Fermion Masses and Mixings

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A Thesis Submitted to

The Maharaja Sayajirao University of Baroda

for the Degree of

Doctor of Philosophy

in

Physics

May 1991

Contents

Acknowledgements

It is my great pleasure to thank all those who have contributed to this work in any way.

I would like to acknowledge the guidance and constant help that my supervisors Profs. J.C. Parikh and M.S. Shah have very kindly extended to me.

I am particularly indebted to Utpal Sarkar with whom I did most of the work that goes towards making up this thesis. You gallantly undertook the onerous task of making a man out of an extremely lazy and stubborn graduate student and were extremely patient even through rather trying times. But most of all I have cherished your insistence that I strike out an independent path from the outset.

I also thank N.D. Hari Dass, Anjan Joshipura, M.V.N. Murthy, Saurabh Rindani and many others from whom I have learnt a lot during the last so many years. And finally I would like to remember the innumerable friends all around the country and especially at the Institute of Mathematical Sciences and the Physical Research Laboratory who have helped me keep sane and actually enjoy my work.

Research Publications

List of Papers on which this thesis is based:

1. B–$\overline{B}$ Mixing, ϵ'/ϵ and Quark Mass Matrices,
   D. Choudhury and U. Sarkar, Phys. Lett. **B 217** (1989) 341.

2. Model Independent Analysis of Quark Mass Matrices,
   D. Choudhury and U. Sarkar, Phys. Rev. **D 39** (1989) 3425.

3. Naturally Light Majorana Neutrinos with no Neutrinoless Double Beta Decay,
   D. Choudhury and U. Sarkar, Phys. Rev. **D 41** (1990) 1591.

4. Large Magnetic Moments For Near Massless Neutrinos,
   D. Choudhury and U. Sarkar, Phys. Lett. **B 235** (1990) 113.

5. Gravitational Helicity Flip for Massive Neutrinos and SN 1987A,
   D. Choudhury, M. V. N. Murthy and N. D. Hari Dass, Classical and Quantum Gravity **6** (1989) L167.

6. Neutrino Magnetic Moments and the 17 keV ν_τ,
   D. Choudhury and U. Sarkar, PRL preprint PRL–TH/90–15 (submitted to Phys. Rev. Lett.)

Synopsis

The standard model of particle physics, though immensely successful in terms of experimental agreement, leaves many questions unanswered. Not the least of which is the problem of fermion masses which are not predicted by the minimal model and are only phenomenologically determined. In principle, a study of this problem can enlighten us about the possible extensions of the standard model that could resolve many of the puzzling issues. In the case of the quark masses and mixings, various aesthetic but partial solutions have been offered. These involve imposing on the quark mass matrices different ansätze motivated by particular models with higher gauge symmetry and/or experimental observations. With neutrinos the situation is even more fluid. While there is hardly any direct evidence for nonzero neutrino masses, yet the potentially rich phenomenology and the dramatic consequences in astrophysical and terrestrial laboratories that their existence would imply has spurred many studies.

The present thesis is divided into two distinct but connected parts that look at some of the questions raised above. The first part deals with the study of quark mass matrices. The most popular ansätze discussed in the literature are those due to Stech and Fritzsch and certain modifications thereof and are examined here for concurrence with constraints coming from three sets of experimental observations. These are the measurement of ϵ_K, the parameter describing the indirect CP violation in the neutral K–system or, in other words, the CP violation in interactions changing strangeness by two units ($\Delta S = 2$), and the more recent measurements of the B_d^0–$\overline{B_d^0}$ mixing parameter x_d (which gives the time–integrated probability of a $\overline{B_d^0}$ appearing in a B_d^0 beam) and the direct ($\Delta S = 1$) CP violation parameter ϵ'_K, *i.e.* the one relevant in K–decays. It is found that while the Stech ansatz can be made consistent with ϵ_K and x_d for low top quark mass ($m_t \sim 45 GeV$, an experimentally consistent value when the work was done but ruled out since) it is completely ruled out by ϵ'_K/ϵ_K [35]. As a corollary, all schemes incorporating the Stech assumptions are proved to be inconsistent. On the other hand the Fritzsch and other Fritzsch-like schemes still do admit many solutions (albeit with a much restricted parameter space) but with different characteristics for different ranges of m_t [35].

These observations naturally led to a model independent analysis of the most general quark mass matrices for three families [38]. It was shown how phenomenological considerations restrict the parameter space to different disjoint sectors. The constraints on the general form that lead to various ansätze were examined and it was shown that all the popular models lie in a particular one of the aforementioned sectors [38]. The analysis also points out the alternative directions future model–building efforts could adopt.

The second part of the thesis deals with the problem of neutrino masses and some related topics. The relation between the Majorana masses of the neutrinos and neutrinoless double beta decay [$(\beta\beta)_{0\nu}$] was reexamined and it was argued that, contrary to naive expectations, the $(\beta\beta)_{0\nu}$ rate does not distinguish between the Dirac and Majorana mass of the physical electron neutrino (ν_e) [53]. It had been held that if the tritium β–decay experiments indicate a neutrino mass larger than that predicted by $(\beta\beta)_{0\nu}$, then ν_e has to be a Dirac particle and models were constructed to incorporate such an eventuality. Based on our analysis, we propose a new scenario wherein the physical ν_e can naturally be a Majorana neutrino without any $(\beta\beta)_{0\nu}$. Some supersymmetric grand unified theories were shown to yield such scenarios naturally [53]. These models turn out to be much simpler and more economical than existing ones predicting light Dirac neutrinos.

A related question is that of the neutrino magnetic moments, a large value of which would offer a solution to the solar neutrino problem (the longstanding discrepancy between the ν_e absorption rates in the Davis experiment and that predicted by the standard solar model) and has the added advantage of explaining the apparent anticorrelation between the solar neutrino flux and the sunspot activity. A model – based on a gauged $SO(3)$ horizontal symmetry – that decouples the magnetic transition moment of the neutrino from its Majorana mass was constructed, thus allowing large magnetic moments for nearly massless neutrinos [66]. The present scheme is most strikingly different from all the others of its genre in that not only does it not depend upon an intact $SU(2)_\nu$ symmetry between neutrinos to suppress their mass while allowing a large magnetic moment (Voloshin mechanism), rather in this case the appearance of the last–mentioned actually hinges on the breaking of the symmetry in question. The model, while treating all fermions on par, also avoids observable Goldstone bosons, excessive fine tuning and extra fermions, embarassments which plagued earlier efforts.

We next study the problem of the gravitational helicity flip of a massive neutrino. If the neutrinos be massive, then interactions that flip their helicities could have dramatic implications for the cooling of hot neutron matter during stellar collapse. With a central core as massive and compact as a neutron star, gravitational helicity flip could play an important role too. A semiclassical analysis of the gravitational scattering problem for low energy neutrinos in the vicinity of a neutron star shows that this mechanism could in fact overwhelm all other known sources of helicity flip! From a study of cooling rates of the supernova SN 1987A, strong bounds were placed on parity violating gravitational interaction strengths [74].

Finally, we integrate the aforementioned ideas in neutrino physics in the quest of a natural model for the recently reported 17 keV neutrino. In the $SU(2)_H$ model that is constructed for the purpose, the required mass scale $\sim O(100\ keV - 1\ MeV)$ is generated radiatively. The crippled see–saw mechanism then naturally leads to a pseudo–Dirac ν_τ that is identified with the new find. This particle can hence be used as a very good probe for gravitational helicity flips. The Majorana neutrinos ν_e and ν_μ remain extremely light (mass $\sim O(10^{-5} - 10^{-4}\ eV)$) but possess a relatively large transition magnetic moment $\mu_{\nu_e\nu_\mu} \sim 10^{-12}\mu_B$. The $(\beta\beta)_{0\nu}$amplitude is extremely small and is consistent with all bounds. The spontaneous lepton number violation in the theory results in a $SU(2)_L$ singlet–doublet Majoron unconstrained either by the LEP results on the Z–decay width or the astrophysical bounds. The ν_τ in the model is very short-lived and primarily decays into ν_μ and the Majoron.

To summarise, during the course of this work we have looked at various aspects of fermion masses and mixings and examined some of their observable consequences. On the hadronic front, certain well–discussed forms for the quark mass matrices were examined in the context of new experimental results, either to rule them out or to severely curtail the limits of their validity. A general model–independent analysis of the problem provides insights into the assumptions involved in the existing models and brings into perspective the course future model–building should adopt. In the leptonic sector, the study of neutrino masses exhibited the independence of the $(\beta\beta)_{0\nu}$ rate and the Majorana or Dirac mass of the ν_e. An economical model that naturally led to light Majorana neutrinos with no $(\beta\beta)_{0\nu}$ was constructed. A novel mechanism was also proposed to generate large magnetic moments for neutrinos while keeping their masses small. The scheme avoided the pitfalls faced by earlier efforts in this direction. An examination of astrophysical consequences of a non–zero neutrino mass exhibited the dominance of gravitational effects in the helicity flipping transitions of low energy neutrinos in the vicinity of a massive dense object. This also led to imposition of a very strict bound on possible parity–violating effects in gravity. Last but not the least, a model that can naturally accomodate the exciting new find of a 17 keV neutrino as well as a relatively large $\mu_{\nu_e\nu_\mu}$ is presented. As phenomenological viability demands consistency of the predictions for neutrino oscillation and $(\beta\beta)_{0\nu}$ rates with the experimental bounds, this study brings into focus the interrelationships of all the issues in neutrino physics discussed here.

Chapter 1

Introduction

While the $V - A$ [1] current-current form of the universal weak-interaction Lagrangian had successfully supplanted the earlier general 'four-fermi theory' [2], it still failed to answer many questions. The very 'weakness' of the interaction strength as reflected by $G_F = 1.16 \times 10^{-5} GeV^{-2}$ in

$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} J_{\mu 1}^{\dagger} J_2^{\mu}$$

where

$$J_\mu = \overline{\psi}_p \gamma_\mu (1 - \gamma_5) \psi_n, \ \overline{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_\nu \ \ etc.$$

is puzzling. The coupling constant has mass dimension -2 and closely parallels $\frac{\alpha}{q^2}$ for the electromagnetic interactions (q_μ being the momentum of the internal photon). In fact, Fermi was tempted to carry on the analogy with QED and describe the weak interactions as being mediated by some vector boson, but he did not pursue the idea any further. However this very structure of the Lagrangian ensures bad high energy behaviour for the cross sections. Clearly for a point interaction as above, just from dimensional arguments one has for the cross section, $\sigma \sim G_F^2 s$ where s is the invariant energy. But this being the $\ell = 0$ mode of a partial wave expansion, one has $\sigma = const/s$. Thus for $s > G_F^{-1}$, one runs into contradiction with unitarity requirements. The cure for this problem has already been hinted at: to formulate the weak interactions so as to be mediated by Intermediate Vector Bosons (IVB). With $\mathcal{L} = g J_\mu^- W^{+\mu}$ and g a dimensionless coupling constant, one has for the amplitude $M \sim g^2 \frac{J_\mu^+ J^{\mu -}}{M_W^2 - q^2}$ and hence for a large M_W, the low energy interaction would adequately be parametrized by $G_F \sim \frac{g^2}{M_W^2}$, whereas at large energy transfers, the cross–section falls off as required.

But even this would fail to please the purist. For, like the original theory, the massive IVB theory too is non–renormalizable. In fact, the only renormalizable theories involving vector bosons as fundamental constituents are those with a local gauge symmetry. But then is there a gauge symmetry associated with the weak interactions, and even if there is, how does one get the IVB's to be massive? These questions were answered by Glashow, Salam and Weinberg [3] and while the crux of their arguments shall be presented in the next chapter, the rest of this chapter is concerned with motivating their solution.

The near equality of the muon decay constant and the vector coupling constants for neutron and pion β–decay, inspite of only the latter receiving strong interation corrections, gives an indication of the symmetry one is looking for. Drawing an analogy from electrodynamics where the equality of the proton and the positron charges is explained by an assumption of equal bare charges and the current conservation law $\partial_\mu A^\mu(x) = 0$, it was proposed that the $\Delta Y = 0$ (Y being the hypercharge) vector currents be part of the divergenceless isospin current of the strong interactions. According to this 'Conserved Vector Current' hypothesis then, the vector current strength would not be renormalized. This also implies that the $\Delta Y = 0$ semileptonic weak interactions and the electromagnetic interactions involving hadrons are related. This is because the electromagnetic current contains the third component of the isospin current. To wit,

$$\begin{aligned} V^\mu(x) &= J^\mu_{1+i2}(x) \\ J^\mu_{em}(x) &= J^\mu_3(x) + J^\mu_Y(x) \end{aligned}$$

where $J^\mu_Y(x)$ denotes the isoscalar hypercharge current [4].

Thus one is led to consider a $SU(2)$ group as the gauge symmetry for the weak interactions with the associated charged gauge bosons identified with the IVB's. Phenomenological reasons dictate that only the left–handed fermion fields transform nontrivially under this group. Hence any attempt to identify the neutral gauge boson with the photon is bound to fail as the electromagnetic interaction is vectorial. The next most economical way is to consider a direct product of the $SU(2)$ with a $U(1)$ and let the photon be a combination of the two neutral gauge bosons. This scheme has the added advantage of suggesting a common origin for the electromagnetic and the weak interactions and is the one favoured experimentally.

Thus was born the electroweak theory, a renormalizable gauge field theory chosen from amongst the many competing ones for better experimental agreement. Alongwith Quantum Chromodynamics (QCD), believed to be the theory explaining the strong nuclear force, this forms the so-called Standard Model (SM) of elementary particle physics, a model expected to explain very well physical processes that involve interaction energies upto at least $\sim O(100\,GeV)$.

However all activities (fortunately) do not cease with the choice and even establishment of a model. Apart from testing its consequences in hitherto uncharted areas, one must also identify its limitations and especially so in the context of delineating questions that it cannot presume to address. One such question of prime importance is that of fermion masses, the resolution to which promises insights into physics beyond the Standard Model. Apart from prescribing a method to obtain the masses, the model is absolutely silent about their relative magnitudes and the strength of their consequences. Although experiments do give you the numbers, yet one strives for a theoretical understanding, the first step to which is the act of model building with certain assumptions. Of course, before one takes any of these models seriously, their validity must be checked *vis. a. vis.* experimental agreement. The first part of this thesis aims to do just that for the case of the quarks.

The scenario for the leptons is even more challenging. Whereas Pauli [5] had proposed the neutrinos to be absolutely massless neutral particles reacting only to the weak nuclear force, the modern attitude is to view them not to be strictly massless but rather lacking any mass in the observationally discernible range. In fact, certain experimental puzzles can be resolved if one does postulate a very small but non-zero mass for the neutrinos. But doing this would open up a plethora of new interactions with perhaps rather startling consequences, and some of these issues we examine in the second part of the current work.

The plan of the rest of this document is as follows. In Chapter 2, we give a brief discussion of the essential features of the SM, followed by an account of the formal aspects of determination of quark masses, mixing in the neutral meson systems and CP violation. Chapter 3 deals with the examination of various ansätze for the quark mass matrices and their phenomenological implications. A model–independent analysis of the problem is also presented. The focus in Chapter 4 is on the question of neutrino masses and their experimental signatures. The extent of neutrinoless double beta decay is estimated for various configurations. It is established that contrary to expectations, this rate is not governed by the Majorana mass of ν_e and models are constructed to demonstrate the naturalness of such scenarios. Chapter 5 deals with some aspects of neutrino physics related to the neutrino mass namely a large magnetic moment on the one hand and the gravitational interactions of fermions on the other. To correlate these results, we propose a model for massive neutrinos to accomodate the new finding of a 17 keV neutrino as well as large transition magnetic moment for the ν_e. We also comment on the gravitational interaction of the new particle. Finally, in the concluding chapter, we summarise the results of the investigations conducted.

Chapter 2

The Standard Model: Some Issues Revisited

In this chapter we first present a short discussion of the standard model — rather its electroweak part only — and some of its prime features. We discuss at some length the problem of the fermion masses in general and subsequently the quark masses in particular. A general account of quark mixing is given and is followed by the specific form for the case of three fermion generations. The experimental limits on these mixings are also presented.

Since color confinement makes it impossible to observe free quarks, a direct measurement of their masses is not possible. However estimates can be made using different and rather involved techniques and the result of such computations are presented without attempting any kind of discussion of the methods.

Finally we move on to a discussion of the neutral meson mixings and CP violation. The general form of the two independent parameters (*viz.* ϵ_K and ϵ'_K) giving a measure of CP violation in the K^0–$\overline{K^0}$ system are derived and the expression for these in the standard model calculated. A similar exercise is performed for the extent of mixing in the B_d^0–$\overline{B_d^0}$ system.

2.1 The Standard Model

As has been pointed out in the last section, the electroweak gauge group to be considered is $SU(2)_L \otimes U(1)_Y$, such that the left handed fermions transform as doublets of the $SU(2)_L$

and that the generator for $U(1)_{em}$ be given as a combination of the two diagonal generators. Hence we have for the charge operator,

$$Q = aT_{3L} + Y. \tag{2.1.1}$$

Considering either the lepton doublet $\binom{\nu}{e}_L$ or the quark doublet $\binom{u}{d}_L$, one immediately has $a = 1$. Keeping in mind the fact that the right handed fermions do not participate in the weak interactions and hence should be singlets under $SU(2)_L$, one gets for the fermions' transformation under the full symmetry group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ to be

$$\begin{aligned}
l'_{iL} &\equiv \begin{pmatrix} \nu' \\ e' \end{pmatrix}_{iL} && (1, 2, -1/2) \\
e'_{iR} & && (1, 1, -1) \\
q'_{iL} &\equiv \begin{pmatrix} u' \\ d' \end{pmatrix}_{iL} && (3, 2, 1/6) \\
u'_{iR} & && (3, 1, 2/3) \\
d'_{iR} & && (3, 1, -1/3)
\end{aligned} \tag{2.1.2}$$

where i denotes the fermion generation.

As the low energy symmetry evinced in nature is only $SU(3)_c \otimes U(1)_{em}$, we must break $SU(2)_L \otimes U(1)_Y$ down to $U(1)_{em}$. The simplest way to do this is to take recourse to spontaneous symmetry breaking involving a complex scalar field ϕ which transforms as $(1, 2, 1/2)$. Then the Lagrangian piece involving ϕ is

$$\mathcal{L}_\phi = (\mathcal{D}_\mu\phi)^\dagger(\mathcal{D}^\mu\phi) - V(\phi) \tag{2.1.3}$$

where

$$\begin{aligned}
\mathcal{D}_\mu\phi &= (\partial_\mu - igW_\mu - ig'B_\mu)\phi, \\
V(\phi) &= \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2, \\
\text{and} \quad W_\mu &= W_\mu^a\tau^a
\end{aligned} \tag{2.1.4}$$

W_μ^a and B_μ being the gauge fields corresponding to $SU(2)_L$ and $U(1)_Y$ and g and g' the respective couplings. The only restrictions imposed on $V(\phi)$ are the requirements of renormalizability and gauge invariance.

For $\mu^2 < 0$, $V(\phi)$ is minimized at $|\phi^\dagger\phi| = -\frac{\mu^2}{2\lambda} \equiv v^2$. Such a non-zero vacuum expectation value (*v.e.v.*) $\langle\phi\rangle = \binom{0}{v/\sqrt{2}}$ leads to a spontaneous breaking of both the $SU(2)_L$

and the $U(1)_Y$ symmetry. However a different $U(1)$ symmetry — which we shall identify as electromagnetism — generated by the combination of the diagonal generators

$$Q = T_{3L} + Y \tag{2.1.5}$$

is still preserved. The three Goldstone bosons due to symmetry breaking [6] are absorbed by three of the massless gauge bosons to appear as their longitudinal component thus rendering the latter massive. The essence of this Higgs [7] mechanism is encapsulated in the following brief discussion.

We reparametrize the scalar field ϕ, writing its four real components in terms of four new ones ξ_i and η by

$$\phi = U(\xi)\begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}$$

where $U(\xi) = e^{-i\xi\cdot T/2v}$ with T_i being any three independent generators of the gauge group that do not annihilate the vacuum.

Now taking advantage of the local gauge invariance of the theory one might as well work with the gauge transformed field

$$\begin{aligned} \phi \quad &\rightarrow \quad \phi' &&= U^{-1}(\xi)\phi = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}, \\ W_\mu \quad &\rightarrow \quad W'_\mu &&= U^{-1}W_\mu U + \frac{i}{g}U^{-1}\partial_\mu U, \\ B_\mu \quad &\rightarrow \quad B'_\mu &&= B_\mu + \frac{i}{g}U^{-1}\partial_\mu U, \end{aligned}$$

and similarly transformed fermion fields.

Then (dropping the primes on the fields),

$$\mathcal{L}_\phi \rightarrow \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{1}{4}\phi^T(g'B_\mu + gW_\mu)^2\phi - V\left(\frac{(v+\eta)^2}{2}\right),$$

and the gauge boson mass term reads

$$\frac{1}{8}v^2\left[(-g'B_\mu + gW_\mu^3)^2 + g^2\{(W_\mu^1)^2 + (W_\mu^2)^2\}\right].$$

Defining

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}}\left[W_\mu^1 \mp iW_\mu^2\right], \\ Z_\mu &= W_\mu^3\cos\theta_W - B_\mu\sin\theta_W, \\ A_\mu &= W_\mu^3\sin\theta_W + B_\mu\cos\theta_W, \end{aligned} \tag{2.1.6}$$

where

$$\theta_W = \tan^{-1}(g'/g), \tag{2.1.7}$$

we get

$$\begin{aligned} m_W &= \tfrac{1}{2} g v, \\ m_Z &= \tfrac{1}{2} g v \sec\theta_W \quad \text{and} \\ m_A &= 0. \end{aligned} \tag{2.1.8}$$

Thus with this special gauge choice (known as the unitary gauge), the bosonic spectrum of the theory consists of a massless and three massive gauge bosons and the single neutral scalar η. The other three degrees of freedom of the ϕ–field have been absorbed by the vector bosons only to appear as the corresponding longitudinal polarizations.

Writing the quark (and similarly for the other fermions) gauge boson coupling term in the new fields, we have

$$\begin{aligned} & g\overline{q'_L}\gamma^\mu(\sqrt{2}W^+_\mu\tau^- + \sqrt{2}W^-_\mu\tau^+ + W^3_\mu\tau^3)q'_L + g'(\overline{q'_L}\gamma^\mu Y q'_L + \overline{q'_R}\gamma^\mu Y q'_R)B_\mu \\ &= \frac{g}{\sqrt{2}}(\overline{u'_L}\gamma^\mu d'_L W^+_\mu + \overline{d'_L}\gamma^\mu u'_L W^-_\mu) + g\cos\theta_W Z_\mu \overline{q'_L}\gamma^\mu(\tan^2\theta_W Y - \tau^3)q'_L \\ &\quad + g\sin\theta_W A_\mu \overline{q'_L}\gamma^\mu Q q'_L. \end{aligned} \tag{2.1.9}$$

We then see that the massless vector field A couples vectorially with the fermion current and hence can be identified with the photon leading to the identification $g\sin\theta_W = e$. The massive gauge bosons, on the other hand, couple only to the chiral currents leading to the left handed weak interactions.

Looking now at the fermion masses, it is immediately apparent that we cannot write bare terms as

$$\overline{\psi}\psi = \overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L$$

is not $SU(2)_L \otimes U(1)_Y$ invariant on account of ψ_L being a SU(2) doublet and ψ_R a singlet. This is not a problem though as we can use the same mechanism to generate fermion masses as for the gauge bosons *i.e.* spontaneous symmetry breaking. Recognising that the Yukawa term $\overline{q'_L}d'_R\phi$ is gauge invariant and of dimension $(mass)^4$, we have

$$\mathcal{L}_{Yuk} = f_d^{ij}\overline{q'_{Li}}d'_{Rj}\phi + f_u^{ij}\overline{q'_{Li}}u'_{Rj}\tilde{\phi} + f_e^{ij}\overline{l'_{Li}}e'_{Rj}\phi, \tag{2.1.10}$$

where $\tilde{\phi} \equiv \tau_2 \phi^*$ and i, j run over the fermion generations *i.e.* $d_1 \equiv d$, $d_2 \equiv s$, $d_3 \equiv b$ *etc.* In the unitary gauge we then have

$$\mathcal{L}_{Yuk} = \left(M_u^{ij} \overline{u'_{Li}} u'_{Rj} + \frac{1}{v} M_u^{ij} \overline{u'_{Li}} u'_{Rj} \eta + H.c. \right) + (u' \to d') + (u' \to e'), \tag{2.1.11}$$

where $M_u^{ij} = v f_u^{ij}$ is the mass matrix for the up-quarks and similarly for the others.

The theory does not specify f_{ij}^u and hence the mass matrices in any way. All structures for f_{ij}^u satisfy the symmetry requirements and these have to be determined only from experiments. In fact, the matrices do not even need to be hermitian and hence cannot be diagonalized by a unitary transformation. All is not lost though. As the left– and right–handed fermions have different $SU(2)_L$ and $U(1)_Y$ quantum numbers, one can define distinct unitary transformation for each. This is equivalent to treating the mass term as an hermitian operator in a $2n$ dimensional space (for n fermion generations) with a block off–diagonal representation and defining a unitary transformation in $U(2n)$ that is block diagonal in the left– and right–handed subspaces.

Now using a result in elementary linear algebra that any nonsingular matrix can be polar decomposed into a product of a positive definite Hermitian matrix and an isometric matrix, we can define

$$u_L \equiv U_L u'_L \quad \text{and} \quad u_R \equiv U_R u'_R, \tag{2.1.12}$$

such that

$$U_L^\dagger M_u U_R = \widehat{M_u} = \text{diagonal and positive definite}. \tag{2.1.13}$$

Thus U_L and U_R diagonalize the hermitian matrices $M_u M_{u^\dagger}$ and $M_u^\dagger M_u$ respectively. Defining similar transformations $D_{L,R}$ for the d–type quarks, we get

$$\mathcal{L}_{mass} = \overline{u_L} \widehat{M_u} u_R + \overline{d_L} \widehat{M_d} d_R + \overline{e_L} \widehat{M_e} e_R + H.c., \tag{2.1.14}$$

and

$$J_\mu^+ = \overline{u_L} \gamma_\mu K d_L + \overline{e_L} \gamma_\mu \nu_L, \tag{2.1.15}$$

where

$$K \equiv U_L^\dagger D_L \tag{2.1.16}$$

is the Cabibbo-Kobayashi-Maskawa (CKM) matrix [8].

At this stage an interpretation of the results is called for. The original primed fields were the eigenstates of the weak hamiltonian (H_{wk}) but not of the full hamiltonian as H_{wk} does not commute with $H_s + H_{em}$. The unprimed states are the eigenstates of the total hamiltonian and hence have well-defined masses. The CKM matrix then represents the modification of the charged current vertices for the physical quarks induced by quark mixing. It should be noted that the corresponding matrix for the leptonic sector is but unity. This is due to the absence of the ν_R and hence a mass term for the neutrinos, as a consequence of which the diagonalization matrix for M_e can be absorbed into the definition of the ν_L.

The mixing matrix K lies in $U(n)$ and hence is described by n^2 real parameters. Recognizing that ${}^nC_2$ of these are nothing but the Euler angles for a real rotation, we find that the complexity is due to the rest of the ${}^{n+1}C_2$ parameters. But of these, $(2n-1)$ are of no physical significance as they can be absorbed by redefining the relative phases of the quark wavefunctions. So at the end of the day we are left with $2n$ quark masses, ${}^nC_2$ real rotations and ${}^{n-1}C_2$ phases in the mixing matrix.

Henceforth we shall specialize to three generations (unless otherwise stated) as most current experimental results favour such a scenario. Then the CKM matrix is 3×3 and parametrized by three angles θ_{ij} and a phase δ which, in this model, is responsible for CP violation. For explicit calculations involving the CKM matrix, we choose the parametrization due to Maiani [9]:

$$K = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23}e^{-i\delta} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{-i\delta} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}, \tag{2.1.17}$$

where $c_{ij} = \cos\theta_{ij}$; $s_{ij} = \sin\theta_{ij}$.

While θ_{12} is very accurately determined from K_{e3} and hyperon decays [10]

$$s_{12} = 0.221 \pm 0.002, \tag{2.1.18}$$

θ_{23} and θ_{13} are rather poorly determined. The value of s_{23} may be extracted from a determination of V_{cb} (since $s_{23} \approx |V_{cb}|$ to a very good approximation) from the semileptonic B-meson partial width, under the assumption that it is given by the W-mediated process to be

$$\Gamma(b \to c l \bar{\nu}_l) = \frac{G_F^2 m_b^5}{192\pi^3} F(m_c^2/m_b^2)|V_{cb}|^2, \tag{2.1.19}$$

where $F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln(x)$ is a phase space factor. Thus

$$s_{23}^2 = \frac{192\pi^3}{G_F^2} \frac{Br(b \to c l \bar{\nu}_l)}{\tau_b m_b^5 F(m_c^2/m_b^2)} \quad . \tag{2.1.20}$$

Using the experimental results for the branching ratio and the B-meson lifetime [11]

$$\mathrm{Br}(b \to c l \bar{\nu}_l) = 0.121 \pm 0.008 \qquad \tau_B = (1.16 \pm 0.16) \times 10^{-12}\ sec, \tag{2.1.21}$$

and the estimates for the quark masses (see section 2.2):

$$m_c = (1.35 \pm 0.05)\ GeV \qquad m_b = (5.3 \pm 0.1)\ GeV,$$

we get [12][1]

$$0.035 \leq s_{23} \leq 0.07 \tag{2.1.22}$$

The charmless B–meson decay width puts a limit [14][2]

$$0.07 \leq s_{13}/s_{23} \leq 0.22. \tag{2.1.23}$$

The CP–violating phase δ is allowed to adopt any value in the range $[0, \pi]$ by the current experimental results.

2.2 Quark Masses

In a renormalizable field theoretic treatment, the coupling constant and masses lose their absolute meaning and become dependent on the momentum scale one is addressing the problem at. This dependence arises from two sources, though the two cannot be demarcated easily.

In quantum field theoretical calculations infinities creep up quite often and are taken care of by what is called a 'regularization' prescription. Though there is *nothing ad hoc* about this program, there do exist different inequivalent schema for this procedure, the

[1]The experimental numbers quoted in this chapter are those used in [35,38]. Since then many of these have been revised. For example, now one has $s_{23} = 0.043^{+0.007}_{-0.009}$ [13].

[2]Current limits [13] are $0.05 \leq s_{13}/s_{23} \leq 0.13$.

difference lying in the amount of the finite part to be subtracted alongwith the divergent piece. Thus it creates a dependence on the momentum scale introduced, that is different in different schemes.

However this implies that the physical quantities would depend on the renormalization scheme adopted and the scale at which it is being performed, a situation clearly unacceptable as starting from a unique Lagrangian, all measurables ought to have a unique value. This then leads to the requirement that under a finite renormalization transformation, physical predictions be invariant. Expressed in a different language, this implies that all renormalized quantities should change with a change of the scale (equivalent to a finite renormalization transformation) in a well-defined fashion and that the functional dependence of measurables on these should change such that their (measurables') value remains the same. These finite renormalizations form a transformation group and the functional relations determining the changes can be expressed as differential equations of evolution known as the Renormalization Group (RG) equations.

When talking of quarks, the relevant theory of course is QCD and the RG equations important in our study are those governing the evolution of the quark masses and the strong coupling constant with the renormalization scale μ:

$$\begin{aligned} \mu\frac{dg}{d\mu} &= \beta(g), \\ \mu\frac{dm_i}{d\mu} &= -\gamma_{m_i}(g)m_i. \end{aligned} \tag{2.2.1}$$

In the modified minimal subtraction ($\overline{MS}$) scheme, the beta function and the anomalous dimension are respectively given by [15]

$$\beta(g) = -\frac{\beta_0}{(4\pi)^2}g^3 - \frac{\beta_1}{(4\pi)^4}g^5 + O(g^7), \tag{2.2.2}$$

and

$$\gamma_m(g) = \frac{\gamma_0}{(4\pi)^2}g^2 + \frac{\gamma_1}{(4\pi)^4}g^4 + O(g^6), \tag{2.2.3}$$

with

$$\begin{aligned} \beta_0 &= (11C_G - 4T_R N_f)/3, \\ \beta_1 &= \left[34C_G^2 - 45C_G + 3C_F)T_R N_f)\right]/3, \\ \gamma_0 &= 6C_F, \\ \gamma_1 &= C_F[9C_F + 97C_G - 20T_R N_f]/3, \end{aligned} \tag{2.2.4}$$

where N_f = number of quark flavours, T_R is given by the normalization of the generators $\left[Tr(T^a T^b) = N_f T_R\right]$ and C_G and C_F are the values of the quadratic Casimir operator on the gluons and the quarks respectively. For $SU(3)$, by convention, $T_R = 1/2$, and $C_G = 3,\ C_F = 4/3$ and hence

$$\begin{aligned}\beta_0 &= 11 - \tfrac{2}{3}N_f, \\ \beta_1 &= 102 - \tfrac{38}{3}N_f, \\ \gamma_0 &= 8, \\ \gamma_1 &= \tfrac{4}{3}\left(101 - \tfrac{10}{3}N_f\right).\end{aligned}$$

The solutions to the differential equations are

$$\alpha_s(\mu) \equiv \frac{g^2(\mu)}{4\pi} = \frac{4\pi}{\beta_0 L}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln L}{L} + O\left(\left(\frac{\ln L}{L}\right)^2\right)\right], \tag{2.2.5}$$

and

$$m_i(\mu) = \overline{m}_i\left(\frac{L}{2}\right)^{-\gamma_0/2\beta_0}\left[1 - \frac{\beta_1\gamma_0}{2\beta_0^3}\frac{1+\ln L}{L} + \frac{\gamma_1}{2\beta_0^2 L} + O\left(\left(\frac{\ln L}{L}\right)^2\right)\right], \tag{2.2.6}$$

where $L = \ln(\mu^2/\Lambda^2)$. Here Λ and $\overline{m}_i$ are the RG–invariant scale parameter and masses, respectively defined through

$$\begin{aligned} e^{-\beta_0 g^2(0)} &= \tfrac{\lambda^2}{\Lambda^2}\left(\ln\tfrac{\lambda^2}{\Lambda^2}\right)^{\beta_1/\beta_0^2} \\ \text{and}\quad m_i(0) &= \overline{m}_i\left(\ln\tfrac{\lambda^2}{\Lambda^2}\right)^{-\gamma_0/2\beta_0}, \end{aligned}$$

λ being the momentum cutoff. This then takes care of the perturbation theory induced cutoff–dependence of the bare couplings. The arbitrary coupling constant $g(0)$ is thus replaced through 'dimensional transmutation' by a dimensionful parameter Λ, which along with the quark masses are the only arbitrary parameters in QCD and would be fixed by experimental data.

In all the above formulae the value of N_f to be used is determined by the energy scale of the problem at hand, with the assumption that all heavier degrees of freedom can be taken to be frozen. The physical mass of a quark is then its value calculated at the same scale. Thus to one loop order, the physical mass of the i'th quark is given by

$$m_i^{\rm phys} = m_i(m_i)\left[1 + \frac{4}{3\pi}\alpha_s(m_i)\right]. \tag{2.2.7}$$

While non-observation of the top-quark puts a lower limit [16] to its mass [3]

$$m_t^{\rm phys} \gtrsim 45\ GeV, \tag{2.2.8}$$

[3] Current bound [17] is $m_t^{\rm phys} \gtrsim 89\ GeV$.

experimental consistency with the radiative corrections in the standard model requires [18]

$$m_t^{\text{phys}} \lesssim 180\ GeV. \tag{2.2.9}$$

Substituting $N_f = 6$ and $\Lambda_{QCD} = 100 MeV$, we have for the above range of interest

$$m_t^{\text{phys}} \approx 0.6 m_t(1\ GeV),$$

which gives

$$75\ GeV \lesssim m_t(1\ GeV) \lesssim 300\ GeV. \tag{2.2.10}$$

The physical masses of the charm and bottom quarks can be calculated to a great degree of accuracy from e^+e^- data by using QCD sum rules for the vacuum polarization amplitude. We have then

$$\begin{aligned} m_c(1\ GeV) &= (1.35 \pm 0.05)\ GeV \\ m_b(1\ GeV) &= (5.3 \pm 0.1)\ GeV. \end{aligned} \tag{2.2.11}$$

The determination of the lighter quark masses involves larger errors. These are best evaluated using chiral perturbation theory and meson and baryon spectroscopy. Though the individual errors are large, restrictions on the ratio of the masses reduce the indeterminacy somewhat:

$$\begin{aligned} m_u &= (5.1 \pm 1.5) MeV \\ m_d &= (8.9 \pm 2.6) MeV \\ m_s &= (175 \pm 55) MeV \\ m_s/m_d &= (19.6 \pm 1.6) \\ m_d/m_u &= (1.76 \pm 0.13) \\ m_s/m_u &= (34.5 \pm 5.1) \\ \frac{m_u - m_d}{m_u + m_d} &= (-0.28 \pm 0.03). \end{aligned} \tag{2.2.12}$$

2.3 *CP* Violation and Neutral Meson Systems

Apart from the usual continuous (gauge) symmetries that lead to conserved Nöther's currents, physical theories most often respect certain discrete symmetries as well. The most common of these are:-

Parity (P): this implies an equivalence of 'left' and 'right' *i.e.* a mirror image of an experiment would yield the same result in the reflected frame of reference as the original would in the initial frame.

Charge conjugation (C): implies invariance under replacing each particle by its antiparticle (*i.e.* reversing all additive quantum numbers).

Time reversal(T): referring to a formal reversal of time flow, this implies invariance under reversal of all momenta, angular momenta *etc.*

Though a theorem due to Lüders and Pauli [19] guarantees that any Lorentz-invariant unitary local field theory is invariant under the combined action CPT (in any order), the individual symmetries are not assured by any deep principle. In fact though gravitational, electromagnetic and the strong interaction seem to respect each of these to a very great degree (for a discussion of possible discrete symmetry violations in gravity, see section 5.2), it was established quite early on that the weak interactions violated both C and P conservation almost maximally. However even they seemed to respect CP and consequently T symmetry. In fact till date the only evidence of CP violation has been seen in the neutral kaon system and there too to a very small extent only. Thus any study of CP violation would demand as a prerequisite a thorough understanding of the K^0–$\overline{K^0}$ system. Also the heavier meson systems are exactly similar and most of the results obtained for the kaon system can easily be extended in a straightforward manner. As for the leptonic sector, CP violation is identically zero in the minimal standard model, but could arise if one were to include massive neutrinos (Chapter 4). Though the issues involved are somewhat different, most of the analysis here trivially follows through.

2.3.1 The K^0–$\overline{K^0}$ system:

The neutral K–mesons K^0 and $\overline{K^0}$ are characterised by definite strangeness values $S = 1$ and -1 respectively and hence are good basis states when one is talking about either the strong or the electromagnetic interactions. This is so because both these interactions do respect strangeness conservation and hence

$$\langle K^0 | H_s + H_{em} | \overline{K^0} \rangle = 0. \tag{2.3.1}$$

However weak interactions do not preserve strangeness and thus can mix K^0 and $\overline{K^0}$. This results in these particles not having well defined masses or weak decay rates. Rather there exist two independent linear combinations of these states namely K_L and K_S that do have precise masses and decay rates. These new states are characterized by the difference in their decay modes and hence their lifetimes. While K_S decays primarily through the 2π mode (a state with CP eigenvalue +1), K_L has many decay channels mostly going to final states with $CP = -1$ *e.g.* 3π or $\pi^{\pm}l^{\mp}\overset{(-)}{\nu}$. Obviously the two new states do not have well defined strangeness.

Working with a choice of basis such that

$$CP|K^0\rangle = -|\overline{K^0}\rangle \quad \text{and} \quad CP|\overline{K^0}\rangle = -|K^0\rangle, \tag{2.3.2}$$

if we define two new states as

$$|K^0_{1,2}\rangle \equiv \frac{1}{\sqrt{2}}\left[|K^0\rangle \pm |\overline{K^0}\rangle\right], \tag{2.3.3}$$

then obviously

$$CP|K^0_1\rangle = -|K^0_1\rangle \quad \text{and} \quad CP|K^0_2\rangle = |K^0_2\rangle. \tag{2.3.4}$$

So if CP were an exact symmetry, this would imply that

$$|K_L\rangle = |K^0_1\rangle \quad \text{and} \quad |K_S\rangle = |K^0_2\rangle. \tag{2.3.5}$$

However in 1964, Cronin *et al.* [20] observed that K_L does decay into the $\pi^+\pi^-$ channel (*i.e.* a $CP = +1$ state) with a branching ratio of 2×10^{-3}. Hence the identification in eqn.(2.3.5) is wrong and we should rather have

$$|K_{L,S}\rangle = N_{L,S}\left[|K^0\rangle \pm e^{i\xi_{L,S}}|\overline{K^0}\rangle\right], \tag{2.3.6}$$

where $\xi_{L,S}$ are complex numbers and $N_{L,S}$ the wavefunction normalizations. Since K^0 and $\overline{K^0}$ both mix and decay, their time evolution is governed by an effective hamiltonian $H = H_s + H_{em} + H_{wk}$ such that

$$i\frac{\mathrm{d}}{\mathrm{dt}}|\psi\rangle = H|\psi\rangle, \tag{2.3.7}$$

where

$$\begin{aligned} |\psi\rangle &= \begin{pmatrix} |K^0\rangle \\ |\overline{K^0}\rangle \end{pmatrix} \\ \text{and } H &= M - \tfrac{i}{2}\Gamma, \end{aligned} \tag{2.3.8}$$

with M and Γ being 2×2 hermitian matrices called the mass and decay matrices respectively. Now CPT invariance demands that

$$\begin{aligned} H_{11} &\equiv \langle K^0|H|K^0\rangle = \langle K^0|(CPT)^{-1}H(CPT)|K^0\rangle \\ &= \langle\overline{K^0}|H|\overline{K^0}\rangle \equiv H_{22} \end{aligned} \tag{2.3.9}$$

or $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. On the other hand CP conservation would require

$$\begin{aligned} H_{12} &\equiv \langle K^0|H|\overline{K^0}\rangle = \langle K^0|(CP)^{-1}H(CP)|\overline{K^0}\rangle \\ &= \langle\overline{K^0}|H|K^0\rangle \equiv H_{21} \end{aligned} \tag{2.3.10}$$

and hence $M_{12} = M_{21}$ and $\Gamma_{12} = \Gamma_{21}$. Now, the eigenvalues of H are

$$E_{L,S} \equiv m_{L,S} - \frac{i}{2}\gamma_{L,S} = \frac{1}{2}\left[H_{11} + H_{22} \pm \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}\right] \tag{2.3.11}$$

and the difference is given by

$$E_L - E_S \equiv \Delta m - \frac{i}{2}\Delta\gamma = \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}. \tag{2.3.12}$$

If $K_{L,S}$ are to be the eigenvectors then we must have

$$e^{i\xi_L} = \frac{E_L - H_{11}}{H_{12}} \quad \text{and} \quad e^{i\xi_S} = \frac{H_{21}}{E_S - H_{22}}.$$

Invariance under CPT then demands that

$$\xi_L = \xi_S \equiv \xi = -\frac{i}{2}\ln\left(\frac{H_{21}}{H_{12}}\right),$$

and CP invariance would guarantee that

$$e^{i\xi} = 1. \tag{2.3.13}$$

However the last relation is phase convention dependent as can be seen by redefining the meson wavefunctions by

$$\begin{aligned} |K^0\rangle &\to |K^0\rangle' = e^{i\alpha}|K^0\rangle, \\ |\overline{K^0}\rangle &\to |\overline{K^0}\rangle' = e^{-i\alpha}|\overline{K^0}\rangle. \end{aligned} \tag{2.3.14}$$

Under this change of basis the diagonal matrix elements of any operator $\mathcal{O}$ remain invariant whereas the off-diagonal elements pick up phases

$$\mathcal{O}_{12} \to \mathcal{O}'_{12} = e^{-2i\alpha}\mathcal{O}_{12} \quad \text{and} \quad \mathcal{O}_{21} \to \mathcal{O}'_{21} = e^{2i\alpha}\mathcal{O}_{21}$$

and hence

$$\xi \to \xi' = \xi + 2\alpha. \tag{2.3.15}$$

Thus the basis invariant condition for CP conservation is that ξ be real [21]. An often used measure of CP violation is given by

$$\epsilon \equiv \frac{1 - e^{i\xi}}{1 + e^{i\xi}} \tag{2.3.16}$$

but this clearly is a phase convention dependent quantity.

We turn next to a discussion of the two pion decays of the neutral K–mesons. Bose statistics demands that the 2π state be in either total isospin $I = 0$ or $I = 2$ state. Hence defining the amplitudes

$$\langle n|H_{wk}| \overset{(-)}{K^0} \rangle = \overset{(-)}{a}_n \, e^{i\delta_n}, \qquad n = 0, 2 \tag{2.3.17}$$

where $|n\rangle \equiv |2\pi; I = n\rangle$ and δ_n is the 2π s–wave phase shift for the $I = n$ state, we have

$$\begin{array}{rll} & CPT \text{ invariance} & \Longrightarrow \quad \overline{a}_n = -a_n^* \\ \text{and} & CP \text{ invariance} & \Longrightarrow \quad a_2/a_0 \ \text{ is real.} \end{array} \tag{2.3.18}$$

Under the phase rotation as described by equation (2.3.14),

$$a_n \longrightarrow a_n' = a_n e^{i\alpha} \tag{2.3.19}$$

and hence the following combinations are phase choice independent

$$\begin{aligned} \epsilon_0 &\equiv \frac{\langle 0|H_{wk}|K_L\rangle}{\langle 0|H_{wk}|K_S\rangle} &&= \frac{a_0 - a_0^* e^{i\xi}}{a_0 + a_0^* e^{i\xi}} \\ \epsilon_2 &\equiv \frac{1}{\sqrt{2}}\frac{\langle 2|H_{wk}|K_L\rangle}{\langle 0|H_{wk}|K_S\rangle} &&= \frac{1}{\sqrt{2}}\frac{a_2 - a_2^* e^{i\xi}}{a_0 + a_0^* e^{i\xi}} e^{i(\delta_2 - \delta_0)} \\ \omega &\equiv \frac{\langle 2|H_{wk}|K_S\rangle}{\langle 0|H_{wk}|K_S\rangle} &&= \frac{a_2 - a_2^* e^{i\xi}}{a_0 + a_0^* e^{i\xi}}. \end{aligned} \tag{2.3.20}$$

The experimentally measurable quantities are

$$\begin{aligned} \eta^{+-} &\equiv \frac{\langle \pi^+\pi^-|H_{wk}|K_L\rangle}{\langle \pi^+\pi^-|H_{wk}|K_S\rangle} = \frac{\epsilon_0 + \epsilon_2}{1 + \omega/\sqrt{2}} = \epsilon_0 + \frac{\epsilon'}{1 + \omega/\sqrt{2}} \\ \text{and} \quad \eta^{00} &\equiv \frac{\langle \pi^0\pi^0|H_{wk}|K_L\rangle}{\langle \pi^0\pi^0|H_{wk}|K_S\rangle} = \frac{\epsilon_0 - 2\epsilon_2}{1 - \sqrt{2}\omega} = \epsilon_0 - \frac{2\epsilon'}{1 - \sqrt{2}\omega} \end{aligned} \tag{2.3.21}$$

where

$$\epsilon' \equiv \epsilon_2 - \frac{\omega\epsilon_0}{\sqrt{2}}. \tag{2.3.22}$$

In terms of the matrix elements of M and Γ then

$$\epsilon_0 = i\frac{Im(M_{12}a_0^2) - iIm(\Gamma_{12}a_0^2)}{Re(a_0^2 M_{12}) - \frac{i}{2}Re(a_0^2\Gamma_{12}) + \frac{|a_0|^2}{2}(\Delta m - \frac{i}{2}\Delta\gamma)} \tag{2.3.23}$$

$$\epsilon' = \frac{i}{\sqrt{2}}\frac{Im(a_2 a_0^*)(\Delta m - \frac{i}{2}\Delta\gamma)e^{i(\delta_2 - \delta_0)}}{Re(a_0^2 M_{12}) - \frac{i}{2}Re(a_0^2\Gamma_{12}) + \frac{|a_0|^2}{2}(\Delta m - \frac{i}{2}\Delta\gamma)} \tag{2.3.24}$$

All analysis till now has been the most general possible. No particular reference to the kaons have been made and all the results would hold equally well for any other neutral meson system. At this stage we would like to specialise to the K^0–$\overline{K^0}$ system and use some experimental results to obtain some approximate but easy to handle relations.

Now experimentlly we have [13]

$$\begin{aligned} m_K &= 0.498\ GeV, & \Delta m_K = 3.5 \times 10^{-15}\ GeV, \\ \Delta\gamma_K &\approx -\gamma_{K_S} = -7.3 \times 10^{-15} GeV. & \end{aligned} \quad (2.3.25)$$

The $\Delta I = 1/2$ rule for K–decays manifests itself in the form of a small suppression factor [22]

$$\omega \approx 0.045. \quad (2.3.26)$$

The dominant contribution to Γ_{12} comes from the 2π intermediate states and more specifically the $I = 0$ state. Thus

$$\Gamma_{12} \sim \langle K^0 | H_{wk}^{\Delta S=1} | 0 \rangle \langle 0 | H_{wk}^{\Delta S=1} | \overline{K^0} \rangle \quad (2.3.27)$$

and hence

$$\frac{Im\Gamma_{12}}{Re\Gamma_{12}} \simeq \frac{Im(a_0^*)^2}{Re(a_0^*)^2}. \quad (2.3.28)$$

Using the experimental values of η^{+-} and η^{00}, alongwith (2.3.26) we then get [13]

$$|\epsilon_0| = 2.3 \times 10^{-3}, \quad (2.3.29)$$

and the phase of ϵ_0 is nearly $\pi/4$. Such a small value of the CP violating effect can be best understood as resulting from

$$Im\Gamma_{12} \ll Re\Gamma_{12} \quad \text{and} \quad ImM_{12} \ll ReM_{12}.$$

Under this approximation eqn.(2.3.12) reduces to

$$\Delta m_K \approx 2ReM_{12} \quad \text{and} \quad \Delta\gamma \approx 2Re\Gamma_{12}. \quad (2.3.30)$$

In the SM, $K^0 \to 2\pi$ decays proceed through the "box diagram" (see section 2.3.2) and with a certain phase choice known as the 'quark phase convention', one can rotate away the phase of a_2 to have

$$a_0 = |a_0| e^{i\theta_0} \quad \text{and} \quad a_2 = \pm |a_2|. \quad (2.3.31)$$

Then using (2.3.25 – 2.3.31) in (2.3.23, 2.3.24) we get

$$\epsilon_K \equiv \epsilon_0 \approx \frac{e^{i\pi/4}}{\sqrt{2}}\left[\frac{ImM_{12}}{\Delta m_K} + \tan\theta_0\right] \tag{2.3.32}$$

$$\epsilon_K' \equiv \epsilon' \approx \mp\frac{1}{\sqrt{2}}\frac{|a_2|}{|a_0|}\sin\theta_0 e^{i(\delta_2-\delta_0+\pi/2)}. \tag{2.3.33}$$

To determine the parameter ϵ_K' one needs to measure η^{+-} and η^{00} to a great degree of accuracy, a task of considerable difficulty. However recently such measurements have been made to yield [23][4]

$$|\epsilon_K'/\epsilon_K| = (3.3 \pm 1.1) \times 10^{-3}. \tag{2.3.34}$$

2.3.2 Sources of CP Violation

The main thrust of the current chapter and the next is to establish a link between the CP violation in the K^0–$\overline{K^0}$ system and the quark mass matrices. But before jumping onto any conclusion, we would rather like to have a quick look at the various possible sources and only then point out the essential simplicity of the CKM picture.

CP violation in a theory satisfying the Lüders–Pauli criteria [19] can be categorised as those

a) violating each of C, P and T;

b) violating P and T but conserving C;

c) violating C and T but conserving P.

As parity violating effects in strong and electromagnetic interactions have been experimentally constrained to less than $O(10^{-5})$ [25,26], such theories obviously cannot explain ϵ_K. Thus if CP violation were to come from these sectors, then they must be of category (c). On the other hand, CP violating effects in the weak interactions are most likely to be of type (a), though H_{wk} might as well have small admixtures of categories (b) and (c). Keeping such considerations in mind, the candidate theories can be classified into four types. Of these, the millistrong and the electromagnetic models require an adequately small part of the corresponding hadronic interaction to be of type (c). The CP violation in $K \to 2\pi$

[4] It must be remembered though that a later experiment [24] gives a value $(-0.5 \pm 1.4) \times 10^{-3}$ *i.e.* consistent with zero.

Figure 2.1: "Box"–diagram generating K^0–$\overline{K^0}$ mixing and ϵ_K in the SM

(which is supposed to occur through an intermediate state with one of the decays being driven by the CP conserving H_{wk}) then arises as a result of an interference of amplitudes. However, experimentally such models are not favoured [26].

Milliweak models require a part ($\sim O(10^{-3})$) of H_{wk} be CP violating, resulting in single–shot $K_L \to 2\pi$, and hence similar effects should be observable elsewhere, say in the B–decays. On the other hand superweak models predict a CP violating $\Delta S = 2$ piece in H_{wk} with $K_L \to 2\pi$ occuring through an intermediate K_S state. In such a case CP violation occurs only in the K^0–$\overline{K^0}$ system. Consistency with the observed value of Δm_K, which arises now as a first order effect requires $g_{sw} \sim 10^{-8}$ and hence the name. The distinguishing feature of this model is that ϵ'_K is identically zero.

In the 3–generation SM, which, for a complex CKM matrix, is a milliweak theory, K^0–$\overline{K^0}$ mixing and $K_L \to 2\pi$ come about because of the 1–loop Feynman diagrams in Figures (2.1) and (2.2) respectively, giving rise to

$$ImM_{12} = \frac{G_F^2}{12\pi^2} f_K^2 m_K m_W^2 B_K \left[\lambda_c^2 \eta_1 S(y_c) + \lambda_t^2 \eta_2 S(y_t) + \lambda_c \lambda_t \eta_3 S(y_c, y_t)\right], \qquad (2.3.35)$$

and

$$\tan\theta_0 = \frac{s_{13}s_{23}}{s_{12}} \sin\delta \left[\frac{150MeV}{m_s(1\ GeV)}\right]^2 \bar{H}, \qquad (2.3.36)$$

where

$$\begin{array}{lclclcl} \lambda_i & \equiv & K_{id}^* K_{is} & & y_i & \equiv & m_i^2/m_W^2 \\ f_K & = & 0.16\ GeV & & m_W & = & 81.8\ GeV. \end{array} \qquad (2.3.37)$$

Figure 2.2: "Penguin"–diagram responsible for ϵ'_K in the SM

Whereas f_K is the pion decay constant, the bag parameter B_K reflects our ignorance of the hadronic matrix elements. If vacuum saturation approximation were correct then one would have $B_K = 1$, but theoretical estimates only put the rather loose bound of $1/3 \leq B_K \leq 1$. The functions $S(x)$ and $S(x, y)$ arise from the loop integral and are given by

$$\begin{aligned} S(x) &= x\left[\tfrac{1}{4} + \tfrac{9}{4(1-x)} - \tfrac{3}{2(1-x)^2}\right] + \tfrac{3}{2}\left[\tfrac{x}{x-1}\right]^3 \ln x \\ S(x,y) &= xy\left[\left\{\tfrac{1}{4} + \tfrac{3}{4(1-x)} - \tfrac{3}{4(1-x)^2}\right\}\tfrac{\ln x}{x-y} - \tfrac{3}{8}\tfrac{1}{1-x}\tfrac{1}{1-y}\right] + (x \leftrightarrow y) \end{aligned}$$

The quantities η_i represent QCD corrections [27]. While η_1 does not depend on m_t and is evaluated to be 0.85, η_2 is essentially independent of m_t for $40\ GeV \lesssim m_t^{\rm phys} \lesssim 130\ GeV$ and $\eta_2 = 0.61$. η_3 and $\bar{H}$ are slowly varying functions of m_t and are approximately 0.25 and 0.37 respectively [28]. However we shall allow for their full variation in our calculations.

2.3.3 The B_d^0–$\overline{B_d^0}$ system

The analysis for the B_d^0–$\overline{B_d^0}$ system proceeds exactly as for the K^0–$\overline{K^0}$ system. But unlike the latter, no trace of CP violation has yet been found here. Instead, we shall concentrate solely on the issue of particle–antiparticle mixing. Defining the time–integrated mixing parameters

$$\begin{aligned} r_d &\equiv \frac{\int_0^\infty |\langle \overline{B_d^0} | B_d^0 \rangle|^2 \mathrm{d}t}{\int_0^\infty |\langle B_d^0 | B_d^0 \rangle|^2 \mathrm{d}t} \\ &= \left| e^{i\xi_B} \right|^2 \frac{(\Delta m_B)^2 + (\Delta\Gamma_B)^2/4}{2\Gamma_B^2 + (\Delta m_B)^2 + (\Delta\Gamma_B)^2/4} \end{aligned} \qquad (2.3.38)$$

(where in the second line we have dropped the subscript d) and similarly for $\bar{r}_d$, we see that CP conservation demands that $\bar{r}_d = r_d$. Experimentally however one cannot directly measure either r_d or $\bar{r}_d$ as generally the mesons are created as pairs. Rather one looks at the dilepton decay modes and defines parameters R_d and A_d which measure mixing and decay asymmetry (and thus CP violation) respectively:

$$R_d = \frac{N^{++} + N^{--}}{N^{+-} + N^{-+}} \qquad A_d = \frac{N^{++} - N^{--}}{N^{++} + N^{+-} + N^{-+} + N^{--}}, \tag{2.3.39}$$

where N's denote the number of dilepton pairs with the associated charges. For the $e^+e^- \to \Upsilon(4S) \to B_d^0\overline{B_d^0}$ process these relations reduce to

$$R_d = \frac{1}{2}(r_d + \bar{r}_d), \qquad A_d = \frac{r_d - \bar{r}_d}{2 + r_d + \bar{r}_d}.$$

For the 3–generation SM with a relatively heavy top, the dominant contribution to r_d comes from the corresponding box–diagram with the top flowing in it. With this simplifying assumption we have

$$x_d \equiv \frac{\Delta m_d}{\Gamma_d} = \frac{2G_F^2}{3\pi^2}\tau_B \eta B_B f_B^2 m_B m_W^2 S(y_t)|K_{td}^* K_{tb}|^2 \tag{2.3.40}$$

where τ_B is the B_d^0 lifetime, f_B the decay constant, B_B the bag parameter and η a QCD correction factor. Experimentally we have [11]

$$r_d = 0.21 \pm 0.08 \qquad \Longrightarrow \qquad x_d = 0.73 \pm 0.18 \tag{2.3.41}$$

and

$$\begin{array}{rclrcl} m_B & = & 5.28\, GeV & \tau_B & = & (1.16 \pm 0.16) \times 10^{-12} s \\ \eta & = & 0.85 & 0.1\, GeV \;\le & f_B\sqrt{B_B} & \le \; 0.2\, GeV. \end{array} \tag{2.3.42}$$

Armed with the resources of this chapter, we can now attack the problem of quark mass matrices and the various ansätze for them. The three experimental inputs discussed here *viz.* ϵ_K, ϵ'_K, and x_d shall be used in the next chapter to check for the phenomenological validity of various models for quark masses.

Chapter 3

Quark Masses And Mixings

As we have seen in the last chapter, the standard model does not provide one with any guideline as to what the fermion masses and the mixings should be, the only criterion for determining these being experimental consistency. But this situation is aesthetically not a very pleasing one and there have been many efforts to formulate models that remove the arbitrariness to some degree. The methodology is quite simple. One imposes certain symmetries on the quark mass matrices to relate at least some of the ten parameters in this sector. Mainly motivated by phenomenological considerations, some of these models can no doubt be looked upon as having arisen from theories with higher gauge symmetries.

In this chapter we start with a brief discussion of some of these models and the motivation behind each. Once the predictions due to each are identified, the next logical step is obviously to check their validity in the light of the current experimental results. Finally we end with a model independent study of the three generation quark mass matrices and identify the various models as special cases.

3.1 Models for Quark Masses and Mixings

3.1.1 Stech Model:

This model [29] was motivated by grand unified theories where the gauge group has a $SU(5)$ subgroup and all the fermions of a generation are contained in an irreducible representation. The fermion masses arise from non-zero vacuum expectation value of Higgs fields transforming under different representations. The assumption was that the mass contributions due to the symmetric Higgs representations dominate and that the antisymmetric representations do not contribute to the up-sector. Such a scenario was to be ensured by suitable discrete symmetries and a proper choice of the higgs couplings. A further choice of hermiticity of the mass matrices restrict their form to

$$M_{u(S)} = \widehat{M_u} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \tag{3.1.1}$$

and

$$M_{d(S)} = \alpha \widehat{M_u} + A, \tag{3.1.2}$$

where α is a constant and A is an antisymmetric matrix.

$M_{d(S)}$ can be brought into a diagonal form by an orthogonal transformation. In this basis A is still hermitian and antisymmetric. It should however be noticed that choosing a particular basis for the CKM matrix would necessitate a unitary transformation by a phase matrix. While this would leave M invariant, A would lose its antisymmetry and would be a hermitian matrix with all diagonal elements zero. In fact the basis independent statement is that $det(A) = 0$.

As shall be shown in section 3.3.4 , this model is characterised by seven parameters and hence we expect three relations between the quark sector parameters. These can be read off from the matrix equation

$$K^{\dagger} M_{d(S)} K = \widehat{M_d} = diag(m_d, m_s, m_b) \tag{3.1.3}$$

The analysis is exactly similar to that employed for the model independent case [section 3.3] and shall not be presented here. The relations one is looking for are

$$\begin{aligned} s_{12}^2 &\approx \left(\frac{m_d}{m_s} - \frac{m_u}{m_c}\right)\left(1 - \frac{m_u m_d}{m_s m_c}\right)^{-1}, \\ s_{23}^2 &\approx \left(\frac{m_s}{m_b} - \frac{m_c}{m_t}\right)\left(1 - \frac{m_s m_c}{m_b m_t}\right)^{-1}, \\ q\cos\delta &\approx -\frac{m_s}{m_b} s_{12}. \end{aligned} \tag{3.1.4}$$

There is indeed a fourth relation claimed by Stech:

$$s_{13}^2 = \frac{m_u}{m_c} s_{23}^2, \tag{3.1.5}$$

but in section 3.3.2 it shall be shown to be not a consequence of the model but to have arisen from a flaw in the analysis.

Though the ansatz looks simple enough, it is very difficult to ensure such a form in viable models. The first such scheme was presented in the context of left–right models [30], but in these the tree level derivations of the Stech model were somewhat spoiled by infinite corrections at higher loops. An alternative model [31] based on a supersymmetric $SO(10)$ theory with softly broken supersymmetry is probably the best candidate available in the literature. The symmetric parts of the mass matrices are unaltered at the one–loop level and the antisymmetric part for the down quarks arises as a one–loop correction and is hence smaller than the tree level terms. The relevant diagrams involve charged color triplet scalars and because of the choice for their quantum numbers give a net antisymmetric contribution. The cornerstone of the model is the proportionality of such corrections with the Majorana mass of the ν_R (for a definition of Majorana neutrinos, see Chapter 4) and hence there are no corresponding diagrams for the up–sector.

3.1.2 The Fritzsch Model:

The Fritzsch model [32] envisages a scenario where, to begin with, only the heaviest quarks in either sector are massive and all others gain mass successively through charged current mixings with the next higher generation. This model was first obtained [32] for a field theory with $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ as the electroweak gauge group and two Higgs fields, on imposition of a certain discrete symmetry. The simplest such construction is given by a scenario where one considers only the matter fields q_{iL} $(2, 1, \frac{1}{6})$, q_{iR} $(1, 2, \frac{1}{6})$ (i being the generation index) and two Higgs fields $\phi_{1,2}(2, 2, 0)$. The number in the parentheses here represent the transformation properties of the field under the gauge group. The most general Yukawa term then reads

$$\mathcal{L}_{Yuk} = \overline{q_{iL}} q_{jR} \left(h_{ij}\phi_1 + g_{ij}\phi_2 \right) + H.c. \tag{3.1.6}$$

Parity invariance obviously requires that the matrices h_{ij} and g_{ij} be hermitian. If we further demand that the Lagrangian should respect the discrete symmetry

$$\begin{array}{lllllll} q_{1L} & \longrightarrow & -iq_{1L} & \quad & q_{1R} & \longrightarrow & iq_{1R} \\ q_{2L} & \longrightarrow & iq_{2L} & & q_{2R} & \longrightarrow & -iq_{2R} \\ q_{3L} & \longrightarrow & q_{3L} & & q_{3R} & \longrightarrow & q_{3R} \\ \phi_1 & \longrightarrow & \phi_1 & & \phi_2 & \longrightarrow & i\phi_2, \end{array} \tag{3.1.7}$$

the only non–zero Yukawa couplings would be

$$h_{21} = h_{12}^*, \quad g_{33}, \quad h_{31} = h_{13}^* \quad \text{and} \quad h_{32} = h_{23}^*,$$

and one obtains the desired form for the mass matrices.

In a basis where the up–quark mass matrix is real, one then has

$$M_{u(F)} = \begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & b_u \\ 0 & b_u & c_u \end{pmatrix} \tag{3.1.8}$$

and

$$M_{d(F)} = \begin{pmatrix} 0 & a_d e^{i\varphi_1} & 0 \\ a_d e^{-i\varphi_1} & 0 & b_d e^{i\varphi_2} \\ 0 & b_d e^{-i\varphi_2} & c_d \end{pmatrix}. \tag{3.1.9}$$

The quark sector is thus characterized by eight parameters and hence two relations between the masses and the CKM matrix parameters are predicted. The form of the mass matrices also imply that the middle (in magnitude) eigenvalue of both $M_{u(F)}$ and $M_{d(F)}$ would have a sign opposite to the other two.

$M_{d(F)}$ can be brought into real form by performing a phase rotation on both the left– and the right–handed down quark fields

$$M_{d(F)} = P^\dagger M'_{d(F)} P, \tag{3.1.10}$$

where

$$P = diag\left(1, e^{i\varphi_1}, e^{i(\varphi_1+\varphi_2)}\right) \tag{3.1.11}$$

and

$$M'_{d(F)} = \begin{pmatrix} 0 & a_d & 0 \\ a_d & 0 & b_d \\ 0 & b_d & c_d \end{pmatrix}. \tag{3.1.12}$$

$M_{u(F)}$ and $M'_{d(F)}$ being real symmetric matrices, can now be diagonalized by orthogonal transformations. For example

$$O_u^T M_{u(F)} O_u = \widehat{M_u} \equiv diag\,(m_u, m_c, m_t) \tag{3.1.13}$$

with

$$O_u = \begin{pmatrix} \frac{1}{N_1} & \frac{1}{N_2} & \frac{1}{N_3} \\ \frac{m_u}{N_1 a_u} & \frac{m_c}{N_2 a_u} & \frac{m_t}{N_3 a_u} \\ \frac{-m_u b_u}{N_1 a_u (m_c + m_t)} & \frac{-m_c b_u}{N_2 a_u (m_u + m_t)} & \frac{-m_t b_u}{N_3 a_u (m_u + m_c)} \end{pmatrix}. \tag{3.1.14}$$

The eigenvalues m_i can be obtained by inverting the relations

$$\begin{aligned} a_u &= (-m_u m_c m_t / c_u)^{1/2} \\ b_u &= [-(m_u + m_c)(m_u + m_t)(m_c + m_t)/c_u]^{1/2} \\ c_u &= (m_u + m_c + m_t)\,, \end{aligned} \tag{3.1.15}$$

and N_i are the normalizations for the eigenvectors of $M_{u(F)}$:

$$\begin{aligned} N_1^2 &= \frac{m_c - m_u}{m_c} \left[1 + \frac{(m_c + m_u) m_u}{(m_t + m_c) m_c} \right] \\ N_2^2 &= \frac{m_u - m_c}{m_u} \left[1 + \frac{(m_u - m_c) m_c}{(m_t + m_u) m_t} \right] \\ N_3^2 &= \frac{m_t^3}{m_u m_c (m_c + m_u)} \left[1 - \frac{m_c^2 + m_c m_u + m_u^2}{m_t^2} + \frac{m_u m_c (m_c + m_u)}{m_t^3} \right]. \end{aligned} \tag{3.1.16}$$

The weak mixing matrix being given by

$$K = O_u^T P^\dagger O_d, \tag{3.1.17}$$

we have

$$\begin{aligned} s_{12} &\approx \left| \sqrt{-\frac{m_d}{m_s}} - e^{-i\varphi_1} \sqrt{-\frac{m_u}{m_c}} \right|, \\ s_{23} &\approx \left| \sqrt{-\frac{m_s}{m_b}} - e^{-i\varphi_2} \sqrt{-\frac{m_c}{m_t}} \right|, \\ s_{13} &\approx \left| \frac{m_s}{m_b} \sqrt{\frac{m_d}{m_b}} - e^{-i\varphi_1} \left(\sqrt{-\frac{m_s}{m_b}} - e^{-i\varphi_2} \sqrt{-\frac{m_c}{m_t}} \right) \right|, \\ \frac{\sin\delta}{\frac{s_{12} s_{23}}{s_{13}} - \cos\delta} &\approx \frac{\sin\varphi_1}{\cos\varphi_1 - \sqrt{\frac{m_d m_c}{m_s m_u}}}. \end{aligned} \tag{3.1.18}$$

The first two of these equations can be used to determine the phases φ_1 and φ_2 and then the last two represent the predictions of the model.

3.1.3 Fritzsch-Shin Model:

The two phases $\varphi_{1,2}$ in the Fritzsch mass matrix are not determined by the imposed discrete symmetry and hence are quite arbitrary. If these could be fixed by some means, the arbitrariness could be reduced somewhat and further relations between the parameters would be predicted. To this end, Shin [33] made a choice for the two phases namely $\varphi_1 = 90°$ and $\varphi_2 = 0$, the hope being that this could be achieved on imposition of further discrete symmetries. Thus the model is now characterised by six parameters and this results in two further constraints on the system over and above those obtained for the general Fritzsch case. For example, all of equations (3.1.18) are now predictions of the model.

3.1.4 Fritzsch-Stech model:

That the Stech and the Fritzsch ansätze are not inconsistent with each other is easy to see. Exploiting this freedom, Gronau, Johnson and Schechter [34] proposed a scenario in which both these sets of assumptions are incorporated. (However no realistic model to achieve this has been constructed.) In a suitable basis the mass matrices are thus given by

$$M_{u(FS)} = \begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & b_u \\ 0 & b_u & c_u \end{pmatrix} \tag{3.1.19}$$

and

$$M_{d(FS)} = \alpha M_{u(FS)} + \begin{pmatrix} 0 & ia & 0 \\ -ia & 0 & ib \\ 0 & -ib & 0 \end{pmatrix} . \tag{3.1.20}$$

Like the Fritzsch–Shin model, this also results in a six parameter family and the predictions over and above either the Stech or the Fritzsch scheme can easily be obtained. (For a detailed account of the same, see Section 3.3.4.)

3.2 Validity of Models

In the last section we had a brief overview of the more popular models to explain the quark masses and mixings. All these ansätze predict some relations between the otherwise free parameters in this sector and hence the most obvious check for the validity of such models would be the comparison of their respective predictions with experimental data. A detailed analysis was performed by Harari & Nir and by Nir [36] where they compared the relations with the restrictions imposed by the CP–violation parameter ϵ_K and the B_d^0–$\overline{B_d^0}$ mixing extent r_d to conclude that while the Stech ansatz was ruled out, the Fritzsch scheme barely survived. However it was pointed out in the literature [12,37] that these authors had put unnecessarily severe constraints on some ill–determined parameters involved in the calculations. In the light of this, we redid the exercise to obtain results quite different from that in ref.[36]. For example, the Stech ansatz was not yet ruled out and the Fritzsch model had much more freedom than claimed [35]. However the newer result on the $\Delta S = 1$ CP–violating parameter ϵ'_K proved to be a very useful constraint to check models by. In the rest of this section [1]we shall describe the method of comparison adopted and the results thereof.[2]

3.2.1 The Stech Model : Consistency Check

As we have seen earlier, the Stech ansatz results in three predictions *viz.* eqns.(3.1.4). The first of the three relations is obviously consistent with the experimental values, while the second, on scanning through the entire range allowed to the masses gives $m_t(1\ GeV) \leq 82.4\ GeV$ thus implying that we need to examine only the range $45\ GeV \leq m_t^{\rm phys} \leq 51.5\ GeV$. An examination of the overlap of ϵ_K and x_d bounds in the q–δ plane (where $q \equiv s_{13}/s_{23}$) for various choices of $B_B f_B^2$ and B_K indicates $q \sim 0.1$ thus requiring δ to be nearly 90° for the Stech scheme to be valid. This indicates a near 'maximal' CP violation in the neutral kaon system as expected from the choice for M_u and M_d. A thorough examination shows that the Stech ansatz agrees with the ϵ_K– and x_d–values only for $m_t^{\rm phys} \sim 51.5\ GeV$, $s_{23} \sim 0.07$, $B_K \sim 0.33$ and $B_B f_B^2 \sim 0.04$. This overlap was absent in the analyses in refs.[36] as they had limited s_{23} to be below 0.05. But even this tenuous agreement is destroyed by the ϵ'_K observation. Noting that $m_t(1\ GeV) \sim 82\ GeV$ and $s_{23} \sim 0.07$ implies $m_s(1\ GeV) \sim 120 MeV$, a substitution of all relevant variables in equation(2.3.33) predicts $|\epsilon'_K/\epsilon_K| \gtrsim 9 \times 10^{-3}$ which is considerably higher than the experimental upper limit of 4.4×10^{-3}.

[1] This section is based on the work in ref. [35]

[2] It must be noted that independent of the contents of this section, the recent improved bounds [17] on the top quark mass ($m_t \gtrsim 89\ GeV$) effectively rules out all the models under discussion

3.2.2 The Fritzsch Model: Consistency Check

As far as this model goes, equation (3.1.18) gives $m_t \leq m_c/(\sqrt{m_s/m_b} - s_{23})^2$ and using the entire range for the other parameters, one gets $m_t(1\ GeV) \leq 223\ GeV$ thus requiring us to look only at the interval $45\ GeV \leq m_t^{\rm phys} \leq 127\ GeV$. To check the validity of the model, we select a combination of m_t, m_d, s_{23}, B_K and $B_B f_B^2$ and look for any overlap in the q–δ plane of the x_d- and ϵ_K- bands and the region allowed by the model. Note that equation (3.1.18) gives rise to two bands (corresponding to the two different relative signs between φ_1 and φ_2, as yet undetermined) independent of δ, whose widths are determined by the error bars on m_i and s_{12} and which may, in some cases, coalesce into one. Furthermore these selections are to be checked for consistency with the ϵ'_K/ϵ_K results.

Our analysis shows that unlike in ref. [36] one does obtain a large number of solutions for this ansatz. Though most of the solutions obtained are for $s_{23} \sim 0.07$, a significant number do exist for $s_{23} \sim 0.059$, an upper bound many authors have quoted. More important the extremal conditions required in the earlier analyses [36] are released. We divide the solution into three broad categories

Large m_t:

For $95\ GeV \leq m_t^{\rm phys} \leq 127\ GeV$, the requirement of $B_B f_B^2$ and x_d being respectively at the top and the bottom of their individual given ranges is relaxed with their ratio being allowed to take central values. But $B_K \leq 0.6$ and $s_{23} \geq 0.06$ are slowly pushed to their respective minimum and maximum as m_t increases. Also m_u, m_d and m_s need to assume almost the lowest values allowed. Small $q(\sim 0.035$–$0.06)$ is favoured while δ is allowed over a considerable range ($40°$–$120°$) with progressively higher values for lower m_t.

But this solution is in contradiction with $q \geq 0.07$, a limit imposed by observed levels of charmless b-decay. So for this range the Fritzsch scheme is effectively ruled out.

Low m_t:

For 70 $GeV \geq m_t^{\rm phys} \geq 45$ GeV, on the other hand $x_d/B_B f_B^2$ needs to be constrained near the lowest value. While B_K, m_d and consequently m_u and m_s have the freedom to assume values close to or slightly below the centre of the range, s_{23} again needs to be larger and larger as one progresses to lower m_t's. q takes on a typically a larger value (0.08 – 0.1) than allowed for large m_t and δ is constrained between 110° and 130°.

Middle m_t:

In this range (70 $GeV \leq m_t^{\rm phys} \leq 95$ GeV) the results are similar to those in ref. [36] and though many more solutions are obtained, we are not detailing them here.

It is to be noted that ϵ'_K results hardly constrain the Fritzsch model solution domains. One of the very few examples where this result did rule out this model is

$$\begin{array}{rclrcll} B_K & = & 0.85 & B_B f_B^2 & = & 0.02 & s_{23} = 0.06 \\ m_d(1\ GeV) & = & 6.3 MeV & m_t^{\rm phys} & = & 90\ GeV & \end{array}$$

The other contraints agreed for $\delta \sim 113°$–$123°$, and $q \sim 0.067$ which would have required $|\epsilon'_K/\epsilon_K| \leq 2.09 \times 10^{-3}$.

3.2.3 Fritzsch–Shin Model: Consistency Check

Since this ansatz is but a special case of the Fritzsch scheme, it stands to reason that the agreement would be narrower. Indeed our check shows that of the three zones we demarcate, the Shin choice is invalid in both the high m_t and the low m_t regions. Even for the middle m_t region, the agreement is very marginal as in ref.[36] and not much improved by relaxing the upper bound on s_{23}. It is however noted that $\varphi_2 = 0$ alone has much better agreement than the Shin ansatz.

3.2.4 Fritzsch–Stech model: Gonsistency Check

That the experimental agreement of such models would be narrower than that of either the Fritzsch or the Stech model is obvious on account of its incorporating the assumptions of both the latter ones. Of particular relevance is the Stech lineage. Hence even without bothering to examine we could safely conclude that this ansatz is phenomenologically inconsistent.

3.2.5 Conclusions

Our analysis has shown that if one takes into consideration the entire range allowed [12] experimentally to K_{cb}, a much wider range of solutions is allowed to the Fritzsch ansatz predictions than claimed hitherto. The necessity of a heap of theoretically and experimentally ill-determined parameters assuming extreme values allowed is removed. But the model fails to take advantage of one concession that a higher value of s_{23} gives it, *i.e.* agreeing for high m_t. As we have seen earlier, $m_t^{\rm phys} > 95$ GeV is ruled out as it entails a value of q smaller than the experimentally allowed minimum. If the limit $m_t^{\rm phys} \leq 55$ GeV [37] is taken seriously, the Fritzsch model would still be in the running. But in that case the Shin modification is totally ruled out. On the other hand, the Stech scheme which was allowed a marginal agreement with the earlier data for $s_{23} \sim 0.07$, is totally ruled out by the $|\epsilon'_K/\epsilon_K|$ results. As a corollary, other models incorporating the Stech ansatz like the Fritzsch–Stech model of Gronau, Johnson and Schechter are automatically invalidated.

3.3 A Model Independent Analysis

The results of the last section demonstrate that none of the current models for quark mass matrices do the job efficiently. This naturally prompts a model independent study [3] of the problem in the hope that such an activity would help us in gaining some insight into the matter at hand and possibly indicate fertile but as yet untapped territory for future model building.

To begin with, we start with the most general mass matrices for three generations. The

[3] This section is based on the work in ref. [38]

arguments of section 2.1 show that by making a phase transformation on the right handed quark fields alone we can make these matrices into hermitian ones with all their eigenvalues to be positive. However such a choice for the basis would, in general, not be consistent with the particular form of the CKM matrix that we have chosen to work with. Hence we shall only demand hermiticity and allow the eigenvalues to take either sign. This need not cause any alarm as in the standard model the sign of the fermion mass has no significance and can be changed by a chiral transformation.

In the basis in which M_u is diagonal, we then have for the most general case

$$M_d = \alpha \widehat{M_u} + A \tag{3.3.1}$$

where

$$A = \begin{pmatrix} 0 & R_1 e^{i\rho_1} & R_2 e^{i\rho_2} \\ R_1 e^{-i\rho_1} & f & R_3 e^{i\rho_3} \\ R_2 e^{-i\rho_2} & R_3 e^{-i\rho_3} & d \end{pmatrix}. \tag{3.3.2}$$

Thus the mass matrices are a ten parameter family determined by the values of m_u, m_c, m_t, α, f, d, $R_{1,2,3}$ and the invariant phase $(\rho_1 + \rho_3 - \rho_2)$. That the other phase combinations are unphysical can easily be seen by making the most general phase redefinitions of the quark wavefunctions. This then leads to $M_d \longrightarrow P_1^\dagger M_d P_2$ where $P_{1,2}$ are some arbitrary phase matrices. While the magnitudes of the individual elements are invariant under this change of basis, their phases are not. The simplest nontrivial combinations resisting change are given by [39] the "cycles" $arg\left[(M_d)_{ij}(M_d^\dagger)_{jk}(M_d)_{kl}(M_d^\dagger)_{li}\right]$ (no summation) and this in the present case simplifies to the expression given earlier.

Though on the face of it this parametrization has no predictive power as we are using ten parameters to relate ten others, in our analysis we would not be using all of them and most of our conclusions would be drawn by considering only the diagonal elements.

On diagonalizing M_d we have

$$K\widehat{M_d}K^\dagger = M_d = \alpha M_u + A$$

where

$$\widehat{M_d} = diag(m_d, m_s, m_b).$$

The diagonal elements of the matrix equation give three relations, of which one is the trace

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condition

$$\alpha = \frac{m_d + m_s + m_b - f - d}{m_u + m_c + m_t} \tag{3.3.3}$$

and the others are

$$\begin{aligned} \alpha m_u &= m_d + c_{13}^2 s_{12}^2 (m_s - m_d) + s_{13}^2 (m_b - m_d) \\ \text{and} \quad \alpha m_c + f &= m_d + \left| c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} \right|^2 + c_{13}^2 s_{23}^2 (m_b - m_d). \end{aligned} \tag{3.3.4}$$

3.3.1 The two generation limit:

As a first approximation we assume that the third generation essentially decouples from the first two – a not too strong assumption as experimentally both s_{23} and s_{13} are small compared to s_{12}. In this limit (3.3.4) reduces to

$$\begin{aligned} \alpha m_u &= m_d + s_{12}^2 (m_s - m_d) \\ \alpha m_c + f &= m_d + c_{12}^2 (m_s - m_d). \end{aligned} \tag{3.3.5}$$

Eliminating α from above, we obtain

$$s_{12}^2 = \frac{\left(1 - \frac{f}{m_s}\right)\frac{m_u}{m_c} - \frac{m_d}{m_s}}{\left(1 + \frac{m_u}{m_c}\right)\left(1 - \frac{m_d}{m_s}\right)} \tag{3.3.6}$$

Using eqns.(2.1.18) and (2.2.12) in (3.3.6) gives an allowed range for f for a given m_d which has been plotted in Figure 3.1. It is seen that for $m_d/m_s < 0$, f assumes small values irrespective of the sign of m_u/m_c, and is consistent with zero. While for $m_d/m_s > 0$, f is comparatively larger and its sign is opposite to that of m_u/m_c.

3.3.2 Back to three generations:

Assuming the two generation limit for s_{12} and using it as an input in eqn. (3.3.4), we have (for $c_{13} \approx 1$),

$$s_{23}^2 = \frac{m_c}{m_t + m_c + m_u} \frac{(m_b - d)(m_c + m_u) + m_t(f - m_s - m_d)}{m_b(m_c + m_u) - (m_d m_c + m_s m_c + f m_u)} \tag{3.3.7}$$

i.e.

$$\begin{aligned} &\frac{d}{m_b}\left[s_{23}^2\left\{\frac{m_b}{m_d}\left(1 + \frac{m_u}{m_c}\right) - \left(1 + \frac{m_s}{m_d} + \frac{f m_u}{m_d m_c}\right)\right\} + \frac{m_s}{m_d}\left(1 - \frac{f}{m_s}\right) + 1\right] \\ &= \left(1 - \frac{m_t}{m_c}\right)\left(1 + \frac{m_u}{m_c}\right)\frac{m_b}{m_d} \\ &\quad - s_{23}^2\left(1 + \frac{m_u}{m_c}\right)\left\{\frac{m_b}{m_d}\left(1 + \frac{m_u}{m_c}\right) - \left(1 + \frac{m_s}{m_d} + \frac{f m_u}{m_d m_c}\right)\right\} \end{aligned}$$

Figure 3.1: The allowed region for f (shaded region) as a function of m_d (see eqn.3.3.6).

$$a) \ \frac{m_s}{m_d} > 0, \quad \frac{m_u}{m_c} > 0 \qquad b) \ \frac{m_s}{m_d} > 0, \quad \frac{m_u}{m_c} < 0$$

$$c) \ \frac{m_s}{m_d} < 0, \quad \frac{m_u}{m_c} > 0 \qquad d) \ \frac{m_s}{m_d} < 0, \quad \frac{m_u}{m_c} < 0$$

All values are calculated at $\mu = 1\ GeV$. m_d has been assumed to be positive. For $m_d < 0, \quad f \rightarrow -f$.

Thus for a given s_{23} we have a linear relation between d and m_t with the slope and intercept depending on the signs of the various mass ratios. In Stech model, for example, d was required to be zero, thus fixing m_t upto error bars due to the experimental uncertainties. A non-zero value of d would unfreeze this restriction and allow for better agreements with the experiments. The allowed regions for d for a fixed m_t has been given in Table 3.1. Similar to the case for f, d takes 'small' values about zero for $m_d/m_s < 0$ while for a positive value of this ratio it is considerably larger and a vanishing value is not consistent with the observations.

$Sign(\frac{m_b}{m_d}, \frac{m_b}{m_d}, m_c)$	Limits on $d(1\ GeV)$ (in GeVs)
$(+,+,+)$	$-3.43m_t + 5.19 < d < -3.27m_t + 5.39$
$(+,+,-)$	$-3.47m_t + 5.19 < d < -3.29m_t + 5.39$
$(+,-,+)$	$-0.15m_t + 5.19 < d < +0.19m_t + 5.19$
$(+,-,-)$	$-0.18m_t + 5.19 < d < +0.15m_t + 5.19$
$(-,+,+)$	$-3.46m_t + 5.21 < d < -3.35m_t + 5.21$
$(-,+,-)$	$-3.44m_t + 5.21 < d < -3.26m_t + 5.41$
$(-,-,+)$	$-0.18m_t + 5.12 < d < +0.16m_t + 5.21$
$(-,-,-)$	$-0.16m_t + 5.21 < d < +0.18m_t + 5.21$

Table 3.1: Limits on $d(1\ GeV)$ in terms of $m_t(1\ GeV)$ as imposed by eqn. (3.3.7). The limits are calculated for positive m_u, m_t, and m_d. For $m_d < 0$, $d \to -d$.

Taking the two generation result to be exact and substituting in eqn.(3.3.4) one obtains

$$\begin{aligned} s_{13}^2 &= \frac{m_u}{m_t + m_c + m_u} \frac{(m_c + m_u)(m_b - d) - m_t(m_s + m_d - f)}{(m_c + m_u)m_b - m_u(m_s + m_d - f)} \\ &\approx \frac{m_u}{m_c}\left(1 - \frac{m_s}{m_b}\right) s_{23}^2 \end{aligned} \quad (3.3.8)$$

This implies that $m_u/m_c > 0$. The analysis and the result are similar to Stech's (Section 3.1.1). An attempt to obtain a better approximation by an iterative procedure (*i.e.* substituting the current expressions for s_{23} and s_{13} in eqns. (3.3.5) and (3.3.6), instead of taking them to be zero and then redoing the same analysis) yields an extra term much smaller in magnitude.

But this result is in direct contradiction to the Fritzsch model (Section 3.1.2) wherein alternate generations have masses of opposite signs. Indeed, if equations (3.1.18) are squared, one obtains an equation similar to (3.3.8), but with an extra term typically larger than the right hand side.

The inconsistency lies in the analysis where one is aiming to solve for three angles from two equations. The relation (3.3.8) is thus shown not to be an outcome of the Stech ansatz but rather arising from an overkill of the equations (3.3.5) and (3.3.6). The best one can achieve without using the off-diagonal terms is an expression for s_{13} in terms of three unknowns m_t, f and d, the measured parameters s_{12} and the other five quark masses:

$$s_{13}^2 = \frac{\alpha m_u - \left[m_d + (m_s - m_d)s_{13}^2\right]}{m_b - \left[m_d + (m_s - m_d)s_{13}^2\right]}. \tag{3.3.9}$$

3.3.3 The off-diagonal terms:

Till now we have used only the diagonal terms of the matrix equation (3.3.1), ignoring the off diagonal terms, inclusion of which would give exact but contentless results. We continue in the same vein but would nevertheless like to look at these relations so as to get an idea of the relative magnitudes of these terms. We have

$$\begin{aligned}
R_1 e^{i\rho_1} &= c_{12}c_{23}c_{13}s_{12}(m_s - m_d) + c_{13}s_{13}s_{23}(m_b - c_{12}^2 c_{23} m_d - s_{12}^2 m_s)e^{-i\delta} \\
R_2 e^{i\rho_2} &= c_{23}c_{13}s_{13}(m_b - s_{12}^2 m_s - c_{12}^2 m_d) + c_{12}c_{13}s_{12}s_{23}(m_d - m_s)e^{i\delta} \\
R_3 e^{i\rho_3} &= c_{12}s_{12}s_{13}(m_d - m_s)\left(c_{23}^2 - s_{23}^2 e^{2i\delta}\right) \\
&\quad + c_{23}s_{23}e^{i\delta}\left[c_{13}^2 m_b + (c_{12}^2 s_{13}^2 - s_{12}^2)m_d - (s_{12}^2 s_{13}^2 - c_{12}^2)m_s\right]
\end{aligned} \tag{3.3.10}$$

The complex phases ρ_1 and ρ_2 are relatively small and lie in the same quadrant as can be seen from the fact that $\tan\rho_1 \tan\rho_2 \approx s_{23}^2$. While $\sin\rho_1$ attains its maximum of 0.12 when m_s and s_{12} assume the lowest allowed values and s_{13}, s_{23}, m_b the highest and $\delta \approx 83°$, $\sin\rho_2$ is maximized to 0.15 by giving $\left|\frac{K_{ub}}{K_{cb}}\right|$ and m_b their lowest values, m_s, s_{12} their highest and putting $\delta \approx 81.5°$. On the other hand $\rho_3 \approx \delta$. Hence this dominates the invariant phase $\rho_1 - \rho_2 + \rho_3$ and most of the CP–violating contribution comes from this term.

3.3.4 Models as special cases of the general form:

In this section we revert to a discussion of the models mentioned earlier. We demonstrate how these models could be obtained from the general mass–matrix on imposing suitable constraints. This would exhibit the restrictions one is pre–imposing on the various parameters and hopefully afford a better understanding of the implications of an ansatz.

Stech ansatz

A straightforward comparison of equations (3.1.2) and (3.3.2) gives the constraints to be

$$\begin{aligned} f &= 0 \\ d &= 0 \\ \rho_1 - \rho_2 + \rho_3 &= 90^\circ \end{aligned} \tag{3.3.11}$$

Using (3.3.11) in (3.3.10) one gets

$$\frac{s_{13}}{s_{23}} \cos\delta \approx 0 \tag{3.3.12}$$

This could be directly seen in the light of the discussion following equation (3.3.10). The Stech ansatz thus restricts the mass matrices to a seven parameter family which predicts near 'maximal' CP violation. One of the three promised predictions is then equation (3.3.12) or equivalently the last of of equations (3.1.4) and the others can be obtained from equations (3.3.6) and (3.3.7) by substituting $f = 0$ and $d = 0$ in them respectively.

The first two conditions obviously restrict the mass matrices to the negative m_s/m_d sector. Also a lower limit on the mass of the d-quark is set:

$$\begin{aligned} m_d(1GeV) &> 7MeV \quad \text{for } m_u/m_c > 0 \\ m_d(1GeV) &> 8.5MeV \quad \text{for } m_u/m_c < 0 \end{aligned} \tag{3.3.13}$$

There exist in the literature certain modifications of the Stech scheme as for example a non-zero d or an invariant phase $\rho_1 - \rho_2 + \rho_3$ different from 90°. Such models have reasonable agreement with experiments at the cost of loss of predictive power.

Fritzsch model

The simplest way to find the constraints to be put on the general form to obtain the Fritzsch form is to rotate M_d with O_u and compare the resultant with $M_{d(F)}$.

$$M_{d(F)} = O_u M_d O_u^T$$

gives the two required constraints:

$$\begin{aligned} 0 &= \frac{a m_u}{N_1^2} + \frac{a m_c + f}{N_2^2} + \frac{a m_t + d}{N_3^2} + \frac{2R_1 \cos\rho_1}{N_1 N_2} + \frac{2R_2 \cos\rho_2}{N_1 N_3} + \frac{2R_3 \cos\rho_3}{N_2 N_3} \\ 0 &= \frac{a m_u}{N_1^2} m_u^2 + \frac{a m_c + f}{N_2^2} m_c^2 + \frac{a m_t + d}{N_3^2} m_t^2 \\ &+ \frac{2R_1 \cos\rho_1}{N_1 N_2} m_u m_c + \frac{2R_2 \cos\rho_2}{N_1 N_3} m_u m_t + \frac{2R_3 \cos\rho_3}{N_2 N_3} m_c m_t \end{aligned} \qquad (3.3.14)$$

Using equations (3.1.16) and (3.3.10) in the above, any two of the ten parameters can be eliminated. For example if f and d are evaluated in terms of the masses and the CKM parameters, then substituting the expressions for them in (3.3.6) and (3.3.7) would give us, say s_{13} and m_t in terms of the others and these would be the predictions of the model.

Fritsch-Shin scheme

Shin's choice for the phases in the Fritzsch mass matrix reduces the parameters by a further two and now we have four predictions. The choice is equivalent to imposing two additional constraints on the general form over and above eqns. (3.3.14):

$$\begin{aligned} 0 &= (m_c^2 - m_u^2)\frac{m_u m_c}{N_1 N_2} R_1 \sin\rho_1 + (m_t^2 - m_u^2)\frac{m_u m_t}{N_1 N_3} R_2 \sin\rho_2 \\ &+ (m_t^2 - m_c^2)\frac{m_c m_t}{N_2 N_3} R_3 \sin\rho_3 \\ 0 &= \frac{a m_u}{N_1^2} m_u + \frac{a m_c + f}{N_2^2} m_c + \frac{a m_t + d}{N_3^2} m_t + \frac{2R_1 \cos\rho_1}{N_1 N_2}(m_u + m_c) \\ &+ \frac{2R_2 \cos\rho_2}{N_1 N_3}(m_u + m_t) + \frac{2R_3 \cos\rho_3}{N_2 N_3}(m_c + m_t) \end{aligned} \qquad (3.3.15)$$

Proceeding in a manner similar to that for the general Fritzsch form, eqns.(3.3.15) give two more relations between the masses, the weak mixing angles and the CP–violating phase.

Fritzsch-Stech matrix

The simplest way to write the two additional constraints that take the general Fritzsch form to the one of interest is to demand that

$$\begin{aligned} Re(a_d) &= \frac{c_d}{c_u} a_u \\ Re(b_d) &= \frac{c_d}{c_u} b_u \end{aligned} \tag{3.3.16}$$

In our language this would look

$$\begin{aligned}
&\frac{(m_u+m_c)(m_u+m_t)(m_c+m_t)}{(m_u+m_c+m_t)^2} \left[\frac{\alpha m_u^3}{N_1^2(m_c+m_t)^2} + \frac{2m_u m_c R_1 \cos\rho_1}{N_1 N_2 (m_u+m_c)(m_c+m_t)} \right. \\
&\quad + \frac{(\alpha m_c + f)m_c^2}{N_2^2(m_u+m_t)^2} + \frac{2m_u m_t R_2 \cos\rho_2}{N_1 N_3 (m_u+m_c)(m_c+m_t)} \\
&\quad \left. + \frac{(\alpha m_t + d)m_t^2}{N_2^2(m_u+m_c)^2} + \frac{2m_c m_t R_3 \cos\rho_3}{N_2 N_3 (m_u+m_c)(m_u+m_t)} \right] \\
&= -\left[\frac{\alpha m_u}{N_1^2} m_u + \frac{\alpha m_c + f}{N_2^2} m_c + \frac{\alpha m_t + d}{N_3^2} m_t + \frac{2R_1 \cos\rho_1}{N_1 N_2}(m_u+m_c) \right. \\
&\quad \left. + \frac{2R_2 \cos\rho_2}{N_1 N_3}(m_u+m_t) + \frac{2R_3 \cos\rho_3}{N_2 N_3}(m_c+m_t) \right] \\
&= \frac{\alpha m_u^3}{N_1^2(m_c+m_t)} + \frac{(\alpha m_c + f)m_c^2}{N_2^2(m_u+m_t)} + \frac{(\alpha m_t + d)m_t^2}{N_2^2(m_u+m_c)} + \frac{m_u m_c (m_u+m_c+2m_t)}{(m_u+m_t)(m_c+m_t)} \frac{R_1 \cos\rho_1}{N_1 N_2} \\
&\quad + \frac{m_c m_t (2m_u+m_c+m_t)}{(m_u+m_c)(m_u+m_t)} \frac{R_2 \cos\rho_2}{N_2 N_3} + \frac{m_u m_t (m_u+2m_c+m_t)}{(m_u+m_c)(m_c+m_t)} \frac{R_3 \cos\rho_3}{N_1 N_3}
\end{aligned} \tag{3.3.17}$$

3.3.5 Conclusions

Our analysis has shown that the parameters involved in the general three-generation quark mass matrix are not fixed by the current experimental data but are allowed a continuous range. However this range is limited to different sectors depending on the relative signs of the mass terms. Most of the large width of these sectors arise due to the large indeterminacy in the masses of the lighter quarks and only to a lesser degree from the inaccuracy of the knowledge of the c–b mixing strength.

From the expressions in Section 3.3.3 we see that the off-diagonal terms in M_d are relatively small. In fact $\left|\frac{R_{1,2}}{m_s}\right| < O(0.1)$ and $\left|\frac{R_3}{m_s}\right| < O(1)$. In the light of this, if one demands that $\left|\frac{f}{m_s}\right|$ and $\left|\frac{d}{m_b}\right|$ not be too large either, then from Fig. 3.1 and Table 3.1 we are limited to the $\frac{m_d}{m_s} < 0$ sector. All the specific models that we have encountered so far lie in this category.

This implies that future model building could take two different courses. The more conservative course, given the moderate success of the current models would be to reexamine the present constraints and offer slight modifications that would alter or extend the models to a degree without drastically changing the basic structure. All these would be expected to lie in the $\frac{m_d}{m_s} < 0$ sector. The other and more radical approach would be to consider an entirely different class of models. This would entail f and d assuming much larger values compared to the other parameters in M_d and would demand a theoretical justification for such a behaviour.

Chapter 4

Neutrino Masses and some Consequences

As opposed to the last two chapters, we shall concentrate here solely on certain aspects of neutrino physics. To begin with, we discuss the different types of mass terms possible for neutrinos and go on to give an outline of the most general case. A short discussion on the non–trivial consequences of neutrino mixing and oscillations follows next. The question of distinguishability of Dirac and Majorana particles leads us to the feature of neutrinoless double beta decay.

In the second part of the current chapter we present a new discussion on the connection between the Majorana mass of the neutrino and the neutrinoless double beta decay $[(\beta\beta)_{0\nu}]$ rate. It is argued that contrary to conventional wisdom, the latter does not distinguish between the Dirac and Majorana mass of the physical electron neutrino (ν_e). Building on this observation, we also identify scenarios where ν_e can naturally be a light Majorana neutrino with no $(\beta\beta)_{0\nu}$, and construct supersymmetric grand unified models that admit such possibilities.

4.1 Neutrino Masses

The question of neutrino masses is on a somewhat different footing than that of quark or charged lepton masses. For one, in the standard model the neutrinos are assumed to be strictly massless. This is forced upon us not by some theoretical constraint but rather by our failure so far to conclusively detect any non-zero mass for the neutrinos. However any negative experimental result is only as good as the resolution limits of the apparatus and in this case a lot of room is still left.

To achieve a symmetry between the leptonic and the hadronic sectors, one would like to consider the case where the neutrino does have a small (but non–zero) mass and look for consequences thereof. In this chapter we would venture to do the same.

As soon as one postulates a non-zero m_ν, one has to go beyond the minimal standard model as the lack of both ν_R's as well as triplet scalars in the SM prevents such a mass term. The simplest way then is to introduce one or more ν_R and as these are gauge singlets, the anomaly cancellation is not affected. One can then have m_ν through the usual method of Yukawa couplings and spontaneous symmetry breaking. However there is more to neutrino mass than just this and rather than duplicate the analysis in Chapter 2, we would take a different track.

To begin with, we digress somewhat to have a quick look at the charge conjugation properties of a spinor field:

$$C : \psi \rightarrow \psi^c \equiv C\overline{\psi}^T , \tag{4.1.1}$$

where C is a matrix in the Dirac space satisfying

$$C\gamma_\mu^T C^{-1} = -\gamma_\mu, \quad C^\dagger C = 1, \quad C^T = -C. \tag{4.1.2}$$

A look at the Dirac equation then shows that

$$\begin{aligned} C: \ \ \psi_L \rightarrow \ \ & (\psi_R)^c = C\overline{\psi_R}^T = P_L\psi^c \\ C: \ \ \psi_R \rightarrow \ \ & (\psi_L)^c = C\overline{\psi_L}^T = P_R\psi^c , \end{aligned} \tag{4.1.3}$$

where $P_{L,R}$ are the left– and right–projection operators respectively. This is to say that the charge conjugation operator takes the state vector of a given particle to that of its antiparticle while preserving the momentum and helicity.

For a neutral left-handed particle we can then write a mass term of the form

$$m\overline{(\psi_L)^c}\psi_L ,$$

where m is either a bare mass term or arises from a *v.e.v.* of some scalar as the case may be. Such a mass term is different from the usual Dirac term as it involves a field of only one helicity and is obviously absent for charged particles as that would violate charge conservation.

In the general case where one has n ν_L fields and m ν_R fields, the most general neutrino mass term in the Lagrangian would read

$$-\mathcal{L}_{\nu\,\mathrm{mass}} = \frac{1}{2}\overline{\nu_L} M_L \nu_L^c + \frac{1}{2}\overline{\nu_R} M_R \nu_R^c + \overline{\nu_L} M_D \nu_R + H.c.\,, \tag{4.1.4}$$

where M_L, M_R and M_D are matrices of dimension $n \times n$, $m \times m$ and $n \times m$ respectively. Now given any two fields ψ and χ,

$$\overline{\psi} M \chi^c = \overline{\psi} M C \overline{\chi}^T = \overline{\chi} M \psi^c. \tag{4.1.5}$$

Hence M_L and M_R are symmetric matrices. Writing

$$n_L \doteq \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \quad \text{and} \quad \mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R^\dagger \end{pmatrix}, \tag{4.1.6}$$

we have

$$-\mathcal{L}_{\nu\,\mathrm{mass}} = \frac{1}{2}\overline{n_L}\mathcal{M} n_L^c.$$

Like M_L and M_R, $\mathcal{M}$ is also a complex symmetric matrix and hence can be "diagonalized" by an unitary matrix U such that

$$U^* \mathcal{M} U = \mathcal{M}_{diag}\,,$$

where U diagonalizes $\mathcal{M}^\dagger \mathcal{M}$ (See section 2.1). Defining

$$\chi = U^T n_L + U^\dagger n_L{}^c, \tag{4.1.7}$$

we have

$$-\mathcal{L}_{\nu\,\mathrm{mass}} = \frac{1}{2}\overline{\chi}\mathcal{M}_{diag}\chi\,. \tag{4.1.8}$$

Obviously $\chi_k^c = \chi_k$ and hence these are Majorana particles [40].

If $M_L = 0 = M_R$, then the eigenvalues of M are either zero ($|m-n|$ in number) or resolve into $min(m,n)$ pairs of the form $\pm m_i$ (m_i are complex). In the second case,

$$m(\overline{\chi_+}\chi_+ - \overline{\chi_-}\chi_-) = m(\overline{\chi_D}\chi_D + H.c.)\,, \tag{4.1.9}$$

where

$$\chi_D \equiv \frac{1}{\sqrt{2}}(\chi_+ + \gamma_5\chi_-). \tag{4.1.10}$$

Thus two Majorana neutrinos with equal and opposite masses but the same CP properties (or equivalently degenerate neutrinos with opposite CP phases) combine to give a Dirac neutrino of the same mass. The number of degrees of freedom obviously remains the same.

At this stage one might ask whether the differences between a Majorana and a Dirac neutrino are limited only to their abstract vector space properties or if there exists any measurable quantity that distinguishes them. A more general question is that regarding the observational consequences of the neutrino mass matrix. We attempt to answer the last question first, not only because it closely parallels the discussion in Chapter 2, but also because the first problem, in a sense, is only a subset of the second.

4.1.1 Neutrino Mixing and Oscillations

Proceeding in a fashion analogous to that for the quarks, we define the charged lepton mass basis by

$$l_L = L_L l'_L \quad \text{and} \quad l_R = L_R l'_R, \tag{4.1.11}$$

where $L_{L,R}$ diagonalize the lepton mass matrix M^l through the biunitary transformation

$$L_L^\dagger M_l L_R = \widehat{M_l}. \tag{4.1.12}$$

Assuming now that all the left handed neutrinos are part of $SU(2)_L$ doublets with hypercharge $Y = -\frac{1}{2}$, and all right handed neutrinos are gauge singlets, we have then, for the relevant charged current

$$J_\mu^+ = \sum_{i,j=1}^{n} \sum_{\alpha=1}^{n+m} \overline{l_{Li}} \gamma_\mu \chi_{L\alpha} (L_L^\dagger)_{ij} (U^*)_{j\alpha} \tag{4.1.13}$$

leading to an effective neutrino mixing matrix (analogous to the CKM matrix) K^ν given by

$$(K^\nu)_{i\alpha} = \sum_{j=1}^{n} (L_L^\dagger)_{ij} (U^*)_{j\alpha}. \tag{4.1.14}$$

Notice that unlike in the hadronic case, we not only have the neutrino–CKM to be non–unitary, but it is rectangular $[n \times (n+m)]$ to boot. One has

$$(K^\nu K^{\nu\dagger})_{ik} = \delta_{ik} \quad \text{but} \quad (K^{\nu\dagger} K^\nu)_{\alpha\beta} = \sum_{k=1}^{n} U^T_{\alpha k} U^*_{k\beta}.$$

The non–orthogonality also manifests itself in the neutral current interactions, the relevant isotriplet part of which is given by

$$J^3_\mu = \sum_{i=1}^{n} \overline{\nu_{iL}} \gamma_\mu \nu_{iL} = \sum_{\alpha,\beta=1}^{n+m} (K^{\nu\dagger} K^\nu)_{\alpha\beta} \overline{\chi_{\alpha L}} \chi_{\beta L}.$$

Parameter counting in this case is slightly different from that in the hadronic sector. K^ν is best recognized as being a rectangular part of a $(n+m) \times (n+m)$ unitary matrix and hence, in the most general case is given by ${}^{n+m}C_2$ angles and ${}^{n+m+1}C_2$ phases. However, we can't proceed as for the quarks and eliminate $2(n+m)-1$ phases by redefinition of wavefunctions, for the Majorana neutrinos obviously cannot absorb phase transformations. At most n phases can be eliminated by redefining only the charged lepton wavefunctions and thus we are left with ${}^{n}C_2 + \frac{m(2n+m+1)}{2}$ CP violating phases. It seems quite logical then that this difference can be exploited to distinguish a Majorana neutrino from a Dirac one, but Schechter and Valle [41] have shown that these extra CP violating effects are always suppressed by an additional factor of $(m_\nu/E_\nu)^2$, where m_ν and E_ν respectively are the mass and energy of the Majorana neutrino taking part in the process. The suppression is easily understood by appreciating that a process dependent on the Majorana mass must have an amplitude proportional to the latter and hence for dimensional reasons there has to be a suppression factor given by the relevant energy scale in the problem.

As in the case of the K^0–$\overline{K^0}$ system, we have, in the general case, a number of neutrinos with possibly all different masses mixing with each other. While the interaction terms in the Lagrangian conserve the individual lepton numbers (for a definition of lepton numbers, see section 4.1.2), the mass terms do not, and in the case of Majorana neutrinos even the total lepton number is not preserved. As a neutrino with definite interaction properties evolves in time, each of its massive modes propagates differently resulting in a periodic variation in their relative proportions in the generic neutrino 'beam'. Analogous to strangeness oscillations for the neutral kaons, we have then the possibility of lepton number oscillations [42].

To start with, we take a quick look at the oscillation of neutrinos in vacuum. In this section we shall adopt a slightly different and unorthodox notation. We extend the definition of flavour eigenstates to include the right–handed neutrinos as well, and shall denote them by $|\nu_i\rangle$ (where $i = 1...N(= n+m)$). Identifying the mass eigenstates as $|\chi_k\rangle$ as before, we have

$$|\nu_i(t=0)\rangle = \sum_{k=1}^{N} U_{ik}|\chi_k(t=0)\rangle \quad \text{and} \quad |\chi_k(t=0)\rangle = \sum_{i=1}^{N} U^*_{ik}|\nu_i(t=0)\rangle \tag{4.1.15}$$

where U is a $N \times N$ unitary matrix for N neutrino species. Then

$$\begin{aligned} |\nu_i(t)\rangle &= \sum_{k=1}^{N} U_{ik} e^{-iE_k t}|\chi_k(t=0)\rangle \\ &= \sum_{k=1}^{N} U_{ik} e^{-iE_k t} \sum_{j=1}^{N} U^*_{jk}|\nu_j(t=0)\rangle \end{aligned} \tag{4.1.16}$$

and thus

$$P_{\nu_i \to \nu_j}(t) \equiv |\langle \nu_j(t=0)|\nu_i(t)\rangle|^2 = \left| \sum_{k=1}^{N} U_{ik} U^*_{jk} e^{-iE_k t} \right|^2 . \tag{4.1.17}$$

Assuming the neutrinos do not decay,

$$\sum_{j=1}^{N} P_{\nu_i \to \nu_j}(t) = 1. \tag{4.1.18}$$

Now

$$CPT \text{ theorem} \implies P_{\overline{\nu}_j \to \overline{\nu}_i}(t) = P_{\nu_i \to \nu_j}(t) \tag{4.1.19}$$

$$\text{while} \quad CP \text{ conservation} \implies P_{\overline{\nu}_i \to \overline{\nu}_j}(t) = P_{\nu_i \to \nu_j}(t). \tag{4.1.20}$$

It is easy to see that in the case of two neutrino species, (4.1.18) and (4.1.19) together imply (4.1.20) and thus to detect CP violation in neutrino oscillations, one requires at least three neutrinos to mix (a not unexpected conclusion). However, henceforth we shall, for the sake of simplicity, assume that leptonic CP violation is absent and hence the matrix U shall be treated to be an orthogonal one.

Although a general study of the neutrino oscillation problem is quite a straightforward one, the physics issues involved are more transparent if one restricts oneself to the simplest possible case, namely that of only two neutrinos, say ν_e and ν_μ. The mixing matrix U then simplifies to

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} .$$

If we further assume that each neutrino is light enough so that we can write for its energy, $E \simeq p + \frac{m^2}{2p}$, where p is the momentum, we have

$$P_{\nu_e \to \nu_\mu}(R) = \frac{1}{2} \sin^2 2\theta \left[1 - \cos\frac{2\pi R}{L}\right] , \tag{4.1.21}$$

where R is the detector distance and

$$L \equiv \frac{4\pi p}{\Delta m^2} = 2.5 \frac{p\,(MeV)}{\Delta m^2\,(eV^2)} \text{ meters} \tag{4.1.22}$$

is the oscillation length. Here $\Delta m^2 = m_1^2 - m_2^2$ is the difference in neutrino mass squares. Thus for oscillations to be visible, one not only needs a non-zero mixing angle θ, but also $R \gtrsim L$. In practice however, it is not easy to recognize oscillation employing a single detector as one must average over the uncertainties in the zone of beam formation and detection *etc.*, leading to

$$\langle P_{\nu_e \to \nu_\mu} \rangle \approx \frac{1}{2} \sin^2 2\theta \quad \text{and} \quad \langle P_{\nu_e \to \nu_e} \rangle \approx 1 - \frac{1}{2} \sin^2 2\theta.$$

One of the prime motives behind the study of neutrino oscillations was the possibility of a resolution of the solar neutrino problem. The problem (real or imaginary, depending on the prejudices of the person concerned) lies in the low solar neutrino count in the Davis experiment [43] as compared to the predictions of the standard solar model [44]. An interesting solution would be to invoke transformation of the solar ν_e to ν_μ or ν_τ (which the Davis experiment cannot detect) while traversing the distance to earth. But the restrictions imposed by the terrestrial experiments on the $\sin 2\theta$–Δm^2 plane rules out a dominant role for vacuum oscillations in this context. A more practical solution lay in considering the effect of matter on neutrino oscillations [45]. While ν_e travelling in matter suffers both charged current (*c.c.*) and neutral current (*n.c.*) interactions, the other species have only the *n.c.* interactions. This induces an additional potential term proportional to the electron density for the ν_e or, equivalently, an extra term in the $(mass)^2$ matrix. With a matter density gradient, as is there in the Sun, this results in an quantum mechanical eigenvalue cross–over problem and consequently, in the adiabatic approximation, in a resonant conversion of ν_e to say, ν_μ [46]. This mechanism could magnify the oscillation effects due to even a small vacuum mixing angle sufficiently enough to explain the rather large discrepancy. But even this mechanism cannot explain the reported anticorrelation [47] between the solar magnetic activity and the observed neutrino flux. An explanation for such a behaviour is found if one ascribes a non–zero magnetic dipole moment to the neutrino thus enabling the solar magnetic field to rotate ν_e to some sterile (in the Davis context) species.

4.1.2 Neutrinoless Double Beta Decay

The most distinguishing feature of a Majorana mass term is the explicit breaking of a symmetry of the Lagrangian that its existence implies. In the absence of such terms, the Lagrangian is invariant under the global transformation

$$l'_{iL} \to e^{i\theta} l'_{iL}, \quad e'_{iR} \to e^{i\theta} e'_{iR}, \quad \nu'_{iR} \to e^{i\theta} \nu'_{iR}. \tag{4.1.23}$$

This obviously leads to an exactly conserved charge L (the lepton number, with values ± 1 for (anti–)leptons and zero for all other particles) with the consequence that the electroweak interactions (and trivially the strong interactions too) preserve the relative abundance of leptons over antileptons. However the individual flavour numbers are not conserved, leading to possible decays like

$$\mu \to e + \gamma, \quad \mu \to 3e, \quad K \to \pi \mu e. \tag{4.1.24}$$

On the other hand, if both the neutrino and the electron mass matrices be simultaneously diagonalizable, or in other words, if the neutrino mixing matrix is but a phase matrix, then the Lagrangian is invariant under independent global transformations

$$l'_{iL} \to e^{i\theta_i} l'_{iL}, \quad e'_{iR} \to e^{i\theta_i} e'_{iR}, \quad \nu'_{iR} \to e^{i\theta_i} \nu'_{iR}. \tag{4.1.25}$$

This leads to individually conserved lepton flavour numbers L_i (the corresponding invariances in the hadronic case are explicitly broken down to a conserved total baryon number by the non–zero quark mixings). In such a case, the interactions as in (4.1.24) are obviously absent.

With the introduction of the Majorana mass term (either M_L or M_R), even the total lepton number no longer remains a symmetry. In fact, a non–zero Majorana mass implies the existence of a propagator of the form $\langle \chi \chi^T \rangle$, leading to L violation by two units. This effect manifests itself most dramatically in neutrinoless double beta decay. It is to be noted, however, that existence of Majorana mass terms need not mean absence of any conserved lepton charge. For it might so happen that a certain (or more) combination(s) of L_i may still be a good symmetry. A particularly simple case is that of two left–handed neutrinos ν_{eL} and $\nu_{\mu L}$ such that the mass matrix reads

$$M_\nu = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix} .$$

The Lagrangian then has a $U(1)$ invariance under which the two leptonic generations being considered have opposite charges, *i.e.*,

$$\begin{array}{ll} e'_{L,R} \to e^{i\theta} e'_{L,R} & \nu'_{eL} \to e^{i\theta}\nu'_{eL} \\ \mu'_{L,R} \to e^{-i\theta}\mu'_{L,R} & \nu'_{\mu L} \to e^{-i\theta}\nu'_{\mu L} \end{array}$$

We have then a Dirac neutrino mass term of the form $m\overline{\nu}\nu$ where $\nu \equiv \nu_{eL} + (\nu_{\mu L})^c$. The conserved charge in this case is given by $L = L_e - L_\mu$ and was introduced by Zeldovich, Konopinsky and Mahmoud [48].

In 1937, Racah [49] pointed out that if the neutrino emitted by a neutron is a Majorana particle, then it can stimulate the decay of a second neutron. Furry [50] then pointed out that this neutrino can be a virtual one thus inventing the process of neutrinoless double beta decay [$(\beta\beta)_{0\nu}$]. We now expect that the $(\beta\beta)_{0\nu}$ experiment will give us the physical Majorana mass of the neutrino. Since a Dirac particle can be thought of as two Majorana particles with opposite CP properties, their contributions to $(\beta\beta)_{0\nu}$ cancel [51,52]. Thus we also expect that $(\beta\beta)_{0\nu}$ experiments will allow us to distinguish between a Dirac and a Majorana particle.

4.2 Naturally Light Majorana Neutrinos with no Neutrinoless Double Beta Decay

In this section [1], from a general analysis of the neutrino mass matrix, we argue that the $(\beta\beta)_{0\nu}$ amplitude does not depend on the physical Dirac or Majorana mass of the electron neutrino. We discuss the situation in which ν_e is a Majorana neutrino (and may even be the only light one) and yet there is no $(\beta\beta)_{0\nu}$. Conversely we also know of situations wherein there is $(\beta\beta)_{0\nu}$ inspite of the ν_e being a Dirac particle. We then proceed to construct certain supersymmetric grand unified theories that naturally have a light Majorana ν_e with no $(\beta\beta)_{0\nu}$ or, on the contrary, a massless ν_e with considerable $(\beta\beta)_{0\nu}$. Thus experimental signature or otherwise of $(\beta\beta)_{0\nu}$ gives very little information about the neutrino masses.

As is evident from the discussion in the last section, a mass matrix of the form in eqn.(4.1.6) in general induces $(\beta\beta)_{0\nu}$. It has been shown [54] that the amplitude for this

[1] Based on the work in ref. [53]

event goes as

$$A\left((\beta\beta)_{0\nu}\right) \propto \langle m\rangle \equiv \sum_{k=1}^{n} (U_{ek})^2\, m_k F(m_k, N) \qquad (4.2.1)$$

where $F(m_k, N) = \langle e^{-m_k r}/r\rangle \langle 1/r\rangle^{-1}$, the average being done over the nucleus N in question. (In this section we would, for the sake of simplicity, assume that the charged lepton mass matrix is diagonal and hence L_L is the identity matrix.) For neutrinos lighter than a few MeV the suppression factor F is nearly one and then one has

$$\langle m\rangle \approx \sum_{k=1}^{n+m} (U_{ek})^2\, m_k = \mathcal{M}_{ee} \qquad (4.2.2)$$

the last equality following from the definition of U. Thus for light neutrinos, $(\beta\beta)_{0\nu}$ level depends only on $\mathcal{M}_{ee}$. (Although this result was obtained by Wolfenstein [51] in 1981, its significance was not quite appreciated.) It is quite independent of whether ν_e is massless or if massive, whether it is of the Dirac or Majorana type.

However if one or more of the ν–species are too heavy to be kinematically produced inside the nucleus, then the effective mass $\langle m\rangle$ gets modified to

$$\langle m\rangle = \sum_{\text{light }\nu} (U_{ek})^2\, m_k F(m_k, N) \approx \sum_{\text{light }\nu} (U_{ek})^2\, m_k. \qquad (4.2.3)$$

The last approximate equality follows under the assumption that the neutrinos are either too heavy to be of kinematic importance or quite light, *i.e.* their masses do not lie in the MeV region. In this case $\mathcal{M}_{ee}$ is no longer a measure of $(\beta\beta)_{0\nu}$.

To consider a concrete case, assume the mass matrix to be of the form

$$M = \begin{pmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{pmatrix} \qquad (4.2.4)$$

where $M_{1,3}$ are real symmetric matrices and one has a hierarchy $M_1 \ll M_2 \ll M_3$ such that $M_1 \sim O(M_2 M_3^{-1} M_2^T)$. M can then be approximately block diagonalized by an orthogonal matrix

$$V = \begin{pmatrix} 1 - \frac{1}{2}\rho^T\rho & \rho^T \\ -\rho & 1 - \frac{1}{2}\rho\rho^T \end{pmatrix} \qquad (4.2.5)$$

where $\rho = M_3^{-1} M_2^T$. Then one has

$$M_{BD} \equiv V^T M V = \begin{pmatrix} \widetilde{m} & 0 \\ 0 & \widetilde{M} \end{pmatrix} + O(\rho^2 M_1) \qquad (4.2.6)$$

where

$$\widetilde{m} = M_1 - M_2\rho \quad \text{and} \quad \widetilde{M} = M_3 + \frac{1}{2}(\rho M_2 + M_2^T\rho^T). \qquad (4.2.7)$$

Let $K_{1,3}$ be orthogonal matrices such that $K = diag(K_1, K_3)$ diagonalizes M_{BD}. Then

$$U \equiv VK = \begin{pmatrix} (1 - \frac{1}{2}\rho^T\rho)K_1 & \rho^T K_3 \\ -\rho K_1 & (1 - \frac{1}{2}\rho\rho^T)K_3 \end{pmatrix} \tag{4.2.8}$$

diagonalizes the mass matrix M. If we assume the eigenvalues of M to be very large, then the effective mass $\langle m \rangle$ for $(\beta\beta)_{0\nu}$ is given by

$$\langle m \rangle = \sum_{k \in \widetilde{m}} (U_{ek})^2 m_k = \widetilde{m}_{11} \tag{4.2.9}$$

Thus if we block diagonalize the mass matrix M into two blocks $\widetilde{m}$ and $\widetilde{M}$, such that the eigenvalues of $\widetilde{M}$ are much larger than $1MeV$ while those of $\widetilde{m}$ are not, then the $(\beta\beta)_{0\nu}$ amplitude depends only upon the element $(\widetilde{m})_{11}$ and not on the actual eigenvalues of $\widetilde{m}$. We are here working in the basis $(\nu_e\, \nu_a\, \nu_b \cdots)$ where ν_a, $\nu_b \ldots$ can either be of different generations or sterile.

Let us now consider a few special cases. If $M \doteq \begin{pmatrix} 0 & m \\ m & m' \end{pmatrix}$ where $m' \gg 1MeV \gg m$, then we get a light Majorana neutrino of mass m^2/m'. In this case $M_{11} = 0$, yet we do get a nonzero contribution to $(\beta\beta)_{0\nu}$. This is the well-known see–saw mechanism [55]. On the other hand, if in the same mass matrix we have $m, m' \lesssim 1MeV$, then M itself describes low energy ν–phenomena and we do not have to take recourse to constructing $\widetilde{m}$. In this case though we have two Majorana particles of masses $\left(m' \pm \sqrt{m'^2 + 4m^2}\right)/2$, yet $A\left((\beta\beta)_{0\nu}\right) = 0$ as $M_{11} = 0$.

We have demonstrated two situations. In both the cases $M_{11} = 0$, but while $(\beta\beta)_{0\nu}$ is present in one, it is absent in the other. In either case the physical neutrino is a Majorana particle. Let us now consider the case when it is a Dirac particle, at least at the tree level. Since our analysis does not depend on the radiative corrections, we shall not talk about loop effects. Consider the mass matrix $M = \begin{pmatrix} m' & m \\ m & -m' \end{pmatrix}$. This corresponds to two Majorana particles of equal masses $\sqrt{m'^2 + m^2}$ with opposite CP properties, and hence they combine to give a two helicity state Dirac neutrino. Both the eigenvalues being equal we can ignore the factor F and write $\langle m \rangle \propto m'$. Now we can have two scenarios : $m = 0$ or $m \neq 0$. Each will predict a Dirac neutrino [56] but in the first there isn't any $(\beta\beta)_{0\nu}$ whereas in the second it does appear.

Although in the simplest example cited above, if the physical neutrino is a Dirac particle, then the contribution to $(\beta\beta)_{0\nu}$ is given by $\langle m \rangle = M_{11}$, this is not the case in general. Consider for example

$$M = \begin{pmatrix} 0 & 0 & 0 & m \\ 0 & 0 & m & 0 \\ 0 & m & m' & 0 \\ m & 0 & 0 & -m' \end{pmatrix}$$

where $m' \gg m$. This mass matrix predicts one light Dirac neutrino of mass m^2/m' and also gives nonzero $(\beta\beta)_{0\nu}$ ($\langle m \rangle = m^2/m'$) although $M_{11} = 0$.

The question we attempt to answer next is the one regarding the naturalness of the above arguments. We have demonstrated many scenarios which, in principle, can exist. But if we cannot get them naturally from any realistic theory, then it does not make much sense.

Models [57,58] were constructed to predict light Dirac neutrinos naturally, which give no $(\beta\beta)_{0\nu}$. The most popular versions start with three additional sterile neutrinos per generation. Then using some symmetry of the theory one gets a mass matrix in the $(\nu_e\,\nu_a\,\nu_b\,\nu_c)$ basis of the form [57]

$$M = \begin{pmatrix} 0 & 0 & A & 0 \\ 0 & 0 & B & C \\ A & B & 0 & 0 \\ 0 & C & 0 & 0 \end{pmatrix}$$

where $B \gg A, C$. This predicts a light Dirac neutrino of mass $AC/B \sim$ a few eV, so that this can explain the ITEP result [59] of $m_\nu \sim 20eV$, as well as the absence of $(\beta\beta)_{0\nu}$ [60].

We shall proceed in a similar fashion to demonstrate a scenario where we have a light Majorana neutrino with $m_\nu \sim 20eV$ but no $(\beta\beta)_{0\nu}$. The model can also accomodate a 17 keV Majorana ν with a small mixing with ν_e (similar to that seen by Simpson [62,63] albeit with a smaller mixing). The numbers are not very special to the model. What we would like to emphasize is that one can obtain light Majorana neutrinos from realistic GUTs naturally which do not admit $(\beta\beta)_{0\nu}$. We also start with three sterile neutrinos alongwith ν_e and seek to get in the $(\nu_e\,\nu_a\,\nu_b\,\nu_c)$ basis a mass matrix of the form

$$M = \begin{pmatrix} \alpha & 0 & 0 & a \\ 0 & 0 & k & 0 \\ 0 & k & 0 & G \\ a & 0 & G & B \end{pmatrix} \qquad (4.2.10)$$

where $G \gg k, B \gg a \gg \alpha$. In fact α can even be zero. This mass matrix can be block diagonalized to $M_{BD} = diag(\widetilde{m}, \widetilde{M})$ with

$$\widetilde{m} = \begin{pmatrix} \alpha & -ak/G \\ -ak/G & k^2B/G^2 \end{pmatrix} \quad \text{and} \quad \widetilde{M} \approx \begin{pmatrix} 0 & G \\ G & B \end{pmatrix} . \tag{4.2.11}$$

Then if $aG \ll kB$, we have four Majorana neutrinos with masses

$$m_1 = \alpha - a^2/B, \quad m_2 = k^2B/G^2, \quad m_{3,4} = B \pm G/2 \tag{4.2.12}$$

With a suitable choice for the 5 parameters appearing in M we can obtain two light neutrinos and two superheavy ones. The ν–less double beta decay amplitude is proportional to $(\widetilde{m})_{11} = \alpha$ as two of the neutrinos are too heavy to be kinematically produced at the ordinary decay energies.

The most interesting aspect of this exercise is the relation between α and m_1 . As has been mentioned earlier, α is a parameter in the mass matrix much smaller than the others. In the explicit models to be considered later, it turns out to be of the order of a^2/B or smaller. Thus we have three possibilities:

a) α of the same order as m_1 : This gives the usual picture of $(\beta\beta)_{0\nu}$ being proportional to the Majorana mass of ν_e;

b) $\alpha \approx 0$: then we get a light Majorana ν without appreciable $(\beta\beta)_{0\nu}$.

c) $\alpha \approx a^2/B$: this leads to a very small Majorana mass but a rather large amount of $(\beta\beta)_{0\nu}$.

We now proceed to present a model based on a supersymmetric $SO(10)$ grand unified theory in which the hierarchy of the parameters that we require appears naturally. We do not aim to construct a complete gauge theory; rather we give an illustration of how we can get light Majorana neutrinos, with no $(\beta\beta)_{0\nu}$. This model is on the same footing as those which predict light Dirac neutrinos. In particular, we require one $U(1)$ global symmetry to get the required form of the mass matrix as compared to the three $U(1)$ symmetries required for a light Dirac neutrino in a similar model.

We shall focus our attention on a single family assuming intergeneration mixing to be small. We start with a $SO(10)$ model with two fermion singlet superfields S and S' in addition to the usual 16-plet matter superfield χ. The symmetry breaking chain being considered is

$$
\begin{aligned}
SO(10) &\xrightarrow{M_{GUT}} SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \\
&\xrightarrow{M_{LR}} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \\
&\xrightarrow{M_{wk}} SU(3)_c \otimes U(1)_Q
\end{aligned}
$$

To give masses to the fermion fields as well as to prompt the last two stages of the above symmetry breaking chain, we introduce four Higgs superfields $\chi(\overline{16})$, $\Phi(10)$, $\Delta(126)$ and $\sigma(1)$ where the numbers in parentheses denote their transformation properties under $SO(10)$. The most general Yukawa coupling allowed then is

$$\mathcal{L}_Y = \overline{\psi^c}\psi(f_1\Phi + f_2\Delta) + \overline{\psi^c}\chi(f_3 S + f_4 S') + (f_5 S S' + f_6 S S + f_7 S' S')\sigma.$$

We impose an additional U(1) global symmetry, the non-trivial transformations under it being

$$\chi \to e^{i\theta}\chi \qquad \sigma \to e^{-i\theta}\sigma \qquad S \to e^{-i\theta} S \qquad S' \to e^{2i\theta} S' \tag{4.2.13}$$

This eliminates the Yukawa couplings given by f_4, f_6 and f_7 as well the bare mass terms for S and S'. Then in the basis $(\nu\ S\ S\ \nu^c)$, the mass matrix reads as in eqn(4.2.10), and with a particular hierarchy of $v.e.v.$s [64]:

$$
\begin{array}{lllllllll}
G & = & f_3\langle\chi\rangle & \sim & O(10^9\, GeV), & B & = & f_2\langle\Delta\rangle & \sim & O(10\, TeV), \\
k & = & f_5\langle\sigma\rangle & \sim & O(10\, TeV), & a & = & f_1\langle\Phi\rangle & \sim & O(10\, MeV) \text{ and} \\
\alpha & \sim & O(10\, eV). & & & & & & &
\end{array}
$$

While the value of G is self-evident, the others need some explanation. The scales of k and B, proportional to M_{SUSY}, appear naturally in a certain class of supersymmetric models where the corresponding scalars remain massless at the tree level, only to gain mass through radiative corrections. a reflects the electroweak breaking scale, assuming a value comparable to the light quark masses. A small value of α (proportional to $\langle\Phi\rangle^2/\langle\Delta\rangle$) is generated due to the features of potential minimization in a left-right symmetric model.

An $SU(5)$ analog of this model can easily be constructed using three singlet fermions $(S_{1,2,3})$ apart from the usual 10 (χ) and 5 (ψ) superfields. The Higgs sector is enlarged to accomodate two singlets (σ_1 and σ_2) and a 5–plet $\tilde{\Phi}$ alongwith the usual 24–plet Σ and the 5–plet Φ. Then the imposition of an $U(1)$ symmetry:

$$
\begin{array}{lll}
S_1 \to e^{i\theta} S_1 & S_2 \to e^{3i\theta} S_2 & S_3 \to e^{2i\theta} S_3 \\
\sigma_1 \to e^{-4i\theta}\sigma_1 & \sigma_2 \to e^{-5i\theta}\sigma_2 & \tilde{\Phi} \to e^{-2i\theta}\tilde{\Phi}
\end{array} \tag{4.2.14}
$$

will give us the Yukawa coupling

$$\mathcal{L}_Y = (f_1\psi\chi + f_2\chi\chi)\Phi + g_1\psi S_3\tilde{\Phi} + g_4 S_2 S_3\sigma_2 + (g_2 S_1 S_2 + g_3 S_3 S_3)\sigma_1$$

This gives a neutrino mass matrix of the form of eqn.(4.2.10) with

$$\begin{aligned} G &= g_4\langle\sigma_2\rangle, & k &= g_2\langle\sigma_1\rangle, \\ B &= g_3\langle\sigma_1\rangle, & a &= g_1\langle\tilde{\Phi}\rangle \end{aligned}$$

with $a \ll k, B \ll G$. It is to be noted that in this case $\alpha = 0$. With a suitable choice (depending on the details of the model concerned) of a, k, B and G we obtain naturally light Majorana neutrinos with no $(\beta\beta)_{0\nu}$.

As an aside we point out that in these scenarios with a careful, but not too unnatural, choice of the Yukawa couplings one could simultaneously accomodate a 25 eV Majorana neutrino with very low $(\beta\beta)_{0\nu}$ rate alongwith a 17 keV neutrino with a small mixing. Also the cosmological constraint on the masses of stable light neutrinos would not pose much of a problem as the keV mass neutrino could be made to decay through Majorons [65].

In summary we argue that the widely held belief that the neutrinoless double beta decay experiments would give us the Majorana mass of the physical electron neutrino is tenable if and only if there is just a single species of ultralight neutrino per generation (as for example in a minimal extension of the standard model with the inclusion of right–handed neutrino singlets) and if the inter-family mixing is non-existent. But in a generic grand unified theory, where there are more than one type of neutrino per generation, it fails to go through. While the absence of $(\beta\beta)_{0\nu}$ cannot say anything about the mass matrix except that $\tilde{m}_{11} = 0$, its presence only confirms the existence of lepton number violation in nature and hence the presence of a Majorana mass term but does not distinguish between a pseudo-Dirac and a Majorana particle. This is exploited in the construction of a supersymmetric grand unified theory in which a Majorana particle that can explain the ITEP results while not succumbing to the $(\beta\beta)_{0\nu}$ constraints is naturally generated.

Chapter 5

On Some Exotic Neutrino Phenomenology

In the previous chapter we have contemplated the changes that need to be made in the minimal standard model so as to incorporate a non-zero neutrino mass. We have also looked at one of the consequences of a possible breakdown of a global lepton number symmetry that a Majorana mass for the neutrino might induce. At this stage one question begs to be asked. Why need we consider a mass for the neutrino at all? Apart from the rhetorical answer "Well, why not? Nothing prevents it anyway.", there is the deeper and more practical reason of its potential to answer many ill-understood problems. Neutrinos being very light (?) and weakly interacting particles, do not manifest themselves too dramatically at ordinary interaction energies, but at the astrophysical scales, they are expected to play a very crucial role. Moreover, in view of the recent spate of results, both experimental and theoretical, in neutrino physics, it is quite conceivable that this field might afford the most accessible testing ground for new physics beyond the standard model.

In this chapter we look at different aspects of "non-standard" neutrino physics. To begin with, we examine the question of a sizable magnetic moment for a very light neutrino. We propose a new mechanism that decouples the question of neutrino masses and magnetic moments and based on this, develop a model which generates a large transition magnetic moment for an ultralight ν_e in a natural way. Next we take a look at the consequences of a non-zero neutrino mass in the context of gravitational interactions. We find that contrary to expectations, for a low-energy neutrino at the vicinity of a supernova the gravitational interaction could be the dominant one. We use this result to put very strong bounds on parity-violating effects in gravity. Finally we move on to present a phenomenologically consistent model for Simpson's $17keV$ neutrino that naturally accomodates a large magnetic moment for the ν_e. We also look at the grtavitational interaction of this neutrino as well as its effect on $(\beta\beta)_{0\nu}$ rates. It is here that the results of the previous exercises are used as inputs to achieve a coherent picture of the problem in its entirety.

5.1 Large Magnetic Moment for Nearly Massless Neutrinos

The question of the compatibility of a large magnetic moment and a very small mass for the neutrinos, apart from being very interesting in itself, is of much importance as a way out of the solar neutrino puzzle [67]. For, the neutrino spin rotation (flavour-changing or otherwise) in conjunction with the matter oscillation effects could lead to a substantial reduction in the ν_e-flux — irrespective of the validity of the adiabatic approximation — thus explaining the discrepancy between the standard solar model prediction [44] and the Davis and Kamiokande results [43]. Moreover a substantial neutrino magnetic moment could play a crucial role in supernova dynamics [68].

That the problem is a non–trivial one is not difficult to appreciate. The magnetic moment term being a non–renormalizable one, cannot occur in the bare Lagrangian and may appear only at the one–loop level or higher. But the very same diagram that gives rise to a non–zero μ_ν also, when the photon line is removed, gives a mass correction. This leads to a proportionality between μ_ν and m_ν with the result that normally one cannot be enhanced while the other is being suppressed. For example, in a minimal extension of the standard model, one gets

$$\mu_\nu \approx 10^{-19} \frac{m_\nu}{1\ eV} \mu_B,$$

and hence it is impossible to generate $\mu_\nu \gtrsim 10^{-12} \mu_B$ (needed for this mechanism to play any meaningful role in the solar context) without being saddled with an unaccepatably large mass for the ν_e.

It was first noticed by Voloshin [69] that if ν_e and ν_e^c transform as a doublet under some $SU(2)_\nu$ symmetry, then while the magnetic moment term is invariant, the mass term behaves as a triplet. This was incorporated in $SU(3)_L \otimes U(1)_Y$ electroweak models [70]. A variant in which $SU(2)_\nu$ was some kind of a horizontal symmetry with $(\nu_e \ \ \nu_\mu)$ as a doublet was also considered [71]. In the limit of exact $SU(2)_\nu$ symmetry then, there exists no mass term but only a nonzero magnetic (transition) moment. The breaking of this symmetry however genarates masses, the proportionality of which to the magnetic moments can be kept down only by imposing certain naturalness conditions.

In this section [1] we aim to generalize Voloshin's argument and see if we can have scenarios wherein the neutrino magnetic moment can exist independent of its mass even after the symmetry breaking, thus rendering the naturalness conditions redundant. We extend the standard model to include a horizontal symmetry that treats all fermions on an equal footing. The lepton number violating higgs (σ) is also responsible for breaking the horizontal symmetry. Thus in the exact symmetry limit, both the Majorana masses and the transition moments vanish. This is so because $\overline{\nu^c}_i \nu_j$ and $\overline{\nu^c}_i \sigma_{\mu\nu} \nu_j F^{\mu\nu}$ both violate lepton number. Since we do not have tree level Majorana or Dirac mass terms for the neutrinos, the origin of both the transition moments and the ν_e–mass lie in the radiative corrections. To one loop order they can be parametrized in terms of dimension five operators with the mass suppression scale being decided by the internal higgs particles in the relevant diagrams. Thus if the couplings and the *v.e.v.*s in the theory could be so chosen that only the antisymmetric terms get any contribution from the diagrams containing $\langle\sigma\rangle$, then the νs would acquire a transition moment while keeping the mass correction zero. In the case where one of the internal higgs flowing in the diagram happens to be a horizontal group singlet, this can be ensured simply by seeing to it that the effective *v.e.v.* structure couples only to antisymmetric combination of the fermions.

The gauge group we consider is $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes O(3)_H$ with the particle representations as under (for the scalars the super and subscripts denote the electric charge and the T_{3H} quantum numbers respectively):

[1] Based on the work in ref. [66]

Fermions

$$
\begin{array}{lclll}
\psi_L & \equiv & \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{pmatrix}_L & (1,2,-1/2,3) & L=1 \\
\psi_L^c & \equiv & (e_L^c \;\; \mu_L^c \;\; \tau_L^c) & (1,1,1,3) & L=-1 \\
Q_L & \equiv & \begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix}_L & (3,2,1/6,3) & L=0 \\
U_L^c & \equiv & (u_L^c \;\; c_L^c \;\; t_L^c) & (\bar{3},1,-2/3,3) & L=0 \\
D_L^c & \equiv & (d_L^c \;\; s_L^c \;\; b_L^c) & (\bar{3},1,-1/3,3) & L=0
\end{array}
\qquad (5.1.1)
$$

Higgs:

$$
\begin{array}{lclll}
\sigma & \equiv & (\sigma_1^0 \;\; \sigma_0^0 \;\; \sigma_{-1}^0) & (1,1,0,3) & L=2 \\
\Sigma & \equiv & (\Sigma_1^0 \;\; \Sigma_0^0 \;\; \Sigma_{-1}^0) & (1,1,0,3) & L=0 \\
\phi_s & \equiv & \begin{pmatrix} \phi_s^0 \\ \phi_s^- \end{pmatrix} & (1,2,-1/2,1) & L=0 \\
\Phi & \equiv & \begin{pmatrix} \phi_1^0 & \phi_0^0 & \phi_{-1}^0 \\ \phi_1^- & \phi_0^- & \phi_{-1}^- \end{pmatrix} & (1,2,-1/2,3) & L=0 \\
H & \equiv & \begin{pmatrix} h_2^0 & h_1^0 & h_0^0 & h_{-1}^0 & h_{-2}^0 \\ h_2^- & h_1^- & h_0^- & h_{-1}^- & h_{-2}^- \end{pmatrix} & (1,2,-1/2,5) & L=0 \\
\widetilde{H} & \equiv & \begin{pmatrix} \tilde{h}_2^0 & \tilde{h}_1^0 & \tilde{h}_0^0 & \tilde{h}_{-1}^0 & \tilde{h}_{-2}^0 \\ \tilde{h}_2^- & \tilde{h}_1^- & \tilde{h}_0^- & \tilde{h}_{-1}^- & \tilde{h}_{-2}^- \end{pmatrix} & (1,2,-1/2,5) & L=0 \\
\eta & \equiv & (\eta_1^{1/3} \;\; \eta_0^{1/3} \;\; \eta_{-1}^{1/3}) & (\bar{3},1,1/3,3) & L=-1 \\
\chi & \equiv & \begin{pmatrix} \chi^{2/3} \\ \chi^{-1/3} \end{pmatrix} & (3,2,1/6,1) & L=-1
\end{array}
\qquad (5.1.2)
$$

Here L denotes the lepton number.

On imposition of a further discrete symmetry

$$
\begin{array}{lclclcl}
\psi_L^c & \rightarrow & -\psi_L^c, & & U_L^c & \rightarrow & -U_L^c \\
\Phi & \rightarrow & -\Phi, & & \widetilde{H} & \rightarrow & -\widetilde{H}
\end{array}
\qquad (5.1.3)
$$

the most general Yukawa term in the Lagrangian would look like

$$
\begin{aligned}
\mathcal{L}_Y = \; & f\overline{\psi_L^c} Q_L \eta + f' D_L^c \psi_L \chi + \psi_L^c \psi_L (g_3^e \Phi + g_5^e \widetilde{H}) \\
+ \; & U_L^c Q_L (g_3^u \Phi + g_5^u \widetilde{H}) + D_L^c Q_L (g_3^d \phi_s + g_5^d H)
\end{aligned}
\qquad (5.1.4)
$$

The global lepton number symmetry will also ensure baryon number conservation, so that even after σ acquires a $v.e.v.$, although lepton number is violated there is no proton decay. Since there is no direct coupling of the fermions with σ, the coupling of the Goldstone boson (Majoron) corresponding to the global lepton number symmetry breaking with the fermions is suppressed by the horizontal scale and thus remains invisible. The strictest bounds [72] on η and χ come from rare K–decay rates leading to

$$\frac{f^2}{m_\eta^2}, \frac{f'^2}{m_\chi^2} \lesssim 10^{-4} G_F \,. \tag{5.1.5}$$

The $O(3)_H$ symmetry can be used to assure that of the three components of σ only σ_{-1} acquires a non-zero $v.e.v.$ The assumption that of the five neutral h_i^0, only the $T_{3H} = -2$ component acquires a $v.e.v.$ is consistent with this. To break the remaining $O(2)_H$ symmetry we choose $\langle \Sigma_0 \rangle \neq 0$. The scale for the horizontal symmetry breaking we choose to be of the order of $10^8 GeV$. The $v.e.v.$s are then

$$\begin{aligned}
\langle \sigma \rangle &= \begin{pmatrix} 0 & 0 & \langle \sigma_{-1}^0 \rangle \end{pmatrix} \\
\langle \Sigma \rangle &= \begin{pmatrix} 0 & \langle \Sigma_0^0 \rangle & 0 \end{pmatrix} \\
\langle \phi_s \rangle &= \begin{pmatrix} \langle \phi_s^0 \rangle \\ 0 \end{pmatrix} \\
\langle H \rangle &= \begin{pmatrix} 0 & 0 & 0 & 0 & \langle h_{-2}^0 \rangle \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} .
\end{aligned} \tag{5.1.6}$$

We have refrained from specifying $\langle \Phi \rangle$ and $\langle \tilde{H} \rangle$ as these give masses only to the up-quark and the charged lepton sectors. Assuming the Yukawa couplings to be unity the most general form of mass matrices is then

$$\begin{pmatrix}
\langle \tilde{h}_2^0 \rangle & -\frac{1}{\sqrt{2}} \langle \tilde{h}_1^0 \rangle - \langle \phi_1^0 \rangle & \frac{1}{\sqrt{6}} \langle \tilde{h}_0^0 \rangle + \langle \phi_0^0 \rangle \\
-\frac{1}{\sqrt{2}} \langle \tilde{h}_1^0 \rangle + \langle \phi_1^0 \rangle & \frac{2}{\sqrt{6}} \langle \tilde{h}_0^0 \rangle & -\frac{1}{\sqrt{2}} \langle \tilde{h}_{-1}^0 \rangle - \langle \phi_{-1}^0 \rangle \\
\frac{1}{\sqrt{6}} \langle \tilde{h}_0^0 \rangle - \langle \phi_0^0 \rangle & -\frac{1}{\sqrt{2}} \langle \tilde{h}_{-1}^0 \rangle + \langle \phi_{-1}^0 \rangle & \langle \tilde{h}_{-2}^0 \rangle
\end{pmatrix} . \tag{5.1.7}$$

Thus there is a wide freedom to choose the Higgs couplings so as to obtain a $v.e.v.$ structure amenable to giving phenomenologically consistent mass matrices in these sectors. We shall not discuss this sector any further.

Figure 5.1: One–loop diagrams leading to possible neutrino mass corrections and transition magnetic moment $\mu_{\nu_e \nu_\mu}$.

The down-quark mass matrix reads

$$M_d = \begin{pmatrix} 0 & 0 & g_1^d\langle\phi_s^0\rangle \\ 0 & -g_1^d\langle\phi_s^0\rangle & 0 \\ g_1^d\langle\phi_s^0\rangle & 0 & g_5^d\langle h_{-2}^0\rangle \end{pmatrix} \tag{5.1.8}$$

For the eigenvalues one then gets the interesting hierarchy

$$m_s^2 = m_d m_b \tag{5.1.9}$$

a relation consistent with experimental data.

Of the multitude of terms in the Higgs potential, the one that interests us the most is $\kappa\phi_s\eta\chi\sigma$, where κ is a dimensionless constant. (Such a coupling for Σ is ruled out by lepton number conservation.) This gives rise to one loop diagrams as in Figure 5.1 that result in a non-zero transition moment of the form

$$\mu_{\nu_e\nu_\mu} = 2e\frac{ff'}{16\pi^2}m_s\frac{\kappa\langle\phi_s^0\rangle\langle\sigma_{-1}^0\rangle}{m_{\eta_{-1}}^2 m_{\chi^{1/3}}^2} \tag{5.1.10}$$

while the mass correction vanishes since the contribution from the two diagrams cancel exactly.

Assuming $\kappa \approx 1$, $\langle\phi_s^0\rangle \sim 100GeV$, $m_\chi \sim 50GeV$ and $m_\eta \sim 10^7 GeV$ and taking f^2/m_η^2, f'^2/m_χ^2 to be at the top of the allowed range one obtains a transition magnetic moment

$$\mu_{\nu_e\nu_\mu} = 10^{-12}\mu_B. \tag{5.1.11}$$

It should be noted that in the limit of exact $O(3)_H$ symmetry, $\mu_{\nu_e\nu_\mu}$ vanishes. But while it appears as a consequence of spontaneous breaking of $O(3)_H$, the mass term still remains zero on account of the absence of either a singlet or a 5-plet term in the effective *v.e.v.* structure. The naturalness condition required in refs.[70,71] to suppress the contribution of the Higgs mass splitting to the ν–masses, is redundant as because of χ being a $O(3)_H$ singlet only one set of scalars appear in the relevant diagrams.

It is easy to see that there are no diagrams giving rise to a mass to ν_τ or transition moments involving it. We have thus obtained a model in which the spontaneous breaking of the horizontal symmetry gives rise to a sole transition moment $\mu_{\nu_e\nu_\mu} \sim 10^{12}\mu_B$ while keeping all the neutrinos massless to 1–loop order without invoking any naturalness condition. In fact, the lepton number violating part of the relevant effective *v.e.v.* structure being a $O(3)_H$ triplet, there would be no radiative corrections to the neutrino mass. Thus the only source of a mass is the introduction of a tree level term as for example through a see–saw like mechanism induced by introduction of singlets. This analysis can easily be extended to the case of more than three genarations or higher symmetry groups. Care need only be taken that the effective lepton number violating higgs *v.e.v.* couples only to the antisymmetric combination(s) of the neutrinos.

Though such a small $\mu_{\nu_e\nu_\mu}$, while consistent with the bounds from supernova neutrino data might seem to be too uninteresting in the solar context, actually it is not so. For, coupled with a very small neutrino mass difference ($10^{-8}eV^2 < \Delta m^2 < 10^{-5}eV^2$) as is natural here, this could play a significant role in a moderately nonadiabatic evolution scenario [73].

Also one does not need to introduce extra higgs to suppress the influence of $\mu_{\nu_e \nu_\mu}$ during a supernova explosion.

5.2 Gravitational Helicity Flip of Neutrinos

If neutrinos did possess a small mass, they could, in principle, make a transition from a helicity state in which they would be dominantly participating in the electroweak process to one where they would have practically no interaction with matter. Such helicity flip mechanisms could drastically affect the evolution of astrophysical systems like neutron stars born in supernova explosions. Features most vulnerable to such helicity mechanisms are the cooling rate and the deleptonisation of the neutron star core. Normally one would expect the electromagnetic interactions of the neutrino (through μ_ν generated at the one–loop level) to give the dominant contribution to such effects. Gaemers *et al.* [75] have however argued that the Z–mediated process could be the more important one.

In this section [2] we wish to concentrate on a different mechanism for flipping the helicity of massive neutrinos that can be potentially important, namely that due to gravitational interactions. Neutron stars appear to be the most favourable candidates to look for such effects. The value of GM/Rc^2 for a typical neutron star of mass M and radius R, is around 0.1 and this is an important quantity which will lead to a sizeable helicity flip due to gravity as we shall see. In what follows, we shall treat the gravitational field in the so–called weak–field limit (also called the linearized approximation). It would however be desirable to formulate the contents of this section within the framework of general relativity.

The coupling of neutrinos to the gravitational field, strictly speaking, requires the introduction of tetrads (vierbeins). However in the weak–field limit the coupling can be described by an external field metric of the form

$$G\overline{\Psi}(P_2)[\gamma_\mu P_\nu + \gamma_\nu P_\mu]\Psi(P_1)h^{\mu\nu}(P_2 - P_1) \tag{5.2.1}$$

$$\text{where} \qquad P = \frac{1}{2}(P_2 + P_1) \qquad g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}. \tag{5.2.2}$$

Here P_1 and P_2 denote the initial and final neutrino momenta and G is the gravitational coupling strength.

[2]Based on the work in ref. [74]

For the moment, we shall assume that only the diagonal components of the stress tensor of the neutron star are important. This yields

$$h^{00} = GM/q^2 \quad \text{and} \quad h^{ij} = \delta^{ij} h^{00}\,, \tag{5.2.3}$$

where $q = |\mathbf{P}_2 - \mathbf{P}_1|$. At first sight this appears to be different from the Schwarzschild metric

$$ds^2 = \left[1 - \frac{r_g}{r}\right] dt^2 - \frac{dr^2}{(1 - r_g/r)} - r^2[\sin^2\theta \, d\phi^2 + d\theta^2]\,, \tag{5.2.4}$$

(where r_g is the Schwarzschild radius) but the metric (5.2.3) is indeed the Schwarzschild metric expressed in the isotropic spherical coordinates [76]:

$$ds^2 = \frac{(1 - r_g/4r)^2}{(1 + r_g/4r)^2} dt^2 - \left[1 + \frac{r_g}{4r}\right]^4 [dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2]. \tag{5.2.5}$$

The S-matrix element for the scattering of a massive neutrino of mass m is then given by

$$S_{fi} = -\frac{i}{V} \frac{m^2}{E_1 E_2} 2\bar{u}(\mathbf{P}_2, \lambda_2) P_\mu \gamma_\nu u(\mathbf{P}_1, \lambda_1) 2\pi\delta(E_2 - E_1) h^{\mu\nu}(q). \tag{5.2.6}$$

The differential cross section for helicity flip is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{\pi^2} \frac{(GMm)^2}{q^2} 2EP(1 - \cos\theta)\,, \tag{5.2.7}$$

where $E(P)$ denote the initial neutrino energy (momentum). We have removed the subscripts since in the absence of recoil $E_1 = E_2 = E$ and $|\mathbf{P}_1| = |\mathbf{P}_2| = P$. The angle of scattering is denoted by θ . The total cross section for helicity flip is then given by

$$\sigma = \frac{(GMm)^2}{\pi} 2\frac{E^2}{P^4} \ln\frac{q_{max}}{q_{min}} \tag{5.2.8}$$

where q_{max} is equal to $2P$. q_{min}^{-1} determines in some sense the maximum impact parameter of the neutrinos. Since we want to determine what happens to the cooling rate and other aspects of the neutron star, we must restrict $q_{min}^{-1} \leq R$. Thus the relevant total cross section is

$$\sigma = \frac{2}{\pi}(GMm)^2 \frac{E^2}{P^4} \ln(2PR) \simeq 0.7 \times 10^{32} \ln(2PR) m^2(keV) \frac{E^2(MeV)}{P^4(MeV)}\, GeV^{-2}. \tag{5.2.9}$$

This is a very large number compared to the typical cross sections one encounters, namely the total cross section for helicity flip due to neutral current interactions [75]:

$$\sigma_z^{flip} \simeq 1.6 \times 10^{-23} m^2(keV)\, GeV^{-2}. \tag{5.2.10}$$

However what is physically relevant is not the cross section itself but the product $n_{\rm scat}\sigma$, where $n_{\rm scat}$ is the number density of scatterers. For the gravitational scattering this is (volume of the star)$^{-1}$. Thus the interesting ratio is

$$\frac{\sigma n_{\rm scat}}{\sigma_z n^z_{\rm scat}} = \frac{\sigma}{N\sigma_z} = \xi \tag{5.2.11}$$

where N is the total number of nucleons $\simeq 10^{57} \times 1.4$ (for a 1.4 solar mass neutron star). Then,

$$\xi = \frac{1}{4} \times 10^{-2} \times \frac{E^2(MeV)}{P^4(MeV)}[\ln 10^{16} P(MeV)R(km)] \simeq 0.1\frac{E^2(MeV)}{P^4(MeV)}. \tag{5.2.12}$$

Thus for low-energy neutrinos ($< O(1\ MeV)$) this can indeed become larger than the standard model effect. Although this mechanism is not of much interest for the cooling rate of newly born neutron stars where the average neutrino energy is $\sim 10\ MeV$ or more, it could be of potential interest when the neutrino temperature drops to $0.5\ MeV$ or so. It should of course be kept in mind that at such temperatures the opacity due to weak interactions is also low. Only detailed investigations can tell whether the gravitational mechanisms play any observable role in late-time neutron star cooling.

By virtue of the fact that

$$u^\dagger_{\lambda'}(\mathbf{P}_2)\gamma_0\gamma_i u_\lambda(\mathbf{P}_1) = 0 \quad \text{for } \lambda \neq \lambda' , \tag{5.2.13}$$

one sees that the rotation of the neutron star, manifested as a non-vanishing g^{0i} in the leading approximation, does not contribute to the effect.

In connection with the phenomenon of helicity flip due to magnetic moment in a magnetic field, Voloshin [79] has proposed the novel idea of a resonant helicity flip. We now demonstrate a similar effect for gravitational helicity flip. In the weak-field approximation the coupling of the neutrino spin S to the gravitational field is given by

$$\frac{3GM}{2R^3}(\mathrm{R} \times \mathrm{v}) \cdot \mathrm{S}. \tag{5.2.14}$$

This can be thought of as if a magnetic moment μ were interacting with a magnetic field H, such that

$$\mu H = \frac{3GM}{2R^2}. \tag{5.2.15}$$

Thus the requirement for an adiabatic resonant helicity flip due to gravity becomes [79]

$$2\frac{GM}{R}\frac{R}{R_f}\frac{1}{R_f} > 2 \times 10^{-2}\ cm^{-1}[\rho/(10^{12}\ g\ cm^{-3})]^{1/2}[80\ km/R_f]^{1/2} , \tag{5.2.16}$$

where R_f is the resonant radius at which the density of nucleons is ρ.

Interestingly, gravitational helicity flip mechanisms combined with our understanding of the cooling rates of newly born neutron stars can severely constrain the possible discrete symmetry violations in gravitation. The implications of the gravitational interactions not conserving discrete symmetries has been studied by Hari Dass [77]. He had also proposed a laboratory experiment based on ultra-cold neutron spin precession that could probe for such effects [78]. In a non–relativistic system interacting with a static non-rotating gravitational object of mass M such discrete symmetry breaking effects can be parametrised by the potential

$$V(r) = \alpha_1 \frac{GM}{r^3} \mathbf{S} \cdot \mathbf{r} + \alpha_2 \frac{GM}{r^2} \mathbf{S} \cdot \mathbf{v} \,. \tag{5.2.17}$$

The first term violates parity and time reversal while the second term violates parity and, through the CPT theorem (the status of this theorem in the context of gravitational interactions is not understood very well), charge conjugation invariance. The existing limits on the parameters are $\alpha_2 < 10^{-6}$ but $\alpha_1 < 10^4$. The experiment proposed by Hari Dass[77] is capable of probing $\alpha_1 \sim 1$ but technically is very hard to perform.

Before calculating the cross sections for helicity flip due to these interactions we present a (special) relativistic generalisation of the above potential. There are many such choices that reduce to the above form in the non-relativistic limit, but the choice is restricted if we demand smoothness in the zero fermion mass limit. Then the only possible term is

$$\alpha_1 \bar{u}(\mathbf{P}_2, \lambda_2) \gamma_5 [\gamma_\mu \sigma_{\nu\alpha} q^\alpha + \gamma_\nu \sigma_{\mu\alpha} q^\alpha] u(\mathbf{P}_1, \lambda_1) h^{\mu\nu}(P_2 - P_1). \tag{5.2.18}$$

In fact it is not possible to write a C violating term that contributes to helicity flip scattering. One need not be alarmed that the stress tensor in equation (5.2.18) does not appear to be conserved. It has been argued on general grounds that discrete symmetry violations in gravitation imply the breakdown of local Lorentz invariance [78] and hence the stress tensor is no longer symmetric. Even though the stress tensor is conserved, its symmetric part is not. It should be stressed that the asymmetry vanishes in the classical (as opposed to the quantum mechanical) limit so there is no conflict with the classical tests of general relativity. The total cross section for helicity flip due to the parity violating interaction is given by

$$\sigma^{\mathrm{pv}} = \alpha_1^2 \frac{8}{\pi} (GMm)^2 \frac{1}{PE} \ln(2PR). \tag{5.2.19}$$

Numerically the total contribution to $n_{\rm scat}\sigma$ from the standard model effect as well as from gravitational interactions (both parity conserving and parity violating) is

$$10^{34} m^2(keV)\left[2.7 + \frac{1+4\alpha_1^2}{3E^2(MeV)}\right] = \sigma_0^{\rm hel.flip} \quad \text{in } GeV^{-2} \tag{5.2.20}$$

assuming $E \simeq P \gg m$. The cooling rates of newly born neutron stars appear to limit this quantity by $< 3.2 \times 10^{37}\, GeV^{-2}$, leading to the constraint

$$m^2(keV)\left[1 + \frac{1+4\alpha_1^2}{8E^2(MeV)}\right] \leq 1.2 \times 10^3. \tag{5.2.21}$$

As noted by Gaemers *et. al.* [75], a limit of 40 keV is obtained for the neutrino mass, independent of the considerations of this section. If the actual mass of the tau neutrino saturates this bound, there will be no room for any parity violations in gravitation. Even if the tau neutrino mass turns out to be as small as as 1 keV, α_1 will be constrained to be smaller than 300 which is already two orders of magnitude better than the existing limits. We have seen that cosmological bounds on the stable neutrino masses require that those neutrinos with masses in the keV range be unstable on a cosmological scale [80]. If, however, the neutrino is stable with a mass of approximately $50\,eV$, the limit imposed on the parity violating parameter α_1 will be similar to the existing limit. On the other hand if the neutrinos are found to be massless no limit on the parity violating gravitational interaction will obtain.

5.3 Model for the 17 keV Neutrino

Signatures of a 17 keV neutrino that mixes with roughly 1% strength with the electron neutrino have recently been reported [62,63]. The observations, if corroborated, seem to call for the tau neutrino to be a 17 keV Dirac one (unless, of course, the new particle is an exotic one altogether) as an identification of the new neutrino with ν_μ is ruled out by the present experimental limits on mixings and the Majorana option negated by the non–observation of neutrinoless double beta decay. But even with this, further problems like the cosmological limit [80] of 100 eV for the masses of stable neutrino species and the aesthetic one of such a bizarre mass hierarchy persist. Various models to accomodate the new find in a phenomenologically viable way have been proposed [81,82] with different degrees of success but none of these address the issue of the reported anticorrelation of the solar neutrino flux with the sunspot activity [47]. As we have seen in the section 5.1, perhaps the only natural way to explain such a phenomenon is the assumption of a non–zero magnetic moment for the neutrino [67].

Thus the problem on hand is to realize a scheme that not only produces the required hierarchy for the neutrino masses consistent with the mixing and decay constraints, but can also account for a substantial (transition) magnetic moment for the nearly massless ν_e. While doing so, care must be taken to ensure that the result is not dependent on a severe fine–tuning of parameters. A particularly appealing solution for the first part of the problem has been proposed by Glashow [81]. The idea is to extend the SM fermion spectrum to include three gauge singlet right handed neutrinos and employ the singlet Majoron scheme [83] to break the global $B - L$ symmetry. The neutrino mass matrix, in the $(\nu_L \ \nu_R)$ basis, then reads

$$\mathcal{M} = \begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix}, \tag{5.3.1}$$

where m gives the Dirac masses and $M = M^T$ is the Majorana mass term. Assuming that M is of rank two (*i.e.* it has one zero eigenvalue), the see–saw mechanism [55] generates the lighter masses to give a spectrum comprising of four Majorana neutrinos, two heavy ones of masses $\sim O(M)$, two light ones of masses $\sim O(m^2/M)$ and a nearly Dirac one of intermediate mass $\sim O(m)$. Taking $m \sim 17\,keV$ and $M \sim 300\,GeV$, one then identifies the Simpson neutrino with the pseudo–Dirac particle — comprised mainly of $\nu_{\tau L}$ and the massless ν_R — and has $m(\nu_e), m(\nu_\mu) \sim O(10^{-3}\,eV)$, values that can explain the solar neutrino puzzle via the MSW mechanism [46]. There however is one catch to this beautiful ansatz, for obtaining a 17 keV Dirac mass term necessitates Yukawa couplings of the order of 10^{-7}, a none–too–pleasing choice.

In this section [3] we marry the concepts outlined above and in Section 5.1 to construct a model with all the required features namely that the neutrino mass matrix should be such that it should accomodate a 17 keV Dirac ν_τ with a 1% mixing with ν_e and be consistent with the $(\beta\beta)_{0\nu}$ and neutrino oscillation experiments. Moreover a relatively large $\mu_{\nu_e\nu_\mu}$ should be present. However, unlike in Glashow's case [81], the neutrino mass matrix is a 5×5 one with M now a 2×2 matrix of rank one. This results in one of the neutrino being exactly massless at the tree level. Further, the Dirac terms for the neutrinos arise as

[3] Based on the work in ref. [84]

a consequence of radiative corrections and are hence naturally kept small.

The gauge group we choose is a slight modification of that in section 5.1 *viz.* $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes SU(2)_H$ with the SM fermions ψ_L, ψ_L^c, Q_L, U_L^c and D_L^c transforming as triplets under the horizontal group. In addition, we include a singlet neutrino field $N_L^c\,[(1,1,0,1)_1]$ with a non–conventional lepton number denoted by the subscript. The scalar sector consists of the SM higgs ϕ_s and in addition the fields $\Phi\,[(1,2,-1/2,3)_0]$, $H, \tilde{H}\,[(1,2,-1/2,5)_0]$ to give masses to the charged fermions; $\sigma\,[(1,1,0,3)_2]$, $\Sigma\,[(1,1,0,2)_8]$ to break the $SU(2)_H$ and lepton number. The neutrino Dirac masses $[\sim O(10keV)]$ are generated by $\tilde{\phi}[(1,2,-1/2,2)_2]$, which acquires *v.e.v.* only through radiative correction through a diagram involving the fields $\xi_1\,[(1,2,-3/2,1)_{-2}]$, $\xi_2\,[(1,1,-1,3)_0]$, $\xi_3\,[(1,2,-3/2,1)_2]$ and $\xi_4\,[(1,1,-1,1)_4]$. We also need two color triplet fields $\eta\,[(\bar{3},1,1/3,3)_{-1}]$, $\chi\,[(3,2,1/6,1)_{-1}]$ to radiatively generate the magnetic moment. The model thus offers charge quantization as there is no gaugeable (*i.e.* anomaly–free) global $U(1)$ symmetry [85].

We impose a further discrete symmetry (to be broken softly in the higgs sector), under which $(\psi_L^c, U_L^c, \Phi, \tilde{H}, \xi_3) \longrightarrow -(\psi_L^c, U_L^c, \Phi, \tilde{H}, \xi_3)$. The most general Yukawa term is then

$$\begin{aligned} \mathcal{L}_Y &= f\psi_L^c Q_L \eta + f' D_L^c \psi_L \chi + \psi_L^c \psi_L (g_3^e \Phi + g_5^e \tilde{H}) + U_L^c Q_L (g_3^u \Phi + g_5^u \tilde{H}) \\ &\quad + D_L^c Q_L (g_3^d \phi_s + g_5^d H) + g_D N_L^c \psi_L \tilde{\phi} + g_M N_L^c N_L^c \sigma^\dagger . \end{aligned} \tag{5.3.2}$$

Instead of writing down the full scalar potential, we rather focus on the terms that are responsible for the physics we seek, namely a radiative *v.e.v.* generation and a one–loop magnetic moment. These are $\lambda_1 \tilde{\phi} \Sigma^\dagger \xi_4 \xi_1^\dagger$, $\sigma \xi_1 \xi_2^\dagger (\lambda_2 H^\dagger + \lambda_3 \phi_s^\dagger)$, $\sigma \xi_2 \xi_3^\dagger (\lambda_4 \tilde{H} + \lambda_5 \Phi)$, $\sigma \xi_3 \xi_4^\dagger (\lambda_6 \tilde{H} + \lambda_7 \Phi)$ and $\eta \chi \sigma (\kappa \phi_s + \kappa' H)$.

A proper choice (always possible) of the higgs couplings along with the $SU(2)_H$ symmetry can be exploited to ensure that only the $T_{3H} = -1$ component of σ and $T_{3H} = -2$ component of H acquire *v.e.v.* and $\tilde{\phi}$ does not acquire any tree level vacuum expectation values. The rest of the scalars may assume any *v.e.v.* consistent with charge conservation. For phenomenological consistency we demand that $\langle\sigma\rangle$, $\langle\Sigma\rangle \sim O(10^6\ GeV)$ and that any other *v.e.v.* be of the order of the electroweak scale or less. The *v.e.v.* of $\tilde{\phi}$ is however not protected by any symmetry and at one–loop level the diagram in Figure 5.2 contributes.

Figure 5.2: One–loop diagram responsible for radiative generation of $\langle\widetilde{\phi}\rangle$

Written symbolically,

$$\langle\widetilde{\phi}\rangle \sim \frac{\lambda^4}{16\pi^2}\frac{m_{Horiz}^4 m_{wk}^3}{m_\xi^4 m_{\widetilde{\phi}}^2} \approx 100\ keV - 1\ MeV, \tag{5.3.3}$$

on assuming that $m_\xi \sim m_{Horiz}$, $m_{\widetilde{\phi}} \sim m_{wk}$ and $\lambda_i \sim O(10^{-1})$.

The neutrino mass matrix is then a 5×5 one of a form similar to that in eqn.(5.3.1) (with M now a 2×2 matrix of rank one) and can be written as

$$\mathcal{M} = \begin{pmatrix} M_1 & M_2 \\ M_2^T & A \end{pmatrix} \tag{5.3.4}$$

where

$$M_1 = \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ a_1 & a_2 & a_3 & 0 \end{pmatrix} \quad \text{and} \quad M_2 = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ 0 \end{pmatrix} \tag{5.3.5}$$

Here $A = g_M\langle\sigma_{-1}^0\rangle$ and the elements a_i, b_i are of the order of $g_D\langle\widetilde{\phi}\rangle$ (the differences arising on account of the Clebsch–Gordon coefficients). As $\mathcal{M}$ is of rank 4, we have one exactly massless neutrino. $\mathcal{M}$ can be approximately block–diagonalized (for details, see Section 4.2) to the form $\begin{pmatrix} \widetilde{m} & 0 \\ 0 & \widetilde{A} \end{pmatrix} + O(\rho^2 M_1)$, where $\rho = A^{-1}M_2^T$, $\widetilde{m} = M_1 - M_2\rho$ and $\widetilde{A} = A + \frac{1}{2}(\rho M_2 + M_2^T\rho^T)$.

The rest of the spectrum then consists of a light Majorana neutrino of the order of $b_i^2/A \sim O(10^{-5}\ GeV)$, a pseudo–Dirac particle with mass $\sim \sqrt{\sum_i a_i^2} \equiv 17\ keV$ (the mass splitting $\sim b_i^2/A$) and a superheavy Majorana one with mass $\approx A \sim 10^4\ GeV$ (we have assumed Yukawa couplings g_D, $g_M \sim O(10^{-2})$).

The neutrino mixing angles are given essentially by the ratios of a_i and b_i apart from the contribution from the electron mixing matrix and can easily be chosen to satisfy the experimental constraints. The contribution to neutrinoless double beta decay amplitude is very small and assuming a diagonal form for the charged lepton mass matrix, is given by [51,53] $(\tilde{m})_{11}$. However the ν_e and ν_μ masses are very small.

The Majoron (ϑ) in our model is mainly comprised of $Im(\sigma)$ and $Im(\Sigma)$ with a small admixture of $Im(\tilde{\phi})$ of the order of $\langle\tilde{\phi}\rangle/M_{Horiz}$ and contributions from other $SU(2)_L$ doublets further suppressed by a factor of $(\langle\tilde{\phi}\rangle/M_{wk})^2$. The coupling of the charged fermions with ϑ is then very small and hence consistent with all astrophysical constraints [86]. Looking at the Dirac terms in the fourth row and column, we see that these arise due to the *v.e.v.* s of different components of $\tilde{\phi}$. As these scalars do not have identical contributions to ϑ even to the leading order, the neutrino mass and the Majoron coupling matrices are not diagonalized simultaneously. This results in the light neutrinos having a substantial non–diagonal coupling with ϑ (of the order of m_ν/M_{Horiz}) and hence affords a decay channel to the tau neutrino of the form $\nu_\tau \to \nu_{\mu,e} + \vartheta$. The ν_τ is then comparatively short–lived, with a lifetime $\sim 10^5\ sec$ and cosmological requirements are easily satisfied[80]. As is easily recognised, this feature is a consequence of the Majoron having contributions both from $SU(2)_L$ doublets and singlets and is absent for the usual singlet Majoron models.

It is curious to note that the results of Section 5.2 can be sharpened in the context of the 17 keV neutrino. For, if it is actually comprised mainly of the $SU(2)_L$ doublet ν_τ, as is being hypothesised, then it satisfies all the criteria to be considered as a probe for the physics in the interior of a neutron star. This would result in the very strong bound (see equations 5.2.17 and 5.2.21) of $\alpha_1 \lesssim O(10)$ on parity violating effects in gravity.

The first set of criteria having been satisfied, we now turn to the problem of the magnetic moments. In fact, it is easy to see that the results of Section 5.1 are carried through without any modifications. We have thus obtained a model that naturally incorporates a 17 keV ν_τ

as well as a sole (neglecting lepton mixing) transition magnetic moment $\mu_{\nu_e \nu_\mu} \sim 10^{-12} \mu_B$ for nearly massless neutrinos. The latter effect obviously vanishes exactly in the limit of $SU(2)_\nu$ symmetry only to appear when the symmetry is broken. Yet the mass term remains identically zero, for the effective *v.e.v.* is antisymmetric in the family space. This effect allows us, unlike many earlier models [70,71], to totally dispense with any naturalness condition to suppress mass generation. Thus to this order, the only contribution to the light neutrino masses come from the see–saw terms which are very small anyway. However, as there is no conserved lepton number, one expects there to be radiative mass generation and the two–loop contribution would typically be of the order of 10^{-4}–10^{-5} eV. That this is so is easily seen from the fact that diagrams similar to those in Fig. 5.1 but with the coupling $\kappa' H \eta \chi \sigma$, contributes to the Majorana mass once the $T_{3H} = 0, -1$ components of H get *v.e.v.*s radiatively. The 17 keV ν_τ in the present model decays dominantly into a ν_μ and a doublet–singlet Majoron.

Chapter 6

Summary

In this concluding chapter we take stock of the achievements catalogued in the earlier parts. We had started out with the objective of understanding the generation of fermion masses in the standard model and its extensions made popular by the possibility of answering questions that the former cannot even deign to ask. As with the case of the objects of our study, the thesis is also divided into two parts. The first half deals with the strongly interacting particles, namely the quarks, whereas the second half is devoted to the study of leptons, or without any loss of generality, aspects of neutrino physics.

To circumvent the the lack of any predictive power of the standard model as far as the quark sector parameters are concerned, various models that go beyond the SM have been proposed in the literature. Based as these are on higher symmetries, they give specific forms of the quark mass matrices resulting in relations between the ten parameters *viz.* the six masses, the three mixing angles and the CP–violating phase. In the first part of this thesis we have looked at the predictions of these models and compared them with the results of some recent experiments. It was found that, contrary to previous claims, the B_d^0–$\overline{B_d^0}$ mixing extent x_d and the $\Delta S = 2$ CP violation parameter ϵ_K do not rule out either the Stech or the Fritzsch model or for that matter even their derivatives. Regions of validity, though narrow, could still be found. However once the $UA1$ result for the direct CP violation parameter ϵ'_K came into the picture, the situation changed completely. The Stech model and all others incorporating the ansatz were ruled out emphatically. While the Fritzsch scheme did survive, the parameter space available to it was curtailed significantly with experimental agreement limited to the case of a none–too–heavy top quark ($m_t \lesssim 90\,GeV$). Later experiments have shown that such a top quark is not allowed in the context of the minimal standard model, thus ringing the death–knell for such models as candidates for low energy physics.

The failure of the existing ansätze for quark masses and mixings led to a model–independent analysis of the problem. Looking at the most general form for such matrices, we found that observational data do not constrain them to a great degree but rather allow them a continuous range that is divided into four disjoint sectors associated with the relative signs of the mass terms. The large width afforded to these parameters is traced to the indeterminacy in the masses of the light quarks and owes comparatively little to the lack of knowledge about the mixings involving the third generation. Surprisingly (?), all the models discussed in the literature lie in the same sector. This could provide a clue to the direction that efficient model–building could adopt in the future.

The first question that we address in our effort to understand the wide field that neutrino physics is, relates to neutrinoless double beta decay [$(\beta\beta)_{0\nu}$]. Conventional wisdom had it that the extent of $(\beta\beta)_{0\nu}$ is specified by the Majorana mass of the electron neutrino ν_e. A reexamination of the problem showed us that this need not be the case and that many different possibilities do exist. In fact the $(\beta\beta)_{0\nu}$ amplitude is determined primarily by only one element of the effective low–energy neutrino mass matrix. This then means that one could have a scenario where the physical ν_e is a Dirac particle but the $(\beta\beta)_{0\nu}$ rate is quite high and conversely a case of a light Majorana ν_e with identically zero $(\beta\beta)_{0\nu}$ rate. Thus the numerous ongoing $(\beta\beta)_{0\nu}$ experiments cannot differentiate between a (pseudo–)Dirac and a Majorana ν_e except in the simplest of cases. The only comment they can make about is that regarding the magnitude of the particular term mentioned before and hence the extent of lepton number violation that could be visible at low energies. We further go on to construct supersymmetric grand unified theories (both $SO(10)$ and $SU(5)$) that naturally accomodate the scenarios that we talk about.

The longstanding discrepancy between the observed levels of solar neutrino flux and the theoretically calculated rates have been a source of embarassment. A possible explanation that has the added advantage of accounting for the reported anticorrelation of the observed flux with the solar magnetic activity is that of a non–zero neutrino magnetic moment (μ_ν). However it is very difficult to generate a large enough μ_ν while keeping the neutrino mass within acceptable limits. Though some models had been proposed in the literature, they all suffered from one weakness or the other. We have proposed a novel mechanism to achieve the same goal. The cornerstone of the ansatz is that to a given order in perturbation theory, the only effective higgs *v.e.v.* term coupling to fermion currents violating lepton number by two units is antisymmetric in the generation space. This leads to a non–zero transition moment while keeping the mass term identically zero. The $O(3)_H$ model that we construct, unlike the others of its genre, neither needs extra fermions nor does it treat the standard model fermions unequally. Moreover, it has the added advantage of avoiding the pitfalls of earlier efforts, namely severe fine tuning of parameters or an unwanted Goldstone boson.

Neutrinos, though seemingly insignificant on account of their very weak interactions and very low mass, are actually of great importance, especially in the astrophysical and the cosmological context. A non–zero mass for the neutrino would lead to interactions flipping its helicity, and as a consequence allow neutron stars to lose energy at a very high rate. Normally one would expect the electroweak interactions to dominate, but we find that gravity could be a strong contender for supremacy in the case of low energy neutrino scattering! Though at the initial stages of stellar collapse the neutrino energy is probably too high for such an eventuality, at the late stages of supernova evolution these effects could be of great importance. Proceeding with the study, we also look at the possible discrete symmetry violations in gravity that a deviation from the geometric theory would allow. It is found that there is only one discrete symmetry (in this case, parity) violating term consistent with special relativity that could lead to helicity flip. Comparing the scattering cross–sections with the limits put by the observed neutrino bursts from the supernova $SN\,1987A$, very strong bounds are put on such interactions.

The aspects of neutrino physics discussed hitherto might seem to be bit disjointed to the casual reader. That this is not so, is brought out by the last topic that we consider. A lot of interest has been generated recently by independent claims of the experimental signature of a 17 keV neutrino that mixes with the ν_e to as great an extent as 1%. The study of $(\beta\beta)_{0\nu}$ clearly demonstrates that it cannot be a Majorana particle unless there are other such particles with exactly the right mixing with ν_e so that the individual contributions to $(\beta\beta)_{0\nu}$ cancel giving an acceptably low level. Accomodating such a (pseudo–)Dirac neutrino quite often leads to problems either with their decay or with unaesthetically small Yukawa couplings. We present a model that avoids these difficulties. The 100 keV scale is generated radiatively and hence there is no need to suppress the Yukawa couplings. The see–saw mechanism assures a superheavy Majorana neutrino, a pseudo–Dirac 17 keV ν_τ and extremely light Majorana ν_e and ν_μ. The ν_τ evades the cosmological problem by decaying very fast into a lighter neutrino and a singlet–doublet Majoron that is not constrained significantly by either the Z–decay width or the various astrophysical bounds. The resolution of the solar neutrino problem is achieved through the radiative generation of a large transition magnetic moment connecting the two ultralight neutrinos. As for the ν_τ, its relatively large mass can be used to better the previous bounds on parity violation in gravitational interactions by as much as three orders of magnitude.

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