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THE FRINGES OF THE THALTEJ RADIOTELESCOPE

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August 1991

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C O N T E N T S

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(A) INTRODUCTION

The Thaltej Radiotelescope (near Ahmedabad) is a correlation interferometer working at a frequency of 103MHz. It consists of a filled aperture full wave dipole array which is 20,000 sq.m in area. The technical details about the telescope are given in Alurkar et al (1987) and in Sharma (1988).

The Thaltej correlation interferometer radiotelescope gives a SIN channel output and a COS channel output. The equations for the envelope, the cosine fringe and the sine fringe are as follows (Sharma, 1988)

$$\text{Envelope } E = \frac{k \sin^2[a\,(\delta - \delta_o)]}{a^2\,(\delta - \delta_o)^2} \qquad (1a)$$

$$\text{cosine fringe} \quad COS = E \cos[b\,(\delta - \delta_o)] \qquad (1b)$$

$$\text{sine fringe} \quad SIN = E \sin[b\,(\delta - \delta_o)] \qquad (1c)$$

where k is a constant of proportionality

a is a constant depending on antenna parameters and it decides half power beamwidth of the beam

b is a constant depending on λ/D and as will be

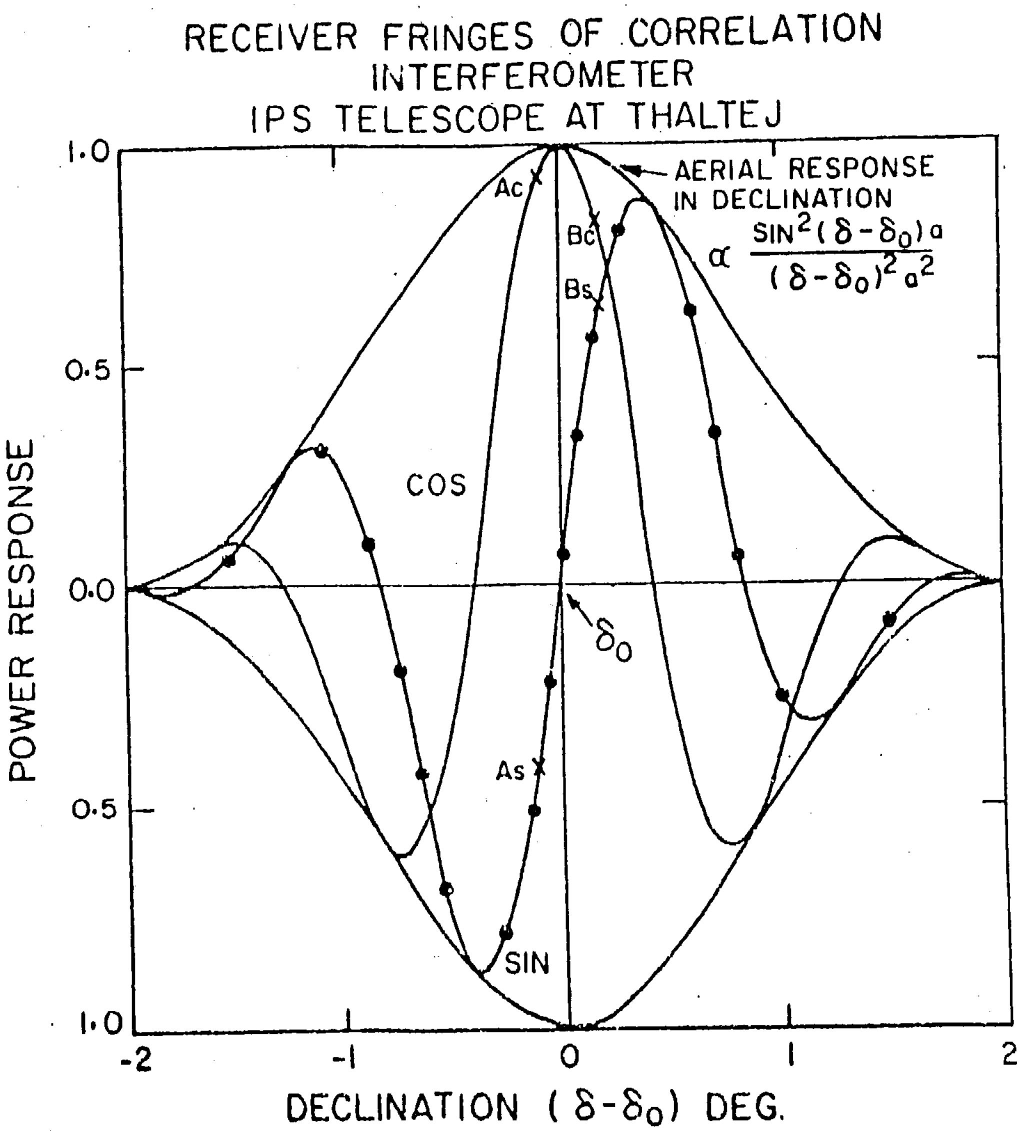


Fig. 1 Receiver fringes of correlation interferometer of radio telescope at Thaltej. A_c, B_c and A_s, B_s are assumed positions of 3C 298 on COS and SIN fringes.

(Alurkar et al., 1985).

shown below its value can be predicted by theory. (λ is wavelength and D is spacing of interferometer)

δ_o is the declination towards which the centre of the beam is pointing (from now onwards δ_o will be referred to as the declination of the centre of the beam)

δ is the declination of the radio source

Uptil now the sign of the sine fringe in eq.(1c) was not known for the Thaltej radio telescope. The aim of this project was to find out whether this sign is negative or positive. (In fig.(1), positive sign has been assumed for plotting sine fringe).

(B) SOME DEFINITIONS

Half power beam width : The half power beamwidth is the angular distance between points at which the power response of the telescope is half of what it is at the centre of the beam. It is denoted by h.

Beam seperation: The peak to peak angular distance between the adjacent beams is the angle between the centres of consecutive beams. It is denoted by Δ.

h and Δ need not be identical though for the Thaltej radio telescope they are quite close. According to Sharma (1988) they are supposed to be

$$h \cong 1.8^{o}$$
$$\Delta \cong 2.0^{o}$$

Δ is independent of the envelope (i.e.aerial response in declination) for a particular beam.and it depends only λ/D as will be shown below .(though it increases slightly with increasing beam number).h however depends on antenna parameters since in the envelope function

$$\text{Envelope } E = \frac{k \sin^{2}[a(\delta - \delta_o)]}{a^{2}(\delta - \delta_o)^{2}}$$

the constant a depends on the antenna parameters.

[C] Theoretical Model

We have made a simple mathematical model ,of the correlation interferometer ,which gives the sign of the sine fringe. Let us consider the radio telescope as a two array interferometer .As shown in fig.(2) D is the physical distance between one row of dipoles in north half and corresponding row in south half of the antenna .The path difference S' between the radiowave reaching A and that reaching B is

$$S' = D \sin \theta \qquad (2)$$

$$\theta = |\delta_A - \delta_B|$$

where δ_A is declination of local zenith. $\delta_A = 23.044^{\circ}$ for Thaltej.

δ_B is declination of radio source

Now in the Thaltej radio telescope a path difference of $\lambda/2$ is added to the output from one of the antennas .This is done in order to shift the beams such that there is no beam pointing straight up towards declination δ_A Hence we get symmetrically placed beams about AC .Thus the net path difference is given by

$$S = D \sin \theta \pm \lambda/2 \qquad (3)$$

If the correlated voltage at A is V_1 and at B is V_2 then let

$$V_1 = A \cos \omega t \qquad (4a)$$

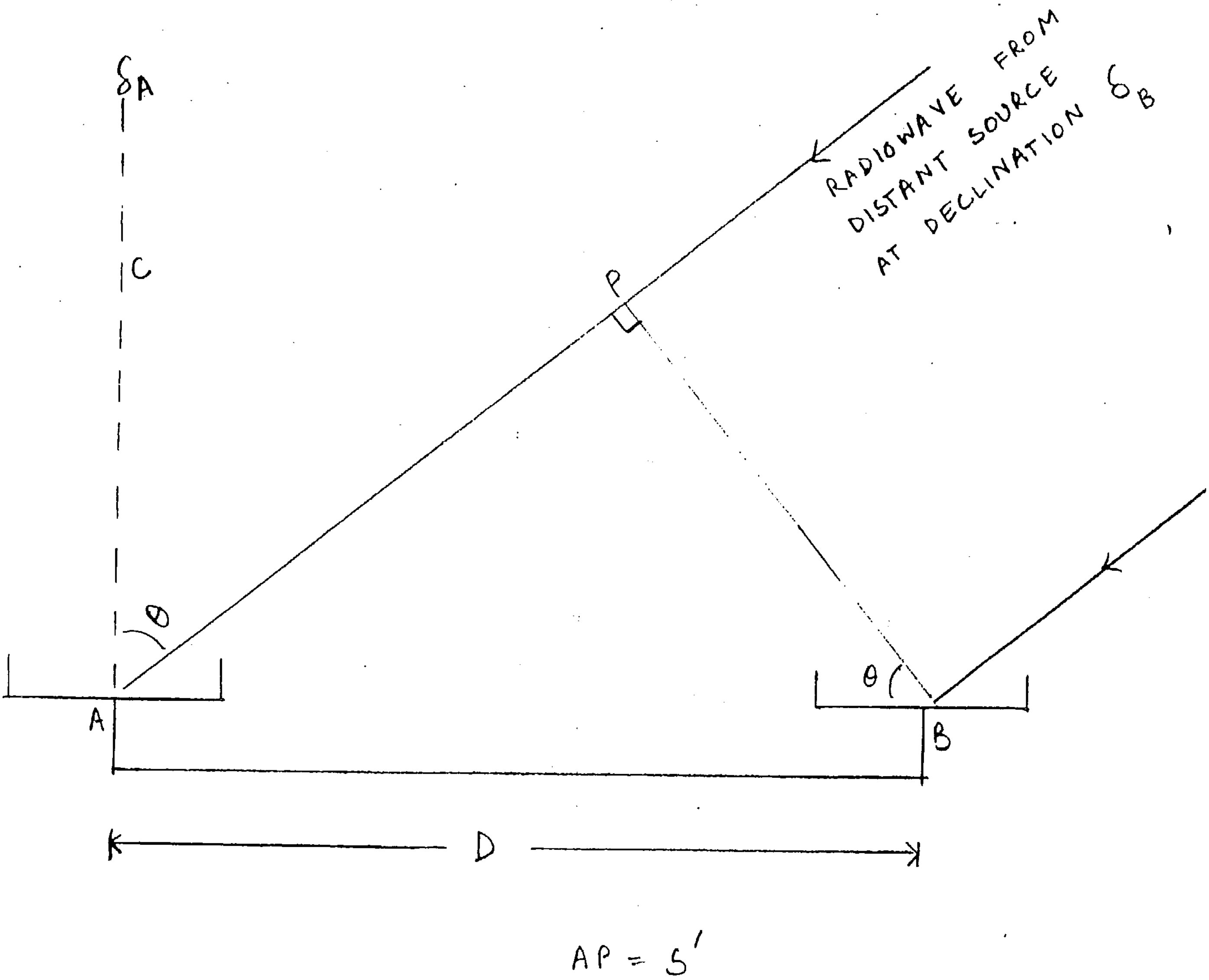
$$V_2 = c\, A \cos(\omega t + \phi) \qquad (4b)$$

where

A is amplitude

ω is angular frequency

ϕ is phase difference



δA
C
θ
A
P
RADIOWAVE FROM
DISTANT SOURCE
AT DECLINATION δB
θ
B
D
AP = S'

FIG. 2

c is a factor introduced to account for any differences in gain between the two antennas (c is close to one).

A path difference of $S = D \sin \theta \pm \lambda/2$ corresponds to a phase difference of ϕ where

$$\phi = 2 \pi/\lambda \; (D \sin \theta \pm \lambda/2) \tag{5}$$

We get

$$V_1 V_2 = c A^2 \cos(\omega t) \cos(\omega t + \phi)$$

$$V_1 V_2 = \frac{c A^2}{2} \left[\cos(2 \omega t + \phi) + \cos(\phi) \right] \tag{6}$$

Hence the output of the COS channel will be proportional to $C(\theta)$ where

$$C(\theta) = \frac{1}{2T} \int_{-T}^{T} V_1 V_2 dt \tag{7}$$

$$C(\theta) = \frac{c A^2}{4T} \int_{-T}^{T} \left[\cos(2 \omega t + \phi) + \cos\phi \right] dt$$

$$C(\theta) = \frac{c A^2}{8 T \omega} \left[\sin(2 \omega T + \phi) + \sin(2 \omega T - \phi) \right] + \frac{c A^2}{2} \cos \phi$$

$$C(\theta) = \frac{c A^2 \sin(2 \omega T) \cos\phi}{4 T \omega} + \frac{c A^2}{2} \cos \phi \tag{8}$$

Now T is the integration time and ω is the angular frequency .For the Thaltej radio telescope

$$T = 0.1 \text{ sec}$$

$$\omega = 2\pi\nu = 6.47 \times 10^{8} \text{ radians/sec}$$

Since the signal is weak ,amplitude A is small .hence first term in eq.(8) is negligible .Thus we get

$$C(\theta) = \frac{c A^{2}}{2} \cos\phi$$

$$C(\theta) = \frac{c A^{2}}{2} \cos\left[\frac{2\pi D \sin\theta}{\lambda} \pm \pi\right] \qquad (9)$$

$$C(\theta) = -\frac{c A^{2}}{2} \cos\left[\frac{2\pi D \sin\theta}{\lambda}\right] \qquad (10)$$

For getting the SIN channel output ,before multiplying V_1 and V_2 ,a phase difference of $\pi/2$ is added or subtracted from V_2 .Adding a phase difference of $\pi/2$ to V_2 is equivalent to subtracting a phase difference of $\pi/2$ from V_1 .Subtracting a phase difference of $\pi/2$ from V_2 is equivalent to adding a phase difference of $\pi/2$ to V_1 .

Hence

$$V_1 = A \cos \omega t$$

$$V_2 = c A \cos(\omega t + \phi \pm \pi/2)$$

If we proceed as above we get the result that the output of the SIN channel will be proportional to $S(\theta)$ where

$$S(\theta) = \frac{c A^{2}}{2} \cos(\phi \pm \pi/2)$$

$$S(\theta) = \frac{c\ A^2}{2} \cos\left[\frac{2\ \pi\ D}{\lambda} \sin\ \theta \pm \pi \pm \pi/2\right]$$

$$S(\theta) = -\frac{c\ A^2}{2} \cos\left[\frac{2\ \pi\ D}{\lambda} \sin\ \theta \pm \pi/2\right] \qquad (11)$$

Hence

$$\frac{SIN}{COS} = \frac{S(\ \theta)}{C(\ \theta)} = \frac{\cos\left[\frac{2\ \pi\ D}{\lambda} \sin\ \theta \pm \pi/2\right]}{\cos\left[\frac{2\ \pi\ D}{\lambda} \sin\ \theta\right]} \qquad (12)$$

If we choose + sign in $\pm\pi/2$ we get

$$\frac{SIN}{COS} = -\frac{\sin\left[\frac{2\ \pi\ D}{\lambda} \sin\ \theta\right]}{\cos\left[\frac{2\ \pi\ D}{\lambda} \sin\ \theta\right]}$$

$$\frac{SIN}{COS} = -\tan\left[\frac{2\ \pi\ D}{\lambda} \sin\ \theta\right] \qquad (13)$$

If we choose − sign in $\pm\pi/2$ we get

$$\frac{SIN}{COS} = +\tan\left[\frac{2\ \pi\ D}{\lambda} \sin\ \theta\right] \qquad (14)$$

This means that the sign of the sine fringe depends on where the difference of $\pi/2$ is added or subtracted .If phase of $\pi/2$ is added to antenna which is in the direction of source (this is equivalent to subtracting phase $\pi/2$ from antenna away from source)then eq.(13) is true .If phase of $\pi/2$ is subtracted from the antenna which is in the direction of source (this is equivalent to adding phase $\pi/2$ to antenna away from source)then eq.(14) is true .For example if antenna A is in north (see fig.(2)), B is in south and source is in south then if phase $+\pi/2$ is added to output from B then eq.(13) is true .

Thus provided we know direction of source and in which antenna phase difference of $\pm\pi/2$ is added we can predict the sign of the sine fringe .Hence the work done by the author gives a theoretical soluton of the problem of the sign of the sine fringe .

(D) THE POSITIONS OF THE CENTRES OF THE BEAMS

If δ_o is the centre of a beam then for maximum sensitivity we need to locate it such that $C(\theta)$ is apositive maximum .Hence from eq.(9) we get that δ_o should satisfy

$$C(\theta) = \frac{c\, A^2}{2} \cos\left[\frac{2\,\pi\, D}{\lambda} \sin\theta \pm \pi\right] = \frac{c\, A^2}{2} \qquad (15)$$

Now

$$\theta = |\delta_o - \delta_A|$$

Hence eq.(15) implies

$$\frac{2\,\pi\, D}{\lambda} \sin|\delta_o - \delta_A| \pm \pi = 2\, n\, \pi$$

where n is an integer .

$$\sin|\delta_o - \delta_A| = (n \mp 1/2)\, \lambda/D \qquad (16)$$

If we choose − sign then n =1,2,3,.... .If we choose + sign then n =0,1,2,3,.... .Choosing − sign we get

$$|\delta_o - \delta_A| = \arcsin\left[(n - 1/2)\, \lambda/D\right] \qquad (17)$$

Hence

$$\delta_o = \delta_A + \arcsin\left[(n - 1/2)\, \lambda/D\right] \quad \text{if } \delta_o > \delta_A \qquad (18)$$

$$\delta_o = \delta_A - \arcsin\left[(n - 1/2)\, \lambda/D\right] \quad \text{if } \delta_o < \delta_A \qquad (19)$$

Here n =1,2,3,... .

The equations (18) and (19) agree with the formula for the position of beam peaks given by Delaney (1962) though Delaney derives the formula for the position of beam peaks by another method

For the Thaltej radio telescope

$\delta_A = 23^{\circ}\ 02'\ 39'' = 23.044^{\circ}$

$\lambda = 2.91$ m

$D = (32)\ (2.6) = 83.2$ m

$\lambda/D = 0.034976 \approx 1/28.6$

Using these values of the constants we have prepared table 1 .From the table we can see that Δ increases gradually with increasing beam number .

TABLE 1

DELTA FOR RIGHT BEAMS

BEAM NO.	DELTA δ_o	Δ
1	24.046	0.000
2	26.050	2.005
3	28.059	2.008
4	30.073	2.015
5	32.097	2.023
6	34.131	2.035
7	36.181	2.049
8	38.247	2.066
9	40.334	2.087
10	42.444	2.111
11	44.583	2.138
12	46.753	2.170
13	48.961	2.207
14	51.210	2.249
15	53.508	2.298
16	55.861	2.353

TABLE 1

DELTA FOR LEFT BEAMS

BEAM NO.	DELTA δ_o	Δ
1	22.042	0.000
2	20.038	2.005
3	18.029	2.008
4	16.015	2.015
5	13.991	2.023
6	11.957	2.035
7	9.907	2.049
8	7.841	2.066
9	5.754	2.087
10	3.644	2.111
11	1.505	2.138
12	-0.665	2.170
13	-2.873	2.207
14	-5.122	2.249
15	-7.420	2.298
16	-9.773	2.353

(E) A USEFUL APPROXIMATION

We have

$$\frac{SIN}{COS} = \pm\tan\left[\frac{2\ \pi\ D}{\lambda}\sin\ \theta\right]$$

Let $\delta_A > \delta_B$ (if $\delta_A < \delta_B$ then discussion is similar).

$$\theta = \delta_B - \delta_A$$
$$\alpha = \delta_B - \delta_o \qquad (20)$$
$$\beta = \delta_o - \delta_A$$

Hence

$$\frac{SIN}{COS} = \pm\tan\left[\frac{2\ \pi\ D}{\lambda}\sin\ (\ \alpha + \beta)\right]$$

$$= \pm\tan\left[\frac{2\ \pi\ D}{\lambda}(\ \sin\ \alpha\ \cos\ \beta + \cos\ \alpha\ \sin\ \beta\ )\right] \qquad (22)$$

Since δ_B is declination of source which is seen in beam whose centre is δ_o we have that $\alpha = \delta_B - \delta_o$ is small .

$$-2^o < \alpha^o < 2^o$$

If α is in radians then to a very good approximation $\sin\ \alpha = \alpha$ and $\cos\ \alpha = 1$.

If we also substitute eq.(16) for $\sin\ \beta$ and simplify then we get

$$\frac{SIN}{COS} = \pm\tan\left[\frac{2\ \pi\ D}{\lambda}(\cos\ \beta)\ \alpha\right] \qquad (23)$$

Hence

$$\frac{SIN}{COS} = \pm\tan\left[b\ (\ \delta_B - \delta_o)\ \right] \qquad (24)$$

Where δ_B is declination of source , δ_o is declination of centre of beam , b is a constant depending on beam number .

If we go from beam 1L to beam 10L then β increases from about 1° to about 20° .The corresponding changes in cos β is from 1.00 to 0.94 . Hence b is remarkably unaffected by the changes in β .It is almost the same for all the beams .For the Thaltej radio telescope ,if we take cos β =1 we get $b = 2\pi D/\lambda = 179.7$ .In fig.(3) we have plotted graph of SIN/COS where

$$\frac{\text{SIN}}{\text{COS}} = \pm\tan\left[\frac{2\pi D}{\lambda}\sin(\alpha+\beta)\right] \qquad (25)$$

β is constant for a particular beam while α varies from -2° to $+2^{\circ}$ for each graph .These graphs provide visual evidence that eq.(25) is remarkably immune to changes in β .Hence eq.(24) is a very good approximation indeed .

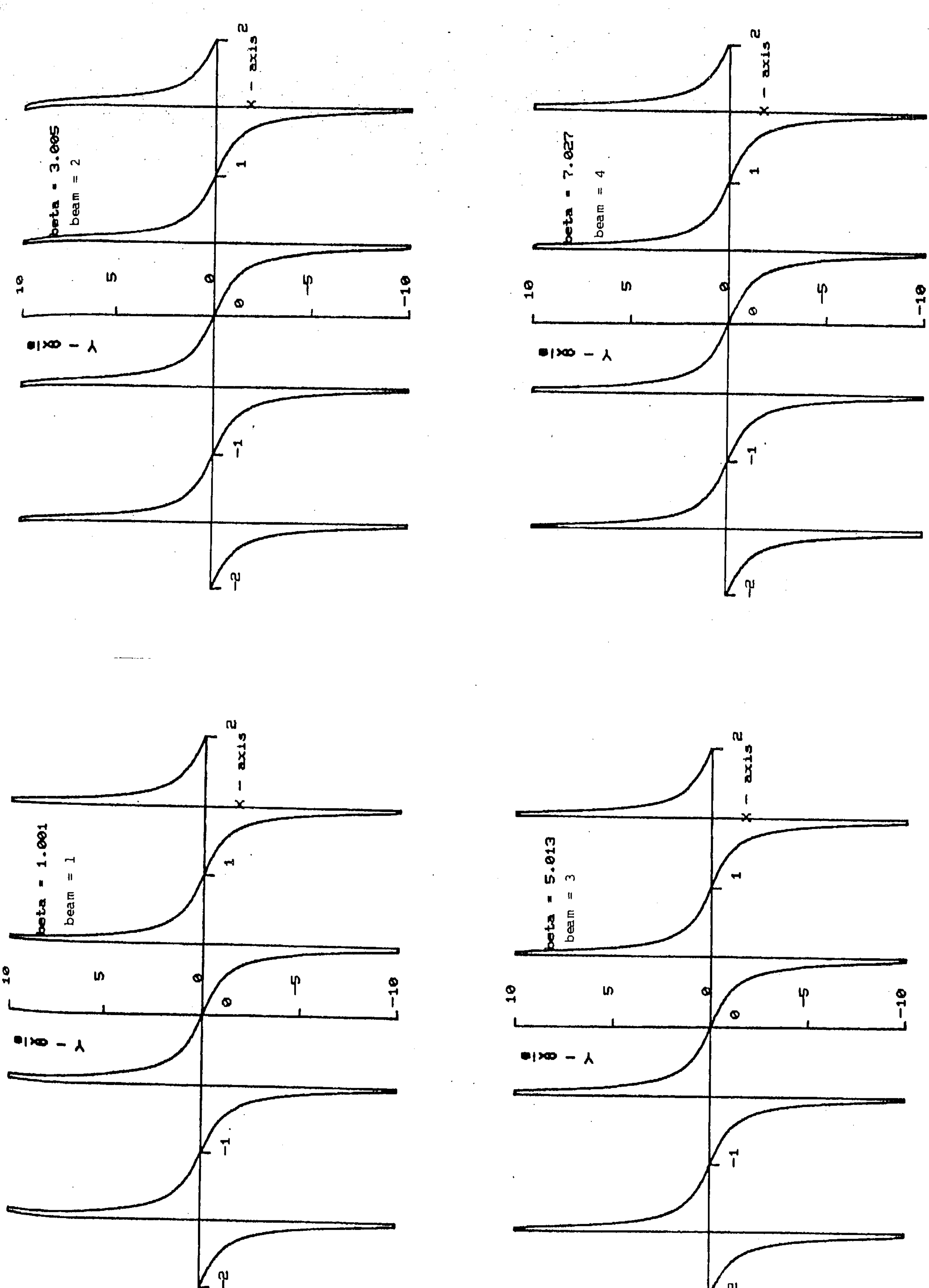

beta = 1.001
beam = 1
beta = 3.005
beam = 2
beta = 5.013
beam = 3
beta = 7.027
beam = 4
X - axis
Y - axis

Fig. 3

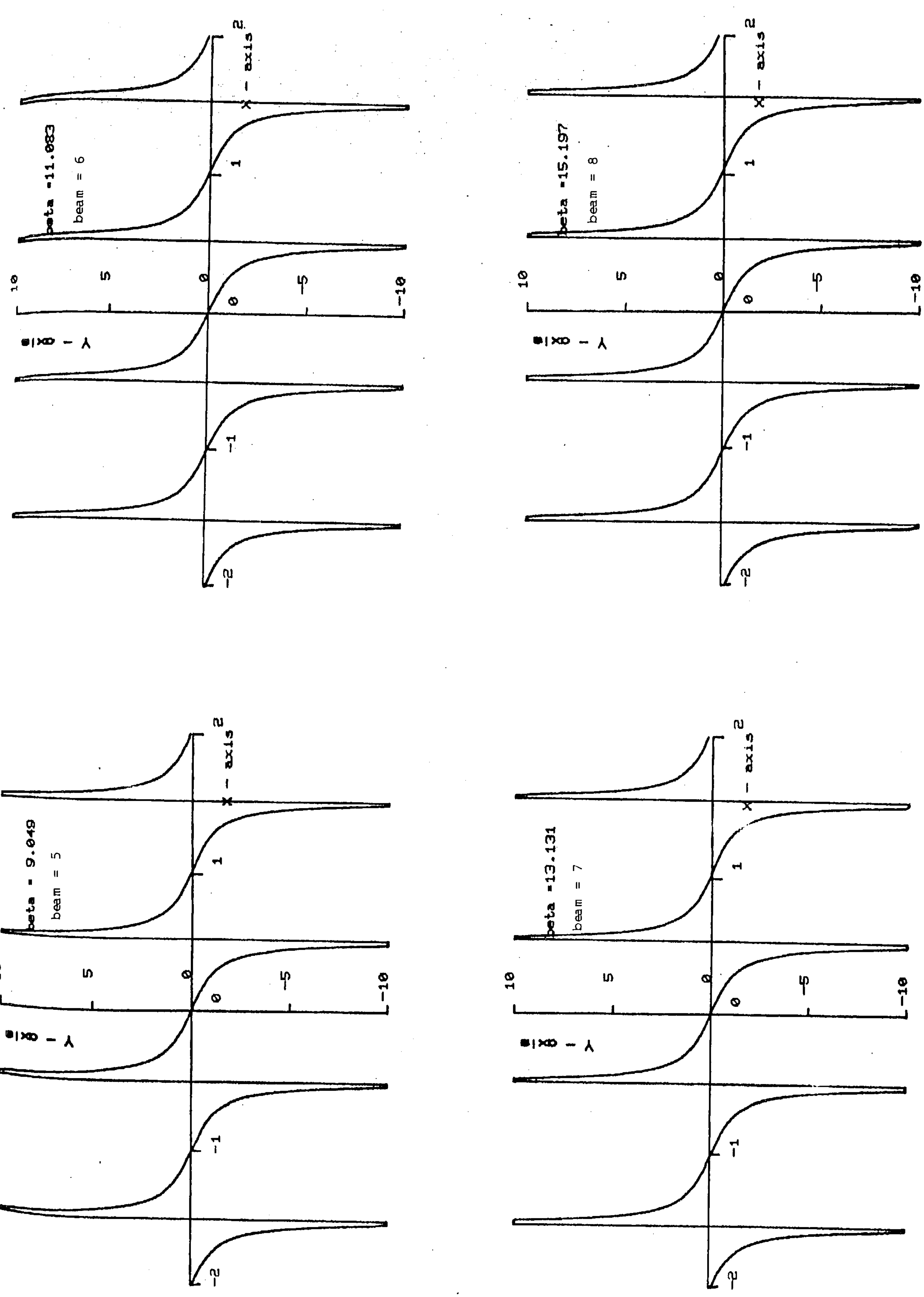

beta -11.083
beam = 6
beta -15.197
beam = 8
beta - 9.049
beam = 5
beta -13.131
beam = 7
Y - axis
X - axis

Fig. 3

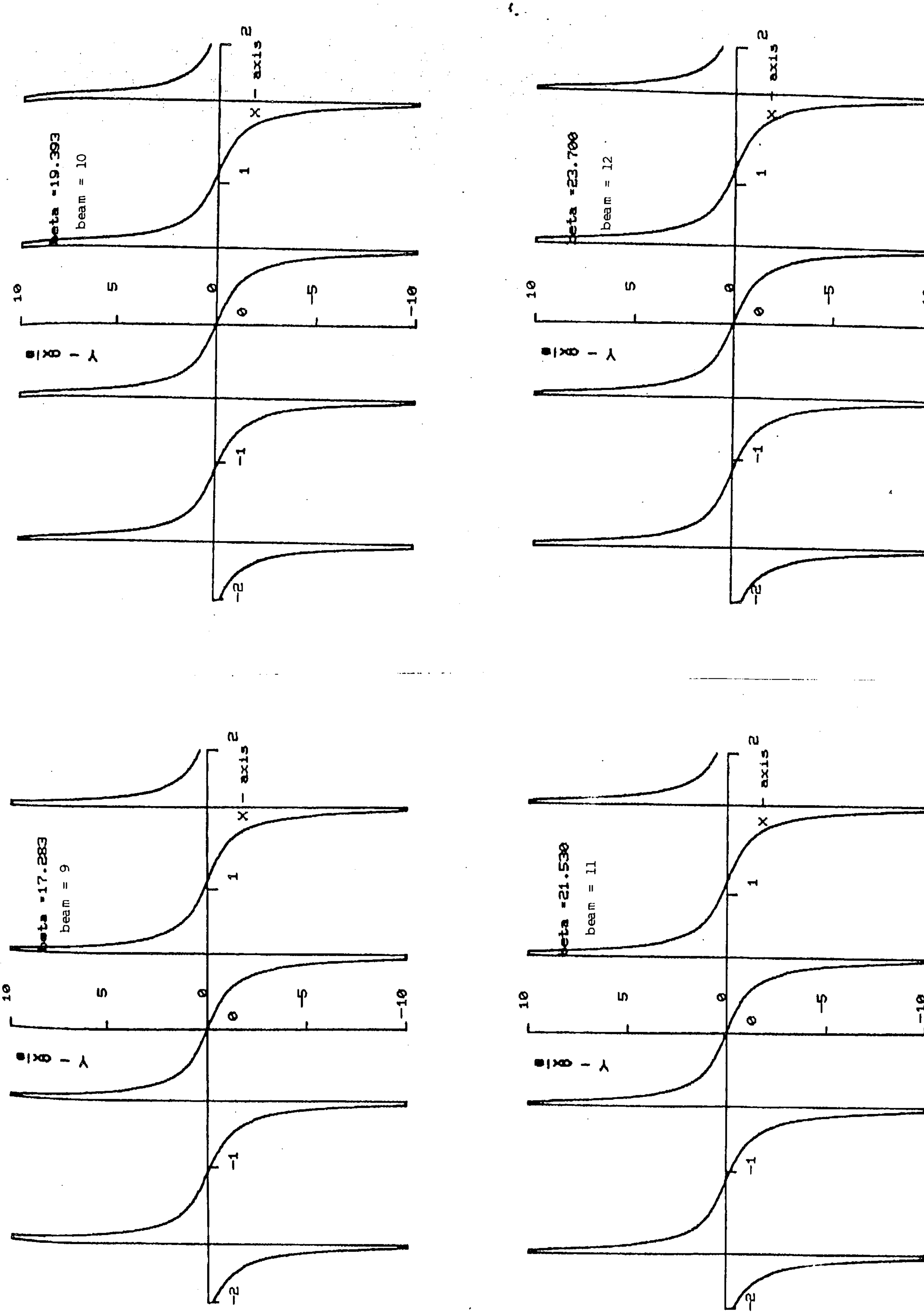

beta =19.393
beam = 10
beta =23.700
beam = 12
beta =17.283
beam = 9
beta =21.530
beam = 11
Y - axis
X - axis
10
5
0
-5
-10
-2
-1
0
1
2

Fig. 3

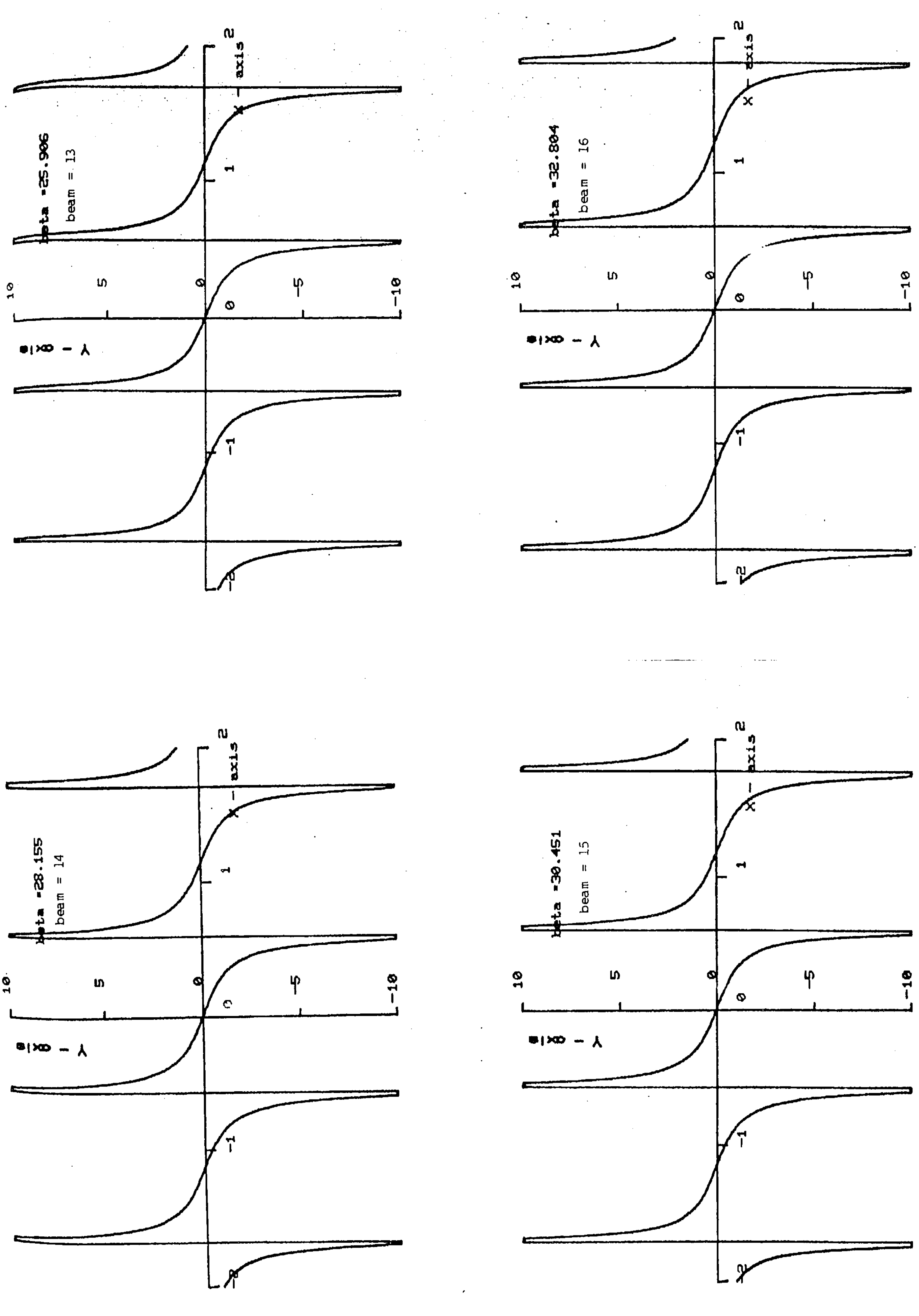

beta =25.906
beam = 13
beta =32.804
beam = 16
beta =28.155
beam = 14
beta =30.451
beam = 15
Y - axis
X - axis

Fig. 3

(F) AN APPLICATION OF THE THEORETICAL MODEL

We present below an procedure to find the equatorial coordinates of a radio source .The right ascension of a given radio source is equal to the sidereal time when maximum deflection is got on COS (or SIN) channel .We can find the declination of the source by using a method based on the theory developed in the previous sections .This method is described below .For a particular radio source , we can find by the theory of the previous sections if the applicable equation for the source is (26a) or (26b) .

$$\frac{SIN}{COS} = -\tan\left[b\ (\ \delta - \delta_o) \right] \qquad (26a)$$

$$\frac{SIN}{COS} = +\tan\left[b\ (\ \delta - \delta_o) \right] \qquad (26b)$$

If C is the maximum COS deflection in the chart record of the transit of a source and S is the corresponding SIN deflection then we can get S/C .We already know b and if we also know δ_o then by substituting average value of S/C (=SIN/COS) in eq.(26) we can get the value of δ i.e. the declination of the source .

While taking the average of S/C ,we ignore the small number of abnormal readings with sign of S/C different from that of the majority .These abnormal readings are due to blobs and holes which change the apparent declination δ' of a source as seen by the antenna .The radiowave from the radio source seems to come from the declination δ' .Hence the value of S/C for that particular reading is different from the usual value of S/C .

A blob is a region of the interplanetary medium having density greater than the ambient .A hole is a region of the interplanetary medium having density less than the ambient .Due to a blob or a hole ,the radiowave from a radio source is refracted and hence apparent position of the radio source changes .This makes the apparent declination δ' of the source different from the usual declination δ .

It may be possible to develop a theory of blobs and holes by which we could calculate the physical parameters of blobs and holes by using the experimentally measurable quantity $\delta' - \delta$.(the declination shift)

(G) AN EXPERIMENTAL METHOD TO TEST THE THEORETICAL MODEL

We made an experimental attempt to test the theoretical model. The sources were divided into sources south of the zenith and sources north of the zenith. We attempted to plot the ratio $S/C \rightarrow \delta - \delta_o$ where

C is maximum COS deflection

S is SIN deflection

δ is declination of source

δ_o is declination in which centre of beam is pointing (which will be referred to as the declination of the beam in the following discussion).

Now for a particular source either eq.(26a) or eq.(26b) is true. For the southern sources any one of the above should hold true and for the northern sources the remaining one should hold true. Hence on plotting an experimental graph we will get a curve corresponding to any one of eqs.(26).

However we could not do an experimental test of eq.(26) due to the following reasons :

1. Readhead(1974) and Purvis(1982) are used at Thaltej to get the values of the declinations of the various sources. However these declinations have uncertainties of about $\pm 0.5^{o}$. The plot of $S/C \rightarrow \delta - \delta_o$ is from $\delta - \delta_o = -2^{o}$ to $\delta - \delta_o = +2^{o}$. Hence an uncertainty of $\pm 0.5^{o}$ in δ is too large for making a experimental graph.

2. To get the values of the declinations δ_o of the centres of the beams we have to use the theoretical model itself (eqs.(18) and (19)) whereas for an experimental verification of the model we need an independent experimental method to determine δ_o .

3. For the southern sources (or northern sources) we need at least 5 sources having $\delta - \delta_o > 0$ and at least 5 sources having $\delta - \delta_o < 0$ in order to get a reliable graph . However the total number of northern sources observed regularly at Thaltej is only 6 . Moreover almost all the observed sources have $\delta - \delta_o > 0$ .. Hence the experimental data is too less to get a reliable graph .

Thus due to the above reasons we could not make an experimental verification of eqs. 26) .

(H) CONCLUSIONS AND SUGGESTIONS

The theory developed in this report implies that the graph of the ratio S/C → $\delta - \delta_o$ is a function of the form

$$\frac{S}{C} = \pm\tan\left[b\,(\delta - \delta_o) \right]$$

where which sign to take in the ± is decided by the discussion in section (C). The constant b is approximately the same for all th beams.

We attempted to test this theoretical result by using the data obtained from the Thaltej radio telescope.However for the reasons given in section (G) the experimental test could not be done.

In order to experimentally verify the theoretical results obtained in this report we suggest that

1. An experimental attempt be made to get values of δ and δ_o with an uncertainty of less than ± 0.1^o.
2. A much larger number of sources should be observed .

ACKNOWLEDGEMENTS

I am very grateful to my guide Dr. H.O. Vats for his valuable guidance and deep interest in my project. I thank him for his cheerful attitude inspite of my frequent interruptions in his own work.

I appreciate Prof. S.K. Alurkar's kind interest and help in this project. I am thankful to Medha Alurkar and to Mr. Janardhan P. for helping in the work related to the computer. I thank Dr. Harish Chandra, Dr. A.K. Sharma, Miss Harsha Machhar, Mr. D. Stephen, Mr. K.J. Shah, Mr. S.L. Kayasth, Mrs. U.K. Modi, Mr. V.R. Chokshi and others at Thaltej IPS Field Station for helping me in various ways. I thank my friends S. Malaviya and R. Goswami for my interesting discussions with them.

Above all I thank my parents for their patient understanding.

Thanks are due to Prof. R.K. Varma, Director, Physical Research Laboratory (PRL) and Dr. U.C. Joshi of the Astronomical Society of India (A.S.I.) for giving me this opportunity to work on this radio astronomy project.

REFERENCES

1. *Alurkar S.K. et al (1985)*
Detection of Large Scale Electron Density Irregularities During IPS Observations at 103 MHz.
Proc. Indian Acad. Sci. (Earth planet. Sci.)
Vol 94, No. 2, July 1985, pp 77-82

2. *Alurkar S.K. et al (1987)*
Three Site Solar Wind Observatory

3. *Christiansen W.N. and Högbom J.A. (1985)*
Radiotelescopes (Second edition) Cambridge University Press.

4. *Delaney W.P. (1962)*
An RF Multiple Beam-Forming Technique
IRE Transactions on Military Electronics, pg 179

5. *Duffet-Smith P. (1979)*
Practical Astronomy with Your Calculator
Cambridge University Press.

6. *Purvis A. et al (*1982*)*
Combridge IPS Survey at 81.5 MHz

7. *Readhead A.C.S. and Hewish A. (1974)*
Fine Structure in Radio Sources at 81.5 MHz -3
Mem. R. astr. Soc. 78, 1-49

8. *Sharma A.K. (1988)*

Studies of Interplanetary Medium Using Interplanetary Scintillation (Thesis) (Gujarat University)

Ch 3 : The Radio Telescope at Thaltej

9. *Thompson A.R. et al (1986)*

Interferometry and Synthesis in Radio Astronomy

Wiley Interscience