Constraints on Particle Physics and Modified Gravity Models from Observations

A THESIS

submitted for the Award of Ph.D. degree from

MOHANLAL SUKHADIA UNIVERSITY

in the

Faculty of Science

by

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MOHANLAL SUKHADIA UNIVERSITY

UDAIPUR

Year of submission: 2015

To

My Loved Ones

DECLARATION

I, **Girish Kumar Chakravarty***, S/O Shri Suresh Prasad Chakravarty, resident of 203, PRL Hostel, PRL Thaltej Campus, Ahmedabad, 380054, hereby declare that the research work incorporated in the present thesis entitled,* **"Constraints on Particle Physics and Modified Gravity Models from Observations"** *is my own work and is original. This work (in part or in full) has not been submitted to any University for the award of a Degree or a Diploma. I have properly acknowledged the material collected from secondary sources wherever required. I solely own the responsibility for the originality of the entire content.*

Date:

(Girish Kumar Chakravarty)

CERTIFICATE

I feel great pleasure in certifying that the thesis entitled, **"Constraints on Particle Physics and Modified Gravity Models from Observations"** embodies a record of the results of investigations carried out by Mr. Girish Kumar Chakravarty under my guidance. He has completed the following requirements as per Ph.D regulations of the University.

(a) Course work as per the university rules.

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(d) Presented his work in the departmental committee.

(e) Published minimum of one research papers in a refereed research journal.

I am satisfied with the analysis, interpretation of results and conclusions drawn. I recommend the submission of thesis.

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Acknowledgements

Firstly, I would like to thank my family members for their continual love and encouragement, and supporting me in all my decisions. I express my sincerest gratitude to my supervisor Prof. Subhendra Mohanty for his kind guidance. It has been a pleasure and a privilege to work with him. His knowledge and experience has always been inspiring, as has been his kind personality and joyful nature.

Secondly, I would like to thank my collaborators Dr. Naveen Kumar Singh, Prof. Gaetano Lambiase and Dr. Suratna Das. Working with them was very fruitful. Thanks to Dr. Akhilesh Nautiyal and Dr. Gaveshna Gupta for various useful discussions I had with them at various points in time. I thank Dr. Gaveshna Gupta also for partially proofreading my thesis.

I would also specially like to thank Prof. Srubabati Goswami for her generous, helping and a very friendly nature towards me. I thank Prof. Jitesh Bhatt and Dr. Namit Mahajan for reviewing my thesis.

My PhD studies at PRL would not have been as easy as it were, if it were not the presence of my friends. Though I have a lot of friends at PRL, past and present, I would like to name a few with whome I had a lot of long interactions : Avdhesh Kumar, Yashpal Singh, Jagat Bisht, Naveen Negi, Dillip Nandy, Monojit Ghosh, Gulab Bambhaniya, Girish Yadav, Arun Pandey, Tanmoy Mondal, Bhaswar Chaterjee, Soumya Rao, Akhilesh Nautiyal, H Zeen Devi, Pankaj Sharma, Vimal Kishore, Arvind Singh, Vineet Goswami, Suman Acharya, Lalit Shukla, Guru Kadam, Gaurav Jaiswal, Subha Bose, Abhaya Swain, Manu George, Rukmani Bai, Kuldeep Suthar, Midhun Madhavan, Lekshmy Rary, Salla Gangy Reddy, Damodar Rao, A Aadhi, Anjali Rao, Arko Roy. I thank them all for various reasons unspecified.

Last but not the least, I would like to thank PRL for providing good research facilities and, a nice hostel room and a beautiful campus for stay, which I always enjoyed.

Girish

ABSTRACT

In this thesis, we discuss the Standard Model of Cosmology and its achievements. We discuss inflation, a period of accelerated expansion in the early universe, in a great detail which is a cornerstone of Modern Cosmology, as it not only solves the problems of initial conditions in the Standard Model but also can provide the mechanism for generation of tiny density fluctuations in the early universe, which are responsible for structures formation : the stars, galaxies and galactic clusters of today. We give a detailed calculation of the power spectra of scalar and tensor perturbations which characterizes the density fluctuations and gravitational waves, respectively, in the universe. We briefly discuss the models of inflation in the framework of Standard Einstein Gravity Theory, Modified Einstein Gravity Theories and Supergravity Theories. We calculate the key inflationary observables : amplitude of the power spectrum of curvature perturbations $\Delta^2_{\mathcal{R}}$, spectral index n_s and tensor-to-scalar ratio r. The recent data from Planck observations give the amplitude and the spectral index as $\ln(10^{10}\Delta^2_{\mathcal{R}}) = 3.089 \pm 0.036$ and $n_s = 0.9666 \pm 0.0062$ at (68% CL), respectively. And *Keck Array*/BICEP2 CMB polarization data combined with Planck analysis of CMB polarization and temperature data put an upper bound on tensor-to-scalar ratio $r_{0.05} < 0.07$ at (95% CL). We fix the parameters of the models under study by using these observed values. Also in order to motivate these models, as they are not generic in the particle physics models, we derive them from a fundamental theory called Supergravity.

We give a Higgs inflation model with a generalized curvature coupling $\xi\phi^a R^b$ which is a generalization of the Higgs inflation model $\xi\phi^2 R$ with $\lambda\phi^4$ potential. In this model, with curvature coupling parameter ξ of order unity, we could resolve the problem of unitarity violation in scalar-graviton scatterings in $\xi\phi^2 R$ Higgs inflation models. And with self-coupling $\lambda \sim 0.1$, the inflaton field in this model can be interpreted as standard model Higgs field. Also we find that, upto slow-roll approximation, the predictions of Jordan and Einstein frames are same and therefore the two frames are equivalent. However, this model predicts large tensor-to-scalar ratio. In the second part of this thesis, we study a power law correction to Einstein gravity $R + \frac{1}{M^2}R^\beta$ as a model of inflation. We find that the two parameter power law model is actually equivalent to generalized non-minimal curvature coupled models with quantum corrected $\lambda\phi^4$ potentials, *i.e.* models of the form $\xi\phi^a R^b + \lambda\phi^{4(1+\gamma)}$. We find that this model predicts large tensor-to-scalar ratio $r \approx 0.22$ compared to the experimental bound $r_{0.05} < 0.07$. Therefore, a large class of curvature coupled models and models with quantum corrected potentials are ruled out. In order to motivate R^β correction to Einstein gravity, we derived it from no-scale supergravity with minimal Wess-Zumino form of superpotential and by adding a power law $(\phi+\bar{\phi})^n$ term to the minimal no-scale SUGRA Kähler potential.

In the last part of this thesis, we present a generalized two-field inflationary scenario where inflaton field is accompanied by a dilaton field and has a non-canonical kinetic term due to the presence of a dilaton field. We show that in such an inflationary scenario, the quartic and quadratic inflaton potentials, which in the context of single field slow-roll inflation are ruled out by the present Planck data, yield the observed scalar spectral index and tensor-to-scalar ratio. This model predicts a tensor-to-scalar ratio of the order of 10^{-2} which can be probed by future B-mode experiments like Keck/BICEP3, LiteBIRD, CMBPol and thus can be put to test in future. In a multifield scenario, the curvature perturbations are not constant on superhorizon scales and isocurvature perturbations are expected to be generated. We show that in the considered two-field scenario, upto slow-roll approximation, the isocurvature perturbations vanish. To motivate such a two-field model, we show that it can be derived from no-scale supergravity with appropriate choice of superpotential and string motivated Kähler potential. Future observations of $B-$mode will verify or rule out models studied in this thesis.

Keywords : FLRW Cosmology, Inflation, CMBR Physics, $B-$mode, Higgs, Modified Gravity, Jordan Frame, Conformal Transformation, Einstein Frame, Supergravity, No-scale Supergravity, WMAP/Planck/*Keck*/BICEP Observations.

LIST OF PUBLICATIONS

Publications contributing to this thesis :

1. *Higgs Inflation in $f(\phi, R)$ Theory*,
   Girish Kumar Chakravarty, Subhendra Mohanty and Naveen Kumar Singh, Int. J. Mod. Phys. D 23, 1450029 (2014), arXiv:1303.3870 [astro-ph.CO].

2. *Power law Starobinsky model inflation from no-scale SUGRA*,
   Girish Kumar Chakravarty and Subhendra Mohanty, Phys. Lett. B 746 (2015) 242, arXiv:1405.1321 [hep-ph].

3. *Moduli assisted two-field inflation from no-scale supergravity*,
   Girish Kumar Chakravarty, Suratna Das, Gaetano Lambiase and Subhendra Mohanty, arXiv:1511.03121 [hep-ph] (Communicated for publication).

List of Abbreviations

ΛCDM	Λ Cold Dark Matter
FLRW	Friedmann–Lemaître–Robertson–Walker
CMBR	Cosmic Microwave Background Radiation
LSS	Last Scattering Surface
DE	Dark Energy
DM	Dark Matter
CDM	Cold Dark Matter
EF	Einstein Frame
JF	Jordan Frame
COBE	Cosmic Background Explorer
WMAP	Wilkinson Microwave Anisotropy Probe
BICEP	Background Imaging of Cosmic Extragalactic Polarization
CMBPol	Cosmic Microwave Background Polarization
SUSY	Supersymmetry
SUGRA	Supergravity
VEV	Vacuum Expectation Value
EW	Electro-weak
LHC	Large Hadron Collider

Contents

List of Tables

List of Figures

Chapter 1

Introduction

"For me, it is far better to grasp the Universe as it really is than to persist in delusion, however satisfying and reassuring."

– Carl Sagan

Today the very basic picture of our observable universe is presented quite accurately by the standard Big Bang model of cosmology, also known as ΛCDM-model or FLRW cosmology. Under the standard Big Bang model, the universe in its early stages is considered to be very hot, uniform in its density and expanding

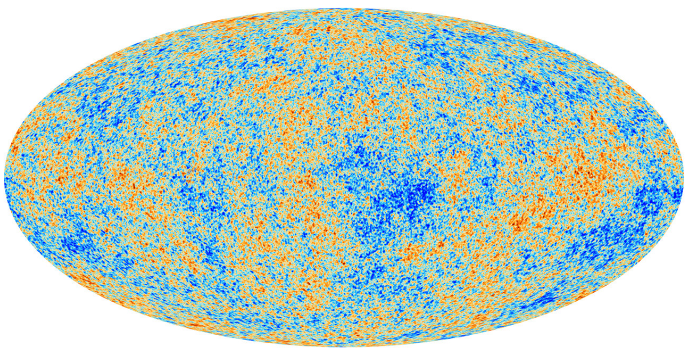

Figure 1.1: The temperature anisotropies map of the Cosmic Microwave Background Radiation (CMBR) as observed by Planck mission. Image credit: [1].

uniformly in all the directions and cooling down at late times. So far, it has passed a large number of increasingly precise tests. It successfully predicts the age, Hubble expansion rate, mass density of the universe and light elemental abundance in the early universe. Also it explains the presence of the Cosmic Microwave Background Radiation (CMBR). The CMBR is a snapshot of the oldest light in our universe leftover after decoupling and imprinted on the Last Scattering Surface (LSS), when the universe was just $3,80,000$ years old. The most remarkable feature of CMBR is its high degree of uniformity everywhere and in all the directions. It has inhomogeneity only at the level of one part in 10^5. Figure 1.1 shows the temperature anisotropy in the CMBR. The Blue spots represents the sky where the temperature is 10^{-5} below the mean temperature $T_0 = 2.725°K$. This corresponds to the regions where photons loose their energy while climbing out of the gravitational potential of the overdense regions in the early universe. Yellow and Red spots represents the underdense regions where the temperature is 10^{-5} above the mean temperature. These tiny inhomogeneities in the early universe are believed to have grown to cosmological scales later in the history of the universe which resulted in structure formation : the stars, galaxies and galactic clusters of today. Current precision measurements of these small inhomogeneities in the CMBR has led to constraining a variety of cosmological parameters and therefore theoretical cosmological models. However, the standard Big Bang model could poorly explain some of the observed characteristics of the universe, *e.g.* why the universe is so uniform and its intrinsic geometry is so flat. These unsolved problems in standard cosmological model are also known as Horizon and Flatness problems. Invocation of a rapid exponential expansion phase, *Inflation*, in the very beginning of the universe could solve these problems. Inflation is the main theme of this thesis. But before we go into the details of inflation, let us briefly discuss the FRLW cosmology, the problems therein and the solutions presented by inflation.

1.1 FLRW Cosmology

Cosmology aims to understand the past, present and future dynamics of our Universe. The combination of high precision observational data and the theoretical understanding of the universe has led to the standard model of cosmology or the ΛCDM (Λ Cold Dark Matter) model. The standard model of cosmology is based on three fundamental assumptions : (i) On the large-scales $\mathcal{O}(10^2 Mpc)$ our universe is Homogeneous and Isotropic. (ii) Gravity is described by the Einstein's General Theory of Relativity. (iii) The basic material constituents of our universe are described by the standard model particles and in addition dark matter and dark energy (or a cosmological constant).

The homogeneity and isotropy of the universe describes the invariance under the spatial translations and rotations. This is also known as the *Cosmological Principle*. We know that the universe at largest scale ($> 100Mpc$) is homogeneous and isotropic from the CMBR observations and expanding from the Type Ia supernova observations. The homogeneity, isotropy and expanding nature of the space-time is mathematically described by the Freidmann-Lemaitre-Robertson-Walker (FLRW) metric given by

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= -dt^2 + a^2(t) \left[\frac{dr^2}{1-\kappa r^2} + r^2 \left(d\theta^2 + sin^2\theta d\phi^2 \right) \right], \end{aligned} \tag{1.1}$$

where $a(t)$ is scale factor which describes the spatial expansion in cosmic time t and (r, θ, ϕ) are the comoving spatial coordinates. κ represents the spatial curvature and can take values $+1, 0, -1$ describing open, flat and closed universe, respectively. In Cartesian coordinates, for flat universe, the above metric can be written as

$$ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2, \tag{1.2}$$

where $\mathbf{x} \equiv (x, y, z)$. In comoving coordinates, the distance dx between any two spatial points is always fixed (or constant) and the physical distance between them is given by $a(t)dx$.

The dynamics of the universe is studied using the Einstein field equations. The field equations can be derived by varying the Einstein Hilbert action w.r.t. metric $g_{\mu\nu}$. The Einstein-Hilbert action is given by

$$S_E = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_p^2 R - 2\Lambda\right] + S_M, \tag{1.3}$$

where g is the determinant of the metric $g_{\mu\nu}$, $d^4x\sqrt{-g}$ is the invariant volume element, R is the Ricci scalar and M_p is the reduced Planck mass $M_p^2 = (8\pi G)^{-1}$. S_M is the additional matter and Λ is the cosmological constant introduced by Einstein in order to explain the static universe.

Demanding the action S_E to be invariant under the infinitesimal change in the metric, *i.e.* $\delta g_{\mu\nu}$, we get the Einstein Field equations as

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu} \tag{1.4}$$

where $G_{\mu\nu}$ is the Einstein tensor :

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} \tag{1.5}$$

and $R_{\mu\nu}$ is the Ricci curvature tensor derived from the metric $g_{\mu\nu}$ and the Ricci scalar $R = g_{\mu\nu}R^{\mu\nu}$. The field equation (1.4) relates the curvature of the space-time, represented by the LHS, to the matter content of the universe through symmetric stress-energy tensor $T_{\mu\nu}$ appearing on the RHS.

In the standard model of cosmology, the matter content of the universe is described by a perfect fluid which is characterised only by the energy density ρ and isotropic pressure p. The stress-energy-momentum tensor for a perfect fluid with energy density ρ and pressure p is given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}, \tag{1.6}$$

where u_μ is the 4-velocity of the fluid in some arbitrary coordinate system x_μ given by

$$u_\mu = \frac{dx_\mu}{d\tau} \tag{1.7}$$

here τ is the proper time of the observer, so that $g^{\mu\nu}u_\mu u_\nu = -1$. If such a fluid is at rest in the geometry described by metric (1.1) and obeys the equation of state $p_i = \omega_i \rho_i$, then from the covariant conservation of the stress-energy-momentum tensor, *i.e.* $\nabla_\mu T^{\mu\nu} = 0$, we find the equation of motion of energy densities in the FLRW universe

$$\dot{\rho}_i + 3H(1+\omega_i)\rho_i = 0, \tag{1.8}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter describing Hubble expansion rate, i represents the various components of the cosmological fluid, *e.g.* matter, radiation and dark energy, and ω_i represents the respective equation of state parameter for different components. For the system as a whole, the total energy density is given by

$$\rho = \sum_i \rho_i \tag{1.9}$$

and total pressure is given by

$$p = \sum_i p_i. \tag{1.10}$$

The solution of the continuity equation (1.8) is given by

$$\rho_i \propto a^{-3(1+\omega_i)}. \tag{1.11}$$

As the universe expands, the matter density, consisting of all non-relativistic matter particles, dilutes as $\rho_{nr} \propto a^{-3}$; and radiation density, consisting of all relativistic particles, dilutes as $\rho_r \propto a^{-4}$, as for pressureless non-relativistic matter $\omega = 0$ and for radiation $\omega = \frac{1}{3}$. For $\omega = -1$ which corresponds to a negative pressure fluid, a strange behavior occurs, the energy density of the universe remains constant as the universe expands. Such an exotic matter is known as *Dark Energy* or *Cosmological Constant* and usually attributed to the present day accelerated expansion of our universe.

In the flat FLRW universe the dynamics of the scale factor $a(t)$ is determined through Friedmann equations which can be derived by solving Einstein field equation (1.4) for the FLRW metric (1.1) and energy-momentum tensor (1.6):

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{\kappa}{a^2}, \tag{1.12}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}, \tag{1.13}$$

According to acceleration equation if $\Lambda = 0$ then the matter and radiation filled universe decelerates which contradicts the observational data from Type-1a supernovae [2, 3], South Pole Telescope [4] and from the measurement of high multipole CMB data [5–7]. These observations have led to the conclusion that the universe is accelerating in its expansion. In the ΛCDM model the present accelerated expansion is achieved with a small positive cosmological constant Λ. Also from equation (1.13) it is implied, that the present accelerated expansion can be achieved if the energy density of the universe is dominated by some unknown exotic matter with negative pressure $p < \frac{-\rho}{3}$ or equation of state parameter $\omega < \frac{-1}{3}$. Such an exotic matter with $\omega < \frac{-1}{3}$ generates repulsive gravity. To consider different matter contributions to the total energy density of the universe, it is common to define the density parameter as

$$\Omega_i \equiv \frac{\rho_i}{\rho_c} \tag{1.14}$$

where ρ_c is the critical density for which the universe is spatially flat *i.e.* from equation (1.12) $\rho_c = \frac{3H^2}{8\pi G}$ for $\kappa = 0$. We define the total density parameter of the universe as

$$\Omega = \sum_i \Omega_i. \tag{1.15}$$

If we divide Friedmann equation (1.12) by H^2, it can be written as

$$\begin{aligned} \sum_i \Omega_i + \Omega_\kappa &= \Omega_m + \Omega_r + \Omega_\Lambda + \Omega_\kappa \\ &= 1 \end{aligned} \tag{1.16}$$

where $\Omega_\Lambda = \frac{\Lambda}{3H^2} = \frac{8\pi G}{3H^2}\rho_\Lambda$ and $\Omega_\kappa = \frac{-\kappa}{H^2 a^2}$ are the dark energy and curvature density parameters respectively. The matter density parameter consists of baryonic matter and non-relativistic cold dark matter (CDM), *i.e.* $\Omega_m = \Omega_b + \Omega_{CDM}$. The recent Planck observations of CMB [7] combined with WMAP polarization data [6] for low multipoles $l < 23$, give the present values of the density parameters at $68\% CL$ as:

$$\Omega_b h^2 = 0.02205 \pm 0.00028, \tag{1.17}$$

$$\Omega_{CDM} h^2 = 0.1199 \pm 0.0027, \tag{1.18}$$

$$\Omega_\Lambda = 0.685^{+0.018}_{-0.016}, \tag{1.19}$$

where h is the dimensionless parameter defined through the present value of the Hubble parameter as

$$\begin{aligned} H_0 &= 100h\ km\ s^{-1} Mpc^{-1} \\ &= 67.3 \pm 1.2\ km\ s^{-1} Mpc^{-1}. \end{aligned} \tag{1.20}$$

Therefore, the present observations suggest that our universe is composed of nearly 4.9% atoms (or baryons), 26.8% (cold) dark matter and 68.3% of dark energy which adds up to approximately 1 in the total density parameter. According to Planck combined with BAO data [7]

$$\Omega_\kappa = 0.000 \pm 0.005 \quad (95\%\ CL). \tag{1.21}$$

Therefore, the observations suggest that the intrinsic geometry of our universe is very close to flat *i.e.* $\kappa \simeq 0$ or the universe is at the critical density. Why the universe is so close to flat geometry or at its critical density is known as *flatness problem*. We will see in the next section, in the discussion of flatness problem that as we go back more and more in the past, the density parameter Ω_κ tends to be closer and closer to zero and hence this problem is also termed as *fine tuning problem*.

1.2 Inflation

1.2.1 The Problem of Initial Conditions

So far we have discussed the ΛCDM model which can describe the evolution of the universe in a great detail. Before we discuss the mathematical description of *inflation*, let us briefly discuss the problem of initial conditions. The conventional model of standard Big Bang cosmology requires a set of fine-tuned initial conditions so that the universe could evolve to its present state. These initial conditions are the assumptions of the extreme flatness and homogeneity in the beginning of the universe. The dramatic flatness of the universe at its beginning can not be predicted or explained by the standard model, instead it must be assumed as an initial condition. Similarly, the large scale homogeneity of the universe is not predicted or explained by the standard model but it must be assumed.

In the late 1970's, cosmologists realised the problem of initial conditions with the ΛCDM model and solution to these problems could be reached at with the invocation of an accelerated expansion phase in the early evolution of the universe. This accelerated expansion phase is termed as *Inflation* [8–16]. The cosmological inflation is believed to took place in the very early universe around 10^{-35} seconds after the Big Bang. Remarkably, inflation not only explains the large scale homogeneity and isotropy of the universe but also widely accepted as responsible for the formation and evolution of the structures in the universe. Inflation can provide the mechanism for producing the tiny density fluctuations which are responsible for seeding the structures in our universe *e.g.* stars, galaxies and galactic clusters.

Mathematically, the accelerated expansion of the FLRW universe or *condition for inflation* can be given as

$$\ddot{a} > 0. \tag{1.22}$$

The second time derivative of the scale factor can easily be related to the time variation of the Hubble parameter as

$$\frac{\ddot{a}}{a} = H^2(1-\epsilon), \tag{1.23}$$

where $\epsilon \equiv -\frac{\dot{H}}{H^2}$. Therefore acceleration $\ddot{a} > 0$ corresponds to

$$\epsilon = -\frac{\dot{H}}{H^2} = -\frac{1}{H}\frac{dH}{dN} < 1, \tag{1.24}$$

here, we have defined $dN = d\ln a = Hdt$, which determines the number of e-foldings in an inflationary expansion. More precisely the number of e-foldings N during inflation in the time interval $t_i < t < t_e$ is given by the integral

$$N = \int_{t_i}^{t_e} Hdt. \tag{1.25}$$

The equation (1.24) implies that the fractional change in the Hubble parameter per e-folding is small. An inflationary scenario where this change is too small $i.e.$ $\epsilon << 1$ is termed as *slow-roll inflation*.

Also from the acceleration equation (1.13), we see that, for inflation, the energy density of the universe must be dominated by a fluid whose equation of state satisfies the condition

$$\omega \equiv \frac{p}{\rho} < -\frac{1}{3}. \tag{1.26}$$

In 1981, Alan Guth discussed the flatness and horizon problems in his paper on inflation and the mechanism to solve them [8]. Below we discuss in detail the Flatness and Horizon problems and their solutions presented by inflation.

1.2.2 Inflation Solves the Horizon Problem

To understand the horizon problem we introduce the comoving (particle) horizon or the conformal time τ defined as the causal horizon or the maximum distance light can travel between the time 0 and t

$$\tau = \int_0^t \frac{dt'}{a(t')} = \int_0^a (aH)^{-1} d(\ln a), \tag{1.27}$$

where $(aH)^{-1}$ is defined as the *comoving Hubble radius*. The comoving Hubble radius determines whether the two regions in the universe can communicate or not $i.e.$ whether they are within each others causal horizon or not. The condition for inflation (1.22) implies that the comoving Hubble radius decreases in time

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0, \tag{1.28}$$

therefore the universe which was within its causal horizon will soon go outside the horizon due to accelerated expansion. The photons from the two different regions in the CMB sky or on the last scattering surface (LSS), that we observe today, were well outside from each others causal horizon when they were first emitted. The last scattering surface is the sphere of points in the CMB sky when the photons were first decoupled from the rest of the matter.

Consider two regions at LSS, if these two regions were so distant apart that they were outside each others particle horizon than there is no possibility that they were interacting. However, if they were outside the comoving Hubble radius but inside the particle horizon then there is a possibility that they communicated at some point of time in the past. We know from the CMB observations that the background temperature is nearly uniform in the universe. From this, one can infer that the two regions on the opposite sides of the universe were in causal contact at LSS. However, when we extrapolate the light rays backward towards LSS, it is found that these regions were not in causal contact. From the temperature of the photons at the time of last scattering $T_{LSS} \sim \ 0.3eV$ and the CMB temperature today $T_0 \sim 10^{-13}GeV$ it can be estimated that the two regions on the LSS which were separated by an angular distance greater than $1°$, which corresponds to multipoles smaller than $l \sim 200$, were not in causal contact. But today we know from the CMB observations that even those regions in the sky which are separated by much larger angular separations than $1°$ share the same temperature. The horizon problem poses the question that when the two distant regions at the LSS were so distant apart that they were not in causal contact, then how could they share the same temperature today.

Inflation is able to solve the horizon problem as it causes the comoving Hubble radius to decrease and particle horizon to be nearly constant during the inflationary expansion. Therefore, this allows more and more regions of the universe to be in causal contact. In other words, inflation caused different regions of the universe, which were in causal contact before inflation, to go outside the particle horizon. Therefore, it is no surprise that today universe is filled with incredibly large number of homogeneous patches even at the large angular separations.

1.2.3 Flatness Problem is Solved by Inflation

The flatness problem states that why the intrinsic geometry of our universe is very close to flat *i.e.* $\kappa \sim 0$ or $\Omega_\kappa \sim 0$. Flatness problem could also be stated as why the present energy density of our universe is at its critical density. To quantify the problem, consider the Friedmann equation (1.12) in the following form

$$\Omega - 1 = \frac{\kappa}{a^2 H^2}. \tag{1.29}$$

Observations suggest that our universe during most of its history underwent a radiation dominated epoch so we consider the the FRW universe to be radiation dominated. From the solution of the continuity equation (1.11) during radiation dominated epoch $\rho \sim a^{-4}$, we can solve Friedmann equation (1.12) to relate scale factor with time

$$a \sim t^{1/2} \quad \Longrightarrow \quad H \sim a^{-2}. \tag{1.30}$$

Substituting the above result into the Friedmann equation (1.29), we get

$$|\Omega - 1| \sim a^2. \tag{1.31}$$

Taking the ratio of equation (1.31) at some very early epoch t_i and present epoch t_0

$$\frac{|\Omega_i - 1|}{|\Omega_0 - 1|} = \frac{a^2(t_i)}{a^2(t_0)}. \tag{1.32}$$

Since the evolution of the scale factor relates to the evolving temperature of the universe as $T \sim a^{-1}$ and we know from the observations that today the universe is close to flat $|\Omega_0 - 1| \sim \mathcal{O}(1)$, therefore using (1.32), we find that at different times *e.g.* at Planck time, BBN and GUT, the deviations from the flatness can be obtained as

$$|\Omega_{Pl} - 1| \leq \mathcal{O}(10^{-61}) \tag{1.33}$$

$$|\Omega_{GUT} - 1| \leq \mathcal{O}(10^{-55}) \tag{1.34}$$

$$|\Omega_{BBN} - 1| \leq \mathcal{O}(10^{-16}). \tag{1.35}$$

We therefore find that as we go more and more into the past, the universe seems incredibly fine-tuned towards flatness.

It is easy to see that inflation is able to solve this problem. If the universe in its early history underwent quasi-exponential expansion in some time interval $t_i < t < t_e$ during which the Hubble rate H is nearly constant then the Friedmann equation (1.29) gives

$$\frac{|\Omega_i - 1|}{|\Omega_e - 1|} \simeq \frac{a(t_e)^2}{a(t_i)^2}. \tag{1.36}$$

Since the scale factor grows quasi-exponentially as the inflation proceeds, therefore $a(t_e) \gg a(t_i)$ (please note that here Ω_i is some initial value of the total density parameter and it does not represent density parameter for different cosmological fluids as defined earlier). We see that for some generic initial density $\Omega_i \neq 1$, the quantity $|\Omega_e - 1|$ is driven incredibly close to zero and hence the universe evolves very close to flatness as the inflation proceeds. A more straight forward argument to solve the flatness problem can be given in terms of comoving Hubble radius. From the Friedmann equation (1.29) it is clear that as the comoving Hubble radius decreases during inflation, it drives the quantity $\Omega - 1$ towards zero and therefore the universe towards flatness.

1.3 Inflation from Modified Gravity Theories

The standard Big-Bang model of cosmology based on radiation and matter dominated epochs can be well described within the framework of Einstein's General Theory of Relativity. However, a rapid development of observational cosmology in nearly last three decades has established that our universe has undergone two phases of cosmic acceleration. The first phase is called inflation which occurred before radiation domination and the second phase is called the late time acceleration which started after matter domination. A scalar field ϕ with a slowly varying potential can be a candidate to source the inflation as well as the late time acceleration (or Dark Energy). While scalar field models of inflation and dark energy in the General Relativity (GR) framework correspond to modification of the energy momentum tensor in the field equations, there exists another approach to explain the early and late time acceleration of the universe. This corresponds to modified gravity theories in which the action is modified compared to GR action. The Lagrangian density for GR is given by $f(R) = R - 2\Lambda$, where R is Ricci scalar and Λ is a cosmological constant. The Λ, which can give an exponential expansion, can not be used for inflation because accelerated expansion that needs to end to connect to the radiation dominated epoch can not be achieved with pure cosmological constant. However, it can be used to explain the late time acceleration since the acceleration today does not need to end.

One of the simplest known modifications to General Relativity is the $f(R)$-gravity in which the Lagrangian density $f(R)$ can be a generic function of Ricci scalar R [17, 18]. A very well known model with $f(R) = R + \frac{1}{M^2}R^2$, $(M > 0)$, is the Starobinsky model of inflation which can lead to an accelerated expansion of the universe due to the presence of the term $\frac{1}{M^2}R^2$ [10]. This model is well consistent with observations of the CMB anisotropies and therefore can be a viable alternative to the scalar field model of inflation. It is well known that the $f(R)$ gravity theories in the metric formalism (in which the field equations are obtained by varying the action w.r.t. the metric $g_{\mu\nu}$) are equivalent to scalar-tensor theory, the Brans-Dicke theory, with the Brans-Dicke parameter ω_{BD} equals to zero [19].

The simple single-field inflation models in which there is no coupling between the scalar field ϕ and the Ricci curvature scalar R in the action are known as *minimally coupled* inflation model, *e.g.* the standard slow-roll inflation model.

There is another class of models with Lagrangian density $f(\phi)R$ which are termed as *non-minimally coupled* inflation models due to the presence of a coupling between field and curvature scalar. A simplest well known model of inflation with non-minimal coupling is the Higgs inflation model where the Higgs scalar ϕ can give rise to a viable inflationary phase as a result of coupling with the curvature scalar of the form $f(\phi)R = R + \xi\phi^2 R$, where ξ is the non-minimal coupling parameter [20]. Interestingly, the Starobinsky model of inflation is shown to be equivalent to Higgs inflation model in the conformal Einstein frame [19, 20]. Both of these models lead to the same scalar potential in Einstein frame which can be shown via conformal transformation of the metric $g_{\mu\nu}$. We will see in the Chapter 2 that a general $f(R)$ and $f(\phi)R$ gravity action can be conformally transformed to an Einstein frame action [19]. These actions may arise naturally in the low energy limit of higher dimensional theories *e.g.* in supergravity, string theory and Kaluza-Klein theories [21, 22]. There is an important difference between $f(R)$ and $f(\phi)R$ theories that unlike the non-minimal $f(\phi)R$ theories, the minimally coupled $f(R)$ theories don't depend on extra scalar fields but relies on the scalar degree of freedom of the EF metric tensor itself which can provide the potential and the kinetic terms of the inflaton in the EF.

In general, any frame in which the action has non-standard form of the gravity sector are termed as *Jordan frame* (JF), *e.g.* $f(R)$ and $f(\phi)R$ gravity actions. On the other, hand one can go to an *Einstein frame* (EF) via conformal transformation of the metric where gravity sector of the action has the standard Einstein-Hilbert form *i.e.* the Ricci scalar in the action is minimally coupled to the field. Also, in such JF theories the conformal transformations lead to an EF action where it has a non-canonical kinetic term in the field. However the non-canonical kinetic term can be converted into canonical form by redefining the JF field into an EF field. Also due to conformal transformation the scalar potentials in both the frames become different and they are related via conformal transformation factor.

The usefulness of conformal transformations of modified gravity theories to an EF lies in the fact, that once we have converted any generic JF action into

EF action, we don't need to perform the background calculation and work out the whole complicated perturbation theory to determine physical observables, instead we can directly use the mathematical expressions for physical observables as obtained in the Einstein frame calculations. It should be noted that physical cosmological observables are unaltered by the conformal transformation procedure [23–26]. However, the quantities that are not physical observables may be altered [27].

In $f(\phi)R$ theories the action is linear in Ricci scalar. However, there are even more generalized models with Lagrangian density of the form $f(\phi, R)$ possible where, along with non-minimal coupling, the Ricci scalar can appear non-linearly. The important point to make here is that both $f(R)$ and $f(\phi)R$ theories can be transformed to an conformal EF. However, in general it is not possible to conformally transform $f(\phi, R)$ action, with non-linear terms in R, to an Einstein frame action. We will discuss this type of scenario in a great detail in Chapter §3 in the context of generalized non-minimal Higgs inflation, where we will show that although exactly it is not possible to conformally transform $f(\phi, R)$ action to an EF but it can be done under the large field approximation. We will study the conformal transformation of $f(R)$ action into EF in more detail in Chapter §4. Exhaustive studies of $f(R)$, $f(\phi)R$ and $f(\phi, R)$ theories can be found in Ref.s [19, 28–34].

1.4 Inflation from Supergravity Theory

Supergravity (SUGRA) is a local version of the $\mathcal{N} = 1$ Supersymmetry (SUSY) in four dimension [35–38]. Supersymmetry is a symmetry which relates fermionic and bosonic degrees of freedom. $\mathcal{N}$ represents the number of independent SUSY transformations and therefore independent SUSY transformation parameters. Global SUSY extension of standard model (SM) of particle physics can not only solve the hierarchy problem but also account for the large amount of dark matter in our universe. SUSY in the context of cosmology is also a welcome tool. If nature is found to be supersymmetric then gravitational sector should be supersymmetric too. The local version of SUSY automatically engages the theory of gravity as spin$-3/2$ gauge field ψ_μ^α, termed as *gravitino*, of the SUGRA transformations has superpartner spin-2 tensor field $g_{\mu\nu}$, termed as *graviton* which can be identified with the metric tensor. Therefore, local SUSY is a perfect landscape for establishing connections between high energy particle physics and cosmology. The basic difference between a local and a global SUSY is that the symmetry transformation parameter in local SUSY is explicitly spacetime dependent.

The presence of many scalar fields in the supersymmetry allows to realize inflation within its framework. Since in models of inflation, the inflationary energy scale is very high and close to the fundamental scale of gravity, the Planck scale $M_p \sim 10^{19} GeV$, where all the fundamental forces are expected to unify, the effects of an unknown theory of quantum gravity can not be neglected. The $\mathcal{N} = 1$ SUGRA in four dimensions may offer an effective description of quantum gravity. Also it is worth noting that SUSY plays a crucial role in the structure of string theory and the low-energy limit of the string theory compactifications include supergravity .

Realizing inflation in SUGRA is not so trivial because of the presence of an exponential factor in the scalar potential of the supergravity. For canonical Kähler potential $\delta^{ij}\phi_i\phi_j^*$, any scalar field or inflaton acquires mass of the order of Hubble parameter and it violates one of the slow-roll condition. Therefore, it is not possible to have nearly flat potential for successful inflation in these models. This problem in realizing inflation in SUGRA is known as η *problem* [39, 40]. This difficulty can not be resolved without invoking some symmetry or fine tuning of the scalar potential. To resolve this problem people tune the Kähler potential and superpotential in SUGRA models to obtain a suitable scalar potential which can provide slow-roll inflation. The Kähler potential must be fixed by the model builder and they are not fixed by the symmetries of the theory. There are no legitimate reasons to justify the choices of the Kähler potentials and superpotentials.

During the development of Starobinsky model of inflation in early 1980, the no-scale SUGRA was also discovered and developed and applied to particle physics problems [41–44]. As required for successful inflation, in SUGRA models of inflation the effective potential should vary slowly enough for a sufficient period over a large range of inflaton field values during inflation. This occurs naturally in no-scale supergravity models [37, 41, 44]. Also in these models the energy scale of the effective potential can be naturally much smaller than $M_p \sim 10^{19} GeV$ as required by CMB observations. These models are *called* no-scale because the scale at which the SUSY breaks is undetermined at the tree level and could be anywhere between the experimental lower limit 1 TeV from the LHC [45] and 10 TeV from the measurement of tensor-to-scalar ratio [46]. These no-scale SUGRA models have an attractive feature that they arise naturally in generic four dimensional reductions of string theory [47] and therefore they were proposed as a framework for constructing models of inflation [48]. There are several inflationary models in the context of no-scale supergravity [37, 41, 44, 49].

1.5 Models of Inflation and Key Inflationary Observables

The models of inflation can be broadly divided into two categories: *large field inflation* and *small field inflation*. The class of models in which during inflation $\phi_s > 1M_p$ are called the large field models. The chaotic potential $V(\phi) = \lambda_n \phi^n$ models and exponential potential $V(\phi) = V_0 e^{\phi}$ models are the large field type models. In the chaotic inflation scenario, first introduced in [50], as the universe exits the Planck era at $t \sim 10^{-43} sec$ the initial value of the inflaton field is set chaotically, *i.e.* it acquires different values in different parts of the universe and the initial displacement of the field from the minimum of the potential is larger than Planck scale. These models usually satisfy $V''(\phi) > 0$. There are another class of models, known as small field inflation, in which the slow-roll trajectory is at the small field values $\phi_s < 1M_p$. In these models the field starts close to an unstable maximum of the potential and rolls down to a stable minimum. An example of small field models is 'new inflation' [51] which arises naturally in the mechanism of spontaneous symmetry breaking. In general, the form of the potential in these models are $V(\phi) = V_0(1 - \phi^n)$ and typically these models satisfy $V''(\phi) < 0$.

A basic difference between the large field and small field models is that the large field models predict large amplitude of gravity waves produced during inflation whereas the small field models predict small amplitudes of gravity waves which are too small to be detected in future observations. In either class of these models the inflation ends as soon as the slow-roll conditions are violated and the field rolls down to the minimum of the potential, oscillates and decays into the standard model particles. The decay process of the fields into standard model particles is known as *reheating* and after this universe eventually enters into the radiation domination phase [52, 53].

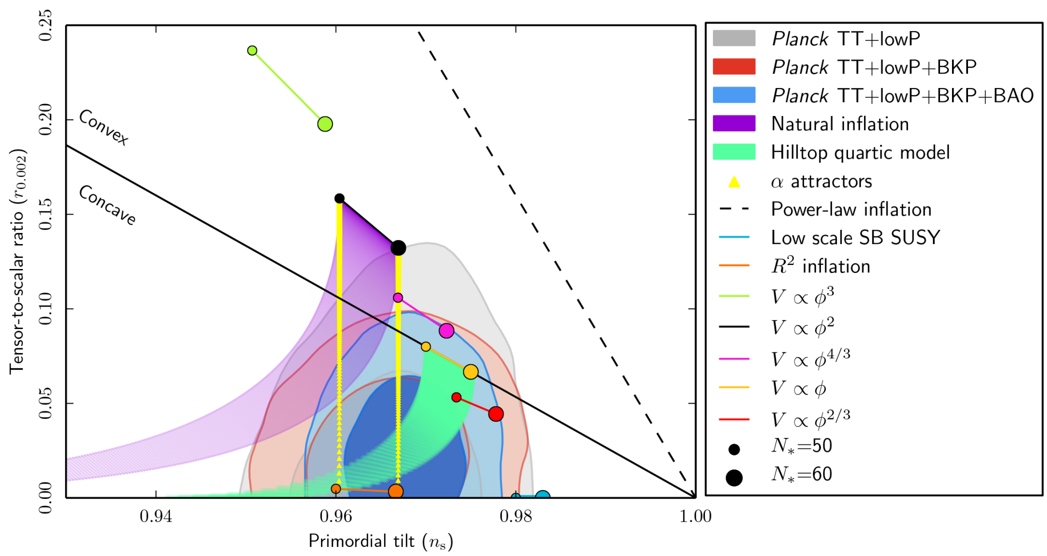

Figure 1.2: The n_s and $r_{0.002}$ values at 68%CL and 95%CL from Planck in combination with other data sets are compared with theoretical predictions of various slow-roll inflation models. Figure credit: [54].

We will see in Chapter §2 that the models of standard slow-roll inflation are typically defined through its scalar potential. For any inflation model in order not to be ruled out, it must predict certain physical quantities in agreement with the observations. These physical quantities/observables are: the amplitude of the power spectrum of curvature perturbations $\Delta^2_{\mathcal{R}}$, spectral index n_s, running of spectral index α_s and tensor-to-scalar ratio r. These observables will be introduced in a great detail in Section §2.2. The latest constraints on the inflationary observables as given by Planck-2015, for the combination *Planck TT + lowP*, are [54]

$$\ln(10^{10}\Delta^2_{\mathcal{R}}) = 3.089 \pm 0.036, \tag{1.37}$$

$$n_s = 0.9666 \pm 0.0062, \tag{1.38}$$

$$r_{0.002} < 0.1. \tag{1.39}$$

The above values are for 7-parameter $\Lambda CDM{+}r$ model, when there is no scale dependence of the scalar and tensor spectral indices. The value of amplitude and spectral index are given at $68\%CL$ at the pivot scale $k = 0.05Mpc^{-1}$. Whereas the upper bound on tensor-to-scalar ratio is determined at $95\%CL$ at $k = 0.002Mpc^{-1}$ [54]. There are numerous models of inflation. For a review on variety of models of inflation we refer the reader to ref. [55] and references therein. The $n_s - r$ predictions, at the lowest order in slow-roll, for various single-field and modified gravity models with e-folds between $50 < N < 60$ are as shown in Fig. 1.2.

Also for 8-parameter $\Lambda CDM{+}r{+}\alpha_s$ model, when there is $k-$dependence of the spectral index or there is a running of the spectral index, the Planck observations give

$$n_s = 0.9667 \pm 0.0132, \tag{1.40}$$

$$r_{0.05} < 0.168. \tag{1.41}$$

The value of amplitude and spectral index are given at $68\%CL$ at the pivot scale $k = 0.05Mpc^{-1}$. Whereas the upper bound on tensor-to-scalar ratio is determined at $95\%CL$ at $k = 0.05Mpc^{-1}$. However, the amplitude of the power spectrum remains the same. Notice that with running, the constraint on tensor-to-scalar ratio is relaxed.

Later, the joint BICEP2/Keck Array and Planck analysis put an upper limit on tensor-to-scalar [56]

$$r_{0.05} < 0.12 \qquad at \;\; 95\% CL. \tag{1.42}$$

Most recently BICEP2/Keck Array Collaboration with its CMB polarization data and combining it with Planck analysis of CMB polarization and temperature data have further improved the bound on r [57]

$$r_{0.05} < 0.07 \qquad at \;\; 95\% CL. \tag{1.43}$$

We will use these observed values to constrain parameters of the models studied in this thesis.

1.6 Notational Clarifications

- ◇ We shall use the metric signature (-, +, +, +) through out this thesis.
- ◇ In Chapter §2 and Chapter §5, field ϕ and potential $V(\phi)$ are the Einstein frame inflaton field and inflaton potential, respectively.
- ◇ In Chapter §3 and Chapter §4, field ϕ and potential $V(\phi)$ are the Jordan frame inflaton field and inflaton potential, respectively. Whereas field χ and potential $U(\chi)$ represent the Einstein frame inflaton field and inflaton potential, respectively. Also in these Chapters, quantities with tilde explicitly represent the quantities in Einstein frame. E.g. $\tilde{R}$ and $\tilde{H}$ are the Ricci scalar and Hubble parameter of Einstein frame, respectively. Whereas the same quantities without tilde, *i.e.* R and H are the Ricci scalar and Hubble parameter of Jordan frame, respectively.
- ◇ Fields ϕ_i, in Sections §4.3 and §5.4 on SUGRA models, are the Superfields. They can be inflaton or can not be, their exact nature is described in these Sections.
- ◇ Some of the Parameters, *e.g.* α, β, γ, λ, ξ, a, b, n, m, are used repeatedly

in different Chapters. Their role/meaning is defined at the model action or in various defining relations in the respective Chapters. Therefore, the parameters of one Chapter should not be confused with the parameters of the other Chapter.

1.7 Outline of the Thesis

In this thesis, we study the single and double field models of inflation in the framework of modified gravity theories. In order to motivate these models, as they are not generic in the particle physics models, we derive them from a fundamental theory *called* supergravity. The main contents of the thesis are organized as follows :

In Chapter §2, we will give the necessary theoretical foundations for the purpose of this thesis. We will mainly discuss the slow-roll inflation, the theory of cosmological perturbations and, the modified gravity and supergravity framework for inflation. We will discuss no-scale supergravity models, a special class of supergravity theories, in order to motivate the modifications to standard Einstein gravity.

In Chapter §3, we will discuss our model of generalized Higgs inflation $\xi\phi^a R^b$ in the framework of $f(\phi, R)$ gravity theory. This model is a generalization of the Higgs inflation model $\xi\phi^2 R$ which has a problem of unitarity violation at the Planck scale energies. We will show that in the generalized Higgs inflationary scenario with a Higgs quartic potential, the problem of unitarity violation and the fine tuning problem of Higgs self-coupling can be resolved.

In Chapter §4, we will discuss our power law Starobinsky model $R+\frac{1}{M^2}R^\beta$ of inflation which is a generalization of Starobinsky model $R+\frac{1}{M^2}R^2$. We will show, unlike Starobinsky model which gives small tensor-to-scalar ratio $r \sim 0.001$, its generalization can produce larger $r \sim 0.2$. We will show that higher order curvature theories like Starobinsky model and power law model can be derived from the more fundamental theories like no-scale supergravity. Also, we will show that generalized Higgs inflation models with quantum corrected $\lambda\phi^4$ potential are equivalent to power law model.

In Chapter §5, we will discuss our two-field model of inflation with two arbitrary parameters. We will show that in this model quartic and quadratic inflaton potentials, which in the context of single-field slow-roll inflation are ruled out by Planck observations, can produce correct observational results. Also, in order to motivate such a two-field model, we will give a derivation of this model from no-scale supergravity.

In Chapter §6, we will summarize and conclude by pointing out the main findings of this thesis.

Chapter 2

Theoretical Foundations

2.1 Scalar Fields as a Source of Inflation

The existence of scalar fields in the very early universe is suggested by our best theories of fundamental interactions in Nature, which predict that the universe went through a succession of phase transitions in its early stages as it expanded and cooled. In general, the phase transition occurs when certain scalar parameters known as Higgs fields acquire a non-zero value or vacuum expectation value (VEV) via a process called spontaneous symmetry breaking. The symmetry is manifest as long as the Higgs fields have not acquired vev and it is spontaneously broken as soon as at least one of the Higgs fields becomes non-zero. Therefore, the existence of scalar fields in the early universe is suggested by the occurrence of phase transitions and therefore provides the motivation for considering them as the source of inflation.

Since the inflation requires a source with negative pressure as depicted in relation (1.26). We will see here that scalar fields can act as a negative pressure source.

The simplest inflation models involves a single scalar field ϕ which in the inflationary context is termed as *inflaton*. The model is described by the following action

$$
\begin{aligned}
S &= \int d^4x\sqrt{-g}\left[-\frac{M_p^2}{2}R+\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+V(\phi)\right] \\
&= \int d^4x\sqrt{-g}\left(\mathcal{L}_{EH}+\mathcal{L}_\phi\right). \qquad (2.1)
\end{aligned}
$$

In this action the inflaton field has a minimal coupling with the gravity and a canonical kinetic term. $V(\phi)$ is the potential of the field due to self-interaction and it can be different in different inflation models. Here we will assume an arbitrary $V(\phi)$. Also in this Chapter from here onwards, for the brevity of the text, we shall work with $M_p = 1$ unit and shall restore it after the Section §2.2.4 for clarity.

The energy momentum tensor of the scalar field can be obtained by varying the action (2.1) w.r.t. the metric $g_{\mu\nu}$ as

$$
\begin{aligned}
T^\phi_{\mu\nu} &= -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_\phi}{\delta g^{\mu\nu}} \\
&= \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}\partial^\rho\phi\partial_\rho\phi+V(\phi)\right]. \qquad (2.2)
\end{aligned}
$$

In principle, scalar fields can be dependent on space and time both *i.e.* $\phi = \phi(t,\mathbf{x})$, however as we know that the universe is homogeneous on largest scales, therefore at the background level, homogeneity implies that scalar field can be described by its time dependence only, *i.e.* $\phi(t,\mathbf{x}) \equiv \phi(t)$. Therefore for the homogeneous background field, the energy momentum tensor for ϕ takes the form of a perfect fluid (1.6) with energy density and pressure for scalar fields given by

$$
\rho_\phi = \frac{\dot\phi^2}{2}+V(\phi), \qquad (2.3)
$$

$$
p_\phi = \frac{\dot\phi^2}{2}-V(\phi), \qquad (2.4)
$$

respectively and hence the resulting equation of state

$$\omega_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{\dot{\phi}^2}{2} - V(\phi)}{\frac{\dot{\phi}^2}{2} + V(\phi)}. \tag{2.5}$$

If the potential energy of the field dominates over its kinetic energy *i.e.* $\dot{\phi}^2 << V(\phi)$, then the above simple relation (2.5) implies that the scalar field can act as a negative pressure source *i.e.* $\omega_\phi < 0$ and can provide accelerated expansion *i.e.* $\omega_\phi < -\frac{1}{3}$.

The equation of motion of the scalar field can be obtained by varying the action (2.1) w.r.t. the field ϕ as

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + \frac{dV}{d\phi} = 0, \tag{2.6}$$

which for the background scalar field gives

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{2.7}$$

and the Friedmann equation for the scalar field can be obtained by varying the action (2.1) w.r.t. metric $g_{\mu\nu}$ as

$$H^2 = \frac{1}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right). \tag{2.8}$$

The equations (2.7) and (2.8) determines the dynamics of the scalar field in a FRW universe.

2.1.1 Slow-roll Inflation

The equations (2.7) and (2.8) can be solved analytically for some specific potentials $V(\phi)$, however in general, an analytical solution is possible only under slow-roll approximation. As discussed above, slow-roll inflation occurs when $\dot{\phi}^2 \ll V(\phi)$ which implies that the field ϕ rolls down the potential slow enough that the potential is nearly constant during inflation. A second order differentiation of the condition $\dot{\phi}^2 \ll V(\phi)$ implies $\ddot{\phi} \ll V'(\phi)$ which ensures that the accelerated expansion is sustained for a sufficient period of time. Under slow-roll approximation equations (2.7) and (2.8) become

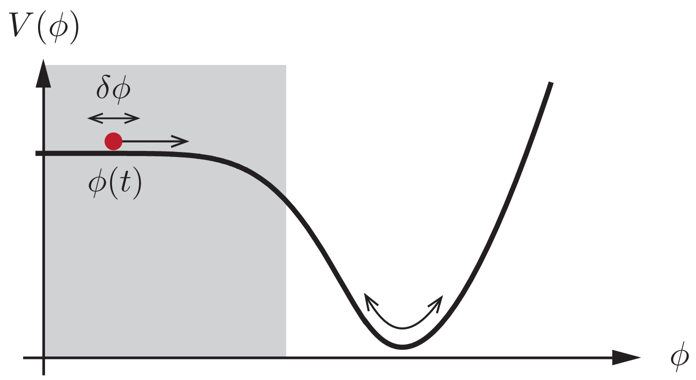

Figure 2.1: A generic example of a slow-roll potential of inflaton. The shaded region corresponds to slow-roll inflation when field rolls on the nearly flat part of the potential. Along with the background evolution, the inflaton $\phi(t)$ experiences spatially varying quantum fluctuations $\delta\phi(t, \mathbf{x})$. Figure credit: [58].

$$3H\dot{\phi} \simeq -V'(\phi), \tag{2.9}$$

$$3H^2 \simeq V(\phi). \tag{2.10}$$

It is worth noting that the time variation of the Hubble parameter and scalar field can be related easily by differentiating equation (2.10) w.r.t. time and combining the result with equation (2.9) as

$$\dot{H} \simeq -\frac{\dot{\phi}^2}{2}. \tag{2.11}$$

The slow-roll conditions $\dot{\phi}^2 \ll V(\phi)$ and $\ddot{\phi} \ll V'(\phi)$ can be put into useful dimensionless parameters as

$$\epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1, \tag{2.12}$$

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \simeq \frac{V''(\phi)}{V(\phi)} \ll 1. \tag{2.13}$$

These two conditions ensures that the potential $V(\phi)$ is sufficiently 'flat' that the field ϕ rolls slowly enough for inflation to occur. A generic example of a slow-roll potential is shown in Fig. 2.1. In the figure the shaded region corresponds to slow-roll inflation when field rolls on the nearly flat part of the potential between the times when the observable modes of present particle horizon size leave the horizon and when inflation ends. After the end of inflation when field has crossed the flat part of the potential, it fast rolls, $\dot{\phi}^2 \approx V(\phi)$, towards the minimum of the potential and then oscillates and decays into the standard model particles.

It is worth considering the case in which $V(\phi)$ is nearly constant during some part of the period of inflation during which Hubble expansion is constant. Solving (2.10), one obtains that during this period scale factor evolves exponentially

$$a(t) \sim e^{Ht}, \tag{2.14}$$

such a spacetime is approximately de-Sitter.

From the acceleration equation (1.23), it is clear that inflation ends when

$$\epsilon(\phi_e) = 1, \tag{2.15}$$

which represents the violation of slow-roll condition $\epsilon \ll 1$ and as soon as this condition is met *i.e.* $\epsilon = 1$, the kinetic energy of the field $\dot{\phi}^2$ becomes comparable to its potential energy $V(\phi)$ and the potential becomes steeper and field speeds up towards the minimum of the potential as shown in the Fig.(2.1).

The number of e-foldings before the inflation ends, as defined in (2.12), is given by

$$N(\phi) = \int_{t_i}^{t_e} H dt = \int_{\phi_i}^{\phi_e} \frac{H}{\dot{\phi}} d\phi \simeq \int_{\phi_e}^{\phi_i} \frac{V}{V'} d\phi, \tag{2.16}$$

where we used the slow-roll equations (2.9) and (2.10). To solve the horizon and flatness problems it is required that the total number of e-foldings during inflation exceeds 60

$$N_{tot} \equiv \ln \frac{a_e}{a_i} \gtrsim 60. \tag{2.17}$$

However the precise value of N_{tot} depends on the energy scale of inflation and details of the reheating after inflation. It is during the slow-roll phase, which lasts nearly $40 - 60$ e-folds before inflation ends (the precise value again is determined by the details of reheating and post-inflationary evolution of the universe), when the quantum fluctuations in the field are imprinted on the CMB and ϕ_i corresponds to the field value when these fluctuations in the CMB are created.

2.2 Cosmological Perturbation Theory

Here we present the calculation of the primordial density fluctuations power spectra generated by quantum fluctuations in the inflaton field during inflation. Observation of the CMB anisotropies $\frac{\delta\rho}{\rho} \sim 10^{-5}$ proves that the early universe was not perfectly uniform in its matter distribution. However, as the observed anisotropies are very small, therefore these can be analyzed in terms of linear quantum fluctuations $\delta\phi(t, \mathbf{x})$ around the homogeneous background. The linear theory of cosmological perturbations is a cornerstone of the modern cosmology. It not only explains the CMB anisotropies but also the formation and evolution of the structures in the universe. The seed of these anisotropies were stretched to astronomical scales because of the superluminal expansion of the cosmic space during inflationary quasi de-Sitter expansion. This theory has been extensively studied in literature; the details can be found in Ref.s [59, 60].

The linear perturbations of the metric $g_{\mu\nu}$ can be decomposed according to their spin w.r.t. a local rotation of the spatial coordinates on the hypersurfaces of constant time into three kinds of perturbations: scalar, vector and tensor. Here we will study only scalar and tensor perturbations in detail. Scalar perturbations explain the CMB temperature anisotropy (or matter density fluctuations) and the seed for the structure formations in the universe. One can see from the Einstein field equation (1.4) that the scalar perturbations which give rise to perturbations in the energy-momentum tensor leads to metric perturbations. On the other hand, metric perturbations back react through the perturbations in the KG equations of motion (2.6) of the field, giving rise to field (or matter) perturbations. Therefore

$$\delta g_{\mu\nu}(t, \mathbf{x}) \Longleftrightarrow \delta\phi(t, \mathbf{x}). \tag{2.18}$$

The tensor perturbations corresponds to primordial gravitational waves which is a generic prediction of inflationary models. Observational constraints on the amplitude of the primordial gravitational waves can be used to eliminate various inflation models.

2.2.1 Linear Perturbations

Linear order perturbations in the metric and field around the homogeneous background solutions of the field $\phi(t)$ and the metric $g_{\mu\nu}(t)$ can be given as

$$\delta\phi(t,\mathbf{x}) = \phi(t,\mathbf{x}) - \phi(t), \tag{2.19}$$

$$\delta g_{\mu\nu}(t,\mathbf{x}) = g_{\mu\nu}(t,\mathbf{x}) - g_{\mu\nu}(t). \tag{2.20}$$

The most general linearly perturbed spatially flat FLRW metric can be written as

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \tag{2.21}$$

$$= -(1+2\Phi)dt^2 + 2a(t)B_i dt dx^i + a(t)^2 \left[(1-2\Psi)\delta_{ij} + 2E_{ij}\right] dx^i dx^j, \tag{2.22}$$

where Φ, Ψ are the scalar perturbations, B_i are the vector perturbations and E_{ij} are the tensor perturbations. According to *SVT decomposition* scalar, vector and tensor perturbations are decoupled during inflation and therefore evolve independently, this is also known as *Decomposition Theorem*. This theorem implies that if some physical process in the early universe sets up tensor perturbations then these do not induce scalar perturbations, on the other hand, evolution of the scalar perturbations is unaffected by the presence of any possible vector and tensor perturbations [61]. The importance of SVT decomposition is that the Einstein equations for scalar, vector and tensor perturbations do not mix at linear order and can therefore be studied separately. In this way the SVT decomposition greatly simplifies the calculations. The vector perturbations are not sourced by inflation and furthermore they quickly decay with the expansion of the universe [59]. Therefore we will ignore vector perturbations and focus on scalar and tensor perturbations only.

According to SVT decomposition of the metric perturbations in real space, the vector B_i can be decomposed into a gradient of as scalar, say B, and divergence free vector, say S_i, as

$$B_i \equiv \partial_i B - S_i, \qquad where \quad \partial^i S_i = 0, \tag{2.23}$$

and similarly, any second rank tensor E_{ij} can be written in terms of a divergence free vector and a traceless and divergence free tensor as

$$E_{ij} \equiv 2\partial_i \partial_j E + 2\partial_{(i} F_{j)} + h_{ij}, \qquad where \quad \partial^i F_i = 0, \quad h^i_i = \partial^i h_{ij} = 0. \tag{2.24}$$

Since the $g_{\mu\nu}$ is a symmetric tensor therefore in 4-dimensions it has 10 independent components or 10 degrees of freedom (d.o.f.). The 10 d.o.f. of metric has been decomposed into 4+4+2 SVT d.o.f., *i.e.* 4 scalar d.o.f. Ψ, Φ, B, E, 2 vector d.o.f. for S_i, F_i vectors each and 2 d.o.f. of tensor h_{ij}.

2.2.2 Gauge Transformation and Gauge Invariance

In general relativity the gauge transformations are the general coordinate transformations from one local reference frame to another. Here we will briefly review the gauge fixing and the behavior of the scalar, vector and tensor perturbations under general coordinate transformation. We will introduce the gauge invariant quantities in next Section §2.2.3. Fixing a gauge in General relativity implies choosing a coordinate system, a slicing of spacetime into constant time hypersurfaces and threading into lines with fixed spatial coordinate $\mathbf{x}$. Now let us consider the infinitesimal coordinate transformations

$$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu, \tag{2.25}$$

where ξ^μ is a spacetime dependent infinitesimal quantity. At a given point on spacetime manifolds, the metric in the new coordinate system $\tilde{x}^\mu$ can be determined using the invariance of the line-element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \tilde{g}_{\mu\nu} d\tilde{x}^\mu d\tilde{x}^\nu. \tag{2.26}$$

or by applying the usual tensor transformation law

$$\tilde{g}_{\mu\nu}(\tilde{x}^\rho) = \frac{\partial x^\mu}{\partial \tilde{x}^\alpha} \frac{\partial x^\nu}{\partial \tilde{x}^\beta} g_{\mu\nu}(x^\rho). \tag{2.27}$$

Consider splitting the metric $g_{\mu\nu}(x^\rho)$ into background and perturbed parts in both x^μ and $\tilde{x}^\mu$ coordinate systems

$$g_{\mu\nu}(x^\rho) = g^{(0)}_{\mu\nu}(x^\rho) + \delta g_{\mu\nu}(x^\rho), \tag{2.28}$$

$$\tilde{g}_{\mu\nu}(\tilde{x}^\rho) = g^{(0)}_{\mu\nu}(\tilde{x}^\rho) + \delta \tilde{g}_{\mu\nu}(\tilde{x}^\rho). \tag{2.29}$$

Note that we have not put tilde over the background metric because due to homogeneity and isotropy, the background forms of the metric tensor (also vectors and scalars) does not change, so that the background quantities behave the same way in the new coordinate system $\tilde{x}^\mu$. Partial differentiation of equation (2.25) gives

$$\frac{\partial x^\mu}{\partial \tilde{x}^\alpha} = \delta^{\mu\alpha} - \frac{\partial \xi^\mu}{\partial \tilde{x}^\alpha}, \tag{2.30}$$

Also using the Taylor expansion, the background metric may be expanded as

$$g^{(0)}_{\mu\nu}(x^\rho) = g^{(0)}_{\mu\nu}(\tilde{x}^\rho - \xi^\rho) \simeq g^{(0)}_{\mu\nu}(\tilde{x}^\rho) - \frac{\partial g^{(0)}_{\alpha\beta}}{\partial \tilde{x}^\rho} \xi^\rho. \tag{2.31}$$

Substituting (2.28) and (2.30) into (2.27) and comparing with (2.29) while using (2.31), we get the transformation law of metric tensor perturbation,

$$\boxed{\delta g_{\alpha\beta}(x^\rho) \rightarrow \delta \tilde{g}_{\alpha\beta}(\tilde{x}^\rho) = \delta g_{\alpha\beta} - \frac{\partial g^{(0)}_{\alpha\beta}(\tilde{x}^\rho)}{\partial \tilde{x}^\rho} \xi^\rho - g^{(0)}_{\alpha\nu}(x^\rho) \frac{\partial \xi^\nu}{\partial \tilde{x}^\beta} - g^{(0)}_{\mu\beta}(x^\rho) \frac{\partial \xi^\mu}{\partial \tilde{x}^\alpha}}. \tag{2.32}$$

Similarly, a 4−vector $u^\mu(x^\rho)$ which follows the tensor transformation law

$$\tilde{u}_\alpha(\tilde{x}^\rho) = \frac{\partial x^\mu}{\partial \tilde{x}^\alpha} u_\mu(x^\rho), \tag{2.33}$$

its perturbation $\delta u_\alpha(x^\rho) = u_\alpha(x^\rho) - u_\alpha^{(0)}(x^\rho)$ transforms as

$$\boxed{\delta u_\alpha(x^\rho) \to \delta\tilde{u}_\alpha(\tilde{x}^\rho) = \delta u_\alpha(x^\rho) - \frac{\partial u_\alpha^{(0)}(\tilde{x}^\rho)}{\partial \tilde{x}^\rho}\xi^\rho - u_\mu^{(0)}(x^\rho)\frac{\partial \xi^\mu}{\partial \tilde{x}^\alpha}}. \tag{2.34}$$

And similarly, a scalar $q(x^\mu)$ which doesn't change under the coordinate transformation

$$\tilde{q}(\tilde{x}^\mu) = q(x^\mu) \tag{2.35}$$

its perturbation $\delta q(x^\mu) = q(x^\mu) - q^{(0)}(x^\mu)$ does, as

$$\boxed{\delta q(x^\mu) \to \delta\tilde{q}(\tilde{x}^\mu) = \delta q(x^\mu) - \frac{\partial q^{(0)}(\tilde{x}^\rho)}{\partial \tilde{x}^\rho}\xi^\rho}. \tag{2.36}$$

Now let us write the temporal and spatial components of the infinitesimal vector $x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu$ as

$$\begin{aligned} t &\to t + \alpha, \\ x^i &\to x^i + \delta^{ij}\partial_j\beta. \end{aligned} \tag{2.37}$$

where $x^\mu \equiv (x^0, x^i) = (t, x^i)$, $\xi^\mu \equiv (\xi^0, \xi^i) = (\alpha, \partial^i\beta)$, α is infinitesimal temporal shift and β is a scalar function.

Using the metric tensor and scalar perturbation transformation laws (2.32) and (2.36), we find that the tensor perturbations h_{ij} are invariant under the gauge transformations (and therefore they already represents gravitational waves in a gauge invariant manner), whereas the scalar perturbations Φ, Ψ, B and E transform as

$$\begin{aligned} \Phi &\to \Phi - \dot{\alpha}, \\ \Psi &\to \Psi + H\alpha, \\ B &\to B + a^{-1}\alpha - a\dot{\beta}, \\ E &\to E - \beta, \end{aligned} \tag{2.38}$$

Thus we find that only α and β contributes to the transformations of the scalar perturbations and we can choose them appropriately (as we are free to choose them) and can impose two conditions on the scalar functions Φ, Ψ, B and E to remove any two of them. This is called the *gauge fixing* or *gauge choice* which corresponds to choosing a *gauge transformation*. It is possible that the freedom in coordinate choice leads to an appearance of fictitious perturbation modes which do not describe any real physical inhomogeneities. However, one can construct gauge invariant quantities which do not depend on choice of coordinate system and represents real inhomogeneities. Two important gauge-invariant quantities were introduced by Bardeen [62]

$$\Phi_B \equiv \Phi - \frac{d}{dt}[a^2(\dot{E} - B/a)], \tag{2.39}$$

$$\Psi_B \equiv \Psi + a^2 H(\dot{E} - B/a). \tag{2.40}$$

The gauge invariance of Φ_B and Ψ_B implies that if they vanish in one particular coordinate system then they will be vanishing in any coordinate system. Such a construction of gauge invariant quantities allows us to distinguish between physical inhomogeneities and fictitious perturbations. If there are metric perturbations present even when both Φ_B and Ψ_B are zero, then they are fictitious perturbations and can be eliminated using change of coordinates.

Using (2.36), we find that the perturbations of the scalar field ϕ transform as

$$\delta\tilde{\phi}(\tilde{x}^\mu) = \delta\phi(x^\mu) - \frac{\partial\phi^{(0)}(x^\rho)}{\partial x^\rho}\xi^\rho. \tag{2.41}$$

Since the background field $\phi^{(0)} = \phi(t)$ is time-dependent only, therefore

$$\delta\tilde{\phi}(\tilde{x}^\mu) = \delta\phi(x^\mu) - \dot{\phi}(t)\alpha, \tag{2.42}$$

where $\alpha = \xi^0$. Also the matter perturbations or the perturbations to the total stress energy tensor $T_{\mu\nu}$ are given in terms of the perturbations of the energy density $\delta\rho$, perturbations of pressure δp and perturbations of momentum density

δq. Under gauge transformation these perturbations transform as

$$\begin{aligned} \delta\rho &\to \delta\rho - \dot{\rho}\alpha, \\ \delta p &\to \delta p - \dot{p}\alpha, \\ \delta q &\to \delta q + (\rho + p)\alpha. \end{aligned} \tag{2.43}$$

2.2.3 Gauge Invariant Variables

We discussed and explained the fictitious and real perturbations in previous Section §2.2.2. In order to avoid the fictitious gauge modes, it is preferable to use the gauge-invariant combinations of the matter and metric perturbations [62]. An important gauge-invariant quantity is the *comoving curvature perturbation* $\mathcal{R}$ [63] defined as

$$\boxed{\mathcal{R} \equiv \Psi - \frac{H}{\rho + p}\delta q}, \tag{2.44}$$

this can also be given in terms of the metric perturbations in the longitudinal gauge (the longitudinal gauge in which $B = E = 0$ is defined in the next Section §2.2.4) as [64]

$$\mathcal{R} = \Psi - \frac{H}{\dot{H}}(\dot{\Psi} + H\Phi), \tag{2.45}$$

and for perfect fluid, from equation (2.73), this can further be simplified to give

$$\mathcal{R} = \Phi - \frac{H}{\dot{H}}(\dot{\Phi} + H\Phi). \tag{2.46}$$

The condition (2.44) can be constructed by considering the slicing of the space-time into constant δq (or constant-ϕ) hyperserfaces which provide the constraint

$$\delta q \to \delta q + (\rho + p)\alpha = 0 \quad \Longrightarrow \quad \alpha = -\frac{\delta q}{\rho + p} \tag{2.47}$$

substituting this α into the metric transformation relation $\Psi \to \Psi + H\alpha$ gives the relation (2.44) for $\mathcal{R}$. Since δq is the scalar $0i-$component of the perturbed energy momentum tensor $T^0_i = \partial_i \delta q$ and during inflation $T^0_i = -\dot{\phi}\partial_i\delta\phi$, therefore both of these relations implies $\delta q = -\dot{\phi}\delta\phi$. Also from (2.3) and (2.4) we have

$\rho + p = \dot{\phi}^2$. Therefore, the comoving curvature perturbations (2.44) during inflation becomes

$$\boxed{\mathcal{R} \simeq \Psi + \frac{H}{\dot{\phi}}\delta\phi}. \tag{2.48}$$

Geometrical interpretation of $\mathcal{R}$ is that it measures the spatial curvature of the comoving hypersurface where $\delta\phi = 0$, *i.e.*

$$\mathcal{R} = \Psi|_{\delta\phi=0}. \tag{2.49}$$

Another important gauge-invariant quantity is *curvature perturbations on constant energy density hypersurfaces* ζ defined as

$$\boxed{-\zeta \equiv \Psi + \frac{H}{\dot{\rho}}\delta\rho}, \tag{2.50}$$

Similar to $\mathcal{R}$, the quantity ζ can be constructed by considering the slicing of the spacetime into constant energy density hyperserfaces which provide the constraint

$$\delta\rho \to \delta\rho - \dot{\rho}\alpha = 0 \quad \Longrightarrow \quad \alpha = \frac{\delta\rho}{\dot{\rho}} \tag{2.51}$$

substituting this α into the metric transformation relation $\Psi \to \Psi + H\alpha$ gives the relation (2.50) for ζ. Since during slow-roll, from equation (2.3), $\delta\rho = \dot{\phi}\delta\dot{\phi} + V'\delta\phi \simeq V'\delta\phi$ and $\dot{\rho} = \dot{\phi}\ddot{\phi} + V'\dot{\phi} \simeq V'\dot{\phi}$, which implies $\frac{\delta\rho}{\dot{\rho}} \simeq \frac{\delta\phi}{\dot{\phi}}$. Therefore $-\zeta$ becomes

$$\boxed{-\zeta \simeq \Psi + \frac{H}{\dot{\phi}}\delta\phi}. \tag{2.52}$$

Geometrical interpretation of ζ is that it measures the spatial curvature of the uniform density hypersurface, *i.e.*

$$-\zeta = \Psi|_{\delta\rho=0}. \tag{2.53}$$

We see that the curvature perturbations $\mathcal{R}$ and ζ by construction are invariant under the gauge transformations (2.37), which can be verified using (2.38) and (2.43) into their expressions (2.44) and (2.50).

Also using the linearized Einstein field equations it can be shown that the gauge invariant curvature perturbations ζ and $\mathcal{R}$ are related as [59]

$$\boxed{-\zeta = \mathcal{R} + \left(\frac{k}{aH}\right)^2 \frac{2\rho}{3(\rho + p)}\Psi_B}, \tag{2.54}$$

which implies that *"on superhorizon scale $k \ll aH$, ζ and $\mathcal{R}$ are equal"*. Also we saw that under slow-roll they are equal, *cf.* equations (2.48) and (2.52).

"The curvature perturbations ζ and $\mathcal{R}$ also share an important property that on superhorizon scales they are conserved for adiabatic matter perturbations".

In general, it is possible that the pressure perturbations (in any gauge) can be split into adiabatic and non-adiabatic (entropic) parts as

$$\begin{aligned} \delta p &\equiv \delta p_{ad} + \delta p_{nad} \\ &= c_s^2 \delta\rho + \delta p_{nad}, \end{aligned} \tag{2.55}$$

where $c_s^2 \equiv \frac{\dot{p}}{\dot{\rho}}$ and the adiabatic pressure perturbations are defined as

$$\delta p_{ad} \equiv \frac{\dot{p}}{\dot{\rho}}\delta\rho, \tag{2.56}$$

which satisfy the condition

$$\frac{\delta p}{\dot{p}} = \frac{\delta\rho}{\dot{\rho}} \tag{2.57}$$

which implies that a given time displacement δt causes the same relative fractional change $\frac{\delta X}{\dot{X}}$ in all scalar quantities $X \equiv (\rho, p, ...)$.

The non-adiabatic part of the pressure perturbations δp_{nad} are defined as

$$\boxed{\delta p_{nad} \equiv \dot{p}\Gamma \equiv \delta p - \frac{\dot{p}}{\dot{\rho}}\delta\rho}, \tag{2.58}$$

where

$$\boxed{\Gamma \equiv \frac{\delta p}{\dot{p}} - \frac{\delta\rho}{\dot{\rho}}}, \tag{2.59}$$

is the *entropy perturbation*, also known as *isocurvature perturbation*. Γ, defined in this way, is gauge-invariant and represents the displacement between hypersurfaces of uniform pressure and uniform density.

Using the perturbed Einstein field equations (2.64), as discussed in detail in the next Section §2.2.4, it can be shown that the evolution of the gauge invariant curvature perturbations in the longitudinal gauge is given by [59, 65]

$$\boxed{\dot{\mathcal{R}} = -\frac{H}{\rho + p}\delta p_{nad} + \left(\frac{k}{aH}\right)^2 \left[\frac{H^2}{3(\rho + p)}\delta q\right]}, \tag{2.60}$$

therefore if there are no non-adiabatic matter perturbations $\delta p_{nad} = 0$ or no isocurvature perturbations $\Gamma = 0$, the curvature perturbations $\mathcal{R}$ (also ζ, *cf.* equation (2.54)) are conserved on superhorizon scales $k \ll aH$.

Physical Interpretation of Adiabatic (Curvature) and Isocurvature Perturbations :

If the curvature perturbations are such that they can not give rise to variations in the relative density between different components of the cosmological fluid (photons, baryons, neutrinos and CDM particles) after inflation, then the curvature perturbations are adiabatic :

$$\delta\left(\frac{n_m}{n_r}\right) = 0 \quad \Longrightarrow \quad \frac{\delta n_m}{n_m} = \frac{\delta n_r}{n_r}, \tag{2.61}$$

where $\delta n = n(t, \mathbf{x}) - n^{(0)}(t)$ and the index m collectively stands for non-relativistic matter components *e.g.* baryons and CDM and index r for relativistic matter components *e.g.* photons and neutrinos. As we know $n_{(m,r)} \propto a^{-3}$, $\rho_m \propto a^{-3}$ and $\rho_r \propto a^{-4}$, condition (2.61) gives

$$\frac{\delta\rho_m}{\rho_m} = \frac{3}{4}\frac{\delta\rho_r}{\rho_r}. \tag{2.62}$$

In single-field slow inflationary scenario, the condition (2.62) holds and therefore the perturbations produced by single-field inflation are purely adiabatic. However, in inflationary models with more than one field, the perturbations are not necessarily adiabatic. If during inflation there are more than one field and all are evolving in time, the fluctuations orthogonal to background trajectory can affect the relative density between different components of the cosmological fluid even if the total density (and therefore spatial curvature) is unperturbed [66]. For example, the relative density perturbations (isocurvature/entropy perturbations) between photon and CDM can be defined as

$$\Gamma_{m\gamma} \equiv \frac{\delta\rho_{cdm}}{\rho_{cdm}} - \frac{3}{4}\frac{\delta\rho_\gamma}{\rho_\gamma}. \tag{2.63}$$

Since adiabatic and isocurvature perturbations give different peak structure in the CMB power spectrum, therefore different type of perturbations can be distinguished from the CMB measurements. In fact, CMB observations suggest that even if the isocurvature perturbations are present, their amplitude is vanishingly small compared to amplitude of the adiabatic (curvature) perturbations [54]. The theoretical predictions of the isocurvature perturbations are extremely model dependent. Not only the presence of more than one scalar field may give rise to entropic perturbations but these may also be generated in non-minimally coupled inflation models [65]. Also the post-inflationary evolution may generate them.

2.2.4 Curvature Perturbation and Scalar Power Spectrum

For a metric with small perturbations, the Einstein tensor $G_{\nu\mu}$ can be written as $G_{\nu\mu} = G^{(0)}_{\nu\mu} + \delta G_{\nu\mu} + ...$, where $\delta G_{\nu\mu}$ represents the terms with linear metric perturbations $\delta g_{\mu\nu}$. The stress energy tensor T^ν_μ can be split in a similar fashion and we get the linearized Einstein field equations

$$\boxed{\delta G^\nu_\mu = 8\pi G\ \delta T^\nu_\mu}. \tag{2.64}$$

The gauge freedom allows to choose the two functions α and β which provides two conditions on the scalar functions Φ, Ψ, B, E and therefore allows to remove any two of them. The gauge freedom greatly simplifies the calculations and knowing the solutions of the gauge-invariant variables, one can calculate the density and metric perturbations in any coordinate system in a simple way [67]. One of many useful gauges is the *conformal Newtonian gauge* or *longitudinal gauge* which is defined by the conditions $B = E = 0$. In this gauge the FRW line element assumes the simple form

$$ds^2 = -(1+2\Phi)dt^2 + a(t)^2(1-2\Psi)\delta_{ij}dx^i dx^j. \tag{2.65}$$

Now we calculate the perturbed Einstein field equations (2.64). For the metric (2.65), the components of the perturbed Einstein tensor can be obtained as

$$\delta G^0_0 = -2\nabla^2\Psi + 6H^2\Phi + 6H\dot{\Psi}, \tag{2.66}$$

$$\delta G^0_i = -2\partial_i(H\Phi + \dot{\Psi}), \tag{2.67}$$

$$\delta G^i_j = \partial^i\partial_j(\Psi - \Phi) + [\nabla^2(\Phi - \Psi) + 2\ddot{\Psi} + (4\dot{H} + 6H^2)\Phi + H(2\dot{\Phi} + 2\dot{\Psi})]\delta^i_j. \tag{2.68}$$

Using the perfect fluid description as defined in equation (1.6) and the stress-energy-momentum tensor for the scalar field ϕ as defined in equation (2.2), the components of the perturbed $T_{\mu\nu}$ are given by

$$\delta T^0_0 = -\delta\rho = \dot{\phi}^2\Phi - \dot{\phi}\delta\dot{\phi} - V'\delta\phi, \tag{2.69}$$

$$\delta T^0_i = \delta q = -\dot{\phi}\partial_i\delta\phi, \tag{2.70}$$

$$\delta T^i_j = \delta p = [-\dot{\phi}^2\Phi + \dot{\phi}\delta\dot{\phi} - V'\delta\phi]\delta^i_j. \tag{2.71}$$

where we have used the relation

$$\delta T^\nu_\mu = \delta(g^{\nu\tau}T_{\mu\tau}) = \delta g^{\nu\tau}T_{\mu\tau} + g^{\nu\tau}\delta T_{\mu\tau}. \tag{2.72}$$

Now we can compute the curvature perturbation $\mathcal{R}$. First we consider the ij-component of the perturbed Einstein field equation (2.64). From equation (2.71), we see that the stress energy tensor has no off-diagonal components, therefore taking the off-diagonal components, *i.e.* $i \neq j$, of the equations (2.68) and (2.71), we have

$$\partial^i\partial_j(\Psi - \Phi) = 0 \quad \Longrightarrow \quad \Phi = \Psi, \tag{2.73}$$

therefore we can work with any of the variable Φ or Ψ, let's work with Ψ. We note that if the spatial part of the stress energy tensor is diagonal, *i.e.* $\delta T^i_j \propto \delta^i_j$, the variable Φ or Ψ can be seen as a generalisation of the Newtonian potential which therefore explains the name *Newtonian gauge* for this choice of coordinate system.

Now considering the diagonal components, *i.e.* $i = j$, of equations (2.68) and (2.71), we get

$$\ddot{\Psi} + 4H\dot{\Psi} + (2\dot{H} + 3H^2)\Psi = -\dot{\phi}^2\Phi + \dot{\phi}\delta\dot{\phi} - V'\delta\phi. \tag{2.74}$$

Since ϕ is background quantity which is only time dependent, therefore equations (2.67) and (2.70) for $0i$-components give

$$\dot{\Psi} + H\Psi = 4\pi G\ \dot{\phi}\delta\phi = \epsilon H^2 \frac{\delta\phi}{\dot{\phi}}, \tag{2.75}$$

where we used the relation for slow-roll parameter $\epsilon = 4\pi G \frac{\dot{\phi}^2}{H^2}$. Similarly the equations (2.66) and (2.69) for 00-component gives

$$\nabla^2\Psi - 3H\dot{\Psi} - 3H^2\Psi = 4\pi G(\dot{\phi}\delta\dot{\phi} - \dot{\phi}^2\Psi + V'\delta\phi). \tag{2.76}$$

For the purpose of analysis, it is convenient to work in terms of the Fourier decomposition of the metric and the field perturbations, and see what happens to a perturbation corresponding to a given comoving spatial scale k with corresponding comoving wavelength $\lambda = \frac{2\pi}{k}$. Using Fourier transformation, we can decompose the perturbations Ψ and $\delta\phi$ into a superposition of plane-wave states with comoving wavevector $\mathbf{k}$:

$$\Psi(t, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \Psi_{\mathbf{k}}(t) e^{i\mathbf{k}.\mathbf{x}}, \tag{2.77}$$

where $\mathbf{x} = (x, y, z)$ and a similar expression holds for $\delta\phi$. The evolution of a mode amplitude $\Psi_{\mathbf{k}}$ or $\delta\phi_{\mathbf{k}}$ depends only on the comoving wavenumber $k = |\mathbf{k}|$ whereas the corresponding actual physical wavenumber is $\frac{k}{a(t)}$ as $\lambda \propto k^{-1} \propto a$.

Using (2.77) it is easy to show that both the perturbations satisfy the Poisson equation:

$$\nabla^2 \Psi_k = -k^2 \Psi_k \ , \qquad \nabla^2 \delta\phi_k = -k^2 \delta\phi_k. \tag{2.78}$$

Using these Poisson's equations for perturbations we can simply work in terms of $\Psi_{\mathbf{k}}$ and $\delta\phi_{\mathbf{k}}$. We now add equations (2.75) and (2.76) to arrive at the equation of motion of gravitational potential Ψ as

$$\ddot{\Psi}_k + \left(H - 2\frac{\ddot{\phi}}{\dot{\phi}} \right) \dot{\Psi}_k + 2\left(\dot{H} - H\frac{\ddot{\phi}}{\dot{\phi}} \right) \Psi_k + \frac{k^2}{a}\Psi_k = 0, \tag{2.79}$$

where we have used the background equation for scalar field $V' \simeq -3H\dot{\phi}$ and the relation $\dot{H} \simeq -4\pi G\ \dot{\phi}^2$. Using the slow-roll parameter relation $\delta = \eta - \epsilon = \frac{-\ddot{\phi}}{H\dot{\phi}}$, the above equation (2.79) can also be given as

$$\ddot{\Psi}_k + H\left(1 - 2\epsilon + 2\eta\right)\dot{\Psi}_k + 2H^2\left(\eta - 2\epsilon\right)\Psi_k + \frac{k^2}{a}\Psi_k = 0, \tag{2.80}$$

Since the slow-roll parameters satisfy $\epsilon \ll 1$ and $\eta \ll 1$, it is easy to infer from the above equation (2.80) that on superhorizon scales $k \ll (aH)$,

$$\dot{\Psi}_k \simeq 2(2\epsilon - \eta)H\Psi_k \quad \Longrightarrow \quad \dot{\Psi}_k \ll H\Psi_k \tag{2.81}$$

which implies that on superhorizon scales the time variations of the perturbations Ψ_k can be safely neglected compared to $H\Psi_k$. This relation holds true for field perturbations as well, *i.e.* $\delta\dot{\phi}_k \ll H\delta\phi_k$. Therefore on superhorizon scales, from equation (2.75), we can relate the gravitational potential and field perturbations as

$$\Psi_k \simeq \epsilon H\frac{\delta\phi}{\dot{\phi}}, \tag{2.82}$$

This can be used to compute the comoving curvature perturbation $\mathcal{R}_k$ on super-horizon scale (2.48) as

$$\mathcal{R}_k \simeq \Psi_k + \frac{H}{\dot{\phi}}\delta\phi_k \simeq (1+\epsilon)\frac{H}{\dot{\phi}}\delta\phi_k \approx \frac{H}{\dot{\phi}}\delta\phi_k. \tag{2.83}$$

Before we go any further, we define a useful quantity known as ***Power Spectrum*** which characterizes the properties of perturbations. Any generic quantity $f(t, \mathbf{x})$ in the Fourier space can be expanded as

$$f(t, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} f_{\mathbf{k}}(t) e^{i\mathbf{k}.\mathbf{x}}, \tag{2.84}$$

and the power spectrum $\mathcal{P}_f(k)$ of the quantity $f_{\mathbf{k}}(t)$ is defined through

$$\langle | f^*_{\mathbf{k}} f_{\mathbf{k}'} | \rangle \equiv \delta^{(3)}(\mathbf{k} - \mathbf{k}') \frac{2\pi^2}{k^3} \mathcal{P}_f(k), \tag{2.85}$$

where $\langle | f^*_{\mathbf{k}} f_{\mathbf{k}'} | \rangle$ implies the vacuum expectation value of the quantity $f_k(t)$ in the vacuum quantum state $|0\rangle$ of the system and $\delta^{(3)}(\mathbf{k} - \mathbf{k}')$ is the three dimensional Kronecker delta function. The definition (2.85) lead to the power spectrum:

$$\mathcal{P}_f(k) = \frac{k^3}{2\pi^2} \langle |f_k|^2 \rangle. \tag{2.86}$$

Therefore, using (2.86), we may write the *power spectrum of comoving curvature perturbation* $\mathcal{R}$ as

$$\boxed{\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \langle |\mathcal{R}_k|^2 \rangle}. \tag{2.87}$$

Hence, using (2.83), the power spectrum of comoving curvature perturbation on superhorizon scale $k \ll (aH)$ becomes

$$\begin{aligned} \mathcal{P}_{\mathcal{R}}(k) &\simeq \frac{k^3}{2\pi^2} \frac{H^2}{\dot{\phi}^2} \langle |\delta\phi_k|^2 \rangle \\ &\simeq \frac{k^3}{4\pi^2 \epsilon M_p^2} \langle |\delta\phi_k|^2 \rangle. \end{aligned} \tag{2.88}$$

Now we are left to calculate the time evolution of the field perturbation mode amplitudes $\delta\phi_k$. Consider perturbing the KG equation of motion (2.7) for scalar field ϕ, *i.e.* taking the variation of KG equation, we get

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \frac{k^2}{a^2}\delta\phi_k + V''\delta\phi_k = -2V'\Psi_k + 4\dot{\phi}\dot{\Psi}_k \tag{2.89}$$

where we have used the background equation (2.9). Since on superhorizon scales

$|2V'\Psi_k| \gg |4\dot{\phi}\dot{\Psi}_k|$ (which follows from the condition $\dot{\Psi}_k \ll H\Psi_k$ upon using the relation $V' \simeq -3H\dot{\phi}$), using equation (2.82) and (2.9), the perturbed KG equation on superhorizon scale can be written as

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + (V'' + 6\epsilon H^2)\delta\phi_k = 0. \tag{2.90}$$

We now replace the variable $\delta\phi_k$ with $\frac{\delta\sigma_k}{a}$ in the above equation (2.90) and write the above equation in conformal time $d\tau = \frac{dt}{a}$ as defined in (2.125). The above equation (2.90) becomes

$$\delta\sigma_k'' - \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\delta\sigma_k = 0, \tag{2.91}$$

where prime denotes the derivatives w.r.t. conformal time τ and

$$\nu^2 = \left(\frac{9}{4} - \frac{m_\phi^2}{H^2}\right) \simeq \frac{9}{4} + 9\epsilon - 3\eta. \tag{2.92}$$

In deriving the above relation (2.91), we have used relations $\eta = \frac{V''}{V} \simeq \frac{m_\phi^2}{3H^2}$ and $\frac{a''}{a} \simeq \frac{1}{\tau^2}(2+3\epsilon) = \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)$ which can be obtained using the definition of the conformal time during quasi de-Sitter expansion. Under quasi de-Sitter expansion during which the Hubble rate is not exactly constant and follow the relation $\dot{H} = -\epsilon H^2$, the definition of conformal time establishes the relation for the scale factor as $a(\tau) = -\frac{1}{H\tau}\frac{1}{1-\epsilon}$.

We see that the perturbed KG equation (2.91) is Bessel equation and its solution can be given in terms of Hankel functions

$$\delta\sigma_k = \sqrt{-\tau}[c_1(k)H_\nu^{(1)}(-k\tau) + c_2(k)H_\nu^{(2)}(-k\tau)], \tag{2.93}$$

where $H_\nu^{(1)}$ and $H_\nu^{(2)}$ are the Hankel's functions of the first and second kind, respectively. We assume that in the ultraviolet regime, *i.e.* on subhorizon scales, $k \gg aH$ ($-k\tau \gg 1$) the solutions matches the plane wave solutions $e^{-ik\tau}/\sqrt{2k}$. The assumption that in the ultraviolet regime when the mode wavelengths are of sub horizon size the modes should behave like plane waves as we expect in the flat

Minkowski spacetime is called the *Bunch-Davies boundary condition*. Knowing that in the limit $-k\tau \gg 1$ Hankel's functions are given by

$$H_\nu^{(1)}(-k\tau \gg 1) \sim \sqrt{\frac{2}{-k\tau\pi}} e^{i\left(-k\tau - \frac{\pi}{2}\nu - \frac{\pi}{4}\right)}, \tag{2.94}$$

$$H_\nu^{(2)}(-k\tau \gg 1) \sim \sqrt{\frac{2}{-k\tau\pi}} e^{i\left(-k\tau - \frac{\pi}{2}\nu - \frac{\pi}{4}\right)}, \tag{2.95}$$

if we set $c_1(k) = \frac{\sqrt{\pi}}{2} e^{i\left(\nu + \frac{1}{2}\right)\frac{\pi}{2}}$ and $c_2(k) = 0$, from equation (2.93) we get the exact solution for $\delta\sigma_k$ as

$$\delta\sigma_k = \frac{\sqrt{\pi}}{2} e^{i\left(\nu + \frac{1}{2}\right)\frac{\pi}{2}} \sqrt{-\tau} H_\nu^{(1)}(-k\tau). \tag{2.96}$$

As we are interested in the modes which have become superhorizon $k \ll aH$ ($-k\tau \ll 1$) during inflation, knowing that in the limit $-k\tau \ll 1$ Hankel's function have solution

$$H_\nu^{(1)}(-k\tau \ll 1) \sim \sqrt{\frac{2}{\pi}} \frac{\Gamma(\nu)}{\Gamma(3/2)} 2^{\nu - \frac{3}{2}} e^{-i\frac{\pi}{2}} (-k\tau)^{-\nu}, \tag{2.97}$$

the solution (2.96) on superhorizon scales becomes

$$\delta\sigma_k \simeq \frac{\Gamma(\nu)}{\Gamma(3/2)} 2^{\nu - \frac{3}{2}} e^{i\left(\nu + \frac{1}{2}\right)\frac{\pi}{2}} \frac{1}{\sqrt{2k}} (-k\tau)^{\frac{1}{2} - \nu}. \tag{2.98}$$

Since $\epsilon \ll 1$ and $\eta \ll 1$, we can set $\nu \sim \frac{3}{2}$ in the factors but will not do the same in the exponent because exponent term $(-k\tau)^{\frac{1}{2} - \nu}$ gives the small scale dependence of the power spectrum of perturbations. Going back to original variable $\delta\phi_k$, we find the the fluctuations on superhorizon scales in cosmic time

$$\boxed{|\delta\phi_k(t)| \simeq \frac{H}{\sqrt{2k^3}} \left(\frac{k}{aH}\right)^{\frac{3}{2} - \nu}}. \tag{2.99}$$

Therefore the power spectrum of fluctuations, from (2.88), becomes

$$\boxed{\mathcal{P}_\mathcal{R}(k) \simeq \frac{1}{8\pi^2\epsilon} \frac{H^2}{M_p^2} \left(\frac{k}{aH}\right)^{n_s - 1} \equiv \Delta_\mathcal{R}^2 \left(\frac{k}{aH}\right)^{n_s - 1}}, \tag{2.100}$$

where we have defined the spectral index n_s of the comoving curvature perturbations, which determines the tilt of the power spectrum or the small deviation of the power spectrum from scale invariance, as

$$\boxed{n_s - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k} = 3 - 2\nu = 2\eta - 6\epsilon}. \tag{2.101}$$

Since the slow-roll parameters ϵ and η are much smaller than unity, therefore $n_s - 1 \simeq 0$ which implies inflation is responsible for producing the curvature perturbations with an almost scale invariant spectrum.

For comparison with the observations, the power spectrum (2.100) can be given as

$$\mathcal{P}_{\mathcal{R}}(k) = \Delta^2_{\mathcal{R}}(k_0) \left(\frac{k}{k_0}\right)^{n_s - 1}, \tag{2.102}$$

where $k_0 = a_0 H_0$ is the pivot scale. The pivot scale corresponds to a wavelength $\lambda_0 \propto k_0^{-1}$ at which the instrument measuring the CMB radiation has the maximum sensitivity. $\Delta^2_{\mathcal{R}}(k_0)$ is the amplitude of the power spectrum at the pivot scale k_0.

It is possible that spectral index may depend on scales k. The running (variation) of the spectral index α_s with modes k is defind as

$$\alpha_s \equiv \frac{dn_s}{d \ln k}, \tag{2.103}$$

the running of the spectral index with scale arises only at the second order in slow-roll parameters and is therefore expected to be very small $\alpha_s = \mathcal{O}(\epsilon^2)$. Using the fact that $\frac{dN}{dt} = H$, at horizon crossing $k = a(t_k)H(t_k)$ we find $\frac{d \ln k}{dt} = H\left(1 + \frac{\dot{H}}{H^2}\right) \approx H$. The α_s can be given as

$$\alpha_s = \frac{dn_s}{d\phi}\frac{d\phi}{dt}\frac{dt}{d \ln k} = \frac{\dot{\phi}}{H}\frac{dn_s}{d\phi} = -\frac{V'}{V}\frac{dn_s}{d\phi}, \tag{2.104}$$

where in the last equality we have used the background slow-roll equations (2.9) and (2.10). Now using spectral index relation (2.101) and the definition of slow-roll parameters in terms of scalar potential, the α_s in terms of slow-roll parameters can be given as

$$\boxed{\alpha_s = 16\epsilon\eta - 24\epsilon^2 - 2\xi}, \tag{2.105}$$

where the slow-roll parameter ξ is defined as $\xi = \frac{V'V'''}{V^2}$.

2.2.5 Tensor Power Spectrum

Along with density fluctuations (or scalar perturbations), inflation also predicts the existence of gravitational waves which are identified with the tensor perturbations in the metric. According to SVT decomposition theorem all perturbations (scalar,vector and tensor) evolve independently. The line element for tensor perturbations around the flat background is given by

$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij})dx^i dx^j, \tag{2.106}$$

where $h_{ij} \ll 1$. The tensor perturbations h_{ij} has 6 d.o.f., but as we studied in §2.2.1 the tensor perturbations are traceless $\delta^{ij}h_{ij} = 0$ and divergence free $\partial^i h_{ij} = 0$, these 4 conditions reduces the tensor d.o.f. to 2 physical d.o.f. which corresponds to 2 polarisations of the gravitational waves, indicated by $s = +, \times$.

For a diagonal stress-energy tensor, as provided by inflaton $T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\mathcal{L}$, the tensor modes do not have any source term in their equation of motion. This statement can be verified very easily by calculating the perturbed Einstein field equations for the tensor perturbations metric (2.106) where we find $\delta R = 0$ and $\delta R_{\mu\nu} = 0$ for all components except $i \neq j$ and $\delta T^i_j = 0$ for $i \neq j$. Therefore we have a glimmer of decomposition theorem and we can state that the e.o.m. for tensor metric perturbations have no scalar (inflaton) source in it and they evolve independent of scalar perturbations. Using the above mentioned perturbed components of Ricci scalar and Ricci curvature into the perturbed field equations (2.64) the e.o.m. (2.111) for tensor metric perturbations h_{ij} can be obtained.

In a more simpler manner the e.o.m. for h_{ij} can be obtained from second order expansion of the Einstein Hilbert action [59, 60]

$$S^{(2)} = \frac{M_p^2}{2}\int dx^4\sqrt{-g}\frac{1}{2}\partial_\rho h_{ij}\partial^\rho h_{ij}, \tag{2.107}$$

$$= \frac{M_p^2}{4}\int d\eta dx^3\frac{a^2}{2}[(h'_{ij})^2 - (\partial_l h_{ij})^2]. \tag{2.108}$$

This is the same actions as for the free massless scalar field in FRW universe. We define the following Fourier expansion

$$h_{ij} = \int \frac{d^3k}{(2\pi)^3}\sum_{s=+,\times} h_{\mathbf{k}}^s(\tau)e_{ij}^s(k)e^{i\mathbf{k}.\mathbf{x}}, \tag{2.109}$$

where e_{ij}^s are the polarozation tensors which satisfy the following properties

$$\begin{aligned} e_{ij}(k) &= e_{ji}(k), \qquad e_{ii}(k) = 0, \\ k^i e_{ij}(k) &= 0, \qquad e_{ij}^s(k)e_{ij}^{s'}(k) = 2\delta_{ss'}. \end{aligned} \tag{2.110}$$

Using (2.109) and (2.110), the action (2.108) leads to the e.o.m. for the quantity $h_{\mathbf{k}}$

$$h_{\mathbf{k}}^{s\prime\prime} + 2\frac{a'}{a}h_{\mathbf{k}}^{s\prime} + k^2 h_{\mathbf{k}}^s = 0. \tag{2.111}$$

Defining the canonically normalized field

$$\nu_{\mathbf{k}}^s \equiv \frac{1}{2}ah_{\mathbf{k}}^s M_p, \tag{2.112}$$

the e.o.m. (2.111) becomes

$$\nu_{\mathbf{k}}^{s\prime\prime} + \left(k^2 - \frac{a''}{a}\right)\nu_{\mathbf{k}}^s = 0, \tag{2.113}$$

where $\frac{a''}{a} = \frac{2}{\eta^2}(2+3\epsilon)$ during quasi de-Sitter epoch when $\dot{H} = -\epsilon H$. On super horizon scale $k \ll aH$, k^2 term in the equation (2.113) can be neglected and then it exactly matches with the Mukhanov-Sasaki equation (2.91) for the massless scalar field in FRW universe during quasi de-Sitter epoch whose solution on superhorizon scales can be given in analogy with the solution (2.99) for $\delta\sigma_k$ as

$$|\nu^s_{\mathbf{k}}| = \frac{1}{M_p}\frac{aH}{\sqrt{2k^3}}\left(\frac{k}{aH}\right)^{\frac{3}{2}-\nu_T}, \tag{2.114}$$

where, we have obtained $\nu_T \simeq \frac{3}{2}+\epsilon$, using the relation $\frac{a''}{a} \simeq \frac{1}{\eta^2}(2+3\epsilon) = \frac{1}{\eta^2}\left(\nu_T^2 - \frac{1}{4}\right)$. Also, since the equation (2.113) or the action (2.108) matches with the equations for massless scalar field, therefore there will be no appearance of slow-roll parameter η in ν_T through m_ϕ in contrast to relation (2.92).

To characterize the tensor perturbations, we define the power spectrum of tensor perturbations as

$$\begin{aligned}\mathcal{P}_T &\equiv \frac{k^3}{2\pi^2}\sum_{s=+,\times}|h^s_{\mathbf{k}}|^2 \\ &= 2\times\frac{k^3}{2\pi^3}\frac{4|\nu^s_{\mathbf{k}}|^2}{a^2},\end{aligned} \tag{2.115}$$

where the factor of 2 is due to the sum over the two polarization states of the gravitational wave and we have used the relation (2.112) for $\nu^s_{\mathbf{k}}$.

Substituting for the solution (2.114), we get the amplitude of the tensor power spectrum on superhorizon scales as

$$\boxed{\mathcal{P}_T = \frac{2}{\pi^2}\frac{H^2}{M_p^2}\left(\frac{k}{aH}\right)^{n_T} \equiv \Delta_T^2\left(\frac{k}{aH}\right)^{n_T}}. \tag{2.116}$$

Similar to scalar spectral index n_s, we can define the *tensor spectral index* n_T as

$$\boxed{n_T \equiv \frac{d\ln\mathcal{P}_T}{d\ln k} = 3 - 2\nu_T = -2\epsilon}. \tag{2.117}$$

Tensor-to-Scalar Ratio and Energy Scale of Inflation :

Amplitude of the tensor perturbations are often normalized relative to the measured amplitude of the scalar perturbations $\Delta^2_{\mathcal{R}} \simeq 1.95\times 10^{-9}$. The *tensor-to-*

scalar ratio r is defined as the ratio of the two amplitudes

$$\boxed{r \equiv \frac{\Delta_T^2}{\Delta_{\mathcal{R}}^2} = 16\epsilon}, \tag{2.118}$$

which determines the relative contribution of the tensor modes to mean squared low multipole CMB anisotropy. In the last equality in above equation (2.118), we have used the amplitude relations (2.100) and (2.116) for scalar and tensor perturbations. Since scalar amplitude is fixed from the observations $\Delta_{\mathcal{R}}^2 \simeq 1.95 \times 10^{-9}$ and, from (2.116), amplitude of the tensor perturbations $\Delta_T^2 \propto H^2 \approx V(\phi)$, therefore the value of tensor-to-scalar ratio is a direct measure of *energy scale of inflation*:

$$\boxed{V(\phi)^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\ GeV}. \tag{2.119}$$

The value of tensor-to-scalar ratio $r > 0.01$ implies inflation occurring at the GUT energy scale $10^{16}\ GeV$.

The Lyth Bound and Large-Field Inflation :

Inflation models which can predict large amplitude of the gravity waves (or large r) are extremely sensitive to super Planckian physics. Here we will derive the Lyth bound which relates the tensor-to-scalar ratio with super Planckian displacement of the inflaton value $\Delta\phi$ during inflation.

During slow-roll inflation, using (2.11) and (2.12), the slow-roll parameter ϵ can be given as

$$\epsilon \simeq \frac{1}{2M_p^2}\frac{\dot{\phi}^2}{H^2} = \frac{1}{2M_p^2}\left(\frac{d\phi}{dN}\right)^2, \tag{2.120}$$

where we have used the relation $N = Hdt = \frac{H}{\dot{\phi}}d\phi$. Therefore the tensor-to-scalar ratio can be directly related to the evolution of the inflaton as a function of e-foldings N

$$r = 16\epsilon \simeq \frac{8}{M_p^2}\left(\frac{d\phi}{dN}\right)^2, \tag{2.121}$$

which implies that the total change in the field during inflation between the

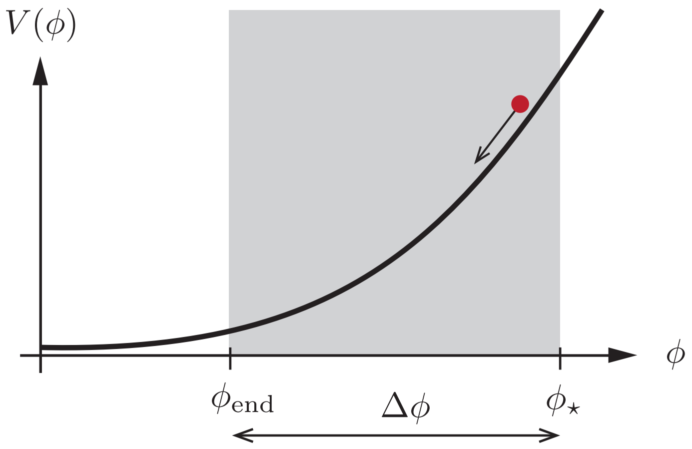

Figure 2.2: The shaded region represents the evolution of the inflaton field from ϕ_* (or ϕ_s), when the observable CMB modes leave the horizon, to ϕ_e, when inflation ends. The total change in the field Δ_ϕ is related to tensor-to-scalar ration r via relation 2.123. Figure credit: [58].

times when observable CMB modes leaves the horizon at $N_{cmb} = N_s$ and the end of inflation at N_e can be given by the following integral

$$\frac{\Delta\phi}{M_p} = \int_{N_s}^{N_e} \sqrt{\frac{r}{8}}\, dN. \tag{2.122}$$

Since during slow-roll inflation r doesn't evolve much with change in N, therefore the above integral, for $\Delta N = N_s - N_e \approx 60$, gives

$$\boxed{\frac{\Delta\phi}{M_p} \simeq \mathcal{O}(1)\left(\frac{r}{0.01}\right)^{1/2}}, \tag{2.123}$$

so the large value of tensor-to-scalar ratio, $r > 0.01$, implies large field inflation $\Delta\phi > M_p$. Or $\Delta\phi > M_p \Rightarrow (\phi_s - \phi_e) > M_p \Rightarrow \phi_s > M_p$, since $\phi_e > 0$, implies inflaton field values are super Planckian during the time observable CMB modes leave the horizon. The evolution of the inflaton field from the time when observable CMB modes leave the horizon to the end of inflation is shown in Fig. 2.2.

We will use the formalism and expressions for power spectrum, spectral index and its running, and tensor-to-scalar ratio derived in this Section §2.2, extensively in the subsequent chapters of this thesis.

2.3 Modified Gravity Framework for Inflation

As we stated in §1.3 that it is possible to transform $f(R)$ and $f(\phi)R$ gravity actions into an Einstein gravity action via conformal transformation of the metric $g_{\mu\nu}$ and redefinition of the field ϕ. To better understand the mechanism of conformal transformations, we take $f(\phi)R$ gravity action as an example and show that it can be recast into Einstein frame action.

Consider a single-field non-minimal scenario given by the action

$$S_J = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]. \quad (2.124)$$

Consider the conformal transformation of the metric

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad \Longrightarrow \quad \tilde{g}^{\mu\nu} = \Omega^{-2}g^{\mu\nu}, \quad (2.125)$$

here and now onwards tilde represents quantities in the Einstein frame. Under conformal transformation (2.125) various metric dependent quantities such as line element ds^2, determinant of the metric g and Ricci scalar R transforms as follows

$$\begin{aligned} ds^2 &= \Omega^{-2}d\tilde{s}^2, \\ \sqrt{-g} &= \Omega^{-4}\sqrt{-\tilde{g}}, \\ R &= \Omega^2\left[\tilde{R} + 6\frac{\tilde{\Box}\Omega}{\Omega} - 12\frac{\tilde{g}^{\mu\nu}\partial_\mu\Omega\partial_\nu\Omega}{\Omega^2}\right]. \end{aligned} \quad (2.126)$$

Using the above transformation relations and choosing $\Omega^2 = f(\phi)$, the action (2.124) can be given as

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[\frac{M_p^2}{2}\tilde{R} - \frac{3M_p^2}{4}\frac{\tilde{g}^{\mu\nu}\partial_\mu f\partial_\nu f}{f^2} - \frac{1}{2}\frac{\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}{f} - \frac{V(\phi)}{f^2}\right]. \quad (2.127)$$

We see that the conformal transformation of the metric leads to gravity sector into Einstein Hilbert form of action, but with a non-canonical kinetic term. To get a canonical kinetic term we redefine field ϕ to $\tilde{\phi}$ through

$$\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi} = \frac{3M_p^2}{4}\frac{\tilde{g}^{\mu\nu}\partial_\mu f\partial_\nu f}{f^2} + \frac{1}{2}\frac{\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}{f}, \tag{2.128}$$

therefore, with the redefined kinetic term (2.128), we get the EF action (2.127) as

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[\frac{M_p^2}{2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi} - \tilde{V}(\tilde{\phi})\right], \tag{2.129}$$

where

$$\tilde{V}(\tilde{\phi}) = \frac{V(\phi(\tilde{\phi}))}{f(\phi(\tilde{\phi}))^2} \tag{2.130}$$

is the EF potential. Using the fact $\partial_\mu f(\phi) = \frac{\partial f}{\partial\phi}\partial_\mu\phi$, the equation (2.128) can be solved to give

$$\frac{\partial\tilde{\phi}}{\partial\phi} = \sqrt{\frac{3M_p^2}{2f^2}\left(\frac{\partial f}{\partial\phi}\right)^2 + \frac{1}{f}}. \tag{2.131}$$

For a given form of $f(\phi)$ the above equation can be integrated to give the EF field $\tilde{\phi}$ in terms of JF field ϕ.

Similarly, in the context of *multi-scalar field inflation with non-minimal coupling* the action can be written as

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2 f(\phi^K)R - \frac{1}{2}G_{IJ}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^K)\right]. \tag{2.132}$$

where $I, J, K = 1, 2, 3, ..., N$ for a model with N scalar fields. Under conformal transformation (2.125), the above action transforms to an Einstein frame action

$$\tilde{S} = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}M_p^2\tilde{R} - \frac{1}{2}\tilde{G}_{IJ}(\phi^K)\tilde{g}^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - \tilde{V}\right], \tag{2.133}$$

where $\tilde{V} = V/f^2$ is the EF potential and

$$\tilde{G}_{IJ} = \frac{1}{f}G_{IJ} + \frac{3M_p^2}{2}\frac{f_{,I}f_{,J}}{f^2}. \tag{2.134}$$

where $f_{,I} = \partial f/\partial\phi^I$.

Such multifield models with action (2.133) where there is no coupling between scalar fields and curvature scalar R but kinetic terms in the fields are non-canonical arise naturally in Higher dimensional theories such as supergravity and string theories. We will study such a model in Chapter §5 with two fields where there is no cross term in the kinetic energy of the fields and illustrate how such a model can be derived from supergravity with an appropriate choice of kähler potential and superpotential. Multifield dynamics in the context of Higgs inflationary scenario have been studied [28, 68] and it is shown that for $N-$fields model which obey an $SO(N)$ gauge symmetry in field space, the multifield effects damp out very quickly at the onset of inflation. And the predictions of these multifield models for observable quantities revert to their familiar single-field forms.

2.4 Supergravity Framework for Inflation

In this section, we briefly summarize the SUGRA results which are relevant for our consideration which is to motivate certain inflation models from SUGRA. Since our primary aim is to derive the Lagrangian for modified gravity and multi field inflation models, for our purpose the most important part of the SUGRA Lagrangian is its scalar part which gives the kinetic as well as the potential term for the inflaton. The chiral multiplet for SUGRA algebra has the field content (ϕ^i, χ^i, F^i), where ϕ^i are the complex scalar fields, χ^i are the Weyl fermions and F^i are complex scalar auxiliary fields.

The scalar part of the SUGRA Lagrangian is determined by three functions, Kähler potential $K(\Phi_i, \Phi_i^*)$, superpotential $W(\Phi_i)$ and gauge kinetic function $f(\Phi_i)$. The superpotential W and gauge kinetic function f are the holomorphic functions of complex scalar fields ϕ_i , while the Kähler potential is not holomorphic and a real function of ϕ_i and their conjugates ϕ_i^* [1].

The interactions or the coupling of all the chiral superfields ϕ^i are determined

[1]A holomorphic function, say $h(z)$, is a complex valued function of one or more complex fields that is complex differentiable at each point z_0 in its domain. They satisfy the Cauchy-Riemann equations of complex analysis or equivalently $\frac{\partial h}{\partial z^*} = \frac{\partial h^*}{\partial z} = 0$ and $\frac{\partial h}{\partial z}|_{z\to z_0} = h'(z_0)$, $\frac{\partial h^*}{\partial z^*}|_{z^*\to z_0^*} = h'(z_0^*)$.

by a real function called *Kähler function*

$$G(\phi_i, \phi_i^*) \equiv K(\phi_i, \phi_i^*) + \ln W(\phi_i) + \ln W^*(\phi_i^*), \tag{2.135}$$

The Kähler functions has a property that they are invariant under the so called Kähler transformations

$$\begin{aligned} W(\phi_i) &\rightarrow e^{-U(\phi_i)} W(\phi_i), & (2.136)\\ K(\phi_i, \phi_i^*) &\rightarrow K(\phi_i, \phi_i^*) + U(\phi_i) + U^*(\phi_i^*), & (2.137) \end{aligned}$$

where $U(\phi_i)$ are arbitrary holomorphic function of the scalar fields ϕ_i. The invariance of $G(\phi_i, \phi_i^*)$ is manifest if $U(\phi_i) = \ln W$. The Kähler transformation sends $W \rightarrow e^{-U} W \equiv 1$.

The canonical SUGRA Lagrangian for the complex scalar fields in curved spacetime is given by

$$e^{-1}\mathcal{L} = -\frac{1}{2}R + \mathcal{L}_{kin} - V(\phi_i, \phi_i^*). \tag{2.138}$$

where $e = \sqrt{-g}$ is the determinant of the tetrad e_μ^a [2].

The first term in eq (2.138) is the familiar vacuum Einstein-Hilbert action and, second and third terms are the kinetic and potential terms, respectively. The Kinetic terms $\mathcal{L}_{kin}$ of the scalar fields are determined in terms of the Kähler

[2]The quantity e_μ^a are called *tetrad* or *vierbeins*, where a is the local Lorentz index and μ is the gauge (curved) index. In order to deal with the spinors in curved spacetime it is necessary to formulate the theory in terms of tetrads which are related to curved spacetime metric as

$$g_{\mu\nu}(x) = e_\mu^a(x) e_\nu^b(x) \eta_{ab},$$

where η_{ab} is the Minkowski spacetime metric and the tetrads e_μ^a are defined as the transformation from a local Lorentz inertial frame $\xi^a(x_0; x)$ at the point x_0 to a general non-inertial frame x^μ, *i.e.* $\xi^a \rightarrow x^\mu$, as

$$e_\mu^a(x_0) \equiv \frac{\partial \xi^a(x_0; x)}{\partial x^\mu}|_{x=x_0}.$$

potential and given by [3]

$$\boxed{\mathcal{L}_{kin} = -K_{ij^*} g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^{*j}}, \tag{2.139}$$

where K_{ij^*} is the Kähler metric given by

$$K_{ij^*} = \frac{\partial^2 K}{\partial \phi^i \partial \phi^{*j}}. \tag{2.140}$$

And the scalar potential $V(\phi_i, \phi_i^*)$ can be split into two different contributions

$$V(\phi_i, \phi_i^*) = V_F(\phi_i, \phi_i^*) + V_D(\phi_i, \phi_i^*), \tag{2.141}$$

referred to as the F-term and D-term potentials. The F-term potential is determined in terms of superpotential W and Kähler potential K as

$$V_F = e^K \left[D_{\phi_i} W K^{ij^*} D_{\phi_j^*} W^* - 3|W|^2 \right], \tag{2.142}$$

where $K^{ij^*} \equiv K_{ij^*}^{-1}$ is the inverse of the Kähler metric K_{ij^*} and

$$D_{\phi_i} W = \frac{\partial W}{\partial \phi_i} + \frac{\partial K}{\partial \phi_i} W. \tag{2.143}$$

The D-term potential is related to gauge symmetry and given in terms of Kähler potential K and gauge kinetic function f,

$$V_D = \frac{1}{2} \sum_a \left[Ref_a(\phi_i) \right]^{-1} g_a^2 D_a^2, \tag{2.144}$$

where the subscript a represents a gauge symmetry, g_a is a gauge coupling constant and T_a is an associated generator. ξ_a is a Fayet-Iliopoulos term which is non-zero only when the gauge symmetry is Abelian, *i.e.* a $U(1)$-symmetry.

It can be shown that the potentials (2.142),(2.144) and the kinetic term

[3] In general in the kinetic term (2.139) the partial derivative ∂_μ is actually a Lorentz covariant derivative $D_\mu \equiv \partial_\mu + \frac{1}{2}\omega_\mu^{ab}\Sigma_{ab}$, where ω_μ^{ab} are a set of gauge fields known as *spin connections* and Σ_{ab} are the generators of the Lorentz $SO(1,3)$ group which signifies the *spin* of the associated gauge fields. Since for scalars $\Sigma_{ab} = 0$, therefore covariant derivative D_μ in the scalar Lagrangian equals a partial derivative ∂_μ.

(2.139) are invariant under the Kähler transformations (2.137). The F-term scalar potential (2.142) in terms of a physically relevant quantity, the Kähler function G (2.135), can also be written as

$$\boxed{V_F = e^G \left[\frac{\partial G}{\partial \phi^i} K^i_{j*} \frac{\partial G}{\partial \phi^*_j} - 3 \right]}. \tag{2.145}$$

We will use this form of the F-term potential in this thesis. Also we will consider only the F-term potential which can produce the desired Lagrangian for the inflation models discussed in the Chapters §4 and §5.

We now briefly consider the $\eta-$*problem* as discussed in §1.4 and the ways to solve it [39, 40, 69]. Consider the canonical Kähler potential

$$K_{ij^*} = \delta^{ij} \phi_i \phi^*_j, \tag{2.146}$$

for which the kinetic term (2.139) of the scalar fields ϕ_i becomes canonical. The F-term potential V_F (2.142) can be written as

$$\begin{aligned} V_F &= e^{\delta^{ij}\phi_i\phi^*_j} \times \left\{ \left[\frac{\partial W}{\partial \phi_i} + \phi^*_i W \right] \left[\frac{\partial W^*}{\partial \phi^*_j} + \phi_j W^* \right] \delta_{ij} - 3|W|^2 \right\} \\ &= V_g + V_g \sum_i |\phi_i|^2 + other\ terms, \end{aligned} \tag{2.147}$$

where V_g is the global SUSY F-term potential given by

$$V_g = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2. \tag{2.148}$$

Since at the background level from Friedmann equation we have $V \simeq V_g \simeq 3H^2$, with little algebraic simplification of equation (2.147), it can be shown that

$$\eta = \frac{V''_F}{V_F} = 1 + \eta_0 + other\ terms, \tag{2.149}$$

where $\eta_0 = V''_g/V_g$ and we find that one of the slow-roll approximation, *i.e.* $\eta << 1$, is violated which is required for successful inflation. $\eta \sim 1$ implies that any scalar field including the one which acts as the inflaton receives the effective mass $V_F'' \simeq 3\eta H^2 \sim H^2$ of the order of Hubble parameter. This is the main problem that makes it difficult to incorporate inflation in SUGRA.

Though there are several ways to get around this problem, in the most widely used method, one uses the Kähler potential other than the canonical one (2.146). If one choose a Kähler potential which is not canonical, the kinetic term (2.139) of scalar field also becomes non-canonical, which however can be made canonical by redefining the scalar field. The canonical normalization of kinetic terms changes the scalar potential to an effective form which could be nearly flat even if it was originally very steep. On the other hand it is also possible to impose some symmetry on Kähler and/or superpotential which can ensure the slow-roll potential necessary for successful inflation [69–71]. The η problem is specific to F-term potential. Successful inflation can also be achieved using the D-term potential, if it can produce positive energy [69].

In the F-term inflation models, the difficulty is not limited to η problem. In the case of canonical Kähler potential the exponential factor is $e^K = e^{\delta^{ij}\Phi_i\Phi_j^*}$, therefore in the large field limit $\phi_i > 1$ (in $M_p = 1$ unit) the potential becomes too steep to give a nearly flat potential suitable for inflation. Thus, it is very difficult to incorporate chaotic inflation models in supergravity, which require inflaton field values larger than unity during inflation. There have been proposed several models of chaotic inflation in SUGRA where the inflaton field can have values larger than unity while producing the nearly flat inflaton potential. In these models either Kähler potentials are fine-tuned without any symmetry reasons [72, 73] or there are models in which the Kähler potential $K(\phi, \phi^*)$ follow Nambu-Goldstone type of shift symmetry $\phi \to \phi + ic$, where c is some real parameter [74, 75].

2.4.1 No-scale SUGRA Models

The main idea of no-scale models is that they are constructed in such a way that the F-term potential V_F vanishes for all values of the scalar fields. Therefore, from (2.142), the condition for a model to be no-scale can be given in terms of the Kähler function as [38]

$$\frac{\partial G}{\partial \phi^i} K^i_{j*} \frac{\partial G}{\partial \phi^*_j} = 3. \tag{2.150}$$

For constant superpotential models, $\partial_{\phi_i} W = 0$, the no-scale condition (2.150) can be given in terms of Kähler potential as

$$\frac{\partial K}{\partial \phi^i} K^i_{j*} \frac{\partial K}{\partial \phi^*_j} = 3. \tag{2.151}$$

Examples of no-scale Kähler potentials with single complex scalar field ϕ and double complex scalar fields $\phi_i = (\phi_1, \phi_2) = (T, \phi)$ which satisfy the no-scale condition (2.151) are

$$K = -3 \ln(T + T^*), \tag{2.152}$$

and

$$K = -3 \ln \left[T + T^* - \frac{\phi \phi^*}{3} \right], \tag{2.153}$$

respectively. It is shown that in the case of no-scale two-field Kähler potential (2.153), if we give vev to T field such that $2\langle ReT \rangle = C$ and $\langle ImT \rangle = 0$, with the following Wess-Zumino choice of superpotential

$$W = \frac{\hat{\mu}}{2} \Phi^2 - \frac{\lambda}{3} \Phi^3, \tag{2.154}$$

where the scalar component of the chiral superfield Φ is ϕ, the Starobinsky inflationary potential for field χ (where field χ arises from redefinition of ϕ due to canonical normalization of the kinetic term for ϕ) can be obtained

$$V_F = \frac{\mu^2}{4} \left(1 - e^{-\sqrt{\frac{2}{3}}\chi} \right)^2, \tag{2.155}$$

with the choice $\lambda = \mu/3$, where $\mu = \hat{\mu}/\sqrt{C/3}$, the vev of the T field is absorbed in the mass scale μ [46]. Since here in deriving Starobinsky model from two-field Kähler potential, the superpotential (2.154) is not constant, such a model defines an *almost no-scale* model.

Let us now briefly discuss the importance of no-scale supergravity. The smallness of some physical quantities *e.g. cosmological constant* Λ can either be explained by some symmetry argument or by fine tuning. One naturally prefers symmetry reasons for the smallness of the physical quantities. From the form of the F-term scalar potential (2.142), it is clear that for the potential to be vanishing either one can choose some parameter inside G, through Kähler potential and/or superpotential, to be fine-tuned or alternatively impose the no-scale condition (2.150). If one imposes the no-scale condition on the model then the fine tuning is no-longer required for obtaining the vanishing vacuum energy. Also the potential becomes flat as required for successful inflation, thereby solving the η problem naturally [37]. To explain vanishingly small vacuum energy and nearly flat inflaton potential we require $V_F > 0$, which from equation (2.145) implies the condition

$$e^G \frac{\partial G}{\partial \phi^i} \neq 0, \tag{2.156}$$

thus breaking the supergravity and generating the mass of gravitino $m_{3/2} = e^{G/2} \neq 0$ which represents the scale of supergravity breaking. Although the mass of gravitino $m_{3/2}$ is non-vanishing but undetermined at the tree level despite the fact that supergravity is broken and the classical potential V_F is vanishing for all values of the scalar fields and therefore it is said to have flat directions [70]. In no-scale SUGRA all the mass scales below the Planck scale are determined with quantum corrections [37, 42–44, 76–78].

The importance of no-scale models lie in the fact, that fine tuning is not needed to achieve positive vanishing cosmological constant and the gravitino mass $m_{3/2}$ can be determined dynamically. Also there are flat directions with gentle slope without fine tuning which are essential for successful inflation.

Chapter 3

Generalized Higgs Inflation Model

3.1 Overview

In this chapter we study a general Higgs inflationary scenario $\xi\phi^a R^b$ in the framework of $f(\phi, R)$ gravity theory. This model is a generalization of the Higgs inflation model $\xi\phi^2 R$. In the light of discoveries by CMS [79] and ATLAS [80] it is of interest to consider the Standard Model Higgs boson as the candidate for inflaton. However in the standard single-field slow-roll inflation with inflaton quartic potential, the idea of considering standard model Higgs as inflaton does not work as the inflaton quartic coupling should be of the order $\lambda \sim 10^{-12}$ to explain the amplitude of CMB perturbations measured by WMAP/Planck [6, 7] while the 125 GeV Higgs has a quartic coupling $\lambda \sim 0.13$ at the electroweak scale which can however go down to smaller values at the Planck scale due to renormalization [81–86]. However just from the standard model renormalization one cannot have the Higgs coupling $\lambda \sim 10^{-12}$ over the entire range of the rolling field $\sim (10-1)M_P$ during inflation and the standard slow-roll inflation with a Higgs field does not give the observed amplitude and spectrum of density perturbations [87]. If the Higgs and top mass are fine tuned then there can be a small kink in the Higgs potential and the universe trapped in this false vacuum can undergo a period of inflation [88–90].

Later a way out of fine tuning the scalar self coupling to unnaturally small values was found out [91–94] and it was shown that if one couples the scalar field to the Ricci scalar $\xi\phi^2 R$ then the effective potential in the Einstein frame becomes a slow roll one with the effective scalar coupling being λ/ξ^2 and the amplitude of the density perturbations constrain this ratio rather than λ, hence ξ can be increased as large as required to get the desired self-coupling λ. Density perturbations from inflation in the curvature coupled theories were calculated in [95, 96]. The equivalence of the density perturbation in Jordan and Einstein frame was shown by Komatsu and Futamase [97] who also calculated the tensor perturbations and showed that the tensor-to-scalar ratio is generically small in $\xi\phi^2 R$ model.

Bezrukov and Shaposhnikov [98] revived the large curvature coupling model to motivate the idea that the standard model Higgs field could serve as the inflaton in the early universe. The amplitude and spectral index of density perturbations observed by WMAP/Planck can be generated by the Higgs field with self coupling $\lambda \sim 0.1$ and curvature coupling $\xi \sim 10^4$ [20, 98–102]. This large value of ξ needed however is seen as a problem as at the time of inflation the Higgs field is at the Planck scale and hence graviton-scalar scatterings due to the curvature coupling of the scalar would become non-unitary [103]. Ways of solving the unitarity violation problem in the Higgs inflation models have been explored in [104–108].

In this paper we assume that the dominant interaction between Higgs field and gravity is through operators of the form

$$\mathcal{L} = \frac{\xi(\mathcal{H}^\dagger \mathcal{H})^{a/2} R^b}{M_p^{a+2b-4}}. \tag{3.1}$$

This form (3.1) of Higgs Curvature interaction has been mentioned in the Ref. [109]. The complete dynamics of the Higgs field involves the role of the Goldstone modes as has been studied in detail in [28, 65, 110]. The multifield dynamics of the Goldstone modes gives rise to sizable non-gaussianity. We will study the dynamics of the Higgs mode and impose a charge conservation and CP symmetry such that the Goldstone modes of the Higgs field do not acquire vevs. We will take the background Higgs field to be

$$\mathcal{H} = \begin{pmatrix} 0 \\ \phi \end{pmatrix} \tag{3.2}$$

where ϕ is the Higgs mode with mass 126 GeV. Our inflation model falls in the class of inflation in $f(\phi, R)$ theories studied in Ref. [19]. Our motivation is that we use the Higgs quartic coupling $\lambda(\mathcal{H}^\dagger\mathcal{H})^2$ where the standard model value of $\lambda(\mu \sim M_P)$ can lie in the range $\lambda = (10^{-5} - 0.1)$ depending on the value of top quark mass [85, 86] or on new physics [111]. We take curvature coupling ξ to be unity and check the possibility of generating the observed density perturbations from Higgs inflation by varying parameters a, b and λ. The non minimal coupling ξ has been taken unity in order to improve the unitarity behavior which increases the natural cutoff scale Λ from $\Lambda \simeq \frac{M_p}{\xi} \simeq 10^{15}$ to $\Lambda \simeq M_p \simeq 10^{19}$.

We derive the curvature perturbation during inflation in two different ways. We derive the perturbations of modified Einsteins field equation in the Jordan frame in presence of the Higgs-curvature interaction terms and derive the amplitude and spectral index of curvature perturbation. We find that to generate the Planck+WP preferred amplitude $\Delta^2_{\mathcal{R}} = 2.1955^{+0.533}_{-0.585} \times 10^{-9}$ and spectral index $n_s = .9603 \pm .0073$ [112] for $\lambda = 10^{-3}$ we should have $a \sim 3.02, b \sim 0.49$ (and for $\lambda = 0.1$ we need $a \sim 3.56, b \sim .22$). In these fits we take $\xi = 1$.

As we discussed in Section §1.3 that in non-minimal $\xi\phi^2 R$ theory we can always make a conformal transformation to the Einstein frame so one can compute the density perturbations either in Einstein frame or Jordan frame and the gauge invariant curvature perturbations should be same in both the frames [95]. In our case with the $\xi\phi^a R^b$ coupling we find that no conformal transformation exists which can in general remove this term (i.e go to an Einstein frame). We find that in the general $\xi\phi^a R^b$ theory such a conformal transformation is only possible if the metric is quasi-de Sitter. The accurate comparison with the experimental data should be made however with the Jordan frame results. Calculation of the curvature perturbation in both Einstein and Jordan frame for the $\xi\phi^2 R$ theory has been done previously in [26, 95, 113, 114].

This chapter is organized as follows: In Section §3.2 we derive the curvature perturbations and tensor perturbations in our theory $\xi\phi^a R^b$ in the Jordan frame and in Section §3.3 we make a conformal transformation to go to the Einstein frame and compute the curvature perturbations. In Section §3.4 we compare the results of the two frames. And finally in the last Section §3.5 we discuss the findings and viability of our considered Higgs inflation model.

3.2 Model in the Jordan Frame

In this section we introduce a scalar-gravity interaction term $\xi\phi^a R^b$ in the action and calculate physical quantities related to the inflationary density perturbations such as the amplitude of curvature perturbation, spectral index and tensor-to-scalar ratio. We start with the action for a scalar field interacting with gravity of the form

$$S_J = \int d^4x\sqrt{-g}\left[-\frac{f(\phi,R)}{2\kappa^2} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right] , \tag{3.3}$$

where we take,

$$\frac{1}{\kappa^2}f(\phi,R) = \frac{1}{\kappa^2}R + \frac{\xi\phi^a R^b}{M_p^{a+2b-4}} \quad ; \quad V(\phi) = \frac{\lambda\phi^4}{4} , \tag{3.4}$$

where $\kappa^2 = 1/M_p^2$ and ξ is a dimensionless coupling constant. Varying the action (3.3) w.r.t $g^{\mu\nu}$ and ϕ we obtain the field equations,

$$\begin{aligned} G_{\mu\nu} &= FR_{\mu\nu} - \frac{1}{2}fg_{\mu\nu} - \nabla_\mu\nabla_\nu F + g_{\mu\nu}\Box F \\ &= \kappa^2\left(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\nabla^\rho\phi\nabla_\rho\phi - Vg_{\mu\nu}\right), \end{aligned} \tag{3.5}$$

$$\Box\phi = V_{,\phi} - \frac{f_{,\phi}}{2\kappa^2}, \tag{3.6}$$

respectively, where $F = \partial f/\partial R = 1 + \frac{\xi b\phi^a R^{b-1}}{M_p^{a+2b-2}}$.

3.2.1 Background Evolution in Quasi de-Sitter Space

For the unperturbed background FRW metric $diag(-1, a^2(t), a^2(t), a^2(t))$ the above Eqs. (3.5) and (3.6) can be solved to give the field equations

$$3FH^2 + \frac{1}{2}(f - RF) + 3H\dot{F} = \kappa^2\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \quad (3.7)$$

$$-2F\dot{H} - \ddot{F} + H\dot{F} = \kappa^2\dot{\phi}^2 \quad (3.8)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} - \frac{f_{,\phi}}{2\kappa^2} = 0\ . \quad (3.9)$$

We assume that the second term of F *i.e.* $\frac{\xi b\phi^a R^{b-1}}{M_p^{a+2b-2}}$ is dominant for some values of a and b. We find this assumption to be valid while solving numerically for the values of a and b in our model which give rise to the experimentally observed density perturbations as discussed in the Section §3.4. From equation (3.7), under this assumption and considering the slow-roll parameters which are defined in equation (3.29) as small, the Hubble parameter in the Jordan frame turns out to be of the form

$$H = \frac{\lambda^{\frac{1}{2b}}}{\sqrt{12}\left[\xi(2-b)\right]^{\frac{1}{2b}}}\left(\frac{\phi}{M_p}\right)^{\frac{4-a}{2b}} M_p\ . \quad (3.10)$$

From equation (3.9) under the slow-roll assumption we get

$$\dot{\phi} = -\frac{\lambda\phi^3}{3H}\left[1 - \frac{a}{2(2-b)}\right]. \quad (3.11)$$

3.2.2 Scalar Field and Metric Perturbations

Now we perturb Eqs. (3.5) and (3.6) by perturbing the scalar field $\phi = \phi(t) + \delta\phi(x,t)$ and the metric [19] as

$$ds^2 = -(1+2\Phi)dt^2 - 2a(t)\partial_i\beta dt dx^i + a^2(t)\left[(1+2\Psi)\delta_{ij} + 2\partial_i\partial_j\gamma\right]dx^i dx^j, \quad (3.12)$$

where, Φ, Ψ, β and γ are scalar perturbations. We derive the Einstein field equations for the $f(R,\phi)$ theory keeping the linear terms in the metric and scalar field perturbations [115, 116]. The component δG_{00} is given by

$$\frac{\triangle}{a^2(t)}\Psi + HA = \frac{-1}{2F}\left[\left(3H^2 + 3\dot{H} + \frac{\triangle}{a^2(t)}\right)\delta F - 3H\delta\dot{F} + \frac{1}{2}\left(2\kappa^2 V_{,\phi} - f_{,\phi}\right)\delta\phi \right.$$
$$\left. + \kappa^2\dot{\phi}\delta\dot{\phi} + (3H\dot{F} - \kappa^2\dot{\phi}^2)\Phi + \dot{F}A\right], \tag{3.13}$$

where $\triangle$ is the Laplacian. And taking the difference $\delta G^i_i - \delta G^0_0$ we get

$$\dot{A} + 2HA + \left(3\dot{H} + \frac{\triangle}{a^2(t)}\right)\Phi = \frac{1}{2F}\left[3\delta\ddot{F} + 3H\delta\dot{F} - \left(6H^2 + \frac{\triangle}{a^2(t)}\right)\delta F \right.$$
$$+ 4\kappa^2\dot{\phi}\delta\dot{\phi} + \left(-2\kappa^2 V_{,\phi} + f_{,\phi}\right)\delta\phi - 3\dot{F}\dot{\Phi}$$
$$\left. - \dot{F}A - \left(4\kappa^2\dot{\phi}^2 + 3H\dot{F} + 6\ddot{F}\right)\Phi\right] \tag{3.14}$$

where $A = 3(H\Phi - \dot{\Psi}) - \triangle B/a^2(t)$ and $B = a(t)(\beta + a(t)\dot{\gamma})$. Here, in arriving the Eqs. (3.13) and (3.14), the leading order Eqs. (3.7) and (3.8) are also used. The other components δG_{i0} and δG_{ij} $(i \neq j)$ of the first order perturbed Einstein equation can be written as

$$H\Phi - \dot{\Psi} = \frac{1}{2F}\left[\kappa^2\dot{\phi}\delta\phi + \delta\dot{F} - H\delta F - \dot{F}\Phi\right], \tag{3.15}$$

and

$$\dot{B} + HB - \Phi - \Psi = \frac{1}{F}\left(\delta F - \dot{F}B\right) \tag{3.16}$$

respectively. The equation of motion of scalar perturbation is

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \left[-\frac{\triangle}{a^2(t)} + \left(\frac{2V_{,\phi} - f_{,\phi}/\kappa^2}{2}\right)\right]\delta\phi \tag{3.17}$$
$$= \dot{\phi}\dot{\Phi} + \left(2\ddot{\phi} + 3H\dot{\phi}\right)\Phi + \dot{\phi}A + \frac{1}{2}F_{,\phi}\left(\frac{\delta R}{\kappa^2}\right), \tag{3.18}$$

where

$$\delta R = -2\left[\dot{A} + 4AH + \left(\frac{\triangle}{a^2(t)} + 3\dot{H}\right)\Phi + 2\frac{\triangle}{a^2(t)}\Psi\right]. \tag{3.19}$$

Now we analyze the curvature perturbation $\mathcal{R} = \Psi - H\delta\phi/\dot{\phi}$ by choosing a gauge where $\delta\phi = 0$ and $\delta R = 0$. This sets $\mathcal{R} = \Psi$ and moreover we have $\delta F = 0$ via $\delta F = (\partial F/\partial\phi)\,\delta\phi + (\partial F/\partial R)\,\delta R$. Under this gauge the equation (3.15) gives,

$$\Phi = \frac{\dot{\mathcal{R}}}{H + \dot{F}/(2F)} \tag{3.20}$$

and hence from equation (3.13) we get

$$A = -\frac{1}{H + \dot{F}/(2F)}\left(\frac{\triangle}{a^2(t)}\mathcal{R} + \frac{\left(3H\dot{F} - \kappa^2\dot{\phi}^2\right)\dot{\mathcal{R}}}{2F\left(H + \dot{F}/(2F)\right)}\right). \tag{3.21}$$

Using equation (3.8) and equation (3.14), we obtain

$$\dot{A} + \left(2H + \frac{\dot{F}}{2F}\right)A + \frac{3\dot{F}}{2F}\dot{\Phi} + \left(\frac{3\ddot{F} + 6H\dot{F} + \kappa^2\dot{\phi}^2}{2F} + \frac{\triangle}{a^2(t)}\right)\Phi = 0. \tag{3.22}$$

Now we may write the differential equation for curvature perturbation by using the above Eqs. (3.20), (3.21) and (3.22) as

$$\ddot{\mathcal{R}} + \frac{(a^3(t)Q_s)^{\cdot}}{a^3(t)Q_s}\dot{\mathcal{R}} + \frac{k^2}{a^2(t)}\mathcal{R} = 0, \tag{3.23}$$

where,

$$Q_s = \frac{\dot{\phi}^2 + 3\dot{F}^2/(2\kappa^2 F)}{\left(H + \dot{F}/(2F)\right)^2}. \tag{3.24}$$

In arriving equation (3.23), equation (3.8) is again used. Now one may re-write the equation (3.23) in terms of variables $\omega = a(t)\sqrt{Q_s}$ and $\sigma_k = \omega\mathcal{R}$ as

$$\sigma_k'' + \left(k^2 - \frac{\omega''}{\omega}\right)\sigma_k = 0, \tag{3.25}$$

where prime denotes the derivative with respect to the conformal time defined as $d\eta = dt/a(t)$ and

$$\frac{\omega''}{\omega} = \frac{a''(t)}{a(t)} + \frac{a'(t)}{a(t)}\frac{Q_s'}{Q_s} + \frac{1}{2}\frac{Q_s''}{Q_s} - \frac{1}{4}\left(\frac{Q_s'}{Q_s}\right)^2 \tag{3.26}$$

under quasi de-Sitter expansion $a(\eta) = \frac{-1}{H\eta(1-\epsilon_1)}$ and hence $\frac{a''(t)}{a(t)} = \frac{1}{\eta^2}\Big[2 + 3\epsilon_1\Big]$ and $a'(t)/a(t) = a(t)H$. Therefore we have

$$\frac{\omega''}{\omega} = \frac{1}{\eta^2}\left[\nu_{\mathcal{R}}^2 - \frac{1}{4}\right] \tag{3.27}$$

where

$$\nu_{\mathcal{R}}^2 = \frac{9}{4}\left[1 + \frac{4}{3}\left(2\epsilon_1 + \epsilon_2 - \epsilon_3 + \epsilon_4\right)\right]. \tag{3.28}$$

In arriving at the above expression we have defined

$$\epsilon_1 = -\frac{\dot H}{H^2}\ ,\ \epsilon_2 = \frac{\ddot\phi}{H\dot\phi}\ ,\ \epsilon_3 = \frac{\dot F}{2HF}\ ,\ \epsilon_4 = \frac{\dot E}{2HE}\ ; \tag{3.29}$$

$$E = F + \frac{3\dot F^2}{2\kappa^2\dot\phi^2} = \frac{Q_s(1+\epsilon_3)^2}{\dot\phi^2/(FH^2)}\ . \tag{3.30}$$

Here ϵ_i are slow-roll parameters and $\dot\epsilon_i$ terms have been neglected. Equation (3.25) then has solutions in the Hankel functions of order ν_R

$$\sigma = \frac{\sqrt{\pi|\eta|}}{2}e^{i(1+2\nu_{\mathcal{R}})\pi/4}\left[c_1\, H^{(1)}_{\nu_{\mathcal{R}}}(k|\eta|) + c_2\, H^{(2)}_{\nu_{\mathcal{R}}}(k|\eta|)\right] \tag{3.31}$$

Applying the Bunch-Davies boundary condition $\sigma(k\eta \to -\infty) = e^{ik\eta}/\sqrt{2k}$, we fix the integration constants $c_1 = 1$ and $c_2 = 0$. Using the relation $H_\nu(k|\eta|) = \frac{-i}{\pi}\Gamma(\nu)\left(\frac{k|\eta|}{2}\right)^{-\nu}$ for the super-horizon modes $k\eta \to 0$, we obtain the expression for the power spectrum of curvature perturbations which is defined as (2.87)

$$\mathcal{P}_{\mathcal{R}} = \frac{k^3}{2\pi^2}\langle|\mathcal{R}|^2\rangle \equiv \Delta_{\mathcal{R}}^2\left(\frac{k}{a(t)H}\right)^{n_{\mathcal{R}}-1}, \tag{3.32}$$

where, the amplitude of the curvature power spectrum turns out to be

$$\Delta_{\mathcal{R}}^2 = \frac{1}{Q_s}\left(\frac{H^2}{4\pi^2}\right) \tag{3.33}$$

and the spectral index is

$$n_{\mathcal{R}} - 1 = 3 - 2\nu_{\mathcal{R}} \simeq -4\epsilon_1 - 2\epsilon_2 + 2\epsilon_3 - 2\epsilon_4 \simeq -6\epsilon_1 \ . \tag{3.34}$$

Using slow-roll parameters, equation (3.24) can be simplified to the form $Q_s \simeq 6F\epsilon_3^2 M_p^2$ with $\frac{\kappa^2\dot{\phi}^2}{FH^2} << 6\epsilon_3^2$ which will be justified later in Section §3.4. In our model of $f(\phi, R)$ coupling we find $\epsilon_1 \approx -\epsilon_3$, $\epsilon_2 \approx -\epsilon_4$ and these relations are used in the calculation of perturbation amplitude and spectral index. Plugging the values H and $\dot{\phi}$ from Eqs. (3.10) and (5.7) into equation (3.29), the slow-roll parameters can be written as

$$\epsilon_1 = b^{-1}(a-4)(2-b)^{(1-b)/b}(a+2b-4)\lambda^{(b-1)/b}\xi^{1/b}\left(\frac{\phi}{M_p}\right)^{\frac{a+2b-4}{b}} \tag{3.35}$$

$$\epsilon_2 = b^{-1}(a+6b-4)\,(2-b)^{(1-b)/b}\,(a+2b-4)\lambda^{(b-1)/b}\xi^{1/b}\left(\frac{\phi}{M_p}\right)^{\frac{a+2b-4}{b}} \tag{3.36}$$

For our model, we can write the expressions for the amplitude of power spectrum and the number of e-folding as

$$\Delta_{\mathcal{R}}^2 = \frac{b[(2-b)/\lambda]^{3-\frac{4}{b}} M_p^{8+\frac{4(a-4)}{b}}\xi^{-\frac{4}{b}}\phi^{-\frac{4(a+2b-4)}{b}}}{288(a-4)^2(a+2b-4)^2\pi^2} \tag{3.37}$$

and

$$N_J = \int_{\phi_J}^{\phi_f}\frac{H}{\dot{\phi}}d\phi = \frac{b[(2-b)/\lambda]^{\frac{b-1}{b}}\xi^{-\frac{1}{b}}}{2(a+2b-4)^2}\left(\frac{\phi}{M_p}\right)^{\frac{4-a-2b}{b}}\Bigg|_{\phi_f}^{\phi_J} \tag{3.38}$$

respectively. Here ϕ_J and ϕ_f are the values of scalar field ϕ at the beginning and the end of inflation respectively.

3.2.3 Tensor Perturbations

Defining the tensor perturbation to the background metric

$$ds^2 = -dt^2 + a(t)^2(\delta_{ij} + h_{ij})dx^i dx^j, \tag{3.39}$$

where the components h_{ij} of the tensor metric perturbations are traceless and divergenceless, *i.e.*,

$$h_{ii} = 0, \quad \partial_i h_{ij} = 0. \tag{3.40}$$

We calculate the components of the perturbed Ricci scalar and Ricci curvature,

$$\delta R_{00} \;=\; 0, \qquad \delta R_{i0} = 0, \qquad \delta R = 0. \tag{3.41}$$

and

$$\delta R_{ij} \;=\; -\frac{1}{2a^2(t)}\nabla^2 h_{ij} + \frac{1}{2}\ddot{h}_{ij} - \frac{\dot{a}}{2a}\dot{h}_{ij} + 2\left(\frac{\dot{a}}{a}\right)^2 h_{ij}. \tag{3.42}$$

Using the above results, we get the perturbed field equation (3.5),

$$\begin{aligned}&\frac{1}{2}Fa^2\ddot{D}_{ij} + \left(\frac{1}{2}\dot{F}a^2 + \frac{3}{2}a\dot{a}F\right)\dot{D}_{ij} - \frac{F}{2}\nabla^2 D_{ij} \\ &= \left[2\frac{\dot{a}}{a}\dot{F} - 2F\left(\frac{\dot{a}}{a}\right)^2 - \frac{\ddot{a}}{a}F + \frac{f}{2} + \ddot{F} + \frac{\kappa^2}{2}\left(\dot{\phi}^2 - 2V\right)\right]a^2 D_{ij},\end{aligned} \tag{3.43}$$

where $D_{ij} = h_{ij}/a^2$. The right hand side of equation (3.43) vanishes by using equations (3.7) and (3.8) and with little simplification we get

$$\ddot{D}_{ij} + \frac{(a^3F)^{\cdot}}{a^3F}\dot{D}_{ij} + \frac{\kappa^2}{a^2}D_{ij} = 0. \tag{3.44}$$

In terms of polarization tensors e^1_{ij} and e^2_{ij}, the tensor D_{ij} can be written as

$$D_{ij} = D_1 e^1_{ij} + D_2 e^2_{ij}. \tag{3.45}$$

For gravity wave propagating in $\hat{z}$ direction, the components of polarization tensor are given by

$$e^1_{xx} = -e^1_{yy} = 1, \quad e^2_{xy} = e^2_{yx} = 1, \quad e^{1,2}_{iz} = e^{1,2}_{zi} = 0. \tag{3.46}$$

So the equation (3.44) can be written as

$$\ddot{D}_\lambda + \frac{(a^3 F)^{\cdot}}{a^3 F}\dot{D}_\lambda + \frac{\kappa^2}{a^2} D_\lambda = 0, \tag{3.47}$$

where $\lambda \equiv 1, 2$ corresponds to two polarizations of gravity wave. Now substituting $z = a\sqrt{F}$ and $v_k = z D_\lambda M_P/\sqrt{2}$, we get

$$v''_\lambda + \left(k^2 - \frac{z''}{z}\right) v_\lambda = 0, \tag{3.48}$$

where, prime $'$ is derivative with respect to conformal time. Summing over all polarization states, the equation (3.48) provides us the amplitude of power spectrum of D_λ as

$$\Delta^2_T = 4 \times \left(\frac{2}{M_p^2}\right) \frac{k^3}{2\pi^2} \frac{1}{a^2 F} v^2_\lambda \simeq \frac{2}{\pi^2} \left(\frac{H}{M_P}\right)^2 \frac{1}{F}. \tag{3.49}$$

So, the ratio of the amplitude of tensor perturbations to scalar perturbations r is given by

$$\begin{aligned} r &\equiv \frac{\Delta^2_T}{\Delta^2_{\mathcal{R}}} \\ &= \frac{8 Q_s}{M_p^2 F} \simeq 48 \epsilon_3^2. \end{aligned} \tag{3.50}$$

where the first equality is true for a generic $f(\phi, R)$ theories in JF and the second approximate equality corresponds to our model $\xi \phi^a R^b$.

3.3 Model in the Einstein Frame

Starting with the considered JF action

$$S_J = \int d^4x\sqrt{-g}\left[-\frac{{M_p}^2}{2}R\left(1+\frac{\xi\phi^a R^{b-1}}{M_p^{a+2b-2}}\right)+\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+\frac{\lambda\phi^4}{4}\right] \tag{3.51}$$

we perform a conformal transformation of the metric $g_{\mu\nu}$ to the Einstein frame metric $\tilde{g}_{\mu\nu}$ which is defined as

$$\tilde{g}_{\mu\nu}(x) \;=\; \Omega^2(x)g_{\mu\nu}(x)\ , \tag{3.52}$$

where the conformal factor is given by

$$\Omega^2 = 1+\frac{\xi\phi^a R^{b-1}}{{M_p}^{a+2b-2}}\ . \tag{3.53}$$

The Ricci scalar transforms as

$$R \;=\; \Omega^2\left[\tilde{R}+6\frac{\tilde{\Box}\Omega}{\Omega}-12\frac{\tilde{\partial}^\mu\Omega\tilde{\partial}_\mu\Omega}{\Omega^2}\right]. \tag{3.54}$$

Here and here after all the quantities with tilde represents quantities in EF. For quasi de-Sitter space we may ignore the second and third terms in the bracket in equation (3.54) which is justified in the equation (3.69). For this assumption we may write equation (3.53) in Einstein frame as

$$\Omega^2 = 1+\frac{\xi^{1+q}\phi^p\tilde{R}^q}{M_p^{p+2q}}\ , \tag{3.55}$$

where, $p = a/(2-b)$ and $q = (b-1)/(2-b)$ and $\tilde{R} = 12\tilde{H}^2 \simeq Constant$. Now via conformal transformation we write the action (3.51) in term of new field χ, which is related to the field ϕ by the relation

$$\frac{d\chi}{d\phi} = \frac{1}{\Omega^2}\left(\Omega^2+\frac{3p^2\xi'^2}{2}\left(\frac{\phi}{M_p}\right)^{2p-2}\right)^{1/2}, \tag{3.56}$$

where $\xi' = \xi^{1+q}(\tilde{R}/M_p^2)^q$. This leads the action JF action to EF action in term of the filed χ:

$$S_E = \int d^4x \left(-\frac{M_p^2}{2}\tilde{R} + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi + U(\chi) \right) , \tag{3.57}$$

where the EF potential is given by

$$U(\chi) = \frac{1}{\Omega^4}\frac{\lambda}{4}\phi(\chi)^4. \tag{3.58}$$

For $\phi >> M_P/\xi'^{1/p}$, equation (3.56) can be integrated to give

$$\phi = \frac{M_p}{\xi'^{1/p}} \exp\left(\sqrt{\frac{2}{3}}\frac{\chi}{M_p p} - \frac{1}{2} \right). \tag{3.59}$$

Considering $\tilde{g}_{\mu\nu} = diag(-M^2(t), \tilde{a}^2(t), \tilde{a}^2(t), \tilde{a}^2(t))$ and varying the action (3.57) with respect to $M(t)$ or $a(t)$ and setting $M = 1$ in the final equation which corresponds FRW metric, we get the Friedmann equation

$$12\tilde{H}^2 - \zeta^{-1}M_p^2\lambda\left(1 + \frac{2q}{p}\right) = 0 \ , \tag{3.60}$$

where

$$\zeta = 12^{4q/p}\left(\frac{\tilde{H}^2}{M_p^2}\right)^{4q/p} \xi^{\frac{4(1+q)}{p}} \exp\left(2\sqrt{\frac{2}{3}}\frac{(p-2)\chi}{pM_p} \right). \tag{3.61}$$

Here we have neglected all the derivative terms of Hubble parameter $\tilde{H}$. This corresponds to slow roll condition, *i.e.*, $\dot{\chi}^2$ is much smaller than potential term. We may write the Hubble parameter from equation (4.1) as

$$\tilde{H} = M_p \frac{\left[(1 + 2q/p)\,\lambda\right]^{\frac{p}{2(p+4q)}}}{\sqrt{12}\ \xi^{\frac{2(1+q)}{p+4q}}} \exp\left[\sqrt{\frac{2}{3}}\left(\frac{2-p}{p+4q}\right)\frac{\chi}{M_p} \right] . \tag{3.62}$$

Now using equations (3.55)and (3.62) into (3.58) we obtain the scalar potential as

$$U(\chi) = \frac{1}{4} M_p^4 \lambda^{\frac{p}{p+4q}} \xi^{-\frac{4(1+q)}{p+4q}} \left(1 + \frac{2q}{p}\right)^{-\frac{4q}{p+4q}} \exp\left[2\sqrt{\frac{2}{3}}\left(\frac{2-p}{p+4q}\right)\frac{\chi}{M_p}\right]. \quad (3.63)$$

Here we have taken the large field approximation $\exp(\sqrt{\frac{2}{3}}\frac{\chi}{M_p}) >> 1$ for $\chi >> M_p$. Also the parameters a and b, which appear through p and q as defined above, in the potential (3.63) are the parameters of the Jordan frame action (3.51) which appear in the Eistein frame potential via conformal transformation of the metric (3.52).

We now compute the spectral index and curvature perturbation using above potential (3.63). The slow-roll parameters for large $\chi >> M_p$ comes out to be

$$\epsilon = \frac{M_p^2}{2}\left(\frac{U'}{U}\right)^2 = \frac{4}{3}\left(\frac{a+2b-4}{a+4b-4}\right)^2, \quad (3.64)$$

$$\eta = M_p^2\left(\frac{U''}{U}\right) = \frac{8}{3}\left(\frac{a+2b-4}{a+4b-4}\right)^2, \quad (3.65)$$

and the amplitude of curvature perturbations (2.100)

$$\begin{aligned}\Delta_{\mathcal{R}}^2 &= \frac{1}{8\pi^2\epsilon}\frac{\tilde{H}^2}{M_p^2} \\ &= \frac{1}{128\pi^2}\left(\frac{y+2}{2y-x+4}\right)^{\frac{x+2y+4}{x}} \lambda^{\frac{2y-x+4}{x}} \xi^{-\frac{4}{x}} \left(\frac{x}{y}\right)^2 e^{-2\sqrt{\frac{2}{3}}\frac{y}{x}\frac{\chi}{M_p}}, \quad (3.66)\end{aligned}$$

where $x = a + 4b - 4$ and $y = a + 2b - 4$.

The spectral index in the term of slow-roll parameters is (2.101)

$$n_s = 1 - 6\epsilon + 2\eta. \quad (3.67)$$

The number of e-folding is calculated as

$$\begin{aligned}N_E &= \int_{\chi_e}^{\chi_0} \frac{U(\chi)}{U'(\chi)} d\chi \\ &= -\frac{1}{2}\sqrt{\frac{3}{2}}\left(\frac{x}{y}\right)\left(\frac{\chi_0 - \chi_e}{M_p}\right) \quad (3.68)\end{aligned}$$

For $\chi_0 \sim 13M_p$ and $\chi_e \sim 1M_p$, the number of e-folding is found to be around 60.

Now from equations (3.53) and (3.59), we can calculate the order of terms like $\ddot{\Omega}/\Omega$ and $(\dot{\Omega}/\Omega)^2$ for $\phi >> \frac{M_p}{\xi^{1/p}}$. For $\lambda = 10^{-3}$ and $\xi = 1$, we get

$$\begin{aligned} \frac{\ddot{\Omega}}{\Omega} &\sim \frac{U}{9M_p^2}(\epsilon + \sqrt{3\epsilon}(\eta - \epsilon)) = 4.1 \times 10^{-11} M_p^2 \text{ and} \\ \left(\frac{\dot{\Omega}}{\Omega}\right)^2 &\sim \frac{U}{9M_p^2}\epsilon = 3.3 \times 10^{-11} M_p^2 \end{aligned} \tag{3.69}$$

whereas the value of curvature scalar $\tilde{R} = 12\tilde{H}^2$ at the same values of parameter is $4.1 \times 10^{-8} M_p^2$. Thus our approximation (*i.e.* for quasi de-Sitter space we can ignore the second and third terms in the bracket in equation (3.54)) made is consistent and may be checked for other values of a and b. We now use the measured values of these CMB anisotropy parameters to get the numerical values for the parameters (a, b, ξ, λ).

3.4 Results and Discussion

From the Planck+WP measurements [112] we know that the curvature perturbation $\Delta^2_{\mathcal{R}} = 2.195^{+0.533}_{-0.585} \times 10^{-9}$, spectral index $n_{\mathcal{R}} = 0.9603 \pm 0.0073$ and the tesnsor to scalar ratio $r < 0.11 (95\% CL)$. For inflation to solve the horizon and flatness problems of standard hot big bang cosmology the number of e-foldings in the Eintein frame N_E is required to be about 60. From equation (3.68) we see that to get 60 e-foldings, the scalar field χ should roll from $13M_p$ to $1M_p$. We compute the curvature perturbation (3.66) and spectral index (3.67) in the Einstein frame and equate the expressions with the Planck+WMAP values to compute the parameters a and b for different values of λ and assume that the curvature coupling parameter $\xi = 1$. Our results for the correlated set of parameters λ, a, b at $\chi_0 = 13M_P$ which give the measured values of $\Delta^2_{\mathcal{R}}$ and n_s are shown in the Table (3.1). $\chi_0 = 13M_P$ is the field value approximately 60 e-folds before the end of inflation. The slow-roll parameters ϵ, η and the Hubble parameter $\tilde{H}$ are ~ 0.02, ~ 0.04 and $\sim 5.8 \times 10^{-5} M_p$ respectively. And for $\epsilon \simeq 0.02$

λ	0.1	10^{-2}	10^{-3}	10^{-4}	10^{-5}
a	3.385	3.026	2.735	2.494	2.292
b	0.277	0.439	0.571	0.679	0.770
a+2b	3.939	3.904	3.877	3.852	3.832

Table 3.1: The values of parameters a and b in the Einstein frame at $\chi_0 = 13M_p$ with $\xi = 1$ for different values of λ. The parameters a and b of the Jordan frame action appears in the Einstein frame potential via conformal transformation of the metric.

the tensor-to-scalar ratio in EF is predicted to be large $r \simeq 0.3$. We see that compared to the $\xi\phi^2 R$ models with large ξ the small deviations of a and b from 2 and 1 respectively can result in a large change in ξ which is 1 in our model compared to the earlier curvature coupling models where $\xi \sim 10^4$.

Next we equate the curvature perturbations and spectral index in the Jordan frame from equation (3.37) and equation (3.34) with the Planck+WMAP data to evaluate the values of the parameters λ, a and b (keeping $\xi = 1$) . The scalar field values ϕ in the Jordan frame corresponding to $\chi_e = 1M_p$ and $\chi_0 = 13M_p$ for different values of λ are displayed in Table (3.2). χ_e is the field value at the end of inflation. Using these values of the range of the roll in ϕ we see that the number of e-foldings N_J in the Jordan frame, corresponding to $N_E = 60$ is $N_J \sim 830$. The values of the parameters λ, a and b which give the required curvature perturbation and spectral index are shown in the Table (3.2). The slow-roll parameters are found to be $\epsilon_1 \simeq -\epsilon_3 \simeq 0.007$ and $\epsilon_2 \simeq -\epsilon_4 \simeq -0.013$ for chosen range of λ. The calculated value for the tensor-to-scalar ratio and Hubble parameter are $r \simeq 0.002$ and $H \sim 10^{-3}M_p$ respectively.

The values of $F = 1 + \frac{\xi b \phi^a R^{b-1}}{M_p^{a+2b-2}}$ are found to be $\sim 10^5$ i.e much larger than unity and hence our assumption of dropping the unity in the expression for F is justified. Also we find that the order of the term $\frac{\kappa^2 \dot{\phi}^2}{FH^2} \sim 10^{-9}$ is much smaller than $6\epsilon_3^2 \sim 10^{-4}$ as assumed in Section §3.2.2.

We find that in the limit $a \simeq 2$ and $b \simeq 1$ the correct value of $\Delta^2_{\mathcal{R}}$ and $n_{\mathcal{R}}$ are obtained for $\lambda \sim 0.1$ only for large value of $\xi \sim 10^4$ as is the prediction of Higgs inflation models $\xi\phi^2 R$ [20, 98, 100, 101].

λ	0.1	10^{-2}	10^{-3}	10^{-4}	10^{-5}
$\phi_f\|_{(\chi e=1M_p)}$	$0.0146M_p$	$0.0253M_p$	$0.044M_p$	$0.077M_p$	$0.134M_p$
$\phi_J\|_{(\chi_0=13M_p)}$	$3.566M_p$	$6.187M_p$	$10.77M_p$	$18.8M_p$	$32.77M_p$
a	3.56398962	3.2751299	3.0257694	2.809561	2.620851
b	0.21800513	0.3624348	0.4871146	0.595217	0.689566
a+2b	3.999999	3.999999	3.999998	3.999995	3.99998

Table 3.2: The values of parameters a and b are evaluated in the Jordan frame at $\xi = 1$ and $\phi_J|_{\chi_0=13M_p}$ for different values of λ.

3.5 Conclusions

We have generalised the curvature coupling models of Higgs inflation to study inflation with a scalar field for a $\frac{\lambda}{4}\phi^4$ potential and a curvature coupling of the form $\frac{\xi\phi^a R^b}{M_p^{a+2b-4}}$. It may be possible to generate a tree level term of this form by choosing a suitable Kahler potential in a $f(\mathcal{R})$ supergravity theory [117–119].

We find that for $\xi = 1$ and λ in the range $(10^{-5} - 0.1)$, the phenomenologically acceptable parameters a and b fall in the ranges $(2.3 - 3.6)$ and $(0.77 - 0.22)$ respectively. We discover an interesting symmetry related to the numerical value of a and b which give the correct amplitude and spectral index. We find that for any value of λ the values of a and b which give the required density perturbations satisfy the relation $(a + 2b) \simeq 4$ as shown in Table(3.2). This means that the curvature coupling term $\frac{\xi\phi^a R^b}{M_p^{a+2b-4}}$ has no dimensional couplings and is scale invariant.

It has been shown that the Higgs self coupling can go from $\lambda = 0.13$ at the electroweak scale for the 125 GeV Higgs to $\lambda = 10^{-5}$ at the Planck scale by tuning the top mass or by introducing extra interactions [85, 86, 111]. This leads us to conclude that the Higgs field may still be a good candidate for being the inflaton in the early universe if one considers a generalized curvature-Higgs coupling of the form $\xi\phi^a R^b$.

The tensor-to-scalar ratio r in this model is large, therefore $\frac{\lambda}{4}\phi^4$ with scalar curvature couplings is ruled out by observational limits on r like the pure $\frac{\lambda}{4}\phi^4$ theory [6, 54, 120]. However, Jordan frame calculations give small $r \simeq 0.002$ which is contradicts the Einstein frame prediction or r. The debate, on which frame is more physical, is still going on and the issue that the observations should be compared with Einstein frame or Jordan frame results is still not settle [121–124].

We find that the values of (a, b) computed with Jordan and Einstein frame calculations of the curvature perturbation and spectral index are comparable but are not identical because unlike the $\xi\phi^2 R$ theory, in the $\xi\phi^a R^b$ theory it is not possible in general to go to an Einstein frame with a conformal transformation. If the space is quasi de-Sitter however such an transformation given by equation (3.55) is possible but the results will differ in the two frames due to the slow-roll approximation. Finally, by requiring the curvature coupling parameter to be of order unity, we have evaded the problem of unitarity violation in scalar-graviton scatterings [103] which plagued the $\xi\phi^2 R$ Higgs inflation models [20, 98, 100, 101].

Chapter 4

Power Law Starobinsky Model of Inflation and its Motivation from No-scale SUGRA

4.1 Overview

In this chapter we study a model $R+\frac{1}{M^2}R^\beta$ of inflation which is a generalization of Starobinsky model $R+\frac{1}{M^2}R^2$ and so we call it as power law Starobinsky model. In the previous Chapter §3 we studied a scalar curvature coupled model $\xi\phi^a R^b$ of inflation. Along with the independent observable predictions of this model, we will show that generalized Higgs inflation model $\xi\phi^a R^b$ is equivalent to power law Starobinsky model. Also we will see that power law model can be motivated/derived from no-scalar supergravity with an appropriate choice of Kähler potential and superpotential.

The Starobinsky model of inflation [9, 10] with an $\frac{1}{M^2}R^2$ interaction term is of interest as it requires no extra scalar fields but relies on the scalar degree of the metric tensor to generate the 'inflaton' potential. The R^2 correction to Einstein gravity is equivalent to scalar-tensor theory with a scalar potential which is an exponentially corrected plateau potential. This model is favored by the Planck constraint on the tensor to scalar ratio which ruled out potentials like $m^2\phi^2$ and

$\lambda\phi^4$ in the context of standard slow-roll inflation. In addition the Starobinsky model could be mapped to the Higgs-inflation models with $\xi\phi^2 R + \lambda\phi^4$ theory [98]. The characteristic feature of the Starobinsky equivalent models was the prediction that the tensor-to-scalar ratio was $r \simeq 10^{-3}$. BICEP2 reported a large value of $r = 0.2^{+0.07}_{-0.05}$ [125] but the recent joint analysis by Planck + BICEP2 + Keck Array give only an upper bound of $r_{0.05} < 0.12(95\%CL)$ [7, 54, 56]. In an analysis of the genus structure of the B-mode polarisation of Planck + BICEP2 data by Colley et al. put the tensor-to-scalar ratio at $r = 0.11 \pm 0.04(68\%CL)$ [126]. In the light of the possibility that r can be larger than the Starobinsky model prediction of $r \sim 0.003$, generalisations of the Starobinsky model are of interest.

We study a general power law $\frac{1}{6M^2}\frac{R^\beta}{M_p^{2\beta-2}}$ correction to the Einstein gravity and compute the scalar and tensor power spectrum as a function of the two dimensionless parameters M and β. It is well known that the $\frac{1}{M^2}R^2$ model is equivalent to the $\xi\phi^2 R + \lambda\phi^4$ Higgs-inflation model as they led to the same scalar potential in the Einstein frame [30, 127]. One can find similar equivalence between generalized Higgs-inflation models and the power law Starobinsky model whose common feature is violation of the global Weyl symmetry. A general scalar curvature coupled $\xi\phi^a R^b$ model was studied in [128]. The quantum correction on ϕ^4-potential in Jordan frame was studied in [129–131] where they have shown the equivalence of the $\xi\phi^2 R + \lambda\phi^{4(1+\gamma)}$ model with $\frac{1}{M^2}R^\beta$ model. The generalized Starobinsky model with R^p correction has been studied in the ref. [55, 132–137]. In general scalar-curvature theories the scalar plays the role of the inflaton after transforming to Einstein frame whereas in pure curvature theories like $R + \frac{1}{M^2}R^\beta$ model the longitudinal part of the graviton is the equivalent scalar in the Einstein frame plays the role of inflaton.

The higher order curvature theories arise naturally in theories of supergravity. The supergravity embedding of the Higgs-inflation [98] does not produce a slow-roll potential in MSSM but a potential suitable for inflation is obtained in NMSSM [138]. The potential in NMSSM however has a tachyonic instability in the direction orthogonal to the slow-roll [139]. This instability can be cured by the addition of quartic terms of the fields in the Kähler potential [140, 141].

In the context of a supergravity embedding of the Starobinsky model, It was shown by Cecotti[142] that quadratic Ricci curvature terms can be derived in a supergravity theory by adding two chiral superfields in the minimal supergravity. A no-scale SUGRA[37, 41, 44] model with a modulus field and the inflation field with a minimal Wess-Zumino superpotential gives the same F-term potential in the Einstein frame as the Starobinsky model [46]. The symmetry principle which can be invoked for the SUGRA generalization of the Starobinsky model is the spontaneous violation of superconformal symmetry [143]. The quadratic curvature can also arise from D-term in a minimal-SUGRA theory with the addition of a vector and chiral supermultiplets [144]. The Starobinsky model has been derived from the D-term potential of a SUGRA model [145–147]. Quartic powers of Ricci curvature in the bosonic Lagrangian can also be obtained in a SUGRA model by the D-term of higher order powers of the field strength superfield [147, 148].

In this chapter we give a SUGRA model for the general power law $\frac{1}{M^2}R^\beta$ model. We show that adding a $(\phi+\bar{\phi})^n$ term to the minimal no-scale Kähler potential and with a Wess-Zumino form of the superpotential $W(\phi)$ yields the same potential in the Einstein frame as the generalized Starobinsky model. In the limit $n=2$ the Starobinsky limit $\beta=2$ is obtained. We derive the relations between the two parameters of the power-law Starobinsky model and the two parameters of our SUGRA model. The interesting point about the generalization is that small deviations from the Starobinsky limit of $n=\beta=2$ can produce large shifts in the values of r. Many SUGRA models have been constructed which can yield a range of r from $10^{-3}-10^{-1}$ by changing the parameters of the Kähler potential and the superpotential [14, 148–164].

Also we show in this chapter that our 2−parameter SUGRA model which we relate to the 2-parameter $\frac{1}{M^2}R^\beta$ power law model is the most economical representation of the 5-parameter scalar-curvature coupled inflation models $\xi\phi^a R^b+\lambda\phi^{4(1+\gamma)}$ in terms of the number of parameters.

This chapter is organized as follows: In the Section §4.2, we calculate an equivalent scalar potential in the Einstein frame for $R + \frac{1}{M^2}R^\beta$ gravity. We find the parameter M and β values which satisfy the observed amplitude $\Delta^2_{\mathcal{R}}$, spectral index n_s and tensor-to-scalar r. We fix model parameters for two cases: one with running of n_s and another without running of n_s. In Section §4.3, we give a SUGRA embedding of the $\frac{1}{M^2}R^\beta$ model with a specific choice of the Kähler potential K and superpotential W. In Section §4.4, we show that the generalized curvature coupling model $\xi\phi^a R^b$ with quantum corrected potential $\lambda\phi^{4(1+\gamma)}$ is equivalent to $R + \frac{1}{M^2}R^\beta$ model and give the relation between the parameters of these two generalized models. Finally we summarize in Section §4.5.

4.2 Power Law Starobinsky Model and its Predictions

We start with a $f(R)$ action of the form [19, 165]

$$S_J = \frac{-M_p^2}{2}\int d^4x\sqrt{-g}\left(R + \frac{1}{6M^2}\frac{R^\beta}{M_p^{2\beta-2}}\right) \tag{4.1}$$

where $M_p^2 = (8\pi G)^{-1}$, g is the determinant of the metric $g_{\mu\nu}$ and M is a dimensionless real parameter. The subscript J refers to Jordan frame which indicates that the gravity sector is not the Einstein gravity form. The action (4.1) can be transformed to an Einstein frame action using the conformal transformation $\tilde{g}_{\mu\nu}(x) = \Omega(x)g_{\mu\nu}(x)$, where Ω is the conformal factor and tilde represents quantities in the Einstein frame. Under conformal transformation the Ricci scalar R in the two frames is related by

$$R = \Omega\Big(\tilde{R} + 3\tilde{\Box}\omega - \frac{3}{2}\tilde{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega\Big) \tag{4.2}$$

where $\omega \equiv \ln\Omega$. If one choose the conformal factor to be $\Omega = F = \frac{\partial f(R)}{\partial R}$ and introduce a new scalar field χ defined by $\Omega \equiv \exp\left(\frac{2\chi}{\sqrt{6}M_p}\right)$, using (4.2), the action

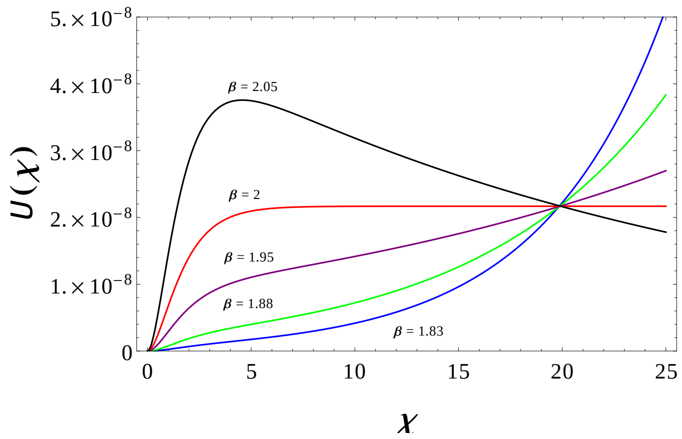

Figure 4.1: The nature of the potential (4.5) for different β values (with $M = 1.7 \times 10^{-4}$). The potential and the field values are in $M_p = 1$ units.

(4.1) gets transform to an Einstein Hilbert form:

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[\frac{-M_p^2}{2}\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + U(\chi)\right] \tag{4.3}$$

where $U(\chi)$ is the Einstein frame potential given by

$$U(\chi) = \frac{(RF(R) - f(R))\, M_p^2}{2F(R)^2} \tag{4.4}$$

which, by using the $f(R)$ form (4.1) and $\Omega = F = \exp\left(\frac{2\chi}{\sqrt{6}M_p}\right)$, can be given explicitly in terms of model parameters M and β as

$$U(\chi) = \frac{(\beta-1)}{2}\left(\frac{6M^2}{\beta^\beta}\right)^{\frac{1}{\beta-1}} \exp\left[\frac{2\chi}{\sqrt{6}}\left(\frac{2-\beta}{\beta-1}\right)\right]\left[1 - \exp\left(\frac{-2\chi}{\sqrt{6}}\right)\right]^{\frac{\beta}{\beta-1}} \tag{4.5}$$

where we have taken $M_p = 1$ and from here onwards we shall work in $M_p = 1$ units. Also we see that in the limit $\beta \to 2$ potential (4.5) reduces to exponentially corrected flat plateau potential of the Starobinsky model.

Assuming large field limit $\chi \gg \frac{\sqrt{6}}{2}$ and $1 < \beta < 2$, the potential (4.5) reduces to

$$U(\chi) \simeq \frac{(\beta-1)}{2}\left(\frac{6M^2}{\beta^\beta}\right)^{\frac{1}{\beta-1}} \exp\left[\frac{2\chi}{\sqrt{6}}\left(\frac{2-\beta}{\beta-1}\right)\right] \tag{4.6}$$

We shall use equation(4.6) later in the Section (4.3) to compare with SUGRA version of the power law potential in the large field limit.

In Fig.5.1 we plot the potential for small deviations from the Starobinsky model value $\beta = 2$. We see that the potential is very flattest for $\beta = 2$ but becomes very steep even with small deviation from Starobinsky model value $\beta = 2$. The scalar curvature perturbation $\Delta^2_{\mathcal{R}} \propto \frac{U(\chi)}{\epsilon}$ is fixed from observations which implies that the magnitude of the potential $U(\chi)$ would have to be larger as ϵ increases for steep potential to maintain the level of observed amplitude $\Delta^2_{\mathcal{R}}$. The tensor perturbation which depends on the magnitude of $U(\chi)$ therefore increases rapidly as β varies from 2. The variation of r with β is shown in the Fig.5.3.

From equation(4.5), in the large field approximation, the slow-roll parameters in Einstein frame can be obtained as

$$\epsilon = \frac{1}{2}\left(\frac{U'}{U}\right)^2 \simeq \frac{1}{3}\left[\frac{\beta(3-2\beta)}{(\beta-1)^2}\exp\left(\frac{-2\chi}{\sqrt{6}}\right)+\frac{\beta-2}{\beta-1}\right]^2, \tag{4.7}$$

$$\eta = \frac{U''}{U} \simeq \frac{-2}{3}\left[\frac{\beta(3-2\beta)^2}{(\beta-1)^3}\exp\left(\frac{-2\chi}{\sqrt{6}}\right)-\frac{(\beta-2)^2}{(\beta-1)^2}\right], \tag{4.8}$$

$$\xi = \frac{U'U'''}{U^2} \simeq \frac{4\sqrt{\epsilon}}{3\sqrt{3}}\left[\frac{\beta(3-2\beta)^3}{(\beta-1)^4}\exp\left(\frac{-2\chi}{\sqrt{6}}\right)+\frac{(\beta-2)^3}{(\beta-1)^3}\right]. \tag{4.9}$$

The field value χ_e at the end of inflation can be fixed from equation(4.7) by using the end of inflation condition $\epsilon \simeq 1$. And the initial scalar field value χ_s corresponding to $N = 60$ e-folds before the end of inflation, when observable CMB modes leave the horizon, can be fixed by using the e-folding expression $N = \int_{\chi_e}^{\chi_s} \frac{U(\chi)}{U'(\chi)} d\chi$.

Under slow-roll approximation we use the standard Einstein frame relations for the amplitude of the curvature perturbation $\Delta^2_{\mathcal{R}} = \frac{1}{24\pi^2}\frac{U^*}{\epsilon^*}$, the spectral index $n_s = 1 - 6\epsilon^* + 2\eta^*$, the running of spectral index $\alpha_s = \frac{dn_s}{d\ln k} = 16\epsilon^*\eta^* - 24(\epsilon^*)^2 - 2\xi^*$ and the tensor-to-scalar ratio $r = 16\epsilon^*$ to fix the parameters of our model. The above mentioned form of the amplitude $\Delta^2_{\mathcal{R}}$ in terms of inflaton potential can be obtained using the slow-roll equation (2.10) into (2.100). Note that the superscript $*$ indicates that the observables are evaluated at the initial field value

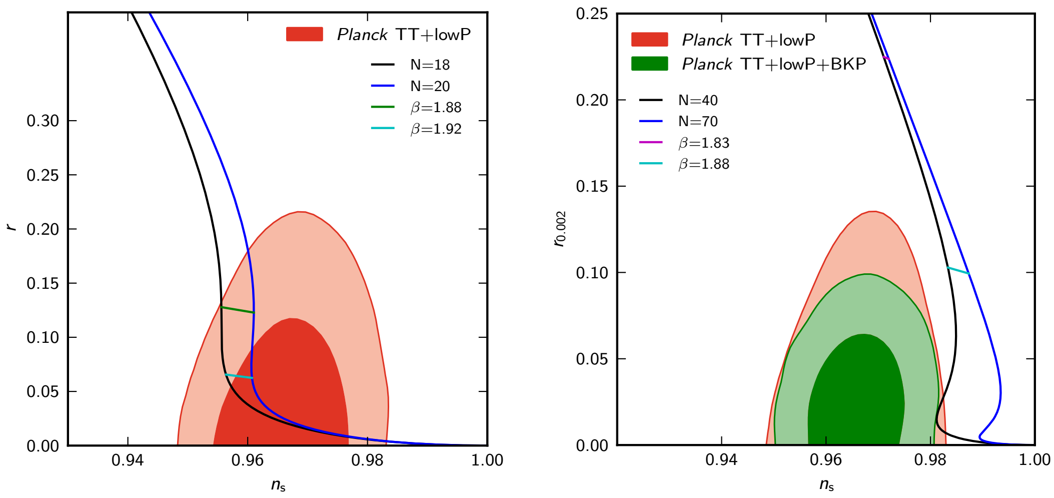

Figure 4.2: The regions of (n_s, r) allowed by Planck-2015 and joint BKP analysis at $68\%CL$ and $95\%CL$ are shown. In the left panel running of n_s is considered and in the right panel there is no running of n_s. The colored contour lines are the predictions for our model for two sets of β and N values corresponding to $M \approx 10^{-4}$ which satisfies the observed amplitude of the CMB power spectrum.

χ_s which is the field value approximately 60 e-folds before the end of inflation.

We know from CMB observations, for 8-parameter ΛCDM+r+α_s model, that if there is a large running of the spectral index $\alpha_s = -0.013^{+0.010}_{-0.009}$ at ($68\%CL$, PlanckTT+lowP) then the amplitude is $10^{10}\ln(\Delta^2_{\mathcal{R}}) = 3.089 \pm 0.072$, the spectral index is $n_s = 0.9667 \pm 0.0132$ and tensor-to-scalar ratio is $r_{0.05} < 0.168$ ($95\%CL$, PlanckTT+lowP) [7, 54, 56]. Also a joint BICEP2/Keck Array and Planck analysis put an upper limit on $r_{0.05} < 0.12(95\%CL)$. Since the scalar potential $U(\chi)$ depends on both the parameters M and β whereas the slow-roll parameters depend only on β, therefore parameter M affects only the scalar amplitude $\Delta^2_{\mathcal{R}} \propto \frac{U(\chi)}{\epsilon}$ whereas r, n_s and α_s which depend only on slow-roll parameters remain unaffected by M. Therefore taking amplitude from the observation and fixing the number of e-foldings N fixes the value of M and β. We find numerically that for the best fit parameter values $\beta \simeq 1.88$ and $M \simeq 1.7 \times 10^{-4}$, the e-foldings turns out to be $N \approx 20$, see Fig.5.2 (left panel). The tensor-to-scalar ratio can be further reduced to $r \approx 0.06$ for $\beta \simeq 1.92$, $M \simeq 10^{-4}$ but e-foldings still comes out to be low $N \approx 20$, see Fig.5.2 (left panel). Therefore constraining model parameters using running data implies that cosmological problems like Horizon

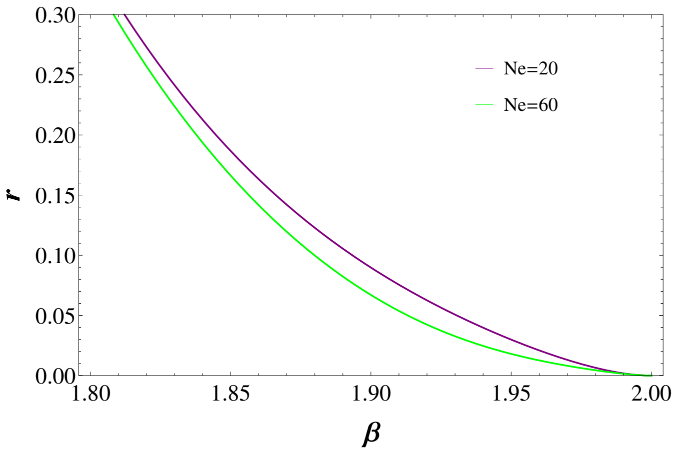

Figure 4.3: The variation of r with β shown for two cases studied in our model: (i) for $N = 20$ when running of n_s is considered and (ii) for $N = 60$ when there is no running of n_s.

and flatness problems which require a minimum of $50 - 60$ e-foldings cannot be solved with the power law generalization of the Starobinsky model.

Also from CMB observations, for 7-parameter $\Lambda CDM{+}r$ model, when there is no scale dependence of the scalar and tensor spectral indices the bound on r becomes tighter $r_{0.002} < 0.1$ ($95\% CL$, PlanckTT+lowP) and the amplitude and the spectral index become $10^{10}\ln(\Delta^2_{\mathcal{R}}) = 3.089 \pm 0.036$ and $n_s = 0.9666 \pm 0.0062$ respectively at ($68\% CL$, PlanckTT+lowP) [7, 54, 56]. We find that the values of $M \simeq 1.7 \times 10^{-4}$ and $\beta \simeq 1.83$ which satisfy the amplitude and the spectral index for $N \approx 60$ gives large $r \approx 0.22$. Also we see that for $\beta \simeq 1.88$ and $M \simeq 1.25 \times 10^{-4}$ tensor-to-scalar ratio can be reduced to $r \simeq 0.1$ but it increases $n_s \simeq 0.985$, see Fig.5.2(lower panel).

4.3 Power Law Starobinsky Model from No-scale SUGRA

In this section we give a SUGRA model of the power law Starobinsky model. We shall derive a model where the scalar potential in the Einstein frame is the same as equation(4.6) which we have shown in the Section §4.2 is equivalent to the power law Starobinsky model $R + \frac{1}{6M^2}R^{\beta}$. The F-term scalar potential in

SUGRA depends upon the combination of the Kähler potential $K(\phi_i)$ and the superpotential $W(\phi_i)$ as $G \equiv K + \ln W + \ln W^*$, where ϕ_i are the chiral superfields whose scalar component are ϕ_i [166]. The effective potential and kinetic term in the Einstein frame are given by (2.145)

$$V = e^G \left[\frac{\partial G}{\partial \phi^i} K^i_{j*} \frac{\partial G}{\partial \phi^*_j} - 3 \right] \tag{4.10}$$

and (2.139)

$$\mathcal{L}_K = K^{j*}_i \partial_\mu \phi^i \partial^\mu \phi^*_j \tag{4.11}$$

respectively, where K^i_{j*} is the inverse of the Kähler metric $K^{j*}_i \equiv \partial^2 K / \partial \phi^i \partial \phi^*_j$.

A no-scale SUGRA model [46] with a choice of the Kähler potential $K = -3 \ln [T + T^* - \phi\phi^*/3]$ and a minimal Wess-Zumino superpotential with a single chiral superfield ϕ

$$W(\phi) = \frac{\mu}{2}\phi^2 - \frac{\lambda}{3}\phi^3 \tag{4.12}$$

gives the same F-term potential in the Einstein frame as the Starobinsky model which give vanishing tensor-to-scalar ratio $r \sim 0.003$ for specific choice $\frac{\lambda}{\mu} = \frac{1}{3}$. A slight change in the ratio $\frac{\lambda}{\mu}$ can increase r upto $r \sim 0.005$ but it gives large $n_s \approx 0.98$.

To get a no-scale SUGRA model corresponding to power law Starobinsky model which can give a larger r, we choose the minimal Wess-Zumino form of the superpotential (5.48) and a minimal no-scale Kähler potential with an added $(\phi + \phi^*)^n$ term as

$$K = -3 \ln \left[T + T^* - \frac{(\phi + \phi^*)^n}{12} \right] \tag{4.13}$$

which can be motivated by a shift symmetry $T \to T + iC$, $\phi \to \phi + iC$ with C real, on the Kähler potential. Here T is a modulus field and ϕ is a matter filed which plays the role of inflaton.

We calculate equation(5.45) and equation(5.46) for chosen Kähler potential (5.47) and superpotential (5.48). We assume that the T field gets a vev $\langle T+T^* \rangle = 2\langle ReT \rangle = c > 0$ and $\langle ImT \rangle = 0$. We write ϕ in terms of its real and imaginary parts $\phi = \phi_1 + i\phi_2$. If we fix the imaginary part of the inflaton field ϕ to be zero then $\phi = \phi^* = \phi_1$ and for simplicity we replace ϕ_1 by ϕ, the effective Lagrangian in the Einstein frame is given by

$$\mathcal{L}_E = \frac{n(2\phi)^{n-2}[c(n-1) + \frac{(2\phi)^n}{12}]}{4[c - \frac{(2\phi)^n}{12}]^2} |\partial_\mu \phi|^2 - \frac{4(2\phi)^{2-n}}{n(n-1)[c - \frac{(2\phi)^n}{12}]^2} \left| \frac{\partial W}{\partial \phi} \right|^2 . \quad (4.14)$$

To make the kinetic term canonical in the $\mathcal{L}_E$, we redefine the field ϕ to χ with

$$\frac{\partial \chi}{\partial \phi} = -\frac{\sqrt{n(2\phi)^{n-2}[c(n-1) + \frac{(2\phi)^n}{12}]}}{2[c - \frac{(2\phi)^n}{12}]} \quad (4.15)$$

Assuming that $n \sim \mathcal{O}(1)$ and the large field limit $(2\phi)^n \gg 12c$ during inflation, integrating equation(4.15) gives

$$\phi \simeq \frac{1}{2} \exp\left(\frac{2\chi}{\sqrt{3n}}\right) \left[1 + \frac{6c(n+1)}{n} \exp\left(\frac{-2n\chi}{\sqrt{3n}}\right)\right] \quad (4.16)$$

Now substituting from equation(5.48) and equation(4.16) into the potential term of equation(4.14) and simplifing, we get the effective scalar potential in the Einstein frame as

$$\begin{aligned} V &= \frac{144\mu^2}{n(n-1)} \exp\left[\frac{2\chi}{\sqrt{6}} \left(\frac{3\sqrt{2}(2-n)}{\sqrt{n}}\right)\right] \\ &\times \left[1 - \frac{2\mu}{\lambda} \exp\left(\frac{-2\chi}{\sqrt{3n}}\right) - \frac{9c(n^2-n-2)}{n} \exp\left(\frac{-2n\chi}{\sqrt{3n}}\right)\right]^2 , \end{aligned} \quad (4.17)$$

which, assuming $1 < n < 2$, in the large field limit $\chi \gg \frac{\sqrt{3n}}{2}$ is equivalent to

$$V \simeq \frac{144\mu^2}{n(n-1)} \exp\left[\frac{2\chi}{\sqrt{6}} \left(\frac{3\sqrt{2}(2-n)}{\sqrt{n}}\right)\right] \quad (4.18)$$

We see that in the limit $n \to 2$ and with the specific choice $\frac{\lambda}{\mu} = \frac{1}{2}$, the potential (4.17) reduces to Starobinsky Model potential.

β	M	n	$\mu = \frac{\lvert\lambda\rvert}{2}$	$\alpha_s = \frac{dn_s}{d\ln k}$
1.83	1.7×10^{-4}	1.93	3.13×10^{-6}	-9.16×10^{-6}
1.88	1.7×10^{-4}	1.96	5.54×10^{-6}	-2.86×10^{-3}
2.00	1.1×10^{-5}	2.00	1.16×10^{-6}	-5.23×10^{-4}

Table 4.1: The SUGRA model parameter values (in $M_p = 1$ unit) for three values of β corresponding to running and without running of spectral index n_s as depicted in Fig.5.2 and for Starobinsky limit $\beta = 2$.

We can now compare the power law potential (4.6) and SUGRA potential (4.18) for inflaton to show the relation between the parameters of the two model. Comparing the constant coefficient and exponent in the two potentials we get

$$\beta = \frac{2\sqrt{n} + 3\sqrt{2}(2-n)}{\sqrt{n} + 3\sqrt{2}(2-n)} \tag{4.19}$$

and

$$M^2 = \frac{\beta^\beta}{6}\left[\frac{288\mu^2}{n(n-1)(\beta-1)}\right]^{\beta-1}. \tag{4.20}$$

Numerically we evaluate the SUGRA model parameter values (in $M_p = 1$ unit) for three values of β corresponding to running and without running of spectral index n_s as depicted in Fig.5.2 and for Starobinsky limit $\beta = 2$. These values are shown in the TABLE[4.1].

4.4 Equivalence of the Power-law Starobinsky Model with Generalized Non-minimally Curvature Coupled Models

In this section we will show that generalized non-minimally coupled Inflation models $\xi\phi^a R^b$ [128] with the quantum corrected ϕ^4-potential [129–131] can be reduced to the power law Starobinsky form. We consider the generalized non-minimal coupling $\xi\phi^a R^b$ and the quantum correction to quartic scalar potential

$\phi^{4(1+\gamma)}$ into the action

$$S_J = \int d^4x\sqrt{-g}\left(-\frac{M_p^2 R}{2} - \frac{\xi\phi^a R^b}{2M_p^{a+2b-4}} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{\lambda\phi^{4(1+\gamma)}}{4M_p^{4\gamma}}\right) \tag{4.21}$$

where the scalar field ϕ is the inflaton field. Since during inflation potential energy of the scalar field is dominant therefore kinetic term in the action S_J can be neglected w.r.t. potential, the action reduces to

$$\int d^4x\sqrt{-g}\left(-\frac{M_p^2 R}{2} - \frac{\xi\phi^a R^b}{2M_p^{a+2b-4}} + \frac{\lambda\phi^{4(1+\gamma)}}{4M_p^{4\gamma}}\right) \tag{4.22}$$

we may integrate out the scalar field through its equation of motion $\frac{\partial L}{\partial\phi} \approx 0$ [132], which implies

$$\phi \approx \left(\frac{\xi a R^b}{2\lambda(1+\gamma)M_p^{a+2b-4(1+\gamma)}}\right)^{\frac{1}{4(1+\gamma)-a}} \tag{4.23}$$

Using equation(4.23) for ϕ, the action (4.22) reduces to power law Starobinsky action

$$\int d^4x\sqrt{-g}\left(\frac{-M_p^2}{2}\right)\left(R + \frac{1}{6M^2}\frac{R^\beta}{M_p^{2\beta-2}}\right) \tag{4.24}$$

where the two parameters β and M of the power law model are identified in terms of a, b, λ, ξ and γ as

$$\beta = \frac{4b(1+\gamma)}{4(1+\gamma)-a} \tag{4.25}$$

and

$$M^2 = \frac{a}{3(4(1+\gamma)-a)\lambda}\left(\frac{2\lambda(1+\gamma)}{\xi a}\right)^{\frac{4(1+\gamma)}{4(1+\gamma)-a}} \tag{4.26}$$

which for $a = 2,\ b = 1, \gamma = 0$, i.e at $\beta = 2$, reduces to Higgs Inflation-Starobinsky case $M_S^2 \approx \frac{\lambda}{3\xi^2} \approx 10^{-10}$. Also with $a = 2, b = 1, \gamma \neq 0$ results of the Higgs inflation models with quantum corrected potential can be obtained [129, 130].

4.5 Conclusions

We have explored a generalization of the Starobinsky model with a $\frac{1}{M^2}R^\beta$ model and fit β and M from CMB data. We find that to fit the amplitude $\Delta^2_{\mathcal{R}}$ and the spectral index n_s (with no running) from observations [7, 54, 56] we require $M \simeq 1.7 \times 10^{-4}$ and $\beta \simeq 1.83$ for $N \approx 60$ but these parameter values gives large $r \approx 0.22$. Also we find that the parameters β and M deviates from $M \approx 10^{-5}$ and $\beta = 2$ of the original Starobinsky model which could fit the amplitude and the spectral index but predicted very small value of $r \sim 10^{-3}$. When large running of the spectral index $\alpha_s \sim 10^{-3}$ is considered we find that the best fit parameter values are $\beta \simeq 1.88$ and $M \simeq 1.7 \times 10^{-4}$ which corresponds to $N \approx 20$. This implies that the standard cosmological problems like Horizon and flatness problems which require a minimum of $50 - 60$ e-foldings cannot be solved with the power law generalization of the Starobinsky model.

We have shown that the 5-parameters generalized non-minimal scalar-curvature coupled inflation models with the quantum correction to quartic scalar potential *i.e.* $\xi\phi^a R^b + \phi^{4(1+\gamma)}$ are actually equivalent to 2-parameter power law Starobinsky model $\frac{1}{M^2}R^\beta$. Therefore we see that in terms of number of parameters the power law model is the most economical parametrization of the class of scalar-curvature models with quantum corrected ϕ^4-potential.

In this paper we have given a SUGRA model for the general power law $\frac{1}{M^2}R^\beta$ model by adding a $(\phi + \bar{\phi})^n$ term to the minimal no-scale Kähler potential and with a Wess-Zumino form of the superpotential $W(\phi)$. In the limit $n = 2$ the Starobinsky limit $\beta = 2$ is obtained. We derive the relations between the two parameters of the power-law Starobinsky model and the two parameters of our SUGRA model. The interesting point about the generalization is that the small deviations from the Starobinsky limit of $n = \beta = 2$ can produce value of larger value of $r \sim 0.1$. Generalizations of the Starobinsky model which can explain a possible larger value of r are therefore of interest.

Chapter 5

Two-Field Model of Inflation and its Motivation from No-scale SUGRA

5.1 Overview

In this chapter we study a two-field model of inflation where inflaton field is accompanied by a dilaton field and has a non-canonical kinetic term due to the presence of the dilaton field. And the effective potential is a product type of potential *i.e.* inflaton potential times the exponential potential of dilaton field. We show that novelty of such an inflationary scenario is that the quartic and quadratic inflaton potentials, which are not favoured by the present Planck data, yield scalar spectral index and tensor-to-scalar ratio which are in accordance with the present data. Such a model yield a tensor-to-scalar ratio of the order of 10^{-2} which can be probed by future $B-$mode experiments like Keck/BICE3, SPT-3G, LiteBIRD, CMBPol, Spider and thus can be put to test in future. The action of such a model in Einstein frame can be generically written as

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}\nabla^\mu\sigma\nabla_\mu\sigma - \frac{1}{2}e^{-\frac{\gamma\sigma}{M_p}}\nabla^\mu\phi\nabla_\mu\phi - e^{-\frac{\beta\sigma}{M_p}}V(\phi)\right] \tag{5.1}$$

where M_p is the reduced Planck mass and β and γ are arbitrary parameters. For brevity of the text we shall use $M_p = 1$ unit from here onwards. This action is equivalent to Brans-Dicke action in Jordan frame [167], as argued in ref. [168], wherein Brans-Dicke parameter ω_{BD} can be identified in terms of the parameter β or γ. The Brans-Dicke action can be derived from Higher-dimensional, induced gravity or string theories. The spectrum of density fluctuations in such a two field models, which include an inflaton field and a dilaton field, are calculated in [168–171] and the parameters of the generalized scalar-tensor theories are constrained. In these model $\gamma = \beta/2$ and they give no better predictions than standard slow-roll inflation for quartic and quadratic potentials. In the limits β and γ tending to zero their predictions converge to standard slow-roll predictions and for any positive values of these parameters with $\beta > \gamma$, the predictions are worse. However, if $\gamma > \beta$ then the model can correctly predict the observables n_s and r. We will see that it is possible to derive a two-field action (5.1) in Supergravity theories with appropriate choice of Kähler potential and superpotential where β and γ can appear as two arbitrary parameters and we can have $\gamma > \beta$. Two field inflation with non-minimal curvature coupling of the inflaton field is studied in [172] and it gives small $r \sim 0.009$.

It is well known that the simplest single-field slow-roll inflation models with most common chaotic potentials, *e.g.* quartic ($\lambda\phi^4$) and quadratic ($m_\phi^2\phi^2$) potentials produce large tensor-to-scalar ratio $r \simeq 0.26$ and $r \simeq 0.13$ respectively, for $\Delta N \approx 60$ where ΔN is the number of e-foldings from the end of inflation. Planck-2015 high-ℓ polarization data puts an upper bound on $r_{0.002} < 0.11\,(95\%\ \text{CL})$ and the predicted amplitude and the spectral index are $10^{10}\ln(\Delta^2_{\mathcal{R}}) = 3.089 \pm 0.036$ and $n_s = 0.9666{\pm}0.0062$ respectively at ($68\%CL$, PlanckTT+lowP) [7, 54]. Joint BKP analysis of B-mode polarization data puts an upper limit $r_{0.05} < 0.12\,(95\%$ CL) [56]. More recently BICEP2/Keck Array CMB polarization experiments combined with Planck analysis of CMB polarization and temperature data have further improved the bound on $r_{0.05} < 0.07\,(95\%\ \text{CL})$[57]. These observations indicate that single-field models with quartic and quadratic potentials are ruled out at $\ln B = -23.3$ and $\ln B = -4.7$ respectively [54]. Also these models predict very small inflaton self coupling ($\lambda \sim 10^{-13}$) and very light inflaton with mass ($m_\phi \sim 10^{13} GeV$) [50] at the GUT scale in order to yield correct scalar amplitude which prohibits the inflaton in these models to be interpreted as the

Standard Model Higgs at GUT scale [98].

The Higgs inflation scenario [98], where the Higgs inflaton field ϕ is non-minimally coupled to the curvature scalar R (coupling term looks like $\xi\phi^2 R$), gives rise to very small tensor-to-scalar ratio $r \sim 0.003$ for $N = 60$ which is consistent with the Joint BKP bound on r and makes it a highly favored inflationary model after the release of PLANCK-2013 data [173]. Even though, such a model encounter the problem of unitarity violation because of very large curvature coupling $\xi \sim 10^4$ [174]. Starobinsky model of inflation with $\frac{1}{M^2}R^2$ correction to Einstein gravity is mathematically equivalent to Higgs inflation model with quartic potential and therefore produces small tensor-to-scalar ratio $r \sim 0.003$ [9, 10]. As we studied in the previous Chapter §4, the generalized non-minimally coupled models with coupling term $\xi\phi^a R^b$ and quantum corrected $\lambda\phi^{4(1+\gamma)}$ potentials, which are equivalent to power law inflation model $R + R^\beta/M^2$ predicts large tensor-to-scalar ratio $r \sim 0.2$ [128–130, 175]. Therefore these models are also ruled out by present status of the data. A study of genus topology and cross-correlation of BICEP2 and Planck B-Modes produced by gravity waves in the early universe put tensor-to-scalar ratio at $r = 0.11 \pm 0.04(68\%CL)$ [126]. Also there is a possibility that future B-mode observations like Keck/BICEP3, SPT-3G, LiteBIRD, CMBPol, Spider may put r around $r \approx 0.1 - 0.01$ and therefore Starobinsky model along with Higgs inflationary scenario might become incompatible with the observations in future. However we will see that the two-field model for certain choice of parameters β and γ values with $\gamma > \beta$ can produce tensor-to-scalar ratio in this range.

In order to motivate the above two-field action with $\gamma > \beta$, we will show that such an action can be derived from no-scale supergravity. The two-field models of inflation with string motivated tree-level no-scale Kähler potential in no-scale supergravity framework are analyzed in Ref.s [155, 160, 161, 176]. With the considered form of Kähler potential and superpotantial, we will derive the two-field action (5.1) where the parameter γ has a fixed value $\gamma = 2\sqrt{2/3}$ and β is arbitrary, therefore we can have $\gamma > \beta$ as required to achieve the correct values of the observables n_s and r.

The chapter is organised as follows : In Section (5.2) we discuss in detail the evolution of background and perturbations in our model and derive the co-moving curvature power spectrum and isocurvature power spectrum along with the observables namely the scalar spectral index and the tensor-to-scalar ratio. In Section (5.3) we consider two different inflaton potentials, namely quadratic and quartic, to show that these two potentials yield scalar spectral index and tensor-to-scalar ratio which are compatible with present observations. In the Section (5.4) in order to motivate two-field model from a fundamental theory we give a SUGRA derivation of this model. In Section (5.5) we will discuss the main results of this chapter and conclude.

5.2 The Model

In this section we would derive the evolution of background and scalar perturbations if one considers the action given in Eq. (5.1) with arbitrary β and γ.

5.2.1 Background Evolution

In a homogeneous and isotropic FLRW universe the variation of the action (5.1) w.r.t. fields and metric yields the equations of motion of the scalar fields and Friedmann equations as

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{\gamma}{2}e^{-\gamma\sigma}\dot{\phi}^2 - \beta e^{-\beta\sigma}V(\phi) = 0 \tag{5.2}$$

$$\ddot{\phi} + 3H\dot{\phi} - \gamma\dot{\sigma}\dot{\phi} + e^{(\gamma-\beta)\sigma}V'(\phi) = 0 \tag{5.3}$$

$$3H^2 = \frac{1}{2}\dot{\sigma}^2 + \frac{1}{2}e^{-\gamma\sigma}\dot{\phi}^2 + e^{-\beta\sigma}V(\phi) \tag{5.4}$$

$$\dot{H} = -\frac{1}{2}\left(\dot{\sigma}^2 + e^{-\gamma\sigma}\dot{\phi}^2\right) \tag{5.5}$$

where an over dot represents derivatives w.r.t. time and prime denotes derivative with respect to ϕ.

In the slow-roll regime when both the fields slow-roll, terms with single and double time derivatives can be neglected therefore the background equations reduces to

$$3H\dot{\sigma} = \beta e^{-\beta\sigma} V(\phi), \tag{5.6}$$

$$3H\dot{\phi} = -e^{(\gamma-\beta)\sigma} V'(\phi), \tag{5.7}$$

$$3H^2 = e^{-\beta\sigma} V(\phi). \tag{5.8}$$

We define the slow-roll parameters for both the fields. Here the full potential $U(\sigma,\phi) = e^{-\beta\sigma}V(\phi)$ can be regarded as the product of potentials of the two fields. We then define the slow-roll parameters as :

$$\begin{aligned}
\epsilon_\phi &= \frac{1}{2}\left(\frac{U_\phi}{U}\right)^2 = \frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2, \\
\eta_\phi &= \frac{U_{\phi\phi}}{U} = \frac{V''(\phi)}{V(\phi)}, \\
\epsilon_\sigma &= \frac{1}{2}\left(\frac{U_\sigma}{U}\right)^2 = \frac{\beta^2}{2}, \\
\eta_\sigma &= \frac{U_{\sigma\sigma}}{U} = \beta^2,
\end{aligned} \tag{5.9}$$

Now to have the σ field slow-roll during inflation we require $\beta^2 \ll 2$.

Using equations (5.8), the evolution of the background fields is given by

$$\sigma = \sigma_0 + \beta \ln\left(\frac{a}{a_0}\right), \tag{5.10}$$

$$\int d\phi \frac{V(\phi)}{V'(\phi)} = -\frac{e^{\gamma\sigma_0}}{\beta\gamma}\left[\left(\frac{a}{a_0}\right)^{\beta\gamma} - 1\right] \tag{5.11}$$

We define

$$f(\phi) \equiv \int d\phi \frac{V(\phi)}{V'(\phi)}, \tag{5.12}$$

For sufficient inflation we need $\frac{a_f}{a_0} \gtrsim e^{\Delta N}$, combining (5.11) and (5.12), yields

$$\frac{1}{\beta\gamma}\ln\left[1 + \beta\gamma e^{-\gamma\sigma_0}(f(\phi_0) - f(\phi_f))\right] \gtrsim \Delta N. \tag{5.13}$$

In the above expressions the subscript $'0'$ and $'f'$ represent the field values at the start and end of inflation respectively and the same notation will be used in the rest of the text.

5.2.2 Linear Perturbations

In this section we will consider the linear perturbations to background fields $\sigma(t) + \delta\sigma(t, x^i)$,$\phi(t) + \delta\phi(t, x^i)$ and metric $g_{\mu\nu}(t) + \delta g_{\mu\nu}(t, x^i)$. In a longitudinal gauge the metric perturbations can be given as

$$ds^2 = -(1 + 2\Phi)dt^2 + a(t)^2(1 - 2\Phi)\delta_{ij}dx^i dx^j \tag{5.14}$$

and the equations of motion of scalar field perturbations are

$$\begin{aligned}
&\delta\ddot{\sigma} + 3H\delta\dot{\sigma} + \left(\frac{k^2}{a^2} - \frac{\gamma^2}{2}e^{-\gamma\sigma}\dot{\phi}^2 + \beta^2 e^{-\beta\sigma}V(\phi)\right)\delta\sigma + \gamma e^{-\gamma\sigma}\dot{\phi}\delta\dot{\phi} - \beta e^{-\beta\sigma}V'(\phi)\delta\phi \\
&= 2\beta e^{-\beta\sigma}V(\phi)\Phi + 4\dot{\Phi}\dot{\sigma}, \qquad (5.15)\\
&\delta\ddot{\phi} + (3H - \gamma\dot{\sigma})\,\delta\dot{\phi} + \left(\frac{k^2}{a^2} + e^{(\gamma-\beta)\sigma}V''(\phi)\right)\delta\phi - \gamma\dot{\phi}\delta\dot{\sigma} + (\gamma - \beta)V'(\phi)e^{(\gamma-\beta)\sigma}\delta\sigma \\
&= -2e^{(\gamma-\beta)\sigma}V'(\phi)\Phi + 4\dot{\Phi}\dot{\phi} \qquad (5.16)
\end{aligned}$$

the ii, 00 and $0i$-components of the perturbed field equations give

$$\begin{aligned}
\ddot{\Phi} + 4H\dot{\Phi} + (3H^2 + \dot{H})\Phi &= \frac{1}{2}\Big[\dot{\sigma}\delta\dot{\sigma} + e^{-\gamma\sigma}\dot{\phi}\delta\dot{\phi} - e^{-\beta\sigma}V'(\phi)\delta\phi \\
&\quad -\beta\left(\frac{1}{4}e^{-\gamma\sigma}\dot{\phi}^2 - e^{-\beta\sigma}V(\phi)\right)\delta\sigma\Big], \qquad (5.17)\\
3H\dot{\Phi} + (3H^2 + \dot{H})\Phi + \frac{k^2}{a^2}\Phi &= \frac{-1}{2}\Big[\dot{\sigma}\delta\dot{\sigma} + e^{-\gamma\sigma}\dot{\phi}\delta\dot{\phi} + e^{-\beta\sigma}V'(\phi)\delta\phi \\
&\quad -\beta\left(\frac{1}{4}e^{-\gamma\sigma}\dot{\phi}^2 + e^{-\beta\sigma}V(\phi)\right)\delta\sigma\Big] \qquad (5.18)\\
\dot{\Phi} + H\Phi &= \frac{1}{2}(\dot{\sigma}\delta\sigma + e^{-\gamma\sigma}\dot{\phi}\delta\phi) \qquad (5.19)
\end{aligned}$$

respectively.

The comoving curvature perturbations on the constant energy density hypersurfaces [169, 170] are (2.46)

$$\mathcal{R} = \Phi - \frac{H}{\dot{H}}\left(\dot{\Phi} + H\Phi\right) = \Phi + H\frac{\dot{\sigma}\delta\sigma + e^{-\gamma\sigma}\dot{\phi}\delta\phi}{\dot{\sigma}^2 + e^{-\gamma\sigma}\dot{\phi}^2} \tag{5.20}$$

Also the time evolution of the comoving curvature perturbations can be obtained by combining equations (5.17), (5.18) and (5.19) as

$$\dot{\mathcal{R}} = \frac{k^2}{a^2}\frac{H^2}{\dot{H}}\Phi + \mathcal{S} \tag{5.21}$$

where $\mathcal{S}$ represents entropy (isocurvature) perturbations given by

$$\mathcal{S} = 2H\frac{e^{-\beta\sigma}(\beta\dot{\sigma}\dot{\phi}^2 V(\phi)e^{-\gamma\sigma} + \dot{\phi}\dot{\sigma}^2 V'(\phi))}{(\dot{\sigma}^2 + e^{-\gamma\sigma}\dot{\phi}^2)^2}\left(\frac{\delta\sigma}{\dot{\sigma}} - \frac{\delta\phi}{\dot{\phi}}\right) \tag{5.22}$$

On super horizon scales ($k \ll aH$) and under slow-roll approximation one can safely ignore terms containing $\dot{\Phi}$ and double-time derivatives to get

$$\Phi = \frac{1}{2H}(\dot{\sigma}\delta\sigma + e^{-\gamma\sigma}\dot{\phi}\delta\phi) = \frac{\beta}{2}\delta\sigma - \frac{V'(\phi)}{2V(\phi)}\delta\phi, \tag{5.23}$$

$$3H\delta\dot{\sigma} + \beta^2 e^{-\beta\sigma}V(\phi)\delta\sigma - \beta e^{-\beta\sigma}V'(\phi)\delta\phi = 2\beta e^{-\beta\sigma}V(\phi)\Phi, \tag{5.24}$$

$$3H\delta\dot{\phi} + e^{(\gamma-\beta)\sigma}V''(\phi)\delta\phi + (\gamma-\beta)e^{(\gamma-\beta)\sigma}V'(\phi)\delta\sigma = -2e^{(\gamma-\beta)\sigma}V'(\phi)\Phi. \tag{5.25}$$

The above equations can be solved to give superhorizon solutions as, [see [168]]

$$\frac{\delta\sigma}{\dot{\sigma}} = \frac{c_1}{H} - \frac{c_3}{H}, \tag{5.26}$$

$$\frac{\delta\phi}{\dot{\phi}} = \frac{c_1}{H} + \frac{c_3}{H}\left(e^{-\gamma\sigma} - 1\right), \tag{5.27}$$

$$\Phi = -c_1\frac{\dot{H}}{H^2} + c_3\left[\frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2(1 - e^{\gamma\sigma}) - \frac{\beta^2}{2}\right] \tag{5.28}$$

where c_1 and c_3 are the time independent integration constants and can be fixed using initial conditions. In the above expression (5.28), terms proportional to

c_1 represent the adiabatic modes while those proportional to c_3 represent the isocurvature modes.

The equations of motion for scalar perturbations can be approximated as equations of motion of free massless scalar field in inflating background for $k \geq aH$ and even in the region $k < aH$ when $H(t_k) \gg |\dot{H}(t_k)|(t - t_k)$ and thus the expectation values of the scalar perturbations for the modes k crossing the Hubble horizon *i.e.* when $k = a(t_k)H(t_k)$ can be given by

$$\langle|\delta\sigma_k|^2\rangle = \frac{H(t_k)^2}{2k^3} \; ; \qquad \langle|\delta\phi_k|^2\rangle = \frac{H(t_k)^2}{2k^3} e^{\gamma\sigma(t_k)} \tag{5.29}$$

where the exponential factor in $\langle|\delta\phi_k|^2\rangle$ is due to the non-canonical kinetic term of the inflaton field in the Einstein frame.

Using background slow-roll equations (5.6)-(5.8), equation (5.20) can be simplified to give the curvature perturbations as

$$\mathcal{R} \simeq \Phi + c_1 - c_3 + c_3 \frac{\frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2}{\frac{\beta^2}{2} + \frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 e^{\gamma\sigma}} \tag{5.30}$$

$$\simeq \Phi + c_1 - c_3 + c_3 \frac{\epsilon_\phi}{\epsilon_\sigma + e^{\gamma\sigma}\epsilon_\phi} \tag{5.31}$$

Since from equation(5.28) it is clear that all the terms in Φ are proportional to $(c_1, c_3)\times$ slow-roll parameters, therefore we will ignore the potential Φ compared to c_1 and c_3. Using equation (5.26) and equation (5.27) we can calculate c_1 and c_3. Substituting for c_1 and c_3 into equation (5.31), the comoving curvature perturbations on super horizon scales becomes

$$\mathcal{R} = H\frac{\delta\phi}{\dot{\phi}} e^{\gamma\sigma} A + H\frac{\delta\sigma}{\dot{\sigma}} B \tag{5.32}$$

where $A = \epsilon_\phi/(\epsilon_\sigma + e^{\gamma\sigma}\epsilon_\phi)$ and $B = \epsilon_\sigma/(\epsilon_\sigma + e^{\gamma\sigma}\epsilon_\phi)$. Therefore, using equation(5.29), the power spectrum of comoving curvature perturbations can be obtained as

$$\mathcal{P}_{\mathcal{R}} = \frac{k^3}{2\pi^2}\langle \mathcal{R}^2\rangle \simeq \frac{H^4}{4\pi^2}\left[\frac{e^{3\gamma\sigma}}{\dot{\phi}^2}A^2 + \frac{1}{\dot{\sigma}^2}B^2\right] \tag{5.33}$$

$$\simeq \frac{V(\phi)e^{-\beta\sigma}}{24\pi^2}\left[\frac{e^{\gamma\sigma}}{\epsilon_\phi}A^2 + \frac{1}{\epsilon_\sigma}B^2\right] \tag{5.34}$$

where $\dot{\sigma}^2$ and $\dot{\phi}^2$ are given by equations (5.6) and (5.7).

In the Einstein frame the tensor power spectrum retains its generic form as

$$\mathcal{P}_{\mathcal{T}} = \frac{8H^2(t_k)}{4\pi^2}, \tag{5.35}$$

which, combining with (5.34), yields the tensor-to-scalar ratio as

$$r \equiv \frac{\mathcal{P}_{\mathcal{T}}}{\mathcal{P}_{\mathcal{R}}} \simeq 16\left[\frac{e^{\gamma\sigma}}{\epsilon_\phi}A^2 + \frac{1}{\epsilon_\sigma}B^2\right]^{-1} \tag{5.36}$$

Now once we have the power spectrum of scalar perturbations (5.33), we can find the scalar spectral index using (2.101)

$$\begin{aligned} n_s - 1 &= \frac{d\ln \mathcal{P}_{\mathcal{R}}(k)}{d\ln k} \\ &= \frac{d\ln \mathcal{P}_{\mathcal{R}}(k)}{dt} \times \frac{dt}{dN} \times \frac{dN}{d\ln k} \\ &\simeq \frac{4\dot{H}}{H^2} + \frac{1}{HP_1}\frac{dP_1}{dt} \end{aligned} \tag{5.37}$$

where $P_1 = (e^{3\gamma\sigma}/\dot{\phi}^2)A^2 + (1/\dot{\sigma}^2)B^2$ and we used the fact $\frac{dt}{dN} = \frac{1}{H}$ and at horizon crossing $k = a(t_k)H(t_k)$,

$$\frac{dN}{d\ln k} \simeq 1 - \frac{\dot{H}}{H^2} \approx 1. \tag{5.38}$$

Using background slow-roll equations (5.5)-(5.8), we calculate $\ddot{\sigma}$ and $\ddot{\phi}$, equation (5.37) can be further simplified to give

$$n_s - 1 \simeq -A\left[(6\epsilon_\phi - 2\eta_\phi)e^{2\gamma\sigma} + \beta(2\beta+\gamma)e^{\gamma\sigma}\right] - B\beta^2 \tag{5.39}$$

It should be noted that in the limit $\beta \to 0$ and $\gamma \to 0$, $A = 1$ and $B = 0$, equations (5.34),(5.36) and (5.37) for power spectrum, tensor-to-scalar ratio and

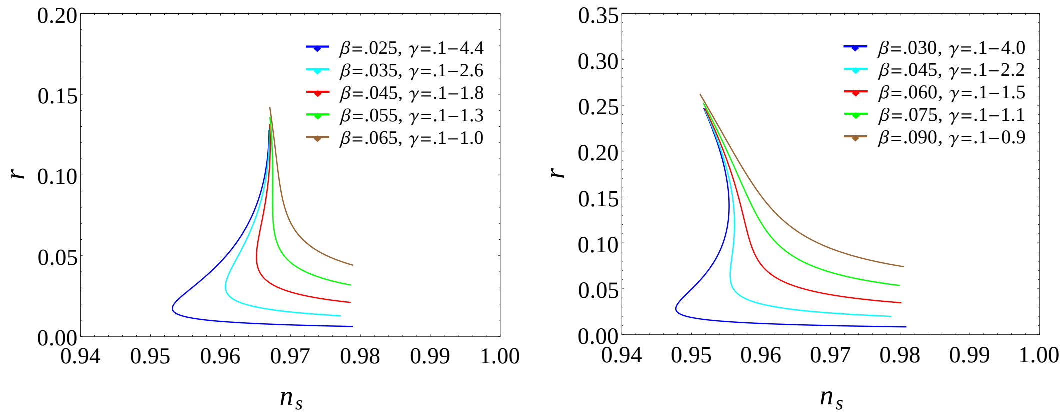

Figure 5.1: $n_s - r$ predictions of our two-field model for quadratic (left panel) and quartic (right panel) potentials are shown. We have taken $\Delta N = 60$ and $\sigma_0 = 0.1$. In both the panels the range of values of γ increases along the curves from top to bottom. It is also clear that as the values of β and γ goes to zero, n_s and r values converges to standard slow-roll inflation predictions.

spectral index respectively reduces to their standard forms in the single-field slow-roll inflation.

Also, using equation (5.29) into equation (5.22), the amplitude of the isocurvature perturbations is given by

$$\begin{aligned} \mathcal{P}_{\mathcal{S}} &= \frac{k^3}{2\pi^2}\langle \mathcal{S}^2 \rangle \\ &= \frac{H^4}{\pi^2}\frac{e^{(\gamma-2\beta)\sigma}\left(\beta e^{-\gamma\sigma}\dot{\phi}V(\phi) + \dot{\sigma}V'(\phi)\right)^2}{(\dot{\sigma}^2 + e^{-\gamma\sigma}\dot{\phi}^2)^3} \end{aligned} \tag{5.40}$$

which vanishes upon using equations (5.6) and (5.7) for $\dot{\sigma}$ and $\dot{\phi}$ respectively. Therefore in this two-field model, up to slow-roll approximation, the isocurvature perturbations vanishes independent of inflaton potential and β, γ values.

5.3 Analysis of the Model with $\lambda_n \phi^n$ Potentials

In this section we analyze the observable parameters when inflaton has a potential $V(\phi) = \frac{\lambda_n}{n}\phi^n$. We define σ_0 and ϕ_0 as the field values ΔN e-folds before the end of inflation and σ_f and ϕ_f field values at the end of inflation. From equation (5.11) we find

$$\sigma_f = \sigma_0 + \beta\Delta N. \tag{5.41}$$

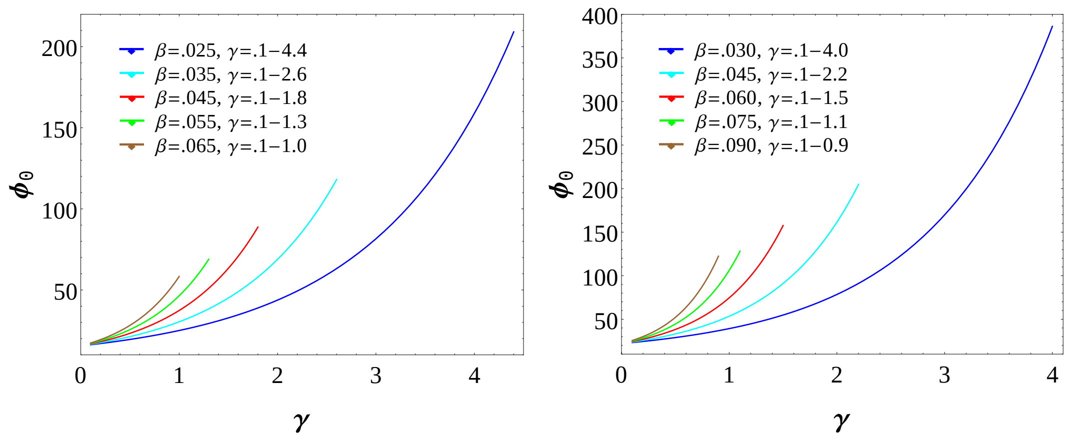

Figure 5.2: The inflaton values during inflation, for quadratic (left panel) and quartic (right panel) potentials, are shown. We have taken $\Delta N = 60$ and $\sigma_0 = 0.1$. In both the panels the range of values of γ increases along the curves from bottom to top. It is also clear that as the values of β and γ goes to zero, the field values converges to standard slow-roll inflation predictions.

And using equation (5.6)-(5.8) we can solve for $\frac{\ddot{a}}{a} = H^2 + \dot{H}$ and setting $\frac{\ddot{a}}{a} = 0$, which is the condition for the end of inflation, we get the inflaton field value at the end of inflation as

$$\phi_f = n e^{\gamma\sigma_f/2}/\sqrt{2-\beta^2}. \tag{5.42}$$

And the field value ϕ_0 is given by equation(5.13) as

$$\begin{aligned}\phi_0^2 &\simeq \phi_f^2 + \frac{2ne^{\gamma\sigma_0}}{\beta\gamma}\left(e^{\beta\gamma\Delta N} - 1\right)\\ &\simeq \frac{ne^{\gamma\sigma_0}\left[e^{\beta\gamma\Delta N}(4-2\beta^2+n\beta\gamma) - 4 + 2\beta^2\right]}{\beta\gamma(2-\beta^2)}\end{aligned} \tag{5.43}$$

Now we substitute ϕ_0 from equation (5.43) into equations (5.34), (5.36) and (5.37) to give n_s, r and $\mathcal{P}_\mathcal{R}$ in terms of σ_0, n, ΔN, β and γ. For $\Delta N = 60$ e-folds and for the choice $\sigma_0 = 0.1$ with various choices of the parameters β and γ, the $n_s - r$ predictions for quadratic ($n = 2$) and quartic ($n = 4$) potentials are shown in the Fig. 5.1. For the above choices of the parameters values, inflaton field values during inflation are shown in the Fig. 5.2. For $\sigma_0 = 0.1$, $\Delta N = 60$ and for the range of the parameter values of (β, γ) as shown in Fig. 5.1, we finds scalar mass in the range $\lambda_2 = m_\phi^2 \sim 10^{-11} - 10^{-14}$ and scalar self coupling in the range $\lambda_4 = \lambda \sim 10^{-13} - 10^{-17}$. E.g. for the choice $\beta = 0.05$ and $\gamma = 0.7$, which can produce $n_s \simeq 0.9666$ and $r \simeq 0.06$, gives $m_\phi \approx 2 \times 10^{-6}$. And for $\beta = 0.06$ and $\gamma = 1$, which gives $n_s \simeq 0.964$ and $r \simeq 0.05$, gives $\lambda \approx 10^{-16}$. Therefore in this two-field scenario with quadratic and quartic potentials, similar to the case of single-field slow-roll inflation, we require light inflaton mass and fine-tuning of the inflaton self-couplings in order to explain the observables. However, unlike the Higgs inflationary scenario which has no fine tuning problem predicts very small tensor-to-scalar ratio $r \sim 10^{-3}$, the two-field model predicts larger tensor-to-scalar ratio $r \sim 10^{-1} - 10^{-2}$.

5.4 Deriving Two-Field Model Action from No-scale Supergravity

In this section we give a derivation of the two-field inflation model from no-scale Supergravity. The F-term scalar potential in EF is determined from Kähler function given in terms of Kähler potential $K(\phi_i, \phi_i^*)$ and superpotential $W(\phi_i)$:

$$G(\phi_i, \phi_i^*) \equiv K(\phi_i, \phi_i^*) + \ln W(\phi_i) + \ln W^*(\phi_i^*) \tag{5.44}$$

where ϕ_i are the chiral superfields. In the supergravity action the effective potential and kinetic term in the Einstein frame are given by (2.145)

$$V = e^G \left[\frac{\partial G}{\partial \phi^i} K^i_{j*} \frac{\partial G}{\partial \phi^*_j} - 3 \right] \tag{5.45}$$

and (2.139)

$$\mathcal{L}_K = K_i^{j*} \partial_\mu \phi^i \partial^\mu \phi^*_j \tag{5.46}$$

respectively, where K^i_{j*} is the inverse of the Kähler metric $K_i^{j*} \equiv \partial^2 K / \partial \phi^i \partial \phi^*_j$.

In no-scale supergravity [46] with Kähler potential $K = -3 \ln \left[T + T^* - \rho\rho^*/3\right]$, where ρ is identified as chiral inflaton superfield and T as complex modulus field, and with a Wess-Zumino superpotential $W(\rho) = \frac{\mu}{2}\rho^2 - \frac{\lambda}{3}\rho^3$, the F-term scalar potential in EF would give Starobinsky-type of inflation potential with the choice $\frac{\lambda}{\mu} = \frac{1}{3}$. Here we consider the Kähler potential of the following form

$$K = -3\ln[T + T^*] + \frac{b\rho\rho^*}{(T + T^*)^\omega}, \tag{5.47}$$

and we identify T as the two component chiral inflaton superfield and ρ as additional matter field with modular weight ω. In typical orbifold string compactifications with three moduli fields the modular weight ω has value 3 [160, 176, 177]. Here we shall treat ω as phenomenological parameter whose value can have small deviation from the canonical value 3 due to string loop corrections to the effective Supergravity action [178]. Also we will see at the end of this section that the parameter b is no new parameter and can be given in terms of the parameter ω.

For the complete specification of the supergravity we assume the following superpotential

$$W = \lambda_m \; \rho \; T^m \tag{5.48}$$

If we assume that the field ρ rapidly goes to zero at the onset of inflation, then from (5.45) and (5.46), for the taken K and W we get

$$V = \frac{\lambda_m^2 T^m T^{*m}}{b(T + T^*)^{3-\omega}}, \qquad \mathcal{L}_K = \frac{3\partial^\mu T \partial_\mu T^*}{(T + T^*)^2} \tag{5.49}$$

We can decompose T field in its real and imaginary parts parametrized by two real fields ϕ and σ respectively as

$$T = e^{-\sqrt{\frac{2}{3}}\sigma} + i\sqrt{\frac{2}{3}}\phi, \tag{5.50}$$

Using (5.50) into (5.49), we get the following forms of the kinetic and the potential terms in the Lagrangian:

$$\mathcal{L}_K = \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + \frac{1}{2}e^{-\gamma\sigma}\partial^\mu\phi\partial_\mu\phi, \tag{5.51}$$

$$V = \frac{\lambda_m^2 2^{(\omega-3)}}{b} e^{-\beta\sigma}\left[e^{\gamma\sigma} + \frac{2}{3}\phi^2\right]^m, \tag{5.52}$$

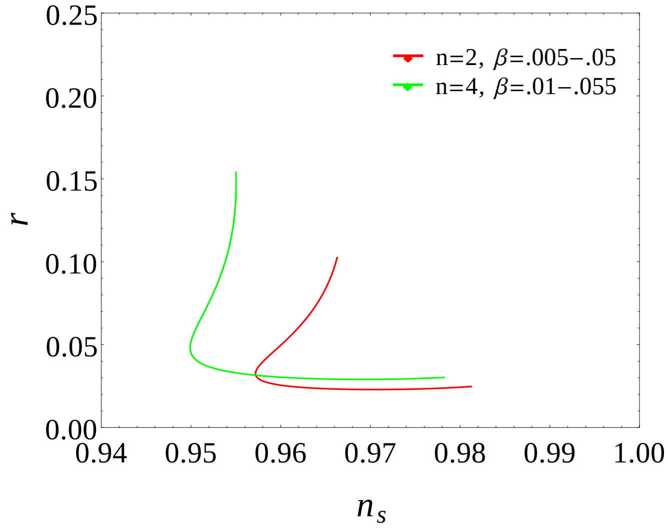

Figure 5.3: The $n_s - r$ predictions for a fixed value of $\gamma = 2\sqrt{2/3}$, for quadratic ($n = 2$) and quartic ($n = 4$) potentials, are shown. The range of values of β increases along the curves from top to bottom.

where $\gamma = 2\sqrt{\frac{2}{3}} \simeq 1.633$ and $\beta = (3-\omega)\sqrt{\frac{2}{3}}$. If during inflation the field σ rolls slow enough compared to inflaton field ϕ then $e^{\gamma\sigma} << \frac{2\phi^2}{3}$ and hence first term inside the bracket in (5.52) may be neglected. Also as per the analysis performed in two-field model, we saw that the field σ during the 60 e-folds inflation, rolls from $\mathcal{O}(0.1M_p)$ to $\mathcal{O}(1M_p)$ and the corresponding change in the inflaton field value during inflation is $\phi \sim \mathcal{O}(10M_p)$ to $\mathcal{O}(1M_p)$. Therefore for $\gamma = 2\sqrt{\frac{2}{3}}$ we find that $e^{\gamma\sigma} << \frac{2\phi^2}{3}$. Therefore, from (5.51) and (5.52), the effective matter field Lagrangian in the EF is given by

$$\mathcal{L}_M = \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + \frac{1}{2}e^{-\gamma\sigma}\partial^\mu\phi\partial_\mu\phi + e^{-\beta\sigma}V(\phi) \tag{5.53}$$

where $V(\phi) = \frac{\lambda_m^2}{2m}\phi^{2m}$ and we set $b = \frac{2^\omega}{6}$ for quadratic potential *i.e.* with $m = \frac{n}{2} = 1$ and $b = \frac{2\times 2^\omega}{9}$ for quartic potential *i.e.* with $m = \frac{n}{2} = 2$. For $\Delta N = 60$ and $\sigma_0 = 0.1$, the $n_s - r$ predictions for a fixed value of $\gamma = 2\sqrt{2/3}$ and with varying β are shown in Fig. 5.3.

5.5 Conclusions

The standard single-field slow-roll inflation models with quartic and quadratic potentials, which generically produce large tensor-to-scalar ratio, are not compatible with the present Planck data. One novel way of making quartic self-coupling of inflaton viable with the present status of the data is by what is called the Higgs inflationary scenario which gives very small $r \simeq 0.003$.

Here we present a two-field inflationary model where inflaton is accompanied by a dilaton field and has a non-canonical kinetic term due to the presence of the dilaton field. And the effective potential is a product type of potential *i.e.* inflaton potential times the exponential potential of dilaton field. We find that, unlike standard single-field slow-roll inflation with quadratic and quartic potentials, the observed $r < 0.11$ and $n_s = 0.966 \pm 0.0062$ can be obtained for certain choice of dilaton field value σ_0 and the parameter values β and γ for 60 e-folds as depicted in the Fig. (5.1). For example, inflaton with quadratic potential yields $n_s \sim 0.9666$ and $r \sim 0.06$ for parameters value $\beta = 0.05$, $\gamma = 0.7$ and inflaton with quartic potential yields $n_s \sim 0.964$ and $r \sim 0.05$ for parameters value $\beta = 0.06$, $\gamma = 1$. This shows that this scenario yields tensor-to-scalar ratio much larger than the generic Higgs inflationary scenario or Starobinsky inflationary scenario. This model for a range of parameters (β, γ) values can produce large tensor-to-scalar ratio $r \sim 10^{-1} - 10^{-2}$ which would definitely be probed by future $B-$mode experiments and thus such a model can be put to test with these future observations. We find that one requires to fine-tune the self-couplings of the inflaton field in order to be in accordance with observations, unlike the Higgs inflationary scenario. Also we find, like in the standard slow-roll inflation, in two-field scenario the fine tuning problem of inflaton self coupling and prediction of very light inflaton mass at GUT scale persists. In the limits $\beta \to 0$ and $\gamma \to 0$ the standard slow-roll inflation predictions can be obtained. We show that, up to slow-roll approximation, the amplitude of the isocurvature perturbations vanishes identically independent of the choice of the parameters values.

Future $B-$mode observations (Keck/BICEP3, SPT-3G, LiteBIRD, CMBPol and others) would further improve constraints on r. These observations aim to probe tensor-to-scalar ratio upto the theoretical limit of 2×10^{-3}, however such a limit is achievable only if the noise can be reduced to $\sim 1\,\mu$K-arcmin and lensing $B-$modes are reduced to 10% if one considers the preset PLANCK data on foreground [179]. Also in view of the study of genus topology and cross-correlation of BICEP2 and Planck B-Modes by Colley et al. [126] we expect r to get settle down somewhere in the range $r \sim 0.1 - 0.01$. Unlike standard slow-roll inflation in which quadratic and quartic potentials are ruled out by the observations, we find that in two-field scenario these potentials are allowed and therefore it is a significant result.

We derived the two-field inflation action from no-scale SUGRA with the considered form of Kähler potential and superpotantial, wherein the parameter γ has a fixed value $\gamma = 2\sqrt{2/3}$ and β appears as an arbitrary parameter. However, we believe that with some string motivated Kähler potential and appropriate choice of superpotential one can derive the two-field action with absolutely arbitrary parameters β and γ. In this model with $\gamma > \beta$, which is required condition to get the correct values of the observables n_s and r, we find that to fit the observables we need $\beta \sim 10^{-2}$, which, from the relation $\beta = (3-\omega)\sqrt{\frac{2}{3}}$, implies a very small deviation of the parameter ω from 3 which may be obtained from the string loop contributions to tree-level supergravity effective action.

Chapter 6

Summary and Conclusions

The idea, that the universe through a period of exponential expansion, called inflation, has proved useful for solving the horizon and flatness problems of standard cosmology and in addition providing an explanation for the scale invariant super-horizon perturbations which are responsible for generating the CMB anisotropies and formation of structures in the universe. A successful theory of inflation requires a flat potential where a scalar field acquires a slow-roll over a sufficiently long period to enable the universe to expand by at least 60 e-foldings during the period of inflation.

There are a wide variety of particle physics models which can provide the slow-roll scalar field 'inflaton' for inflation [55, 180]. From the observations of CMB anisotropy spectrum by COBE, WMAP and Planck [5–7], it is not yet possible to pin down a specific particle physics model as the one responsible for inflation. Though all of the above experiments gave tighter and tighter constraints on inflationary observables, *e.g.* power spectrum and spectral index, which allowed several models of inflation to be ruled out but still there is a large degeneracy in inflation models. The 2015 data from Planck observation gives the amplitude, spectral index and tensor-to-scalar ratio as $10^{10}\ln(\Delta^2_{\mathcal{R}}) = 3.089 \pm 0.036$, $n_s = 0.9666 \pm 0.0062$ at (68% CL) and $r_{0.002} < 0.11$ at (95%CL), respectively [54]. Also the latest results by BKP collaboration put r at $r_{0.05} < 0.07$ at (95%CL)[57]. Therefore, all those models which can produce the correct amplitude of CMB power spectrum and its spectral tilt along with producing $r < 0.07$ are allowed.

The future $B-$mode observations are expected to fix r which will allow many existing inflation models to be ruled out.

In this thesis we study a single-field generalized non-minimal model of inflation, a power law model of inflation and a two-field model of inflation with a non-canonical kinetic term. We calculate the key inflationary observables : amplitude of the power spectrum of curvature perturbations, spectral index and its running, tensor-to-scalar ratio and amplitude of isocurvature perturbations. And fix the parameters of the models using the measured values of these observables. Also we motivate these models from a fundamental theory called no-scale supergravity.

In Chapter §1, we briefly discussed the standard model of cosmology, its successes and inflation as a solution to problems therein. Also, we discuss inflation from modified gravity theories and no-scale supergravity theory. In Chapter §2, we discussed the necessary theoretical foundations for the purpose of this thesis.

In Chapter §3, we studied a generalized curvature coupled Higgs inflation model $\xi\phi^a R^b$ which is a generalization of the Higgs inflation model $\xi\phi^2 R$ with $\lambda\phi^4$ potential. We find that if the Higgs self coupling λ is in the range $(10^{-5} - 0.1)$, parameter a in the range $(2.3 - 3.6)$ and b in the range $(0.77 - 0.22)$ at the Planck scale, one can have a viable inflation model even for $\xi \simeq 1$. $\lambda \sim 0.1$ in this model solve the fine tuning problem of Higgs self-coupling in the standard slow-roll inflation which predict $\lambda \sim 10^{-12}$. The tensor-to-scalar ratio r in this model in EF is large $r \simeq 0.3$, therefore model with generalized scalar-curvature couplings is ruled out by observational limits on r like the pure $\frac{\lambda}{4}\phi^4$ theory. However, with independent calculations in JF gives small $r \simeq 0.002$ which is allowed from the observations. Therefore, JF result contradicts the EF result. The observations should be compared with EF results or JF results is still matter of debate. However in this model, by requiring the curvature coupling parameter to be of order unity, we have evaded the problem of unitarity violation in scalar-graviton scatterings which plague the $\xi\phi^2 R$ Higgs inflation models. Therefore, the Higgs field may still be a good candidate for being the inflaton in the early universe if one considers higher dimensional curvature coupling. Such Higher dimensional curvature couplings $\xi\phi^a R^b$ may be generated at tree level by choosing a suitable Kähler potential in a $f(\mathcal{R})$ supergravity theory [117–119]. Also we find that, upto slow-roll approximation, for the same set of parameter values (ξ, λ) the set of (a,b) values is nearly the same in Jordan and Einstein frames. Therefore, the Einstein and Jordan frames are equivalent. In this model, we found a symmetry $a + 2b \approx 4$ which holds true in both the frame and it implies that the curvature coupling ξ is nearly scale invariant.

In the following Chapter §4, we studied a power law $\frac{1}{M^2}R^\beta$ correction to Einstein gravity as a model for inflation. The interesting feature of this form of generalization is that small deviations from the Starobinsky limit $\beta = 2$ can change the value of tensor-to-scalar ratio from $r \sim \mathcal{O}(10^{-3})$ to $r \sim \mathcal{O}(0.1)$. We find that this model predicts large tensor-to-scale $r \approx 0.22$ as indicated by BICEP2 measurements, for the value of $\beta \approx 1.83$ and $M \sim 10^{-4}$. Also we showed that the general R^β model can be obtained from a SUGRA construction with minimal Wess-Zumino form of superpotential and by adding a power law $(\phi + \bar{\phi})^n$ term to the minimal no-scale SUGRA Kähler potential. We further showed that this two parameter power law generalization of the Starobinsky model is equivalent to generalized non-minimal curvature coupled models with quantum corrected $\lambda\phi^4$ potentials *i.e.* models of the form $\xi\phi^a R^b + \lambda\phi^{4(1+\gamma)}$, and thus the power law Starobinsky model is the most economical parametrization of such models. Since such a power law correction to Einstein gravity generates large amplitude of gravity waves, therefore they are ruled out by the current status of the observation data by Planck and BKP Collaboration.

In Chapter §5, we analyze a two-field model where one field is the standard inflaton and the other field is dilaton. The inflaton field has a non-canonical kinetic term and its potential is exponentially coupled to the dilaton field. Such a model can be derived from fundamental theories like Supergravity or Generalized Einstein gravity theories in conformal frame. In earlier works with this type of action, the parameters β and γ are related as $\beta = 2\gamma$ and they give no better predictions than the standard slow-roll inflation with quartic and quadratic potentials. In our model, we keep these parameters independent of each other.

We show that in this model, unlike standard slow-roll inflation, with quartic and quadratic potentials for the inflaton field leads to a viable inflation where it is a possible to get $r \sim 0.1 - 0.01$ which may be measured in upcoming $B-$mode polarization experiments. Also we find in two-field scenario the fine tuning problem of inflaton self coupling and prediction of very light inflaton mass at GUT scale persists. Also in the limits $\beta \to 0$ and $\gamma \to 0$ for quadratic and quartic potentials the predictions of this model coincide with the predictions of the standard slow-roll inflation. The presence of two fields give rise to a possibility that on super horizon scales the curvature perturbations are not constant and generate isocurvature perturbations. We find that, up to slow-roll approximation, the amplitude of the isocurvature perturbations vanishes. In order to motivate the considered two-field model with arbitrary parameters β and γ values, we give a derivations of this model from no-scale supergravity. Though in the SUGRA derivation of this model the parameters β and γ are independent but the parameter γ has a fixed value. Therefore the SUGRA version of two-field model has a limited predictions capability. However, we believe that with some string motivated Kähler potential and appropriate choice of superpotential, one can obtain the two-field action with absolutely arbitrary nature of the parameters β and γ. The main finding of this work is that, that unlike standard slow-roll inflation in which quadratic and quartic potentials are ruled out by the observations, in a generalized two-field scenario these potentials are allowed which in our view is a significant result.

In short, we can conclude that currently tensor-to-scalar ratio is an extremely important inflationary parameter in view of validating and ruling out models of inflation. The observation of primordial B-modes (CMB polarization) will provide the constraint on r. The B-modes are the signature of inflationary tensor modes, precise observations of which will provide a most distinctive confirmation of occurrence of an inflationary era in the early universe. There are several ongoing experiments for $B-$mode detection and all hope to observe these signals from the inflationary era. There are several experiments, *e.g.* ground based (Keck/BICEP3, SPT-3G, AdvACT, CLASS, Simons Array), balloons based (Spider,EBEX) and satellites based (CMBPol, LiteBIRD and COrE). These observational experiments will be taking into account the recent Planck data on polarized dust. And they aim to probe the tensor-to-scale ratio at the level of $r \sim 10^{-3}$ which is a theoretically motivated limit [179]. Also, the high precision measurements of small-scale temperature anisotropies along with the observations of $B-$mode will not only test the inflationary hypothesis but allow to remove a large degeneracy in the models of inflation. The theoretical community is eagerly waiting the results from these observational experiments.

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Publications attached with the thesis

1. *Higgs Inflation in $f(\phi, R)$ Theory*,
   Girish Kumar Chakravarty, Subhendra Mohanty and Naveen Kumar Singh, Int. J. Mod. Phys. D 23, 1450029 (2014), arXiv:1303.3870 [astro-ph.CO].

2. *Power law Starobinsky model inflation from no-scale SUGRA*,
   Girish Kumar Chakravarty and Subhendra Mohanty, Phys. Lett. B 746 (2015) 242, arXiv:1405.1321 [hep-ph].

3. *Moduli assisted two-field inflation from no-scale supergravity*,
   Girish Kumar Chakravarty, Suratna Das, Gaetano Lambiase and Subhendra Mohanty, arXiv:1511.03121 [hep-ph] (Communicated for publication).

International Journal of Modern Physics D
Vol. 23, No. 4 (2014) 1450029 (15 pages)

DOI: 10.1142/S0218271814500291

HIGGS INFLATION IN $f(\Phi, R)$ THEORY

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Received 22 August 2013
Revised 19 December 2013
Accepted 6 January 2014
Published 6 February 2014

We generalize the scalar-curvature coupling model $\xi\Phi^2 R$ of Higgs inflation to $\xi\Phi^a R^b$ to study inflation. We compute the amplitude and spectral index of curvature perturbations generated during inflation and fix the parameters of the model by comparing these with the Planck + WP data. We find that if the scalar self-coupling λ is in the range 10^{-5}–0.1, parameter a in the range 2.3–3.6 and b in the range 0.77–0.22 at the Planck scale, one can have a viable inflation model even for $\xi \simeq 1$. The tensor to scalar ratio r in this model is small and our model with scalar-curvature couplings is not ruled out by observational limits on r unlike the pure $\frac{\lambda}{4}\Phi^4$ theory. By requiring the curvature coupling parameter to be of order unity, we have evaded the problem of unitarity violation in scalar-graviton scatterings which plague the $\xi\Phi^2 R$ Higgs inflation models. We conclude that the Higgs field may still be a good candidate for being the inflaton in the early universe if one considers higher-dimensional curvature coupling.

Keywords: Higgs inflation; CMB spectrum; nonminimal coupling; Jordan frame; Einstein frame; perturbations.

PACS Number(s): 98.80.Cq, 14.80.Bn

1. Introduction

The idea that the universe through a period of exponential expansion, called inflation[1–9] has proved useful for solving the horizon and flatness problems of standard cosmology and in addition providing an explanation for the scale invariant super-horizon perturbations which are responsible for generating the CMB anisotropies and formation of structures in the universe. A successful theory of inflation requires a flat potential where a scalar field acquires a slow-roll over a sufficiently long period to enable the universe to expand by at least 60 e-foldings during the period of inflation. There is a wide variety of particle physics models which can provide the slow roll scalar field "inflaton" for inflation.[10] From the observations of CMB anisotropy spectrum by COBE and WMAP[11] it is not yet possible to pin down a specific particle physics model as the one responsible for inflation. In the light of recent discoveries by CMS[12] and ATLAS[13] it is of interest to consider the Standard Model Higgs boson as the candidate for inflaton. On the face of it the idea does not work as the inflaton quartic coupling should be of the order $\lambda \sim 10^{-12}$ to explain the amplitude of CMB perturbations measured by WMAP[11] while the 125 GeV Higgs has a quartic coupling $\lambda \sim 0.13$ at the electroweak scale which can however go down to smaller values at the Planck scale due to renormalization.[14–19] However, just from the standard model renormalization one cannot have the Higgs coupling $\lambda \sim 10^{-12}$ over the entire range of the rolling field $(10-1)M_P$ during inflation and the standard slow roll inflation with a Higgs field does not give the observed amplitude and spectrum of density perturbations.[20] If the Higgs and top mass are fine tuned then there can be a small kink in the Higgs potential and the universe trapped in this false vacuum can undergo a period of inflation.[21–23]

Later a way out of fine tuning the scalar self-coupling to unnaturally small values was found out[24–27] and it was shown that if one couples the scalar field to the Ricci scalar $\xi\Phi^2 R$ then the effective potential in the Einstein frame becomes a slow roll one with the effective scalar coupling being λ/ξ^2 and the amplitude of the density perturbations constrain this ratio rather than λ, hence ξ can be increased as large as required to get the desired self-coupling λ. Density perturbations from inflation in the curvature coupled theories were calculated in Refs. 28 and 29. The equivalence of the density perturbation in Jordan and Einstein frame was shown by Komatsu and Futamase[30] who also calculated the tensor perturbations and showed that the tensor to scalar ratio is generically small in $\xi\Phi^2 R$ model.

Bezrukov and Shaposhnikov[31] revived the large curvature coupling model to motivate the idea that the standard model Higgs field could serve as the inflaton in the early universe. The amplitude and spectral index of density perturbations observed by WMAP can be generated by the Higgs field with self-coupling $\lambda \sim 0.1$ and curvature coupling $\xi \sim 10^4$ (Refs. 31–36). This large value of ξ needed however is seen as a problem as at the time of inflation the Higgs field is at the Planck scale and hence graviton-scalar scatterings due to the curvature coupling of the scalar would become nonunitary.[37] Ways of solving the unitarity violation problem in the Higgs inflation models have been explored in Refs. 38–43.

In this paper, we assume that the dominant interaction between Higgs field and gravity is through operators of the form

$$\mathcal{L} = \frac{\xi(\mathcal{H}^\dagger\mathcal{H})^{a/2}R^b}{M_p^{a+2b-4}}. \quad (1)$$

This form (1) of Higgs curvature interaction has been mentioned in Ref. 44. The complete dynamics of the Higgs field involves the role of the Goldstone modes as has been studied in detail in Refs. 45–47. The multifield dynamics of the

Goldstone modes gives rise to sizable nongaussianity. We will study the dynamics of the Higgs mode and impose a charge conservation and CP symmetry such that the Goldstone modes of the Higgs field do not acquire vevs. We will take the background Higgs field to be

$$\mathcal{H} = \begin{pmatrix} 0 \\ \Phi \end{pmatrix}, \tag{2}$$

where Φ is the Higgs mode with mass 126 GeV. Our inflation model falls in the class of inflation in $f(\Phi, R)$ theories studied in Ref. 48. Our motivation is that we use the Higgs quartic coupling $\lambda(\mathcal{H}^\dagger\mathcal{H})^2$ where the standard model value of $\lambda(\mu \sim M_P)$ can lie in the range $\lambda = (10^{-5}$–$0.1)$ depending on the value of top quark mass[18,19] or on new physics.[49] We take curvature coupling ξ to be unity and check the possibility of generating the observed density perturbations from Higgs inflation by varying parameters a, b and λ. The nonminimal coupling ξ has been taken to be unity in order to improve the unitarity behavior which increases the natural cutoff scale Λ from $\Lambda \simeq \frac{M_p}{\xi} \simeq 10^{15}$ to $\Lambda \simeq M_p \simeq 10^{19}$.

We derive the curvature perturbation during inflation in two different ways. We derive the perturbations of modified Einstein's field equation in the Jordan frame in presence of the Higgs-curvature interaction terms and derive the amplitude and spectral index of curvature perturbation. We find that to generate the Planck+WP preferred amplitude $\Delta^2_{\mathcal{R}} = 2.1955^{+0.533}_{-0.585} \times 10^{-9}$ and spectral index $n_s = 0.9603 \pm 0.0073$ (Ref. 50) for $\lambda = 10^{-3}$ we should have $a \sim 3.02, b \sim 0.49$ (and for $\lambda = 0.1$ we need $a \sim 3.56, b \sim .22$). In these fits we take $\xi = 1$.

In the $\xi\Phi^2 R$ theory we can always make a conformal transformation to the Einstein frame so one can compute the density perturbations either in Einstein frame or Jordan frame and the gauge invariant curvature perturbations should be same in both the frames.[28] In our case with the $\xi\Phi^a R^b$ coupling we find that no conformal transformation exists which can in general remove this term (i.e. go to an Einstein frame). We find that in the general $\xi\Phi^a R^b$ theory such a conformal transformation is only possible if the metric is quasi-de Sitter. The accurate comparison with the experimental data should be made however with the Jordan frame results.

Calculation of the curvature perturbation in both Einstein and Jordan frame for the $\xi\Phi^2 R$ theory has been done previously in Refs. 28 and 51–53. In Sec. 2, we derive the curvature perturbations and tensor perturbation in our theory in the Jordan frame and in Sec. 3 we make a conformal transformation to go to the Einstein frame and compute the curvature perturbations. Finally, in Sec. 5 we compare the results of the two frames and discuss the viability of our considered Higgs inflation model.

2. Calculation in the Jordan Frame

In this section, we introduce a scalar-gravity interaction term $f(\Phi, R)$ in the action and calculate physical quantities related to the inflationary density perturbations such as the spectral index, curvature perturbation and tensor-to-scalar ratio. We start with the action for a scalar field interacting with gravity of the form

$$S = \int d^4x\sqrt{-g}\left[-\frac{f(\Phi,R)}{2\kappa^2} + \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + V(\Phi)\right], \tag{3}$$

where we take,

$$\frac{1}{\kappa^2}f(\Phi,R) = \frac{1}{\kappa^2}R + \frac{\xi\Phi^a R^b}{M_p^{a+2b-4}}; \quad V(\Phi) = \frac{\lambda\Phi^4}{4}, \tag{4}$$

where $\kappa^2 = 1/M_p^2$ and ξ is a dimensionless coupling constant. Varying the action (3) with respect to $g^{\mu\nu}$ and Φ we obtain the field equations,

$$\begin{aligned} G_{\mu\nu} &= FR_{\mu\nu} - \frac{1}{2}fg_{\mu\nu} - \nabla_\mu\nabla_\nu F + g_{\mu\nu}\Box F \\ &= \kappa^2\left(\nabla_\mu\Phi\nabla_\nu\Phi - \frac{1}{2}g_{\mu\nu}\nabla^\rho\Phi\nabla_\rho\Phi - Vg_{\mu\nu}\right), \end{aligned} \tag{5}$$

$$\Box\Phi = V_{,\Phi} - \frac{f_{,\Phi}}{2\kappa^2}, \tag{6}$$

where $F = \partial f/\partial R = 1 + \frac{\xi b\Phi^a R^{b-1}}{M_p^{a+2b-2}}$.

2.1. *Background quasi de-Sitter solution*

For the unperturbed background FRW metric diag$(-1, a^2(t), a^2(t), a^2(t))$ and scalar field $\Phi = \phi(t)$, the above Eqs. (5) and (6) reduce to the form

$$3FH^2 + \frac{1}{2}(f - RF) + 3H\dot{F} = \kappa^2\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \tag{7}$$

$$-2F\dot{H} - \ddot{F} + H\dot{F} = \kappa^2\dot{\phi}^2, \tag{8}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} - \frac{f_{,\phi}}{2\kappa^2} = 0. \tag{9}$$

Now, we assume the second term of F i.e. $\frac{\xi b\phi^a R^{b-1}}{M_p^{a+2b-2}}$ is dominant for some values of a and b. We find this assumption to be valid while solving numerically for the values of a and b in our model which give rise to the experimentally observed density perturbations as discussed in Sec. 4. From Eq. (7), under this assumption and considering the slow roll parameters which are defined in Eq. (28) as small, the Hubble parameter in the Jordan frame turns out to be of the form

$$H = \frac{\lambda^{\frac{1}{2b}}}{\sqrt{12}[\xi(2-b)]^{\frac{1}{2b}}}\left(\frac{\phi}{M_p}\right)^{\frac{4-a}{2b}} M_p. \tag{10}$$

From Eq. (9) under the slow roll assumption we get

$$\dot{\phi} = -\frac{\lambda\phi^3}{3H}\left[1 - \frac{a}{2(2-b)}\right]. \tag{11}$$

2.2. *Scalar field and metric perturbations*

Now, we perturb Eqs. (5) and (6) by perturbing the scalar field $\Phi = \phi(t) + \delta\phi(x,t)$ and the metric as

$$ds^2 = -(1+2\alpha)dt^2 - 2a(t)(\partial_i\beta)dtdx^i + a^2(t)(\delta_{ij}(1+2\psi) + 2\partial_i\partial_j\gamma)dx^idx^j, \quad (12)$$

where, α, ψ, β and γ are scalar perturbations. We derive the Einstein equations for the $f(R,\phi)$ theory[54,55] keeping the first-order terms in the metric and scalar field perturbations. The component δG_{00} is given by

$$\frac{\triangle}{a^2(t)}\psi + HA = \frac{-1}{2F}\left[\left(3H^2 + 3\dot{H} + \frac{\triangle}{a^2(t)}\right)\delta F - 3H\delta\dot{F} + \frac{1}{2}(2\kappa^2 V_{,\phi} - f_{,\phi})\delta\phi + \kappa^2\dot{\phi}\dot{\delta\phi} + (3H\dot{F} - \kappa^2\dot{\phi}^2)\alpha + \dot{F}A\right] \quad (13)$$

and taking the difference $\delta G^i_i - \delta G^0_0$ we get

$$\dot{A} + 2HA + \left(3\dot{H} + \frac{\triangle}{a^2(t)}\right)\alpha = \frac{1}{2F}\left[3\delta\ddot{F} + 3H\delta\dot{F} - \left(6H^2 + \frac{\triangle}{a^2(t)}\right)\delta F + 4\kappa^2\dot{\phi}\dot{\delta\phi} + (-2\kappa^2 V_{,\phi} + f_{,\phi})\delta\phi - 3\dot{F}\dot{\alpha} - \dot{F}A - (4\kappa^2\dot{\phi}^2 + 3H\dot{F} + 6\ddot{F})\alpha\right], \quad (14)$$

where $A = 3(H\alpha - \dot{\psi}) - \triangle\chi/a^2(t)$ and $\chi = a(t)(\beta + a\dot{\gamma})$. Here, to arrive at Eqs. (13) and (14), the leading order Eqs. (7) and (8) are also used. The other components δG_{i0} and δG_{ij} $(i \neq j)$ of the first-order perturbed Einstein equation can be written as

$$H\alpha - \dot{\psi} = \frac{1}{2F}[\kappa^2\dot{\phi}\delta\phi + \delta\dot{F} - H\delta F - \dot{F}\alpha] \quad (15)$$

and

$$\dot{\chi} + H\chi - \alpha - \psi = \frac{1}{F}(\delta F - \dot{F}\chi), \quad (16)$$

respectively. The equation of motion of scalar perturbation is

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} + \left[-\frac{\triangle}{a^2(t)} + \left(\frac{2V_{,\phi} - \frac{f_{,\phi}}{\kappa^2}}{2}\right)\right]\delta\phi = \dot{\phi}\dot{\alpha} + (2\ddot{\phi} + 3H\dot{\phi})\alpha + \dot{\phi}A + \frac{1}{2}F_{,\phi}\left(\frac{\delta R}{\kappa^2}\right), \quad (17)$$

where

$$\delta R = -2\left[\dot{A} + 4AH + \left(\frac{\triangle}{a^2(t)} + 3\dot{H}\right)\alpha + 2\frac{\triangle}{a^2(t)}\psi\right]. \quad (18)$$

Now, we analyze the curvature perturbation $\mathcal{R} = \psi - H\delta\phi/\dot{\phi}$ by choosing a gauge where $\delta\phi = 0$ and $\delta R = 0$. This sets $\mathcal{R} = \psi$ and moreover we have $\delta F = 0$ via

$\delta F = (\partial F/\partial\phi)\delta\phi + (\partial F/\partial R)\delta R$. Under this gauge Eq. (15) gives,

$$\alpha = \frac{\dot{\mathcal{R}}}{H + \frac{\dot{F}}{2F}} \tag{19}$$

and hence from Eq. (13) we get

$$A = -\frac{1}{H + \frac{\dot{F}}{(2F)}} \left(\frac{\triangle}{a^2(t)}\mathcal{R} + \frac{(3H\dot{F} - \kappa^2\dot{\phi}^2)\dot{\mathcal{R}}}{2F\left(H + \frac{\dot{F}}{2F}\right)} \right). \tag{20}$$

Using Eqs. (8) and (14), we obtain

$$\dot{A} + \left(2H + \frac{\dot{F}}{2F}\right) A + \frac{3\dot{F}}{2F}\dot{\alpha} + \left(\frac{3\ddot{F} + 6H\dot{F} + \kappa^2\dot{\phi}^2}{2F} + \frac{\triangle}{a^2(t)} \right) \alpha = 0. \tag{21}$$

Now, we may write the differential equation for curvature perturbation by using the above Eqs. (19)–(21) as

$$\ddot{\mathcal{R}} + \frac{(a^3(t)Q_s)^{\cdot}}{a^3(t)Q_s}\dot{\mathcal{R}} + \frac{k^2}{a^2(t)}\mathcal{R} = 0, \tag{22}$$

where

$$Q_s = \frac{\dot{\phi}^2 + \frac{3\dot{F}^2}{2\kappa^2 F}}{\left(H + \frac{\dot{F}}{2F}\right)^2}. \tag{23}$$

To arrive at Eq. (22), Eq. (8) is again used. Now, one may rewrite Eq. (22) in terms of variables $\omega = a\sqrt{Q_s}$ and $\sigma_k = \omega\mathcal{R}$ as

$$\sigma_k'' + \left(k^2 - \frac{\omega''}{\omega}\right)\sigma_k = 0, \tag{24}$$

where prime denotes the derivative with respect to the conformal time defined as $d\eta = dt/a(t)$ and

$$\frac{\omega''}{\omega} = \frac{a''(t)}{a(t)} + \frac{a'(t)}{a(t)}\frac{Q_s'}{Q_s} + \frac{1}{2}\frac{Q_s''}{Q_s} - \frac{1}{4}\left(\frac{Q_s'}{Q_s}\right)^2, \tag{25}$$

under quasi de-Sitter expansion $a(\eta) = \frac{-1}{H\eta(1-\epsilon_1)}$ and hence $\frac{a''(t)}{a(t)} = \frac{1}{\eta^2}[2 + 3\epsilon_1]$ and $a'(t)/a(t) = a(t)H$. Therefore, we have

$$\frac{\omega''}{\omega} = \frac{1}{\eta^2}\left[\nu_{\mathcal{R}}^2 - \frac{1}{4}\right], \tag{26}$$

where

$$\nu_{\mathcal{R}}^2 = \frac{9}{4}\left[1 + \frac{4}{3}(2\epsilon_1 + \epsilon_2 - \epsilon_3 + \epsilon_4)\right]. \tag{27}$$

In arriving at the above expression we have defined

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{\ddot{\phi}}{H\dot{\phi}}, \quad \epsilon_3 = \frac{\dot{F}}{2HF}, \quad \epsilon_4 = \frac{\dot{E}}{2HE}; \tag{28}$$

$$E = F + \frac{3\dot{F}^2}{2\kappa^2\dot{\phi}^2} = \frac{Q_s(1+\epsilon_3)^2}{\frac{\dot{\phi}^2}{FH^2}}. \tag{29}$$

Here, ϵ_i are slow roll parameters and $\dot{\epsilon_i}$ terms have been neglected. Equation (24) then has solutions in the Hankel functions of order ν_R

$$\sigma = \frac{\sqrt{\pi|\eta|}}{2} e^{i(1+2\nu_{\mathcal{R}})\pi/4}[c_1\, H^{(1)}_{\nu_{\mathcal{R}}}(k|\eta|) + c_2\, H^{(2)}_{\nu_{\mathcal{R}}}(k|\eta|)]. \tag{30}$$

Applying the Bunch–Davies boundary condition $\sigma(k\eta \to -\infty) = e^{ik\eta}/\sqrt{2k}$ we fix the integration constants $c_1 = 1$ and $c_2 = 0$. Using the relation $H_\nu(k|\eta|) = \frac{-i}{\pi}\Gamma(\nu)(\frac{k|\eta|}{2})^{-\nu}$ for the super-horizon modes $k\eta \to 0$, we obtain the expression for the power spectrum for curvature perturbations, defined as

$$\mathcal{P}_{\mathcal{R}} = \frac{4\pi k^3}{(2\pi)^3}|\mathcal{R}|^2 \equiv \Delta^2_{\mathcal{R}} \left(\frac{k}{a(t)H}\right)^{n_{\mathcal{R}}-1}. \tag{31}$$

The amplitude of the curvature power spectrum turns out to be

$$\Delta_{\mathcal{R}} = \frac{1}{\sqrt{Q_s}}\left(\frac{H}{2\pi}\right) \tag{32}$$

and the spectral index is

$$n_{\mathcal{R}} - 1 = 3 - 2\nu_{\mathcal{R}} \simeq -4\epsilon_1 - 2\epsilon_2 + 2\epsilon_3 - 2\epsilon_4 \simeq -6\epsilon_1. \tag{33}$$

Using slow roll parameters, Eq. (23) can be simplified to the form $Q_s \simeq 6F\epsilon_3^2 M_p^2$ with $\frac{\kappa^2\dot{\phi}^2}{FH^2} \ll 6\epsilon_3^2$ which will be justified later in Sec. 4. In our model of $f(\Phi, R)$ coupling we find $\epsilon_1 \approx -\epsilon_3$, $\epsilon_2 \approx -\epsilon_4$ and these relations are used in the calculation of perturbation amplitude and spectral index. Plugging the values H and $\dot{\phi}$ from Eqs. (10) and (11) into Eq. (28), the slow roll parameters can be written as

$$\epsilon_1 = b^{-1}(a-4)(2-b)^{(1-b)/b}(a+2b-4)\lambda^{(b-1)/b}\xi^{1/b}\left(\frac{\phi}{M_p}\right)^{\frac{a+2b-4}{b}}, \tag{34}$$

$$\epsilon_2 = b^{-1}(a+6b-4)(2-b)^{(1-b)/b}(a+2b-4)\lambda^{(b-1)/b}\xi^{1/b}\left(\frac{\phi}{M_p}\right)^{\frac{a+2b-4}{b}}. \tag{35}$$

For our model, we can write the expressions for the amplitude of power spectrum and the number of e-folding as

$$\Delta^2_{\mathcal{R}} = \frac{b\left[\frac{(2-b)}{\lambda}\right]^{3-\frac{4}{b}} M_p^{8+\frac{4(a-4)}{b}}\xi^{-\frac{4}{b}}\phi^{-\frac{4(a+2b-4)}{b}}}{288(a-4)^2(a+2b-4)^2\pi^2} \tag{36}$$

and

$$N_J = \int_{\phi_J}^{\phi_f} \frac{H}{\dot{\phi}} d\phi = \frac{b\left[\frac{(2-b)}{\lambda}\right]^{\frac{b-1}{b}} \xi^{-\frac{1}{b}}}{2(a+2b-4)^2} \left(\frac{\phi}{M_p}\right)^{\frac{4-a-2b}{b}} \Bigg|_{\phi_f}^{\phi_J} \tag{37}$$

respectively. Here, ϕ_J and ϕ_f are the values of scalar field ϕ at the beginning and the end of inflation respectively.

2.3. *Tensor perturbations*

We define the perturbation of metric as follows

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \quad \text{and} \quad g^{\mu\nu} = \bar{g}^{\mu\nu} + h^{\mu\nu}, \tag{38}$$

where $\bar{g}_{\mu\nu}$ is background metric and

$$h^{ij} = -\frac{1}{a^4(t)} h_{ij}, \quad h^{i0} = \frac{1}{a^2(t)} h_{i0}, \quad h^{00} = -h_{00}. \tag{39}$$

Now, to get the equation of tensor perturbation, we set $h_{i0} = h_{00} = 0$ in the calculation. From the decomposition theorem, the nonzero spatial components h_{ij} are traceless and divergenceless, i.e.

$$h_{ii} = 0, \quad \partial_i h_{ij} = 0. \tag{40}$$

Using Eqs. (39) and (40), we obtain

$$\delta R_{00} = 0, \quad \delta R_{i0} = 0, \tag{41}$$

$$\delta R_{ij} = -\frac{1}{2a^2(t)} \nabla^2 h_{ij} + \frac{1}{2}\ddot{h}_{ij} - \frac{\dot{a}}{2a}\dot{h}_{ij} + 2\left(\frac{\dot{a}}{a}\right)^2 h_{ij}, \quad \delta R = 0. \tag{42}$$

So, perturbing Eq. (5), we obtain

$$\begin{aligned} &\frac{1}{2} F a^2 \ddot{D}_{ij} + \left(\frac{1}{2}\dot{F} a^2 + \frac{3}{2} a\dot{a} F\right) \dot{D}_{ij} - \frac{F}{2}\nabla^2 D_{ij} \\ &\quad = \left[2\frac{\dot{a}}{a}\dot{F} - 2F\left(\frac{\dot{a}}{a}\right)^2 - \frac{\ddot{a}}{a}F + \frac{f}{2} + \ddot{F} + \frac{\kappa^2}{2}(\dot{\phi}^2 - 2V)\right] a^2 D_{ij}, \end{aligned} \tag{43}$$

where $D_{ij} = h_{ij}/a^2$. The right-hand side of Eq. (43) vanishes by Eqs. (7) and (8). Thus, we have

$$\ddot{D}_{ij} + \frac{(a^3 F)^{\cdot}}{a^3 F} \dot{D}_{ij} + \frac{\kappa^2}{a^2} D_{ij} = 0. \tag{44}$$

In the terms of polarization tensors e^1_{ij} and e^2_{ij}, the tensor D_{ij} is written as

$$D_{ij} = D_1 e^1_{ij} + D_2 e^2_{ij}. \tag{45}$$

For gravity wave propagating in $\hat{z}$ direction, the components of polarization tensor are given by

$$e^1_{xx} = -e^1_{yy} = 1, \quad e^2_{xy} = e^2_{yx} = 1, \quad e^{1,2}_{iz} = e^{1,2}_{zi} = 0. \tag{46}$$

So Eq. (44) can be written as

$$\ddot{D}_\lambda + \frac{(a^3F)^{\cdot}}{a^3F}\dot{D}_\lambda + \frac{\kappa^2}{a^2}D_\lambda = 0, \tag{47}$$

where $\lambda \equiv 1, 2$. Now substituting $z = a\sqrt{F}$ and $v_k = zD_\lambda M_P/\sqrt{2}$, we get

$$v''_\lambda + \left(k^2 - \frac{z''}{z}\right)v_\lambda = 0, \tag{48}$$

where, prime $'$ is derivative with respect to conformal time. Summing over all polarization states, Eq. (48) provides us the amplitude of power spectrum of D_λ as

$$P_T = 4 \times \left(\frac{2}{M_p^2}\right)\frac{\kappa^3}{2\pi^2}\frac{1}{a^2F}v_\lambda^2 \simeq \frac{2}{\pi^2}\left(\frac{H}{M_P}\right)^2\frac{1}{F}. \tag{49}$$

So, the ratio of the amplitude of tensor perturbations to scalar perturbations r for $f(\Phi, R)$ theories is given by

$$r \simeq \frac{8\kappa^2 Q_s}{F} \simeq 48\epsilon_3^2. \tag{50}$$

3. Calculation in the Einstein Frame

Starting with the considered action

$$S = \int d^4x\sqrt{-g}\left[-\frac{{M_p}^2}{2}R\left(1 + \frac{\xi\Phi^a R^{b-1}}{M_p^{a+2b-2}}\right) + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{\lambda\Phi^4}{4}\right], \tag{51}$$

we perform a conformal transformation of the metric $g_{\mu\nu}$ to the Einstein frame metric $\tilde{g}_{\mu\nu}$which is defined as

$$\tilde{g}_{\mu\nu}(x) = \Omega^2(x)g_{\mu\nu}(x)\ , \tag{52}$$

where

$$\Omega^2 = 1 + \frac{\xi\Phi^a R^{b-1}}{{M_p}^{a+2b-2}}. \tag{53}$$

The Ricci scalar transform as

$$R = \Omega^2\left[\tilde{R} + 6\frac{\tilde{\Box}\Omega}{\Omega} - 12\frac{\tilde{\partial}^\mu\Omega\tilde{\partial}_\mu\Omega}{\Omega^2}\right]. \tag{54}$$

For quasi de-Sitter space we can ignore the second and third terms in the bracket in Eq. (54) which is justified in Eq. (67). For this slow roll case, we can write Eq. (53)

in Einstein frame as

$$\Omega^2 = 1 + \frac{\xi^{1+\beta}\Phi^\alpha \tilde{R}^\beta}{M_p^{\alpha+2\beta}}, \tag{55}$$

where, $\alpha = a/(2-b)$ and $\beta = (b-1)/(2-b)$. Now, we write the action (51) in terms of new field ϕ_E, which is related to the field Φ by the relation

$$\frac{d\phi_E}{d\Phi} = \frac{1}{\Omega^2}\left(\Omega^2 + \frac{3\alpha^2\xi'^2}{2}\left(\frac{\Phi}{M_p}\right)^{2\alpha-2}\right)^{1/2}, \tag{56}$$

where $\xi' = \xi^{1+\beta}(\tilde{R}/M_p^2)^\beta$. This leads the action in terms of ϕ_E as follows

$$S_E = \int d^4x \left(-\frac{M_p^2}{2}\tilde{R} + \frac{1}{2}\tilde{D}^\mu\phi_E\tilde{D}_\mu\phi_E + U(\phi_E)\right), \tag{57}$$

where

$$U(\phi_E) = \frac{1}{\Omega^4}\frac{\lambda}{4}\Phi(\phi_E)^4. \tag{58}$$

For $\Phi \gg M_P/\xi'^{1/\alpha}$, Eq. (56) can be integrated to give

$$\Phi = \frac{M_p}{\xi'^{1/\alpha}}\exp\left(\sqrt{\frac{2}{3}}\frac{\phi_E}{M_p\alpha} - \frac{1}{2}\right). \tag{59}$$

Considering $\tilde{g}_{\mu\nu} = \mathrm{diag}(-M^2(t), \tilde{a}^2(t), \tilde{a}^2(t), \tilde{a}^2(t))$ and varying the action (57) with respect to $M(t)$ or $a(t)$ and setting $M = 1$ in the final equation which corresponds to FRW metric, we get the Friedman equation

$$12\tilde{H}^2 - \zeta^{-1}M_p^2\lambda\left(1 + \frac{2\beta}{\alpha}\right) = 0, \tag{60}$$

where

$$\zeta = 12^{4\beta/\alpha}\left(\frac{\tilde{H}^2}{M_p^2}\right)^{4\beta/\alpha}\xi^{\frac{4(1+\beta)}{\alpha}}\exp\left(2\sqrt{\frac{2}{3}}\frac{(\alpha-2)\phi_E}{\alpha M_p}\right). \tag{61}$$

Here, we have neglected all the derivative terms of Hubble parameter $\tilde{H}$. This corresponds to slow roll condition, i.e. $\dot{\phi_E}^2$ is much smaller than potential term. We may write the Hubble parameter from Eq. (60) as

$$\tilde{H} = M_p\frac{\left[\left(1 + \frac{2\beta}{\alpha}\right)\lambda\right]^{\frac{\alpha}{2(\alpha+4\beta)}}}{\sqrt{12}\xi^{\frac{2(1+\beta)}{\alpha+4\beta}}}\exp\left[\sqrt{\frac{2}{3}}\left(\frac{2-\alpha}{\alpha+4\beta}\right)\frac{\phi_E}{M_p}\right]. \tag{62}$$

Now, using Eqs. (62) and (58) we obtain

$$U(\phi_E) = \frac{1}{4}M_p^4\lambda^{\frac{\alpha}{\alpha+4\beta}}\xi^{-\frac{4(1+\beta)}{\alpha+4\beta}}\left(1 + \frac{2\beta}{\alpha}\right)^{-\frac{4\beta}{\alpha+4\beta}}\exp\left[2\sqrt{\frac{2}{3}}\left(\frac{2-\alpha}{\alpha+4\beta}\right)\frac{\phi_E}{M_p}\right]. \tag{63}$$

Here, we have taken the approximation $\exp(\sqrt{\frac{2}{3}}\frac{\phi_E}{M_p}) \gg 1$ for $\phi_E \gg M_p$. We now compute the spectral index and curvature perturbation using above potential (63).

The slow roll parameters for large $\phi_E \gg M_p$ comes out to be

$$\epsilon = \frac{M_p^2}{2}\left(\frac{U'}{U}\right)^2 = \frac{4}{3}\left(\frac{a+2b-4}{a+4b-4}\right)^2; \quad \eta = M_p^2\left(\frac{U''}{U}\right) = \frac{8}{3}\left(\frac{a+2b-4}{a+4b-4}\right)^2 \tag{64}$$

and the curvature perturbation

$$\begin{aligned}\Delta_{\mathcal{R}} &= \frac{3H^3}{2\pi U'(\phi_E)} \\ &= \frac{1}{8\sqrt{2}\,\pi}\left(\frac{y+2}{2y-x+4}\right)^{\frac{x+2y+4}{2x}} \lambda^{\frac{2y-x+4}{2x}} \xi^{-\frac{2}{x}} \left(\frac{x}{y}\right) \exp\left(-\sqrt{\frac{2}{3}}\frac{y\phi_E}{xM_p}\right),\end{aligned} \tag{65}$$

where $x = a + 4b - 4$ and $y = a + 2b - 4$.

The spectral index in the term of slow roll parameters is $n_s = 1 - 6\epsilon + 2\eta$.

The number of e-folding is calculated as

$$\begin{aligned}N_E &= \int_{\phi_{E_e}}^{\phi_{E_0}} \frac{U(\phi_E)}{U'(\phi_E)} d\phi_E \\ &= -\frac{1}{2}\sqrt{\frac{3}{2}}\left(\frac{x}{y}\right)\left(\frac{\phi_{E_0} - \phi_{E_e}}{M_p}\right).\end{aligned} \tag{66}$$

For $\phi_{E_0} \sim 13M_p$ and $\phi_{E_e} \sim 1M_p$, the number of e-folding is found to be around 60. The slow roll parameters ϵ, η and the Hubble parameter H are nearly independent of λ and are ~ 0.02, ~ 0.04 and $\sim 5.8 \times 10^{-5} M_p$, respectively.

Now from Eqs. (53) and (59), we can calculate the order of terms like $\ddot{\Omega}/\Omega$ and $(\dot{\Omega}/\Omega)^2$ for $\phi \gg \frac{M_p}{\xi'^{1/\alpha}}$. For $\lambda = 10^{-3}$ and $\xi = 1$,

$$\begin{aligned}\frac{\ddot{\Omega}}{\Omega} &\sim \frac{U}{9M_p^2}(\epsilon + \sqrt{3\epsilon}(\eta - \epsilon)) = 4.1 \times 10^{-11} M_p^2 \quad \text{and} \\ \left(\frac{\dot{\Omega}}{\Omega}\right)^2 &\sim \frac{U}{9M_p^2}\epsilon = 3.3 \times 10^{-11} M_p^2,\end{aligned} \tag{67}$$

respectively, whereas the value of curvature scalar $\tilde{R} = 12\tilde{H}^2$ at the same values of parameter is $4.1 \times 10^{-8} M_p^2$. Thus, our approximation (i.e. for quasi de-Sitter space we can ignore the second and third terms in the bracket in Eq. (54)) made is consistent and may be checked for other values of a and b.

We now use the measured values of these CMB anisotropy parameters to get the numerical values for the parameters (a, b, ξ, λ).

4. Comparison with Data

From the Planck + WP measurements[50] we know that the curvature perturbation $\Delta_{\mathcal{R}}^2 = 2.195^{+0.533}_{-0.585} \times 10^{-9}$, spectral index $n_{\mathcal{R}} = 0.9603 \pm 0.0073$ and the tensor to scalar ratio $r < 0.11$(95%CL). For inflation to solve the horizon and flatness problems of standard hot big bang cosmology the number of e-foldings in the Einstein frame N_E is required to be about 60. From Eq. (66), we see that to get 60 e-foldings, the scalar field ϕ_E should roll from $13M_p$ to $1M_p$. We compute the curvature perturbation (65) and spectral index in the Einstein frame and equate the expressions with the Planck + WMAP values to compute the parameters a and b for different values of λ and assume that the curvature coupling parameter $\xi = 1$. Our results for the correlated set of parameters λ, a, b at $\phi_E = 13M_P$ which give the measured values of $\Delta^2_{\mathcal{R}}$ and n_s are shown in Table 1. We see that compared to the $\xi\phi^2 R$ models with large ξ the small deviations of a and b from 2 and 1 respectively can result in a large change in ξ which is 1 in our model compared to the earlier curvature coupling models where $\xi \sim 10^4$.

Next, we equate the curvature perturbations and spectral index in the Jordan frame from Eqs. (36) and (33) with the Planck + WMAP data to evaluate the values of the parameters λ, a and b (keeping $\xi = 1$) . The scalar field values Φ in the Jordan frame corresponding to $\phi_E = 1M_p$ and $13M_P$ for different values of λ are displayed in Table 2. Using these values of the range of the roll in Φ we see that the number of e-foldings N_J in the Jordan frame, corresponding to $N_E = 60$ is $N_J \sim 830$. The values of the parameters λ, a and b which give the required curvature perturbation and spectral index are shown in Table 2. The slow roll parameters are found to be $\epsilon_1 \simeq -\epsilon_3 \simeq 0.007$ and $\epsilon_2 \simeq -\epsilon_4 \simeq -0.013$ for chosen range of λ. The calculated value for the tensor to scalar ratio and Hubble parameter are $r \simeq 0.002$ and $H \sim 10^{-3}M_p$, respectively.

The values of $F = 1 + \frac{\xi b \Phi^a R^{b-1}}{M_p^{a+2b-2}}$ are found to be $\sim 10^5$ i.e. much larger than unity and hence our assumption of dropping the unity in the expression for F is justified. Also we find that the order of the term $\frac{\kappa^2 \dot{\phi}^2}{FH^2} \sim 10^{-9}$ is much smaller than $6\epsilon_3^2 \sim 10^{-4}$ as assumed in Sec. 2.2.

Table 1. The values of parameters (a, b) in the Einstein frame at $\phi_E = 13M_p$ with $\xi = 1$ for different values of λ.

λ	0.1	10^{-2}	10^{-3}	10^{-4}	10^{-5}
a	3.385	3.026	2.735	2.494	2.292
b	0.277	0.439	0.571	0.679	0.770
$a + 2b$	3.939	3.904	3.877	3.852	3.832

Table 2. The values of parameters (a, b) are evaluated in the Jordan frame at $\xi = 1$ and $\phi_J|_{\phi_E=13M_p}$ for different values of λ.

λ	0.1	10^{-2}	10^{-3}	10^{-4}	10^{-5}
$\phi_f\|_{(\phi_E=1M_p)}$	$0.0146M_p$	$0.0253M_p$	$0.044M_p$	$0.077M_p$	$0.134M_p$
$\phi_J\|_{(\phi_E=13M_p)}$	$3.566M_p$	$6.187M_p$	$10.77M_p$	$18.8M_p$	$32.77M_p$
a	3.56398962	3.27512990	3.02576940	2.80956100	2.62085100
b	0.21800513	0.36243484	0.48711456	0.59521700	0.68956620
$a + 2b$	3.999999	3.999999	3.999998	3.999995	3.99998

We find that in the limit $a \simeq 2$ and $b \simeq 1$ the correct value of $\Delta^2_{\mathcal{R}}$ and $n_{\mathcal{R}}$ are obtained for $\lambda \sim 0.1$ only for large value of $\xi \sim 10^4$. Our Jordan frame calculation in this limit is consistent with the results of Refs. 31 and 33–35, who do the calculation in the Einstein frame.

5. Discussion and Conclusion

We have generalized the curvature coupling models of Higgs inflation to study inflation with a scalar field for a $\frac{\lambda}{4}\Phi^4$ potential and a curvature coupling of the form $\frac{\xi\Phi^a R^b}{M_p^{a+2b-4}}$. It may be possible to generate a tree level term of this form by choosing a suitable Kahler potential in a $f(\mathcal{R})$ supergravity theory.[56–58]

We find that for $\xi = 1$ and λ in the range $(10^{-5}$–$0.1)$, the phenomenologically acceptable parameters a and b fall in the ranges (2.3–3.6) and (0.77–0.22) respectively. We discover an interesting symmetry related to the numerical value of a and b which give the correct amplitude and spectral index. We find that for any value of λ the values of a and b which give the required density perturbations satisfy the relation $(a + 2b) \simeq 4$ as shown in Table 2. This means that the curvature coupling term $\frac{\xi\Phi^a R^b}{M_p^{a+2b-4}}$ has no dimensional couplings and is scale invariant.

It has been shown that the Higgs self-coupling can go from $\lambda = 0.13$ at the electroweak scale for the 125 GeV Higgs to $\lambda = 10^{-5}$ at the Planck scale by tuning the top mass or by introducing extra interactions.[18,19,49] This leads us to conclude that the Higgs field may still be a good candidate for being the inflaton in the early universe if one considers a generalized curvature-Higgs coupling of the form $\xi\Phi^a R^b$.

The tensor to scalar ratio r in this model is small and the $\frac{\lambda}{4}\Phi^4$ with scalar curvature couplings is not ruled out by observational limits on r unlike the pure $\frac{\lambda}{4}\Phi^4$ theory.[11,59]

We find that the values of (a, b) computed with Jordan and Einstein frame calculations of the curvature perturbation and spectral index are comparable but are not identical because unlike the $\xi\Phi^2 R$ theory, in the $\xi\Phi^a R^b$ theory it is not possible in general to go to an Einstein frame with a conformal transformation. If the space is quasi de-Sitter however such an transformation given by Eq. (55) is possible but the results will differ in the two frames due to the slow roll approximation. Finally, by requiring the curvature coupling parameter to be of order unity, we have evaded the problem of unitarity violation in scalar-graviton scatterings[37] which plagued the $\xi\Phi^2 R$ Higgs inflation models.[31,33–35]

Acknowledgment

We thank the anonymous referees for their constructive suggestions.

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Physics Letters B 746 (2015) 242–247

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Power law Starobinsky model of inflation from no-scale SUGRA

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ARTICLE INFO

Article history:
Received 27 January 2015
Received in revised form 2 April 2015
Accepted 26 April 2015
Available online 30 April 2015
Editor: G.F. Giudice

Keywords:
Inflation
CMB
B-mode
Starobinsky model
$f(R)$-theory
Supergravity

ABSTRACT

We consider a power law $\frac{1}{M^2}R^\beta$ correction to Einstein gravity as a model for inflation. The interesting feature of this form of generalization is that small deviations from the Starobinsky limit $\beta = 2$ can change the value of tensor-to-scalar ratio from $r \sim \mathcal{O}(10^{-3})$ to $r \sim \mathcal{O}(0.1)$. We find that in order to get large tensor perturbation $r \approx 0.1$ as indicated by BKP measurements, we require the value of $\beta \approx 1.83$ thereby breaking global Weyl symmetry. We show that the general R^β model can be obtained from a SUGRA construction by adding a power law $(\Phi + \bar{\Phi})^n$ term to the minimal no-scale SUGRA Kähler potential. We further show that this two-parameter power law generalization of the Starobinsky model is equivalent to generalized non-minimal curvature coupled models of the form $\xi\phi^a R^b + \lambda\phi^{4(1+\gamma)}$ and thus the power law Starobinsky model is the most economical parametrization of such models.

1. Introduction

The Starobinsky model of inflation [1,2] with a $\frac{1}{M^2}R^2$ interaction term is of interest as it requires no extra scalar fields but relies on the scalar degree of the metric tensor to generate the 'inflaton' potential. The R^2 Starobinsky model gives rise to a 'plateau potential' of the inflaton when transformed to the Einstein frame. This model was favored by the Planck constraint on the tensor-to-scalar ratio which ruled out potentials like $m^2\phi^2$ and $\lambda\phi^4$. In addition the Starobinsky model could be mapped to the Higgs-inflation models with $\xi\phi^2 R + \lambda\phi^4$ theory [3]. The characteristic feature of the Starobinsky equivalent models was the prediction that the tensor-to-scalar ratio was $r \simeq 10^{-3}$. BICEP2 reported a large value of $r = 0.2^{+0.07}_{-0.05}$ [4] but the recent joint analysis by Planck+BICEP2+Keck Array gives only an upper bound of $r_{0.05} < 0.12(95\% CL)$ [5–7]. In an analysis of the genus structure of the B-mode polarization of Planck+BICEP2 data Colley et al. put the tensor-to-scalar ratio at $r = 0.11 \pm 0.04(68\% CL)$ [8]. In the light of the possibility that r can be larger than the Starobinsky model prediction of $4 \sim 0.003$, generalizations of the Starobinsky model are of interest.

We study a general power law $\frac{1}{6M^2}\frac{R^\beta}{M_p^{2\beta-2}}$ correction to the Einstein gravity and compute the scalar and tensor power spectrum as a function of the two dimensionless parameters M and β. It is well known that the $\frac{1}{M^2}R^2$ model is equivalent to the $\xi\phi^2 R + \lambda\phi^4$ Higgs-inflation model as they lead to the same scalar potential in the Einstein frame [9,10]. One can find similar equivalence between generalized Higgs-inflation models and the power law Starobinsky model whose common feature is violation of the global Weyl symmetry. A general scalar-curvature coupled $\xi\phi^a R^b$ model was studied in [11]. The quantum correction on ϕ^4-potential in Jordan frame was studied in [12–14] where the equivalence of the $\xi\phi^2 R + \lambda\phi^{4(1+\gamma)}$ model with $\frac{1}{M^2}R^\beta$ model was shown. The generalized Starobinsky model with R^p correction has been studied in Refs. [15–21]. In general scalar-curvature theories the scalar plays the role of the inflaton after transforming to Einstein frame whereas in pure curvature theories like $R + \frac{1}{M^2}R^\beta$ model the longitudinal part of the graviton is the equivalent scalar in the Einstein frame plays the role of inflaton.

The higher order curvature theories arise naturally in theories of supergravity. The supergravity embedding of the Higgs-inflation [3] does not produce a slow roll potential in MSSM but a potential suitable for inflation is obtained in NMSSM [22]. The potential in NMSSM however has a tachyonic instability in the direction orthogonal to the slow roll [23]. This instability can be cured by the addition of quartic terms of the fields in the Kähler potential [24, 25].

In the context of a supergravity embedding of the Starobinsky model, it was shown by Cecotti [26] that quadratic Ricci curvature

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http://dx.doi.org/10.1016/j.physletb.2015.04.056
0370-2693/

terms can be derived in a supergravity theory by adding two chiral superfields in the minimal supergravity. A no-scale SUGRA [27–29] model with a modulus field and the inflation field with a minimal Wess–Zumino superpotential gives the same F-term potential in the Einstein frame as the Starobinsky model [30]. The symmetry principle which can be invoked for the SUGRA generalization of the Starobinsky model is the spontaneous violation of superconformal symmetry [31]. The quadratic curvature can also arise from D-term in a minimal-SUGRA theory with the addition of a vector and chiral supermultiplets [32]. The Starobinsky model has been derived from the D-term potential of a SUGRA model [33–35]. Quartic powers of Ricci curvature in the bosonic Lagrangian can also be obtained in a SUGRA model by the D-term of higher order powers of the field strength superfield [35,36].

In this paper we give a SUGRA model for the general power law $\frac{1}{M^2}R^\beta$ model. We show that adding a $(\Phi+\bar{\Phi})^n$ term to the minimal no-scale Kähler potential and with a Wess–Zumino form of the superpotential $W(\Phi)$ yields the same potential in the Einstein frame as the generalized Starobinsky model. In the limit $n=2$ the Starobinsky limit $\beta=2$ is obtained. We derive the relations between the two parameters of the power law Starobinsky model and the two parameters of our SUGRA model. The interesting point about the generalization is that small deviations from the Starobinsky limit of $n=\beta=2$ can produce large shifts in the values of r. Many SUGRA models have been constructed which can yield a range of r from 10^{-3} to 10^{-1} by changing the parameters of the Kähler potential and the superpotential [36–53].

We also show in this paper that our 2-parameter SUGRA model which we relate to the 2-parameter $\frac{1}{M^2}R^\beta$ model is the most economical representation of the 5-parameter scalar-curvature coupled inflation models $\xi\phi^a R^b+\lambda\phi^{4(1+\gamma)}$ in terms of the number of parameters.

The organization of this paper is as follows: In Section 2, we calculate an equivalent scalar potential in the Einstein frame for $R+\frac{1}{M^2}R^\beta$ gravity. We then find the parameter M and β values which satisfy the observed amplitude $\Delta^2_{\mathcal{R}}$, spectral index n_s and tensor to scalar r. We fix model parameters for two cases: one with running of n_s and another without running of n_s. In Section 3, we give a SUGRA embedding of the $\frac{1}{M^2}R^\beta$ model with a specific choice of the Kähler potential K and superpotential W. In Section 4, we show that the generalized curvature coupling model $\xi\phi^a R^b+\lambda\phi^{4(1+\gamma)}$ is equivalent to $R+\frac{1}{M^2}R^\beta$ model and give the relation between the parameters of these two generalized models. Finally we conclude in Section 5.

2. Power law Starobinsky model

We start with an $f(R)$ action of the form [54,55]

$$S_J=\frac{-M_p^2}{2}\int d^4x\sqrt{-g}\left(R+\frac{1}{6M^2}\frac{R^\beta}{M_p^{2\beta-2}}\right) \quad (1)$$

where $M_p^2=(8\pi G)^{-1}$, g is the determinant of the metric $g_{\mu\nu}$ and M is a dimensionless real parameter. The subscript J refers to Jordan frame which indicates that the gravity sector is not the Einstein gravity form. The action (1) can be transformed to an Einstein frame action using the conformal transformation $\tilde{g}_{\mu\nu}(x)=\Omega(x)g_{\mu\nu}(x)$, where Ω is the conformal factor and tilde represents quantities in the Einstein frame. Under conformal transformation the Ricci scalar R in the two frames is related by

$$R=\Omega\left(\tilde{R}+3\tilde{\Box}\omega-\frac{3}{2}\tilde{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega\right) \quad (2)$$

where $\omega\equiv\ln\Omega$. If one choose the conformal factor to be $\Omega=F=\frac{\partial f(R)}{\partial R}$ and introduce a new scalar field χ defined by $\Omega\equiv\exp\left(\frac{2\chi}{\sqrt{6}M_p}\right)$, using (2), the action (1) gets transform to an Einstein Hilbert form:

$$S_E=\int d^4x\sqrt{-\tilde{g}}\left[\frac{-M_p^2}{2}\tilde{R}+\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi+U(\chi)\right] \quad (3)$$

where $U(\chi)$ is the Einstein frame potential given by

$$U(\chi)=\frac{(RF(R)-f(R))\,M_p^2}{2F(R)^2} \quad (4)$$

which, by using the $f(R)$ form (1) and $\Omega=F=\exp\left(\frac{2\chi}{\sqrt{6}M_p}\right)$, can be given explicitly in terms of model parameters M and β as

$$U(\chi)=\frac{(\beta-1)}{2}\left(\frac{6M^2}{\beta^\beta}\right)^{\frac{1}{\beta-1}}\exp\left[\frac{2\chi}{\sqrt{6}}\left(\frac{2-\beta}{\beta-1}\right)\right]\times\left[1-\exp\left(\frac{-2\chi}{\sqrt{6}}\right)\right]^{\frac{\beta}{\beta-1}} \quad (5)$$

where we have taken $M_p=1$ and from here onwards we shall work in $M_p=1$ units. Also we see that in the limit $\beta\to2$ potential (5) reduces to exponentially corrected flat plateau potential of the Starobinsky model.

Assuming large field limit $\chi\gg\frac{\sqrt{6}}{2}$ and $1<\beta<2$, the potential (5) reduces to

$$U(\chi)\simeq\frac{(\beta-1)}{2}\left(\frac{6M^2}{\beta^\beta}\right)^{\frac{1}{\beta-1}}\exp\left[\frac{2\chi}{\sqrt{6}}\left(\frac{2-\beta}{\beta-1}\right)\right] \quad (6)$$

We shall use Eq. (6) later in Section 3 to compare with SUGRA version of the power law potential in the large field limit.

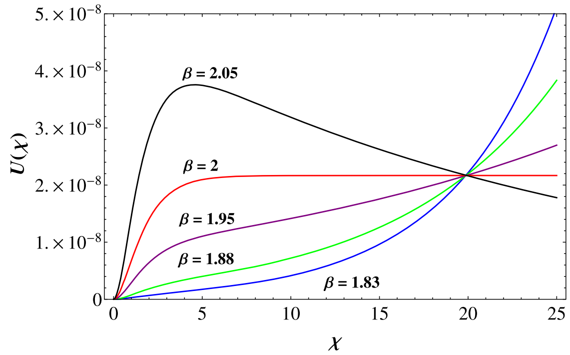

Fig. 1. The nature of the potential (5) for different β values (with $M=1.7\times10^{-4}$). The potential and the field values are in $M_p=1$ units.

In Fig. 1 we plot the potential for small deviations from the Starobinsky model value $\beta=2$. We see that the potential is flattest for $\beta=2$ but becomes very steep even with small deviation from Starobinsky model value $\beta=2$. The scalar-curvature perturbation $\Delta^2_{\mathcal{R}}\propto\frac{U(\chi)}{\epsilon}$ is fixed from observations which implies that the magnitude of the potential $U(\chi)$ would have to be larger as ϵ increases for steep potential. The tensor perturbation which depends on the magnitude of $U(\chi)$ therefore increases rapidly as β varies from 2. The variation of r with β is shown in Fig. 3.

From Eq. (5), in the large field approximation, the slow roll parameters in Einstein frame can be obtained as

$$\epsilon=\frac{1}{2}\left(\frac{U'}{U}\right)^2\simeq\frac{1}{3}\left[\frac{\beta(3-2\beta)}{(\beta-1)^2}\exp\left(\frac{-2\chi}{\sqrt{6}}\right)+\frac{\beta-2}{\beta-1}\right]^2 \quad (7)$$

$$\eta = \frac{U''}{U} \simeq \frac{-2}{3}\left[\frac{\beta(3-2\beta)^2}{(\beta-1)^3}\exp\left(\frac{-2\chi}{\sqrt{6}}\right) - \frac{(\beta-2)^2}{(\beta-1)^2}\right] \tag{8}$$

$$\xi = \frac{U'U'''}{U^2} \simeq \frac{4\sqrt{\epsilon}}{3\sqrt{3}}\left[\frac{\beta(3-2\beta)^3}{(\beta-1)^4}\exp\left(\frac{-2\chi}{\sqrt{6}}\right) + \frac{(\beta-2)^3}{(\beta-1)^3}\right] \tag{9}$$

The field value χ_e at the end of inflation can be fixed from Eq. (7) by using the end of inflation condition $\epsilon \simeq 1$. And the initial scalar field value χ_s corresponding to $N = 60$ e-folds before the end of inflation, when observable CMB modes leave the horizon, can be fixed by using the e-folding expression $N = \int_{\chi_e}^{\chi_s} \frac{U(\chi)}{U'(\chi)} d\chi$.

Under slow roll approximation we use the standard Einstein frame relations for the amplitude of the curvature perturbation $\Delta^2_{\mathcal{R}} = \frac{1}{24\pi^2}\frac{U^*}{\epsilon^*}$, the spectral index $n_s = 1 - 6\epsilon^* + 2\eta^*$, the running of spectral index $\alpha_s = \frac{dn_s}{d\ln k} = 16\epsilon^*\eta^* - 24(\epsilon^*)^2 - 2\xi^*$ and the tensor-to-scalar ratio $r = 16\epsilon^*$ to fix the parameters of our model. Note that the superscript $*$ indicates that the observables are evaluated at the initial field value χ_s.

We know from CMB observations, for 8-parameter $\Lambda CDM + r + \alpha_s$ model, that if there is a large running of the spectral index $\alpha_s = -0.013^{+0.010}_{-0.009}$ at (68% *CL*, PlanckTT+lowP) then the amplitude is $10^{10}\ln(\Delta^2_{\mathcal{R}}) = 3.089 \pm 0.072$, the spectral index is $n_s = 0.9667 \pm 0.0132$ and tensor-to-scalar ratio is $r_{0.05} < 0.168$ (95% *CL*, PlanckTT+lowP) [5–7]. Also a joint BICEP2/Keck Array and Planck analysis put an upper limit on $r_{0.05} < 0.12$(95% *CL*). Since the scalar potential $U(\chi)$ depends on both the parameters M and β whereas the slow roll parameters depend only on β, therefore parameter M affects only the scalar amplitude $\Delta^2_{\mathcal{R}} \propto \frac{U(\chi)}{\epsilon}$ whereas r, n_s and α_s which depend only on slow roll parameters remain unaffected by M. Therefore taking amplitude from the observation and fixing the number of e-foldings N fixes the values of M and β. We find numerically that for the best fit parameter values $\beta \simeq 1.88$ and $M \simeq 1.7 \times 10^{-4}$, the e-foldings turn out to be $N \approx 20$. The tensor-to-scalar ratio can be further reduced to $r \approx 0.06$ for $\beta \simeq 1.92$, $M \simeq 10^{-4}$ but e-foldings still come out to be low $N \approx 20$, see Fig. 2 (upper panel). Therefore constraining model parameters using running data imply that cosmological problems like horizon and flatness problems which require a minimum of 50–60 e-foldings cannot be solved with the power law generalization of the Starobinsky model.

Also from CMB observations, for 7-parameter $\Lambda CDM + r$ model, when there is no scale dependence of the scalar and tensor spectral indices the bound on r becomes tighter $r_{0.002} < 0.1$ (95% *CL*, PlanckTT+lowP) and the amplitude and the spectral index become $10^{10}\ln(\Delta^2_{\mathcal{R}}) = 3.089 \pm 0.036$ and $n_s = 0.9666 \pm 0.0062$ respectively at (68% *CL*, PlanckTT+lowP) [5–7]. We find that the values of $M \simeq 1.7 \times 10^{-4}$ and $\beta \simeq 1.83$ which satisfy the amplitude and the spectral index for $N \approx 60$ give large $r \approx 0.22$. Also we see that for $\beta \simeq 1.88$ and $M \simeq 1.25 \times 10^{-4}$ tensor-to-scalar ratio can be reduced to $r \simeq 0.1$ but it increases $n_s \simeq 0.985$, see Fig. 2 (lower panel).

3. Power law Starobinsky model from supergravity

In this section we give a SUGRA model of the power law Starobinsky model. We shall derive a model where the scalar potential in the Einstein frame is the same as Eq. (6) which we have shown in Section 2 is equivalent to the power law Starobinsky model $R + \frac{1}{6M^2}R^\beta$. The F-term scalar potential in SUGRA depends

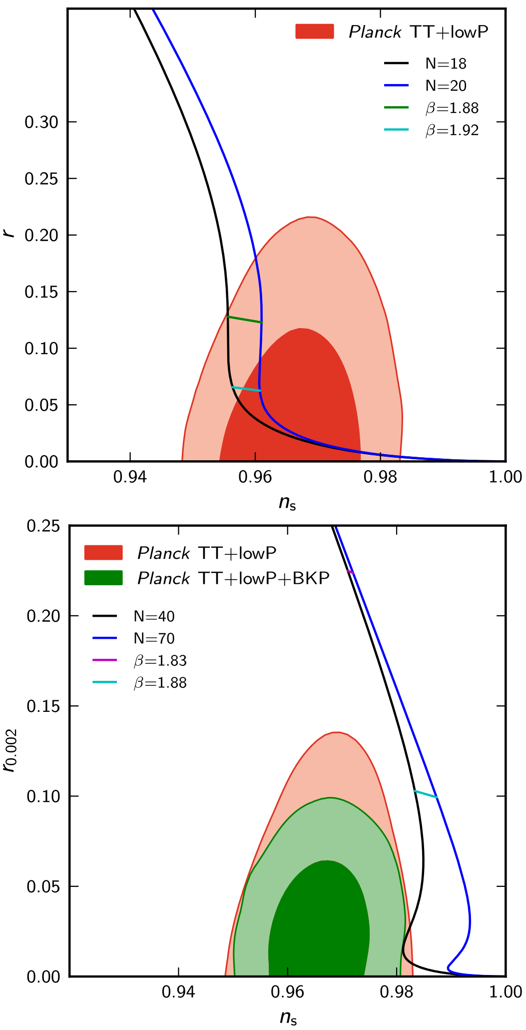

Fig. 2. The regions of (n_s, r) allowed by Planck-2015 and joint BKP analysis at 68% *CL* and 95% *CL* are shown. In the upper panel running of n_s is considered and in the lower panel there is no running of n_s. The colored contour lines are the predictions for our model for two sets of β and N values corresponding to $M \approx 10^{-4}$ which satisfies the observed amplitude of the CMB power spectrum. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

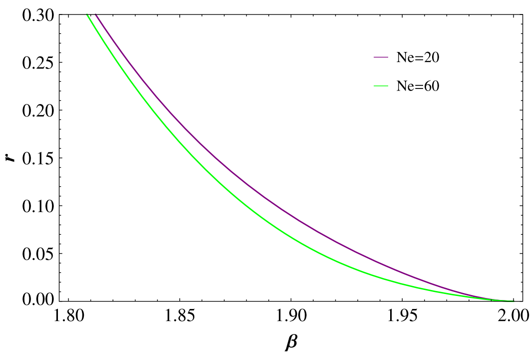

Fig. 3. The variation of r with β shown for two cases studied in our model: (i) for $N = 20$ when running of n_s is considered and (ii) for $N = 60$ when there is no running of n_s.

upon the combination of the Kähler potential $K(\Phi_i)$ and the superpotential $W(\Phi_i)$ as $G \equiv K + \ln W + \ln W^*$, where Φ_i are the chiral superfields whose scalar components are ϕ_i [56]. The effective potential and kinetic term in the Einstein frame are given by

$$V = e^G \left[\frac{\partial G}{\partial \phi^i} K^i_{j*} \frac{\partial G}{\partial \phi^*_j} - 3 \right] \tag{10}$$

and

$$\mathcal{L}_K = K_i^{j*} \partial_\mu \phi^i \partial^\mu \phi^*_j \tag{11}$$

respectively, where K^i_{j*} is the inverse of the Kähler metric $K_i^{j*} \equiv \partial^2 K / \partial \phi^i \partial \phi^*_j$.

A no-scale SUGRA model [30] with a choice of the Kähler potential $K = -3\ln[T + T^* - \phi\phi^*/3]$ and a minimal Wess–Zumino superpotential with a single chiral superfield Φ

$$W(\Phi) = \frac{\mu}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3 \tag{12}$$

give the same F-term potential in the Einstein frame as the Starobinsky model which gives vanishing tensor-to-scalar ratio $r \sim 0.003$ for specific choice $\frac{\lambda}{\mu} = \frac{1}{3}$. A slight change in the ratio $\frac{\lambda}{\mu}$ can increase r up to $r \sim 0.005$ but it gives large $n_s \approx 0.98$.

To get a no-scale SUGRA model corresponding to power law Starobinsky model which can give a larger r, we choose the minimal Wess–Zumino form of the superpotential (12) and a minimal no-scale Kähler potential with an added $(\phi + \phi^*)^n$ term as

$$K = -3\ln\left[T + T^* - \frac{(\phi + \phi^*)^n}{12}\right] \tag{13}$$

which can be motivated by a shift symmetry $T \to T + iC$, $\phi \to \phi + iC$ with C real, on the Kähler potential. Here T is a modulus field and ϕ is a matter filed which plays the role of inflaton.

We calculate Eq. (10) and Eq. (11) for chosen Kähler potential (13) and superpotential (12). We assume that the T field gets a vev $\langle T + T^* \rangle = 2\langle \mathrm{Re}\, T \rangle = c > 0$ and $\langle \mathrm{Im}\, T \rangle = 0$. We write ϕ in terms of its real and imaginary parts $\phi = \phi_1 + i\phi_2$. If we fix the imaginary part of the inflaton field ϕ to be zero, then $\phi = \phi^* = \phi_1$ and for simplicity we replace ϕ_1 by ϕ; the effective Lagrangian in the Einstein frame is given by

$$\mathcal{L}_E = \frac{n(2\phi)^{n-2}[c(n-1) + \frac{(2\phi)^n}{12}]}{4[c - \frac{(2\phi)^n}{12}]^2} |\partial_\mu \phi|^2 - \frac{4(2\phi)^{2-n}}{n(n-1)[c - \frac{(2\phi)^n}{12}]^2} \left|\frac{\partial W}{\partial \phi}\right|^2 \tag{14}$$

To make the kinetic term canonical in the $\mathcal{L}_E$, we redefine the field ϕ to χ with

$$\frac{\partial \chi}{\partial \phi} = -\frac{\sqrt{n(2\phi)^{n-2}[c(n-1) + \frac{(2\phi)^n}{12}]}}{2[c - \frac{(2\phi)^n}{12}]} \tag{15}$$

Assuming that $n \sim \mathcal{O}(1)$ and the large field limit $(2\phi)^n \gg 12c$ during inflation, integrating Eq. (15) gives

$$\phi \simeq \frac{1}{2}\exp\left(\frac{2\chi}{\sqrt{3n}}\right)\left[1 + \frac{6c(n+1)}{n}\exp\left(\frac{-2n\chi}{\sqrt{3n}}\right)\right] \tag{16}$$

Now substituting from Eq. (12) and Eq. (16) into the potential term of Eq. (14) and simplifying, we get the effective scalar potential in the Einstein frame as

$$V = \frac{144\mu^2}{n(n-1)}\exp\left[\frac{2\chi}{\sqrt{6}}\left(\frac{3\sqrt{2}(2-n)}{\sqrt{n}}\right)\right] \times \left[1 - \frac{2\mu}{\lambda}\exp\left(\frac{-2\chi}{\sqrt{3n}}\right) - \frac{9c(n^2-n-2)}{n} \times \exp\left(\frac{-2n\chi}{\sqrt{3n}}\right)\right]^2 \tag{17}$$

Table 1
The SUGRA model parameter values (in $M_p = 1$ unit) for three values of β corresponding to running and without running of spectral index n_s as depicted in Fig. 2 and for Starobinsky limit $\beta = 2$.

β	M	n	$\mu = \frac{\lvert\lambda\rvert}{2}$	$\alpha_s = \frac{dn_s}{d\ln k}$
1.83	1.7×10^{-4}	1.93	3.13×10^{-6}	-9.16×10^{-6}
1.88	1.7×10^{-4}	1.96	5.54×10^{-6}	-2.86×10^{-3}
2.00	1.1×10^{-5}	2.00	1.16×10^{-6}	-5.23×10^{-4}

which, assuming $1 < n < 2$, in the large field limit $\chi \gg \frac{\sqrt{3n}}{2}$ is equivalent to

$$V \simeq \frac{144\mu^2}{n(n-1)}\exp\left[\frac{2\chi}{\sqrt{6}}\left(\frac{3\sqrt{2}(2-n)}{\sqrt{n}}\right)\right] \tag{18}$$

We see that in the limit $n \to 2$ and with the specific choice $\frac{\lambda}{\mu} = \frac{1}{2}$, the potential (17) reduces to Starobinsky model potential.

We can now compare the power law potential (6) and SUGRA potential (18) for inflaton to show the relation between the parameters of the two models. Comparing the constant coefficient and exponent in the two potentials, we get

$$\beta = \frac{2\sqrt{n} + 3\sqrt{2}(2-n)}{\sqrt{n} + 3\sqrt{2}(2-n)} \tag{19}$$

and

$$M^2 = \frac{\beta^\beta}{6}\left[\frac{288\mu^2}{n(n-1)(\beta-1)}\right]^{\beta-1}. \tag{20}$$

Numerically we find the SUGRA model parameter values (in $M_p = 1$ unit) for three values of β corresponding to running and without running of spectral index n_s as depicted in Fig. 2 and for Starobinsky limit $\beta = 2$ as shown in Table 1.

4. Equivalence of the power law Starobinsky model with generalized non-minimally curvature coupled models

In this section we will show that generalized non-minimally coupled inflation models $\xi\Phi^a R^b$ [11] with the quantum corrected Φ^4-potential [12–14] can be reduced to the power law Starobinsky form. We consider the generalized non-minimal coupling $\xi\Phi^a R^b$ and the quantum correction to quartic scalar potential $\Phi^{4(1+\gamma)}$ into the action

$$S_J = \int d^4x\sqrt{-g}\left(-\frac{M_p^2 R}{2} - \frac{\xi\Phi^a R^b}{2M_p^{a+2b-4}} + \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{\lambda\Phi^{4(1+\gamma)}}{4M_p^{4\gamma}}\right) \tag{21}$$

where the scalar field Φ is the inflaton field. Since during inflation potential energy of the scalar field is dominant, therefore kinetic term in the action S_J can be neglected w.r.t. potential, the action reduces to

$$\int d^4x\sqrt{-g}\left(-\frac{M_p^2 R}{2} - \frac{\xi\Phi^a R^b}{2M_p^{a+2b-4}} + \frac{\lambda\Phi^{4(1+\gamma)}}{4M_p^{4\gamma}}\right) \tag{22}$$

we may integrate out the scalar field through its equation of motion $\frac{\partial L}{\partial \Phi} \approx 0$ [15], which implies

$$\Phi \approx \left(\frac{\xi a R^b}{2\lambda(1+\gamma)M_p^{a+2b-4(1+\gamma)}}\right)^{\frac{1}{4(1+\gamma)-a}} \tag{23}$$

Using Eq. (23) for Φ, the action (22) reduces to power law Starobinsky action

$$\int d^4x\sqrt{-g}\left(\frac{-M_p^2}{2}\right)\left(R+\frac{1}{6M^2}\frac{R^\beta}{M_p^{2\beta-2}}\right) \tag{24}$$

where the two parameters β and M of the power law model are identified in terms of a, b, λ, ξ and γ as

$$\beta=\frac{4b(1+\gamma)}{4(1+\gamma)-a} \tag{25}$$

and

$$M^2=\frac{a}{3(4(1+\gamma)-a)\lambda}\left(\frac{2\lambda(1+\gamma)}{\xi a}\right)^{\frac{4(1+\gamma)}{4(1+\gamma)-a}} \tag{26}$$

which for $a=2$, $b=1$, $\gamma=0$, i.e., at $\beta=2$, reduces to Higgs-inflation Starobinsky case $M_S^2\approx\frac{\lambda}{3\xi^2}\approx 10^{-10}$. Also with $a=2$, $b=1$, $\gamma\neq 0$ results of Refs. [12,13] are obtained.

5. Conclusion

We have explored a generalization of the Starobinsky model with a $\frac{1}{M^2}R^\beta$ model and fit β and M from CMB data. We find that to fit the amplitude $\Delta^2_{\mathcal{R}}$ and the spectral index n_s (with no running) from observations [5–7] we require $M\simeq 1.7\times 10^{-4}$ and $\beta\simeq 1.83$ for $N\approx 60$ but these parameter values give large $r\approx 0.22$. Also we find that the parameters β and M deviate from the $M\approx 10^{-5}$ and $\beta=2$ of the original Starobinsky model which could fit the amplitude and the spectral index but predicted very small value of $r\sim 10^{-3}$. When large running of the spectral index $\alpha_s\sim 10^{-3}$ is considered, we find that the best fit parameter values are $\beta\simeq 1.88$ and $M\simeq 1.7\times 10^{-4}$ which gives $N\approx 20$. This implies that the standard cosmological problems like Horizon and flatness problems which require a minimum of 50–60 e-foldings cannot be solved with the power law generalization of the Starobinsky model.

We have shown that the 5-parameter generalized non-minimal scalar-curvature coupled inflation models with the quantum correction to quartic scalar potential, i.e., $\xi\Phi^aR^b+\Phi^{4(1+\gamma)}$ are actually equivalent to 2-parameter power law Starobinsky model $\frac{1}{M^2}R^\beta$. Therefore we see that in terms of number of parameters the power law model is the most economical parametrization of the class of scalar-curvature models with quantum corrected Φ^4-potential.

In this paper we have given a SUGRA model for the general power law $\frac{1}{M^2}R^\beta$ model by adding a $(\Phi+\bar{\Phi})^n$ term to the minimal no-scale Kähler potential and with a Wess–Zumino form of the superpotential $W(\Phi)$. In the limit $n=2$ the Starobinsky limit $\beta=2$ is obtained. We derive the relations between the two parameters of the power law Starobinsky model and the two parameters of our SUGRA model. The interesting point about the generalization is that the small deviations from the Starobinsky limit of $n=\beta=2$ can produce value of $r\sim 0.1$ which is consistent with the joint Planck+BICEP2+Keck Array upper bound on $r<0.12(95\%\,CL)$. Generalizations of the Starobinsky model which can explain a possible larger value of r are therefore of interest.

Acknowledgement

We thank Akhilesh Nautiyal for valuable discussions.

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Moduli assisted two-field inflation from no-scale supergravity

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Abstract

We present a two-field inflationary scenario where inflaton field is accompanied by a dilaton field and has a non-canonical kinetic term due to the presence of the dilaton field. We show that novelty of such an inflationary scenario is that the quartic and quadratic inflaton potentials, which in standard single-field inflation models are ruled out by the present Planck data, yield scalar spectral index and tensor-to-scalar ratio in accordance with the present data. Such a model yield tensor-to-scalar ratio of the order of 10^{-2} which can be probed by future $B-$mode experiments like Keck/BICEP3, CMBPol, COrE, LiteBIRD and thus can be put to test in future. In a multifield scenario the curvature perturbations are not constant on superhorizon scales and isocurvature perturbations are expected to be generated. We show that in the considered two-field scenario, upto slow-roll approximation, the isocurvature perturbations vanish. To motivate such a two-field model, we show that it can be derived from no-scale supergravity with appropriate choice of superpotential and string motivated Kähler potential.

Keywords: Two-field Inflation, CMB, B-mode, No-scale Supergravity

1. Introduction

Inflation [1, 2, 3, 4, 5], a rapid accelerated expanding phase in the very early universe, provides a mechanism of generation of tiny density fluctuations, in an otherwise homogeneous and isotropic universe, which later evolve into large scale structures like galaxies and clusters of galaxies in the universe. It also explains the observed nearly scale invariant spectrum of these density fluctuations on superhorizon scales. Parameters, like scalar amplitude, scalar spectral index and tensor-to-scalar ratio, predicted by inflationary paradigm are being very accurately determined by the high-precision CMBR observations, like WMAP and Planck. The recent Planck-2015 high-ℓ polarization data puts an upper bound on the tensor-to-scalar ratio as $r_{0.002} < 0.11\,(95\%$ CL) and predicts the scalar amplitude and the scalar spectral index as $10^{10}\ln(\Delta^2_{\mathcal{R}}) = 3.089 \pm 0.036$ and $n_s = 0.9666 \pm 0.0062$ respectively at (68% CL, PlanckTT+lowP) [6, 7]. Joint BKP analysis of B-mode polarization data also puts a similar upper limit on tensor-to-scalar ratio as $r_{0.05} < 0.12\,(95\%$ CL) [8]. Recently BICEP2/Keck Array CMB polarization experiments combined with Planck analysis of CMB polarization and temperature data have further improved the bound on $r_{0.05} < 0.07\,(95\%$ CL)[9].

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There is a plethora of inflationary models which explain the key inflationary parameters like scalar amplitude, scalar spectral index and tensor-to-scalar ratio as observed in the CMB measurements. But it is well known that the simplest single field slow-roll inflation models with most common chaotic potentials, *e.g.* quartic ($\lambda\phi^4$) and quadratic ($m_\phi^2\phi^2$) potentials, produce large tensor-to-scalar ratio $r \simeq 0.26$ and $r \simeq 0.13$ respectively, for $\Delta N \approx 60$ where ΔN is the number of e-foldings from the end of inflation. The tensor-to-scalar ratio and the spectral index vary with ΔN in such scenarios as

$$n_s \simeq 1 - \frac{3}{\Delta N}\ ; \qquad r \simeq \frac{16}{\Delta N} \tag{1}$$

for quartic potential, and

$$n_s \simeq 1 - \frac{2}{\Delta N}\ ; \qquad r \simeq \frac{8}{\Delta N} \tag{2}$$

for quadratic potential. Thus single field models with quartic and quadratic potentials are ruled out at $\ln B = -23.3$ and $\ln B = -4.7$ respectively [6].

One novel way of making the quartic potential of inflaton viable is through what is now known as the Higgs inflation scenario [10], where the inflaton field ϕ is non-minimally coupled to the curvature scalar R (coupling term looks like $\xi\phi^2 R$). This gives rise to very small tensor-to-scalar ratio $r \sim 0.003$ for $N = 60$ which makes the scenario sit in the 'sweet spot' of the $n_s - r$ plot obtained by Planck, thus making it a highly favored inflationary model after the release of PLANCK-2013 data [11]. Even though, such a model might encounter the problem of unitarity violation because of very large curvature coupling $\xi \sim 10^4$ [12]. Starobinsky model of inflation with R^2 correction to Einstein gravity is mathematically equivalent to Higgs inflation model with quartic potential and therefore produces small tensor-to-scalar ratio $r \sim 0.003$ in a similar fashion [13, 14]. Higgs-inflation or Starobinsky inflation model can be put to test with future proposed $B-$mode experiments like Keck/BICEP3, CMBPol, COrE, LiteBIRD which aim to probe tensor-to-scalar ratio upto the theoretical limit of 2×10^{-3}, though such a limit is achievable only if the noise can be reduced to $\sim 1\,\mu$K-arcmin and lensing $B-$modes are reduced to 10% if one considers the preset PLANCK data on foreground [15]. On the other hand, generalized non-minimally coupled models with coupling term $\xi\phi^a R^b$ and quantum corrected quartic potential $\lambda\phi^{4(1+\gamma)}$, which are equivalent to power law inflation model $R + R^\beta$, are studied in [16, 17, 18, 19] which produce large $r \sim 0.2$. Such high tensor-to-scalar ratio is now disfavored by present status of the data [6, 7].

In this work we study a two-field inflationary model where the inflaton field ϕ is assisted by a dilaton field σ and has a non-canonical kinetic term due to the presence of the dilaton field. The action of such a model can be generically written as

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}\nabla^\mu\sigma\nabla_\mu\sigma - \frac{1}{2}e^{-\frac{\gamma\sigma}{M_p}}\nabla^\mu\phi\nabla_\mu\phi - e^{-\frac{\beta\sigma}{M_p}}V(\phi)\right], \qquad (3)$$

where $M_{\rm Pl}$ is the reduced Planck mass[1] and β and γ are arbitrary parameters. Such an action can be achieved by a conformal transformation from a Jordan frame where the action is either depicted by Brans-Dicke theory or where the dilaton field has a non-minimal coupling to gravity [21, 22, 23, 24]. The Brans-Dicke parameter ω_D or the non-minimal coupling parameter ξ present in the Jordan frame can be identified in terms of the parameters β and γ in the Einstein frame (EF). But, if the above action has such an origin in the Jordan frame then the parameters β and γ, which one gets after the conformal transformation to EF, can not remain as two independent parameters. Instead, in such cases these two parameters β and γ remain to be related, as they originate from the same term in Jordan frame namely the Brans-Dicke term or the non-minimal coupling term, where generically one finds $\beta = 2\gamma$. And with $\beta = 2\gamma$, which implies $\gamma < \beta$, for any choices of the parameter β values this model predicts larger tensor-to-scalar ratio than standard slow-roll inflation for quadratic and quartic inflaton potential. However, in the limit $\beta \to 0$ the prediction of n_s and r are exactly the same as the standard slow-roll inflation which is disfavored by the observations. Therefore, in this work we explore the other possibility where we treat both β and γ to be arbitrary and thus one can have $\gamma > \beta$.

We will see that this two-field scenario with $\gamma > \beta$ can bring down the tensor-to-scalar ratio considerably such that quartic and quadratic inflaton potentials are allowed by the observations. Also such a scenario, produces larger tensor-to-scalar ratio than generic Higgs inflation or Starobinsky inflation scenario which may be ruled out by future $B-$mode observations even if the theoretical limit of $r \sim 2 \times 10^{-3}$ cannot be achieved by these observations.

In order to motivate the above two-field action with $\gamma > \beta$, we will show that such an action can be derived from no-scale supergravity. We consider no-scale supergravity [26, 27, 28] as a natural framework for deriving our two-field model. No-scale supergravity is a special class of $\mathcal{N} = 1$ local supersymmetry in 4-dimensions, i.e. supergravity, which addresses the η-problem in supergravity [29, 30]. Also these theories naturally arises in the low-energy limit of the string theory compactifications [31]. The two field models of inflation with string motivated tree-level no-scale Kähler potential in no-scale supergravity framework are analyzed in Ref.s [32, 33, 34, 35]. With the considered form of Kähler potential and superpotantial, we will derive the two field action (3) where the parameter γ has a fixed value $\gamma = 2\sqrt{2/3}$ and β is arbitrary, therefore we can have $\gamma > \beta$ as required to achieve the correct values of the observables n_s and r.

The paper is organized as follows : In Section (2) we discuss in detail the evolution of background and perturbations in our model and derive the comoving curvature power spectrum and isocurvature power spectrum along with the observables namely the scalar spectral index and the tensor-to-scalar ratio. In Section (3) we consider two different inflaton potentials, namely quadratic and quartic, to show that these two potentials yield scalar spectral index and tensor-to-scalar ratio which are compatible with present observations. In Section (4) we give a SUGRA derivation of the considered two-field model. In Section (5) we would discuss the main results obtained in this paper and conclude.

2. The Model

In this section we would derive the evolution of background and scalar perturbations if one considers the action given in Eq. (3) with arbitrary β and γ.

2.1. Background Evolution

Let us first discuss the evolutions of the background fields and the background FRW metric in this setup. The equations of motion of the scalar fields σ and ϕ and the

[1]The signature of the metric has been taken here as (-, +, +, +). For the brevity of the text we shall use $M_p = 1$ unit from here onwards.

Friedmann equations are given as

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{\gamma}{2}e^{-\gamma\sigma}\dot{\phi}^2 - \beta e^{-\beta\sigma}V(\phi) = 0 \quad (4)$$

$$\ddot{\phi} + 3H\dot{\phi} - \gamma\dot{\sigma}\dot{\phi} + e^{(\gamma-\beta)\sigma}V'(\phi) = 0 \quad (5)$$

$$3H^2 = \frac{1}{2}\dot{\sigma}^2 + \frac{1}{2}e^{-\gamma\sigma}\dot{\phi}^2 + e^{-\beta\sigma}V(\phi) \quad (6)$$

$$\dot{H} = -\frac{1}{2}\left(\dot{\sigma}^2 + e^{-\gamma\sigma}\dot{\phi}^2\right) \quad (7)$$

where an over dot represents derivatives w.r.t. time and prime denotes derivative with respect to ϕ. In the slow-roll regime when both the fields slow-roll, terms with double time derivatives can be neglected therefore the background equations reduces to

$$3H\dot{\sigma} = \beta e^{-\beta\sigma}V(\phi), \quad (8)$$

$$3H\dot{\phi} = -e^{(\gamma-\beta)\sigma}V'(\phi), \quad (9)$$

$$3H^2 = e^{-\beta\sigma}V(\phi). \quad (10)$$

We define the slow-roll parameters for both the fields. Here the full potential $U(\sigma,\phi) = e^{-\beta\sigma}V(\phi)$ can be regarded as the product of potentials of the two fields. We then define the slow-roll parameters as :

$$\begin{aligned}
\epsilon_\phi &= \frac{1}{2}\left(\frac{U_\phi}{U}\right)^2 = \frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2, \\
\eta_\phi &= \frac{U_{\phi\phi}}{U} = \frac{V''(\phi)}{V(\phi)}, \\
\epsilon_\sigma &= \frac{1}{2}\left(\frac{U_\sigma}{U}\right)^2 = \frac{\beta^2}{2}, \\
\eta_\sigma &= \frac{U_{\sigma\sigma}}{U} = \beta^2. \quad (11)
\end{aligned}$$

Now to have the σ field slow-roll during inflation we require $\beta^2 \ll 2$.

Using Eq.s (10), the evolution of the background fields is given by

$$\sigma = \sigma_0 + \beta\ln\left(\frac{a}{a_0}\right), \quad (12)$$

$$\int d\phi\frac{V(\phi)}{V'(\phi)} = -\frac{e^{\gamma\sigma_0}}{\beta\gamma}\left[\left(\frac{a}{a_0}\right)^{\beta\gamma} - 1\right]. \quad (13)$$

Defining

$$f(\phi) \equiv \int d\phi\frac{V(\phi)}{V'(\phi)}, \quad (14)$$

and requiring $\frac{a_f}{a_0} \gtrsim e^{\Delta N}$ for sufficient inflation one gets

$$\frac{1}{\beta\gamma}\ln\left[1 + \beta\gamma e^{-\gamma\sigma_0}(f(\phi_0) - f(\phi_f))\right] \gtrsim \Delta N. \quad (15)$$

In the above expressions the subscript 0 and f represent the field values at the beginning and end of inflation respectively and the same notation will be used in the rest of the text.

2.2. Linear Perturbations

Let us now study the evolution of the scalar perturbations in this theory. In a longitudinal gauge the metric perturbations can be given as

$$ds^2 = -(1+2\Phi)dt^2 + a(t)^2(1-2\Phi)\delta_{ij}dx^i dx^j. \quad (16)$$

The equations of motion of scalar field perturbations in this gauge are given as

$$\delta\ddot{\sigma} + 3H\delta\dot{\sigma} + \left(\frac{k^2}{a^2} - \frac{\gamma^2}{2}e^{-\gamma\sigma}\dot{\phi}^2 + \beta^2 e^{-\beta\sigma}V(\phi)\right)\delta\sigma$$
$$+\gamma e^{-\gamma\sigma}\dot{\phi}\delta\dot{\phi} - \beta e^{-\beta\sigma}V'(\phi)\delta\phi = 2\beta e^{-\beta\sigma}V(\phi)\Phi + 4\dot{\Phi}\dot{\sigma}, \quad (17)$$

$$\delta\ddot{\phi} + (3H - \gamma\dot{\sigma})\,\delta\dot{\phi} + \left(\frac{k^2}{a^2} + e^{(\gamma-\beta)\sigma}V''(\phi)\right)\delta\phi - \gamma\dot{\phi}\delta\dot{\sigma}$$
$$+(\gamma-\beta)V'(\phi)e^{(\gamma-\beta)\sigma}\delta\sigma = -2e^{(\gamma-\beta)\sigma}V'(\phi)\Phi + 4\dot{\Phi}\dot{\phi}. \quad (18)$$

The ii, 00 and $0i$-components of the perturbed field equations give

$$\ddot{\Phi} + 4H\dot{\Phi} + (3H^2 + \dot{H})\Phi = \frac{1}{2}\Big[\dot{\sigma}\delta\dot{\sigma} + e^{-\gamma\sigma}\dot{\phi}\delta\dot{\phi}$$
$$-e^{-\beta\sigma}V'(\phi)\delta\phi - \beta\left(\frac{1}{4}e^{-\gamma\sigma}\dot{\phi}^2 - e^{-\beta\sigma}V(\phi)\right)\delta\sigma\Big], \quad (19)$$

$$3H\dot{\Phi} + (3H^2 + \dot{H})\Phi + \frac{k^2}{a^2}\Phi = \frac{-1}{2}\Big[\dot{\sigma}\delta\dot{\sigma} + e^{-\gamma\sigma}\dot{\phi}\delta\dot{\phi}$$
$$+e^{-\beta\sigma}V'(\phi)\delta\phi - \beta\left(\frac{1}{4}e^{-\gamma\sigma}\dot{\phi}^2 + e^{-\beta\sigma}V(\phi)\right)\delta\sigma\Big], \quad (20)$$

$$\dot{\Phi} + H\Phi = \frac{1}{2}(\dot{\sigma}\delta\sigma + e^{-\gamma\sigma}\dot{\phi}\delta\phi) \quad (21)$$

respectively.

The comoving curvature perturbation on constant energy density hypersurfaces [22, 23] is

$$\mathcal{R} = \Phi - \frac{H}{\dot{H}}\left(\dot{\Phi} + H\Phi\right) = \Phi + H\frac{\dot{\sigma}\delta\sigma + e^{-\gamma\sigma}\dot{\phi}\delta\phi}{\dot{\sigma}^2 + e^{-\gamma\sigma}\dot{\phi}^2}, \quad (22)$$

while its time derivative can be obtained by combining eq.s (19), (20) and (21) as

$$\dot{\mathcal{R}} = \frac{k^2}{a^2}\frac{H^2}{\dot{H}}\Phi + \mathcal{S}, \quad (23)$$

where $\mathcal{S}$ represents entropy (isocurvature) perturbations given by

$$\mathcal{S} = 2H\frac{e^{-\beta\sigma}(\beta\dot{\sigma}\dot{\phi}^2 V(\phi)e^{-\gamma\sigma} + \dot{\phi}\dot{\sigma}^2 V'(\phi))}{(\dot{\sigma}^2 + e^{-\gamma\sigma}\dot{\phi}^2)^2}\left(\frac{\delta\sigma}{\dot{\sigma}} - \frac{\delta\phi}{\dot{\phi}}\right). \quad (24)$$

On super horizon scales ($k \ll aH$) and under slow-roll approximation one can safely ignore terms containing $\dot{\Phi}$ and double-time derivatives to get

$$\Phi = \frac{1}{2H}(\dot{\sigma}\delta\sigma + e^{-\gamma\sigma}\dot{\phi}\delta\phi) = \frac{\beta}{2}\delta\sigma - \frac{V'(\phi)}{2V(\phi)}\delta\phi,$$
$$3H\delta\dot{\sigma} + \beta^2 e^{-\beta\sigma}V(\phi)\delta\sigma - \beta e^{-\beta\sigma}V'(\phi)\delta\phi = 2\beta e^{-\beta\sigma}V(\phi)\Phi,$$
$$3H\delta\dot{\phi} + e^{(\gamma-\beta)\sigma}V''(\phi)\delta\phi + (\gamma-\beta)e^{(\gamma-\beta)\sigma}V'(\phi)\delta\sigma$$
$$= -2e^{(\gamma-\beta)\sigma}V'(\phi)\Phi. \quad (25)$$

The above equations can be solved to give superhorizon solutions as, [see [21]]

$$\frac{\delta\sigma}{\dot{\sigma}} = \frac{c_1}{H} - \frac{c_3}{H}, \quad (26)$$

$$\frac{\delta\phi}{\dot{\phi}} = \frac{c_1}{H} + \frac{c_3}{H}\left(e^{-\gamma\sigma} - 1\right), \quad (27)$$

$$\Phi = -c_1\frac{\dot{H}}{H^2} + c_3\left[\frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2(1-e^{\gamma\sigma}) - \frac{\beta^2}{2}\right], \quad (28)$$

where c_1 and c_3 are the time independent integration constants and can be fixed using initial conditions. In the above expression (28), terms proportional to c_1 represent the adiabatic modes while those proportional to c_3 represent the isocurvature modes.

The equations of motion for scalar perturbations can be approximated as equations of motion of free massless scalar field in inflating background for $k \geq aH$ and even in the region $k < aH$ when $H(t_k) \gg |\dot{H}(t_k)|(t-t_k)$ and thus the expectation values of the scalar perturbations for the modes k crossing the Hubble horizon i.e. when $k = a(t_k)H(t_k)$ can be given by

$$\delta\sigma(\mathbf{k}) = \frac{H(t_k)}{\sqrt{2k^3}}\varepsilon_\sigma(\mathbf{k}), \qquad \delta\phi(\mathbf{k}) = \frac{H(t_k)}{\sqrt{2k^3}}e^{\frac{\beta\sigma(t_k)}{4}}\varepsilon_\phi(\mathbf{k}), \quad (29)$$

where the exponential factor in $\delta\phi(\mathbf{k})$ is due to the non-canonical kinetic term of the inflaton field and $\varepsilon_\sigma(\mathbf{k})$ and $\varepsilon_\phi(\mathbf{k})$ are classical random Gaussian quantities.

Using background slow-roll eq.s (8)-(10), eq. (22) can be simplified to give the curvature perturbations as

$$\mathcal{R} \simeq \Phi + c_1 - c_3 + c_3\frac{\frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2}{\frac{\beta^2}{2} + \frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 e^{\gamma\sigma}} \quad (30)$$

$$\simeq \Phi + c_1 - c_3 + c_3\frac{\epsilon_\phi}{\epsilon_\sigma + e^{\gamma\sigma}\epsilon_\phi}. \quad (31)$$

Since from eq.(28) it is clear that all the terms in Φ are proportional to $(c_1, c_3)\times$ slow-roll parameters, therefore we will ignore the potential Φ compared to c_1 and c_3. Using eq. (26) and eq. (27) we can calculate c_1 and c_3. Substituting for c_1 and c_3 into eq. (31), the comoving curvature perturbation on super horizon scales becomes

$$\mathcal{R} = H\frac{\delta\phi}{\dot{\phi}}e^{\gamma\sigma}A + H\frac{\delta\sigma}{\dot{\sigma}}B, \quad (32)$$

where $A = \epsilon_\phi/(\epsilon_\sigma + e^{\gamma\sigma}\epsilon_\phi)$ and $B = \epsilon_\sigma/(\epsilon_\sigma + e^{\gamma\sigma}\epsilon_\phi)$. Therefore, using eq.(29), the power spectrum of comoving curvature perturbations can be obtained as

$$\mathcal{P}_\mathcal{R} = \frac{k^3}{2\pi^2}\langle\mathcal{R}^2\rangle \simeq \frac{H^4}{4\pi^2}\left[\frac{e^{3\gamma\sigma}}{\dot{\phi}^2}A^2 + \frac{1}{\dot{\sigma}^2}B^2\right] \quad (33)$$

$$\simeq \frac{V(\phi)e^{-\beta\sigma}}{24\pi^2}\left[\frac{e^{\gamma\sigma}}{\epsilon_\phi}A^2 + \frac{1}{\epsilon_\sigma}B^2\right], \quad (34)$$

where in the second equality one replaces $\dot{\sigma}^2$ and $\dot{\phi}^2$ using eq.s (8) and (9).

On the other hand, the tensor power spectrum retains its generic form as

$$\mathcal{P}_\mathcal{T} = \frac{8H^2(t_k)}{4\pi^2}, \quad (35)$$

which, combining with (34), yields the tensor-to-scalar ratio as

$$r \equiv \frac{\mathcal{P}_\mathcal{T}}{\mathcal{P}_\mathcal{R}} \simeq 16\left[\frac{e^{\gamma\sigma}}{\epsilon_\phi}A^2 + \frac{1}{\epsilon_\sigma}B^2\right]^{-1}. \quad (36)$$

Now once we have the power spectrum of scalar perturbations (33), we can find the scalar spectral index using

$$n_s - 1 = \frac{d\ln\mathcal{P}_\mathcal{R}(k)}{d\ln k} = \frac{d\ln\mathcal{P}_\mathcal{R}(k)}{dt}\times\frac{dt}{dN}\times\frac{dN}{d\ln k}$$

$$\simeq \frac{4\dot{H}}{H^2} + \frac{1}{HP_1}\frac{dP_1}{dt}, \quad (37)$$

where $P_1 = (e^{3\gamma\sigma}/\dot{\phi}^2)A^2 + (1/\dot{\sigma}^2)B^2$ and we used $\frac{dt}{dN} = \frac{1}{H}$ and at horizon crossing $k = a(t_k)H(t_k)$, $\frac{dN}{d\ln k} \simeq 1 - \frac{\dot{H}}{H^2} \approx 1$. Using background slow-roll eq.s (7)-(10), we calculate $\ddot{\sigma}$ and $\ddot{\phi}$, eq. (37) can be further simplified to give

$$n_s - 1 \simeq -A\left[(6\epsilon_\phi - 2\eta_\phi)e^{2\gamma\sigma} + \beta(2\beta+\gamma)e^{\gamma\sigma}\right] - B\beta^2. \quad (38)$$

It can be noted that in the limit $\beta \to 0$ and $\gamma \to 0$ (i.e. $A = 1$ and $B = 0$) eq.s (34), (36) and (37) for power spectrum, tensor-to-scalar ratio and spectral index respectively reduces to their standard forms in the single field slow-roll inflation.

The amplitude of the isocurvature perturbations is given by

$$\mathcal{P}_\mathcal{S} = \frac{k^3}{2\pi^2}\langle\mathcal{S}^2\rangle$$

$$= \frac{H^4}{\pi^2}\frac{e^{(\gamma-2\beta)\sigma}\left(\beta e^{-\gamma\sigma}\dot{\phi}V(\phi) + \dot{\sigma}V'(\phi)\right)^2}{(\dot{\sigma}^2 + e^{-\gamma\sigma}\dot{\phi}^2)^3}, \quad (39)$$

which vanishes upon using eq.s (8) and (9) for $\dot{\sigma}$ and $\dot{\phi}$ respectively. Therefore in this two field model, up to slow-roll approximation, the isocurvature perturbations vanishes independent of inflaton potential and β, γ values.

3. Analysis of the Model with $\lambda_n\phi^n$ Potentials

In this section we analyze the observable parameters when inflaton has a potential $V(\phi) = \frac{\lambda_n}{n}\phi^n$. We define σ_0 and ϕ_0 as the field values ΔN e-folds before the end of inflation and σ_f and ϕ_f as the field values at the end of inflation. From eq. (13) we find $\sigma_f = \sigma_0 + \beta\Delta N$. Using eq. (8)-(10) we can solve for $\frac{\ddot{a}}{a} = H^2 + \dot{H}$ and setting $\frac{\ddot{a}}{a} = 0$, which is the condition for the end of inflation, we get the inflaton field value at the end of inflation as $\phi_f =$

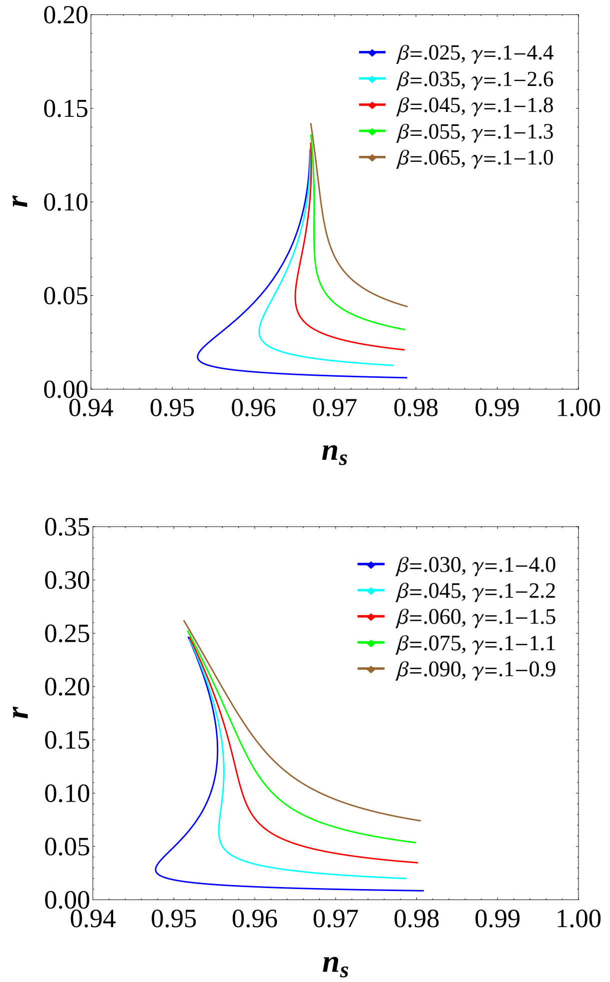

Figure 1: $n_s - r$ predictions of our two field model for quadratic (upper panel) and quartic (lower panel) potentials are shown. We have taken $\Delta N = 60$ and $\sigma_0 = 0.1$. In both the panels the range of values of γ increases along the curves from top to bottom. It is also manifest that as the values of β and γ goes to zero, n_s and r values converges to standard slow-roll inflation predictions.

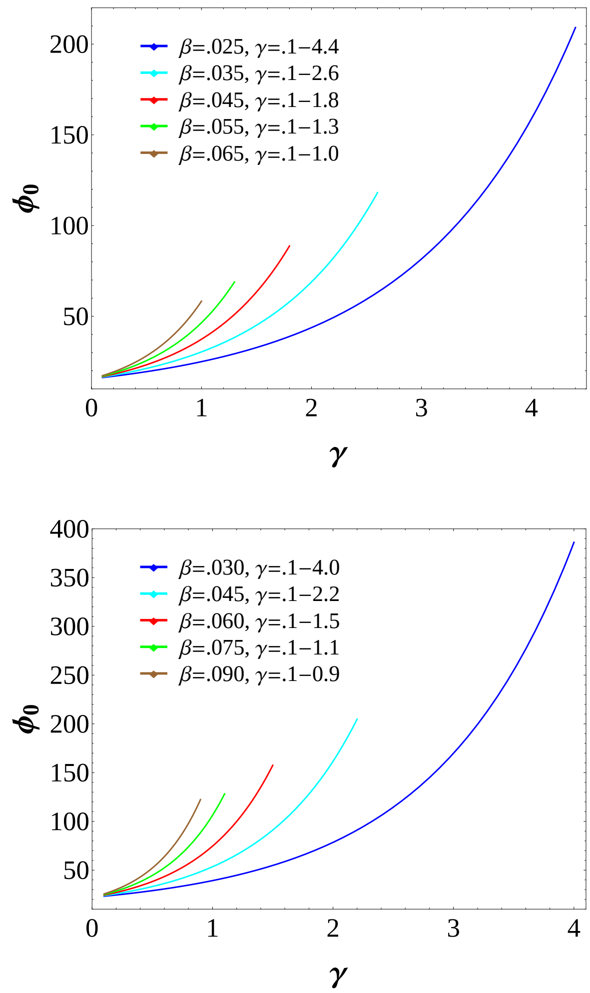

Figure 2: The inflaton values during inflation, for quadratic (upper panel) and quartic (lower panel) potentials, are shown. We have taken $\Delta N = 60$ and $\sigma_0 = 0.1$. In both the panels the range of values of γ increases along the curves from bottom to top. It is also manifest that as the values of β and γ goes to zero, the field values converges to standard slow-roll inflation predictions.

$ne^{\gamma\sigma_f/2}/\sqrt{2-\beta^2}$. The field value ϕ_0 can be expressed in terms of ϕ_f and σ_0 as given by eq.(15) as

$$\phi_0^2 \simeq \phi_f^2 + \frac{2ne^{\gamma\sigma_0}}{\beta\gamma}\left(e^{\beta\gamma\Delta N} - 1\right)$$
$$\simeq \frac{ne^{\gamma\sigma_0}\left[e^{\beta\gamma\Delta N}(4 - 2\beta^2 + n\beta\gamma) - 4 + 2\beta^2\right]}{\beta\gamma(2-\beta^2)}, \quad (40)$$

where, in the last equality, we used the expression for ϕ_f and σ_f as obtained above. Now we substitute ϕ_0 from eq. (40) into eq.s (34), (36) and (37) to give n_s, r and $\mathcal{P}_\mathcal{R}$ in terms of σ_0, n, ΔN, β and γ. For $\Delta N = 60$ e-folds and for the choice $\sigma_0 = 0.1$ with various choices of the parameters β and γ, the $n_s - r$ predictions for quadratic ($n = 2$) and quartic ($n = 4$) potentials are shown in the Fig. 1. For the above choices of the parameters values, inflaton field values during inflation are shown in Fig. 2. For $\sigma_0 = 0.1$, $\Delta N = 60$ and for the range of the parameter values of (β, γ) as shown in Fig. 1, we find scalar mass in the range $\lambda_2 = m_\phi^2 \sim 10^{-11}$–$10^{-14}$ and scalar self coupling in the range $\lambda_4 = \lambda \sim 10^{-13} - 10^{-17}$. E.g. for the choice $\beta = 0.05$ and $\gamma = 0.7$, which can produce $n_s \simeq 0.9666$ and $r \simeq 0.06$, gives $m_\phi \approx 2 \times 10^{-6}$. And for $\beta = 0.06$ and $\gamma = 1$, which gives $n_s \simeq 0.964$ and $r \simeq 0.05$, gives $\lambda \approx 10^{-16}$. Therefore in this two field scenario with quadratic and quartic potentials, similar to the case of single field slow-roll inflation, we require light inflaton mass and fine-tuning of the inflaton self-couplings in order to explain the observables. However, unlike the Higgs inflationary scenario, which has no fine tuning problem predicts very small tensor to scalar ratio $r \sim 10^{-3}$, the two field model predicts larger tensor to scalar ratio $r \sim 10^{-1} - 10^{-2}$.

4. Deriving the Model Action from No-scale Supergravity

In this section we give a derivation of the two field inflation model from no-scale Supergravity. The F-term scalar potential in Einstein Frame (EF) is determined from Kähler function given in terms of Kähler potential $K(\phi_i, \phi_i^*)$ and superpotential $W(\phi_i)$:

$$G(\phi_i, \phi_i^*) \equiv K(\phi_i, \phi_i^*) + \ln W(\phi_i) + \ln W^*(\phi_i^*), \quad (41)$$

where ϕ_i are the chiral superfields. In the supergravity action the effective potential and kinetic term in the Einstein frame are given by

$$V = e^G \left[\frac{\partial G}{\partial \phi^i} K^i_{j*} \frac{\partial G}{\partial \phi^*_j} - 3 \right], \quad (42)$$

and

$$\mathcal{L}_K = K_i^{j*} \partial_\mu \phi^i \partial^\mu \phi^*_j \quad (43)$$

respectively, where K^i_{j*} is the inverse of the Kähler metric $K_i^{j*} \equiv \partial^2 K / \partial \phi^i \partial \phi^*_j$.

In no-scale supergravity [36] with Kähler potential $K = -3 \ln [T + T^* - \rho\rho^*/3]$, where ρ is identified as chiral inflaton superfield and T as complex modulus field, and with a Wess-Zumino superpotential $W(\rho) = \frac{\mu}{2}\rho^2 - \frac{\lambda}{3}\rho^3$, the F-term scalar potential in EF would give Starobinsky-type of inflation potential with the choice $\frac{\lambda}{\mu} = \frac{1}{3}$. Here we consider the Kähler potential of the following form

$$K = -3 \ln[T + T^*] + \frac{b\rho\rho^*}{(T + T^*)^\omega}, \quad (44)$$

and we identify T as the two component chiral inflaton superfield and ρ as additional matter field with modular weight ω. In typical orbifold string compactifications with three moduli fields the modular weight ω has value 3 [37, 32, 33]. Here we shall treat ω as phenomenological parameter whose value can have small deviation from the canonical value 3 due to string loop corrections to the effective supergravity action [38]. Also we will see at the end of this section that the parameter b is no new parameter and can be given in terms of the parameter ω.

For the complete specification of the supergravity we assume the following superpotential

$$W = \lambda_m \; \rho \; T^m. \quad (45)$$

If we assume that the field ρ rapidly goes to zero at the onset of inflation, then from (42) and (43), for the taken K and W we get

$$V = \frac{\lambda_m^2 T^m T^{*m}}{b(T + T^*)^{3-\omega}}, \qquad \mathcal{L}_K = \frac{3\partial^\mu T \partial_\mu T^*}{(T + T^*)^2}. \quad (46)$$

We can decompose T field in its real and imaginary parts parametrized by two real fields ϕ and σ respectively as

$$T = e^{-\sqrt{\frac{2}{3}}\sigma} + i\sqrt{\frac{2}{3}}\phi. \quad (47)$$

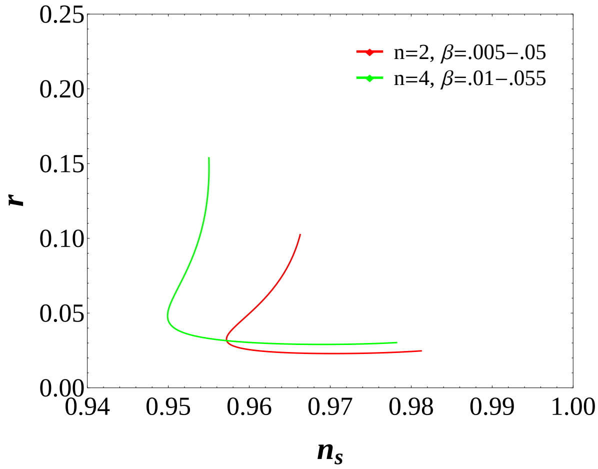

Figure 3: The $n_s - r$ predictions for a fixed value of $\gamma = 2\sqrt{2/3}$, for quadratic ($n = 2$) and quartic ($n = 4$) potentials, are shown. The range of values of β increases along the curves from top to bottom.

Using (47) into (46), we get the following forms of the kinetic and the potential terms in the Lagrangian:

$$\mathcal{L}_K = \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma + \frac{1}{2} e^{-\gamma\sigma} \partial^\mu \phi \partial_\mu \phi, \quad (48)$$

$$V = \frac{\lambda_m^2 2^{(\omega-3)}}{b} e^{-\beta\sigma} \left[e^{\gamma\sigma} + \frac{2}{3}\phi^2 \right]^m, \quad (49)$$

where $\gamma = 2\sqrt{\frac{2}{3}} \simeq 1.633$ and $\beta = (3-\omega)\sqrt{\frac{2}{3}}$. If during inflation the field σ evolves slow enough compared to inflaton field ϕ, then $e^{\gamma\sigma} << \frac{2\phi^2}{3}$ and hence first term inside the bracket in (49) may be neglected. Also as per the analysis performed in two-field model with quartic and quadratic potentials, we saw that the field σ during the 60 e-folds inflation, rolls from $\mathcal{O}(0.1\, M_p)$ to $\mathcal{O}(1\, M_p)$ and the corresponding change in the inflaton field value during inflation is $\phi \sim \mathcal{O}(10\, M_p)$ to $\mathcal{O}(1\, M_p)$. Therefore for $\gamma = 2\sqrt{\frac{2}{3}}$ we find that $e^{\gamma\sigma} << \frac{2\phi^2}{3}$. Hence, from (48) and (49), the effective matter Lagrangian in the EF can be given as

$$\mathcal{L}_M = \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma + \frac{1}{2} e^{-\gamma\sigma} \partial^\mu \phi \partial_\mu \phi + e^{-\beta\sigma} V(\phi) \quad (50)$$

where $V(\phi) = \frac{\lambda_m^2}{2m}\phi^{2m}$ and we set $b = \frac{2^\omega}{6}$ for quadratic potential i.e. with $m = \frac{n}{2} = 1$ and $b = \frac{2 \times 2^\omega}{9}$ for quartic potential i.e. with $m = \frac{n}{2} = 2$. For $\Delta N = 60$ and $\sigma_0 = 0.1$, the $n_s - r$ predictions for a fixed value of $\gamma = 2\sqrt{2/3}$ and with varying β are shown in Fig. 3.

5. Conclusions

The standard single field slow-roll inflation models with quartic and quadratic potentials, which generically produce large tensor-to-scalar ratio, are not compatible with the present Planck data. One novel way of making quartic self-coupling of inflaton viable with the present status of the data is by what is called the Higgs inflationary scenario which gives very small $r \simeq 0.003$. Though future $B-$mode detection experiments aim to reach the sensitivity where such low tensor-to-scalar ratio can be probed, there might be experimental limitations to reach such a low limit.

Here we present a two-field inflationary model where inflaton is accompanied by a dilaton field and has a non-canonical kinetic term due to the presence of the dilaton field. We find that, unlike standard single field slow-roll inflation with quadratic and quartic potentials, the observed $r < 0.11$ and $n_s = 0.966 \pm 0.0062$ can be obtained for certain choice of dilaton field value σ_0 and the parameter values β and γ for 60 e-folds as depicted in the Fig. (1). For example, inflaton with quadratic potential yields $n_s \sim 0.9666$ and $r \sim 0.06$ for parameters value $\beta = 0.05$, $\gamma = 0.7$ and inflaton with quartic potential yields $n_s \sim 0.964$ and $r \sim 0.05$ for parameters value $\beta = 0.06$, $\gamma = 1$. This shows that this scenario yields tensor-to-scalar ratio much larger than the generic Higgs inflationary scenario or Starobinsky inflationary scenario. This model for a range of parameters (β, γ) values can produce large tensor-to-scalar ratio $r \sim 10^{-1} - 10^{-2}$ consistent with the current bound on $r_{0.05} < 0.07\,(95\%\ \text{CL})$. Also r in this range would definitely be probed by future $B-$mode experiments and thus such a model can be put to test with these future observations. We also find that one requires to fine-tune the self-couplings of the inflaton field in order to be in accordance with observations, unlike the Higgs inflationary scenario. We show that, up to slow roll approximation, the amplitude of the isocurvature perturbations vanishes identically independent of the choice of the parameters values.

We derived the two-field inflation action from no-scale SUGRA with the considered form of Kähler potential and superpotential, wherein the parameter γ has a fixed value $\gamma = 2\sqrt{2/3}$ and β appears as an arbitrary parameter. However, we believe that with some string motivated Kähler potential and appropriate choice of superpotential one can derive the two-field action with absolutely arbitrary parameters β and γ. In this model with $\gamma > \beta$, which is the required condition to get the correct values of the observables n_s and r, we find that to fit the observables we need $\beta \sim 10^{-2}$, which, from the relation $\beta = (3 - \omega)\sqrt{\frac{2}{3}}$, implies a very small deviation of the parameter ω from 3 which may be obtained from the string loop contributions to tree-level supergravity effective action.

Acknowledgement

Work of SD is supported by Department of Science and Technology, Government of India under the Grant Agreement number IFA13-PH-77 (INSPIRE Faculty Award).

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