PARAMETRIC INTERACTION OF WAVES IN A MAGNETIZED PLASMA

BY

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Ā paritoṣādvidusāṁ na sādhu manye
prayogavijñānam

Balavadapi śikṣitānāmātmanyapraty ayaṁ
cetaḥ.

(Prelude, Abhijñānaśakuntalam)

[I] do not consider [my] knowledge of representation sound until [it causes] the satisfaction of the learned. The mind of even the highly instructed [persons] is void of confidence in itself.

DEDICATED

TO MY MOTHER

and

IN SWEET MEMORY

OF MY FATHER

C O N T E N T S

....

C E R T I F I C A T E

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

S. Bujarbarua.

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A B S T R A C T

In this thesis we have studied some aspects of parametric interaction processes in a magnetized plasma which have applications in thermo-nuclear fusion devices and in ionospheric modification schemes. A detailed study has been made of parametric scattering instabilities in a magnetized plasma and their implications for a laser produced plasma discussed. A particular case of scattering instability, viz. the filamentation instability, which leads to self focussing of light, has been studied in a high β magnetized plasma, and interesting modifications arising from magnetic field perturbations discussed. A study has also been made of decay instabilities with the pump wave frequency near three natural resonances in a magnetized plasma viz (1) the lower hybrid resonance (2) the ion-ion hybrid resonance and (3) the upper hybrid resonance. The first two decay processes are relevant for plasma heating schemes in thermo-nuclear fusion devices like tokamak etc., while the third decay process has been studied in the context of an iono-spheric modification experiment. In this experiment it is suggested that a ground based transmitter could be used to artificially trigger or suppress the equatorial F-region irregularities.

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CHAPTER 1

INTRODUCTION

1.1 Brief History

The field of plasma Physics deals with the study of matter consisting of a conglomeration of positive ions and electrons which strive to maintain macroscopic charge neutrality. This many body system displays unique features due to a variety of collective states it can support and the investigation of these states is an important aspect of plasma research.

For any dynamical system the determination of its equilibrium configuration and the subsequent stability of the established equilibrium is necessary from an application point of view. It is known that for a many body system with a large number of degrees of freedom, the number of stable configurations reduces drastically so that it is well recognized that a plasma displays a variety of instabilities. An instability arises because of the existence of a source of free energy in the system.

In a plasma the sources of free energy are the various currents set up in the equilibrium configuration because of the geometry of the set up and background stationary fields (differential $\underset{\sim}{E}X\underset{\sim}{B}$ drifts in the partially ionized plasma, the gravitational, curvature and gradient drifts are a few such reservoirs of energy). An uneven distribution of energy in the various degrees of freedom (measured by the temperature anisotropy) can also trigger an instability. On the other hand there is a class of instabilities in which the source of energy can be introduced through an external agency. By choosing appropriate field configurations (electric and magnetic), currents can be induced which can lead to the instability of a certain class of perturbations. If the equilibrium is a dynamic type due to a steady state normal mode of the system, the instability induced on some other modes (which bear a specific relationship to the initial mode) is referred to as a parametric instability.

More specifically parametric instabilities (also known as parametric excitation of waves) are produced by an external electric field oscillating at a definite frequency or by an intense beam of radiation traversing a plasma. Physically it corresponds to a basic process in non-linear mode coupling. It is an important mechanism of non linear mode conversion from electromagnetic to electrostatic and from high frequency to low frequency waves. Many experimental observations of anomalous absorption and scattering of electromagnetic waves can be attributed to a parametric excitation phenomenon. Recently study of parametric excitation phenomenon in a plasma has been an active field of research because of its important role in anomalous heating of plasmas such as laser heating, lower hybrid resonance heating and so on, and in ionospheric modification schemes.

Parametric excitation can also be defined as an amplification of an oscillation due to a periodic modulation of a parameter that characterizes the oscillation. A plasma may be regarded as an ensemble of an infinite number of oscillators i.e. elementary modes. In the linear (small amplitude) approximation these oscillators become harmonic and independent of each other. However, the non-linear properties of the medium lead to coupling between the oscillator modes. If there is a modulating wave (we shall call it a pump wave) in the plasma, it will non-linearly couple to some natural modes of the plasma and under certain conditions parametrically excite atleast a pair of waves.

Thus it has been discovered that the simplest type of parametric instability in a plasma is the decay instability. This consists in the simultaneous growth, in the presence of a pump wave of frequency ω_0 and wave vector $\underset{\sim}{k}_0$ of a pair of waves $(\omega_1, \underset{\sim}{k}_1)$ and $(\omega_2, \underset{\sim}{k}_2)$ satisfying the conditions $\omega_0 = \omega_1 + \omega_2$ $\underset{\sim}{k}_0 = \underset{\sim}{k}_1 + \underset{\sim}{k}_2$. This is basically a mode-conversion process. If the pump wave is supplied by some external agencies, then the mode conversion results in a deposition of the external pump energy into the plasma. Further, if the excited plasma waves subsequently accelerate the plasma particles, the energy deposited by the pump wave can be converted to thermal energy of the particles. In this way, parametric excitation can act as an efficient mechanism for heating the plasma.

The first theoretically predicted type of parametric instability was the instability of a longitudinal electron plasma wave[1,2]. In this process, waves 1 and 2 are respectively another electron plasma wave and an ion acoustic wave. This process is often referred to as plasmon $\rightarrow$ plasmon $+$ phonon and was first experimentally observed by Franklin et al[3] in 1971 in a thermally ionized sodium plasma column. The linear theory of the parametric instabilities is well developed and has been reviewed by several authors[4,5].

Perkins et al[6] reviewed the linear and non-linear theories of parametric instabilities with their applications to ionospheric modification scheme while Nishikawa[7] gave a general formalism of parametric instabilities in the presence of damping and calculated the growth rates and the threshold of the parametric instability i.e. the pump wave amplitude above which the instability sets in.

Study of parametric interaction of waves can be broadly divided into three catagories viz. (1) the decay instability (2) the scattering instability and (3) the filamentation instability. When an electromagnetic (EM) or an electrostatic (ES) pump wave interacts with the plasma and decays into two ES modes, which grow at the expense of the pump wave, then the process is known as decay instability. This process is very efficient for heating the plasma as the ES waves can easily transfer energy to the plasma particles by wave-particle interactions. When an EM wave decays into another EM wave and a low frequency ES wave, the process is known as scattering instability. In this process there is very little energy transfer from the incident EM wave to the plasma, as the scattered EM wave, because of its high phase velocity, goes out of the system without interacting with the plasma particles. Filamentation instability is a special class of scattering instability and takes place when the EM pump wave travels perpendicular to the low frequency ES (or mixed ES-EM) mode. This gives rise to the interesting phenomenon of self focussing of light.

An unmagnetized plasma supports three natural modes of propagation viz. (1) an EM wave with the dispersion relation

$$\omega^2 = \omega_{pe}^2 + c^2k^2 \tag{1.1}$$

(2) a high frequency ES plasma wave with a frequency

$$\omega = \left(\omega_{pe}^2 + 3k^2v_e^2\right)^{1/2} \tag{1.2}$$

and (3) a low frequency electrostatic ion acoustic wave with a frequency,

$$\omega = kC_s = k\left(T_e/M\right)^{1/2} \tag{1.3}$$

In the above expressions $\omega_{pe} = \left(4\pi e^2 n_0/m\right)^{1/2}$ is the electron plasma frequency, $v_e = \left(T_e/m\right)^{1/2}$ and $C_s = \left(T_e/M\right)^{1/2}$ are respectively the electron thermal speed and acoustic speed; T_e is the electron temperature in energy units; m and M are the masses of electron and ion respectively and other symbols have their usual meaning. The EM wave suffers only collisional damping while the two ES waves suffer an additional collisionless damping due to wave-particle interactions.

If an external EM pump wave is applied in such an unmagnetized plasma, it can lead to the following decay processes due to the parametric interaction of the collective modes in the plasma.

1) When the frequency ω_0 of the pump wave is near the plasma frequency ω_{pe} it may decay into an electron plasma wave (plasmon) and an ion acoustic wave (phonon), leading to the anomalous absorption of the pump wave.

2) When $\omega_0 \simeq 2\omega_{pe}$, the pump wave may decay into two plasmons.[8]

3) When $\omega_0 \gtrsim 2\omega_{pe}$, the pump wave may decay into a photon (scattered EM wave) and a plasmon, leading to an anomalous scattering of the EM wave off the electron plasma wave. This process is referred to as the Raman scattering[9].

4) The pump wave may decay into a photon and a phonon leading to an anomalous scattering of the EM wave off the ion acoustic wave when $\omega_0 > \omega_{pe}$ This process is called the Brillouin scattering[9].

The dominant non-linear coupling terms responsible for the above parametric interaction processes are (a) the non-linear current density arising due to the density fluctuations in one of the decay waves, interacting with the oscillating electron velocities due to the pump wave and (b) a ponderomotive force produced by the pump wave and one of the excited waves.

However, most laboratory and natural plasmas exist in the presence of magnetic fields and therefore it is important to investigate the effect of a magnetic field on the parametric interaction problem. In a magnetized plasma, there is a greater variety of natural modes of oscillation which makes the problem considerably complex and interesting. One can also choose various pump frequencies corresponding to various natural resonances, which exist in such a plasma and thereby have several different alternatives for decay waves. Another qualitatively new feature introduced by a magnetic field is that many of the excited modes have now additional sources of damping e.g. collisionless cyclotron damping etc., arising due to wave particle interactions in a magnetic field. This makes the parametric excitations, in a magnetized plasma more attractive for plasma heatings.

1.2 Scope of the thesis

The present thesis aims at studying some aspects of the parametric interaction phenomenon in a magnetized plasma. We have studied some aspects of all the three different parametric processes discussed above.

In the second chapter we have studied the stimulated scattering process in the presence of a background magnetic field. The ES modes which we have considered are (1) the electron Bernstein modes, (2) lower hybrid waves and (3) drift waves. We have calculated the growth rates and threshold powers for each of the processes and have found them to be comparable to those obtained for unmagnetized plasma. We have also considered the convective saturation effects of the density inhomogeneity in the plasma. The calculations were motivated by recent experiments in laser produced plasmas where mega gauss magnetic fields were observed. In such a situation it is important to consider the various low frequency modes that a magnetic field can support.

In the third chapter we have investigated the filamentation instability of high frequency EM waves in an inhomogeneous, magnetized plasma with $1 \gg \beta \gg m/M$ where β is the ratio of the plasma pressure to the magnetic pressure. For sufficiently long parallel wavelength in such plasmas, the transverse modulation of the EM wave gets coupled to drift-Alfven waves and leads to a new kind of filamentation instability. We have calculated the growth rates and threshold powers for this instability under different conditions. Our estimates of threshold power and growth rate etc. are found to be comparable to those obtained for excitation of ES modes and hence this could be a competing process in a realistic situation with finite β. We also note that our dispersion relation does not permit a purely growing solution in contrast to homogeneous unmagnetized plasma calculations. Both the inhomogeneity and the magnetic field effects contribute towards a small real part of the frequency and thus cause the density modulations to have a finite phase velocity.

In the next three chapters we have studied some cases of parametric decay instabilities. The lower hybrid resonance has received special importance in recent times for heating processes. In the fourth chapter we have pointed out the possibility of excitation of a low frequency 'cold' ion Bernstein mode by a lower hybrid pump in a cold, homogeneous, magnetized plasma. We have calculated the growth rate and the threshold power for this process. The threshold is found to be minimum when the pump wave is applied perpendicular to the plane containing the magnetic field and the wave vector of the excited mode. Numerical estimates with typical laboratory plasma parameters show that the threshold power in this case is quite low and thus the 'cold' ion Bernstein mode can be easily excited in such a situation.

In a two ion species, magnetized plasma it is possible to exploit the ion-ion hybrid resonance for the decay instability. In the fifth chapter, we have pointed out three different possibilities for a decay of this type where the pump frequency is near the ion-ion hybrid frequency. (1) The pump wave can decay into a 'cold' ion Bernstein mode and another ion-ion hybrid wave when the phase velocity, of the 'cold' ion Bernstein mode, parallel to the magnetic field exceeds the electron thermal velocity. (2) It can decay into a low frequency ion acoustic wave propagating parallel to the ambient magnetic field and another ion-ion hybrid wave when the parallel phase velocity of the acoustic mode is much less than the electron thermal velocity but much greater than the ion thermal velocity. (3) It can decay into two ion ion hybrid waves when the pump frequency is more than twice the ion-ion hybrid frequency. In the first two cases we have taken the wavelength of the pump wave to be infinitely large so that we could use the dipole approximation. For the decay into two ion-ion hybrid waves, frequency and wave vector matching conditions do not allow infinitely large pump wavelength so that dipole approximation cannot be used in this case. Our theory for the decay into 'cold' ion Bernstein and into low frequency acoustic waves, is quite general and applies to any masses of the two ion species whereas for the decay into two ion-ion hybrid waves we have used the approximation $\Omega_1^2 \gg \omega^2 \gg \Omega_2^2$ where Ω_1 and Ω_2 are the cyclotron frequencies of the ion species 1 and 2 respectively. Thus for this case there ia s restriction on the masses of the ion species and the theory cannot be applied to the small mass differences of the ion species. We have obtained the estimates of threshold powers and growth rates for all the three processes. Numerical estimates with typical laboratory plasma parameters show that they are well within the regime of present power densities envisaged for r.f.heating experiments and thus the instabilities considered in our work can be easily exploited in an experimental situation.

Finally in the sixth chapter we have studied the influence of a large amplitude electric field oscillating near the upper hybrid frequency on the natural stability of the Rayleigh Taylor and collisional drift modes. It has been proposed recently that the equatorial F-region irregularities are due to the Rayleigh-Taylor and collisional drift instabilities. It has been theoretically suggested that the large scale irregularities ranging from few kilometers down to a few hundred meters are due to R-T instability whereas the small scale irregularities ranging from few hundred meters down to a few tens of meters are due to collisional drift instability. We have studied the influence of an external wave, with frequency near the upper hybrid frequency, on the R-T and collisional drift modes in the F-region of the ionosphere and have found that the natural growth rates, of these instabilities, can either be enhanced or depleted by the pump wave under different conditions. We have also calculated the threshold power to excite (or suppress) these instabilities when the ionospheric conditions are below (or above) the natural instability level. We have suggested an ionospheric modification experiment where a ground based transmitter could be used to control the equatorial F-region irregularities.

Briefly, therefore, we have studied some aspects of parametric instabilities in a magnetized plasma and pointed out some interesting effects which can be of relevence to plasma heating schemes and to ionospheric modification experiments.

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CHAPTER 2

STIMULATED SCATTERING OF EM WAVES IN A MAGNETIZED PLASMA

2.1 Introduction

An important problem in parametric heating of plasmas by an intense EM wave is that of enhanced (or stimulated) scattering of incident EM waves by some ES waves. Physically we can understand the process as follows. The incident EM field induces electron oscillations through the Lorentz force. The electrons are initially driven along the electric field vector but then develop a longitudinal component through the $\underset{\sim}{V} \times \underset{\sim}{B}$ force, where $\underset{\sim}{V}$ is the velocity of the electrons and $\underset{\sim}{B}$ is the magnetic field of the EM wave. The ions do not respond directly to the field due to their large mass. Thus local charge imbalances are generated which tend to be restored to neutrality by the opposing coulomb interaction. On a macroscopic scale, the density oscillations are coupled to the pump field by the ponderomotive force density which varies essentially as the gradient of the intensity. For suitable phase matching between the incident and the scattered waves growth is induced in slowly moving density waves above a threshold intensity determined by the dissipation of the interacting waves.

In the absence of the pump wave, an unmagnetized plasma supports two natural ES modes of propagation. They are (1) the high frequency ES plasma wave and (2) the low frequency ES ion-acoustic wave. Accordingly in an unmagnetized plasma we can get two types of scattering processes. When EM pump wave is scattered by a plasma wave, the process is known as stimulated Raman scattering (SRS) and when it is scattered by an ion acoustic wave, the process is known as stimulated Brillouin scattering (SBS).

Historically study of scattering of EM waves by a plasma started from the work of W.E. Gordon[1], in 1958, who predicted that if a powerful beam of radio waves with a frequency well above the penetration frequency were sent vertically through the ionosphere, an extremely small, but still measurable amount of power would be scattered back to the ground from the randomly distributed free electrons in the ionosphere. Later on Dougherty and Farley[2], and Salpeter[3] also studied the problem of incoherent scattering of radio waves by a plasma and Goldman and DuBois[4] studied the incoherent scattering of light from plasmas.

SRS in plasmas was studied theoretically by Comisar[5] in 1966. Gorbunov[6], using one dimensional fluid and kinetic descriptions, calculated threshold powers and growth rates for the backward SBS process. A unified formation for the stimulated scattering of an EM wave off ES waves, in a homogeneous unmagnetized plasma, was derived by Drake[7] et al in 1974. These authers derived a general dispersion relation for such a scattering process and discussed various instabilities including SRS, SBS, Compton scattering, and filamentation and modulational instabilities. These instabilities have direct relevance to laser produced plasmas as these instabilities can be excited in the under dense region of the laser produced plasma resulting in a partially reflecting induced dielectric mirror which can lead to substantial reflection of the incident radiation.

Recently Stamper et al[8] have observed intense spontaneously generated magnetic fields in the laser produced plasmas. Rough estimates of the field strength in the plasma, by numerical simulation, has shown that fields of the order of mega gauss are produced[9]. These intense fields can completely modify the spectrum of electrostatic modes in the laser produced plasma. At the same time, if the laser frequency is much greater than the electron cyclotron frequency, these magnetic fields will not significantly influence the propagation characteristics of the incident and scattered EM waves. It is, therefore, conceivable that in a magnetized plasma stimulated scattering can occur from a large number of ES modes and it is important to assess the relative potentialities of each one of them. In this chapter we shall discuss the scattering of the incident EM wave off various electrostatic modes occurring in magnetized plasma. The low frequency ES modes which we have considered are (1) the Bernstein modes (2) the lower hybrid modes and (3) the drift modes. We have calculated the threshold powers and the maximum growth rates for scattering off each of these modes.

These growth rates correspond to usual idealized infinite medium linear theory and for a realistic situation we must consider the stabilizing mechanisms present in the system. On the level of linear theory we may consider three stabilizing effects, viz. (1) damping of the product waves, (2) finite pump wave band width and (3) plasma inhomogeneity. The effect of damping of decay waves is to stabilize the decay if $\nu_1 \nu_2 > \gamma_0^2$ where ν_1 and ν_2 are the damping rates of the decay products (in the present case the low frequency ES mode and the scattered EM wave) and γ_0 is the growth rate obtained in the idealized case. If the damping is classical (i.e. collisional and Landau damping) then this does not appear to be an important mechanism of stabilization for laser fusion conditions. Also if γ_0 is greater than the band width $\delta\omega$ then the finite band width effect is not important for stabilization.

Thus the principal linear stabilization mechanism appears to result from the inhomogeneity effect. If the plasma is infinite but inhomogeneous, then the matching conditions $\omega_0 = \omega_1 + \omega_2$ and $\underset{\sim}{k}_0 = \underset{\sim}{k}_1 + \underset{\sim}{k}_2$ can be satisfied only over a finite region, as the frequencies and the wave vectors of the interacting waves depend on the plasma density through the dispersion relation of the waves. As a wave travels away from the point of perfect matching, it grows until the phase mismatch is so large that the proper phasing for growth is lost. The wave equation has a turning point there and the wave propagates without growth from there on. A turning point occurs when the phase mismatch $\int K dx \simeq K l_t$ is of order 1, where $K = k_0 - k_1 - k_2$ is the phase mismatch and l_t is the length of the interacting region. Thus the decay must occur before the product waves have time to convect out of the interaction region. A detailed consideration[10] shows that this results in a growth of fluctuations by a factor e^{α} where $\alpha = 2\pi\gamma_0^2/V_1 V_2 K'$ is the amplification factor; V_1 and V_2 are the group velocities of the decay products and $K' = dK/dx$.

Liu[11] et al studied SRS and SBS in an inhomogeneous plasma and calculated the amplification factors for these scattering processes. In this chapter we have calculated the amplification factors for the different scattering processes in a magnetized plasma mentioned above.

In section 2.2 we shall briefly discuss the general dispersion relation for scattering of an EM wave off a ES mode. In section 2.3 we shall outline the derivation of the amplification factor α. In section 2.4 we shall present estimates[12] of threshold powers, maximum growth rates and amplification factors for the instabilities under consideration. Section 2.5 gives a discussion of our results and compares them with those obtained by other authors for similar calculations.

2.2 Derivation of the General Dispersion Relation.

Let us consider a large amplitude, plane polarized EM pump wave

$$\underset{\sim}{E}_0 = 2E_0\hat{e}_0\cos(\underset{\sim}{k}_0\cdot\underset{\sim}{x} - \omega_0 t) \tag{2.1}$$

propagating in a homogeneous plasma. We assume (ω_0, k_0) to satisfy the linear dispersion relation,

$$\omega_0^2 = \omega_{pe}^2 + c^2k_0^2 \tag{2.2}$$

In equilibrium, electrons oscillate with high velocity in the incident electric field $\underset{\sim}{E}_0$ with the ions forming a stationary background. If there is a density perturbation $(\omega, \underset{\sim}{k})$ associated with an electrostatic wave, then the electron density perturbation δn_e will be driven by $\underset{\sim}{E}_0$ leading to currents at $\omega \pm l\omega_0$ and $\underset{\sim}{k} \pm l\underset{\sim}{k}_0$ where l is an integer. These currents will generate mixed ES-EM side band modes at $\omega \pm l\omega_0, \underset{\sim}{k} \pm l\underset{\sim}{k}_0$ which will in turn interact with the pump wave field producing a ponderomotive bunching force $\sim\underset{\sim}{\nabla}E^2$ that amplifies the density perturbation. Thus there is a positive feedback system that will lead to instability of the original density perturbation and the side band modes.

The Fourier transformed wave equation for the side band modes $\omega_{\pm} = \omega \pm \omega_0$, $\underset{\sim}{k}_{\pm} = \underset{\sim}{k} \pm \underset{\sim}{k}_0$ can be written as

$$\left[\left(k_{\pm}^2 - \frac{\omega_{\pm}^2}{c^2}\right)\underset{\approx}{I} - \underset{\sim}{k}_{\pm}\underset{\sim}{k}_{\pm}\right] \cdot \underset{\sim}{E}_{\pm} = \frac{4\pi i \omega_{\pm}}{c^2}\underset{\sim}{J}_{\pm} \qquad (2.3)$$

where we have considered the lowest order coupling $\ell = 1$ which is justified when $eE_0/m\omega_0 c \ll 1$ (here e is the unit charge, c is the velocity of light). In equation (2.3) $\underset{\approx}{I}$ is the unit dyadic and $\underset{\sim}{E}_{\pm} \equiv \underset{\sim}{E}(\omega_{\pm}, \underset{\sim}{k}_{\pm})$. The current density perturbation $\underset{\sim}{J}_{\pm}$ arises from the linear response of electrons to $\underset{\sim}{E}_{\pm}$ and from the coupling between the oscillating velocity produced by the pump and the electron density perturbation produced by the electrostatic wave. Substituting these terms for $\underset{\sim}{J}_{\pm}$ in equation (2.3) and rearranging we get,

$$\left[\left(k_{\pm}^2 - \frac{\omega_{\pm}^2}{c^2}\epsilon_{\pm}\right)\underset{\approx}{I} - \underset{\sim}{k}_{\pm}\underset{\sim}{k}_{\pm}\right] \cdot \underset{\sim}{E}_{\pm} = -\frac{\omega_{pe}^2\, \delta n_e(\omega, \underset{\sim}{k})}{c^2 n_0}\underset{\sim}{E}_{0\pm} \qquad (2.4)$$

where $\epsilon_{\pm} = 1 - \omega_{pe}^2/\omega_{\pm}^2$ is the linear dielectric constant at frequencies $\omega_{\pm}$, $\omega_{pe} = (4\pi e^2 n_0/m)^{1/2}$ is the plasma frequency, n_0 is the equilibrium density and $\underset{\sim}{E}_{0\pm} = \hat{e}_0 E(\pm\omega_0, \pm k_0)$ refer to the components of the pump wave. We have assumed here $\omega_0 \gg \omega$ and $\omega_0/\omega_{pe} \gg 1$ (or if ω_0/ω_{pe} is arbitrary then $k\lambda_D \ll 1$ where λ_D is the Debye wavelength). Inverting equation (2.4) we get,

$$-\underset{\sim}{E}_{\pm} = \omega_{pe}^{2} \frac{\delta n_e(\underset{\sim}{k},\omega)}{n_0}\left[\left(\underset{\approx}{I} - \frac{\underset{\sim}{k}_{\pm}\underset{\sim}{k}_{\pm}}{k_{\pm}^{2}}\right)\Big/ D_{\pm} - \frac{\underset{\sim}{k}_{\pm}\underset{\sim}{k}_{\pm}}{k_{\pm}^{2}\omega_{\pm}^{2}\epsilon_{\pm}}\right]\cdot \underset{\sim}{E}_{0\pm} \tag{2.5}$$

where $D_{\pm} = k_{\pm}^{2}c^{2} - \omega_{\pm}^{2}\epsilon_{\pm} = c^{2}k^{2} \pm 2\underset{\sim}{k}\cdot\underset{\sim}{k}_0 c^{2} \mp 2\omega\omega_0 - \omega^{2}$.

$$\tag{2.6}$$

In order to calculate $\delta n_e(\underset{\sim}{k},\omega)$ we first calculate the low frequency force F_ω on the electrons, due to the presence of two high frequency waves, which is given by the gradient of the ponderomotive potential Ψ_ω i.e.

$$\underset{\sim}{F}_{\omega} = -\underset{\sim}{\nabla}\Psi_\omega = -\left(e^{2}/m\omega_0^{2}\right)\underset{\sim}{\nabla}\left(\underset{\sim}{E}_{0+}\cdot\underset{\sim}{E}_{-} + \underset{\sim}{E}_{0-}\cdot\underset{\sim}{E}_{+}\right) \tag{2.7}$$

Inserting this force term in the Vlasov equation and solving it for electron density perturbation we get

$$\delta n_e(\underset{\sim}{k},\omega) = \frac{\chi_e(\underset{\sim}{k},\omega)}{4\pi e}\, i\underset{\sim}{k}\cdot\left[\underset{\sim}{E}(\underset{\sim}{k},\omega) + \frac{i\underset{\sim}{k}}{e}\Psi(\underset{\sim}{k},\omega)\right] \tag{2.8}$$

where $\chi_e(\underset{\sim}{k},\omega)$ and $\underset{\sim}{E}(\underset{\sim}{k},\omega)$ denote the electron susceptibility and the self consistent electric field respectively. We can calculate the ion density perturbation in the same way but without the ponderomotive force since it is smaller than the electron term by the mass ratio. Inserting this expression for the ion perturbation into Poisson's equation, we can solve for the electron density fluctuation

$$\delta n_e(\underset{\sim}{k},\omega) = -\frac{1+\chi_i(\underset{\sim}{k},\omega)}{4\pi e}\, i\underset{\sim}{k}\cdot\underset{\sim}{E}(\underset{\sim}{k},\omega). \tag{2.9}$$

where $\chi_i(\underset{\sim}{k},\omega)$ denotes the ion susceptibility. One can eliminate $\underset{\sim}{E}(\underset{\sim}{k},\omega)$ between equations (2.8) and (2.9) and use equation (2.7) to get

$$\delta n_e(\underset{\sim}{k},\omega) = -\frac{k^2}{4\pi m\omega_0^2}\frac{\chi_e(\underset{\sim}{k},\omega)}{\epsilon(\underset{\sim}{k},\omega)}\left(1+\chi_i(\underset{\sim}{k},\omega)\right)$$
$$\times\left(\underset{\sim}{E}_{0+}\cdot\underset{\sim}{E}_- + \underset{\sim}{E}_{0-}\cdot\underset{\sim}{E}_+\right) \tag{2.10}$$

using equations (2.5) and (2.10) we get the general dispersion relation,

$$\frac{1}{\chi_e(\underset{\sim}{k},\omega)} + \frac{1}{1+\chi_i(\underset{\sim}{k},\omega)} = k^2\left[\frac{|\underset{\sim}{k}_-\times\underset{\sim}{V}_0|^2}{k_-^2 D_-} - \frac{|\underset{\sim}{k}_-\cdot\underset{\sim}{V}_0|^2}{k_-^2\omega_-^2\epsilon_-}\right.$$
$$\left. + \frac{|\underset{\sim}{k}_+\times\underset{\sim}{V}_0|^2}{k_+^2 D_+} - \frac{|\underset{\sim}{k}_+\cdot\underset{\sim}{V}_0|^2}{k_+^2\omega_+^2\epsilon_+}\right] \tag{2.11}$$

where $V_0 = eE_0/m\omega_0$.

This dispersion relation describes the parametric coupling of a low frequency electrostatic mode at $(\omega, \underset{\sim}{k})$ and two high frequency mixed ES-EM side bands at $(\omega\pm\omega_0, \underset{\sim}{k}\pm\underset{\sim}{k}_0)$. The $\underset{\sim}{k}\times\underset{\sim}{V}_0$ terms in equation (2.11) arise from the EM components of the side band modes and the $\underset{\sim}{k}\cdot\underset{\sim}{V}_0$ terms from the ES components, when $\omega_0\sim\omega_{pe}$, $\epsilon_\pm\simeq 0$ and the side bands are predominantly electrostatic. If $k\lambda_D \ll 1$ then this reduced form of equation (2.11) correctly describes the parametric excitation of two ES waves by an incident EM wave. On the other hand when $D_\pm\simeq 0$, we find $\epsilon_\pm\neq 0$. In this case, the side bands are predominantly EM and this represents the scattering process where the EM pump wave

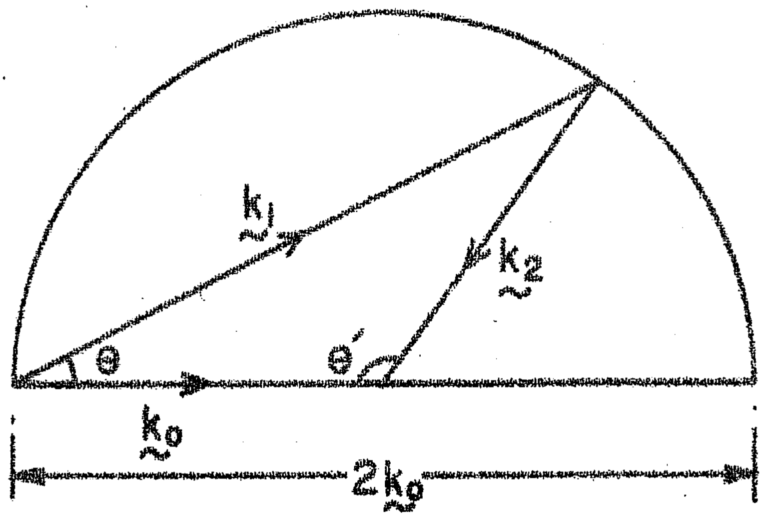

Figure 2.1 - Wave vector diagram for non forward scattering. Here $\underset{\sim}{k}_0$, $\underset{\sim}{k}_1$ and $\underset{\sim}{k}_2$ $(= \underset{\sim}{k}_0 - \underset{\sim}{k}_1)$ are the EM pump wave vector, electrostatic wave vector and the scattered EM wave vector respectively.

excites an ES wave and new EM waves at shifted frequencies. Let us consider a case where $D_- \approx 0$ and $D_+ \neq 0$ i.e. the Stoke's components are resonant and anti-Stoke's components are non-resonant. In such a situation equation (2.11) reduces to

$$\frac{1}{\chi_e(\omega, \underset{\sim}{k})} + \frac{1}{1+\chi_i(\omega, \underset{\sim}{k})} = k^2 \frac{|\underset{\sim}{k} \times V_0|^2}{k_-^2 D_-} \tag{2.12}$$

Equation (2.12) is valid for $\omega \ll c^2 \underset{\sim}{k} \cdot \underset{\sim}{k}_0 / \omega_0$ and breaks down for small $\underset{\sim}{k}$ or if $\underset{\sim}{k} \perp \underset{\sim}{k}_0$. For $\omega_0 \gg \omega$ we may write

$$D(\omega_-, \underset{\sim}{k}_-) = c^2 k_-^2 - \omega_-^2 + \omega_{pe}^2 \simeq 2\omega_0 \left(\omega - \frac{c^2 \underset{\sim}{k} \cdot \underset{\sim}{k}_0}{\omega_0} + \frac{c^2 k^2}{2\omega_0} \right) \tag{2.13}$$

The last two terms in the parenthesis are of order ω_0 and therefore must roughly cancel for D_- to be small. This condition implies that $k \simeq 2k_0 \cos\theta$, where θ is the angle between $\underset{\sim}{k}$ and $\underset{\sim}{k}_0$. Figure 2.1 expresses this result in a geometrical form. Thus substituting $2k_0 \cos\theta$ for k everywhere except D_- in equation (2.12) we get

$$\frac{1}{\chi_e(\underset{\sim}{k}, \omega)} + \frac{1}{1+\chi_i(\underset{\sim}{k}, \omega)} = \frac{2k_0^2 V_0^2}{\omega_0(\omega - \Delta\omega)} \Psi^2(\theta, \phi) \tag{2.14a}$$

where
$$\Psi = |\sin\phi| \cos\theta . \tag{2.14b}$$

$$\sin^2\phi = |\underset{\sim}{k} \times \underset{\sim}{V}_0|^2 / k^2 V_0^2 \tag{2.14c}$$

$$\Delta\omega = c^2 \underset{\sim}{k}\cdot\underset{\sim}{k}_0/\omega_0 - c^2k^2/2\omega_0 = \underset{\sim}{k}\cdot\underset{\sim}{v}_g - \delta \tag{2.14d}$$

$$\underset{\sim}{v}_g = c^2\underset{\sim}{k}_0/\omega_0 \ ; \qquad \delta = c^2k^2/2\omega_0 \tag{2.14e}$$

The dispersion relation (2.14a) is quite general and the basic form of this equation will not change when a magnetic field or density, temperature gradients are introduced provided these additions only influence the low frequency modes at frequency ω and leave the higher frequency modes unaffected. The correct dispersion relation can be obtained by substituting the appropriate electron and ion susceptibility tensors $\underset{\approx}{\chi}_e$ and $\underset{\approx}{\chi}_i$ in equation (2.14a). Equation (2.14a) can be rewritten as

$$\frac{(\omega - \Delta\omega)}{\omega_0}\epsilon_\ell = 2\chi_e(1+\chi_i)\frac{V_0^2}{c^2}\Psi^2 \tag{2.15}$$

The linear dielectric function $\epsilon_\ell(\omega_r + i\gamma)$ can be expanded near ω giving

$$\epsilon_\ell = i\gamma\left.\frac{\partial\epsilon_{\ell r}}{\partial\omega}\right|_{\omega=\omega_\ell} \tag{2.16}$$

Here ω_r is the real part of the frequency and γ is the growth rate. $\epsilon_{\ell r}$ is the real part of the dielectric function. Substituting equation (2.16) into (2.15) we get

$$(\gamma+\Gamma_\ell)(\gamma+\Gamma_-) = -2\chi_e(1+\chi_i)\frac{\omega_0 V_0^2}{c^2}\Psi^2\Big/\left(\frac{\partial\epsilon_{\ell r}}{\partial\omega}\right)_{\omega=\omega_\ell}. \tag{2.17}$$

where we have introduced Γ_ℓ and Γ_- to denote the phenomenological damping rates of the free ES and the free EM waves respectively. We have also taken $\omega = \omega_\ell + i\gamma$ and $\Delta\omega = \omega_\ell$

The maximum growth rate is obtained from equation (2.17) for $\gamma_0 \gg \Gamma_\ell, \Gamma_-$, and is given by

$$\gamma_0^2 = - \frac{2\chi_e(\omega)(1+\chi_i(\omega))\omega_0}{\left(\frac{\partial \epsilon_{\ell r}}{\partial \omega}\right)_{\omega=\omega_\ell}} \frac{V_0^2}{c^2} \Psi^2 \tag{2.18}$$

The threshold power is readily obtained by setting $\gamma = 0$ in equation (2.17).

$$\frac{V_{OT}^2}{c^2} = -\frac{1}{2\Psi^2} \frac{\Gamma_\ell}{\omega_\ell} \frac{\Gamma_-}{\omega_0} \left(\frac{\partial \omega_\ell \epsilon_{\ell r}}{\partial \omega}\right)_{\omega=\omega_\ell} \frac{1}{\chi_e(1+\chi_i)} \tag{2.19}$$

In section (2.4) we shall present estimates of maximum growth rates and threshold powers for different ES modes, in a magnetized plasma, using equations (2.18) and (2.19) respectively.

2.3 Derivation of the amplification factor α.

In an inhomogeneous plasma, the $\underset{\sim}{k}$-matching condition can holdonly locally and the mismatch $K = k_{0x}(x) - k_{1x}(x) - k_{2x}(x)$ develops due to the spatially dependent quantities e.g. density, temperature which occur in the dispersion relations determining $k(\omega, x)$. This mismatch then localizes the region of resonant interaction.

For simplicity, we consider a plasma slab with density gradient in the x-direction. In the weak pump case, the equations for the amplitudes a_1, a_2 of the decay waves in an inhomogeneous medium are,

$$\frac{\partial a_1}{\partial t} + V_1 \frac{\partial a_1}{\partial x} = \gamma_0 a_2^* \exp\left(i\int_0^x K\,dx\right) \qquad (2.20)$$

and

$$\frac{\partial a_2^*}{\partial t} + V_2 \frac{\partial a_2^*}{\partial x} = \gamma_0 a_1 \exp\left(-i\int_0^x K\,dx\right) \qquad (2.21)$$

The coupling factor γ_0 is taken to be the growth rate in the absence of damping for the homogeneous medium. Laplace transforming in time and neglecting initial values, eliminating a_2^* and putting

$$a_1 = \xi \exp\left[i\int_0^x \tfrac{1}{2}K\,dx - \tfrac{1}{2}p\left(\frac{1}{V_1} + \frac{1}{V_2}\right)x\right] \qquad (2.22)$$

we easily find that

$$\frac{\partial^2 \xi}{\partial x^2} + \left[\frac{1}{4}\left\{K - ip\left(\frac{1}{V_1} - \frac{1}{V_2}\right)\right\}^2 + \frac{i}{2}\frac{dK}{dx} - \frac{\gamma_0^2}{V_1 V_2}\right]\xi = 0 \qquad (2.23)$$

where p is the Laplace transformed variable.

Equation (2.23) is an eigen value equation for ξ with p its eigen value. If there is a well behaved solution with $\mathrm{Re}\,p_0 > 0$ then p_0 will correspond to the eigen value for a temporally growing mode with growth rate given by $\mathrm{Re}\,p_0$. This mode is generally localized within certain regions and grows exponentially in time until some non linear mechanism limits the growth. If no such p_0 exists, then only spatial amplification over a given source at the thermal level, is possible. By spatial amplification we mean that a thermal source, due to spontaneous emission of waves, can grow for a limited period of time and stops growing once a maximum level is reached. This level of growth eventually reached is limited principally by the duration of interaction, which in turn, is determined by the wave propagation out of the interaction region where the phase matching conditions are approximately satisfied. Since K increases with x, the possible behaviour at infinity is

$$a_I = \exp(-p/V_1)x$$

and
$$a_{II} = \exp\left[i\int_0^x K\,dx - (p/V_2)x\right] \tag{2.24}$$

for $V_1 V_2 > 0$ both solutions are badly behaved at either plus or minus infinity for $Re\,p > 0$ and no temporally growing modes are possible. With $V_1 V_2 < 0$ (decay modes propagating oppositely) normal modes with $Re\,p > 0$ are possible, provided the solutions well behaved at $\pm\infty$ can be joined at $x = 0$.

For $K = K'(0)x$ we make a transformation $K'(0)x - ip(1/V_1 - 1/V_2) = K'(0)\chi'$ reducing equation (2.23) to the parabolic cylinder equation. Since this equation has no well behaved solution for $Re\,p > 0$ only spatial amplification is possible. We choose $p = \epsilon$ where ϵ is a small positive number to give the proper behaviour at infinity. Considering $|\gamma_0^2/V_1 V_2| \gg K'$ for sizable amplification equation (2.23) can be written as

$$\frac{d^2\xi}{dx'^2} + \left(\frac{1}{4}K'x'^2 - \gamma_0^2/V_1V_2\right)\xi = \delta(x') \tag{2.25}$$

where we have put the source at $x' = 0$.

Let us consider the case $V_1V_2 > 0$ then from the boundary conditions we know that $\xi = 0$ for $x' < 0$ (since the amplification takes place from $x' = 0$). Beyond the turning point $x_t = 2\gamma_0/K'(V_1V_2)^{1/2}$ the solution is oscillatory while between 0 and x_t it has the approximate form $\sinh\int_0^{x_t}[\gamma_0^2/V_1V_2 - K'^2x'^2/4]^{1/2}dx$ Thus there is a net e-folding given by

$$\int_0^{x_t}\left[\gamma_0^2/V_1V_2 - \frac{1}{4}K'^2x'^2\right]^{1/2}dx' = \pi\gamma_0^2/2V_1V_2K' \tag{2.26}$$

Putting the source term at $-x_t$ we get for the e-folding of intensity as

$$I = I_0\exp\left(2\pi\gamma_0^2/V_1V_2K'\right) \tag{2.27}$$

where I_0 is the non-driven, thermal source intensity. For effective growth $\alpha = 2\pi\gamma_0^2/V_1V_2K' > 1$ which means that the wave must grow substantially during the time it propagates to the point where the phase mismatch is substantial. The same result can be obtained for $V_1V_2 < 0$ case also.

2.4 Calculation of growth rates, thresholds and amplification factors.

In this section we shall calculate the growth rates, threshold powers and amplification factors for different electrostatic modes using equations (2.18), (2.19) and (2.27). We take the geometry as follows: the magnetic field $\underset{\sim}{B}_0$ is in the z- direction; there is a density gradient along the x- direction; the pump wave is applied along the x- direction.

2.4.1 Bernstein modes.

The Bernstein modes are polarized with their electric vector nearly parallel to wave vector $\underset{\sim}{k}$ and they are almost pure longitudinal waves. These waves are the counterpart of the field free plasma waves $\omega = \omega_{pe}$ and $\omega = kC_s$ and in the limit of the magnetic field $\underset{\sim}{B}_0$ going to zero, they reduce to the Langmuir Oscillations at high frequency and to ion sound waves at low frequency. The Bernstein modes propagate in frequency ranges that lie between harmonics of the cyclotron frequency. The lowest frequency for which propagation occurs lies above the electron cyclotron frequency $\Omega_e (= eB_0/mc)$ i.e. $2\Omega_e > \omega > \Omega_e$. The exact location of the modes is a function of density, temperature and field strength. The general dispersion relation for the Bernstein modes is given by

$$k^2 = \sum_n \sum_\alpha \left[\frac{2n^2 \omega_{p\alpha}^2 \Omega_\alpha^2}{\omega^2 - n^2 \Omega_\alpha^2} \frac{m_\alpha}{T_\alpha} I_n\left(\frac{k^2 T_\alpha}{\Omega_\alpha^2 m_\alpha}\right) \exp\left(-\frac{k^2 T_\alpha}{\Omega_\alpha^2 m_\alpha}\right)\right] \tag{2.28}$$

where α denotes electrons or ions, n is an integer, T_α is the temperature measured in energy units, I_n is the modified Bessel's function of the first kind, $I_n(z) = \exp(-i\pi n/2) J_n(e^{i\pi/2} z)$. When the plasma temperature is low or when long waves are being studied, the Bessel function I_n can be expanded in powers of ka_α, a_α being the gyro-radius of species α. We shall consider here two special cases.

(i) Low density plasma

When the plasma density is low, all the Bernstein modes occur very close to cyclostron harmonics and the explicit solutions of equation (2.28) are

$$\omega^2 = n^2 \Omega_e^2 (1+\beta_n) \tag{2.29}$$

where $\beta_n = \dfrac{2\omega_{pe}^2}{\Omega_e^2} \dfrac{1}{k^2 a_e^2} I_n(k^2 a_e^2) \exp(-k^2 a_e^2)$

$$a_e^2 = T_e/m_e \Omega_e^2 \quad \text{and} \quad \omega_{pe}^2 \ll \Omega_e^2$$

For this mode, the ion and the electron susceptibilities are given by,

$$\begin{aligned} \chi_i(\omega) &\simeq 0 \\ \chi_e(\omega) &= -n^2 \Omega_e^2 (1+\beta_n)/\omega^2 \end{aligned} \tag{2.30}$$

Therefore

$$\chi_e(\omega_\ell) = -1 \tag{2.31}$$

The dielectric function ϵ is given by

$$\epsilon(\omega) = 1 - n^2\Omega_e^2(1+\beta_n)/\omega^2 \tag{2.32}$$

Therefore
$$\left(\frac{\partial\epsilon}{\partial\omega}\right)_{\omega=\omega_\ell} \simeq \frac{2}{n\Omega_e} \tag{2.33}$$

Substituting equations (2.30) to (2.33) in equations (2.18) and (2.19) we get the expressions for growth rate and the threshold power for the scattering of an incident pump wave off a Bernstein mode at low density as

$$\gamma_0^2 = V_0^2\omega_0\Omega_e\Psi^2/c^2 \tag{2.34}$$

and
$$\frac{V_{0T}^2}{c^2} = \frac{1}{\Psi^2}\frac{\Gamma}{\omega_0}\frac{\Gamma_\ell}{\Omega_e}. \tag{2.35}$$

where we have taken $n = 1$.

In order to calculate the amplification factor α we need to calculate the group velocities V_{1x} and V_{2x} of the electrostatic wave and scattered EM wave respectively and K' the derivative of the phase mismatch. From equation (2.29) we have

$$V_{1x} = \partial\omega/\partial k_{1x} = -\omega_{pe}^2 a_e^2 k_{1x}/\Omega_e \tag{2.36}$$

From the dispersion relation of the scattered EM wave viz.

$$\omega_2^2 = \omega_{pe}^2 + c^2k_2^2 \simeq c^2k_2^2 \tag{2.37}$$

we have
$$V_{2x} = \partial\omega_2/\partial k_{2x} = c^2k_{2x}/\omega_0 \tag{2.38}$$

Also equation (2.29) can be written as

$$\omega^2 = \Omega_e^2 + \omega_{pe}^2\left(1 - k_1^2 a_e^2\right),$$ from which

$$k_{1x} = \left[\left(\omega_{pe}^2 + \Omega_e^2 - \omega^2 - \omega_{pe}^2 a_e^2 k_{1y}^2\right)/\omega_{pe}^2 a_e^2\right]^{1/2} \tag{2.39}$$

using equation (2.37) we have for the scattered wave vector

$$k_{2x} = \left[\left(\omega_0^2 - \omega_{pe}^2 - c^2 k_{2y}^2\right)/c^2\right]^{1/2} \tag{2.40}$$

And for the pump wave we get

$$k_{0x} = \left[\left(\omega_0^2 - \omega_{pe}^2\right)/c^2\right]^{1/2} \tag{2.41}$$

Substituting equations (2.39) to (2.41) in the expression

$$K' = \frac{d}{dx}\left(k_{0x} - k_{1x} - k_{2x}\right) \tag{2.42}$$

we have

$$K' = \frac{\omega_{pe}^2 \sin^2\theta'/2}{L_n c^2 k_{0x}\cos\theta'}\left\{1 + \frac{\omega_0^2}{\omega_{pe}^2}\cos\theta' - \frac{c^2(1 - a_e^2 k_{1y}^2)\cos\theta'}{4\omega_{pe}^2 a_e^2 \sin^4\theta'/2}\right\} \tag{2.43}$$

where θ' is the angle between $\underset{\sim}{k}_0$ and $\underset{\sim}{k}_2$ and we have written $L_n = (1/\omega_{pe}^2)(d\omega_{pe}^2/dx)$. Substituting equations (2.34), (2.36), (2.38) and (2.43) in the expression

$$\alpha = 2\pi\gamma_0^2 / V_{1x}V_{2x}K' \tag{2.44}$$

we get for the spatial amplification factor

$$\alpha = \frac{\pi V_0^2 \sin^2\phi\, k_0 L_n \Omega_e^4}{v_e^2 \sin^2\theta'/2\, \omega_{pe}^4} \Big/ \left\{1 + \frac{\omega_0^2\cos\theta'}{\omega_{pe}^2} - \frac{c^2(1 - a_e^2 k_{1y}^2)\cos\theta'}{4\omega_{pe}^2 a_e^2 \sin^4\theta'/2}\right\} \tag{2.45}$$

where $v_e = (T_e/m)^{1/2}$ is the electron thermal speed.

We shall now consider two special cases:

Case (a) : When $\theta' = 90^0$ i.e. for side scattering equation (2.45) becomes

$$\alpha_{90^0} = 2\pi\left(\frac{V_0}{v_e}\right)^2 (k_0 L_n)\left(\frac{\Omega_e}{\omega_{pe}}\right)^4 \sin^2\phi \tag{2.46}$$

Case (b) : When $\theta' = 180^0$ i.e. for back scattering equation (2.45) becomes

$$\alpha_{180^0} = 4\pi\left(\frac{V_0}{c}\right)^2 (k_0 L_n)\left(\frac{\Omega_e}{\omega_{pe}}\right)^2 \sin^2\phi \tag{2.47}$$

(ii) High density plasma

When the plasma density is large and magnetic field strength is small, the lowest frequency mode is given by,

$$\omega_1 = 2\Omega_e\left(1 - \frac{3}{8}\frac{\omega_{pe}^2}{\Omega_e^2}\frac{k^2 T_e}{m}\Big/(\omega_{pe}^2 - 3\Omega_e^2)\right) \tag{2.48}$$

for $\omega_{pe}^2 > 3\Omega_e^2$ and $k^2 a_e^2 \ll 1$.

For this mode the ion and the electron susceptibilities are given by,

$$\chi_i(\omega) \simeq 0\,;\quad \chi_e(\omega) = -\frac{2\Omega_e}{\omega}\left(1 - \frac{\omega_{pe}^2}{(\omega_{pe}^2 - 3\Omega_e^2)}\frac{3}{16}k^2 a_e^2\right) \tag{2.49}$$

Therefore $$\chi_e(\omega_1) = -1 . \tag{2.50}$$

The dielelectric function ϵ is given by

$$\epsilon(\omega) = 1 - \frac{2\Omega_e}{\omega}\left(1 - \frac{\omega_{pe}^2}{(\omega_{pe}^2 - 3\Omega_e^2)}\frac{3}{16}k^2 a_e^2\right) \tag{2.51}$$

Therefore $$\left(\frac{\partial\epsilon}{\partial\omega}\right)_{\omega=\omega_l} = \frac{1}{2\Omega_e} \qquad (2.52)$$

using equations (2.49) to (2.52) we get the following growth rate and the threshold power for the scattering of an incident EM wave off a Bernstein mode at high density,

$$\gamma_0^2 = 2V_0^2\omega_0\Omega_e\Psi^2/c^2 \qquad (2.53)$$

and $$\frac{V_{0T}^2}{c^2} = \frac{1}{2\Psi^2}\frac{\Gamma}{\omega_0}\frac{\Gamma_l}{\omega_l} \qquad (2.54)$$

Next we shall calculate the spatial amplification factor α for this case. The group velocity V_{1x} is given by

$$V_{1x} = -3\Omega_e\omega_{pe}^2 k_{1x}a_e^2/4\left(\omega_{pe}^2 - 3\Omega_e^2\right) \qquad (2.55)$$

The wave vector k_{1x} is calculated to be

$$k_{1x} = \sqrt{\frac{8}{3\Omega_e}}\frac{1}{a_e}\left[2\Omega_e\left(1-\frac{3\Omega_e^2}{\omega_{pe}^2}\right) - \omega_1\left(1-\frac{3\Omega_e^2}{\omega_{pe}^2}\right) - \frac{3}{8}\Omega_e k_{1y}^2 a_e^2\right]^{1/2} \qquad (2.56)$$

Substituting equations (2.40), (2.41) and (2.56) in equation (2.42) we get,

$$K' = \frac{\omega_{pe}^2\sin^2\theta/2}{L_n c^2 k_{0x}\cos\theta'}\left\{1 - \frac{3\Omega_e^2\omega_0^2\cos\theta'}{\omega_{pe}^2\sin^2\frac{\theta'}{2}\left(\omega_{pe}^2 - 3\Omega_e^2\right)}\right\} \qquad (2.57)$$

Substituting equations (2.38), (2.53), (2.55) and (2.57) in equation (2.44) we get the following expression for the spatial amplification factor,

$$\alpha = \frac{16\pi V_0^2\omega_0\sin^2\phi\left(\omega_{pe}^2 - 3\Omega_e^2\right) cL_n}{3c^2 2\omega_{pe}^4 a_e^2\sin^2\frac{\theta'}{2}\left\{1 - 3\Omega_e^2\omega_0^2/\omega_{pe}^2\sin^2\frac{\theta'}{2}\left(\omega_{pe}^2 - 3\Omega_e^2\right)\right\}} \qquad (2.58)$$

Thus for side scattering we get,

$$\alpha_{90^\circ} = \frac{16\pi}{3}\left(\frac{V_0}{v_e}\right)^2 k_0 L_n \left(\frac{\Omega_e}{\omega_{pe}}\right)^2 \frac{(\omega_{pe}^2 - 3\Omega_e^2)}{\omega_{pe}^2} \sin^2\phi \quad (2.59)$$

And for back scattering we get

$$\alpha_{180^\circ} = \frac{8\pi}{9}\left(\frac{V_0}{v_e}\right)^2 k_0 L_n \frac{(\omega_{pe}^2 - 3\Omega_e^2)^2}{\omega_{pe}^2 \omega_0^2} \sin^2\phi \quad (2.60)$$

2.4.2 Lower hybrid waves:

These electrostatic waves propagate almost perpendicular to the magnetic field $\underset{\sim}{B_0}$ having frequency lying between electron cyclotron and ion cyclotron frequencies. They are known as hybrid waves because the frequency of these waves depends both on the plasma density and the magnetic field. When the phase velocity of lower hybrid wave along the magnetic field is greater than the electron thermal velocity, ie. for cold plasma approximation, the dispersion relation for the lower hybrid mode is given by,

$$\omega^2 = \frac{\omega_{pi}^2}{1 + \omega_{pe}^2/\Omega_e^2}\left(1 + \frac{k_z^2}{k^2}\frac{M}{m}\right) \quad (2.61)$$

where $\omega_{pi} = (4\pi e^2 n_0/M)^{1/2}$ is the ion plasma frequency and k_z refers to the wave vector parallel to the magnetic field such that $k_z \ll k$. Ion and electron susceptibilities are given by

$$\chi_i(\omega) = -\frac{\omega_{pi}^2}{\omega^2}\ ; \ \chi_e(\omega) = \frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pe}^2}{\omega^2}\frac{k_z^2}{k^2}, \quad (2.62),(2.63)$$

The dielectric function ϵ is given by

$$\epsilon(\omega) = 1 - \frac{\omega_{pe}^2}{\omega^2}\beta + \frac{\omega_{pe}^2}{\Omega_e^2} \qquad (2.64)$$

where $\beta = m/M + k_z^2/k^2$, and we have assumed, $\Omega_e^2 \gg \omega^2 \gg \Omega_i^2$. From equation (2.64) we have

$$\left(\frac{\partial \epsilon}{\partial \omega}\right)_{\omega=\omega_\ell} = \frac{2}{\omega_{pe}\beta^{1/2}}\left(1 + \omega_{pe}^2/\Omega_e^2\right)^{3/2} \qquad (2.65)$$

Substituting equations (2.62), (2.63) and (2.65) into equations (2.18) and (2.19) we get, after some algebra, the following expressions for growth rate and the threshold power for the scattering of an incident EM wave off a lower hybrid wave as,

$$\gamma_0^2 = \frac{V_0^2}{c^2}\Psi^2\omega_0\omega_{pe}\left(\frac{k_z^2}{k^2} - \frac{\omega_{pi}^2}{\Omega_e^2}\right)^2 \Big/ \left(1 + \frac{\omega_{pe}^2}{\Omega_e^2}\right)^{3/2} \times \left(\frac{k_z^2}{k^2} + \frac{m}{M}\right)^{3/2} \qquad (2.66)$$

$$\frac{V_{0T}^2}{c^2} = \frac{1}{\Psi^2}\frac{\Gamma}{\omega_0}\frac{\Gamma_\ell}{\omega_{pe}}\left(1 + \frac{\omega_{pe}^2}{\Omega_e^2}\right)^{3/2}\left(\frac{k_z^2}{k^2} + \frac{m}{M}\right)^{3/2} \Big/ \left(\frac{k_z^2}{k^2} - \frac{\omega_{pi}^2}{\Omega_e^2}\right)^2 \qquad (2.67)$$

For $\Omega_e^2 \gg \omega_{pe}^2$ and $\frac{k_z^2}{k^2}\frac{M}{m} \gg 1$, equations (2.66) and (2.67) become

$$\gamma_0^2 = \frac{V_0^2}{c^2}\omega_0\omega_{pe}\frac{k_z}{k}\Psi^2 \qquad (2.66a)$$

$$\frac{V_{0T}^2}{c^2} = \frac{1}{\Psi^2}\frac{\Gamma}{\omega_0}\frac{\Gamma_\ell}{\omega_{pe}}\frac{k}{k_z} \qquad (2.67a)$$

Next we shall calculate the amplification factor for the lower hybrid mode. The group velocity V_{1x} is given by

$$V_{1x} = \frac{\partial\omega}{\partial k_{1x}} = -\frac{k_z^2 k_{1x}}{k_1^4} \frac{\omega_{pe}^2}{1+\omega_{pe}^2/\Omega_e^2} \frac{1}{\omega_{LH}} \tag{2.68}$$

where $$\omega_{LH}^2 = \omega_{pe}^2\beta \Big/ \left(1+\omega_{pe}^2/\Omega_e^2\right) \tag{2.69}$$

Again from equation (2.61) we have,

$$k_{1x} = \left\{ \frac{k_z^2\omega_{pe}^2}{\omega^2\left(1+\omega_{pe}^2/\Omega_e^2\right)-\omega_{pi}^2} - k_{1y}^2 \right\}^{1/2} \tag{2.70}$$

Substituting equation (2.40), (2.41) and (2.70) in (2.42) we get,

$$K = \frac{\omega_{pe}^2 \sin^2\theta'/2}{L_n c^2 k_{0x}} \left\{ \frac{1}{\cos\theta'} - \frac{\omega_0^2 \eta}{\omega_{pe}^2 \sin^2\theta'/2} \right\} \tag{2.71}$$

where $$\eta = 1 - \frac{k^2}{k_z^2}\left(\frac{\omega^2}{\Omega_e^2} - \frac{m}{M}\right)$$

Now substituting equations (2.38), (2.66), (2.68) and (2.71) into equation (2.44) we get for the amplification factor for the lower hybrid mode as,

$$\alpha = \frac{4\pi V_0^2}{c^2} \frac{\left(k_z^2/k^2 - \omega_{pi}^2/\Omega_e^2\right)^2 c k_0^2 L_n \omega_0 \sin^2\phi\, k_1^2/k_z^2}{\omega_{pe}^2\left(1+\frac{\omega_{pe}^2}{\Omega_e^2}\right)\left(\frac{k_z^2}{k^2}+\frac{m}{M}\right)\left(1-\frac{\omega_0^2\cos\theta'\eta}{\omega_{pe}^2\sin^2\theta'/2}\right)} \tag{2.72}$$

For side scattering equation (2.72) becomes,

$$\alpha_{90^\circ} = \frac{4\pi V_0^2}{c^2}\frac{\left(k_z^2/k^2 - \omega_{pi}^2/\Omega_e^2\right)^2 k_0 L_n}{\left(1+\omega_{pe}^2/\Omega_e^2\right)\left(k_z^2/k^2 + m/M\right)}\frac{\omega_0^2}{\omega_{pe}^2}\frac{k_1^2}{k_z^2}\sin^2\phi. \tag{2.73}$$

And for back scattering equation (2.72) becomes,

$$\alpha_{180^\circ} = \frac{4\pi V_0^2}{c^2}\frac{\left(k_z^2/k^2 - \omega_{pi}^2/\Omega_e^2\right)^2 k_0 L_n}{\left(1 + k^2 m/k_z^2 M\right)^2}\frac{k_1^4}{k_z^4}\sin^2\phi \tag{2.74}$$

For $\Omega_e^2 \gg \omega_{pe}^2$ and $k_z^2 M/k^2 m \gg 1$, we get the following expressions for α_{90° and α_{180° as

$$\alpha_{90^\circ} = \frac{4\pi V_0^2}{c^2}\left(k_0 L_n\right)\left(\omega_0^2/\omega_{pe}^2\right)\sin^2\phi. \tag{2.75}$$

and

$$\alpha_{180^\circ} = \frac{4\pi V_0^2}{c^2}\left(k_0 L_n\right)\sin^2\phi. \tag{2.76}$$

2.4.3 Drift waves

When the density inhomogeneity is perpendicular to the magnetic field, then drift waves propagate almost perpendicular to the magnetic field. The density inhomogeneity scale length L_n is assumed to be greater than the wavelength of perturbation. The phase parallel to the magnetic field is assumed to be greater than the ion thermal velocity v_i but smaller than the electron thermal velocity v_e. The frequency of the drift mode is given by,

$$\omega = \frac{\omega_* \beta'}{2-\beta'} \tag{2.77}$$

where $\omega_* = -\frac{k_{1y} T_e}{m \Omega_e L_n}$; $\beta' = 1 - b_i$; $b_i = \frac{k^2 v_i^2}{\Omega_i^2} \ll 1$.

For this mode the ion and the electron susceptabilities are

$$\chi_i(\omega) = \frac{k_D^2}{k^2}\left(1 - \frac{(\omega+\omega_*)\beta'}{\omega}\right) \tag{2.78}$$

so that $$\chi_i(\omega_\ell) = -k_D^2/k^2 \tag{2.79}$$

and $$\chi_e(\omega_\ell) = k_D^2/k^2 \tag{2.79a}$$

The dielectric function is given by

$$\epsilon = 1 + \frac{k_D^2}{k^2}\left(2 - \beta' - \frac{\omega_* \beta'}{\omega}\right); \quad k_D^2/k^2 \gg 1 \tag{2.80}$$

Thus we have

$$\left(\frac{\partial \epsilon}{\partial \omega}\right)_{\omega=\omega_\ell} = \left(k_D^2/k^2\right)\left(2-\beta'\right)/\omega_\ell \tag{2.81}$$

Substituting equations (2.79) to (2.81) into (2.18) and (2.19) we get the values of growth rate and threshold power as

$$\gamma_0^2 = \frac{V_0^2}{v_e^2}\frac{\omega_\ell}{\omega_0}\omega_{pe}^2 \sin^2\phi. \tag{2.82}$$

and $$\frac{V_{0T}^2}{c^2} = \frac{2}{\sin^2\phi}\frac{\Gamma}{\omega_0}\frac{\Gamma_\ell}{v_\ell}\frac{k_0^2}{k_D^2} \tag{2.83}$$

Now as the laser produced plasma is expanding with a blow off velocity U, which is comparable to the phase velocity of the drift wave, the frequency of the drift wave will be doppler shifted and can be written as

$$\omega_1 = \frac{\omega_* \beta'}{2-\beta'} - k_{1x} U(r) = \frac{k_{1y} v_d \beta'}{2-\beta'} - k_{1x} U(r) \qquad (2.84)$$

where $v_d = -T_e/m\Omega_e L_n$ is the drift velocity. The group velocity is given by

$$V_1 = -U(r) \qquad (2.85)$$

and $$k_{1x} = (\omega_1 - k_{1y} v_d)/U(r) \qquad (2.86)$$

Then K' is calculated to be

$$K' = \frac{\omega_{pe}^2 \sin^2\theta'/2}{L_n c^2 k_{0x}} \left\{ \frac{1}{\cos\theta'} + \frac{2\omega_0^2}{\omega_{pe}^2} \frac{L_n}{L_U} \right\} \qquad (2.87)$$

where $$L_U = \frac{1}{U} \frac{dU}{dx} \qquad (2.88)$$

Substituting equations (2.38), (2.82), (2.85) and (2.87) in equation (2.44) we get the amplification factor for the drift wave as

$$\alpha = 2\pi \frac{V_0^2}{v_e^2} \frac{\omega_1 L_n}{U} \frac{\sin^2\phi}{\sin^2\theta'/2} \Big/ \left\{ 1 + \frac{2\cos\theta' \omega_0^2}{\omega_{pe}^2} \frac{L_n}{L_U} \right\} \qquad (2.89)$$

For side scattering we have

$$\alpha_{90^\circ} = 4\pi \frac{V_0^2}{v^2} (k_0 L_n) \sin^2\phi \qquad (2.90)$$

and for back scattering we get,

$$\alpha_{180^\circ} = 2\pi \frac{V_0^2}{v_e^2} (k_0 L_U) \frac{\omega_{pe}^2}{\omega_0^2} \sin^2\phi \qquad (2.91)$$

2.5 Conclusion and discussion.

In this chapter we have calculated the threshold powers, maximum growth rates and amplification factors for scattering of an EM wave by three different electrostatic modes in a magnetized plasma. Our results are summarised in Table 2.1. Recently there has also been other calculations, done independently, in the same general area. Yu et al[13] have calculated growth rates and threshold powers for scattering of an EM wave by upper hybrid modes, lower hybrid modes and drift modes. Their results for lower hybrid and drift modes agree with those of our's. Lee[14] considered a two ion species plasma and calculated growth rates and threshold powers for backscattering of an EM wave from a ion ion hybrid wave and found that the threshold was much greater, and the growth rate much smaller, compared to stimulated scattering from upper and lower hybrid waves. Lee[15] also considered stimulated scattering of EM waves of circular polarization incident on a magnetized plasma from ion waves propagating parallel to the external magnetic field. He[16] also studied the stimulated scattering of EM ordinary waves from electron plasma waves at the upper hybrid frequency and found that the threshold intensity required for stimulated scattering was higher in a magnetized plasma than in an unmagnetized plasma and that it increased with increasing magnetic field.

In our calculations we have considered scattering from only one ion mode viz. drift modes. It is to be noted that the growth rates for scattering off most low frequency ion modes are comparable to each other. Another interesting feature is the very low threshold field for the hybrid and the Bernstein modes because of their weak damping. These modes however require long parallel wavelengths and might, therefore, be prevented by finite geometry effects.

It therefore appears unjustified to ignore the evolution of magnetized modes in laser plasma situations - especially while considering the non-linear saturation levels of the scattered electromagnetic waves. In conclusion we suggest that the inclusion of spontaneously generated magnetic field is essential for a realistic estimate of the stimulated scattering of laser beams in pellet fusion systems.

TABLE 2.1 - Growth rates, threshold powers, amplification factors for scattering of an EM pump wave off different ES modes.

Electrostatic Mode	Threshold Power V_{0T}^2/c^2	Maximum Growth Rate γ_0^2	Spatial Amplification factor α	
			Sidescatter $(\theta' = 90^\circ)$	Backscatter $(\theta' = 180^\circ)$
Drift Wave	$\frac{2}{\sin^2\phi} \frac{\Gamma}{\omega_0} \frac{\Gamma_\ell}{\omega_\ell} \frac{k_0^2}{k_D^2}$	$\frac{V_0^2}{v_e^2} \frac{\omega_\ell}{\omega_0} \omega_{pe}^2 \sin^2\phi$	$4\pi \frac{V_0^2}{v_e^2} (k_0 L_n) \times \sin^2\phi$	$2\pi \frac{V_0^2}{v_e^2} (k_0 L_u) \frac{\omega_{pe}^2}{\omega_0^2} \sin^2\phi$
Electron Bernstein Modes. (n = 1) (i) $\omega_{pe}^2 \ll \Omega_e^2$	$\frac{1}{\Psi^2} \frac{\Gamma}{\omega_0} \frac{\Gamma_\ell}{\Omega_e}$	$\frac{V_0^2}{c^2} \omega_0 \Omega_e \Psi^2$	$2\pi \frac{V_0^2}{v_e^2} (k_0 L_n) \times \left(\frac{\Omega_e}{\omega_{pe}}\right)^4 \sin^2\phi$	$4\pi \frac{V_0^2}{c^2} (k_0 L_n) \times \left(\frac{\Omega_e}{\omega_{pe}}\right)^2 \sin^2\phi$
(ii) $\omega_{pe}^2 > 3\Omega_e^2$	$\frac{1}{2\Psi^2} \frac{\Gamma}{\omega_0} \frac{\Gamma_\ell}{\Omega_e}$	$\frac{2V_0^2}{c^2} \omega_0 \Omega_e \Psi^2$	$\frac{16\pi}{3} \frac{(k_0 L_n)}{\omega_{pe}^2 - 3\Omega_e^2} (\Omega_e \omega_0/\omega_{pe})^2 P$	$\frac{8\pi}{9} (k_0 L_n) P$

(Table continued)

Lower Hybrid Waves.	$\frac{1}{\Psi^2}\frac{\Gamma}{\omega_0}\frac{\Gamma_i}{\omega_{pe}}\cdot\frac{A^{3/2}}{Q^2}$	$\frac{V_0^2}{c^2}\omega_0\omega_{pe}\frac{Q^2}{A^{3/2}}\Psi^2$	$4\pi\frac{V_0^2}{c^2}\frac{Q^2}{A}(k_0L_n)$ $\frac{\omega_0^2}{\omega_{pe}^2}\frac{k^2}{k_z^2}\sin^2\phi$	$4\pi\frac{V_0^2}{c^2}k_0L_n\frac{Q^2}{2A^2}$ $\times\left(1+\frac{\omega_{pe}^2}{\Omega_e^2}\right)\sin^2\phi$
$\Omega_e^2 \gg \omega_{pe}^2$ $\frac{k_z^2}{k^2} \gg \frac{m}{M}$	$\frac{1}{\Psi^2}\frac{\Gamma}{\omega_0}\frac{\Gamma_i}{\omega_{pe}}\cdot\frac{k}{k_z}$	$\frac{V_0^2}{c^2}\omega_0\omega_{pe}\frac{k_z}{k}\Psi^2$	$4\pi\frac{V_0^2}{c^2}(k_0L_n)\frac{\omega_0^2}{\omega_{pe}^2}$ $\sin^2\phi$	$4\pi\frac{V_0^2}{c^2}(k_0L_n)\sin^2\phi$
Remarks	$L_n = \frac{1}{\omega_{pe}^2}\frac{d}{dx}\omega_{pe}^2$, $L_U = \frac{1}{U}\frac{dU}{dx}$, U is the blow off velocity. $P = \frac{(\omega_{pe}^2 - 3\Omega_e^2)^2}{\omega_0^2\omega_{pe}^2}\frac{V_0^2}{V_e^2}\sin^2\phi$; $Q = \left(\frac{k_z^2}{k^2} - \frac{\omega_{pi}^2}{\Omega_e^2}\right)$; $A = \left(1+\frac{\omega_{pe}^2}{\Omega_e^2}\right)\left(\frac{k_z^2}{k^2}+\frac{m}{M}\right)$			

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CHAPTER 3

FILAMENTATION INSTABILITY OF EM WAVES IN A FINITE β INHOMOGENEOUS PLASMA

3.1 Introduction

In chapter 2 we have discussed the stimulated scattering of an EM wave by different ES modes in arbitrary directions, in a magnetized plasma. In this chapter we shall discuss another type of instability, known as filamentation or modulational instability, which leads to growing density fluctuations in the direction perpendicular to the incident EM wave. The saturated state of such an instability results in light filamentation leading to self focussing of light. We can understand the process physically as follows: When the wave vector $\underset{\sim}{k}_0$ of the incident EM wave lies along the perturbed density striations (i.e. the wave vector $\underset{\sim}{k}_1$ of the density perturbation lies perpendicular to $\underset{\sim}{k}_0$), then the refraction caused by the density perturbation channels the light beam into less dense regions. The resulting ponderomotive force pushes plasma away from such regions and increases the density perturbations. In the steady state the ponderomotive force balances the electron pressure, and regions of high plasma density with adjacent regions of high EM energy density are formed. This is known as filamentation or self focussing of light.

This phenomenon can be considered as a parametric scattering instability in which the ES (or mixed ES-EM) wave vector $\underset{\sim}{k}_1$ is at right angles to $\underset{\sim}{k}_0$. Since $\underset{\sim}{k}_1$ is small, in this limit, the anti-Stoke's process $\underset{\sim}{k}_2 = \underset{\sim}{k}_0 + \underset{\sim}{k}_1$ is indistinguishable from the Stoke's process $\underset{\sim}{k}_2 = \underset{\sim}{k}_0 - \underset{\sim}{k}_1$, discussed in the last chapter. Thus both D_- and D_+ in equation (2.11) are resonant and both must be taken into account. Filamentation is, therefore, a four wave, rather than a three wave interaction. Self focussing of light in plasmas, was first studied by Askar'yan(1) in 1962. Later on Palmer(2) and Kaw et al(3) investigated such an effect.

In an inhomogeneous magnetized plasma, the low frequency plasma mode to which a high frequency EM wave may parametrically couple, is an ES drift wave, and such a process is discussed in the second chapter. However in plasmas, characterized by sufficiently large values of β ($1 \gg \beta \gg m/M$ where β is the ratio of the particle kinetic pressure to magnetic pressure and m and M are the masses of the electron and ion respectively), low frequency drift perturbations with sufficiently long parallel wavelengths tend to be mixed EM-ES modes because of coupling to Alfven waves (and the resultant importance of magnetic field perturbations).

Thus in an inhomogeneous magnetized plasma with finite β filamentation and other stimulated scattering instabilities can result because of coupling to drift waves or drift-Alfven waves depending on how closely the angle between the direction of propagation of low frequency mode and the magnetic field approaches 90°. This chapter is devoted to a detailed investigation of the latter type of coupling viz. coupling to drift-Alfven waves. To our knowledge this is the first piece of work on stimulated scattering instability (of which filamentation is a special case) in which the magnetic perturbations of the low-frequency mode have been taken into account.[4] Drake et al[5] had investigated the filamentation instability of EM waves in a homogeneous, un-magnetized plasma and Yu et al[6] studied the same in a magnetized plasma.

In section 3.2 we have derived a dispersion relation for scattering of an incident EM wave by a drift-Alfven wave in an inhomogeneous magnetized plasma. In section 3.3 we have solved this dispersion relation for filamentation instability and estimates of growth rates and threshold powers for this instability are presented for different cases. Finally section 3.4 is devoted to discussion and conclusion.

3.2 Derivation of Dispersion Relation

We consider a weakly inhomogeneous plasma with a uniform background magnetic field of intensity $\underset{\sim}{B}_0$. In the simple slab geometry the density gradient is chosen along the x-axis $\left(\nabla n_0/n_0 = (K, 0, 0)\right)$ and the background magnetic field lies along the Z-axis $\left(\underset{\sim}{B}_0 = (0, 0, B_0)\right)$. We shall closely follow the method used in the second chapter, to derive the dispersion relation. We also restrict our attention to drift-Alfven waves propagating in a layer of constant density i.e. localized in a plane normal to the density gradient.

Let a plane polarized EM wave $\underset{\sim}{E}_0 = 2E_0 \hat{e}_z \cos(k_0 x - \omega_0 t)$ be incident along the x-axis. The polarization is chosen such that the electric wave vector $\underset{\sim}{\tilde{E}}_0$ and magnetic wave vector $\underset{\sim}{\tilde{B}}_0$ are along z and - y axes respectively (ordinary wave). We assume $\omega_0 \gg \omega_{pe}$ so that $k_0 \simeq \omega_0/c$ is nearly independent of x. For a high frequency pump the electrons oscillate with an equilibrium velocity $\underset{\sim}{V}_0 \left(= -ie\underset{\sim}{E}_0/m\omega_0\right)$, whereas ions which do not respond to fast fields continue to provide a stationary neutralizing background. Such an equilibrium can be unstable to low frequency perturbations corresponding to ES modes or in general to mixed EM-ES modes. Thus electron motion in the pump wave can couple with the density fluctuations in a low frequency mode $(\omega, \underset{\sim}{k})$ and produce currents and EM fields at the high frequency side bands $(\omega_\pm = \omega \pm \omega_0, \underset{\sim}{k}_\pm = \underset{\sim}{k} \pm \underset{\sim}{k}_0)$. The side band modes interact back with the pump mode to produce a ponderomotive force at frequency ω, which can enhance the original perturbation and thus lead to an instability. We shall consider the case where the low frequency perturbation corresponds to a drift-Alfven mode excited

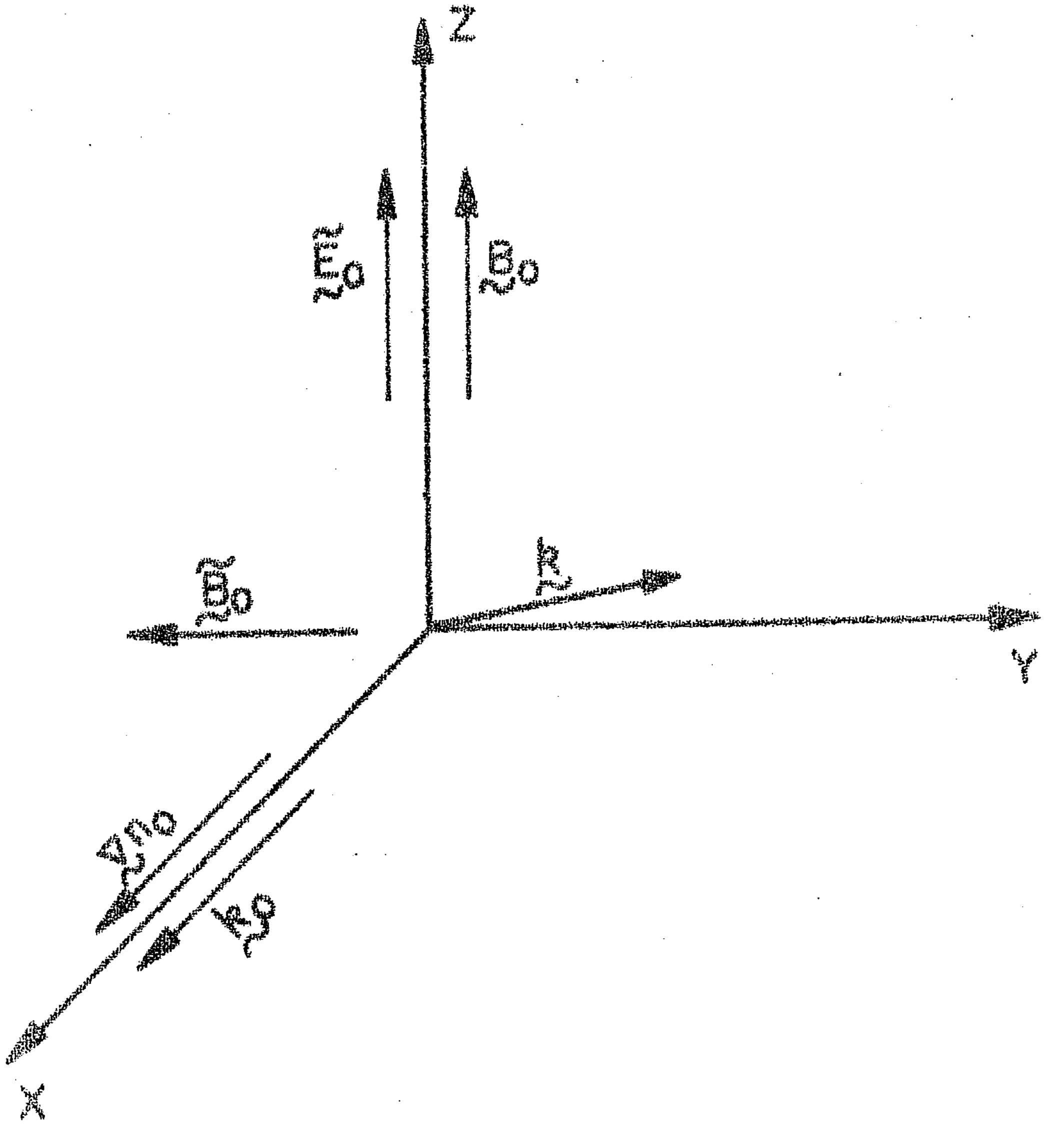

Figure 3.1 - The Co-ordinate system used

in the y-z plane such that $\omega \ll \omega_0$.

The Fourier transformed wave equation, for the side bands, is given by

$$\left[\left(k_\pm^2 - \omega_\pm^2/c^2\right)\underset{\approx}{I} - \underset{\sim}{k}_\pm \underset{\sim}{k}_\pm\right]\cdot \underset{\sim}{E}^\pm = 4\pi i \omega_\pm \underset{\sim}{J}_\pm / c^2 \qquad (3.1)$$

where $\underset{\approx}{I}$ denotes the unit dyadic and $\underset{\sim}{E}^\pm = \underset{\sim}{E}\ (\omega_\pm, \underset{\sim}{k}_\pm)$.
The current density perturbation $\underset{\sim}{J}_\pm$ arises from the linear response of electrons to $\underset{\sim}{E}^\pm$ and the non linear coupling between the electron quiver velocity and the electron density fluctuations produced by the drift-Alfven mode. Thus

$$\underset{\sim}{J}_\pm = - e n_0 \underset{\sim}{V}_e^\pm - e n_e \underset{\sim}{V}_0^\pm \qquad (3.2)$$

Here $\underset{\sim}{V}_e^\pm$ is the velocity of the electrons at frequency $\omega \pm \omega_0$ and $\underset{\sim}{V}_0^\pm = \underset{\sim}{V}_0\left(\pm\omega_0, \pm \underset{\sim}{k}_0\right)$ refers to the equilibrium velocity of the electrons, n_0 and n_e are equilibrium and perturbed electron densities respectively. We neglect ion contributions, since the pump has a high frequency. We can calculate $\underset{\sim}{V}_e^\pm$ from the equation of motion which is given by,

$$\frac{\partial \underset{\sim}{V}_e^\pm}{\partial t} = - \frac{e}{m}\underset{\sim}{E}^\pm - \frac{e}{mc}\underset{\sim}{V}_0^\pm \times \underset{\sim}{B}_1 - \frac{e}{mc}\underset{\sim}{V}_e^\pm \times \underset{\sim}{B}_0 \qquad (3.3)$$

$\underset{\sim}{B}_1$ is the perturbed magnetic field at frequency ω and can be simply expressed as,

$$\omega \underset{\sim}{B}_1/c = \left(k_y E_z - k_z E_y\right)\hat{e}_x \qquad (3.4)$$

since we restrict ourselves to perturbations in the y-z plane ($k_x = 0$); we shall also assume $k_y \gg k_z$. The assumption of localized perturbations in the x-direction leads to considerable simplification because we can Fourier analyse in the y-z plane. We solve for $\underset{\sim}{V}_e^{\pm}$ from equation (3.3) and substitute in equation (3.2). The linear part of $\underset{\sim}{J}_{\pm}$ can be absorbed on the L.H.S. of equation (3.1) and for $\omega_{\pm} \gg \Omega_e$ a simplified equation can be written down,

$$\left[\left(k_{\pm}^2 - \frac{\omega_{\pm}^2}{c^2}\underset{\approx}{\epsilon}_{\pm}\right)\underset{\approx}{I} - \underset{\sim}{k}_{\pm}\underset{\sim}{k}_{\pm}\right]\cdot\underset{\sim}{E}^{\pm} = -\frac{4\pi i e}{c^2}\omega_{\pm} n_e \underset{\sim}{V}_0^{\pm} + k_y\omega_{pe}^2 V_0^{\pm}\underset{\approx}{A}\cdot\underset{\sim}{E}/\omega c^2 \qquad (3.5)$$

where $\underset{\approx}{\epsilon}_{\pm}(\omega_{\pm}, \underset{\sim}{k}_{\pm})$ is the dielectric tensor for a magnetized electron plasma given by,

$$\underset{\approx}{\epsilon}_{\pm} = \begin{pmatrix} 1-\omega_{pe}^2/\omega_{\pm}^2 & i\omega_{pe}^2\Omega_e/\omega_{\pm}^3 & 0 \\ -i\omega_{pe}^2\Omega_e/\omega_{\pm}^3 & 1-\omega_{pe}^2/\omega_{\pm}^2 & 0 \\ 0 & 0 & 1-\omega_{pe}^2/\omega_{\pm}^2 \end{pmatrix}$$

and $\underset{\approx}{A}$ is given by (3.6)

$$\underset{\approx}{A} = \begin{pmatrix} 0 & -ik_z\Omega_e/k_y\omega_{\pm} & i\Omega_e/\omega_{\pm} \\ 0 & k_z/k_y & -1 \\ 0 & 0 & 0 \end{pmatrix} \qquad (3.7)$$

We now need to express the low frequency quantities n_e and $\underset{\sim}{E}$ in terms of $\underset{\sim}{E}^{\pm}$ and $\underset{\sim}{E}_0^{\pm}$ to be able to substitute in equation (3.5) and obtain a dispersion relation. We assume that the low frequency perturbations evolve according to the fluid equations as given below,

$$\partial n_j/\partial t = -\nabla\cdot(n_j \underset{\sim}{V}_j) \qquad (3.8)$$

$$\frac{\partial \underset{\sim}{V}_j}{\partial t} + (\underset{\sim}{V}_j \cdot \underset{\sim}{\nabla}) \underset{\sim}{V}_j = \frac{e_j}{m_j} (\underset{\sim}{E} + \underset{\sim}{V}_j \times \underset{\sim}{B}/c) - T_j \underset{\sim}{\nabla} \log n_j / m_j \quad (3.9)$$

$$\underset{\sim}{\nabla} \times \underset{\sim}{E} = -\frac{1}{c} \partial \underset{\sim}{B}/\partial t \; ; \quad \underset{\sim}{\nabla} \cdot \underset{\sim}{E} = 4\pi \sum_j e_j n_j \quad (3.10), (3.11)$$

$$\underset{\sim}{\nabla} \times \underset{\sim}{B} = 4\pi \underset{\sim}{J}/c + \partial \underset{\sim}{E}/c\partial t \; ; \quad \underset{\sim}{\nabla} \cdot \underset{\sim}{B} = 0 \quad (3.12), (3.13)$$

where suffix j stands for electrons or ions and other symbols have their usual meaning. For $\beta > m/M$ the electron thermal speed v_e exceeds the Alfven speed $C_A \left(= B_o/\sqrt{4\pi M n_o}\right)$ and it can be assumed that $\omega/k_z \lesssim C_A \ll v_e$ so that the electrons reach equilibrium along the field lines. Neglecting electron inertia (since $\omega \ll \Omega_e$), the z-component of equation (3.9) can be written as

$$m\left\{(\underset{\sim}{V}_e^+ \cdot \underset{\sim}{\nabla}) V_o^- + (\underset{\sim}{V}_e^- \cdot \underset{\sim}{\nabla}) V_o^+ + (\underset{\sim}{V}_o^+ \cdot \underset{\sim}{\nabla}) V_{ez}^- + (\underset{\sim}{V}_o^- \cdot \underset{\sim}{\nabla}) V_{ez}^+\right\}$$

$$= -eE_z + \frac{e}{c}\left(V_{ex}^+ \widetilde{B}_o^- + V_{ex}^- \widetilde{B}_o^+\right) + \frac{e}{c} V_{dy} B_{1x} - \frac{T_e}{n_o}\frac{\partial n_e}{\partial z} \quad (3.14)$$

where $\underset{\sim}{V}_d = -T_e \kappa \hat{e}_y / m\Omega_e$ is the electron drift velocity arising out of the density gradient and $\widetilde{\underset{\sim}{B}}_o^{\pm} \left(\omega_{o\pm}, \underset{\sim}{k}_{o\pm}\right)$ is assumed to be in the -y direction. Remembering that the pump wave vector $\underset{\sim}{k}_{o\pm}$ is in the x-direction and the quiver velocity $\underset{\sim}{V}_o^{\pm} (\omega_{o\pm})$ is in the z-direction we Fourier analyse equation (3.14) and solve it for n_e, the electron density perturbation and thus leading to,

$$\frac{n_e}{n_o} = \frac{ieE_z}{k_z T_e} - \frac{ieV_{dy}k_y}{\omega T_e}\left(\frac{E_z}{k_z} - \frac{E_y}{k_y}\right) + \frac{im}{k_z T_e} a_{NL} \quad (3.15)$$

where a_{NL} is the z-component of the non-linear ponderomotive acceleration given by,

$$a_{NL} = V_{ex}^{+}\left(ik_0^{-}V_0^{-} + e\tilde{B}_0^{-}/mc\right) + V_{ex}^{-}\left(ik_0^{+}V_0^{+} + e\tilde{B}_0^{+}/mc\right)$$
$$+ iV_0^{+}k_z^{-}V_{ez}^{-} + iV_0^{-}k_z^{+}V_{ez}^{+} \qquad (3.16)$$

Now from Maxwell's equation it can be shown that

$$ik_0^{\pm}V_0^{\pm} = -e\tilde{B}_0^{\pm}/mc$$

Also we have $k_z^{-} = k_z^{+} = k_z$ (since k_0 is in the x direction). Substituting these conditions in equation (3.16) we can rewrite equation (3.15) as

$$\frac{n_e}{n_0} = \frac{ieE_z}{k_z T_e} - \frac{ik_y V_{dy} e}{\omega T_e}\left(\frac{E_z}{k_z} - \frac{E_y}{k_y}\right) - \frac{m}{T_e}\left(V_0^{+}V_{ez}^{-} + V_0^{-}V_{ez}^{+}\right) \qquad (3.17)$$

The corresponding expression for the ion density perturbation can be obtained by solving the linearized ion equations with no contributions from $\underset{\sim}{B}_1$ and $\underset{\sim}{V}_0^{\pm}$,

$$n_i/n_0 = (ie/M\omega)\left(-kE_y/\Omega_i + k_z E_z/\omega\right) \qquad (3.18)$$

Substituting for n_i and n_e in the Poisson's equation gives us one relation between E_y and E_z,

$$\left(1 + \frac{\omega_* k_D^2}{\omega k_y^2} + \frac{k_D^2 C_0^2 k}{\omega\Omega_i}\right)k_y E_y + \left(1 - \frac{k_D^2 C_0^2}{\omega^2} + \frac{k_D^2}{k_z^2}\left(1+\frac{\omega_*}{\omega}\right)\right)k_z E_z$$
$$= -\left(imk_D^2/e\right)\left(V_0^{+}V_{ez}^{-} + V_0^{-}V_{ez}^{+}\right) \qquad (3.19)$$

where $k_D = \omega_{pe}/v_e$ is the Debye wave number, $C_s = (T_e/M)^{1/2}$ is the ion accoustic speed and $\omega_* = -k_y V_{dy} = k_y T_e K/m\Omega_e$.
A second relation between E_y and E_z can be obtained from the relation,

$$\frac{4\pi}{c} J_z = (\underset{\sim}{\nabla} \times \underset{\sim}{B}_1) \cdot \hat{e}_z = \frac{icK_z k_y^2}{\omega}\left(\frac{E_y}{k_y} - \frac{E_z}{k_z}\right) \qquad (3.20)$$

where J_z is to be calculated from the fluid equations.
For the long wavelength drift-Alfven waves one can assume quasi neutrality$(n_e \simeq n_i)$ and subtracting the continuity equation for electrons from that for ions, we get an expression for J_z,

$$ik_z J_z = -e(V_{ix} - V_{ex})\frac{dn_0}{dx} - ien_0 k_y (V_{iy} - V_{ey}) \qquad (3.21)$$

The ion velocities V_{ix} and V_{iy} can be written directly from the equation of motion (equation 3.9) with no contributions from $\underset{\sim}{B}_1$ and $\underset{\sim}{V}_0^{\pm}$ as

$$V_{ix} = \frac{e}{m\Omega_e}\left(1 - \frac{k_y^2 T_i}{M\Omega_i^2}\right)E_y \; ; \; V_{iy} = -\frac{ic}{B_0}\frac{\omega}{\Omega_i}E_y . \qquad (3.22)$$

where the second term within the parenthesis in V_{ix} arises because of finite ion-Larmor radius corrections.
In order to calculate electron velocities V_{ex} and V_{ey} let us write down the x and y components of equation (3.9) for electrons,

$$(\underset{\sim}{V}_0^+ \cdot \underset{\sim}{\nabla}) V_{ex}^- + (\underset{\sim}{V}_0^- \cdot \underset{\sim}{\nabla}) V_{ex}^+ = -\frac{e}{mc} V_{ey} B_0 + \frac{e}{mc}\left(V_{ez}^+ \tilde{B}_0^- + V_{ez}^- \tilde{B}_0^+ + V_0^+ B_y^- + V_0^- B_y^+\right) \qquad (3.23)$$

$$(\underset{\sim}{V}_0^+\cdot\underset{\sim}{\nabla})V_{ey}^- + (\underset{\sim}{V}_0^-\cdot\underset{\sim}{\nabla})V_{ey}^+ = -\frac{e}{m}E_y + \frac{e}{mc}\left(V_{ex}B_0 - V_0^+B_x^- - V_0^-B_x^+\right) \tag{3.24}$$

Here $\underset{\sim}{B}^\pm(\omega_\pm, \underset{\sim}{k}_\pm)$ refers tothe magnetic field of the scattered waves. We can express $\underset{\sim}{B}^\pm(\omega_\pm, \underset{\sim}{k}_\pm)$ in terms of $\underset{\sim}{E}^\pm(\omega_\pm, k_\pm)$ by Maxwell's equations which can again be expressed in terms of $V_e^\pm(\omega_\pm, \underset{\sim}{k}_\pm)$ by equation (3.3) (It is to be noted that for $\omega_\pm \gg \Omega_e$ only the first term on the R.H.S. of equation (3.3) is important). We can thus simplify the terms proportional to the ponderomotive force in equations (3.23) and (3.24) which can now be written simply as

$$V_{ex} = \frac{e}{m\Omega_e}E_y + \frac{ik_y}{\Omega_e}\left(V_0^+V_{ez}^- + V_0^-V_{ez}^+\right) \;;\; V_{ey} \simeq 0 \tag{3.25}$$

Substituting equations (3.22) and (3.25) in (3.21) yields,

$$ik_zJ_z = -\frac{en_0ck_y}{B_0\Omega_i}\left(\omega - \omega_*\frac{T_i}{T_e}\right)E_y + \frac{ien_0}{\Omega_e}k_yK\left(V_0^+V_{ez}^- + V_0^-V_{ez}^+\right) \tag{3.26}$$

From equations (3.20) and (3.26) we get

$$\left[1 - \omega^2(1 - \omega_*T_i/\omega T_e)/k_z^2C_A^2\right]E_y - k_yE_z/k_z$$
$$= iK\omega\omega_{pe}^2 m\left(V_0^+V_{ez}^- + V_0^-V_{ez}^+\right)/ec\Omega_e k_z^2 . \tag{3.27}$$

We now solve for E_y and E_z from equations (3.19) and (3.27) and get,

$$E_y = \frac{im}{e}\frac{\left(V_0^+V_{ez}^- + V_0^-V_{ez}^+\right)}{\phi}\left[-\frac{K\omega\omega_{pe}^2}{c^2\Omega_e}\left\{1 - \frac{k_D^2C_0^2}{\omega^2} + \frac{k_D^2}{k_z^2}\left(1 + \frac{\omega_*}{\omega}\right)\right\} + k_yk_D^2\right] \tag{3.28}$$

$$E_z = -\frac{im}{e}\frac{(V_0^+ V_{ez}^- + V_0^- V_{ez}^+)}{\phi}\left[-\frac{\kappa\omega\omega_{pe}^2}{c^2\Omega_e}\frac{k_y}{k_z}\left(1+\frac{\omega_*}{\omega}\frac{k_D^2}{k^2}\right) + \frac{\omega^2 k_D^2}{k_z^2 C_A^2}\left(1-\frac{\omega_*}{\omega}\frac{T_i}{T_e}\right) - k_z k_D^2\right] \quad (3.29)$$

where ϕ is given by

$$\phi = k_D^2\left(1+\frac{\omega_*}{\omega}\right)\left[\frac{\omega^2}{k_z^2 C_A^2}\left(1-\frac{\omega_*}{\omega}\frac{T_i}{T_e}\right)-1\right] + k_y^2\left(\frac{k_D^2 C_s^2 \kappa}{\omega\Omega_i k_y} - \frac{\omega_*}{\omega}\frac{k_D^2}{k_y^2} - 1\right) + k_z^2\left(1-\frac{k_D^2 C_s^2}{\omega^2}\right)\left[\frac{\omega^2}{k_z^2 C_A^2}\left(1-\frac{\omega_*}{\omega}\frac{T_i}{T_e}\right)-1\right] \quad (3.30)$$

We now return to equation (3.5) to substitute for n_e and $\underset{\sim}{E}$ in terms of the pump amplitude. It is convenient to effect a simplification at this stage by neglecting the second term on the R.H.S. of equation (3.5). For a sufficiently large pump frequency $(\omega_0 \gg \Omega_e k_z/\kappa)$ this is quite justified. Equation (3.5) can be easily inverted now (specially since $\omega_0 \gg \Omega_e$ and $\underset{\approx}{\epsilon}$ is approximated by a scalar) to give,

$$\underset{\sim}{E}^{\pm} = -\omega_{pe}^2 \frac{n_e}{n_0}\left[\left(\underset{\approx}{I} - \frac{\underset{\sim}{k}_\pm \underset{\sim}{k}_\pm}{k_\pm^2}\right)\Big/ D_\pm - \frac{\underset{\sim}{k}_\pm \underset{\sim}{k}_\pm}{k_\pm^2(\omega_\pm^2-\omega_{pe}^2)}\right]\cdot \underset{\sim}{E}_0^{\pm} \quad (3.31)$$

where

$$D_\pm = c^2 k_\pm^2 - \omega_\pm^2 + \omega_{pe}^2 .$$

Now we have to substitute for n_e/n_0 to get the dispersion relation. Substituting equations (3.28) and (3.29) in (3.17) we get,

$$n_e/n_0 = -e^2 k_D^2 (\underset{\sim}{E}_0^+ \cdot \underset{\sim}{E}^- + \underset{\sim}{E}_0^- \cdot \underset{\sim}{E}^+)\psi / m T_e \phi\, \omega_0^2 . \quad (3.32)$$

where $\psi = 1 + \frac{\omega_* \kappa}{\Omega_e k_y} \frac{\omega_{pe}^2}{c^2 k_D^2}\left(1 - \frac{k_D^2 c_s^2}{\omega^2}\right) - \frac{\omega}{k_z^2 c_A^2}(\omega + \omega_*)$

$$\times\left(1 - \frac{\omega_*}{\omega}\frac{T_i}{T_e}\right) + \frac{\omega_{pe}^2}{c^2 k_z^2}\frac{\kappa}{k_y}\frac{\omega_*(\omega+\omega_*)}{\omega\Omega_e} + \frac{k_y \kappa \omega_{pe}^2(\omega+\omega_*)}{k_z^2 c^2 k_D^2 \Omega_e}\left(1 + \frac{\omega_* k_D^2}{\omega k_y^2}\right) \tag{3.33}$$

Substituting equation (3.32) in (3.31) we have,

$$\underset{\sim}{E}^{\pm} = \frac{\omega_{pe}^2 k_D^4 \psi(\underset{\sim}{E}_0^+ \cdot \underset{\sim}{E}^- + \underset{\sim}{E}_0^- \cdot \underset{\sim}{E}^+)}{4\pi\omega_0^2 n_0 m \phi}\left[\left(\underset{\sim}{I} - \frac{\underset{\sim}{k}_\pm \underset{\sim}{k}_\pm}{k_\pm^2}\right)\Big/ D_\pm - \frac{\underset{\sim}{k}_\pm \underset{\sim}{k}_\pm}{k_\pm^2(\omega_\pm^2 - \omega_{pe}^2)}\right] \cdot \underset{\sim}{E}_0^{\pm} \tag{3.34}$$

$$\underset{\sim}{E}_0^+ \cdot \underset{\sim}{E}^- = \frac{\omega_{pe}^2 k_D^4 \psi(\underset{\sim}{E}_0^+ \cdot \underset{\sim}{E}^- + \underset{\sim}{E}_0^- \cdot \underset{\sim}{E}^+)}{4\pi\omega_0^2 n_0 m \phi}\left[\frac{|\underset{\sim}{k}_- \times \underset{\sim}{E}_0|^2}{k_-^2 D_-} - \frac{|\underset{\sim}{k}_- \cdot \underset{\sim}{E}_0|^2}{k_-^2(\omega_-^2 - \omega_{pe}^2)}\right] \tag{3.35}$$

$$\underset{\sim}{E}_0^- \cdot \underset{\sim}{E}^+ = \frac{\omega_{pe}^2 k_D^4 \psi(\underset{\sim}{E}_0^+ \cdot \underset{\sim}{E}^- + \underset{\sim}{E}_0^- \cdot \underset{\sim}{E}^+)}{4\pi\omega_0^2 n_0 m \phi}\left[\frac{|\underset{\sim}{k}_+ \times \underset{\sim}{E}_0|^2}{k_+^2 D_+} - \frac{|\underset{\sim}{k}_+ \cdot \underset{\sim}{E}_0|^2}{k_+^2(\omega_+^2 - \omega_{pe}^2)}\right] \tag{3.36}$$

Adding equations (3.35) and (3.36) we get the dispersion relation,

$$\phi = \frac{\omega_{pe}^2 k_D^4 \psi}{4\pi\omega_0^2 n_0 m}\left[\frac{|\underset{\sim}{k}_+ \times \underset{\sim}{E}_0|^2}{k_+^2 D_+} - \frac{|\underset{\sim}{k}_+ \cdot \underset{\sim}{E}_0|^2}{k_+^2(\omega_+^2 - \omega_{pe}^2)} + \frac{|\underset{\sim}{k}_- \times \underset{\sim}{E}_0|^2}{k_-^2 D_-} - \frac{|\underset{\sim}{k}_- \cdot \underset{\sim}{E}_0|^2}{k_-^2(\omega_-^2 - \omega_{pe}^2)}\right] \tag{3.37}$$

Equation (3.37) is the generalized dispersion relation for the drift-Alfven waves in the presence of the pump wave. In the electrostatic limit andfor $\kappa \to 0$ it reduces to the dispersion relation (2.11) obtained in the second Chapter.

3.3 Filamentation Instability

We shall now solve equation (3.37) in the limit when both D_+ and D_- are equally important and the other two terms are negligible because the denominators $(\omega_\pm^2 - \omega_{pe}^2)$ are non-resonant. Since $\omega_0 \gg \Omega_e$ we also have $\underset{\sim}{k} \cdot \underset{\sim}{v}_g = 0$ where $\underset{\sim}{v}_g = c^2 \underset{\sim}{k}_0 / \omega_0$ is the group velocity of the pump wave. For $\omega < \omega_*$ and retaining only the leading terms equation (3.37) can be simplified to

$$\omega(\omega + i\nu) - \omega\omega_* \frac{T_i}{T_e} - k_z^2 C_A^2 = - \frac{K\omega\delta}{(\omega + i\Gamma)^2 - \delta^2} \quad (3.38)$$

where $\delta = c^2 k^2 / \omega_0$ and $K = k_D^2 \omega_* |V_0|^2 / \omega_0$ is a measure of the applied power. We have introduced the parameters ν and Γ to represent the natural damping rates of the low frequency wave and pump wave respectively, in a phenomenological manner. Writing $\omega = x + iy$ we can separate equation (3.38) into its real and imaginary parts

$$x^2 - y^2 - \nu y - \omega_* \frac{T_i}{T_e} x - k_z^2 C_A^2 = - \frac{Kx\delta(x^2 - y^2 - \Gamma^2 - \delta^2)}{F(x,y)} \quad (3.39)$$

and

$$2xy + \nu x - \omega_* y \frac{T_i}{T_e} = \frac{K\delta(2\Gamma(x^2 + y^2) + x^2 y + \Gamma^2 y + y^3 + y\delta^2)}{F(x,y)} \quad (3.40)$$

where

$$F(x,y) = (x^2 - y^2 - \Gamma^2 - \delta^2 - 2y\Gamma)^2 + 4x^2(y + \Gamma)^2.$$

It is straight forward to obtain an estimate of the threshold power $\left(K=K_c,\ y\to 0\right)$ from the above equations. For the drift-Alfven branch we get,

$$K_c = \frac{\nu}{2\delta\Gamma x_c}\left[\left(x_c^2-\Gamma^2-\delta^2\right)^2+4\Gamma^2x^2\right] \qquad (3.41)$$

with the critical frequency x_c given by,

$$|x_c| \simeq \frac{k_z^2 C_A^2 T_e}{\omega_* T_i} + \frac{\nu T_e}{2\Gamma\omega_* T_i}\left(\Gamma^2+\delta^2\right) \qquad (3.42)$$

We note that our dispersion relation does not permit a purely growing solution. Thus the density modulations excited in this instance do not form a stationary pattern but possess a slow time dependence given by x_c. The threshold expression (3.41) can be further minimized with respect to δ to obtain the minimum power necessary for parametric instability. This occurs for $\delta \simeq x_c$ and yields

$$K_m = \frac{2\nu\Gamma}{\delta}\ \frac{k_z^2 C_A^2}{\omega_*} \qquad (3.43)$$

One can expect the same result in terms of a minimum field amplitude E_m,

$$E_m^2/8\pi n_0 T_e = \nu\Gamma\omega_0^3 k_z^2 C_A^2/\delta\omega_*^2\omega_{pe}^2 \qquad (3.44)$$

Just above the minimum thresholds, equations (3.39) and (3.40) can be linearized to give,

$$|\Delta x| \simeq 2\nu\Gamma\left(K-K_m\right)/\omega_* K_m \qquad (3.45)$$

$$y = 2\Gamma\left(K-K_m\right)/K_m \qquad (3.46)$$

$\Delta x = x - x_c$ is the frequency shift in the real part of ω and y is the growth rate.

Finally for large applied powers (much beyond threshold) one would expect $x, y \gg \omega_*, \nu, \Gamma$ and equation (3.38) may be approximately written as

$$\omega(\omega^2 - \delta^2) = -K\delta \tag{3.47}$$

It is possible to estimate the order of magnitude of the maximum growth rate (or the maximum frequency shift) by a dimensional analysis of equation (3.47). The maximum occurs for $\delta \sim x_m, y_m$ and comes out to be

$$x_{max}, y_{max} \sim (K)^{1/2} \tag{3.48}$$

Thus the maximum growth rate is of the order of $k_D V_0 (\omega_*/\omega_0)^{1/2}$ and is comparable to growth rates obtained for modulational instabilities corresponding to various electrostatic modes(6).

3.4 Discussion

We have derived a general dispersion relation for stimulated scatteringof an EM wave off drift-Alfven waves in a finite β inhomogeneous plasma. The special case of filamentation instability which leads to self-focussing of an intense EM wave, has been investigated in detail. Our estimates of threshold power and growth rate etc. are found to be comparable to those obtained for excitation of ES modes and this could be a competingprocess in a realistic situation with finite β. We also note that for $\underset{\sim}{k} \cdot \underset{\sim}{v}_g = 0$ our dispersion relation does not permit a purely growing solution in contrast to the homogeneous unmagnetized plasma calculations[5]. Both the inhomogencity and the magnetic field effects contribute towards a small real part of the frequency and thus cause the density modulations to have a finite phase velocity.

Our results could be of significance on experiments with laser fusion plasmas. Recent numerical simulation[7] and laboratory experiments[8] have shown that intense spontaneously generated magnetic fields $(\sim 10^6 G)$ may be present in the interiorof laser produced plasmas. Such plasmas have sufficiently high values of β (in the kilo-volt range) to make coupling to drift-Alfven waves important. It is therefore likely that self focussing in such plasmas is governed by process discussed in this chapter.

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CHAPTER 4

DECAY INSTABILITY AT LOWER HYBRID RESONANCE

4.1 Introduction

In chapters 2 and 3 we have discussed parametric scattering instabilities in a magnetized plasma. In the next three chapters we shall discuss the parametric decay instability where both the decay modes are ES waves. Parametric decay instability is considered to be very useful because of its role in anomalous heating of plasmas, such as laser heating, lower hybrid resonance heating and so on. In most fusion devices like tokamaks and stellarators etc. ohmic heating cannot raise the plasma temperature to the desired value of ten kilo electron volts or so. This is because the electron ion collision frequency (and hence the electrical resistivity) is a rapidly decreasing function of electron temperature. It is hoped therefore that non-ohmic process, such as anomalous absorption due to parametric processes will provide the additional heating.

Let us now try to understand physically what we mean by anomalous heating of plasmas. Imagine a pump wave of frequency ω_0 and wave vector $\underset{\sim}{k}_0$ imposed on a collisionless plasma. If the wavelength of the pump wave is very large (i.e. small $\underset{\sim}{k}_0$), its phase velocity may be more than the electron thermal speed and this mode will be essentially undamped. Now if there is a low frequency short wavelength $(\omega, \underset{\sim}{k})$ perturbation in the plasma, such that $\omega \ll \omega_0$ and $k \gg k_0$, it will non-linearly interact with the pump wave leading to the generation of side band components at $\omega \pm \omega_0 \simeq \omega_0$ and $\underset{\sim}{k} \pm \underset{\sim}{k}_0 \simeq \underset{\sim}{k}$, which will have a considerably smaller phase velocity viz. ω/k. When the pump wave frequency is close to one of the natural frequencies of the plasma, say a lower hybrid frequency, it drives the natural oscillations resonantly and their amplitude becomes quite large. These large amplitude side-band modes are, however, heavily damped by Landau damping because of their low phase velocity. Thus the pump wave which decays into other modes, ultimately gets absorbed in the plasma. This is known as anomalous absorption of the pump wave leading to anomalous heating of the plasma. Anomalous heating takes place preferentially for that species of plasma particles which interacts strongly with the excited high frequency ES modes. Thus if the applied oscillating electric field is near the electron plasma frequency, anomalous heating of electrons takes place. Any excessive heating of ions, in this case, takes place indirectly and is not, therefore, very efficient for fusion. For direct anomalous heating of ions, one should work with a pump field at a lower frequency such as magnetosonic, ion cyclotron, lower hybrid and ion ion hybrid frequency etc. It should also be noted that high power is much more easily available at lower frequencies. That is why anomalous heating for fusion purposes, a great deal of attention is being given to frequencies like the lower hybrid resonance.

In this chapter we shall study the anomalous heating of plasma by decay instability at lower hybrid resonance. Decay instability at lower hybrid resonance has been widely discussed in the literature. The theory of linear conversion process around the lower hybrid frequency was first developed by Stix(1) and by Piliya and Federov(2). They showed that a long wavelength EM wave at the lower hybrid frequency can be linearly converted into short wavelength ES modes which then get heavily absorbed by linear Landau damping. Kindel et al(3) for the first time predicted theoretically and observed in computer simulation experiments, the parametric instability and heating of electrons and ions by a lower hybrid pump. They considered the decay of the lower hybrid pump into another lower hybrid wave and an ion acoustic wave. Soon after Hooke and Bernabei(4) made experimental observations of parametric instabilities near the lower hybrid resonance frequency, where the low frequency waves were found to be ion acoustic waves.

Excitation of an ion cyclotron wave and a lower hybrid wave by a lower hybrid pump was discussed theoretically by Chu et al[5]. Sundaram and Kaw[6] have investigated the effects of pump at lower hybrid resonance on the excitation and suppression of drift instabilities in an inhomogeneous plasma. In a two ion species plasma, the excitation of ion ion hybrid wave by a lower hybrid pump has been discussed by Kaw and Lee[7] and by Ott et al[8].

Chang and Porkolab[9] have reported experimental observations of a new type of parametric instability which involves the excitation of lower hybrid waves and non resonant low frequency modes (ion quasi modes) when the pump r.f. field is near the lower hybrid frequency; according to them, the non resonant instability is the dominant one for pump frequencies between 1 to 3 times the lower hybrid frequency. Porkolab[10] has theoretically discussed the excitation of the ion quasi modes and the lower hybrid modes by a lower hybrid pump. Recently Ott[11] has discussed the decay of two lower hybrid waves by a pump field near the lower hybrid frequency. The widely used dipole approximation can not be made in this problem and the pump wave frequency should be more than twice the lower hybrid frequency for this instability to occur. Satya et al[12] have shown that a long wavelength oscillating electric field at lower hybrid frequency can be anomalously absorbed in a plasma with short wavelength low frequency fluctuations because of their coupling to short wavelength lower hybrid modes which are damped. Thus anomalous absorption of lower hybrid waves can result even if the wave is not intense enough to excite parametric instabilities, the only requirement is that there should be significant amount of low frequency fluctuations in the medium.

In the present chapter we shall discuss the excitation of a very low frequency 'cold' ion Bernstein mode, propagating almost normal to the magnetic field in a cold, homogeneous plasma, by a pump at a frequency near the lower hybrid wave. In section 4.2 we shall first discuss the linear dispersion relation for 'cold' ion Bernstein mode(13) and then derive a dispersion relation for parametric excitation of this mode by a lower hybrid pump. In section 4.3 we shall present estimates of growth rates and threshold powers, for this decay instability, under different conditions. In section 4.4 we shall discuss our results and point out some applications to our results.

4.2 Dispersion relation

We consider a cold, homogeneous, magnetized plasma with the magnetic field $\underset{\sim}{B}_0$ along the z- direction. By cold plasma we mean the phase velocity of the perturbation parallel to the magnetic field is much greater than the electron thermal speed v_e. The ion and the electron susceptibilities $\chi_i(\omega)$ and $\chi_e(\omega)$ in such a plasma is given as

$$\chi_i(\omega) = -\frac{\omega_{pi}^2}{\omega}\left(\frac{\bar{\omega}}{\bar{\omega}^2-\Omega_i^2}+\frac{1}{\bar{\omega}}\frac{k_{\parallel}^2}{k^2}\right) \qquad (4.1)$$

and
$$\chi_e(\omega) = -\frac{\omega_{pe}^2}{\omega}\left(\frac{\bar{\omega}'}{\bar{\omega}'^2-\Omega_e^2}+\frac{1}{\bar{\omega}'}\frac{k_{\parallel}^2}{k^2}\right) \qquad (4.2)$$

where $\bar{\omega} = \omega + i\nu_i$ and $\bar{\omega}' = \omega + i\nu_e$; $\nu_j\,(j=e,i)$ represents the phenomenological damping rate for the species concerned. $k_{\parallel}$ is the wave vector parallel to the magnetic field such that $k_{\parallel} \ll k$. The linear dispersion relation is given by

$$\epsilon(\omega) = 1 + \chi_i(\omega) + \chi_e(\omega) = 0 \qquad (4.3)$$

we shall consider the wave frequency, ω, to be much smaller than the ion cyclotron frequency $\omega^2 \ll \Omega_i^2$

For such low frequency waves we can ignore unity in equation (4.3) and substituting for $\chi_i(\omega)$ and $\chi_e(\omega)$ from equations (4.1) and (4.2) in (4.3) we get,

$$\left(\frac{\omega_{pi}^2}{\Omega_i^2}\frac{\bar{\omega}}{\omega}+\frac{\omega_{pe}^2}{\Omega_e^2}\frac{\bar{\omega}'}{\omega}\right)-\frac{k_{\parallel}^2}{k^2}\left(\frac{\omega_{pi}^2}{\omega\bar{\omega}}+\frac{\omega_{pe}^2}{\omega\bar{\omega}'}\right)=0 \qquad (4.4)$$

From equation (4.4) we see that ion motion in the perpendicular direction is the dominant one, whereas electron motion dominates in the parallel direction. Keeping only dominant terms in equation (4.4) we get the linear dispersion relation for 'cold' ion Bernstein modes as,

$$(\omega + i\nu_i)(\omega + i\nu_e) = \Omega_e \Omega_i k_{\parallel}^2/k^2 \tag{4.5}$$

Thus from equation (4.5) we see that if $\omega^2 \ll \Omega_i^2$ then we must have $k_{\parallel}^2/k^2 \ll m/M$ writing $\omega = \omega_L - i\nu_L$ where ω_L and ν_L are the real frequency and damping rate respectively of the low frequency mode under consideration, and equating real and imaginary parts in equation (4.5) we get,

$$\nu_L = (\nu_e + \nu_i)/2 \tag{4.6}$$

and $$\omega_L^2 = \Omega_e \Omega_i k_{\parallel}^2/k^2 - (\nu_e - \nu_i)^2/4 . \tag{4.7}$$

Thus for this mode to be undamped we require that,

$$m/M \gg k_{\parallel}^2/k^2 \gg (\nu_e - \nu_i)^2/4\Omega_e\Omega_i \tag{4.8}$$

under this assumption the dispersion relation (4.5) can be written as,

$$\omega_L^2 = \Omega_e \Omega_i k_{\parallel}^2/k^2 \tag{4.9}$$

It is to be noted that in a cold, homogeneous, magnetized plasma with a single species of ions this is the only low frequency mode $(\omega < \Omega_i)$ propagating in a small cone perpendicular to the magnetic field. Therefore the excitation of this mode by a high frequency pump should be important.

We now consider the influence of a lower hybrid pump wave on this low frequency mode. An oscillating electric field $\underset{\sim}{E} = \underset{\sim}{E}_0 \cos \omega_0 t$ at the lower hybrid frequency ω_0 is applied across the magnetic field. The pump wavelength is considered to be much larger than the wavelength of perturbations, so that we can assume the applied electric field to be spatially uniform (dipole approximation). We will study the case when such a pump decays into another lower hybrid mode $(\omega_H, \underset{\sim}{k}_H)$ and the low frequency 'cold' ion Bernstein mode $(\omega_L, \underset{\sim}{k}_L)$ satisfying the resonant conditions.

$$\begin{aligned} \omega_0 &= \omega_H + \omega_L \\ \underset{\sim}{k}_0 &= 0 = \underset{\sim}{k}_H + \underset{\sim}{k}_L \end{aligned} \tag{4.10}$$

where $(\omega_L, \underset{\sim}{k}_L)$ satisfy dispersion relation (4.9). We now need to obtain a general dispersion relation that will contain the appropriate non-linear corrections to depict the parametric decay process. A standard method, as developed by Arnush et al[14], is to transform to the oscillating frame of each species (to account for the influence of the pump wave on them) and obtain relations connecting the density perturbations and electric field fluctuations. Finally use is made of the Poisson's equation after inverting each quantity back to the laboratory frame. This results in a chain of coupled equations for the perturbed quantities and the determinant of its co-efficients set to zero gives the dispersion relation. The determinant which is of infinite order can be suitably truncated to a 3 x 3 determinant, by limiting oneself to the lowest order side bands ($\omega \pm l\omega_0$; $l = 0, 1$ and $\omega \ll \omega_0$) and using the smallness of the excursion length of the two species under the influence of the pump wave compared to the wavelength of perturbations $(\underset{\sim}{k} \cdot \underset{\sim}{R}_j \ll 1)$. Since the details of the method have already been published we will write down here the final dispersion relation(10)

$$\epsilon(\omega) = -J_1^2(\rho)\,\chi_i(\omega)\,\chi_e(\omega)\left[\frac{1}{\epsilon(\omega+\omega_0)} + \frac{1}{\epsilon(\omega-\omega_0)}\right] \tag{4.11}$$

where

$$\epsilon(\omega+n\omega_0) = 1 + \chi_i(\omega+n\omega_0) + \chi_e(\omega+n\omega_0) \tag{4.12}$$

and

$$\chi_i(\omega+n\omega_0) = -\frac{\omega_{pi}^2}{\omega+n\omega_0}\left[\frac{\bar{\omega}+n\omega_0}{(\bar{\omega}+n\omega_0)^2-\Omega_i^2} - \frac{k_{\parallel}^2}{k^2}\frac{1}{\bar{\omega}+n\omega_0}\right] \tag{4.13}$$

$$\chi_e(\omega+n\omega_0) = -\frac{\omega_{pe}^2}{\omega+n\omega_0}\left[\frac{\bar{\omega}'+n\omega_0}{(\bar{\omega}'+n\omega_0)^2-\Omega_e^2} - \frac{k_{\parallel}^2}{k^2}\frac{1}{\bar{\omega}'+n\omega_0}\right]$$

Here $n = 0, \pm 1$; also we have (4.14)

$\rho = (\lambda^2+\mu^2)^{1/2} \ll 1$; λ and μ are defined by

$$\underset{\sim}{k}\cdot(\underset{\sim}{R}_{oi} - \underset{\sim}{R}_{oe}) = \lambda \sin\omega_0 t + \mu\cos\omega_0 t \tag{4.15}$$

R_{oj} is the excursion length for the j^{th} species under the influence of the pump wave, λ and μ are the components of the excursion along the wave vector $\underset{\sim}{k}$; they have respectively phase differences of $\pi/2$ and zero to the applied field. J_1 is the Bessel function of the first order.

We shall next calculate $\epsilon(\omega \pm \omega_0)$. Substituting equations (4.13) and (4.14) in equation (4.12) we get

$$\epsilon(\omega \pm \omega_0) = 1 - \omega_{pi}^2 / (\omega \pm \omega_0)(\omega \pm \omega_0 + i\nu_i)$$
$$+ \omega_{pe}^2 (\omega \pm \omega_0 + i\nu_e) / \Omega_e^2 (\omega \pm \omega_0) - k_{\parallel}^2 \omega_{pe}^2 / \{k^2 (\omega \pm \omega_0) \times (\omega \pm \omega_0 + i\nu_e)\}$$
$$= \omega_{pi}^2 \left\{ \frac{1}{\omega_{LH}^2} - \frac{1}{(\omega \pm \omega_0)^2} \right\} - \frac{k_{\parallel}^2}{k^2} \frac{\omega_{pe}^2}{\omega_0^2} \pm 2i\omega_{pi}^2 \nu_H / \omega_0^3 \qquad (4.16)$$

where ν_H is the lower hybrid damping frequency defined as,

$$2\nu_H = \nu_i + \nu_e \left(\omega_0^2 / \Omega_e \Omega_i + k_{\parallel}^2 M / k^2 m \right) \qquad (4.17)$$

and ω_{LH} is defined as,

$$\omega_{LH}^2 = \omega_{pi}^2 / (1 + \omega_{pe}^2 / \Omega_e^2) \qquad (4.18)$$

Now equation (4.16) can be rewritten as,

$$\epsilon(\omega \pm \omega_0) = \omega_{pi}^2 \left((\omega \pm \omega_0)^2 - \omega_H^2 \right) / \omega_0^4 \pm 2i\omega_{pi}^2 \nu_H / \omega_0^3$$
$$= 2\omega_{pi}^2 (\delta \pm \omega \pm i\nu_H) / \omega_0^3 \qquad (4.19)$$

where $\delta = \omega_0 - \omega_H$ is the frequency mismatch of pump frequency ω_0 from the lower hybrid frequency ω_H

defined as

$$\omega_H^2 = \frac{\omega_{pi}^2}{1+\omega_{pe}^2/\Omega_e^2}\left(1+\frac{k_{\parallel}^2}{k^2}\frac{M}{m}\right) \tag{4.20}$$

Again for the 'cold' ion Bernstein mode we have the ion and the electron susceptibilities $\chi_i(\omega)$ and $\chi_e(\omega)$ defined as,

$$\chi_i(\omega) = \frac{\omega_{pi}^2}{\Omega_i^2}\frac{\bar{\omega}}{\omega} \quad \text{and} \quad \chi_e(\omega) = -\frac{k_{\parallel}^2}{k^2}\frac{\omega_{pe}^2}{\omega\bar{\omega}}, \tag{4.21}$$

Substituting equations (4.19) and (4.21) in equation (4.11), the dispersion relation is reduced to a simpler form,

$$\omega^2 + 2i\omega\nu_L - \omega_L^2 = \frac{K\delta}{\delta^2-(\omega+i\nu_H)^2} \tag{4.22}$$

where

$$K = J_1^2(\rho)\,\omega_0^3\,k_{\parallel}^2 M/k^2 m\,. \tag{4.23}$$

K is proportional to the applied power and can be defined as a threshold parameter. For $\rho = 0$; $K = 0$ and equation (4.22) reduces to the linear dispersion relation discussed above. We shall now solve equation (4.22) in various limits.

4.3 Estimates of threshold powers and growth rates:

We can have two types of solution of equation (4.22) viz. (1) a purely growing solution when $Re\,\omega = 0$ and

(2) an oscillating growing solution when $Re\,\omega \neq 0$.

4.3.1 Purely growing instability.

In this case the real part of the frequency ω vanishes and we can write $\omega = iy$; y being real. Substututing this in equation (4.22) we have

$$y^2 + 2y\nu_L + \omega_L^2 = -\frac{K\delta}{\delta^2 + (y+\nu_H)^2} \tag{4.24}$$

This equation has a solution only for $\delta < 0$. Thus a purely growing mode can be excited by the pump when its frequency ω_0 is less than the lower hybrid frequency ω_H The threshold power in this case can be obtained by putting $y \to 0$ in equation (4.24). Thus we get

$$K = -\omega_L^2(\nu_H^2 + \delta^2)/\delta \tag{4.25}$$

This is independent of the low frequency damping rate ν_L, and assumes the minimum value

$$K_m = 2\omega_L^2 \nu_H \tag{4.26}$$

for $\delta = -\nu_H$. It may be of interest to express the minimum threshold in terms of applied electric field. In the Appendix we have calculated the values of ρ, the argument of the Bessel function, for the field $\underset{\sim}{E}_0$ applied in three different directions and found that the value of ρ is largest when $\underset{\sim}{E}_0$ is applied in the $\underset{\sim}{k} \times \underset{\sim}{B}_0$ direction. Thus equations (A.18), (4.23) and (4.26) give the minimum threshold power for purely growing instability as

$$E_{0m}^{2} = \frac{8m^{2}\nu_{H}\Omega_{e}^{2}\Omega_{i}^{2}}{e^{2}k^{2}\omega_{0}} \tag{4.27}$$

The maximum growth rate near the threshold region is given by

$$\gamma_{m} = \frac{K - K_{m}}{2(\omega_{L}^{2} + 2\nu_{L}\nu_{H})} \tag{4.28}$$

For $\gamma \gg \nu_{L}$, ν_{H} and ω_{L} we have $\gamma_{m} = (K/3)^{1/3}$ at the frequency $\omega_{0} = \omega_{H} - (K/2)^{1/3}$

In the intermediate region two cases arise. When $\omega_{L} \gg \nu_{H}$ and $\omega_{L}^{3} \gg K \gg K_{m}$ we get $\gamma_{m} \sim K/2\omega_{L}^{2}$ at the frequency $\omega_{0} \sim \omega_{H} - K/2\omega_{L}^{2}$. When $\omega_{L} \ll \nu_{H}$, and $\nu_{H}^{3} \gg K \gg K_{m}$, we get $\gamma_{m} \sim (K/2\nu_{H})^{1/2}$ at the frequency $\omega_{0} \sim \omega_{H} - \nu_{H}$.

4.3.2 Oscillatory growing solution

When the real part of the frequency is not equal to zero, we then write $\omega = x + iy$ and $x \neq 0$. The dispersion relation (4.22) can be seperated into real and imaginary parts giving,

$$x^{2} - y^{2} - 2y\nu_{L} - \omega_{L}^{2} = K\delta\left(\delta^{2} - x^{2} + (y+\nu_{H})^{2}\right)/F(x,y) \tag{4.29}$$

and
$$2x(y+\nu_{L}) = 2xK\delta(y+\nu_{H})/F(x,y) \tag{4.30}$$

where
$$F(x,y) = \left(x^{2} - \delta^{2} - (y+\nu_{H})^{2}\right)^{2} + 4x^{2}(y+\nu_{H})^{2} \tag{4.31}$$

From equation (4.30) we see that for $x \neq 0$, a growing solution $(y > 0)$ is possible only when $\delta > 0$; in other words an oscillatory growing solution is possible only when the frequency of the pump ω_0 is greater than the lower hybrid frequency ω_H.

The threshold can be found again by putting $y = 0$. The frequency $x = x_c$ at the threshold can be obtained from equations (4.29) to (4.31) giving

$$x_c = \pm \left\{ \frac{1}{\nu_L + \nu_H} \left(\nu_H \omega_L^2 + \nu_L (\nu_H^2 + \delta^2) \right) \right\}^{1/2} \qquad (4.32)$$

and the threshold given by,

$$K_c(\delta) = \frac{\nu_H \nu_L}{\delta} \left\{ 4\delta^2 + \left(\nu_H^2 + 2\nu_H \nu_L + \omega_L^2 - \delta^2\right)^2 / (\nu_L + \nu_H)^2 \right\} \qquad (4.33)$$

Thus from (4.33), the threshold is zero, if any one of the modes is undamped (i.e. ν_H or $\nu_L \sim 0$). Obviously the threshold power is a function of δ. The value of optimum δ for minimum threshold is determined by the condition $dK_c/d\delta = 0$. This condition can be written as

$$\delta^2 = -\frac{1}{3}\left((\nu_L + \nu_H)^2 - \omega_L^2 + \nu_L^2\right) \pm \frac{1}{6}\left\{ 4\left((\nu_L + \nu_H)^2 - \omega_L^2 + \nu_L^2\right)^2 + 12\left((\nu_L + \nu_H)^2 + \omega_L^2 - \nu_L^2\right) \right\}^{1/2} \qquad (4.34)$$

we shall investigate these conditions in some limiting cases.

Case (a): When $\omega_L \gg \nu_H$ we get from (4.32) and (4.33)

$$K_c(\delta) = \frac{\nu_H \nu_L}{\delta}\left(4\delta^2 + \frac{\omega_L^2 - \delta^2}{(\nu_L+\nu_H)^2}\right) \tag{4.35}$$

$$\varkappa_c = \pm\,\omega_L \left\{1 + \frac{\nu_L}{\nu_L+\nu_H}\left((\delta/\omega_L)^2 - 1\right)\right\}^{1/2} \tag{4.36}$$

The minimum value of the threshold power is given by

$$K_m = 4\omega_L \nu_L \nu_H \tag{4.37}$$

at the frequency $\delta \sim \omega_L$. When the electric field $\underset{\sim}{E_0}$ is applied in the $\underset{\sim}{k} \times \underset{\sim}{B_0}$ direction, the minimum threshold power can be written in terms of $\underset{\sim}{E_0}$ as

$$E_{02}^2 = 16\nu_L \nu_H \Omega_e^{3/2} \Omega_i^{3/2} m^2 / e^2 \omega_0 k_{\parallel} k \tag{4.38}$$

Case (b): Let us consider the case when $\nu_H \gg \omega_L \gg \nu_L$. In this case, we have from equations (4.33) and (4.34)

$$K_c(\delta) = \nu_L (\nu_H^2 + \delta^2)^2 / \nu_H \delta \tag{4.39}$$

$$\varkappa_c \simeq \pm\left\{\omega_L^2 + \nu_L \nu_H \left(1 + \delta^2/\nu_H^2\right)\right\}^{1/2} \tag{4.40}$$

The minimum valueof the threshold is given by

$$K_m = \frac{\sqrt{3}\,16}{9} \nu_L \nu_H^2 \tag{4.41}$$

which is attained at the frequency $\delta \sim \nu_H/\sqrt{3}$. In terms of the electric field, the minimum threshold in this case can be written as,

$$E_{03}^2 = \frac{\sqrt{3}\,64}{9} \frac{\nu_L \nu_H^2 \Omega_e \Omega_i m^2}{e^2 \omega_0 k_{\parallel}^2} \tag{4.42}$$

Finally we calculate the growth rate well above threshold. The maximum growth rate in this case is given by

$$\gamma_m = \frac{\sqrt{3}}{2}\left(\frac{K}{4}\right)^{1/3} \tag{4.43}$$

at the frequency $\delta \sim \left(\frac{K}{4}\right)^{1/3}$. (4.44)

4.4 Discussion and conclusion

In the present chapter we have pointed out the possibility of parametric excitation of a low frequency 'cold' ion Bernstein mode, propagating almost perpendicular to the magnetic field in a cold, homogeneous, magnetized plasma, by a lower hybrid pump wave. We have calculated the growth rates and threshold powers for different cases viz., for purely growing and oscillatory growing instabilities(15). Since the 'cold' ion Bernstein mode propagates almost perpendicular to the magnetic field, its phase velocity parallel to the magnetic field tends to be large and in some cases it may be comparable to the Alfven speed $C_A (= B_o/(4\pi M n_o)^{1/2})$. In such situations the electrostatic approximation is not valid and one should consider the electromagnetic effects also. In order that the Alfven speed is larger than the parallel phase velocity (for which the ES approximation is valid), the theory presented in this chapter is applicable to a plasma with a high magnetic field and low density. Also for the cold plasma approximation to be valid, the plasma temperature should not be too high.

For an order of magnitude calculation for the threshold power, we choose some typical parameters characteristic of a laboratory plasma: $n_o = 10^{12}$ per c.c, $Te = 10^2$ e.v., B_o = 25 KG, $M = 3.3 \times 10^{-24}$ gm., $\omega_L = 10^7$ rad./sec., $\nu_H = 10^6$ rad./sec., $\nu_L = 10^5$ rad./sec., the perpendicular wavelength $\lambda_\perp$ = .1cm. and parallel wavelength $\lambda_{\parallel}$ = 100 cm. Using these values we have numerically estimated the threshold power for the excitation of 'cold' ion Bernstein mode for the case $\omega_L \gg \nu_H$ and $\underset{\sim}{E}_0$ in the direction parallel to $\underset{\sim}{k} \times \underset{\sim}{B}_0$. This gives $\underset{\sim}{E}_0 \sim 80$ volt/cm. which is well within the regime of present power densities envisaged for r.f. heating experiments and thus the instabilities considered in this chapter can be easily exploited in an experimental situation.

APPENDIX 'A'

We shall calculate the argument ρ of the Bessel function J_1 in equation (4.23). By definition

$$\rho = (\lambda^2 + \mu^2)^{1/2} \tag{A.1}$$

and

$$\underset{\sim}{k} \cdot (\underset{\sim}{R}_{oe} - \underset{\sim}{R}_{oi}) = \lambda \sin\omega_o t + \mu \cos\omega_o t \tag{A.2}$$

Thus in order to calculate λ and μ we need to calculate R_{oe} and R_{oi}, the electron and ion excursion lengths respectively. We use the following fluid equations for the ions and the electrons,

$$\frac{\partial \underset{\sim}{v}_{io}}{\partial t} = \frac{e}{M} \underset{\sim}{E}_o \cos\omega_o t + \underset{\sim}{v}_{io} \times \underset{\sim}{\Omega}_i \tag{A.3}$$

$$\frac{\partial \underset{\sim}{v}_{eo}}{\partial t} = -\frac{e}{m} \underset{\sim}{E}_o \cos\omega_o t - \underset{\sim}{v}_{eo} \times \underset{\sim}{\Omega}_e \tag{A.4}$$

where $\underset{\sim}{v}_{io}$ and $\underset{\sim}{v}_{eo}$ are respectively the ion and the electron velocities under the influence of the pump wave field. The parallel and perpendicular velocities for the electrons and ions can be solved from equations (A-4) and (A-3) giving

$$v_{eo\parallel} = -\frac{e}{m\omega_o} E_{o\parallel} \sin\omega_o t \tag{A.5}$$

$$\underset{\sim}{v}_{eo\perp} = -\frac{e\omega_o}{m} \frac{\underset{\sim}{E}_{o\perp} \sin\omega_o t}{\omega_o^2 - \Omega_e^2} - \frac{e}{m} \frac{\underset{\sim}{E}_{o\perp} \times \underset{\sim}{\Omega}_e}{\omega_o^2 - \Omega_e^2} \cos\omega_o t \tag{A.6}$$

$$v_{io\parallel} = \frac{e}{M\omega_o} E_{o\parallel} \sin\omega_o t \tag{A.7}$$

$$\underset{\sim}{v}_{io\perp} = \frac{e\omega_o}{M} \frac{\underset{\sim}{E}_{o\perp} \sin\omega_o t}{\omega_o^2 - \Omega_i^2} - \frac{e}{M} \frac{\underset{\sim}{E}_{o\perp} \times \underset{\sim}{\Omega}_i}{\omega_o^2 - \Omega_i^2} \cos\omega_o t . \tag{A.8}$$

For the lower hybrid frequency ω_0 we have $\Omega_i^2 \ll \omega_0^2 \ll \Omega_e^2$. Using this approximation and integrating equations (A-5) to (A-8) once we get,

$$R_{e0\parallel} = (e/m\omega_0^2) E_{0\parallel} \cos\omega_0 t \quad (A.9)$$

$$\underset{\sim}{R}_{e0\perp} = -(e/m\Omega_e^2) \underset{\sim}{E}_{0\perp} \cos\omega_0 t + (e/m\Omega_e^2\omega_0) \underset{\sim}{E}_{0\perp} \times \underset{\sim}{\Omega}_e \sin\omega_0 t \quad (A.10)$$

$$R_{i0\parallel} = -(e/M\omega_0^2) E_{0\parallel} \cos\omega_0 t \quad (A.11)$$

$$\underset{\sim}{R}_{i0\perp} = -(e/M\omega_0^2) \underset{\sim}{E}_{0\perp} \cos\omega_0 t - (e/M\omega_0^3) \underset{\sim}{E}_{0\perp} \times \underset{\sim}{\Omega}_i \sin\omega_0 t \quad (A.12)$$

using equations (A-2) and (A-9) to (A-12) we get the expressions for λ and μ as

$$\lambda = (e/m\Omega_e^2\omega_0)\, \underset{\sim}{k}_\perp \cdot \underset{\sim}{E}_{0\perp} \times \underset{\sim}{\Omega}_e + (e/M\omega_0^3)\, \underset{\sim}{k}_\perp \cdot \underset{\sim}{E}_{0\perp} \times \underset{\sim}{\Omega}_i \quad (A.13)$$

$$\mu = e k_\parallel E_{0\parallel} (1 + m/M)/\omega_0^2 m - e\, \underset{\sim}{k}_\perp \cdot \underset{\sim}{E}_{0\perp} (1 - m\Omega_e^2/M\omega_0^2)/m\Omega_e^2 \quad (A.14)$$

Now we fix our co-ordinate system such that $\underset{\sim}{B}_0$ is along z-axis and $\underset{\sim}{k}$ is in the y-z plane so that $k_x = 0$. Then substituting (A-13) and (A-14) in (A-1) we get,

$$p^2 = \frac{e^2 k_y^2 E_{0x}^2 \Omega_e^2}{m^2 \Omega_e^4 \omega_0^2} + \frac{e^2 k_y^2 E_{0y}^2}{M^2 \omega_0^4} + \frac{e^2 k_\parallel^2 E_{0\parallel}^2}{m^2 \omega_0^4} + \frac{e^2 k_y^2 E_{0y}^2}{m^2 \Omega_e^4}$$
$$+ \frac{2 E_{0y} E_{0\parallel} e^2 k_y k_\parallel}{M m \omega_0^4} - \frac{2 e^2 k_y^2 E_{0y}^2}{M m \omega_0^2 \Omega_e^2} - \frac{2 e^2 k_\parallel k_y E_{0y} E_{0\parallel}}{m^2 \omega_0^2 \Omega_e^2} \quad (A.15)$$

We shall now calculate f^2 for some special cases.

Case (a): When $E_{0\parallel} = E_0$ and $\underset{\sim}{E}_{0\perp} = 0$, then (A-15) gives

$$f^2 = e^2 k_\parallel^2 E_0^2 / m^2 \omega_0^4 \tag{A.16}$$

Case (b): When $E_{0y} = E_0$ and $E_{0x} = E_{0\parallel} = 0$, then we get from (A-15)

$$f^2 = e^2 k_y^2 E_0^2 / M^2 \omega_{pi}^4 \tag{A.17}$$

Case (c): When $E_{0x} = E_0$ and $E_{0y} = E_{0\parallel} = 0$, then we get from (A-15)

$$f^2 = e^2 k_y^2 E_0^2 / \Omega_i^2 \omega_0^2 M^2 \tag{A.18}$$

It is easy to see from equations (A-16) to (A-18) that for lower hybrid frequency, f^2 is largest when $\underset{\sim}{E}_0$ is applied in the $\underset{\sim}{k} \times \underset{\sim}{B}_0$ direction given by equation (A-18).

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CHAPTER 5

DECAY INSTABILITY AT ION-ION HYBRID RESONANCE

5.1 Introduction

In chapter 4 we discussed the decay instability of a lower hybrid wave in a cold homogeneous magnetized plasma with a single species of ions. However, normally, the thermonuclear plasmas consist of two ion species viz. deuterium and tritium or other combinations like LiD, CH_2 and $C_{36}D_{74}$, and therefore for realistic situation, one should consider a plasma with two species of ions. In a two species plasma, resonance at ion-ion hybrid frequency has attained considerable importance for plasma heating by oscillating r.f. fields[1-3].

The ion-ion hybrid resonance was first pointed out by Buchsbaum[4] (and is hence referred to as the Buchsbaum resonance). Buchsbaum pointed out that when a high density plasma column in an axial magnetic field possesses two (or more) species of ions of different charge to mass ratios, there exists a plasma resonance condition which involves only the ion cyclotron frequencies. At resonance, the two species oscillate transversely to the static magnetic field and 180 degree out of phase with each other, while the electrons remain relatively motionless. The ratio of the ion oscillatory energy to that of the electrons is of the order of the ratio of the ion to electron masses. This resonance was later on experimentally observed by Haas[5], Toyama[6] and by Tarasenko et al[7].

Generally, for a parametric decay type of situation, the ion ion hybrid mode is taken to be one of the low frequency decay modes (i.e. the difference mode between high frequency pump wave and a high frequency decay mode). The possibility of exciting such an ion ion hybrid mode, by applying external oscillating fields at the lower hybrid frequency was first studied by Kaw and Lee[1] and later by Ott et al[2]. The possibility of stimulated back scattering of EM waves from ion ion hybrid waves in a magnetized plasma, was theoretically studied by Lee[8]. He showed that although such a process is theoretically possible, it is unlikely to be of importance in present day experiments of heating plasma with intense EM waves, as the threshold, for such a process, is much greater and growth rate much smaller than the corresponding stimulated scattering processes from upper and lower hybrid waves. Plasma heating by linear conversion of ion ion hybrid waves, was recently studied by Klima et. al.[9] and by Longinov et.al.[10].

One could also consider a situation where the pump wave itself oscillates at the ion ion hybrid frequency and decays into an ion ion hybrid mode and some other lower frequency modes. This has some advantage like greater power availability for the pump wave (compared to higher frequency pump waves) and relatively higher efficiency for ion heating. In a recent paper Satya et. al[3] have considered such a scheme for an inhomogeneous plasma where the low frequency mode involved is the drift mode.

In this chapter we wish to study some other cases which can occur in a homogeneous plasma. In particular we have in mind two very low frequency modes which have received only scant attention in such a context. One such mode is an ion acoustic mode propagating parallel to the ambient magnetic field. For a warm, homogeneous two ion species plasma with $v_e \gg \omega/k_{\parallel} \gg v_j$ (where v_e and v_j $(j = 1,2)$ represent the thermal velocities of the electrons and the two ion species respectively) this mode obeys the dispersion relation

$$\omega^2 = k_{\parallel}^2 \left(\alpha c_1^2 + (1-\alpha) c_2^2\right) \tag{5.1}$$

Here $c_j^2 = T_e/M_j$ and $\alpha \left(= n_{01}/n_0\right)$ and $(1-\alpha)$ $\left(= n_{02}/n_0\right)$ are the concentrations of the species 1 and 2 respectively.

The other low frequency mode we have considered is the 'cold' ion Bernstein mode, discussed in the fourth chapter. In a two ion species plasma, the dispersion relation for this mode is written as

$$\omega^2 \simeq \frac{k_{\parallel}^2}{k^2} \frac{\Omega_e \Omega_1 \Omega_2}{\alpha\Omega_2 + (1-\alpha)\Omega_1} \tag{5.2}$$

where Ω_j, $j = e, 1, 2$ are the cyclotron frequencies. This mode propagates nearly perpendicular tothe ambient magnetic field $\left(k_{\parallel}^2/k^2 \ll m/M_2\right.$, m and M_2 are the masses of the electron and ion of species 2 respectively such that $M_1 < M_2$) and also satisfies the relation $\omega/k_{\parallel} \gg v_e$

A third possibility (which is analogous to the decay of plasma waves at a quarter critical density) is the decay of the pump wave into two ion-ion hybrid modes of lower frequencies. In order to satisfy the resonant conditions for such a decay process, it is necessary to take account of the spatial inhomogeneity of the pump wave (the widely used dipole approximation is not valid) and the k- spectrum of the waves is restricted.

In the following sections we have studied the various possibilities and obtained quantitative expressions for the threshold powers and growth rates. Sections 5.2.1 and 5.2.2 are devoted to considering the 'cold' ion Bernstein and the ion acoustic modes respectively and the calculations have been made under the dipole approximation. In section 5.3 we have considered the decay of the pump into two ion-ion hybrid modes and the mathematical formalism is necessarily different from the other two cases, to account for the inhomogeneity in the pump mode. Section 5.4 contains a brief discussion of our results and their applications.

5.2 Derivation of the dispersion relation

Let us consider a homogeneous plasma consisting of electrons and two ion species embedded in a uniform magnetic field of strength $\underset{\sim}{B}_0 = (0,0,B_0)$. Let us represent a large amplitude uniform pump wave by $\underset{\sim}{E} = \underset{\sim}{E}_0 \cos\omega_0 t$ where ω_0 is close to the Buchsbaum frequency ω_B given by

$$\omega_B = \left(\frac{\omega_{p1}^2 \Omega_2^2 + \omega_{p2}^2 \Omega_1^2}{\omega_{p1}^2 + \omega_{p2}^2} \right)^{1/2} \qquad (5.3)$$

where ω_{pj} is the plasma frequency of the j^{th} ion species. Thus at equilibrium we have a homogeneous plasma with electrons and the two ion species oscillating at the frequency ω_0 under the influence of the pump field. We now wish to study the stability of this equilibrium against low frequency perturbations characterized by $(\omega, \underset{\sim}{k})$. In general, if this low frequency perturbation corresponds to a natural mode $(\omega_L, \underset{\sim}{k}_L)$ of the plasma, then the pump wave will decay into another ion ion hybrid mode $(\omega_H, \underset{\sim}{k}_H)$ and the low frequency mode $(\omega_L, \underset{\sim}{k}_L)$ satisfying the resonant conditions.

$$\begin{aligned} \omega_0 &= \omega_H + \omega_L \\ \underset{\sim}{k}_0 &= 0 = \underset{\sim}{k}_H + \underset{\sim}{k}_L \end{aligned} \qquad (5.4)$$

We now require a general dispersion relation that depicts the appropriate non-linear corrections of the parametric decay process. To this extent a general dispersion relation was derived by Satya et.al.[3] and a brief account of the derivation of the dispersion relation will be discussed in the Appendix B. We shall write the final dispersion relation in the form

$$\epsilon(\omega) = J_1^2(r_{12})\,\chi_1(\omega)\,\chi_2(\omega)\,\chi_e(\omega)\left[\frac{1}{\epsilon(\omega+\omega_0)}+\frac{1}{\epsilon(\omega-\omega_0)}\right] \tag{5.5}$$

where $\epsilon(\omega+n\omega_0) = 1+\chi_1(\omega+n\omega_0)+\chi_2(\omega+n\omega_0)$
$$+\chi_e(\omega+n\omega_0)$$

$n = 0, \pm 1$ and the χ_j's represent the susceptibility functions of the j th species. The argument of the Bessel function J_1 is given by $r_{12}=(\lambda^2+\mu^2)^{1/2}$ with λ and μ defined by,

$$\underset{\sim}{k}\cdot\left(\underset{\sim}{R}_{01}-\underset{\sim}{R}_{02}\right)=\lambda\sin\omega_0 t+\mu\cos\omega_0 t$$

$\underset{\sim}{R}_{oj}$ $(j = 1,2)$ is the excursion length for the j^{th} ion species under the influence of the pump field. For $r_{12} = 0$ equation (5.5) reduces to $\epsilon(\omega)=0$ which is the linear dispersion relation of the low frequency mode. The R.H.S. of equation (5.5) gives the non linear corrections for the coupling of the pump wave and the ion ion hybrid mode.

5.2.1 Decay into 'Cold' Ion-Bernstein Mode.

First we consider a cold, homogeneous, magnetized plasma and solve equation (5.5) by suitably substituting for the various expressions. Using a simple three fluid model (electrons and two ion species), the susceptibilities for the low frequency mode can be written as

$$\chi_j(\omega) = \omega_{pj}^2\left(1+i\nu_i/\omega\right)/\Omega_j^2 \tag{5.6a}$$

$$\chi_e(\omega) = -\frac{k_{\parallel}^2}{k^2}\,\omega_{pe}^2\Big/\omega^2\left(1+i\nu_e/\omega\right) \tag{5.6b}$$

where $\nu_j\,(j = e, 1, 2)$ represent the collision frequency of the j^{th} species (it can also be used to mock up the Landau effects phenomenologically). In the derivation of expressions (5.6a, b), the perpendicular electron and the parallel ion motions are neglected and we have retained only the principal contributions that constitute the 'cold' ion Bernstein mode. This mode propagates nearly perpendicular to the ambient magnetic field and satisfies the condition

$$\frac{(\nu_e - \nu_{eff})^2}{4\Omega_e\Omega_{eff}} \ll \frac{k_{\parallel}^2}{k^2} \ll \frac{m}{M_2}$$

ν_{eff} and Ω_{eff} are defined as

$$\nu_{eff} = \frac{\nu_1\Omega_2\alpha + \nu_2\Omega_1(1-\alpha)}{\Omega_2\alpha + \Omega_1(1-\alpha)}\;;\; \Omega_{eff} = \frac{\Omega_1\Omega_2}{\Omega_2\alpha + \Omega_1(1-\alpha)} \tag{5.7}$$

Substituting equation (5.6) in (5.5) we get

$$\omega^2 + 2i\omega\nu_L - \omega_L^2 = -J_1^2(\gamma_{12})\frac{\omega_{pe}^4\alpha(1-\alpha)\Omega_{eff}}{\Omega_e\Omega_1\Omega_2}\frac{k_{\parallel}^2}{k^2}$$

$$\times\left[\frac{1}{\epsilon(\omega+\omega_0)}+\frac{1}{\epsilon(\omega-\omega_0)}\right] \qquad (5.8)$$

where $\nu_L = (\nu_e + \nu_{eff})/2$ is the linear damping rate of the low frequency mode. We have

$$\chi_j(\omega\pm\omega_0) = -\frac{\omega_{pj}^2}{(\omega\pm\omega_0+i\nu_j)^2-\Omega_j^2}\,\frac{\omega\pm\omega_0+i\nu_j'}{\omega\pm\omega_0} \qquad (5.9)$$

For $|\omega+i\nu_j| \ll |\omega-\Omega_j|$ we can write equation((5.9) as

$$\chi_j(\omega\pm\omega_0) \simeq \frac{\omega_{pj}^2}{\omega_0^2-\Omega_j^2}\left[1\mp\frac{\omega}{\omega_0-\Omega_j}\mp\frac{\nu_j\Omega_j}{\omega_0(\omega_0-\Omega_j)}\right] \qquad (5.10)$$

using equation (5.10), we can express $\epsilon(\omega\pm\omega_0)$ as

$$\epsilon(\omega\pm\omega_0) = (\delta\pm\omega\pm i\nu_H)D \qquad (5.11)$$

where

$$D = \sum_j \frac{\omega_{pj}^2}{(\omega_0^2-\Omega_j^2)(\omega_0-\Omega_j)} \qquad (5.12)$$

δ is an approximate measure of the deviation of the pump frequency ω_0 from the Buchsbaum frequency ω_B and is defined as

$$\delta = -\frac{(\omega_{p1}^2+\omega_{p2}^2)(\omega_0^2-\omega_B^2)}{D(\omega_0^2-\Omega_1^2)(\omega_0^2-\Omega_2^2)} \qquad (5.13)$$

Similarly, ν_H is the effective damping rate of ion-ion hybrid mode and is given by,

$$\nu_H = \frac{1}{D}\sum_j \frac{\nu_j \Omega_j \omega_{pj}^2}{\omega_0(\omega_0-\Omega_j)(\omega_0^2-\Omega_j^2)} \tag{5.14}$$

Substituting (5.11) in (5.8) we get,

$$\left(\omega^2 + 2i\omega\nu_L - \omega_L^2\right)\left((\omega+i\nu_H)^2 - \delta^2\right) = K\delta \tag{5.15}$$

where
$$K = \frac{2J_1^2(r_{12})\,\omega_{pe}^4\,\alpha(1-\alpha)\,\Omega_{eff}}{\Omega_e\Omega_1\Omega_2 D}\frac{k_{\parallel}^2}{k^2} \tag{5.16}$$

and is proportional to the applied power through the argument of the Bessel function. We shall now solve equation (5.15) for two types of solutions viz. purely growing and oscillatory growing modes.

5.2.1a Purely growing mode

In this case the real part of the frequency vanishes and we can write $\omega = iy$, y being real. Substituting this in equation (5.15) we get,

$$y^2 + 2y\nu_L + \omega_L^2 = \frac{K\delta}{(y+\nu_H)^2+\delta^2} \tag{5.17}$$

Equation (5.17) admits solutions only for $\delta > 0$; thus a purely growing mode can only be excited by the pump when its frequency is slightly higher than the Buchsbaum frequency. For $y \to 0$ we can easily obtain the threshold power for excitation as

$$K_c = \omega_L^2(\nu_H^2+\delta^2)/\delta \tag{5.18}$$

This is independent of the low frequency damping rate ν_L and assumes the minimum value,

$$K_{min} = 2\omega_L^2 \nu_H \quad \text{at} \quad \delta = \nu_H \tag{5.19}$$

In order to get the actual pump power it is more instructive to express the above relation in terms of the field amplitudes. However, since the general expression in that case is quite cumbersome, we will write it down for special orientations of the electric vector of the pump wave.

Thus for $\underset{\sim}{E}_0$ parallel to the magnetic field $\underset{\sim}{B}_0$ we get the threshold electric field as,

$$E_{OB}^2 = \frac{4\nu_H \Omega_e^2 \Omega_1 \Omega_2 \omega_0^4 M_1^2 M_2^2 D}{e^2 \omega_{pe}^2 \alpha(1-\alpha)(M_1 - M_2)^2 k_{\parallel}^2} \tag{5.20}$$

For $\underset{\sim}{E}_0 \perp \underset{\sim}{B}_0$ and $\underset{\sim}{E}_0 \parallel \underset{\sim}{k}_\perp$ we get,

$$E_{OB}'^2 = E_{OB}^2 \, \frac{k_{\parallel}^2 (\omega_0^2 - \Omega_1^2)^2 (\omega_0^2 - \Omega_2^2)^2}{k^2 \omega_0^4 (\omega_0^2 + \Omega_1 \Omega_2)^2} \tag{5.21}$$

And for $\underset{\sim}{E}_0 \parallel \underset{\sim}{k} \times \underset{\sim}{B}_0$ we get

$$E_{OB}''^2 = E_{OB}'^2 M_1 M_2 (\omega_0^2 + \Omega_1 \Omega_2)^2 / (M_1 + M_2)^2 \Omega_1 \Omega_2 \omega_0^2 \tag{5.22}$$

Next, we can write down the maximum growth rate near the threshold region as

$$\gamma_{max} \simeq \frac{K - K_{min}}{2(\omega_L^2 + 2\nu_L \nu_H)} \tag{5.23}$$

Far from threshold $(\gamma > \nu_L, \nu_H, \omega_L)$ we get the usual cube root law[11]

$$\gamma_{max} \simeq (K/3)^{1/3} \quad \text{at} \quad \delta \simeq (K/2)^{1/3} \tag{5.24}$$

In the intermediate region we get the following behaviour,

For $\omega_L \gg \nu_H$ and $\omega_L^3 \gg K \gg K_{min}$, we have

$$\gamma_{max} \sim K/2\omega_L^2 \quad \text{at} \quad \delta \simeq K/2\omega_L^2 \tag{5.25}$$

For $\omega_L \ll \nu_H$ and $\nu_H^3 \gg K \gg K_{min}$, we have

$$\gamma_{max} \sim (K/2\nu_H)^{1/2} \quad \text{at} \quad \delta \simeq \nu_H \tag{5.26}$$

5.2.1b Oscillatory growing solution

When $\omega = x + iy$ and $x \neq 0$ then the dispersion relation (5.15) can be separated into real and imaginary parts to give

$$x^2 - y^2 - 2y\nu_L - \omega_L^2 = K\delta\{x^2 - (y+\nu_H)^2 - \delta^2\}/F(x,y) \tag{5.27}$$

$$2x(y+\nu_L) = -2Kx\delta(y+\nu_H)/F(x,y) \tag{5.28}$$

where $$F(x,y) = \{x^2 - \delta^2 - (y+\nu_H)^2\}^2 + 4x^2(y+\nu_H)^2 \tag{5.29}$$

From equation (5.28) we find that for $x \neq 0$, δ has to be negative to give a positive growth rate. The threshold for this instability can again be found by letting $y \to 0$. The frequency $x = x_c$ at the threshold power can be obtained from equations (5.27) to (5.29) and is given by

$$x_c = \pm \left\{ \frac{1}{\nu_H + \nu_L} \left(\nu_L(\nu_H^2 + \delta^2) + \nu_H \omega_L^2 \right) \right\}^{1/2} \tag{5.30}$$

and the threshold power is

$$K_c(\delta) = -\frac{\nu_H \nu_L}{\delta} \left\{ 4\delta^2 + \frac{(\nu_H^2 + 2\nu_H\nu_L + \omega_L^2 - \delta^2)^2}{(\nu_H + \nu_L)^2} \right\} \tag{5.31}$$

Expression (5.31) can be minimized with respect to δ and the expression for K_{min} examined in various limits. We have listed these results in Table 5.1, where for the sake of completeness we have also included the results for the purely growing case. The behaviour of the maximum growth rate is analogous to case (5.2.1a) and far from threshold value the usual cube root law is found to hold.

5.2.2. Decay into Ion - Acoustic Mode

The low frequency ion-acoustic mode propagating parallel to the magnetic field can be easily excited in a warm plasma for $v_e \gg \omega/k_{\parallel} \gg v_j$ and $k_{\parallel}/k \gg \omega/\Omega_j$. The pertinent susceptibilities are now given as

$$\chi_j = -\frac{k_{\parallel}^2}{k^2} \frac{\omega_{pj}^2}{\omega(\omega + i\nu_j)} \tag{5.32}$$

$$\chi_e = \frac{\omega_{pe}^2}{k^2 v_e^2} \tag{5.33}$$

Substituting equations (5.32) and (5.33) into the general dispersion relation (5.5) we get

$$\left(\omega^2+2i\omega\nu_L-\omega_L^2\right)\left(\delta^2-(\omega+i\nu_H)^2\right)=K\delta \tag{5.34}$$

where ω_L and ν_H are defined in equations (5.1) and (5.14) respectively. ν_L and K are given by

$$\nu_L = k_{\parallel}^2\left(c_1^2\alpha\nu_1+c_2^2(1-\alpha)\nu_2\right)/2\omega_L^2 \tag{5.35}$$

$$K = 2J_1^2(r_{12})\,\frac{\omega_{pe}^4\alpha(1-\alpha)}{\omega_L^2 D}\,\frac{m^2}{M_1M_2}\,\frac{k_{\parallel}^4}{k^4} \tag{5.36}$$

The analysis is now exactly analogous to that of section 5.2.1 and the results are displayed in Table 5.1. It is to be noted, however, that for the purely growing mode one requires $\delta<0$ (i.e. $\omega_0<\omega_B$) for instability in contrast to the 'cold' ion Bernstein case where the condition was $\delta>0$; whereas for the oscillatory growing solution the reverse conditions hold good for both the cases i.e. $\delta>0$ for ion acoustic mode and $\delta<0$ for the 'cold' ion Bernstein mode.

5.3 Decay into Two Ion-Ion Hybrid Modes

We now consider a decay instability in which the incident pump wave decays into two lower frequency

TABLE 5.1 - Threshold powers for decay of an ion ion hybrid wave into 'cold' ion Bernstein mode and into slow acoustic mode.

		'Cold' ion Bernstein mode $\underset{\sim}{E}_0 \parallel \underset{\sim}{B}_0$	Slow ion acoustic mode $\underset{\sim}{E}_0 \parallel \underset{\sim}{B}_0$
$Re\,\omega \neq 0$ Oscillatory Growing Instability	$\omega_L \gg \nu_H$	$E_{0B}^2 = \dfrac{8\nu_L \nu_H \Omega_1 \Omega_2 \Omega_e^{3/2} \omega_0^4 D}{e^2 \omega_{pe}^4 \alpha(1-\alpha) \Omega_{eff}^{1/2}} \times \dfrac{M_1^2 M_2^2}{(M_1-M_2)^2} \dfrac{k}{k_{\parallel}^3}$	$E_{0A}^2 = \dfrac{8\left(c_1^2\alpha + (1-\alpha)c_2^2\right)^{3/2} \nu_L \nu_H \omega_0^4}{e^2 \omega_{pe}^4 \alpha(1-\alpha)} \times \dfrac{M_1^3 M_2^3 D}{m^2 (M-M_2)^2} \dfrac{k^4}{k_{\parallel}^3}$
	$\nu_H \gg \omega_L$	$E_{0B}^2 = \dfrac{32\sqrt{3}}{9} \dfrac{\nu_L \nu_H^2 \Omega_1 \Omega_2 \Omega_e \omega_0^4 D}{e^2 \omega_{pe}^4 \alpha(1-\alpha) \Omega_{eff}} \times \dfrac{M_1^2 M_2^2}{(M_1-M_2)^2} \dfrac{k^2}{k_{\parallel}^4}$	$E_{0A}^2 = \dfrac{32\sqrt{3}}{9} \dfrac{\left(c_1^2\alpha + (1-\alpha)c_2^2\right)}{e^2 \omega_{pe}^4 \alpha(1-\alpha)} \times \dfrac{\nu_L \nu_H^2 \omega_0^4 M_1^3 M_2^3 D}{m^2 (M_1-M_2)^2} \dfrac{k^4}{k_{\parallel}^4}$

(Table Continued)

$Re\,\omega = 0$ Purely growing Instability	$E_{0B}^2 = \dfrac{4\nu_H \Omega_e^2 \Omega_1 \Omega_2 \omega_0^4 M_1^2 M_2^2 D}{e^2 \omega_{pe}^4 \alpha(1-\alpha)(M_1-M_2)^2 k_{\parallel}^2}$	$E_{0A}^2 = \dfrac{4\left(\alpha c_1^2 + c_2^2(1-\alpha)\right)\nu_H \omega_0^4 D}{e^2 \omega_{pe}^4 \alpha(1-\alpha)} \times \dfrac{M_1^3 M_2^3}{m^2 (M_1-M_2)^2} \dfrac{k^4}{k_{\parallel}^2}$
Remarks	When $\underset{\sim}{E}_0 \parallel \underset{\sim}{k}_{\perp}$ $E_{0\,A,B}'^2 = E_{0\,A,B}^2\, \xi$ Where $\xi = \dfrac{k_{\parallel}^2}{k^2} \dfrac{(\omega_0^2-\Omega_1^2)^2(\omega_0^2-\Omega_2^2)^2}{\omega_0^4(\omega_0^2+\Omega_1\Omega_2)^2}$ When $\underset{\sim}{E}_0 \parallel \underset{\sim}{k} \times \underset{\sim}{B}_0$ $E_{0\,A,B}''^2 = E_{0\,A,B}'^2\, \zeta$ Where $\zeta = \dfrac{(\omega_0^2+\Omega_1\Omega_2)^2}{\Omega_1\Omega_2\omega_0^2} \dfrac{M_1 M_2}{(M_1+M_2)^2}$	

ion-ion hybrid waves which can subsequently get absorbed in the plasma and thus lead to plasma heating. An instability of this kind is always dependent upon finite pump wave length, and it is not possible to satisfy the wave number and frequency matching conditions under the dipole approximation. In order to introduce the finite pump wavelength, it is necessary to take account of the electron motion in the ion-ion hybrid mode. Since electron cyclotron frequency, Ω_e is much greater than the ion ion hybrid frequency, electrons will be tightly bound to the magnetic field and only their motion parallel to the magnetic field will be important. For the ions, on the other hand, the perpendicular motion is the important one. Thus the susceptibilities for the ions and the electrons in this case are given by,

$$\chi_j = -\frac{k_\perp^2}{k^2}\frac{\omega_{pj}^2}{\omega^2-\Omega_j^2} \tag{5.37}$$

and $$\chi_e = -k_\parallel^2\omega_{pe}^2/k^2\omega^2$$

where $k_\perp$ refers to the wave vector perpendicular to the magnetic field. The dispersion relation for the ion-ion hybrid mode can be written down as

$$-\frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega^2}-\frac{k_\perp^2}{k^2}\left(\frac{\omega_{p1}^2}{\omega^2-\Omega_1^2}+\frac{\omega_{p2}^2}{\omega^2-\Omega_2^2}\right)=0 \tag{5.38}$$

where we have assumed quasi neutrality. Equation (5.38) can be expressed as a fourth order algebric equation in ω. The exact solution of this fourth order equation is quite involved and as such we will make an approximation right in the beginning. We will consider a plasma consisting of a light ion species and another heavy ion species such that $\Omega_1^2 \gg \omega^2 \gg \Omega_2^2$. Under this approximation equation (5.38) can be solved easily to give

$$\omega^2 = \omega_B^2\left(1 + \frac{k_{\parallel}^2}{k^2}\beta\right) \tag{5.39}$$

and $$\beta = M_2/m(1-\alpha) \tag{5.40}$$

In deriving (5.39) we get the following condition for the angle of propagation θ,

$$\frac{M_2}{M_1}\frac{\alpha}{1-\alpha} \gg \beta\cos^2\theta \gg \frac{\alpha}{1-\alpha}\frac{M_1}{M_2} \tag{5.41}$$

where $\cos\theta = k_{\parallel}/k$.

Now the pump wave and the other two decay modes have to satisfy the frequency and the wave vector matching conditions. We denote the pump wave by suffix a and the two decay modes by b and c respectively. Then the wave vector matching condition is

$$\underset{\sim}{k}_a = \underset{\sim}{k}_b + \underset{\sim}{k}_c \tag{5.42}$$

From equation (5.39 the frequency matching condition can be written as

$$\left(1+\frac{k_{\parallel a}^{2}}{k_{a}^{2}}\beta\right)^{1/2}=\left(1+\frac{k_{\parallel b}^{2}}{k_{b}^{2}}\beta\right)^{1/2}+\left(1+\frac{k_{\parallel c}^{2}}{k_{c}^{2}}\beta\right)^{1/2} \quad (5.43)$$

In order for (5.43) to be satisfied $k_{\parallel a}^{2}\beta/k_{a}^{2}$ has to be greater than 3, that is $\omega_a > 2\omega_\beta$. It has been shown by Ott[12], for the case of lower hybrid waves (where similar equations occur) that solutions to (5.42) and (5.43) exist if the condition $\omega_a > 2\omega_\beta$ is satisfied.

To calculate the growth rate γ_0 we note that it is no longer possible to use the general dispersion relation (5.5) as it was derived under the dipole approximation. Dispersion relations with finite pump wavelength have been derived previously for particular cases. Jackson[13] has treated the case of decay into two plasma waves at quarter critical density for an EM pump wave oscillating at twice the plasma frequency. His results are, however, valid only for $|k_0/k| \ll 1$ (where $\underset{\sim}{k}_0$ and $\underset{\sim}{k}$ are the wave vectors of the pump wave and one of the decay modes respectively) and for an EM pump. Similarly the generalized dispersion relation of Drake et. al.[14] suffers from the fact that the nonlinearity arising in the continuity equation ($\underset{\sim}{V}_0 \cdot \underset{\sim}{\nabla} n$) has not been included and these are

are important when the decay waves are both ES. For magnetized plasmas, the recent analysis of Ott[12] for decay of a lower hybrid mode is most complete in this sense, and since the ion-ion hybrid mode has close analogies with the lower hybrid mode, it is most convenient to use the method of Ott for our analysis.

Basically we assume that the two modes $(\omega_l, \underset{\sim}{k}_l)$ and $(\omega_c, \underset{\sim}{k}_c)$ couple to produce second order density perturbations $n^{(2)}(\omega_a, \underset{\sim}{k}_a)$ with $\omega_a = \omega_l + \omega_c$ and $\underset{\sim}{k}_a = \underset{\sim}{k}_l + \underset{\sim}{k}_c$. In this ordering scheme the first order perturbations $n^{(1)}(\omega_a, \underset{\sim}{k}_a)$, if considered alone would lead to the linear dispersion relation, while the inclusion of second order perturbations would give the proper non-linear correlations to the dispersion relation describing parametric decay process.

In order to calculate the growth rate γ_0 we first consider a cold, homogeneous and collisionless plasma. The effects of collisions will be introduced later. For the ion-ion hybrid mode under consideration (i.e. with one light ion species and the other heavy ion species), one of the ion species (the heavier one; we refer here by mass M_2) can be considered to be unmagnetized while the lighter ion species (with mass M_1) and the electrons are magnetized. For the heavy ion species we write down the continuity equation and the equation of motion as follows,

$$\frac{\partial n_2}{\partial t} + \nabla \cdot (n_2 \mathbf{V}_2) = 0 \tag{5.44}$$

$$\frac{\partial \mathbf{V}_2}{\partial t} + \mathbf{V}_2 \cdot \nabla \mathbf{V}_2 = -\frac{e}{M_2} \nabla \phi \tag{5.45}$$

where n_2 and V_2 are the perturbations in density and the velocity for the ion species 2, and ϕ is the perturbed potential. From equations (5.44) and (5.45) we can calculate the second order density perturbation $n_2^{(2)}$ for the ion species under consideration and we get,

$$n_2^{(2)}(\omega_a, \mathbf{k}_a) = \eta_2(\omega_a, \mathbf{k}_a)\,\phi(\omega_b, \mathbf{k}_b)\,\phi(\omega_c, k_c) \tag{5.46}$$

where

$$\eta_2(\omega_a, \mathbf{k}_a) = \frac{n_{02} e^2}{M_2^2 \omega_a^2 \omega_b^2 \omega_c^2}\Big[\omega_a \omega_c\, \mathbf{k}_a \cdot \mathbf{k}_c k_b^2 + \omega_b \omega_c (\mathbf{k}_a \cdot \mathbf{k}_b)(\mathbf{k}_b \cdot \mathbf{k}_c) + (a \leftrightarrow b)\Big] \tag{5.47}$$

n_{02} is the equilibrium density for the ion species 2. The electrons and the light ion species are magnetized so that their motion perpendicular to the magnetic field consists of the polarization drift and the $\mathbf{E}\mathbf{X}\mathbf{B}$ drift and the total velocity can be written as

$$\mathbf{V}_j = \mathbf{V}_{\parallel j} + \mathbf{V}_{Pj} + \mathbf{V}_{E\times Bj} \tag{5.48}$$

where suffix j refers to either electrons or ion species 1. The parallel and the perpendicular velocities for the j-Th species can be written down as

$$\frac{\partial V_{\parallel j}}{\partial t} + \mathbf{V}_j \cdot \nabla V_{\parallel j} = -\frac{e_j}{M_j}\frac{\partial \phi}{\partial z} \tag{5.49}$$

$$\underset{\sim}{V}_{E\times Bj} = -\frac{c}{B_0}\underset{\sim}{\nabla}\phi\times\hat{z}_0 \tag{5.50}$$

$$\underset{\sim}{V}_{Pj} = -\frac{c^2 M_j}{e_j B_0^2}\left(\frac{\partial}{\partial t}+\underset{\sim}{V}_j\cdot\underset{\sim}{\nabla}\right)\underset{\sim}{\nabla}_\perp\phi \tag{5.51}$$

where $\hat{z}_0$ is a unit vector along the z axis. For $j\equiv 1$ $e_j = e$ and $M_j = M_1$ and for $j\equiv$ electrons $e_j = -e$ and $M_j = m$. The continuity equation for the j^{th} species is written as

$$\frac{\partial n_j}{\partial t}+\underset{\sim}{\nabla}\cdot\left(n_j\underset{\sim}{V}_j\right) = 0 \tag{5.52}$$

using equations (5.49) to (5.52) the second order density perturbations for the j^{th} species can be obtained

$$n_j^{(2)}(\omega_a,\underset{\sim}{k}_a) = \eta_j(\omega_a,\underset{\sim}{k}_a)\,\phi(\omega_b,\underset{\sim}{k}_b)\,\phi(\omega_c,\underset{\sim}{k}_c) \tag{5.53}$$

where

$$\eta_j(\omega_a,\underset{\sim}{k}_a) = \frac{n_{oj}e_j^2}{M_j^2\omega_a}\Bigg[\frac{k_{za}k_{zb}k_{zc}}{\omega_b\omega_c}\left(\frac{k_{za}}{\omega_a}+\frac{k_{zb}}{\omega_b}+\frac{k_{zc}}{\omega_c}\right)$$
$$-\frac{i}{\Omega_j}\left(\hat{z}_0\cdot\underset{\sim}{k}_b\times\underset{\sim}{k}_c\right)\left(\frac{k_{zb}}{\omega_b}-\frac{k_{zc}}{\omega_c}\right)\left(\frac{k_{za}}{\omega_a}+\frac{k_{zb}}{\omega_b}+\frac{k_{zc}}{\omega_c}\right)$$
$$+\frac{1}{\Omega_j^2\omega_a}\left\{k_{za}^2\,\underset{\sim}{k}_{\perp b}\cdot\underset{\sim}{k}_{\perp c}-\frac{\omega_b^2}{\omega_a^2}k_{zb}^2\,\underset{\sim}{k}_{\perp a}\cdot k_{\perp c}+(a\leftrightarrow b)\right\}+\frac{\omega}{\Omega_j^4}\left(k_{\perp b}\times k_{\perp c}\right)^2\Bigg] \tag{5.54}$$

The first term in (5.54) arises due to the terms quadratic in the motion parallel to the field lines; the second term is due to the coupling of the $\underset{\sim}{E}\times\underset{\sim}{B}$ drift with the motion parallel to the field lines; the third term is due to the coupling of polarization drift with the motion parallel to the field lines; and the fourth term is due to the terms quadratic in the polarization drift.

Now, we shall make use of the Poisson's equation for the second order density perturbations which can be written down as,

$$i\left\{\frac{\partial}{\partial\omega}\underset{\sim}{k}\cdot\underset{\approx}{\epsilon}\cdot\underset{\sim}{k}\right\}_{\omega_l,c}\frac{\partial\phi(\omega_l,c)}{\partial t}=4\pi\rho^{(2)}(\omega_l,c) \qquad (5.55)$$

where $\underset{\approx}{\epsilon}$ is the dielectric function and $\rho^{(2)}$ is given by

$$\rho^{(2)}=e\left(n_1^{(2)}+n_2^{(2)}-n_e^{(2)}\right) \qquad (5.56)$$

From equations (5.55) and (5.56) we obtain the growth rate as

$$\gamma_0^2=\frac{16\pi^2e^2|\phi(\omega_a)|^2\left|\eta_1(\omega_l,\underset{\sim}{k}_l)+\eta_2(\omega_l,\underset{\sim}{k}_l)-\eta_e(\omega_l,\underset{\sim}{k}_l)\right|^2}{\left(\frac{\partial}{\partial\omega}k^2\epsilon\right)_{\omega_l}\left(\frac{\partial}{\partial\omega}k^2\epsilon\right)_{\omega_c}} \qquad (5.57)$$

In deriving equation (5.57) we have made use of the symmetry relations $\eta_{1,2,e}(\omega_l,\underset{\sim}{k}_l)=\eta_{1,2,e}(\omega_c,\underset{\sim}{k}_c)$ If we consider the case when $k_{\parallel}/k\sim\beta^{-1/2}$ then from equations (5.47) and (5.54) we see that the most important term arises from the electron density perturbation. The second term in η_e is greater than η_1 and η_2 and the other three terms in η_e. Retaining only this term we rewrite (5.57) as

$$\gamma_0^2=\left|\frac{k_a^2e\phi(\omega_a,\underset{\sim}{k}_a)}{2\omega_a m}\right|^2\frac{\omega_{pe}^4\Omega_1^4\left|\hat{z}_0\cdot\underset{\sim}{k}_l\times\underset{\sim}{k}_c\right|^2}{\omega_{p1}^4\Omega_e^2\omega_l\omega_c k_a^4k_l^2k_c^2}$$

$$\left(k_{zl}\omega_c-k_{zc}\omega_l\right)^2\left(\frac{k_{za}}{\omega_a}+\frac{k_{zl}}{\omega_l}+\frac{k_{zc}}{\omega_c}\right)^2 \qquad (5.58)$$

Setting $k_{a,b,c} \sim \beta^{1/2} k_{z\,a,b,c}$, we obtain an order of magnitude estimate for γ_0

$$\gamma_0 \sim \omega_a (k_z r_z) \left(\frac{M_2}{\alpha(1-\alpha) M_1} \right)^{1/2} \tag{5.59}$$

where r_z is the electron excursion length parallel to the magnetic field. To obtain the threshold for the pump power, we need to include damping effects in our analysis. In a homogeneous medium the growth rate in the presence of damping is given by

$$\gamma = \frac{1}{2} \left\{ 4\gamma_0^2 + (\nu_b - \nu_c)^2 \right\}^{1/2} - \frac{1}{2}(\nu_b + \nu_c) \tag{5.60}$$

where ν_b and ν_c are damping rates of the decay modes b and c respectively. The instability threshold thus becomes $\gamma_0^2 > \nu_b \nu_c$. The threshold value for the applied field in this case is given by

$$E_{0r}^2 \sim \frac{\nu_b \nu_c \omega_0^4 \omega_{p1}^4 M_1^2}{e^2 \omega_{pe}^4 \Omega_1^2} \frac{k^2}{k_{\parallel}^4} . \tag{5.61}$$

5.4 Discussion and conclusion

We have investigated in the present chapter the decay instability at the ion ion hybrid resonance in a homogeneous magnetized plasma. Two low frequency modes considered for this purpose are the slow ion-acoustic mode propagating parallel to the magnetic field and the 'cold' ion-Bernstein mode propagating nearly perpendicular to the magnetic field. We have calculated the instability growth rates and threshold powers for the onset of the instability for each case and our results are tabulated in Table 5.1. Figures 5.1 and 5.2 show the variation of the growth rate with ρ^* which is proportional to the applied power, for the ion-acoustic and the 'cold' ion-Bernstein modes respectively. It is easily seen from the figures that the growth rate is higher in ion-acoustic case than the ion-Bernstein case.

A third possibility of decay into two lower frequency ion-ion hybrid waves has also been considered and the growth rate and the threshold power calculated. In this case our analysis is applicable to a plasma with two ion species of considerable mass difference. This restricts the angle of propagation, of the decay modes, with respect to the magnetic field. In this case the instability is found to be dependent on the finiteness of the pump wavelength and the dominant contribution is due to the pump induced electron motion parallel to the magnetic field.

Our present calculations, complement in a sense the earlier work of Satya et.al[3], where the decay of an ion-ion hybrid mode to another ion ion hybrid mode and a drift mode was considered. In fact the growth rates and the threshold powers are quite comparable and therefore

$^*\rho = \gamma_{12}$ defined in equation(5.5).

in an actual experiment one has to evaluate the various competing possibilities. Finally for an order of magnitude estimate we choose some typical parameters characteristic of a laboratory plasma: $n_o = 10^{12}$ per c.c, $\alpha = .5$ $Te = 10^2$ e.v., $B_o = 25$KG. For the 'cold' ion Bernstein and the acoustic mode we consider a deuterium-tritium plasma so that $M_1 = 3.3 \times 10^{-24}$ gm, $M_2 = 5 \times 10^{-24}$ gm. Also we have considered the case $\omega_L \gg \nu_H$ and have taken $\omega_L = 10^7$ rad/sec., $\nu_H = 10^6$ rad/sec., $\nu_L = 10^5$ rad/sec. and $\lambda = .1$ cm. Thus the threshold electric field $\underset{\sim}{E}_o$ in the direction $\underset{\sim}{K} \times \underset{\sim}{B}_o$ is calculated for the 'cold' ion-Bernstein mode and the slow acoustic mode, to be respectively, $E_{oB} \sim 30$ volt/cm and $E_{oA} \sim 15$ volt/cm. For the case of decay into two ion-ion hybrid waves we consider a deutrium-oxygen plasma so that $M_2 = 2.6 \times 10^{-23}$ gm; and the threshold electric field is calculated to be $E_{OI} = 80$ volt/cm.

Thus the above values for the threshold electric fields are well within the regime of present power densities envisaged for r.f. heating experiments and thus the instabilities considered in our work can be easily exploited in an experimental situation.

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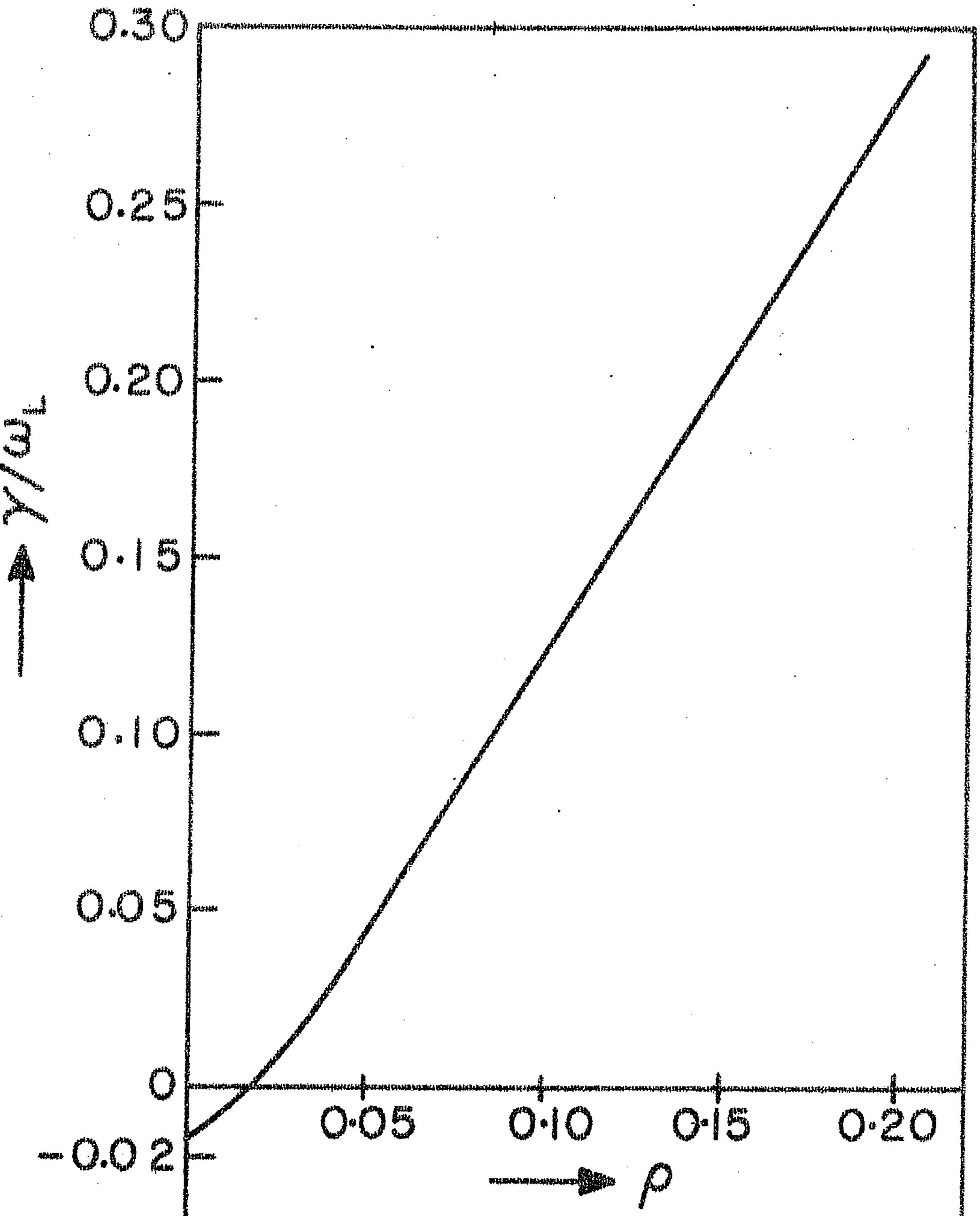

Figure 5.1 - Variation of growth rate γ/ω_L versus ρ which is proportional to the applied power; for the slow ion acoustic case. We have used, here, $\delta/\omega_L = 1.0$, $\nu_H/\omega_L = 0.1$, $\nu_L/\omega_L = 0.01$ and $\omega_L = 10^7$ rad./sec.

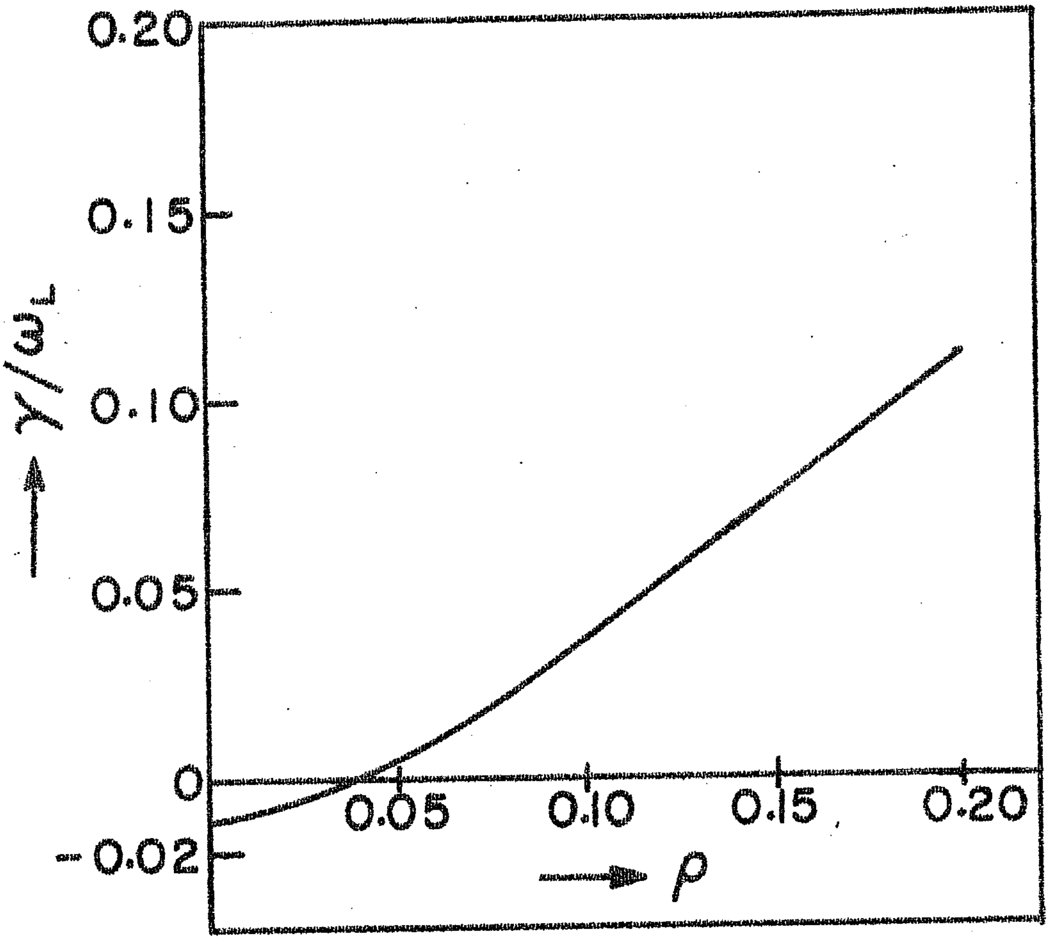

Figure 5.2 - Variation of growth rate γ/ω_L versus ρ which is proportional to the applied power, for the 'cold' ion Bernstein case. We have used $\delta/\omega_L = 1.0$, $\nu_H/\omega_L = 0.1$, $\nu_L/\omega_L = 0.01$ and $\omega_L = 10^7$ rad./sec.

APPENDIX B

B.1 Derivation of the general dispersion relation.

To derive a general dispersion relation the standard technique (as developed by Arnush et. al.[15]) is to transform to the oscillatory frame of each species (to account for the influence of the pump wave on them). Thus if $\underset{\sim}{r}$ and $\underset{\sim}{r}_j'$ are the radius vectors in the laboratory and the oscillating frames respectively we have

$$\underset{\sim}{r}_j' = \underset{\sim}{r} - \underset{\sim}{R}_{oj}(t) \text{ where } \frac{d}{dt}\underset{\sim}{R}_{oj}(t) = \underset{\sim}{V}_{oj} \; ; \quad \underset{\sim}{R}_{oj} \text{ and } \underset{\sim}{V}_{oj}$$

are the excursion lengths and equilibrium velocities of the j^{th} species under the influence of the applied electric field. Thus if $N_j(\underset{\sim}{r}, t)$ and $\bar{N}_j'(\underset{\sim}{r}_j', t)$ are the density perturbations for the j^{th} species in the laboratory and the oscillating frames respectively, then we have

$$\bar{N}_j(\underset{\sim}{r}_j', t) = N_j(\underset{\sim}{r}, t) = N_j(\underset{\sim}{r}_j' + \underset{\sim}{R}_{oj}, t) \qquad \text{(B.1)}$$

We now define

$$\underset{\sim}{k}\,.\,\underset{\sim}{R}_{oj} = \lambda_j \sin\omega_o t + \mu_j \cos\omega_o t \qquad \text{(B.2)}$$

where μ_j and λ_j are the components of the excursion along the wave vector $\underset{\sim}{k}$; they have phase differences of zero and $\pi/2$ respectively to the applied field. Fourier transformation of equation (B.1) gives

$$\bar{N}_j(\underset{\sim}{k},\omega)=\sum_{m,l} i^l J_m(\lambda_j)\, J_l(\mu_j)\, N_j\left(\underset{\sim}{k},\omega+(m-l)\omega_0\right) \tag{B.3}$$

The inverse relation expressing the fluctuations in the oscillating frame of a given species to the laboratory frame can be written as

$$N_j(\underset{\sim}{k},\omega)=\sum_{m',l'} i^{l'} J_{m'}(-\lambda_j)\, J_{l'}(-\mu_j)\, \bar{N}_j\left(\underset{\sim}{k},\omega+(m'-l')\omega_0\right) \tag{B.4}$$

In deriving equations (B.3) and (B.4) we have made use of the identity

$$e^{-ip\sin(\omega_0 t+\alpha)}=\sum_m J_m(p)\, e^{-im(\omega_0 t+\alpha)}$$

The Poisson's equation is given by

$$i\underset{\sim}{k}.\underset{\sim}{E} = 4\pi e(N_1+N_2-N_e) \tag{B.5}$$

We now define the susceptibilities χ_j in the oscillating frame as

$$\bar{N}_j = -ik\chi_j \bar{E}_j/4\pi e_j \tag{B.6}$$

where $\underset{\sim}{E}$ and $\underset{\sim}{E_j}$ are the electric fields in the laboratory and the oscillating frames respectively. e_j is the charge corresponding to the jth species. Equation (B.6) can be used to express the density fluctuations in terms of the electric fields. The electric fields are further transformed to the laboratory frame by means of relation (B.3). This gives,

$$-\frac{1+\chi_1(\omega)}{\chi_1(\omega)}\bar{N}_1(\omega) = \sum_{n=-\infty}^{\infty} J_n(r_{12})\, e^{in\phi_{12}}\, \bar{N}_2(\omega+n\omega_0) + \sum_{n=-\infty}^{\infty} J_n(r_{1e})\, e^{in\phi_{1e}}\, \bar{N}_e(\omega+n\omega_0) \tag{B.7}$$

In equation (B.7) the following definitions are used

$$\lambda = \lambda_j - \lambda_k = r_{jk}\cos\phi_{jk}$$
$$\mu = \mu_j - \mu_k = r_{jk}\sin\phi_{jk}$$

From symmetry considerations we get two more relations connecting the density fluctuations of the different plasma components in their respective oscillating frames.

$$-\frac{1+\chi_2(\omega)}{\chi_2(\omega)}\bar{N}_2(\omega) = \sum_{n=-\infty}^{\infty} J_n(r_{21})\, e^{in\phi_{21}}\, \bar{N}_1(\omega+n\omega_0) + \sum_{n=-\infty}^{\infty} J_n(r_{2e})\, e^{in\phi_{2e}}\, \bar{N}_e(\omega+n\omega_0) \tag{B.8}$$

$$-\frac{1+\chi_e(\omega)}{\chi_e(\omega)}\bar{N}_e(\omega) = \sum_{n=-\infty}^{\infty} J_n(r_{e1})\, e^{in\phi_{e1}}\, \bar{N}_1(\omega+n\omega_0) + \sum_{n=-\infty}^{\infty} J_n(r_{e2})\, e^{in\phi_{e2}}\, \bar{N}_2(\omega+n\omega_0) \tag{B.9}$$

The co-efficients of $\overline{N}_j$ in equations (B.7) to (B.9) form an infinite matric whose determinant set to zero gives us the dispersion relation. To truncate the infinite determinant we assume that the particle excursions are much smaller than the perturbation wavelengths so that the argument of the Bessel functions become very small, that is

$$J_{\pm n}(r_{ij}) \ll 1 \quad n \gtrsim 2 \text{ as } |r_{ij}| \ll 1$$

Also for $\omega \ll \omega_0$ higher order side band terms proportional to $\omega \pm \omega_0$, $n \gtrsim 2$ are non resonant and can be ignored. This reduces the infinite determinant to a 7 x 7 one and this determinant can be solved to get the dispersion relation as,

$$\epsilon(\omega) = J_1^2(r_{12})\,\chi_1(\omega)\,\chi_2(\omega)\,\chi_e(\omega)\left[\frac{1}{\epsilon(\omega+\omega_0)} + \frac{1}{\epsilon(\omega-\omega_0)}\right] \tag{5.5}$$

It is to be noted, from equation (5.5) that only the relative streaming between the two ion species can lead to coupling to ion ion hybrid modes. Physically electron streaming effects drop out of the problem because electron density fluctuations at $\omega \pm \omega_0$ do not couple to ion ion hybrid waves (note that $\chi_e(\omega \pm \omega_0) \simeq 0$).

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CHAPTER 6.

PARAMETRIC CONTROL OF EQUATORIAL SPREAD-F.

6.1 Introduction

In the foregoing chapters, we have discussed a few problems on parametric interaction of waves and have obtained some interesting results which have wide applications in fusion schemes like laser fusion, tokamak etc. In this chapter we shall discuss a parametric interaction process which has a direct relevance to ionosphere. In recent years, ionospheric F-region irregularities have been extensively studied both theoretically and experimentally. These irregularities, known as spread-F (named so because of the diffused trace on the ionogram), were first observed in 1938 by Booker and Wells(1). Subsequently many experimental observations have been made by ground based low frequency radio techniques, UHF radar techniques and _in situ_ rocket and satellite measurements(2-5). The occurrence of spread-F predominates in the two regions - at the equatorial latitudes and high latitudes. It is a relatively rare event in the region between 20° to 40° geometric latitudes. The equatorial and high latitude spread-F are similar kind of phenomena and both involve field aligned irregularities of electron density in the F-region, but it seems likely that the irregularities are produced by different physical mechanisms in the two cases. This is due to some morphological features which distinguishes them as two types. For example the correlation of spread-F with geomagnetic activity is positive at high latitudes and negative at the equator.

In the present chapter we are concerned only with the equatorial spread-F. The equatorial spread-F is a night time phenomenon and generally onsets in the evening hours within half an hour after the sun set. The experimental observations show that these irregularities are essentially field aligned with perpendicular wavelengths varying from a few meters upto a few kilometers.

So far various properties of F-region irregularities have been the subject of both theoretical and experimental investigations by many workers in the recent past and a vast amount of data using various techniques have been gathered, revealing a wealth of information about the occurrence, behaviour and properties of equatorial spread-F irregularities. But even then the processes responsible for producing them and many of their fundamental features are not yet completely understood. The difficulty lies in the fact that so many parameters are to be considered in the study of ionospheric plasma that it becomes almost impossible to take them all into account simultaneously. Because of this complexity, it seems reasonable to believe that there may be many different physical mechanisms operating individually or simultaneously, which are responsible for the generation of spread-F irregularities observed with a wide range of scale sizes perpendicular to the magnetic field and with a characteristic growth rate of more than a few seconds to $\sim$ 30 minutes. Farley et al[2] had reviewed most of the theories on the equatorial spread-F that have been advanced in the past and had listed the reasons pointing out why none of the existing theories of spread-F could adequately account for all of the observed characteristics.

Very recently Balsley et al[6] and Hudson and Kennel[7,8] have come forward with the theories of spread-F irregularities which are dependent on gravity and electron density gradient. In fact, Dungey[9] was the first to suggest that the source of the equatorial F-region irregularities could be a gravitational instability of the underside of the F-layer. In the night time, the gradient of electron density is often steep, below the F-region maxima, and the layer resembles a slab of inhomogeneous plasma supported against gravity by magnetic field, thus fulfilling broadly the light fluid (lower density) supporting a heavy fluid (higher density) description of the classical Rayleigh Taylor instability. Such a configuration is well known to be unstable. With the help of this instability many properties of the long wavelength irregularities (for $\lambda_\perp \gg a_i$ where $\lambda_\perp$ and a_i are the perpendicular wavelength and ion Larmor radius respectively) below the F-region maximum (where the background density gradient ∇n_0 is antiparallel to the acceleration due to gravity) can be explained[6,7]. The R-T instability has a finite-Larmor -radius (FLR) cut off (i.e. stabilized due to FLR effects) at shorter wavelengths of the order of a few hundred meters so that the irregularity scale sizes below a few hundred meters cannot be explained by the R-T instability. Hudson and Kennel[8] have recently proposed that for perpendicular wavelengths below a few hundred meters down to a few tens of meters (so that $\lambda_\perp \gg a_i$ is still satisfied), some of the observations of the equatorial spread-F irregularities could be explained by the density gradient driven collisional drift instability. In contrast to the R-T instability which is stabilized by FLR effects, the collisional drift instability is found to be destabilized by the FLR effects. Further from the analysis of Hudson and Kennel[7,8], it is found that collisions reduce the maximum R-T growth rate and effectively eliminate the collisionless FLR cut off. They inferred that below the F-peak where spread-F predominates, both the drift and

R-T modes, growing on the same zero order density gradient, contribute to the total spread-F wavelength spectrum, with the R-T mode dominating at long wavelengths (of the order of few kilometers down to a few hundred meters) and collisional drift mode at short wavelengths (of the order of few hundred meters down to a few tens of meters) above the ion Larmor radius.

In all the observations discussed above, the experimenters have been restricted to a passive role of observing the instability as it naturally occurs and have not exerted any control over the instabilities. Since in recent times, ionospheric modification experiments[10] have achieved interesting results (particularly for high latitude spread-F) we expect that a similar approach for equatorial F-region irregularities would also yield valuable insight. With such an experiment in mind, we have carried out a theoretical investigation of the influence of a large amplitude oscillating electric field on the dispersion characteristics of both the R-T mode and the collisional drift mode propagating in a plasma that is characteristic of the F-region. The external pump wave can parametrically excite (or stabilize) both the plasma instabilities mentioned above, when the natural conditions in the ionospheric F-layer are below (or above) the linear instability threshold. This suggests the possibility of experimental observation of plasma instabilities in the F-layer under controlled conditions.

Our method of analysis is very similar to that used by Lee et. al.[11] in their investigations of the E-region irregularities. We shall see in later sections that the conditions for significant parametric interaction are (1) the pump wave frequency should be near the local upper hybrid frequency of the F-layer and (2) there should be a finite electric field component of the pump wave perpendicular to the background static magnetic field $\underset{\sim}{B}_0$. The plan of this chapter is as follows. In section 6.2, we have investigated the modification of the Rayleigh Taylor instability due to the pump wave and obtained the modified growth rate and the threshold power for this case. Section 6.3 contains these results for the collisional drift instability case. In section 6.4, we have studied the propagation characteristics of the pump wave in the magnetic equatorial region and we have concluded that the above two conditions can best be met by an ordinary mode circularly polarized pump wave, transmitted at oblique incidence to the ionosphere in the magnetic meridian plane. In section 6.5 we have summarized our results and discussed the validity of the assumptions used in our theory.

We shall treat the problem by using fluid theory which restricts our analysis to perpendicular wavelengths greater than the ion Larmor radius and parallel wavelengths greater than the electron mean free path $\lambda_e = c_e / (\nu_{ei} + \nu_{en})$, which depends on the electron thermal speed c_e and the sum of the electron-ion and electron neutral collision frequencies ν_{ei} and ν_{en} respectively.

6.2 Modification of the Rayleigh Taylor Instability

We shall consider the equatorial geometry. There is a constant vertical density gradient in the positive direction and a gravitational field in the negative direction. The earth's magnetic field $\underset{\sim}{B}_0$ is in the y-direction (north-south). It is straight-forward to derive the main features of the natural instabilities suggested in this region by considering a simplified picture of the F-region of the ionosphere. Thus we shall assume a scalar pressure and neglect the temperature fluctuations. The electrons and ions then satisfy the following set of fluid equations:

$$\frac{\partial N_j}{\partial t} + \underset{\sim}{\nabla} \cdot (N_j \underset{\sim}{V}_j) = 0 \tag{5.1}$$

$$N_j M_j \left(\frac{\partial}{\partial t} + \underset{\sim}{V}_j \cdot \underset{\sim}{\nabla} \right) \underset{\sim}{V}_j = \pm e N_j \left(\underset{\sim}{E} + \frac{1}{c} \underset{\sim}{V}_j \times \underset{\sim}{B}_0 \right) + M_j N_j \underset{\sim}{g} - \underset{\sim}{\nabla} P_j + \underset{\sim}{R}_{ei} + \underset{\sim}{R}_{jn} \tag{5.2}$$

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{E} = 4\pi e (N_i - N_e) \tag{5.3}$$

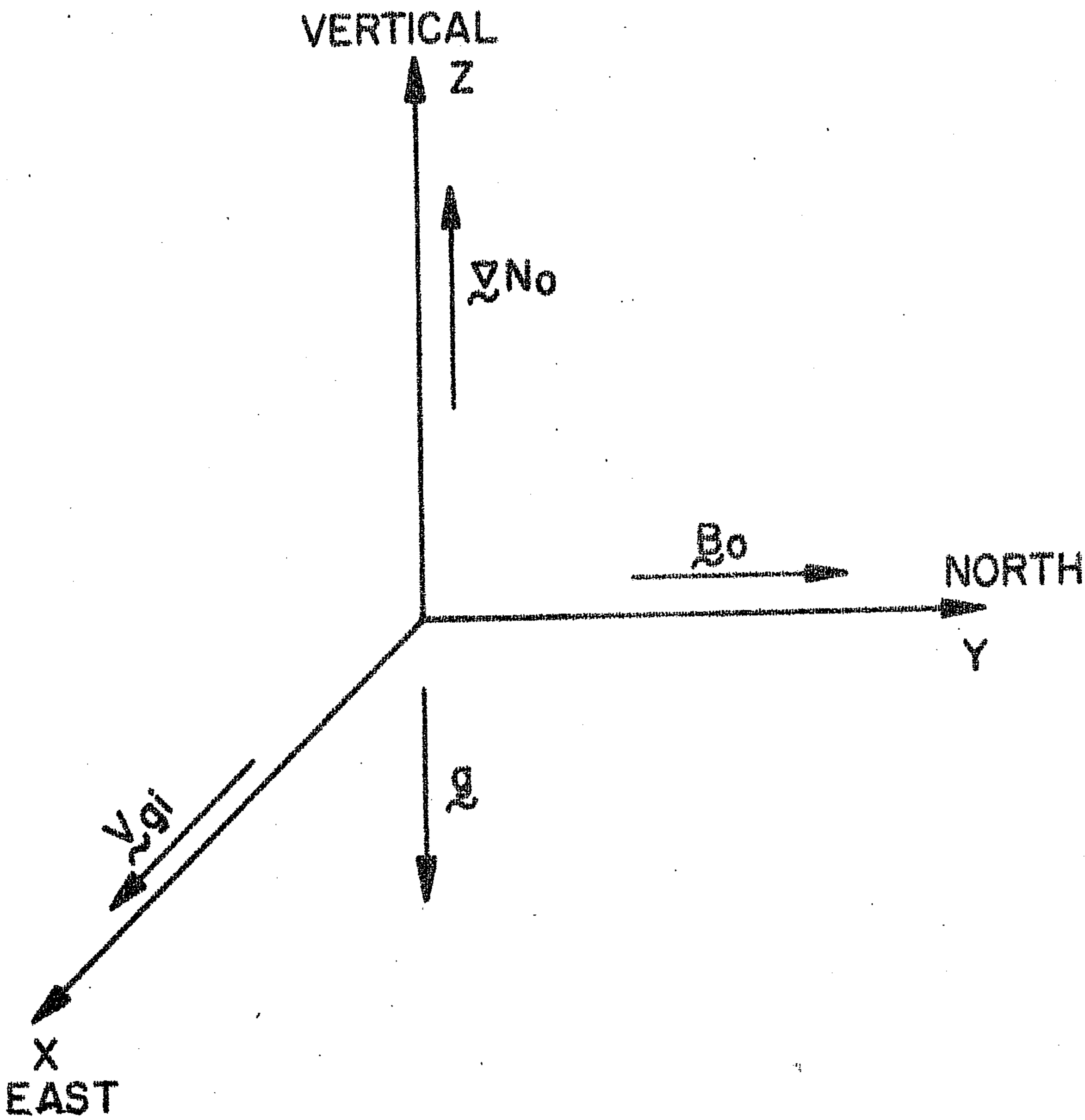

Figure 6.1 - Co-ordinate system appropriate for the magnetic dip equator

where $P_j = N_j T_j$ (6.4a)

$$\underset{\sim}{E} = \underset{\sim}{E}_0(\omega_0, t) + \underset{\sim}{E}'(x, t) \tag{6.4b}$$

$$\underset{\sim}{R}_{ei} = -\underset{\sim}{R}_{ie} = -M_e N_e \nu_{ei}(\underset{\sim}{V}_e - \underset{\sim}{V}_i) \tag{6.4c}$$

$$\underset{\sim}{R}_{en} = N_e M_e \nu_{en} \underset{\sim}{V}_e \tag{6.4d}$$

$$\underset{\sim}{R}_{in} = N_i M_i \nu_{in} \underset{\sim}{V}_i \tag{6.4e}$$

and $M_e = m, \; M_i = M$ (6.4f)

In equations (6.1) to (6.4) N_j and T_j are the number density and temperature of the jth species. $\underset{\sim}{E}_0(\omega_0, t)$ is the electric field component of the large amplitude pump wave of frequency ω_0 and $\underset{\sim}{E}'(x, t)$ is the excited electrostatic field. We shall take the electric field of the large amplitude wave to be approximately spatially uniform (dipole approximation) which is justified when the wavelength of the perturbation is much less than that of the pump wave. ν_{ei}, ν_{en} and ν_{in} are the electron-ion, electron-neutral and ion neutral collision frequencies respectively. The other symbols have their usual meanings.

The equilibrium velocities $\underset{\sim}{V}_{jg}$ and $\underset{\sim}{V}_{j0}$ of the jth species in the presence of gravity and pump field respectively are given by,

$$0 = \pm \frac{e N_0}{c} \underset{\sim}{V}_{jg} \times \underset{\sim}{B}_0 + M_j N_0 \underset{\sim}{g} \tag{6.5}$$

and $$\frac{d}{dt}\underset{\sim}{V}_{jo} = \pm \frac{e}{M_j}\left(\underset{\sim}{E}_o + \frac{1}{c}\underset{\sim}{V}_{jo} \times \underset{\sim}{B}_o\right) \tag{6.6}$$

In equations (6.5) and (6.6) we have neglected the collision terms since in the F-region of interest the collision frequencies are much smaller than the cyclotron frequencies of the particles. From equation (6.5) we can approximate the ion drift as $\underset{\sim}{V}_{ig} = (cM_i/eB_o^2)\,\underset{\sim}{g} \times \underset{\sim}{B}_o$ and neglect the electron drift since it is m/M times smaller than the ion drift.

Next we write down the linearized equations governing the perturbed quantities.

The continuity equation is

$$\frac{\partial}{\partial t}N_j' + (\underset{\sim}{V}_{jg} + \underset{\sim}{V}_{jo})\cdot\underset{\sim}{\nabla}N_j' + N_o\underset{\sim}{\nabla}\cdot\underset{\sim}{V}_j' + \underset{\sim}{V}_j'\cdot\underset{\sim}{\nabla}N_o = 0 \tag{6.7a}$$

The equation of motion for the ions is

$$N_o\left(\partial\underset{\sim}{V}_i'/\partial t + (\underset{\sim}{V}_{ig} + \underset{\sim}{V}_{io})\cdot\underset{\sim}{\nabla}\underset{\sim}{V}_i'\right) = eN_o\left(\underset{\sim}{E}' + \frac{1}{c}\underset{\sim}{V}_i' \times \underset{\sim}{B}_o\right)/M - \underset{\sim}{\nabla}P_i'/M - \nu_{in}N_o\underset{\sim}{V}_i' \tag{6.7}$$

and that for the electrons is

$$N_o\left(\partial\underset{\sim}{V}_e'/\partial t + \underset{\sim}{V}_{eo}\cdot\underset{\sim}{\nabla}\underset{\sim}{V}_e'\right) = -eN_o\left(\underset{\sim}{E}' + \frac{1}{c}\underset{\sim}{V}_e' \times \underset{\sim}{B}_o\right)/m - \underset{\sim}{\nabla}P_e'/m - \nu_e N_o \underset{\sim}{V}_e' \tag{6.8}$$

and the Poisson's equation is written as,

$$\underset{\sim}{\nabla}\cdot\underset{\sim}{E}' = 4\pi e(N_i' - N_e') \tag{6.9}$$

In the above equations the prime denotes the perturbed quantities. In equation (6.7) we have retained only the ion-neutral collision term since the ion electron collision frequency is much smaller than the ion neutral collision frequency. In equation (6.8) $\nu_e = \nu_{en} + \nu_{ei}$. It is to be noted that for the Rayleigh-Taylor mode we shall consider wave propagation perpendicular to $\underset{\sim}{B}_0$ and for $\Omega_e \gg \nu_e$ (which is true for the F-region of interest) we shall neglect the collision term in (6.8) for this mode. However for the upper hybrid mode the collision term is important (as will be evident later) and, therefore, we have written this term in equation (6.8). Since ion motions are neglected at the upper hybrid frequency, we have written the collisional term in equation (6.8) without the ion velocity. In order to solve equations (6.6) to (6.9) we transform to the oscillating frames of the two species. If $\underset{\sim}{r}$ and $\underset{\sim}{r}'$ are the radius vectors in the laboratory and the oscillating frame respectively then we have

$$\underset{\sim}{r}_j' = \underset{\sim}{r} - \underset{\sim}{R}_j(t) \tag{6.10}$$

where $\underset{\sim}{R}_j(t)$ is the excursion length for the jth species under the influence of the pump wave and is given by,

$$\frac{d}{dt} R_j(t) = \underset{\sim}{V}_{j0}(t) \tag{6.11}$$

Let us now define the perturbed quantities in the oscillating frame as,

$$\begin{aligned} N_j'(\underset{\sim}{r}, t) &= \tilde{N}_j'(\underset{\sim}{r}_j', t) \\ \underset{\sim}{V}_j'(\underset{\sim}{r}, t) &= \tilde{\underset{\sim}{V}}_j'(\underset{\sim}{r}_j', t) \\ \underset{\sim}{E}'(\underset{\sim}{r}, t) &= \tilde{\underset{\sim}{E}}_j'(\underset{\sim}{r}_j', t) \end{aligned} \tag{6.12}$$

using equations (6.10) to (6.12) we can write the linearized equations (6.6) to (6.8) in oscillating frame as

$$\frac{\partial}{\partial t}\tilde{N}_j' + \underset{\sim}{V}_{jg}\cdot\underset{\sim}{\nabla}\tilde{N}_j' + N_0\underset{\sim}{\nabla}\cdot\tilde{\underset{\sim}{V}}_j' + \tilde{\underset{\sim}{V}}_j'\cdot\underset{\sim}{\nabla}N_0 = 0 \qquad (6.13)$$

$$N_0\left(\frac{\partial\tilde{\underset{\sim}{V}}_i'}{\partial t} + \underset{\sim}{V}_{ig}\cdot\underset{\sim}{\nabla}\tilde{\underset{\sim}{V}}_i'\right) = \frac{eN_0}{M}\left(\tilde{\underset{\sim}{E}}_i' + \frac{1}{c}\tilde{\underset{\sim}{V}}_i'\times\underset{\sim}{B}_0\right) - \frac{\underset{\sim}{\nabla}\tilde{P}_i'}{M} - \nu_{in}N_0\tilde{\underset{\sim}{V}}_i' \qquad (6.14)$$

and

$$N_0\frac{\partial\tilde{\underset{\sim}{V}}_e'}{\partial t} = -\frac{eN_0}{m}\left(\tilde{\underset{\sim}{E}}_e' + \frac{1}{c}\tilde{\underset{\sim}{V}}_e'\times\underset{\sim}{B}_0\right) - \frac{\underset{\sim}{\nabla}\tilde{P}_e'}{m} - \nu_e N_0\tilde{\underset{\sim}{V}}_e' \qquad (6.15)$$

The susceptibilities χ_i and χ_e are defined in the oscillating frame as

$$\tilde{N}_i' = -ik\chi_i\tilde{E}_i'/4\pi e \quad \text{and} \quad \tilde{N}_e' = ik\chi_e\tilde{E}_e'/4\pi e \qquad (6.16)$$

For the Rayleigh Taylor mode we shall take $k_y = 0$. Then we can readily obtain from equations (6.13) to (6.15) the following expressions for the susceptibilities viz.

$$\chi_i(\omega) = \omega_{pi}^2\left(\bar{\omega} + \frac{\Omega_i k_x}{k^2L}\right)\Big/\left(\tilde{\omega}\Omega_i^2 + k^2c_i^2\bar{\omega}\right) \qquad (6.17)$$

and

$$\chi_e(\omega) = -\omega_{pe}^2\left(\tilde{\omega}' - \frac{\Omega_e k_x}{k^2L}\right)\Big/\left\{\left(\tilde{\omega}'^2 - \Omega_e^2\right)\omega - k^2c_e^2\tilde{\omega}'\right\} \qquad (6.18)$$

where

$$\bar{\omega} = \omega - \underset{\sim}{k}\cdot\underset{\sim}{V}_{ig} + i\nu_{in}\,;\quad \tilde{\omega} = \omega - \underset{\sim}{k}\cdot\underset{\sim}{V}_{ig}\,;\quad \tilde{\omega}' = \omega + i\nu_e$$

$$c_j^2 = \frac{T_j}{M_j}\,;\quad \omega_{pj}^2 = 4\pi e^2N_0/M_j\,;\quad \frac{1}{L} = \frac{1}{N_0}\frac{dN_0}{dz} \quad \text{and}$$

$$\Omega_j = eB_0/M_jc.$$

Now it is necessary to transform back to the laboratory frame in order to use the Poisson's equation. We shall discuss the details of this transformation in the Appendix(C). Since we shall be considering large amplitude pump waves with frequencies matching the upper hybrid frequency, we will have $\omega_0 \gg \omega_{pi}$ and can neglect the effect of the oscillating field on the ions, With this approximation the dispersion relation which includes the effect of the large amplitude EM wave on electrons can be compactly written as (from Appendix C.1).

$$H_i N_i'(\underset{\sim}{k},\omega) = \int \frac{d\omega'}{2\pi} \tilde{\Delta}(\underset{\sim}{k},\omega-\omega') \frac{1}{H_e(\omega')} \int \frac{d\omega''}{2\pi} \Delta(\underset{\sim}{k},\omega'-\omega'') \times N_i'(\underset{\sim}{k},\omega'') \qquad (6.19)$$

where

$$H_i = -(1+\chi_i)/\chi_i \quad \text{and} \quad H_e = -(1+\chi_e)/\chi_e \qquad (6.19a)$$

with $$\tilde{\Delta}(\underset{\sim}{k},\omega) = \Delta(-\underset{\sim}{k},\omega) \qquad (6.20)$$

In the appendix (C.3), we have derived an explicit expression for the transformation operator $\Delta(\underset{\sim}{k},\omega)$. Substituting for $\Delta(\underset{\sim}{k},\omega)$ in equation (6.19) we get

$$H_i(\omega) N_i'(\underset{\sim}{k},\omega) = \sum_{m,n} \frac{J_n(\rho) J_m(\rho) e^{i(m-n)\beta} N_i'(\underset{\sim}{k},\omega+(m-n)\omega_0)}{H_e(\omega+m\omega_0)} \qquad (6.21)$$

where $\rho = |\underset{\sim}{k} \cdot \underset{\sim}{R}_{oe}|$ and β is defined in (6.13). Since ions do not respond at high frequencies (at freq. ω_0 and its multiples) because of their large mass, we can write

$$N_i'(\underset{\sim}{k}, \omega + (m-n)\omega_0) \simeq 0 \quad \text{for} \quad m \neq n \tag{6.22}$$

From equations (6.21) and (6.22) we have

$$H_i(\omega) = \sum_m J_m^2(\rho) / H_e(\omega + m\omega_0) \tag{6.23}$$

We can truncate the infinite terms on the R.H.S. of equation (6.23) by assuming $\rho \ll 1$ i.e. for the electron excursion length in the pump wave field much smaller than the perturbation wavelength. In this approximation, terms for $m \gtrsim 2$ are neglected. Further these terms involve a nonresonant response of electrons to the pump wave. Under these conditions equation (6.23) reduces to

$$1 + \chi_i + \chi_e = -J_1^2(\rho)\chi_i(1+\chi_e)\left[\frac{1}{H_e(\omega+\omega_0)} + \frac{1}{H_e(\omega-\omega_0)}\right] \tag{6.24}$$

In the absence of the pump wave i.e. for $\rho = 0$, equation (6.24) becomes

$$1 + \chi_i + \chi_e = 0 \tag{6.25}$$

Substituting for χ_i and χ_e from equations (6.17) and (6.18) and taking $\Omega_e \gg \omega, \nu_e$ and kC_e we have

$$\frac{\omega_{pi}^2(\bar{\omega}+\Omega_i k_x/k^2L)}{\tilde{\omega}\Omega_i^2+k^2c_i^2\bar{\omega}}-\frac{\omega_{pe}^2k_x}{\omega\Omega_e k^2L}=0 \qquad (6.26)$$

where unity has been neglected compared to other terms. Equation (6.26) gives after some simplification,

$$(\omega+\omega_{*i})(\omega+i\nu_{in})+g/L=0 \qquad (6.27)$$

where $\omega_{*i}=-k_xT_i/M\Omega_iL$. (6.28)

Equation (6.27) is the dispersion relation for the collisional Rayleigh-Taylor mode. Natural instability of this mode in the F-region has been studied in great detail by Hudson and Kennel[7] and by Haerendal[12]. In what follows we shall discuss the effect of the external pump wave on the natural instability of this mode.

For a finite amplitude of the applied oscillating electric field, equation (6.24) gives

$$(\omega+\omega_{*i})(\omega+i\nu_{in})+\frac{g}{L}=J_1^2(\rho)\frac{\omega_{pi}^2}{k^2L^2}\left[\frac{1}{H_e(\omega+\omega_0)}+\frac{1}{H_e(\omega-\omega_0)}\right] \qquad (6.29)$$

where we have taken $\bar{\omega}<\Omega_i/kL$. Using equations (6.18) and (6.19a) we calculate the function within the square bracket and equation (6.29) can be rewritten as

$$\omega^2+(\omega_{*i}+i\nu_{in})\omega+(i\nu_{in}\omega_{*i}+\frac{g}{L}+A)=0 \qquad (6.30)$$

where $$A = J_1^2(\rho)\,\frac{2\omega_{pi}^2\omega_{pe}^2\,\delta}{k^2L^2(\delta^2+\nu_e^2\omega_{pe}^2)} \tag{6.31}$$

and $\delta = \omega_{UH}^2 - \omega_0^2$ is a measure of the frequency mismatch. The upper hybrid frequency ω_{UH} is defined as

$$\omega_{UH} = \left(\omega_{pe}^2 + \Omega_e^2 + k^2c_e^2 + \nu_e^2\right)^{1/2} \tag{6.32}$$

From equation (6.31) we find that for $\nu_e \to 0$ and $\omega_0 \approx \omega_{UH}$ i.e. $\delta \approx 0$, 'A' becomes infinitely large. This explains the inclusion of ν_e for the upper hybrid mode.

The unstable root of equation (6.30) is given by

$$\omega_+ = -\tfrac{1}{2}(\omega_{*i} + i\nu_{in}) + \left[(\omega_{*i} - i\nu_{in})^2 - 4(g/L + A)\right]^{1/2}/2 \tag{6.33}$$

Equation (6.33) will, in general, give the real frequency and the growth rate. We shall discuss equation (6.33) in two special limits.

<u>Case (1)</u> : The collisional limit: When $\omega_{*i} \approx 0$ we have from (6.33)

$$\omega_+ = -i\nu_{in}/2 + i\left[\nu_{in}^2 + 4(g/L + A)\right]^{1/2}/2 \tag{6.34}$$

In this limit the growth rate is independent of the finite larmour radius (FLR) effects. This limit corresponds to that obtained by Balsley et al[6].

<u>Case (2)</u> : The collisionless limit: When $\nu_{in} \to 0$ we get from equation (6.33)

$$\omega_+ = -\omega_{*i}/2 + \left[\omega_{*i}^2 - 4(g/L + A)\right]^{1/2}/2 \tag{6.35}$$

In both the limits, the last term within the square brackets of equations (6.34) and (6.35) will product a destabilizing effect when $\delta > 0$ i.e. when $\omega_{UH} > \omega_0$ whereas it will produce a stabilizing effect when $\delta < 0$ i.e. $\omega_{UH} < \omega_0$. We shall now calculate the modified growth rate and threshold for this excitation.

'A' is maximum when $\delta = \pm \omega_{pe} \nu_e$. From equations (6.31) and (6.35) we have the modified growth rate of the Rayleigh-Taylor instability as

$$\gamma = \frac{1}{2}\left[\frac{4g}{L} - \omega_{*i}^2 \pm \frac{4 J_1^2(P)\, \omega_{pi}^2 \omega_{pe}}{k^2 L^2 \nu_e}\right]^{1/2} \qquad (6.36)$$

The external oscillating electric field merely appears to produce a pressure term that enhances or depletes the normal thermokinetic pressure term proportional to ω_{*i}^2. Physically, the electric field of the pump wave and the electric field of the upper hybrid wave together produce a ∇E^2 type pressure term at the low beat frequency, the phase being determined by the sign of δ. This is what produces a stabilizing or destabilizing effect on the Rayleigh-Taylor mode.

6.2.1 Calculation of Threshold Power

Equation (6.36) can also yield information about the threshold power i.e. the minimum power necessary to trigger off the instability. For that, one needs to solve (6.36) for ρ when $\gamma = 0$. This gives us,

$$\rho^2 = \pm\left(\frac{\nu_e}{\omega_{pe}}\right) k^2 L^2 \left(4g/L - \omega_{xi}^2\right)/\omega_{pi}^2 \qquad (6.37)$$

ρ is a measure of the particle displacement under the influence of the electric field and can be expressed in terms of the field amplitudes as (See Appendix C2).

$$\rho^2 = \frac{e^2}{m^2(\omega_0^2 - \Omega_e^2)^2}\Big[k_x^2 \epsilon_x^2 + k_z^2 \epsilon_z^2 + \frac{\Omega_e^2}{\omega_0^2}\left(k_x^2 \epsilon_z^2 + k_z^2 \epsilon_x^2\right) + 2\frac{\Omega_e}{\omega_0}\epsilon_x \epsilon_z \sin\psi \left(k_x^2 + k_z^2\right) + 2 k_x k_z \epsilon_x \epsilon_z \cos\psi \left(1 - \frac{\Omega_e^2}{\omega_0^2}\right)\Big] \qquad (6.38)$$

where ϵ 'S are the amplitudes of the electric field intensity of the pump wave and ψ is the phase difference between the x and z components of the electric vector. In Section 6.4 we shall see that under the optimum conditions for parametric excitation, the pump wave is circularly polarized O-mode so that $\epsilon_x \simeq \epsilon_z$ and $\psi = -\pi/2$. Also we can write $\omega_0^2 \approx \omega_{pe}^2 + \Omega_e^2$. Thus equation (6.38) becomes,

$$\frac{c\epsilon_x^2}{8\pi} = \frac{c\,\omega_{pe}^2 N_0 m}{2k^2(1 - \Omega_e/\omega_0)^2}\rho^2 \qquad (6.39)$$

Substituting for ρ^2 from (6.37) in (6.38), the minimum required energy flux for stabilization and debilization is

$$\frac{c\epsilon_x^2}{8\pi} = \pm\left(\frac{\nu_e}{\omega_{pe}}\right)\frac{2cN_0 M g L}{(1 - \Omega_e/\omega_0)^2}\left(1 - \frac{k_x^2 T_i^2}{4gM^2\Omega_i^2 L}\right) \qquad (6.40)$$

For the plus sign in (6.40), k_x is defined by $\delta = +\omega_{pe}\nu_e$ and for the corresponding minus sign, k_x is defined by $\delta = -\omega_{pe}\nu_e$.

The high frequency pump wave passes through an altitude region where its frequency nearly matches the local upper hybrid frequency. This is the region where irregularities are influenced most. If the plasma parameters are only a function of height, then our analysis shows that for each height there is a critical wave number k_{xc} defined by $\delta = \omega_{UH}^2 - \omega_0^2 = 0$ i.e.

$$k_{xc}^2 C_e^2 = \omega_0^2 - (\omega_{pe}^2 + \Omega_e^2 + \nu_e^2). \tag{6.41}$$

From the conditions of destabilization and stabilization and from equation (6.41) we see that irregularities with $k_x > k_{xc}$ can be destabilized and those with $k_x < k_{xc}$ can be stabilized. However, whether a given k_x (or a given range of k_x) will be stabilized or destabilized depends on the strength of the high frequency pump wave. The new marginal stability condition is defined by

$$\frac{4g}{L} - \frac{k_x^2 C_i^4}{\Omega_i^2 L^2} + \frac{2\omega_{pi}^2 \omega_{pe}^2 \delta}{k^2 L^2(\delta^2 + \nu_e^2 \omega_{pe}^2)} \rho^2 = 0 \tag{6.42}$$

At a given height (i.e. for fixed set of values of C_i, Ω_i, L, ν_e and N_0) and for a given k_x one can determine, the minimum ρ required for stabilization and destabilization, from equation (6.42).

6.3 Modification of the Collisional Drift Instability.

In this section we shall discuss the effect of the pump wave on the collisional drift instability. This instability has been proposed to be one of the candidates responsible for the equatorial spread-F, by Hudson and Kennel[8] for short wavelength irregularities of the order of a few tens of meters upto a few hundred meters. The basic equations are similar to equations (6.6) to (6.9) of section 6.2, except that the effect of gravity is ignored here for such short wavelengths under consideration. We take the wave propagation to be perpendicular to the density gradient i.e. $\underset{\sim}{k} = \underset{\sim}{k}_x + \underset{\sim}{k}_y$ such that $k_x \gg k_y$. Since equatorial geometry permits very long parallel wavelengths we shall consider the limit $k_y \lambda_e < (m/M)^{1/2}$. In this limit the temperature fluctuations and finite heat conduction effects are unimportant and can be neglected. Further for the collisional drift mode, the ion velocity parallel to the magnetic field is ignored which is justified for $\omega / k_y \gg C_i$. In the parallel electron motion ν_e is important and therefore has been included. Under these assumptions the ion and the electron susceptibilities in the oscillating frame are

$$\chi_i(\omega) = \omega_{pi}^2 \left(\tilde{\omega}'' + \frac{\Omega_i}{k_x L} \right) \Big/ \Omega_i^2 \left(\omega + \tilde{\omega}'' \frac{T_i}{T_e} b \right) \tag{6.43}$$

and

$$\chi_e(\omega) = \frac{-\omega_{pe}^2\{\tilde{\omega}'^2 + (\tilde{\omega}'^2 - \Omega_e^2)k_y^2/k^2 - \tilde{\omega}'\Omega_e/k_x L\}}{\omega\tilde{\omega}'(\tilde{\omega}'^2 - \Omega_e^2) - k^2 c_e^2 \tilde{\omega}'^2 + k_y^2 c_e^2 \Omega_e^2} \tag{6.44}$$

where

$$b = \frac{k^2 c_s^2}{\Omega_i^2}, \quad c_s^2 = \frac{T_e}{M} \quad \text{and} \quad \tilde{\omega}'' = \omega + i\nu_{in}$$

As in section 6.2, for $\rho = 0$ we can obtain the linear dispersion relation for the collisional drift mode by substituting equations (6.43) and (6.44) in equation (6.25), neglecting unity for low frequency waves. Then we get

$$\frac{\omega_{pi}^2\left(\tilde{\omega}'' + \frac{\Omega_i}{k_x L}\right)}{\Omega_i^2\left(\omega + \tilde{\omega}''\frac{T_i}{T_e}b\right)} - \frac{\omega_{pe}^2\{\tilde{\omega}' + (\tilde{\omega}'^2 - \Omega_e^2)k_y^2/k^2 - \tilde{\omega}'\Omega_e/k_x L\}}{\omega\tilde{\omega}'(\tilde{\omega}'^2 - \Omega_e^2) - k^2 c_e^2 \tilde{\omega}'^2 + k_y^2 c_e^2 \Omega_e^2} = 0 \tag{6.45}$$

For $\omega \ll \nu_e \ll \Omega_e$ and $\omega/\Omega_e \ll k_y/k$, equation (6.45) is simplified as

$$b(\omega + \omega_{*i})(\omega + i\nu_{in}) + i\nu_{\parallel}\{(1 + b\rho')\omega + ib\rho'\nu_{in} - \omega_{*e}\} = 0 \tag{6.46}$$

where $\nu_{\parallel} = k_y^2 c_e^2/\nu_e$; $\omega_{*e} = -k_x T_e/m\Omega_e L$

and $\rho' = 1 + T_i/T_e$.

Equation (6.46) gives the dispersion relation for the collisional drift mode. This equation resembles to that obtained by Hudson and Kennel[8] in the isothermal limit.

For a finite amplitude of the applied oscillating electric field equations (6.24), (6.43) and (6.44) give,

$$b\left(\omega+\omega_{*e}\frac{T_i}{T_e}\right)\left(\omega+i\nu_{in}\right)+i\nu_{\parallel}\left\{\left(1+b\rho'\right)\omega+ib\rho'\nu_{in}-\omega_{*e}\right\}$$
$$=-i\nu_{\parallel}J_1^2(\rho)\frac{\omega_{pi}^2}{\Omega_i R_x L}\left(1-\frac{i\omega_{*e}}{\nu_{\parallel}}\right)\left[\frac{1}{H_e(\omega+\omega_0)}+\frac{1}{H(\omega-\omega_0)}\right] \tag{6.47}$$

Again we calculate the functions within the square bracket in (6.47), using equations (6.19a) and (6.44). Then the modified dispersion relation becomes,

$$b\left(\omega+\omega_{*e}\frac{T_i}{T_e}\right)\left(\omega+i\nu_{in}\right)+i\nu_{\parallel}\left\{\left(1+b\rho'\right)\omega+ib\rho'\nu_{in}-\omega_{*e}\right\}$$
$$=-i\nu_{\parallel}J_1^2(\rho)\frac{2\omega_{pi}^2\omega_{pe}^2\delta}{\Omega_i R_x L\left(\delta^2+\nu_e^2\omega_{pe}^2\right)}\left(1-\frac{i\omega_{*e}}{\nu_{\parallel}}\right) \tag{6.48}$$

Equation (6.48) can be rewritten as

$$\omega\left(1+\rho' b\right)-\omega_{*e}+\frac{b\nu_{in}}{\nu_{\parallel}}\left(\omega+\omega_{*e}\frac{T_i}{T_e}\right)-A'$$
$$=\frac{ib}{\nu_{\parallel}}\left(\omega^2+\omega\omega_{*e}\frac{T_i}{T_e}-\rho'\nu_{in}\nu_{\parallel}-\frac{A'\omega_{*e}}{b}\right) \tag{6.49}$$

where $$A'=2J_1^2(\rho)\,\omega_{pi}^2\omega_{pe}^2\delta\Big/\Omega_i R_x L\left(\delta^2+\nu_e^2\omega_{pe}^2\right) \tag{6.50}$$

To the lowest order R.H.S. of equation (6.49) can be neglected. Then the real frequency can be written as

$$\omega_R = \omega_{*e} + A' \qquad (6.51)$$

where we have neglected $\rho' b$, $b\nu_{in}/\nu_{\parallel}$ compared to unity. Equation (6.51) shows that the real frequency gets shifted in the presence of the pump wave. Substituting ω_R for ω in R.H.S. of equation (6.49) we get an expression for the modified growth rate given by,

$$\gamma = \frac{\rho' b}{\nu_{\parallel}}\left(\omega_{*e}^2 - \nu_{in}\nu_{\parallel} - A'\omega_{*e}/\rho' b\right) \qquad (6.52)$$

As in the case of Rayleigh-Taylor instability, in this case also, the last term of the R.H.S. of equation (6.52) will produce a destabilizing effect when $\delta > 0$ i.e. when $\omega_{UH} > \omega_0$, and a stabilizing effect when $\delta < 0$ i.e. when $\omega_{UH} < \omega_0$. At maximum A', the growth rate becomes

$$\gamma = \frac{b\rho'}{\nu_{\parallel}}\left(\omega_{*e}^2 - \nu_{in}\nu_{\parallel} \pm J_1^2(\rho)\,\omega_{pi}^2\omega_{pe}\omega_{*e}/\rho' b\,\nu_e\Omega_i k_x L\right) \qquad (6.53)$$

6.3.1 Calculation of threshold power

In this case ρ is calculated to be (See Appendix C.2)

$$\rho^2 = \frac{e^2}{m^2(\omega_0^2 - \Omega_e^2)^2}\left[k_x^2\epsilon_x^2 + 2k_x^2\epsilon_x\epsilon_z\left(\frac{\Omega_e}{\omega_0}\right)\sin\psi + k_x^2\epsilon_z^2\left(\frac{\Omega_e}{\omega_0}\right)^2\right] \qquad (6.54)$$

As in section 6.2, for $\epsilon_x \simeq \epsilon_z$ $\psi = -\pi/2$, we get

$$\frac{c\epsilon_x^2}{8\pi} = \frac{c^2\omega_{pe}^2 N_0 m}{2k^2(1-\Omega_e/\omega_0)^2}\rho^2 \tag{6.55}$$

using equations (6.53) and (6.55) we get the minimum required energy flux for stabilization and destabilization as

$$\frac{c\epsilon_x^2}{8\pi} = \pm\left(\frac{\nu_e}{\omega_{pe}}\right)\frac{2cN_0\rho' T_e^2 k_x^2}{M\Omega_i^2\left(1-\frac{\Omega_e}{\omega_0}\right)^2}\left(1-\frac{\nu_{in}k_y^2\Omega_e^2 mL^2}{\nu_e k_x^2 T_e}\right) \tag{6.56}$$

And the new marginal stability criteria is given by

$$\frac{k_x^2 c_e^4}{\Omega_e^2 L^2} - \frac{\nu_{in}}{\nu_e}k_y^2 c_e^2 + \frac{\omega_{pi}^2\omega_{pe}^2\delta}{k_x^2 L^2(\delta^2+\nu_e^2\omega_{pe}^2)}\frac{\rho^2}{2} = 0 \tag{6.57}$$

As already discussed in the previous section, we can calculate from equation (6.57), the minimum ρ required for stabilization or destabilization for a given k_x at a given height.

We summarize the results of sections 6.2 and 6.3 by saying that an external pump wave can be used to parametrically stabilize or destabilize the F-region plasma modes provided that (1) $\delta/\omega_0 \ll 1$ i.e. the wave freq. is very close to the local upper hybrid freq. and (2) $\epsilon_\perp$ is finite i.e. the electric field has a finite component perpendicular to the static magnetic field.

6.4 Propagation Characteristics of the Pump Wave.

In sections 6.2 and 6.3 we have seen that parametric stabilization and destabilization, of the naturally occuring Rayleigh-Taylor and collisional drift modes in the F-region of the ionosphere, are possible under some conditions which are summarized in the previous section. In order to satisfy those conditions, it is necessary to consider the optimum experimental geometry for which the parametric coupling can take place most favourably. As we can see from the previous sections that our analysis is applicable to equatorial region only, where the density gradient and the magnetic field are normal to each other, so that the experiment has tobe performed for the equatorial region. Since the frequency of the pump wave is near the local upper hybrid frequency, we cannot use an extra-ordinary mode (X-mode) pump wave for parametric interaction as an X-mode will be reflected at the right hand cut off before it ever reaches the excitation region near the upper hybrid frequency. On the other hand, at the magnetic equator an ordinary mode (O-mode) pump wave transmitted at vertical or oblique incidence to the ionosphere and perpendicular to the magnetic meridian plane will be plane polarized with its electric vector parallel to the static magnetic field. Furthermore, this polarization remains unaltered throughout its entire transmission path. Parametric stabilization or destabilization of the Rayleigh-Taylor and collisional drift modes, by this mode of transmission is also not possible, since an electric field component of the pump wave perpendicular to the magnetic field is one of the necessary conditions.

It thus appears that the transmission of the pump wave in the magnetic meridian plane but at oblique incidence to the ionosphere is the best geometry for achieving parametric coupling. The WKB treatment of the general problem of oblique propagation in a horizontally stratified magnetized plasma has been given by Ginzburg(13). For the case of N-S propagation at the magnetic equator, Ginzburg's propagation formulas for the O-mode can be written as,

$$q^2 = (1-S^2) + \left\{ u\Lambda(1-S^2) - 2\Lambda(1-\Lambda) + \left[u^2\Lambda^2(1-S^2)^2 + 4u\Lambda^2(1-\Lambda)S^2\right]^{1/2} \right\} \Big/ 2(1-u-\Lambda) \tag{6.58}$$

$$\epsilon_y(z,y) = \frac{\text{constant}}{\left[q(G^2-1)+SH\right]^{1/2}} \exp\left[\pm i\frac{\omega}{c}\left(Sy + \int_0^z q\,dz'\right)\right] \tag{6.59}$$

$$\epsilon_x = G\epsilon_y \quad \text{and} \quad \epsilon_z = H\epsilon_y \tag{6.60}$$

where $G = -i(1-\Lambda-q^2)\Lambda u^{1/2} \Big/ \left[(1-u-\Lambda)-(1-u)\times(S^2+q^2)\right]Sq$

$$
\begin{aligned}
&H = -(1-\Lambda-q^2)/Sq \\
&\Lambda = \omega_{pe}^2/\omega_0^2, \quad u = \Omega_e^2/\omega_0^2 \\
&S^2 = (c^2/\omega_0^2)k_{py}^2 = \sin^2\theta_0, \quad q^2 = (c^2/\omega_0^2)k_{pz}^2
\end{aligned} \tag{6.61}
$$

In the above equations k_{py} and k_{pz} are the y and z components of the local propagation vector of the large amplitude pump wave and θ_0 is the angle of incidence. For the general case of oblique incidence, q is a solution of a quartic equation, and reflection occurs at $dq/d\Lambda = \infty$[14]. For the case of N-S propagation at the magnetic equator, the quartic equation for q reduces to a quadratic equation for q^2, and reflection at $dq/d\Lambda = \infty$ for the O-mode occurs at $q^2 = 0$.

Booker[15] obtained an expression for the critical electron density (in terms of the electron plasma frequency normalized to the pump wave frequency) as a function of the angle of incidence, required for reflection of the O-mode for N-S transmission in the equatorial region, given by

$$\Lambda = (1-\sin^2\theta_0)\left(1+\frac{\Omega_e}{\omega_0}\right) \text{ when } \tan^{-1}\left(\frac{\Omega_e}{\omega_0}\right)^{1/2} \leq \theta_0 \leq \pi/2 \tag{6.62}$$

Figure 6.2, gives a plot of Λ against $\sin^2\theta_0$. In section 6.2 we have shown that the value of 'A' is maximum, i.e. parametric interaction occurs most favourably, when the frequency mismatch $\delta_m = \pm\omega_{pe}\nu_e$, which can be written as $\omega_0 \simeq \omega_{UH} \pm \nu_e/2$. For stabilization or destabilization of a selected range of the optimum angle of incidence θ_m is determined from figure 6.2

For angles of incidence much larger than θ_m the pump wave will be reflected before it reaches the region of favourable frequency matching. For angles of incidence much smaller than θ_m the pump wave will be reflected at a height too far above the region of favourable frequency matching to take maximum advantage of any amplitude swelling of the pump wave near its reflection point.

Next we shall find expressions for the swelling of the electric field amplitudes of the pump wave by expressing the pump wave field components in terms of the incident energy flux. We can write down the time averaged energy flux from Maxwell's equations as,

$$\underset{\sim}{S} = (c^2/8\pi\omega)\left[\underset{\sim}{\epsilon}^* \times (\underset{\sim}{k}_P \times \underset{\sim}{\epsilon})\right] \qquad (6.63)$$

using equations (6.60), (6.61) and (6.63) we get,

$$S_z = (c/8\pi)\left[(|G|^2+1)q - Hs\right]|\epsilon_y|^2$$

$$S_y = (c/8\pi)\left[(|G|^2+|H|^2)s - Hq\right]|\epsilon_y|^2 \qquad (6.64)$$

and
$$S = (S_y^2 + S_z^2)^{1/2}$$

From the conservation of z-component of the energy flux we have,

$$S_z = S_{0z} = S_0\left[1 + S_{0y}^2/S_{0z}^2\right]^{-1/2} = \alpha_0 S_0 \qquad (6.65)$$

where α_0 is given by

$$\alpha_0 = \left\{1 + \left[(|G_0|^2+|H_0|^2)s_0 - H_0 q_0\right]^2 \Big/ \left[(|G_0|^2+1)q_0 - H_0 s_0\right]^2\right\}^{-1/2} \qquad (6.66)$$

In (6.65), S_o is the incident flux. In equation (6.66) all quantities with subscript 0 are to be evaluated at the base of the ionosphere. Now from equations (6.60), (6.64) and (6.65) we write

$$
\begin{aligned}
c|\epsilon_y|^2/8\pi &= \alpha_y S_0 \\
c|\epsilon_x|^2/8\pi &= \alpha_x S_0 \\
c|\epsilon_z|^2/8\pi &= \alpha_z S_0
\end{aligned}
\qquad (6.67)
$$

where

$$\alpha_y = \frac{\alpha_0}{[(|G|^2+1)q - H\zeta]}, \; \alpha_x = |G|^2\alpha_y, \; \alpha_z = |H|^2\alpha_y$$

Figure 6.3 is a plot of the amplitude swelling factors $\alpha_x, \alpha_y, \alpha_z$ against Λ. The swelling of the electric field components of the pump wave near the reflection is due to a combination of energy density build up and polarization change. From figure 6.3 we see that near the reflection point α_x and α_z are equal and α_y goes to zero, which shows that the pump wave becomes circularly polarized about $\underset{\sim}{B}_0$ near the reflection point.

We shall next calculate the minimum threshold energy flux for parametric instability of Rayleigh-Taylor and collisional drift modes. From equations (6.40) and (6.67) we get for the minimum threshold energy flux for Rayleigh Taylor instability as

$$S_{0,R.T} = 30 \times \alpha_x^{-1}\left(1 - \frac{k_x^2 T_i^2}{49 M^2 \Omega_i^2 L}\right) \mu w/m^2 \qquad (6.68)$$

Again from equations (6.56) and (6.67) the minimum threshold energy flux for collisional drift mode is

$$S_{O,C.D} = 20 \times \alpha_x^{-1}\left(1 - \frac{\nu_{in} k_y^2 \Omega_e^2 m L^2}{\nu_e k_x^2 T_e}\right) \mu\omega/m^2 \tag{6.69}$$

In obtaining the above estimates the following numerical values have been used: $\omega_0 = 1.84 \times 10^7$ rad/sec., $\omega_{pe} = 1.78 \times 10^7$ rad/sec., $\Omega_e = 5 \times 10^6$ rad/sec., $N_o = 10^5$ Cm^{-3}, $\nu_e = 10^3$ Cm^{-1}, $T_e = 10^3$°K, $M = 3 \times 10^{-23}$ gms, $L = 3 \times 10^6$ cm. For the collisional drift instability we have used a typical perpendicular wavelength $\lambda_\perp = 50$ meters.

Our numerical estimates for minimum threshold energy flux may be compared with the typical energy flux used in recent ionospheric modification experiments. In the Boulder F-layer modification experiment(10), the incident energy flux was of the order of $50\mu\omega/m^2$, which is very close to our numerical results. Thus sources as strong as the Boulder transmitter might be adequate to produce parametric stabilization and destabilization effects in the equatorial F-region.

6.5 Discussion and Conclusion

We have investigated the effect of a high frequency electric field, oscillating near the upper hybrid frequency, on the dispersion characteristics of the low frequency Rayleigh-Taylor mode and the collisional drift mode and obtained conditions for artificially triggering them or suppressing them. We have used fluid theory for the study of parametric interaction processes which is justified for perpendicular wavelengths greater than the ion Larmor radius and parallel wavelengths greater than λ_e, the electron mean path. We have considered a simplified picture and have retained only the important terms contributing to the natural instabilities of the modes under consideration, since we are mainly interested in how the natural instabilities are affected by the pump wave. We have considered an isothermal plasma for the drift mode which is justified for long parallel wavelengths $\lambda_{\parallel}$, such that

$$\lambda_{\parallel} > \lambda_e (M/m)^{1/2}$$

In our analysis, we have used the slab geometry instead of a dipole geometry and thus have neglected the field-line curvature. This is due to the fact that in the ionosphere, the plasma density is locally altitude-dependent rather than field-line aligned. Further, the modes with parallel wavelengths extending out of the region of the sharpest density gradient to the regions of weaker gradients, will not grow as fast as those with parallel wavelengths confined locally to the regions of maximum gradient. Therefore, there is a maximimum limit imposed on the parallel wavelengths for the largest growing modes in the system. The meaning of this limit in the context of equatorial F-region ionosphere becomes immediately clear when we realise that due to the field-line curvature, field lines at the equator soon enter the regions of varying density gradients. Thus the drift mode, which has a finite k_y , will be localized along the field lines to the region of the maximum density gradient. This is in contrast to the R-T flute mode, which being the lowest order perturbation of the entire flux tube of field lines, has no variation along the field lines. The maximum parallel wavelength, to which the drift mode is limited is given by[16]

$$\lambda_{\parallel}^{max} = 2 (L R_E)^{1/2}$$

where R_E is the radius of the earth.

In figure 6.4 we have shown the experimental geometry. We have taken a typical height of 300 Km below the peak of the F-region, for parametric interaction. It is shown that for the most favourable parametric coupling at a height of 300 Km, the pump wave should be incident in the N-S direction (in the magnetic meridian plane) from a place about 180 Kms. north or south of the region to be modified. Although we have taken a typical height of 300 Km for the irregularities, it should be noted that the drift mode can grow on the top side as well as on the bottom side of the F-peak, whereas the Rayleigh-Taylor mode, which requires the density gradient to be antiparallel to gravity, grows only on the bottom side of the F-peak.

In our analysis we have neglected the energy loss due to propagation of the excited waves out of the excited regions. Such effects have been discussed by Perkins and Flick[17] and Fejer and Leer[18]. Propagation losses can limit the net amplification of the waves. Consequently, the power threshold for parametric amplification, obtained above, should be regarded as a lower limit to the power required to produce an observable amplification.

It seems appropriate to conclude therefore that an ionospheric modification experiment to study the equatorial spread-F phenomenon is both a feasible and a desirable one.

..

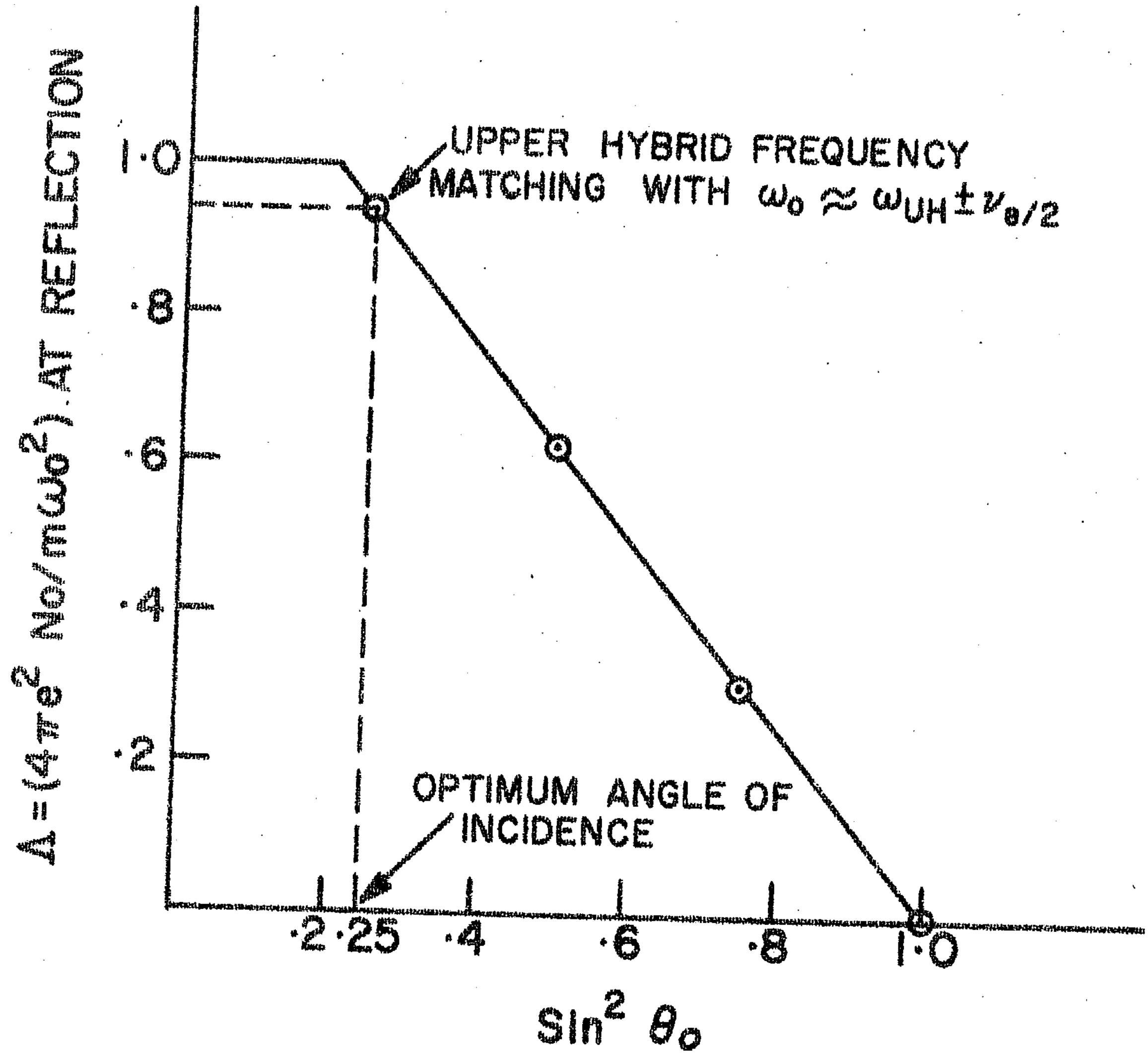

Figure 6.2 - Variation of electron density in units of plasma frequency squared ($\Delta = \omega_{pe}^2 / \omega_o^2$), required for reflection of an O-mode pump wave in the magnetic meridian plane at the magnetic dip equator, versus $\sin^2\theta_o$ where θ_o is the angle of incidence. We have used $\omega_o = 1.84 \times 10^7$ rad./sec., $n_o = 10^5$/c.c., $\nu_e = 10^3$ sec.$^{-1}$, and $\Omega_e = 4.6 \times 10^6$ rad./sec.

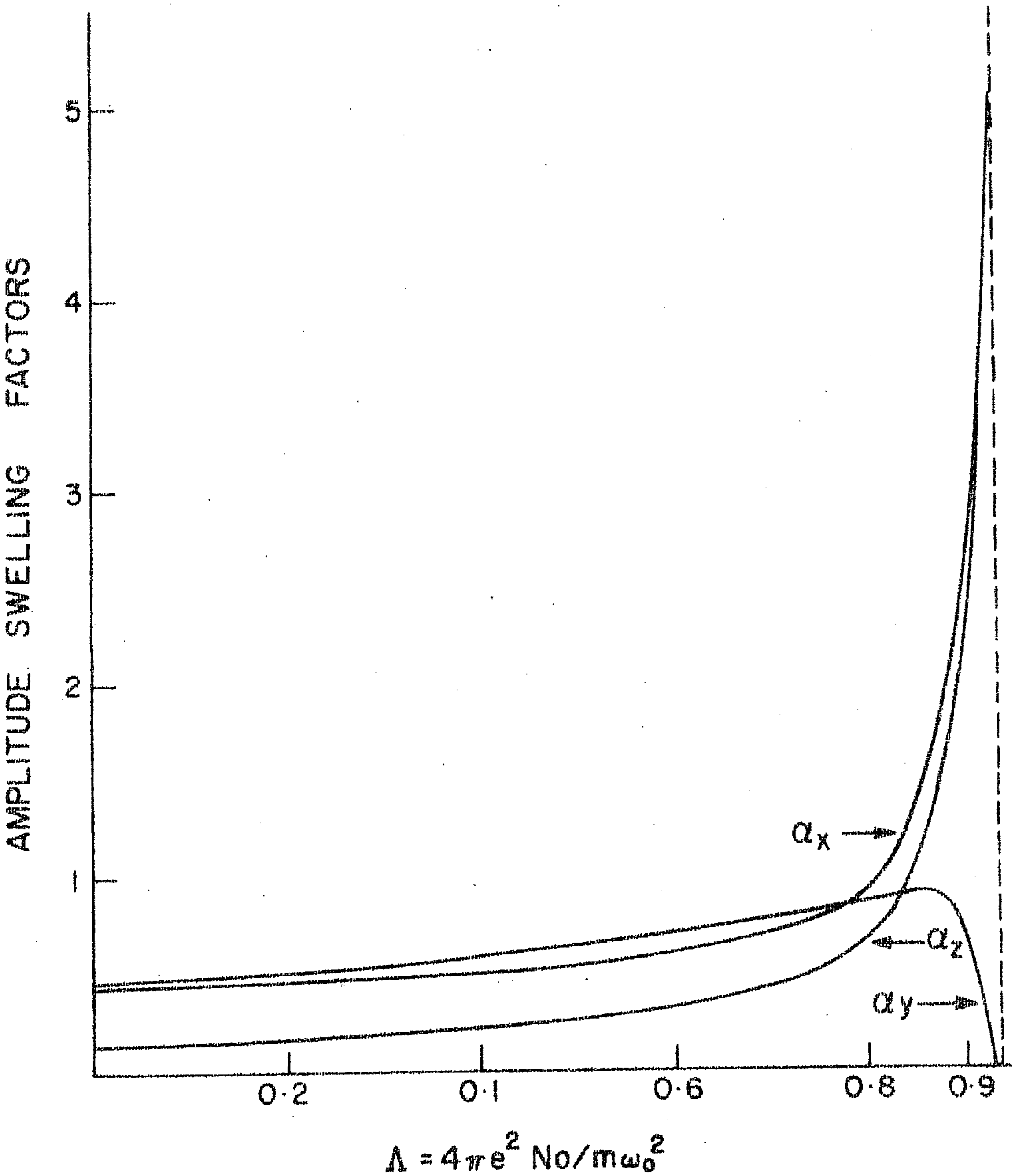

Figure 6.3 - Variation of amplitude swelling factors $(\alpha_x, \alpha_y, \alpha_z)$ versus Λ $(=\omega_{pe}^2/\omega_o^2)$. Here x and z are pump electric-field components perpendicular, and y that parallel, to the background magnetic field. Near the reflection point (Λ =.94), the pump wave is approximately circularly polarized about the magnetic field.

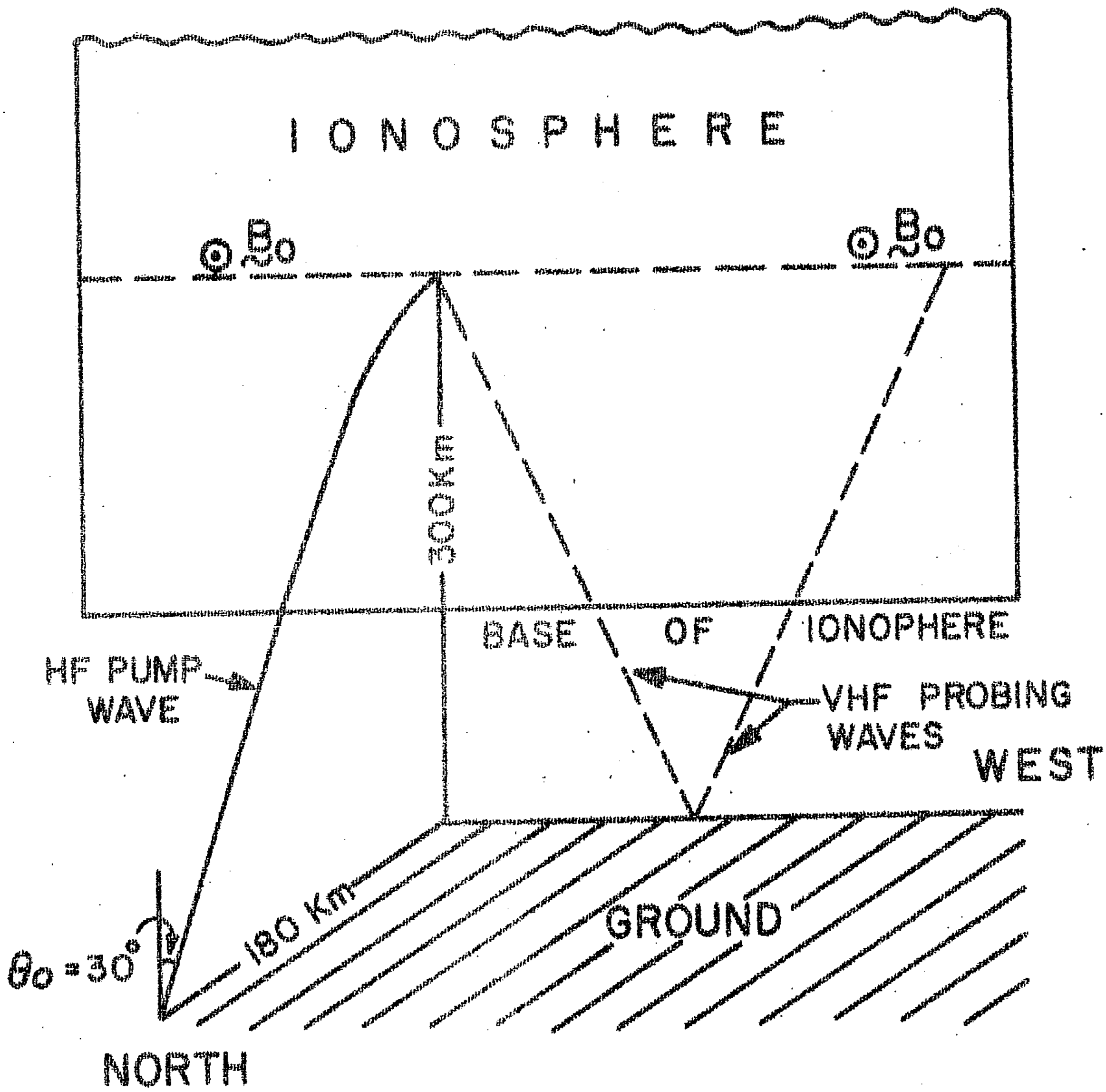

Figure 6.4 - Sketch of the experimental geometry. The pump wave is transmitted in the N-S direction (in the magnetic meridian plane) and at an oblique angle of incidence to the ionosphere. Dianostics can be made with VHF back scattering radars transmitted in the E-W direction. Correlation studies can be made between signals returned from the unperturbed region and the region of parametric interaction.

APPENDIX C

C.1 Derivation of Equation (6.19)

We have by definition

$$\tilde{N}_j'(\underset{\sim}{r},t) = N_j'(\underset{\sim}{r}+\underset{\sim}{R}_j(t),t) \tag{C.1}$$

By taking Fourier and Laplace transforms of both sides of equation (C1), we get two relations between the Fourier and Laplace transforms of the above two quantities as follows:

$$\tilde{N}_j'(\underset{\sim}{k},\omega) = \frac{1}{2\pi}\int \Delta_j(\underset{\sim}{k},\omega-\omega')N_j'(\underset{\sim}{k},\omega')\,d\omega' \equiv \Delta_j\cdot N_j' \tag{C.2}$$

$$N_j'(\underset{\sim}{k},\omega) = \frac{1}{2\pi}\int \tilde{\Delta}_j(\underset{\sim}{k},\omega-\omega')\tilde{N}_j'(\underset{\sim}{k},\omega')\,d\omega' \equiv \tilde{\Delta}_j\tilde{N}_j' \tag{C.3}$$

where $\Delta_j(\underset{\sim}{k},\omega) = \int_0^\infty dt\,\exp\left(-i\underset{\sim}{k}\cdot\underset{\sim}{R}_j(t)+i\omega t\right)$ (C.4)

and $\tilde{\Delta}_j(\underset{\sim}{k},\omega) = \Delta_j(-\underset{\sim}{k},\omega)$

Since we have taken $\omega_0 \gg \omega_{pi}$ in our analysis, the effect of the pump wave on the ions can be neglected and thus $\tilde{N}_i' = N_i'$. From Poisson's equation we can show that

$$H_i N_i' = -N_e' \tag{C.5}$$

where

$$H_i = -(1+\chi_i)/\chi_i \tag{C.6}$$

from equation (C.2) and Poisson's equation we have

$$\Delta_e . N_e' = \tilde{N}_e' = \chi_e (\Delta_e . N_i' - \Delta_e . N_e') \qquad (C.7)$$

Now using equations (C.3) and (C.5) we can write (C.7) as

$$H_i N_i' = \tilde{\Delta}_e . \left[\frac{1}{H_e}\right] \Delta_e . N_i' \qquad (C.8)$$

where $$H_e = -(1+\chi_e)/\chi_e \qquad (C.9)$$

Writing equation (C.8) explicitly we get

$$H_i(\underset{\sim}{k},\omega) N_i'(\underset{\sim}{k},\omega) = \int \frac{d\omega'}{2\pi} \tilde{\Delta}_e(\underset{\sim}{k},\omega-\omega') \frac{1}{H_e(\omega')} \times \int \frac{d\omega''}{2\pi} \Delta_e(\underset{\sim}{k},\omega'-\omega'') N_i'(\underset{\sim}{k},\omega'') \qquad (6.19)$$

C.2 Calculation of ρ^2

The electron excursion length Re, under the influence of the pump wave field, satisfies the following equation

$$\frac{d^2 \underset{\sim}{R}_e}{dt^2} = -\frac{e}{m} \underset{\sim}{E}_0(\omega_0, t) - \frac{e}{mc}\left(\frac{d\underset{\sim}{R}_e}{dt}\right) \times \underset{\sim}{B}_0 \qquad (C.10)$$

We have already discussed that the parametric coupling occurs most favourably near the reflection point where $\epsilon_y \simeq 0$ so that we can express $\underset{\sim}{E}_0$ near the reflection point as,

$$\underset{\sim}{E}_0 \doteq \frac{\epsilon_x}{-2i} \exp(-i\omega_0 t)\hat{e}_x + \frac{\epsilon_z}{-2i} \exp(-i\psi - i\omega_0 t)\hat{e}_z + c.c. \qquad (C.11)$$

where c.c. is the complex conjugate and

$$R_e = \underset{\sim}{R}_{oe} \exp(-i\omega_o t) + c.c. \qquad (C.12)$$

Ψ is the phase difference between the x and z components.

Now let us define

$$\underset{\sim}{k} \cdot \underset{\sim}{R}_e = \rho \sin(\omega_o t + \beta) \qquad (C.13)$$

For the Rayleigh-Taylor instability $(\underset{\sim}{k} = \underset{\sim}{k}_x + \underset{\sim}{k}_z)$ case equations (C.10) to (C.13) give,

$$\rho \cos\beta = \frac{e}{m(\omega_o^2 - \Omega_e^2)} \left(k_x \epsilon_x + k_x \epsilon_z \frac{\Omega_e}{\omega_o} \sin\Psi + k_z \epsilon_z \cos\Psi \right) \qquad (C.14)$$

$$\rho \sin\beta = \frac{e}{m(\omega_o^2 - \Omega_e^2)} \left(-k_x \epsilon_z \frac{\Omega_e}{\omega_o} \cos\Psi + k_z \epsilon_z \sin\Psi + k_z \epsilon_x \frac{\Omega_e}{\omega_o} \right) \qquad (C.15)$$

From equations (C.14) and (C.15), we immediately write down the expression for ρ^2 for the R-T case as

$$\rho^2 = \frac{e^2}{m^2(\omega_o^2 - \Omega_e^2)^2} \left\{ \left(k_x^2 \epsilon_x^2 + k_z^2 \epsilon_z^2\right) + \frac{\Omega_e^2}{\omega_o^2}\left(k_x^2 \epsilon_z^2 + k_z^2 \epsilon_x^2\right) + 2\frac{\Omega_e}{\omega_o}\left(k_x^2 + k_z^2\right) \epsilon_x \epsilon_z \sin\Psi + 2 k_x k_z \epsilon_x \epsilon_z \cos\Psi \times \left(1 - \frac{\Omega_e^2}{\omega_o^2}\right) \right\} \qquad (6.38)$$

For the collisional drift instability $(\underset{\sim}{k} = \underset{\sim}{k}_x + \underset{\sim}{k}_y)$ case equations (C.10) to (C.13) give

$$\rho \cos\beta = \frac{e}{m(\omega_o^2 - \Omega_e^2)} \left(k_x \epsilon_x + k_x \epsilon_z \left(\frac{\Omega_e}{\omega_o}\right) \sin\Psi \right) \qquad (C.16)$$

$$\rho \sin\beta = -\frac{e}{m(\omega_0^2-\Omega_e^2)} k_x \epsilon_z \left(\frac{\Omega_e}{\omega_0}\right) \cos\psi \qquad (C.17)$$

Thus the expression for ρ^2 for the collisional drift instability case is given by

$$\rho^2 = \frac{e^2}{m^2(\omega_0^2-\Omega_e^2)^2}\left[k_x^2\epsilon_x^2 + 2k_x^2\epsilon_z\epsilon_x\left(\frac{\Omega_e}{\omega_0}\right)\sin\psi + k_x^2\epsilon_z^2\left(\frac{\Omega_e}{\omega_0}\right)^2\right] \qquad (6.54)$$

C.3 Calculation of $\Delta_e(\underset{\sim}{k},\omega)$

By definition

$$\Delta_e(\underset{\sim}{k},\omega) = \int_0^\infty dt\, \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{R}_e + i\omega t) \qquad (C.18)$$

From equation (C.13) and using the identify

$$\exp[-i\rho\sin(\omega_0 t+\beta)] = \sum_n J_n(\rho)\exp[-in\omega_0 t - in\beta] \qquad (C.19)$$

equation (C.18) gives

$$\Delta_e(\underset{\sim}{k},\omega) = \sum_n i\,\frac{J_n(\rho)\,e^{-in\beta}}{(\omega-n\omega_0)} \qquad (C.20)$$

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