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SOME STUDIES OF ELECTRON–ATOM INELASTIC COLLISION AND HIGH ENERGY ELECTRON CAPTURE

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Certificate

This is to certify that the thesis entitled SOME STUDIES OF ELECTRON–ATOM INELASTIC COLLISION AND HIGH ENERGY ELECTRON CAPTURE submitted by Shri Himadri Shekhar Chakraborty who got his name registered on 13th of April, 1992 for the award of Ph.D. (Science) degree of Jadavpur University, is absolutely based upon his own work under the supervisions of Dr. D.P. Dewangan and Prof. D.P. Sural and that neither this thesis nor any part of it has been submitted for any degree/diploma or any other academic award anywhere before.

1.

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For

my sweet and loving Mom

and

for

the fondly cherished memory of my Dad

who was

one of the greatest dads

in the

universe

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A Summary of the Thesis

In inelastic scattering of structureless charged particle by neutral atoms in the intermediate and high energy regime the Glauber approximation [Gla59] has been used with reasonable success [GeT74]. One important and useful aspect of the method which emerges from the inherent properties of the approximation is that the Glauber predictions are expected to be progressively accurate with increasing initial and final principal quantum numbers of the target. The success of the method has motivated its extension [IsC75, NaT75, ThF76] to the case of charged particle and ionic target collision, where a net residual Coulombic interaction plays an important role. In doing so, people have treated the residual Coulomb interaction as a limit of an appropriately chosen short range potential [Gla59, GeT74, ThF76]. This new approximation, a modification of the Glauber by incorporating the effects of the long range Coulomb interaction potential, is known as the Coulomb–Glauber approximation.

We derive [DeC91] *a new one-dimensional integral representation* of the scattering amplitude in the Coulomb–Glauber approximation for arbitrary $nlm \rightarrow n'l'm'$ excitation of hydrogenic ions by the impact of structureless charged particles. With the target nuclear charge $+1$ this formula reduces straightaway to the Glauber scattering amplitude of hydrogen atom; this is a useful feature as it enables the method to work even for atomic hydrogen. An attractive feature of the integral representation is that it incorporates all detailed structures of initial and final state wave functions via the form factor. As a result the method remains free from parametric differentiations. We evaluate the form factor in a closed form for arbitrary $nlm \rightarrow n'l'm'$ transition by elementary method and program. Gauss quadrature is employed to perform the one–dimensional numerical integration to obtain the amplitudes and hence the differential cross sections. In performing the numerical integration in several segments over the whole range, care has been taken to use appropriate analytic continuation formulas of the Gauss hypergeometric functions to obtain rapid convergence. We carry out a further one–dimensional integration over the physical range of the momentum transfer to compute the total cross sections. In our computer program all q–independent coefficients of the form factor are calculated and stored once for all to compute a cross section for a given transition. Care has been taken to make the program efficient, economical and workable for the transitions involving higher excited states.

In order to test our method, we calculated the Glauber cross sections for several transitions, from the ground state to excited states as well as from excited states to excited states of hydrogen by electron impact. We found that all our Glauber cross sections for electron–hydrogen scattering agreed with the results reported previously [Tos77, SuS81].

As a test of the computer program, we calculated the cross sections for electron–impact excitation of hydrogenic ions (of $Z_T = 2$ to $Z_T = 10$) from the ground $1s$ state to various excited states. We have seen that our cross sections for $n = 2$ and $n = 3$ excitation of various

ions agreed with those obtained earlier by Thomas [Tho78]. We also performed calculations to reproduce several cross sections for $1s \rightarrow ns$ and $1s \rightarrow np$ excitation of hydrogenic ions reported by Das and Sil [DaS84b]. To our knowledge, no previous calculations are available in the literature for excited state to excited state transition of hydrogen–like ions by the impact of charged particles in the Coulomb–Glauber approximation. We present in this thesis some cross sections for electron scattering by HeII and BeIV.

The study of the dependence of the scattering amplitude for excitation of hydrogenic ions on the nuclear charge Z_T is of considerable interest, since its understanding can simplify the calculations of the cross sections over a range of energy and nuclear charge. We investigate [ChD94] the Z_T–dependence of the scattering amplitude in the Coulomb–Glauber approximation for arbitrary $nlm \rightarrow n'l'm'$ excitation of hydrogenic ions by the impact of structureless particles of charge Z_P and give a closed form expression of the Coulomb–Glauber amplitude in the limit of infinite Z_T. The closed form expressions of the limiting amplitude for $1s \rightarrow n' = 2$ and $1s \rightarrow n' = 3$ transitions derived previously [Tho78 and ThF76] are special cases of our general formula. From the general formula, one can also obtain explicit forms for excited state to excited state transitions. As illustrations, we give in this thesis explicit expressions of the limiting amplitude for the $2s \rightarrow 3s$ and $2s \rightarrow 3p$ transitions. We have written a program for evaluating the limiting amplitude and have verified that the present results agree with those obtained earlier by Das and Sil [DaS84b] and by Thomas [Tho78]. We have computed the CG cross sections for $Z_T = 2$ to $Z_T = 20$ by using the one–dimensional integral representation of the CG amplitude [DeC91] mentioned above. In some tables, we list cross sections for electron–impact excitation for the selected transitions such as $6s \rightarrow 7s$, $7s \rightarrow 8p$, $4d \rightarrow 7f$ and $3p \rightarrow 8p$ as a function of Z_T and energy (some of them are also presented graphically). It is seen that the Coulomb–Glauber results approach their limiting values as Z_T and E increase. However, how fast they tend to their limiting values is dependent on the characteristics of the initial and final states. In a separate table we compare the collision strengths in the Coulomb–Glauber approximation with those of the Coulomb–Born approximation [BHT70] for $Z_T = \infty$. As expected, the predictions of the two models are found to differ significantly, particularly at lower energies.

We also examine the behavior of the Coulomb–Glauber amplitude for large and small momentum transfers for arbitrary $nlm \rightarrow n'l'm'$ excitation. While the small q behavior of the amplitude shows an explicit dependence on both the orbital (l) and magnetic (m) quantum numbers of initial and final target states the large q behavior is free from l but exhibits an m–dependence.

We carry out the final integration over all energies to calculate the rates as a function of electron temperature. Our computer code can be used to generate collision data of astrophysical interest fairly efficiently. Systematic calculations are made to compute the rate coefficients for all transitions among the angular momentum sublevels of principal quantum numbers $n = 1, 2, 3, 4, 5, 6, 7$ and 8 for the targets HI, HeII and CVI over a wide range of electron temperatures [CSD93, CDS94]. We select 30 temperatures in the ranges as following: (1) for HI from $10^{3.4}$K

to $10^{7.0}$K, (2) for HeII from $10^{3.6}$K to $10^{8.0}$K and (3) for CVI from $10^{4.2}$K to $10^{10.0}$K. For all the cases several plots are given to compare the present rate coefficient results with that of the R–matrix calculations by Aggarwal *et al.* [ABB91, ABK91, AgK91a]. It is also shown that the formula developed by Kastner [Kas80] for ionic targets underestimates considerably both present and the R–matrix rate coefficients in large range of electron temperature. We tabulate our complete set of results for HeII and CVI.

We also present in this thesis a closed form expression [DeC89, DMDC90] for the B1B cross sections for $1s \rightarrow 1s$ capture. Our method makes the evaluation of the cross section a rather simple matter. We have programmed our final result and our numerical results agree perfectly with extensive calculations of previous authors [BGS86]. We remark that using the parametric differentiation technique of this work, one can also obtain closed form expressions for capture involving changes in angular momentum states.

Finally we mention that we have used for our computations the Work–Stations IBM RS6000/580, hp 9000/720 and Apollo 3500 of the Computer Center at P.R.L.

Chapter 1

Introduction

This thesis deals with two different topics in atomic collision physics . The first topic, which constitutes the major portion of the thesis, is concerned with the collisions of electrons with neutral hydrogen and hydrogenic ions in their excited states. The second topic is the charge transfer in energetic ion–atom collision in which a closed form expression for $1s \rightarrow 1s$ capture cross sections in boundary–corrected first Born approximation is obtained. In this chapter we describe the motivation behind our work.

1.1 Collisions of electrons with hydrogenic systems

Several astrophysical objects such as galaxies, gas clouds, stars etc. are observed spectroscopically. It is important to analyze their spectra in order to understand many properties of the plasmas in those objects. These kinds of analysis are usually carried out by the construction of the models. Because the observed photons are produced by atomic processes (and by pair–productions in very energetic environments), the collisions of electrons with atoms and ions play an important role in these studies [Sea75].

To interpret the observed spectra a good description of the atomic level population is needed. By considering the rates of radiative and collisional transitions between all states of the constituent atomic species, one obtains equations relating the level populations at each point to the local radiation field. The equation of radiative transfer relates the radiation field at a given point to the level populations everywhere in the object. Thus a large nonlinear system of equations must be solved. In this analysis, therefore, a knowledge of the transition rate coefficients between the levels of the model atom becomes essential.

Atomic hydrogen and several ions of the same isoelectronic sequence as hydrogen are of particular interest for the spectral diagnostics of many astrophysical plasmas. In the following we describe briefly the astrophysical importance of some of them:

(i) HI : Atomic hydrogen is the most abundant element in the astrophysical plasmas. In various applications (say for examples (a) [KrM78] and [CoD86] for active galactic nuclei, (b)

[DrU80] for the emission–line spectrum from a slab of hydrogen at moderate to high densities, (c) [BhU86] for Wolf–Rayet spectra) a hydrogen model atom with several principal levels is needed to interpret the spectra over a wide range of temperatures. The choice of how many levels should be split into l–sublevels is more dependent on the particular problem at hand and, especially, on the relevant density range.

(ii) HeII : Emission–lines of HeII are observed in a variety of solar and astrophysical objects. The future space missions, particularly, the SOHO (Solar and Heliospheric Observatory) are planned to observe its lines. Spectral analysis have been carried out to interpret the HeII lines for several Wolf–Rayet star spectra [CaB70, WiW78, BhU86].

(iii) CVI : Carbon is a cosmically abundant element with abundance approximately 8.7 on a scale where the abundance of atomic hydrogen is 12.0. It is observed in various ionization stages and the lines of CVI are of particular interest in the diagnosis and modeling of astrophysical plasmas such as solar corona and chromosphere [Kat76]. The lines of CVI is also of potential interest in the forthcoming project SOHO.

(iv) NeX : In astrophysical plasmas NeX is an important element [Kat76]. For the analysis of its spectra in several celestial objects rate coefficients for transitions upto the $n = 10$ levels [AgK91b] become important.

(v) CaXX : Calcium is also an important and abundant (with abundance 6.3 in the scale with that of HI being 12.0) coronal ion and the lines of CaXX have been observed in solar plasma [Kat76]. The temperature range of interest for emission lines of CaXX is expected to be above 10^5K [Kat76].

Apart from these, there are many other hydrogenic ions such as SeXIV, FeXXVI etc. find applications in astrophysics [Kat76].

A few specific examples of the theoretical modeling in astrophysics may be cited here. Bhatia and Underhill [BhU86] have analyzed the emission lines of HI, HeI and HeII in the spectra of the Wolf–Rayet stars HD 50896, HD 191765, HD 192103 and HD 192163, and of the O stars ζ Pap and α Cam. In their analysis, the model HeII and HI atoms consist of the first 14 and 16 levels respectively. Nine Wolf–Rayet stars spectra obtained in ultraviolet observations are interpreted by Willis and Wilson [WiW78]. In their work the HeII model ion is represented by its lowest 30 bound levels apart from a continuum. Also, the observed line intensities from the WN 6 stars HD 192163 and HD 191765 are used by Castor and Blerkom [CaB70] to infer the level populations of HeII ions in their envelopes. In this analysis the calculation of the populations of the first 30 levels of HeII is performed in which the equations of statistical equilibrium are solved with all radiative and collisional transitions included.

For such astrophysical applications, cross sections for the excited state to excited state transitions are needed. A good source to obtain the electron–atom and electron–ion collision data is

from laboratory experiments. Unfortunately the situation about the experimental results in the context of their various applications is not very promising. Most of the available data on the collision cross sections, to our knowledge, are confined to the transitions from the ground state upto a few low–lying levels. Some of them may be referred here: for $1s \rightarrow 2s$ and $1s \rightarrow 2p$ excitations of HI the reported measurements are [KOF70] (see Fig. 2.2A) and [LCS68] (see Fig. 2.2B) respectively; for $1s \rightarrow 3(s,p,d)$ excitations of HI Mahan *et al.* [Mah74, MGS76] reported their observations; and for $1s \rightarrow 2s$ and $1s \rightarrow 2p$ excitations of HeII the measurements are [DoP73] (see Fig. 2.6A) and [DZI74] (see Fig. 2.6C) respectively. Primarily because of the difficulties in preparing the experimental samples, the data for transitions involving higher excited states are meager. As a consequence, one depends on theoretical calculations.

A variety of theoretical methods for calculating electron–atom scattering cross section have been developed (see Chapter 2). In close–coupling methods, the total wave function is expanded in a complete set of target eigenstates. Therefore, the close–coupling predictions are, in principle, exact. However, in practice, one truncates the sum and retains some manageable number of *important* states to evaluate the amplitude. The effect of the continuum is often accounted for by including pseudostates in the basis set (we discuss this in Section 2.3). It is known that the close–coupling calculations for excited state to excited state transitions—where a large number of basis states (both exact atomic states and pseudostates) are needed to account for couplings among a number of nearby levels having small energy–spacings—become calculationally complicated. However, 15–state R–matrix calculations have been reported in a series of papers by Aggarwal and his co–workers [ABB91, ABK91, AgK91a, AgK91b, AgK92a, AgK92b, AgK93] for atomic hydrogen and several hydrogenic ions for all transitions between $n = 1,2,3,4$ and 5 states. Perturbative methods like the first order Born and Coulomb–Born approximations are valid for high energies. Semiclassical methods [PeR70, PeR71a,b, Ric73] have proved to be useful for higher level of hydrogenic systems, but in general they give results for summed $n \rightarrow n'$ cross sections.

Because of these reasons no theoretical calculations of state to state ($nlm \rightarrow n'l'm'$) cross sections for higher excited state transitions (say for $n, n' > 5$) of hydrogenic ions, to our knowledge, are available in the literature. On the other hand, as mentioned above, in astrophysical plasma diagnostics, the model ions are sometimes represented by as large as 30 lowest bound levels [WiW78, CaB70]. Obviously, this is far from an ideal situation and workers in this field usually take resort to either some ad hoc formulas (examples: [Reg62], [Joh72], [GPL76], [Kas80] etc.) or some numerical approximations [GNP87, GiP89] available in the literature. In these cases, the results are estimated from simplistic assumptions and/or by extrapolating available experimental or theoretical data of various qualities. Since the validity of these formulas are not tested properly, their applications may induce errors in the analysis.

Therefore, realizing the need for collision cross sections and rate coefficients for a wide selection of target excited states, this project has been undertaken. We employ the Glauber and the Coulomb–Glauber approximations. The Glauber approximation has been used over many

years in the electron–atom collisions in intermediate energies with reasonable success. However, because of a large number of atomic levels having approximately the same electronic energy, the underlying assumptions of the Glauber approximation can be better justified for excited state to excited state transitions than the excitations from the ground state. As a result, its predictions are expected to be good for transitions involving the excited states. The Coulomb–Glauber approximation is an extension of the Glauber to include the residual Coulomb potential. One may thus expect similar success as of the Glauber from the Coulomb–Glauber predictions.

1.2 Analytic evaluation of capture cross sections in boundary–corrected first Born approximation

Charge transfer in high energy ion–atom collisions has received considerable attention since the early years of quantum mechanics. The first quantum mechanical model of energetic charge transfer was developed by Oppenheimer [Opp28] in 1928 in the wave treatment, and by Brinkman and Kramers [BrK30] in 1930 in the impact parameter treatment. This first–order model, the OBK1 approximation, is based on the standard T–matrix formalism (valid for short–range interactions) from which the nucleus–nucleus interaction is dropped. From that time onwards, for more than 50 years, the general thinking prevailing in the field was that the standard T–matrix formalism provided an exact theoretical framework for developing perturbative models of energetic charge exchange and the effect of the infinitely long–range of the Coulomb interactions involved in the problem was innocuous and unimportant. When it became clear that the standard first Born (OBK1) approximation failed to give a good description of experiments, attempts were focussed on developing more sophisticated higher–order and multiple scattering models of this formalism. This general thinking remained deeply entrenched in the field even after Cheshire [Che64] in 1964 and Belkić *et al.* [BGS79] in 1979 had stressed that a sound theoretical approach should satisfy the Coulomb boundary conditions.

The thinking changed in 1985 when Dewangan and Eichler [DeE85, DeE87b] discovered that, owing to an improper treatment of the long–range Coulomb potential, the standard T–matrix formalism gives rise to singularities and, therefore, various perturbative models developed within this formalism are theoretically unsound. It was further shown that a natural way to cure the singularity and to obtain divergence–free perturbation expansions, consists in treating the Coulomb interactions properly from the outset. These findings focused attention on the Coulomb boundary conditions and led to the development of an approach, the boundary–corrected formalism, which is consistent with the Coulomb boundary conditions. The first– and second–order Born approximations, the boundary–corrected first Born (see e.g. [BGH86, BST87, DeE86, DeE87a]) and second Born approximations (see e.g. [Bel88, DecE89a, DecE89b, DeB88]), of this formalism have been studied in many papers and this has clarified several long standing points of perturbation theory (for a discussion, see e.g. [BrD88, BrM92, DeE94]). The boundary–corrected first

Born or B1B approximation has been applied to a wide variety of ion–atom collision systems. It is established that, in contrast to the OBK1 approximation, it gives a remarkably good description of the integral cross section in the intermediate and high–energy regions where the Thomas double–scattering mechanism is not important. The applications of the boundary–corrected formalism and other approaches have been discussed in detail in [BrM92, DeE94].

In all the previous applications [BGH86, BST87, DeE86, DeE87a], the capture probability amplitude of the B1B approximation was reduced to a one–dimensional integral expression which was then used in the numerical computations of the cross sections. We have obtained closed form expressions for the the B1B amplitudes and cross sections for $1s \rightarrow 1s$ capture for arbitrary target and projectile nuclear charges, Z_T and Z_P [DeC89, DMDC90]. Chapter 6 gives an account of the derivation.

We use atomic units throughout this article unless we state otherwise.

Chapter 2

A Brief Outline of Some Theoretical Methods in Electron Atom Scattering

In this chapter we outline some of the theoretical methods that have been employed to study the scattering of electrons by atoms. We describe in some detail the Glauber and the Coulomb–Glauber approximations.

2.1 Notations

Let us consider a nonrelativistic collision of a structureless charged particle of charge Z_P and mass m_p with an N–electron atom. The nuclear charge and the mass of the target atom are denoted by Z_T and M_T respectively. We treat the incident particle distinguishable i.e. the spin–dependent interaction is neglected. We may write the Hamiltonian $\mathcal{H}$ of the system in the center of mass frame as

$$\mathcal{H} = \mathcal{K}_P + \mathcal{H}_a + \mathcal{V}, \tag{2.1}$$

where

$$\mathcal{K}_P \equiv -\frac{1}{2M}\nabla^2_{R_{rel}} \tag{2.2}$$

$$\mathcal{H}_a \equiv \sum_{I=1}^{N}\left(-\frac{1}{2}\nabla^2_{r_I} - Z_T/r_I\right) + \frac{1}{2}\sum_{I,J=1;I\neq J}^{N}\frac{1}{|\mathbf{r}_I - \mathbf{r}_J|} \tag{2.3}$$

and

$$\begin{aligned}\mathcal{V} &\equiv \mathcal{V}(\mathbf{R}_{rel},\zeta)\\ &\equiv Z_P\left(\frac{Z_T}{R_{rel}} - \sum_{I=1}^{N}\frac{1}{|\mathbf{R}_{rel} - \mathbf{r}_I|}\right)\end{aligned} \tag{2.4}$$

in which M ($\equiv m_P M_T/(m_P + M_T)$) is the reduced mass of the projectile–target system; $\mathbf{R}_{rel}$ is the relative position vector of the projectile and $\mathbf{r}_I$ labels the position vector of the I–th atomic electron with respect to the atomic nucleus. The symbols $\mathcal{K}_P$, $\mathcal{H}_a$ and $\mathcal{V}$ represent respectively the kinetic energy of the projectile, atomic Hamiltonian and interaction potential seen by the projectile. ζ stands for the collection of internal coordinates required to specify the atomic eigenstates. If we neglect the spin coordinates then $\zeta \equiv (\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \ldots, \mathbf{r}_N)$. The total outgoing wave function $\Psi^{(+)}$ of the $(N+1)$–particle system satisfies the time–independent Schrödinger equation

$$\mathcal{H}\Psi^{(+)} = \mathcal{E}\Psi^{(+)}. \tag{2.5}$$

In Eq. (2.5), $\mathcal{E} = \frac{1}{2M}K_i^2 + \epsilon_i = \frac{1}{2M}K_f^2 + \epsilon_f$ where $\mathbf{K}_i$ and $\mathbf{K}_f$ are the relative incident and final momenta, and ϵ_i and ϵ_f are the initial and final energy eigenvalues of the target.

If we consider the target atom to be neutral (i.e. $Z_T = N$ in Eq. (2.4)) then $\mathcal{V}$ is of short range. Here by short range we mean that the potential falls off faster than the characteristic $[R_{rel}]^{-1}$ decay of the Coulomb potential. In this case the solution of Eq. (2.5) satisfies the outgoing wave boundary conditions (see for examples [Bra83, Mes62])

$$\lim_{R_{rel}\to\infty} \Psi^{(+)} \sim \exp(i\mathbf{K}_i \cdot \mathbf{R}_{rel})\phi_i(\zeta) + \sum_n \phi_n(\zeta)\frac{\exp(iK_n R_{rel})}{R_{rel}} A(i \to n; \Theta) \tag{2.6}$$

where the sum runs over all open channels and ϕ_i denotes the normalized wave function of the initial bound state of the target atom. In (2.6) $\mathbf{K}_n$ is the relative momentum of the projectile with respect to the target in its excited state $\phi_n(\zeta)$ and $A(i \to n; \Theta)$ is the scattering amplitude where Θ is the angle of scattering. Choosing ϕ_f to be the final bound state wave function, the amplitude for transition from the initial state i to the final state f is given by

$$A(i \to f; \Theta) = -\frac{M}{2\pi} T_{fi} \tag{2.7}$$

where

$$T_{fi} = \left\langle \exp(i\mathbf{K}_f \cdot \mathbf{R}_{rel}\phi_f \,|\mathcal{V}|\, \Psi^{(+)} \right\rangle \tag{2.8}$$

is known as the transition amplitude or the T–matrix elements.

Let us suppose now that the target is an ion i.e. $Z_T \neq N$. The target is a positive or a negative ion accordingly as $Z_T > N$ or $Z_T < N$. We cast the interaction potential $\mathcal{V}_{ion}(\mathbf{R}_{rel}, \zeta)$ in this case as follows:

$$\mathcal{V}(\mathbf{R}_{rel}, \zeta) = \mathcal{V}_{neutral}(\mathbf{R}_{rel}, \zeta) + \mathcal{V}_{Coulomb}(\mathbf{R}_{rel}), \tag{2.9}$$

where

$$\mathcal{V}_{neutral}(\mathbf{R}_{rel}, \zeta) \equiv Z_P \left(\frac{N}{R_{rel}} - \sum_{I=1}^{N} \frac{1}{|\mathbf{R}_{rel} - \mathbf{r}_I|} \right) \tag{2.10}$$

$$\mathcal{V}_{Coulomb}(\mathbf{R}_{rel}) \equiv \frac{Z_P(Z_T - N)}{R_{rel}}. \tag{2.11}$$

Since $\mathcal{V}$ now contains a residual Coulomb term $\mathcal{V}_{Coulomb}$, the Schrödinger equation (2.5) satisfies the Coulomb boundary condition (see for examples [Bra83, MoM65])

$$\begin{aligned}\lim_{R_{rel}\to\infty} \Psi^{(+)} \sim\ & \exp[i\mathbf{K}_i \cdot \mathbf{R}_{rel} + a\log(K_i R_{rel} - \mathbf{K}_i \cdot \mathbf{R}_{rel})]\phi_i(\zeta) \\ & + \sum_n \phi_n(\zeta)\frac{\exp[iK_n R_{rel} - a\log(2K_n R_{rel})]}{R_{rel}} A(i \to n;\Theta)\end{aligned} \tag{2.12}$$

where $a = iZ_P(Z_T - N)/K_i$. Here the scattering amplitude for a final state $\phi_f(\zeta)$ can be written as

$$\begin{aligned}A(i \to f;\Theta) &= -\frac{M}{2\pi} T_{fi} \\ &= -\frac{M}{2\pi}\left\langle \Phi^{(-)}_{\mathbf{K}_f}\phi_f \left| \mathcal{V}_{neutral} \right| \Psi^{(+)} \right\rangle\end{aligned} \tag{2.13}$$

in which $\Phi^{(-)}_{\mathbf{K}_f}$ is the outgoing projectile–state in the field of an ion with charge $(Z_T - N)$ and is given by [Bra83, OMK78]

$$\begin{aligned}\Phi^{(-)}_{\mathbf{K}_f} &= \exp\left(\pi\frac{Z_P(Z_T - N)}{K_f}\right)\Gamma(1+b)\exp(i\mathbf{K}_f \cdot \mathbf{R}_{rel}) \\ &\quad \times {}_1F_1[-b, 1; -i(K_f R_{rel} + \mathbf{K}_f \cdot \mathbf{R}_{rel})]\end{aligned} \tag{2.14}$$

where $b = iZ_P(Z_T - N)/K_f$.

The differential cross section for excitation $i \to f$ is obtained by the basic relation

$$\frac{d\sigma}{d\Omega}(i \to f;\, \Theta,\, E) = \frac{K_f}{K_i}\left|A(i \to f;\, \Theta,\, E)\right|^2 \tag{2.15}$$

where $d\Omega$ is the element of solid angle subtended at the center of mass of the projectile–target system. Affecting a change of variable from Θ to q, where $\mathbf{q} = \mathbf{K}_i - \mathbf{K}_f$ is the momentum transfer, a further integration over the physical range of $\mathbf{q}$ (from $q = |K_i - K_f|$ i.e. the forward scattering as $\Theta = 0$ to $q = K_i + K_f$ i.e. the backward scattering as $\Theta = \pi$) gives the total cross section

$$\sigma(i \to f;\, E) = \frac{2\pi}{K_i^2}\int_{|K_i - K_f|}^{K_i + K_f} dq\, q\left|A(i \to f;\, q,\, E)\right|^2. \tag{2.16}$$

It is extremely difficult to solve Eq. (2.5) exactly. Therefore, many approximation methods have been developed. These methods can be broadly divided into two groups, non–perturbative (Section 2.2) and perturbative (Section 2.3) methods. However, approximate schemes which include ingredients of both non–perturbative and perturbative methods have also been developed. Some of these methods are described below.

2.2 Non–perturbative methods

Several non–perturbative methods to solve Eq. (2.5) are available in the literature. In the close–coupling method the exact wave function $\Psi^{(+)}$ is expanded in a complete set of target eigenfunctions:

$$\Psi^{(+)} = \sum_{n=0}^{\infty} \phi_n(\zeta) F_n^{(+)}(\mathbf{R}_{rel}). \tag{2.17}$$

Substituting (2.17) in (2.5) gives the infinity of coupled equations

$$\left(\Delta_{rel}^2 + K_n^2\right) F_n^{(+)}(\mathbf{R}_{rel}) = 2M \sum_{m=0}^{\infty} \langle \phi_n(\zeta) | \mathcal{V} | \phi_m(\zeta) \rangle F_m^{(+)}(\mathbf{R}_{rel}) \qquad n = 0, 1, 2, \cdots. \tag{2.18}$$

In practice different methods are employed to solve Eq. (2.18).

The first is to truncate the sum over the target eigenstates in Eq. (2.17) to retain a finite number of states:

$$\Psi_t^{(+)} = \sum_{n=0}^{N_{max}} \phi_n(\zeta) F_n^{(+)}(\mathbf{R}_{rel}). \tag{2.19}$$

The coupled equations thus obtained are solved by direct numerical methods or indirectly, by variational or R–matrix methods.

The essential idea of the R–matrix (RM) method [BuR75] is that the configuration space describing the scattered particle and the target is divided into two regions. In the internal region ($R_{rel} < a$) the interaction is many–body and strong and the collision process is difficult to calculate. In the external region, on the other hand, the interaction is week and in many cases is exactly solvable in terms of plane waves or of Coulomb waves. In the internal region a complete discrete set of states is defined by imposing logarithmic boundary conditions on the surface of this region. This R–matrix basis is then used to expand the collision wave function. From this and the known solution in the external region the scattering matrix (S–matrix) and the cross sections are calculated. In transitions involving a few low-lying states of the atom, an appropriate boundary is one which just envelopes the charge distribution of these states.

Here we briefly discuss some results obtained by these methods relevant to our work. A close–coupling calculation for transitions among all the levels of $n = 1, 2$ and 3 states of HeII are those of Hummer and Storey [HuS87] (see Figs. 2.6A and 2.6C for excitations to $n = 2$ states). They have performed six–state close–coupling calculations for partial waves with $L \leq 12$ using the IMPACT code of Crees *et al.* [CSW78] and the unitarized distorted–wave method of Eissner and Seaton [EiS72] for $13 \leq L \leq 23$. Fon and co–workers have recently studied the elastic scattering of electrons from hydrogen [FRA93] and the electron impact excitation of $n = 2$ states of hydrogen [FAR92] by applying a 15–state R-matrix method. Electron collision cross sections and excitation rate coefficients for all transitions between the $n = 1, 2, 3, 4$ and 5 levels of atomic hydrogen [ABB91] ((i) cross sections: see Figs. 2.2A and 2.2B for $n = 2$ state excitations, Fig. 2.4

for elastic $2s$ transition, and Figs. 2.5A–F for all $n = 2 \rightarrow n' = 3$ excitations; (ii) rate coefficients: see Figs. 5.1A–C for excitations $n = 2 \rightarrow n' = 3$, $n = 2 \rightarrow n' = 4$ and $n = 2 \rightarrow n' = 5$), and some hydrogenic ions, namely, HeII [ABK91] ((i) cross sections: see Figs. 2.6A and 2.6C for excitations of $n = 2$ states; (ii) rate coefficients: see Figs. 5.2A–C and 5.3A–C for all transitions among $n = 1, 2, 3$ and 4 states), CVI [AgK91a] (rate coefficients: see Figs. 5.4A–D, 5.5A–C and 5.6A–C for all transitions between the states $n = 1, 2, 3, 4$ and 5), NeX [AgK91b], SiXIV [AgK92a], CaXX [AgK92b] and FeXXVI [AgK93] are computed by Aggarwal *et al.* using the R–matrix program [BBL78] with 15 basis states.

In the intermediate energy range, however, the inclusion of the continuum in close–coupling methods is important. This can be done in several ways.

The effect of the continuum is included, for example, by augmenting the target eigenfunction expansion (2.19) with the addition of pseudostates [Bra83]. van Wyngaarden and Walters [vaW86] have approximated the continuum through the use of pseudo–states in their calculation for hydrogen. In that work the wave function was expanded in terms of three exact hydrogen states plus 18 pseudostates of angular momentum up to 2. For higher angular momenta, van Wyngaarden and Walters used an approximation for the second order distorted wave amplitude. Recently Callaway and Unnikrishnan [CaU93] report the cross sections by the coupled pseudostate calculation for excitations of the $n = 2$ (see Figs. 2.2A and 2.2B) and $n = 3$ states of hydrogen. Their 17–state basis contains all exact atomic states through $n = 3$ plus $4f$, and 10 pseudostates ($5s$, $3p$ and $2d$). For small values of the total angular momentum ($L \leq 3$), their computation employs the algebraic variational method [Cal78] and for larger L it uses the linear algebraic integral equation method. Wakid and Callaway [WaC80] have studied the $1s \rightarrow 2s$ transitions in HeII in the intermediate energy range (see Fig. 2.6B). They have employed the algebraic variational method with a small pseudostate basis. For $1s \rightarrow 2s$ and $1s \rightarrow 2p$ excitations of the ions, namely, CVI and OVIII, Abu–Salbi and Callaway [AbC81] have reported their study using a pseudostate expansion in which they solve the ensuing coupled integro–differential equations by a variational approach. In this work the Coulomb–Born approximation with exchange has been invoked to obtain the results for high energy and high–L. A multichannel extension of a hybrid algebraic variational method [ObN73] was applied by Morgan [Mor80] to study the electron impact inelastic scattering cross sections for the targets, namely, HeII, LiIII, CVI and NeX. Quantal close–coupling calculations of electron (proton) impact excitation of the $n = 2$ levels in the hydrogenic ions HeII, CVI, MgXII, SXVI and ArXVIII are reported by Zygelman and Dalgarno (ZyD87).

Another way to include the continuum is the optical potential method [Wal84, Bra83, BrM77]. This method is a way of rewriting equations (2.18) in a finite set of equations which contains only the channels of interest. We give below a treatment, in short, following Walters [Wal84].

If we are interested in the channels which involve the eigenstates correspond to $n = 1, 2, \cdots, N_{max}$, then we need only that part of $\Psi^{(+)}$ which contains these states, $\Psi^{P(+)}$ say.

This component may be extracted from $\Psi^{(+)}$ by using the projection operator

$$P \equiv \sum_{n=0}^{N_{max}} |\phi_n(\zeta)\rangle\langle\phi_n(\zeta)| \tag{2.20}$$

i.e.

$$\begin{aligned}\Psi^{P(+)} &= P\Psi^{(+)} \\ &= \sum_{n=0}^{N_{max}} F_n^{(+)}(\mathbf{R}_{rel})\phi_n(\zeta).\end{aligned} \tag{2.21}$$

It is shown [Gel69] that $\Psi^{P(+)}$ satisfies the equation

$$(\mathcal{K}_P + \mathcal{H}_a + \mathcal{V}_{\rm OPT} - \mathcal{E})\Psi^{P(+)} = 0, \tag{2.22}$$

where

$$\mathcal{V}_{\rm OPT} = P\mathcal{V}P + \lim_{\eta\to 0+} P\mathcal{V}Q[Q(\mathcal{E}+i\eta-\mathcal{K}_P-\mathcal{H}_a-\mathcal{V})Q]^{-1}Q\mathcal{V}P \tag{2.23}$$

in which

$$Q = 1 - P. \tag{2.24}$$

$\mathcal{V}_{\rm OPT}$ is the optical potential for the P–space. Using (2.21), Eq. (2.22) may be written in the coupled form

$$\begin{aligned}(\Delta_{rel}^2 + K_n^2)F_n^{(+)}(\mathbf{R}_{rel}) &= 2M\sum_{m=0}^{N_{max}} \langle\phi_n(\zeta)|\mathcal{V}_{\rm OPT}|\phi_m(\zeta)\rangle F_m^{(+)}(\mathbf{R}_{rel}) \\ & \qquad n = 0, 1, \cdots, N_{max}.\end{aligned} \tag{2.25}$$

This is a finite set of coupled equations involving only the states of interest.

Bransden *et al.* [BrC72, BrM77] have developed a second order optical potential method in which the potential $\mathcal{V}_{\rm OPT}$ is evaluated in a closure approximation. In a subsequent publication [BCS72] this method was applied to collision of electrons and protons with atomic hydrogen to calculate cross sections for elastic scattering, and $1s \to 2s$, $1s \to 2p$ and $1s \to 3p$ excitations. For all transitions among the $n = 1, 2, 3$ states of hydrogen, Callaway and his co–workers [CUO87] have carried out the calculation in which the resulting equations from a six–state close–coupling expansion are supplemented by an optical potential to represent contributions from states not explicitly included (see Fig. 2.4 for elastic $2s$ transition and Figs. 2.5A–F for $n = 2$ to $n' = 3$ excitations). In their calculation the optical potential is evaluated through a pseudostate expansion. An optical–potential study of electron impact excitations of HeII is carried out by Unnikrishnan *et al.* [UCO91] where they report excitation cross sections for transitions from the $1s$, $2s$ and $2p$ states in the energy range 4.4–14.71 Ry (see Figs. 2.6A and 2.6C for $1s \to 2s$ and $1s \to 2p$ transitions). They employ a six–state close–coupling expansion together with an optical potential representing the higher bound and continuum states.

The close–coupling calculation to explicitly include all contributing bound and continuum states is the recent work of Bray *et al.* They have reported their convergent close–coupling calculation for the $n = 2$ excitations of neutral hydrogen [BrS92] and HeII [BMW93]. These are optical model calculations in which P–space contains $1s$, $2s$ and $2p$ states and Q–space is everything else. In the evaluation of the optical potential for Q–space, Bray *et al.* summed over the exact remaining bound states and integrated over the Coulomb continuum states to numerical convergence. For bound states, angular momenta through 2 were required for convergence and for the continuum angular momenta through 15 were required.

For excitations to the $n = 2$ levels [SSW89] (see Figs. 2.2B and 2.3) and to the $n = 3$ levels [SOB93] of ground state hydrogen, Scott *et al.* have performed the calculation using the intermediate energy R–matrix method [BNS87]. In these calculations, the basis states used in the internal region were constructed from the bound $1s$, $2s$ and $2p$ hydrogen atom orbitals plus a set of L^2 functions. For intermediate angular momenta, pseudostate close–coupling results were used (basis set consisted of three exact states and six pseudostates) and for high angular momenta approximate second Born amplitude were used.

2.3 Perturbative methods

If $\mathcal{V}$ is of short range then the Schrödinger equation (2.5) together with the boundary condition (2.6) can be recast in the form of an integral equation, the Lippmann–Schwinger equation

$$\Psi^{(+)} = \exp(i\mathbf{K}_i \cdot \mathbf{R}_{rel})\phi_i + G_0^+ \mathcal{V}\Psi^{(+)} \tag{2.26}$$

where

$$G_0^{\pm} \equiv \lim_{\eta \to 0+} (\mathcal{E} - \mathcal{K}_P - \mathcal{H}_a \pm i\eta)^{-1} \tag{2.27}$$

is the Green's function. A formal series solution of Eq. (2.26) in powers of $\mathcal{V}$ is obtained by iteration:

$$\Psi^{(+)} = [1 + G_0^+\mathcal{V} + (G_0^+\mathcal{V})^2 + \cdots + (G_0^+\mathcal{V})^p + \cdots]\exp(i\mathbf{K}_i \cdot \mathbf{R}_{rel})\phi_i. \tag{2.28}$$

This is the Born series for the wave function. Using (2.28) in (2.7) gives the Born expansion of the scattering amplitude:

$$\begin{aligned} A(i \to f;\Theta) &= -\frac{M}{2\pi}\Big\langle \exp(i\mathbf{K}_f \cdot \mathbf{R}_{rel})\phi_f \Big| [\mathcal{V} + \mathcal{V}G_0^+\mathcal{V} + \mathcal{V}(G_0^+\mathcal{V})^2 \\ &\quad + \cdots + \mathcal{V}(G_0^+\mathcal{V})^p + \cdots]\Big| \exp(i\mathbf{K}_i \cdot \mathbf{R}_{rel})\phi_i \Big\rangle \\ &= \sum_{p=1}^{\infty} A_{\text{Born}}^{(p)}(i \to f). \end{aligned} \tag{2.29}$$

When the infinite series (2.29) is truncated after the n–th term we obtain the n–th Born amplitude:

$$A_{\text{n-thBA}}(i \to f) = \sum_{p=1}^{n} A_{\text{Born}}^{(p)}(i \to f). \tag{2.30}$$

From Eq. (2.30) the first Born amplitude is given by

$$\begin{aligned} A_{\rm FBA}(i \to f) &= A^{(1)}_{\rm Born}(i \to f) \\ &= -\frac{M}{2\pi} \langle \exp(i\mathbf{K}_f \cdot \mathbf{R}_{rel}) \phi_f \, |\mathcal{V}| \exp(i\mathbf{K}_i \cdot \mathbf{R}_{rel}) \phi_i \rangle \\ &= -\frac{M}{2\pi} \int d\mathbf{R}_{rel} \, d\zeta \exp(i\mathbf{q} \cdot \mathbf{R}_{rel}) \phi^*_f(\zeta) \mathcal{V} \phi_i(\zeta) \end{aligned} \tag{2.31}$$

where $*$ stands for complex conjugate. The first Born amplitude (2.31) is related to the form factor

$$G_{fi}(\mathbf{q}) = \int d\zeta \, \phi^*_f(\zeta) \sum_{I=1}^{N} \exp(i\mathbf{q} \cdot \mathbf{r}_I) \phi_i(\zeta) \tag{2.32}$$

by the relation

$$A_{\rm FBA}(i \to f) = 2Z_P M \frac{G_{fi}(\mathbf{q}) - N\delta_{fi}}{q^2}. \tag{2.33}$$

The first Born approximation is valid in high energies.

Another first order approximation is obtained if the first Born approximation to the reactance matrix or the K–matrix elements (see page 165 of [Bra83] for the definition of the reactance matrix) are computed, rather than that to the T–matrix elements (Eq. (2.8)). This method produces a unitary S–matrix [Bra83] and is known as the unitarized Born approximation. Couplings between the channels are, to some extent, allowed in this method. For a range of energies above the ionization threshold, electron impact cross sections are calculated among all inelastic transitions between hydrogen atom states whose principal quantum numbers are less than or equal to 4 (see Figs. 2.2A, 2.2B for $n = 2$ state excitations, Fig. 2.4 for elastic $2s$ transition, and Figs. 2.5A–F for $n = 2$ to $n' = 3$ excitations) by Whelan [Whe86a, Whe86b, Whe84] in the unitarized Born approximation where high–L problem is treated using the Bethe approximation [Whe86a] to the reactance matrix.

The second Born amplitude can be written, from (2.30), as

$$A_{\rm SBA}(i \to f) = A^{(1)}_{\rm Born}(i \to f) + A^{(2)}_{\rm Born}(i \to f). \tag{2.34}$$

Many studies, using the second Born approximation for electron scattering with atom, are there in the literature (for references, see [Wal84]). Buckley and Walters [BuW74, BuW75] have shown that the third–order second Born approximation, in which the second Born cross section is evaluated correct to only third order in the interaction potential, gives poor result for both elastic and inelastic scattering of electrons and positrons by helium atom in ground state. The full second Born approximation, in which cross sections are calculated by squaring the modulus of the full second Born amplitude, is better but still not satisfactory. These authors corrected the simplified second Born approximation [HoM68] and applied the corrected theory to (i) the elastic scattering of electrons and positrons by the 1^1s ground state [BuW74] and (ii) the electron and positron impact excitations of the $1^1s \to 2^1s$ transition [BuW75] in helium. The exact second

order Born calculation for electron excitation of the $2s$ state of hydrogen is due to Ermolaev and Walters [ErW80].

If the interaction $\mathcal{V}$ is split into two pieces as

$$\mathcal{V} = \mathcal{V}_1 + \mathcal{V}_2 \tag{2.35}$$

then the scattering amplitude can be rewritten as [Wal84]

$$A(i \to ; \Theta) = -\frac{M}{2\pi}\left[\left\langle \exp(i\mathbf{K}_f \cdot \mathbf{R}_{rel})\phi_f \left| \mathcal{V}_1 \right| \psi_i^{(+)}(\mathbf{K}_i)\right\rangle + \left\langle \psi_f^{(-)}(\mathbf{K}_f) \left| \mathcal{V}_2 \right| \Psi^{(+)}\right\rangle\right] \tag{2.36}$$

where

$$\psi_n^{(\pm)}(\mathbf{K}) = \exp(i\mathbf{K} \cdot \mathbf{R}_{rel})\phi_m(\zeta) + G_0^{\pm}\mathcal{V}_1\psi_n^{\pm}(\mathbf{K}) \tag{2.37}$$

in which $G_0^{\pm}$ is given by (2.27). The Lippmann–Schwinger equation (2.26) for $\Psi^{(+)}$ can then be rewritten as

$$\Psi^{(+)} = \psi_i^{(+)}(\mathbf{K}_i) + G_1^{+}\mathcal{V}_2\Psi^{(+)}, \tag{2.38}$$

where

$$G_1^{\pm} = \lim_{\eta \to 0+} (\mathcal{E} - \mathcal{K}_P - \mathcal{H}_a - \mathcal{V}_1 \pm i\eta)^{-1}. \tag{2.39}$$

Using (2.38) in Eq. (2.36) the scattering amplitude can be expanded as a power series on $\mathcal{V}_2$ [Wal84]:

$$\begin{aligned} A(i \to f; \Theta) = -\frac{M}{2\pi}\Big[&\left\langle \exp(i\mathbf{K}_f \cdot \mathbf{R}_{rel})\phi_f \left| \mathcal{V}_1 \right| \psi_i^{(+)}(\mathbf{K}_i)\right\rangle + \left\langle \psi_f^{(-)}(\mathbf{K}_f) \right| \Big(\mathcal{V}_2 + \mathcal{V}_2 G_1^{+}\mathcal{V}_2 \\ &+ \mathcal{V}_2 G_1^{+}\mathcal{V}_2 G_1^{+}\mathcal{V}_2 + \cdots + \mathcal{V}_2 \left(G_1^{+}\mathcal{V}_2\right)^n + \cdots\Big) \left| \psi_i^{+}(\mathbf{K}_i)\right\rangle\Big]. \end{aligned} \tag{2.40}$$

This is called the distorted wave Born series for the scattering amplitude. Truncating (2.40) to a finite number of terms one obtains a distorted wave approximation [Wal84, Bra83]. The nature of the approximation depends on the choice of $\mathcal{V}_1$. It may be noticed that the choice $\mathcal{V}_1 = 0$ gives the plane wave Born series (2.29).

The scattering amplitude in the distorted wave first Born approximation is given by

$$\begin{aligned} A_{\text{DWFBA}}(i \to f) = -\frac{M}{2\pi}\Big[&\left\langle \exp(i\mathbf{K}_f \cdot \mathbf{R}_{rel})\phi_f \left| \mathcal{V}_1 \right| \psi_i^{(+)}(\mathbf{K}_i)\right\rangle \\ &+ \left\langle \psi_f^{(-)}(\mathbf{K}_f) \left| \mathcal{V}_2 \right| \psi_i^{+}(\mathbf{K}_i)\right\rangle\Big]. \end{aligned} \tag{2.41}$$

Calculations are there in the distorted wave first Born approximation. For a detailed account we refer to [Cal80, BrM77, BrM78, Win78, MaW83] and the references therein.

The distorted wave second Born amplitude can be written as

$$\begin{aligned} A_{\text{DWFBA}}(i \to f) = -\frac{M}{2\pi}\Big[&\left\langle \exp(i\mathbf{K}_f \cdot \mathbf{R}_{rel})\phi_f \left| \mathcal{V}_1 \right| \psi_i^{(+)}(\mathbf{K}_i)\right\rangle + \left\langle \psi_f^{(-)}(\mathbf{K}_f) \left| \mathcal{V}_2 \right| \psi_i^{+}(\mathbf{K}_i)\right\rangle \\ &+ \left\langle \psi_f^{(-)}(\mathbf{K}_f) \right| \mathcal{V}_2 G_1^{+}\mathcal{V}_2 \left| \psi_i^{+}(\mathbf{K}_i)\right\rangle\Big]. \end{aligned} \tag{2.42}$$

There have also been a number of calculations in the distorted wave second Born approximation (for references, see [Wal84]). We cite a couple of them here. The distorted wave second order Born approximation has been applied by Kingston and Walters [KiW80] to the elastic scattering of electrons by ground state atomic hydrogen and to the electron impact excitation of the $1s \rightarrow 2s$ and $1s \rightarrow 2p$ transitions. For the elastic scattering these authors introduced a new method of choosing the average energy in the closure approximation to the second born amplitude. The full second order distorted–wave calculations are carried out by Madison and his co–workers [Mad84, MBM91]; calculations have been made for $n = 2$ state excitations of hydrogen atom.

It is sometime useful to make a perturbative expansion of (2.23) in powers of $\mathcal{V}$ as

$$\begin{aligned} \mathcal{V}_{\rm OPT} &= P\mathcal{V}P + P\mathcal{V}Q(\mathcal{E}+i\eta-\mathcal{K}_P-\mathcal{H}_a)^{-1}Q\mathcal{V}P \\ &\quad + P\mathcal{V}Q(\mathcal{E}+i\eta-\mathcal{K}_P-\mathcal{H}_a)^{-1}Q\mathcal{V}Q(\mathcal{E}+i\eta-\mathcal{K}_P-\mathcal{H}_a)^{-1}Q\mathcal{V}P+\cdots \end{aligned} \tag{2.43}$$

where the limit $\eta \rightarrow 0+$ is understood. For a description of the optical potential perturbative methods we refer to [Wal84].

A distorted wave treatment developed by McDowell and co–workers using Eq. (2.43) is known as the distorted wave polarized orbital model [BrM77]. In this model the target is distorted by the incident particle and at great distances this distortion takes the form of a polarization. Here one approximates the full optical potential $\mathcal{V}_{\rm OPT}$ (2.43) by its expansion to second order and neglect the absorptive part. This model is used by McDowell *et al.* [MMM73] to calculate cross sections for $1s \rightarrow ns$ $(n = 2,3,4,5)$ in hydrogen and HeII from threshold to 20 Ry. As an improvement of this work a coupled–state distorted–wave polarized–orbital model is employed to calculate cross sections for $1s \rightarrow 2s$ [MMN74] and $1s \rightarrow np$ [MMM75] of both hydrogen (see Figs. 2.2B and 2.3 for ground state excitations to $n = 2$ states) and HeII (see Figs. 2.6A, 2.6B and 2.6C for ground state excitations to $n = 2$ states). Also recently, the coupled–channel distorted–wave optical potential calculations for electron–hydrogen scattering are performed by Bray *et al.* [BMM90].

There are many other distorted wave methods available in the literature (see for references [BrM77]).

When $\mathcal{V}$ has a residual Coulomb part (see Eq. (2.9)), the scattering amplitude is given by Eq. (2.13). In this case, a Coulomb–Born expansion of the amplitude can be found where the motion of the incident projectile is described in the zeroth approximation by an eigenstate of the Hamiltonian $H = \mathcal{K}_P + \mathcal{V}_{Coulomb}$. Consequently, in the collision the incident electron undergoes a transition from one eigenstate of H to another, owing to the interaction $\mathcal{V}_{neutral}$. A first order amplitude, the Coulomb–Born amplitude is then given by

$$A_{\rm CB}(i \rightarrow f) = -\frac{M}{2\pi}\int d\mathbf{R}_{rel}d\zeta\,\Phi^{(-)}_{\mathbf{K}_f}{}^{*}(\mathbf{R}_{rel})\phi_f^*\mathcal{V}_{neutral}\,\phi_i\Phi^{(+)}_{\mathbf{K}_i}(\mathbf{R}_{rel}) \tag{2.44}$$

where

$$\begin{aligned} \Phi^{(+)}_{\mathbf{K}_i} &= \exp\left(\pi\frac{Z_P(Z_T-N)}{K_f}\right)\Gamma(1-a)\exp(i\mathbf{K}_i\cdot\mathbf{R}_{rel}) \\ &\quad \times {}_1F_1[a,1;i(K_iR_{rel}-\mathbf{K}_f\cdot\mathbf{R}_{rel})]. \end{aligned} \tag{2.45}$$

Burgess *et al.* [BHT70] have used the Coulomb–Born approximation to study the electron scattering from HeII (see Figs. 2.6B and 2.6C). Oh *et al.* [OMK78] investigated the excitation of positive ions by charged particle impact in the Coulomb–Born approximation and gave closed form expression for $1s \rightarrow 2s$ and $1s \rightarrow 2p$ amplitudes. Studies of electron impact excitation of the ground state of hydrogenic ions using the Coulomb–Born approximation have also been carried out by several other workers [MiS76, MiS78, SRS78, DSS83]; calculations were made to evaluate cross section numerically.

The Glauber and the Coulomb–Glauber approximations are also the perturbative methods. But since the present work is an application of these approximations, we devote a separate section to describe them in some detail.

2.4 The Glauber and Coulomb–Glauber approximation

Glauber approximation is one of the several eikonal approximations for the scattering amplitude. The derivation of the amplitude was given by Glauber [Gla59] in the case of potential scattering. The use of the Glauber approximation in atomic collisions in the intermediate and high energy regions has given useful results.

2.4.1 The Glauber amplitude

We give the Glauber amplitude formula. We do not intend to go into detailed mathematical accounts of the derivation of the formula; they are beautifully presented in various review articles [GeT74, JoQ74, ByJ77a CLF79]. We rather highlight its main assumptions [Wal84].

All important atomic states have the same energy

The center–of–mass frame Lippmann–Schwinger integral equation (2.26) can be re–written as

$$\begin{aligned} \Psi^{(+)}(\mathbf{R}_{rel},\zeta) &= \exp(i\mathbf{K}_i\cdot\mathbf{R}_{rel})\phi_i(\zeta) - \frac{M}{2\pi}\int d\zeta' d\mathbf{R}'_{rel}\sum_m \frac{\exp(iK_m|\mathbf{R}_{rel}-\mathbf{R}'_{rel}|)}{|\mathbf{R}_{rel}-\mathbf{R}'_{rel}|} \\ &\times \phi_m(\zeta)\phi_m^*(\zeta')\mathcal{V}(\mathbf{R}'_{rel},\zeta')\Psi^{(+)}(\mathbf{R}'_{rel},\zeta'). \end{aligned} \tag{2.46}$$

In Eq. (2.46) the summation over m extends over the complete set of intermediate states, both discrete and continuous of the target atom;

$$\frac{K_m^2}{2M} + \epsilon_m = \frac{K_i^2}{2M} + \epsilon_i, \tag{2.47}$$

where ϵ_m is the energy of the state ϕ_m.

The first important assumption is to set $K_m \simeq K_i$ for all m in (2.46). This becomes a reasonable assumption when (i) the projectile energy is sufficiently high and (ii) the transition occurs between the excited states of the target. Replacing K_m throughout by K_i and applying the closure relation

$$\sum_m \phi_m(\zeta)\phi_m^*(\zeta') = \delta(\zeta-\zeta') \tag{2.48}$$

we immediately simplify Eq. (2.46) to

$$\Psi^{(+)}(\mathbf{R}_{rel},\zeta) = \exp(i\mathbf{K}_i\cdot\mathbf{R}_{rel})\phi_i(\zeta) - \frac{M}{2\pi}\int d\mathbf{R}'_{rel}\frac{\exp(iK_i|\mathbf{R}_{rel}-\mathbf{R}'_{rel}|)}{|\mathbf{R}_{rel}-\mathbf{R}'_{rel}|} \times \mathcal{V}(\mathbf{R}'_{rel},\zeta)\Psi^{(+)}(\mathbf{R}'_{rel},\zeta). \quad (2.49)$$

In the above equation the internal coordinates ζ of the target composite are present as parameters; the initial eigenstate ϕ_i and the interaction $\mathcal{V}$ determine the ζ–dependence of $\Psi^{(+)}$. Therefore, the ϕ_i–dependence of Eq. (2.49) can be *exactly* removed by the substitution $\Psi^{(+)}(\mathbf{R}_{rel},\zeta) = \Psi(\mathbf{R}_{rel},\zeta)\phi_i(\zeta)$. We thus obtain

$$\Psi(\mathbf{R}_{rel},\zeta) = \exp(i\mathbf{K}_i\cdot\mathbf{R}_{rel}) - \frac{M}{2\pi}\int d\mathbf{R}'_{rel}\frac{\exp(iK_i|\mathbf{R}_{rel}-\mathbf{R}'_{rel}|)}{|\mathbf{R}_{rel}-\mathbf{R}'_{rel}|} \times \mathcal{V}(\mathbf{R}'_{rel},\zeta)\Psi(\mathbf{R}'_{rel},\zeta). \quad (2.50)$$

Evidently, this is the Lippmann–Schwinger equation for scattering of a particle by the potential $\mathcal{V}(\mathbf{R}'_{rel},\zeta)$.

Forward scattering

Here one assumes that the major contribution to the amplitude comes from forward scattering. This essentially implies that the intermediate momentum $-i\nabla_{rel}$, does not differ much from $\mathbf{K}_i$. As given by Glauber [Gla59], the solution of Eq. (2.50) is

$$\Psi(\mathbf{R}_{rel},\zeta) = \exp(i\mathbf{K}_i\cdot\mathbf{R}_{rel})\exp\left(-\frac{i}{v}\int_{-\infty}^{z}dz'\mathcal{V}(\mathbf{R}'_{rel}(x,y,z')),\zeta)\right) \quad (2.51)$$

where the incident momentum $\mathbf{K}_i = M\mathbf{v}$, in which $\mathbf{v}$ is the initial relative velocity in the center–of–mass system; it is understood that in (2.51) the Z–axis is in the direction of $\mathbf{K}_i$. Thus the incident plane wave sweeps virtually undeviated through the region of interaction and emerges suffering only a position–dependent change of phase. Substituting Eq. (2.51) in the definition (2.7) of scattering amplitude we obtain

$$A_{\mathrm{G}}(i\to f) = -\frac{1}{2\pi}\int d\mathbf{R}_{rel}d\zeta\exp(-iK_f\cdot\mathbf{R}_{rel})\phi_f^*(\zeta)\mathcal{V}(\mathbf{R}_{rel},\zeta) \times \exp(i\mathbf{K}_i\cdot\mathbf{R}_{rel})\phi_i(\zeta)\exp\left(-\frac{i}{v}\int_{-\infty}^{z}dz'\mathcal{V}(\mathbf{R}'_{rel}(x,y,z')),\zeta)\right). \quad (2.52)$$

Another important assumption of the Glauber approximation is that the momentum transfer, $\mathbf{q}\equiv\mathbf{K}_i-\mathbf{K}_f$, is perpendicular to the direction of $\mathbf{K}_i$. In this case the integral over z in (2.52) may be performed to obtain the final form of the Glauber amplitude for $i\to f$ excitation as

$$A_{\mathrm{G}}(i\to f) = \frac{iK_i}{2\pi}\int d\mathbf{b}\,d\zeta\exp(i\mathbf{q}\cdot\mathbf{b})\phi_f^*(\zeta)[1-\exp(i\chi)]\phi_i(\zeta), \quad (2.53)$$

wherein

$$\chi(\mathbf{b},\zeta) = -\frac{1}{v}\int_{-\infty}^{\infty}dz\mathcal{V}(\mathbf{R}_{rel},\zeta) \quad (2.54)$$

is the Glauber's eikonal phase–shift function. $\mathbf{b}$ is the projection of $\mathbf{R}_{rel}(\equiv x, y, z)$ onto the plane perpendicular to the Z–axis. The plane containing $\mathbf{q}$ and $\mathbf{b}$ defines the XY plane.

We insert Eq. (2.4) (with $Z_T = N$, for neutral target) into Eq. (2.54). Then, $\chi(\mathbf{b}, \zeta)$ reduces to

$$\chi(\mathbf{b}, \zeta) = 2\eta \sum_{I=1}^{N} \ln\left(\frac{|\mathbf{b} - \mathbf{s}_I|}{b}\right). \tag{2.55}$$

We thus find

$$\exp(i\chi) = \prod_{I=1}^{N} \left(\frac{|\mathbf{b} - \mathbf{s}_I|}{b}\right)^{2i\eta}, \tag{2.56}$$

where $\eta = -Z_P/v$. The two–dimensional vector $\mathbf{s}_I$ is the projection of $\mathbf{r}_I$ onto the XY–plane.

Apart from the derivation given by Glauber [Gla59], there are several other ways of deriving the eikonal result (2.51). For examples, Eq. (2.51) can be obtained by applying semi–classical approximation to the Schrödinger equation [GeT74]. Eq. (2.51) can also be obtained by linearizing the free propagator

$$g_0 = \frac{\exp(iK_i|\mathbf{R}_{rel} - \mathbf{R}'_{rel}|)}{|\mathbf{R}_{rel} - \mathbf{R}'_{rel}|} \tag{2.57}$$

in (2.50), after expressing it in the Fourier integral representation

$$g_0 = \frac{1}{2\pi^2} \lim_{\eta \to 0} \int d\tau \frac{\exp[i\tau \cdot (\mathbf{R}_{rel} - \mathbf{R}'_{rel})]}{\tau^2 - K_i^2 - i\eta} \tag{2.58}$$

where τ is the conjugate of $\mathbf{K}_i$ [GeT74].

A number of applications of the Glauber approximation to electron–atom scattering [GhS69, TTB69, GhS70, GSS70, TBG70, GhS71, BhG71, SuS81, ThG71, ThG73, Tos77] have been made. The Glauber scattering amplitudes for the excitation of the ns and np levels of atomic hydrogen by electrons (or protons) incident upon the ground state have been obtained in closed form by Thomas and Gerjuoy [ThG71] (see Figs. 2.2A, 2.2B and 2.3 for $n = 2$ excitations). The same authors [ThG73] have obtained a closed form Glauber cross section in p–H and e^-–H collisions for $1s \to 1s$ elastic scattering. Toshima [Tos77] employed the Glauber approximation technique for electron (positron) impact excitation of hydrogen atom and obtained a closed form expression of the amplitude for any arbitrary $nlm \to n'l'm'$ transition (see Fig. 2.4 for elastic $2s$ scattering and Figs. 2.5A–F for $n = 2 \to n' = 3$ excitations).

The Glauber amplitude (2.53) can be expanded in powers of $\mathcal{V}$ [ByJ77a, Wal84] as follows:

$$A_G(i \to f) = \sum_{n=1}^{\infty} A^{(n)}(i \to f) \tag{2.59}$$

where

$$A^{(n)}(i \to f) = \frac{K_i}{2\pi} \frac{i^{n-1}}{n!} \int d\mathbf{b} d\zeta \exp(i\mathbf{q} \cdot \mathbf{b}) \phi_f^*(\zeta) \chi^n \phi_i(\zeta). \tag{2.60}$$

The term $A^{(n)}(i \to f)$ is the n–th order in $\mathcal{V}$. The series (2.59) is called the Glauber series. This is to note here that the first term $A^{(1)}(i \to f)$ in this series is identically equal to the first Born amplitude (2.31), remembering that $\mathbf{q}$ is perpendicular to the Z–direction so that $\mathbf{q}\cdot\mathbf{b} = \mathbf{q}\cdot\mathbf{R}_{rel}$ in Eq. (2.60).

On the other hand, the higher terms in Eq. (2.59) are not identical to the corresponding terms ($A^{(n)}_{\rm Born}$, $n = 2, 3, \cdots$) of the Born series (2.29). Considering ϕ_i and ϕ_f real (s–states), it can be seen [ByJ77a, Wal84] that the terms in the Glauber series (2.59), starting from the second order term, are alternately imaginary and real whereas the corresponding Born terms are complex. Byron and Joachain [ByJ77b, ByJ74, ByJ75] have analyzed the leading behavior (in K_i) of the terms of the Glauber and Born series for $e^{\pm}$ scattering from the ground states of hydrogen and helium. Their results apply for $s \to s$ transitions. It is seen that the Glauber terms $A^{(n)}$ give in each order the leading piece of $A^{(n)}_{\rm Born}$ (which is of the order of K_i^{1-n}) at all q. The only exception is in second order at small q where $A^{(2)}$ diverges as $\log q$ as $q \to 0$.

Byron and Joachain [ByJ73] have suggested a *practical* approximation, called Eikonal Born Series (EBS) approximation for including the leading contribution from the third Born term. The EBS scattering amplitude is given by

$$A_{\rm EBS}(i \to f) \;=\; A^{(1)} + A^{(2)}_{\rm Born} + A^{(3)} \tag{2.61}$$

where the third Born term is replaced by the term $A^{(3)}$. Here the second Born term $A^{(2)}_{\rm Born}$ is evaluated by the closure approximation. To improve the Glauber amplitude in the second order a modified Glauber approximation (MGA) [ByJ75, Gie84] was suggested. The amplitude in MGA is written as

$$A_{\rm MGA}(i \to f) \;=\; A_{\rm G} + A^{(2)}_{\rm Born} - A^{(2)} \tag{2.62}$$

where the second term of the Glauber series (2.59) is replaced by the second Born term. An improvement upon the Glauber approximation was also made by the (first–order) Wallace approximation [Wal84]. The Wallace approximation has been improved by correcting it in second order. This leads to the unitarized eikonal Born series approximation [BJP81, BJP82].

In Fig. 2.1A we plot the Glauber differential cross sections for elastic scattering of electrons by H($1s$) [ByJ77b] and K($4s$) [Wal73] at 50 and 54.4 eV respectively to compare them with corresponding experimental data (for H [Wil75] and for K [BNT79]). It is seen that for hydrogen the Glauber results (dashed curve) lie well below the experimental points (filled circles), by a factor of approximately 2.5. On the contrary, for K, Glauber cross sections (solid curve) show good agreement with experiments (filled triangles). It is also found that for elastic scattering by Li($2s$) and Na($3s$) Glauber results compare experimental data very well in the forward angles, while for He(1^1s) case the accord with measurements is as bad as H($1s$) [Wal84].

The Glauber approximation, therefore, predicts strikingly good results for alkali atoms in contrast to those for H($1s$) and He(1^1s). This difference between the alkali metals on the one hand and H($1s$) and He(1^1s) on the other lies in atomic energy differences. The energy separation

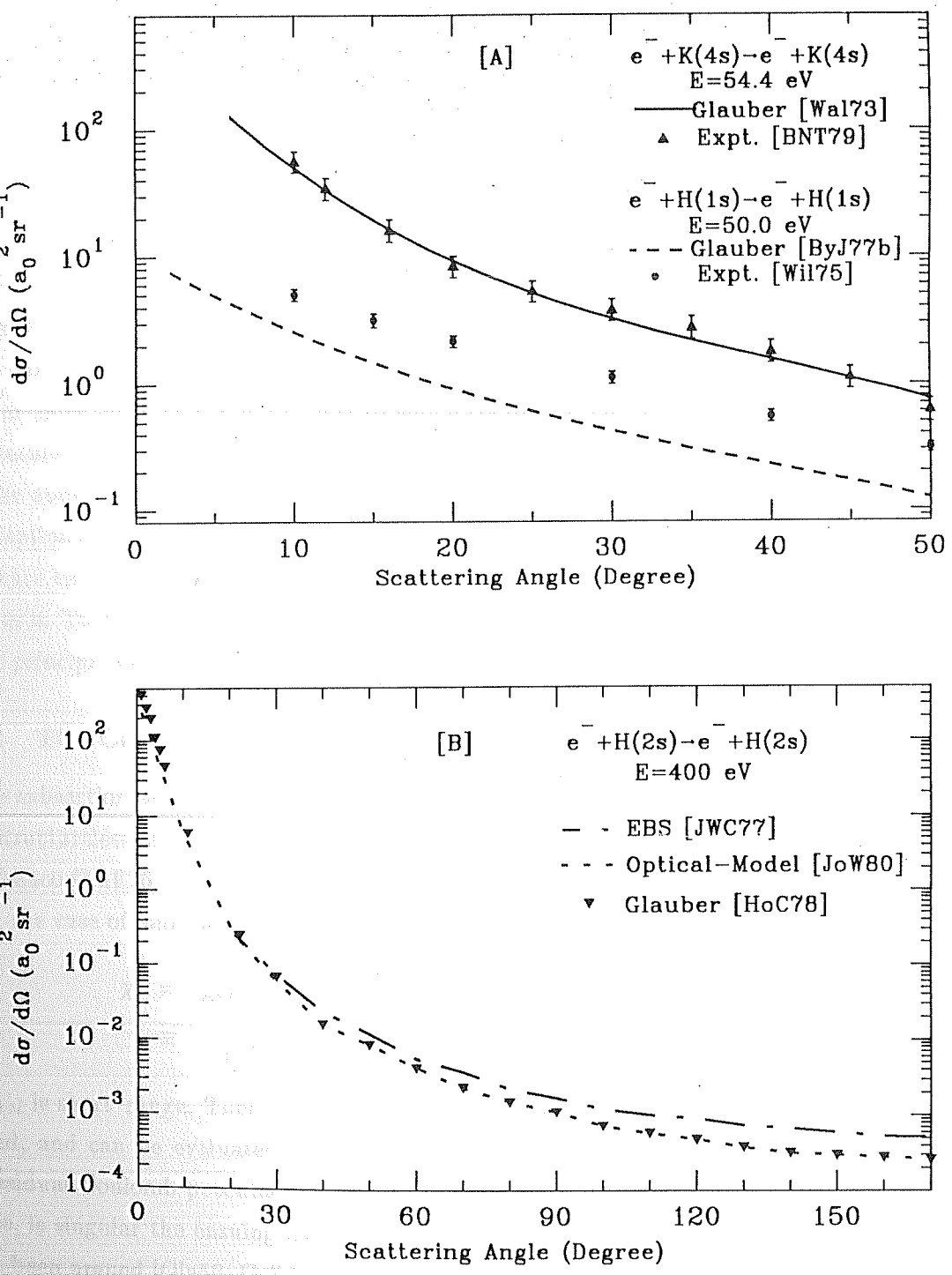


Figure 2.1: [A]: Glauber differential cross section for elastic scattering of electrons by H($1s$) at 50 eV and K($4s$) at 54.4 eV as a function of scattering angle is compared with experimental data. [B]: Glauber differential cross section for elastic scattering of electrons by H($2s$) at 400 eV is compared with other theoretical predictions. Both the figures are reproduced from [Wal84].

between the $1s$ state of hydrogen and the first excited state is 0.375 a.u. In K the first excited state is $4p$ which is only 0.06 a.u. above the ground state; also the main coupling of the ground state is with the $4p$ level. This obviously points to the first assumption, *all important atomic states have the same energy*, being the principal explanation behind the success of the Glauber approximation for the alkali metals and of its failure for H and He.

In Fig. 2.1B we plot Glauber differential cross sections (filled triangles) [HoC78] for elastic electron–scattering from H($2s$) at 400 eV along with the corresponding results from EBS approximation (short–long dashed curve) [JWC77] and an optical model close–coupling calculation (solid curve) [JoW80]. It is seen that for angles from about 10^o upto 180^o the Glauber cross sections agree quite well with the optical model results (which are considered to be exact), while EBS results show poor agreement.

The above comparisons illustrate the point that when the assumptions of the Glauber approximation are satisfied one may expect reasonable results. In atomic energy spectra the energy difference between two consecutive levels decreases with the increase of the principal quantum number n. Therefore, the Glauber results are expected to be increasingly better with the increase of the principal quantum numbers of both initial and final states.

2.4.2 The Coulomb–Glauber amplitude

In this subsection we briefly discuss how the Glauber approximation is modified for the scattering of a structureless charged particle by an ion in the line of the procedure adopted by Thomas and Franco [ThF76].

In the case of ionic target we have, from Eq. (2.9),

$$\begin{aligned} \chi &= \chi_{neutral} + \chi_{Coulomb} \\ &\equiv -\frac{1}{v}\int_{-\infty}^{\infty} dz\, \mathcal{V}_{neutral}(\mathbf{R}_{rel},\zeta) - \frac{1}{v}\int_{-\infty}^{\infty} dz\, \mathcal{V}_{Coulomb}(\mathbf{R}_{rel}). \end{aligned} \tag{2.63}$$

$\mathcal{V}_{neutral}$ is short–range. Therefore, the corresponding phase–shift function $\chi_{neutral}(\mathbf{b},\zeta)$ is well–defined, and can be evaluated in the form as shown earlier (refer to Eq. (2.55)). $\mathcal{V}_{Coulomb}$ is the residual Coulomb potential; consequently since z–integration over it, due to its long–range nature, is singular the ensuing amplitude (2.53) is expected to be ambiguous. Nevertheless, as it has been argued [Gla59, GeT74], we may still use Eq. (2.53) to define the Glauber amplitude for the charged–particle–ion–target collisions provided $\mathcal{V}_{Coulomb}$ is treated as the limit of an appropriately chosen short–range potential, e.g. cut–off Coulomb or Yukawa potential. Thus $\mathcal{V}_{Coulomb}$ can be modulated by a screening factor $\exp(-\epsilon R_{rel})$ to obtain

$$\mathcal{V}^{\epsilon}_{Coulomb}(\mathbf{R}_{rel}) \equiv \frac{Z_P(Z_T-N)}{R_{rel}}\exp(-\epsilon R_{rel}) \tag{2.64}$$

and consequently the corresponding phase–shift function $\chi^{\epsilon}_{Coulomb}$ can be represented by

$$\chi^{\epsilon}_{Coulomb}(\mathbf{b},\zeta) = -\frac{Z_P(Z_T-N)}{v}\int_{-\infty}^{\infty} dz \frac{\exp(-\epsilon R_{rel})}{R_{rel}}$$

$$= -\frac{Z_P(Z_T-N)}{v}\int_{-\infty}^{\infty} dz \frac{\exp\left(-\epsilon(b^2+z^2)^{1/2}\right)}{(b^2+z^2)^{1/2}}. \qquad (2.65)$$

Substituting $z = b\sinh\kappa$ and using the formula (7) on the page 182 of [Wat52] we then obtain

$$\chi^{\epsilon}_{Coulomb}(\mathbf{b},\zeta) = 2\alpha K_0(\epsilon b) \qquad (2.66)$$

where $\alpha = -Z_P(Z_T-N)/v$ and K_0 is the modified Bessel function.

Therefore, if we allow the interaction to be comprised of the short-range term (2.10) and an Yukawa-like term (2.64) the Glauber amplitude becomes convergent and may be given using Eqs. (2.53), (2.54), (2.56) and (2.66), by

$$A^{\epsilon}_{ion}(i\to f) = \frac{iK_i}{2\pi}\int d\mathbf{b}\, d\zeta \exp(i\mathbf{q}\cdot\mathbf{b})\phi_f^*(\zeta)\left[1-\exp(2i\alpha K_0(\epsilon b)+i\chi)\right]\phi_i. \qquad (2.67)$$

Now, because we are concerned with the inelastic transition i.e. $i \neq f$ we may manipulate Eq. (2.67) using the orthogonality of the electronic wave functions ϕ_i and ϕ_f to write

$$A^{\epsilon}_{ion}(i\to f) = \frac{iK_i}{2\pi}\int d\mathbf{b}\, d\zeta \exp(i\mathbf{q}\cdot\mathbf{b})\exp(2i\alpha K_0(\epsilon b)\phi_f^*\left[1-\exp(i\chi)\right]\phi_i. \qquad (2.68)$$

Here, we have made use of the fact that $\chi^{\epsilon}_{Coulomb}$ is independent of ζ.

The Coulomb-Glauber (CG) amplitude is given by the limit

$$A_{\mathrm{CG}}(i\to f;\mathbf{q}) = \lim_{\epsilon\to 0} A^{\epsilon}_{ion}(i\to f;\mathbf{q}). \qquad (2.69)$$

Since $\lim_{z\to\infty} K_0(z) = -[\gamma+\log(z/2)+O(z)]$ (page 69 of [MOS66]) we have

$$\exp(2i\alpha K_0(\epsilon b)) \sim \exp\{-2i\alpha[\ln(\epsilon/2)+\gamma]\}b^{-2i\alpha} \qquad (2.70)$$

where γ is the Euler's constant. We substitute (2.70) into (2.68) and neglect the inconsequential overall constant phase factor $\exp\{-2i\alpha[\ln(\epsilon/2)+\gamma]\}$. The final form of the CG amplitude for an inelastic scattering of a structureless charged particle by an ion is given, to within an overall phase factor, by

$$A_{\mathrm{CG}}(i\to f) = \frac{iK_i}{2\pi}\int d\mathbf{b}\, d\zeta \exp(i\mathbf{q}\cdot\mathbf{b})b^{-2i\alpha}\phi_f^*(\zeta)[1-\exp(i\chi)]\phi_i(\zeta) \qquad (2.71)$$

with χ represented by Eq. (2.55). Comparing Eqs. (2.53) and (2.71) we may observe that due to the residual Coulomb interaction, an extra phase factor $b^{-2i\alpha}$ appears in the integrand of Eq. (2.71). It may be noteworthy that with $Z_T = N$ the Coulomb-Glauber formula (2.71) reduces to the Glauber formula (2.53).

In a quantum-mechanical treatment of scattering of a charged particle by an ionic target, the relative motion in both the initial and final channels is represented by a Coulomb wave in the field of the residual Coulomb potential (refer to Eq. (2.6)). Such a treatment predicts finite (nonzero) value of the total excitation cross section for a bound-bound transition at the threshold [MoM65]. On the other hand in the foregoing treatment of the scattering problem the

final state of the outgoing charged particle has been treated in the derivation of Eq. (2.71) (as in the derivation of Glauber amplitude formula Eq. (2.53)) as a plane wave rather than a Coulomb wave in the residual Coulomb field. Nevertheless, it is found that the Coulomb–Glauber approximation reproduces exactly the Coulomb scattering amplitude *at all angles* [Gla59, GeT74] even though it is obtained by assuming that scattering near the forward directions provide dominant contributions.

It will be seen from the figures in the following section (Section 2.5) that both the Glauber and the Coulomb–Glauber results start showing good accord with the experiments when electron impact energy is increased approximately three times the excitation threshold. Therefore, the incorrect threshold behavior of the Coulomb–Glauber amplitude, as described above, becomes inconsequential and hence poses no serious limitations in the utility of the results.

The CG approximation has been used by several authors [DaS84a, DaS84b, IsC75, NaT75, Tho78, ThF76] to study excitation of ions in the ground state. Thomas and Franco [ThF76] have evaluated the CG amplitude for $1s \rightarrow nlm$ excitations in a closed form. In a later publication Thomas [Tho78] has applied this analytic result to the $n = 2$ (see Figs. 2.6A–C) and $n = 3$ excitations of hydrogenic ions by incident electrons for various values of the target–ion nuclear charge. Das and Sil [DaS84a, b] have calculated the CG cross sections for any arbitrary ns and np state excitations of hydrogenic ions in ground level by deriving a one-dimensional integral for the CG amplitude.

2.5 Comparisons among experiments and various theories: total cross sections

In the previous sections we have discussed a few theoretical methods and also mentioned some results obtained by these methods. In this section we compare the predictions of the Glauber and the Coulomb–Glauber approximations with those obtained by other methods and experiments.

2.5.1 Collisions of electrons with atomic hydrogen

The total cross sections calculated via various theoretical approximations for the excitation of the $2s$ and $2p$ state of atomic hydrogen by electron impact are compared with experimental data. We also discuss several theoretical results for $2s \rightarrow 2s$ elastic transition and for all inelastic transitions among the states $n = 2(s, p)$ and $n' = 3(s, p, d)$.

Excitation from the ground state

(i) $1s \rightarrow 2s$ excitation

Fig. 2.2A plots cross section versus impact energy (E) as obtained from the first Born approximation (FBA) (short–dashed curve); the unitarized Born approximation (UBA) [Whe86b]

(short–long–dashed curve); the recent coupled pseudostate (CPS) calculation [CaU93] (long–dashed curve); the 15–state R–matrix (RM) calculation [ABB91] (filled diamonds) with exchange being considered (calculation is limited in low energies); the Glauber approximation [ThG71] (solid curve); and finally the measurements of Kauppila *et al.* [KOF70] normalized to FBA at 200 eV (filled circles with vertical error bars).

The measurement plotted in Fig. 2.2A do not distinguish between H($2s$) atoms created by $1s \rightarrow 2s$ excitation and those produced by radiative cascading to H($2s$) after excitation to higher levels. Therefore the effective $\sigma(1s \rightarrow 2s)$ observed in the experiment quoted in Fig. 2.2A must be

$$\sigma^{eff}(1s \rightarrow 2s) \;=\; \sigma(1s \rightarrow 2s) + \sum_j P(j \rightarrow 2s)\sigma(1s \rightarrow j) \tag{2.72}$$

which is summed over all energetically accessible levels j lying above H($2s$), with $P(j \rightarrow 2s)$ the probability of cascading to H($2s$) after initial excitation to H(j). The predominant cascade mechanism to H($2s$) is via excitation to H($3p$) [TBG70], i.e. the largest term in the above sum corresponds to $j = 3p$. Thus we may rewrite (2.72) in the form

$$\sigma^{eff}(1s \rightarrow 2s) \;=\; \sigma(1s \rightarrow 2s) + \gamma\sigma(1s \rightarrow 3p), \tag{2.73}$$

where it is generally agreed [KOF70, TBG70], on the basis of Hummer and Seaton's [HuS61] arguments, that $\gamma = 0.23$ over a broad range of energies. Hence, to plot all the curves in Fig. 2.2A we used $\sigma^{eff}(1s \rightarrow 2s)$ obtained from the relation (2.73).

It is seen from Fig. 2.2A that in the incident energy ≥ 30 eV Glauber results show the best agreement (within 20 percent overestimation) with the accurate CPS calculations. In the energy below 30 eV Glauber results largely underestimate all other theoretical calculations.

The agreement between the experimental observations of Kauppila *et al.* [KOF70] and the Glauber theoretical values is quite good in the energy range above 30 eV. It is seen that except for the CPS calculation the Glauber approximation is the only theoretical estimate which is reasonably close to the experimental data in the energy range approximately greater than 30 eV. All other theories predict $\sigma(1s \rightarrow 2s)$ which are higher.

In a separate graph (Fig. 2.3) we present the cross sections calculated by some theoretical methods for direct $1s \rightarrow 2s$ excitation (without the cascade contribution). Aside from FBA and Glauber [ThG71] (which have the same legends as in Fig. 2.2A), the cross sections estimated by the coupled–state distorted wave polarized–orbital (CSDWPO) calculation [MMN74] (medium–dashed curve) and the intermediate energy R–matrix (IERM) calculation [SSW89] (short–short–long dashed curve) are plotted in this figure. We notice that at E larger than 30 eV the Glauber predictions are in excellent agreement with CSDWPO and show reasonable agreement with the recent IERM calculations deferring by $\leq 15\%$.

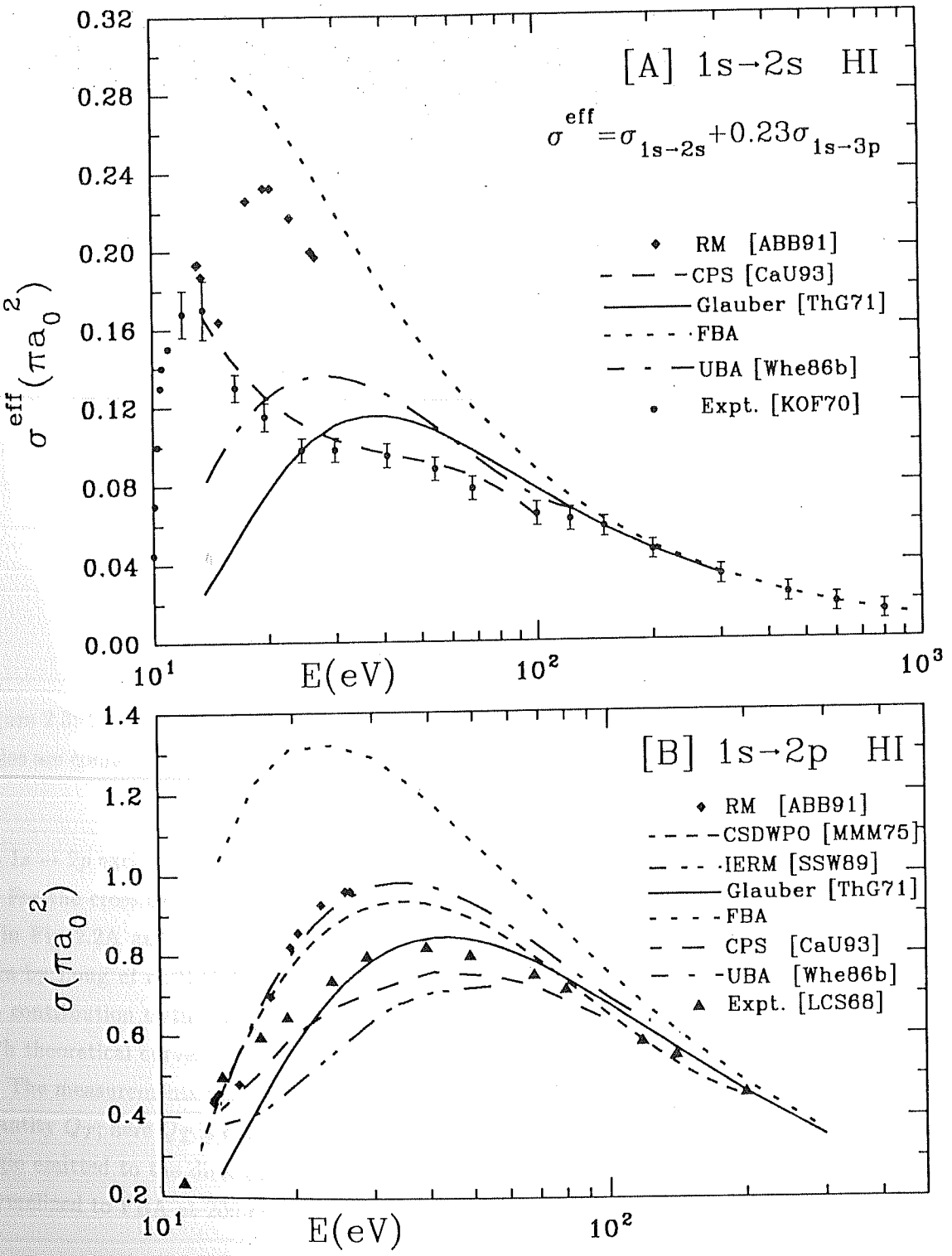


Figure 2.2: Integrated cross section σ, in units of πa_0^2, from several theories and experiments is plotted as a function of impact energy E (eV) for ground state excitations of atomic hydrogen. [A]: $1s \to 2s$ transition (cascade contributions included), [B]: $1s \to 2p$ transition (cascade contributions excluded).

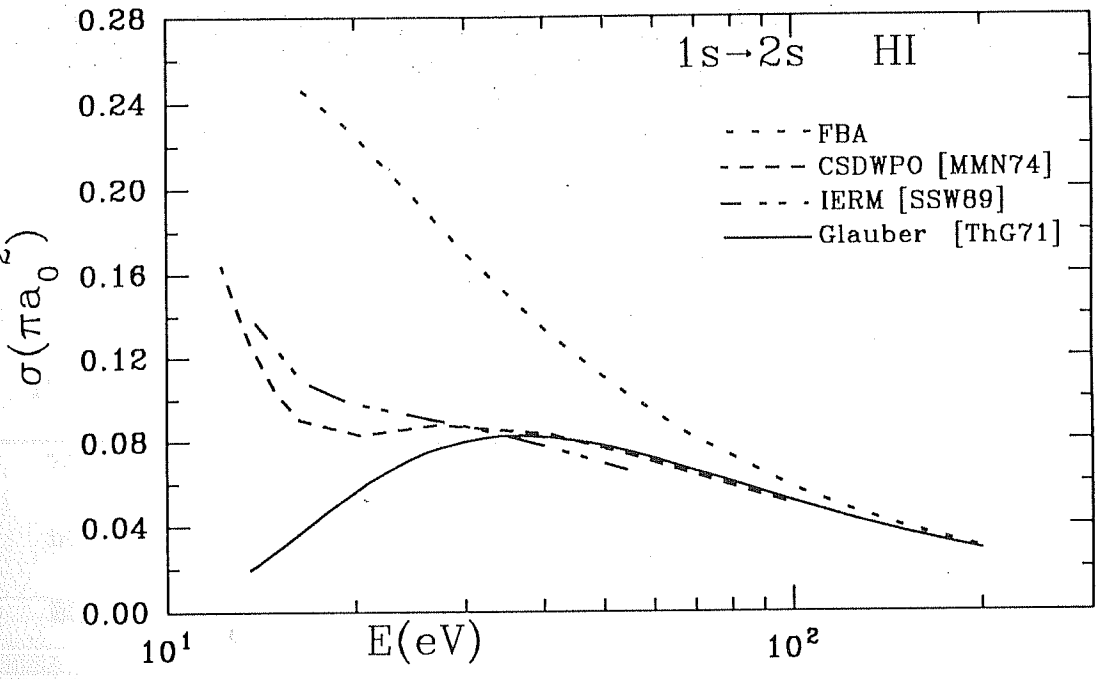


Figure 2.3: Glauber predicted integrated cross sections σ, in units of πa_0^2, for several impact energies are compared with some theoretical predictions for $1s \rightarrow 2s$ excitation of atomic hydrogen.

(ii) $1s \rightarrow 2p$ excitation

For the cross section $\sigma(1s \rightarrow 2p)$ Fig. 2.2B shows, apart from all the theoretical predictions as in Fig. 2.2A and Fig. 2.3 with their sources and descriptions being same, the experimental data by Long *et al.* [LCS68] (filled triangles). Because the cascading is estimated to give only 2% contribution to the observed $\sigma(1s \rightarrow 2p)$, in Fig. 2.2B we compare the observed data points with theoretical curves uncorrected for cascading.

The measurements plotted in Fig. 2.2B is calculated from the reported [LCS68] values of the quantity Q_T; here Q_T is defined by the 4π times the number of Lyman–α photons per unit solid angle emitted to the direction perpendicular to the direction of incident electron beam. Q_T is normalized to FBA at 200 eV. The total cross section σ is obtained from the relation

$$\sigma = \left(1 - \frac{1}{3}P\right) Q_T \tag{2.74}$$

where, by definition, the polarization fraction P can be expressed as

$$P = (I_{\parallel} - I_{\perp})/(I_{\parallel} + I_{\perp}) \tag{2.75}$$

in which $I_{\parallel}$ and $I_{\perp}$ are the observed intensities at 90° to the electron beam axis, of the Lyman–α components having electric vectors parallel and perpendicular respectively to the electron beam axis. Ott *et al.* [OKF70] have reported the measured polarization fraction for various energies. We use those data along with the Q_T of [LCS68] to compute the cross section $\sigma(1s \rightarrow 2p)$.

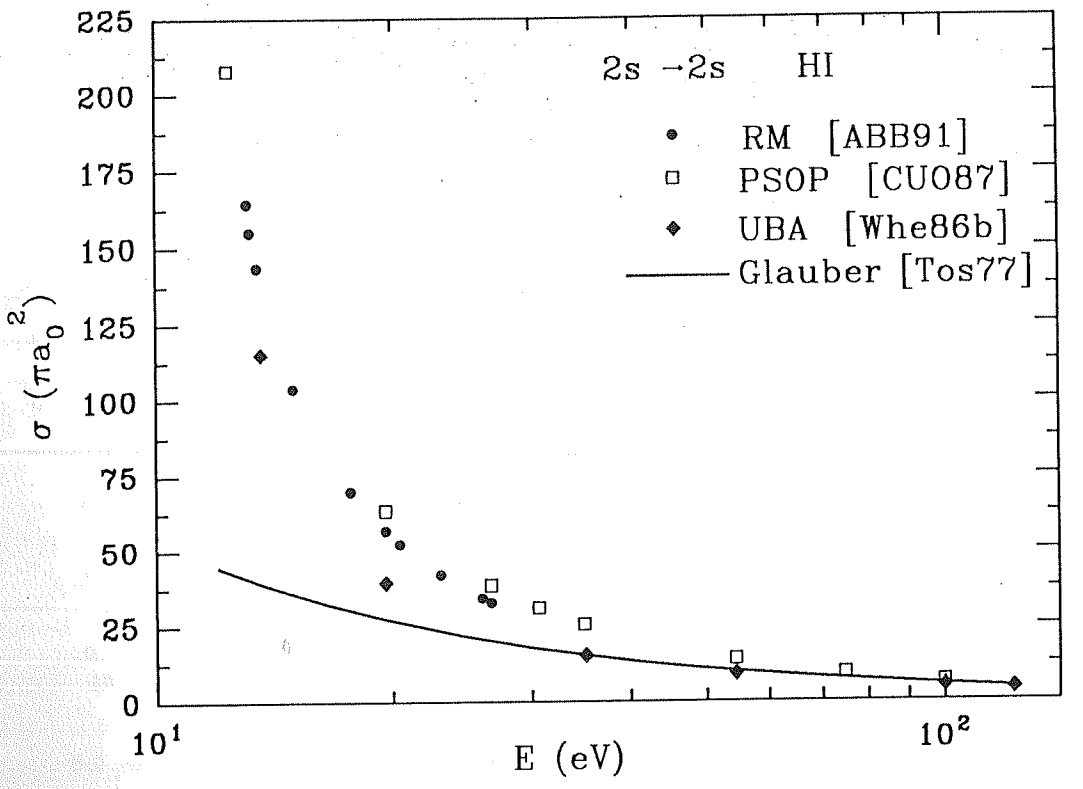


Figure 2.4: Integrated cross section σ, in units of πa_0^2, from several theories is plotted as a function of impact energy E (eV) for $2s \rightarrow 2s$ transition of atomic hydrogen.

Again we see that the Glauber approximation gives the best overall agreement with the experiment. In contrast, the IERM and the CPS results underestimate the experimental data whereas the other theoretical results included in Fig. 2.2B overestimate them

Excited state to excited state transitions

In Fig. 2.4 we plot $\sigma(2s \rightarrow 2s)$ versus impact energy E estimated via the Glauber approximation and several recent theoretical calculations. In Figs. 2.5A, 2.5B, 2.5C, 2.5D, 2.5E and 2.5F we plot $\sigma(2l \rightarrow 3l')$ versus impact energy E for all inelastic transitions involving the orbital angular momentum levels of $n = 2$ and $n = 3$ atomic states. We treat these as the representative cases of excited state to excited state transitions. In all these figures, solid curves are the Glauber data [Tos77]. The unitarized Born (UBA) results [Whe86b] are represented by filled diamonds. Hollow squares denote the calculations by the pseudostate optical potential (PSOP) method [CUO87]. We also present the 15-state R-matrix (RM) results [ABB91], which are only available at lower energies, via filled circles. In Figs. 2.5A–F, short dashed curves are the predictions by FBA.

For all these transitions we see a large discrepancies between the predictions of different theories. As a result, it is difficult to make any definite statement about the accuracies of these calculations. However, we have seen (Fig. 2.1B) that the Glauber approximation gives a very good description of the differential cross sections for $2s \rightarrow 2s$ transition. One, therefore, expects

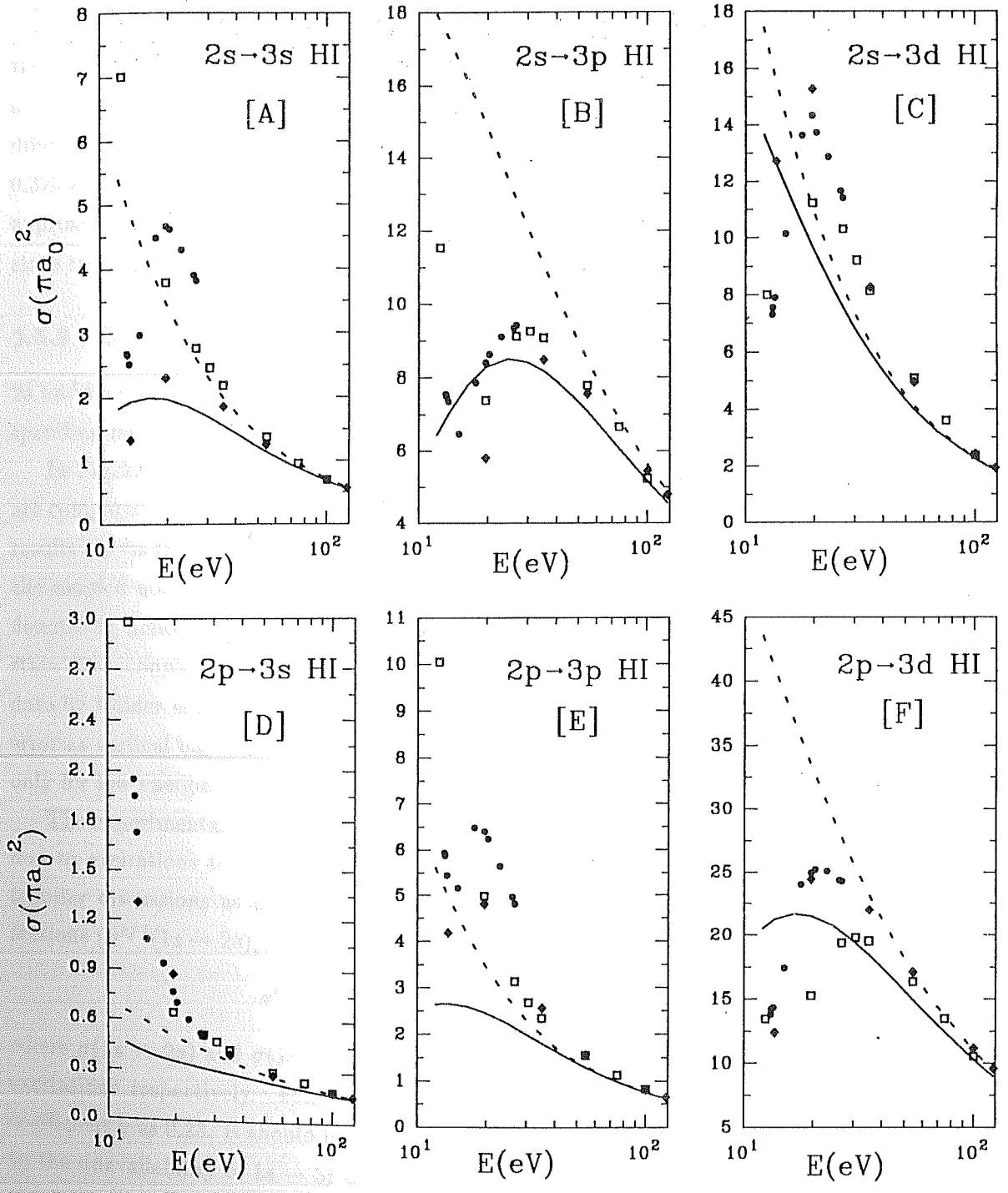


Figure 2.5: Integrated cross section σ, in units of πa_0^2, from several theories is plotted as a function of impact energy E (eV) for all inelastic transitions between $n = 2(s,p)$ and $n' = 3(s,p,d)$ levels of atomic hydrogen. Solid curves are the Glauber calculations [Tos77]; short–dashed curves are the first Born calculations; filled diamonds are the unitarized Born results [Whe86b]; hollow squares are the pseudostate optical potential data [CUO87]; and filled circles are by the R–matrix method [ABB91].

that the Glauber integrated cross sections for $2s \rightarrow 2s$ transition may also be reliable, despite their disagreement with other results. Furthermore, as pointed out previously, the assumptions of the Glauber approximation suit better in the case of $n = 2 \rightarrow n' = 3$ transitions (where the difference between the energy levels is 0.0694 a.u. as compare to the first excitation threshold 0.375 a.u.) than the excitations from the ground state. Therefore, we expect that the Glauber approximation would provide a good description of cross sections for excited state to excited state transitions.

2.5.2 Collisions of electrons with helium ion

$2s$ and $2p$ excitation cross section, calculated by various theories are compared with the corresponding measurements for singly ionized helium (HeII)[1] (i) $1s \rightarrow 2s$ excitation

In Fig. 2.6A the integrated cross sections obtained from several theoretical approximations are compared with experiment. Solid line represents the CG calculations [Tho78]. Other plotted results are via the optical potential (OP) calculations [UCO91], represented by long dashed curve; the coupled–state distorted wave polarized–orbital (CSDWPO) calculations [MMN74, MMM75] denoted by hollow squares; the R–matrix (RM) calculations [ABK91], as filled diamonds; the six–state close–coupling (6CC) calculations [HuS87] given by hollow circles; finally the experimental data by Dolder *et al.* [DoP73] represented by filled circles with 90% confidence limits of random error as vertical bars. The cross sections by the RM, 6CC and CSDWPO methods are available only for low energies.

The experimental results shown in the Fig. 2.6A include an appreciable cascade contribution due to excitations to higher–lying states followed by radiative decay to the $2s$ state of HeII (similar discussions as in connection to the Fig. 2.2A hold here). Hence all theoretical cross sections ($\sigma^{eff}(1s \rightarrow 2s)$) in Fig. 2.6A are computed by using the relation

$$\sigma^{eff}(1s \rightarrow 2s) \quad = \quad \sigma(1s \rightarrow 2s) + \gamma\sigma(1s \rightarrow 3p) \tag{2.76}$$

where $\sigma(1s \rightarrow 2s)$ and $\sigma(1s \rightarrow 3p)$ represents the cross sections for the $1s \rightarrow 2s$ and $1s \rightarrow 3p$ excitations respectively. Following Thomas [Tho78] we have used the value of the cascade coefficient γ as 0.23. It should be pointed out here that in the case of the CSDWPO results, due to the unavailability of $1s \rightarrow 3p$ excitation data in the energy range of present interest, we use the 'unorthogonalized' distorted wave polarized–orbital predictions [MMM75] for $\sigma(1s \rightarrow 3p)$. The experimental data are normalized to the CG cross section at $E = 1000$ eV.

It is seen from Fig. 2.6A that at $E \geq 75$ eV CG compares reasonably well with experiment, even though the agreement of OP with the measurements is superior. In this energy range CG and OP predictions agree within 15%. At the energies lower than 75 eV neither of this methods give good result. It is also seen that for say $E < 90$ eV the methods like the RM and 6CC give poor agreement with data.

[1] Experimental data are only available for the $2s$ and $2p$ excitations of HeII. To our knowledge, no experiment with ions, other than HeII, in the hydrogen isoelectronic sequence has so far been reported.

In Fig. 2.6B we plot cross sections for $1s \rightarrow 2s$ excitation by some approximations such as (i) the algebraic variational (AV) calculations with a small pseudostate basis [WaC80] (short–short–long dashed curve), (ii) the CSDWPO calculations [MMN74] (medium dashed curve), (iii) the three–state close–coupling (3CC) calculations [BMS64] (short–long dashed curve), (iv) the unitarized Coulomb–Born (UCBII) calculations [BHT70] (short–dashed curve) and finally (v) the CG approximation [Tho78] (solid curve). It is seen that CG lies consistently below other methods over the entire range from threshold to, approximately, $E = 400$ eV (this is not applicable for the AV method, as no data using this method are available for higher energies, say, $E \geq 100$ eV). This essentially indicates that none of these methods can show better agreement with the experiment than that of the CG at energies greater than 30 eV, since experimental points in Fig. 2.5A are always lower than the CG curve over this energy range.

(ii) $1s \rightarrow 2p$ excitation

In Fig. 2.6C the experimental [DZI74] $1s \rightarrow 2p$ integrated cross sections (filled circles) are compared with various theoretical predictions including the CG. The sources and descriptions of all theoretical plots are similar to $1s \rightarrow 2s$ case; we only specify that for the CSDWPO we have chosen the legend used in Fig. 2.6B instead of Fig. 2.6A. Cascade correction which make a comparatively small contribution to the observed data, have been ignored in the theoretical curves in Fig. 2.6C. The experimental points are normalized at 217 eV to UCBII and the error bars on the data reflect 90% confidence limits on the relative measurements.

For incident energy greater than 100 eV, the CG predictions as well as 3CC and UCBII are in reasonably good accord with experiment. At energies $\geq$ 70 eV CSDWPO cross sections are lower than the CG cross sections by $\leq$15%. On the scale of the figure the UCBII cross sections are virtually indistinguishable from CG above approximately 165 eV. Also above 100 eV, where CG fits the experiment, OP predictions are higher than CG by $\leq$20%. It may also be noted that below 100 eV the agreement of CG with RM and 6CC in this case is better than that for the $1s \rightarrow 2s$ case.

2.6 Conclusion

We have seen (Figs. 2.2A and 2.2B) that for $1s \rightarrow 2s$ and $1s \rightarrow 2p$ excitations, the Glauber approximation gives a good description of the experimental integrated cross sections. The Glauber prediction for these transitions compare very well with the predicted results of other methods such as the IERM and the CPS. Similarly for $1s \rightarrow 2s$ and $1s \rightarrow 2p$ excitations of HeII (see Figs. 2.6A–C), CG method performs reasonably well and its predictions are in good agreement with the experiment.

There exists large disagreement between various close–coupling results for excited state to excited state transitions as pointed out in the discussions of the Figs. 2.4 and 2.5A–F. Unfortunately experimental measurements of the cross sections are not available for excited state

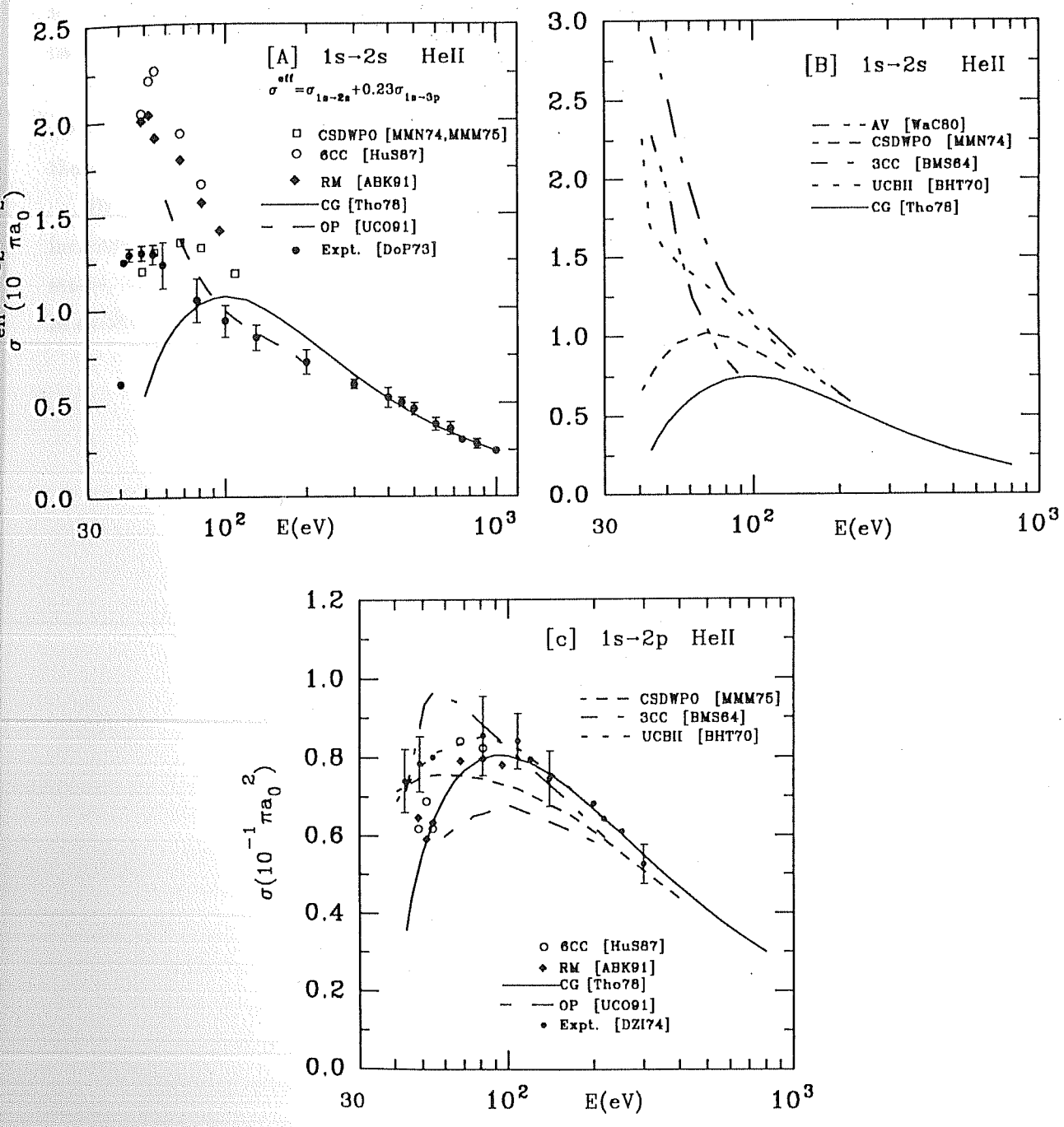


Figure 2.6: Integrated cross section σ, in units of πa_0^2, from several theories and experiments is plotted as a function of impact energy E (eV) for ground state excitations of singly ionized helium. [A]: $1s \to 2s$ transition (cascade contributions included), [B]: $1s \to 2s$ transition (all theoretical predictions), [C]: $1s \to 2p$ (cascade contributions excluded).

to excited state transitions. Therefore, the capability of various methods for these transitions cannot be readily tested.

We may remark that the perturbative methods such as distorted wave, optical potential etc. are theoretically attractive. But because of a great amount of labour needed in the calculations, these methods can not be easily applied for transitions involving excited states.

We have already mentioned (see Section 2.4) that the Glauber results are expected to be increasingly better with the increase of the principal quantum numbers of both initial and final states. This has motivated us to apply the Glauber and the Coulomb–Glauber approximations in excited state to excited state transitions.

Chapter 3

Coulomb–Glauber Cross Sections for Arbitrary $nlm \rightarrow n'l'm'$ Transition

The Coulomb–Glauber (CG) amplitude for arbitrary $nlm \rightarrow n'l'm'$ excitation of a hydrogenic ion ($N = 1$) of charge Z_T induced by the impact of a structureless charged particle of charge Z_P is given, to within a overall phase factor, by

$$A(nlm \rightarrow n'l'm') = \frac{iK_i}{2\pi} \int d\mathbf{b} d\mathbf{r} \, \exp(i\mathbf{q}\cdot\mathbf{b})\, b^{-2i\alpha} \phi^*_{n'l'm'}(\mathbf{r}) \times \left[1 - \left(\frac{|\mathbf{b}-\mathbf{s}|}{b}\right)^{2i\eta}\right] \phi_{nlm}(\mathbf{r}). \quad (3.1)$$

For convenience, we re–explain all the quantities used in the expression (3.1). The initial momentum is $\mathbf{K}_i = M\mathbf{v}$, where $\mathbf{v}$ denotes the initial relative velocity in the center–of–mass system, and M is the reduced mass of the collision–system; we represent $\eta = -Z_P/v$ and $\alpha = -Z_P(Z_T - 1)/v$. In Eq. (3.1), $\phi_{nlm}(\mathbf{r})$ and $\phi_{n'l'm'}(\mathbf{r})$ denote the initial and final bound–state wave functions of the electron; $\mathbf{r}$ is the coordinate vector of the electron measured with respect to the target nucleus; the momentum transfer is given by $\mathbf{q} = \mathbf{K}_i - \mathbf{K}_f$, where the final momentum $\mathbf{K}_f = M\mathbf{v}'$, in which $\mathbf{v}'$ is the final relative velocity.

The scattering amplitude (3.1) is derived by taking the Z axis to be along a direction perpendicular to $\mathbf{q}$. In (3.1), $\mathbf{b}$ is the projection of $\mathbf{R}_{rel}$, the relative position vector of the projectile, onto the plane perpendicular to the Z axis. Let the plane containing $\mathbf{q}$ and $\mathbf{b}$ define the XY plane. Then the two–dimensional vector $\mathbf{s}$ is the projection of $\mathbf{r}$ onto the XY plane. We may note that if we put $Z_T = 1$, the scattering amplitude (3.1) reduces to the Glauber scattering amplitude for $nlm \rightarrow n'l'm'$ excitation of hydrogen atom.

The bound state wave functions in (3.1) can be written as $\phi_{nlm}(\mathbf{r}) = Y_l^m(\hat{\mathbf{r}}) R_{nl}(r)$ and $\phi_{n'l'm'}(\mathbf{r}) = Y_{l'}^{m'}(\hat{\mathbf{r}}) R_{n'l'}(r)$, where Y_L^M denotes the spherical harmonic of degree L and order M. We employ the Rose convention [Ros57] for Y_L^M so that

$$Y_L^M(\hat{\mathbf{r}}) = \left(\frac{2L+1}{4\pi}\right)^{1/2} \left(\frac{(L-M)!}{(L+M)!}\right)^{1/2} \exp(iM\varphi)\, P_L^M(\cos\vartheta). \quad (3.2)$$

Here P_L^M is the associated Legendre polynomial (defined in [MOS66]).

From the symmetry of the problem, the Coulomb–Glauber amplitude (3.1) remains invariant under the transformation $Z \to -Z$ (i.e. $\vartheta \to \pi - \vartheta$). Therefore, using the parity of the associated Legendre polynomial

$$P_L^M(\cos\vartheta) \to P_L^M(-\cos\vartheta) \quad = \quad (-1)^{L+M} P_L^M(\cos\vartheta) \tag{3.3}$$

we see that the scattering amplitude $A(nlm \to n'l'm')$ vanishes for $(l + l' + m + m')$ = an *odd integer*, a result welknown for the Glauber amplitude (see e.g. [ThG71]).

It may be mentioned that the Glauber approximation predicts equal cross sections for electron and positron scattering. This can be easily seen when ϕ_i and ϕ_f are both s–states. In this case ϕ_i and ϕ_f may both be chosen real; replacing $\mathcal{V}$ to $-\mathcal{V}$ in Eq.(3.1) is then to change $A(nlm \to n'l'm')$ to its complex conjugate without therefore affecting its modulus.

The CG method has received considerable attention. Thomas and Franco [ThF76] have evaluated the Coulomb–Glauber amplitude for $1s \to nlm$ excitation in a closed form involving successive parametric differentiations of certain expressions containing Meijer G functions. As is known, the technique of successive parametric differentiation, in general, becomes tedious and impractical as the principal quantum number n increases. Das and Sil [DaS84a, b] have developed a method to study arbitrary $1s \to ns$ and $1s \to np$ excitation. However, their method also relies on the technique of successive parametric differentiation for generating spherical harmonic functions and it remains to be seen whether it can be extended to the excitation of states with large orbital angular momentum l.

In Section 3.1 we obtain an elegant *one–dimensional integral representation* of the CG–amplitude for $nlm \to n'l'm'$ transition which proves to be useful for efficient computations.

3.1 Derivation of a new one–dimensional integral representation of the Coulomb–Glauber amplitude

The integral in Eq.(3.1) is five–dimensional. We have reduced it into a new and convenient one–dimensional form [DeC91]. A detail account of our derivation is presented below:

We start by treating the integral over $\mathbf{r}$ of the expression (3.1), namely,

$$U(\mathbf{b}) \quad = \quad \int d\mathbf{r}\, \phi^*_{n'l'm'}(\mathbf{r}) \left[1 - \left(\frac{|\mathbf{b}-\mathbf{s}|}{b}\right)^{2i\eta}\right] \phi_{nlm}(\mathbf{r}). \tag{3.4}$$

We write

$$\phi_{nlm}(\mathbf{r})\phi^*_{n'l'm'}(\mathbf{r}) \quad = \quad R_{nl}(r) R_{n'l'}(r) \sum_{\lambda=l_{min}}^{l+l'} a_\lambda Y_\lambda^{\mu *}(\hat{\mathbf{r}}) \tag{3.5}$$

where $\mu = m' - m$ and

$$a_\lambda \quad \equiv \quad \left[\frac{(2\lambda+1)(2l+1)}{4\pi(2l'+1)}\right]^{1/2} C(\lambda, l, l'; 0, 0, 0)\; C(\lambda, l, l'; \mu, m, m') \tag{3.6}$$

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in which $C(l,l_1,l_2;m,m_1,m_2,)$ is the Clebsch–Gordan coefficient. Although in general in the sum over λ, the lower value of λ is $l_{min}=|l-l'|$, for the case when the quantum numbers of the initial and final states are such that $|\mu|\equiv|m'-m|>|l-l'|$, $l_{min}=|\mu|$, and the sum over λ runs from $l_{min}=|\mu|$ to $l+l'$. To proceed further, we employ a very useful integral representation of Thomas and Franco [ThF76], which after some rearrangement can be expressed in the following elegant form:

$$\int d\hat{\mathbf{r}}\, Y_l^{m*}(\hat{\mathbf{r}})\left(\frac{|\mathbf{b}-\mathbf{s}|}{b}\right)^{2i\eta} = -4\pi c_0 i^{l-m}\, Y_l^{m*}(\hat{\mathbf{b}})\int_0^\infty dt\, t^{-2i\eta}\frac{d}{dt}\left[J_m(t)\, j_l\left(\frac{rt}{b}\right)\right] \quad (3.7)$$

in which $c_0=2^{2i\eta}\frac{\Gamma(1+i\eta)}{\Gamma(1-i\eta)}$, $J_m(z)$ is the Bessel function of order m and $j_l(z)$ is the spherical Bessel function of order l. It is assumed that the parameter η is complex, it contains an infinitesimal *negative* imaginary part, $-\epsilon$, so that $\eta\to\eta-i\epsilon$. It is understood that ϵ is to be put equal to zero after performing all appropriate operations.

We substitute Eq. (3.5) in Eq. (3.4) and perform the angular integration over $\hat{\mathbf{r}}$ by using Eq. (3.7). We then obtain

$$\begin{aligned} U(\mathbf{b}) &= \delta_{fi}+i^{-\mu}c_0\int_0^\infty dt\, t^{-2i\eta} \\ &\times \frac{d}{dt}\left[J_\mu(t)\left\{4\pi\sum_\lambda i^\lambda a_\lambda Y_\lambda^{\mu*}(\hat{\mathbf{b}})\int_0^\infty dr\, r^2 R_{n'l'}(r)\, j_\lambda\left(\frac{rt}{b}\right)R_{nl}(r)\right\}\right]. \end{aligned} \quad (3.8)$$

For brevity here and in the following, the set of quantum numbers nlm and $n'l'm'$ specifying the initial and final bound states, are collectively denoted by i and f, respectively. The Kronecker delta, δ_{fi}, in the right hand side of Eq. (3.8) comes from the orthonormality of the bound state wave functions. An important step in the simplification of the expression (3.8) is to identify that the quantity inside $\{\cdots\}$ is the form factor (2.32). This can be seen as follows. Replacing q by $t\mathbf{b}/b^2$ we rewrite Eq. (2.32) (with $N=1$) as

$$G_{fi}\left(\frac{t\mathbf{b}}{b^2}\right) = \int d\mathbf{r}\,\phi^*_{n'l'm'}(\mathbf{r})\,\exp\left(i\frac{t}{b^2}\mathbf{b}.\mathbf{r}\right)\phi_{nlm}(\mathbf{r}). \quad (3.9)$$

We substitute Eq. (3.5) in Eq. (3.9), and expand $\exp\left(i\frac{t}{b^2}\mathbf{b}\cdot\mathbf{r}\right)$ in a complete set of spherical harmonics,

$$\exp\left(i\frac{t}{b^2}\mathbf{b}\cdot\mathbf{r}\right) = 4\pi\sum_{L,M} i^L\, Y_L^M(\hat{\mathbf{r}})\, Y_L^{M*}(\hat{\mathbf{b}})\, j_L\left(\frac{rt}{b}\right). \quad (3.10)$$

Applying the orthonormality property of the spherical harmonics we carry out $\hat{\mathbf{r}}$–integration to find the required result

$$G_{fi}\left(\frac{t\mathbf{b}}{b^2}\right) = 4\pi\sum_\lambda i^\lambda a_\lambda Y_\lambda^{\mu*}(\hat{\mathbf{b}})\int_0^\infty dr\, r^2 R_{n'l'}(r)\, j_\lambda\left(\frac{rt}{b}\right)R_{nl}(r). \quad (3.11)$$

By making use of the identity

$$c_0\int_0^\infty dt\, t^{-2i\eta}\frac{d}{dt}J_0(t) = -1 \quad (3.12)$$

we may write Eq. (3.8) as

$$U(\mathbf{b}) = i^{-\mu} c_0 \int_0^\infty dt\, t^{-2i\eta} \frac{d}{dt} \left\{ J_\mu(t) \left[G_{fi}\left(\frac{t\mathbf{b}}{b^2}\right) - \delta_{fi} \right] \right\}. \tag{3.13}$$

Integrating Eq. (3.13) by parts, we get

$$U(\mathbf{b}) = 2\eta\, i^{1-\mu} c_0 \int_0^\infty dt\, t^{-2i\eta-1} J_\mu(t) \left[G_{fi}\left(\frac{t\mathbf{b}}{b^2}\right) - \delta_{fi} \right]. \tag{3.14}$$

Substituting Eq. (3.14) in Eq. (3.1) and changing the order of integration, we obtain

$$\begin{aligned} A(nlm \to n'l'm') &= -2\eta\, i^{-\mu} c_0 \frac{K_i}{2\pi} \int_0^\infty dt\, t^{-2i\eta-1} J_\mu(t) \\ &\times \int d\mathbf{b}\, b^{-2i\alpha} \exp(i\mathbf{q}\cdot\mathbf{b}) \left[G_{fi}\left(\frac{t\mathbf{b}}{b^2}\right) - \delta_{fi} \right]. \end{aligned} \tag{3.15}$$

Since, by definition, $\mathbf{b}$ lies on the XY plane, we have $\mathbf{b} \equiv (b, \vartheta_b = \pi/2, \varphi_b)$. It is important to note that the form factor $G_{fi}(t\mathbf{b}/b^2)$ depends on the variable φ_b only through the factor $\exp(-i\mu\varphi_b)$, a result which can be deduced from the presence of $Y_\lambda^{\mu *}(\hat{\mathbf{b}})$ in Eq. (3.8). We define the reduced form factor

$$g_{fi}\left(\frac{t}{b}\right) = \exp(i\mu\varphi_b)\; G_{fi}\left(\frac{t\mathbf{b}}{b^2}\right). \tag{3.16}$$

Obviously, $g_{fi}(t/b) = G_{fi}(t/b, \vartheta_b = \pi/2, \varphi_b = 0)$, since $t\mathbf{b}/b^2 \equiv (t/b, \vartheta_b = \pi/2, \varphi_b)$. Substituting Eq. (3.16) in Eq. (3.15), we carry out φ_b integration to obtain

$$\begin{aligned} A(nlm \to n'l'm') &= -2\eta K_i c_0 \exp(-i\mu\varphi_q) \int_0^\infty dt\, t^{-2i\eta-1} J_\mu(t) \\ &\times \int_0^\infty db\, b^{-2i\alpha+1} J_\mu(qb) \left[g_{fi}\left(\frac{t}{b}\right) - \delta_{fi} \right] \end{aligned} \tag{3.17}$$

where φ_q is the azimuthal angle of the momentum transfer vector $\mathbf{q}$. We mention here that we shall make use of this two-dimensional integral representation (3.17) of the CG amplitude in Chapter 3 for investigating the Z_T-dependence of the amplitude.

Here, we remark that by putting $\alpha = 0$ in Eq. (3.17), and by using the explicit expression of the reduced form factor $g_{fi}(t/b)$ for some transition, say $1s \to 1s$ of hydrogen, one can analytically evaluate the b integral followed by the t integral. The closed form expression of the Glauber amplitude thus obtained agrees with that given previously by Thomas and Gerjuoy [ThG71].

The two-dimensional integral (3.17) can be reduced to a one-dimensional integral as follows. We substitute $t = by$ in Eq. (3.17) to get

$$\begin{aligned} A(nlm \to n'l'm') &= -2\eta K_i c_0 \exp(-i\mu\varphi_q) \int_0^\infty dy\, y^{-2i\eta-1} [g_{fi}(y) - \delta_{fi}] \\ &\times \int_0^\infty db\, b^{-2i\eta-2i\alpha+1} J_{|\mu|}(qb) J_{|\mu|}(by) \end{aligned} \tag{3.18}$$

where we have made use of the fact that for integer μ, $J_\mu(qb)J_\mu(by) = J_{|\mu|}(qb)J_{|\mu|}(by)$, with $|\mu| = \mathrm{mod}(m'-m)$. By using the formula 6.576(2) on p.693 of Gradshtein and Rhyzhik [GrR80]

and remembering that η is complex, we evaluate the b integral in Eq. (3.18). We then substitute $y = qx$ and after some simple algebra we obtain

$$A(nlm \to n'l'm') = -\frac{\eta K_i c_0 q^{2i\alpha-2}\Gamma(|\mu|-i\eta-i\alpha+1)}{2^{2i\eta+2i\alpha-2}\Gamma(|\mu|+1)\Gamma(i\eta+i\alpha)} \exp(-i\mu\varphi_q)\, I_{fi} \tag{3.19}$$

where

$$\begin{aligned} I_{fi} &= \int_0^\infty dx \frac{x^{-2i\eta+|\mu|-1}}{(1+x)^{2|\mu|-2i\alpha-2i\eta+2}}[g_{fi}(qx) - \delta_{fi}] \\ &\times \; {}_2F_1\left(|\mu| - i\eta - i\alpha + 1,\, |\mu| + \frac{1}{2};\, 2|\mu| + 1;\, \frac{4x}{(1+x)^2}\right). \end{aligned} \tag{3.20}$$

We thus reduce the original multi-dimensional integral expression (3.1) of the CG amplitude for arbitrary $nlm \to n'l'm'$ transition to a one-dimensional integral form.

The one-dimensional integral representation of the CG amplitude (Eqs. (3.19) and (3.20)) contains the reduced form factor g_{fi} explicitly. The form factor G_{fi} (see Eq. (2.32)) is connected to g_{fi} by the relation (3.16). Evidently Eq. (3.19) incorporates all detailed structures of both initial and final state wave functions via the reduced form factor. One can readily derive an analytic expression of the reduced form factor for an arbitrary $nlm \to n'l'm'$ transition.

3.2 A closed-form expression of the form factor

There have been studies (see for example [FlM79]) on the form factor to investigate its behavior. We give in the following a simple evaluation of G_{fi} for an arbitrary $nlm \to n'l'm'$ transition.

As previous (refer to Section 3.1), we write the orthonormal hydrogenic wave functions ϕ_i and ϕ_f as $\phi_i(\mathbf{r}) = Y_l^m(\hat{\mathbf{r}})R_{nl}(r)$ and $\phi_f(\mathbf{r}) = Y_{l'}^{m'}(\hat{\mathbf{r}})R_{n'l'}(r)$. The complete expressions for the radial part of the wave functions may be given by

$$R_{nl}(r) = N_i \sum_{j_1=0}^{n-l-1} \mathcal{D}_{j_1}^{(i)} r^{l+j_1} \exp(-\beta_i r) \tag{3.21}$$

and

$$R_{n'l'}(r) = N_f \sum_{j_2=0}^{n'-l'-1} \mathcal{D}_{j_2}^{(f)} r^{l'+j_2} \exp(-\beta_f r). \tag{3.22}$$

The abbreviations used in (3.21) and (3.22) are

$$N_i \equiv \frac{2^{l+1}\beta_i^{l+3/2}}{(2l+1)!}\left[\frac{(n+l)!}{n(n-l-1)!}\right]^{1/2}, \tag{3.23}$$

$$N_f \equiv \frac{2^{l'+1}\beta_f^{l'+3/2}}{(2l'+1)!}\left[\frac{(n'+l')!}{n'(n'-l'-1)!}\right]^{1/2} \tag{3.24}$$

and

$$\mathcal{D}_{j_1}^{(i)} \equiv \frac{(-n+l+1)_{j_1}}{(j_1)!\,(2l+1)_{j_1}}(2\beta_i)^{j_1}, \tag{3.25}$$

$$\mathcal{D}_{j_2}^{(f)} \equiv \frac{(-n'+l'+1)_{j_2}}{(j_2)!\,(2l'+1)_{j_2}}(2\beta_f)^{j_2} \tag{3.26}$$

in which $\beta_i = Z_T/n$ and $\beta_f = Z_T/n'$; $(a)_j$ denotes the Pochhammer's symbol. Using Eqs. (3.21) and (3.22) we then written

$$\phi_i(\mathbf{r})\phi_f^*(\mathbf{r}) = N_i N_f \sum_{\lambda=l_{min}}^{l+l'} a_\lambda Y_\lambda^{\mu*}(\hat{\mathbf{r}}) \sum_{j=0}^{n+n'-l-l'-2} b_j r^{l+l'+j} \exp(-\beta r) \tag{3.27}$$

where $\mu = m' - m$ and a_λ has already been defined by Eq. (3.6); we also define $\beta = \beta_i + \beta_f$ and

$$\begin{aligned} b_j &\equiv \sum_{k=0}^{j} \mathcal{D}_k^{(i)}\,\mathcal{D}_{j-k}^{(f)} \\ &= \sum_{k=0}^{j} \mathcal{D}_{j-k}^{(i)}\,\mathcal{D}_k^{(f)}. \end{aligned} \tag{3.28}$$

To obtain Eq. (3.27) we have made use of the identity

$$\sum_{j_1=0}^{n-l-1} \sum_{j_2=0}^{n'-l'-1} \mathcal{D}_{j_1}^{(i)}\,\mathcal{D}_{j_2}^{(f)}\, r^{j_1+j_2} = \sum_{j=0}^{n+n'-l-l'-2} b_j\, r^j. \tag{3.29}$$

Substituting Eq. (3.27) into Eq. (2.32) (with $N = 1$) we obtain the form factor for a general momentum transfer $\mathbf{P}$ as

$$\begin{aligned} G_{fi}(\mathbf{P}) &= N_i N_f \sum_{\lambda=l_{min}}^{l+l'} a_\lambda \sum_{j=0}^{n+n'-l-l'-2} b_j \\ &\times \int d\mathbf{r}\, Y_\lambda^{\mu*}(\hat{\mathbf{r}}) \exp(i\mathbf{P}\cdot\mathbf{r})\, r^{l+l'+j} \exp(-\beta r). \end{aligned} \tag{3.30}$$

We expand the quantity $\exp(i\mathbf{P}\cdot\mathbf{r})$ in a complete set of spherical harmonic basis as Eq. (3.10) and apply the orthonormality property of the spherical harmonics to perform the $\hat{\mathbf{r}}$–integration in Eq. (3.30) leaving

$$G_{fi}(\mathbf{P}) = 4\pi N_i N_f \sum_{\lambda=l_{min}}^{l+l'} i^\lambda a_\lambda Y_\lambda^{\mu*}(\hat{\mathbf{p}}) \sum_{j=0}^{n+n'-l-l'-2} b_j S(P) \tag{3.31}$$

where

$$S(P) = \int_0^\infty dr\, r^{l+l'+j+2} j_\lambda(Pr) \exp(-\beta r). \tag{3.32}$$

We use the relation $j_L(z) = \sqrt{\pi/(2z)}J_{L+1/2}$ and apply the standard integration formula 6.621(1) on page 711 of [GrR80]. Affecting this we then obtain

$$\begin{aligned} S(P) &= \frac{\sqrt{\pi}}{2}\left(\frac{P}{2}\right)^\lambda \frac{\Gamma(\lambda+l+l'+j+3)}{\Gamma(\lambda+3/2)} \frac{\beta^{l+l'-\lambda+j+1}}{(\beta^2+P^2)^{l+l'+j+2}} \\ &\times {}_2F_1\left(\frac{\lambda-l-l'-j-1}{2}, \frac{\lambda-l-l'-j}{2}; \lambda+\frac{3}{2}; -\frac{P^2}{\beta^2}\right). \end{aligned} \tag{3.33}$$

It may be noticed here that the Gauss hypergeometric function in the above equation is necessarily a polynomial since the quantity $(\lambda - l - l' - j)$ is either zero or a negative integer. Substituting Eq. (3.33) in Eq. (3.31) we obtain a closed form expression of the form factor for an arbitrary $nlm \rightarrow n'l'm'$ transition. After some rearrangement, this can be given by

$$\begin{aligned} G_{fi}(\mathbf{P}) &= N_0 \sum_{\lambda=l_{min}}^{l+l'} A_\lambda \sum_{j=0}^{n+n'-l-l'-2} B_{\lambda,j} \frac{P^\lambda}{(P^2+\beta^2)^{l+l'+j+2}} \\ &\times {}_2F_1\left(\frac{\lambda-l-l'-j-1}{2}, \frac{\lambda-l-l'-j}{2}; \lambda+\frac{3}{2}; -\frac{P^2}{\beta^2}\right) \end{aligned} \tag{3.34}$$

where

$$\begin{aligned} N_0 &= \frac{\pi\, 2^{l+l'+2}}{(2l+1)!\,(2l'+1)!} (\beta_i)^{l+3/2} (\beta_f)^{l'+3/2} \\ &\times \sqrt{\left(\frac{(2l+1)\,(n+l)!\,(n'+l')!}{nn'\,(2l'+1)\,(n-l-1)!\,(n'-l'-1)!}\right)}, \end{aligned} \tag{3.35}$$

$$\begin{aligned} A_\lambda &= \frac{\sqrt{2\lambda+1}}{\Gamma(\lambda+3/2)} C(\lambda,l,l';0,0,0)\, C(\lambda,l,l';\mu,m,m') \\ &\times \left(\frac{i}{2}\right)^\lambda Y_\lambda^{\mu *}(\hat{\mathbf{P}}) \end{aligned} \tag{3.36}$$

and

$$\begin{aligned} B_{\lambda,j} &= 2^j \sum_{k=0}^{j} \frac{(-n+l+1)_{j-k}\,(\beta_i)^{j-k}\,(-n'+l'+1)_k\,(\beta_f)^k}{(j-k)!\,(2l+2)_{j-k}\,k!\,(2l'+2)_k} \\ &\times \Gamma(\lambda+l+l'+j+3)\,\beta^{l+l'-\lambda+j+1}. \end{aligned} \tag{3.37}$$

We obtain the corresponding expression for the reduced form factor $g_{fi}(P)$ by the relation (3.16). Since $Y_\lambda^{\mu *}(\vartheta_P = \pi/2, \varphi_P = 0) = 0$ if $\lambda - \mu$ is an odd integer, we see from Eqs. (3.34) and (3.36) that the reduced form factor incorporates the selection rule, namely, the CG amplitude $A(nlm \rightarrow n'l'm')$ vanishes if $(l + l' + m + m')$ = an *odd integer*. The analytical expression (Eq. (3.34)–(3.37)) of the form factor for an arbitrary $nlm \rightarrow n'l'm$ transition of a target of nuclear charge Z_T is used to program the reduced form factor g_{fi}. We have found the simpler forms of G_{fi} for small and large P (see Eqs. (4.42) and (4.46)), and introduced them in the computer program.

3.3 Computations of the CG amplitudes

We numerically integrate the one–dimensional integral expression (3.20) to calculate the Coulomb–Glauber amplitude for an inelastic collision as a function of the scattering angle Θ. It may be noted that Gauss hypergeometric series ${}_2F_1(\cdots)$ appearing in Eq. (3.20) diverges at the point $x = 1$. This divergence may be explicitly exhibited by using the formula 15.3.3 on page

559 of [AbS70],

$$
\begin{aligned}
{}_2F_1(\cdots) &= \left|\frac{1-x}{1+x}\right|^{2i\eta+2i\alpha-1} \\
&\times {}_2F_1\left(|\mu|+i\eta+i\alpha, |\mu|+\frac{1}{2}; 2|\mu|+1; \frac{4x}{(1+x)^2}\right). \qquad (3.38)
\end{aligned}
$$

Consequently, we may write the x–integral of Eq. (3.19) as

$$
I_{fi} = \int_0^\infty dx\, |1-x|^{2i\eta+2i\alpha-1} L(x) \qquad (3.39)
$$

where

$$
\begin{aligned}
L(x) &= \frac{x^{-2i\eta+|\mu|-1}}{(1+x)^{2|\mu|+1}} [g_{fi}(qx) - \delta_{fi}] \\
&\times {}_2F_1\left(|\mu|+i\eta+i\alpha, |\mu|+\frac{1}{2}; 2|\mu|+1; \frac{4x}{(1+x)^2}\right). \qquad (3.40)
\end{aligned}
$$

We see from Eqs. (3.39) and (3.40) that the integrand is singular at $x = 1$. Since η, by assumption, has an infinitesimal negative imaginary part, $-\epsilon$, it is easily verified that for finite $\Delta > 0$

,

$$
\int_{1-\Delta}^{1+\Delta} dx\, |1-x|^{2i\eta+2i\alpha-1} = \frac{\Delta^{2i\eta+2i\alpha}}{i\eta+i\alpha}. \qquad (3.41)
$$

Therefore, we may write

$$
\begin{aligned}
\int_{1-\Delta}^{1+\Delta} dx\, |1-x|^{2i\eta+2i\alpha-1} L(x) &= \frac{\Delta^{2i\eta+2i\alpha}}{i\eta+i\alpha} L(1) \\
&+ \int_{1-\Delta}^{1+\Delta} dx\, |1-x|^{2i\eta+2i\alpha-1} (L(x) - L(1)). \qquad (3.42)
\end{aligned}
$$

Now, ϵ may be put equal to zero. Obviously the integrand of the x–integral on the right–hand side of Eq. (3.42) is finite at $x = 1$ and hence the right–hand side may be evaluated numerically without difficulty.

We numerically calculated the integral I_{fi} by dividing the range of integration into three parts, $(0, 1-\Delta)$, $(1-\Delta, 1+\Delta)$ and $(1+\Delta, \infty)$. For numerical calculation of the x–integral in the range $(1-\Delta, 1+\Delta)$, we used Eq. (3.42). In performing the numerical integration in all other parts the form of Eq. (3.20) was retained. Care was taken to use appropriate analytical continuation formulas of Gauss hypergeometric functions to obtain rapid convergence. In fact, we have further divided the first part $(0, 1-\Delta)$ into two segments as $(0, \Delta_1)$ and $(\Delta_1, 1-\Delta)$, and the third part $(1+\Delta, \infty)$ into three segments as $(1+\Delta, \Delta_2)$, (Δ_2, χ) and (χ, ∞). In the following, we list the formulas, which were employed to ensure faster convergence in each of the six segments, for the Gauss hypergeometric functions

$$
{}_2F_1^{(1)}(\cdots) \equiv {}_2F_1\left(|\mu|-i\eta-i\alpha+1, |\mu|+\frac{1}{2}; 2|\mu|+1; \frac{4x}{(1+x)^2}\right) \qquad (3.43)
$$

in Eq. (3.20) and

$$
{}_2F_1^{(2)}(\cdots) \equiv {}_2F_1\left(i\eta+i\alpha+|\mu|, |\mu|+\frac{1}{2}; 2|\mu|+1; \frac{4x}{(1+x)^2}\right) \qquad (3.44)
$$

in Eq. (3.42).

1. In the range $(0, \Delta_1)$:

$$
\begin{aligned}
{}_2F_1^{(1)}(\cdots) &= \left(\frac{1}{1+x}\right)^{2i(\eta+\alpha)-2|\mu|-2} \left(1-x^2\right)^{2i(\eta+\alpha)-1} \\
&\times \; {}_2F_1\left(i\eta+i\alpha,\, i\eta+i\alpha+|\mu|;\, |\mu|+1;\, x^2\right). \qquad (3.45)
\end{aligned}
$$

We have used the formulas 15.3.17 on page 560 and 15.3.3 on page 559 of [AbS70] to derive Eq. (3.45).

2. In the ranges $(\Delta_1, 1-\Delta)$ and $(1+\Delta, \Delta_2)$:

$$
\begin{aligned}
{}_2F_1^{(1)}(\cdots) &= \frac{\Gamma(2|\mu|+1)\,\Gamma(i\eta+i\alpha-1/2)}{\Gamma(|\mu|+i\eta+i\alpha)\,\Gamma(|\mu|+1/2)} \left[\frac{(1+x)^2}{4x}\right]^{2|\mu|} \\
&\times \; {}_2F_1\left(\frac{1}{2}-|\mu|,\, -i\eta-i\alpha-|\mu|+1;\, -i\eta-i\alpha+\frac{3}{2};\, \left|\frac{1-x}{1+x}\right|^2\right) \\
&+ \frac{\Gamma(2|\mu|+1)\,\Gamma(-i\eta-i\alpha+1/2)}{\Gamma(|\mu|-i\eta-i\alpha+1)\,\Gamma(|\mu|+1/2)} \left|\frac{1-x}{1+x}\right|^{2i(\eta+\alpha)-1} \left[\frac{(1+x)^2}{4x}\right]^{2|\mu|} \\
&\times \; {}_2F_1\left(\frac{1}{2}-|\mu|,\, i\eta+i\alpha-|\mu|;\, i\eta+i\alpha+\frac{1}{2};\, \left|\frac{1-x}{1+x}\right|^2\right). \qquad (3.46)
\end{aligned}
$$

Here we have made use of the formulas 15.3.6 and 15.3.3 on page 559 of [AbS70] to deduce the above equation.

3. In the ranges (Δ_2, χ) and (χ, ∞):

$$
\begin{aligned}
{}_2F_1^{(1)}(\cdots) &= \left(\frac{x}{1+x}\right)^{2i(\eta+\alpha)-2|\mu|-2} \left(1-\frac{1}{x^2}\right)^{2i(\eta+\alpha)-1} \\
&\times \; {}_2F_1\left(i\eta+i\alpha,\, i\eta+i\alpha+|\mu|;\, |\mu|+1;\, \frac{1}{x^2}\right). \qquad (3.47)
\end{aligned}
$$

To derive Eq. (3.47) formulas 15.3.17 on page 560 and 15.3.3 on page 559 of [AbS70] are used.

4. In the range $(1-\Delta, 1+\Delta)$:

$$
\begin{aligned}
{}_2F_1^{(2)}(\cdots) &= \frac{\Gamma(2|\mu|+1)\,\Gamma(i\eta+i\alpha-1/2)}{\Gamma(|\mu|+i\eta+i\alpha)\,\Gamma(|\mu|+1/2)} \left|\frac{1+x}{1-x}\right|^{2i(\eta+\alpha)-1} \left[\frac{(1+x)^2}{4x}\right]^{2|\mu|} \\
&\times \; {}_2F_1\left(\frac{1}{2}-|\mu|,\, -i\eta-i\alpha-|\mu|+1;\, -i\eta-i\alpha+\frac{3}{2};\, \left|\frac{1-x}{1+x}\right|^2\right) \\
&+ \frac{\Gamma(2|\mu|+1)\,\Gamma(-i\eta-i\alpha+1/2)}{\Gamma(|\mu|-i\eta-i\alpha+1)\,\Gamma(|\mu|+1/2)} \left[\frac{(1+x)^2}{4x}\right]^{2|\mu|} \\
&\times \; {}_2F_1\left(\frac{1}{2}-|\mu|,\, i\eta+i\alpha-|\mu|;\, i\eta+i\alpha+\frac{1}{2};\, \left|\frac{1-x}{1+x}\right|^2\right). \qquad (3.48)
\end{aligned}
$$

Formulas 15.3.6 and 15.3.3 on page 559 of [AbS70] are used here to obtain Eq. (3.48).

Table 3.1. Typical values of some parameters used to perform x–integration numerically.

Δ	Δ_1	Δ_2	χ	∞
0.05	0.2	2.4	15	100

In Table 3.1 we give typical values of Δ, Δ_1, Δ_2, χ and ∞ used in the program. From the expression of I_{fi} (Eq. (3.20)) it is evident that the integrand is an oscillatory function which decays with large x. We have applied Gauss quadrature to perform the x–integration.

We carry out a further integration over the amplitudes to calculate the total cross section, using the relation (2.16). It is noted from Eq. (3.19) and Eq. (3.20) that except the reduced form factor all other quantities are independent of q. Therefore, for a transition, our computer program calculates in the beginning all q–independent quantities and stores them; these numbers are used subsequently to generate differential cross sections for several q's in order to compute the total cross section. Also, we avoid repeated calculation by storing all q–independent coefficients of the form factor in our program. Gauss quadrature is employed to perform the q–integration.

3.4 Results and discussions

For $Z_T = 1$, Eq. (3.19) gives a *new integral representation* of the Glauber amplitude for arbitrary $nlm \rightarrow n'l'm'$ excitation of hydrogen by the impact of charged particles. In order to test our method, we calculated the Glauber cross sections for several transitions, from the ground state to excited states as well as from excited states to excited states of hydrogen by electron-impact ($Z_P = -1$). We found that all our Glauber cross sections obtained by numerical evaluation of Eq. (3.19) for electron-hydrogen scattering agreed with the results reported previously [Tos77, SuS81].

As a test of the computer program, we calculated the cross sections for electron-impact ($Z_P = -1$) excitation of hydrogen-like ions (of $Z_T = 2$ to $Z_T = 10$) from the ground $1s$ state to various excited states. We found that our cross sections for n=2 and n=3 excitation of various ions agreed with those obtained earlier by Thomas [Tho78]. We also performed calculations to reproduce several cross sections for $1s \rightarrow ns$ and $1s \rightarrow np$ excitation of hydrogen-like ions reported by Das and Sil [DaS84b].

We have calculated cross sections for inelastic electron scattering by atomic hydrogen and hydrogenic ions for several excitations. To our knowledge, no previous calculations are available in the literature for excited state to excited state transition of hydrogen–like ions by the impact of charged particles in the Coulomb–Glauber approximation. As examples, in Table 3.2, we list a few cross sections for $n = 2 \rightarrow n = 3$ transitions of helium ($Z_T = 2$) and beryllium ($Z_T = 4$) ions by electron-impact.

In Chapter 5 we apply the CG approximation to calculate the rate coefficients for the electron impact excitations of HeII and CVI ions.

Table 3.2. Scaled total cross sections $Z_T^4 n'^3 \sigma$ (in units of πa_0^2) in the Coulomb-modified Glauber approximation for electron-impact excitation of hydrogen-like ions with $Z_T = 2$ and $Z_T = 4$ tabulated as a function of incident energies (in units of $\frac{1}{2} Z_T^2 (1/n^2 - 1/n'^2)$).

Charge on target nucleus	Transitions	Scaled cross sections $Z_T^4 n'^3 \sigma$	
		E=1.2	E=2.
$Z_T = 2$	$1s - 2p$	6.77	1.02(+1)†
	$2s - 3s$	1.78(+2)	1.71(+2)
	$2s - 3p$	2.82(+2)	4.18(+2)
	$2s - 3d$	7.79(+2)	9.29(+2)
	$2p - 3p$	1.95(+2)	2.02(+2)
$Z_T = 4$	$1s - 2p$	1.22(+1)	1.43(+1)
	$2s - 3s$	2.34(+2)	2.34(+2)
	$2s - 3p$	1.07(+3)	1.30(+3)
	$2s - 3d$	1.16(+3)	1.16(+3)
	$2p - 3p$	2.73(+2)	2.70(+2)

† The number inside the parenthesis denotes the power of ten by which the entry is to be multiplied

Chapter 4

Behavior of the Coulomb–Glauber Amplitude for Infinitely Large Nuclear Charge, and Large and Small Momentum Transfer

The study of the dependence of the scattering amplitude for excitation of hydrogenic ions on the nuclear charge Z_T is of considerable interest, since its understanding can simplify the calculations of the cross sections over a range of energy and nuclear charge. We investigate [ChD94] in this chapter the Z_T–dependence of the scattering amplitude in the Coulomb–Glauber approximation [IsC75, NaT75, ThF76] for arbitrary $nlm \rightarrow n'l'm'$ excitation of hydrogenic ions by the impact of structureless particles of charge Z_P and derive a closed form expression of the CG amplitude in the limit of infinite Z_T. The Z_T–dependence of the CG cross section has also been studied previously but these studies are confined to the excitation of the ground $1s$ state. Thomas and Franco [ThF76] and Thomas [Tho78] have evaluated closed form expressions of the CG amplitude in the large Z_T limit for excitation from the ground $1s$ state to the excited $n' = 2$ and $n' = 3$ states. Das and Sil [DaS84a, b] have computed cross sections for $1s \rightarrow n's$ and $1s \rightarrow n'p$ excitation of hydrogenic ions for n' and Z_T both ranging from 2 to infinity. The Z_T–dependence of the Coulomb–Born amplitude has also been studied for several transitions (see e.g. [BHT70], [MiS78], [SRS78]).

The asymptotic behavior of the scattering amplitude at large and small values of q is of practical interest when the limiting values of q are contained within its physical range ($|K_i - K_f| \leq q \leq K_i + K_f$). As we only consider the inelastic scattering ($K_i \neq K_f$), $q > 0$; however, $K_i - K_f$ can be small if K_i is sufficiently large. Correspondingly, $K_i + K_f$ is large, when K_i is large. Thus at large K_i, the limiting behavior of the amplitude at small q determines the scattering at small angles near the forward direction. Similarly, the large–q behavior of the amplitude determines the scattering at wide scattering angles. Thomas and Franco [ThF76] has

examined the behavior of the CG amplitude for large and small q in the ground state excitation of the hydrogenic ions to any arbitrary $n'l'm'$ states. We also present in this chapter our study on the dependence of the CG scattering amplitude on q for $nlm \to n'l'm'$ transition.

4.1 Closed form expression of the CG amplitude for arbitrary $nlm \to n'l'm'$ excitation in the limit of infinite Z_T

4.1.1 Derivation

We have already obtained in the previous chapter the two- and one-dimensional (refer to Eqs. (3.18) and (3.19) respectively) integral representations of the CG amplitude for arbitrary $nlm \to n'l'm'$ excitation. The starting point of the present investigation is the two-dimensional integral representation of the CG amplitude given in Eq. (3.18), which we rewrite as

$$\begin{aligned} A(nlm \to n'l'm') &= -2^{2i\eta+1}\eta K_i \frac{\Gamma(1+i\eta)}{\Gamma(1-i\eta)} \exp(-i\mu\varphi_q) \\ &\times \int_0^\infty dy\, y^{-2i\eta-1}[g_{fi}(Z_T, y) - \delta_{fi}] \\ &\times \int_0^\infty db\, b^{-2i\eta-2i\alpha+1} J_{|\mu|}(qb) J_{|\mu|}(by). \end{aligned} \tag{4.1}$$

Since the atomic wave function $\phi_{nlm}(\mathbf{r})$ of a hydrogenic ion of charge Z_T depends on r only through rZ_T, it is seen that $G_{fi}(Z_T, \mathbf{Q}) = G_{fi}(1, \mathbf{Q}/Z_T)$ and hence $g_{fi}(Z_T, y) = g_{fi}(1, y/Z_T)$. Thus, the (reduced) form factor of a hydrogenic ion of nuclear charge Z_T at a given $\mathbf{Q}$ has the same value as the (reduced) form factor of hydrogen at $\mathbf{Q}/Z_T$.

The special form of the Z_T-dependence of the reduced form factor suggests that the change of the variables of integration from y and b to $y = Z_T p$ and $b = \rho/Z_T$ may be used for studying the Z_T dependence of the amplitude (4.1). Affecting this, we obtain

$$A(nlm \to n'l'm') = Z_T^{2i\overline{\eta}(1-1/Z_T)-2}\, \overline{A}(nlm \to n'l'm') \tag{4.2}$$

in which

$$\begin{aligned} \overline{A}(nlm \to n'l'm') &= -2^{2i\overline{\eta}/Z_T+1}\overline{\eta}\,\overline{K_i} \frac{\Gamma(1+i\overline{\eta}/Z_T)}{\Gamma(1-i\overline{\eta}/Z_T)} \exp(-i\mu\varphi_q) \\ &\times \int_0^\infty dp\, (p)^{-2i\overline{\eta}/Z_T-1} [g_{fi}(1,p) - \delta_{fi}] \\ &\times \int_0^\infty d\rho\, (\rho)^{-2i\overline{\eta}+1} J_{|\mu|}(\overline{q}\rho) J_{|\mu|}(\rho p) \end{aligned} \tag{4.3}$$

where $\overline{K_i} = K_i/Z_T$, $\overline{\eta} = \alpha + \eta = Z_P M/\overline{K_i}$ and $g_{fi}(1,p)$ is the reduced form factor for excitation of hydrogen atom. The expressions (4.2) and (4.3) are exact.

The amplitude $\overline{A}(nlm \to n'l'm')$ depends on the parameters, $\overline{K_i}$, $\overline{\eta}$ and $\overline{q}$. Let us briefly discuss the significance of these parameters. Obviously, as the nuclear charge Z_T increases, the electronic binding energy $Z_T^2/(2n^2)$ also increases. The energy conservation relation,

$$\frac{K_i^2}{2M} - \frac{K_f^2}{2M} = Z_T^2 \left(\frac{1}{2n^2} - \frac{1}{2n'^2}\right) \tag{4.4}$$

shows that the initial and final momenta, K_i and K_f, involved in an arbitrary $nlm \to n'l'm'$ transition of a hydrogenic ion of charge Z_T are related to the corresponding momenta for the hydrogenic ion of charge $Z_T = 1$ by $\overline{K_i} = K_i/Z_T$ and $\overline{K_f} = K_f/Z_T$. Hence the parameter $\overline{\eta} = Z_P/(\overline{K_i}/M) = Z_P/\overline{v}$, where $\overline{v} = \overline{K_i}/M$, refers to excitation of the hydrogenic ion of charge $Z_T = 1$. Furthermore, for a fixed scattering angle Θ in the center of mass frame, $q^2 = K_i^2 + K_f^2 - 2K_iK_f\cos\Theta = Z_T^2(\overline{K_i}^2 + \overline{K_f}^2 - 2\overline{K_i}\,\overline{K_f}\cos\Theta) = Z_T^2\overline{q}^2$. Thus, if $\mathbf{q}$ is the momentum transfer vector in the $nlm \to n'l'm'$ excitation of the hydrogenic ion of charge Z_T, the corresponding momentum transfer for the hydrogenic ion of charge $Z_T = 1$ is $\overline{\mathbf{q}} = \mathbf{q}/Z_T$. Thus, $\overline{K_i}, \overline{\eta}$ and $\overline{q}$ can be interpreted as the values of K_i, η and q for the special case when $Z_T = 1$ and $g_{fi}(1,p)$ is the reduced form factor for excitation of hydrogen atom.

Eq. (4.3) shows that the CG amplitude $A(nlm \to n'l'm')$ depends on Z_T through the term $\overline{\eta}/Z_T = Z_P/(\overline{v}Z_T)$. Thus for a fixed $\overline{v}$, it depends on the ratio Z_P/Z_T. For $Z_T\overline{v} \gg Z_P$, the amplitude $A(nlm \to n'l'm')$ can be expanded in the powers of $Z_P/(\overline{v}Z_T)$. In the limit as $Z_P/(\overline{v}Z_T) \to 0$, the expression (4.3) reduces to

$$\begin{aligned}\overline{A}_{as}(nlm \to n'l'm') &= -2\overline{\eta}\,\overline{K_i}\exp(-i\mu\varphi_q)\int_0^\infty d\rho\ (\rho)^{-2i\overline{\eta}+1}\,J_{|\mu|}(\overline{q}\rho) \\ &\times \int_0^\infty dp\,\frac{[g_{fi}(1,p)-\delta_{fi}]}{p}J_{|\mu|}(\rho p) \qquad (4.5)\end{aligned}$$

where we have changed the order of integration. Since $Z_P(\geq 1)$ and $\overline{v}$ are finite, this is also the limiting expression for $Z_T \to \infty$. Evidently, the limiting amplitude $\overline{A}_{as}(nlm \to n'l'm')$ is a function of $\overline{q}$ and $\overline{\eta}$ and does not depend on charge Z_T explicitly. It must be stressed here that although values of $\overline{K_i}, \overline{\eta}$ and $\overline{q}$ correspond to the the $nlm \to n'l'm'$ excitation of hydrogen ($Z_T = 1$), the limiting amplitude $\overline{A}_{as}(nlm \to n'l'm')$ is *not* identical to the Glauber amplitude for excitation of hydrogen atom.

In the limit of infinite nuclear charge Z_T, Eq. (4.2) reduces to

$$\lim_{Z_T\to\infty} A(nlm \to n'l'm') = Z_T^{2i\overline{\eta}-2}\overline{A}_{as}(nlm \to n'l'm'). \qquad (4.6)$$

Thus, for large target nuclear charge, the cross section of the Coulomb–Glauber approximation varies as $\sim Z_T^{-4}$. This dependence is the same as in the Coulomb–Born approximation (see e.g. [BHT70]).

To evaluate the integral (4.5), we need the explicit expression of the reduced form factor $g_{fi}(1,p)$ for an arbitrary $nlm \to n'l'm'$ excitation. Earlier (see Eqs. (3.34–3.37) we have obtained an analytic expression for $G_{fi}(1,p)$ by elementary method. Using Eq. (3.16) an expression for $g_{fi}(1,p)$ then is given by

$$\begin{aligned}g_{fi}(1,p) &= \overline{N_0}\sum_{\lambda=l_{min}}^{l+l'} A'_\lambda \sum_{j=0}^{n+n'-l-l'-2}\overline{B}_{\lambda,j}\frac{p^\lambda}{\left(p^2+\overline{\beta}^2\right)^{l+l'+j+2}} \\ &\times\ {}_2F_1\left(\frac{\lambda-l-l'-j-1}{2},\frac{\lambda-l-l'-j}{2};\lambda+\frac{3}{2};-\frac{p^2}{\overline{\beta}^2}\right) \qquad (4.7)\end{aligned}$$

where

$$\overline{N_0} = \frac{\pi\, 2^{l+l'+2}}{(2l+1)!\,(2l'+1)!}\left(\overline{\beta_i}\right)^{l+3/2}\left(\overline{\beta_f}\right)^{l'+3/2} \times \sqrt{\left(\frac{(2l+1)\,(n+l)!\,(n'+l')!}{nn'\,(2l'+1)\,(n-l-1)!\,(n'-l'-1)!}\right)}, \tag{4.8}$$

$$A'_\lambda = \sqrt{\frac{1}{4\pi}\frac{(\lambda-\mu)!}{(\lambda+\mu)!}\frac{2\lambda+1}{\Gamma(\lambda+3/2)}}C(\lambda,l,l';0,0,0)\,C(\lambda,l,l';\mu,m,m') \times \left(\frac{i}{2}\right)^\lambda P_\lambda^\mu(0) \tag{4.9}$$

and

$$\overline{B}_{\lambda,j} = 2^j\sum_{k=0}^{j}\frac{(-n+l+1)_{j-k}\,(\overline{\beta_i})^{j-k}\,(-n'+l'+1)_k\,(\overline{\beta_f})^k}{(j-k)!\,(2l+2)_{j-k}\,k!\,(2l'+2)_k} \times \Gamma(\lambda+l+l'+j+3)\,\overline{\beta}^{l+l'-\lambda+j+1} \tag{4.10}$$

In Eqs. (4.7), (4.8) and (4.10) $\overline{\beta_i} = 1/n$, $\overline{\beta_f} = 1/n'$ and $\overline{\beta} = \overline{\beta_i} + \overline{\beta_f} = 1/n + 1/n'$. Expanding the Gaussian hypergeometric function in Eq. (4.7) in powers of p^2 we re-express g_{fi} as

$$g_{fi}(1,p) = \overline{N_0}\sum_{\lambda=l_{min}}^{l+l'}A'_\lambda\sum_{j=0}^{n+n'-l-l'-2}\overline{B}_{\lambda,j}\frac{p^\lambda}{\left(p^2+\overline{\beta}^2\right)^{l+l'+j+2}}\sum_{s=0}^{t}\overline{f}_{\lambda,j,s}p^{2s} \tag{4.11}$$

where

$$\overline{f}_{\lambda,j,s} = (-)^s\frac{\left(\frac{\lambda-l-l'-j-1}{2}\right)_s\left(\frac{\lambda-l-l'-j}{2}\right)_s}{\left(\lambda+\frac{3}{2}\right)_s\,s!}\left(\overline{\beta}\right)^{-2s}. \tag{4.12}$$

In Eq. (4.11), $t = (-\lambda+l+l'+j)/2$ if $\lambda-l-l'-j$ is zero or even, otherwise $t = (-\lambda+l+l'+j+1)/2$.

Before proceeding further, we mention here a complication which arises when the expression (4.11) is directly inserted into (4.5). In this case, we obtain

$$\overline{A}_{as}(nlm\to n'l'm') = -2\overline{\eta}\,\overline{K_i}\,\overline{N_0}\exp(-i\mu\varphi_q)\sum_{\lambda=l_{min}}^{l+l'}A'_\lambda\sum_{j=0}^{n+n'-l-l'-2}\overline{B}_{\lambda,j} \times \sum_{s=0}^{t}\overline{f}_{\lambda,j,s}\int_0^\infty d\rho\,(\rho)^{-2i\overline{\eta}+1}\,J_{|\mu|}(\overline{q}\rho) \times \int_0^\infty dp\frac{p^{\lambda+2s-1}}{\left(p^2+\overline{\beta}^2\right)^{l+l'+j+2}}J_{|\mu|}(\rho p). \tag{4.13}$$

To see the source of the complication, we consider the p-integration, namely,

$$I(\rho) = \int_0^\infty dp\frac{p^{\lambda+2s-1}}{\left(p^2+\overline{\beta}^2\right)^{l+l'+j+2}}J_{|\mu|}(\rho p). \tag{4.14}$$

For some (e.g. $ns\to n's$) transitions, the expansion (4.11) contains terms (corresponding to both $\lambda = 0$ (and hence $|\mu| = 0$) and $s = 0$) and which tend to a constant as $p\to 0$. For each of these

terms, the integrand of (4.14) behaves as $1/p$ causing the integral (4.14) to diverge. However, since $g_{fi}(1,p=0) \equiv <\varphi_f|\varphi_i> = \delta_{fi}$, the reduced form factor for an inelastic transition vanishes at $p=0$, and hence these divergent terms must cancel themselves. Therefore, in this method, one needs to keep track of singularities to obtain an expression for the amplitude.

This complication can be avoided if we remove, at the outset, the terms from (4.11) which become constants at $p=0$. Evidently, these terms are the nonvanishing terms of $g_{fi}(1,p=0)$. Upon putting $p=0$ in Eq. (4.11), we obtain

$$\overline{N_0} \sum_{\lambda=l_{min}}^{l+l'} A'_\lambda \sum_{j=0}^{n+n'-l-l'-2} \frac{\overline{B}_{\lambda,j}}{\left(\overline{\beta}\right)^{2(l+l'+j+2)}} \delta_{\lambda,0}\delta_{\mu,0} = \delta_{fi}. \tag{4.15}$$

Using this result in Eq. (4.11) and carrying out some simple algebra, we obtain

$$\begin{aligned} g_{fi}(1,p) &\equiv g_{fi}(1,p) - \delta_{fi} \\ &= \overline{N_0} \sum_{\lambda=l_{min}}^{l+l'} A'_\lambda \sum_{j=0}^{n+n'-l-l'-2} \overline{B}_{\lambda,j} \frac{p^\lambda}{\left(p^2+\overline{\beta}^2\right)^{l+l'+j+2}} \overline{g}_{\lambda,j}(p) \end{aligned} \tag{4.16}$$

where

$$\overline{g}_{\lambda,j}(p) = \sum_{s=\delta_{\lambda,0}}^{t} \overline{f}_{\lambda,j,s} p^{2s} - \delta_{\lambda,0} \sum_{s=1}^{l+l'+j+2} \overline{g}_{j,s} p^{2s} \tag{4.17}$$

where the symbol $s=\delta_{\lambda,0}$ means that the $s=1$ if $\lambda=0$ otherwise $s=0$ and

$$\overline{g}_{j,s} = {}^{l+l'+j+2}C_s \left(\overline{\beta}\right)^{-2s}. \tag{4.18}$$

We see that each term in the expansion (4.16) vanishes as $p \to 0$. As a result, the singular form of the integral (4.14) does not occur.

Substituting (4.16) in (4.5), we obtain

$$\begin{aligned} \overline{A}_{as}(nlm \to n'l'm') &= -2\overline{\eta}\overline{K_i}\,\overline{N_0} \exp(-i\mu\varphi_q) \sum_{\lambda=l_{min}}^{l+l'} A'_\lambda \\ &\times \sum_{j=0}^{n+n'-l-l'-2} \overline{B}_{\lambda,j} \left[\sum_{s=\delta_{\lambda,0}}^{t} \overline{f}_{\lambda,j,s} - \delta_{\lambda,0} \sum_{s=1}^{l+l'+j+2} \overline{g}_{j,s} \right] \\ &\times \int_0^\infty d\rho\, I(\rho)\,(\rho)^{-2i\overline{\eta}+1}\, J_{|\mu|}(\overline{q}\rho) \end{aligned} \tag{4.19}$$

where $I(\rho)$ is given by Eq. (4.14). Evidently, this expression differs from (4.13) only in the term for $\lambda=0$.

It may be mentioned here that the p-integration in (4.14) can be performed in terms of the generalized hypergeometric function ${}_1F_2(\cdots)$ with the help of the formula 6.565(8) on page 687 of Gradshtein and Ryzhik [GrR80]. However, we adopt an alternative (and perhaps simpler) method to obtain a closed form expression for the limiting amplitude $\overline{A}_{as}$ in terms of the Gauss hypergeometric functions ${}_2F_1$.

To perform the p–integration, we reduce (4.14) into a series of integrals of the form

$$I(\rho) = \int_0^\infty dp \frac{p^{\gamma+1}}{\left(p^2+\overline{\beta}^2\right)^\zeta} J_\gamma(\rho p). \tag{4.20}$$

This integral can be evaluated with the help of the formula 6.565(4) on page 686 of [GrR80], namely,

$$\int_0^\infty dx \frac{J_\nu(bx)x^{\nu+1}}{(x^2+a^2)^{\mu+1}} = \frac{a^{\nu-\mu}\, b^\mu}{2^\mu\, \Gamma(\mu+1)} K_{\nu-\mu}(ab)$$

$$\text{provided} \quad \left[-1 < \text{Re}\ \nu < \text{Re}\left(2\mu+\frac{3}{2}\right), \quad a>0, \quad b>0 \right] \tag{4.21}$$

where $K_\gamma(z)$ is the modified Bessel function.

In the case when $\lambda = |\mu|$ and $s = 0$, the integrand of (4.14) is of the form $p^{|\mu|-1}J_{|\mu|}(\rho p)/\left(p^2+\overline{\beta}^2\right)^{l+l'+j+2}$. To reduce this integral into the form (4.20), we integrate it by parts $|\mu|$ times with the help of the formula 9.1.30(2), with $k=1$, on page 361 of [AbS70], and obtain

$$\begin{aligned} I_1(\rho) &= \frac{1}{2}\frac{(|\mu|-1)!}{\overline{\beta}^{2(l+l'+j+2)}}\left(\frac{\rho}{2}\right)^{-|\mu|} \\ &- (l+l'+j+2)\sum_{r=1}^{|\mu|}(|\mu|-r+1)_{r-1}\left(\frac{\rho}{2}\right)^{-r} \\ &\times \int_0^\infty dp \frac{p^{|\mu|-r+1}}{\left(p^2+\overline{\beta}^2\right)^{l+l'+j+3}} J_{|\mu|-r}(\rho p). \end{aligned} \tag{4.22}$$

In all other cases, $\lambda > |\mu|$, and we rearrange the integrand of (4.14) as

$$\frac{p^{\lambda+2s-1}}{\left(p^2+\overline{\beta}^2\right)^{l+l'+j+2}} J_{|\mu|}(\rho p) = p^{|\mu|+1}\frac{\left[\left(p^2+\overline{\beta}^2\right)-\overline{\beta}^2\right]^\nu}{\left(p^2+\overline{\beta}^2\right)^{l+l'+j+2}} J_{|\mu|}(\rho p) \tag{4.23}$$

where $\nu = (\lambda - |\mu| + 2s - 2)/2$. Since the CG amplitude vanishes when $l+l'+m+m'$ and hence $\lambda+|\mu|$ is an odd number, ν is either zero or a positive integer. Making the binomial expansion of $\left[\left(p^2+\overline{\beta}^2\right)-\overline{\beta}^2\right]^\nu$ in (4.23), and using the resulting expression in (4.14), we obtain

$$I_2(\rho) = \sum_{r=0}^{\nu}(-)^r\, {}^\nu C_r\, \overline{\beta}^{2r} \int_0^\infty dp \frac{p^{|\mu|+1}}{\left(p^2+\overline{\beta}^2\right)^{l+l'+j-\nu+r+2}} J_{|\mu|}(\rho p) \tag{4.24}$$

where ${}^nC_m$ is the binomial coefficient.

The p–integrals in both (4.22) and (4.24) are evaluated with the help of the formula (4.21). We finally get

$$\begin{aligned} I_1(\rho) &= \frac{1}{2}\frac{(|\mu|-1)!}{\overline{\beta}^{2(l+l'+j+2)}}\left(\frac{\rho}{2}\right)^{-|\mu|} \\ &- \frac{1}{\Gamma(l+l'+j+2)}\sum_{r=1}^{|\mu|}\overline{\beta}^{|\mu|-l-l'-j-r-2}(|\mu|-r+1)_{r-1} \\ &\times \left(\frac{\rho}{2}\right)^{l+l'+j-r+2} K_{|\mu|-l-l'-j-r-2}(\overline{\beta}\rho) \end{aligned} \tag{4.25}$$

and

$$I_2(\rho) = \sum_{r=0}^{\nu} \frac{(-)^r \, {}^{\nu}C_r}{\Gamma(l+l'+j-\nu+r+2)} \overline{\beta}^{|\mu|-l-l'-j+\nu+r-1} \times \left(\frac{\rho}{2}\right)^{l+l'+j-\nu+r+1} K_{|\mu|-l-l'-j+\nu-r-1}(\overline{\beta}\rho). \tag{4.26}$$

We use the results (4.25) and (4.26) in (4.19). The expression (4.25) leads to the ρ–integrals of the forms

$$W_1(\overline{q}) = \int_0^{\infty} d\rho \, (\rho)^{-2i\overline{\eta}-|\mu|+1} \, J_{|\mu|}(\overline{q}\rho) \tag{4.27}$$

and

$$W_2(\overline{q}) = \int_0^{\infty} d\rho \, (\rho)^{-2i\overline{\eta}+l+l'+j-r+3} \, J_{|\mu|}(\overline{q}\rho) \, K_{|\mu|-l-l'-j-r-2}(\overline{\beta}\rho). \tag{4.28}$$

The expression (4.26) gives rise to the ρ–integrals of the similar form to (4.28).

The integrals of the forms (4.27) and (4.28) are evaluated by making use of the formulas 6.561(14) and 6.576(3) on the pages 684 and 693 of [GrR80], respectively. Upon rearranging these results, we obtain the final closed form expression of the CG amplitude in terms of the Gauss hypergeometric functions,

$$\overline{A}_{as} = -2\overline{\eta}\,\overline{K_i}\,\overline{N_0} \exp(-i\mu\varphi_q) \sum_{\lambda=l_{min}}^{l+l'} A'_{\lambda} \sum_{j=0}^{n+n'-l-l'-2} \overline{B}_{\lambda,j} \Theta_j(\lambda) \tag{4.29}$$

in which

$$\Theta_j(\lambda=0) = \sum_{s=1}^{t} \overline{f}_{0,j,s} U_{0,j,s} - \sum_{s=1}^{l+l'+j+2} \overline{g}_{j,s} U_{0,j,s} \tag{4.30}$$

and

$$\Theta_j(\lambda \neq 0) = \sum_{s=0}^{t} \overline{f}_{\lambda,j,s} D_{\lambda,j,s} \tag{4.31}$$

where $D_{\lambda,j,s} = V_{|\mu|,j}$ if $\lambda = |\mu|$ and $s = 0$, and otherwise $D_{\lambda,j,s} = U_{\lambda,j,s}$. The explicit expressions for $U_{\lambda,j,s}$ and $V_{|\mu|,j}$ are

$$U_{\lambda,j,s} = \left(\frac{\overline{\beta}}{2}\right)^{2i\overline{\eta}} \left(\overline{\beta}\right)^{-2(l+l'+j-\nu+2)} \frac{\Gamma(-i\overline{\eta}+|\mu|+1)}{\Gamma(|\mu|+1)} \overline{q}^{|\mu|} \times \sum_{r=0}^{\nu} (-)^r \, {}^{\nu}C_r \frac{\Gamma(-i\overline{\eta}+l+l'+j-\nu+r+2)}{\Gamma(l+l'+j-\nu+r+2)} \times {}_2F_1\left(-i\overline{\eta}+|\mu|+1, -i\overline{\eta}+l+l'+j-\nu+r+2; |\mu|+1; -X^2\right) \tag{4.32}$$

$$V_{|\mu|,j} = 2^{-2i\overline{\eta}} \frac{(|\mu|-1)!}{\left(\overline{\beta}\right)^{2(l+l'+j+2)}} \frac{\Gamma(-i\overline{\eta}+1)}{\Gamma(i\overline{\eta}+|\mu|)} \overline{q}^{2i\overline{\eta}+|\mu|-2}$$

$$
\begin{aligned}
&- \left(\frac{\overline{\beta}}{2}\right)^{2i\overline{\eta}} \left(\overline{\beta}\right)^{-2(l+l'+j+3)} \frac{\Gamma(-i\overline{\eta}+l+l'+j+3)}{\Gamma(|\mu|+1)\Gamma(l+l'+j+2)} \overline{q}^{|\mu|} \\
&\times \sum_{r=1}^{|\mu|} (|\mu|-r+1)_{r-1}\, \Gamma(-i\overline{\eta}+|\mu|-r+1) \\
&\times {}_2F_1\left(-i\overline{\eta}+|\mu|-r+1, -i\overline{\eta}+l+l'+j+3; |\mu|+1; -X^2\right) \qquad (4.33)
\end{aligned}
$$

where $X = \overline{q}/\overline{\beta}$.

4.1.2 Results and discussions

From the general formula (4.29)–(4.33), we obtain the closed form expressions of the limiting amplitude $\overline{A}_{as}$ for $1s \to 2s$ and $1s \to 2p$ transitions, which are as follows:

$$
\begin{aligned}
\overline{A}_{as}(1s \to 2s) &= -\overline{K_i}\overline{\eta} 2^{-2i\overline{\eta}+4} \frac{1}{3} \left(\frac{\sqrt{2}}{3}\right)^3 [\Gamma(-i\overline{\eta}+1)]^2 \overline{\beta}^{2i\overline{\eta}-2} \\
&\times \left\{2(i\overline{\eta}-1)(i\overline{\eta}-2) {}_2F_1\left(-i\overline{\eta}+1, -i\overline{\eta}+1; 1; -X^2\right)\right. \\
&+ 4(i\overline{\eta}-2)(-i\overline{\eta}+1)^2 X^2 {}_2F_1\left(-i\overline{\eta}+2, -i\overline{\eta}+2; 2; -X^2\right) \\
&+ \left.(-i\overline{\eta}+1)^2(-i\overline{\eta}+2)^2 X^4 {}_2F_1\left(-i\overline{\eta}+3, -i\overline{\eta}+3; 3; -X^2\right)\right\} \Big/_{\overline{\beta}=3/2} \qquad (4.34)
\end{aligned}
$$

and

$$
\begin{aligned}
\overline{A}_{as}(1s \to 2p_\pm) &= \pm \frac{i}{\sqrt{2}} \overline{K_i}\overline{\eta} \exp\left(\mp i\varphi_q\right) \left(\frac{\sqrt{2}}{3}\right)^3 2^{-2i\overline{\eta}+4} \left(\overline{\beta}\right)^{-2} \overline{q}^{2i\overline{\eta}-1} \Gamma(-i\overline{\eta}+1) \\
&\times \left\{\frac{2}{\Gamma(i\overline{\eta}+1)} - X^{-2i\overline{\eta}+2} \Gamma(-i\overline{\eta}+4)\right. \\
&\times \left. {}_2F_1\left(-i\overline{\eta}+1, -i\overline{\eta}+4; 2; -X^2\right)\right\} \Big/_{\overline{\beta}=3/2} . \qquad (4.35)
\end{aligned}
$$

These results are derived previously by Thomas and Franco [ThF76]. We have verified that the present formula reproduces the results obtained by Thomas [Tho78] for $1s \to n' = 3$ transitions. From the general formula, one can also obtain explicit forms for excited state to excited state transitions. As an illustration, we give the explicit expressions of the limiting amplitude $\overline{A}_{as}$ for the $2s \to 3s$ and $2s \to 3p$ transitions,

$$
\begin{aligned}
\overline{A}_{as}(2s \to 3s) &= -\left(\frac{2}{3}\right)^{3/2} \overline{\eta}\,\overline{K_i} \left(\frac{\overline{\beta}}{2}\right)^{2i\overline{\eta}} \frac{1}{\overline{\beta}^5} \Gamma(-i\overline{\eta}+1) \left[\left(\frac{40}{9}\frac{1}{\overline{\beta}^3} - \frac{88}{9}\frac{1}{\overline{\beta}^2} + \frac{7}{\overline{\beta}} - 2\right)\right. \\
&\times \left\{\Gamma(-i\overline{\eta}+1)\, {}_2F_1\left(-i\overline{\eta}+1, -i\overline{\eta}+1; 1; -X^2\right)\right. \\
&+ \left.\Gamma(-i\overline{\eta}+2)\, {}_2F_1\left(-i\overline{\eta}+1, -i\overline{\eta}+2; 1; -X^2\right)\right\} \\
&+ \left(\frac{20}{9}\frac{1}{\overline{\beta}^2} - \frac{44}{9}\frac{1}{\overline{\beta}} + \frac{14}{3}\right) \frac{1}{\overline{\beta}} \Gamma(-i\overline{\eta}+3)\, {}_2F_1\left(-i\overline{\eta}+1, -i\overline{\eta}+3; 1; -X^2\right) \\
&+ \left(\frac{16}{27}\frac{1}{\overline{\beta}} - \frac{88}{27}\right) \frac{1}{\overline{\beta}^2} \Gamma(-i\overline{\eta}+4)\, {}_2F_1\left(-i\overline{\eta}+1, -i\overline{\eta}+4; 1; -X^2\right)
\end{aligned}
$$

$$+ \quad \frac{16}{27}\frac{1}{\overline{\beta}^3}\Gamma(-i\overline{\eta}+5)\,{}_2F_1\left(-i\overline{\eta}+1,-i\overline{\eta}+5;1;-X^2\right)\Bigg]\Bigg/_{\overline{\beta}=5/6} \tag{4.36}$$

and

$$\begin{aligned}
\overline{A}_{as}(2s\rightarrow 3p_\pm) &= \pm i\sqrt{2}\frac{4}{81}\overline{\eta}\,\overline{K_i}\exp\left(\mp i\varphi_q\right)\cdot \\
&\times \Bigg[2^{-2i\overline{\eta}+2}\frac{\Gamma(-i\overline{\eta}+1)}{\Gamma(i\overline{\eta}+1)}\left(\frac{15}{\overline{\beta}^2}-\frac{20}{\overline{\beta}}+6\right)\frac{\overline{q}^{2i\overline{\eta}-1}}{\overline{\beta}^5} \\
&- \Gamma(-i\overline{\eta}+1)\left(\frac{\overline{\beta}}{2}\right)^{2i\overline{\eta}}\overline{q} \\
&\times \Bigg\{\frac{12}{\overline{\beta}^7}\Gamma(-i\overline{\eta}+4)\,{}_2F_1\left(-i\overline{\eta}+1,-i\overline{\eta}+4;2;-X^2\right) \\
&- \frac{40}{3}\frac{1}{\overline{\beta}^8}\Gamma(-i\overline{\eta}+5)\,{}_2F_1\left(-i\overline{\eta}+1,-i\overline{\eta}+5;2;-X^2\right) \\
&+ \frac{5}{2}\frac{1}{\overline{\beta}^9}\Gamma(-i\overline{\eta}+6)\,{}_2F_1\left(-i\overline{\eta}+1,-i\overline{\eta}+6;2;-X^2\right)\Bigg\} \\
&+ \Gamma(-i\overline{\eta}+2)\left(\frac{\overline{\beta}}{2}\right)^{2i\overline{\eta}}\overline{q} \\
&\times \Bigg\{\frac{8}{3}\frac{1}{\overline{\beta}^8}\Gamma(-i\overline{\eta}+4)\,{}_2F_1\left(-i\overline{\eta}+2,-i\overline{\eta}+4;2;-X^2\right) \\
&- \frac{3}{2}\frac{1}{\overline{\beta}^9}\Gamma(-i\overline{\eta}+5)\,{}_2F_1\left(-i\overline{\eta}+2,-i\overline{\eta}+5;2;-X^2\right)\Bigg\}\Bigg]\Bigg/_{\overline{\beta}=5/6}.
\end{aligned} \tag{4.37}$$

We have written a program for evaluating the limiting amplitude $\overline{A}_{as}(nlm\rightarrow n'l'm')$ and have performed calculations to reproduce the results obtained by Thomas [Tho78]. We have also verified that the present cross sections for the $1s\rightarrow ns$ and $1s\rightarrow np$ excitation agree with those obtained earlier by Das and Sil [DaS84b] for $Z_T=\infty$.

In Tables 4.1–4.4, we list cross sections for electron–impact excitation for some selected transitions as a function of Z_T and energy. Figs. (4.1) and (4.2) show the scaled cross sections ${Z_T}^4\sigma$ for $7s\rightarrow 8p$ and $3p\rightarrow 8p$ excitations as a function of $(2Ea_0^2/{Z_T}^2)$ for various values of Z_T. The CG cross sections for $Z_T=2$ to $Z_T=20$ are computed by using the one–dimensional integral representation of the CG amplitude (see Eqs. (3.19) and (3.20). The CG cross sections for $Z_T=\infty$ are obtained from the limiting amplitudes $\overline{A}_{as}(nlm\rightarrow n'l'm')$. It is seen that the CG results approach their limiting values as Z_T and E increase. However, how fast they tend to their limiting values is independent on the characteristics of the initial and final states.

Table 4.5 compares the collision strengths of the CG approximation with those of the Coulomb-Born approximation [BHT70] for $Z_T=\infty$. The predictions of the two models differ significantly, particularly at lower energies. This is not an unexpected result since the physical contents of these two models differ.

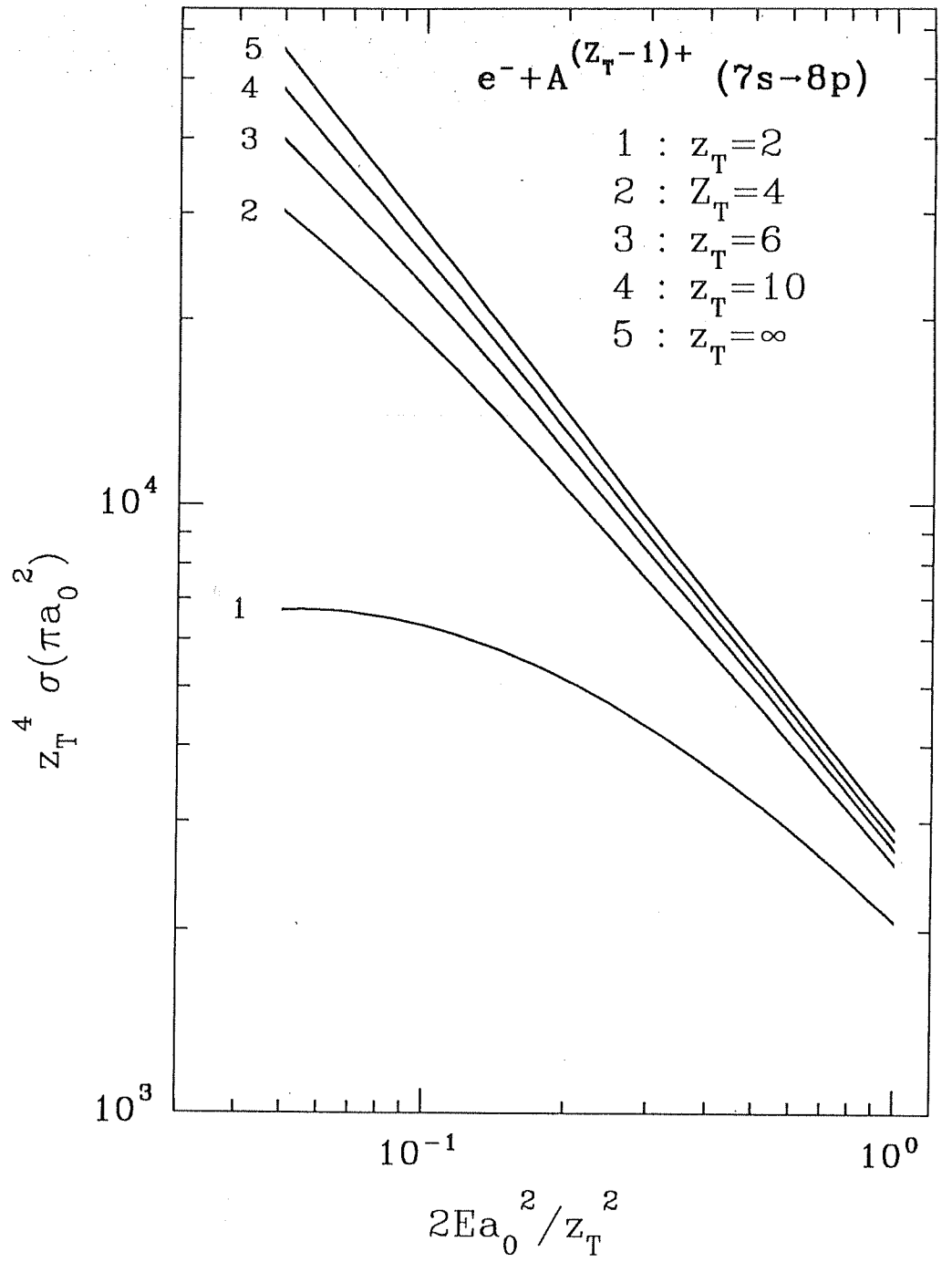


Figure 4.1: Scaled CG cross section $Z_T{}^4\sigma$ (in units of πa_0^2) for $7s \rightarrow 8p$ excitation is plotted as a function of $(2Ea_0^2/Z_T{}^2)$, for different values of Z_T.

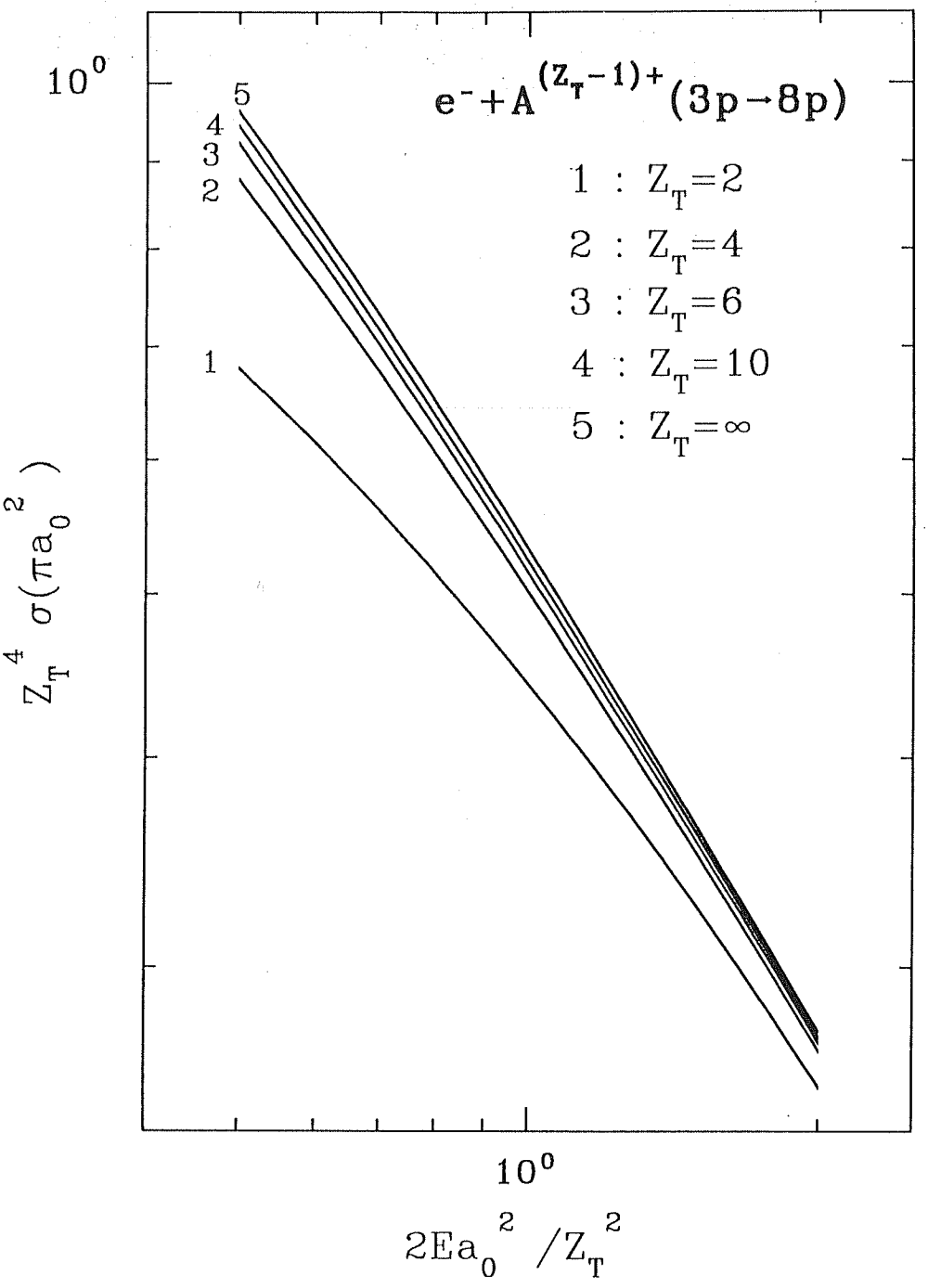


Figure 4.2: Scaled CG cross section $Z_T{}^4\sigma$ (in units of πa_0^2) for $3p \rightarrow 8p$ excitation is plotted as a function of $(2Ea_0^2/Z_T{}^2)$, for different values of Z_T.

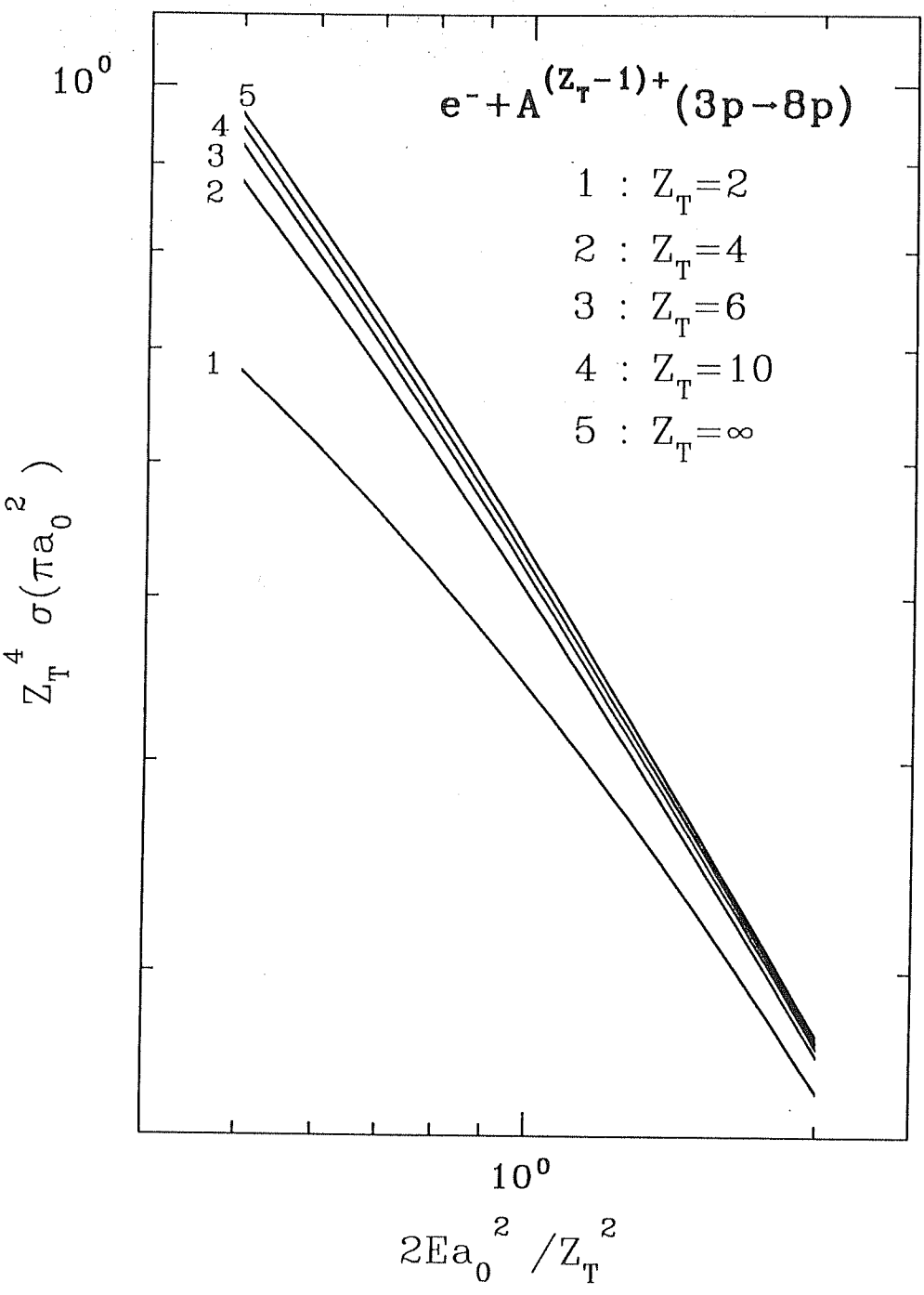


Figure 4.2: Scaled CG cross section $Z_T{}^4\sigma$ (in units of πa_0^2) for $3p \rightarrow 8p$ excitation is plotted as a function of $(2Ea_0^2/Z_T{}^2)$, for different values of Z_T.

The scaled Coulomb-Glauber cross sections $Z_T^4\sigma$ (in units of πa_0^2) for electron-impact excitations of hydrogen-like ions are listed in the following four tables (each for a transition) as a function of Z_T for various impact energies (in units of ${Z_T}^2/2a_0^2$).

Table 4.1 Transition: $6s \rightarrow 7s$

E	$Z_T = 2$	$Z_T = 4$	$Z_T = 6$	$Z_T = 10$	$Z_T = 15$	$Z_T = 20$	$Z_T = \infty$
0.075	0.277(+4)†	0.365(+4)	0.353(+4)	0.339(+4)	0.333(+4)	0.330(+4)	0.326(+4)
0.10	0.219(+4)	0.281(+4)	0.275(+4)	0.270(+4)	0.267(+4)	0.266(+4)	0.265(+4)
0.15	0.152(+4)	0.191(+4)	0.190(+4)	0.188(+4)	0.188(+4)	0.188(+4)	0.188(+4)
0.50	0.516(+3)	0.580(+3)	0.585(+3)	0.586(+3)	0.587(+3)	0.587(+3)	0.587(+3)
0.75	0.356(+3)	0.386(+3)	0.390(+3)	0.391(+3)	0.391(+3)	0.391(+3)	0.392(+3)
1.00	0.274(+3)	0.290(+3)	0.293(+3)	0.293(+3)	0.294(+3)	0.294(+3)	0.294(+3)
2.00	0.143(+3)	0.146(+3)	0.147(+3)	0.147(+3)	0.147(+3)	0.147(+3)	0.147(+3)
4.00	0.728(+2)	0.734(+2)	0.736(+2)	0.737(+2)	0.737(+2)	0.737(+2)	0.737(+2)

Table 4.2 Transition: $7s \rightarrow 8p$

E	$Z_T = 2$	$Z_T = 4$	$Z_T = 6$	$Z_T = 10$	$Z_T = 15$	$Z_T = 20$	$Z_T = \infty$
0.05	0.671(+4)	0.302(+5)	0.397(+5)	0.479(+5)	0.515(+5)	0.529(+5)	0.555(+5)
0.075	0.661(+4)	0.230(+5)	0.286(+5)	0.331(+5)	0.350(+5)	0.358(+5)	0.373(+5)
0.10	0.625(+4)	0.185(+5)	0.224(+5)	0.253(+5)	0.265(+5)	0.270(+5)	0.281(+5)
0.15	0.570(+4)	0.135(+5)	0.157(+5)	0.172(+5)	0.179(+5)	0.181(+5)	0.187(+5)
0.50	0.323(+4)	0.474(+4)	0.516(+4)	0.544(+4)	0.555(+4)	0.560(+4)	0.573(+4)
0.75	0.252(+4)	0.331(+4)	0.353(+4)	0.368(+4)	0.375(+4)	0.378(+4)	0.386(+4)
1.00	0.207(+4)	0.257(+4)	0.271(+4)	0.281(+4)	0.285(+4)	0.287(+4)	0.293(+4)
2.00	0.126(+4)	0.140(+4)	0.145(+4)	0.148(+4)	0.150(+4)	0.150(+4)	0.152(+4)

Table 4.3 Transition: $4d \rightarrow 7f$

E	$Z_T = 2$	$Z_T = 4$	$Z_T = 6$	$Z_T = 10$	$Z_T = 15$	$Z_T = 20$	$Z_T = \infty$
0.40	0.407(+2)	0.666(+2)	0.741(+2)	0.788(+2)	0.806(+2)	0.814(+2)	0.832(+2)
0.45	0.385(+2)	0.601(+2)	0.663(+2)	0.703(+2)	0.719(+2)	0.725(+2)	0.742(+2)
0.50	0.366(+2)	0.548(+2)	0.601(+2)	0.635(+2)	0.648(+2)	0.654(+2)	0.669(+2)
0.75	0.291(+2)	0.385(+2)	0.411(+2)	0.430(+2)	0.437(+2)	0.441(+2)	0.450(+2)
1.00	0.241(+2)	0.299(+2)	0.315(+2)	0.327(+2)	0.332(+2)	0.334(+2)	0.341(+2)
2.00	0.147(+2)	0.163(+2)	0.168(+2)	0.172(+2)	0.174(+2)	0.175(+2)	0.177(+2)
4.00	0.855(+1)	0.901(+1)	0.916(+1)	0.927(+1)	0.932(+1)	0.935(+1)	0.942(+1)
6.00	0.617(+1)	0.638(+1)	0.645(+1)	0.650(+1)	0.653(+1)	0.654(+2)	0.658(+1)

†The number inside the parenthesis denotes the power of ten by which the entry is to be multiplied

Table 4.4 Transition: $3p \rightarrow 8p$

E	$Z_T = 2$	$Z_T = 4$	$Z_T = 6$	$Z_T = 10$	$Z_T = 15$	$Z_T = 20$	$Z_T = \infty$
0.50	0.679(+0)	0.878(+0)	0.920(+0)	0.942(+0)	0.950(+0)	0.953(+0)	0.961(+0)
0.75	0.536(+0)	0.638(+0)	0.661(+0)	0.674(+0)	0.679(+0)	0.681(+0)	0.688(+0)
1.00	0.442(+0)	0.501(+0)	0.514(+0)	0.522(+0)	0.525(+0)	0.527(+0)	0.531(+0)
1.50	0.325(+0)	0.349(+0)	0.355(+0)	0.359(+0)	0.361(+0)	0.361(+0)	0.363(+0)
2.00	0.255(+0)	0.268(+0)	0.271(+0)	0.273(+0)	0.274(+0)	0.274(+0)	0.275(+0)
3.00	0.178(+0)	0.182(+0)	0.184(+0)	0.184(+0)	0.185(+0)	0.185(+0)	0.185(+0)
4.00	0.136(+0)	0.138(+0)	0.139(+0)	0.139(+0)	0.139(+0)	0.139(+0)	0.139(+0)
6.00	0.920(-1)	0.928(-1)	0.930(-1)	0.932(-1)	0.933(-1)	0.933(-1)	0.933(-1)

Table 4.5 In the case of transitions: $1s \rightarrow 2s$ and $1s \rightarrow 2p$, scaled collision strength $(2l+1)EZ_T^4\sigma$ (where the cross section σ is in units of πa_0^2) for $Z_T = \infty$, estimated in the Coulomb-Glauber (CG) approximation, is compared with the predictions of Coulomb-Born (CB) by Burgess et al. [BHT70] for different values of impact energy E (in units of ${Z_T}^2/2a_0^2$).

E	$1s \rightarrow 2s$		$1s \rightarrow 2p$	
	CG	CB	CG	CB
0.80	0.545(-1)	0.436(+0)	0.126(+1)	0.179(+1)
0.90	0.104(+0)	0.440(+0)	0.202(+1)	0.193(+1)
1.00	0.142(+0)	0.443(+0)	0.246(+1)	0.206(+1)
1.50	0.264(+0)	0.451(+0)	0.345(+1)	0.262(+1)
2.00	0.324(+0)	0.454(+0)	0.394(+1)	0.307(+1)
3.00	0.377(+0)	0.455(+0)	0.457(+1)	0.378(+1)
4.00	0.399(+0)	0.454(+0)	0.503(+1)	0.433(+1)

4.2 Small– and large–q behavior of the Coulomb–Glauber amplitude

Before examining the behavior of the CG amplitude for small and large momentum transfer, we discuss briefly the dependence of the form factor and hence the first Born amplitude on q.

When q is small, we write the form factor, by employing the representation (4.16) for the quantity $[g_{fi}(q) - \delta_{fi}]$, to obtain:

$$G_{fi}(\mathbf{q}) - \delta_{fi} = N_0 \sum_{\lambda=l_{min}}^{l+l'} A_\lambda \sum_{j=0}^{n+n'-l-l'-2} B_{\lambda,j} \frac{q^\lambda}{(q^2+\beta^2)^{l+l'+j+2}} \mathcal{G}_{\lambda,j}(qx) \tag{4.38}$$

where

$$\mathcal{G}_{\lambda,j}(qx) = \sum_{s=\delta_{\lambda,0}}^{t} f_{\lambda,j,s} q^{2s} - \delta_{\lambda,0} \sum_{s=1}^{l+l'+j+2} g_{j,s} q^{2s}. \tag{4.39}$$

Here N_0, A_λ and $B_{\lambda,j}$ are given by Eqs. (3.35), (3.36) and (3.37) respectively; and

$$f_{\lambda,j,s} = (-)^s \frac{\left(\frac{\lambda-l-l'-j-1}{2}\right)_s \left(\frac{\lambda-l-l'-j}{2}\right)_s}{\left(\lambda+\frac{3}{2}\right)_s s!} \beta^{-2s} \tag{4.40}$$

and

$$g_{j,s} = {}^{l+l'+j+2}C_s \, \beta^{-2s} \tag{4.41}$$

in which $\beta = \beta_i + \beta_f = Z_T/n + Z_T/n'$. Since we are to study the behavior of the Born amplitude at $q \to 0$, we consider the term corresponds to $\lambda = l_{min}$ in (4.38); also, for the same reason, we retain the lowest–order terms from both the series in Eq. (4.39). Thus we have for small q

$$G_{fi}(\mathbf{q}) - \delta_{fi} \simeq N_0 A_{l_{min}} \sum_{j=0}^{n+n'-l-l'-2} B_{l_{min},j} \frac{q^{l_{min}}}{(q^2+\beta^2)^{l+l'+j+2}} \mathcal{G}^{(0)}_{l_{min},j}(q) \tag{4.42}$$

where

$$\mathcal{G}^{(0)}_{l_{min},j}(q) = f_{l_{min},j,\delta_{l_{min},0}} \left(q^2\right)^{\delta_{l_{min},0}} - \delta_{l_{min},0} \, g_{j,1} \, q^2. \tag{4.43}$$

To investigate the large–q behavior of the form factor, we begin by using the formula 15.3.5 on page 559 of [AbS70] to rewrite the form factor (3.34) as,

$$\begin{aligned} G_{fi}(\mathbf{q}) &= N_0 \sum_{\lambda=l_{min}}^{l+l'} A_\lambda \, q^\lambda \sum_{j=0}^{n+n'-l-l'-2} B_{\lambda,j} \frac{\beta^{\lambda-l-l'-j}}{\left(\sqrt{q^2+\beta^2}\right)^{\lambda+l+l'+j+4}} \\ &\times {}_2F_1\left(\frac{\lambda-l-l'-j}{2}, \frac{\lambda+l+l'+j+4}{2}; \lambda+\frac{3}{2}; \frac{q^2}{q^2+\beta^2}\right). \end{aligned} \tag{4.44}$$

Now, for q large enough in compare to β, we can approximate the expression (4.44) to obtain

$$\begin{aligned} G_{fi}(\mathbf{q}) &\simeq N_0 \sum_{\lambda=l_{min}}^{l+l'} A_\lambda \sum_{j=0}^{n+n'-l-l'-2} B_{\lambda,j} \beta^{\lambda-l-l'-j} \\ &\times {}_2F_1\left(\frac{\lambda-l-l'-j}{2}, \frac{\lambda+l+l'+j+4}{2}; \lambda+\frac{3}{2}; 1\right) \left(\frac{1}{q}\right)^{l+l'+j+4}. \end{aligned} \tag{4.45}$$

Expression (4.45) is a polynomial of increasing power of $(1/q)$. Obviously, for large q we may further approximate G_{fi} by retaining only the leading terms (i.e. the terms correspond to $j = 0$ in Eq. (4.45)). Thus for large momentum transfer q we finally have

$$G_{fi}(\mathbf{q}) \simeq D_1 \left(\frac{1}{q}\right)^{l+l'+4} \tag{4.46}$$

where

$$\begin{aligned} D_1 &= N_0 \sum_{\lambda=l_{min}}^{l+l'} A_\lambda \Gamma(\lambda + l + l' + 3) \\ &\times {}_2F_1\left(\frac{\lambda - l - l'}{2}, \frac{\lambda + l + l' + 4}{2}; \lambda + \frac{3}{2}; 1\right) \end{aligned} \tag{4.47}$$

in which we have used the explicit expression of $B_{\lambda,0}$ obtained from Eq. (3.37).

Therefore, using the result (4.42) in Eq. (2.33) (with $N = 1$), we find that for small q

$$A_{\rm FBA}(i \to f) \simeq 2Z_P M N_0 A_{l_{min}} \sum_{j=0}^{n+n'-l-l'-2} B_{l_{min},j} \frac{q^{l_{min}-2}}{(q^2+\beta^2)^{l+l'+j+2}} \mathcal{G}^{(0)}_{l_{min},j}(q). \tag{4.48}$$

Similarly from (2.33) (with $N = 1$) and (4.46) the approximated form of the first Born amplitude for large–q is

$$A_{\rm FBA}(i \to f) \simeq \frac{2Z_P M}{q^2}\left[-\delta_{fi} + D_1 \left(\frac{1}{q}\right)^{l+l'+4}\right]. \tag{4.49}$$

Obviously in the case of elastic scattering ($i = f$) the first Born amplitude shows a constant q^{-2} behavior for large q.

In the following, we derive the behaviors of the CG amplitude for small and large momentum transfer.

The behavior of the CG amplitude in the limit $q \to 0$

Using the formula 15.3.3 on page 559 of [AbS70] to transform the hypergeometric function ${}_2F_1$ in Eq. (3.20) we rewrite the equation as

$$\begin{aligned} I_{fi} &= \int_0^\infty dx\, x^{-2i\eta+|\mu|-1} \frac{|1-x|^{2i\eta+2i\alpha-1}}{(1+x)^{2|\mu|+1}} [g_{fi}(qx) - \delta_{fi}] \\ &\times {}_2F_1\left(|\mu| + i\eta + i\alpha, |\mu| + \frac{1}{2}; 2|\mu| + 1; \frac{4x}{(1+x)^2}\right). \end{aligned} \tag{4.50}$$

For small–q we use the approximated form (4.42) of the form factor in Eq. (4.50). At this point it may be convenient to consider two cases (i) $l_{min} \neq 0$ and (ii) $l_{min} = 0$ separately:

$$\begin{aligned} I_{fi}^{l_{min}\neq 0} &= N_0 A'_{l_{min}} q^{l_{min}} \sum_{j=0}^{n+n'-l-l'-2} B_{l_{min},j}\, f_{l_{min},j,0} \\ &\times \int_0^\infty dx x^{-2i\eta+l_{min}+|\mu|-1} \frac{|1-x|^{2i\alpha+2i\eta-1}}{(1+x)^{2|\mu|+1}(q^2x^2+\beta^2)^{l+l'+j+2}} \\ &\times {}_2F_1\left(|\mu| + i\eta + i\alpha, |\mu| + \frac{1}{2}; 2|\mu| + 1; \frac{4x}{(1+x)^2}\right) \end{aligned} \tag{4.51}$$

and

$$I_{fi}^{l_{min}=0} = N_0 A_0' q^2 \sum_{j=0}^{n+n'-l-l'-2} B_{0,j}\,(f_{0,j,1} - g_{1,1}) \times \int_0^\infty dx x^{-2i\eta+|\mu|+1} \frac{|1-x|^{2i\alpha+2i\eta-1}}{(1+x)^{2|\mu|+1}\,(q^2x^2+\beta^2)^{l+l'+j+2}} \times {}_2F_1\left(|\mu|+i\eta+i\alpha, |\mu|+\frac{1}{2}; 2|\mu|+1; \frac{4x}{(1+x)^2}\right). \quad (4.52)$$

Let us first concentrate on the x–integral in the Eq. (4.51), namely,

$$F = \int_0^\infty dx x^{-2i\eta+l_{min}+|\mu|-1} \frac{|1-x|^{2i\alpha+2i\eta-1}}{(1+x)^{2|\mu|+1}\,(q^2x^2+\beta^2)^{l+l'+j+2}} \times {}_2F_1\left(|\mu|+i\eta+i\alpha, |\mu|+\frac{1}{2}; 2|\mu|+1; \frac{4x}{(1+x)^2}\right). \quad (4.53)$$

We divide the range of integration into two parts as

$$F \equiv F_1 + F_2 = \int_0^h dx[\cdots] + \int_h^\infty dx[\cdots] \quad (4.54)$$

where h is any positive finite quantity. For convenience we choose $h = 1$. It is seen that F_1 is finite $q = 0$. However, if we put $q = 0$ in F_2, the integrand behaves as $x^{2i\alpha+l_{min}-|\mu|-3}$ for large x. This causes F_2 to diverge at $q = 0$ for $l_{min} \geq |\mu| + 3$. In contrast, the integral F is well–defined. Therefore, to examine the q–dependence of F_2 we substitute $x = 1 + t/q$ and obtain

$$F_2 = q^{-2i\alpha-l_{min}+|\mu|+2} \int_0^\infty dt \frac{t^{2i\eta+2i\alpha-1}}{[(t+q)^2+\beta^2]^{l+l'+j+2}} \times \frac{(t+q)^{-2i\eta+l_{min}+|\mu|-1}}{(t+2q)^{2|\mu|+1}} {}_2F_1\left(|\mu|+i\eta+i\alpha, |\mu|+\frac{1}{2}; 2|\mu|+1; \frac{4q(t+q)}{(t+2q)^2}\right). \quad (4.55)$$

Next, we expand all quantities, except $(t+2q)^{-2|\mu|-1}$, of the integrand in powers of q and retain only the leading terms. Thus, for small q we obtain the well–defined integral

$$F_2 \simeq q^{-2i\alpha-l_{min}+|\mu|+2} \int_0^\infty dt \frac{t^{2i\alpha+l_{min}+|\mu|-2}}{(t+2q)^{2|\mu|+1}\,(t^2+\beta^2)^{l+l'+j+2}}. \quad (4.56)$$

We have evaluated this integral by the method of contour integration. The leading terms of this integral are given by obtain

$$F_2 \simeq \frac{2\pi i\, q^{-2i\alpha-l_{min}+|\mu|+2}}{1-\exp[2\pi i(2i\alpha+l_{min}+|\mu|-2)]} \times \left[a\frac{\beta^{-2(l+l'+j+2)}}{(2|\mu|)!} q^{2i\alpha+l_{min}-|\mu|-2} + f_1(i\beta) + f_2(-i\beta)\right] \quad (4.57)$$

where

$$a = (2i\alpha + l_{min} - |\mu| - 1)_{2|\mu|}\,(-2)^{2i\alpha+l_{min}-|\mu|-2}, \quad (4.58)$$

and $f_1(i\beta)$ and $f_2(-i\beta)$ are the residues of the poles $x = \pm i\beta$ of order $l + l' + j + 2$ respectively. Thus from the Eqs. (3.20), (4.51), (4.53), (4.54) and (4.57), the CG amplitude for small–q is given by,

$$A(nlm \to n'l'm') \simeq d_1\, q^{2i\alpha + l_{min} - 2} + d_2\, q^{|\mu|} \tag{4.59}$$

where the quantities d_1 and d_2 are independent of q. We thus see that the behavior of the CG amplitude at small q depends on the relative values of l_{min} and $|\mu|$,

(i) If $l_{min} < |\mu| + 2$, then

$$A(nlm \to n'l'm') \propto q^{2i\alpha + l_{min} - 2}, \tag{4.60}$$

(ii) if $l_{min} = |\mu| + 2$, then

$$A(nlm \to n'l'm') \propto q^{|\mu|} F(q) \tag{4.61}$$

where $F(q) = d_1 q^{2i\alpha} + d_2$ is bounded but does not approach a well defined limit as $q \to 0$, and

(iii) if $l_{min} > |\mu| + 2$, then

$$A(nlm \to n'l'm') \propto q^{|\mu|}. \tag{4.62}$$

Now in the remaining case i.e. when $l_{min} = 0$, we consider the integral in Eq. (4.52) and following the similar analysis as above we can verify that as $q \to 0$

$$A(nlm \to n'l'm') \propto f(q) \tag{4.63}$$

where $f(q)$ is also a bounded function of q but does not tend to a definite limit as $q \to 0$.

The behavior of the CG amplitude in the limit $q \to \infty$

In order to investigate the large–q behavior of the CG amplitude we begin with Eq. (4.50) and use the substitution $qx = y$ to obtain

$$\begin{aligned} I_{fi} &= q^{-2i\alpha + |\mu| + 2} \int_0^\infty dy\, y^{-2i\eta + |\mu| - 1} \frac{|q - y|^{2i\eta + 2i\alpha - 1}}{(q + y)^{2|\mu| + 1}} \\ &\quad \times [g_{fi}(y) - \delta_{fi}]\, {}_2F_1\left(|\mu| + i\eta + i\alpha,\, |\mu| + \frac{1}{2};\, 2|\mu| + 1;\, \frac{4yq}{(q+y)^2}\right). \end{aligned} \tag{4.64}$$

With q being very large we may approximate I_{fi} as

$$I_{fi} \simeq q^{2i\eta - |\mu|} W \tag{4.65}$$

where

$$W = \int_0^\infty dy\, y^{-2i\eta + |\mu| - 1} [g_{fi}(y) - \delta_{fi}]. \tag{4.66}$$

Therefore, from Eqs. (3.20) and (4.65) we find that for $q \to \infty$

$$A(nlm \to n'l'm') \;=\; d\, q^{2i\eta+2i\alpha-|\mu|-2} \tag{4.67}$$

where

$$d \;=\; -\frac{\eta K_i c_0 q^{2i\alpha-2}\Gamma(|\mu| - i\eta - i\alpha + 1)}{2^{2i\eta+2i\alpha-2}\Gamma(|\mu|+1)\Gamma(i\eta + i\alpha)} \exp(-i\mu\varphi_q)\, W. \tag{4.68}$$

The large– and small–q behaviors of the CG amplitude derived by Thomas and Franco [ThF76] in the case of $1s \to nlm$ transitions are the special cases of our general results.

Chapter 5

Calculations of the Coulomb–Glauber Rate Coefficients

One of the underlying assumptions of the Glauber and the Coulomb–Glauber approximations (Section 2.4) is that all important atomic states have the same energy. Therefore (as we have also discussed in the same section), because of a large number of atomic levels having approximately the same electronic energy, the Glauber and the CG approximations can be better justified for excited state to excited state transitions. This has prompted us to use these approximations to calculate the rate coefficients for a wide selection of the target states.

In this chapter we present our rate coefficient results. Before going to discuss these results we define the rate coefficient in the following section.

5.1 Rate coefficient

Let us consider two atomic states i and f characterized by their principal, orbital and magnetic quantum numbers as, $i \equiv (n, l, m)$ and $f \equiv (n', l', m')$. The collision strength $\Omega(i \rightarrow f)$ for excitation from i to f is related to the cross section $\sigma(i \rightarrow f)$ by

$$\sigma(i \rightarrow f) = \frac{\Omega(i \rightarrow f)}{E_i w_i} \pi a_0^2, \tag{5.1}$$

where w_i is the statistical weight of the initial state, E_i is the energy of the incident electron in Ry relative to the initial state. The rate coefficient for excitation from i to f, q_{fi} may be written as [HuS87]

$$q_{fi} = 8.629 \times 10^{-6} \frac{\Gamma(i \rightarrow f; T_e)}{w_i T_e^{1/2}} \exp\left(-\frac{\Delta_{fi}}{kT_e}\right) \text{cm}^3 \text{ s}^{-1}, \tag{5.2}$$

where $\Delta_{fi} = \epsilon_f - \epsilon_i$ is the energy of the transition and $\Gamma(i \rightarrow f; T_e)$ is an effective collision strength obtained by averaging $\Omega(i \rightarrow f)$ over a Maxwellian distribution of electron velocities,

$$\Gamma(i \rightarrow f; T_e) = \int_0^\infty \Omega(i \rightarrow f) \exp\left(-\frac{E_f}{kT_e}\right) d\left(\frac{E_f}{kT_e}\right) \tag{5.3}$$

in which E_f is the electron energy leaving the target in the final state f.

The rate coefficient for $nl \rightarrow n'l'$ transition is obtained by summing over all m' levels and averaging over all m levels:

$$q(nl \rightarrow n'l') = \frac{1}{2l+1} \sum_{mm'} q_{fi}. \tag{5.4}$$

The total rate coefficient $q(n \rightarrow n')$ for n to n' transition is written as

$$\begin{aligned} q(n \rightarrow n') &= \frac{1}{n^2} \sum_{ll'} (2l+1) q(nl \rightarrow n'l') \\ &= \frac{1}{n^2} \sum_{ll'} \sum_{mm'} q_{fi}. \end{aligned} \tag{5.5}$$

In astrophysical applications, due to nonavailability of theoretical results for excited state to excited state ($nlm \rightarrow n'l'm'$) transitions, either some ad hoc formulas (examples: [Reg62], [Joh72], [GPL76], [Kas80] etc.) or some numerical approximations [GNP87, GiP89] available in the literature have often been used to estimate the rate coefficients. We discuss below one such rate coefficient formula for ions developed by Kastner [Kas80].

The plane wave Bethe approximation for excitation of ions leads to the cross section [Gai66]

$$\sigma(i \rightarrow f) = \frac{F(i \rightarrow f)}{(\Delta_{fi})^2} \sigma_0 \tag{5.6}$$

where the transition oscillator strength $F(i \rightarrow f)$ for an N–electron atom can be given by,

$$F(i \rightarrow f) = \frac{2\Delta_{fi}}{q^2} \left| \left\langle \phi_f(\zeta) \left| \sum_{I=1}^{N} \exp(i\mathbf{q} \cdot \mathbf{r}_I \right| \phi_i(\zeta) \right\rangle \right|^2 \tag{5.7}$$

and

$$\sigma_0 = \frac{2\Delta_{fi}}{E} \ln \left[\frac{E^{1/2} + (E - \Delta_{fi})^{1/2}}{E^{1/2} - (E - \Delta_{fi})^{1/2}} \right], \tag{5.8}$$

with E and Δ_{fi} in atomic units. Gailitis [Gai66] improved this approximation by including Coulomb wave functions, showing that cross section were then non–zero at threshold as they should be for excitation of positive ions. Kastner [Kas80] has shown that this improved Bethe–Coulomb formula is reasonably well–simulated by using the following simple form of σ_0:

$$\sigma_0 = \frac{2}{x} \ln(4x - 1) \tag{5.9}$$

where $x = E/\Delta_{fi}$. The corresponding electron excitation rate coefficient is given by Kastner as

$$q_{fi} = \frac{2.58 \times 10^{-8}}{(\Delta)^2} F_{fi} y T_e^{1/2} \exp(-y/4) \left[1.099 \exp(-3y/4) + E_i\left(\frac{3y}{4}\right) \right] cm^3\, s^{-1}, \tag{5.10}$$

in which $\Delta = 27.21\Delta_{fi}$ is the transition energy in eV, $y = 1.16 \times 10^4 \Delta / T_e$ and $E_i(z)$ is the exponential integral function (see [AbS70]).

For the transition $n \rightarrow n'$ of the hydrogenic system Kastner has used the formula [All58]

$$F(n \rightarrow n') = \frac{3.92}{2(n')^2} \left[\frac{1}{(n')^2} - \frac{1}{n^2} \right]^{-3} \left(\frac{1}{nn'} \right)^3 \tag{5.11}$$

for the oscillator strength.

5.2 Results and discussions

We have computed [CSD93, CDS94] the rate coefficients for a large number of transitions over a wide selection of electron temperature for HI, HeII and CVI using the Glauber and the Coulomb–Glauber cross sections. Our calculation of rate coefficient $q(nl \rightarrow n'l')$, Eq. (5.4), requires a sum over all final magnetic states m' and an averaging over the initial magnetic state m. Here, of course, we omit the transitions which are forbidden by the Glauber selection rule (see Chapter 3) i.e. for the cases when $l + l' + m + m'$ is odd the Glauber amplitude vanishes. Again for a fixed n and n', $(n \times n')$ number of such rate coefficients $q(nl \rightarrow n'l')$ had to be calculated.

We have considered all inelastic state to state $nlm \rightarrow n'l'm'$ transitions for which $n, n' \leq 8$. We have selected 30 electron temperature values in the ranges as follows: (1) for HI from $10^{3.4}$K to $10^{7.0}$K, (2) for HeII from $10^{3.6}$K to $10^{8.0}$K and (3) for CVI from $10^{4.2}$K to $10^{10.0}$K. Therefore, for one hydrogenic system we have computed rates for 546 transitions $(nl \rightarrow n'l')$ at 30 temperatures. The work needed a considerable amount of computational management. We have made use of our computer code (as described in Chapter 3) to calculate the integrated cross section $\sigma(nlm \rightarrow n'l'm')$ needed to carry out the integral (5.3) numerically. In the following three subsections we discuss our results for the targets HI, HeII and CVI.

5.2.1 HI

In Figs. 5.1A, 5.1B and 5.1C, we plot the total rate coefficients (5.5) versus electron temperature calculated via the present Glauber approximation method (solid curves), the R–matrix calculations [ABB91] (long–short dashed curves) and the first Born approximation (dashed curves) for HI. The transitions considered are $n = 2 \rightarrow n' = 3, 4, 5$.

The R–matrix calculations shown in the figures are based on 15 state expansion. It is known that this method provides resonance structure in the low energy region and, therefore, is preferable for use in this region. However, for excited state to excited state transitions due to the strong coupling of many atomic levels of approximately the same energy, 15 basis state may not be sufficient to provide good estimation. The RM rate coefficients are reported upto the electron energy 50,000 K ($\sim$ 5 eV). For $n = 2 \rightarrow n' = 3$ transition the RM rates show good agreement with the first Born rates in all temperatures plotted which is not the case for the other transitions. The CG approximation are expected to be better suited for excited state to excited state transitions than the ground state excitations (see Section 2.4). It is seen from the figures that the Glauber results are lower than both the RM and FBA results for all the cases plotted. As expected, the Glauber and FBA rates become indistinguishable at the higher temperatures. We have found that at temperatures T_e approximately $\geq 10^{6.5}$K Glauber and first Born data become almost equal.

5.2.2 HeII

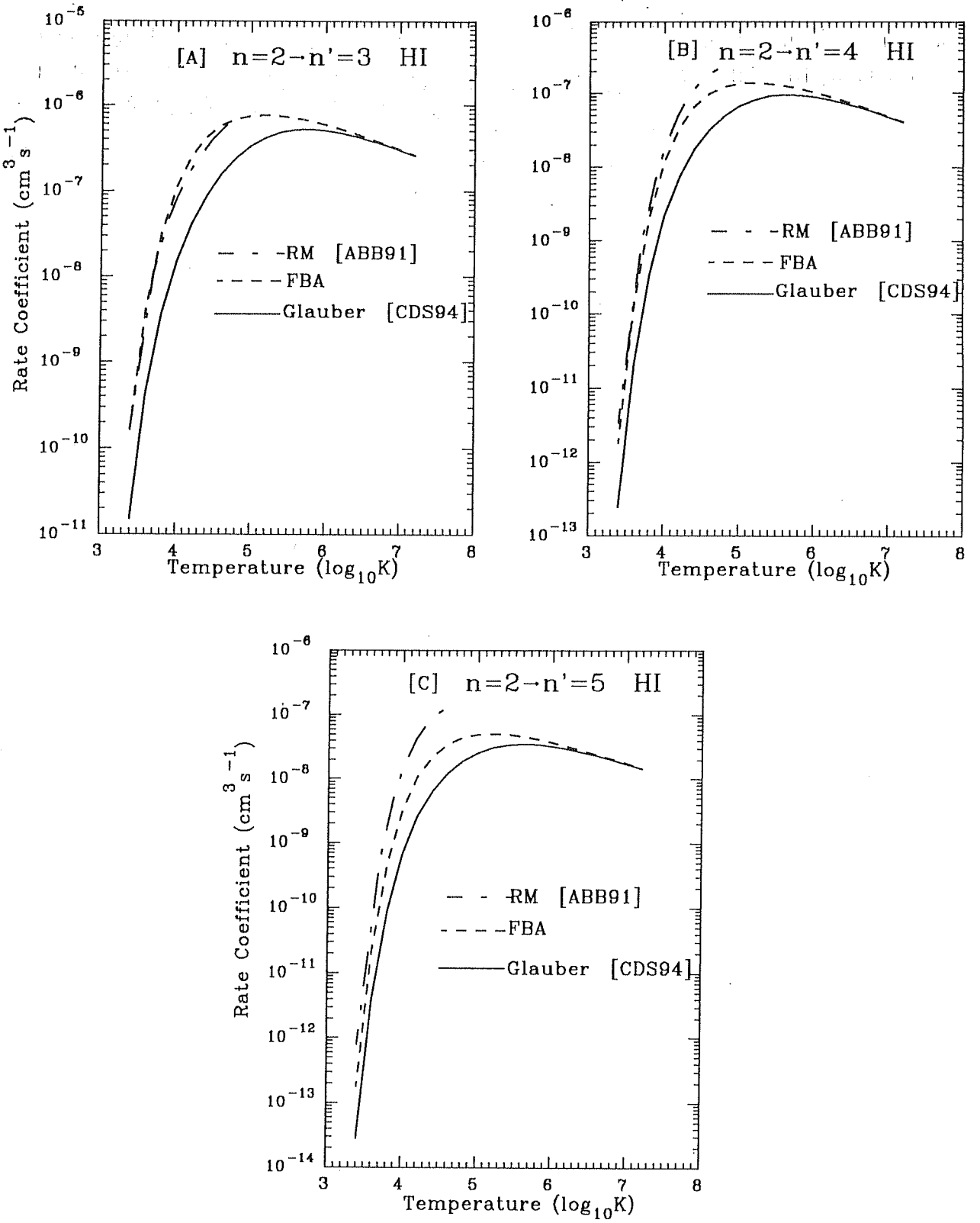


Figure 5.1: Rate coefficients (in units of $\mathrm{cm}^3\mathrm{s}^{-1}$) versus electron temperature (in units of $\log_{10} K$) for HI. Transitions: $n = 2 \rightarrow n' = 3, 4, 5$. Present results: solid curves; R–matrix results: long–short dashed curves; first Born results: dashed curves.

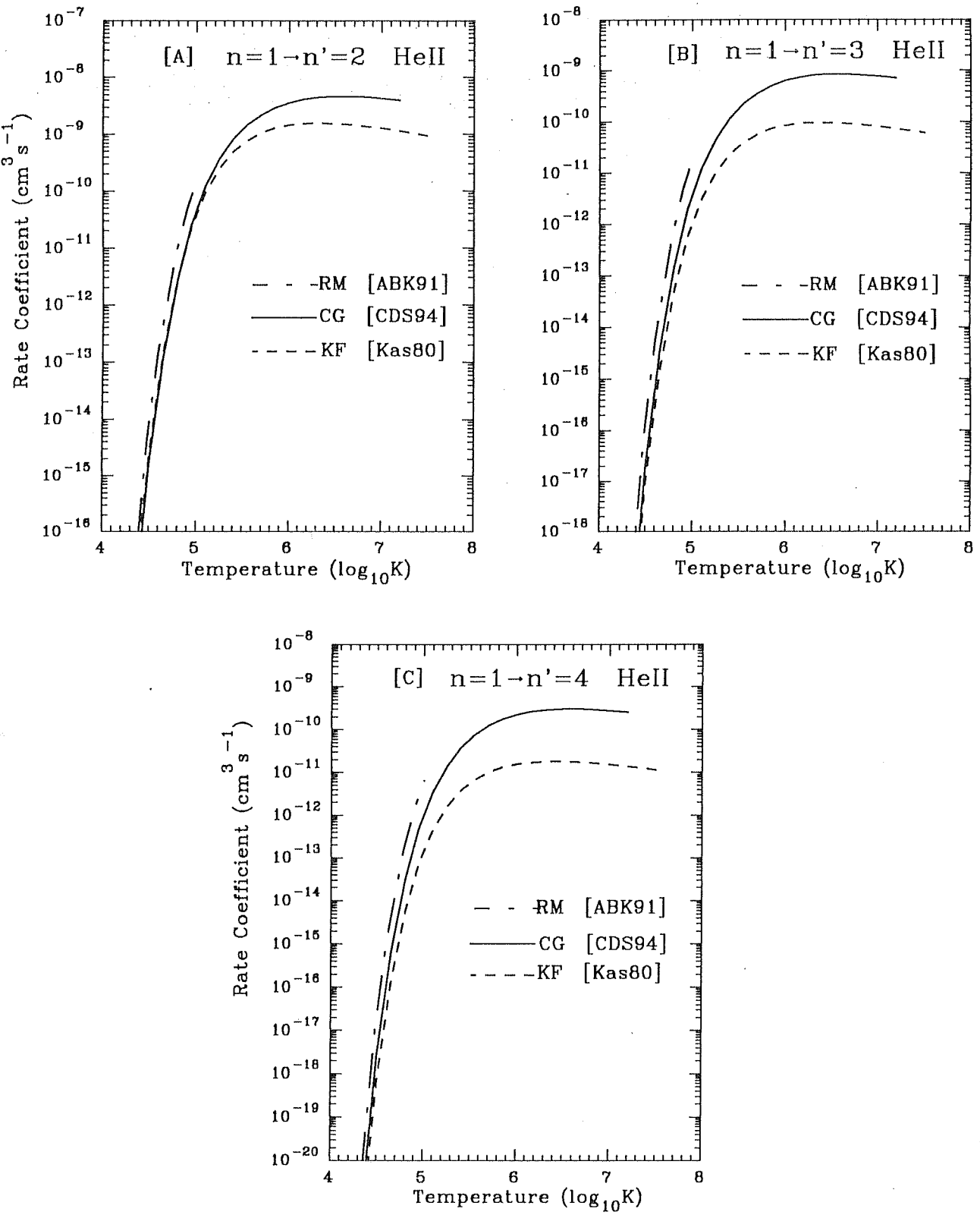


Figure 5.2: Electron impact excitation rate coefficients (in units of cm^3s^{-1}) versus electron temperature (in units of $\log_{10} K$) are plotted for HeII to compare present results (solid curves) with R-matrix data (long–short dashed curves) and the data from Kastner's formula (dashed curves) for all transitions between $n = 1 \rightarrow n' = 2, 3, 4$.

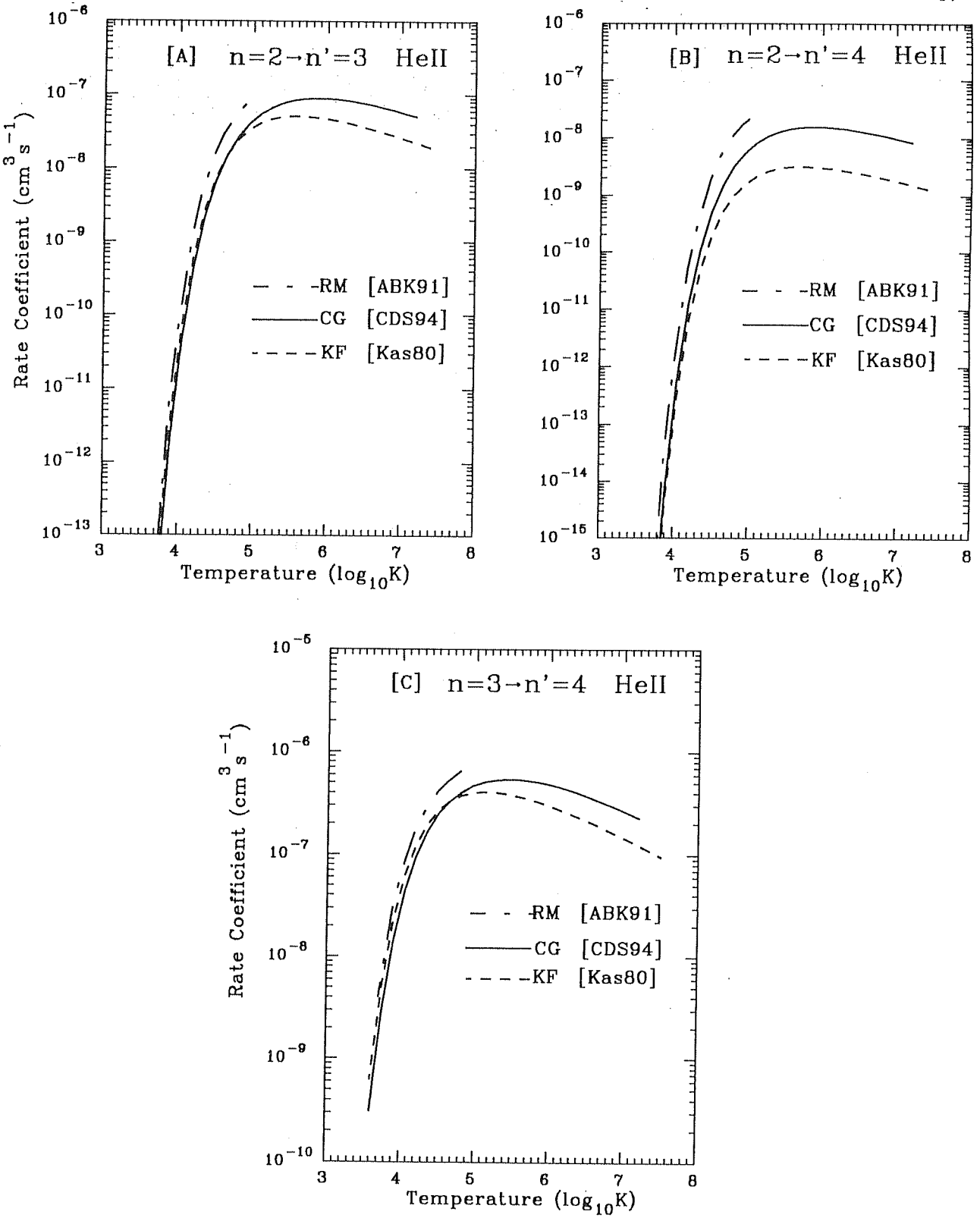


Figure 5.3: Rate coefficients versus electron temperature for HeII. Transitions: $n = 2 \rightarrow n' = 3, 4$ and $n = 3 \rightarrow n' = 4$.

To show how the present Coulomb–Glauber transition rate coefficients compare with the R–matrix calculations of Aggarwal *et al.* [ABK91] we give some figures (Figs. 5.2A, 5.2B and 5.2C, and Figs. 5.3A, 5.3B and 5.3C) in the case of electron impact excitations of HeII for all transitions between $n = 1, 2, 3$ and 4. The solid curves denote the present results. The long–short dashed curves are the rates obtained by the R-matrix method [ABK91]. We also plot as dashed curves the corresponding estimations (KF) by the formula (5.10) given by Kastner [Kas86].

The R–matrix rate coefficients are calculated upto $10^{5.0}$K considering a fifteen state expansion. At low energies where there are resonances, the R–matrix data are expected to give better estimation. However, in this method, for the excited state to excited state transitions, a 15 state expansion may not be sufficient to provide good result. On the other hand, (see Section 2.4) the CG approximation are expected to be better suited for the excited state to excited state transitions than the ground state excitations. We see that all three results show qualitatively the same shape and have maxima at similar temperatures. It is interesting to note that for the excitation $n = 3 \rightarrow n' = 4$ (Fig. 5.3C) in the range approximately $T_e \geq 10^{4.8}$K, the CG and RM results differ by less than 30%.

For all transitions plotted, the Kastner's data [Kas80] for the rate coefficients are considerably lower than CG rate coefficients over a large range of energy suggesting that the CG results are preferable for use in the astrophysical modeling.

We tabulate a complete set of rates $q(nl \rightarrow n'l')$ (Eq. (5.4)) for HeII ion (Table 5.1) for all $nl \rightarrow n'l'$ transitions for which $n, n' \leq 8$ at fifteen electron temperatures.

5.2.3 CVI

In some more figures (Figs. 5.4A, 5.4B, 5.4C and 5.4D, Figs. 5.5A, 5.5B and 5.5C, and Figs. 5.6A, 5.6B and 5.6C) the present rates are compared with the R–matrix [AgK91a] rates and the rates obtained from the Kastner's formula (5.10) for all transitions between $n = 1, 2, 3, 4$, and 5 of CVI. As earlier the CG rates are presented by the solid curves; the long–short dashed curves represent the rates obtained in the RM method [AgK91a]; and the dashed curves denote the corresponding estimations by the formula (5.10) given by Kastner [Kas86].

The R–matrix rate coefficients for CVI are reported for upto $T_e = 10^{6.2}$K ($\sim 10^{2.2}$ eV). We have discussed previously that the quality of the Glauber results for the excited state transitions improves in compare to that for ground state excitations. Here also, as earlier, all the three results have the similar shape and have maxima at similar temperatures. It may be noted that for the excitation $n = 4 \rightarrow n' = 5$ (Fig. 5.6C) in the range approximately $T_e \geq 10^5.0$K and also for $n = 3 \rightarrow n' = 4$ (Fig. 5.6A) transition in the range $T_e \geq 10^{5.5}$K the CG and RM results agree within 20%. Again for all transitions shown in the figures, the Kastner's rates [Kas80] are much lower than the CG rates over a wide range of energy.

We also list a complete set of electron impact rate coefficients for CVI ion (Table 5.2). Rates for all inelastic transitions $nl \rightarrow n'l'$ are given for $n, n' \leq 8$ at fifteen electron temperatures.

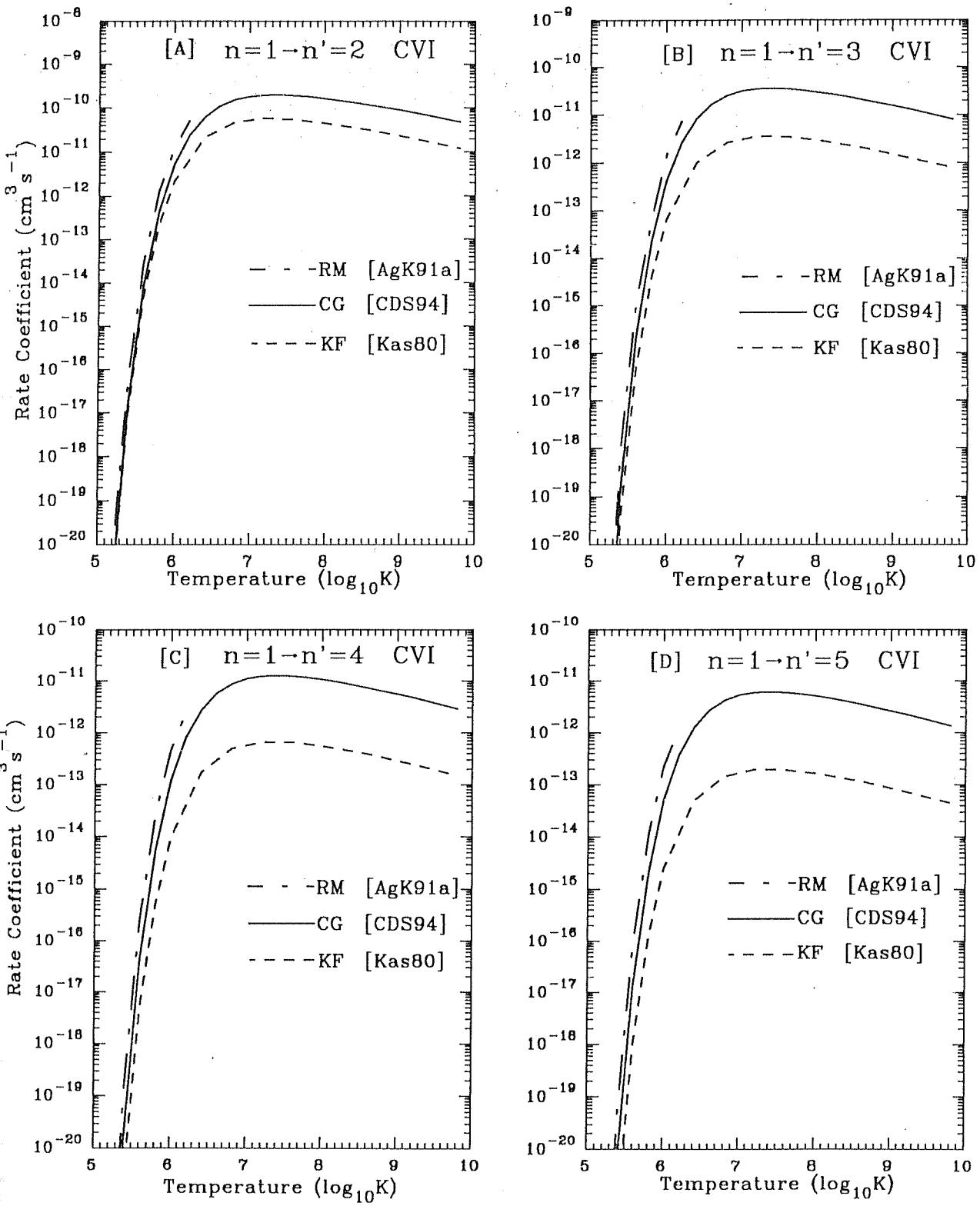


Figure 5.4: Electron impact excitation rate coefficients (in units of cm^3s^{-1}) versus electron temperature (in units of $\log_{10} K$) are plotted for CVI to compare present results (solid curves) with R-matrix data (long-short dashed curves) and the data from Kastner's formula (dashed curves) for all transitions between $n = 1 \rightarrow n' = 2, 3, 4, 5$.

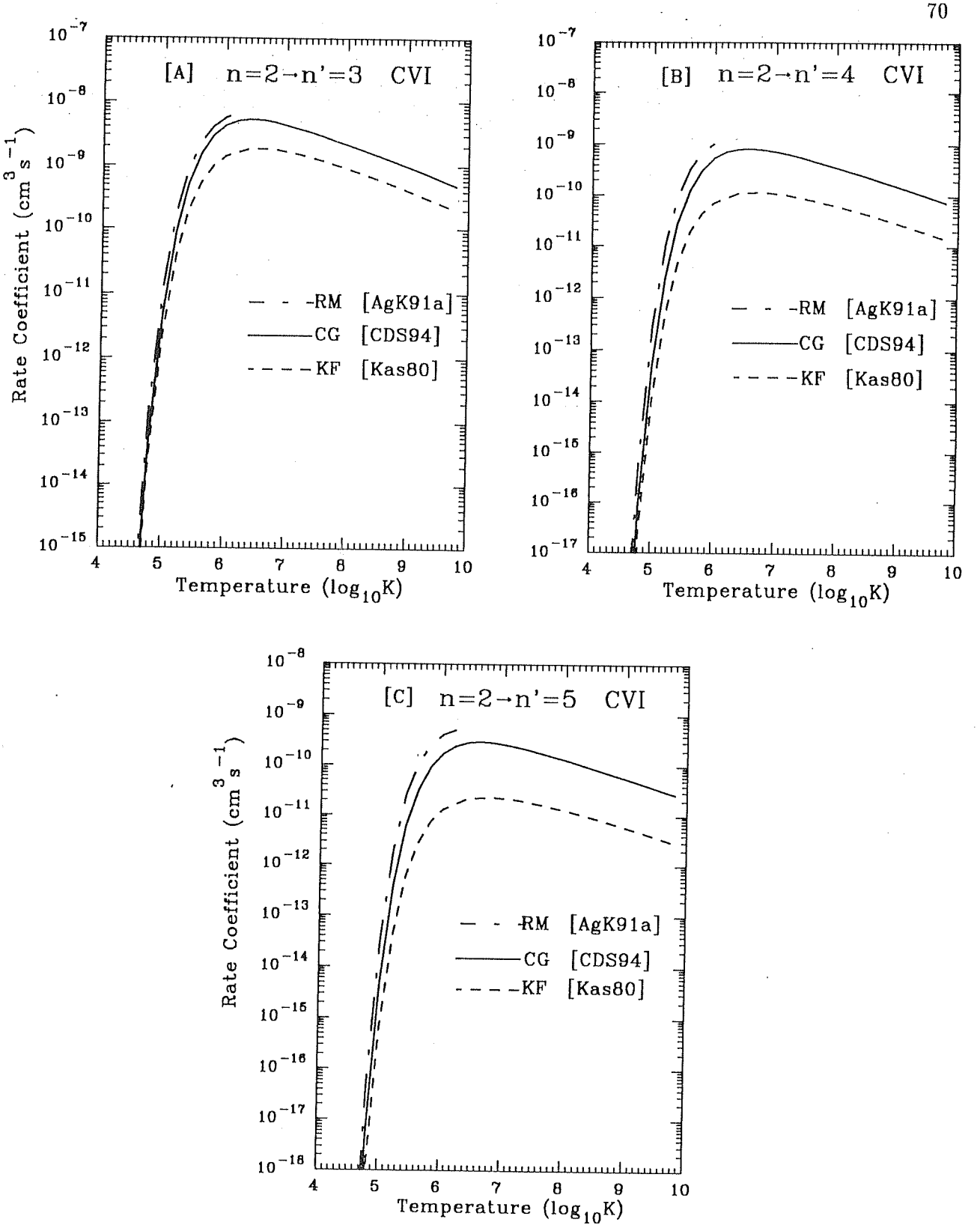


Figure 5.5: Rate coefficients versus electron temperature for CVI. Transitions: $n = 2 \rightarrow n' = 3, 4, 5$.

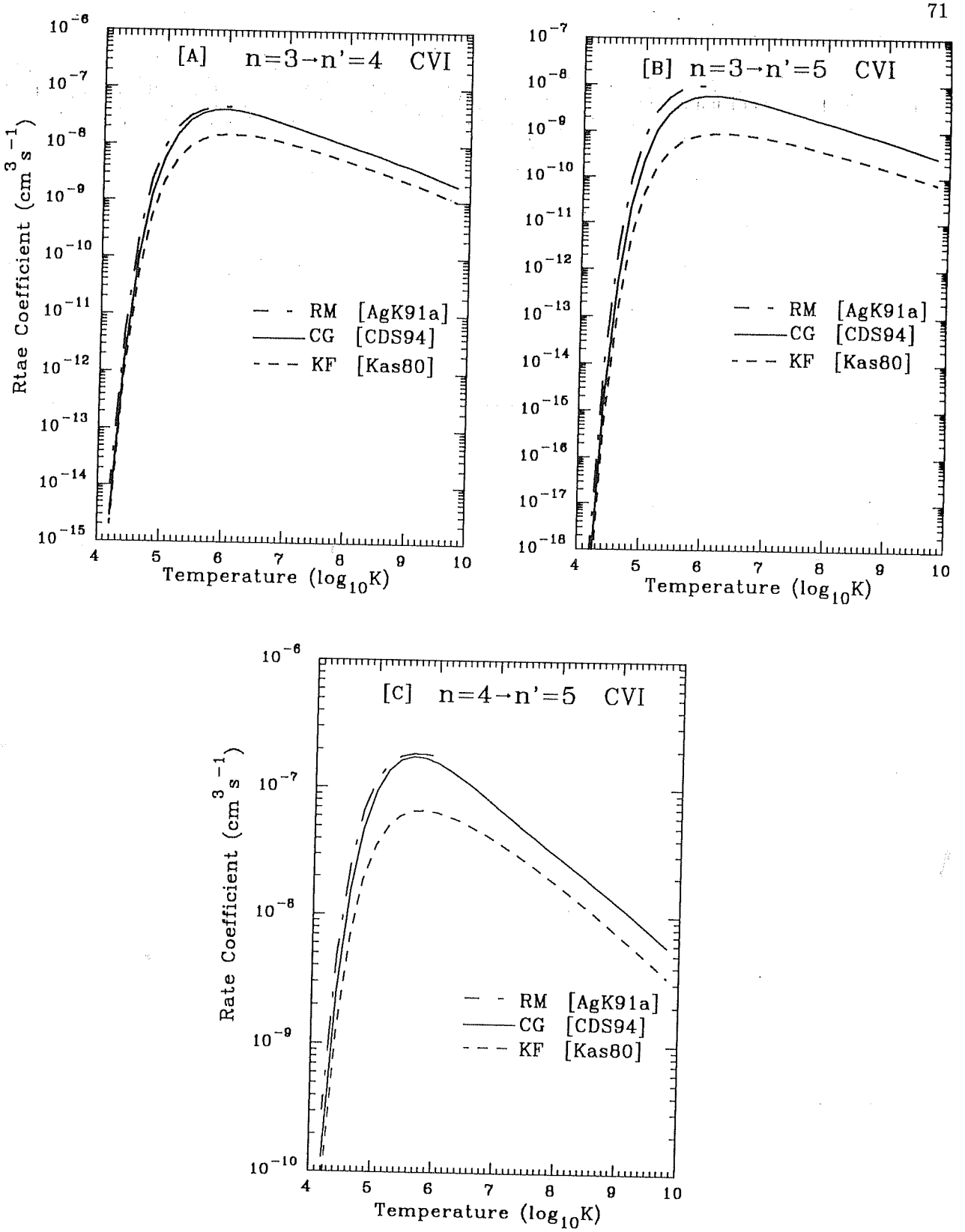


Figure 5.6: Rate coefficients versus electron temperature for CVI. Transitions:$n = 3 \rightarrow n' = 4, 5$ and $n = 4 \rightarrow n' = 5$.

5.3 Conclusion

Previous rate coefficient results [ABB91, ABK91, AgK91a, AgK91b, AgK92a, AgK92b, AgK93] obtained by the 15–state R–matrix calculations are confined within the levels $n, n' \leq 5$ of hydrogen and several other hydrogenic systems. A reasonably large basis set (including pseudostates) is needed in the close coupling methods to obtain a good description of experiments for the excitation from the ground state. For excited state to excited transitions, there are a considerably large number of nearby levels which are strongly coupled and, therefore, they are needed to be included in the close coupling methods. We have seen in the Section 2.5 (see Figs. 2.4 and 2.5A–F) for both $2s \rightarrow 2s$ and $2(s,p) \rightarrow 3(s,p,d)$ transitions of hydrogen that the cross sections calculated in several methods such as the R–matrix and the pseudostate optical potential differ considerably from each other. We are not aware of any study on the convergence of these methods for excited state to excited state transitions. Therefore, it is difficult to comment on their accuracies. On the other hand, as has been pointed out, the Glauber and the Coulomb–Glauber results are expected to be reasonably good for transitions involving high lying levels. Our CG rate coefficients[1] listed in the thesis are complete and for extended range of temperature so that they can be used consistently.

Our computer program can calculate the Coulomb–Glauber rate coefficients for electron impact excitation of several other hydrogenic ions (see Chapter 1) of astrophysical interest. We hope that our results of rate coefficients involving excited states will be useful in astrophysical plasma diagnostics.

[1]Apart from the rates tabulated here we have results at many other temperatures also. Those can be obtained on request.

Table 5.1 Coulomb-Glauber rate coefficients as a function of electron temperature for excitations of HeII.

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
1s-2s	.776E-61	.955E-46	.461E-35	.169E-27	.384E-22	.239E-18	.117E-15	.941E-14	.213E-12	.197E-11	.301E-10	.122E-09	.244E-09	.323E-09	.335E-09	.270E-09	.182E-09	.111E-09
1s-2p	.962E-60	.118E-44	.572E-34	.210E-26	.476E-21	.296E-17	.144E-14	.115E-12	.259E-11	.236E-10	.348E-09	.137E-08	.274E-08	.382E-08	.433E-08	.421E-08	.353E-08	.271E-08
1s-3s	.343E-71	.264E-53	.122E-40	.113E-31	.253E-25	.798E-21	.123E-17	.227E-15	.923E-14	.130E-12	.335E-11	.179E-10	.414E-10	.592E-10	.638E-10	.528E-10	.357E-10	.215E-10
1s-3p	.469E-70	.361E-52	.166E-39	.153E-30	.342E-24	.107E-19	.163E-16	.294E-14	.117E-12	.160E-11	.384E-10	.192E-09	.430E-09	.629E-09	.727E-09	.709E-09	.591E-09	.450E-09
1s-3d	.481E-71	.370E-53	.170E-40	.157E-31	.349E-25	.109E-20	.166E-17	.297E-15	.117E-13	.159E-12	.370E-11	.176E-10	.367E-10	.488E-10	.503E-10	.405E-10	.276E-10	.173E-10
1s-4s	.541E-75	.396E-56	.902E-43	.259E-33	.129E-26	.719E-22	.166E-18	.407E-16	.203E-14	.331E-13	.102E-11	.601E-11	.146E-10	.214E-10	.234E-10	.195E-10	.132E-10	.794E-11
1s-4p	.760E-74	.555E-55	.126E-41	.360E-32	.178E-25	.981E-21	.224E-17	.536E-15	.260E-13	.410E-12	.117E-10	.635E-10	.148E-09	.220E-09	.257E-09	.251E-09	.209E-09	.158E-09
1s-4d	.103E-74	.749E-56	.170E-42	.484E-33	.240E-26	.132E-21	.299E-18	.713E-16	.344E-14	.537E-13	.148E-11	.769E-11	.167E-10	.227E-10	.236E-10	.191E-10	.131E-10	.819E-11
1s-4f	.266E-76	.194E-57	.441E-44	.125E-34	.620E-28	.340E-23	.771E-20	.183E-17	.877E-16	.135E-14	.362E-13	.178E-12	.361E-12	.454E-12	.440E-12	.327E-12	.211E-12	.128E-12
1s-5s	.713E-77	.147E-57	.714E-44	.345E-34	.250E-27	.181E-22	.505E-19	.141E-16	.774E-15	.135E-13	.453E-12	.278E-11	.689E-11	.102E-10	.113E-10	.942E-11	.639E-11	.383E-11
1s-5p	.101E-75	.208E-56	.101E-42	.484E-33	.348E-26	.249E-21	.683E-18	.187E-15	.995E-14	.167E-12	.515E-11	.291E-10	.692E-10	.104E-09	.121E-09	.119E-09	.986E-10	.748E-10
1s-5f	.571E-78	.117E-58	.566E-45	.272E-35	.194E-28	.139E-23	.378E-20	.102E-17	.538E-16	.887E-15	.257E-13	.131E-12	.271E-12	.345E-12	.335E-12	.250E-12	.162E-12	.976E-13
1s-5g	.503E-80	.103E-60	.498E-47	.239E-37	.171E-30	.121E-25	.330E-22	.888E-20	.464E-18	.758E-17	.214E-15	.106E-14	.211E-14	.261E-14	.248E-14	.181E-14	.116E-14	.696E-15
1s-6s	.542E-78	.198E-58	.150E-44	.963E-35	.853E-28	.713E-23	.220E-19	.660E-17	.382E-15	.690E-14	.242E-12	.152E-11	.382E-11	.571E-11	.630E-11	.528E-11	.359E-11	.215E-11
1s-6p	.776E-77	.282E-57	.212E-43	.136E-33	.119E-26	.984E-22	.299E-18	.876E-16	.491E-14	.855E-13	.275E-11	.159E-10	.381E-10	.575E-10	.673E-10	.658E-10	.547E-10	.414E-10
1s-6d	.122E-77	.443E-58	.334E-44	.213E-34	.187E-27	.154E-22	.465E-19	.136E-16	.757E-15	.130E-13	.408E-12	.224E-11	.502E-11	.692E-11	.726E-11	.591E-11	.404E-11	.253E-11
1s-6f	.536E-79	.195E-59	.147E-45	.935E-36	.819E-29	.673E-24	.203E-20	.589E-18	.326E-16	.557E-15	.168E-13	.880E-13	.184E-12	.235E-12	.229E-12	.171E-12	.111E-12	.667E-13
1s-6g	.726E-81	.263E-61	.198E-47	.126E-37	.110E-30	.904E-26	.272E-22	.785E-20	.432E-18	.730E-17	.215E-15	.109E-14	.220E-14	.272E-14	.260E-14	.190E-14	.122E-14	.730E-15
1s-6h	.353E-83	.128E-63	.961E-50	.611E-40	.533E-33	.435E-28	.130E-24	.374E-22	.204E-20	.342E-19	.978E-18	.477E-17	.925E-17	.111E-16	.103E-16	.734E-17	.465E-17	.276E-17
1s-7s	.573E-165	.116E-108	.391E-73	.102E-50	.144E-36	.122E-27	.532E-22	.195E-18	.352E-16	.952E-15	.290E-13	.108E-12	.158E-12	.149E-12	.113E-12	.632E-13	.327E-13	.165E-13
1s-7p	.994E-164	.201E-107	.673E-72	.175E-49	.242E-35	.202E-26	.857E-21	.302E-17	.518E-15	.131E-13	.346E-12	.116E-11	.168E-11	.172E-11	.150E-11	.107E-11	.694E-12	.429E-12
1s-7d	.118E-164	.239E-108	.803E-73	.209E-50	.290E-36	.242E-27	.103E-21	.363E-18	.624E-16	.158E-14	.417E-13	.136E-12	.185E-12	.168E-12	.127E-12	.715E-13	.372E-13	.189E-13
1s-7f	.503E-166	.101E-109	.340E-74	.884E-52	.122E-37	.102E-28	.431E-23	.151E-19	.258E-17	.645E-16	.163E-14	.500E-14	.629E-14	.536E-14	.385E-14	.207E-14	.106E-14	.531E-15
1s-7g	.750E-168	.151E-111	.507E-76	.132E-53	.182E-39	.152E-30	.641E-25	.224E-21	.381E-19	.948E-18	.234E-16	.702E-16	.863E-16	.723E-16	.515E-16	.275E-16	.140E-16	.705E-17
1s-7h	.564E-170	.114E-113	.381E-78	.988E-56	.136E-41	.113E-32	.476E-27	.165E-23	.278E-21	.682E-20	.162E-18	.465E-18	.550E-18	.449E-18	.315E-18	.167E-18	.849E-19	.427E-19
1s-7i	.136E-172	.274E-116	.917E-81	.237E-58	.327E-44	.271E-35	.113E-29	.392E-26	.653E-24	.159E-22	.370E-21	.104E-20	.122E-20	.996E-21	.699E-21	.371E-21	.189E-21	.950E-22

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
1s-8s	.681-166	.260-109	.131E-73	.440E-51	.723E-37	.679E-28	.315E-22	.120E-18	.223E-16	.612E-15	.190E-13	.712E-13	.104E-12	.983E-13	.748E-13	.418E-13	.217E-13	.109E-13
1s-8p	.118-164	.448-108	.224E-72	.749E-50	.122E-35	.112E-26	.507E-21	.186E-17	.327E-15	.841E-14	.225E-12	.763E-12	.111E-11	.113E-11	.987E-12	.702E-12	.457E-12	.282E-12
1s-8d	.143-165	.545-109	.273E-73	.912E-51	.148E-36	.137E-27	.619E-22	.228E-18	.402E-16	.104E-14	.277E-13	.914E-13	.124E-12	.113E-12	.855E-13	.481E-13	.251E-13	.127E-13
1s-8f	.639-167	.243-110	.121E-74	.406E-52	.659E-38	.607E-29	.273E-23	.999E-20	.174E-17	.444E-16	.114E-14	.352E-14	.444E-14	.379E-14	.272E-14	.146E-14	.746E-15	.375E-15
1s-8g	.106-168	.403-112	.201E-76	.672E-54	.109E-39	.100E-30	.452E-25	.165E-21	.287E-19	.724E-18	.182E-16	.549E-16	.677E-16	.567E-16	.404E-16	.216E-16	.110E-16	.554E-17
1s-8h	.989-171	.376-114	.188E-78	.626E-56	.101E-41	.930E-33	.416E-27	.151E-23	.259E-21	.647E-20	.156E-18	.452E-18	.536E-18	.438E-18	.307E-18	.163E-18	.828E-19	.416E-19
1s-8i	.392-173	.149-116	.743E-81	.247E-58	.400E-44	.366E-35	.163E-29	.587E-26	.100E-23	.248E-22	.587E-21	.167E-20	.196E-20	.160E-20	.112E-20	.595E-21	.303E-21	.152E-21
1s-8j	.638-176	.242-119	.121E-83	.402E-61	.649E-47	.592E-38	.262E-32	.939E-29	.159E-26	.388E-25	.896E-24	.247E-23	.284E-23	.229E-23	.159E-23	.844E-24	.429E-24	.216E-24
2s-3s	.480E-17	.286E-14	.265E-12	.646E-11	.612E-10	.295E-09	.878E-09	.185E-08	.309E-08	.435E-08	.638E-08	.739E-08	.739E-08	.662E-08	.546E-08	.369E-08	.233E-08	.142E-08
2s-3p	.687E-17	.417E-14	.395E-12	.997E-11	.987E-10	.505E-09	.162E-08	.378E-08	.702E-08	.112E-07	.208E-07	.298E-07	.359E-07	.380E-07	.369E-07	.314E-07	.246E-07	.181E-07
2s-3d	.206E-16	.123E-13	.115E-11	.286E-10	.277E-09	.137E-08	.419E-08	.911E-08	.155E-07	.220E-07	.313E-07	.334E-07	.302E-07	.248E-07	.192E-07	.122E-07	.751E-08	.452E-08
2p-3s	.483E-18	.289E-15	.269E-13	.665E-12	.641E-11	.316E-10	.971E-10	.213E-09	.370E-09	.546E-09	.879E-09	.112E-08	.125E-08	.127E-08	.120E-08	.100E-08	.778E-09	.571E-09
2p-3p	.525E-17	.313E-14	.290E-12	.710E-11	.676E-10	.328E-09	.986E-09	.211E-08	.356E-08	.508E-08	.752E-08	.867E-08	.854E-08	.752E-08	.613E-08	.409E-08	.257E-08	.156E-08
2p-3d	.258E-16	.155E-13	.146E-11	.365E-10	.359E-09	.182E-08	.578E-08	.132E-07	.239E-07	.365E-07	.615E-07	.789E-07	.859E-07	.842E-07	.769E-07	.616E-07	.462E-07	.331E-07
2s-4s	.343E-21	.194E-17	.885E-15	.667E-13	.140E-11	.119E-10	.528E-10	.149E-09	.304E-09	.500E-09	.906E-09	.120E-08	.130E-08	.122E-08	.103E-08	.712E-09	.453E-09	.276E-09
2s-4p	.665E-21	.382E-17	.177E-14	.137E-12	.298E-11	.264E-10	.124E-09	.376E-09	.836E-09	.150E-08	.324E-08	.501E-08	.627E-08	.677E-08	.661E-08	.564E-08	.439E-08	.323E-08
2s-4d	.113E-20	.666E-17	.311E-14	.240E-12	.519E-11	.453E-10	.208E-09	.602E-09	.125E-08	.206E-08	.347E-08	.400E-08	.375E-08	.312E-08	.242E-08	.154E-08	.933E-09	.542E-09
2s-4f	.692E-21	.395E-17	.182E-14	.140E-12	.301E-11	.262E-10	.121E-09	.349E-09	.729E-09	.120E-08	.201E-08	.229E-08	.210E-08	.171E-08	.130E-08	.818E-09	.496E-09	.297E-09
2p-4s	.387E-22	.224E-18	.105E-15	.814E-14	.178E-12	.159E-11	.747E-11	.223E-10	.484E-10	.835E-10	.159E-09	.215E-09	.240E-09	.241E-09	.224E-09	.183E-09	.140E-09	.101E-09
2p-4p	.368E-21	.208E-17	.950E-15	.718E-13	.152E-11	.130E-10	.587E-10	.168E-09	.351E-09	.588E-09	.109E-08	.143E-08	.153E-08	.141E-08	.117E-08	.795E-09	.502E-09	.305E-09
2p-4d	.183E-20	.105E-16	.484E-14	.374E-12	.817E-11	.727E-10	.344E-09	.104E-08	.228E-08	.401E-08	.798E-08	.111E-07	.124E-07	.122E-07	.111E-07	.878E-08	.649E-08	.460E-08
2p-4f	.532E-21	.304E-17	.141E-14	.108E-12	.234E-11	.206E-10	.955E-10	.282E-09	.603E-09	.102E-08	.185E-08	.231E-08	.231E-08	.203E-08	.164E-08	.109E-08	.684E-09	.415E-09
2s-5s	0.688E-43	.791E-31	.320E-23	.199E-18	.209E-15	.167E-13	.266E-12	.155E-11	.479E-11	.986E-11	.197E-10	.220E-10	.184E-10	.133E-10	.896E-11	.466E-11	.236E-11	.119E-11
2s-5p	0.625E-42	.723E-30	.295E-22	.186E-17	.197E-14	.157E-12	.245E-11	.135E-10	.386E-10	.722E-10	.124E-09	.133E-09	.118E-09	.962E-10	.755E-10	.499E-10	.314E-10	.190E-10
2s-5d	0.266E-42	.308E-30	.126E-22	.799E-18	.850E-15	.682E-13	.107E-11	.586E-11	.165E-10	.304E-10	.487E-10	.468E-10	.357E-10	.245E-10	.160E-10	.818E-11	.412E-11	.207E-11
2s-5f	0.266E-42	.307E-30	.125E-22	.787E-18	.828E-15	.656E-13	.101E-11	.545E-11	.151E-10	.271E-10	.414E-10	.381E-10	.281E-10	.189E-10	.123E-10	.623E-11	.313E-11	.157E-11
2s-5g	0.255E-43	.293E-31	.119E-23	.739E-19	.767E-16	.597E-14	.901E-13	.476E-12	.129E-11	.227E-11	.341E-11	.313E-11	.232E-11	.156E-11	.102E-11	.518E-12	.261E-12	.131E-12
2p-5s	0.171E-44	.213E-32	.968E-25	.714E-20	.939E-17	.987E-15	.209E-13	.157E-12	.592E-12	.139E-11	.322E-11	.400E-11	.374E-11	.309E-11	.241E-11	.157E-11	.971E-12	.581E-12
2p-5p	0.914E-43	.105E-30	.426E-23	.266E-18	.278E-15	.220E-13	.344E-12	.194E-11	.581E-11	.116E-10	.224E-10	.248E-10	.206E-10	.148E-10	.994E-11	.516E-11	.261E-11	.131E-11

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
2p-5d	0.123E-41	.143E-29	.583E-22	.367E-17	.389E-14	.310E-12	.484E-11	.265E-10	.749E-10	.138E-09	.224E-09	.224E-09	.185E-09	.142E-09	.105E-09	.657E-10	.397E-10	.233E-10
2p-5f	0.287E-42	.331E-30	.135E-22	.846E-18	.890E-15	.705E-13	.109E-11	.593E-11	.166E-10	.304E-10	.490E-10	.479E-10	.373E-10	.261E-10	.173E-10	.897E-11	.455E-11	.228E-11
2p-5g	0.181E-43	.209E-31	.848E-24	.532E-19	.559E-16	.441E-14	.678E-13	.365E-12	.101E-11	.182E-11	.280E-11	.261E-11	.194E-11	.131E-11	.854E-12	.435E-12	.219E-12	.110E-12
2s-6s	.402E-45	.233E-32	.262E-24	.311E-19	.489E-16	.506E-14	.953E-13	.617E-12	.205E-11	.443E-11	.932E-11	.107E-10	.908E-11	.660E-11	.446E-11	.232E-11	.118E-11	.591E-12
2s-6p	.373E-44	.218E-31	.246E-23	.296E-18	.470E-15	.486E-13	.893E-12	.546E-11	.167E-10	.325E-10	.585E-10	.642E-10	.574E-10	.471E-10	.370E-10	.245E-10	.154E-10	.929E-11
2s-6d	.146E-44	.858E-32	.977E-24	.118E-18	.189E-15	.197E-13	.363E-12	.222E-11	.673E-11	.129E-10	.218E-10	.214E-10	.165E-10	.113E-10	.743E-11	.380E-11	.191E-11	.959E-12
2s-6f	.175E-44	.102E-31	.115E-23	.138E-18	.218E-15	.224E-13	.406E-12	.244E-11	.721E-11	.135E-10	.216E-10	.203E-10	.151E-10	.102E-10	.660E-11	.336E-11	.169E-11	.846E-12
2s-6g	.255E-45	.148E-32	.167E-24	.198E-19	.309E-16	.311E-14	.552E-13	.324E-12	.937E-12	.172E-11	.270E-11	.252E-11	.189E-11	.128E-11	.831E-12	.424E-12	.213E-12	.107E-12
2s-6h	.861E-47	.500E-34	.560E-26	.663E-21	.103E-17	.104E-15	.184E-14	.108E-13	.311E-13	.569E-13	.879E-13	.806E-13	.592E-13	.396E-13	.256E-13	.130E-13	.656E-14	.329E-14
2p-6s	.857E-47	.544E-34	.698E-26	.100E-20	.204E-17	.285E-15	.730E-14	.621E-13	.253E-12	.628E-12	.154E-11	.196E-11	.185E-11	.153E-11	.119E-11	.768E-12	.475E-12	.283E-12
2p-6p	.539E-45	.313E-32	.352E-24	.418E-19	.657E-16	.673E-14	.124E-12	.779E-12	.250E-11	.523E-11	.107E-10	.121E-10	.102E-10	.737E-11	.496E-11	.257E-11	.130E-11	.654E-12
2p-6d	.709E-44	.414E-31	.469E-23	.564E-18	.898E-15	.928E-13	.170E-11	.104E-10	.314E-10	.604E-10	.103E-09	.105E-09	.870E-10	.667E-10	.496E-10	.308E-10	.186E-10	.109E-10
2p-6f	.190E-44	.111E-31	.125E-23	.150E-18	.237E-15	.243E-13	.443E-12	.268E-11	.803E-11	.154E-10	.259E-10	.259E-10	.203E-10	.142E-10	.948E-11	.491E-11	.249E-11	.125E-11
2p-6g	.182E-45	.106E-32	.120E-24	.143E-19	.226E-16	.231E-14	.418E-13	.250E-12	.741E-12	.139E-11	.225E-11	.213E-11	.160E-11	.109E-11	.707E-12	.360E-12	.182E-12	.909E-13
2p-6h	.484E-47	.281E-34	.316E-26	.375E-21	.587E-18	.593E-16	.106E-14	.625E-14	.182E-13	.338E-13	.536E-13	.504E-13	.378E-13	.256E-13	.167E-13	.852E-14	.430E-14	.215E-14
2s-7s	.159E-46	.245E-33	.509E-25	.892E-20	.180E-16	.217E-14	.451E-13	.312E-12	.108E-11	.240E-11	.523E-11	.611E-11	.521E-11	.380E-11	.257E-11	.134E-11	.680E-12	.341E-12
2s-7p	.150E-45	.232E-32	.486E-24	.860E-19	.175E-15	.211E-13	.427E-12	.278E-11	.883E-11	.177E-10	.328E-10	.364E-10	.328E-10	.269E-10	.212E-10	.140E-10	.879E-11	.531E-11
2s-7d	.561E-46	.872E-33	.184E-24	.328E-19	.673E-16	.819E-14	.167E-12	.109E-11	.345E-11	.681E-11	.118E-10	.118E-10	.912E-11	.629E-11	.413E-11	.211E-11	.106E-11	.533E-12
2s-7f	.735E-46	.114E-32	.238E-24	.421E-19	.852E-16	.102E-13	.205E-12	.131E-11	.403E-11	.775E-11	.127E-10	.121E-10	.903E-11	.610E-11	.396E-11	.202E-11	.101E-11	.508E-12
2s-7g	.129E-46	.199E-33	.415E-25	.727E-20	.145E-16	.171E-14	.335E-13	.209E-12	.629E-12	.119E-11	.191E-11	.181E-11	.136E-11	.920E-12	.599E-12	.305E-12	.154E-12	.771E-13
2s-7h	.692E-48	.107E-34	.221E-26	.386E-21	.769E-18	.904E-16	.177E-14	.110E-13	.330E-13	.621E-13	.984E-13	.913E-13	.673E-13	.452E-13	.292E-13	.149E-13	.749E-14	.375E-14
2s-7i	.116E-49	.179E-36	.369E-28	.641E-23	.126E-19	.146E-17	.282E-16	.173E-15	.510E-15	.945E-15	.147E-14	.136E-14	.101E-14	.677E-15	.439E-15	.224E-15	.113E-15	.566E-16
2p-7s	.317E-48	.536E-35	.128E-26	.275E-21	.724E-18	.120E-15	.343E-14	.314E-13	.134E-12	.343E-12	.871E-12	.112E-11	.106E-11	.877E-12	.682E-12	.440E-12	.271E-12	.162E-12
2p-7p	.214E-46	.330E-33	.687E-25	.121E-19	.242E-16	.290E-14	.590E-13	.395E-12	.132E-11	.285E-11	.599E-11	.692E-11	.586E-11	.425E-11	.286E-11	.149E-11	.754E-12	.378E-12
2p-7d	.278E-45	.431E-32	.905E-24	.161E-18	.327E-15	.395E-13	.800E-12	.520E-11	.164E-10	.323E-10	.567E-10	.585E-10	.487E-10	.373E-10	.277E-10	.172E-10	.104E-10	.606E-11
2p-7f	.807E-46	.125E-32	.261E-24	.461E-19	.932E-16	.112E-13	.225E-12	.145E-11	.451E-11	.886E-11	.154E-10	.155E-10	.123E-10	.862E-11	.574E-11	.297E-11	.151E-11	.757E-12
2p-7g	.927E-47	.143E-33	.300E-25	.529E-20	.107E-16	.127E-14	.255E-13	.162E-12	.500E-12	.963E-12	.160E-11	.154E-11	.116E-11	.788E-12	.513E-12	.262E-12	.132E-12	.660E-13
2p-7h	.390E-48	.601E-35	.125E-26	.219E-21	.438E-18	.518E-16	.102E-14	.640E-14	.194E-13	.370E-13	.603E-13	.574E-13	.432E-13	.294E-13	.192E-13	.978E-14	.493E-14	.247E-14
2p-7i	.563E-50	.867E-37	.180E-28	.316E-23	.631E-20	.744E-18	.146E-16	.914E-16	.276E-15	.521E-15	.832E-15	.775E-15	.572E-15	.384E-15	.249E-15	.127E-15	.638E-16	.320E-16

Table 5.1 continued

Exc. $ni - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
2s-8s	.177E-47	.514E-34	.159E-25	.359E-20	.848E-17	.113E-14	.252E-13	.181E-12	.647E-12	.146E-11	.326E-11	.384E-11	.329E-11	.241E-11	.163E-11	.850E-12	.431E-12	.217E-12
2s-8p	.168E-46	.490E-33	.153E-24	.349E-19	.832E-16	.111E-13	.240E-12	.163E-11	.530E-11	.108E-10	.204E-10	.229E-10	.206E-10	.170E-10	.133E-10	.881E-11	.554E-11	.334E-11
2s-8d	.612E-47	.179E-33	.564E-25	.130E-19	.312E-16	.421E-14	.917E-13	.624E-12	.202E-11	.407E-11	.721E-11	.725E-11	.563E-11	.389E-11	.255E-11	.131E-11	.658E-12	.330E-12
2s-8f	.850E-47	.248E-33	.774E-25	.176E-19	.418E-16	.554E-14	.118E-12	.787E-12	.249E-11	.488E-11	.816E-11	.779E-11	.584E-11	.395E-11	.257E-11	.131E-11	.657E-12	.329E-12
2s-8g	.166E-47	.482E-34	.150E-25	.337E-20	.789E-17	.103E-14	.215E-13	.139E-12	.431E-12	.827E-12	.135E-11	.129E-11	.970E-12	.659E-12	.430E-12	.219E-12	.110E-12	.553E-13
2s-8h	.110E-48	.318E-35	.985E-27	.221E-21	.517E-18	.671E-16	.140E-14	.907E-14	.279E-13	.534E-13	.861E-13	.804E-13	.595E-13	.400E-13	.259E-13	.132E-13	.663E-14	.332E-14
2s-8i	.302E-50	.872E-37	.269E-28	.600E-23	.139E-19	.178E-17	.365E-16	.233E-15	.705E-15	.133E-14	.211E-14	.196E-14	.145E-14	.980E-15	.636E-15	.324E-15	.163E-15	.820E-16
2s-8j	.276E-52	.797E-39	.245E-30	.545E-25	.126E-21	.161E-19	.329E-18	.209E-17	.634E-17	.119E-16	.188E-16	.173E-16	.127E-16	.847E-17	.548E-17	.279E-17	.141E-17	.705E-18
2p-8s	.342E-49	.109E-35	.392E-27	.109E-21	.338E-18	.623E-16	.191E-14	.183E-13	.807E-13	.210E-12	.545E-12	.706E-12	.671E-12	.555E-12	.431E-12	.277E-12	.171E-12	.102E-12
2p-8p	.239E-47	.695E-34	.216E-25	.487E-20	.115E-16	.152E-14	.330E-13	.230E-12	.792E-12	.174E-11	.373E-11	.436E-11	.371E-11	.270E-11	.182E-11	.945E-12	.479E-12	.240E-12
2p-8d	.308E-46	.901E-33	.282E-24	.644E-19	.154E-15	.205E-13	.444E-12	.300E-11	.971E-11	.195E-10	.349E-10	.363E-10	.303E-10	.232E-10	.172E-10	.107E-10	.641E-11	.375E-11
2p-8f	.936E-47	.273E-33	.852E-25	.194E-19	.459E-16	.609E-14	.130E-12	.875E-12	.280E-11	.560E-11	.990E-11	.101E-10	.797E-11	.561E-11	.374E-11	.194E-11	.982E-12	.493E-12
2p-8g	.119E-47	.347E-34	.108E-25	.246E-20	.581E-17	.768E-15	.164E-13	.109E-12	.343E-12	.673E-12	.114E-11	.110E-11	.833E-12	.567E-12	.370E-12	.188E-12	.949E-13	.476E-13
2p-8h	.619E-49	.180E-35	.557E-27	.126E-21	.295E-18	.385E-16	.809E-15	.529E-14	.165E-13	.319E-13	.529E-13	.508E-13	.384E-13	.261E-13	.170E-13	.869E-14	.438E-14	.220E-14
2p-8i	.146E-50	.424E-37	.131E-28	.296E-23	.694E-20	.905E-18	.190E-16	.124E-15	.383E-15	.735E-15	.119E-14	.112E-14	.830E-15	.558E-15	.362E-15	.184E-15	.927E-16	.465E-16
2p-8j	.127E-52	.366E-39	.113E-30	.253E-25	.588E-22	.758E-20	.157E-18	.100E-17	.306E-17	.581E-17	.929E-17	.866E-17	.642E-17	.432E-17	.281E-17	.143E-17	.722E-18	.362E-18
3s-4s	.513E-10	.463E-09	.222E-08	.668E-08	.144E-07	.243E-07	.345E-07	.429E-07	.487E-07	.519E-07	.524E-07	.477E-07	.409E-07	.331E-07	.257E-07	.165E-07	.102E-07	.615E-08
3s-4p	.349E-10	.328E-09	.165E-08	.530E-08	.124E-07	.232E-07	.373E-07	.536E-07	.716E-07	.902E-07	.126E-06	.152E-06	.163E-06	.160E-06	.146E-06	.118E-06	.892E-07	.642E-07
3s-4d	.965E-10	.881E-09	.429E-08	.132E-07	.294E-07	.516E-07	.762E-07	.990E-07	.117E-06	.128E-06	.132E-06	.117E-06	.960E-07	.744E-07	.557E-07	.347E-07	.211E-07	.126E-07
3s-4f	.135E-09	.118E-08	.548E-08	.159E-07	.330E-07	.539E-07	.738E-07	.889E-07	.976E-07	.100E-06	.912E-07	.742E-07	.566E-07	.416E-07	.300E-07	.181E-07	.108E-07	.644E-08
3p-4s	.900E-11	.776E-10	.352E-09	.101E-08	.207E-08	.339E-08	.474E-08	.599E-08	.709E-08	.805E-08	.969E-08	.108E-07	.112E-07	.108E-07	.984E-08	.790E-08	.595E-08	.428E-08
3p-4p	.588E-10	.531E-09	.255E-08	.769E-08	.167E-07	.284E-07	.406E-07	.510E-07	.587E-07	.631E-07	.644E-07	.587E-07	.499E-07	.400E-07	.309E-07	.197E-07	.121E-07	.729E-08
3p-4d	.747E-10	.708E-09	.359E-08	.115E-07	.270E-07	.499E-07	.788E-07	.111E-06	.143E-06	.174E-06	.223E-06	.251E-06	.255E-06	.240E-06	.214E-06	.167E-06	.124E-06	.878E-07
3p-4f	.175E-09	.156E-08	.743E-08	.222E-07	.477E-07	.807E-07	.115E-06	.143E-06	.162E-06	.172E-06	.167E-06	.143E-06	.114E-06	.864E-07	.639E-07	.394E-07	.238E-07	.142E-07
3d-4s	.965E-12	.838E-11	.388E-10	.114E-09	.248E-09	.433E-09	.650E-09	.869E-09	.107E-08	.122E-08	.135E-08	.128E-08	.108E-08	.854E-09	.645E-09	.404E-09	.246E-09	.147E-09
3d-4p	.810E-11	.709E-10	.327E-09	.948E-09	.198E-08	.326E-08	.456E-08	.570E-08	.657E-08	.717E-08	.764E-08	.740E-08	.675E-08	.588E-08	.496E-08	.367E-08	.262E-08	.181E-08
3d-4d	.572E-10	.512E-09	.244E-08	.726E-08	.155E-07	.260E-07	.367E-07	.454E-07	.515E-07	.546E-07	.546E-07	.490E-07	.412E-07	.327E-07	.250E-07	.159E-07	.973E-08	.584E-08
3d-4f	.242E-09	.225E-08	.112E-07	.352E-07	.803E-07	.145E-06	.223E-06	.304E-06	.379E-06	.444E-06	.529E-06	.556E-06	.533E-06	.481E-06	.414E-06	.314E-06	.227E-06	.158E-06
3s-5s	.815E-20	.972E-16	.354E-13	.140E-11	.137E-10	.584E-10	.140E-09	.239E-09	.327E-09	.386E-09	.395E-09	.319E-09	.228E-09	.153E-09	.991E-10	.506E-10	.255E-10	.128E-10
3s-5p	.418E-19	.506E-15	.188E-12	.762E-11	.771E-10	.341E-09	.824E-09	.138E-08	.182E-08	.206E-08	.201E-08	.165E-08	.128E-08	.971E-09	.732E-09	.464E-09	.285E-09	.160E-09

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
3s-5d	.227E-19	.273E-15	.101E-12	.406E-11	.406E-10	.176E-09	.414E-09	.675E-09	.864E-09	.944E-09	.851E-09	.632E-09	.430E-09	.281E-09	.180E-09	.911E-10	.457E-10	.229E-10
3s-5f	.243E-20	.306E-16	.121E-13	.527E-12	.589E-11	.291E-10	.776E-10	.141E-09	.198E-09	.231E-09	.225E-09	.171E-09	.117E-09	.761E-10	.486E-10	.245E-10	.123E-10	.616E-11
3s-5g	.141E-19	.171E-15	.640E-13	.260E-11	.261E-10	.114E-09	.267E-09	.432E-09	.548E-09	.593E-09	.527E-09	.388E-09	.261E-09	.169E-09	.108E-09	.546E-10	.274E-10	.137E-10
3p-5s	.110E-20	.136E-16	.521E-14	.220E-12	.240E-11	.118E-10	.318E-10	.598E-10	.875E-10	.107E-09	.117E-09	.102E-09	.806E-10	.619E-10	.466E-10	.294E-10	.179E-10	.100E-10
3p-5p	.107E-19	.127E-15	.463E-13	.183E-11	.180E-10	.764E-10	.182E-09	.306E-09	.414E-09	.483E-09	.487E-09	.389E-09	.276E-09	.185E-09	.120E-09	.610E-10	.307E-10	.154E-10
3p-5d	.701E-19	.844E-15	.312E-12	.126E-10	.127E-09	.554E-09	.132E-08	.219E-08	.284E-08	.316E-08	.298E-08	.236E-08	.176E-08	.130E-08	.958E-09	.593E-09	.357E-09	.198E-09
3p-5f	.869E-20	.107E-15	.405E-13	.169E-11	.176E-10	.804E-10	.198E-09	.337E-09	.447E-09	.502E-09	.470E-09	.356E-09	.244E-09	.159E-09	.102E-09	.516E-10	.259E-10	.130E-10
3p-5g	.189E-19	.229E-15	.849E-13	.343E-11	.343E-10	.149E-09	.352E-09	.573E-09	.730E-09	.792E-09	.702E-09	.511E-09	.342E-09	.221E-09	.141E-09	.709E-10	.356E-10	.178E-10
3d-5s	.219E-21	.260E-17	.952E-15	.382E-13	.391E-12	.179E-11	.457E-11	.816E-11	.114E-10	.134E-10	.134E-10	.106E-10	.738E-11	.488E-11	.315E-11	.160E-11	.802E-12	.402E-12
3d-5p	.735E-21	.888E-17	.331E-14	.136E-12	.146E-11	.705E-11	.193E-10	.368E-10	.546E-10	.674E-10	.721E-10	.598E-10	.447E-10	.320E-10	.226E-10	.132E-10	.756E-11	.410E-11
3d-5d	.805E-20	.964E-16	.353E-13	.140E-11	.138E-10	.584E-10	.138E-09	.230E-09	.310E-09	.363E-09	.372E-09	.301E-09	.214E-09	.143E-09	.928E-10	.472E-10	.238E-10	.119E-10
3d-5f	.111E-18	.133E-14	.491E-12	.198E-10	.198E-09	.866E-09	.206E-08	.340E-08	.440E-08	.486E-08	.449E-08	.347E-08	.252E-08	.180E-08	.129E-08	.774E-09	.455E-09	.250E-09
3d-5g	.285E-19	.342E-15	.126E-12	.504E-11	.499E-10	.214E-09	.499E-09	.808E-09	.103E-08	.112E-08	.102E-08	.763E-09	.524E-09	.345E-09	.222E-09	.113E-09	.569E-10	.285E-10
3s-6s	.334E-22	.201E-17	.202E-14	.148E-12	.220E-11	.126E-10	.357E-10	.683E-10	.102E-09	.126E-09	.139E-09	.116E-09	.840E-10	.568E-10	.371E-10	.190E-10	.955E-11	.479E-11
3s-6p	.174E-21	.106E-16	.109E-13	.818E-12	.126E-10	.751E-10	.214E-09	.400E-09	.567E-09	.674E-09	.697E-09	.589E-09	.462E-09	.353E-09	.266E-09	.169E-09	.104E-09	.588E-10
3s-6d	.859E-22	.523E-17	.538E-14	.405E-12	.623E-11	.368E-10	.104E-09	.191E-09	.264E-09	.304E-09	.290E-09	.221E-09	.152E-09	.995E-10	.639E-10	.323E-10	.162E-10	.814E-11
3s-6f	.776E-23	.495E-18	.543E-15	.445E-13	.794E-12	.544E-11	.176E-10	.367E-10	.563E-10	.699E-10	.726E-10	.571E-10	.395E-10	.259E-10	.166E-10	.837E-11	.420E-11	.210E-11
3s-6g	.658E-22	.402E-17	.415E-14	.313E-12	.480E-11	.283E-10	.790E-10	.143E-09	.195E-09	.222E-09	.208E-09	.157E-09	.107E-09	.694E-10	.444E-10	.224E-10	.113E-10	.564E-11
3s-6h	.201E-22	.121E-17	.123E-14	.910E-13	.136E-11	.776E-11	.212E-10	.378E-10	.509E-10	.572E-10	.525E-10	.388E-10	.260E-10	.168E-10	.107E-10	.540E-11	.272E-11	.136E-11
3p-6s	.352E-23	.220E-18	.235E-15	.186E-13	.322E-12	.215E-11	.713E-11	.155E-10	.251E-10	.331E-10	.393E-10	.356E-10	.287E-10	.221E-10	.167E-10	.105E-10	.639E-11	.360E-11
3p-6p	.433E-22	.261E-17	.263E-14	.194E-12	.289E-11	.166E-10	.468E-10	.888E-10	.130E-09	.160E-09	.173E-09	.143E-09	.103E-09	.694E-10	.452E-10	.231E-10	.116E-10	.582E-11
3p-6d	.286E-21	.174E-16	.178E-13	.133E-11	.204E-10	.121E-09	.341E-09	.630E-09	.882E-09	.103E-08	.102E-08	.827E-09	.623E-09	.461E-09	.338E-09	.209E-09	.125E-09	.701E-10
3p-6f	.223E-22	.139E-17	.147E-14	.115E-12	.189E-11	.120E-10	.364E-10	.710E-10	.104E-09	.125E-09	.126E-09	.993E-10	.693E-10	.457E-10	.294E-10	.149E-10	.748E-11	.375E-11
3p-6g	.929E-22	.564E-17	.578E-14	.434E-12	.663E-11	.389E-10	.109E-09	.199E-09	.273E-09	.311E-09	.290E-09	.216E-09	.146E-09	.945E-10	.603E-10	.304E-10	.153E-10	.764E-11
3p-6h	.198E-22	.119E-17	.121E-14	.898E-13	.134E-11	.767E-11	.209E-10	.370E-10	.497E-10	.557E-10	.512E-10	.382E-10	.258E-10	.168E-10	.107E-10	.542E-11	.272E-11	.136E-11
3d-6s	.737E-24	.449E-19	.464E-16	.357E-14	.594E-13	.383E-12	.122E-11	.255E-11	.396E-11	.499E-11	.539E-11	.437E-11	.310E-11	.206E-11	.133E-11	.675E-12	.339E-12	.170E-12
3d-6p	.270E-23	.164E-18	.169E-15	.130E-13	.219E-12	.144E-11	.483E-11	.107E-10	.175E-10	.231E-10	.266E-10	.227E-10	.171E-10	.122E-10	.851E-11	.489E-11	.276E-11	.149E-11
3d-6d	.343E-22	.206E-17	.208E-14	.153E-12	.227E-11	.129E-10	.357E-10	.666E-10	.967E-10	.119E-09	.130E-09	.109E-09	.789E-10	.531E-10	.345E-10	.176E-10	.886E-11	.444E-11
3d-6f	.412E-21	.249E-16	.254E-13	.190E-11	.291E-10	.171E-09	.482E-09	.887E-09	.123E-08	.143E-08	.139E-08	.109E-08	.793E-09	.566E-09	.403E-09	.240E-09	.140E-09	.772E-10
3d-6g	.155E-21	.938E-17	.954E-14	.709E-12	.107E-10	.618E-10	.171E-09	.309E-09	.423E-09	.485E-09	.463E-09	.355E-09	.246E-09	.162E-09	.105E-09	.533E-10	.269E-10	.135E-10
3d-6h	.155E-22	.938E-18	.954E-15	.709E-13	.107E-11	.618E-11	.171E-10	.307E-10	.417E-10	.471E-10	.437E-10	.325E-10	.220E-10	.143E-10	.911E-11	.459E-11	.231E-11	.116E-11

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
3s-7s	.109E-23	.173E-18	.326E-15	.370E-13	.713E-12	.449E-11	.141E-10	.291E-10	.452E-10	.582E-10	.663E-10	.566E-10	.415E-10	.282E-10	.184E-10	.944E-11	.476E-11	.239E-11
3s-7p	.579E-23	.932E-18	.180E-14	.212E-12	.424E-11	.274E-10	.862E-10	.172E-09	.254E-09	.310E-09	.332E-09	.286E-09	.226E-09	.173E-09	.131E-09	.832E-10	.510E-10	.290E-10
3s-7d	.270E-23	.435E-18	.845E-15	.999E-13	.200E-11	.130E-10	.407E-10	.802E-10	.117E-09	.138E-09	.137E-09	.106E-09	.732E-10	.481E-10	.309E-10	.156E-10	.787E-11	.394E-11
3s-7f	.250E-24	.425E-19	.895E-16	.118E-13	.271E-12	.202E-11	.732E-11	.163E-10	.262E-10	.335E-10	.360E-10	.287E-10	.200E-10	.131E-10	.843E-11	.426E-11	.214E-11	.107E-11
3s-7g	.220E-23	.355E-18	.690E-15	.815E-13	.163E-11	.105E-10	.324E-10	.628E-10	.898E-10	.105E-09	.102E-09	.776E-10	.531E-10	.347E-10	.222E-10	.112E-10	.564E-11	.282E-11
3s-7h	.110E-23	.175E-18	.334E-15	.384E-13	.744E-12	.463E-11	.140E-10	.266E-10	.374E-10	.432E-10	.408E-10	.305E-10	.206E-10	.133E-10	.850E-11	.429E-11	.216E-11	.108E-11
3s-7i	.957E-25	.152E-19	.286E-16	.325E-14	.621E-13	.380E-12	.113E-11	.210E-11	.290E-11	.329E-11	.307E-11	.229E-11	.155E-11	.100E-11	.642E-12	.324E-12	.163E-12	.818E-13
3p-7s	.100E-24	.165E-19	.324E-16	.375E-14	.884E-13	.712E-12	.266E-11	.627E-11	.108E-10	.148E-10	.184E-10	.171E-10	.140E-10	.108E-10	.810E-11	.510E-11	.310E-11	.175E-11
3p-7p	.141E-23	.223E-18	.412E-15	.436E-13	.855E-12	.594E-11	.187E-10	.380E-10	.585E-10	.744E-10	.834E-10	.703E-10	.512E-10	.346E-10	.226E-10	.115E-10	.582E-11	.291E-11
3p-7d	.940E-23	.149E-17	.280E-14	.302E-12	.610E-11	.435E-10	.136E-09	.269E-09	.393E-09	.472E-09	.484E-09	.397E-09	.301E-09	.222E-09	.163E-09	.100E-09	.602E-10	.338E-10
3p-7f	.555E-24	.918E-19	.186E-15	.233E-13	.505E-12	.357E-11	.123E-10	.262E-10	.407E-10	.511E-10	.544E-10	.439E-10	.311E-10	.206E-10	.133E-10	.675E-11	.340E-11	.170E-11
3p-7g	.320E-23	.514E-18	.992E-15	.116E-12	.231E-11	.148E-10	.459E-10	.895E-10	.129E-09	.151E-09	.145E-09	.110E-09	.745E-10	.484E-10	.309E-10	.156E-10	.782E-11	.392E-11
3p-7h	.109E-23	.175E-18	.333E-15	.385E-13	.747E-12	.466E-11	.141E-10	.266E-10	.372E-10	.428E-10	.406E-10	.306E-10	.209E-10	.136E-10	.868E-11	.439E-11	.221E-11	.111E-11
3p-7i	.779E-25	.124E-19	.234E-16	.267E-14	.513E-13	.317E-12	.950E-12	.179E-11	.250E-11	.286E-11	.268E-11	.200E-11	.135E-11	.869E-12	.554E-12	.280E-12	.141E-12	.705E-13
3d-7s	.211E-25	.338E-20	.646E-17	.735E-15	.171E-13	.136E-12	.499E-12	.114E-11	.188E-11	.247E-11	.278E-11	.230E-11	.164E-11	.109E-11	.707E-12	.359E-12	.180E-12	.903E-13
3d-7p	.843E-25	.134E-19	.254E-16	.284E-14	.647E-13	.509E-12	.192E-11	.461E-11	.800E-11	.110E-10	.132E-10	.115E-10	.864E-11	.615E-11	.427E-11	.243E-11	.136E-11	.735E-12
3d-7d	.114E-23	.179E-18	.331E-15	.349E-13	.676E-12	.465E-11	.143E-10	.284E-10	.432E-10	.549E-10	.625E-10	.533E-10	.389E-10	.263E-10	.172E-10	.875E-11	.441E-11	.221E-11
3d-7f	.129E-22	.204E-17	.382E-14	.409E-12	.824E-11	.584E-10	.183E-09	.359E-09	.523E-09	.623E-09	.623E-09	.494E-09	.361E-09	.257E-09	.182E-09	.108E-09	.628E-10	.347E-10
3d-7g	.571E-23	.913E-18	.175E-14	.202E-12	.395E-11	.249E-10	.763E-10	.147E-09	.211E-09	.248E-09	.245E-09	.190E-09	.133E-09	.878E-10	.568E-10	.289E-10	.146E-10	.729E-11
3d-7h	.888E-24	.142E-18	.271E-15	.314E-13	.615E-12	.388E-11	.119E-10	.228E-10	.323E-10	.376E-10	.360E-10	.271E-10	.184E-10	.120E-10	.766E-11	.387E-11	.194E-11	.973E-12
3d-7i	.439E-25	.699E-20	.132E-16	.152E-14	.293E-13	.181E-12	.544E-12	.103E-11	.144E-11	.165E-11	.157E-11	.118E-11	.806E-12	.525E-12	.336E-12	.170E-12	.854E-13	.428E-13
3s-8s	.108E-24	.310E-19	.851E-16	.113E-13	.267E-12	.210E-11	.709E-11	.152E-10	.245E-10	.322E-10	.376E-10	.325E-10	.240E-10	.163E-10	.107E-10	.548E-11	.277E-11	.139E-11
3s-8p	.579E-24	.169E-18	.474E-15	.646E-13	.160E-11	.130E-10	.437E-10	.907E-10	.138E-09	.172E-09	.188E-09	.163E-09	.130E-09	.995E-10	.752E-10	.479E-10	.294E-10	.167E-10
3s-8d	.260E-24	.760E-19	.214E-15	.293E-13	.735E-12	.601E-11	.203E-10	.419E-10	.627E-10	.759E-10	.768E-10	.600E-10	.417E-10	.274E-10	.176E-10	.893E-11	.449E-11	.225E-11
3s-8f	.258E-25	.785E-20	.236E-16	.360E-14	.110E-12	.101E-11	.390E-11	.908E-11	.150E-10	.195E-10	.213E-10	.171E-10	.120E-10	.788E-11	.505E-11	.255E-11	.128E-11	.642E-12
3s-8g	.217E-24	.635E-19	.179E-15	.245E-13	.607E-12	.494E-11	.164E-10	.333E-10	.489E-10	.583E-10	.577E-10	.444E-10	.305E-10	.200E-10	.128E-10	.646E-11	.325E-11	.163E-11
3s-8h	.137E-24	.398E-19	.110E-15	.147E-13	.349E-12	.274E-11	.884E-11	.175E-10	.253E-10	.298E-10	.287E-10	.216E-10	.146E-10	.948E-11	.605E-11	.305E-11	.154E-11	.769E-12
3s-8i	.195E-25	.562E-20	.154E-16	.204E-14	.474E-13	.367E-12	.116E-11	.225E-11	.319E-11	.369E-11	.351E-11	.264E-11	.179E-11	.116E-11	.742E-12	.375E-12	.189E-12	.946E-13
3s-8j	.751E-27	.215E-21	.585E-18	.767E-16	.176E-14	.134E-13	.421E-13	.813E-13	.115E-12	.133E-12	.125E-12	.933E-13	.627E-13	.405E-13	.258E-13	.130E-13	.657E-14	.329E-14
3p-8s	.911E-26	.275E-20	.807E-17	.120E-14	.353E-13	.320E-12	.129E-11	.320E-11	.570E-11	.803E-11	.103E-10	.973E-11	.798E-11	.617E-11	.464E-11	.292E-11	.177E-11	.100E-11
3p-8p	.139E-24	.403E-19	.111E-15	.148E-13	.352E-12	.279E-11	.939E-11	.200E-10	.318E-10	.413E-10	.475E-10	.405E-10	.297E-10	.201E-10	.132E-10	.672E-11	.339E-11	.170E-11

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
3p-8d	.933E-24	.272E-18	.759E-15	.103E-12	.254E-11	.205E-10	.687E-10	.142E-09	.213E-09	.260E-09	.272E-09	.225E-09	.171E-09	.127E-09	.927E-10	.571E-10	.342E-10	.192E-10
3p-8f	.458E-25	.137E-19	.401E-16	.590E-14	.170E-12	.151E-11	.566E-11	.128E-10	.208E-10	.267E-10	.294E-10	.242E-10	.172E-10	.115E-10	.743E-11	.378E-11	.190E-11	.951E-12
3p-8g	.322E-24	.940E-19	.263E-15	.357E-13	.879E-12	.710E-11	.236E-10	.481E-10	.711E-10	.850E-10	.837E-10	.638E-10	.434E-10	.283E-10	.180E-10	.911E-11	.457E-11	.229E-11
3p-8h	.137E-24	.399E-19	.111E-15	.148E-13	.353E-12	.279E-11	.897E-11	.177E-10	.255E-10	.299E-10	.288E-10	.219E-10	.150E-10	.977E-11	.626E-11	.316E-11	.159E-11	.796E-12
3p-8i	.160E-25	.461E-20	.127E-16	.168E-14	.395E-13	.308E-12	.985E-12	.194E-11	.278E-11	.324E-11	.309E-11	.232E-11	.157E-11	.102E-11	.647E-12	.327E-12	.164E-12	.824E-13
3p-8j	.552E-27	.159E-21	.434E-18	.573E-16	.133E-14	.102E-13	.321E-13	.621E-13	.877E-13	.101E-12	.957E-13	.718E-13	.486E-13	.315E-13	.201E-13	.102E-13	.513E-14	.257E-14
3d-8s	.188E-26	.554E-21	.159E-17	.236E-15	.705E-14	.636E-13	.255E-12	.618E-12	.106E-11	.143E-11	.165E-11	.138E-11	.988E-12	.660E-12	.427E-12	.217E-12	.109E-12	.546E-13
3d-8p	.819E-26	.239E-20	.672E-17	.962E-15	.271E-13	.239E-12	.972E-12	.246E-11	.442E-11	.621E-11	.765E-11	.672E-11	.507E-11	.360E-11	.250E-11	.141E-11	.788E-12	.424E-12
3d-8d	.113E-24	.326E-19	.898E-16	.119E-13	.280E-12	.219E-11	.717E-11	.149E-10	.234E-10	.304E-10	.354E-10	.306E-10	.225E-10	.152E-10	.994E-11	.508E-11	.256E-11	.128E-11
3d-8f	.124E-23	.360E-18	.100E-14	.135E-12	.332E-11	.267E-10	.891E-10	.183E-09	.274E-09	.333E-09	.339E-09	.272E-09	.198E-09	.141E-09	.999E-10	.590E-10	.343E-10	.189E-10
3d-8g	.601E-24	.174E-18	.484E-15	.651E-13	.157E-11	.125E-10	.408E-10	.822E-10	.121E-09	.145E-09	.146E-09	.114E-09	.800E-10	.531E-10	.343E-10	.175E-10	.881E-11	.441E-11
3d-8h	.114E-24	.331E-19	.918E-16	.124E-13	.298E-12	.237E-11	.774E-11	.155E-10	.227E-10	.268E-10	.262E-10	.199E-10	.136E-10	.884E-11	.565E-11	.285E-11	.143E-11	.718E-12
3d-8i	.912E-26	.264E-20	.727E-17	.969E-15	.229E-13	.179E-12	.574E-12	.113E-11	.163E-11	.191E-11	.184E-11	.140E-11	.957E-12	.624E-12	.400E-12	.202E-12	.102E-12	.509E-13
3d-8j	.243E-27	.702E-22	.193E-18	.256E-16	.603E-15	.471E-14	.151E-13	.296E-13	.425E-13	.496E-13	.474E-13	.356E-13	.240E-13	.156E-13	.993E-14	.502E-14	.252E-14	.126E-14
4s-5s	.105E-07	.299E-07	.623E-07	.104E-06	.146E-06	.183E-06	.208E-06	.221E-06	.223E-06	.218E-06	.194E-06	.163E-06	.131E-06	.102E-06	.767E-07	.482E-07	.294E-07	.176E-07
4s-5p	.694E-08	.194E-07	.410E-07	.711E-07	.108E-06	.151E-06	.198E-06	.249E-06	.301E-06	.352E-06	.440E-06	.489E-06	.494E-06	.464E-06	.412E-06	.321E-06	.237E-06	.168E-06
4s-5d	.123E-07	.362E-07	.781E-07	.136E-06	.200E-06	.263E-06	.314E-06	.351E-06	.369E-06	.372E-06	.342E-06	.285E-06	.224E-06	.169E-06	.125E-06	.768E-07	.464E-07	.277E-07
4s-5f	.205E-07	.558E-07	.112E-06	.178E-06	.241E-06	.288E-06	.316E-06	.323E-06	.315E-06	.296E-06	.241E-06	.184E-06	.136E-06	.982E-07	.703E-07	.422E-07	.252E-07	.150E-07
4s-5g	.151E-07	.395E-07	.755E-07	.115E-06	.149E-06	.171E-06	.181E-06	.180E-06	.171E-06	.157E-06	.125E-06	.949E-07	.698E-07	.505E-07	.361E-07	.216E-07	.129E-07	.769E-08
4p-5s	.240E-08	.625E-08	.119E-07	.182E-07	.239E-07	.284E-07	.319E-07	.348E-07	.376E-07	.403E-07	.456E-07	.487E-07	.485E-07	.452E-07	.401E-07	.313E-07	.231E-07	.164E-07
4p-5p	.122E-07	.347E-07	.723E-07	.121E-06	.171E-06	.216E-06	.247E-06	.266E-06	.271E-06	.267E-06	.240E-06	.201E-06	.161E-06	.124E-06	.934E-07	.584E-07	.356E-07	.213E-07
4p-5d	.849E-08	.256E-07	.578E-07	.106E-06	.168E-06	.238E-06	.313E-06	.387E-06	.458E-06	.520E-06	.612E-06	.648E-06	.631E-06	.577E-06	.503E-06	.385E-06	.281E-06	.197E-06
4p-5f	.233E-07	.655E-07	.135E-06	.224E-06	.315E-06	.395E-06	.451E-06	.483E-06	.490E-06	.479E-06	.420E-06	.339E-06	.260E-06	.194E-06	.142E-06	.866E-07	.521E-07	.311E-07
4p-5g	.198E-07	.531E-07	.104E-06	.163E-06	.216E-06	.254E-06	.275E-06	.278E-06	.268E-06	.249E-06	.201E-06	.152E-06	.112E-06	.808E-07	.578E-07	.346E-07	.206E-07	.123E-07
4d-5s	.378E-09	.997E-09	.195E-08	.313E-08	.438E-08	.559E-08	.662E-08	.743E-08	.795E-08	.816E-08	.778E-08	.668E-08	.535E-08	.409E-08	.303E-08	.188E-08	.113E-08	.677E-09
4d-5p	.277E-08	.733E-08	.142E-07	.220E-07	.292E-07	.345E-07	.382E-07	.402E-07	.412E-07	.415E-07	.408E-07	.387E-07	.353E-07	.310E-07	.263E-07	.196E-07	.140E-07	.971E-08
4d-5d	.130E-07	.367E-07	.756E-07	.125E-06	.175E-06	.218E-06	.246E-06	.261E-06	.264E-06	.257E-06	.227E-06	.188E-06	.149E-06	.114E-06	.852E-07	.531E-07	.323E-07	.193E-07
4d-5f	.173E-07	.525E-07	.119E-06	.218E-06	.342E-06	.480E-06	.622E-06	.754E-06	.872E-06	.968E-06	.109E-05	.111E-05	.104E-05	.932E-06	.800E-06	.602E-06	.434E-06	.302E-06
4d-5g	.333E-07	.927E-07	.188E-06	.307E-06	.423E-06	.520E-06	.581E-06	.610E-06	.608E-06	.584E-06	.498E-06	.395E-06	.300E-06	.221E-06	.161E-06	.979E-07	.588E-07	.350E-07
4f-5s	.589E-10	.162E-09	.336E-09	.561E-09	.808E-09	.104E-08	.123E-08	.136E-08	.142E-08	.141E-08	.125E-08	.101E-08	.768E-09	.564E-09	.407E-09	.246E-09	.147E-09	.871E-10
4f-5p	.334E-09	.890E-09	.174E-08	.276E-08	.377E-08	.465E-08	.533E-08	.578E-08	.600E-08	.599E-08	.547E-08	.456E-08	.356E-08	.268E-08	.196E-08	.120E-08	.721E-09	.429E-09

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
4f-5d	.185E-08	.489E-08	.950E-08	.146E-07	.193E-07	.227E-07	.249E-07	.258E-07	.259E-07	.254E-07	.234E-07	.205E-07	.174E-07	.144E-07	.117E-07	.823E-08	.565E-08	.380E-08
4f-5f	.114E-07	.320E-07	.654E-07	.107E-06	.148E-06	.182E-06	.203E-06	.214E-06	.214E-06	.207E-06	.181E-06	.148E-06	.117E-06	.887E-07	.659E-07	.409E-07	.248E-07	.148E-07
4f-5g	.490E-07	.146E-06	.323E-06	.574E-06	.875E-06	.119E-05	.149E-05	.173E-05	.193E-05	.207E-05	.217E-05	.209E-05	.190E-05	.164E-05	.138E-05	.101E-05	.715E-06	.492E-06
4s-6s	.238E-09	.118E-08	.364E-08	.800E-08	.138E-07	.198E-07	.250E-07	.287E-07	.307E-07	.313E-07	.297E-07	.262E-07	.219E-07	.174E-07	.133E-07	.847E-08	.520E-08	.312E-08
4s-6p	.180E-09	.898E-09	.283E-08	.650E-08	.120E-07	.188E-07	.266E-07	.348E-07	.432E-07	.518E-07	.672E-07	.772E-07	.797E-07	.756E-07	.676E-07	.529E-07	.391E-07	.277E-07
4s-6d	.343E-09	.172E-08	.537E-08	.120E-07	.213E-07	.319E-07	.421E-07	.504E-07	.561E-07	.588E-07	.567E-07	.486E-07	.387E-07	.294E-07	.217E-07	.134E-07	.807E-08	.482E-08
4s-6f	.353E-09	.168E-08	.500E-08	.106E-07	.176E-07	.246E-07	.301E-07	.337E-07	.351E-07	.347E-07	.303E-07	.240E-07	.180E-07	.132E-07	.949E-08	.572E-08	.341E-08	.203E-08
4s-6g	.380E-10	.205E-09	.704E-09	.173E-08	.333E-08	.529E-08	.725E-08	.886E-08	.991E-08	.103E-07	.975E-08	.811E-08	.628E-08	.466E-08	.339E-08	.206E-08	.123E-08	.731E-09
4s-6h	.659E-09	.312E-08	.915E-08	.190E-07	.308E-07	.418E-07	.499E-07	.542E-07	.551E-07	.532E-07	.449E-07	.349E-07	.259E-07	.188E-07	.135E-07	.808E-08	.482E-08	.287E-08
4p-6s	.582E-10	.272E-09	.791E-09	.163E-08	.267E-08	.372E-08	.464E-08	.538E-08	.598E-08	.647E-08	.726E-08	.770E-08	.764E-08	.711E-08	.628E-08	.487E-08	.358E-08	.253E-08
4p-6p	.281E-09	.139E-08	.430E-08	.947E-08	.164E-07	.238E-07	.305E-07	.354E-07	.384E-07	.396E-07	.381E-07	.336E-07	.279E-07	.220E-07	.167E-07	.105E-07	.644E-08	.387E-08
4p-6d	.251E-09	.130E-08	.426E-08	.101E-07	.190E-07	.303E-07	.427E-07	.555E-07	.677E-07	.790E-07	.964E-07	.104E-06	.103E-06	.942E-07	.820E-07	.626E-07	.455E-07	.318E-07
4p-6f	.488E-09	.238E-08	.723E-08	.157E-07	.267E-07	.382E-07	.482E-07	.552E-07	.588E-07	.594E-07	.538E-07	.442E-07	.342E-07	.255E-07	.186E-07	.114E-07	.681E-08	.406E-08
4p-6g	.677E-10	.334E-09	.103E-08	.232E-08	.412E-08	.621E-08	.827E-08	.100E-07	.112E-07	.118E-07	.113E-07	.952E-08	.742E-08	.554E-08	.402E-08	.245E-08	.146E-08	.870E-09
4p-6h	.698E-09	.335E-08	.996E-08	.210E-07	.346E-07	.476E-07	.574E-07	.629E-07	.643E-07	.624E-07	.532E-07	.418E-07	.313E-07	.229E-07	.165E-07	.992E-08	.593E-08	.353E-08
4d-6s	.994E-11	.464E-10	.136E-09	.293E-09	.507E-09	.759E-09	.102E-08	.127E-08	.146E-08	.159E-08	.163E-08	.146E-08	.119E-08	.920E-09	.684E-09	.425E-09	.256E-09	.153E-09
4d-6p	.649E-10	.308E-09	.911E-09	.192E-08	.321E-08	.454E-08	.572E-08	.664E-08	.727E-08	.761E-08	.763E-08	.711E-08	.629E-08	.535E-08	.442E-08	.318E-08	.222E-08	.151E-08
4d-6d	.291E-09	.144E-08	.442E-08	.969E-08	.166E-07	.239E-07	.302E-07	.346E-07	.371E-07	.378E-07	.358E-07	.311E-07	.255E-07	.200E-07	.151E-07	.947E-08	.577E-08	.346E-08
4d-6f	.509E-09	.263E-08	.858E-08	.201E-07	.371E-07	.579E-07	.801E-07	.102E-06	.121E-06	.138E-06	.160E-06	.164E-06	.155E-06	.138E-06	.117E-06	.873E-07	.623E-07	.431E-07
4d-6g	.263E-09	.127E-08	.380E-08	.811E-08	.136E-07	.191E-07	.236E-07	.266E-07	.279E-07	.279E-07	.249E-07	.203E-07	.157E-07	.116E-07	.848E-08	.515E-08	.309E-08	.184E-08
4d-6h	.747E-09	.365E-08	.111E-07	.240E-07	.406E-07	.576E-07	.718E-07	.813E-07	.856E-07	.854E-07	.757E-07	.607E-07	.461E-07	.338E-07	.244E-07	.147E-07	.879E-08	.523E-08
4f-6s	.147E-11	.746E-11	.242E-10	.570E-10	.107E-09	.170E-09	.236E-09	.296E-09	.339E-09	.362E-09	.351E-09	.296E-09	.230E-09	.171E-09	.124E-09	.755E-10	.449E-10	.266E-10
4f-6p	.767E-11	.378E-10	.117E-09	.262E-09	.467E-09	.707E-09	.949E-09	.116E-08	.132E-08	.141E-08	.139E-08	.120E-08	.952E-09	.720E-09	.529E-09	.324E-09	.194E-09	.115E-09
4f-6d	.386E-10	.184E-09	.549E-09	.117E-08	.197E-08	.282E-08	.360E-08	.424E-08	.468E-08	.491E-08	.481E-08	.425E-08	.352E-08	.281E-08	.218E-08	.147E-08	.962E-09	.623E-09
4f-6f	.222E-09	.110E-08	.336E-08	.734E-08	.125E-07	.178E-07	.222E-07	.253E-07	.270E-07	.275E-07	.261E-07	.228E-07	.187E-07	.145E-07	.109E-07	.683E-08	.416E-08	.249E-08
4f-6g	.976E-09	.497E-08	.158E-07	.362E-07	.654E-07	.998E-07	.135E-06	.167E-06	.194E-06	.214E-06	.232E-06	.224E-06	.201E-06	.171E-06	.141E-06	.102E-06	.706E-07	.479E-07
4f-6h	.639E-09	.321E-08	.101E-07	.226E-07	.399E-07	.592E-07	.773E-07	.917E-07	.101E-06	.106E-06	.102E-06	.877E-07	.705E-07	.541E-07	.403E-07	.250E-07	.152E-07	.907E-08
4s-7s	.254E-10	.177E-09	.691E-09	.180E-08	.348E-08	.547E-08	.736E-08	.885E-08	.980E-08	.103E-07	.101E-07	.917E-08	.781E-08	.630E-08	.485E-08	.310E-08	.191E-08	.115E-08
4s-7p	.204E-10	.143E-09	.574E-09	.157E-08	.327E-08	.561E-08	.837E-08	.113E-07	.144E-07	.174E-07	.231E-07	.269E-07	.281E-07	.268E-07	.240E-07	.189E-07	.139E-07	.989E-08
4s-7d	.398E-10	.279E-09	.110E-08	.291E-08	.576E-08	.934E-08	.131E-07	.164E-07	.188E-07	.202E-07	.201E-07	.175E-07	.140E-07	.107E-07	.791E-08	.488E-08	.294E-08	.176E-08

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
4s-7f	.306E-10	.207E-09	.788E-09	.200E-08	.380E-08	.588E-08	.781E-08	.929E-08	.102E-07	.104E-07	.957E-08	.783E-08	.599E-08	.442E-08	.319E-08	.193E-08	.115E-08	.686E-09
4s-7g	.830E-11	.607E-10	.255E-09	.723E-09	.153E-08	.260E-08	.373E-08	.471E-08	.539E-08	.571E-08	.546E-08	.457E-08	.355E-08	.264E-08	.192E-08	.117E-08	.697E-09	.414E-09
4s-7h	.540E-10	.364E-09	.138E-08	.344E-08	.639E-08	.958E-08	.123E-07	.142E-07	.150E-07	.150E-07	.132E-07	.105E-07	.790E-08	.577E-08	.415E-08	.249E-08	.149E-08	.887E-09
4s-7i	.366E-10	.245E-09	.921E-09	.228E-08	.419E-08	.621E-08	.787E-08	.892E-08	.933E-08	.919E-08	.796E-08	.629E-08	.473E-08	.345E-08	.248E-08	.149E-08	.893E-09	.532E-09
4p-7s	.624E-11	.415E-10	.155E-09	.384E-09	.720E-09	.111E-08	.148E-08	.181E-08	.207E-08	.228E-08	.257E-08	.272E-08	.268E-08	.249E-08	.219E-08	.169E-08	.124E-08	.874E-09
4p-7p	.301E-10	.209E-09	.821E-09	.214E-08	.419E-08	.665E-08	.906E-08	.111E-07	.125E-07	.132E-07	.133E-07	.120E-07	.101E-07	.807E-08	.616E-08	.391E-08	.239E-08	.144E-08
4p-7d	.303E-10	.218E-09	.903E-09	.252E-08	.531E-08	.914E-08	.136E-07	.183E-07	.228E-07	.270E-07	.335E-07	.367E-07	.363E-07	.334E-07	.291E-07	.221E-07	.160E-07	.112E-07
4p-7f	.481E-10	.329E-09	.127E-08	.326E-08	.628E-08	.981E-08	.132E-07	.158E-07	.174E-07	.180E-07	.168E-07	.140E-07	.109E-07	.818E-08	.597E-08	.364E-08	.218E-08	.130E-08
4p-7g	.686E-11	.498E-10	.206E-09	.577E-09	.122E-08	.211E-08	.312E-08	.409E-08	.486E-08	.534E-08	.543E-08	.472E-08	.375E-08	.283E-08	.206E-08	.126E-08	.754E-09	.449E-09
4p-7h	.631E-10	.429E-09	.164E-08	.415E-08	.780E-08	.118E-07	.153E-07	.177E-07	.188E-07	.188E-07	.166E-07	.133E-07	.101E-07	.744E-08	.537E-08	.324E-08	.194E-08	.115E-08
4p-7i	.342E-10	.231E-09	.877E-09	.220E-08	.409E-08	.614E-08	.789E-08	.907E-08	.959E-08	.955E-08	.838E-08	.665E-08	.500E-08	.365E-08	.262E-08	.157E-08	.940E-09	.560E-09
4d-7s	.106E-11	.709E-11	.271E-10	.702E-10	.140E-09	.233E-09	.340E-09	.447E-09	.540E-09	.606E-09	.646E-09	.588E-09	.485E-09	.376E-09	.280E-09	.174E-09	.105E-09	.626E-10
4d-7p	.685E-11	.463E-10	.176E-09	.447E-09	.857E-09	.135E-08	.185E-08	.228E-08	.261E-08	.282E-08	.290E-08	.270E-08	.236E-08	.199E-08	.162E-08	.115E-08	.786E-09	.528E-09
4d-7d	.306E-10	.212E-09	.831E-09	.216E-08	.420E-08	.661E-08	.891E-08	.107E-07	.119E-07	.125E-07	.124E-07	.110E-07	.923E-08	.730E-08	.554E-08	.349E-08	.213E-08	.128E-08
4d-7f	.601E-10	.432E-09	.177E-08	.487E-08	.101E-07	.169E-07	.246E-07	.323E-07	.394E-07	.455E-07	.535E-07	.555E-07	.525E-07	.465E-07	.394E-07	.291E-07	.207E-07	.142E-07
4d-7g	.157E-10	.107E-09	.411E-09	.106E-08	.204E-08	.321E-08	.436E-08	.533E-08	.601E-08	.638E-08	.632E-08	.553E-08	.447E-08	.344E-08	.255E-08	.158E-08	.955E-09	.570E-09
4d-7h	.812E-10	.559E-09	.217E-08	.559E-08	.107E-07	.167E-07	.222E-07	.264E-07	.288E-07	.295E-07	.270E-07	.221E-07	.169E-07	.124E-07	.899E-08	.544E-08	.325E-08	.193E-08
4d-7i	.283E-10	.194E-09	.748E-09	.191E-08	.363E-08	.557E-08	.730E-08	.854E-08	.917E-08	.925E-08	.829E-08	.670E-08	.511E-08	.376E-08	.272E-08	.165E-08	.986E-09	.587E-09
4f-7s	.166E-12	.122E-11	.516E-11	.149E-10	.326E-10	.579E-10	.878E-10	.117E-09	.141E-09	.155E-09	.157E-09	.134E-09	.106E-09	.791E-10	.576E-10	.351E-10	.209E-10	.123E-10
4f-7p	.773E-12	.554E-11	.228E-10	.632E-10	.133E-09	.228E-09	.336E-09	.441E-09	.526E-09	.581E-09	.597E-09	.524E-09	.420E-09	.319E-09	.234E-09	.144E-09	.860E-10	.511E-10
4f-7d	.391E-11	.266E-10	.102E-09	.262E-09	.508E-09	.816E-09	.114E-08	.145E-08	.169E-08	.185E-08	.189E-08	.169E-08	.140E-08	.110E-08	.849E-09	.558E-09	.359E-09	.228E-09
4f-7f	.217E-10	.151E-09	.589E-09	.153E-08	.296E-08	.462E-08	.618E-08	.741E-08	.824E-08	.869E-08	.868E-08	.783E-08	.655E-08	.516E-08	.390E-08	.245E-08	.149E-08	.892E-09
4f-7g	.970E-10	.689E-09	.277E-08	.744E-08	.150E-07	.247E-07	.353E-07	.455E-07	.544E-07	.612E-07	.678E-07	.662E-07	.592E-07	.502E-07	.410E-07	.291E-07	.200E-07	.135E-07
4f-7h	.891E-10	.626E-09	.249E-08	.659E-08	.131E-07	.211E-07	.293E-07	.363E-07	.414E-07	.442E-07	.439E-07	.385E-07	.313E-07	.241E-07	.180E-07	.112E-07	.678E-08	.405E-08
4f-7i	.167E-10	.116E-09	.457E-09	.119E-08	.233E-08	.369E-08	.502E-08	.608E-08	.677E-08	.707E-08	.670E-08	.561E-08	.437E-08	.325E-08	.236E-08	.144E-08	.860E-09	.512E-09
4s-8s	.563E-11	.488E-10	.223E-09	.647E-09	.136E-08	.225E-08	.316E-08	.392E-08	.444E-08	.473E-08	.478E-08	.440E-08	.379E-08	.308E-08	.238E-08	.153E-08	.942E-09	.567E-09
4s-8p	.464E-11	.407E-10	.192E-09	.587E-09	.133E-08	.241E-08	.375E-08	.521E-08	.671E-08	.821E-08	.110E-07	.129E-07	.136E-07	.130E-07	.117E-07	.916E-08	.678E-08	.481E-08
4s-8d	.919E-11	.802E-10	.370E-09	.109E-08	.233E-08	.399E-08	.580E-08	.746E-08	.875E-08	.955E-08	.969E-08	.851E-08	.687E-08	.526E-08	.389E-08	.240E-08	.145E-08	.863E-09
4s-8f	.599E-11	.505E-10	.226E-09	.645E-09	.134E-08	.222E-08	.312E-08	.387E-08	.438E-08	.461E-08	.439E-08	.366E-08	.284E-08	.211E-08	.153E-08	.928E-09	.554E-09	.329E-09
4s-8g	.300E-11	.266E-10	.127E-09	.388E-09	.862E-09	.151E-08	.222E-08	.284E-08	.328E-08	.350E-08	.337E-08	.283E-08	.220E-08	.164E-08	.119E-08	.724E-09	.433E-09	.257E-09
4s-8h	.919E-11	.783E-10	.351E-09	.993E-09	.202E-08	.326E-08	.442E-08	.530E-08	.580E-08	.593E-08	.538E-08	.435E-08	.330E-08	.242E-08	.175E-08	.105E-08	.628E-09	.374E-09

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
4s-8i	.116E-10	.974E-10	.430E-09	.119E-08	.239E-08	.376E-08	.498E-08	.583E-08	.623E-08	.625E-08	.553E-08	.443E-08	.335E-08	.245E-08	.176E-08	.106E-08	.636E-09	.379E-09
4s-8j	.195E-11	.162E-10	.710E-10	.196E-09	.388E-09	.606E-09	.800E-09	.934E-09	.997E-09	.999E-09	.881E-09	.701E-09	.526E-09	.383E-09	.275E-09	.165E-09	.986E-10	.588E-10
4p-8s	.138E-11	.115E-10	.504E-10	.141E-09	.288E-09	.473E-09	.667E-09	.842E-09	.985E-09	.110E-08	.125E-08	.131E-08	.129E-08	.120E-08	.105E-08	.812E-09	.593E-09	.417E-09
4p-8p	.670E-11	.581E-10	.267E-09	.777E-09	.164E-08	.276E-08	.393E-08	.494E-08	.570E-08	.616E-08	.634E-08	.583E-08	.496E-08	.398E-08	.305E-08	.194E-08	.119E-08	.714E-09
4p-8d	.722E-11	.646E-10	.311E-09	.965E-09	.219E-08	.398E-08	.614E-08	.844E-08	.107E-07	.128E-07	.161E-07	.177E-07	.176E-07	.161E-07	.141E-07	.107E-07	.774E-08	.540E-08
4p-8f	.994E-11	.853E-10	.387E-09	.111E-08	.232E-08	.385E-08	.539E-08	.667E-08	.753E-08	.792E-08	.757E-08	.638E-08	.500E-08	.376E-08	.274E-08	.167E-08	.100E-08	.598E-09
4p-8g	.223E-11	.200E-10	.962E-10	.297E-09	.670E-09	.121E-08	.183E-08	.244E-08	.294E-08	.326E-08	.334E-08	.293E-08	.233E-08	.176E-08	.128E-08	.785E-09	.469E-09	.279E-09
4p-8h	.115E-10	.987E-10	.445E-09	.127E-08	.261E-08	.423E-08	.575E-08	.690E-08	.754E-08	.770E-08	.700E-08	.570E-08	.436E-08	.322E-08	.233E-08	.141E-08	.841E-09	.501E-09
4p-8i	.111E-10	.943E-10	.420E-09	.118E-08	.239E-08	.381E-08	.512E-08	.607E-08	.658E-08	.666E-08	.598E-08	.480E-08	.363E-08	.266E-08	.191E-08	.115E-08	.687E-09	.409E-09
4p-8j	.166E-11	.140E-10	.618E-10	.172E-09	.346E-09	.546E-09	.726E-09	.852E-09	.914E-09	.918E-09	.814E-09	.652E-09	.493E-09	.361E-09	.260E-09	.156E-09	.936E-10	.558E-10
4d-8s	.243E-12	.203E-11	.909E-11	.266E-10	.579E-10	.103E-09	.158E-09	.215E-09	.267E-09	.305E-09	.332E-09	.306E-09	.254E-09	.197E-09	.147E-09	.913E-10	.550E-10	.329E-10
4d-8p	.149E-11	.127E-10	.567E-10	.163E-09	.341E-09	.577E-09	.834E-09	.107E-08	.126E-08	.139E-08	.146E-08	.136E-08	.119E-08	.992E-09	.801E-09	.562E-09	.382E-09	.255E-09
4d-8d	.672E-11	.582E-10	.267E-09	.776E-09	.163E-08	.272E-08	.383E-08	.478E-08	.544E-08	.583E-08	.589E-08	.535E-08	.452E-08	.359E-08	.274E-08	.173E-08	.106E-08	.634E-09
4d-8f	.141E-10	.126E-09	.599E-09	.183E-08	.407E-08	.720E-08	.108E-07	.146E-07	.180E-07	.210E-07	.250E-07	.261E-07	.247E-07	.218E-07	.185E-07	.136E-07	.961E-08	.659E-08
4d-8g	.262E-11	.226E-10	.104E-09	.304E-09	.651E-09	.111E-08	.163E-08	.212E-08	.252E-08	.280E-08	.297E-08	.273E-08	.228E-08	.178E-08	.134E-08	.842E-09	.511E-09	.306E-09
4d-8h	.170E-10	.146E-09	.664E-09	.192E-08	.400E-08	.660E-08	.919E-08	.113E-07	.126E-07	.132E-07	.123E-07	.102E-07	.783E-08	.578E-08	.419E-08	.253E-08	.151E-08	.902E-09
4d-8i	.980E-11	.839E-10	.379E-09	.109E-08	.224E-08	.365E-08	.500E-08	.603E-08	.663E-08	.680E-08	.623E-08	.509E-08	.390E-08	.289E-08	.209E-08	.127E-08	.758E-09	.452E-09
4d-8j	.116E-11	.984E-11	.442E-10	.125E-09	.256E-09	.413E-09	.560E-09	.671E-09	.732E-09	.747E-09	.677E-09	.548E-09	.416E-09	.305E-09	.219E-09	.132E-09	.790E-10	.471E-10
4f-8s	.367E-13	.346E-12	.177E-11	.584E-11	.141E-10	.270E-10	.432E-10	.599E-10	.739E-10	.832E-10	.859E-10	.746E-10	.589E-10	.443E-10	.323E-10	.197E-10	.117E-10	.691E-11
4f-8p	.171E-12	.155E-11	.760E-11	.241E-10	.559E-10	.104E-09	.162E-09	.221E-09	.271E-09	.306E-09	.321E-09	.285E-09	.229E-09	.175E-09	.128E-09	.788E-10	.471E-10	.280E-10
4f-8d	.837E-12	.714E-11	.323E-10	.935E-10	.199E-09	.345E-09	.513E-09	.681E-09	.823E-09	.922E-09	.972E-09	.877E-09	.725E-09	.571E-09	.436E-09	.284E-09	.181E-09	.114E-09
4f-8f	.457E-11	.396E-10	.182E-09	.528E-09	.111E-08	.184E-08	.257E-08	.319E-08	.364E-08	.393E-08	.405E-08	.373E-08	.315E-08	.250E-08	.190E-08	.119E-08	.727E-09	.435E-09
4f-8g	.206E-10	.182E-09	.851E-09	.254E-08	.552E-08	.959E-08	.142E-07	.188E-07	.229E-07	.261E-07	.295E-07	.289E-07	.259E-07	.219E-07	.178E-07	.125E-07	.858E-08	.574E-08
4f-8h	.217E-10	.190E-09	.883E-09	.261E-08	.561E-08	.956E-08	.138E-07	.176E-07	.205E-07	.223E-07	.226E-07	.200E-07	.164E-07	.126E-07	.943E-08	.587E-08	.356E-08	.213E-08
4f-8i	.636E-11	.551E-10	.253E-09	.739E-09	.156E-08	.262E-08	.371E-08	.464E-08	.529E-08	.561E-08	.543E-08	.461E-08	.361E-08	.269E-08	.196E-08	.119E-08	.715E-09	.426E-09
4f-8j	.524E-12	.451E-11	.206E-10	.595E-10	.124E-09	.205E-09	.285E-09	.348E-09	.388E-09	.403E-09	.376E-09	.311E-09	.241E-09	.179E-09	.130E-09	.789E-10	.473E-10	.282E-10
5s-6s	.137E-06	.248E-06	.378E-06	.500E-06	.602E-06	.665E-06	.695E-06	.689E-06	.659E-06	.617E-06	.515E-06	.412E-06	.320E-06	.242E-06	.180E-06	.111E-06	.674E-07	.404E-07
5s-6p	.933E-07	.160E-06	.240E-06	.330E-06	.427E-06	.535E-06	.648E-06	.770E-06	.890E-06	.100E-05	.117E-05	.123E-05	.119E-05	.109E-05	.942E-06	.719E-06	.523E-06	.367E-06
5s-6d	.123E-06	.233E-06	.372E-06	.519E-06	.660E-06	.769E-06	.848E-06	.880E-06	.877E-06	.844E-06	.726E-06	.582E-06	.446E-06	.332E-06	.243E-06	.148E-06	.892E-07	.532E-07
5s-6f	.214E-06	.363E-06	.514E-06	.642E-06	.724E-06	.763E-06	.756E-06	.724E-06	.670E-06	.606E-06	.472E-06	.352E-06	.256E-06	.184E-06	.131E-06	.783E-07	.467E-07	.278E-07
5s-6g	.206E-06	.330E-06	.446E-06	.527E-06	.569E-06	.573E-06	.550E-06	.508E-06	.459E-06	.406E-06	.306E-06	.225E-06	.163E-06	.117E-06	.830E-07	.496E-07	.296E-07	.176E-07

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
5s-6h	.990E-07	.155E-06	.205E-06	.239E-06	.255E-06	.255E-06	.245E-06	.227E-06	.205E-06	.182E-06	.138E-06	.101E-06	.732E-07	.523E-07	.371E-07	.221E-07	.132E-07	.785E-08
5p-6s	.340E-07	.543E-07	.725E-07	.869E-07	.966E-07	.104E-06	.110E-06	.117E-06	.125E-06	.133E-06	.147E-06	.152E-06	.146E-06	.133E-06	.116E-06	.881E-07	.641E-07	.450E-07
5p-6p	.163E-06	.292E-06	.445E-06	.589E-06	.712E-06	.791E-06	.833E-06	.833E-06	.803E-06	.756E-06	.636E-06	.509E-06	.394E-06	.298E-06	.220E-06	.136E-06	.823E-07	.492E-07
5p-6d	.974E-07	.183E-06	.296E-06	.430E-06	.576E-06	.729E-06	.877E-06	.102E-05	.116E-05	.127E-05	.141E-05	.143E-05	.135E-05	.120E-05	.103E-05	.778E-06	.560E-06	.391E-06
5p-6f	.208E-06	.374E-06	.567E-06	.752E-06	.910E-06	.102E-05	.108E-05	.108E-05	.105E-05	.987E-06	.819E-06	.641E-06	.484E-06	.357E-06	.259E-06	.157E-06	.944E-07	.563E-07
5p-6g	.258E-06	.426E-06	.586E-06	.712E-06	.785E-06	.809E-06	.788E-06	.743E-06	.679E-06	.608E-06	.467E-06	.345E-06	.250E-06	.179E-06	.127E-06	.761E-07	.453E-07	.270E-07
5p-6h	.131E-06	.207E-06	.277E-06	.326E-06	.350E-06	.351E-06	.337E-06	.312E-06	.282E-06	.249E-06	.189E-06	.139E-06	.101E-06	.722E-07	.514E-07	.307E-07	.183E-07	.109E-07
5d-6s	.789E-08	.125E-07	.173E-07	.214E-07	.250E-07	.278E-07	.296E-07	.304E-07	.302E-07	.293E-07	.257E-07	.209E-07	.162E-07	.122E-07	.894E-08	.548E-08	.330E-08	.197E-08
5d-6p	.445E-07	.723E-07	.979E-07	.118E-06	.130E-06	.137E-06	.139E-06	.140E-06	.140E-06	.139E-06	.136E-06	.129E-06	.117E-06	.102E-06	.861E-07	.638E-07	.455E-07	.314E-07
5d-6d	.173E-06	.308E-06	.465E-06	.609E-06	.728E-06	.800E-06	.834E-06	.825E-06	.789E-06	.737E-06	.613E-06	.485E-06	.374E-06	.281E-06	.207E-06	.127E-06	.771E-07	.461E-07
5d-6f	.141E-06	.276E-06	.462E-06	.687E-06	.930E-06	.118E-05	.142E-05	.164E-05	.183E-05	.198E-05	.214E-05	.212E-05	.196E-05	.173E-05	.147E-05	.110E-05	.784E-06	.543E-06
5d-6g	.333E-06	.582E-06	.862E-06	.111E-05	.132E-05	.144E-05	.149E-05	.147E-05	.140E-05	.130E-05	.106E-05	.814E-06	.608E-06	.446E-06	.322E-06	.195E-06	.117E-06	.697E-07
5d-6h	.219E-06	.359E-06	.491E-06	.592E-06	.648E-06	.664E-06	.643E-06	.604E-06	.550E-06	.491E-06	.376E-06	.278E-06	.201E-06	.144E-06	.102E-06	.610E-07	.364E-07	.216E-07
5f-6s	.158E-08	.272E-08	.395E-08	.513E-08	.609E-08	.676E-08	.710E-08	.712E-08	.689E-08	.645E-08	.526E-08	.402E-08	.297E-08	.216E-08	.154E-08	.928E-09	.552E-09	.328E-09
5f-6p	.904E-08	.144E-07	.198E-07	.242E-07	.274E-07	.294E-07	.302E-07	.300E-07	.289E-07	.274E-07	.231E-07	.183E-07	.140E-07	.104E-07	.754E-08	.459E-08	.275E-08	.164E-08
5f-6d	.393E-07	.638E-07	.860E-07	.103E-06	.112E-06	.116E-06	.115E-06	.112E-06	.109E-06	.104E-06	.941E-07	.836E-07	.722E-07	.607E-07	.498E-07	.358E-07	.250E-07	.170E-07
5f-6f	.177E-06	.312E-06	.465E-06	.602E-06	.712E-06	.774E-06	.798E-06	.783E-06	.743E-06	.690E-06	.567E-06	.445E-06	.340E-06	.254E-06	.187E-06	.115E-06	.693E-07	.414E-07
5f-6g	.290E-06	.567E-06	.935E-06	.137E-05	.182E-05	.226E-05	.264E-05	.298E-05	.324E-05	.342E-05	.354E-05	.340E-05	.307E-05	.265E-05	.222E-05	.163E-05	.116E-05	.797E-06
5f-6h	.419E-06	.721E-06	.105E-05	.133E-05	.154E-05	.165E-05	.169E-05	.164E-05	.154E-05	.141E-05	.112E-05	.854E-06	.634E-06	.461E-06	.332E-06	.201E-06	.120E-06	.716E-07
5g-6s	.351E-09	.587E-09	.835E-09	.105E-08	.121E-08	.131E-08	.136E-08	.135E-08	.130E-08	.121E-08	.989E-09	.759E-09	.564E-09	.410E-09	.294E-09	.177E-09	.105E-09	.623E-10
5g-6p	.139E-08	.229E-08	.322E-08	.405E-08	.469E-08	.512E-08	.534E-08	.534E-08	.517E-08	.484E-08	.395E-08	.303E-08	.225E-08	.163E-08	.117E-08	.702E-09	.417E-09	.248E-09
5g-6d	.506E-08	.806E-08	.109E-07	.132E-07	.147E-07	.156E-07	.158E-07	.157E-07	.150E-07	.141E-07	.118E-07	.930E-08	.703E-08	.517E-08	.374E-08	.227E-08	.136E-08	.807E-09
5g-6f	.230E-07	.376E-07	.510E-07	.612E-07	.668E-07	.689E-07	.684E-07	.661E-07	.631E-07	.592E-07	.509E-07	.426E-07	.348E-07	.279E-07	.220E-07	.151E-07	.101E-07	.670E-08
5g-6g	.149E-06	.260E-06	.383E-06	.490E-06	.571E-06	.613E-06	.625E-06	.607E-06	.571E-06	.527E-06	.429E-06	.334E-06	.254E-06	.189E-06	.138E-06	.845E-07	.509E-07	.304E-07
5g-6h	.736E-06	.140E-05	.222E-05	.314E-05	.401E-05	.480E-05	.541E-05	.590E-05	.620E-05	.635E-05	.623E-05	.573E-05	.502E-05	.424E-05	.349E-05	.252E-05	.176E-05	.120E-05
5s-7s	.655E-08	.164E-07	.314E-07	.490E-07	.659E-07	.795E-07	.879E-07	.917E-07	.912E-07	.882E-07	.778E-07	.648E-07	.520E-07	.402E-07	.302E-07	.189E-07	.115E-07	.690E-08
5s-7p	.530E-08	.127E-07	.239E-07	.380E-07	.539E-07	.708E-07	.882E-07	.106E-06	.125E-06	.142E-06	.172E-06	.187E-06	.185E-06	.170E-06	.149E-06	.114E-06	.829E-07	.582E-07
5s-7d	.737E-08	.187E-07	.367E-07	.590E-07	.827E-07	.104E-06	.121E-06	.133E-06	.138E-06	.137E-06	.123E-06	.101E-06	.783E-07	.586E-07	.429E-07	.263E-07	.158E-07	.942E-08
5s-7f	.101E-07	.236E-07	.420E-07	.618E-07	.790E-07	.913E-07	.978E-07	.988E-07	.957E-07	.894E-07	.725E-07	.553E-07	.408E-07	.295E-07	.211E-07	.127E-07	.754E-08	.449E-08
5s-7g	.377E-08	.880E-08	.158E-07	.235E-07	.305E-07	.356E-07	.385E-07	.393E-07	.383E-07	.361E-07	.297E-07	.230E-07	.171E-07	.125E-07	.896E-08	.539E-08	.321E-08	.191E-08
5s-7h	.330E-08	.812E-08	.152E-07	.232E-07	.306E-07	.360E-07	.391E-07	.399E-07	.387E-07	.363E-07	.295E-07	.225E-07	.166E-07	.120E-07	.856E-08	.513E-08	.305E-08	.182E-08

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
5s-7i	.159E-07	.362E-07	.625E-07	.886E-07	.109E-06	.120E-06	.123E-06	.120E-06	.112E-06	.102E-06	.804E-07	.603E-07	.440E-07	.316E-07	.225E-07	.135E-07	.804E-08	.479E-08
5p-7s	.190E-08	.431E-08	.747E-08	.107E-07	.135E-07	.155E-07	.170E-07	.181E-07	.191E-07	.200E-07	.218E-07	.225E-07	.218E-07	.199E-07	.173E-07	.131E-07	.954E-08	.667E-08
5p-7p	.796E-08	.198E-07	.377E-07	.589E-07	.796E-07	.968E-07	.108E-06	.114E-06	.115E-06	.112E-06	.998E-07	.832E-07	.663E-07	.510E-07	.381E-07	.237E-07	.144E-07	.863E-08
5p-7d	.567E-08	.146E-07	.291E-07	.489E-07	.723E-07	.974E-07	.123E-06	.147E-06	.170E-06	.190E-06	.217E-06	.225E-06	.215E-06	.192E-06	.165E-06	.124E-06	.893E-07	.621E-07
5p-7f	.112E-07	.275E-07	.517E-07	.796E-07	.107E-06	.128E-06	.143E-06	.149E-06	.149E-06	.144E-06	.123E-06	.973E-07	.739E-07	.545E-07	.395E-07	.240E-07	.144E-07	.858E-08
5p-7g	.634E-08	.146E-07	.257E-07	.373E-07	.473E-07	.543E-07	.580E-07	.585E-07	.567E-07	.531E-07	.433E-07	.332E-07	.245E-07	.178E-07	.127E-07	.765E-08	.456E-08	.271E-08
5p-7h	.269E-08	.671E-08	.128E-07	.201E-07	.273E-07	.332E-07	.369E-07	.384E-07	.380E-07	.361E-07	.300E-07	.233E-07	.174E-07	.126E-07	.908E-08	.546E-08	.326E-08	.194E-08
5p-7i	.174E-07	.401E-07	.702E-07	.101E-06	.125E-06	.139E-06	.144E-06	.142E-06	.134E-06	.122E-06	.960E-07	.720E-07	.525E-07	.377E-07	.268E-07	.160E-07	.955E-08	.569E-08
5d-7s	.445E-09	.100E-08	.175E-08	.256E-08	.336E-08	.409E-08	.470E-08	.516E-08	.542E-08	.549E-08	.512E-08	.431E-08	.341E-08	.258E-08	.190E-08	.117E-08	.704E-09	.419E-09
5d-7p	.238E-08	.550E-08	.972E-08	.142E-07	.181E-07	.210E-07	.230E-07	.240E-07	.243E-07	.242E-07	.232E-07	.214E-07	.190E-07	.162E-07	.134E-07	.970E-08	.679E-08	.463E-08
5d-7d	.842E-08	.209E-07	.396E-07	.614E-07	.823E-07	.990E-07	.109E-06	.114E-06	.114E-06	.110E-06	.966E-07	.797E-07	.630E-07	.481E-07	.358E-07	.222E-07	.135E-07	.806E-08
5d-7f	.862E-08	.227E-07	.462E-07	.786E-07	.117E-06	.157E-06	.197E-06	.234E-06	.267E-06	.293E-06	.324E-06	.325E-06	.302E-06	.265E-06	.225E-06	.166E-06	.118E-06	.815E-07
5d-7g	.127E-07	.304E-07	.557E-07	.835E-07	.109E-06	.127E-06	.137E-06	.140E-06	.136E-06	.128E-06	.106E-06	.823E-07	.615E-07	.449E-07	.324E-07	.195E-07	.117E-07	.696E-08
5d-7h	.182E-08	.450E-08	.865E-08	.139E-07	.195E-07	.249E-07	.293E-07	.323E-07	.336E-07	.334E-07	.296E-07	.239E-07	.182E-07	.134E-07	.968E-08	.586E-08	.350E-08	.208E-08
5d-7i	.205E-07	.481E-07	.862E-07	.126E-06	.160E-06	.181E-06	.190E-06	.188E-06	.178E-06	.164E-06	.130E-06	.983E-07	.722E-07	.521E-07	.373E-07	.223E-07	.133E-07	.794E-08
5f-7s	.837E-10	.203E-09	.389E-09	.630E-09	.903E-09	.117E-08	.139E-08	.154E-08	.160E-08	.158E-08	.138E-08	.110E-08	.826E-09	.605E-09	.436E-09	.263E-09	.156E-09	.930E-10
5f-7p	.473E-09	.110E-08	.198E-08	.299E-08	.397E-08	.485E-08	.551E-08	.594E-08	.612E-08	.606E-08	.541E-08	.442E-08	.342E-08	.255E-08	.186E-08	.113E-08	.678E-09	.403E-09
5f-7d	.198E-08	.462E-08	.821E-08	.120E-07	.154E-07	.179E-07	.195E-07	.201E-07	.202E-07	.197E-07	.178E-07	.153E-07	.128E-07	.104E-07	.821E-08	.565E-08	.380E-08	.251E-08
5f-7f	.825E-08	.203E-07	.382E-07	.587E-07	.778E-07	.927E-07	.101E-06	.105E-06	.104E-06	.997E-07	.866E-07	.709E-07	.556E-07	.423E-07	.313E-07	.194E-07	.117E-07	.701E-08
5f-7g	.165E-07	.431E-07	.863E-07	.144E-06	.209E-06	.275E-06	.336E-06	.389E-06	.431E-06	.461E-06	.482E-06	.462E-06	.415E-06	.355E-06	.295E-06	.213E-06	.150E-06	.102E-06
5f-7h	.490E-08	.114E-07	.202E-07	.295E-07	.374E-07	.430E-07	.460E-07	.470E-07	.462E-07	.442E-07	.379E-07	.305E-07	.234E-07	.175E-07	.127E-07	.778E-08	.467E-08	.278E-08
5f-7i	.243E-07	.586E-07	.107E-06	.162E-06	.211E-06	.247E-06	.268E-06	.273E-06	.266E-06	.250E-06	.205E-06	.157E-06	.116E-06	.843E-07	.604E-07	.363E-07	.216E-07	.129E-07
5g-7s	.196E-10	.507E-10	.101E-09	.166E-09	.237E-09	.303E-09	.354E-09	.386E-09	.396E-09	.388E-09	.336E-09	.265E-09	.200E-09	.146E-09	.105E-09	.635E-10	.377E-10	.223E-10
5g-7p	.715E-10	.181E-09	.354E-09	.577E-09	.825E-09	.106E-08	.126E-08	.139E-08	.144E-08	.142E-08	.124E-08	.982E-09	.739E-09	.542E-09	.390E-09	.235E-09	.140E-09	.830E-10
5g-7d	.233E-09	.559E-09	.103E-08	.160E-08	.219E-08	.273E-08	.316E-08	.345E-08	.358E-08	.356E-08	.317E-08	.257E-08	.196E-08	.145E-08	.105E-08	.640E-09	.382E-09	.227E-09
5g-7f	.995E-09	.235E-08	.424E-08	.630E-08	.823E-08	.977E-08	.109E-07	.116E-07	.118E-07	.116E-07	.103E-07	.859E-08	.683E-08	.528E-08	.401E-08	.261E-08	.167E-08	.106E-08
5g-7g	.577E-08	.142E-07	.266E-07	.407E-07	.535E-07	.632E-07	.686E-07	.707E-07	.699E-07	.673E-07	.588E-07	.483E-07	.378E-07	.287E-07	.212E-07	.130E-07	.789E-08	.471E-08
5g-7h	.271E-07	.692E-07	.135E-06	.220E-06	.311E-06	.399E-06	.476E-06	.535E-06	.577E-06	.598E-06	.590E-06	.538E-06	.463E-06	.384E-06	.310E-06	.219E-06	.150E-06	.101E-06
5g-7i	.218E-07	.553E-07	.107E-06	.171E-06	.235E-06	.291E-06	.331E-06	.354E-06	.360E-06	.353E-06	.312E-06	.255E-06	.197E-06	.148E-06	.109E-06	.667E-07	.402E-07	.240E-07
5s-8s	.104E-08	.321E-08	.706E-08	.123E-07	.179E-07	.229E-07	.266E-07	.287E-07	.294E-07	.289E-07	.262E-07	.224E-07	.182E-07	.143E-07	.108E-07	.678E-08	.414E-08	.248E-08
5s-8p	.892E-09	.268E-08	.588E-08	.104E-07	.160E-07	.221E-07	.283E-07	.346E-07	.409E-07	.472E-07	.579E-07	.638E-07	.638E-07	.590E-07	.518E-07	.397E-07	.290E-07	.203E-07

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
5s-8d	.131E-08	.408E-08	.913E-08	.162E-07	.243E-07	.323E-07	.391E-07	.441E-07	.469E-07	.476E-07	.439E-07	.365E-07	.285E-07	.214E-07	.157E-07	.963E-08	.579E-08	.345E-08
5s-8f	.152E-08	.444E-08	.933E-08	.154E-07	.214E-07	.263E-07	.297E-07	.311E-07	.310E-07	.296E-07	.247E-07	.192E-07	.143E-07	.104E-07	.747E-08	.449E-08	.268E-08	.159E-08
5s-8g	.334E-09	.104E-08	.235E-08	.426E-08	.650E-08	.871E-08	.105E-07	.117E-07	.121E-07	.120E-07	.105E-07	.842E-08	.638E-08	.469E-08	.339E-08	.204E-08	.122E-08	.725E-09
5s-8h	.108E-08	.323E-08	.688E-08	.115E-07	.161E-07	.198E-07	.220E-07	.229E-07	.225E-07	.213E-07	.175E-07	.134E-07	.990E-08	.717E-08	.512E-08	.307E-08	.183E-08	.109E-08
5s-8i	.104E-08	.305E-08	.640E-08	.105E-07	.145E-07	.177E-07	.195E-07	.201E-07	.197E-07	.186E-07	.152E-07	.117E-07	.865E-08	.627E-08	.449E-08	.269E-08	.160E-08	.955E-09
5s-8j	.211E-08	.605E-08	.123E-07	.196E-07	.263E-07	.310E-07	.333E-07	.336E-07	.322E-07	.298E-07	.238E-07	.180E-07	.132E-07	.946E-08	.673E-08	.402E-08	.240E-08	.143E-08
5p-8s	.320E-09	.913E-09	.186E-08	.300E-08	.411E-08	.502E-08	.569E-08	.618E-08	.655E-08	.688E-08	.744E-08	.766E-08	.741E-08	.675E-08	.586E-08	.445E-08	.322E-08	.225E-08
5p-8p	.126E-08	.390E-08	.858E-08	.150E-07	.218E-07	.282E-07	.331E-07	.362E-07	.376E-07	.375E-07	.344E-07	.293E-07	.237E-07	.184E-07	.138E-07	.864E-08	.525E-08	.315E-08
5p-8d	.984E-09	.313E-08	.730E-08	.136E-07	.217E-07	.307E-07	.400E-07	.488E-07	.570E-07	.642E-07	.745E-07	.780E-07	.749E-07	.673E-07	.578E-07	.434E-07	.312E-07	.216E-07
5p-8f	.184E-08	.558E-08	.120E-07	.206E-07	.296E-07	.376E-07	.435E-07	.470E-07	.480E-07	.470E-07	.411E-07	.329E-07	.251E-07	.186E-07	.135E-07	.818E-08	.490E-08	.292E-08
5p-8g	.645E-09	.189E-08	.401E-08	.672E-08	.960E-08	.122E-07	.141E-07	.153E-07	.156E-07	.153E-07	.133E-07	.106E-07	.805E-08	.591E-08	.426E-08	.257E-08	.153E-08	.911E-09
5p-8h	.102E-08	.305E-08	.657E-08	.111E-07	.158E-07	.198E-07	.225E-07	.237E-07	.236E-07	.225E-07	.188E-07	.146E-07	.109E-07	.793E-08	.570E-08	.343E-08	.204E-08	.122E-08
5p-8i	.132E-08	.389E-08	.816E-08	.134E-07	.185E-07	.225E-07	.248E-07	.257E-07	.252E-07	.237E-07	.194E-07	.149E-07	.110E-07	.794E-08	.567E-08	.340E-08	.202E-08	.121E-08
5p-8j	.208E-08	.603E-08	.124E-07	.200E-07	.270E-07	.320E-07	.346E-07	.349E-07	.336E-07	.311E-07	.250E-07	.190E-07	.139E-07	.101E-07	.718E-08	.429E-08	.257E-08	.153E-08
5d-8s	.695E-10	.200E-09	.415E-09	.692E-09	.999E-09	.131E-08	.159E-08	.182E-08	.198E-08	.205E-08	.197E-08	.169E-08	.135E-08	.103E-08	.756E-09	.467E-09	.281E-09	.167E-09
5d-8p	.395E-09	.115E-08	.239E-08	.394E-08	.551E-08	.686E-08	.785E-08	.849E-08	.879E-08	.887E-08	.853E-08	.778E-08	.680E-08	.574E-08	.470E-08	.335E-08	.232E-08	.157E-08
5d-8d	.133E-08	.409E-08	.898E-08	.156E-07	.226E-07	.290E-07	.337E-07	.365E-07	.375E-07	.370E-07	.334E-07	.281E-07	.225E-07	.173E-07	.130E-07	.808E-08	.491E-08	.294E-08
5d-8f	.154E-08	.500E-08	.118E-07	.222E-07	.353E-07	.495E-07	.639E-07	.771E-07	.888E-07	.983E-07	.110E-06	.111E-06	.103E-06	.906E-07	.765E-07	.564E-07	.399E-07	.275E-07
5d-8g	.164E-08	.488E-08	.103E-07	.173E-07	.242E-07	.300E-07	.339E-07	.357E-07	.357E-07	.343E-07	.291E-07	.229E-07	.173E-07	.127E-07	.917E-08	.554E-08	.331E-08	.197E-08
5d-8h	.700E-09	.215E-08	.479E-08	.841E-08	.125E-07	.165E-07	.197E-07	.218E-07	.228E-07	.227E-07	.201E-07	.162E-07	.123E-07	.906E-08	.654E-08	.396E-08	.236E-08	.140E-08
5d-8i	.204E-08	.604E-08	.127E-07	.211E-07	.292E-07	.356E-07	.393E-07	.405E-07	.396E-07	.373E-07	.306E-07	.236E-07	.175E-07	.127E-07	.910E-08	.546E-08	.326E-08	.194E-08
5d-8j	.202E-08	.595E-08	.125E-07	.204E-07	.280E-07	.337E-07	.369E-07	.377E-07	.367E-07	.343E-07	.278E-07	.212E-07	.156E-07	.112E-07	.802E-08	.480E-08	.286E-08	.171E-08
5f-8s	.144E-10	.436E-10	.989E-10	.183E-09	.292E-09	.411E-09	.521E-09	.605E-09	.652E-09	.661E-09	.597E-09	.483E-09	.367E-09	.271E-09	.195E-09	.118E-09	.702E-10	.418E-10
5f-8p	.707E-10	.212E-09	.461E-09	.808E-09	.121E-08	.162E-08	.198E-08	.225E-08	.241E-08	.244E-08	.225E-08	.186E-08	.145E-08	.108E-08	.791E-09	.483E-09	.289E-09	.171E-09
5f-8d	.315E-09	.928E-09	.196E-08	.326E-08	.459E-08	.575E-08	.662E-08	.718E-08	.741E-08	.739E-08	.681E-08	.585E-08	.481E-08	.384E-08	.299E-08	.202E-08	.133E-08	.867E-09
5f-8f	.125E-08	.383E-08	.836E-08	.144E-07	.208E-07	.264E-07	.303E-07	.326E-07	.333E-07	.327E-07	.293E-07	.245E-07	.196E-07	.150E-07	.112E-07	.693E-08	.421E-08	.251E-08
5f-8g	.281E-08	.904E-08	.211E-07	.386E-07	.600E-07	.821E-07	.103E-06	.121E-06	.136E-06	.146E-06	.154E-06	.148E-06	.133E-06	.113E-06	.931E-07	.668E-07	.465E-07	.315E-07
5f-8h	.428E-09	.128E-08	.275E-08	.471E-08	.690E-08	.908E-08	.111E-07	.127E-07	.139E-07	.145E-07	.143E-07	.125E-07	.102E-07	.792E-08	.592E-08	.369E-08	.223E-08	.134E-08
5f-8i	.336E-08	.101E-07	.218E-07	.367E-07	.521E-07	.648E-07	.735E-07	.774E-07	.773E-07	.739E-07	.618E-07	.481E-07	.358E-07	.260E-07	.187E-07	.112E-07	.669E-08	.398E-08
5f-8j	.177E-08	.530E-08	.113E-07	.190E-07	.266E-07	.327E-07	.365E-07	.379E-07	.373E-07	.353E-07	.291E-07	.225E-07	.168E-07	.122E-07	.874E-08	.526E-08	.314E-08	.187E-08
5g-8s	.316E-11	.109E-10	.271E-10	.528E-10	.854E-10	.120E-09	.150E-09	.172E-09	.183E-09	.184E-09	.163E-09	.131E-09	.992E-10	.729E-10	.526E-10	.318E-10	.188E-10	.111E-10
5g-8p	.118E-10	.391E-10	.938E-10	.179E-09	.289E-09	.408E-09	.516E-09	.598E-09	.642E-09	.650E-09	.583E-09	.470E-09	.356E-09	.263E-09	.189E-09	.114E-09	.679E-10	.404E-10

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
5g-8d	.340E-10	.106E-09	.242E-09	.443E-09	.691E-09	.955E-09	.120E-08	.139E-08	.150E-08	.153E-08	.140E-08	.115E-08	.887E-09	.659E-09	.478E-09	.290E-09	.173E-09	.103E-09
5g-8f	.150E-09	.446E-09	.954E-09	.161E-08	.233E-08	.302E-08	.362E-08	.407E-08	.432E-08	.439E-08	.407E-08	.341E-08	.270E-08	.207E-08	.155E-08	.989E-09	.620E-09	.386E-09
5g-8g	.789E-09	.242E-08	.529E-08	.910E-08	.131E-07	.166E-07	.190E-07	.205E-07	.210E-07	.208E-07	.189E-07	.160E-07	.127E-07	.975E-08	.725E-08	.448E-08	.271E-08	.162E-08
5g-8h	.381E-08	.120E-07	.273E-07	.488E-07	.740E-07	.991E-07	.122E-06	.140E-06	.153E-06	.161E-06	.161E-06	.147E-06	.126E-06	.103E-06	.827E-07	.575E-07	.390E-07	.259E-07
5g-8i	.436E-08	.136E-07	.304E-07	.536E-07	.793E-07	.104E-06	.123E-06	.136E-06	.141E-06	.141E-06	.128E-06	.106E-06	.824E-07	.621E-07	.456E-07	.280E-07	.169E-07	.101E-07
5g-8j	.108E-08	.331E-08	.729E-08	.126E-07	.183E-07	.233E-07	.271E-07	.292E-07	.297E-07	.289E-07	.250E-07	.198E-07	.150E-07	.110E-07	.790E-08	.477E-08	.285E-08	.169E-08
6s-7s	.661E-06	.976E-06	.128E-05	.152E-05	.167E-05	.172E-05	.171E-05	.163E-05	.152E-05	.139E-05	.112E-05	.870E-06	.660E-06	.492E-06	.361E-06	.221E-06	.134E-06	.799E-07
6s-7p	.454E-06	.617E-06	.788E-06	.978E-06	.119E-05	.143E-05	.167E-05	.193E-05	.216E-05	.236E-05	.261E-05	.264E-05	.248E-05	.220E-05	.188E-05	.141E-05	.102E-05	.706E-06
6s-7d	.539E-06	.828E-06	.114E-05	.142E-05	.164E-05	.178E-05	.184E-05	.183E-05	.176E-05	.165E-05	.136E-05	.106E-05	.802E-06	.590E-06	.429E-06	.260E-06	.156E-06	.932E-07
6s-7f	.833E-06	.113E-05	.136E-05	.151E-05	.156E-05	.154E-05	.147E-05	.136E-05	.122E-05	.108E-05	.819E-06	.599E-06	.433E-06	.310E-06	.220E-06	.132E-06	.783E-07	.466E-07
6s-7g	.838E-06	.107E-05	.122E-05	.128E-05	.127E-05	.120E-05	.110E-05	.978E-06	.859E-06	.746E-06	.551E-06	.401E-06	.287E-06	.205E-06	.146E-06	.868E-07	.517E-07	.308E-07
6s-7h	.602E-06	.739E-06	.820E-06	.839E-06	.813E-06	.758E-06	.686E-06	.610E-06	.535E-06	.464E-06	.341E-06	.246E-06	.176E-06	.125E-06	.888E-07	.529E-07	.315E-07	.188E-07
6s-7i	.216E-06	.265E-06	.293E-06	.301E-06	.293E-06	.274E-06	.251E-06	.225E-06	.198E-06	.173E-06	.128E-06	.932E-07	.670E-07	.478E-07	.339E-07	.202E-07	.120E-07	.716E-08
6p-7s	.166E-06	.208E-06	.235E-06	.253E-06	.266E-06	.279E-06	.294E-06	.312E-06	.332E-06	.351E-06	.378E-06	.378E-06	.355E-06	.316E-06	.270E-06	.202E-06	.146E-06	.101E-06
6p-7p	.791E-06	.116E-05	.152E-05	.180E-05	.199E-05	.207E-05	.206E-05	.198E-05	.186E-05	.171E-05	.138E-05	.108E-05	.815E-06	.605E-06	.444E-06	.272E-06	.164E-06	.978E-07
6p-7d	.456E-06	.675E-06	.921E-06	.120E-05	.148E-05	.177E-05	.205E-05	.231E-05	.252E-05	.270E-05	.287E-05	.281E-05	.259E-05	.227E-05	.192E-05	.142E-05	.102E-05	.703E-06
6p-7f	.780E-06	.115E-05	.151E-05	.180E-05	.200E-05	.210E-05	.211E-05	.205E-05	.193E-05	.177E-05	.142E-05	.110E-05	.817E-06	.598E-06	.432E-06	.261E-06	.157E-06	.933E-07
6p-7g	.104E-05	.136E-05	.157E-05	.169E-05	.171E-05	.165E-05	.155E-05	.141E-05	.125E-05	.110E-05	.824E-06	.599E-06	.431E-06	.308E-06	.219E-06	.130E-06	.776E-07	.462E-07
6p-7h	.737E-06	.924E-06	.104E-05	.108E-05	.107E-05	.100E-05	.913E-06	.813E-06	.712E-06	.618E-06	.455E-06	.331E-06	.237E-06	.169E-06	.120E-06	.714E-07	.426E-07	.253E-07
6p-7i	.281E-06	.345E-06	.382E-06	.392E-06	.382E-06	.358E-06	.326E-06	.292E-06	.257E-06	.224E-06	.166E-06	.120E-06	.861E-07	.613E-07	.434E-07	.259E-07	.154E-07	.917E-08
6d-7s	.444E-07	.571E-07	.684E-07	.767E-07	.827E-07	.861E-07	.870E-07	.855E-07	.821E-07	.769E-07	.641E-07	.507E-07	.385E-07	.285E-07	.208E-07	.127E-07	.759E-08	.453E-08
6d-7p	.237E-06	.300E-06	.341E-06	.365E-06	.373E-06	.374E-06	.372E-06	.370E-06	.367E-06	.366E-06	.357E-06	.335E-06	.302E-06	.261E-06	.219E-06	.160E-06	.114E-06	.784E-07
6d-7d	.830E-06	.121E-05	.157E-05	.184E-05	.201E-05	.207E-05	.205E-05	.196E-05	.183E-05	.167E-05	.134E-05	.103E-05	.780E-06	.578E-06	.422E-06	.258E-06	.156E-06	.928E-07
6d-7f	.550E-06	.864E-06	.124E-05	.166E-05	.210E-05	.254E-05	.294E-05	.329E-05	.358E-05	.378E-05	.395E-05	.381E-05	.347E-05	.302E-05	.254E-05	.187E-05	.133E-05	.917E-06
6d-7g	.117E-05	.168E-05	.216E-05	.252E-05	.275E-05	.284E-05	.280E-05	.268E-05	.249E-05	.227E-05	.180E-05	.137E-05	.101E-05	.737E-06	.531E-06	.321E-06	.192E-06	.114E-06
6d-7h	.116E-05	.149E-05	.171E-05	.182E-05	.182E-05	.174E-05	.162E-05	.146E-05	.130E-05	.114E-05	.845E-06	.612E-06	.440E-06	.314E-06	.223E-06	.133E-06	.791E-07	.470E-07
6d-7i	.438E-06	.548E-06	.619E-06	.642E-06	.632E-06	.594E-06	.542E-06	.484E-06	.425E-06	.369E-06	.273E-06	.199E-06	.143E-06	.102E-06	.722E-07	.431E-07	.257E-07	.153E-07
6f-7s	.128E-07	.169E-07	.201E-07	.225E-07	.238E-07	.241E-07	.236E-07	.222E-07	.205E-07	.184E-07	.143E-07	.106E-07	.773E-08	.557E-08	.397E-08	.238E-08	.141E-08	.840E-09
6f-7p	.583E-07	.746E-07	.877E-07	.953E-07	.992E-07	.993E-07	.967E-07	.921E-07	.861E-07	.790E-07	.638E-07	.494E-07	.371E-07	.272E-07	.197E-07	.119E-07	.715E-08	.426E-08
6f-7d	.231E-06	.293E-06	.329E-06	.347E-06	.348E-06	.339E-06	.327E-06	.313E-06	.300E-06	.288E-06	.264E-06	.237E-06	.206E-06	.174E-06	.143E-06	.103E-06	.723E-07	.492E-07
6f-7f	.859E-06	.124E-05	.159E-05	.185E-05	.200E-05	.205E-05	.201E-05	.191E-05	.177E-05	.161E-05	.128E-05	.985E-06	.740E-06	.546E-06	.398E-06	.243E-06	.146E-06	.874E-07

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
6f-7g	.872E-06	.141E-05	.204E-05	.274E-05	.345E-05	.413E-05	.472E-05	.520E-05	.555E-05	.578E-05	.587E-05	.553E-05	.496E-05	.426E-05	.355E-05	.259E-05	.183E-05	.126E-05
6f-7h	.168E-05	.236E-05	.299E-05	.342E-05	.366E-05	.371E-05	.361E-05	.340E-05	.313E-05	.283E-05	.220E-05	.166E-05	.122E-05	.884E-06	.635E-06	.383E-06	.229E-06	.136E-06
6f-7i	.895E-06	.115E-05	.130E-05	.138E-05	.137E-05	.131E-05	.121E-05	.109E-05	.964E-06	.841E-06	.625E-06	.453E-06	.325E-06	.232E-06	.164E-06	.980E-07	.584E-07	.347E-07
6g-7s	.325E-08	.439E-08	.534E-08	.593E-08	.620E-08	.617E-08	.593E-08	.554E-08	.506E-08	.455E-08	.351E-08	.262E-08	.191E-08	.138E-08	.984E-09	.588E-09	.350E-09	.208E-09
6g-7p	.139E-07	.181E-07	.211E-07	.232E-07	.242E-07	.242E-07	.235E-07	.221E-07	.203E-07	.183E-07	.141E-07	.105E-07	.765E-08	.551E-08	.393E-08	.235E-08	.140E-08	.830E-09
6g-7d	.448E-07	.567E-07	.657E-07	.702E-07	.717E-07	.706E-07	.677E-07	.636E-07	.588E-07	.535E-07	.426E-07	.326E-07	.243E-07	.177E-07	.128E-07	.773E-08	.462E-08	.275E-08
6g-7f	.186E-06	.236E-06	.265E-06	.278E-06	.277E-06	.267E-06	.253E-06	.237E-06	.221E-06	.206E-06	.178E-06	.151E-06	.126E-06	.103E-06	.828E-07	.579E-07	.397E-07	.266E-07
6g-7g	.850E-06	.121E-05	.154E-05	.177E-05	.190E-05	.192E-05	.187E-05	.176E-05	.162E-05	.147E-05	.116E-05	.883E-06	.659E-06	.485E-06	.352E-06	.214E-06	.129E-06	.769E-07
6g-7h	.175E-05	.277E-05	.390E-05	.511E-05	.626E-05	.728E-05	.810E-05	.871E-05	.908E-05	.925E-05	.906E-05	.831E-05	.730E-05	.618E-05	.509E-05	.368E-05	.258E-05	.176E-05
6g-7i	.188E-05	.260E-05	.323E-05	.362E-05	.381E-05	.380E-05	.364E-05	.339E-05	.309E-05	.276E-05	.213E-05	.159E-05	.116E-05	.838E-06	.601E-06	.361E-06	.216E-06	.128E-06
6h-7s	.787E-09	.101E-08	.116E-08	.127E-08	.132E-08	.133E-08	.130E-08	.124E-08	.115E-08	.104E-08	.811E-09	.607E-09	.444E-09	.320E-09	.228E-09	.137E-09	.809E-10	.479E-10
6h-7p	.278E-08	.358E-08	.420E-08	.458E-08	.476E-08	.478E-08	.465E-08	.441E-08	.409E-08	.370E-08	.290E-08	.218E-08	.159E-08	.115E-08	.822E-09	.492E-09	.292E-09	.174E-09
6h-7d	.719E-08	.915E-08	.106E-07	.115E-07	.120E-07	.121E-07	.118E-07	.112E-07	.103E-07	.939E-08	.733E-08	.549E-08	.401E-08	.289E-08	.206E-08	.124E-08	.734E-09	.436E-09
6h-7f	.224E-07	.283E-07	.326E-07	.346E-07	.351E-07	.344E-07	.330E-07	.310E-07	.287E-07	.261E-07	.207E-07	.158E-07	.117E-07	.850E-08	.611E-08	.368E-08	.219E-08	.130E-08
6h-7g	.105E-06	.134E-06	.151E-06	.159E-06	.158E-06	.153E-06	.144E-06	.134E-06	.123E-06	.112E-06	.926E-07	.750E-07	.597E-07	.469E-07	.364E-07	.245E-07	.162E-07	.106E-07
6h-7h	.687E-06	.966E-06	.121E-05	.137E-05	.145E-05	.145E-05	.140E-05	.131E-05	.119E-05	.107E-05	.838E-06	.635E-06	.472E-06	.346E-06	.251E-06	.152E-06	.913E-07	.545E-07
6h-7i	.401E-05	.609E-05	.824E-05	.104E-04	.122E-04	.137E-04	.148E-04	.154E-04	.156E-04	.155E-04	.145E-04	.129E-04	.111E-04	.920E-05	.748E-05	.532E-05	.369E-05	.250E-05
6s-8s	.443E-07	.798E-07	.122E-06	.161E-06	.194E-06	.214E-06	.224E-06	.222E-06	.212E-06	.199E-06	.167E-06	.134E-06	.105E-06	.796E-07	.591E-07	.366E-07	.222E-07	.133E-07
6s-8p	.380E-07	.639E-07	.934E-07	.124E-06	.156E-06	.188E-06	.221E-06	.256E-06	.291E-06	.324E-06	.373E-06	.388E-06	.372E-06	.334E-06	.288E-06	.217E-06	.156E-06	.109E-06
6s-8d	.441E-07	.805E-07	.125E-06	.171E-06	.214E-06	.248E-06	.273E-06	.283E-06	.281E-06	.270E-06	.232E-06	.184E-06	.141E-06	.104E-06	.758E-07	.462E-07	.277E-07	.165E-07
6s-8f	.636E-07	.106E-06	.148E-06	.183E-06	.206E-06	.217E-06	.215E-06	.207E-06	.193E-06	.175E-06	.137E-06	.103E-06	.750E-07	.539E-07	.385E-07	.230E-07	.137E-07	.815E-08
6s-8g	.414E-07	.658E-07	.892E-07	.107E-06	.117E-06	.120E-06	.118E-06	.112E-06	.103E-06	.923E-07	.715E-07	.534E-07	.391E-07	.281E-07	.201E-07	.120E-07	.716E-08	.426E-08
6s-8h	.101E-07	.184E-07	.282E-07	.377E-07	.454E-07	.503E-07	.523E-07	.517E-07	.492E-07	.453E-07	.361E-07	.272E-07	.200E-07	.144E-07	.103E-07	.616E-08	.366E-08	.218E-08
6s-8i	.452E-07	.732E-07	.995E-07	.119E-06	.130E-06	.132E-06	.127E-06	.119E-06	.108E-06	.958E-07	.728E-07	.537E-07	.389E-07	.278E-07	.198E-07	.118E-07	.703E-08	.419E-08
6s-8j	.801E-07	.126E-06	.167E-06	.196E-06	.210E-06	.211E-06	.202E-06	.187E-06	.169E-06	.150E-06	.113E-06	.832E-07	.599E-07	.427E-07	.303E-07	.180E-07	.108E-07	.641E-08
6p-8s	.139E-07	.222E-07	.297E-07	.355E-07	.393E-07	.417E-07	.434E-07	.451E-07	.472E-07	.494E-07	.531E-07	.538E-07	.509E-07	.455E-07	.390E-07	.292E-07	.210E-07	.146E-07
6p-8p	.553E-07	.982E-07	.149E-06	.196E-06	.237E-06	.263E-06	.277E-06	.277E-06	.268E-06	.253E-06	.214E-06	.172E-06	.134E-06	.101E-06	.748E-07	.462E-07	.279E-07	.167E-07
6p-8d	.375E-07	.679E-07	.106E-06	.150E-06	.195E-06	.240E-06	.283E-06	.324E-06	.360E-06	.392E-06	.430E-06	.431E-06	.401E-06	.353E-06	.300E-06	.222E-06	.158E-06	.109E-06
6p-8f	.634E-07	.112E-06	.166E-06	.218E-06	.261E-06	.290E-06	.306E-06	.306E-06	.295E-06	.277E-06	.228E-06	.177E-06	.133E-06	.973E-07	.703E-07	.426E-07	.255E-07	.152E-07
6p-8g	.573E-07	.928E-07	.126E-06	.152E-06	.167E-06	.172E-06	.169E-06	.160E-06	.148E-06	.133E-06	.103E-06	.770E-07	.560E-07	.402E-07	.287E-07	.172E-07	.102E-07	.607E-08
6p-8h	.131E-07	.225E-07	.330E-07	.427E-07	.504E-07	.553E-07	.572E-07	.565E-07	.538E-07	.498E-07	.400E-07	.305E-07	.226E-07	.164E-07	.117E-07	.703E-08	.418E-08	.249E-08

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
6p-8i	.443E-07	.726E-07	.100E-06	.121E-06	.134E-06	.138E-06	.134E-06	.127E-06	.116E-06	.104E-06	.794E-07	.586E-07	.425E-07	.304E-07	.216E-07	.129E-07	.768E-08	.457E-08
6p-8j	.904E-07	.143E-06	.191E-06	.225E-06	.241E-06	.242E-06	.232E-06	.214E-06	.193E-06	.171E-06	.130E-06	.955E-07	.690E-07	.493E-07	.350E-07	.209E-07	.125E-07	.742E-08
6d-8s	.404E-08	.631E-08	.853E-08	.104E-07	.120E-07	.132E-07	.140E-07	.145E-07	.144E-07	.140E-07	.123E-07	.999E-08	.773E-08	.578E-08	.422E-08	.258E-08	.155E-08	.924E-09
6d-8p	.191E-07	.310E-07	.421E-07	.511E-07	.567E-07	.598E-07	.609E-07	.608E-07	.603E-07	.593E-07	.564E-07	.519E-07	.459E-07	.391E-07	.324E-07	.234E-07	.164E-07	.112E-07
6d-8d	.584E-07	.103E-06	.156E-06	.204E-06	.244E-06	.269E-06	.281E-06	.278E-06	.267E-06	.250E-06	.209E-06	.167E-06	.129E-06	.969E-07	.714E-07	.439E-07	.265E-07	.159E-07
6d-8f	.468E-07	.889E-07	.145E-06	.209E-06	.278E-06	.346E-06	.408E-06	.466E-06	.515E-06	.554E-06	.593E-06	.581E-06	.532E-06	.463E-06	.388E-06	.285E-06	.202E-06	.139E-06
6d-8g	.829E-07	.142E-06	.207E-06	.263E-06	.306E-06	.330E-06	.340E-06	.332E-06	.314E-06	.289E-06	.231E-06	.176E-06	.131E-06	.950E-07	.683E-07	.412E-07	.246E-07	.146E-07
6d-8h	.282E-07	.453E-07	.614E-07	.745E-07	.829E-07	.872E-07	.875E-07	.849E-07	.800E-07	.736E-07	.589E-07	.447E-07	.329E-07	.238E-07	.170E-07	.102E-07	.610E-08	.363E-08
6d-8i	.376E-07	.637E-07	.909E-07	.114E-06	.130E-06	.138E-06	.139E-06	.133E-06	.124E-06	.112E-06	.873E-07	.654E-07	.477E-07	.344E-07	.246E-07	.147E-07	.877E-08	.522E-08
6d-8j	.112E-06	.180E-06	.243E-06	.288E-06	.312E-06	.316E-06	.305E-06	.284E-06	.258E-06	.229E-06	.174E-06	.128E-06	.923E-07	.659E-07	.468E-07	.279E-07	.166E-07	.990E-08
6f-8s	.994E-09	.167E-08	.241E-08	.316E-08	.382E-08	.434E-08	.464E-08	.473E-08	.463E-08	.436E-08	.358E-08	.276E-08	.204E-08	.148E-08	.106E-08	.640E-09	.380E-09	.225E-09
6f-8p	.505E-08	.810E-08	.112E-07	.138E-07	.159E-07	.172E-07	.180E-07	.181E-07	.176E-07	.167E-07	.141E-07	.112E-07	.851E-08	.628E-08	.456E-08	.277E-08	.166E-08	.986E-09
6f-8d	.180E-07	.294E-07	.401E-07	.487E-07	.538E-07	.563E-07	.566E-07	.554E-07	.536E-07	.511E-07	.455E-07	.394E-07	.332E-07	.272E-07	.218E-07	.152E-07	.104E-07	.691E-08
6f-8f	.603E-07	.106E-06	.158E-06	.205E-06	.243E-06	.265E-06	.274E-06	.270E-06	.257E-06	.239E-06	.198E-06	.157E-06	.121E-06	.903E-07	.664E-07	.407E-07	.246E-07	.147E-07
6f-8g	.750E-07	.144E-06	.236E-06	.341E-06	.447E-06	.550E-06	.638E-06	.717E-06	.777E-06	.819E-06	.844E-06	.802E-06	.716E-06	.612E-06	.507E-06	.367E-06	.257E-06	.176E-06
6f-8h	.777E-07	.130E-06	.183E-06	.226E-06	.255E-06	.268E-06	.268E-06	.255E-06	.236E-06	.214E-06	.168E-06	.126E-06	.926E-07	.670E-07	.480E-07	.288E-07	.172E-07	.102E-07
6f-8i	.226E-07	.398E-07	.597E-07	.793E-07	.959E-07	.108E-06	.114E-06	.116E-06	.113E-06	.106E-06	.879E-07	.680E-07	.505E-07	.367E-07	.264E-07	.159E-07	.945E-08	.562E-08
6f-8j	.145E-06	.239E-06	.331E-06	.401E-06	.443E-06	.455E-06	.445E-06	.417E-06	.380E-06	.339E-06	.259E-06	.192E-06	.139E-06	.100E-06	.713E-07	.426E-07	.254E-07	.151E-07
6g-8s	.266E-09	.475E-09	.732E-09	.996E-09	.123E-08	.140E-08	.149E-08	.151E-08	.146E-08	.136E-08	.111E-08	.847E-09	.627E-09	.455E-09	.327E-09	.195E-09	.116E-09	.691E-10
6g-8p	.106E-08	.183E-08	.272E-08	.362E-08	.441E-08	.503E-08	.539E-08	.548E-08	.534E-08	.502E-08	.410E-08	.314E-08	.232E-08	.168E-08	.121E-08	.725E-09	.430E-09	.255E-09
6g-8d	.363E-08	.593E-08	.830E-08	.104E-07	.120E-07	.131E-07	.136E-07	.136E-07	.132E-07	.125E-07	.104E-07	.818E-08	.617E-08	.453E-08	.327E-08	.198E-08	.118E-08	.702E-09
6g-8f	.134E-07	.222E-07	.305E-07	.372E-07	.413E-07	.434E-07	.438E-07	.428E-07	.411E-07	.387E-07	.331E-07	.273E-07	.219E-07	.172E-07	.133E-07	.890E-08	.585E-08	.379E-08
6g-8g	.563E-07	.982E-07	.145E-06	.186E-06	.219E-06	.236E-06	.242E-06	.236E-06	.223E-06	.207E-06	.170E-06	.134E-06	.103E-06	.765E-07	.561E-07	.343E-07	.207E-07	.123E-07
6g-8h	.134E-06	.253E-06	.402E-06	.567E-06	.723E-06	.866E-06	.977E-06	.107E-05	.113E-05	.116E-05	.114E-05	.104E-05	.901E-06	.753E-06	.613E-06	.436E-06	.302E-06	.204E-06
6g-8i	.232E-07	.373E-07	.512E-07	.622E-07	.702E-07	.751E-07	.776E-07	.781E-07	.767E-07	.738E-07	.645E-07	.527E-07	.411E-07	.309E-07	.227E-07	.139E-07	.840E-08	.501E-08
6g-8j	.188E-06	.321E-06	.458E-06	.576E-06	.655E-06	.695E-06	.693E-06	.668E-06	.622E-06	.565E-06	.444E-06	.332E-06	.242E-06	.174E-06	.124E-06	.745E-07	.444E-07	.264E-07
6h-8s	.757E-10	.137E-09	.209E-09	.282E-09	.346E-09	.394E-09	.421E-09	.428E-09	.415E-09	.389E-09	.316E-09	.242E-09	.179E-09	.130E-09	.928E-10	.557E-10	.328E-10	.194E-10
6h-8p	.254E-09	.456E-09	.697E-09	.943E-09	.116E-08	.133E-08	.142E-08	.145E-08	.141E-08	.133E-08	.108E-08	.831E-09	.616E-09	.448E-09	.321E-09	.192E-09	.114E-09	.677E-10
6h-8d	.550E-09	.972E-09	.147E-08	.199E-08	.247E-08	.285E-08	.311E-08	.320E-08	.315E-08	.298E-08	.245E-08	.188E-08	.139E-08	.101E-08	.727E-09	.436E-09	.259E-09	.154E-09
6h-8f	.156E-08	.263E-08	.381E-08	.493E-08	.590E-08	.664E-08	.710E-08	.727E-08	.715E-08	.681E-08	.571E-08	.446E-08	.335E-08	.245E-08	.176E-08	.106E-08	.633E-09	.376E-09
6h-8g	.636E-08	.106E-07	.148E-07	.184E-07	.210E-07	.227E-07	.235E-07	.235E-07	.229E-07	.217E-07	.184E-07	.147E-07	.114E-07	.866E-08	.646E-08	.413E-08	.260E-08	.163E-08
6h-8h	.373E-07	.648E-07	.953E-07	.122E-06	.141E-06	.151E-06	.154E-06	.150E-06	.142E-06	.132E-06	.109E-06	.861E-07	.658E-07	.489E-07	.358E-07	.218E-07	.131E-07	.783E-08

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
6h-8i	.193E-06	.356E-06	.549E-06	.753E-06	.935E-06	.109E-05	.120E-05	.128E-05	.131E-05	.131E-05	.123E-05	.108E-05	.901E-06	.733E-06	.585E-06	.407E-06	.276E-06	.184E-06
6h-8j	.180E-06	.325E-06	.497E-06	.664E-06	.809E-06	.909E-06	.970E-06	.979E-06	.953E-06	.901E-06	.753E-06	.593E-06	.450E-06	.333E-06	.242E-06	.147E-06	.885E-07	.528E-07
7s-8s	.211E-05	.275E-05	.328E-05	.361E-05	.374E-05	.372E-05	.355E-05	.332E-05	.305E-05	.274E-05	.214E-05	.163E-05	.122E-05	.896E-06	.653E-06	.398E-06	.239E-06	.143E-06
7s-8p	.133E-05	.163E-05	.197E-05	.235E-05	.278E-05	.325E-05	.372E-05	.418E-05	.456E-05	.487E-05	.516E-05	.504E-05	.463E-05	.404E-05	.341E-05	.251E-05	.180E-05	.124E-05
7s-8d	.161E-05	.218E-05	.274E-05	.316E-05	.344E-05	.356E-05	.352E-05	.339E-05	.319E-05	.292E-05	.234E-05	.180E-05	.134E-05	.976E-06	.705E-06	.427E-06	.256E-06	.152E-06
7s-8f	.207E-05	.252E-05	.279E-05	.291E-05	.289E-05	.274E-05	.255E-05	.229E-05	.202E-05	.177E-05	.131E-05	.953E-06	.685E-06	.488E-06	.347E-06	.207E-06	.123E-06	.733E-07
7s-8g	.215E-05	.239E-05	.248E-05	.242E-05	.227E-05	.207E-05	.183E-05	.161E-05	.140E-05	.120E-05	.881E-06	.635E-06	.453E-06	.323E-06	.229E-06	.136E-06	.812E-07	.483E-07
7s-8h	.170E-05	.183E-05	.185E-05	.177E-05	.163E-05	.147E-05	.130E-05	.113E-05	.976E-06	.836E-06	.606E-06	.434E-06	.310E-06	.220E-06	.156E-06	.926E-07	.551E-07	.328E-07
7s-8i	.956E-06	.102E-05	.102E-05	.970E-06	.892E-06	.801E-06	.707E-06	.616E-06	.533E-06	.458E-06	.332E-06	.239E-06	.171E-06	.121E-06	.859E-07	.511E-07	.305E-07	.182E-07
7s-8j	.283E-06	.306E-06	.310E-06	.299E-06	.280E-06	.255E-06	.228E-06	.201E-06	.175E-06	.151E-06	.111E-06	.798E-07	.570E-07	.405E-07	.287E-07	.171E-07	.102E-07	.605E-08
7p-8s	.467E-06	.521E-06	.559E-06	.582E-06	.609E-06	.640E-06	.678E-06	.721E-06	.760E-06	.796E-06	.832E-06	.810E-06	.743E-06	.650E-06	.548E-06	.404E-06	.289E-06	.200E-06
7p-8p	.253E-05	.327E-05	.390E-05	.431E-05	.448E-05	.448E-05	.430E-05	.404E-05	.372E-05	.336E-05	.264E-05	.201E-05	.150E-05	.111E-05	.803E-06	.489E-06	.294E-06	.175E-06
7p-8d	.136E-05	.179E-05	.225E-05	.275E-05	.326E-05	.376E-05	.423E-05	.463E-05	.495E-05	.517E-05	.530E-05	.505E-05	.456E-05	.394E-05	.329E-05	.241E-05	.171E-05	.118E-05
7p-8f	.208E-05	.271E-05	.326E-05	.363E-05	.382E-05	.385E-05	.372E-05	.352E-05	.326E-05	.295E-05	.232E-05	.176E-05	.130E-05	.942E-06	.678E-06	.409E-06	.245E-06	.146E-06
7p-8g	.248E-05	.289E-05	.310E-05	.314E-05	.305E-05	.284E-05	.260E-05	.232E-05	.203E-05	.176E-05	.130E-05	.937E-06	.672E-06	.478E-06	.339E-06	.202E-06	.121E-06	.716E-07
7p-8h	.210E-05	.230E-05	.235E-05	.227E-05	.211E-05	.191E-05	.168E-05	.147E-05	.128E-05	.110E-05	.798E-06	.574E-06	.409E-06	.291E-06	.206E-06	.123E-06	.732E-07	.436E-07
7p-8i	.117E-05	.126E-05	.128E-05	.122E-05	.113E-05	.102E-05	.901E-06	.785E-06	.677E-06	.581E-06	.421E-06	.302E-06	.216E-06	.153E-06	.108E-06	.644E-07	.384E-07	.228E-07
7p-8j	.356E-06	.382E-06	.384E-06	.369E-06	.342E-06	.310E-06	.276E-06	.243E-06	.211E-06	.182E-06	.134E-06	.965E-07	.691E-07	.491E-07	.348E-07	.207E-07	.123E-07	.735E-08
7d-8s	.148E-06	.171E-06	.190E-06	.202E-06	.208E-06	.209E-06	.205E-06	.195E-06	.183E-06	.168E-06	.135E-06	.104E-06	.781E-07	.573E-07	.415E-07	.252E-07	.151E-07	.899E-08
7d-8p	.704E-06	.792E-06	.842E-06	.855E-06	.857E-06	.848E-06	.841E-06	.837E-06	.831E-06	.827E-06	.801E-06	.744E-06	.661E-06	.566E-06	.471E-06	.342E-06	.242E-06	.166E-06
7d-8d	.263E-05	.338E-05	.399E-05	.436E-05	.450E-05	.446E-05	.425E-05	.397E-05	.365E-05	.328E-05	.256E-05	.194E-05	.144E-05	.106E-05	.769E-06	.468E-06	.281E-06	.168E-06
7d-8f	.153E-05	.213E-05	.279E-05	.352E-05	.425E-05	.496E-05	.560E-05	.612E-05	.650E-05	.675E-05	.684E-05	.645E-05	.578E-05	.496E-05	.414E-05	.301E-05	.213E-05	.147E-05
7d-8g	.288E-05	.368E-05	.435E-05	.477E-05	.494E-05	.492E-05	.470E-05	.440E-05	.404E-05	.363E-05	.282E-05	.212E-05	.156E-05	.113E-05	.811E-06	.488E-06	.292E-06	.174E-06
7d-8h	.294E-05	.338E-05	.358E-05	.358E-05	.344E-05	.318E-05	.289E-05	.256E-05	.223E-05	.193E-05	.142E-05	.102E-05	.731E-06	.520E-06	.369E-06	.220E-06	.131E-06	.778E-07
7d-8i	.179E-05	.195E-05	.198E-05	.191E-05	.177E-05	.160E-05	.141E-05	.123E-05	.107E-05	.916E-06	.667E-06	.480E-06	.342E-06	.243E-06	.172E-06	.103E-06	.612E-07	.364E-07
7d-8j	.536E-06	.579E-06	.586E-06	.562E-06	.523E-06	.473E-06	.421E-06	.369E-06	.320E-06	.275E-06	.201E-06	.144E-06	.103E-06	.733E-07	.519E-07	.309E-07	.184E-07	.109E-07
7f-8s	.503E-07	.584E-07	.636E-07	.655E-07	.651E-07	.622E-07	.580E-07	.528E-07	.470E-07	.414E-07	.312E-07	.228E-07	.165E-07	.118E-07	.840E-08	.502E-08	.298E-08	.177E-08
7f-8p	.212E-06	.239E-06	.255E-06	.262E-06	.258E-06	.250E-06	.237E-06	.220E-06	.202E-06	.182E-06	.143E-06	.108E-06	.803E-07	.586E-07	.422E-07	.255E-07	.153E-07	.908E-08
7f-8d	.721E-06	.804E-06	.842E-06	.836E-06	.816E-06	.781E-06	.747E-06	.717E-06	.688E-06	.665E-06	.615E-06	.553E-06	.481E-06	.406E-06	.334E-06	.239E-06	.168E-06	.114E-06
7f-8f	.270E-05	.344E-05	.403E-05	.438E-05	.448E-05	.442E-05	.420E-05	.390E-05	.357E-05	.320E-05	.249E-05	.188E-05	.139E-05	.102E-05	.739E-06	.449E-06	.270E-06	.161E-06
7f-8g	.208E-05	.303E-05	.405E-05	.518E-05	.627E-05	.728E-05	.815E-05	.881E-05	.925E-05	.950E-05	.944E-05	.877E-05	.777E-05	.662E-05	.548E-05	.397E-05	.280E-05	.192E-05

Table 5.1 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.25	5.55	5.85	6.15	6.45	6.90	7.35	7.80
7f-8h	.410E-05	.513E-05	.596E-05	.641E-05	.654E-05	.642E-05	.606E-05	.562E-05	.512E-05	.457E-05	.351E-05	.262E-05	.191E-05	.138E-05	.991E-06	.596E-06	.356E-06	.212E-06
7f-8i	.306E-05	.347E-05	.365E-05	.362E-05	.345E-05	.317E-05	.287E-05	.253E-05	.220E-05	.190E-05	.139E-05	.999E-06	.714E-06	.508E-06	.360E-06	.214E-06	.128E-06	.760E-07
7f-8j	.996E-06	.108E-05	.110E-05	.106E-05	.982E-06	.889E-06	.786E-06	.688E-06	.598E-06	.514E-06	.375E-06	.270E-06	.193E-06	.137E-06	.974E-07	.580E-07	.346E-07	.206E-07
7g-8s	.155E-07	.180E-07	.194E-07	.197E-07	.190E-07	.179E-07	.164E-07	.148E-07	.132E-07	.115E-07	.863E-08	.633E-08	.457E-08	.328E-08	.233E-08	.139E-08	.828E-09	.492E-09
7g-8p	.629E-07	.714E-07	.761E-07	.770E-07	.753E-07	.712E-07	.658E-07	.595E-07	.528E-07	.465E-07	.348E-07	.254E-07	.184E-07	.131E-07	.935E-08	.558E-08	.332E-08	.197E-08
7g-8d	.186E-06	.207E-06	.215E-06	.216E-06	.209E-06	.198E-06	.185E-06	.169E-06	.154E-06	.137E-06	.106E-06	.801E-07	.590E-07	.429E-07	.309E-07	.186E-07	.111E-07	.661E-08
7g-8f	.652E-06	.725E-06	.755E-06	.742E-06	.714E-06	.670E-06	.626E-06	.584E-06	.543E-06	.510E-06	.448E-06	.387E-06	.327E-06	.270E-06	.218E-06	.153E-06	.106E-06	.713E-07
7g-8g	.275E-05	.346E-05	.401E-05	.432E-05	.439E-05	.430E-05	.406E-05	.376E-05	.342E-05	.306E-05	.236E-05	.177E-05	.131E-05	.956E-06	.691E-06	.419E-06	.252E-06	.150E-06
7g-8h	.343E-05	.500E-05	.664E-05	.837E-05	.997E-05	.114E-04	.125E-04	.133E-04	.137E-04	.139E-04	.134E-04	.122E-04	.107E-04	.903E-05	.743E-05	.533E-05	.374E-05	.255E-05
7g-8i	.541E-05	.664E-05	.755E-05	.799E-05	.801E-05	.776E-05	.724E-05	.664E-05	.600E-05	.532E-05	.405E-05	.300E-05	.218E-05	.157E-05	.112E-05	.674E-06	.403E-06	.240E-06
7g-8j	.218E-05	.245E-05	.256E-05	.252E-05	.239E-05	.219E-05	.197E-05	.174E-05	.150E-05	.130E-05	.947E-06	.682E-06	.487E-06	.346E-06	.246E-06	.146E-06	.871E-07	.518E-07
7h-8s	.453E-08	.514E-08	.547E-08	.555E-08	.543E-08	.517E-08	.480E-08	.436E-08	.390E-08	.343E-08	.257E-08	.189E-08	.137E-08	.978E-09	.696E-09	.415E-09	.246E-09	.146E-09
7h-8p	.163E-07	.186E-07	.198E-07	.201E-07	.196E-07	.185E-07	.172E-07	.156E-07	.140E-07	.123E-07	.927E-08	.681E-08	.494E-08	.354E-08	.252E-08	.150E-08	.893E-09	.530E-09
7h-8d	.434E-07	.486E-07	.515E-07	.517E-07	.505E-07	.478E-07	.442E-07	.402E-07	.358E-07	.315E-07	.237E-07	.174E-07	.125E-07	.898E-08	.640E-08	.382E-08	.227E-08	.135E-08
7h-8f	.128E-06	.142E-06	.147E-06	.146E-06	.140E-06	.131E-06	.121E-06	.110E-06	.993E-07	.882E-07	.677E-07	.506E-07	.371E-07	.269E-07	.193E-07	.116E-07	.692E-08	.412E-08
7h-8g	.506E-06	.563E-06	.586E-06	.574E-06	.548E-06	.508E-06	.467E-06	.427E-06	.388E-06	.355E-06	.297E-06	.246E-06	.200E-06	.160E-06	.126E-06	.866E-07	.586E-07	.389E-07
7h-8h	.264E-05	.329E-05	.376E-05	.401E-05	.403E-05	.391E-05	.367E-05	.337E-05	.305E-05	.271E-05	.208E-05	.155E-05	.114E-05	.830E-06	.599E-06	.363E-06	.217E-06	.130E-06
7h-8i	.642E-05	.913E-05	.118E-04	.144E-04	.167E-04	.186E-04	.200E-04	.207E-04	.210E-04	.208E-04	.196E-04	.175E-04	.150E-04	.125E-04	.102E-04	.725E-05	.506E-05	.343E-05
7h-8j	.550E-05	.662E-05	.739E-05	.769E-05	.759E-05	.725E-05	.669E-05	.608E-05	.545E-05	.480E-05	.362E-05	.266E-05	.193E-05	.138E-05	.988E-06	.592E-06	.354E-06	.211E-06
7i-8s	.111E-08	.123E-08	.129E-08	.131E-08	.129E-08	.125E-08	.118E-08	.109E-08	.988E-09	.880E-09	.671E-09	.497E-09	.361E-09	.260E-09	.185E-09	.110E-09	.650E-10	.386E-10
7i-8p	.382E-08	.423E-08	.445E-08	.451E-08	.446E-08	.430E-08	.407E-08	.376E-08	.340E-08	.302E-08	.230E-08	.170E-08	.124E-08	.889E-09	.633E-09	.378E-09	.224E-09	.133E-09
7i-8d	.841E-08	.940E-08	.989E-08	.100E-07	.985E-08	.946E-08	.892E-08	.822E-08	.745E-08	.663E-08	.505E-08	.374E-08	.272E-08	.196E-08	.139E-08	.833E-09	.494E-09	.293E-09
7i-8f	.205E-07	.226E-07	.237E-07	.237E-07	.232E-07	.222E-07	.207E-07	.190E-07	.171E-07	.152E-07	.116E-07	.852E-08	.618E-08	.444E-08	.316E-08	.189E-08	.112E-08	.665E-09
7i-8g	.618E-07	.682E-07	.702E-07	.693E-07	.662E-07	.622E-07	.576E-07	.525E-07	.474E-07	.421E-07	.323E-07	.241E-07	.176E-07	.127E-07	.911E-08	.547E-08	.326E-08	.193E-08
7i-8h	.285E-06	.317E-06	.329E-06	.321E-06	.305E-06	.282E-06	.257E-06	.233E-06	.209E-06	.188E-06	.151E-06	.119E-06	.932E-07	.722E-07	.553E-07	.366E-07	.241E-07	.156E-07
7i-8i	.206E-05	.252E-05	.284E-05	.299E-05	.296E-05	.284E-05	.264E-05	.241E-05	.217E-05	.191E-05	.145E-05	.108E-05	.791E-06	.573E-06	.413E-06	.249E-06	.149E-06	.889E-07
7i-8j	.132E-04	.180E-04	.223E-04	.263E-04	.295E-04	.317E-04	.332E-04	.335E-04	.331E-04	.323E-04	.293E-04	.255E-04	.215E-04	.177E-04	.143E-04	.100E-04	.692E-05	.466E-05

Table 5.2 Coulomb-Glauber rate coefficients as a function of electron temperature for excitations of CVI.

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
1s-2s	.321E-127	.412E-84	.655E-57	.954E-40	.647E-29	.444E-22	.922E-18	.493E-15	.263E-13	.326E-12	.437E-11	.115E-10	.143E-10	.126E-10	.930E-11	.512E-11	.264E-11	.133E-11
1s-2p	.613E-126	.785E-83	.124E-55	.179E-38	.120E-27	.809E-21	.164E-16	.843E-14	.428E-12	.500E-11	.590E-10	.143E-09	.184E-09	.181E-09	.157E-09	.113E-09	.744E-10	.463E-10
1s-3s	.131E-149	.162E-98	.281E-66	.618E-46	.425E-33	.538E-25	.704E-20	.121E-16	.135E-14	.268E-13	.591E-12	.192E-11	.265E-11	.242E-11	.182E-11	.101E-11	.524E-12	.264E-12
1s-3p	.236E-148	.291E-97	.501E-65	.109E-44	.742E-32	.923E-24	.118E-18	.194E-15	.206E-13	.384E-12	.737E-11	.218E-10	.301E-10	.303E-10	.263E-10	.188E-10	.123E-10	.763E-11
1s-3d	.174E-149	.214E-98	.369E-66	.807E-46	.549E-33	.684E-25	.873E-20	.144E-16	.154E-14	.288E-13	.554E-12	.160E-11	.205E-11	.183E-11	.138E-11	.770E-12	.401E-12	.203E-12
1s-4s	.126E-157	.973E-104	.975E-70	.278E-48	.962E-35	.338E-26	.843E-21	.217E-17	.314E-15	.736E-14	.193E-12	.675E-12	.960E-12	.892E-12	.675E-12	.376E-12	.195E-12	.983E-13
1s-4p	.222E-156	.171E-102	.170E-68	.482E-47	.165E-33	.569E-25	.138E-19	.342E-16	.469E-14	.103E-12	.235E-11	.743E-11	.105E-10	.107E-10	.930E-11	.662E-11	.433E-11	.267E-11
1s-4d	.219E-157	.168E-103	.168E-69	.475E-48	.163E-34	.562E-26	.137E-20	.339E-17	.467E-15	.103E-13	.234E-12	.723E-12	.955E-12	.862E-12	.650E-12	.365E-12	.190E-12	.962E-13
1s-4f	.504E-159	.387E-105	.386E-71	.109E-49	.373E-36	.128E-27	.311E-22	.769E-19	.105E-16	.228E-15	.497E-14	.144E-13	.177E-13	.149E-13	.107E-13	.573E-14	.292E-14	.147E-14
1s-5s	.188E-161	.285E-106	.187E-71	.175E-49	.128E-35	.719E-27	.241E-21	.751E-18	.122E-15	.310E-14	.880E-13	.318E-12	.459E-12	.430E-12	.326E-12	.182E-12	.942E-13	.476E-13
1s-5p	.329E-160	.496E-105	.324E-70	.300E-48	.217E-34	.120E-25	.392E-20	.117E-16	.181E-14	.430E-13	.106E-11	.346E-11	.496E-11	.504E-11	.440E-11	.313E-11	.204E-11	.126E-11
1s-5d	.360E-161	.544E-106	.356E-71	.329E-49	.238E-35	.132E-26	.432E-21	.130E-17	.201E-15	.478E-14	.117E-12	.374E-12	.501E-12	.454E-12	.343E-12	.192E-12	.100E-12	.508E-13
1s-5f	.121E-162	.182E-107	.119E-72	.110E-50	.794E-37	.439E-28	.143E-22	.426E-19	.654E-17	.154E-15	.362E-14	.108E-13	.135E-13	.114E-13	.819E-14	.439E-14	.224E-14	.113E-14
1s-5g	.968E-165	.146E-109	.954E-75	.882E-53	.637E-39	.352E-30	.115E-24	.340E-21	.521E-19	.121E-17	.281E-16	.819E-16	.995E-16	.830E-16	.590E-16	.315E-16	.161E-16	.807E-17
1s-6s	.131E-163	.999E-108	.182E-72	.323E-50	.355E-36	.258E-27	.102E-21	.352E-18	.611E-16	.161E-14	.478E-13	.176E-12	.256E-12	.241E-12	.183E-12	.102E-12	.529E-13	.267E-13
1s-6p	.227E-162	.173E-106	.314E-71	.553E-49	.600E-35	.429E-26	.165E-20	.546E-17	.901E-15	.223E-13	.572E-12	.190E-11	.274E-11	.280E-11	.244E-11	.174E-11	.113E-11	.699E-12
1s-6d	.263E-163	.200E-107	.363E-72	.640E-50	.695E-36	.498E-27	.192E-21	.637E-18	.105E-15	.261E-14	.669E-13	.217E-12	.292E-12	.266E-12	.201E-12	.113E-12	.588E-13	.298E-13
1s-6f	.103E-164	.782E-109	.142E-73	.250E-51	.271E-37	.193E-28	.741E-23	.245E-19	.401E-17	.978E-16	.241E-14	.732E-14	.917E-14	.780E-14	.560E-14	.301E-14	.153E-14	.771E-15
1s-6g	.127E-166	.965E-111	.175E-75	.308E-53	.334E-39	.238E-30	.911E-25	.300E-21	.490E-19	.119E-17	.287E-16	.850E-16	.104E-15	.870E-16	.619E-16	.331E-16	.168E-16	.847E-17
1s-6h	.597E-169	.454E-113	.822E-78	.145E-55	.156E-41	.111E-32	.423E-27	.138E-23	.224E-21	.535E-20	.124E-18	.352E-18	.415E-18	.338E-18	.237E-18	.126E-18	.639E-19	.321E-19
1s-7s	.573E-165	.116E-108	.391E-73	.102E-50	.144E-36	.122E-27	.532E-22	.195E-18	.352E-16	.952E-15	.290E-13	.108E-12	.158E-12	.149E-12	.113E-12	.632E-13	.327E-13	.165E-13
1s-7p	.994E-164	.201E-107	.673E-72	.175E-49	.242E-35	.202E-26	.857E-21	.302E-17	.518E-15	.131E-13	.346E-12	.116E-11	.168E-11	.172E-11	.150E-11	.107E-11	.694E-12	.429E-12
1s-7d	.118E-164	.239E-108	.803E-73	.209E-50	.290E-36	.242E-27	.103E-21	.363E-18	.624E-16	.158E-14	.417E-13	.136E-12	.185E-12	.168E-12	.127E-12	.715E-13	.372E-13	.189E-13
1s-7f	.503E-166	.101E-109	.340E-74	.884E-52	.122E-37	.102E-28	.431E-23	.151E-19	.258E-17	.645E-16	.163E-14	.500E-14	.629E-14	.536E-14	.385E-14	.207E-14	.106E-14	.531E-15
1s-7g	.750E-168	.151E-111	.507E-76	.132E-53	.182E-39	.152E-30	.641E-25	.224E-21	.381E-19	.948E-18	.234E-16	.702E-16	.863E-16	.723E-16	.515E-16	.275E-16	.140E-16	.705E-17
1s-7h	.564E-170	.114E-113	.381E-78	.988E-56	.136E-41	.113E-32	.476E-27	.165E-23	.278E-21	.682E-20	.162E-18	.465E-18	.550E-18	.449E-18	.315E-18	.167E-18	.849E-19	.427E-19
1s-7i	.136E-172	.274E-116	.917E-81	.237E-58	.327E-44	.271E-35	.113E-29	.392E-26	.653E-24	.159E-22	.370E-21	.104E-20	.122E-20	.996E-21	.699E-21	.371E-21	.189E-21	.950E-22

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
1s-8s	.681-166	.260-109	.131E-73	.440E-51	.723E-37	.679E-28	.315E-22	.120E-18	.223E-16	.612E-15	.190E-13	.712E-13	.104E-12	.983E-13	.748E-13	.418E-13	.217E-13	.109E-13
1s-8p	.118-164	.448-108	.224E-72	.749E-50	.122E-35	.112E-26	.507E-21	.186E-17	.327E-15	.841E-14	.225E-12	.763E-12	.111E-11	.113E-11	.987E-12	.702E-12	.457E-12	.282E-12
1s-8d	.143-165	.545-109	.273E-73	.912E-51	.148E-36	.137E-27	.619E-22	.228E-18	.402E-16	.104E-14	.277E-13	.914E-13	.124E-12	.113E-12	.855E-13	.481E-13	.251E-13	.127E-13
1s-8f	.639-167	.243-110	.121E-74	.406E-52	.659E-38	.607E-29	.273E-23	.999E-20	.174E-17	.444E-16	.114E-14	.352E-14	.444E-14	.379E-14	.272E-14	.146E-14	.746E-15	.375E-15
1s-8g	.106-168	.403-112	.201E-76	.672E-54	.109E-39	.100E-30	.452E-25	.165E-21	.287E-19	.724E-18	.182E-16	.549E-16	.677E-16	.567E-16	.404E-16	.216E-16	.110E-16	.554E-17
1s-8h	.989-171	.376-114	.188E-78	.626E-56	.101E-41	.930E-33	.416E-27	.151E-23	.259E-21	.647E-20	.156E-18	.452E-18	.536E-18	.438E-18	.307E-18	.163E-18	.828E-19	.416E-19
1s-8i	.392-173	.149-116	.743E-81	.247E-58	.400E-44	.366E-35	.163E-29	.587E-26	.100E-23	.248E-22	.587E-21	.167E-20	.196E-20	.160E-20	.112E-20	.595E-21	.303E-21	.152E-21
1s-8j	.638-176	.242-119	.121E-83	.402E-61	.649E-47	.592E-38	.262E-32	.939E-29	.159E-26	.388E-25	.896E-24	.247E-23	.284E-23	.229E-23	.159E-23	.844E-24	.429E-24	.216E-24
2s-3s	.166E-30	.156E-22	.166E-17	.244E-14	.240E-12	.427E-11	.260E-10	.809E-10	.165E-09	.254E-09	.361E-09	.343E-09	.265E-09	.185E-09	.122E-09	.631E-10	.319E-10	.160E-10
2s-3p	.141E-29	.133E-21	.142E-16	.210E-13	.206E-11	.366E-10	.219E-09	.654E-09	.125E-08	.180E-08	.231E-08	.217E-08	.182E-08	.146E-08	.115E-08	.761E-09	.481E-09	.292E-09
2s-3d	.986E-30	.929E-22	.993E-17	.147E-13	.144E-11	.255E-10	.152E-09	.448E-09	.845E-09	.119E-08	.144E-08	.123E-08	.893E-09	.601E-09	.392E-09	.200E-09	.101E-09	.506E-10
2p-3s	.141E-31	.134E-23	.147E-18	.227E-15	.238E-13	.463E-12	.313E-11	.108E-10	.238E-10	.390E-10	.605E-10	.634E-10	.560E-10	.459E-10	.362E-10	.240E-10	.152E-10	.919E-11
2p-3p	.204E-30	.192E-22	.204E-17	.299E-14	.293E-12	.518E-11	.312E-10	.951E-10	.190E-09	.287E-09	.400E-09	.379E-09	.292E-09	.203E-09	.134E-09	.691E-10	.349E-10	.175E-10
2p3d	.351E-29	.330E-21	.353E-16	.521E-13	.512E-11	.906E-10	.541E-09	.160E-08	.304E-08	.432E-08	.533E-08	.478E-08	.381E-08	.292E-08	.220E-08	.140E-08	.862E-09	.514E-09
2s-4s	.654E-39	.382E-28	.236E-21	.449E-17	.222E-14	.110E-12	.130E-11	.619E-11	.168E-10	.316E-10	.567E-10	.604E-10	.493E-10	.353E-10	.236E-10	.123E-10	.621E-11	.312E-11
2s-4p	.575E-38	.338E-27	.210E-20	.404E-16	.202E-13	.100E-11	.116E-10	.528E-10	.133E-09	.229E-09	.358E-09	.369E-09	.322E-09	.262E-09	.205E-09	.136E-09	.856E-10	.518E-10
2s-4d	.283E-38	.167E-27	.104E-20	.201E-16	.100E-13	.499E-12	.574E-11	.259E-10	.643E-10	.109E-09	.159E-09	.147E-09	.110E-09	.751E-10	.490E-10	.250E-10	.126E-10	.632E-11
2s-4f	.188E-38	.110E-27	.686E-21	.131E-16	.652E-14	.321E-12	.365E-11	.163E-10	.398E-10	.659E-10	.924E-10	.821E-10	.598E-10	.400E-10	.259E-10	.131E-10	.660E-11	.331E-11
2p-4s	.236E-40	.145E-29	.975E-23	.210E-18	.123E-15	.757E-14	.112E-12	.660E-12	.212E-11	.447E-11	.924E-11	.109E-10	.101E-10	.831E-11	.650E-11	.426E-11	.265E-11	.160E-11
2p-4p	.852E-39	.498E-28	.308E-21	.586E-17	.289E-14	.142E-12	.165E-11	.765E-11	.201E-10	.368E-10	.640E-10	.676E-10	.549E-10	.391E-10	.261E-10	.135E-10	.684E-11	.343E-11
2p-4d	.121E-37	.713E-27	.444E-20	.853E-16	.426E-13	.211E-11	.243E-10	.110E-09	.274E-09	.464E-09	.690E-09	.667E-09	.543E-09	.416E-09	.311E-09	.195E-09	.119E-09	.701E-10
2p-4f	.198E-38	.116E-27	.722E-21	.138E-16	.686E-14	.338E-12	.385E-11	.173E-10	.428E-10	.723E-10	.107E-09	.101E-09	.771E-10	.536E-10	.355E-10	.184E-10	.931E-11	.467E-11
2s-5s	0.688E-43	.791E-31	.320E-23	.199E-18	.209E-15	.167E-13	.266E-12	.155E-11	.479E-11	.986E-11	.197E-10	.220E-10	.184E-10	.133E-10	.896E-11	.466E-11	.236E-11	.119E-11
2s-5p	0.625E-42	.723E-30	.295E-22	.186E-17	.197E-14	.157E-12	.245E-11	.135E-10	.386E-10	.722E-10	.124E-09	.133E-09	.118E-09	.962E-10	.755E-10	.499E-10	.314E-10	.190E-10
2s-5d	0.266E-42	.308E-30	.126E-22	.799E-18	.850E-15	.682E-13	.107E-11	.586E-11	.165E-10	.304E-10	.487E-10	.468E-10	.357E-10	.245E-10	.160E-10	.818E-11	.412E-11	.207E-11
2s-5f	0.266E-42	.307E-30	.125E-22	.787E-18	.828E-15	.656E-13	.101E-11	.545E-11	.151E-10	.271E-10	.414E-10	.381E-10	.281E-10	.189E-10	.123E-10	.623E-11	.313E-11	.157E-11
2s-5g	0.255E-43	.293E-31	.119E-23	.739E-19	.767E-16	.597E-14	.901E-13	.476E-12	.129E-11	.227E-11	.341E-11	.313E-11	.232E-11	.156E-11	.102E-11	.518E-12	.261E-12	.131E-12
2p-5s	0.171E-44	.213E-32	.968E-25	.714E-20	.939E-17	.987E-15	.209E-13	.157E-12	.592E-12	.139E-11	.322E-11	.400E-11	.374E-11	.309E-11	.241E-11	.157E-11	.971E-12	.581E-12
2p-5p	0.914E-43	.105E-30	.426E-23	.266E-18	.278E-15	.220E-13	.344E-12	.194E-11	.581E-11	.116E-10	.224E-10	.248E-10	.206E-10	.148E-10	.994E-11	.516E-11	.261E-11	.131E-11

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
2p-5d	0.123E-41	.143E-29	.583E-22	.367E-17	.389E-14	.310E-12	.484E-11	.265E-10	.749E-10	.138E-09	.224E-09	.224E-09	.185E-09	.142E-09	.105E-09	.657E-10	.397E-10	.233E-10
2p-5f	0.287E-42	.331E-30	.135E-22	.846E-18	.890E-15	.705E-13	.109E-11	.593E-11	.166E-10	.304E-10	.490E-10	.479E-10	.373E-10	.261E-10	.173E-10	.897E-11	.455E-11	.228E-11
2p-5g	0.181E-43	.209E-31	.848E-24	.532E-19	.559E-16	.441E-14	.678E-13	.365E-12	.101E-11	.182E-11	.280E-11	.261E-11	.194E-11	.131E-11	.854E-12	.435E-12	.219E-12	.110E-12
2s-6s	.402E-45	.233E-32	.262E-24	.311E-19	.489E-16	.506E-14	.953E-13	.617E-12	.205E-11	.443E-11	.932E-11	.107E-10	.908E-11	.660E-11	.446E-11	.232E-11	.118E-11	.591E-12
2s-6p	.373E-44	.218E-31	.246E-23	.296E-18	.470E-15	.486E-13	.893E-12	.546E-11	.167E-10	.325E-10	.585E-10	.642E-10	.574E-10	.471E-10	.370E-10	.245E-10	.154E-10	.929E-11
2s-6d	.146E-44	.858E-32	.977E-24	.118E-18	.189E-15	.197E-13	.363E-12	.222E-11	.673E-11	.129E-10	.218E-10	.214E-10	.165E-10	.113E-10	.743E-11	.380E-11	.191E-11	.959E-12
2s-6f	.175E-44	.102E-31	.115E-23	.138E-18	.218E-15	.224E-13	.406E-12	.244E-11	.721E-11	.135E-10	.216E-10	.203E-10	.151E-10	.102E-10	.660E-11	.336E-11	.169E-11	.846E-12
2s-6g	.255E-45	.148E-32	.167E-24	.198E-19	.309E-16	.311E-14	.552E-13	.324E-12	.937E-12	.172E-11	.270E-11	.252E-11	.189E-11	.128E-11	.831E-12	.424E-12	.213E-12	.107E-12
2s-6h	.861E-47	.500E-34	.560E-26	.663E-21	.103E-17	.104E-15	.184E-14	.108E-13	.311E-13	.569E-13	.879E-13	.806E-13	.592E-13	.396E-13	.256E-13	.130E-13	.656E-14	.329E-14
2p-6s	.857E-47	.544E-34	.698E-26	.100E-20	.204E-17	.285E-15	.730E-14	.621E-13	.253E-12	.628E-12	.154E-11	.196E-11	.185E-11	.153E-11	.119E-11	.768E-12	.475E-12	.283E-12
2p-6p	.539E-45	.313E-32	.352E-24	.418E-19	.657E-16	.673E-14	.124E-12	.779E-12	.250E-11	.523E-11	.107E-10	.121E-10	.102E-10	.737E-11	.496E-11	.257E-11	.130E-11	.654E-12
2p-6d	.709E-44	.414E-31	.469E-23	.564E-18	.898E-15	.928E-13	.170E-11	.104E-10	.314E-10	.604E-10	.103E-09	.105E-09	.870E-10	.667E-10	.496E-10	.308E-10	.186E-10	.109E-10
2p-6f	.190E-44	.111E-31	.125E-23	.150E-18	.237E-15	.243E-13	.443E-12	.268E-11	.803E-11	.154E-10	.259E-10	.259E-10	.203E-10	.142E-10	.948E-11	.491E-11	.249E-11	.125E-11
2p-6g	.182E-45	.106E-32	.120E-24	.143E-19	.226E-16	.231E-14	.418E-13	.250E-12	.741E-12	.139E-11	.225E-11	.213E-11	.160E-11	.109E-11	.707E-12	.360E-12	.182E-12	.909E-13
2p-6h	.484E-47	.281E-34	.316E-26	.375E-21	.587E-18	.593E-16	.106E-14	.625E-14	.182E-13	.338E-13	.536E-13	.504E-13	.378E-13	.256E-13	.167E-13	.852E-14	.430E-14	.215E-14
2s-7s	.159E-46	.245E-33	.509E-25	.892E-20	.180E-16	.217E-14	.451E-13	.312E-12	.108E-11	.240E-11	.523E-11	.611E-11	.521E-11	.380E-11	.257E-11	.134E-11	.680E-12	.341E-12
2s-7p	.150E-45	.232E-32	.486E-24	.860E-19	.175E-15	.211E-13	.427E-12	.278E-11	.883E-11	.177E-10	.328E-10	.364E-10	.328E-10	.269E-10	.212E-10	.140E-10	.879E-11	.531E-11
2s-7d	.561E-46	.872E-33	.184E-24	.328E-19	.673E-16	.819E-14	.167E-12	.109E-11	.345E-11	.681E-11	.118E-10	.118E-10	.912E-11	.629E-11	.413E-11	.211E-11	.106E-11	.533E-12
2s-7f	.735E-46	.114E-32	.238E-24	.421E-19	.852E-16	.102E-13	.205E-12	.131E-11	.403E-11	.775E-11	.127E-10	.121E-10	.903E-11	.610E-11	.396E-11	.202E-11	.101E-11	.508E-12
2s-7g	.129E-46	.199E-33	.415E-25	.727E-20	.145E-16	.171E-14	.335E-13	.209E-12	.629E-12	.119E-11	.191E-11	.181E-11	.136E-11	.920E-12	.599E-12	.305E-12	.154E-12	.771E-13
2s-7h	.692E-48	.107E-34	.221E-26	.386E-21	.769E-18	.904E-16	.177E-14	.110E-13	.330E-13	.621E-13	.984E-13	.913E-13	.673E-13	.452E-13	.292E-13	.149E-13	.749E-14	.375E-14
2s-7i	.116E-49	.179E-36	.369E-28	.641E-23	.126E-19	.146E-17	.282E-16	.173E-15	.510E-15	.945E-15	.147E-14	.136E-14	.101E-14	.677E-15	.439E-15	.224E-15	.113E-15	.566E-16
2p-7s	.317E-48	.536E-35	.128E-26	.275E-21	.724E-18	.120E-15	.343E-14	.314E-13	.134E-12	.343E-12	.871E-12	.112E-11	.106E-11	.877E-12	.682E-12	.440E-12	.271E-12	.162E-12
2p-7p	.214E-46	.330E-33	.687E-25	.121E-19	.242E-16	.290E-14	.590E-13	.395E-12	.132E-11	.285E-11	.599E-11	.692E-11	.586E-11	.425E-11	.286E-11	.149E-11	.754E-12	.378E-12
2p-7d	.278E-45	.431E-32	.905E-24	.161E-18	.327E-15	.395E-13	.800E-12	.520E-11	.164E-10	.323E-10	.567E-10	.585E-10	.487E-10	.373E-10	.277E-10	.172E-10	.104E-10	.606E-11
2p-7f	.807E-46	.125E-32	.261E-24	.461E-19	.932E-16	.112E-13	.225E-12	.145E-11	.451E-11	.886E-11	.154E-10	.155E-10	.123E-10	.862E-11	.574E-11	.297E-11	.151E-11	.757E-12
2p-7g	.927E-47	.143E-33	.300E-25	.529E-20	.107E-16	.127E-14	.255E-13	.162E-12	.500E-12	.963E-12	.160E-11	.154E-11	.116E-11	.788E-12	.513E-12	.262E-12	.132E-12	.660E-13
2p-7h	.390E-48	.601E-35	.125E-26	.219E-21	.438E-18	.518E-16	.102E-14	.640E-14	.194E-13	.370E-13	.603E-13	.574E-13	.432E-13	.294E-13	.192E-13	.978E-14	.493E-14	.247E-14
2p-7i	.563E-50	.867E-37	.180E-28	.316E-23	.631E-20	.744E-18	.146E-16	.914E-16	.276E-15	.521E-15	.832E-15	.775E-15	.572E-15	.384E-15	.249E-15	.127E-15	.638E-16	.320E-16

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
2s-8s	.177E-47	.514E-34	.159E-25	.359E-20	.848E-17	.113E-14	.252E-13	.181E-12	.647E-12	.146E-11	.326E-11	.384E-11	.329E-11	.241E-11	.163E-11	.850E-12	.431E-12	.217E-12
2s-8p	.168E-46	.490E-33	.153E-24	.349E-19	.832E-16	.111E-13	.240E-12	.163E-11	.530E-11	.108E-10	.204E-10	.229E-10	.206E-10	.170E-10	.133E-10	.881E-11	.554E-11	.334E-11
2s-8d	.612E-47	.179E-33	.564E-25	.130E-19	.312E-16	.421E-14	.917E-13	.624E-12	.202E-11	.407E-11	.721E-11	.725E-11	.563E-11	.389E-11	.255E-11	.131E-11	.658E-12	.330E-12
2s-8f	.850E-47	.248E-33	.774E-25	.176E-19	.418E-16	.554E-14	.118E-12	.787E-12	.249E-11	.488E-11	.816E-11	.779E-11	.584E-11	.395E-11	.257E-11	.131E-11	.657E-12	.329E-12
2s-8g	.166E-47	.482E-34	.150E-25	.337E-20	.789E-17	.103E-14	.215E-13	.139E-12	.431E-12	.827E-12	.135E-11	.129E-11	.970E-12	.659E-12	.430E-12	.219E-12	.110E-12	.553E-13
2s-8h	.110E-48	.318E-35	.985E-27	.221E-21	.517E-18	.671E-16	.140E-14	.907E-14	.279E-13	.534E-13	.861E-13	.804E-13	.595E-13	.400E-13	.259E-13	.132E-13	.663E-14	.332E-14
2s-8i	.302E-50	.872E-37	.269E-28	.600E-23	.139E-19	.178E-17	.365E-16	.233E-15	.705E-15	.133E-14	.211E-14	.196E-14	.145E-14	.980E-15	.636E-15	.324E-15	.163E-15	.820E-16
2s-8j	.276E-52	.797E-39	.245E-30	.545E-25	.126E-21	.161E-19	.329E-18	.209E-17	.634E-17	.119E-16	.188E-16	.173E-16	.127E-16	.847E-17	.548E-17	.279E-17	.141E-17	.705E-18
2p-8s	.342E-49	.109E-35	.392E-27	.109E-21	.338E-18	.623E-16	.191E-14	.183E-13	.807E-13	.210E-12	.545E-12	.706E-12	.671E-12	.555E-12	.431E-12	.277E-12	.171E-12	.102E-12
2p-8p	.239E-47	.695E-34	.216E-25	.487E-20	.115E-16	.152E-14	.330E-13	.230E-12	.792E-12	.174E-11	.373E-11	.436E-11	.371E-11	.270E-11	.182E-11	.945E-12	.479E-12	.240E-12
2p-8d	.308E-46	.901E-33	.282E-24	.644E-19	.154E-15	.205E-13	.444E-12	.300E-11	.971E-11	.195E-10	.349E-10	.363E-10	.303E-10	.232E-10	.172E-10	.107E-10	.641E-11	.375E-11
2p-8f	.936E-47	.273E-33	.852E-25	.194E-19	.459E-16	.609E-14	.130E-12	.875E-12	.280E-11	.560E-11	.990E-11	.101E-10	.797E-11	.561E-11	.374E-11	.194E-11	.982E-12	.493E-12
2p-8g	.119E-47	.347E-34	.108E-25	.246E-20	.581E-17	.768E-15	.164E-13	.109E-12	.343E-12	.673E-12	.114E-11	.110E-11	.833E-12	.567E-12	.370E-12	.188E-12	.949E-13	.476E-13
2p-8h	.619E-49	.180E-35	.557E-27	.126E-21	.295E-18	.385E-16	.809E-15	.529E-14	.165E-13	.319E-13	.529E-13	.508E-13	.384E-13	.261E-13	.170E-13	.869E-14	.438E-14	.220E-14
2p-8i	.146E-50	.424E-37	.131E-28	.296E-23	.694E-20	.905E-18	.190E-16	.124E-15	.383E-15	.735E-15	.119E-14	.112E-14	.830E-15	.558E-15	.362E-15	.184E-15	.927E-16	.465E-16
2p-8j	.127E-52	.366E-39	.113E-30	.253E-25	.588E-22	.758E-20	.157E-18	.100E-17	.306E-17	.581E-17	.929E-17	.866E-17	.642E-17	.432E-17	.281E-17	.143E-17	.722E-18	.362E-18
3s-4s	.185E-15	.112E-12	.618E-11	.751E-10	.348E-09	.894E-09	.156E-08	.216E-08	.255E-08	.271E-08	.244E-08	.184E-08	.127E-08	.834E-09	.538E-09	.273E-09	.138E-09	.689E-10
3s-4p	.957E-15	.582E-12	.328E-10	.408E-09	.194E-08	.512E-08	.901E-08	.123E-07	.141E-07	.145E-07	.127E-07	.991E-08	.749E-08	.565E-08	.424E-08	.267E-08	.164E-08	.895E-09
3s-4d	.581E-15	.351E-12	.195E-10	.238E-09	.110E-08	.280E-08	.475E-08	.623E-08	.692E-08	.689E-08	.561E-08	.399E-08	.266E-08	.173E-08	.111E-08	.559E-09	.281E-09	.141E-09
3s-4f	.159E-15	.102E-12	.608E-11	.806E-10	.407E-09	.113E-08	.208E-08	.291E-08	.340E-08	.349E-08	.293E-08	.209E-08	.139E-08	.891E-09	.567E-09	.285E-09	.143E-09	.717E-10
3p-4s	.411E-16	.252E-13	.144E-11	.183E-10	.900E-10	.249E-09	.464E-09	.669E-09	.809E-09	.867E-09	.800E-09	.643E-09	.494E-09	.375E-09	.283E-09	.178E-09	.109E-09	.596E-10
3p-4p	.238E-15	.143E-12	.793E-11	.960E-10	.443E-09	.113E-08	.196E-08	.267E-08	.312E-08	.328E-08	.292E-08	.219E-08	.150E-08	.989E-09	.637E-09	.324E-09	.163E-09	.816E-10
3p-4d	.159E-14	.965E-12	.541E-10	.670E-09	.317E-08	.828E-08	.144E-07	.194E-07	.221E-07	.225E-07	.193E-07	.147E-07	.109E-07	.804E-08	.593E-08	.367E-08	.222E-08	.121E-08
3p-4f	.500E-15	.309E-12	.177E-10	.222E-09	.106E-08	.281E-08	.491E-08	.660E-08	.748E-08	.756E-08	.628E-08	.450E-08	.301E-08	.195E-08	.125E-08	.630E-09	.317E-09	.159E-09
3d-4s	.577E-17	.338E-14	.180E-12	.212E-11	.958E-11	.244E-10	.423E-10	.579E-10	.674E-10	.701E-10	.609E-10	.449E-10	.306E-10	.200E-10	.129E-10	.651E-11	.327E-11	.164E-11
3d-4p	.204E-16	.124E-13	.702E-12	.886E-11	.435E-10	.121E-09	.229E-09	.337E-09	.415E-09	.451E-09	.417E-09	.327E-09	.242E-09	.176E-09	.127E-09	.762E-10	.449E-10	.242E-10
3d-4d	.161E-15	.985E-13	.554E-11	.685E-10	.322E-09	.835E-09	.146E-08	.201E-08	.238E-08	.253E-08	.230E-08	.175E-08	.121E-08	.794E-09	.511E-09	.260E-09	.130E-09	.654E-10
3d-4f	.316E-14	.192E-11	.108E-09	.135E-08	.638E-08	.167E-07	.292E-07	.393E-07	.446E-07	.453E-07	.385E-07	.289E-07	.209E-07	.152E-07	.110E-07	.665E-08	.396E-08	.214E-08
3s-5s	.815E-20	.972E-16	.354E-13	.140E-11	.137E-10	.584E-10	.140E-09	.239E-09	.327E-09	.386E-09	.395E-09	.319E-09	.228E-09	.153E-09	.991E-10	.506E-10	.255E-10	.128E-10
3s-5p	.418E-19	.506E-15	.188E-12	.762E-11	.771E-10	.341E-09	.824E-09	.138E-08	.182E-08	.206E-08	.201E-08	.165E-08	.128E-08	.971E-09	.732E-09	.464E-09	.285E-09	.160E-09

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
3s-5d	.227E-19	.273E-15	.101E-12	.406E-11	.406E-10	.176E-09	.414E-09	.675E-09	.864E-09	.944E-09	.851E-09	.632E-09	.430E-09	.281E-09	.180E-09	.911E-10	.457E-10	.229E-10
3s-5f	.243E-20	.306E-16	.121E-13	.527E-12	.589E-11	.291E-10	.776E-10	.141E-09	.198E-09	.231E-09	.225E-09	.171E-09	.117E-09	.761E-10	.486E-10	.245E-10	.123E-10	.616E-11
3s-5g	.141E-19	.171E-15	.640E-13	.260E-11	.261E-10	.114E-09	.267E-09	.432E-09	.548E-09	.593E-09	.527E-09	.388E-09	.261E-09	.169E-09	.108E-09	.546E-10	.274E-10	.137E-10
3p-5s	.110E-20	.136E-16	.521E-14	.220E-12	.240E-11	.118E-10	.318E-10	.598E-10	.875E-10	.107E-09	.117E-09	.102E-09	.806E-10	.619E-10	.466E-10	.294E-10	.179E-10	.100E-10
3p-5p	.107E-19	.127E-15	.463E-13	.183E-11	.180E-10	.764E-10	.182E-09	.306E-09	.414E-09	.483E-09	.487E-09	.389E-09	.276E-09	.185E-09	.120E-09	.610E-10	.307E-10	.154E-10
3p-5d	.701E-19	.844E-15	.312E-12	.126E-10	.127E-09	.554E-09	.132E-08	.219E-08	.284E-08	.316E-08	.298E-08	.236E-08	.176E-08	.130E-08	.958E-09	.593E-09	.357E-09	.198E-09
3p-5f	.869E-20	.107E-15	.405E-13	.169E-11	.176E-10	.804E-10	.198E-09	.337E-09	.447E-09	.502E-09	.470E-09	.356E-09	.244E-09	.159E-09	.102E-09	.516E-10	.259E-10	.130E-10
3p-5g	.189E-19	.229E-15	.849E-13	.343E-11	.343E-10	.149E-09	.352E-09	.573E-09	.730E-09	.792E-09	.702E-09	.511E-09	.342E-09	.221E-09	.141E-09	.709E-10	.356E-10	.178E-10
3d-5s	.219E-21	.260E-17	.952E-15	.382E-13	.391E-12	.179E-11	.457E-11	.816E-11	.114E-10	.134E-10	.134E-10	.106E-10	.738E-11	.488E-11	.315E-11	.160E-11	.802E-12	.402E-12
3d-5p	.735E-21	.888E-17	.331E-14	.136E-12	.146E-11	.705E-11	.193E-10	.368E-10	.546E-10	.674E-10	.721E-10	.598E-10	.447E-10	.320E-10	.226E-10	.132E-10	.756E-11	.410E-11
3d-5d	.805E-20	.964E-16	.353E-13	.140E-11	.138E-10	.584E-10	.138E-09	.230E-09	.310E-09	.363E-09	.372E-09	.301E-09	.214E-09	.143E-09	.928E-10	.472E-10	.238E-10	.119E-10
3d-5f	.111E-18	.133E-14	.491E-12	.198E-10	.198E-09	.866E-09	.206E-08	.340E-08	.440E-08	.486E-08	.449E-08	.347E-08	.252E-08	.180E-08	.129E-08	.774E-09	.455E-09	.250E-09
3d-5g	.285E-19	.342E-15	.126E-12	.504E-11	.499E-10	.214E-09	.499E-09	.808E-09	.103E-08	.112E-08	.102E-08	.763E-09	.524E-09	.345E-09	.222E-09	.113E-09	.569E-10	.285E-10
3s-6s	.334E-22	.201E-17	.202E-14	.148E-12	.220E-11	.126E-10	.357E-10	.683E-10	.102E-09	.126E-09	.139E-09	.116E-09	.840E-10	.568E-10	.371E-10	.190E-10	.955E-11	.479E-11
3s-6p	.174E-21	.106E-16	.109E-13	.818E-12	.126E-10	.751E-10	.214E-09	.400E-09	.567E-09	.674E-09	.697E-09	.589E-09	.462E-09	.353E-09	.266E-09	.169E-09	.104E-09	.588E-10
3s-6d	.859E-22	.523E-17	.538E-14	.405E-12	.623E-11	.368E-10	.104E-09	.191E-09	.264E-09	.304E-09	.290E-09	.221E-09	.152E-09	.995E-10	.639E-10	.323E-10	.162E-10	.814E-11
3s-6f	.776E-23	.495E-18	.543E-15	.445E-13	.794E-12	.544E-11	.176E-10	.367E-10	.563E-10	.699E-10	.726E-10	.571E-10	.395E-10	.259E-10	.166E-10	.837E-11	.420E-11	.210E-11
3s-6g	.658E-22	.402E-17	.415E-14	.313E-12	.480E-11	.283E-10	.790E-10	.143E-09	.195E-09	.222E-09	.208E-09	.157E-09	.107E-09	.694E-10	.444E-10	.224E-10	.113E-10	.564E-11
3s-6h	.201E-22	.121E-17	.123E-14	.910E-13	.136E-11	.776E-11	.212E-10	.378E-10	.509E-10	.572E-10	.525E-10	.388E-10	.260E-10	.168E-10	.107E-10	.540E-11	.272E-11	.136E-11
3p-6s	.352E-23	.220E-18	.235E-15	.186E-13	.322E-12	.215E-11	.713E-11	.155E-10	.251E-10	.331E-10	.393E-10	.356E-10	.287E-10	.221E-10	.167E-10	.105E-10	.639E-11	.360E-11
3p-6p	.433E-22	.261E-17	.263E-14	.194E-12	.289E-11	.166E-10	.468E-10	.888E-10	.130E-09	.160E-09	.173E-09	.143E-09	.103E-09	.694E-10	.452E-10	.231E-10	.116E-10	.582E-11
3p-6d	.286E-21	.174E-16	.178E-13	.133E-11	.204E-10	.121E-09	.341E-09	.630E-09	.882E-09	.103E-08	.102E-08	.827E-09	.623E-09	.461E-09	.338E-09	.209E-09	.125E-09	.701E-10
3p-6f	.223E-22	.139E-17	.147E-14	.115E-12	.189E-11	.120E-10	.364E-10	.710E-10	.104E-09	.125E-09	.126E-09	.993E-10	.693E-10	.457E-10	.294E-10	.149E-10	.748E-11	.375E-11
3p-6g	.929E-22	.564E-17	.578E-14	.434E-12	.663E-11	.389E-10	.109E-09	.199E-09	.273E-09	.311E-09	.290E-09	.216E-09	.146E-09	.945E-10	.603E-10	.304E-10	.153E-10	.764E-11
3p-6h	.198E-22	.119E-17	.121E-14	.898E-13	.134E-11	.767E-11	.209E-10	.370E-10	.497E-10	.557E-10	.512E-10	.382E-10	.258E-10	.168E-10	.107E-10	.542E-11	.272E-11	.136E-11
3d-6s	.737E-24	.449E-19	.464E-16	.357E-14	.594E-13	.383E-12	.122E-11	.255E-11	.396E-11	.499E-11	.539E-11	.437E-11	.310E-11	.206E-11	.133E-11	.675E-12	.339E-12	.170E-12
3d-6p	.270E-23	.164E-18	.169E-15	.130E-13	.219E-12	.144E-11	.483E-11	.107E-10	.175E-10	.231E-10	.266E-10	.227E-10	.171E-10	.122E-10	.851E-11	.489E-11	.276E-11	.149E-11
3d-6d	.343E-22	.206E-17	.208E-14	.153E-12	.227E-11	.129E-10	.357E-10	.666E-10	.967E-10	.119E-09	.130E-09	.109E-09	.789E-10	.531E-10	.345E-10	.176E-10	.886E-11	.444E-11
3d-6f	.412E-21	.249E-16	.254E-13	.190E-11	.291E-10	.171E-09	.482E-09	.887E-09	.123E-08	.143E-08	.139E-08	.109E-08	.793E-09	.566E-09	.403E-09	.240E-09	.140E-09	.772E-10
3d-6g	.155E-21	.938E-17	.954E-14	.709E-12	.107E-10	.618E-10	.171E-09	.309E-09	.423E-09	.485E-09	.463E-09	.355E-09	.246E-09	.162E-09	.105E-09	.533E-10	.269E-10	.135E-10
3d-6h	.155E-22	.938E-18	.954E-15	.709E-13	.107E-11	.618E-11	.171E-10	.307E-10	.417E-10	.471E-10	.437E-10	.325E-10	.220E-10	.143E-10	.911E-11	.459E-11	.231E-11	.116E-11

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
3s-7s	.109E-23	.173E-18	.326E-15	.370E-13	.713E-12	.449E-11	.141E-10	.291E-10	.452E-10	.582E-10	.663E-10	.566E-10	.415E-10	.282E-10	.184E-10	.944E-11	.476E-11	.239E-11
3s-7p	.579E-23	.932E-18	.180E-14	.212E-12	.424E-11	.274E-10	.862E-10	.172E-09	.254E-09	.310E-09	.332E-09	.286E-09	.226E-09	.173E-09	.131E-09	.832E-10	.510E-10	.290E-10
3s-7d	.270E-23	.435E-18	.845E-15	.999E-13	.200E-11	.130E-10	.407E-10	.802E-10	.117E-09	.138E-09	.137E-09	.106E-09	.732E-10	.481E-10	.309E-10	.156E-10	.787E-11	.394E-11
3s-7f	.250E-24	.425E-19	.895E-16	.118E-13	.271E-12	.202E-11	.732E-11	.163E-10	.262E-10	.335E-10	.360E-10	.287E-10	.200E-10	.131E-10	.843E-11	.426E-11	.214E-11	.107E-11
3s-7g	.220E-23	.355E-18	.690E-15	.815E-13	.163E-11	.105E-10	.324E-10	.628E-10	.898E-10	.105E-09	.102E-09	.776E-10	.531E-10	.347E-10	.222E-10	.112E-10	.564E-11	.282E-11
3s-7h	.110E-23	.175E-18	.334E-15	.384E-13	.744E-12	.463E-11	.140E-10	.266E-10	.374E-10	.432E-10	.408E-10	.305E-10	.206E-10	.133E-10	.850E-11	.429E-11	.216E-11	.108E-11
3s-7i	.957E-25	.152E-19	.286E-16	.325E-14	.621E-13	.380E-12	.113E-11	.210E-11	.290E-11	.329E-11	.307E-11	.229E-11	.155E-11	.100E-11	.642E-12	.324E-12	.163E-12	.818E-13
3p-7s	.100E-24	.165E-19	.324E-16	.375E-14	.884E-13	.712E-12	.266E-11	.627E-11	.108E-10	.148E-10	.184E-10	.171E-10	.140E-10	.108E-10	.810E-11	.510E-11	.310E-11	.175E-11
3p-7p	.141E-23	.223E-18	.412E-15	.436E-13	.855E-12	.594E-11	.187E-10	.380E-10	.585E-10	.744E-10	.834E-10	.703E-10	.512E-10	.346E-10	.226E-10	.115E-10	.582E-11	.291E-11
3p-7d	.940E-23	.149E-17	.280E-14	.302E-12	.610E-11	.435E-10	.136E-09	.269E-09	.393E-09	.472E-09	.484E-09	.397E-09	.301E-09	.222E-09	.163E-09	.100E-09	.602E-10	.338E-10
3p-7f	.555E-24	.918E-19	.186E-15	.233E-13	.505E-12	.357E-11	.123E-10	.262E-10	.407E-10	.511E-10	.544E-10	.439E-10	.311E-10	.206E-10	.133E-10	.675E-11	.340E-11	.170E-11
3p-7g	.320E-23	.514E-18	.992E-15	.116E-12	.231E-11	.148E-10	.459E-10	.895E-10	.129E-09	.151E-09	.145E-09	.110E-09	.745E-10	.484E-10	.309E-10	.156E-10	.782E-11	.392E-11
3p-7h	.109E-23	.175E-18	.333E-15	.385E-13	.747E-12	.466E-11	.141E-10	.266E-10	.372E-10	.428E-10	.406E-10	.306E-10	.209E-10	.136E-10	.868E-11	.439E-11	.221E-11	.111E-11
3p-7i	.779E-25	.124E-19	.234E-16	.267E-14	.513E-13	.317E-12	.950E-12	.179E-11	.250E-11	.286E-11	.268E-11	.200E-11	.135E-11	.869E-12	.554E-12	.280E-12	.141E-12	.705E-13
3d-7s	.211E-25	.338E-20	.646E-17	.735E-15	.171E-13	.136E-12	.499E-12	.114E-11	.188E-11	.247E-11	.278E-11	.230E-11	.164E-11	.109E-11	.707E-12	.359E-12	.180E-12	.903E-13
3d-7p	.843E-25	.134E-19	.254E-16	.284E-14	.647E-13	.509E-12	.192E-11	.461E-11	.800E-11	.110E-10	.132E-10	.115E-10	.864E-11	.615E-11	.427E-11	.243E-11	.136E-11	.735E-12
3d-7d	.114E-23	.179E-18	.331E-15	.349E-13	.676E-12	.465E-11	.143E-10	.284E-10	.432E-10	.549E-10	.625E-10	.533E-10	.389E-10	.263E-10	.172E-10	.875E-11	.441E-11	.221E-11
3d-7f	.129E-22	.204E-17	.382E-14	.409E-12	.824E-11	.584E-10	.183E-09	.359E-09	.523E-09	.623E-09	.623E-09	.494E-09	.361E-09	.257E-09	.182E-09	.108E-09	.628E-10	.347E-10
3d-7g	.571E-23	.913E-18	.175E-14	.202E-12	.395E-11	.249E-10	.763E-10	.147E-09	.211E-09	.248E-09	.245E-09	.190E-09	.133E-09	.878E-10	.568E-10	.289E-10	.146E-10	.729E-11
3d-7h	.888E-24	.142E-18	.271E-15	.314E-13	.615E-12	.388E-11	.119E-10	.228E-10	.323E-10	.376E-10	.360E-10	.271E-10	.184E-10	.120E-10	.766E-11	.387E-11	.194E-11	.973E-12
3d-7i	.439E-25	.699E-20	.132E-16	.152E-14	.293E-13	.181E-12	.544E-12	.103E-11	.144E-11	.165E-11	.157E-11	.118E-11	.806E-12	.525E-12	.336E-12	.170E-12	.854E-13	.428E-13
3s-8s	.108E-24	.310E-19	.851E-16	.113E-13	.267E-12	.210E-11	.709E-11	.152E-10	.245E-10	.322E-10	.376E-10	.325E-10	.240E-10	.163E-10	.107E-10	.548E-11	.277E-11	.139E-11
3s-8p	.579E-24	.169E-18	.474E-15	.646E-13	.160E-11	.130E-10	.437E-10	.907E-10	.138E-09	.172E-09	.188E-09	.163E-09	.130E-09	.995E-10	.752E-10	.479E-10	.294E-10	.167E-10
3s-8d	.260E-24	.760E-19	.214E-15	.293E-13	.735E-12	.601E-11	.203E-10	.419E-10	.627E-10	.759E-10	.768E-10	.600E-10	.417E-10	.274E-10	.176E-10	.893E-11	.449E-11	.225E-11
3s-8f	.258E-25	.785E-20	.236E-16	.360E-14	.110E-12	.101E-11	.390E-11	.908E-11	.150E-10	.195E-10	.213E-10	.171E-10	.120E-10	.788E-11	.505E-11	.255E-11	.128E-11	.642E-12
3s-8g	.217E-24	.635E-19	.179E-15	.245E-13	.607E-12	.494E-11	.164E-10	.333E-10	.489E-10	.583E-10	.577E-10	.444E-10	.305E-10	.200E-10	.128E-10	.646E-11	.325E-11	.163E-11
3s-8h	.137E-24	.398E-19	.110E-15	.147E-13	.349E-12	.274E-11	.884E-11	.175E-10	.253E-10	.298E-10	.287E-10	.216E-10	.146E-10	.948E-11	.605E-11	.305E-11	.154E-11	.769E-12
3s-8i	.195E-25	.562E-20	.154E-16	.204E-14	.474E-13	.367E-12	.116E-11	.225E-11	.319E-11	.369E-11	.351E-11	.264E-11	.179E-11	.116E-11	.742E-12	.375E-12	.189E-12	.946E-13
3s-8j	.751E-27	.215E-21	.585E-18	.767E-16	.176E-14	.134E-13	.421E-13	.813E-13	.115E-12	.133E-12	.125E-12	.933E-13	.627E-13	.405E-13	.258E-13	.130E-13	.657E-14	.329E-14
3p-8s	.911E-26	.275E-20	.807E-17	.120E-14	.353E-13	.320E-12	.129E-11	.320E-11	.570E-11	.803E-11	.103E-10	.973E-11	.798E-11	.617E-11	.464E-11	.292E-11	.177E-11	.100E-11
3p-8p	.139E-24	.403E-19	.111E-15	.148E-13	.352E-12	.279E-11	.939E-11	.200E-10	.318E-10	.413E-10	.475E-10	.405E-10	.297E-10	.201E-10	.132E-10	.672E-11	.339E-11	.170E-11

3.2 continued

ι'	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
i	.933E-24	.272E-18	.759E-15	.103E-12	.254E-11	.205E-10	.687E-10	.142E-09	.213E-09	.260E-09	.272E-09	.225E-09	.171E-09	.127E-09	.927E-10	.571E-10	.342E-10	.192E-10
	.458E-25	.137E-19	.401E-16	.590E-14	.170E-12	.151E-11	.566E-11	.128E-10	.208E-10	.267E-10	.294E-10	.242E-10	.172E-10	.115E-10	.743E-11	.378E-11	.190E-11	.951E-12
;	.322E-24	.940E-19	.263E-15	.357E-13	.879E-12	.710E-11	.236E-10	.481E-10	.711E-10	.850E-10	.837E-10	.638E-10	.434E-10	.283E-10	.180E-10	.911E-11	.457E-11	.229E-11
ı	.137E-24	.399E-19	.111E-15	.148E-13	.353E-12	.279E-11	.897E-11	.177E-10	.255E-10	.299E-10	.288E-10	.219E-10	.150E-10	.977E-11	.626E-11	.316E-11	.159E-11	.796E-12
	.160E-25	.461E-20	.127E-16	.168E-14	.395E-13	.308E-12	.985E-12	.194E-11	.278E-11	.324E-11	.309E-11	.232E-11	.157E-11	.102E-11	.647E-12	.327E-12	.164E-12	.824E-13
j	.552E-27	.159E-21	.434E-18	.573E-16	.133E-14	.102E-13	.321E-13	.621E-13	.877E-13	.101E-12	.957E-13	.718E-13	.486E-13	.315E-13	.201E-13	.102E-13	.513E-14	.257E-14
s	.188E-26	.554E-21	.159E-17	.236E-15	.705E-14	.636E-13	.255E-12	.618E-12	.106E-11	.143E-11	.165E-11	.138E-11	.988E-12	.660E-12	.427E-12	.217E-12	.109E-12	.546E-13
)	.819E-26	.239E-20	.672E-17	.962E-15	.271E-13	.239E-12	.972E-12	.246E-11	.442E-11	.621E-11	.765E-11	.672E-11	.507E-11	.360E-11	.250E-11	.141E-11	.788E-12	.424E-12
i	.113E-24	.326E-19	.898E-16	.119E-13	.280E-12	.219E-11	.717E-11	.149E-10	.234E-10	.304E-10	.354E-10	.306E-10	.225E-10	.152E-10	.994E-11	.508E-11	.256E-11	.128E-11
	.124E-23	.360E-18	.100E-14	.135E-12	.332E-11	.267E-10	.891E-10	.183E-09	.274E-09	.333E-09	.339E-09	.272E-09	.198E-09	.141E-09	.999E-10	.590E-10	.343E-10	.189E-10
;	.601E-24	.174E-18	.484E-15	.651E-13	.157E-11	.125E-10	.408E-10	.822E-10	.121E-09	.145E-09	.146E-09	.114E-09	.800E-10	.531E-10	.343E-10	.175E-10	.881E-11	.441E-11
ı	.114E-24	.331E-19	.918E-16	.124E-13	.298E-12	.237E-11	.774E-11	.155E-10	.227E-10	.268E-10	.262E-10	.199E-10	.136E-10	.884E-11	.565E-11	.285E-11	.143E-11	.718E-12
i	.912E-26	.264E-20	.727E-17	.969E-15	.229E-13	.179E-12	.574E-12	.113E-11	.163E-11	.191E-11	.184E-11	.140E-11	.957E-12	.624E-12	.400E-12	.202E-12	.102E-12	.509E-13
	.243E-27	.702E-22	.193E-18	.256E-16	.603E-15	.471E-14	.151E-13	.296E-13	.425E-13	.496E-13	.474E-13	.356E-13	.240E-13	.156E-13	.993E-14	.502E-14	.252E-14	.126E-14
s	.994E-11	.186E-09	.113E-08	.340E-08	.643E-08	.920E-08	.110E-07	.117E-07	.116E-07	.108E-07	.825E-08	.569E-08	.374E-08	.241E-08	.154E-08	.780E-09	.393E-09	.197E-09
)	.402E-10	.758E-09	.473E-08	.146E-07	.289E-07	.430E-07	.527E-07	.567E-07	.559E-07	.519E-07	.401E-07	.293E-07	.212E-07	.155E-07	.114E-07	.714E-08	.435E-08	.257E-08
i	.222E-10	.409E-09	.248E-08	.735E-08	.138E-07	.194E-07	.224E-07	.229E-07	.216E-07	.192E-07	.138E-07	.922E-08	.598E-08	.383E-08	.243E-08	.123E-08	.616E-09	.309E-09
	.659E-11	.134E-09	.890E-09	.290E-08	.594E-08	.900E-08	.111E-07	.118E-07	.115E-07	.104E-07	.757E-08	.507E-08	.327E-08	.208E-08	.132E-08	.662E-09	.332E-09	.166E-09
	.130E-11	.317E-10	.252E-09	.958E-09	.221E-08	.366E-08	.478E-08	.534E-08	.534E-08	.496E-08	.374E-08	.255E-08	.166E-08	.105E-08	.667E-09	.333E-09	.169E-09	.849E-10
	.308E-11	.584E-10	.369E-09	.117E-08	.237E-08	.365E-08	.464E-08	.514E-08	.518E-08	.488E-08	.384E-08	.282E-08	.205E-08	.151E-08	.111E-08	.695E-09	.423E-09	.250E-09
p	.129E-10	.239E-09	.145E-08	.432E-08	.815E-08	.116E-07	.138E-07	.146E-07	.143E-07	.132E-07	.100E-07	.690E-08	.453E-08	.291E-08	.186E-08	.942E-09	.474E-09	.238E-09
l	.538E-10	.101E-08	.630E-08	.194E-07	.379E-07	.558E-07	.678E-07	.723E-07	.706E-07	.651E-07	.498E-07	.360E-07	.258E-07	.187E-07	.136E-07	.840E-08	.507E-08	.298E-08
f	.202E-10	.384E-09	.240E-08	.733E-08	.142E-07	.204E-07	.240E-07	.249E-07	.237E-07	.213E-07	.154E-07	.104E-07	.672E-08	.429E-08	.273E-08	.137E-08	.691E-09	.346E-09
g	.337E-11	.737E-10	.531E-09	.186E-08	.405E-08	.645E-08	.824E-08	.906E-08	.897E-08	.826E-08	.612E-08	.413E-08	.267E-08	.170E-08	.107E-08	.536E-09	.270E-09	.136E-09
s	.513E-12	.907E-11	.525E-10	.150E-09	.278E-09	.394E-09	.468E-09	.494E-09	.480E-09	.438E-09	.325E-09	.221E-09	.145E-09	.930E-10	.592E-10	.298E-10	.150E-10	.752E-11
p	.195E-11	.370E-10	.233E-09	.737E-09	.150E-08	.234E-08	.302E-08	.339E-08	.344E-08	.325E-08	.255E-08	.185E-08	.132E-08	.945E-09	.681E-09	.413E-09	.247E-09	.144E-09
l	.104E-10	.196E-09	.121E-08	.364E-08	.695E-08	.999E-08	.120E-07	.128E-07	.126E-07	.117E-07	.900E-08	.623E-08	.410E-08	.263E-08	.168E-08	.850E-09	.429E-09	.215E-09
f	.892E-10	.168E-08	.105E-07	.321E-07	.628E-07	.920E-07	.111E-06	.118E-06	.115E-06	.106E-06	.802E-07	.574E-07	.408E-07	.292E-07	.211E-07	.129E-07	.772E-08	.451E-08
g	.178E-10	.350E-09	.226E-08	.718E-08	.143E-07	.212E-07	.255E-07	.269E-07	.259E-07	.235E-07	.172E-07	.116E-07	.756E-08	.483E-08	.306E-08	.154E-08	.776E-09	.389E-09
	.447E-13	.832E-12	.504E-11	.151E-10	.291E-10	.432E-10	.540E-10	.595E-10	.598E-10	.559E-10	.424E-10	.289E-10	.187E-10	.118E-10	.724E-11	.350E-11	.179E-11	.920E-12
	.224E-12	.411E-11	.247E-10	.739E-10	.143E-09	.211E-09	.261E-09	.286E-09	.287E-09	.268E-09	.204E-09	.140E-09	.913E-10	.580E-10	.361E-10	.177E-10	.899E-11	.461E-11

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
4f-5d	.718E-12	.140E-10	.904E-10	.293E-09	.615E-09	.983E-09	.130E-08	.149E-08	.154E-08	.146E-08	.115E-08	.820E-09	.571E-09	.397E-09	.276E-09	.161E-09	.934E-10	.534E-10
4f-5f	.655E-11	.127E-09	.810E-09	.251E-08	.490E-08	.714E-08	.864E-08	.932E-08	.931E-08	.876E-08	.683E-08	.476E-08	.313E-08	.201E-08	.128E-08	.649E-09	.328E-09	.165E-09
4f-5g	.152E-09	.290E-08	.182E-07	.561E-07	.110E-06	.161E-06	.195E-06	.207E-06	.201E-06	.184E-06	.139E-06	.990E-07	.695E-07	.492E-07	.350E-07	.210E-07	.125E-07	.721E-08
4s-6s	.178E-13	.169E-11	.288E-10	.165E-09	.471E-09	.881E-09	.126E-08	.152E-08	.164E-08	.162E-08	.134E-08	.964E-09	.647E-09	.420E-09	.270E-09	.138E-09	.695E-10	.349E-10
4s-6p	.682E-13	.662E-11	.117E-09	.704E-09	.212E-08	.416E-08	.610E-08	.738E-08	.786E-08	.769E-08	.631E-08	.473E-08	.347E-08	.256E-08	.188E-08	.118E-08	.717E-09	.424E-09
4s-6d	.382E-13	.363E-11	.625E-10	.363E-09	.105E-08	.197E-08	.275E-08	.317E-08	.323E-08	.303E-08	.230E-08	.158E-08	.103E-08	.664E-09	.423E-09	.213E-09	.107E-09	.536E-10
4s-6f	.796E-14	.797E-12	.148E-10	.938E-10	.300E-09	.624E-09	.952E-09	.118E-08	.127E-08	.123E-08	.968E-09	.670E-09	.439E-09	.280E-09	.178E-09	.889E-10	.447E-10	.225E-10
4s-6g	.625E-14	.567E-12	.933E-11	.523E-10	.149E-09	.278E-09	.392E-09	.460E-09	.476E-09	.452E-09	.349E-09	.240E-09	.157E-09	.100E-09	.617E-10	.299E-10	.153E-10	.781E-11
4s-6h	.145E-13	.148E-11	.275E-10	.173E-09	.533E-09	.106E-08	.154E-08	.184E-08	.192E-08	.182E-08	.140E-08	.958E-09	.623E-09	.396E-09	.249E-09	.123E-09	.626E-10	.316E-10
4p-6s	.381E-14	.379E-12	.692E-11	.434E-10	.138E-09	.290E-09	.454E-09	.581E-09	.648E-09	.655E-09	.560E-09	.429E-09	.317E-09	.234E-09	.172E-09	.107E-09	.651E-10	.383E-10
4p-6p	.236E-13	.222E-11	.379E-10	.217E-09	.620E-09	.116E-08	.165E-08	.197E-08	.210E-08	.206E-08	.168E-08	.120E-08	.805E-09	.522E-09	.335E-09	.170E-09	.858E-10	.431E-10
4p-6d	.973E-13	.937E-11	.164E-09	.978E-09	.291E-08	.563E-08	.813E-08	.970E-08	.102E-07	.985E-08	.792E-08	.583E-08	.420E-08	.304E-08	.221E-08	.136E-08	.816E-09	.478E-09
4p-6f	.253E-13	.245E-11	.431E-10	.258E-09	.770E-09	.149E-08	.214E-08	.252E-08	.262E-08	.248E-08	.192E-08	.133E-08	.870E-09	.558E-09	.354E-09	.178E-09	.897E-10	.450E-10
4p-6g	.539E-14	.498E-12	.841E-11	.489E-10	.145E-09	.284E-09	.419E-09	.510E-09	.542E-09	.525E-09	.413E-09	.285E-09	.187E-09	.119E-09	.736E-10	.358E-10	.183E-10	.935E-11
4p-6h	.203E-13	.203E-11	.370E-10	.227E-09	.692E-09	.135E-08	.193E-08	.227E-08	.234E-08	.221E-08	.170E-08	.117E-08	.761E-09	.485E-09	.306E-09	.152E-09	.772E-10	.389E-10
4d-6s	.919E-15	.849E-13	.142E-11	.811E-11	.237E-10	.461E-10	.681E-10	.831E-10	.888E-10	.864E-10	.689E-10	.486E-10	.323E-10	.208E-10	.133E-10	.662E-11	.335E-11	.169E-11
4d-6p	.282E-14	.275E-12	.491E-11	.303E-10	.958E-10	.203E-09	.323E-09	.420E-09	.472E-09	.479E-09	.406E-09	.302E-09	.216E-09	.153E-09	.108E-09	.640E-10	.375E-10	.216E-10
4d-6d	.199E-13	.189E-11	.323E-10	.185E-09	.531E-09	.994E-09	.142E-08	.170E-08	.183E-08	.180E-08	.149E-08	.107E-08	.718E-09	.465E-09	.298E-09	.151E-09	.765E-10	.385E-10
4d-6f	.152E-12	.146E-10	.255E-09	.151E-08	.447E-08	.859E-08	.123E-07	.146E-07	.152E-07	.146E-07	.116E-07	.841E-08	.597E-08	.425E-08	.304E-08	.184E-08	.109E-08	.634E-09
4d-6g	.666E-14	.666E-12	.123E-10	.779E-10	.249E-09	.516E-09	.788E-09	.980E-09	.106E-08	.104E-08	.835E-09	.591E-09	.393E-09	.254E-09	.162E-09	.815E-10	.395E-10	.201E-10
4d-6h	.359E-13	.349E-11	.619E-10	.370E-09	.110E-08	.211E-08	.301E-08	.352E-08	.361E-08	.340E-08	.257E-08	.175E-08	.114E-08	.724E-09	.457E-09	.229E-09	.115E-09	.579E-10
4f-6s	.170E-15	.154E-13	.253E-12	.142E-11	.408E-11	.786E-11	.117E-10	.144E-10	.157E-10	.155E-10	.125E-10	.865E-11	.562E-11	.353E-11	.211E-11	.980E-12	.508E-12	.269E-12
4f-6p	.648E-15	.599E-13	.100E-11	.572E-11	.167E-10	.325E-10	.483E-10	.598E-10	.650E-10	.642E-10	.521E-10	.366E-10	.241E-10	.152E-10	.931E-11	.444E-11	.228E-11	.120E-11
4f-6d	.158E-14	.153E-12	.271E-11	.164E-10	.511E-10	.106E-09	.168E-09	.219E-09	.246E-09	.249E-09	.208E-09	.151E-09	.104E-09	.699E-10	.467E-10	.258E-10	.145E-10	.809E-11
4f-6f	.131E-13	.126E-11	.217E-10	.127E-09	.366E-09	.686E-09	.978E-09	.118E-08	.127E-08	.127E-08	.106E-08	.767E-09	.514E-09	.331E-09	.212E-09	.107E-09	.543E-10	.275E-10
4f-6g	.189E-12	.182E-10	.317E-09	.187E-08	.554E-08	.106E-07	.153E-07	.181E-07	.188E-07	.180E-07	.142E-07	.102E-07	.709E-08	.495E-08	.348E-08	.205E-08	.119E-08	.684E-09
4f-6h	.679E-13	.650E-11	.113E-09	.658E-09	.191E-08	.358E-08	.500E-08	.578E-08	.590E-08	.555E-08	.424E-08	.293E-08	.193E-08	.124E-08	.791E-09	.400E-09	.201E-09	.101E-09
4s-7s	.405E-15	.102E-12	.325E-11	.276E-10	.101E-09	.224E-09	.357E-09	.464E-09	.526E-09	.540E-09	.468E-09	.345E-09	.235E-09	.153E-09	.987E-10	.504E-10	.255E-10	.128E-10
4s-7p	.155E-14	.401E-12	.133E-10	.118E-09	.460E-09	.106E-08	.173E-08	.225E-08	.251E-08	.254E-08	.217E-08	.166E-08	.123E-08	.910E-09	.672E-09	.420E-09	.256E-09	.151E-09
4s-7d	.834E-15	.212E-12	.687E-11	.600E-10	.226E-09	.506E-09	.794E-09	.990E-09	.106E-08	.103E-08	.813E-09	.567E-09	.375E-09	.241E-09	.154E-09	.774E-10	.389E-10	.195E-10

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
4s-7f	.131E-15	.357E-13	.127E-11	.125E-10	.535E-10	.137E-09	.241E-09	.328E-09	.376E-09	.381E-09	.315E-09	.223E-09	.147E-09	.941E-10	.596E-10	.297E-10	.150E-10	.755E-11
4s-7g	.225E-15	.561E-13	.177E-11	.151E-10	.560E-10	.124E-09	.193E-09	.240E-09	.257E-09	.248E-09	.195E-09	.136E-09	.891E-10	.568E-10	.353E-10	.173E-10	.878E-11	.447E-11
4s-7h	.283E-15	.763E-13	.263E-11	.245E-10	.977E-10	.230E-09	.377E-09	.485E-09	.533E-09	.526E-09	.420E-09	.292E-09	.191E-09	.121E-09	.749E-10	.362E-10	.187E-10	.952E-11
4s-7i	.253E-15	.660E-13	.218E-11	.194E-10	.736E-10	.165E-09	.257E-09	.317E-09	.338E-09	.325E-09	.254E-09	.175E-09	.115E-09	.728E-10	.456E-10	.225E-10	.115E-10	.582E-11
4p-7s	.696E-16	.187E-13	.648E-12	.614E-11	.256E-10	.649E-10	.115E-09	.162E-09	.192E-09	.204E-09	.185E-09	.146E-09	.110E-09	.811E-10	.597E-10	.371E-10	.225E-10	.132E-10
4p-7p	.531E-15	.134E-12	.427E-11	.364E-10	.134E-09	.298E-09	.473E-09	.611E-09	.686E-09	.698E-09	.597E-09	.436E-09	.295E-09	.192E-09	.124E-09	.628E-10	.318E-10	.160E-10
4p-7d	.225E-14	.580E-12	.190E-10	.168E-09	.642E-09	.147E-08	.235E-08	.301E-08	.331E-08	.330E-08	.275E-08	.205E-08	.148E-08	.107E-08	.779E-09	.478E-09	.287E-09	.168E-09
4p-7f	.451E-15	.117E-12	.389E-11	.352E-10	.139E-09	.325E-09	.529E-09	.680E-09	.746E-09	.736E-09	.594E-09	.418E-09	.277E-09	.178E-09	.113E-09	.566E-10	.286E-10	.144E-10
4p-7g	.208E-15	.520E-13	.165E-11	.142E-10	.537E-10	.122E-09	.195E-09	.248E-09	.270E-09	.265E-09	.211E-09	.147E-09	.964E-10	.613E-10	.382E-10	.187E-10	.949E-11	.484E-11
4p-7h	.420E-15	.111E-12	.376E-11	.343E-10	.135E-09	.313E-09	.503E-09	.638E-09	.693E-09	.677E-09	.539E-09	.376E-09	.247E-09	.157E-09	.980E-10	.482E-10	.246E-10	.125E-10
4p-7i	.278E-15	.721E-13	.236E-11	.208E-10	.785E-10	.175E-09	.273E-09	.339E-09	.361E-09	.348E-09	.271E-09	.186E-09	.121E-09	.771E-10	.484E-10	.239E-10	.122E-10	.615E-11
4d-7s	.176E-16	.438E-14	.140E-12	.123E-11	.483E-11	.118E-10	.202E-10	.275E-10	.315E-10	.322E-10	.271E-10	.195E-10	.131E-10	.849E-11	.538E-11	.267E-11	.136E-11	.685E-12
4d-7p	.527E-16	.139E-13	.469E-12	.436E-11	.182E-10	.471E-10	.860E-10	.123E-09	.149E-09	.158E-09	.142E-09	.108E-09	.779E-10	.549E-10	.383E-10	.223E-10	.130E-10	.740E-11
4d-7d	.452E-15	.114E-12	.366E-11	.313E-10	.116E-09	.256E-09	.407E-09	.526E-09	.595E-09	.610E-09	.527E-09	.387E-09	.262E-09	.170E-09	.109E-09	.555E-10	.282E-10	.142E-10
4d-7f	.339E-14	.868E-12	.282E-10	.248E-09	.945E-09	.215E-08	.343E-08	.435E-08	.476E-08	.470E-08	.384E-08	.281E-08	.200E-08	.142E-08	.101E-08	.607E-09	.359E-09	.208E-09
4d-7g	.117E-15	.304E-13	.103E-11	.962E-11	.399E-10	.100E-09	.175E-09	.239E-09	.276E-09	.283E-09	.240E-09	.174E-09	.118E-09	.760E-10	.479E-10	.237E-10	.121E-10	.617E-11
4d-7h	.850E-15	.220E-12	.726E-11	.645E-10	.247E-09	.563E-09	.894E-09	.113E-08	.121E-08	.118E-08	.927E-09	.639E-09	.418E-09	.266E-09	.167E-09	.833E-10	.421E-10	.213E-10
4d-7i	.308E-15	.791E-13	.257E-11	.225E-10	.843E-10	.186E-09	.288E-09	.353E-09	.374E-09	.359E-09	.279E-09	.193E-09	.126E-09	.805E-10	.508E-10	.254E-10	.128E-10	.647E-11
4f-7s	.415E-17	.103E-14	.320E-13	.274E-12	.104E-11	.245E-11	.415E-11	.563E-11	.651E-11	.669E-11	.561E-11	.396E-11	.258E-11	.162E-11	.945E-12	.427E-12	.224E-12	.121E-12
4f-7p	.152E-16	.377E-14	.119E-12	.103E-11	.399E-11	.949E-11	.162E-10	.220E-10	.255E-10	.262E-10	.223E-10	.159E-10	.106E-10	.665E-11	.401E-11	.188E-11	.970E-12	.518E-12
4f-7d	.373E-16	.959E-14	.313E-12	.278E-11	.111E-10	.275E-10	.492E-10	.699E-10	.839E-10	.888E-10	.780E-10	.574E-10	.395E-10	.263E-10	.173E-10	.923E-11	.510E-11	.284E-11
4f-7f	.314E-15	.798E-13	.256E-11	.219E-10	.807E-10	.177E-09	.278E-09	.358E-09	.405E-09	.418E-09	.365E-09	.269E-09	.182E-09	.118E-09	.750E-10	.379E-10	.193E-10	.979E-11
4f-7g	.380E-14	.968E-12	.313E-10	.273E-09	.103E-08	.234E-08	.372E-08	.472E-08	.514E-08	.506E-08	.409E-08	.295E-08	.205E-08	.142E-08	.991E-09	.578E-09	.334E-09	.190E-09
4f-7h	.199E-14	.506E-12	.164E-10	.142E-09	.531E-09	.117E-08	.182E-08	.226E-08	.242E-08	.234E-08	.185E-08	.129E-08	.857E-09	.552E-09	.352E-09	.178E-09	.897E-10	.451E-10
4f-7i	.275E-15	.702E-13	.227E-11	.197E-10	.738E-10	.164E-09	.254E-09	.313E-09	.332E-09	.319E-09	.247E-09	.170E-09	.111E-09	.708E-10	.448E-10	.224E-10	.113E-10	.567E-11
4s-8s	.330E-16	.158E-13	.750E-12	.822E-11	.355E-10	.874E-10	.150E-09	.204E-09	.239E-09	.251E-09	.224E-09	.168E-09	.115E-09	.753E-10	.486E-10	.248E-10	.126E-10	.632E-11
4s-8p	.127E-15	.623E-13	.308E-11	.356E-10	.162E-09	.419E-09	.730E-09	.990E-09	.114E-08	.118E-08	.103E-08	.800E-09	.596E-09	.442E-09	.326E-09	.204E-09	.124E-09	.734E-10
4s-8d	.661E-16	.320E-13	.156E-11	.177E-10	.791E-10	.199E-09	.336E-09	.440E-09	.488E-09	.484E-09	.392E-09	.276E-09	.184E-09	.118E-09	.754E-10	.380E-10	.191E-10	.959E-11
4s-8f	.858E-17	.449E-14	.245E-12	.321E-11	.168E-10	.496E-10	.955E-10	.139E-09	.166E-09	.173E-09	.148E-09	.106E-09	.703E-10	.450E-10	.284E-10	.140E-10	.713E-11	.360E-11
4s-8g	.215E-16	.103E-13	.499E-12	.562E-11	.250E-10	.624E-10	.105E-09	.137E-09	.151E-09	.149E-09	.120E-09	.837E-10	.552E-10	.353E-10	.220E-10	.109E-10	.549E-11	.279E-11
4s-8h	.193E-16	.981E-14	.507E-12	.612E-11	.290E-10	.769E-10	.136E-09	.185E-09	.210E-09	.212E-09	.174E-09	.122E-09	.801E-10	.508E-10	.308E-10	.145E-10	.759E-11	.392E-11

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
4s-8i	.311E-16	.153E-13	.755E-12	.863E-11	.386E-10	.962E-10	.161E-09	.208E-09	.227E-09	.223E-09	.178E-09	.124E-09	.812E-10	.517E-10	.321E-10	.156E-10	.804E-11	.409E-11
4s-8j	.593E-17	.286E-14	.138E-12	.153E-11	.667E-11	.162E-10	.266E-10	.340E-10	.369E-10	.359E-10	.283E-10	.195E-10	.127E-10	.807E-11	.503E-11	.247E-11	.126E-11	.641E-12
4p-8s	.503E-17	.257E-14	.134E-12	.165E-11	.821E-11	.235E-10	.453E-10	.672E-10	.832E-10	.909E-10	.858E-10	.690E-10	.522E-10	.388E-10	.285E-10	.177E-10	.107E-10	.630E-11
4p-8p	.432E-16	.206E-13	.985E-12	.109E-10	.472E-10	.117E-09	.200E-09	.271E-09	.314E-09	.328E-09	.288E-09	.214E-09	.146E-09	.952E-10	.612E-10	.311E-10	.158E-10	.794E-11
4p-8d	.187E-15	.909E-13	.444E-11	.187E-15	.909E-13	.444E-11	.507E-10	.229E-09	.582E-09	.100E-08	.151E-08	.145E-08	.115E-08	.842E-09	.609E-09	.376E-09	.230E-09	.138E-09
4p-8f	.308E-16	.153E-13	.770E-12	.917E-11	.433E-10	.115E-09	.204E-09	.278E-09	.316E-09	.321E-09	.266E-09	.190E-09	.127E-09	.814E-10	.516E-10	.258E-10	.131E-10	.657E-11
4p-8g	.214E-16	.102E-13	.492E-12	.555E-11	.249E-10	.631E-10	.108E-09	.144E-09	.161E-09	.160E-09	.130E-09	.911E-10	.600E-10	.382E-10	.239E-10	.117E-10	.594E-11	.303E-11
4p-8h	.295E-16	.147E-13	.748E-12	.887E-11	.414E-10	.108E-09	.187E-09	.250E-09	.281E-09	.281E-09	.229E-09	.161E-09	.106E-09	.677E-10	.418E-10	.202E-10	.104E-10	.535E-11
4p-8i	.349E-16	.171E-13	.836E-12	.949E-11	.422E-10	.105E-09	.175E-09	.227E-09	.250E-09	.245E-09	.195E-09	.135E-09	.883E-10	.561E-10	.350E-10	.172E-10	.876E-11	.445E-11
4p-8j	.563E-17	.271E-14	.131E-12	.146E-11	.635E-11	.154E-10	.252E-10	.320E-10	.346E-10	.336E-10	.265E-10	.184E-10	.120E-10	.763E-11	.477E-11	.236E-11	.120E-11	.610E-12
4d-8s	.125E-17	.597E-15	.289E-13	.335E-12	.160E-11	.453E-11	.862E-11	.125E-10	.151E-10	.159E-10	.138E-10	.101E-10	.685E-11	.443E-11	.280E-11	.138E-11	.706E-12	.357E-12
4d-8p	.385E-17	.192E-14	.977E-13	.119E-11	.592E-11	.175E-10	.350E-10	.534E-10	.672E-10	.736E-10	.683E-10	.529E-10	.382E-10	.268E-10	.186E-10	.107E-10	.619E-11	.352E-11
4d-8d	.366E-16	.176E-13	.843E-12	.933E-11	.407E-10	.101E-09	.172E-09	.233E-09	.272E-09	.285E-09	.254E-09	.189E-09	.129E-09	.840E-10	.539E-10	.274E-10	.139E-10	.702E-11
4d-8f	.273E-15	.132E-12	.642E-11	.729E-10	.327E-09	.828E-09	.142E-08	.188E-08	.212E-08	.214E-08	.178E-08	.131E-08	.935E-09	.663E-09	.471E-09	.282E-09	.166E-09	.960E-10
4d-8g	.107E-16	.526E-14	.264E-12	.316E-11	.153E-10	.426E-10	.796E-10	.114E-09	.136E-09	.142E-09	.124E-09	.917E-10	.625E-10	.406E-10	.256E-10	.127E-10	.645E-11	.330E-11
4d-8h	.664E-16	.325E-13	.160E-11	.183E-10	.829E-10	.211E-09	.361E-09	.477E-09	.532E-09	.528E-09	.425E-09	.296E-09	.194E-09	.123E-09	.774E-10	.383E-10	.194E-10	.985E-11
4d-8i	.406E-16	.197E-13	.959E-12	.108E-10	.477E-10	.118E-09	.194E-09	.250E-09	.272E-09	.266E-09	.211E-09	.147E-09	.967E-10	.617E-10	.388E-10	.193E-10	.979E-11	.496E-11
4d-8j	.476E-17	.229E-14	.110E-12	.122E-11	.532E-11	.130E-10	.213E-10	.272E-10	.295E-10	.288E-10	.226E-10	.156E-10	.102E-10	.648E-11	.407E-11	.202E-11	.102E-11	.517E-12
4f-8s	.333E-18	.157E-15	.746E-14	.843E-13	.388E-12	.105E-11	.195E-11	.282E-11	.340E-11	.358E-11	.309E-11	.220E-11	.144E-11	.899E-12	.517E-12	.229E-12	.121E-12	.664E-13
4f-8p	.120E-17	.570E-15	.273E-13	.311E-12	.145E-11	.394E-11	.736E-11	.107E-10	.129E-10	.137E-10	.120E-10	.864E-11	.573E-11	.360E-11	.215E-11	.993E-12	.516E-12	.350E-12
4f-8d	.306E-17	.148E-14	.716E-13	.819E-12	.384E-11	.108E-10	.209E-10	.316E-10	.396E-10	.431E-10	.391E-10	.291E-10	.201E-10	.133E-10	.860E-11	.452E-11	.248E-11	.138E-11
4f-8f	.262E-16	.125E-13	.599E-12	.660E-11	.285E-10	.692E-10	.116E-09	.157E-09	.183E-09	.193E-09	.173E-09	.129E-09	.879E-10	.568E-10	.363E-10	.183E-10	.932E-11	.475E-11
4f-8g	.289E-15	.139E-12	.669E-11	.753E-10	.335E-09	.843E-09	.144E-08	.190E-08	.214E-08	.214E-08	.177E-08	.128E-08	.887E-09	.613E-09	.425E-09	.246E-09	.141E-09	.801E-10
4f-8h	.178E-15	.857E-13	.414E-11	.465E-10	.205E-09	.505E-09	.840E-09	.109E-08	.120E-08	.118E-08	.957E-09	.675E-09	.449E-09	.289E-09	.185E-09	.931E-10	.470E-10	.236E-10
4f-8i	.391E-16	.189E-13	.912E-12	.102E-10	.450E-10	.111E-09	.185E-09	.239E-09	.261E-09	.256E-09	.202E-09	.140E-09	.920E-10	.588E-10	.371E-10	.186E-10	.935E-11	.471E-11
4f-8j	.283E-17	.136E-14	.653E-13	.724E-12	.315E-11	.764E-11	.125E-10	.159E-10	.172E-10	.168E-10	.132E-10	.920E-11	.604E-11	.386E-11	.244E-11	.122E-11	.617E-12	.311E-12
5s-6s	.124E-08	.586E-08	.149E-07	.252E-07	.332E-07	.373E-07	.380E-07	.361E-07	.328E-07	.287E-07	.203E-07	.135E-07	.871E-08	.554E-08	.353E-08	.178E-08	.898E-09	.450E-09
5s-6p	.408E-08	.195E-07	.510E-07	.912E-07	.127E-06	.151E-06	.161E-06	.158E-06	.146E-06	.131E-06	.963E-07	.683E-07	.485E-07	.349E-07	.254E-07	.156E-07	.940E-08	.552E-08
5s-6d	.216E-08	.996E-08	.248E-07	.416E-07	.540E-07	.596E-07	.592E-07	.545E-07	.478E-07	.407E-07	.277E-07	.181E-07	.116E-07	.739E-08	.469E-08	.236E-08	.118E-08	.593E-09
5s-6f	.671E-09	.346E-08	.956E-08	.175E-07	.245E-07	.286E-07	.295E-07	.280E-07	.250E-07	.214E-07	.147E-07	.957E-08	.612E-08	.387E-08	.245E-08	.123E-08	.616E-09	.309E-09
5s-6g	.193E-09	.119E-08	.383E-08	.794E-08	.122E-07	.152E-07	.164E-07	.161E-07	.147E-07	.129E-07	.904E-08	.598E-08	.384E-08	.243E-08	.154E-08	.774E-09	.389E-09	.195E-09

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
5s-6h	.431E-10	.326E-09	.122E-08	.281E-08	.462E-08	.610E-08	.686E-08	.691E-08	.645E-08	.571E-08	.405E-08	.267E-08	.172E-08	.108E-08	.681E-09	.335E-09	.171E-09	.864E-10
5p-6s	.413E-09	.199E-08	.532E-08	.974E-08	.140E-07	.171E-07	.186E-07	.187E-07	.176E-07	.158E-07	.117E-07	.835E-08	.594E-08	.428E-08	.311E-08	.191E-08	.115E-08	.677E-09
5p-6p	.159E-08	.743E-08	.187E-07	.317E-07	.415E-07	.466E-07	.473E-07	.448E-07	.404E-07	.352E-07	.248E-07	.165E-07	.106E-07	.677E-08	.431E-08	.217E-08	.109E-08	.548E-09
5p-6d	.486E-08	.232E-07	.604E-07	.107E-06	.148E-06	.174E-06	.183E-06	.178E-06	.164E-06	.145E-06	.106E-06	.751E-07	.529E-07	.378E-07	.273E-07	.167E-07	.997E-08	.582E-08
5p-6f	.197E-08	.938E-08	.240E-07	.413E-07	.548E-07	.613E-07	.615E-07	.570E-07	.503E-07	.429E-07	.293E-07	.192E-07	.123E-07	.782E-08	.496E-08	.249E-08	.125E-08	.627E-09
5p-6g	.465E-09	.256E-08	.748E-08	.144E-07	.211E-07	.255E-07	.270E-07	.260E-07	.236E-07	.204E-07	.141E-07	.927E-08	.593E-08	.376E-08	.238E-08	.119E-08	.598E-09	.300E-09
5p-6h	.813E-10	.558E-09	.194E-08	.427E-08	.683E-08	.879E-08	.971E-08	.965E-08	.893E-08	.787E-08	.558E-08	.369E-08	.237E-08	.150E-08	.943E-09	.467E-09	.237E-09	.120E-09
5d-6s	.716E-10	.319E-09	.769E-09	.127E-08	.167E-08	.189E-08	.194E-08	.184E-08	.166E-08	.144E-08	.100E-08	.662E-09	.427E-09	.273E-09	.173E-09	.870E-10	.437E-10	.219E-10
5d-6p	.297E-09	.144E-08	.387E-08	.712E-08	.103E-07	.127E-07	.140E-07	.141E-07	.133E-07	.119E-07	.883E-08	.622E-08	.437E-08	.310E-08	.222E-08	.134E-08	.798E-09	.463E-09
5d-6d	.138E-08	.655E-08	.167E-07	.285E-07	.377E-07	.425E-07	.433E-07	.412E-07	.373E-07	.327E-07	.232E-07	.154E-07	.995E-08	.633E-08	.403E-08	.203E-08	.102E-08	.513E-09
5d-6f	.725E-08	.346E-07	.900E-07	.159E-06	.218E-06	.255E-06	.266E-06	.258E-06	.236E-06	.209E-06	.152E-06	.107E-06	.750E-07	.533E-07	.383E-07	.232E-07	.138E-07	.805E-08
5d-6g	.212E-08	.103E-07	.271E-07	.476E-07	.643E-07	.731E-07	.740E-07	.692E-07	.614E-07	.526E-07	.362E-07	.237E-07	.152E-07	.966E-08	.612E-08	.308E-08	.155E-08	.775E-09
5d-6h	.283E-09	.165E-08	.511E-08	.103E-07	.155E-07	.192E-07	.207E-07	.202E-07	.185E-07	.161E-07	.113E-07	.741E-08	.475E-08	.300E-08	.189E-08	.942E-09	.475E-09	.239E-09
5f-6s	.828E-11	.393E-10	.101E-09	.174E-09	.240E-09	.285E-09	.302E-09	.297E-09	.274E-09	.241E-09	.169E-09	.112E-09	.719E-10	.454E-10	.286E-10	.141E-10	.715E-11	.361E-11
5f-6p	.442E-10	.204E-09	.508E-09	.874E-09	.119E-08	.140E-08	.149E-08	.146E-08	.134E-08	.117E-08	.829E-09	.551E-09	.356E-09	.226E-09	.143E-09	.710E-10	.359E-10	.181E-10
5f-6d	.149E-09	.742E-09	.204E-08	.384E-08	.568E-08	.717E-08	.800E-08	.814E-08	.774E-08	.697E-08	.515E-08	.360E-08	.249E-08	.174E-08	.123E-08	.725E-09	.424E-09	.244E-09
5f-6f	.113E-08	.545E-08	.141E-07	.244E-07	.325E-07	.369E-07	.378E-07	.361E-07	.329E-07	.289E-07	.207E-07	.138E-07	.892E-08	.567E-08	.361E-08	.182E-08	.918E-09	.461E-09
5f-6g	.114E-07	.544E-07	.141E-06	.249E-06	.341E-06	.397E-06	.413E-06	.398E-06	.363E-06	.320E-06	.232E-06	.162E-06	.113E-06	.797E-07	.567E-07	.341E-07	.202E-07	.117E-07
5f-6h	.181E-08	.910E-08	.247E-07	.446E-07	.617E-07	.714E-07	.732E-07	.692E-07	.619E-07	.534E-07	.370E-07	.243E-07	.156E-07	.990E-08	.627E-08	.315E-08	.158E-08	.795E-09
5g-6s	.843E-12	.466E-11	.133E-10	.254E-10	.372E-10	.464E-10	.512E-10	.518E-10	.489E-10	.437E-10	.315E-10	.208E-10	.132E-10	.815E-11	.488E-11	.223E-11	.117E-11	.631E-12
5g-6p	.381E-11	.200E-10	.562E-10	.105E-09	.153E-09	.189E-09	.207E-09	.210E-09	.198E-09	.176E-09	.126E-09	.832E-10	.530E-10	.328E-10	.198E-10	.923E-11	.479E-11	.255E-11
5g-6d	.145E-10	.717E-10	.192E-09	.351E-09	.503E-09	.618E-09	.679E-09	.679E-09	.635E-09	.565E-09	.405E-09	.269E-09	.173E-09	.109E-09	.671E-10	.324E-10	.166E-10	.859E-11
5g-6f	.546E-10	.287E-09	.825E-09	.160E-08	.242E-08	.311E-08	.351E-08	.361E-08	.346E-08	.312E-08	.230E-08	.158E-08	.108E-08	.731E-09	.500E-09	.284E-09	.162E-09	.914E-10
5g-6g	.715E-09	.354E-08	.942E-08	.166E-07	.225E-07	.258E-07	.266E-07	.257E-07	.236E-07	.209E-07	.151E-07	.101E-07	.654E-08	.416E-08	.264E-08	.133E-08	.672E-09	.338E-09
5g-6h	.180E-07	.867E-07	.227E-06	.401E-06	.549E-06	.638E-06	.662E-06	.635E-06	.579E-06	.508E-06	.366E-06	.254E-06	.176E-06	.123E-06	.867E-07	.514E-07	.301E-07	.173E-07
5s-7s	.128E-10	.160E-09	.753E-09	.189E-08	.320E-08	.425E-08	.485E-08	.501E-08	.482E-08	.440E-08	.325E-08	.223E-08	.146E-08	.943E-09	.600E-09	.303E-09	.153E-09	.766E-10
5s-7p	.417E-10	.539E-09	.266E-08	.710E-08	.128E-07	.180E-07	.212E-07	.223E-07	.216E-07	.197E-07	.150E-07	.107E-07	.766E-08	.553E-08	.402E-08	.247E-08	.149E-08	.875E-09
5s-7d	.235E-10	.291E-09	.137E-08	.345E-08	.584E-08	.766E-08	.850E-08	.842E-08	.774E-08	.678E-08	.475E-08	.315E-08	.204E-08	.130E-08	.827E-09	.414E-09	.208E-09	.104E-09
5s-7f	.603E-11	.806E-10	.413E-09	.115E-08	.213E-08	.304E-08	.360E-08	.374E-08	.356E-08	.317E-08	.224E-08	.149E-08	.972E-09	.620E-09	.393E-09	.189E-09	.955E-10	.464E-10
5s-7g	.257E-11	.320E-10	.157E-09	.429E-09	.806E-09	.117E-08	.142E-08	.150E-08	.144E-08	.130E-08	.925E-09	.628E-09	.410E-09	.261E-09	.159E-09	.648E-10	.323E-10	.147E-10
5s-7h	.442E-11	.547E-10	.257E-09	.650E-09	.111E-08	.147E-08	.166E-08	.165E-08	.153E-08	.133E-08	.925E-09	.612E-09	.394E-09	.250E-09	.155E-09	.736E-10	.369E-10	.180E-10

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
5s-7i	.484E-11	.726E-10	.406E-09	.119E-08	.227E-08	.326E-08	.384E-08	.397E-08	.375E-08	.332E-08	.237E-08	.156E-08	.102E-08	.650E-09	.392E-09	.157E-09	.784E-10	.361E-10
5p-7s	.354E-11	.469E-10	.238E-09	.660E-09	.124E-08	.182E-08	.224E-08	.242E-08	.240E-08	.223E-08	.171E-08	.124E-08	.884E-09	.638E-09	.463E-09	.284E-09	.171E-09	.100E-09
5p-7p	.171E-10	.212E-09	.992E-09	.249E-08	.420E-08	.557E-08	.631E-08	.647E-08	.617E-08	.559E-08	.414E-08	.281E-08	.184E-08	.117E-08	.750E-09	.380E-09	.191E-09	.961E-10
5p-7d	.528E-10	.677E-09	.330E-08	.870E-08	.155E-07	.214E-07	.249E-07	.259E-07	.248E-07	.225E-07	.169E-07	.120E-07	.847E-08	.605E-08	.435E-08	.265E-08	.158E-08	.923E-09
5p-7f	.176E-10	.223E-09	.107E-08	.279E-08	.486E-08	.653E-08	.739E-08	.742E-08	.690E-08	.610E-08	.434E-08	.289E-08	.186E-08	.119E-08	.753E-09	.378E-09	.190E-09	.953E-10
5p-7g	.294E-11	.387E-10	.199E-09	.567E-09	.109E-08	.162E-08	.198E-08	.212E-08	.206E-08	.187E-08	.136E-08	.913E-09	.591E-09	.375E-09	.236E-09	.116E-09	.590E-10	.298E-10
5p-7h	.510E-11	.624E-10	.289E-09	.722E-09	.122E-08	.159E-08	.177E-08	.175E-08	.161E-08	.141E-08	.994E-09	.658E-09	.423E-09	.267E-09	.167E-09	.826E-10	.418E-10	.212E-10
5p-7i	.654E-11	.948E-10	.516E-09	.148E-08	.278E-08	.395E-08	.465E-08	.481E-08	.456E-08	.407E-08	.292E-08	.193E-08	.124E-08	.785E-09	.491E-09	.241E-09	.123E-09	.624E-10
5d-7s	.911E-12	.109E-10	.495E-10	.123E-09	.211E-09	.285E-09	.329E-09	.336E-09	.318E-09	.285E-09	.206E-09	.139E-09	.905E-10	.578E-10	.368E-10	.184E-10	.926E-11	.465E-11
5d-7p	.288E-11	.375E-10	.188E-09	.515E-09	.970E-09	.143E-08	.177E-08	.193E-08	.192E-08	.177E-08	.135E-08	.960E-09	.672E-09	.472E-09	.334E-09	.198E-09	.116E-09	.667E-10
5d-7d	.150E-10	.187E-09	.881E-09	.222E-08	.377E-08	.502E-08	.572E-08	.589E-08	.565E-08	.515E-08	.384E-08	.262E-08	.171E-08	.109E-08	.699E-09	.353E-09	.178E-09	.896E-10
5d-7f	.776E-10	.988E-09	.479E-08	.126E-07	.222E-07	.304E-07	.352E-07	.362E-07	.345E-07	.312E-07	.232E-07	.164E-07	.115E-07	.811E-08	.579E-08	.349E-08	.207E-08	.120E-08
5d-7g	.108E-10	.140E-09	.698E-09	.189E-08	.341E-08	.476E-08	.553E-08	.569E-08	.538E-08	.482E-08	.348E-08	.233E-08	.151E-08	.958E-09	.606E-09	.303E-09	.153E-09	.770E-10
5d-7h	.566E-11	.000E+00	.311E-09	.769E-09	.129E-08	.170E-08	.190E-08	.189E-08	.174E-08	.153E-08	.107E-08	.709E-09	.455E-09	.287E-09	.179E-09	.883E-10	.447E-10	.227E-10
5d-7i	.112E-10	.155E-09	.810E-09	.225E-08	.411E-08	.573E-08	.662E-08	.676E-08	.634E-08	.564E-08	.403E-08	.268E-08	.173E-08	.109E-08	.687E-09	.341E-09	.173E-09	.875E-10
5f-7s	.184E-12	.217E-11	.977E-11	.243E-10	.422E-10	.586E-10	.695E-10	.732E-10	.708E-10	.641E-10	.467E-10	.313E-10	.202E-10	.127E-10	.796E-11	.382E-11	.197E-11	.101E-11
5f-7p	.747E-12	.894E-11	.409E-10	.103E-09	.180E-09	.250E-09	.296E-09	.310E-09	.298E-09	.270E-09	.198E-09	.133E-09	.869E-10	.551E-10	.345E-10	.169E-10	.862E-11	.438E-11
5f-7d	.178E-11	.230E-10	.115E-09	.313E-09	.592E-09	.883E-09	.110E-08	.121E-08	.120E-08	.111E-08	.842E-09	.588E-09	.402E-09	.274E-09	.188E-09	.107E-09	.609E-10	.343E-10
5f-7f	.121E-10	.152E-09	.721E-09	.183E-08	.313E-08	.419E-08	.480E-08	.497E-08	.481E-08	.441E-08	.332E-08	.227E-08	.149E-08	.949E-09	.605E-09	.306E-09	.155E-09	.778E-10
5f-7g	.107E-09	.136E-08	.657E-08	.172E-07	.301E-07	.411E-07	.474E-07	.486E-07	.462E-07	.416E-07	.307E-07	.215E-07	.149E-07	.104E-07	.736E-08	.437E-08	.256E-08	.147E-08
5f-7h	.330E-11	.428E-10	.215E-09	.592E-09	.111E-08	.161E-08	.195E-08	.206E-08	.200E-08	.182E-08	.134E-08	.910E-09	.594E-09	.378E-09	.238E-09	.118E-09	.598E-10	.304E-10
5f-7i	.231E-10	.305E-09	.152E-08	.407E-08	.725E-08	.991E-08	.113E-07	.114E-07	.107E-07	.942E-08	.665E-08	.439E-08	.282E-08	.179E-08	.113E-08	.562E-09	.283E-09	.143E-09
5g-7s	.378E-13	.461E-12	.214E-11	.539E-11	.936E-11	.130E-10	.156E-10	.166E-10	.162E-10	.149E-10	.110E-10	.736E-11	.464E-11	.283E-11	.162E-11	.695E-12	.375E-12	.212E-12
5g-7p	.141E-12	.172E-11	.794E-11	.201E-10	.350E-10	.489E-10	.585E-10	.625E-10	.612E-10	.560E-10	.413E-10	.274E-10	.174E-10	.106E-10	.619E-11	.274E-11	.145E-11	.811E-12
5g-7d	.371E-12	.455E-11	.212E-10	.540E-10	.947E-10	.132E-09	.158E-09	.168E-09	.163E-09	.149E-09	.111E-09	.743E-10	.478E-10	.296E-10	.178E-10	.831E-11	.432E-11	.232E-11
5g-7f	.957E-12	.125E-10	.615E-10	.166E-09	.306E-09	.450E-09	.559E-09	.611E-09	.607E-09	.561E-09	.420E-09	.287E-09	.191E-09	.125E-09	.817E-10	.434E-10	.239E-10	.132E-10
5g-7g	.705E-11	.912E-10	.444E-09	.115E-08	.199E-08	.268E-08	.308E-08	.322E-08	.314E-08	.291E-08	.222E-08	.152E-08	.996E-09	.633E-09	.403E-09	.203E-09	.103E-09	.519E-10
5g-7h	.112E-09	.143E-08	.695E-08	.182E-07	.320E-07	.438E-07	.505E-07	.518E-07	.492E-07	.443E-07	.326E-07	.226E-07	.155E-07	.107E-07	.744E-08	.433E-08	.250E-08	.142E-08
5g-7i	.517E-10	.659E-09	.317E-08	.817E-08	.140E-07	.187E-07	.209E-07	.208E-07	.193E-07	.170E-07	.120E-07	.802E-08	.519E-08	.331E-08	.210E-08	.106E-08	.532E-09	.267E-09
5s-8s	.732E-12	.174E-10	.123E-09	.400E-09	.799E-09	.118E-08	.145E-08	.158E-08	.158E-08	.148E-08	.113E-08	.790E-09	.523E-09	.338E-09	.215E-09	.109E-09	.548E-10	.275E-10
5s-8p	.235E-11	.578E-10	.430E-09	.150E-08	.320E-08	.501E-08	.637E-08	.701E-08	.701E-08	.656E-08	.510E-08	.370E-08	.266E-08	.193E-08	.140E-08	.863E-09	.520E-09	.305E-09

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
5s-8d	.132E-11	.313E-10	.224E-09	.742E-09	.150E-08	.222E-08	.267E-08	.278E-08	.265E-08	.238E-08	.170E-08	.114E-08	.744E-09	.477E-09	.302E-09	.151E-09	.758E-10	.379E-10
5s-8f	.294E-12	.753E-11	.592E-10	.219E-09	.496E-09	.811E-09	.105E-08	.117E-08	.116E-08	.106E-08	.771E-09	.523E-09	.342E-09	.219E-09	.138E-09	.631E-10	.319E-10	.150E-10
5s-8g	.195E-12	.457E-11	.330E-10	.114E-09	.244E-09	.387E-09	.491E-09	.536E-09	.528E-09	.487E-09	.351E-09	.239E-09	.156E-09	.993E-10	.587E-10	.238E-10	.115E-10	.550E-11
5s-8h	.349E-12	.848E-11	.623E-10	.213E-09	.445E-09	.677E-09	.831E-09	.883E-09	.849E-09	.764E-09	.540E-09	.364E-09	.235E-09	.149E-09	.929E-10	.414E-10	.209E-10	.972E-11
5s-8i	.189E-12	.490E-11	.387E-10	.142E-09	.317E-09	.510E-09	.651E-09	.711E-09	.696E-09	.630E-09	.455E-09	.303E-09	.199E-09	.126E-09	.705E-10	.232E-10	.108E-10	.504E-11
5s-8j	.384E-12	.996E-11	.770E-10	.272E-09	.579E-09	.888E-09	.109E-08	.116E-08	.111E-08	.993E-09	.710E-09	.467E-09	.305E-09	.194E-09	.115E-09	.440E-10	.217E-10	.997E-11
5p-8s	.169E-12	.431E-11	.335E-10	.123E-09	.278E-09	.462E-09	.620E-09	.712E-09	.735E-09	.704E-09	.559E-09	.411E-09	.297E-09	.215E-09	.156E-09	.948E-10	.571E-10	.333E-10
5p-8p	.975E-12	.231E-10	.163E-09	.532E-09	.106E-08	.158E-08	.193E-08	.208E-08	.206E-08	.193E-08	.145E-08	.101E-08	.666E-09	.429E-09	.272E-09	.136E-09	.685E-10	.342E-10
5p-8d	.308E-11	.750E-10	.551E-09	.189E-08	.397E-08	.613E-08	.767E-08	.833E-08	.822E-08	.761E-08	.582E-08	.417E-08	.296E-08	.211E-08	.152E-08	.922E-09	.550E-09	.320E-09
5p-8f	.883E-12	.215E-10	.158E-09	.541E-09	.113E-08	.173E-08	.212E-08	.225E-08	.218E-08	.197E-08	.143E-08	.962E-09	.627E-09	.402E-09	.254E-09	.123E-09	.620E-10	.306E-10
5p-8g	.182E-12	.439E-11	.329E-10	.119E-09	.267E-09	.439E-09	.577E-09	.647E-09	.648E-09	.602E-09	.439E-09	.300E-09	.195E-09	.124E-09	.753E-10	.313E-10	.155E-10	.720E-11
5p-8h	.431E-12	.104E-10	.751E-10	.253E-09	.521E-09	.780E-09	.943E-09	.988E-09	.942E-09	.843E-09	.605E-09	.404E-09	.262E-09	.167E-09	.104E-09	.484E-10	.243E-10	.117E-10
5p-8i	.243E-12	.632E-11	.499E-10	.183E-09	.407E-09	.651E-09	.831E-09	.910E-09	.895E-09	.817E-09	.591E-09	.392E-09	.255E-09	.161E-09	.930E-10	.336E-10	.161E-10	.746E-11
5p-8j	.436E-12	.111E-10	.851E-10	.299E-09	.631E-09	.960E-09	.117E-08	.123E-08	.118E-08	.105E-08	.753E-09	.500E-09	.326E-09	.207E-09	.125E-09	.505E-10	.251E-10	.117E-10
5d-8s	.477E-13	.111E-11	.774E-11	.258E-10	.539E-10	.841E-10	.106E-09	.117E-09	.116E-09	.107E-09	.782E-10	.543E-10	.355E-10	.229E-10	.142E-10	.640E-11	.319E-11	.152E-11
5d-8p	.141E-12	.352E-11	.269E-10	.976E-10	.222E-09	.374E-09	.508E-09	.589E-09	.610E-09	.585E-09	.457E-09	.329E-09	.232E-09	.162E-09	.113E-09	.647E-10	.377E-10	.214E-10
5d-8d	.860E-12	.205E-10	.145E-09	.476E-09	.957E-09	.142E-08	.175E-08	.190E-08	.189E-08	.178E-08	.136E-08	.943E-09	.621E-09	.400E-09	.253E-09	.125E-09	.632E-10	.315E-10
5d-8f	.448E-11	.109E-09	.792E-09	.269E-08	.562E-08	.860E-08	.107E-07	.115E-07	.113E-07	.104E-07	.783E-08	.555E-08	.389E-08	.275E-08	.196E-08	.117E-08	.693E-09	.401E-09
5d-8g	.404E-12	.101E-10	.769E-10	.278E-09	.616E-09	.990E-09	.127E-08	.139E-08	.138E-08	.127E-08	.941E-09	.641E-09	.421E-09	.270E-09	.168E-09	.781E-10	.393E-10	.190E-10
5d-8h	.570E-12	.134E-10	.951E-10	.314E-09	.636E-09	.942E-09	.113E-08	.118E-08	.112E-08	.998E-09	.711E-09	.471E-09	.305E-09	.194E-09	.121E-09	.577E-10	.288E-10	.143E-10
5d-8i	.439E-12	.113E-10	.879E-10	.317E-09	.693E-09	.109E-08	.137E-08	.147E-08	.144E-08	.131E-08	.951E-09	.636E-09	.413E-09	.262E-09	.158E-09	.657E-10	.326E-10	.153E-10
5d-8j	.533E-12	.134E-10	.100E-09	.346E-09	.722E-09	.109E-08	.133E-08	.140E-08	.134E-08	.120E-08	.858E-09	.567E-09	.367E-09	.233E-09	.143E-09	.624E-10	.312E-10	.148E-10
5f-8s	.110E-13	.252E-12	.175E-11	.585E-11	.124E-10	.199E-10	.260E-10	.292E-10	.295E-10	.278E-10	.209E-10	.138E-10	.902E-11	.568E-11	.326E-11	.122E-11	.582E-12	.275E-12
5f-8p	.418E-13	.972E-12	.686E-11	.232E-10	.497E-10	.798E-10	.104E-09	.116E-09	.117E-09	.110E-09	.815E-10	.561E-10	.365E-10	.231E-10	.139E-10	.568E-11	.278E-11	.131E-11
5f-8d	.949E-13	.234E-11	.176E-10	.630E-10	.143E-09	.243E-09	.333E-09	.388E-09	.403E-09	.386E-09	.299E-09	.210E-09	.144E-09	.972E-10	.649E-10	.343E-10	.193E-10	.105E-10
5f-8f	.699E-12	.167E-10	.119E-09	.391E-09	.788E-09	.117E-08	.145E-08	.157E-08	.158E-08	.150E-08	.117E-08	.809E-09	.532E-09	.342E-09	.215E-09	.105E-09	.531E-10	.264E-10
5f-8g	.583E-11	.141E-09	.102E-08	.345E-08	.716E-08	.109E-07	.135E-07	.144E-07	.141E-07	.129E-07	.965E-08	.677E-08	.469E-08	.327E-08	.229E-08	.135E-08	.785E-09	.449E-09
5f-8h	.389E-12	.920E-11	.658E-10	.221E-09	.460E-09	.705E-09	.875E-09	.944E-09	.924E-09	.846E-09	.625E-09	.427E-09	.281E-09	.180E-09	.113E-09	.527E-10	.263E-10	.129E-10
5f-8i	.116E-11	.287E-10	.215E-09	.745E-09	.157E-08	.242E-08	.299E-08	.319E-08	.308E-08	.279E-08	.200E-08	.133E-08	.860E-09	.546E-09	.340E-09	.158E-09	.792E-10	.384E-10
5f-8j	.647E-12	.159E-10	.117E-09	.399E-09	.821E-09	.122E-08	.147E-08	.154E-08	.146E-08	.131E-08	.935E-09	.622E-09	.403E-09	.257E-09	.160E-09	.747E-10	.376E-10	.183E-10
5g-8s	.314E-14	.718E-13	.495E-12	.161E-11	.332E-11	.517E-11	.666E-11	.748E-11	.750E-11	.688E-11	.528E-11	.354E-11	.221E-11	.136E-11	.704E-12	.203E-12	.853E-13	.411E-13
5g-8p	.111E-13	.255E-12	.176E-11	.574E-11	.119E-10	.186E-10	.242E-10	.273E-10	.278E-10	.262E-10	.201E-10	.131E-10	.822E-11	.502E-11	.266E-11	.825E-12	.362E-12	.175E-12

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
5g-8d	.266E-13	.619E-12	.432E-11	.143E-10	.297E-10	.467E-10	.602E-10	.676E-10	.688E-10	.652E-10	.503E-10	.335E-10	.213E-10	.131E-10	.742E-11	.267E-11	.125E-11	.594E-12
5g-8f	.661E-13	.159E-11	.115E-10	.393E-10	.844E-10	.138E-09	.184E-09	.212E-09	.218E-09	.207E-09	.159E-09	.109E-09	.723E-10	.467E-10	.298E-10	.153E-10	.830E-11	.457E-11
5g-8g	.435E-12	.104E-10	.748E-10	.247E-09	.496E-09	.736E-09	.904E-09	.987E-09	.995E-09	.941E-09	.738E-09	.513E-09	.338E-09	.215E-09	.136E-09	.683E-10	.348E-10	.131E-10
5g-8h	.533E-11	.128E-09	.929E-09	.313E-08	.647E-08	.983E-08	.121E-07	.130E-07	.127E-07	.116E-07	.869E-08	.604E-08	.412E-08	.281E-08	.194E-08	.111E-08	.637E-09	.360E-09
5g-8i	.362E-11	.872E-10	.630E-09	.211E-08	.429E-08	.635E-08	.763E-08	.798E-08	.763E-08	.686E-08	.498E-08	.334E-08	.217E-08	.139E-08	.880E-09	.443E-09	.223E-09	.112E-09
5g-8j	.644E-12	.156E-10	.113E-09	.378E-09	.772E-09	.115E-08	.138E-08	.143E-08	.136E-08	.121E-08	.868E-09	.577E-09	.371E-09	.235E-09	.149E-09	.742E-10	.374E-10	.188E-10
6s-7s	.180E-07	.437E-07	.721E-07	.926E-07	.101E-06	.101E-06	.943E-07	.841E-07	.727E-07	.613E-07	.416E-07	.270E-07	.174E-07	.110E-07	.701E-08	.352E-08	.177E-08	.889E-09
6s-7p	.484E-07	.121E-06	.212E-06	.294E-06	.348E-06	.372E-06	.369E-06	.346E-06	.312E-06	.272E-06	.197E-06	.138E-06	.965E-07	.688E-07	.495E-07	.301E-07	.180E-07	.105E-07
6s-7d	.257E-07	.609E-07	.987E-07	.125E-06	.136E-06	.133E-06	.122E-06	.106E-06	.903E-07	.748E-07	.496E-07	.320E-07	.204E-07	.130E-07	.821E-08	.411E-08	.206E-08	.103E-08
6s-7f	.847E-08	.224E-07	.396E-07	.542E-07	.622E-07	.635E-07	.599E-07	.532E-07	.454E-07	.378E-07	.251E-07	.161E-07	.103E-07	.651E-08	.411E-08	.205E-08	.103E-08	.515E-09
6s-7g	.295E-08	.913E-08	.184E-07	.278E-07	.342E-07	.368E-07	.357E-07	.326E-07	.284E-07	.240E-07	.162E-07	.105E-07	.675E-08	.428E-08	.270E-08	.132E-08	.658E-09	.322E-09
6s-7h	.104E-08	.374E-08	.849E-08	.139E-07	.183E-07	.206E-07	.207E-07	.193E-07	.171E-07	.145E-07	.987E-08	.640E-08	.411E-08	.260E-08	.162E-08	.733E-09	.352E-09	.166E-09
6s-7i	.270E-09	.112E-08	.276E-08	.477E-08	.647E-08	.743E-08	.759E-08	.716E-08	.638E-08	.547E-08	.373E-08	.243E-08	.155E-08	.976E-09	.580E-09	.230E-09	.104E-09	.476E-10
6p-7s	.586E-08	.150E-07	.269E-07	.381E-07	.463E-07	.506E-07	.512E-07	.486E-07	.441E-07	.388E-07	.281E-07	.197E-07	.138E-07	.987E-08	.710E-08	.432E-08	.258E-08	.151E-08
6p-7p	.227E-07	.547E-07	.898E-07	.115E-06	.126E-06	.125E-06	.117E-06	.104E-06	.896E-07	.754E-07	.510E-07	.332E-07	.213E-07	.135E-07	.858E-08	.431E-08	.216E-08	.108E-08
6p-7d	.538E-07	.134E-06	.233E-06	.319E-06	.373E-06	.395E-06	.388E-06	.361E-06	.323E-06	.281E-06	.202E-06	.140E-06	.980E-07	.695E-07	.497E-07	.300E-07	.179E-07	.104E-07
6p-7f	.234E-07	.568E-07	.942E-07	.122E-06	.134E-06	.132E-06	.121E-06	.106E-06	.903E-07	.750E-07	.497E-07	.321E-07	.205E-07	.130E-07	.822E-08	.412E-08	.207E-08	.103E-08
6p-7g	.656E-08	.182E-07	.339E-07	.481E-07	.570E-07	.596E-07	.571E-07	.513E-07	.442E-07	.370E-07	.247E-07	.160E-07	.102E-07	.644E-08	.407E-08	.201E-08	.100E-08	.498E-09
6p-7h	.182E-08	.606E-08	.130E-07	.205E-07	.261E-07	.287E-07	.283E-07	.262E-07	.230E-07	.195E-07	.133E-07	.865E-08	.554E-08	.351E-08	.220E-08	.103E-08	.505E-09	.244E-09
6p-7i	.403E-09	.158E-08	.376E-08	.637E-08	.852E-08	.972E-08	.988E-08	.930E-08	.828E-08	.708E-08	.480E-08	.312E-08	.200E-08	.126E-08	.759E-09	.319E-09	.148E-09	.691E-10
6d-7s	.107E-08	.249E-08	.399E-08	.510E-08	.564E-08	.570E-08	.540E-08	.483E-08	.418E-08	.351E-08	.236E-08	.153E-08	.987E-09	.628E-09	.398E-09	.197E-09	.979E-10	.483E-10
6d-7p	.465E-08	.120E-07	.217E-07	.309E-07	.378E-07	.415E-07	.421E-07	.400E-07	.363E-07	.318E-07	.229E-07	.159E-07	.111E-07	.783E-08	.558E-08	.334E-08	.198E-08	.114E-08
6d-7d	.206E-07	.501E-07	.830E-07	.107E-06	.118E-06	.117E-06	.110E-06	.978E-07	.845E-07	.713E-07	.483E-07	.314E-07	.202E-07	.128E-07	.814E-08	.408E-08	.205E-08	.103E-08
6d-7f	.750E-07	.187E-06	.322E-06	.438E-06	.509E-06	.535E-06	.522E-06	.484E-06	.431E-06	.374E-06	.267E-06	.185E-06	.129E-06	.913E-07	.651E-07	.392E-07	.232E-07	.135E-07
6d-7g	.265E-07	.653E-07	.110E-06	.144E-06	.160E-06	.159E-06	.147E-06	.129E-06	.110E-06	.916E-07	.609E-07	.393E-07	.251E-07	.159E-07	.101E-07	.503E-08	.252E-08	.126E-08
6d-7h	.556E-08	.161E-07	.309E-07	.452E-07	.547E-07	.582E-07	.564E-07	.512E-07	.444E-07	.374E-07	.251E-07	.162E-07	.103E-07	.655E-08	.412E-08	.201E-08	.999E-09	.493E-09
6d-7i	.915E-09	.323E-08	.718E-08	.116E-07	.150E-07	.168E-07	.167E-07	.156E-07	.138E-07	.117E-07	.798E-08	.518E-08	.332E-08	.210E-08	.129E-08	.588E-09	.283E-09	.136E-09
6f-7s	.142E-09	.350E-09	.594E-09	.798E-09	.930E-09	.983E-09	.966E-09	.886E-09	.774E-09	.654E-09	.441E-09	.287E-09	.184E-09	.117E-09	.731E-10	.333E-10	.160E-10	.763E-11
6f-7p	.789E-09	.189E-08	.313E-08	.414E-08	.476E-08	.496E-08	.481E-08	.439E-08	.384E-08	.325E-08	.220E-08	.144E-08	.926E-09	.590E-09	.371E-09	.178E-09	.876E-10	.428E-10
6f-7d	.276E-08	.731E-08	.134E-07	.195E-07	.243E-07	.270E-07	.276E-07	.264E-07	.240E-07	.210E-07	.151E-07	.104E-07	.720E-08	.504E-08	.355E-08	.210E-08	.123E-08	.704E-09
6f-7f	.185E-07	.453E-07	.756E-07	.981E-07	.108E-06	.108E-06	.102E-06	.909E-07	.788E-07	.666E-07	.453E-07	.295E-07	.190E-07	.121E-07	.765E-08	.383E-08	.192E-08	.964E-09

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$) 4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
6f-7g	.110E-06	.274E-06	.471E-06	.637E-06	.737E-06	.769E-06	.747E-06	.688E-06	.611E-06	.528E-06	.376E-06	.260E-06	.180E-06	.127E-06	.899E-07	.538E-07	.317E-07	.183E-07
6f-7h	.284E-07	.715E-07	.123E-06	.164E-06	.184E-06	.185E-06	.172E-06	.152E-06	.130E-06	.109E-06	.724E-07	.468E-07	.299E-07	.190E-07	.120E-07	.598E-08	.299E-08	.150E-08
6f-7i	.342E-08	.104E-07	.207E-07	.311E-07	.383E-07	.414E-07	.405E-07	.371E-07	.324E-07	.274E-07	.184E-07	.119E-07	.762E-08	.481E-08	.301E-08	.145E-08	.717E-09	.352E-09
6g-7s	.219E-10	.606E-10	.112E-09	.161E-09	.199E-09	.219E-09	.220E-09	.207E-09	.184E-09	.158E-09	.108E-09	.701E-10	.454E-10	.288E-10	.180E-10	.829E-11	.419E-11	.199E-11
6g-7p	.987E-10	.261E-09	.472E-09	.669E-09	.816E-09	.894E-09	.897E-09	.840E-09	.746E-09	.637E-09	.431E-09	.282E-09	.181E-09	.115E-09	.721E-10	.340E-10	.171E-10	.833E-11
6g-7d	.387E-09	.969E-09	.168E-08	.232E-08	.277E-08	.298E-08	.295E-08	.274E-08	.243E-08	.207E-08	.141E-08	.926E-09	.597E-09	.381E-09	.240E-09	.117E-09	.586E-10	.290E-10
6g-7f	.145E-08	.397E-08	.748E-08	.111E-07	.139E-07	.157E-07	.161E-07	.154E-07	.140E-07	.123E-07	.877E-08	.600E-08	.411E-08	.283E-08	.197E-08	.114E-08	.656E-09	.371E-09
6g-7g	.151E-07	.377E-07	.637E-07	.834E-07	.926E-07	.930E-07	.876E-07	.787E-07	.685E-07	.581E-07	.397E-07	.260E-07	.167E-07	.106E-07	.676E-08	.339E-08	.170E-08	.854E-09
6g-7h	.165E-06	.410E-06	.702E-06	.948E-06	.109E-05	.113E-05	.110E-05	.100E-05	.888E-06	.765E-06	.542E-06	.373E-06	.257E-06	.180E-06	.127E-06	.753E-07	.442E-07	.254E-07
6g-7i	.234E-07	.605E-07	.107E-06	.145E-06	.165E-06	.168E-06	.158E-06	.141E-06	.121E-06	.102E-06	.680E-07	.441E-07	.282E-07	.179E-07	.113E-07	.565E-08	.284E-08	.142E-08
6h-7s	.315E-11	.101E-10	.207E-10	.318E-10	.408E-10	.463E-10	.475E-10	.458E-10	.418E-10	.367E-10	.246E-10	.164E-10	.104E-10	.640E-11	.373E-11	.143E-11	.690E-12	.325E-12
6h-7p	.123E-10	.384E-10	.769E-10	.117E-09	.148E-09	.168E-09	.172E-09	.165E-09	.150E-09	.130E-09	.898E-10	.581E-10	.373E-10	.233E-10	.137E-10	.551E-11	.268E-11	.128E-11
6h-7d	.356E-10	.105E-09	.205E-09	.305E-09	.384E-09	.432E-09	.443E-09	.423E-09	.380E-09	.328E-09	.225E-09	.146E-09	.939E-10	.592E-10	.356E-10	.154E-10	.754E-11	.368E-11
6h-7f	.130E-09	.355E-09	.660E-09	.960E-09	.119E-08	.131E-08	.133E-08	.126E-08	.113E-08	.976E-09	.669E-09	.439E-09	.283E-09	.179E-09	.111E-09	.514E-10	.255E-10	.126E-10
6h-7g	.564E-09	.163E-08	.320E-08	.482E-08	.614E-08	.696E-08	.718E-08	.691E-08	.629E-08	.550E-08	.389E-08	.263E-08	.177E-08	.120E-08	.810E-09	.449E-09	.252E-09	.139E-09
6h-7h	.956E-08	.244E-07	.421E-07	.559E-07	.627E-07	.634E-07	.600E-07	.544E-07	.477E-07	.407E-07	.279E-07	.184E-07	.118E-07	.754E-08	.478E-08	.239E-08	.120E-08	.601E-09
6h-7i	.247E-06	.618E-06	.106E-05	.143E-05	.165E-05	.171E-05	.164E-05	.150E-05	.132E-05	.114E-05	.799E-06	.547E-06	.375E-06	.260E-06	.182E-06	.107E-06	.624E-07	.356E-07
6s-8s	.456E-09	.208E-08	.515E-08	.857E-08	.111E-07	.124E-07	.126E-07	.120E-07	.108E-07	.939E-08	.662E-08	.439E-08	.285E-08	.183E-08	.116E-08	.586E-09	.295E-09	.148E-09
6s-8p	.133E-08	.630E-08	.165E-07	.294E-07	.409E-07	.485E-07	.514E-07	.502E-07	.463E-07	.411E-07	.300E-07	.210E-07	.148E-07	.106E-07	.761E-08	.463E-08	.277E-08	.162E-08
6s-8d	.735E-09	.330E-08	.805E-08	.133E-07	.172E-07	.189E-07	.187E-07	.172E-07	.150E-07	.127E-07	.863E-08	.561E-08	.361E-08	.230E-08	.146E-08	.730E-09	.367E-09	.183E-09
6s-8f	.214E-09	.105E-08	.282E-08	.511E-08	.715E-08	.837E-08	.864E-08	.819E-08	.731E-08	.624E-08	.428E-08	.278E-08	.179E-08	.114E-08	.719E-09	.359E-09	.180E-09	.902E-10
6s-8g	.826E-10	.406E-09	.112E-08	.215E-08	.316E-08	.385E-08	.411E-08	.398E-08	.360E-08	.311E-08	.217E-08	.142E-08	.925E-09	.590E-09	.374E-09	.182E-09	.921E-10	.455E-10
6s-8h	.838E-10	.368E-09	.898E-09	.152E-08	.202E-08	.230E-08	.235E-08	.221E-08	.196E-08	.166E-08	.113E-08	.734E-09	.476E-09	.303E-09	.191E-09	.885E-10	.448E-10	.213E-10
6s-8i	.110E-09	.557E-09	.152E-08	.276E-08	.382E-08	.441E-08	.451E-08	.423E-08	.376E-08	.320E-08	.219E-08	.143E-08	.916E-09	.579E-09	.365E-09	.179E-09	.904E-10	.449E-10
6s-8j	.924E-10	.554E-09	.172E-08	.342E-08	.506E-08	.614E-08	.648E-08	.624E-08	.563E-08	.486E-08	.334E-08	.218E-08	.140E-08	.885E-09	.555E-09	.250E-09	.127E-09	.596E-10
6p-8s	.145E-09	.705E-09	.189E-08	.348E-08	.502E-08	.614E-08	.666E-08	.663E-08	.619E-08	.552E-08	.405E-08	.284E-08	.200E-08	.143E-08	.103E-08	.623E-09	.372E-09	.217E-09
6p-8p	.606E-09	.274E-08	.673E-08	.112E-07	.145E-07	.161E-07	.163E-07	.154E-07	.138E-07	.120E-07	.837E-08	.554E-08	.360E-08	.230E-08	.146E-08	.736E-09	.370E-09	.185E-09
6p-8d	.154E-08	.725E-08	.187E-07	.330E-07	.454E-07	.531E-07	.555E-07	.537E-07	.492E-07	.434E-07	.315E-07	.219E-07	.153E-07	.108E-07	.776E-08	.468E-08	.278E-08	.161E-08
6p-8f	.578E-09	.266E-08	.668E-08	.114E-07	.150E-07	.168E-07	.168E-07	.155E-07	.137E-07	.116E-07	.791E-08	.516E-08	.332E-08	.211E-08	.134E-08	.670E-09	.336E-09	.168E-09
6p-8g	.119E-09	.599E-09	.168E-08	.319E-08	.466E-08	.567E-08	.601E-08	.582E-08	.527E-08	.455E-08	.314E-08	.205E-08	.133E-08	.844E-09	.534E-09	.263E-09	.132E-09	.655E-10
6p-8h	.905E-10	.397E-09	.968E-09	.165E-08	.221E-08	.254E-08	.261E-08	.246E-08	.219E-08	.187E-08	.127E-08	.836E-09	.541E-09	.345E-09	.217E-09	.102E-09	.516E-10	.250E-10

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$) 4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
6p-8i	.135E-09	.662E-09	.176E-08	.314E-08	.430E-08	.494E-08	.503E-08	.471E-08	.417E-08	.355E-08	.241E-08	.157E-08	.100E-08	.635E-09	.400E-09	.197E-09	.987E-10	.491E-10
6p-8j	.116E-09	.677E-09	.206E-08	.406E-08	.595E-08	.717E-08	.752E-08	.720E-08	.648E-08	.557E-08	.384E-08	.251E-08	.161E-08	.102E-08	.642E-09	.296E-09	.150E-09	.718E-10
6d-8s	.368E-10	.159E-09	.377E-09	.623E-09	.816E-09	.924E-09	.948E-09	.891E-09	.793E-09	.679E-09	.468E-09	.308E-09	.200E-09	.128E-09	.814E-10	.405E-10	.204E-10	.102E-10
6d-8p	.126E-09	.604E-09	.161E-08	.294E-08	.425E-08	.522E-08	.568E-08	.565E-08	.527E-08	.467E-08	.339E-08	.235E-08	.163E-08	.114E-08	.806E-09	.476E-09	.279E-09	.160E-09
6d-8d	.545E-09	.248E-08	.615E-08	.103E-07	.134E-07	.150E-07	.152E-07	.144E-07	.130E-07	.113E-07	.793E-08	.526E-08	.341E-08	.218E-08	.139E-08	.698E-09	.351E-09	.176E-09
6d-8f	.213E-08	.998E-08	.256E-07	.448E-07	.611E-07	.709E-07	.737E-07	.709E-07	.647E-07	.568E-07	.410E-07	.285E-07	.198E-07	.139E-07	.992E-08	.594E-08	.351E-08	.203E-08
6d-8g	.464E-09	.219E-08	.566E-08	.992E-08	.134E-07	.153E-07	.156E-07	.146E-07	.129E-07	.111E-07	.759E-08	.497E-08	.320E-08	.204E-08	.129E-08	.642E-09	.323E-09	.161E-09
6d-8h	.923E-10	.423E-09	.110E-08	.198E-08	.281E-08	.338E-08	.357E-08	.345E-08	.313E-08	.270E-08	.186E-08	.122E-08	.789E-09	.503E-09	.316E-09	.153E-09	.766E-10	.377E-10
6d-8i	.189E-09	.887E-09	.227E-08	.391E-08	.520E-08	.584E-08	.586E-08	.541E-08	.476E-08	.404E-08	.274E-08	.178E-08	.114E-08	.725E-09	.456E-09	.225E-09	.113E-09	.562E-10
6d-8j	.180E-09	.100E-08	.296E-08	.568E-08	.821E-08	.980E-08	.102E-07	.979E-08	.879E-08	.755E-08	.516E-08	.336E-08	.216E-08	.137E-08	.861E-09	.410E-09	.207E-09	.101E-09
6f-8s	.772E-11	.333E-10	.804E-10	.135E-09	.183E-09	.214E-09	.223E-09	.215E-09	.195E-09	.168E-09	.115E-09	.759E-10	.490E-10	.313E-10	.198E-10	.942E-11	.478E-11	.229E-11
6f-8p	.327E-10	.143E-09	.344E-09	.582E-09	.786E-09	.916E-09	.962E-09	.921E-09	.829E-09	.714E-09	.494E-09	.327E-09	.213E-09	.136E-09	.864E-10	.420E-10	.211E-10	.104E-10
6f-8d	.828E-10	.397E-09	.106E-08	.195E-08	.284E-08	.353E-08	.387E-08	.387E-08	.361E-08	.320E-08	.230E-08	.158E-08	.108E-08	.743E-09	.514E-09	.295E-09	.169E-09	.956E-10
6f-8f	.473E-09	.218E-08	.543E-08	.914E-08	.120E-07	.135E-07	.137E-07	.131E-07	.118E-07	.103E-07	.729E-08	.486E-08	.316E-08	.202E-08	.128E-08	.644E-09	.324E-09	.162E-09
6f-8g	.293E-08	.137E-07	.349E-07	.608E-07	.824E-07	.951E-07	.984E-07	.941E-07	.855E-07	.749E-07	.538E-07	.372E-07	.257E-07	.179E-07	.127E-07	.752E-08	.441E-08	.253E-08
6f-8h	.232E-09	.115E-08	.313E-08	.576E-08	.816E-08	.967E-08	.101E-07	.969E-08	.873E-08	.754E-08	.523E-08	.344E-08	.222E-08	.142E-08	.897E-09	.444E-09	.223E-09	.111E-09
6f-8i	.257E-09	.115E-08	.281E-08	.467E-08	.605E-08	.668E-08	.661E-08	.605E-08	.528E-08	.445E-08	.299E-08	.194E-08	.124E-08	.783E-09	.493E-09	.243E-09	.122E-09	.606E-10
6f-8j	.338E-09	.178E-08	.503E-08	.936E-08	.132E-07	.154E-07	.159E-07	.150E-07	.134E-07	.115E-07	.786E-08	.514E-08	.330E-08	.210E-08	.132E-08	.646E-09	.325E-09	.161E-09
6g-8s	.197E-11	.880E-11	.216E-10	.373E-10	.516E-10	.615E-10	.657E-10	.638E-10	.581E-10	.506E-10	.351E-10	.230E-10	.150E-10	.956E-11	.588E-11	.250E-11	.125E-11	.573E-12
6g-8p	.746E-11	.329E-10	.812E-10	.140E-09	.193E-09	.230E-09	.244E-09	.238E-09	.218E-09	.190E-09	.130E-09	.861E-10	.555E-10	.353E-10	.217E-10	.967E-11	.484E-11	.231E-11
6g-8d	.201E-10	.891E-10	.219E-09	.377E-09	.520E-09	.619E-09	.660E-09	.641E-09	.583E-09	.507E-09	.350E-09	.234E-09	.152E-09	.969E-10	.604E-10	.284E-10	.142E-10	.695E-11
6g-8f	.520E-10	.249E-09	.661E-09	.121E-08	.177E-08	.220E-08	.242E-08	.242E-08	.226E-08	.200E-08	.142E-08	.962E-09	.646E-09	.435E-09	.293E-09	.161E-09	.894E-10	.491E-10
6g-8g	.363E-09	.170E-08	.429E-08	.730E-08	.965E-08	.109E-07	.112E-07	.107E-07	.977E-08	.858E-08	.608E-08	.408E-08	.265E-08	.170E-08	.108E-08	.539E-09	.271E-09	.136E-09
6g-8h	.364E-08	.170E-07	.435E-07	.757E-07	.102E-06	.118E-06	.121E-06	.116E-06	.105E-06	.915E-07	.654E-07	.449E-07	.308E-07	.213E-07	.149E-07	.875E-08	.508E-08	.290E-08
6g-8i	.151E-09	.685E-09	.171E-08	.297E-08	.405E-08	.472E-08	.492E-08	.469E-08	.421E-08	.363E-08	.251E-08	.166E-08	.108E-08	.690E-09	.436E-09	.213E-09	.107E-09	.531E-10
6g-8j	.776E-09	.383E-08	.102E-07	.183E-07	.250E-07	.286E-07	.291E-07	.273E-07	.241E-07	.205E-07	.139E-07	.906E-08	.580E-08	.368E-08	.232E-08	.115E-08	.578E-09	.288E-09
6h-8s	.468E-12	.221E-11	.564E-11	.983E-11	.136E-10	.164E-10	.178E-10	.176E-10	.164E-10	.147E-10	.100E-10	.659E-11	.416E-11	.252E-11	.142E-11	.485E-12	.224E-12	.108E-12
6h-8p	.165E-11	.775E-11	.196E-10	.341E-10	.471E-10	.564E-10	.609E-10	.601E-10	.556E-10	.491E-10	.347E-10	.224E-10	.144E-10	.891E-11	.504E-11	.182E-11	.857E-12	.405E-12
6h-8d	.382E-11	.178E-10	.453E-10	.787E-10	.109E-09	.131E-09	.140E-09	.138E-09	.128E-09	.112E-09	.781E-10	.512E-10	.327E-10	.205E-10	.119E-10	.468E-11	.225E-11	.108E-11
6h-8f	.918E-11	.428E-10	.108E-09	.189E-09	.262E-09	.314E-09	.339E-09	.334E-09	.307E-09	.271E-09	.190E-09	.125E-09	.807E-10	.510E-10	.306E-10	.132E-10	.648E-11	.318E-11
6h-8g	.258E-10	.126E-09	.336E-09	.611E-09	.879E-09	.109E-08	.119E-08	.119E-08	.111E-08	.986E-09	.695E-09	.465E-09	.306E-09	.201E-09	.130E-09	.659E-10	.352E-10	.187E-10
6h-8h	.199E-09	.967E-09	.251E-08	.435E-08	.581E-08	.662E-08	.684E-08	.662E-08	.609E-08	.541E-08	.385E-08	.259E-08	.169E-08	.108E-08	.681E-09	.337E-09	.169E-09	.845E-10

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
6h-8i	.344E-08	.163E-07	.419E-07	.731E-07	.989E-07	.114E-06	.117E-06	.112E-06	.101E-06	.880E-07	.625E-07	.427E-07	.291E-07	.199E-07	.138E-07	.792E-08	.454E-08	.256E-08
6h-8j	.195E-08	.914E-08	.232E-07	.397E-07	.523E-07	.582E-07	.581E-07	.537E-07	.473E-07	.402E-07	.274E-07	.179E-07	.115E-07	.732E-08	.464E-08	.233E-08	.117E-08	.585E-09
7s-8s	.990E-07	.166E-06	.216E-06	.237E-06	.234E-06	.217E-06	.193E-06	.166E-06	.139E-06	.115E-06	.762E-07	.491E-07	.313E-07	.198E-07	.126E-07	.632E-08	.317E-08	.159E-08
7s-8p	.229E-06	.410E-06	.580E-06	.701E-06	.760E-06	.765E-06	.728E-06	.664E-06	.587E-06	.507E-06	.361E-06	.250E-06	.174E-06	.123E-06	.881E-07	.531E-07	.315E-07	.183E-07
7s-8d	.121E-06	.198E-06	.254E-06	.276E-06	.271E-06	.248E-06	.219E-06	.185E-06	.154E-06	.126E-06	.822E-07	.527E-07	.335E-07	.212E-07	.134E-07	.674E-08	.338E-08	.169E-08
7s-8f	.424E-07	.765E-07	.106E-06	.122E-06	.124E-06	.117E-06	.105E-06	.898E-07	.749E-07	.614E-07	.400E-07	.256E-07	.162E-07	.103E-07	.648E-08	.324E-08	.163E-08	.813E-09
7s-8g	.166E-07	.346E-07	.530E-07	.661E-07	.713E-07	.699E-07	.639E-07	.559E-07	.474E-07	.393E-07	.260E-07	.168E-07	.107E-07	.674E-08	.426E-08	.213E-08	.107E-08	.535E-09
7s-8h	.723E-08	.170E-07	.286E-07	.383E-07	.437E-07	.445E-07	.417E-07	.372E-07	.319E-07	.266E-07	.177E-07	.114E-07	.725E-08	.458E-08	.289E-08	.143E-08	.717E-09	.355E-09
7s-8i	.286E-08	.745E-08	.136E-07	.191E-07	.225E-07	.234E-07	.222E-07	.199E-07	.172E-07	.144E-07	.967E-08	.626E-08	.399E-08	.252E-08	.159E-08	.760E-09	.373E-09	.179E-09
7s-8j	.780E-09	.220E-08	.417E-08	.599E-08	.717E-08	.757E-08	.725E-08	.660E-08	.572E-08	.480E-08	.320E-08	.207E-08	.132E-08	.832E-09	.514E-09	.223E-09	.105E-09	.491E-10
7p-8s	.320E-07	.588E-07	.852E-07	.105E-06	.117E-06	.119E-06	.115E-06	.105E-06	.937E-07	.812E-07	.579E-07	.402E-07	.280E-07	.198E-07	.142E-07	.855E-08	.507E-08	.295E-08
7p-8p	.123E-06	.206E-06	.266E-06	.292E-06	.289E-06	.267E-06	.238E-06	.204E-06	.171E-06	.142E-06	.935E-07	.603E-07	.384E-07	.244E-07	.154E-07	.775E-08	.389E-08	.195E-08
7p-8d	.241E-06	.428E-06	.598E-06	.714E-06	.766E-06	.763E-06	.722E-06	.654E-06	.575E-06	.495E-06	.350E-06	.242E-06	.168E-06	.118E-06	.841E-07	.504E-07	.298E-07	.173E-07
7p-8f	.109E-06	.182E-06	.237E-06	.261E-06	.258E-06	.237E-06	.209E-06	.177E-06	.147E-06	.121E-06	.788E-07	.505E-07	.321E-07	.203E-07	.129E-07	.645E-08	.323E-08	.162E-08
7p-8g	.345E-07	.651E-07	.930E-07	.111E-06	.115E-06	.111E-06	.100E-06	.863E-07	.724E-07	.596E-07	.390E-07	.250E-07	.159E-07	.100E-07	.633E-08	.317E-08	.159E-08	.794E-09
7p-8h	.120E-07	.263E-07	.423E-07	.547E-07	.606E-07	.605E-07	.560E-07	.494E-07	.421E-07	.351E-07	.234E-07	.151E-07	.961E-08	.607E-08	.383E-08	.191E-08	.954E-09	.474E-09
7p-8i	.417E-08	.103E-07	.182E-07	.250E-07	.291E-07	.301E-07	.284E-07	.255E-07	.220E-07	.184E-07	.122E-07	.790E-08	.504E-08	.318E-08	.200E-08	.970E-09	.479E-09	.233E-09
7p-8j	.103E-08	.281E-08	.521E-08	.743E-08	.882E-08	.925E-08	.882E-08	.799E-08	.692E-08	.582E-08	.389E-08	.252E-08	.161E-08	.101E-08	.625E-09	.281E-09	.134E-09	.635E-10
7d-8s	.602E-08	.980E-08	.126E-07	.139E-07	.140E-07	.133E-07	.120E-07	.104E-07	.875E-08	.724E-08	.479E-08	.309E-08	.197E-08	.125E-08	.793E-09	.398E-09	.200E-09	.998E-10
7d-8p	.275E-07	.509E-07	.741E-07	.919E-07	.102E-06	.104E-06	.100E-06	.921E-07	.816E-07	.705E-07	.501E-07	.345E-07	.239E-07	.168E-07	.119E-07	.710E-08	.418E-08	.241E-08
7d-8d	.114E-06	.192E-06	.250E-06	.276E-06	.273E-06	.253E-06	.226E-06	.194E-06	.163E-06	.135E-06	.894E-07	.576E-07	.367E-07	.233E-07	.148E-07	.741E-08	.372E-08	.186E-08
7d-8f	.318E-06	.562E-06	.780E-06	.924E-06	.984E-06	.976E-06	.919E-06	.829E-06	.727E-06	.624E-06	.440E-06	.303E-06	.210E-06	.148E-06	.105E-06	.627E-07	.370E-07	.214E-07
7d-8g	.124E-06	.211E-06	.277E-06	.307E-06	.305E-06	.282E-06	.249E-06	.212E-06	.176E-06	.144E-06	.940E-07	.603E-07	.383E-07	.243E-07	.153E-07	.769E-08	.386E-08	.193E-08
7d-8h	.330E-07	.640E-07	.939E-07	.114E-06	.121E-06	.117E-06	.107E-06	.926E-07	.780E-07	.643E-07	.423E-07	.271E-07	.172E-07	.109E-07	.687E-08	.342E-08	.171E-08	.856E-09
7d-8i	.879E-08	.200E-07	.330E-07	.435E-07	.489E-07	.494E-07	.460E-07	.409E-07	.350E-07	.292E-07	.195E-07	.126E-07	.802E-08	.507E-08	.320E-08	.157E-08	.783E-09	.387E-09
7d-8j	.182E-08	.467E-08	.834E-08	.116E-07	.136E-07	.141E-07	.134E-07	.121E-07	.104E-07	.875E-08	.584E-08	.377E-08	.241E-08	.152E-08	.942E-09	.440E-09	.215E-09	.104E-09
7f-8s	.873E-09	.151E-08	.205E-08	.239E-08	.253E-08	.249E-08	.232E-08	.173E-08	.204E-08	.144E-08	.957E-09	.614E-09	.392E-09	.248E-09	.157E-09	.776E-10	.387E-10	.191E-10
7f-8p	.495E-08	.831E-08	.110E-07	.126E-07	.131E-07	.127E-07	.117E-07	.102E-07	.871E-08	.725E-08	.483E-08	.312E-08	.199E-08	.127E-08	.802E-09	.399E-09	.199E-09	.991E-10
7f-8d	.182E-07	.344E-07	.510E-07	.641E-07	.718E-07	.740E-07	.715E-07	.657E-07	.583E-07	.503E-07	.356E-07	.245E-07	.168E-07	.117E-07	.828E-08	.489E-08	.286E-08	.164E-08
7f-8f	.107E-06	.181E-06	.236E-06	.261E-06	.260E-06	.241E-06	.215E-06	.185E-06	.156E-06	.129E-06	.856E-07	.552E-07	.352E-07	.223E-07	.141E-07	.710E-08	.356E-08	.178E-08
7f-8g	.444E-06	.780E-06	.108E-05	.127E-05	.134E-05	.132E-05	.124E-05	.111E-05	.973E-06	.832E-06	.585E-06	.403E-06	.278E-06	.195E-06	.138E-06	.821E-07	.483E-07	.278E-07

Table 5.2 continued

Exc. $nl - n'l'$	Temperature ($\log_{10} K$)																	
	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.4	6.8	7.2	7.6	8.0	8.6	9.2	9.8
7f-8h	.143E-06	.245E-06	.326E-06	.366E-06	.366E-06	.340E-06	.301E-06	.257E-06	.214E-06	.175E-06	.115E-06	.735E-07	.467E-07	.296E-07	.187E-07	.936E-08	.470E-08	.235E-08
7f-8i	.286E-07	.570E-07	.856E-07	.106E-06	.114E-06	.112E-06	.102E-06	.892E-07	.755E-07	.624E-07	.412E-07	.264E-07	.168E-07	.106E-07	.670E-08	.333E-08	.166E-08	.830E-09
7f-8j	.446E-08	.105E-07	.177E-07	.236E-07	.268E-07	.273E-07	.255E-07	.228E-07	.196E-07	.164E-07	.110E-07	.709E-08	.452E-08	.286E-08	.179E-08	.864E-09	.427E-09	.211E-09
7g-8s	.160E-09	.308E-09	.453E-09	.565E-09	.626E-09	.637E-09	.608E-09	.543E-09	.467E-09	.390E-09	.261E-09	.169E-09	.108E-09	.684E-10	.430E-10	.203E-10	.992E-11	.471E-11
7g-8p	.735E-09	.134E-08	.192E-08	.235E-08	.258E-08	.261E-08	.247E-08	.220E-08	.189E-08	.158E-08	.105E-08	.678E-09	.435E-09	.276E-09	.173E-09	.829E-10	.408E-10	.199E-10
7g-8d	.292E-08	.508E-08	.698E-08	.827E-08	.884E-08	.876E-08	.818E-08	.727E-08	.624E-08	.522E-08	.350E-08	.226E-08	.145E-08	.921E-09	.581E-09	.285E-09	.142E-09	.704E-10
7g-8f	.113E-07	.218E-07	.328E-07	.417E-07	.471E-07	.489E-07	.473E-07	.435E-07	.385E-07	.332E-07	.234E-07	.159E-07	.109E-07	.753E-08	.525E-08	.306E-08	.177E-08	.101E-08
7g-8g	.958E-07	.163E-06	.215E-06	.239E-06	.238E-06	.222E-06	.199E-06	.171E-06	.145E-06	.120E-06	.797E-07	.514E-07	.328E-07	.208E-07	.132E-07	.661E-08	.332E-08	.166E-08
7g-8h	.632E-06	.111E-05	.152E-05	.178E-05	.187E-05	.183E-05	.171E-05	.153E-05	.133E-05	.114E-05	.796E-06	.545E-06	.375E-06	.262E-06	.184E-06	.109E-06	.640E-07	.368E-07
7g-8i	.149E-06	.261E-06	.352E-06	.400E-06	.404E-06	.378E-06	.336E-06	.287E-06	.240E-06	.197E-06	.129E-06	.830E-07	.528E-07	.334E-07	.211E-07	.106E-07	.530E-08	.265E-08
7g-8j	.172E-07	.354E-07	.545E-07	.685E-07	.747E-07	.740E-07	.681E-07	.600E-07	.510E-07	.423E-07	.280E-07	.180E-07	.114E-07	.723E-08	.455E-08	.224E-08	.112E-08	.558E-09
7h-8s	.361E-10	.748E-10	.116E-09	.151E-09	.173E-09	.181E-09	.174E-09	.158E-09	.137E-09	.115E-09	.762E-10	.496E-10	.320E-10	.203E-10	.123E-10	.511E-11	.235E-11	.109E-11
7h-8p	.136E-09	.279E-09	.430E-09	.555E-09	.632E-09	.656E-09	.633E-09	.573E-09	.497E-09	.416E-09	.277E-09	.180E-09	.116E-09	.731E-10	.445E-10	.194E-10	.915E-11	.431E-11
7h-8d	.397E-09	.769E-09	.115E-08	.146E-08	.165E-08	.170E-08	.163E-08	.147E-08	.127E-08	.107E-08	.713E-09	.460E-09	.295E-09	.187E-09	.115E-09	.528E-10	.255E-10	.124E-10
7h-8f	.145E-08	.266E-08	.382E-08	.468E-08	.515E-08	.521E-08	.494E-08	.444E-08	.384E-08	.322E-08	.217E-08	.140E-08	.902E-09	.573E-09	.358E-09	.172E-09	.849E-10	.420E-10
7h-8g	.630E-08	.125E-07	.191E-07	.245E-07	.278E-07	.289E-07	.280E-07	.258E-07	.228E-07	.196E-07	.137E-07	.924E-08	.624E-08	.425E-08	.292E-08	.166E-08	.946E-09	.531E-09
7h-8h	.783E-07	.135E-06	.180E-06	.201E-06	.201E-06	.188E-06	.169E-06	.146E-06	.124E-06	.103E-06	.686E-07	.443E-07	.283E-07	.180E-07	.114E-07	.570E-08	.286E-08	.143E-08
7h-8i	.906E-06	.158E-05	.217E-05	.253E-05	.264E-05	.258E-05	.240E-05	.214E-05	.185E-05	.158E-05	.110E-05	.749E-06	.513E-06	.356E-06	.250E-06	.147E-06	.857E-07	.491E-07
7h-8j	.119E-06	.212E-06	.292E-06	.336E-06	.343E-06	.324E-06	.290E-06	.249E-06	.209E-06	.172E-06	.113E-06	.728E-07	.463E-07	.293E-07	.185E-07	.926E-08	.464E-08	.232E-08
7i-8s	.688E-11	.159E-10	.263E-10	.353E-10	.415E-10	.442E-10	.431E-10	.400E-10	.353E-10	.300E-10	.202E-10	.127E-10	.819E-11	.492E-11	.272E-11	.894E-12	.363E-12	.169E-12
7i-8p	.244E-10	.555E-10	.915E-10	.122E-09	.143E-09	.153E-09	.148E-09	.138E-09	.121E-09	.103E-09	.686E-10	.444E-10	.282E-10	.173E-10	.956E-11	.335E-11	.139E-11	.639E-12
7i-8d	.572E-10	.128E-09	.207E-09	.275E-09	.319E-09	.338E-09	.330E-09	.305E-09	.268E-09	.228E-09	.152E-09	.988E-10	.627E-10	.389E-10	.220E-10	.839E-11	.368E-11	.171E-11
7i-8f	.148E-09	.313E-09	.495E-09	.648E-09	.749E-09	.789E-09	.763E-09	.702E-09	.615E-09	.520E-09	.348E-09	.225E-09	.144E-09	.897E-10	.528E-10	.220E-10	.102E-10	.489E-11
7i-8g	.517E-09	.103E-08	.156E-08	.198E-08	.224E-08	.232E-08	.223E-08	.204E-08	.178E-08	.150E-08	.101E-08	.655E-09	.420E-09	.266E-09	.163E-09	.741E-10	.357E-10	.175E-10
7i-8h	.261E-08	.538E-08	.837E-08	.108E-07	.124E-07	.129E-07	.125E-07	.115E-07	.102E-07	.871E-08	.603E-08	.403E-08	.269E-08	.180E-08	.121E-08	.658E-09	.364E-09	.200E-09
7i-8i	.494E-07	.869E-07	.117E-06	.132E-06	.133E-06	.125E-06	.113E-06	.983E-07	.837E-07	.699E-07	.468E-07	.303E-07	.194E-07	.123E-07	.781E-08	.390E-08	.195E-08	.978E-09
7i-8j	.131E-05	.229E-05	.313E-05	.364E-05	.379E-05	.369E-05	.341E-05	.303E-05	.262E-05	.222E-05	.154E-05	.105E-05	.713E-06	.492E-06	.343E-06	.200E-06	.116E-06	.660E-07

Chapter 6

Analytic Evaluation of the B1B Cross Sections

Section 6.1 of this chapter reviews the derivation of the one–dimensional integral representation of the B1B amplitude. Section 6.2 gives the derivation of a closed form expression for the $1s \rightarrow 1s$ capture amplitude and cross section.

6.1 One–dimensional integral representation of the B1B amplitude

Dewangan and Eichler [DeE86] have given a derivation to reduce the B1B amplitude for $1s \rightarrow 2s$ capture into a one–dimensional integral form. We present their derivation here.

We consider the problem of one electron in the field of two bare nuclei (target charge Z_T and projectile charge Z_P). If we adopt the impact parameter picture and let $\mathbf{R}(t) = \mathbf{b} + \mathbf{v}t$ describe the relative motion of the projectile (with velocity $\mathbf{v}$ and impact parameter $\mathbf{b}$) with respect to the target nucleus, the total electronic wave function has the asymptotic behavior

$$\lim_{t \rightarrow -\infty} \Psi_i^+(t) \;=\; \Phi_i(t) \exp(-i\sigma_i(t)) \tag{6.1}$$

and

$$\lim_{t \rightarrow \infty} \Psi_f^-(t) \;=\; \Phi_f(t) \exp(i\sigma_f(t)). \tag{6.2}$$

Here, $\sigma_i(t) = (Z_P/v)\log(R - vt)$ and $\sigma_f(t) = (Z_T/v)\log(R + vt)$ are the asymptotic Coulomb phases, $\Phi_i(t) = \phi_i(\mathbf{r}_T)\exp(-i\epsilon_i t)$, $\Phi_f(t) = \phi_f(\mathbf{r}_P)\exp(-i\epsilon_i t + i\mathbf{v}\cdot\mathbf{r}_T - \frac{1}{2}iv^2 t)$ and $\epsilon_{i,f}$ and $\phi_{i,f}$ are the target and projectile electronic binding energies and wave functions; $\mathbf{r}_T$ and $\mathbf{r}_P$ are position of the electron with respect to the target and the projectile nucleus. Following Belkić *et al.* [BGS79], Dewangan and Eichler [DeE85] have formulated a rigorous capture amplitude that satisfies the correct boundary conditions as

$$B(\infty) \;=\; -i \int_{-\infty}^{\infty} dt \, < \Phi_f(t) \exp(i\sigma_f(t) | Z_T/R - Z_T/r_T | \Psi_i^+(t) > \, . \tag{6.3}$$

A 'boundary corrected' first-order approximation of (6.3) is given by

$$A^{(0)}_{\rm B1B} = -i\int_{-\infty}^{\infty} dt < \Phi_f(t)\exp(i\sigma_f(t)|Z_T/R - Z_T/r_T|\Phi_i\exp(-i\sigma_i(t) > . \qquad (6.4)$$

This amplitude presents the post version of the theory. The prior form of the amplitude, in which Z_T and r_T are replaced by Z_P and r_P, can be shown [DeE94] to be identical to the post form provided that exact eigenfunctions ϕ_f, ϕ_i are used. The expression (6.4) is based on a purely electronic Hamiltonian and is adequate for calculating total cross sections. If, however, we want to account for the deflection of the projectile caused by the internuclear repulsion (see e.g. [BeS76, HEL82]) we have to introduce the term Z_PZ_T/R into the Hamiltonian. As is welknown, such a term may be removed by the phase transformations $\exp[i\nu\log(R-vt)]$ and $\exp[-i\nu\log(R+vt)]$ with $\nu = Z_PZ_T/v$ for the initial and final states, respectively. The transition amplitude based on the full Hamiltonian including the internuclear interaction may then be written as

$$\begin{aligned} A_{\rm B1B} &= -i\int_{-\infty}^{\infty} dt < \Phi_f(t)\exp[-i\nu_T\log(R+vt)]|Z_T/R \\ &\quad -Z_T/r_T|\Phi_i(t)\exp[i\nu_P\log(R-vt)] > \end{aligned} \qquad (6.5)$$

where $\nu_T = Z_T(Z_P-1)/v$ and $\nu_P = Z_P(Z_T-1)/v$. We immediately notice that $\exp[i\nu\log(R+vt)]\exp[i\nu\log(R-vt)] = b^{2i\nu}$ is independent of time and hence

$$A_{\rm B1B} = b^{2i\nu}A^{(0)}_{\rm B1B}. \qquad (6.6)$$

The phase factor $b^{2i\nu}$ describes the Rutherford projectile deflection within the optical eikonal approximation (see [BGS79]) but has no effect on the total cross section.

To describe $1s \rightarrow 1s$ transition let us know assume hydrogenic wave functions $\phi_i(\mathbf{r}_T) = \alpha^{3/2}\pi^{-1/2}\exp(-\alpha\mathbf{r}_T)$ and $\phi_f(\mathbf{r}_P) = \beta^{3/2}\pi^{-1/2}\exp(-\beta\mathbf{r}_T)$ where α and β are to be treated as parameters. After performing all the operations these parameters are set to be equal to Z_T and Z_P respectively. Decomposing (6.5) into partial amplitudes for Z_T/R and Z_T/r_T, respectively, according to $A_{\rm B1B} = A^{(1)}_{\rm B1B} + A^{(2)}_{\rm B1B}$ and substituting the $1s$ wave functions, we obtain (with $Z = vt$) for the Z_T/r_T term

$$\begin{aligned} A^{(2)}_{\rm B1B}(b) &= i\frac{\alpha^{3/2}\beta^{3/2}}{\pi}\frac{Z_T}{v}\int_{-\infty}^{\infty} dZ\exp\left[i\left(\frac{\epsilon_f-\epsilon_i}{v}+\frac{v}{2}\right)Z\right] \\ &\quad \times\exp[i\nu_T\log(R+Z)]\exp[i\nu_P\log(R-Z)]E(\mathbf{R}) \end{aligned} \qquad (6.7)$$

where

$$E(\mathbf{R}) = -\frac{\partial}{\partial\beta}\left(\int d^3r_T\frac{\exp(-\alpha r_T-\beta r_P-i\mathbf{v}\cdot\mathbf{r}_T)}{r_Tr_P}\right) \qquad (6.8)$$

is essentially an exchange matrix element. Following standard techniques [McC70] Eq. (6.8) can be cast into the form

$$E(\mathbf{R}) = -2\pi\frac{\partial}{\partial\beta}\left(\int_0^1 dy\frac{\exp(-\Delta R-iy\mathbf{v}\cdot\mathbf{R})}{\Delta}\right) \qquad (6.9)$$

with

$$\Delta^2(\alpha,\beta) = v^2y(1-y)+\alpha^2y+\beta^2(1-y). \tag{6.10}$$

Substituting (6.9) into (6.7) we obtain

$$\begin{aligned} A^{(2)}_{\mathrm{B1B}}(b) &= -i\alpha^{3/2}\beta^{3/2}\frac{2Z_T}{v}\frac{\partial}{\partial\beta}\left(\int_{-\infty}^{\infty}dZ\int_0^1 dy\exp[i\nu_P\log(R-Z)]\right. \\ &\quad \left.\times\exp[i\nu_T\log(R+Z)]\exp(-\Delta R+i\mathbf{J}\cdot\mathbf{R})\Delta^{-1}\right) \end{aligned} \tag{6.11}$$

where $\mathbf{J}=[(\epsilon_f-\epsilon_i)/v+v(\frac{1}{2}-y)]\hat{\mathbf{v}}$. In order to proceed further we define the Fourier transform

$$F^{(2)}(\mathbf{q}) = \frac{1}{2\pi}\int d^2b\exp(i\mathbf{q}\cdot\mathbf{b})A^{(2)}_{\mathrm{B1B}}(b) \tag{6.12}$$

and insert (6.11) into (6.12). With $\mathbf{p}=\mathbf{J}+\mathbf{q}$ we obtain

$$\begin{aligned} F^{(2)}(\mathbf{q}) &= -\frac{iZ_T}{\pi v}\alpha^{3/2}\beta^{3/2}\frac{\partial}{\partial\beta}\left[\int_0^1 dy\left(-\frac{1}{\Delta}\frac{\partial}{\partial\Delta}\right)\right. \\ &\quad \times\left(\int d^3R\frac{-\exp(-\Delta R+i\mathbf{p}\cdot\mathbf{R})}{R}\exp[i\nu_P\log(R-Z)]\right. \\ &\quad \left.\left.\times\exp[i\nu_T\log(R+Z)]\right)\right]. \end{aligned} \tag{6.13}$$

It turns out that the integral over R can be performed analytically, so that we are left with a one dimensional integral representation. In a similar fashion one may treat the matrix element of Z_T/R in (6.5) and derive the Fourier transform $F^{(1)}(\mathbf{q})$ of $A^{(1)}_{\mathrm{B1B}}(b)$. The cross section is then expressed by the complete Fourier transform $F(\mathbf{q})=F^{(1)}(\mathbf{q})+F^{(2)}(\mathbf{q})$ as

$$\sigma = \int d^2q|F(\mathbf{q})|^2 \tag{6.14}$$

with

$$F(\mathbf{q}) = N_0\int_0^1 dy\frac{\partial}{\partial\beta}\left(-\frac{1}{\Delta}\frac{\partial}{\partial\Delta}+\frac{\partial}{\partial\alpha}\frac{1}{\Delta}\right)L_0(\Delta(\alpha,\beta)). \tag{6.15}$$

Here the following abbreviations are used:

$$N_0 = -i2^{2+i\nu_P+i\nu_T}{Z_P}^{3/2}{Z_T}^{5/2}\Gamma(1+i\nu_P)\Gamma(1+i\nu_T)/v \tag{6.16}$$

and

$$L_0(\Delta(\alpha,\beta)) = \frac{(\Delta^2+p^2)^{-i\nu_P-i\nu_T-1}}{(\Delta-ip_z)^{-i\nu_P}(\Delta+ip_z)^{-i\nu_T}}{}_2F_1[-i\nu_P,-i\nu_T;1;-p_b^2/(\Delta^2+p_z^2)] \tag{6.17}$$

where $p_z=\mathbf{p}\cdot\hat{\mathbf{v}}=(\epsilon_f-\epsilon_i)/v+v(\frac{1}{2}-y)$ and $\mathbf{p}_b=\mathbf{p}\cdot\hat{\mathbf{b}}=\mathbf{q}$.

6.2 Analytic evaluation of the B1B cross sections for $1s\rightarrow 1s$ capture

We now give a closed form expressions both for the B1B amplitude and the cross section for the $1s\rightarrow 1s$ capture [DeC89, DMDC90]. We also discuss how the present technique can be extended to obtain an analytic expression for a general $nlm\rightarrow n'l'm'$ capture.

The B1B amplitude for the $1s \rightarrow 1s$ capture is given by Eq. (6.5). Noting that $\exp[i\nu_P \log(R-Z)+i\nu_T \log(R+Z)] = b^{2i\nu_T} \exp[i\eta \log(R-Z)]$ with $\eta = \nu_P - \nu_T = (Z_T - Z_P)/v$, Eq. (6.5) becomes

$$A_{\mathrm{B1B}}(\mathbf{b}) = b^{2i\nu_T} C_{\mathrm{B1B}}(\mathbf{b}) \tag{6.18}$$

where

$$C_{\mathrm{B1B}}(\mathbf{b}) = -\frac{i}{v}\int_{-\infty}^{\infty} dZ \exp[i\eta \log(R-Z)] < \Phi_f | Z_T/R - Z_T/r_T | \Phi_i > . \tag{6.19}$$

Introducing the two-dimensional Fourier transform $f(\mathbf{q})$ (see Eq. (6.12)) for $C_{\mathrm{B1B}}(\mathbf{b})$ the total cross section is given by (6.14) with $F(\mathbf{q})$ substituted by $f(\mathbf{q})$. Evidently the internuclear potential does not contribute to the total cross section as it is also pointed out previously. The expression for $f(\mathbf{q})$ is obtained by putting $\nu_T = 0$ and $\nu_P = \eta$ in Eq. (6.15) and we have

$$f(\mathbf{q}) = N\int_0^1 dy \frac{\partial}{\partial\beta}\left(-\frac{1}{\Delta}\frac{\partial}{\partial\Delta} + \frac{\partial}{\partial\alpha}\frac{1}{\Delta}\right) T^{-i\eta-1} X(0) \tag{6.20}$$

where

$$N = -i2^{2+i\eta} {Z_P}^{3/2} {Z_T}^{5/2} \Gamma(1+i\eta)/v \tag{6.21}$$

$$X(k) = (\Delta - ip_z)^{i\eta+k} \qquad k = 0, \pm 1, \pm 2, \ldots \tag{6.22}$$

$$T = \Delta^2 + p^2 \tag{6.23}$$

$$\Delta^2 = v^2 y(1-y) + \alpha^2 y + \beta^2 (1-y) \ . \tag{6.24}$$

$$\mathbf{p} = \mathbf{q} + \mathbf{p}_z \tag{6.25}$$

$$\begin{aligned} \mathbf{p}_z &= -vy + [v/2 + (\epsilon_f - \epsilon_i)/v] \\ &\equiv -vy + s. \end{aligned} \tag{6.26}$$

As mentioned earlier, α and β are to be treated as parameters in Eq. (6.20) and only after performing the indicated operations are they to be set equal to Z_T and Z_P respectively.

It is important to note that for $\alpha = Z_T$ and $\beta = Z_P$, $T = T_0 = q^2 + (\beta^2 + s^2) = q^2 + a^2$, a quantity independent of the variable y. We take advantage of this fact by only partly performing differentiation in (6.20) and then taking out all T_0-dependent factors from the y integral. After some algebra Eq. (6.20) can be written as

$$f(\mathbf{q}) = N T_0^{-i\eta}\left(\frac{A_1}{T_0} + \frac{A_2}{T_0^2} + \frac{A_3}{T_0^3}\right) \tag{6.27}$$

where

$$A_1 = I_{11} - i\eta I_{12} \tag{6.28}$$

$$A_2 = -4I_{21} + 4(i\eta + 1)I_{22} \tag{6.29}$$

$$A_3 = 4(i\eta + 2)[Z_{T31} - Z_P(i\eta + 1)[I_{32} - I_{33}]]. \tag{6.30}$$

The I_{ij} represent the remaining integrals that can be evaluated in closed forms. In order to give the closed form expressions for these integrals we will need the following definitions:

$$\lambda(m) = \frac{a_2^{i\eta+m} - a_1^{i\eta+m}}{i\eta + m} \qquad m = 0, \pm 1, \pm 2 \ldots \tag{6.31}$$

with

$$a_1 = Z_P - is \tag{6.32}$$

and

$$a_2 = Z_T + i(v - s). \tag{6.33}$$

We are now in the position to write (after the partial differentiations, α and β are replaced by Z_T and Z_P respectively)

$$\begin{aligned} I_{11} &= \frac{\partial^2}{\partial\alpha\partial\beta}\int_0^1 dy \frac{X(0)}{\Delta} \\ &= -\frac{1}{v^2}[Z_T a_1^{i\eta-2} + Z_P a_2^{i\eta-2}] + 2\frac{Z_T Z_P}{iv^3}\lambda(-2) \end{aligned} \tag{6.34}$$

$$\begin{aligned} I_{12} &= \frac{\partial}{\partial\beta}\int_0^1 dy \frac{X(-1)}{\Delta} \\ &= \frac{i}{v}a_1^{i\eta-2} - \frac{Z_P}{v^2}\lambda(-2) \end{aligned} \tag{6.35}$$

$$\begin{aligned} I_{21} &= \frac{\partial^2}{\partial\alpha\partial\beta}\int_0^1 dy X(1) \\ &= -\frac{Z_T}{v^2}[a^2 a_1^{i\eta-2} - isa_1^{i\eta-1}] - \frac{Z_P}{v^2}[a^2 a_2^{i\eta-2} + i(v-s)a_2^{i\eta-1}] \\ &\quad + \frac{Z_T Z_P}{iv^3}[3a^2\lambda(-2) + 2i(v-2s)\lambda(-1) - \lambda(0)] \end{aligned} \tag{6.36}$$

$$\begin{aligned} I_{22} &= \frac{\partial}{\partial\beta}\int_0^1 dy X(0) \\ &= \frac{i}{2v}[a^2 a_1^{i\eta-2} + a_1^{i\eta}] - \frac{Z_P}{v^2}[a^2\lambda(-2) + i(v-s)\lambda(-1)] \end{aligned} \tag{6.37}$$

$$\begin{aligned} I_{31} &= \frac{\partial}{\partial\beta}\int_0^1 dy X(1) \\ &= -\frac{1}{4v^2}[a^4 a_1^{i\eta-2} - 2isa^2 a_1^{i\eta-1} - 2isa_1^{i\eta+1} - a_1^{i\eta+2}] \\ &\quad + \frac{Z_P}{4iv^3}[3a^4\lambda(-2) + 4i(v-2s)a^2\lambda(-1) - (4s^2 - 4sv + 2a^2)\lambda(0) - \lambda(2)] \end{aligned} \tag{6.38}$$

$$I_{32} = \int_0^1 dy X(0)$$
$$= \frac{1}{2iv}[a^2\lambda(-1) + \lambda(1)] \tag{6.39}$$

$$I_{33} = \int_0^1 dy\, yX(0)$$
$$= \frac{1}{4v^2}[a^4\lambda(-2) - 2isa^2\lambda(-1) - 2is\lambda(1) - \lambda(2)]. \tag{6.40}$$

Since the coefficient A_n are independent of q, the integral over the two-dimensional vector $\mathbf{q}$ appearing in (6.14), replacing $F(\mathbf{q})$ by $f(\mathbf{q})$, is easily evaluated leaving the cross section as

$$\sigma = \frac{\pi|N|^2}{a^{10}}[a^8|A_1|^2 + \frac{1}{2}a^6(A_1A_2^* + A_1^*A_2)$$
$$+\frac{1}{3}a^4(|A_2|^2 + A_1A_3^* + A_1^*A_3) + \frac{1}{4}a^2(A_2A_3^* + A_2^*A_3) + \frac{1}{5}|A_3|^2]. \tag{6.41}$$

One evidence for the correctness of the analysis is to reproduce the property of microreversibility that the cross section must obey, namely

$$\sigma(Z_T, 1s \to Z_P, 1s) = \sigma(Z_P, 1s \to Z_T, 1s). \tag{6.42}$$

With a bit of patience, one can show *analytically* that the closed form expressions do indeed possess the property (6.42). We have programmed Eq. (6.41) and our numerical results satisfy the relationship (6.42) in all the cases treated. They also agree completely with the extensive calculations of previous authors [BST87].

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