LINEAR AND NONLINEAR THEORY OF IONOSPHERIC IRREGULARITIES

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TO MY PARENTS

C E R T I F I C A T E

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

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September 1, 1976.

CONTENTS

STATEMENT

In this thesis, we have studied some linear and nonlinear properties of processes occurring in the ionospheric plasma system and have discussed the results in the light of experimental observations of ionospheric irregularities. In the equatorial F region, the long scalesize irregularities follow a typical k^{-2} power law and at times develop coherently with asymmetric shapes. The k^{-2} power law is explained by means of a two-step theory in which a turbulent generation of short scalesizes takes place on the gradients of long scalesizes, if the gradients (representing amplitude to scalesize ratio) exceed a certain critical value. This leads to the k^{-2} law. Further, it is shown that the two major instabilities generally believed to be responsible for the large scalesize irregularities, in the equatorial spread F, the Rayleigh - Taylor and collisional drift instabilities, may nonlinearly evolve as coherent nonsinusoidal structures, as observed, due to finite - larmor - radius and ion - viscosity effects, respectively. For the equatorial electrojet region, a generalized fluid theory is developed for the excitation of mixed electrostatic - electromagnetic (es - em) modes by retaining e.m. effects (which become important at long wavelengths, so that $ck/\omega_{pe} \lesssim 1$), in the analysis. A new low - frequency mixed e.s. - e.m. instability is discovered, which has a lower threshold than the ion - acoustic speed and which requires a finite wave - vector, $k_{\parallel}$. It should be observable as phase - correlated density - cum - magnetic field fluctuations in the equatorial electrojet. The two - step theory for the generation of small scalesize type II equatorial electrojet irregularities has been treated by using a nonlinear mode - coupling calculation and it is shown that the small scalesizes can grow at the expense of large amplitude large scalesizes via two - stream or cross - field instabilities. Next we have shown that field - aligned currents, frequently observed in the auroral F and E regions, can destabilize ion - cyclotron waves in a collisional medium (like the ionospheric E and F regions) due to dissipative effects in the parallel electron motion. It is also shown that the electrostatic ion - cyclotron waves possess some propagating sawtooth-like solutions at small but finite amplitudes. Finally, we show that the cross - field current driven resistive ion - acoustic waves, which may be of relevance to the auroral E region, can have special steady state nonharmonic solutions due to finite ion - viscosity effects.

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CHAPTER 1

INTRODUCTION

The earth's ionosphere has been an object of intense study for several decades now. Successively improved experimental techniques and theoretical ideas have considerably enhanced our understanding of the structure and nature of the ionosphere. Such studies are of interest even today because of their continued widespread applications to disciplines like radio communications, radio astronomy, planetary atmospheric studies, etc.

The gross structure of equilibrium ionosphere in normal quiet conditions and the physical processes involved in its formation are well understood now. The ionosphere is, technically defining, that part of earth's atmosphere which is partially ionized by the solar radiations in the daytime. Normally bottom of the ionosphere is defined to be at 70 kms. altitude and the top is loosely assumed to be at $\sim$ 1000 kms. An ionization density versus altitude profile, plotted for daytime conditions, would normally show a smooth rise in the density from 70 kms. upto $\sim$ 85 kms. when a peak is observed in the curve. After this peak, the rise is slowed down. This region is called the D region. The density slowly rises to form another peak at around 100 kms. and this part is called the E region. After the E region peak the rise continues until at around 200 kms. a third peak, denoting the maximum density in the ionosphere, is observed. This part of ionosphere is called the F region. Beyond the F peak, density decreases until the upper boundary of ionosphere merges into the plasmasphere. In the present work, we shall be interested only in the phenomena occuring in E and F regions.

The aforediscussed daytime ionospheric profile is determined by various factors like the production and loss mechanisms, the distribution of neutrals with respect to height and the intensity of incoming solar radiation etc. The production mechanism is the photoionization of neutrals by solar uv radiation in the daytime. The chief loss processes are losses due to recombination and diffusion, with the latter being important only at F region altitudes. In the nighttime, as the photoionization is "switched off", recombination losses eat up most of the lower ionosphere. The D region disappears while the E region ionization is depleted considerably. In the F region, recombination process is very slow and therefore F region survives even in the nighttime, although the peak now shifts to higher heights like 400 kms. The existence of peaks in the ionosphere, imply equilibrium ionization density gradients in the medium and is one of its characteristic features. Another feature of the equilibrium ionosphere is the existence of currents in it. One such current system flows in the E region and is widely known as the S_q current system. The physical origin of this current system lies in the fact that the ionospheric conductivity transverse to the earth's magnetic field displays a maximum at the E region heights, with much smaller values below and above it. Thus, when the tidal winds carry the neutral atmosphere across the earth's magnetic field, emf and currents are induced in the embedded ionization owing to its high transverse conductivity. This current system is locally enhanced at the equator where the special geometric configuration of nearly horizontal magnetic field lines causes an inhibition of vertical current, which results into a polarization electric field. The resulting enhanced current system is commonly known as the equatorial electrojet. In the auroral regions, where the magnetic field lines are nearly vertical, the precipitation of charged particles along the field lines during the auroral activity, constitutes the flow of intense field-aligned currents, all the way from the top of ionosphere down to E-region altitudes.

The numerous experimental observations on ionosphere (using experimental techniques like ionosondes, VHF back - scatter radar, rocket and satellite in-situ probes, radio star scintillation measurements, etc.) have established that the ionization density variation with altitude is often not smooth, but instead, displays enhancements or depletions superimposed on a smooth background distribution. These are termed as 'ionization irregularities'. Ionization irregularities are observed in all regions of ionosphere, at all geomagnetic latitudes, at any time of the day and with certain characteristic patterns of geographical and temporal variation. The irregularities are characteristically field - aligned and display a wide range of spatial scale - sizes e.g. from a few meters to a few kms. This thesis is devoted to an understanding of the origin and certain observed features of the E and F region irregularities, using principles of plasma physics.

Note that the background ionosphere can be regarded as an essentially unbounded plasma medium which remains temporally steady on the characteristic time - scales of the irregularities. The irregularities are then caused by density fluctuations associated with low - frequency electrostatic waves in the medium. Such electrostatic waves would be driven by the sources of free energy in the medium. The obvious sources of free energy in the equilibrium ionosphere are density gradients, electric fields and the associated currents. Consequently, several linear plasma instability theories have been developed in order to explain the ionospheric irregularity observations. In a linear theory one perturbs the equilibrium configuration with an infinitesmally small perturbation and studies its stability properties. Following such a procedure we can obtain information regarding conditions when the instability is likely to occur and the typical growth time in which it is likely to develope into saturated state. Such theories developed for the E and F regions have been successful in accounting for many observed conditions of onset and time scales of growth of irregularities[20].

As an example, in the equatorial E region, current-driven two-stream instability and density gradient-driven gradient drift instability have been developed to explain the observations of small scalesizes (~ a few meters) and large scalesizes (~ a few hundred meters) respectively. The two-stream instability occurs when the relative drift between electrons and ions exceeds the ion sound speed in the medium and is invoked to explain the type I echoes of VHF backscatter radar[18]. The gradient drift instability can occur when the relative electron drift is smaller than the ion sound speed and requires the applied electric field and the gradient to be in the same direction[57, 59]. Note however, that none of these linear theories can explain the excitation of small scale irregularities, causing type II scatter echoes, in the equatorial electrojet when the electron drift speed is less than the ion-acoustic speed.[73]

In the equatorial F region, a sharp density gradient is formed on the lower side in the post-sunset hours due to recombination losses and absence of photoionization. This results in the excitation of gradient - driven plasma instabilities. The Rayleigh - Taylor instability of a plasma possessing a density gradient antiparallel to a gravitational force and perpendicular to the earth's magnetic field is believed to generate large scalesizes of the order of a few kms.[28] The collisional drift instability driven by the electron-ion coulomb collisions in the parallel electron motion, has been invoked to explain the generation of scalesizes of the order of a few hundred meters or less[33]. In this region too, the generation of small scalesizes responsible for VHF backscatter echoes can not be explained by any linear theory directly. Furthermore, above the Fpeak no satisfactory linear theory seems to be capable of explaining the generation of large scalesizes. In the top side F region, the density gradient, much feebler as compared to that on the bottom side, points in the same direction as gravity and hence is stable for Rayleigh - Taylor instability.

In the auroral regions too, the two-stream and gradient drift instabilities have been invoked for E-region where cross-field currents, known as auroral electrojet, flow[27]. A field - aligned current driven ion - cyclotron wave[12] and a cross-field current-driven resistive ion acoustic wave[38] have also been cited in the literature as possible sources of irregularities in the auroral ionosphere.

Recent rocket-borne and satellite-borne in-situ measurements of ionization irregularities have yielded new important information regarding the spectra and in some cases, the shapes of the irregularities. As these observations correspond to the irregularities being in a saturated state of evolution, nonlinear theories need to be worked out to explain these observations. The development of nonlinear theory usually implies going beyond the small amplitude approximation, taking mode-coupling effects into account, etc. Some work has already been started on these lines by many workers. Thus, in order to explain the rocket - observed saw - tooth structure[56] of the vertical electron density profile in the equatorial E region, it has been suggested that mode-coupling of linearly unstable two-dimensional perturbation to linearly damped spatial structures in the vertical direction, ultimately leads to the saturation of cross-field instability and appearance of a total saw - tooth profile in the vertical direction[63]. Sato[67] and Rogister[60] have considered a quasi-linear shielding of electric field as a stabilising mechanism of two-stream instability and Rogister[61] has suggested a mode-coupling to linearly damped modes as the saturation mechanism for one-dimensional cross-field instability. To explain the generation of small scalesizes in equatorial electrojet associated with type II echoes, Sudan et al.[73] have proposed a two-step 'multi-linear' mechanism, in which small scalesizes grow linearly on the top of large amplitude large scalesizes due to cross-field or two-stream instability mechanisms. Analogously, for the equatorial spread F, small scalesize generation has been attributed to the kinetic drift instability, due to inverse Landau damping by electron motion parallel to the field lines, occuring on the top of some linearly driven large amplitude large scalesize mode.[32]

Little work has been done to explain the observations regarding spectra and shapes of F region irregularities. The satellite observations have shown that the large scalesizes in the F region have a power spectra going as k^{-2} and because most of the time they are observed like intense noise, these kind of irregularities have been called turbulent irregularities.[16] Sometimes instead of turbulent ones, a different class of irregularities has been observed which shows that the power is confined to a leading scalesize and its sub-multiples. These irregularities have distorted sinusoidal shapes and have been termed as 'wavelike'[16]. Later, rocket observations have further confirmed these findings[41].

The first part of our thesis is therefore devoted to the development of nonlinear theories which offer a possible interpretation of the observations of equatorial F-region irregularities. An interpretation of the observed k^{-2} law is proposed, using a two-step theory. A mode-coupling calculation shows that the collisionless and collisional drift instabilities excite small-scale perturbations, if the gradient associated with a large amplitude long scalelength wave (which is the ratio of amplitude to scalesize) exceeds a certain critical value. Thus in the equilibrium situation, this ratio remains constant over a range of long wavelengths, and rests near the threshold value. This provides a natural explanation for the k^{-2} power law. This calculation also shows that the secondary drift modes do not grow on a large scalelength drift mode because the appropriate mode coupling terms cancel out. Therefore in the equatorial F region, Rayleigh - Taylor instability driven long wavelengths are probably responsible for the two-step process.

The nonlinear development of a linearly unstable situation into a turbulent or a coherent state is largely determined by the initial conditions. If initially, many modes with different scale-lengths are excited, then a turbulent state is likely to result whereas if only one mode is preferentially excited linearly, then coherent mode can evolve. In the latter event, nonlinear effects lead to a steepening of the coherent mode and can generate a saw-tooth shaped structure. The power distribution between the fundamental scale-size and its sub-multiples in this case also follows a k^{-2} law. Observationally, the difference between a turbulent k^{-2} spectrum and a coherent k^{-2} spectrum would be the presence of intense phase correlations between various k-modes in the latter case. However, such phase correlation measurements have not yet been made and so it is not possible to decide which (if any one at all!) of the two nonlinear mechanisms dominates the picture. It is even likely that both effects are simultaneously operating, the two-step process leading to generation of new structures normal to the original gradient and the coherent steepening effects leading to saw-tooth shapes.

In a separate section we have therefore developed a coherent nonlinear theory for the Rayleigh-Taylor instability. Carrying out the calculation in the one-dimensional approximation, we show that ion inertial effects lead to a nonlinear coupling of the linearly unstable mode to finite Larmor radius (FLR) stabilized oscillatory modes, leading to the evolution of a stationary coherent steepened structure. It is quite clear that a more realistic calculation would be a two-dimensional one, where the $(\underset{\sim}{E}\times\underset{\sim}{B}).\nabla n$ term in the continuity equation generates the important nonlinear effects. However, such a calculation cannot be done analytically and has been postponed for future numerical work.

In the last section of the first part, we study the nonlinear coherent evolution of the collisional drift instability under the steady-state approximation as applicable to the equatorial F region situation. It is found that due to the ion viscosity effects a finite amplitude propagating wave can be set up in the medium which can have nonsinusoidal shapes.

In the second part of our thesis, we have dealt with the irregularities associated with the equatorial electrojet. It is well known that at large wavelengths, such that $ck \lesssim \omega_{pe}$, electromagnetic effects on streaming instabilities become important. Past investigations for the instabilities in the electrojet have been made only for the electrostatic perturbations. In the present study, we have developed a generalized fluid treatment for cross-field current driven electrostatic-electromagnetic instabilities in a weakly ionized inhomogeneous magnetoplasma[39]. We discover a new low frequency es-em instability for modes, satisfying $ck \lesssim \omega_{pe}$, which have a finite $k_{\parallel}$ (parallel-propagation-vector). This instability can be excited even when the relative drift between electrons and ions is smaller than the ion sound speed; in the electrojet, it would excite almost field-aligned density-cum-magnetic field perturbations of large scalesizes ($\sim$ 500 meters).

Next we have considered the two-step process of Sudan et al.[73] for the generation of small scalesize type II irregularities, by adopting a nonlinear mode-coupling approach. The results of earlier workers are corroborated by our treatment also, viz., the small scalesizes grow in the presence of large amplitude, large scalesizes, even if the electron drift velocity is smaller than the ion-sound speed.

In the third part of our work, we have concentrated on the phenomena taking place in the auroral ionosphere. First, we have investigated the destabilizing effect of field-aligned currents, frequently observed in these regions during activity period on an obliquely propagating ion cyclotron mode in a collisional plasma system such as the ionosphere. It is found that, these modes grow if a threshold current is exceeded, with the dissipative mechanisms like electron thermal conductivity, resistivity etc. in the parallel electron motion, causing the growth. We consider the cases of both a fully ionized plasma as well as a weakly ionized plasma and apply these results to upper F - and E - regions, respectively. In the upper F - region, effects due to coulomb collisions predominate over the charged particle neutral collisions in the wavelength range of interest and hence the assumption of F-region as a fully - ionized system is used in our work. In both cases, the observed currents[40, 8] are found to exceed the threshold conditions and drive the instability. Observational evidence of ion-cyclotron waves in ionosphere vis-a-vis these results has been discussed.

Next, we have investigated the propagation characteristics of an obliquely propagating electrostatic ion-cyclotron wave at small but finite amplitudes in a collisionless plasma. It is found that the nonlinearity associated with the ion-inertial motion for this wave deforms the wave shape to a saw-tooth-like shape. This study was made in analogy to similar studies made for two other low frequency electrostatic modes of a warm collisionless plasma, ion-acoustic wave[65] and a drift wave[50] and was intended as a first step towards an eventual development of a nonlinear theory for the collisional ion-cyclotron instability, which is a topic of future study.

Lastly, we also investigated the nonlinear coherent evolution of a cross-field current-driven resistive ion-acoustic waves in a weakly-ionized magnetoplasma. This instability has been cited as of relevance to the auroral E-region irregularities[38]. This instability requires a finite parallel wave vector and is driven by the dissipative mechanisms like resistivity, thermal conductivity, etc. in the parallel electron motion. Inclusion of ion-viscosity leads to some special steady-state solutions of the resistive ion-acoustic waves with nonsinusoidal shapes.

We recapitulate all the major results of our work in the last (fifth) chapter.

CHAPTER 2

IRREGULARITIES IN EQUATORIAL F REGION

The irregularities in equatorial F region have been studied both experimentally and theoretically for several decades now. The experimental investigations have been carried out using various techniques like the ionosonde, VHF backscatter radar and more recent rocket - and satellite - borne probe measurements. As a consequence, a wealth of observational data exists regarding various features of these irregularities. There have also been numerous theoretical attempts to understand the generation mechanism of these irregularities in the past. Johnson and Hulburt[36] and Dungey[15] suggested a gravitational (flute) instability of the underside of F layer (as the density gradient and gravity are antiparallel to each other there) which would lead to plasma density irregularities in the medium. Martyn[46] suggested an $\underset{\sim}{E}\times\underset{\sim}{B}$ instability of an inhomogeneous plasma layer drifting across the confining magnetic field due to an $\underset{\sim}{E}\times\underset{\sim}{B}$ drift, in which perturbations on the underside of F layer would grow if it were moving upwards and vice versa. Dagg[11] considered the convection of irregularities generated in the moderate latitude E region to the equatorial F region along the highly conducting equipotential magnetic field lines of earth. In a more quantitative treatment, Farley[17] later showed that the minimum scalesizes that could be transported this way, would be of the order of a few kilometers. Dessler[13] postulated that the hydromagnetic waves generated in the magnetosphere and propagating into the earth's ionosphere could be responsible for the Spread F irregularities.

The backscatter radar measurements of equatorial spread F have revealed many new features of these irregularities. Farley et al.[19] have critically reviewed the above theories in the light of these observations and have found that none of the theories could satisfactorily explain the experimental data. For example, the Rayleigh - Taylor instability [Dungey, Johnson and Hulburt] could not explain the generation of irregularities above the F peak, where gravity and the density gradient are in the parallel direction. Martyn's theory of $\underset{\sim}{E}\times\underset{\sim}{B}$ instability could not be accepted because irregularities on the underside of F layer were observed even when it was moving downwards while according to the theory, in this case, the perturbations should have been damped away. Dagg's theory of convection of irregularities from the high - latitude E region to equatorial F region failed because it could account for only scalesizes of the order of a kilometer or more, whereas the radar echoes are caused by small scalesizes of the order of 3 meters. Similarly, Dessler's theory of irregularities due to hydromagnetic waves was able to explain only scalesizes of the order of tens of kilometers or more.

Following Farley et al.[19], it was recognized that probably a single instability mechanism may not be responsible for producing equatorial spread F irregularities in the whole observed range of scalesizes, namely, from a few kms. down to a few meters. Haerendal[28, 5] has proposed that most of the characteristic features of the large scalesize spread F irregularities could be explained by a collisional modification of the theories proposed by Johnson and Hulburt[36] and Dungey[15] viz., a collision dominated gravity induced Rayleigh - Taylor instability. Haerendal has further modified these earlier treatments by taking the integrated means of plasma quantities in unit area cross-section flux tubes of earth's magnetic field lines at the equator, averaged over the entire length of the dipole field lines. This theory is capable of explaining generation of irregularities even above the F peak. This is because, since the length of the flux tube increases with altitude, the plasma content in it also increases with altitude, and one obtains a positive density gradient even above the F peak. In this framework, the F region is shown to be unstable upto the altitudes of approximately 100 kms. higher than the F peak. Hudson and Kennel[38] have recently shown that the collisional drift instability has a significant growth rate for wavelengths of the order of 100 meters or less, and therefore would contribute to the total spectrum of spread F irregularities. For the generation of small scalesizes of the order of a few meters, a two-step theory has been proposed[32] in which short wavelength secondary perturbations grow on the gradients of large amplitude large scalesize primary irregularities due to the collisionless drift instability, driven by inverse electron - Landau damping along the magnetic field lines.

The current understanding of equatorial F region irregularities may thus be summarized as follows. For the underside of F region and upto 100 kms above the F peak, the collisional Rayleigh - Taylor instability is the source of irregularities of scalesizes ranging from a few kms. down to a few hundred meters. The collisional drift instability is probably responsible for the scalesizes of the order of a few hundred meters or less and the small-scale irregularities of the order of a few meters are generated by the kinetic drift instability in a two-stage mechanism. The excitation mechanism for large scalesizes at the heights greater than 100 kms. above F peak is yet not understood at all.

Recent observations by the in-situ probe measurements have provided more information about the irregularities, like shapes, spectra etc. Such observations mostly correspond to the saturated state of irregularities and therefore require the consideration of nonlinear effects in the theoretical treatment, which are developed to interpret them. In the present chapter, we present nonlinear theories to explain two characteristic features of equatorial F region irregularities. First, their property of obeying a typical k^{-2} power law at large scalesizes[16] and the second, their occasional development into steepened saw tooth - shaped structures[16, 41]. In the following sections, first we present a two-step theory interpretation of the k^{-2} law and then we present nonlinear theories of coherent development of the Rayleigh - Taylor and collisional drift instabilities in the context of F-region situation.

2.1 TWO - STEP PROCESSES

Recent experimental observations have shown that large scalesize F region irregularities obey a k^{-2} power law over two orders of magnitudes of scalesizes and amplitudes of density fluctuation[16]. This essentially means that the gradients in the waves remain constant over this range, or in other words, that the amplitude increases almost linearly with the scalesize. In the present section, we show that this value of gradient associated with the irregularities corresponds to a critical value of gradient in the medium, which if exceeded, makes the system unstable to the growth of small scalesize perturbations due to the drift instability. This is a kind of two - stage process in which the first stage was the growth of the large wavelength perturbations itself due to some linear instability (like the Rayleigh - Taylor instability). The eventual state in this case is a turbulent one in which smaller scalesizes are generated on the top of larger scalesizes and this process leads to the amplitude saturation of the larger scalesizes in such a way that they follow a power law going as k^{-2}. The two-step theory has already been proposed as a mechanism to generate small scalesizes, responsible for VHF backscatter, in equatorial spread F[32]. The experimental support for the two-step process in equatorial spread F seems to be coming from the observation that the small scalesize irregularities are always detected by the VHF radar only after the occurrence of spread F on the ionograms has been reported, with a certain time lag between the two events[19].

We have taken a fresh look at the two-step process here. We point out that a two-step process is like a nonlinear mode-mode coupling and should not be treated like a linear instability. Thus a careful investigation reveals that we cannot have a short wavelength drift wave growing on a long wavelength drift wave because the appropriate mode-coupling terms cancel each other. Physically this seems to be related to the fact that in a drift wave the total first order macroscopic velocity of electrons perpendicular to the magnetic field is zero i.e. the first order electron diamagnetic drift which would support secondary drift modes is absent. The important two-step process in the F-region must therefore be a drift wave growing on the density gradients of a Rayleigh - Taylor mode (or some other mode).

Next we show that there exists a threshold value of the density gradient in the system corresponding to the equatorial F region, which if exceeded would result in the onset of the collisional drift instability. Thus on the equilibrium situation, the gradients associated with the longer wavelengths lie near this critical value and hence the property that the amplitudes are proportional to the scalesize. Thus the two-step process offers a qualitative explanation of the k^{-2} spectral law for the long scalelength irregularities in a natural way.

We now wish to describe the excitation of secondary drift waves in the F region by collisional drift instability mechanism in the presence of a large amplitude long wavelength wave. Our starting equations are the two fluid equations for ions and electrons which are valid for perturbations with $k_\perp \gamma_{Li} \ll 1$, $k_z \lambda^e_{mfp} \ll 1$ etc. We use a co-ordinate system (appropriate to the equatorial region) in which the z-axis points along the magnetic field of the earth, x-axis points along the west - east direction. We ignore the equilibrium background density gradient and also the effect of gravity on the drift - dissipative instability which is justified for excitation of not too large wavelengths (viz., greater than the ion larmor radius but of the order of a few tens of meters).

The electron equations are

$$\frac{\partial n_e}{\partial t} + \nabla_\perp \cdot (n_e \underset{\sim}{v}_{e\perp}) + \frac{\partial}{\partial z}(n_e v_{ez}) = 0 \qquad (1)$$

$$0 \simeq -\nabla_\perp p_e - e n_e \left[-\nabla_\perp \phi + \frac{\underset{\sim}{v}_{e\perp} \times \underset{\sim}{B}_o}{c} \right] \qquad (2)$$

$$0 \simeq -\frac{\partial p_e}{\partial z} + e n_e \frac{\partial \phi}{\partial z} - m_e n_e \nu_e v_{ez} \qquad (3)$$

Here ν_e refers to the coulomb collisions between electrons and ions and the rest of the symbols have their usual meanings. The parallel and perpendicular electron velocities are given by

$$n v_{ez} = -\frac{T_e}{m\nu_e}\frac{\partial n}{\partial z} + \frac{T_e}{m\nu_e} n_e \frac{\partial \psi}{\partial z} \tag{4}$$

$$n v_{\perp e} = \frac{T_e}{m\Omega_e}\left[\nabla_\perp n \times \hat{z} - n \nabla_\perp \psi \times \hat{z}\right] \tag{5}$$

where $\psi = e\phi/T_e$.

Here we neglected the electron inertia and assumed $\omega << k_z v_{Te}$. In the limit of no collisions, equation (4) above expresses the well known Boltzmann - type relationship between the density and potential perturbations in the wave,

$$\frac{\tilde{n}}{n_o} \simeq \psi$$

where we used a linearized version of the equation (4). Substitution of this relation in the equation (5) immediately shows that the macroscopic electron velocity across the magnetic field remains almost equal to zero in a drift wave. On substituting the above velocities in the electron equation of continuity, we obtain

$$\frac{\partial n_e}{\partial t} - \frac{T_e}{m\nu_e}\frac{\partial^2 n_e}{\partial z^2} + \frac{T_e n_e}{m\nu_e}\frac{\partial^2 \psi}{\partial z^2} + \frac{T_e}{m\nu_e}\frac{\partial n_e}{\partial z}\frac{\partial \psi}{\partial z} - \frac{cT_e}{eB_o}(\nabla_\perp \psi \times \hat{e}_z)\cdot\nabla n_e = 0$$

or

$$\frac{\partial}{\partial z}\left[\frac{\partial n_e}{\partial z} - n_e\frac{\partial \psi}{\partial z}\right] - \frac{m\nu_e}{T_e}\left[\frac{\partial n_e}{\partial t} - \frac{cT_e}{eB_o}(\nabla_\perp \psi \times \hat{e}_z)\cdot\nabla n_e\right] = 0 \tag{6}$$

The first term in big brackets above is the usual Boltzmann - equilibration and the second term gives the departure of the density distribution from it due to resistive parallel motion.

The ion equations are

$$\frac{\partial n_i}{\partial t} + \nabla_\perp \cdot (n_i \underset{\sim}{v}_{i\perp}) = 0 \qquad (7)$$

$$M n_i \left[\frac{\partial}{\partial t} + \underset{\sim}{v}_{i\perp}\cdot\nabla\right] \underset{\sim}{v}_{i\perp} = -\nabla_\perp p_i + e n_i \left[-\nabla_\perp \phi + \frac{\underset{\sim}{v}_{i\perp} \times B_0}{c}\right]$$
$$- M n \nu_{in} \underset{\sim}{v}_{i\perp} + m \nu_e n_i (\underset{\sim}{v}_e - \underset{\sim}{v}_i) + M n_i \underset{\sim}{g} + \nu_\perp n_i \nabla_\perp^2 \underset{\sim}{v}_{i\perp} \qquad (8)$$

where $\nu_\perp = 0.3 \frac{T_i}{\Omega_i^2} \nu_{ii}$ and represents the viscosity effects while ν_{in} refers to the ion collision frequency with the neutrals. We shall ignore the viscosity effects for simplicity in this treatment and also the effects of gravity on drift waves here and elsewhere. Further the ion parallel motion is neglected which is valid for $\omega \gg k_z C_s$. We assume that ions are basically $\underset{\sim}{E}\times\underset{\sim}{B}$ drifted but we retain their inertial effects in $\underset{\sim}{v}_{i\perp}$ by using iterative procedure, which is equivalent to the assumption $\omega \ll \Omega_i$. Thus the ion velocity is given by

$$n \underset{\sim}{v}_{i\perp} = -\frac{T_i}{M\Omega_i} \nabla n \times \hat{z} - \frac{T_e}{M\Omega_i} n \nabla \psi' \times \hat{z} - n\left(\frac{\partial}{\partial t} + \nu_{in} + \underset{\sim}{v}_i \cdot \nabla\right)\left[\frac{T_i}{M\Omega_i^2}\frac{\nabla n_i}{n_i} + \frac{T_e}{M\Omega_i^2}\nabla\psi\right] \qquad (9)$$

Using it in equation (7) we get

$$\frac{\partial n_i}{\partial t} - \frac{T_e}{M\Omega_i}(\nabla\psi \times \hat{e}_z)\cdot\nabla n_i - \left[\frac{\partial}{\partial t}\frac{T_i \nabla^2 n_i}{M\Omega_i^2} + \frac{\partial}{\partial t}\frac{T_e \nabla\cdot(n_i \nabla\psi)}{M\Omega_i^2}\right] = 0 \qquad (10)$$

where ν_{in} has been neglected for simplicity.

In (10) the second term describes generation of ion density fluctuations due to EXB motion of ions along the perpendicular density gradient. The third term gives the contribution due to finite ion inertia. Electrons set up a Boltzmann-distribution along the field lines (see equation 6) and neutralise any charge bunching due to ions; this leads to the drift wave. The higher order corrections to equations (6) and (10) (viz. the last terms) are primarily responsible for the instability of these waves.

We now split the total plasma quantities into zero-order quantities and perturbations,

$$n \simeq n_0 + \delta n_0 + \tilde{n}$$
$$\psi \simeq \delta\psi_0 + \tilde{\psi} \tag{11}$$

where terms with 'δ' refer to the large - amplitude wave and terms with tilde refer to the perturbations. Equations (6) and (10) are then linearized with respect to the perturbations and can be written as

$$\frac{\partial \tilde{n}_e}{\partial t} - D_e \frac{\partial^2 \tilde{n}_e}{\partial z^2} + n_0 D_e \frac{\partial^2 \tilde{\psi}}{\partial z^2} = \frac{cT_e}{eB_0}\left[(\nabla_\perp \delta\psi_0 \times \hat{e}_z)\cdot\nabla\tilde{n}_e + (\nabla_\perp\tilde{\psi}\times\hat{e}_z)\cdot\nabla\delta n_0\right]$$

with $D_e = T_e/m_e\nu_e$ and

$$\frac{\partial}{\partial t}\left[\left(1 - \frac{T_i\nabla^2}{M\Omega_i^2}\right)\tilde{n}_i - \frac{T_e n_0}{M\Omega_i^2}\nabla^2\tilde{\psi}\right] = \frac{cT_e}{eB_0}\left[(\nabla\tilde{\psi}\times\hat{e}_z)\cdot\nabla\delta n_0 + (\nabla\delta\psi_0\times\hat{e}_z)\cdot\nabla\tilde{n}_i\right]$$

where terms on the right hand side describe mode - coupling between the large amplitude mode and the perturbations. We assume that the two vary as $\delta f_0 \sim \delta \bar{f}_0 \left[\exp (i \underset{\sim}{k}_0 \cdot \underset{\sim}{r} - i \omega_0 t) + c.c. \right]$ and $\exp \left[i \underset{\sim}{k} \cdot \underset{\sim}{r} - i \omega t \right]$ respectively. Also, we have ignored the nonlinearity in the parallel electron motion which will be justified later on.

The above equations now become

$$\frac{\tilde{n}_e}{n_0}\left[1 - i\frac{\omega}{D_e k_z^2}\right] = \tilde{\Psi} + \frac{cT_e}{eB_0}\frac{1}{D_e k_z^2}\Big[\delta\bar{\Psi}_0 (\underset{\sim}{k}_0 \times \hat{e}_z)\cdot \underset{\sim}{k} \left(\frac{\tilde{n}_{e+}}{n_0} - \frac{\tilde{n}_{e-}}{n_0}\right) + \frac{\delta\bar{n}_0}{n_0} (\underset{\sim}{k} \times \hat{e}_z)\cdot \underset{\sim}{k}_0 (\tilde{\Psi}_+ - \tilde{\Psi}_-)\Big] \tag{12}$$

and

$$\left[i\omega(1+b)\right]\frac{\tilde{n}_i}{n_0} + \left[-i\omega\frac{T_e}{T_i}b\right]\tilde{\Psi} = \frac{cT_e}{eB_0}(\underset{\sim}{k}\times\hat{e}_z)\cdot\underset{\sim}{k}_0\frac{\delta\bar{n}_0}{n_0}\left[\tilde{\Psi}_- - \tilde{\Psi}_+\right] + \frac{cT_e}{eB_0}(\underset{\sim}{k}_0\times\hat{e}_z)\cdot\underset{\sim}{k}\,\bar{\Psi}_0\left[\frac{\tilde{n}_{i-}}{n_0} - \frac{\tilde{n}_{i+}}{n_0}\right] \tag{13}$$

where

$$b = \frac{T_i k_\perp^2}{M\Omega_i^2}\ ; \quad \tilde{n}_\pm, \tilde{\Psi}_\pm \equiv \tilde{n}_{k\pm k_0}, \tilde{\Psi}_{k\pm k_0}$$

To the lowest order equation (12) reproduces

$$\frac{\tilde{n}_e}{n_0} \simeq \tilde{\Psi} \tag{14}$$

which we substitute in equation (13) by invoking the quasi-neutrality condition, namely $\tilde{n}_e \simeq \tilde{n}_i$. This leads to

$$i\omega_s \tilde{\Psi}_s = \left[\frac{cT_e}{eB_0}(\underset{\sim}{k}\times\hat{e}_z)\cdot\underset{\sim}{k}_0\left(\frac{\delta\bar{n}_0}{n_0} - \delta\bar{\Psi}_0\right)\right]\times\left[\tilde{\Psi}_{s-1} - \tilde{\Psi}_{s+1}\right] \tag{15}$$

where the terms proportional to ion inertial effects ($\sim b$) have been ignored in (13), and here s refers to the s-th fourier - mode of perturbations. We see from the equation (15) that if for the large amplitude mode, the relation $\frac{\delta\tilde{n}_0}{n_0} \simeq \delta\tilde{\psi}_0$, is satisfied then the perturbed potential $\tilde{\psi}_s$ vanishes. This Boltzmann-type relationship between density and potential perturbations is satisfied for all the drift modes (see (14)) and consequently the second order drift modes on them will be absent. We know that the drift modes travel with the perpendicular diamagnetic drift of electron fluid due to a pressure gradient. In the present case this drift is always equal and opposite to the corresponding $\underset{\sim}{E_0} \times \underset{\sim}{B_0}$ drift. The resultant net velocity of electrons perpendicular to $\underset{\sim}{B_0}$ is thus zero[9] and thence no higher order drift wave perturbations result. This is not the case with a Rayleigh - Taylor type of flute mode. As can be seen from equation (6), for an R - T mode, the linear relation between density and potential perturbations may be expressed as

$$\frac{\delta\bar{n}_0}{n_0} = \left(-\frac{k_{0y}}{\omega_0}\frac{T_e}{M\Omega_i L}\right)\delta\bar{\psi}_0 = \left(\frac{\omega_{*0}}{\omega_0}\right)\delta\bar{\psi}_0 \qquad (16)$$

It may be noted here that for R - T modes, $k_{z0} = 0$. Thus we were justified in neglecting the parallel electron motion nonlinearities earlier in the calculations. Thus the conclusion is that the two step process will take place on the waves for which the relation (14) is not satisfied. In the equatorial F region, below the F peak, the R - T mode seems to be responsible for such a process and generating smaller wave-length drift modes. Equation (15) leads to the infinite series

$$
\begin{aligned}
i\omega\,\Psi &= \alpha\,(\Psi_{-1} - \Psi_{+1}) \\
i\,\omega_{\pm 1}\Psi_{\pm 1} &= \alpha\,(\pm\Psi \mp \Psi_{\pm 2}) \\
i\,\omega_{\pm 2}\Psi_{\pm 2} &= \alpha\,(\pm\Psi_{\pm 1} \mp \Psi_{\pm 3})
\end{aligned}
\tag{17}
$$

The infinite determinant of this series can be written as

$$\left[i\omega + \frac{\alpha^2}{i\omega_{-1} + \dfrac{\alpha^2}{i\omega_{-2} + \cdots\infty}} + \frac{\alpha^2}{i\omega_{+1} + \dfrac{\alpha^2}{i\omega_{+2} + \cdots\infty}} \right]\Psi = 0$$

For the nontrivial solutions of the above equation, one requires that

$$i\omega + \frac{\alpha^2}{i\omega_{-1} + \dfrac{\alpha^2}{\cdots\infty}} + \frac{\alpha^2}{i\omega_{+1} + \dfrac{\alpha^2}{\cdots\infty}} = 0 \tag{18}$$

This is the dispersion relation for the secondary drift waves. As our large amplitude mode is an R - T mode, we can assume $\omega_0 \simeq 0$, and write

$$i\omega + \frac{2\alpha^2}{i\omega + \dfrac{\alpha^2}{\cdots\infty}} = 0$$

which yields, after some algebra,

$$\omega \simeq \pm \ \omega_{*} \tag{19}$$

where $$\omega_{*} \simeq k_x \left(\frac{2cT_e}{eB_o\tilde{L}} \right) \simeq k_x v_d$$

and $\tilde{L}^{-1} \simeq k_{oy} \frac{\delta \bar{n}_o}{n_o}$, the first order density gradient associated with the large mode.

In deriving (19) we assumed that $\underset{\sim}{k} = k_x \hat{e}_x$ and $\underset{\sim}{k}_o = k_{oy} \hat{e}_y$. If higher order effects like resistive electron parallel motion, ion inertia, etc. are included then the growth rate for these drift modes can be written as

$$\omega_{Im} \simeq \frac{b}{(1+b)} \left[(1+b) \frac{\omega_R^2 m \nu_e}{k_z^2 \ T_e} - \nu_{in} - 3b\nu_{ii} \right] \tag{20}$$

The assumption made here was that the nonlinearities do not significantly modify the smaller effects which lead to the growth or damping of the drift waves.

From (20), we see that the growth condition is, approximately

$$2 \frac{k_x}{k_z} \cdot \sqrt{\frac{\nu_e}{(\nu_{in} + 3b\nu_{ii})} \frac{T_e}{m\Omega_e^2}} \cdot \frac{1}{\tilde{L}} > 1 \tag{21}$$

We see that the growth depends, among the other factors, on the ratio of perpendicular to parallel wavenumbers and on the driving gradient. We find that the ratio k_x/k_z in the equatorial F-region is fixed to a maximum value by the following considerations. Due to the curvature of earth's magnetic field lines, there exists a geometrical constraint on the minimum parallel wavenumber that can exist in the system[33]. In principle, the perpendicular wavenumber can be as large as γ_{Li}^{-1} but our fluid analysis is not valid in this domain. Therefore the maximum wavenumber is restricted by the condition $k_x \gamma_{Li} \ll 1$ for the collisional drift instability. Thus the ratio k_x / k_z for the F region situation is maximum for certain modes and these modes grow fastest in the system, independent of all other parameters. From the condition (21) we see that these modes require a minimum Lcrit^{-1}, in order to grow. The density gradient associated with a wave is roughly the ratio of fluctuation amplitude to the wavelength. Therefore if a large - amplitude mode has an amplitude such that

$$\frac{\text{Amplitude}}{\text{Scalesize}} \simeq L_W^{-1} > \text{Lcrit}^{-1}$$

then, the collisional drift waves would become unstable and would grow at the expense of the large - amplitude mode. This would continue until the amplitude of the wave has decreased such that $L_W \simeq$ Lcrit, when the large - amplitude mode is just stable. Thus any wavelength can have only a maximum amplitude, as given by $\sim$ Lcrit^{-1} x scalesize, in the stable situation. Therefore the power in a mode can be represented as

$$\left\langle \left(\frac{\delta \tilde{n}}{n_o}\right)^2 \right\rangle \simeq \tilde{\lambda}^2 L_{crit}^{-2} \propto k^{-2} \tag{22}$$

The recent observations by Dyson et al.[16] do indeed show that this kind of spectral law is obeyed by the irregularities in the F region over a large number of scalesizes. They have reported two sets of observations - one below F peak and for the equatorial latitudes and the other one above the F peak and for high latitudes. We shall confine our discussion mainly to the equatorial observations. It seems certain now that the Rayleigh - Taylor instability is the main source of irregularities at large wavelengths for the equatorial spread F[5,33]. This mechanism is operative only below the F peak and has a cut-off at shorter wavelengths due to finite-larmor-radius (FLR) stabilization effects. We suggest that the range of scalesizes observed by Dyson et al. corresponds to this instability. This range is 7 km. down to 70 meters. The FLR cut-off at shorter wavelengths depends on the background density gradient and would correspond to a higher minimum $\lambda_\perp$ if the gradient is sharper and vice-versa. It is known that the gradient varies from day to day and the spread F seems to be stronger on the day when the gradient is weaker[41]. This supports the Rayleigh - Taylor instability model for longer wavelength spread F irregularities and also states that the FLR limit on minimum $\lambda_\perp$ obtained by Hudson and Kennel[33] for the Rayleigh - Taylor instability is not unique and can vary from day to day. Now after arguing that the whole range of irregularities from 7 kms. to 70 ms. can be due to the Rayleigh - Taylor instability mechanism alone, we further suggest that their amplitude growth is limited by the two-step process. Once they exceed this amplitude, the collisional drift modes are excited and the surplus energy goes into the shorter λ's thus limiting the power in a mode to a value governed by the expression (22). A rough estimate of numerical value of Lcrit given by (22) may also be made. For the typical values of the parameters for annighttime altitude of $\sim$ 425 kms. $\left[\nu_e \sim 10^2\ s^{-1},\ \nu_{in} \sim .1\ s^{-1},\ \nu_{ii} \sim 6\ s^{-1},\ r_{Le} \sim \sqrt{\frac{T_e}{m\,\Omega_e^2}} \sim 5\ \text{cms} \right]$ the Lcrit required by a mode with $\lambda_z \sim$ 100 kms., $\lambda_\perp \sim$ 50 ms. (which corresponds to a 'fastest' growing mode), is of the order of a few kilometers, as calculated from (21). This agrees well with the observations of Dyson et al.[16] who report a constant gradient of $\sim$ 3 km. with these irregularities. A precise agreement between the two values is not to be expected as the parameter values vary in the ionosphere from day to day and exact values of these parameters on the day of the irregularity observations are uncertain.

We have not said anything about the observations of Dyson et al. for high latitude and high altitude ($\sim$ 700 kms.) region, so far. It is certain that the mechanism responsible for these irregularities is not the Rayleigh - Taylor instability because the density gradient and gravity point in the same direction there. Though we do not know the mechanism behind the generation of these large-scale irregularities, we suggest that the amplitude - limiting mechanism seems to be the two - step process in this case also (so long as these modes are not modes satisfying the $\frac{\delta \bar{n}_0}{n_0} \sim \delta \bar{\psi}_0$ relation). These modes also generate collisionless as well as collisional drift modes of smaller wavelength by the two-step process and their amplitude and power is governed by the relations (21) and (22) respectively.

Though all along we have mentioned only the collisional drift instability as the mechanism responsible for cascading down the energy from large λ's to short λ's, the collisionless drift instability is an equally possible competing mechanism for it, as postulated by Hudson et al.[32]. Thus in the actual situation both mechanisms must be operating and the large-amplitude mode generates short $\lambda_\perp$'s ($k_\perp \gamma_{Li} \gtrsim 1$) by the collisionless drift instability and $\lambda_\perp$'s such that ($k_\perp \gamma_{Li} \ll 1$) by the collisional drift instability, thereby the power residing in a given scalesize being governed by the law (22).

DISCUSSION

In the above, we have shown that the k^{-2} power law can be interpreted in terms of a two-step theory wherein a turbulent generation of secondary irregularities takes place by the large scalesizes. This situation is likely to occur when initially many large scalesize modes are excited. We could also have considered a coherent nonlinear theory in which a single mode is excited and, on attaining large amplitudes, develops into steepened profiles due to nonlinearities. We shall, in fact, show in the next sections that the Rayleigh - Taylor instability and collisional drift instability, both can evolve into such coherent nonlinear waves due to finite - Larmor - radius stabilization and ion viscosity effects, respectively. The final profile in these cases can be saw-tooth shaped, which as we know, would also give a k^{-2} power law for the various component scalesizes. However, experimentally they correspond to very different physical situations.

In the first case the various excited k's are randomly phased with respect to each other whereas in the second case, the k's are completely phase correlated. An experimental observation which measures the phase correlations between the various k's might therefore be very useful in deciding which of the two is the dominant nonlinear effect.

2.2 COHERENT NONLINEAR RAYLEIGH - TAYLOR MODE

Recent theoretical and experimental work on the equatorial spread F irregularities has, by and large, emphasized the role of Rayleigh - Taylor (R - T) instability in generating large scalesize irregularities in the nighttime lowerside equatorial F region[28, 41]. These irregularities normally obey a k^{-2} power spectra[16] and occasionally develop as coherent modes with steepened shapes (which apparently corresponds to the nonlinear stage of irregularities). In this section, we have, therefore developed a nonlinear theory for the R - T instability, in order to explain these features. The set of one-dimensional (1-d) equations describing this instability is solved in the steady state approximation and is shown to possess special periodic propagating solutions. This could be of direct relevance to the cited experimental observations. A more realistic calculation for the R - T instability, however, would be a two-dimensional one wherein the term $\underset{\sim}{E} \times \underset{\sim}{B} \cdot \nabla n$ in the continuity equation gives the most important nonlinear effects (in contrast to the ion-inertial nonlinearity which is dominant in the 1-d case). But an analytic solution of this problem cannot be obtained and hence a numerical investigation has been planned in future for this case.

The geometry of the situation corresponding to the equatorial F region has already been described earlier and the equations corresponding to this problem were presented in the two-fluid form in the last section (equations (1-3) and (7,8) of the last section for electrons and ions respectively). The collisions are neglected in the electron equations and perturbations are supposed to be only in the one-dimension, namely, along the y-axis. For such a case, the electron equations of continuity and motion can be combined to give

$$\frac{\partial n_e}{\partial t} - \frac{cT_e}{eB_0}\frac{\partial \psi}{\partial y}\frac{dn}{dx} = 0 \tag{1}$$

This equation shows that the electron density changes take place locally because of $\underset{\sim}{E} X \underset{\sim}{B}$ convection of electrons along the density gradient. For the ion equation of motion, we ignore collisions and viscosity effects and write the velocity as

$$\begin{aligned} \underset{\sim}{v}_i = & -\frac{T_i}{M\Omega_i}\frac{\nabla n_i \times \hat{z}}{n_i} - \frac{T_e}{M\Omega_i}\nabla\psi \times \hat{z} + \frac{\underset{\sim}{g} \times \hat{z}}{\Omega_i} \\ & -\frac{1}{\Omega_i}\frac{\partial}{\partial t}\left[-\frac{T_i}{M\Omega_i}\frac{(\nabla n_i \times \hat{z})\times\hat{z}}{n_i} - \frac{T_e}{M\Omega_i}(\nabla\psi \times \hat{z})\times\hat{z}\right] \\ & -\frac{1}{\Omega_i}(\underset{\sim}{v}_i \cdot \nabla)\underset{\sim}{v}_i \times \hat{z} \end{aligned} \tag{2}$$

Here we ignore the last term (it represents second order inertial effects) and then substitute the ion velocity in the continuity equation. This leads to

$$g\frac{\partial n}{\partial y} - \frac{\partial}{\partial y}\left(\frac{n}{dn/dx}\frac{\partial^2 n}{\partial t^2}\right) - \frac{T_i}{M\Omega_i}\frac{\partial}{\partial y}\left[n\frac{\partial}{\partial t}\left(\frac{1}{n}\frac{\partial n}{\partial y}\right)\right] = 0 \tag{3}$$

where we have eliminated ψ with the help of (1). In this equation, first term comes from the effect of d.c. drift due to gravitational force on ions, the second term gives the frequency of oscillation of density fluctuation at a point due to the $\underset{\sim}{E} \times \underset{\sim}{B}$ convection of plasma and the third term represents the effect of finite ion pressure and inertia. The first two terms describe the usual purely growing Rayleigh - Taylor instability and the third term in the linear limit, represents FLR stabilization effects[37]. This last term is a nonlinear term, with nonlinearity coming from the ion - inertial motion. We look for special steady state solutions and assume that the quantities depend only on $\xi = y - ut$. This leads to

$$\left[v_D - (1+N)u \right] \frac{d^2 N}{d\xi^2} - \frac{v_D}{(1+N)} \left(\frac{dN}{d\xi} \right)^2 + \frac{g}{uL} N = 0 \tag{4}$$

where we have split $n = n_0 + \tilde{n}$, and $N = \dfrac{\tilde{n}}{n_0}$.

Also $v_D = T_i / M \Omega_i L$ with $L = \left(\dfrac{1}{n} \dfrac{dn}{dx} \right)^{-1}$.

Assuming $N \ll 1$ (in agreement with the observations of Dyson et al.), we write the above equation as

$$(v_D - u) \frac{d^2 N}{d\xi^2} - v_D \left(\frac{dN}{d\xi} \right)^2 + \frac{g}{uL} N = 0 \tag{5}$$

Introducing the scaling,

$$q = \sqrt{\frac{2g}{uL(v_D - u)}} \quad \text{and} \quad p = \frac{2 v_D}{(v_D - u)} N \tag{6}$$

Then equation (5) becomes

$$2\frac{d^2 p}{dq^2} - \left(\frac{dp}{dq}\right)^2 + p = 0 \tag{7}$$

Here it must be borne in mind that we have assumed $v_D > u$. This equation has been discussed in detail in the literature[1] earlier. The first integral of (7) is

$$\left(\frac{dp}{dq}\right)^2 = Ce^{p} + p + 1 \tag{8}$$

where C is a constant of integration. One can readily verify that periodic solutions are possible only for $-1 < C < 0$. For $C \simeq -1$ and $p \ll 1$, we may write

$$\left(\frac{dp}{dq}\right)^2 \simeq -p^2/2 \tag{9}$$

This corresponds to a situation in which small amplitude sinusoidal oscillations exist in the system.

The case $C \simeq -0$ corresponds to large values of p_M and $(dp/dq)_M$ where M denotes the maximum values of these quantities. [p_M is obtained from (8) by putting $dp/dq = 0$ and $(dp/dq)_M$ from (7) by putting $d^2p/dq^2 = 0$] . In this limit the solutions are strongly nonharmonic and have a triangular saw-tooth structure. The condition $p_M \gg 1$ corresponds to

$$\left(\frac{\tilde{n}}{n_o}\right)_M \gg \frac{v_D - u}{2v_D} \tag{10}$$

and can readily be verified for $v_D \simeq u$. This solution may represent a qualitative explanation for the steepened irregularity structures observed by Dyson et al.[16] and Kelley et al.[41].

In the above analysis if FLR term is omitted, we do not obtain the periodic solutions for R-T modes. Thus the evolution of purely growing R-T mode into a nonlinear propagating wave is highly dependent on FLR effects and may be understood as follows. The linearly unstable wavelength gets coupled to shorter FLR - stable oscillatory wavelengths. This nonlinear interaction results in the introduction of a frequency shift in the frequency of the unstable wave-length and in the saturated state, this frequency shift is adequate to overcome the destabilizing term and provide an oscillatory character to the mode. Because of this frequency shift the wave acquires a phase velocity which for weakly nonlinear case is smaller than v_D and for a strongly nonlinear case (when many shorter FLR scalesizes are generated and the wave profile becomes nonsinusoidal) it becomes of the order of v_D. This latter case probably corresponds to the experimental observations of Kelley et al.[41] who have observed the evolution of irregularities into steepened structures with asymmetric slopes on the either side on a time scale of the order of 30 minutes. This is the typical growth time of R - T modes in equatorial F region and hence would be the time required by modes to evolve nonlinearly. The asymmetry in slopes on the other side may be due to the presence of a westward - pointing background gradient existing in the medium at evening hours[19]. The superposition of this gradient on the wave density profile would give an asymmetric steepening on the eastward edge as observed. The findings of Dyson et al. who observe coherent wavelike structures with power residing in a fundamental scalesize (~ 7 kms., a typical scalesize to be excited by R - T instability) and its sub-multiples seem to corroborate above assertions. The fact that these structures are observed only at the equator lends further credence to the theory as at equator the R-T instability is believed to be the source of irregularities whereas at higher latitudes a different mechanism seems to operate[53].

2.3 COHERENT NONLINEAR COLLISIONAL DRIFT WAVE

It has been recently shown that the linear collisional drift instability is a likely mechanism to generate irregularities (with scalesizes of the order of a few hundred meters or less) in the equatorial spread F[33]. Recent experimental observations of these irregularities show that often the irregularities develop as coherent steepened large amplitude waves[16,41]. Thus, it is of interest to examine the nonlinear development of the collisional drift instability as applicable to the F region situation. Shut'ko[70] has developed a nonlinear theory of coherent drift waves to explain the observation of almost harmonic large amplitude collisional drift waves in the fully-ionized Q-machine plasma[30] under steady state conditions. In this theory, a mode coupling of linearly unstable fundamental mode to its viscous - damped harmonics leads to the establishment of a finite-amplitude wave in the stationary situation. The nonlinearity in this treatment is associated with the parallel motion of ions, which imposes restriction on the validity of this treatment in the wavenumber space, due to the following reason. The effect of ion motion parallel to the magnetic field describes the ion-acoustic wave effects on the drift wave and is important for short parallel wavelengths (so that $\omega/k_{\parallel} \sim v_{Ti}$). If the parallel wavelengths are large, such that $\omega/k_{\parallel} \gg v_{Ti}$

(or C_s, if $T_e \gg T_i$), then one may ignore the ion parallel motion as it is unimportant for the drift wave. In this regime, Shut'ko's analysis is not valid and we have carried out an alternate calculation in which the important nonlinearity comes from the electron set of equations and the ion parallel motion is neglected. The rest of the development remains similar to Shutko's and we also find that due to nonlinear mode - coupling to damped higher harmonics, a finite amplitude wave gets established in the medium. Since the important nonlinearity in Shut'ko's theory represents ion-acoustic effects on drift waves, we call it as the nonlinear drift-acoustic wave theory, in contrast to our treatment which we term as the nonlinear drift wave theory. In order to apply these results to the equatorial F region, the above theories are modified to take damping due to ion - neutral collisions into account. We first present Shut'ko's analysis of the nonlinear drift - acoustic waves (modified to take ν_{in} into account), then present our treatment of the nonlinear drift - modes and finally discuss the relevance of these results in the context of recent observations of coherent nonharmonic structures of the equatorial spread F irregularities.

The basic equations used in Shut'ko's theory and the relevant geometry has already been discussed in the previous sections of this chapter. We follow Shut'ko's treatment closely, but introduce the momentum lost by ions to neutrals in the ion equation of motion. The resulting damping due to ν_{in} may become important compared to viscous damping due to ν_{ii} which goes as $\sim b\,\nu_{ii}$ where $b = k_{\perp}^{2}\, r_{Li}^{2}$ and hence depends on the transverse wavelengths. Therefore for large wavelengths, damping due to ν_{in} may be larger than that due to ν_{ii}, even though $\nu_{in} << \nu_{ii}$. Further we assume cold ions for simplicity. Following Shut'ko[70], we write

$$n \simeq n_0(\underset{\sim}{r})\, n'(\underset{\sim}{r}, t)$$

where $n'(\underset{\sim}{r}, t)$ is a new unknown dimensionless function and $n_0(\underset{\sim}{r})$ is the stationary average density with a known variation in the configuration space. The parallel motion of electrons leads to, in the limit of no collisions,

$$\ln n' = \rho = \psi \tag{1}$$

which is the quasi-static electron pressure balance. The deviation from this relationship arising due to ν_{ei} is determined in the linear limit of electron set of equations. For that, we introduce a correction factor n_1, defined as

$$\rho = \psi + n_1$$

or (2)

$$n' = \exp(\psi + n_1)$$

The perpendicular electron motion is obtained in collision-free limit $(\nu_{ei} \simeq 0)$ and on substituting for v_{ez} and $v_{e\perp}$ in the electron continuity equation, we get a relation for n_1

$$\left[\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t} - D_e \frac{\partial^2}{\partial z^2}\right) - c_s^2 \frac{\partial^2}{\partial z^2}\right] n_1 = \left[-\frac{\partial^2}{\partial t^2} - v_{dr} \frac{\partial^2}{\partial t \partial y} + c_s^2 \frac{\partial^2}{\partial z^2}\right] \psi \quad (3)$$

where $c_s^2 = T_e/M$; $v_{dr} = \frac{c_s^2}{\Omega_i} \partial \rho_0/\partial x$ are the ion sound and the electron diamagnetic drift velocities respectively. In deriving the above equation we have used relations (1) and (2).

The parallel ion motion is given by

$$\frac{\partial V_{zi}}{\partial t} + V_{zi} \frac{\partial V_{zi}}{\partial z} = -c_s^2 \frac{\partial \phi}{\partial z} \quad (4)$$

The perpendicular ion velocity is given by equation (9) of section 2.1. Use of $v_{i\perp}$ and v_{zi} in the continuity equation gives

$$\frac{\partial \psi}{\partial t} + v_{dr} \frac{\partial \psi}{\partial y} - \frac{c_s^2}{\Omega_i^2} \nabla_\perp^2 \left[\frac{\partial}{\partial t} - \frac{\nu_\perp \nabla_\perp^2}{M} + \nu_{in}\right] \psi + \frac{\partial n_1}{\partial t} + v_{zi}\, \partial \psi/\partial z \quad (5)$$
$$+ \partial v_{zi}/\partial z = 0$$

One looks for a solution of equations (3) - (5) in the form of a stationary wave as $t \to \infty$, such that all the quantities depend only on one variable $\xi = \underset{\sim}{k} . \underset{\sim}{r} - \omega t$, where $\underset{\sim}{k}$ and ω are the mean wave vector and frequency.

The set of equations, finally leads to an ordinary differential equation

$$b \frac{k_\perp^2 \nu_\perp}{M} \frac{d^4 v_{zi}}{d\xi^4} + b\left[\left(\frac{\omega\omega_*}{D_e k_z^2} - \nu_{in}\right)\frac{d^2}{d\xi^2}\right] v_{zi} \quad (6)$$
$$+ \frac{d}{d\xi}\left[(-\omega + \omega_*) v_{zi} + \frac{2\omega - \omega_*}{2\omega} v_{zi}^2 k_z\right] = 0$$

The first term above gives the viscous damping of the wave, the second term gives growth (reduced from the value obtained by Shut'ko[70] due to the introduction of the additional ν_{in} damping); in the third term, the linear part describes the drift waves and the nonlinear part leads to the mode-mode coupling of linearly growing mode to the higher damped harmonics. One further assumes that the viscosity is high, so that the basic contribution to the wave profile comes from only a first few harmonics and seeks a solution of equation (10) in the form of a summation of three harmonics:

$$v_{zi} = v_1 \sin \xi + v_2 \sin 2\xi + v_3 \sin 3\xi \qquad (7)$$

In the zeroth approximation, one gets

$$\omega = \omega_* \qquad (8)$$

On equating the coefficients of $\sin \xi$, $\sin 2\xi$, and $\sin 3\xi$ to zero and using (8), three equations are obtained which determine the amplitudes v_1, v_2, v_3. These equations are

$$k_z v_1 \left[\frac{b\omega_*^2}{D_e k_z^2} - b\left(\nu_{in} + \frac{\nu_\perp k_\perp^2}{m_i}\right)\right] + v_2 \frac{k_z^2}{2} (v_1 + v_3) = 0 \qquad (9)$$

$$k_z v_2 \left[4 \frac{b\omega_*^2}{D_e k_z^2} - 4b\left(\nu_{in} + 4\frac{\nu_\perp k_\perp^2}{m_i}\right)\right] - v_1 \frac{k_z^2}{2} (v_1 - 2v_3) = 0 \qquad (10)$$

$$k_z v_3 \left[9 \frac{b\omega_*^2}{D_e k_z^2} - 9b\left(\nu_{in} + 9\frac{\nu_\perp k_\perp^2}{m_i}\right)\right] - \frac{3k_z^2}{2} v_1 v_2 = 0 \qquad (11)$$

One assumes that the viscosity lies in the following range

$$1 > \nu_\perp k_\perp^2 D_e k_z^2 / \omega_*^2 \geq 1/4 \qquad (12)$$

Then near the upper limit of the above inequality i.e. near the threshold of excitation of the first harmonic, from (9) - (11), we obtain

$$k_z^2 v_1^2 = \frac{48\, b^2 \omega^2}{D_e k_z^2} \left(\frac{\omega_*^2}{D_e k_z^2} - \nu_\perp k_\perp^2 \right) \tag{13}$$

$$k_z^2 v_2^2 = 4 b^2 \left(\frac{\omega_*^2}{D_e k_z^2} - \nu_\perp k_\perp^2 \right)^2 \tag{14}$$

This shows that the nonlinear generation of higher harmonics leads to the amplitude saturation of the wave. In the viscosity range as defined by (12), we find that the wave profile is almost harmonic, as the amplitude of the second harmonic is down by an order of magnitude than that of the first harmonic. In the ionospheric situations, the viscosity is not as high and thus the saturation of the instability may take place after many harmonics have been excited. This would correspond to a non-harmonic profile of the wave.

The nonlinearity in the above theory comes from the ion motion parallel to the magnetic field. It is well known that a drift wave is like an accelerated ion acoustic wave and for large parallel wave numbers (k_z), goes over into an ion acoustic wave[37]. The inclusion of parallel ion motion represents the ion - acoustic effects on a drift wave. In the above nonlinear treatment, thus the nonlinear ion - acoustic wave effects were responsible for the stabilization of the drift instability. This is why we called the saturated wave as the nonlinear drift-acoustic mode.

In the ionosphere, normally modes with very small parallel wavenumbers are likely to exist. Therefore a treatment is required for ionospheric applications which would be valid for small parallel wave numbers also. In the following we present an analysis in which the parallel ion motion is not considered. The nonlinearity comes from the electron set of equations. We call the resulting nonlinear stable modes as the nonlinear drift mode to distinguish it from the nonlinear drift - acoustic mode.

The basic equations used and the geometry of the system have already been described earlier in this section. Only the ion parallel motion is now neglected, which is justified for low enough parallel wave numbers (k_z) so that the parallel phase velocity much exceeds the ion sound speed, C_s (or the ion thermal velocity v_{Ti}, if $T_e \simeq T_i$). Other assumptions used are the same as before. The electron nonlinear equation remains exactly as (4) but there are some modifications in the ion equation due to the retention of viscosity and the assumption of cold ions. The electron equation can be written as

$$\frac{\partial \tilde{n}}{\partial t} - \frac{T_e}{m\nu_e}\frac{\partial^2 \tilde{n}}{\partial z^2} + D_e \frac{\partial}{\partial z}\left(n_e \frac{\partial \psi}{\partial z}\right) - \frac{T_e}{m\Omega_e}\frac{dn}{dx}\frac{\partial \psi}{\partial y} = 0 \qquad (15)$$

and the ion equation as

$$\frac{\partial \tilde{n}}{\partial t} - \frac{T_e}{M\Omega_i}\frac{\partial \psi}{\partial y}\frac{dn}{dx} + \frac{\partial}{\partial y}\left(-n\nu_{in}\frac{T_e}{M\Omega_i^2}\frac{\partial \psi}{\partial y} + n\frac{\nu_\perp}{M^2}\frac{T_e}{\Omega_i^2}\frac{\partial^3 \psi}{\partial y^3} - n\frac{T_e}{M\Omega_i^2}\frac{\partial^2 \psi}{\partial t \partial y}\right) = 0 \qquad (16)$$

where $n = n_0 + \tilde{n}$ and $\tilde{n}$ is the perturbed density.

Now we assume that as the time progresses on, in the asymptotic limit of $t \to \infty$, a time stationary wave is set up in the system and we transform the above equations to the propagating wave frame in which all the quantities vary with respect to a single variable $\xi = k_y y + k_z z - \omega t$. Then (15) becomes

$$\frac{d}{d\xi}\left[-\omega \tilde{n} - \frac{T_e k_z^2}{m\nu_e}\frac{d\tilde{n}}{d\xi} + \frac{T_e k_z^2}{m\nu_e} n \frac{d\psi}{d\xi} - \frac{T_e k_y}{m\Omega_e}\frac{dn}{dx}\psi\right] = 0$$

which on one integration leads to

$$\left[-\omega + \frac{T_e k_z^2}{m\nu_e}\frac{d\psi}{d\xi}\right]N - \frac{T_e k_z^2}{m\nu_e}\frac{dN}{d\xi} = \frac{T_e k_y}{m\Omega_e L}\psi - \frac{T_e k_z^2}{m\nu_e} d\psi/d\xi \qquad (17)$$

where $L = \left(\frac{1}{n_0}\frac{dn_0}{dx}\right)^{-1}$; $N = \frac{\tilde{n}}{n_0}$ and $b = T_e k_y^2 / M\Omega_i^2$

Similarly the ion equation (16) after an ~~introduction~~ integration gives

$$\left[-\omega - \nu_{in} b \frac{d\psi}{d\xi} + b\frac{\nu_\perp}{M} k_y^2 \frac{d^3\psi}{d\xi^3} + \omega b \frac{d^2\psi}{d\xi^2}\right] N$$

$$= \frac{k_y T_e}{M\Omega_i L}\psi + \nu_{in} b \frac{d\psi}{d\xi} - \frac{\nu_\perp}{M} b k_y^2 \frac{d^3\psi}{d\xi^3} - b\omega\frac{d^2\psi}{d\xi^2} \qquad (18)$$

Equations (17) and (18) form the set which describes the nonlinear collisional drift waves. The nonlinear terms are the second term on the l.h.s. of (17) and the last three terms on the l.h.s. of (18). We find that for the F-region parameters, the dominant nonlinearity is the one in electron equation (17). The typical parameters are $r_{Li} = \sqrt{\frac{T_e}{M\Omega_i^2}} \sim 5$ meters, $\Omega_i = 2\times 10^2 \, s^{-1}$, $v_{Te} \sim 2.5\times 10^7 \, cm.s^{-1}$, $v_{Ti} \sim 1\times 10^5 \, cms.s^{-1}$, $\nu_{ei} \sim 10^3 \, s^{-1}$, $\lambda_{mfp}^{e} \sim 250$ meters, at an altitude of $\sim$ 400 kms. For $\lambda_\perp \sim$ 300 to 100 meters, b varies from 10^{-2} to 10^{-1}. The nonlinearities in the ion equation go as $\sim$ b when compared with the linear terms in the same equation. A similar comparison in the case of electrons gives

$$\frac{T_e k_z^2}{m \nu_e \omega} \sim k_z^2 \left(\lambda^e_{mfp}\right)^2 \frac{\nu_e}{\omega} \sim 10$$

where $\omega \sim 10^{-2}\ s^{-1}$ for $\lambda_\perp \sim$ 300 meters.

Clearly the electron nonlinearity is important for this range of parameters and we retain only this nonlinearity in the further analysis.

Then the ions are treated linearly and (18) can be written as

$$-\omega N = \frac{k_y T_e}{M \Omega_i L} \psi + b \nu_{in} \frac{d\psi}{d\xi} - b \frac{\nu_\perp}{M} k_y^2 \frac{d^3 \psi}{d\xi^3} - b\omega \frac{d^2 \psi}{d\xi^2} \qquad (19)$$

Using (19) to substitute N from it in (17), leads us finally to

$$\left(\frac{b \nu_\perp k_y^2}{M}\right) \frac{d^4 \psi}{d\xi^4} + b\left(\frac{m \nu_e \omega_*^2}{T_e k_z^2} - \nu_{in}\right) \frac{d^2 \psi}{d\xi^2} + \frac{d}{d\xi}\left[(-\omega + \omega_*)\psi + \frac{k_y v_D}{2} \psi^2\right] = 0 \qquad (20)$$

where we omitted some nonlinear terms going as $\sim b$. Here $\omega_* = -k_y v_D$ and $v_D = T_e / M\Omega_i L$. As $N = \psi$ for the drift waves in the lowest order, (20) can be written as

$$\left(\frac{b \nu_\perp k_y^2}{M}\right) \frac{d^4 N}{d\xi^4} + b\left(\frac{m \nu_e \omega_*^2}{T_e k_z^2} - \nu_{in}\right) \frac{d^2 N}{d\xi^2} + \frac{d}{d\xi}\left[(-\omega + \omega_*) N + \frac{k_y v_D}{2} N^2\right] = 0 \qquad (21)$$

A comparison of this equation with equation (6) obtained for the nonlinear drift - acoustic mode, shows that they are nearly identical. Hence in the present case of the nonlinear drift waves we get similar solutions as in the earlier case. Analysis of (21) therefore becomes similar to the one presented after equation (6). The result in the present case is again the establishment of a finite amplitude drift wave in the medium due to nonlinear coupling and consequent generation of higher harmonics which are linearly damped due to viscosity. If the viscosity is high, the wave profile is almost harmonic, otherwise it is steepened as many harmonics would be generated in the low viscosity case.

The two theories presented above, though different in approximations used, finally arive at a similar result. In both the cases the drift instability evolves into nonlinear coherent propagating modes. As the approximations involved in the two approaches are different, we compare the limits of validity of the two cases for equatorial F region situation. Thus on comparing the nonlinear terms in (6) and (21), we find

$$\frac{k_z v_z}{\omega} : \psi = \frac{e\phi}{T_e}$$

or

$$\frac{k_z v_z}{\omega}\,\frac{T}{e\phi} = \frac{k_z^2 e\phi}{M\omega^2}\,\frac{T}{e\phi} = \frac{k_z^2 C_s^2}{\omega^2} \sim \left(\frac{k_z^2}{k_y^2}\right)\left(\frac{L}{\gamma_{Li}}\right)^2$$

The validity of the two theories thus depends upon the values of k_z. For large k_z's (such that $\frac{k_z}{k} > \frac{\gamma_{Li}}{L}$) the drift - acoustic mode theory holds good while in the opposite limit of small k_z's (so that $\frac{k_z}{k} < \frac{\gamma_{Li}}{L}$), the drift mode theory is valid. We note here that for $k_z/k > \gamma_{Li}/L$, the drift mode is more like an ion sound wave[37]. In terms of the F region parameters, this corresponds to the parallel scalesizes $(\lambda_z) \lesssim$ 200 kms. (for $\lambda_\perp \sim$ 100 meters) whereas the drift theory case corresponds to $\lambda_z \gtrsim$ 200 kms.

It is generally believed that the spread F irregularities are likely to have very long scalelengths parallel to the magnetic field in the nighttime F region. In such a case, the nonlinear drift wave theory for long parallel wavelengths presented above would apply. But other perturbations with shorter λ_z's would also be possible in the medium and would be of the drift-acoustic type. A clear cut distinction between such two types of irregularities is thus only possible through the measurements of parallel scalesizes of the irregularities.

Finally, we comment on the profile of the nonlinear wave in light of the recent in-situ measurements of equatorial spread F irregularities. Such measurements have reported steepened wavelike structures of density in the medium[41]. The longer scalesizes ($\sim$ a few kms.) in equatorial spread F are due to R-T mode while medium scalesizes ($\sim$ a few hundred meters) are due to the collisional drift instability. In this section, we have noted the possibility of getting a steepened wave profile for the collisional drift instability case, when due to low viscosity, many harmonics are generated nonlinearly. This may qualitatively explain the observations of such structures at medium scalesizes[41]. For the case of long scalelengths, it was shown in the last section that R-T mode also has such non-sinusoidal nonlinear solutions.

CHAPTER 3

IRREGULARITIES IN EQUATORIAL ELECTROJET

The irregularities in the equatorial E region have been studied for a considerable period now using ionosonde, VHF backscatter radar, rocket-borne in-situ, langmuir probes etc.[20]. It is more or less established that the physical situation in the daytime E region corresponds to a partially ionized plasma with magnetized electrons ($\nu_e << \Omega_e$) and collisional ions ($\nu_i >> \Omega_i$). A vertical polarization electric field pointing upwards drives the electrons in the $\underset{\sim}{E} \times \underset{\sim}{B}$ direction with respect to ions and this constitutes the so-called electrojet current. In the nighttime, the electric field and hence the current reverse in the direction. Similar physical situations are encountered elsewhere also like auroral electrojets, MHD generators, certain type of gas discharges etc. The cross - field current in these systems is known to result in the excitation of low-frequency two-stream and cross-field instabilities. The two-stream instability[18,7] occurs when the electron drift velocity V_d exceeds the ion-acoustic speed C_s and favours the excitation of small scalesizes (of the order of a few meters) associated with the type I VHF radar echoes. The cross-field instability[71,31] requires a density gradient parallel to the polarization electric field for excitation and has got a lower threshold than that of the two-stream instability. It favours the excitation of large scalesizes and is believed to be causally associated with the type II radar echoes[59, 77].

In the present chapter, we have developed a generalized fluid treatment for the excitation of electrostatic - electromagnetic waves by a cross-field current in a partially ionized inhomogeneous plasma system as described above. Next we discuss the relevance of the results to observable phenomena in the electrojet region of ionosphere. Finally, we have also examined the two - step theory of generation of type II small scalesize irregularities in equatorial electrojet, by using a mode - coupling nonlinear treatment.

3.1 GENERAL THEORY OF EXCITATION OF ELECTROSTATIC - ELECTROMAGNETIC IRREGULARITIES BY A CROSS - FIELD CURRENT

It is generally recognized in plasma physics that electromagnetic effects can no longer be ignored on streaming instabilities at long wavelengths, when $ck \lesssim \omega_{pe}$. Thus given a physical situation, it is of interest to investigate the excitation of mixed electrostatic - electromagnetic modes by a cross - field current. In this section, we present a generalized fluid treatment for the excitation of electrostatic - electromagnetic (es - em) instabilities in such a system. We use the entire set of Maxwell's equations in order to account for the electromagnetic effects and derive a generalized dispersion relation. This dispersion relation is then analysed in the various limits of the parameter ck/ω_{pe}. The limit $ck/\omega_{pe} \gg 1$, corresponds to the electrostatic limit in which we recover the earlier results of the two-stream and cross-field instabilities. For $ck/\omega_{pe} \lesssim 1$, we discover a new mixed es-em instability for perturbations which have a finite parallel wave vector ($k_{\parallel}$). In what follows, we first present the mathematical analysis for the general case and then analyse each limit separately. Finally we discuss their observational significance in the equatorial electrojet situation.

The basic equations used in the analysis are the two fluid equations describing the perturbed motions of electrons and ions. This is justified when the Doppler shifted wave frequency in the electron frame $(\omega - \underset{\sim}{k}\cdot\underset{\sim}{u}_o) \ll \nu_e$ and $\omega < \nu_i$ and also the inequalities $k_\perp r_{Le} \ll 1$, $k \lambda^{e,i}_{mfp} \ll 1$ are satisfied. We choose a coordinate system in which the z-axis is along the magnetic field, the x-axis is the direction of inhomogeneity and y-axis is the direction of equilibrium electron drift with respect to ions. In the equatorial E region, these correspond to northward, downward and westward directions respectively in the daytime. The entire set of Maxwell's equations is used. Further we also take the zero-order gradient in the steady magnetic field into account. This gradient is a result of an equilibrium current flowing in the system, and is given by Maxwell's equation[3]

$$\frac{\partial B_o}{\partial x} = \frac{4\pi}{c} e n_o u_o \tag{1}$$

where u_o is the equilibrium electron drift velocity.

The basic equations are

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \underset{\sim}{v}_\alpha) = 0 \tag{2A}$$

$$0 \simeq -\nabla p_e - e n_e \left[\underset{\sim}{E} + \frac{\underset{\sim}{v} \times \underset{\sim}{B}_o}{c} + \frac{\underset{\sim}{u}_o \times \underset{\sim}{\tilde{B}}}{c} \right] - m_e n_e \nu_e \underset{\sim}{v}_e \tag{2B}$$

$$M n_i \left(\frac{\partial}{\partial t} + \underset{\sim}{V} \cdot \nabla \right) \underset{\sim}{V} = -\nabla p_i + e n_i \underset{\sim}{E} - M n_i \nu_i \underset{\sim}{V} \tag{2C}$$

and the Maxwell's equations

$$\nabla \times \underset{\sim}{E} = -\frac{1}{c}\frac{\partial \underset{\sim}{B}}{\partial t} \quad ; \nabla \times \underset{\sim}{B} = \frac{4\pi}{c}\underset{\sim}{J} + \frac{1}{c}\frac{\partial \underset{\sim}{E}}{\partial t} \tag{2D}$$

The Maxwell's equations may be combined to yield

$$\nabla^2 \underset{\sim}{E} - \nabla(\nabla \cdot \underset{\sim}{E}) = \frac{4\pi}{c^2}\frac{\partial \underset{\sim}{J}}{\partial t} \tag{2E}$$

which is generally known as the wave equation. In the above, electron inertia and the displacement current have been ignored due to the low frequencies we are concerned with. The perturbed current is given

$$\underset{\sim}{J} = e n_0 (\underset{\sim}{V} - \underset{\sim}{v}) - e\tilde{n}\underset{\sim}{u}_0 \tag{3}$$

We follow the standard procedure of obtaining the dispersion relation : express the perturbed currents (3) in terms of wave electric fields and substitute it in (2E), which finally yields the dispersion relation. For this, we obtain perturbed ion and electron velocities and the perturbed density from linearized equations of continuity and motion and substitute them in (3), assuming a harmonic time dependence of the form exp $(-i\omega t)$ for perturbations. Substitution of $\underset{\sim}{J}$ in (2E) yields a vector equation in $\underset{\sim}{E}$. We fourier analyse in y, z directions and eliminate all components of $\underset{\sim}{E}$ except one and in the process take account of the inhomogeneities of n_0 and B_0 along x-axis. Finally the dispersion relation is obtained by fourier analysing in the x-direction also, which is justified for $k_x \gg \frac{d(\ln n_0)}{dx}, \frac{d(\ln B_0)}{dx}$.

For simplicity of presentation, we shall further describe the procedure only for cold plasmas. The introduction of finite temperature primarily leads to diffusion damping of modes propagating almost perpendicular to the magnetic field and these corrections will be introduced in the final results.

The perturbed ion and electron velocities can be written as

$$\underset{\sim}{V} = e\underset{\sim}{E} / M(\nu_i - i\omega) \tag{4}$$

$$\underset{\sim}{v} = -\frac{c}{eB_o}(\underset{\sim}{F}_{\perp} \times \hat{e}_z) + \frac{\nu_e}{m\Omega_e^2}\underset{\sim}{F}_{\perp} + \hat{e}_z \frac{\underset{\sim}{F}_{\parallel}}{m\nu_e} \tag{5}$$

where

$$\underset{\sim}{F} = -e\underset{\sim}{E} + \frac{ie}{\omega}\underset{\sim}{u}_o \times (\nabla \times \underset{\sim}{E}) \tag{6}$$

is the net linearized electromagnetic force on the electron fluid (the second term arising from the magnetic field perturbation $\tilde{\underset{\sim}{B}}$ interacting with equilibrium electron flow) and $\hat{e}_z$ is the unit vector along the z-axis. Ions were assumed collisional ($\nu_i \gg \Omega_i$) in deriving (4) and hence the effect of $\underset{\sim}{B}_o$ was ignored on their motion, whereas assumption of magnetized electrons ($\nu_e \ll \Omega_e$) leads to the appearance of the Hall, the Pederson, and the direct mobilities respectively on right side of (5).

The ion - continuity equation gives

$$\frac{\tilde{n}_i}{n_o} = -\frac{i}{\omega}\left[\nabla \cdot \underset{\sim}{V} + \underset{\sim}{V} \cdot \frac{\nabla n_o}{n_o}\right] \tag{7}$$

We assume quasi - neutrality now, $\tilde{n}_e \simeq \tilde{n}_i$, and substitute for $\tilde{n}$, $\underset{\sim}{v}$, and $\underset{\sim}{V}$ from (7), (5) and (4) respectively in J's expression in (3), which on being substituted in (2) leads to

$$\nabla(\nabla\cdot\underset{\sim}{E}) - \nabla^2\underset{\sim}{E} = \frac{4\pi e n_0}{c^2}(i\omega)\Big[\frac{e\underset{\sim}{E}}{M(\nu_i - i\omega)} + \frac{e}{m\nu_e}\Big\{E_z + \Big(\frac{\underset{\sim}{u}_0}{i\omega}\times\nabla\times\underset{\sim}{E}\Big)\cdot\hat{e}_z\Big\}\hat{e}_z - \frac{c}{B_0}\Big(\underset{\sim}{E} + \frac{\underset{\sim}{u}_0}{i\omega}\times\nabla\times\underset{\sim}{E}\Big)\times\hat{e}_z + \frac{e\nu_e}{m\Omega_e^2}\Big(\underset{\sim}{E} + \frac{\underset{\sim}{u}_0}{i\omega}\times\nabla\times\underset{\sim}{E}\Big)\Big] - \underset{\sim}{u}_0\frac{\omega_{pi}^2}{c^2\nu_i}\Big[\nabla\cdot\underset{\sim}{E} + \underset{\sim}{E}\cdot\frac{\nabla n_0}{n_0}\Big] \quad (8)$$

This equation may be fourier analysed now in y, z directions as $\sim \exp i(k_y y + k_z z)$ and the three components : x^{th}, y^{th}, z^{th} be written down. For example, the z^{th} component leads to

$$E_z = \frac{i k_z}{D k_\perp^2}\frac{\partial E_x}{\partial x} - \frac{1}{D k_\perp^2}\Big(k_z k_y + i\frac{\omega_p^2}{c^2}\frac{u_0 k_z}{\nu_e}\Big)E_y$$

where $$D = -1 + i\frac{\tilde{\omega}}{\nu_e}\frac{\omega_{pe}^2}{c^2 k_\perp^2}$$

This equation may be used to eliminate E_z and thus the x-component of (8) leads to a relation between E_y and E_x,

$$X\, E_x = Z\, E_y$$

where $$X = -k^2 + i\frac{\omega_{pe}^2}{c^2}\Big(\frac{m}{M}\frac{\omega}{\tilde{\nu}_i} + \frac{\nu_e\tilde{\omega}}{\Omega_e^2}\Big)$$

and $$Z = i\frac{\omega}{\Omega_e}\frac{\omega_{pe}^2}{c^2} - \frac{\omega k_y}{L k_\perp^2}\frac{\omega_{pe}^2}{c^2}\frac{k_z^2}{D^2 k_\perp^2 \nu_e}$$

Substitution of E_x and E_z, thus obtained, in the y^{th} component leads to the dispersion relation following a fourier analysis in the x-axis also. The dispersion formula is

$$\Big[-k_z^2 + \frac{\omega_{pe}^2}{c^2}\left(i\frac{\tilde{\omega}}{\tilde{\nu}_i}\frac{m}{M} + \frac{i\omega\nu_e}{\Omega_e^2}\right) + \frac{k_y}{D k_\perp^2}\left(i k^2 + \frac{\omega_{pe}^2\tilde{\omega}}{c^2\nu_e}\right)\times$$
$$\frac{\partial}{\partial x}\left(\frac{Z}{X}\right) + \frac{\omega_{pe}^2}{c^2}\frac{Z}{X}\left(i\frac{\tilde{\omega}}{\Omega_e} + \frac{m}{M}\frac{u_o}{L\tilde{\nu}_i}\right) + \frac{k_z^2}{D}\Big(1+$$
$$+ i\frac{\omega_{pe}^2}{c^2}\frac{u_o k_y}{k_\perp^2\nu_e}\Big)\left(-1 + i\frac{\omega_{pi}^2}{c^2 k_\perp^2}\frac{u_o k_y}{\tilde{\nu}_i}\right)\Big] = 0 \qquad (9)$$

where we assumed $k_\perp \gg k_z$ and ignored higher order terms in $(d\ln B_o/dx)$ and $(d\ln n_o/dx)$.

Now we analyse the dispersion relation in various limits. First we consider the case when $k_z = 0$. Then the electrostatic limit, $ck/\omega_p \gg 1$ of (9) leads to the recovery of two-stream and cross-field instability driven modes. To this end, we further assume $c^2 k_\perp^2/\omega_{pe}^2 \gg m/M$, a condition which restricts our results to $k_\perp^{-1} \lesssim 20$ kms., (which poses no serious problem to our treatment as, at such large wavelengths, the local approximation used by us viz. $k_x \gg \frac{d\ln n_o}{dx} \sim L^{-1}$ would no longer be valid, since $L \sim 10$ kms.).

We can then replace X by $-k^2$ in (9), and write the dispersion relation

$$i\frac{\tilde{\omega}}{\tilde{\nu}_i}\frac{m}{M} + i\frac{\omega\nu_e}{\Omega_e^2} - \frac{\omega}{\Omega_e}\frac{k_y}{k_\perp^2 L} + \frac{\omega_{pe}^2}{c^2k^2}\frac{\omega^2}{\Omega_e^2} = 0 \qquad (10)$$

Now we assume, $\frac{ck}{\omega_{pe}} \gg 1$, and write

$$\tilde{\omega} + \frac{\omega\nu_e\tilde{\nu}_i}{\Omega_e\Omega_i} + i\frac{k_y}{k_\perp^2 L}\frac{\omega\tilde{\nu}_i}{\Omega_i} = 0 \qquad (11)$$

For $\omega = \omega_R + i\gamma$ with $|\gamma| << \omega_R$, this equation yields

$$\omega_R \simeq u_o k_y / (1 + \frac{\nu_e \nu_i}{\Omega_e \Omega_i}) \tag{12a}$$

and

$$\gamma \simeq (1 + \frac{\nu_e \nu_i}{\Omega_e \Omega_i})^{-1} \left[\frac{\nu_e \omega_R^2}{\Omega_e \Omega_i} - \frac{k_y}{k_\perp^2 L} \frac{\nu_i}{\Omega_i} \omega_R \right] \tag{12b}$$

The first term gives the two-stream contribution[18] and the second one gives the cross-field contribution[59] to growth rate of electrostatic perturbations in the above (one should remember that $L \sim (\frac{1}{n_o} \frac{dn_o}{dx})^{-1}$ is negative).

When one introduces finite temperature effects, the growth rate is modified to

$$\gamma \simeq (1 + \frac{\nu_e \nu_i}{\Omega_e \Omega_i})^{-1} \left[\frac{\nu_e \omega_R^2}{\Omega_e \Omega_i} - \frac{k_y \nu_i}{k_\perp^2 L \Omega_i} \omega_R - \frac{\nu_e k^2}{\Omega_e \Omega_i} \frac{T_e + T_i}{M} \right] \tag{12c}$$

We find that the temperature term leads to a diffusive damping of the perturbations and imposes a certain threshold condition on the drift velocity for excitation. The characteristic damping time $\sim \left(\frac{k^2 \nu_e}{\Omega_e \Omega_i} c_s^2 \right)^{-1}$ can be interpreted as the time required to destroy a fluctuation with scale-length k^{-1} by cross-field electron diffusion. Due to ambipolar effects, the slower electron diffusion dominates over the ion diffusion.

Because of its k-dependence, the diffusion damping increases with k (i.e. higher for shorter wavelengths) and decreases with k decreasing (less for longer wavelengths). For very large wavelengths, then recombination also becomes an important damping mechanism for the perturbations. Thus in the daytime, for long wave cross-field instability driven perturbations in the electrojet, recombination damping should be taken into account to calculate actual growth rates etc. Introduction of this modifies the above growth rate to

$$\gamma = \left(1+\frac{\nu_e\nu_i}{\Omega_e\Omega_i}\right)^{-1}\left[\frac{\nu_e\omega_R^2}{\Omega_e\Omega_i} - \frac{k_y\nu_i}{k_\perp^2 L\,\Omega_i}\,\omega_R - \frac{\nu_e k^2 c_s^2}{\Omega_e\Omega_i}\right] - \nu_R \qquad (12d)$$

where $\nu_R = 2\alpha n_0$, α being the recombination rate.

Retention of electromagnetic corrections for this case modifies the cold plasma growth rate to

$$\gamma \simeq \left(1+\frac{\nu_e\nu_i}{\Omega_e\Omega_i}\right)^{-1}\left[\left(\nu_e+\nu_i\frac{\omega_p^2}{c^2k^2}\right)\frac{\omega_R^2}{\Omega_e\Omega_i} - \frac{k_y}{k_\perp^2 L}\frac{\nu_i}{\Omega_i}\,\omega_R\right] \qquad (13)$$

It shows that electromagnetic corrections to these instabilities are further destabilizing and become important for $\frac{ck}{\omega_{pe}} \simeq \sqrt{\frac{\nu_i}{\nu_e}}$. For the electrojet parameters, though, gradient - drift instability term always dominates over these terms for long wavelengths, hence these corrections are unimportant in this situation.

The two-stream instability is driven by ion inertia, and the resistive Pederson motion of electrons is essential to obtain this instability. In equatorial electrojet, for $v_d > C_s$, it excites the wavelengths typically of the order of 3 meters and causes the type I radar echoes. At such wavelengths, the contribution due to the gradient term to growth is not significant[69]. In some cases, this term may reduce the threshold of excitation of type I irregularities to a value of electron drift v_d, a little lesser than C_s [21]. Recently this instability has been identified in the laboratory plasmas[35].

The gradient - driven cross-field instability preferentially excites scalesizes of the order of a few hundred meters. This instability is analogous to the gravitational instability of an inhomogeneous plasma layer, supported by a magnetic field against gravity.[64] In the cross-field case, the instability takes place when the plasma density gradient is supported by the magnetic field against increasing electrostatic potential and collisions provide the required cross-field relative mobility between ions and electrons. In the linear limit, ion inertia is not important for excitation of this instability. It has been well established that this instability occurs in the equatorial electrojet by rocket experiments of Prakash et al.[56], which have been further supported by radar measurements[22], and it has been identified recently in a laboratory - simulation experiment of electrojet situation[68].

Next we consider modes with $k_z \neq 0$. We consider $k_\perp \gg k_z$, $k_z^2/k^2 > \frac{m}{M}$, $O(1) \gtrsim \frac{Ck}{\omega_{pe}} \gg \frac{m}{M}$, $\nu_i \gg \omega$, $\frac{\nu_e \nu_i}{\Omega_e \Omega_i} \ll 1$, then we can write

$$Z \simeq \frac{i\omega\,\omega_{pe}^2}{\Omega_e c^2}, \quad X \simeq -k^2, \quad \tilde{\nu}_i (= \nu_i - i\omega) \simeq \nu_i,$$

$$\frac{\partial}{\partial x}\left(\frac{Z}{X}\right) \simeq -i\,\frac{\omega\,\omega_{pe}^2}{\Omega_e c^2 k^2}\left(\frac{1}{L} - \frac{u_0}{\Omega_e}\frac{\omega_{pe}^2}{c^2}\right).$$

Then the dispersion relation can be written as

$$\left[-k_z^2 + i\,\frac{\tilde{\omega}}{\nu_i}\,\frac{\omega_{pi}^2}{c^2} + \frac{\omega_{pe}^4}{c^4 k^2}\,\frac{\omega\tilde{\omega}}{\Omega_e^2}\right] - \frac{\omega\,\omega_{pe}^2}{\Omega_e c^2 k^2}\left[\frac{k_y}{L} - \frac{k_y u_0}{\Omega_e}\,\frac{\omega_{pe}^2}{c^2}\right] - \frac{k_z^2}{D}\left(1 + i\,\frac{\omega_{pe}^2}{c^2}\,\frac{u_0 k_y}{\nu_e k_\perp^2}\right) \simeq 0$$

where terms of the order $\left(\frac{m}{M}\frac{u_0}{L\nu_i}\right), \left(\frac{m}{M}\right)\left(\frac{u_0 k_y}{\nu_i}\right)$ were ignored. On substituting for D, and after some algebra we get the dispersion relation

$$k_z^2 + \frac{m}{M}\frac{\nu_e\tilde{\omega}}{\nu_i\omega}k_\perp^2 - i\,\frac{\omega_{pi}^2}{c^2}\,\frac{\tilde{\omega}^2}{\omega\nu_i} = \frac{\omega_{pe}^4\,\tilde{\omega}}{c^4 k^2\,\Omega_e^2}\left(\omega - \frac{c^2 k_y\,\Omega_e}{\omega_{pe}^2\,L}\right)\left(1 + i\,\frac{c^2 k_\perp^2\,\nu_e}{\omega_{pe}^2\,\tilde{\omega}}\right) \tag{14}$$

We further assume that $L \to \infty$ and neglect the right hand side of the above as, $\frac{\nu_e\nu_i}{\Omega_e\Omega_i} << 1$ and $\omega\tilde{\omega}/\Omega_e^2$ is small, then the above equation becomes

$$k_z^2 + \frac{\tilde{\omega}}{\omega}\left[\frac{m}{M}\frac{\nu_e}{\nu_i}k^2 - i\,\frac{\tilde{\omega}\,\omega_{pi}^2}{\nu_i c^2}\right] \simeq 0 \tag{15A}$$

We now introduce temperature effects and the recombination damping (as we are considering long wavelengths), then the above equation becomes

$$k_z^2 + \frac{\tilde{\omega} + i\nu_R}{\omega + i\nu_{eff}}\left[\frac{m}{M}\frac{\nu_e}{\nu_i}k^2 - i\,\frac{\tilde{\omega}\,\omega_{pi}^2}{\nu_i c^2}\right] \simeq 0 \tag{15}$$

where $$\nu_{eff} = \nu_R + \frac{k^2 (T_e + T_i)}{M \nu_i}$$

Substituting $\omega = \omega_R + i\gamma$ with $|\gamma| << \omega_R$, then we get

$$\omega_R \simeq u_o k_y \left(\frac{m}{M}\frac{\nu_e}{\nu_i}\right) \Big/ \left(\frac{k_z^2}{k^2} + \frac{m\nu_e}{M\nu_i}\right) \tag{16a}$$

and
$$\gamma \simeq \left(\frac{k_z^2}{k^2} + \frac{m\nu_e}{M\nu_i}\right)^{-1} \left[\frac{\omega_{pi}^2}{c^2 k^2 \nu_i} \frac{(k_z^2/k^2)^2 u_o^2 k_y^2}{\left(\frac{k_z^2}{k^2} + \frac{m\nu_e}{M\nu_i}\right)^2} - \nu_{eff}\left(\frac{k_z^2}{k^2} + \frac{m\nu_e}{M\nu_i}\frac{\nu_R}{\nu_{eff}}\right)\right] \tag{16b}$$

This is a new instability for almost perpendicularly propagating electromagnetic - electrostatic modes. This instability can be obtained only when the electromagnetic effects (at $ck/\omega_{pe} \lesssim 1$) are retained, and a finite parallel propagation ($k_\parallel$) is assumed. It can occur in homogeneous systems ($L \to \infty$) and, unlike the two - stream instability, does not require ion inertia to drive it. In electrojet situation, it can be observed as density-cum-magnetic field perturbations and requires a lower threshold electron velocity for excitation than C_s. Such instabilities have been studied for semi-conductor plasmas recently[34]. We discuss this instability in greater detail in the next section.

Inclusion of finite temperature in (14), with $\frac{ck}{\omega_{pe}} \gg 1$, and assuming $k_z^2/k^2 \gg \frac{m}{M}$, $L \to \infty$, leads to

$$\omega^2 + i\omega\nu_i - k^2 C_s^2 = -i\left(\frac{m}{M}\right)\left(k^2/k_z^2\right)\nu_e\tilde{\omega} \tag{17}$$

which, on writing $\omega = \omega_R + i\gamma$; $\gamma << \omega_R$ leads to

$$\omega_R \simeq k\, c_s \tag{18a}$$

$$\gamma \simeq -\frac{\nu_i}{2} - \left(\frac{m}{M}\right)\left(\frac{k^2}{k_z^2}\right)\nu_e\left(1 - \frac{\underset{\sim}{k}\cdot\underset{\sim}{v}_d}{\omega}\right) \tag{18b}$$

Thus unstable ion - acoustic waves result if $\underset{\sim}{v}_d$ exceeds the ion-acoustic speed[38]. This is a negative - energy wave instability (Hasegawa, 1971)[29]. Physically, it arises due to the diffusion damping effects in electron motion parallel to the magnetic field, and hence requires a finite k_z for its existence. The Pederson motion for electrons is not important in this case. Strictly, one should use kinetic equation for ions, as $\omega > \nu_i$, for the instability. But if $T_e >> T_i$ (so that the ion Landau damping effects are small), ion fluid equations are adequate for the analysis. The relevance of this instability to some non-field-aligned structures observed in ratio aurora has been pointed out by Kaw[38]. We have also studied the nonlinear development of this instability, which shall be presented in the next (4th) chapter.

3.2 A NEW ELECTROSTATIC – ELECTROMAGNETIC MODE

In the last section, it has been shown that a cross-field current driven mixed electrostatic – electromagnetic instability can occur in a weakly ionized magnetoplasma for the perturbations with $Ck/\omega_{pe} \lesssim 1$, and a non-zero $k_{\parallel}$. The real frequency of these modes and their growth rate is given by expressions (16a) and (16b) respectively, of last section. The threshold velocity required for excitation turns out to be smaller than C_s, in this case. It can be calculated by putting $\gamma = 0$, in (16b), and we obtain

$$k_y^2 u_{0T}^2 = \frac{c^2 \nu_i \nu_{eff}}{\omega_{pi}^2 k_z^4} \left(k_z^2 + k^2 \frac{m\nu_e}{M\nu_i} \right)^2 \left(k_z^2 + k^2 \frac{m\nu_e \nu_R}{M\nu_i \nu_{eff}} \right) \quad (1)$$

One can derive an expression for the minimum threshold velocity by first minimizing the right side of above equation with respect to $(k_z/k)^2$, treating k as a given quantity. This minimization occurs for

$$k_z^2/k^2 \simeq \frac{m\nu_e}{M\nu_i} \quad (2)$$

and gives

$$u_{0T} \simeq \frac{c}{\omega_{pe}} (\nu_e \nu_{eff})^{1/2} \quad (3)$$

Now we notice that the least ν_{eff} is ν_R, and is for waves with

$$k \lesssim \left(\frac{\nu_i \nu_R}{c_s^2} \right)^{1/2} \tag{4}$$

giving an absolute minimum threshold velocity of magnitude

$$u_{Tm} \simeq \frac{c}{\omega_{pe}} \left(\nu_e \nu_R \right)^{1/2} \tag{5}$$

As $\nu_R = 2\alpha n_0$, we find that u_{Tm} is independent of equilibrium density n_0. The growth rate well above threshold is obtained from (16b) of last section and (2), and is given by

$$\gamma \simeq \frac{\omega_{pe}^2}{\nu_e} \frac{u_0^2}{c^2} \tag{6}$$

The effect of background density gradient on this instability can be determined from the relation (14) of the previous section. We introduce finite temperature and recombination damping effects in that equation and assume that $\omega << \dfrac{c^2 k_y \Omega_e}{\omega_{pe}^2 L}$, which essentially means that $\left(\dfrac{d \ln B_0}{d \ln n_0}\right) << 1$ (or $L_B >> L$ where L_B is the scalesize of magnetic field gradient), thereby ignoring magnetic field gradient effects from the equation. Then the resulting expression leads to the modified minimum threshold velocity

$$\bar{u}_{Tm} = u_{Tm} \left[\sqrt{1+\beta^2} + \beta \right] \tag{7}$$

where u_{Tm} is given by (5) and

$$\beta = \frac{M\,\omega_{pe}}{2\,mck}\,\frac{\sqrt{\nu_e \nu_{eff}}}{\Omega_e}\,\frac{1}{kL} \tag{8}$$

As one would expect, the gradient is destabilizing for $\beta < 0$ (due to negative L, i.e. gradient pointing upwards in the daytime) and stabilizing for $\beta > 0$.

Physically this instability is like a resistive negative energy wave instability in which the electron stream interacts with a wave moving slower than it and transfers energy to it through collisional interaction. The physics of the instability becomes clearer from a simple derivation of the dispersion relation which we present here. For simplicity, we assume $L, L_B \to \infty$, $k_x = 0$ and write linearized perturbed currents J_y and J_z as

$$J_y \simeq -e\tilde{n}u_0 + \frac{e^2 n_0}{M\nu_i}\left(E_y - \frac{T_i}{e}\,ik_y\frac{\tilde{n}}{n_0}\right) \tag{9}$$

$$J_z \simeq \frac{e^2 n_0}{m\nu_e}\left(\frac{\tilde{\omega}}{\omega}E_z + \frac{k_z u_0}{\omega}E_y + \frac{T_e}{e}\,ik_z\frac{\tilde{n}}{n_0}\right) \tag{10}$$

where we have included temperature corrections. The parallel current is dominated by collisional electron motion (we have ignored corresponding ion motion). The important contributions to the perpendicular current are from electron density perturbations carried by the equilibrium electron drift and collisional ion motion along the electric field. The x component of electric field and J_x are both negligible in this approximation $(k_x = 0)$.

The ion continuity equation gives density perturbations

$$\frac{\tilde{n}}{n_o} \simeq \frac{e k_y E_y}{M \nu_i (\omega + i \nu_R + i k_y^2 T_i / M \nu_i)} \tag{11}$$

where it was assumed that $k_y E_y$ dominates in $\underset{\sim}{k} \cdot \underset{\sim}{E}$ as $k_z << k_\perp$.
The y^{th} and z^{th} components of the wave equation now become

$$\left[k_z^2 - \frac{i\omega}{\nu_i} \frac{\omega_{pi}^2}{c^2} \frac{(\tilde{\omega} + i \nu_R)}{(\omega + i \nu_R + i k^2 T_i / M \nu_i)} \right] E_y = k_y k_z E_z \tag{12}$$

and

$$\left[k_y^2 - i \frac{\omega_{pe}^2 \tilde{\omega}}{c^2 \nu_e} \right] E_z = k_y k_z E_y \left[1 - \frac{\omega}{\nu_e} \frac{\omega_{pe}^2 \, T_e}{M c^2 \nu_i (\omega + i \nu_R + i \frac{k^2 T_i}{M \nu_i})} + i \frac{u_o k_y}{\nu_e} \frac{\omega_{pe}^2}{c^2 k_y^2} \right] \tag{13}$$

The dispersion relation can be easily obtained from the eliminant of these two equations. The instability is closely related to the collisional perpendicular motion of ions and the electron streaming which carries the density perturbations with velocity u_o ($> \omega / k_y$) through the term $\tilde{n} u_o$ in (9). The electron streaming also enters the problem by giving a $\underset{\sim}{u}_o \times \underset{\sim}{\tilde{B}}$ component of parallel force on electrons and emphasizes the electromagnetic character of the instability. The energy exchange between the stream and the wave is obtained by computing $\overline{\underset{\sim}{J} \cdot \underset{\sim}{E}}$, the time averaged work done per unit time by the wave on particles. Thus a negative $\overline{\underset{\sim}{J} \cdot \underset{\sim}{E}}$ means vice-versa and represents a gain in wave energy at the expense of particle kinetic energy, or in other words, a wave growth. To focus our attention only on the processes leading to instability, we put temperature effects and ν_R equal to zero (as they only cause damping), and write from (9) and (11)

$$\overline{J_y E_y} = \frac{e^2 n_o}{M \nu_i} \overline{E_y^2} - e u_o \overline{\tilde{n} E_y} \tag{14}$$

$$\doteq \frac{e^2 n_o}{M \nu_i} \frac{\tilde{\omega}}{\omega} \overline{E_y^2} \tag{15}$$

From (15) we recognize that if $\omega < k_y u_o$, $\overline{J_y E_y}$ can be negative and this term can lead to amplification. The amplification results from the second term on right of (14) and hence because of energy exchange between the electron stream and the wave. The resistive nature of the instability is borne out by the fact that collisions are essential to this energy exchange process (otherwise V and hence n would always be 90° out of phase with E_y and no net energy exchange would result). Using (10) and (12) one can show

$$\overline{J_z E_z} = \frac{e^2 n_o}{m \nu_e} \overline{E_z^2} \Big/ \left\{ 1 + \left(\frac{\omega_{pe}^2}{c^2 k^2} \right)^2 \frac{k_y^2 u_o^2}{\nu_e^2} \right\} \tag{16}$$

Thus the parallel motion of electrons, both due to wave electric field (E_z) and wave magnetic field ($\underset{\sim}{u_o} \times \underset{\sim}{\tilde{B}}$), dissipates the wave.

We see from the above that introduction of finite k_z has two effects : (i) it produces a magnetic field perturbation and slows the wave down to a speed less than u_0 (eqn. 16 (a) of last section) and is responsible for its amplification through (15), (ii) it induces a parallel motion of electrons which dissipates the wave. These competing effects explain the optimization of the wave growth with k_z/k. Equations (15) and (16) can be used to obtain the growth rate of the instability. We denote the wave energy density by W and write

$$\frac{dW}{dt} = -\overline{\underset{\sim}{J}\cdot\underset{\sim}{E}}$$

$$= \frac{e^2 n_0}{M\nu_i}\,\overline{E_y^2}\,\frac{M}{m}\,\frac{\nu_i}{\nu_e}\,\frac{k_z^2}{k^2}\left[1-\frac{k^2}{k_z^2}\,\frac{\overline{E_z^2}}{\overline{E_y^2}}\,\frac{1}{1+\frac{\omega_p^2 u_0^2}{c^2\nu_e^2}\frac{\omega_p^2}{c^2k^2}}\right] \quad (17)$$

where we used (16a) of the last section to substitute for $\tilde{\omega}/\omega$. Here, W is the sum of electromagnetic field energy density of the wave and kinetic energy density of the particle motions (under the influence of the wave fields). It can be readily verified that the dominant contribution comes from the magnetic field energy, $B_x^2/8\pi$, so that

$$W \simeq \frac{\overline{B_x^2}}{8\pi} \simeq \frac{\overline{E_y^2}}{8\pi}\,\frac{\omega_{pe}^2}{c^2k^2}\,\frac{\omega_{pi}^2}{\nu_e\nu_i} \quad (18)$$

where we have used Maxwell's equation $\nabla\times\underset{\sim}{B}=\frac{4\pi}{c}\underset{\sim}{J}$'s y^{th} component to express B_x in terms of J_y, which was expressed in terms of E_y as in (15). For the optimum case, $\frac{M\nu_i}{m\nu_e}\frac{k_z^2}{k^2}\sim 1$, we replace $\frac{dW}{dt}$ by $2\gamma W$ and assume that $\frac{\omega_{pe}^2}{\nu_e^2}\frac{u_0^2}{c^2}\frac{\omega_{pe}^2}{c^2k^2} << 1$, we then find

$$\gamma \simeq \frac{u_0^2}{c^2}\,\frac{\omega_{pe}^2}{\nu_e}$$

which is just the growth rate (6) derived earlier.

Thus we can summarise that long wave perturbations travelling obliquely to the magnetic field in a system carrying a cross-field current get destabilised if u_0 is sufficiently high. We now calculate the threshold velocity etc. for the equatorial electrojet situation where the conditions necessary for the onset of this instability are normally satisfied. The typical parameters for daytime altitude of ~ 105 kms., are : $\nu_e \simeq 2 \times 10^4\ s^{-1}$, $\nu_i \simeq 1.5 \times 10^3\ s^{-1}$, $B_0 \simeq 0.3$ gauss; $\omega_{pe} \simeq 2 \times 10^7\ s^{-1}$, $\frac{m}{M} \simeq 2 \times 10^{-5}$, $C_s \simeq 3.6 \times 10^4$ cms. s^{-1}, $\nu_R = 2 \times \alpha n_0 \simeq 10^{-2}$ to $10^{-3}\ s^{-1}$ where $\alpha = 10^{-7}$ to $10^{-8}\ cm^{-3}\ s^{-1}$, is the conventional recombination coefficient[58].

One can then obtain (using equations (2) - (6)), the following results for the characteristic properties of this instability :

Typical scalesize	$\left(\frac{2\pi}{k}\right)$	$\gtrsim$	500 meters
Typical aspect angle	k_z/k	$\simeq$	1°
Typical threshold velocity	u_{Tm}	$\simeq$	200 meters s^{-1}
Typical Growth time	γ^{-1}	$\simeq$	1 minute.

All these parameters put this instability in a range amenable to detection by the in-situ rocket-borne Langmuir probe and magnetometer measurements. In general one would expect to see long scalelength correlated fluctuations in electron density and magnetic field. The existing measurements do not exclude the possibility of such irregularities. It may be noted here that the effect of the density gradient (typical daytime value $L \simeq 7$ kms.) on this instability is unimportant for the typical ionospheric parameters given above.

A crucial parameter in any measurement of this instability would be magnetic field perturbation $\tilde{\underset{\sim}{B}}$. A convenient component for measurement at the equator is the vertical component since the background vertical field is small. The linearized theory given above indicates

$$\frac{\tilde{B}_x}{B_o} \simeq \frac{4\pi}{c}\frac{J_y}{B_o} = \frac{u_o}{c}\left(\frac{\omega_{pe}^2}{ck_z\Omega_e}\right)\left(\frac{\tilde{n}}{n_o}\right)$$

where we have used equations (9) and (11) (with damping agents ignored) to express J_y in terms of $\tilde{n}$. For $k_z/k \sim 10^{-2}$, $ck/\omega_{pe} \sim 10^{-1}$ and ionospheric parameters given above, one gets

$$\frac{\tilde{B}_x}{B_o} \simeq 10^{-3}\,(\tilde{n}/n_o)$$

Thus if the above linear relations hold good, a 10% density fluctuation should have a magnetic field fluctuation of order few gammas associated with it; the detection of such field strengths is well within the reach of present magnetometer technology.

Now we speculate over the final saturated nonlinear state of this instability. The biggest open question is what saturates the linear growth in the nonlinear limit. It is conceivable that the instability saturates when the oscillating electron fluid velocities ($\tilde{\underset{\sim}{v}}$) associated with the wave electric fields become as large as the zero - order electron streaming , u_o. But if one can show that, due to some mechanism, the growth is not inhibited at this level then some interesting consequences follow. In this case, one would have locally oscillating velocities of the order of or greater than the ion-sound speed even though the zero - order electron velocity is smaller. In such a situation, small scalelength perturbations due to two-stream instability would be excited in these local regions. This might explain some observations of small scale irregularities when the electrojet current is sub-critical[55].

3.3 TWO - STEP PROCESSES

The linear theories of equatorial electrojet discussed so far, fail to account for the observations of type II radar echoes in the equatorial electrojet. These echoes (scattered by small scalesizes, i.e. of the order of a few meters) are observed even when the equilibrium electron drift velocity v_d is smaller than the ion acoustic speed, C_s [4]. In this situation, the two stream instability cannot be excited because the condition of instability requires that $v_d > C_s$. The cross-field instability, which can be excited even when $v_d < C_s$, favours the generation of large scalesizes (viz. of the order of hundred meters), in the medium. Sudan et al.[73] have proposed a two - step generation mechanism to interpret these irregularities. According to this mechanism, once the large scalesize irregularities (which can be driven linearly by the cross- field instability, when $v_d < C_s$, in the first step) attain some finite amplitudes, the density gradients and electron fluid velocities associated with them can then 'linearly' excite. small scale irregularities in the second step by cross-field or two-stream instability mechanisms.

In this section, we treat the two-step mechanism as a nonlinear mode-coupling process, as has been done in the last chapter for the case of two - step process in equatorial spread F. We assume that in the equilibrium situation in the system, a finite amplitude wave is set up and then we consider interaction of it with small amplitude small scalesize perturbations. The excitation of the latter due to a.c. electric fields and density gradients associated with the large amplitude wave is shown to take place, and this result, in general, is in agreement with the theory of Sudan et al. For simplicity, only the two-stream excitation of secondary modes is considered and is shown to take place even if the equilibrium electron drift v_d is smaller than C_s.

We consider a coordinate system in which for the E-region situation, the x-axis is aligned to the magnetic field lines, y-axis is westwards and the z-axis is vertically upwards.

The equations of continuity and motion for the ions and electrons can be written as

$$\frac{\partial \tilde{n}}{\partial t} + (n_0 + \tilde{n}) \nabla \cdot \underset{\sim}{v}_{\alpha} + (\underset{\sim}{v}_{\alpha 0} + \underset{\sim}{v}_{\alpha}) \cdot \nabla (n_0 + \tilde{n}) = 0 \qquad (1)$$

and

$$0 = -T_{\alpha} \nabla (n_0 + \tilde{n}) + q_{\alpha} e (n_0 + \tilde{n}) \left[-\nabla \phi + \frac{\underset{\sim}{v}_{\alpha} \times \underset{\sim}{B}_0}{c} \right] - m_{\alpha} (n_0 + \tilde{n}) \nu_{\alpha} \underset{\sim}{v}_{\alpha} \qquad (2)$$

where $q_{\alpha} = \pm 1$ (sign of charge).
The equation of motion leads to ,

$$\underset{\sim}{v}_{\alpha} = -\frac{\nu_{\alpha}}{(\nu_{\alpha}^2 + \Omega_{\alpha}^2)} \left[S_{\alpha}^2 \frac{\nabla (n_0 + \tilde{n})}{(n_0 + \tilde{n})} + \frac{q_{\alpha} e}{m_{\alpha}} \nabla \tilde{\phi} \right] - \frac{q_{\alpha} \Omega_{\alpha}}{(\nu_{\alpha}^2 + \Omega_{\alpha}^2)} \left[S_{\alpha}^2 \frac{\nabla (n_0 + \tilde{n})}{(n_0 + \tilde{n})} + \frac{q_{\alpha} e \nabla \phi}{m_{\alpha}} \right] \times \hat{e}_x \qquad (3)$$

where $$S_\alpha^2 = T_\alpha / m_\alpha \, .$$

The equation of continuity can be written as

$$\left(\frac{\partial}{\partial t} + v_{\alpha oy} \frac{\partial}{\partial y} \right) \tilde{n} + (n_o + \tilde{n}) \left(\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) + v_{\alpha y} \frac{\partial \tilde{n}}{\partial y} + v_{\alpha z} \frac{\partial \tilde{n}}{\partial z} + \frac{v_{\alpha z} n_o}{L} = 0 \qquad (4)$$

On substituting for $v_{\alpha y}$ and $v_{\alpha z}$ in the above equation we get,

$$\left(\frac{\partial}{\partial t} + v_{\alpha zo} \frac{\partial}{\partial z} + v_{\alpha yo} \frac{\partial}{\partial y} \right) \tilde{n} - \frac{\nu_\alpha}{(\nu_\alpha^2 + \Omega_\alpha^2)} \Big[S_\alpha^2 \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \tilde{n} + \frac{q_\alpha e}{m_\alpha} \frac{\partial}{\partial y} \left\{ (n_o + \tilde{n}) \frac{\partial \phi}{\partial y} \right\} + \frac{q_\alpha e}{m_\alpha} \frac{\partial}{\partial z} \left\{ (n_o + \tilde{n}) \frac{\partial \phi}{\partial z} \right\} \Big] + \frac{e \Omega_\alpha}{m_\alpha (\nu_\alpha^2 + \Omega_\alpha^2)} \left[\frac{\partial (n_o + \tilde{n})}{\partial z} \frac{\partial \phi}{\partial y} - \frac{\partial (n_o + \tilde{n})}{\partial y} \frac{\partial \phi}{\partial z} \right] = 0 \qquad (5)$$

We ignore the background density gradient $\nabla_z n_o$ and the equilibrium electron drift v_d, in our analysis for the small scalesize perturbations. Further, we assume that the primary large amplitude large scalesize wave is propagating in the y-direction. Then from (3), we have

$$\delta \underset{\sim}{v}_o = \frac{c\, \delta \underset{\sim}{E}_o \times \underset{\sim}{B}_o}{B_o^2} = - \hat{e}_z \frac{c\, \delta E_{oy}}{B_o} \qquad (6)$$

and similarly

$$\frac{\delta n_o}{n_o} \doteq \frac{\underset{\sim}{k} \cdot \delta \underset{\sim}{v}_o}{\tilde{\omega}} = \frac{\underset{\sim}{k} \cdot \delta \underset{\sim}{V}_o}{\omega} = \frac{e k_o \delta E_{oy}}{M \omega_o \nu_i} = \left(\frac{e k_o}{M \omega_o \nu_i} \right) \delta E_{oy} \qquad (7)$$

We shall assume that $\delta \underset{\sim}{v}_o$, $\frac{\delta n_o}{n_o}$ vary as $\cos(k_{oy} y - \omega_o t)$.

Equation (5) above describes the mode - coupling effects between this wave and the small scale perturbations which we assume to be in the z-direction. For ions, we ignore their zero-order wave associated motion, nonlinearity and the temperature term (which contributes to the diffusion damping). We fourier analyse the perturbations as $\sim \exp i(\underset{\sim}{k}\cdot\underset{\sim}{r}-\omega t)$ and write

$$\phi_{k,\omega} = i\,\frac{M\,(\nu_i - i\omega)\omega}{e\,k^2}\,N_{\omega,k} \tag{8}$$

where $N = \tilde{n}/n_0$ and the ion inertia has been included. In the electron equation, we retain the Pederson motion only in the linear form, and write

$$\left(\frac{\partial}{\partial t} + \tilde{v}_{\alpha z o}\frac{\partial}{\partial z}\right)\tilde{n} + \frac{e}{m}\frac{\nu_e}{\Omega_e^2}\,n_0\frac{\partial^2\phi}{\partial z^2} - \frac{e}{m\,\Omega_e}\frac{\partial \tilde{n}_0}{\partial y}\frac{\partial\phi}{\partial z} = 0 \tag{9}$$

One can see clearly that the density perturbation $\tilde{n}$ varies locally at a point in time due to $(v_{0z}\nabla_z\tilde{n})$ and $(v_z\nabla_y\tilde{n}_0)$ terms in the electron equation (9). The first term describes a streaming effect akin to the $(\underset{\sim}{v}_d\cdot\underset{\sim}{k})$ effect in the case of primary modes (arising due to equilibrium electron streaming) and the second term is like $(\underset{\sim}{v}_{E\times B}\cdot\nabla n_0)$ term which gives the linear cross-field instability of the primary modes (due to the background density gradient). In the case of an a.c. $(\nabla\tilde{n}_0)$, note that this gradient term directly does not give a growth, and the phase relationships are such that it contributes to the frequency at which the local density fluctuates temporally at a point, due to the perturbation. The last term is due to electron pederson motion and is essential to obtain the two-stream instability.

The fourier analysis leads to

$$-i\omega N = i k \overline{v}_c (N_- + N_+) - \frac{e k k_o}{m \Omega_e}\left(\frac{\delta \overline{n}_o}{n_o}\right)(\phi_- - \phi_+) + \frac{e \nu_e k^2}{m \Omega_e^2}\phi \quad (10)$$

where $\pm$ terms correspond to $(k \pm k_o)$ fourier components.

Equation (8) is used to eliminate ϕ's,

$$N = -\overline{v}_o \frac{k}{\omega}(N_- + N_+) + \frac{\tilde{\nu}_i}{\Omega_i} k_o \frac{\delta \overline{n}_o}{n_o}\frac{k}{\omega}\left(\frac{\omega_-}{k_-^2}N_- - \frac{\omega_+}{k_+^2}\right)N_+$$
$$- \frac{\nu_e}{\Omega_e^2}\frac{M}{m}\tilde{\nu}_i N$$

where

$$\tilde{\nu}_i = \nu_i - i\omega$$

or

$$\left(1 + \frac{\nu_e \tilde{\nu}_i}{\Omega_e \Omega_i}\right)N = -\frac{k}{\omega}\overline{v}_o (N_- + N_+) + k_o \frac{\delta \overline{n}_o}{n_o}\frac{k}{\omega}\frac{\tilde{\nu}_i}{\Omega_i}\left(\frac{\omega_-}{k_-^2}N_- - \frac{\omega_+}{k_+^2}N_+\right) \quad (11)$$

In a reference frame, moving with the equilibrium electron drift velocity v_d, the frequency ω_o of the large scalesize modes is nearly zero (since their dispersion relation is $\omega_o \simeq k_o v_d$). In this reference frame, then, (11) becomes

$$\left(1 + \frac{\nu_e \tilde{\nu}_i}{\Omega_e \Omega_i}\right)N = -\frac{k}{\omega}\overline{v}_o (N_- + N_+) + k_o \frac{\delta \overline{n}_o}{n_o}\frac{k}{\omega}\frac{\tilde{\nu}_i}{\Omega_i}\left(\frac{\omega}{k_-^2}N_- - \frac{\omega}{k_+^2}N_+\right) \quad (12)$$

This leads to the coupling of ω, k modes to ω, $k \pm k_o$ modes and so on. The infinite series thus obtained, can be easily summed up. For simplicity, we assume $k \gg k_o$. Then the dispersion relation obtained from (12) is of the form

$$1 - \cfrac{\frac{2}{\left(1+\frac{\nu_e\tilde{\nu}_i}{\Omega_e\Omega_i}\right)^2}\left(\bar{v}_0\frac{k}{\omega}+\frac{\tilde{\nu}_i}{\Omega_i\tilde{L}k}\right)\left(\bar{v}_0\frac{k}{\omega}-\frac{\tilde{\nu}_i}{\Omega_i\tilde{L}k}\right)}{1-\cfrac{\frac{1}{\left(1+\frac{\nu_e\tilde{\nu}_i}{\Omega_e\Omega_i}\right)^2}\left(\bar{v}_0\frac{k}{\omega}+\frac{\tilde{\nu}_i}{\Omega_i\tilde{L}k}\right)\left(\bar{v}_0\frac{k}{\omega}-\frac{\tilde{\nu}_i}{\Omega_i\tilde{L}k}\right)}{1-\cdots\infty}} = 0 \qquad (13)$$

where

$$\tilde{L} = \left(k_0\frac{\delta\tilde{n}_0}{n_0}\right)^{-1}$$

This yields to

$$\omega^2\left(1+\frac{\nu_e\tilde{\nu}_i}{\Omega_e\Omega_i}\right)^2\left(1+\frac{4\tilde{\nu}_i^2}{\Omega_i^2\tilde{L}^2k^2}\right) = 4k^2\bar{v}_0^2 \qquad (14)$$

or

$$\omega^2 = 4k^2\bar{v}_0^2 \Big/ \left(1+\frac{2\nu_e\tilde{\nu}_i}{\Omega_e\Omega_i}+\frac{4\tilde{\nu}_i^2}{\Omega_i^2\tilde{L}^2k^2}\right) \qquad (15a)$$

or

$$\omega \simeq 2k\bar{v}_0 \Big/ \left(1+\frac{\nu_e\tilde{\nu}_i}{\Omega_e\Omega_i}+\frac{2\tilde{\nu}_i^2}{\Omega_i^2\tilde{L}^2k^2}\right) \qquad (15b)$$

For the small scalesizes of the order of a few meters and the typical equatorial electrojet parameters, the last term in the denominator above can be neglected

Splitting $\omega = \omega_R + i\gamma$, with $\gamma << \omega_R$, we get

$$\omega_R \simeq 2k\bar{v}_0 \Big/ \left(1+\frac{\nu_e\nu_i}{\Omega_e\Omega_i}\right) \qquad (16)$$

and

$$\gamma \simeq \frac{\nu_e}{\Omega_e\Omega_i}\left(\frac{1}{1+\frac{2\nu_e\nu_i}{\Omega_e\Omega_i}}\right)\left(\omega_R^2 - k^2C_s^2\right) \qquad (17)$$

where in the growth rate the diffusion damping has been introduced. Equation (16) describes the frequency (in laboratory frame) of perturbations carried by the oscillating electron drifts, associated with the large amplitude wave.

From (17) we can derive the condition of growth for the second step two-stream instability. One finds that the growth occurs when

$$\omega_R > k\, C_s$$

or

$$\frac{2\, k \bar{v}_o}{(1 + \nu_e \nu_i / \Omega_e \Omega_i)} > k\, C_s \tag{18}$$

But from equation (7), we know that

$$\bar{v}_o \simeq \frac{c\, \delta\bar{E}_o}{B_o} \simeq \frac{\nu_i}{\Omega_i} \frac{v_d}{(1 + \nu_e \nu_i / \Omega_e \Omega_i)} \frac{\delta\bar{n}_o}{n_o} \tag{19}$$

Substituting it in (18), the growth condition can be written as

$$\frac{\nu_i}{\Omega_i} \frac{v_d}{\left(1 + \frac{\nu_e \nu_i}{\Omega_e \Omega_i}\right)^2} \frac{\delta\bar{n}_o}{n_o} > C_s / 2 \tag{20}$$

which, in general, agrees with the growth condition obtained by Sudan et al.[73] (see their relation (42)). The only difference is the factor 2 which appears above in (20). For, $\nu_e \simeq 4 \times 10^4\ s^{-1}$, $\nu_i \simeq 2.5 \times 10^3\ s^{-1}$, $\Omega_i = 90$ rads., $\Omega_e = 5 \times 10^6$ rads., $\lambda_o = 10^4\ v_d^{-1/2}$ cm, $C_s^2 = 10^9\ cm^2\ s^{-2}$, $\delta\bar{n}_o/n_o \simeq 0.4$, $v_d \simeq 100\ m\ s^{-1}$, we find that the wavelengths to grow in the medium would be $\lambda \gtrsim 2$ meters, due to the second step two-stream excitation of secondary irregularities.

CHAPTER 4

IRREGULARITIES IN AURORAL IONOSPHERE

In this chapter we consider the plasma processes relevant to the auroral ionosphere. It is well known that during the activity period, intense field - aligned currents flow in the auroral E and F regions due to precipitating fluxes of energetic charged particles. In addition to these vertical currents, horizontal cross -field currents flow in the auroral E-region, known as the auroral electrojet. We first investigate the destabilizing effects of a field - aligned current on ion - cyclotron waves in the F and E regions. We then study the effect of nonlinearity on the propagation characteristics of ion - cyclotron modes. Finally we investigate the nonlinear behaviour of a resistive ion - acoustic instability which may be driven by the auroral electrojets.

4.1 CURRENT DRIVEN ION – CYCLOTRON WAVES IN F – REGION

It is well known that a relative streaming between electrons and ions can drive ion – acoustic[24] and electrostatic ion – cyclotron[14] waves unstable in a collisionless plasma. The growth takes place when the relative drift velocity exceeds a threshold value. The damping increment due to electron Landau damping becomes negative in a kind of Doppler effect (and thus causes growth) while the threshold drift velocity for excitation is determined by the dissipative mechanism like ion – Landau damping. The ion – acoustic waves require $v_d > c_s$ and $T_e \gg T_i$ for growth, because at $T_e \simeq T_i$, the ion – Landau damping becomes too heavy and the threshold required for excitation becomes of the order of v_{Te}. But ion – cyclotron waves can be excited even at $T_e \simeq T_i$ with lower drift velocities (than v_{Te}) because, this mode being a slow mode, its parallel phase velocity (ω/k_z) remains smaller than v_{Te}. The effect of weak collisions (which become important as the modal wavelength λ approaches the electron mean freepath λ^e_{mfp} from the lower side, i.e. from $\lambda << \lambda^e_{mfp}$ (which is the collisionless approximation)), on the ion – acoustic instability has been analysed by using a Fokker – Planck collision term[44, 47] or a BGK – type model collision term[72]. It was concluded that the electron – ion collisions (ν_{ei}) facilitate the growth of the wave whereas ion – ion collisions (ν_{ii}) hamper it. Physically, the enhancement of growth due to ν_{ei} is due to the inverse energy dependence of the coulomb collision cross-section due to which the diffusion of particles in velocity space decreases with the increasing velocity. Thus the $e-i$ collisions diffuse more particles from the region which is responsible for the damping (viz. the particles having velocity smaller than the wave phase speed) than from the region which causes the growth (viz. the particles having velocity greater than the wave phase speed) thereby facilitating the excitation. The effect of ion - ion collisions is two - fold. First, their effect is to slow the wave down which results into enhanced ion - Landau damping and the second effect is the viscous drag which also damps the wave. Similar conclusions were reached when the effect of weak ion - ion collisions on the ion - cyclotron instability was investigated[54,76].

The strongly collisional limit, when $\lambda \gg \lambda^{e}_{mfp}$, for the ion - acoustic instability has been investigated using the two - fluid equations and it has been found that the dissipative effects like thermal conductivity, viscosity etc. in the electron equations lead to instability in this case while the ion dissipative effects damp the wave[43, 10, 62, 74]. This instability has been interpreted as the growth of a negative energy wave carried by the electron stream, moving faster than the wave phase velocity, due to the dissipative mechanisms[74].

In the present work, we have studied the effects of strong collisions on the current - driven ion - cyclotron instability. We use the two-fluid equations to describe the motion of electrons and ions across the magnetic field and the parallel electron motion. (Ion motion parallel to the magnetic field is unimportant for this wave). We find that besides the expected destabilization coming from dissipation due to thermal conductivity etc. in the electron equations, the electrical resistivity also contributes to the growth of the ion - cyclotron waves. The dissipation in the ion motion has a stabilizing influence on this instability. A new feature of this instability is the appearance of $k_\perp^2 / k_z^2$ factor in the growth rate due to the characteristic obliqueness of the ion - cyclotron modes which makes the growth rate of this instability much greater than that of a current - driven collisional ion - acoustic wave.

In a current carrying collisional plasma, one must in general, take into account the time dependence of the equilibrium temperatures due to Joule heating etc. Thus Sundaram and Kaw[74] have shown that when this effect is included for the ion acoustic wave instability, the growth rate is severely modified. The reason for this is that for the ion - acoustic wave, the real part of the frequency is a strong function of temperature and therefore the time dependence of the wave frequency (due to

Joule heating etc.) is comparable to the growth rate of the wave. We shall not meet this difficulty with the ion - cyclotron waves, since the wave frequency is only a weak function of temperature.

As this work was nearing completion, we came across the work of Milic, who has also studied the effect of collisions on the current driven electrostatic ion - cyclotron waves by using a Landau collision term in the kinetic equations[49]. He considers the case of long parallel wavelengths ($\lambda_z \gg \lambda^e_{mfp}$) with perpendicular wavelengths ranging from short ($\lambda_\perp \lesssim r_{Li}$) to long ($\lambda_\perp \gg r_{Li}$) with respect to r_{Li} , the ion Larmor radius, and solves the equations numerically. It is shown in his work that as v_d is increased, the instability first sets in at long wavelengths and then at shorter wavelengths. He also finds that ν_{ei} facilitates the growth whereas ν_{ii} hampers it. Note here that the physical mechanisms responsible for the instability are more easily discernible from the fluid analysis than from the kinetic treatment.

We have also studied the effect of parallel current on an obliquely travelling electromagnetic ion - cyclotron wave in a collisional plasma. Since this mode is essentially a mixed (electrostatic - electromagnetic) mode, such a wave has density perturbations associated with it, and therefore the usual dissipative mechanisms like resistivity in the parallel electron motion, again lead to a wave amplification when v_d exceeds a certain threshold value, due to similar physical reasons. We have considered only the cold plasma case in the analysis as the inclusion of temperature effects does not drastically alter the results.

The auroral ionosphere is very often characterized by intense field - aligned currents, caused by precipitating charged particle fluxes, during the activity period. In a recent measurement, ion - cyclotron waves were observed in association with such currents[40]. We shall show later that the observed currents are strong enough to cause the collisional excitation of ion - cyclotron waves in the medium.

Electrostatic Ion-Cyclotron Instability

We use two - fluid equations for the dynamics of electrons and ions across the magnetic field (assumed to be along the z-axis) and for the electron motion parallel to the magnetic field, which restricts our results to the following limits :

$\Omega_e, \nu_e \gg \omega$, $\nu_i \gg (\omega - \Omega_i)$, $k_\perp r_{Li} \ll 1$, $k_z \lambda^e_{mfp} \ll 1$ etc. The parallel ion motion is ignored which is justified for $\omega / k_z \gg C_s$. We assume that perturbations propagate in the xz-plane and ignore viscosity effects in our treatment for simplicity. The equations, in the form given by Braginskii[6] are

the equation of continuity

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \underset{\sim}{v}_\alpha) = 0 \tag{1a}$$

the equation of momentum transfer

$$m_\alpha n_\alpha \left(\frac{\partial}{\partial t} + \underset{\sim}{v}_\alpha \cdot \nabla\right) \underset{\sim}{v}_\alpha = -\nabla p_\alpha + q n_\alpha \left(\underset{\sim}{E} + \frac{\underset{\sim}{v}_\alpha \times \underset{\sim}{B}_0}{c}\right) + \underset{\sim}{R}_\alpha \tag{1b}$$

and the equation of conservation of energy

$$\frac{3}{2} n_\alpha \left(\frac{\partial}{\partial t} + \underset{\sim}{v}_\alpha \cdot \nabla\right) T_\alpha + p_\alpha \nabla \cdot \underset{\sim}{v}_\alpha = -\nabla \cdot \underset{\sim}{q}_\alpha + Q_\alpha \tag{1c}$$

where

$$\underset{\sim}{R}_e \simeq -\underset{\sim}{R}_i = -m_e n_e \nu_e (c_{er_\parallel} \underset{\sim}{u}_\parallel + c_{er_\perp} \underset{\sim}{u}_\perp) - c_{et_\parallel} n_e \nabla_\parallel T_e - c_{et_\perp} \frac{n_e \nu_e}{\Omega_e} (\hat{z} \times \nabla T_e) \tag{1d}$$

$$\underset{\sim}{q}_e = c_{et_\parallel} n_e T_e \underset{\sim}{u}_\parallel + c_{et_\perp} \frac{n_e T_e \nu_e}{\Omega_e} [\hat{z} \times \underset{\sim}{u}] - \chi_\parallel^e \nabla_\parallel T_e - \chi_\perp^e \nabla_\perp T_e - \frac{5}{2} \frac{e n_e T_e}{eB} [\hat{z} \times \nabla T_e] \tag{1e}$$

$$\underset{\sim}{q}_i = -\chi_\parallel^i \nabla_\parallel T_i - \chi_\perp^i \nabla_\perp T_i + \frac{5}{2} \frac{c n_i T_i}{eB} [\hat{z} \times \nabla T_i] \tag{1f}$$

where

$$\left[\chi_\parallel^\alpha = c_{\alpha\chi_\parallel} \frac{n_\alpha T_\alpha}{m_\alpha \nu_\alpha} \; ; \; \chi_\perp^\alpha = c_{\alpha\chi_\perp} \frac{n_\alpha T_\alpha \nu_\alpha}{m_\alpha \Omega_\alpha^2} \right]$$

$$Q_e = -\underset{\sim}{R} \cdot \underset{\sim}{u} - Q_\Delta \tag{1g}$$

and

$$Q_i = Q_\Delta = 3 \frac{m_e}{m_i} n_e \nu_e (T_e - T_i) \tag{1h}$$

Here m_e and m_i are the electron and ion masses respectively, and

$$\nu_{ei} \equiv \nu_e = \frac{4\sqrt{2\pi}\,\lambda e^4 Z^2}{3\sqrt{m_e}} \frac{n_i}{T_e^{3/2}} ; \quad \nu_{ii} \equiv \nu_i = \frac{4\sqrt{\pi}\,\lambda e^4 Z^4 n_i}{3\sqrt{m_i}\;T_i^{3/2}}$$

The numerical coefficients (c's) of frictional and thermal forces etc. for electrons and ions are tabulated by Braginskii[6].

The perturbed quantities are assumed to vary as $\sim \exp(i\omega t - i\underset{\sim}{k}\cdot\underset{\sim}{r})$. Electron inertia is ignored, which is justified for $\omega \ll \Omega_e, k_z v_{Te}$ and quasi-neutrality is assumed, which restricts the results to $\omega \ll \omega_{pi}$, $k\lambda_D \ll 1$ where ω_{pi} is the ion plasma frequency and λ_D is the Debye length (which is comparable for ions and electrons for a plasma with $T_e \simeq T_i$). The system is perturbed as

$$n = n_o + \tilde{n}, \quad E = 0 + \tilde{E} = -\nabla\tilde{\phi} \quad \text{etc.}$$

The equations are linearized for small amplitude perturbations. Thus the parallel motion of electrons is given by the zth component of (1b):

$$i k_z (\tilde{n} T_{eo} + n_o \tilde{T}_e) - i e n_o k_z \phi + i\, c_{et\parallel} n_o k_z \tilde{T}_e - m_e n_o \nu_e c_{er\parallel} v_z = 0 \qquad (2)$$

where we have neglected the parallel ion motion.

The above equation can be rewritten as

$$\frac{e\phi}{T_e} \simeq \frac{\tilde{n}}{n_o}\left(1 + i\, c_{er\parallel} \frac{m \nu_e \tilde{\omega}}{T_{eo} k_z^2}\right) + \frac{\tilde{T}_e}{T_{eo}}(1 + c_{et\parallel}) \qquad (3)$$

where we have made use of the linearised electron equation of continuity and ignored $(\text{div.}\, \underset{\sim}{v}_{\perp e})$ effects under the approximation

$\nu_e << \Omega_e$.

For the case of no collisions and no temperature perturbations, the above equation reduces to

$$\frac{e\phi}{T_e} \simeq \frac{\tilde{n}_o}{n_o} \qquad (4)$$

which expresses the familiar quasi-static pressure balance; a consequence of low frequency nature of the wave which makes the electrons behave like a "mass - less" fluid and establish such a balance at any instant. The phase shift introduced by collisions between $\tilde{n}$ and $\tilde{\phi}$, as can be seen from the second term ($C_{e\pi\parallel}$ term) on the right hand side of equation (3), ultimately leads to the resistive contribution to the growth of ion - cyclotron waves, as we shall see later.

Similarly other components of electron and ion motion can be written down. One can eliminate all the perturbed velocities from the set of equations of motion and continuity for ions and electrons and write down one equation in perturbed variables $\tilde{n}$, $\tilde{T}_e$ and $\tilde{T}_i$,

$$\left[\frac{T_{eo}+T_{io}}{T_{eo}} + \frac{m_i(\Omega_i^2-\omega^2)}{T_{eo}\,k_\perp^2} + i\,C_{e\pi\parallel}\frac{m_e\nu_e\tilde{\omega}}{T_{eo}\,k_z^2}\right]\frac{\tilde{n}}{n_o}$$
$$+\left[\frac{T_{io}}{T_{eo}} - i\,C_{eT\perp}\frac{T_{io}}{T_{eo}}\frac{m_e}{m_i}\frac{\nu_e}{\omega}\frac{k^2}{k_z^2}\right]\frac{\tilde{T}_i}{T_{io}} + (1+C_{eT\parallel})\frac{\tilde{T}_e}{T_{eo}} = 0 \qquad (5)$$

where $\tilde{\omega} = \omega - k_z v_d$.

The linearised ion - energy equation can be written as,

$$\frac{\tilde{n}}{n_o} + \left(-\frac{3}{2} + i\, C_{i\chi\parallel} \frac{T_{io} k_z^2}{m_i \nu_i \omega}\right) \frac{\tilde{T}_i}{T_{io}} = 0 \qquad (6)$$

and similarly the electron - energy equation, as

$$\left[(1 + C_{et\parallel}) - i\, 2 C_{er\parallel} \frac{m_e \nu_e V_d}{T_{eo} k_z}\right] \frac{\tilde{n}}{n_o} + \left[-i\,(C_{et\perp} + C_{et\parallel} C_{er\perp}) \frac{\nu_e}{\Omega_e} \frac{k^2 T_{io}}{\tilde{\omega}\, \Omega_i m_i} - 2 C_{er\parallel} C_{er\perp} \frac{k^2}{k_z^2} \frac{\nu_e^2}{\Omega_e^2} \frac{T_{io}}{T_{eo}} \frac{V_d k_z}{\tilde{\omega}}\right] \frac{\tilde{T}_i}{T_{io}} + \left[-\frac{3}{2} + i\, C_{e\chi\parallel} \frac{k_z^2 T_{eo}}{m_e \nu_e \tilde{\omega}}\right] \frac{\tilde{T}_e}{T_{eo}} = 0 \qquad (7)$$

Here we have neglected the collisional heat transfer from electrons to ions which is valid for not too low frequencies, $\omega \gg \nu_e \frac{m_e}{m_i}$.

The eliminant of equations (5) - (7) is an equation quartic in ω , which can be written as

$$\omega^4 + A\omega^3 + B\omega^2 + C\omega + D = 0 \qquad (8)$$

This equation describes the non-trivial solutions for the set of equations (5 - 7) and determines the characteristic modes in the system, their dispersion properties and their damping or growth. The four modes of the system are, two electrostatic ion-cyclotron modes and two damped diffusion modes due to perturbations in the electron and ion temperatures. We shall be interested in the ion - cyclotron mode here. Writing $\omega = \omega_R + i\gamma$ with $|\gamma| \ll \omega_R$, we solve (8) for the real and imaginary parts of the frequency. The solution may be written as

$$\omega_R^2 \simeq \Omega_i^2 + k_\perp^2 C_s^2 \tag{9a}$$

$$\gamma \simeq \frac{1}{2C_{e\chi\parallel}} \frac{k_\perp^2}{k_z^2} \frac{m_e}{m_i} \nu_e \left(1 - \frac{k_z v_d}{\omega_R}\right) + \frac{C_{e\eta\parallel}}{2} \frac{k_\perp^2}{k_z^2} \frac{m_e}{m_i} \nu_e \left(1 - \frac{k_z v_d}{\omega_R}\right)$$
$$+ \frac{C_{et\parallel}(2 + C_{et\parallel})}{2C_{e\chi\parallel}} \frac{k_\perp^2}{k_z^2} \frac{m_e}{m_i} \nu_e \left(1 - \frac{k_z v_d}{\omega_R}\right) + \frac{2}{9} C_{i\chi\parallel} \frac{T_{io}^2 k_\perp^2 k_z^2}{m_i^2 \nu_i \omega_R^2} \tag{9b}$$

In deriving the above growth rate, we made use of the assumption

$$1 > \lambda_{mfp}^{e\,2} k_z^2 \gg \frac{m_e}{m_i} \frac{k_\perp^2}{k_z^2}$$

In (9b), the contributions to the growth due to parallel electron thermal conductivity, parallel resistivity and parallel thermal force are described by the first, second and third terms respectively while the last term gives the damping due to ion - viscosity. The first three terms are stabilizing when $v_d = 0$, and become destabilizing when

$$v_d > \frac{\omega}{k_z}$$

Of course, the overall excitation of the wave does not take place at this value of v_d, but at a higher value which is determined by the ion viscous damping.

On comparing the growth rate (9b) for ion - cyclotron waves, with the one for ion - acoustic waves, obtained by Rognlein and Self[62], we notice some new features in the present case. First, we find that the second and third terms of (9b) were not obtained in the case of ion - acoustic waves, or in other words, resistivity does not lead to the ion wave growth. This may be understood physically as follows. In an ion - acoustic wave , the perturbed motion of ions and electrons are nearly equal and thus collisions between them do not amount to any net momentum transfer between them (this momentum transfer between electrons and ions leads to the frictional and thermal forces). In an ion - cyclotron wave, on the other hand, the parallel motion of electrons along the lines of force is much larger than that of ions and so electrons lose momentum to ions. Thus, parallel resistivity and parallel thermal force can lead to a dissipative effect on the ion - cyclotron wave (but not on the ion - acoustic wave) when electrons are not drifting. When the electron drift exceeds a critical value, these dissipative effects contribute to destabilization of the ion - cyclotron wave along with the other usual terms. The second new feature in (9b) is the appearance of the large factor $k_\perp^2/k_z^2$ $(\gg 1)$ in the growth rate, which makes the growth rate of ion - cyclotron waves much larger than that of parallel propagating ion acoustic waves in the same system. Physically this is connected with the fact that the ion - cyclotron wave propagates almost across the steady magnetic field direction and its field energy in the perpendicular direction is $k_\perp^2/k_z^2$ times the energy in parallel fields. An exchange of energies between the drifting electrons and the parallel electric fields can therefore lead to a stronger growth of this instability. Finally, we remark that the ion - cyclotron instability is unaffected by the non-stationary nature of the equilibrium temperatures of the system (arising due to Joule heating etc.), in contrast to the case of the ion - acoustic waves.[74] This is due to the fact that frequency of ion - cyclotron wave is weakly dependent on temperature (through small $k_\perp^2 C_s^2$ term) and remains always nearly equal to Ω_i, and hence temperature variation introduces only weak corrections.

Electromagnetic Ion - Cyclotron Instability

We now go over to the case of an electromagnetic ion - cyclotron wave propagating obliquely to the magnetic field in a collisional plasma with a parallel relative streaming between electrons and ions, v_d. We shall show that, the parallel resistivity leads to the excitation of this mode also when v_d exceeds a certain threshold. The case considered here is of a cold plasma, for the sake of simplicity. The equations relevant to the problem are,

the Maxwell's equations

$$\nabla \times \underset{\sim}{E} = -\frac{1}{c}\frac{\partial \underset{\sim}{B}}{\partial t}, \quad \nabla \times \underset{\sim}{B} = \frac{4\pi}{c}\underset{\sim}{J} \tag{10a}$$

where the current is given by

$$\underset{\sim}{J} = n_0 e(\underset{\sim}{v}_i - \underset{\sim}{v}_e) - \tilde{n} e \underset{\sim}{v}_d \tag{10b}$$

The displacement current was ignored above because of low frequencies of interest.

The continuity equation is

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \underset{\sim}{v}_\alpha) = 0 \qquad (10c)$$

Momentum transfer equation for ions is

$$m_i n_o \frac{\partial \underset{\sim}{v}_i}{\partial t} = e n_o \left[\underset{\sim}{E} + \frac{\underset{\sim}{v}_i \times \underset{\sim}{B}_o}{c} \right] \qquad (10d)$$

and the same for electrons is

$$0 \simeq -e n_o \left[\underset{\sim}{E} + \frac{\underset{\sim}{v}_e \times \underset{\sim}{B}_o}{c} + \frac{\underset{\sim}{v}_d \times \underset{\sim}{B}}{c} \right] - m_e n_o \nu_e \underset{\sim}{v}_e \qquad (10c)$$

As before, our coordinate system has $\underset{\sim}{v}_d$ and B_o parallel to the z-axis and the perturbations are assumed to vary as $\sim$ exp $(i\omega t - i\underset{\sim}{k}\cdot\underset{\sim}{r})$ with $\underset{\sim}{k}$ in the xz - plane. The electron inertia was neglected because of low frequencies concerned and the quasi-neutrality would be assumed for similar reasons.

The perturbed electron velocities can be written as

$$v_x = \frac{\tilde{\omega}}{\omega} \frac{cE_y}{B_o}$$

, where $\tilde{\omega} = \omega - k_z v_d$

$$v_y = -\frac{\tilde{\omega}}{\omega}\frac{eE_x}{m\Omega_e} - \frac{v_d k_z}{\omega}\frac{eE_z}{m\Omega_e}$$

and $$v_z = -\frac{eE_z}{m\nu_e}$$

Similarly, the ion velocities,

$$V_x = \frac{1}{(1-\Omega_i^2/\omega^2)}\left[-i\,\frac{eE_x}{M\omega} - \frac{\Omega_i}{\omega}\,\frac{eE_y}{M\omega}\right]$$

$$V_y = \frac{1}{(1-\Omega_i^2/\omega^2)}\left[\frac{\Omega_i\, eE_x}{M\omega^2} - i\,\frac{eE_y}{M\omega}\right]$$

Now the perturbed currents (occurring in 10 (a), and defined by 10 (b)) can be expressed in terms of the perturbed wave fields, using continuity equation to express the perturbed density. Thus the three components of the wave equation (obtained on eliminating B from set 10 (a)), can be written as

$$\left[-(1-\frac{\Omega_i^2}{\omega^2})k_z^2 + \frac{\omega_{pi}^2}{c^2}\right]E_x - i\,\frac{\omega_{pi}^2}{c^2}\left[\frac{\Omega_i}{\omega} + \frac{\tilde{\omega}}{\Omega_i}(1-\frac{\Omega_i^2}{\omega^2})\right]E_y + (1-\frac{\Omega_i^2}{\omega^2})\,k_x k_z E_z = 0 \,,$$

$$i\,\frac{\omega_{pi}^2}{c^2}\left[\frac{\Omega_i}{\omega} + \frac{\tilde{\omega}}{\Omega_i}(1-\frac{\Omega_i^2}{\omega^2})\right]E_x + \left[-k^2(1-\frac{\Omega_i^2}{\omega^2}) + \frac{\omega_{pi}^2}{c^2}\right]E_y + i\,\frac{\omega_{pi}^2}{c^2}\,\frac{\omega}{\Omega_i}\,(1-\frac{\Omega_i^2}{\omega^2})\,\frac{v_d k_z}{\omega}\,E_z = 0,$$

$$k_x k_z E_x - i\,\frac{\omega_p^2}{c^2}\,\frac{v_d k_x}{\Omega_e}\,\frac{\omega}{\tilde{\omega}}\,E_y + \left[-k_x^2 + i\,\frac{\omega_p^2}{c^2}\,\frac{\omega^2}{\tilde{\omega}\nu_e}\right]E_z = 0$$

The dispersion relation is obtained on putting the determinant of the above equations to zero, and is found to be

$$k^2 k_z^2(\omega^2 - \Omega_i^2) - (k^2 + k_z^2)\frac{\omega_{pi}^2}{c^2}\,\omega^2 + \frac{\omega_{pi}^4}{c^4}\,v_d^2 k_z^2 - \frac{\omega_{pi}^4}{c^4}\,\frac{\omega^2\tilde{\omega}^2}{\Omega_i^2} - i\,k^2 k_x^2\,\frac{m_e}{m_i}\,\nu_e\tilde{\omega} = 0$$

where it was further assumed that

$$\frac{m_i}{m_e} \gg \frac{\omega_p}{ck} \quad , \quad \frac{\omega_p}{ck}\,\frac{k_z}{k_x}$$

The dispersion relation, for $\omega \simeq \Omega_i$ and $\omega = \omega_R + i\gamma$ with $|\gamma| << \omega_R$, yields the growth rate

$$\gamma \simeq \frac{k_x^2}{2k_z^2}\,\frac{m_e}{m_i}\,\nu_e\,\frac{\tilde{\omega}}{\omega} = \frac{k_\perp^2}{2k_\parallel^2}\,\frac{m_e}{m_i}\,\nu_e\left(1 - \frac{v_d k_z}{\omega_R}\right)$$

The growth rate obtained for the electromagnetic case can be compared to that of electrostatic case and on doing so, we find that the resistive contribution for the latter case is exactly identical to the one presented above. We therefore conclude from this that the other dissipative effects like electron viscosity, electron thermal conductivity, etc., will also contribute to the destabilization of the electromagnetic ion-cyclotron mode due to similar physical effects.

It is of interest to point out here that the current driven electromagnetic ion - cyclotron instability in a collisionless plasma was considered by Forslund et al.[23]. The effect of weak collisions on this instability has not been considered, but from the foregoing analysis we can infer that the electron - ion collisions would facilitate the excitation whereas ion - ion collisions would hamper it.

<u>Application to F - region</u>

We have discussed in the preceding sections how an ion -

cyclotron wave becomes unstable due to dissipative effects in parallel electron motion in a fully - ionized collisional plasma, if a threshold current is exceeded along the field lines. Recent observations in the auroral F region have shown that strong field - aligned currents flow in those regions during the period of auroral activity. One such observation is due to Kelley et al.[40] who have carried out in-situ rocket measurements in this region. We shall point out here that ion-cyclotron waves are potentially unstable in such a system due to collisional effects.

Kelley et al.[40] have, in fact, observed strong monochromatic a.c. electric fields in the regions where an intense field - aligned current (due to an experimentally observed ion - beam) was also flowing. They have attributed these oscillating electric fields to ion - cyclotron waves, excited due to current - driven ion - cyclotron instability in a collisionless plasma[14]. We show here that the possibility of this excitation being collisional in lieu of collisionless, is more likely because of the following reasons. The dimensions of the extent of current observed by Kelley et al.[40], are roughly $\sim$ 200 meters across earth's magnetic field $\underset{\sim}{B}_o$, and $\sim$ tens of kilometers along $\underset{\sim}{B}_o$. This was at an altitude of $\sim$ 450 kms., where coulomb collisions predominate over charged particle - neutral collisions in the wavelength range of interest, and the system may be considered as fully ionized. At those altitudes, the collision frequencies are $\nu_{ei} \sim 10^3\ s^{-1}$, $\nu_{ii} \sim 10\ s^{-1}$ and electron mean free path is $\lambda^e_{mfp} \sim 100$ meters. The ion parameters are, $\Omega_i \sim 10^2\ s^{-1}$, $r_{Li} \sim 10$ meters etc. The importance of collisions for ion - cyclotron instability in such a system is self-evident when we find that $\omega(\simeq \Omega_i) < \nu_{ei}$ and the possible parallel wavelengths in the system $\lambda_{\parallel} \gg \lambda^e_{mfp}$. Excitation of collisional ion - cyclotron waves by the observed current is possible as the inequalities $k_z \lambda^e_{mfp} \ll 1$, $k_\perp r_{Li} \ll 1$ etc. are readily satisfied at these heights. The dominance of collisional excitation over the collisionless excitation has been shown by the numerical calculations of Milic[49]. There it was found that, as the electron drift velocity is gradually increased, the instability first sets in at long parallel wavelengths (with $k_z \lambda^e_{mfp} \ll 1$) and in this regime also, the longer transverse wavelengths ($k_\perp r_{Li} \ll 1$) are excited first. Clearly the collisional mechanism is important for such wavelengths. The threshold relative electron drift needed to excite the perturbations is obtained from (9b) and can be written as,

$$v_d > \frac{\omega_R}{k_z} \left[1 + \frac{4}{9} \frac{C_{i\chi\parallel}}{\left(C_{e\chi\parallel} + \frac{(1+C_{et\parallel})^2}{C_{e\chi\parallel}} \right)} \frac{T_{io}^2}{m_e m_i} \frac{k_z^4}{\nu_e \nu_i \omega_R^2} \right]$$

Substitution of relevant parameters in the above expression shows that a relative drift of a few kms s^{-1} between electrons and ions in auroral F region would readily excite the long wavelength ion - cyclotron waves (satisfying $k_z \lambda^e_{mfp} \ll 1$ etc.) due to collisional effects. Kelley et. al.[40]'s measurements indicate that the relative drifts exceeding this value were present during the observation. Therefore we conclude that their observation of ion - cyclotron waves corresponds to a current - driven collisional excitation of these modes.

4.2 COLLISIONAL ION - CYCLOTRON WAVES IN E REGION

In the last section, we have studied the destabilizing properties of a field - aligned current on an obliquely travelling ion - cyclotron wave in a collisional fully - ionized plasma. We extend this analysis now to the case of a weakly ionized plasma in the present section. The high - altitude auroral E region ($\gtrsim$ 120 kms.) is one example of such a situation. As before, we are interested in the limit of strong collisions, and describe the ion and electron dynamics by fluid equations. It is shown that ion - cyclotron waves again can be destabilized by a field - aligned current in the system if a certain threshold current is exceeded. (A cross-field current can also excite this instability, though in this case thresholds required for the onset of the instability may be much higher.) The instability arises due to the dissipative effects like resistivity etc., in the parallel motion of electrons (as in the fully ionized case) whereas the dissipative effects in the ion equations like ion - neutral collisions ν_{in}, viscosity ν_{ii} etc., damp the wave. Towards the closing stages of this work, we learnt about the work of Milic' and Sünder [48] who have used a kinetic theory treatment with a Boltzmann - collision integral to analyse the effect of collisions on ion - cyclotron instability in a partially ionized plasma. They also have concluded that electron - neutral collisions facilitate the growth of the wave while the ion - neutral collisions provide the stabilizing effects. Similar results were obtained by Kindel and Kennel[42] who investigated the effect of weak collisions on the ion - cyclotron instability using a BGK - type of model collision term. We apply our results to the E region situation in the end and show that ion - cyclotron waves can be excited by the observed field - aligned currents in that region.

The mathematical analysis of the problem follows the fully - ionized case closely. As before, the two-fluid equations are used for transverse electron and ion motion, and for the longitudinal electron motion, valid for $k_\perp \gamma_{Le} << 1$, $\omega (\simeq \Omega_i) << \nu_{en}$, $k_\perp \gamma_{Li} << 1$, $(\omega - \Omega_i) << \nu_{in}$, $k_z \lambda^e_{mfp} << 1$. Here the magnetic field and equilibrium electron drift v_d, are assumed to be in the z-direction. The parallel motion of ions is ignored which is justified for $\omega >> k_z C_s$. In the momentum transfer equations for ions and electrons, terms due to coulomb collisions (ν_{ei}) are replaced by the charged particle - neutral collision terms (ν_{en} and ν_{in}). The temperature fluctuations and viscosity effects are ignored for simplicity. The perturbations are assumed to vary as $\sim \exp(i \underset{\sim}{k} \cdot \underset{\sim}{r} - i\omega t)$. Then the perturbed ion velocity can be written as

$$\left(1 - \frac{\Omega_i^2}{\bar{\omega}^2}\right) \underset{\sim}{v} = \frac{\underset{\sim}{k} T_i}{M \bar{\omega}} \left(\frac{\tilde{n}_i}{n_o} + \frac{T_e}{T_i} \frac{e\phi}{T_e} \right) + i \frac{T_i}{M \bar{\omega}^2} \underset{\sim}{k} \times \underset{\sim}{\Omega}_i \left(\frac{\tilde{n}_i}{n_o} + \frac{e\phi}{T_i} \right)$$

where $\bar{\omega} \simeq \omega + i\nu_{in}$, $\tilde{n}_i$ is perturbation in the ion density and ϕ is the perturbed potential derivable from $\underset{\sim}{\tilde{E}} = -\nabla \phi$.

The terms along $\underset{\sim}{k}$ describe inertial response of ions to the perturbation electric field in the above equation while $\underset{\sim}{E}\times\underset{\sim}{B}$ drift is described by the other term.

The density fluctuations are obtained from the continuity equation

$$\frac{\tilde{n}_i}{n_o} \simeq \frac{k^2}{(\bar{\omega}^2-\Omega_i^2)} \frac{T_e}{M} \frac{\bar{\omega}}{\omega} \frac{e\phi}{T_e} \Bigg/ \left(1 - \frac{T_i k^2}{M} \frac{\bar{\omega}}{\omega(\bar{\omega}^2-\Omega_i^2)}\right) \tag{1}$$

The parallel electron motion gives

$$v_{ez} = -i k_z \frac{T_e}{m\nu_e}\left(\frac{\tilde{n}_e}{n_o} - \frac{e\phi}{T_e}\right)$$

The perpendicular electron motion does not contribute to the electron equation in the limit $\nu_e \to 0$, which is justified for $\Omega_e \gg \nu_e$. Thus we write the electron continuity equation as

$$\frac{\tilde{n}_e}{n_o} \simeq \frac{i k_z^2 T_e}{m\nu_e(\omega - \underset{\sim}{k}\cdot\underset{\sim}{v}_d)} \frac{e\phi}{T_e} \Bigg/ \left[1 + i\frac{T_e k_z^2}{m\nu_e(\omega-\underset{\sim}{k}\cdot\underset{\sim}{v}_d)}\right] \tag{2}$$

We now assume quasi-neutrality $\tilde{n}_e \simeq \tilde{n}_i$, and equate the two expressions (1) and (2) which gives the dispersion equation,

$$\omega^2 \simeq \Omega_i^2 + c_s^2 k_\perp^2 + \nu_i^2 + \frac{m}{M}\frac{k_\perp^2}{k_z^2}\nu_e\nu_i\frac{(\omega - \underset{\sim}{k}\cdot\underset{\sim}{v}_d)}{\omega}$$
$$- i\frac{m}{M}\frac{k_\perp^2}{k_z^2}\nu_e(\omega-\underset{\sim}{k}\cdot\underset{\sim}{v}_d) + i\frac{\nu_i}{\omega}c_s^2 k_\perp^2 - i\omega\nu_i \tag{3}$$

Splitting $\omega = \omega_R + i\gamma$ with $|\gamma| \ll \omega_R$ the above equation yields :

$$\omega_R^2 \simeq \Omega_i^2 + k_\perp^2 C_s^2 \tag{4}$$

and
$$\gamma \simeq -\frac{m}{2M}\frac{k_\perp^2}{k_z^2}\nu_e\left(1-\frac{\underset{\sim}{k}\cdot\underset{\sim}{v}_d}{\omega_R}\right)-\nu_i/2 \tag{5}$$

We obtain the condition for growth from equation (5),

$$\underset{\sim}{k}\cdot\underset{\sim}{v}_d > \omega_R\left(1+\frac{M}{m}\frac{k_z^2}{k_\perp^2}\frac{\nu_i}{\nu_e}\right) \tag{6}$$

where it was assumed that $\nu_i < \omega$. The above result (6) shows that both, v_d along the field lines or across it, can give a growth of ion - cyclotron wave perturbations. Though in a weakly ionized plasma, a reasonable value of cross - field v_d (due to an applied external electric field E_o , perpendicular to $\underset{\sim}{B}_o$), is realised only when $\nu_i > \Omega_i$ and $\Omega_e \gg \nu_e$. However in the present case, $\omega > \nu_i$, and therefore $\Omega_i(\sim\omega) > \nu_i$. This condition is satisfied at high altitudes ($\gtrsim$ 120 kms.) in E region. Hence the threshold $v_d\,(E_o)$ required for the instability may be considerably high.

A field - aligned electron streaming can easily excite this instability once the threshold condition (6) is satisfied. The importance of collisional excitation in a system, like E region [where $\nu_e > \omega(\sim\Omega_i)$ for ion-cyclotron waves] , is provided by the kinetic theory calculations of Milić and Sünder[48]. They show that in a system unstable to ion - cyclotron perturbations due to a field - aligned current, the instability first sets in at the longer perpendicular and parallel wavelengths (i.e. $k_\perp \gamma_{Li} \ll 1$, $k_z \lambda^e_{mfp} \ll 1$) due to collisional effects and as the electron drift is augmented, it is subsequently followed by the onset at shorter parallel and perpendicular wavelengths due to collisionless mechanism.

The field aligned currents observed in the auroral E regions[52,8] during the auroral activity have been considered by D'Angelo[12] to excite electrostatic ion cyclotron waves. But to estimate the effect of collisions on this instability in the weakly ionized E region plasma, he has considered the work of Levine and Kuckes[45] and Varma and Bhadra[76] (who have investigated the effects of collisions on this instability for a fully - ionized plasma), and has also used the results of Kindel and Kennel[42] who investigated the effect of weak collisions on this instability. We have explicitly shown above that in a partially ionized case, ion - cyclotron waves can be excited by a field - aligned current in the limit of strong collisions. The threshold drift velocity for the excitation is given as

$$v_d > \frac{\omega_R}{k_z}\left(1 + \frac{M}{m}\frac{k_z^2}{k_\perp^2}\frac{\nu_{in}}{\nu_{en}}\right) \qquad (7)$$

For the high altitude E region (at $\gtrsim$ 120 kms., so that $\Omega_i > \nu_i$), we have $\Omega_i \simeq 2\text{x}10^2\ s^{-1}$, $\nu_i < \Omega_i$, $\nu_e \simeq 10^4\ s^{-1}$ etc. and $k_z << k_\perp$ with $\lambda_z > \lambda^e_{m+p} \sim 100$ meters and $\lambda_\perp > \gamma_{Li} \sim 5$ meters which gives the condition that v_d should exceed a few kms. s^{-1} in order to cause growth. This threshold is often exceeded in the E region as some recent measurements indicate. The typical scalesizes thus excited would be much larger than γ_{Li} (~ 5 meters) and hence would not be amenable to detection by the VHF backscatter radar. In-situ electrostatic probes can be used to detect these nearly field - aligned irregularities at high altitudes ($\gtrsim$ 130 kms.), with the scalesizes of tens to hundreds of meters across the earth's magnetic field.

4.3 COHERENT NONLINEAR ION – CYCLOTRON WAVES IN A COLLISIONLESS PLASMA

We have seen in second chapter that nonlinearities can lead to a saturation of instabilities, with the result that the driven modes temporally evolve in the system as finite amplitude propagating modes. Such nonlinear studies are not confined only to the unstable situations but have also been carried out in the cases wherein the sources or sinks of energy in the system (i.e. the instability and damping mechanisms) are not considered. In these studies then, one essentially investigates normal modes of a plasma system in which the amplitude by which one "perturbs" the equilibrium situation (which leads to the appearance of normal modes in the linear limit) is no more regarded as infinitesimally small. Such investigations have been made for two low frequency electrostatic modes of a collisionless warm plasma, the ion – acoustic wave and the drift wave. It has been shown that at small but finite amplitudes an ion – acoustic wave has special solitary wave solutions[65], and a drift wave (in an inhomogeneous magneto-plasma) has both shock or solitary wave solutions[50, 75]. A third low frequency electrostatic normal mode of a warm plasma is an obliquely propagating ion – cyclotron wave, and such studies have not been made in the past for this case. In this section, we have therefore investigated the propagation characteristics of a nonlinear electrostatic ion – cyclotron wave in a collisionless plasma. Ion - cyclotron waves are of interest in the auroral ionosphere where the observed field - aligned currents can excite them through the collisional instability, as has been discussed in the previous sections. The present study, therefore, is intended to serve as a first step towards ultimately developing a nonlinear theory of collisional ion - cyclotron instability which has been left as a topic for future study.

We consider a situation in which the magnetic field is along the z - axis, and the electrostatic perturbations are confined to the xz plane. For this low frequency mode, the electrons behave like a 'massless' fluid and move rapidly along the field - lines, responding instantaneously to any perturbation. Thus if the parallel phase velocity of the wave is smaller than the electron thermal speed, this fast motion results in a Boltzmann - like density distribution of particles in the wave potential at any instant. On the other hand, ions undergo two kinds of motion in response to a perturbation electric field - the divergence - free $\underset{\sim}{E}\times\underset{\sim}{B}$ motion (just as in the case of a drift wave) and the inertial motion along E which is not divergence - free. Thus a charge density appears across the magnetic field direction due to ion motion which is destroyed rapidly by the fast electron motion along the field lines. This results in the almost perpendicular propagation of the electrostatic ion - cyclotron waves. The parallel ion motion and the perpendicular electron motion are inconsequential for this mode and hence will be ignored. Then the parallel electron equation is

$$0 = -T_e \frac{\partial n_e}{\partial z} + n_e T_e \frac{\partial \Psi}{\partial z} \quad (1)$$

where $\Psi = e\phi/T_e$, is the normalized potential of the wave.

This immediately gives

$$n_e \simeq n_o e^{\Psi} \quad (2)$$

The ion equation of motion is

$$\frac{\partial \underset{\sim}{V}_\perp}{\partial t} + \underset{\sim}{V}_\perp \cdot \nabla \underset{\sim}{V}_\perp = \frac{T_e}{M} \nabla_\perp \Psi + \Omega_i \underset{\sim}{V}_\perp \times \hat{z} \quad (3)$$

where cold ions are assumed for simplicity.

This equation gives

$$\frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} = -C_s^2 \frac{\partial \Psi}{\partial x} + \Omega_i V_y \quad (4)$$

and $$\frac{\partial V_y}{\partial t} + V_x \frac{\partial V_y}{\partial x} = -\Omega_i V_x$$

where $$C_s^2 = T_e/M .$$

The set (4) yields, on elimination of V_y

$$\left(\frac{\partial}{\partial t} + V_x \frac{\partial}{\partial x}\right)^2 V_x + \Omega_i^2 V_x + C_s^2 \left(\frac{\partial}{\partial t} + V_x \frac{\partial}{\partial x}\right) \frac{\partial \Psi}{\partial x} = 0 \quad (5)$$

The ion - continuity equation is

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i V_x) = 0 \quad (6)$$

We assume that a propagating stationary wave is established in the system and all dependent variables depend on a single variable, $\xi = k_x x + k_z z - \omega t$, which corresponds to a frame moving with the wave phase velocity. Then, in place of the above equation, one gets

$$-\omega \frac{dn}{d\xi} + k_x \frac{d}{d\xi}(n V_x) = 0$$

or

$$\frac{d}{d\xi}(-\omega n + n k_x V_x) = 0$$

or

$$-\omega n + n k_x V_x = \text{constant} = -\omega n_o$$

for the boundary condition that when $V_x = 0$, $n = n_o$ and where $n \simeq n_o + \tilde{n}$.

Then

$$V_x = \frac{\tilde{n}}{(n_o + \tilde{n})} \frac{\omega}{k_x} \qquad (7)$$

Equation (2), in the small amplitude limit gives

$$\frac{\tilde{n}}{n_o} \simeq \Psi \qquad (8)$$

Here we have made use of the quasi-neutrality assumption $\tilde{n}_e \simeq \tilde{n}_i \simeq \tilde{n}$.

We substitute for Ψ in (5) from (8) and use the ξ- transformation to write

$$(k_x V_x - \omega)\frac{d}{d\xi}\left[(k_x V_x - \omega)\frac{dV_x}{d\xi}\right] + \Omega_i^2 V_x + \frac{c_s^2}{n_o}(k_x V_x - \omega)\frac{d^2\tilde{n}}{d\xi^2} = 0 \qquad (9)$$

Use of (7) eliminates V_x and we get

$$\left[\frac{\omega^2}{k_x^2 \Omega_i^2 (1+N)^3} - \frac{c_s^2}{\Omega_i^2}\right]\frac{d^2 N}{d\xi^2} - \frac{3\omega^2}{k_x^2 \Omega_i^2 (1+N)^4}\left(\frac{dN}{d\xi}\right)^2 + N = 0 \qquad (10)$$

where $N = \frac{\tilde{n}}{n_o}$. We assume that the waves have small but finite amplitude and that $\frac{\omega}{k_x} \simeq u > C_s$, so that

$$\left[\frac{(\omega^2 - c_s^2 k_x^2)}{k_x^2 \, \Omega_i^2} \right] \frac{d^2 N}{d \xi^2} - \frac{3\omega^2}{k_x^2 \Omega_i^2} \left(\frac{dN}{d\xi} \right)^2 + N = 0$$

or

$$\alpha \frac{d^2 N}{d \xi^2} - \beta \left(\frac{dN}{d\xi} \right)^2 + N = 0 \tag{11}$$

Using the scaling $p = \frac{2\beta}{\alpha} N ; \; q = \sqrt{\frac{2}{\alpha}} \, \xi$, we write

$$2 \frac{d^2 p}{d q^2} - \left(\frac{dp}{dq} \right)^2 + p = 0 \tag{12}$$

An investigation of this equation reveals that for $p << 1$, sinusoidal solutions emerge[1]. In this essentially linear limit, we get back the familiar dispersion relation of electrostatic ion cyclotron waves

$$\omega^2 \simeq \Omega_i^2 + k_x^2 C_s^2 \tag{13}$$

In the nonlinear limit, when $p \gg 1$, we obtain saw-tooth shaped solutions[1]. Physically, the nonlinearity associated with the ion - inertial motion gives rise to the nonlinear distortion (steepening) of the wave profile from a sinusoidal shape to a sawtooth shape.

4.4 COHERENT NONLINEAR ION ACOUSTIC WAVES IN A MAGNETISED PLASMA

Ion waves in a collisional plasma can be excited by a current due to isothermal electrons in a fully ionized plasma[10, 62] or due to resistive electron motion in a weakly ionized plasma[26]. The nonlinear evolution of such growing waves has been studied by several people by invoking mode - coupling to higher damped harmonics of such modes. A mode - coupling of this kind introduces a nonlinear damping in the linearly unstable system. Thus a situation is achieved when the linear growth and the nonlinear damping balance each other, so that the modes attain a time - stationary amplitude. The ultimate wave profile in this case is distorted from a sinusoid due to the harmonic contribution to it. Investigations have been carried out in the past for the case of unstable ion sound waves in a fully ionized plasma[2, 51], and in a weakly ionized plasma[1]. In the present section, we show that ion waves in a weakly ionized magnetoplasma, which can be destablised by a cross-field current[38], do also evolve into nonlinear finite amplitude waves with a distorted profile, due to similar physical reasons. This instability may be of relevance to the auroral E region[38]. This study, thus, completes our plan of the nonlinear evolution of low - frequency instabilities, with the Rayleigh - Taylor and the collisional drift instabilities having already been studied in the second chapter.

We choose a coordinate system with the magnetic field along the z-axis, the equilibrium electron drift along the y-axis and perturbations in the yz - plane. The fluid equations are used for ions and electrons. For ions, this is justified if $T_e \gg T_i$, so that the kinetic effects (ion Landau damping, which arises when $\omega \sim \nu_i$ and $k\, \lambda^i_{mfp} \sim 1$) are unimportant. The ion - viscosity term in the momentum transfer equations is added, which is important for short wavelengths. Ions are considered unmagnetized $(\nu_i \gg \Omega_i)$ and electrons magnetized $(\nu_e << \Omega_e)$. The parallel wavevector is assumed to be large enough so that the parallel wave phase velocity (ω / k_z) is smaller than the electron thermal speed, v_{Te}. In this case, the electrons set up a Boltzmann-like density distribution instantaneously in the wave potential. But electron - neutral collisions impede this motion and cause dissipation of the perturbations. The electron parallel motion can be written down as

$$n v_{ez} = - \frac{T_e}{m \nu_e} \left(\frac{\partial n}{\partial z} - n \frac{\partial \psi}{\partial z} \right) \qquad (1)$$

where $\psi = e\phi / T_e$,

In the collisionless case $(\nu_e \to 0)$ we get the Boltzmann-like density distribution

$$n = e^{\psi} \tag{2}$$

In the perpendicular electron motion, the pederson mobility is ignored (as $\nu_e \ll \Omega_e$)[38] and we can write

$$n \underset{\sim}{v}_{e\perp} = -\frac{T_e}{m\Omega_e} \nabla_\perp n \times \hat{z} + n \frac{T_e}{m\Omega_e} \nabla_\perp \psi \times \hat{z} \tag{3}$$

The ion continuity equation can be written as

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial y}(nV) = 0$$

where we have ignored the parallel ion motion V_z. This amounts to assuming that the wave is propagating at a large angle to the magnetic field. Further we assume that the unstable waves have some special steady state solutions and that all quantities in the wave frame depend only upon one variable $\xi = k_y y + k_z z - \omega t$. Transforming the above equation to the ξ - frame, we get

$$-\omega \frac{dn}{d\xi} + k_y \frac{d}{d\xi}(nV) = 0$$

or

$$V = \frac{\tilde{n}}{(n_0 + \tilde{n})} \frac{\omega}{k_y} \tag{4}$$

where

$$n = n_0 + \tilde{n}$$

The ion equation of motion is

$$\left(\frac{\partial}{\partial t} + V_y \frac{\partial}{\partial y}\right) V_y = -\frac{T_i}{M} \frac{\nabla n}{n} - \frac{e\nabla\phi}{M} - \nu_{in} V + \mu \frac{\partial^2 V}{\partial y^2}$$

where the last term on right side represents the viscosity effects and μ is the viscosity coefficient. Under the ξ - transformation the above equation becomes

$$\left(-\omega\frac{dV}{d\xi}+\nu_i V-\mu k_y^2\frac{d^2V}{d\xi^2}+Vk_y\frac{dV}{d\xi}\right)+\frac{T_i}{Mn}k_y\frac{dn}{d\xi}=-\frac{T_e}{M}k_y\frac{d\psi}{d\xi} \quad (5)$$

The electron continuity equation is

$$\frac{\partial\tilde{n}}{\partial t}+v_d\frac{\partial\tilde{n}}{\partial y}+\nabla_\perp\cdot(n\underset{\sim}{v}_{e\perp})+\frac{\partial}{\partial z}(nv_{ez})=0$$

It can be easily verified from (3) that $\nabla_\perp\cdot(n\underset{\sim}{v}_{e\perp})=0$.

Using (1) and the ξ - transformation, we write above equation as

$$-(\omega-k_yv_d)\frac{d\tilde{n}}{d\xi}-\frac{T_e}{m\nu_e}k_z^2\frac{d}{d\xi}\left[\frac{d\tilde{n}}{d\xi}-n\frac{d\psi}{d\xi}\right]=0 \qquad (6)$$

On substituting for $\frac{d\psi}{d\xi}$ from (5) and for V from (4), we finally get

$$\frac{d}{d\xi}\Big[\left(\nu_i+\frac{m}{M}\frac{k_y^2}{k_z^2}\frac{\tilde{\omega}}{\omega}\right)N+\frac{c_s^2k_y^2}{\omega}\frac{dN}{d\xi}-\omega\frac{d}{d\xi}\frac{N}{(1+N)}$$
$$-\mu k_y^2(1+N)\frac{d^2}{d\xi^2}\frac{N}{(1+N)}\Big]=0 \qquad (7)$$

where

$$C_s^2=\frac{T_e+T_i}{M}\;,\quad\frac{\tilde{n}}{n_0}=N\quad\text{and}\quad\tilde{\omega}=\omega-k_yv_d\,.$$

Finally we get, after an integration and some algebra

$$\frac{\mu k_y^2}{(1+N)}\frac{d^2N}{d\xi^2}-\frac{2\mu k_y^2}{(1+N)^2}\left(\frac{dN}{d\xi}\right)^2-\left(\frac{m}{M}\frac{k_y^2}{k_z^2}\frac{\tilde{\omega}}{\omega}\nu_e+\nu_i\right)N$$
$$-\frac{(C_s^2k_y^2-\omega^2)}{\omega\,(1+N)^2}\frac{dN}{d\xi}-\frac{N(2+N)C_s^2k_y^2}{\omega\,(1+N)^2}\frac{dN}{d\xi}=0 \qquad (8)$$

We make use of the linear disperson relation for the ion - acoustic waves, $\omega=k_yC_s$, in the above and further assume that they are small but finite amplitude waves so that $N << 1$ $(\tilde{n} << n_0)$. The above equation is then reduced to

$$\mu k_y^2 \frac{d^2N}{d\xi^2} - 2\mu k_y^2 \left(\frac{dN}{d\xi}\right)^2 + \left(\frac{m}{M}\frac{k_y^2}{k_z^2}\frac{\tilde{\omega}}{\omega}\nu_e - \nu_i\right)N - 2N\frac{c_s^2 k_y^2}{\omega}\frac{dN}{d\xi} = 0 \quad (9)$$

where the fact that $\tilde{\omega}/\omega < 1$ for instability, has been used. The above equation readily yields the linear growth[38] in the linear limit, with additional viscous damping of the modes. The two nonlinear terms in the above equation become important in two different limits. First, if $T_e >> T_i$ so that the nonlinearity associated with the viscosity can be ignored (as viscosity effects would be relatively less important for small T_i), then we have

$$\mu k_y^2 \frac{d^2N}{d\xi^2} - \frac{2c_s^2 k_y^2}{\omega} N\frac{dN}{d\xi} + \left(\frac{m}{M}\frac{k_y^2}{k_z^2}\frac{\tilde{\omega}}{\omega}\nu_e - \nu_i\right)N = 0 \quad (10)$$

The above equation is like a steady - state Burger's equation (with a growth term added to it) and has got stationary saw-tooth shaped solutions[51]. Second in the opposite limit, when $\omega >> \nu_i$, so that k is high (as $\omega \sim k c_s$) and the viscosity effects become important, we have

$$\mu k_y^2 \frac{d^2N}{d\xi^2} - 2\mu k_y^2 \left(\frac{dN}{d\xi}\right)^2 + \left(\frac{m}{M}\frac{k_y^2}{k_z^2}\frac{\tilde{\omega}}{\omega}\nu_e - \nu_i\right)N = 0 \quad (11)$$

This equation also possesses steepened non-sinusoidal solutions for finite amplitude waves*. In both the cases discussed above, however, the ~~vosicosity~~ viscosity is quite essential to get the nonlinear solutions. This is intimately related to the physical description of the nonlinear stabilization of the instability that we considered earlier in this section. There it was stated that due to nonlinearity, viscously damped higher harmonics get coupled to the linearly growing mode and provide a nonlinear sink for the energy input at the fundamental end. The steady state is established when the inflow balances the dissipation due to viscosity and the wave-profile is distorted because nonlinearly several harmonics are generated.

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* See for example sections 2.2 and 4.3 in this thesis and also reference 1.

CHAPTER 5

SUMMARY

In this chapter we summarise all the main results of our thesis. The work has been divided into three parts, one dealing with the equatorial spread F, the second with the equatorial electrojet irregularities and the third with the plasma processes related to the auroral ionosphere.

The part on the equatorial spread F is mainly devoted to interpreting some recent in-situ experimental observations of these irregularities regarding spectra, shapes, etc. We have shown that the observed k^{-2} power spectra of large scalesizes can be interpreted in terms of a two-step theory in which small scalesizes grow on the gradients of large scalesizes in a second - stage drift instability. It has been shown that the excitation of drift instability requires a critical gradient in the medium and if the gradient associated with the large scalesizes (roughly the ratio between the amplitude and the scalesizes) is greater than this value, a turbulent generation of shorter scalesizes follows. Thus in the equilibrium state, these gradients for a wide range of scalesizes remain near this constant value and the k^{-2} law follows as a natural consequence of this. We treat the two-step mechanism as a nonlinear mode - coupling process and find that a second stage drift instability cannot take place on the large scalesize drift modes. Next we have studied the nonlinear coherent development of the Rayleigh-Taylor and collisional drift instabilities, generally believed to generate large scalesizes in the equatorial spread F, in order to explain some observed large amplitude coherent steepened structures. The R – T instability is shown to evolve as a finite – amplitude sawtooth like propagating wave due to finite – Larmor radius effects, in the one-dimensional approximation. Such coherently evolved irregularities would also show a k^{-2} – like power spectra and may contribute to the overall observation of k^{-2} spectra. It is evident that a two-dimensional analysis of this problem is more appropriate, but it is not possible to solve it analytically. A numerical investigation of this case is postponed for future research. The collisional drift instability has been considered next, and it is found that due to ion – viscosity, this instability may also saturate as a coherent finite – amplitude wave with a distorted shape.

Second part deals with the irregularities in the equatorial electrojet. The past investigations of plasma instabilities in this case, like the two stream and cross-field instabilities, have been carried out using electrostatic approximation, thus neglecting electromagnetic effects on these instabilities which become important at long wavelengths, i.e., when the parameter $ck/\omega_{pe} \lesssim 1$. We have, therefore, developed a generalized fluid theory for the excitation of mixed electrostatic - electromagnetic modes by a cross-field current in a partially ionized plasma. We discover a new low - frequency mixed es - em instability (for $ck/\omega_{pe} \lesssim 1$ and with a nonzero $k_{\parallel}$) in such a system which requires a lower threshold drift velocity compared to C_s . This instability would be observable as nearly field - aligned phase - correlated large scalesize density - cum - magnetic field fluctuations in the equatorial electrojet. Next, in this part, we have investigated the two-step theory of the equatorial electrojet for the excitation of short scalesizes on the gradients of large scalesize irregularities due to second step two - stream or cross-field instabilities. We use a nonlinear mode-coupling approach and essentially recover old results which were obtained through a 'multilinear' analysis.

In the third part, we consider the situation prevailing in the auroral ionosphere during the auroral activity period. Intense vertical field - aligned currents flow up and down in the medium due to charged particle precipitation both in F and E regions and are extended upto the magnetosphere. In E - region, they are believed to be connected by horizontal cross - field currents known as auroral electrojet. We investigate the destabilizing effect of a field - aligned current on an obliquely propagating ion - cyclotron wave in collisional media, corresponding to auroral F and E regions, and show that the wave excitation can take place in both the regions, the excitation mechanism being dissipative processes, like resistivity, thermal conductivity, etc., in the parallel electron motion. Ion - cyclotron waves have been observed in the auroral F region and we have discussed our results in light of these observations. In E region also, ion - cyclotron waves would be observable at high altitudes (greater than 120 kms.) as nearly field - aligned long scalesize irregularities. Nonlinear propagation characteristics of electrostatic ion - cyclotron waves in a collisionless plasma are studied next and it is shown that they have sawtooth shapes at small but finite amplitudes. An extension of this analysis for the collisional ion - cyclotron instability is a topic of future study. Lastly, we study the nonlinear coherent saturation of a cross - field current - driven resistive ion - acoustic instability. This instability may have relevance in the auroral electrojet and we show that some special nonsinusoidal solutions are obtained in this case due to ion viscosity in the nonlinear steady state approximation.

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