H_{21}}{H_{12}}} \tag{3.9}
$$

Thus under CPT we have

$$
\frac{1-\epsilon_1}{1+\epsilon_1} = \frac{1-\epsilon_2}{1+\epsilon_2} \tag{3.10}
$$

giving

$$
\epsilon_1 = \epsilon_2 = \epsilon = \frac{\sqrt{H_{12}} - \sqrt{H_{21}}}{\sqrt{H_{12}} + \sqrt{H_{21}}}
$$

ϵ is the CP violating parameter in $(K^0 - \bar{K}^0)$ oscillation. From eq.(3.3) we see that $|K_L\rangle$ and $|K_S\rangle$ are not orthogonal states

$$
\langle K_L | K_S \rangle = \epsilon_1 + \epsilon_2^* = 2Re(\epsilon)
$$

The decay modes K_S^0 and K_L^0 into two pions can be expressed in terms of the decay amplitudes of K^0 and $\bar{K}^0$ into two pions. The final state pions are bosons and hence have to be symmetric. Thus the final pion states can have isospin $I = 2$ or 0

Let

$$
\begin{aligned}
A_0 &= \langle \pi\pi, I = 0 | H_W | K^0 \rangle \\
A_2 &= \langle \pi\pi, I = 2 | H_W | K^0 \rangle
\end{aligned}
$$

Given A_0 and A_2 one can obtain the amplitudes of $K_L^0 \to \pi^+\pi^-$, $K_S^0 \to \pi^+\pi^-$ It can be shown that the ratio of these amplitudes is

$$
\eta^{+-} = \frac{\langle \pi^+\pi^- | H_W | K_L^0 \rangle}{\langle \pi^+\pi^- | H_W | K_S^0 \rangle} = \epsilon + \epsilon' \tag{3.11}
$$

where ϵ' is proportional to the phase difference between A_2 and A_0. ϵ' is a new CP violating parameter. Even if K_L^0 and K_S^0 are orthogonal, one can have CP violating decay $K_L^0 \to \pi^+\pi^-$ if ϵ' is nonzero.

3.2 Fermion mass structure

The lepton and the quark masses show hierarchical structure. For e.g in the quark sector

$$\frac{m_u}{m_c} \sim O(\lambda^4) \quad ; \quad \frac{m_u}{m_t} \sim O(\lambda^8)$$
$$\frac{m_d}{m_s} \sim O(\lambda^2) \quad ; \quad \frac{m_d}{m_b} \sim O(\lambda^4)$$

In the charged lepton sector

$$\frac{m_e}{m_\mu} \sim O(\lambda^3) \; ; \frac{m_e}{m_\tau} \sim O(\lambda^4) \tag{3.12}$$

Here the value of λ is equal to the Cabbibo angle $\theta_C \sim 0.21$ Such hierarchical structure indicates specific texture in the quark and the lepton mass matrices. The quark mixing angles are also determined from these textures. Frogatt and Nielson [3] investigated whether such textures at low scales can be obtained through renormalisation group evolution of the Yukawa couplings at high scales where all the entries in the Yukawa matrix are of the same order. But they found that such hierarchical structure cannot be obtained just from evolution through renormalisation group equations. One has to impose specific selection rules to get suitable textures. Such selection rules can be obtained by imposing discrete symmetries [4]. However Frogatt and Nielson employed continuous family symmetries to obtain such textures [3, 5]. Most of the fermions that we know today are massless to start with. This can be achieved by imposing a singlet Higgs scalar η_0 in the model with zero horizontal charge R. The vacuum expectation value of η_0 is very high and generates only super heavy fermion masses which have vectorial assignment of R-charge. These fermions attain very high mass and are not observed at the presently attainable energies. The light fermions are assigned different charges for left and right handed components. They are massless in the limit of the exact conservation of the horizontal symmetry. However if an additional singlet Higgs η_1 is introduced with one unit of horizontal charge (R=1) then the light fermions can get mass through coupling with η_1. The following non-renormalizable terms give mass to the light fermions.

$$-\mathcal{L}_Y = g_{ij}\bar{\psi}_{iL}\psi_{jR}\Phi_0 \left(\frac{\eta_1}{\langle\eta_0\rangle}\right)^{n_{ij}} \tag{3.13}$$

Φ_0 is the usual Higgs doublet of Standard model with zero horizontal charge R. g_{ij} are the Yukawa couplings and are complex numbers of order one. This term gives a non-zero entry in the $(i,j)^{th}$ position of the mass matrix if the R-charge of ψ_{iL} and ψ_{jR} differ by n_{ij}. The mass matrix in this way gets hierarchical entries in terms of a small parameter $\epsilon = \frac{\langle n_1 \rangle}{\langle n_0 \rangle}$ given by

$$M_{ij} \approx g_{ij}\epsilon^{n_{ij}}$$

Let the R-charge of the right handed fermions be a_i and that of left handed fermions be b_i. Then $n_{ij} = b_j - a_i$. With this the mass ratio of two fermions is given as [3]

$$\frac{m_i}{m_j} = \epsilon^{(b_i - a_i)-(b_j - a_j)} = \epsilon^{(b_i - b_j)-(a_i - a_j)} \tag{3.14}$$

If the charge assignments are such that

$$b_i > b_j \quad \text{and} \quad a_i < a_j \quad \text{for } i < j \tag{3.15}$$

then the fermion masses get a hierarchical pattern as ϵ is a small parameter. The elements of the unitary matrix U which diagonalize the mass squared matrices $M^\dagger M$ are given by[3]

$$U_{ij} \approx \omega_{ij}\epsilon^{|b_i - b_j|} \approx \left(\frac{m_i}{m_j}\right)^{C_{ij}} \tag{3.16}$$

where

$$C_{ij} = \left(1 - \frac{a_i - a_j}{b_i - b_j}\right)^{-1} \tag{3.17}$$

and ω_{ij} are complex quantities related to g_{ij} and is expected to be of $O(1)$. With the charge assignments in eq.(3.15) we have

$$\frac{a_i - a_j}{b_i - b_j} < 0$$

In particular $\frac{a_i - a_j}{b_i - b_j} = -1$ leads to $C_{ij} = \frac{1}{2}$ and from eq.(3.16)

$$U_{ij} \approx \sqrt{\frac{m_i}{m_j}} \tag{3.18}$$

This is what is needed to give the correct Cabbibo mixing angle θ_C in the down quark sector $\theta_C \approx \sqrt{\frac{m_d}{m_s}}$.

The breaking of the $U(1)_H$ symmetry by non renormalizable terms give a hierarchical texture of fermion mass matrices in terms of powers of the small parameter ϵ. This texture then gives the correct hierarchical pattern in the masses. The mixing angles are also consistent with the experimental observations. However this mechanism does not give any restrictions on the CP violation. Thus from this model one cannot understand the smallness of the CP violation. We discuss in detail a similar model in section 3.4 which in addition to giving right hierarchies and mixing also conserves CP. The inclusion of non renormalizable terms then violate CP by the desired amount.

3.3 Origin of CP violation

Various models exist to explain the CP violation. We will discuss sources of CP violation in gauge theories. There are two sectors from which CP violation can come. The weak charged current sector violates CP due to the complex phase in the CKM matrix. Also, CP can be spontaneously violated in models with extended Higgs sector.

The CP violating parameter ϵ measured from the $K^0 - \bar{K}^0$ decays can be expressed completely in terms of the mixing parameters and the phase in the CKM matrix [6]. As a result it is expected to be small. The final state phase shifts in the $K^0 \to 2\pi$ decays were calculated taking into account the CKM matrix [7, 8]. Thus ϵ' in eq(3.11) is also shown to be related to the CKM matrix and is expected to be small.

Even if the complex phase in the CKM matrix is zero there can be CP violation provided one extends the Higgs sector. In a model with two Higgs doublets, Lee [9] showed that even though CP can be arranged to be conserved by the Lagrangian it is spontaneously violated by the ground state. Let us consider this model in some detail. The most general potential for two Higgs doublet in a gauged $SU(2) \times U(1)$ theory can be written as

$$V(\Phi_1, \Phi_2) = -m_1^2\Phi_1^\dagger\Phi_1 - m_2^2\Phi_2^\dagger\Phi_2 + a_{11}(\Phi_1^\dagger\Phi_1)^2 + a_{22}(\Phi_2^\dagger\Phi_2)^2$$

$$+a_{12}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+b_{12}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\left[(\Phi_1^\dagger\Phi_2)\{d_{12}(\Phi_1^\dagger\Phi_2)+e_{12}(\Phi_1^\dagger\Phi_1)+f_{12}(\Phi_2^\dagger\Phi_2)\}+H.c.\right] \quad (3.19)$$

The Vacuum has a degeneracy due to the $SU(2)$ symmetry. One can choose the vacuum such that the vacuum expectation value of Φ_1 and Φ_2 is the following

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ v_1 e^{i\theta} \end{pmatrix} \quad , \quad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \quad (3.20)$$

where

$$\cos\theta = -\frac{e_{12}v_1^2 + f_{12}v_2^2}{4d_{12}v_1v_2} \quad (3.21)$$

Under CP let Φ_i transform as follows

$$(CP)\Phi_i(CP)^{-1} = e^{i\alpha_i}\Phi_i \quad (3.22)$$

From (3.19) it is clear that if $V(\Phi_1, \Phi_2)$ is invariant under CP then

$$\alpha_1 - \alpha_2 = 0 \quad (3.23)$$

Under CP the vacuum transforms as

$$CP|0\rangle = |0'\rangle \quad (3.24)$$

As CP is antiunitary

$$\langle 0'|(CP)\Phi_i(CP)^{-1}|0'\rangle = \langle 0|\Phi_i|0\rangle^* \quad (3.25)$$

For a vacuum invariant under CP we have $|0'\rangle = |0\rangle$. Thus we get from (3.25) and (3.22)

$$e^{i\alpha_i}\langle 0|\Phi_i|0\rangle = \langle 0|\Phi_i|0\rangle^* \quad (3.26)$$

From (3.20) we get

$$\alpha_1 = -2\theta \quad \text{and} \quad \alpha_2 = 0 \quad (3.27)$$

We see that these values of α_1 and α_2 are incompatible with eq(3.23) unless $\theta = 0$. Thus one can have a suitable range of parameters in the Higgs potential for which CP is spontaneously broken by any choice of the ground state. The two Higgs doublet model naturally generates mixing in the fermions and gives flavor changing neutral Higgs current (FCNH). The experimental bounds on flavor changing neutral processes then give the lower bound on the Higgs mass to be $O(\text{TeV})$ in order to sufficiently suppress these processes. Such a heavy Higgs would generate a very small CP violation and this would not contribute significantly to the observed amount of CP violation. The problem of FCNH is eliminated in a class of model proposed by Weinberg [10]. In these models with two generations of quarks and with one Higgs or two Higgs doublets CP is conserved in all sectors of the theory. CP conservation here results from a careful choice of discrete symmetries that eliminates certain terms in the Higgs potential and the Yukawa couplings with the fermions[10, 11]. For e.g. consider the following discrete symmetry

$$\Phi_1 \to -\Phi_1 \ , \ \Phi_2 \to \Phi_2 \ , \ d_{Ri} \to -d_{Ri} \ , \ u_{Ri} \to u_{Ri} \tag{3.28}$$

Such a symmetry ensures that only Φ_1 couples to the down quark while Φ_2 couples to the up quark. Thus flavor changing neutral current is naturally eliminated in this model. This discrete symmetry would also require $e_{12} = f_{12} = 0$ in the Higgs potential in eq(3.19). The condition on the CP phases of Φ_1 and Φ_2 given in eq.(3.23) also changes to

$$\alpha_1 - \alpha_2 = n\pi \tag{3.29}$$

From eq.(3.21) we see that now $\theta = (2m + 1)\pi/2$ Thus from (3.27) we see that it is possible to have a CP conserving ground state if $n = 2m + 1$. Thus there is no spontaneous CP violation in this case. However if there are three Higgs fields in the model then CP is spontaneously violated [10]. Even if there are more than two generations of quarks, one can have CP conservation in the gauge vector boson sector if one imposes 'Natural flavor conservation' (NFC) in the model [12]. Consider the Yukawa interaction in the Lagrangian given by

$$-\mathcal{L}_Y = \bar{\psi}_{iL}\Gamma^{\alpha}_{ij}\Phi^{\alpha}\psi_{jR} \tag{3.30}$$

The CP invariance imposes Γ^{α}_{ij} to be real. NFC restricts Γ^{α} in such a way that all of them can be diagonalized simultaneously by biorthogonal transformation. After spontaneous symmetry breaking the neutral component of Φ^{α} acquires a vacuum expectation value

$$\langle \Phi^{0\alpha} \rangle = v^{\alpha}$$

where v^{α} can be complex in general. But due to the property of NFC the mass matrix given by $\Gamma^{\alpha}v^{\alpha}$ can be diagonalized by the above mentioned biorthogonal transformations through the matrices O_L and $O_R P$ where P is a diagonal phase matrix [12, 11]. This happens for both the up and the down sectors. This ensures that the CKM matrix given by $(O_L^u)^T O_L^d$ is real where superscript u and d stand for up and down sector.

NFC can be achieved by suitable set of discrete symmetries. In these class of models spontaneous CP violation can only come from the Higgs sector if one has more than two Higgs doublet in the model [12, 11]. CP is violated here by the charged Higgs boson exchange. The contribution of the charged Higgs mediated process to the neutron electric dipole moment (e.d.m) was found to be in excess of the experimental bound [10]. However if one considers the contribution from the neutral Higgs exchange processes [13], then the neutron e.d.m was shown to be possible within the experimental bound. Liu and Wolfenstein [14] studied a two Higgs doublet model where the discrete symmetries necessary for NFC is broken explicitly in the Lagrangian by a small amount. The discrete symmetry is also broken by another small parameter that makes e_{12} and f_{12} zero in the Higgs potential (3.19). This gives spontaneous CP violation as discussed above. As NFC is broken by a small parameter FCNH is suppressed and the Higgs now can be light $O(100\text{GeV})$. This gives CP violation in the flavor changing neutral Higgs current(FCNH). In another two Higgs doublet model [15] discrete symmetries are introduced so that they don't completely eliminate FCNH but gives a suppressed and hierarchical nature to it. As a consequence Higgs as light as 10 GeV is consistent with the observed $K_L - K_S$ mass difference. CP is violated both in the charged and the neutral Higgs sector.

The approximate conservation of CP can be naturally understood if it arises as an automatic symmetry of the renormalizable Lagrangian. We studied [16] a specific model with this feature. In this model the horizontal symmetry gives the necessary hierarchical pattern and the mixing in the mass matrices as discussed in section 3.2. CP turns out to be an automatic symmetry of the renormalizable Lagrangian. The violation of CP is due to non-renormalizable interactions and is related to the breaking of $U(1)_H$ at a high scale making it naturally small. The model is discussed in detail in the next section.

3.4 Automatic CP invariance

In this model we impose a horizontal symmetry on the Lagrangian that gives Fritzsch structure [17] for the quark mass matrices and also leads automatically to a CP invariant Lagrangian. In realistic case, one needs CP violation as well as deviations from the Fritzsch structure [18]. Both these occur through non-renormalizable interactions when the horizontal symmetry is broken at very high scale. The smallness of CP violation in this case is thus intimately linked to the scale of horizontal symmetry breaking.

We consider an extension of $SU(3) \otimes SU(2) \otimes U(1)$ to a left-right symmetric theory [19]. In addition to the $G_{LR} = SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{(B-L)}$ group we need to impose a horizontal symmetry $U(1)_H$ and the Peccei-Quinn [20] (PQ) symmetry $U(1)_{PQ}$ in order to get a fully CP invariant theory.

The $U(1)_H$ is a gauged horizontal symmetry which is chosen to obtain texture zeroes in the quark mass matrices. The choice of $U(1)_H$ is constrained by the requirement of anomaly cancellation. Anomalies are seen to cancel if one chooses the $U(1)_H$ charges (1, 0, -1) for the left handed quark fields denoted in the weak basis by q'_{iL}. The corresponding right handed fields are chosen to have opposite $U(1)_H$ values. We need to introduce three bi-doublet Higgs fields Φ_α with the $U(1)_H$ charges (1, -1, -2). These Higgs fields are needed in order to obtain essentially real but non-trivial quark mass matrices with non-vanishing masses and mixing angles.

The $U(1)_{PQ}$ is a global Peccei Quinn symmetry which serves dual purpose here. It allows rotation of the strong CP violating angle θ [20] and it also forbids some crucial couplings in the Yukawa and Higgs sectors. Under the PQ symmetry, $q'_{iR} \to e^{i\beta} q'_{iR}$ and $\Phi_\alpha \to e^{-i\beta}\Phi_\alpha$. Rest of the fields remain invariant. Given this choice, the most general $G \equiv G_{LR} \otimes U(1)_{PQ} \otimes U(1)_H$ invariant Yukawa couplings can be written as

$$-\mathcal{L}_Y = \overline{q}'_L \Gamma_\alpha \Phi_\alpha q'_R + H.C. \tag{3.31}$$

with

$$\Gamma_1 = \begin{pmatrix} 0 & a & 0 \\ a^* & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b \\ 0 & b^* & 0 \end{pmatrix} ; \Gamma_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c \end{pmatrix} ; \quad (3.32)$$

We have imposed here the conventional discrete parity [19] $q'_L \leftrightarrow q'_R$ and $\Phi_\alpha \leftrightarrow \Phi^\dagger_\alpha$. CP is not imposed as a symmetry and hence the couplings a, b appearing in Γ_α are complex in general. But their phases can be rotated away leaving a CP invariant Lagrangian. In order to show this, we first concentrate on the G invariant scalar potential for the fields Φ_α and $\tilde{\Phi}_\alpha = \tau_2 \Phi^*_\alpha \tau_2$:

$$\begin{aligned} V_1(\Phi) \;=\; & \mu^2_\alpha tr(\Phi^\dagger_\alpha \Phi_\alpha) + \lambda_\alpha \{tr(\Phi^\dagger_\alpha \Phi_\alpha)\}^2 \\ & + \lambda_{1\alpha,\beta} tr(\Phi^\dagger_\alpha \tilde{\Phi}_\beta) tr(\tilde{\Phi}^\dagger_\alpha \Phi_\beta) \\ & + \rho_{1\alpha} tr(\Phi_\alpha \Phi^\dagger_\alpha \Phi_\alpha \Phi^\dagger_\alpha) + \rho_{2\alpha} tr(\Phi^\dagger_\alpha \tilde{\Phi}_\alpha \tilde{\Phi}^\dagger_\alpha \Phi_\alpha) + \rho_{3\alpha} tr(\Phi_\alpha \tilde{\Phi}^\dagger_\alpha \tilde{\Phi}_\alpha \Phi^\dagger_\alpha) \\ & + \sum_{\alpha \neq \beta} \{\lambda_{2\alpha,\beta} tr(\Phi^\dagger_\alpha \Phi_\beta) tr(\Phi^\dagger_\beta \Phi_\alpha) + \lambda_{3\alpha,\beta} tr(\Phi^\dagger_\alpha \Phi_\alpha) tr(\Phi^\dagger_\beta \Phi_\beta) \\ & + \delta_{1\alpha\beta} tr(\Phi^\dagger_\alpha \Phi_\beta \Phi^\dagger_\beta \Phi_\alpha) + \delta'_{1\alpha\beta} tr(\Phi^\dagger_\beta \Phi_\beta \Phi^\dagger_\alpha \Phi_\alpha) \\ & + \delta_{2\alpha\beta} tr(\Phi^\dagger_\alpha \tilde{\Phi}_\beta \tilde{\Phi}^\dagger_\alpha \Phi_\beta) + \delta'_{2\alpha\beta} tr(\tilde{\Phi}^\dagger_\alpha \tilde{\Phi}_\beta \Phi^\dagger_\alpha \Phi_\beta) \\ & + \delta_{3\alpha\beta} tr(\Phi_\alpha \tilde{\Phi}^\dagger_\beta \tilde{\Phi}_\beta \Phi^\dagger_\alpha) + \delta'_{3\alpha\beta} tr(\Phi_\alpha \Phi^\dagger_\alpha \tilde{\Phi}_\beta \tilde{\Phi}^\dagger_\beta)\} \end{aligned} \quad (3.33)$$

The combined requirement of hermiticity and $U(1)_H \otimes U(1)_{PQ}$ symmetry forces all the parameters of $V_1(\Phi)$ to be real [21]. As a consequence, CP appears as a symmetry of $V_1(\Phi)$ although this was not imposed. One could choose a CP conserving minimum for a suitable range of parameters :

$$\langle \Phi_\alpha \rangle \equiv \begin{bmatrix} \kappa_{\alpha u} & 0 \\ 0 & \kappa_{\alpha d} \end{bmatrix} \quad (3.34)$$

where $\kappa_{\alpha u}$ and $\kappa_{\alpha d}$ are real. Eqs.(3.32) and (3.34) imply the following quark mass matrices:

$$M_{u,d} = \begin{bmatrix} 0 & a\kappa_{1u,d} & 0 \\ a^*\kappa_{1u,d} & 0 & b\kappa_{2u,d} \\ 0 & b^*\kappa_{2u,d} & c\kappa_{3u,d} \end{bmatrix} \quad (3.35)$$

Note that the M_u and M_d allow for general up and down quark masses in spite of the correlated structures. However because of this correlation, M_u and M_d can be simultaneously made real with a diagonal phase matrix P :

$$\widehat{M}_{u,d} \equiv P M_{u,d} P^\dagger \;=\; \begin{bmatrix} 0 & |a|\kappa_{1u,d} & 0 \\ |a|\kappa_{1u,d} & 0 & |b|\kappa_{2u,d} \\ 0 & |b|\kappa_{2u,d} & |c|\kappa_{3u,d} \end{bmatrix} \quad (3.36)$$

Phases in P can be easily related to that in a and b. $\widehat{M}_{u,d}$ are diagonalized by orthogonal matrices

$$O_{u,d}\widehat{M}_{u,d}O^T_{u,d} = \text{diag}(m_{iu,d})$$

Let us now discuss the CP properties of the model. Because of the fact that both M_u and M_d can be made real by the same phase matrix P, the Kobayashi Maskawa matrices in the left as well as the right handed sectors are real. The reality of $\kappa_{\alpha u,d}$ also imply that the $W_L - W_R$ mixing is real. Hence gauge interactions are CP conserving. Moreover the matrix P appearing in eq.(3.36) in fact make the individual Yukawa couplings real, i.e.

$$P\Gamma_\alpha P^\dagger = |\Gamma_\alpha| \tag{3.37}$$

for every α. This has the consequence that the couplings of the neutral and charged Higgses to the mass eigenstates of quarks also become real. As a result, the Higgs interactions would also conserve CP as long as mixing among the Higgs fields is CP conserving. This is assured by the CP invariance of $V_1(\Phi)$ and reality of $\langle\Phi_\alpha\rangle$. It follows from the above arguments that the model presented so far is in fact CP conserving although one did not impose it anywhere.

We have not yet introduced fields needed to break $SU(2)_R \otimes U(1)_{PQ} \otimes U(1)_H$. This can be done without spoiling the automatic CP invariance obtained above. As a concrete example let us introduce the conventional [19] $SU(2)$ triplet Higgses $\Delta_{L,R}$ with zero $U(1)_H$ and $U(1)_{PQ}$ charges. The breaking of the PQ symmetry by $\langle\Phi_\alpha\rangle$ generates a weak scale axion. We need to introduce a $G_{L,R} = SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ singlet σ in order to make this axion invisible [22]. σ is taken to transform under PQ symmetry as $\sigma \to e^{-i\beta}\sigma$ and remains invariant under $U(1)_H$. Finally, we introduce a $G_{L,R}$ singlet field η_H with $U(1)_H$ charge -2 and transforming under the PQ symmetry as $\eta_H \to e^{-2i\beta}\eta_H$. The most general Higgs potential involving these fields and their couplings to Φ fields can be written as:

$$\begin{aligned} V_2 &= \mu_{22}tr(\Phi_2\tilde{\Phi}_2^\dagger)\eta_H^* + \delta_{12}tr(\Phi_1\tilde{\Phi}_2^\dagger)\sigma^{*2} + \text{H.c} \\ &\quad + V(\Delta) + V(\Delta\text{-}\Phi) + V(\eta_H\text{-}\sigma\text{-}\Delta) \end{aligned}$$

where

$$V(\Delta) = \mu^2\left[tr(\Delta_L^\dagger\Delta_L) + tr(\Delta_R^\dagger\Delta_R)\right]$$

$$
\begin{aligned}
& +\rho_1\left[\left[tr(\Delta_L^\dagger\Delta_L)\right]^2+\left[tr(\Delta_R^\dagger\Delta_R)\right]^2\right] \\
& +\rho_2 tr(\Delta_L^\dagger\Delta_L)tr(\Delta_R^\dagger\Delta_R) \\
& +\rho_3\left[tr(\Delta_L^\dagger\Delta_L)^2+tr(\Delta_R^\dagger\Delta_R)^2\right] \\
& +\rho_4\left[tr(\Delta_L^\dagger\Delta_L^\dagger)tr(\Delta_L\Delta_L)+tr(\Delta_R^\dagger\Delta_R^\dagger)tr(\Delta_R\Delta_R)\right] \\
& +\rho_5\left[tr(\Delta_L^\dagger\Delta_L^\dagger\Delta_L\Delta_L)+tr(\Delta_R^\dagger\Delta_R^\dagger\Delta_R\Delta_R)\right] \\
V(\Delta\text{-}\Phi) \;=\; & \left\{\lambda_{1\alpha}tr(\Phi_\alpha^\dagger\Phi_\alpha+\lambda_{2\alpha}tr(\tilde{\Phi}_\alpha^\dagger\tilde{\Phi}_\alpha\right\}\left[tr(\Delta_L^\dagger\Delta_L)+tr(\Delta_R^\dagger\Delta_R)\right] \\
& +\lambda_{3\alpha}\left[tr(\Phi_\alpha^\dagger\Phi_\alpha\Delta_R^\dagger\Delta_R)+tr(\Phi_\alpha\Phi_\alpha^\dagger\Delta_L^\dagger\Delta_L)\right] \\
& +\lambda_{4\alpha}\left[tr(\tilde{\Phi}_\alpha^\dagger\tilde{\Phi}_\alpha\Delta_R^\dagger\Delta_R)+tr(\tilde{\Phi}_\alpha\tilde{\Phi}_\alpha^\dagger\Delta_L^\dagger\Delta_L)\right] \\
& +\lambda_{5\alpha}\left[tr(\Delta_L^\dagger\Phi_\alpha\Delta_R\Phi_\alpha^\dagger)+tr(\Delta_R^\dagger\Phi_\alpha^\dagger\Delta_L\Phi_\alpha)\right] \\
& +\lambda_{6\alpha}\left[tr(\Delta_L^\dagger\tilde{\Phi}_\alpha\Delta_R\tilde{\Phi}_\alpha^\dagger)+tr(\Delta_R^\dagger\tilde{\Phi}_\alpha^\dagger\Delta_L\tilde{\Phi}_\alpha)\right] \\
V(\eta_H\text{-}\sigma\text{-}\Delta) \;=\; & \lambda_\eta(\eta_H^*\eta_H)^2+\lambda_{\eta\sigma}(\eta_H^*\eta_H\sigma^*\sigma) \\
& +(\mu_\eta^2\eta_H^*\eta_H+\mu_\sigma^2\sigma^*\sigma)\left[1+\chi_1 tr(\Delta_L^\dagger\Delta_L)+\chi_2 tr(\Delta_R^\dagger\Delta_R)\right]
\end{aligned}
$$

It can be shown that the parts $V(\Delta)$, $V(\Delta\text{-}\Phi)$ and $V(\eta_H\text{-}\sigma\text{-}\Delta)$ contain only real couplings. The only complex couplings possible are μ_{22} and δ_{12}. But their phases can be absorbed into redefining σ and η_H without effecting reality of other parameters in V_2. Thus the above V_2 is automatically CP conserving just like V_1 of eq.(3.33). V_1 and V_2 together constitute the complete scalar potential of the model.

Now we consider nonrenormalizable Yukawa couplings in the Lagrangian. As discussed in section 2 they can give an understanding of the textures of the fermion masses. In the present context, such terms would also induce naturally small CP violation. In fact the model presented above allows the following general dim-5 terms resulting in fermion masses:

$$
-\mathcal{L}_{NR}=\frac{1}{M}\bar{q}_L\Gamma'_\alpha\tilde{\Phi}_\alpha q_R\eta_H+\text{H.C.} \tag{3.38}
$$

Here M is some heavy mass scale which we take to be the Planck scale M_P. The textures for Γ'_α are dictated by the $U(1)_H$ symmetry. The contribution of $\mathcal{L}_{NR}$ to quark masses depends upon the parameter $\epsilon\equiv\frac{\langle\eta_H\rangle}{M_P}$.

The M_u and M_d following from eqs. (3.35) and (3.38) can be written as [23] :

$$M_{u,d} = \begin{bmatrix} 0 & a\kappa_{1u,d} & \epsilon\kappa_{3d,u}(\Gamma'_3)_{13} \\ a^*\kappa_{1u,d} & \epsilon\kappa_{3d,u}(\Gamma'_3)_{22} & b\kappa_{2u,d} + \epsilon\kappa_{2d,u}(\Gamma'_2)_{23} \\ \epsilon\kappa_{3d,u}(\Gamma'_3)^*_{13} & b^*\kappa_{2u,d} + \epsilon\kappa_{2d,u}(\Gamma'_2)^*_{23} & c\kappa_{3u,d} \end{bmatrix}$$

The non-renormalizable contribution signified by ϵ works in a dual way here. Firstly the presence of ϵ no longer makes it possible to rotate away the phase from $M_{u,d}$ and hence from the KM matrix. Secondly it also modifies the Fritzsch texture obtained in the above example. This is a welcome feature in view of the fact that the Fritzsch ansatz is found to be inconsistent [18] with the large top mass. The texture of $M_{u,d}$ obtained above retains the successful predictions of the original ansatz and is also consistent phenomenologically.

Note that the original Fritzsch ansatz implies that in the limit $\epsilon \to 0$,

$$|a\kappa_{1u}| \sim \sqrt{m_u m_c}\,;\quad |b\kappa_{2u}| \sim \sqrt{m_c m_t}\,;\quad |c\kappa_{3u}| \sim m_t\,;$$

$$|a\kappa_{1d}| \sim \sqrt{m_d m_s}\,;\quad |b\kappa_{2d}| \sim \sqrt{m_s m_b}\,;\quad |c\kappa_{3d}| \sim m_b$$

It follows therefore that $|\kappa_{2,3d}| \ll |\kappa_{2,3u}|$. Hence the presence of ϵ terms alters the structure of M_d more significantly than that of M_u. To a good approximation one may take M_u as in eq.(3.36) and M_d as follows

$$M_d \sim \begin{bmatrix} 0 & |a|\kappa_{1d} & \epsilon\kappa_{3u}\delta_1 e^{i\alpha} \\ |a|\kappa_{1d} & \epsilon\kappa_{3u}\delta_2 & |b|\kappa_{2d} \\ \epsilon\kappa_{3u}\delta_1 e^{-i\alpha} & |b|\kappa_{2d} & c\kappa_{3d} \end{bmatrix} \quad (3.39)$$

As before, we have redefined the quark fields and absorbed the phases of (12) and (23) elements. But this now leaves phases in terms involving ϵ.

Since the matrix diagonalising M_u is completely fixed in terms of up-quark masses, we can express M_d of eq.(3.39) in terms of the known parameters as

$$M_d = O_u^T K \text{diag}(m_d, -m_s, m_b) K^\dagger O_u$$

where K is the KM matrix in the Wolfenstein parameterization [24]. Comparing above M_d with the R.H.S of eq.(3.39) implies the successful relation

$$\lambda = \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}}$$

Moreover the other parameters also get fixed in terms of the masses and mixing angles. Specifically,

$$a\kappa_{1d} \approx -\sqrt{m_d m_s} \;\; ; \;\; b\kappa_{2d} \approx -m_b\lambda^2\left(A + \frac{1}{\lambda^2}\sqrt{\frac{m_c}{m_t}}\right) \;\; ; \;\; c\kappa_{3d} \approx m_b;$$
$$\epsilon\kappa_{3u}\delta_2 \approx -m_s(1-\lambda^2) + m_b\left(\lambda^2 A + \sqrt{\frac{m_c}{m_t}}\right)^2;$$
$$\epsilon\kappa_{3u}\delta_1 \cos\alpha \approx m_b A\lambda^3\left(\rho - \frac{1}{\lambda}\sqrt{\frac{m_u}{m_c}}\right) \;\; ; \;\; \epsilon\kappa_{3u}\delta_1 \sin\alpha \approx m_b A\lambda^3\eta$$

where A, ρ and η are parameters in Wolfenstein matrix [24]. The exact value of ϵ depends upon other parameters. If one chooses Yukawa couplings $c, \delta_2 \sim O(1)$ then $\epsilon \sim \frac{m_s}{m_t} \sim 10^{-3}$. Consistency then requires $\delta_1 \sim 10^{-2}$ in this case. For $\epsilon \sim 10^{-3}$, the $U(1)_H$ symmetry breaking scale is required to be of the order of 10^{16} GeV [25] if the scale of the non-renormalizable terms is set by the Planck mass.

3.5 Conclusion

In this chapter we addressed the problem of CP violation in weak interactions. CP is violated in nature and is most significantly observable in the K^0 $\bar{K}^0$ decays. Two parameters ϵ and ϵ' are identified which measure the amount of CP violation in these systems. The observed values of ϵ and ϵ' are extremely small $O(10^{-3})$. We discussed models that relate these parameters to the parameters in the CKM matrix and gives an understanding of their smallness. CP can be violated through the Higgs exchange reactions. Here the magnitude of CP violation is related to the scale of spontaneous symmetry breaking and hence it is small. However in these models CP is imposed on the Lagrangian through Natural flavor conservation. We presented a model where the horizontal symmetry which gives the required Fritzsch structure to the quark mass matrices also gives a CP conserving theory. The horizontal symmetry is broken at a very high scale. The nonrenormalizable terms which are introduced in the theory generates the right structure of fermion masses at low scales due to the spontaneous breaking of the horizontal symmetry. The CP violation here also comes from the non renormalizable terms and its smallness is closely linked to the scale of horizontal symmetry breaking.

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Chapter 4

Confrontation of Horizontal Symmetry with Experiments

In chapters 2 and 3 we discussed the role of horizontal symmetry in giving structure to the fermion masses and mixing. Gauged horizontal symmetries need additional gauge bosons to be introduced. The extended gauge symmetry we study is the simplest one i.e $U(1)$. The extra gauge boson in the model is thus chargeless and is reffered to as an extra Z or Z'. Such an extra $U(1)$ can be embedded in Grand unified theories like $SO(10)$ or E_6 [1]. The phenomenology of extra Z depends upon the model considered.

Standard model accommodates quark mixing through the CKM matrix. However the CKM matrix is completely arbitrary. The horizontal symmetries acting on the quark sector gives a structure to the quark mass matrices and hence to the CKM matrix. As we have seen in chapter three one can get hierarchy in masses, relations between the masses and mixing and the smallness of CP violation all related to the horizontal symmetry. The situation is different in the lepton sector. The neutrinos are massless in the Standard model. Hence there cannot be any relative flavor mixing between the charged leptons and the neutrinos. This has the effect of lepton flavors being globally conserved. The bounds on lepton flavor violation are very stringent. Gauged horizontal symmetry that acts on leptons and cause flavor mixing are heavily constrained by these bounds. This chapter discusses the phenomenology of extended gauge models that act on leptons. Bounds are obtained on Z' mass which mediates flavor violating processes. Section 4.1 discusses flavor mixing and flavor changing neutral current(FCNC). Section 4.2 discusses the mixing between Z and Z'. Section 4.3 discusses an extended gauge model with and without additional scalar doublets. In section 4.4 the expressions for flavor changing neutral processes are obtained with a specific choice of horizontal symmetry. Section 4.5 discusses a detailed phenomenology of the model discussed in section 4.4. Section 4.6 presents results and conclusions.

4.1 Rare processes

Flavor violating processes or rare processes are of interest since a long time. In this section we will discuss rare processes induced by horizontal gauge bosons in the leptonic sector.

The horizontal symmetry can be vector like or flavor chiral type [2]. In a flavor vector theory both the right and the left handed fields are assigned the same horizontal charge whereas in a flavor chiral theory they are opposite. Let the left handed component of the i^{th} generation of fermion get a horizontal charge x_i. Let us denote it as a diagonal matrix $X_L = diag(x_1, x_2, x_3)$. The right handed charges are denoted by a similar diagonal matrix X_R. With additional Higgs doublets transforming non-trivially under the horizontal symmetry, the mass matrix assumes a non-diagonal structure which leads to fermion mixing.

Few observations can be made about the general structure of the mass matrices in a flavor vector and flavor chiral theories. In a flavor vector theory, the diagonal entries in the mass matrix come from a single Higgs doublet with horizontal charge 0. The offdiagonal entries m_{ij} and m_{ji} necessarily has to come from two different Higgs with different horizontal charges. viz $x_i - x_j$ and $x_j - x_i$. This makes the mass matrices necessarily non-hermitian. However in a flavor chiral theory $X_L = -X_R$. A single Higgs doublet is not sufficient to give all the diagonal entries in the mass matrix. Moreover m_{ij} and m_{ji} get contribution from the same Higgs. With a further restriction $x_i \neq x_j$ for $i \neq j$, the mass matrix one gets has the canonical structure or the Fritzsch structure [2]. In general one can have a combination of flavor vector and flavor chiral charges. In such cases X_L and X_R are arbitrary.

As mentioned earlier there is an extra gauge boson Z' in the theory. As we are interested in the flavor violation in the leptonic sector, let us look at the leptonic current coupling to Z' in the following Lagrangian.

$$\mathcal{L}_{Z'} = \frac{g'}{\cos\theta} \left\{ \bar{e}'_L X_L \gamma_\mu e'_L + \bar{e}'_R X_R \gamma_\mu e'_R \right\} Z'_\mu \tag{4.1}$$

where $e'_{L,R}$ are column vectors in generation space and θ is the weak mixing angle, introduced here purely for notational convenience. The coupling of the physical (i.e. mass eigenstate) fermions to Z' depends upon the structure of the mass matrix $\mathcal{M}_l$ which is non-diagonal as discussed above. $\mathcal{M}_l$ is non-hermitian in general and can be diagonalized by bi-unitary transformation

$$U_L \mathcal{M}_l U_R^\dagger = \text{diag.}(m_e, m_\mu, m_\tau) \tag{4.2}$$

$$e_{L,R} = U_{L,R} e'_{L,R} \tag{4.3}$$

$\mathcal{L}'_Z$ then assumes the following form in terms of the mass eigenstates:

$$\mathcal{L}_{Z'} = \frac{g'}{cos\theta} (\kappa_{Lij} \bar{e}_{iL} \gamma_\mu e_{jL} + \kappa_{Rij} \bar{e}_{iR} \gamma_\mu e_{jR}) Z'^\mu \tag{4.4}$$

where

$$\kappa_a \equiv U_a X_a U_a^\dagger \qquad a = L, R. \tag{4.5}$$

Eq.(4.4) represents the general form of the Z' interactions in all the $SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ models under study. κ_a are in general non-diagonal and hence Z' mediated interactions are flavor changing. The structure of κ_a depends upon the choice of the Higgs field in the model. Consider a model with only the Standard model Higgs with $U(1)_X$ charge zero and vectorial assignment of X to the leptons. As discussed above [2] we will only have a diagonal mass matrix $\mathcal{M}_l$ and hence there will be no lepton mixing here. The only way to have non-diagonal $\mathcal{M}_l$ with the standard Higgs is to assign non-vectorial X charges to the leptons. Suppose the charge assignments are x_{Li} and x_{Ri}. Let the charge matrix be given by

$$Q_{ij} = x_{Li} - x_{Rj}$$

To have a non-zero determinant of the mass matrix $\mathcal{M}_l$ we must have $Q_{ij} = 0$ in at least three appropriate positions (i, j). This restrictions demands x_{Ri} to be a permutation of x_{Li}. An inverse permutation or redefinition of right handed fields would diagonalize the mass matrix $\mathcal{M}_l$ and make the charge assignments on the physical leptons vectorial. We have in this case $U_L = 1$ and U_R corresponding to a permutation to give $\kappa_L = \kappa_R = X_L$ in eq.(4.5). The Z' coupling to leptons thus become like the coupling of Standard model Z and there will be no flavor mixing. However when an additional doublet with nonzero $U(1)_X$ charge is introduced the structure of the mass matrix will become more complex. The bi-unitary diagonalization in this case does not in general correspond to simple permutation as above and hence we will have flavor mixing in general. The additional doublet also causes the Z and the Z' to mix. We will discuss the important role played by the additional doublet Higgs in bringing out flavor changing processes in section 4.3.

Different models are specified by the choice of X and the Higgs fields which determine $\mathcal{M}_l$ and hence $U_{L,R}$.

4.2 $Z - Z'$ Mixing

In the presence of an additional Higgs doublet Φ_2 with non-zero horizontal charge, x, the Z' will mix with the Standard model Z to produce two physical mass eigenstates Z_1 and Z_2. The photon does not mix with Z' [3]. We show this in the following. The part of the Lagrangian for the Higgs scalar relevant here is

$$\begin{aligned} \mathcal{L}_{Higgs} &= \left|\left(\partial_\mu + \frac{i}{2}(g_1\tau \cdot W_\mu + g_2 Y B_\mu)\right)\Phi_1\right|^2 \\ &+ \left|\left(\partial_\mu + \frac{i}{2}(g_1\tau \cdot W_\mu + g_2 Y B_\mu + g' X B'_\mu)\right)\Phi_2\right|^2 \end{aligned} \tag{4.6}$$

Here B'_μ is the gauge field corresponding to $U(1)_X$. After spontaneous symmetry breaking the mass terms for neutral gauge bosons can be obtained from the following

$$\mathcal{L}_{mass} = \frac{1}{4}\left(g_1 W_\mu^3 + g_2 B_\mu\right)^2 \langle\Phi_1\rangle^2 + \frac{1}{4}\left(g_1 W_\mu^3 + g_2 B_\mu + g' x B'_\mu\right)^2 \langle\Phi_2\rangle^2 \tag{4.7}$$

The mass matrix for the neutral gauge bosons W_μ^3, B_μ and B'_{mu} is thus

$$\mathcal{M}_0^2 = \frac{1}{4}\begin{bmatrix} g_1^2(\langle\Phi_1\rangle^2 + \langle\Phi_2\rangle^2) & g_1g_2(\langle\Phi_1\rangle^2 + \langle\Phi_2\rangle^2) & g_1g'x\langle\Phi_2\rangle^2 \\ g_1g_2(\langle\Phi_1\rangle^2 + \langle\Phi_2\rangle^2) & g_2^2(\langle\Phi_1\rangle^2 + \langle\Phi_2\rangle^2) & g_2g'x\langle\Phi_2\rangle^2 \\ g_1g'x\langle\Phi_2\rangle^2 & g_2g'x\langle\Phi_2\rangle^2 & g'^2x^2\langle\Phi_2\rangle^2 \end{bmatrix} \tag{4.8}$$

The above mass matrix can be diagonalized by the following orthogonal transformation

$$\mathcal{M}_{diag} = O^T\mathcal{M}_0^2O \tag{4.9}$$

where

$$O = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \tag{4.10}$$

The first factor of O is the same as the one occurring in Standard model mixing the photon and the standard model Z. The angle θ called the weak mixing angle is given by $\tan\theta = g_1/g_2$. After block diagonalization by the first factor of O the massless photon gets decoupled from the Z and Z' and the transformed mass matrix is of the form

$$\mathcal{M}_Z^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_Z^2 & \delta M^2 \\ 0 & \delta M^2 & M_{Z'}^2 \end{pmatrix}$$

where

$$M_Z^2 = \frac{1}{4}g^2[\langle\Phi_1\rangle^2 + \langle\Phi_2\rangle^2]; \qquad M_{Z'}^2 = \left(\frac{xg'}{2}\right)^2[\langle\Phi_2\rangle^2]$$

$$\frac{\delta M^2}{M_Z^2} = \frac{2xg'}{g}\sin^2\beta \quad \text{where} \quad \sin^2\beta = \frac{\langle\Phi_2\rangle^2}{\langle\Phi_1\rangle^2 + \langle\Phi_2\rangle^2} \tag{4.11}$$

Here

$$g = \sqrt{g_1^2 + g_2^2}$$

The mixing angle ϕ between Z and Z' is then given by

$$\tan^2\phi = \frac{M_Z^2 - M_1^2}{M_2^2 - M_Z^2} \tag{4.12}$$

In addition, one has

$$M_1^2\cos^2\phi + M_2^2\sin^2\phi = \frac{M_W^2}{\cos^2\theta} \quad \text{and} \quad \sin\phi\cos\phi = \frac{\delta M^2}{M_2^2 - M_1^2} \tag{4.13}$$

$\theta(M_W)$ being the Weinberg angle (W-mass) at the tree level.

In standard model one defines a parameter called the ρ parameter as

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta}$$

With the choice of the Higgs in the standard model, the ρ parameter is equal to 1. However due to the presence of an additional gauge boson that mixes with the standard model Z the ρ parameter differs from 1 and is given by

$$\begin{aligned} \rho_M &= \frac{M_W^2}{M_1^2 \cos^2\theta} \\ &= \frac{1 + \tan^2\phi \frac{M_2^2}{M_1^2}}{1 + \tan^2\phi} \end{aligned} \tag{4.14}$$

One can see from (4.14) that in the absence of the extra Z or its mixing $\rho_M = 1$.

4.3 Gauging lepton numbers

4.3.1 Without additional doublet Higgs

In the minimal standard model, right handed neutrinos are not introduced. Thus neutrinos are massless and lepton numbers corresponding to each generation are globally conserved. The conserved lepton numbers correspond to global symmetries $U(1)_{L_e, L_\mu, L_\tau}$. However these global symmetries cannot be gauged as it would introduce anomalies in the theory. Introduction of right handed neutrinos however allow neutrinos to be massive and give possibility to gauge family lepton number symmetry. It was shown in ref.[4] that though individual lepton numbers cannot be gauged without expanding the fermionic content of the standard model, a linear combination $X = \alpha L_e + \beta L_\mu + \gamma L_\tau$ can be gauged . The condition of anomaly cancellation restricts the values of α, β and γ through the following equation

$$\begin{aligned} \alpha + \beta + \gamma &= 0 \\ \alpha^3 + \beta^3 + \gamma^3 &= 0 \end{aligned} \tag{4.15}$$

It may be noted that in this model $U(1)_X$ acts vectorially on the leptons. This is a very simple gauged horizontal symmetry as it does not require additional fermions to be introduced in order to cancel the anomalies. The only solution to the set of equations above give the following possibilities for X.

$$L_1 = L_e - L_\mu, \quad L_2 = L_e - L_\tau, \quad L_3 = L_\mu - L_\tau \tag{4.16}$$

However only one of L_i can be gauged at a time as $L_i^2 L_j$ anomalies are necessarily non-zero for $i \neq j$. The $U(1)_X$ symmetry is broken by a singlet Higgs η. No additional doublet scalars are added to the theory. Hence there will be no $Z - Z'$ mixing as can be seen from eqs. (4.11-4.13). There is no flavor mixing and flavor violating rare processes in this model though the usual leptonic processes gets modified. The bounds on the coupling constants of $U(1)_X$, g' and the Z' mass M_Z' were obtained from forward backward asymmetry A_{FB} of the neutral leptonic processes $e^+e^- \to \mu^+\mu^-$, $\tau^+\tau^-$ and the cross sections of these processes relative to QED given by

$$R_l = \frac{\sigma(e^+e^- \to l^+l^-)}{\sigma_{QED}(e^+e^- \to l^+l^-)}; \quad l = \mu, \tau \tag{4.17}$$

Bounds are also obtained from $\nu - e$ scattering experiments. The results for the two models with $X = L_1$ and $X = L_2$ are as follows [4]:

For L_1 the best fit values occur at $g' = 0$ and $M_{Z'} \to \infty$. Thus the L_1 model does not give any improved agreement between theory and experiments. For L_2 model the best fit parameters are $g' = 0.0005g$ and $M_{Z'} = 58$GeV. The Z' resonance in A_{FB} and R_τ shows a very narrow lineshape. The energy interval affected by Z' is rather small and there is possibility that such a narrow peak could be skipped without disturbing standard model prediction in the TRISTAN LEP window.

4.3.2 With additional doublet Higgs

In this subsection we discuss models where the horizontal symmetries are linear combination of lepton numbers like in ref[4]. The fermionic content is also same as in the standard model. We introduce an additional Higgs doublet. This changes the phenomenology of model in subsection 4.3.1 significantly [3]. To be general, we consider non-vectorial assignment of horizontal charges to fermions although the X assignment of members of a given $SU(2)_L$ doublet have to be identical. However we will show that any non-vectorial assignment is equivalent to a vectorial one in this case. For notational convenience let us write X-charges in terms of diagonal matrices in the generation space.

$$X_{L,R} = \mathrm{diag}(\alpha_1, \alpha_2, \alpha_3)_{L,R}$$

X_L determine the $U(1)_X$ assignment of the leptonic doublet while X_R that of the charged right-handed leptons. Like in eq.(4.15) the possible choices of α_{iL} and α_{iR} are restricted due to anomaly cancellation which require:

$$\begin{array}{rcll} \sum_i \alpha_{iL} & = & \sum_i \alpha_{iR} & = \; 0 \\ \sum_i \alpha_{iL}^2 & = & \sum_i \alpha_{iR}^2 & \\ 2\sum_i \alpha_{iL}^3 & = & \sum_i \alpha_{iR}^3 & \end{array} \tag{4.18}$$

These constraints can be satisfied by taking any two of α_{iL} and α_{iR} to be ± 1 and the third to be zero. Such non-vectorial assignments reduce to vectorial assignments under permutation of leptonic flavors amongst right handed or left handed fields. This redefinition can be always done since the initial choice of basis is arbitrary. The physical charge assignment reduce to simple vectorial one $X_L = X_R = X$. In this case the restrictions in eq.(4.18) reduces to those in eq.(4.15). The allowed X is restricted as in ref[4] either to $L_e - L_\mu$, $L_e - L_\tau$ or $L_\tau - L_\mu$. The current coupled to $U(1)_X$ boson Z' is vectorial when expressed in the weak basis in this case. But the structure of physical current coupled to mass eigenstates of fermions depends upon the choice of Higgs fields. In the event of only one Higgs doublet neutral under $U(1)_X$, the charged leptonic mass matrix is diagonal and the physical current coupled to Z' is vectorial. When one introduces more Higgs fields transforming non-trivially under $U(1)_X$, the $U(1)_X$ no-longer remains vectorial.

The structure of the current associated with the new Z' can be written as,

$$\mathcal{L}_{Z'} = \frac{g'}{\cos\theta} \left\{ \bar{e}'_L X \gamma_\mu e'_L + \bar{e}'_R X \gamma_\mu e'_R \right\} Z'^\mu \tag{4.19}$$

The coupling of the physical (i.e. mass eigenstate) fermions to Z' depend upon the structure of the mass matrix $\mathcal{M}_l$ for the charged leptons. This is dictated by the charge matrix Q whose $(i,j)^{th}$ element correspond to the X-charge of bilinear $\bar{e}'_{iL}e'_{jR}$. For example, we have in case of $L_e - L_\tau$

$$Q = \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix} \quad (4.20)$$

The possible structures of mass matrices follow from that of Q. In particular, a Higgs field with charge $-Q_{ij}$ would contribute to the $(i,j)^{th}$ element of the mass matrix $\mathcal{M}_l$. Note that the two different fields contribute to the $(\mathcal{M}_l)_{ij}$ and $(\mathcal{M}_l)_{ji}$. Hence $\mathcal{M}_l$ is necessarily non-hermitian except when it is diagonal with only one Higgs doublet carrying zero charge under $U(1)_X$. In this case, the weak basis $e'_{L,R}$ coincide with the mass basis $e_{L,R}$ and Z' couples to a vector current corresponding to X. When one introduces one or more additional doublets transforming non-trivially under $U(1)_X$ then $\mathcal{M}_l$ is necessarily non-hermitian and can be diagonalized by a bi-unitary transformation as shown in eq(4.3). However now the charge assignment being vectorial κ_a in eq.(4.5) is given as

$$\kappa_a = U_a X U_a^\dagger, \quad a = L, R \quad (4.21)$$

The current coupled to Z' is non-vectorial as $\kappa_L \neq \kappa_R$. To prove this explicitly , we write $U_R = U_L V$, V being a unitary matrix different from I. Then $U_L X U_L^\dagger = U_R X U_R^\dagger$ only if $VX = XV$. This is not possible because of the restricted structure of X.

4.4 $SU(2)_L \otimes U(1)_Y \otimes U(1)_{L_e - L_\tau}$ Model

We choose $X = L_e - L_\tau$ as an example and study the consequences in detail. The fermion content is like standard model while two Higgs doublets $\Phi_{1,2}$ and an $SU(2)_L \otimes U(1)_Y$ singlet η are introduced. X charges of Φ_1 and Φ_2 are chosen to be 0 and +2 respectively. The field η is assumed to carry some non-zero charge under $U(1)_X$ and it is solely introduced to provide a different mass scale characteristic of the $U(1)_X$ breaking.

The quark sector of the model remains the same as in the SM model while lepton couplings to the neutral Higgs fields are given by the following:

$$\begin{aligned} -\mathcal{L}_Y &= h_{ii}\bar{e}'_{iL}e'_{iR}\phi_1^0 + h_{13}\bar{e}'_{1L}e'_{3R}\Phi_2^0 \\ &\equiv \frac{m_i}{\langle\Phi_1^0\rangle}\bar{e}'_{iL}e'_{iR}\phi_1^0 + \frac{\delta}{\langle\phi_2^0\rangle}\bar{e}'_{1L}e'_{3R}\phi_2^0 + h.c \end{aligned} \tag{4.22}$$

This leads to the following mass matrix $\mathcal{M}_l$:

$$\mathcal{M}_l = \begin{pmatrix} m_1 & 0 & \delta \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \tag{4.23}$$

Let $U_{L,R}$ diagonalize $\mathcal{M}_l$, i.e.

$$U_L\mathcal{M}_lU_R^\dagger = \text{diag}(m_e, m_\mu, m_\tau)$$

where

$$\begin{aligned} m_\mu^2 &= m_2^2 \\ m_e^2 &= \frac{1}{2}\left\{m_1^2 + m_3^2 + \delta^2 + \left[(m_1^2 - m_3^2)^2 + 2\delta^2(m_1^2 + m_3^2) + \delta^4\right]^{\frac{1}{2}}\right\} \\ m_\tau^2 &= \frac{1}{2}\left\{m_1^2 + m_3^2 + \delta^2 - \left[(m_1^2 - m_3^2)^2 + 2\delta^2(m_1^2 + m_3^2) + \delta^4\right]^{\frac{1}{2}}\right\} \end{aligned}$$

$$U_{L,R} = \begin{bmatrix} \cos\theta_{L,R} & 0 & \sin\theta_{L,R} \\ 0 & 1 & 0 \\ -\sin\theta_{L,R} & 0 & \cos\theta_{L,R} \end{bmatrix} \tag{4.24}$$

The mixing angles $\theta_{L,R}$ are given by:

$$\sin 2\theta_L = -\frac{2\delta m_3}{m_\tau^2 - m_e^2} \qquad \sin 2\theta_R = -\frac{2\delta m_1}{m_\tau^2 - m_e^2}$$

As we will soon see, the $\theta_{L,R}$ are constrained to be quite small. It is therefore appropriate to work in the approximation $\delta < m_1, m_3$. In this limit,

$$\sin 2\theta_R \approx -\frac{2\delta m_e}{m_\tau^2} \qquad \sin 2\theta_L \approx -\frac{2\delta}{m_\tau} \tag{4.25}$$

The parameters κ_{aij} ($a = L, R$) determining the couplings of Z' to leptons through eq.(4.21) are explicitly given in the present case by

$$\begin{aligned} \kappa_{a11} &= \cos 2\theta_a = -\kappa_{a33} \\ \kappa_{a13} &= -\sin 2\theta_a \\ \kappa_{a2i} &= 0 \qquad i = 1, 2, 3 \end{aligned} \tag{4.26}$$

The quark sector of the model remains the same as in the SM model while lepton couplings to the neutral Higgs fields are given by the following:

$$\begin{aligned} -\mathcal{L}_Y &= h_{ii}\bar{e}'_{iL}e'_{iR}\phi_1^0 + h_{13}\bar{e}'_{1L}e'_{3R}\Phi_2^0 \\ &\equiv \frac{m_i}{\langle\Phi_1^0\rangle}\bar{e}'_{iL}e'_{iR}\phi_1^0 + \frac{\delta}{\langle\phi_2^0\rangle}\bar{e}'_{1L}e'_{3R}\phi_2^0 + h.c \end{aligned} \tag{4.22}$$

This leads to the following mass matrix $\mathcal{M}_l$:

$$\mathcal{M}_l = \begin{pmatrix} m_1 & 0 & \delta \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \tag{4.23}$$

Let $U_{L,R}$ diagonalize $\mathcal{M}_l$, i.e.

$$U_L\mathcal{M}_lU_R^\dagger = \text{diag}(m_e, m_\mu, m_\tau)$$

where

$$\begin{aligned} m_\mu^2 &= m_2^2 \\ m_e^2 &= \frac{1}{2}\left\{m_1^2 + m_3^2 + \delta^2 + \left[(m_1^2 - m_3^2)^2 + 2\delta^2(m_1^2 + m_3^2) + \delta^4\right]^{\frac{1}{2}}\right\} \\ m_\tau^2 &= \frac{1}{2}\left\{m_1^2 + m_3^2 + \delta^2 - \left[(m_1^2 - m_3^2)^2 + 2\delta^2(m_1^2 + m_3^2) + \delta^4\right]^{\frac{1}{2}}\right\} \end{aligned}$$

$$U_{L,R} = \begin{bmatrix} \cos\theta_{L,R} & 0 & \sin\theta_{L,R} \\ 0 & 1 & 0 \\ -\sin\theta_{L,R} & 0 & \cos\theta_{L,R} \end{bmatrix} \tag{4.24}$$

The mixing angles $\theta_{L,R}$ are given by:

$$\sin 2\theta_L = -\frac{2\delta m_3}{m_\tau^2 - m_e^2} \qquad \sin 2\theta_R = -\frac{2\delta m_1}{m_\tau^2 - m_e^2}$$

As we will soon see, the $\theta_{L,R}$ are constrained to be quite small. It is therefore appropriate to work in the approximation $\delta < m_1, m_3$. In this limit,

$$\sin 2\theta_R \approx -\frac{2\delta m_e}{m_\tau^2} \qquad \sin 2\theta_L \approx -\frac{2\delta}{m_\tau} \tag{4.25}$$

The parameters κ_{aij} ($a = L, R$) determining the couplings of Z' to leptons through eq.(4.21) are explicitly given in the present case by

$$\begin{aligned} \kappa_{a11} &= \cos 2\theta_a = -\kappa_{a33} \\ \kappa_{a13} &= -\sin 2\theta_a \\ \kappa_{a2i} &= 0 \qquad i = 1, 2, 3 \end{aligned} \tag{4.26}$$

Since one of the doublets carry non-zero $U(1)_X$ charge, the Z' will mix with the conventional Z boson to produce two mass eigenstates $Z_{1,2}$.

$$\begin{aligned} Z &= \cos\phi\ Z_1 + \sin\phi\ Z_2 \\ Z' &= -\sin\phi\ Z_1 + \cos\phi\ Z_2 \end{aligned} \tag{4.27}$$

The couplings of the neutral gauge boson $Z_{1,2}$ to the leptons are now given by

$$\mathcal{L}_Z = \frac{g}{\cos\theta}\left\{\sum_{m=1,2} F_{Lmij}\bar{e}_{iL}\gamma_\mu e_{jL} Z_m^\mu + L \leftrightarrow R\right\} \tag{4.28}$$

where

$$\begin{aligned} F_{L1ij} &= \cos\phi(-\frac{1}{2} + \sin^2\theta)\delta_{ij} - \sin\phi\frac{g'}{g}\kappa_{Lij} \\ F_{R1ij} &= \cos\phi\sin^2\theta\delta_{ij} - \sin\phi\frac{g'}{g}\kappa_{Rij} \\ F_{L2ij} &= \sin\phi(-\frac{1}{2} + \sin^2\theta)\delta_{ij} + \cos\phi\frac{g'}{g}\kappa_{Lij} \\ F_{R2ij} &= \sin\phi\ \sin^2\theta\delta_{ij} + \cos\phi\frac{g'}{g}\kappa_{Rij} \end{aligned}$$

As would be expected, eqs.(4.23) and (4.26) show that the muon number is exactly conserved in the model. This is a consequence of the fact that both the Z' interactions as well as the mass matrix, eq.(4.23), respect this symmetry. When $\delta << m_\tau$, the flavor violations and departure from vectorial symmetry are very small. Moreover, these departures are more suppressed in the right-handed sector compared to the left-handed sector.

The generalization to other models in this category is obvious. One could construct another model with additional Higgs carrying $L_e - L_\tau$ charge -2 instead of $+2$. In this case $(\mathcal{M}_l)_{31}$ will be non-zero instead of $(\mathcal{M}_l)_{13}$ as in eq.(4.23). All the couplings of this model are then obtained by interchange of $\theta_L \leftrightarrow \theta_R$ in eq.(4.26). In addition to these two models with $L_e - L_\tau$ symmetry, one could construct pair of models each with symmetry $L_e - L_\mu$ and $L_\mu - L_\tau$. These are respectively characterized by an unbroken L_τ and L_e.

In addition to the flavor violations induced by Z', there exists other flavor violations associated with the Higgs fields. These arise in a well-known [5] manner whenever the fermions with the same charge obtain their masses from two different Higgses as in eq.(4.23). Using eqns.(4.23-4.24) it follows that

$$-\mathcal{L}_{FCNC} = \delta\left(\frac{\Phi_1^0}{\langle\Phi_1^0\rangle} - \frac{\Phi_2^0}{\langle\Phi_2^0\rangle}\right)\{\cos\theta_L \sin\theta_R e_L \tau_R + \cos\theta_R \sin\theta_L \tau_L e_R\} + h.c.$$

It follows from eq.(4.25) that these flavor violations are of $O\left(\delta^2/m_\tau < \phi_{1,2}^0 >\right)$ and hence would be suppressed in the limit $\delta << m_\tau$ compared to Z' induced flavor violations unless the associated Higgs is much lighter than Z'. We shall therefore concentrate on the Z' induced flavor violations in the next section.

4.5 Phenomenology of $SU(2)_L \otimes U(1)_Y \otimes U(1)_{L_e - L_\tau}$

We shall now explore the phenomenological consequences of the $SU(2)_L \times U(1)_Y \times U(1)_X$ models. The extra Z-boson associated with $U(1)_X$ change the phenomenology of the SM in two ways. The Z' contribute to the known processes induced by the Z boson. In addition, in the present case, Z' induce new flavor violating processes. The detailed phenomenology will depend upon the model. We shall take the model presented in the last section as an illustrative example and work out consequences within that model.

In the absence of additional Higgs, the Z' induced flavor violation disappears. Moreover the Z' does not mix with the ordinary Z. In this case Z' makes its effect felt by contributing to known processes like $e^+e^- \to \mu^+\mu^-$ scattering. The detailed restrictions on the relevant parameters by LEP results have been worked out in ref. [4] for this case. These restrictions continue to hold in the present case. But additionally one gets more stringent restrictions due to flavor violations and Z-Z' mixing. We shall concentrate on these in the following.

The phenomenology of models with extra Z boson is extensively discussed in the literature [5, 6]. The present class of models have characteristic differences arising due to the fact that Z' couples only to leptons. In other models, an important restriction on the Z' mass arises from the direct experimental observations at the hadronic colliders. These restrictions though model dependent strongly constrain the Z' mass. For example in the left-right symmetric model [7], the search in pp collisions imply [8] $M_{Z_{LR}} > 310$ GeV. Similar restrictions are not applicable here since Z' couples only to leptons. Its production at the hadronic colliders arise only through mixing with the ordinary Z and is therefore highly suppressed. The Z' mass as well as its mixing with Z is constrained in the present case by (a) the observations at LEP and (b) the observed limits on the leptonic flavor violations. We discuss them in turn.

4.5.1 Constraints from the LEP data

We closely follow the analysis of ref. [5] in deriving constraints on the relevant parameters from observations at LEP. These constraints have been derived in two different ways. The observations of the ratio M_W/M_1 and the Z-mass M_1, at CDF and LEP respectively, constrain the ρ parameter and lead to restrictions on M_2 and $\tan\phi$. Other method is to use the fact that the extra Z induce changes in observables like width to fermions, peak cross section in e^+e^- collisions etc. One could then make a detailed fit to the LEP data and derive constraints on M_2 and ϕ.

The mixing between Z and Z' change the tree level relation between the W and the Z mass. Specifically,

$$\frac{M_W^2}{\rho_M M_1^2} = \cos^2\theta$$

θ being the tree level weak mixing angle. The parameter ρ_M is given by eq.(4.14). One could eliminate $\cos^2\theta$ in favor of G_F, α and M_1 to obtain

$$\frac{M_W^2}{\rho_M M_1^2} = \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\mu^2}{\rho_M M_1^2}}\right) \tag{4.29}$$

where

$$\mu = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_F}} = (37.280\text{GeV})$$

These restrictions are valid at the tree level. Since the extra Z induced effects are comparable to the radiative corrections in the standard model, one must incorporate the later. This has been done in ref. [5], assuming that the radiative corrections induced by Z_2 are negligible. The radiative corrections of SM are included using the improved Born approximation which changes eq.(4.29) to the following:

$$\frac{M_W^2}{\rho M_1^2} = \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\mu^2}{\rho M_1^2(1-\Delta\alpha)}}\right) \tag{4.30}$$

where the ρ parameter is now

$$\rho = \frac{\rho_M}{1 - \Delta\rho_T}$$

with

$$\Delta\rho_T \simeq 3\frac{G_F m_t^2}{8\pi^2\sqrt{2}} \quad \text{and} \quad \Delta\alpha = 0.0602 + \frac{40}{9}\frac{\alpha}{\pi}\ln\frac{M_1(\text{GeV})}{92} \pm 0.0009$$

The determination of M_1 from the data is fairly insensitive to the presence of Z'. Hence the CDF result on $\frac{M_W}{M_1} = 0.779$ together with the LEP result on the Z-mass M_1 can be used to obtain $\rho = 1.005 \pm 0.003$ in eq.(4.30). This implies at 1σ

$$\Delta\rho_M = \rho_M - 1 \leq 0.008 - 0.003\left(\frac{m_t(\text{GeV})}{100}\right)^2 \tag{4.31}$$

In addition to this restriction, $\Delta\rho_M$ can also be constrained [5, 6] by the other observables at LEP. Specifically, the presence of Z' would change the three leptonic widths $\Gamma_{e\,\mu\,\tau}$ as well as the hadronic width Γ_h of the Z_1. These changes can be parameterized [5] in terms of $\Delta\rho_M$ and mixing angle ϕ:

$$d\Gamma_i = A_i\Delta\rho_M + B_i\phi \tag{4.32}$$

In our case

$$\begin{aligned} A_i &= 4N_c\rho_f\left[(T_{3Li} - \sin^2\theta_f Q_i)^2 + T_{3Li}^2 + \frac{4\sin^2\theta_f\cos^2\theta_f}{\cos 2\theta_f}Q_i(T_{3Li} - \sin^2\theta_f Q_i)\right] \\ B_i &= 8N_c\rho_f\left[(T_{3Li} - \sin^2\theta_f Q_i)g'_{Vi} - T_{3Li}g'_{Ai}\right] \end{aligned}$$

where

$$g'_{Vi} = \frac{g'}{g}(\kappa_{Lii} + \kappa_{Rii}); \quad g'_{Ai} = \frac{g'}{g}(\kappa_{Lii} - \kappa_{Rii});$$

$$\rho_f \equiv \frac{\rho}{\rho_M}; \quad \sin^2\theta_f = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\mu^2}{\rho_f M_1^2(1-\Delta_\alpha)}}$$

$N_c = 3(1+\frac{\alpha_s}{\pi})$ for quarks and 1 for leptons. Fermionic width Γ_i of Z_1 have been extracted from the LEP data in a model independent way. We use the values derived in ref. [9] to constrain $\Delta\rho_M$ and ϕ. Specifically,

$$\begin{aligned}\Gamma_e &= 82.6 \pm 0.7\text{MeV} \\ \Gamma_\mu &= 83.6 \pm 1.1\text{MeV} \qquad\qquad \Gamma_h = 1.741 \pm 0.015\text{MeV} \\ \Gamma_\tau &= 83.1 \pm 1.2\text{MeV}\end{aligned}$$

We use these values and determine the best values for $\Delta\rho_M$ and ϕ appearing in eq.(4.32) through a least square fit. This gives (for $m_t = 150$GeV) at 1σ:

$$\Delta\rho_M = -0.0018 \pm 0.004 \qquad\qquad \phi = 0.0094 \pm 0.012 \tag{4.33}$$

The value of $\Delta\rho_M$ as determined by eq.(4.33) is less stringent than following from eq.(4.31) derived on the basis of the CDF result on $\frac{M_W}{M_1}$. We shall therefore use the values given by eq.(4.31) for $\Delta\rho_M$ in the next section to constrain the parameters of the model.

4.5.2 Constraints from the rare processes

As already discussed, the model of section 4.4 contains flavor violations involving τ and e. The muon number is exactly conserved in the model. As a consequence one expects the following rare processes to occur in the model:

- $Z_{1,2} \to e\tau$
- $\tau \to eee$
- $\tau \to e\mu\mu$

The branching ratios for these processes can be easily worked out and are given by:

$$\frac{\Gamma(\tau \to eee)}{\Gamma(\tau \to \nu_\tau \nu_e e)} = 16 M_1^4 \left\{ (g^e_{LL})^2 + (g^e_{RR})^2 + \frac{1}{2}\left((g^e_{LR})^2 + (g^e_{RL})^2\right) \right\}$$

$$\frac{\Gamma(\tau \to e\mu\mu)}{\Gamma(\tau \to \nu_\tau \nu_e e)} = 4M_1^4\{(g_{LL}^\mu)^2 + (g_{RR}^\mu)^2 + (g_{LR}^\mu)^2 + (g_{RL}^\mu)^2\}$$

$$\Gamma(Z \to \tau e) = \frac{G_F M_1^3}{3\sqrt{2}\pi}\{(F_{L1}^{\tau e})^2 + (F_{R1}^{\tau e})^2\}$$

where

$$g_{LL}^m = \frac{F_{L1}^{\tau e} F_{L1}^{mm}}{M_1^2} + \frac{F_{L2}^{\tau e} F_{L2}^{mm}}{M_2^2} \qquad g_{LR}^m = \frac{F_{L1}^{\tau e} F_{R1}^{mm}}{M_1^2} + \frac{F_{L2}^{\tau e} F_{R2}^{mm}}{M_2^2}$$

$m = e, \mu$. g_{RR} and g_{RL} are obtained by $L \leftrightarrow R$ interchange in above equation. The difference in the rates for the $\tau \to eee$ and $\tau \to e\mu\mu$ arise due to both the s and t channel $Z_{1,2}$ exchanges contributing to the former. In addition to constraints from the LEP discussed earlier the rare decays also provide important constraints on the model. The specific constraints are [8] given by the following:

$$\begin{aligned}
Br(Z \to e^+\mu^-) &< 2.4 \times 10^{-5} \\
Br(Z \to e^+\tau^-) &< 3.4 \times 10^{-5} \\
Br(Z \to \mu^+\tau^-) &< 4.8 \times 10^{-5} \\
Br(\tau \to eee) &< 2.7 \times 10^{-5} \\
Br(\tau \to e\mu\mu) &< 2.7 \times 10^{-5} \\
Br(\tau \to \mu\mu\mu) &< 1.7 \times 10^{-5}
\end{aligned}$$

The basic parameters of models are mixing angles $\theta_{L,R}$, Z_2 mass M_2, Z-Z' mixing angle ϕ and the $U(1)_X$ gauge coupling g'. Both the Z-Z' mixing and the flavor violation arise in the model from the presence of the additional doublet ϕ_2. Thus both are related to the parameter $\tan\beta = \langle\phi_2\rangle/\langle\phi_1\rangle$. Relation between ϕ and β follows from eq.(4.11) and (4.12)

$$\sin\phi \sim 4C\left(\frac{M_1}{M_2}\right)^2 \sin^2\beta \tag{4.34}$$

where

$$C \approx \frac{g'}{g}\left(1 - \frac{M_1^2}{M_2^2}\right)^{-1} \sim O(1)$$

The $\theta_{L,R}$ also goes to zero when $\beta \to 0$. If one assumes that the flavor violating Yukawa coupling h_{13} in eq.(4.23) is of the same order as flavor conserving one (namely h_{33}) then $\delta \approx m_\tau \tan\beta$ and hence from eq.(4.4)

$$\sin 2\theta_L \approx -2\tan\beta \tag{4.35}$$

The existing limits on the $Br(\tau \to eee)$ as well as $Z \to e\tau$ imply restrictions on the parameters β and M_2. These are displayed in fig.1 assuming $h_{13} = h_{33}$. Analogous constraints also follow from the process $\tau \to e\mu\mu$. This process is comparatively suppressed in the present case and hence imply much weaker constraints. This is not displayed in the figure for simplicity. The same parameters are also constrained by $\Delta\rho_M$ and ϕ(see eqs.(4.14) and (4.34)).

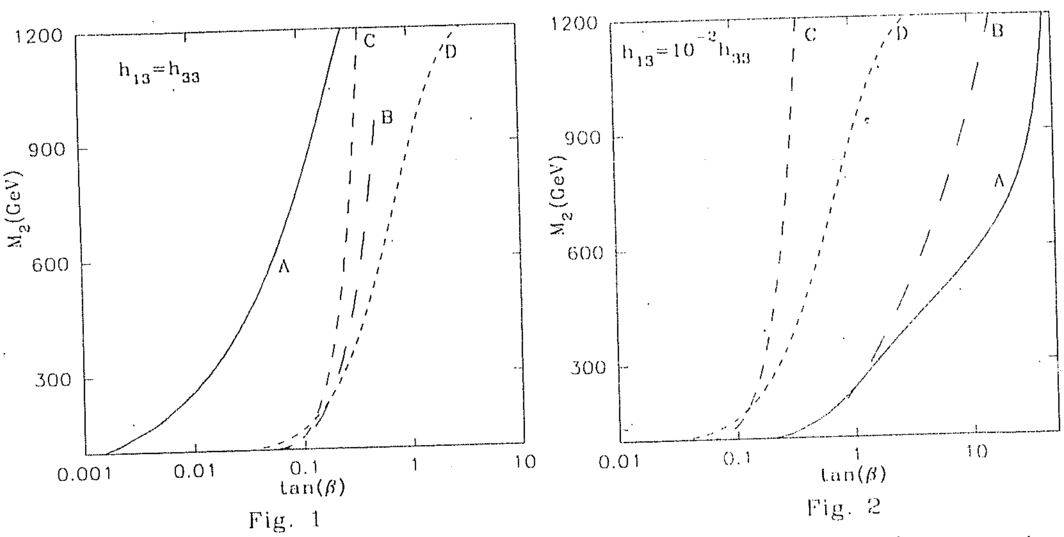

Fig. 1: The allowed region in the M_2-$\tan\beta$ plane implied by various constraints: Curve(A) is a contour for $Br(\tau \to eee) = 2.7 \times 10^{-5}$; (B) for $Br(Z \to e\tau) = 3.4 \times 10^{-5}$; (C) for $\Delta\rho_M = 0.00125$; and (D) for $\phi = 0.021$. These curves are for $h_{13} = h_{33}$ (see text). Region to the left of the curves is allowed.
Fig. 2: Same as figure 1 except that $h_{13} = 10^{-2}h_{33}$.

It follows that the strongest constraints on the parameters are implied by the rare decay $\tau \to eee$. Hence the process $\tau \to eee$ is allowed by the LEP data to occur at a rate consistent with the present experimental precision. Improvement in the limits for this process would either imply more stringent restrictions on β and M_2 or one should be able to see this decay in future. Fig.1 was based on the assumption of equal Yukawa couplings, $h_{13} = h_{33}$, in eq.(4.23). For comparison we also display in fig.2 limits on β and M_2 in case of $h_{13} = 10^{-2}h_{33}$. Reduction in the value of h_{13} strongly suppresses the flavor violating couplings of τ. $\Delta\rho_M$ and ϕ remain unchanged. As a result, now the LEP data imply stronger restrictions on $\tan\beta$ and M_2. In this case, the LEP observations already rule out possibility of seeing flavor violation in future experiments which are expected to provide improved limits on $\tau \to eee$.

It is clear from fig.1 and 2 that as long as $M_2 < O(TeV)$, $\tan\beta$ is restricted to be $< O(0.1-0.5)$. Hence the vacuum expectation value of the field ϕ_2 responsible for flavor violations is strongly constrained in the model. Likewise, low values of M_2 (e.g. 400GeV) are possible only if $\tan\beta$ is chosen small (0.03 in case of $h_{13} = h_{33}$, and 0.3 in case of $h_{13} = 10^{-2}h_{33}$).

Although we restricted ourselves to the $L_e - L_\tau$ model, the analogous constraints would follow in models with $X = L_e - L_\mu$ or $L_\mu - L_\tau$. In particular, one would expect very severe constraint if $L_e - L_\mu$ is gauged since $\mu \to eee$ is much severely constrained experimentally.

4.6 Summary

Gauged horizontal symmetries $U(1)_X$ can give suitable structures to the fermion mass matrices that generate correct hierarchical structure and mixing. The choice of $U(1)_X$ to be gauged are restricted by the requirement of anomaly cancellation without extending the fermionic sector of the standard model. These are thus the simplest gauge extension of the standard model. These models are prototypes of more general horizontal symmetries [10].

In a specific case where $U(1)_X$ couples only to leptons all possible choices are categorized. Constraints on additional gauge bosons are obtained. In a model by He et. al.[4] no additional doublet Higgs is introduced. In this model flavor mixing and $Z - Z'$ mixing is absent. The only restriction on Z' mass comes from forward backward asymmetry of leptonic processes and the total cross sections of $e^+e^- \to l^+l^-$ relative to QED contributions. In the case where $X = L_e - L_\mu$ there is no improvement in fits with experiment, compared to standard Model. For $X = L_e - L_\tau$, however the best fit is obtained at $M_{Z'} = 58GeV$ for a reasonable strength of $U(1)_X$ coupling, $g' = 0.0005g$.

However when additional Higgs doublets are introduced with non-zero $U(1)_X$

charges suitable restrictions are obtained on mixing matrices. Moreover, they also give rise to interesting flavor violations thus providing window into the existence of such symmetry. The mixing of the $U(1)_X$ gauge boson Z' with the ordinary Z is correlated in these models to the flavor violation. In fact both these features originate from the existence of the Higgs doublet carrying non-zero $U(1)_X$ charge. As a result the observations at LEP could indirectly provide important constraints on flavor violations. Detailed study presented here shows that under reasonable assumptions on relevant Yukawa couplings, the LEP observations do allow sizeable flavor violations and it is possible to obtain rate for $\tau \rightarrow eee$ near its present experimental limit. In contrast, the lepton flavor violating decays of Z are considerably suppressed in these models.

We presented here models in which $U(1)_X$ acts only on leptons. Models with $U(1)_X$ acting on quarks [11] or both can be analogously studied. A systematic study of these horizontal models and restrictions on flavor violations in these models in the light of LEP observations would be interesting in its own right.

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Chapter 5

Conclusions

The Standard Model of the Strong and electroweak interaction has been very successful in describing physics around and below the scale of $O(M_W)$. The internal consistency of this model has been verified in various experiments to a high degree of precision. Grand Unified Theory attempts to embed the three distinct gauge groups of Standard model into a single large gauge group with a single coupling constant. There exists three identical families of fermions that form representation of Standard Model. Fermions from different families can mix in principle. This happens because the physical or mass states of fermions are in general not the same as the weak states which form the families. As Standard Model and GUT operates identically on all the families it does not give any restrictions on fermion masses and mixing.

Fermions exhibit specific pattern in their masses. Quarks and charged lepton masses show hierarchical patterns. Experiments allow neutrinos to be massive though very small. However unlike other fermions they may be required to be nearly degenerate. The mixing in fermions is related to the hierarchy in their masses. Phenomenological ansatz for mass matrices like the Fritzsch ansatz are quite successful in generating these patterns. These ansatz basically try to minimize the number of arbitrary parameters in the general mass matrices and then obtain relation between them. However it is desirable to have an underlying symmetry that would constrain the Lagrangian to produce Fritzsch like structure for mass matrices. These symmetries operate on family space of fermions and are reffered to as Horizontal symmetries. In this thesis we have studied the effect of an abelian horizontal symmetry on various sectors of the standard model.

Though neutrinos are massless within Standard Model one can generate neutrino masses in certain extensions of standard model like the left-right symmetric models. In these models both Dirac and Majorana mass terms for neutrinos can be generated. In such a case Seesaw mechanism makes the effective mass of the neutrinos very small while assuming the existence of very heavy right handed neutrinos in the theory. The usual Seesaw scheme display the same hierarchy in neutrino masses as that of other fermions. The solution to the solar and the atmospheric neutrino problem require (mass)^2 difference between two neutrino flavors to be $\sim O(10^{-2}\text{-}10^{-6}eV^2)$. The hierarchies along with the constraints of such small (mass)^2 differences $\sim 10^{-2}\text{-}10^{-6}eV^2$ make the masses of even the heaviest neutrinos unobservably small $O(0.1\text{eV})$. It is found that the hierarchy in neutrino masses is sensitive to the structure of the heavy right handed neutrino mass matrix $\mathcal{M}_R$. Obvious way to remove hierarchy in neutrino mass matrix is to consider a diagonal $\mathcal{M}_R$ as in usual Seesaw but impose suitable hierarchy in its diagonal entries. This is however very unnatural assumption. We have shown that such unnatural hierarchy can be avoided by considering non-diagonal structure for $\mathcal{M}_R$. Such non-diagonal structure is obtained by imposing horizontal symmetries. In a specific model as an example we have shown that for the entries of elements of $\mathcal{M}_R$ in the intermediate scale $\sim 10^9$GeV all the neutrino masses turn out to be in the eV range. These neutrinos can together provide the hot component of the Dark matter which requires $\sum m_{\nu_i} \leq 7$eV. The (mass)^2 difference between the two of the neutrinos is around $\sim 10^{-5}\text{eV}^2$. This falls in the range required to solve the solar neutrino problem through the MSW mechanism. The mixing of neutrinos with such small (mass)^2 difference is nearly maximal. However the horizontal symmetry also causes mixing in the charged lepton sector. The relative mixing of the neutrinos with respect to that in the charged leptons need not be too large . The large angle solution for solar neutrino problem needs $\sin^2 2\theta_{e\tau} \sim 0.65\text{-}0.85$. This would require the mixing in the charged leptonic sector to be $\phi = 10\text{-}20^0$. This amount of lepton mixing leads to an observable signal for neutrinoless double beta decay for $m_{\nu_e} \sim O(\text{eV})$.

We also studied the possibility of horizontal symmetry being related to the smallness of CP violation in weak decays. Violation of CP is observed in K^0-$\bar{K}^0$ oscillation. Two parameters ϵ and ϵ' are identified as measures of CP violation in K^0-$\bar{K}^0$ system. These parameters are related to the complex phase in the CKM matrix which describes the quark mixing. The quark mixing angles are approximately given by the square root of the ratio of their masses. Due to hierarchy in quark masses these mixing angles are small. Frogatt and Nielsen found that such hierarchies in the mass matrix at low energies cannot be generated by the renormalisation group evolution of Yukawa couplings at high scales where all the entries are of the same order. Some kind of selection rule or horizontal symmetry is imperative. They introduced two singlet Higgses η_0 and η_1 in addition to the Standard model doublets. η_0 has very high v.e.v and generates only superheavy fermion masses. η_1 generates the masses of the ordinary fermions through non-renormalisable Yukawa couplings.The hierarchies in quark masses are obtained in terms of powers of a small parameter $\epsilon = \langle\eta_1\rangle/\langle\eta_0\rangle$. CP is not a symmetry of the model and hence CP violation is not restricted to be small.

The smallness of CP violation indicates a link with spontaneous symmetry breaking mechanism. In a two Higgs doublet model Lee showed that CP can be arranged to be conserved by the Lagrangian. But the choice of the ground state spontaneously violates CP. However these models contain flavor changing neutral Higgs (FCNH) current which constraints the Higgs mass to be O(TeV). This suppresses the CP violation far too smaller than the observed amount. It was shown by several authors that in Lee type model if FCNH are either eliminated or suppressed by discrete symmetries then CP violation obtained can be of the desired amount.

We studied a model where CP is arranged to be conserved by renormalisable Lagrangian. We considered the left-right symmetric extension of the Standard model with a horizontal symmetry $U(1)_H$ and a Peccei Quinn symmetry $U(1)_{PQ}$. Three bidoublet Higgs are introduced in addition to the triplet Higgses, Δ_L and Δ_R of the left right symmetric model. The quark mass matrices generated by the bi-doublet Higgses have the required Fritzsch structure. The horizontal symmetry is broken by a singlet Higgs, η_H. With all these fields CP is arranged to be conserved by the renormalisable Lagrangian. The breaking of horizontal symmetry at a high scale generates at low scales non-renormalisable effective Yukawa couplings. These terms contribute to the mass matrices in terms of a small parameter $\epsilon = \langle\eta_H\rangle/M$. Here M is taken to be the Planck scale. The modification of the mass matrices by ϵ also generates a complex phase in the CKM matrix. Thus in this theory the strength of CP violation is linked to the horizontal symmetry breaking scale. With the Yukawa couplings of $O(1)$, ϵ is predicted to be $\epsilon \approx m_s/m_t \approx 10^{-3}$. This requires $U(1)_H$ symmetry breaking scale to be 10^{16}GeV.

Gauged horizontal symmetry requires additional gauge bosons . The masses of these gauge bosons reffered to as Z' depends upon the horizontal symmetry breaking scale. If this scale is not very high then the effects of Z' can be obtained in various processes. We studied horizontal symmetry that acts only on the leptonic sector. The fermionic content of the model is not extended. This restricts the $U(1)_H$ to be generated by $L_e - L_\mu$, $L_\mu - L_\tau$ and $L_e - L_\tau$ due to the restriction of anomaly cancellation. We showed the role of additional doublet Higgs with non-zero $U(1)_H$ charge in bringing out flavor mixing and the Z-Z' mixing. Thus flavor changing neutral currents are possible in this model and mediated by Z'. In a similar model considered by He et al. without additional Higgs doublet, neither flavor mixing nor Z-Z' mixing is possible. They constrain the gauge coupling g' and the Z' mass $M_{Z'}$ through forward backward asymmetry, A_{FB} and the relative cross section R_l of leptonic processes given by

$$R_l = \frac{\sigma(e^+e^- \to l^+l^-)}{\sigma_{QED}(e^+e^- \to l^+l^-)}; \quad l = \mu, \tau$$

For $U(1)_{L_e - L_\tau}$ they obtain the following best fit values for parameters

$$g' = 0.0005g \qquad \text{and} \qquad M_{Z'} = 58GeV.$$

In our model the major constraint comes from rare processes. Due to the presence of an extra Z the ρ parameter is different from 1. The deviation, $\Delta\rho_M$, of ρ from 1 lead to restriction on the Z' mass M_2 and $\tan\phi$ where ϕ is the mixing angle between Z and Z'. With the CDF and LEP measurement of M_W/M_1 and M_1 respectively the constraint obtained is

$$\Delta\rho_M < 0.00125 \;\; \text{for } m_t = 150\text{GeV}$$

They are also constrained by the changes in the Z width to fermions. For $m_t = 150$ GeV at 1σ the constraints are

$$\Delta\rho_M < 0.0022 \qquad \text{and} \qquad \phi < 0.021$$

The restrictions on $\Delta\rho_M$ and ϕ gives restrictions on M_2- $\tan\beta$ parameter space where $\tan\beta$ is the ratio of v.e.v of the two doublet Higgses.

The constraint on rare processes $\tau \to eee$, $\tau \to e\mu\mu$ and $Z \to e\tau$ also limit the allowed region in M_2- $\tan\beta$ parameter space. These constraints are shown in fig. 1 and fig. 2 of chapter 4 with the Yukawa couplings $h_{13} = h_{33}$ and $h_{13} = 10^{-2}h_{33}$. It is found that for $h_{13} = h_{33}$ the strongest constraint is given by the rare decay $\tau \to eee$. Thus the LEP data allows the rare process $\tau \to eee$ to occur at the present experimental precision. For $h_{13} = 10^{-2}h_{33}$ the LEP data is found to suppress all rare processes. This is expected as h_{13} is a off diagonal Yukawa coupling due to which flavor mixing occurs. The allowed parameter space shows that if $M_2 <$ O(TeV) then $\tan\beta < O(0.1\text{-}0.5)$. Thus the v.e.v. of the additional Higgs doublet is constrained to be smaller than that of the Standard model Higgs if M_2 is not allowed to be unobservably large.

All our considerations throughout the thesis have been in the context of Standard model or its generalization to the left-right symmetric theory. But it is well-known that horizontal symmetries can be defined even at the GUT scale where they can restrict the structure of fermion masses and mixing. Such generalization is quite natural within the Frogatt-Nielson mechanism adopted in chapter 3 since breaking of $U(1)_H$ is required to take around the GUT scale. Likewise neutrino masses and Seesaw mechanism also become more natural in the context of $SO(10)$ theory. Many of the considerations in chapters 2 and 3 can be generalized to include GUT such as $SO(10)$. In contrast the content of chapter 4 is based on the assumption of relatively low breaking of horizontal symmetry. Moreover the purely lepton number like symmetries discussed there are also not easy to incorporate in GUT which treat quarks and leptons on similar footing. In contrast, the horizontal symmetries of the type considered in chapter 4 are more amenable to experimental tests through rare processes and ultimately through discovery of horizontal gauge bosons if they exist. Experiments will have the final say as in most phenomenological studies.