A STUDY OF INSTABILITIES AND TURBULENCE IN MIRROR PLASMAS

BY

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چلو اچھا ہوا کام آ گئی دیوانگی اپنی
وگرنہ ہم زمانے بھر کو سمجھانے کہاں جاتے ۔

قتیل —

चलो अच्छा हुआ काम आ गई दीवानगी अपनी
वगरना हम ज़माने भर को समझाने कहां जाते.

— क़तील

TO MA AND PAPA

CERTIFICATE

I hereby declare that the work presented in this thesis is original and has not formed the basis for the award of any degree or diploma by any University or Institute.

Avinash
(Avinash)

Certified by

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(R. K. Varma)

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ABSTRACT

The thesis work identifies certain physical process which may be playing important role in the observations of Base-Ball II, Constance II, 2XIIB and β -2 mirror machines.

In Base-Ball II mirror experiment (1975) the anomalous losses of particles from the trap were related to bursts of azimuthal potential perturbations. In an effort to understand these losses we have identified these potential perturbations with mode described by Varma (1967). We have investigated the non-linear stability of these oscillations in the presence of ion-cyclotron-oscillations seen in Base-Ball II. The coupling is envisaged through non-linear Landau damping. We find that Varma mode exhibits a bursting instability on a time scale of $\sim 400 \mu s$ as observed in the experiments.

In 2XIIB and Constance II mirror machines (1978, 1979, 1980) Electron Beams (EB) were injected parallel to the magnetic field lines to suppress Drift Cyclotron Loss Cone (DCLC) turbulence. In our work we have given a model for the various observations of 2XIIB and Constance II which is based on the resonant damping of EB induced Langmuir plasmons on DCLC modes. The effect of resonant damping was studied by adiabatic approximation. We have also derived a quasi-linear theory for these experiments where we have set up a closed set of equations to calculate ion life time, electron and ion temperature, fluctuation level, etc.

We have further studied the effect of EB induced Langmuir waves on High Frequency Convective Loss Cone modes (HFCLC) which have been predicted for mirror plasmas. In the high (ω, k) region of HFCLC spectrum we find that these modes are strongly coupled through non-linear Landau damping to Langmuir waves and beam electrons. Mainly the growth rates are affected i.e. they are strongly damped.

In the low (ω, k) region of HFCLC spectrum we find that growth length of these modes are affected. The growth lengths are reduced by about five times. This can be quite dangerous mirror confinement. If these modes are convectively stabilized (as in 2XIIB) then the electron beam will again destabilize them.

Based on the anomalous resistivity we have given another mechanism for the saturation of DCLC turbulence in 2XIIB. Invoking perturbed orbit formalism we have shown that ion-diffusion in 2XIIB generates enough anomalous resistivity to stabilize DCLC modes. The saturated fluctuations level calculated from this agrees well with the experimentally observed fluctuation level. It also explains depression of the fluctuation level and improvement in the ion life time with warm plasma streams.

We have investigated the effect of neutral beam generated Lower Hybrid Waves (LHW) on DCLC modes. We have shown that DCLC modes in the presence of LHW are strongly damped. This reduces the ion-diffusion in velocity space and thus improves the ion life time inside the trap.

We have investigated the possibility of suppressing the unstable DCLC modes by feedback circuits. We have shown that using material probes as suppression and sensor grids and again of $\sim 50\,\Omega_i$ + 90° phase shift, the critical density gradient required for the DCLC instability can be pushed up by as much as two orders of magnitude.

CHAPTER I

INTRODUCTION

A BRIEF HISTORY OF MAGNETIC MIRROR SYSTEMS

The concept of magnetic mirror reactor was suggested in early fifties independently by R.F. Post in U.S. and G.I. Budker in U.S.S.R. The plasma confinement in the mirror trap is brought about due to the reflection of particles from the strong regions of magnetic field ('mirrors') at the end of device. These reflect only particles of large pitch angles; particles with smaller pitch angles freely fly out of the device, so that a conical 'hole' in the velocity space is formed. Ions leave out when they scatter in angle by classical columbic collisions to small pitch angles at which they can no longer be reflected by the mirrors. Electrons tend to scatter more rapidly than ions so that their density tends to drop slightly below that of the ion-density. The resulting charge difference gives rise to a positive ambipolar potential which holds the electrons electrostatically such that electron losses are kept equal to ion-losses. The ion-confinement τ in the simple mirror is then determined roughly by the diffusion time in pitch angle

$$\tau = \tau_{ii} < \theta_{inj} - \theta_{LC} >^2 \qquad (1)$$

where τ_{ii} is the time for an ion to diffuse one-radian in pitch angle and where θ_{inj} and θ_{LC} are the ion injection and loss angles respectively. Magnetic mirror systems are also called 'open traps' as in this system the magnetic lines of force leave the plasma volume and pass through the material walls. This is contrary to the closed lines systems where lines of force do not leave the volume but are rather confined to a family of nested topologically toroidal flux surfaces.

In ideal collisional circumstances (that is when the non-classical effects have been suppressed) τ_{ii} is given by

$$\tau_{ii} = E_i^{3/2}/n \qquad (2)$$

where E_i is the average ion-energy and n is the density. From equation (2) it follows that the Lawson's product $n\tau$ for mirror reactors will be independent of the size, and the shape of the plasma. However, when the effects of mirror ratio R and the ambipolar potential ϕ ($\sim$ typically 30 to 40 percent of E_i) are considered the Lawson's product turns out to be

$$n\tau = K \times 10^{10}\, E_i^{3/2}\,[\mathrm{keV}]\,\log R_{eff.} \tag{3}$$

where

$$R_{eff.} = \frac{R}{[1 + q\phi/m_i E_i]} \tag{4}$$

where m_i and q are the mass and charge of an ion. And K is a phenomenological constant which takes into account other effects like wave diffusion, electron drag, etc. It was suggested that to fuel the mirror reactor and to sustain the plasma against the collisional end losses, high power neutral beams must be injected perpendicular to the field lines (1-3). The Q of a mirror reactor which is defined as the ratio of thermal power output to the neutral beam power injected, is then given by

$$Q = \frac{n\tau \langle \sigma v \rangle E_{nucl}}{4 E_0} \tag{5}$$

where $\langle \sigma v \rangle$ is the averaged reaction rate, E_{nucl} is fusion energy released per D-T reaction ($\simeq$ 17.6 MeV), E_0 is the injection energy. Taking into account only the classical effects the Q for a mirror reactor turns out to be $\sim$ 1.1 x log R_{eff}, which is a near unity value (4,5). The implication of near unity value of Q is that, in order to have economic advantage, in D-T mirror reactors, a large fraction of the output power will have to be recirculated (re-injected into the machine) with high efficiency; the thermal conversion fails on the measure of the required efficiency (6). Because of these considerations, mirror machines appeared in the early sixties, to have doubtful prospects as fusion reactors. Later Post et al suggested that because of the low values of Q mirror reactors could function as power amplifiers. In order to achieve economical net electric output, the scheme of 'Direct Conversion' was suggested, by which the end losses could be turned into an advantage. The principle, in brief, is that the plasma streaming out at the ends is first spread by diverging magnetic fields until its density is reduced to the extent that the Debye length becomes large enough to permit the penetration of electric fields. The ions and the electrons can then be collected on separate collectors to get net electrical power (7-9).

Inspite of the somewhat poor prospects, the studies on open traps were nevertheless conducted in a large number of laboratories and occupied atleast as important a place in the overall fusion research as the work on closed traps (because at that time, the prospects of mastering the fusion energy still seemed very remote). Some important considerations in favour of the mirror traps were their technical simplicity and flexibility and the possibility of conducting a wide range of physical experiments on them. In the period from 1960 to 1965 much of what forms now the basis of plasma physics was derived from the open traps. In fact it can be said that open traps passed through a golden age in the early sixties; at the IAEA Conference held at Salzburg (1961) and Culham (1965) more than a quarter of papers were devoted to them (22).

The result of Ioffe's team (12) on the possibility of suppressing flute instability by generating magnetic well created a particular gush of enthusiasm. It had been well known since the work of Rosenbluth and Longmire (1957) and that of Kadomtsev (1961) that the plasma in a simple axisymmetric mirror machine was strongly susceptible to the flute instability. Although in many moderate scale mirror traps this instability did not occur, presumably, because of the "line tying" due to the presence of cold plasma at the ends), it was feared that the position would worsen in the case of very hot plasma with high- β . In the light of Ioffe's result this problem could be regarded as solved. Soon other magnetic systems were reported which generated min- β . Of these the system which gained most currency was that of the Yin-Yang type proposed at Livermore.

A new hazard was discovered in 1965-66 through the studies of Post and Rosenbluth (13) who pointed out the existence of a dangerous class of micro-instabilities associated with the presence of a loss-cone. The instabilities studied by Post and Rosenbluth were electrostatic and could occur in a low-pressure plasma, their growth rate lying in between ion-cyclotron and ion-plasma frequency. To suppress the loss cone instabilities, Post in 1967 (14) proposed the addition of a group of hot ions to the plasma. In experiments carried out later it was shown that this measure was in fact very effective.

Nevertheless, from 1968, open traps appeared to gradually lose ground. This was due to the fact that in their plasma parameters and especially in confinement time open traps were progressively lagging behind Tokamaks which made rapid advances through 1969-73. Consolidation of this point of view was promoted by the preliminary development of designs for commercial fusion reactors of various types - it seemed that because of the low Q value open traps would not give acceptable energy characteristics for fusion power plants.

There were some hopeful developments, however, in Golovin's Laboratory. Chuyanov, Arsenin (15) and coworkers conducted interesting experiments on the suppression of MHD instabilities by feedback; experiments carried out by Ioffe, Kanaev, and Yushamanov on the PR-6, PR-7 (16) devices contributed to a better understanding of the mechanisms of suppression of loss-cone instabilities; at Livermore, the 2XIIB was built which played a decisive role in the enhancement of mirror trap research. Various laboratories continued persistent search for the method of increasing the Q-value in open systems. In particular, the concept of multi-mirror confinement was proposed and experimentally tried at Novosibirsk and Berkeley.

A sharp turn for the better occured in the years 1975 and 1976 which were noteworthy for two important events: first the 2XIIB machine (17) was commissioned at Livermore and second the concept of ambipolar trap or tandem mirror was proposed by Dimov and coworkers (18) at Novosibirsk and by Fowler and Logan of Livermore (19). The first of these events demonstrated that the experimentors had at their disposal a technology equal to the task of obtaining in open traps a plasma with fusion parameters ($n \sim 10^{14}$ cm^{-3}), T_i = 10 KeV): high power (5 MW) neutral beam injectors with sufficient operating reliability, a vacuum technology based on gettering which ensured sufficient plasma purity under condition of high-energy neutral injection; plasma beams ensuring neutral capture at the initial stage and suppression of loss-cone instabilities at later stages (22). The second event was the proposal of the idea of the ambipolar traps which atleast in principle, enabled the Q value to be raised considerably for the same range of plasma parameters that are conventional for open traps, and on the basis of the technology of the type used for 2XIIB. A number of large devices were built in order to verify the idea of the ambipolar trap. The first to be commissioned, in May 1978, was the Gamma-6 device at the University of Tsukuba, which was followed at the end of 1978 by TMX at Livermore and in May 1980 by Phaedrus at the University of Wisconsin. In June 1981 the AMBAL device is planned to go into operation at the Novosibirsk Institute of Nuclear Physics and in the autumn of that year TMX-V at Livermore.

Experiments on Gamma-6 and TMX (20) have demonstrated with sufficient reliability the influence of the ambipolar potential on plasma life-time in the central mirror trap. Besides, the experiments on TMX showed the feasibility of MHD-stable equilibria at finite plasma pressure in the central mirror trap.

On the other hand, some of the results obtained on TMX put one on one's guard, for in the regimes which are most promising at first sight corresponding to maximum electron temperature the confinement time does not increase, as is predicted by the classical theory (21) but decreases sharply. This is due to the development, in the end mirror trap, of fluctuations which propagate to the central mirror trap and cause the growth of 'ion tails', which then easily comes out of the trap. The fluctuations are evidently associated with the development of drift-cyclotron-loss-cone (DCLC) instability. As has been pointed above, previous experiments on PR-6, PR-7 and 2XIIB showed that there were a number of methods of suppressing loss cone instabilities. However, these instabilities are still far from being controlled - so far it has been possible to achieve stabilization only for small T_e/T_i ratio not exceeding $\simeq 10^{-2}$ whereas in reactor this ratio is expected to be higher than 10^{-1}. Since a rise in electron temperature is bound to be accompanied by a deterioration in the confinement of low-energy ions, loss-cone instabilities may become more hazardous.

Many things remain unclear about the mechanism of excitation of these instabilities under real experimental conditions; in particular, there is no simple explanation for the frequently observed regularity of oscillations, nor is it understood why their frequency is close to the ion-cyclotron frequency corresponding to the minimum of magnetic field. As to other physical problems such as enhanced transverse transport in a non-axisymmetric magnetic field or the ballooning instability in the central mirror trap, these can certainly be solved by an appropriate choice of magnetic field geometry, perhaps at the cost of a more complicated system.

Reactor calculations made so far on the basis of the ambipolar trap show that a level of Q = 5-10 can fairly easily be attained in such machines though this involves using 0.5-1 MeV ion injectors in the end mirror traps and increasing the magnetic field in the mirrors to 150 KG. In economic characteristics such a system is inferior to the tokamak reactors if it is a pure fusion reactor that we are considering. However, where fusion-fission hybrid systems are concerned the ambipolar trap is even today competitive (22) with tokamaks. In the ambipolar trap research, the efforts are being directed towards improvement of the longitudinal ion-life time in the central trap and towards lowering the injection energy requirements in the end traps. Both these conditions are met by the concept of 'thermal barriers' suggested by Baldwin and Logan in 1979 (23). The essence of the idea is to break down the thermal contact between the electrons of the central and end mirror traps, after which by some method or other the end-trap electrons can be heated to a temperature considerably exceeding the temperature of central trap electrons. Then the electrostatic potential hump in the end traps can be raised substantially for a given ion density. The most natural method of breaking contact between the two electron populations is to include an additional cell with low density plasma between the central and the end traps. Maintenance of the minimum density in this cell requires that the captured ions should be continuously 'pumped out' of it; this can be achieved by charge exchange on a specially oriented streams of neutrals or by using the drift effect in a non-axisymmetric magnetic field. Another method of creating the minimum density is to use inclined ion injection (sloshing ions) (24). The ion distribution functions obtained in this case appear to be less susceptible to loss-cone instabilities, although two-stream instabilities may represent some danger to them. However, even if thermal barrier works well and the longitudinal life time ceases to limit the Q factor the anomalous transverse losses from the central trap, associated with the development of drift instabilities may still remain as an important problem. In coming years one hopes to have solutions to these problems of ambipolar traps. Of great interest in this respect will be the experiments to be carried out on the TMX-V device which is now being built at Livermore (22). Very recently a method based on slashing electrons has also been proposed for creating thermal barrier (23).

In the preceeding pages we have followed the development, so to say, from the axisymmetric mirror traps, to the mirror traps with min-B onward to the ambipolar traps. However, this line was never the only one. There were significant developments in other directions too e.g. reversed field traps, multiple mirror trap, anti-mirror trap, spindle cusps systems wigh high frequency plugging of mirror etc. Thus in the words of Ryatov (27) we can say that on the whole "open systems are now experiencing a period of fresh growth. Having retained their traditional advantages i.e. geometric simplicity, the possibility of attaining high-β values and steady state operating conditions, they have a good chance in the next few years to rid themselves of their main fault i.e. the short confinement time due to loss-cone excited instabilities and turbulence".

I.2. SCOPE OF THESIS:

As stated in the previous section, at present, loss cone generated instabilities seems to be plaguing the performance of the open traps. Because of the instabilities plasma readily becomes turbulent. This turbulence enhances the particle losses and thus adversely affects the performance of mirror traps as fusion reactors. Hence from the point of view of evaluating and improving upon the performance of mirror traps, it becomes necessary to identify and understand various physical processes which may be related to the turbulence. The primary aim of the thesis is to study these various instabilities, turbulence and associated transport processes and further to suggest ways and means of suppressing them in some cases.

In Chapter II we have investigated the mechanism of anomalous particle losses in Base-Ball II mirror experiment. In this experiment the start up was obtained without the target plasma i.e. by firing energetic neutral beams in vacuum. The observations indicated that because of r-f activity the density build up was limited to $\sim 10^9$ cm^{-3}. The r-f bursts occurred typically over a time scale of a few hundred μs. In the experiment ion-cyclotron oscillations and oscillations at ∇B -drift frequency were also noticed. In an effort to explain the anomalous particle losses we have identified the mode at ∇B -drift frequency with the Varma mode. We have further shown that in the presence of ω_{ci} -oscillation the Varma mode becomes nonlinearly unstable. It exhibits a periodic bursting behaviour on the time scale of few hundreds of μs. This periodic increase of amplitude increases the scattering into the loss-cone which thereby gives rise to bursts of particles.

In Chapter III we have investigated the phenomenon of suppression of DCLC turbulence by parallel injection of an electron beam in the context of Constance II and 2XIIB mirror experiments. We have shown that the parallel injection of electron beam gives rise to a spectrum of Langmuir waves. These Langmuir waves, on the time scale of DCLC waves, act as quasi-particles and hence can exhibit resonant damping or growth depending upon the slope of the plasmon distribution function at the resonance point. In this Chapter we derive a non-linear dispersion relation for DCLC modes in the presence of these quasi-particles and show that the modified growth rate depends upon the beam power. In the quasi-linear theory we have derived a closed set of equations to calculate the electron and ion temperatures, fluctuation level, ion life times etc. We have shown that scattering of Langmuir waves by DCLC waves is quite small hence the effect of Langmuir waves on DCLC modes will last for a long time. We have further shown that in the presence of electron beam with power greater than a threshold value, the ion diffusion is reduced and hence the ion life time is increased. Our results agree very well with the observations of 2XIIB and Constance II.

In Chapter IV we have investigated the non-linear instability of high frequency convective loss cone (HFCLC) modes in the presence of electron beam induced Langmuir waves. We have shown that Langmuir waves can be strongly coupled to HFCLC by non-linear Landau damping. In this

Chapter we derive an appropriate set of kinetic wave equations for LW and HFCLC waves. The non-linear growth rate shows a dependence on the beam velocity and has a negative sign. Thus in the presence of electron beam HFCLC waves are heavily damped which may explain their absence in Constance II.

In Chapter V we have investigated the interaction of LW on low frequency wave number region of HFCLC spectrum. To study this interaction the adiabatic approximation is employed. It turns out that in this region the growth lengths are strongly affected. Typically for Constance II parameter they are reduced by about five times. This could be very dangerous for electron beam injected plasmas, because normally HFCLC modes are stabilized by axial convection. But if electron beams are employed for the purpose of controlling DCLC turbulence then these modes will be again destablized. We have further shown that in both the cases i.e. when the electron gun and plasma gun are fired simultaneously and the case when the firing of electron gun is delayed the growth lengths are reduced by about five times hence this harmful effect seems to be inevitable.

In Chapter VI we have investigated the process of saturation of DCLC turbulence in 2XIIB. Using perturbed orbit formalism, we have calculated the anomalous resistivity due to DCLC turbulence itself. We have shown that it induces enough dissipation to saturate the growth of DCLC modes. We have calculated the fluctuation level which agrees with the fluctuation level observed in 2XIIB. The model also explains the depression in fluctuation level and improvement in ion life time when warm plasma streams are employed.

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In Chapter VII we have investigated the suppression of DCLC instability by feedback circuits. Using material probes as suppressor and sensor we have shown that with gain of $\sim 50\,\Omega_i$, and a phase-different of $+90^{\circ}$ the critical density gradient required to excite the DCLC instability can be pushed by atleast two orders of magnitude. The minimum plasma radius can be brought down in the same proportion i.e. from 500 a_i to 3 a_i, where a_i is the ion larmour radius. Hence the constraint imposed by DCLC turbulence on mirror plasma radius is almost removed.

In Chapter VIII we have investigated some favourable effects of neutral beam induced lower hybrid waves (LHW) on DCLC modes. We have shown that for a modest level of LHW turbulence, the DCLC modes are damped. In the quasi-linear theory we have shown that in the presence of LHW, the hot ion-diffusion is decreased and hence the ion-life time inside the trap is increased.

In Chapter IX we have summarised our work and have given future possible directions for the extensions of this work.

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CHAPTER II

ANAMOLOUS LOSS OF PARTICLES IN BASE-BALL II MIRROR EXPERIMENT

1. Introduction:

The present concept of simple mirror fusion reactor envisages the perpendicular injection of energetic neutral beams (500 MW) for the purpose of fueling the reactor plasma in a steady state magnetic field. A general problem associated with this kind of reactor is to develop a method of 'start up', that is to procure the initial plasma inside the machine. The aim of Livermore Base-Ball II experiment was to develop a method of start up which the can be extended to the reactor regimes. In general there are following methods of procuring the initial plasma:

(i) Initially the transient target plasma is obtained inside the machine. This can be done in several ways like by laser irradiation of solid pellat, placed inside the machine (1,2) by passing cold plasma filament along the field lines (3) by using arc discharges, pulsed deuterium discharges (4) etc. or by injecting the plasma directly, which is heated and trapped by ECRH (5). Further the density and temperature build up are obtained by shooting neutral beams perpendicular to the field lines.

(ii) This method consists in obtaining a build up from high vacuum (6,7). For this purpose neutral beams are injected into vacuum in mirror machine. The beam gets Lorentz ionised to provide a target plasma which is then heated by charge exchange and maintained by ionization of the injected beam.

In one of the experiments done on Base-Ball II machine the build up was obtained by the 2nd method; injecting powerful neutral beams (2 KW) in high vacuum (7). The observations indicated that in the process of density build up, beyond a certain threshold, ion-cyclotron oscillations were excited. The onset of the instability was observed to depend on beam geometry and instability signals are usually absent shortly after the beam injection ceases. The draw back of this technique is that anomalous losses increase as the injected beam current increases limiting the density to $3\text{-}5\times10^{9}$ cm^{-3}. The spatial and temporal distribution of anomalous losses is related to potential perturbation rotating at ion ∇B B drift frequency. In this Chapter we have studied in some detail the mechanism of this anomalous loss of particles from the trap. In this connection we identify the mode near ∇ B-drift frequency with the mode described by Varma (8) for simple mirror geometry. We find that this mode in the presence of ion-cyclotron fluctuations becomes nonlinearly unstable and exhibits a periodic bursting behaviour very similar to the periodic bursts of azimuthal potential perturbations observed in the experiment. We have calculated the nonlinear growth time which agrees fairly well with the observed time of 50-100μs between the two bursts.

As the present investigation concerns Base-Ball II experiment it will be appropriate to briefly describe the experimental set up and observations. This is done in Section 1. In this section we have also discussed our reasons to identify the observed azimuthal potential perturbation with the Varma mode. Section 2 contains a brief description of the linear theory of the Varma mode, a description of Simon and Wing's no linear theory of the Varma mode, which, as we shall see, is important in the present context and the energy properties of the Varma mode. In Section 3 we have briefly discussed the model which we have chosen for the ion-cyclotron oscillations. Section 4 contains the non-linear instability theory of the Varma mode in the context of Base-Ball II mirror experiment, while in the last section we have summarized our results.

2. (i) Experiment:

The confining field of Base-Ball II experiment is a quadrupole magnetic well of depth 2.1 generated by a super conducting Base-Ball seam windings. This magnet has been operated at central fields upto 1.5 T. The mean diameter of the magnet is 1.2 m, resulting in a plasma radius of 0.1m and a volume of $\simeq$ 10 lit. The mirror separation on axis is 90 cm. To prevent the loss of plasma ions by charge-exchange a high vacuum is maintained in the plasma region. In this experiment several square meters of cryogenic pumping surfaces, at temperatures down to 2°K provide high speed pumping for all ambient gases. The base pressure is $\simeq 10^{-9}$ P_{α}. A single neutral beam of either hydrogen or deuterium atom was injected normal to the magnetic axis for a period of seconds. The beam energy was usually in the range of 0.5 to 5 KeV with some measurement upto 20 KeV. Beam currents were adjustable with the maximum beam current ranging upto 0.1 A equivalent at 20 KeV.

(ii) Observations:

In the experiment collisional plasma regime was examined by injecting hydrogen atom at low energy (0.5 - 2 KeV) under good vacuum conditions. The observations showed that a stable plasma at a density of $\simeq 10^9$ cm^{-3} was obtained in which classical scattering losses dominated over the charge exchange losses by the ratio of 6 to 1. Alongwith a near classical confinement time the r-f activity indicated the presence of an ion-cyclotron instability. It was noticed that as the density was increased, the anomalous losses increase rapidly. The repetitive bursts of ions each lasting for 50-100 micro-seconds were observed. In the analysis of the end-loss signals, in a typical case with high current injection the loss due to instability exceeded the classical losses, including the charge exchange, by more than a factor of four. The plasma density which could be achieved with the available beam power was limited to 3 to 5×10^9 cm^{-3}.

In the experiment low frequency oscillations at ion- ∇B drift frequency were also observed. The electrostatic probes displaced in azimuth around the central plasma region detected perturbations of plasma potential rotating generally in the direction of ion-drift. The signals were of two types. One was of a very small amplitude (perturbed potential of the order of 0.5 V) near the ∇ B drift frequency. These types of oscillations have been seen in stable plasmas as well. In experiment they were followed for as long as several minutes during the decay of the plasma. The other type of oscillations were also near ∇B frequency. These signals are large amplitude (20-50 V perturbed ϕ) and exhibit a bursting behaviour. The amplitude rises to a full value in typically 50-100 μ s seconds. Apparently the anomalous particles losses are due to these modes. Energetic bursts of protons and electrons are emitted from the rotating region of perturbed potential starting promptly with the potential increase in each bursts. The origin of this mode is not clear. If suitable density and temperature gradients are included in the fluid equations for min-B geometry, the characteristics of these modes can be probably reproduced. However, there are no apriori reasons to expect such gradients in the experiment. The authors of the experiments associate the stable small amplitude mode with the Varma mode. This may not be quite correct. It has been pointed out that the non-linear effects like wave-coupling through non-linear Landau damping plays an important role in the development of turbulence in the mirror plasma (9,10). Thus Varma mode which is linearly stable may not be so in nonlinear regime. In fact Simon and Weng (11) have shown that in the presence of other flute modes, in simple mirror geometry, the Varma mode becomes nonlinearly explosive. They correlate this behaviour of the Varma mode with the observed instability in 'Alice' and 'Phoenix' (12,13). In Base-Ball II which has a magnetic well the Varma mode cannot be driven unstable by other flute modes, as all of them are linearly stable. But as we shall show here that in the presence of ion-cyclotron oscillations the Varma mode is again driven unstable. It exhibists a periodic bursting behaviour. Hence it is more likely that in the experimental observations the large amplitude signal at ∇ B-drift frequency which exhibits a periodic bursting behaviour, rather than the small amplitude which is stable, is Varma's mode.

3. The Varma Mode:

Experiments on 'Alice' and 'Phoenix' (12,13) have indicated that plasma is capable of collective behaviour event at such low densities as 10^8 cm^{-3}. As the plasma is built up particle by particle from almost zero density by injecting neutrals atoms, the system becomes unstable at the density of about $\simeq 10^9$. It was noted that in addition to the usual flute modes, a density independent mode was seen both in 'Alice' as well as 'Phoenix'. Post (12) has given a theory for the phenomenon observed in 'Alice' which is worked out for plane geometry and approximate identification with the cylindrical geometry of the morror machine is made by introducing appropriate boundary conditions. His theory, which amounts to using an equivalent gravitational acceleration 'g' for the force due to the inhomogeneous magnetic field and is worked out for δ -function velocity distribution predicts a two branch curve for the frequency versus density upto a critical density where the two branches merge. While Kadomtsev (14) has discussed the problem for a mirror machine self-consistently using an equation for transverse motion averaged over the line of force. However, the use of 'effective gravity' may give results which may not be wholly correct. In certain circumstances, the use of an effective gravity not only gives results which are quantitatively incorrect, but also suppresses some information which is contained in the actual magnetic gradient force ($= - \mu \nabla B$) which depends upon the velocity of the particle. Varma (8) improved the theoretical treatment by relaxing the constraint of constant gravity. By allowing for variation of μ and using a closure equation for μ of the type

$$\frac{d\mu}{dt} = 0 \tag{1}$$

alongwith the usual set of fluid equation, it was shown that dispersion relation for the flute mode is of the form

$$1 = -\frac{\omega_{pi}^2}{\Omega_i}\frac{|\sigma|}{k''}\left\{\frac{|\sigma'/\sigma| + 2}{\omega'' + k''|V_{Di}|} - \frac{k''|V_{Di}|}{(\omega'' + k''|V_{Di}|)^2}\right.$$

$$\qquad - \frac{\omega_{pe}^2}{\Omega_e}\frac{|\sigma|}{k''}\left[\frac{|\sigma'/\sigma| + 2}{\omega''}\right] \tag{2}$$

where (ω'', k'') are the frequency and wave numbers of flute mode, $\sigma = -\frac{1}{B}\frac{dB}{dx}\hat{z}$ is the magnetic field gradient, $\sigma' = \frac{1}{n}\frac{dn}{dx}$ is the density gradient, $|V_{Di}| = \frac{1}{2}\sigma u_{\perp 0}^2/\Omega$ is the ∇B-drift, $u_{\perp 0}$ is the velocity with which N -B particles are injected etc. Equation (2) is cubic in ω rather than the usual quadratic.

Defining

$$\nu = \frac{\omega''}{m W_i} \qquad W_i = |V_{Di}|/r_0$$

and

$$q = \frac{\omega_{pi}^2 r_0}{|\Omega_i| m^2 |V_{Di}|}$$

[r_0 is the radius of plasma]

we can put equation (2) in a more tractable form as

$$\nu^3 + 2\nu + \nu - q\nu - \nu = 0 \tag{3}$$

In the limit $|\sigma\gamma_0| << 1$, this can be solved to give the three roots as

$$\nu = -1$$

$$\nu = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1+4q}$$

ν = -1 is the root which can be identified with the density independent mode observed in 'Alice' and 'Phoenix'.

Later Simon and Weng (11) examined the nonlinear stability of the Varma mode in the presence of the other flute modes. They found that if the injection current is increased beyond a threshold one of the lower modes starts growing. Because of the nonlinear coupling it triggers an explosive growth of the Varma mode and shuts off itself. The Varma mode then grows on its own until inhibited by some conservation process. This behaviour was identified by them with the sudden explosion of the amplitude signal when the injection current was increased beyond the threshold. In min-B geometry such a coupling is not possible as all the modes are linearly stable.

The energy of the Varma mode in weakly dissipative dieletric media is given by

$$W = \epsilon_0 \nu \frac{\partial}{\partial \nu} \epsilon'' \langle E''^2 \rangle / 2 \qquad (4)$$

where E'' is the electric field amplitude of the Varma mode and ϵ'' is the dielectric constant given by

$$\epsilon'' = 1 - \frac{q}{\nu(\nu+1)} - \frac{q|\sigma|\gamma_0}{(\nu+1)^2} \qquad (5)$$

In order to calculate the energy of the Varma mode we will have to retain terms of the order $\sigma\gamma_0$ while solving for roots from equation (2) (In obtaining equation (3) they were neglected). Thus including the first order correction in $\sigma\gamma_0$ to the density independent root we have

$$\nu = [-1 - \sigma\gamma_0] \tag{6}$$

Using this value of ν in equation (5), we have the energy of Varma mode as

$$W = \epsilon_0 \frac{3q}{|\sigma\gamma_0|^2} \frac{\langle E'' \rangle^2}{2} \tag{7}$$

Now as $q > 0$, W is > 0. Hence the Varma mode is always a positive energy mode.

4. <u>Model for Ion Cyclotron Oscillations</u>:

In this section we will try to model the linear properties of ion-cyclotron oscillations observed in the experiment. In the mechanism of excitation of these oscillations electron Landau damping plays a very important role (12). In mirror machines, because of Loss Cone in velocity space a mirror confined plasma develops a positive ambipolar potential ϕ_M. In this case the electrons are retained electrostatically having a truncated Maxwellian distribution extending upto an energy corresponding to the positive potential ϕ_M. The anisotropic distribution tries to drive the ion-Berstein mode with finite $k_\parallel$ unstable. But in situations involving freely streaming electrons, strong Landau damping occurs when $\omega / k_\parallel$ matches the electron thermal velocity. The frequency of the n^{th}

harmonic is given by

$$\omega = n\Omega_i = \omega_{pe} K_{\parallel}/K \qquad (K_{\parallel} \ll K_{\perp})$$

Hence the resonance condition for Landau damping requires

$$\frac{1}{2} m v_{\parallel}^2 = \frac{1}{2} m \left(\omega_{pe}^2 / K_{\perp}^2 \right) \tag{8}$$

when the electrons are held in by an electrostatic potential, as in the present experiment, then in the central region of the plasma, there exists a range of possible electron energies given by

$$\frac{1}{2} m v_{\parallel}^2 \leq e \phi_m \tag{9}$$

From these considerations it is clear that threshold for the instability is the point when the density becomes so that

$$\frac{1}{2} m \left(\frac{\omega_{pe}^2}{K_{\perp}^2} \right) > e \phi_m$$

or

$$\frac{\omega_{pi}^2}{\Omega_i^2} > (K_{\perp} a_i)^2 \frac{e \phi_m}{T_i} \tag{10}$$

where a_i is the ion gyro radius and T_i is the ion temperature. This equation suggests that with increase in the electron temperature and the concommittant increase in ϕ_m ($\simeq 3\ T_e$), the density threshold should increase linearly. This agrees well with the observed increase of ω_{pi}^2/Ω_i^2 with $e\phi_m/v_{thi}$ in Phoenix II, Base-Ball I & II etc. (7,12).

To find the energy characteristics of these oscillations we write the general dispersion relation for electrostatic mode as (15)

$$\epsilon = 1 - \frac{2\pi\omega_{pi}^2}{k^2}\int \frac{\sum_n J_n^2\left[k_{\parallel}v_{\parallel}\frac{\partial f_{io}}{\partial v_{\parallel}^2} + n\Omega\frac{\partial f_{io}}{\partial v_{\perp}^2}\right]dv_{\perp}^2 dv_{\parallel}}{(k_{\parallel}v_{\parallel} - \omega + n\Omega_i)}$$

$$- \frac{2\pi\omega_{pe}^2}{k^2}\int \frac{\sum_n J_n^2\left[k_{\parallel}v_{\parallel}\frac{\partial f_{eo}}{\partial v_{\parallel}^2} + n\Omega_e\frac{\partial f_{eo}}{\partial v_{\perp}^2}\right]dv_{\perp}^2 dv_{\parallel}}{(k_{\parallel}v_{\parallel} - \omega - n\Omega_e)}$$

(11)

where f_{eo} and f_{io} are the zero order electron and ion distribution functions respectively. In the plasma formed by neutral beam injection the electrons are generally cold ($T_e \simeq$ a few eV) hence we may use cold electron approximation i.e. $k^2 a_e^2 << 1$. In the ion term we may neglect the term $\partial f_{io}/\partial v_{\parallel}^2$ as compared to $\partial f_{io}/\partial v_{\perp}^2$ for the reason that in plasma formed from neutral beam injection there is a large temperature anisotropy i.e. $T_{\perp} >> T_{\parallel}$. Under this approximation equation (11) becomes

$$\epsilon = 1 - \frac{\omega_{pe}^2 k_{\parallel}^2}{k^2\omega^2} - \frac{2\pi\omega_{pi}^2}{k^2}\int \sum_n \frac{J_n^2 n\Omega_i \frac{\partial f_{io}}{\partial v_{\perp}^2} dv_{\perp}^2 dv_{\parallel}}{(k_{\parallel}v_{\parallel} - \omega + n\Omega_i)}$$

(12)

where $\omega \simeq \omega_{pe}k_{\parallel}/k \simeq n\Omega_i$. It can be shown that for the n^{th} harmonic only the n^{th} term in the summation will contribute significantly. Hence we retain only the n^{th} term in the summation. The condition for the instability for the n^{th} mode from the linear theory is

$$\int_0^{\infty} J_n^2 \frac{\partial f_{io}}{\partial v_\perp^2} dv_\perp^2 > 0 \tag{13}$$

To find out the energy characteristics of the n^{th} harmonic we differentiate ϵ with respect to ω to get

$$\frac{\partial \epsilon}{\partial \omega} = 2\frac{\omega_{pe}^2 k_{\parallel}^2}{k^2 \omega^3} - \frac{2\pi \omega_{pi}^2}{k^2} n\Omega_i \int_{-\infty}^{\infty}\int_0^{\infty} \frac{J_n^2 \frac{\partial f_{io}}{\partial v_\perp^2} dv_\perp^2 dv_{\parallel}}{(k_{\parallel}v_{\parallel} - \omega + n\Omega_i)^2} \tag{14}$$

For given ω and k, $\partial\epsilon/\partial\omega$ can be evaluated. In order to evaluate the energy of the wave, let us take ω to be positive so that the energy of the wave is given by sign of $\partial\epsilon/\partial\omega$ which can be evaluated for a given value of ω and K from equation (14). We may evaluate it for some typical values of ω and K. Accordingly typically for $\omega \sim \omega_{pe} k_{\parallel}/k \sim \Omega_i$ the first term is $\simeq 1/\Omega_i$, while the lowest bound on second term $\simeq 1/k^2 \lambda_{Di}^2 \Omega_i$. In the experimental observations $k\langle a_i \rangle$ was typically in the range 1 to 1.5. This yields $k^2\lambda_{Di}^2 \simeq$ 0.4 to 0.5 < 1. Hence the waves observed in the experiment were negative energy waves.

5. Non-linear Stability of the Varma Mode:

In this section we investigate the nonlinear instability of the Varma-mode in the framework of 'weak turbulence theory'. The validity of the application of this theory requires that $W = \langle E^2\rangle/8\pi / nkT$ be $\ll 1$ where $\langle E^2\rangle/8\pi$ is the average field energy density and nkT

is the thermal energy density of particles. In the experiment (7) the amplitude of the Varms mode ranges from 25 to 50 V in which case for an ion temperature of 1 KeV, $W \simeq 1/20 < 1$. Under this condition the perturbation expansion of f and E are made in terms of W as the smallness parameter

$$f = f_0 + W f_1 + W^2 f_2 + \cdots\cdots$$

$$E_k = W E_k^{(1)} + W^2 E_k^{(2)} + \cdots\cdots \tag{15}$$

where we assume that the equilibrium distribution function gives a weak instability. Then using Vlasov-Maxwell's equation once the solution is obtained to a certain order (in this case to the third order), appropriate statistical averages using random phase approximation are performed over spatially uniform ensemble to obtain a set of coupled equation for the spectral energy density. Based on this approach, the comprehensive treatment of the theories of plasma turbulence have been developed by a nunber of authors (16,17,18,19). Our aim in this Chapter is to obtain in explicit form, the matrix elements for nonlinear Landau damping of the Varma mode and the ion-cyclotron mode. This will be done using Porkolab and Change formalism (20) based on expansions of Vlasov's equation and methods of characteristics. The resonance condition for such an interaction is given by

$$\omega - \omega'' - (k_{\parallel} - k_{\parallel}'')\, v_{\parallel} = m\Omega_i \tag{16}$$

where (ω, k), (ω'', k'') are the frequency and the wave number of ion cyclotron and the Varma mode respectively. It should be noted that selection rules of non-resonant interaction like non-linear Landau damping are more easily satisfied than those for resonant interaction and hence they are the dominant processes in the development of turbulence. In fact it can be shown that in the matrix element of non-linear Landau damping, there are two terms of opposite sign. One of the terms represents space charge effects (i.e. scattering from dressed particle) while the other term represents the ponderomotive force due to the beat wave (four wave scatter from the bare particle). Normally, these terms cancel making this interaction ineffective i.e. terms of order $O(k^2 \lambda_{Di}^2)$ cancel [the first surviving term is $O(k^4 \lambda_{Di}^4)$ (21)] except for short wave-length modes in the absence of magnetic field or for waves travelling perpendicular to the magnetic field. The present case belongs to the later type.

We start from the Vlasov-Maxwell's equations which for electrostatic waves are

$$\frac{\partial f_k}{\partial t} + \vec{v}\cdot\frac{\partial f_k}{\partial \vec{x}} + \frac{q}{m}\left(\frac{\vec{v}\times\vec{B}_0}{c}\right)\cdot\frac{\partial f_k}{\partial \vec{v}} + \frac{q}{m}\vec{E}_k\cdot\frac{\partial f_0}{\partial \vec{v}}$$

$$= -\frac{q}{m}\sum_{k'\neq 0}\vec{E}_{k-k'}\cdot\frac{\partial f_{k'}}{\partial \vec{v}} \qquad \text{--(17)}$$

$$i\vec{k}\cdot\vec{E}_k = 4\pi\sum_{i,e} q\int d^3v\, f_k \qquad \text{--(18)}$$

$$\vec{E}_k = -\vec{\nabla}\phi_k = -i\vec{k}\,\phi_k \qquad \text{--(19)}$$

The notations are standard. Here all quantities are expanded in Fourier series namely

$$\vec{E}(x,t) = \sum_k \vec{E}_k \, e^{i(\vec{k}\cdot\vec{x}-\omega t)} \tag{20}$$

where the f_k and E_k are the Fourier components of the distribution function f and electric field E defined through

$$f(x,t) = g + \sum_{k'\neq 0} f_{k'} \, e^{i(k'\cdot x-\omega t)} \tag{21}$$

K = 0 term has been treated separately according to the usual quasi-linear treatment. Thus $f_{k=0}(v,t) = g$ varies slowly in time. In this work we are interested in mode coupling term on the right hand side of equation (17). In the following we shall make use of the reality condition $\omega_{-k} = -\omega_k^*$, $\phi_{-k} = \phi_k^*$.

As stated before, we expand ϕ_k and f_k in a perturbation expansion. Since we are mainly interested in nonlinear Landau damping we will need perturbation solution valid upto fourth order in ϕ_k. The iterative solution of equation (17) to equation (19) can then be written to first, second and third order in ϕ_k as follows.

In first order we have:

$$\epsilon(\omega,\vec{k})\,\phi_k^{(1)}(t) = 0 \tag{22}$$

where $\epsilon(\omega,\vec{k})$ is the quasi-linear dielectric function given later.

To second order we have:

$$\epsilon(\omega,\vec{k})\,\phi_k^{(2)}(t) = -\sum_j \sum_{k'} \frac{4\pi q^3 n_0}{k^2 m_j^2} \int d^3v \int_0^\infty d\tau \int_0^\infty d\tau'$$

$$G_k(\tau) \times \Big[(\vec{k}-\vec{k}')\cdot \frac{\partial}{\partial \vec{v}(\tau)} G_{k'}(\tau')\, \vec{k}'\cdot \frac{\partial g}{\partial \vec{v}(\tau')}$$

$$+\ \vec{k}'\cdot \frac{\partial}{\partial \vec{v}(\tau)} G_{k-k'}(\tau')(\vec{k}-\vec{k}')\cdot \frac{\partial g}{\partial \vec{v}(\tau)} \Big] \phi_{k'}\, \phi_{k-k'}$$

$$\text{-----(23)}$$

To third order:

$$\epsilon(\omega,\vec{k})\,\phi_k^{(3)}(t) = -\sum_j \sum_{k'} \frac{4\pi q^3 n_0^2}{k^2 m_j^2} \Big\{ \int d^3v \int_0^\infty d\tau$$

$$\int_0^\infty d\tau'\, G_k(\tau)(\vec{k}-\vec{k}')\cdot \frac{\partial}{\partial \vec{v}(\tau)} G_{k'}(\tau')\, \vec{k}'\cdot \frac{\partial g}{\partial \vec{v}(\tau')} \Big[\phi_{k-k'}^{(2)}\, \phi_{k'}$$

$$+\ \phi_{k-k'}\, \phi_{k'}^{(2)} \Big]$$

$$+\Big[\sum_{k''} \frac{iq}{m_j} \int_0^\infty d\tau \int_0^\infty d\tau' \int_0^\infty d\tau''\, G_{\vec{k}}(\tau)\,(\vec{k}-\vec{k}')\cdot \frac{\partial}{\partial \vec{v}(\tau)} G_{\vec{k}'}(\tau)$$

$$(\vec{k}'-\vec{k}'')\cdot \frac{\partial}{\partial \vec{v}(\tau')} G_{\vec{k}''}(\tau'')\, \vec{k}''\cdot \frac{\partial g}{\partial \vec{v}(\tau'')} \Big]$$

$$\cdot\ \phi_{k-k'}\, \phi_{k'-k''}\, \phi_{k''}$$

$$\text{---(24)}$$

where $G_k(\tau)$ is the Green's function for the propagation along the exact orbits, M_j is the particle mass, n_o is the equilibrium density. In the weak turbulence approximation, we have

$$G_k(\tau) = e^{i[\vec{k}\cdot(\vec{x}'-\vec{x})+\omega\tau]} \qquad (25)$$

where x' designates the unperturbed particle orbits. We recall that in equation (24) there are two types of terms, namely those associated with the resonant mode-mode coupling (i.e. $\epsilon(\omega,k) = 0$) and those due to non-resonant wave-wave scattering (i.e. $\epsilon(\omega,k) \neq 0$). The latter also are also called 'virtual waves' or 'quasi-modes'. To obtain a wave-kinetic equation valid upto fourth order in ϕ_k, we substitute the appropriate expression for virtual waves $\phi_{k'}^{(2)}$ and $\phi_{k-k'}^{(2)}$ from equation (23) in equation (24). The last term of equation (24) is the so-called four-wave-scattering term. This term also contributes to the non-linear Landau damping through the resonance condition mentioned before. In the random phase-approximation, equation (23) and equation (24) contribute to the same order in ϕ_k in the wave-kinetic equation which is constructed from the wave equation

$$\epsilon(\omega,\vec{k})\phi_k(t) = \epsilon(\omega,\vec{k})\left[\phi_k^{(2)}(t) + \phi_k^{(3)}(t)\right] \qquad (26)$$

Utilizing the random phase approximation expanding $\epsilon(\omega,k)$ in the usual manner and transforming to time-co-ordinates, equation (26) becomes (Appendix E)

$$\frac{\partial \epsilon(\omega,\vec{k})}{\partial \omega}\frac{\partial \phi_k(t)}{\partial t} = \gamma_k \frac{\partial \epsilon(\omega,k)}{\partial \omega}\phi_k$$

$$+\sum_{k'}\Big\{ i A_{k-k',k'}\,\phi_{k-k'}\,\phi_{k'} + \mathrm{Im}\Big[B_{k,k'}\,\phi_k\,|\phi_{k'}|^2$$

$$+ C_{k,k-k'}\,\phi_k\,|\phi_{k-k'}|^2$$

$$+ D_{k,k-k'}\,\phi_k\,|\phi_{k-k'}|^2\Big]\Big\}$$

(27)

where γ_k is the quasi-linear growth rate, A is the resonant mode coupling co-efficient given by the right hand side of equation (23) and the terms B, C and D are the contribution from equation (24).

We are here interested in obtaining compact expression for the latter terms. Using previous equations, we obtain the following expressions:

$$B_{k,k'} = -\sum_j \frac{16\pi^2 q_j^6 n_0^2}{M_j^4\, k^2 (k-k')^2\, \epsilon[\omega-\omega', \vec{k}-\vec{k}']}$$

$$\cdot\Big\{\int d^3v \int_0^\infty d\tau \int_0^\infty d\tau'\, G_{k-k'}(\tau)\times\Big[\vec{k}\cdot\frac{\partial}{\partial \vec{V}(\tau)}G_{-k'}(\tau')$$

$$\vec{k}'\cdot\frac{\partial g}{\partial \vec{V}(\tau')} + \vec{k}'\cdot\frac{\partial}{\partial \vec{V}(\tau)}G_k(\tau')\;\vec{k}\cdot\frac{\partial g}{\partial \vec{V}(\tau')}\Big]$$

$$\times\int d^3v\int_0^\infty d\tau\int_0^\infty d\tau'\, G_k(\tau)\,(\vec{k}-\vec{k}')$$

$$\cdot\frac{\partial}{\partial \vec{V}(\tau)}G_{k'}(\tau')\,\vec{k}'\cdot\frac{\partial g}{\partial \vec{V}(\tau')}\Big\}$$

--- (28)

$$C_{k,k-k'} = -\sum_j \frac{16\pi^2 q^6 n_0^2}{m_j^4 \, k^2 \, k'^2 \, \epsilon(\omega', \vec{k}')}$$

$$\times \left\{ \int d^3v \int_0^\infty d\tau \int_0^\infty d\tau' \, G_{k'}(\tau) \times \left[(\vec{k}-\vec{k}') \cdot \frac{\partial}{\partial \vec{V}(\tau)} \right.\right.$$

$$\left. G_k(\tau') \, \vec{k} \cdot \frac{\partial g}{\partial \vec{V}(\tau')} + \vec{k} \cdot \frac{\partial}{\partial \vec{V}(\tau)} G_{-(k-k')} (\vec{k}-\vec{k}') \cdot \frac{\partial g}{\partial \vec{V}(\tau')} \right]$$

$$\times \int d^3v \int_0^\infty d\tau \int_0^\infty d\tau' \, G_k(\tau) (\vec{k}-\vec{k}')$$

$$\left. \cdot \frac{\partial}{\partial \vec{V}(\tau)} G_{k'}(\tau') \, \vec{k}' \cdot \frac{\partial g}{\partial \vec{V}(\tau')} \right\}$$

---- (29)

$$D_{k,k-k'} = \sum_j \frac{i \, 4\pi q^4 n_0}{k^2 \, m_j^3} \int d^3v \int_0^\infty d\tau \int_0^\infty d\tau' \int_0^\infty d\tau''$$

$$G_k(\tau) (\vec{k}-\vec{k}') \cdot \frac{\partial}{\partial \vec{V}(\tau)} G_{k'}(\tau') \cdot$$

$$\left[(\vec{k}-\vec{k}') \cdot \frac{\partial}{\partial \vec{V}(\tau')} G_k(\tau'') \, \vec{k} \cdot \frac{\partial g}{\partial \vec{V}(\tau'')} \right.$$

$$\left. + \vec{k} \cdot \frac{\partial}{\partial \vec{V}(\tau')} G_{-(k-k')}(\tau'') \, (\vec{k}-\vec{k}') \cdot \frac{\partial g}{\partial \vec{V}(\tau'')} \right]$$

---- (30)

where we normalized so that $\int d^3v\, g(v) = 1$ or

$$\int_{-\infty}^{\infty} dv_{\parallel} \int_{0}^{\infty} v_{\perp}\, dv_{\perp}\, F_0(v_{\perp}) = 1$$

(where $2\pi g = F_0$). In order to perform the indicated integrations we use the cylindrical co-ordinates in velocity space with the magnetic ~~field in Z-direction. In particular, we have~~ $\cancel{\vec{B} = [0, 0, B]}$,
$k = [k_z, k_\theta, k_\gamma]$, $k' = [k'_z, k'_\theta, k'_\gamma]$ and we put $k - k' = k''$
$\omega - \omega' = \omega''$. For orbits we have

$$V(\tau) = \{v_{\perp} \cos[\Omega\tau + \theta(\tau)], v_{\perp} \sin[\Omega\tau + \theta(\tau)], v_{\parallel}\}$$

$$y' = y + \frac{v_{\perp}}{\Omega}\left[\cos(\Omega\tau + \theta) - \cos\theta\right]$$

$$z' = [z - v_{\parallel}\tau] \tag{31}$$

and hence

$$\vec{k}\cdot\frac{\partial}{\partial \vec{V}(\tau)} = k_{\perp}\sin(\Omega\tau + \theta)\frac{\partial}{\partial V_{\perp}} + \frac{k_{\perp}\cos(\Omega\tau + \theta)}{V_{\perp}}\frac{\partial}{\partial\theta} + k_{\parallel}\frac{\partial}{\partial V_{\parallel}} \tag{32}$$

where Ω = qB/mc is the cyclotron frequency. For simplicity we assume co-linear propagation in the direction perpendicular to $\vec{B}$. Note that in the foregoing $\theta(\tau) = \theta$, $\theta(\tau') = (\theta + \Omega\tau)$, $\theta(\tau'') = \theta + \Omega(\tau + \tau')$ and so the integration of C_k, and $\vec{V}$ over τ, τ' and τ'' are inter-related. This is a consequence of the time dependent orbits of particles in the magnetic field. Following the method outlined by Porkolab and Chang we can perform these tedious integrations to get explicit expressions for $B_{k,k''}$, $C_{k,k''}$ and $D_{k,k''}$. This is done in Appendix A. In order to obtain the wave-kinetic equation for the mode (ω, k) we multiply equation (27) by ϕ_k^* and average over the initial phases and obtain

$$\frac{\partial N_k}{\partial t} = 2\gamma_k N_k + \sum_{k''} \Big\{ S_k L_{k,k''} N_{k,k''} + \text{Complex Conjugate} \Big\} \quad (33)$$

In equation (33) the resonant mode-mode coupling contribution has been dropped as this process is not important here. $L_{k\,k''}$ is the coupling co-efficient given by

$$L_{k,k''} = \frac{16\pi}{\left|\frac{\partial\epsilon}{\partial\omega}\right| \left|\frac{\partial\epsilon''}{\partial\omega''}\right| |k''|^2} \,\mathrm{Im}\left[B_{k\,k''} + C_{k\,k''} + D_{k,k''} \right] \quad \text{--- (34)}$$

where $N_k = \frac{k^2}{8\pi} |\phi_k|^2 \frac{\partial\epsilon}{\partial\omega}$ and $S_k = \mathrm{Sgn}\left[\frac{\partial\epsilon}{\partial\omega}\right]$

It is understood that $|\partial\epsilon/\partial\omega| = \frac{\partial\epsilon}{\partial\omega}\Big|_{\omega/k}$ etc. Using expressions for $B_{k\,k''}$, $C_{k\,k''}$ and $D_{k\,k''}$ from the Appendix A we can write $L_{k\,k'}$ in the useful form as

$$L_{kk''}^{(m)} = \sum_j \frac{4\pi \omega_{pj}^2 m\Omega_j}{\left|\frac{\partial\epsilon}{\partial\omega}\right|\left|\frac{\partial\epsilon''}{\partial\omega''}\right| k^2 k''^2 m_j n_0} \int_{-\infty}^{\infty} dv_{\parallel} \int_0^{\infty} dv_{\perp}$$

$$\frac{\partial F_0}{\partial v_\perp}\,\delta(\omega' - m\Omega_j - k'_\parallel v_\parallel) \times \left| \sum_p \frac{k_\perp k''_\perp J_p(x)\, J_{p-m}(x'')}{(\omega - p\Omega - k'_\parallel v_\parallel)^2 - \Omega_j^2} \right.$$

$$\left. - \frac{\omega_{pj}^2}{k'^2} J_m(x') \frac{H_{k,k'',k'}}{\epsilon'(\omega',\vec{k}')} \right|^2 \quad \text{-- (35)}$$

where $H_{k,k'',k'}$ and $\epsilon'(\omega',\vec{k}')$ are given by the following equations

$$H_{k,k'',k'} = \sum_s \sum_l \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp\, \tilde{Y}_{l,s} \frac{\partial F_0}{\partial v_\perp} \times \frac{k_\perp k'_\perp k''_\perp}{(\omega - s\Omega_j - k_\parallel v_\parallel)^2 - \Omega_j^2}$$

$$\times \left[\frac{(s-l)\Omega_j/k''_\perp}{\omega'' - (s-l)\Omega_j - k''_\parallel v_\parallel} + \frac{l\Omega_j/k'_\perp}{\omega' - l\Omega_j - k'_\parallel v_\parallel} \right]$$

$$\text{--- (36)}$$

where

$$\tilde{Y}_{n,p}(v_\perp) = J_n(k'_\perp v_\perp/\Omega)\, J_{p-n}(k''_\perp v_\perp/\Omega)\, J_p(k_\perp v_\perp/\Omega)$$

and

$$\epsilon(\omega,k) = 1 + \sum_j \frac{\omega_{pj}^2}{k^2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp\, v_\perp \times \frac{J_n^2(k_\perp v_\perp/\Omega_j)}{[\omega - m\Omega_j - k_\parallel v_\parallel]}$$

$$\times \left[\frac{m\Omega_j}{k_\perp} \frac{\partial F_0}{\partial v_\perp} + k_\parallel \frac{\partial F_0}{\partial v_\parallel} \right]$$

$$\text{---- (37)}$$

As explained in Appendix B it should be noted that we shall be interested mainly in the imaginary term associated with the resonance condition (16) and hence the poles associated with ω and ω'' will be neglected. Thus in the foregoing expressions we split the integrals with ω' poles in the form

$$(\omega' - m\Omega_j - k'_{\parallel} v_{\parallel})^{-1} = P(\omega' - m\Omega_j - k'_{\parallel} v_{\parallel})^{-1} - i\pi\delta(\omega' - m\Omega_j - k'_{\parallel} v_{\parallel})$$

where P denotes the principle value part.

Finally we call attention to the importance of symmetry relation which will be needed shortly

$$L^{(-m)}_{k'',k} = -L^{(m)}_{k,k''} \tag{38}$$

Thus relation can be easily obtained from equation (35). Thus using equation (33) we can write the kinetic wave equation for (ω_k, k) mode as

$$\frac{\partial N_k}{\partial t} = \gamma_k N_k + \sum_{k''}\sum_{m} S_k L^{(m)}_{k,k''} N_{k''} N_k \tag{39}$$

where $L_{k,k''}$ is given by equation (35).

Proceeding in a similar manner and using equation (38), we can derive a kinetic wave equation for $(\omega_{k''}, k'')$ modes as

$$\frac{\partial N_{k''}}{\partial t} = -\gamma_{k''} N_{k''} - \sum_{k}\sum_{m} S_{k''} L^{(m)}_{kk''} N_k N_{k''} \tag{40}$$

Now we identify (ω_k, k) modes with ion-cyclotron oscillations described in Section 5 and $(\omega_{k''}, k'')$ modes with the Varma mode described in Section 3. From equation (35) it is easy to see that the matrix element $L^{(m)}_{K,k''}$ depends upon $K'_{\parallel} = (K_{\parallel} - K''_{\parallel})$ which remains finite as long as one of the modes has finite $K_{\parallel}$. Hence even if we assume a perfect Varma flute mode, the nonlinear instability will not be significantly affected. However, a finite $K_{\parallel}$ in Varma mode may give rise to damping due to electrons which is given by the first term on the right hand side of equation (40).

To proceed further let us know find out the sign of matrix element $L^{(m)}_{K,k''}$. Accordingly we have

$$L^{(m)}_{K,K''} = \frac{4\pi\,\omega_{pi}^{4}\,\Omega_i}{\left|\frac{\partial \varepsilon}{\partial \omega}\right|\left|\frac{\partial \epsilon''}{\partial \omega''}\right| k^2 k''^2 m_i n_0} \int_{-\infty}^{\infty} du_{\parallel}\, \delta(\omega' - \Omega_i - k'_{\parallel} u_{\parallel})$$

$$\times \int_0^{\infty} du_{\perp} \frac{\partial F_{oi}}{\partial u_{\perp}} \times \left| \sum_p \frac{k_{\perp} k''_{\perp} J_p(x) J_{p-m}(x'')}{(\omega - p\Omega_i - k_{\parallel} u_{\parallel})^2 - \Omega_i^2} - \frac{\omega_{pi}^2}{k'^2} \frac{J_m(x') H_{k k'' k'}}{\epsilon'(\omega', \vec{k}')} \right|^2$$

---- (41)

It should be noted that in equation (41) summation over species and m has already been performed. Since in the experiment under discussion $\omega = \Omega_i$ [~10 MHz range], $\omega'' =$ kHz range and modes are nearly flute (i.e. $K_{\parallel}, K''_{\parallel} = 0$).

$\omega' \simeq \Omega_i$ in which case because of δ-function the only finite term would be m = 1 in the ion species. Now as $(\omega', \vec{k}')$ is a damped mode (quasi-mode) of the system $\epsilon'(\omega', k') > 0$ in which case we may neglect the second term as compared to the first under the integral sign in equation (41). Hence equation (41) becomes

$$L^{(1)}_{k,k''} = \frac{4\pi\omega_{pi}^4 \Omega_i}{|\partial\epsilon/\partial\omega| \, |\partial\epsilon''/\partial\omega''| \, k^2 k''^2 m_i n_0} \int_{-\infty}^{\infty} du_{\parallel}$$

$$\delta(\omega' - k'_{\parallel} u_{\parallel} - \Omega_i) \times \int du_{\perp} \frac{\partial F_{0i}}{\partial u_{\perp}} \left| \sum_p \frac{k_{\perp} k''_{\perp} J_p(x') J_{p-1}(x'')}{(\omega - p\Omega_i - k_{\parallel} u_{\parallel})^2 - \Omega_i^2} \right|^2 \qquad (42)$$

In p summation p=2 will have dominant contribution as $\omega \simeq \Omega_i$ and $k_{\parallel} \simeq 0$. For $J_2(x'')$ we may use the Bessel's identity in which case the matrix element $L^{(1)}_{k,k''}$ finally becomes

$$L^{(1)}_{k,k''} = \frac{4\pi\omega_{pi}^4 \Omega_i}{\left|\frac{\partial\epsilon}{\partial\omega}\right| \left|\frac{\partial\epsilon''}{\partial\omega''}\right| m_i n_0 \Omega_i^4} \times \int_0^{\infty} du_{\perp} \frac{\partial F_{0i}}{\partial u_{\perp}} \left| \frac{4}{x} J_1(x) J_1(x'') - J_0(x) J_1(x'') \right|^2 \qquad (43)$$

where $x = k_{\perp} u_{\perp}/\Omega_i$ $\quad x'' = k''_{\perp} u_{\perp}/\Omega_i$

In equation (43), it is to see that as J_0 and J_1 are out of phase by nearly 90° the $J_0 J_1$ will give negligible contribution to the $dv_\perp$ integral. While for distribution appropriate to plasma formed by neutral beam injection in vacuum, i.e.

$$F_{oi}(v) = \frac{\pi^{1/2}}{\alpha_{\parallel}^{1/2}\ \bar{u}_{\perp 0}}\ \delta(v_\perp - u_{\perp 0})\, e^{-v_\parallel^2/\alpha_\parallel^2} \tag{44}$$

the term $x_i^2 = k_\perp^2 v_\perp^2/\Omega_i^2$ tends to weight the positive (rising) portion of $\partial F_{oi}/\partial v_\perp$ over the negative (falling) portion of $\partial F_{oi}/\partial v_\perp$. Hence the integral in equation (41) is positive and hence $L^{(1)}_{k,k''}$ is $>$ 0 for the case under consideration. In Section 4 we have already shown that for the observed ion-cyclotron mode $\partial\epsilon/\partial\omega < 0$ for positive ω. Thus $S_k < 0$ in equation (39). Let us now find out $S_{k''}$. We recall that $S_{k''}$ represents the sign of $\partial\epsilon''/\partial\omega''$ at $\nu = \omega''/m\omega_i = -1$. Then

$$\frac{\partial\epsilon''}{\partial\omega''} = \left(\frac{\partial\epsilon''}{\partial\nu}\right)\frac{1}{m\omega_i} \tag{45}$$

From equation (5) we have

$$\frac{\partial\epsilon''}{\partial\nu} = -\frac{3q}{|\sigma\gamma_0|^2} \tag{46}$$

Then for positive m $\partial\epsilon''/\partial\omega''$ bears a negative sign. Hence $S_{k''}$ in equation (40) is $<$ 0. Hence the wave-kinetic equation (39) and equation (40) become

$$\frac{\partial N_k}{\partial t} = \left[\gamma_k - \sum_{k''} |L^{(1)}_{kk''}|\, N_{k''}\right] N_k \tag{47}$$

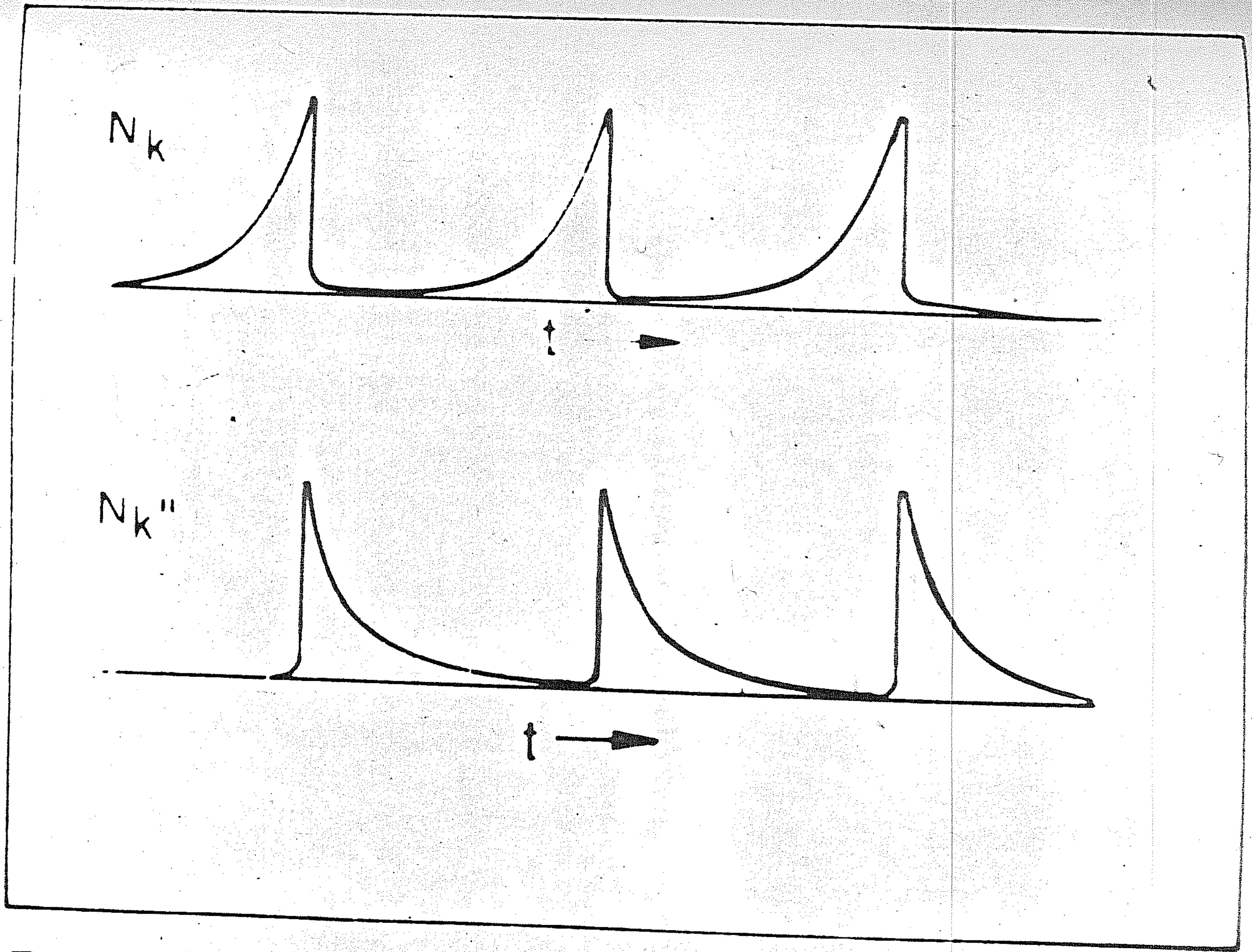

FIG. 1

Fig. 1   *Plot of energy densities of Varma mode ($N_{K''}$) and ion-cyclotron mode (N_K) with time.*

$$\frac{\partial N_{k''}}{\partial t} = \left[-\gamma_{k''} + \sum_{k} |L^{(1)}_{k,k''}| N_k \right] N_{k''} \tag{48}$$

Thus coupling gives rise to the following very interesting situation: As the beam injection current is increased beyond a certain critical density the electron Landau damping vanishes and the ion-cyclotron oscillations are linearly excited as represented by the first term in equation (46). Thus N_k grows and when $N_k L^{(1)}_{k,k''}$ becomes larger than $\gamma_{k''}$ then Varma mode is nonlinearly excited. Hence $N_{k''}$ grows and when $N_{k''} L^{(1)}_{k,k''}$ becomes larger than γ_k the ion-cyclotron oscillations are nonlinearly damped, and hence N_k starts decreasing and when $\gamma_{k''} > L^{(1)}_{k,k''} N_k$ Varma mode also becomes damped and starts decreasing. Thus we see that because of the coupling the energy of Varma mode and ion-cyclotron mode bursts periodically (Fig.1).

This is quite similar to the periodic bursts of Varma-mode and ion-cyclotron mode observed in Base-Ball II. We will now estimate the nonlinear time-scale of the bursts and compare it with the experimental value. Here we consider the time between bursts to be a meaningful measure of the time for the wave amplitude to grow from a very small to a very large level. In the model this time is given by the nonlinear growth time which is the inverse of the nonlinear growth-rate $\gamma_{NLk''}$. From equation (42) and equation (48) $\gamma_{k''NL}$ for Varma mode is given by

$$\gamma_{k''NL} = \frac{4\pi\,\omega_{pi}^{4} \sum_{k} \Omega_i N_k}{\left|\frac{\partial\epsilon}{\partial\omega}\right| \left|\frac{\partial\epsilon''}{\partial\omega''}\right| k^2 k''^2 m_i n_0}$$

$$\times \int_{-\infty}^{\infty} du_{\parallel} \int_{0}^{\infty} du_{\perp} \frac{\partial F_{oi}}{\partial u_{\perp}} \delta(\omega' - \Omega - k_{\parallel}' u_{\parallel})$$

$$\times \left| \frac{k_{\perp} k_{\perp}'' J_2(x) J_1(x'')}{(\omega - 2\Omega_i - k_{\parallel} u_{\parallel})^2 - \Omega_i^2} \right|^2$$

(49)

For F_{oi} we choose a distribution function characterising the neutral beam injection, namely,

$$F_{oi} = \frac{\sqrt{\pi}}{(\alpha_{\parallel})^{1/2}} \frac{1}{u_{\perp o}} \delta(u_{\perp} - u_{\perp o})\, e^{-u_{\parallel}^2/\alpha_{\parallel}^2}$$

which satisfies the normalisation condition

$$\int_{-\infty}^{\infty} du_{\parallel} \int_{0}^{\infty} u_{\perp} du_{\perp} F_{oi} = 1$$

where $u_{\perp o}$ is the velocity of the beam

From equation (14)

$$\frac{\partial\epsilon}{\partial\omega} \sim \frac{1}{\Omega_i} \tag{50}$$

while from equation (5) we have

$$\frac{\partial\epsilon''}{\partial\omega''} = \frac{3\,\omega_{pi}^2}{k''^2 |V_{Di}|^2} \left(\frac{\sigma'}{\sigma}\right)^2 \frac{1}{k''\gamma_0} \frac{1}{\Omega_i}$$

where $$V_{Di} = -\frac{U_{thi}^2}{\Omega_i}|\sigma|$$

From equation (49), we may write $\gamma_{k''NL}$ as

$$\gamma_{k''NL} = \frac{4\pi \dfrac{\omega_{pi}^4}{\Omega_i^4}}{\left|\dfrac{\partial\epsilon}{\partial\omega}\right|\left|\dfrac{\partial\epsilon''}{\partial\omega''}\right|} \times \frac{1}{\Omega_i} W. \tag{51}$$

where $W = \sum_k N_k \Omega_i / n_0 m_i U_{10}^2$ i.e. the ratio of energy in waves to that in particles. For Base-Ball II parameters i.e. $n_p = 4\text{x}10^9\ \text{cm}^{-3}$ (n_p is the plasma density), $T_i = 2$ KeV (T_i is the ion temperature), $\omega_{pi} \simeq 10^7$ Hz, $U_{thi} \simeq 3.09\text{x}10^7$ cm/sec, $\sigma = 1/50\ \text{cm}^{-1}$, $\sigma' = 1/5\ \text{cm}^{-1}$, $\Omega_i \simeq 10^7$ Hz, we have $V_{Di} = 2\text{x}10^6$ cm/sec. To our knowledge there is no experimental data on the wave number of low frequency fields. Typically for azimuthal mode number equal to 1 or 2 we expect $k'' = 1$. For these parameters

$$\frac{\partial\epsilon''}{\partial\omega''} = \frac{1.25\times10^2}{\Omega_i} \tag{52}$$

It should be noted that energy of ion-cyclotron oscillations W also fluctuates. Hence to get the order of time scale of nonlinear interaction we may choose a modest average value of $W = \sqrt{m_e/m_i}$ under the weak-turbulence approximation. Substituting for various terms on the right hand side of equation (51), we get the order of nonlinear interaction time scale of Varma mode as

$$\tau_{k''NL} \simeq \frac{1}{\gamma_{k''NL}} \simeq 400\,\Omega_i^{-1} \qquad (53)$$

For $\Omega_i = 10^7$ Hz, $\tau_{k''NL}$ turns out to be $\simeq$ 40 μs which agrees fairly well with the observed bursting time of 50-100 μs for Varma mode.

This periodic bursting of these oscillations periodically increases the scattering of particles into the loss cone. From the loss cone particles are lost thereby giving rise to bursts of particles.

7. Discussion and Conclusions:

The model developed in the preceeding pages we have shown that the stability of the Varma mode is affected in the presence of the ion-cyclotron modes observed in the experiment. It no longer remains stable but exhibits a periodic bursting instability on the time scale of $\simeq 400\,\Omega_i^{-1}$ ($\simeq$ 40 μs). Hence the identification made previously by O. A. Anderson et al of a small amplitude in KHz frequency range with the Varma mode may not be correct. Rather the large amplitude wave exhibiting a bursting instability on the time scale of 500-100 μs is more likely to be the Varma mode. The small amplitude mode may be one of the other two flute modes whose coupling efficiency to ion-cyclotron fluctuations on account of its low fluctuation level may be small (i.e. the matrix element $L_{k,k''}$ of this coupling may be quite small as compared with the matrix element for coupling between Varma mode and the ion-cyclotron mode. It should be noted that the nonlinear mechanism in the present experiment i.e. Base-Ball II and the previous ones like 'Alice' or 'Phoenix' etc. are slightly different. As has been pointed out by Simon and

Weng, in 'Alice' and 'Phoenix' explosive instability of the Varma mode is responsible for the observations. While in the present case it is the periodic bursting instability responsible for the anomalous losses.

APPENDIX A

$L_{k,k''}$

By substituting the orbits given by equation (31) and equation (32) into equation (28) and equation (29), the time integration can be performed to give the following compact expression for $\left[B_{kk''} + C_{kk''} \right]$

$$B_{kk''} + C_{kk''} = \sum_j \frac{\omega_{pj}^6 \, H_{k,k'';k'} \, H_{k',k,-k''}}{4\pi n_0 m_j k^2 k'^2 \, \epsilon(\omega', k')} \qquad \text{--- (1)}$$

where

$$H_{k,k'',-k'} = \sum_s \sum_\ell \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp v_\perp \, Y_{\ell,s}(v_\perp)$$

$$\times \left[\frac{k_\perp \left\{ k_\perp' Z_{s-\ell}(k'') + k_\perp'' Z_\ell(k') \right\}}{(\omega - s\Omega_j - k_\parallel v_\parallel)^2 - \Omega_j^2} + \frac{k_\parallel \left\{ k_\parallel' Z_{s-\ell}(k'') + k_\parallel'' Z_\ell(k') \right\}}{(\omega - s\Omega_j - k_\parallel v_\parallel)^2} \right] \qquad \text{--- (2)}$$

and

$$H_{k',k,-k''} = -\sum_n \sum_p \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp v_\perp \tilde{Y}_{n,p}(v_\perp)$$

$$\times \left\{ \frac{k_\perp' \left[k_\perp'' Z_p(k) - k_\perp Z_{p-n}(k'') \right]}{(\omega' - n\Omega_j - k_\parallel' v_\parallel)^2 - \Omega_j^2} + \frac{k_\parallel' \left[k_\parallel'' Z_p(k) - k_\parallel Z_{p-n}(k'') \right]}{(\omega' - n\Omega_j - k_\parallel' v_\parallel)^2} \right\} \quad (3)$$

where s.1,n, and p are all possible integers, from $-\infty$ to $+\infty$, and $\omega_p^2 = 4\pi n_0 q^2/m$. Here we have defined

$$\tilde{Y}_{n,p}(v_\perp) = J_n(k_\perp' v_\perp/\Omega)\, J_{p-n}(k_\perp'' v_\perp/\Omega)\, J_p(k_\perp' v_\perp/\Omega)$$

$$Z_n(k) = \frac{\frac{n\Omega}{v_\perp}\frac{\partial F_0}{\partial v_\perp} + k_\parallel \frac{\partial F_0}{\partial v_\parallel}}{(\omega - n\Omega - k_\parallel v_\parallel)}$$

$$Z_n(k') = Z_n(k \to k',\ \omega \to \omega') \quad (4)$$

$$F_0(v) = F_0(v_\perp)\, F_0(v_\parallel) \quad (5)$$

and

$$\epsilon(\omega, k) = 1 + \sum_j \frac{\omega_{pj}^2}{k^2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dv_{\parallel} \int_0^{\infty} dv_{\perp} v_{\perp}$$

$$\times \frac{J_n^2 \left(k_\perp v_\perp / \Omega_j \right)}{\left(\omega - n\Omega_j - k_{\parallel} v_{\parallel} \right)} \left[\frac{n\Omega_j}{v_\perp} \frac{\partial F_0}{\partial v_\perp} + k_{\parallel} \frac{\partial F_0}{\partial v_{\parallel}} \right] \tag{6}$$

And of course as $(\omega', \vec{k}')$ is a quasi-mode $\epsilon'(\omega', \vec{k}') \neq 0.$ Equation (1) gives the contribution to the scattering from the shielding cloud which is characteristic of plasma. It is also called the nonlinear scattering term. As a remark in passing we note that resonant mode coupling coefficient in equation (27) is given by

$$A_{k,k'',k'} = - \sum_j \frac{q\, \omega_{pj}^2}{k^2 m_j} H_{k,k'',k'} \tag{7}$$

The derivation of four-wave coupling coefficient is as follows: It may be written in the following form:

$$D_{k,k''} = \sum_j \frac{\omega_p^4 \, T_{k,k''}}{4\pi m_j n_0 k^2} \tag{8}$$

where

$$T_{k,k''} = \sum_n \sum_b \sum_c \int_{-\infty}^{\infty} dv_{\parallel} \int_0^{\infty} dv_j v_\perp W(v_j) \vec{k} \cdot D \cdot \vec{k}'' \tag{9}$$

Here n, b, and c are all integers in the range $(-\infty, \infty)$ and

$$W(v_j) = J_n\left(\frac{k_\perp v_\perp}{\Omega}\right) J_b\left(\frac{k_j v_\perp}{\Omega}\right) J_{b-c}\left(\frac{k_\perp'' v_\perp}{\Omega}\right) J_{n-c}\left(\frac{k_\perp' v_\perp}{\Omega}\right) \qquad ---(10)$$

and

$$\mathcal{D} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & D_{\perp,\perp} & D_{\perp,\parallel} \\ 0 & D_{\parallel,\perp} & D_{\parallel,\parallel} \end{bmatrix} \tag{11}$$

so that

$$\vec{k}\cdot D\cdot\vec{k}'' = k_\perp k_\perp'' D_{\perp\perp} + k_\perp k_\parallel'' D_{\perp\parallel} + k_\parallel k_\perp'' D_\parallel D_\perp + k_\parallel k_\parallel'' D_{\parallel,\parallel} \quad --- (12)$$

The expression for different matrix element can be derived as follows: Substituting the orbits in equation (30), we can split the integral with three terms so that

$$D_{k,k''} = D_i + D_{ii} + D_{iii} \quad - (13)$$

where

$$D_i = \int d^3v \int d\tau \, D_a k_\perp'' \sin\theta' \frac{\partial D_b}{\partial v_\perp} \quad -- (14)$$

$$D_{ii} = \int d^3v \int d\tau \, D_a k_\perp'' \cos\theta' v_\perp^{-1} \frac{\partial D_b}{\partial \theta'} \quad --- (15)$$

$$D_{iii} = \int d^3v \int_0^\infty d\tau \, D_a k_\parallel'' \frac{\partial D_b}{\partial v_\parallel} \quad --- (16)$$

in equation (14) to equation (16), we have

$$D_a = i \exp\left[i k_\perp v_\perp/\Omega \, (\cos\theta' - \cos\theta) + i(\omega - k_\parallel v_\parallel)\tau \right] \qquad \text{---(17)}$$

and after integrating over τ'' in equation (30) we obtain

$$D_b = \frac{i}{2\pi} \sum_c \sum_d \sum_m \sum_n \int_0^\infty d\tau' \, J_c\left(\frac{k_\perp' v_\perp}{\Omega}\right) J_d\left(\frac{k_\perp' v_\perp}{\Omega}\right)$$

$$\times \exp\left[i(c-d)(\tfrac{1}{2}\pi - \theta') + i(\omega' - c\Omega - k_\parallel' v_\parallel)\tau' \right]$$

$$\Big\{ \Big[k_\perp'' \sin(\theta' + \Omega\tau') \frac{\partial}{\partial v_\perp} + \frac{i k_\perp''}{v_\perp}(m-n) \times \cos(\theta' + \Omega\tau')$$

$$+ k_\parallel'' \frac{\partial}{\partial v_\parallel} \Big] \Big[J_n\left(\frac{k_\perp v_\perp}{\Omega}\right) J_m\left(\frac{k_\perp v_\perp}{\Omega}\right) \times \exp\big[-i(m-n)$$

$$(\tfrac{1}{2}\pi - \theta' - \Omega\tau')\big] \Big] Z_n(k) - \Big[k_\perp \sin(\theta' + \Omega\tau') \frac{\partial}{\partial v_\perp} +$$

$$\frac{i k_\perp}{v_\perp}(n-m) \times \cos(\theta' + \Omega\tau') + k_\parallel \frac{\partial}{\partial v_\parallel} \Big] \times$$

$$\times \Big[J_m\left(\frac{k_\perp'' v_\perp}{\Omega}\right) J_n\left(\frac{k_\perp'' v_\perp}{\Omega}\right) \times \exp\big[-i(n-m)$$

$$\times (\tfrac{1}{2}\pi - \theta' - \Omega\tau') \big] Z_m(k'') \Big\}$$

$$\text{----(18)}$$

where the expression for $Z_n(k)$ is given before by equation (3). We can now integrate over τ', substitute back into equation (14) to equation (16) expand the exponential in equation (17) in a double-Bessel function series and integrate over τ. Finally we integrate over the velocity space angle θ integrate by parts over $v_\perp$ repeatedly and obtain

$$
\begin{aligned}
D_i = -\sum_a \sum_b \sum_c \sum_d \sum_m \sum_n \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp \frac{k_\perp''/2}{(\omega - b\Omega - k_\parallel v_\parallel}
\Big\{ & J_m(k_\perp v_\perp/\Omega)\, J_n(k_\perp v_\perp/\Omega) \times \Big[- Z_n(k)\, \tfrac{1}{2} k_\perp'' \frac{\left[\frac{\partial}{\partial v_\perp} + (m-n)/v_\perp\right]\varphi_a}{\omega' - [c-m+n-1]\Omega - k_\parallel' v_\parallel} \\
& \times (\delta_{p-2} + \delta_p) + Z_n(k)\, \tfrac{1}{2} k_\perp'' \left[-\frac{\partial}{\partial v_\perp} + \frac{(m-n)}{v_\perp} \right] \varphi_a (\delta_p + \delta_{p+2}) \\
& + \frac{k_\parallel'' \varphi_a (\delta_{p-1} + \delta_{p+1})}{\omega' - (c-m+n)\Omega - k_\parallel' v_\parallel} \frac{\partial}{\partial v_\parallel} Z_n(k) \Big] \\
& - \Big[- Z_m(k'') \frac{\tfrac{1}{2} k_\perp \left[\frac{\partial}{\partial v_\perp} + \frac{(m-n)}{v_\perp} \right] \varphi_a}{\omega' - [c-n+m-1]\Omega - k_\parallel' v_\parallel} (\delta_{p-2} + \delta_p) \\
& + \frac{Z_m(k'')\, \tfrac{1}{2} k_\perp \left[-\frac{\partial}{\partial v_\perp} + (n-m)/v_\perp \right] \varphi_a}{\omega' - (c-n+m)\Omega - k_\parallel' v_\parallel} (\delta_p + \delta_{p+2}) \\
& + \frac{k_\parallel \varphi_a (\delta_{p-1} + \delta_{p+1})}{\omega' - (c-n+m)\Omega - k_\parallel' v_\parallel} \frac{\partial}{\partial v_\parallel} Z_m(k'') \Big] \Big\}
\end{aligned}
\tag{19}
$$

where $\rho = (a-b+c-d+n-m)$ and

$$\varphi_a = J_c\left(\frac{k_\perp' v_\perp}{\Omega}\right) J_d\left(\frac{k_\perp' v_\perp}{\Omega}\right) \frac{\partial}{\partial v_\perp}\left[v_\perp J_a\left(\frac{k_\perp v_\perp}{\Omega}\right) J_b\left(\frac{k_\perp v_\perp}{\Omega}\right)\right]$$

$$D_{ii} = \sum_a \sum_b \sum_c \sum_d \sum_m \sum_n \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp \frac{k_\perp''/2}{(\omega - b\Omega - k_\parallel v_\parallel)}$$

$$\Big\{ J_m(k_\perp v_\perp/\Omega)\, J_n(k_\perp v_\perp/\Omega)$$

$$\times \left[\frac{\frac{1}{2} Z_n(k)\, k_\perp'' \left[\frac{\partial}{\partial v_\perp} + \frac{(m-n)}{v_\perp}\right] \varphi_b\, (c-d+n-m-1)\,(-\delta_{\rho+2} + \delta_\rho)}{\omega' - (c-m+n-1)\Omega - k_\parallel' v_\parallel}\right.$$

$$+ \frac{\frac{1}{2} Z_n(k)\, k_\perp'' \left(-\frac{\partial}{\partial v_\perp} + \frac{(m-n)}{v_\perp}\right) \varphi_b\, (c-d+n-m+1)\,(+\delta_\rho - \delta_{\rho+2})}{\omega' - (c-m+n-1)\Omega - k_\parallel' v_\parallel}$$

$$\left. + \frac{k_\parallel''\,(c-d+n-m)\,(\delta_{\rho-1} - \delta_{\rho+1})\, \varphi_b \left(\frac{\partial}{\partial v_\parallel}\right) Z_n(k)}{\omega' - (c-m+n)\Omega - k_\parallel' v_\parallel}\right]$$

$$- \left[\frac{\frac{1}{2} Z_m(k'')\, k_\perp \left[\frac{\partial}{\partial v_\perp} + \frac{(n-m)}{v_\perp}\right] \varphi_b\, (c-d+m-n-1) \times (-\delta_{\rho-2} + \delta_\rho)}{\omega' - (c-n+m-1)\Omega - k_\parallel' v_\parallel}\right.$$

$$+ \frac{\frac{1}{2} Z_m(k'')\, k_\perp \left(-\frac{\partial}{\partial v_\perp} + \frac{(n-m)}{v_\perp}\right) \varphi_b\, (c-d+m-n+1)\,(+\delta_\rho - \delta_{\rho+2})}{\omega' - (c-n+m-1) - k_\parallel' v_\parallel}$$

$$+ \frac{k_\parallel\,(c-d+m-n)\,(\delta_{\rho-1} - \delta_{\rho+1})\, \varphi_a \frac{\partial}{\partial v_\parallel} Z_m(k'')}{\omega' - (c-n+m)\Omega - k_\parallel' v_\parallel}$$

— (20)

Where $\quad \varphi_b = J_a\left(\frac{k_\perp v_\perp}{\Omega}\right) J_b\left(\frac{k_\perp v_\perp}{\Omega}\right) J_c\left(\frac{k_\perp' v_\perp}{\Omega}\right) J_d\left(\frac{k_\perp' v_\perp}{\Omega}\right)$

$$D_{iii} = \sum_a \sum_b \sum_c \sum_d \sum_m \sum_n \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp \, k_\parallel''$$

$$\Big\{ Z_n(k) J_n\left(\frac{k_\perp v_\perp}{\Omega}\right) J_m\left(\frac{k_\perp v_\perp}{\Omega}\right) \times \Bigg[\frac{\frac{1}{2} k_\perp'' \left[\frac{\partial}{\partial v_\perp} + \frac{(m-n)}{v_\perp}\right] \varphi_c \delta_{p-1}}{\omega' - (c-m+n-1)\Omega - k_\parallel' v_\parallel}$$

$$- \frac{\frac{1}{2} k_\perp'' \left[-\frac{\partial}{\partial v_\perp} + \frac{(m-n)}{v_\perp}\right] \varphi_c \delta_{p+1}}{\omega' - (c-m+n-1)\Omega - k_\parallel' v_\parallel}$$

$$+ k_\parallel'' \frac{\partial}{\partial v_\parallel} \frac{\varphi_c \delta_p}{\omega' - (c-m+n)\Omega - k_\parallel' v_\parallel} \Bigg] \left[\frac{\partial}{\partial v_\parallel} (\omega - b\Omega - k_\parallel v_\parallel)^{-1}\right]$$

$$- \Bigg[\frac{\frac{1}{2} k_\perp \left[\frac{\partial}{\partial v_\perp} + \frac{(n-m)}{v_\perp}\right] \varphi_c \delta_{p-1}}{\omega' - (c-n+m-1)\Omega - k_\parallel' v_\parallel}$$

$$- \frac{\frac{1}{2} k_\perp \left[-\frac{\partial}{\partial v_\perp} + \frac{(n-m)}{v_\perp}\right] \varphi_c \delta_{p+1}}{\omega' - (c-n+m-1)\Omega - k_\parallel' v_\parallel}$$

$$+ k_\parallel \frac{\partial}{\partial v_\parallel} \frac{\varphi_c \delta_p}{\omega' - (c-n+m)\Omega - k_\parallel' v_\parallel} \left[\frac{\partial}{\partial v_\parallel} (\omega - b\Omega - k_\parallel v_\parallel)^{-1}\right] \Bigg] \qquad (21)$$

where $\varphi_c = v_\perp \varphi_b$. We further note that in the foregoing expressions the derivative $\partial/\partial v_\perp$ and $\partial/\partial v_\parallel$ operate on all quantities to their right. To proceed further, we add D_i and D_{ii}

and begin to sum over indices with the help of the kronecker deltas. Then a considerable amount of manipulation with the Bessel function identities is necessary to combine the large numbers of terms. Furthermore integration by parts over $U_{\parallel}$ is performed wherever convenient. Thus after a considerable amount of tedious algebra, we obtain following expressions for the matrix elements:

$$D_{\perp\perp} = \left\{ \sum_{\gamma=\pm 1} \frac{1}{2\Omega} \frac{\gamma k_{\perp} \left[k_{\perp}'' Z_n(k) - k_{\perp} Z_{n-c}(k'') \right]}{\left[(\omega - (b+\gamma)\Omega - k_{\parallel} U_{\parallel})^2 - \Omega^2 \right] \left[\omega' - (c+\gamma)\Omega - k_{\parallel}' U_{\parallel} \right]} \right.$$

$$\left. - \frac{k_{\perp}'' \left[k_{\perp}'' Z_n(k) - k_{\perp} Z_{n-c}(k'') \right]}{\left[(\omega - b\Omega - k_{\parallel} U_{\parallel})^2 - \Omega^2 \right] \left[(\omega' - c\Omega - k_{\parallel}' U_{\parallel})^2 - \Omega^2 \right]} \right\}$$

--- (22)

$$D_{\perp,\parallel} = \left\{ \sum_{\gamma=\pm 1} \frac{1}{2\Omega} \frac{\gamma k_{\parallel} \left[k_{\perp}'' Z_n(k) - k_{\perp} Z_{n-c}(k'') \right]}{\left[\omega - (b+\gamma)\Omega - k_{\parallel} U_{\parallel} \right]^2 \left[\omega' - (c+\gamma)\Omega - k_{\parallel}' U_{\parallel} \right]} \right.$$

$$\left. - \frac{k_{\perp}'' k_{\parallel}}{k_{\perp}} \frac{\left[k_{\perp}'' Z_n(k) - k_{\perp} Z_{n-c}(k'') \right]}{(\omega - b\Omega - k_{\parallel} U_{\parallel})^2 \left[(\omega' - c\Omega - k_{\parallel}' U_{\parallel})^2 - \Omega^2 \right]} \right\}$$

--- (23)

$$D_{\parallel,\perp} = -\frac{k_\perp}{k_\parallel}\left[(\omega - b\Omega - k_\parallel U_\parallel)^2 - \Omega^2\right]^{-1}\left[\omega' - c\Omega - k_\parallel'' U_\parallel\right]^{-1}$$

$$\frac{\partial}{\partial U_\parallel}\left[\, k_\parallel'' Z_n(k) - k_\parallel Z_{n-c}(k'')\right]$$

---(24)

$$D_{\parallel,\parallel} = -1\ (\omega - b\Omega - k_\parallel U_\parallel)^{-2}\,(\omega' - c\Omega - k_\parallel U_\parallel)^{-1}$$

$$\frac{d}{dU_\parallel}\left[\, k_\parallel'' Z_n(k) - k_\parallel Z_{n-c}(k'')\right]$$

---(25)

Here $Z_n(k)$ has been defined before. Equation (4) is also called 'compton scattering term' or scattering from bare particle.

We may now write equation (34) in the following form

$$L_{k,k''} = \frac{4\omega_p^4}{|\partial\epsilon/\partial\omega|\,|\partial\epsilon''/\partial\omega''|\,k''^2 k^2\, m n_0} \times$$

$$\mathrm{Im}\left[\, T_{k,k''} + \frac{\omega_p^2}{k'^2\,\epsilon(\omega',k')} \times H_{k,k'',k}\, H_{k',k,-k''}\right]$$

---(26)

where $\omega' = (\omega - \omega'')$, $k' = (k - k'')$. From equation (26) we see that only Im part of the matrix element is required. In particular, we shall be interested mainly in the imaginary terms associated with the resonance condition (16) of the text and hence the poles associated with ω and ω'' will be neglected. Thus in the foregoing expressions we split the integrals with ω' poles in the form

$$(\omega' - n\Omega - k' v_{\parallel})^{-1} = P(\omega' - n\Omega - k_{\parallel}' v_{\parallel})^{-1} - i\pi\,\delta(\omega' - n\Omega - k_{\parallel}' v_{\parallel}) \tag{27}$$

where P denotes the principle value part. Utilizing this equation, we will now determine $L_{k,k''}$ for the case under consideration. To do this we make following assumptions:

$$k_{\parallel} < k_{\perp}\ , \quad k_{\parallel}'' < k_{\perp}''\ , \quad k_{\parallel}' \left\langle \frac{\partial F_0}{\partial v_{\parallel}} \right\rangle \ll n\Omega \left\langle v_{\perp}^{-1} \frac{\partial F_0}{\partial v_{\perp}} \right\rangle \tag{28}$$

Under these approximations we retain $k_{\parallel}'$ only at poles so that we have following relation for the matrix elements H.

$$H_{k,k'',k'} = + \sum_{s} \sum_{\ell} \int_{-\infty}^{\infty} dv_{\parallel} \int_{0}^{\infty} dv_{\perp}\ \tilde{Y}_{\ell,s} \frac{\partial F_0}{\partial s} \times \frac{k_{\perp} k_{\perp}' k_{\perp}''}{(\omega - s\Omega - k_{\parallel} v_{\parallel})^2 - \Omega^2}$$

$$\times \left[\frac{(s-\ell)\Omega / k_{\perp}''}{\omega'' - (s-\ell)\Omega - k_{\parallel}'' v_{\parallel}} + \frac{\ell\Omega / k_{\perp}'}{\omega' - \ell\Omega - k_{\parallel}' v_{\parallel}} \right] \tag{29}$$

$$H_{k',k,-k''} = -\sum_n \sum_p \int_{-\infty}^{\infty} du_{\parallel} \int_0^{\infty} du_{\perp} \, \tilde{Y}_{n,p}(u_{\perp}) \frac{\partial F_0}{\partial u_{\perp}}$$

$$\frac{k_{\perp} k_{\perp}' k_{\perp}''}{[(\omega' - n\Omega - k_{\parallel}' u_{\parallel})^2 - \Omega^2]}$$

$$\times \left[\frac{p\Omega/k_{\perp}}{(\omega - p\Omega - k_{\parallel} u_{\parallel})} - \frac{(p-n)\Omega/k_{\perp}''}{\omega'' - (p-n)\Omega - k_{\parallel}'' u_{\parallel}} \right] \quad ---(30)$$

After this we write down the following symmetry relations:

$$H_{k,k'',k'} = H_{k',k,-k''} , \quad H_{k,k'',k'} = H_{k,k',k''} \quad (31)$$

These relations are proved in Apprndix D. Furthermore in Appendix B, we have proved the relation

$$\text{Im}\, H_{k,k'',k'} = \text{Im}\, H_{k',k,-k''} \quad ---(32)$$

Equation (31) and equation (32) allows us to write

$$H_{k,k'',k'} H_{k',k,-k''} = H^2_{k,k'',k'} \quad ---(33)$$

$\text{Im}\, T_{kk''}$ is obtained in Appendix C. To proceed further, we can easily establish the relationship

$$\mathrm{Im}\left[\frac{\omega_p^2}{k'^2}\frac{H^2_{k,k'',k'}}{\epsilon'(\omega,k')}\right] = \frac{\omega_p^2}{k'^2}\left[-\left|\frac{H_{k,k'',k'}}{\epsilon(\omega',k')}\right|^2\right.$$

$$\times \mathrm{Im}\,\epsilon'(\omega',k') + 2\mathrm{Re}\left[\frac{H_{k,k'',k'}}{\epsilon'(\omega',k')}\right]^2$$

$$\times \mathrm{Im}\,H_{k,k'',k'} \qquad \text{---(34)}$$

From equation (29), we obtain the imaginary part of $H^{(n)}_{k,k'',k'}$ as follows:

$$\mathrm{Im}\,H^{(n)}_{k,k'',k'} = -\sum_s \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp\, \tilde{Y}_{m,s}(v_\perp)$$

$$\times \frac{k_\perp k''_\perp\, m\Omega\, \pi\, \delta(\omega' - m\Omega - k'_\parallel v_\parallel)}{(\omega - s\Omega - k_\parallel v_\parallel)^2 - \Omega^2} \qquad \text{---(35)}$$

And from equation (6), we obtain $\mathrm{Im}\,\epsilon'(\omega',k')$ as

$$\mathrm{Im}\,\epsilon'(\omega',k') = -\pi \sum_j \frac{\omega_{pj}^2}{k'^2}\int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp \frac{\partial F_0}{\partial v_\perp}$$

$$m\Omega\, J_m^2\left(\frac{k'_\perp v_\perp}{\Omega}\right)\delta(\omega' - m\Omega - k_\parallel v_\parallel) \qquad \text{---(36)}$$

Then substituting equation (34) and the result of Appendix B in equation (26) and further using equation (35) and equation (36) for $\operatorname{Im} \epsilon(\omega', k')$ and $\operatorname{Im} H_{k\,k''\,k'}$ and after some algebra we have the following simple and useful form of $L^{(m)}_{k,k''}$

$$L^{(m)}_{k,k''} = \sum_{j} \frac{4\pi\,\omega_{pj}^{4}\, m\Omega}{|\partial\epsilon/\partial\omega|\,|\partial\epsilon''/\partial\omega''|\, k^2 k''^2\, m\, n_0}$$

$$\times \int_{-\infty}^{\infty} dv_{\parallel} \int_{0}^{\infty} dv_{\perp}\, \frac{\partial F_0}{\partial v_{\perp}}\, \delta(\omega' - m\Omega - k_{\parallel}' v_{\parallel})$$

$$\times \left| \sum_{p} \frac{k_{\perp} k_{\perp}''\, J_p(x)\, J_{p-m}(x)}{(\omega - p\Omega - k_{\parallel} v_{\parallel})^2 - \Omega^2} - \frac{\omega_p^2}{k'^2}\, J_m(x') \times \frac{H_{k,k'',k'}}{\epsilon'(\omega', k')} \right|^2$$

(37)

where $H_{k,k'',k'}$ and $\epsilon(\omega', k')$ in equation (37) are given by equation (29) and equation (6) respectively.

APPENDIX B

To get $\mathrm{Im}\, H_{k',k,-k''}$ we may proceed as follows.

We write

$$\left[(\omega' - n\Omega - k'_{\parallel} v_{\parallel})^2 - \Omega^2 \right]^{-1}$$

$$= -(2\Omega)^{-1} \left\{ \left[\omega' - (n-1)\Omega - k'_{\parallel} v_{\parallel} \right]^{-1} \right.$$

$$\left. - \left[\omega' - (n+1)\Omega - k'_{\parallel} v_{\parallel} \right]^{-1} \right\} \tag{1}$$

Then we let n $\longrightarrow$ n+1 and n-1 and obtain

$$H_{k',k,-k''} = + \frac{1}{2} (k_\perp k'_\perp k''_\perp)$$

$$\times \sum_n \sum_p \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp \frac{\partial F_0/\partial v_\perp \, J_p(x)}{(\omega' - n\Omega - k'_\parallel v_\parallel)}$$

$$\times \left[\frac{p/k_\perp \left\{ J_{n+1}(x') J_{p-n-1}(x'') - J_{n-1}(x') J_{p-n+1}(x'') \right\}}{(\omega - p\Omega - k_\parallel v_\parallel)} \right.$$

$$- \frac{\left\{ (p-n-1)/k''_\perp \right\} J_{n+1}(x') J_{p-n-1}(x'')}{\omega'' - (p-n-1)\Omega - k''_\parallel v_\parallel}$$

$$\left. + \frac{\left[(p-n+1)/k''_\perp \right] J_{n-1}(x') J_{p-n+1}(x'')}{\omega'' - (p-n+1)\Omega - k''_\parallel v_\parallel} \right]$$

where we have used the notation

$$x = k_\perp v_\perp / \Omega$$

Now using the resonance condition we pick up the pole n=m, write

$$\omega'' - (p-n)\Omega - k_\parallel'' v_\parallel = (\omega - p\Omega - k_\parallel v_\parallel)$$

in the last two terms let $p \mp 1 \longrightarrow p$ and obtain

$$\mathrm{Im}\, H^{(m)}_{k',k,-k''} = \frac{1}{2}(-i\pi k_\perp')$$

$$\times \sum_p \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp \frac{\partial F_0}{\partial v_\perp} \frac{\delta(\omega' - m\Omega - k_\parallel' v_\parallel)}{(\omega - p\Omega - k_\parallel v_\parallel)}$$

$$\times \Big\{ k_\perp'' p J_p(x) \left[J_{m+1}(x') J_{p-m-1}(x'') - J_{m-1}(x') J_{p-m+1}(x'') \right]$$

$$- k_\perp (p-m) J_{p-m}(x'') \left[J_{m+1}(x') J_{p+1}(x) \right.$$

$$\left. - J_{m-1}(x') J_{p-1}(x) \right] \Big\}$$

Now we employ the identity

$$2pJ_p(x) = x\left[J_{p-1}(x) + J_{p+1}(x)\right] \quad \text{--(2)}$$

to split the $pJ_p(x)$ and $(p-m)J_{p-m}(x'')$, then collect terms and finally obtain

$$\mathrm{Im}\, H_{k',k,-k''} = \frac{1}{4}\left(-i\pi\, k_\perp k_\perp''\right)$$

$$\times \int_{-\infty}^{\infty} dv_\parallel \int_{0}^{\infty} dv_\perp \frac{\partial F_0}{\partial v_\perp} \frac{m\,\delta(\omega' - m\Omega - k_\parallel' v_\parallel)}{(\omega - p\Omega - k_\parallel v_\parallel)}$$

$$J_m(x') \times \Big[J_{p-1}(x') J_{p-m-1}(x'')$$

$$- J_{p+1}(x) J_{p-m+1}(x'') \Big]$$

Now let $p \mp 1 \longrightarrow p$ in the first and second terms respectively and get

$$\mathrm{Im}\, H^{(m)}_{k',k,-k''} = -\pi\, k_\perp k_\perp'' \sum_p \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp \frac{\partial F_0}{\partial v_\perp}$$

$$\times \frac{m\Omega\, J_m(x')\, J_{p-m}(x'')\, J_p(x)\, \delta(\omega' - m\Omega - k_\parallel' v_\parallel)}{(\omega - p\Omega - k_\parallel v_\parallel)^2 - \Omega^2}$$

(3)

APPENDIX C

Derivation of $\mathrm{Im}\, D_{\perp,\perp}$

Splitting terms into partial fractions we can write the equation (22) of Appendix A as

$$\sum k_\perp k_\perp'' D W = \sum_n \sum_b \sum_c \frac{1}{2} \sum_{\gamma = \pm 1}$$

$$\frac{\left[k_\perp J_n(x) J_{b-\gamma}(x) J_{b-c-\gamma}(x'') J_{n-c}(x'') - k_\perp'' J_n(x) J_b(x) J_{b-c}(x'') J_{n-c}(x'') \right]}{\left(\omega - b\Omega - k_\parallel v_\parallel\right)^2 - \Omega^2}$$

$$\times \left[\frac{\gamma k_\perp k_\perp''}{\omega' - (c+\gamma)\Omega - k_\parallel' v_\parallel} \right]$$

$$\times \left[\frac{n k_\perp''}{(\omega - n\Omega - k_\parallel v_\parallel)} - \frac{(n-c) k_\perp}{\omega'' - (n-c)\Omega - k_\parallel'' v_\parallel} \right] \frac{\partial F_0}{\partial v_\perp} \tag{1}$$

Now we let $c \longrightarrow (c - \gamma)$ and in the last term $(n - c) \longrightarrow n$. Then, we write out all the terms perform a series of manipulation with the Bessel-function identity i.e.

$$2 p J_p(x) = x \left[J_{p-1}(x) + J_{p+1}(x) \right]$$

Collect terms and obtain

$$\sum k_\perp k_\perp'' u_\perp D_{\perp,\perp} W = \sum_n \sum_b \sum_c \left\{ \frac{n\Omega k_\perp k_\perp''^2 J_n(x)}{(\omega - n\Omega - k_{\parallel} u_{\parallel})} \right.$$

$$\times \left[c J_{n-c+1}(x'') J_b(x) J_{b-c}(x'') + (n-c) J_{n-c}(x'') \frac{\mathcal{E}_j}{x''} \right]$$

$$- \frac{(n-c)\Omega k_\perp'' k_\perp^2}{\omega'' - (n-c)\Omega - k_{\parallel}'' u_{\parallel}} J_{n-c}(x'') \times$$

$$\left. \times \left[c J_{n-1}(x) J_b(x) J_{b-c}(x'') + n J_n(x) \mathcal{E}_j / x \right] \right\}$$

$$\times \frac{1/u_\perp \, \partial F_0 / \partial u_\perp}{(\omega' - c\Omega - k_{\parallel}' u_{\parallel}) \left[(\omega - b\Omega - k_{\parallel} u_{\parallel})^2 - \Omega^2 \right]}$$

where $\mathcal{E}_j = -x J_{b+1}(x) J_{b-c}(x'') + x'' J_b(x) J_{b-c-1}(x'')$

Now we use the resonance condition equation (16) of the text to obtain the imaginary part and hence pick C=m from the summation and obtain

$$\mathrm{Im} \sum k_\perp k_\perp'' u_\perp D_{\perp,\perp} W$$

$$= - \sum_n \sum_b \pi (1/u_\perp) \frac{\partial F_0}{\partial u_\perp} k_\perp k_\perp'' \Omega J_{b-m}(x'') J_b(x)$$

$$\frac{\left[k_\perp'' n m J_n(x) J_{n-m+1}(x') - k_\perp m (n-m) J_{n-1}(x) J_{n-m}(x'') \right] \times \left[\delta(\omega' - m\Omega - k_{\parallel}' u_{\parallel}) \right]}{\left[(\omega - b\Omega - k_{\parallel} u_{\parallel})^2 - \Omega^2 \right] \left[\omega - n\Omega - k_{\parallel} u_{\parallel} \right]}$$

where $\delta(\ ) = \delta(\omega' - m\Omega - k_{\parallel}' v_{\parallel})$ and the terms with e_{c_s} cancel out.

Splitting $n J_n(x)$ and $(n-c) J_{n-c}(x'')$ according to the Bessel function identity collecting terms gives us, for the square bracket

$$\left(\Omega x x''/2v_\perp\right)\left[J_{n+1}(x) J_{n-m+1}(x'') - J_{n-m-1}(x'')\right.$$
$$\left. \times J_{n-1}(x)\right]$$

Now we let n ⟶ n∓1 in the first and second terms respectively, combine the poles $[\omega - (n \pm 1)\Omega - k_{\parallel} v_{\parallel}]^{-1}$ and obtain

$$\mathrm{Im} \sum_n \sum_b k_\perp k_\perp'' v_\perp W D_{\perp,\perp}$$

$$= \sum_n \sum_b \frac{\pi m \Omega k_\perp^2 k_\perp''^2 W(v_\perp)\ \partial F_0/\partial v_\perp\ \delta(\omega' - m\Omega - k_\parallel' v_\parallel)}{[(\omega - b\Omega - k_\parallel v_\parallel)^2 - \Omega^2][(\omega - n\Omega - k_\parallel v_\parallel)^2 - \Omega^2]} \qquad \text{---(2)}$$

where $W(v_\perp)$ is defined in the text.

⟶s

... (1) and

... y from the

... equation (2), we proceed as

APPENDIX D

We wish to show the following relations:

$$H_{k,k'',k'} = H_{k,k',k''} \tag{1}$$

$$H_{k',k,-k''} = H_{k,k'',k'} \tag{2}$$

where $H_{k,k'',k'}$ is as defined in the text and $H_{k',k,-k''}$ is written in the following form

$$H_{k',k,-k''} = -\sum_{\ell}\sum_{s} k_\perp k_\perp' k_\perp''$$

$$\times \int_{-\infty}^{\infty} dv_\parallel \int_0^\infty dv_\perp \frac{\partial F_0}{\partial v_\perp} \frac{J_s(x)\, J_{s-\ell}(x'')\, J_\ell(x')}{(\omega' - \ell\Omega - k_\parallel' v_\parallel)^2 - \Omega^2}$$

$$\times \left[\frac{s\Omega/k_\perp}{(\omega - s\Omega - k_\parallel v_\parallel)} - \frac{(s-\ell)\Omega/k_\perp''}{[\omega'' - (s-\ell)\Omega - k_\parallel'' v_\parallel]} \right] \tag{3}$$

Equation (3) follows from equation (30) of Appendix A by letting $p \longrightarrow s$ and $n \longrightarrow \ell$. All other symmetry relations follow from equation (1) and equation (2). We see that equation (1) follows immediately from the form $H_{k,k'',k'}$. In order to show equation (2), we proceed as follows: In equation (3) we write

$$[(\omega' - l\Omega - k_{\parallel}' v_{\parallel})^2 - \Omega^2]^{-1} = (2\Omega)^{-1}$$
$$[(\bar{\omega}' - l\Omega - \Omega)^{-1}$$
$$- (\bar{\omega}' - l\Omega + \Omega)^{-1}] \quad (4)$$

where we have used the notation $\bar{\omega}' = (\omega' - k_{\parallel}' v_{\parallel})$. Similarly, in the following we shall abbreviate $\bar{\omega} = (\omega - k_{\parallel} v_{\parallel})$ and $\bar{\omega}'' = (\omega'' - k_{\parallel}'' v_{\parallel})$. Now we let $l \longrightarrow l \pm 1$, $s \longrightarrow s \pm 1$ and obtain

$$H_{k', k, -k''} = - \sum_{l} \sum_{s} \frac{1}{2} (k_\perp k_\perp' k_\perp'') \int_{-\infty}^{\infty} dv_{\parallel} \int_{0}^{\infty} dv_\perp$$

$$\frac{\partial F_0}{\partial v_\perp} \frac{J_{s-l}(x'')}{(\bar{\omega}' - n\Omega)} \times$$

$$\times \Big\{ \frac{[\bar{\omega} - (s+1)\Omega](s-1) J_{l-1}(x') J_{s-1}(x) - [\bar{\omega}' - (s-1)\Omega](s+1) \times J_{l+1}(x') J_{s+1}(x)}{k_\perp [(\bar{\omega} - s\Omega)^2 - \Omega^2]}$$

$$- (s-l) \frac{[J_{l-1}(x') J_{s-1}(x) - J_{l+1}(x') J_{s+1}(x)]}{k_\perp'' [\bar{\omega}'' - (s-l)\Omega]} \Big\}$$

where in the first term we cross multiplical by the denominators. Now in the first we can apply the Bessel's function density mentioned in Appendix B to rewrite the Bessel function $J_{l\pm 1}(x')$ in terms of $J_l(x')$, $J_{l\pm 2}(x')$ and $J_{s\pm 1}(x)$. After regrouping terms we obtain

$$H_{k',k,-k''} = H^{(1)}_{k',k,-k''} + H^{(2)}_{k',k,-k''} \tag{5}$$

where

$$H^{(1)}_{k',k,-k''} = \sum_{\ell} \sum_{s} k_\perp k_\perp' k_\perp'' \int_{-\infty}^{\infty} dv_\parallel \int_{0}^{\infty} dv_\perp$$

$$\frac{\dfrac{\partial F_0}{\partial v_\perp} J_s(x)\, J_{s-\ell}(x'')\, J_\ell(x')\, \ell\Omega / k_\perp}{\left[(\bar{\omega} - s\Omega)^2 - \Omega^2\right]\left[\bar{\omega}' - \ell\Omega\right]}$$

(6)

$$H^{(2)}_{k',k,-k''} = - \sum_{\ell} \sum_{s} \frac{1}{2}(k_\perp k_\perp' k_\perp'') \int_{-\infty}^{\infty} dv_\parallel \int_{0}^{\infty} dv_\perp$$

$$\frac{\partial F_0}{\partial v_\perp} \frac{J_{s-\ell}(x'')}{(\bar{\omega}' - \ell\Omega)} \left\{ a - b - c \right\}$$

— — (7)

where

$$a = (\bar{\omega} - s\Omega) \frac{\left[(s-1) J_{s-1}(x)\, J_{\ell-1}(x') - (s+1) J_{s+1}(x)\, J_{\ell+1}(x')\right]}{k_\perp \left[(\bar{\omega} - s\Omega)^2 - \Omega^2\right]}$$

$$b = \Omega \Big\{ (\ell k_\perp / k_\perp') J_\ell(x') \left[J_{s-2}(x) + J_{s+2}(x) \right] - \left[(s-1)/k_\perp \right] J_{s-1}(x) J_{\ell+1}(x'') - \left[(s+1)/k_\perp \right] J_{s+\ell}(x) J_{\ell-1}(x') \Big\} \Big/ \left[(\bar{\omega} - s\Omega)^2 - \Omega^2 \right]$$

$$c = \frac{(s-\ell)\left[J_{s-1}(x) J_{\ell-1}(x') - J_{s+1}(x) J_{\ell+1}(x') \right]}{k_\perp'' \left[\bar{\omega}'' - (s-\ell)\Omega \right]}$$

Applying the previous mentioned Bessel's identity in (A) and (B) we may rewrite the terms $(s \mp 1) J_{s \mp 1}(x)/k_\perp$ and $\ell J_\ell(x')/k_\perp'$ and obtain

$$\left(\frac{v}{\Omega}\right) \Big\{ \left[\bar{\omega} - (s-1)\Omega \right] \left[J_s(x) J_{\ell-1}(x') - J_{s+2}(x) J_{\ell+1}(x') \right] + \left[\bar{\omega} - (s+1)\Omega \right] \left[J_{s-2}(x) J_{\ell-1}(x') - J_s(x) J_{\ell+1}(x') \right] \Big\}$$

Now we break up the denominator $\left[(\bar{\omega} - s\Omega)^2 - \Omega^2 \right]$ according to the relation (4), let $s \longrightarrow s \mp 1$, collect terms and regroup them as follows:

$$H^{(2)}_{k, k', -k''} = H^{(2a)}_{k, k', -k''} + H^{(2b)}_{k, k', -k''} \qquad (8)$$

where

$$H^{(2a)}_{k,k',-k''} = -\sum_{\ell}\sum_{s} \frac{k_\perp k_\perp' k_\perp''}{(4\Omega)} \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp \frac{v_\perp \frac{\partial F_0}{\partial v_\perp}}{(\bar{\omega}' - \ell\Omega)}$$

$$\times \Big\{ \left[(\bar{\omega} - s\Omega)^{-1} - (\omega'' - (s-\ell)\Omega)^{-1} \right]$$

$$\left[J_{s-\ell-1}(x'') + J_{s-\ell+1}(x'') \right]$$

$$\times \left[J_{s-1}(x) J_{\ell-1}(x') - J_{s+1}(x) J_{\ell+1}(x') \right] \Big\}$$

$$\text{---(9)}$$

and

$$H^{(2b)}_{k,k',-k''} = -\sum_{\ell}\sum_{s} \frac{k_\perp k_\perp''}{4\Omega} \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp \frac{\frac{\partial F_0}{\partial v_\perp}}{(\bar{\omega}' - \ell\Omega)}$$

$$\times \Big\{ (s-2) J_{s-2}(x) J_{s-\ell-1}(x'') J_{\ell-1}(x')$$

$$- s J_s(x) \times \left[J_{s-\ell+1}(x'') J_{\ell-1}(x') + J_{s-\ell-1}(x'') J_{\ell+1}(x') \right]$$

$$+ (s+2) J_{s+2}(x) J_{s-\ell+1}(x'') J_{\ell+1}(x') \Big\}$$

$$\text{--(10)}$$

Now using $\omega' = \omega - \omega''$, $k_\parallel' = k_\parallel - k_\parallel''$ we have
$\bar{\omega}'' - \Omega(s-\ell) - (\bar{\omega} - s\Omega) = -(\bar{\omega}' - \ell\Omega)$ and in equation (9),

we have

$$\left[(\bar{\omega}' - l\Omega)^{-1} \right] \left[(\bar{\omega} - s\Omega)^{-1} - (\bar{\omega}'' - (s-l)\Omega)^{-1} \right]$$

$$= - \left[(\bar{\omega} - s\Omega)(\bar{\omega}'' - (s-l)\Omega \right]^{-1}$$

Now let $l, s \longrightarrow l \mp 1$ in equation (9) and using the Bessel function identity, we obtain

$$H^{(2a)}_{k', k, -k''} = \sum_s \sum_l k_\perp k_\perp' k_\perp'' \int_{-\infty}^{\infty} dv_{\parallel} \int_0^{\infty} dv_\perp \frac{\partial F_0}{\partial v_\perp}$$

$$\times \frac{J_s(x) J_{s-l}(x'') J_l(x')}{(\bar{\omega} - s\Omega)^2 - \Omega^2} \times \frac{(s-l)\Omega / k_\perp''}{\bar{\omega}'' - (s-l)\Omega}$$

— — —(11)

where once more we used equation (9) for combining the $\left[\bar{\omega} - (p \mp 1)\Omega \right]$ poles. It is now straight forward to show by a long series of manipulations using the Bessel function identity and resumming with the index S- that the Bessel functions in equation (10) cancel identically, and we have

$$H^{(2b)}_{k, k', -k''} = 0 \qquad (12)$$

Adding equations (6), (11) and (12), we obtain

$$H_{k',k,-k''} = \sum_{\ell} \sum_{s} k_\perp k_\perp' k_\perp'' \int_{-\infty}^{\infty} du_\parallel \int_0^\infty du_\perp \frac{\partial F_0}{\partial u_\perp}$$

$$\times \frac{J_s(x)\, J_{s-\ell}(x'')\, J_\ell(x')}{(\omega - s\Omega - k_\parallel u_\parallel)^2 - \Omega^2}$$

$$\times \left[\frac{\ell\Omega/k_\perp}{(\omega' - \ell\Omega - k_\parallel' u_\parallel)} + \frac{(s-\ell)\Omega/k_\perp''}{(\omega'' - (s-\ell)\Omega - k_\parallel'' u_\parallel)} \right.$$

— — —(13)

which is identically equal to $H_{k,k',k''}$. Hence the symmetric relation (2) is proven.

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CHAPTER III

THEORY OF THE SUPPRESSION OF DRIFT CYCLOTRON LOSS CONE INSTABILITY BY ELECTRON BEAMS

1. Introduction

Recently it has been shown that by parallel injection of an electron beam (EB) in mirror machines that the DCLC turbulence can be controlled efficiently (1-5). The success of this technique in Constance II initiated the same experiments in 2XIIB, β -2 (4) from the point of view of assessing its applicability to ultimately the end plugs of MFTF-5 and future tandem mirror reactors. The mechanism usually suggested for the suppression of DCLC turbulence is the one based on appearance of hot electrons in the late stages of beam plasma interaction. For reasons discussed in Section 2, we think this mechanism to be unsatisfactory. In this article we propose an alternative model for the observations of Constance II and 2XIIB which is based on the process of resonant damping by Langmuir plasmons.

In Section 3, our studies reveal that this process leads to a modification in the growth rates of linear DCLC modes which depends upon the beam power. In Section 4, we have investigated the saturation mechanism of these modified DCLC modes. A closed set of equations are obtained to study the evolution of the distribution function to calculate fluctuation level, final electron and ion temperatures, ion life time etc. In Section 5, we have explained varioss observations of Constance II and 2 XIIB using these equations.

2. Proposed Mechanism

The usual mechanism suggested for the suppression of DCLC turbulence is, in brief, as follows: If the injected beam is warm enough i.e. $\Delta V_b / V_b \gtrsim (n_b/n_p)^{1/3}$ (where V_b is the beam velocity, n_b is the beam density, n_p is the plasma density) with the spread Δ in the wave number K given by $\Delta \simeq \frac{\omega_{pe}}{V_b} \left(n_b/n_p \right)^{1/3}$ (ω_{pe} is the electron plasma frequency) centred around $K_0 = \omega_{pe}/V_b$ and electron energy content given by approximately by one-third of the total beam kinetic energy, is produced (6). The quasi-linear theory fails to explain the saturation of these unstable waves (7-10). However, when strong turbulent effects are considered the saturation is explained. Accordingly it has been shown by a number of authors (11-20) that when the amplitude of the waves becomes sufficiently large, following non-linear processes can occur.

(i) Quasilinear effects on the particle distribution function. However, as these effects are important for very short wavelengths i.e. $K \lambda_D > 0.2$ they are generally not considered.

(ii) Two plasmon absorption and emission processes which are one step higher order processes in $q\phi/T$ (where T and q are the particle temperature and charge and ϕ is the wave potential) than the quasilinear effects. There are two such processes.

(a) Induced scattering caused by electrons i.e.

$$e + l = e' + l' \tag{1}$$

(where e and l represent the electrons and langmuir wave respectively). The process is important in short wave-length region given by $1/\lambda_D > k > 1/\lambda_D \, (m_e/m_i)^{1/5}$. The time scale of the process is $\simeq (k^2 \lambda_{De}^2 W_k \, \omega_{pe})^{-1}$ (where λ_{De} is the electron debye length and W_k is the ratio of energy in langmuir waves to that in particles).

(b) Scattering of Langmuir waves by ions

$$i + l = i' + l' \tag{2}$$

(i denotes an ion). The process is important in the long wave length region given by $1/\lambda_D (m_e/m_i)^{1/2} < k < 1/\lambda_D \, (m_e/m_i)^{1/5}$. The characteristic time scale of this process is $[(m_e/m_i)^{1/2} W_k \, \omega_{pe}]^{-1}$ (m_e and m_i are the electrons and ion masses respectively).

(iii) Four plasmon process given by

$$l + l' = l'' + l''' \tag{3}$$

The process is operative in small k region given by $k \lesssim 1/\lambda_{Di} \, (m_e/m_i)^{1/2} \equiv k^*$. The characteristic time scale of this process is $\simeq [\omega_{pe} W_k^2 \, k^2 \lambda_{De}^2]^{-1}$.

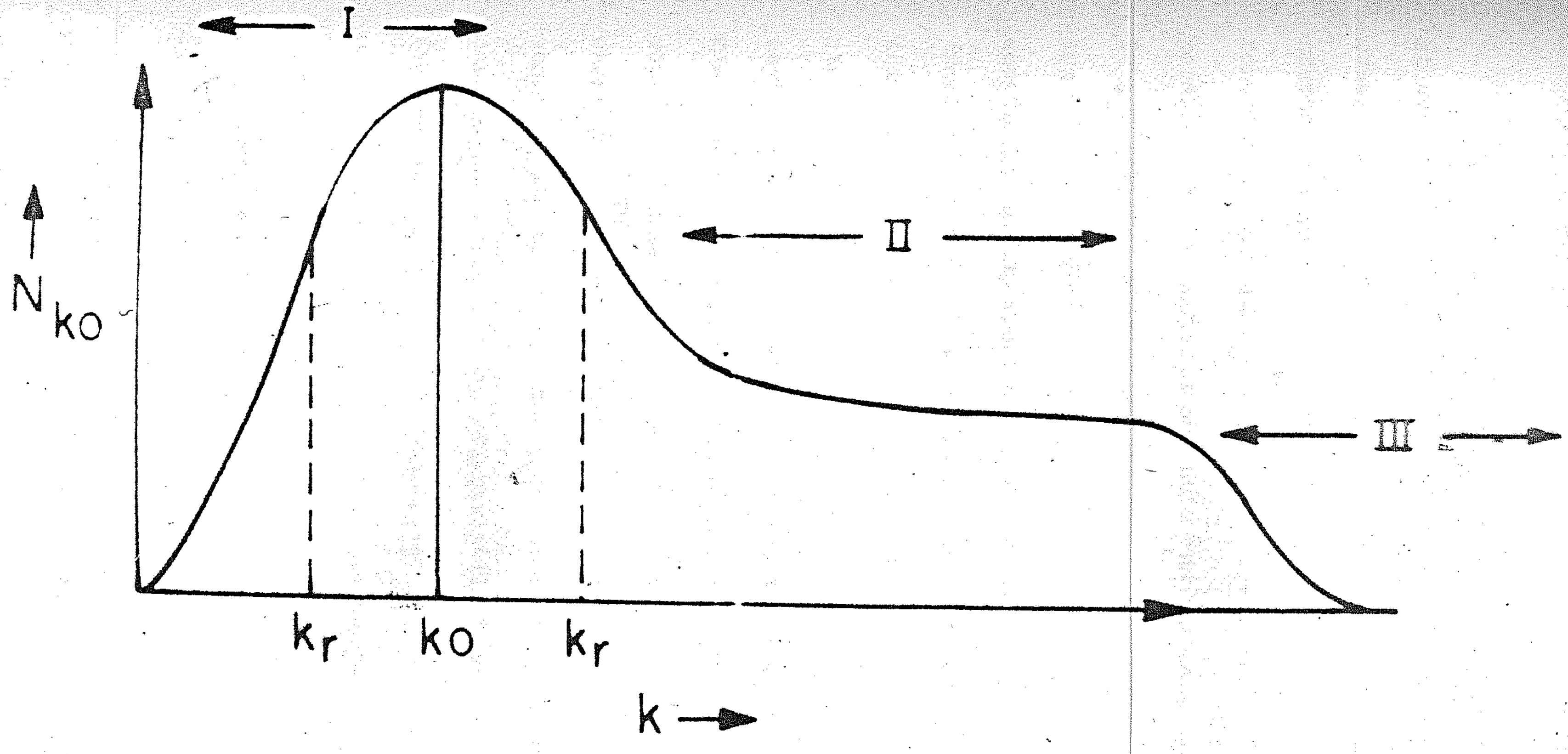

FIG.1. STATIONARY SPECTRUM OF LANGMUIR TURBULENCE

Let us assume that the generation of Langmuir wave takes place at sufficiently large K, so that by these non-linear processes the energy is transferred to smaller K and as a result the spectrum starts broadening. The rate of this transfer decreases with K so that for some K the rate of influx of energy and that of absorption is comparable. The transfer stops here and the wave-energy starts accumulating. In general, the stationary spectrum formed by these processes has following features. For $K > K^* \simeq 1/\lambda_{Di} (m_e/m_i)^{1/5}$ where electron scattering is important the spectrum is Kolmogorov type i.e. $W_K \propto 1/K^{5/3}$ (part III in fig.1); In regions where ion scattering is important W_K still decreases with K but less rapidly (part II in fig.1). In the region $K < K^* \simeq 1/\lambda_{Di} (m_e/m_i)^{1/5}$, where plasmon-plasmon scattering is important the spectrum goes through a maximum before going to zero at K = 0 (part I in fig.1). The main scale of the turbulence K_0 i.e. where the maximum energy occurs is given by $K_0 = K^* \left(\frac{\nu_c^2 K^*}{\alpha Q} \right)^{1/2(\nu-1)}$ where $\alpha = \frac{\pi}{27} \frac{\omega_{pe}^2}{n_0 m_i v_{The}^4 [1 + T_e/T_i]^2}$, $\nu_c = \frac{\omega_{pe}}{n_0 \lambda_{De}^3} \ln \Lambda$, ν is about $\simeq 3.86$ and Q is the total power of generation source which in this case is the electron beam. Clearly the main scale of turbulence will move towards smaller K as the beam power is increased. This pile up of wave-energy in lower K is called the condensation of Langmuir turbulence in $K \simeq 0$ state. As a consequence of this pile up of wave energy oscillating two stream instability is excited which opposes this process of "condensation" by generating larger K (7,13). As numerical simulations have shown (12,14), these processes ultimately lead to the state of 'spiky turbulence' where there are localised regions of strong electric fields.

$[W_K = \langle E_K^2/8\pi \rangle / n_0 K T_e \leq 1]$. The generation of large leads to a 'collapse' of Langmuir turbulence by electron Landau damping and a small population of super thermal electrons (T_e > 20 KeV) appears as a tail in the electron distribution function (21). These electrons get trapped in the mid-plane to reduce the ambipolar potential which helps to fill in the 'hole' in the ion distribution and thus quench the loss cone instabilities. The observed enhancement at low beam powers ($\leqslant$ 42 KW in Constance II was explained by the authors (1) by stating that in this case, the bulk heating of electrons takes place. This increases the ambipolar potential, which in turn, widens the 'hole' in the ion distribution leading thereby to the enhancement of the instability. The experimental findings of this paper however do not support this conjecture. In fact, as discussed below, the conditions and the observations of the experiments on Constance II clearly contradict the basic requirements for the quenching of the DCLC instabilities by hot electrons. As established by Ioffe et al (22) in their experiment on quenching of DCLC instabilities by micro-wave heating of electrons, that in order that quenching occurs by hot electrons certain conditions must be satisfied namely,

(i) The density of the plasma should be in the regime $\omega_{pe} \ll \omega_{ce}$ In this regime strong heating of a few electrons is more dominant. At higher densities i.e. in the regime $\omega_{pe} \gtrsim \omega_{ce}$ the bulk heating of electrons is more dominant which leads to the enhancement of the instability. Thus with the decreasing plasma density from the regime $\omega_{pe} \gtrsim \omega_{ce}$ to $\omega_{pe} \ll \omega_{ce}$ a gradual transition from the enhancement to suppression of the instability should be observed.

(ii) The floating potential should get inverted from the normal positive to the negative potential for the quenching to take place. It has been shown both by Ioffe et al (22) and Kanaev et al (23) that the floating potential and the loss cone instabilities are closely connected and a quenching of the instability by hot electrons must be simultaneously accompanied by a drop in the positive potential of the plasma.

Now firstly in Constance II experiment densities are large i.e. $n \simeq 10^{13}\ cm^{-3}$ so that $\omega_{pe} \gtrsim \omega_{ce}$ and hence the bulk heating will be more important which the authors report; T_e increased from 10 eV to 20 eV. Secondly in the same experiment a gradual transition from the enhancement to suppression was observed with the increasing beam power. In 2XIIB the stabilization did appear with the density decay (4), but the density was always in the regime $\omega_{pe} \gtrsim \omega_{ce}$. And lastly in Constance II no sharp drop in the potential was observed. All this seems to suggest that some mechanism other than that of the tail formation due to a few hot electrons is responsible for these observations.

In a recent paper (24) we have proposed an alternative scheme which envisages a direct coupling between Langmuir turbulence and the DCLC instability. The mechanism in brief is as follows: The injection of an electron-beam gives rise to a spectrum of Langmuir waves. By dominant non-linear processes mentioned earlier this spectrum broadens in which case the wave-plasmon interactions or more precisely the effect of plasmon Landau resonances on DCLC modes become important. We have taken into account these processes to explain the observations of Constance II and 2XIIB experiments. Our previous studies had revealed that these processes lead to a modification in the growth rates of linear DCLC modes which depends upon the beam power. In this Chapter we give a more complete model including a saturation mechanism for these modified DCLC modes. Our previous treatment of the modification in the linear properties of DCLC modes (24) was to some extent arbitrary because the nonlinear evolution of Langmuir turbulence was not taken into account. For the stationary spectrum of Langmuir turbulence a Gaussian was used which is, to some extent, arbitrary. Hence in the following section we present a more exact treatment using the nonlinearly evolved Langmuir turbulenece spectrum. Section 4 deals with the saturation mechanism of these modified DCLC modes.

3. Modification in the Linear Properties of DCLC Modes:

For our theoretical model we consider a slab geometry with the Z-axis along the mirror axis. In general, in the regime $\omega_{pe}/\omega_{ce} \gtrsim 1$ where all the present day mirrors lie, the injection of an electron beam along the field line will produce plasmons which will travel in all directions with respect to the field lines. The purpose of this analysis is to bring out the essential features of a possible physical process involved in the experiment. To this end we will make certain assumptions which may have to be dispensed with for more quantitative accuracy. Thus we consider the nonlinearly evolved and saturated spectra of Langmuir waves in the Y-Z plane. Let the main scale of turbulence lie at an angle θ_0 from the Y-axis. To study the effect of this population of the Langmuir plasmons on DCLC mode we notice that the DCLC waves and the Langmuir waves have widely different dispersion characteristics i.e. $\omega_k \simeq \omega_{pe} > \Omega \sim \omega_{ci}$ [ω_k (Ω) is the frequency of Langmuir (DCLC) waves). Hence we can make use of the adiabatic approximation due to Vedenov et al (25) which satisfactorily takes into account the incoherent interaction between waves of widely different dispersion characteristics. This approximation envisages that the high frequency waves behave as a group of particles (or quasi-particles) on the time scale of low frequency waves. Hence by Landau resonance which may lead to damping or growth, this population of quasiparticles can alter the temporal behaviour of the low frequency modes. The approximation considers $\omega_k - \omega_{pe} > \Omega - \omega_{ci}$, $k >> q$, where $K(q)$ is the wave number of Langmuir waves (DCLC). A similar idea was earlier proposed by Camac et al (26). There are number of authors who have successfully used this approximation in different physical situations (27-30). Recently Pozzoli and Ryotov (31) have furnished a rather general treatment for studying the modulational instability induced by Langmuir turbulence in a magnetic field. Unfortunately, though their treatment is general, it cannot be used in the present case for explaining the experimental observations of Constance II and 2XIIB. They consider an ion acoustic like mode (which is quite different from a DCLC mode) and envisage the effect of langmuir turbulence on this mode by an effective temperature denoted by 'T_{eff}', which directly depends upon the beam velocity.

However, from the turbulence term they drop the term Ω/q. Thus in their case the instability results when $T_{eff} > T_e$ (T_e is the electron temperature) which is contrary to the experimental observation that for high beam velocity, suppression of the instability is observed. It will be shown here that the term Ω/q in the turbulence term is very important for explaining the experimental observations and hence should be retained.

Here the effect of turbulence is envisaged through Langmuir plasmons Landau damping or growth on DCLC mode, which gives a simple explanation of experimental observations. For these reasons we consider the problem in terms of the simple model furnished by Vedenov et al (25). It should be noted that Sakai et al (32) have pointed the incomplete nature of the interaction as envisaged by Vedenov et al. Nevertheless we will use Vedenov et al's treatment since the interactions retained by them are sufficient for the purpose of our analysis to explain the experimental observations. We have shown later that the complete interaction as envisaged by Sakai et al will alter the results only quantitatively. The evolution of the plasmon distribution function will be studied by a wave-kinetic equation as developed by Vedenov et al (25).

$$\frac{\partial N_k}{\partial t} + \vec{V_g} \cdot \nabla N_k - \frac{\partial \omega_k}{\partial \vec{r}} \cdot \frac{\partial N_k}{\partial \vec{k}} = 0 \qquad (4)$$

where N_k is the plasmon distribution function and $\vec{V_g} = \partial \omega_k / \partial \vec{k}$ is the group velocity of the plasma waves. In general the velocity of the plasmons (which is same as the group velocity $\vec{V_g}$) contained in the momentum state K is not along K (31). But as an approximation (which is discussed at the end) we may take the velocity of the plasmon to be $\vec{V_g} \simeq \vec{k}\, \lambda_{De}\, v_{the}$ (28). It should be noted that the wave kinetic equation (4) is valid only when the spread in the group velocities is so large that the convective term $\vec{V_g} \cdot \nabla N_k$ dominates the effect of diffraction i.e. the term containing $\partial V_g / \partial \vec{k}$. In brief the effect of HFT arises as follows : The low frequency density perturbation creates a perturbation in the plasmon density. The gradient of this plasmon density gives rise to a ponderomotive force (PF) which reacts back on the low frequency waves to modify its propagation characteristics. Alternatively, a Langmuir plasmon decays into another plasmon and a DCLC wave. The PF on ions is m /m_i times smaller than that on electrons and hence will be neglected. The plasmon distribution function N_k is perturbed as follows: $N_k = N_{k0} + \tilde{n}_k$ where N_{k0} is the equilibrium distribution function (normalised as $\int N_{k0} dk = 1$). From the wave kinetic equation (4) $\tilde{n}_k$ is given as

$$\tilde{n}_k = \frac{\partial \omega_k}{\partial \vec{r}} \cdot \frac{\partial N_{k0}}{\partial \vec{k}} \Big/ i(\vec{q}\cdot\vec{V}_g - \Omega) \qquad (5)$$

The dependence of $\tilde{n}_k$ on the low frequency density perturbation comes from the term $\partial \omega_k / \partial \vec{r}$. The plasma waves in the regime $\omega_{pe} > \omega_{ce}$ are given by $\omega_k^2 = \omega_{pe}^2 + k^2 v_{the}^2$, which gives $\partial \omega_k / \partial \vec{r} \simeq i/_2 \, \vec{q} \, \omega_k n_{e1}/n_0$ where $\tilde{n}_{e1}$ is the low frequency perturbation. The wave vector $\vec{q}$ for the DCLC will be taken in the y-direction with the density gradient $\epsilon = \frac{1}{n}\frac{dn}{dx}\hat{x}$ in the X-direction.

For DCLC modes we will use the model given by Post and Rosenbluth (34). According to this model DCLC modes are electrostatic flute modes which arise because of the resonance between the positive energy electron drift mode and the negative energy ion Bernstein mode; the typical phase velocity of these modes as seen in 2XIIB (38) experiment is approximately the ion thermal veolcity. It should be noted that this model for DCLC modes ignores certain effects which may be needed for quantitative accuracy in a given physical situation such as electromagnetic effects to the electron contribution (35), ion drift term (36), and temperature gradients (37).

To calculate the PF on an electron the modified equation of motion is

$$m_e \frac{\partial \vec{V}}{\partial t} + m_e \vec{V}_e \cdot \nabla V = e\vec{E} - \frac{e}{c}\vec{V} \times \vec{B}_o \tag{6}$$

where now in the equation of motion the nonlinear term $m\,(\vec{V}\cdot\nabla\vec{V})$ has been retained. In equation (6) $\vec{V} = \vec{V}_s + \vec{V}_f$, where $\vec{V}_s$ is the low frequency component sustained by DCLC while V_f is the high frequency component sustained by Langmuir turbulence. The total P.F. on an electron due to the whole spectrum can be obtained by averaging equation (6) over an ensemble of plasmons.(40)

$$m_e \langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle = -e\vec{E}_s - e\vec{V}_s \times \vec{B}_o \tag{7}$$

where V_f is governed by the high frequency equation

$$m_e \frac{d\vec{V}_f}{dt} = -e\vec{E}_f - e\vec{V}_f \times \vec{B}_o \tag{8}$$

In equation (7), $\vec{E}_s$ is the perturbed E-field due to DCLC hence $\vec{E}_s = |E_s|\,\hat{Y}$ while in equation (8), $\vec{E}_f$ is the perturbed E-field due to HFT hence $\vec{E}_f = |E_{fy}|\,\hat{Y} + |E_{fz}|\,\hat{Z}$. From equation (6) V_{sx} and V_{sy} can be written as

$$V_{sx} = \frac{\omega_{ce}^2 E_s/B_o}{(\omega_{ce}^2 - \Omega^2)} - \frac{\omega_{ce}^2 \langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_x}{i\Omega(\omega_{ce}^2 - \Omega^2)} + \frac{\omega_{ce} \langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y}{(\omega_{ce}^2 - \Omega^2)} + \frac{\langle \vec{V}_f \cdot \nabla V_f \rangle_x}{i\Omega} \tag{9}$$

$$V_{sy} = \frac{i\Omega\,\omega_{ce} E_s/B_o}{(\omega_{ce}^2 - \Omega^2)} - \frac{\omega_{ce}\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_x}{(\omega_{ce}^2 - \Omega^2)} + \frac{i\Omega\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y}{(\omega_{ce}^2 - \Omega^2)} \tag{10}$$

The ordering relevant to the present case is $\Omega(\sim \omega_{ci}) < \omega_{ce} < \omega_k (\sim \omega_{pe})$. With this ordering it can be shown from equation (8) that P.F. in X-direction i.e. $m_e\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_x$ is ω_{ce}/ω_{pe} times $<$ the P.F. in Y-direction i.e. $m_e\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y$ and hence may be neglected. In the linearized equation of continuity, therefore, V_{sy} may be neglected and in V_{sx} only $\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y$ term, apart from the E_s/B_o drift, may be retained. Thus the modified linearized equation of continuity gives

$$-i\Omega n_{e1} + n_o \epsilon \left[E_s/B_o + \frac{\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y}{\omega_{ce}} \right] = 0 \tag{11}$$

The term $\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y$ can be written as

$$\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y = \sum_k \left[\frac{1}{2}\frac{\partial |V_{fy}|^2}{\partial y} + V_{fx}\frac{\partial V_{fy}}{\partial x} \right] \tag{12}$$

From equation (8) V_{fy} and V_{fx} are given in terms of the high frequency field E_f as

$$V_{fx} = \frac{\omega_{ce}^2\, E_{fy}/B_o}{(\omega_{ce}^2 - \omega_k^2)} \tag{13}$$

$$V_{fy} = \frac{i\,\omega_k\,\omega_{ce}\, E_{fy}/B_o}{(\omega_{ce}^2 - \omega_k^2)} \tag{14}$$

The second term in equation (12) can be neglected for the reason that HFT lies in Y-Z plane hence $\partial V_{fy}/\partial x \propto \partial E_{fy}/\partial x = 0$. Hence equation (12) becomes

$$\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y = \sum_k \frac{1}{2} \frac{\partial}{\partial y} |V_{fy}|^2 = \sum_k \frac{1}{2} \frac{\omega_k^2 e^2}{m_e^2} \times \frac{1}{(\omega_{ce}^2 - \omega_k^2)^2} \cdot \frac{\partial}{\partial y} |E_{fy}|^2 \qquad (15)$$

$E_{fy} = E_f \sin\theta$, where θ is the angle between the Z-axis and K. Since $|E_{fk}|^2/8\pi = N_o N_k \omega_k$ (where N_o is the plasmon density) we can write equation (15) as

$$\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y = \sum_k \frac{4\pi\, \omega_k^3\, e^2 \sin^2\theta\, N_o}{m_e^2 (\omega_{ce}^2 - \omega_k^2)^2} \frac{\partial \tilde{n}_k}{\partial y} \qquad (16)$$

$\tilde{n}_k$ may be substituted from equation (4) in which case equation (16) becomes

$$\langle V_f \cdot \nabla V_f \rangle_y = \sum_k i \frac{4\pi \omega_k^3 e^2 \sin^2\theta}{m_e^2 (\omega_{ce}^2 - \omega_k^2)^2} \frac{\omega_{pe} N_o}{2 n_o} \frac{\vec{q} \cdot \nabla_k N_{ko}}{(\vec{q} \cdot \vec{V}_g - \Omega)} \tilde{n}_{e1} \qquad (17)$$

Equation (17) may be used to substitute for $\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y$ in equation (11) to get the modified electron density perturbation as

$$\tilde{n}_{e1} = -i n_o \epsilon E_s/B_o \left[1 - \frac{q\, n_o \epsilon}{\omega_{ce} - \Omega} \frac{4\pi e^2}{m_e^2} \frac{\omega_k^3 N_o}{(\omega_k^2 - \omega_{ce}^2)^2} \frac{\omega_{pe}}{2 n_o} \cdot \int \frac{\sin^2\theta\, \vec{q} \cdot \vec{\nabla} N_{ko}}{(\vec{V}_g \cdot \vec{q} - \Omega)} d\vec{k} \right]^{-1} \qquad (18)$$

where now in the turbulence term (i.e. the second term in the denominator) the summation is replaced by an integration in the K-plane. The integration has to be done in accordance with Landau's prescription. As ions are not affected by the P.F. the perturbed ion density $\tilde{n}_{i1}$ can be calculated in the manner shown by Post et al (34). The ions and electrons perturbed densities $\tilde{n}_{i1}$ and $\tilde{n}_{e1}$ can be substituted in Poisson's equation to get the modified dispersion relation as

$$1 = \frac{\Omega^*/\Omega}{[1 - L/\Omega]} - b \sum_{n=-\infty}^{\infty} \frac{\Omega}{(\Omega - n\omega_{ci})} \tag{19}$$

where $\Omega^* = \frac{\omega_{pe}^2}{\omega_{ce}} \frac{\epsilon}{|q|}$ $\quad b = \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{1}{(q\langle a_i \rangle)^3}$

$$a_i = \frac{v_{thi}}{\omega_{ci}}$$

The integration in K-plane has to be done using stationary spectrum which, in the presence of magnetic field will be a function of K and θ In the high density limit $\omega_{pe} > \omega_{ce}$, the effect of magnetic field on Langmuir oscillation is not very significant and hence to a good degree of accuracy we may use for N_{k0} the stationary Langmuir spectrum calculated by Tsytovich et al (20) in the absence of the magnetic field given in fig.1.

The turbulence term L in equation (19) is given by

$$L = \frac{q n_0 \epsilon}{\omega_{ce}} \frac{4\pi e^2}{m_e^2} \frac{\omega_k^3 N_0}{(\omega_k^2 - \omega_{ce}^2)^2} \frac{\omega_{pe}}{2 n_0} \int \frac{\sin^2\theta\, \vec{q} \cdot \frac{\partial N_{K_0}(\vec{K},\theta)}{\partial \vec{K}} \, d\vec{k}}{(\vec{V}_g \cdot \vec{q} - \Omega)} \tag{20}$$

The integration in K-plane has to be done along Landau's contour according to the resonance condition $V_g \cdot q = \Omega$. Doing this we have

$$L = L_r + i L_i \tag{21}$$

where

$$L_r = q^2 \Omega^* \frac{\omega_k^3 \omega_{pe}}{(\omega_k^2 - \omega_{pe}^2)^2} W_k \oint \frac{\vec{q} \cdot \frac{\partial N_{K_0}(\vec{K},\theta)}{\partial \vec{K}} \, d\vec{k}}{\left[\vec{K} \cdot \vec{q} - \frac{\Omega}{\lambda_{De} U_{the} q}\right]} \quad \text{---(a)}$$

$$L_i = q^2 \Omega^* \frac{\omega_k^3 \omega_{pe}}{(\omega_k^2 - \omega_{pe}^2)^2} W_k \int_0^{2\pi}\int_0^{\infty} \sin^2\theta\, \vec{q} \cdot \frac{\partial N_{K_0}(K,\theta)}{\partial \vec{K}} \, \delta\left(\vec{q} \cdot \vec{K} - \frac{\Omega}{\lambda_{De} U_{the} q}\right) \cdot K \, dK \, d\theta \quad \text{---(b)}$$

(22)

The real part of L i.e. L_r is due to nonresonant plasmons satisfying $V_g < \Omega/q$. While the imaginary part of L i.e L_i is due to the resonant plasmons satisfying the condition $V_g > \Omega/q$ and is important in our analysis. The θ integration in L_i integral can be done to give

$$L_i = q^2 \Omega^* \frac{\omega_k^3 \omega_{pe}}{(\omega_k^2 - \omega_{ce}^2)^2} W_k k_\gamma \int_{k_\gamma}^{\infty} \frac{\frac{1}{k^3} \frac{\partial N_{k0}(k,\theta)}{\partial k}\Big|_{\sin\theta = k_\gamma/k} dk}{[1-(k_\gamma/k)^2]} \tag{23}$$

where $$k_\gamma = \frac{\Omega}{q\, v_{the}\, \lambda_{De}} \tag{24}$$

According to Sakai et al (32) in the turbulence term there should be another factor which is proportional to the intensity of the spectrum N_{k0}. For reasons explained later this term will have to be dropped here. According to the experimental observations the DCLC wave spectrum is a narrow band spectrum i.e. $\Delta q/q \simeq 1/10$ (40) with maximum power in the fundamental mode at $\Omega \sim \omega_{ci}$. To look for the stability of this mode we retain n=1 term in the summation in equation (19) and solve the resultant quadratic equation to get the modified Ω as

$$\Omega = \frac{1}{2(1+b)} \Big\{ \omega_{ci} + \Omega^* - (1+b)L \pm \Big[\{\omega_{ci} + \Omega^* - (1+b)L\}^2 - 4\omega_{ci}\{\Omega^* - L(1+b)\} \Big]^{1/2} \tag{24}$$

For L = 0 the dispersion relation (24) reduces to that obtained by Post et al (32) where long wavelengths characterized by b >> 1 are unstable. We will now evaluate the real and the imaginary part of Ω i.e. Ω_γ and γ. In the experimental observations $2.9 < q_\perp a_i < 6.1$ hence b ranges from 1.69×10^3 to 3.55×10^3. While $\Omega^* = \frac{\omega_{pe}^2}{\omega_{ce}} \epsilon/q_\perp \simeq 10^{10}$ rad/sec, for $\omega_{pe} \simeq 3.34\times10^{11}$ rad/sec, $\omega_{ce} \simeq 1.13\times10^{11}$ rad/sec, $\epsilon = 1/7\ cm^{-1}$, $q_\perp \sim 2\ cm^{-1}$ (for $a_2 = 2.57$ cm) and $\omega_{ci} = 3.07\times10^7$ rad/sec. Since $|L|$ arises from the nonlinear effects we may use the ordering $|L| < \Omega_\gamma \sim \omega_{ci} \ll \Omega^*$. In this ordering equation (24) gives $\Omega_\gamma \sim \Omega^*/2b$ which for the experimental parameter given above is of the same order as ω_{ci} thus justifying the ordering $\Omega_\gamma \sim \omega_{ci}$. To evaluate the growth rate γ we note that in the ordering we are considering the first term under the radical $\sim \Omega^{*2}$ while the second term is $\sim 4\omega_{ci}\Omega^* b$. For experimental parameters given above $4\omega_{ci}\Omega^* b > \Omega^{*2}$. Hence we may make binomial expansion in powers of $\Omega^*/4\omega_{ci} b$. For the parameters given above $\Omega^*/4\omega_{ci} b \sim 0.1$, in which case we may retain the leading term to get the growth rate as

$$\gamma = -i\frac{L_i}{2} \pm i\,\frac{(\omega_{ci}\Omega^* b)^{1/2}}{b} \tag{25}$$

In the absence of beam the growth rate $\sim (\omega_{ci}\Omega^* b)^{1/2}/b \simeq 0.3 \times 10^7$ rad/sec. This agrees well within a factor of 3 with the observed growth rate in 2XIIB (38), i.e. $\gamma \simeq .02\,\omega_{ci}$ to $.03\,\omega_{ci}$ for $\omega_{ci} \sim 3.07\times10^7$
$.1\times10^7$ rad/sec. The discripency of a factor of 3 may be attributed to the fact that effects mentioned earlier e.g. electromagnetic effects to electron contribution, the ion drift term which are stabilizing are not included in the present analysis. Let us now see how the stability of these modes is affected by the beam power. We recall from equation (22b) that L_i is given by

$$L_i = q^2\Omega^* \frac{\omega_k^3\omega_{pe}}{(\omega_k^2-\omega_{pe}^2)^2} W_k K_\gamma^3 \int_{K_\gamma}^{\infty} \frac{\frac{1}{k^3}\frac{\partial N_{ko}(k,\theta)}{\partial k}\Big|_{\sin\theta = K_\gamma/k} dk}{\left[1-K_\gamma^2/k^2\right]^{1/2}} \tag{26}$$

In the high density limit $\omega_{pe} > \omega_{ce}$ we may assume the θ dependence of N_{ko} to be weak $\left[\frac{\partial N_{ko}}{\partial \theta} \simeq 0\right]$

From fig.1 N_{k_0} has a maximum at K_0, hence the necessary conditions for the sign of L_i are

$$K_0 > K_\gamma \qquad \text{negative } L_i \qquad \text{(a)}$$

$$K_0 < K_\gamma \qquad \text{positive } L_i \qquad \text{(b)} \tag{27}$$

where as mentioned before K_0 is the main scale of turbulence given by (20)

$$K_0 = K^* \left(\frac{2 \nu_e^2 K^*}{\alpha Q} \right)^{\frac{1}{2(\nu-1)}} \tag{28}$$

where $K^* = \dfrac{1}{\lambda_{Di}} \left(\dfrac{m_e}{m_i} \right)^{1/2}$, $\nu_e = \dfrac{\omega_{pe}}{n_0 \lambda_{De}^3} \ln \Lambda$,

$$\alpha = \frac{\pi}{27} \frac{1}{(1 + T_e/T_i)^2} \frac{\omega_{pe}^3}{n_0 m_i U_{the}^4}$$

and Q is the total power of the generation source which in this case is the electron beam. From this expression we see that with the change in beam power the entire spectrum can be made to shift. It is this shift of Langmuir turbulence spectrum relative to the wave number of the DCLC mode which leads to the damping or growth according to the sign of L_i which through conditions (27) a and b depend on the beam power. This is quite possible in Constance II experiment where an increase in beam power from 30 KW to 50 KW changed the enhancement of DCLC into damping. Thus $K_0 > K_\gamma$ which may correspond to beam power $>$ 42 KW, L_i bears a negative sign and the growth rates of unstable modes are enhanced. For $K_0 < K_\gamma$ which may correspond to beam powers $>$ 42 KW, L_i bears a positive sign consequently the growth rates of unstable modes are reduced.

Let us now estimate typically the strength of Langmuir turbulence which is needed so that the damping induced by plasmons is greater than the growth due to ion distribution. From equation (26) we may write L_i typically as $[\omega_K/\omega_{ce} \gtrsim 1]$

$$L_i \simeq \Omega^* W_K (q/\Delta)^2 \tag{29}$$

As the Langmuir spectrum is broad we may take $\Delta \simeq K_0 \simeq K^* \left(\frac{2\nu_e^2 K^*}{\alpha \Omega}\right)^{\frac{1}{2(\nu-1)}}$ and $q \sim U_{thi}/\omega_{ci}$ typically. Then for the damping induced by plasmons to be greater than the growth induced by ion distribution the required level of Langmuir turbulence should be $\gtrsim W_c$ where

$$W_c = \frac{2\gamma}{\Omega^*}\left(\frac{K_0}{q}\right)^2 \tag{30}$$

For the parameters of 2XIIB $K_0 = .18\ \text{cm}^{-1}$, $q \simeq .4\ \text{cm}^{-1}$ and for Constance II parameters $K_0 = 2$ and $q = 2$. Typically γ observed in 2XIIB is $\gamma \simeq 0.3\times10^7$ Hz (38), which gives $W_c \simeq 10^{-4}$ for Constance II and 2XIIB. This is a very modest level of Langmuir turbulence. The actual level of Langmuir turbulence excited by the beam is much higher than this value. Thus we see that the electron beam induced resonant damping is sufficient to overcome the growth due to ions.

We can now see that if we were to consider the complete interaction as pointed out by Sakai et al, then in the expression for L, there would be another additive term proportional to $\int N_{K_0}\, \delta(\Omega - \vec{q}\cdot\vec{V}_q)\, d\vec{K}$. This term is always positive, hence would lead to growth. Depending on the shape of the spectrum, this term can be of the same order as the term proportional to $\int \frac{\partial N_{K_0}}{\partial K}\, \delta(\Omega - \vec{V}_q\cdot q)\, dK$. But clearly the inclusion of the term proportional to N_{k_0} will only slightly alter the transition point i.e. the beam power where enhancement changes to damping. The explanation still comes essentially from the fact that with the increase in beam power, $\partial N_{k_0}/\partial k$ changes sign. It should be noted that inclusion of effects like ion drift, temperature gradient and complete interaction as pointed by Sakai et al should provide better quantitative agreement with the experimental observations. Also more realistic expression for N_{k_0} should be used. Pozzoli and Ryotov (31) have shown that in general, the group velocity of the Langmuir waves in a magnetic field is given by

$$\vec{V}_g = \frac{\omega_{ce}^2}{k\omega_{pe}} \sin^2\theta\, \hat{k} + \frac{\omega_{ce}^2 \omega_{pe}^2}{k^3 \omega_{pe} c^2} \hat{k} + 3k\lambda_{De} v_{the} \hat{k} - \frac{\omega_{ce}^2}{k\omega_{pe}} \sin\theta\, \hat{z} \qquad (31)$$

where $\hat{k}$ is a unit vector along $\vec{k}$ while $\hat{z}$ is the unit vector along B_0. In equation (25) we may evaluate the order of each term. Thus typically for $k = k_0 \simeq \omega_{pe}/V_b$, the second term is of the order $\simeq \frac{\omega_{ce}^2}{\omega_{pe}^2} \frac{V_b^2}{c^2} V_b$ which for $\omega_{ce} < \omega_{pe}$ and $V_b << c$ can be dropped. The first and the last terms are of the order $\omega_{ce}^2/\omega_{pe}^2 \, V_b$ and hence can be dropped in the **high density** regime $\omega_{ce} < \omega_{pe}$ where all the present day mirror machine lie. Hence $\vec{V}_g \simeq \vec{k}\lambda_{De} v_{the}$

Thus it follows from this section that in the presence of an electron beam the Langmuir turbulence modifies the linear properties of the DCLC waves. However, saturated DCLC waves are observed in the experiment. Thus the considerations made so far may not provide a full explanation of the experimental observations. To accomplish this we must investigate the saturation mechanism of these modified DCLC modes which is done in the next section.

4. Saturation Mechanism of Modified DCLC Mode:

In this section we investigate the nonlinear saturation mechanism of the modified DCLC modes. The saturation of DCLC modes will occur when the growth due to ions is balanced by plasmon damping. The injection of electron beam, with power greater than a certain value reduces the ion diffusion and thereby improves the ion life time inside the trap.

The important velocity transport processes causing the diffusion in velocity space are:

(i) Diffusion in velocity space by DCLC waves

(ii) Ion-Ion scattering

(iii) Charge-exchange replacement by cold gas atoms

(iv) Loss of unconfined plasma

Keeping these processes in consideration and the fact that we are considering flute modes ($q_{\parallel}$ = 0) we may integrate the distribution function over $v_{\parallel}$ and write down the following equation for the time evolution of the resulting reduced distribution function $F(v_{\perp}, t)$ as

$$\frac{\partial F(v_{\perp},t)}{\partial t} = \frac{\partial}{\partial v_{\perp}}\left[\frac{v_{\perp} F(v_{\perp},t)}{\tau_{ii}}\right] + \frac{\partial}{\partial v_{\perp}}\left[\frac{v_{\perp} F(v_{\perp},t)}{\tau_{ie}}\right] + \frac{\partial}{\partial v_{\perp}}\left[D(v_{\perp},t)\frac{\partial F}{\partial v_{\perp}}(v_{\perp},t)\right] - \nu_t F(v_{\perp},t) - \frac{F(v_{\perp},t)}{\tau_{cxg}} + g(v_{\perp})\int \frac{F(v_{\perp},t)}{\tau_{cxg}}\, dv_{\perp} \tag{32}$$

In equation (32) τ_{ie} is the electron drag time $= 1.5 \times 10^2 \, (T_e \, (\text{KeV}))^{3/2}/n_0 \ln \Lambda$, τ_{ii} is the ion-ion scattering time $= \dfrac{2.5 \times 10^{10} \times E^{3/2} \text{ KeV}}{n_0 \ln \Lambda}$ ($\ln \Lambda$ is the Spitzer's factor $\simeq 10$). The last two terms represent the charge exchange with the background gas (40), $\tau_{cxg} \simeq n R_p / 2 \phi_g$, R_p is the plasma radius while, ϕ_g is the background flux, ν_t is approximately the inverse transit time of the untrapped plasma. We take

$$\nu_t = 1/\tau_T \qquad v_\perp < v_h$$

$$\nu_t = 0 \qquad v_\perp > v_h \tag{33}$$

where $v_h^2 = 2e\phi/m_i$ is the parameter which measures the size of the 'hole' in the ion distribution function due to ambipolar potential ϕ. It is roughly $\simeq 3\,T_e$ (35) hence $v_h^2 = 3T_e/m_i = C_s^2$, $\tau_T = L_p/C_s$ where L_p is the axial length of the mirror plasma and C_s is the ion sound velocity. It should be noted that if the axial length of the plasma as measured in the experiment turns out to be much less than the distance between the mirrors, then it can be safely assumed that the pitch angle distribution is peaked at angles nearly perpendicular to the magnetic field in which case the ion-ion scattering is not an important velocity transport process. Unfortunately in the published papers (1-5), the axial length of the plasma has not been made available, hence it is difficult to decide ion-ion scattering constitutes an important velocity transport process or not. However as the ratio of τ_{ii} to τ_{ie} is $\simeq 1/10^2 \, (T_i/T_e)^{3/2}$ (T_i and T_e are the ion and electron temperatures) which for the experimental parameters is < 1. We may presently neglect ion-ion scattering; of course, for greater quantitative accuracy it may have to be included. The velocity diffusion coefficient $D(v_\perp, t)$ due to fluctuating DCLC fields is given by

$$D(v_\perp,t) = \frac{\pi e^2}{m_i^2 v_\perp^2} \sum_n n^2 \omega_{ci}^2 \int \frac{dq}{(2\pi)} \frac{|E_q|^2}{q^2} J_n^2\left(\frac{q_\perp v_\perp}{\omega_{ci}}\right) \cdot \frac{\Delta\Omega}{(\Omega - n\omega_{ci})^2 + (\Delta\Omega)^2} \tag{34}$$

(where γ is the growth rate of DCLC waves in the absence of the beam). Apart from the linear growth rate, the spectral width $\Delta\Omega$ may contain frequency shift due to other nonlinear effects. In the absence of beam the velocity space diffusion continues to fill the loss cone, till the transit time loss of the unconfined plasma is balanced by the turbulent diffusion. This leads to the formation of a plateau i.e. $\partial F/\partial v_\perp = 0$ and the DCLC instability saturates. The equation for the time evolution of DCLC fluctuation energy $W_q(t)$ is

$$\frac{dW_q(t)}{dt} = [\gamma_1(t) + \gamma_2(t)]\, W_q \tag{35}$$

where $\gamma_2(t)$ is the damping or growth due to plasmon resonance. From equations (23) and (25) we have

$$\gamma_2 = L_i/2 = q^2 \Omega^* \frac{\omega_k^3 \omega_{pe}}{(\omega_k^2 - \omega_{ce}^2)^2} W_k K_\gamma^3 \int_{K_\gamma}^{\infty} \frac{\frac{1}{k^3} \left.\frac{\partial N_{k0}(k,\theta)}{\partial k}\right|_t}{\left[1 - (k_\gamma/k)^2\right]} dk \Bigg|_{\sin\theta = k_\gamma/k} \tag{36}$$

while $\gamma_1(t)$ in equation (35) is the linear growth rate of DCLC modes in the absence of the beam. To close the set of equations we need equations describing the time evolution of the electron temperature and the plasmon distribution function $N_{k0}(k,t)$

The evolution of electron temperature can be described by (36)

$$\frac{d}{dt}\left(\tfrac{3}{2} n T_e\right) = \frac{n_H T_i}{\tau_{ie}} - \eta\, T_e\, J \tag{37}$$

where $J = \nu_t \int_0^{v_h} F(v_\perp, t)\, v_\perp\, dv_\perp$ is the flux of the lost particles, n_H and T_i are the hot ions density and average energy respectively. Equation (37) envisages that the electron energy increases when energetic ions cool on slower electrons by ion electron collisions (1st term) and is drained off by electrons escaping from the ends (represented by the second term). η is a parameter which measures the energy expanded per electron. It will be atleast $\simeq \phi/T_e$ in addition to any other mechanism of energy loss. If process such as ionization of the background gas etc. are included η turns out to be $\simeq 8$ for good vacuum. For given F and n_H at $t = t_0$, T_e at $t = t_0$ can be evaluated from this equation. Now we proceed to find an equation which describes the time evolution of $N_{k0}(k,t)$. From experimental point of view only the case where electron beam suppresses the DCLC turbulence are important. As stated before this is caused by the negative slope of Langmuir turbulence spectrum where mostly scattering due to ions is important. This implies that the equation for the evolution of N_{k0} must contain terms corresponding to scattering by ions, scattering by DCLC waves and the linear growth. Hence

$$\frac{\partial N_{k}}{\partial t} = \alpha N_{\vec{k}} \frac{\partial N_{\vec{k}}}{\partial \vec{k}} + \int \Big[\sum N_{\vec{k}+\vec{q}} \cdot n_{\vec{q}} P(\vec{k}+\vec{q} \rightarrow \vec{k}) \, \delta(\omega_{\vec{k}+\vec{q}} - \omega_{\vec{k}} - \Omega) - N_{\vec{k}} \, n_{\vec{q}} \, P(\vec{k} \rightarrow \vec{k}+\vec{q}) \, \delta(\omega_{\vec{k}+\vec{q}} - \omega_{\vec{k}} - \Omega) \Big] d\vec{q} + \gamma_k N_k \tag{38}$$

where $$\alpha = \frac{\pi \, \omega_{pe}^3}{27 n_0 m_i \, U_{the}^4 [1 + T_i/T_e]^2}$$

The second term in equation (38) gives the rate of change of N_k due to direct and inverse DCLC emission and absorption processes by Langmuir plasmons, n_q is the DCLC Plasmon distribution function and $P(\vec{k}+\vec{q} \rightarrow \vec{k})$ and $P(\vec{k} \rightarrow \vec{k}+\vec{q})$ represents the transition probability from the state $\vec{k}+\vec{q}$ to $\vec{K}$ and vice-verse and are assumed to be equal. For $q/k << 1$ i.e. the adiabatic approximation we may expand N_{k+q} around N_k to give (25) equation (33) as

$$\frac{\partial N_{\vec{k}}(k,t)}{\partial t} = \alpha N_{\vec{k}} \frac{\partial N_{\vec{k}}}{\partial \vec{k}} + \frac{\partial}{\partial k_\alpha} D_{\alpha\beta} \frac{\partial N_{\vec{k}}(t)}{\partial k_\beta} + \gamma_{\vec{k}} N_{\vec{k}} \tag{39}$$

where $D_{\alpha\beta}(k,t)$ is diffusion tensor in K-space given by

$$D_{\alpha\beta}(k,t) = \int n_q(t)\, q_\alpha q_\beta P(k+q \rightarrow k)\, \delta(\Omega - \vec{q}\cdot \frac{\partial \omega_k}{\partial \vec{k}}) \cdot d\vec{q}$$

(40)

To calculate the transition probability $P(\vec{k}+\vec{q} \rightarrow \vec{k})$ we consider the following process. A Langmuir plasma having momentum $\vec{K}$, absorbs and emits a DCLC wave resonantly according to the condition $\Omega = \vec{V}_g \cdot \vec{q}$ The momentum conservation implies

$$\vec{k}' = \vec{k} + \vec{q} \tag{41}$$

Then the rate of change of DCLC plasmon distribution function due to this process is given by

$$\frac{\partial n_q}{\partial t} = \int \Big[N_{\vec{k}+\vec{q}}\, P\, n_{\vec{q}}\, \delta(\omega_{\vec{k}+\vec{q}} - \omega_{\vec{k}} - \Omega) - N_{\vec{k}}\, P\, n_{\vec{q}}\, \delta(\omega_{\vec{k}+\vec{q}} - \omega_{\vec{k}} - \Omega)\Big] dk$$

(42)

Expanding the R.H.S. of this equation for $q/k \ll 1$, we have

$$\frac{\partial n_q}{\partial t} = n_q \int P\, \vec{q}\cdot \frac{\partial N_k}{\partial \vec{k}}\, \delta(\vec{q}\cdot \frac{\partial \omega_k}{\partial \vec{k}} - \Omega)\, d\vec{k} \tag{43}$$

From equation (22b) and (25) we know that the damping of DCLC waves by Langmuir plasmons is given by

$$\gamma_2 = Li/2 = \pi/2\, \Omega^* \frac{q^2}{\partial n_0 m_e} \frac{\omega_k^3 \omega_{pe}}{(\omega_k^2 - \omega_{ce}^2)^2} \int \sin^2\theta\, \vec{q}\cdot \frac{\partial N_{k0}}{\partial \vec{k}} \cdot \delta(\vec{q}\cdot \frac{\partial N_k}{\partial \vec{k}} - \Omega)\, d\vec{k}$$

(44)

In equation (43) $\frac{1}{n_q}\frac{\partial n_q}{\partial t}$ can be put $= 2\gamma_2$ which when compared with equation (44) gives

$$P(\vec{k}+\vec{q} \rightleftharpoons \vec{k}) = \frac{\pi}{2}\frac{\omega_{pe}^2}{\omega_{ce}}\frac{\epsilon}{T} q \times \frac{1}{2 n_0 m_e}\frac{\omega_k^3 \omega_{pe}}{(\omega_k^2-\omega_{pe}^2)^2}\sin^2\theta$$

(45)

This expression for $P(\vec{k}+\vec{q} \rightleftharpoons \vec{k})$ can be used in equation (40) for calculating Langmuir plasmon diffusion coefficient.

Thus we have the following set of closed equations for studying the time evolution of $F(v_\perp, t)$

$$\frac{\partial F(v_\perp,t)}{\partial t} = \frac{\partial}{\partial v_\perp}\left(\frac{v_\perp F}{\tau_{ie}}\right) + \frac{\partial}{\partial v_\perp}\left(\frac{v_\perp F}{\tau_{ii}}\right) + \frac{\partial}{\partial v_\perp}\left[D\frac{\partial F}{\partial v_\perp}\right] - \nu_t F - \frac{F}{\tau_{cxy}} + g(v_\perp)\int_0^\infty dv_\perp \frac{F(v_\perp,t)}{\tau_{cxy}}$$

--- (46)

$$D(v_\perp,t) = \frac{\pi e^2}{m_i^2 v_\perp^2}\sum_n n^2\omega_{ci}^2\int\frac{dq}{(2\pi)}\frac{\mathcal{E}_q(t)}{q^2} J_n^2\left(q v_\perp/\omega_{ci}\right)\frac{\Delta\Omega}{(\Omega-n\omega_{ci})^2+(\Delta\Omega)^2}$$

--- (47)

$$\frac{\partial \mathcal{E}_q(t)}{\partial t} = [\gamma_1(t)+\gamma_2(t)]\,\mathcal{E}_q(t)$$

--- (48)

$$\gamma_1(t) = \frac{\omega_{ci}\sqrt{x}}{(1+x)}, \quad x = \frac{8\pi^2 n_0 c^2}{q^2 m_i}\int \frac{\partial F(v_\perp, t)}{\partial v_\perp^2}\, dv_\perp^2\, dv_\parallel \qquad \text{---(49)}$$

$$\gamma_2(t) = \frac{\pi}{2}\,\frac{\Omega^* q^2}{n_0 m_e}\,\frac{\omega_k^3\,\omega_{pe}}{(\omega_k^2-\omega_{pe}^2)^2}\int \sin^2\theta\ \vec{q}\cdot\frac{\partial N_k(t)}{\partial \vec{k}} \cdot \delta\left(\vec{q}\cdot\frac{\partial \omega_k}{\partial \vec{k}} - \Omega\right) d\vec{k}$$

$$\text{---(50)}$$

$$\frac{\partial N_k(t)}{\partial t} = \alpha N_k \frac{\partial N_k}{\partial k} + \frac{\partial}{\partial k_\alpha} D_{\alpha\beta}\frac{\partial N_k(t)}{\partial k_\beta} + \gamma_k N_k \qquad \text{---(51)}$$

$$D_{\alpha\beta} = \int n_q(t)\, q_\alpha q_\beta\, P(\vec{k}+\vec{q}\rightarrow\vec{k}) \cdot \delta\left(\Omega - \vec{q}\cdot\frac{\partial \omega_k}{\partial \vec{k}}\right) d\vec{q} \qquad \text{---(52)}$$

$$P(\vec{k}+\vec{q} \rightleftharpoons \vec{k}) = \frac{\pi}{2}\,\frac{\omega_{pe}^2}{\omega_{ce}}\,\frac{\epsilon}{1}\times\frac{q}{n_0 m_e}\,\frac{\omega_k^3\,\omega_{pe}}{(\omega_k^2-\omega_{pe}^2)^2} \times \sin^2\theta \qquad \text{---(53)}$$

$$\frac{d}{dt}\left(\tfrac{3}{2} n T_e\right) = n_H T_i/\tau_{ie} - \eta T_e J \qquad \text{---(54)}$$

where
$$\alpha = \frac{\pi\,\omega_{pe}^3}{27\, n_0 m_i\, v_{the}^4\left[1+(T_i/T_e)\right]^2}$$

$$\tau_{ie} = \frac{1.5\times10^{12}\times[T_e(\text{keV})]^{3/2}}{n_0 \ln\Lambda}, \quad \tau_{ii} = \frac{2.5\times10^{11}[T_i(\text{keV})]^{3/2}}{n_0\ln\Lambda}$$

5. Discussion:

Unfortunately in the published papers (1–5) important parameters like background gas flux ϕ_g, the axial length and the radius of mirror plasma which are needed in the numerical integration of equations (46)–(54) have not been made available. Also the observations like energy diffusion rates in different energy channels etc. which would have been useful in comparing it with the rates calculated from equations (39) – (47) have not been made available. For these reasons we have not undertaken the numerical integration of this set of equations to calculate the saturated fluctuation level, final electron and ion life time etc. However, as we will see, it may not be necessary here. Qualitatively we can extract enough information from this set of equations to understand the experimental observations.

To begin with we calculate the K-space diffusion coefficient of Langmuir waves by DCLC waves. This can be obtained from equation (52). As the DCLC is taken to be in Y-direction only, the diffusion tensor will have only one non-zero component which after performing q-integration is given by

$$D_{\alpha\beta}(k,t) = \frac{\pi}{2} n_q \frac{\omega_{pe}^2}{\omega_{ce}} \frac{\epsilon}{n_0 m_e} \frac{\omega_k^3 \omega_{pe}}{(\omega_k^2 - \omega_{ce}^2)^2} \left(\frac{\Omega}{V_g}\right)^3 \times \frac{1}{\sin^2\theta} \times \frac{1}{V_g} \tag{55}$$

where $q = \Omega / V_g$. We may put this equation in the following form

$$D_{\alpha\beta}(k,t) = \frac{\pi}{2} W_q \frac{q}{\Delta q} \frac{\omega_k^3 \omega_{pe}}{(\omega_k^2 - \omega_{ce}^2)^2} \left(\frac{\Omega^*}{\omega_{ci}}\right) \frac{v_{the}^2 \Omega^3}{V_g^4} \times \frac{1}{\sin^2\theta}$$

— — —(56)

where $W_q = n_q \omega_{ci} \Delta q / n_0 K T_e$ i.e. the normalised energy density of DCLC waves. It should be noted that approximation of $\int \mathcal{E}_q dq$ by $n_q \omega_{ci} \Delta q$ may be valid because DCLC turbulent spectra as observed in 2XIIB is narrow i.e. $\Delta q / q \simeq$ .02 (40) and is peaked around ω_{ci} . Using expression for α we may write the ratio of DCLC scattering term and ion scattering term in equation (51) as

$$\frac{D_{\alpha\beta}}{\alpha N_k \Delta k} \simeq \frac{27}{2} \frac{W_q}{W_k} \frac{q}{\Delta q} \left(\frac{\Omega}{\omega_{pe}}\right)^3 \left(\frac{v_{the}}{V_g}\right)^4 \frac{\Omega^*}{\Omega} \frac{\omega_k^3 \omega_{pe}}{(\omega_k^2 - \omega_{ce}^2)^2} \times \frac{1}{\sin^2\theta} \times \frac{m_i}{m_e}$$

— — — (57)

At resonance $V_g \simeq \Omega / q \simeq v_{thi}$ $(\sin\theta \simeq 1)$, and in high density regime $\omega_k > \omega_{ce}$ and $\Omega \simeq \omega_{ci}$ hence

$$\frac{D_{\alpha\beta}}{\alpha N_k \Delta k} \simeq \frac{27}{2} \frac{W_q}{W_k} \left(\frac{q}{\Delta q}\right) \left(\frac{\omega_{ci}}{\omega_{pe}}\right)^3 \left(\frac{v_{the}}{v_{thi}}\right)^4 \frac{\Omega^*}{\omega_{ci}} \cdot \frac{m_i}{m_e}$$

(58)

In 2XIIB we take the average plasma density $n_p \simeq$ .5x10^{13} cm^{-3}, magnetic field at the mid-plane $\simeq$ 6.4 KG, ion temperature $T_i \simeq$ 13 KeV and electron temperature $T_e \simeq$ 175 eV (38). For these parameters we have $\omega_{pe} \simeq 3.34\times10^{11}$ rad/sec, $\omega_{ce} \simeq 1.13\times10^{11}$ rad/sec $v_{thi} \simeq 7.89\times10^7$ cm/sec, $v_{the} \simeq 5.54\times10^8$ cm/sec $\omega_{ci} \simeq$ 3.07x10^7 ad/sec and

$\Omega^* = \frac{\omega_{pe}^2}{\omega_{ce}} \epsilon / q \simeq 10^{10}$ rad/sec for $\epsilon = \frac{1}{n}\frac{dn}{dx} \simeq 1/7$ cm^{-1} and $q = 2$ cm^{-1} (In 2XIIB $2.9 < q < a_i > < 6.1$, where a_i is the ion gyroradius $\simeq 2.57$ cm). In Constance II we take $n_p \simeq 2\text{x}10^{13}$ cm^{-3}, $T_i = 400$ eV $T_e \simeq 10$ eV (1). For these parameters we have $\omega_{pe} \simeq 2.51\text{x}10^{11}$ rad/sec. $\omega_{ce} \simeq 4.9\text{x}10^{10}$ rad/sec, $v_{thi} = 1.38\text{x}10^7$ cm/sec, $v_{the} \simeq 1.32\text{x}10^8$ cm/sec. In 2XIIB the potential fluctuation ranged from 10 - 50 V. This gives $W_q \simeq 8\text{x}10^{-8}$. In weak turbulence limit the energy in Langmuir spectrum would be atleast one-third of the total beam energy (6). Hence minimum value of $W_k = \frac{1}{3}\frac{n_b m_e V_b^2}{n_p m_e v_{the}^2}$ where n_b and V_b are the beam density and velocity. In Constance II $n_b \simeq 10^{10}$ cm^{-3}, $V_b \simeq 3\text{x}10^9$ cm/sec. This gives $W_k \simeq 1/10$ atleast. Thus for typical experimental parameters $(\omega_{ci}/\omega_{pe}) = 1/10^4$, $\frac{v_{the}}{v_{thi}} = 10$ $m_i/m_e \simeq 10^3$ $\Omega^*/\omega_{ci} \simeq 10^3$ $W_q/W_k = 1/10^7$ etc. $\frac{D_{\alpha\beta}}{\alpha N_k \Delta k} << 1$

Thus on account of the low level of DCLC turbulence the scattering due to DCLC waves is not as efficient as that due to ions; hence the damping or growth induced by resonant plasmons will persist for long enough time and the saturation of DCLC amplitudes will occur when $\gamma_1 = -\gamma_2$.

In 2XIIB the enhancement which was earlier seen in Constance II was not seen (1,4). This may be understood as follows: We recall that K_0 for 2XIIB $\simeq .18$ cm^{-1} while K_γ i.e. K at which the resonance occur can be found from the equation

$$V_g = K_\gamma \frac{v_{the}^2}{\omega_{pe}} \simeq \Omega/q \simeq v_{thi} \tag{59}$$

For 2XIIB parameters $K_\gamma \simeq .86$ cm^{-1}, thus $K_\gamma/K_0 >> 1$. Because of this the DCLC mode will lie in the far right of the Langmuir wave spectrum indicated in fig.1 and hence the positive slope which leads to enhancement of the turbulence is inaccessible in the experiment. The observation that for beam power $\simeq$ 15 KW the stabilization was ineffective may be due to the fact that the resonant K i.e. K_γ lies in between $K^* \simeq 10$ and $K^{**} \simeq 110$ where mainly ion scattering is important. The slope in this part of spectrum is generally small - almost flat - and hence resonant damping due to plasmons is not expected to be very effective. If at this stage the density decays then Langmuir wave generation region given by $K = \frac{\omega_{pe}}{V_b}$ will move to the left i.e. towards low K in fig.1. Consequently, the whole spectrum would shift to the left and it may shift enough so that the mode which was previously lying in part II of fig.1 will now lie on part III. In this region the slope is significant i.e.

$N_k \propto \frac{1}{k^{5/3}}$ hence the resonant damping will be effective and stabilization will reappear. This may explain why in 2XIIB, the stabilization occured when the density delayed from 7×10^{-13} cm^{-3} to 1×10^{13} cm^{-3}. It should be noted this stabilization cannot be attributed to the appearence of hot electrons (22). This requires that density should decay from the regime $\omega_{pe} \gtrsim \omega_{ce}$ to $\omega_{pe} \ll \omega_{ce}$ ($\simeq 10^{12}$ cm^{-1} for 2XIIB). However, in 2XIIB during the decay, the density always remained in the regime $\omega_{pe} \gtrsim \omega_{ce}$. It is also clear that once the electron gun is turned off the plasmon distribution would relax under the scattering by DCLC waves. The relaxation time τ_R, then would give the typical time for which the damping hence the stabilization would be effective after the gun turn off. The typical relaxation time is given by

$$\tau_R \simeq \frac{\langle K_\gamma \rangle^2}{D_{\alpha\beta}(K_\gamma)} \qquad (60)$$

where typically $D_{\alpha\beta}$ is given by equation (56) as

$$D_{\alpha\beta} = \frac{\pi}{2} W_q \frac{q}{\Delta q} \frac{\omega_k^3 \omega_{pe}}{(\omega_k^2 \omega_{ce})^2} \frac{\Omega^*}{\omega_{ci}} \frac{v_{the}^2 \Omega^3}{v_{thi}^2} \tag{61}$$

In 2XIIB $W_q \simeq$ 8x10^{-8}, then for $\omega_k \gg \omega_{ce}$, $\Omega \sim \omega_{ci}$ and $V_g \simeq v_{thi}$, $D_{\alpha\beta}$ turns out to be $\simeq$ 1.45x10^7 cm^{-2} sec^{-1}. Using this value of $D_{\alpha\beta}$ and the value of K_γ given before, $\tau_R \simeq$ 150μs. The fig.4 in the ref.4 which is a plot of rf activity with time indicated that effect of beam lasted for about 3 to 4 hundreds of μs after the beam turn off. This is within a factor of two of the relaxation time τ_R (calculated above).

The turbulence level of Constance II was not made available in the published references (1,2,3). However, referee of the present paper indicated in his comments that in Constance II the effect of beam lasted for 200μs after the gun turn off. We can use this information to calculate the turbulence level in Constance II. We may calculate K_γ for Constance II from equation (59). Then for $\tau_R \simeq$ 200 μs and the value of K_γ, we may calculate the required $D_{\alpha\beta}$ from equation (61). This turns out to be equal to $\simeq$ 2x10^8 cm^{-2} sec^{-1}. Then using the expression for $D_{\alpha\beta}$ i.e.

$$D_{\alpha\beta} = 2\times10^8 = \frac{\pi}{2} W_q \frac{q}{\Delta q} \frac{v_{the}^2 \omega_{ci}^3}{v_{thi}^4} \frac{\Omega^*}{\omega_{ci}} \tag{62}$$

we may calculate W_q which is equal to 6 to 7x10^{-8}. This is quite reasonable for machines like Constance II, 2XIIB. It can be seen that turbulence level in Constance II is slightly less than the level observed in 2XIIB where observed W_q was $\simeq$ 8x10^{-8}.

As is clear from equation (47), the velocity space diffusion coefficient depends on the energy in DCLC turbulence. As shown before, for the beam powers less than a certain value, the DCLC modes see a positive slopes of both the ion distribution as well as the Langmuir plasmon distribution. Hence both of them dump energy into the DCLC turbulence. This increase in the energy of DCLC turbulence, increases the velocity space diffusion and thus reduces the ion life time inside the trap. For beam powers greater than a critical value Langmuir plasmons de-energise the DCLC turbulence. This reduces the energy of DCLC turbulence and thereby improves the ion life time as observed.

And lastly, electron temperature modelling is consistent with the Constance II. For steady state equation (54) gives

$$T_e\,[\mathrm{KeV}] = 2 \times 10^5 \times n_0^{1/4} \times T_i^{1/2}\,(\mathrm{keV}) \tag{63}$$

In Constance II, T_i = 400 eV, $n_0 \simeq 10^{13}$ cm^{-3}. These parameters yield $T_e \simeq$ 20 eV which agrees well with the observed $T_e \simeq$ 10 eV. For 2XIIB Baldwin et al have already shown that this modelling is consistent with the experiment (36).

In summary we have developed a model for the saturation of DCLC waves in the presence of an electron beam. In section 3 we have shown that the presence of an electron beam modifies the linear growth rates of DCLC modes. If the beam power is less than a critical value the growth rates are enhanced while for beam power above the critical value the growth rates are reduced. In section 4 a closed set of equations are obtained to study the time evolution of the given distribution function in the presence of an electron beam and to calculate the final ion and electron temperature life time, fluctuation level etc. Further we have shown that scattering of Langmuir waves by DCLC waves is negligible as compared to the scattering caused by ions. Hence the damping induced by Langmuir plasmons persists and the saturation of DCLC turbulence occurs when the growth due to ions is balanced by the damping due to Langmuir plasmons. For beam powers greater than a critical value the diffusion in velocity spaces decreases. Hence the hot ion life time improves.

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APPENDIX A

DERIVATION OF THE LIOUVILLE EQUATION FOR PLASMONS

To derive the kinetic equation for the plasmons in terms of number of quasi-particles, we start from the system of equations

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \frac{\partial f_\alpha}{\partial \vec{x}} + \frac{e_\alpha}{m_\alpha} \vec{E}_\alpha \cdot \frac{\partial f_\alpha}{\partial \vec{v}} = 0 \tag{1}$$

$$\nabla \cdot \vec{E} = \sum_\alpha 4\pi e_\alpha \int f_\alpha \, d\vec{v} \tag{2}$$

where α denotes the species. Suppose the electric field $\vec{E}$ as well as the oscillatory part corresponding to plasma oscillations, has a part which varies slowly in space and in time

$$\vec{E}(\vec{r},t) = \vec{E}_0(\vec{r},t) - \nabla \sum_l \Phi_l(\vec{r},t)\, e^{iS_l(\vec{r},t)} \tag{3}$$

where the summation over l denotes summing over all possible initial values of the eikonal $S_l(\vec{r},0)$

A similar division is also valid for the particle distribution function

$$f_\alpha(\vec{v},\vec{r},t) = f_{0\alpha}(\vec{v},\vec{r},t) + \sum_l F_{\alpha l}(\vec{v},\vec{r},t)\, e^{iS_l(\vec{r},t)} \tag{4}$$

Let us split the initial equation into fast and slow oscillatory parts

$$\left(\frac{\partial}{\partial t} + \vec{V}\cdot\frac{\partial}{\partial \vec{r}}\right)F_{\alpha l} - i(W_l - \vec{K}_l\cdot\vec{V})F_{\alpha l} + \frac{e_\alpha}{m_\alpha}\vec{E}_0\cdot\frac{\partial F_{\alpha l}}{\partial \vec{V}}$$

$$- \frac{e_\alpha}{m_\alpha}\left(\frac{\partial \phi_l}{\partial \vec{r}} + i\vec{K}_l\phi_l\right)\cdot\frac{\partial f_{0\alpha}}{\partial \vec{V}} = 0 \qquad \text{---(5)}$$

$$\frac{\partial^2 \phi_l}{\partial r^2} + 2i\vec{K}_l\cdot\frac{\partial \phi_l}{\partial \vec{r}} + i(\nabla\cdot\vec{K}_l)\phi_l - K_l^2\phi_l$$

$$+ \sum_\alpha 4\pi e_\alpha \int F_{\alpha l}\, d\vec{V} = 0 \qquad \text{---(6)}$$

$$\frac{\partial f_{0\alpha}}{\partial t} + \vec{V}\cdot\frac{\partial f_{0\alpha}}{\partial \vec{r}} + \frac{e_\alpha}{m_\alpha}\vec{E}_0\cdot\frac{\partial f_{0\alpha}}{\partial \vec{V}} = 0$$

$$W_l = -\frac{\partial S_l}{\partial t}, \quad \vec{K}_l = \frac{\partial S_l}{\partial \vec{r}} \qquad \text{---(7)}$$

We shall assume that in equation for $F_{\alpha l}$

$$\frac{e_\alpha}{m_\alpha}\vec{E}_0\cdot\frac{\partial F_{\alpha l}}{\partial \vec{V}},\ \left(\frac{\partial}{\partial t} + \vec{V}\cdot\frac{\partial}{\partial \vec{r}}\right)F_{\alpha l} \ll (W_l - \vec{K}_l\cdot\vec{V})F_{\alpha l}$$

Then, solving the equation for $F_{\alpha l}$ to an accuracy to the first order in the parameters

$$\frac{1}{W_l}\frac{\partial}{\partial t}, \qquad \frac{1}{\vec{K}_l}\cdot\frac{\partial}{\partial \vec{r}}$$

and substituting these in the equation for the potential, we obtain, to this approximation a differential equation for ϕ_ℓ

If we define explicit expression for the quantities

$$\epsilon = 1 + \sum_\alpha \frac{4\pi e_\alpha^2}{k_\ell^2 m_\alpha} \int \frac{\vec{k}_\ell \cdot \frac{\partial f_{0\alpha}}{\partial \vec{v}} \, d\vec{v}}{(\omega_\ell - \vec{k}_\ell \cdot \vec{v})}$$

$$\frac{\partial}{\partial \vec{k}_\ell}(k_\ell^2 \epsilon) = 2k_\ell + \sum_\alpha \left\{ \frac{4\pi e_\alpha^2}{m_\alpha} \int \frac{\frac{\partial f_{0\alpha}}{\partial \vec{v}} \, d\vec{v}}{(\omega_\ell - \vec{k}_\ell \cdot \vec{v})} + \frac{4\pi e_\alpha^2}{m_\alpha} \int \frac{\vec{k}_\ell \cdot \frac{\partial f_{0\alpha}}{\partial \vec{v}}}{(\omega_\ell - \vec{k}_\ell \cdot \vec{v})^2} \vec{v} \, d\vec{v} \right\}$$

$$\frac{\partial}{\partial \omega_\ell}(k_\ell^2 \epsilon) = -\sum_\alpha \frac{4\pi e_\alpha^2}{m_\alpha} \int \frac{\vec{k}_\ell \cdot \frac{\partial f_{0\alpha}}{\partial \vec{v}} \, d\vec{v}}{(\omega_\ell - \vec{k}_\ell \cdot \vec{v})^2}$$

(8)

and similar expressions for

$$\frac{\partial}{\partial t} \cdot \frac{\partial}{\partial \omega_\ell}(k_\ell^2 \epsilon)$$

$$\nabla \cdot \frac{\partial}{\partial \vec{k}_\ell}(k_\ell^2 \epsilon)$$

then the equation for ϕ_l can be transformed into

$$k_l^2 \epsilon \phi_l = i \frac{\partial}{\partial k_l} (k_l^2 \epsilon) \cdot \frac{\partial \phi_l}{\partial \vec{r}} + \frac{i}{2} \nabla \cdot \frac{\partial}{\partial \vec{k}_l} (k_l^2 \epsilon) \cdot \phi_l$$

$$- i \frac{\partial}{\partial \omega_l} (k_l^2 \epsilon) \cdot \frac{\partial \phi_l}{\partial t} - \frac{1}{2} \frac{\partial}{\partial t} \frac{\partial}{\partial \omega_l} (k_l^2 \epsilon) \cdot \phi_l$$

$$+ \frac{i}{2} \phi_l \sum_\alpha \frac{4\pi e_\alpha^2}{m_\alpha} \int \frac{d\vec{v}}{(\omega_l - \vec{k}_l \cdot \vec{v})^2} \vec{k}_l \cdot \frac{\partial}{\partial \vec{v}} \left[\frac{\partial f_{o\alpha}}{\partial t} + \vec{v} \cdot \frac{\partial f_{o\alpha}}{\partial \vec{r}} + \frac{e_\alpha}{m_\alpha} \vec{E}_o \cdot \frac{\partial f_{o\alpha}}{\partial \vec{v}} \right] \qquad \text{---(9)}$$

and the last term in brackets equals to zero because of equation (7).

If we consider the damping of the plasma waves to be small and neglect the imaginary part of ϵ, after separating the real and imaginary parts in equation (9), we obtain the following equation for $|\phi_l|^2$

$$\frac{\partial}{\partial t} \left[\frac{\partial}{\partial \omega_l} (k_l^2 \epsilon) |\phi_l|^2 \right] - \nabla \cdot \left[\frac{\partial}{\partial \vec{k}_l} (k_l^2 \epsilon) |\phi_l|^2 \right] = 0 \qquad (10)$$

where $k_l^2 \epsilon = 0$

If we introduce the quantity

and use the identity

$$\frac{\partial}{\partial \vec{K}_\ell}(K_\ell^2 \epsilon) + \frac{\partial}{\partial \omega_\ell}(K_\ell^2 \epsilon)\cdot \frac{\partial \omega_\ell}{\partial \vec{K}_\ell} = 0$$

equation (10) can be written in the following form

$$\frac{\partial n_1}{\partial t} + \nabla \cdot \left(\frac{\partial \omega_\ell}{\partial \vec{K}_\ell} n_\ell \right) = 0 \tag{11}$$

We note that

$$\frac{\partial}{\partial t}\int d\vec{r}\, n_\ell = 0$$

Therefore n_ℓ represents the number density of particles in the six-D space $[\vec{r}, \vec{K}]$

$$n_k = \sum_\ell n_\ell\, \delta[\vec{K} - \vec{K}_\ell(\vec{r}, t)] \tag{12}$$

The equation of n_k is readily obtained by using the equation for n_ℓ deduced above and the relationship $-\partial \omega_\ell/\partial \vec{r} = \partial K_\ell/\partial \vec{r}$.
Differentiating n_k with time we have

$$\frac{\partial n_k}{\partial t} = -\sum_\ell \frac{\partial}{\partial \vec{r}} \cdot \left[\frac{\partial \omega_\ell}{\partial k_\ell} n_\ell\right] \delta(\vec{k}-\vec{k}_\ell) - \sum_\ell n_\ell \frac{\partial}{\partial \vec{k}} (\vec{k}-\vec{k}_\ell) \cdot \frac{\partial k_\ell}{\partial t}$$

$$= -\frac{\partial}{\partial \vec{r}} \cdot \sum_\ell \left(\frac{\partial \omega_\ell}{\partial k_\ell}\right)_{\vec{r}} n_\ell\, \delta(\vec{k}-\vec{k}_\ell) + \frac{\partial}{\partial \vec{k}} \cdot \sum_\ell \left(\frac{\partial \omega_\ell}{\partial \vec{r}}\right)_{\vec{k}_\ell}$$

$$n_\ell\, \delta(\vec{k}-\vec{k}_\ell) = -\frac{\partial}{\partial \vec{r}} \cdot \left[\frac{\partial \omega_k}{\partial \vec{k}} n_k\right] + \frac{\partial}{\partial \vec{k}} \cdot \left[\frac{\partial \omega_k}{\partial \vec{r}} n_k\right]$$

where $\omega_k = \omega_\ell(\vec{k}_\ell, \vec{r}, t)\big|_{k_\ell = k}$ carrying out the differentiation with respect to $\vec{r}$ and $\vec{k}$ we obtain the following equation for the plasmon number density .

$$\frac{\partial n_k}{\partial t} + \frac{\partial \omega_k}{\partial \vec{k}} \cdot \frac{\partial n_k}{\partial \vec{r}} - \frac{\partial \omega_k}{\partial \vec{r}} \cdot \frac{\partial n_k}{\partial \vec{k}} = 0 \tag{13}$$

CHAPTER IV

STABILIZATION OF CONVECTIVE MODES BY NON-LINEAR LANDAU DAMPING IN ELECTRON BEAM INJECTED MIRRORS

1. Introduction:

The instability of High Frequency Convective Loss Cone (HFCLC) has been predicted for all present-day mirror machines (1). These modes have a finite phase velocity along the field lines hence the concept of growth length in a plasma of a finite extent like that of mirror, becomes important. It has been argued that in the absence of wave-reflections at the mirror throat these modes will restrict the machine length, to a few hundred ion gyro-radii a_i. For machines longer than this critical length, these modes will grow to a significant level. The electrostatic dispersion relation for these modes in the limit of cold fluid electrons and straight line ion orbit is given by (1)

$$k_{\parallel}''^{2} + k''^{2}\left[\omega''^{2}\left\{\frac{1}{\omega_{pe}^{2}} + \frac{1}{\omega_{ce}^{2}}\right\} - \frac{m_e}{m_i} y^2 F(y)\right] = 0 \tag{1}$$

where $(\omega, k_{\parallel}'', k'')$ are the frequency, parallel wave number and full wave number of HFCLC modes. ω_{pe} and ω_{ce} are the plasma and cyclotron frequency of electrons, m_e and m_i are the masses of electrons and ions and $F(y)$ is given by

$$F(y) = -2\int_0^\infty dx\,\frac{\partial \psi}{\partial x}\,\frac{1}{(1 - x/y^2)^{1/2}} \tag{2}$$

where $x = v_\perp^2/v_{thi}^2$, $y = \frac{\omega''}{k'' v_{thi}}$, v_{thi} is the thermal velocity of ions, $\psi = C\int_{-\infty}^{\infty} f_0(v_\perp^2, v_\parallel^2)\,dv_\parallel$ $f_0(v_\perp^2, v_\parallel^2)$ is the distribution function of mirror plasma and C is to be chosen so that $\int_0^\infty \psi\,dx = 1$. From equation (1), under the assumption of the weak-reflection at the morror throat, the critical length L of the machine turns out to be

$$L = 10^4 \left(m_e/m_i\right)^{1/2} a_i \tag{3}$$

Thus the electrostatic modes impose a limit that the machine length should not be greater than 300 to 500 a_i.

The electromagnetic dispersion relation for these modes in the limit of cold fluid electrons and straight line orbits is (2)

$$+\frac{\omega_{pe}^2}{\omega_{ce}^2}\left(1+\frac{\omega_{pe}^2}{k''^2c^2}\right)-\frac{k_{\parallel}''^2\omega_{pe}^2}{k''^2\,\omega''^2}\left(1+\frac{\omega_{pe}^2}{k''^2c^2}\right)^{-1}$$

$$+\frac{\omega_{pi}^2}{k''^2}\int_0^{\infty}2v_\perp dv_\perp\frac{\partial f_0}{\partial v_\perp^2}\left(1-\frac{k_\perp''^2v_\perp^2}{\omega''^2}\right)^{-1/2}=0 \qquad (4)$$

In this equation various quantities are assumed to vary in the Z-direction (along the field lines) while the variation perpendicular to field lines are ignored. It should be noted that the cold electron approximation is valid when $\omega''/k_{\parallel}''v_{the} \gg 1$ for $\frac{\omega_{pe}}{k''c} \lesssim$ 1 or when (3) $\beta_e \ll 1$, $\omega_{pe}/k''c \gg 1$ (β_e is the ratio of electron pressure to magnetic pressure). Near the ends of the machine $\omega_{pe} \longrightarrow 0$ so $k_{\parallel}'' \longrightarrow \infty$; before this happens $\omega''/k_{\parallel}''v_{the} < 1$ and electron landau damping occurs. So we expect the wave to be completely observed when it reaches the end of the machine. Actually we require that $dk_{\parallel}''/dz \ll k''^2$ as $\omega_{pe}(z) \longrightarrow 0$ which means that $\omega_{pe}(z)$ cannot go to zero abruptly but this condition is satisfied for a collisional distribution (3,4). However, there are possibilities that there may be some wave-reflection at the mirror throat. Aamodt and Book (5) pointed out that even a gradual change in $k_{\parallel}''(z)$ with $dk_{\parallel}''/dz \ll k''^2$ can cause an exponentially small fraction of the incident wave energy to be reflected. This is a fraction of order exp($-k_{\parallel}''a$) where $a = k_{\parallel}''\left(dk_{\parallel}''/dz\right)^{-1}$. Since the wave grows exponentially large by a factor exp ($-\mathrm{Im}\,k_{\parallel}''L$), in travelling the length L of the machine the mode would become absolutely unstable if $-\mathrm{Im}\,k_{\parallel}''L > k_{\parallel}''a$ [$\mathrm{Im}\,k_{\parallel}'' < 0$ for unstable mode since $\partial\omega''/\partial k_{\parallel}'' > 0$] . For the fastest growing modes $k_\perp \simeq \omega_{pi}/v_{thi}\left(1+\omega_{pe}^2/\omega_{ce}^2\right)^{-1/2}$ and $\omega/k_\perp v_{thi} \simeq 1$ (for a mirror ratio of 2 or so), which may be shown to imply that $a \gtrsim L$ (assuming slowly varying magnetic field) and $-\mathrm{Im}\,k_{\parallel}'' < k_{\parallel}''$. Thus these modes do not exhibit absolute instability).

Berk et al (6) showed that non-local reflection mechanism can be more important than the local reflection considered by Aamodt and Book (5) when $\omega''/k_{\parallel}'' v_{the} < 1$. This non-local reflection depends on the electrons retaining memory of the original wave perturbation after they reflect from the end of the mirror machine, and the reflection co-efficient is of order $\exp(-\omega \Delta\tau)$, where $\Delta\tau$ is the range of turn around time for electrons. Typically $\Delta\tau = a/v_{the}$, so that non-local reflection is more important than local reflection just when $\omega''/k_{\parallel}'' v_{the} \lesssim 1$ but in special cases (e.g. electrons trapped in parabolic potential well) $\Delta\tau$ may be much smaller and the reflection coefficient much greater. In any case the reflection is never much less than $\exp(-k_{\parallel}'' a)$. For a typical mirror machine, it was estimated that the non-local reflection might cause absolute instability when the machine was longer than about half the critical length calculated in the absence of wave reflection. Electromagnetic effects become important when $\omega_{pe}/k'' c < 1$ (2). Recent calculations (7) give a critical length of about 50 a_i. At low frequencies $\omega'' \simeq \omega_{ci}$, the straight line orbit approximation breaks down and reflection occurs near those values of Z where ω'' is an integral multiple of $\omega_{ci}(Z)$. Thus an absolute instability, called the negative energy instability, can occur at low frequencies if the mirror machine is longer than a critical length (8).

Very recently a calculation by Gerver (9) has shown that HFCLC modes are absolutely unstable which impose a critical length of about a few ion gyro-radii a_i (9). In 2XIIB these modes were not observed and it was conjectured that they were convectively stabilized by warm plasma streams.

Very recently in Constance II, electron beams were injected in mirror machines parallel to field lines to control Drift Cyclotron Loss Cone (DCLC) turbulence (10, 11, 12, 13). The passage of the electron beam through mirror plasma gives rise to Langmuir waves (LW) as has been observed in Constance II. Thus it becomes important to investigate the properties of HFCLC modes in the presence of these electron stream generated langmuir waves. In this Chapter, we investigate the effect of LW on the high frequency ($\omega \simeq \omega''$, where ω is frequency of LW) part of HFCLC spectrum. In the following Chapter, we will investigate the effect of LW on low frequency i.e. $\omega > \omega''$ part of HFCLC spectrum.

2. Anomalous Resistivity due to Scattering by Beam Electrons:

The injection of electron streams (ES) along the field lines produces Langmuir waves (LW) with frequency $\omega \simeq \omega_{pe} k_{\parallel}/k_{\perp}$ (where $k_{\parallel}$ and $k_{\perp}$ are the parallel and perpendicular wave numbers of LW). In Constance II (10) oscillations at 30 GHz were observed which correspond to Langmuir wave frequency for Constance II plasma. In the same experiment the electron stream velocity $V_s \sim 10^9$ cm/sec (corresponding to beam power from 30 - 50 KW) and the plasma electron temperature T_e was about $\sim$ 10 eV, so that $\sqrt{T_e/m_i} = U_{the} \simeq 10^8$ cm/sec $< V_s \simeq 10^9$ cm/sec. This implies that instability excited is of gentle bump type $\omega/k_{\parallel} < V_s$ (14) where positive energy Langmuir waves are driven unstable by negative dissipation of stream electrons (15). In the presence of these oscillations the HFCLC modes (ω'', k'') are strongly coupled by stream electrons which satisfy the resonance condition

$$\omega - \omega'' \simeq (k_{\parallel} - k''_{\parallel}) V_s \tag{5}$$

We have shown here that scattering of electrons by Langmuir waves according to equation (5) produces enough anomalous resistivity to stabilize the HFCLC modes.

To study this coupling we consider a slab mirror geometry with parallel magnetic field lines in Z-direction. This is justified as we are concerned in regions around the mid-plane where lines are nearly parallel. In this geometry we consider Langmuir waves $[\omega, k]$ and HFCLC waves $[\omega'', k'']$ having phase velocities $\omega/k_{\parallel}$ and $\omega''/k''_{\parallel}$ in Z-direction. The stream-electrons are also streaming with an average velocity V_s, in Z-direction. It is well known that in real situations, the resonance conditions for non-resonant interactions, like the nonlinear Landau damping etc. (equation (5)) are more easily satisfied than those for resonant interactions like parametric decay etc. Hence non-resonant interactions are important in the nonlinear evolution of a system.

Accordingly, to study the coupling given by equation (4), we use the kinetic wave equations obtained by Porkolab and Change (16)

$$\frac{\partial N_k}{\partial t} = \omega_i N_k + \sum_{k''} S_k L_{kk''} N_{k''} N_k \tag{6}$$

$$\frac{\partial N_{k''}}{\partial t} = \omega''_i N_{k''} - \sum_{k} S_{k''} L_{kk''} N_k N_{k''} \tag{7}$$

where

$$L_{kk''} = \frac{4\pi\,\omega_{pe}^4}{\left|\frac{\partial \epsilon}{\partial \omega}\right|\left|\frac{\partial \epsilon''}{\partial \omega''}\right| k^2 k''^2 m_e n_s} \int_{-\infty}^{\infty} d v_{\parallel}\, k'_{\parallel}(\omega - k'_{\parallel} v_{\parallel}) \cdot \frac{\partial F_s}{\partial v_{\parallel}} \left| \frac{k_{\parallel} k''_{\parallel}}{(\omega - k_{\parallel} v_{\parallel})^2} \right|^2 \qquad (8)$$

$$N_k = \frac{k^2}{8\pi} |\phi_k|^2 \frac{\partial \epsilon}{\partial \omega}, \quad N_{k''} = \frac{k''^2}{8\pi} |\phi_{k''}|^2 \frac{\partial \epsilon''}{\partial \omega''}$$

$$S_k = \mathrm{sgn}\left(\frac{\partial \epsilon}{\partial \omega}\right) \qquad S_{k''} = \mathrm{sgn}\left(\frac{\partial \epsilon''}{\partial \omega''}\right)$$

$$\omega' = \omega - \omega'' \qquad k'_{\parallel} = k_{\parallel} - k''_{\parallel}$$

ω_i' and ω_i'' are the linear growth rates of LW and HFCLC waves respectively, ϵ and ϵ'' are the dielectric constants of LW and HFCLC modes, ϕ_k and $\phi_{k''}$ are their fluctuation amplitudes. F_s is the distribution function of stream electrons which we take to be

$$F_s = \frac{1}{\sqrt{\pi}\, v_{ths}} e^{-(v_{\parallel} - V_s)^2 / 2 v_{ths}^2} \qquad (9)$$

where v_{ths} is the thermal spread, V_s is the average streaming velocity and n_s is the density of streaming electrons. Following points should be noted about equation (6) to equation (9). Firstly that in the expression for $L_{kk''}$, we have neglected the term representing the scattering from the dressed particles. This is because it is proportional to $[\epsilon'(\omega',k')]^{-1}$ i.e. the dielectric constant of the beat wave (16). Since the beat-wave in this coupling is heavily damped by streaming electrons $\epsilon'(\omega',k') >> 0$ hence we can neglect this term. In equation (8). Secondly since the beat-wave is damped on streaming electrons, the coupling coefficient $L_{kk''}$ given by equation (8) depends only on the distribution of streaming electron given by equation (9). And thirdly since electrons are tightly bound to the field lines and to a first approximation they behave as if $B_o \to \infty$ (B_o is the strength of the magnetic field). This implies $k^2 a_e^2 << 1$ (a_e is the gyro-radius of electrons). Hence we use the coupling equation (6) and (7) in the absence of the magnetic field.

The HFCLC waves are positive energy waves hence $S_{k''} > 0$ (17). Further we may drop the second term as compared to the first on the R.H.S. of equation (6) under the $|L_{kk''} N_{k''}| < \omega_i$. This is justified as we will show later that the Langmuir waves keep HFCLC mode heavily damped in which case this approximation is justified. Hence the coupling equations after performing the velocity integration in $L_{kk''}$ using equation (9) becomes

$$\frac{\partial N_k}{\partial t} = \omega_i N_k \qquad (10)$$

$$\frac{\partial N_{k''}}{\partial t} = \left(\omega_i'' - \sum_k L_{kk''} N_k\right) N_{k''} \qquad (11)$$

where now $L_{kk''}$ is given by

$$L_{kk''} = -\frac{4\pi\,\omega_{pe}^4}{\left|\frac{\partial\epsilon}{\partial\omega}\right|\left|\frac{\partial\epsilon''}{\partial\omega''}\right| k^2 k''^2 m_e n_s U_{ths}} \times \frac{1}{U_{ths}^2}$$

$$\times\left(\frac{\omega'}{k_{\parallel}'} - V_s\right)\frac{k_{\parallel}^2}{k_{\parallel}''^2} \times \frac{1}{\left(\omega/k_{\parallel} - \omega'/k_{\parallel}'\right)^4}$$

(12)

Let us now find the sign of $L_{kk''}$. Since $\omega > \omega' = \omega - \omega''$ and we take $k_{\parallel} \sim k_{\parallel}'' \sim k_{\parallel}'$, we have $\omega/k_{\parallel} > \omega'/k_{\parallel}'$ and as $V_s > \omega/k_{\parallel}$ for gentle-bump instability, we have $V_s > \omega'/k_{\parallel}'$ which makes $L_{kk''} > 0$. Thus the Langmuir modes keep HFCLC modes nonlinearly damped through the second term in equation (11).

We will now estimate the nonlinear growth rate given by $\sum_k L_{kk''} N_k$ and show that even for modest level of turbulence allowed in weak turbulence theories, it is greater than or is of the same order as the linear growth ω_i''. From the linear theory of HFCLC mode (1) $\omega_i'' \lesssim \omega_{pi}$. Using $\omega/k_{\parallel} > \omega'/k_{\parallel}'$ and $\omega'/k_{\parallel}' < V_s$ $k_{\parallel} \simeq k_{\parallel}''$ etc., we have

$$\sum_k L_{kk''} N_k \simeq \frac{4\pi \sum_k N_k \omega_{pe}}{n_s m_e U_{ths}^2} \frac{\omega_{pe}^3}{\left|\frac{\partial\epsilon}{\partial\omega}\right|\left|\frac{\partial\epsilon''}{\partial\omega''}\right|} \frac{V_s}{U_{ths}}$$

$$\times\left(\frac{k_{\parallel}}{\omega}\right)^4 \qquad --- (13)$$

Further using $\omega/k_{\parallel} \simeq \frac{\omega_{pe}}{k}$, $\partial\epsilon/\partial\omega \sim \frac{1}{\omega_{pe}}$, $\frac{\partial\epsilon''}{\partial\omega''} \sim \frac{1}{\omega_{pi}}$ etc
we have

$$\sum_{k} L_{kk''} N_k \simeq 4\pi W \frac{V_s}{U_{ths}} \omega_{pi} \tag{14}$$

where $W = \sum_{k} N_k \omega_{pe} / n_s m_e U_{ths}^2$ i.e. the ratio of energy in Langmuir waves to the thermal energy of particles. Now typically $U_{ths} = V_s (n_s/n_p)^{1/3}$ where n_p is the main plasma density. In Constance II $n_s \simeq 10^{10}$ cm^{-3}, $n_p \simeq 10^{13}$ cm^{-3}, so that $(n_p/n_s)^{1/3} \simeq V_s/U_{ths} \simeq 10$. Thus even for a modest level of Langmuir turbulence i.e. $W \simeq \sqrt{m_e/m_i}\ (\sim 1/40)$ the nonlinear growth rate $\sum_{k} L_{kk''} N_k \simeq \pi\, \omega_{pi} > \omega_{pi} \gtrsim \omega_i''$

3. <u>Discussion:</u>

Thus we see that the scattering of electrons from Langmuir waves produces enough anomalous resistivity to damp the HFCLC modes in the part of the spectrum i.e. $\omega'' \sim \omega$, $k'' \sim k$.

In the end we note that this mechanism of stabilization of HFCLC modes is independent of conditions at the mirror throat i.e. the presence or the absence of wave reflection at the throat, because the time scale of nonlinear interaction $\left|\sum_{k} L_{kk''} N_k\right|^{-1}$ is much faster than the rate of convection. That is, the HFCLC waves are damped on a time scale which is much smaller than the time scale on which they travel from any point on the axis to the throat. The typical time scale of nonlinear damping of HFCLC waves is $\tau_{NL} \simeq \left|\sum L_{kk''} N_k\right|^{-1} \sim \omega_{pi}^{-1} \simeq 10^{-9}$ sec for Constance II. While the transit time scale of HFCLC is $\tau_T \simeq L k_{\parallel}''/\omega''$

where L is the length of the machine (L $\simeq$ 100 cm for Constance II). Typically, for HFCLC modes $\frac{\omega''}{k''_{\parallel}} \simeq \frac{\omega_{pe}}{k''} \sim \frac{\omega_{pe}}{\omega_{pi}} v_{thi} \sim v_{the}$ where v_{the} and v_{thi} are the thermal velocity of main plasma electrons and ions. In Constance II, $v_{the} \simeq 10^8$ cm/sec, which yields $\tau_T = \frac{L k''_{\parallel}}{\omega''} \simeq \frac{L}{v_{the}} \simeq 10^{-6}$ sec. Hence $\tau_{NL} << \tau_T$.

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CHAPTER V

CRITICAL LENGTHS IN ELECTRON BEAM INJECTED MIRROR MACHINES

1. Introduction:

In the preceeding chapter we had studied the interaction between HFCLC waves and electron beam induced Langmuir waves in the regime $\omega'' (\simeq \omega_{pe} k_{\parallel}''/k'') \simeq \omega (\simeq \omega_{pe} k_{\parallel}/k)$, $k_{\parallel}'' = k_{\parallel}$ etc. It was shown there that scattering of beam electrons from Langmuir waves produces enough anomalous resistivity to stabilize the HFCLC modes. The coupling was envisaged according to the equation

$$\frac{\omega - \omega''}{k_{\parallel} - k_{\parallel}''} = V_{\parallel} \tag{a}$$

In this chapter we will study the interaction of HFCLC waves and Langmuir waves (electron beam induced) in the regime $\omega \sim \omega_{pe} k_{\parallel}/k \sim \omega_{pe} \gg \omega'' \sim \omega_{pe} \frac{k_{\parallel}''}{k''} \sim \omega_{pi}$ $[k_{\parallel} \sim k,\ k_{\parallel}'' \ll k'']$ and $k_{\parallel} > k_{\parallel}''$. This regime is appropriate to those Langmuir waves which travel almost along the field lines [so that - $k_{\parallel} \sim k$] . Clearly then equation (a) for this regime becomes

$$\frac{\omega}{k_{\parallel}} = v_{\parallel}$$

which simply expresses the resonance of Langmuir waves with electrons i.e. the HFCLC get decoupled from the Langmuir waves. Thus we see that in this regime the coupling between HFCLC waves and Langmuir waves by non-linear landau damping becomes weak. However, as we show in this Chapter the Langmuir waves and HFCLC waves can still interact in this regime. The appropriate technique to study the interaction in this regime is the 'adiabatic approximation' which considers interaction between waves of widely different properties i.e. $\omega >> \omega''$ etc.

In Chapter IV it was shown that in the interactions considered in the regime $\omega \simeq \omega''$, the spectral features of Langmuir waves do not play any role. However, in the regime considered here the spectral features of Langmuir waves play an important role. The magnitude of the effect of Langmuir waves on HFCLC waves varies in its course of non-linear evolution. Thus, while studying the interaction it becomes important to take into consideration the non-linear evolution of Langmuir waves.

The non-linear evolution of Langmuir turbulence has already been discussed in general in Chapter III. However, for the sake of continuity we briefly discuss here some of its features relevant to the present problem. Accordingly, the general scenario is as follows: If the injected beam is warm enough i.e. $\Delta V_b/V_b \gtrsim (n_b/n_p)^{1/3}$

(V_b , n_b are the beam velocity and density, n_p is the plasma density), a narrow spectrum of wave-number spread $K\lambda_D = \frac{\Delta V_b}{V_b}\frac{U_{the}}{V_b}$ (where U_{the} is thermal velocity of electrons) centred around $K_o = \omega_{pe}/V_b$ is generated. The quasi-linear theory fails to explain the saturation of this unstable spectrum (1,2). It is explained when strong turbulence effects are taken into account. Accordingly it has been shown by a number of authors (1-5) that, when amplitude becomes large enough, various non-linear processes like parametric decay, oscillating two stream instability etc. can cause scattering in K-space. Computer simulation (6,7) have shown that these processes lead to the formation of 'spiky turbulence', where there are localised structures of intense electric fields ($W = \langle E^2\rangle/8\pi n_o kT_e \simeq 1.5$, $\langle E^2/8\pi\rangle$ is the wave energy density). It has also been shown numerically by solving appropriate kinetic-wave equations, that these processes lead to a significant broadening of the plasmon spectrum ($\Delta \sim K_o$ where Δ is the width of plasmon spectrum), which can be approximated by a Gaussian centred around K_o (8). The non-linear evolution of Langmuir turbulence proceeds on the time scale of a few tens of $1000\,\omega_{pe}^{-1}$ i.e. a few μ s for typical experimental situations. Now, in experiments employing parallel injection of electron beam, the electron gun and the plasma gun are fired simultaneously. It takes a few μ s ($\simeq$ a few ion transit time) for the ion distribution to shape itself to develop the hole, so that the loss cone instabilities are triggered a few μ s after the firing of the plasma gun. Hence in such experiments we expect a broad and energetic ($\Delta \simeq K_o$, $W \lesssim 1$) spectrum of Langmuir waves to come into existence by the time HFCLC waves are excited. We wish to inveestigate the effect of this spectrum on HFCLC modes. Later, we will investigate the experimental situation in which narrow spectrum of Langmuir turbulence interacts with HFCLC waves.

2. The effect of Langmuir Spectrum on HFCLC Waves:

In our theoretical model we consider a low- β mirror plasma with the Z-axis along the mirror axis. In this machine we consider a HFCLC mode (ω'' , $\vec{k}''$) which is mostly in Y-direction i.e. $k_\perp'' > k_\parallel''$ with frequency in the vicinity of $\omega'' \simeq \omega_{pe} k_\parallel''/k_\perp'' \simeq \omega_{pi} \simeq k_\perp'' v_{thi}$ (where ω_{pi} is the ion plasma frequency and v_{thi} is the ion thermal velocity). In the same geometry, we consider a packet of Langmuir waves (ω, k) in Y-Z plane, mostly along Z-direction i.e. $k_\parallel \approx k$. Let the packet be inclined at a small angle θ from the field lines and centred around $k_0 = \omega_{pe}/v_b$

The evolution of the Langmuir plasmon distribution function will be studied by a wave-kinetic equation developed by Vedenov et al (9).

$$\frac{dN_k}{dt} = \frac{\partial N_k}{\partial t} + \vec{V_g}\cdot\nabla N_k - \frac{\partial \omega_k}{\partial \vec{r}}\cdot\frac{\partial N_k}{\partial \vec{k}} = 0 \tag{1}$$

where $\vec{V_g} = \partial\omega/\partial\vec{k}$ is the group velocity of the Langmuir waves. In brief the effect of Langmuir turbulence arises as follows i.e. the low frequency perturbation creates a perturbation in the plasmon density. The gradient of this plasmon density gives rise to a ponderomotive force (P.F.) which reacts back on the low frequency waves (HFCLC) to modify its characteristics. The P.F. on ions is m_e/m_i times smaller than that on electrons. hence dropped.

The plasma distribution function is perturbed as follows:

$$N_k = N_{k0} + n_k \tag{2}$$

where N_{k0} is the equilibrium distribution function normalised as

$$\int N_{k0} d\vec{k} = 1 \tag{3}$$

The space and time dependence of is given by

$$n_k = \tilde{n}_k \, e^{i(\vec{k}'' \cdot \vec{x} - \omega'' t)} \tag{4}$$

where (ω'', k'') is the low frequency mode. From equations (1), (2) and (3), we have

$$\tilde{n}_k = \frac{\dfrac{\partial \omega_k}{\partial \vec{r}} \cdot \dfrac{\partial N_{k0}}{\partial \vec{k}}}{i(\vec{k}'' \cdot \vec{V}_g - \omega'')} \tag{5}$$

The dependence of $\tilde{n}_k$ on the low frequency density perturbation comes from the term $\partial\omega_k/\partial\vec{r}$. For plasma waves in the regime $\omega_{pe}/\omega_{ce} > 1$

$$\omega_k^2 = \omega_{pe}^2 + k^2 v_{the}^2 \tag{6}$$

which gives

$$\frac{\partial \omega_k}{\partial \vec{r}} = \frac{i}{2} k'' \omega_{pe} \frac{\tilde{n}_{e1}}{n_0} \tag{7}$$

where $\tilde{n}_{e1}$ is the low frequency density perturbation. Following Post and Rosenbluth we shall use the straight line orbit approximation (10) i.e. on the time scale of the growth of this instability $[\sim \omega_{pi}^{-1}]$ the ion motion is taken to be rectilinear and mainly perpendicular to the field lines while electron motion is mainly along the field lines. Hence we shall include the effect of ponderomotive force only in the electron equation for the motion parallel to the magnetic field, which thus becomes

$$m_e \frac{\partial v_z}{\partial t} = -eE_z - m_e v_z \frac{\partial v_z}{\partial z} \tag{8}$$

In equation (8), we write

$$v_z = \tilde{v}_{zs} + \tilde{v}_{zf}, \quad E_z = \tilde{E}_{zs} + \tilde{E}_{zf} \tag{9}$$

where $\tilde{v}_{zs}$ and $\tilde{E}_{zs}$ are parallel velocity and electric field sustained by HFCLC (identified as the low frequency perturbation as compared to the Langmuir field frequency) while $\tilde{v}_{zf}$ and $\tilde{E}_{zf}$ are those sustained by Langmuir waves identified as the high frequency field. Averaging equation (8) over an ensemble of random set of Langmuir waves

$$m_e \frac{\partial \tilde{v}_{zs}}{\partial t} = -e\tilde{E}_{zs} - m_e \sum_{\vec{k}} \frac{1}{2} \frac{\partial |v_{zf\vec{k}}|^2}{\partial z} \tag{10}$$

where $\tilde{v}_{zf}$ may be evaluated from the following equation

$$m_e \frac{d\tilde{v}_{zf}}{dt} = -e\tilde{E}_{zf} \tag{11}$$

From equation (11) $\tilde{v}_{zf}$ may be evaluated and substituted in equation (10) which becomes

$$m_e \frac{\partial \tilde{v}_{zs}}{\partial t} = -e\tilde{E}_{zs} - \sum_{\vec{k}} \frac{e^2}{2m_e\omega_{\vec{k}}^2} \frac{\partial |\tilde{E}_{zfk}|^2}{\partial z} \tag{12}$$

where $\tilde{E}_{zf}$, the z-component of the Langmuir field is assumed to oscillate with frequencies $\omega_{\vec{k}}$, $E_{zf} \sim e^{-i\omega_{\vec{k}} t} E_{zfk}$

If $\vec{E}_f$ is the Langmuir field amplitude in a direction θ with respect to Z-axis, then

$$E^2_{zf\vec{k}} = E^2_{fk} \cos^2\theta , \quad \langle E_{fk} \rangle^2 / 8\pi = N_{\vec{k}} N_0 \omega$$

where N_0 is the average plasmon density. Using these relations in equation (12), we have:

$$m_e \frac{d\tilde{v}_{zs}}{dt} = -eE_{zs} - \sum_{\vec{k}} \frac{4\pi e^2}{m_e} \frac{1}{\omega} \cos^2\theta \, N_0 \frac{\partial \tilde{n}_k}{\partial z} \tag{13}$$

Now from equation (5), we have $\tilde{n}_k$ as

$$\tilde{n}_{\vec{k}} = \tfrac{1}{2} \vec{k}'' \, \tilde{n}_{e1}/n_0 \cdot \frac{\partial N_{k0}}{\partial \vec{k}} \Big/ i(\vec{k}'' \cdot \vec{V}_g - \omega'') \tag{14}$$

Using this expression for $\tilde{n}_k$ in equation (13), we obtain:

$$m_e \frac{d\tilde{v}_{zs}}{dt} = -e\tilde{E}_{zs} - \sum_{\vec{k}} \frac{4\pi e^2}{m_e} \frac{\cos^2\theta \, N_0}{n_0} \frac{\frac{1}{2} \vec{k}'' \cdot \frac{\partial N_{\vec{k}0}}{\partial \vec{k}}}{i(\vec{k}'' \cdot \vec{V}_g - \omega'')} \times \frac{\partial \tilde{n}_{e1}}{\partial z} \tag{15}$$

which gives

$$m_e \frac{d\tilde{v}_{zs}}{dt} = -e\vec{E}_{zs} - \sum_{k} \frac{4\pi e^2}{2m_e n_0} \cos^2\theta \, N_0 \frac{\vec{k}'' \cdot \frac{\partial N_{k0}}{\partial \vec{k}} \, i k''_z \, \tilde{n}_{e1}}{(\vec{k}'' \cdot \vec{V}_g - \omega'')} \tag{16}$$

From this equation we can calculate $\tilde{V}_{zs}$ which is given as (after using $\tilde{E}_{zs} = -i k''_z \tilde{\Phi}_1$)

$$\tilde{V}_{zs} = -\frac{e}{m_e} \frac{k''_z \Phi_1}{\omega''} + \sum_{\vec{k}} \frac{4\pi e^2}{2m_e n_0} \frac{\cos^2\theta \, N_0}{\omega'' \, m_e^2} k''_z \frac{\vec{k}'' \cdot \frac{\partial N_{k0}}{\partial \vec{k}}}{(\vec{k}'' \cdot \vec{V}_g - \omega'')} \tilde{n}_{e1} \tag{17}$$

Next, the equations for the perpendicular motion of the electrons give (taking the perturbation in the perpendicular motion to be on the slow time scale of the HFCLC modes)

$$-i\omega'' m_e \tilde{v}_{xs} = -i\, U_{ys} B_0 \qquad \text{---(18)}$$

$$-i\omega'' m_e \tilde{U}_{ys} = i e k''_y \tilde{\Phi}_1 + e \tilde{U}_{xs} B_0 \qquad \text{---(19)}$$

From these equations we have

$$\tilde{U}_{ys} = \frac{-e k''_z \tilde{\Phi}_1 / m_e \omega''}{\left[1 - \omega_{ce}^2 / \omega''^2 \right]} \qquad (20)$$

Also from the equation of continuity on the slow time scale of HFCLC mode we have

$$\frac{\partial n_{e1}}{\partial t} = -i n_0 k''_y \tilde{U}_{ys} - i n_0 k''_z \tilde{U}_{zs} \qquad (21)$$

Substituting for $\tilde{U}_{ys}$ and $\tilde{U}_{zs}$ from equation (20) and equation (17), we have the electron density perturbation as

$$\tilde{n}_{e1} = \left[\frac{-\dfrac{n_0 k_y''^2 \phi_1}{m_e \omega''^2}}{[1 - \omega_{ce}^2/\omega''^2]} - \frac{e}{m_e}\frac{k_z''^2}{\omega''^2}\phi_1 n_0 \right]$$

$$\times \left[1 - \frac{k_z''^2}{\omega''^2} n_0 A \int \frac{\cos^2\theta\, \vec{k}'' \cdot \dfrac{\partial N_{k0}}{\partial \vec{k}}\, d\vec{k}}{(\vec{k}'' \cdot \vec{V}_g - \omega'')} \right]^{-1}$$

(22)

where $A = 4\pi e^2/2 m_e^2 n_0$. The summation over K-plane has been replaced by integration which has to be done according to Landau's prescription. The ion density perturbation is unaffected by Langmuir waves and hence can be calculated in the manner shown by Post et al (10). Using electron density perturbation and ion density perturbation in the Poisson's equation we get the modified dispersion relation for the HFCLC modes as

$$1 = \frac{\dfrac{\omega_{pe}^2 k_z''^2}{\omega''^2} - \dfrac{\omega_{pe}^2}{\omega_{ce}^2}}{\left[1 + W\left(\dfrac{\omega^*}{\omega''}\right)^2 \left(\dfrac{k''}{\Delta}\right)^2 \left\{1 + \xi_\xi Z(\xi_\xi)\right\}\right]}$$

$$+ \frac{\omega_{pi}^2}{k''^2} \int \vec{k}'' \cdot \vec{V}_\perp \frac{\partial F_i}{\partial \vec{v}} d\vec{v} \Big/ (\omega_r'' - \vec{k}'' \cdot \vec{v} + i\omega_i'') \quad (23)$$

where in the electron term, the integration in the K-plane has been performed by modelling $N_{k0} = \frac{1}{\sqrt{2\pi}\,\Delta} \exp\left[-(\vec{k} - \vec{k}_0)^2/2\Delta^2\right]$, where Δ is the width of the spectrum. Thus $Z(\xi_\xi)$ is the plasma dispersion function with $\vec{V}_0 = \vec{k}_0 \lambda_{De} v_{the}$, $\xi_\xi = \frac{\vec{k}_0 \cdot \vec{k}''}{\Delta k''}\left(\frac{\omega''}{\vec{k}'' \cdot \vec{V}_0} - 1\right)$

$$W = N_0 \omega_{pe} / n_0 k T_e \; , \quad \omega^{*2} = \omega_{pe}^2 k_z''^2 / k_y''^2 \; (k_\perp'' = k_y'')$$

$(k_z'' = k_\parallel'')$ etc. In writing down the electron we have also used the approximation

$\omega_{ce}^2 >> \omega''^2$ which gives $\omega_{pe}^2 / (\omega''^2 - \omega_{ce}^2) \approx - \omega_{pe}^2 / \omega_{ce}^2$

In the ion term under small growth rate $[\omega_i'']$ approximation, the principle value and the pole part can be separated and the ϕ integration (ϕ is the angle between $\vec{k''}$ and $\vec{V_\perp}$) in the pole part can be performed to give equation (23) as (11)

$$1 = \frac{\dfrac{\omega_{pe}^2 k_z''^2}{\omega''^2 k_y''^2} - \dfrac{\omega_{pe}^2}{\omega_{ce}^2}}{\left[1 + W \cdot \left(\dfrac{\omega^*}{\omega''}\right)^2 \left(\dfrac{k''}{A}\right)^2 \left\{1 + \xi_g Z(\xi_g)\right\}\right]} - \frac{\omega_{pi}^2}{k''^2 U_{thi}^2} Z_2(y)$$

$$+ i \frac{\omega_{pi}^2}{k''^2 U_{thi}^2} Z_1(y) \qquad \text{----} (24)$$

where $y = \omega'' / k'' U_{thi} \qquad x = V_\perp^2 / U_{thi}^2$

$$Z_1(y) = [\operatorname{sgn} \omega''] \int_y^\infty \frac{\partial g(x)/\partial x}{(x/y^2 - 1)} dx$$

$$Z_2(y) = \int_0^y \frac{\partial g(x)/\partial x \, dx}{[1 - x/y^2]^{1/2}} \qquad g(x) = 2\pi U_{thi}^2 \int_0^\infty F_i(U_\perp^2, U_\parallel^2) dU_\parallel$$

$$\int_0^\infty dx \, g(x) = 2$$

F_i is the loss cone ion distribution (i.e. $F_i(v_\perp = 0) = 0$). Here the resonance condition is $x > y^2$ or $V_\perp > \omega''/k_y''$, in which case only Z_1 term is important and Z_2 may be neglected. $\cos^2\theta$ has been put $\simeq 1$. In the absence of Langmuir turbulence W = 0, equation (24) reduces that obtained by Post and Rosenbluth. At the ends $\omega_{pe} \to 0$ and hence $|k_{\parallel}''| \to \infty$ but before this, the condition for electron Landau damping i.e. $\omega''/k_{\parallel}'' v_{the} < 1$ is satisfied. Hence we assume in this analysis that waves are effectively absorbed at the (10). $|F(y)|$ for collisional equilibrium distribution function is $\lesssim 1$ (10). In such cases the critical length can be evaluated by assuming real ω'', $\omega''/k'' v_{thi} \lesssim 1$ and solving for $\mathrm{Im}\, k_{\parallel}''$ by making binomial expansion in equation (24) which gives [$k_{\parallel}''$ is same as k_z'' and k_y'' is same as $k_\perp''$]

$$\mathrm{Im}\, k_{\parallel}'' = -\frac{1}{2}\left(\frac{m_e}{m_i}\right)^{1/2} \cdot y Z_1(y) \frac{\omega_{pi}}{v_{thi}}\left(\frac{\omega_{pe}}{\omega_{ce}}\right)^{-1} \qquad (25)$$

For growth $y Z_1(y) > 0$. We assume that the critical length is about 10 times the growth length calculated from equation (25). Hence the critical length comes out to be

$$L \simeq 20\left(\frac{m_i}{m_e}\right)^{1/2} \cdot \frac{\frac{v_{thi}}{\omega_{pi}}\left(\frac{\omega_{pe}}{\omega_{ce}}\right)}{|y Z_1(y)|} \qquad (26)$$

where $\omega_{pe}^2/\omega_{ce}^2 > 1$ has been used which is the typical density regime of present day machines. It should be noted that $Z_1(y)$ is a smooth function of Y and acquires positive and negative value with maximum value of order unity on both sides. We will now evaluate the critical length in the presence of the electron beam.

3. Electron Gun and Plasma Gun Fired Simultaneously:

In this case as stated earlier the HFCLC waves encounter a broad spectrum of Langmuir waves, hence we may use $\xi_\ell \simeq \frac{\sin\theta\, k_0}{\Delta}$, $\left(\frac{\omega''}{\vec{k}''\cdot\vec{V}_0}-1\right) << 1$ so that $Z(\xi_\ell)$ can be approximated as $Z = i\sqrt{\pi}$

In this limit the Landau resonance between the wave and the Langmuir plasmons becomes significant which brings about substantial changes in the temporal and spatial growth of HFCLC modes. We further assume $\omega'' \simeq \omega^* \simeq \frac{\omega_{pe} k_{\shortparallel}''}{k''}$ and $W\left(k''/\Delta\right)^2 >> 1$ (typically as will be shown later). Using this we may write $k_{\shortparallel}''^2$ from equation (24) as

$$k_{\shortparallel}''^2 = \frac{\omega''^2 k''^2}{\omega_{pe}^2}\Big[1 + a + T + i\sqrt{\pi}\, T\, \xi_\ell - i\frac{\omega_{pi}^2}{k''^2 v_{thi}^2} Z_1$$

$$- iT\frac{\omega_{pi}^2}{k''^2 v_{thi}^2} Z_1 + \sqrt{\pi}\, T\, \xi_\ell \frac{\omega_{pi}^2}{k''^2 v_{thi}^2} Z_1\, \xi_\ell\Big] \tag{27}$$

where we have put $a = \omega_{pe}^2/\omega_{ce}^2$ and $T = W\left(k''/\Delta\right)^2$

In the bracket we may neglect unity in comparison with (a+T) and take (a+T) and evaluate the square root to give $k_{\shortparallel}''$ as

$$k_{\shortparallel}'' = \frac{\omega'' k''}{\omega_{pe}}(a+T)^{1/2}\Big[1 + i\sqrt{\pi}\frac{T}{(a+T)}\xi_\ell - i\frac{\omega_{pi}^2}{k''^2 v_{thi}^2} Z_1 \frac{(1+T)}{(a+T)}$$

$$+ \sqrt{\pi}\frac{T}{(a+T)}\frac{\omega_{pi}^2}{k''^2 v_{thi}^2} Z_1 \xi_\ell\Big]^{1/2} \tag{28}$$

Since a,T >> 1, $\epsilon_g << 1$ $\omega_{pi}^2/k''^2 v_{thi}^2 \leq 1$, $Z_1 \leq 1$ etc. the three terms other than unity in bracket are less than unity in which case we may make a binomial expansion to get the $\text{Im}\, k_{\parallel}''$ as

$$\text{Im}\, k_{\parallel}'' = \frac{\omega'' k''}{2\omega_{pe}} (a+T)^{1/2} \left[\frac{\sqrt{\pi}\, T \epsilon_g}{(a+T)} - \frac{\omega_{pi}^2}{k''^2 v_{thi}^2} \frac{Z_1(1+T)}{(a+T)} \right] \tag{29}$$

The first term in equation (29) is smaller than the second on account of ϵ_g hence we neglect it to get $\text{Im}\, k_{\parallel}''$ as

$$\text{Im}\, k_{\parallel}'' = -\frac{1}{2} \frac{\omega'' k''}{\omega_{pe}} (a+T)^{1/2} \frac{\omega_{pi}^2}{k''^2 v_{thi}^2} \frac{Z_1(1+T)}{(a+T)} \tag{30}$$

which can be rearranged as

$$\text{Im}\, k_{\parallel}'' = -\frac{1}{2} \left(m_e/m_i\right)^{1/2} \frac{y Z_1(y)}{\frac{v_{thi}}{\omega_{pi}}} \frac{\sqrt{a}\, T}{\frac{\omega_{pe}}{\omega_{ce}}(a+T)^{1/2}} \tag{31}$$

Thus all those HFCLC modes which have their perpendicular phase velocities $\left[y = \frac{\omega''}{k'' v_{thi}} \right]$ such that $y Z_1(y)$ is > 0 will grow spatially with a growth length given by equation (31). The modified critical length for a mirror machine where these modes are excited is accordingly given by

$$L_m \simeq \frac{20 \left(m_i/m_e\right)^{1/2} \frac{1}{|y Z_1(y)|} \frac{v_{thi}}{\omega_{pi}} \left(\frac{\omega_{pe}}{\omega_{ce}}\right)}{\sqrt{a}\, T/(a+T)^{1/2}} \tag{32}$$

Using equation 26) we may write equation (32) as

$$L_m = \frac{L/\sqrt{aT}}{(a+T)^{1/2}} \tag{33}$$

where L is the critical length in the absence of the beam. Since $\sqrt{aT}/(a+T)^{1/2} > 1$ we see that the critical length is reduced thereby endangering the open-ended confinement.

We now make a numerical estimate of this reduction of critical length in actual experimental situations. In Constance II mirror machine (12), the electron beam was injected to suppress DCLC fluctuations. Hollow beam (1 cm diameter, 0.1 cm thick) of 8 KV, 7A was injected in a plasma produced by the T_i-washer gun with following parameter: $n_p = 2\times10^{13}$ cm^{-3}, $T_i = 400$ eV, $T_e = 10$ eV. In the experiment β of the plasma was $\simeq 4\times10^{-3}$, a low-β plasma. The critical length in the absence of wave reflection at the ends was about 200 cm, while with the wave reflection (due to different mechanism mentioned earlier), the critical length was a few cms. In the experiment, without the beam, no oscillations at ω_{pi} were observed which implies that there was no significant wave-reflection at the ends and that the waves were damped by the electrons at the throats before they could grow to a significant level. We can apply our results as we have assumed a low-β plasma and the absence of wave-reflections at the ends. During the injection strong signals at 30 GHz were observed which confirmed the existence of Langmuir turbulence in the machine ($\omega_{pe} \simeq$ 30 GHz for $n_p \simeq 10^{13}$ cm^{-3}). In the experiment electron gun and plasma gun were fired simultaneously hence as stated earlier HFCLC modes will encounter a broad, energetic spectrum with $\Delta \simeq K_o$, $W \simeq 0.2$.

From the electron beam parameter, the beam density comes out to be $n_b \simeq 10^{10}\ \mathrm{cm}^{-3}$, $V_b = 5\times10^9$ cm/sec. As typically $k'' \simeq \omega_{pi}/v_{thi}$ $\Delta \simeq k_o \simeq \omega_{pe}/V_b$, $W \leq 0.2$, $W(q/\Delta)^2 = 20$ and $a = \omega_{pe}^2/\omega_{ce}^2 \simeq 10$ so that $\sqrt{a} \simeq 3$. Thus $\sqrt{a}\,T/(a+T)^{1/2} \simeq 12$ which gives $L_m \simeq L/12$. From this it is clear that in these experiments the critical length is expected to be reduced by atleast an order of magnitude i.e. from a few hundred to a few tens of cm. This can seriously jeopardize the mirror confinement.

4. Electron Gun Fired After the Plasma Gun:

Let us now see whether this situation can be salvaged by delaying the injection of the beam. Such situations are generally encountered in mirror experiments employing relativistic electron beam for heating purposes (13). If the injection of electron beam is delayed by a few μ s after the plasma gun turn off, then the HFCLC modes will encounter the Langmuir wave-spectrum in its initial stage. In this stage the spectrum is narrow, the width is typically given by $\simeq \omega_{pe}/V_b\,(n_b/n_p)^{1/3}$ and contains roughly 1/3 energy of the beam (11). Hence $W = \frac{1}{3}\frac{n_b m_e V_b^2}{n_p m_e v_{the}^2}$

In equation (24) this situation is characterized by $\xi_g = \frac{k_o \sin\theta}{\Delta}\left(\frac{\omega''}{\vec{k}''\cdot\vec{V}_g} - 1\right) \gg 1$. In this limit $Z(\xi_g)$ is given as

$$Z(\xi_g) = \left[-1/\xi_g - 1/2\xi_g^3 \cdots\right] \tag{34}$$

Using this approximation in equation (24) we have $k_\parallel''^2$ as

$$k_\parallel''^2 = \frac{\omega''^2 k''^2}{\omega_{pe}^2}\left[X\left\{1 + \frac{\omega_{pe}^2}{\omega_{ce}^2 X}\right\} - i\frac{\omega_{pi}^2}{k''^2 v_{thi}^2} Z_1 X\right] \tag{35}$$

where $$X = \left[1 - W\left(\frac{\omega^*}{\omega''}\right)^2\left(\frac{k''}{\Delta}\right)^2\frac{1}{2\varrho_i^2}\right]$$

Typically as we will show later $\frac{\omega_{pe}^2}{\omega_{ce}^2 |X|}$ is $\gtrsim$ 1 in which case we may write K_{11}'' (after evaluating the square root as

$$K_{11}'' = \frac{\omega'' k''}{\omega_{pe}} \frac{\omega_{pe}}{\omega_{ce}}\left[1 - i\frac{\omega_{pi}^2}{k''^2 v_{thi}^2} Z_1(y) X\right]^{1/2} \tag{36}$$

Hence after binomial expansion

$$\text{Im}\, K_{11}'' = \frac{1}{2}\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \frac{\omega_{pi}}{v_{thi}} y Z_1(y)\left(\frac{\omega_{pe}}{\omega_{ce}}\right)^{-1}|X| \tag{37}$$

which gives the modified critical length L_m as

$$L_m = L/|X| \tag{38}$$

Let us now evaluate $|x|$ for the parameters of Constance II. Using the expressions for W and ϱ_g given before we have for Constance II parameters $W = 1/10$, $\varrho_g = 10$ $\left(k''/\Delta\right)^2 \simeq 10^2$ so that $W\left(k''/\Delta\right)^2 \frac{1}{2\varrho_g^2} \simeq 5$ so that $|x| \simeq 4$ and as $\omega_{pe}^2/\omega_{ce}^2 \gtrsim 10$ we have $\omega_{pe}^2/\omega_{ce}^2 |X| \simeq 2.5 \gtrsim 1$ thus justifying our approximation stated before. We remark that this approximation will all the more hold for future generation of high density mirror machines. From equation (38) we have that typically for present day mirror machine with electron beam injection $L_m \simeq 1/4$, the critical length is reduced by about a factor of four. This is not as dangerous as the previous case where the two guns were fired simultaneously but reduction of the critical length by four times can seriously risk the mirror confinement.

5. Discussion:

From these calculations we see that there exists a definite risk of the worsening of mirror-confinement in the machines which employ parallel injection of electron beam for the purpose of controlling drift cyclotron loss cone turbulence or for heating purposes (REB). If the plasma and the beam gun are fired simultaneously the critical length is reduced by about an order of magnitude, this can seriously jeopardize the confinement. On the other hand we see that if we delay the injection of electron beam i.e. fire it a few μ s after the plasma gun firing the critical length is still reduced by about four to five times. Thus we do not gain much by way of improving the confinment or eliminating the danger associated with the previous case. On the other hand, there is yet another strong reason against delaying the injection of the electron beam. There is a strong possibility that the very purpose of electron beam injection is defeated. As stated earlier, after the firing of plasma gun, it takes a few ion transit time (a few μ s) for DCLC instabilities to get triggered. These instabilities saturate very quickly (on the time scale of ω_{ci}^{-1}, about 1/10 of 1 μs for typical real situations) by diffusion in velocity space (14). In that process they push substantial amount of plasma in the loss cone from where it is lost. By injecting electron beams one tries to inhibit this process by super thermal electrons which are generated a few μs after the injection of the beam. Thus if electron beam injection is delayed by a few μs then it may so happen that by the time the hot electrons appear DCLC turbulence has already developed and saturated resulting in the concommittant particle loss. For precisely these reasons it has been found safer to fire the two guns simultaneously. Thus we see that by delaying the injection of electron beam we do not gain much by way of improving the confinement on the contrary we run the risk of defeating the very purpose of electron beam injection. Clearly we see that this harmful effect of electron beam injection is unavoidable hence serious. We remark that if from the reactor point of view we do a more exact calculation including all the mechanism for wave-reflection and high- β effects the critical length will be further reduced making the confinement still worse. This calculation shows that even in the simplest case electron beam can be harmful enough.

In view of this discussion we suggest the technique of electron cyclotron resonance heating employed by Ioffe et al for creating the hot electrons to suppress the DCLC instabilities. It is much safer and free from these complications.

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CHAPTER VI

ON STABILIZATION OF ION-CYCLOTRON TURBULENCE IN MIRRORS

1. Introduction:

In mirror machines such as the 2XIIB (1) at Livermore and PR-6, PR-7 at Kurcnatov, fluctuations at the ion-cyclotron frequency ω_{ci} have been observed. The observed waves have frequencies less than the vacuum field ion-cyclotron frequency ω_{ci}, wave numbers in the range $3 < k_\perp \rho_i < 6$ (ρ_i being the ion gyro-radius), phase velocities very close to the ion thermal velocity and propagates in the direction of ion-dimagnetic drift. These waves could be the drift-cyclotron loss cone (DCLC) mode which is predicted to be unstable (3,4) in mirror machines.

The saturation mechanism of these modes is not quite clear as yet. The observations show a strong correlation of ion-energy diffusion with the build up in the amplitude of the oscillations. This suggests a quasi-linear type of process as a possible saturation mechanism. Such a mechanism was in fact proposed by Galeev (5) in which the unstable DCLC modes saturate by plateau formation due to the partial filling of the loss cone as a consequence of velocity space-diffusion. This theory predicted ion-life times inside the trap to be a few axial bounce periods which were not in agreement with the observed life times of a few hundred bounce periods and much longer life times when warm plasma streams were employed. This theory was improved by Baldwin et al (4) who argued that mirror plasmas formed by neutral beam injection do not necessarily fill the entire phase-space available to them, but are peaked at pitch angles nearly perpendicular to the magnetic field. The predictions of this theory agree well with the observations of 2XIIB. However there are other mechanisms which do not invoke the velocity space diffusion. Gerver (6) and Aamodt et al (7) have proposed that for measured plasma lengths in 2XIIB the short wave-lengths are stabilized by axial convection. For plasma lengths longer than the 2XIIB plasma, Aamodt (8), Timofeev (9), Smith et al (10) and Menyuk (11) propose ion trapping as the saturation mechanism for a coherent short wave-length mode. Hasegawa (12) proposed anomalous resistivity arising from the scattering of electrons from the low-frequency oscillations (ω_{ci} /10) as the saturation mechanism of these unstable waves. Rosenbluth et al (13) and Aamodt (14) have proposed a mechanism in which a nonlinear frequency shift leads to the detuning of resonance between ion cyclotron wave and ion-drift wave which saturates the drift cyclotron instability.

In this Chapter we propose a saturation mechanism for the modes observed in 2XIIB at $\omega \sim \omega_{ci}$, $K \sim n R_p/\rho_i^2$ where R_p is the plasma radius and n is an integer. The physical process considered here is the velocity space diffusion as indicated by the experimental data and we use the renormalised plasma turbulence theory for the analysis. This process is distinct from the coherent mechanism proposed by Rosenbluth et al (13) as well as the theories based on loss cone filling and the plateau formation (4,5). The saturated fluctuation level calculated here agrees well with the observations.

Calculations:-

The plasma in a mirror machine is inhomogeneous and supports the electron drift mode at $\sim \omega_{pe}/k_\perp R_p$ where ω_{pe} is electron plasma frequency, and the ion drift mode at $\omega = \omega_{*i} = K_\perp U_{thi}^2/\omega_{ci} R_p$ where U_{thi} is the thermal velocity of ions. These are positive energy modes and when they are coupled to the negative energy ion - Bernstein mode the instability at $\omega \sim \omega_{ci}$ is excited. The appropriate dispersion relation for these modes with $K_{\parallel} = 0$ is (4)

$$1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} + \frac{\omega_{pe}^4}{\omega_{ce}^2 k_\perp^2 c^2} - \frac{\omega_{pe}^2}{\omega_{ce} R_p k_\perp} + \frac{\omega_{pi}^2}{k_\perp^2} f_{oi}(v_\perp = 0)$$
$$+ \frac{\omega_{pi}^2}{k_\perp^2} \sum_{n=-\infty}^{\infty} \frac{\omega}{(\omega - n\omega_{ci})} \int_0^\infty dv_\perp v_\perp J_n^2 (k_\perp v_\perp/\omega_{ci}) \left[2\frac{\partial f_{oi}}{\partial v_\perp^2} + \frac{\omega_{*i}}{\omega} \frac{f_{oi}}{U_{thi}^2} \right] = 0 \quad (1)$$

where f_{0i} is the ion-distribution function. It should be noted that in the above dispersion relation the effects due to the temperature gradients (19) the magnetic perturbations (20) etc. have been neglected. We envisage the saturation of the observed instability by the diffusion in velocity space arising from the perturbation of the particle orbits by the growing waves. This process is described by the perturbed orbit formalism (16,18) and is akin to the quasilinear diffusion process. The hitherto mentioned agreement between the quasilinear theory and 2XIIB observations show the preponderance of velocity space diffusion and this forms the basis of present study. With the procedure adopted here the saturation level of the instability is calculated. From the linear dispersion relation it is seen that the electron contributions do not exhibit kinetic nature and hence the perturbation of the electron orbits by the growing waves is not likely to play an important role. On the other hand the contribution of the ions being kinetic, the perturbed ion orbits will be important. Consequently we shall consider only the ion trajectories to be perturbed by the fluctuations and this leads to a diffusion of ions in the velocity space. The linear relation for a mode in the plasma is obtained by integrating over the unperturbed orbits. The nonlinear dispersion relation which includes the average effect of the fluctuating fields on the particle may be obtained by using the perturbed orbit formalism (15,16). As the name indicates in this formalism the integration is over the perturbed orbits and it is averaged over a set of random fluctuating fields. With this procedure, the nonlinear dispersion relation for $\omega \sim \omega_{ci}$ mode with $k_{\parallel} = 0$ is obtained.

$$1+\frac{\omega_{pe}^2}{\omega_{ce}^2}-\frac{\omega_{pe}^2}{\omega\omega_{ce}k_\perp R_p}=-i\frac{\omega_{pi}^2}{k_\perp^2}\sum_{n=-\infty}^{\infty}\int d\vec{v}\,J_n^2\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right)$$

$$\times\left[\frac{n\omega_{ci}}{v_\perp}\frac{\partial f_{oi}}{\partial v_\perp}+\frac{\omega_{*i}}{v_{thi}^2}f_{oi}\right]$$

$$\times\int_{-\infty}^{0}d\tau\, e^{-i(\omega-n\omega_{ci})\tau+\frac{1}{3}k_\perp^2D_i\tau^3}$$

(2)

In the ion contribution to equation (2) i.e. the right hand side D_i is the coefficient of diffusion in velocity space and is defined by (17)

$$D_i(\vec{v})=\frac{8\pi q^2}{m^2}\sum_{n=-\infty}^{\infty}n^2\omega_{ci}^2\int d\vec{k}\,\frac{|E_k|^2}{k^2}J_n^2\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right)$$

$$\times\int_{-\infty}^{0}d\tau\, e^{-i(\omega-n\omega_{ci})+\frac{1}{3}k_\perp^2D_i(v)\tau^3}$$

(3)

Equations (2) and (3) constitute a coupled set of equations which can be iterated for a given spectrum of modes to give the effect of perturbed ion orbits. Although the diffusion coefficient D_i is, in general, a function of the velocity, we assume the v dependence of D_i to be weak and use a constant D in the following analysis. Also in the orbit integral in equations (2) and (3) we approximate the term $\frac{1}{3}k_\perp^2D_i\tau^3$ by $(\frac{1}{3}k_\perp^2D_i)^{1/3}\tau$ as this term does not contribute for large τ

As ions are hot we may use the approximation $k_\perp v_\perp/\omega_{ci} \gg 1$ for simplyfying Bessel's functions. Then calculations on the same lines as (4) yield

$$1+\frac{\omega_{pe}^2}{\omega_{ce}^2}+\frac{\omega_{pe}^4}{\omega_{ce}^2 k_\perp^2 c^2}-\frac{\omega_{pe}^2}{\omega\,\omega_{ce} k_\perp R_p}$$

$$= -\frac{\omega_{pi}^2\,\omega_{ci}}{\pi k_\perp^3 v_{Thi}^3}\times\sum_{n=-\infty}^{\infty}\frac{\omega+i\left(\frac{1}{3}k_\perp^2 D_i\right)^{1/3}-\omega_{*i}}{\omega-n\omega_{ci}+i\left(\frac{1}{3}k_\perp^2 D_i\right)^{1/3}} \tag{4}$$

where we define $\quad v_{Thi}^{-3} = \int_0^\infty \frac{f_{oi}}{v_\perp^3}\,2\pi v_\perp\,dv_\perp$

We shall now consider only the fundamental mode (n=1) $\omega \simeq \omega_{ci}$ which contains maximum power in the observed spectrum (18). Defining the quantities

$$a = 1+\frac{\omega_{pe}^2}{\omega_{ce}^2}+\frac{\omega_{pe}^4}{\omega_{ce}^2 k_\perp^2 c^2}$$

$$b = \frac{\omega_{pi}^2}{\omega_{ci}^2}\,\frac{1}{(k\langle\rho_i\rangle)^3} \qquad d = \frac{\omega_{pe}^2}{\omega_{ce}}\,\frac{1}{k_\perp}\,\frac{1}{R_p}$$

$$c = i\left[\frac{1}{3}k_\perp^2 D_i\right]^{1/3}$$

the nonlinear dispersion relation equation (4) may be reduced to the form

$$a-\frac{d}{\omega}+\frac{b(\omega+c-\omega_{*i})}{(\omega-\omega_{ci}+c)}=0 \tag{5}$$

which can be further reduced to the quadratic equation

$$(a+b)\omega^2 - \left[a\omega_{ci} + d + b\omega_{*i} - C\{a+b\}\right]\omega + \omega_{*i}(\omega_{ci} - C) = 0$$

(6)

The roots of equation (6) are

$$\omega = \frac{a\omega_{ci}}{2(a+b)} + \frac{b\omega_{*i}}{2(a+b)} + \frac{d}{2(a+b)} - C \pm \frac{1}{2(a+b)} \times \left[\left\{a\omega_{ci} + d + b\omega_{*i} - C(a+b)\right\}^2 - 4(a+b)\,d(\omega_{ci} - C)\right]^{1/2}$$

(7)

For 2XIIB parameters and the observed range of $K_\perp \rho_i$ from 2.9 to 6.1, we have b in the range 1.69×10^3 to 3.55×10^3, $a \simeq 10^3$, $d \simeq 1 \times 10^{11}$. Then the square root term under the approximation $\omega - \omega_{ci} >> C$ (C being the nonlinear effect), yields the growth rate γ of the linear instability.

$$\gamma = \left[4(a+b)\,d\omega_{ci} - \left\{a\omega_{ci} + d + b\omega_{*i}\right\}^2\right]^{1/2}$$

With $\omega = \omega_r + i\omega_i$, the imaginary part of equation (7) is

$$\omega_i = -\tfrac{1}{2}\left(\tfrac{1}{3}k_\perp^2 D_i\right)^{1/3} + \gamma$$

(8)

From this equation, it is clear that as the linear unstable mode with growth rate γ grows, the velocity space diffusion starts becoming important and hence the perturbed orbit effects which lead to the damping term in equation (8) also start becoming important. When the diffusion is sufficient to make the damping term of the same order as γ the saturation occurs. In 2XIIB the observed ion-energy diffusion coefficient is about 10^{20} cm^2/sec^3 in terms of velocity space diffusion. If we use $k_\perp \sim 2.37$ cm^{-1}, corresponding to $k_\perp \rho_i \sim 6.1$ and $\rho_i \sim 2.57$ cm, then the damping term is $\simeq 3\times10^6$ sec^{-1}, while the observed growth rate in 2XIIB is $\simeq 0.02\omega_{ci} \simeq 2\times10^6$ sec^{-1} for $\omega_{ci} \simeq 10^7$ sec^{-1}. Thus we see that energy diffusion in 2XIIB is sufficient to saturate the growth of the observed modes.

Now let us calculate the fluctuation level. Accordingly the condition for this is given by

$$-\frac{1}{2}\left(k_\perp^2 D_i\right)^{1/2} + \gamma = 0 \tag{9}$$

The diffusion coefficient to the lowest order from equation (3) is

$$D_i = \frac{8\pi e^2}{m_i^2 U_\perp^2} \sum_n \int dk \, \frac{\varepsilon_k}{k^2} \, n^2 \omega_{ci}^2 \, \frac{J_n^2(k_\perp U_\perp/\omega_{ci})\,\gamma}{(\omega - n\omega_{ci})^2 + \gamma^2} \tag{10}$$

where $\varepsilon_k = |E_k|^2/8\pi$ is the energy density of the K^{th} mode. Also for ions $k_\perp \rho_i > 1$ so that J_n^2 can be approximated as

$$J_n^2 \simeq \frac{1}{\pi} \frac{\omega_{ci}}{k_\perp U_\perp} \tag{11}$$

Typically we may evaluate D_i at $U = U_{thi}$ so that D_i is given by

$$D_i = \left[\frac{\omega_{pi}^2 \omega_{ci}^3}{v_{thi}^3 n_0 T_i \gamma} \int dk \frac{\mathcal{E}_k}{k^3} \right] v_{thi}^2 \tag{12}$$

or

$$D_i \sim \frac{2}{\pi} \frac{\omega_{pi}^2 v_{thi}^2}{(k_\perp \langle \rho_i \rangle)^3 \gamma} \frac{W}{n_0 T_i} \tag{13}$$

where $W = \int \mathcal{E}_k dk$ is the energy density of ion-cyclotron loss cone turbulence, T_i is the ion temperature and $k_\perp$ is some typical wave number. From equation (9) and equation (13), the saturation level is given by

$$\frac{W}{n_0 T_i} \sim \frac{12\pi \, k_\perp \rho_i \, \gamma^4}{\omega_{pi}^2 \, \omega_{ci}^2} \tag{14}$$

Using experimental parameters of 2XIIB i.e. $\omega_{pi} \sim 5.52 \times 10^9$ Hz[1] $\omega_{ci} \sim 10^7$ Hz, $k_\perp \rho_i \sim 6.1$, $\gamma = \omega_{ci}/50$, etc., we get $W/n_0 T_i \sim 8 \times 10^{-8}$, which corresponds to about 18 to 20 V in terms of root mean square potential. This agrees well with the observed potential fluctuations which varies from 10 to 50V without the stream stabilization (18).

And finally we briefly discuss the case of saturation in the presence of warm plasma streams. Equation (9) shows a strong dependence of the fluctuation level on γ . Hence a small decrease in γ can cause an appreciable decrease in the fluctuation level. This may be one reason why the fluctuation level is reduced when a small quantity of warm plasma is added. It can be shown from the linear dispersion relation (1) that with the addition of a small quantity of warm plasma the linear growth rates are reduced. Then according to equation (16) this can bring down by an appreciable amount the fluctuation level. The saturation condition equation (9) gives the ion life time as

$$\tau_i \simeq \frac{v_{thi}^2}{D_i} \simeq \frac{k_\perp^2 v_{thi}^2}{(2\gamma)^3}$$

This shows that a small decrease in the growth rate due to warm plasma injection will significantly bring down the ion life time in the trap as has been observed.

3. Conclusion:

To conclude we have shown that in 2XIIB the energy diffusion is sufficient to saturate the ion-cyclotron loss cone turbulence by perturbed orbit effect. The fluctuation level calculated from here agrees well with the observations. We have also shown that apart from loss cone filling, the perturbed orbit effect may be another important factor in bringing down the fluctuation level and improving the ion life time when warm plasma is added. In this work, we have not discussed the problem of sporadic bursts which have been associated with other physical processes (21).

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CHAPTER VII

FEED BACK STABILIZATION OF DRIFT CYCLOTRON LOSS CONE INSTABILITY BY MODULATED ELECTRON SOURCES

1. Introduction:

The drift cyclotron loss cone instability (DCLC) which arises because of the resonance between the positive energy electron drift mode and the negative energy ion Bernstein mode has been conclusively identified in high density mirror machines like PR-6, PR-7 (Kanaev and Yushmanov 1974, 1975), 2XIIB (Simomen 1976), etc. These modes occur at Ω_i, and its harmonics (Ω_i, ion gyro frequency) and have growth rates roughly of the same order. They require a critical density gradient (CDG) to become unstable and thus set a minimum limit on the mirror plasma radius (Post and Rosenbluth 1966) ($R = 200\ a_i$, R mirror plasma radius, a_i ion gyro radius). In this paper we have examined the feedback stabilization of these dangerous modes by modulated electron sources. This method has been suggested and used before for the stabilization of low frequency drift instabilities and drift temperature instabilities, etc. (Simonen 1969; Kitao 1971; Lakhina and Sen 1974). Here we have shown that when the feedback differs by $+90^{\circ}$ in phase from the unstable perturbation then the CDG increases approximately linearly with the gain. Typically with a feedback gain of $\sim 50\Omega_i$, the CDG can be pushed by as much as two orders of magnitude, thereby considerably improving the stability of mirror plasmas against DCLC instability.

2. Calculations:

We consider a high density hot ion plasma i.e. $\omega_{pe} \gtrsim \Omega_e, T_i >> T_e$ (ω_{pe} electron plasma frequency, Ω_e electron gyro frequency, T_i ion temperature, T_e electron temperature). This plasma is embedded in a mirror magnetic field $\vec{B} = B_0 \hat{z}$ and has a density gradient

$$\frac{1}{n}\frac{dn}{dx}\hat{x} = \vec{\epsilon}$$

The feedback system consists of an instability amplitude sensing probe whose signals are amplified, phase-shifted and impressed on the suppressor probe which, with appropriate negative DC bias, modulates the local electron flow to the probe in giving rise to modulated electron sources. Althou in any real experiment such sources would be present only at finite points in the plasma, nevertheless we assume in this model that sources are distributed uniformly throughout the plasma and they respond linearly to the local density perturbation. Such an assumption yields results in good agreement with the experiment (Furth and Rutherford 1969). Thus we represent the source in the form

$$S = -i\omega_f n_{e1} \qquad (1)$$

where the amplitude and the argument of $i\omega_f$ represent the gain and the phase of the feedback, n_{e1} is the local electron density perturbation. This source is included in Vlasov's electron equation.

Following the standard procedure outlined in Sen and Sundaram (1976) and using the Vlasov-Poisson system of equations with appropriate source term in electron dynamics the dispersion relation for the electrostatic, flute modes in an inhomogeneous, low-β plasma can be written as

$$1 + \chi_e' + \chi_i = 0 \qquad (2)$$

χ_i and χ_e' are ion and electron susceptibilities and are given by

$$\chi_i = -\frac{1}{k^2 d_i^2}\left[1 - \sum_{n=-\infty}^{\infty} \int \frac{J_n^2(b_i^2)(\omega + \omega_{Ni}) f_{oi}}{(\omega - n\Omega_i)} d\vec{v}\right]$$

$$\chi_e = \frac{-\frac{1}{k^2 d_e^2}\left[1 - \sum_{n=-\infty}^{\infty} \int \frac{J_n^2(b_e^2)(\omega + \omega_{Ne}) f_{oe}\, d\vec{v}}{(\omega - n\Omega_e)}\right]}{\left[1 - \frac{\omega_f}{\omega} \sum_{n=-\infty}^{\infty} I_n(b_e^2)\exp[-b_e^2]\right]}$$

$$b_j^2 = k_\perp^2 v_j^2 / 2\Omega_j^2 , \quad V_j = (2T_j/m_j)^{1/2}, \quad d_j = \left[\frac{T_j}{4\pi e^2 n_j}\right]^{1/2}$$

$$\Omega_j = \frac{Q_j B_0}{m_j c} , \quad \omega_{NJ} = k_\perp \frac{T_j}{m_j} \frac{1}{\Omega_j} \frac{1}{N} \frac{dN}{dx}$$

$$\omega_f = \omega_{fr} + i\,\omega_{fi} , \quad i\omega_f = R e^{i\theta}$$

Thus, in the dispersion relation (2) the new feature is reflected in χ_e' wherein now we have a term in the denominator describing the source characteristics. Since we only consider the electron sources, the ion term is unaffected.

We shall consider the effect of this term on DCLC wave spectrum. For this we make use of the following approximations (Post and Rosenbluth 1966) $k_\perp v_e/\Omega_e \ll 1$, $\omega \ll \Omega_e$ for electrons and $[\epsilon V_\perp^2 k_\perp/\Omega_i \omega] \gg 1$, $k_\perp v_i/\Omega_i \gg 1$ for ions. With these approximations and taking a loss cone distribution for ions $f_{oi}(v_\perp = 0) = 0$ in equation (2) we arrive at the modified dispersion relation for the DCLC modes as

$$1 + \frac{\omega_{pe}^2}{\Omega_e^2} = \frac{\omega_{pe}\,\epsilon/\Omega_e\,\omega|k|}{[1 - \omega_f/\omega]} - \frac{\omega_{pi}^2\,\Omega_i^2}{k_\perp^3 v_i^3 \pi} \sum_{n=-\infty}^{\infty} \frac{\omega}{(\omega - n\Omega_i)} \tag{3}$$

Using $\sum_{n=-\infty}^{\infty} 1/(x+n) = \pi \cot \pi x$ and putting $W = \frac{\omega}{\Omega_i} \times \pi$

and $W_f = \frac{\omega_f}{\Omega_i} \times \pi$ equation (3) can be put in the form

$$W^2 \cot W + W[\beta - W_f \cot W] - \beta W_f$$
$$= \beta^{2/3} \epsilon \langle a_i \rangle \pi^{4/3} \left(\frac{m}{M} + \frac{\Omega_i^2}{\omega_{pi}^2} \right)^{-2/3}$$

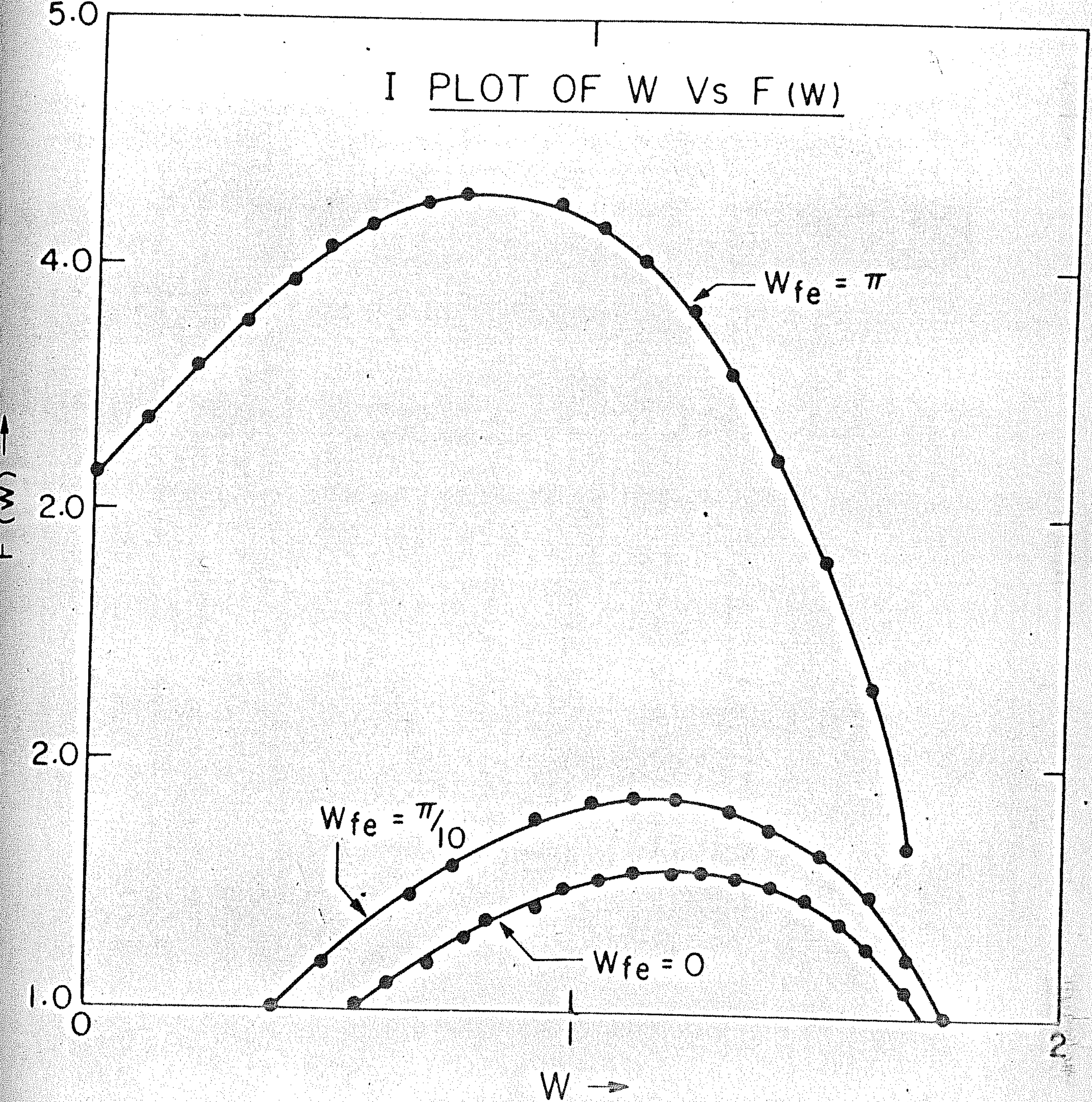
where $\beta = \pi (W \langle a_i \rangle)^3 \left(\frac{m}{M} + \frac{\Omega_i^2}{\omega_{pi}^2} \right) > 0$ ——— (4)

For a given density gradient $\epsilon \langle a_i \rangle$ and mode number defined by β, (4) will give the real and the imaginary of the frequency defined by W. But as (4) is transcendental, real and imaginary part of W cannot be evaluated analytically. However, there is yet another method of examining the stability through (4). For W_f = 0 equation (4) reduces to the well-known dispersion relation for DCLC modes obtained in Post and Rosenbluth (1966). It is shown that the left side of (4) admits a saddle point with respect to W and β which gives rise to a critical density gradient $\epsilon_c \langle a_i \rangle$. For $\epsilon \langle a_i \rangle < \epsilon_c \langle a_i \rangle$, all the modes defined by β are stable, while for $\epsilon \langle a_i \rangle > \epsilon_c \langle a_i \rangle$, some of the β are unstable giving rise to the DCLC instability. The idea here is to examine the stability by evaluating CDG in the presence of the feedback i.e. for a finite W_f. Such an analysis is possible only when W_f is real i.e. for $\theta = \pm 90^\circ$. For other values of θ, W_f becomes complex which makes the evaluation of CDG quite difficult. Accordingly, we proceed to evaluate the CDG for different gains in phase $\theta = \pm 90^\circ$.

(i) $\theta = +90^\circ \quad [\omega_{fr} = -\omega_f\,, \ \omega_{fi} = 0]$

In this case (4) becomes

$$W^2 \cot W + W[\beta + W_f \cot W] + \beta W_f = \beta^{2/3} \epsilon \langle a_i \rangle \pi^{2/3}$$
$$\times \left(\frac{m}{M} + \frac{\Omega_i^2}{\omega_{pi}^2} \right)^{-2/3} \quad \text{--- (5)}$$


I PLOT OF W Vs F (W)
5.0
4.0
2.0
2.0
1.0
0
1
2
W
F (W)
$W_{fe} = \pi$
$W_{fe} = \pi/10$
$W_{fe} = 0$

<u>Fig. 1</u> *Plot of W vs. F (W) for fixed values of* W_f.

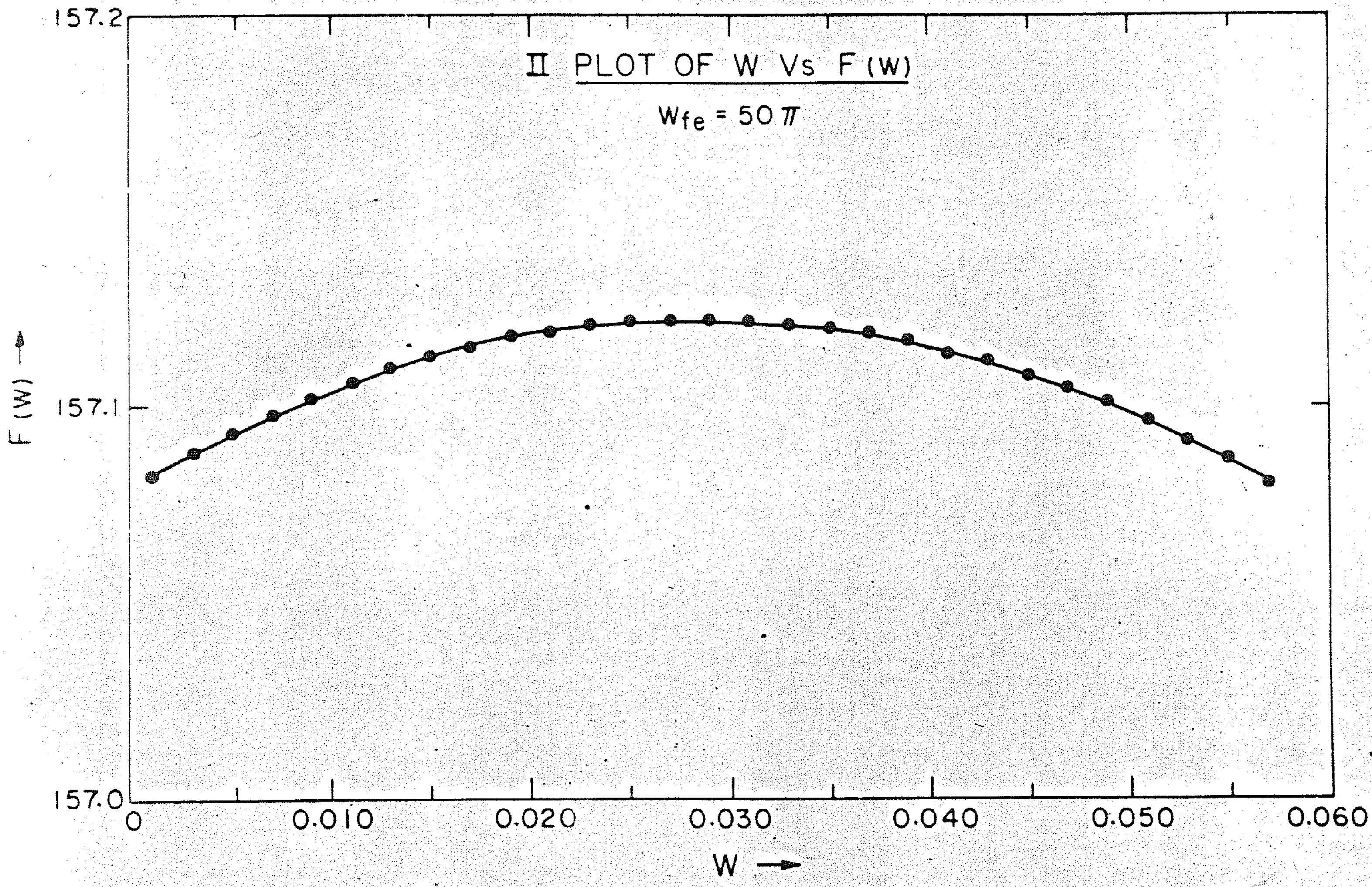

II PLOT OF W Vs F (W)
Wfe = 50 π
157.2
157.1
157.0
F (W)
0
0.010
0.020
0.030
0.040
0.050
0.060
W

Fig. 2 Plot of W vs. $F(W)$ for fixed values of W_f

We shall use a conventional numerical method to estimate CDG in the presence of the feedback source term, following Post and Rosenbluth (1966). In order to determine the saddle points at which critical density gradient occurs, we adopt the procedure described below:

A particular value of W_f is chosen in (5) and a function $f[W, \beta, W_f]$ is defined as

$$f[W, \beta, W_f] = \frac{W^2 \cot W + W[\beta + W_f \cot W] + \beta W_f}{\beta^{2/3} \times \pi^{2/3}} \tag{6}$$

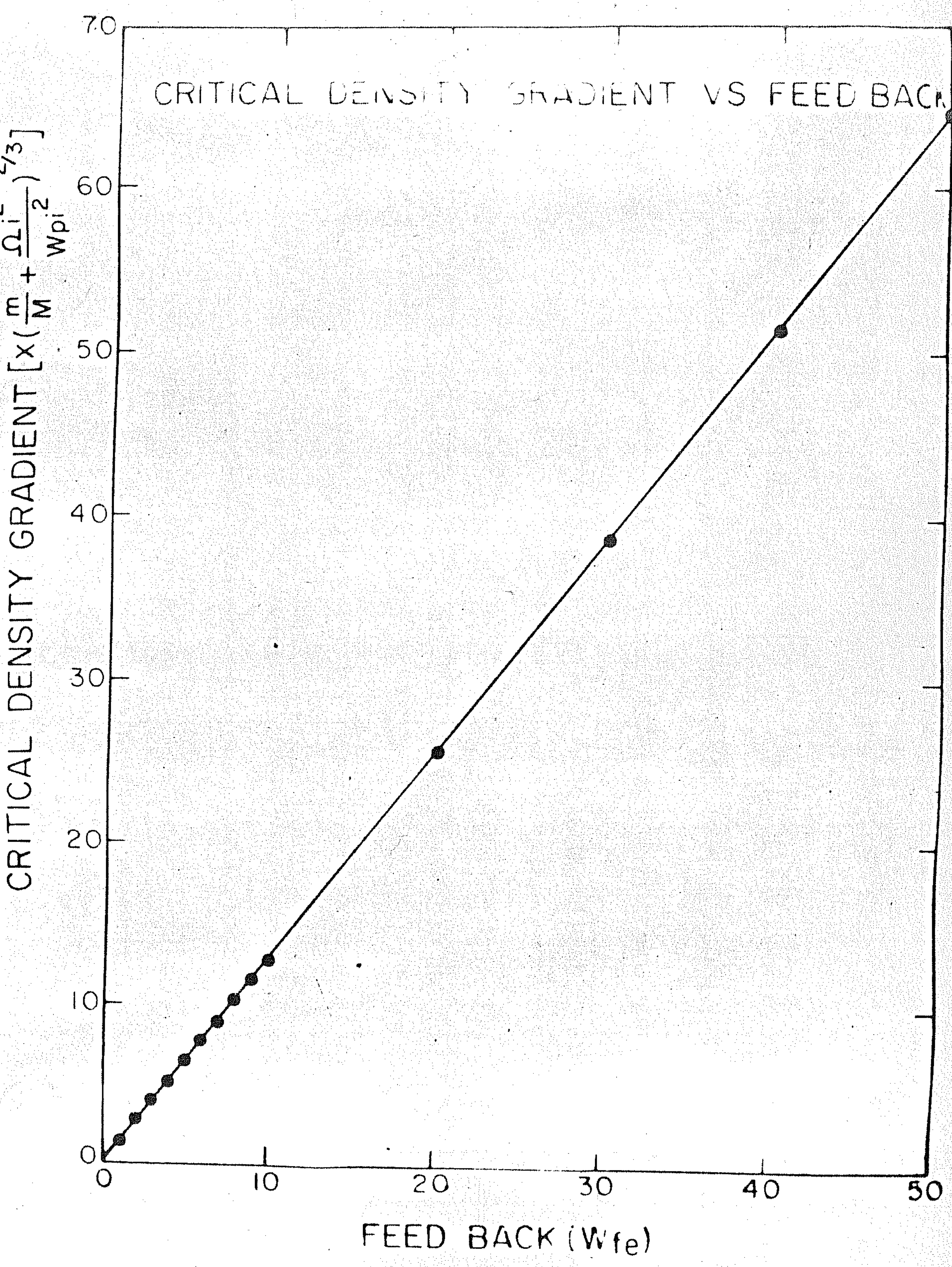
We plot in figures 1 and 2, f as a function of W for different values of W_f and β (the values of β are not given in the figure). The plot shows a maximum with respect to W between 0 and π. The existence of a minimum with respect to β is seen from the following equation.

$$\left.\frac{d^2 f}{d\beta^2}\right|_{\substack{W_1 \\ \beta_1}} = \frac{1}{3}\beta_1^{-5/3}[W_1 + W_f] > 0 \tag{7}$$

As the quantities β_1, W_1 and W_f are all positive, there is a minimum with respect to β. This implies the existence of a saddle point $[W_1 \beta_1]$ which can be located by simultaneously solving the following set of non-linear equations.

$$\left[\frac{df}{d\beta}\right]_{\substack{W_1 \\ \beta_1}} = \beta_1 - 2W_1 \cot W_1 = 0 \tag{8}$$

$$\left[\frac{df}{dW}\right]_{\substack{W_1 \\ \beta_1}} = \sin W_1 \cos W_1 \left[1 + \frac{W_1 + W_f}{3W_1}\right] - \frac{W_1 + W_f}{3} = 0 \tag{9}$$



CRITICAL DENSITY GRADIENT VS FEED BACK
CRITICAL DENSITY GRADIENT $\left[\times\left(\frac{m}{M}+\frac{\Omega_i^2}{W_{pi}^2}\right)^{2/3}\right]$
FEED BACK (W_{fe})
0
10
20
30
40
50
60
70

Fig. 3 *Plot of feedback gain vs. critical density gradient*

From (8) and (9) we obtain the saddle point coordinates W_1 and β_1 as a function of W_f. The CDG for a particular value of W_f is then given by

$$\frac{f[\beta_1[W_f],\ W_1(W_f),\ W_f]}{\pi^{2/3}}\left(\frac{m}{M}+\frac{\Omega_i^2}{\omega_{pi}^2}\right)^{2/3} = \epsilon_c \langle a_1 \rangle \qquad (10)$$

where $f[\beta_1(W_f), W_1(W_f), W_f]$ is to be evaluated from (6) which can be rewritten as

$$f[W_1 \beta_1 W_f] = \frac{W_1^2 \operatorname{Cot} W_1 + \beta_1 W_1}{\beta_1^{2/3}\ \pi^{4/3}} + \left[\frac{W_1 \operatorname{Cot} W_1 + \beta_1}{\beta^{2/3} \times \pi^{4/3}}\right] W_f \qquad \text{--(11)}$$

Such calculations have been carried out for different values of W_f. The results are tabulated in table 1. The first column gives the value of W_f, the second gives the corresponding CDG. In figure 3, CDG is plotted against Gain. The plot shows that CDG increases linearly with in the range 0-50 π. It must be mentioned here that this linearity is only approximate because it so turns out that the value of the first term and the coefficient of W_f in the second term in (11) does not change much for different saddle point coordinates $W_1(W_f)$ and $\beta_1(W_f)$ because of which (11) approximately represents a straight line with f and W_f as the two variables. It must be mentioned that (though it appears) the straight line does not pass through the origin because even in the absence of the feedback there is a CDG. This is also obvious from (11).

Table 1

Feedback gain vs. critical density gradient

Gain	CDG
0	0.38
3π	3.961
4π	5.231
5π	6.50
6π	7.79
7π	9.07
8π	10.35
9π	11.64
10π	12.76
20π	25.82
30π	38.72
40π	51.52
50π	64.52

From the table it is clear that with a gain of $\sim 50\Omega_i$, the CDG can be increased by as much as two orders of magnitude, thereby conside ably improving the stability of the mirror plasma against DCLC instability. As shown in Post and Rosenbluth (1966) the minimum plasma radius is given b

$$R_{min} = \frac{\pi^{4/3}}{f[W_1,\beta_1]} \left(M/m\right)^{4/3} a_i \,\psi\left(\Omega_e^2/\omega_{pe}^2\right) \qquad \text{---(12)}$$

where $\bar{\Psi}$ is a function of Ω_e^2/ω_{pe}^2 and is tabulated in Post and Rosenbluth (1966) for different values of its argument. It is clear from (12) that R_{min} goes inversely as the gain of the feedback. Typically in the present-day mirror machines where $\Omega_e^2/\omega_{pe}^2 \sim 1$, $a_i \sim 1$ cm, with a gain of $\sim 50\,\Omega_i$, R_{min} can be brought down from 500 cm to 3 cm, thereby almost removing the constraint imposed by DCLC instability on the radius of the mirror plasma.

(ii) $\theta = -90^\circ$ ($W_{fr} = W_f$, $W_{fi} = 0$). In this case, as follows from (7) the sign of $d^2f/d\beta^2$ is not fixed. For $W > W_f$ $d^2f/d\beta^2$ is positive while for $W < W_f$, $d^2/d\beta^2$ is negative. Thus the saddle point and the consequent CDG does not exist, and the effect of the feedback on the overall DCLC spectrum cannot be evaluated. The stability of a particular K in such phase angles should be evaluated numerically from the dispersion relation (4).

3. Discussion:

We have shown here that the stability of the mirror plasma against DCLC instability can be considerably improved by modulating electron sources at ion gyro frequencies and at a + 90° phase difference from the unstable perturbation. The question of the number of feedback loops and their location has to be decided by the experiment. For instance in Simonen's (1969) experiment on quenching of drift instabilities in Q-machines by modulated electron sources only one feedback loop consisting of two Langmuir probes located in the region of maximum wave amplitude was sufficient to bring about a considerable improvement in the density build-up, confinement time, etc. In our problem, however, more than one loop may be required as DCLC modes are not localized and in fact are spread over a larger plasma cross-section. In the case of mirror plasmas, as the probe would be in actual contact with the hot plasma some complication may arise due to the heating and sputtering with the hot plasma of the probe. But we do not expect these effects to be very important as mirror plasma experiments are pulsed (a few msec) and their thermal energy content is very low (a few calories).

Arsenin et al (1968 abc) have reported stabilization of m=1 flute mode and ion cyclotron instabilities in the low density plasmas ($\sim 10^7/cm^3$) by placing feedback loops at the radial boundaries to appropriately control them. This method cannot be very effective for the suppression of drift instabilities, the signals have to be injected in the plasma to modulate the particle sources.

Our calculations mainly highlight the importance of feedback systems in stabilizing DCLC mode and provide an upper limiting value of the feedback gain. Considerations such as warm plasma streams (Baldwin et al 1976; Gerver 1976) in the loss region of velocity space and the compressional perturbations of magnetic field (Tang 1972) will significantly lower the limit for the gain and thereby will make the stabilizing action of modulated electron sources easier.

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CHAPTER VIII

EFFECT OF LOWER HYBRID TURBULENCE ON DRIFT CYCLOTRON LOSS CONE INSTABILITY

1. Introduction:

The injection of high energy neutral beams in mirror machines is of considerable interest as it has been envisaged as an efficient method of fueling and heating the target plasma in future mirror reactors (1,2,3). Theoretical speculations on plausible microinstabilities associated with such systems have also been reported (4). The upshot is that, the most easily excitable instability is that of Lower Hybrid Waves (LHW), which is produced by a relative drift of electrons and ions across the field. The neutral beams in its interaction with the target plasma yields ions and electrons. If then, there is sufficient relative motion between the species because of their different degrees of magnetisation then the lower hybrid waves become unstable.

The threshold for this instability is $V_0 > v_{thi}$ (5) (V_0 is the beam velocity, v_{thi} is thermal velocity of ions). In mirror machines this condition is very likely to be satisfied as the hot plasma is formed mostly by the charge exchange between the low energy ions of target plasma and the energetic neutral particle of the beam. Recently many groups have reported ion beam driven LHW instability using perpendicular injection (6,7). In Chang's experiment (8) waves at $\omega \sim \omega_{LH}$ (ω_{LH} is the LHW frequency; $\omega_{LH} \sim \omega_{pi}$) were observed by perpendicular injection of an ion beam with $V_0 = 15\, v_{thi}$. In Burnent V steady state mirror strong radial electric field were produced by electron beam injection. This led to a relative motion between the ions and electrons and consequently strong oscillations at $\sim \omega_{pi}$ were observed. Cordey et al (9) have shown that during the initial stages of neutral beam injection the hot particle distribution function is peaked and hence is unstable to perturbations at $\sim \omega_{pi}$. They have also shown that the neutral beam injection can give rise to large scale radial electric field which in turn can destabilize LH waves. Quite apart from these considerations, in 2XIIB the threshold for flute type lower hybrid drift instability [$a_i/L > \left(\frac{m_e}{m_i} + \frac{\omega_{ci}^2}{\omega_{pi}^2}\right)^{1/4}$), where a_i is the ion gyro-radius and L is the length of the machine) is exceeded hence one expects a continuous spectrum of lower hybrid waves superimposed on a discrete spectrum of ion-cyclotron oscillations (15). Thus it appears that a modest level of LHW turbulence in future neutral beam injected mirror machines seems to be inevitable. The positive aspect of these oscillations is that they provide an efficient mechanism of energy transfer from the beam to the particles and thus lead to a strong heating of electrons and ions (8). Now it has been shown that in the presence of a background turbulence of these oscillations the normal modes of plasma are drastically modified (10,11). In this Chapter we propose to investigate the effect of a modest level of this turbulence on the most important and the most deterrent normal mode of the mirror plasma i.e. the drift cyclotron loss cone (DCLC) instability in realistic situations. Recently Shaing (12) et al have reported an investigation of the effect of LHW pump on DCLC instability. However, the result of their investigations may not be applicable to the actual experimental situation in the mirror machine for the following reason. They hace considered a parametric coupling between a LHW and a DCLC mode, which is a coherent interaction between two waves. But in real situation the interaction is not expected to be coherent. The condition for the saturation of a coherent wave (by particle trapping) is $\tau_{ac} \sim (k\,\delta v_p) >> \tau_g$ where τ_{ac} is the auto-correlation time, τ_g is the linear growth time, K is the typical wave number of LHW waves, and δv_p is the spread in the phase-velocity of the waves, while the condition for saturation of broad spectrum is $\tau_g \gtrsim \tau_{ac}$. Now τ_g for LHW $\simeq (\omega_{pi}\,\eta^{1/3})^{-1}$ where $\eta_b = n_b/n_p$ [n_b is the beam density, n_p is the target plasma density) while typically $K = \frac{\omega_{pi}}{v_0}$ and $\delta v_p \simeq v_T \simeq v_0 \eta_b^{1/3}$ where v_0 is the beam velocity and v_T is the thermal spread of the beam. Hence $\tau_{ac} \simeq \left(\frac{\omega_{pi}}{v_0} v_0 \eta_b^{1/3}\right)^{-1}$ $\simeq \frac{1}{\omega_{pi}\,\eta_b^{1/3}} \simeq \tau_g$, in which case the saturation of a broad spectrum of LHW by quasilinear diffusion in velocity space is more likely. Indeed in Chang's experiment the observed spectrum of LHW was quite broad. In order to study the incoherent interaction we notice that in experimental situations LHW spectrum is broad while DCLC wave spectrum is narrow (14) and secondly that the dispersion characteristics of two waves are widely apart i.e. $\omega_l \sim \omega_{pi} \gg \Omega \sim \omega_{ci}$ [$\omega_k(\Omega)$ is the frequency of LHW (DCLC)]. Hence we will use 'Vedenov technique(16) called 'adiabatic approximation' which satisfactorily takes into account the incoherent interaction between waves of widely different properties. In this method one treats the high frequency microturbulence as a wave packet with a distribution in K -space that satisfies a wave-kinetic equation. One then studies the motion of these wave packets in a medium varying slowly in space and time, the variation being due to the low frequency long-wave length wave. The reverse influence of the high frequency turbulence on low frequency waves comes through the average electric field pressure ∇E^2 which modifies the electron dynamics.

Our studies reveal that in the presence of LHW the DCLC waves are strongly damped. Thus the presence of a lower hybrid turbulence will have a two fold advantage in mirror machines, firstly it will provide an efficient mechanism of the energy transfer from beam to particles for heating purposes and secondly it will help in controlling the DCLC turbulence, and improving the ion life time inside the machine.

In Section 2 we have calculated the effect of LHW turbulence on linear growth rates of DCLC waves. In Sec.3 we have discussed our our results.

2. Coupling of DCLC Waves to Lower Hybdird Waves:

For a theoretical model we consider a slab gemoetry with Z-axis along the mirror axis. The plasma has a density gradient $\frac{1}{n}\frac{dn}{dx} = \epsilon \hat{x}$. The DCLC-wave-spectrum observed generally in high density mirror machines like 2XIIB is a narrow spectrum (14), with maximum power in the fundamental mode at ω_{ci} . We may approximate this by considering a single DCLC mode in Y-direction with frequency and wave number given by Ω and $\vec{q}$ respectively. We take the LHW turbulence in Y-Z plane with frequency and wave number ω and $\vec{K}$ respectively where $\vec{K} = K_\perp \hat{y} + K_\parallel \hat{z}$ and $K_\parallel^2/K_\perp^2 \sim \frac{m_e}{m_i} \ll 1$ This turbulence may be either because of injection of energetic neutra-beam perpendicular to the field lines or because the threshold for flute types lower hybrid instability has been exceeded in the mirror plasma i.e. $a_i/L > \left(\frac{m_e}{m_i} + \frac{\omega_{ci}^2}{\omega_{pi}^2}\right)^{1/4}$ (For 2XIIB $a_i/L \simeq 1/2$ and $\left(m_e/m_i + \omega_{ci}^2/\omega_{pi}^2\right)^{1/4} \sim 0.1$). To consider the effect of these modes on DCLC waves we consider a broad spectrum of LHW which has saturated by diffusion in vellcity space. The spectrum is taken to be peaked around $K_0 = \frac{\omega_{LH}}{V_0}$ (where ω_{LH} is the LHW frequency). The LHW obey the linear dispersion relation

$$\omega_K^2 = \omega_{LH}^2 \left[1 + \frac{K_\parallel^2}{K_\perp^2} \frac{m_i}{m_e} \right] \tag{1}$$

where $\omega_{LH}^2 = \omega_{pi}^2 \left[1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \right]$ and $\frac{K_\parallel^2}{K_\perp^2} \sim \frac{m_e}{m_i} \ll 1$

We will now envoke the adiabatic approximation due to Vedenov et al. In this approximation the evolution of Lower Hybrid (LH) plasmon distribution function N_K is studied by the following equation (16)

$$\frac{\partial N_K}{\partial t} + \vec{V}_g \cdot \nabla N_K - \frac{\partial \omega_{\vec{K}}}{\partial \vec{r}} \cdot \frac{\partial N_{\vec{K}}}{\partial \vec{K}} = 0 \tag{2}$$

where $\frac{\partial \omega_{\vec{k}}}{\partial \vec{k}} = \vec{V}_g$ is the group velocity of LHW and from (1)

$\vec{V}_g \sim \partial \omega_{LH} \, k_{\parallel}^2/k_{\perp}^3 \, m_i/m_e \sim \partial \omega_{LH}/k_{\perp}$ as $k_{\parallel}^2/k_{\perp}^2 \simeq m_e/m_i$

It should be noted that the wave-kinetic equation (2) is valid only when the spread in the group velocities is so large that the convective term $\vec{V}_g \cdot \nabla N_{\vec{k}}$ dominates the effect of diffraction i.e. the term containing $\partial V_g/\partial \vec{k}$. In brief the effect of LHT arises as follows: The low frequency perturbation creates a perturbation in the plasmon density. The gradient of this plasmon density gives rise to a ponderomotive force (PF) which reacts back on the low frequency oscillations to modify its characteristics. The effect can also be viewed by considering the group of plasmons as a group of particles (or quasi-particles) on the time scale of low density perturbation. Hence depending upon the slope of plasmon distribution at the resonance point, the Landau resonance between the DCLC wave and plasmon distribution may lead to a growth or damping of the DCLC wave. the PF on ions is m_e/m_i times smaller than that on electrons and hence will be dropped. The plasmon distribution function is perturbed as $N_k = N_{k0} + n_k$ (where N_{k0} is the equilibrium distribution function normalised as $\int N_{\vec{k}0} d\vec{k} = 1$). From the wave-kinetic equation (2)

$$\tilde{n}_k = \frac{\frac{\partial \omega_{\vec{k}}}{\partial \vec{r}} \cdot \frac{\partial N_{\vec{k}0}}{\partial \vec{k}}}{i(\vec{q} \cdot \vec{V}_g - \Omega)} , \quad n_k = \tilde{n}_k e^{i(\vec{q}\cdot\vec{r} - \Omega t)} \tag{3}$$

The dependence of $\tilde{n}_k$ on the low frequency density perturbation comes from the term $\frac{\partial \omega_{\vec{k}}}{\partial \vec{r}}$; for LHW $\omega_k^2 \simeq \omega_{LH}^2 \simeq \omega_{pi}^2$ so that $\frac{\partial \omega_k}{\partial \vec{r}} \simeq i\vec{q}\, \frac{\omega_k}{2}\, \tilde{n}_1/n_0$ where $\tilde{n}_1$ is the low frequency perturbation.

For DCLC modes we will use the model given by Post and Rosenbluth (21). According to this model the DCLC modes are electrostatic flute modes which arise because of resonance between the positive energy electron drift mode and negative energy ion Bernstein mode. The typical frequency and the phase-velocity of these modes as seen in 2XIIB (17) is $\Omega \simeq \omega_{ci}$ and $\Omega/q \sim v_{thi}$ respectively. It should be noted that this model for DCLC Modes ignores certain effects which may be needed for more quantitative accuracy, such as electromagnetic effects to the electron contribution (18) ion drift term (19) temperature gradient (20).

To calculate the PF on an electron the modified e uation of motion is

$$m_e \frac{\partial \vec{V}}{\partial t} + m_e \vec{V} \cdot \nabla \vec{V} = - e\vec{E} - \frac{e}{c} \vec{V} \times \vec{B}_0 \tag{4}$$

where now in the equation of motion the nonlinear term ($\vec{V} \cdot \nabla \vec{V}$) has been retained. In equation (4) $\vec{V} = \vec{V}_s + \vec{V}_f$ where $\vec{V}_s$ is the low frequency part of $\vec{V}$ sustained by DCLC mode, and $\vec{V}_f$ is the high frequency part of $\vec{V}$ sustained by LHW. The electron sees the effect averaged over many periods of high frequency oscillations. Hence taking the average over many high frequency oscillation period (13)

$$m_e \langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle = - e \vec{E}_s - \frac{e \vec{V}_s \times \vec{B}_0}{c} \tag{5}$$

where $\vec{V}_f$ is given by the high frequency equation

$$m_e \frac{d\vec{V}_f}{dt} = - e \vec{E}_f - \frac{e \vec{V}_f \times \vec{B}_0}{c} \tag{6}$$

In equation (5) $\vec{E}_s = |E_s|\hat{y}$ is the low frequency perturbed electric field, while in equation (6) $\vec{E}_f = |E_{fy}|\hat{y} + |E_{fz}|\hat{z}$, $\frac{E_{fz}}{E_{fy}} \ll 1$ is high frequency perturbed E-field. From equation (5) V_{sx} can be written as

$$V_{sx} = \frac{E_s}{B_o} + \frac{\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y}{\omega_{ce}} \tag{7}$$

Here we have neglected the inertial term due to slow mode as $\Omega \ll \omega_{ce}$

Then the modified equation of continuity becomes

$$-i\Omega n_{e1} + \epsilon n_o \left[\frac{E_s}{B_o} + \frac{\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y}{\omega_{ce}} \right] = 0 \tag{8}$$

the term $\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y$ can be written as

$$\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y = \sum_{\vec{k}} \left[\frac{1}{2} \frac{\partial}{\partial y} |V_{fy}(\vec{k})|^2 + V_{fz}(\vec{k}) \frac{\partial V_{fy}(\vec{k})}{\partial x} \right] \tag{9}$$

where $V_{fy}(\vec{k})$, $V_{fx}(\vec{k})$ are fourier amplitude given by

$$V_{fx}(\vec{k}) = \frac{\omega_{ce}^2 \, E_{fy}/B_o}{(-\omega_k^2 + \omega_{ce}^2)} \tag{10}$$

$$V_{fy}(\vec{k}) = \frac{-i\, e/m_e \, \omega_k \, E_{fy}(\vec{k})}{(\omega_k^2 - \omega_{ce}^2)} \tag{11}$$

Since LHW lie in Y-Z plane $\partial V_{fy}(\vec{k}) / \partial x = 0$

Hence we have $$\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y = \sum_{\vec{k}} \frac{1}{2} \frac{\partial}{\partial y} |V_{fy}(\vec{k})|^2$$

$$= \sum_{\vec{k}} \frac{\frac{1}{2} \frac{e^2}{m_e^2} \omega_k^2 \frac{\partial}{\partial y} | E_{fy}(\vec{k}) |^2}{(\omega_k^2 - \omega_{ce}^2)^2} \tag{12}$$

Substituting $E_{fy}(k) = E_f \cos\theta$ where θ is the angle between Y and K we have

$$\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y = \sum_{\vec{k}} \frac{\frac{1}{2} \frac{e^2}{m_e^2} \cos^2\theta\, \omega_k^2 \frac{\partial}{\partial y} |E_f(\vec{k})|^2}{(\omega_k^2 - \omega_{ce}^2)^2} \tag{13}$$

Since $|E_f(\vec{k})|^2/8\pi = N_0 \omega_k n_k$ where N_0 is the average LH plasmon density and $n_k = N_{k0} + n_{k1}$. Hence equation (13) can be written as

$$\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y = \sum_{\vec{k}} \frac{4\pi e^2}{m_e^2} \frac{\cos^2\theta\, \omega_k^3 N_0}{(\omega_k^2 - \omega_{ce}^2)^2} \frac{\partial \tilde{n}_{k1}}{\partial y} \tag{14}$$

Substituting $\tilde{n}_{k1}$ from equation (3) we have

$$\langle \vec{V}_f \cdot \nabla \vec{V}_f \rangle_y = \sum_{\vec{k}} \frac{4\pi e^2}{m_e^2} \frac{\cos^2\theta\, \omega_k^3 N_0}{(\omega_k^2 - \omega_{ce}^2)^2} \frac{\omega_k}{\partial n_0} \frac{\vec{q} \cdot \nabla N_{\vec{k}0}}{(\vec{V}_g \cdot \vec{q} - \Omega)} \frac{\partial \tilde{n}_1}{\partial y}$$

where $n_1 = \tilde{n}_1 e^{i(qy - \Omega t)}$ so that $\partial \tilde{n}_1/\partial y = iq\tilde{n}_1$ (15)

While evaluating the dispersion relation for DCLC modes we will make use of quasi-neutrality condition. This is consistent with the fact that DCLC instability is for long wavelengths i.e.

$b = \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{1}{q\langle a_i\rangle^3} \gg 1$ and the adiabatic approximation which consider: $q \ll K$. Using the quasi-neutrality condition the dispersion relation for DCLC mode in the absence of LHW comes out as

$$0 = \frac{\Omega^*}{\Omega} - b \sum_{n=-\infty}^{\infty} \frac{\Omega}{(\Omega - n\omega_{ci})} \tag{16}$$

where $b = \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{1}{(q\langle a_i\rangle)^3}$ $\quad \Omega^* = \frac{\omega_{pe}^2}{\omega_{ce}} \frac{\epsilon}{|q|}$

Near $\Omega - \omega_{ci}$ this equation predicts unstable roots for b $\gg$ 1 with growth rate $\sim (\omega_{ci}\Omega^* b)^{1/2}/b$. In 2XIIB (17) the typical unstable mode had following parameter; $\omega_{pi}^2/\omega_{ci}^2 \simeq 10^4$, $q\langle a_i\rangle \sim 3$, $\Omega \sim \omega_{ci} \sim 3\times10^7$ rad/sec. For this mode equation (16) would predict a growth rate $\sim 0.3\times10^7$ rad/sec, which agrees fairly well with the observed growth rate $\sim 0.1\times10^7$ rad/sec. Hence using $\tilde{n}_{e1} = \tilde{n}_1$ in equation (15) and substituting in equation (8) we have the modified electron density perturbation as

$$\tilde{n}_{e1} = \frac{-i\, E_s/B_o\, \epsilon\, n_o/\Omega}{\left[1 - \frac{\cos^2\theta}{\omega_{ce}} \frac{4\pi e^2}{m_e^2} \frac{n_o q}{\Omega} \frac{\epsilon}{1} \cdot \frac{\omega_K^4}{(\omega_K^2 - \omega_{ce}^2)^2} \times \frac{1}{2n_o} \int \frac{\vec{q}\cdot\nabla N_{\vec{K}o}\, d\vec{K}}{(\vec{q}\cdot\vec{V}_g - \Omega)}\right]} \tag{17}$$

where now in the turbulence term the summation has been replaced by an integration. As ions are not affected by the PF, $\tilde{n}_{i1}$ can be calculated in the way shown by Post et al (21). Then using the quasi-neutrality condition $\tilde{n}_{i1} = \tilde{n}_{e1} = \tilde{n}_1$ we have the modified dispersion relation for DCLC modes as

$$\frac{\Omega^*/\Omega}{[1 - T/\Omega]} = b \sum_{n=-\infty}^{\infty} \frac{\Omega}{(\Omega + n\omega_{ci})} \tag{18}$$

where
$$T = \frac{\cos^2\theta}{2} q^2 \frac{\Omega^*}{\omega_{pi}} W U_{the}^2 \frac{\omega_K^4}{(\omega_K^2 - \omega_c^2)^2} \times \int \frac{\vec{q} \cdot \frac{\partial N_{K0}}{\partial \vec{K}} \, d\vec{K}}{(\vec{V}_g \cdot \vec{q} - \Omega)} \tag{19}$$

$$W = \frac{N_0 \omega_{pi}}{n_0 K T_e} \quad \text{etc.}$$

The integration in the K-plane has to be performed according to Landau's prescription. Hence the turbulence will give a Cauchy's principle value given by T_R and a pole term given by T_i due to resonance of LH plasmons with DCLC modes. The resonance condition is given by $V_g(K) = \frac{\Omega}{q \cos\theta}$. As V_g typically $\sim V_0$ $> \Omega/q \sim U_{thi}$ ($V_0 > U_{thi}$ is the threshold for LHW instability) and $\cos\theta < 1$, the resonance condition will hold and the damping or growth coming from this resonance may be significant. The damping or growth will depend upon the slope of LH plasmon distribution at K given by $V_g(k) \left(= \frac{\omega_{LH}}{k}\right) = \Omega / q \cos\theta$. To perform the integration in K-plane we model the LH plasmon distribution function in Y-Z plane by a broad two dimensional gaussian spectrum given by

$$N_{k_0} = \frac{1}{2\pi\Delta^2} \exp\left[-\frac{(K_\perp - K_{\perp 0})^2}{2\Delta^2} - \frac{(K_\parallel - K_{\parallel 0})^2}{2\Delta^2} \right] \tag{20}$$

where $K_0^2 = K_{\perp 0}^2 + K_{\parallel 0}^2$ $(K_{\perp 0} \gg K_{\parallel 0})$ and Δ is the width of the spectrum. Since we have assumed a broad spectrum we take $\Delta \sim K_0$. From equation (19) T_γ and T_i can be written as

$$T_\gamma = \frac{\cos^2\theta}{2}\, q^3 \frac{\Omega^*}{\omega_{pi}} W \frac{\omega_K^4}{(\omega_K^2 - \omega_{ce}^2)^2} U_{the}^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial N_{k_0}}{\partial K_\perp} \frac{dK_\perp dK_\parallel}{(V_g q \cos\theta - \Omega)} \tag{21}$$

To do the pole integration we make the approximation that V_g is a function of $K_\perp$ through $V_g = \omega_{LH}/K_\perp$. In this case $K_\parallel$ integral can be done directly using equation (20) while $K_\perp$ integral will have a pole at $K_{\perp\gamma} = \omega_{LH} q/\Omega$. Hence

$$T_i = -\frac{\pi}{2} \cos^2\theta\, q^3 \frac{\Omega^*}{\omega_{pi}} W \frac{\omega_K^4}{(\omega_K^2 - \omega_{ce}^2)^2} U_{the}^2 \frac{q\,\omega_{pi}}{\Omega^2}$$

$$\frac{(K_{\perp\gamma} - K_0)}{\Delta^2} \frac{1}{\sqrt{2\pi}\Delta} \exp\left[-\frac{(K_{\perp\gamma} - K_{\perp 0})^2}{2\Delta^2} \right] \tag{22}$$

and

$$K_{\perp\gamma} \sim \frac{\omega_{LH} q}{\Omega}, \quad K_{\perp 0} \sim K_0 \sim \frac{\omega_{LH}}{V_0} \sim \frac{\omega_{pi}}{V_0} \tag{23}$$

In the experimental observations the maximum power is seen to be concentrated in the fundamental mode at $\Omega \sim \omega_{ci}$ (14). To look for the stability of this mode we put $T = T_r + i T_i$, retain $n = 1$ term in the summation and solve the resultant quadratic equation for Ω as

$$\Omega = \frac{\Omega^*}{2b} + \frac{T_r}{2} + i\frac{T_i}{2} \pm \left[\frac{(\Omega^* + bT)^2 - 4\Omega^* \omega_{ci} b}{2b}\right]^{1/2} \tag{24}$$

Now to examine the DCLC spectrum in the presence of LHW, we evaluate the relative orders of T_r and T_i as follows. We write T_r as

$$T_r = \frac{\cos\theta}{2} \, q \, \frac{\Omega^*}{\omega_{pi}} W U_{the}^2 \frac{\omega_k^4}{(\omega_k^2 - \omega_{ce}^2)^2} \iint_{-\infty}^{\infty} \frac{\frac{\partial N_{ko}}{\partial k_\perp} dk_\perp dk_\parallel}{(V_g - \Omega/q \cos\theta)} \tag{25}$$

In equation (25) $k_\parallel / k_\perp \ll 1$, $\theta \simeq 0$ and $\cos\theta \sim 1$. And as $V_g \sim V_0 > \Omega/q \sim U_{thi}$, we may expand the denominator in powers of Ω/qV_g and retain the leading term. For N_{ko} we may substitute from equation (20) and perform the $k_\parallel$ integration directl For performing $k_\perp$ integration we put $V_g = \frac{\omega_{LH}}{k_\perp} = \frac{\omega_{pi}}{k_\perp}$ in which case T_r becomes

$$T_r \sim W \frac{q^2 U_{the}^2}{\Omega^2} \left(\frac{\Omega \Omega^*}{\omega_{pi}^2}\right) \frac{\omega_k^4}{(\omega_k^2 - \omega_{ce}^2)^2} \Omega \tag{26}$$

For typical parameters of present day high density mirrors e.g. 2XIIB, PR-7, etc., $\omega_k > \omega_{ce}$, $\Omega/q \sim U_{thi} \sim 10^7$ cm/sec, $\Omega^* = 10^{10}$
then $\sim$ 5x10^8 cm/sec, $\omega_{ci}/\omega_{pi} \simeq 1/10^2$ ($\omega_{ci} \sim 3\text{x}10^7$ rad/sec . , $\epsilon \sim 1/7 \text{ cm}^{-1}$
$\omega_{pi} \sim 1.6\text{x}10^9$ rad/sec for plasma density $\sim 10^{13}$ cm^{-3}) and a modest

level of LHW turbulence allowed in weak turbulence theory which we are considering i.e. $W = (m_e/m_i)$ we have $T_r \simeq \omega_{ci}/10 < \omega_{ci}$. On the other hand in expression for T_i in equation (22) $k_{\perp r} \sim \omega_{LH} q/\Omega \simeq \frac{\omega_{pi}}{v_{thi}}$ is always $> k_0 \sim \frac{\omega_{pi}}{V_0}$ as $V_0 > v_{thi}$ is the threshold for LHW. Hence T_i will always bear a negative sign and cause damping (as we will show later). In an order of magnitude sense

$$T_i \sim \left(q/\Delta\right)^3 \frac{\omega_k^4}{(\omega_k^2 - \omega_{ce}^2)^2} W\left(\frac{v_{the} q}{\Omega}\right)^2 \frac{k_{\perp r}}{q_-} \cdot \frac{\Omega^*}{\omega_{ci}} \Omega \tag{27}$$

For $\Delta \sim k_0 \sim \omega_{pi}/V_0$, $k_{\perp r} \sim \omega_{LH} q/\Omega \sim \frac{\omega_{pi}}{v_{thi}}$ $V_0 \gtrsim v_{thi}$

$W = m_e/m_i$, $T_i \simeq \frac{\omega_{ci}}{10} < \omega_{ci}$. Hence $|T| < \omega_{ci}$. Typically DCLC oscillations are excited with $q_\perp a_i \gtrsim 1$; in 2XIIB 2.9 $< q_\perp a_i <$ 6.1. For these wave numbers $b \simeq \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{1}{(q_\perp a_i)^3} \sim 3\times10^3$. We will now examine the stability of these wave numbers in the presence of a weak LHW turbulence. In equation (24) if we put $T = 0$, we get the dispersion relation obtained by Post and Rosenbluth (21) which is

$$\Omega = \frac{\Omega^*}{2b} \pm \frac{1}{2b}\left[\Omega^{*2} - 4\omega_{ci}\Omega^* b\right]^{1/2} \tag{28}$$

For typical 2XIIB parameters i.e. $\Omega^* \simeq 10^{10}$ rad/sec, $b \simeq 3\times10^3$, $\omega_{ci} \simeq 3\times10^7$ rad/sec. Equation (28) gives the real part of DCLC frequency $\Omega_r \simeq \omega_{ci}$. For these wave numbers $4\omega_{ci}\Omega^* b > \Omega^{*2}$ under the radical in equation (28), hence they are unstable with a typical growth rate $\gamma \simeq (\omega_{ci}\Omega^* b)^{1/2}/b$ $\simeq .3\times10^7$ rad/sec. The growth rate observed in 2XIIB $\simeq$ .02 to .03 ω_{ci} $\simeq .1\times10^7$ rad/sec. The discrepency of a factor of 3 may be attributed to the fact the the effects mentioned earlier i.e. electromagnetic effects in the electron contribution, ion drift term which are stabilizing are not included. In equation (24) we note that for typical parameters $\Omega^* \gtrsim b|T|$ and $\Omega^{*2} < 4\Omega^*\omega_{ci}b$ hence the modified growth rate in the presence of LHW turbulence is given by

$$\gamma = -\frac{T_i}{2} \pm \frac{(\omega_{ci}\Omega^* b)^{1/2}}{b} \tag{29}$$

The damping induced by LHW plasmons is $\sim T_i/2 \sim \omega_{ci}/20 \simeq 0.15\times10^7$ rad/sec which of the same order as $(\omega_{ci}\Omega^* b)^{1/2}/b \sim 3\times10^7 \frac{rad}{sec}$. Thus we see that a low level of LHW turbulence can bring about a significant stabilization of DCLC modes.

<u>Discussion</u>:

These findings are contrary to the findings of Shiang et al who find stabilization only in the range $\omega_{LH} < \Omega < \omega^*$ where $\omega^* = \frac{1}{2}[\omega_1 + (\omega_1^2 + 4\omega_{LH}^2)]^{1/2}$ and $\omega_1 = \frac{\omega_{LH}^2}{\omega_{ci}}\frac{\epsilon}{|\eta|}$ They also find a region of enhancement. In our case, we do not find a region of enhancement. It follows from here that Lower Hybrid plasmon damping has significant stabilizing influence on the observed wave number range of DCLC spectrum. It should be noted that this damping cannot be converted into growth by shifting the turbulent spectrum so that $k_{\perp T} < k_0$. This will require $V_0 < v_{thi}$ which violates the condition for the excitation of lower hybrid modes.

Thus it follows that presence of lower hybrid turbulence in mirrors has two fold advantage firstly it leads to an efficient heating of ions and electrons and secondly it helps in reducing the fluctuation level due to DCLC modes which will help in improving the ion life time inside the trap.

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CHAPTER IX

SUMMARY AND CONCLUSIONS

What has been thrown:

In the community of physicists, plasma physicists are especially known for their knack of circumventing the hurdles by throwing away terms under the garb of "suitable approximations". As such there is nothing wrong in throwing away things (as long as you do not hurt others with it) but sometimes it may so happen that in this process of throwing one may throw away the baby with the bath. As a safeguard against such a folley it is required from a plasma physicist that he should clearly identify what he has thrown and show that atleast he has not thrown away the baby with the bath.

The calculations presented in this thesis are no different from others in this respect. Lot of terms have been thrown away. Hence in this Chapter we will make an attempt to identify what have we thrown, how much has been lost in this throwing and how can it be picked up again if the need arises again.

Clearly in the calculations presented in this work we did not wish to reproduce exactly the observations of β-2, 2XIIB, Base-Ball II, Constance II. Our aims have been rather modest. Our aim was to identify certain physical processes which we think are responsible for the observations of 2XIIB, Constance II, Base-Ball II and β-2 and further to evaluate their relative importance in some simplified situations. To this end approximations have been made which will certainly have to be dispensed with far more quantitative accuracy. In this respect these calculations are to be regarded as only an initial step towards a more complete calculations. It is on this aspect that the merit of these calculations require judgement.

As can be clearly seen that the general direction of the work presented in this thesis is in studying various normal modes of mirror plasmas, their non-linear couplings, their suppression or enhancement in the presence of electron beams or lower hybrid waves, their nonlinear saturation and their suppression by feedback circuits. In the second chapter we have studied the nonlinear instability of a low frequency flute mode first predicted by Varma (1) for mirror plasmas. In our calculations we have studied the nonlinear instability of this mode in the presence of ion-cyclotron oscillation which were seen alongwith

Varma mode in Base-Ball II (2). The interesting feature of this coupling between the Varma mode and ion-cyclotron mode is that the kinetic wave equations yield a solution which periodically bursts in time. The time scale of these bursts turns out to be 400 Ω_i^{-1} (Ω_i is the ion-cyclotron frequency) which is quite similar to the bursts of large amplitude flute mode seen in Base-Ball II. This is in contrast to the results obtained by Simon and Weng (3) who studied the nonlinear instability of Varma mode in the presence of the other flute modes and got an explosive solution for the Varma mode which they correlated to the instability observed in 'Alice' and 'Phoenix'. In Base Ball II such a coupling is not possible as all the flute modes are linearly stable. However, this work suffers from incompleteness on following accounts, firstly we have not made any effort to identify the other small amplitude stable flute mode (0.5 V) observed in Base-Ball II. We suspect that this could be one of the other flute modes. Probably because of its low energy content the coupling of this mode to the ion-cyclotron mode is weak and hence it remains stable. Of course such a statement can be made only after a calculation has been made to show that the matrix element of the coupling of this mode with ion-cyclotron mode is small as compared to the matrix element of coupling between the Varma mode and the ion-cyclotron mode. Secondly, in Section 3 of Chapter III the linear theory of the Varma mode is worked out for a perfect flute mode i.e. $K_{\parallel}^{"} = 0$ while in the nonlinear instability calculation we have considered the Varma-mode with finite $K_{\parallel}^{"}$. Qualitatively a finite $K_{\parallel}^{"}$ would give rise to linear damping by particles which has been included in the calculations. But as far as nonlinear instability is concerned, it will not be aff ted as it depends on $k_{\parallel}' = k_{\parallel} - k_{\parallel}''$ which remains finite as long as one of $k_{\parallel}$'s remain finite. N ertheless to have a better calculation, the linear theory of the Varma mode with finite $k_{\parallel}''$ should be examined. These considerations will be taken up in future.

In Chapters III, IV and V, we have examined the effect of electron beam (EB) induced Langmuir waves on DCLC or HFCLC instabilities. It should be pointed at the outset that in all our calculations regarding DCLC or HFCLC instabilities we have made use of the electrostatic approximation and have neglected the effect of temperature gradients. Baldwin (4) and Catto et al (5) have pointed out their importance in experimental situations. They have a stabilizing effect. However, we have dropped them from our calculations as we wanted to consider the worst possible situations. For better quantative accuracy they certainly will have to be co sidered.

In Chapter III we have identified a physical process which we think plays an important role in suppression or enhancement of DCLC turbulence in electron beam injected mirrors i.e. the process of resonant damping by electron beam induced Langmuir plasmons. Of late this technique has achieved special attention as it provides an efficient method of suppressing DCLC turbulence in the end plugs of tandem mirrors (6). The usual warm plasma stabilization method may not be quite useful here because it cools the electrons which leads to a reduction of the ambipolar potential of the plugs and thus worsens the confinement in the central cell (7). The electron beam method does not suffer from this defect, as we have shown that by resonant damping it controls the DCLC turbulence and due to collapse of Langmuir turbulence it heats up the electrons which lead to an increase in the ambipolar potential and thus a better confinement for the central cell ions. The only defect of this method is that for certain range of beam powers it enhances the turbulence. In Chapter III, we have outlined an approach by which this range can be identified and avoided. In the same chapter we have given a set of closed equations to study the time evolution of the ion distribution function and to calculate some important parameters like final electron and ion temperatures, ion life times , final fluctuation level. However, this work suffers from a certain degree of incompleteness on the following account: firstly we have not undertaken the numerical integration of these equations to calculate parameters mentioned above but rather by qualitative arguments and estimates we have made attempts to explain the observations from these equations. One reason why such a study was not taken up was the lack of efficient computing facilities at the institute and secondly parameters like background gas flux, evolution of electron and ion temperatures, ion life time, diffusion rates in different energy channels which would have been required for integration and confirmation of these equations were not made available in the published references.

The other source of error in the calculation may be that for the stationary spectrum we have used the stationary spectrum calculated by Tsytovich et al (8) in the absence of the magnetic field. This has been done under the approximation $\omega_{pe} > \omega_{ce}$ where the effect of field on particle motion is not very strong. Thus while a more exact calculation of the stationary spectrum may very well offset the quantitative behaviour, it will not alter much the qualitative features of the process considered in the model (i.e. the resonant damping by Langmuir plasmons).

Hence on the whole it can be said about this calculation that to obtain a better confirmation (or refutation?) of the model we must integrate the closed set of equations given in Chapter III and calculate the final fluctuation level, electron and ion temperatures, ion life time and compare them with the experimental values (if they can be achieved). Such a programme will be taken up in future here.

In the Fourth and Fifth Chapters, we have studied the effect of electron beam induced Langmuir waves on another loss cone generated instability i.e. the HFCLC instability. We find that while in the high frequency part of HFCLC, Langmuir waves have favourable effects i.e. they generate sufficient anomalous resistivity to stabilize the HFCLC mode, in the low frequency range, their effect is harmful for the confinement; they tend to reduce the critical lengths. In our estimates regarding the reduction of critical lengths we have not included the important reflections mechanism like reflection due to corrections to WKB approximation (9), incoherent bouncing of electrons (10), turning points due to ion-cyclotron resonances (11), etc. M. Gerver (12) has pointed that all these mechanisms alongwith high- β effect tend to reduce the critical length. On this account our estimates about critical length may not be of experimental interest. But neither they were meant to be so. All we wanted to show from these estimates was that even in simple situation (low- β effect, and absence of wave reflections) the electron beam is harmful enough. If the various wave-reflection mechanisms are considered the situation will become worse.

In the Sixth Chapter, we have given a simple calculation to show the importance of orbit diffusion effects in the saturation of DCLC turbulence in mirrors. Hitherto all the theories put forward for the saturation of turbulence in mirrors are based on perturbation schemes. One of the unsatisfactory features of any perturbation expansion schemes in general is that they do not furnish a prescription for the truncation of the series. Because of this it becomes difficult to decide upto what orders one must retain terms in order to explain a physical process. Thus on one hand we have Galeev (13) and Baldwin et al (14) who think that the first order effects like plateau formation etc. are enough to explain the saturation of DCLC turbulence in mirrors. On the other hand we have Rosenbluth and Aamodt et al (15) who think that one has to go to third order effects like detuning of resonance in order to explain the saturation of turbulence in mirrors. Perturbed orbit formalism does not suffer from such defect as it is not based on any perturbation scheme. This interesting result borne out by the calculations is that damping due to perturbed orbit effects is sufficient to overcome the growth due to ion distribution. These results are not very surprising especially in the light of the fact that velocity space diffusion which is responsible for the diffusion of trajectories is very strong in mirrors i.e. 100 eV/1 μs (7). In the same chapter we have also shown that in perturbed orbit formalism the fluctuation level is proportional to fourth power of the linear growth while ion life time is inversely proportional to third power of the linear growth. So that in the presence of the warm plasma where the linear growth is reduced by a small amount, fluctuation level may decrease by a large amount and ion life time may increase significantly. This points out the importance of the effect of orbit diffusion in stabilization of DCLC turbulence in the presence of warm plasma streams. However, this calculation may suffer from incompleteness because perturbed orbit theory itself is not a complete theory. In it, the operator is normalised but vertex normalisation is not taken care of. The theory which satisfactorily takes into account operator and vertex normalisation is the "Direct Interaction Approximation". A study of ion-cyclotron turbulence under this approximation is being considered at present.

In Chapter VII, we have pointed out the efficiency of damping of DCLC turbulence by lower hybrid turbulence. A modest level of lower hybrid turbulence is going to be inevitable in future neutral beam injected mirrors. In this case this turbulence may serve a double purpose i.e. it will lead to an efficient heating of electrons and ions (16) and it will help in controlling the DCLC turbulence. As stated before for more reliable estimates the effects of magnetic perturbations and temperature gradients should be included.

In Chapter VIII, we have investigated the possibility of suppressing the DCLC turbulence by feedback circuits. In these calculations we have made use of the assumption that electron sources are present uniformly in the plasma. This is not a very correct assumption because metal probes (which act as sources) are present at finite points only. However, as Rutherford (17) has pointed out, this assumption gives results which agree fairly well with the experimental observations. The other source of incompleteness is that we have not investigated the stability in phase angles other than 90°. This has to be done numerically and may be taken up in future.

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