STRUCTURE OF MAGNETO FLUID DISK AROUND A COMPACT OBJECT

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by

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CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institution.

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Summary of the thesis

Accretion disks are the most popular models to explain high luminosity (10^{36} $ergs - s^{-1}$ to 10^{47} $ergs - s^{-1}$) objects like quasars, X-ray binaries, AGNs etc. Disks can be thin or thick depending on their geometrical shapes. More quantitatively, if $(h/r) \simeq c_s/v_{virial} \ll 1$ the disk is considered to be thin. Here h is the scale height normal to the orbital plane, r is the radial distance from the center of the central star, and c_s is the internal sound speed. On the other hand, if the internal pressure builds up so that $c_s \simeq (GM/r)^{1/2}$, the disks become geometrically thick, with $h \simeq r$. Thin disk models are well studied and using these models many observable quantities can be calculated. Such models are based fairly on observational basis. On the contrary, the study of thick accretion disks is not well developed.

In this thesis, we have studied the structure and stability of a pressure-supported, magnetized thick non-accreting disk equilibria. Such studies are important due to follwing reasons:

(i) The study of thick disks is important as such structures may presumably be formed in nature, e.g., around AGNs and protostars. Thin disk can go to thick disk configuration when the infalling gas is hot or gets heated up by electromagnetic waves or radiation pressure. Such studies are also useful from theoretical point of view, as they can give better insight into the thin disk model approximations and allows one to treat intermediate cases.

(ii) However, the study of thick disk equilibrium, in general, is extremely complex and far from complete. There remain many uncertainties pertaining to their structure and stability. For example, the radiation pressure-supported thick disk is subjected to various kinds of powerful instabilities and therefore, the existence of steady state radiation supported thick disks is doubtful. However, there is another possibility of thick disk, e.g., ion-supported tori which may not be subjected to these kind of instabilities. The detailed study of such models is not carried out so far.

(iii) Magnetic fields may have significant dynamical effects in the disk environment

depending on the concrete physical situations. For example, they can generate and collimate jets, removing angular momentum from the disk in the form winds, they can be source of coronal heating, they can also influence spin-up and spin-down rates of the central star. In fact, it is found that the large scale ordered magnetic field may exert a powerful torque onto the central star and which in turn changes its period.

(iv) Though it is a general belief that strong gravitational field of the central object may not influence the physics of accretion disks, there are some interesting effects related with the presence of both strong gravity and magnetic field. For example, the strong magnetic field in the Schwarzschild geometry can bring the inner edge of the disk arbitrarily close to the surface of the compact object, it can also influence many electromagnetic processes like pair-production near the vicinity of the star's surface.
Therefore, we have considered a pressure-supported, magnetized disk equilibria without invoking any thin or thick disk approximations. We use ideal magnetohydrodynamic (MHD) frame work to describe the disk equilibria. The disk is usually believed to be filled with plasma or with a highly ionized gas. The macroscopic behaviour of such a state can be analyzed by the MHD approximation since the Larmor radii of ionized species are much smaller than the size of the disk. Moreover, molecular and magnetic Reynold numbers are usually very large for a typical accretion disk scenario ranging from 10^{14} for white dwarf disk to 10^{26} for an AGN disk. Therefore, molecular viscosity and resistivity can be neglected. The effects of turbulance or dissipative forces such as anomalous viscosity and anomalos resistivity is regarded small compared to the long ranged order equilibrium forces. Such effects may be introduced perturbatively over the equilibrium force balance.

The general study of thick accretion disk is very difficult. To begin with, we have considered that the disk is having only azimuthal motion and therefore, the corresponding equilibria considered by us are non-accreting. Generally, in accretion disk scenario, the radial inflow velocity is much less than the azimuthal velocity. Thus, the solutions describing the accretion may be constructed by incorporating radial velocity perturbatively.

Though we have considered the disk dynamics within the ideal MHD framework, the external magnetic field of the central object considered to be penetrating inside the disk. From the studies of well known thin magnetically threaded disk models, it is conjectured that external field can penetrate into the disk *via* non-linear processes like Kelvin-Helmholtz instability, turbulent dissipation etc. In our work, we have used the basic set of covariant form of general relativistic magnetohydrodynamic equations which govern the flow of the stationary (i.e. $\partial_t \equiv 0$), axisymmetric (i.e. $\partial_\varphi \equiv 0$) pressure-supported magnetized fluid disk in a state of azimuthal motion only with the following assumptions: The self gravity of the disk is negligible. The geometry of the spacetime is described by the Schwarzschild metric and the electromagnetic fields do not modify the spacetime structure.

Under the above mentioned assumptions, We have obtained two classes of solutions: (i) when the MHD flow velocity becomes quasi-Keplerian in the flat space limit and (ii) when the MHD flow velocity becomes rigid rotation type (with the star) in the flat space limit. In the Newtonian limit, the solution having a quasi-Keplerian azimuthal velocity are found to depend upon two parameters α and β. The parameter α signifies the ratio of the gravitational potential energy to the bulk kinetic energy of a fluid element. The parameter β signifies the ratio of the toroidal magnetic field strength to poloidal magnetic field strength. Analyses of the pressure profile show that the equilibrium solutions are physically plausible for certain values of toroidal magnetic field allowed by the inequality relation satisfied by α and β. Furthermore, these kind of solutions support non-barotropicity. In the general relativistic case the above solutions indicate that the strength of the toroidal magnetic field decreases near the surface of the compact object. Analyses of magnetic field line topology show that toroidal field generates very high shear and thus, indicate that this kind of equilibrium might be unstable.

The solutions having rigid rotation type velocity profile are found to be less sensitive to the parameters α and β. The pressure profiles show that these solutions support a barotropic equilibrium in the asymptotic limt. But, the barotropicity may be violated very near to the compact object because of the strong gravity. Also the effect of strong gravity can cause a departure from the rigid rotation type behaviour of the velocity profile.

In closing, the importance of the solution in the context of a neutron star is discussed.

Moreover, the discussion of future work based on the obtained solutions is provided in the concluding chapter of the thesis.

Research Publications

1. Structure of a fluid disk around a magnetized compact object in the presence of self-consistent toroidal magnetic field, D. Banerjee, J. R. Bhatt, A. C. Das, and A. R. Prasanna, 1995, *Astrophys. J.*, **449**, 789-799.

2. Axisymmetric magnetohydrodynamic equilibrium around a magnetized compact object, D. Banerjee, J. R. Bhatt, A. C. Das, and A. R. Prasanna, 1997, *Astrophys. J.*, **472**, 1st January issue.

Chapter 1

Introduction

Capture of ambient matter gravitationally by compact objects is called accretion. When the matter is falling onto the surface of the compact object either directly or through spiraling-in process, due to viscous dissipation most of the energy of the ordered bulk motion is converted into its internal energy which in turn being converted into electromagnetic radiation. However, the accretion depends mainly on the nature of the effective potential which the flow experiences near the central compact object. The effective potential of a rotating gas with specific angular momentum λ in the presence of a Newtonian star is given by

$$\phi_N(r) = -\frac{1}{r} + \frac{1}{2}\frac{\lambda^2}{r^2}. \tag{1.1}$$

For small r, the rotational term causes the potential to diverge to positive infinity at the origin at $r = 0$. It can be easily shown that only those matter, around a Newtonian star, with angular momentum less than the Keplerian value on the surface of the star

$$l_{Kep,N} = R^{1/2} \tag{1.2}$$

can accrete. For the case of black hole accretion, the effective potential of the fluid around a non-rotating black hole (Schwarzschild solution), in the units where the gravitational constant G, the central mass M and the velocity of light c are all unity ($G = M = c = 1$), is given by (Chakrabarti, 1996)

$$\phi_S(r) = \left[\frac{1 - 2/r}{1 + \frac{(1-2/r)\,l^2}{r^2}}\right]^{1/2} \tag{1.3}$$

If the specific angular momentum of the flow is less than that of the marginally stable value (i.e., $\tilde{l} < l_{ms}$), the flow can fall onto the black hole without any barrier. If the angular momentum is higher than the marginally bound value (i.e., $\tilde{l} > l_{mb}$), the potential barrier is higher than 1 - the rest mass of the particle. It tells that matter must have significant energy to begin with for the accretion to take place. Thus, matter must have significant radial velocity or thermal energy at a large distance. In any case, since the potential turns around and passes through zero, matter can always be made to accrete when pushed sufficiently hard. Matter having angular momentum between the marginally bound and marginally stable values would form accretion disks.

One of the important difference between the fluid dynamics around a black hole and that around a Newtonian star is as follows : As the matter accretes on a Newtonian star, it can hit the surface of the star subsonically (i.e., when the radial velocity of matter is less than the adiabatic sound speed) or supersonically (i.e., when the radial velocity of matter is greater than the adiabatic sound speed) depending on the location of the star surface. But in case of black hole accretion, the flow is supersonic on the horizon since the velocity of sound (even for the steepest equation of state) is less than the flow velocity on the horizon, which is the velocity of light. Hence, it is evident that the black hole accretion is necessarily transonic.

Following order-of-magnitude estimates show the role of gravitational potential energy in accretion process.

Suppose that the matter falls freely onto the surface of the star of mass M_x and radius R_x so that a unit mass has a kinetic energy $G\,M_x\,/\,R_x$. If the amount of matter falling per unit time on the surface is $\dot{M}$, then the total luminosity of the accreting object is given by (Lipunov, 1992)

$$L_{acc.} = \dot{M}\,\frac{G\,M_x}{R_x} \equiv \eta\,\dot{M}\,c^2\,; \tag{1.4}$$

where, $\dot{M}$ being the accretion rate and η being the accretion efficiency (i.e., efficiency of conversion of matter into energy). One can write

$$\eta = \frac{1}{2}\,\frac{R_g}{R_x}\,; \tag{1.5}$$

where, R_g is the gravitational radius defined as $R_g = \dfrac{2\,G\,M_x}{c^2}$.

For a neutron star, $R_x = 10\,km.$ and $R_g = 3\,km.$, which means that $\eta = 10\,\%$. This is about 100 times the efficiency of nuclear fusion reactions. Thus, accretion is a process that can be considerably more efficient as a cosmic energy source and could thus play a central role in understanding the central engine which is widely believed to power most luminous objects, for which the nuclear source of energy of the stars are wholly inadequate.

It is evident from the eq. (1.5) that the efficiency of accretion as an energy generation mechanism is strongly dependent on the compactness of the central accreting object. Furthermore, for a fixed value of the compactness, the luminosity of an accreting system depends on the mass accreting rate (i.e. $\dot{M}$). In case of high luminosity scenarios, the accretion rate may be controlled by the transfer of momentum outwardly from the radiation to the accreting material by scattering and absorption. Under certain circumstances, there can be a maximum luminosity for a given mass, usually referred to as the Eddington luminosity which can be obtained with the following assumptions : Accretion flow is assumed to be steady and spherically symmetric. Accreting material is assumed to be mainly hydrogen and to be fully ionized. In these situations, Eddington luminosity is defined for a spherically symmetric Newtonian star of mass $M = 1\,M_\odot$ as :

$$\begin{aligned} L_{Edd.} &= 4\,\pi\,G\,M_x\,m_p\,c\,/\,\sigma_s \\ &\cong 1.3\,\times\,10^{38}\,(M_x\,/\,M_\odot)\,erg\,s^{-1} \end{aligned} \tag{1.6}$$

where, m_p is the proton mass, σ_s is the Thompson scattering cross-section, and $M_\odot$ denotes the solar mass. Though it is a Newtonian concept and the computation is done using spherical geometry, the definition is generally used unchanged in measuring luminosity of accreting matter around a black hole. For black hole accretion, the Eddington luminosity can go up to $10^{47}\ \ erg\ s^{-1}$. The luminosity of the quasars and active galaxies have indeed been observed to be sometimes as high as $10^{47}\,erg\,s^{-1}$ and the usual explanation for such a high energy output is that the energy is mostly coming from the gravitational binding energy of matter accreting onto a massive black hole with $M_x \sim 10^9\,M_\odot$.

The physics of accretion flow onto a compact star and the emitted radiation pattern are, in general, extremely difficult and intricate in nature. Flow behavior depends on many physical parameters like flow geometry, for e.g., if the fluid possess intrinsic angular momentum, the flow will be either two- or three-dimensional, depending upon the flow symmetry. In simple cases, the flow pattern may be spherical when there is no mean motion of the fluid far from the stationary compact star or disk-like as in axisymmetric flow of the fluid with intrinsic angular momentum. Secondly it depends on dominant heating and cooling mechanisms that characterize the accreting plasma. If the fluid is optically thick to emitted radiation, the net heating and cooling rates depend on the radiation field which is to be evaluated self-consistently. Third, the magnetic field play a dominant role in controlling the plasma motion near the surface of the magnetized compact objects. Fourth, the effect of radiation pressure which influences the geometry of the flow pattern near the compact source. Finally, one has to understand the flow boundary conditions both at large distances where the fluid " joins on " to the ambient medium, and at the stellar surface, where the fluid merges into the star. The study of gas dynamic flow of matter near central compact objects were first started by Hoyle, Bondi, and Mac Crea in connection with the problem of interaction of ordinary stars with interstellar matter.

Generally there are two important kinds of accretion processes are being studied in detail and are frequently being realized in practical model building. They are (i) Spherically symmetric accretion where the accreting star practically does not move relative to the medium: $v_\infty \ll a_\infty$ and the matter constituting the medium does not possess any significant angular momentum and (ii) Disk accretion where the total angular momentum of matter is sufficient to form an accretion disk, around the central compact object.

1.1 Spherical Accretion

Though the process of accretion as a source of energetics gained popularity in the seventies, its importance was earlier recognized in the context of cosmology nearly five decades ago. Bondi and Hoyle (1944) studied the physics of stationary spherical accretion of gas (at rest at infinity) onto a spherical Newtonian star with the accreting gas having a polytropic equation of state (i.e., $P = K\rho^{\gamma}$, where K is a constant measuring the entropy of the flow, and γ is the adiabatic index which is the ratio of the specific heats and is assumed to be constant throughout the flow). In terms of the density of the flow at infinity ρ_{∞}, the Bondi mass flux $\dot{M}$ is given as (Bondi, 1952),

$$\dot{M}_c = \frac{1}{4}\frac{\rho_{\infty}}{a_{\infty}{}^3}\left(\frac{n}{n - 3/2}\right)^{n - 3/2} \tag{1.7}$$

This above equation shows that accretion from the interstellar medium is unlikely to be an observable phenomenon; reasonable values would be $a_{\infty} = 10\ Km\ s^{-1}$, $\rho_{\infty} = 10^{-24}\ g\ cm^{-3}$, corresponding to a temperature of about $10^4\ K$ and number density near 1 particle cm^{-3} give mass accretion rate of $10^{11}\ \ g\ s^{-1}$. This mass accretion rate on to a neutron star yields L_{acc} only of the order $2 \times 10^{31}\ erg\ s^{-1}$; at a typical distance of 1 *kpc* this gives far too low a flux to be detected. Bondi also pointed out that the star's gravitational pull seriously influences the flow behavior of the gas only inside the accretion radius $R_{acc.}$ and inflow velocity must become supersonic near the stellar surface. Thus, in general, the efficiency of the spherical accretion solutions were not found to be sufficiently high to explain the high observed luminosity from active galaxies. Furthermore, this kind of accretion process cannot reproduce the observed bump in the ultraviolet in the continuum. Therefore, it is very much essential to look for more realistic accretion flows which posses angular momentum and magnetic field.

1.2 Accretion Disk

In most of the astrophysical situations, the infalling matter would have sufficiently high angular momentum and thus form a disk. In the previous section we have discussed the properties of spherically symmetric accretion flows. In these flows, the infall velocity was very high and therefore, for a given accretion rate, the density was very low. Thus, these flows were found to be inefficient in converting the gravitational potential energy of the infalling matter into radiation. But, if the flow has high angular momentum, the inflow velocity becomes very small and the density becomes much higher. As a result the infall time becomes higher and hence, viscosity has time to dissipate angular momentum (except in regions very close to the black hole) and energy. As matter loses its angular momentum, it sinks deeper into the potential well of the central compact object and radiates more efficiently. For example, the accretion disk around a Schwarzschild black hole can radiate up to six percent of the gravitational potential energy of the infalling matter as compared to its rest mass energy and in case of Kerr black hole, the efficiency can go up to forty percent depending upon the rotation parameter. However, the actual efficiency depends on quantities such as viscosity parameter and the cooling process inside the disk. This energy is released in the entire electromagnetic spectrum and the success of the disk model on its ability to describe the way this energy is distributed in various frequency bands. The temperature and density distributions as well as the geometrical shape of the disk governs the nature of the emerging radiation spectrum. These, in turn, depend on the outer boundary condition like the rate of matter supply, the specific angular momentum, and energy content of the matter.

1.2.1 Observational evidences of Accretion Disks

It was first mentioned by Zel'dovich (1964) that the most favorable condition for accretion to a relativistic star is encountered in the case when this star forms a pair with a normal star. This assumption is in excellent agreement with observations. With the capability to observe from space-based telescopes, binary stellar X-ray sources have been a subject of intensive observational and theoretical study during the past two decades. There are more than 170 sources of this type in the Galaxy with the majority of them consisting of a neutron star and a relatively unevolved companion. Of these, about 100 systems have companions of high mass ($M \gtrsim 10\,M_\odot$), and the remainder have stellar companions of low mass ($\lesssim 2\,M_\odot$). For review of the observational data and the interpretation of these X-ray sources see the article by Nagase (1989) for the high-mass systems and by Lewin, van Paradijs, and Taam (1993) for low-mass systems. Though X-rays had been observed in some discrete sources in the pre-satellite days, the satellite astronomy brought in a rich haul of observations and particularly the UHURU satellite discovered the sources Cen X-3, Her X-1 (Giacconi, *et al.*, 1971; Schreier *et al.*, 1972; Tananbaum *et al.*, 1972) and with a balloon-borne instrument (Lewin *et al.*, 1971). These objects exhibiting eclipses and periodic Doppler variations of the pulsations which really confirmed the speculation that close binaries could be X-ray sources as a result of mass accretion (Hayakawa and Matsuoka, 1964). However, the most dramatic observations concerning X-ray sources came in 1985 with the discovery of quasi-periodic oscillations (QPO) in X-ray binaries (Van der Klis *et al.*, 1985), a phenomena which still eludes explanation. It was almost immediately recognized that the source of energy to drive the X-ray pulsations was accretion of matter onto the magnetic polar caps of a rotating neutron star. The accreted matter is transferred to the neutron star from a relatively normal binary companion. Furthermore, SAS-3 X-ray astronomy satellite and *Hakucho* satellite discovered X-ray bursts (see review Joss and Rappaport, 1984). There is now persuasive evidence that these sources, too, are neutron stars in close binary system with relatively weak surface magnetic field ($\ll 10^{12}\ G$). As weak field is unable to direct the flow of the accreting matter it leads to no X-ray pulsation. Therefore, the lack of funneling of the accretion flow alters the properties of the neutron-star surface layers in such a way that the freshly accreted matter may undergo strong thermonuclear flashes. It is these flashes that result in the emission of X-ray bursts. Recent observations have provided considerable insight into the structure of the accretion disk, an issue intimately linked to the details of the mechanism that precesses the disk. In the context of compact stars in the binary systems; for example, the $35-$day X-ray modulation of Hercules X-1 is widely attributed to disk precession, and a variety of other X-ray sources are now known to have long-term periods probably also due to disk precession (Lang *et al.*, 1981; Priedhorsky and Terrell, 1984). In the case of SS 433, the amplitude and shape of the 164-day photometric component of the light curve indicate that the disk is present and it is remarkably thick (Anderson *et al.*, 1983a,b; Bochkarev *et al.*, 1980); a disk half-thickness to radius ratio of $2/3$ has been suggested. This conclusion is supported by an entirely independent observation, namely the existence of the 6.3-day nodding motion (Katz *et al.*, 1982). The disk must be extremely viscous to rapidly transmit the nodding from the outside where significant torque is applied to the interior, where the jets originate and are seen to respond to the motion (Katz, 1980). Again, the existence of nodding motion provides independent confirmation of the extent of the disk (i.e., with dimensions of order $10^{12}\ cm$), because the torque can be significant only on a rather large structure.

Furthermore, the cataclysmic variables (i.e. a binary having a white dwarf with a companion star with low mass ($\sim\ 0.1\,M_\odot$ to $\sim\ 1\,M_\odot$) and close to the lower main sequence) show strong emission lines due to the phasing of their periodic red- and blue shifts and can be strongly identified as coming from the vicinity of the accreting component of the binary system. In this scenario, the emission lines are found to be double peaked with extensive wings which is expected for line emission from a rotating disk of optically thin gas. The extensive (broad) wings are due to the Keplerian motion of the gas. Similar to the line emissions seen in Cataclysmic variables, one would expect that if the accretion disk around a black hole also emits lines, they should show double horned patterns (Smak, 1980; Horne and Marsh, 1986). If there is a non-axisymmetric feature on the disk, the double horned pattern need not be symmetric (Chakrabarti and Wiita, 1994; Bunk *et al.*, 1990).

In the context of AGNs, a dominant feature of the continuum spectrum is a hump in the spectrum near blue region known as the 'big blue bump'. This emission feature is observed in radio-weak sources as well as in many radio-loud quasars. In galactic black hole candidates, the spectra in the soft-state also has the bump in soft X-rays. The 'big blue bump' in the spectrum of many QSOs as well as the soft X-ray bump in galactic black hole candidates are usually attributed to the quasi-thermal emission from accretion disks (Cowley, 1992; Shields, 1978; Malkan, 1983, and Malkan and Sargent, 1982) and good fits to this part of the spectrum can be obtained from standard, optically thick but geometrically thin 'α' disk models (Sun and Malkan, 1989; Laor, 1990), thick accretion disks (Madau, 1988), and disks incorporating shock waves in the disk (Chakrabarti and Wiita, 1992) among others. Sun and Malkan (1989) fits 60 quasars and AGNs from IR to UV region of the continuum by using standard accretion disk models (Shakura and Sunyaev, 1973; Novikov and Thorne, 1973) around black holes. The relativistic effects of disk inclination, including Doppler boosting, gravitational focusing and gravitational red-shift on the observed spectra for both Kerr and Schwarzschild black holes are considered. Some of the active galactic nuclei, especially those which are radio-quiet, show strong evidence of X-ray emission. *ROSAT* in its all sky survey has identified numerous active galactic nuclei in the soft X-ray range. Based on the analysis of the Ginga data, one of the at least four (model dependent) components has been tentatively assumed to be present (Pounds *et al.*, 1990; Nandra and Pounds, 1994): fluorescent Fe-K emission line at 6.4 keV. The more recent analysis of Ginga data (Titarchuk, 1994) tells that the X-ray is considered to be of thermal origin. Thus, the observation of X-rays may testify to the existence of compact hot source of energetics electrons as well as soft photon source from the underlying pre-shock cool accretion disks. The energetic electrons could also be produced from magnetic corona of the accretion disks (Chakrabarti and D'Silva, 1994; D'Silva and Chakrabarti, 1994).

1.2.2 Formation of Accretion Disk

However, the detailed study of interacting binary systems has revealed the importance of angular momentum in accretion. In many situations, the transferred material can not simply fall onto the stellar surface directly until it has rid itself of most of its angular momentum and thus, it leads to the formation of accretion disk. It is well known in the case of binary systems, that the normal stars can lose their mass mainly in two ways (we are dealing with slow processes only and not with the cataclysmic processes like supernova bursts etc.): (i) In the form of a quasi-spherical stellar wind; such phenomena are observed practically in all stars, starting from the Sun and ending with the massive super giants; (ii) In the form of gas jets upon the filling of a Roche lobe by normal star. Apparently, there exists another flow regime which characterizes rapidly rotating stars (e.g., Be-stars), viz., overflow in the form of disk-shaped shells. Possibly there also exists a strongly transient regime in which matter is ejected in the form of individual gas clusters. In the binary system, where one of the components is a compact object, i.e., a white dwarf, neutron star or a black hole, the companion is stripped of its matter due to the tidal effects. To understand the circumstance in which an accretion disk may form around a compact object, we consider a binary system with component masses M_1 and M_2 having angular velocity Ω. So in the case of circular orbits in the reference system rigidly connected with a component of a binary system, there exits an effective scalar potential Φ_{eff} describing the gravitational and centrifugal force. In the plane of the orbit, this potential corresponding Newtonian system can be presented in the form

$$\Phi_{eff} = -\frac{M_1}{|\mathbf{r} - \mathbf{r}_1|} - \frac{M_2}{|\mathbf{r} - \mathbf{r}_2|} - \frac{1}{2}(\boldsymbol{\Omega} \times \mathbf{r}). \tag{1.8}$$

where $\Omega = 2\pi / T$ is the angular velocity of rotation in the binary system. The total energy of the particle moving in the potential Φ_{eff} is given by

$$\Phi_{eff} + \frac{v^2}{2} = const = E_0. \tag{1.9}$$

The constant E_0 in the above equation is determined by the total energy of the particle at a certain instant of time. The region of possible motion of the particle is determined by the equality $v^2 / 2 \geq 0$. In this case, we obtain from the above equation the equivalent relation

$$\Phi_{eff} \leq E_0. \tag{1.10}$$

Consequently, the equipotential surface $\Phi_{eff} = E_0$ (Hill's surface) limits the region of possible trajectories of a moving particle with energy E_0. For a certain energy $E_R = \Phi_R$, Hill's surfaces around adjacent stars come in contact and form the Roche lobe.

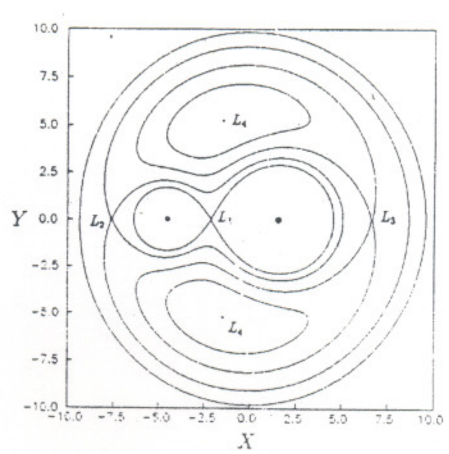

Fig. 1.1: Equipotential surfaces of a compact binary system with mass ratio $M_1/M_2 = 1/3$. Distances are in units of GM_1/c^2. Five points of four distinct types marked as L_1, L_2, L_3 and L_4 are the so-called Lagrange points where Φ_{eff} is locally or globally an extremum. Roche lobe overflow occurs when matter from M_2 fills its lobe (right section of the figure-of-eight formed by the innermost contour) and passes through L_1 to the star M_1 on the left (Chakrabarti, 1996).

The point of contact (inner Lagrangian point) can be determined from the condition $d\,\Phi_{eff} / d\,x = 0$ (the resultant of all the forces is equal to zero) as shown in the *Fig. 1.1*.

The shape of the Hill's surfaces is independent of the absolute values of the component masses and depends only on their ratio ($q = M_1 / M_2$).

At a certain stage of its evolution, a normal star fills its Roche lobe and begins to intensely flow over to the neighboring component through the inner Lagrangian point. The matter flowing from the normal star has an enormous angular momentum due to the orbital motion. Since the angular momentum is conserved as the accreting star is approached, the centrifugal acceleration of the infalling matter increases in accordance with the law $v^2 / R \propto R^{-3}$, i.e., more rapidly than the acceleration due to gravity. Therefore, at certain distance the matter enters some orbit. The following batches of the infalling matter having nearly the same initial conditions fall on the same orbit. A ring of increasing density is formed by the gas. Because of the collisions of gas elements, shocks, viscous dissipation, turbulence cells, etc., some of the energy of the ordered bulk orbital motion about the primary will be converted into internal (heat) energy. Eventually, some amount of this energy is radiated and therefore lost from the gas. The only way the gas can meet this drain of energy is by spiraling-in into the gravitational potential well of the compact object and hence loss of angular momentum. However, in the absence of external torque, this can only occur by transfer of angular momentum outwards by internal torques. But, it is worth noting that the viscous time scale (i.e. $t_{visc} \sim R^2 / \nu \sim R / v_R$ where, R being the radial extent, ν being the coefficient of the turbulent viscosity, and v_R being the radial inflow velocity of the fluid) on which the orbiting gas can redistribute its angular momentum is normally much longer than both the time scales over which it loses energy by radiative cooling, $t_{rad.}$ defined as $t_{rad.} \sim \mathcal{M}^{-2}\, t_{visc}$ where, $\mathcal{M}$ being the Mach number (i.e. $\mathcal{M} = v_\varphi / c_s$, c_s being the speed of sound), and the dynamical (i.e. orbital) time scale $t_{dyn.}$ defined as $t_{dyn.} \sim R / v_\varphi \sim \Omega_k{}^{-1}$ where, Ω_k is the Keplerian angular velocity and v_φ the azimuthal velocity of the fluid. For example, in case of standard α-disk models where α is assumed to be ≤ 1, there exists a well-defined hierarchy of time scales $t_{dyn.} \leq t_{rad.} \ll t_{visc.}$. It was found that the dynamical and thermal time scales were of the order of minutes, and the viscous time scale of the order of days to weeks for typical parameters. Since the removal of the angular momentum process operates on slower time scales as compared to free fall time, the infalling gas with sufficiently high angular momentum can form a disk like structure around a central gravitating body. If the viscous stresses disappear at the inner edge of the disk, the matter flows inwards from outside, and the angular momentum is transported outwards from inside. In another context, some matter from the winds of the companion will also be accreted by the primary, and the Keplerian flow could become sub- or super-Keplerian close to the compact objects due to terms such as advection, pressure and cooling efficiency which are neglected in standard disks. Therefore, flows close to the compact object will be an admixture of Keplerian and sub-Keplerian matter.

Depending on their geometrical shapes, the disk can be classified into thin or thick disk.

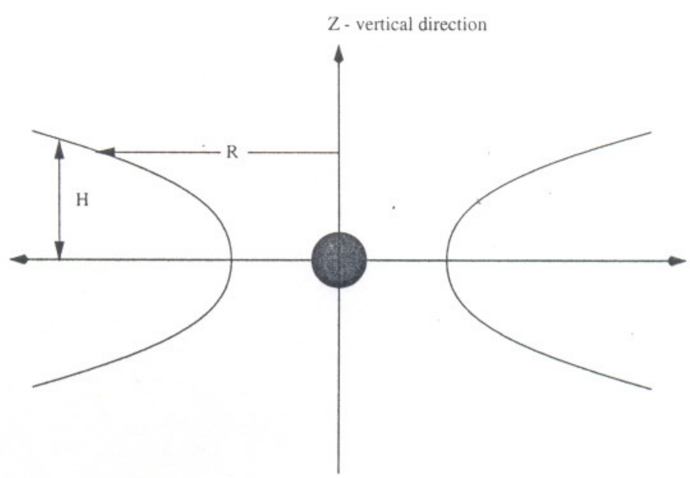

Fig. 1.2: This is a schematic diagram of the accretion disk around the central star with H as the scale height defined along the z- direction and R defined as the radius of the central star.

It is useful to introduce a vertical scale height parameter H defined as $H \sim (R^3/GM)(\partial_z P/\rho)$ to provide such classification more quantitatively.

1.2.3 Thin Accretion Disk

For the case when $H \ll R$, the disk is called thin disk and the model equations governing the stationary disk flow become the set of ordinary differential equations.

Disk in this regime is well studied and using these models many observable quantities can be calculated. Such models are based fairly on observational basis (Pringle and Rees, 1972; Shakura and Sunyaev, 1973). Subsequently several improvements were made from these early models which may be found in a detailed review by Lightman et al. (1978). All these models come under the nomenclature of standard accretion disk model (**SADM**) or α model as they assume the same viscosity law (i.e., $P = \alpha\, t_{r\,\phi}$). However, the weak point in modeling the thin accretion disks is the specification of viscosity law. As the ordinary molecular viscosity is almost certainly irrelevant in disk scenarios, it is conjectured that the turbulent or magnetic viscosity may play a key role in the disk dynamics though their nature is not fully well understood. Although the radial dependence of the viscosity law does not alter the energetics for a disk with fixed $\dot{M}$, it may have a influence on the column density through the disk which in turn change the emitted spectrum.

1.3 Standard Thin Disk Model

The standard disk model was originally conceived to describe Roche lobe accretion in a binary system.

1.3.1 Model equations

In this model, the heat generated by the viscous stress is radiated out and hence the disk becomes cool which means $k\,T \ll G\,M\,m_p\,/\,r$ contrary to spherical accretion where the temperature is virial which means $k\,T \sim G\,M\,m_p\,/\,r$. Therefore, the thin disk is highly non-adiabatic. In the thin disk limit, the vertical velocity component is negligible as compared to radial and azimuthal velocity components. Equations in two directions are decoupled and hence vertical equation can be solved independently of the radial equation. The accretion rate is assumed to be much lower compared to the Eddington rate and pressure is neglected so that the radial force balance equation governs the specific angular momentum distribution to become Keplerian.

The specific angular momentum of a Keplerian circular orbit of radius r around a

Newtonian star is given by

$$\tilde{l} = (G\,M\,r)^{1/2}. \tag{1.11}$$

Considering $2\,H$ to be the vertical scale height and Σ the surface density of the disk at the radius r, the surface density can be expressed as

$$\Sigma \equiv \int_{-H}^{H} \rho\, d\, z. \tag{1.12}$$

where the density ρ is computed on the mid-plane of the disk. By replacing the integral of products by the product of the averages, one can write the above integral as,

$$\Sigma \approx 2\,H\,\rho. \tag{1.13}$$

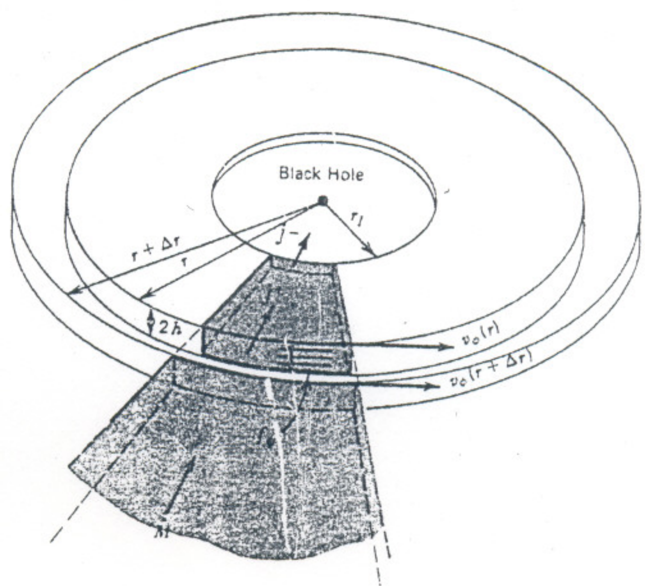

Fig. 1.3: Two annular sections of a thin accretion disk are drawn at radii r and $r + \delta r$ to illustrate how matter is accreted from $r + \delta r$ to r after angular momentum is transported from r to $r + \delta r$ through the action of the viscous stress f_ϕ. The figure is reproduced from Shapiro and Teukolsky (1983).

For a Keplerian disk, the stress tensor is expressed as,

$$t_{r\,\phi} = \eta\, r\, \frac{d\,\Omega}{d\,r} = -\frac{3}{2}\,\eta\,\Omega, \tag{1.14}$$

where, $\Omega^2 = G\,M\,/\,r^3$ is the Keplerian angular velocity. The viscous stress f_ϕ exerted in the ϕ direction by one fluid element at r to its adjacent fluid element at $r + d\,r$ (*Fig. 1.3*) is related to the stress tensor according to $f_\phi = -\,t_{r\,\phi}$ and hence,

$$f_\phi = -\,t_{r\,\phi} = \frac{3}{2}\,\eta\,\Omega = \frac{3}{2}\,\eta\,(G\,M\,/\,r^3)^{1/2}. \tag{1.15}$$

where η is the dynamic viscosity coefficient. In order to obtain the steady state disk structure, one has to solve four conservation equations describing the conservations of the rest mass, the specific angular momentum, the specific energy and the vertical momentum balance condition. In addition, a viscosity law must be specified which transports angular momentum outwards allowing matter to fall in.

Using the conservation of energy, one can obtain the luminosity of the disk by integrating over the radial extent of the disk and is given as,

$$L_{disk} = \int_{r_{in}}^{\infty} 2\,F(r)\,\times\,2\,\pi\,r\,d\,r = \frac{1}{2}\,\frac{G\,M\,\dot{M}}{r_{in}} = \frac{1}{2}\,L_{acc}. \tag{1.16}$$

where $F(r)$ denotes flux defined as $H\,Q^+$ where Q^+ is the measure of the rate of heat generated by the viscosity. Q^+ defined as $Q^+ \sim (t_{r\,\phi})^2\,/\,\eta$, where η is the dynamic viscosity coefficient.

It is worth noting that the above luminosity expression for the disk is exactly half the potential energy of the matter at the inner edge of the disk as well as half of the accretion luminosity. The reason for this magical factor is due to the assumption of the Keplerian distribution. If there is no energy loss, the rotational velocity at the inner edge should be $\frac{1}{2}\,v_\phi{}^2 = G\,M\,/\,r$ but instead it is found to be $\frac{1}{2}\,v_{\phi\,K}{}^2 = G\,M\,/\,2\,r$ because of the choice of angular momentum distribution at the inner edge of the disk. Therefore, half of the energy must come out of the disk irrespective of the physical viscosity in order to maintain a Keplerian disk. This above relation for L_{disk} is valid only for the case when there is no transport of angular momentum at the inner edge of the disk. This clearly shows that having the disk inner edge closer to the central compact object would increase the luminosity. The other half of the accretion luminosity is still available to be radiated from the boundary layer itself, which is therefore just as important as the disk for the total emission. For the case of non-rotating black hole, dividing the relation (1.16) by $\dot{M}\,c^2$, the efficiency of energy liberation during disk accretion is $\eta \simeq 1\,/\,12$, i.e., $\sim 8\%$. For the case of rotating black hole, the efficiency sharply increases to $\sim 42\,\%$ (Bardeen et al., 1972).

Since the disk is thin, the vertical velocity component must be smaller compared to other velocity components and thus one can ignore the advection term containing v_z in the z-component of the Euler equation :

$$\frac{1}{\rho}\,\frac{d\,P}{d\,z} = -\,\frac{G\,M}{r^2}\,\frac{z}{r}. \tag{1.17}$$

By setting $\Delta\,P \sim P$ and $\Delta\,z \sim H$, one can obtain,

$$H \sim \frac{a_s}{\Omega}. \tag{1.18}$$

Thus,

$$\frac{H}{r} \sim \frac{a_s}{v_\phi}. \tag{1.19}$$

It is clear from the above relation that in the thin disk condition $H(r) \ll r$ boils down to assuming that the flow is subsonic with respect to the azimuthal velocity or the azimuthal velocity v_ϕ is Keplerian and highly supersonic (i.e. $H \sim \mathcal{M}^{-1}\,r$, where $\mathcal{M}$ is the Mach number) which clearly puts a condition on the temperature of the disk and hence, ultimately, on the cooling mechanism. Hence the thermal energy of the gas in thin disks is much lower than the gravitational energy (i.e. the entire energy is concentrated in the kinetic energy of rotation) which tells that equilibrium along the azimuthal direction does not involve the pressure gradient because the gravitational force is balanced by the centrifugal force. Furthermore, the radial drift velocity v_r and vertical scale-height H are self-consistently small (i.e. $v_r \sim \alpha\,\mathcal{M}^{-1}\,a_s$).

It is generally accepted that the exact nature of the physical viscous mechanisms is very poorly understood in the accretion disk scenarios. However, the molecular and radiative viscosities are usually too small for a cool Keplerian accretion disk. So, one of the possibilities is to consider small scale turbulent dissipation. In this case, the coefficient of viscosity is given by

$$\eta \sim \rho\,v_{turb}\,l_{turb}, \tag{1.20}$$

where, v_{turb} is the velocity of turbulent eddies relative to the mean gas motion and l_{turb} is the size of the largest turbulent eddies. In case of supersonic turbulence, the shocks dissipate energy into heat and enforce $v_{turb} \leq c_s$. Since the largest size of the eddies are bounded by the disk thickness, $l_{turb} \leq H$ and thus the viscous stress is bounded by,

$$f_\phi = -t_{r\phi} \leq \rho v_s H \Omega \sim \rho {c_s}^2 \sim P. \tag{1.21}$$

Therefore, in general,

$$f_\phi = \alpha P, \tag{1.22}$$

with $\alpha < 1$. Here, P is the pressure in the equatorial plane and α is a dimensionless parameter called the turbulence parameter.This prescription has some basis in phenomenological models of turbulent (Lynden-Bell and Pringle, 1974) and magnetic (Lynden-Bell, 1969, Eardley and Lightman, 1976 etc.) viscosities, which predict values of α in the range $\sim (10^{-3} - 0.1)$. If $\alpha < 1$ then the inflow is subsonic with respect to the gas in the disk and disk will be in hydrostatic equilibrium in the vertical direction. However, this generally used prescription is inadequate in describing flows which include discontinuity such as shock waves.

1.3.2 Stability of a Thin Disk

Thin disks are subject to a variety of instabilities. Some of these arise from the viscosity model (Lightman and Eardley, 1974), while others are related to the processes by which thin disks cool, and are particularly dangerous for disks supported vertically by radiation pressure (Pringle *et al.*, 1973; Pringle, 1981; Shakura and Sunyaev, 1976; Piran, 1978). All these above instabilities are secular, i.e., they grow over times considerably longer than the orbital time. Consider, for example, a radiation pressure and Thomson scattering dominated inner disk of variable half thickness H. More generally, an equilibrium thin disk will be thermally unstable if

$$\left(\frac{\partial (Q^+ - Q^-)}{\partial H} \right)_\Sigma > 0 \tag{1.23}$$

(Shakura and Sunyaev, 1973; Pringle, 1981; Frank *et al.*, 1992). The time scale for thermal instability to develop is the local cooling time

$$t_{th} \sim \frac{2\pi}{\Omega \alpha} \tag{1.24}$$

A thermally unstable disk will develop a corrugated surface, where the amplitude of the corrugations is limited by non-linear effects. Their general effect is to cause the disk to break up into rings. Many types of equilibrium disks are thermally unstable. It is worth to note that the assumption that the viscous heating is described by the α model with constant α is crucial and different prescriptions can render our particular example stable.

An alternative class of instabilities can develop more slowly on the time-scale associated with mass accretion. If the disk adjusts to thermal equilibrium, then $Q^+ = Q^-$ holds inside the disk. If the accretion rate increases, then the local density can readjust to a new equilibrium; if not, then the disk is subject to viscous instability. The criterion for viscous stability is then given by

$$\frac{d\,(\nu\,\Sigma)}{d\,\Sigma} > 0 \tag{1.25}$$

(Pringle, 1981; Frank *et al.*, 1992). When this inequality is reversed, then the disk will depart further from equilibrium and become unstable. If there is viscous instability, then perturbations will grow on the mass accretion time-scale

$$t_{vis} \sim \frac{r}{v} \sim t_{th}\left(\frac{r}{H}\right)^2 \tag{1.26}$$

It is also important to know about the non-linear development of these instabilities and to understand whether or not they are likely to disrupt global mass accretion flow.

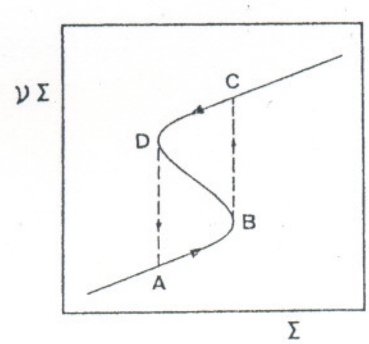

Fig. 1.4: Mass accretion rate at a given radius as a function of the local surface density. When $d(\nu\,\Sigma)/d\Sigma > 0$, *the disk is stable to small perturbations in the surface density. When the inequality is reversed, the disk is unstable. If mass is accreting at an unstable rate the disk may follow the limit cycle variation ABCD instead of accreting steadily (Blandford, 1990).*

One exception is a class of disks where relationship between the mass accretion rate and the surface density has the form depicted in the *Fig. 1.4.* If the rate at which mass is supplied to the disk lies within an unstable range, then the disk may alternate between two stable states following a limit cycle through this evolution. This limit cycle behavior is held responsible for dwarf nova outbursts in cataclysmic variables. In this case a portion of the disk associated with an ionization zone is believed to have an unstable $\dot{M}(\Sigma)$ curve. Similar behavior may occur in AGN accretion disks (Lin and Shields, 1986; Clarke and Shields, 1989).

Other instabilities operate on a dynamical time although the unstable conditions evolve over an inflow time, e.g., convective instabilities (particularly for radiation-dominated disks: Cunningham, 1973; Shakura and Sunyaev, 1976). These instabilities can be avoided if the viscous stress scales as the gas pressure rather than the radiation pressure (Sakimoto and Coroniti, 1981; Meier, 1979). Thin disks in a Keplerian potential are believed to be dynamically stable. However, when a disk is sufficiently massive that its self-gravitation becomes important, then the disk can become dynamically unstable. Finally, certain dynamical instabilities have been suggested as the basis of the angular momentum transport, e.g., the Kelvin-Helmholtz instability leading to inflow in the Gunn (1977) model for NGC 4278 and the weak Jeans instability in a disk which is marginally self-gravitating in the vertical direction (Paczyński and Rozyczka, 1977; Bailey and Clube, 1978).

1.3.3 Magnetized Accretion Disks

So far we have discussed the structure and stability of the thin accretion disk without considering the role of magnetic fields. Magnetic fields are ubiquitous in most astrophysical systems and might be generated in the accreting plasma due to dynamical processes. It is thought that accretion disks, whether in star-forming regions, in cataclysmic variables, X-ray sources or in the centers of active galactic nuclei are likely to possess magnetic fields. Magnetic fields could play varied and important role in the accretion disk scenarios depending on the concrete physical situations under considerations. Magnetic fields may also have a big effect on the radiation spectrum emerging from a realistic thin disk. For example, energy transported by magnetic buoyancy into a hot corona could dominate the (approximately blackbody) radiation from the dense part of the disk. magnetic flares in the corona may accelerate relativistic electrons that radiate non-thermally. In case of AGN disks, there is no ultimate repository from the angular momentum of disks carried outward by viscous tress. Blandford (1984) has emphasized that if the magnetic field were sufficiently well ordered, a coronal wind (rather than outward transfer via viscosity within the disk itself) could be the main sink for the angular momentum of accreted material (Blandford, 1976; Blandford and Payne, 1982). In the context of extragalactic radio sources, formation and collimation of extragalactic jets (for a recent review, see Spruit, 1996), could onset varied magneto-hydrodynamic instabilities which can influence many radiation mechanisms which in turn may explain important observed phenomena like variability, polarization, pulse modulations etc. It can also be responsible for generating viscosity (turbulent) which may redistribute the angular momentum in a more efficient way. In many accretion disk scenarios the central objects possess intrinsic magnetic fields. The observations from the X-ray satellites substantiate the evidence of the existence of X-ray stars. These stars are broadly classified into two categories: (a) those emitting pulsed X-ray radiation (X-ray pulsars) and usually belonging to a comparatively younger stellar population in the Galaxy (plane component), and (b) the sources of non-periodic varying radiation (e.g., X-ray bursters) belonging to the galactic bulge, a quasi-spherical sub-systems with radius on the order of 5 kpc. All these stars are undoubtedly binary systems. The existence of pulsations can be attributed to very powerful magnetic fields. From the observations of cyclotron spectral features in X-ray pulsar systems, estimates for the surface magnetic field strengths of neutron stars have been inferred to be in the range of $\sim 1-4 \times 10^{12}$ Gauss. The magnetic field strength on neutron stars in the galactic bulge is in the range of $10^{8} - 10^{11}$ Gauss. These estimates have been inferred from the interpretation of the intensity-dependent quasi-periodic oscillations in the bright sources (Lewin *et al.*, 1988) in terms of the magnetospheric beat-frequency modulated accretion model of Alper and Shaham (1985) and Lamb *et al.* (1985).

1.3.4 Survey of Magnetospheric Models

A star with such a strong magnetic field has a magnetosphere which can be hundreds of times the size of the star itself. Many workers studied the structure of the magnetosphere of a neutron star in the vacuum approximation, i.e., under the assumption that matter does not penetrate the magnetosphere. Such magnetospheres can indeed exist for some neutron stars in the propeller or georotator regimes. However, it is clear that in some powerful X-ray sources (like X-ray pulsars), matter penetrates to the surface across thousands of kilometers of magnetic field strata. After all, it is the interaction of plasma and the magnetic field in the magneto sphere that is responsible for a number of observational facts, such as (1) liberation of energy of the accreting matter on the surface and its emission (anisotropic in a powerful magnetic field): circumpolar region; (2) exchange of angular momentum between a star and accreting matter, leading to a change in the rotational period of the neutron star (Alfvén zone– transition layer); (3) determination of the time evolution of a source by a valve at the magnetosphere boundary. The first ideas on the passage of the plasma across the magnetosphere boundary were put forth by Shvartsman (see, Lipunov, 1992) and he suggested that plasma may pass through the magnetosphere boundary due to magnetohydrodynamic instabilities.

Stimulated by the observations of these binary X-ray sources, it has been recognized that the structure of the magnetosphere surrounding an accreting neutron star and the structure and evolution of the accretion disk in the system are central for providing a physical interpretation of the observed phenomena. Some theoretical progress has been made in understanding the dynamics of accretion within the magnetosphere and the topology. However, in the context of magnetospheric models around magnetized compact objects, Lamb, Pethick and Pines (1973) were the first to assume that the plasma entering the magnetosphere would be rapidly threaded by the magnetic field and showed that the plasma would then be forced to flow along the field lines. Later on Ghosh, Lamb, and Pethick (1977, hereafter GLP) have investigated accretion of matter by a rotating magnetic neutron star, assuming that the magnetic field of the star has a symmetry axis which is aligned with the rotation axis, that the accreting matter becomes threaded by the stellar magnetic field near the magnetosphere boundary, and that the star is not rapidly rotating. They have shown that for bright X-ray sources the flow of matter within the Alfvén surface is well described by the equations of magnetohydrodynamics, and that the matter there moves along field lines when viewed in the corotating frame with respect to the star. They found that matter inside the Alfvén surface rotates in a sense opposite to that of the net angular momentum flux toward the star if the star is rotating slowly. Using a simple model for the transition region between the magnetosphere and the exterior flow, they have illustrated how matching the interior and exterior flows determines the angular momentum flux, and therefore whether the neutron star is spinning up or spinning down. They have also explored the special case of Keplerian disk flow outside the magnetosphere. Their calculations show that if $0 < \Omega_s r_A^2 / l < 1$, then any energy dissipated in the disk beyond that provided by the release of the gravitational binding energy of the matter flowing through the disk comes neither from the matter flowing through the disk nor from the rotational energy of the star but from the energy released by matter in the transition zone between the disk and the magnetosphere.

Scharlemann (1978) emphasized the importance of the shape of the field lines threading the plasma in controlling the flow of plasma from the inner edge of the disk to the neutron star. By assuming the stellar magnetic field is completely excluded from the disk by screening currents which are represented by a current ring of radius R_{CF}, (i.e., Chapman-Ferraro radius estimated by equating the pressure of the dipole magnetic field to the gas pressure of the accreting matter) Scharlemann discussed the possible mechanisms *viz.* Kelvin-Helmholtz (K-H) and Rayleigh-Taylor (R-T) instabilities through which the plasma can flow to the stellar surface either along the field lines or through the equatorial magnetosphere. Ichimaru (1978) proposed a model of disk accretion in which the inner radius of the disk is determined by the static pressure balance condition but modified to take into account of effective gravity and centrifugal force acting on the plasma in the boundary layer between the disk and the magneto sphere of the compact object. However, all these models are mostly qualitative in nature and several questions have been raised on the validity of the inherent assumptions. The most important of all, concerns the complete screening of the stellar magnetic field from the disk.

However, the pulse periods for a number of X-ray pulsars have been sufficiently well measured over the past decade to provide important information regarding the torques exerted on the neutron stars by the accreting material. From the pulse period histories of these eight sources, it is apparent that the "spin-up" trend first noted in Her

X-1 and Cen X-3 (Giacconi, 1974; Gursky and Schreier, 1975) is very prominent in at least five of them. The trend in most of the X-ray pulsars toward a secular decrease in pulse period can be understood in terms of the torques exerted by the matter accreting onto the neutron star. These torques can be readily calculated for the case where the matter has roughly circular Keplerian velocities at the magnetopause of the neutron star, as would be the case if the accretion is mediated by a disk. These models are known as magnetically threaded disk models (hereafter MTD).

There have been extensive studies on this subject (Pringle and Rees, 1972; Ghosh and Lamb, 1978, 1979a,b, (hereafter GL); Lipunov, 1992; Wang, 1995; Li and Wang, 1996). The principal uncertainty in the calculation of the net torque N appears to lie in the behavior of the toroidal field, $B_\phi\,(r)$. Such calculations (Pringle and Rees, 1972; Lamb *et al.*, 1973; GL, 1979a,b, and references therein) show that the rate of change $\dot{P}$ of the intrinsic pulse period P is related to the X-ray luminosity and the physical properties of the neutron star:

$$\frac{\dot{P}}{P} \simeq -3 \times 10^{-5} \left(\frac{\xi\, v_r}{v_{ff}}\right)^{1/7} \left(\frac{M}{M_\odot}\right)^{-10/7} \left(\frac{R}{10\,Km}\right)^{6/7} \left(\frac{R_g}{10\,Km}\right)^{-2}$$

$$\times \left(\frac{\mu}{10^{30}\,G\,cm^3}\right)^{2/7} \left(\frac{L_x}{10^{37}\,erg\,s^{-1}}\right)^{6/7} \left(\frac{P}{1\,s}\right) yr^{-1}. \qquad (1.27)$$

Here, ξ is the fractional solid angle subtended at the neutron star by the infalling matter at the magnetopause; $v_r\,/\,v_{ff}$ is the ratio of the average radial infall velocity of a particle to its free-fall velocity just outside the magnetopause; M, R, R_g, and μ are the mass, radius, radius of gyration, and magnetic dipole moment of the neutron star, respectively; and L_x is the accretion-driven luminosity. The quantity $(\xi\, v_r\,/\,v_{ff})^{1/7}$ is not expected to differ greatly from unity (see, for example, Lamb *et al.*, 1973). The overall minus sign in the above equation is explicitly for the case where the sense of the orbital angular momentum in the accreting matter is the same as that of the rotation of the neutron star. For simplicity, one can rewrite the above equation as,

$$\frac{\dot{P}}{P} = -3 \times 10^{-5} f \left(\frac{P}{1\,s}\right) \left(\frac{L_x}{10^{37}\,erg\,s^{-1}}\right)^{6/7}. \qquad (1.28)$$

(Rappaport and Joss, 1977), where the dimensionless function f is expected to be of order unity for a neutron star and contains parameters that are not yet measurable for most or all of the X-ray pulsars. Therefore, it actually seems that the pulse-period changes in binary X-ray pulsars are a sensitive diagnostic of the process of accretion onto a neutron star in the presence of the its magnetic field. Measurements of the magnitude and sign of $\dot{P} / P$ may also be used to infer intrinsic properties of the source such as its dipole moment, and may even provide information about the nature of the binary system and its evolution.

One of the main conclusions of GL model is that in disk accretion the transition zone is not thin, and recognition of this is an important step in understanding the period behavior of Her X-1, Cen X-3, and 4U 0900-40. However, the principal uncertainty in the calculation of the net torque N appears to lie in the behavior of the toroidal field, $B_\phi(R)$. In their model the toroidal magnetic field B_φ is generated by the shear motion between the disk and the star; but the amplification to take place on a time-scale $\tau_w \sim |\gamma\,(\Omega_s - \Omega_K)|^{-1}$ (with γ of order one), is limited by reconnection between the field lines above and below the symmetry plane of the disk (where B_ϕ changes sign). The latter process occurs on a time scale $\tau_d \sim h\,/\,(\xi\,|v_{A\,\phi}|)$, where $v_{A\,\phi} \equiv B_\phi\,/\,(4\,\pi\,\rho)^{1/2}$, ρ denotes the plasma mass density, and ξ is a numerical factor which may be considerably less than unity (cf. Priest, 1981). In this scenario, lines of force become twisted and a corresponding stress is exerted on the neutron star. They have found that the torque can have both positive and negative contributions, with near-cancellation occurring under some conditions. The model was applied to Her X-1, which is observed to spin-up on a time scale much longer than expected from naive dimensional estimates, as well as to other binary X-ray pulsars for which measurements of $\dot{P}$ were available. However, the detailed predictions of the model depend on assumptions about the amount of distortion undergone by the magnetic field threading the disk.

However, the *Ansatz* (i.e., generation and amplification of wounded magnetic field) is physically untenable for the following reason. According to GL (1979a), the structure of the outer zone $r > r_0$ resembles that of an undisturbed, thermally-supported accretion disk (see, e.g., the review of Pringle, 1981). If this is the case, the vertical pressure scale-height (or half-thickness) of the disk is related to the isothermal sound speed $c_s = (p\,/\,\rho)^{1/2}$ through $h \sim c_s\,/\,\Omega_K$. This means that the pressure of the wound field, which they did not include in their vertical hydrostatic balance equation, greatly exceeds the thermal pressure p beyond $R \sim R_c$ (where R_c is denoted as corotation radius defined as $R_c \equiv (GM / \Omega_s^2)^{1/3}$ where Ω_s is the angular velocity of the neutron star defined as $\Omega_s = 2\pi / P$) and would disrupt the disk. On the other hand, their model requires that a substantial (negative) contribution to the torque on a fast rotator like Her X-1 originate from the region $r \gg r_c$ (see GL, 1979b).

However, Wang (1987) has presented a model for the accretion disk torque acting on a neutron star, based on a more realistic treatment of the magnetic stresses than has been considered by GL. Although, his approximations are still crude to allow an accurate determination of dipole moment, the model generally indicates higher values of dipole moment than that of GL. The differences arises because, for given μ (dipole moment) and η (dipole screening factor), GL treatment predicts a larger spindown contribution to the torque from the asymptotic region beyond r_c: the winding law that they adopted leads to too much amplification of the toroidal field. In other respects, the predictions of the two models are in qualitative agreement. We know a useful parameter often adopted in diagnosis of the magnetic field-accretion disk interaction is the "fastness parameter" which is defined as $\omega_s \equiv \Omega_s / \Omega(r_0) \equiv (r_0 / r_c)^{3/2}$, where Ω_s and Ω are the angular velocities of the star and the disk plasma respectively, r_c is the corotation radius of the star. Li and Wang (1996, hereafter LW) have shown that the torque exerted on a rotating, magnetized star by magnetically threaded accretion disk is depending on the "fastness parameter", ω_s. In equilibrium state for which there is no torque exerted on the star, the fastness parameter reaches its critical value ω_c. Recently Wang (1995) obtained that the value of ω_c lies in the range of $0.875 - 0.95$ But LW argue that an uncertainty of a factor of 4 exists in the torque calculation at the inner edge of the disk. Together with uncertainties relating to the strength of the toroidal field, LW find that the value of ω_c can range between 0.71 and 0.85 for different physically plausible conditions inside the accretion disk. These results are found to be larger than those of GL (1979a, b), but smaller than those of Wang (1995), implying that the field-threaded disk is moderately effective in braking the star. Combining Wang's results, LW conclude that the value of ω_c should lie in the range of $0.71 - 0.95$. Hence LW support the idea that the MTD model is incompatible with the critical fastness parameter whose value is significantly less than 1 (Wang, 1995). LW results present constraints on the magnetically threaded disk model and seem to be consistent with observations of the binary X-ray pulsar $4U1626-67$, which belongs to a low-mass X-ray binary (McClintock *et al.*, 1977; Levine *et al.*, 1988), in which the neutron star accretes matter from its companion through a disk. The pulse period of $4U1626-67$ is ~ 7.7 seconds and the period measurements obtained before 1990 demonstrated a constant spin-up of $-1.6 \times 10^{-3}\ s\ yr^{-1}$ (Nagase, 1989). Recent observations with ART-P / Granat (Lutovinov *et al.*, 1994) and BATSE / CGRO (Bildsten *et al.*, 1994) showed that the period derivative changed sign in 1991. This fact reflects that $4U1626-67$ was close to the equilibrium state for which the net torque vanishes. If the beat frequency model (Alper and Shaham, 1985; Lamb *et al.*, 1985) is applied for the QPO of $4U1626-67$, the rotation frequency of the disk plasma at the inner edge can be estimated to be $0.17\ Hz$ and the critical fastness parameter $\omega_c \simeq 0.76$ which lies in LW estimated range of ω_c.

Lovelace *et al.* (1986) and Mobarry and Lovelace (1986) have proposed a general theory for both Newtonian and relativistic ideal MHD flows around a rotating magnetized neutron star and Schwarzschild black hole and they derived a virial equation and discussed the stability of the motion of the charged test particle in the presence of an electromagnetic field but without presenting any specific equilibrium solution. The importance of general relativity in discussing the structure and stability of magnetospheric plasma around a compact object is explained by Prasanna (1991). In a completely different formalism, Prasanna and Chakrabarty (1981) and Chakrabarty and Prasanna (1981) analyzed structure and stability of fluid disks around a Schwarzschild black hole and observed that (i) the inner edge of the disk can not lie within $4\,m$, (ii) if the inner edge of the disk lies within $4\,m$ and $6\,m$, then the outer edge must lie beyond $2\,a\,/\,(a-4)$, where a is the radius of the inner edge defined in units of m ($m = M\,G\,/\,c^2$), (iii) there exists no restriction on outer edge, if the inner edge is at or beyond $6\,m$, (iv) in the case of pressureless disk, the structure is stable if the inner edge is greater than $6\,m$, (v) an ordinary perfect fluid disk rotating around central source is stable under radial perturbations. The dynamics of accretion disk and its emerging flux in the presence of electromagnetic fields on curved space time for several special cases of azimuthal velocity distributions was obtained by Prasanna and Bhaskaran (1989) and Bhaskaran and Prasanna (1989). A subsequent analysis by Bhaskaran and Prasanna (1990) including the radial velocity component of the flow revealed the inter-dependence of different physical parameters like outer density, seed magnetic field and finite conductivity on the continuous pressure distributions of the disk configurations. An analysis for disk around slowly rotating compact object was also carried out by Bhaskaran *et al.* (1990) which demonstrated the influence of co- and counter rotation at the inner edge of the disk. However, all these analyses were confined to thin disk limit (i.e., $\theta = \pi / 2$). In another context, Anzer *et al.* (1987) computed the changes in the vertical structure of the accretion disk brought about by an external magnetic field and showed that high probability exists for the occurrence of instabilities at the magnetospheric boundary due to the inversion of the density profile.

Directed outflow is a ubiquitous feature of active galactic nuclei, and it is also seen in some small-scale prototypes of AGNs in our own Galaxy (e.g., SS 433). Over the past twenty years radio observations have shown us unexpected phenomena which have revolutionized our ideas about the structure and energetics of active galactic nuclei. Specifically, they have revealed that many galactic nuclei produce what appear to be collimated jets of plasma which traverse the vast distances spanned by the extended radio emission. The *Hubble Space Telescope* for the first time enabled optical and UV images of jets to be obtained with spatial resolution comparable to radio interferometric techniques. Jets may be significantly more common than presently realized given the serendipitous discovery by *HST* of a new, previously unsuspected optical nonthermal synchrotron jet in 3C 264. In the realm of jets the magnetic fields have long figured prominently in models designed to account for the double nature of extragalactic radio sources. Magnetic fields can operate in two different ways, *passively* in defining a channel along which a stream of high-energy particles can flow, and *actively* if the field itself carries a major component of the power leaving the nucleus.

It is generally believed that the origin of the bipolar outflows and jets are closely related to the properties of magnetized accretion disk around a compact object. Any disk structure near a black hole provides a pair of preferred directions along the rotation axis; moreover, within the Lense-Thirring effect's domain of influence, this axis is maintained steady by the hole's gyroscopic effect. These outflows and jets can extract angular momentum very efficiently from the disk with the attractive attribute of a self-confining toroidal field when there is no binary companion. It had been shown that angular momentum transport in a bipolar outflow is consistent with its loss from the proto-stellar disk (e.g., Pudritz and Norman, 1986; Königl, 1989; Contopoulos and Lovelace, 1994). Jets may be produced electromagnetically. The potential difference across a disk threaded by open magnetic field lines can exceed 10^{20} V (Lovelace, 1976; Blandford, 1976), and this is available for accelerating high-energy particles which will produce an electron-positron cascade and ultimately a relativistic jet that carries away the binding energy and angular momentum of the accreting gas. Since plasma can easily flow from the disk into the magnetosphere, a hydromagnetic description is probably more appropriate than the force-free approximation (Blandford and Payne, 1982 , hereafter BP; Phinney, 1983; Ustyugova *et al.*, 1995 and references therein). In this description the jet mechanism relies upon hydromagnetic stresses exerted by a magnetized accretion disk to fling gas outward centrifugally (Blandford, 1990). The gas will be tied to the magnetic field and its inertia will cause the magnetic field lines to be bent backwards creating a toroidal component. Now this toroidal component of magnetic field has an associated "hoop" stress which can act to collimate the poloidal flow of plasma.

BP (1982) have developed a model self-consistently for the infinitesimally, thin, Keplerian, magnetized disk whose solutions also produces collimated radio jets. In this model, matter is ejected using a 'sling-shot' mechanism if the poloidal field line, emerging from the disk, is sufficiently bent outwards and making an angle θ with the outward radial direction. Plasma that is attached to this field line will behave somewhat like a 'bead' on a wire, and it is straightforward to show that if the disk is in Keplerian orbit and $\theta \lesssim 60°$, then centrifugal force will exceed gravity and gas will flow away from the disk surface. The natural assumption to make in this case is that the particle stresses, ρv^2, are in rough equipartition with the magnetic and gravitational stresses. Therefore, the plasma in the magnetosphere is streaming outward at roughly the Alfvén speed $B\,(4\,\pi\,\rho)^{-1/2} \sim (G\,M\,/\,r)^{1/2}$ rather than the speed of light, as is the case with force-free assumption. This piece of work shows that collimated bipolar jets may be formed from a Keplerian disk. In real physical situation, the disk need not be Keplerian and the potential will be modified due to curvature effect very near to the black hole and as a result much higher inclination than $60°$ will be required in order to fling out matter from the strong gravitational field.

In the absence of resistivity, magnetic field lines can be thought of as being frozen into the plasma (e.g., Parker, 1979). If no field is entrained from the ambient medium, the magnetic flux in the jet is conserved along the jet trajectory, although the field strength may be amplified by internal shear. In the absence of a velocity gradient across the jet, the magnetic field should become predominantly toroidal far from the source of the jet, regardless of its configuration closer in (Begelman *et al.*, 1984). The presence of toroidal field B_ϕ has threé general consequences. First, there can be a magnetic tension associated with it, i.e., large enough to collimate the outflow. Second, there will be an associated Poynting flux of energy $\sim\ (B_\phi^2\,/\,4\,\pi)\,v_j$. This energy flux is in a form suitable for driving particle acceleration at large distances from the central source, in particular within the radio components. Third, as the magnetic field is not entirely toroidal but retains a poloidal component B_p that decays with distance as d^{-2}, there is also a flux of electromagnetic angular momentum associated with the outflow $\sim\ B_p\,B_\phi\,d\,/\,4\,\pi$.

1.3.5 Stability of Magnetized Accretion Disks

However, the magnetized thin disk also suffers from instability like magnetic shearing instabilities. Although the hydrodynamic shear flow is stable as long as the specific angular momentum is increasing outwards, the corresponding ideal magneto hydrodynamic shear flow becomes unstable as soon as the angular velocity decreases outwards in the presence of a weak magnetic field. A magneto hydrodynamic shearing instability was first found by Velikhov (1959) and independently by Chandrasekhar (1960, 1961). In fact, Safronov (1972) had hinted the existence of the above mentioned instability in the accretion disk scenario where azimuthal field is more important than the one of the vertical field. Thereafter, Balbus and Hawley (1991) found a magnetic shearing instability that affects a magnetized thin Keplerian flow.

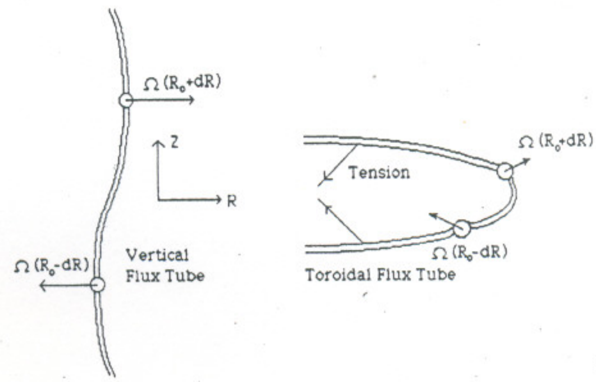

Fig. 1.5: Schematic diagram showing the origin of Balbus-Hawley instability inside a differentially rotating disk in presence of vertical and toroidal flux tubes (Chakrabarti, 1996).

Their analysis assumed a continuous magnetic field, but it can be shown that this instability also holds for magnetic flux tubes. Furthermore, Balbus and Hawley (1992a,b) have emphasized that the growth rate is related to Oort's A-constant, and by numerical simulations in two dimensions (Hawley and Balbus, 1991). The driving force behind the magnetic shearing instability is the centrifugal force which enhances the sinusoidal perturbation and causes strong bending as well as stretching of the field lines as shown in the *Fig. 1.5*. The bending of the field line introduces two different effects. The first effect is stabilizing the perturbed field line at $R = R_0$ by pulling it back due to magnetic tension and the second effect is destabilizing by forcibly rotating the displaced field line at its original angular frequency $\Omega_K = \Omega_K(R_0) \equiv \Omega_{K0}$ due to magnetic tension.

The stretching makes the tube buoyant so that the perturbed parts rise out of the disk. The Coriolis force affects the radial displacement by twisting the tube path. Therefore, the joint effect of buoyancy ("lift") and Coriolis force ("twist") results in "wiggling" of the tube path. The wiggles and the uppermost parts of the tube are regions of high curvature and torsion implying strong currents and magnetic field twisting respectively. If the resistivity are taken into account, the tube will be disrupted by kinking instabilities and reconnection. The instability criterion is given as,

$$k^2\, v_{A0}^2 \,-\, 3\,\Omega_{K0}^2 \,<\, 0. \tag{1.29}$$

if the magnetic field is sufficiently weak or the wavelength sufficiently long. The above criterion can be translated into a lower limit of the perturbation wavelength:

$$\Lambda \,>\, \Lambda_{crit} \,=\, \frac{2\,\pi}{sqrt3}\,\frac{v_{A0}}{\Omega_{K0}}. \tag{1.30}$$

For a sufficiently weak magnetic field the wavelength becomes comparable to the length scale of magnetic diffusion and the instability is no longer ideal. At the other limit of a strong magnetic field the length scale is comparable to the scale height of the disk so that the local analysis does not apply.

However, the validity of this local analysis was questioned by Knobloch (1992) who showed that the simple-minded derivation of a dispersion relation from a local analysis can yield erroneous results in shear flows if applied carelessly. The proper way to analyze instabilities in shear flows is given by an eigenvalue-problem for the frequency, which introduces a dependence on the radial boundary conditions. Knobloch could not exclude the possibility of an instability even though he suggested that it would be overstable instead of exponentially growing because of the stabilizing influence of the toroidal magnetic field. Dubrulle and Knobloch (1993) derived a stability criterion for axisymmetric perturbations

$$\frac{1}{R^2}\,\frac{\partial}{\partial R}\left(R^2\,\frac{B_\theta^2}{4\,\pi\,\rho}\right) \,-\, R^2\,\frac{\partial}{\partial R}\left(\frac{v}{R^2}\right) \,>\, 0, \tag{1.31}$$

where v is the equilibrium azimuthal velocity, which in general is not Keplerian because the equilibrium state is affected by the presence of the azimuthal field. The growth rate of the most unstable mode is of the order of Ω_K^{-1} but decreases as the azimuthal field becomes stronger. Similar stability criteria have been derived recently from an interchange method (Christodoulou *et al.* 1995). Balbus and Hawley (1991, 1992a,b) did not find the stabilizing effect of the toroidal field, as they did not take into account the tension of the toroidal field, $B_\theta^2\,/\,R$ which is the stabilizing force. The global stability of magnetized accretion disks has been analyzed by Kumar *et al.* (1994), Coleman *et al.* (1995), Gammie and Balbus (1994) and in particular by Curry *et al.* (1994). The single most important result of all the papers fortunately is that the instability still exists in the global analysis for a fairly large range of parameters, although the azimuthal magnetic field does have a stabilizing influence.

1.4 Thick Accretion Disks

So far we have discussed theoretical work on thin disk structure primarily aimed at understanding cataclysmic variables, X-ray binaries, etc., but it is also relevant in the context of active galactic nuclei. In all disks, the thermal balance of the outer parts is likely to be controlled by irradiation (causing photoionization, Compton heating, etc.) from the central region. Even where such disks exist, they could be embedded in hotter quasi-spherical structures. Thus, there may be no distinct demarcation between thin disks and the toroidal structures (i.e., thick disks or tori). Disks become geometrically thick, with $H \simeq R$, if the internal pressure builds up so that $c_s \simeq (GM/r)^{1/2}$ (Rees, 1984). This can happen if gas passing through a thin disk reaches a radius within which the internal pressure builds up- either because it is unable to cool in an inflow time (i.e., the material is unable to radiate the energy dissipated by viscous friction which then remains as internal energy) or because the radiation-pressure force is competitive with gravity. In a thick disk or torus the pressure provides substantial support in the radial as well as the vertical direction, and the angular momentum distribution (now a function of z as well as r) may be far from Keplerian which in turn becomes (within certain constraints) a free parameter. Nevertheless, the analogy with thin accretion disks still applies if slow inflow occurs as a result of the viscous transfer of angular momentum. However, in thick disks the viscous stress must be considered in two directions. The stresses determine the distribution of both angular momentum and enthalpy, and therefore the shape of the isobars inside the disk; internal circulation patterns may be important for energy transport. There always exists a pressure maximum at $r = r_{max}$ in the equatorial plane. The angular velocity becomes sub-Keplerian for $r > r_{max}$ and becomes faster than Keplerian for $r < r_{max}$.

However, the above fact has enabled Paczyński and Wiita (1980), Jaroszyński, Abramowicz, Paczyński (1980), Abramowicz, Calvani, and Nobili (1980), and Wiita (1982) to construct global models of thick accretion disks without reference to the viscosity. They have exploited an important simplifying feature: the shape of a torus depends only on its surface distribution of angular momentum. If the angular velocity Ω is taken to be the function of angular momentum $\mathcal{L}$, then the binding energy U is given implicitly by

$$dU\,/\,U \;=\; \Omega\, d\mathcal{L}\,/\,(c^2 \;-\; \mathcal{L}\,\Omega^2). \tag{1.32}$$

For uniform $\mathcal{L}$, the binding energy U is then constant over the whole surface of the torus and thus for each value of $\mathcal{L}$, a family of such tori parameterized by the surface binding energy can be obtained. The tori will 'puff up' when $U \;\rightarrow\; 0$ and hence a part of the surface close to the rotation axis becomes paraboloidal in shape. The gravitational field remain Newtonian throughout the torus except near the hole where relativistic effects play an important role if $\mathcal{L} \;\simeq\; \mathcal{L}_{min}$, the angular momentum of the smallest stable orbit. For $\mathcal{L}_{min} \;<\; \mathcal{L} \;<\; \mathcal{L}_0$, the binding energy U of the torus exactly equals the binding energy of the unstable orbit of angular momentum $\mathcal{L}$. There is then a cusp-like inner edge, across which material can spill over into the hole (just as material leaves a star that just fills its Roche lobe in a binary system). This particular relation between U and $\mathcal{L}$ would approximately prevail at the inner edge of any torus where quasi-steady accretion is going on. Although self-consistency and stability considerations do somewhat constrain the acceptable forms of the angular momentum distribution (Abramowicz, Calvani, and Nobili, 1980), this model allows considerable freedom to specify the shape of the surface arbitrarily, and yield virtually no information about the interior. More generally, one can consider tori having angular velocity Ω goes as some power of $\mathcal{L}$ (Phinney, 1983). These tori can exist in all cases where the angular momentum increases with Ω at a slower rate compared to Keplerian. In this context the funnels tend to be conical rather than paraboloidal if the rotation law is close to Keplerian and they extend closer to $r \;=\; r_g$ when the black hole is rapidly rotating. Accretion flows possessing high internal pressures could resemble such tori if the viscosity were low enough to provide essentially circular flow and provided with stable configuration.

A generic feature of accretion tori is that they liberate energy per gram of infalling matter less efficiently as compared to thin disks. The efficiency is given by the binding energy of the material at the cusp which depends on the angular momentum profile described by the eq. (1.32). But for a constant angular momentum $\mathcal{L}$ torus of outer radius r_{out}, it is $(r_{out} / r_g)^{-1}$, which implies very low efficiency for large tori.

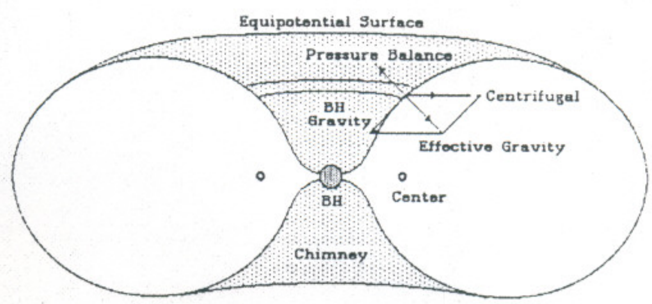

Fig. 1.6: An equipotential surface of a barotropic thick accretion disk is schematically shown along with the chimney (or funnel) along the axis. The directions of the gravitational force, centrifugal force, the net force along the effective gravity, and the force due to pressure gradient are shown with arrows. An equipotential surface is formed by the tangent vector normal to the directions of the pressure balance. (Reproduced from Chakrabarti, 1996)

Before we discuss various thick disk models it is worth to sketch a cartoon picture to indicate forces acting on a blob of matter inside a thick accretion disk. The *Fig. 1.6.* shows schematically an equipotential surface. The gravitational force acts radially inwards and the centrifugal force acts in a direction normal to the angular momentum vector. Addition of these two vectors produces a net force along the effective gravity. In order to remain in hydrostatic equilibrium, a force of equal magnitude due to the pressure gradient must act opposite to this direction. The tangents drawn normal to the pressure balance produce the equipotential surface.

However, tori may be supported by either radiation pressure or gas pressure. A radiation-pressure supported thin disk has a constant thickness $\sim r_{tr}$, the trapping radius defined as $r_{tr} = (\dot{M} / \dot{M}_{Edd})\, r_g$ where $\dot{M}_{Edd} = L_{Edd} / c^2$ is the critical accretion rate associated with the Eddington limit; hence it becomes geometrically thick at $4\, r_g < r \lesssim r_{tr}$, when the accretion rate exceeds the critical value $\dot{M}_{Edd}$. Indeed, in any configuration supported in this way, not only the total luminosity but its distribution over the surface is determined by the form of the isobars. Tori having long narrow funnels have the property that their total luminosity can exceed L_{Edd} by a logarithmic factor (Sikora, 1981). Most of this radiation escapes along the funnel, where centrifugal effects make the surface gravity (and hence the leakage of radiation) much larger than over the rest of the surface. If the accretion powers such a torus, then $\dot{m} \times$ (efficiency) $\gtrsim 10$. However, radiation tori are similar in many respects to the massive objects originally postulated by Hoyle and Fowler (1963a,b) and may suffer the same fate (i.e., dynamically unstable). There are possible axisymmetric local instabilities caused by unfavorable entropy and angular momentum gradients (e.g., Seguin, 1975; Kandrup, 1982). These presumably evolve to create marginally stable convection zones just as in a star. But more threatening are non-axisymmetric instabilities. Papaloizou and Pringle (1984) have demonstrated that a toroidal configuration known to be marginally stable to axisymmetric disturbances possesses global, non-axisymmetric dynamical instabilities. More detailed studies suggest that the modes are most damaging when

$$\frac{d\,\ell n\,\Omega}{d\,\ell n\,r} < -3^{1/2}. \tag{1.33}$$

As they are dynamical, these modes grow on the time scale of a few orbital periods.

$$t_{dyn} \sim \Omega^{-1} \sim \left(\frac{r}{H}\right)^2 \alpha^{-1}\, t_{vis}. \tag{1.34}$$

For a thick disk with $\alpha \sim 0.1$, as commonly assumed, they grow somewhat more rapidly than viscous modes. The tori will apparently destroy itself in a few orbital periods unless nonlinear terms can saturate the instability at a low amplitude. It is not yet known if this is the general property of these tori. Furthermore, it is not at all clear that tori can evolve towards stable or marginally stable states even if they exist, nor that the rate of internal energy generation through viscous dissipation can always be balanced by the transport. Numerical simulations (Hawley, 1989), exhibit the formation of counter-rotating *planets* as the non-linear evolution of this instability. These structures may be highly dissipative and destroy the disk. If this happens, then the concept of highly optically and geometrically thick torus, may be invalidated and in particular the fluid approximation may no longer be appropriate. Alternatively, (i) if the radial velocity is sufficiently large (i.e., inclusion of accretion flow) , particularly in the vicinity of the cusp, the equilibrium flow may convect the waves inwards and inhibits their reflection (Blaes, 1987; Hawley, 1991; Gat and Livio, 1992). The waves might then be maintained at a level of marginal stability transporting angular momentum just fast enough to limit the wave growth. In this sense, they may act like an effective viscosity and ultimately drive a steady disk flow, (ii) inclusion of self-gravity (Goodman and Narayan, 1988) could stabilize the tori. It was pointed out that the growth rate of the instability was too low to be of any astrophysical significance and decreases with increasing width of the torus. More numerical simulations are undoubtedly needed to settle this controversial matter.

A gas-pressure-dominated disk would become thick in the opposite limit, that of low accretion rate. Like for spherically symmetric inflow, the cooling time scale and even the electron-ion coupling time can be longer than the free-fall time in case of flow with angular momentum, provided that $\dot{m}$ is low enough. In this model the cooling time varies inversely with $\dot{M}$, while the infall time $\sim \alpha^{-1}\, t_{free-fall}$ should be insensitive to $\dot{M}$ for a turbulent or magnetic viscosity (Rees *et al.*, 1982) and the characteristic density for a given $\dot{m}$ is higher by α^{-1}. The condition for electron-ion coupling to be ineffective in the inner parts of a torus is

$$\dot{m}\,\alpha^{-2} < 50. \tag{1.35}$$

When the above equation holds, the ion can remain at the virial temperature even if synchrotron and Compton processes allows the the electrons to cool, and the disk swells up into a torus. The dominant viscosity is likely to be magnetic. Though the estimates of the magnetic viscosity are very uncertain, Eardley and Lightman (1976) suggest that α falls in the range $0.01 - 1.0$. So eq. (1.35) should definitely be fulfilled for sufficiently low accretion rates. An accretion flow where $\dot{m}$ is small, and where the radiative efficiency is low, it may seem that the model is doubly unpromising for any powerful galactic nucleus. Even though it may not radiate much directly, the torus around a spinning black hole offers an environment where the Blandford-Znajek (1977) process could operate and torus in turn serve as a catalyst for tapping the hole's latent spin energy. However, if there exist pair-productions or other collective plasma processess, it enhances the ion-electron coupling yielding to rapid cooling of protons (Sikora and Zbyszewska, 1985; Takahara and Kusunose, 1985; Begelman *et al.*, 1987). As a result, in the presence of strong magnetic field ion torus may be cooled rapidly and get deflated. then the restoring force can bring back the cool disk to the hot torus configuration and therefore it leads to instability which may be responsible for the case of AGN variabilities (Begelman *et al.*, 1987). In the case of gas pressure dominated thick disk, the gas may be completely photo-dissociated into protons and neutrons. Protons may be accreted onto the black hole as they experience the magnetic viscosity whereas neutrons may orbit around the hole till they decay and as a consequence, neutron torus may be formed which combines with incoming matter give rise to neutron-rich elements in the galaxy (Hogan and Applegate, 1987).

However, the likely presence of magnetic fields in both types of tori (i.e., radiation-dominated torus and ion-supported torus) may prompt a question whether this kind of configurations are dynamically stable. Therefore, global dynamical stability of magnetized tori has been a part of active research (Goldreich *et al.*, 1986, hereafter GGN; Blandford, 1990) and is not being completely investigated yet. With the recent renewed interest in magnetic processes spurred by the Balbus-Hawley (1991) instability (the local version of the Velikhov-Chandrasekhar instability), it seems natural to consider whether a global, non-axisymmetric counterpart exists in tori. In fact, Hawley (1991) has commented that the viability of thick disk may very well rest with this magnetic instability. Interestingly, Curry and Pudritz (1996) showed that thick and thin disks are equally susceptible to the magnetic version of the Papaloizou and Pringle (1984) instability, so applications need not be limited to the AGN context. They have shown that any differentially rotating disk threaded by even a weak magnetic field should be susceptible to the instabilities. They have demonstrated the existence of globally unstable, non-axisymmetric modes in incompressible MHD cylinders and, by extension, in astrophysical disks. In closing, since the instability acts for all allowable angular momentum distributions, both thick (i.e., radiation pressure supported and ion-supported tori) and thin disks should be equally affected.

Unlike magnetized thin disk models no similar models for thick magnetized disk have been developed which are associated with the production of jets. However, Chakrabarty and Prasanna (1982) showed the possible existence of thick disk structures due to the interaction of magnetic and intense gravitational fields. This analysis also revealed that the formation of cusp is possible only when the gravitational field is described by the general relativistic formalism and not in the Newtonian theory.

Furthermore, no detail study of the structure, stability of the pressure-supported magnetized disk is available in the literature. These models may help in understanding the various instabilities which can give better insight in various radiation mechanisms generated near the surface of a magnetized compact objects like neutron star or white dwarf.

1.5 Objective of the thesis

In the previous section we have discussed, in some detail, the standard model of thin accretion disk and the role of magnetic field in the disk dynamics. We have also pointed out that there are many theoretical difficulties in constructing a detail model of thick accretion disk. For instance, radiation-supported thick accretion disk models are subjected to powerful instabilities and therefore, the existence of a steady-state, radiation-supported thick disk structure may be doubtful. Despite these odds, study of thick accretion disk configuration may be important due to the following reasons:

(i) Thick disk steady-state structure may actually occur in the nature, e.g., around a proto-star or AGNs. In fact it was pointed out by Rees *et al.* (1982) that ion-supported thick disk structure may not be subjected to the powerful instability as suffered by a radiation dominated thick disk structures.

(ii) Studies of thick accretion disk are also interesting from theoretical point of view as they can provide valuable insight in the approximations invoked in thin accretion disk models. Also such studies might be useful in handling the intermediate cases, i.e., when the disk is neither thin nor thick. Therefore, in the work done by us in this thesis, we set out to study pressure-supported disk configuration and their stability without invoking any thin disk approximation.

In all the accretion disk models, the release of the gravitational potential energy of the infalling matter in form of radiation is the main source of the observed high luminosity in various astrophysical scenarios. It is generally beleived that though the most characteristic accretion phenomena taking place around a compact object having a strong gravitational field, the effects of general relativity do not play a crucial role in the physics of accretion disk. Therefore, most of the accretion disk models use the Newtonian framework to describe the disk structure and dynamics. However, the effects of general relativity have found to have following consequences:

(a) It can change the disk morphology,

(b) It can affect the inner radius of the disk (together with magnetic field),

(c) It can influence the signals coming out near the vicinity of the compact object.

(a) It is found that non-Newtonian stiffening of the gravitational potential close to the event horizon causes the equipotential surfaces for a given angular momentum distribution to form a toroidal cusp close to the event horizon. Thus, the cusp formation is purely a general relativistic effect.

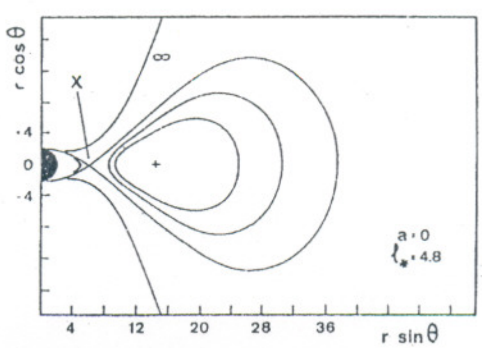

Fig. 1.7: Nested equipotential surfaces for a barotropic radiation torus in orbit about a massive black hole. In this example, it is assumed that the angular momentum is constant. Note the presence of a neutrally stable ring, X. Matter that fills the equipotential surface passing through X, is able to spill through onto the black hole. (Adopted from Blandford, 1990)

In the case of a thick disk, the angular momentum distribution intersects with the Keplerian distribution at two locations, r_{in} and r_c, the inner edge and the center of the disk respectively. Therefore, the equipotential surfaces of a thick accretion disk around a black hole possess cusp very similar to the Lagrange point in a Roche lobe overflow (*Fig. 1.7*). In this case, matter fills the closed potential and forms the thick accretion disk, and the excess matter is accreted to the black hole through the cusp. The cusp is formed at r_{in} between the marginally bound ($2r_g$) and marginally stable ($3r_g$) orbits. The separatrix passing through this cusp is a limiting surface for matter orbiting in a stationary torus shown in the *Fig. 1.7.* If a thick accretion disk develops, it is expected that the torus will expand until its innermost radius approaches the cusp. At this point there are no pressure gradients and the gravitational orbit is unstable. On passing through the cusp, matter will spill through onto the black hole with little further emission. On the contrary, in the Newtonian analysis, the angular momentum in a thick disk is monotonic and intersects this distribution only once, and therefore, no cusp is expected to form in a thick disk in Newtonian geometry.

(b) It is pointed out by Znajek (1976) that the strong gravitational field of the comapct object together with the magnetic field can support disk equilibrium in the close vicinity of the surface of the compact star. This conclusion also made by Prasanna and Varma (1977) while studying single particle trajectories in a magnetized Schwarzschild geometry.

(c) Gonthier and Harding (1994) have recently pointed out that the electromagnetic signals can be influenced by the curvature effect.

In many accretion disk scenarios the central objects possess an intrinsic magnetic field. Also the magnetic field might be generated in the accreting plasma due to dynamical processes. For example, although black holes do not have an intrinsic magnetic field, the analysis of Galeev *et al.* (1979) showed that as a result of stretching of the interstellar magnetic fields, the field gets amplified and becomes dynamically important. Magnetic fields could play varied and important role in the accretion disk scenarios depending on the concrete physical situations under considerations. For example, in the absence of the magnetic field, the transformation of kinetic energy into radiation is very small but this gets enhanced in the presence of the magnetic field due to the intense synchrotron radiation (Bisnovatyi-Kogan, 1979). They could be a source for coronal heating, formation and collimation of extragalactic jets (for a recent review see Spruit, 1996) etc. There is a alternative proposal involving magnetic fields in which jets are electrodynamically accelerated due to the unipolar induction dynamo effect, has also been considered (Lovelace *et al.*, 1976; Blandford, 1976; Blandford and Znajek, 1977). The strong intrinsic magnetic field, for the case of a neutron star, can influence many crucial parameters of the accretion disk like its inner edge and torque. This in turn can influence the spin-up and spin-down rates of the neutron star (Ghosh and lamb, 1978, 1979a,b, Lipunov, 1992). In these works, the authors have considered thin accretion disk around a neutron star within the Newtonian framework.

Most of the models of magnetized accretion disk, incorporate large scale ordered magnetic field and are successful in explaining many observational features. However, it was shown by Balbus and Hawley (1991) that accretion disks are dynamically unstable to axisymmetric shear perturbation when a weak magnetic field is present in the vertical direction. This instability could destroy the disk and may cast doubt about some models of magnetized accretion disks. But, it has been pointed out recently by Knobloch (1992) that such an instability is absent in the case when a toroidal component of the magnetic field is present.

Therefore, in this thesis, one investigates the structure and stability of a thick magnetized, pressure-supported disk and also analyze the role of general relativistic effects. The rest of the thesis is divided into the following chapters. The equilibrium configuration both in Newtonian and general relativistic framework and stability analyses are being discussed in detail in Chapter2. The conclusions and future work are summarized in Chapter3.

Chapter 2

Axisymmetric Magnetohydrodynamic Equilibrium Around a Magnetized Compact Object

2.1 Introduction

In this chapter, we discuss axisymmetric magnetohydrodynamic (MHD) equilibrium around a magnetized non-rotating compact object both in Newtonian and Schwarzschild background in detail. We first write basic set of MHD equations in general relativistic covariant form and subsequently apply them to study axisymmetric equilibrium solutions.

As mentioned in Chap. 1, though there have been numerous discussions of the magnetospheric theory with considerable progress, only a few of them attempts to obtain self-consistent equilibrium configurations of a thick pressure-supported disk in the presence of both poloidal and toroidal magnetic fields around a magnetized compact object. Furthermore, most of the the investigators have considered thin accretion disk around a neutron star or a black hole. Prasanna *et al.* (1989) and Tripathy *et al.* (1990) have studied, for various velocity profiles, both thin and thick magnetofluid disk equilibrium around a magnetized compact object in the Newtonian as well as in the curved background in detail. In these investigations authors have, however, not considered the effects of toroidal magnetic field in the disk dynamics. As we have mentioned in the Chap. 1 the toroidal magnetic field in the accretion disk provides the stabilizing influence over the magnetic shearing instability which arises due to shear in the accretion flows. The stabilizing effect stems out from the tension of the toroidal field which is the stabilizing force.

As it has been discussed in the previous chapter, our main interest is to examine equilibrium structure and stability of a pressure-supported magnetized thick disk. In what follows, we begin our study by considering a magnetically threaded disk (hereafter MTD) equilibria, around a central compact object, in which the magnetic field of the central star is penetrating inside the disk. This kind of models have been considered by earlier researchers (Li and Wang, 1996; Wang, 1995; Ghosh and Lamb, 1978, 1979a,b; Ghosh *et al.*, 1977; Lamb *et al.*, 1973) around a magnetized neutron star and such models are proved to be successful in explaining many observational facts like spin-up and spin-down rates of the observed pulsating sources. In these studies, authors have considered a thin magnetized Keplerian disk. Though in these works, accretion disk is described by highly conducting plasma, the stellar magnetic field penetrates the inner part of the disk *via* non-linear processes such as Kelvin-Helmholtz instability, turbulent diffusion, and reconnection.

The matter surrounding a neutron star is almost always in the form of a plasma at high temperature and hence has a high electric conductivity. The conductivity of a completely ionized plasma is estimated by the expression (Pikel'ner, 1966)

$$\lambda_c \approx 10^7 \, T_e^{3/2} \ s^{-1} \tag{2.1}$$

where T_e is the electron temperature. For $T_e \approx T_{ff} \approx 10^8 - 10^{10} \ K$, $\lambda_e \approx 10^{19} - 10^{22} \ cm^{-1}$, which is higher than the conductivity of copper. Therefore, Ideal magnetohydrodynamic (MHD) frame work can describe equilibrium accretion disk to a fairly good approximation (Lovelace *et al.*, 1986).

The effects of general relativity and the magnetic field could play a central role in determining important disk parameters like disk inner edge, influencing the emergent radiation coming from the inner edge of the disk as mentioned earlier. Since the effect of strong gravitational field is important only in the vicinity of the compact object, we are going to investigate the equilibrium structure of magnetized accretion disk solutions in the presence of strong gravitational field and a toroidal magnetic field in the corotation regime without invoking any thin disk approximations. It is possible to show that the solutions of the velocity profile satisfy the Ferraro's iso-rotation law in the corresponding Newtonian limit.

The complete solutions of magnetically threaded thick accretion disk is quite difficult to obtain. Therefore, in the present work, we first construct non-accreting (i.e., velocity flow is only in azimuthal direction) magnetized thick disk structures. The thick accretion disk model can be constructed by incorporating radial velocity profile perturbatively in such equilibria. The relevance of these solutions in the context of a neutron star in Schwarzschild geometry are discussed.

2.2 Formalism

The basic set of general relativistic magnetohydrodynamic equations is as follows :
Particle conservation :

$$(n U^i)_{;i} = 0; \tag{2.2}$$

where, n is the proper baryon number density, i.e., number of baryons per unit three-dimensional volume in the rest frame and U^i is the time-like fluid four-velocity with the normalization condition defined as

$$g_{ij} U^i U^j = +1; \tag{2.3}$$

where, g_{ij} is the general metric tensor defining the background geometry ;

$$ds^2 = g_{ij}\, dx^i\, dx^j . \tag{2.4}$$

Energy-momentum conservation :

$$(T^{ij}{}_{matt.} + T^{ij}{}_{e.m.})_{;j} = 0; \tag{2.5}$$

where, $T^{ij}{}_{matt.}$ and $T^{ij}{}_{e.m.}$ are energy-momentum tensor for the fluid and electromagnetic stress tensor respectively. Here, $T^{ij}{}_{matt.}$ for the general fluid is given by (heat conduction assumed to be negligible)

$$T^{ij}{}_{matt.} = (n \mathcal{F}/c^2) U^i U^j - \bar{P} g^{ij} + 2\eta_s \xi^{ij}; \tag{2.6}$$

where, $\mathcal{F} \equiv (\rho + \bar{P}) / n$ is the proper enthalpy per particle, ρ is the proper internal energy density and $\bar{P}$ is the proper pressure defined as

$$\bar{P} = P - (\eta_b - \frac{2}{3} \eta_s) U^i{}_{;i} ; \tag{2.7}$$

where, η_b and η_s are the coefficients of bulk and shear viscosity. ξ^{ij} is the conductivity tensor defined as

$$\xi^{ij} = \frac{1}{2} (U^{i;k} h^j{}_k + U^{j;k} h^i{}_k) - \frac{1}{3} U^k{}_{;k} h^{ij} ; \tag{2.8}$$

with h^{ij} being the projection tensor defined as

$$h^{ij} = g^{ij} - U^i U^j . \tag{2.9}$$

The electromagnetic stress tensor $T^{ij}{}_{e.m.}$ is related to the electromagnetic field tensor F_{ij} through

$$T^{ij}{}_{e.m.} = F^{ik} F^j{}_k - \frac{1}{4} g^{ij} F_{kl} F^{kl} ; \tag{2.10}$$

where, the antisymmetric field tensor F_{ij} is defined through a vector potential as :

$$F_{ij} = A_{j\,;i} - A_{i\,;j} . \tag{2.11}$$

The covariant form of the Maxwell's equations can be written as :

$$F^{ij}{}_{;j} = (4 \pi / c) J^i ; \tag{2.12}$$

$$F_{(ij;k)} = 0 ; \tag{2.13}$$

with

$$(T^{ij}{}_{e.m.})_{;j} = \frac{1}{c} F^i{}_j J^j ; \tag{2.14}$$

where, J^j is the plasma four current density defined through the covariant form of Ohm's law as :

$$J^i = c \rho_e U^i + \sigma^{ij} F_{jk} U^k ; \tag{2.15}$$

where, ρ_e is the charge density measured locally and σ^{ij} is the electrical conductivity tensor of the plasma fluid.

In the present work, we will consider the complete set of dynamical equations which govern the magnetohydrodynamic flow of the fluid disk on a given curved background with the following assumptions:

(i) The fluid disk is not massive in comparison with the central compact object such that the spacetime structure supporting the disk is entirely determined by the central body. Thus, the self-gravity of the disk is considered to be negligible.

(ii) The energy of the electromagnetic field is regarded to be negligible as compared to the energy associated with the mass of the central object. Thus, the electromagnetic fields do not influence the geometry but, they can be modified by the geometry of the central object. Further, we consider the central object to possess a poloidal magnetic field (Ginzburg and Ozernoi, 1965; see also Prasanna and Varma, 1977; Wasserman and Shapiro, 1983) which has the usual dipolar form in the asymptotic limit of the Schwarzschild metric.

(iii) Though the central compact object may be in general rotating, the Schwarzschild geometry is used to describe the spacetime structure. This is because it is assumed that the angular momentum parameter $a \ll 1$ and which, indeed, seems to be the case for most of the pulsars (e.g. Gonthier and Harding, 1994).

Molecular and magnetic Reynold numbers, defined by $\mathcal{R}_{mol} = (V\,L\,/\,\nu_{mol})$ and $\mathcal{R}_{mag} = (V\,L\,/\,\nu_{mag})$ respectively, are usually very large for a typical accretion disk scenario. Here, V, L, and $\nu_{mol\,(mag)}$ are typical flow velocity, flow length, and the coefficients of the molecular (magnetic) viscosity respectively. Indeed, accretion disk are characterized by very large Reynolds numbers ranging from 10^{14} for a white dwarf disk to 10^{26} for an AGN disk (Dubrulle and Knobloch, 1992). This allows one to neglect molecular viscosity and resistivity terms from eqs. (2.6) and (2.15). The effect of turbulence or dissipative forces such as anomalous viscosity and resistivity is regarded as small compared to the long ranged ordered equilibrium forces. Such effects may be introduced perturbatively over the equilibrium force balance (e.g. Lovelace *et al.*, 1986).

The Schwarzschild metric is defined by:

$$ds^2 = \left(1 - \frac{2m}{r}\right) c^2 dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2); \qquad (2.16)$$

where, $m = GM / c^2$ with G as universal gravitation constant, M is the mass of the central object and c is the velocity of light. In this notation, $2m$ is the Schwarzschild radius. We express the dynamical equations in terms of physical quantities by writing them in the orthonormal tetrad frame appropriate to the Schwarzschild metric ;

$$\lambda^i_{(a)} = diag\left[\left(1 - \frac{2m}{r}\right)^{-1/2}, \left(1 - \frac{2m}{r}\right)^{1/2}, \frac{1}{r}, \frac{1}{r\sin\theta}\right]; \qquad (2.17)$$

satisfying

$$\lambda^i{}_{(a)} \lambda^j{}_{(b)} g_{ij} = \eta_{(a)(b)}; \qquad (2.18)$$

where, $\eta_{(a)(b)}$ is the metric tensor defined in Local Lorentz Frame. All the global variables are then defined in Local Lorentz Frame as,

$$F_{(a)(b)} = \lambda^i_{(a)} \lambda^j_{(b)} F_{ij}; \qquad (2.19)$$

$$J_{(a)} = \lambda^i_{(a)} J_i; \qquad (2.20)$$

$$E_{(a)} = F_{(a)(t)}; \qquad (2.21)$$

$$B_{(a)} = \epsilon_{abc} F_{(b)(c)}; \qquad (2.22)$$

where, ϵ_{abc} is the Levi-civita symbol. Using these definitions, the electromagnetic field components can be explicitly written as,

$$F_{(r)(\theta)} = \frac{1}{r}\left(1 - \frac{2m}{r}\right)^{1/2} F_{r\theta}; \qquad (2.23)$$

$$F_{(r)(t)} = F_{rt}; \qquad (2.24)$$

$$F_{(\theta)(\phi)} = \frac{1}{r^2 \sin\theta} F_{\theta\phi}; \qquad (2.25)$$

$$F_{(\theta)(t)} = \frac{1}{r}\left(1 - \frac{2m}{r}\right)^{-1/2} F_{\theta t}; \qquad (2.26)$$

$$F_{(\phi)(r)} = \frac{1}{r\sin\theta}\left(1 - \frac{2m}{r}\right)^{1/2} F_{\phi r}; \qquad (2.27)$$

$$F_{(\phi)(t)} = \frac{1}{r\sin\theta}\left(1 - \frac{2m}{r}\right)^{-1/2} F_{\phi t}. \qquad (2.28)$$

Using the same tetrad eqn. (2.17), one can also express the spatial 3-velocity V_α, defined through the relation $U_\alpha = U_0 \, (V_\alpha \, / \, c)$, in terms of local Lorentz components as given by,

$$V_{(r)} = \left(1 - \frac{2\,m}{r}\right)^{-1} V_r \, ; \tag{2.29}$$

$$V_{(\theta)} = r \left(1 - \frac{2\,m}{r}\right)^{-1/2} V_\theta \, ; \tag{2.30}$$

$$V_{(\varphi)} = r \, \sin\theta \left(1 - \frac{2\,m}{r}\right)^{-1/2} V_\varphi \, . \tag{2.31}$$

Further, we assume, as a first approximation, that the radial and meridional components of the velocity are zero i.e. $V_{(r)} = V_{(\theta)} = 0$, it implies that $\nabla \cdot \mathbf{V} = \mathbf{0}$ is identically satisfied. Next, as the poloidal components of the field are external, the ring current density is absent inside the disk (i.e. $J_{(\varphi)} = 0$). In the ideal MHD limit the equation (2.15) reduces to

$$\mathbf{E}_p = -\frac{1}{c} \left(\mathbf{V} \times \mathbf{B}_p \right) ; \tag{2.32}$$

where, subscript p represents the poloidal component of the fields. Eq. (2.32) has similar form as that of the Ohm's law in the Newtonian MHD equation. As a consequence of stationarity and axisymmetry, the toroidal component of the electric field is zero.

In order to study the problem in a dimensionless form, we introduce the following three parameters : B_0, B_1, and V_0 signifying the strength of the external magnetic field on the surface of the neutron star, the strength of the toroidal magnetic field at the inner edge of the disk, and the velocity of the fluid at the inner edge of the disk respectively. We choose the geometric unit system, i.e. $G = c = 1$. The fundamental unit of length is m and the Schwarzschild coordinate r is normalized with respect to m by introducing a dimensionless variable $\tilde{r} = r \, / \, m$. By regarding all the quantities with tilde as dimensionless, all the MHD variables can be written as follows : poloidal magnetic field $B_{(P)} = B_0 \, \tilde{B}_{(P)}$, toroidal magnetic field $B_{(T)} = B_1 \, \tilde{B}_{(T)}$, velocity field $V_{(\varphi)} = V_0 \, \tilde{V}_{(\varphi)}$, poloidal electric field $E_{(P)} = E_0 \, \tilde{E}_{(P)}$, poloidal current density $J_{(P)} = J_0 \, \tilde{J}_{(P)}$, proper enthalpy $\mathcal{F} = \mathcal{F}_0 \, \tilde{\mathcal{F}}$, and proper pressure $P = P_0 \, \tilde{P}$. Here, $E_0 = V_0 \, B_0$, $J_0 = B_1 \, / \, 4\,\pi\, m$, $\mathcal{F}_0 = (1 \, / \, 4\,\pi) \, (\, B_0 \, / \, V_0 \,)^2$, and $P_0 = {B_0}^2 \, / \, 4\,\pi$. With these substitutions, all the basic equations of the problem reduce to dimensionless form and contain two dimensionless parameters (α and β) whose meaning will be clarified later.

Now using the eqns. (2.13) and (2.32), one gets,

$$\tilde{r}\left(1-\frac{2}{\tilde{r}}\right)^{1/2}\tilde{B}_{(r)}\frac{\partial\tilde{V}_{(\varphi)}}{\partial\tilde{r}}+\tilde{B}_{(\theta)}\frac{\partial\tilde{V}_{(\varphi)}}{\partial\theta}+\tilde{V}_{(\varphi)}\left[\left(1-\frac{2}{\tilde{r}}\right)^{1/2}\tilde{B}_{(r)}\right.$$

$$\left.+\frac{1}{\tilde{r}}\left(1-\frac{2}{\tilde{r}}\right)^{-1/2}\tilde{B}_{(r)}+\tilde{r}\left(1-\frac{2}{\tilde{r}}\right)^{1/2}\frac{\partial\tilde{B}_{(r)}}{\partial\tilde{r}}+\frac{\partial\tilde{B}_{(\theta)}}{\partial\theta}\right]=0\,; \qquad (2.33)$$

the solution of which will give the azimuthal component of the velocity field in the presence of the external poloidal magnetic field. From the equation (2.12), one can get

$$\tilde{J}_{(t)} = -\frac{1}{\tilde{r}^2}\left(1-\frac{2}{\tilde{r}}\right)^{1/2}\frac{\partial}{\partial\tilde{r}}\left[\tilde{r}^2\,\tilde{E}_{(r)}\right]+\frac{1}{\tilde{r}\,\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\,\tilde{E}_{(\theta)}\right]\,; \qquad (2.34)$$

$$\tilde{J}_{(r)} = -\frac{1}{\tilde{r}\,\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\,\tilde{B}_{(\varphi)}\right]\,; \qquad (2.35)$$

$$\tilde{J}_{(\theta)} = \frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}\left[\tilde{r}\left(1-\frac{2}{\tilde{r}}\right)^{1/2}\tilde{B}_{(\varphi)}\right]. \qquad (2.36)$$

For the present case, the azimuthal component of the eq. (2.5) reads

$$\tilde{B}_{(r)}\,\tilde{J}_{(\theta)}-\tilde{B}_{(\theta)}\,\tilde{J}_{(r)}=0\,; \qquad (2.37)$$

which constrains the poloidal components of the current density. $\tilde{B}_{(\varphi)}$ can be determined from eq. (2.37) by eliminating the current densities in the equation (2.37) by using equations (2.35) and (2.36), *viz.*

$$\tilde{r}\left(1-\frac{2}{\tilde{r}}\right)^{1/2}\tilde{B}_{(r)}\frac{\partial\tilde{B}_{(\varphi)}}{\partial\tilde{r}}+\tilde{B}_{(\theta)}\frac{\partial\tilde{B}_{(\varphi)}}{\partial\theta}+\tilde{B}_{(\varphi)}\left[\left(1-\frac{2}{\tilde{r}}\right)^{1/2}\tilde{B}_{(r)}\right.$$

$$\left.+\frac{1}{\tilde{r}}\left(1-\frac{2}{\tilde{r}}\right)^{-1/2}\tilde{B}_{(r)}+\cot\theta\,\tilde{B}_{(\theta)}\right]=0\,. \qquad (2.38)$$

Equation (2.38) clearly tells that the toroidal component of the magnetic field is coupled only to the external poloidal magnetic field. The poloidal components of the Euler equation (2.5) are the following :

$$\frac{\partial\tilde{P}}{\partial\tilde{r}} = -\tilde{\mathcal{A}}\left\{\left(1-\frac{2}{\tilde{r}}\right)^{-1}\tilde{\mathcal{F}}'\left[\frac{\alpha}{\tilde{r}^2}-\left(1-\frac{2}{\tilde{r}}\right)\frac{\tilde{V}^2_{(\varphi)}}{\tilde{r}}\right]\right.$$

$$\left.-\left(1-\frac{2}{\tilde{r}}\right)^{-1/2}\left[\gamma^2\,\tilde{E}_{(r)}\,\tilde{J}_{(t)}+\beta^2\,\tilde{B}_{(\varphi)}\,\tilde{J}_{(\theta)}\right]\right\}\,; \qquad (2.39)$$

$$\frac{\partial \widetilde{P}}{\partial \theta} = \widetilde{\mathcal{A}} \left\{ \cot\theta \, \widetilde{V}^2_{(\varphi)} \, \widetilde{\mathcal{F}}' - \left[\gamma^2 \, \widetilde{r} \, \widetilde{E}_{(\theta)} \, \widetilde{J}_{(t)} - \beta^2 \, \widetilde{r} \, \widetilde{B}_{(\varphi)} \, \widetilde{J}_{(r)} \right] \right\} ; \tag{2.40}$$

where, $\widetilde{\mathcal{F}}'$ is defined as $\widetilde{\mathcal{F}}' = \left(1 - \gamma^2 \, \widetilde{V}^2_{(\varphi)}\right)^{-1} \widetilde{\mathcal{F}}$ with $\gamma = V_0 \, / \, c$ and $\widetilde{\mathcal{A}} = {B_0}^2 \, / \, 4\,\pi\, P_0$. The two dimensionless parameters, defined as $\alpha = 1 \, / \, {V_0}^2$ which signifies, in this normalization scheme, the ratio of gravitational energy of the fluid element at the inner edge of the disk due to the central object to its kinetic energy and $\beta = B_1 \, / \, B_0$ which denotes the ratio of the field strength of the toroidal to poloidal magnetic fields respectively.

2.3 Possible Equilibrium Solutions

We first solve equation (2.38) and its solutions describe the form of the toroidal magnetic field in the presence of external poloidal field. This equation is a first order homogeneous partial differential equation whose solution is obtained by using the method used by Ginzburg and Ozernoi (1965) *viz.* the flat space solution (BBDP) is multiplied by a scalar function $\widetilde{h}(\widetilde{r})$ of the Schwarzschild coordinate $\widetilde{r}$ to take account of general relativistic effects. The partial differential equation will reduce to an ordinary differential equation for $\widetilde{h}(\widetilde{r})$ with $\widetilde{r}$ as an independent variable. Next, we impose the $\widetilde{r}$ as an independent variable. Next, we impose the condition that as $\widetilde{r} \to \infty$, $\widetilde{h}(\widetilde{r}) \to 1$. This will ensure that in the asymptotic limit the solution will reduce to that of the flat space. The components of the external magnetic field of the central object in Schwarzschild coordinate can be written in dimensionless form as :

$$\widetilde{B}_{(r)} = \frac{2}{\widetilde{r}^3} \left\{ \left(\frac{-3\widetilde{r}^3}{8} \right) \widetilde{B}_g \right\} \cos\theta ; \tag{2.41}$$

$$\widetilde{B}_{(\theta)} = \frac{1}{\widetilde{r}^3} \left\{ \left(\frac{3\widetilde{r}^3}{4} \right) \widetilde{B}_h \right\} \left(1 - \frac{2}{\widetilde{r}}\right)^{1/2} \sin\theta ; \tag{2.42}$$

where, the dimensionless variables $\widetilde{B}_g$ and $\widetilde{B}_h$ are defined as :

$$\widetilde{B}_g = \ell n \left(1 - \frac{2}{\widetilde{r}}\right) + \frac{2}{\widetilde{r}} \left(1 + \frac{1}{\widetilde{r}}\right) ; \tag{2.43}$$

$$\widetilde{B}_h = \ell n \left(1 - \frac{2}{\widetilde{r}}\right) + \frac{1}{\widetilde{r}} \left(1 - \frac{2}{\widetilde{r}}\right)^{-1} + \frac{1}{\widetilde{r}} ; \tag{2.44}$$

which approach to unity in the asymptotic limit. By substituting $\widetilde{B}_{(r)}$ and $\widetilde{B}_{(\theta)}$ into eq. (2.38), the solution of the toroidal component of the magnetic field is found to be

$$\widetilde{B}_{(\varphi)} = \frac{1}{\tilde{r}^{7/2}} \left\{ \left(\frac{-3\tilde{r}^3}{8} \right) \widetilde{B}_g \right\}^{5/2} \left(1 - \frac{2}{\tilde{r}} \right)^{-1/2} \sin^4 \theta \tag{2.45}$$

The above solution reduces to Newtonian solution of BBDP in the asymptotic limit. As we mentioned in the introduction, we have considered two kinds of solutions which in the flat space limit reduce to (i) quasi-Keplerian velocity profile, (ii) rigid rotation velocity profile, and are furnished below :

2.3.1 Quasi-Keplerian velocity profile

By substituting the explicit forms of $\widetilde{B}_{(r)}$ and $\widetilde{B}_{(\theta)}$ into eq. (2.33), the azimuthal component of the velocity field is found to be

$$\widetilde{V}_{(\varphi)} = \frac{1}{\tilde{r}^{1/2}} \left\{ \left(\frac{-3\tilde{r}^3}{8} \right) \widetilde{B}_g \right\}^{3/2} \left(1 - \frac{2}{\tilde{r}} \right)^{-1/2} \sin^4 \theta \tag{2.46}$$

The above solution also reduces to Newtonian one (BBDP, 1995) in the asymptotic limit. At $\theta = \pi/2$ plane, eq.(2.46) shows departure from the Keplerian profile due to the general relativistic effect. The poloidal components of the electric field are found from the eq. (2.32) :

$$\widetilde{E}_{(r)} = \frac{1}{\tilde{r}^{7/2}} \left\{ \left(\frac{-3\tilde{r}^3}{8} \right) \widetilde{B}_g \right\}^{3/2} \left\{ \left(\frac{3\tilde{r}^3}{4} \right) \widetilde{B}_h \right\} \sin^5 \theta\,; \tag{2.47}$$

$$\widetilde{E}_{(\theta)} = -\frac{2}{\tilde{r}^{7/2}} \left\{ \left(\frac{-3\tilde{r}^3}{8} \right) \widetilde{B}_g \right\}^{5/2} \left(1 - \frac{2}{\tilde{r}} \right)^{-1/2} \sin^4 \theta \cos \theta. \tag{2.48}$$

The components of the current density **J** are found to be

$$\widetilde{J}_{(r)} = -\frac{2}{\tilde{r}^{9/2}} \left[\left(-\frac{3\,\tilde{r}^3}{8} \right) \widetilde{B}_g \right]^{5/2} \left(1 - \frac{2}{\tilde{r}} \right)^{-1/2} \sin^3 \theta \cos \theta\,; \tag{2.49}$$

$$\widetilde{J}_{(\theta)} = -\frac{1}{\tilde{r}^{9/2}} \left[\left(-\frac{3\,\tilde{r}^3}{8} \right) \widetilde{B}_g \right]^{3/2} \left[\left(\frac{3\,\tilde{r}^3}{4} \right) \widetilde{B}_h \right] \sin^4 \theta\,. \tag{2.50}$$

and the charge density $\widetilde{J}_{(t)}$ is obtained by substituting the expressions for the poloidal components of the electric field in the eq. (2.34) to get,

$$\tilde{J}_{(t)} = \frac{2}{\tilde{r}^{9/2}} \left[\frac{3}{4} \left\{ \left(\frac{-3\,\tilde{r}^3}{8} \right) \tilde{B}_g \right\}^{1/2} \left\{ \left(\frac{3\,\tilde{r}^3}{4} \right) \tilde{B}_h \right\}^2 \left(1 - \frac{2}{\tilde{r}} \right)^{1/2} \right.$$

$$- \tilde{r} \left\{ \left(\frac{-3\,\tilde{r}^3}{8} \right) \tilde{B}_g \right\}^{3/2} \left\{ \left(\frac{3\,\tilde{r}^3}{4} \right) \tilde{B}_h \right\}' \left(1 - \frac{2}{\tilde{r}} \right)^{1/2}$$

$$\left. + \left\{ \left(\frac{3\,\tilde{r}^3}{8} \right) \tilde{B}_g \right\}^{5/2} \left(1 - \frac{2}{\tilde{r}} \right)^{-1/2} \left(5 \sin^{-2}\theta - 6 \right) \right] \sin^5\theta\,; \qquad (2.51)$$

where, the prime stands for ordinary derivative with respect to $\tilde{r}$.

The Euler equations (2.39) and (2.40) are then used to determine the proper pressure and the proper enthalpy for given $\tilde{V}_{(\varphi)}$ and $\tilde{B}_{(\varphi)}$. We assume the bulk motion of the fluid satisfies $V_0 \ll c$ whereas the internal motion can still be relativistic. This assumption is consistent with the accretion disk scenario because the infalling gas can convert its bulk kinetic energy into its internal energy. Consequently, the contribution of the term due to electric field is neglected in the Euler's equation. The correct solution of eqs. (2.39) and (2.40) must satisfy the compatibility condition $\left[\frac{\partial}{\partial\theta} \left(\frac{\partial \tilde{P}}{\partial \tilde{r}} \right) = \frac{\partial}{\partial \tilde{r}} \left(\frac{\partial \tilde{P}}{\partial \theta} \right) \right]$ from which an equation for proper enthalpy $\tilde{\mathcal{F}}$ is determined, *viz.*

$$\cot\theta\, \tilde{V}_{(\varphi)}^2 \frac{\partial \tilde{\mathcal{F}}}{\partial \tilde{r}} + \left(1 - \frac{2}{\tilde{r}} \right)^{-1} \left[\frac{\alpha}{\tilde{r}^2} - \left(1 - \frac{2}{\tilde{r}} \right) \frac{\tilde{V}_{(\varphi)}^2}{\tilde{r}} \right] \frac{\partial \tilde{\mathcal{F}}}{\partial \theta}$$

$$+ 2\, \tilde{\mathcal{F}}\, \tilde{V}_{(\varphi)} \left[\cot\theta\, \frac{\partial \tilde{V}_{(\varphi)}}{\partial \tilde{r}} - \frac{1}{\tilde{r}} \frac{\partial \tilde{V}_{(\varphi)}}{\partial \theta} \right]$$

$$= -\,\beta^2 \left[\frac{\partial}{\partial \tilde{r}} \left(\tilde{r}\, \tilde{B}_{(\varphi)}\, \tilde{J}_{(r)} \right) + \left(1 - \frac{2}{\tilde{r}} \right)^{-1/2} \frac{\partial}{\partial \theta} \left(\tilde{B}_{(\varphi)}\, \tilde{J}_{(\theta)} \right) \right] \qquad (2.52)$$

We see that the proper enthalpy stratification does not depend on the external poloidal field. It is determined by the azimuthal components of the velocity and magnetic fields only. After substituting for $\tilde{B}_{(\varphi)}$, $\tilde{V}_{(\varphi)}$, $\tilde{J}_{(r)}$, and $\tilde{J}_{(\theta)}$, one could get the solution of the eq. (2.52) by using the characteristic method. The solution is found to be

$$\tilde{\mathcal{F}} = \frac{1}{\tilde{r}^6} \left[\frac{15}{8} \left(a_1\,\alpha - \sin^8\theta \right)^{-15/8} + \beta^2\, a_2 \right] \left\{ \left(\frac{-3\tilde{r}^3}{8} \right) \tilde{B}_g \right\}^{-3} \left(1 - \frac{2}{\tilde{r}} \right); \qquad (2.53)$$

where, a_1 and a_2 are defined as :

$$a_1 = \frac{8\,\mathcal{I}_1}{\tilde{r}^8}\,; \qquad (2.54)$$

$$a_2 \;=\; -\,5\,\mathcal{I}_2\,\tilde{r}^{15}\,; \tag{2.55}$$

where, $\mathcal{I}_1$ and $\mathcal{I}_2$ are two integrals defined as :

$$\mathcal{I}_1 = \int \tilde{r}^{7}\left\{\left(\frac{-3\tilde{r}^3}{8}\right)\tilde{B}_g\right\}^{-3} d\tilde{r}\,; \tag{2.56}$$

$$\mathcal{I}_2 = \int \tilde{r}^{-16}\left[\left\{\left(\frac{-3\tilde{r}^3}{8}\right)\tilde{B}_g\right\}^{4}\left\{\left(\frac{3\tilde{r}^3}{4}\right)\tilde{B}_h\right\}\right.$$

$$\left.+\,2\left\{\left(\frac{-3\,\tilde{r}}{8}\right)\tilde{B}_g\right\}^{5}\left\{1+\frac{1}{\tilde{r}}\left(1-\frac{2}{\tilde{r}}\right)^{-1}\right\}\right]\left(1-\frac{2}{\tilde{r}}\right)^{-1} d\tilde{r}\,. \tag{2.57}$$

One can see from the above that a_1 and a_2 tend to unity in the asymptotic limit. Finally, after substituting for $\tilde{\mathcal{F}}$, $\tilde{V}_{(\varphi)}$, and $\tilde{B}_{(\varphi)}$ into the eq. (2.39) and integrating over $\tilde{r}$ we obtain *viz.*

$$\tilde{P} \;=\; \frac{1}{\tilde{r}^7}\left[\frac{15}{8}\left(a_1\,\alpha\;-\;\sin^8\theta\right)^{-7/8} + \beta^2\left(a_3\,\alpha\;-\;3.5\,a_4\,\sin^8\theta\right)\right]; \tag{2.58}$$

where, the variables a_3 and a_4 are defined as :

$$a_3 \;=\; 35\,\mathcal{I}_3\,\tilde{r}^{7}\,; \tag{2.59}$$

$$a_4 \;=\; \frac{(5\,\tilde{S}(\tilde{r})\;-\;a_2)}{4}\,; \tag{2.60}$$

with

$$\tilde{S}(\tilde{r}) \;=\; \left\{\left(\frac{-3\tilde{r}^3}{8}\right)\tilde{B}_g\right\}^{5}\left(1\;-\;\frac{2}{\tilde{r}}\right)^{-1}\,; \tag{2.61}$$

$\mathcal{I}_3$ being the integral defined as

$$\mathcal{I}_3 \;=\; \int \tilde{r}^{7}\left\{\left(\frac{-3\tilde{r}^3}{8}\right)\tilde{B}_g\right\}^{-3}\mathcal{I}_2\,d\tilde{r}\,. \tag{2.62}$$

2.3.2 Solutions obtained in Newtonian limit

By assuming the proper enthalpy function represents only the rest mass energy density, it can be shown that the eqs. (2.45)-(2.46), (2.53) & (2.58) will be reduced to the expressions for the toroidal magnetic field, azimuthal velocity field, the density and the pressure respectively in the Newtonian limit (BBDP, 1995). The derived solutions in the Newtonian limit are given below:

$$B_\varphi = B_1 \left(\frac{r_{in}}{r}\right)^{7/2} \sin^4\theta \,; \tag{2.63}$$

$$V_\varphi = V_0 \left(\frac{r_{in}}{r}\right)^{1/2} \sin^4\theta \,; \tag{2.64}$$

$$\rho_m = \rho_{m_0} \left(\frac{r_{in}}{r}\right)^{6} \left[\frac{15}{8}(\alpha - \sin^8\theta)^{-15/8} + \beta^2\right] ; \tag{2.65}$$

$$P = \frac{B_0^2}{28\pi} \left(\frac{r_{in}}{r}\right)^{7} \left[\frac{15}{8}(\alpha - sin^8\theta)^{-7/8} + \beta^2(\alpha - \frac{7}{2}sin^8\theta)\right] . \tag{2.66}$$

where, the set of above solutions are normalised r_{in} which is denoted as the inner edge of the disk. In our case, α has a natural lower bound $\alpha > 1$ as the plasma pressure P and the matter density ρ_m should be nowhere singular.

To study the magnetic field configuration within the disk, the solutions of magnetic lines-of-force equations,

$$\frac{dr}{B_r} = \frac{r d\theta}{B_\theta} = \frac{r \sin\theta d\phi}{B_\varphi} \,; \tag{2.67}$$

should be analyzed. In order to visualise the field line structure, it is useful to transform over to a Cartesian frame through the usual relations ($X = r\sin\theta\cos\phi, Y = r\sin\theta\sin\phi, Z = r\cos\theta$) and then to obtain the corresponding parametric equations that generate the curves, *viz.*

$$X = r_{in} \cos\left[\phi_0 - \beta\left(\frac{r_{in}}{R}\right)^3 \cos\theta\right] \sin^3\theta; \tag{2.68}$$

$$Y = r_{in} \sin\left[\phi_0 - \beta\left(\frac{r_{in}}{R}\right)^3 \cos\theta\right] \sin^3\theta \,; \tag{2.69}$$

$$Z = r_{in} \sin^2\theta \cos\theta \,; \tag{2.70}$$

where ϕ_0 is a constant of integration(due to the azimuthal symmetry it can be set to zero, without any loss of generality) and R defined as the radius of the compact object. From the eqs. (2.68)-(2.70), one can see that self-consistently generated toroidal magnetic field can affect magnetic field line structures in the disk and this will be discussed in detail in the next section.

2.3.3 Stability Analysis in Newtonian limit

The general question of stability of these equilibrium solutions is extremely difficult and beyond the scope of present paper. Therefore, we have considered a simplified stability problem under the following assumptions:

(**1**) The structure of the disk is assumed to have cylindrical symmetry and the gravitational potential of the compact object does not change appreciably along the z-axis. Hence, only the gravitational potential along the radial direction affects the perturbations through its effect on the equilibrium values of the magnetic field and fluid rotation.

(**2**) We assume only the B_z component of the magnetic field to be non-zero.
With these assumptions, the equilibrium configuration is generally that of a cylinder with an annular cross-section and all the physical variables governing the flow are functions of only the radial cylidrical coordinate. Therefore, our analysis is valid only around the z=0 plane and in the limit $B_\varphi \to 0$. In this regime of parameter space, one can draw an analogy with the stability analysis of the Couette flow carried out by Chandrasekhar (1961), but with two differences: *viz* the magnetic field and density are not constant. We would like to note that although the equilibrium density profiles are not constant, we have assumed the perturbations in the velocity fields to be solenoidal. This is quite compatible with incompressibility assumption. Incompressible fluids can support density stratified profiles in the external field like gravity and the global stability of such equilibria had been analyzed in the literatures (e.g. Chandrasekhar 1961).

Following the analysis of Chandrasekhar (1961) we consider, with appropriate modifications (i.e. $\mathbf{B} = [0, 0, B_z(r)]$ and $\rho_m \neq$ constant) the resulting simpler set of equations that allow the stationary solutions:

$$u_r = u_z = 0, \quad V_\varphi = V(r) = r\Omega(r),$$

$$B_r = B_\varphi = 0, \quad B_z = B_z(r), \tag{2.71}$$

$$\rho_m = \rho_m(r), \quad \text{and} \quad P = P(r)$$

where $V(r)$, $B_z(r)$, $\rho_m(r)$, and $P(r)$ are now *arbitrary functions* of r and $\theta = \frac{\pi}{2}$
Let us consider an infinitesimal perturbation of the flow represented by the solution (2.71),

$$u_r, \quad V + u_\varphi, \quad u_z, \quad b_r, \quad b_\varphi, \quad B_z(r) + b_z, \quad \rho_m(r) + \rho_{m_1}, \quad \text{and} \quad P(r) + P_1. \tag{2.72}$$

The linear equations governing these perturbations are easily found to be:

$$\frac{\partial \rho_{m_1}}{\partial t} + \mathbf{u} \cdot \nabla \rho_m = 0, \tag{2.73}$$

$$\frac{\partial \mathbf{b}}{\partial t} - \nabla \times (\mathbf{V} \times \mathbf{b}) - \nabla \times (\mathbf{u} \times \mathbf{B}) = 0, \tag{2.74}$$

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{V} + \frac{\nabla P_1}{\rho_m} - \left(\frac{\nabla P}{{\rho_m}^2}\right) \rho_{m_1} & - \\ \left(\frac{1}{4\pi \rho_m}\right) [(\nabla \times \mathbf{B}) \times \mathbf{b} + (\nabla \times \mathbf{b}) \times \mathbf{B}] & = 0. \end{aligned} \tag{2.75}$$

In addition to these equations, one takes into account the conditions for the solenoidal character of u and b.

In accordance with the general procedure, one analyzes the disturbance in terms of axisymmetric normal modes. We assume the various quantities describing the perturbation to have a (t,z)-dependence as given by:

$$exp\,[i(\omega t + kz)], \tag{2.76}$$

where ω is a constant (which can be complex) representing frequency and k is the wave number of the disturbance in the z-direction.
Introducing Lagrange variables ξ_r, ξ_φ, and ξ_z related to u_r, u_φ, and u_z by

$$u_r = i\omega\xi_r, \quad u_\varphi = i\omega\xi_\varphi - \left(\frac{dV}{dr} - \frac{V}{r}\right)\xi_r, \quad \text{and} \quad u_z = i\omega\xi_z, \tag{2.77}$$

For the 'incompressible' perturbations,

$$\nabla \cdot \xi = 0. \tag{2.78}$$

Equation (2.74) gives,

$$b_r = ikB_z\xi_r, \quad b_\varphi = ikB_z\xi_\varphi, \tag{2.79}$$

and

$$b_z = ikB_z\xi_z - \frac{dB_z}{dr}\xi_r. \tag{2.80}$$

Then from the equations (2.73) and (2.75), one gets,

$$\rho_{m_1} = -\left(\frac{d\rho_m}{dr}\right)\xi_r, \tag{2.81}$$

$$\left[\left(\omega^2 - 2r\Omega\frac{d\Omega}{dr} - \Omega_B^2\right) - \frac{1}{{\rho_m}^2}\left(\frac{d\rho_m}{dr}\right)\left(\frac{d\widetilde{P}}{dr}\right)\right]\xi_r + 2i\omega\Omega\xi_\varphi = \frac{1}{\rho_m}\frac{d\widetilde{P}_1}{dr}, \tag{2.82}$$

$$\left(\omega^2 - \Omega_B^2\right)\xi_\varphi - 2i\omega\Omega\xi_r = 0, \tag{2.83}$$

$$\left(\omega^2 - \Omega_B^2\right)\xi_z - \left(\frac{ik}{8\pi\rho_m}\frac{dB_z^2}{dr}\right)\xi_r = \frac{ik}{\rho_m}\widetilde{P}_1. \tag{2.84}$$

where Ω_B^2 is the Alfven frequency defined as $\Omega_B^2 = k^2\ \dfrac{B_z^2}{4\pi\rho_m}$, and $\widetilde{P} = P_0 + \dfrac{B_z^2}{8\pi}$, and $\widetilde{P}_1 = P_1 + \dfrac{\mathbf{B}\cdot\mathbf{b}}{4\pi}$.

Eliminating ξ_φ, ξ_z, and $\widetilde{P}_1$ between these equations, one finds,

$$\left[\left(\omega^2 - 2r\Omega\frac{d\Omega}{dr} - \Omega_B^2\right) - \left(\frac{1}{{\rho_m}^2}\right)(D\rho_m)(D\widetilde{P})\right]\xi_r - \left(\frac{4\omega^2\Omega^2}{\omega^2 - \Omega_B^2}\right)\xi_r =$$

$$\left(\frac{1}{\rho_m}\right)D\left[\left(\frac{\rho_m}{k^2}\right)\left\{(\omega^2 - \Omega_B^2)D_\star\xi_r - \left(\frac{k^2}{8\pi\rho_m}\right)(DB_z^2)\xi_r\right\}\right] \tag{2.85}$$

where $D_\star = D + 1/r = d/dr + 1/r$. In the present case the Alfven frequency turns out to be a constant because ρ_m and B_z^2 have the same radial dependence (i.e. r^{-6})and is renamed as Ω_A.radial dependence (i.e. r^{-6})and is renamed as Ω_A. So, equation (2.85) can be written as

$$\left[(\omega^2 - \Omega_A^2) - \Phi(r) - \frac{4\Omega^2\Omega_A^2}{(\omega^2 - \Omega_A^2)} - \frac{1}{{\rho_m}^2}(D\rho_m)(D\widetilde{P}) + \frac{1}{\rho_m}\ D^2(\frac{B_z^2}{8\pi})\right]\xi_r =$$

$$\frac{(\omega^2 - \Omega_A^2)}{k^2}(DD_\star\xi_r) + \frac{(\omega^2 - \Omega_A^2)}{k^2}(D\ln\rho_m)D_\star\xi_r - \frac{1}{\rho_m}D(\frac{B_z^2}{8\pi})(D\xi_r) \tag{2.86}$$

where $\Phi(r)$ is Rayleigh discriminant and is defined as $\Phi(r) = 2r\Omega\frac{d\Omega}{dr} + 4\Omega^2$. Defining $\omega^2 - \Omega_A^2 = \kappa$, one has then,

$$\begin{aligned}\kappa(DD_\star - k^2)\xi_r &= -k^2\left[\Phi(r) + \frac{4\Omega^2\Omega_A^2}{\kappa} + \frac{1}{{\rho_m}^2}(D\rho_m)(D\tilde{P}) + \frac{\kappa}{k^2 r}D\ \ln\ \rho_m\frac{1}{\rho_m}D(\frac{B_z^2}{8\pi})\right]\xi_r \\ &+ \frac{1}{\rho_m}\left[D(\frac{B_z^2}{8\pi})k^2 - \kappa(D\rho_m)\right](D\xi_r) \qquad (2.87)\end{aligned}$$

If the fluid is confined between two rigid coaxial cylinders of radii r_{in} and r_{out}, we must require that the radial component of the velocity vanishes at the edges. Thus, the solution of equation (2.87) must be sought which satisfies the boundary conditions

$$\xi_r = 0 \ \text{ for } \ r = r_{in} \text{ and } r_{out} \qquad (2.88)$$

Equation (2.87) together with the boundary conditions (2.88) constitute a characteristic value problem for κ.

After doing some algebraic manipulation and integrating over the range of r, one can obtain,

$$\begin{aligned}\kappa\int_{r_{in}}^{r_{out}} r\{|D_k\xi_r|^2 + k^2|\xi_r|^2\}dr &= k^2\int_{r_{in}}^{r_{out}} r\left\{\Phi(r) + \frac{4\Omega_A^2\Omega^2\kappa^\star}{|\kappa|^2}\right\}|\xi_r|^2 dr \\ &+\kappa\int_{r_{in}}^{r_{out}}(D\ \ln\ \rho_m)|\xi_r|^2 dr \\ &+k^2\int_{r_{in}}^{r_{out}} r\frac{D\ \ln\ \rho_m)}{\rho_m}(D\tilde{P})|\xi_r|^2 dr \\ &-\frac{\Omega_A^2}{2}\int_{r_{in}}^{r_{out}}\frac{r}{\rho_m}(D^2\rho_m)|\xi_r|^2 dr \\ &-\frac{\Omega_A^2}{2}\int_{r_{in}}^{r_{out}} r(D\ \ln\ \rho_m)\xi_r^\star(D\xi_r)dr \\ &+\kappa\int_{r_{in}}^{r_{out}} r(D\ \ln\ \rho_m)\xi^\star(D\xi_r)dr. \qquad (2.89)\end{aligned}$$

Assuming ξ_r to be real, the imaginary part of the equation (2.89) gives,

$$Im(\kappa) = 0\,; \qquad (2.90)$$

In view of the reality of the characteristic values of κ, we can rewrite equation (2.89) in the form

$$\kappa^2 I_1 - \kappa k^2 I_2 - 4\Omega_A^2 k^2 I_3 = 0\,; \qquad (2.91)$$

where

$$I_1 = \int_{r_{in}}^{r_{out}} r\left\{|D_\star \xi_r|^2 + \left(k^2 - \frac{D\ \ln\ \rho_m}{r}\right)|\xi_r|^2\right\} dr, \tag{2.92}$$

$$I_2 = \int_{r_{in}}^{r_{out}} r\left\{\left(\Phi(r) + \frac{1}{\rho_m}(D\ \ln\ \rho_m)(D\widetilde{P}) - \frac{V_A^2}{2}\frac{(D^2\rho_m)}{\rho_m}\right)|\xi_r|^2\right\} dr, \tag{2.93}$$

and

$$I_3 = \int_{r_{in}}^{r_{out}} r\Omega^2|\xi_r|^2 dr \tag{2.94}$$

Now extracting the root of the equation (2.91), we have

$$\kappa = \omega^2 - \Omega_A^2 = \left[k^2 I_2 \pm \sqrt{(k^4 I_2^2 + 16k^2\Omega_A^2 I_3 I_1)}\right]\frac{1}{2I_1}. \tag{2.95}$$

For stability it is necessary that

$$\omega^2 = \Omega_A^2 +, \frac{1}{2I_1}\left\{k^2 I_2 - \sqrt{(k^4 I_2^2 + 16k^2\Omega_A^2 I_1 I_3)}\right\} > 0. \tag{2.96}$$

which gives the condition :

$$V_A^2 I_1 > -\int_{r_{in}}^{r_{out}} r\left\{r\frac{d\Omega^2}{dr} + \frac{1}{\rho_m}(D\ \ln\ \rho_m)(D\widetilde{P}) - \frac{V_A^2}{2}\frac{D^2\rho_m}{\rho_m}\right\}|\xi_r|^2 dr \tag{2.97}$$

This condition is satisfied if the bracketed expression is positive. We then obtain following condition to be satisfied by the equilibrium solutions for stability,

$$\frac{D(\ell n\rho_m)(D\widetilde{P})}{\rho_m} > \left(\frac{V_A^2}{2}\right)\left(\frac{D^2\rho_m}{\rho_m}\right) - r\frac{d\Omega^2}{dr} \tag{2.98}$$

where D and D^2 are the operators defined as $d\,/\,dr$ and $d^2\,/\,dr^2$ respectively. V_A is the Alfven velocity, ρ_m and $\widetilde{P}$ are the background matter density and hydrostatic pressure, respectively and Ω is the angular velocity of the flow. The above criterion when used with the equilibrium solutions obtained earlier shows that for stability the ratio of r to r_{in} (= Alfven radius),

$$\frac{r}{r_{in}} < \frac{15}{8}\left(\frac{r_{in}}{R}\right)^6\left[2(\alpha-1)^{-7/8} - (\alpha-1)^{-15/8}\right], \tag{2.99}$$

which is satisfied for all values of $\alpha > 1.5$.

2.3.4 Rigid rotation velocity profile

Similarly, by substituting the solutions of $\widetilde{B}_{(r)}$ and $\widetilde{B}_{(\theta)}$ in eq. (2.33), the toroidal component of the velocity field is obtained to be

$$\widetilde{V}_{(\varphi)} = \left(1 - \frac{2}{\tilde{r}}\right)^{-1/2}\tilde{r}\sin\theta\,; \tag{2.100}$$

where, the above solution reduces to rigid rotation velocity field of the Newtonian type in the asymptotic limit. The presence of strong gravitational field introduces 'non-rigidness' in the velocity profile. Between $\tilde{r} = 2$ to 3, velocity no longer increases with distance. However, for a neutron star having mass around one $M_{\odot}$, the inner edge of the disk lies far away and thus, this behavior of velocity profile, between $\tilde{r} = 2$ to 3, does not arise. The poloidal components of the electric field are found from the eq. (2.32) *viz.*

$$\tilde{E}_{(r)} = \frac{1}{\tilde{r}^2}\left\{\left(\frac{3\,\tilde{r}^3}{4}\right)\tilde{B}_h\right\}\sin^2\theta\,; \tag{2.101}$$

$$\tilde{E}_{(\theta)} = -\frac{2}{\tilde{r}^2}\left\{\left(\frac{-\,3\,\tilde{r}^3}{8}\right)\tilde{B}_g\right\}\sin\theta\,\cos\theta. \tag{2.102}$$

Since, the form of $\tilde{B}_{\varphi}$ is the same as that of the previous case, the current density expressions are identical and given by the eqs. (2.49) & (2.50). Now, we consider the Euler equation to determine the equilibrium proper pressure and enthalpy for the given solutions of $\tilde{V}_{(\varphi)}$ and $\tilde{B}_{(\varphi)}$. Like in the previous case, we consider $V_0 \ll c$ and ignore the electric field contribution in the Euler equation. However, in the vicinity of the light cylinder this assumption may be violated. The compatibility condition on the pressure derivatives then give a differential equation to determine the proper enthalpy. This equation is the same as eq. (2.52). After substituting for $\tilde{B}_{(\varphi)}$, $\tilde{V}_{(\varphi)}$, $\tilde{J}_{(r)}$, and $\tilde{J}_{(\theta)}$, one could obtain the solution of the eq. (2.52) by using the characteristic method. The solution is found to be

$$\mathcal{F} = \frac{1}{4\pi}\left(\frac{B_0}{V_0}\right)^2\left[(-2\beta^2)\left(1-\frac{2}{\tilde{r}}\right)\left(\delta^3\mathcal{I}_1 - 6\alpha\delta^2\mathcal{I}_2 + 12\alpha^2\delta\mathcal{I}_3 - 8\alpha^3\mathcal{I}_4\right)+1\right]; \tag{2.103}$$

where, δ is the integration constant defined as $\delta = 2\,\alpha\,/\,\tilde{r} + \tilde{r}^2\,\sin^2\theta$ and $\mathcal{I}_1$, $\mathcal{I}_2$, $\mathcal{I}_3$, & $\mathcal{I}_4$ are the four integrals defined as :

$$\mathcal{I}_1 = \int \frac{1}{\tilde{r}^{16}}\,\tilde{Q}(\tilde{r})\,d\,\tilde{r}\,; \tag{2.104}$$

$$\mathcal{I}_2 = \int \frac{1}{\tilde{r}^{17}}\,\tilde{Q}(\tilde{r})\,d\,\tilde{r}\,; \tag{2.105}$$

$$\mathcal{I}_3 = \int \frac{1}{\tilde{r}^{18}}\,\tilde{Q}(\tilde{r})\,d\,\tilde{r}\,; \tag{2.106}$$

$$\mathcal{I}_4 = \int \frac{1}{\tilde{r}^{19}}\,\tilde{Q}(\tilde{r})\,d\,\tilde{r}\,; \tag{2.107}$$

where, $\widetilde{Q}(\widetilde{r})$ is defined as :

$$\widetilde{Q}(\widetilde{r}) = \left(1-\frac{2}{\widetilde{r}}\right)^{-1}\left\{\left(\frac{-3\widetilde{r}^3}{8}\right)\widetilde{B}_g\right\}^4\left[2\left\{\left(\frac{-3\widetilde{r}}{8}\right)\widetilde{B}_g\right\}+\left\{\left(\frac{3\widetilde{r}^3}{4}\right)\widetilde{B}_h\right\}+\right.$$

$$\left.\frac{2}{\widetilde{r}}\left\{\left(\frac{-3\widetilde{r}^3}{8}\right)\widetilde{B}_g\right\}\left(1-\frac{2}{\widetilde{r}}\right)^{-1}\right]. \qquad (2.108)$$

Eq. (2.103) can be reduced to the following form in the asymptotic limit

$$\mathcal{F} = \frac{1}{4\pi}\left(\frac{B_0}{V_0}\right)^2\left[\beta^2\left\{\frac{\alpha^3}{255}\frac{1}{\widetilde{r}^{18}}+\frac{3\alpha^2}{85}\frac{\sin^2\theta}{\widetilde{r}^{15}}+\frac{3\alpha}{20}\frac{\sin^4\theta}{\widetilde{r}^{12}}+\frac{2}{5}\frac{\sin^6\theta}{\widetilde{r}^9}\right\}+1\right]. \qquad (2.109)$$

Finally, after substituting for $\widetilde{\mathcal{F}}$, $\widetilde{V}_{(\varphi)}$ and $\widetilde{B}_{(\varphi)}$ in the eq. (2.39) and integrating over $\widetilde{r}$, we obtain *viz.*

$$P = \frac{{B_0}^2}{4\pi}\left[2\,\beta^2\,(\,\alpha\, b_1\, - \, b_2\,\sin^2\theta - \frac{b_3}{2}\,\sin^8\theta)\right]; \qquad (2.110)$$

where, b_1, b_2, and b_3 are defined as :

$$b_1 \;=\; \int\frac{1}{\widetilde{r}^2}\left[(\delta^3\mathcal{I}_1 - 6\alpha\delta^2\mathcal{I}_2 + 12\alpha^2\delta\mathcal{I}_3 - 8\alpha^3\mathcal{I}_4) - \frac{1}{2\beta^2}\left(1-\frac{2}{\widetilde{r}}\right)^{-1}\right]d\widetilde{r}; \quad (2.111)$$

$$b_2 \;=\; \int\widetilde{r}\left[\left(1-\frac{2}{\widetilde{r}}\right)(\delta^3\mathcal{I}_1 - 6\alpha\delta^2\mathcal{I}_2 + 12\alpha^2\delta\mathcal{I}_3 - 8\alpha^3\mathcal{I}_4) - \frac{1}{2\beta^2}\right]d\widetilde{r}; \qquad (2.112)$$

$$b_3 \;=\; \int\frac{1}{\widetilde{r}^8}\left(1-\frac{2}{\widetilde{r}}\right)^{-1}\left\{\left(\frac{-3\,\widetilde{r}^3}{8}\right)\widetilde{B}_g\right\}^4\left\{\left(\frac{3\,\widetilde{r}^3}{4}\right)\widetilde{B}_h\right\}d\,\widetilde{r}; \qquad (2.113)$$

The solution of the proper pressure given in the eq. (2.110) reduces to the following form in the asymptotic limit as:

$$P = \frac{{B_0}^2}{4\pi}\left[\left(\frac{\alpha}{\widetilde{r}}+\frac{\widetilde{r}^2}{2}\sin^2\theta\right)+\beta^2\left(\frac{\alpha^4}{4845}\frac{1}{\widetilde{r}^{19}}\;+\;\frac{\alpha^3}{510}\frac{\sin^2\theta}{\widetilde{r}^{16}}+\frac{39\alpha^2}{22100}\frac{\sin^4\theta}{\widetilde{r}^{13}}\right.\right.$$

$$\left.\left.+\;\frac{\alpha}{40}\frac{\sin^6\theta}{\widetilde{r}^{10}}-\frac{1}{5}\frac{\sin^8\theta}{\widetilde{r}^7}\right)\right] \qquad (2.114)$$

2.4 Discussion and Conclusions

2.4.1 Quasi-Keplerian velocity profile

We first consider the case when the flow velocity is quasi-Keplerian in the flat space limit. This class of solutions is a general relativistic generalisation of the Newtonian solutions obtained by BBDP (1995). It is worth noting here that in the flat space limit, the pressure and enthalpy profiles do not satisfy the barotropic condition even when the $\mathbf{J} \times \mathbf{B}$ term is zero. This is because when the velocity profile is the quasi-Keplerian, the $(\mathbf{V} \cdot \nabla)(\nabla \times \mathbf{V})$ term is not zero. Further, the inclusion of the $\mathbf{J} \times \mathbf{B}$ term in the Euler's equation can introduce additional non-barotropicity. From the flat space limit of eqs. (2.53) & (2.58), one can see that the non-zero toroidal magnetic field ($\beta \neq 0$) introduces additional inhomogeneities in the pressure and enthalpy profiles. The strength of these inhomogeneities is of order β, which for $\beta \sim 1$ is of the same order as the equilibrium without any $\mathbf{J} \times \mathbf{B}$ force.

The solutions obtained in the Newtonian limit depend crucially on the two dimensionless parameters α defined as the ratio of the gravitational energy to the kinetic energy of a fluid element at $r = r_{in}$ and β defined as the ratio of the toroidal magnetic field to the poloidal magnetic field strengths.

The vertical (meridional) nature of the pressure profile described by the eq. (2.66) shows a quite complicated behavior. First, the pressure function does not necessarily have a global maximum at $\theta = \pi/2$ for all the values of parameters α and β leading to unphysical equilibria. Second, it becomes negative for certain values of α and β. Obviously these features are unwanted. In order to ensure that the plasma pressure P is positive everywhere within the disk and that it has a local maximum at $\theta = \pi/2$, the parameters α and β have to satisfy the following inequalities :

$$\beta^2 < 3.67\,\alpha^{-15/8}\,; \tag{2.115}$$

$$\beta^2 < \frac{15}{32}(\alpha - 1)^{-15/8} \tag{2.116}$$

The parameter space (α, β) allowed by these above inequalities is depicted in the *Fig. 2.1* by the shaded region with α ranging from 1 to 3.5. For $\alpha \geq 1.1$, the strongest constraint comes from equation (2.116). It can also be seen from *Fig. 2.1* that the maximum value of β must be less than 2, while the parameter α does not have any upper bound. However, for large valueα does not have any upper bound. However, for large values of α (say $\alpha > 3.5$), maximum allowed value of β becomes negligibly small to have any importance for the kind of equilibrium that we have considered.

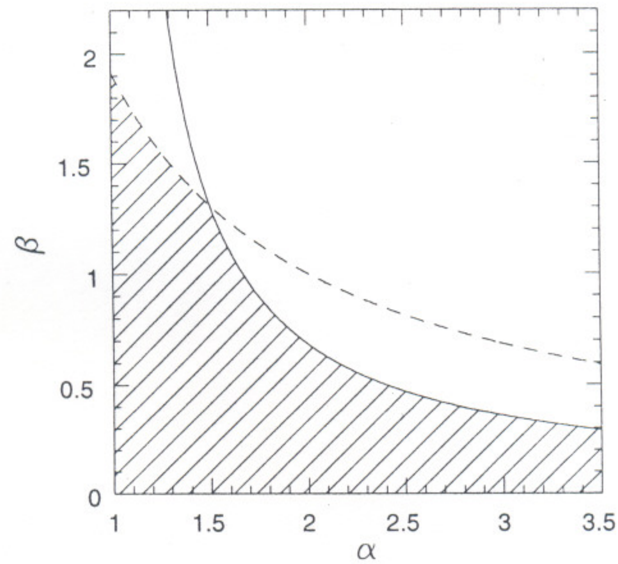

Fig. 2.1: subtended below the dashed line shows the parameter space allowed by the inequality (2.116). The area below the solid line shows the parameter space described by the inequality (2.115). The shaded region shows the allowed parameter space by both the inequalities.

Vertical (meridional) structures of equilibrium plasma density $\rho_m(\theta) \equiv \frac{15}{8}(\alpha - \sin^8\theta)^{-15/8} + \beta^2$ and pressure profiles $P(\theta) \equiv \frac{15}{8}(\alpha - sin^8\theta)^{-7/8} + \beta^2(\alpha - \frac{7}{2}sin^8\theta)$ are shown in *Figs. (2.2)-(2.5)*. We first study the case when the toroidal field is absent (i.e., $\beta = 0$) and it is shown in shown in *Figs. (2.2)-(2.3)* given below. Under this condition, as there is no azimuthal plasma current density (i.e. $J_\varphi = 0$), the effect of external dipole field does not play any direct role in the disk equilibridipole field does not play any direct role in the disk equilibrium. One can see that both pressure and density profiles show global maxima at $\theta = \pi/2$ plane for all the values of parameter α. It should also be stated that the pressure profile is always positive for $\beta = 0$. One can also observe from the following figures that, for the case $\alpha \to 1$, both pressure and density profiles have sharp gradients in the θ- direction indicating rapid decrease in matter density off the equatorial plane. Such solutions may, however, be unstable since $\alpha < 1.5$ (sub-section 2.3.3).

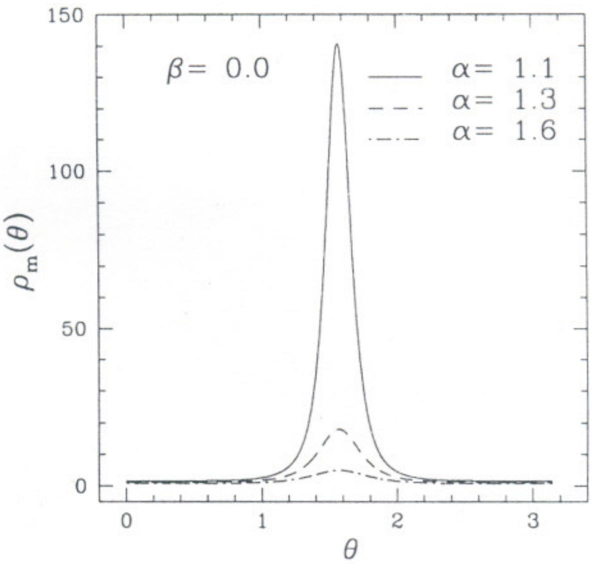

fig. 2.2: The variation of density along the meridional direction as a function of θ from $\theta_{min} = 0$ to $\theta_{max} = \pi$ for $\beta = 0$ and for three different values of α.

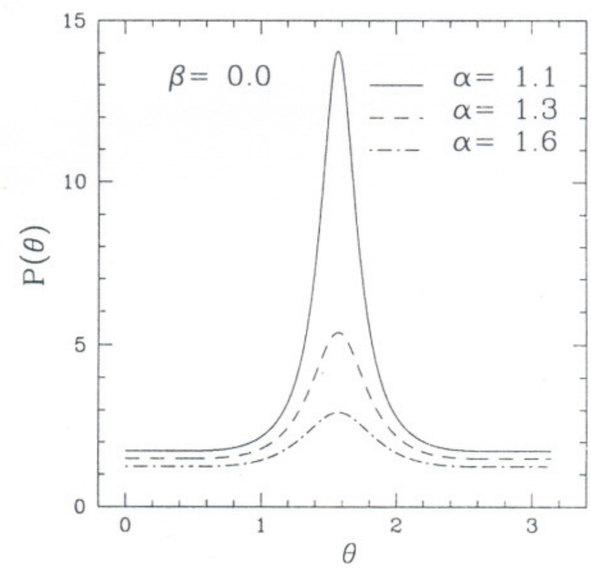

fig. 2.3: The variation of pressure along the meridional direction as a function of θ from $\theta_{min} = 0$ to $\theta_{max} = \pi$ for $\beta = 0$ and for three different values of α.

Next we consider the case in the presence of toroidal magnetic field (i.e. $\beta \neq 0$) and its effect is shown in the *Figs. (2.4)-(2.5)* given below. *Figs. (2.4)-(2.5)* depict the pressure profiles in meridional plane with various values of β. One can clearly notice that the inclusion of toroidal field leads to distinct qualitative changes in the pressure profiles compared to $\beta = 0$ case.

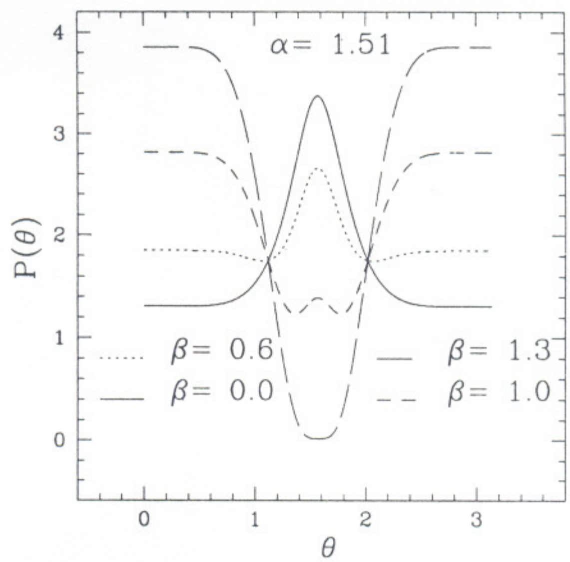

Fig. 2.4: The variation of pressure along the meridional direction as a function of θ from $\theta_{min} = 0$ to $\theta_{max} = \pi$ for a fixed value of $\alpha = 1.51$ and for four different values of β.

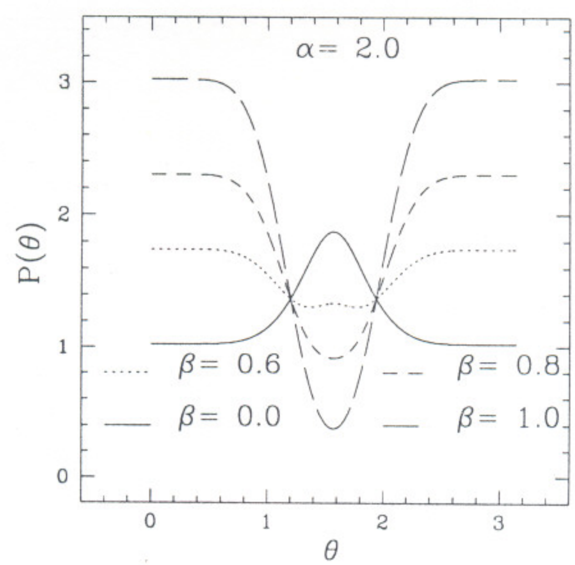

Fig. 2.5: The variation of pressure along the meridional direction as a function of θ from $\theta_{min} = 0$ to $\theta_{max} = \pi$ for a fixed value of $\alpha = 2.0$ and for four different values of β.

First, due to the presence of toroidal magnetic field, the value of the pressure maximum has been reduced. Secondly, the pressure function shows two local minima on either side of $\theta = \pi/2$ plane. This feature could be because the toroidal field assists the plasma pressure gradient in balancing the θ component of centrifugal force, making thus the pressure gradients less steeper in this region. After the point $\theta = \sin^{-1}\left((2\alpha/7)^{1/8}\right)$ from either side of the $\theta = \pi/2$ plane, the isotropic part of the pressure profile dominates over the plasma pressure with $\beta = 0$ while for a higher value of $\beta \rightarrow 1$ it shows a large gradient in the θ- direction. These features are found to be valid for all allowed values of α and β depicted in the parameter space (*Fig. 2.1*). On the contrary, density profiles do not exhibit any change with non-zero β value except for changes the back-ground values.

We next consider the magnetic field topology. For the present problem, as there is axisymmetry, we have the toroidal component of magnetic field $\mathbf{B_T}$ satisfying $\nabla \cdot \mathbf{B_T} = \mathbf{0}$, where $\mathbf{B_T} = (0, 0, B_\varphi)$ and the poloidal component of the magnetic field $\mathbf{B_P}$ separately satisfying $\nabla \cdot \mathbf{B_P} = \mathbf{0}$ with $\mathbf{B_P} = (B_r, B_\theta, 0)$.

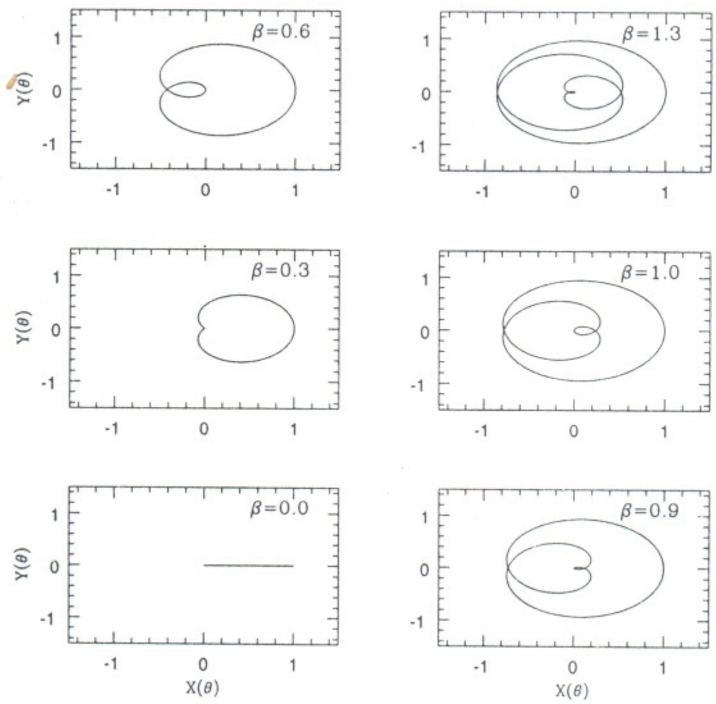

Fig. 2.6: The projection of a magnetic line of force within the disk on the X-Y plane runs from $\theta_{min} = 0$ *to* $\theta_{max} = \pi$*, at* $r = r_{in} = 14m$ *and* $R = 7m$ *for six different values of* $\beta = B_1/B_0$.

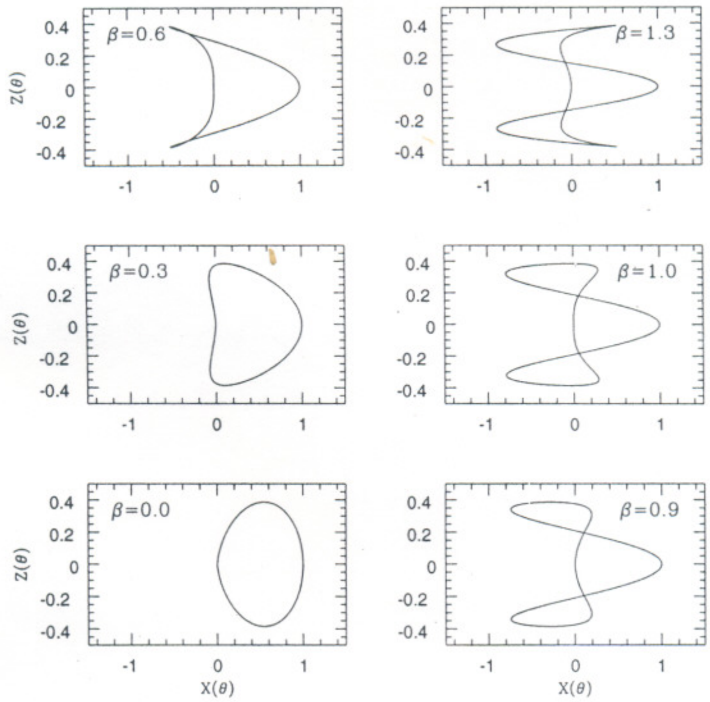

Fig. 2.7: The projection of a magnetic line of force within the disk on the X-Z plane runs from $\theta_{min} = 0$ to $\theta_{max} = \pi$, at $r = r_{in} = 14m$ and $R = 7m$ for six different values of $\beta = B_1/B_0$.

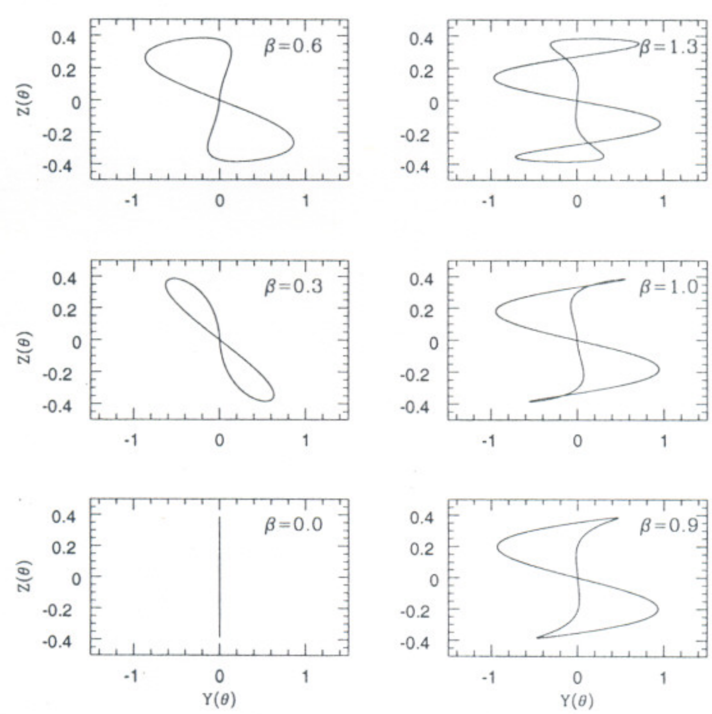

Fig. 2.8: The projection of a magnetic line of force within the disk on the Y-Z plane runs from $\theta_{min} = 0$ to $\theta_{max} = \pi$, at $r = r_{in} = 14m$ and $R = 7m$ for six different values of $\beta = B_1/B_0$.

Thus, B_φ has a loop type structure in the $r-\phi$ plane of the disk. With the help of eqs. (2.68)-(2.70), we can study the general magnetic field line structure which is represented in *Figs. (2.6)-(2.8)* depicting the projection of a line-of-force in the $X-Y$, $X-Z$, and $Y-Z$ planes, respectively, for various values of β. In *Fig. 2.6*, for $\beta = 0$, when the external dipole field axis is aligned along z-axis, one can see the projection of a dipole field line as a straight line in the $X-Y$ plane. For $\beta \neq 0$, the structure forms a loop caused by the B_ϕ component. As the value of β increases, one can clearly visualise that the line-of-force is being streched more and the size of the loop beccomes larger and, in order to accomodate the line of force, kinks developa. In *Fig. 2.7*, the projection of the dipole field line represents a loop structure on $X-Z$ plane, the inclusion of the toroidal field only deforms the loop by stretching and compressing. Finally, in *Fig. 2.8*, for the case $\beta = 0$, the projection of the pure dipole field is a stright line on $Y-Z$ plane. For $\beta \neq 0$, the toroidal field line also has straight line projection that makes a right angle with the dipolar field line. Thus, one can clearly notice two lobes on the $Y-Z$ plane and these lobes grow in size as β increases and tilt in the horizental direction.

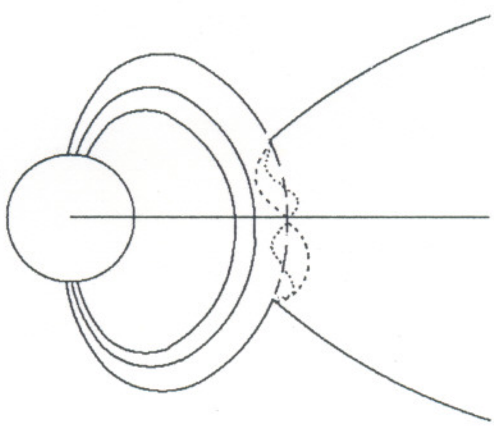

Fig. 2.9: A schematic diagram of magnetic lines of force at the inner edge of the disk with and without toroidal magnetic field. A dashed line depicts for $\beta = 0$, the dot-dashed line depicts for $\beta_1 \neq 0$ and the dotted line depicts $\beta_2 \neq 0$ where $\beta_2 > \beta_1$.

This kind of field line topology clearly indicates shear in the magnetic field lines which is shown schematically in *Fig. 2.9* which is shown above. The structure of the magnetic field (depicted in *Fig. 2.9*) shows that the shear which produces kinks in the field lines increases with the strength of the toroidal magnetic field compared to the poloidal field strength. Such lines might store sufficient free energy which potentially can drive the instabilities. In fact, a very recent study (Kumar *et al.*, 1994) has shown that in the case of a thin disk while the vertical fields tend to stabilize the disk, the toroidal field destabilises the disk, especially when the ratio of toroidal to poloidal field is large. In our case as well, the kinks in the field lines increase with increasing β and the role of the toroidal magnetic field becomes very important in disscussion of thick magnetized accretion disks. We have found that the obtained equilibria do not support arbitrarily high toroidal magnetic field (i.e. $\beta < 2$). Further, it was shown that, the plasma pressure gradient changes sign somewhere between the polar and equatorial planes due to the presence of this toroidal magnetic field. This structure might give rise to instabilities.

Another important aspect to be considered is the generalisation of the Newtonian analyses to general relativistic formalism wherein the spacetime curvature produced by the strong gravitational field of the central compact object would modify the magnetic fields and may introduce new features. From the general relativistic solutions obtained for a pressure-supported magnetised thick disk it is clear that the pressure function also does not necessarily have a global maximum at $\theta = \pi/2$ plane for all values of parameters α and β, leading to unphysical equilibria. Second, it also becomes negative for certain values of α and β. Obviously these features are undesirable to have physically meaningful disk solutions. Hence, in order to ensure that the plasma pressure $\tilde{P}$ is positive everywhere within the disk and that it has a local maximum at $\theta = \pi/2$ plane, the parameters α and β have to satisfy the following inequalities :

$$\beta^2 < \frac{3.67}{a_4}\left[\frac{7}{5}a_1 - \frac{2}{5}\left(\frac{a_3}{a_4}\right)\right]^{-15/8}\alpha^{-15/8}; \tag{2.117}$$

$$\beta^2 < \frac{15}{32}\frac{(a_1\alpha - 1)^{-15/8}}{a_4}. \tag{2.118}$$

Unlike in the Newtonian case, in the general relativistic situation, the above inequalities are functions of the Schwarzschild coordinate $\tilde{r}$. It can be shown that in the asymptotic limit (i.e. $\tilde{r} \to \infty$), these inequalities reduce to those of Newtonian analyses.

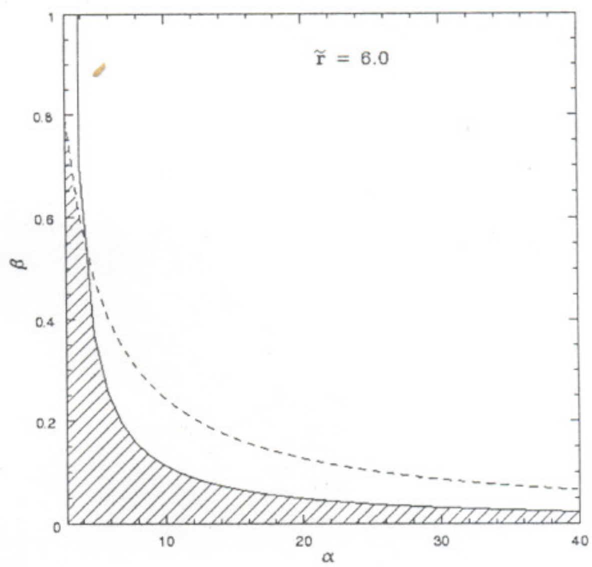

Fig. 2.10: The parameter space (α, β) at the Schwarzschild coordinate $\tilde{r} = 6.0$ shown for the case when the plasma azimuthal velocity profile is quasi-Keplerian in the asymptotic limit. The areas below the dashed and solid lines show the domain of values of α and β allowed by the inequalities (2.117) and (2.118) respectively. The shaded region shows the common parameter space allowed by both the inequalities.

We would like to recall from (BBDP 1995) that the parameter β has an upper bound around $\beta \sim 2$, beyond which positivity and local maximum at $\theta = \pi/2$ plane are violated. Also the parameter α has a natural lower bound $\alpha > 1$. Thus, the rotational state of the disk and the toroidal component of the magnetic field get related due to positivity and local maximum criteria on the pressure profile at $\theta = \pi/2$ plane. In the general relativistic situation, the lower bound on the parameter α is $\alpha_{min} = 1 / a_1$ and the upper bound on the parameter β is β_{max} given by

$$\beta^2{}_{max} = \frac{3.67}{a_4} \left[\frac{7}{5} a_1 - \frac{2}{5} \left(\frac{a_3}{a_4} \right) \right]^{-15/8} a_1^{15/8}; \tag{2.119}$$

are the functions of the Schwarzschild coordinate $\tilde{r}$. *Fig. 2.10* shows the allowed values of the parameter space (α & β) denoted by the shaded region for the value of the Schwarzschild coordinate $\tilde{r} = 6$.

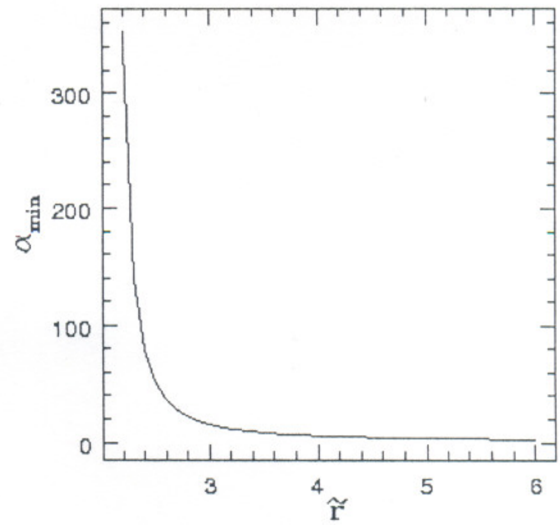

Fig. 2.11: The parameter α has the minimum allowed value $\alpha_{min} = 1 / a_1$ where, a_1 defined in eq. (2.54) is a function of the Schwarzschild coordinate $\tilde{r}$. α_{min} is plotted as a function of $\tilde{r}$.

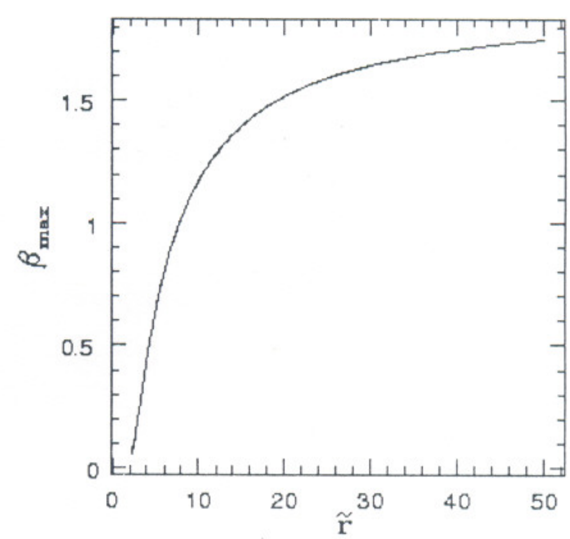

Fig. 2.12: The parameter β has the maximum allowed value β_{max} [defined by the eq. (2.119)] is a function of the Schwarzschild coordinate $\tilde{r}$. β_{max} is plotted as a function of $\tilde{r}$.

Figs. (2.11)-(2.12) describe the variation of α_{min} and β_{max} as the functions of the Schwarzschild coordinate $\tilde{r}$ respectively. From *Fig. 2.11*, it is clear that α_{min} increases to a very high value as the inner edge of the disk comes closer to the star's surface. This shows that the gravitational potential energy of the fluid element at the inner edge of the disk due to the central compact object becomes very large as compared to its kinetic energy. *Fig. 2.12* clearly shows that the parameter β_{max} drops to a very low value as the inner edge of the disk comes nearer to the stellar surface. Thus, the relative strength of self-consistently generated toroidal magnetic field decreases very fast compared to the external poloidal magnetic field. As a consequence of smaller β_{max} to retain the positivity and local maximum of the pressure at $\theta = \pi/2$ plane, the field line structure is largely determined by the external poloidal field. Moreover, there is no significant qualitative departure from the field line topology as examined in the Newtonian analyses. *Fig. 2.13* shows the meridional (vertical) structure of the pressure profile for the allowed values of the parameters (α & β). Unlike BBDP, the vertical structure of the pressure is a function of the Schwarzschild coordinate $\tilde{r}$. Due to the presence of toroidal magnetic field, the local maximum of the pressure has been reduced at $\theta = \pi/2$ plane.

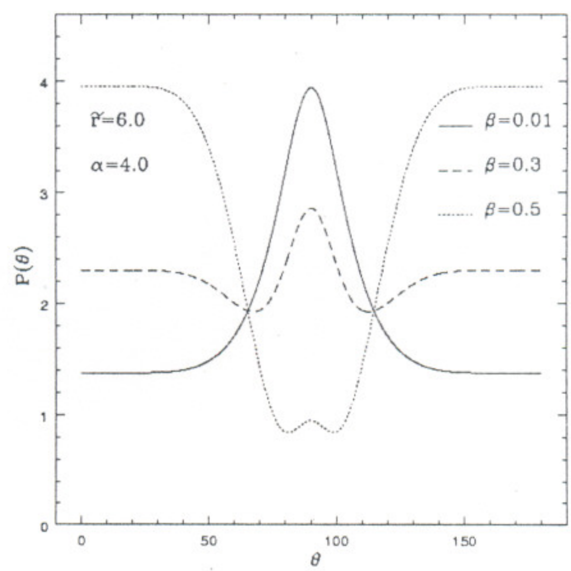

Fig. 2.13: The variation of pressure along the meridional direction as a function of θ from $\theta_{min} = 0$ to $\theta_{max} = \pi$ at Schwarzschild coordinate $\tilde{r} = 6.0$ for a fixed value of $\alpha = 4.0$ and for three different values of β.

Moreover, the pressure shows two minima on the either side of the $\theta = \pi/2$ plane. This feature is qualitatively same as that of Newtonian thick disk problem treated in

BBDP (1995).

2.4.2 Rigid rotation velocity profiles

In order to see the nature of equilibrium described by the proper pressure [eq. (2.110)] and the proper enthalpy [eq. (2.103)] profiles, we consider their flat space limit. In this limit they are described by eqs. (2.114) & (2.109) respectively. For the case when there is no $\mathbf{J} \times \mathbf{B}$ force on the plasma, i.e., $\beta = 0$, one can see from eq. (2.114) that density is a constant. Under this condition, criteria for the barotropic equilibrium, i.e., $\nabla \mathbf{P} \times \nabla \mathcal{F} = 0$ is satisfied by the pressure and density. It can also be seen from eqs. (2.114) & (2.109) that the finite β term introduces an additional $\tilde{r}$ & θ dependence but, its strength is extremely small for the entire region satisfying $\tilde{r} \gg 1$. Thus, non-barotropic behavior introduced by the toroidal magnetic field is quite small. In the general relativistic case magnetic field is quite small. In the general relativistic case even when $\beta \rightarrow 0$, the proper enthalpy does not become constant, [see eq. (2.103)] which may lead to an additional source of 'non-barotropicity' near the surface of the compact object.

We also find that, unlike the pressure and enthalpy profiles for the quasi-Keplerian velocity, the positivity of the pressure and local maximum at $\theta = \pi/2$ plane is maintained for any range of parameter α and β. The criterion for the global stability for axisymmetric perturbations with stratified enthalpy and inhomogeneous magnetic field was obtained in BBDP (1995). It can be shown that in the Newtonian limit with $\beta \rightarrow 0$, the enthalpy becomes constant and the stability criterion is satisfied.

The solutions that we have examined here may be important in the following respect : In this paper, we have obtained the solutions for non-accreting, pressure-confined, and magnetized disk equilibria around a magnetized compact object without invoking any thin disk approximations. Such solutions can be used to study magnetic torque exerted by a thick disk on the central star by incorporating the effects of accretion flow and finite resistivity perturbatively. Moreover, the presence of plasma equilibrium may influence the electromagnetic processes near a magnetized central object (e.g. Michel, 1982). Such solutions may also be employed to study the effect of general relativistic plasma on such processes.

Chapter 3

Summary and Conclusions

We have studied axisymmetric, non-accreting pressure-supported thick disk MHD equilibria around a compact object. Specifically we have analyzed two cases of importance:

(i) The solutions having a quasi-Keplerian azimuthal velocity profile are found to depend upon two parameters α and β. The parameter α signifies the ratio of the gravitational potential energy to the bulk kinetic energy of a fluid element. The parameter β signifies the ratio of the toroidal magnetic field strength to poloidal magnetic field strength. Analyses of the pressure profile show that such solutions cannot support a toroidal magnetic field of the arbitrary strength but the maximum strength of the toroidal field can be determined from the inequality relation satisfied by α and β. We have also found that this kind of solution support a non-barotropic distributions.

Analyses of the general relativistic solutions indicate that the strength of the toroidal field decreases significantly near the surface of the compact object due to relativistic effects, and the magnetic field line structure shows that the generation of toroidal magnetic field makes magnetic field lines highly sheared in this kind of equilibria. Such field line structure may indicate that this kind of equilibrium might be unstable.

(ii) The solutions having rigid rotation type azimuthal velocity profile are found to depend less sensitively on the parameters α and β. The pressure and the density profiles obtained by solving the relevant equations show that such configurations, in the asymptotic limit, support a barotropic equilibrium. However, the effect of strong gravity may violate the barotropic nature of the equilibrium. Also the effect of strong gravity can cause a departure from the rigid rotation type behavior of the velocity profile. This solution can also have relevance as a model of plasma magnetosphere around magnetized compact object like a neutron star.

So far we have completed a study of non-accreting thick disk equilibrium and some aspects of their stability. Such study can be logically continued, as future work, in the following directions.

(1) The interpretation of most pulsating X-ray sources as accreting neutron star based on the qualitative features of their spectra and their secular spin-up rates. If the accreting star possesses a sufficiently strong magnetic field, interaction between the stellar magnetic fields and the surrounding accretion disk significantly influences the structure and dynamics of the disk and spin evolution of the star. The qualitative features of disk accretion by rotating magnetic neutron stars were first described by Pringle and Rees (1972), Lamb, Pethick and Pines (1973), and Ghosh, Lamb, and Pethick (1977). These authors have argued that a slowly rotating neutron star accreting matter from a Keplerian disk should spin up as a consequence of the torque exerted on the star by the accreting matter. These models have been used to calculate the bounds on the accretion torque. The period changes observed in these X-ray sources are generally consistent with the above theoretical estimates. Furthermore, an understanding of this phenomenon would provide an important tool for exploring other important problems, such as the characteristics of mass transfer in binaries and properties of neutron stars (e.g., star's dipole field, size of its moment of inertia, relative inertial moments of the crust and superfluid neutron core etc.).

The progress in understanding these observations has been stymied in part by the absence of a detailed self-consistent quantitative model of disk accretion by magnetic neutron stars. However, generally two kinds of quantitative models have been proposed for the magnetic field-disk interaction. These models can be used to calculate torque exerted on the star by the accreting matter and explain the changes observed in the pulsation periods of these X-ray sources. Some investigators (e.g., Anzer and

Börner, 1980, 1983; Arons *et al.*, 1985; Aly, 1986) considered that the disk plasma is infinitely conducting, and the surface currents on the disk exclude the stellar magnetic field. In this case, the torque N exerted on the star by the disk only results from the material stress, $N = \dot{M}\,(G\,M\,r_{in})^{1/2}$, where M and $\dot{M}$ are the mass and mass accretion rate of the star respectively, r_{in} the inner edge of the accretion disk and G the Newtonian constant of Gravity.

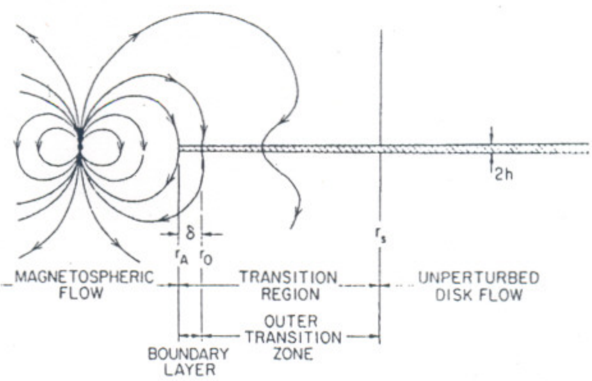

Fig. 3.1: Schematic picture of the accretion flow. Beyond the radius r_s at which the stellar magnetic field is completely screened, the disk flow, of vertical thickness $2h$, is unperturbed by the magnetosphere. In the transition region between r_s and r_A, disk flow changes into magnetospheric flow. The transition region divides into two parts, an outer transition zone where viscous stresses dominate magnetic stresses, and a boundary layer of width $\delta \ll r_A$ where magnetic stresses dominate (Ghosh and Lamb, 1978).

On the other hand, Ghosh and Lamb (1978, 1979a,b, hereafter GL) developed a magnetically threaded disk (MTD) model with the assumption that the effective resistivity arises due to flow conditions. In this model, the disk is partially threaded by the magnetic field B_z *via* the non-linear mechanisms like the Kelvin- Helmholtz instability, turbulent diffusion, and reconnection with small- scale fields within the disk etc. They have adopted cylindrical coordinate (R, ϕ, z) centered-on the star and the disk is located on the $z = 0$ plane, which is perpendicular to the star's spin and magnetic axes.

The total torque exerted by the accretion disk is contributed by both the material stress and the magnetic stress in the case of axisymmetric disk accretion can be expressed as,

$$N = N_0 + \int_{r_0}^{r_s} \left[B_\phi(r) \, B_z(r) \right]_{z=h} r^2 \, dr \,, \tag{3.1}$$

where $N_0 = \dot{M} \, (G \, M \, r_0)^{1/2}$ denotes the rate at which angular momentum is transported inward past the point $r = r_0$ by matter within the disk while the second term represents the effect of magnetic stresses (torque) acting outside r_0 (i.e., outside the co-rotation radius) across the surfaces of the disk, $z = \pm h$ (with equal contributions coming from top and bottom), which can be positive or negative, determined by the spin period, the magnetic moment and the mass accretion rate of the star as shown in the *Fig. 3.1*. So, the star can be spun down even while accretion occurs. r_0 is defined as the radius at which the angular velocity of the plasma begins to depart significantly from the Keplerian behavior. By assumption, the magnetic field disrupts the disk flow inside r_0, and all of the matter is eventually accreted onto the neutron star.

The toroidal component of the magnetic field in eq. (3.1) changes sign as it crosses the equatorial plane (i.e. $\theta = \pi/2$). Such a B_ϕ arises due to coupling of plasma, its azimuthal motion in the poloidal field with the stellar magnetic field. In order to calculate B_ϕ, which changes its sign near the equator, we need to know the radial velocity profile v_r also. One can use the present solution to calculate corresponding v_r and B_ϕ perturbatively. This can allow us to calculate the magnetic torque from the accretion disk by analytical means. However, this will be a rather simplified model of the magnetic torque calculation in case of a thick disk configuration. For the proper treatment of the transition region (GL, 1978, 1979a,b) one has to carry out a detailed numerical study.

(**2**) General relativistic effects near neutron stars have previously been explored in the context of emission models for X-ray pulsars and gamma-ray bursts. Recently Gonthier and Harding (1994) examined the importance of general relativistic corrections to the production of gamma rays near the surface of a neutron star. They have found that the curved spacetime metric significantly increases the magnitude of the magnetic field. Therefore, the attenuation coefficients of curvature radiation for pair production in a magnetic field can be increased by factors as large as 100. As a result, the survival distance of $1Gev$ photons for pair production, is decreased by a factor of 2 for $B \sim 10^{12}\ G$. In this work, space near the neutron star surface assumed to contain zero matter density. However, the presence of plasma equilibrium can strongly influence the electromagnetic processes near the neutron star (Gonthier and Harding, 1994). Therefore, the axisymmetric MHD equilibrium carried out by us can be used to study the effects of a plasma equilibrium on electromagnetic process. In particular, it would be of interest to investigate how single particle moving in such a equilibrium will radiate.

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