# Neutrino Masses And Leptogenesis

## A THESIS

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*by*

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**UDAIPUR**

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# Certificate

*I feel great pleasure in certifying that the thesis entitled "Neutrino Masses And Leptogenesis" embodies a record of the results of investigations carried out by Mr. Santosh Kumar Singh under my guidance. I am satisfied with the analysis of data, interpretation of results and conclusions drawn.*

*He has completed the residential requirement as per rules.*

*I recommend the submission of thesis.*

Date:

Prof. Utpal Sarkar
Physical Research Laboratory

# Declaration

*I hereby declare that the work incorporated in the present thesis entitled "Neutrino Masses And Leptogenesis" is my own work and is original. This work (in part or in full) has not been submitted to any University for the award of a Degree or a Diploma.*

Signature of the Candidate.

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# Abstract

Neutrinos are massless in the Standard Model (SM )and there is no CP violation in the leptonic sector. Robust evidences of neutrino oscillations have confirmed that the neutrinos do have masses, although small. The understanding of neutrino masses requires new physics beyond SM. As the neutrino is charge-neutral particle, it can be a Majorana fermion where neutrino will be its own antiparticle. Seesaw mechanism provides a natural suppression of neutrino masses by introducing a lepton number violating source at some high scale. In this thesis, we will study neutrino masses in various seesaw scenarios and Grand unified theories after a short review of experimental status of neutrino masses and mixing. Then we study the role of neutrinos in two very important astrophysical and cosmological problems, 1) baryon asymmetry of the present universe and 2) cosmic coincidence problem for dark energy.

If the neutrinos are Majorana fermions, the low energy neutrino mass matrix comes out to be symmetric. Assuming neutrinos to be Majorana fermions we have constructed rephasing invariant measures of CP violation with elements of the neutrino mass matrix, in the basis in which the charged lepton mass matrix is diagonal. We have applied our approach to study CP violation in all the phenomenologically acceptable 3-generation two-zero texture neutrino mass matrices and have shown that in any of these cases there is only one CP phase which contributes to the neutrino oscillation experiment and there are no Majorana phases.

An attractive explanation for tiny neutrino masses and small matter antimatter asymmetry of the present Universe lies in leptogenesis. At present the *size* of the lepton asymmetry is precisely known, while the *sign* is not known yet. We have determined the sign of this asymmetry in the framework of two right-handed neutrino models by relating the leptogenesis phase with the low energy CP violating phases appearing in the leptonic mixing matrix. It has been shown that the knowledge of low energy lepton-number violating rephasing invariants can indeed determine the sign of the present matter antimatter asymmetry of the Universe and hence indirectly probing the light physical neutrinos to be of Majorana type.

One of the very interesting model of dark energy in current days is based on proposal of mass varying neutrinos. We have proposed a left-right symmetric model that can accommodate this neutrino dark energy proposal. Type III seesaw mechanism is implemented to give masses to the neutrinos. Unlike earlier models of mass varying neutrinos, in the present model the mass parameter that depends on the scalar field (acceleron) remains very light *naturally*. The model is then embedded in an SO(10) Grand Unified Theory and the allowed symmetry breaking scales are determined by the condition of the gauge coupling unification. The neutrino masses are studied in detail in this model, which shows that at least 3 left-right gauge singlet fermions are required for consistent understanding of the observed low energy neutrino mass spectrum.

# Contents

# Publications

The content of the thesis is based on following work:

1. **Title - Leptogenesis and low energy CP phases with two heavy neutrinos.**
   **Author:** Kaushik Bhattacharya, Narendra Sahu, Utpal Sarkar, Santosh K. Singh (Ahmedabad, Phys. Res. Lab)
   **Published** in **Phys.Rev.D74:093001,2006.**
   **e-Print:** hep-ph/0607272

2. **Title - CP violation in neutrino mass matrix.**
   **Author:** Utpal Sarkar, Santosh K. Singh (Ahmedabad, Phys. Res. Lab)
   **Published** in **Nucl.Phys.B771:28-39,2007.**
   **e-Print:** hep-ph/0608030

3. **Title - Left-right symmetric model of neutrino dark energy.**
   **Author :** Jitesh R. Bhatt (Ahmedabad, Phys. Res. Lab) , Pei-Hong Gu (ICTP, Trieste) , Utpal Sarkar, Santosh K. Singh (Ahmedabad, Phys. Res. Lab)
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   **e-Print:** arXiv:0711.2728 [hep-ph]

4. **Title - Neutrino Dark Energy in Grand Unified Theories.**
   **Author :** Jitesh R. Bhatt (Ahmedabad, Phys. Res. Lab) , Pei-Hong Gu (ICTP, Trieste) , Utpal Sarkar, Santosh K. Singh (Ahmedabad, Phys. Res. Lab)
   **e-Print:** arXiv:0812.1895 [hep-ph] (**communicated**)

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# Chapter 1

# Introduction

Although the existence of neutrinos was first proposed in 1930 by Pauli to explain the continuous energy spectrum of the electron coming from the nuclear beta decay process and was confirmed in 1956 by Cowan and Reines [4], the fact that they have masses too could only be verified conclusively in the last two decades. The obvious reason behind the difficulty to observe the neutrinos lies in the fact that they are very weakly interacting particles with very small masses. They interact with other particles through gravitational and weak interactions. However, their gravitational influence comes out to be very small compared to their weak interaction.

As the neutrinos interact through weak interaction, it promises to play some special role in particle physics which can not be shared by any other particle. The properties of many astrophysical objects are addressed by detecting and studying the light coming from them. However, the light has to travel through the different galactic and intergalactic spaces before coming to earth and, hence, is quite perturbed. The situation is different with the case of neutrinos which get almost no perturbation while coming from source to us. So it promises to carry all the original information and can be analyzed to reproduce the more accurate property of the astrophysical object. This has provided enough motivation and speculation to explore the feasibility of telescope based on neutrino detection.

This thesis is divided in two parts. The first part concerns about both general and model dependent study of masses and mixings of neutrinos. Within the Standard Model (SM) the neutrinos are massless and are described by two component Weyl spinors. This is because of the absence of right-handed neutrinos in the SM. Hence there is no CP violation in the leptonic sector within the SM. We review some main features of the SM in chapter 2 and discuss fermion masses in detail.

Although neutrinos were thought to be massless initially, current evidences suggest that neutrinos are massive, and they mix with each other. The atmospheric neutrino problem [5] , solar neutrinos [6–8] and laboratory neutrino oscillation experiments [9–11] have provided measurements of three mixing angles, as well as two mass squared differences. But we do not know the absolute values of neutrino masses and there is no evidence of CP (charge conjugation+parity) violation in leptonic sector till now.

To accommodate the neutrino masses, one need to go beyond the SM. One possible extension of the SM is to introduce three right-handed neutrinos in SM Lagrangian and to provide Dirac masses to the neutrinos. Another fact about the neutrino is that it has no electric charge which endows it with certain properties not shared by the charged fermions of the SM, i.e., it can be its own antiparticle without violating electric charge conservation. In that case, the neutrino is called a Majorana fermion. We first explore the possibility of writing a model independent Lorentz invariant mass term for both Dirac and Majorana neutrinos in chapter 3 and then discuss various neutrino experiments related to both flavor oscillations of neutrinos and direct detection of neutrino masses.

The symmetric nature of the mass matrix of the Majorana neutrinos at the low energy requires only one unitary matrix to diagonalize it, which is not possible in the case of Dirac neutrinos. So, in the basis where the charged lepton mass matrix is real and diagonal, all the information about mixing angles and CP violating phases remains in the neutrino mass matrix. At the same time, the diagonalizing unitary matrix turns out to be the mixing matrix in the leptonic sector and usually called as $U_{PMNS}$ mixing matrix [12–14].

In the literature, the question of CP violation in the leptonic sector is usually discussed by studying the neutrino mixing matrix [15–17]. The trouble in defining the measures of CP violation in this way is that it requires the neutrino mass matrix to be diagonalized by some unitary matrix first. In chapter 4, we try to construct independent measures of CP violation, directly from the low energy neutrino mass matrix elements for any number of generations without restricting our analysis to any specific origin of the neutrino masses even if there are zero entries present in the neutrino mass matrix.

Analogy from the quark sector suggests that neutrinos should have masses more or less of the order of charged lepton masses. But very small masses of neutrinos ($\sim eV$) requires some new mechanism to explain why neutrinos are so light compared to the charged leptons. Assuming the neutrinos are of Majorana type, natural suppression of neutrino masses can be explained by the elegant seesaw mechanism [18–20]. Three types of seesaw mechanism have been discussed in the literature for the purpose. Chapter 5 is devoted to the detailed discussion of the various seesaw mechanisms along with their realization in the left-right symmetric extension of SM.

The theory of Grand Unification has emerged as the most elegant and attractive scenario to go beyond SM. Out of the many attractive features of Grand Unified

Theory (GUT), one is that the fermion mass matrices which look independent of each other in the SM, get related in the GUT framework. This along with the recent data of neutrino masses and mixing has triggered the study of the neutrino masses in various , especially predictive, GUT models. In chapter 6, we review both $SU(5)$ and $SO(10)$ GUTs and study various relations between fermion mass matrices while keeping main focus around neutrinos.

The second part of the thesis mainly concerns about the role of neutrinos to address two very challenging astrophysical and cosmological problems 1) cosmic coincidence of dark energy and 2) current baryon asymmetry of the universe.

Present observations reveal that the dark energy (DE) contributes about 70% to the total density of our universe with equation of state $\omega = -0.98 \pm 0.12$, indicating that we are living in an accelerated universe [21–23]. The DE can be expressed as $E^4$ and this correspond to the energy scale $E \sim 3 \times 10^{-3}$eV. But the results from Cosmic Microwave Background reveal that universe was dominated by matter at red shift ($z$=1100) with equation of state $\omega = 0$, i.e., the acceleration of the universe is a fairly recent phenomena. This is known as the cosmic coincidence problem.

DE can be invoked by introducing a cosmological constant in the Einstein's equation, but it faces severe fine-tuning problem as its scale lies nowhere near the scale of DE. So one assumes that it is zero by some mechanism and tries to explore other ways to explain current acceleration of the universe. One possibility is to introduce what are called tracking scalar fields rolling slowly in flat potentials which track the matter or radiation energy density during matter or radiation dominated epochs being sub-dominant and become dominant only at the current phase of the universe [24–26]. The fact that only known scale near the scale of DE is scale of neutrino masses suggests that neutrinos can play an important role in solving cosmic coincidence problem. In an interesting possibility, a coupled system of neutrino and a light scalar field can behave like DE after neutrinos becomes non-relativistic [27]. Chapter 8 is devoted to the discussion of Quintessence and neutrino dark energy (NDE) models along with the review of the observational evidences of current acceleration of the universe.

In the original model of NDE, the SM is extended by including singlet right-handed neutrinos and giving Majorana masses to the neutrinos which vary with a scalar field, the acceleron. Naturalness requires the Majorana masses of the right-handed neutrinos also to be in the range of eV, so the main motivation of the seesaw mechanism is lost. Also the mechanism cannot be embedded into a left-right symmetric model. In chapter 11, we try to construct a left-right symmetric model of NDE keeping naturalness in mind. We then proceed further to study the embedding of the left-right symmetric model in SO(10) GUT.

Understanding the origin of matter is one of the fundamental questions, the answer to which is most likely going to come from particle physics. Seesaw mechanism is at the heart of particle physics for this purpose, since it not only explains the smallness of neutrino masses but also provides a natural solution to the baryon asymmetry of the universe through leptogenesis [28]. Leptogenesis is a way to understand the baryon asymmetry of the universe by first creating lepton asymmetry in the early universe. Chapter 9 discusses the basic structure of leptogenesis along with some other mechanism for baryogenesis.

Although seesaw mechanism connects low energy neutrino mass matrix with high energy Yukawa couplings, still there is no one to one correspondence between low energy parameter to high energy parameters in general. However, one would like to connect the low energy measures of CP violation (expressed in conventional form) to leptogenesis by choosing a suitable parametrization where it is transparent that which phases are responsible for leptonic CP violation at low energy and which ones are relevant for leptogenesis so that the viability of leptogenesis can be addressed if we can measure strength of CP violation at low energy in neutrino oscillation experiments. With the same motivation in mind, we try to connect leptogenesis to low energy CP measures in chapter 10 in type I seesaw scenario with two right-handed neutrinos where we expect less number of unknown parameters.

Finally, the last chapter on conclusion summarizes our main work and the related results.

# Part I

# Neutrino masses and CP invariants

# Chapter 2

# Standard Model and fermion masses

To the best of our present knowledge, the nature seems to be equipped with four kinds of interactions (1) gravitational, (2) strong, (3) electromagnetic and (4) weak. The gravitational interaction is universal, in the sense that it exists between any two particles having energy and momentum. It is responsible for the formation of Galaxies, stars and planets. So far , a consistent renormalizable quantum theory of gravity is still awaited.

The strong interaction manifests itself in the nucleon-nucleon interactions inside the nucleus. Actually, the nucleons are not the elementary particles but consist of quarks which, up to now, are believed to be the basic building blocks of baryons. The strong interaction, in its fundamental form, is mediated by gluons between the quarks and its effective manifestation is the one, which is observed as a very strong nuclear force between the protons and the neutrons. The theory describing the strong interaction is known as quantum chromodynamics (QCD) which is based on a non-Abelian $SU(3)_c$ gauge symmetry.

The electromagnetic interaction is mediated by photons between any two particles carrying nonzero electric charge. It is best described by a theory known as quantum electrodynamics (QED) based on an Abelian $U(1)_Q$ gauge symmetry. $Q$ corresponds to the charge of a particle. The need for the weak interaction arises for the understanding of the nuclear beta decay. Initially, a six dimensional effective operator corresponding to four fermions was used to address the weak interaction. However, this four fermion operator badly fails to address the interaction at higher energies. Later, it was realized that it can emerge from a more fundamental renormalizable theory. This new renormalizable theory, known as the electroweak theory, has passed all the experimental tests up to now and is a very essential part of the Standard Model (SM).

The SM of particle physics has been constructed to address all the three interactions, other than gravity, on one platform. It consist of two distinct parts one strong and the other one electroweak. We will be concentrating mostly on the electroweak part in this chapter.

## 2.1 The model

The SM is based on the local gauge symmetry with the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ ($G_{321}$) where suffix c corresponds to the color, L corresponds to the left-handed (for fermions) and Y corresponds to the hypercharge. The $SU(3)_c$ part describes the strong interaction and the $SU(2)_L \times U(1)_Y$ part describes the electroweak interaction. The later is spontaneously broken to the electromagnetic interaction $U(1)_Q$ below the electroweak scale (100 GeV). So the weak and the electromagnetic interaction originate from the more fundamental electroweak interaction. The charge generator $Q$ of $U(1)_Q$ is related to $T_{3L}$, the diagonal generator of $SU(2)_L$, and $Y$ , the generator of $U(1)_Y$, as

$$Q = T_{3L} + Y\,.$$

The fermion sector of the SM comes in three generations all having the same quantum numbers under the SM gauge group. They belong to the fundamental representation of the SM gauge group $G_{321}$. Moreover, the SM is left-right asymmetric, i.e., its left-handed fermions belong to different representations as compared to the right-handed ones. The left-handed fermions are doublets under the group $SU(2)_L$, while the right-handed fermions are singlets. Also the fermions can be classified into two types, quarks and leptons. Quarks participate in both strong and electroweak interactions while leptons participate only in electroweak interactions. The quantum number assignments of the quarks are given as

$$\begin{aligned} Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} &\equiv [3,2,1/6] \\ u_R, c_R, t_R &\equiv [3,1,2/3] \\ d_R, s_R, b_R &\equiv [3,1,-1/3]\,. \end{aligned} \tag{2.1}$$

Similarly the quantum numbers for the leptons are assigned as

$$\begin{aligned} \ell_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} &\equiv [1,2,-1/2] \\ e_R, \mu_R, \tau_R &\equiv [1,2,-1]\,. \end{aligned} \tag{2.2}$$

The right-handed counterpart of the left-handed neutrinos are absent in the model, which leads to massless neutrinos in the SM.

| Gauge Bosons | Symbols | Gauge Group | Quantum No |
|---|---|---|---|
| Gluons | $G_\mu^a$ | $SU(3)_c$ | [8, 1, 0] |
| Non-Abelian weak boson | $W_\mu^i$ | $SU(2)_L$ | [1, 3, 0] |
| Abelian bosons | $B_\mu$ | $U(1)_Y$ | [1, 1, 1] |

Table 2.1: SM gauge bosons

The SM interactions are mediated by spin one bosons known as gauge bosons. The eight gauge bosons mediating strong interaction, called gluons, belong to the adjoint representation of $SU(3)_c$. Similarly, three gauge bosons correspond to the adjoint representation of $SU(2)_L$. Also, $U(1)_Y$ has one more gauge boson. The transformation properties of all the gauge bosons are given in table 2.1.

In addition to the fermions and the gauge bosons, we need a Higgs scalar to break the SM gauge symmetry. Its transformation property is given by the quantum numbers

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \equiv [1, 2, 1/2] \,.$$

The kinetic term as well as the self interaction of gauge bosons are given by the following part of the Lagrangian

$$\mathcal{L}_{gauge} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \,,$$

where

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_3 f^{abc} G^b_\mu G^c_\nu \\ W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_2 \varepsilon^{ijk} W^j_\mu W^k_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \,. \end{aligned}$$

$f^{abc}$ and $\varepsilon^{ijk}$ represent the structure constant of the Lie groups $SU(3)_c$ and $SU(2)_L$ respectively. $g_1, g_2$ and $g_3$ are the three gauge coupling constants of the groups $U(1)_Y, SU(2)_L$ and $SU(3)_c$ respectively.

The couplings of the gauge bosons with the fermions and the Higgs boson arise by simply imposing the local gauge symmetry on the free fermions and the Higgs boson. The local gauge invariance requires partial derivative ($\partial_\mu$) present in the kinetic term of the fermions and the Higgs boson to be replaced by covariant derivative ($D_\mu$). The most general covariant derivative is related to the partial derivative as

$$D_\mu = \partial_\mu - i \left( g_1 Y B_\mu + g_2 \tau^i W^i_\mu + g_3 \lambda^a G^a_\mu \right) \,, \tag{2.3}$$

where $Y$, $\tau^i$ and $\lambda^a$ are the generators of the groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ respectively. $Y$ represents the hypercharge of the particle. However, all the terms inside the bracket in equation 2.3 are present only when the matter particle transforms non-trivially under all the three groups. If a particle is singlet under any of the three groups, the corresponding term will vanish in the covariant derivative. For example, the leptons are singlet under $SU(3)_c$ and so the term associated with the coupling constant $g_3$ will vanish while writing the covariant derivative for the leptons. Similarly, the terms associated with $g_2$ will vanish for all right-handed fermions.

The kinetic term for the fermions and the Higgs boson in the SM also includes their interaction with the SM gauge bosons. The relevant part of the Lagrangian is given as

$$\begin{aligned} \mathcal{L} &= i\overline{Q_L}\not{D}Q_L + i\overline{u_R}\not{D}u_R + i\overline{d_R}\not{D}d_R \\ &+ i\overline{\ell_L}\not{D}\ell_L + i\overline{e_R}\not{D}e_R \\ &+ \left(D_\mu\phi\right)^\dagger\left(D^\mu\phi\right), \end{aligned}$$

where $\not{D} = D_\mu\gamma^\mu$.

## 2.2 Spontaneous symmetry breaking

We know that the weak interaction is a finite range interaction while the electromagnetic interaction is an infinite range interaction. As the photons are massless spin one bosons, it can carry the electromagnetic forces up to infinite range. On the other hand, the finite range dynamics of the weak interaction requires rather massive force carriers. So, three out of four gauge bosons corresponding to the electroweak part of the SM should get appropriate masses.

But, the SM gauge symmetry forbids mass term for any of the gauge bosons. However, if we introduce gauge boson masses by hand, it will spoil gauge invariance and renormalizability. The mechanism to provide masses to the relevant gauge bosons without destroying the renormalizability of the theory, known as Higgs mechanism, is based on spontaneous breaking of SM gauge symmetry.

The spontaneous breaking of a given symmetry is achieved by writing the Higgs potential such that the minima of the potential correspond to a finite vacuum expectation value(vev) of the Higgs field. In other words, the ground state of the theory does not respect the underlying gauge symmetry and breaks it spontaneously. Mass terms for the gauge boson are automatically generated once the theory is expanded around the new vacuum. However, the renormalizability of the theory is still maintained due to the underlying gauge symmetry.

The Higgs potential in the SM is given as

$$\mathcal{L}(\phi) = -\frac{\mu^2}{2}\phi^\dagger\phi + \frac{\lambda}{4}\left(\phi^\dagger\phi\right)^2 ., \tag{2.4}$$

where $\mu^2, \lambda > 0$. The potential is bounded from down and ensures finite vev

$$< \phi^\dagger\phi >= v^2 = \mu^2/\lambda .$$

The gauge invariance allows to choose the following vacuum for the theory

$$< \phi >= \begin{pmatrix} 0 \\ v \end{pmatrix} .$$

The electroweak gauge bosons obtains their mass terms by expanding the kinetic term of the Higgs boson $\left|D_\mu\phi\right|^2$ around the vacuum given above $\phi^{phys} = \phi - < \phi >$ as

$$\begin{aligned} \triangle\mathcal{L} &= \left|D_\mu < \phi >\right|^2 \\ &= \frac{1}{2}\frac{v^2}{4}\left[g_2\left(W_\mu^1\right)^2 + g_2\left(W_\mu^2\right)^2 + \left(-g_2W_\mu^3 + g_1B_\mu\right)^2\right] . \end{aligned} \tag{2.5}$$

As a result, the spontaneous symmetry breaking leads to the following charged gauge boson

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp W_\mu^2\right)$$

with mass $M_{W^\pm} = gv/2$. However, the presence of the mixed terms of the other two gauge bosons $\left(W_\mu^3, B_\mu\right)$ in the third term of the expression 2.5 reveals that these two bosons are not in their mass basis in the broken theory. However, the physical gauge bosons $\left(Z_\mu, A_\mu\right)$ can be simply written as the following linear combination of the original fields $\left(W_\mu^3, B_\mu\right)$ parametrized by weak mixing angle $\theta_W$ as

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix},$$

where $\sin\theta_W = \frac{g_1}{(g_1+g_2)}$. The physical gauge boson $A_\mu$ remains massless while $Z_\mu$ becomes massive with mass $M_Z = M_W/\cos\theta_W$ .

It is straightforward to identify $A_\mu$as the usual gauge field corresponding to the photon, responsible for the electromagnetic interaction, leading to the following expression for elementary electric charge (e)

$$e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} = g_2 \cos\theta_W \,.$$

The vacuum expectation value (*vev*) is estimated by using the measured value of Fermi constant $G_F$ at low energy corresponding to four fermion interaction:

$$v^2 = \frac{1}{\sqrt{2}G_F} = (246\,GeV)^2 \,.$$

The interaction of the physical gauge bosons with the fermions is described as

$$\mathcal{L}_{gauge} = -\frac{g}{\sqrt{2}}\left(J^{\mu}_{W^+} W^+_{\mu} + J^{\mu}_{W^-} W^-_{\mu}\right) \quad (2.6)$$
$$-\frac{g_1 g_2}{e} J^{\mu}_Z Z_{\mu} - e J^{\mu}_Q A_{\mu} \,, \quad (2.7)$$

where the charged current $J^{\mu}_{W^\pm}$ in the first row, the neutral current $J^{\mu}_Z$ and the electromagnetic current $J^{\mu}_Q$ in the second row are written as

$$\begin{aligned} J^{\mu}_{W^\pm} &= \overline{\psi_{Li}}\gamma^{\mu}(\tau_1 \pm i\tau_2)\psi_{Li} \\ &= \overline{Q_{Li}}\gamma^{\mu}(\tau_1 \pm i\tau_2) Q_{Li} + \overline{\ell_{Li}}\gamma^{\mu}(\tau_1 \pm i\tau_2)\ell_{Li} \\ &= \overline{u_{Li}}\gamma^{\mu} d_{Li} + \overline{d_{Li}}\gamma^{\mu} u_{Li} + \overline{e_{Li}}\gamma^{\mu}\nu_{Li} + \overline{\nu_{Li}}\gamma^{\mu} e_{Li} \quad (2.8) \\ J^{\mu}_Z &= \overline{\psi_i}\gamma^{\mu}\frac{1}{2}(c_V - c_A\gamma_5)\psi_i \\ J^{\mu}_Q &= \overline{\psi_i}\gamma^{\mu} Q \psi_i \,, \quad (2.9) \end{aligned}$$

where $c_V = \tau_3 - 2Q\sin^2\theta_W$ and $c_A = \tau_3$, $i$ represents the generation indices and $\psi$ represents both left-handed and right-handed fermions.

## 2.3 Fermion masses and mixing

Like the case of gauge bosons in SM, the pure mass term of the SM fermions are also not invariant under SM gauge group simply because the left-handed fermions are doublets under $SU(2)_L$while the right-handed ones are singlets. However, the Higgs mechanism can again be implemented to realize realistic masses for quarks and leptons.

The left-handed fermions along with the right-handed fermions can couple to the Higgs doublet to form invariant Yukawa terms in the SM. The relevant part of the Lagrangian, called as Yukawa sector, is written as

$$\mathcal{L}_Y = \overline{Q_{Li}}\,\tilde{\phi}\,(Y_u)_{ij}u_{Rj} + \overline{Q_{Li}}\,\phi\,(Y_d)_{ij}d_{Rj} + \overline{\ell_{Li}}\,\phi\,(Y_\ell)_{ij}e_{Rj}\,,$$

where $\tilde{\phi} = i\tau_2\phi^*$ and i, j, k are the generation indices. The masses for all the SM fermions except neutrinos arise naturally when the Yukawa sector of the SM is expanded around the minima of the Higgs potential $<\phi>= v$.

$$\begin{aligned}\mathcal{L}_m &= \overline{Q_{Li}} < \tilde{\phi} > (Y_u)_{ij}u_{Rj} + \overline{Q_{Li}} < \phi > (Y_d)_{ij}d_{Rj} + \overline{\ell_{Li}} < \phi > (Y_\ell)_{ij}e_{Rj}\\ &= \overline{u_{Li}}\,v\,(Y_u)_{ij}u_{Rj} + \overline{d_{Li}}\,v\,(Y_d)_{ij}d_{Rj} + \overline{e_{Li}}\,v\,(Y_\ell)_{ij}e_{Rj}\,,\end{aligned} \tag{2.10}$$

where $u_{L/R}$ represents the up-type left/right-handed quarks, $d_{L/R}$ represents the down-type left/right-handed quarks and $e_{L/R}$ represent the left/right-handed charged leptons. The type of mass term appearing in the expression 2.10 is known as Dirac type mass term for the fermions. In the next chapter, we will find out that it is possible to write an another type of Lorentz invariant mass term, known as Majorana mass term, for charge neutral fermions like neutrinos. Now the mass matrices for the up-type quarks $(M_u)$, down-type quarks $(M_d)$ and charged leptons $(M_\ell)$, in the weak basis (as given in expression 2.1 and 2.2), can be written as

$$\begin{aligned} M_u &= vY_u \\ M_d &= vY_d \\ M_\ell &= vY_\ell\,. \end{aligned}$$

Obviously there is no mass matrix for the neutrinos because it is not possible to write the corresponding Yukawa term in the absence of right-handed neutrinos.

The quark and the charged lepton mass matrices are not diagonal in general. This fact leads to what is called as fermion mixing in SM. Let us first consider the quarks. In the three generation scenario, both up and down type quark mass matrices are general $3\times3$ complex matrices having nine magnitudes and nine phases. They need be diagonalized to go to their mass basis in order to provide them definite masses. Two unitary matrices are required to diagonalize each of the matrices. It is logical as the left-handed and the right-handed fermions belongs to the different representations the SM gauge group and their rotation, needed to take from the weak basis to the physical basis, can be quite independent. Let $U_{L/R}$ be the unitary rotation that connects weak basis of left/right-handed up-type quarks to their mass basis, i.e. $u_{Li} = (U_L)_{ia}u_{La}$ and $u_{Ri} = (U_R)_{ia}u_{Ra}$ where $u_{La/Ra}$ represents the left/right-handed up type quarks in their mass basis, we expect

$$U_L^\dagger M_u U_R = M_u^{Diag}\,,$$

where $M_u^{Diag}$ is the $3 \times 3$ real and diagonal matrix with the diagonal entries corresponding to the physical masses of three up-type quarks. Similar approach to diagonalize the down-type quark mass matrix can be adopted with the rotation matrix $D_L$ corresponding to the left-handed and and $D_R$ to the right-handed down quarks as

$$D_L^\dagger M_d D_R = M_d^{Diag} \,.$$

One may think that the quarks can be rotated from the weak basis to the mass basis without any physical consequences. But it not true due to the presence of charged current interaction term as shown in expression 2.8. Although the charge current interaction term is diagonal in the weak basis $(u_{Li}, d_{Li})$, that we started with, it no longer remains same once it is rewritten in the physical basis. For example,

$$\begin{aligned} \left(J^\mu_{W^+} W^+_\mu\right) &= \overline{u_{Li}} \gamma^\mu d_{Li} W^+_\mu + h.c. \\ &\to \overline{(U_L)_{ia} u_{La}} \gamma^\mu (D_L)_{ib} d_{Lb} W^+_\mu + h.c. \\ &\to \overline{u_{La}} \gamma^\mu \left(U_L^\dagger D_L\right)_{ab} d_{Lb} W^+_\mu + h.c. \end{aligned}$$

The combined unitary matrix $U_L^\dagger D_L$ is called as Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix

$$V = U_L^\dagger D_L \,.$$

However, the observable CKM mixing matrix can only infer about the combined effect of the unitary rotation matrices of left-handed up and down type quarks and not the individuals. Even similar kind of physical consequence is not possible for the rotation corresponding to the right-handed quarks simply because they do not appear in the charge current interaction term. Although they appear in neutral current interaction term, any such combined effect of rotations for both left and right-handed quarks exactly cancels as the same type of quarks (either up or down) are present in any of the Neutral current terms.

The unitary CKM matrix has three magnitude and six phases for three generation. However, five of the phases can be absorbed in the redefinition of the quarks. The single unremovable complex phase produces CP violation in the quark sector which has been confirmed experimentally.

Unlike the quark sector, there is no such mixing possible in the charge current interaction in the leptonic sector. The absence of mass matrix for neutrinos forbids neutrinos to obtain a fixed mass basis. So the left-handed unitary matrix for diagonalizing the charged lepton mass matrix, which appears in the charged current interaction term, can be absorbed in the redefinition of the weak basis of the neutrinos leading to no observable effect. However, we have now firm evidences that neutrinos do have masses which can be accommodated only when we go beyond SM and look for some mass generating mechanism.

# Chapter 3

# Neutrino masses and mixing

There exists three flavors of active left-handed neutrinos with no right-handed partner in the SM leading to no masses for the neutrinos. However, the robust evidences of neutrino oscillations have given first push to go beyond SM to allow masses for the neutrinos to explain the oscillations. However, the charge neutrality of the neutrinos opens up two possibilities for the nature of their masses. They can either posses Dirac masses with separate particle and antiparticle identity like any other charged fermions or can have Majorana masses which means no difference between neutrinos are antineutrinos like photons. While all the parameters entering in the neutrino oscillations are fairly known, the nature and the absolute scale of their masses are still to be probed through the experiments involving the beta decay and the neutrinoless double beta-decay processes.

## 3.1 Neutrino masses: Dirac and Majorana

The basic difference between a Dirac and Majorana neutrino comes from the fact that the Dirac neutrino respects the lepton-number conservation while the Majorana neutrino violates it by two units. Allowing the lepton-number violating processes makes the Majorana case to be more interesting and phenomenologically rich to study.

A study of the Lorentz transformation property of a four component Dirac fermion reveals that it consists of two independent two-component Weyl spinors. Decomposing the four component spinor under the group $SU(2) \times SU(2)^*$, locally isomorphic to the Lorentz group $SO(3,1)$, reads

$$\psi(4) \equiv \psi_L(2,1) + \psi_R(1,2)\,,$$

where $\psi_L$and $\psi_R$ correspond to left-handed and right-handed component of the Dirac fermion, respectively . The two independent Weyl spinors can be projected out by the chiral projection operator $\frac{1}{2}(1\pm\gamma_5)$as

$$\begin{aligned}\psi_L &= \frac{1}{2}(1+\gamma_5)\,\psi \\ \psi_R &= \frac{1}{2}(1-\gamma_5)\,\psi .\end{aligned}$$

The decomposition is invariant under the Lorentz group as $\psi_L$ and $\psi_R$ transform as two separate irreducible representations. However, the two representation matrices transforming $\psi_L$ $(U_L)$ and $\psi_R$ $(U_R)$ , in the chiral representations of $\gamma$ matrices, can be related as

$$U_L = \left(U_R^{-1}\right)^{\dagger}\gamma_0 . \tag{3.1}$$

The discrete parity transformation $(\mathcal{P})$ takes the left and the right-handed Weyl spinors into each other as

$$\psi_L \overset{\mathcal{P}}{\longleftrightarrow} \psi_R .$$

Obviously, the SM is not invariant under parity transformation as its left-handed fermion members are doublets under $SU(2)_L$ while the right-handed ones are singlets. So the a Lagrangian for a Dirac particle in terms of its Weyl components can be written as

$$\mathcal{L}_D = i\overline{\psi_L}\gamma^{\mu}\partial_{\mu}\psi_L + i\overline{\psi_R}\gamma^{\mu}\partial_{\mu}\psi_R - m_D\left(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L\right) .$$

The mass term can be shown to be invariant under Lorentz transformation using the relation 3.1. It is apparent that in the absence of mass term, $\psi_L$and $\psi_R$ are just the two independent Weyl spinors with no physical connection. It is only the mass term which connects the two Weyl spinors. In other words the combination of two Weyl spinors connected with the mass term is interpreted what is known as a Dirac fermion. Moreover, one can easily notice that the Lagrangian is invariant under a $U(1)_Q$ symmetry where both $\psi_L$ and $\psi_R$ have the same quantum numbers for charge ($Q$) while their c-conjugate $\left(\psi_{L/R}\right)^c$ correspond to ($-Q$) charge. It means a Dirac mass term can be constructed only with those two Weyl fermions which have same charges if there exists a $U(1)_Q$ symmetry, local or global.

However, it is also possible to construct a Lorentz invariant mass term for a single Weyl spinor as

$$\begin{aligned}\mathcal{L}_M &= i\overline{\psi_L}\gamma^\mu\partial_\mu\psi_L - \frac{1}{2}M\left(\overline{(\psi_L)^c}\psi_L + h.c.\right)\\ &= i\overline{\psi_L}\gamma^\mu\partial_\mu\psi_L - \frac{1}{2}M\left((\psi_L)^T C\psi_L + h.c.\right),\end{aligned}$$

where $(\psi_L)^c$ is the c-conjugate of $\psi_L$ $\left(= C\,\overline{(\psi_L)^c}^T\right)$. Although this mass term is invariant under Lorentz transformation (can be shown using expression 3.1), it can be written only for a neutral particle if there exist a $U(1)_Q$ symmetry in the theory. In our nature, we have the remnant $U(1)_Q$ symmetry corresponding to the electromagnetic interaction . All the SM fermions except neutrinos are charged particles and, hence, can be provided only the Dirac masses by writing the Dirac mass term with the help of two Weyl spinors having same charges. However, a charge neutral Weyl spinor like the SM left-handed neutrino can have a Lorentz invariant Majorana mass term. However, such a mass term for the neutrino is allowed under the remnant $U(1)_Q$ gauge symmetry, it is forbidden under the SM gauge symmetry. This is because the Yukawa couplings, which can provide the Majorana mass terms for the neutrinos in the broken symmetry, are forbidden under SM gauge group.

The SM does not allow both Dirac and Majorana masses for the neutrinos simply because the left-handed neutrinos do not have the right-handed partners to write the Dirac mass term and the Majorana masses for the left-handed neutrino are not possible to write. However, several extensions of SM, which we have to do anyway to explain neutrino masses, are able to provide Dirac masses or Majorana masses or both for the neutrinos.

## 3.2 Neutrino mixing

In the previous section, we concluded that the neutrinos can be given either the Dirac or the Majorana masses due to its charge neutrality. However, generation of neutrino masses requires new physics beyond SM. In the present section, we will assume neutrinos to be massive by some or other mechanism and study only its phenomenological consequences.

Let us first consider the case of Dirac neutrinos which can be trivially accommodated in the SM by introducing the singlet right-handed neutrinos. From the standpoint of mass generating mechanism, this will provide an equal footing for both quarks and leptons. Moreover, the generic concept of mixing in the quark sector can be comfortably borrowed to explain mixing in the leptonic sector.

Like the quark mass matrix, the charged lepton mass matrix can in general be diagonalized by a bi-unitary transformation. Without loss of generality we can assume the charged lepton mass matrix to be real, positive and diagonal to start with. In other words, the flavor basis of the charged leptons can be safely assumed to be same as its mass basis.

The charged-current interaction term in the leptonic sector is given by

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\, \bar{e}_{Li}\gamma^{\mu}\nu_{Li}W_{\mu}^{-} + h.c. \tag{3.2}$$

in the mass basis ($e_{Li}, i = e, \mu, \tau$) of charged leptons , *i.e.*, the flavor states $e, \mu, \tau$ correspond to physical states with our assumption. The flavor of a neutrino produced in association with a charged lepton is always same as the flavor of the charged lepton. However, the flavor states of the neutrinos need not be same as its mass states and so the neutrino mass matrix, in the chosen basis, is not real and diagonal in general.

Like the quark sector, the Dirac neutrino mass matrix can be diagonalized by two unitary matrices, one for rotating the left-handed neutrinos and other for the right-handed one. The one for the left-handed neutrinos appears in the charged current interaction term (3.2) and solely represents the mixing matrix in the leptonic sector in the chosen mass basis for charged leptons. This unitary mixing matrix, like quarks, has three magnitudes and one physical phase.

However, the situation is slightly different in the case of Majorana neutrinos. For the case of Majorana neutrinos, the neutrino mass term for the three generation scenario can be written as

$$\mathcal{L}_M = m_{\nu ij}\ \nu_{iL}^T\, C\ \nu_{jL} \tag{3.3}$$

Unlike the Dirac case, the Majorana neutrino mass matrix comes out to be a $3 \times 3$ complex as well as symmetric matrix. The symmetric nature of the mass matrix of the Majorana neutrinos at the low energy requires only one unitary matrix to diagonalize it, which is not possible in the case of the Dirac neutrinos. If $U$ is the diagonalizing neutrino mass matrix $m_\nu$, then we have

$$U^\dagger\ m_\nu\ U^* = K_P^2\ m_\nu^{Diag}\ ., \tag{3.4}$$

where $m_\nu^{Diag} = \mathrm{diag}[m_1,\ m_2,\ m_3]$ is a real diagonal matrix and $K_P$ is a diagonal phase matrix. The unitary matrix $U_{ia}$ (with $i, j = e, \mu, \tau$ and $a, b = 1, 2, 3$) relates the physical neutrino states $\nu_a$ (with masses $m_a$) to the weak states

$$\nu_a = U_{ai}\ \nu_i + {K_{Paa}}^2\ U_{ai}^*\ \nu_i^c\,, \tag{3.5}$$

so that the physical neutrinos satisfy the Majorana condition

$$\nu = K_P^2\ \nu^c. \tag{3.6}$$

The unitary neutrino mixing matrix $U$ is able to diagonalize the $m_\nu$ but it does not guarantee the real mass eigenvalues. However, the unitary matrix $U$ combined with the Majorana phase matrix $K_P$ can serve the purpose. This combined form $U$ and $K_P$ is known as the the neutrino PMNS mixing matrix $U_{PMNS}$ [12–14], The $U_{PMNS}$ matrix can diagonalize the $m_\nu$ with real eigenvalues and is related to $U$ and $K_P$ as

$$U_{PMNS} \quad = \quad U\, K_P\,.$$

The standard PDG (Particle Data Group) parametrization [29] of the PMNS mixing matrix is given as

$$U_{PMNS} = R_{23}(\theta_{23}) R_{13}(\theta_{13}, \delta_{13}) R_{12}(\theta_{12}) U_{ph}\,, \tag{3.7}$$

where $U_{ph} = \mathrm{diag.}\left(1, e^{i\eta}, e^{i(\xi+\delta_{13})}\right)$ , $\delta_{13} \in [-\pi, \pi]$ and $R_{ij}(\theta_{ij})$ is the rotation matrix in the $(i, j)$ plane of the neutrino mass matrix. The expanded form of the PDG parametrization reads as

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} .U_{ph}, \tag{3.8}$$

where $c_{ij}$, $s_{ij}$ stands for $\cos\theta_{ij}$ and $\sin\theta_{ij}$ respectively.

It is straightforward to observe that the PMNS mixing matrix in the case of Majorana neutrinos is different from the mixing matrix one in the case of Dirac neutrinos (or the *CKM* mixing matrix for quarks) in the sense that it is endowed with two extra CP violating phases $\eta$ and $\xi$ [15]. In the Dirac case, these two phases can be absorbed in the redefinition of the neutrino fields and, hence, turn out to be unphysical. However, the same kind of absorption is not possible in the case of Majorana neutrinos due to the Majorana condition given in expression 3.6 and one end up with the two more physical phases usually called as Majorana phases. The two physical phases in $U_{ph}$ acts as the new sources of CP violation. We will study their CP violating nature in detail in the next chapter. However, the two phases are not relevant for the neutrino oscillation because the parameter entering in the neutrino oscillation belongs to the matrix $U$ only. So the theory of neutrino oscillation can be formulated in terms of matrix $U$ without worrying about the Majorana phases.

## 3.3 Neutrino oscillations and evidences

From the basic concepts of quantum theory, we know that the state that do not evolve with time are the physical states or mass eigenstates. A state described by a linear combination of mass eigenstates does not remain invariant and do evolve with time. Similar situation happens to occur in the case of massive neutrinos.

The neutrino participates in charged-current interaction with definite flavor same as the flavor of the associated charged lepton. So the flavor of the neutrino is decided by the flavor of the charged lepton involved in the same charge-current interaction. However, the fact that the flavor states of the neutrinos are not the mass eigenstates leads to the phenomena of neutrino flavor oscillation. If a neutrino of a particular flavor is allowed to travel a distance before its detection, it can evolve in between and the flavor of the detected neutrino may not correspond to the original flavor of the neutrino.

The neutrinos in its physical state $\mid \nu_a >$ with masses $m_a$ and energy $E_a$ evolves with time $t$ by a phase only as

$$\mid \nu_a(t) >= e^{-iE_a t} \mid \nu_a(0) > \ .$$

The time evolution of a neutrino of a particular flavor $i$ at $t = 0$ will be given as

$$\mid \nu_i(t) >= \sum_a e^{-iE_a t} U^*_{ia} \mid \nu_i(0) > \ .$$

The probability amplitude of detecting a neutrino with flavor $j$ after time t can be written as

$$< \nu_j \mid \nu_i(t) >= \sum_a e^{-iE_a t} U^*_{ia} U_{ja} \, .$$

For the ultra-relativistic neutrino, one can have the following approximation

$$E_a \simeq E + \frac{m_a^2}{2E} \, .$$

Now replacing $t$ by $L$ the distance traveled by the neutrino, the probability amplitude turns out to be

$$< \nu_j \mid \nu_i(L) >= \sum_a e^{-i\left(E + m_a^2 L/E\right)} U^*_{ia} U_{ja} \, .$$

The probability of the process will be just the squire of the probability amplitude

$$P\left(\nu_i \to \nu_j\right) \equiv <\nu_j \mid \nu_i(L)>^2 \;=\; \delta_{ij} - 4\sum_{a>b} \mathrm{Re}\left(U^*_{ia}U_{ja}U_{ib}U^*_{jb}\right)\sin^2\left[\frac{\triangle m^2_{ab}L}{4E}\right]$$
$$+2\sum_{a>b} \mathrm{Im}\left(U^*_{ia}U_{ja}U_{ib}U^*_{jb}\right)\sin\left[\frac{\triangle m^2_{ab}L}{2E}\right],$$

where $\triangle m^2_{ab} = m^2_a - m^2_b$. It is straightforward to realize that the necessary condition for the neutrinos to oscillation is that their mass square differences should be finite.

### 3.3.1 Solar neutrinos

The first indication of the solar neutrino oscillation was noticed long back in 1967 in Homestake experiment by Davis and his collaborators [30, 31]. It was an radiochemical experiment to detect the solar neutrino flux using the inverse beta-decay process

$$\nu_e + {}^{37}\mathrm{Cl} \to {}^{37}\mathrm{Ar} + e^- .$$

By counting the Argon atoms by the radiochemical methods, the experiment found noticeable deficit in the observed electron neutrino flux compared to the flux predicted by the standard solar model (SSM). However, it was not clear at that time whether the SSM has to be revised or some new physics is needed to explain the observed deficit. Later, GALLEX [32] and SAGE [33] experiments also based on radiochemical techniques , but with lower energy threshold of neutrinos, produced similar results.

However, these radiochemical experiments were not able to provide the directional correlation of the incoming flux from the sun. The situation was revolutionized when a real time experiment, the Kamiokande (1987 to 1996) [34], based on Cherenkov techniques was able to provide directional as well as energy information of the incoming neutrinos. This is possible by observing the Cerenkov radiation produced by the relativistic electrons scattered elastically by the high energy neutrinos

$$\nu_\alpha + e^- \to \nu_\alpha + e^- .$$

However, the scattering is dominantly sensitive to the electron neutrinos because its cross-section is six times higher compared to other two flavor neutrinos. Since the threshold energy for neutrinos was much higher, about 7 MeV, only the $^8B$ neutrinos from the sun were possible to be detected in Kamiokande. The experiment used kilo tons of water in a cylindrical tank surrounded by thousands of photomultiplier tubes (PMTs) to detect the Cherenkov radiation. Based on $^8B$ neutrinos detection, Kamiokande reported the measured neutrino flux to be around half of what is expected from the SSM. So the Kamiokande was first to confirm the disappearance of electron neutrinos in the solar neutrino flux.

Later, the role of Kamiokande was replaced by Super-Kamiokande [6, 35–38] which started in 1996 with more sensitivity and lower value of threshold energy for the neutrinos( around 5 MeV). However, it was based on similar experimental techniques. The first phase of the experiment took data till July 2001 and measured the $^8B$ neutrino flux to be $(2.35 \pm 0.02\,(\mathrm{stat}) \pm 0.08\,(\mathrm{sys})) \times 10^6\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$ [39] which is less than half of what is expected from the SSM.

However, all the above experiments fails to address the question whether the incoming total solar neutrino flux is actually same as expected. The amount of the disappearance of electron neutrinos can only be then predicted with confidence. The confusion about the reliability of the SSM was removed when Sudbury Neutrino Observatory (SNO) measured the total neutrino flux in all flavors coming from sun [7, 8, 40, 41]. The SNO is also a real time experiment based on Cherenkov techniques but uses heavy water $(D_2O)$ instead of the ordinary water. Using heavy water, the SNO was able to detect both electron neutrinos through the charged interaction as well as all active neutrino flavors through the neutral current interaction along with the interactions involving scattering:

$$\begin{aligned}
\nu_e + d &\rightarrow p + p + e^- \quad (\mathrm{charged\,current})\,, \\
\nu_\alpha + d &\rightarrow \nu_\alpha + p + n \quad (\mathrm{neutral\,current})\,, \\
\nu_e + e^- &\rightarrow \nu_e + e^- \quad (\mathrm{elastic\,scattering})\,.
\end{aligned}$$

While charge current process is only sensitive to the electron neutrino, the neutral current interaction is independent of the neutrino flavor. So the SNO could measure both the electron neutrino flux and the total neutrino flux coming from the sun independently. The ratio of the two fluxes is expected to be 1 in the absence of any oscillation which is in contradiction to what has been measured [8]

$$\frac{\mathrm{Flux}^{CC}}{\mathrm{Flux}^{NC}} = 0.306 \pm 0.026(\mathrm{stat}) \pm 0.024(\mathrm{syst})\,.$$

The ratio confirms that solar neutrinos undergo oscillation independent of SSM.

### 3.3.2 Atmospheric neutrinos

The collision of the nuclei of the upper atmosphere with the cosmic ray protons produces what are called as atmospheric neutrinos. They production comes from the following chain process:

$$
\begin{aligned}
p + X \rightarrow \pi^{\pm} \quad &+ \quad Y \\
\pi^{\pm} \quad &\rightarrow \quad \mu^{\pm} + \nu_{\mu}\,(\overline{\nu_{\mu}}) \\
& \quad\quad \mu^{\pm} \rightarrow e^{\pm} + \nu_e\,(\overline{\nu_e}) + \overline{\nu_{\mu}}\,(\nu_{\mu})\ .
\end{aligned}
$$

This production of the high energy neutrinos, above a few GeV, in the upper atmosphere turns out to practically uniform around the earth. A detector on the earth surface can detect neutrinos both down-going and up-going. The down-going neutrinos travel only few tens of kilometer while the up-going neutrinos coming from the opposite side of the globe travel distance of about several thousand kilometers. In the absence of any neutrino oscillation, the flux of both up and down-going neutrinos of a given flavor would be expected to remain same. Moreover, the ratio of the muon neutrino flux to that of electron neutrino flux can be predicted to be around 2 by just looking at the chain reaction.

However, if neutrinos do oscillate one would expect asymmetry in the observed amount of up and down-going fluxes, defined as $A_{\alpha} = \left(\frac{U-D}{U+D}\right)_{\alpha}$, simply because the neutrinos coming from the other end of the globe travel enough distances to oscillate into some other flavors. In fact the asymmetry has been observed in the experiments, first reported by Super-Kamiokande collaboration in 1998 [5]. They measured the following up-down asymmetry in muon and electron neutrino fluxes for multi-GeV events

$$
\begin{aligned}
A_{\mu} &= -0.296 \pm 0.048 \pm 0.01\,, \\
A_{e} &= -0.036 \pm 0.067 \pm 0.02\,.
\end{aligned}
$$

While there is a clear asymmetry in the muon neutrino flux, the asymmetry in the electron neutrino flux is practically zero. This observed asymmetry in muon neutrino flux was the first clear evidence of atmospheric muon neutrinos oscillation to the tau neutrinos.

### 3.3.3 Reactor neutrinos

Nuclear reactors are rich sources of artificial electron antineutrinos $(\overline{\nu_e})$. The antineutrinos are mainly produced due to the nuclear fission of the isotopes of Uranium and Plutonium. The energy spectra and flux of $\overline{\nu_e}$ are estimated by studying its correlation with thermal power of the reactor. The reactor neutrino experiments basically looks for some deficit in the expected $\overline{\nu_e}$ flux at certain distance from the reactor. In order to obtain observable oscillation at short distances, one relies on measuring the disappearance of $\overline{\nu_e}$ of few MeV energy.

The reactor $\overline{\nu_e}$ are detected through inverse neutron decay reaction

$$\overline{\nu_e} + p \to n + e^+ .$$

The produced positron soon annihilates with the surrounding electron and the generated energy can be seen in the scintillator detector. The $\overline{\nu_e}$ detection is finally confirmed once the positron detection is consequently followed by neutron-capture signal. Neglecting the small recoil of the neutron, the energy of the incoming $\overline{\nu_e}$ can be easily correlated with the energy of positron to get threshold energy of $\overline{\nu_e}$ to be around 1.8 GeV.

The first such reactor experiment was CHOOZ experiment [11, 42, 43] which was located near CHOOZ power plant in Ardennes, France at about 1 km distance. It started taking data in April 1997 up to July 1998 and measured no noticeable disappearance of $\overline{\nu_e}$.

An another such experiment is KamLAND (Kamioka liquid scintillator Antineutrino Detector) experiment [9, 44]which is located in the same cavity of Kamioka mine where Kamiokande experiment was functioning. It has been designed to detect the $\overline{\nu_e}$ coming from several reactors in Japan at an average distance of 180 km. Based on the data taken between 2002 to 2004, KamLAND showed a clear disappearance of $\overline{\nu_e}$ with the observed ratio of measured flux to the expected one

$$R = 0.658 \pm 0.044 \pm 0.047 ,$$

indicating towards the clear phenomena of neutrino oscillations.

### 3.3.4 Accelerator neutrinos

An another source of artificial neutrinos are due to the accelerators where it is possible to get controlled and directional beam of the neutrinos. The accelerator neutrino beam along its axis do not undergo much loss in its flux intensity unlike the case of reactor neutrinos where the antineutrino flux is isotropic and decreases rapidly with the distance. The neutrino beam in the accelerators comes from the decay of pions in flight produced by the collision of accelerated protons on a fixed target:

$$
\begin{aligned}
p + \text{target} \rightarrow \pi^{\pm} \quad &+ \quad Y \\
\pi^{\pm} \quad &\rightarrow \quad \mu^{\pm} + \nu_{\mu}\left(\overline{\nu_{\mu}}\right) \\
& \qquad \mu^{\pm} \rightarrow e^{\pm} + \nu_{e}\left(\overline{\nu_{e}}\right) + \overline{\nu_{\mu}}\left(\nu_{\mu}\right) .
\end{aligned}
$$

In fact the nature of the neutrino production in accelerators is similar to the neutrino production in the Atmosphere. So the accelerator based experiments can serve as an confirming test of the atmospheric neutrino oscillation. The detector is situated at the distance of several hundred kilometers from the accelerators in the path of the intense neutrino beam.

The first of such an accelerator neutrino experiment has been long baseline K2K experiment [10, 45, 46] with a distance gap of 250 kilometers from the KEK laboratory to the Super-Kamiokande detector in the Kamioka mine. The first phase of K2K experiment started from June 1999 to July 2001. The second phase collected the data from January 2003 to February 2004. Results of the K2K experiment are found in very good agreement with the results of Atmospheric neutrino experiments. An another similar type of experiment is Main Injector Neutrino oscillation Search (MINOS) [47–49] working in the same range of L/E. In addition to the study of muon neutrino disappearance, it is also looking for electron neutrino appearance. To date, MINOS is also in good agreement with the results of atmospheric neutrino experiments.

### 3.3.5 Global fit of neutrino oscillation data

Based on the parametrization of neutrino mixing matrix in the expression 3.8, the global fit to the parameters has been estimated at 90% CL as [29]:

$$
\begin{aligned}
\Delta m_{21}^2 = m_2^2 - m_1^2 &= (7.59 \pm 0.20) \times 10^{-5}\ \text{eV}^2 \\
\Delta m_{32}^2 = m_3^2 - m_2^2 &= (2.43 \pm 0.13) \times 10^{-3}\ \text{eV}^2 \\
\sin^2(2\theta_{12}) &= 0.87 \pm 0.03 \\
\sin^2(2\theta_{23}) &> 0.92 \\
\sin^2(2\theta_{13}) &< 0.19. \qquad (3.9)
\end{aligned}
$$

Moreover, there are two possible configuration of hierarchy, normal and inverted hierarchy, in the neutrino mass spectrum as shown in figure 3.1.

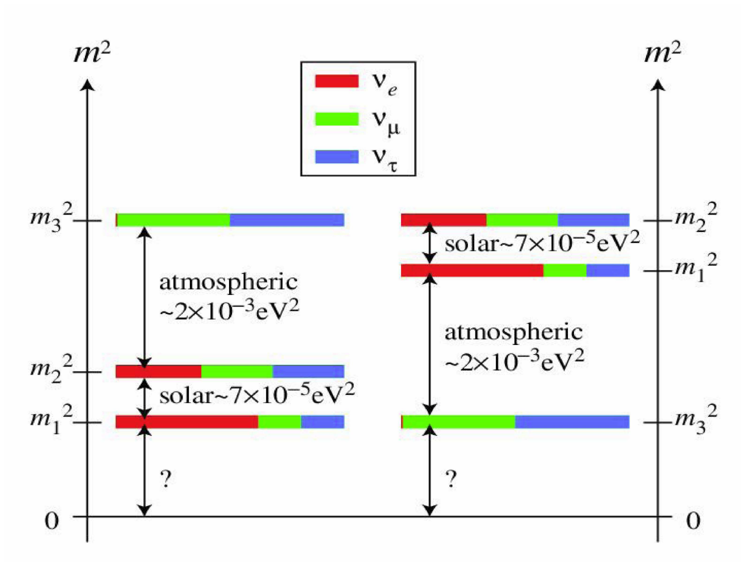


Figure 3.1: Two configuration for neutrino mass spectrum [1]

## 3.4 Direct detection of neutrino masses

The neutrino oscillation experiments are able to provide information about the mixing angles and the mass squired difference. However, they do not infer anything about the absolute mass scale of the three neutrinos. It is here where the non-oscillation experiments play important role. The issue can be addressed by experiments like beta decay and neutrinoless double-beta decay.

### 3.4.1 Beta decay

Nuclear beta decay can serve as the most sensitive method to measure the mass of the electron neutrinos:

$$\begin{aligned} (A,Z) &\rightarrow (A,Z+1)+e^-+\overline{\nu_e} \\ n &\rightarrow p+e^-+\overline{\nu_e}\,. \end{aligned}$$

By measuring the end point of the spectral distribution of electron in the nuclear beta decay, one can infer about absolute mass of the electron neutrinos. The Tritium beta decay is believed to be the most suited candidate for the purpose from two considerations (1) it has the smallest Q-value $(=18.574)$KeV among all known beta decays and (2) the atomic structure of the tritium atom is less complicated compared to the other heavier atoms facilitating the accurate calculation of the atomic effect.

$$^3H \rightarrow ^3He+e^-+\overline{\nu_e}\,.$$

Based on the tritium beta decay experiment, Mainz [50] and Troitzk [51] have provided the best constraint on the electron neutrino mass both at 95% confidence level:

$$\begin{aligned} m_{\nu_e} &< 2.3\,\mathrm{eV} \quad (\mathrm{Mainz}) \\ m_{\nu_e} &< 2.5\,\mathrm{eV} \quad (\mathrm{Troitzk})\,. \end{aligned}$$

A future beta decay experiment KATRIN, a joint collaboration of Mainz and Troitzk, is scheduled to start in 2010 and expected to achieve sensitivity of 0.2 eV to electron neutrino mass.

### 3.4.2 Neutrinoless double beta decay

If neutrinos are Majorana fermions in nature, the bounds from the beta decay experiments can be improved by observing the allowed neutrinoless double beta decay process. Although extremely weak double-neutrino beta decay $(2\nu\beta\beta)$ has been observed in several candidates, a clear evidence of the neutrinoless double decay $(0\nu\beta\beta)$ is still missing. A $2\nu\beta\beta$ occurs when an even-even nucleus decays to an another even-even nucleus where the single beta decay is energetically forbidden.

$$\begin{aligned} (A,Z) &\rightarrow (A,Z+2)+e^-+e^-+\overline{\nu_e}+\overline{\nu_e} \\ n+n &\rightarrow p+p+e^-+e^-+\overline{\nu_e}+\overline{\nu_e} \qquad (2\nu\beta\beta)\,. \end{aligned}$$

However, the Majorana neutrinos can allow the double beta decay without any emission of the neutrinos.

$$\begin{aligned} (A,Z) &\rightarrow (A,Z+2)+e^-+e^- \\ n+n &\rightarrow p+p+e^-+e^- \qquad (0\nu\beta\beta)\,. \end{aligned}$$

This $0\nu\beta\beta$ decay is an extremely rare process and is very difficult to observe. However, observation of any such process would be very interesting for particle physics as it would not only confirm the Majorana nature for the neutrinos but would also imply lepton-number violation in nature by two units. The amplitude of the process is proportional to the effective mass

$$< m > = \sum_i U_{ei}^2 m_i = m_{ee}\,,$$

where $m_{ee}$ is the (1,1) element of the neutrino mass matrix written in the mass basis of charged leptons.

The most sensitive experiment on $0\nu\beta\beta$ decay has been carried out by Heidelberg-Moscow group using 11 kg of enriched $^{76}Ge$. It looked for the decay of $^{76}Ge$ to $^{76}Se$ through the following $0\nu\beta\beta$ process

$$^{76}Ge \rightarrow ^{76}Se + 2e^{-}\,.$$

By analyzing the data accumulated from August 1990 to May 2003, the group has claimed to observe 29 events of $0\nu\beta\beta$ decays corresponding to the following range of the effective Majorana neutrino mass with 99% confidence level[52–54]:

$$m_{ee} = (0.1 - 0.9)\ \mathrm{eV}\,.$$

However, the interpretation of the data is still controversial and confirmation of the result is still awaited.

# Chapter 4

# CP violation in neutrino mass matrix

In the SM there is only one source of CP violation, which is in the charged-current mixing matrix in the quark sector. The charged-current mixing matrix in the quark sector contains one CP phase, which has been observed. It is not possible to identify the position of the CP phase, since it is possible to make any phase transformations to the quarks. However, it is possible to define a rephasing invariant quantity as product of elements of the mixing matrix that remains invariant under any rephasing of the quarks [55–58]. This is known as Jarlskog invariant.

In the leptonic sector, SM does not allow any CP violation. If one considers extensions of the SM to accommodate the observed neutrino masses, then there can be several CP phases [15, 59–62]. In the simplest scenario of three generations, there could be one CP phase in the mixing matrix in the leptonic sector, similar to the quark sector. In addition, if neutrinos are Majorana particles they can have two more Majorana CP phases[59]. In this case it is possible to work in a parametrization, in which all the three CP phases could be in the charged-current mixing matrix in the leptonic sector. One of these CP phase will contribute to the neutrino oscillation experiments, while the other two will contribute to lepton-number violating process like neutrinoless double beta decay. A natural explanation for the smallness of the neutrino masses comes from the seesaw mechanism [18–20].

The CP phases in the leptonic sector has been studied and rephasing invariants for both lepton-number conserving as well as lepton-number violating CP violation have been constructed [15–17]. In the present chapter we try to study this question only in terms of neutrino masses. Since neutrinos are produced only through weak interactions, it is possible to work in the weak interaction basis, in which the charged lepton mass matrix is diagonal. The neutrino mass matrix in this basis will then contain all the information about CP violation. We try to find rephasing invariant combinations of the neutrino mass elements, so that with those invariants some general comments can be made about CP violation in the model without deriving the structure of the charged-current mixing matrix.

## 4.1 CP violation in the quark sector

We briefly review the rephasing invariants in terms of the mixing matrices and then show how the same results can be obtained from the mass matrix without taking the trouble of diagonalizing them in the leptonic sector. Consider first the quark sector, where the up and the down quark mass matrices are diagonalized by the bi-unitary transformations. Then, from the discussion of section 2.3, we know that the charged current interactions in terms of the physical fields will contain the Kobayashi-Cabibbo-Maskawa mixing matrix

$$V = U_L^\dagger D_L.$$

Since the right-handed fields are singlets under the SM interactions, they do not enter in the charged current interactions. In any physical processes, only this CKM mixing matrix would appear and hence the matrices $U_R$ and $D_R$ becomes redundant. So, the up and down quark masses have much more freedom and the physical observables that can determine the $V_{\alpha i}$ cannot infer about the up and down quark masses uniquely.

For the CP violation, one needs to further consider the rephasing of the left-handed fields. Any phase transformation to the up and down quarks will also transform the CKM matrix

$$V_{\alpha i} \to e^{i(d_\alpha - u_i)}.$$

However, if there is any CP phase in the CKM matrix, which cannot be removed by any phase transformations of the up and the down quarks, should be present in the following rephasing invariant known as Jarlskog invariant [55–58]

$$J_{\alpha i \beta j} = \mathrm{Im}[V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i}]. \tag{4.1}$$

Thus if the Jarlskog invariant is a measure of CP violation in the quark sector and a non-vanishing Jarlskog invariant would imply CP violation in the quark mixing. It is apparent from the definition that any phase transformations to the up and down quarks cannot change $J_{\alpha i \beta j}$. In a three generation scenario there can be only one such invariant and hence the CKM matrix can have only one CP phase, which is invariant under rephasing of the up and the down quarks.

## 4.2 CP violation in the leptonic sector

From the discussion of the neutrino mixing matrix in the section 10.1, we know that the Unitary matrix $U$ gives the mixing of the neutrinos and hence neutrino oscillations and $K_P$ is the Majorana phase matrix containing the Majorana phases, which are the new sources of CP violation entering due to the Majorana nature of the neutrinos. The unitary matrix $U$ also can contain CP violating phases, which should be observed in the neutrino oscillation experiments. We call these phases in the mixing matrix $U$ as *Dirac phases* to distinguish them from the *Majorana phases*. The main difference between a Majorana phase and a Dirac phase is that the Majorana phases do not affect any lepton-number conserving process like neutrino oscillations. On the other hand, the Dirac phases may contribute to both lepton-number conserving as well as lepton-number violating processes.

From the above discussions it is apparent that the information about the CP phases can be obtained from either $U$ and $K_P$ or only from the mass matrix $M_\nu$. In the literature the question of CP violation is usually discussed by studying $U$ and $K_P$. In this chapter we point out that it is possible to study the question of CP phases only by studying the neutrino mass matrix $m_\nu$. In particular, the information about CP violation is conveniently obtained from the rephasing invariant combinations of neutrino mass elements. When the neutrino masses originate from seesaw mechanism, the question of CP violation has been studied in details and similar invariants have been constructed [61, 62]. Our approach is different in the sense that we are working with only effective low energy neutrino mass matrix without restricting the analysis to any specific origin of the neutrino masses. Our results are general and applicable to any models of neutrino masses.

Consider the transformation of different quantities under the rephasing of the neutrinos

$$\begin{aligned}
\nu_a &\to e^{i\delta_a}\nu_a \\
\ell_i &\to e^{i\eta_i}\ell_i \\
U_{ia} &\to e^{-i(\eta_i-\delta_a)}U_{ia} \\
(K_P)_a &\to e^{i\delta_a}(K_P)_a .
\end{aligned} \tag{4.2}$$

From these transformations it is possible to construct the rephasing invariants [15–17]

$$s_{iab} = U_{ia}U_{ib}^*(K_P)_a^*(K_P)_b . \tag{4.3}$$

In the three generation case there will be three independent rephasing invariant measures in case of Majorana neutrinos. There is another rephasing invariant which is similar to the Jarlskog invariant in the quark sector,

$$t_{iajb} = U_{ia}U_{jb}U_{ib}^*U_{ja}^*, \tag{4.4}$$

so that $J_{iajb} = \text{Im } t_{iajb}$ and $S_{iab} = \text{Im } s_{iab}$ becomes the measure of CP violation.

$J_{iajb}$ contains the information about the Dirac phase, while $S_{iab}$ contains information about both Dirac as well as Majorana phases. One can then use the relation

$$t_{iajb} = s_{iab} \cdot s_{iba}$$

to eliminate the invariants $J$'s or else keep the $J$'s as independent measures and reduce the number of independent $S$'s. One convenient choice for the independent measures is the independent $t_{iajb}$'s and $s_{1ab}$'s. In the three generation case there is only one $t_{iajb}$ and two $s_{1ab}$'s.

The advantage of this parametrization is that the measure $J_{iajb}$ provides the measure of the CP violation in any lepton-number conserving process like neutrino oscillation experiment, while the measures $S_{1ab}$ corresponds to CP violation in lepton-number violating interactions like the neutrinoless double beta decay or scattering processes like $W^- + W^- \rightarrow \ell_i^- + \ell_j^-$ also. Moreover, $J_{iajb}$ enters into the lepton-number violation processes also.

Since only one $t_{iajb}$ is independent, one can define the measure of CP violation $J_{CP}$ in neutrino oscillation as the imaginary part of any one of the invariants of $t_{iajb}$'s using expression 4.4

$$J_{CP} = \text{Im}\left[U_{e1}U_{\mu 2}U_{e2}^{*}U_{\mu 1}^{*}\right] . \tag{4.5}$$

Similarly, other two independent measures of CP violation, $J_1$ and $J_2$, can be constructed as

$$\begin{aligned} J_1 &= Im\left[U_{e1}U_{e2}^{*}(K_P)_{11}^{*}(K_P)_{22}\right] \\ J_2 &= Im\left[U_{e1}U_{e3}^{*}(K_P)_{11}^{*}(K_P)_{33}\right] . \end{aligned} \tag{4.6}$$

So while rephasing invariant CP violating quantity $J_{CP}$ only appears in the lepton-number conserving processes, like neutrino oscillations, all three $J_{CP}$, $J_1$ and $J_2$ appears in the lepton-number violating processes, like neutrinoless double beta decay. We will be using these construction of CP violating measures in some of the chapters later.

## 4.3 Rephasing invariants with neutrino masses

We shall now proceed to construct such measures of CP violation in terms of the mass matrix itself. The rephasing invariant measures with the mixing matrix can allow all the rephasing invariants non-vanishing even when there is only one Dirac phase. However, in the present formalism, the number of rephasing invariants is same as the number of CP phases. So, we can find out if there is any Majorana phase or not. Since the neutrino mass matrix is diagonalized by a single unitary matrix, the mass matrix contains all the information about the PMNS mixing matrix and also the mass eigenstates. However, this is not obvious with the CP phase. When the neutrinos are given a phase transformation, the mass matrix will be transformed the same way. Since the we are working in the weak basis, any transformation to the charged leptons can be transformed to the mixing matrix and in turn to the neutrino masses. Thus the phase transformation to the mass matrix will become

$$\begin{aligned} \nu_i &\rightarrow e^{i\delta_i}\nu_i \\ \ell_i &\rightarrow e^{i\eta_i}\ell_i \\ M_{\nu ij} &\rightarrow e^{i(\delta_i+\delta_j-\eta_i-\eta_j)}M_{\nu ij} \\ &\rightarrow e^{i(\alpha_i+\alpha_j)}M_{\nu ij}. \end{aligned} \tag{4.7}$$

Where $\alpha_i = \delta_i - \eta_i$.

Consider the transformation $E \rightarrow XE$, where X is the phase transformation to the charged leptons. The mixing matrix will transform as $U \rightarrow X^*U$. However, in equation 3.4 this transformation can be interpreted as a transformation to the mass matrix, $m_\nu \rightarrow X^* m_\nu X^*$. Thus any rephasing invariant measure constructed with only the mass matrix will contain the information about CP violation.

Unlike the mixing matrices, the mass matrix is not unitary and instead it is symmetric. We write the elements of the mass matrix $m_\nu$ as $m_{ij}$ and try to construct the rephasing invariants in terms of $m_{ij}$. This analysis do not depend on the origin of neutrino masses. We work with the neutrino mass matrix after integrating out any heavier degrees of freedom and in the weak basis. Any quadratic terms that can be constructed from the elements of the neutrino mass matrix are all real, $m^*_{ij}m_{ij} = |m_{ij}|^2$, as expected. Let us next consider the quartic terms

$$I_{ijkl} = m_{ij}m_{kl}m^*_{il}m^*_{kj}. \tag{4.8}$$

It is easy to check that any three factors of the above quartic invariant can be made real by appropriate rephasing, but fourth one will remain complex. Since there are $n$ rephasing phases $(\delta_i)$, one can get $n$ number of linear equations to make mass elements of the mass matrix to be real. So $n$ number of entries (excluding symmetric elements) of the mass matrix can be made real, but positions of the mass entries can not be chosen randomly. That is the reason why all the above rephasing quartic invariants can not be made real in general. An $n \times n$ symmetric matrix has $n(n+1)/2$ independent entries and so it has the same number of phases. By appropriate rephasing, as argued above, $n$ independent phases can be removed. Then, one is left with $n(n-1)/2$ number of independent phases.

To find out the minimal set of rephasing invariants we list some of the transitive and conjugation properties of the invariants:

$$\begin{aligned} I_{ijpl} I_{pjkl} &= |m_{pj} m_{pl}|^2 I_{ijkl} \\ I_{ijkp} I_{ipkl} &= |m_{ip} m_{kp}|^2 I_{ijkl} \end{aligned}$$

and

$$I_{ijkl} = I^*_{ilkj} = I_{klij} = I^*_{kjil} \tag{4.9}$$

Using these relations it can be shown that all the $I_{ijkl}$ are not independent and they can be expressed in terms of a subset of these invariants $I_{ij\alpha\alpha}$ and the quadratic invariants as

$$I_{ijkl} = \frac{I_{ij\alpha\alpha} I_{kl\alpha\alpha} I^*_{li\alpha\alpha} I^*_{kj\alpha\alpha}}{|m_{\alpha\alpha}|^4 |m_{i\alpha} m_{j\alpha} m_{k\alpha} m_{l\alpha}|^2} \tag{4.10}$$

Where $i, j \neq \alpha$ and $\alpha = 1, 2, ..., n$, where $n$ is the number of generations. On the other hand, any quartics of the form $I_{ij\alpha\alpha}$ can be expressed in terms of $I_{\beta\beta\alpha\alpha}$ as

$$\text{Im}\,[I_{ij\alpha\alpha}] = -\text{Im}\,[I_{i\alpha\alpha j}] = -\frac{\text{Im}[I_{ii\alpha\alpha} \cdot I_{\alpha\alpha jj} \cdot I_{iijj}]}{\text{Re}\,[I_{i\alpha\alpha j}]\; (|m_{ii}|^2\; |m_{jj}|^2)}. \tag{4.11}$$

Thus we can express all other invariants in terms of $I_{iijj}$ and hence consider them to be of fundamental importance. However, when there are texture zeroes in the neutrino mass matrix, some or all of these invariants $I_{iijj}$ could be vanishing. In that case, it is convenient to use the $I_{ij\alpha\alpha}$ as the measure of CP violation. For the present we shall concentrate on the more general case with neutrino mass matrices without any texture zeroes, when the simplest rephasing invariants are $I_{iijj}$.

We can thus define the independent CP violating measures as

$$I_{ij} = \text{Im}\,[I_{iijj}] = \text{Im}\,[I_{iijj}] = \text{Im}\,[m_{ii} m_{jj} m^*_{ij} m^*_{ji}], \qquad (i < j) \tag{4.12}$$

These are the minimal set of CP violating measures one can construct and this gives the independent CP violating quantities. Since $I_{ij}$ satisfies

$$I_{ij} = I_{ji} \quad \text{and} \quad I_{ii} = 0,$$

there are $n(n-1)/2$ independent measures for $n$ generations.

We elaborate with some examples starting with a 2-generation scenario. There are three $I_{ijkl}$, two of which are real: $I_{1211} = |m_{11} m_{12}|^2$; and $I_{1222} = |m_{12} m_{22}|^2$. The third one can have imaginary phase, which is $I_{12} = \text{Im}[I_{1122}] = \text{Im}\,[m_{11} m_{22} m^*_{12} m^*_{21}]$. In the 3-generation case there are thus three independent measures $I_{12}, I_{13}, I_{23}$. Imaginary phases in all other quartics $I_{ijkl}$ are related to only these three independent measures. For example,

$$I^2_{1223} = \frac{I^*_{12} \cdot I^*_{23} \cdot I_{13}}{|m_{11}|^2 \; |m_{33}|^2}.$$

Similarly, for 4-generations there will be six rephasing invariant independent phases, which are $I_{12}, I_{23}, I_{31}, I_{14}, I_{24}, I_{34}$.

The above arguments have been stated without considering any texture zeroes in the mass matrix. If any element of the mass matrix is zero, then these discussions have to be generalized. It is because some quartic invariants can become undefined because of vanishing denominator of the right hand side of the expression 4.10 and 4.11. In that case one needs to consider all possible invariants $I_{ijkl}$, which could be non-vanishing. In addition, even if all the quartic invariants vanish, the product of six mass matrix elements of the form

$$I_{ijklpq} = m_{ij} \; m_{kl} \; m_{pq} \; m^*_{il} \; m^*_{kq} \; m^*_{pj}$$

could be non-vanishing and can contribute to CP violation. When there are no texture zeroes, the product of six mass elements do not contain any new information about CP phases, they are related to the quartic invariants

$$I_{ijklpq} = \frac{I_{ijkl} \;\; I_{pqkj}}{|m_{kj}|^2}. \tag{4.13}$$

Other products of six mass elements are of the form, $m_{ij} \; m_{kl} \; m_{pq} \; m^*_{il} \; m^*_{kj} \; m^*_{pq} = |m_{pq}|^2 \; I_{ijkl}$ or $|m_{ij} \; m_{kl} \; m_{pq}|^2$.

We summarize this section by restating that when all elements of the neutrino mass matrix are non-vanishing, $I_{ij}, \; (i < j)$ gives the total number of Dirac and Majorana phases. If some of the elements of the mass matrix vanishes, then either $I_{ijkl}$ or $I_{ijklpq}$ could also represent some of the independent phases.

## 4.4 CP violation in lepton number conserving processes

The rephasing invariant independent phases contained in $I_{ij}, i < j$, are inclusive of the Dirac phases as well as the Majorana phases. We shall now identify the rephasing invariant measures, which is independent of the Majorana phases, which would enter in the neutrino oscillation experiments. The mass matrix ($m_\nu$) in terms of the diagonal mass matrix ($\hat{m}_\nu$) can be expressed following equation 3.4 as

$$m_\nu = U^* \; K_P^2 \; \hat{m}_\nu \; U^\dagger.$$

Thus the products

$$\tilde{M} = (m_\nu^\dagger\, m_\nu) = (m_\nu\, m_\nu^\dagger)^* = U\, \hat{m}_\nu^2\, U^\dagger \tag{4.14}$$

are independent of the Majorana phases $K_P$ and any rephasing invariant measure constructed with elements $\tilde{m}_{ij}$ of $\tilde{M}$ will contain only the Dirac phases and hence should contribute to any lepton-number conserving processes.

The mass-squared elements $\tilde{m}_{ij}$ transforms under rephasing of the neutrinos and charged leptons as

$$\tilde{m}_{ij} \longrightarrow e^{i(\alpha_i - \alpha_j)}\tilde{m}_{ij}. \tag{4.15}$$

Since the mass-squared matrix $\tilde{m}_\nu$ is Hermitian, $\tilde{M}_\nu^\dagger = \tilde{M}_\nu$, the mass elements satisfy

$$\tilde{m}_{ij} = \tilde{m}_{ji}^*. \tag{4.16}$$

Thus the simplest rephasing invariant that can be constructed from the mass-squared matrix $\tilde{M}_\nu$ is just $\tilde{m}_{11}$. However, from equation 4.16 it is obvious that this is a real quantity. The next possible rephasing invariant would be a quadratic term, but even that is also real

$$\tilde{m}_{ij}\tilde{m}_{ji} = \tilde{m}_{ij}\tilde{m}_{ij}^* = |\tilde{m}_{ij}|^2.$$

Thus the simplest rephasing invariant combination that can contain the complex CP phase is of the form

$$\mathcal{J}_{ijk} = \tilde{m}_{ij}\,\tilde{m}_{jk}\,\tilde{m}_{ki} \qquad (i \neq j \neq k). \tag{4.17}$$

$\mathrm{Im}[\mathcal{J}_{ijk}]$ are antisymmetric under interchange of any two indices and hence vanishes when any two of the indices are same. We can express $\mathcal{J}_{ijk}$ in terms of $M$ matrix elements as,

$$\begin{aligned}\mathcal{J}_{ijk} &= \tilde{m}_{ij}\tilde{m}_{jk}\tilde{m}_{kl} \\ &= \left(\sum_\alpha m_{i\alpha}^* m_{j\alpha}\right)\left(\sum_\beta m_{j\beta}^* m_{k\beta}\right)\left(\sum_\gamma m_{k\gamma}^* m_{l\gamma}\right)\end{aligned} \tag{4.18}$$

Where $\sum_\alpha m_{i\alpha}^* m_{j\alpha}$ can be interpreted as scalar product of $i$th and $j$th row. A similar invariant was constructed in the case of seesaw model of neutrino masses in ref. [**?** ], although the approach to the problem is completely different. In this expression, if any one scalar product vanishes then number of independent rephasing measure $\mathrm{Im}[\mathcal{J}_{ijk}]$ which are independent of the Majorana phases will be reduced by one.

It is possible to express all the rephasing invariants containing the Dirac phases

$\mathcal{J}_{ijk}$ in terms of a minimal set of $\frac{(n-1)(n-2)}{2}$ invariants $\mathcal{J}_{ijn}$, $(i<j<n)$ as

$$\mathcal{J}_{ijk} = \frac{\mathcal{J}_{ijn}\mathcal{J}_{jkn}\mathcal{J}_{kin}}{|m_{in}|\ |m_{jn}|\ |m_{kn}|} \tag{4.19}$$

where $n$ is the index corresponding to the number of generations. Thus we define the measures of CP violation in lepton-number conserving processes as

$$J_{ijn} = \mathrm{Im}[\mathcal{J}_{ijn}] \qquad (i<j<n). \tag{4.20}$$

These invariants $\mathrm{Im}[\mathcal{J}_{ijk}]$ are not independent of the invariants $I_{ijkl}$ and can be expressed as

$$\mathcal{J}_{ijk} = \sum_{a,b,c} \frac{I_{iajb}\cdot I_{kaic}}{|m_{ia}|^2}. \tag{4.21}$$

So, the independent measures $I_{ij}$ include these independent measures of Dirac CP phases $\mathrm{Im}[J_{ijn}]$, $(i<j<n)$.

There are $n(n-1)/2$ phases present in $\tilde{M}$ for $n$ generations, but all of them are not independent. $(n-1)$ of these phases can be removed by redefining the phases of the leptons. That leaves $\frac{n(n-1)}{2} - n = \frac{(n-1)(n-2)}{2} = ^{(n-1)}C_2$ independent phases in $\tilde{M}$. This is the number of Dirac phases and may be observed in neutrino oscillation experiments. Let us assume that some particular $n-1$ entries are made real with appropriate rephasing. We can take all possible pair-product of these real entries. To have non-real rephasing invariant $\mathcal{J}_{ijk}$, one will have to multiply pair-product with some complex entry. For each real pair-product there correspond only one complex entry so that there product is a complex rephasing invariant defined as in equation 4.17. So number of all possible pair of real entries will give the number of non vanishing rephasing measures independent of Majorana phases which is $^{(n-1)}C_2 = \frac{(n-1)(n-2)}{2}$. This number is same as the number of physical phases present in $\tilde{M}$ as it has been analyzed earlier.

In the 2-generation case there is only one CP phase which is a Majorana phase. Which implies there should not be any non-vanishing $\mathcal{J}_{ijk}$, which is trivial to check. In the 3-generation case there is only one Dirac CP measure, which is

$$\mathcal{J}_{123} = \tilde{m}_{12}\ \tilde{m}_{23}\ \tilde{m}_{31} = \sum_{a,b,c} \left[m^*_{a1}\ m_{a2}\ m^*_{b2}\ m_{b3}\ m^*_{c3} m_{c1}\right]. \tag{4.22}$$

Thus given a neutrino mass matrix one can readily say if this mass matrix will imply CP violation in the neutrino oscillation experiments.

In the 4-generation case there are three CP phases in the PMNS mixing matrix and 3-Majorana phase. The independent rephasing invariants of Dirac phases will be given as $\mathcal{J}_{124}$, $\mathcal{J}_{134}$ and $\mathcal{J}_{234}$ . One dependent rephasing invariant is $\mathcal{J}_{123}$ which can be expressed as

$$\mathcal{J}_{123} = \frac{\mathcal{J}_{124}\mathcal{J}_{234}\mathcal{J}^*_{134}}{|\tilde{m}_{14}\tilde{m}_{24}\tilde{m}_{34}|^2}.$$

In general, these invariants satisfy

$$\mathcal{J}_{ijk}\mathcal{J}^*_{ikl}\mathcal{J}_{ilj} = |\tilde{m}_{ij}\tilde{m}_{ik}\tilde{m}_{il}|^2\mathcal{J}_{jkl} \tag{4.23}$$

for $n$ generations, where $i, j, k = 1, 2, ..., n$.

We summarize this section by restating for $n$-generation neutrino mass matrix without any texture zeroes, the rephasing invariants corresponding to the Dirac phase are $J_{ijn}$, $(i < j < n)$. If there are texture zeroes, then some of the $J_{ijk}$, $(i < j < k,\ k \neq n)$ could also be independent.

## 4.5 Texture zeroes

In case neutrino mass matrix contains zero entries in all the columns, it is convenient to define basic independent quartic invariants to the rephasing of charged leptons in slightly different form as,

$$\mathcal{R}_{ijnn} = \lim_{|m_{in}|,|m_{jn}|,|m_{nn}|\to 0} \frac{m_{ij}m^*_{in}m^*_{jn}m_{nn}}{|m_{in}|\ |m_{jn}|\ |m_{nn}|} \quad (i \leq j \text{ and } i, j \neq n) \tag{4.24}$$

Limit has to be taken for all $n$th column elements. Any other such quartic invariant can be expressed in terms of these independent rephasing invariants $\mathcal{R}_{ijnn}$ as,

$$\mathcal{R}_{ijkl} = \mathcal{R}_{ijnn}\mathcal{R}_{klnn}\mathcal{R}^*_{linn}\mathcal{R}^*_{kjnn} \tag{4.25}$$

Advantage of defining the independent rephasing(to the rephasing of charged lepton) invariants $\mathcal{R}_{ijnn}$ as the limiting case is that the expressions do not become undefined due to presence of vanishing denominators. Let us write the invariants in a different form as,

$$\mathcal{R}_{ijnn} = |m_{ij}|e^{i(\theta_{ij}+\theta_{nn}-\theta_{in}-\theta_{jn})} \quad (i, j\ \neq n \textit{ and } i \leq j)$$

Where $\theta_{kn}$ is the phase present at $(k, n)$ entry of the mass matrix. If there are some zero entries present in $n$th column, then the corresponding phases present in expression of $\mathcal{R}_{ijnn}$ must be unphysical. So a subset of the set of these basic independent invariants will be having the unphysical phases associated with the zero entries of the $n$th column. We can not define CP measures corresponding to the invariants having unphysical phases by extracting the imaginary part of the invariants, although we can define CP measure as usual for rest of the invariants. All the invariants defined above are invariants corresponding to the rephasing of charged leptons but not rephasing of the unphysical phase corresponding to zero entries present in the $n$th column. We will have to construct the full rephasing invariants to all unphysical phases in terms above invariants. This can be done by multiplying two or more invariants having unphysical phases in such a way that all of the unphysical phases cancel out. It turns out that eliminating one of these unphysical phases corresponding to the zero entries in $n$th column reduces the number of full rephasing invariants by one. Also, one independent full rephasing invariant (and so one CP measure) vanishes corresponding to the zero entry present in other than $n$th column (or $n$th row). Thus the number of independent CP measures $N_{CP}$ defined as the imaginary part of these full rephasing invariants for neutrino mass matrix having $p$ zero entries and $q$ zero rows (all the row entries are vanishing) for $n$ generations is given by

$$N_{CP} = \frac{n(n-1)}{2} - p + q \tag{4.26}$$

This is equal to the number of physical phases present in the matrix which can not be removed. In the same way we can study the mass-squared matrices $\tilde{M}$ and write down the number of rephasing invariant measures independent of Majorana phases $\widetilde{N}_{CP}$ is given as

$$\widetilde{N}_{CP} = \frac{(n-1)(n-2)}{2} - r + s$$

where $r$ is the number of zero entries in $\tilde{M}$ and $s$ is the number of those rows whose all the entries excluding diagonal one are zero. It should be noticed that above relation of $\tilde{N_{CP}}$ is only valid if $N_{CP}$ is not zero.

## 4.6 Application to texture two-zero mass matrices

In this section, we discuss a potential application of our formulation in context of neutrino mass matrix with two zero textures[63]. The study of all possible neutrino mass matrices with two zero entries has revealed that only seven such matrices are phenomenologically allowed in light of existing data on neutrino masses and mixing [63, 64] . At the same time, the three texture zero mass matrices are found to be inconsistent. The question of realization of the two texture zeros in see saw context has been discussed in [65, 66]. The possibility of the origin of textures zeros is GUT scenarios has been addressed in [67–69]. Another such possibility that has been considered in literature is by invoking some flavor symmetry [70–72].

With our present formalism, we shall now study a class of 3-generation neutrino mass matrices with two-zero textures, which has been listed in ref. [63]. There are seven such mass matrices that are consistent with present information about

Table 4.1: Phenomenologically allowed two texture neutrino mass matrices

| Pattern | Texture of $m_\nu$ | Mass spectrum |
|---|---|---|
| $A_1$ | $\begin{pmatrix} \mathbf{0} & \mathbf{0} & \times \\ \mathbf{0} & \times & \times \\ \times & \times & \times \end{pmatrix}$ | hierarchical |
| $A_2$ | $\begin{pmatrix} \mathbf{0} & \times & \mathbf{0} \\ \times & \times & \times \\ \mathbf{0} & \times & \times \end{pmatrix}$ | hierarchical |
| $B_1$ | $\begin{pmatrix} \times & \times & \mathbf{0} \\ \times & \mathbf{0} & \times \\ \mathbf{0} & \times & \times \end{pmatrix}$ | quasi-degenerate |
| $B_2$ | $\begin{pmatrix} \times & \mathbf{0} & \times \\ \mathbf{0} & \times & \times \\ \times & \times & \mathbf{0} \end{pmatrix}$ | quasi-degenerate |
| $B_3$ | $\begin{pmatrix} \times & \mathbf{0} & \times \\ \mathbf{0} & \mathbf{0} & \times \\ \times & \times & \times \end{pmatrix}$ | quasi-degenerate |
| $B_4$ | $\begin{pmatrix} \times & \times & \mathbf{0} \\ \times & \times & \times \\ \mathbf{0} & \times & \mathbf{0} \end{pmatrix}$ | quasi-degenerate |
| $C$ | $\begin{pmatrix} \times & \times & \times \\ \times & \mathbf{0} & \times \\ \times & \times & \mathbf{0} \end{pmatrix}$ | quasi-degenerate |

neutrino masses listed in the table

From our discussions in the previous section, there can be only one CP phase in all these cases. We shall now identify the rephasing invariants in all the cases. Although all these matrices differ in phenomenology, as far as CP violation is concerned, the interchange of the indices $(2 \leftrightarrow 3)$ will not change any discussion. So, we shall not explicitly discuss the models $A_2, B_2, B_4$, which can be obtained by changing the indices $(2 \leftrightarrow 3)$ from the matrices $A_1, B_1, B_3$ respectively.

**Case $A_1$:**

There is only one non-vanishing $I_{ij}$, which is $I_{23}$. The lepton-number conserving rephasing invariant measure $J_{123}$ is given by

$$\begin{aligned} [J_{123}] &= \text{Im}\,[(m_{31}^* m_{32})(m_{22}^* m_{23} + m_{32}^* m_{33})(m_{33}^* m_{31})] \\ &= |m_{31}|^2 I_{23} \end{aligned} \tag{4.27}$$

Thus there is only one Dirac CP phase in this case, which will contribute to the lepton-number conserving processes. The same result is valid for $A_2$.

**Case** $B_1$**:**

In this case all the measures $I_{ij}$ are vanishing. Even the invariants of the form $I_{ij}$ are all vanishing. However, there is one CP phase as discussed in the previous section. The invariant $I_{122133}$ is non-vanishing, which cannot be related to to the lower invariants by $I_{122133} = I_{1221} \cdot I_{3322}/|m_{22}^2$, since $m_{22} = 0$. The lepton-number conserving invariant is related to this invariant by

$$\begin{aligned} [J_{123}] &= \text{Im}\,[(m_{11}^* m_{12})(m_{32}^* m_{33})(m_{23}^* m_{21})] \\ &= \text{Im}[I_{122133}]. \end{aligned} \tag{4.28}$$

Again there are no Majorana CP phase. The analysis is same for the case $B_2$.

**Case** $B_3$**:**

There is only one non-vanishing CP violating measure $I_{13}$, which is related to the lepton-number conserving measure by

$$\begin{aligned} [J_{123}] &= \text{Im}\,[(m_{31}^* m_{32})(m_{32}^* m_{33})(m_{13}^* m_{11} + m_{33}^* m_{31})] \\ &= |m_{32}|^2 I_{13}. \end{aligned} \tag{4.29}$$

There are no more CP phase left in addition to the one entering in lepton-number conserving processes. Replacing the indices $(2 \leftrightarrow 3)$ we get for the case $B_4$ a similar relation $[J_{123}] = |m_{32}|^2 I_{12}$.

**Case** $C$**:**

This is the most interesting case. There are no CP violating measures of the form $I_{ij}$, although the invariant $I_{1123}$ is non-vanishing. So, there is one CP phase in this case, as expected. This is related to the CP violating measure that affects lepton-number conserving processes by

$$\begin{aligned} J_{123} &= \text{Im}[(m_{11}^* m_{12} + m_{31}^* m_{32})(m_{12}^* m_{13})(m_{13}^* m_{11} + m_{23}^* m_{21})] \\ &= |m_{12}|^2 I_{1123} + |m_{13}|^2 I_{1123}^*. \end{aligned}$$

Although this shows that the phase is a Dirac phase, in the special case of $m_{12} = m_{13}$, there will not be any CP violation in the neutrino oscillation experiments. This can be verified from the fact that for $m_{12} = m_{13}$ the third mixing angle and hence $U_{13}$ vanishes. In this case the CP violation can originate from a Majorana phase, since $J_{123}$ vanishes even when $\text{Im}_{1123}$ is non-vanishing.

Another way to understand this is to write the mass matrix in a different basis.

When $m_{12} = m_{13}$, we can write the mass matrix $C$ as

$$\begin{pmatrix} X & X & 0 \\ X & X & 0 \\ 0 & 0 & X \end{pmatrix}.$$

In this case the third generation decouples from the rest and we know that for two generation there is only a Majorana phase, which corresponds to non-vanishing $I_{12}$ and there is no Dirac phase, as we stated above. This is the only example of two-zero texture mass matrices where the CP violating phase could be a Majorana phase, but this mass matrix is not allowed phenomenologically.

Thus there are no phenomenologically acceptable two-zero texture neutrino mass matrices, which has any Majorana phase. The only CP phase possible in any two-zero texture 3-generation mass matrix is of Dirac type and should allow CP violation in neutrino oscillation experiments.

# Chapter 5

# Seesaw mechanism of neutrino masses

Observations of neutrino flavor oscillations from solar [6–8], atmospheric [5] and laboratory neutrino experiments [9–11] have provided firm evidences that the neutrinos have masses, although very small. To accommodate the neutrino masses, one needs to go beyond the SM of particle physics by extending either the fermion sector or the Higgs sector of the SM. One simple way is to introduce right-handed singlet neutrinos in the fermionic sector and generate the Dirac mass terms through the Yukawa couplings. The inability to observe the right-handed neutrinos can be attributed to its singlet nature as this ensures absence of any coupling of this singlets with SM gauge bosons. From the standpoint of mass generating mechanism, this will provide an equal footing for both quarks and leptons. Moreover, the generic concept of mixing in the quark sector can be comfortably borrowed to explain mixing in the leptonic sector.

However, the situation turns out to be quite uncomfortable when we compare the mass scale of neutrinos with that of charged leptons. Natural analogy from the quark sector suggests that the neutrinos should have masses more or less of the order of charged lepton masses. However, the observed very small masses of the neutrinos ($\sim eV$) are quite far from expected (figure 5.1). Obviously, the picture is not able to provide a natural framework to explain the very lightness of the neutrino masses so far as origin of all SM fermion masses is expected to come from some common fundamental structure. This along with considerations of both charge neutrality of neutrinos and lack of any evidence for the right-handed neutrinos below the weak scale have produced much motivation to consider the possibility of neutrinos being Majorana fermions, in the literature.

A Majorana fermion is characterized by the feature that it is its own antiparticle. In fact, it is possible to generate Majorana masses in the elegant seesaw framework by introducing a lepton-number violating source at some high scale [18–20]. The

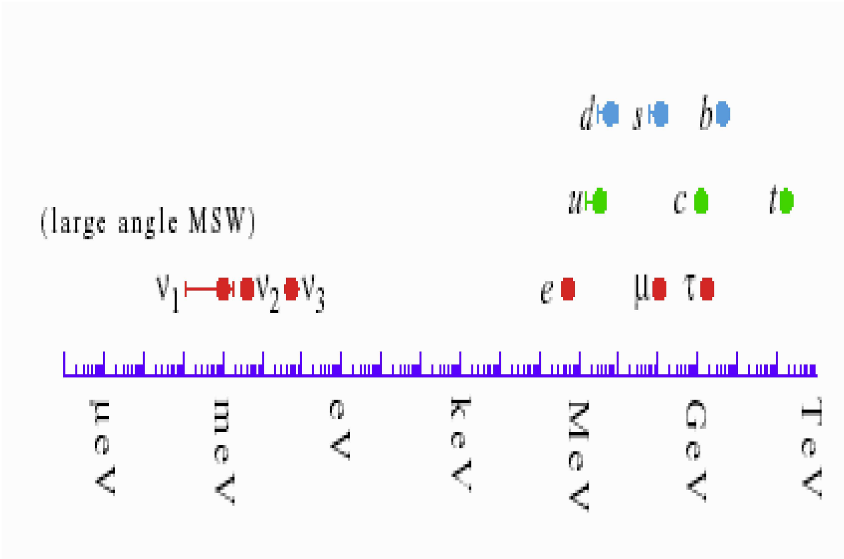


Figure 5.1: Lightness of neutrinos

seesaw mechanism not only provides a natural way to realize the suppression of neutrino masses but also comes with an extra feature that it can explain the current baryon asymmetry of the universe through creating the lepton asymmetry at the early universe which is possible due to presence of the lepton-number violating source term [28, 73–83]. Depending on the different models and their matter contents, several seesaw realizations exist in the literature with the common desired feature that all of them can provide three light neutrinos required for the consistent understanding of combined neutrino oscillation data from all the experiments.

## 5.1 Type I seesaw

As pointed out above, a naturally motivated extension of the SM would be to add right-handed neutrinos to its fermionic content. Like any other charged fermion, they get Dirac masses through the Yukawa couplings of the right-handed neutrinos with the electroweak lepton doublets and Higgs doublet. One can also write the Majorana mass term for these singlet right-handed neutrino as there is no prior reason to neglect it while writing the most general Lagrangian allowed by the SM gauge group. However, this term will break the lepton number by two units $\Delta L = 2$.

The characteristic feature of this seesaw scenario is that the gauge singlet fermions (right-handed neutrinos) can have a natural Majorana mass scale much larger than the electroweak scale, which in turn, leads to natural suppression of the neutrino masses. The relevant part of Lagrangian will be given as

$$\mathcal{L} \;=\; \left(\frac{1}{2}\overline{(N_{R\alpha})^c}(M_R)_{\alpha\beta}N_{R\beta} + \overline{\ell_{Li}}\phi(Y_\ell)_{ij}e_{Rj} + \overline{\ell_{Li}}\tilde{\phi}(Y_\nu)_{i\alpha}N_{R\alpha} + H.C.\right), \quad (5.1)$$

where $\tilde{\phi} = \tau^2\phi^*$ and $i$, $j$ runs from 1 to 3, representing the left-handed fields. $\alpha$ represent the right-handed neutrino indices. $\ell_{Li}$ represents the $SU(2)_L \times U(1)_Y$ doublets, $e_{Ri}$ and $N_{R\alpha}$ are the right-handed singlets of the theory.

Before we proceed further, we would like to make an important remark that it is possible to add any reasonable numbers of right-handed neutrinos except one which is not able to account for the observed non-degenerate mass spectrum of the light neutrinos. After the electroweak symmetry breaking, the terms relevant for the neutrino masses can be written as

$$-\mathcal{L}_{\nu mass} = \frac{1}{2}\begin{pmatrix} \bar{\nu}_{Li} & \overline{(N_{R\alpha})^c} \end{pmatrix}\begin{pmatrix} 0 & (M_{\nu D})_{ij} \\ (M^T_{\nu D})_{ij} & (M_R)_{\alpha\beta} \end{pmatrix}\begin{pmatrix} (\nu_{Lj})^c \\ N_{R\beta} \end{pmatrix},$$

where $M_{\nu D} = Y_\nu v$ is the Dirac mass matrix of the neutrinos, $v$ is the *vev* of the SM Higgs and $M_R$ is the mass matrix of the right-handed neutrinos. Since the Majorana mass matrix is symmetric and $M_R \gg M_{\nu D}$, the whole mass matrix can be block-diagonalized as following

$$\text{Block}-\text{diagonalized}\;\begin{pmatrix} 0 & M_{\nu D} \\ M^T_{\nu D} & M_R \end{pmatrix} \Rightarrow \begin{pmatrix} m_\nu & 0 \\ 0 & M_R \end{pmatrix},$$

where

$$m_\nu = -M_{\nu D}M_R^{-1}M^T_{\nu D}, \qquad . \quad (5.2)$$

After the block-diagonalization, one gets an effective $3\times 3$ low energy neutrino mass matrix $m_\nu$ representing three light Majorana neutrinos and $M_R$ representing the heavy Majorana neutrinos. This way of making particles light at the expense of making another one heavy is known as seesaw mechanism and this particular one is called as type I seesaw. The mass scale of $m_\nu$ is suppressed by the scale of $M_R$. If we take scale of $M_{\nu D}$ roughly similar to the scale of $m_\tau$, the upper bound on neutrino masses can be converted to the lower bound of $M_R$ as

$$M_R > 10^9 GeV\,.$$

Such a high scale is beyond the reach of being tested in the next generation of colliders.

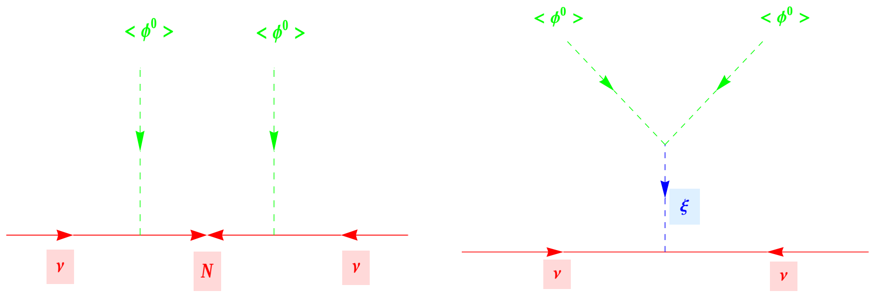


Figure 5.2: Type I (left) and type II (right) seesaw realizations

## 5.2 Type II seesaw

In the type I seesaw scenario, we extended the fermionic sector of the SM by adding two or more right-handed neutrinos. It predicts three light Majorana neutrinos below the electroweak scale. However, an important point that need to be realized is that the number of degree of freedom of three light Majorana neutrinos are same to that of three left-handed Weyl neutrinos. So, one may like to explore the possibility to work without introducing the right-handed neutrinos if somehow the Majorana mass term for the left-handed neutrinos can be allowed to write. It is in fact possible by extending the Higgs content of SM, instead of fermionic content, by adding a Higgs triplet ($\xi \equiv (3,1,1)$) with hypercharge $Y=1$. This Higgs triplet can couple to the lepton doublet and is able to give consistent masses to the neutrinos provided it gets a very tiny *vev*. The part of the Lagrangian responsible for the neutrino masses is given as:

$$\mathcal{L} = f_{ij}\ell_{Li}^T C(i\tau_2)(\vec{\tau}\cdot\vec{\xi})\ell_{Lj} + h.c.,$$

where C represent the inverse of charge conjugation operator. The triplet $\xi$ can be assigned lepton number $L=-2$ so that the above Yukawa terms respect the lepton-number conservation. Now, if the $\xi$ gets a non zero *vev*, the lepton number will be broken by two units leading the Majorana masses for the left-handed neutrinos at the low energy

$$m_\nu = <\xi^0> f\,.$$

Expecting the Yukawa couplings $f_{ij}$ to be of order one, we need to look for an another seesaw mechanism which can naturally provide a very small *vev* for the triplet. The most general Higgs potential of one doublet $\phi = (\phi^+,\phi^0)$ and one triplet

$\xi = (\xi^{++}, \xi^{+}, \xi^{0})$ is given as:

$$\begin{aligned} V &= m^2\phi^\dagger\phi + M^2\xi^\dagger\xi \\ &+ \frac{1}{2}\lambda_1(\phi^\dagger\phi)^2 + \frac{1}{2}\lambda_2(\xi^\dagger\xi)^2 + \lambda_3(\phi^\dagger\phi)(\xi^\dagger\xi) \\ &+ \mu(\bar{\xi}^0\phi^0\phi^0 + \sqrt{2}\xi^-\phi^+\phi^0 + \xi^{--}\phi^+\phi^+) + h.c. \end{aligned} \quad (5.3)$$

Obviously, the coupling $\mu\xi\phi\phi$ violates lepton number explicitly. In the initial triplet model [84, 85], this term is avoided by imposing the lepton-number symmetry. The spontaneous breaking of the lepton number leads to a goldstone boson called Majoron. The model predicts substantial decay of $Z$ boson into the Majoron and $Re(\xi^0)$ which has been ruled out by measured decay width of $Z$ boson. The significance of $\mu\xi\phi\phi$ term was first appreciated by [78] where it was pointed out that the presence of this term will not only help in avoiding any Majoran at low energy but also provides an small induced *vev* to the triplet. If doublet gets *vev* as $\langle\phi^0\rangle = v$ and triplet gets as $\langle\xi^0\rangle = u$, then one can show that $u$ gets an induced *vev* through the figure 5.2, suppressed by square of the mass of the triplet as

$$u = -\frac{\mu v^2}{M^2},$$

where M is the mass of the triplet. So if the mass of the triplet is very large, one can naturally suppress the mass of the neutrinos, since the neutrino mass matrix is proportional to $u$. This mechanism is called as Type II seesaw mechanism for generating light neutrinos.

## 5.3 Type III seesaw

There exists two versions of seesaw mechanism named as type III seesaw in the literature. In the first one, two or more $SU(2)_L$ fermionic triplets are added to the SM. It can couple to the triplet combination of lepton doublet and Higgs doublet and can provide the Majorana mass to the neutrinos after the $\phi$ acquires *vev*. In the second version, a different type of fermionic SM singlets provide the three light Majorana neutrinos along with the right-handed singlet neutrinos.

### 5.3.1 Type III seesaw with fermionic triplet

In this seesaw scenario, fermionic $SU(2)_L$ triplets ($T_F$) with $Y = 0$ are added to the fermionic content of the SM [86]. SM gauge group allows the Majorana mass term for the fermion triplets and their couplings with the lepton doublets and the Higgs doublet are given as

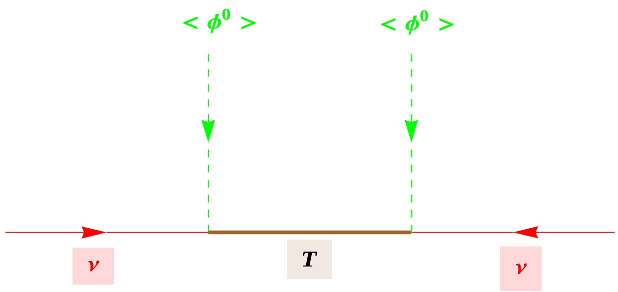


Figure 5.3: Type III seesaw realization with fermionic triplet

Table 5.1: Three types of seesaw mechanism

| Type of seesaw | Corresponding d=5 SM operator |
|---|---|
| Type I | $\frac{1}{\Lambda}(\ell^T_{Li}C\tau_2\phi)(\phi^T\tau_2\ell_{Lj})$ |
| Type II | $\frac{1}{\Lambda}(\ell^T_{Li}C\tau_2\overrightarrow{\tau}\ell_{Lj}).(\phi^T\tau_2\overrightarrow{\tau}\phi)$ |
| Type III | $\frac{1}{\Lambda}(\ell^T_{Li}C\tau_2\overrightarrow{\tau}\phi).(\phi\tau_2\overrightarrow{\tau}\ell_{Lj})$ |

$$\mathcal{L} = M_{Tij}\overrightarrow{T_{Li}}C\overrightarrow{T_{Lj}} + Y_{Tij}\ell^T_{Li}C(i\tau_2)\overrightarrow{\tau}.\overrightarrow{T_{Lj}}\phi\,.$$

If triplet is heavy, i.e., $M_T \gg v$, the neutrino mass matrix can be written in the same way as in type I seesaw case

$$m_\nu = -v^2 Y_T M_T^{-1} Y_T\,.$$

The low energy neutrino mass matrix takes the same structure as in type I seesaw (figure 5.3) . So, we need at least two fermionic triplets to account for the evidence of the non-degenerate spectrum of the neutrino masses.

Before we proceed to the next version of type III seesaw, we would like to make the remark that the three seesaw scenario discussed so far can be, actually, described by a five dimensional SM operator after integrating out all the heavy degrees of freedom in the model (table 5.1).

Although the three operators in table 5.1 appears to be different from each other, they can all be shown to be equivalent. There are only three possible tree-level realizations of this low energy operator which correspond to the three types of seesaw mechanism mentioned so far [87]. The difference is expected to arise only when the scale $\Lambda$ is probed nearby. Interesting study of all possible $d = 6$ operators has been performed in [88].

### 5.3.2 Type III seesaw with fermionic singlet

In this scenario, the SM fermionic content is extended by adding three SM singlet right-handed neutrinos ($N_R$) and three other singlet Weyl fermions ($S_R$). One may ask how to discriminate all the singlets into two groups. The right-handed singlets are characterized be the feature of providing Dirac masses to the neutrinos of the correct order and that they are prevented to acquire Majorana masses like the left-handed neutrinos. On the other hand, the $S_R$ singlets are allowed to have the Majorana masses. The coupling of both left-handed as well as right-handed neutrinos are allowed with $S_R$. This new mechanism is also called Type III seesaw mechanism in the literature. The part of the Lagrangian which gives masses to the neutrinos is given as

$$\mathcal{L}_{\nu\, mass} = (\nu_{iL}, N^c_{iL}, S^c_{mL}) \begin{pmatrix} 0 & (M_{\nu D})_{ij} & F_{in} u \\ (M^T_{\nu D})_{ij} & 0 & F_{in}\Omega \\ F^T_{mj} u & F^T_{mj}\Omega & M_{mn} \end{pmatrix} \begin{pmatrix} \nu_{jL} \\ N^c_{jL} \\ S^c_{nL} \end{pmatrix} . \tag{5.4}$$

The low energy effective neutrino mass matrix will be given as

$$m_\nu = -\left(M_{\nu D} + M^T_{\nu D}\right) \frac{u}{\Omega} - M_{\nu D} \left(F \Omega M^{-1} F^T \Omega\right) M^T_{\nu D} . \tag{5.5}$$

The second term is known as double seesaw contribution [89, 90]. The first term in the mass matrix is the type III seesaw contribution [91]. This seesaw scenario has got a simple realization in the left-right symmetric model that we are going to discuss in the next section.

## 5.4 Seesaw mechanism in left-right symmetric model

A very attractive extension of the SM, very different from the above, would be to enlarge the gauge group of the SM such that the broken left-right parity is restored at some high scale. This kind of model is known as left-right symmetric model in the literature and has been extensively studied. The model has got historic importance as it provides nonzero small neutrino masses and embeds seesaw in it simply by demanding the left-right symmetry breaking at some appropriate high scale.

In the model, the SM gauge group is extended to a left-right symmetric gauge group, $G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)}$ [92–96]. The left-right symmetric nature of the model immediately predicts three right-handed neutrinos. The electric charge is defined in terms of the generators of the group as:

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2} = T_{3L} + Y ., \tag{5.6}$$

where $Y = T_{3R} + \frac{B-L}{2}$. The transformation properties of the quarks and the leptons under the left-right symmetric gauge group are given as:

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \equiv [3,2,1,\frac{1}{6}] \qquad Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \equiv [3,1,2,\frac{1}{6}]$$
$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \equiv [1,2,1,-\frac{1}{2}] \qquad \ell_R = \begin{pmatrix} N_R \\ e_R \end{pmatrix} \equiv [1,1,2,-\frac{1}{2}]$$

The gauge boson (excluding gluons) sector consist of two triplets and one singlets, other than QCD gauge bosons, as:

$$W_{\mu L} = \begin{pmatrix} W^+_{L\mu} \\ W^0_{L\mu} \\ W^-_{L\mu} \end{pmatrix} \equiv (1,3,1,0), \quad W_{\mu R} = \begin{pmatrix} W^+_{R\mu} \\ W^0_{R\mu} \\ W^-_{R\mu} \end{pmatrix} \equiv (1,1,3,0), \quad B_{\mu(B-L)} \equiv (1,1,1,0)$$

The right-handed neutrino $N_R$ is present in all the left-right symmetric model, which is dictated by the structure of the fermion representation and the gauge group. As the name suggests, the left-right symmetric model is characterized by demanding invariance of the Lagrangian under following left-right parity transformation :

$$SU(2)_L \leftrightarrow SU(2)_R$$

$$Q_L \leftrightarrow Q_R$$

$$\ell_L \leftrightarrow \ell_R$$

$$W_{\mu L} \leftrightarrow W_{\mu R}$$

The next important thing that need to be realized is the breaking of left-right gauge group to the SM group:

$$
\begin{aligned}
& SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} \ [G_{LR} \, \text{or} \, G_{322D}] \\
& \stackrel{M_R}{\longrightarrow} SU(3)_c \times SU(2)_L \times U(1)_Y \qquad [G_{\text{std}} \, \text{or} \, G_{321}] \\
& \stackrel{m_W}{\longrightarrow} SU(3)_c \times U(1)_Q \qquad\qquad [G_{\text{em}}] \, .
\end{aligned}
$$

For the purpose, one need a scalar Higgs transforming non-trivially under $SU(2)_R$. Giving a *vev*, with a scale high enough compared to the weak scale, to the scalar along the SM singlet direction will serve the purpose. This scalar has to be singlet under $SU(2)_L$ as the SM gauge group remains unbroken in the first step. Introducing this scalar to break the left-right group, consistency demands the need for an another scalar which transforms trivially under $SU(2)_R$ but non-trivially under $SU(2)_L$. Before assigning the quantum numbers to these left-handed and right-handed Higgs fields, we discuss some interesting feature of the potential minimization involving the two fields. As it would be convenient to think in terms of the *vev* values, let us assign $u_L$ as a *vev* to the left-handed Higgs and $u_R$ to the right-handed one. Keeping left-right symmetry in mind, one can write the potential as

$$
V = -\frac{\mu^2}{2}\left(u_L^2 + u_R^2\right) + \frac{\lambda}{4}\left(u_L^2 + u_R^2\right)^2 + \frac{(g-\lambda)}{2} u_L^2 u_R^2 \, ,
$$

where $\mu^2 > 0$ and $\lambda > 0$ to ensure that the potential is bounded from the below and one of the solution for the extremum $u_L = u_R = 0$ is maxima and not the minima of the potential. The linear terms such as $u_L u_R$ and $(u_L + u_R)$ are forbidden within the Higgs content chosen, but can appear in general when we add some more Higgs fields specially to break the SM gauge group. The last term is crucial in deciding the symmetry breaking pattern as the first two term are blind to the direction of the symmetry breaking.

For the case $g < \lambda$, one gets the solution for minima of the potential as $u_L = u_R \neq 0$. This is unacceptable as we see the broken left-right parity in nature. However, the case $g > \lambda$ provides two equally probable solutions $u_L = 0, u_R \neq 0$ or $u_R = 0, u_L \neq 0$ one of which is phenomenologically reasonable. So the left-right symmetric potential automatically produces a symmetry breaking pattern where left-right symmetry breaking can be naturally realized. The gauge bosons related to the broken $SU(2)_R$ gauge group acquire the masses with the scale similar to the scale of $u_R$ by absorbing the goldstone mode through the usual Higgs mechanism.

Now to break the electroweak symmetry, one introduces a bi-double ($\Phi$) with the quantum number $(1,2,2,0)$ with respect to the left-right model given as:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \; : \; (1,2,2,0)$$

Assigning *vev* to the neutral component of $\Phi$ field not only leads to electroweak breaking but also provides Dirac masses to all the fermions including neutrinos in the model as obvious from the following coupling of $\Phi$ with the fermions:

$$-\mathcal{L}_m = Y_{\ell ij}\overline{\ell_{Li}}\Phi\ell_{Rj} + \widetilde{Y}_{\ell ij}\overline{\ell_{Li}}\widetilde{\Phi}\ell_{Rj} + Y_{Qij}\overline{Q_{Li}}\Phi Q_{Rj} + \widetilde{Y}_{Qij}\overline{Q_{Li}}\widetilde{\Phi}Q_{Rj} + h.c.\,,$$

where $\widetilde{\Phi} = i\tau_2\Phi i\tau_2$. Although the model automatically predicts the Dirac masses for the neutrinos, the problem of realizing naturally small masses for neutrinos still remains unsolved unless one tries to embed the seesaw framework in the model. There do exist two well studied seesaw realizations in the left-right symmetric model in the literature.

### 5.4.1 Left-right type (I+II) seesaw realization

So far we have just talked about the scalar fields needed to break the left-right symmetry but have not specified them. There are two well studied choices of Higgs multiplets used to implement the breaking of the left-right symmetric gauge group. One of them which we discuss here is characterized by introducing following set of Higgs fields:

$$\xi_L \;(1,3,1,1)\;,\; \xi_R\;(1,1,3,1)\,, \tag{5.7}$$

Giving large *vev* to $\xi_R$ will spontaneously break the left-right group spontaneously and will violate the $B-L$ quantum number by two units or the lepton number by two units. As it was discussed above, the coupling $\gamma Tr\left(\Phi^\dagger\vec{\xi_L}\cdot\vec{\tau}\Phi\vec{\xi_R}\cdot\vec{\tau}\right)$ will allow the term in $\gamma v^2 u_L u_R$ which is linear in $u_L$ in the potential. So the field $\xi_L$ will acquire an induced *vev* after the electroweak symmetry breaking which is related to $u_R$ and $v$ as follows:

$$u_L = \gamma\frac{v^2}{u_R}$$

So if $u_R$is taken to be large enough compared to the weak scale, $u_L$ will tend to very small value compared to the weak scale. The relation is very crucial as far as the seesaw embedding is concerned. It is obvious to see that the $SU(2)_L$ triplet $\xi_L$ will provide very small Majorana masses to the left-handed neutrinos as in type II seesaw scenario. At the same time, $\xi_R$ will provide large Majorana masses to the right-handed neutrinos. The part of the Lagrangian relevant for the neutrino masses can be given as:

$$\mathcal{L} = f_{Lij}\ell_{Li}^{T}C(i\tau_2)(\overrightarrow{\tau}\cdot\overrightarrow{\xi_L})\ell_{Lj} + f_{Rij}\ell_{Ri}^{T}C(i\tau_2)(\overrightarrow{\tau}\times\overrightarrow{\xi_R})^{\dagger}\ell_{Rj} + Y_{\ell ij}\overline{\ell_{Li}}\Phi\ell_{Rj} + \widetilde{Y}_{\ell ij}\overline{\ell_{Li}}\widetilde{\Phi}\ell_{Rj}$$

Therefor, in the left-right symmetric model both the terms corresponding to Type I and Type II seesaw mechanism appear simultaneously. After assigning *vev* to both Higgs fields, above equation turns out to be:

$$\mathcal{L}_{\nu\, mass} = (\nu_L, N_L^c)\begin{pmatrix} M_L & M_{\nu D} \\ M_{\nu D}^T & M_R \end{pmatrix}\begin{pmatrix} \nu_L \\ N_L^c \end{pmatrix}, \tag{5.8}$$

where $M_L = u_L f$, $M_R = u_R f$ are the mass matrix corresponding to the Majorana mass term of the left-handed ($\nu_L$) and the right-handed ($N_L^c$) neutrinos and $M_{\nu D}$ is the mass matrix corresponding to the Dirac mass term of the neutrinos. Block diagonalizing the above matrix, the low energy neutrino mass matrix is given as

$$\begin{aligned} m_\nu &= M_L - M_{\nu D}M_R^{-1}M_{\nu D}^T \\ &= u_L f - \frac{v^2}{u_R} Y f^{-1} Y^T \end{aligned}$$

As $\gamma \approx 1$, the scale of both the terms surprisingly comes out to be of same order in the left-right symmetric model. Hence the corresponding contributions to the neutrino masses due to the type I and II seesaw terms are automatically suppressed as soon as we demand a relatively high scale breaking of the left-right gauge group. So the left-right symmetric model provides a natural realization of both type(I+II) seesaw mechanism consistent with each other.

### 5.4.2 Left-right type III seesaw realization

An alternate scenario of breaking the left-right gauge group can also be achieved by the following set of Higgs field:

$$\chi_L\ (1,2,1,1/2)\ ,\ \chi_R\ (1,1,2,1/2)\ , \tag{5.9}$$

Let us denote their *vev* values by $v_L$ and $v_R$. Obviously, providing a large *vev* to $\chi_R$ will break the left-right gauge group at the desired high scale. Like the previous scenario, $\chi_L$ can again acquire an very small induced *vev* due to the presence of the coupling of $\chi_L$ with $\chi_R$ and $\phi$ leading to the same kind of relation:

$$v_L = \gamma' \frac{v^2}{v_R}$$

Both the scalars $\chi_{L,R}$ break the $B-L$ number or the lepton number by one unit, so will not be able to provide the Majorana masses to either of the neutrinos. However, one can induce the Majorana masses for the neutrinos by introducing three fermions $S_R$, singlet under the left-right gauge group having very small Majorana masses.

$$S_R \equiv (1, 1, 1, 0)$$

These singlets will interact with the neutrinos and will generate the Majorana masses for the neutrinos. The terms contributing to the neutrino masses are:

$$\mathcal{L} = M_{mn} (S_L^c)_m^T C S_{Ln}^c + F_{Ljm} \ell_{Li}^T C \chi_L^\star S_{Lm}^c + F_{Rim} (\ell^c)_{Li}^T C \chi_R S_{Lm}^c + Y_{\ell ij} \overline{\ell_{Li}} \Phi \ell_{Rj} + \widetilde{Y}_{\ell ij} \overline{\ell_{Li}} \widetilde{\Phi} \ell_{Rj} + h.c.$$

Assigning the *vev*s, we get

$$\mathcal{L}_{\nu\, mass} = (\nu_{iL}, N_{iL}^c, S_{mL}^c) \begin{pmatrix} 0 & (M_{\nu D})_{ij} & F_{in} v_L \\ (M_{\nu D}^T)_{ij} & 0 & F_{in} v_R \\ F_{mj}^T v_L & F_{mj}^T v_R & M_{mn} \end{pmatrix} \begin{pmatrix} \nu_{jL} \\ N_{jL}^c \\ S_{nL}^c \end{pmatrix}, \tag{5.10}$$

which is similar to the expression 5.4. So it is also possible to realize the type III seesaw mechanism of the neutrino masses in the left-right symmetric scenario by using $\chi_L$ and $\chi_R$ Higgs fields which simultaneously play the role of breaking the left-right parity.

In both the left-right symmetry breaking scenario, we started with conserved left-right parity. One can also have left-right symmetric models starting with explicitly broken parity. In an other class of left-right symmetric models, the parity is spontaneously broken by introducing a singlet Higgs field odd under the left-right parity transformation. The detailed discussion of all these cases has been outlined in [97].

# Chapter 6

# Grand unified theories and neutrino masses

The idea of Grand Unified Theories (GUTs) has emerged as an attractive possibility to go beyond the SM . It promises to unify the three different gauge coupling constants of the SM. The basic idea is that the three coupling constants vary differently with respect to the energy scale and their renormalization group running shows that they tend to meet at some very high energy scale ($\sim 10^{15}$GeV) known as the GUT scale. Some new physics is expected to appear at this scale which can be described by a bigger gauge group with single coupling constant, i.e., the grand unified group.

GUTs provide a natural platform to address some of the most appealing issues which is not possible elsewhere. It reduces the number of required particle irreducible multiplets in the SM into lesser number of irreducible multiplets under the grand group. As a consequence the ad-hoc looking assignment of the quantum number to the SM fermions gets a predictive framework, for example the charge quantization remains no more a surprise in GUTs. Another very attractive feature is that the fermion mass matrices, looking independent of each other in the SM, get related in the GUT framework.

One of the common characteristic features of the GUTs is that it predicts lepton and baryon number violating interactions mediated by exchange of either a super heavy gauge boson or a super heavy Higgs boson with correct quantum number allowed by $d = 6$ dimensional operator of the SM. This arises from the fact that quarks and leptons share their quantum numbers from the same multiplet in GUTs. In particular, the gauge mediated interaction does not depend on the chosen Higgs content or the symmetry breaking pattern and so promises to provide a model independent test of GUTs. However, the GUT scale masses of the heavy gauge bosons make it almost impossible to observe any such clean event. So one needs to look for the signal in the process with relatively very small background to be practically observed. One such kinetically allowed process is the decay of proton to pions and leptons.

Although, no such decay events have been observed to date, its lifetime is being more and more tightly constrained. The current experimental lower bound of the partial life time for $p \to e^+\pi^0$ is $\tau_p > 8.2\times 10^{33}$ years and for $p \to \mu^+\pi^0$ is $\tau_p > 6.6\times 10^{33}$ years [98]. The theoretical decay rate of the proton can be estimated as[99]:

$$\Gamma_p \simeq \alpha_{GUT}^2 \frac{m_p^5}{M_{X,Y}^4}\,.$$

This can be used to estimate the lower limit of the Heavy gauge boson masses. If the mass scale of super heavy gauge bosons are given as $M_X \simeq 10^n$GeV, the above proton decay bound is equivalent to

$$\kappa = \left(\frac{\alpha_{GUT}}{45}\right) \times 10^{2(n-15)} \gtrsim 11.8\,. \tag{6.1}$$

This also provides the lower bound on GUT scale as $M_X \gtrsim 10^{15.5}$ as $\alpha_{GUT}$ does not vary substantially.

In this chapter, we discuss the basic structure of two GUT scenarios. The first one is $SU(5)$ GUT which has got the historical importance as it was the first GUT model proposed [100]. The rank of the $SU(5)$ group is same as the rank of SM gauge group and so it is the smallest GUT gauge group to accommodate SM gauge group. Its non-supersymmetric minimal version, which was initially proposed, has got very tight constraint on parameter space from the negative results of the proton decay experiments and moreover does not unify the three gauge coupling constant. However, several extensions have been studied in literature and we will discuss few of them which are interesting from the point of view of the fermion masses and mixing specially in leptonic sector.

Out of the higher rank gauge groups containing the SM gauge group as a subgroup, the rank five semi-simple group $SO(10)$ has emerged as a very attractive candidate for GUTs. The most interesting fact in favor of $SO(10)$ GUT is that it can accommodate the entire SM fermion content in a single 16-dimensional complex irreducible spinor representation including right-handed neutrino, with three copies for the three families. Its all irreducible representations are anomaly free providing a natural predictive framework to understand fermion masses and mixing. It is also most preferable GUT framework to naturally embed seesaw mechanism within itself. In addition, the left-right symmetric gauge group can also be embedded in $SO(10)$ GUT.

## 6.1 SU(5) GUTs

In $SU(5)$ GUT, all the SM fermions of each generation are accommodated in fundamental $\overline{5}$ and antisymmetric 10-dimensional irreducible representations as follows:

$$\psi_5 = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}_L , \quad \psi_{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & e^c \\ & & & & 0 \end{pmatrix}_L ,$$

where 1, 2, 3 represents the color indices. If $U$ represent the general $SU(5)$ gauge transformation, above multiplets will transform as

$$\psi_5 \to U^\star \psi_5 , \quad \psi_{10} \to U\, \psi_{10}\, U^T .$$

The gauge boson ($A_\mu$) of the $SU(5)$ will be represented by its 24 dimensional adjoint representation. The SM gauge bosons can be easily identified from its decomposition under SM gauge group as

$$A_\mu(24) \equiv G_\mu(8,1,0) + W_\mu(1,3,0) + B_\mu(1,1,0) + X_\mu(3,2,-5/6) + Y_\mu\left(\overline{3},2,5/6\right) , \tag{6.2}$$

where $X$ and $Y$ gauge bosons are the additional gauge bosons other than the SM gauge bosons. Spontaneous symmetry breaking of the $SU(5)$ group to the SM group automatically generates the GUT scale masses for these additional gauge bosons through Higgs mechanism. The nontrivial color and flavor characteristics of $X$ and $Y$ allow them to couple to the quark-quark and the quark-lepton states. These are the characteristic super heavy gauge bosons which can lead to both $B$ and $L$ violating processes as discussed earlier.

### 6.1.1 Spontaneous symmetry breaking

The spontaneous symmetry breaking of $SU(5)$ gauge group to the SM gauge group is achieved by introducing an adjoint Higgs. As it can be seen in the expression 6.2 of adjoint decomposition, it has a singlet in the direction of SM group. Assigning a *vev* toward this singlet direction will serve the purpose. Let $\Theta = \sum_{i=1}^{24} \Theta_i T_i$ represents the adjoint Higgs transforming as $\Theta \to U\Theta U^\dagger$, where $T_i$'s are the $SU(5)$ generators and $\Theta_i$'s here represent the components of the adjoint Higgs. Then the most general Higgs potential can be written as (with the discrete symmetry $\Theta \to -\Theta$) :

$$V(\Theta) = -\frac{\mu_\Theta^2}{2}\mathrm{Tr}\,\Theta^2 + \frac{1}{4}g_\Theta\left(\mathrm{Tr}\,\Theta^2\right)^2 + \frac{1}{2}\lambda_\Theta \mathrm{Tr}\,\Theta^4 . \tag{6.3}$$

The *vev* along the singlet direction of the SM corresponds to the *vev* value $\langle\Theta\rangle = \mathrm{V}\ \mathrm{Diag}\,(1, 1, 1, -3/2, -3/2)$.

The Higgs component having quantum number $(3, 2, 5/6)$or $h.c.$ which has the same quantum number as that of $SU(5)$ gauge bosons corresponding to the broken generators under the SM gauge group. These are goldstone modes eaten up by the corresponding gauge bosons after spontaneous symmetry breaking. In turn, these gauge bosons become heavy by getting masses of GUT scale. The remaining Higgs components too can be shown to acquire GUT scale masses provided the following conditions on parameter space are satisfied: $\lambda_\Theta > 0$, $15g_\Theta + 7\lambda_\Theta > 0$.

The further breaking of the SM gauge group can be achieved by a 5-dimensional Higgs multiplet $H$. From its decomposition $5 = (3,1,-3/2) + \phi(1,2,1)$, one can easily identify the SM Higgs $\phi$ . The color triplet component can mediate the scalar driven proton decay and so need to be made heavy. The combined $SU(5)$ invariant potential can be given as

$$\begin{aligned} V(\Theta, H) \;\; = \;\; & -\frac{\mu_\Theta^2}{2}\mathrm{Tr}\,\Theta^2 + \frac{1}{4}g_\Theta\left(\mathrm{Tr}\,\Theta^2\right)^2 + \frac{1}{2}\lambda_\Theta \mathrm{Tr}\,\Theta^4 \\ & -\frac{\mu_H^2}{2}H^\dagger H + \frac{\lambda_h}{4}\left(H^\dagger H\right)^2 \\ & +\alpha H^\dagger H\,\mathrm{Tr}\,\Theta^2 - \beta H^\dagger \Theta^2 H , \end{aligned}$$

where we have some additional conditions $\lambda_h > 0\,, \beta > 0$. All the terms other than the $\beta$ term are insensitive to the direction of $\langle H\rangle$. Any *vev* assignment to the color triplet must be avoided in order to retain the color gauge group and that is what the condition $\beta > 0$ ensures by demanding the potential minimization. With the condition, the $H$ field can acquire a *vev* along the direction of the charge neutral component.

But we soon encounter a form of hierarchy problem what is called as doublet-triplet splitting problem. Both the mixed terms contribute to the masses of doublet and color triplet and the two contributions comes out to be two linearly independent combination of $\alpha$ and $\beta$ couplings times $v$ with GUT scale. Anyway we desire heavy color triplet to avoid fast proton decay but at the same time we require a light doublet Higgs for electroweak breaking. Both the requirement can be simultaneously achieved only by a huge fine-tuning between the parameters of at least 26 order of magnitude.

### 6.1.2 Gauge coupling unification

It was only after the realization of the fact that strength of an interaction is not an absolute concept but varies with the energy scale of the interaction that led to the idea of unification of all the coupling constants. The running of the couplings is described by the following one loop renormalization group equation:

$$\alpha_i^{-1}(M_\mu) = \alpha_i^{-1}(M_0) - 2a_i M_{\mu 0}\,,$$

where $M_{\mu 0} = \ln\left(\frac{M_\mu}{M_0}\right)$ and $\alpha_i(M_\mu)$ are the values of coupling constants at desired scale $M_\mu$ and $\alpha(M_0)$ are known values at scale $M_0$. The $a_i$ is the beta function which contains the contributions from the gauge bosons, fermions and scalars in the model

$$a_i = \frac{1}{4\pi}\left[-\frac{11}{3}C_i(\text{Vectors}) + \frac{2}{3}(\text{Weyl Fermions}) + \frac{1}{3}C_i(\text{Complex scalars})\right].$$

Above the electroweak scale, the beta functions corresponding to different coupling constants are given as

$$\begin{aligned} a_{1Y} &= \frac{41}{40\pi}\,, \\ a_{2L} &= -\frac{19}{24\pi}\,, \\ a_{3c} &= -\frac{7}{4\pi}\,. \end{aligned}$$

We write down the individual evaluation equation of the couplings systematically as

$$\begin{aligned} \alpha_{1Y}^{-1}(M_\mu) &= \alpha_{1Y}^{-1}(M_W) - 2a_{1Y}\,M_{\mu W} \\ \alpha_{2L}^{-1}(M_\mu) &= \alpha_{2L}^{-1}(M_W) - 2a_{2L}\,M_{\mu W} \\ \alpha_{3c}^{-1}(M_\mu) &= \alpha_{3c}^{-1}(M_W) - 2a_{3c}\,M_{\mu W}\,. \end{aligned}$$

We choose initial starting values of the above three coupling constants ( central values) at scale $M_W\,(=100\text{GeV})$ to be the experimental values which are $\alpha_{1Y}^{-1}(M_\mu) = 59.38$, $\alpha_{2L}^{-1}(M_W) = 29.93$, and $\alpha_{3c}^{-1}(M_W) = 8.47$. The evolution of the couplings have been plotted in figure 6.1

Obviously the couplings do not meet at a point. $\alpha_2$ and $\alpha_3$ meat at energy scale around $10^{17}$ GeV which is preferable as a GUT scale to avoid fast proton decay. But

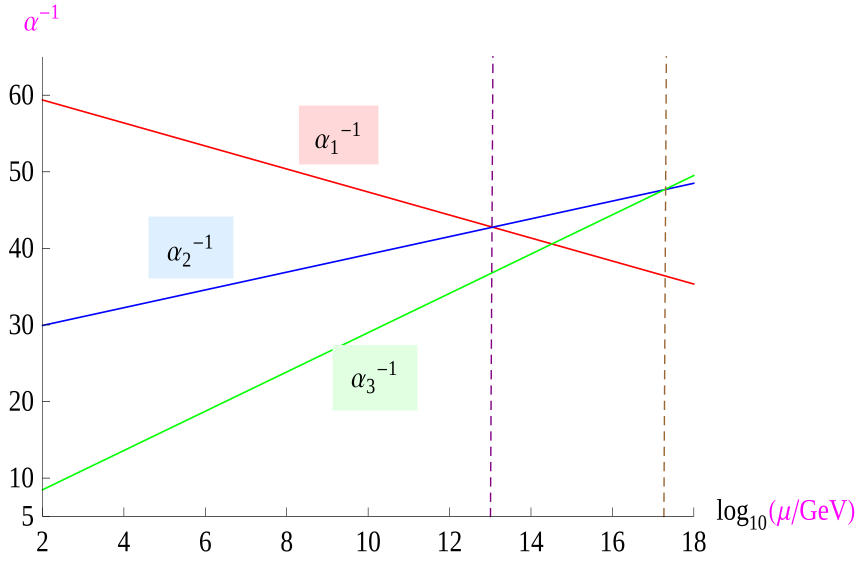


Figure 6.1: Evolution of coupling constants in $SU(5)$ GUT

$\alpha_1$and $\alpha_2$meet too early than what is desirable. So the $SU(5)$ GUT in its minimal version is not viable simply because it is unable to provide unification. One needs to go beyond the minimal version to look for some other sources contributing to beta function in order to achieve unification. Moreover, the minimal version is also not able to produce neutrino masses which we will discuss next.

### 6.1.3 Neutrino masses

The $SU(5)$ GUT with minimal set of Higgs bosons as above allows following renormalizable Yukawa couplings:

$$\begin{aligned}\mathcal{L} &= (Y_d)_{ij} \quad (\psi_5^{\alpha})_i C\left(\psi_{10\alpha\beta}\right)_j H^{\beta} \\ &+ \quad (Y_u)_{ij}\varepsilon^{\alpha\beta\gamma\delta r}\psi_{10\alpha\beta}C\psi_{10\gamma\delta}H_r\,.\end{aligned}$$

With the *vev* assignment to $H$ along the neutral direction $< H >= (0,0,0,0,v)$, the general structure of the fermion mass matrices emerges as

$$\begin{aligned}M_d &= M_l = vY_d \\ M_u &= vY_u\,.\end{aligned} \tag{6.4}$$

The equality of the mass matrix of down type quark and charged lepton are not mere coincidence. Its has to be there from the fact that ■ does not play any role to produce the fermion masses and is only breaks the GUT group to SM group. The *vev* pattern of $H$ is such that it can break $SU(5)$ to $SU(4)\times U(1)$ and not to the SM. So the equality of the two matrices is the consequence of $SU(4)$ symmetry which is still intact in Yukawa sector.

However, above mass matrix structure is not consistent with current data on masses of both quarks and charged leptons even if we forget about neutrinos. Adding singlet right-handed neutrinos can provide masses to the neutrinos, but it will not improve the situation for the charged fermions simply because the relations in 6.4 are left unchanged. So one needs to go to non-trivial extensions of minimal $SU(5)$ not only for consistent understanding of fermion masses but also to look for sources which can ensure the unification of the three coupling constants. Following decomposition property can help to chose a Higgs multiplet which can couple with the fermions to construct invariant Yukawa terms:

$$\begin{aligned} \overline{5}\times\overline{5} &= \overline{10}+\overline{15} \\ \overline{5}\times 10 &= 5+45 \\ 10\times 10 &= 5+\overline{45}+\overline{50} \end{aligned} \tag{6.5}$$

Only 5, 15 and 45 have a component that is electrically neutral and color singlet and so only they can be given *vev*s. In what follows, we discuss three different scenarios of realization of neutrino masses in $SU(5)$ GUT.

### Type I seesaw realization in $SU(5)$ GUT

In this scenario, the Higgs sector of Georgi Glashow model [100] is extended by adding a 45-dimensional Higgs $\Sigma$ [101]. From expression 6.5, it is clear that assigning *vev* will affect both the mass matrix relations of 6.4. The Yukawa sector is given as

$$\begin{aligned} \mathcal{L}_Y &= (Y_{\nu D})_{ij}\,(\psi_5^{\alpha})_i C\,(\psi_1)_j H_\alpha + M_{ij}\,(\psi_1)_i C\,(\psi_1)_j \\ &+ (Y_d)_{ij}\,(\psi_5^{\alpha})_i C\left(\psi_{10\alpha\beta}\right)_j H^\beta + (Y_u)_{ij}\epsilon^{\alpha\beta\gamma\delta r}\psi_{10\alpha\beta} C\psi_{10\gamma\delta} H_r \\ &+ (Y_d')_{ij}\,(\psi_5^{\alpha})_i C\left(\psi_{10\beta\delta}\right)_j \Sigma_\alpha^{\beta\delta} + (Y_u')_{ij}\epsilon^{\alpha\beta\gamma\delta r}\psi_{10\alpha\beta} C\psi_{10\gamma\sigma}\Sigma_{\delta r}^{\sigma}\,. \end{aligned} \tag{6.6}$$

The first two terms provide Dirac and Majorana masses for neutrinos and thus lead to usual type I seesaw scenario if scale of Majorana mass matrix is high enough. The next two terms are similar to the minimal case. The last two terms, which include 45 dimensional multiplet $\Sigma$, correct the bad relation 6.4 for the charged fermions present in the minimal version. The $\Sigma$ can acquire *vev* in the charge neutral and color singlet direction in the following way

$$< \Sigma_{\gamma}^{\alpha\beta} >= \delta^{\beta 5} v' \left( \delta_{\gamma}^{\alpha} - 4\delta_{\gamma 4}\delta^{\alpha 4} \right) .$$

Corresponding new mass matrix relations come out be

$$\begin{aligned} M_d &= Y_d v + Y_d' v' \\ M_l &= Y_d v - 3Y_d' v' \\ M_u &= Y_u v + Y_u' v' . \end{aligned}$$

Obviously, we get enough parameter space to fit all the charged fermion data on masses and mixing.

However, various split multiplets of 24 and 45 Higgs under the SM gauge group have to be given appropriate intermediate mass scales (between GUT scale and electroweak scale) in order to achieve unification at desired scale[102, 103]. In doing so, one leads to leptoquarks below the GUT scale which can mediate fast proton decay. The recent studies [104] show that the model can be almost ruled out by imposing the current experimental bound on Higgs mediated proton decay. Moreover, the absence of any relation between neutrino masses and charged fermion masses makes the model uninteresting.

**Type II seesaw realization in $SU(5)$GUT**

Unlike the previous case, here we do not extend the fermionic sector but try to generate the Majorana masses for neutrinos solely by extending the Higgs sector. From the decomposition in expression 6.5, it is clear that 15 dimensional multiplet can serve the purpose. Looking at its decomposition $15 = (1,3,1) \oplus (3,2,1/6) \oplus (6,1,-2/3)$, one is provided with familiar $SU(2)_L$ triplet $(1,3,1)$ to realize type II seesaw by assigning it an appropriate *vev*. However, the unification constraints does not depend much on the mass scale of the the triplet $(1,3,1)$ and so its scale is not constrained and can lie anywhere in principle.

Unlike the the case in Type I seesaw, 45 dimensional Higgs is not required as the 15 dimensional multiplet can provide successful unification alone. The model is again dependent on the split mass scales of the different component of 15 dimensional Higgs for the unification and predicts a light leptoquark $(3,2,1/6)$ around TeV scale making the model relevant at LHC to be ruled out [105].

Although the model provides the Majorana masses for the neutrinos, it is highly dependent on the non-renormalizable operator in the Yukawa sector to generate the consistent masses for other charged fermions. To retain the renormalizability along with the desired mass structure for the fermions, the model is extended by adding 45 dimensional Higgs [104]. Obviously, the structure of mass matrices for charged fermions remains same as in the case of type I seesaw simply because the Yukawa terms responsible for generating these masses are identical. However, the Yukawa terms for generating the neutrino masses change as

$$\mathcal{L}_{Y\nu} = Y_{ij}\,(\psi_5^{\alpha})_i C \left(\psi_5^{\beta}\right)_j 15_{\alpha\beta}\,,$$

which provides the Majorana mass for the neutrinos after 15 Higgs acquires the *vev* as $m_\nu =< 15 > Y$ through type II seesaw mechanism.

### Type I+III seesaw realization in $SU(5)$GUT

This very interesting realization was first proposed in [106] by demanding the extension of the fermionic sector of minimal $SU(5)$[100] by a 24 dimensional fermionic adjoint. Looking at the decomposition of the adjoint representation 6.2, it is straight forward to identify (i) the SM fermionic singlet which can serve in type I seesaw realization of neutrino masses and (ii) the SM fermionic triplet needed for type III seesaw realization. Since we are adding just one fermionic adjoint, we have only one fermionic singlet and one triplet. Once the $SU(5)$ GUT is broken, one ends up with the following Yukawa terms relevant for neutrino masses:

$$\mathcal{L}_{Y\nu} = \ell_i C\left(Y_S^i 1_F + Y_T^i 3_F\right) H(1,2,1) + \frac{M_S}{2} 1_F C\, 1_F + \frac{M_T}{2} 3_F C\, 3_F\,,$$

where $M_S$ and $M_T$ represent the Majorana masses for the singlet and the triplet fermion.

After the electroweak symmetry breaking, this leads to following mass structure of neutrinos

$$(m_\nu)_{ij} = v^2 \left(\frac{Y_S^i Y_S^j}{M_S} + \frac{Y_T^i Y_T^j}{M_T}\right)\,.$$

The above mass structure immediately leads to one massless neutrino. Although the unification does not need the singlet fermion, it is very sensitive to the triplet one. The detailed unification study predicts the mass scale of the triplet to be less than one TeV which makes the model phenomenologically relevant for LHC.

However, the consistent understanding of the charged fermion masses requires higher dimensional operator in the Yukawa sector. Its renormalized version has been studied in [107] by adding a again 45 dimensional Higgs multiplet. Obviously, the mass matrix structures of charged fermions again come out to be same as in type I and II case.

## 6.2 $SO(10)$ GUTs

One of the compelling feature of $SO(10)$ GUT is that its 16-dimensional complex representation alone is able to accommodate the quantum degrees of freedom of all the SM fermions.

$$
\begin{aligned}
\psi_L(16) &= Q_L\,(3,2,1/6) \oplus u_L^c\,(\overline{3},1,-2/3) \oplus d_L^c\,(\overline{3},1,1/3) \\
&\quad \oplus \ell_L\,(1,2,-1/2) \oplus e_L^c\,(1,1,1) \\
&\quad \oplus N_L^c\,(1,1,0) \qquad\qquad .
\end{aligned}
$$

The first row represents the quarks, second row correspond to the leptons. A SM singlet appearing in the third row is an additional fermion which can be interpreted as the right-handed neutrino. So $SO(10)$ GUT predicts the right-handed neutrino from the model itself rather than putting it by hand like in some models of $SU(5)$ GUT.

The similar prediction in left-right symmetric model [92–96] is not just a matter of coincidence but is a natural consequence of the fact that the left-right gauge group is a maximal subgroup of the $SO(10)$.

$$
G_{3221D} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} \times D \subset SO(10)\,,
$$

where $D$ is a discrete symmetry usually called as left-right parity or D-parity [108] under which we have a symmetry transformation over the fermions as $\psi_{Li} \to \psi^c_{Li}$. While discussing the left-right symmetric model in the previous chapter, the left-right parity, i.e. the D-parity, has been assumed to be intact and is broken only at the time of spontaneous breaking of the left-right gauge group. However, in the $SO(10)$ GUT, it is possible to break the D-parity even before the left-right gauge group is broken.

An another group, which can appear between the GUT scale and the left-right scale or the SM scale, is the well celebrated Pati-Salam gauge group [94] where the two groups $SU(3)_c$ and $U(1)_{(B-L)}$ are unified to a bigger group $SU(4)$. The new gauge group treats lepton number as a quark with fourth color. This was the first bold attempt to unify the quarks and the leptons by putting them in the same multiplet.

$$
G_{422D} \equiv SU(4) \times SU(2)_L \times SU(2)_R \times D \subset SO(10)\ .
$$

The decomposition of the 16-plet fermion representation under this gauge group reads as

$$
\begin{aligned}
16 &\equiv (4,2,1) \oplus (\overline{4},1,2) \\
\overline{16} &\equiv (\overline{4},2,1) \oplus (4,1,2)\,.
\end{aligned}
$$

All the SM left-handed fermions including the SM singlet can be accommodated in the spinor representation of $SO(10)$ decomposed under Pati-Salam group as

$$
\psi_{16L} \equiv \begin{pmatrix} u_1 & u_2 & u_3 & \nu_e \\ d_1 & d_2 & d_3 & e^- \end{pmatrix}_L \oplus \begin{pmatrix} u_1^c & u_2^c & u_3^c & \nu_e^c \\ d_1^c & d_2^c & d_3^c & e^+ \end{pmatrix}_L .
$$

The columns represent the color degrees of freedom of $SU(4)$ and the rows correspond to quantum degrees of freedom of left-right gauge group. Similarly, the $\overline{16}$ plet can incorporate all the SM right-handed fermions including SM singlet.

So far we have concentrated mainly around the fermion assignment in the model and have not said much about the gauge sector. The gauge bosons belong to the adjoint 45-dimensional 2nd rank antisymmetric representation of $SO(10)$ with the decomposition under $G_{3221}$ given as

$$
\begin{aligned}
A_\mu(45) \equiv\; & G_{\mu 3c}(8,1,1;0) \oplus W_{\mu L}(1,3,1;0) \oplus W_{\mu R}(1,1,3;0) \oplus B_{\mu(B-L)}(1,1,1,0) \\
& \oplus (3,1,1;\frac{4}{3}) \oplus (\overline{3},1,1;-\frac{4}{3}) \\
& \oplus (3,2,2;\frac{2}{3}) \oplus (\overline{3},2,2;-\frac{2}{3}) \qquad . \qquad (6.7)
\end{aligned}
$$

The gauge bosons in the first row belong to the left-right model and should not acquire masses at the scale of $SO(10)$ breaking if left-right scale is an intermediate scale of the model . The gauge bosons belonging to the last two rows can mediate the proton decay and should become heavy at the scale of grand unification to avoid the fast proton decay like the case in $SU(5)$ GUT.

### 6.2.1 Symmetry breaking pattern

The most encouraging argument in favor of $SO(10)$ over $SU(5)$ GUT is that it allows several intermediate breaking steps before one finally gets down to SM. This helps to achieve relatively natural unification unlike the case in $SU(5)$ where the unification requires ad-hoc mass scale assignment to the split components of a given Higgs representation without any justification. The presence of intermediate breaking scales provides new sources to the beta functions and their scales can be easily

| Intermediate gauge group | Symbolic Representation |
|---|---|
| $SU(4) \times SU(2)_L \times SU(2)_R \times D$ | $G_{422D}$ |
| $SU(4) \times SU(2)_L \times SU(2)_R$ | $G_{422}$ |
| $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} \times D$ | $G_{3221D}$ |
| $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)}$ | $G_{3221}$ |
| $SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{(B-L)}$ | $G_{3211}$ |
| $SU(3)_c \times SU(2)_L \times U(1)_Y$ | $G_{321}$ |

Table 6.1: Different intermediate Gauge Groups

determined by using unification constraints.

A desired symmetry breaking chains in $SO(10)$ GUT with one or more intermediate steps are realized by choosing suitable Higgs combinations. However, all possible chains may not turn out to be consistent with the existing experimental data available.

Realization of $SO(10)$ breaking to left-right group can take place in two ways. One way is to use 210 dimensional Higgs which decomposes under Pati-Salam gauge group as

$$\begin{aligned} 210 \;&\equiv\; (1,1,1) \oplus (15,1,1) \oplus (6,2,2) \oplus (15,3,1) \\ &\oplus (15,1,3) \oplus (10,2,2) \oplus (\overline{10},2,2)\,. \end{aligned}$$

Giving *vev* towards the singlet direction will lead to Pati-Salam gauge group. However, the D-parity is not respected by the singlet and is broken at the GUT scale. For further breaking, we have component $(15,1,1)$ of 210 which has a singlet under the left-right gauge group. So it can be given *vev* to further break the Pati-Salam group to the left-right group but without D-parity. However, D-parity is intact if $SO(10)$ is directly broken to left-right gauge group by giving appropriate *vev* to the $(15,1,1)_{210}$ component.

Another way to achieve $SO(10)$ breaking to the left-right group is possible by choosing a combination of $(54+45)$-dimensional Higgs with the decomposition of 54 Higgs under Pati-Salam group as

$$54 \equiv (1,1,1) \oplus (1,3,3) \oplus (20,1,1) \oplus (6,2,2)\,.$$

Unlike the case of 210, breaking of $SO(10)$ to Pati -Salam group using the 54-dimensional Higgs does not break D-parity. However, 54 alone does not serve the purpose of further breaking to the left-right group. So an additional 45 dimensional Higgs Field, which has a singlet direction under the left-right group (6.7), is needed along with 54 Higgs. This singlet direction also respects D-parity and can be assigned a *vev* to break the Pati-Salam group down to the left-right group with D-parity intact .

The next stage of the breaking of the left-right group down to the SM group requires additional Higgs multiplets. The new multiplets can be some suitable combinations of Higgs with dimensions 10, 16, 120,126 of $SO(10)$. Some of the multiplets can help in both breaking the left-right group and generating masses for the fermions. Now the breaking of SM can take place by giving an electroweak scale *vev* to the left-right bi-doublet present in some $SO(10)$ Higgs multiplets. This also produces the usual Dirac masses for fermions.

Other than unification, the intermediate mass scales lead to several interesting phenomenological consequences. For example the breaking of $(B-L)$ gauge symmetry provides lepton-number violating sources which can produce, on one hand, Majorana masses to neutrinos [19] and lepton asymmetry in early universe, on the other hand. Moreover, the breaking scale will also enable us to determine the scale of light Majorana neutrinos and possible amount of lepton asymmetry that can be created. The fermion mass relation is also affected by the breaking pattern.

Among another consequences, one is prediction of oscillation between neutron and antineutron [19]. A low scale breaking of $(B-L)$ can lead to a possibility of practical experimental detection of such oscillation. To end the discussion, we would like to emphasize that with the knowledge of intermediate scales and the Higgs content of the model, one is able to decide the unification scale and can predict the proton decay width in various channels. The consistency of the prediction with the current bounds will be finally the true test of a given $SO(10)$model.

### 6.2.2 Yukawa sector and neutrino masses

Although the set of Higgs fields required for breaking the $SO(10)$ group can be quite complicated, the Higgs fields that can appear in the Yukawa sector to address the question of fermion masses is rather simple so far as renormalizability of the model is demanded. The Higgs bosons that can couple to the matter are

$$16 \times 16 \equiv 10 + 120 + \overline{126}\,. \tag{6.8}$$

The Higgs fields needed to generate fermion masses will have to belong to one or more of 10, 120 and $\overline{126}$-dimensional Higgs of $SO(10)$.

$$\begin{aligned} \mathcal{L}_Y &= (Y_{10})_{ij}\, \psi_{16i} BC\Gamma^{\alpha}\psi_{16j} 10_{\alpha} + (Y_{120})_{ij}\, \psi_{16i} BC\Gamma^{\alpha}\Gamma^{\beta}\Gamma^{\gamma}\psi_{16j} 120_{\alpha\beta\gamma} \\ &+ (Y_{126})_{ij}\, \psi_{16i} BC\Gamma^{\alpha}\Gamma^{\beta}\Gamma^{\gamma}\Gamma^{\delta}\Gamma^{\omega}\psi_{16j} \overline{126}_{\alpha\beta\gamma\delta\omega}\,, \end{aligned} \tag{6.9}$$

where $\Gamma^\alpha$ are the analogs of the Dirac gamma matrices for $SO(10)$ and $B$ is the analog of the conjugate matrix for the spinors of $SO(10)$. Due to the properties of the $\Gamma$ matrices, the Yukawa couplings corresponding to 10 and $\overline{126}$ Higgs are symmetric under interchange of generation indices $i$ and $j$ while one belonging to 120 is antisymmetric.

Any one of the three Higgs field can produce fermion masses by acquiring appropriate *vev*. However the contribution may not be always sufficient enough to account for the current data on fermion masses and mixing. For example, let us consider the possibility to generate the fermion masses by taking the real 10 dimensional Higgs field alone. The 10 dimensional field has a left-right bi-doublet component which can provide Dirac masses to all the fermions as well as help in breaking the SM gauge group.

$$10 \stackrel{G_{3221}}{\longrightarrow} \Phi(1,2,2,0) \oplus (3,1,1,-\frac{1}{3}) \oplus (\overline{3},1,1,\frac{1}{3})\,.$$

It is straightforward to argue that the Yukawa couplings of this 10 pelt Higgs would be same for all the fermions (up/down quarks and neutral/charged leptons) leading to a common mass matrix:

$$M_\ell = M_{\nu D} = M_u = M_d =< 10 > Y_{10}\,. \tag{6.10}$$

Extrapolation of this mass matrix relations to weak-scale is not able to account for the fermion data as it predicts the mass matrix structure for up type quarks and down type quarks to be same although different from the common mass matrix corresponding to neutral and charged leptons by a factor of three:

$$M_u(Z) \simeq M_d(Z) \simeq 3M_{\nu D}(Z) \simeq 3M_\ell.$$

The unrealistic degeneracy between the up and the down type quarks can be avoided by complexifying the real 10 plet Higgs. However, one still can not avoid relation between the mass matrices of up type quark and neutrinos $M_u(Z) = 3M_{\nu D}(Z)$ which can be simply ruled out using the current upper bound on the neutrino masses. So, to accommodate the existing data on fermion masses and mixing, one needs to add some new Higgs fields relevant in the Yukawa sector.

### Yukawa sector with 10+$\overline{126}$ Higgs

This combination of the two Higgs fields was first proposed in [109] as a predictive scenario for generating the fermion masses. The decomposition of additional $\overline{126}$ Higgs under Pati-Salam gauge group come out to be

$$\overline{126} = (15,2,2) \oplus (10,1,3) \oplus \left(\overline{10},3,1\right) \oplus (6,1,1) \, .$$

Further decomposition of the above components at the left-right symmetric scale reveals that $(15,2,2)_{126}$ contains usual left-right bi-doublet. The another split Higgs field $(10,1,3)_{126}$ contains a left-right triplet with quantum numbers $(1,1,3,-1)$ which can be traced back to be the complex conjugate of the triplet $\xi_R(1,1,3,1)$ (see expression 5.7) that we have taken while discussing the type I+II seesaw realization in the left-right symmetric model (subsection 5.4.1) in the previous chapter. Since the combination $126+\overline{126}$ has to be present to provide *vev* to their neutral components, analogs of both the fields $\xi_R^*(1,1,3,-1)$ and $\xi_R(1,1,3,1)$ are present in the model.

This component has a SM singlet and can break the left-right group to SM group. One may wonder about the left counter field $\xi_L(1,3,1,1)$ of $\xi_R(1,1,3,1)$ in the model which can be easily identified in the decomposition of $(\overline{10},3,1)_{126}$ under the left-right gauge group. So the presence of both the complex fields $\xi_L$ and $\xi_R$ with the same quantum numbers as in expression 5.7 makes the discussion same as in the left-right model.

So the discussion about breaking mechanism of the left-right model to the SM in the subsection 5.4.1 can be safely borrowed to conclude that this $SO(10)$ model is able to provide light Majorana masses to left-handed neutrinos, heavy Majorana masses to right-handed neutrinos along with usual Dirac masses due to left-right bi-doublet present in both 10 and $\overline{126}$ Higgs leading to type I+II seesaw realization.

However, the $\overline{126}$ Higgs field is contributing to fermion masses in two ways. On one hand, its $(15,2,2)_{126}$ component is contributing to Dirac masses to fermions along with $(1,2,2)_{10}$ Higgs component and on the other hand, its $(10,1,3)_{126}$ and $(10,3,1)_{126}$ component generate Majorana masses for neutrinos. As a consequence, we expect an obvious relation between the Majorana mass matrix of neutrinos and the Dirac mass matrix contribution due to 126 Higgs field which is in addition to what we have in a general left-right model due to $SO(10)$ GUT.

$$\begin{aligned}
M_d &= k_u Y_{10} + k_u' Y_{126} \, , \\
M_u &= k_d Y_{10} + k_d' Y_{126} \, , \\
M_\ell &= k_u Y_{10} - 3k_u' Y_{126} \, , \\
M_{\nu D} &= k_d Y_{10} - 3k_d' Y_{126} \, , \\
M_L &= u_L Y_{126} \, , \\
M_R &= u_R Y_{126} \, , \qquad (6.11)
\end{aligned}$$

where $k_{u/d} =< 1,2,2 >^{u/d}_{10}$ and $k' =< 15,2,2 >^{u/d}_{\overline{126}}$ . $M_{L/R}$ are the Majorana mass matrix for left/right-handed neutrinos and $u_{L/R}$ are the *vevs* of the $\xi_{L/R}$ fields present in 126 dimensional Higgs.

The Yukawa coupling with the complex conjugate of 10 dimensional complex Higgs field has been prevented by imposing the Peccei-Quinn symmetry to provide a predictive framework. The low energy neutrino mass matrix in the model can be simply written as

$$m_\nu = M_L - M_{\nu D} M_R^{-1} M_{\nu D}^T .$$

The realistic model will require a suitable admixture of type I and type II seesaw term to explain the mixing and masses of both charged and neutral fermions.

### Yukawa sector with 120+$\overline{126}$ Higgs

In this model, complex 10 dimensional Higgs field is replaced by a complex 120 dimensional Higgs [110, 111]. Unlike the case of complex 10 Higgs, the 120 Higgs has two left-right bi-doublet. However, due to presence of $\overline{126}$, there will not be any major change in the above discussion as far as neutrino masses are concerned. The model will again lead to the type I+II seesaw realization of neutrino masses. The only difference would be in the expressions of Dirac mass matrices of the fermions simply because the contribution due to 120 Higgs to Dirac mass matrices would be antisymmetric under interchange of generation. This means that there are only three complex Yukawa couplings on top of Yukawa couplings corresponding to $\overline{126}$ Higgs in the model. A two generation study of the model reveals that atmospheric mixing angle should be as far as possible from the maximal value[111]. The degeneracy or hierarchy of the neutrino spectrum is controlled by the relation:

$$\frac{m_3^2 - m_2^2}{m_3^2 + m_2^2} = \frac{\cos 2\theta_{23}}{1 - \sin^2 2\theta_{23}/2} .$$

### Yukawa sector with 10+$\left(16+\overline{16}\right)$ Higgs

In this class of models, the fermionic sector of $SO(10)$ GUT is extended by adding two or more $SO(10)$ singlet fermions. The natural extension would be addition of one singlet fermion per generation. The complex 10 Higgs will produce Dirac masses as usual. However, the combination $\left(16+\overline{16}\right)$ instead of 126 would change the whole discussion about neutrino masses.

Looking at the decomposition of the new Higgs fields under left-right gauge group will make it obvious that it has all the ingredients to realize type III seesaw mechanism for generating light neutrino masses.

$$\begin{aligned} 16 &= \chi_L^*(1,2,1,-\frac{1}{2}) \oplus \chi_R(1,1,2,\frac{1}{2}) \\ &\oplus (3,2,1,\frac{1}{6}) \oplus (\overline{3},1,2,-\frac{1}{6}) \\ \overline{16} &= \chi_L(1,2,1,\frac{1}{2}) \oplus \chi_R^*(1,1,2,-\frac{1}{2}) \\ &\oplus (3,1,2,\frac{1}{6}) \oplus (\overline{3},2,1,-\frac{1}{6}) \quad , \end{aligned}$$

where $\chi_{L/R}$ correspond to the same left-right doublet fields, required to break the left-right gauge group to the SM group, as while addressing the type III seesaw scenario in subsection 5.4.2 in the previous chapter. We would like to postpone a detailed study of this scenario to the chapter 11 where we would slightly extend the model to address the dark energy of the universe within the framework of $SO(10)$ GUTs.

# Part II

# Neutrinos and astroparticle

# Chapter 7

# Standard cosmology

The subject of cosmology deals with the study of the various phases of the evolution of our universe. Our present understanding of the universe is based on two basic postulates. First, the universe is homogeneous and isotropic at large scale and the second, the evolution of the universe is governed by Einstein's general relativity. A model independent study based on these two postulates reveals that the universe becomes more and more dense and hot as we go back in the past compared to that of present universe.

Proper understanding of the evolution of the universe needs input from the particle content and their interactions at various phases of the universe. According to our current knowledge of particle physics, we are able to extrapolate back up to the temperature corresponding to the electroweak scale. However, further extrapolation requires input from new physics and is highly model dependent. Since many of the models beyond the SMs are out of reach of the colliders in the near future, cosmology can serve as an indirect probe to test these models as the universe is expected to carry certain characteristic imprints of the new physics while it was undergoing the hot and dense phase corresponding to the scale of the new physics.

The standard big-bang model of cosmology appears to predict that the universe started from a kind of singularity with infinite dense and hot region. However, the conclusion can not be drawn without knowing the correct quantum theory of gravity. The quantum effect of gravity can not be ignored above the temperature corresponding to the Planck scale and due to this very reason, theory of evolution of the universe before the time corresponding to Planck scale is not possible to formulate. Hence the phenomenology of the universe can be only studied below the Planck scale.

## 7.1 FRW cosmology

The universe based on the symmetry of homogeneity and isotropy is well described by the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right],$$

where $a(t)$ is scale factor with cosmic time $t$. The coordinates $r$, $\theta$ and $\phi$ are known as comoving coordinates. $K$ represents the spatial curvature of the universe.

The dynamics of the universe is decided by the time dependence of the scale factor which can be determined once we input this metric into the Einstein's equation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}\,, \tag{7.1}$$

along with specifying the matter content of the universe characterized by the energy momentum tensor $T_{\mu\nu}$ and cosmological constant $\Lambda$. For a homogeneous and isotropic fluid the energy momentum tensor can be written as

$$T^{\mu}_{\nu} = \mathrm{Diag}\left(-\rho, p, p, p\right)\,, \tag{7.2}$$

where $\rho$ and $p$ are energy density and pressure density of the fluid respectively. The conservation of energy momentum tensor implies the following continuity relation

$$d\left(\rho a^3\right) = -p\, d\left(a^3\right)\,. \tag{7.3}$$

If we define equation of state as $p = \omega\rho$, where $\omega$ is time independent, the energy density can be shown to vary according to the following equation (using relation 7.3)

$$\rho = a^{-3(1+\omega)}\,. \tag{7.4}$$

To determine the evolution of the scale factor with time, we need to input the FRW metric and energy momentum tensor in the Einstein's equation which leads to two expressions as follows

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{K_r}{a^2}\,, \tag{7.5}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}\,, \tag{7.6}$$

where $H$ is the Hubble parameter or the expansion rate of the universe and $K_r$ rep-

| Description | Radiation domination | Matter Domination | DE Domination |
|---|---|---|---|
| $\omega$ | $\frac{1}{3}$ | $0$ | $-1$ |
| Eq. of state | $p = \frac{1}{3}\rho$ | $p = 0$ | $p = -\rho$ |
| $\rho \propto a^{-3(1+\omega)}$ | $\rho \propto a^{-4}$ | $\rho \propto a^{-3}$ | $\rho \propto const$ |
| $H = \frac{2}{3(1+w)t}$ | $H = \frac{1}{2t}$ | $H = \frac{2}{3t}$ | $H = const$ |
| $\rho \propto t^{\frac{2}{3(1+\omega)}}$ | $a \propto t^{1/2}$ | $a \propto t^{2/3}$ | $a \propto e^{Ht}$ |

Table 7.1: Expansion dynamics of the universe

resents the spatial curvature of the universe. In subsection 6.1.3 of chapter 6, we used the notation $H$ for 5-dimensional $SU(5)$ Higgs, but we will be using it for representing the expansion rate of the universe ($H = \left(\frac{\dot{a}}{a}\right)^2$) for rest of the discussion in this thesis.

From the expression 7.5, it is obvious to notice that the cosmological constant $\Lambda$ can be treated as a energy component of the universe which remains uniform and constant its energy density without any effect of the expansion of the universe. The energy component is called as the dark energy (DE) in a more general sense. Using the relation 7.4, we can easily find $\omega = -1$ for such an energy component of the universe which implies a fluid with negative pressure with equation of state

$$p = -\rho_{DE}\,,$$

where $\rho_{DE}$ is the energy density of the DE given as $\rho_{DE(\Lambda)} = \Lambda/8\pi G$.

Now we rewrite equation 7.5 as

$$\Omega(t) - 1 = \Omega_M(t) + \Omega_R(t) + \Omega_{DE}(t) - 1 = \frac{K_r}{(aH)^2}\,, \tag{7.7}$$

where $\Omega(t) = \rho(t)/\rho_c(t)$ with the critical energy density $\rho_c(t) = 3H(t)^2/8\pi G$. Moreover, $\Omega_M(t)\,(=\rho_M/\rho_c)$, $\Omega_R(t)\,(=\rho_R/\rho_c)$ and $\Omega_{DE}(t)\,(=\rho_{DE}/\rho_c)$ correspond to the ratio of energy density of matter, radiation and DE to the critical energy density of the universe, respectively.

As the $H^2a^2$ remains positive, the value of $\Omega$ determines the spatial geometry of the universe

$$\Omega > 1 \quad \text{or } \rho > \rho_c \quad \Rightarrow K_r = +1 \quad \text{Closed}, \tag{7.8}$$

$$\Omega = 1 \quad \text{or } \rho = \rho_c \quad \Rightarrow K_r = \;\; 0 \quad \text{Flat}, \tag{7.9}$$

$$\Omega < 1 \quad \text{or } \rho < \rho_c \quad \Rightarrow K_r = -1 \quad \text{Open Universe}. \tag{7.10}$$

However, the current observation shows that $\Omega^0(t = t_0)$, where $t_0$ represents the age of the universe, to be very close to one [29]

$$\Omega^0 = 1.011\,(12)\;,$$

which implies that our universe can be treated as flat. So we will consider our universe to be flat for the rest of the discussion.

Let $H_0$ represents the current value of the Hubble's parameter written as $H_0 = 100\,h\,\mathrm{Km\,s^{-1}\,Mpc^{-1}}$, where $h$ is a dimensionless quantity. Then the scale factor dependence of the Hubble parameter can be given as

$$H^2 = H_0^2 \sum_i \Omega_i^{(0)} a^{-3(1+w_i)}\,, \tag{7.11}$$

where $i = R, M, DE$. The expansion of the universe is parametrized in terms of the cosmological redshift ($z$) in the light coming from remote astrophysical objects. It is related to the scale factor as

$$1+z = \frac{a_0}{a(t)}\,,$$

where $a_0$ is the current value of the scale factor which is conventionally taken to be one without loss of generality. Now the expression 7.11 for Hubble's constant can be rewritten in terms of $z$ as

$$H^2 = H_0^2 \sum_i \Omega_i^{(0)} (1+z)^{3(1+w_i)}\,.$$

The observed luminosity distance $(d_L)$ of any astrophysical object at redshift $z$ can be related to the present value of the $\Omega_i^0$ as

$$\begin{aligned} d_L &= (1+z)\int_0^z \frac{dz'}{H(z')} \\ &= \frac{(1+z)}{H_0}\int_0^z \frac{dz'}{\sqrt{\sum_i \Omega_i^{(0)}(1+z')^{3(1+w_i)}}}\,. \end{aligned} \tag{7.12}$$

Similarly the age of the universe $(t_0)$ can also be related to the $\Omega$'s as

$$\begin{aligned} t_0 = \int_0^{t_0} dt &= \int_0^\infty \frac{dz'}{(1+z')H(z')} \\ &= \frac{dz'}{H_0\,(1+z')\sqrt{\sum_i \Omega_i^{(0)}(1+z')^{3(1+w_i)}}}\,. \end{aligned} \tag{7.13}$$

These relations will be useful when we discuss the DE part later in detail.

## 7.2 Thermodynamics of the early universe

From the discussion of the previous section, we easily realize that the scale factor $a$ decreases as we go back towards the past of our universe. As a consequence, the energy densities of matter and radiation are expected to become more and more dense with relatively high temperature. So although the current universe is out of thermal equilibrium, the same can not be expected in the early universe. In spite of the expansion of the universe, all the particles in the early universe remain in the thermal equilibrium due to the their rapid interactions with each other. Thus, our universe expands like a gaseous fluid in thermal equilibrium with a varying temperature $T$. The energy density $(\rho)$, number density $(n)$ and pressure $p$ of a weakly interacting gas with temperature $T$ and internal degree of freedom $g$ are given as

$$n = g \int \frac{d^3\mathbf{p}}{(2\pi)^3} f(\mathbf{p}) , \tag{7.14}$$

$$\rho = g \int \frac{d^3\mathbf{p}}{(2\pi)^3} E(\mathbf{p}) f(\mathbf{p}) , \tag{7.15}$$

$$p = g \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{|\mathbf{p}|^2}{3E} f(\mathbf{p}) , \tag{7.16}$$

where $E^2 = |\mathbf{p}|^2 + m^2$ and $f(p)$ is the phase space distribution function given as

$$f(\mathbf{p}) = \frac{1}{e^{(E-\mu)/T} \pm 1} \begin{cases} -1 & \text{Bose} - \text{Einstein} \\ +1 & \text{Fermi} - \text{Dirac} \end{cases} \tag{7.17}$$

and $\mu$ is the chemical potential.

To discuss about entropy, we write the second law of thermodynamics

$$T\,dS = dU + p\,dV ,$$

where $U$ can be written as $U = \rho V = \rho a^3$. Now using the relation 7.3, we can show that

$$T\frac{dS}{dt} = \frac{d}{dt}\left(\rho a^3\right) + p\frac{d}{dt}\left(a^3\right) = 0 ,$$

i.e., the entropy remains conserved during the expansion of the universe, $dS = 0$. So the evolution of the universe is an adiabatic process and the result is also valid even in those epochs where the equation of state changes, like in the epoch of matter-radiation transition.

Moreover, the rapid interactions between the particles in the early universe en-

| | Relativistic boson | Relativistic fermion | Non-relativistic particles |
|---|---|---|---|
| $n$ | $\frac{\zeta(3)}{\pi^2} g T^3$ | $\frac{3}{4} \frac{\zeta(3)}{\pi^2} g T^3$ | $g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T}$ |
| $\rho$ | $\frac{\pi^2}{30} g T^4$ | $\frac{7}{8} \frac{\pi^2}{30} g T^4$ | $m n$ |
| $p$ | $\frac{1}{3}\rho$ | $\frac{1}{3}\rho$ | $n T \, (\ll \rho)$ |
| $s$ | $\frac{2\pi^2}{45} g T^3$ | $\frac{7}{8} \frac{2\pi^2}{45} g T^3$ | $\left(\frac{m-\mu}{T} + \frac{5}{2}\right) n$ |

Table 7.2: $\rho$, $n$, $p$ and entropy density $s$ for different particles.

sure that the local thermal equilibrium was maintained during most of the early history of the early universe. In this case, the entropy per comoving volume $(S)$ is expected to remain constant and can be written as

$$S = \left(\frac{\rho + p}{T}\right) a^3 = s\, a^3 = \text{constant}. \tag{7.18}$$

where $s$ is the corresponding entropy density. A list of expressions for $n$, $\rho$, $p$ and $s$ are given in the table 7.2 for relativistic fermions, bosons and non-relativistic particles.

From the table 7.2, it is clear that the number, energy and entropy density of a non-relativistic particles decreases exponentially as a function of temperature. So the contribution to $\rho$, $p$ and $s$ comes mainly from relativistic species. Ignoring the contribution from Non-relativistic species, we can write

$$\begin{aligned} \rho &= \frac{\pi^2}{30} g_* T^4, \\ p &= \frac{1}{3}\rho \\ s &= \frac{2\pi^2}{45} g_{*s} T^3, \end{aligned}$$

where $T$ represents temperature of the photons. $g_*(T)$ and $g_{*s}(T)$ represent the net relativistic degrees of freedom for energy density and entropy density, respectively, and given as

$$\begin{aligned} g_*(T) &= \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4, \\ g_{*s}(T) &= \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^3. \end{aligned}$$

Summation extends over all fermion and bosons with temperature $T_i$ and internal degree of freedom $g_i$. The relation for $\rho$ can be used to write the following behavior of Hubble parameter $H$ with respect to temperature $T$ using the equation 7.5

$$H(T) = 1.66\sqrt{g_*}\frac{T^2}{M_{Pl}}, \tag{7.19}$$

where we have ignored curvature and $\Lambda$ term in the radiation dominated early universe.

Since the total entropy per comoving volume is constant, we can write from equation 7.18

$$g_{*S}T^3a^3 = \text{constant},$$

which implies $a \sim T^{-1}$. The conservation of $S$ also implies the relation $a^3 \propto s^{-1}$ which can be used to define total number of particles of a given species in a comoving volume like the Baryon number of the universe as

$$Y_B = \frac{(n_B - n_{\bar{B}})}{s},$$

which is expected to remain constant so long as baryon number violating processes are occurring very slowly. It is useful to note that the entropy density is proportional to the number density of relativistic particles and hence can be related to the number density of photons $(n_\gamma)$ as $s = 1.80 g_{*S} n_\gamma$. Today, we have $s = 7.1 n_\gamma$. However, since $g_{*S}$ is a function of temperature in general, the relation should be taken with some caution.

The time variation of the number density of a particular species in the expanding universe is crudely described by following equation

$$\frac{dn}{dt} = -3Hn + \Gamma n,$$

where $\Gamma$ is the interaction rate of the particle $\Gamma = n'\sigma v$. Here $n'$ is the number density of the target particle and $\sigma v$ is the cross section multiplied by relative velocity of the particles. So the evolution of the number density of a given species is determined by its interaction rate $\Gamma$ as long as the interaction rate $\Gamma$ is greater than expansion rate $H$ of the universe. However, if the expansion rate begins dominating over the interaction rate, the evolution will be solely governed by $H$ and the species will go out of equilibrium. From the expression 7.19 for $H(T)$, the condition for a species going out of equilibrium turns out to be

$$\Gamma(T) < 1.66\sqrt{g_*}\frac{T^2}{M_{Pl}}. \tag{7.20}$$

This condition is very important for explaining the current baryon asymmetry of the universe that we will discuss later.

## 7.3 Brief thermal history of the universe

To the best of our current understanding, we can extrapolate our present knowledge of the universe up to the Planck scale. Above the Planck scale the quantum behavior gravity starts dominating and it is not possible to speculate anything above this scale due to the lack of our knowledge of quantum theory of gravity. A consistent theory of gravity can only answer whether universe started with a big-bang or the scenario was completely different. So we begin our discussion from the Planck scale.

The next important scale below the Planck scale is the GUT scale. During the grand unification phase transition, rearrangement of the vacuum provided heavy masses to many of its gauge bosons and Higgs bosons, out of which only a few could provide baryon or lepton-number violating processes. The heavy particles soon decayed into the lighter particles and their recreation dropped due to the lack of light particles with sufficient energy below the grand unification phase transition. The next important event is believed to took place around temperature corresponding to $10^{8-10}$GeV when the lepton-number violating interactions created lepton asymmetry in the early universe which was later converted to baryon asymmetry. The left-right symmetry breaking is also expected to be in the same epoch if it is at all existing in nature at some high scale.

The next important phase transition that occurred was the electroweak phase transition at around 300 GeV when three of four electroweak gauge bosons became massive leading to almost frozen weak interactions. At the temperature around 100 to 300 MeV, the phase transition associated with chiral symmetry breaking is expected to take place. During this epoch the strongly interacting quarks were confined to color singlet states like baryon and mesons.

After the electroweak phase transition, the heavy $W^{\pm}$ and $Z$ gauge bosons started decaying and became decoupled because their reproduction started decreasing due to their weak inverse decay rate. The strength of weak interactions turned out to be very small since it became inversely proportional to the square of the masses of the three heavy gauge bosons. Since the neutrinos are neutral particles and they interact with other particles only through the weak interactions, their interaction rate decreased rapidly. Finally, their interaction rate became smaller than the expansion rate of the universe at temperature around 1 MeV and they decoupled from the rest of the relativistic plasma. Below 1 MeV the temperature of the decoupled neutrinos $(T_\nu)$ kept on scaling as $a^{-1}$. This very weak cosmic neutrino background (CNB) is one of the very robust predictions of the big-bang cosmology, although still waiting to be confirmed. The expected current contribution of the energy density of the the CNB to the total energy density is [29]

$$\Omega_{\nu}^{0} = (0.001 - 0.05)\,.$$

With this contribution, the upper bound on sum of all three neutrino masses has been estimated as [112]

$$\sum_{i} m_{\nu} < 0.67\text{eV}\,.$$

which is an independent upper bound on the mass scale of the light neutrinos other than the beta decay bound.

The next important and established event in the thermal history of universe was nucleosynthesis. The protons and neutrons, formed after chiral symmetry breaking, got together and synthesized primordial light nuclei at around 1 MeV temperature. The predictions of nucleosynthesis about the amount of produced light nuclei fit very well with the observations. So the primordial nucleosynthesis provides the earliest test of the standard cosmology.

After the neutrino decoupling, the temperature of the universe dropped below the mass of the electron and all the $e^{\pm}$ pairs were annihilated with only $e^{-}$ left along with light nuclei and photons. Soon after, the electrons and the light nuclei specially protons paired to form neutral atoms and universe became opaque to the photons. The interaction of the photons with $e^{-}$ and other nuclei became very small and the radiation decoupled from matter. This radiation is the cosmic microwave background radiation (CMBR) that we observe today. The present temperature $(T_0)$ and number density $(n_{\gamma})$ of the CMBR have been measured to be [29]

$$\begin{aligned} T_0 &= 2.725(1)\,\text{K}\,. \\ n_{\gamma} &= 410.5\,\text{cm}^{-3}\,. \end{aligned}$$

Although the CMBR is almost isotropic and homogeneous, it has tiny fluctuations in its temperature[112] in different directions. This anisotropy of the CMBR temperature provides important information needed to explain the large scale structure formation in the universe.

Before ending this chapter , we would like to mention about two very important issues in connection to the current state of our universe. The first is that our universe has recently entered $(z < 0.67)$ into the phase of acceleration which needs the domination of an DE density over the matter energy density. Although the cosmological constant can address this issue, as we pointed out earlier, it soon runs into severe fine-tuning problem. We will discuss the related problems in the next chapter in detail and will review some alternatives to attack the the problem from an entirely different angle.

The other important issue that need to be address is the current baryon asymmetry of the universe. There is no prior reason to assume that universe started with such an asymmetry. So one need to find out some consistent mechanism to create this asymmetry in some early phase of the evolution of the universe which remains up to now in spite of some washout effects. A very popular mechanism of creating baryon asymmetry of the present universe is by first creating lepton asymmetry in early universe. The mechanism is known as leptogenesis which we will discuss in detail in chapter 9.

# Chapter 8

# Dark energy

Dark energy (DE) is a component of energy of our with equation of state $\omega = -1$ with negative constant pressure. It can appear in the most general form of Einstein's equation through the cosmological constant term. It does affect the evolution of the scale factor as can be easily seen from the expression

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}.$$

The first term in the right-hand side remains positive for radiation and matter components while the second term is positive. So in the absence of cosmological constant, the expansion rate of the universe will decelerate, i.e., $a > 0$ and $\ddot{a} < 0$. If we extrapolate back, it can be easily realized that universe must have started from $a = 0$. We can assume this point of the universe as the starting point with $t = 0$. The age of the universe, denoted with $t_0$, will be then just estimated from this reference point.

However, if there exist a cosmological constant, the second term can dominate over the first term at some phase of the universe as the energy density of both radiation and matter keeps on decreasing with the expansion of the universe. Consequently the universe can switch from the decelerating phase to an accelerating phase of its evolution. It is in fact the situation at present where there are enough and growing evidences that universe has entered into an accelerating phase, although, recently.

## 8.1 Observational evidences

The first direct evidence of the current acceleration phase of the universe has come from the study of the observed luminosity distance and redshift of the Type Ia supernovae [21, 22]. Astrophysical studies have shown that the type Ia supernova can be treated as an ideal standard candle and their luminosity distance $(d_L)$ can be

| $h$ | $\Omega^0_M$ | $\Omega^0_{\text{Baryonic}}$ | $\Omega^0_{\text{Dark Matter}}$ | $\Omega^0_{DE}$ | $\omega_{DE(\Lambda)}$ |
|---|---|---|---|---|---|
| 0.73(3) | 0.27(2) | ≈0.0425 | ≈0.20 | 0.73(3) | -0.97(7) |

Table 8.1: The best fit values for cosmological parameters

estimated by measuring the apparent magnitude. Knowing both $d_L$ and $z$, one can estimate the present values of $\Omega_i$'s using the expression 7.12.

The first evidence came in 1998 from two groups Supernova Cosmology Project (SCP) and High-$z$ Supernova Team (HSST). Assuming a flat universe and DE in the form of $\Lambda$, they found that about 70% of the total energy density of the present universe is in the form of DE. In 2004 HSST [113] found that the matter density (visible and dark matter) of the present universe is

$$\Omega^0_M = 0.29^{+0.05}_{-0.03},$$

and showed that universe exhibits transition from deceleration phase to acceleration phase (at 99% confidence level) at redshift around $z = 0.67$. It is important to note that the current acceleration of the universe is fairly recent phenomena if one compares it on cosmological time scale.

Another independent evidence for the presence of DE comes from the age analysis of the universe. The fact that the age of the universe $(t_0)$ should be greater than the age of the oldest stellar objects $(t_s)$ observed leads towards the need for the presence of DE. The study of some oldest stellar objects [114, 115] suggests that the age of the universe should be grater than $11-12$ Gyr. However, the estimated value of $t_0$ using the equation 7.13, in the absence of DE, comes out to be quite low $t_0 = 8-10\,\text{Gyr}$. If one includes the cosmological constant the theoretical age of the universe can be shown to increase. For the values $\Omega^0_M = 0.3$ and $\Omega^0_{DE} = 0.7$, the age of the universe is easily estimated to be $t_0 = 13.1$Gyr which is consistent with the age of the oldest stellar object.

The DE dominated universe has also got independent support from observations related to CMBR from WMAP [23] and large scale structuring (LSS) from SDSS [116–118]. In the figure 8.1, the allowed region in the $\Omega^0_M - \Omega^0_{DE(\Lambda)}$ plane by the supernovae, CMBR and LSS data has been shown . It is obvious that common overlapping regions of CMBR and LSS together are able to confirm the existence of the DE without using the supernova data. However, the supernova data helps in constraining the parameter space when it is included. All the three data sets can be simultaneously explained only when one assumes DE to be the dominant component of energy of the present universe. Based on the cosmological model with $\Lambda$, we list the best fit values of the relevant cosmological parameters from PDG [29] in table 8.1.

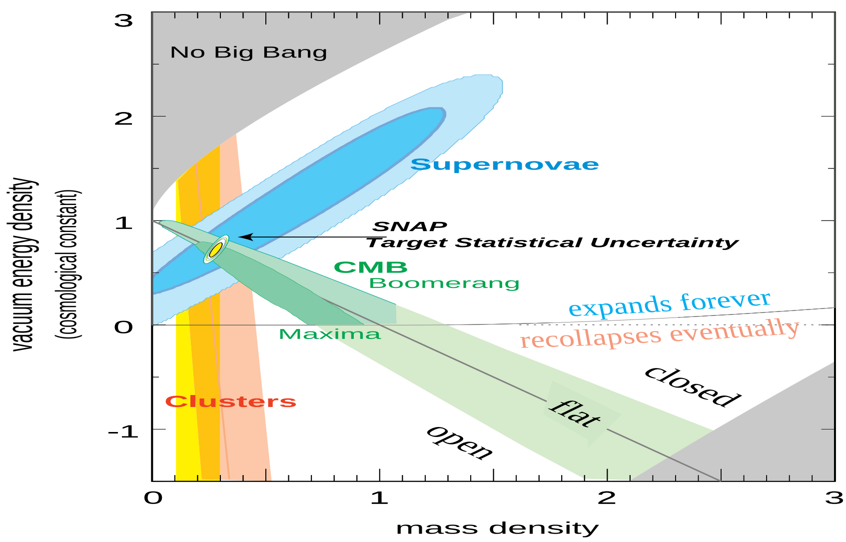


Figure 8.1: The allowed regions in $\Omega_M^0 - \Omega_{DE(\Lambda)}^0$ plane from the observation from CMBR, LSS and SN Ia[2].

## 8.2 Problem with cosmological constant

So far we realized that the DE induced by cosmological constant term in the Einstein's equation can solve the the puzzle of the current acceleration of the universe. However, it is interesting to note that $\Lambda$ was not initially proposed to explain the acceleration of the universe. Einstein introduced it in 1917 to achieve an static universe. Later, Hubble's observation related to the expansion of the universe made Einstein to drop this term. Its importance was realized only recently to explain the accelerating universe.

Although the DE model in form of $\Lambda$ is able to satisfy all the astrophysical data, it also leads to two very uncomfortable problems (1) fine-tuning problem and (2) cosmic coincidence problem.

### 8.2.1 Finetunning problem

The cosmological constant runs through severe fine-tuning problem when we try to realize it from the stand point of particle physics. From particle physics point of view, the cosmological constant naturally arises as zero point energy or vacuum energy density $(\rho_{vac})$. The observed energy density corresponding to the cosmological constant comes out to be

$$\rho_\Lambda = 10^{-47}\mathrm{GeV}^4 \simeq (10^{-3}\,\mathrm{eV})^4 \,.$$

On the other hand the vacuum energy density arising from zero point energy of a quantum field with mass $m$ is given as

$$\rho_{vac} = \int_0^\infty \frac{d^3k}{(2\pi)^3}\sqrt{k^2+m^2}\,. \tag{8.1}$$

Obviously one gets infinite contribution to the vacuum energy density. However, the theory may be considered to be valid up to some upper cutoff scale say $k_{max}$. Then the relation 8.1 gives finite energy density

$$\rho_{vac} \simeq \frac{k_{max}^4}{16\pi^2}\,.$$

Now if we take the natural cutoff of the energy scale to be Planck scale, we see that the $\rho_{vac}$ comes out to be extremely large value compared to observed value of $\rho_\Lambda$ by 123 order of magnitude.

$$\rho_{vac} \simeq \begin{cases} 10^{74}\mathrm{GeV}^4 & \text{for } k_{max} = 10^{19}\mathrm{GeV}\ (\text{Planck Scale}) \\ 10^{10}\mathrm{GeV}^4 & \text{for } k_{max} = 10^{3}\mathrm{GeV}\ (\text{SUSY Breaking Scale})\,. \end{cases}$$

However, if our nature possess supersymmetry (SUSY), the cutoff scale can be brought down to the scale of SUSY breaking as the total contribution above the SUSY breaking scale is exactly canceled due to equal and opposite contribution of fermions and their super partner bosons or vice-verse. The current data from particle physics indicates the SUSY scale to lie around TeV scale. Unfortunately, the corresponding estimates of the vacuum energy density is again much larger compared to the observed $\rho_\Lambda$. So, to achieve the observed value of DE energy density, one will have to rely on very severe fine-tuning between the two contributions from two independent quantities, cosmological constant and the zero point energy, which seems to be quite an unnatural scenario.

### 8.2.2 Cosmic Coincidence problem

The fact that universe has entered into the acceleration phase very recently (after $z < 0.67$) has led to what is referred as the cosmic coincidence problem. In other words, the problem can be phrased as why the DE component of the universe has dominated now and why the order of $\Omega^0_{DE}$ is of the same order of $\Omega^0_M$. So although these two energy densities vary differently at different epochs, they are of the same order of magnitude today.

The problem becomes even worse with cosmological models which include $\Lambda$. Since the DE due to cosmological constant remains constant throughout the evolution of the universe, $\Lambda$ needs to be properly tuned so that it dominates only today.

For example, the ratios of DE density to the radiation density at Planck time and at the time of electroweak phase transition are many order of magnitude less than compared to the present ratio of DE density to matter density $(\sim 1)$

$$\frac{\rho_\Lambda}{\rho_r} \simeq \begin{cases} 10^{-123} & (\text{Planck Scale}) \\ 10^{-55} & (\text{Electroweak Scale}) \, . \end{cases}$$

So an huge fine-tuning of the initial condition is required to ensure the ratio $\rho_\Lambda^0/\rho_M^0 \sim 1$ today.

With the problems discussed above, cosmological constant does not seem to be a natural candidate for DE to explain the accelerating universe. This has led many others to presume the cosmological constant to be zero, instead of small, by some unknown mechanism and to explain the presence of DE by dynamics of some light scalar field. This scenario is known as time varying DE scenario. For the rest of the discussion we will be moving around the models with time varying DE only.

## 8.3 Varying DE models

As the name suggests the time varying DE scenarios allow equation of state to vary with time with the constraint that only its current value is close to $-1$. A successful cosmological model should admit (1) a radiation dominated era, (2) followed by a long matter dominated era and (3) the finally the present accelerated era. So a dynamical DE should have the feature that it is sub-dominated during radiation and matter dominated era and becomes dominant only today.

### 8.3.1 Quintessence models

The relatively old and popular models of time varying DE scenario are through dynamics of a scalar field, known as quintessence model [119, 120]. In this model the energy density of DE is attributed to the energy density of a quintessence described by an ordinary scalar field ($Q$), minimally coupled to gravity. Neglecting the spatial curvature, one can obtain the energy density and pressure density of the scalar field as

$$\rho_Q = \frac{1}{2}\dot{Q}^2 + V(Q) \, , \tag{8.2}$$

$$p_Q = \frac{1}{2}\dot{Q}^2 - V(Q) \, . \tag{8.3}$$

The corresponding equation of motion and equation of sate are given as

$$\ddot{Q}+3H\dot{Q} = -\frac{dV}{dQ}, \tag{8.4}$$

$$\omega_Q = \frac{\frac{1}{2}\dot{Q}^2 - V(Q)}{\frac{1}{2}\dot{Q}^2 + V(Q)}. \tag{8.5}$$

Since the potential of the scalar field determines the dynamics of the scalar field, there exists variety of quintessence models with different cosmological consequence. Also, the equation of state of $Q$ can vary in the range $-1 \leq \omega_Q \leq +1$.

In the limit of vanishing kinetic energy of $\dot{Q}^2 < V(Q)$, the equation of state approaches towards $\omega_Q = -1$, the value required for the current acceleration of the universe. However, one would like to obtain a natural quintessence solution for cosmological constant problem with the desired feature that it can explain the late time acceleration of the universe starting from wide range of initial conditions and equation of states. However, the energy density of DE should be sub-dominant in the radiation and the matter dominated era specially during big-bang nucleosynthesis where its dominant contribution can substantially spoil the nucleosynthesis predictions. It is only recently that DE has started dominating over matter density.

In fact, there exist solutions where the DE density follows the background energy density of the universe. The solutions are known as scaling solutions [121–123] where the ratio of the energy density of the $Q$ to the background energy density $(\rho_B)$ becomes almost constant

$$\frac{\rho_Q}{\rho_B} = \text{constant}. \tag{8.6}$$

This is possible when there exists what are called as fixed or critical points working as attractors for the corresponding scaling solutions. An simple exponential potential has fixed points corresponding to a scaling solution as well as a non-scaling solution. The non-scaling solution can provide late time acceleration of the universe. The parameters of the potential are required to be adjusted such that the scaling solutions correspond to the sub-dominant DE density during radiation and matter dominated era.

However, the system should exit from the initial scaling solution to enter the accelerated phase. This is possible with tracker fields where the DE tracks some component of matter so that DE exit the scaling solution to finally enter the accelerating phase of the universe [24–26]. The tracking solutions are not the fixed point solutions like in case of scaling solutions and so the energy density of the tracker field changes according to the background energy density.

In an another interesting scenario, the DE is made to track the energy density of

CNB. This scenario is also interesting from the fact that it can naturally avoid some of the unwanted issues related to the previous case where DE is allowed to track Dark matter or baryonic matter.

### 8.3.2 Neutrino models of DE

A very interesting coincidence which is worth noting that the scale of DE $\rho_{DE} \simeq (10^{-3}\,\mathrm{eV})^4$ is almost close as the scale of neutrino masses. Motivated with this fact, Fradon, Nelson and Weiner suggested a scenario where a coupled system of CNB and a scalar field behaves like DE [27]. The coupling is introduced by assuming that the mass of the neutrino $m_\nu$ varies as a function of the scalar field, called acceleron ($\mathcal{A}$).

However, the coupling between $\mathcal{A}$ and the neutrinos appears only when the neutrinos in the CNB become non-relativistic. The equation governing the evolution of the neutrino energy density $\rho_\nu$ and scalar field $\mathcal{A}$ can be obtained as

$$
\begin{aligned}
\rho_\nu + 3H\,(\rho_\nu + p_\nu) &= \frac{d\ln m_\nu}{d\mathcal{A}}\dot{\mathcal{A}}\,(\rho_\nu - 3p_\nu)\,, \\
\ddot{\mathcal{A}} + 2H\dot{\mathcal{A}} + a^2\frac{dV_0}{d\mathcal{A}} &= -a^2\frac{d\ln m_\nu}{d\mathcal{A}}\,(\rho_\nu - 3p_\nu)\,,
\end{aligned}
\tag{8.7}
$$

where $V$ is the potential of the acceleron field. As the relation $\rho_\nu = 3p_\nu$ is satisfied for the relativistic neutrinos, the right hand side of the above two expressions vanishes leading to the fact that the relativistic neutrinos and the acceleron field evolve independent of each other. Only when the neutrinos become non-relativistic, the two components begin affecting the evolution of each other.

For the non-relativistic CNB the energy density will be given as $\rho_\nu = m_\nu n_\nu$ with pressure equals to zero. When the neutrinos become non-relativistic, effective potential ($V$) of the combined system of neutrinos and $\mathcal{A}$ will be written as

$$
V\,(m_\nu) = m_\nu n_\nu + V_0(m_\nu)\,,
$$

where both $V$ and $V_0$ has been expressed as function of $m_\nu$ instead of $\mathcal{A}$ for convenience. It is always possible to do so as the neutrino mass and $\mathcal{A}$ is related by some functional form. Even if $V_0$ does not have any minimum, the first term induces an instantaneous minimum for the effective potential $V$ and the acceleron field $\mathcal{A}$ relaxes at this minima satisfying the condition

$$
V'\,(m_\nu) = n_\nu + V_0'(m_\nu) = 0\,,
\tag{8.8}
$$

where $V'\,(m_\nu)$ denotes derivative of the effective potential $V$ w.r.t. $m_\nu$.

However, the minima is not fixed but evolves slowly with the number density $n_\nu$ of CNB. The minima shifts towards the larger values of $m_\nu$ as $n_\nu$ dilutes. So the neutrino mass is decided by the competition between the two terms in the effective potential. Neglecting the kinetic energy of the acceleron field, the equation of state of the combined $(\nu+\mathcal{A})$ system can be written as

$$\begin{aligned}\omega &= \frac{\text{Pressure}}{\text{Density}} = \frac{p_{\mathcal{A}}}{\rho_{\mathcal{A}}+\rho_\nu} \\ &= \frac{-V_0}{V_0+m_\nu n_\nu} = -1+\frac{m_\nu n_\nu}{V} \\ &= -1-\frac{m_\nu V_0'(m_\nu)}{V} = -1+\frac{\Omega_\nu}{\Omega_\nu+\Omega_{\mathcal{A}}}.\end{aligned}$$

The observed late time value of $\omega \approx -1$ implies that the energy density of the CNB is a small fraction of the total energy density. This corresponds to condition, $V_0'(m_\nu) \ll V(m_\nu)$ which can be achieved by choosing a flat potential for $\mathcal{A}$ or rather steep dependence of $m_\nu$ on $\mathcal{A}$. With this condition, the combined $(\nu+\mathcal{A})$ system behaves like DE with equation of state $\omega = -1$ and causes the desired acceleration of the universe. Since the neutrinos in CNB are expected to become non-relativistic now, the question of why DE dominated today gets a natural answer in the mass varying neutrino scenario.

Now we shall discuss the neutrino mass varying model in simplified one generation type I seesaw scenario. Since the right-handed neutrino is the SM gauge singlet, it can be allowed to vary as a function of $\mathcal{A}$. The mass of light neutrinos are given by the famous seesaw formula

$$m_\nu(\mathcal{A}) = \frac{M_{\nu D}^2}{M_R(\mathcal{A})}.$$

Let us take the following flat potential for the model

$$V_0(\mathcal{A}) = \Lambda^4 \log\left[1+M_R(\mathcal{A})/\mu\right], \tag{8.9}$$

where it is assumed that $M_R(\mathcal{A})/\mu \gg 1$ as the right-handed neutrino Majorana mass scale is very high. $\Lambda$ is chosen to be equal to the characteristic energy scale of DE, i.e., $\Lambda \sim 10^{-3}$eV. We rewriting the above equation in terms of $m_\nu$ using the seesaw formula as

$$V_0(m_\nu) = \Lambda^4 \log\left(\frac{m_0}{m_\nu}\right),$$

where $m_0 = M_{\nu D}^2/\mu$. Using equation 8.8, we get $n_\nu = \Lambda^4/m_\nu$ which implies a constant energy density for non-relativistic CNB:

$$m_\nu n_\nu = \Lambda^4 \,.$$

Hence the equation of sate can be simply written as

$$\begin{aligned}
\omega &= -1 + \frac{m_\nu n_\nu}{V} \\
&= -1 + \left[1 + \log\left(\frac{m_0}{m_\nu}\right)\right] ,^{-1}
\end{aligned}$$

which is almost equal to $-1$ as the second term vanishes because the scale of $m_\nu$is much smaller than the scale of $m_0$. So the coupled $(\nu + \mathcal{A})$ system behaves like DE.

In the type I seesaw scenario, it is possible to vary the SM singlet right-handed neutrino with the acceleron field. However it is not possible to do the same if this right-handed neutrino emerges from left-right symmetric model or GUT models. The right-handed neutrino comes with right-handed charged leptons in left-right symmetric model and with other SM fermions in GUT models and so transforms non-trivially under the related gauge group. In chapter 11, we will try to construct a left-right symmetric model for neutrino DE and try to embed it in $SO(10)$ GUT scenario.

# Chapter 9

# Baryon asymmetry through lepton asymmetry

To best of our understanding, the everything all around us such as earth, moon, our solar system, galaxies and clusters, is made up of matter. In fact, our own existence is because we live in a universe where the matter dominates over antimatter. This most challenging problem of cosmology can only be properly addressed when we use our knowledge of particle physics.

But one may ask why we are not taking the baryon asymmetry as an initial condition the universe started with. The first argument is that there is no reason to assume that universe would have started with an asymmetry in the number density of a particle with a given quantum number and its antiparticle . The second argument comes from the fact that the observed asymmetry is a very very small number. In a volume containing $10^{10}$ photons, we get around 6 matter particles in excess to the antimatter particles. One will have to accept a very unnatural fine-tuning of the initial condition of the very early universe to explain this current number. So one assumes this initial value to be zero and try to invoke some particle physics mechanism to explain the puzzle.

The baryon asymmetry of the universe can be defined and estimated in two equivalent ways, first in terms of entropy density and the second in terms of photon number density as follows

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = \frac{n_B}{n_\gamma} = (6.12 \pm 19) \times 10^{-10} \tag{9.1}$$

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} = (8.61 \pm 0.26) \times 10^{-11} \,. \tag{9.2}$$

The numbers have been taken from [29]. From the relation between current entropy density and the photon density $n_\gamma = s/7.1$, as pointed out in the chapter of

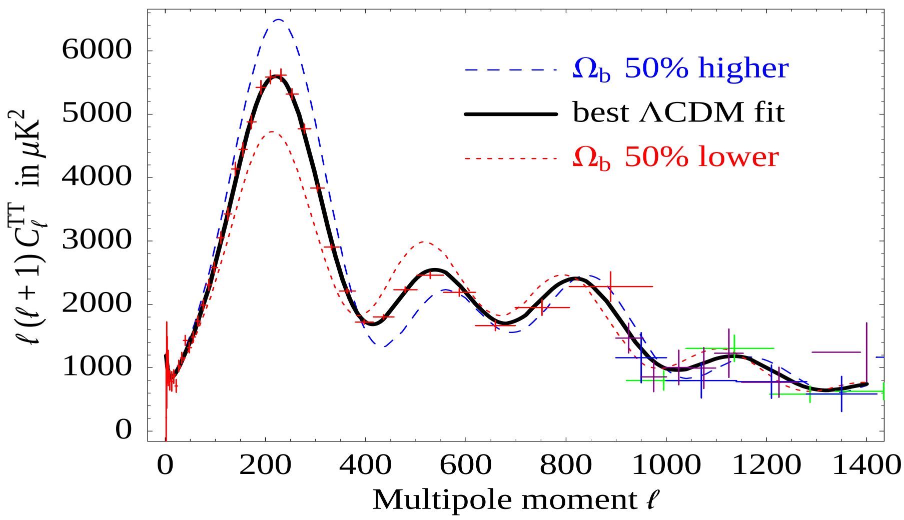


Figure 9.1: Dependence of CMBR anisotropy on $\eta_B$

standard cosmology, we can easily relate the above two values as $Y_B = \eta_B/7.1$.

The observed values of the current baryon asymmetry has got support from two independent sources. First one comes from big bang nucleosynthesis. The predicted primordial abundances of light nuclei, like $D$, $^3He$, $^4He$ and $^7Li$ , are very sensitive to the amount of baryon matter present at the time of nucleosynthesis. In fact, this baryon asymmetry has to be generated before the nucleosynthesis in order to have the successful synthesis of the observed light nuclei in correct amount. One obtains the following range of $\eta_B$, although small but non-zero, which is consistent with all the four abundances [29]

$$4.7 \times 10^{-10} \leq \eta_B \leq 6.5 \times 10^{-10} \quad (95\%) \, .$$

The other independent support comes from the study of two point correlation function of the anisotropy observed in the temperature of CMBR. The small temperature anisotropies are usually analyzed by decomposing the signal, coming from a direction characterized by angles $\theta$ and $\phi$, into spherical harmonics $(Y_{lm})$ as

$$\frac{\triangle T\,(\theta,\phi)}{T} = \sum_{l,m} a_{lm} Y_{l,m} \, ,$$

where $a_{lm}$'s are the expansion coefficients. The study of anisotropy in the CMBR temperature can be translated to the study of CMBR angular power spectrum given as $C_l =< |a_{lm}|^2 >$. As the position and relative height of the acoustic peaks of the CMBR anisotropy spectrum are sensitive to $\Omega_B$, a fit of the power spectrum helps to extract the value of parameter corresponding to the baryon to photon ratio along with other parameters. The figure 9.1 (figure taken from [3]) shows the variation of the CMBR power spectrum fitting when $\eta_B$ is taken to be up to 50% more and less of the central value for best fit. A fit to recent WMAP 5 year data along with Type Ia supernova and SDSS data provides the following value for the ratio of baryon energy density to the total energy density of the universe as [112]

$$\Omega_B h^2 = 0.02267^{+0.00058}_{-0.00059} .$$

However, we will be using the value from PDG given in expression 9.1 or 9.2 for future discussion.

## 9.1 Baryogenesis

With the clear evidence of presence of baryon asymmetry at the time of big bang nucleosynthesis, we now turn to look for some mechanism for creating baryon asymmetry in the early universe. The mechanism is known as baryogenesis. To generate a small baryon asymmetry starting from baryon symmetric universe, one need to satisfy three conditions given by Sakharov in 1967 [124].

### 9.1.1 Sakharov's conditions

**Baryon number violation**

The presence of baryon number violating interactions is very first condition for generating baryon asymmetry from the baryon symmetric universe. If baryon number (B) is conserved the interaction that generate a baryon from a state $B = 0$ will also generate an antibaryon leading to vanishing net baryon number. Since we have assumed that the universe is neutral to any conserved charge to start with, this would imply that the number density of the particles with nonzero baryon number would be same as the number density of the antiparticles.

**C and CP violation**

Now we assume that the first condition for baryon number violation is satisfied. Then Sakharov's second condition demands both C and CP to be violated also. Let a initial state $i$ goes to final state $j$ where baryon number is violated. Suppose C is conserved in the process $i \to j$, then we can immediately conclude that the number of left-handed particles with nonzero baryon number will be generated in equal amount to the number of corresponding left-handed antiparticles. So C violation is necessary condition to see a net baryon number violation.

Now suppose if C is violated but CP is conserved, then the number of left-handed particles generated will be compensated by the same number of right-handed antiparticles generated. Same will be true with right-handed particles with nonzero baryon number and corresponding left-handed antiparticles. However, if both C and CP are violated we expect asymmetry in the number of particles and the antiparticles created.

**Departure from thermal equilibrium**

Since particle and antiparticle has opposite baryon number, B is odd under C transformation but even under parity (P) and time reversal (T) transformations. So B is odd under CPT transformation. The thermal expectation value of the baryon number $< B >_T$ is written as

$$< B >= \frac{\mathrm{Tr}\left[B e^{-\beta \hat{H}}\right]}{\mathrm{Tr}\left[e^{-\beta \hat{H}}\right]},$$

where $\hat{H}$ is Hamiltonian of the system and $\beta = 1/T$.

Since the Hamiltonian $\hat{H}$ commute with CPT, we can show that the expectation value of the baryon number in thermal equilibrium comes out to be zero as

$$\begin{aligned} < B >_T &= \frac{\mathrm{Tr}\left[B e^{-\beta \hat{H}}\right]}{\mathrm{Tr}\left[e^{-\beta \hat{H}}\right]} = \frac{\mathrm{Tr}\left[CPT\, CPT^{-1} B e^{-\beta \hat{H}}\right]}{\mathrm{Tr}\left[e^{-\beta \hat{H}}\right]} \\ &= \frac{\mathrm{Tr}\left[CPT^{-1} BCPT\, e^{-\beta \hat{H}}\right]}{\mathrm{Tr}\left[e^{-\beta \hat{H}}\right]} = -\frac{\mathrm{Tr}\left[B e^{-\beta \hat{H}}\right]}{\mathrm{Tr}\left[e^{-\beta \hat{H}}\right]} \\ &= - < B >_T . \end{aligned}$$

Thus an averaged baryon asymmetry vanishes in thermal equilibrium. So departure from thermal equilibrium is also required to achieve any non-trivial amount of baryon asymmetry. This is the third Sakharov's condition.

### 9.1.2 Connecting baryon and lepton asymmetry

As the baryon number violation is a necessary condition for baryogenesis, let us discuss the baryon-number ($B$) and lepton-number ($L$) violation in the SM itself. In SM, both baryon number and lepton number are accidental global symmetries at classical level, i.e., it is not possible to violate these symmetries at tree-level. However, quantum corrections violate them leading to global $B$ and $L$ anomaly [125, 126]. In other words the current associated with $B$ and $L$ do not vanish after including the quantum corrections. Further analysis shows that although $B$ and $L$

are anomalous individually, a combination $B-L$ is anomaly free. However, the other linearly independent combination $B+L$ still remains anomalous.

The $B+L$ violation basically arises due to the vacuum structure of the Non-Abelian nature of the gauge theory of SM. In a non-Abelian gauge theory there are infinitely many degenerate ground states, which are characterized by so-called Chern-Simon number as $\triangle N_{cs} = \pm 1, \pm 2, ...$, separated by a potential barrier whose height is given by what is called as sphaleron energy. For the three generation of fermions, the vacuum to vacuum transition can lead to the following changes in $B$ and $L$ in terms $N_{cs}$

$$\begin{aligned} \triangle B = \triangle L &= 3\triangle N_{cs} \\ &= \pm 3n\,, \end{aligned}$$

where n is an positive integer. However, in classical approximation, such vacuum to vacuum tunneling in SM comes out to be exponentially suppressed $\left(O\left(10^{-165}\right)\right)$ and extremely small [125, 126] .

The situation changes drastically when one studies the same phenomena in thermal bath. In 1985, Kuzmin, Rubakov and Shaposhnikov [127] pointed out that it is possible to make transition from one vacuum to other by thermal fluctuation over the barrier in the thermal bath of early universe. When temperature is larger than the height of the barrier , the sphaleron mediated $B+L$ violating interactions can become so strong that they are in equilibrium in the expanding universe. In fact, it can be shown that sphalerons are in equilibrium for the range of temperature starting from $10^2$ GeV to $10^{12}$ GeV.

As the $B+L$ violating interactions are in equilibrium along with other interactions above the temperature corresponding to the electroweak phase transition, one would expect some relation between the baryon asymmetry and lepton asymmetry if they are nonzero. If the asymmetry in number density of a given particle $(n_i)$ and its antiparticle $(\bar{n_i})$ is very small compared to the total number density, the asymmetry is related to the chemical potential $(\mu_i)$ of the particle as

$$n_i - \bar{n_i} = n_d \frac{gT^3}{6}\left(\frac{\mu_i}{T}\right).$$

At high temperature, quarks, leptons and Higgs not only interact by gauge and Yukawa couplings but also by sphaleron processes in addition. All these processes in equilibrium put various constraints on the chemical potential of different particles and so this can lead towards some relation in the chemical potentials. Since the $B$ and $L$ are function of their chemical potentials, we expect some relation among them in the thermal bath . In fact a detail analysis of the chemical potentials [128, 129] provides us the following relation between B and L, above electroweak phase transition, for three generation scenario:

$$\begin{aligned} B &= p\,(B-L) \\ L &= (p-1)\,(B-L) \qquad (9.3) \\ B+L &= (2p-1)\,(B-L)\,, \qquad (9.4) \end{aligned}$$

where $p = (24+4N_H)/(66+13N_H)$ and $N_H$ is number of Higgs bi-doublets. It is important to notice that any nonzero value for $B$, $L$ or $B+L$ generated by any mechanism can not survive unless $B-L$ is also generated. So any such asymmetry in $B+L$ will be washed out before electroweak phase transition for vanishing $B-L$.

### 9.1.3 Mechanism for baryogenesis

Keeping Sakharov's condition in mind, we now discuss the following three interesting mechanism for baryogenesis.

#### Electroweak baryogenesis

This mechanism is very interesting from the fact that it does not require completely unknown and new physics beyond SM but tries to answer the baryon asymmetry using our knowledge of SM [130, 131]. The first Sakharov condition for baryon number violation is easily satisfied as baryon number is not conserved at quantum level. Considering the second condition, C violation is quite explicit in SM. Moreover CP violation is also established in the quark sector, although the amount of violation is rather small. The last condition regarding departure from thermal equilibrium can be provided by strong first order electroweak phase transition.

However, the phase transition for SM is not very strong and even the CP violation in SM is not enough to create baryon asymmetry of correct amount. Thus a viable model of electroweak baryogenesis need a modification in the Higgs sector such that nature of phase transition changes and new sources of CP violation is generated. In one of the extension of SM where two Higgs doublet are present, one get more parameter in the Higgs potential which can provide new sources for CP violation [132]. However, the predictability of electroweak baryogenesis in such extension of SM is lost.

### GUT baryogenesis

The GUT baryogenesis was the first natural implementation of Sakharov's conditions to create baryon asymmetry [133–135]. The B violation are very natural in GUT scenario as quarks and leptons share quantum numbers from the same multiplet. Since the fermions belongs to the chiral representation, C is maximally violated. Furthermore, sufficient amount of CP violation can be incorporated in GUT as there exists many complex phases in the couplings of heavy bosons, whose decay violates baryon number. Moreover, departure from thermal equilibrium can also be easily satisfied as the expansion rate of the universe at the unification scale was sufficiently high compared to the decay rate of heavy bosons to baryon number violating states.

Although GUT baryogenesis appeared to be quite a natural scenario, it soon ran into washout problems. In $SU(5)$ GUT both B and L are violated but the combination B-L remains conserved globally. Even in $SO(10)$ GUT, B-L is the part of the gauge group and is broken at some intermediate scale. So baryon number violation at GUT scale in both the GUT scenarios conserves B-L. But we saw in the previous section that any baryon asymmetry will be washed out before electroweak phase transition unless B-L is nonzero. So the GUT scale baryogenesis in the B-L conserving scenario is not possible to explain the current baryon asymmetry of the universe [127]. However, breaking of the B-L gauge symmetry in $SO(10)$ GUT at some intermediate scale can provide lepton asymmetric universe which can be later converted to baryon asymmetry. This mechanism is known as baryogenesis via leptogenesis and can be a part of $SO(10)$ GUT framework. However, this mechanism does not need to be a part of $SO(10)$ GUT in general as it can also work independently in some rather simple extensions of SM.

### Baryogenesis via leptogenesis

This is the most popular mechanism at present to generate baryon asymmetry by first creating lepton asymmetry. The mechanism, proposed by Fukugita and Yanagida in 1986 [28], has got enough scientific attention due to its simplicity. The lepton asymmetry can be created by introducing some Lepton number violating source term at some appropriate high scale. This is in fact the situation in seesaw framework [18–20] where a lepton-number violating term is introduced explicitly to suppress the low energy neutrino masses naturally.

So seesaw mechanism not only provides very tiny masses to neutrinos but also promises to create lepton asymmetry in early universe. This lepton asymmetry is partially converted to baryon asymmetry due to sphaleron process before electroweak phase transition leading to the current baryon asymmetric universe. The baryon asymmetry can be estimated in terms of the lepton asymmetry using relation 9.3 as

$$B = \left(\frac{p}{p-1}\right) L \,. \tag{9.5}$$

Although leptogenesis is possible in all the seesaw scenarios, we will only discuss the details of leptogenesis mechanism in type I seesaw.

## 9.2 Leptogenesis

Leptogenesis is a simple and elegant mechanism to generate lepton asymmetry in the early universe before electroweak phase transition. In type I seesaw scenario, the lepton asymmetry is created by out of equilibrium decay of heavy Majorana neutrinos to leptons and antileptons in different amount [28] . The first Sakharov condition is easily satisfied due to presence of Majorana mass term of the right-handed neutrinos which violates lepton number by two units [124]. C violation is also maximal as in the SM. CP violation can also be incorporated by assuming unremovable phases present in the complex Yukawa couplings of right-handed neutrinos with lepton doublets and Higgs doublet.

Let us write the part of the Lagrangian again relevant for leptogenesis from section 10.1

$$\mathcal{L} \;=\; \left(\frac{1}{2}\overline{(N_{R\alpha})^c}(M_R)_{\alpha\beta}N_{R\beta} + \overline{\ell_{Li}}\phi(Y_\ell)_{ij}e_{Rj} + \overline{\ell_{Li}}\tilde{\phi}(Y_\nu)_{i\alpha}N_{R\alpha} + H.C.\right), \tag{9.6}$$

where the symbols represent the similar thing as in the section 10.1. In a mass basis where the right-handed neutrinos are real and diagonal the Majorana neutrinos are defined as $N_i = \frac{1}{\sqrt{2}}(N_{Ri} \pm N_{Ri}^c)$ with masses $M_i$'s.

Due to the presence of CP violation in the model, the Majorana heavy right-handed neutrino $N_i$ decays into lepton+Higgs and antilepton+antiHiggs in different proportions. In the mass basis of right-handed neutrinos, the CP asymmetry factor is defined as

$$\varepsilon_i = \frac{\Gamma_i\left(N_i \to \ell\phi\right) - \overline{\Gamma}_i\left(N_i \to \overline{\ell\phi}\right)}{\Gamma_i\left(N_i \to \ell\phi\right) + \overline{\Gamma}_i\left(N_i \to \overline{\ell\phi}\right)}\,, \tag{9.7}$$

where $\Gamma_i$ is the decay rate of $N_i$. The denominator of the right-hand side of the above expression is the average decay rate of the $N_i$ given as

$$\Gamma_{D_i} = \frac{1}{8\pi}\left(Y_\nu^\dagger Y_\nu\right)_{ii} M_i = \frac{1}{8\pi v^2}\left(M_{\nu D}^\dagger M_{\nu D}\right)_{ii} M_i\,, \tag{9.8}$$

where $v$ is the *vev* of SM Higgs $\phi$ after electroweak symmetry breaking and $M_{\nu D}$ is the usual Dirac mass matrix for neutrinos.

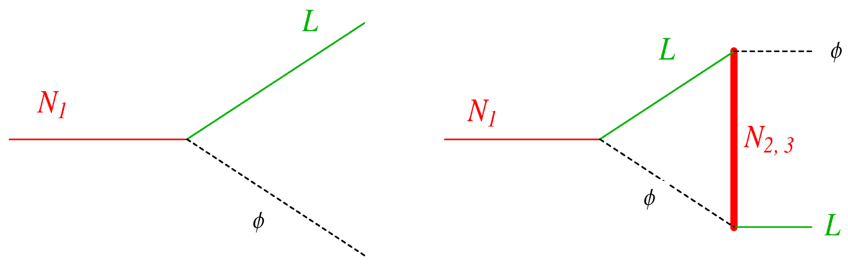


Figure 9.2: The tree-level and vertex decay diagrams [3].

In a special case of normal mass hierarchy ($M_1 << M_{2,3}$) in the right-handed neutrino sector the lepton asymmetry created by decay of $N_{2,3}$ is wiped out due to presence of $N_1$ and final lepton asymmetry is created only due to the decay of the lightest right-handed neutrino, $N_1$.

Now let us define the lepton asymmetry $Y_L$ similar to the baryon asymmetry as

$$Y_L = \frac{n_L - n_{\bar{L}}}{s}.$$

While $N_1$ is in thermal equilibrium with the background particles, $Y_L$ remains zero. As temperature of the universe drops below the mass of the $N_1$, it starts to decouple and then to decay generating lepton asymmetry. If the equilibrium number density per comoving volume of $N_1$ is $Y_N^0$ before out of equilibrium decay, the final lepton asymmetry, in crude sense, after the complete decay of the right-handed neutrinos would be given as

$$Y_L = \varepsilon_1 Y_N^0 d\,, \tag{9.9}$$

where $d$ is the dilution factor or efficiency factor which arises due to the competitions between decay rate $\Gamma_1$ and expansion rate of the universe $H$ at $T \simeq M_1$.

The equilibrium number density $Y_N^0$ before the decay is of order of $10^{-3}$ and dilution factor $d \simeq 0.1$. For SM, the $Y_B$ is related to $Y_L$ as

$$\begin{aligned} Y_B &= -0.55 Y_L \\ &= -0.55 \varepsilon_1 Y_N^0 d\,. \end{aligned} \tag{9.10}$$

Using the observed value of $Y_B$ as given in expression 9.2, above relation correspond to the following required value for $\varepsilon_1$

$$\varepsilon_1 \gtrsim 10^{-6} - 10^{-7}.$$

We will use the typical value for $\varepsilon$ to be $10^{-6}$ for some of our analysis. However, it is worth discussing CP violation in some detail.

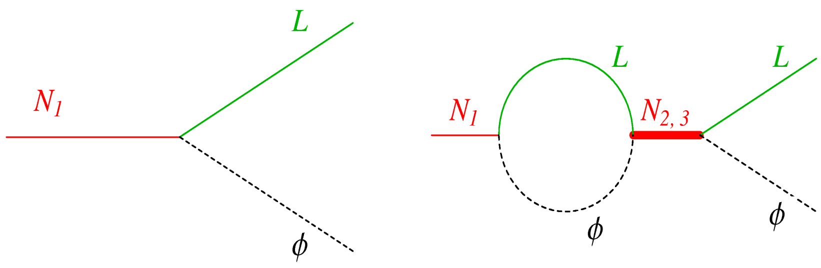


Figure 9.3: The tree-level and self-energy decay diagrams [3].

### 9.2.1 CP asymmetry

There are two independent sources of CP violation in the decay process of heavy right-handed neutrinos in type I seesaw scenario. The first one arises from the interference of tree-level and vertex diagram as shown in figure 9.2. This decay type source for CP violation was used for thermal leptogenesis in the initial literature [28, 73, 75, 136, 137]. The corresponding asymmetry factor can be calculated as [138]

$$\varepsilon_i = \frac{1}{8\pi}\frac{1}{\left(Y_\nu^\dagger Y_\nu\right)_{ii}}\sum_k \mathrm{Im}\left[\left(Y_\nu^\dagger Y_\nu\right)^2_{ik}\right] f\left(\frac{M_k^2}{M_i^2}\right),$$

where

$$f(x) = \sqrt{x}\left[1-(1+x)\ln\left(\frac{1+x}{x}\right)\right].$$

The another source of CP violation [76, 77] comes from the interference of tree-level decay diagram of right-handed neutrinos with self-energy diagram as shown in figure 9.3. The two heavy neutrinos in the self-energy diagram belong to two different generations and so CP violation is essentially coming from the Majorana mass matrix of right-handed neutrinos [76, 77, 139–142]. This oscillation type source of CP violation was first studied in [76, 77] to generate the lepton asymmetry .

The corresponding asymmetry factor can be estimated as

$$\varepsilon_i = \frac{1}{8\pi}\frac{1}{\left(Y_\nu^\dagger Y_\nu\right)_{ii}}\sum_k \mathrm{Im}\left[\left(Y_\nu^\dagger Y_\nu\right)^2_{ik}\right] g\left(\frac{M_k^2}{M_i^2}\right),$$

where

$$g(x) = \frac{\sqrt{x}}{1-x}.$$

For the comparison between the two sources of CP violation we just take only two heavy right-handed neutrinos. The only uncommon factors in the expression

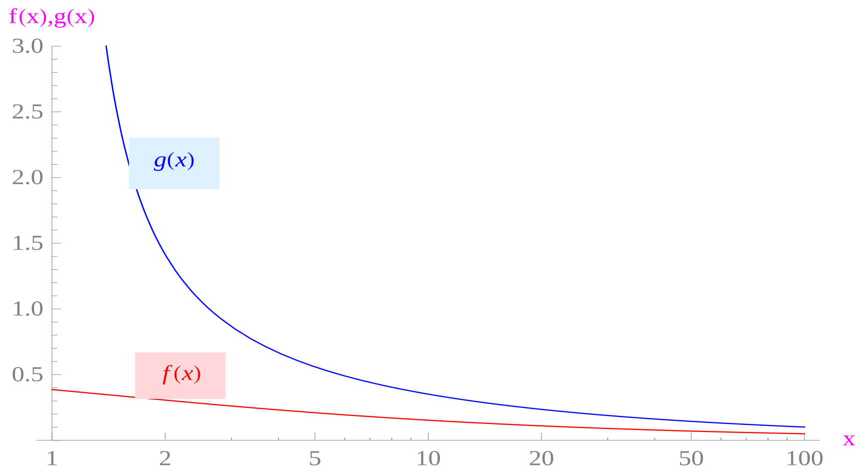


Figure 9.4: The comparison of $f(x)$ and $g(x)$ as a function of $x = (M_2/M_1)^2$

of CP parameter due to the two sources are the function $f(x)$ and $g(x)$, where $x = (M_2/M_1)^2$. So we plot the two functions of $x$ in figure 9.4 to compare the CP asymmetry factors from the two sources. When the mass ratio of the two heavy neutrinos is large the two CP asymmetry factors are comparable. However, for almost degenerate heavy neutrinos, the self-energy contribution to the CP violation is very large leading to the resonance effect. Consequently the amount of lepton asymmetry generated through the self-energy diagram can be many orders of magnitude larger than the asymmetry generated trough the vertex diagram. This way of resonant production of lepton asymmetry is called as resonant leptogenesis [76, 77, 139–142].

Now the total CP asymmetry will be determined by the sum of the CP asymmetry coming from the two independent sources.

For the hierarchical structure of right-handed neutrino masses ,i.e. for $M_1 << M_{2,3}$ , the only relevant total asymmetry factor is $\varepsilon_1$ (not all $\varepsilon_i$'s) which can be approximated as

$$\varepsilon_1 = -\frac{3}{16\pi}\frac{1}{\left(Y_\nu^\dagger Y_\nu\right)_{11}}\sum_k \mathrm{Im}\left[\left(Y_\nu^\dagger Y_\nu\right)^2_{1k}\right]\frac{M_1}{M_k}. \tag{9.11}$$

This expression will be used in the next chapter where we consider type I seesaw scenario with two right-handed neutrinos with hierarchical mass structure.

### 9.2.2 Basic leptogenesis mechanism

In this subsection, we will study the basic mechanism of leptogenesis. The time evolution of the number density per comoving volume $Y_N$ of the lightest heavy Majorana neutrino and the corresponding lepton asymmetry $Y_L$ generated in the expanding universe are determined by solving full set of Boltzmann equations. The main processes that can affect the $Y_L$ in the thermal bath include

1. Decay of $N_1$

$$N_1 \to \ell + \phi \qquad N_1 \to \overline{\ell} + \overline{\phi}\,.$$

2. Inverse decay of $N_1$

$$\ell + \phi \to N_1 \qquad \overline{\ell} + \overline{\phi} \to N_1\,.$$

3. Higgs mediated 2-2 scattering $(\triangle L = 1)$

$$\begin{array}{llll} N_1\,\ell \longleftrightarrow \overline{t}\,Q & , & N_1\,\overline{\ell} \longleftrightarrow t\,\overline{Q} & (\mathrm{s-channel}) \\ N_1\,t \longleftrightarrow \overline{\ell}\,Q & , & N_1\,\overline{t} \longleftrightarrow \ell\,\overline{Q} & (\mathrm{t-channel}) \\ N_1\,Q \longleftrightarrow \ell\,t & , & N_1\,\overline{Q} \longleftrightarrow \overline{\ell}\,\overline{t} & (\mathrm{u-channel})\,. \end{array}$$

4. $N$ mediated 2-2 scattering $(\triangle L = 2)$

$$\begin{array}{llll} \ell\phi \longleftrightarrow \overline{\ell\phi} & , & & (\mathrm{s\,and\,t-channel}) \\ \ell\ell \longleftrightarrow \overline{\phi}\,\overline{\phi} & , & \overline{\ell\ell} \longleftrightarrow \phi\phi & (\mathrm{t\,and\,u-channel})\,. \end{array}$$

Now just to illustrate a simple mechanism of leptogenesis, we will only consider the processes involving decay and inverse decay. The on-shell contribution of the s-channel of $N_1$ mediated $2-2$ scattering has also been included for consistency. We shall see that the decay and the inverse decay together are able to describe qualitatively many features of the full solution. The final Boltzmann equation governing the evolution of $Y_N$ and $Y_L$ takes the following form

$$\frac{dY_N(z)}{dz} = -D(z)\,(Y_N(z) - Y_N^{eq}(z))\,, \tag{9.12}$$

$$\frac{dY_L(z)}{dz} = \varepsilon_1\,D(z)\,(Y_N(z) - Y_N^{eq}(z)) - W_{ID}(z)\,Y_L(z)\,, \tag{9.13}$$

where we have defined $z = M_1/T$ . In this chapter, we will use the notation $z$ for this definition only instead of earlier definition for the redshift used in the chapter 7
.

$Y_N^{eq}$ is the equilibrium number density per comoving volume of $N_1$ and is given as

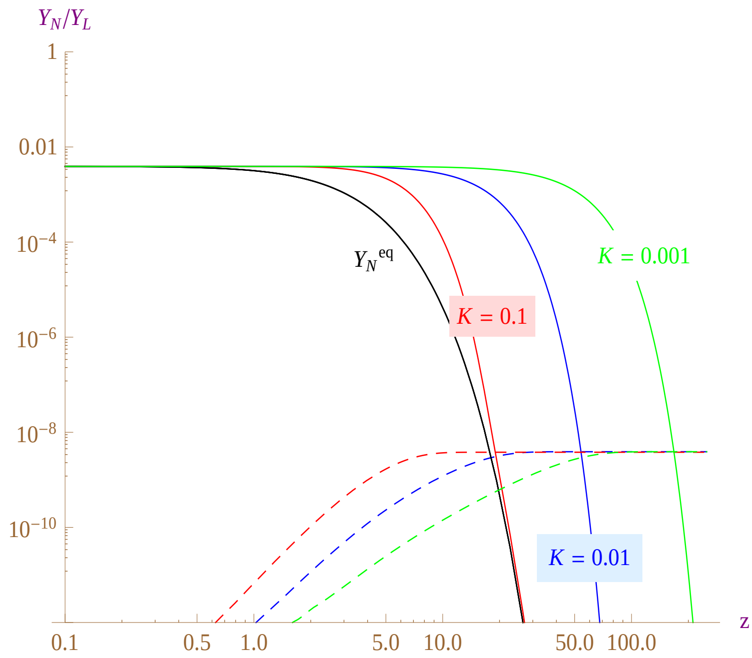


Figure 9.5: Evolution of $Y_N$ and $Y_L$ for the thermal initial abundance of $N_1$

$$
\begin{aligned}
Y_N^{eq} &= \frac{n_N^{eq}}{s} = \frac{\left(\frac{3\zeta(3)}{4\pi^2} M_1^2 T \mathcal{K}_2(M_1/T)\right)}{\left(\frac{2\pi^2}{45} g_{*s} T^3\right)} \\
&= \frac{45}{2\pi^4 g_{*s}} \frac{3\zeta(3)}{4} z^2 \mathcal{K}_2(z)
\end{aligned}
$$

where $\mathcal{K}_\gamma(z)$ is the modified Bessel function of the second kind. $D(z)$ accounts for strength of decay and inverse decay processes and is given as

$$
D(z) = zK \frac{\mathcal{K}_1(z)}{\mathcal{K}_2(z)},
$$

where parameter $K$ is a measure of how fast the decay rate is in comparison with the expansion rate of the universe at temperature $T = M_1$

$$
K = \frac{\Gamma_{D_1}}{H(M_1)}. \tag{9.14}
$$

The $W_{ID}$ term determines the washout amount of the lepton asymmetry created due to the decay of $N_1$

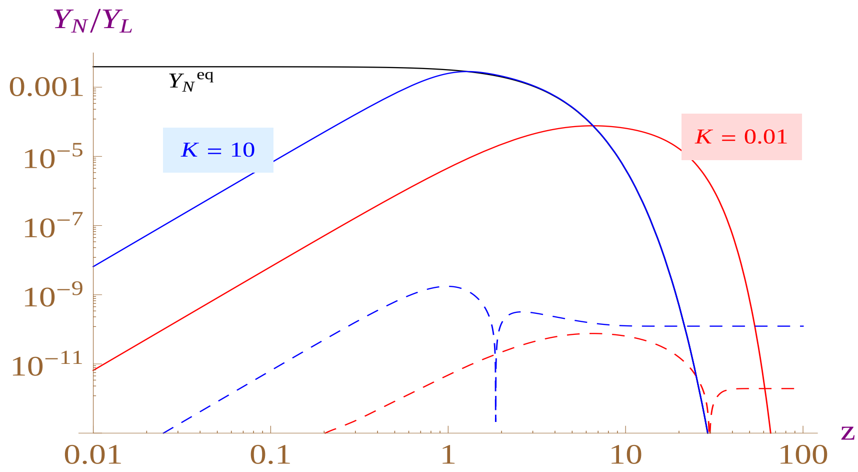


Figure 9.6: Evolution of $Y_N$ and $|Y_L|$ for the zero initial abundance of $N_1$

$$W_{ID}(z) = \frac{1}{2}\frac{Y_N^{eq}}{Y_\ell^{eq}} D(z) = \frac{1}{4} z^3 K \mathcal{K}_1(z)\,,. \tag{9.15}$$

Now let us discuss the numerical solution of the set of equations 9.12 and 9.13 with two kinds of typical initial conditions

1. Thermal initial abundance of $N_1$, i.e., at $z \ll 1$, $Y_N = Y_N^{eq}$ and $Y_L = 0$,
2. Zero initial abundance of $N_1$, i.e., at $z \ll 1$, $Y_N = 0$ and $Y_L = 0$.

The numerical solution for the thermal initial abundance of $N_1$ is shown in figure 9.5 for different values of parameter $K$. The black curve correspond to the equilibrium value of $Y_N$. The $Y_N$ evolution (solid curve) goes far away from the equilibrium as we choose smaller values for $K$. This is obviously expected as the decay of $N_1$ corresponding to the one with relatively smaller ratio of decay rate to expansion rate is more likely to run into out of equilibrium. The lepton asymmetry $Y_L$ (dashed curve) is simultaneously generated as the out of equilibrium decay of $N_1$ takes place. It is straight forward to observe that the final asymmetry comes out to be same for all the values taken for $K$. It is only true for small values of $K$. For large values of $K$, the final asymmetry will be moderately washed out simply because $N_1$ will remain in thermal equilibrium for longer time even at temperature far below $M_1$.

In the case of zero initial abundance of $N_1$, $Y_N$ is first generated from zero through the inverse decay and scattering until it reaches the equilibrium value. Afterward, it again starts decaying in out of equilibrium when temperature drops below its mass. The numerical solution corresponding to evolution of both $Y_N$ (solid curve) and $|Y_L|$ (dashed curve) have been shown in figure 9.6. If $K \gg 1$, both inverse decay and decay are strong and so thermalization is fast and $Y_N$ achieves its equilibrium value sooner at high temperature. However, if $K \ll 1$, $Y_N$ attains its state of thermal equilibrium late at relatively lower temperature and so a relatively smaller number density of $N_1$ per comoving volume is generated before it decays. Consequently, the generated lepton asymmetry due to out of equilibrium decay of $N_1$ is also lowered due to smaller abundance achieved by $N_1$.

The interesting feature with the second initial condition is that lepton asymmetry $Y_L$ starts generating even before $Y_N$ achieves its equilibrium value. Simple reason is that the CP violating effects are also present in the inverse decay process of leptons, i.e., in the creation of $N_1$. consequently, a lepton asymmetry is created in the direction opposite to the one created due the decay of $N_1$. Once $Y_N$ achieves its equilibrium value, further creation of $Y_L$ in opposite direction stops and then washed out when subsequent out of equilibrium decay of $N_1$ starts taking place. However, lepton asymmetry created by decay can overcome the one created by inverse decay with some final net lepton asymmetry after cancellation. Now for $K \gg 1$, the decay of $N_1$ is fast and $Y_N$ remains close to the equilibrium value while for $K \ll 1$, decay is far out of equilibrium which is similar to the behavior as in the case of thermal initial abundancy.

### 9.2.3 Lower bound on $M_1$

The lower bound on $M_1$ for sufficient leptogenesis comes from Davidson-Ibarra bound on $|\varepsilon_1|$ [143]. For a hierarchical structure of heavy Majorana neutrino masses and normal hierarchy for light neutrinos, the bound is given as

$$|\varepsilon_1| \lesssim \frac{3}{16\pi}\frac{M_1\,(m_3 - m_1)}{v^2}\,.$$

Using relation 9.10, one can convert the the upper bound on $|\varepsilon_1|$ to the lower bound on $M_1$ as follows

$$\begin{aligned} M_1 &\gtrsim \frac{16\pi v^2}{3\sqrt{\triangle m_{31}^2}}\left(\frac{Y_B}{0.55\,Y_N^0}\right)\left(\frac{1}{d}\right) \\ &\gtrsim 2.06\times 10^{16}\left(\frac{8.61\times 10^{-11}}{0.55\times 3.9\times 10^{-3}}\right)\left(\frac{1}{d}\right)\text{GeV} \\ &\gtrsim 8.25\times 10^{8}\,\text{GeV}\left(\frac{1}{d}\right). \end{aligned}$$

As the dilution or efficiency factor can be utmost of order 1, the bound on $M_1$ can be approximately given as

$$M_1 \gtrsim 10^9\text{GeV}\,.$$

So the above bound should be satisfied in order to generate sufficient amount of baryon asymmetry in our universe. The bound remains same for the inverted hierarchical structure of light neutrino masses.

# Chapter 10

# Connecting leptogenesis to low energy CP violation

The goal of the present neutrino oscillation experiments is to determine the nine degrees of freedom in the low energy neutrino mass matrix. They are parametrized by three masses, three mixing angles and three CP violating phases out of which two are Majorana and one is Dirac. At present the neutrino oscillation experiments able to measure the two mass square differences, the solar and the atmospheric, and three mixing angles with varying degrees of precision, while there is no information about the phases.

In the present chapter we limit ourselves to the case of type I seesaw models. Although we call them right-handed neutrinos, in the extensions of the SM they are just singlet fermions that transform trivially under the SM gauge group. So, there is no apparent reasons for the number of heavy singlet neutrinos to be same as the number of left-handed neutrinos. So, for the main part of our discussions we restrict ourselves to only two right-handed neutrinos. These results will also be true when there are three right-handed neutrinos, but the third right-handed neutrino do not mix with the other two neutrinos. We start with three right-handed neutrinos and after some general comments work mostly with two right-handed neutrinos.

While there is no information about the absolute mass scales of the physical neutrinos, the currently discovered tiny mass scales; the atmospheric neutrino mass ($\Delta_{atm} = \sqrt{|m_3^2 - m_2^2|}$) in the $\nu_\mu - \nu_\tau$ oscillation and the solar neutrino mass ($\Delta_\odot = \sqrt{m_2^2 - m_1^2}$) in the $\nu_e - \nu_\mu$ oscillation, can be explained by adding at least two right-handed neutrinos to the SM Lagrangian. However, with two right-handed neutrinos the seesaw mechanism predicts one of the physical light neutrino mass to be exactly zero which is permissible within the current knowledge of neutrino masses and mixing.

As discussed in previous chapter, the Majorana mass of the right-handed neutrino violates $L$-number and hence is a natural source of lepton-asymmetry in the early Universe [28]. A partial lepton-asymmetry is then converted to baryon asymmetry through the non-perturbative sphaleron processes, unsuppressed above the electroweak phase transition.

It is legitimate to ask if there are any connecting links between leptogenesis and the CP violation in the low energy leptonic sector, in particular neutrino oscillation and neutrinoless double beta decay. In the context of three right-handed neutrino models several attempts have been taken in the literature to connect the CP violation in leptogenesis and neutrino oscillations [60, 61, 144, 145]. It is found that there are almost no links between these two phenomena unless one considers special assumptions [146, 147]. In fact it is shown that leptogenesis can be possible irrespective of the CP violation at low energy [148]. On the other hand, in the two right-handed neutrino models there is a ray of hope connecting leptogenesis with the CP violation in neutrino oscillation [149, 150] and neutrinoless double beta decay processes.

While the magnitude of CP violation is fairly known in the quark sector, it is completely shaded in the leptonic sector of the SM. Therefore, searching for CP violation in the leptonic sector is of great interest in the present days. It has been pointed out that the Dirac phase, being involved in the L conserving processes, can be measured in the long baseline neutrino oscillation experiments [151–154], while the Majorana phase, being involved in the L violating processes, can be investigated in the neutrinoless double beta decay [155, 156] processes.

At present the magnitude of baryon-asymmetry is precisely known, while the sign of this asymmetry is not known yet. However, by knowing the CP violating phases in the leptonic mixing matrix one can determine the sign of the baryon-asymmetry. This is the study taken up in this work. We consider a minimal extension of the SM by including two singlet right-handed neutrinos which are sufficient to explain the present knowledge of neutrino masses and mixings. We adopt a general parametrization of the neutrino Dirac Yukawa coupling and give the possible links between the CP violation in leptogenesis and neutrino oscillation, CP violation in neutrinoless double beta decay and leptogenesis. It is shown that the knowledge of low energy CP violating rephasing invariants can indeed determine the sign of the baryon-asymmetry since the size of this asymmetry is known precisely.

## 10.1 Type I seesaw and parameter counting

In this section we discuss about number of independent parameters both magnitude and phases in type I seesaw framework. The write the corresponding part of the

Lagrangian from the section

$$\mathcal{L} - \left(\frac{1}{2}\overline{(N_{R\alpha})^c}(M_R)_{\alpha\beta}N_{R\beta} + \overline{\ell_{Li}}\phi(Y_e)_{ij}\ell_{Rj} + \overline{\ell_{Li}}\tilde{\phi}(Y_\nu)_{i\alpha}N_{R\alpha} + H.C.\right) \tag{10.1}$$

where notation are similar to one in the section. The low energy neutrino mass matrix comes out to be

$$m_\nu = -M_{\nu D}M_R^{-1}M_{\nu D}^T\,, \tag{10.2}$$

Without loss of generality we consider $M_R$ to be diagonal and in this basis $M_{\nu D}$ contains rest of the physical parameters that appears in $m_\nu$.

As discussed in section , the diagonalization of $m_\nu$, through the lepton flavor mixing matrix $U_{PMNS}$ [12–14], gives us three masses of the physical neutrinos. Its eigenvalues are given by

$$m_\nu^{Diag} \equiv \text{diag.}(m_1, m_2, m_3) = U_{PMNS}^\dagger m_\nu U_{PMNS}^*\,, \tag{10.3}$$

where the masses $m_i$ are real and positive. The standard PDG parametrization of the PMNS mixing matrix $U_{PMNS}$ are described in the section . Let us write it again for convenience

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} .U_{ph}\,,$$

where notations are similar as in section. The two physical phases $\eta$ and $\xi$, present in $U_{ph}$, associated with the Majorana character of neutrinos are not relevant for neutrino oscillations. Thus we see that there are three phases in the low energy effective theory responsible for CP violation. However, these phases may not give rise to CP violation at high energy regime, in particular, leptogenesis to our interest. In the following we study this in the framework of three and than two right-handed neutrino models.

In general if $n$ and $n'$ are the number of generations of the left and right-handed neutrinos that take part in the seesaw then the total number of parameters in the effective theory is estimated to be [157]

$$N_{\text{moduli}} = n + n' + nn' , \quad (10.4)$$

$$N_{\text{phase}} = n(n'-1) . \quad (10.5)$$

For $n = 3$ and $n' = 3$, $N_{\text{moduli}} = 15$ and $N_{\text{phase}} = 6$, which in the effective theory manifests as three masses of charged leptons, three masses of right-handed neutrinos and remaining 15 parameters including nine moduli and six phases in the Dirac mass matrix $M_{\nu D}$ in a basis where the charged lepton mass matrix is real and diagonal. equation

In the bi-unitary parametrization the mass matrix $M_{\nu D}$ can be given as

$$M_{\nu D} = U_L^\dagger m_D^{diag} U_R , \quad (10.6)$$

where $U_L$ and $U_R$ are $3\times 3$ unitary matrices. $U_L$ diagonalizes the left-handed sector while $U_R$ is the diagonalizing matrix of $M_{\nu D}^\dagger M_{\nu D}$. Any arbitrary $3\times 3$ unitary matrix $U'$ can be written as

$$U' = e^{i\varphi} P_1 \widetilde{U} P_2 , \quad (10.7)$$

where $\varphi$ is an overall phase and

$$P_1 = \text{diag.}(1, e^{-i\alpha_1}, e^{-i\alpha_2}) , \quad (10.8)$$

$$P_2 = \text{diag.}(1, e^{-i\beta_1}, e^{-i\beta_2}) , \quad (10.9)$$

are phase matrices. $\widetilde{U}$ is a CKM like matrix parametrized by three angles and one embedded phase. Now using equation 10.7 in equation 10.6 we get

$$M_{\nu D} = e^{i(-\varphi_L+\varphi_R)} P_{2L}^\dagger \widetilde{U}_L^\dagger P_{1L}^\dagger m_D^{diag} P_{1R} \widetilde{U}_R P_{2R} . \quad (10.10)$$

Without loss of generality three of the left phases can be absorbed in the redefinition of charged lepton fields. As a result the effective Dirac mass matrix turns out to be

$$M_{\nu D} = \widetilde{U}_L^\dagger P_3 m_D^{diag} \widetilde{U}_R P_{2R} , \quad (10.11)$$

where $P_3 = P_{1L}^\dagger P_{1R}$ is an effective phase matrix. Thus in the models with three right-handed neutrinos $M_{\nu D}$ contains 15 parameters.

In leptogenesis, the CP asymmetry comes in a form $M_{\nu D}^\dagger M_{\nu D}$, which contains $P_{2R}$ and $\widetilde{U}_R$, *i.e.*,

$$M_{\nu D}^\dagger M_{\nu D} = P_{2R}^\dagger \widetilde{U}_R^\dagger (m_D^{diag})^2 \widetilde{U}_R P_{2R} , \quad (10.12)$$

and hence is independent of $P_3$ and $\widetilde{U}_L$. Although it would be good to know the exact relationship of the phases in $P_{2R}$ and $\widetilde{U}_R$ with the phases appearing in the $U_{PMNS}$ matrix but that is not possible. So, we try with some special cases.

**Case-I:** Let us first consider the case, when $\widetilde{U}_R$ is a diagonal matrix. This is the case when the right-handed neutrino Majorana mass matrix is diagonal to start with. The mass matrix can still contain Majorana phases. In that case, $\widetilde{U}_R$ and $m_D^{diag}$ will commute and hence $M_{\nu D}^\dagger M_{\nu D}$ will be real and there will not be any leptogenesis. This already tells us that the phases in leptogenesis crucially depends on the mixing of the right-handed physical neutrinos. Even in this case there will be CP violation at low energy as we shall see below. The light neutrino mass matrix is given by

$$m_\nu = -\widetilde{U}_L^\dagger (P_3)^2 (\widetilde{U}_R)^2 (P_{2R})^2 (m_D^{diag})^2 M_R^{-1} \widetilde{U}_L^*$$

so that the PMNS matrix will become

$$U_{PMNS} = \widetilde{U}_L^\dagger P_3 P_{2R} \,.$$

Thus both the Dirac and Majorana phases at low energy are non-vanishing.

**Case-II:** We shall now consider another special case when there is no leptogenesis. If the diagonal Dirac neutrino mass matrix is proportional to a unit matrix, i.e., $m_D = m \cdot I$ ($I$ is the identity matrix), again there is no leptogenesis,

$$M_{\nu D}^\dagger M_{\nu D} = m^2 \cdot I \,.$$

In this case the light neutrino mass matrix becomes

$$m_\nu = -\widetilde{U}_L^\dagger P_3 \widetilde{U}_R P_{2R} m^2 M_R^{-1} P_{2R} \widetilde{U}_R^T P_3 \widetilde{U}_L^* \,,$$

so that the PMNS matrix can be read off to be

$$U_{PMNS} = \widetilde{U}_L^\dagger P_3 \widetilde{U}_R P_{2R} \,.$$

Even in this case the Dirac and Majorana phases are present.

Thus in both these examples, even if CP violation is observed at low energy neutrino experiments, this CP violation may not be related to leptogenesis. Since it is not possible to make any further progress with three heavy neutrinos, we shall now restrict ourselves to models with two heavy neutrinos.

## 10.2 Parametrization of $M_{\nu D}$ in two right-handed neutrino models

From now on we shall work with only two right-handed neutrinos. This result will be applicable when there are only two heavy neutrinos or when there are three heavy neutrinos but one of them do not mix with others and heavier than the other right-handed neutrinos and hence its contribution to the light neutrinos is also negligible. In the present case where we have $n = 3$ and $n' = 2$, from equation 10.4 and 10.5, we get $N_{\text{moduli}} = 11$ and $N_{\text{phase}} = 3$. The 14 parameters in the effective theory manifest them as three masses of charged leptons, two masses of right-handed neutrinos and remaining nine parameters including six moduli and three phases appear in the Dirac mass matrix $M_{\nu D}$.

There are various textures and their phenomenological implications of $M_{\nu D}$ in the two right-handed neutrino models that have been considered in the literature [158–160]. In this chapter a general parametrization of the $3 \times 2$ mass matrix of the Dirac neutrinos is considered. This is given by

$$M_{\nu D} = vY_\nu = vU'Y_{2RH}\,, \tag{10.13}$$

where $U'$ is an arbitrary Unitary matrix and the Yukawa coupling of the two right-handed neutrino model is given as

$$Y_{2RH} = \begin{pmatrix} 0 & x \\ z & ye^{-i\theta} \\ 0 & 0 \end{pmatrix}. \tag{10.14}$$

A derivation of equation 10.14 is given in the appendix **??**. However, we declare that the texture of $Y_{2RH}$ is not unique. By choosing appropriately the $U$ matrix one can place $x, y, z$ at different positions so as to get the different textures of $Y_{2RH}$ . Using 10.7 in equation 10.13 we get

$$M_{\nu D} = v\widetilde{U}P_2Y_{2RH}\,, \tag{10.15}$$

where $\widetilde{U}$ contains four parameters including three moduli and one phase, $P_2$ contains two phases and $Y_{2RH}$ contains four parameters including three moduli and one phase which all together makes ten parameters in $M_{\nu D}$. However, by multiplying the phase matrix $P_2$ with $Y_{2RH}$ one can see that one of the phases in the phase matrix $P_2$, i.e., $\beta_2$ becomes redundant and can be dropped without loss of generality. As a result the total number of effective parameters is actually nine and hence consistent with our counting.

Substituting $M_{\nu D}$, given by equation 10.15, in equation 11.9 we can calculate the effective neutrino mass matrix, $m_\nu$. The diagonalization of $m_\nu$, through the lepton flavor mixing matrix $U_{PMNS}$ , then gives us two non-zero masses of the physical neutrinos while setting one of the mass to be exactly zero as shown in the following section.

## 10.3 Neutrino masses and mixings in two right-handed neutrino models

The unitary matrix $\widetilde{U}$, appearing in equation 10.15, can be parametrized as [1]

$$\widetilde{U} = R_{23}(\Theta_{23})R_{13}(\Theta_{13},\delta'_{13})R_{12}(\Theta_{12})\,. \tag{10.16}$$

It turns out that this parametrization is useful in determining the leptonic mixing matrix in two right-handed neutrino models. Now from equations. 11.9 and 10.15 we get the effective neutrino mass matrix to be

$$\begin{aligned} m_\nu &= -v^2\widetilde{U}P_2Y_{2RH}M_R^{-1}Y_{2RH}^T P_2\widetilde{U}^T \\ &= -v^2\widetilde{U}P_2XP_2\widetilde{U}^T\,, \end{aligned} \tag{10.17}$$

where

$$X = Y_{2RH}M_R^{-1}Y_{2RH}^T\,. \tag{10.18}$$

For simplicity of the calculation let us take $e^{-i\theta}$ common from 2nd row of $Y_{2RH}$ matrix given by equation 10.14 and absorb it in $P_2$ by redefining $\beta_1$ as $(\beta_1+\theta)\to\beta_1$. As a result opposite phase will reappear with $z$. Then the matrix $Y_{2RH}$ turns out to be

$$Y_{2RH} = \begin{pmatrix} 0 & x \\ ze^{i\theta} & y \\ 0 & 0 \end{pmatrix}. \tag{10.19}$$

Using equation 10.19 in the above equation 10.18 we get

$$X = \begin{pmatrix} \frac{x^2}{M_2} & \frac{xy}{M_2} & 0 \\ \frac{xy}{M_2} & \frac{y^2}{M_2}+\frac{z^2e^{2i\theta}}{M_1} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{10.20}$$

[1] This parametrization is usually used for determining the leptonic mixing matrix in the PDG parametrization. Here we have used it for parametrizing $m_D$.

In writing the above equation we have used a diagonal basis of the right-handed neutrinos where $M_R = \text{diag.}(M_1, M_2)$. For simplicity, we absorb $M_1$ and $M_2$ in $x, y$ and $z$ as $\frac{x}{\sqrt{M_2}} \to a$, $\frac{y}{\sqrt{M_2}} \to b$ and $\frac{z}{\sqrt{M_1}} \to c$. So $X$ can be rewritten as:

$$X = \begin{pmatrix} a^2 & ab & 0 \\ ab & b^2 + c^2 e^{2i\theta} & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{10.21}$$

Looking to the effective neutrino mass matrix as given by equation 10.17 we can guess that the diagonalizing matrix would be of the form

$$U_{PMNS} = \widetilde{U} K_u \,, \tag{10.22}$$

where $K_u$ is an unitary matrix. Using equations 10.3 and 10.22 in equation 10.17 we see that

$$m_\nu^{Diag} = -K_u^\dagger P_2 X P_2 K_u^* \,, \tag{10.23}$$

which implies that $K_u$ would diagonalize the matrix $P_2 X P_2$. From the structure of $X$ it is clear that one of the light physical neutrinos must be massless. The matrix $K_u$ can be parametrized as

$$K_u = P_2 R_{12}(\omega, \phi) P \,, \tag{10.24}$$

where $P = \text{diag.}(e^{i\eta_1/2}, e^{i\eta_2/2}, 1)$ and

$$R_{12}(\omega, \phi) = \begin{pmatrix} \cos\omega & e^{i\phi}\sin\omega & 0 \\ -e^{-i\phi}\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{10.25}$$

with

$$\tan 2\omega = \left[ \frac{2ab\left(a^4 + b^4 + c^4 + 2a^2b^2 + 2b^2c^2\cos 2\theta + 2c^2a^2 cos2\theta\right)^{1/2}}{(-a^4 + b^4 + c^4 + 2b^2c^2\cos 2\theta)} \right] \tag{10.26}$$

and

$$\tan\phi = \left[ \frac{-c^2 \sin 2\theta}{a^2 + b^2 + c^2\cos 2\theta} \right] . \tag{10.27}$$

Since $R_{12}(\omega, \phi)$ diagonalizes the matrix $X$ the resulting diagonal matrix will have complex eigenvalues in general. However, by choosing appropriately the phases of $P$ one can make the eigenvalues of $X$ real. Using equations 10.26 and 10.27 we get the eigenvalues $\{\lambda_1, \lambda_2, \lambda_3\}$ of $X$ to be

$$\lambda_1 = a^2 - abe^{i\phi}\tan\omega\,, \quad \lambda_2 = e^{-2i\phi}(a^2 + abe^{i\phi}\cot\omega) \quad \text{and} \quad \lambda_3 = 0 \tag{10.28}$$

The absolute masses of the physical neutrinos are then given by $\{m_1 = v^2|\lambda_1|, m_2 = v^2|\lambda_2|, m_3 = 0\}$. The MSW effect in the solar neutrino oscillation experiments indicates that $m_2 > m_1$. The corresponding mass scale, giving rise to the $\nu_e - \nu_\mu$ oscillation, is given by

$$\Delta m^2_{\odot} \equiv m_2^2 - m_1^2 = v^4(|\lambda_2|^2 - |\lambda_1|^2)\,. \tag{10.29}$$

Using equation 10.28 in the above equation we get the solar neutrino mass scale to be

$$\begin{aligned} \Delta m^2_{\odot} &= v^4\left\{\left[(a^2+b^2)^2 + c^4 + 2b^2c^2\cos 2\theta\right]^2 - 4a^4c^4\right\}^{1/2} \\ &\simeq 8\times 10^{-5} eV^2\,. \end{aligned} \tag{10.30}$$

The atmospheric mass scale, on the other hand, is given by

$$\Delta m^2_{atm} \equiv |m_2^2 - m_3^2| = v^4(|\lambda_2|^2 - |\lambda_3|^2)\,. \tag{10.31}$$

Using equation 10.28 in the above equation we get the atmospheric mass scale to be

$$\begin{aligned} \Delta m^2_{atm} &= \frac{v^4}{2}\Big((a^2+b^2)^2 + c^4 + 2b^2c^2\cos 2\theta \\ &\quad + \left\{\left((a^2+b^2)^2 + c^4 + 2b^2c^2\cos 2\theta\right)^2 - 4a^4c^4\right\}^{1/2}\Big)\,, \\ &\simeq 2\times 10^{-3} eV^2\,. \end{aligned} \tag{10.32}$$

These equations may be inverted to obtain

$$\begin{aligned} v^4\left((a^2+b^2)^2 + c^4 + 2b^2c^2\cos 2\theta\right) &= 2\Delta m^2_{atm} - \Delta m^2_{\odot} \\ a^4c^4v^8 &= \Delta m^2_{atm}(\Delta m^2_{atm} - \Delta m^2_{\odot})\,. \end{aligned} \tag{10.33}$$

Using equations 10.9 and 10.25 in equation 10.24 we can rewrite the matrix $K$

as

$$K_u = R_{12}(\omega,\phi+\beta_1)P'$$
$$= \begin{pmatrix} \cos\omega & e^{i(\phi+\beta_1)}\sin\omega & 0 \\ -e^{-i(\phi+\beta_1)}\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (10.34)$$
$$\begin{pmatrix} e^{i\eta_1/2} & 0 & 0 \\ 0 & e^{i(\eta_2/2-\beta_1)} & 0 \\ 0 & 0 & e^{-i\beta_2} \end{pmatrix}. \quad (10.35)$$

Thus using equations 10.35 and 10.16 in equation 10.22 the PMNS matrix $U_{PMNS}$ is given as

$$U_{PMNS} = R_{23}(\Theta_{23})R_{13}(\Theta_{13},\delta'_{13})R_{12}(\Theta_{12})R_{12}(w,\phi+\beta_1)P', \quad (10.36)$$

where

$$R_{12}(\Theta_{12})R_{12}(\omega,\phi+\beta_1) = \begin{pmatrix} \cos\Theta'_{12}e^{i\rho_1} & \sin\Theta'_{12}e^{i\rho_2} & 0 \\ -\sin\Theta'_{12}e^{-i\rho_2} & \cos\Theta'_{12}e^{-i\rho_1} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} e^{i\left(\frac{\rho_1+\rho_2}{2}\right)} & 0 & 0 \\ 0 & e^{-i\left(\frac{\rho_1+\rho_2}{2}\right)} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\Theta'_{12} & \sin\Theta'_{12} & 0 \\ -\sin\Theta'_{12} & \cos\Theta'_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (10.37)$$
$$\begin{pmatrix} e^{i\left(\frac{\rho_1-\rho_2}{2}\right)} & 0 & 0 \\ 0 & e^{-i\left(\frac{\rho_1-\rho_2}{2}\right)} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (10.38)$$

In the above equation we have

$$\cos 2\Theta'_{12} = \cos 2\omega\cos 2\Theta_{12} - \cos(\phi+\beta_1)\sin 2\omega\sin 2\Theta_{12}, \quad (10.39)$$
$$\sin(\rho_2-\rho_1) = \sin(\phi+\beta_1)\tan\omega\left[\cot 2\Theta'_{12} + \frac{\cos 2\Theta_{12}}{\sin 2\Theta'_{12}}\right], \quad (10.40)$$
$$\sin(\rho_1+\rho_2) = \frac{\sin 2\omega\sin(\phi+\beta_1)}{\sin 2\Theta'_{12}}. \quad (10.41)$$

For further simplification of the PMNS matrix 10.36 we now compute the matrix product $R_{12}(\Theta_{12})K_u = R_{12}(\Theta_{12})R_{12}(\omega, \phi+\beta_1)P'$ which is given as

$$R_{12}(\Theta_{12})R_{12}(\omega,\phi+\beta_1)P' = e^{i(\frac{\eta_1}{2}-\rho_2)}\begin{pmatrix} e^{i(\rho_1+\rho_2)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\Theta'_{12} & \sin\Theta'_{12} & 0 \\ -\sin\Theta'_{12} & \cos\Theta'_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i(\rho_2-\rho_1+(\eta_2-\eta_1)/2-\beta_1)} & 0 \\ 0 & 0 & e^{-i(\beta_2-\rho_2+\frac{\eta_1}{2})} \end{pmatrix} \quad (10.42)$$

Using equation 10.42 in equation 10.36 the $U_{PMNS}$ matrix can be rewritten as:

$$\begin{aligned} U_{PMNS} &= \tilde{U}K_u \\ &= R_{23}(\Theta_{23})R_{13}(\Theta_{13},\delta_{13})R_{12}(\Theta'_{12}) \\ &\quad \text{diag.}(1, e^{i(\rho_2-\rho_1+(\eta_2-\eta_1)/2-\beta_1)}, e^{-i(\beta_2-\rho_2+\frac{\eta_1}{2})}) \\ &= U\,.\,K_P\,, \end{aligned} \quad (10.43)$$

where $U$ is the CKM like matrix and $K_P$ is the Majorana phase matrix. The effective $CP$ violating phase in the $V$ matrix is given by

$$\delta_{13} = \delta'_{13} + (\rho_1+\rho_2)\,. \quad (10.44)$$

Note that in writing equation 10.43 the overall phase $e^{i(\frac{\eta_1}{2}-\rho_2)}$ has been taken out. Moreover, we absorb the unphysical phase matrix diag.$(1, e^{-(\rho_1+\rho_2)}, e^{-(\rho_1+\rho_2)})$ into the redefinition of charged lepton fields. From equations 3.8, 10.16 and 10.44 we see that, for the chosen parametrization of $Y_{2RH}$, two of the mixing angles $\Theta_{23}$ and $\Theta_{13}$ remains same as of the $(2-3)$ and $(1-3)$ mixing angles in PDG parametrization of the leptonic mixing matrix. Thus we can write $\Theta_{23} \equiv \theta_{23}$ and $\Theta_{13} \equiv \theta_{13}$. Since $\Theta_{12}$ gets modified to $\Theta'_{12}$ , we can write $\Theta'_{12} = \theta_{12}$. Moreover, the modified CP violating phase $\delta_{13}$ is given by equation 10.44. We will the global fit values for the neutrino mixing angles as listed in 3.9 for our analysis.

## 10.4 Leptogenesis in two right-handed neutrino models

We have discussed leptogenesis in type I seesaw scenario in the previous chapter with three right-handed neutrinos. However, the discussion remains same for two right-handed neutrinos . We assume a normal mass hierarchy ($M_1 << M_2$) in the right-handed neutrino sector then the final lepton-asymmetry is given by the decay of the lighter right-handed neutrino, $N_1$. Using the expression 9.11 for CP asymme-

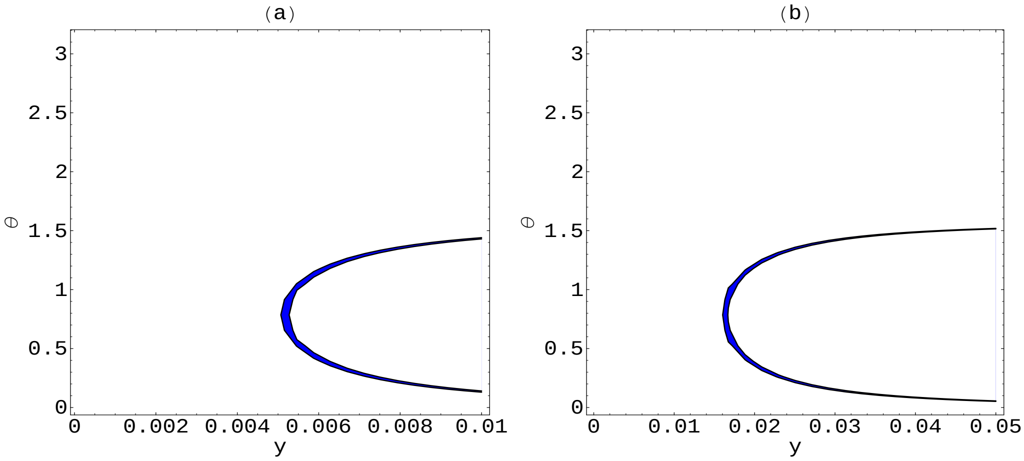


Figure 10.1: The allowed values of $y$ are shown against θ (in rad) for the observed matter antimatter asymmetry, given by equation **??**, with (a)$\frac{M_1}{M_2} = 0.1$ and (b)$\frac{M_1}{M_2} = 0.01$.

try parameter, arising from the decay of $N_1$, is then given by

$$\varepsilon_1 = \frac{-3}{16\pi v^2}\left(\frac{M_1}{M_2}\right)\frac{\mathrm{Im}[(M^{\dagger}_{\nu D}M_{\nu D})_{12}]^2}{(M^{\dagger}_{\nu D}M_{\nu D})_{11}}. \tag{10.45}$$

Using equations 10.15 and 10.14 in the above equation 10.45 we get

$$\varepsilon_1 = \frac{-3}{16\pi}\left(\frac{M_1}{M_2}\right)y^2\sin 2\theta. \tag{10.46}$$

From the above equation 10.46 it is clear that if $\theta = 0$ then there is no CP violation in leptogenesis. Therefore, θ can be thought of the phase associated with $M_i$ in a basis where $M_i$'s are complex. Moreover, θ always hangs around $y$. So $y = 0$ implies no leptogenesis. We will discuss more about later while we compare the CP violation in leptogenesis, neutrino oscillation and neutrinoless double beta decay processes.

Now using the relation 9.9 in the previous chapter, we can write the final lepton asymmetry generated as

$$Y_L = -5.97\times 10^{-5}\frac{M_1}{M_2}\left(\frac{Y_N^0 d}{10^{-3}}\right)y^2\sin 2\theta. \tag{10.47}$$

A part of the lepton-asymmetry is then transferred to the baryon-asymmetry via the sphaleron processes which are unsuppressed above the electroweak phase transition. Taking into account the particle content in the $SM$, the relation between baryon and lepton-asymmetries are described in expression 9.10. Using this expression, we can write

$$Y_B \simeq 3.28\times 10^{-5}\frac{M_1}{M_2}\left(\frac{Y_N^0 d}{10^{-3}}\right)y^2\sin 2\theta. \tag{10.48}$$

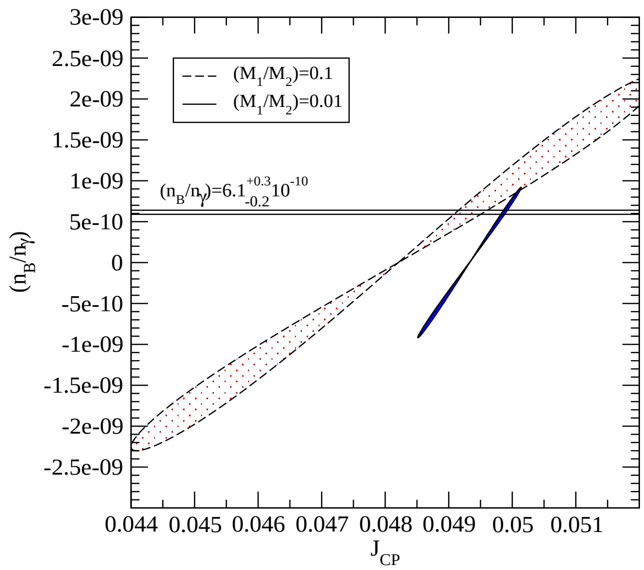


Figure 10.2: The overlapping region in the $n_B/n_\gamma - J_{CP}$ plane is shown as $\theta$ (in rad) varies from 0 to $\pi$ with $\Theta_{23} = \pi/4$, $\Theta_{13} = 13^\circ$, $\delta'_{13} = \beta_1 = \pi/2$ and $z = x = 0.01$. The dashed line is obtained for $\Theta_{12} = 33.5^\circ$, $y = 0.01$ and $\frac{M_1}{M_2} = 0.1$, while the solid line is obtained for $\Theta_{12} = 33.8^\circ$, $y = 0.02$ and $\frac{M_1}{M_2} = 0.01$.

The observed baryon-asymmetry can also be given as

$$\frac{n_B}{n_\gamma} = 7.1 Y_B = 2.3 \times 10^{-4} \frac{M_1}{M_2} \left( \frac{Y_N^0 d}{10^{-3}} \right) y^2 \sin 2\theta \,. \tag{10.49}$$

Comparing the above equation 10.49 with the observed matter antimatter asymmetry, we get

$$y^2 \sin 2\theta = (2.57 \quad - \quad 2.78) \times 10^{-6} \frac{M_2}{M_1} \left( \frac{10^{-3}}{Y_N^0 d} \right) . \tag{10.50}$$

We have shown the allowed values of $y$ in figure 10.1, using $(Y_N^0 d) = 10^{-3}$, for hierarchical right-handed neutrinos in the $y - \theta$ plane. It is shown in figure 10.1(a) that for $(M_1/M_2) = 0.1$ the minimum allowed value of $y$ is $5 \times 10^{-3}$. However, this value is lifted up to $1.7 \times 10^{-2}$ for $(M_1/M_2) = 0.01$ as shown in figure 10.1(b).

## 10.5 CP violation in leptogenesis and neutrino oscillation

It has been pointed out that the Dirac phase $\delta_{13}$ can be measured in the long baseline neutrino oscillation experiments [151–154]. In that case the CP violation arises from the difference of transition probability $\Delta P = P_{\nu_e \to \nu_\mu} - P_{\bar{\nu}_e \to \bar{\nu}_\mu}$. It can be shown that the transition probability $\Delta P$ is proportional to the leptonic Jarlskog invariant $J_{CP}$ defined In expression 4.5 of the section 4.2 .

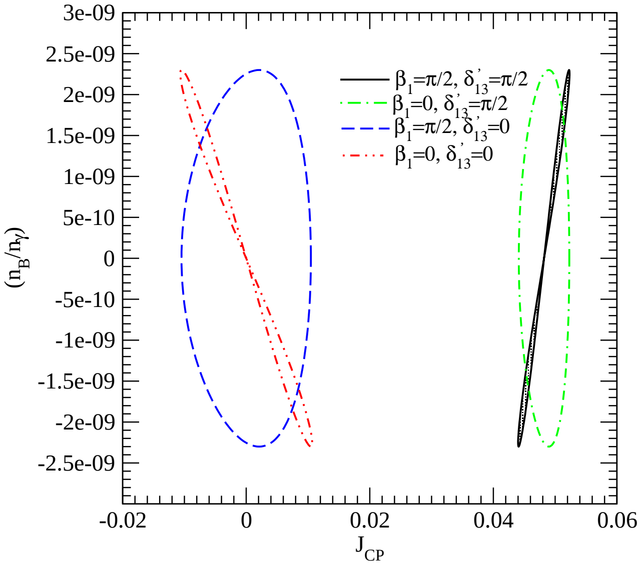


Figure 10.3: The variation of $n_B/n_\gamma$ is shown against $J_{CP}$ for different values of $\beta_1$ and $\delta'_{13}$ as $\theta$ (in rad) varies from 0 to $\pi$. We have chosen $\Theta_{23} = \pi/4$, $\Theta_{13} = 13^\circ$, $\Theta_{12} = 33.5^\circ$, $x = y = z = 0.01$ and $\frac{M_1}{M_2} = 0.1$.

Using expression 4.5, the rephasing invariant $J_{CP}$ can be rewritten as

$$J_{CP} = \frac{1}{8} \sin 2\Theta'_{12} \sin 2\Theta_{23} \sin 2\Theta_{13} \cos\Theta_{13} \sin(\delta'_{13} + \rho_1 + \rho_2) \,. \tag{10.51}$$

Now using equations 10.26, 10.27, 10.39 and 10.41 in the above equation 10.51 we get

$$\begin{aligned} J_{CP} &= \frac{1}{8} \frac{\sin 2\Theta_{23} \sin 2\Theta_{13} \cos\Theta_{13}}{\sqrt{\left[(a^2+b^2)^2 + c^4 + 2b^2c^2\cos 2\theta\right]^2 - 4a^4c^4}} \\ &\times \left[2ab\cos\delta'_{13}\{-c^2\sin 2\theta\cos\beta_1 + (a^2+b^2+c^2\cos 2\theta)\sin\beta_1\}\right. \\ &+ 2ab\cos 2\Theta_{12}\sin\delta'_{13}\{(a^2+b^2+c^2\cos 2\theta)\cos\beta_1 + c^2\sin 2\theta\sin\beta_1\} \\ &+ \left.\sin\delta'_{13}\sin 2\Theta_{12}(-a^4+b^4+c^4+2b^2c^2\cos 2\theta)\right] \,. \end{aligned} \tag{10.52}$$

From the above equation 10.52 it is obvious that $J_{CP} = 0$ only if both $\sin\delta'_{13} = 0$ and $b = 0$, while only $b = 0$ (equivalently $y = 0$) implies the condition for “no leptogenesis". This indicates that there is no one-to-one correspondence between the CP violation in neutrino oscillation and the CP violation in leptogenesis, even in the two right-handed neutrino models. However, it is interesting to see the common regions in the plane of $(n_B/n_\gamma)$ versus $J_{CP}$. This is shown in figure 10.2 by taking a typical set of parameters. The main aim is to illustrate the maximal contrast between the positive and negative values of $n_B/n_\gamma$ for a given set of values of $J_{CP}$. This helps us in determining the sign of the asymmetry by knowing the size of the asymmetry. From the figure 10.2 it is obvious that for the given set of parameters the positive sign of the asymmetry allows the values of $J_{CP}$ in the range $0.049-0.0495$ for $(M_1/M_2) = 0.1$. However, this range is significantly reduced for $(M_1/M_2) = 0.01$. On the other hand, the negative sign of the asymmetry allows the values of $J_{CP}$ in the range $0.0465-0.047$ for $(M_1/M_2) = 0.1$ which is further reduced for $(M_1/M_2) = 0.01$. In this figure the value of $\Theta_{12}$ is used from figure 10.4 where we have shown the allowed values of $\Theta_{12}$ as $\theta$ varies from 0 to $\pi$. Note that the above results are true for a non-zero $\Theta_{13}$. Consequently the allowed range of values of $J_{CP}$ may vary depending on the values of $\Theta_{13}$. Thus we anticipate that in the two right-handed neutrino models a knowledge of $J_{CP}$ can predict the sign of matter antimatter asymmetry of the Universe. We should note that the predictive power of the model depends on the CP violating phases $\beta_1$ and $\delta'_{13}$. This can be visible from figure 10.3 where we have shown the variation of $n_B/n_\gamma$ with $J_{CP}$ for different values of $\beta_1$ and $\delta'_{13}$. In particular, for the choice ($\beta_1 = \pi/2$, $\delta'_{13} = 0$) and ($\beta_1 = 0$, $\delta'_{13} = \pi/2$), the contrast between the positive and negative values of $n_B/n_\gamma$ is almost zero for a given set of values of $J_{CP}$. On the other hand, for the choice ($\beta_1 = \pi/2$, $\delta'_{13} = \pi/2$) and ($\beta_1 = 0$, $\delta'_{13} = 0$), the contrast between the positive and negative values of $n_B/n_\gamma$ is maximal and can be chosen for the present purpose.

## 10.6 CP violation in leptogenesis and neutrinoless double-beta decay

The observation of the neutrinoless double beta decay would provide direct evidence for the violation of total $L$-number in the low energy effective theory and hence probing the left-handed physical neutrinos to be Majorana type. Note that the $L$-number violation at high energy scale is a necessary criteria for leptogenesis. In the canonical seesaw models this is conspired by assuming that the right-handed neutrinos are Majorana in nature. However, this assumption doesn't ensure that the left-handed physical neutrinos are Majorana type. Assuming that the physical neutrinos are of Majorana type we investigate the connecting links between the two $L$-number violating phenomena occurring at two different energy scales.

In the low energy effective theory with three generations of left-handed fermions, apart from the $J_{CP}$, one can write two more rephasing invariants $J_1$ and $J_2$ as discussed in section 4.2. However, in the models with two right-handed neutrinos one of the eigen values of the physical light neutrino mass matrix is exactly zero. Therefore, the corresponding phase in the Majorana phase matrix can always be chosen so as to set one of the lepton-number violating CP violating rephasing invariant to zero. In the present case $m_3 = 0$ and hence the corresponding phase is arbitrary. This is ensured through $\beta_2$ which is redundant and pointed out in equation

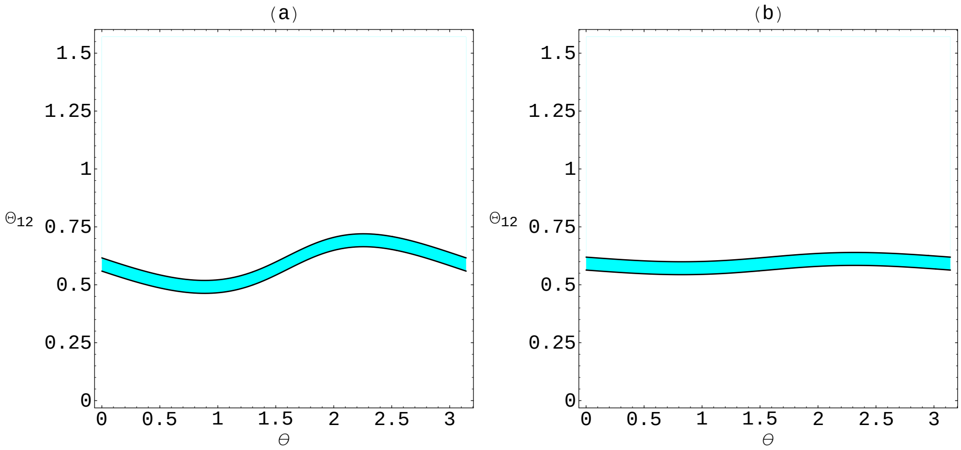


Figure 10.4: The allowed range of $\Theta_{12}$ (in rad) in equation 10.55 is shown as $\theta$ (in rad) varies from 0 to $\pi$ for $\Theta'_{12} = (33.9 \pm 1.6)^\circ$, $\beta_1 = \pi/2$, $x = z = 0.01$ (a) $y = 0.01$ and $(M_1/M_2) = 0.1$, and (b) $y = 0.02$ and $(M_1/M_2) = 0.01$.

10.15. Therefore, from equation 10.43 we can write the only $L$-number violating CP violating rephasing invariant as:

$$\begin{aligned} J &= \mathrm{Im}\left[V_{e1}V^*_{e2}(V_{ph})^*_{11}(V_{ph})_{22}\right] \\ &= -\frac{1}{2}\sin 2\Theta'_{12}\cos^2\Theta_{13}\sin(\rho_2-\rho_1+\frac{(\eta_2-\eta_1)}{2}-\beta_1)\,. \end{aligned} \tag{10.53}$$

Using equation 10.42 the above equation 10.53 can be rewritten as

$$\begin{aligned} J &= -\frac{\cos^2\Theta_{13}}{2}\frac{1}{[(a^2+b^2)^2+c^4+2b^2c^2\cos 2\theta+2c^2a^2\cos 2\theta]} \\ &\times \left[\sin 2\Theta_{12}\cos\theta\{-c^2\sin 2\theta\cos\beta_1+(a^2+b^2+c^2\cos 2\theta)\sin\beta_1\}\right. \\ &\quad \times\sqrt{(a^2+b^2)^2+c^4+2c^2a^2+2b^2c^2\cos 2\theta} \\ &+ \sin 2\Theta_{12}\sin\theta\{c^2\sin 2\theta\sin\beta_1+(a^2+b^2+c^2\cos 2\theta)\cos\beta_1\} \\ &\quad \times\frac{(-a^4+b^4+c^4+2b^2c^2\cos 2\theta)}{\sqrt{(a^2+b^2)^2+c^4+2c^2a^2+2b^2c^2\cos 2\theta}} \\ &+ \left.\cos 2\Theta_{12}\sin\theta\frac{2ab\{(a^2+b^2)^2+c^4+2b^2c^2\cos 2\theta+2c^2a^2\cos 2\theta\}}{\sqrt{(a^2+b^2)^2+c^4+2c^2a^2+2b^2c^2\cos 2\theta}}\right] \end{aligned} \tag{10.54}$$

In the above equation 10.54 the allowed values of $\Theta_{12}$ is obtained from

$$\begin{aligned} \cos\Theta'_{12} &= \left[\frac{1}{2}\left[1+\frac{(-a^4+b^4+c^4+2b^2c^2\cos 2\theta)\cos 2\Theta_{12}}{\sqrt{((a^2+b^2)^2+c^4+2b^2c^2\cos 2\theta)^2-4a^4c^4}}\right.\right. \\ &\quad \left.\left. -\sin 2\Theta_{12}\frac{2ab\{c^2\sin 2\theta\sin\beta_1+(a^2+b^2+c^2\cos 2\theta)\cos\beta_1\}}{\sqrt{((a^2+b^2)^2+c^4+2b^2c^2\cos 2\theta)^2-4a^4c^4}}\right]\right]^{1/2} \end{aligned} \tag{10.55}$$

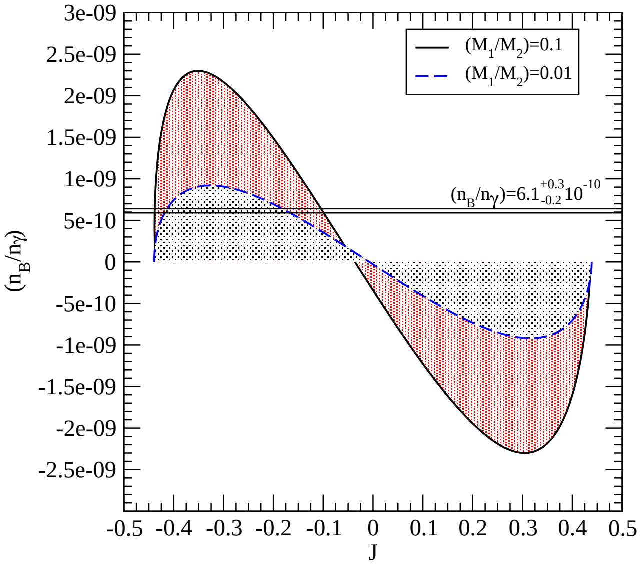


Figure 10.5: The overlapping region in the $\frac{n_B}{n_\gamma} - J$ plane is shown as $\theta$ (in rad) varies from 0 to $\pi$ with $\Theta_{13} = 13^\circ$, $\beta_1 = \pi/2$ and $x = z = 0.01$. The solid line is obtained for $y = 0.01$ and $\Theta_{12} = 33.5^\circ$, while the dashed line is obtained with $y = 0.02$ and $\Theta_{12} = 33.8^\circ$.

by fixing $\Theta'_{12} = (33.9 \pm 1.6)^\circ$. This is shown in figure 10.4.

From equation 10.54 one can see that $J \neq 0$ as $\theta \rightarrow 0$ which is the condition for “no leptogenesis". Thus we see that there is no one-to-one correspondence between the two $L$-number violating processes occurring at two different energy scales. However, it is always interesting to see the overlapping regions in the plane of $\frac{n_B}{n_\gamma}$ versus $J$ as $\theta$ varies from 0 to $\pi$. This is shown in figure 10.5 for a typical set of parameters. From figure 10.5 one can see that for positive sign of the $B$-asymmetry the values of $J$ lie in between $-0.45$ to $-0.1$ for $(M_1/M_2) = 0.1$. This range is further reduced to $(-0.4 - -0.15)$ for $(M_1/M_2) = 0.01$. On the other hand, for the negative sign of the $B$-asymmetry the values of $J$ lie in the range $(0.05 - 0.45)$ for $(M_1/M_2) = 0.1$ and in the range $(0.15 - 0.4)$ for $(M_1/M_2) = 0.01$. Thus we see that within the allowed range of parameters the contrast between the positive and negative values of $\frac{n_B}{n_\gamma}$ is maximum for a given set of values of $J$. Therefore, we expect a knowledge of $J$ can precisely determine the sign of $B$-asymmetry since the value of $n_B/n_\gamma$ is known. Finally we note that, unlike $J_{CP}$, $J$ remains non-vanishing even if $\Theta_{13} = 0$ [2]. Now the remaining question to be addressed is how $n_B/n_\gamma$ varies with respect to $J$ for different values of $\beta_1$. This is shown in figure 10.6 for a given set of parameters. One can see that for $\beta_1 = 0$ and $\beta_1 = \pi$ both positive and negative values of $n_B/n_\gamma$ correspond to the same set of values of $J$ which is unwelcome for determination of sign of the asymmetry. On the other hand for $\beta_1 \neq 0, \pi$ one can

[2]In three generations there are two of them. See for example the paper by Y. Liu and U. Sarkar in ref. [15]

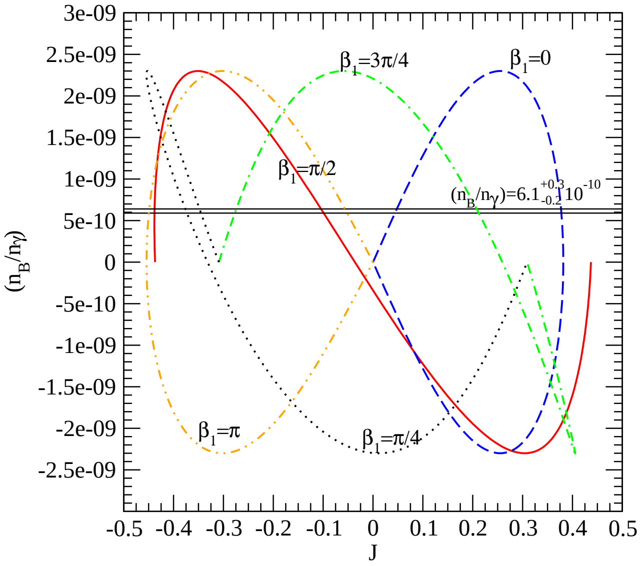


Figure 10.6: The variation of $\frac{n_B}{n_\gamma}$ with $J$

The variation of $\frac{n_B}{n_\gamma}$ is shown against $J$ for different values of $\beta_1$ as $\theta$ (in rad) varies from 0 to $\pi$. We have chosen $\Theta_{13} = 13^\circ$, $\Theta_{12} = 33.5^\circ$, $x = y = z = 0.01$ and $\frac{M_1}{M_2} = 0.1$.

have maximal contrast between the positive and negative values of $n_B/n_\gamma$ for the given set of values of $J$ and hence can be chosen for the present purpose.

# Chapter 11

# Neutrino dark energy in SO(10) GUT model

As discussed in chapter 8, the original models of neutrino dark energy (NDE) or the mass varying neutrinos (mavans) [27, 161, 162] are based on type I seesaw mechanism by allowing the Majorana mass of the right-handed neutrinos to vary with the acceleron field. The basic mechanism is discussed in subsection 8.3.2 of chapter 8. However, this kind of model is not complete and several problems are pointed out in [27, 163]. Some of the problems have been also solved in subsequent works [164–168], but more studies are required to make this model fully consistent.

In type I seesaw models of NDE, the Majorana masses of the right-handed neutrinos varies with the acceleron field and that relates the scale of DE with the light neutrino masses. Naturalness requires the Majorana masses of the right-handed neutrinos also to be in the range of eV and the main motivation of the seesaw mechanism to naturally suppress the low energy neutrino masses is lost. So the smallness of the light neutrino masses cannot be attributed to a large lepton-number violating mass scale in this theory. Moreover, the neutrino Dirac masses cannot be made to vary with the acceleron field, since that will then allow coupling of the acceleron field with the charged leptons and a natural scale for the DE will then be the mass of the heaviest charged lepton. For the same reason, this mechanism cannot be embedded into a left-right symmetric model, in which the $SU(2)_R$ group relates the right-handed neutrinos to the right-handed charged leptons.

The problem with the smallness of the mass parameter that depends on the acceleron field can be softened in the NDE models in type II seesaw scenario [168]. In the NDE model with the triplet Higgs scalars, the coefficient of this trilinear scalar coupling with mass dimension varies with the acceleron field, and naturalness allows this parameter to be as large as a few hundred GeV. Although the scale of this mass parameter predicts new signals in the TeV range, there is no symmetry that makes this scale natural.

In this chapter, we construct a left-right symmetric model for the NDE proposal and try to embed this model in $SO(10)$ GUT. The most important feature of this model is that the mass parameter varying with the acceleron field remains small naturally and the scale of DE is related to the neutrino masses. This is the only NDE model that can be embedded into a grand unified theory, without relating the scale of DE to the charged fermion masses.

## 11.1 Left-right symmetric model for NDE

We have discussed the basic structure of left-right symmetric model in detail in chapter 5. In addition to the basic gauge and fermion structure in the left-right symmetric model, we introduce a singlet field $S_R$. Although there is no $S_L$, the model is consistent with left-right parity operation, since the field $S_R$ transform to its $CP$ conjugate state under the left-right parity as: $S_R \leftrightarrow {S^c}_L$. This also ensures that the Majorana mass term is invariant under the parity transformation, because this field $S_R$ transform under the transformation $SU(2)_L \leftrightarrow SU(2)_R$ to itself $S_R \equiv (1,1,1,0) \leftrightarrow (1,1,1,0)$.

The various left-right symmetric models differ from each other in choice of Higgs bosons and symmetry breaking chains . In the present model, the content of the Higgs sector will be chosen according to the following desired symmetry breaking pattern[169, 170]:

$$
\begin{aligned}
& SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} && [G_{3221D}] \\
& \stackrel{M_R}{\rightarrow} SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{(B-L)} && [G_{3211}] \\
& \stackrel{m_r}{\rightarrow} SU(3)_c \times SU(2)_L \times U(1)_Y && [G_{321}] \\
& \stackrel{m_W}{\rightarrow} SU(3)_c \times U(1)_Q && [G_{\rm em}] \,.
\end{aligned}
$$

Breaking of the left-right symmetric group to $G_{3211}$ requires a right triplet Higgs scalars $\Delta_R$ transforming as $\Delta_R \equiv (1,1,3,0)$. The triplet does not change the rank of the gauge group and only breaks $SU(2)_R \rightarrow U(1)_R$. Since it does not carry any $U(1)_{B-L}$ quantum number, it cannot give any Majorana masses to the neutral fermions. For the next symmetry breaking stage, $U(1)_R \times U(1)_{B-L} \rightarrow U(1)_Y$, we introduce an $SU(2)_R$ doublet Higgs scalar field $\chi_R \equiv (1,1,2,1/2)$ [91]. The *vev* of $\chi_R$ could also break $[G_{3221D}] \rightarrow [G_{321}]$, if the field $\Delta_R$ were not present. The left-right parity would then require the existence of the fields $\Delta_L \equiv (1,3,1,0)$ and $\chi_L \equiv (1,2,1,1/2)$. Finally, the SM symmetry breaking is mediated by a bi-doublet field $\Phi \equiv (1,2,2,0)$, like in any other left-right symmetric model. This field has the Yukawa interaction with the SM fermions and provide Dirac masses to all of them.

We shall introduce one more Higgs bi-doublet scalar $\Psi \equiv (1,2,2,0)$ that is required to write a few desired terms in our model. We also introduce another singlet scalar field $\eta \equiv (1,1,1,0)$, which acquires a tiny *vev* of the order of the light neutrino masses and generate the mass scale for the DE naturally.

Now we write down the explicit forms of all the scalar fields in terms of their components as

$$\Delta_L = \begin{pmatrix} \Delta_L^0 & \Delta_L^+ \\ \Delta_L^- & -\Delta_L^0 \end{pmatrix} \quad , \quad \Delta_R = \begin{pmatrix} \Delta_R^0 & \Delta_R^+ \\ \Delta_R^- & -\Delta_R^0 \end{pmatrix} ,$$

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \quad , \quad \Psi = \begin{pmatrix} \psi_1^0 & \psi_1^+ \\ \psi_2^- & \psi_2^0 \end{pmatrix} ,$$

$$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix} \quad , \quad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} ,$$

The most general scalar potential has to be constructed in such a way that they respect the left-right parity transformation of the scalar fields listed below:

$$\begin{aligned} \chi_L \leftrightarrow \chi_R \quad &, \quad \Delta_L \leftrightarrow \Delta_R \\ \Phi \leftrightarrow \Phi^\dagger \quad &, \quad \Psi \leftrightarrow \Psi^\dagger \\ . \quad & \quad \eta \leftrightarrow \eta \,. \end{aligned}$$

Under the left-right gauge group transformation, the Higgs fields transform as

$$\begin{aligned} \Delta_L \to U_L\, \Delta_L\, U_L^\dagger \quad &, \quad \Delta_R \to U_R\, \Delta_R\, U_R^\dagger \\ \Phi \to U_L\, \Phi\, U_R^\dagger \quad &, \quad \Psi \to U_L\, \Psi\, U_R^\dagger \\ \chi_L \to U_L\, \chi_L \quad &, \quad \chi_R \to U_R\, \chi_R \\ & \quad \eta \to \eta \,. \end{aligned}$$

In order to write down the scalar potential we also construct the fields $\tau^2\Phi^*\tau^2$ and $\tau^2\Psi^*\tau^2$ from $\Phi$ and $\Psi$ which transform in the same ways as $\Phi$ and $\Psi$. For convenience, we represent $\Phi$ as $\phi_1$, $\tau^2\Phi\tau^2$ as $\phi_2$ (and similarly for $\Psi$) from now on.

## 11.2 Potential minimization

We first write down the most general renormalizable gauge invariant scalar potential respecting left-right parity and study details of potential minimization. Besides left-right parity, we impose following $Z_4$ symmetry on only the Higgs potential to avoid few undesired terms

$$\begin{aligned}
\chi_L \to i\chi_L\,, &\quad \chi_R \to -i\chi_R\,,\\
\Delta_L \to -\Delta_L\,, &\quad \Delta_R \to -\Delta_R\,,\\
\Phi \to \Phi\,, &\quad \Psi \to -\Psi\,,\\
&\quad \eta \to \eta\,.
\end{aligned}$$

We write the the Higgs potential as a sum of of various parts and write down each part separately as:

$$\begin{aligned}
V &= V_\phi + V_\psi + V_\Delta + V_\eta + V_\chi + V_{\Delta\phi\psi} + V_{\chi\phi\psi} + V_{\eta\chi\Delta\phi\psi}\\
V_\phi &= -\sum_{i,j} \frac{\mu^2_{\phi ij}}{2}\, tr(\phi_i^\dagger \phi_j) + \sum_{i,j,k,l} \frac{\lambda_{\phi ijkl}}{4}\, tr(\phi_i^\dagger \phi_j)\, tr(\phi_k^\dagger \phi_l)\\
&\quad + \sum_{i,j,k,l} \frac{\Lambda_{\phi ijkl}}{4}\, tr(\phi_i^\dagger \phi_j \phi_k^\dagger \phi_l)\\
V_\psi &= -\sum_{i,j} \frac{\mu^2_{\psi ij}}{2}\, tr(\psi_i^\dagger \psi_j) + \sum_{i,j,k,l} \frac{\lambda_{\psi ijkl}}{4}\, tr(\psi_i^\dagger \psi_j)\, tr(\psi_k^\dagger \psi_l)\\
&\quad + \sum_{i,j,k,l} \frac{\Lambda_{\psi ijkl}}{4}\, tr(\psi_i^\dagger \psi_j \psi_k^\dagger \psi_l)\\
V_\Delta &= -\frac{\mu^2_\Delta}{2}\, [tr(\Delta_L \Delta_L) + tr(\Delta_R \Delta_R)] + \frac{\lambda_\Delta}{4}\, [tr(\Delta_L \Delta_L)^2 + tr(\Delta_R \Delta_R)^2]\\
&\quad + \frac{\Lambda_\Delta}{4}\, [tr(\Delta_L \Delta_L \Delta_L \Delta_L) + tr(\Delta_R \Delta_R \Delta_R \Delta_R)]\\
&\quad + \frac{g_\Delta}{2}\, [tr(\Delta_L \Delta_L)\, tr(\Delta_R \Delta_R)]\\
V_\eta &= \frac{M^2_\eta}{2}\, \eta^2 + \frac{\lambda_\eta}{4}\, \eta^4\\
V_\chi &= -\frac{\mu^2_\chi}{2}\, [\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R] + \frac{\lambda_\chi}{4}\, [(\chi_L^\dagger \chi_L)^2 + (\chi_R^\dagger \chi_R)^2]\\
&\quad + \frac{g_\chi}{2}\, [\chi_L^\dagger \chi_L\, \chi_R^\dagger \chi_R]
\end{aligned}$$

$$
\begin{aligned}
V_{\Delta\phi\psi} &= \sum_{i,j} \alpha_{\phi ij}\, [\Delta_L\Delta_L + \Delta_R\Delta_R]\, tr(\phi_i^\dagger \phi_j) \\
&+ \sum_{i,j} \alpha_{\psi ij}\, [\Delta_L\Delta_L + \Delta_R\Delta_R]\, tr(\psi_i^\dagger \psi_j) \\
&+ \sum_{i,j} \beta_{\phi ij}\, [\, tr(\Delta_L\Delta_L\phi_i\phi_j^\dagger) + tr(\Delta_R\Delta_R\phi_i^\dagger\phi_j)] \\
&+ \sum_{i,j} \beta_{\psi ij}\, [\, tr(\Delta_L\Delta_L\psi_i\psi_j^\dagger + tr(\Delta_R\Delta_R\psi_i^\dagger\psi_j)] \\
&+ \sum_{i,j} h_{\Delta\phi ij}\, tr(\Delta_L\phi_i\Delta_R\phi_j^\dagger) + \sum_{i,j} h_{\Delta\psi ij} tr(\Delta_L\psi_i\Delta_R\psi_j^\dagger) \\
V_{\chi\phi\psi} &= \sum_{i,j} h_{\phi\chi ij}\, [\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R]\, tr(\phi_i^\dagger\phi_j) \\
&+ \sum_{i,j} h_{\psi\chi ij}\, [\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R]\, tr(\psi_i^\dagger\psi_j) \\
V_{\eta\chi\Delta\phi\psi} &= \Big(h_{\eta\chi}\, [\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R] + h_{\eta\Delta}\, [tr(\Delta_L^\dagger\Delta_L) + tr(\Delta_R^\dagger\Delta_R)]\Big)\ \eta^2 \\
&+ \left(\sum_{i,j} h_{\eta\phi ij}\, tr(\phi_i^\dagger\phi_j) + \sum_{i,j} h_{\eta\psi ij}\, tr(\psi_i^\dagger\psi_j)\right)\ \eta^2 \\
&+ \sum_{i,j} h_{\eta ij}\, \eta\, [tr(\phi_i^\dagger\Delta_L\psi_j) + tr(\phi_i\Delta_R\psi_j^\dagger) + h.c.] \\
&+ \sum_{i} h_{\chi i}\, \eta\, [\chi_L^\dagger\phi_i\chi_R + h.c.]\,.
\end{aligned}
$$

We parametrize the true minima of the potential by giving vevs to different scalar fields as follows.

$$
\phi_1 = \begin{pmatrix} v & 0 \\ 0 & v' \end{pmatrix}, \phi_2 = \begin{pmatrix} v' & 0 \\ 0 & v \end{pmatrix}, \qquad \psi_1 = \begin{pmatrix} w & 0 \\ 0 & w' \end{pmatrix},\ \psi_2 = \begin{pmatrix} w' & 0 \\ 0 & w \end{pmatrix},
$$
$$
\chi_L = \begin{pmatrix} 0 \\ v_L \end{pmatrix},\ \chi_R = \begin{pmatrix} 0 \\ v_R \end{pmatrix}, \qquad \Delta_L = \begin{pmatrix} u_L & 0 \\ 0 & -u_L \end{pmatrix},\ \Delta_R = \begin{pmatrix} u_R & 0 \\ 0 & -u_R \end{pmatrix}
$$
$$
\eta = u\,.
$$

Since the phenomenological consistency requires $v \gg v'$and $w \gg w'$, we ignore potential terms involving $v'$and $w'$ and write down the general scalar potential in terms of *vev*s of different scalar fields

$$\begin{aligned}
V \; = \; & -\frac{\mu_\phi^2}{2}\, v^2 + \frac{\lambda_\phi}{4}\, v^4 - \frac{\mu_\psi^2}{2}\, w^2 + \frac{\lambda_\psi}{4}\, w^4 \\
& -\frac{\mu_\Delta^2}{2}\, (u_L^2 + u_R^2) + \frac{\lambda_\Delta}{4}\, (u_L^4 + u_R^4) \\
& +\frac{M_\eta^2}{2}\, u^2 + \frac{\lambda_\eta}{4}\, u^4 \\
& -\frac{\mu_\chi^2}{2}\, (v_L^2 + v_R^2) + \frac{\lambda_\chi}{4}\, (v_L^4 + v_R^4) + \frac{g_\chi}{2}\, (v_L^2\, v_R^2) \\
& +[(\alpha_\phi + \beta_\phi) v^2 + (\alpha_\psi + \beta_\psi) w^2]\, (u_L^2 + u_R^2) + (h_{\Delta\phi} v^2 + h_{\Delta\psi} w^2)\, u_L u_R \\
& +(h_{\phi\chi} v^2 + h_{\psi\chi} w^2)\, (v_L^2 + v_R^2) \\
& +[h_{\eta\chi}(v_L^2 + v_R^2) + h_{\eta\Delta}(u_L^2 + u_R^2) + h_{\eta\phi} v^2 + h_{\eta\psi} w^2]\, u^2 \\
& +h_\eta\, u(u_L + u_R) v w + h_\chi\, u(v_L v_R) v\,.
\end{aligned}$$

For convenience, we have replaced $\lambda_\phi + \Lambda_\phi \to \lambda_\phi,\;\; \lambda_\psi + \Lambda_\psi \to \lambda_\psi,\;\; \lambda_\Delta + \Lambda_\Delta \to \lambda_\Delta$. The minimization of the potential is studied by taking partial derivatives with respect to *vev*s of all Higgs fields and then separately equating them to zero. Solving all such equations will provide us the desired values. One of the minimization conditions $v_L \left(\frac{\partial V}{\partial v_R}\right) - v_R \left(\frac{\partial V}{\partial v_L}\right) = 0$ leads to the following relation between $v_L$ and $v_R$:

$$(v_R^2 - v_L^2)\;\left[(\lambda_\chi - g_\chi) v_L v_R - h_\chi u v\right] = 0\,.$$

Since $(v_R^2 = v_L^2)$ is not desirable phenomenologically, we chose

$$v_L v_R = \frac{h_\chi u v}{(\lambda_\chi - g_\chi)}\,. \tag{11.1}$$

Using above relation in an another minimization condition $v_L \left(\frac{\partial V}{\partial v_R}\right) + v_R \left(\frac{\partial V}{\partial v_L}\right) = 0$, we get

$$v_L^2 + v_R^2 = -\frac{\mu_\chi^2}{\lambda_\chi}\,. \tag{11.2}$$

Parametrizing $v_L = A\, \sin\theta$, $v_R = A\, \cos\theta$ and putting them in the two equations 11.1 and 11.2 , we find $A = -\mu_\chi^2/\lambda_\chi$ $\sin 2\theta = 2\theta = \frac{2h_\chi u v}{(\lambda_\chi - g_\chi)}$ since $\mu_\chi$ is a large number compared to the numerator. So we get

$$\begin{aligned}
v_R \; &= \; A = \sqrt[2]{-\mu_\chi^2/\lambda_\chi}\,, \\
v_L \; &= \; A\theta = \frac{\lambda_\chi h_\chi}{(g_\chi - \lambda_\chi)}\, \frac{u v v_R}{\mu_\chi^2}\,.
\end{aligned}$$

We have chosen the parametrization of $v_L$ and $v_R$ in such a way that $v_R$ gets value equal to breaking scale of $G_{3211}$ and $v_L$ gets a very small value. We could have done other way around but that is not what is phenomenologically allowed. Proceeding with the same kind of analysis for $u_L$ and $u_R$, i.e., using two minimization conditions $u_L\left(\frac{\partial V}{\partial u_R}\right) - u_R\left(\frac{\partial V}{\partial u_L}\right) = 0$ and $u_L\left(\frac{\partial V}{\partial u_R}\right) + u_R\left(\frac{\partial V}{\partial u_L}\right) = 0$, we get

$$\begin{aligned} u_R &= \sqrt[2]{-\mu_\Delta^2/\lambda_\Delta}\,, \\ u_L &= \frac{\lambda_\Delta h_\Delta}{(g_\Delta - \lambda_\Delta)}\,\frac{(h_{\Delta\phi}v^2 + h_{\Delta\psi}w^2)u_R}{\mu_\Delta^2}\,. \end{aligned}$$

Using equation 11.1, the $\eta$ field can be shown to get *vev* only by term $h_\eta u(u_L + u_R)$ as only this term is linear in $u$. The term $h_\chi u(v_L v_R)v$ does not remain linear in $u$ after we substitute the value of $v_L v_R$ from equation 11.1. Since the mass term for $\eta$ field is large and positive, we expect very small *vev*. So we can ignore some of the terms in the potential while solving for $u$ and can easily obtain

$$u = \frac{h_\eta v w(u_L + u_R)}{M_\eta^2 - (h_{\eta\Delta}\mu_\Delta^2/\lambda_\Delta) - (h_{\eta\chi}\mu_\chi^2/\lambda_\chi)}\,.$$

After analyzing the complete scalar potential, we find a consistent solution with ordering

$$u_R \gg v_R > v > w \gg u \gg v_L\,. \tag{11.3}$$

At this stage we can assume the different mass scales to explain the model. However, when we embed this model in an $SO(10)$ grand unified theory, the gauge coupling unification will impose strong constraints on the different symmetry breaking scales. The left-right parity and the $SU(2)_R$ breaking scale will come out to be above $10^{11}$ GeV. So, we shall assume $u_R \sim 10^{11}$ GeV. We also assume $m_\eta \sim m_\Delta \sim u_R$. However, it will be possible to keep the $G_{3211}$ symmetry breaking scale to be very low, and hence, we shall assume $m_\chi \sim v_R \sim$ TeV. We find the remaining mass scales to be $v \sim m_w \sim 100$ GeV, $u \sim u_L \sim$ eV and $v_L \sim 10^{-2}$ eV.

## 11.3 Embedding the model in $SO(10)$GUT

We shall study here the embedding of the present model with all its Higgs content in $SO(10)$ GUT. We consider the following breaking pattern of $SO(10)$ gauge group to first Pati-Salam gauge group $SU(4) \times SU(2)_L \times SU(2)_R$, next to the left-right gauge group and then to the SM gauge group

$$\begin{aligned}
SO(10) &\xrightarrow{M_U} SU(4)\times SU(2)_L\times SU(2)_R \quad [G_{422D}]\\
&\xrightarrow{M_I} SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_{(B-L)} \quad [G_{3221D}]\\
&\xrightarrow{M_R} SU(3)_c\times SU(2)_L\times U(1)_R\times U(1)_{(B-L)} \quad [G_{3211}]\\
&\xrightarrow{m_r} SU(3)_c\times SU(2)_L\times U(1)_Y \quad [G_{321}]\\
&\xrightarrow{m_W} SU(3)_c\times U(1)_Q \quad [G_{\rm em}]\,.
\end{aligned}$$

In the discussion of chapter 6, we saw that the Higgs multiplets which can provide the masses for all the SM fermions are limited as $16\times 16 = 10_s + 120_a + \overline{126}_s$. For convenience, let us write how a 10 dimensional Higgs field $H_\Phi$ decomposes under left-right gauge group as

$$H_\Phi(10) = \Phi(1,2,2,0)\oplus(3,1,1,-\frac{1}{3})\oplus(\overline{3},1,1,\frac{1}{3})\,.$$

One can easily identify the bi-doublet $\Phi(1,2,2,0)$ appearing in the left-right model contained in $H_\Phi(10)$. To include another bi-doublet $\Psi(1,2,2,0)$ present in the model, we introduce a second 10-dimensional Higgs field $H_\Psi(10)$.

Although the fermion and gauge sector of the $SO(10)$ GUT model are quite simple, the Higgs sector is quite complicated since it is not only required for generating fermion Masses, but an appropriate Higgs content is also needed for systematic and consistent breaking of the $SO(10)$ gauge group down to the SM gauge group in one or more steps. To break $SO(10)$ gauge group to the Pati-Salam gauge group, one requires Higgs fields either $S(54)$ or $\Upsilon(210)$, both having singlet under Pati-Salam decomposition. Although we have discussed some of these issues in subsection 6.2.1 of chapter 6, we discuss some part again for the sake of completeness.

Giving *vev* to either of the two fields in the singlet direction will serve the purpose of the desired breaking. The $(15,1,1)$ of $\Upsilon$ also has a singlet under the left-right gauge group which can acquire *vev* to break the Pati-Salam group to the left-right group. The $(15,3,1)$ and $(15,1,3)$ Higgs multiplets of $\Upsilon$ also contain the fields $\Delta_L(1,3,1,0)$ and $\Delta_R(1,1,3,0)$ present the left-right model. However, the $\Upsilon$ singlet under Pati-Salam gauge group is odd under D-Parity. If we give *vev* to $\Upsilon$ singlet, the left-right symmetry will be broken at unification scale itself. Since our model is left-right symmetric, we must avoid D-parity breaking until left-right group is broken.

However, the singlet in $S(54)$ field under Pati-Salam gauge group does respect the D-parity and so can be used to break the GUT group to the Pati-Salam gauge group. As discussed in subsection 6.2.1, a 45-dimensional Higgs field $A(45)$ is needed along with $S(54)$ for breaking the GUT group to the left-right group. The

Higgs field $A(45)$ has the decomposition under the left-right group as

$$\begin{aligned}A(45) &= (1,1,1,0)\oplus\Delta_L(1,3,1,0)\oplus\Delta_R(1,1,3,;0)\\ &\oplus(3,1,1,\frac{4}{3})\oplus(\overline{3},1,1,-\frac{4}{3})\oplus(8,1,1,0)\\ &\oplus(3,2,2,\frac{2}{3})\oplus(\overline{3},2,2,-\frac{2}{3})\,.\end{aligned}$$

The first row of the above decomposition is of our interest as it contains the fields $\Delta_L(1,3,1,0)$ and $\Delta_R(1,1,3,0)$ of our model along with the left-right group singlet. Moreover, the singlet is even under D-parity and so the left-right symmetry is unbroken until $\Delta_R$ acquires *vev* along the singlet direction to the SM gauge group. So a combination of (45+54)-dimensional Higgs fields serves our purpose to break the GUT group to left-right group without violating the D-parity and we will work with this combination for the rest of the discussion.

The fields $\chi_L\ \left(1,2,2,\frac{1}{2}\right)$ and $\chi_R\left(1,1,2,\frac{1}{2}\right)$ are still left to be embedded in some tensors of $SO(10)$. The desired quantum numbers indicate that they can be embedded in the spinorial Higgs representation $\left(C(16)\oplus\overline{C(16)}\right)$. Decomposition of the $16\oplus\overline{16}$ spinor representation under left-right group are given as

$$\begin{aligned}16 &= \chi_L^*(1,2,1,-\frac{1}{2})\oplus\chi_R(1,1,2,\frac{1}{2})\\ &\oplus(3,2,1,\frac{1}{6})\oplus(\overline{3},1,2,-\frac{1}{6})\end{aligned}$$

$$\begin{aligned}\overline{16} &= \chi_L(1,2,1,\frac{1}{2})\oplus\chi_R^*(1,1,2,-\frac{1}{2})\\ &\oplus(3,1,2,\frac{1}{6})\oplus(\overline{3},2,1,-\frac{1}{6})\quad ,\end{aligned}$$

Having embedded all the Higgs fields of our model into $SO(10)$ tensor fields, we now study the renormalization group evolution of the various coupling constants.

## 11.4 Gauge coupling evolution

In the present section, we will be studying the set of two-loop renormalization group (RG) equations for the evolution of the coupling constants and will be verifying the consistency of the chosen *vev* for different Higgs fields in the context of $SO(10)$ GUT. For simplicity, we assume that the scale $M_U$ and $M_C$ are very close and we ignore the evolution of the coupling constants between the two scales. This is quite preferable as we will see later that the unification scale is very tightly constrained by the current proton decay bound [98] and any substantial difference between the two breaking scales would make it even worse. We start with the following equation for the two-loop evaluation of the coupling constant $\alpha_i$

$$\frac{d\alpha_i^{-1}(t)}{dt} = -\frac{a_i}{2\pi} - \frac{b_{ij}}{8\pi^2}\left(\frac{1}{\alpha_j^{-1}}\right) \tag{11.4}$$

where $t = \ln\left(M_\mu\right)$ and $M_\mu$ is the desired energy scale where the couplings constants, $\alpha_i$'s, are be determined. . The $a_i$'s and $b_{ij}$'s are the one-loop and two-loop beta functions governing the evolution of $\alpha_i$'s and include the contributions from gauge bosons, fermions and scalars in the model.

The fermion contribution to the beta function is taken right from the starting, the electroweak scale (100GeV). The contributions of the gauge bosons to beta functions are straightforward to compute as one can easily determine the expected mass scales of the heavy gauge bosons corresponding to any given gauge group. However, the contribution coming from the Higgs content is not so clear because the heavy Higgs modes can have various possible mass spectrum. We will use the extended survival hypothesis to fix this uncertainty. The extended survival hypothesis is based on the assumption that only minimal number of fine-tunings of the parameters in the Higgs potential are imposed to ensure the hierarchy in various gauge boson masses. According to the extended survival hypothesis, only those scalar multiplets are present at any given intermediate breaking scale $M_I$ of a intermediate gauge group $G_I$ which are either required for breaking the gauge group $G_I$ or needed to further break any other intermediate gauge group below scale $M_I$. Rest of the scalars are stuck at the unification scale.

A list of Higgs multiplets surviving at the breaking scale of a intermediate group $G_I$, using the extended survival hypothesis, are given in table. A list of both one-loop and two-loop beta coefficients, which include all the contributions, that govern the evolution above the breaking scale of $G_I$ to the next intermediate scale are also listed.

Since our model contains intermediate steps, we require appropriate matching conditions at the corresponding breaking scales. For the tow-loop RG running of the coupling constants, the matching conditions have been derived in [171, 172]. Suppose a gauge group $G$ is spontaneously broken into a sub-group $\prod_i G_i$ with several individual factor $G_i$, then the following matching condition need to be satisfied for the two-loop analysis

$$\alpha_G^{-1}(M_I) - \frac{C(G)}{12\pi} = \alpha_{G_i}^{-1}(M_I) - \frac{C(G_i)}{12\pi}, \tag{11.5}$$

where $C(G/G_i)$ is the quadratic Casimir invariant for the group $G/G_i$.

| Group $G_I$ | Higgs content | a | b |
|---|---|---|---|
| $G_{321}$ | $\left(1,2,\frac{1}{2}\right)_{10} \oplus \left(1,2,-\frac{1}{2}\right)_{10}$<br>$\left(1,2,\frac{1}{2}\right)_{10'} \oplus \left(1,2,-\frac{1}{2}\right)_{10'}$ | $\begin{pmatrix} -7 \\ -3 \\ \frac{21}{5} \end{pmatrix}$ | $\begin{pmatrix} -26 & \frac{9}{2} & \frac{11}{10} \\ 12 & 8 & \frac{6}{5} \\ \frac{44}{5} & \frac{18}{5} & \frac{104}{25} \end{pmatrix}$ |
| $G_{3211}$ | $\left(1,2,\frac{1}{2}0\right)_{10} \oplus \left(1,2,-\frac{1}{2}0\right)_{10}$<br>$\left(1,2,,\frac{1}{2}0\right)_{10'} \oplus \left(1,2,-\frac{1}{2},0\right)_{10'}$<br>$\left(1,1,-\frac{1}{2},\frac{1}{2}\right)_{16} + \left(1,1,\frac{1}{2},-\frac{1}{2}\right)_{\overline{16}}$ | $\begin{pmatrix} -7 \\ -3 \\ \frac{53}{12} \\ \frac{33}{8} \end{pmatrix}$ | $\begin{pmatrix} -26 & \frac{9}{2} & \frac{3}{2} & \frac{1}{2} \\ 12 & 8 & 1 & \frac{3}{2} \\ 12 & 3 & \frac{17}{4} & \frac{15}{8} \\ 4 & \frac{9}{2} & \frac{15}{8} & \frac{65}{16} \end{pmatrix}$ |
| $G_{3221D}$ | $(1,2,2,0)_{10}$<br>$(1,2,2,0)_{10'}$<br>$\left(1,2,1,-\frac{1}{2}\right)_{16} \oplus \left(1,2,1,\frac{1}{2}\right)_{\overline{16}}$<br>$\left(1,1,2,\frac{1}{2}\right)_{16} \oplus \left(1,1,2,-\frac{1}{2}\right)_{\overline{16}}$<br>$(1,1,3,0)_{45}$<br>$(1,3,1,0)_{45}$ | $\begin{pmatrix} -7 \\ -\frac{5}{2} \\ -\frac{5}{2} \\ \frac{9}{2} \end{pmatrix}$ | $\begin{pmatrix} -26 & \frac{9}{2} & \frac{9}{2} & \frac{1}{2} \\ 12 & \frac{39}{2} & 3 & \frac{9}{4} \\ 12 & 3 & \frac{39}{2} & \frac{9}{4} \\ 4 & \frac{27}{4} & \frac{27}{4} & \frac{23}{4} \end{pmatrix}$ |

Table 11.1: Higgs multiplets at different intermediate breaking scales along with the both one-loop and two-loop beta coefficients, including all the contributions from fermions, gauge bosons and Higgs bosons, which govern the evolution of coupling constants above breaking scale of $G_I$ to the next breaking scale.

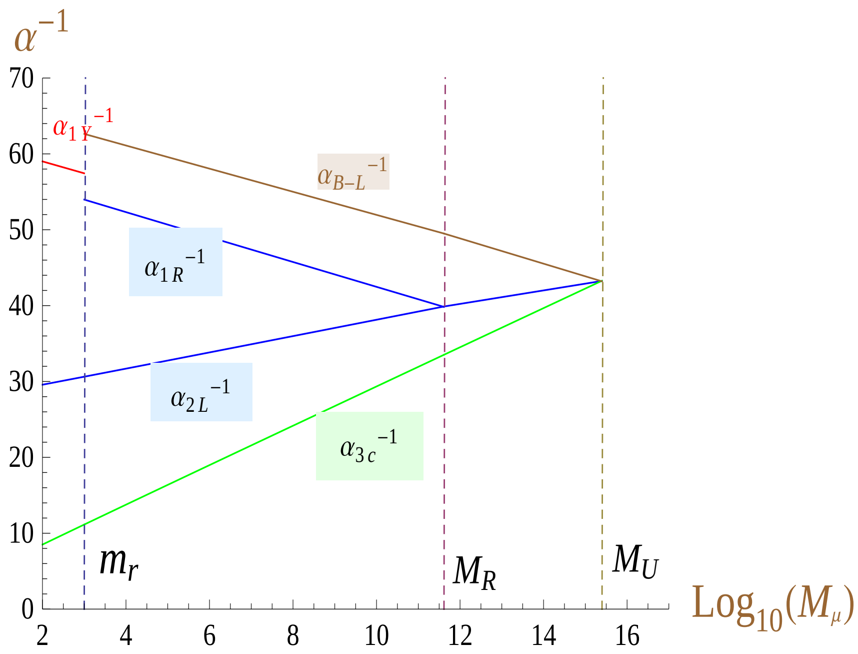


Figure 11.1: Evolution of coupling constants

The boundary conditions at various breaking scales, using the expression 11.5, can be written as

1. At scale $m_r$:

$$\alpha_{1Y}^{-1}(m_r) = \frac{3}{5}\alpha_{1R}^{-1}(m_r) + \frac{2}{5}\alpha_{1(B-L)}^{-1}(m_r).$$

2. At scale $M_R$:

$$\begin{aligned} \alpha_{1R}^{-1}(M_R) &= \alpha_{2R}^{-1}(M_R) - \frac{2}{12\pi}, \\ \alpha_{2R}^{-1}(M_R) &= \alpha_{2L}^{-1}(M_R). \end{aligned}$$

3. At the unification scale $M_U$

$$\begin{aligned} \alpha_{2L}^{-1}(M_U) - \frac{2}{12\pi} &= \alpha_{2R}^{-1}(M_U) - \frac{2}{12\pi} \\ &= \alpha_{U}^{-1}(M_U) - \frac{8}{12\pi}, \\ \alpha_{3c}^{-1}(M_U) - \frac{3}{12\pi} &= \alpha_{U}^{-1}(M_U) - \frac{8}{12\pi}, \\ \alpha_{B-L}^{-1}(M_U) &= \alpha_{U}^{-1}(M_U) - \frac{8}{12\pi}. \end{aligned}$$

The matching conditions at the unification scale have been written by assuming the Pati-Salam scale to be almost close to the unification scale.

Using the above boundary conditions we have numerically solved the equation

| SO(10) Higgs Representation | Higgs multiplets contributing to threshold uncertainty (Decomposed under $G_{3211}$) | $\{a_{3c}, a_{2L}, a_{1R}, a_{1(B-L)}\}$ |
|---|---|---|
| 16 | $(1,1,\frac{1}{2},\frac{1}{2})_{16} \oplus (1,1,-\frac{1}{2},-\frac{1}{2})_{\overline{16}}$<br>$(1,2,0,-\frac{1}{2})_{16} \oplus (1,2,0,\frac{1}{2})_{\overline{16}}$ | $\{0, 1, \frac{1}{2}, \frac{9}{4}\}$ |
| 45 | $(1,3,1,0)_{45}$ | $\{0, 2, 0, 0\}$ |

Table 11.2: Threshold contribution at left-right breaking scale

11.4 for the two-loop RG evolution for all the coupling constants. We have taken the breaking scale of the gauge group $G_{3211}$ to be around 1TeV. The unification scale comes out to be $M_U = 10^{15.4}$GeV and the corresponding coupling constant is found as $\alpha_U^{-1}(M_U) = 43.4$. Also the breaking scale of left-right symmetric gauge group, i.e., $G_{3221D}$ turns out to be $M_R = 10^{11.6}$GeV. The running of the various coupling constants with energy scale are shown in figure 11.1.

However, we find that the scale of the unification along with the $\alpha_U^{-1}$ are not satisfying the most recent bounds on proton decay, although very close to the limit. From our discussion about proton decay in the chapter 6, we know that the most recent proton decay bound [98] is equivalent to (from expression 6.1 )

$$\kappa = \left(\frac{\alpha_{GUT}}{45}\right) \times 10^{2(n-15)} \gtrsim 11.8\,., \tag{11.6}$$

where $M_U \simeq 10^n$GeV. What we obtain for the value of $\kappa$ in our analysis is $\kappa = 6.07$. This is below the lower limit allowed by the proton decay bound as specified in the right-hand side of the expression 11.6. However, the value of $\kappa$ is very close to the allowed lower limit and so we will try to explore the viability of our model by allowing threshold uncertainty in the Higgs spectrum at various intermediate breaking scales. It is important to remark at this point that we could get the the value of $\kappa$ to be so close to the limit only when we optimized certain degrees of freedom in the Higgs sector. For instance, the Higgs-bi-doublet $\Phi$ has been assumed to arise from a real 10-dimensional $SO(10)$ Higgs $H_\Phi$. So $\Phi$ would not be equivalent to two SM Higgs doublets at the electroweak scale but will be equivalent to only one such doublet. Similar assumption has been also taken for $\Psi$. However, we would like to emphasize that the results and discussion of the potential minimization part will remain almost same.

The threshold uncertainty in the Higgs spectrum arises form the fact that the Higgs bosons becoming heavy at a given breaking scale may not get exactly same masses equal to the energy corresponding to the breaking scale. However, the Higgs mass spectrum is expected to be scattered around the energy of the breaking scale

| SO(10) Higgs Representation | Higgs multiplets contributing to threshold uncertainty (Decomposed under $G_{3221D}$) | $\{a_{3c}, a_{2L}, a_{2R}, a_{1(B-L)}\}$ |
|---|---|---|
| 10 | $\left(3, 1, 1 - \frac{1}{3}\right)_{10} \oplus \left(\overline{3}, 1, 1, \frac{1}{3}\right)_{10}$ <br> $\left(3, 1, 1 - \frac{1}{3}\right)_{10'} \oplus \left(\overline{3}, 1, 1, \frac{1}{3}\right)_{10'}$ | $\{2, 0, 0, 2\}$ |
| 16 | $\left(3, 2, 1, \frac{1}{6}\right)_{16} \oplus \left(\overline{3}, 2, 1, -\frac{1}{6}\right)_{\overline{16}}$ <br> $\left(\overline{3}, 1, 2, -\frac{1}{6}\right)_{16} \oplus \left(3, 1, 2, \frac{1}{6}\right)_{\overline{16}}$ | $\{4, 3, 3, 1\}$ |
| 45 | $\left(3, 2, 2, -\frac{1}{3}\right)_{45} \oplus \left(\overline{3}, 2, 2, -\frac{1}{3}\right)_{45}$ <br> $(8, 1, 1, 0)_{45}$ | $\{7, 6, 6, 4\}$ |
| 54 | $\left(6, 1, 1, -\frac{2}{3}\right)_{54} \oplus \left(\overline{6}, 1, 1, \frac{2}{3}\right)_{54}$ <br> $(1, 3, 3, 0)_{54}$ <br> $(8, 1, 1, 0)_{54}$ | $\{8, 6, 6, 8\}$ |

Table 11.3: Threshold contribution at the unification scale

within an small width. For our analysis, we follow a similar approach as discussed in [173]. We assume that the masses of the Higgs bosons are scattered around any breaking scale within the factor of $\frac{1}{30}$ to 30. So if the mass of a Higgs multiplet around a given breaking scale $M_I$ is $M_H$, then we expect

$$\frac{1}{30} \lesssim \frac{M_H}{M_I} \lesssim 30\,.$$

To include the threshold uncertainty at a given breaking scale, we need to slightly modify our matching conditions at that scale. The matching condition given in expression 11.5 is modified as

$$\alpha_G^{-1}(M_I) - \frac{C(G)}{12\pi} = \alpha_{G_i}^{-1}(M_I) - \frac{C(G_i)}{12\pi} - \frac{\lambda_i}{12\pi},$$

where $\lambda_i = a_i \ln\frac{M_H}{M_I}$. So the threshold uncertainty has been included in the matching condition due to presence of the term involving $\ln(M_H/M_I)$.

To avoid any over estimation of the threshold uncertainty we assume that all the Higgs multiplets, belonging to a single common irreducible Higgs representation of $SO(10)$, becoming heavy at a given breaking scale will have the same mass scale around the breaking scale.

The threshold uncertainty at the breaking scale of gauge group $G_{3211}$is vanishing. The Higgs multiplets, coming from different $SO(10)$ irreducible Higgs, contributing to the threshold uncertainty at remaining two intermediate scales, the left-right breaking scale and the unification scale, are listed in the table 11.2 and 11.3, respectively. The corresponding calculated beta-coefficients, $(a_i)$'s, which in-

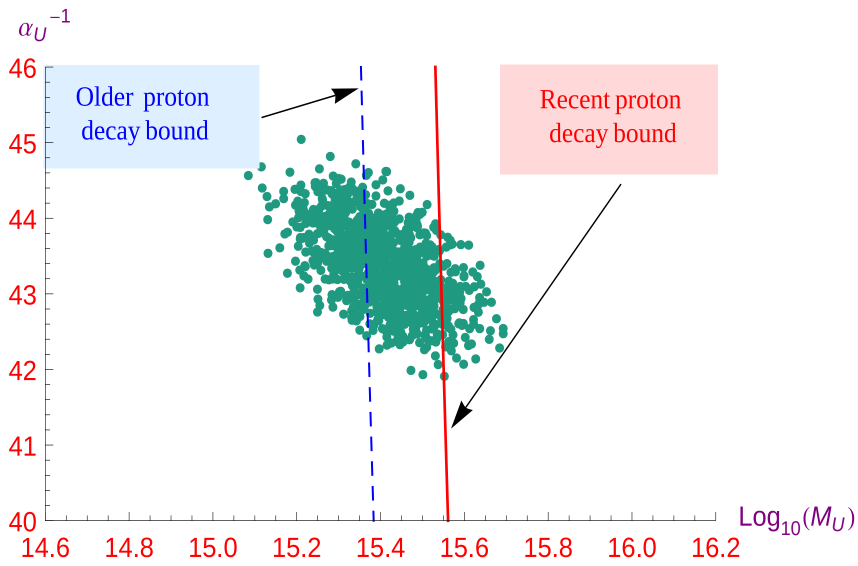


Figure 11.2: Threshold uncertainty in the unification scale.

clude the contribution from all the Higgs multiplets coming from the same $SO(10)$ irreducible representation (as their masses are assumed to be same), are also shown for the two breaking scales.

Now using these calculated $a_i$'s and including uncertainty in $M_H/M_I$, as discussed before, we have shown a scattered-plot between coupling constant $\alpha_U^{-1}$ and the corresponding unification scale $M_U$ in figure 11.2. We have numerically obtained the values for $\alpha_U^{-1}$ and $M_U$ for randomly chosen values for $M_H/M_I$ between the range $\left(\frac{1}{30} - 30\right)$. The random values for all the Higgs multiplets belonging to the same $SO(10)$ irreducible Higgs are taken to be same at one particular breaking scale but different at the other breaking scale.

Moreover, we have also plotted the curve corresponding to the most recent proton decay bound (Red solid curve) [98] and relatively older proton decay bound (blue dashed curve) [174] in figure 11.2 to show the allowed region in $\alpha_U^{-1}$-$M_U$ plane. Only the right part of the curve is allowed by the bound. It is worth noting that the allowed parameter space is more and more constrained as more updated data on proton decay bound is available. However, we get a reasonable allowed region in the figure 11.2, although small, even after allowing the most conservative threshold uncertainty. So we expect our model to be satisfactory within the tolerable amount of threshold uncertainty as far as proton decay bound is concerned.

## 11.5 Yukawa sector and neutrino masses

In the present section, we discuss the origin of neutrino masses in the model. Before proceeding further we would like to make it clear that the discussion about neutrino masses in the present section will only move around the left-right symmetric model with few inputs from the $SO(10)$ GUT in motivating about certain patterns for taken Dirac mass matrices for fermions in our analysis. Moreover, the discussion will be mainly focused on the matrix structure of low energy neutrino mass matrix allowed with certain assumptions. We will also argue, in what follows, that the consistent neutrino mass spectrum is not possible within picture of one or two SO(10) singlet fermions $S$. We start by writing the Yukawa sector of the model as

$$\begin{aligned} \mathcal{L}_Y &= Y_{ij}\,\overline{\ell_{Li}}\,\ell_{Rj}\Phi + Y'_{ij}\,\overline{\ell_{Li}}\,\ell_{Rj}\Psi + (F_L)_{in}\overline{S_{Rn}}\,\ell_{Li}\chi_L + (F_R)_{in}\overline{S^c_{Ln}}\ell_{Ri}\,\chi_R && (11.7)\\ &+ \frac{1}{2}M_{mn}\eta\overline{S^c{}_{Lm}}S_{Rn} && (11.8)\end{aligned}$$

The Yukawa couplings $Y$ and $Y'$ are $3\times3$ matrix, while $F_L$ and $F_R$ are $3\times n$ matrices, if we assume that there are $n$ singlet fermions $S$. So $M$ is a $n\times n$ matrix. Our study of consistent embedding of the model in $SO(10)$ GUT requires same structure for both $F_L$ and $F_R$ up to the scale of left-right symmetry breaking which, after RG running, can produce small difference at the weak scale. For the present discussion we assume it to be small enough so that it can be safely ignored.

The Dirac masses for all the SM fermions including neutrinos are generated form the the first two terms by giving *vev* to the bi-doublets as in any other left-right symmetric model. Since $\Phi$ and $\Psi$ are coming from two independent and real $SO(10)$ 10-dimensional Higgs, the Dirac mass matrix for neutrinos and charged leptons are independent. However, the Dirac mass matrix for the up-type quarks have the same structure as the Dirac mass matrix for the neutrinos and similarly the Dirac mass matrix for the down-type quarks will have similar structure as the Dirac mass matrix for the charged leptons (simply because all SM fermions are assigned to a multiplet of $SO(10)$ GUT). Although these similarities in the structures are exact only at the GUT scale, we expect some of its features to be more or less same even at the low scale. So we can well assume that the Dirac mass matrix of the neutrinos would almost appear diagonal in the basis where the charged lepton mass matrix is diagonal. The assumption is based on the observation that the up-type and down-type quarks are simultaneously diagonal in the a basis as the quark mixing matrix is very close to unity. So we borrow the pattern from the quark sector to the lepton sector where the structure of Dirac mass matrix of the neutrinos is not directly known unless neutrinos are Dirac fermions. We expect the following pattern of the Dirac mass matrix of neutrinos in the diagonal basis of the charged leptons

$$M_{\nu D} = vY_{lepton}\left(\frac{m_t}{m_b}\right) = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}\left(\frac{m_t}{m_b}\right) \simeq v \begin{pmatrix} 0.0001 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.3 \end{pmatrix},$$

where $m_t$ and $m_b$ are masses of top and bottom quarks and $m_e$, $m_\mu$, $m_\tau$ are masses of electron, muon and tau leptons.

The part of the Lagrangian relevant for the neutrino mass generation is given as follows,

$$\begin{aligned} \mathcal{L}_{\nu\ mass} &= \begin{pmatrix} \nu & N^c, & S^c \end{pmatrix}_L . X . \begin{pmatrix} \nu \\ N^c \\ S^c \end{pmatrix}_L + H.C. \\ &= \begin{pmatrix} \nu_i & N_i^c & S_m^c \end{pmatrix}_L \begin{pmatrix} 0 & Y_{ij}v & F_{in}v_L \\ \left(Y_{ij}\right)^T v & 0 & F_{in}v_R \\ F_{mj}^T v_L & F_{mj}^T v_R & M_{mn}u \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j^c \\ S_n^c \end{pmatrix}_L + H.C. \end{aligned}$$

We can easily identify this structure by the type III seesaw structure given in expression 5.4. In fact the our discussion here is just the extension of the previous discussions in subsection 6.2.2 and section 5.4.2.

Our first task is to analyze the mass spectrum provided by the matrix $X$ in case of one generation of all fermions. We write the eigenvalue equation as (eigenvalue: $\lambda$):

$$\lambda^3 - Mu\,\lambda^2 - F^2 v_R^2 \lambda - 2YF^2 v v_L v_R - MY^2 u v^2 = 0$$

**Case 1:** $\lambda >> v$**,** we get

$$\lambda\,(\lambda + Fv_R)\;(\lambda - Fv_R) = 0$$

The above eigenvalue equation predicts two TeV scale Majorana fermions. The massless solution contradicts with the condition we started with, and so is unphysical.

**Case 2:** $\lambda << v$**,** we get

$$\lambda = -\frac{2Yvv_L}{v_R} + \frac{MY^2uv^2}{F^2v_R^2},$$

$$\lambda = -\frac{2Yvv_L}{v_R} + \frac{MY^2uv^2}{F^2v_R^2} \tag{11.9}$$

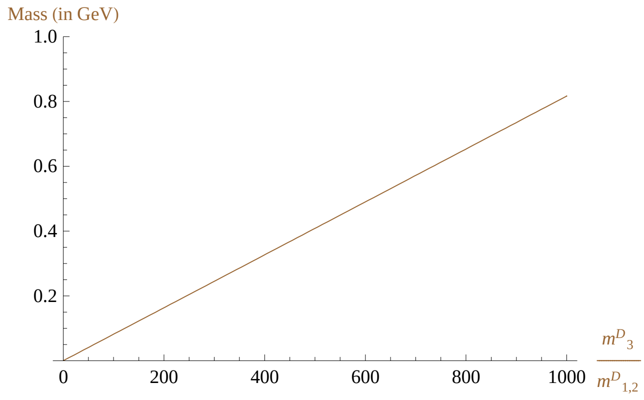


Figure 11.3: Variation of mass

which is of order of eV. So the two Majorana fermions pick up masses of the order as high as TeV and one remains sufficiently light (~eV) to be identified as light neutrino.

To make the discussion some more general, we take three generations for all the SM fermions including the left and right handed neutrinos but only one generation for the singlet $S$. We look for a possibility whether it can account for the existing picture of three light active neutrinos. To search for any such possibility, we try to find out the mass spectrum, within this scenario, by solving for the eigenvalues of the matrix $X$. To simplify further, we take all the eigenvalues of the matrix $M_{\nu D}$ to be same with a common value equal to the largest one for initial analysis. This enable us to factor out $\left(\lambda^2 - z^2 v^2\right)^2$ from the algebraic expression of $\mathrm{Det}\left(X\right)$ predicting four Majorana fermions of scale around 10GeV. The rest of the factors have got the same form as the expression of determinant in case of one generation of all SM fermions, as discussed earlier, leading to the two TeV and one eV scale Majorana fermions. The scenario provides us only one light neutrino and, hence, can not account for the observed neutrino mass spectrum. To explore the effect of some possible hierarchy present in the eigenvalues of the Dirac mass matrix of the neutrino like one present in the charged lepton mass matrix, we take two of the eigenvalues to be same and vary their scale below the third one. We are still able to explicitly get two of the Majorana fermions having mass scale equal to $m_e\left(\frac{m_t}{m_b}\right)$. One may think that the remaining two Majorana fermions might get mass scale as light as eV leading to three light neutrinos. To rule out any such possibility, we have plotted the masses of the two remaining Majorana fermions (which comes out to be same) with the ratio of the two mass scales of the eigenvalues of the Dirac mass mass matrix of the neutrinos in figure 11.3. We find that the masses do not go below the lightest mass scale of the eigenvalues of $m_{\nu D}$. So in two generation scenario of $S$ fermions, there is not much progress except we get two eV scale Majorana fermions which is still not sufficient.

We now turn to the case of three generation for $S$ fermions. The basic way to get the low energy neutrino mass matrix has been outlined in [91] which is given as

$$
\begin{aligned}
m_\nu &= -\left(\frac{v v_L}{v_R}\right)\left(Y+Y^T\right)+\left(\frac{u v^2}{v_R^2}\right) Y\left(F M^{-1} F^T\right)^{-1} Y^T , \\
&= -\left(\frac{v v_L}{v_R}\right)\left[\left(Y+Y^T\right)+r Y\left(F M^{-1} F^T\right)^{-1} Y^T\right] , \qquad (11.10)
\end{aligned}
$$

as we have $uv^2 = r\, v v_L v_R$ in our model (expression 11.1 ) where $r = \left(\lambda_\chi - g_\chi\right)/h_\chi$.

The first term is the type-III seesaw contribution [91] and the second term is the double seesaw contribution. With the choice of the *vevs*, it is obvious that this scenario provides us with three eV neutrinos.

Now we will try to explore the limits of the expression 11.10 for low energy neutrino mass matrix to check its consistency with current data on neutrino masses and mixing by allowing some very simple form for matrix $M$.

In its most general form, it is straight forward to argue that $m_\nu$ can accommodate the existing data on neutrino masses and mixing simply due to the presence of enough number parameters in $F$ and $M$ unless type III term dominates significantly. An interesting thing would be to consider some simpler form of the neutrino mass matrix by reducing appropriate number of parameters with some tolerable assumptions. The basic idea is to explore the possibility of any such simpler structure in light of the current neutrino oscillation data.

We start with the assumption that the three singlet fermions $S$ are blind to their generation within themselves leading to the following democratic structure of matrix $M$ :

$$
M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} u
$$

The structure allows us to believe that there is no induced mixing between the left-right neutrinos and the singlets. So, $F$ matrix can be written as product of a unitary matrix and a diagonal matrix. The unitary matrix connects the basis of the democratic structure to the basis where the charged lepton mass matrix becomes diagonal. To have some more simplicity, we are driven to assume that the two basis are identical, i.e., the unitary mass matrix is identity matrix. It leads to the following structure of the low energy neutrino mass matrix:

$$m_\nu = \frac{v v_L}{v_R}\begin{pmatrix} \alpha^2 - 2\frac{m_t}{m_b}m_e & \alpha\beta & \alpha\gamma \\ \alpha\beta & \beta^2 - 2\frac{m_t}{m_b}m_\mu & \beta\gamma \\ \alpha\gamma & \beta\gamma & \gamma^2 - 2\frac{m_t}{m_b}m_\tau \end{pmatrix},$$

where $\alpha$, $\beta$ and $\gamma$ are the final parameters appearing in the neutrino mass matrix after absorbing all the parameters present in $F$, $M$ and $Y$. We take the following familiar tri-bimaximal form of [175] of the $U_{PMNS}$ mixing matrix for our discussion and attempt to diagonalize $m_\nu$ having above structure:

$$U_{PMNS} = U_{tbm} = \frac{1}{\sqrt{6}}\begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix},$$

where $\theta_{23} = \pi/4$, $\theta_{13} = 0$, and $\sin^2\theta_{12} = 1/3$.

We attempt to diagonalize $m_\nu$ with the tri-biamaximal form of the mixing matrix which requires the following relation of the parameters $\alpha$, $\beta$ and $\gamma$ with masses of the charged leptons as:

$$\begin{aligned} \alpha &= 0 \\ \beta &\simeq \frac{m_\mu}{\sqrt{\frac{m_b}{2m_t}\left(m_\tau + m_\mu\right)}} \simeq 0.05 \\ \gamma &\simeq -\frac{m_\tau}{\sqrt{\frac{m_b}{2m_t}\left(m_\tau + m_\mu\right)}} \simeq -0.75 \end{aligned}$$

The diagonal neutrino mass matrix comes out of the form:

$$m_\nu^{Daig} \simeq -\frac{2m_t}{m_b}\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_e & 0 \\ 0 & 0 & 2\frac{m_\mu m_\tau}{(m_\mu + m_\tau)} \end{pmatrix}\left(\frac{v v_L}{v_R}\right)$$

So the present form of $m_\nu$ and $U_{PMNS}$ produces degenerate masses for the two light neutrinos which is likely to be cured once we slightly deviate from tri-bimaximal form of $U_{PMNS}$. The deviation can be realized either by taking non-maximal value of $\theta_{23}$ or non vanishing value of $\theta_{13}$ or both. We take only non-zero value of $\theta_{13}$ to be the sole realization of the deviation for our purpose. The deviated form of tri-baimaximal matrix for very small value of $\theta_{13}$ can be parametrized as:

$$U_{PMNS} = \frac{1}{\sqrt{6}}\begin{pmatrix} 2 & \sqrt{2} & \theta_{13} \\ -1 - \sqrt{2}\theta_{13} & \sqrt{2} - \theta_{13} & \sqrt{3} \\ 1 - \sqrt{2}\theta_{13} & -\sqrt{2} - \theta_{13} & \sqrt{3} \end{pmatrix}$$

While trying to diagonalize the $m_\nu$, numerical methods are used to find out the desired values of the free parameters. We find that the degeneracy encountered in the case of tri-bimaximal mixing matrix disappears as soon as finite value of $\theta_{13}$ is introduced. This finite value is determined by imposing the condition $\Delta m^2_{21}/\Delta m^2_{31} \simeq 0.033$ which leads to following value of $\sin\theta_{13}$

$$\sin\theta_{13} = 0.11.$$

The value is well within the allowed value for $\theta_{13}$ from oscillation data. The correct scale of the mass square differences is easily achieved by adjusting the over all scale of the neutrino mass matrix. The corresponding values of the other parameters come out to be

$$\begin{aligned} \alpha &= 0.02 \\ \beta &= 0.06 \\ \gamma &= -0.75 \end{aligned}$$

The point we would like to emphasize is that even the simple structure of the mass matrix taken in our analysis is able to account for the existing framework of three active light neutrinos even though the assumptions may not correspond to any real underlying symmetry.

## 11.6 Dark energy in the model

We shall now discuss the implementation of the NDE mechanism in our model. For simplicity, we consider only one-generation scenario. We assume that the singlet mass $M_s = M\langle\eta\rangle = M\,u$ varies with the acceleron field $\mathcal{A}$, so that the neutrino mass becomes a dynamical quantity. This gives the coupling between the neutrinos and the acceleron, which stops the dynamical evolution of the acceleron fields when the neutrinos become non-relativistic. When the neutrinos become non-relativistic the dependence of $M_s$ on $\mathcal{A}$ governs the dynamics of the DE.

To compare the neutrino mass scale with the DE scale we write the effective potential using the Coleman-Weinberg type [176]

$$V_0 = \Lambda^4 \log(1 + |\bar{\mu}/M_s(\mathcal{A})|)\,. \tag{11.11}$$

The parameter $\Lambda$ $\left(\sim 10^{-3}\mathrm{eV}\right)$ is chosen to fit the DE scale. This type of potentials are extensively used in the DE literature [27, 166]. We can write the effective low-energy Lagrangian in our model

$$-\mathcal{L}_{eff} = M_s(\mathcal{A})\frac{Y^2}{F^2}\frac{v^2}{v_R^2}\,\nu_i\nu_j + H.c. + \Lambda^4\log(1+|\bar{\mu}/M_s(\mathcal{A})|)\,, \tag{11.12}$$

From the choices we have made about the *vev*s, we have retained only the dominant double seesaw term 11.9 in the effective Lagrangian. As $u \sim O(\text{eV})$, the mass parameter $M_s$ is of the order of eV. Since the ratio $(v/v_R)^2 \sim 10^{-2} - 10^{-3}$, the Yukawa couplings coupling to be of order unity. Thus the first two terms in equation 11.12 are comparable to the last term describing the DE potential.

The Majorana mass of neutrino varies with the acceleron field through the parameter $M_s$ and the mass scale of this parameter remains near the scale of DE *naturally*. The interesting feature of our model is that we do not need any unnaturally small Yukawa couplings or symmetry breaking scale to achieve this naturalness requirement. Also the variation of $M_s$ does not affect charged fermion masses in the model. Moreover, the electroweak symmetry breaking scale $v$ and the $U(1)_R$ breaking scales are comparable and hence the new gauge boson corresponding to the group $U(1)_R$ will have usual mixing with $Z$ and should be accessible at LHC.

Since the local minimum of the potential relates the neutrino mass to a derivative of the acceleron potential, the value of the acceleron field gets related to the neutrino mass. The acceleron field provide an effective attractive force between the neutrinos. When the this effective force is stronger than the gravity, perturbations in the neutrino-acceleron fluid become unstable. The source of the free-energy comes from the attractive interaction between the neutrino and the acceleron field. The instability is similar to that of the Jeans instability found in a self-gravitating system. The instability can lead to inhomogeneity and structure formation; the instability would grow till the degeneracy pressure of the neutrinos would arrest the growth. The final state of the instability would produce neutrino lumps or nuggets [166, 177]. The neutrino lumps would then behave as dark matter and will not affect the dynamics of the acceleron field [178]. This instability is a generic feature of MaVaNs scenario, however it can be suppressed if the neutrino become superfluid [179] or if the MaVaNs perturbations become nonadiabatic.

# Chapter 12

# Conclusion

The charge neutrality of neutrinos opens up two possibility for their fermionic nature. They can be either Dirac fermions with separate particle-antiparticle identity or Majorana fermions where it is its own antiparticle. The second possibility is phenomenologically very rich and several seesaw mechanism exist in literature to naturally explain the tiny masses of neutrinos.

In the case of Majorana neutrinos, the neutrino mass matrix is symmetric and contains all the information regarding CP violating phases in the basis where charged lepton mass matrix is diagonal. To explore this fact, we have constructed rephasing invariant measures of CP violation with elements of the neutrino masses in the weak basis. For an $n$-generation scenario, in the absence of any texture zeroes there are $n(n-1)/2$ independent measures of CP violation, given by

$$I_{ij} = \text{Im}\,[m_{ii} m_{jj} m^*_{ij} m^*_{ji}] \qquad (i < j)$$

which corresponds to $n(n-1)/2$ independent CP violating phases. Only $(n-1)(n-2)/2$ of these phases of CP violation can contribute to the neutrino oscillation experiments and are independent of the Majorana phases for which the rephasing invariant measures of CP violation can be defined as

$$J_{ijn} = \sum_{a,b,c} \text{Im}\left[(m^*_{ia}\, m_{ja})\,(m^*_{jb}\, m_{nb})\,(m^*_{nc}\, m_{ic})\right] \qquad (i < j < n).$$

We then defined invariants for mass matrices with texture zeroes and elaborated with some examples. We studied all the phenomenologically acceptable 3-generation two-zero texture neutrino mass matrices. We showed that there are no Majorana phase in any of the allowed cases.

The neutrinos are likely to play a very important role in explaining the baryon asymmetry of the present universe [28, 73]. Many interesting models, beyond the SM, proposed to accommodate the neutrino masses come with an extra feature that they can create lepton asymmetry in the early universe which can be converted to baryon asymmetry in the later phase of the universe. The elegant seesaw framework is the most preferable scenario for the the mechanism known as baryogenesis via leptogenesis.

We have studied the connecting links between the CP violating phases giving rise to leptogenesis, occurring at a high energy scale, and the CP violating phases appearing in the low energy phenomena, i.e., neutrino oscillation and neutrinoless double beta decay processes. This is studied in the framework of two right-handed neutrino models. The low energy leptonic CP violation is studied in a rephasing invariant formalism. It is shown that there are only two rephasing invariants; (1) the lepton-number conserving CP violating rephasing invariant $J_{CP}$ which can be determined in the future long-baseline neutrino oscillation experiments, (2) the lepton-number violating CP violating rephasing invariant $J$ which can be determined in the neutrinoless double beta decay experiments. It is found that there is no one-to-one correspondence between these two CP violating phenomena, occurring at two different energy scales, even though the number of parameters involving in the seesaw is exactly same as the number of low energy observable parameters. However, in a suitable parameter space we have shown that the overlapping regions in the plane of $n_B/n_\gamma$ versus $J_{CP}$ and $n_B/n_\gamma$ versus $J$ can indeed determine the *sign* of the matter antimatter asymmetry of the present Universe assuming that the *size* of the asymmetry is precisely known.

Neutrinos are also expected to address the problems of DE [27]. NDE models are able to answer the cosmic coincidence puzzle of DE. Although such models have been constructed in seesaw scenario, there were no left-right or GUT models to accommodate the NDE proposal. For the first time, we have constructed a left-right symmetric model of NDE that can be embedded in an $SO(10)$ GUT. After discussing the Higgs content needed for the model, details of potential minimization have been carried out considering all possible allowed terms. The complete analysis allows the desired ordering of the *vev*s. Then we have studied the embedding of this left-right symmetric model in $SO(10)$ GUT. We show that $SO(10)$ GUT with Higgs multiplets $S(54)$, $A(45)$, two $H(10)$, $C(16) \oplus \overline{C(16)}$, $\eta(1)$ along with an additional fermion singlet is able to accommodate the left-right symmetric model.

We have studied the RG running of various couplings constant and have found that the desired assignment for *vev* values for different Higgs fields is consistent with the gauge unification. Then the origin and possible structure of neutrino masses has been discussed in detail. It has been shown that generation of three light active neutrinos of eV scale is not possible in scenario with one or two $SO(10)$ singlets fermions. Then we have described the implementation of NDE in the model. The model allows the mass parameter of the left-right group singlet, which varies with the acceleron field, to have the same scale as the scale of DE satisfying the desired naturalness requirement.

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