CONSTRAINTS ON UNIFIED THEORIES IN THE LIGHT OF NEW EXPERIMENTAL RESULTS

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Biswajoy Brahmachari. M.Sc.
Physical Research Laboratory
Navrangpura,
Ahmedabad 380009
India

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Dedicated to
The great masters of science of the ancient India

CERTIFICATE

I Declare that the works presented in this thesis are original and has not formed the basis for the award of any degree or diploma by any University or Institution.

Biswajoy Brahmachari
(Author)

Certified by

Dr. Utpal Sarkar
(Guide) Physical Research Laboratory,
Ahmedabad-380009, India

Dr. M. S. Shah
(Co Guide)
Department of Physics
M. S. University of Baroda
Baroda, India.

Publications on which this thesis is based

1.Higher dimensional operators to the rescue minimal SU(5)
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2. Ruling out low energy left-right symmetry in unified theories.
B. Brahmachari, U. Sarkar and K. Sridhar, Phys. Lett. **B 273** (1992), 105

3. Nonperturbative unification in the light of LEP Results
B. Brahmachari, U. Sarkar and K. Sridhar, Mod. Phys. Lett. **A8** (1993), 3349

4. Higgs effect in SU(15) GUT
B. Brahmachari,R.B. Mann, U. Sarkar and T.G Steele, Phys. Rev. **D 45** (1992) 2467.

5. Consequences of supersymmetric SU(15) grand unification
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6. Low energy grand unification with SU(16)
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Synopsis

Standard Model(SM) is a gauge theory invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$. The interaction is mediated by eight gluons of the strong interaction sector and the four gauge bosons $W^{\pm}, Z^0$ and γ of the electroweak sector. In nature $SU(2)_L \times U(1)_Y$ symmetry is broken and so $SU(3)_c \times U(1)_{em}$ symmetry is preserved at low energy. The most popular method to induce this symmetry breaking is to assume the existence of a complex doublet higgs field, which upon acquiring vacuum expectation value breaks the electroweak symmetry. Out of the four real degrees of freedom of the complex doublet three get absorbed by the gauge bosons of electroweak sector as longitudinal degrees of freedom and hence three gauge bosons become massive. The fourth degree of freedom is expected to be detected by the experiment. From LEP searches the present lower bound on its mass is 63 GeV[1].

In SM left handed fields transform as doublets and right handed fields as singlets of the $SU(2)_L$ group. On the other hand quarks transform as triplets and leptons transform as singlets of the $SU(3)_c$ group. The right handed neutrino is not included in SM to render the neutrino absolutely massless. Hence there are fifteen fermions per generation. There exists three generations making a total of fortyfive fermions. Out of all these fermions the top quark is yet to be discovered by direct experimental search though there are a number of indirect (theoretical as well as experimental) evidences of the existence of the Top quark. At the Fermilab Tevatron the Top quark decays which are allowed by SM are studied. The present lower bound on the mass of the top quark is 108 GeV from CDF collaboration and about 10 GeV less from the D0 collaboration[2].

In spite of great experimental successes the Standard Model cannot be given the status of a complete theory and there is strong motivation for the study of the physics beyond the Standard Model. In this thesis we have concentrated on GUTS and supersymmetric GUTS as studies beyond the Standard Model. GUTS offer the possibility of a simple but unified description of strong and electroweak interactions. Typically in these models all the interactions arise out of a single Lagrangian which is locally invariant under the gauge transformations of a single simple lie group called the unification group. The fermions are put into some irreducible representation of the GUT group. A large spectrum of GUTS are proposed in the literature which can be classified by the unification group. All these theoretically very attractive models have at least one common prediction namely the decay of proton.

This thesis looks at the constraints on unification theories vis-a-vis the recent experimental results coming from high LEP results. The work may be subdivided into three broad classes. First part deals with comparing the experimental results with the predictions of the known models of unification. The second part deals with a new paradigm of GUT namely the low energy unification. In the third part we look at the predictions that one may obtain from the study of the evolution of the yukawa couplings in supersymmetric GUTS.

We have shown that the stringent bounds placed on the weak mixing angle from a recent analysis of measurements at LEP rule out any possibility of the survival of Left-Right symmetry at low energies in Grand Unified Theories and partially unified theories. The lowest mass scale to which the left-right symmetry survives is 10^9 GeV. This scale is driven up even higher with the inclusion of the higgs contributions to beta functions, and also in the supersymmetric versions of the theories[3].

Recently it has been shown that using the precise values of the weak mixing angle and the strong coupling constant at weak scale an unique intersection point of the strong weak and hypercharge couplings is not obtained in one step unification assuming only SM at low energy and with one higgs doublet. Furthermore it has also been shown that the criterion of unique intersection of the couplings at the unification scale is satisfied in the minimal supersymmetric standard model, predicting an unification scale around 10^{16} GeV and the supersymmetry scale around 1 TeV[4]. In this context we consider the modification of the minimal $SU(5)$ lagrangian due to higher dimensional operators, arising from quantum gravity effects or from spontaneous compactification of extra dimensions in Kaluza-Klein theories. Due to the presence of these operators the strong weak and hypercharge couplings do not meet at all at the unification scale, M_U, and the magnitude of the mismatch are directly related to the couplings of the higher dimensional operator. In particular, we consider five and six dimensional operators and show that large range of values of couplings of these operators are compatible with the latest values of the weak mixing angle and the strong coupling constant derived from LEP. Experimental constraints on M_U coming from proton lifetime is also satisfied[5]. We have also studied the nonterturbarive unification scenario first proposed by Maiani Parisi and Petronzio[10] in the context of the LEP data. We see that the supersymmetric version of the theory with five fermion generations is still consistent with the LEP data though the nonsupersymmetric version is ruled out.

Recently a new paradigm of GUT has evolved with an interesting possibility that unification is achieved at a low scale which means the absence of the big desert while at the same time the experimental constraints including that of the proton lifetime remain satisfied[6]. A GUT model based on $SU(15)$ or $SU(16)$ offers such a possibility. We have studied the possibility of achieving low unification scale GUT models based on $SU(16)$[7]. Baryon number symmetry being an explicit local gauge symmetry the gauge boson mediated proton decay is absent. We have considered in detail a number of breaking chains and the higgs representations giving rise to the desired symmetry breaking, and identified one chain giving low energy unification. These higgs field representations are constructed in such a way that higgs mediated proton decay is absent. At the end we have indicated the very rich low energy physics obtainable from these models which includes quark-lepton ununified symmetry and chiral color symmetry. Phenomenological implications of these low energy groups are also studied.

The $SU(15)$ GUT model was already existing in the literature when we started working on such model building exercises. Our result [8]is that when one includes the higgs field contributions in the beta functions and assume the well known extended survival hypothesis, it rules out the symmetry breaking chains which exist in the literature. On the other hand it predicts new phenomenologically more interesting symmetry breaking chains.

Supersymmetry offers a very interesting theoretical possibility which places fermions and bosons at an equal footing via its symmetry transformations laws. Though supersymmetry itself can solve the problem of gauge hierarchy it is nevertheless an interesting proposition to endow the $SU(15)$ GUT model with supersymmetric transformation laws and see the consequences. This is simply because supersymmetry is a rich symmetry and nature seems to use all the symmetries available to her. We have considered the supersymmetric version of the $SU(15)$ GUT[9] and applied the constraints coming from the LEP experiments. We have attempted to ask the question that if supersymmetry is discovered in near future how is it going to affect the new paradigm of low energy unification of the $SU(15)$ GUT. We find that the low energy unification with $SU(15)$ in the supersymmetric framework is not allowed. Most of the symmetry breaking chains do not allow for low energy unification, and a few symmetry breaking chains which low energy unification fail to satisfy the perturbative unification constraint (coupling constant less than unity). Hence the signals of the existence of supersymmetry in future colliders will rule out the possibility of low energy unification.

The perfect unification of gauge couplings[4] around the scale 2×10^{16} GeV in the presence of supersymmetry above the TeV range is often considered as a serious hint about the existence of supersymmetry in nature. We have considered the evolutions of the yukawa couplings of the Minimal Supersymmetric Standard Model (MSSM) assuming that there is an underlying perturbative theory upto the scale of unification of the gauge couplings (M_U). Demanding that the Top quark yukawa coupling has to remain perturbative upto the scale M_U we have calculated the upper bound of the top quark yukawa coupling at the low energy scale. This in turn gives a lower bound on the quantity $tan\beta$ of MSSM which is defined as the ratio of the vacuum expectation values of the higgs fields that couples to the top quark to that which couples to the down quark. Such bound on the value of $tan\beta$ is expected to influence the search of higgs bosons in the MSSM [11] [12]. Numerically this bound is very similar to the bound obtained from the fixed point solution of the top quark Yukawa coupling.

In summary in the first part we have considered the presently available experimental data and worked out some of its consequences in the context of unified models. In the second part we have done some model building exercises which avoids the hierarchy problem and does not suffer from fast proton decay. At the end we have studied the evolution of yukawa couplings in the context of supersymmetric GUTS and have given lower bound on the parameter $tan\beta$ of the MSSM.

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Contents

Chapter 1

Introduction

An understanding of how the laws of nature work requires a theory of the elementary particles of matter interacting with each other. Equivalently it requires a theory of the basic forces of nature. Four such forces exist, and until recently a different kind of theory was needed for each of them. Two of them, electromagnetism and gravitation are long range; hence they are familiar to us. The remaining forces which are simply called strong forces and weak forces cannot be felt directly as their influence extends only within the nucleus. The strong force binds together the protons and the neutrons in the nucleus and in another context it binds together the quarks which are the constituents of protons and the neutrons. The weak force is responsible for the decay of certain the neutrons.

The long standing problem of physics is to construct a master theory which takes into account all the known forces. One imagines that such a theory would reveal some deep connection between the theories which apparently look so diverse. In recent past one crucial step towards that goal has been taken. The weak force and the electromagnetism can now be understood in terms of a single theory - this unified theory of electroweak interactions form part of the standard model. Thus one thinks that in near future one can formulate a single theory of strong, weak and electromagnetic theory which is popularly named as grand unified theory (GUT).

The first step towards the construction of the unified theory is the demonstration that the weak, the strong and the electromagnetic forces could all be described in terms of the theories of the same kind namely the gauge theories. The three forces remain distinct but they can be seen to operate through the same mechanism. This hints that the grand unified theory which is intended to incorporate (and not supplement) the established theories is also a gauge theory.

In the following section we shall review the status of the standard electroweak theory which we intend to embed in a grand unified version. Latter in the section following it we shall briefly review the rudiments of the prototype grand unified theory namely the minimal SU(5) GUT. Afterwards we shall go on to review the inclusion of left-right symmetry and supersymmetry in the framework of unification.

1.1 Basic ideas of Standard Model

Standard Model(SM) is a gauge theory invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$. The interaction is mediated by eight gluons of the strong interaction sector and the four gauge bosons $W^{\pm}$, Z^0 and γ of the electroweak sector. Here we have labeled the electroweak gauge bosons by their electric charge as they are not eigenstates of $SU(2)_L \times U(1)_Y$ whereas they are eigenstates of electric charge. Which immediately means that in nature $SU(2)_L \times U(1)_Y$ symmetry is broken and $U(1)_{em}$ symmetry is preserved.

The objective is to formulate a renormalized gauge theory of electroweak interactions incorporating massive gauge bosons. This is achieved by spontaneously breaking the gauge symmetry. The experimental data suggests that the interactions are invariant under weak isospin $SU(2)_L$ and the weak hypercharge $U(1)_Y$ transformations.

We first write down the basic interactions in gauge invariant form. First, we write, an isotriplet of weak currents J_μ coupled to the three vector bosons W_μ in an $SU(2)_L$ invariant way,

$$-ig\ J_\mu \, . \, W^\mu = -ig\bar{\chi}_L\gamma_\mu\, T\, . \, W^\mu \chi_L, \tag{1.1}$$

next, a weak hypercharge current coupled to the fourth vector boson B^μ in a $U(1)_Y$ invariant way,

$$-i\frac{g'}{2}\ J_\mu^Y B^\mu = -ig'\bar{\psi}\gamma_\mu \frac{Y}{2}\psi B^\mu. \tag{1.2}$$

The operators T and B are the generators of the $SU(2)_L$ and $U(1)_Y$ transformations.

The left handed fermions form isospin doublets χ_L and the right handed fermions are the isosinglets ψ_R. For example, for the electron and its neutrino we have

$$\chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad with \quad T = 1/2,\ Y = -1,$$
$$\psi_R = (\, e^- \,)_R \quad with \quad T = 0, \quad Y = -2.$$

Whereas for quarks the assignment is,

$$\chi_L = \begin{pmatrix} u \\ d \end{pmatrix}_L ; \qquad \psi_R = u_R \ or\ d_R.$$

The electromagnetic interaction is embedded in such a way that the generator of the $U(1)_{em}$ is related to the generators of $SU(2)_L$ and $U(1)_Y$ by the following relationship,

$$Q = T^3 + \frac{Y}{2}.$$

In other words, the electromagnetic current is a combination of the two neutral currents J_μ^3 and J_μ^Y. The two neutral gauge fields. The interaction in the neutral current sector can be written as

$$-ig\ J_\mu^3 W^{3\mu} - i\frac{g'}{2}\ J_\mu^Y B^\mu = -iej_\mu^{em} A^\mu - \frac{ie}{sin\theta_w\ cos\theta_w}[J_\mu^3 - sin^2\theta_w j_\mu^{em}]Z^\mu. \tag{1.3}$$

We have written the above equation in terms of the two physical fields,

$$\begin{aligned} A_\mu &= cos\theta_w\ B_\mu + sin\theta_w W_\mu^3, \\ Z_\mu &= -sin\theta_w\ B_\mu + cos\theta_w W_\mu^3. \end{aligned}$$

The requirement that electromagnetic interaction must appear in the right hand side of eqn(1.3) has fixed a relationship between (g, g') and (e, θ_w), namely,

$$e = g\ sin\theta_w = g' cos\theta_w.$$

The weak mixing angle $\sin^2\theta_w$ is one of the important parameters of the standard model, which gives us the amount of mixing of the two neutral gauge bosons with photon and Z_μ.

The Higgs mechanism is incorporated to make the electroweak gauge bosons massive. To do this, the scalar doublet ϕ is introduced. The most economical choice is the isospin doublet with hypercharge $Y = 1$:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}.$$

The relevant part of the $SU(2)_L \times U(1)_Y$ invariant Lagrangian for the gauge boson masses is,

$$\mathcal{L}_1 = [(i\partial_\mu - g\,T\,.\,W_\mu - g'\frac{Y}{2}B_\mu)\phi]^2 - V(\phi).$$

The $[\ ]^2$ above is defined as $[\ ]^\dagger[\ ]$. $V(\phi)$ is the Higgs potential given by,

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2.$$

When the coefficient of μ^2 becomes negative, the Higgs scalar acquires a vacuum expectation value (vev) to make the gauge bosons massive. The vev ϕ_0 has the following form,

$$\phi_0 = \sqrt{\frac{1}{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}.$$

The gauge boson masses are obtained by shifting the variable, $\phi = \phi' - \phi_0$ the vev ϕ_0 in the Lagrangian $\mathcal{L}_1$ and identifying terms with ϕ_0. The relevant term is,

$$\left|(-ig\frac{\tau}{2}\,.\,W_\mu - i\frac{g'}{2}B_\mu)\phi\right|^2$$
$$= (\frac{1}{2}vg)^2\, W_\mu^+ W^{-\mu} \;+\; \frac{1}{8}v^2\,(\,W_\mu^3 \quad B_\mu\,)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}.$$

Now it is easy to see that the gauge bosons $W^\pm$, Z, and the photon A has the following masses,

$$M_W = \frac{1}{2}vg; \quad M_A = 0; \quad M_Z = \frac{1}{2}v\sqrt{g^2 + g'^2}.$$

In the standard model Lagrangian the fermion mass term of the form $m\bar{\psi}\psi$ is forbidden by gauge invariance. An interesting feature of the standard model is that the same Higgs doublet that gives masses to the gauge bosons is also sufficient to give masses to the fermions. For example, to generate the mass for the electron let us consider the following Yukawa part of the Lagrangian:

$$\mathcal{L}_2 = -h_e\Big[(\,\bar{\nu_e} \quad \bar{e}\,)_L \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} e_R + \bar{e}_R\,(\,\phi^- \quad \phi_0\,)\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L\Big].$$

Now we can read off the mass of the electron when the Higgs field gets a vev,

$$m_e = \frac{h_e v}{\sqrt{2}}.$$

The quark masses are generated in the same way. The only novel feature is that to generate a mass for the upper quark of a quark doublet we must construct a new doublet,

$$\phi_c = -i\tau_2\phi^* = \begin{pmatrix} -\phi^0 \\ \phi^- \end{pmatrix}.$$

Due to the special properties of SU(2) ϕ_c transforms identically to ϕ. Hence this minimal choice of Higgs fields can give mass to both up and down type quarks. However in case of supersymmetry, we shall see that such minimal choice no longer works. One needs in that case two Higgs doublets to give masses to up and down type of quarks.

Standard model describes a tremendous chunk of physics. The strong interactions are responsible for the structure of the nuclei and for the most part of what happens in the high energy collisions. The weak interactions are responsible for the nuclear transmutations. The electromagnetism is of course responsible for, "All of Chemistry and most of Physics". SM is highly successful phenomenologically. It is consistent with all available experimental data. From present experimental inputs both model dependent [2] and independent [3] precision tests have been done and they indicate no deviation from Standard Model.

Despite all these successes the Standard Model cannot be given the status of a complete theory and there is strong motivation for the study of the physics beyond the Standard Model. First, it has a large number of parameters which has to be determined from the experiments (gauge couplings, fermion masses etc). The assignment of quantum numbers is partly arbitrary and it has to be explained from a better theory (Charge Quantization). The Higgs sector is not properly understood as there are problems in the fixed point structure of the ϕ^4 theory (Triviality Problem). There is no fundamental symmetry in the SM which will prevent the Higgs boson from getting large quadratic corrections to its mass from radiative processes (Hierarchy Problem). Now to explain charge quantization one has to embed the SM into a larger group and the fermions in some irreducible representation of the bigger group[4]. Then from the remormalization group analysis one can study the unification of forces [10]. These ideas lead to the study of Grand Unified Theories (GUTS). In a different attempt to explain the hierarchy one may assume that there is a scale Λ above which the Higgs boson is not a fundamental particle. Hence this scale will act as a cut-off scale in the quadratically divergent integrals. This idea lead to the study of the Technicolor and Condensate models[6] where one aims to have a spontaneous symmetry breaking with the gauge interactions alone: there is no elementary scalar with its self couplings and Yukawa couplings. On the other hand there may be fermionic-bosonic symmetry due to which large quadratic divergences cancel upto all orders and one is left with a finite theory. Studies in this direction leads to Supersymmetry. Out of these three major directions to go beyond the standard model Technicolor is less favored by the precision electroweak measurements [3].

1.2 Minimal SU(5) GUT

Grand Unified Theories (GUTs) [7] offer the possibility of a simple but unified description of strong and electroweak interactions. Typically in these models at some high energy all the interactions arise out of a single Lagrangian which is locally invariant under the gauge transformations of a single simple Lie group called the unification group. A large spectrum of GUTs are proposed in the literature which is broadly classified by the unification group. In the minimal $SU(5)$ model all interactions unite at a single step at an energy around 10^{14} GeV therefore predicting the absence of any new physics between the standard electroweak breaking scale(M_z) and the unification scale(M_U) while the $SO(10)$ model admits intermediate breakings of symmetry. On the other hand there are models which are inspired by superstring theories, one of them postulates the exceptional group E_6 as the unifying group. This specific model predicts atleast 12 exotic fermions on top of the 15 standard fermions.

All these theoretically very attractive models have atleast one common prediction namely the decay of proton. There has been a desperate search by the experimentalists to see the signature of proton decay for the last decade and a half. Contrary to the theoretical beliefs proton decay has not been discovered. At present the lower limit of the half life of proton is a whopping 10^{32} years.

The simplest model of unification existing in the literature is the SU(5) GUT [1]. Although the simplest nonsupersymmetric version of SU(5) is ruled out by LEP data and limit on proton lifetime (although the effect of gravity may still make it a viable one) the idea of grand unification is very much alive and supposed to be the most popular extension of standard model. To explain the basic concepts of grand unification we briefly describe the SU(5) GUT which is conceptually easiest to understand.

The major success of the Glashow Weinberg Salam (GWS) model of electroweak interactions has motivated physicists to look for a unified theory of strong and electroweak interactions. According to the present view the strong interaction can be understood in terms of Quantum Chromodynamics (QCD), which is an unbroken gauge theory based on the gauge group $SU(3)_c$. The fundamental representation of this group contains the three colored states of a quark.

The color gauge group $SU(3)_c$ is orthogonal to the WS electroweak symmetry group $SU(2)_L \times U(1)_Y$. Thus a broken gauge theory based on the semi-simple group $SU(3)_c \times SU(2)_L \times U(1)_Y$ with a coupling strengths of g_3, g and g', respectively, for the three factors, can explain the strong and electroweak interactions at low energies. The quantum numbers of the fermions under the group $SU(3)_c \times SU(2)_L \times U(1)_Y$ are given in Table 1.1

$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(3, 2, 1/3)$
$\begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$	$(1, 2, -1)$
u_L^c	$(\bar{3}, 1, -4/3)$
d_L^c	$(\bar{3}, 1, 2/3)$
e_L^+	$(1, 1, 2)$

Table 1.1: fermions and their $SU(3)_c \times SU(2)_L \times U(1)_Y$ transformation laws

These particles form a family or generation. We have expressed all the fields in a left handed basis, using $\psi_L^c = c\,\bar{\psi}_R^T$. The results for those particles can be extended to other generations also, which will not be discussed here. The right-handed neutrino ν_R or the left handed antineutrino ν_L^c will also be included in some models.

[1] This discussion on minimal SU(5) follows Ref [13]

However, one notices that the theory is not truly unified in the sense that the gauge coupling strengths g_3, g_2 and g_1 are different. Georgi and Glashow first suggested the idea of embedding the the standard group $SU(3)_c \times SU(2)_L \times U(1)_Y$ into a simple group, namely SU(5), such that at very high energies the symmetry group SU(5) breaks down to the standard group. The fermions are contained in the SU(5) representations $\bar{5}$ and 10 respectively. The explicit form of the fermion representations are:

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}_L \qquad 10 = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}_L .$$

These representations decompose under the standard model gauge group as,

$$\begin{aligned} \bar{5} &= (\bar{3}, 1, \frac{2}{3}) + (1, 2, -1) \\ 10 &= (\bar{3}, 1, -\frac{4}{3}) + (3, 2, \frac{1}{3}) + (1, 1, 2) \end{aligned}$$

The transformation properties of the 24 gauge bosons of the SU(5) model are as follows,

$$24 = (8,1,0) + (1,3,0) + (1,1,0) + (3,2,-5/3) + (\bar{3},2,5/3).$$

The first term represents the gluons and the second and third term represents a linear combination of the $W^{\pm}$, Z boson and the photon. The gauge bosons (3,2,-5/3) and (3,2,5/3) are introduced by the SU(5) GUT and they acquire masses at the scale M_U, i.e, the scale at which the SU(5) symmetry is broken. These gauge bosons couple to diquarks and leptoquarks. They can mediate the processes like $p \longrightarrow e^+\pi^0$ and $n \longrightarrow e^+\pi^-$. The lifetime of such nucleon decay will be given by,

$$\tau_p \sim \alpha_5^{-2} \frac{M_U^4}{m_N^5}, \tag{1.4}$$

where α_5 is the SU(5) coupling constant, m_N stands for the nucleon mass. We shall now estimate M_U following Georgi Quinn and Weinberg [10].

If we use the same normalization conventions for different gauge groups, then the invariance under SU(5) implies that the coupling constants g_5, g_3, g_2 and, g_1 associated with the groups SU(5), SU(3) SU(2) and U(1) respectively are equal when SU(5) is broken at a scale M_U, i.e,

$$g_5(M_U) = g_3(M_U) = g_2(M_U) = g_1(M_U). \tag{1.5}$$

When the SU(5) symmetry is exact the charge Q and the SU(2) and U(1) generators T and T_0 are given by,

$$Q = T_3 + cT_O.$$

The gauge coupling constants g_2 and g_1 must also be related to the usual $SU(2)_L$ and $U(1)_Y$ coupling constants of the WS model, g and g' by,

$$g_2 = g \;\; and \;\; g_1 = c\, g'.$$

Now we can write

$$Tr[Q^2] = (1 + c^2)Tr[T_3^2].$$

For n_F generations of fermions we have,

$$Tr[T_3^2] = 2\ n_f \quad and \quad Tr[Q^2] = \frac{16}{3}n_f$$

This immediately gives us the value of c,

$$c^2 = \frac{5}{3}.$$

Thus we get the value of the Weinberg angle at the scale Q,

$$sin^2\theta_w(Q) = \frac{g'^2(Q)}{g^2(Q) + g'^2(Q)}.$$

Hence, for a scale Q larger than or equal to the unification scale M_U we have,

$$sin^2\theta_w = \frac{c^2}{1 + c^2} = \frac{3}{8}. \tag{1.6}$$

and also,

$$\frac{\alpha}{\alpha_s}(Q) = \frac{e^2/4\pi}{g_3^2/4\pi} = \frac{e^2}{g_3^2} = \frac{g^2 g'^2}{g_3^2(g^2 + g'^2)}.$$

Hence, for a scale Q larger than or equal to the unification scale M_U we have,

$$\frac{\alpha}{\alpha_s} = \frac{3}{8}.$$

Once the value of $sin^2\theta_w$ and $\frac{\alpha}{\alpha_s}$ are calculated in terms of the normalized coupling constants at the SU(5) symmetry level, it is a straight forward job to calculate $sin^2\theta_w$ and $\frac{\alpha}{\alpha_s}$ at the present energies. We use the standard renormalization group equations,

$$\alpha^{-1}(q) = \alpha^{-1}(Q) + 2\ b\ M_{Qq}. \tag{1.7}$$

where in equation (1.7) b is defined as,

$$b = -\frac{1}{16\pi^2}[\frac{11}{3}N - \frac{4}{3}T_R - \frac{1}{6}T_s], \tag{1.8}$$

and, M_{Qq} is defined as $4\pi\ ln\frac{Q}{q}$. The three terms in the r.h.s. of equation (1.8) represent the gauge boson, the fermion and the scalar contributions respectively. For the U(1) groups there is no nonabelian gauge coupling and hence the first term is absent. thus combining the boundary conditions given in equation (1.5) and in in equation(1.6) we can write,

$$sin^2\theta_w(Q^2) = \frac{3}{8}(1 - \frac{55\alpha}{18\pi}ln\frac{M_U^2}{Q^2}),$$

$$\text{and} \quad \frac{\alpha(Q^2)}{\alpha_s(Q^2)} = \frac{3}{8}(1 - \frac{11\alpha}{2\pi}ln\frac{M_U^2}{Q^2}).$$

For $\alpha_s = 0.12$ the value of $sin^2\theta_w$ is predicted to be 0.21 which is lower than the experimentally observed value of 0.23±.002 approximately. The proton lifetime is also predicted to be too low to be consistent with the experimental lower bound on τ_P. Due to these reasons it is believed that the minimal version of the SU(5) model is ruled out. However we shall see in this thesis that including the effects of gravity induced higher dimensional operators one can still make the model consistent with experimental results. Certain other non-minimal extensions of this model like extending it to include new fermions at some intermediate scale can also make the model consistent with experimental results.

1.3 Inclusion of left-right symmetry

One important feature of the minimal SU(5) GUT is that it suggests the existence of a desert above the electroweak scale. Actually, the minimal SU(5) model predicts no new mass scale between the electroweak scale and the unification scale. This depressing feature is absent in more extended models of unification like SO(10) GUT. This model admits intermediate mass scales in which parity is restored.

The nature of the weak interactions appears to be very well described by the celebrated V-A theory, which enjoys sound phenonenological success at the low energies. The standard $SU(2)_L \times U(1)_Y$ gauge theory provides a sound mathematical basis of the V-A theory. An alternative approach to the electroweak interactions has been proposed by Pati and Salam according to which the basic weak Lagrangian is invariant under the space reflections. This involves both V-A as well as V+A interactions. The gauge group of this model is left right symmetric and it is broken to standard model spontaneously by Higgs mechanism.

$$\begin{aligned} SU(2)_L \times SU(2)_R \times U(1)_{B-L} & \stackrel{M_R}{\longrightarrow} SU(2)_L \times U(1)_Y \\ & \stackrel{M_Z}{\longrightarrow} U(1)_{em}. \end{aligned} \tag{1.9}$$

The electromagnetic charge is related to the diagonal generators of the left-right symmetric model as,

$$Q = T_L^3 + T_R^3 + \frac{B-L}{2}.$$

The weak Lagrangian prior to the symmetry breaking is given by,

$$\mathcal{L}_{weak} = \frac{g}{\sqrt{2}}(J_{\mu L} \cdot W_L^\mu + J_{\mu R} \cdot W_R^\mu)$$

here, $J_{\mu L} = J_{\mu R}(\gamma_5 \rightarrow -\gamma_5)$. The noninvariance of the vacuum under space reflections results in $M_{W_R} >> M_{W_L}$, and, as a result, all the low energy weak processes appear the same as in the $SU(2)_L \times U(1)_Y$ theory, with small corrections undetectable in experimental results.

In left right symmetric models, since both left handed and right handed helicities of the neutrino are included, the neutrino naturally has a mass. It has been shown that if the neutrino is a Majorana particle, one can obtain the qualitative relation

$$m_{\nu_e} \simeq O(\frac{1}{m_{W_R}})$$

This relation provides a deeper physical insight into the connection between the small neutrino mass and the maximality of parity violation.

SO(10) is the minimal left-right symmetric grand unified theory. Actually, the whole $SU(4)_c \times SU(2)_L \times SU(2)_R$ of Pati and Salam can be embedded into SO(10). The adjoint representation of SO(10) is 45 dimensional, hence there are 45 gauge bosons. We have 21 gauge bosons associated with the group $SU(2)_L \times SU(2)_R \times SU(4)$ and 24 superheavy guage bosons which become massive at the scale of breaking of SO(10).

The symmetry breaking pattern of the SO(10) model is the following,

$$\begin{aligned} SO(10) \; & \underset{M_U}{\longrightarrow} \; SU(2)_L \times SU(2)_R \times SU(4)_c \; \equiv G_{PS} \\ & \underset{M_1}{\longrightarrow} \; SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \; \equiv G_{LR} \\ & \underset{M_R}{\longrightarrow} \; SU(2)_L \times SU(3)_c \times U(1)_Y \; \equiv G_{std} \\ & \underset{M_W}{\longrightarrow} \; SU(3)_c \times U(1)_Q \end{aligned} \tag{1.10}$$

The first stage of symmetry breaking at the scale M_U is conventionally done by the the vev of 45 dimensional scalar which has a singlet under the lower symmetry group. In the next stage the breaking at the right handed scale M_R is achieved by the vev of a $SU(2)_R$ triplet field Δ_R which transforms as (1,3,1) under G_{PS} and contained in 126 of SO(10). This scalar also contains the left and handed triplets $\Delta_L = (3,1,1)$ under G_{PS}. Finally in the minimal SO(10) model the 10 dimensional scalar contains the usual $SU(2)_L$ doublet which breaks the electroweak symmetry and gives masses to the scalars.

The mass scales of this symmetry breaking chain can be calculated using the renormalization group equations. The intermediate symmetry breaking scales M_I and M_R reduces the hierarchy problem of the SU(5) GUT. For low M_R there can also be interesting phenomenological consequences.

The fermions transform under the 16 dimensional spinorial representation of SO(10). This multiplet contains the right handed neutrino on top of the standard 15 fermions. Let us at this stage state the transformation properties of the fermions under the group $SU(2)_L \times SU(2)_R \times SU(4)_c$,

$$\begin{aligned} \psi_L &= \begin{pmatrix} \nu_L \\ e^-{}_L \end{pmatrix} : (2,1,4) \; ; \; \psi_R = \begin{pmatrix} \nu_R \\ e^-{}_R \end{pmatrix} : (2,1,4) \\ Q_L &= \begin{pmatrix} u_L \\ d_L \end{pmatrix} : (2,1,4) \; ; \; Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} : (2,1,4) \end{aligned} \tag{1.11}$$

Also let us state the scalar fields which may acquire vev,

$$\begin{aligned} \phi_1 &\equiv (2,2,1) \; ; \; \tilde{\phi} \equiv \tau_2 \phi_1^* \tau_2 \\ \Delta_L &\equiv (3,1,10) \; ; \; \Delta_R \equiv (1,3,10) \end{aligned}$$

The vacuum expectation values of the fields have the following form:

$$\begin{aligned} \langle \phi \rangle &= \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} \; ; \; \langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix} \\ \langle \tilde{\phi} \rangle &= \begin{pmatrix} k' & 0 \\ 0 & k \end{pmatrix} \; ; \; \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \end{aligned}$$

Now we are in a position to discuss the physics of neutrino masses in left-right symmetric models. The fermions acquire masses through the Yukawa terms in the Lagrangian when the

Higgs fields acquire vev. The Yukawa part in the Lagrangian written in terms of fermionic and Higgs fields is given by,

$$L_Y = y_1(\bar{f}_L f_R \phi_1) + y_2(\bar{f}_L f_R \phi_2) + y_3(\bar{f}_L^c f_L \Delta_L + \bar{f}_R^c f_R \Delta_R), \tag{1.12}$$

where, y_i (i=1,3) are the Yukawa couplings. With this notation neutrino mass matrix written in the basis (ν_L, ν_L^c) is

$$M = \begin{pmatrix} m_{M_L} & m_D \\ m_D & m_{M_R} \end{pmatrix} \tag{1.13}$$

where m_{M_L} (m_{M_R}) is the left (right) handed Majorana mass term whereas m_D is the Dirac mass term. These terms can be related to the Yukawa couplings and vevs through the following relations;

$$\begin{aligned} m_{M_L} &= y_3 v_L, \\ m_D &= (y_1 + y_2)(k + k'), \\ m_{M_R} &= y_3 v_R. \end{aligned} \tag{1.14}$$

Diagonalizing the mass matrix we obtain the mass eigenvalues. Now let us consider the simplifying assumption that all the Yukawa couplings are of order "h" and the vev k' is much smaller than the *vev* s k. Under this assumption the eigenvalues become,

$$\begin{aligned} m_1 &= y_3 v_R, \\ m_2 &= m_{M_L} - \frac{M_D^2}{m_{M_R}} = y_3 v_L - \frac{h^2 k^2}{y_3 v_R}. \end{aligned}$$

We substitute for v_L from the see-saw condition to get,

$$m_2 = y_3 \frac{\beta k^2}{v_R} - \frac{h^2 k^2}{y_3 v_R}. \tag{1.15}$$

Here β is a function of couplings. We notice that the second term in the right hand side is suppressed by the square of the Yukawa coupling. Due to this the first term dominates. If we want to make the first term small compared to the second we need to fine tune the parameters. Hence one has to fine tune such that $\beta k^2 \simeq 0$ to get acceptable value of the the light neutrino mass.

1.4 Extension to include supersymmetry

Supersymmetry (SUSY) implies that every fermion has a bosoic partner, and vice versa, with the same quantum numbers but with spin differing by one half. Since no such partners have been found in low energy, SUSY has to be broken at the present energy scale but could be restored at and above some higher scale M_{SUSY}.

The primary theoretical motivation for SUSY is that it stabilizes divergent loop contributions to the scalar mass, because fermion and boson loops contribute with opposite signs and largely cancel. This cures the naturalness problem in the SM so long as $M_{SUSY} < O(1TeV)$, where otherwise the Higgs mass would require fine tuning of parameters.

In addition to these general motivations for SUSY there exists several significant motivations for studying SUSY-GUTS. Firstly the ordinary GUTs don't predict satisfactory convergence of the gauge couplings at the scale M_U if there are no intermediate symmetry breaking scale between M_Z and M_U and no new particles (in other words if the couplings converge it does not predict the correct vale of $sin^2\theta_w$ at low energy) however the convergence can be achieved if SUSY partners contribute to the Renormalization Group Equations (RGE). Secondly proton decay is too rapid in non supersymmetric minimal SU(5) GUTs whereas in SUSY GUTs because of the higher value of M_U the it is acceptable. Finally in SUSY-GUT models the Higgs field naturally develops a vacuum expectation value for a heavy top mass.

To understand the basic SUSY algebra, let us consider the following toy example. Here is a field theory where the Lagrangian is given by [2],

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{mass}, \tag{1.16}$$

where, the terms in the r.h.s. has the following explicit form,

$$\begin{aligned} \mathcal{L}_{kin} &= \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{i}{2}\bar{\psi}\partial_\mu\gamma^\mu\psi + \frac{1}{2}(F^2 + G^2), \\ \mathcal{L}_{mass} &= -m\Big[\frac{1}{2}\bar{\psi}\psi + GA + FB\Big]. \end{aligned}$$

Here A, B, F, G are real scalar fields and ψ is a self conjugate, Majorana spinor field. Since ψ is its own charge conjugate we have,

$$\psi = C\bar{\psi}^T.$$

Wess and Zumino [7] observed that under the transformations,

$$\begin{aligned} \delta A &= i\bar{\alpha}\gamma_5\psi, \\ \delta B &= -\bar{\alpha}\psi, \\ \delta\psi &= F\alpha - iG\gamma_5\alpha + \partial_\mu\gamma^\mu\gamma_5 A\alpha + i\partial_\mu\gamma^\mu B\alpha, \\ \delta F &= -i\bar{\alpha}\partial^\mu\gamma_\mu\psi, \\ \delta G &= -\bar{\alpha}\gamma_5\partial^\mu\gamma_\mu\psi, \end{aligned} \tag{1.17}$$

Lagrangian density (1.16) changes by a total time derivative so that the equations of motion are unchanged. We observe that these transformations mix bosons and fermions and so are supersymmetry transformations. The parameter of the transformation, α is thus itself spinorial. Further, to preserve the reality of A and B as well as the Majorana nature of ψ α itself has to have the Majorana nature.

In order to understand the SUSY transformations further, we consider the effect of two successive SUSY transformations with parameters α_1 and α_2.

$$(\delta_2\delta_1 - \delta_1\delta_2)A = -2i\bar{\alpha}_1\gamma_\mu\partial^\mu A\alpha_2. \tag{1.18}$$

In order to find the algebra obeyed by the generator of supersymmetry, we write as usual $\delta = \bar{\alpha}Q$. We note that Q must be a Majorana generator. Now Eqn.(1.18) can be written as,

$$\begin{aligned} (\bar{\alpha_2}Q\ \bar{\alpha_1}Q - \bar{\alpha_1}Q\ \bar{\alpha_2}Q) &= \bar{\alpha_1}_a\alpha_{2b}(\bar{Q}_bQ_a + Q_a\bar{Q}_b)A, \\ &= -2i\bar{\alpha_1}_a\alpha_{2b}(\partial_\mu\gamma^\mu A)_{ab}. \end{aligned} \tag{1.19}$$

[2]The discussion on SUSY algebra closely follows [8]

Here, a, b are spinor indices. To obtain the first equality we have used the fact that the parameter α_{ia} anticommutes with themselves as well as the charges Q_α. Since (1.19) is true for arbitrary values of of the parameters we are led to conclude that the SUSY charges obey the following anticommutation relation,

$$\left[Q_a, \bar{Q}_b\right]_+ = 2(\gamma_\mu P^\mu)_{ab}.$$

Where, P^μ is the translation generator of the Poincare group and the curly brackets denote an anticommutator. This shows that supersymmetry is a space time symmetry and not an internal symmetry. which, in turn, leads us to examine the commutators of Q_a with the generators of the Poincare group. If we identify P^0 with the Hamiltonian, conservation of Q_a implies,

$$\left[Q_a, P^0\right] = 0, \tag{1.20}$$

from covariance,

$$\left[Q_a, P^\mu\right] = 0. \tag{1.21}$$

Notice that the supersymmetry algebra described above contains commutators as well as anticommutators and so is not a Lie algebra. Such algebras are called graded Lie algebras. In fact, Haag, Lopuzanski and Sohnius [9] have shown that the algebra we have derived above is the most general graded Lie algebra compatible with certain rather reasonable physical assumptions.

We note that the the relation (1.20) means that all states but the zero energy one (the vacuum) come in degenerate pairs with one member of the pair being bosonic and the other fermionic. This implies that in a supersymmetric theory, every particle has a partner with the same mass but spin different by $\frac{1}{2}$. Finally we note that supersymmetry being a space time symmetry is independent of any internal symmetry; i.e., the generators of internal symmetry transformations and supersymmetry transformations commute. As a result a particle and its SUSY partner have identical internal symmetry quantum numbers such as electric charge, color, isospin and so on.

In order to construct the Minimal Supersymmetric Standard Model (MSSM), we have to introduce a partner for every particle of the standard model, with a spin differing by $\frac{1}{2}$. Thus the SUSY partners of the matter fermions must have spin zero or spin one. Since the only way we know to consistently introduce interactions of the spin one particles is to make them gauge bosons we are led to consider spin zero bosons as their partners.

In the electroweak symmetry breaking sector, in the standard model, the $SU(2)_L \times U(1)_Y$ symmetry is broken by a single Higgs field. Moreover the same field can give masses to both the upper and lower member of a fermion multiplet. Technically this is possible in SM because both the scalar ϕ and its conjugate ϕ^c can couple to fermions in a gauge invariant way. In SUSY, however, these Yukawa couplings come from the superpotential which in turn cannot depend both on a field and its complex conjugate. As a result, a doublet Higgs scalar can give rise to the mass of either the upper partner or the lower partner of a multiplet but not both.

The MSSM thus contains the particles of the standard model with two Higgs doublets and the spin one bosons which mediate the interactions as shown in the first row of the Table 1.2. The second row contains the super partners of the fields shown in the first row. These are combined to form the super multiplets which are shown in the third row. Thus $L_i(Q_i)$

and $\bar{E}_i(\bar{U}_i, \bar{D}_i)$ are the left handed lepton (quark) and antilepton (antiquark) singlet chiral superfields, where i, refers to the generation index. Similarly $H_{1,2}$ are the chiral superfields representing the the two Higgs doublets and and G is the vector superfield representing the gluon. The Yukawa part of the Lagrangian contains the Higgs coupling terms responsible for the lepton and quark masses.

$$L_Y = h^{\tau}_{ij}(L_i H_1 \bar{E}_j)_F + h^b_{ij}(Q_i H_1 \bar{D}_j)_F + h^t_{ij}(Q_i H_2 \bar{U}_j)_F. \tag{1.22}$$

$ordinary fields$	l_i	$\bar{e}_i$	q_i	$\bar{u}_i$	$\bar{d}_i$	h_1	h_2	g	W	Z	γ
$superpartners$	$\tilde{l}_i$	$\tilde{\bar{e}}_i$	$\tilde{q}_i$	$\tilde{\bar{u}}_i$	$\tilde{\bar{d}}_i$	$\tilde{h}_1$	$\tilde{h}_2$	$\tilde{g}$	$\tilde{W}$	$\tilde{Z}$	$\tilde{\gamma}$
$super fields$	L_i	$\bar{E}_i$	Q_i	$\bar{U}_i$	$\bar{D}_i$	H_1	H_2	G	..	..	..

Table 1.2: first row - ordinary fields, second row - corresponding superpartners and third row - superfields

In the evolution of the gauge couplings of the MSSM the superpartners also contribute. This changes the expression of the coefficient b of equation (1.8). The new expression for b in the SUSY case is,

$$b = -\frac{1}{16\pi^2}[3N - T_R - T_s]. \tag{1.23}$$

In the LEP experiments at CERN, Geneva, the gauge couplings of the standard model are very precisely measured at the Z-peak. If we evolve the couplings using the supersymmetric beta functions with a symmetry breaking scale of around one TeV the couplings meet at an unique intersection point at the scale of around 10^{16} GeV. This is considered as a hint for the existence of SUSY above the TeV region.

The supersymmetric GUT has another advantage over the nonsupersymmetric GUT. As the unification in the SUSY GUT is achieved at a scale of around 10^{16} GeV the proton decay rate is consistent with the experimental lower bound on proton lifetime. On the other hand the nonsupersymmetric GUT the proton decay is predicted to be too fast.

The supersymmetric GUTs predict the correct electroweak symmetry breaking scale. This happens because the top quark Yukawa coupling contributes to the evolution of the Higgs mass that couples to the top quark. As a result for a heavy top the Higgs mass becomes negative at the electroweak region. It can be shown that for reasonable choices in the parameter space it predicts the correct electroweak symmetry breaking.

Independently of the GUT relations among the Yukawa couplings, h^{τ}, h^b and h^t (for the time being let us consider only the third generation for simplicity) must be bounded from above at the low energies otherwise they would blow up before reaching the unification scale M_U. Using this fact one can obtain an upper bound on the top mass in the SUSY GUTs [10].

$$\begin{aligned} m_t &= h^t v_2, \\ &\lesssim (174GeV)\, h^t_{max} \frac{1}{\sqrt{1 + \frac{1}{tan^2\beta}}}, \\ &\lesssim 135, 170, 180, 190\ (GeV)\ \ (for\ tan\beta\ =\ 1, 2, 3, \infty). \end{aligned} \tag{1.24}$$

Here v_2 is the vev of the scalar field H_2 and $tan\beta = \frac{\langle H_2 \rangle}{\langle H_1 \rangle}$. This upper bound is remarkably consistent with the value of the top quark mass predicted from the precision electroweak measurements.

Thus we see that the introduction of SUSY is one of the major candidate theories to solve the gauge hierarchy problem. In the next section we shall introduce another new paradigm of unification models which can also solve the hierarchy problem based on SU(15) and SU(16).

1.5 Low energy unification

One of the most important prediction of GUTs is proton decay. In models such as SU(5) or $SO(10)$, the coupling constants for the SU(3)$_c$, $SU(2)_L$ and $U(1)_Y$ groups evolve to $M_U \sim O(10^{14})$ GeV or higher before they get unified. Proton decay in these models are predicted to occur with a rate of the order of the present experimental limit for such M_U. Recently it has been observed that at least one symmetry breaking chain of a GUT based on the group SU(15) can be unified at a very low energy $M_u \sim O(10^7)$ GeV[17]. Because baryon number B is a gauge symmetry in this model, proton decay can be suppressed, and certain possible Higgs structure has been proposed to this end. This was true for SU(16) GUT[11] also, where it was also known that the proton lifetime can be large. Low energy unification makes these models free from problems of grand unified monopoles[18] and the gauge hierarchy problem is also much less severe.

The drawback of SU(15) and SU(16) GUT compared to SU(5) is that the anomaly cancellation is incomplete without mirror fermions, although the latter could be in principle, replaced by other compensating fermions in higher mass scales in a more general theory based on, e.g., superstrings. If the chiral anomalies are canceled by mirror fermions, it is assumed that such mirror fermions have masses of the order of $O(M_W)$. If they appear as mass degenerate sets, they do not affect the renormalization group determinations of the unification and other mass scales, and more generally need not affect the low energy physics.

The fermions, belong to the 15 dimensional fundamental representation.

$$15_L = (\, u_1, u_2, u_3, d_1, d_2, d_3, \bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{d}_1, \bar{d}_2, \bar{d}_3, e^+, \nu_e, e^- \,)_L \qquad (1.25)$$

The 15 of SU(15) decomposes to (12,1)+(1,3) under the group $SU(12)_{quark} \times SU(3)_{lepton}$. Next the subgroup $SU(12)$ breaks to $SU(6)_L \times SU(6)_R \times U(1)_B$ and 12 of $SU(12)$ breaks to (6,1)+(1,6). In these two mass scales the leptoquark and the diquark gauge bosons become massive. Now as the leptoquark and diquarks have different baryon number (note here $U(1)_B$ can be identified as gauged baryon number) they do not mix and hence there is no gauge boson mediated proton decay.

We denote the group SU$(n)^q_L\times$SU$(m)^l_R\times$U$(1)_X$ as n^q_L–m^l_R–1_X where the subscript implies either the charge of the U(1) group or that right (left) handed particles are non-singlets under SU(n) (SU(m)) and the superscript q (l) means that only quarks (leptons) transform under this group. For the breaking $G_i \longrightarrow G_{i-1}$, the G_{i-1} singlet component of the Higgs scalars acquires a vev at a scale M_i. We consider the pattern $G_1[15] \xrightarrow{\langle\phi_1\rangle} G_2[12^q\text{–}3^l] \xrightarrow{\langle\phi_2\rangle} G_3[6_L\text{–}6_R\text{–}1_B\text{–}3_l] \xrightarrow{\langle\phi_3\rangle} G_4[3_{cL}\text{–}2^q_L\text{–}6_R\text{- -}1_B\text{–}3_l] \xrightarrow{\langle\phi_4\rangle} G_5[3_{cL}\text{–}2^q_L\text{–}3_R\text{–}1_R\text{–}1_B\text{–}2^l_L\text{–}1^l_Y] \xrightarrow{\langle\phi_5\rangle} G_6[3_c\text{–}2_L\text{–}1_B\text{–}1_{Y'}] \xrightarrow{\langle\phi_6\rangle} G_7[3_c\text{–}2_L\text{–}1_Y] \xrightarrow{\langle\phi_7\rangle} G_8[3_c\text{–}1_Q]$, with $\langle\phi_i\rangle = M_i$.

This scenario of symmetry breaking of the $SU(15)$ GUT has some more interesting features. It is essential for the low energy unification to have the chiral color $SU(3)_{cL} \times SU(3)_{cR}$ group and the quark-lepton un-unified group $SU(2)_L^q \times SU(2)_L^l$ survive till very low energy, for the gauge coupling constants to evolve very fast and get unified at an energy scale $M_U \sim O(10^8)$ GeV. Thus the existence of these groups and the lepto-quarks are some of the essential critereons of the low energy unification, which can be tested in the laboratory in near future. Furthermore, any signature of supersymmetry will rule out the possibility of low energy unification [19]. If we find signatures of the low energy unification and supersymmetry both, then our understanding of the grand unification has to be revised completely.

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Chapter 2

LEP Constraints on Unified Theories

2.1 Prelude

Since 1989 LEP has been producing electron positron collisions at various center of mass energies centered around the peak of the resonance production of the Z_0 bosons. The four experiments ALEPH, DELPHI, OPAL and L3 has till now collected about five million events. The analysis of this data on most varied subjects have been performed. The two most publicized results are the very precise measurements of the number of light neutrino and the absence of the standard neutral Higgs bosons in the energy range upto 63 GeV.

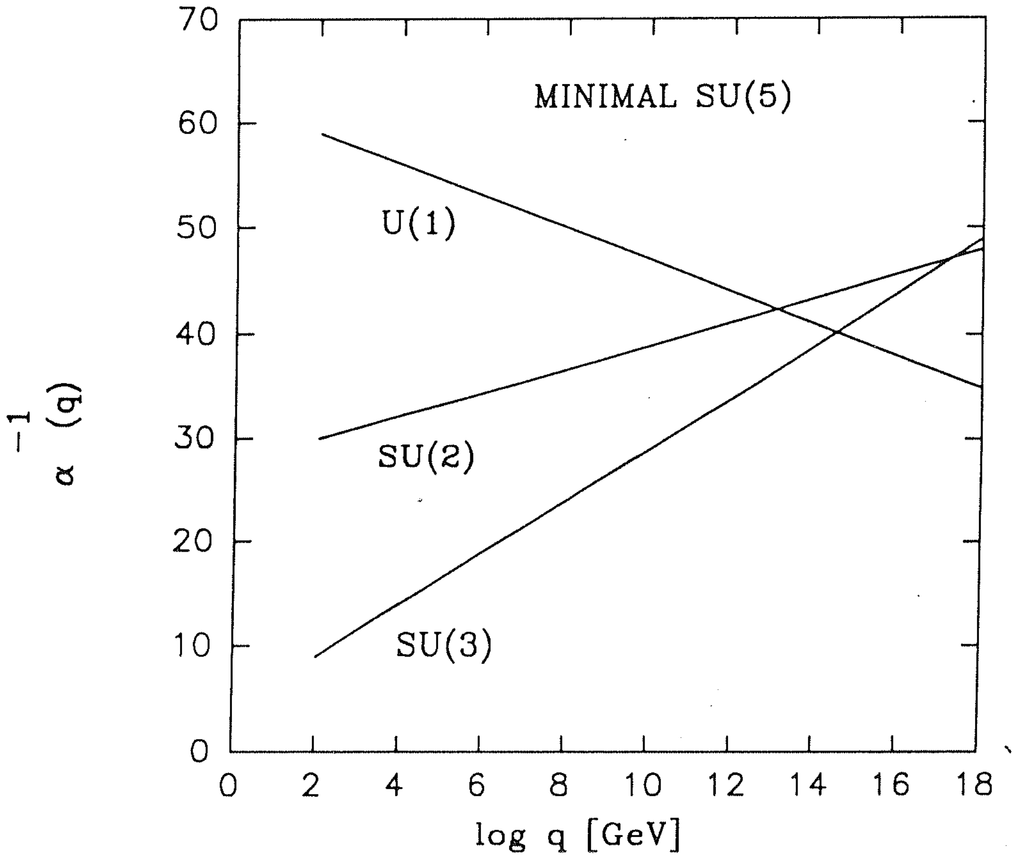
Recent interest in Grand Unified Theories has been motivated by the precision data that has emerged from LEP in the past years [1]. The measurements of the Z mass and width and also the jet cross sections and energy-energy correlations provide very accurate values of $\sin^2\theta_W$ and α_s at the scale M_Z. These precision values of $\sin^2\theta_W$ and α_s, when evolved using the renormalization group equations, can be used to put strong constraints on unified theories [2], [3], [4], [5].

In particular, in the analysis of Refs.[2],[4], it has been shown that using the recent experimental values

$$\begin{aligned} \sin^2\theta_W &= 0.2333 \pm 0.0008 \\ \alpha_s &= 0.113 \pm 0.005 \end{aligned} \tag{2.1}$$

a unique intersection point of the $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ couplings is not obtained in the standard model with one Higgs doublet (See Figure 1). Further in Ref.[2], it is shown that the criterion of unique intersection of the couplings at the unification scale is satisfied in the minimal supersymmetric extension of the standard model, and an unification scale of around 10^{16} GeV is obtained with the supersymmetry scale around 1 TeV (See Figure 2).

There are other ways of modifying the minimal standard model in order to get consistency with the data and the solutions that immediately suggest themselves are the inclusion of the effects of additional fermion generations or Higgs particles. However, the addition of new fermion generations changes the slopes of all the three couplings equally because the fermions contribute the same amount to the beta function coefficients of the $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ groups. As for the Higgs, it has been shown [2] that, in the non-supersymmetric



MINIMAL SU(5)
U(1)
SU(2)
SU(3)
70
60
50
40
30
20
10
0
0
2
4
6
8
10
12
14
16
18
log q [GeV]
α⁻¹ (q)

FIGURE 1

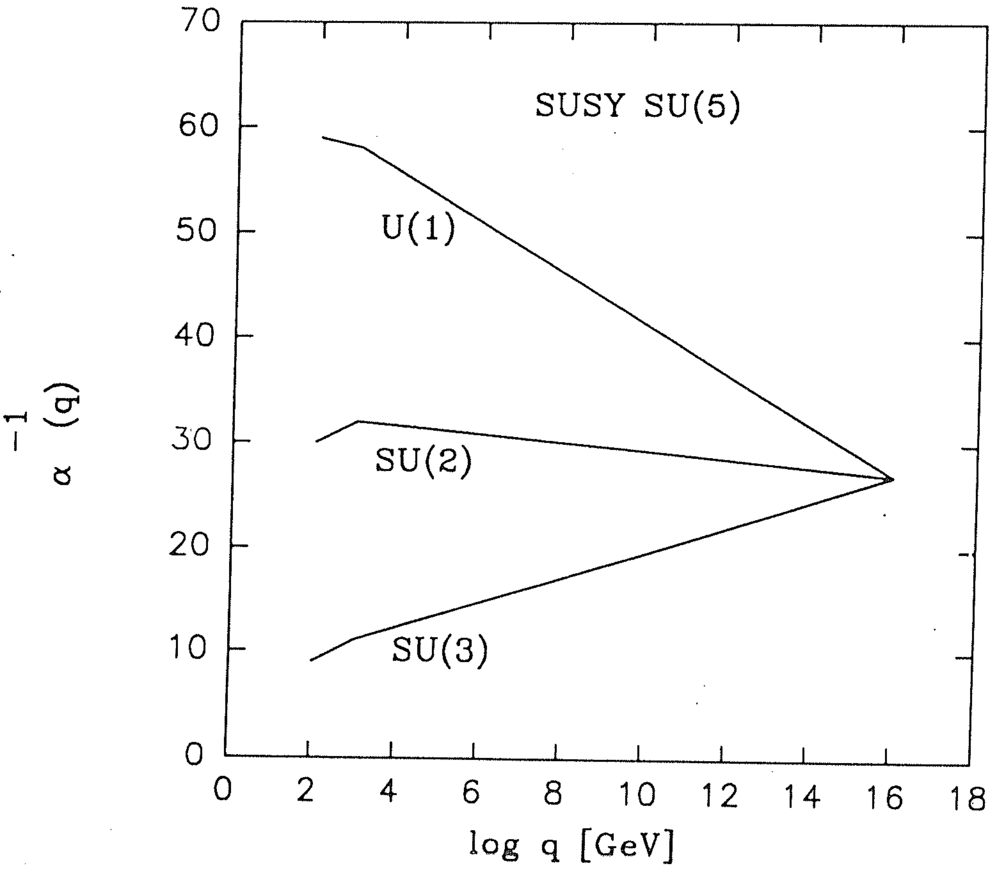

70
60
50
40
30
20
10
0
α^{-1} (q)
0
2
4
6
8
10
12
14
16
18
log q [GeV]
SUSY SU(5)
U(1)
SU(2)
SU(3)

FIGURE 2

case, with six or more Higgs doublets it is possible to obtain a unique intersection point for the couplings, but the value of the unification scale is too low and is ruled out by the measured value of the proton lifetime, $\tau_p = 5.5 \times 10^{32}$ years[6].

The failure of these attempts seems to indicate that the possibility of any unification group breaking in one step to the minimal non-supersymmetric standard model is, indeed, ruled out by present experimental data. Non-minimal (but non-supersymmetric) extensions of the standard model have also been studied [10] where it has been shown that either the introduction of a pair of leptoquarks at a scale of around 100 GeV or of a split **45** dimensional multiplet can also satisfy the unification constraints. The consensus seems to be that if we demand coupling constant unification, then there must be some new physics between the electroweak scale and the unification scale. The possibility of a desert between presently available energies and the unification scale seems to be ruled out. However the minimal left-right symmetric GUTs are consistent with the LEP results. We will see in this chapter that the LEP results put strong lower bounds on the right handed breaking scales of the left-right symmetric GUTS.

2.2 Effects of Higher Dimensional Operators

In this section, we show that this is not a necessary conclusion [1]. This we do by considering the presence of higher dimensional operators in the $SU(5)$ invariant Lagrangian. Such operators, scaled by powers of the Planck mass, arise due to quantum gravity effects [11] or due to spontaneous compactification of the extra spatial dimensions in Kaluza-Klein theories [12]. Such non-renormalizable terms involving fermion and Higgs fields have been used to show that the predictions of the minimal $SU(5)$ model for the fermion masses can be made consistent with observations [13]. Similar terms in the gauge part of the Lagrangian involving the gauge and Higgs scalar fields imply modifications in the gauge coupling constants at the unification scale [11], [14], [15]. We present, in the following, an analysis of the modification of the coupling constants at the unification scale due to the presence of five- and six-dimensional operators in the Lagrangian. We then check whether there is a consistent choice of couplings of the higher dimensional operators which yield $\sin^2\theta_W$, the unification scale, M_U, and the $SU(5)$ coupling constant α_G such that the experimental constraints from LEP and those coming from the measurement of the proton lifetime are simultaneously satisfied.

We start with a $SU(5)$ invariant Lagrangian which breaks at a scale M_U into $SU(3)_c \times SU(2)_L \times U(1)_Y$ due to a scalar Higgs field, ϕ, which transforms under the **24**-dimensional adjoint representation of $SU(5)$. This Lagrangian in the domain of energies $M_U \leq E \leq M_{Pl}$ (where M_{Pl} denotes the Planck mass) is given as a combination of the usual four dimensional terms and the new higher dimensional terms which have been induced by the physics beyond the Planck scale (or compactification scale). In principle, such non-renormalizable operators can be induced even due to the presence of a group G' which breaks to $SU(5)$ at a scale above the unification scale. We note here that the compactification scale can be even two orders of magnitude below the Planck scale in Kaluza-Klein theories [16]. The Lagrangian can be written as

$$L = L_0 + \sum_{n=1} L^{(n)} \tag{2.2}$$

[1]This section follows Ref. [9]

where

$$L_0 = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \tag{2.3}$$

and the sum in Eq. 2.2 runs over all possible higher dimensional operators. We write down the five- and six-dimensional operators explicitly as

$$L^{(1)} = -\frac{1}{2}\frac{\eta^{(1)}}{M_{Pl}}\mathrm{Tr}(F_{\mu\nu}\phi F^{\mu\nu}) \tag{2.4}$$

$$\begin{aligned} L^{(2)} = & -\tfrac{1}{2}\tfrac{1}{M_{Pl}^2}\Big[\eta_a^{(2)}\{\mathrm{Tr}(F_{\mu\nu}\phi^2F^{\mu\nu}) + \mathrm{Tr}(F_{\mu\nu}\phi F^{\mu\nu}\phi)\} + \\ & \eta_b^{(2)}\mathrm{Tr}(\phi^2)\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \eta_c^{(2)}\mathrm{Tr}(F^{\mu\nu}\phi)\mathrm{Tr}(F_{\mu\nu}\phi)\Big] \end{aligned} \tag{2.5}$$

where,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \tag{2.6}$$

$$A_\mu = A_\mu^i\frac{\lambda_i}{2} \tag{2.7}$$

with,

$$\mathrm{Tr}(\lambda_i\lambda_j) = \frac{1}{2}\delta_{ij} \tag{2.8}$$

In the above equations $\eta^{(n)}$ specify the couplings of the higher dimensional operators. Since

$$\mathrm{Tr}(F_{\mu\nu}\phi^2F^{\mu\nu}) = \mathrm{Tr}(F_{\mu\nu}\phi F^{\mu\nu}\phi) \tag{2.9}$$

we have used the same coupling $\eta_a^{(2)}$for both these operators in Eq. 2.5.

At the unification scale M_U, the Higgs field acquires the vacuum expectation value

$$\langle\phi\rangle = \frac{1}{\sqrt{15}}\phi_0\mathrm{diag}(1,1,1,-\frac{3}{2},-\frac{3}{2}) \tag{2.10}$$

The $SU(5)$ symmetry breaks at this scale because of the non-invariance of the Higgs vacuum expectation value under the $SU(5)$ symmetry. The magnitude of $\langle\phi\rangle$ is itself proportional to the unification scale, M_U, and, hence, one can replace the Higgs field appearing in Eq.2.5 by its vacuum expectation value (ignoring the small fluctuations of the Higgs field around $\langle\phi\rangle$). With this replacement one obtains the following $SU(3)\times SU(2)\times U(1)$ invariant Lagrangian:

$$\begin{aligned} -\tfrac{1}{2}(1+\epsilon_C)\mathrm{Tr}(F_{\mu\nu}^{(3)}F^{(3)\mu\nu}) - \tfrac{1}{2}(1+\epsilon_L)\mathrm{Tr}(F_{\mu\nu}^{(2)}F^{(2)\mu\nu}) \\ -\tfrac{1}{2}(1+\epsilon_Y)\mathrm{Tr}(F_{\mu\nu}^{(1)}F^{(1)\mu\nu}) \end{aligned} \tag{2.11}$$

Thus, even in the presence of the higher dimensional operators that we have considered above, we obtain the usual $SU(3)\times SU(2)\times U(1)$ invariant Lagrangian merely scaled by constant factors $(1+\epsilon_i)$ $i = C, L, Y$. Defining the physical gauge fields below the unification scale to be

$$A_i' = A_i(1+\epsilon_i)^{\frac{1}{2}} \tag{2.12}$$

we recover the usual $SU(3)\times SU(2)\times U(1)$ invariant Lagrangian with modified coupling constants

$$\begin{aligned} g_3^2(M_U) &= \bar{g}_3^2(M_U)(1+\epsilon_C)^{-1} \\ g_2^2(M_U) &= \bar{g}_2^2(M_U)(1+\epsilon_L)^{-1} \\ g_1^2(M_U) &= \bar{g}_1^2(M_U)(1+\epsilon_Y)^{-1} \end{aligned} \tag{2.13}$$

The couplings $\bar{g}_i$ are the couplings that would have appeared in the absence of the higher dimensional operators, whereas the g_i are the physical couplings which are evolved down to lower scales.

It is expedient to introduce the parameter $\epsilon^{(n)}$ associated with a given operator of dimension $n+4$ in the following way:

$$\epsilon^{(n)} = \left[\frac{1}{\sqrt{15}}\frac{\phi_0}{M_{Pl}}\right]^n \eta^{(n)} \tag{2.14}$$

The vacuum expectation value ϕ_0 is related to M_U by

$$\phi_0 = \left[\frac{6}{5\pi\alpha_G}\right]^{\frac{1}{2}} M_U \tag{2.15}$$

where $\alpha_G = g_0^2/4\pi$ is the GUT coupling. We then have

$$\epsilon^{(n)} = \left[\left\{\frac{2}{25\pi\alpha_G}\right\}^{\frac{1}{2}}\frac{M_U}{M_{Pl}}\right]^n \eta^{(n)} \tag{2.16}$$

The change in the coupling constants are then related to the $\epsilon^{(n)}$s through the following equations:

$$\begin{aligned} \epsilon_C &= \epsilon^{(1)} + \epsilon_a^{(2)} + \frac{15}{2}\epsilon_b^{(2)} + \ldots \\ \epsilon_L &= -\frac{3}{2}\epsilon^{(1)} + \frac{9}{4}\epsilon_a^{(2)} + \frac{15}{2}\epsilon_b^{(2)} + \ldots \\ \epsilon_Y &= -\frac{1}{2}\epsilon^{(1)} + \frac{7}{4}\epsilon_a^{(2)} + \frac{15}{4}\epsilon_b^{(2)} + \frac{7}{8}\epsilon_c^{(2)} + \ldots \end{aligned} \tag{2.17}$$

The ellipsis in the above equations denote the contribution of operators with dimension greater than six.

As shown above, the effect of the higher dimensional operators is to modify the gauge coupling constants. The unification scale, M_U, is defined, as usual, through the boundary condition

$$\bar{g}_3^2 = \bar{g}_2^2 = \bar{g}_1^2 = g_0^2 \tag{2.18}$$

In the presence of the higher dimensional operators, the couplings $\bar{g}_i$ are not the physical couplings g_i but are related to them via the relations in Eq. 2.13. The result is that the following modified boundary condition is required to be satisfied at the unification scale

$$g_3^2(1+\epsilon_C) = g_2^2(1+\epsilon_L) = g_1^2(1+\epsilon_Y) = g_0^2 \tag{2.19}$$

The crucial point is that the physical couplings at the unification scale are g_3^2, g_2^2 and g_1^2 and these are the quantities that are evolved down to lower energy scales. The condition of equality in Eq.2.19 is, however, not on these physical couplings but on the "bare" couplings $g_i^2(1+\epsilon_i)$. The mismatch of the physical couplings at the unification scale can, therefore, be interpreted as due to the higher dimensional operators. With this in mind, one may use the standard one-loop renormalization group equations[7]

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M_U) + \frac{b_i}{2\pi}\ln(\frac{M_U}{M_Z}) \tag{2.20}$$

with the beta function coefficients given by

$$b_1 = \frac{41}{10}, \quad b_2 = -\frac{19}{6}, \quad b_3 = -7 \tag{2.21}$$

where we have taken $N_f = 3$ and $N_{\rm Higgs} = 1$. Solving the RG equations yield

$$\ln(\frac{M_U}{M_Z}) = \frac{6}{67\alpha}\frac{1}{D}\left[\left\{1-\frac{8}{3}\frac{\alpha}{\alpha_s}\right\}+\left\{\epsilon_C - \frac{5\epsilon_Y+3\epsilon_L}{3}\frac{\alpha}{\alpha_s}\right\}\right] \tag{2.22}$$

$$\sin^2\theta_W = \frac{1}{D}\left[\sin^2\theta_W^{(5)} - \frac{19}{134}\epsilon_C + \frac{1}{67}\left\{21+\frac{41}{2}\frac{\alpha}{\alpha_s}\right\}\epsilon_L + \frac{95}{402}\frac{\alpha}{\alpha_s}\epsilon_Y\right] \tag{2.23}$$

$$\frac{1}{\alpha_G} = \frac{3}{67}\frac{1}{D}\left[\frac{11}{3\alpha_s}+\frac{7}{\alpha}\right] \tag{2.24}$$

$$D = 1 + \frac{1}{67}(11\epsilon_C + 21\epsilon_L + 35\epsilon_Y) \tag{2.25}$$

where $\sin^2\theta_W^{(5)}$ is the usual minimal $SU(5)$ prediction

$$\sin^2\theta_W^{(5)} = \frac{23}{134} + \frac{109}{201}\frac{\alpha}{\alpha_s} \tag{2.26}$$

With the above equations at hand, we now consider whether it is possible to obtain a consistent choice of the parameters ϵ_C, ϵ_L and ϵ_Y such that we can satisfy the constraints on $\sin^2\theta_W$ and M_U from present experiments. First we restrict ourselves to five- dimensional operators only and try to see whether these operators alone can provide the required numerical values for $\sin^2\theta_W$ and M_U. The restriction to five-dimensional operators implies from Eq.2.17 the following relations:

$$\epsilon_C = \epsilon^{(1)}; \quad \epsilon_L = -\frac{3}{2}\epsilon^{(1)}; \quad \epsilon_Y = -\frac{1}{2}\epsilon^{(1)} \tag{2.27}$$

We use the values of $\sin^2\theta_W$ and α_s derived from LEP data, given in Eq.2.1, and $\alpha = 1/127.9$. Since ϵ_C, ϵ_L and ϵ_Y are all determined in terms of a single parameter $\epsilon^{(1)}$, specifying the value of $\sin^2\theta_W$ in Eq.2.23 at the scale M_Z fixes up these parameters uniquely. For the central value of $\sin^2\theta_W$ $(= 0.2333)$, we obtain the solution $\epsilon^{(1)} = -0.0441$, which from Eq.2.22 gives $M_U = 3.8\times 10^{13}$ GeV. The corresponding value of $\alpha_G = 0.0245$. Using

$$\tau_p \approx \frac{1}{\alpha_G^2}\frac{M_U^4}{M_p^5} \tag{2.28}$$

(where M_p is the proton mass), we find that the value of M_U is too low to be consistent with the experimental limits on proton lifetime. We also find that by varying $\sin^2\theta_W$ over the allowed range, the values of M_U and α_G do not change appreciably. Thus, we find that it is not possible to obtain a consistent solution with five-dimensional operators alone.

We would now like to check whether it is possible to obtain a consistent solution if we admit both five- and six-dimensional operators. Then, from Eq.2.17, we see that ϵ_C, ϵ_L and ϵ_Y are now independent parameters. By feeding in the value of $\sin^2\theta_W$, we obtain one constraint on these three parameters. The other constraint that these parameters need to satisfy is, of course, the proton lifetime constraint. From Eq.2.28, we see that the proton lifetime is controlled by both M_U and α_G. There is a further constraint that we impose on the parameters. If we require that successive terms in the sum in Eq.2.17 be scaled by inverse powers of M_{Pl}, then this can be ensured by requiring that $|\eta^{(2)}_{a,b,c}| \leq |\eta^{(1)}|$ and with $\eta^{(1)}$ not too large. Restricting ourselves to the space of parameters that satisfy these constraints, we present some sample values of the parameters in Table 2.1.

ϵ_L	ϵ_C	ϵ_Y	$(M_U)_{\rm min}$	α_G
-0.85	-0.86 – -0.845	-0.893 – -0.864	10^{17}	10^{-3}
-0.90	-0.913 – -0.905	-0.926 – -0.897	10^{17}	10^{-3}
-0.95	-0.956 – -0.953	-0.963 – -0.949	10^{16}	10^{-4}
-0.99	-0.9913 – -0.9906	-0.9926 – -0.9897	10^{16}	10^{-4}

Table 2.1: The ranges of the various parameters obtained with the central value of $\sin^2\theta_W = 0.2333$

We thus find that if we include both five- and six-dimensional operators, then there are a whole range of parameters that are consistent with the values of $\sin^2\theta_W$ and M_U that are required for agreement with experiment. In earlier papers [11], [14], where the effect of only five-dimensional operators was considered, the value of $\sin^2\theta_W$ obtained is too small to be in conformity with the latest values. The effect of six-dimensional operators was also included in Ref.[15]. Our work goes beyond the analysis presented in Ref.[15] in that we have included a more general set of six-dimensional operators. The effect of the extra operators that we have considered cannot *a priori* be neglected. Even if we restrict ourselves to the $\mathrm{Tr}(F_{\mu\nu}\phi^2F^{\mu\nu})$ operator, as in Ref.[15], we have checked that for the range of parameters chosen in Table. II of Ref.[15], the values of $\sin^2\theta_W$ obtained are not in conformity with the LEP results.

Our analysis thus shows that by including the effects of higher dimensional operators arising due to quantum gravity or spontaneous compactification of extra spatial dimensions in Kaluza-Klein theories, it is possible to show that the predictions of a minimal $SU(5)$ GUT is in conformity with the latest LEP values of $\sin^2\theta_W$ and α_s, and also with the experimental constraints on proton lifetime.

2.3 Constraints on Left-Right Symmetric GUTs

An important class of GUTs are the left-right symmetric models where the Standard Model comes from a larger group with a $SU(2)_L \times SU(2)_R$ symmetry [2]. One interesting possibility is that the left-right symmetry survives to relatively low energies, and would therefore have testable consequences at current experiments or in experiments planned in the near future.

Earlier phenomenological studies [17] [18] [19] have indicated that it is indeed possible to have a low value for the left-right symmetry breaking scale, M_R, in various grand unified

[2]This section follows Ref. [5]

theories with intermediate mass scales and in partially unified theories. In these analyses, however, values of $\sin^2\theta_W$ from as low as 0.21 to as high as 0.28 were considered.

Such a large variation in $\sin^2\theta_W$ was expected from the existence of a extra neutral gauge boson, Z', coming from either a $U(1)_R$ or $SU(2)_R$ symmetry. Due to the mixing of the Z with the Z' the usual ρ parameter, defined as

$$\rho \equiv \frac{M_{W^\pm}^2}{M_Z^2 \cos^2\theta_W} \tag{2.29}$$

changes by a positive quantity, $\Delta\rho_M$, given as [20]

$$\Delta\rho_M = \sin^2\xi \left[\frac{M_{Z'}^2}{M_Z^2} - 1\right] \tag{2.30}$$

where ξ is the mixing angle. From the definitions of $\sin^2\theta_W$ and ρ one can see that

$$\delta(\sin^2\theta_W) \approx -\frac{\sin^2\theta_W \cos^2\theta_W}{\cos 2\theta_W}\Delta\rho_M \tag{2.31}$$

The quantity $\Delta\rho_M$ is determined entirely by the measured value of M_W/M_Z, as can be seen clearly from Eq. 2.30. The results of a recent fit [20] to the LEP data [1] yield the following bound:

$$\Delta\rho_M \leq 0.010 - 0.003\left[\frac{m_t(GeV)}{100}\right]^2 \tag{2.32}$$

where the dependence on the top mass, m_t, comes through the radiative corrections. Using this value of $\Delta\rho_M$ we get $\delta(\sin^2\theta_W) \approx 3.32 \times 10^{-3}$ which is well within the quoted errors on $\sin^2\theta_W$. It is with this very stringently bound value of $\sin^2\theta_W$ that we wish to study whether a low value for the left-right symmetry breaking scale, M_R, is allowed.

In this section, we will first assume the existence of precisely such a mass scale and study the evolution of the couplings *via* the renormalization group (RG) equations at the one-loop level. To keep matters simple at first, we consider a symmetry breaking scheme without any other intermediate mass scale other than M_R and neither do we specify the GUT group, G. We take $M_R \approx 1$ TeV and try to determine if there is any unification point below the Planck scale ($\approx 10^{19}$ GeV). The central values for the couplings at the scale M_Z(= 91.176 GeV), obtained from α_s, $\sin^2\theta_W$ and α [2] [3]

$$\alpha_1(M_Z) = 0.016887;\ \alpha_2(M_Z) = 0.03322;\ \alpha_3(M_Z) = 0.11 \tag{2.33}$$

are evolved to the scale M_R. Using the matching conditions of the coupling constants at M_R

$$\begin{aligned} \alpha_{1Y}^{-1}(M_R) &= \frac{3}{5}\alpha_{2R}^{-1}(M_R) + \frac{2}{5}\alpha_{1(B-L)}^{-1}(M_R) \\ \alpha_{2L}^{-1}(M_R) &= \alpha_{2R}^{-1}(M_R) \end{aligned} \tag{2.34}$$

the evolution equations become

$$\begin{aligned} \alpha_{1(B-L)}^{-1}(q) &= \frac{5}{2}\alpha_{1Y}^{-1}(M_Z) - \frac{3}{2}\alpha_{2L}^{-1}(M_Z) - \\ &\quad 5b_{1Y}M_{RZ} + 3b_{2L}M_{RZ} - 2b_{1(B-L)}M_{qR} \\ \alpha_{2L,R}^{-1}(q) &= \alpha_{2L}^{-1}(M_Z) - 2b_{2L}M_{RZ} - 2b_2 M_{qR} \\ \alpha_{3c}^{-1}(q) &= \alpha_{3c}^{-1}(M_Z) - 2b_{3c}M_{RZ} - 2b_{3c}M_{qR} \end{aligned} \tag{2.35}$$

where $M_{ij} \equiv 4\pi \ln(M_i/M_j)$ and the beta function coefficients, b_i's are given as:

$$b_1 = \frac{4}{(4\pi)^2};\; b_2 = -\frac{1}{(4\pi)^2}\frac{10}{3};\; b_3 = -\frac{7}{(4\pi)^2} \qquad (2.36)$$

Here we have taken the number of fermion families, $n_f = 3$. In Fig. 3 we have plotted the gauge coupling constants $\alpha^{-1}_{1(B-L)}$, $\alpha^{-1}_{2L,R}$ and α^{-1}_{3c} as a function of energy, taking $M_R = 1$ TeV. We see from the Figure 3 that as a result of choosing a low value for M_R, there is no unification point even when we evolve upto a scale as high as 10^{19} GeV. The $SU(3)_c$ and $SU(2)_{L,R}$ do intersect at about 10^{17} GeV, but the $U(1)$ scale remains much too high for any possibility of unification to exist. We stress that the discrepancy is of such a large magnitude that the inclusion of the experimental errors on the input values of the coupling constants at M_Z will not redeem the situation and therefore this analysis using the central values should suffice to illustrate the point.

The above illustrative exercise certainly indicates that a low value of M_R is inconsistent with grand unification with no intermediate scales. To investigate this more thoroughly and to consider, in particular, the effect of introducing intermediate mass scales we will study various unification schemes with left-right symmetry, in detail. The traditional model of left-right symmetry is based on $SO(10)$, [17] E_6 [21] or $SU(16)$ [22] unification groups. Recently a very interesting proposal for unification starting from a $SU(15)$ group has been made [23]. Since baryon number is a local symmetry in this model, it is possible to suppress proton decay and allow unification at very low scales ($\approx 10^9$ GeV). This model is not left-right symmetric but a simple extension of this model which uses a $SU(16)$ unification group [24] is left-right symmetric. In this work, we consider the $SO(10)$, E_6 and $SU(16)$ based-models. Other than these we also consider partially unified models, which are left-right symmetric [17], where one starts from the semi-simple group $SU(4)_c \times SU(2)_L \times SU(2)_R$ instead of a simple group. These models are constrained by the value of $\sin^2\theta_W$ but not by the value of α_s and consequently there is more freedom in these models than in the grand unified models.

The detailed breaking chain that gives a intermediate left-right symmetry starting from a $SO(10)$ model is as follows [10]:

$$\begin{aligned} SO(10) \quad & \xrightarrow{M_U} SU(4) \times SU(2)_L \times SU(2)_R \\ & \xrightarrow{M_1} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} \\ & \xrightarrow{M_R} SU(3)_c \times SU(2)_L \times U(1)_Y \\ & \xrightarrow{M_W} SU(3)_c \times U(1)_{em} \end{aligned}$$

The matching conditions at M_R are precisely the same as those given in Eq. 2.34. The weak breaking scale, M_W, is taken to be 250 GeV in our computations. The renormalization group equations for this breaking chain imply the following relations between the standard model coupling constants and the unification coupling constants:

$$\begin{aligned} \alpha^{-1}_{1Y}(M_W) &= \alpha^{-1}_{SO(10)}(M_U) + (\frac{6}{5}b_{2R} + \frac{4}{5}b_4)M_{U1} + \\ & \quad (\frac{6}{5}b_{2R} + \frac{4}{5}b_{1(B-L)})M_{1R} + 2b_{1Y}M_{RW} \\ \alpha^{-1}_{2L}(M_W) &= \alpha^{-1}_{SO(10)}(M_U) + 2b_{2L}M_{U1} + 2b_{2L}M_{1R} + 2b_{2L}M_{RW} \\ \alpha^{-1}_{3c}(M_W) &= \alpha^{-1}_{SO(10)}(M_U) + 2b_4M_{U1} + 2b_{3c}M_{1R} + 2b_{3c}M_{RW} \end{aligned} \qquad (2.37)$$

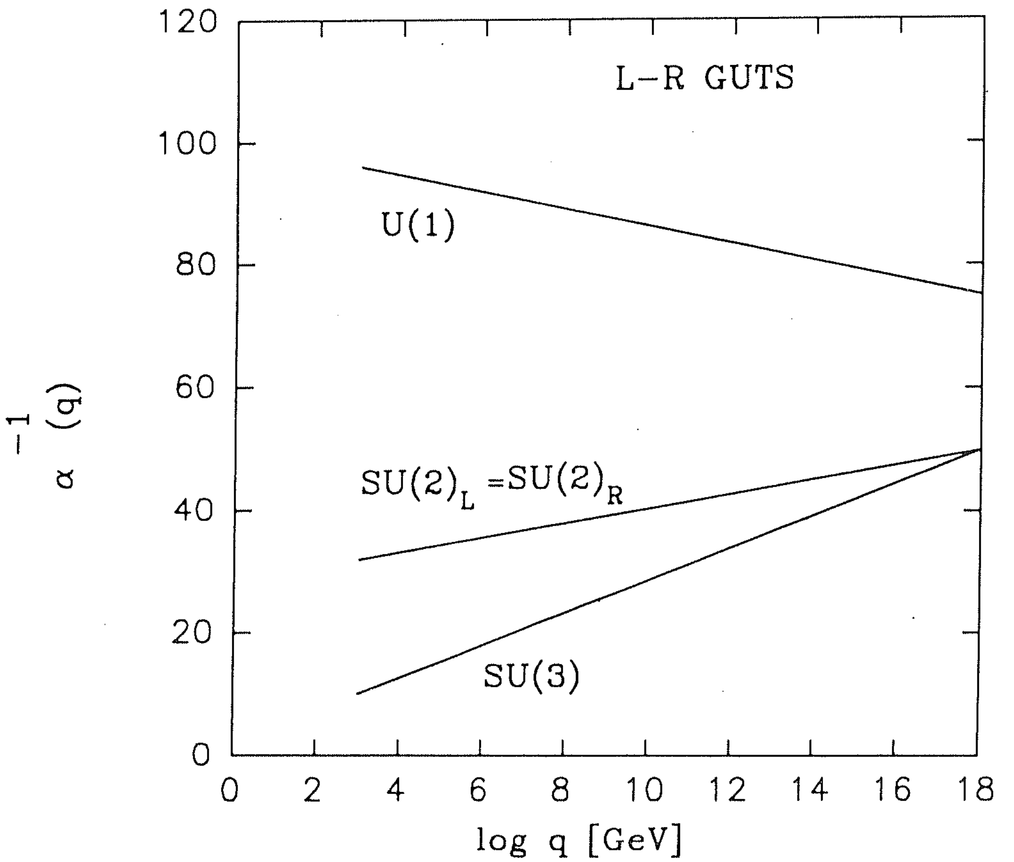

L-R GUTS
U(1)
SU(2)$_L$ =SU(2)$_R$
SU(3)
α^{-1} (q)
log q [GeV]
0
20
40
60
80
100
120
0
2
4
6
8
10
12
14
16
18

FIGURE 3

The analysis for the E_6-based theories is very similar to that of the $SO(10)$-based theories. In the E_6 case, however, there are two independent routes which lead to the Standard Model group. In one of these, the E_6 group goes to the Standard Model group *via* $SU(4)_c \times SU(2)_L \times SU(2)_R$ (exactly as in the $SO(10)$ case). The other possibility, which is of our interest, is

$$
\begin{aligned}
E_6 \overset{M_U}{\longrightarrow} & SU(3)_c \times SU(3)_L \times SU(3)_R \\
\overset{M_1}{\longrightarrow} & SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} \\
\overset{M_R}{\longrightarrow} & SU(3)_c \times SU(2)_L \times U(1)_Y \\
\overset{M_W}{\longrightarrow} & SU(3)_c \times U(1)_{em}
\end{aligned}
$$

The renormalization group equations for this case are

$$
\begin{aligned}
\alpha_{1Y}^{-1}(M_W) &= \alpha_{E_6}^{-1}(M_U) + (\frac{8}{5}b_{3R} + \frac{2}{5}b_{3L})M_{U1} + \\
& (\frac{6}{5}b_{2R} + \frac{4}{5}b_{1(B-L)})M_{1R} + 2b_{1Y}M_{RW} \\
\alpha_{2L}^{-1}(M_W) &= \alpha_{E_6}^{-1}(M_U) + 2b_{3L}M_{U1} + 2b_{2L}M_{1R} + 2b_{2L}M_{RW} \\
\alpha_{3c}^{-1}(M_W) &= \alpha_{E_6}^{-1}(M_U) + 2b_{3c}M_{U1} + 2b_{3c}M_{1R} + 2b_{3c}M_{RW}
\end{aligned}
\tag{2.38}
$$

The M_{ij}'s are as defined earlier and the beta function coefficients are

$$
b_1 = 0;\ b_2 = -\frac{1}{(4\pi)^2}\frac{22}{3};\ b_3 = -\frac{1}{(4\pi)^2}11;\ b_4 = -\frac{1}{(4\pi)^2}\frac{44}{3}
\tag{2.39}
$$

Note that the fermionic contribution to the beta functions have not been written down in the above equations since our intention is to use these in the equations for $\sin^2\theta_W$ and α_s, where the fermionic contributions cancel exactly. The linear combinations of the couplings that yield $\sin^2\theta_W$ and α_s are the following:

$$
\begin{aligned}
\sin^2\theta_W &= \frac{3}{8} - \frac{5}{8}\alpha(\alpha_{1Y}^{-1} - \alpha_{2L}^{-1}) \\
1 - \frac{8}{3}\frac{\alpha}{\alpha_s} &= \alpha(\alpha_{2L}^{-1} + \frac{5}{3}\alpha_{1Y}^{-1} - \frac{8}{3}\alpha_{3c}^{-1})
\end{aligned}
\tag{2.40}
$$

Using the experimental numbers, $\sin^2\theta_W = 0.236$ and $\alpha_s = 0.11$ [2][3], the above relations reduce to the following:

$$
\alpha_{1Y}^{-1} - \alpha_{2L}^{-1} = 29.097
\tag{2.41}
$$

and

$$
\alpha_{2L}^{-1} + \frac{5}{3}\alpha_{1Y}^{-1} - \frac{8}{3}\alpha_{3c}^{-1} = 104.755
\tag{2.42}
$$

Substituting the expressions for the couplings from Eq. 2.37 and in Eq. 2.38 and solving we get the same solutions for both the $SO(10)$ and E_6 cases

$$
\begin{aligned}
m_{U1} &= -26.1741 + \frac{3}{2}m_{RW} \\
m_{12} &= 88.5009 - 4m_{RW}
\end{aligned}
\tag{2.43}
$$

where $m_{ij} = M_{ij}/4\pi$. Using the fact that m_{U1} is positive, one can readily see that the minimum value of m_{RW} is 17.4494 i.e.

$$M_R|_{min} \equiv M_W exp(m_{RW}) \tag{2.44}$$

which means that M_R cannot be lower than 10^9 GeV, in both the $SO(10)$ and E_6 models.

Now consider left-right symmetry coming from $SU(16)$ as the Grand unification group. $SU(16)$ can break to the left-right symmetric group *via* a number of chains. But all the symmetry breaking chains which proceed *via* the $SU(4) \times SU(2)_L \times SU(2)_R$ group will give a lower bound on M_R, similar to what happens in the case of $SO(10)$ or E_6 models. We will, therefore, not discuss the chains which have an intermediate $SU(4) \times SU(2)_L \times SU(2)_R$ group.

It was noticed in the $SU(15)$ GUT that if $SU(3)_L \times SU(2)^q_L \times SU(3)_R \times U(1)_B \times SU(2)^l_L \times U(1)_l$ group breaks at the un-unification scale M_1, then the condition for low energy unification is that M_1 has also to be lowered. This idea can be extended to the $SU(16)$ GUT and it can be seen that lowering the un-unification scale one can achieve low energy unification. We shall now try to explore this scenario and see if we can have low-energy left-right symmetry in $SU(16)$ GUT with low energy unification. We shall assume that the Higgs structure is such that proton decay is suppressed. The detailed breaking chain we consider is the following:

$$\begin{aligned}
SU(16) \quad & \xrightarrow{M_U} SU(3)_L \times SU(2)^q_L \times SU(3)_R \times SU(2)^q_R \times U(1)_B \times \\
& \qquad SU(2)^l_L \times SU(2)^l_R \times U(1)_{lep} \\
& \xrightarrow{M_1} SU(3)_c \times SU(2)^{q+l}_L \times SU(2)^{q+l}_R \times U(1)_{(B-L)} \\
& \xrightarrow{M_R} SU(3)_c \times SU(2)_L \times U(1)_Y \\
& \xrightarrow{M_W} SU(3)_c \times U(1)_{em}
\end{aligned}$$

In the above breaking chain, $U(1)_B$ is proportional to the baryon number and $U(1)_{lep}$ is proportional to lepton number both properly normalized. The matching conditions at M_R are again those given in Eq. 2.34 with appropriate modifications to account for the quark-lepton un-unification group while those at the scale M_1 for the $U(1)$ groups can be easily seen to be

$$\alpha^{-1}_{1(B-L)}(M_1) = \frac{1}{4}\alpha^{-1}_{1B}(M_1) + \frac{3}{4}\alpha^{-1}_{1lep}(M_1) \tag{2.45}$$

From the fermion transformation properties at different levels, we can check that $SU(3)_L$ and $SU(3)_R$ are normalized to 1, $SU(2)^q_L$ and $SU(2)^q_R$ are normalized to 3/2, and $SU(2)^l_L$ and $SU(2)^l_R$ are normalized to 1/2. All other groups are normalized to 2. The beta function coefficients properly normalized are

$$b_{1(B-L)} = 0;\ b_{1Y} = 0;\ b_{1B} = 0;\ b_{1lep} = 0$$

$$b^q_{2L} = b^q_{2R} = -\frac{1}{(4\pi)^2}\frac{88}{9};\ b^l_{2L} = b^l_{2R} = -\frac{1}{(4\pi)^2}\frac{88}{3};\ b_2 = -\frac{1}{(4\pi)^2}\frac{22}{3}$$

$$b_3 = -\frac{1}{(4\pi)^2}11;\ b_{3L} = -\frac{1}{(4\pi)^2}22;\ b_{3R} = -\frac{1}{(4\pi)^2}22 \tag{2.46}$$

The values of the beta function coefficients given above are, as before, without the fermionic contributions since these cancel exactly in the expressions for $\sin^2\theta_W$ and α_s. The evolution equations for the $SU(16)$ case are as follows:

$$\begin{aligned}
\alpha_{1Y}^{-1}(M_W) &= \alpha_{SU(16)}^{-1}(M_U) + (\frac{9}{10}b_{2R}^q + \frac{3}{10}b_{2R}^l + \frac{1}{5}b_{1B} + \frac{6}{10}b_{1lep}) \\
&\quad M_{U1} + (\frac{6}{5}b_2 + \frac{4}{5}b_{1(B-L)})M_{1R} + 2b_{1Y}M_{RW} \\
\alpha_{2L}^{-1}(M_W) &= \alpha_{SU(16)}^{-1}(M_U) + (\frac{3}{2}b_{2L}^q + \frac{1}{2}b_{2L}^l)M_{U1} + 2b_2M_{1R} \\
&\quad +2b_2M_{RW} \\
\alpha_{3c}^{-1}(M_W) &= \alpha_{SU(16)}^{-1}(M_U) + (b_{3L} + b_{3R})M_{U1} + 2b_3(M_{1R} + M_{RW}) \qquad (2.47)
\end{aligned}$$

As in the $SO(10)$ case, we use Eqs. 2.41 and 2.42 and solve for the ratios of the mass scales. The resulting equations are

$$\begin{aligned}
m_{RW} &= 17.4494 \\
m_{1R} &= 18.7031 - 2m_{U1} \qquad (2.48)
\end{aligned}$$

In this case, therefore, we see that we get a fixed value of $m_{RW} = 17.4494$, which is precisely the value of the minimum of m_{RW} in the $SO(10)$ case. This implies a value of $\approx 10^9$ for the left-right symmetry breaking scale, M_R.

Finally, we investigate the possibility of left-right symmetry from a partially unified model [17]. The detailed breaking chain is as follows:

$$\begin{aligned}
&SU(4) \times SU(2)_L \times SU(2)_R \xrightarrow{M_1} \\
&SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} \\
&\xrightarrow{M_R} SU(3)_c \times SU(2)_L \times U(1)_Y \\
&\xrightarrow{M_W} SU(3)_c \times U(1)_{em}
\end{aligned}$$

The matching condition at M_R is the same as in the preceding examples. The evolution equations in this case read as follows:

$$\begin{aligned}
\alpha_{1Y}^{-1}(M_W) &= \frac{3}{5}\alpha_2^{-1}(M_1) + \frac{2}{5}\alpha_4^{-1}(M_1) + \\
&\quad (\frac{6}{5}b_{2R} + \frac{4}{5}b_{1(B-L)})M_{1R} + 2b_{1Y}M_{RW} \\
\alpha_{2L}^{-1}(M_W) &= \alpha_2^{-1}(M_1) + 2b_{2L}M_{1R} + 2b_{2L}M_{RW} \\
\alpha_{3c}^{-1}(M_W) &= \alpha_4^{-1}(M_1)2b_{3c}M_{1R} + 2b_{3c}M_{RW} \qquad (2.49)
\end{aligned}$$

In writing the above equations we have used the fact that, due to the left-right symmetry beyond the scale M_R, $\alpha_{2L} = \alpha_{2R} = \alpha_2$. The beta function coefficients remain the same as before. Note that the important difference in this case is that the single coupling at the unification scale which appeared in the unified models, is now replaced by the disparate coupling strengths at the scale of partial unification. To express the couplings at the scale M_W in terms of $\sin^2\theta_W$ and α_s an appropriate linear combination needs to be constructed such that the different couplings at the partial unification scale are exactly canceled. The resulting equation is

$$\frac{1}{4\pi}\left[\frac{(1-2\sin^2\theta_W)}{\alpha}-\frac{2}{3\alpha_s}\right] = 2(b_{2R}-b_{2L}+\frac{2}{3}b_{1(B-L)}-\frac{2}{3}b_3)M_{1R} + 2(\frac{5}{3}b_{1Y}-b_{2L}-\frac{2}{3}b_3)M_{RW} \tag{2.50}$$

Using the experimental values of $\sin^2\theta_W$ and α_s, we get

$$53.74 - 2m_{RW} = m_{2R} \tag{2.51}$$

To get the minimum of M_R in this case, we also use the constraint that M_2 should be less than (or equal to!) the Planck scale, 10^{19} GeV i.e.

$$m_{RW} + m_{2R} \leq 38.23 \tag{2.52}$$

From these equations, we get $m_{RW} \geq 15.51$ which implies a lower bound on M_R equal to $\approx 10^9$ GeV.

We now consider the effect of including the Higgs contribution in the renormalization group equations. The specific Higgs representations under different symmetry groups are given in Table (2.2).

$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)}$	$SU(3)_c \times SU(2)_L \times U(1)_Y$
$1,1,3,+\sqrt{\frac{3}{2}}$	$1,1,0$
$1,2,2,0$	$1,2,-\frac{1}{2}\sqrt{\frac{3}{5}}$
$1,2,2,0$	$1,2,+\frac{1}{2}\sqrt{\frac{3}{5}}$
$1,3,1,-\sqrt{\frac{3}{2}}$	$1,3,-\sqrt{\frac{3}{5}}$

Table 2.2: The explicit Higgs representations under the left-right symmetric group and the electroweak group.

Using these representations, the beta function coefficients are given as

$$b_{1(B-L)} = \frac{1}{(4\pi)^2}\frac{3}{2};\ b_{1Y} = \frac{1}{(4\pi)^2}\frac{2}{5};\ b_{2L} = b_{2R} = -\frac{1}{(4\pi)^2}\frac{20}{3};$$

$$b_{2L}^{ew} = -\frac{1}{(4\pi)^2}\frac{41}{6};\ b_3 = -\frac{1}{(4\pi)^2}11 \tag{2.53}$$

Solving as before, we find that with the Higgs contribution the lower bound of M_R increases to $\approx 10^{11}$ GeV.

We have also studied the effect on M_R of including supersymmetry. The essential difference in the analysis is that the beta function coefficients are modified. Taking into account these modifications, we find that in the supersymmetric case the lower bound on M_R is $\approx 10^{11}$ GeV.

In conclusion, the most recent experimental data provide very strong constraints on left-right symmetric models. We have shown that if a left-right symmetric group coming from either a grand unified or partially unified group breaks at an intermediate mass scale, M_R, then the tightly constrained values of $\sin^2\theta_W$ and α_s can be used to put a lower bound on the value of M_R. This lower bound is $\approx 10^9$ GeV, irrespective of the unification group. Grand unified theories and partially unified theories, therefore, completely rule out the possibility of seeing the right handed partners of $W^\pm$ at the energies available in current experiments or those planned in the near future. Conversely, the discovery of these particles at such energies can be used to refute unification models. It is of importance to note, however, that our analysis puts no constraints whatsoever on the existence of extra Z at low energies, as an extra $U(1)_R$ can survive down to electroweak breaking scales. The inclusion of the Higgs or supersymmetry increases the lower bound on M_R.

2.4 Contraints on Non-Perturbative Unification

The compatibility of the simple supersymmetric GUT with no intermediate breaking scales and the couplings determined from LEP is remarkable. Nonetheless, it is important to study other models, which are alternatives to grand unification, and see whether they are viable in the light of the available experimental information on couplings [3].

An interesting alternative to GUTs was proposed by Maiani, Parisi and Petronzio [26] several years ago. In this scheme, the couplings enter a non-perturbative phase at a high energy scale, i.e. the theory is asymptotically divergent. Starting from the renormalization group equation for a coupling α,

$$\frac{d\alpha}{dt} = \beta(\alpha), \tag{2.54}$$

where $\beta(\alpha)$ is the beta function and $t = \ln(Q^2/\mu^2)$, μ being some reference scale, we obtain

$$t = \int_{\alpha(\mu)}^{\alpha(Q^2)} \frac{d\alpha}{\beta(\alpha)}. \tag{2.55}$$

For $\beta(\alpha) > 0$ (asymptotically divergent theory) there is a value of t, given by

$$t = \int_{\alpha(\mu)}^{\infty} \frac{d\alpha}{\beta(\alpha)} < \infty, \tag{2.56}$$

for which $\alpha \to \infty$. If perturbation theory is to be valid at all energy scales, we require $\alpha(\mu) = 0$, so that $t_c = \infty$, $\alpha(\mu) = 0$ is the infra-red fixed point. But if $\alpha(\mu) \neq 0$ but small, i.e. it is sufficiently close to the infra-red fixed point, then there is a finite cut-off in energy beyond which the theory is non-perturbative.

In Ref. [26], it was assumed that the standard SU(3)×SU(2)×U(1) theory, due to new fermion generations that get switched on around the weak scale $\Lambda_F = 250$ GeV, is asymptotically divergent beyond Λ_F. The couplings $\alpha_{1,2,3}$ are sufficiently close to zero at Λ_F but not quite zero. As a consequence, the theory is cut off at a scale Λ. At this scale, the most interesting situation is that not just one but all three couplings are large, i.e. of O(1). In fact, it has been shown [27] that such a non-perturbative scenario exhibits a "trapping" mechanism, whereby if one of the couplings grows large, the other couplings will also increase. This effect, by means of which all three couplings are large and of the same order of magnitude at Λ, leads to what is called non-perturbative unification. In Ref. [26] the cut-off scale Λ was assumed

[3] This section follows Ref. [25]

to be the Planck scale; however, in subsequent studies [28, 29], Λ was determined to be of the order of 10^{15} –10^{17} GeV. Since the low-energy couplings are close to the infra-red fixed point, they are insensitive to the values of the couplings at the scale Λ.

One natural extension of the above scenario is the inclusion of supersymmetry. This was first considered in Ref. [28], and was later discussed in Refs. [29, 30]. Other than solving the hierarchy problem, the inclusion of supersymmetry is attractive because it provides a framework for the existence of new particles needed to make the theory asymptotically divergent. In the case of the simplest $N = 1$ supersymmetric extension of the scenario, it suffices to consider $n_f = 5$, where n_f is the number of fermion generations.

In this section, we use the recent LEP values to check whether any strong constraints on the non-perturbative unification scenario can be obtained. The values of $\sin^2\theta_W$ and α_s from LEP are very precise compared to that available from older experiments. One strong constraint is on the number of extra chiral generations. The present limit on the oblique parameters S, T and U allows only three chiral fermion generations, while the vectorial generations are not constrained. Thus in addition to the three chiral fermion generations we are allowed to have only an even number of generations.

We shall first specify the supersymmetric non-perturbative unification scenario in detail. While discussing the results we shall also comment on the results of the non-supersymmetric case. We consider an SU(3)×SU(2)×U(1) supersymmetric gauge theory with the assumption that an $N = 1$ supersymmetry holds above the scale Λ_s. We assume $n_f = 5$ supersymmetric generations and two Higgs supermultiplets. In the discussion of the non-supersymmetric case we shall consider one Higgs scalar and $n_f = 8$ and 9. From the requirement that the Yukawa couplings do not become arbitrarily large, a bound on the fermion masses can be obtained [31, 32]. This bound is that fermion masses are, in general, smaller than 200–250 GeV. We assume that the extra fermion generations, which are required for the theory to be asymptotically divergent, are of the order of 250 GeV in mass.

Having specified the theory we can now address the question of the evolution of the three couplings. The two-loop renormalization group equations for the couplings are given by the following coupled differential equations:

$$\mu\frac{d\alpha_i(\mu)}{d\mu} = \frac{1}{2\pi}\left[a_i + \frac{b_{ij}}{4\pi}\alpha_j(\mu) + \frac{b_{ik}}{4\pi}\alpha_k(\mu)\right]\alpha_i^2(\mu) + \frac{2b_{ij}}{(4\pi)^2}\alpha_i^3(\mu), \tag{2.57}$$

where i, j, $k = 1$, 2, 3 and $i \neq j \neq k$, and a_i and b_{ij} are the one- and two-loop beta function coefficients. In the range of energies between M_Z and the supersymmetric threshold, M_s, we use the non-supersymmetric beta functions to evolve the couplings, whereas from M_s onward the supersymmetric beta functions are effective. We retrieve the result for the non-supersymmetric scenario by taking $M_s = \Lambda_{MPP}$ and large n_f.

In the non-supersymmetric case the one-loop beta function coefficients are [33]

$$b_j = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + n_f \begin{pmatrix} \frac{20}{9} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + n_h \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ 0 \end{pmatrix} \tag{2.58}$$

while the two-loop beta functions are

$$a_{ij} = -\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{136}{3} & 0 \\ 0 & 0 & 102 \end{pmatrix} + n_f \begin{pmatrix} \frac{95}{27} & 1 & \frac{44}{9} \\ \frac{1}{3} & \frac{49}{3} & 4 \\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} + n_h \begin{pmatrix} \frac{1}{2} & \frac{13}{6} & 0 \\ \frac{1}{2} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.59}$$

In the supersymmetric case the one-loop beta functions take the form [33]

$$b_j = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + n_f \begin{pmatrix} \frac{10}{3} \\ 2 \\ 2 \end{pmatrix} + n_h \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} \tag{2.60}$$

while the two-loop beta functions are

$$a_{ij} = -\begin{pmatrix} 0 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 54 \end{pmatrix} + n_f \begin{pmatrix} \frac{190}{27} & 2 & \frac{88}{9} \\ \frac{2}{3} & 14 & 8 \\ \frac{11}{9} & 3 & \frac{68}{3} \end{pmatrix} + n_h \begin{pmatrix} \frac{1}{2} & \frac{3}{2} & 0 \\ \frac{1}{2} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.61}$$

In all these equations, n_f and n_h denote the number of fermion generations and the number of Higgs doublets respectively.

We integrate the coupled differential equations in Eq. (2.57) numerically, with the initial values of the three couplings $\alpha_{1,2,3}$ taken to be of O(1) at the unification scale Λ. What we do in practice is to evolve downwards using the renormalization group equations for several values of Λ, and check what the predicted values of the couplings at the scale M_Z are. The extra fermion generations are assumed to contribute to the beta functions for all energies greater than 250 GeV.

We shall first comment on the non-supersymmetric scenario and then present our main result, namely the supersymmetric extension. In this case we find that for $n_f \leq 8$, $\alpha_2(M_z)$ remains too small, and that $\alpha_{1,2}(M_z)$ falls within the experimental bound for $n_f \geq 9$. But for $n_f \geq 9$ the strong coupling constant evolves extremely fast and $\alpha_3(M_z)$ becomes too large. Thus the precision LEP data rule out the non-supersymmetric scenario completely.

The results of the computation for the supersymmetric version are shown in Figure 4, where $\alpha_{1,2,3}(M_Z)$ are shown as a function of Λ. The solid, dashed and dotted curves are for $M_s =$ 250 GeV, 1.2 TeV and 5 TeV, respectively. The horizontal lines in the Figure 4 show the upper and lower bounds on the couplings at M_Z as determined by the LEP experiment. These are as follows [34]:

$$\begin{aligned} \alpha_1 &= 0.0101322 \pm 0.000024 \\ \alpha_2 &= 0.03322 \pm 0.00025 \\ \alpha_3 &= 0.120 \pm 0.006. \end{aligned} \tag{2.62}$$

It is clear from the Figure 4 that the non-perturbative unification scheme is certainly viable if we have $M_s = 1.2$ TeV and Λ close to 0.78×10^{17} GeV. We have checked that the range of values allowed is $M_s = 1.2 \pm 0.2$ TeV and $\Lambda = (0.7\text{–}0.8) \times 10^{17}$ GeV. We have also checked that the couplings at M_Z are not sensitive to the choice of the couplings at Λ. We have checked this by varying these from 0.75 to 10.

Let us now summarize our results of this section. We have studied the non-perturbative unification scenario first proposed by Maiani, Parisi and Petronzio. We point out that the non-supersymmetric version of this scenario is ruled out by LEP data. However, the supersymmetric extension of this scenario remains a viable alternative to conventional grand unified theories and is capable of predicting the precision values of couplings determined from LEP. Our numerical results show that the non-perturbative scale, Λ, at which all couplings are large, is around $0.7\text{–}0.8 \times 10^{17}$ GeV, with the supersymmetric threshold M_s around 1.0–1.4 TeV. If the scale M_s gets either larger or smaller it is then not possible to reproduce the

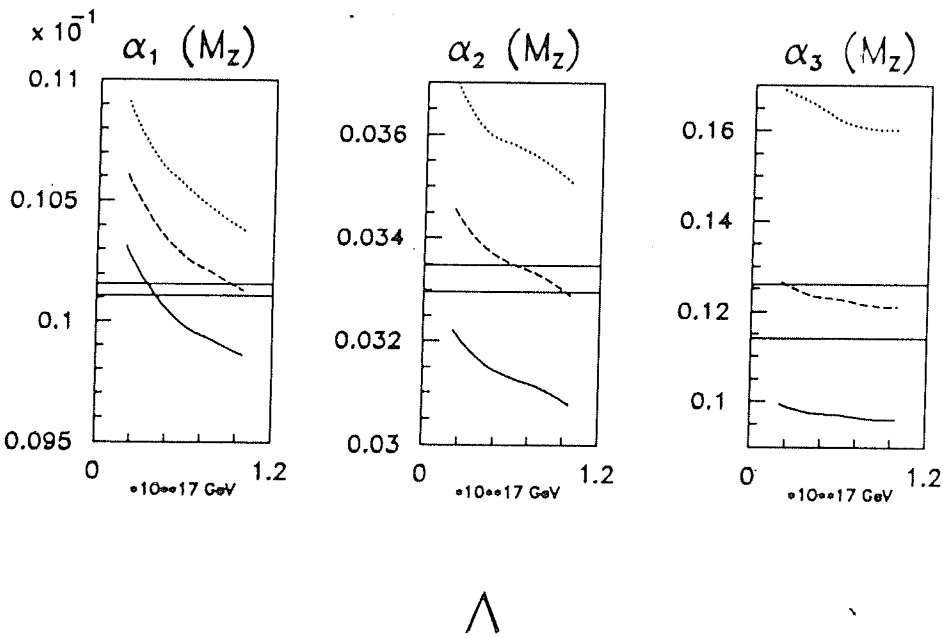

x 10⁻¹
α₁ (M_Z)
0.11
0.105
0.1
0.095
0
1.2
*10**17 GeV
α₂ (M_Z)
0.036
0.034
0.032
0.03
0
1.2
*10**17 GeV
α₃ (M_Z)
0.16
0.14
0.12
0.1
0
1.2
*10**17 GeV
Λ

FIGURE 4

values of the couplings at M_Z. We should note that the agreement with the data is obtained only for a constrained range of parameters of this scenario. In principle, the effect of higher-order corrections could be large and this may ruin the agreement. It is also likely that more accurate measurements of the strong coupling α_3 at low energies may be sufficient to either put strong constraints or completely rule out this scenario. It is nevertheless interesting that this scenario, at the two-loop level, is a possible alternative to conventional grand unification.

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Chapter 3

Low energy unification

3.1 Higgs Effect in SU(15) GUT

Recently a new paradigm of GUT models have evolved[1, 2, 6, 7] following an observation that at least one symmetry breaking chain of a GUT based on the group SU(15) can be unified at a very low energy $M_u \sim O(10^7)$ GeV[1]. Because baryon number B is a gauge symmetry in this model, proton decay can be suppressed, and one possible Higgs structure has been proposed to this end[2]. Low energy unification makes these models free from problems of grand unified monopoles[6] and the gauge hierarchy problem is also much less severe [1].

All the present activity on SU(15) GUT relys on two important claims, namely, (i) there exists at least one symmetry breaking pattern of SU(15) grand unification, where the gauge coupling constants evolve very fast and can be unified at an energy scale $M_u \sim O(10^7)$ GeV and (ii) there exists at least one choice of Higgs fields which can (a) allow the above symmetry breaking chain, (b) forbid any gauge boson mediated proton decay, (c) suppress Higgs mediated proton decay and (d) make this low energy unification consistent with the nonobservation of proton decay.

Here we analyze these two claims. We discuss in a general way proton decay and the choice of Higgs fields required for any symmetry breaking in these GUTs along with their effect on the evolution of the gauge coupling constants. We find this cannot be neglected: for SU(15), unification below $M_u \sim O(10^{14})$ GeV is impossible for the breaking pattern proposed by Frampton and Kephart [FK][2]. However other interesting patterns exist which yield unification at $\sim 10^9 GeV$ and violate baryon number symmetry $U(1)_B$ at about the electroweak breaking scale, although there is no proton decay. The low energy ($\sim 250 GeV$) symmetry includes phenomenologically interesting chiral color symmetry[8] and quark-lepton un-unified electroweak symmetry[9].

Our notation is the following. When we write the semisimple group $SU(n)^q_L \times SU(m)^l_R \times U(1)_X$ the subscript implies either the charge of the U(1) group or that right (left) handed particles are non-singlets under SU(n) (SU(m)) and the superscript q (l) means that only quarks (leptons) transform under this group. The gauge coupling constants of the groups $SU(n)^q_L$ and $U(1)_X$ will be written as $\alpha_{nqL} = \frac{g^2_{nqL}}{4\pi}$ and $\alpha_{1X} = \frac{g^2_{1X}}{4\pi}$ respectively. For the breaking $G_i \longrightarrow G_{i-1}$, the G_{i-1} singlet component of the Higgs ϕ_i acquires a vev at a scale M_i. 1^n

[1] This section is based on Ref. [3]

denotes a totally antisymmetric n^{th} rank tensor; hence $1^m 1^n$ denotes a Young tableaux of m and n in the first and second columns respectively.

$$
\begin{aligned}
G_1[SU(15)] &\xrightarrow{\langle\phi_1\rangle} G_2[SU(12)^q \times SU(3)^l] \\
&\xrightarrow{\langle\phi_2\rangle} G_3[SU(6)_L \times SU(6)_R \times U(1)_B \times SU(3)^l] \\
&\xrightarrow{\langle\phi_3\rangle} G_4[SU(3)_{cL} \times SU(2)_L^q \times SU(6)_R \times U(1)_B \times SU(3)^l] \\
&\xrightarrow{\langle\phi_4\rangle} G_5[SU(3)_{cL} \times SU(2)_L^q \times SU(3)_R \times U(1)_R \times U(1)_B \times SU(2)^l \times U(1)_Y^l] \\
&\xrightarrow{\langle\phi_5\rangle} G_6[SU(3)_c \times SU(2)_L \times U(1)_B \times U(1)_Y'] \\
&\xrightarrow{\langle\phi_6\rangle} G_7[SU(3)_c \times SU(2)_L \times U(1)_Y] \\
&\xrightarrow{\langle\phi_7\rangle} G_8[SU(3)_c \times \times U(1)_Q]
\end{aligned}
\tag{3.1}
$$

$\langle\phi_i\rangle = M_i$. We shall denote this pattern by {1234567}; the pattern of ref. [1] is {1267}, for which $M_3 = M_4 = M_5 = M_6$.

We turn next to the Higgs fields required to ensure this pattern, taking minimal representations whenever possible. Our Higgs structure is very similar to that of [FK][2]. We choose ϕ_1 to be a 1^3, *i.e.*, a **455**-plet. The G_2 singlet component of ϕ_1 can then acquire a vev to break the group $G_1 \longrightarrow G_2$. The vev of the G_3 singlet component of $1^{14}1$ (**224**-plet) can break G_2, leaving $U(1)_B$ unbroken. Breaking $SU(6)_L$ to its special maximal subalgebra $SU(3)_{cL} \times SU(2)_L^q$ requires a somewhat large Higgs representation. Although self-conjugate representations can break any group to its maximal subalgebra, in this case the adjoint representation does not work and the next higher dimensional self-conjugate representation is required. These are contained in the self-conjugate representations of the higher groups, and the particular $SU(6)_L \longrightarrow SU(3)_{cL} \times SU(2)_L^q$ symmetry breaking can be accomplished with a **10800** dimensional ($1^{13}1^2$) Higgs of $SU(15)$ which is contained in $\mathbf{105} \otimes \overline{\mathbf{105}}$. This is the lowest dimensional Higgs to break $G_3 \longrightarrow G_4$; [FK] considered a **14175**-plet ($1^{14}1^{14}11$) $\subset \mathbf{120} \otimes \overline{\mathbf{120}}$ *i.e.* the next-highest one. Appropriate components of the adjoint (**224**-plet) can break $G_4 \longrightarrow G_5$. For the next stage a 1^3 ($\phi_5 \equiv \mathbf{455} - plet$) can be used; this breaks global lepton number in addition to the local groups.

The surviving group is now G_6 $[SU(3)_c \times SU(2)_L \times U(1)_{Y'} - 1_B]$. Note that $U(1)_{Y'}$ is orthogonal to $U(1)_B$, while the hypercharge Y in the standard model does not commute with B. In fact Y is a linear combination of B and Y'. [FK] break G_6 with a 1^5 (**3003**-plet) by giving a vev to the $Y = 0$ component labeled (10,11,12,13,14). To find out whether there exists any lower dimensional Higgs representation one can check that it is not possible to write any B-violating operator only with the fermions invariant under G_6. However with a 1^3 (**455**-dimensional) or a 1^4 (**1365**-dimensional) Higgs field there exists a G_6-invariant B-violating dimension-7 operator. But under G_7 one can write down B-violating dimension-6 operators only with fermions. Hence one can have ϕ_6 = **455** or **1365**. Both have B and Y' nonzero; the $Y = 0$ component can acquire a vev. Either of ϕ_7 = **105** or a **120** can be used to break the standard electroweak symmetry; [FK] had considered both for this purpose, but this is not necessary.

Considering next proton decay, since quark-lepton unification is broken at a scale M_1, the lepto-quark gauge bosons (X_μ) acquire a mass $\approx M_1$, while the di-quark bosons (Y_μ) acquire mass at a scale where the quark-antiquark unification is broken ($\approx M_2$). Since $U(1)_B$ is a local gauge symmetry X_μ and Y_μ do not mix at this level. These transform under G_3 as $X_\mu \equiv [(6,1,\frac{1}{3},\overline{3})+ (1,\overline{6},-\frac{1}{3},\overline{3}) + (1,6,\frac{1}{3},3) + (\overline{6},1,-\frac{1}{3},3)]$ and $Y_\mu \equiv [(6,6,\frac{2}{3},1) + (\overline{6},\overline{6},-\frac{2}{3},1)]$, with $m_X^2 \sim \langle\phi_1\rangle$ and $m_Y^2 \sim \langle\phi_2\rangle$. The mixing between X_μ and Y_μ takes place when the Higgs fields ϕ_a and ϕ_b acquire vevs in the term $X_\mu \phi_a Y^\mu \phi_b \subset D_\mu \phi_a D^\mu \phi_b$. Since X_μ and Y_μ carry different B, the mixing can occur only at M_6, suppressing the amplitude for gauge boson mediated proton decay $\sim O(\frac{M_5 M_6}{M_1^2 M_2^2})$. Thus if $M_1 \approx M_2 \approx M_u$ and $M_5 \approx M_6 \approx 10^2$ GeV, then $M_u \geq 10^9$ GeV from the present limit on the proton lifetime.

Now both X_μ and Y_μ are contained in the SU(15) gauge boson G_μ, which transforms as a self-adjoint **224**-plet of SU(15). As a result, the SU(15) multiplets $\phi_a(\supset \Phi_a)$ and $\phi_b(\supset \Phi_b)$ can allow the coupling $X^\mu \Phi_a Y_\mu \Phi_b$ iff $\phi_a = \phi_b^\dagger (\equiv \phi)$. If only one component of ϕ acquires a vev, *i.e.* the Higgs multiplet which breaks $U(1)_B$ takes part in no other symmetry breaking, then $\langle\Phi_a\rangle$ and $\langle\Phi_b\rangle = \langle\Phi_a^\dagger\rangle$ will carry equal and opposite B, forbidding mixing between X_μ and Y_μ. Gauge boson mediated proton decay is then absent, at least to this order. Couplings of $\phi_a^\dagger$ with other Higgs fields will determine the higher order terms. Since ϕ_a is the Higgs field which breaks $U(1)_B$, in our case $\phi_a = \phi_6 = \mathbf{1365}$. The couplings of $\mathbf{1365}^\dagger$ of the form $\langle\mathbf{1365}\rangle\langle\mathbf{1365}^\dagger\rangle$ with other Higgs fields cannot have any B-violating effect. If we also consider $\phi_7 = \mathbf{120}$ then the only $U(1)_B$-breaking term is of the form $\langle\mathbf{1365}\rangle\langle\mathbf{1365}\rangle\langle\mathbf{1365}\rangle\langle\mathbf{455}\rangle$, for which $B = 3$. Thus this also cannot contribute to proton decay. Since there is no linear coupling of **1365** with other Higgses, in this scenario there is absolutely no gauge boson mediated proton decay with $\phi_6 = \mathbf{1365}$ and $\phi_7 = \mathbf{120}$. If different components of the same Higgs field (which break $U(1)_B$) acquire vevs, then there can be gauge boson mediated proton decay: for example if $\phi_6 = \mathbf{455}$, then since $\phi_5 = \mathbf{455}$, mixing between X_μ and Y_μ will occur. The amplitude will be proportional to $\sim \frac{\langle\phi_5\rangle\langle\phi_6\rangle}{M_1^2 M_2^2}$, which is not suppressed by Yukawa couplings.

There is no straightforward way to understand the Higgs mediated proton decay; such processes will depend on the choice of all the Higgs fields in the theory. For ϕ_6, the types of operators which can lead to proton decay are of the form $\psi\psi\psi\psi\langle\phi_6\rangle$. But the Higgs fields necessary to couple the fermions with $\phi_6 = \mathbf{1365}$ are **105** dimensional, and ϕ_6 does not have any linear couplings with combinations of other Higgs fields; hence this operator cannot give rise to proton decay. Considering higher dimensional operators, with one ϕ_6 there does not exist any other higher dimensional operator, and as a result there is also no Higgs mediated proton decay for this choice. Hence to avoid proton decay we choose $\phi_6 = \mathbf{1365}$ and $\phi_7 = \mathbf{120}$.

We next compute the effect of the Higgs fields considered in the evolution of the coupling constants[10]. We use the one-loop renormalization group equations which have the form,

$$\mu \frac{d\alpha_i(\mu)}{d\mu} = 2\beta_i \alpha_i^2(\mu) \tag{3.2}$$

where $\alpha_i = \frac{g_i^2}{4\pi}$, the β-functions are defined as, $\beta_i = -\frac{b_i}{(4\pi)}$, and $b_i = T_g[i] - \frac{4}{3}T_f[i] - \frac{1}{6}T_s[i]$, corresponding to the contributions from gauge bosons, fermions and Higgs scalars, respectively. The fermionic contributions to the various subgroups are the same and are given by $T_f = n_f$, where n_f is the number of generations; these cancel out in the equation of $\sin^2\theta_w$ and $(1 - \frac{8}{3}\frac{\alpha}{\alpha_s})$. The gauge contributions are

$$T_g[12] = 176; \quad 2T_g[3_{cL}] = 2T_g[3_R] = T_g[3'] = 4T_g[3_c] = 44;$$

$$T_g[6_L] = T_g[6_R] = 88; \quad 3T_g[2_L^q] = T_g[2_L^l] = 4T_g[2_L] = \frac{88}{3}; \tag{3.3}$$

with $T_g = 0$ for all $U(1)$ groups. For our choice of Higgs the T_s are given in Table 3.1.

To include the Higgs contributions we assumed the extended survival hypothesis[11] and the Applequist-Carrazone decoupling theorem[12] (standard assumptions made in calculating Higgs effects in evolution of coupling constants).

$M_1 \to M_2$	$M_2 \to M_3$	$M_3 \to M_4$	$M_4 \to M_5$	$M_5 \to M_6$	$M_6 \to M_7$
$[12] = 3052$	$[6_L] = 264$	$[3_{cL}] = [2_L^q] = 48$	$[3_{cL}] = [2_L^q] = [3_R] = 18$	$[3_c] = 0$	$[3_c] = 0$
$[3_l] = 608$	$[6_R] = 114$	$[6_R] = 114$	$[1_R] = 18.33$	$[2_L] = 0.5$	$[2_L] = 0.5$
	$[1_B] = 93$	$[1_B] = 93$	$[1_B] = 1.5$	$[1_B] = 1.5$	$[1_Y] = .3$
	$[3_l] = 136$	$[3_l] = 136$	$[1_l] = 13.33$	$[1_{Y'}] = .5$	
			$[2_L^l] = 36$		

Table 3.1: Contributions to $T_s[n]$ at various scales

Denoting $\alpha_G^{-1}(M_J)$ by $\mathcal{A}_G(J)$, we employ the appropriate boundary conditions: (i) $\mathcal{A}_{12}(1) = \mathcal{A}_{3l}(1) = \mathcal{A}_{15}(1)$, (ii) $\mathcal{A}_{6L}(2) = \mathcal{A}_{6R}(2) = \mathcal{A}_{1B}(2) = \mathcal{A}_{12}(2)$, (iii) $\mathcal{A}_{3cL}(3) = \mathcal{A}_{2qL}(3) = \mathcal{A}_{6L}(3)$, (iv) $\mathcal{A}_{1R}(4) = \mathcal{A}_{3R}(4) = \mathcal{A}_{6R}(4)$ and $\mathcal{A}_{2l}(4) = \mathcal{A}_{1l}(4) = \mathcal{A}_{3l}(4)$, (v) $\mathcal{A}_{3c}(5) = \frac{1}{2}\mathcal{A}_{3cL}(5) + \frac{1}{2}\mathcal{A}_{3R}(5)$; $\mathcal{A}_{2L}(5) = \frac{3}{4}\mathcal{A}_{2qL}(5) + \frac{1}{4}\mathcal{A}_{2l}(5)$ and $\mathcal{A}_{1Y'}(5) = \frac{1}{2}\mathcal{A}_{1R}(5) + \frac{1}{2}\mathcal{A}_{1l}(5)$, (vi) $\mathcal{A}_{1Y}(6) = \frac{9}{10}\mathcal{A}_{1Y'}(6) + \frac{1}{10}\mathcal{A}_{1B}(6)$. With this information we can relate the SU(15) coupling constants (at energy $M_u \approx M_1$) to the low energy ($M_7 \approx M_w \approx 10^2 GeV$) $SU(3)_c \times SU(2)_L \times U(1)_Y$ coupling constants:

$$\begin{aligned}
\alpha_{3c}^{-1}(M_w) &= \alpha_{15}^{-1}(M_1) + 2\beta_{12} ln(\frac{M_1}{M_2}) + (\beta_{6L} + \beta_{6R}) ln(\frac{M_2}{M_3}) \\
&+ (\beta_{3cL} + \beta_{6R}) ln(\frac{M_3}{M_4}) + (\beta_{3cL} + \beta_{3R}) ln(\frac{M_4}{M_5}) \\
&+ 2\beta_{3c} ln(\frac{M_5}{M_6}) + 2\beta_{3c} ln(\frac{M_6}{M_w})
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
\alpha_{2L}^{-1}(M_w) &= \alpha_{15}^{-1}(M_1) + (\frac{3}{2}\beta_{12} + \frac{1}{2}\beta_{3l}) ln(\frac{M_1}{M_2}) + (\frac{3}{2}\beta_{6L} + \frac{1}{2}\beta_{3l}) ln(\frac{M_2}{M_3}) \\
&+ (\frac{3}{2}\beta_{2qL} + \frac{1}{2}\beta_{3l}) ln(\frac{M_3}{M_4}) + (\frac{3}{2}\beta_{2qL} + \frac{1}{2}\beta_{2lL}) ln(\frac{M_4}{M_5}) \\
&+ 2\beta_{2L} ln(\frac{M_5}{M_6}) + 2\beta_{2L} ln(\frac{M_6}{M_w})
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
\alpha_{1Y}^{-1}(M_w) &= \alpha_{15}^{-1}(M_1) + (\frac{11}{10}\beta_{12} + \frac{9}{10}\beta_{3l}) ln(\frac{M_1}{M_2}) \\
&+ (\frac{9}{10}\beta_{6R} + \frac{1}{5}\beta_{1B} + \frac{9}{10}\beta_{3l}) ln(\frac{M_2}{M_3}) \\
&+ (\frac{9}{10}\beta_{6R} + \frac{1}{5}\beta_{1B} + \frac{9}{10}\beta_{3l}) ln(\frac{M_3}{M_4}) \\
&+ (\frac{9}{10}\beta_{1R} + \frac{1}{5}\beta_{1B} + \frac{9}{10}\beta_{1l}) ln(\frac{M_4}{M_5})
\end{aligned}$$

$$+(\frac{9}{5}\beta_{1Y'}+\frac{1}{5}\beta_{1B})ln(\frac{M_5}{M_6})+2\beta_{1Y}ln(\frac{M_6}{M_w}) \tag{3.6}$$

The relevant linear combinations are those which yield

$$\sin^2(\theta_w)=\frac{3}{8}-\frac{5}{8}\alpha(\alpha_{1Y}^{-1}-\alpha_{2L}^{-1}) \quad and \quad (1-\frac{8}{3}\frac{\alpha}{\alpha_s})=\alpha(\alpha_{2L}^{-1}+\frac{5}{3}\alpha_{1Y}^{-1}-\frac{8}{3}\alpha_{3c}^{-1}) \tag{3.7}$$

namely,

$$\begin{aligned} 2.4(16\pi^2) &= (52.8-162.9h)ln(M_{12})+(35.2-36.7h)ln(M_{23})+(17.2h-82.1)ln(M_{34}) \\ &\quad +(29.3-2.7h)ln(M_{45})+(14.7+.1h)ln(M_{56})+(14.7-.1h)ln(M_{6w}) \quad (3.8) \\ 8.3(16\pi^2) &= (264-814.7h)ln(M_{12})+(117.3-23h)ln(M_{23})+(58.7-19h)ln(M_{34}) \\ &\quad +(88-.5h)ln(M_{45})+(44+.5h)ln(M_{56})+(44+.3h)ln(M_{6w}) \quad (3.9) \end{aligned}$$

where $h=0$ denotes the pure gauge case and $h=1$ includes Higgs effects. Here $M_{ij}\equiv M_i/M_j$ and the current experimental values[13] of $\sin^2\theta_w(=.233)$ and $\alpha_s(=.11)$ have been used.

For the pattern $\{1267\}$ the unification scale $M_1\approx M_u\approx 10^7 GeV$ in the pure gauge case, which is the [FL] result[1]. Large gauge contributions to the evolution equations enhance the coefficients of the first two terms; as a result unification is reached faster than in the usual GUTs like SU(5) (for which $M_1=M_2=M_A=M_u\sim O(10^{14})GeV$). However when Higgs effects are included ($h=1$) we find no solution to (3.8,3.9) for the $\{1267\}$ scenario other than $M_1=M_u\geq O(10^{14})GeV$, forbidding the low energy unification of [FL].

For $h=1$ we find three other interesting three-stage patterns: (A) $\{2467\}$ with $M_1=M_2=M_u$; $M_3=M_4=M_x$; $M_5=M_6=M_y$; (B) $\{3467\}$ with $M_1=M_2=M_3=M_u$; $M_4=M_x$; $M_5=M_6=M_y$; and (C) $\{2567\}$ with $M_1=M_2=M_u$; $M_3=M_4=M_5=M_x$; $M_6=M_y$ each having a 1-parameter family of solutions for M_y. (Although (C) does not have full unification at low energy, it does have interesting TeV physics.) Sample values are given in Table 3.2.

	{2467}		{3467}		{2567}	
M_y	M_u	M_x	M_u	M_x	M_u	M_x
250	7.91×10^8	2.96×10^8	8.87×10^8	3.50×10^2	1.97×10^{14}	1.77×10^3
500	1.11×10^9	4.06×10^8	1.25×10^9	7.05×10^2	1.98×10^{14}	3.53×10^3
1000	1.56×10^9	5.56×10^8	1.76×10^9	1.42×10^3	1.98×10^{14}	7.05×10^3
1500	1.91×10^9	6.68×10^8	2.15×10^9	2.15×10^3	1.99×10^{14}	1.06×10^4

Table 3.2: Mass scales (in GeV) for patterns (A)-(C)

The most interesting pattern is $\{3467\}$, which has both low energy unification at $\sim 10^9$ GeV and interesting TeV physics. We can decouple the electroweak breaking scale with the other symmetry breakings and have TeV scale chiral color symmetry and the quark-lepton un-unified electroweak symmetry breaking, which will raise the unification scale a little. The existence of chiral color symmetry at the TeV scale or lower will imply the presence of axigluons, whose phenomenological consequences have been studied[14]. The presence of the un-unified electroweak symmetry at low energy will imply the existence of extra charged and neutral gauge bosons, whose mixing with the Z-boson will affect various asymmetry parameters in the e^+e^- deep-inelastic scattering[15].

To summarize, we have shown that Higgs fields play a significant role in the evolution of gauge coupling constants in GUTs where baryon number is a symmetry. The consistency of the symmetry breaking scenario presented here with present-day proton decay data along with its interesting TeV scale physics make it a model worthy of further investigation.

3.2 Implications of SUSY SU(15) GUT

Supersymmetry offers a very interesting theoretical possibility which places fermions and bosons at equal footing via its transformation laws. Though supersymmetry itself can solve the problem of gauge hierarchy it is nevertheless an interesting proposition to endow the SU(15) GUT model with supersymmetric transformation laws and see the consequences. This is simply because supersymmetry is a rich symmetry by itself and nature seems to use all the symmetries available to her. Particularly, if one wants to unify these theories with gravity without causing naturalness problem, then supersymmetric version seems more promising. Another important aspect of checking the consistency of the supersymmetric $SU(15)$ GUT is to find out whether experimental findings of supersymmetry will still allow the possibility of low energy unification.

In the suspersymmetric version of the theory [2]every particle will imply the presence of its supersymmetric partner. When all this new particles run in the loops they will alter the renormalization procedure of the conventional theory and hence the beta functions. The supersymmetric beta functions (to one loop order) which will control the evolution of the coupling constants given by the following expression [16]. Let us also note here that due to unequal normalizations of the generators at different stages of the symmetry breaking chain in the calculations of the mass scales one has to multiply the beta functions with proper normalization factors

$$\beta(N) = -\frac{1}{(4\pi)}\left[3N - T - n_f\right] \qquad (3.10)$$

Here, N stands for the SU(N) group of which the coupling constant is under consideration, T denotes the contribution of the scalar loops and n_f stands for the number of fermion generations which is constrained to be three by LEP data.

We use the one-loop renormalization group equations which have the form,

$$\mu\frac{d\alpha(\mu)}{d\mu} = 2\beta\alpha^2(\mu) \qquad (3.11)$$

Where $\alpha(\mu)$ stands for the value of the coupling constant at the energy scale μ

Solving the renormalization group equations (3.6) using the combinations of the couplings given in the equation (3.7) and the Higgs contributions given in Table(3.3) we can find out the unification scale. Afterwards using the value of the unification scale as an input we can find the value of α_{15} at the unification scale using the expressions of α_3. We have calculated

[2]This section is based on Ref. [5]

$M_1 \to M_2$	$M_2 \to M_3$	$M_3 \to M_4$	$M_4 \to M_5$	$M_5 \to M_6$	$M_6 \to M_7$
$[12] = -2908$	$[6_L] = -192$	$[3_{cL}] = -30$	$[3_{cL}] = 0$	$[3_c] = 9$	$[3_c] = 11$
$[3_l] = -572$	$[6_R] = -42$	$[6_R] = -42$	$[1_R] = -18.3$	$[2_L] = 5$	$[2_L] = 7.2$
	$[1_B] = -93$	$[1_B] = -93$	$[1_B] = -1.5$	$[1_B] = -1.5$	$[1_Y] = -.02$
	$[3_l] = -100$	$[3_l] = -100$	$[1_l] = -13.3$	$[1_{Y'}] = -0.5$	
		$[2^q_L] = -40$	$[2^l_L] = -15.6$		
			$[2^q_L] = -10$		
			$[3_R] = 0$		

Table 3.3: Value of [3N-T] at various scales with proper normalizations

these quantities for all possible chains coming from SU(15) GUT. None of the chains can give a consistent low energy unification scheme. For a few breaking chain where unification is apparently achieved at a scale around 10^{12} GeV the value of α_{15} at the unification scale becomes undefined hence forbidding a consistent perturbative unification scheme. What it means is that the coupling constants evolve so fast that they become more than unity much before the unification scale. Then the α^{-1} evolves to zero and the coupling constants becomes undefined.

To outline the procedure of solutions in brief let us set the notation that $m_{ij}=ln\frac{m_i}{m_j}$. Now solving in the case when $m_{12} = 0, m_{34} = 0, m_{56} = 0$ we get by solving for m_{23} and m_{45}:

$$
\begin{aligned}
m_{23} &= -1.35 + 0.005 m_{67} \\
m_{45} &= 17.78 + 0.60 m_{67}.
\end{aligned}
$$

Now for $m_i > m_j$, m_{ij} has to be positive definite which immediately sets the bound $m_{67} = 27$ when $m_{23} = 0$ hence for the breaking chain 2467 m_6 has the minimum value of the order of 10^{14} GeV. Furthermore by using the minimum value of m_{67} in the second equation we can see that the minimum value of m_{45} is 34. Hence forbidding any unification of coupling at all (within the plank scale).

Similarly let us consider the case when $m_{12} = 0, m_{23} = 0, m_{56} = 0$ we get by solving for m_{34} and m_{45}:

$$
\begin{aligned}
m_{34} &= 11.2 - 0.41 m_{67} \\
m_{45} &= -7.03 + 0.31 m_{67}.
\end{aligned}
$$

Now in this case to make m_{45} at least positive m_{67} has to be atleast 22.67 and hence m_{34} has to be atleast 1.9 which leads to an apparent unification scale of approximately 10^{12} GeV. But if one checks the value of the inverse of the coupling constant at the unification scale using the α_3 equation, for example, one sees that it has crossed the value zero and has become negative. Hence for the chain 3467 there is no consistent unification framework. In this simple way all possible symmetry breaking chains can be analyzed.

To see the result for the chain 1367 let us solve the equations for m_{12} and m_{23} in terms of m_{67}, making all other m_{ij}s vanish. The solutions are:

$$
\begin{aligned}
m_{12} &= -0.27 + 0.009 m_{67} \\
m_{23} &= -0.558 + 0.023 m_{67}
\end{aligned}
$$

this means that to make m_{12} positive, m_{67} has a minimum value of 30. Which immediately means that the scale m_6 is at least 10^{15} GeV, and the unification scale is even higher. For the chain 2567 we will solve for m_{23} and m_{56} and making all the other m_{ij}s vanish. The solutions are:

$$m_{12} = 0.26 - 0.004m_{67}$$
$$m_{23} = 34.03 - 1.16m_{67}.$$

Here though m_{67} can be low yet the unification of couplings still occur at a very high value. This is simply because as m_{67} becomes smaller the value of m_{23} increases. Similarly, for most of the chains, we find the unification scale becomes larger than 10^{14} GeV, and the possibility of low energy unification is lost. For these chains we have first calculated by taking the supersymmetry breaking scale to be same as M_6. Taking M_{susy} to be lower, or around the TeV scale, we find the situation worsens.

In the Table 3.4 we state a sample of these values for those chains which was considered earlier in the nonsupersymmetric model, and some more sample chains.

Breaking Chain	Unification Scale	$\alpha_{15}^{-1}(M_U)$
2467	No Unification	–
3467	4.63 10^{12}	Undefined
2567	Greater than 10^{14}	–
1367	Greater than 10^{14}	–
4567	Greater than 10^{14}	–

Table 3.4: Mass scales in GeV

In this section we have attempted to ask the question that if supersymmetry is discovered in near future how is it going to affect the new paradigm of the low energy unification of the SU(15) GUT model. These conclusions will also be true for the $SU(16)$ GUT, with similar symmetry breaking chains. We find that the low energy unification with SU(15) in the supersymmetric framework is not allowed. Most of the symmetry breaking chains do not allow for low energy unifications, and a few symmetry breaking chains which allow low energy unification fails to satisfy the perturbative unification constraint (coupling constants to be less than one). Hence the signals of the existence of supersymmetry in future colliders will rule out the possibility of low energy unification.

The scenario of symmetry breaking in nonsupersymmetric SU(15) GUT, which allows low energy unification, has some interesting features. It is essential for the low energy unification to have chiral color $SU(3)_{cL} \times SU(3)_{cR}$ group and the quark-lepton ununified group $SU(2)^q_L \times SU(2)^l_L$ survive till very low energy, for the gauge coupling constants to evolve very fast and get united at an energy scale around 10^8 GeV. Thus the existence of these groups and the leptoquarks are some of the essential critereons of the low energy unification, which can be tested in the laboratory in near future. Thus any signatures of these groups may seriously question the existence of supersymmetry and if the signatures of the low energy unification and also that of supersymmetry are found, then it will cast a serious question on our understanding of the grand unification scenario.

3.3 Low energy unification with SU(16)

We have already discussed a new paradigm of low energy unification in which we have considered $SU(15)$ as the unification group[1]. Here we extend the idea to the left-right symmetric version of such a theory. We show that retaining all the good features of $SU(15)$ we can also incorporate left-right symmetry in intermediate stages. Unlike the $SU(15)$ GUT here lepton number is also a local gauge symmetry which may survive to a low energy scale. Right handed neutrino can be accommodated naturally as all the fermions transform in the fundamental representation of $SU(16)$.

At the level of highest symmetry the theory is invariant under the gauge group $SU(16)$ [3]. At and above this level the coupling constant is that of the group $SU(16)$. With the decrease in energy, the group goes through a number of symmetry breaking phases, and the theory becomes least symmetric at the present energies with the residual symmetry of $SU(3)$ color and the symmetry of electromagnetic interactions. It is noteworthy that the baryon number symmetry remains exact upto a very low energy scale of a few TeV. This makes the proton stable in the sense that the gauge boson mediated proton decay is absent. Interestingly the completely un-unified symmetry group of the quarks and leptons also appears at a low energy scale together with the chiral color symmetry. The appearance of this group at a comparatively low scale makes this model worthy of phenomenological studies[8],[9],[14].

Here to begin with we give the breaking chain that can give rise to the standard model. We note here that there can be in general a number of chains of descent to the standard model.

$$
\begin{aligned}
SU(16) &\xrightarrow{M_U} G[SU(12) \times SU(4)^l] \\
&\xrightarrow{M_1} G_1[SU(6)_L \times SU(6)_R \times U(1)_B \times SU(4)^l] \\
&\xrightarrow{M_2} G_2[SU(3)_L \times SU(2)^q_L \times SU(6)_R \times U(1)_B \times SU(4)^l] \\
&\xrightarrow{M_3} G_3[SU(3)_L \times SU(2)^q_L \times SU(3)_R \times U(1)^q_R \times U(1)_B \times SU(2)^l_L \times SU(2)^l_R \times U(1)^{lep}] \\
&\xrightarrow{M_4} G_4[SU(3)_L \times SU(2)^q_L \times SU(2)^l_L \times SU(3)_R \times U(1)_R \times U(1)_B \times U(1)^l] \\
&\xrightarrow{M_5} G_5[SU(3)_c \times SU(2)_L \times U(1)_B \times U(1)_h] \\
&\xrightarrow{M_6} G_6[SU(3)_c \times SU(2)_L \times U(1)_Y] \\
&\xrightarrow{M_z} G_7[SU(3)_c \times U(1)_{em}]
\end{aligned}
$$

Here the superscript q or l denotes that quarks or leptons have nontrivial transformation law under these groups and the subscripts L and R mean so for the left and right handed fermions. The subscript c stands for the color gauge group of QCD.

In a previous section we have shown that in $SU(15)$ GUT the effect of Higgs bosons play a significant role in the evolution of the coupling constants with increasing energy and hence on the values of the mass scales. This is due to the presence of high dimensional Higgs fields required to obtain the desired symmetry breaking pattern. The influence of the Higgs fields on the evolution of coupling constants can be so serious that they can alter the symmetry breaking pattern altogether. In $SU(16)$ GUT The symmetry breaking pattern is very similar to that of its $SU(15)$ counterpart. Hence in $SU(16)$ or in $SU(15)$ GUT the Higgs effects must

[3]This section is based on Ref. [4]

be taken seriously. Here we shall consider the Higgs fields required to obtain the breaking chain and their contribution in the renormalization group equations in detail.

The Higgs structure is similar to that we proposed for $SU(15)$ GUT. We denote 1^n as the totally antisymmetric nth rank tensor and 1^n1^m as the representation which has m and n vertical boxes in the first and second columns of its Young's tableau. For the transition from the group G_1 to group G_2 the G_2 singlet component of the Higgs field should acquire vacuum expectation value. Turning to the specific case of $SU(16)$ we note that at the scale M_U the breaking can be achieved by giving the vacuum expectation value to the $SU(12)\times SU(4)$ singlet component of 1^4. Using the exactly same procedure we see that the breaking at the scale M_1 can be done by $1^{14}1$ which leaves $U(1)_B$ unbroken. At the scale M_2 the breaking of $SU(6)_L$ to its special maximal subalgebra requires a somewhat large dimensional Higgs field representation. We use the **14144** dimensional Higgs field $1^{14}1^2$ to break this group. As a passing comment we note here that this Higgs field will contribute significantly to the beta functions of the renormalization group equations and make its presence strongly felt in the determination of the mass scales. The group $SU(4)^l$ can be broken by a Higgs field which transforms as a 15-plet under $SU(4)^l$ and which is contained in **255** under $SU(16)$. At the stage M_3 the breaking of $SU(6)_R$ to $SU(3)_R\times U(1)_R$ is a bit complicated. **255** breaks $SU(6)_R$ to $SU(3)\times SU(3)\times U(1)_R$ and subsequently the two $SU(2)_L$ groups of the quark and leptonic sectors respectively are glued by $1^{14}1^2$. The breaking of the lepton number local gauge symmetry $U(1)^{lep}$ can be achieved by either **16** or the two index symmetric Higgs field of dimension **136**. In the first case it carries a lepton number one unit and in the second case it carries that of two units. We shall see that the choice of specific Higgs field shall give interesting difference of physics in the context of neutrino oscillations. At the scale M_5 the breaking is done by the 1^4 Higgs field which is **1820** dimensional. The baryon number is broken by either 1^5 or 1^6. In both the cases we get interesting physics. As an example in the first case we get processes where baryon number changes by 3 units and in the second case it changes by 2 units. It is welknown that to give masses to the fermions vacuum expectation value has to be given to the component $(1,2,-\frac{1}{2})$ which is contained in either 1^2 or 11. These Higgs Field representations are summarized in Table A

Let us now turn our attention to the group theoretic transformation properties of the fermions under the different symmetry groups in the symmetry breaking scheme. A minimal left-right symmetric theory should have at least one right handed neutrino (ν_R) on top of the standard quarks which includes three left handed doublets and six right handed singlets under the weak interaction gauge group $SU(2)_L$ and three leptons namely one left handed doublet and one right handed singlet. At grand unification energies and above this sixteen fermions should transform under some representation of the unification group. This requirement makes $SU(16)$ a very natural choice of the unification gauge group which has a 16 dimensional fundamental representation. In the model the fermions transform under the fundamental representation of $SU(16)$. Now as the energy becomes lower the symmetry breakings occur and the transformation properties of the fermions change in each symmetry breaking scale. In the following we summarize these transformation properties. We use the notation that (m,n) is a representation which transforms under the semisimple group $SU(M)\times SU(N)$ as a m-plate under the former group and as a n-plate under the the later group.

$$
\begin{aligned}
SU(16) &\longrightarrow 16 \\
G &\longrightarrow (12,1)+(1,4) \\
G_1 &\longrightarrow (1,\bar{6},n,1)+(6,1,-n,1)+(1,1,0,4)
\end{aligned}
$$

$SU(16)$	G	G_1	G_2	G_3	G_4	G_5	G_6
1820							
255	(143,1)						
14144	(4212,1)	(189,1,0,1)					
255	(1,15)	(1,1,0,15)	(1,1,0,1,15)				
255	(143,1)	(1,35,0,1)	(1,1,35,0,1)				
14144	(4212,1)	(1,189,0,1)	(1,1,189,0,1)				
136	(1,10)	(1,1,0,10)	(1,1,1,0,10)	$(1,1,1,0,0,1,3,\frac{2}{\sqrt{8}})$			
16	(1,4)	(1,1,0,4)	(1,1,1,0,4)	$(1,1,1,0,0,1,2,\frac{1}{\sqrt{8}})$			
1820	(66,6)	$(6,\bar{6},0,6)$	$(3,2,\bar{6},0,6)$	$(3,2,\bar{3},-\frac{1}{\sqrt{12}},0,2,2,0)$	$(3,2,\bar{3},-\frac{1}{\sqrt{12}},0,2,\frac{1}{\sqrt{12}})$		
$4368(1^5)$	(220,6)	$(1,\bar{20},\frac{3}{2\sqrt{6}},6)$	$(1,1,\bar{20},\frac{3}{2\sqrt{6}},6)$	$(1,1,1,\frac{1}{\sqrt{12}},\frac{3}{2\sqrt{6}},1,1,-\frac{2}{\sqrt{8}})$	$(1,1,1,\frac{1}{\sqrt{12}},\frac{3}{2\sqrt{6}},1,-\frac{2}{\sqrt{12}})$	$(1,1,\frac{3}{2\sqrt{6}},-\frac{1}{\sqrt{24}})$	
136	(78,1)	$(6,\bar{6},0,1)$	$(3,2,\bar{6},0,1)$	$(3,2,\bar{3},\frac{1}{\sqrt{12}},0,1,1,0)$	$(3,2,\bar{3},\frac{1}{12},0,1,0)$	$(1,2,0,\frac{1}{\sqrt{24}})$	$(1,2,-\frac{1}{2}\sqrt{\frac{3}{20}})$

TABLE A

$$
\begin{aligned}
G_2 &\longrightarrow (1,1,\bar{6},n,1)+(3,2,1,-n,1)+(1,1,1,0,4)\\
G_3 &\longrightarrow (1,1,\bar{3},p,n,1,1,0)+(1,1,\bar{3},-p,n,1,1,0)+(3,2,1,0,-n,1,1,0)\\
&\quad +(1,1,1,0,0,1,2,m)+(1,1,1,0,0,2,1,-m)\\
G_4 &\longrightarrow (1,1,\bar{3},p,n,0)+(1,1,\bar{3},-p,n,0)+(3,2,1,0,-n,0)+(1,1,1,0,0,-2\sqrt{\frac{2}{3}}m)\\
&\quad +(1,2,1,0,0,\sqrt{\frac{2}{3}}m)+(1,1,1,0,0,0)\\
G_5 &\longrightarrow (\bar{3},1,n,n)+(\bar{3},1,n,-n)+(3,2,-n,0)+(1,2,0,n)+(1,1,0,-2n)+(1,1,0,0)\\
G_6 &\longrightarrow (\bar{3},1,-\frac{2}{3}K)+(\bar{3},1,\frac{1}{3}K)+(3,2,\frac{1}{6}K)+(1,1,K)+(1,1,-\frac{1}{2}K)+(1,1,0)
\end{aligned}
$$

Here the $U(1)$ normalization are defined in terms of

$$
n=\frac{1}{2\sqrt{6}};\quad m=\frac{1}{2\sqrt{2}};\quad p=\frac{1}{2\sqrt{3}};\quad K=\sqrt{\frac{3}{20}}.
$$

We know that in the electroweak breaking scale M_Z the generators of electromagnetic symmetry group $U(1)_{em}$ arises out as a linear combination of the generator of the $U(1)$ part of the weak isospin group $SU(2)_L$ and that of the weak hypercharge $U(1)_Y$ by the following equation,

$$
Q=T_L^3+Y. \tag{3.12}
$$

Let us call this equation as the $U(1)$ *matching condition* at the sca'. M_Z. Similarly at the various symmetry breaking scales in the above breaking chain we have used different matching conditions for the groups. These matching conditions are stated below.

At the scale M_4 the lepton number symmetry breaks as the generator of $U(1)^{lep}$ and the diagonal generator of $SU(2)_R^l$ mixes with each other in the following way to generate the group $U(1)^l$,

$$
Y^l=\sqrt{\frac{1}{3}}T_{2_R^l}^3+\sqrt{\frac{2}{3}}Y^{lep}. \tag{3.13}
$$

At the scale M_5, $U(1)_R$ and $U(1)^l$ breaks to make $U(1)_h$.

$$
Y_h=\sqrt{\frac{1}{2}}Y_R+\sqrt{\frac{1}{2}}Y^l. \tag{3.14}
$$

At the scale M_6, baryon number cease to be a local gauge symmetry and conventional hypercharge appears from the linear combination of $U(1)_B$ and $U(1)_h$.

$$
Y=-\sqrt{\frac{1}{10}}Y_B-\sqrt{\frac{9}{10}}Y_h. \tag{3.15}
$$

Now we briefly touch two more mathematically involved topics . To begin with we note that the generators of $SU(16)$ and that of the standard model groups cannot be normalized in the same way. We proceed further by giving a short discussion of the process of calculating the contribution of the Higgs fields to the beta functions. Let us fix that all the generators of $SU(16)$ are normalized to 2. In that case at the standard model energies the generators of $SU(3)_C$ and $SU(2)_L$ automatically becomes the generators of $SU(16)$. In contrast the generators of $U(1)_Y$ are normalized to $\frac{1}{2}$. So in the renormalization group equations we have to multiply the beta function corresponding to $U(1)_Y$ group by the appropriate factor of 4. Similarly it is easy to see that all other $U(1)$ groups in the symmetry breaking chain has to be multiplied by 4. Turning to the non-abelian groups it can be checked that the group $SU(2)_L^q$ in all stages is normalized to $\frac{3}{2}$ hence to treat it at par with all other groups one has to multiply the beta function corresponding to this by a factor of $\frac{4}{3}$. $SU(3)_L$ and $SU(3)_R$ in all the stages are normalized to 1 hence one finds the aforesaid factor to be 2. To complete the discussion on the normalization factors we note that all other groups are normalized to $\frac{1}{2}$ hence the relevant factor is 4

At this point let us turn our attention to the expression of the beta function for the group $SU(N)$

$$b(N) = -\frac{1}{(4\pi)^2}\left[\frac{11}{3}N - \frac{1}{6}T - \frac{4}{3}n_f\right] \tag{3.16}$$

For $U(1)$ groups N vanishes. Here n_f denotes the number of families of fermions and $T(R)$ denotes the contribution of the Higgs fields which transform nontrivialy under the group under consideration. To calculate T we have followed the following sum rule[18]:

Suppose R_i and r_i $(i = 1, 2, ..)$ are different representations of a group $SU(N)$, which when vectorically multiplied satisfies the following relation.

$$R_1 \times R_2 = \sum_{i=1} r_i \tag{3.17}$$

Also let for the representation of dimension r, the contribution to the renormalization group equation is $T(R)$. Then,

$$T(R_1 \times R_2) = R_2 T(R_1) + R_1 T(R_2) = \sum_{i=1} T(r_i) \tag{3.18}$$

To use these equations one uses the following information to start with

$$\begin{aligned}
T(N) &= \frac{1}{2}, \\
T(N^2-1) &= N, \\
T\left[\frac{N(N-1)}{2}\right] &= \frac{N-2}{2}, \\
T\left[\frac{N(N+1)}{2}\right] &= \frac{N+2}{2}, \\
T(1) &= 0.
\end{aligned}$$

As an example consider 3 and $\bar{3}$ representations of $SU(3)$. When vectorically multiplied they give

$$3 \times \bar{3} = 1 + 8,$$

so using the sumrule

$$T(8) = 3T(3) + 3T(\bar{3}) - T(1) = 3.$$

To evaluate the mass scales we use the standard procedure of evolving the couplings with energy. The energy dependence of the couplings with energy[10]. The energy dependence of the couplings are completely determined by the particle content of the theory and their couplings inside the loop diagrams of the guage bosons. This is expressed by the renormalization group equation. The one-loop RG equation is given by the following equation.

$$\mu \frac{d}{d\mu} \alpha(\mu) = 2\ b\ \alpha^2(\mu), \tag{3.19}$$

where

$$\alpha = \frac{g^2}{4\pi}. \tag{3.20}$$

Using the above information and the matching conditions given with each symmetry breaking chain one can relate the $SU(16)$ coupling constant $\alpha_{SU(16)}$ with the standard model couplings α_{3c}, α_{2L} and α_{1Y} at the scale M_Z. At this point let us remember that there are three quark doublets and one leptonic doublet under the group $SU(2)_L$ in the standard model hence in the evolution of coupling α_{2L} the quark and leptonic groups $SU(2)^q_L$ and $SU(2)^l_L$ do not contribute equally to the standard model group $SU(2)_L$ instead they contribute with a relative factor 3.

$$\begin{aligned}
g_{3c}^{-2}(M_z) &= g_{SU(16)}^{-2}(M_U) + \\
&\quad 2b_{12}M_{U1} + (b_{6L} + b_{6R})M_{12} + (b_{3L} + b_{6R})M_{23} + \\
&\quad (b_{3L} + b_{3R})M_{34} + (b_{3L} + b_{3R})M_{45} + 2b_{3c}M_{56} + 2b_{3c}M_{6z} \\
g_{2L}^{-2}(M_z) &= g_{SU(16)}^{-2}(M_U) + \\
&\quad (\frac{3}{2}b_{12} + \frac{1}{2}b_4^l)M_{U1} + (\frac{3}{2}b_{6L} + \frac{1}{2}b_4^l)M_{12} + \\
&\quad (\frac{3}{2}b_{2L}^q + \frac{1}{2}b_4^l)M_{23} + (\frac{3}{2}b_{2L}^q + \frac{1}{2}b_{2L}^l)M_{34} + (\frac{3}{2}b_{2L}^q + \frac{1}{2}b_{2L}^l)M_{45} + \\
&\quad 2b_{2L}M_{56} + 2b_{2L}M_{6z} \\
g_{1Y}^{-2}(M_z) &= g_{SU(16)}^{-2}(M_U) + \\
&\quad (\frac{11}{10}b_{12} + \frac{9}{10}b_4^l)M_{U1} + (\frac{9}{10}b_{6R} + \frac{1}{5}b_{1B} + \frac{9}{10}b_4^l)M_{12} + \\
&\quad (\frac{9}{10}b_{6R} + \frac{1}{5}b_{1B} + \frac{9}{10}b_4^l)M_{23} + (\frac{9}{10}b_{1R} + \frac{1}{5}b_{1B} + \frac{6}{10}b_1^{lep} + \frac{3}{10}b_{2R}^l +)M_{34} + \\
&\quad (\frac{9}{10}b_{1R}^q + \frac{1}{5}b_{1B} + \frac{9}{10}b_1^l)M_{45} + \\
&\quad (\frac{9}{5}b_{1h} + \frac{1}{5}b_{1B})M_{56} + 2b_{1Y}M_{6z}
\end{aligned} \tag{3.21}$$

Here M_{ij} is defined as $ln(\frac{M_i}{M_j})$

To calculate the mass scales we also have to know the numerical values of the beta function coefficients. To know them one has to know the contribution of the Higgs scalars to the beta functions (T). In the Table 3.5 we give these values.

With the quantities $g_{1Y}^{-2}(M_z)$ $g_{2L}^{-2}(M_z)$ and $g_{3c}^{-2}(M_z)$ at hand one can construct two different linear combinations with them to form the experimentally measured quantities at the energy

G	G_1	G_2	G_3	G_4	G_5	G_6
$[12] = 1492$	$[6_L] = 69$	$[3_L] = 42$	$[3_L] = 15$	$[3_L] = 9$	$[3_c] = 0$	$[3_c] = 0$
$[4^l] = 293$	$[6_R] = 93$	$[2^q_L] = 63$	$[2^q_L] = 22.5$	$[2^q_L] = 13.5$	$[2_L] = 0.5$	$[2_L] = 0.5$
	$[1_B] = 45$	$[6_R] = 93$	$[3_R] = 15$	$[3_R] = 9$	$[1_B] = .375$	$[1_Y] = .075$
	$[4^l] = 63$	$[1_B] = 45$	$[1^q_R] = 7.58$	$[1^q_R] = 3.16$	$[1_h] = .083$	
		$[4^l] = 63$	$[1_B] = .375$	$[1_B] = .375$		
			$[2^l_L] = 18$	$[2^l_L] = 9$		
			$[1^{lep}] = 2$	$[1_l] = 3.16$		

Table 3.5: Contributions of the Higgs scalar to the R-G equation at various energy scales

scale M_z.It easy to check that the following relations hold between them.

$$\begin{aligned} sin^2(\theta_w) &= \frac{3}{8} - \frac{5}{8}e^2(g_{1Y}^{-2} - g_{2L}^{-2}), \\ 1 - \frac{8}{3}\frac{\alpha}{\alpha_s} &= e^2(g_{2L}^{-2} + \frac{5}{3}g_{1Y}^{-2} - \frac{8}{3}g_{3c}^{-2}). \end{aligned} \tag{3.22}$$

From the present experimental measurements at LEP the value of $Sin^2(\theta_w)$ and α_s has been very accurately measured.We use for our purpose the following values[2] of them and the $U(1)$ coupling α at the scale M_z

$$\begin{aligned} sin^2(\theta_w) &= .233, \\ \alpha_s &= .11, \\ \alpha &= \frac{1}{127.9.} \end{aligned} \tag{3.23}$$

Having these informations at hand one can straightaway go to calculate the mass scales of symmetry breaking.

Let us discuss the calculation of the first chain in some detail. Let us now assume that $M_4 = M_3 = M_A$. This means that the groups $SU(6)_L$ $SU(6)_R$ and $SU(4)^l$ happens to break at the same scale. Similarly let us also assume that $M_4 = M_5 = M_B$. Solving for M_{U1} and M_{B6} in terms of the other variables one gets,

$$\begin{aligned} M_{U1} &= -.28 - .10M_{1A} - .10M_{6z} + .04M_{AB} \\ M_{B6} &= 19.80 - 4.81M_{1A} - 2.93M_{6z} - .21M_{AB} \end{aligned} \tag{3.24}$$

As the symmetry breaking at M_U precedes thet at M_1, M_{U1} is at least positive. So from the first equation one infers that for a specific set of values of the other parameters in the right-hand side there is a minimum value to M_{AB}.Varying the parameters of the equations one gets the following subset of the solution set allowed by the equations.Taking M_z to be around 91 GeV one can also calculate the unification scale and the scaleM_6 where the completely un-unified symmetry of the quarks and leptons and the chiral color symmetry is broken.We note that as the parameter M_{AB} increases i.e. as the separation between the scale M_A and the scale M_B increases the scale M_B comes down. Results are summarized in Table 3.6.

M_{AB}	M_{1A}	M_{6z}	M_{B6}	M_{U1}	M_B	M_U
7	0	0	18.4	0	10^9	10^{12}
9.5	1	0	12.9	0	10^8	10^{12}
10.75	1.5	0	10.3	0	10^7	10^{11}
12	2	0	8.7	0	10^6	10^{11}
14.5	3	0	2.3	0	10^4	10^{11}

Table 3.6: Mass scales of SU(16) GUT

3.3.1 Proton Decay

Having the mass scales and Higgs structure in hand we proceed in this paper to discuss the issue of proton decay now. In all the breaking chains that we have considered here, the quark lepton unification is broken at the scale M_U while the quark antiquark unification is broken at the scale M_1. As a result the leptoquark gauge bosons (X_μ) will acquire mass at the scale M_U while the diquark gauge bosons (Y_μ) acquire mass at the scale M_1. Under the group G_1 their transformation properties are

$$\begin{aligned} X_\mu &\Longrightarrow (6,1,-B,\bar{4}) + (1,\bar{6},B,\bar{4}) + \\ &\quad\; (\bar{6},1,B,4) + (1,6,-B,4) \\ Y_\mu &\Longrightarrow (6,6,-2B,1) + (\bar{6},\bar{6},2B,1) \end{aligned} \tag{3.25}$$

where B is defined as,

$$B = \frac{1}{2\sqrt{6}} \tag{3.26}$$

Now $U(1)_B$ being an explicit local gauge symmetry of the model, X_μ and Y_μ contains different " Barion Numbers " and hence cannot mix directly to form an $SU(16)$ invariant operator.

The mixing can be induced indirectly through the term $D_\mu\phi_a D^\mu\phi_b$, where D_μ is the covariant derivative of the $SU(16)$ invariant theory.$D_\mu\phi_a D^\mu\phi_b$ will contain a term $X_\mu\phi_a X^\mu\phi_b$. When ϕ_a and ϕ_b acquires vacuum expectation value the mixing between X_μ and Y^μ occurs. But this can occur only at the scale M_6 hence the amplitude is suppressed by a factor of $O\frac{(M_5 M_6)}{(M_1^2 M_2^2)}$.

To see how the gauge bosons couple to the Higgs fields we note that all the gauge bosons at the $SU(16)$ level transform under the **224** dimensional adjoint representation. We also note the following tensor product at the $SU(16)$ level

$$224 \times 224 = 1 + 224 + 224 + 14175 + 10800 + 12376 + 12376 \tag{3.27}$$

Being the product of two selfconjugate representations all the terms in the right hand side are selfconjugate which couples to only self conjugate representations. From the Table A that the the Higgs field that carries Baryon Number is 1^5. So the only Higgs field which can induce a Baryon Number violating effect is 1^5 which is **4368** dimensional.

The only self conjugate combination made up with 1^5s is $< 4368 >< \bar{4368} >$ which again corries no baryon number hence not giving rise to any baryon number violating process[17].

To see the Higgsfield mediated proton decay at first we note that the fermions are in the **16** dimensional fundamental representation. To give mass to the fermions the coupling of the form $\bar{\psi_L}^c \psi_L \phi$ must exist. The minimum dimensional Higgs field which can do the job is **120**. This field can give rise to Higgs madiated proton decay if 1^6 breaks the Baryon Number due to the presence of the term $< 1^6 >< 1^6 >< 1^2 >< 1^2 >$ in the Lagrangian. In that case we can choose **136** to give mass to the fermions. In our choice 1^5 breaks the baryon number hence it does not couple to **120**. Hence there is no Higgs mediated proton decay.

3.3.2 $N - \overline{N}$ Oscillations

Let us consider the $SU(16)$ level operator $< 1^5 >< 1^5 >< 1^5 >< 16 >$. This forms a singlet under $SU(16)$ and hence allowed in the Lagrangian. This term give rise to $\Delta B = 3$ processes. If instead we choose **136** to break the lepton Number symmetry, then this process vanishes.

We have already noted that if 1^6 breaks the Baryon Number symmetry then one has to choose **136** to give mass to the fermions; here we note that then the term $< 1^{14}1^2 >< 136 >< 136 >< 1^6 >< 1^6 >$ will be be allowed in the Lagrangian which may give rise to $\Delta B = 3$ processes. As the term is of dimension five it will be suppressed by M_U. With 1^2 we can construct the $SU(16)$ level operator $< 1^5 >< 1^5 >< 1^4 >< 1^2 >$ which can break the Baryon Number by two units and hence give rise to gauge boson mediated $N - \bar{N}$ oscillations. To see the Higgs field mediated processes we note that if **120** dimensional Higgs field couples to the fermions and 1^6 breaks the Baryon number then the operator $< 1\bar{2}0 >< 1\bar{2}0 >< 1\bar{2}0 >< 1^6 >$ can give rise to Higgs field mediated N-$\bar{N}$ oscillations.

In this paper we have seen that there exists one possible breaking chain in a Grand Unified Theory based on the group $SU(16)$ where a unification scale of the order of 10^{11} GeV is possible. There exists a very low energy scale (M_B) which may be almost anywhere between the unification scale and the electroweak scale where completely ununified symmetry of quarks and leptons may exist together with chiral color symmetry. The scale M_B comes lower when the separation between the scale M_A and the scale M_B is increased. Qualitatively we understand it in the following way. The beta function coefficients can be looked into as the slope of the lines if one plots the inverse coupling constants with respect to energy. It can be easily checked that as at the $SU(16)$ level all the fermions transform under the fundamental representation of the group and in the other levels they transform in a more complicated way under the various groups in the intermediate stages, all the groups cannot be normalized in the same way. To compensate for the mismatch in the normalization the beta function coefficients has to be multiplied by appropriate factors. Because of this the slope of the curves representing the inverse couplings also gets multiplied by the appropriate factors and the couplings get united earlier giving rise to low energy unification.

We have also seen that this model satisfies the experimental constraints coming from proton decay experiments in the sense that proton decay is suppressed. We have shown that there exists atleast one choice of the Higgs sector where there is no Higgs mediated proton decay either.

For some specific choice of the Higgs fields there may exist interesting physical consequences like the $N - \bar{N}$ oscillation. There is also the possibility of having the sea-saw mechanism to give Majorana mass to the neutrinos and this also may have observable consequences.

Last but not the least we emphasize again that there exists very rich low energy physics coming from this model hence keeping in mind the forthcoming high-energy experiments at Superconducting Super Collider, CERN Large Hadron Collider and other places this model is worthy of further investigation.

3.4 Appendix

3.4.1 SU(16) Tensor Products

$$\begin{aligned}
16 \times 16 &= 120_a + 136_s \\
\bar{16} \times 16 &= 1 + 255 \\
16 \times 120 &= 560_a + 1360 \\
\bar{120} \times 120 &= 1 + 255 + 14144 \\
\bar{136} \times 136 &= 1 + 255 + 18240 \\
560_a \times 16 &= 1820_a + 7140 \\
1820_a \times 16 &= 4368_a + 24752
\end{aligned} \tag{3.28}$$

3.4.2 SU(16) Branching Rules

$$\begin{aligned}
SU(16) &\Longrightarrow SU(12)\times SU(4) \\
16 &= (12,1) + (1,4) \\
136 &= (78,1) + (12,4) + (1,10) \\
120 &= (66,1) + (12,4) + (1,6) \\
255 &= (143,1) + (12,\bar{4}) + (\bar{12},4) + \\
&\quad (1,15) + (1,1) \\
560 &= (220,1) + (66,4) + (12,6) + \\
&\quad (1,\bar{4}) \\
1820 &= (495,1) + (220,4) + (66,6) + \\
&\quad (12,\bar{4}) + (1,1) \\
14144 &= (1,1) + (1,35) + (12,\bar{4}) + \\
&\quad (12,\bar{20}) + (\bar{12},4) + (\bar{12},20) + \\
&\quad (\bar{66},6) + (66,\bar{6}) + (143,1) + \\
&\quad (143,15) + (70,\bar{4}) + (\bar{780},4) + \\
&\quad (4212,1)
\end{aligned} \tag{3.29}$$

3.4.3 SU(12) Tensor Products

$$12 \times 12 = 66_a + 78_s$$

$$
\begin{aligned}
\bar{12}\times 12 &= 1+143\\
12\times 66 &= 220_a+572\\
\bar{78}\times 78 &= 1+143+5940\\
\bar{66}\times 66 &= 1+143+4212\\
220_a\times 12 &= 495+2145\\
495_a\times 12 &= 792+5148
\end{aligned}
\tag{3.30}
$$

3.4.4 SU(12) Branching Rules

$$
\begin{aligned}
SU(12) &\Longrightarrow SU(6)\times SU(6)\times U(1)\\
12 &= (6,1,-B)+(1,\bar{6},B)\\
66 &= (15,1,-2B)+(1,\bar{15},2B)+(6,\bar{6},0)\\
78 &= (21,1,-2B)+(1,\bar{21},2B)+(6,\bar{6},0)\\
143 &= (35,1,0)+(\bar{6},\bar{6},2B)+(6,6,-2B)+\\
&\quad (1,1,0)+(1,35,0)\\
220 &= (20,1,-3B)+(1,\bar{20},3B)+(6,\bar{15},B)+\\
&\quad (15,\bar{6},-B)\\
495 &= (15,1,-4B)+(20,\bar{6},-2B)+(15,\bar{15},0)+\\
&\quad (6,\bar{20},2B)+(1,\bar{15},4B)\\
792 &= (\bar{6},1,-5B)+(15,\bar{6},-3B)+(20,\bar{15},-B)+\\
&\quad (15,\bar{20},B)+(6,\bar{15},3B)+(1,6,5B)\\
572 &= (70,1,-3B)+(15,\bar{6},-B)+(6,\bar{15},B)+\\
&\quad (21,\bar{6},-B)+(6,\bar{21},B)+(1,\bar{70},3B)\\
4212 &= (189,1,0)+(15,15,-4B)+(6,6,-2B)+\\
&\quad (84,6,-2B)+(\bar{15},\bar{15},4B)+(1,35,0)+\\
&\quad (1,189,0)+(\bar{6},\bar{84},2B)+(\bar{84},\bar{6},2B)+\\
&\quad (6,84,-2B)+(1,1,0)+(35,1,0)+\\
&\quad (35,35,0)+(\bar{6},\bar{6},2B)
\end{aligned}
\tag{3.31}
$$

3.4.5 SU(6) Branching Rules

$$
\begin{aligned}
SU(6) &\Longrightarrow SU(3)\times SU(2)\\
6 &= (3,2)\\
15 &= (6,1)+(\bar{3},3)\\
20 &= (1,4)+(8,2)\\
21 &= (\bar{3},1)+(6,3)
\end{aligned}
$$

$$
\begin{aligned}
35 &= (1,3)+(8,1)+(8,3) \\
70 &= (1,2)+(8,4)+(8,2) \\
&\quad (10,2)
\end{aligned}
\tag{3.32}
$$

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Chapter 4

Evolution of the Yukawa Couplings of MSSM

Minimal Supersymmetric Standard Model(MSSM) is arguably the most promising extension of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ Standard Model. The model introduces one superpartner to all the fermionic scalar and gauge fields and demands invariance of the Lagrangian under the supersymmetric transformations of the fields. The most interesting feature of the model is that due to the cancellation of infinities between scalar and fermion loops the model does not suffer from the gauge hierarchy problem. Presently much interest is generated in this model as it leads to successful gauge coupling unification when the supersymmetry breaking scale is around 1 TeV[1]. On the other hand the model introduces more free parameters like the masses of the superpertners and the quantity $tan\beta$ defined as the ratio of the vacuum expectation values of the the two Higgs scalars present to give masses to the up and down type quarks respectively.

Recently a lot of effort has gone into trying to constrain these free parameters of MSSM by embedding into a grand unified framework. Particularly in one approach [10] one assumes a GUT group of SO(10), E_6 or and at the same time assumes Georgi-Jarlskog[9] form of mass matrices at the unification scale. In a second approach [7] it is assumed that at the unification scale $y_b = y_\tau$ but y_t does not have to satisfy such an unification condition. In a complete two loop analysis it is shown that in this case $tan\beta$ can have two solutions one of which is substantially larger than the other. In yet another approach [8] it is assumed that in a SO(10) GUT framework the third family fermions get mass from the operator $\mathbf{16} \times \mathbf{16} \times \mathbf{10}$ and hence at the unification scale $y_b = y_\tau = y_t$. In this approach one gets a large value of $tan\beta$.

Our aim is to find a lower bound on the quantity $tan\beta = \frac{v_2}{v_1}$ without imposing any specific boundary condition on the Yukawa couplings at the GUT scale. To do that we first note that the Yukawa coupling $y_i(m_t)$ s are related to the corresponding fermion masses $m_i(m_t)$ and $tan\beta$ (See the exact relation below). Where we have called the top quark mass as m_t. Now let us consider the specific case of the coupling y_t. By solving the Renormalization Group (RG) equations of the Yukawa couplings one can find out the maximum value of the top quark Yukawa coupling at the scale m_t for which the top Yukawa coupling will remain perturbative in the entire range upto M_U which is the unification scale of the gauge couplings (From the proton decay experiments we know that the lifetime of proton is more than 10^{32} years which puts lower bound on the scale M_U. We take $M_U = 2 \times 10^{16}$ GeV[2]). This will give an upper bound on y_t. Now for fixed value of m_t this will give a lower bound on $tan\beta$. Hence now by varying m_t in the entire parameter space of interest one gets an absolute lower bound of the quantity $tan\beta$.

Our starting point is the following Lagrangian

$$\mathcal{L}_{int} = \sqrt{4\pi y_\tau}\ (L_3 H_1 \bar{E}_3)_F + \sqrt{4\pi y_b}\ (Q_3 H_1 \bar{D}_3)_F + \sqrt{4\pi y_t}\ (U_3 \bar{H}_2 Q_3)_F \qquad (4.1)$$

Where y_τ,y_t and y_b are tau lepton top quark and bottom quark Yukawa couplings respectively. Whereas Q_3, $\bar{D}_3$, $\bar{U}_3$, $\bar{E}_3$, L_3 are the chiral fields and the subscript signifies the generation to which they belong. H_1 and H_2 signifies the scalar fields which couple to down and up type quarks respectively. We describe the transformation properties and the anomalous dimensions of the fields in the Table 4.1. α_i is defined as $\frac{g_i^2}{4\pi}$, where g_i s are the gauge couplings.

Field	*Quantum number*	*Anomalous Dimension*
L_3	$(1,2,-\frac{1}{2})$	$\frac{1}{4\pi}[y_\tau - \frac{3}{2}\alpha_2 - \frac{3}{10}\alpha_y]$
$\bar{E}_3$	$(1,1,1)$	$\frac{1}{4\pi}[2y_\tau - \frac{6}{5}\alpha_y]$
$\bar{D}_3$	$(\bar{3},1,\frac{1}{3})$	$\frac{1}{4\pi}[2y_b - \frac{8}{3}\alpha_3 - \frac{4}{30}\alpha_y]$
$\bar{U}_3$	$(\bar{3},1,-\frac{2}{3})$	$\frac{1}{4\pi}[2y_t - \frac{8}{3}\alpha_3 - \frac{8}{15}\alpha_y]$
Q_3	$(3,2,\frac{1}{6})$	$\frac{1}{4\pi}[y_t + y_b - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{1}{30}\alpha_y]$
H_1	$(1,2,-\frac{1}{2})$	$\frac{1}{4\pi}[y_\tau + 3y_b - \frac{3}{2}\alpha_2 - \frac{3}{10}\alpha_y]$
H_2	$(1,2,\frac{1}{2})$	$\frac{1}{4\pi}[y_t - \frac{3}{2}\alpha_2 - \frac{3}{10}\alpha_y]$

Table 4.1: Transformation properties and anomalous dimensions

From the anomalous dimensions one can immediately write down the evolution equations of the Yukawa couplings. The variable t is defined as $t = \frac{1}{2\pi}\ ln\ \mu$ (GeV) [5].

$$\frac{\partial \alpha_y}{\partial t} = \alpha_y^2[2n_f + \frac{3}{5}] \qquad (4.2)$$

$$\frac{\partial \alpha_2}{\partial t} = \alpha_2^2[2n_f - 6 + 1] \qquad (4.3)$$

$$\frac{\partial \alpha_3}{\partial t} = \alpha_3^2[2n_f - 9] \qquad (4.4)$$

$$\frac{\partial y_\tau}{\partial t} = y_\tau[\gamma_{L_3} + \gamma_{H_1} + \gamma_{\bar{E}_3}] = y_\tau[4y_\tau + 3y_b - 3\alpha_2 - \frac{9}{5}\alpha_y] \qquad (4.5)$$

$$\frac{\partial y_b}{\partial t} = y_b[\gamma_{Q_3} + \gamma_{H_1} + \gamma_{\bar{D}_3}] = y_b[y_\tau + y_t + 6y_b - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{17}{15}\alpha_y] \qquad (4.6)$$

$$\frac{\partial y_t}{\partial t} = y_t[\gamma_{U_3} + \gamma_{H_2} + \gamma_{\bar{Q}_3}] = y_t[6y_t + y_b - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{15}\alpha_y] \qquad (4.7)$$

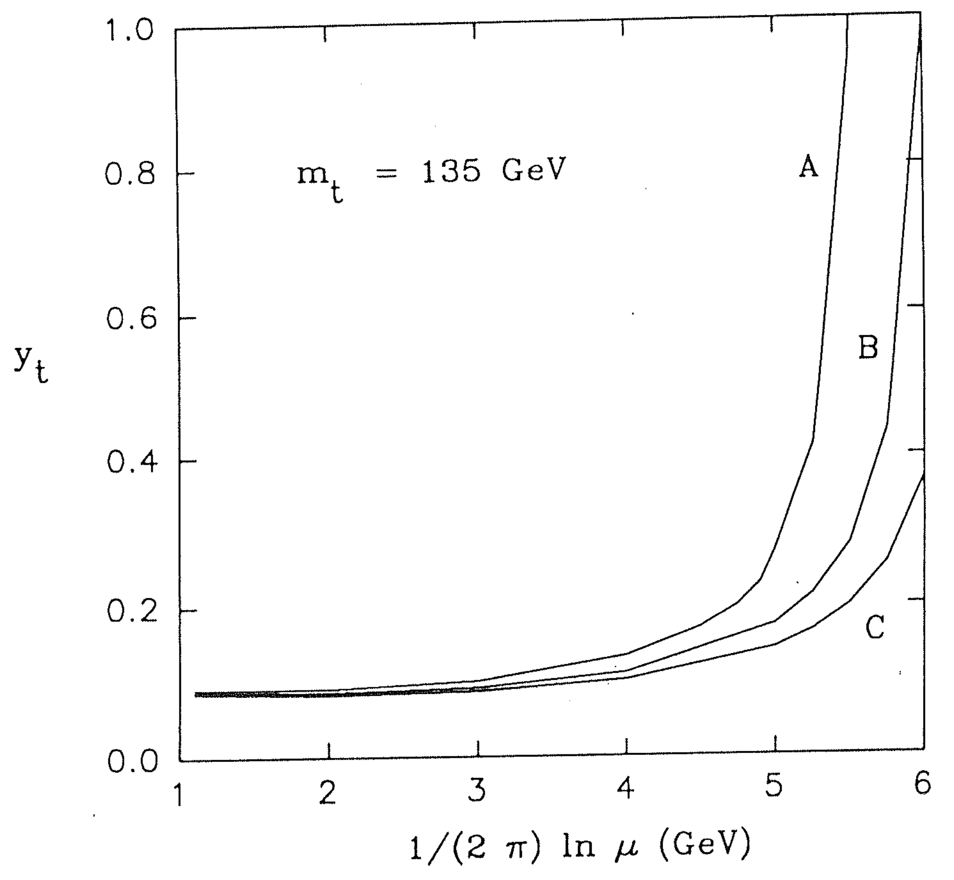

FIGURE 5

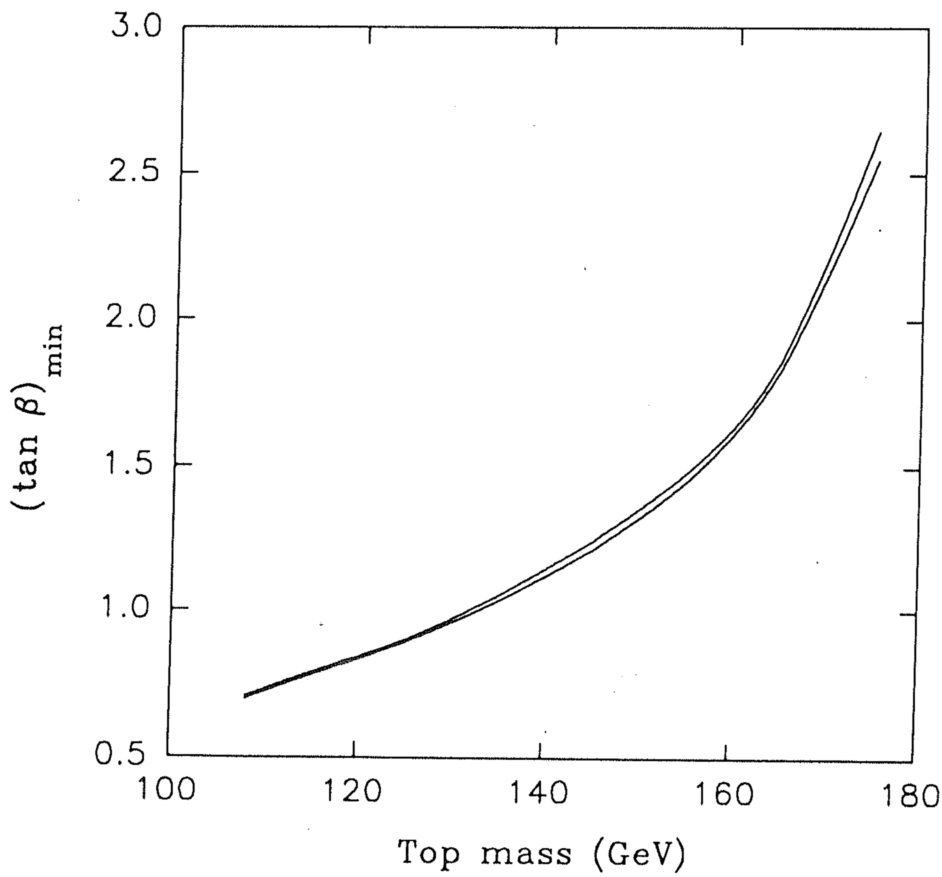

FIGURE 6

While solving this set of differential equations we have made the following inputs of experimental numbers.

$$\sqrt{4\pi y_t(m_t)} = \frac{m_t(m_t)\sqrt{1+tan^2\beta}}{174\tan\beta} \tag{4.8}$$

$$\sqrt{4\pi y_b(m_t)} = \frac{m_b(m_b)\sqrt{1+tan^2\beta}}{174\ \eta_b} \tag{4.9}$$

$$\sqrt{4\pi y_\tau(m_t)} = \frac{m_\tau(m_\tau)\sqrt{1+tan^2\beta}}{174} \tag{4.10}$$

η_b takes into account the 3-loop Q.C.D. plus 1-loop Q.E.D. evolution of y_b from the bottom mass energy scale to the top mass energy scale [4]. The value of η_b depends on the value of α_s. we have used

$$\begin{aligned} \alpha_s(M_z) &= 0.123 \pm .004 \\ sin^2\theta(M_z) &= 0.2334 \pm .0008 \\ \frac{1}{\alpha_{em}(M_z)} &= 127 \pm 0.2 \end{aligned}$$

y_τ does not vary much in this interval as tau lepton does not carry color. Once the values of the couplings y_t, y_b and y_τ are specified at the top mass scale they can be evolved to the supersymmetry breaking scale M_{susy} using the non-supersymmetric renormalization group equations [6]. In our calculation we have considered two cases in one case M_{susy} is taken to be 1 TeV and M_{susy} is taken to be m_t in the other. From 1 TeV to M_U we have used the supersymmetric evolution equation which we have described above. We have used $m_b(m_b) = 4.25 \pm 0.15$ GeV and $m_\tau(m_\tau) = 1.777$ GeV. The top quark mass is taken in the range 108[3] to 175 GeV. We have not assumed any unification of the Yukawa couplings at the GUT scale. The minimum value of $tan\beta$ is 0.70 and it is achieved when m_t is minimum that is 108 GeV.In Figure 5 have plotted the evolution of y_t when m_t is 135 GeV. Curve A represents the case when $tan\beta$ is 1.01 which is lower than the lower bound 1.03 (see Table 4.2) for bounds. Hence we see that curve A reaches the nonperturbative region earlier than the scale M_U. On the other hand curve C which represents $tan\beta = 1.05$ becomes nonperturbative after the scale M_U. Curve B is obtained when $tan\beta = 1.03$. The variation of the lower bound with respect to m_t is plotted in Figure 6.

To conclude we have asked the question that "what is the minimum value of $tan\beta$ that can be achieved without assuming any specific boundary conditions on the Yukawa couplings at the GUT scale?". We have assumed that there is a perturbative supersymmetric theory upto the scale of 2×10^{16} GeV though we have not assumed any specific model of grand unification. We have seen that the requirement that all the Yukawa couplings should be in the perturbative domain upto M_U forces $tan\beta$ to be atleast 0.70 for $m_t = 108$ GeV. This lower limit rises with higher values of m_t. We have checked that if we have $M_{susy} = m_t$ the lower bound does not vary much rather it stays at 0.71 for $m_t = 108$ GeV. In Figure 6 the upper curve is for the case when $M_{susy} = m_t$. We have also checked that the lower bound remains insensitive to the variation the bottom quark mass in the range 4.10 to 4.40 GeV. It is interesting to note that evenif we increase M_{susy} upto 10 TeV the lower bound on $tan\beta$ still remains just above 0.71 when m_t is 108 GeV and for other values of m_t it remains just above the lower bound for $M_{susy} = m_t$ case. As $tan\beta$ is a free parameter in the MSSM we consider such a bound

m_t	$M_{susy} = 1TeV$	$M_{susy} = m_t$
108	0.70	0.71
115	0.78	0.79
125	0.89	0.90
135	1.03	1.05
145	1.21	1.24
155	1.45	1.48
165	1.84	1.87
175	2.25	2.65

Table 4.2: Lower Bounds on $tan\beta$ for $M_{susy} = 1$ TeV and $M_{susy} = m_t$

as important. As a practical example it will have important implications in the search of supersymmetric Higgs bosons in colliders[7].

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Chapter 5

Conclusions

To conclude, the works represented in this thesis may be divided into three parts. In the first part we have studied the constraints imposed by the LEP data on various GUT models. In the second part we have considered the new paradigm of low energy unification and found various interesting results. In the third part we have worked out some constraints on the parameter space of MSSM using the evolution of Yukawa couplings. In the following we summarize this three parts.

5.1 LEP Constraints on Unified Models

Our analysis shows that by including the effects of higher dimensional operators arising due to quantum gravity or spontaneous compactification of extra spatial dimensions in Kaluza-Klein theories (or due to a group G' which breaks to $SU(5)$ above the unification scale), it is possible to show that the predictions of a minimal $SU(5)$ GUT is in conformity with the latest LEP values of $\sin^2\theta_W$ and α_s, and also with the experimental constraints on proton lifetime.

The most recent experimental data provide very strong constraints on left-right symmetric models. We have shown that if a left-right symmetric group coming from either a grand unified or partially unified group breaks at an intermediate mass scale, M_R, then the tightly constrained values of $\sin^2\theta_W$ and α_s can be used to put a lower bound on the value of M_R. This lower bound is $\approx 10^9$ GeV, irrespective of the unification group. Grand unified theories and partially unified theories, therefore, completely rule out the possibility of seeing the right handed partners of $W^\pm$ at the energies available in current experiments or those planned in the near future. Conversely, the discovery of these particles at such energies can be used to refute unification models. It is of importance to note, however, that our analysis puts no constraints whatsoever on the existence of extra Z at low energies, as an extra $U(1)_R$ can survive down to electroweak breaking scales. The inclusion of the Higgs or supersymmetry increases the lower bound on M_R.

We have studied the non-perturbative unification scenario first proposed by Maiani, Parisi and Petronzio. We point out that the non-supersymmetric version of this scenario is ruled out by LEP data. However, the supersymmetric extension of this scenario remains a viable alternative to conventional grand unified theories and is capable of predicting the precision values of couplings determined from LEP. Our numerical results show that the non-perturbative scale, Λ, at which all couplings are large, is around 0.7–0.8×10^{17} GeV, with the supersymmetric threshold M_s around 1.0–1.4 TeV. If the scale M_s gets either larger or smaller it is then not possible to reproduce the values of the couplings at M_Z. We should note that the agreement with the data is obtained only for a constrained range of parameters of this scenario. In principle, the effect of higher-order corrections could be large and this may ruin the agreement. It is also likely that more accurate measurements of the strong coupling α_3 at low energies may be sufficient to either put strong constraints or completely rule out this scenario. It is nevertheless interesting that this scenario, at the two-loop level, is a possible alternative to conventional grand unification.

5.2 Studies on Low Energy Unification

We have shown that Higgs fields play a significant role in the evolution of gauge coupling constants in GUTs where baryon number is a symmetry. The consistency of the symmetry breaking scenario presented here with present-day proton decay data along with its interesting TeV scale physics make SU(15) GUT a model worthy of further investigation. The most interesting pattern is {3467} (see section 3.1), which has both low energy unification at $\sim 10^9$ GeV and interesting TeV physics. We can decouple the electroweak breaking scale with the other symmetry breakings and have TeV scale chiral color symmetry and the quark-lepton un-unified electroweak symmetry breaking, which will raise the unification scale a little. The existence of chiral color symmetry at the TeV scale or lower will imply the presence of axigluons, whose phenomenological consequences have been studied in the literature. The presence of the un-unified electroweak symmetry at low energy will imply the existence of extra charged and neutral gauge bosons, whose mixing with the Z-boson will affect various asymmetry parameters in the e^+e^- deep-inelastic scattering.

The scenario of symmetry breaking in nonsupersymmetric SU(15) GUT, which allows low energy unification, has some interesting features. It is essential for the low energy unification to have chiral color $SU(3)_{cL}\otimes SU(3)_{cR}$ group and the quark-lepton ununified group $SU(2)^q_L\otimes SU(2)^l_L$ survive till very low energy, for the gauge coupling constants to evolve very fast and get united at an energy scale around 10^8 GeV. Thus the existence of these groups and the leptoquarks are some of the essential critereons of the low energy unification, which can be tested in the laboratory in near future. Thus any signatures of these groups may seriously question the existence of supersymmetry and if the signatures of the low energy unification and also that of supersymmetry are found, then it will cast a serious question on our understanding of the grand unification scenario.

We have seen that there exists one possible breaking chain in a Grand Unified Theory based on the group $SU(16)$ where a unification scale of the order of 10^{11} GeV is possible. There exists a very low energy scale (M_B) which may be almost anywhere between the unification scale and the electroweak scale where completely ununified symmetry of quarks and leptons may exist together with chiral color symmetry. The scale M_B comes lower when the separation between the scale M_A and the scale M_B is increased. Qualitatively we understand it in the following way. The beta function coefficients can be looked into as the slope of the lines if one plots the inverse coupling constants with respect to energy. It can be easily checked that as at the $SU(16)$ level all the fermions transform under the fundamental representation of the group and in the other levels they transform in a more complicated way under the various groups in the intermediate stages all the groups cannot be normalized in the same way. To compensate for the mismatch in the normalizations the beta function coefficients has to be multiplied by appropriate factors. Due to that the slope of the curves representing the inverse couplings also gets multiplied by the appropriate factors and the couplings get united earlier giving rise to low energy unification. We have also seen that this model satisfies the experimental constraints coming from proton decay experiments in the sense that proton decay is suppressed. We have shown that there exists atleast one choice of the Higgs sector where there is no Higgs mediated proton decay either. For some specific choice of the Higgs fields there may exist interesting phisical consequences like the $N - \bar{N}$ oscillation. There is also the possibility of having the sea-saw mechanism to give Majorana mass to the neutrinos and this also may have observable consequences. Last but not the least we emphasize again that there exists very rich low energy physics coming from this model hence keeping in mind the forthcoming high-energy experiments at SSC,LHC and other places this model is worthy of further investigation.

5.3 Evolution of Yukawa Couplings

We have asked the question that "what is the minimum value of $tan\beta$ that can be achieved without assuming any specific boundary conditions on the Yukawa couplings at the GUT scale?". We have assumed that there is a perturbative supersymmetric theory upto the scale of 2×10^{16} GeV though we have not assumed any specific model of grand unification. We have seen that the requirement that all the Yukawa couplings should be in the perturbative domain upto M_U forces $tan\beta$ to be atleast 0.70 for $m_t = 108$ GeV. This lower limit rises with higher values of m_t. We have checked that if we have $M_{susy} = m_t$ the lower bound does not vary much. It stays at 0.71 for $m_t = 108$ GeV. In Figure 6 the upper curve is for the case when $M_{susy} = m_t$. We have also checked that the lower bound remains insensitive to the variation the bottom quark mass in the range 4.10 to 4.40 GeV. It is interesting to note that evenif we increase M_{susy} upto 10 TeV the lower bound on $tan\beta$ still remains just above 0.71 when m_t is 108 GeV and for other values of m_t it remains just above the lower bound for $M_{susy} = m_t$ case. As $tan\beta$ is a free parameter in the MSSM we consider such a bound as important. As a practical example it will have important implications in the search of supersymmetric Higgs bosons in colliders.