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TABLES OF GROUP REPRESENTATIONS FOR THE SIX LIMITING SYMMETRIES IN gIBM

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12. Abstract	: Algebraic description of the band structures and related properties generated by pairing, quadrupole and hexadecapole collective modes in nuclei can be provided by the six dynamical symmetries of the interacting boson model including g bosons (gIBM). The group structure of gIBM is U(15) and dynamical symmetries correspond to the subgroups (or subgroup chains) G in $U(15) \supset G \supset O(3)$. We tabulate the irreducible representations of groups, that occur in the group reduction problem, in gIBM for boson numbers $\mathcal{N} \leqslant 15$.
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1. Introduction

The interacting boson model (IBM) was proposed by Arima and Iachello[1] with the aim of unifying quadrupole vibrational and rotational collective motion in nuclei. To this end the active nucleons in a even-even nucleus are replaced by bosons. As Iachello put it "the bosons are correlated pairs of fermions, but the correlations are so large that the pairs loose their memory of being fermions". Thus the boson number N is half the number of valamce nucleons (particles/holes) and is assumed to be conserved. The bosons are allowed to occupy single particle orbits with angular momenta $l = 0$ (s-bosons) or $l = 2$ (d-bosons). The (sd)-bosons ystem is allowed to interact, and 1 + 2 body Hamiltonian (H) in the model has just six parameters for reproducing the spectra of even-even nuclei. At a phenomenological level, the model was remarkably successful; the success of the model and its various extensions are well documented[2,3]. The relationship of IBM to the (microscopic shell model and the (geometric) collective models is fairly well understood[2]

The single most important (perhaps the beautiful) aspects of IBM is that it has a group structure which allows for the introduction of dynamical symmetries. With (sd) degrees of freedom, a single boson in IBM has six degrees of freedom. As a result, all the states of - bosons grow out of the totally symmetric irreducible representation (IRR) $\{\mathcal{N}\}$ of U(6), the unitary group in six dimensions. The dynamical symmetries of IBM-H correspond to the subgroups G in the chain $U(6) \supset G \supset O(3)$, O(3) generating the angular momentum of the $\mathcal{N}$-bosons. In the symmetry limits, IBM is analytically tractable, one can predict typical band structures and derive selection rules etc. The interpolation between any two symmetry limits can be accomplished by considering the Hamiltonian to be a mixure of the casimir operators of the groups in the two symmetry limits. Thus the characteristics of transitional nuclei can be studied with equal ease in IBM, see ref.4). The important first step in the subject of dynamical symmetries is to identify the various subgroups G and determine their IRR. For IBM both these problems are solved. There are three and only three subgroup chains in IBM and they correspond to the well known U(5)[5], SU(3)[6] and O(6)[7] limits. The IRR of the groups in these chains can be given in closed analytic form, see section 2. The dynamical symmetries of IBM gave far reaching insights[2,3] into structure of the quadrupole collective states in nuclei.

The microscopic theories of IBM on one hand and the quest for an algebraic description of hexadecapole deformation in nuclei on the other hand, led one to consider g-bosons in IBM. We call the interacting boson model with s,d and g-bosons, the gIBM. We reserve the name IBM for the interacting boson model with only s and d bosons. In the gIBM all the states corresponding to $\mathcal{M}$ -bosons grow out of the totally symmetric partition, or the irreducible representation $\{\mathcal{M}\}$ of U(15), the unitary group in fifteen dimensions. The dynamical symmetries of the gIBM Hamiltonian correspond to various subgroups (or subgroup chains) G in $U(15) \supset G \supset O(3)$, O(3) generating the angular momentum of the $\mathcal{M}$ -bosons. We enumerate the various dynamical symmetries of gIBM-H using some general principles which can be used for a boson system with $l = 0, 2, 4, 6, ---$. The corresponding group chains and their generators are given in section-3. Compact expressions for the IRR of the groups in gIBM are no longer feasible, except in some special cases. Due to this reason we undertook to tabulate the IRR for particle numbers of interest, and this is the main aim of this report. In section-4 the methods of obtaining the IRR are given. In section-5 the casimir operators of the various groups in gIBM are constructed. In the concluding section-6, the present status of gIBM is reviewed and some future directions are pointed out. In Appendix-A some simple group reduction rules are given and they are used in section-3. Finally in Appendix-B the IRR of the groups in gIBM are given in tables 1-13.

2. Dynamical Symmetries in IBM

One of the most important developments in nuclear structure physics in the last decade is the recognition that there are three dynamical symmetries in IBM, the corresponding group chains are shown in fig.1. The U(5), SU(3) and O(6) limits geometrically correspond to a quadrupole vibrator, regid rotor and a γ-unstable oscillator respectively; see ref.2). For the sake of completeness and continuity, although well known, we will give in this section the irreducible representations of the groups in the three limits and also the energy formulas.

2.1. U(5) Limit

In the U(5) limit, the group chain and the basis states are giveb by[5),]

$$\left| \begin{matrix} U(6) & U(5) & O(5) & O(3) \\ \{M\} & \{n_d\} & [v] & n_\Delta L \end{matrix} \right\rangle \qquad (1)$$

In the U(5) IRR $\{n_d\}^*$, n_d counts the number of d-bosons and in the O(5) IRR $[v]$, v is the d-boson seniority quantum number. Thus we have,

$$n_d = M,\ M-1,\ ---,\ 0$$

$$v = n_d,\ n_d-2,\ ---,\ 0 \text{ or } 1. \qquad (2)$$

The O(3) IRR L for a given O(5) IRR $[v]$ can occur more than once (multiplicity), thus we need an extra lebel n_Δ,

$$v = 3n_\Delta + \lambda \;, \quad \lambda \geq 0$$

then $L = \lambda, \lambda+1, ----, 2\lambda-2, 2\lambda$. (3)

The Hamiltonian and the energy formula in the U(5) limit are given by

$$H\,(U(5)) = a_0(\vec{M}) + a_1 C_1(U(5)) + a_2 M^* C_1(U(5)) + + a_3 C_2(U(5)) + a_4 C_2(O(5)) + a_5 C_2(O(3)) \qquad (4)$$

and

$$E(M, n_d, v, n_\Delta L) = a_0(M) + a_1 n_d + a_2 M n_d + a_3 n_d(n_d+4) + a_4 v(v+3) + a_5 L(L+1) \qquad (5)$$

where $\vec{M}$ is the operator that generates M and $a_0(M) = x_0 + y_0 M + z_0 \binom{M}{2}$ and it contributes only to the binding energy; x_0, y_0 and z_0 are constants. In (4) $C_1(G)$ and $C_2(G)$ are rank-1 and rank-2 (quadratic) casimir operators of the group G.

2.2. SU(3) Limit

In the SU(3) limit, the group chain and the basis states are[6],

$$\left| \begin{matrix} U(6) & SU(3) & O(3) \\ \{M\} & (\lambda\mu) & k\,L \end{matrix} \right\rangle \qquad (6)$$

---

* We follow Wybourne's notation[8] $\{f\}$, $[f]$ and $\langle f\rangle$ in denoting the IRR of Unitary, orthogonal and symplectic groups respectively.

The SU(3) IRR $(\lambda\mu)$ contained in the IRR $\{M\}$ of U(6) is given by the following expression,

$$(\lambda\mu) = \sum \left(2(f_1 - f_2), 2(f_2 - f_3) \right) \tag{7}$$

where $\{f_1\ f_2\ f_3\}$ is a partition of the integer M, $M = f_1 + f_2 + f_3$ and $f_1 \geqslant f_2 \geqslant f_3 \geqslant 0$. Some of the highest weight ($\xi = 2\lambda + \mu$ larger) SU(3) IRR are $(2M, 0)$, $(2M-4, 2)$, $(2M-8, 4)$, $(2M-6, 0)$ ---. The problem of obtaining the L-content of $(\lambda\mu)$ was solved by Elliott[9),

with $\nu_1 = \max(\lambda, \mu)$

$\nu_2 = \min(\lambda, \mu)$

we have $k = \nu_2, \nu_2 - 2, --, 0$ or 1

and $L = k, k+1, ---, k+\nu_1$ for $k \neq 0$

$\quad = \nu_1, \nu_1 - 2, ---, 0$ or 1, for $k = 0$. (8)

Thus the SU(3) IRR $(2M, 0)$ contains $L = 0, 2, 4, ---, 2M$ and $(2M-4, 2)$ has $k = 0$ (β), 2 (γ) bands with band cut off at $2M-4$ and $2M-2$ respectively. The Hamiltonian and the energy formula in this limit are given by,

$$H(SU(3)) = a_0(\vec{M}) + b_1\, C_2(SU(3)) + b_2\, C_2(O(3)) \tag{9}$$

and

$$E(M, (\lambda\mu), kL) = a_0(M) + b_1(\lambda^2 + \mu^2 + \lambda\mu + 3(\lambda+\mu)) + b_2\, L(L+1) \tag{10}$$

2.3. O(6) Limit

Finally the group chain and the basis states in the O(6) limit are given by[7],

$$\left| \begin{matrix} U(6) & O(6) & O(5) & O(3) \\ \{M\} & [\sigma] & [\gamma] & \nu_\Delta L \end{matrix} \right\rangle \qquad (11)$$

The generalized seniority quantum number σ is given by

$$\sigma = M, M-2, ---, 0 \text{ or } 1$$

and $$\gamma = \sigma, \sigma-1, ---, 0 \qquad (12)$$

The reduction of $[\gamma] \rightarrow L$ is given by (4). The Hamiltonian and the energy formula in this limit are,

$$H(O(6)) = a_0(\vec{M}) + c_1 C_2(O(6)) + c_2 C_2(O(5)) + c_3 C_2(O(3)) \qquad (13)$$

and
$$E(M, \sigma, \gamma, \nu_\Delta L) = a_0(M) + c_1 \sigma(\sigma+4) + c_2 \gamma(\gamma+3) + c_3 L(L+1) \qquad (14)$$

The algebra of the groups in fig.(1) is given in detail in ref.(5) - (7).

3. The gIBM and its six limiting Symmetries

The maximum symmetry group (number conserving) for the gIBM-H is U(15), the unitary group in fifteen dimensions. The fifteen dimensions correspond to the fifteen single particle states, one coming from the s-orbit, five coming from the d-orbit and nine coming from the g-orbit. The generators of the U(15) group are $(b_l^{+} b_l)^{L}$, where $b_0^{+} = s^{+}$, $b_2^{+} = d^{+}$, $b_4^{+} = g^{+}$ and $b_l = (b_l^{+})^{+}$. In order to catalogue the limiting symmetries of the gIBM-H, we should know all the subgroups in the chain $U(15) \supset G \supset O(3)$. Note that the gIBM boson number $\mathcal{N}$ and the angular momentum quantum number L label the IRR of U(15) and O(3) groups respectively. We will discuss the general methods of generating subgroups for any boson system with $l = 0, 2, 4, --, k_0$. Note that the l-values correspond to oscillator major shell quantum number k_0 and the IRR $\{1\}$ of the oscillator group $U((k_0+1)*(k_0+2)/2) = U(N_0)$ contains the above l-values, $\{1\}_{U(N_0)} \rightarrow l = 0, 2, 4, ----, k_0$.

3.1. U(6) ⊕ U(9) and U(14) Limits

Given the single boson orbits with $l = 0, 2, ---, k_0$, we can combine them into unitary orbits, then we have the direct sum sub-group $U(N_o) \supset U(N_a) \oplus U(N_b)$, $N_a = \sum_{i=1}^{k} (2l_i+1)$ and $N_b = N_o - N_a$. For example in gIBM one can have (sd,g), (s,dg), (sg,d) orbit combinations, which then give $U(15) \supset U_{sd}(6) \oplus U_g(9)$, $U(15) \supset U_s(1) \oplus U_{dg}(14)$ and $U(15) \supset U_d(5) \oplus U_{sg}(10)$. As the (sg,d) orbit combination is unphysical, we will ignore* it. Thus we can generate two

* As a matter of principle one should deal with this symmetry also. The group chains we obtain here are the ones corresponding to $U(10) \supset U(9) \supset O(9) \supset O(3)$ and $U(10) \supset O(10) \supset O(9) \supset O(3)$. These are easy to handle and we will not discuss them in this report.

limiting symmetries using the method of unitary orbits. Now we have (I) the weak coupling or $U_{sd}(6) \oplus U_g(9)$ limit and (II) the U(14) limit. The complete subgroup chains in these two limits can take various forms. In the $U_{sd}(6) \oplus U_g(9)$ limit, with the three subgroup chains shown in fig.1 and the O(9) subgroup (generating g-boson seniority quantum number) of $U_g(9)$, we have

$$U(15) \supset U_{sd}(6) \oplus U_g(9) \supset \left[U_{sd}(6) \supset U_d(5) \supset O_d(5) \supset O_{sd}(3)\right] \oplus \left[U_g(9) \supset O_g(9) \supset O_g(3)\right] \supset O_{sdg}(3) \qquad (I.1)$$

$$U(15) \supset U_{sd}(6) \oplus U_g(9) \supset \left[U_{sd}(6) \supset SU_{sd}(3) \supset O_{sd}(3)\right] \oplus \left[U_g(9) \supset O_g(9) \supset O_g(3)\right] \supset O_{sdg}(3) \qquad (I.2)$$

and

$$U(15) \supset U_{sd}(6) \oplus U_g(9) \supset \left[U_{sd}(6) \supset O_{sd}(6) \supset O_d(5) \supset O_{sd}(3)\right] \oplus \left[U_g(9) \supset O_g(9) \supset O_g(3)\right] \supset O_{sdg}(3) \,, \qquad (I.3)$$

From now on we will drop the suffixes (sd), g, (sdg) etc. when there is no ambiguity. Chains (I.1)-(I.3) are useful. when we couple one or two g-bosons to the IBM-core nucleus. In the U(14) limit two subgroup chains are possible and they are

$$U(15) \supset U(14) \supset O(14) \supset O(5) \supset O(3) \qquad (II)$$

$$U(15) \supset U(14) \supset O(14) \supset O_d(5) \oplus O_g(9) \supset O_d(3) \oplus O_g(3) \supset O(3) \qquad (II.1)$$

The generators of the subgroups of $U_{sd}(6)$ are well known, see ref. 5–7). The generators of $U_g(9)$ are $(g^+ \tilde{g})^{L_0 = 0-8}$ and the generators of $O_g(9)$ and $O_g(3)$ are $(g^+ \tilde{g})^{1,3,5,7}$ and $(g^+ \tilde{g})^1$ respectively. Similarly the generators of U(14) and O(14) groups in (II) and (II.1) are,

$$U(14): (d^+\tilde{d})^{0-4}, (g^+\tilde{g})^{0-8}, 2^{-1/2}(d^+\tilde{g} + g^+\tilde{d})^{2-6}, \quad 2^{-1/2}(d^+\tilde{g} - g^+\tilde{d})^{2-6}, \tag{15}$$

$$O(14): (d^+\tilde{d})^{1,3}, (g^+\tilde{g})^{1,3,5,7}, 2^{-1/2}(d^+\tilde{g} \pm g^+\tilde{d})^{2-6} \tag{16}$$

In (16), we have to choose +/– sign, thus there are two equivalent O(14) subgroups and this corresponds to two choices of (dg) pair condensed states,

$$\frac{1}{\sqrt{14}}\left[\sqrt{5}\,(d^+d^+)^0 \pm \sqrt{9}\,(g^+g^+)^0\right]|0\rangle \tag{17}$$

We will choose the + sign in (16) and we will comment on the alternative choice at the end. The generators of the O(5) group in (16) are,

$$\mathcal{G}_\mu^{L_0=1} = \sqrt{10}\left\{(d^+\tilde{d})^1_\mu + \sqrt{6}\,(g^+\tilde{g})^1_\mu\right\} \tag{18}$$

and

$$\mathcal{G}_\mu^{L_0=3} = -\frac{4}{7}(d^+\tilde{d})^3_\mu + \frac{3\sqrt{11}}{14}(g^+\tilde{g})^3_\mu + \frac{3}{7}\sqrt{\frac{5}{2}}\,(d^+\tilde{g} + g^+\tilde{d})^3_\mu \tag{19}$$

Finally $\mathcal{G}_\mu^{L_0=1}$ generates the $O_{sdg}(3)$ group.

In the absence of g-bosons, the unitary orbit method generates only one subgroup, $U(6) \supset U_d(5) \oplus U_s(1)$, and it is the U(5) limit of IBM. Thus the $U(6) \oplus U(9)$ and the U(14) limits of gIBM converge to the U(5) limit of IBM, as g-bosons are removed.

3.2. U(5) and SU(3) Limits

We can think of the l-values, $l = 0, 2, 4, --, k_0$, to be that of a two boson system with each "pseudo" boson carrying angular momentum $l = k_0/2$ and this give raise to the subgroup $U(N_0) \supset U(k_0 + 1)$. For gIBM, this mode of generation of a subgroup gives the (III) U(5) limit and the complete group chain here is,

$$U(15) \supset U(5) \supset O(5) \supset O(3) \qquad \text{(III)}$$

The generators of the U(5) group are $G_\mu^{L_o} = 0-4$

$$G_\mu^{L_o} = \sum_{\substack{\ell_1, \ell_2 \\ = 0,2,4}} \sqrt{(2\ell_1+1)(2\ell_2+2)}\, (-1)^{L_o} \begin{Bmatrix} \ell_1 & \ell_2 & L_o \\ 2 & 2 & 2 \end{Bmatrix} * \left(b^+_{\ell_1}, \tilde{b}_{\ell_2}\right)^{L_o}_\mu \qquad (20)$$

The O(5) group in (III) is same as that of the O(5) group in (II). The generators of the O(5) groups are $G_\mu^{L_o} = 1, 3$ and that of the O(3) group are $G_\mu^{L_o} = 1$. Note that $G_\mu^{L_o=3} = g_\mu^{L_o=3}$ and $G_\mu^{L_o=1} = (2\sqrt{10})^{-1} g_\mu^{L_o=1}$ of eq. (18) and (19). See table-A for the explicit form of $G_\mu^{L_o}$.

Alternatively, for any value of k_0, we have Elliott's[9] SU(3) subgroup. The group chain here is

$$U(15) \supset SU(3) \supset O(3) \qquad \text{(IV)}$$

and the generators of the SU(3) group are[9],

$$L^1_\mu(E) = \sum_{\ell=0}^{k_0} \sqrt{\ell(\ell+1)(2\ell+1)}\,(b^\dagger_\ell \tilde{b}_\ell)^1_\mu \tag{21}$$

and

$$Q^2_\mu(E) = \sum_\ell \Bigg[-(2k_0+3)\sqrt{\frac{\ell(\ell+1)(2\ell+1)}{(2\ell-1)(2\ell+3)}}\,(b^\dagger_\ell \tilde{b}_\ell)^2$$

$$+ \sqrt{\frac{6(\ell+1)(\ell+2)(k_0-\ell)(k_0+\ell+3)}{(2\ell+3)}} \ast$$

$$\left\{ (b^\dagger_\ell \tilde{b}_{\ell+2})^2 + (b^\dagger_{\ell+2}\tilde{b}_\ell)^2 \right\} \Bigg] \tag{22}$$

For gIBM $k_0=4$, then the generators of the SU(3) group in (IV) are,

$$L^1_\mu = \frac{1}{\sqrt{3}} L^1_\mu(E) = g^{L_0=1}_\mu$$

$$Q^2_\mu(S) = \frac{1}{\sqrt{15}} Q^2_\mu(E) = \Big[4\sqrt{\tfrac{7}{15}}\,(s^\dagger\tilde{d}+d^\dagger\tilde{s})^2_\mu$$

$$+\frac{36}{\sqrt{105}}(d^\dagger\tilde{g}+g^\dagger\tilde{d})^2_\mu - 11\sqrt{\tfrac{2}{21}}(d^\dagger\tilde{d})^2_\mu - 2\sqrt{\tfrac{33}{7}}(g^\dagger\tilde{g})^2_\mu \tag{23}$$

It is interesting to note that for the (sd) boson IBM, $l=k_0/2=1$. Thus the above two modes of generating subgroups yield only the SU(3) limit. Thus the U(5) and SU(3) limits of gIBM converge to the SU(3) limit of IBM.

3.3. O(15) and U(6) Limits

One obvious subgroup of $U(N_0)$ that we can construct is the generalized seniority[10] group $O(N_0)$. This leads to the (V) O(15) limit of gIBM and the complete subgroup chains in this limit are,

$$U(15) \supset O(15) \supset O(14) \supset O(5) \supset O(3) \qquad \text{(V)}$$

$$U(15) \supset O(15) \supset [O_{sd}(6) \oplus O_g(9)] \supset [O_d(5) \supset O_d(3)] \oplus [O_g(9) \supset O_g(3)] \supset O(3) \qquad \text{(V.1)}$$

$$U(15) \supset O(15) \supset O(14) \supset O_d(5) \oplus O_g(9) \supset O_d(3) \oplus O_g(3) \supset O(3) \qquad \text{(V.2)}$$

Note that the O(5) groups in (II), (III) and (V) are the same. The generators of the O(15) group are,

$$(d^\dagger \tilde{d})^{1,3}_\mu \,,\ (g^\dagger \tilde{g})^{1,3,5,7}_\mu \,,\ (d^\dagger \tilde{s} + \varepsilon_1 s^\dagger \tilde{d})^{2}_\mu \,,\ (g^\dagger \tilde{s} + \varepsilon_2 s^\dagger \tilde{g})^{4}_\mu \ \text{and}\ (d^\dagger \tilde{g} + \varepsilon_3 g^\dagger \tilde{d})^{2-6}_\mu \qquad (24)$$

There are four equivalent O(15) groups corresponding to the choice of the phases $(\varepsilon_1, \varepsilon_2, \varepsilon_3) = (+,+,-), (+,-,+), (-,+,+)$ and $(-,-,-)$ and they correspond to the four different zero coupled pair condensed states,

$$\frac{1}{\sqrt{15}}\left[(s^\dagger s^\dagger)^0 \pm \sqrt{5}\,(d^\dagger d^\dagger)^0 \pm \sqrt{9}\,(g^\dagger g^\dagger)^0\right] \tag{25}$$

The generators of all the other groups in (V), (V.1) and (V.2) are given before.

Finally we can think of the l values, l = 0,2,4, ---,k_0, to be that of two fermions, with each "pseudo" fermion carrying angular momentum $\tilde{j} = (k_0+1)/2$ giving raise to the group chain $U(N_0) \supset U(k_0+2) \supset Sp(k_0+2) \supset O(3)$. For gIBM, j = 5/2 and this leads to the (VI) U(6) limit. The subgroup chain in the U(6) limit is

$$U(15) \supset U(6) \supset Sp(6) \supset O(3) \tag{VI}$$

As in the case of U(5) limit, the generators of the U(6) group in the U(6) limit are,

$$h_\mu^{L_0=0-5} = \sum_{\substack{\ell_1,\ell_2\\=0,2,4}} \left[(2\ell_1+1)(2\ell_2+1)\right]^{1/2} (-1)^{L_0} \begin{Bmatrix} \ell_1 & \ell_2 & L_0 \\ 5/2 & 5/2 & 5/2 \end{Bmatrix} \left(b_{\ell_1}^\dagger \tilde{b}_{\ell_2}\right)_\mu^{L_0} \tag{26}$$

The generators of the Sp(6) group are $h_\mu^{L_0=1,3,5}$ and $h_\mu^{L_0=1} = 70^{-1/2}\, g^{L_0=1}$ generates the O(3) group in (VI). See table-B for the explicit form of $h_\mu^{L_0}$.

The generalized seniority group in IBM is O(6) while $\tilde{j} = 3/2$ give the U(4) limit. As U(4) $\backsim$ O(6), the above two methods yield only one limit, the O(6) limit in IBM. Thus the O(15) and U(6) limits of gIBM converge to the O(6) limit of IBM.

To conclude this section, we showed that there are in general six limiting symmetries in gIBM and they are pairwise equivalent to the three limits of IBM. The implications and consequences of this remarkable correspondence are not yet explored.

4. Methods of obtaining the irreducible representations

The methods of obtaining the irreducible representations of groups in chains (I) - (VI), for the six limiting symmetries of gIBM will be discussed in this section; fig.2 shows the six limits and the groups involved. Our problem is one of determining the IRR contained in the totally symmetric IRR $\{\mathcal{M}\}$ of U(15). Note that the IRR of the chains (II.1), (V.1) and (V.2) can be obtained easily, once the IRR of the groups in (I) - (VI) are known. A preliminary report on the limiting symmetries and the methods of obtaining the IRR of the groups was given by the author in ref.11).

4.1. $U(6) \oplus U(9)$ Limit

In the $U(6) \oplus U(9)$ or the weak coupling limit, the IRR of U(6) and U(9) groups are labelled by m_{sd} and m_g respectively where $\mathcal{M} = m_{sd} + m_g$ and $m_{sd}, m_g \geq 0$. The reduction of the chain $U(6) \supset G \supset O(3)$ was already described in section-2. For the chain $U(9) \supset O(9) \supset O(3)$, the basis states are denoted by,

$$\left| \begin{matrix} U(9) & O(9) & O(3) \\ \{m_g\} & [V_g] & \alpha\, L_g \end{matrix} \right\rangle \qquad (27)$$

Using (A.3) we have,

$$V_g = m_g,\ m_g-2, ---,\ 0 \text{ or } 1 \qquad (28)$$

The reduction of $[V_g] \to L_g$ can be accomplished in several ways. Littlewood[8] gave a general formula for reducing a totally symmetric IRR $\{F\}$ of U(N) to that of O(3). Using (28) which gives $[V_g] = \{V_g\} - \{V_g-2\}$, it is easy to obtain $[V_g] \to L = 4V_g,\ 4V_g-2, 4V_g-3, ---$.

We will describe an alternative method of obtaining the L content of U(N) IRR $\{M\}$. This procedure is powerful, easy to implement for machine calculations and it can be used with ease even in the case of many l-orbits. Let us consider the group U(N), $N = \sum_{\ell=\ell_1}^{\ell_k} (2\ell+1)$ and address the question of reducing the IRR $\{M\}$ of U(N) to the IRR O(3) in $U(N) \supset O(3)$, with the association $\{1\}_{U(N)} \to L = l_1, l_2, --, l_k$. To this end we generate the l_z-orbits and the single particle spectrum for the $\vec{l}_z$-operator. For the examples of $U_{sd}(6)$, $U_g(9)$ and $U_{sdg}(15)$ groups, the single particle spectrum for the $\vec{\ell}_z$ operator is shown in fig.3. The spectrum is equidistant and each level has a definite l_z-eigenvalue m_l with degeneracy

d_i. Now distributing the given number of bosons M in the l_z-orbits in all possible ways we can find the degeneracy d(m) of a given total m-value. Say $(n_1, n_2, ---, n_k)$ is a distribution of M Bosons so that $M = \sum_{i=1}^{k} n_i$ and $m = \sum_{i=1}^{k} m_i n_i$. For this distribution, the degeneracy of the m-value is $\prod_{i=1}^{k} \binom{d_i + n_i - 1}{n_i}$, Thus the degeneracy of the $\sum l_z(i)$ eigenvalue m is,

$$d(m) = \sum_{\{n_1 n_2 --- n_k\}} \prod_{i=1}^{k} \binom{d_i + n_i - 1}{n_i} \tag{29}$$

Where the summation is over all configurations $(n_1, n_2, ---, n_k)$, such that $m = \sum_{i=1}^{k} m_i n_i$. Now the simple difference formula

$$d(L) = d(m=L) - d(m=L+1) \tag{30}$$

give the degeneracy of a given L value. T his is nothing but the multiplicity of the IRR L of O(3) in the reduction of the IRR $\{M\}$ of U(N). It is easy to extend the above procedure for two rowed IRR of U(N), which may be useful when F-spin is included in IBM. Finally we mention that the above procedure was used for Fermions in ref.12,13).

4.2. U(14) Limit

In the U(14) limit, the basis states are given by

$$\left| \begin{matrix} U(15) & U(14) & O(14) & O(5) & O(3) \\ \{M\} & \{m_{dg}\} & [v_{dg}] & \beta[v_1 v_2] & \alpha L \end{matrix} \right\rangle \tag{31}$$

Using (A.1) and (A.3) we have

$$m_{dg} = M, M-1, ---, 0$$

$$v_{dg} = m_{dg}, m_{dg}-2, ----, 0 \text{ or } 1. \qquad (32)$$

The reduction of 0914) $\supset$ O(5) and O(5) $\supset$ O(3) are more involved and we will discuss them in section-4.3. In general for $v_{dg} \geqslant 3$ we have,

$$[v_{dg}] \rightarrow [v_1 v_2] = [2v_{dg}, 0] \oplus [2v_{dg}-2, 2] \oplus [2v_{dg}-2, 0] \oplus [2v_{dg}-3, 1] \oplus [2v_{dg}-4, 4] \oplus ---- \qquad (33)$$

For $v_{dg} \leqslant 2$ we have $[0] \rightarrow [0,0]$, $[1] \rightarrow [2,0]$ and $[2] \rightarrow [4,0] \oplus [2,2] \oplus [2,0]$.

4.3 U(5) Limit

In the U(5) limit, the basis states are,

$$\left| \begin{matrix} U(15) & U(5) & O(5) & O(3) \\ \{M\} & \{f\} = \{f_1 f_2 f_3 f_4 f_5\} & \beta[\tau_1 \tau_2] & \alpha L \end{matrix} \right\rangle \qquad (34)$$

Using (A.4), the U(5) IRR $\{f\}$ in $\{M\}$ are given by

$$\{f\} = \sum \{2m_1, 2m_2, 2m_3, 2m_4, 2m_5\} \oplus \qquad (35)$$

Where $\{m_1, m_2, m_3, m_4, m_5\}$ is a partition of the integer M. Thus in general we have,

$$\{M\} \rightarrow \{f\} = \{2M\} \oplus \{2M-2, 2\} \oplus \{2M-4, 4\} \oplus \{2M-4, 2, 2\} \oplus \qquad (36)$$

In (34) α and β denote the multiplicity of the IRR L and $[\tau_1 \tau_2]$ respectively. We obtained the O(5) IRR $\beta[\tau_1 \tau_2]$ in U(5) $\supset$ O(5) using the following procedure. Using Littlewood's rules[8)] we can write the IRR $\{f\}$ of U(5) as a determinent involving only totally symmetric IRR of U(5). Moreover as $\{f_1\ f_2\ f_3\ f_4\ f_5\} \sim \{f_1-f_5, f_2-f_5, f_3-f_5, f_4-f_5\}$ we have to deal, at the most, with a 4 x 4 determinant,

$$\{g_1 g_2 g_3 g_4\} = \begin{vmatrix} \{g_1\} & \{g_1+1\} & \{g_1+2\} & \{g_1+3\} \\ \{g_2-1\} & \{g_2\} & \{g_2+1\} & \{g_2+2\} \\ \{g_3-2\} & \{g_3-1\} & \{g_3\} & \{g_3+1\} \\ \{g_4-3\} & \{g_4-2\} & \{g_4-1\} & \{g_4\} \end{vmatrix} \qquad (37)$$

The reduction of $\{f_r\}$ to IRR $[v_r]$ of O(5) is given by (A.3),

$$v_r = f_r,\ f_r-2,\ ---,\ 0 \text{ or } 1 \qquad (38)$$

Thus in the determinant (37), we can replace the totally symmetric IRR $\{g_k\}$ by O(5) IRR $[v_r]$ and then the problem is one of performing the Kronecker products of O(5) IRR. This can be accomplished by the following two theorems.

(1) $$[m] \times [n] = \sum_{\substack{k=0-n \\ r=0-(n-k)}} [m-n+k+2r, k] \qquad (39)$$

(2) Using Littlewood's result (see eq.(86a), (86b) in ref(8)),

$$[PQ]\times[R] = \sum k_{\{PQ\}\{R\}\{\rho\}}[\rho] \tag{40}$$

Where $k_{\{PQ\}\{R\}\{\rho\}}$ is defined by

$$\sum \Gamma_{\{\xi\}\{\lambda\}\{PQ\}}\Gamma_{\{\xi\}\{\mu\}\{R\}}\{\lambda\}\{\mu\} \tag{41}$$

In (41) $\{\quad\}$ are U(5) IRR and $\Gamma_{\{a\}\{b\}\{c\}}$ is the number of times $\{c\}$ appears in the Kronecker product $\{a\}\times\{b\}$. Using the fact that $\{R\}$ is a totally symmetric IRR and $\{\xi\}\{\lambda\}\rightarrow\{PQ\}$ in (41), we have

$$\{\xi\} = \{0\}\oplus\{1\}\oplus\{2\}\oplus ---- \oplus\{R\}$$

$$\{\mu\} = \{R-\xi\}$$

$$\Gamma_{\{\xi\}\{\mu\}\{R\}} = 1$$

$$\{\lambda\} = \{\lambda_1\lambda_2\}$$

$$= \sum\{P-r',Q-r''\}\;;\quad \begin{array}{l} r'+r'' = \xi \\ P-r' \geqslant Q-r'' \\ P-Q+r''-r' \geqslant r'' \end{array} \tag{42}$$

and $\Gamma_{\{\xi\}\{\lambda\}\{PQ\}} = 1$

Now, using the simple result that,

$$\{\lambda_1 \lambda_2\} \times \{\mu\} = \sum_{s,t=0}^{\mu} \{\lambda_1 + s, \lambda_2 + t, \mu - s - t\}$$

where $t \leq \lambda_1 - \lambda_2$

$\mu - s - t \leq \lambda_2$

and $\mu \geq s + t$

and (41), we obtain $k_{\{PQ\}\{R\}\{\rho\}}$ in (40). Finally, converting the irregular three rowed IRR of O(5) into two rowed regular IRR using the result,

$$\begin{aligned} [\lambda_1 \lambda_2 \lambda_3] &= 0 \quad \text{if } \lambda_3 \geq 2 \\ &= [\lambda_1 \lambda_2] \quad \text{for } \lambda_3 = 0,1 \end{aligned} \qquad (44)$$

we obtain $[PQ] \times [R] \rightarrow [P'Q']$. This procedure was used to generate all the needed $U(5) \supset O(5)$ reductions. This calculation is also needed in reducing the chains (II) and (V). For example, generating all the O(5) IRR in the U(5) IRR given in (35), we obtain the reduction of $\{M\}$ of U(15) to O(5) IRR. Now, using (32) we obtain $[v_{dg}] \rightarrow [v_1 v_2]$ in $O(14) \supset O(5)$. This procedure is followed in generating table-13. For the lowest two U(5) IRR we have,

$$\{2M\} \rightarrow [\gamma_1 \gamma_2] = \sum_{r=0}^{M} [2M-2r, 0]$$

and $\{2M-2,2\} \rightarrow [\gamma_1,\gamma_2] = [2M-2,0] \oplus [0,0] \oplus$

$$\sum_{\lambda=1}^{M-2} [2M-2\lambda-2,0]^2 \oplus \sum_{\lambda=1}^{M-2} [2M-2\lambda-1,1] \oplus \sum_{\lambda=1}^{M-1} [2M-2\lambda,2]^2 \qquad (45)$$

The reduction of O(5) IRR $[\gamma_1,\gamma_2]$ to that of O(3) can be achieved by repeated use of (39) and the reduction of $[\gamma] \rightarrow$ L given in (3). For example, we have from (39),

$$[m] \times [1] = [m+1] \oplus [m,1] \oplus [m-1] \qquad (46)$$

$$[m] \times [2] = [m+2] \oplus [m+1,1] \oplus [m,2] \oplus [m] \oplus [m-1,1] \oplus [m-2] \qquad (47)$$

Knowing $[m] \rightarrow$ L and using (46), we obtain $[m,1] \rightarrow$ L. Now using $[m] \rightarrow$ L, $[m, 1] \rightarrow$ L and (47), we obtain $[m,2] \rightarrow$ L and so on. From (45), we see that the important O(5) IRR, we encounter in the U(5) limit, are of $[m]$, $[m,1]$ and $[m,2]$ type and in tables 6,7 we give their reduction to O(3) IRR L. We point out that Van Isacker et.al.[14] gave the general result for $[m,1] \rightarrow$ L,

m,1	L = 2m+1, 2m,---, 3	
	m, m-1,---, 1	$(m > 0)$
	m+2, m+1,---,5	$(m > 2)$
	m+3, m+2,---,7	$(m > 3)$
	----------.	(48)

From table-7 we have, for $m > 4$

$$[m,2] \rightarrow L = 2m+2,\ 2m+1,\ (2m)^2,\ (2m-1)^3, --- . \qquad (49)$$

For m $\leq$ 4 we have, $[22] \rightarrow 6,4,3,2,0$, $[32] \rightarrow 8,7,6,5^2$ ---- and $[42] \rightarrow 10,9,8^2,7^2$, ----.

4.4. SU(3) Limit

In the SU(3) limit the basis states are denoted by

$$\left| \begin{matrix} U(15) & SU(3) & O(3) \\ \{M\} & \beta(\lambda\mu) & kL \end{matrix} \right\rangle \quad (50)$$

The method of obtaining the SU(3) IRR in $\{M\}$ was described in detail in ref.15). Note that $\{1\}_{U(15)} \rightarrow (40)_{SU(3)}$. The reduction of $(\lambda\mu) \rightarrow (k)\ L$ is already given in (8). Some of the lowlying SU(3) IRR for m $\geq$ 5 are

$$\{M\} \rightarrow (\lambda\mu) = (4M,0) \oplus (4M-4,2) \oplus (4M-6,3) \oplus (4M-8,4)^2 \oplus (4M-6,0) \oplus (4M-10,5) \oplus (4M-9,3) \oplus (4M-8,1) \oplus ---- \quad (51)$$

Some of the SU(3) IRR in (51) do not exist for $M \leq 4$. For $\{4\}$ the IRR (65) do not exist, $\{2\} \rightarrow (80) \oplus (42) \oplus (04)$ and $\{3\} \rightarrow (120) \oplus (82) \oplus (63) \oplus (44) \oplus (60)$ + ----.

4.5. O(15) Limit

For the O(15) limit, the basis states are,

$$\left| \begin{matrix} U(15) & O(15) & O(14) & O(5) & O(3) \\ \{M\} & [\sigma] & [\tau] & \beta[\pi_1\pi_2] & \alpha L \end{matrix} \right\rangle \quad (52)$$

Using (A.2) and (A.3) we obtain the σ, γ quantum numbers,

$$\sigma = M, M-2, ----, 0 \text{ or } 1$$

and $$\gamma = \sigma, \sigma-1, ----, 0. \qquad (53)$$

The reduction of O(14) IRR $[\gamma]$ to O(5) IRR $[\pi_1 \pi_2]$ and $[\pi_1 \pi_2] \rightarrow$ L is already described in (3.2) and (3.3).

4.6. U(6) Limit

In the U(6) limit, the basis states are denoted by,

$$\left| \begin{array}{cccc} U(15) & U(6) & Sp(6) & O(3) \\ \{M\} & \{F\} & \beta\langle\lambda_1\lambda_2\lambda_3\rangle & \alpha L \\ & =\{F_1 F_2 F_3 F_4 F_5 F_6\} & & \end{array} \right\rangle \qquad (54)$$

The U(6) IRR $\{F\}$ can be obtained from (A.5). We have the association $\{1\}_{U(15)} \rightarrow \{1^2\}_{U(6)}$. With $\{f_1 f_2 --- f_M\}$ a three columned partition of the integer M, we write $\{2f_1, 2f_2, ---, 2f_M\}$ as $\{P^p\ Q^q\ R^r\ S^s\ T^t\ U^u\}$. Note that $6 \geqslant P \geqslant Q \geqslant R \geqslant S \geqslant T \geqslant U \geqslant 0$. Then,

$$\{F\} = \{F_1\ F_2\ F_3\ F_4\ F_5\ F_6$$
$$= \sum \{p+q+r+s+t+u,\ p+q+r+s+t,\ p+q+r+s,\ p+q+r,\ p+q,\ p\} \oplus \qquad (55)$$

and in table-9 we give all the U(6) IRR for $M \leqslant 15$. The lowlying U(6) IRR are,

$$\{F\} = \{M, M\} \oplus \{M-1, M-1, 1, 1\} \oplus \{M-2, M-2, 2, 2\} \oplus \qquad (56)$$

The reduction of $U(6) \supset Sp(6)$ follows the same steps that are used for the $U(5) \supset O(5)$ reduction described in (3.3). As there are not yet any real applications of U(6) limit, we did not write general programmes for reducing $U(6) \supset Sp(6)$. However the branching rules for the lowest two IRR in (56) are given below (see eq. (80) of ref. 8)),

$$\{M,M\}_{U(6)} \rightarrow \langle \lambda_1 \lambda_2 \lambda_3 \rangle_{Sp(6)} = \langle M,M,0\rangle \oplus \langle M-1,M-1,0\rangle \oplus \cdots \oplus \langle 0,0,0\rangle \qquad (57)$$

and

$$\{M-1,M-1,1,1\} \rightarrow \langle \lambda_1 \lambda_2 \lambda_3 \rangle = \langle M-1,M-1,0\rangle \oplus \sum_{\lambda=1}^{M-2} \langle \lambda,\lambda,0\rangle^2 \oplus \sum_{\lambda=2}^{M-1} \langle \lambda,\lambda-1,1\rangle \oplus \langle 0,0,0\rangle . \qquad (58)$$

For the reduction of the chain $Sp(6) \supset O(3)$, we adopted the following procedure which is easy to implement on a mechine. With the association $\{1\}_{U(6)} \rightarrow j = 5/2$, we can reduce $\{m\}_{U(6)} \rightarrow J$ using the method described in (4.1). Now expanding $\{F\}$ of U(6) into a determinant involving only totally symmetric IRR (see eq. (37)), we obtain $\{F\} \rightarrow L$. The expressions (57) and (58) yield a recursive procedure for reducing $\langle m,m,0\rangle$, $\langle m,m-1,1\rangle \rightarrow L$ and we can extend this procedure for any $\langle \lambda_1, \lambda_2 \lambda_3\rangle$. As can be seen from (57) and (58), we need to reduce only $\langle m,m,0\rangle$ and $\langle m,m-1,1\rangle$ type IRR of Sp(6) to IRR of

O(3), for the lowest two U(6) IRR in (56). The results for $M \leq 15$ are given in table-11 and in general we have,

$$\langle M\,M\,0\rangle \to L = (4M) \oplus (4M-2)^2 \oplus (4M-3)^2 \oplus (4M-4)^4 \oplus \cdots \quad \text{for } M \geq 4 \qquad (59)$$

and

$$\langle M, M-1, 1\rangle \to L = (4M-1) \oplus (4M-2)^2 \oplus (4M-3)^3 \oplus (4M-4)^6 \oplus \cdots \quad \text{for } M \geq 5. \qquad (60)$$

For $M \leq 3$, we have $\langle 110\rangle \to 4,2$, $\langle 220\rangle \to 8,6^2,5,4^2,\cdots$ and $\langle 330\rangle \to 12,10^2,9^2,8^3,\cdots$. Similarly for $M \leq 4$ we have $\langle 211\rangle \to 7,6,5,4,\cdots$, $\langle 321\rangle \to 11,10^2,9^2,8^4,\cdots$ and $\langle 421\rangle \to 15,14^2,13^3,12^5,\cdots$.

Before concluding this section, we point out that a stringent test for the reductions given in tables 1-13 is provided by the dimensionalities. In the reduction of a IRR X of G to IRRs Y_i of H in $G \supset H \Rightarrow X \to \sum Y_i \oplus$. Then the dimensionality d (X) of X should be equal to the sum of the dimensionalities of Y_i; if Y_i appear α times in the reduction, then $d(X) = \sum [\alpha_i * d(Y_i)]$ For all the reductions given in tables 1-13 we checked the dimensionality relations; see table-C for the dimensionality formulas.

4. Casimir Operators and Energy Spectra

The gIBM-H when it is expressible interms of the Casimir operators of the groups appearing in a symmetry group chain (i.e.) in the dynamical symmetry limits, we can write down the energy formula. With the generators given in section-3 we can construct the Casimir operators of the various groups in the six limits of gIBM. The matrix elements of the Casimir operators of U(N), Sp(N) and O(N) are well known[16]. With the usual assumption that the H is a 1+2 body operator, we have to deal only with the linear and quadratic Casimir operators.

In the $U_{sd}(6) \oplus U_g(9)$ limit, the Casimir operators of the subgroups of U(6) are given in ref.(5)-(7), and the Casimir operator of U(9) is the g-boson number operator n_g. The Casimir operator of O(9) can be replaced by the g-boson pairing operator $P_2(O(9))$, the explicit form of $P_2(O(9))$ is given ahead. Now the operator form of H in the U(6) ⊕ U(9) limit is,

$$\begin{aligned} H_I = {} & a_0(\vec{N}) + \alpha_0 H_{IBM} + a_1 \vec{n}_g \\ & + a_2 \vec{n}_{sd} \vec{n}_g + a_3 \binom{\vec{n}_g}{2} \\ & + a_4 P_2(O(9)) + a_5 C_2(O_g(3)) \\ & + a_6 C_2(O_{sdg}(3)) \end{aligned} \qquad (61)$$

and the corresponding energy formula is,

$$E_I(M, m_{sd}, m_g, V_g, \alpha L_g, L)$$
$$= a_0(M) + \gamma_0 \langle H_{IBM} \rangle^{m_{sd}} + a_1 m_g$$
$$+ a_2 m_{sd} * m_g + a_3 \binom{m_g}{2}$$
$$+ \frac{a_4}{4}(m_g - V_g)(m_g + V_g + 7) + a_5 L_g(L_g+1)$$
$$+ a_6 L(L+1) \quad (62)$$

where H_{IBM} corresponding to the three limits of IBM are given in (4), (9) and (13) and the energy expressions $\langle H_{IBM} \rangle^{m_{sd}}$ are given in (5), (10) and (14) respectively. An additional term $\beta\, m_d\, m_g$ should be added to (62) in the U(5) limit of IBM. The term $a_0\,(M)$ contributes only to binding energies, $a_0(M) = x_0 + y_0 M + z_0 \binom{M}{2}$ and (x_0, y_0, z_0) are constants.

In the U(14) limit, the Hamiltonian can be written as

$$H_{II} = b_0(\vec{M}) + b_1 \vec{n}_{dg} + b_2 \vec{M} * \vec{n}_{dg}$$
$$+ b_3 \binom{\vec{n}_{dg}}{2} + b_4 P_2(O(14))$$
$$+ b_5 C_2(O(5)) + b_6 C_2(O(3)).$$
$$(63)$$

In (63) $b_0(\vec{M})$ is similar to $a_0(\vec{M})$ in (62). The O(N) pairing operator is defined as follows. Given the l-orbits in which the bosons are allowed to occupy, the 2-boson pair condensed state can be written as

$$\frac{1}{\sqrt{(\sum \Omega_\ell)/2}} S_+|0\rangle = \frac{1}{\sqrt{\sum \Omega_\ell}} \sum_\ell \varepsilon_\ell \Omega_\ell^{1/2} (b_\ell^+ b_\ell^+)^0 |0\rangle \tag{64}$$

Where ε_ℓ is a phase factor; see (17) and (25). For O(14) group l = 2,4. Now the pairing operator P_2 is simply given as

$$P_2(O(N)) = S_+ S_- \; ; \quad S_- = (S_+)^+ \tag{65}$$

For m bosons with generalized seniority v, $\{m\}$ and $[v]$ denoting the IRR of U(N) and O(N) with $N = \sum_\ell (2l+1)$, the matrix elements of P_2 are given by

$$\langle P_2(O(N)) \rangle^{mv} = \frac{1}{4}(m-v)(m+v+N-2) \, . \tag{66}$$

In terms of the O(5) and O(3) generates given in (18),(19) and (20), we have

$$C_2(O(5)) = 8\,[G^1 \cdot G^1 + G^3 \cdot G^3]$$

and $$C_2(O(3)) = 40\, G^1 \cdot G^1 = g^1 \cdot g^1 \tag{67}$$

With $[v_1 \; v_2]$ denoting the IRR of O(5), we have for the matrix elements of the above Casimir operators,

$$\langle C_2(O(5)) \rangle^{[v_1 v_2]} = [v_1(v_1+3) + v_2(v_2+1)]$$

and $$\langle C_2(O(3))\rangle^{L} = L(L+1) \tag{68}$$

Now the energy formula in the U(14) limit is,

$$\begin{aligned} E_{II}&(M, m_{dg}, v_{dg}, \beta[v_1 v_2], \alpha L) \\ &= b_0(M) + b_1 m_{dg} + b_2 M * m_{dg} \\ &+ b_3\binom{m_{dg}}{2} + \frac{b_4}{4}(m_{dg}-v_{dg})(m_{dg}+v_{dg}+12) \\ &+ b_5[v_1(v_1+3)+v_2(v_2+1)] + b_6 L(L+1) \end{aligned} \tag{69}$$

In the U(5) limit we can write down the H as,

$$\begin{aligned} H_{III} = c_0(\vec{M}) + c_1 C_2(U(5)) + c_2 C_2(O(5)) \\ + c_3 C_2(O(3)). \end{aligned} \tag{70}$$

We already gave the Casimir operators of O(5) and O(3) in (67),(68). The U(5) Casimir operator is given interms of the U(5) generators in (20).

$$C_2(U(5)) = \frac{5}{3}\sum_{L_0=0}^{5} G^{L_0}\cdot G^{L_0} \tag{71}$$

and $$\langle C_2(U(5))\rangle^{\{f\}} = \sum_{i=1}^{5} f_i(f_i+6-2i). \tag{72}$$

The energy formula in the U(5) limit is then given by

$$\begin{aligned} E_{III}(M, \{f\}, \beta[\tau_1\tau_2], \alpha L) = c_0(M) + c_1\left[\sum_{i=1}^{5} f_i(f_i+6-2i)\right] \\ + c_2[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)] + c_3 L(L+1) \end{aligned} \tag{73}$$

In the SU(3) limit, the SU(3) Casimir operator is,

$$C_2(SU(3)) = \frac{3}{4}\left[Q(S)\cdot Q(S) + L\cdot L\right] \qquad (74)$$

and the operators Q(S), L are given in (23). Now the Hamiltonian and the energy formula in the SU(3) limit are,

$$H_{IV} = d_0(\vec{M}) + d_1 C_2(SU(3)) + d_2 C_2(O(3))$$

and

$$E_{IV}(M, \beta(\lambda\mu), kL) = d_0(M) + d_1\left[\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)\right] + d_2 L(L+1) \qquad (75)$$

In the O(15) limit, the O(15) Casimir operator is replaced by the O(15) pairing operator with 1 = 0,2,4 in (64)-(66). Since m_{dg} is not a good quantum number in this limit, we cannot use O(14) pairing operator, but instead we have to use the O(14) Casimir operator, $C_2(O(14)) = n_{dg}(n_{dg}+12) - 4P_2(O(14))$ where $n_{dg} = \sum d_m^+ d_m + \sum g_m^+ g_m$. Now the Hamiltonian and the energy formula in the O(15) limit are

$$H_{V} = e_0(\vec{M}) + e_1 P_2(O(15)) + e_2 C_2(O(14)) + e_3 C_2(O(5)) + e_4 C_2(O(3))$$

and

$$E_{\overline{V}}(M,\sigma,\tau,\beta[\pi_1\pi_2],\alpha L) = e_0(M) + \frac{e_1}{4}(M-\sigma)(M+\sigma+13) + e_2\tau(\tau+12) + e_3[\pi_1(\pi_1+3)+\pi_2(\pi_2+1)] + e_4 L(L+1) \quad (76)$$

Finally in the U(6) limit, in terms of the operators given in (26), the U(6) and Sp(6) Casimir operators are,

$$C_2(U(6)) = \frac{48}{5}\sum_{L_0=0}^{5} h^{L_0}\cdot h^{L_0}$$

and $$C_2(Sp(6)) = 8\sum_{L_0=1,3,5} h^{L_0}\cdot h^{L_0} \quad (77)$$

Their matrix elements are simply given as,

$$\langle C_2(U(6))\rangle^{\{F\}} = \sum_{i=1}^{6} F_i(F_i-2i+7)$$

and $$\langle C_2(Sp(6))\rangle^{\langle\lambda_1\lambda_2\lambda_3\rangle} = [\lambda_1(\lambda_1+6)+\lambda_2(\lambda_2+4)+\lambda_3(\lambda_3+2)] \quad (78)$$

Now the Hamiltonian and the energy formula in the U(6) limit are,

$$H_{\overline{VI}} = f_0(\vec{M}) + f_1 C_2(U(6)) + f_2 C_2(Sp(6)) + f_3 C_2(O(3))$$

and

$$E(M,\{F\},\beta\langle\lambda_1\lambda_2\lambda_3\rangle,\alpha L) = f_0(M) + f_1\left[\sum_{i=1}^{6} F_i(F_i-2i+7)\right] + f_2[\lambda_1(\lambda_1+6)+\lambda_2(\lambda_2+4) + \lambda_3(\lambda_3+2)] + f_3 L(L+1) \quad (79)$$

With the representations of the groups in the six limits, given in tables 1-13 and the energy formula given above, one can easily construct the typical spectra that appear in gIBM. The real problem is one of finding empirical examples for each of the six limits.

5. Present status of gIBM and future outlook

The success of the IBM in correlating the properties of even-even nuclei, naturally led one to ask questions on the finer details of the model and its basic foundations, investigations on the microscopic foundations, limitations and extensions of the model are undertaken. Several authors developed various microscopic theories[17,18] of IBM and the most significant outcome of all these studies is that the $l = 4^+$ pairs are unavoidable in IBM. In the language of IBM, $l = 4^+$ pairs are equivalent to g-bosons and in the collective models it is equivalent to including hexadecapole (deformation) degree of freedom. From their microscopic theories, Arima et. al[18] pointed out that hexadepole degree of freedom in addition to monopole and quadrupole degrees of freedom is important and sufficient to reproduce physical quantities[1]. Studies based on Hartree Fock theories[19] clearly pointed out the importance of g-bosons for rotational nuclei. Several authors developed perturbative methods to take into account the g-boson effects; see ref. 18,20). As, for example, Von Egmond and Allart[18] pointed out, the renormalization to (sd) boson H parameters are often large, especially when boson number is large, which happens for SU(3) nuclei. In general g-boson effects are important at the level of 20%. In any case, in the final nanlysis, it is the experimental data that has to answer the question of importance of g-bosons in IBM. Data shows that the E_4 properties, structure of highlying 4^+ levels, angular momentum cutt-off in the rotational bands, appearance of $k = 1^+, 3^+$ bands etc. does not seem to be explainable without explicit inclusion of g-bosons. In order to estimate quantitatively the role of g-bosons, one has to derive signatures of g -bosons effects and perform detailed studies in gIBM. This project was undertaken by various groups and we will describe the progress made so far, here below.

The most general 1+2 body H in gIBM contain too many (35) parameters and secondly the H matrix dimensionalities grow fast, for $\mathcal{N}=12, L=2; d \sim 1100$ in gIBM as against 100 in IBM. As a result simplified gIBM H and gIBM basis are chosen, essentially using the U(6) $\oplus$ U(9) limit, for performing studies with g-bosons. On the otherhand one can construct the dynamical symmetries of gIBM and derive analytic results for various observables and also obtain selection rules, typical spectra etc. These allow for a rapid analysis of data. Keeping in mind the power of this later approach, we solved the first step, (viz) construction of the dynamical symmetries and derived the IRR of the groups involved.

The hexadecapole moments and the E4 decay properties of nuclear levels, obviously yield direct information on g-bosons in IBM. Data on these observables is fast accumulating[22]. Recently Baker[23] analyzed the E4 transition amplitudes of the first 4^+ levels,

$$M^4_{04_1} = \langle 0_1^+ \| T^{E_4} \| 4_1^+ \rangle \quad (80)$$

in A $\sim$ 150-190 nuclei and found that the observed values cannot be explained by the simple IBM E4 operator, $T^{E_4} = \alpha\,(d^{+}\tilde{d})^4$. The maxima and minima in the M^4_{04} values could not be reproduced by IBM. Similarly Baker et. al[24] studied the E4 decay of the first three 4^+ states in ^{192}OS nucleus. Experimentally

$$M^4_{04_1} / M^4_{04_2} / M^4_{04_3} \simeq -2000/\pm 1180/\pm 1100 \; efm^4$$

They found that IBM cannot describe the observed values, particularly for the 4_3^+ state the predicted value is too small. With sensible values for the parameters in a simplified gIBM H and the gIBM E2, E4-operators, they could describe the E2 and E4 properties of ^{192}OS. They conclude that the 4_3^+ state is mostly a g-boson state, However it appears that a better description needs mixing of severla g-bosons. Besides these, studies on E4, E2 properties of levels in ^{156}Gd and ^{104}Ru are made, by coupling a g-boson to the IBM core. In ^{156}Gd the SU(3) core was used and in ^{104}Ru O(6) and U(5) cores are used. Goldhorn et. al.[25] measured hexadecapole (E4) strength distribution by populating 4^+ levels below $\sim$ 3 MeV in ^{156}Gd, and they show that SU(3) core + one g-boson calculations in general fail to reproduce the observed fragmentation. Van Isacker et.al[26)] calculated the E2 and E4 properties of the hexadecapole $k = 4^+$ band at $\sim$ 1.5 MeV in ^{156}Gd. Similarly the properties of lowlying (compared to 2 q.p. excitation energy) 4^+ levels in ^{104}Ru are studied by Heyde et.al[27)]. The general conclusion of these studies is that one has to perform calculations with several g-bosons in IBM, for a good description of data.

One of the important feature of IBM is the finite boson number. As a result in IBM description one has angular momentum cut-off in the spectra. This poses serious problems for SU(3) nuclei, as several rotational nuclei have bands extending beyond SU(3) cut-off value. Inclusion of g-boson degree of freedom, extends the L cut-off value $L_c = 2N$ for IBM to $L_c = 4N$ in gIBM. A good study on the high spin (vibrational like) states is due to Dukelsky et.al[28)] who analyzed the decay properties, B(E1)/B(E2) ratio, of the $k=0_1^+$ and 5_1^- bands in ^{218}Ra using a completely aligned basis with s,d,g and f bosons. They use

$$|n_s;\ n_d L_d = 2n_d;\ n_g L_g = 4n_g;\ n_f L_f = 3n_f;\ L = 2n_d + 4n_g + 3n_f\rangle$$

basis, with $n_s + n_d + n_g + n_f = 5, n_f = 0$ or 1. The quadrupole operator is taken to be $Q = Q_2(S)$ of (23) and the E1-operator is

$$T^{E1} = \alpha_1 (d^\dagger \tilde{f} + f^\dagger \tilde{d})^1 + \beta_1 (g^\dagger \tilde{f} + f^\dagger \tilde{g})^1 , \quad (81)$$

They obtained good agreement with data for L upto l=14$^+$, 17$^-$. With (sd) bosons these high spins are not possible. Leaving aside the questions on the free parameters in the calculations, one can conclude that the above analysis give a good evidence for the g-bosons effects. We wonder whether the U(5) limit of gIBM is relevant here.

In order to overcome the problems of large spaces in gIBM, Pittel et.al[29)] proposed a selfconstant Hartree-Bose approach to make calculations with g-bosons. They applied this approach to study the location and M1 properties of 1$^+$ levels. As it is well known, in IBM-1 the L=1$^+$ levels are strictly forebidden and as a result one cannot have odd-K rotational bands; see eq.(8). However in p-n IBM one can get 1$^+$ levels and the traditional description of 1$^+$ levels/bands is in terms of 2 q.p. excitations, see ref.30). One has to study[31)] in detail the g-boson, p-n IBM and 2 q.p. description of 1$^+$ levels/odd-K bands. Although Pittel et.al[29)] are reasonably successful in describing the isovector-M1 L=1$^+$ state at ~3MeV in ^{156}Gd in terms of g-bosons, there is little doubt that the proper description of this 1$^+$ level is in terms of the p-n IBM.

An alternative to the HB approach, for studies in large spaces we encounter in gIBM, is the spectral averaging methods, see Ref. 10,12 and references therein for descriptions of this approach. Once reasonable gIBM-H are available, they might very well prove to be useful. Some applications of these methods are already made for Boson systems in ref. 10). In these methods the multiplicities of the IRR play an important role.

The knowledge on the structure of gIBM-H is very sparse. We feel that H that interpolate various dynamical symmetries of gIBM might be the one to be used in calculations. One such model Hamiltonian is

$$H_M = \alpha\, Q_2 \cdot Q_2 + \beta\, Q_4 \cdot Q_4 + \gamma\, L \cdot L \tag{82}$$

where

$$Q_2 = (d^\dagger \tilde{d})^2 + \alpha_1 (g^\dagger \tilde{g})^2 + \alpha_2 (d^\dagger \tilde{s} + s^\dagger \tilde{d})^2 + \alpha_3 (g^\dagger \tilde{d} + d^\dagger \tilde{g})^2 \tag{83}$$

and

$$Q_4 = (d^\dagger \tilde{d})^4 + \beta_1 (g^\dagger \tilde{g})^4 + \beta_2 (d^\dagger \tilde{g} + g^\dagger \tilde{d})^4 + \beta_3 (g^\dagger \tilde{s} + s^\dagger \tilde{g})^4 \tag{84}$$

above H interpolates the U(5)-SU(3)-U(6) limits of gIBM. When α's and β's are such that $Q_2 = \alpha_0 G^2$ and $Q_4 = \alpha_0 G^4$, then H_M is expressible in terms of U(5), O(5) Casimir operators, similarly when $Q_2 = \alpha_0' \Lambda^2$ and $Q_4 = \alpha_0' \Lambda^4$ then H_M is expressible in terms of U(6) and Sp(6) Casimir operators. Finally for $Q_2 = \alpha_0'' Q_2(S)$ and $\beta = 0$, H_M is expressible in terms of SU(3) Casimir operator. A slightly better (realistic) H could be,

$$H_M' = \varepsilon_d n_d + \varepsilon_g n_g + \alpha Q_2 \cdot Q_2 + \beta Q_4 \cdot Q_4 + \gamma L \cdot L \quad (85)$$

Studies using (82) and (85) are clearly called for.

By invoking dynamical symmetries, gIBM becomes analytically tractable and also as the symmetry limits have predictive power, one can accomplish a lot more in this approach than with purely numerical calculations described above. Of the six limiting symmetries of gIBM, the simplest one is the U(6) ⊕ U(9) limit. For nuclei with good IBM description, this limit is what can think of using, and we already described the work done in this frame work. While some preliminary work was done on the U(5)[32,33] and U(14) limits, no attention was paid so far to the O(15) and U(6) limits. With all nuclear physicists familiar with Elliott's work, till now it is the SU(3) limit that received considerable attention; see ref.19,21, 31,33-35). We should add that no compelling evidence is seen so far for the goodness of any of the limiting symmetries except perhaps for the SU(3) limit. From Elliott's work[9] one already has expressions for spectra, quadrupole moments and B(E2) values etc. In the SU(3) limit one has a definite prediction[31,34] that there are odd-K bands which are forebidden in IBM, the lowest of them coming from the IRR (4M-6,3) in (51) which has $k=1^+$ and 3^+ bands. The intrinsic structure of these bands is given in ref.31) and they appear close to the (β,γ) bands. Experimentally one sees only (lowlying) 3^+ bands but very rarely (1^+, 3^+) bands. This suggests that the SU(3) symmetry may be a broken symmetry. Some detailed calculations with the framework of the SU(3) limit are performed for deformed nuclei. Yoshinaga et.al[35] and Wu and Zhou[35] studied $_{68}$Er, Wu[34] studied $_{72}$HF and Suguna et.al[35] studied $_{74}$W isotopes. Of all these Yoshinaga et.al[35] analysis of $^{168}_{68}$Er nucleus was rather exhaustive. Within the framework of the SU(3) limit of gIBM including several (20) SU(3) IRR and a mixing interaction with four parameters, they could solve many problems appearing in the lowlying positive parity states of ^{168}Er. For example the problem of anhormonicity regarding the excitation energy of the first $k=4^+$ band relative to that of the first $k=2^+$ band is solved. The lowest $k=3^+$ band is predicted at a reasonable position and they also identify the $k=1^+$ band at 3.39 MeV with the theoretically predicted state at 3.8 MeV which is predominently made up of (2M-6, 3) IRR. All these studies show the importance of g-bosons and provide evidence for the relevance of the SU(3) limit of gIBM. Another indirect evidencee for the same comes from g-boson occupencies. It was shown by Kota[21] that upto $L=\mathcal{M}$ ($\sim$16 typically) the fraction of g-bosons in the ground state rotational (SU(3)) band is $\sim$20-25% amd it is in direct correlation with the empirical fact that[22] hexadecapole deformations are about 20% of the quadrupole deformations and also the microscopic theories, as we pointed out earlier, predict that the g-bosons are important at the level of 20%. In the SU(3) limit, the ground state rotational band has cut-off at L= 4M against 2M in IBM. Thus one is tempted to describe the high spin states with g-bosons. While this may have some relevance, the coupling of 2q.p. excitations appear to be very important. The backbending phenomena in high spin states of $_{56}Ba$, $_{58}Ce$ and $_{34}Ge$ isotopes appear to be well described[36] by coupling of $h_{11/2}$ and $g_{9/2}$ pairs respectively to the IBM core. All the studies on the SU(3) limit clearly show that rotational nuclei provide a good ground for establishing the role of g-bosons. More work on these nuclei is very much essential. Meanwhile Barrett and Halse[37] extended gIBM to p-n gIBM and tried to study the 1^+ levels in ^{156}Gd in the SU(3) limit of the p-n IBM. Finally we[21] extended gIBM to gIBFM by constructing the supersymmetry scheme U(15/30) where gIBM core nucleus is coupled to a j=1/2, 3/2, 5/2, 7/2 and 9/2 particle and this model is applied to ^{185}W.

In order to establish the importance of g-bosons and the symmetry limits of gIBM, work in several directions remain to be done. A careful analysis of experimental data, especially on hexadecapole moments and B(E4) is called for. One has to look for systematics in the data, new experiments to fill gaps if any should be done. Closed analytic expressions for various observables, in the six limits of gIBM, for Q_2, Q_4, B(E2), B(E4) etc. should be derived. In addition changes in the spectra and other properties with the addition of symmetry breaking terms should be worked out. In order to learn about the regions of the periodic table, where the various symmetry limits are applicable and also to understand their physical content one has to study the geometric correspondence for the six limits, here an extension of the results of ref.38) is needed. As we stated before, the six limits of gIBM go pairwise into the three limits of IBM. Here the geometric correspondence might reveal something interesting and useful. A formal problem that one should solve in gIBM is to prove that there exist only six limiting symmetries, besides the seventh one that comes from the (sg,d) orbit combination in the unitary orbit method. One should also decide on the appropriate phases to be chosen in seniority schemes (i.e) the $U(14) \supset O(14)$ and O(15) limits, see (17) and (25). In passing we remark that a similar phase problem occurs in the other limits and the spectral averaging methods are best suited for solving this problem, see ref.10,39). In IBM also this problem is present, however in gIBM it appears that it has a special role to play. Besides, studies on even-even nuclei, the g-boson effects in odd-A nuclei should be looked for. One can construct several supersymmetry (SYSY) schemes in gIBFM, the interesting boson-fermion model with g-bosons. For example one can have a spinor symmetry by coupling a j=5/2 particle to the U(6) limit of gIBM. In ref. 21), we already discussed a U(15/30) SUSY scheme.

Finally we conclude that, with the piecewise evidence accumulating in favour of the importance of g-bosons in nuclei, the studies on the dynamical symmetries of gIBM acquire new importance. The first step towards this goal is completed in this report and the remaining analytic work on the six limits of gIBM is under progress. It will be very exciting to find examples for the six limits of gIBM. With the dynamical symmetries of gIBM providing algebraic description of the hexadecapole deformation, one is in sight of understanding the manifestations and the role of this collective mode in nuclei.

TABLE-A U(5) GENERATORS

$$G^{L}_{0} = 0 \quad = \sqrt{\tfrac{1}{5}} \left\{ (s^{+}\tilde{s})^{0} + \sqrt{5}\,(d^{+}\tilde{d})^{0} + \sqrt{9}\,(g^{+}\tilde{g})^{0} \right\}$$

$$G^{L}_{0} = 1 \quad = \tfrac{1}{2} \left\{ (d^{+}\tilde{d})^{1} + \sqrt{6}\,(g^{+}\tilde{g})^{1} \right\}$$

$$G^{L}_{0} = 2 \quad = \sqrt{\tfrac{1}{5}} \left\{ (s^{+}\tilde{d} + d^{+}\tilde{s})^{2} - \frac{3\sqrt{5}}{14}(d^{+}\tilde{d})^{2} \right.$$

$$\left. + 6/7\,(d^{+}\tilde{g} + g^{+}\tilde{d})^{2} + \frac{3}{14}\sqrt{5.22}\,(g^{+}\tilde{g})^{2} \right.$$

$$G^{L}_{0} = 3 \quad = -\tfrac{4}{7} \left\{ (d^{+}\tilde{d})^{3} - \tfrac{3}{8}\sqrt{10}\,(g^{+}\tilde{d} + d^{+}\tilde{g})^{3} \right.$$

$$\left. - \tfrac{3}{8}\sqrt{11}\,(g^{+}\tilde{g})^{3} \right.$$

$$G^{L}_{0} = 4 \quad = \sqrt{\tfrac{1}{5}} \left\{ (g^{+}\tilde{s} + s^{+}\tilde{g})^{4} + \frac{2\sqrt{5}}{7}(d^{+}\tilde{d})^{4} \right.$$

$$\left. \tfrac{5}{7}\sqrt{\tfrac{11}{2}}\,(d^{+}\tilde{g} + g^{+}\tilde{d})^{4} + \frac{\sqrt{11.13}}{14}(g^{+}\tilde{g})^{4} \right.$$

TABLE-B U(6) GENERATORS

$$h^{L_0=0} = \frac{-1}{\sqrt{6}} \left\{ (s^{+} \tilde{s})^{0} + \sqrt{5}\,(d^{+} \tilde{d})^{0} + \sqrt{9}\,(g^{+} \tilde{g})^{0} \right\}$$

$$h^{L_0=1} = \frac{-1}{\sqrt{7}} \left\{ (d^{+} \tilde{d})^{1} + \sqrt{6}\,(g^{+} \tilde{g})^{1} \right\}$$

$$h^{L_0=2} = \frac{-1}{\sqrt{6}} (d^{+} \tilde{s} + s^{+} \tilde{d})^{2} + \frac{5}{7\sqrt{6}} (d^{+} \tilde{d})^{2} - \frac{9}{14\sqrt{2}} (d^{+} \tilde{g} + g^{+} \tilde{d})^{2} - \frac{\sqrt{33}}{14} (g^{+} \tilde{g})^{2}$$

$$h^{L_0=3} = \frac{9}{14\sqrt{2}} (d^{+} \tilde{d})^{3} - \frac{5\sqrt{3}}{14} (d^{+} \tilde{g} + g^{+} \tilde{d})^{3} + \frac{1}{14}\sqrt{\frac{11}{2}}\, (g^{+} \tilde{g})^{3}$$

$$h^{L_0=4} = -\frac{1}{\sqrt{6}} (s^{+} \tilde{g} + g^{+} \tilde{s})^{4} - \frac{3}{14}\sqrt{\frac{5}{2}}\, (d^{+} \tilde{d})^{4} - \frac{1}{14}\sqrt{\frac{55}{3}}\, (d^{+} g + g^{+} d)^{4} + \frac{1}{14}\sqrt{\frac{11.13}{2}}\, (g^{+} \tilde{g})^{4}$$

$$h^{l_0=5} = \frac{1}{2}\sqrt{\frac{15}{14}}\, (g^{+} \tilde{d} + d^{+} \tilde{g})^{5} + \frac{1}{2}\sqrt{\frac{13}{7}}\, (g^{+} \tilde{g})^{5}$$

TABLE-C DIMENSIONALITIES

GROUP	IRR	DIMENSIONALITY
U(N)	$\{m\}$	$\binom{N+m-1}{m}$
U(N)	$\{f_1 f_2 \ldots\}$	$\prod_{i<j} \frac{f_i - f_j + j - i}{j-i}$
O(N)	$[m]$	$\binom{N+m-3}{m} \frac{2m+N-2}{N-2}$
O(5)	$[\tau_1\ \tau_2]$	$[(\tau_1-\tau_2+1)(\tau_1+\tau_2+2) \times (2\tau_1+3)(2\tau_2+1)/6]$
Sp(6)	$\langle\lambda_1\ \lambda_2\ \lambda_3\rangle$	$[(\lambda_1+3)(\lambda_2+2)(\lambda_1+1) * (\lambda_1-\lambda_2+1)(\lambda_1-\lambda_3+2)(\lambda_2-\lambda_3+1) * (\lambda_1+\lambda_2+5)(\lambda_1+\lambda_3+4) * (\lambda_2+\lambda_3+3)/720]$
SU(3)	$(\lambda\mu)$	$(\lambda+1)(\mu+1)(\lambda+\mu+2)/2$

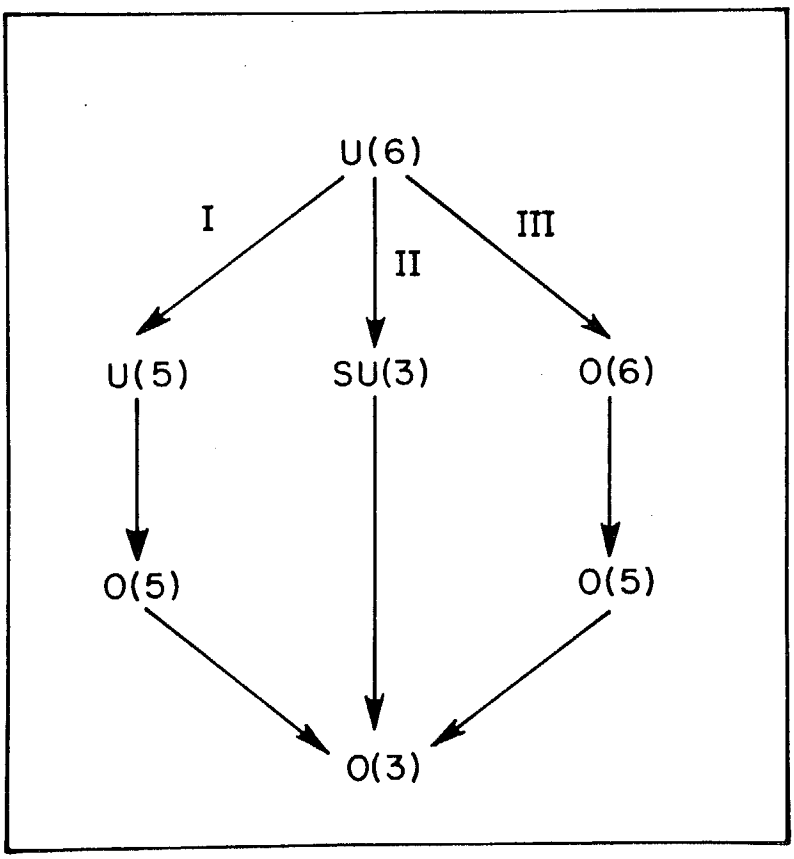

Fig-1 Symmetry group chains in IBM

Symmetry group chains in gIBM

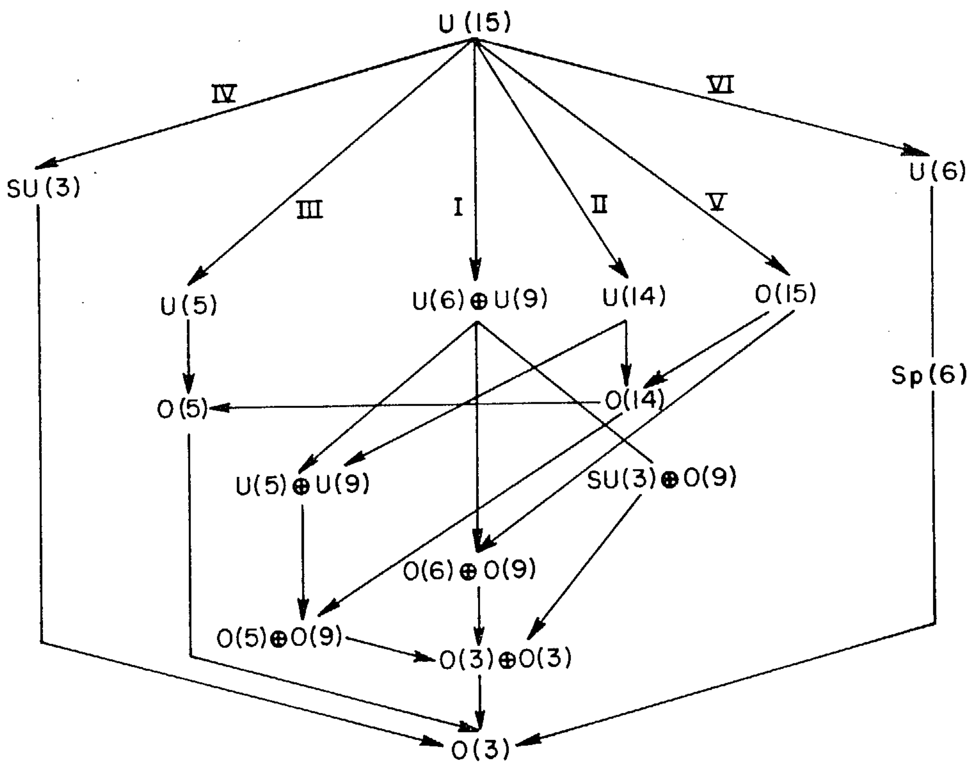

Fig-2

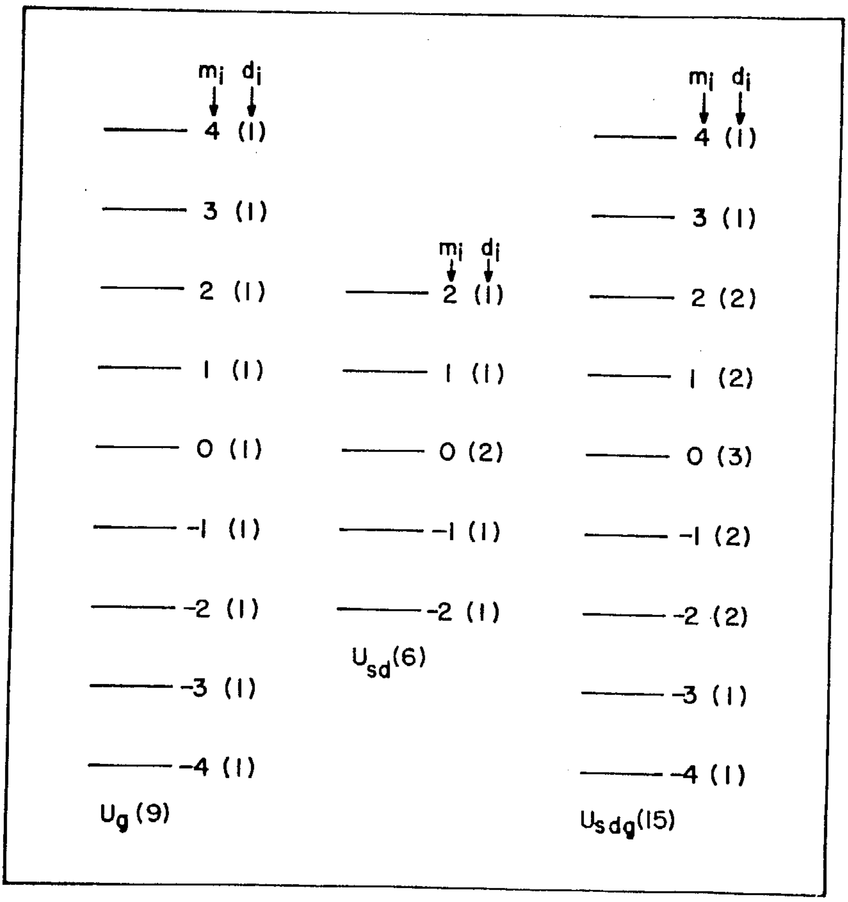

Fig-3 Single particle spectrum for the $\vec{\ell_z}$ operator

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APPENDIX - A

Some simple group-subgroup reduction rules

The reduction of the IRR $\{m\}$ of U(N) to IRR $\{m'\}$ of U(N-1) in $U(N) \supset U(N-1)$ is,

$$m' = m, m-1, ----, 0 \tag{A.1}$$

Similarly for the chain $O(N) \supset O(N-1)$, $[\sigma] \to [\sigma']$ such that

$$\sigma' = \sigma, \sigma-1, ----, 0 \tag{A.2}$$

In the chain $U(N) \supset O(N)$, the IRR $\{m\} \to [v]$ where

$$v = m, m-2, ---, 0 \text{ or } 1 \tag{A.3}$$

Eg. (A.1) and (A.3) follow from the well known notions of configurations and seniority in nuclear shell model and (A.2) can be derived from (A.1) and (A.3).

For the subgroup chain $U(N) \supset U(\mathcal{M}_l)$ with $N = \sum_{l=0,2,----,k_0} (2l+1)$ and $\mathcal{M}_l = k_0+1$, we have the association $\{1\}_{U(N)} \to \{2\}_{U(\mathcal{M}_l)}$. Then the IRR $\{\mathcal{M}\}$ of (U(N) reduces to IRR $\{f\}$ of $U(\mathcal{M}_l)$ where

$$\{\mathcal{M}\}_{U(N)} \to \{f\}_{U(\mathcal{M}_l)} = \sum \{2f_1, 2f_2, -----, 2f_{\mathcal{M}}\} \tag{A.4}$$

where $\{f_1\ f_2\ ---, f_{\mathcal{M}}\}$ is a partition of the integer $\mathcal{M}$ with no more than $\mathcal{M}_l$-parts. The proof of (A.4), though not explicit, is given in Littlewood's article. We arrived at the same from dimensionality considerations. Note that (7) follows directly from (A.4).

For the chain $U(N) \supset U(M_2)$ with $M_2 = k_0 + 2$, we have the association $\{1\}_{U(N)} \rightarrow \{1^2\}_{U(M_2)}$, then the IRR of $U(M_2)$ in $\{M\}$ of $U(N)$ are given by

$$\sum \{2f_1, 2f_2, ---, 2f_M\} \tag{A.5}$$

Where the summation is over all partitions $\{f_1 f_2 ---- f_M\}$ of the integer M with no more than $(k_0+2)/2$ columns. $\{\tilde{f}\}$ in (A.5) is the conjugate IRR (rows replaced by columns). The proof of (A.5) follows from the following theorem of Littlewood[40] where the notion of "plethysm" is used. We can write (A.5) as $\{1^2\} \otimes \{m\} \rightarrow \{k\}$ and similarly (A.4) can be written as $\{2\} \otimes \{m\} \rightarrow \{k'\}$. If $\{\lambda\}$ is a partition of the integer r and

$$\{\lambda\} \otimes \{m\} = \sum \{\nu\}$$

then if r is even $\{\tilde{\lambda}\} \otimes \{m\} = \sum \{\tilde{\nu}\}$

if r is odd $\{\tilde{\lambda}\} \otimes \{\tilde{m}\} = \sum \{\tilde{\nu}\}$ (A.6)

Putting $\{\lambda\} = \{2\}$ and using (A.4) and (A.6) we get (A.5).

APPENDIX-B: TABLES OF IRREDUCIBLE REPRESENTATIONS

For the group subgroup chain $G \supset H$, the problem is one of determining the IRR of H contained in a given IRR of G. In tables 1-13 we present tabulations for various $G \supset H$ chains we encounter in gIBM. As the results are generated on a computer we used the following conventions for the IRR and their multiplicities. The U(N), O(N), Sp(N) IRR are denoted by $\{f_1 f_2 --\}$, $[f_1 f_2 --]$ and $\langle f_1 f_2 -- \rangle$ respectively. The SU(3) IRR and the O(3) IRR L are denoted by $(\lambda \mu)$ and (L) respectively. A gap is inserted between $f_1, f_2, f_3 --$ when there is more than one f_i needed for specifying a IRR. For example $\{12\ 10\ \ 9\}$ means $f_1=12$, $f_2=10, f_3=9$ and $f_4=0, f_5=0, ---$, while $\{15\}$ means $f_1=15$, $f_2=0, f_3=0, --$. In the case of $(\lambda \mu)$ IRR a gap is inserted between λ and μ. The multiplicity of the IRR is given in front of the IRR. For example in table-1, in the reduction $\{6\} \rightarrow L=8$, the multiplicity of L=8 is four. Similarly from table-8, the reduction of $\{10\} \rightarrow (\lambda \mu)_{SU(3)}$ the multiplicity of the SU(3) IRR (12 5) is 46 and it is given as 46(12 5). The contents of tables 1-13 are as follows.

Table-1	$\{\mathcal{M}\}_{U(6)} \rightarrow L$, $\mathcal{M}=1-15$
Table-2	$\{\mathcal{M}\}_{U(9)} \rightarrow L$, $\mathcal{M}=1-15$

Table-5 $\{2M, 0\}_{U(5)} \rightarrow [v_1 v_2]_{O(5)}$, $M = 1\text{–}15$

$\{2M-2, 2\}_{U(5)} \rightarrow [v_1 v_2]_{O(5)}$, $M = 1\text{–}15$

Table-6 $[M, 0]_{O(5)} \rightarrow L$, $M = 0\text{–}30$

$[M, 1]_{O(5)} \rightarrow L$, $M = 1\text{–}29$

Table-7 $[M, 2]_{O(5)} \rightarrow L$, $M = 2\text{–}28$

Table-8 $\{M\}_{U(15)} \rightarrow (\lambda \mu)_{SU(3)}$, $M = 1\text{–}15$

Table-9 $\{M\}_{U(15)} \rightarrow \{f_1 f_2 f_3 f_4 f_5 f_6\}_{U(6)}$, $M = 1\text{–}15$

Table-10 $\{M, M\}_{U(6)} \rightarrow \langle \lambda_1 \lambda_2 \lambda_3 \rangle_{Sp(6)}$, $M = 1\text{–}15$

$\{M-1, M-1, 1, 1\}_{U(6)} \rightarrow \langle \lambda_1 \lambda_2 \lambda_3 \rangle_{Sp(6)}$, $M = 2\text{–}15$

Table-11 $\langle \lambda \lambda 0 \rangle_{Sp(6)} \rightarrow L$, $\lambda = 1\text{–}155$

$\langle \lambda, \lambda-1, 1 \rangle_{Sp(6)} \rightarrow L$, $\lambda = 2\text{–}15$

Table-12 $[v]_{O(15)} \rightarrow [v_1 v_2]_{O(5)}$, $v = 1\text{–}15$

Table-13 $[v]_{O(14)} \rightarrow [v_1 v_2]_{O(5)}$, $v = 1\text{–}15$

Table-1 Reduction of U(6) IRR {M} into IRR (L) of O(3)

{1}	1(0)	1(2)					
{2}	2(0)	2(2)	1(4)				
{3}	3(0)	3(2)	1(3)	2(4)	1(6)		
{4}	4(0)	5(2)	1(3)	4(4)	1(5)	2(6)	1(8)
{5}	5(0)	7(2)	2(3)	6(4)	2(5)	4(6)	1(7)
	2(8)	1(10)					
{6}	7(0)	9(2)	3(3)	9(4)	3(5)	7(6)	2(7)
	4(8)	1(9)	2(10)	1(12)			
{7}	8(0)	12(2)	4(3)	12(4)	5(5)	10(6)	4(7)
	7(8)	2(9)	4(10)	1(11)	2(12)	1(14)	
{8}	10(0)	15(2)	5(3)	16(4)	7(5)	14(6)	6(7)
	11(8)	4(9)	7(10)	2(11)	4(12)	1(13)	2(14)
	1(16)						
{9}	12(0)	18(2)	7(3)	20(4)	9(5)	19(6)	9(7)
	15(8)	7(9)	11(10)	4(11)	7(12)	2(13)	4(14)
	1(15)	2(16)	1(18)				
{10}	14(0)	22(2)	8(3)	25(4)	12(5)	24(6)	12(7)
	21(8)	10(9)	16(10)	7(11)	11(12)	4(13)	7(14)
	2(15)	4(16)	1(17)	2(18)	1(20)		
{11}	16(0)	26(2)	10(3)	30(4)	15(5)	30(6)	16(7)
	27(8)	14(9)	22(10)	11(11)	16(12)	7(13)	11(14)
	4(15)	7(16)	2(17)	4(18)	1(19)	2(20)	1(22)
{12}	19(0)	30(2)	12(3)	36(4)	18(5)	37(6)	20(7)
	34(8)	19(9)	29(10)	15(11)	23(12)	11(13)	16(14)
	7(15)	11(16)	4(17)	7(18)	2(19)	4(20)	1(21)
	2(22)	1(24)					
{13}	21(0)	35(2)	14(3)	42(4)	22(5)	44(6)	25(7)
	42(8)	24(9)	37(10)	21(11)	30(12)	16(13)	23(14)

	11(15)	16(16)	7(17)	11(18)	4(19)	7(20)	2(21)
	4(22)	1(23)	2(24)	1(26)			
{14}	24(0)	40(2)	16(3)	49(4)	26(5)	52(6)	30(7)
	51(8)	30(9)	46(10)	27(11)	39(12)	22(13)	31(14)
	16(15)	23(16)	11(17)	16(18)	7(19)	11(20)	4(21)
	7(22)	2(23)	4(24)	1(25)	2(26)	1(28)	
{15}	27(0)	45(2)	19(3)	56(4)	30(5)	61(6)	36(7)
	60(8)	37(9)	56(10)	34(11)	49(12)	29(13)	40(14)
	23(15)	31(16)	16(17)	23(18)	11(19)	16(20)	7(21)
	11(22)	4(23)	7(24)	2(25)	4(26)	1(27)	2(28)
	1(30)						

Table-2 Reduction of U(9) IRR {M} into IRR (L) of O(3)

{1}	1(4)							
{2}	1(0)	1(2)	1(4)	1(6)	1(8)			
{3}	1(0)	1(2)	1(3)	2(4)	1(5)	2(6)	1(7)	1(8)
	1(9)	1(10)	1(12)					
{4}	2(0)	3(2)	1(3)	4(4)	2(5)	4(6)	2(7)	4(8)
	2(9)	3(10)	1(11)	2(12)	1(13)	1(14)	1(16)	
{5}	2(0)	1(1)	4(2)	3(3)	6(4)	5(5)	7(6)	5(7)
	7(8)	5(9)	6(10)	4(11)	5(12)	3(13)	3(14)	2(15)
	2(16)	1(17)	1(18)	1(20)				
{6}	4(0)	1(1)	7(2)	5(3)	11(4)	7(5)	13(6)	9(7)
	13(8)	10(9)	12(10)	8(11)	11(12)	7(13)	8(14)	5(15)
	6(16)	3(17)	4(18)	2(19)	2(20)	1(21)	1(22)	1(24)
{7}	4(0)	3(1)	10(2)	9(3)	16(4)	14(5)	19(6)	17(7)
	21(8)	18(9)	21(10)	17(11)	19(12)	15(13)	16(14)	12(15)
	13(16)	9(17)	9(18)	6(19)	6(20)	4(21)	4(22)	2(23)
	2(24)	1(25)	1(26)	1(28)				
{8}	7(0)	4(1)	16(2)	13(3)	25(4)	21(5)	31(6)	26(7)
	35(8)	29(9)	35(10)	29(11)	34(12)	27(13)	30(14)	23(15)
	25(16)	19(17)	20(18)	14(19)	15(20)	10(21)	10(22)	6(23)
	7(24)	4(25)	4(26)	2(27)	2(28)	1(29)	1(30)	1(32)
{9}	8(0)	8(1)	21(2)	22(3)	35(4)	33(5)	45(6)	42(7)
	51(8)	48(9)	54(10)	48(11)	54(12)	47(13)	49(14)	43(15)
	44(16)	36(17)	37(18)	29(19)	29(20)	23(21)	22(22)	16(23)
	16(24)	11(25)	10(26)	7(27)	7(28)	4(29)	4(30)	2(31)
	2(32)	1(33)	1(34)	1(36)				
{10}	12(0)	10(1)	32(2)	30(3)	51(4)	48(5)	66(6)	61(7)
	77(8)	70(9)	83(10)	74(11)	84(12)	74(13)	80(14)	69(15)
	74(16)	62(17)	64(18)	53(19)	54(20)	43(21)	43(22)	33(23)
	33(24)	25(25)	24(26)	17(27)	17(28)	11(29)	11(30)	7(31)
	7(32)	4(33)	4(34)	2(35)	2(36)	1(37)	1(38)	1(40)
{11}	13(0)	17(1)	42(2)	45(3)	69(4)	70(5)	91(6)	90(7)
	108(8)	105(9)	118(10)	112(11)	123(12)	114(13)	121(14)	110(15)

	114(16)	102(17)	103(18)	90(19)	90(20)	77(21)	75(22)	63(23)
	61(24)	49(25)	47(26)	37(27)	35(28)	27(29)	25(30)	18(31)
	17(32)	12(33)	11(34)	7(35)	7(36)	4(37)	4(38)	2(39)
	2(40)	1(41)	1(42)	1(44)				
{12}	20(0)	22(1)	58(2)	61(3)	96(4)	95(5)	128(6)	124(7)
	152(8)	147(9)	169(10)	159(11)	179(12)	166(13)	178(14)	164(15)
	173(16)	155(17)	161(18)	142(19)	144(20)	126(21)	125(22)	106(23)
	106(24)	88(25)	85(26)	70(27)	67(28)	53(29)	51(30)	39(31)
	37(32)	28(33)	26(34)	18(35)	18(36)	12(37)	11(38)	7(39)
	7(40)	4(41)	4(42)	2(43)	2(44)	1(45)	1(46)	1(48)
{13}	22(0)	33(1)	75(2)	85(3)	126(4)	133(5)	169(6)	173(7)
	205(8)	205(9)	231(10)	226(11)	247(12)	238(13)	252(14)	239(15)
	249(16)	232(17)	236(18)	217(19)	218(20)	197(21)	195(22)	173(23)
	169(24)	148(25)	142(26)	122(27)	117(28)	98(29)	92(30)	76(31)
	71(32)	57(33)	53(34)	41(35)	38(36)	29(37)	26(38)	19(39)
	18(40)	12(41)	11(42)	7(43)	7(44)	4(45)	4(46)	2(47)
	2(48)	1(49)	1(50)	1(52)				
{14}	31(0)	42(1)	101(2)	111(3)	168(4)	175(5)	227(6)	230(7)
	277(8)	275(9)	314(10)	307(11)	340(12)	327(13)	352(14)	334(15)
	352(16)	330(17)	341(18)	315(19)	321(20)	293(21)	293(22)	264(23)
	262(24)	232(25)	227(26)	198(27)	192(28)	165(29)	158(30)	133(31)
	127(32)	105(33)	98(34)	80(35)	75(36)	59(37)	55(38)	42(39)
	39(40)	29(41)	27(42)	19(43)	18(44)	12(45)	11(46)	7(47)
	7(48)	4(49)	4(50)	2(51)	2(52)	1(53)	1(54)	1(56)
{15}	36(0)	59(1)	126(2)	150(3)	215(4)	233(5)	294(6)	307(7)
	360(8)	369(9)	414(10)	414(11)	453(12)	446(13)	473(14)	462(15)
	481(16)	461(17)	474(18)	448(19)	453(20)	425(21)	423(22)	390(23)
	386(24)	352(25)	342(26)	309(27)	298(28)	264(29)	253(30)	221(31)
	209(32)	181(33)	169(34)	143(35)	134(36)	111(37)	102(38)	84(39)
	77(40)	61(41)	56(42)	43(43)	39(44)	30(45)	27(46)	19(47)
	18(48)	12(49)	11(50)	7(51)	7(52)	4(53)	4(54)	2(55)
	2(56)	1(57)	1(58)	1(60)				

Table-3 Reduction of U(15) IRR {M} into IRR (L) of O(3)

{1}	1(0)	1(2)	1(4)			
{2}	3(0)	4(2)	1(3)	4(4)	1(5)	2(6)
	1(8)					
{3}	7(0)	2(1)	11(2)	7(3)	13(4)	6(5)
	10(6)	4(7)	5(8)	2(9)	2(10)	1(12)
{4}	15(0)	8(1)	32(2)	22(3)	40(4)	26(5)
	34(6)	20(7)	24(8)	12(9)	13(10)	5(11)
	6(12)	2(13)	2(14)	1(16)		
{5}	31(0)	27(1)	79(2)	68(3)	105(4)	84(5)
	102(6)	74(7)	81(8)	53(9)	53(10)	31(11)
	30(12)	15(13)	14(14)	6(15)	6(16)	2(17)
	2(18)	1(20)				
{6}	66(0)	73(1)	184(2)	179(3)	261(4)	227(5)
	274(6)	221(7)	236(8)	179(9)	175(10)	120(11)
	115(12)	72(13)	64(14)	37(15)	33(16)	16(17)
	15(18)	6(19)	6(20)	2(21)	2(22)	1(24)
{7}	126(0)	181(1)	406(2)	428(3)	597(4)	566(5)
	660(6)	582(7)	614(8)	506(9)	496(10)	383(11)
	356(12)	258(13)	229(14)	154(15)	134(16)	83(17)
	70(18)	40(19)	34(20)	17(21)	15(22)	6(23)
	6(24)	2(25)	2(26)	1(28)		
{8}	246(0)	406(1)	851(2)	949(3)	1292(4)	1287(5)
	1490(6)	1381(7)	1460(8)	1275(9)	1257(10)	1040(11)
	976(12)	765(13)	687(14)	510(15)	444(16)	312(17)
	263(18)	173(19)	145(20)	89(21)	73(22)	41(23)
	35(24)	17(25)	15(26)	6(27)	6(28)	2(29)
	2(30)	1(32)				
{9}	460(0)	860(1)	1696(2)	1991(3)	2647(4)	2743(5)
	3158(6)	3048(7)	3216(8)	2943(9)	2917(10)	2537(11)
	2404(12)	1998(13)	1812(14)	1445(15)	1268(16)	964(17)
	824(18)	598(19)	498(20)	346(21)	282(22)	184(23)
	151(24)	92(25)	74(26)	42(27)	35(28)	17(29)
	15(30)	6(31)	6(32)	2(33)	2(34)	1(36)
{10}	839(0)	1714(1)	3267(2)	3947(3)	5194(4)	5537(5)

	6352(6)	6312(7)	6691(8)	6308(9)	6314(10)	5694(11)
	5450(12)	4723(13)	4349(14)	3628(15)	3238(16)	2602(17)
	2254(18)	1746(19)	1478(20)	1101(21)	912(22)	652(23)
	532(24)	365(25)	293(26)	190(27)	154(28)	93(29)
	75(30)	42(31)	35(32)	17(33)	15(34)	6(35)
	6(36)	2(37)	2(38)	1(40)		
{11}	1486(0)	3275(1)	6051(2)	7513(3)	9782(4)	10669(5)
	12217(6)	12425(7)	13210(8)	12777(9)	12876(10)	11951(11)
	11552(12)	10336(13)	9642(14)	8341(15)	7547(16)	6323(17)
	5565(18)	4517(19)	3883(20)	3054(21)	2567(22)	1956(23)
	1615(24)	1189(25)	966(26)	686(27)	551(28)	376(29)
	299(30)	193(31)	155(32)	94(33)	75(34)	42(35)
	35(36)	17(37)	15(38)	6(39)	6(40)	2(41)
	2(42)	1(44)				
{12}	2591(0)	6002(1)	10861(2)	13754(3)	17800(4)	19733(5)
	22613(6)	23395(7)	24976(8)	24632(9)	25008(10)	23712(11)
	23162(12)	21233(13)	20050(14)	17834(15)	16366(16)	14137(17)
	12647(18)	10627(19)	9286(20)	7600(21)	6499(22)	5173(23)
	4348(24)	3367(25)	2777(26)	2093(27)	1703(28)	1243(29)
	1000(30)	705(31)	562(32)	382(33)	302(34)	194(35)
	156(36)	94(37)	75(38)	42(39)	35(40)	17(41)
	15(42)	6(43)	6(44)	2(45)	2(46)	1(48)
{13}	4381(0)	10645(1)	18933(2)	24356(3)	31366(4)	35272(5)
	40409(6)	42427(7)	45459(8)	45540(9)	46540(10)	44930(11)
	44251(12)	41395(13)	39516(14)	35933(15)	33392(16)	29579(17)
	26823(18)	23181(19)	20566(20)	17352(21)	15086(22)	12433(23)
	10612(24)	8546(25)	7169(26)	5638(27)	4661(28)	3577(29)
	2914(30)	2181(31)	1757(32)	1277(33)	1019(34)	716(35)
	568(36)	385(37)	303(38)	195(39)	156(40)	94(41)
	75(42)	42(43)	35(44)	17(45)	15(46)	6(47)
	6(48)	2(49)	2(50)	1(52)		
{14}	7286(0)	18296(1)	32149(2)	41860(3)	53748(4)	61092(5)
	70068(6)	74402(7)	80031(8)	81192(9)	83488(10)	81757(11)
	81181(12)	77163(13)	74401(14)	68878(15)	64744(16)	58509(17)
	53750(18)	47485(19)	42732(20)	36945(21)	32610(22)	27612(23)
	23948(24)	19864(25)	16945(26)	13772(27)	11573(28)	9216(29)
	7634(30)	5951(31)	4871(32)	3714(33)	3002(34)	2235(35)
	1791(36)	1296(37)	1030(38)	722(39)	571(40)	386(41)
	304(42)	195(43)	156(44)	94(45)	75(46)	42(47)
	35(48)	17(49)	15(50)	6(51)	6(52)	2(53)
	2(54)	1(56)				
{15}	11856(0)	30635(1)	53257(2)	70090(3)	89748(4)	102947(5)
	118194(6)	126719(7)	136748(8)	140242(9)	144976(10)	143685(11)
	143692(12)	138437(13)	134632(14)	126546(15)	120144(16)	110398(17)
	102588(18)	92314(19)	84110(20)	74215(21)	66400(22)	57478(23)
	50579(24)	42984(25)	37222(26)	31073(27)	26517(28)	21737(29)

18300(30)	14733(31)	12243(32)	9681(33)	7947(34)	6161(35)
5008(36)	3802(37)	3056(38)	2269(39)	1810(40)	1307(41)
1036(42)	725(43)	572(44)	387(45)	304(46)	195(47)
156(48)	94(49)	75(50)	42(51)	35(52)	17(53)
15(54)	6(55)	6(56)	2(57)	2(58)	1(60)

Table-4 Reduction of U(15) IRR {M} into IRR {pq} of U(5)

{M}				
{1}	{2 0 0 0 0}			
{2}	{4 0 0 0 0}	{2 2 0 0 0}		
{3}	{6 0 0 0 0}	{4 2 0 0 0}	{2 2 2 0 0}	
{4}	{8 0 0 0 0}	{6 2 0 0 0}	{4 4 0 0 0}	{4 2 2 0 0}
	{2 2 2 2 0}			
{5}	{10 0 0 0 0}	{8 2 0 0 0}	{6 4 0 0 0}	{6 2 2 0 0}
	{4 4 2 0 0}	{4 2 2 2 0}	{2 2 2 2 2}	
{6}	{12 0 0 0 0}	{10 2 0 0 0}	{8 4 0 0 0}	{8 2 2 0 0}
	{6 6 0 0 0}	{6 4 2 0 0}	{6 2 2 2 0}	{4 4 4 0 0}
	{4 4 2 2 0}	{4 2 2 2 2}		
{7}	{14 0 0 0 0}	{12 2 0 0 0}	{10 4 0 0 0}	{10 2 2 0 0}
	{8 6 0 0 0}	{8 4 2 0 0}	{8 2 2 2 0}	{6 6 2 0 0}
	{6 4 4 0 0}	{6 4 2 2 0}	{6 2 2 2 2}	{4 4 4 2 0}
	{4 4 2 2 2}			
{8}	{16 0 0 0 0}	{14 2 0 0 0}	{12 4 0 0 0}	{12 2 2 0 0}
	{10 6 0 0 0}	{10 4 2 0 0}	{10 2 2 2 0}	{8 8 0 0 0}
	{8 6 2 0 0}	{8 4 4 0 0}	{8 4 2 2 0}	{8 2 2 2 2}
	{6 6 4 0 0}	{6 6 2 2 0}	{6 4 4 2 0}	{6 4 2 2 2}
	{4 4 4 4 0}	{4 4 4 2 2}		
{9}	{18 0 0 0 0}	{16 2 0 0 0}	{14 4 0 0 0}	{14 2 2 0 0}
	{12 6 0 0 0}	{12 4 2 0 0}	{12 2 2 2 0}	{10 8 0 0 0}
	{10 6 2 0 0}	{10 4 4 0 0}	{10 4 2 2 0}	{10 2 2 2 2}
	{8 8 2 0 0}	{8 6 4 0 0}	{8 6 2 2 0}	{8 4 4 2 0}
	{8 4 2 2 2}	{6 6 6 0 0}	{6 6 4 2 0}	{6 6 2 2 2}
	{6 4 4 4 0}	{6 4 4 2 2}	{4 4 4 4 2}	
{10}	{20 0 0 0 0}	{18 2 0 0 0}	{16 4 0 0 0}	{16 2 2 0 0}
	{14 6 0 0 0}	{14 4 2 0 0}	{14 2 2 2 0}	{12 8 0 0 0}
	{12 6 2 0 0}	{12 4 4 0 0}	{12 4 2 2 0}	{12 2 2 2 2}
	{10 10 0 0 0}	{10 8 2 0 0}	{10 6 4 0 0}	{10 6 2 2 0}
	{10 4 4 2 0}	{10 4 2 2 2}	{8 8 4 0 0}	{8 8 2 2 0}
	{8 6 6 0 0}	{8 6 4 2 0}	{8 6 2 2 2}	{8 4 4 4 0}
	{8 4 4 2 2}	{6 6 6 2 0}	{6 6 4 4 0}	{6 6 4 2 2}
	{6 4 4 4 2}	{4 4 4 4 4}		

{11}	{22 0 0 0 0}	{20 2 0 0 0}	{18 4 0 0 0}	{18 2 2 0 0}
	{16 6 0 0 0}	{16 4 2 0 0}	{16 2 2 2 0}	{14 8 0 0 0}
	{14 6 2 0 0}	{14 4 4 0 0}	{14 4 2 2 0}	{14 2 2 2 2}
	{12 10 0 0 0}	{12 8 2 0 0}	{12 6 4 0 0}	{12 6 2 2 0}
	{12 4 4 2 0}	{12 4 2 2 2}	{10 10 2 0 0}	{10 8 4 0 0}
	{10 8 2 2 0}	{10 6 6 0 0}	{10 6 4 2 0}	{10 6 2 2 2}
	{10 4 4 4 0}	{10 4 4 2 2}	{ 8 8 6 0 0}	{ 8 8 4 2 0}
	{ 8 8 2 2 2}	{ 8 6 6 2 0}	{ 8 6 4 4 0}	{ 8 6 4 2 2}
	{ 8 4 4 4 2}	{ 6 6 6 4 0}	{ 6 6 6 2 2}	{ 6 6 4 4 2}
	{ 6 4 4 4 4}			
{12}	{24 0 0 0 0}	{22 2 0 0 0}	{20 4 0 0 0}	{20 2 2 0 0}
	{18 6 0 0 0}	{18 4 2 0 0}	{18 2 2 2 0}	{16 8 0 0 0}
	{16 6 2 0 0}	{16 4 4 0 0}	{16 4 2 2 0}	{16 2 2 2 2}
	{14 10 0 0 0}	{14 8 2 0 0}	{14 6 4 0 0}	{14 6 2 2 0}
	{14 4 4 2 0}	{14 4 2 2 2}	{12 12 0 0 0}	{12 10 2 0 0}
	{12 8 4 0 0}	{12 8 2 2 0}	{12 6 6 0 0}	{12 6 4 2 0}
	{12 6 2 2 2}	{12 4 4 4 0}	{12 4 4 2 2}	{10 10 4 0 0}
	{10 10 2 2 0}	{10 8 6 0 0}	{10 8 4 2 0}	{10 8 2 2 2}
	{10 6 6 2 0}	{10 6 4 4 0}	{10 6 4 2 2}	{10 4 4 4 2}
	{ 8 8 8 0 0}	{ 8 8 6 2 0}	{ 8 8 4 4 0}	{ 8 8 4 2 2}
	{ 8 6 6 4 0}	{ 8 6 6 2 2}	{ 8 6 4 4 2}	{ 8 4 4 4 4}
	{ 6 6 6 6 0}	{ 6 6 6 4 2}	{ 6 6 4 4 4}	
{13}	{26 0 0 0 0}	{24 2 0 0 0}	{22 4 0 0 0}	{22 2 2 0 0}
	{20 6 0 0 0}	{20 4 2 0 0}	{20 2 2 2 0}	{18 8 0 0 0}
	{18 6 2 0 0}	{18 4 4 0 0}	{18 4 2 2 0}	{18 2 2 2 2}
	{16 10 0 0 0}	{16 8 2 0 0}	{16 6 4 0 0}	{16 6 2 2 0}
	{16 4 4 2 0}	{16 4 2 2 2}	{14 12 0 0 0}	{14 10 2 0 0}
	{14 8 4 0 0}	{14 8 2 2 0}	{14 6 6 0 0}	{14 6 4 2 0}
	{14 6 2 2 2}	{14 4 4 4 0}	{14 4 4 2 2}	{12 12 2 0 0}
	{12 10 4 0 0}	{12 10 2 2 0}	{12 8 6 0 0}	{12 8 4 2 0}
	{12 8 2 2 2}	{12 6 6 2 0}	{12 6 4 4 0}	{12 6 4 2 2}
	{12 4 4 4 2}	{10 10 6 0 0}	{10 10 4 2 0}	{10 10 2 2 2}
	{10 8 8 0 0}	{10 8 6 2 0}	{10 8 4 4 0}	{10 8 4 2 2}
	{10 6 6 4 0}	{10 6 6 2 2}	{10 6 4 4 2}	{10 4 4 4 4}
	{ 8 8 8 2 0}	{ 8 8 6 4 0}	{ 8 8 6 2 2}	{ 8 8 4 4 2}
	{ 8 6 6 6 0}	{ 8 6 6 4 2}	{ 8 6 4 4 4}	{ 6 6 6 6 2}
	{ 6 6 6 4 4}			
{14}	{28 0 0 0 0}	{26 2 0 0 0}	{24 4 0 0 0}	{24 2 2 0 0}
	{22 6 0 0 0}	{22 4 2 0 0}	{22 2 2 2 0}	{20 8 0 0 0}
	{20 6 2 0 0}	{20 4 4 0 0}	{20 4 2 2 0}	{20 2 2 2 2}
	{18 10 0 0 0}	{18 8 2 0 0}	{18 6 4 0 0}	{18 6 2 2 0}
	{18 4 4 2 0}	{18 4 2 2 2}	{16 12 0 0 0}	{16 10 2 0 0}
	{16 8 4 0 0}	{16 8 2 2 0}	{16 6 6 0 0}	{16 6 4 2 0}
	{16 6 2 2 2}	{16 4 4 4 0}	{16 4 4 2 2}	{14 14 0 0 0}
	{14 12 2 0 0}	{14 10 4 0 0}	{14 10 2 2 0}	{14 8 6 0 0}
	{14 8 4 2 0}	{14 8 2 2 2}	{14 6 6 2 0}	{14 6 4 4 0}
	{14 6 4 2 2}	{14 4 4 4 2}	{12 12 4 0 0}	{12 12 2 2 0}
	{12 10 6 0 0}	{12 10 4 2 0}	{12 10 2 2 2}	{12 8 8 0 0}
	{12 8 6 2 0}	{12 8 4 4 0}	{12 8 4 2 2}	{12 6 6 4 0}

{12 6 6 2 2} {12 6 4 4 2} {12 4 4 4 4} {10 10 8 0 0}
{10 10 6 2 0} {10 10 4 4 0} {10 10 4 2 2} {10 8 8 2 0}
{10 8 6 4 0} {10 8 6 2 2} {10 8 4 4 2} {10 6 6 6 0}
{10 6 6 4 2} {10 6 4 4 4} { 8 8 8 4 0} { 8 8 8 2 2}
{ 8 8 6 6 0} { 8 8 6 4 2} { 8 8 4 4 4} { 8 6 6 6 2}
{ 8 6 6 4 4} { 6 6 6 6 4}

{15}

{30 0 0 0 0} {28 2 0 0 0} {26 4 0 0 0} {26 2 2 0 0}
{24 6 0 0 0} {24 4 2 0 0} {24 2 2 2 0} {22 8 0 0 0}
{22 6 2 0 0} {22 4 4 0 0} {22 4 2 2 0} {22 2 2 2 2}
{20 10 0 0 0} {20 8 2 0 0} {20 6 4 0 0} {20 6 2 2 0}
{20 4 4 2 0} {20 4 2 2 2} {18 12 0 0 0} {18 10 2 0 0}
{18 8 4 0 0} {18 8 2 2 0} {18 6 6 0 0} {18 6 4 2 0}
{18 6 2 2 2} {18 4 4 4 0} {18 4 4 2 2} {16 14 0 0 0}
{16 12 2 0 0} {16 10 4 0 0} {16 10 2 2 0} {16 8 6 0 0}
{16 8 4 2 0} {16 8 2 2 2} {16 6 6 2 0} {16 6 4 4 0}
{16 6 4 2 2} {16 4 4 4 2} {14 14 2 0 0} {14 12 4 0 0}
{14 12 2 2 0} {14 10 6 0 0} {14 10 4 2 0} {14 10 2 2 2}
{14 8 8 0 0} {14 8 6 2 0} {14 8 4 4 0} {14 8 4 2 2}
{14 6 6 4 0} {14 6 6 2 2} {14 6 4 4 2} {14 4 4 4 4}
{12 12 6 0 0} {12 12 4 2 0} {12 12 2 2 2} {12 10 8 0 0}
{12 10 6 2 0} {12 10 4 4 0} {12 10 4 2 2} {12 8 8 2 0}
{12 8 6 4 0} {12 8 6 2 2} {12 8 4 4 2} {12 6 6 6 0}
{12 6 6 4 2} {12 6 4 4 4} {10 10 10 0 0} {10 10 8 2 0}
{10 10 6 4 0} {10 10 6 2 2} {10 10 4 4 2} {10 8 8 4 0}
{10 8 8 2 2} {10 8 6 6 0} {10 8 6 4 2} {10 8 4 4 4}
{10 6 6 6 2} {10 6 6 4 4} { 8 8 8 6 0} { 8 8 8 4 2}
{ 8 8 6 6 2} { 8 8 6 4 4} { 8 6 6 6 4} { 6 6 6 6 6}

Table-5 Reduction of U(5) IRR {M N} into IRR [P Q] of O(5)

{0 0}	[0 0]
{2 0}	[2 0] [0 0]
{4 0}	[4 0] [2 0] [0 0]
{2 2}	1[2 0] 1[0 0] 1[2 2]
{6 0}	[6 0] [4 0] [2 0] [0 0]
{4 2}	1[4 0] 2[2 0] 1[0 0] 1[3 1] 1[4 2] 1[2 2]
{8 0}	[8 0] [6 0] [4 0] [2 0] [0 0]
{6 2}	1[6 0] 2[4 0] 2[2 0] 1[0 0] 1[5 1] 1[3 1] 1[6 2] 1[4 2] 1[2 2]
{10 0}	[10 0] [8 0] [6 0] [4 0] [2 0] [0 0]
{8 2}	1[8 0] 2[6 0] 2[4 0] 2[2 0] 1[0 0] 1[7 1] 1[5 1] 1[3 1] 1[8 2] 1[6 2] 1[4 2] 1[2 2]
{12 0}	[12 0] [10 0] [8 0] [6 0] [4 0] [2 0] [0 0]
{10 2}	1[10 0] 2[8 0] 2[6 0] 2[4 0] 2[2 0] 1[0 0] 1[9 1] 1[7 1] 1[5 1] 1[3 1] 1[10 2] 1[8 2] 1[6 2] 1[4 2] 1[2 2]
{14 0}	[14 0] [12 0] [10 0] [8 0] [6 0] [4 0] [2 0] [0 0]
{12 2}	1[12 0] 2[10 0] 2[8 0] 2[6 0] 2[4 0] 2[2 0] 1[0 0] 1[11 1] 1[9 1] 1[7 1] 1[5 1] 1[3 1] 1[12 2] 1[10 2] 1[8 2] 1[6 2] 1[4 2] 1[2 2]
{16 0}	[16 0] [14 0] [12 0] [10 0] [8 0] [6 0] [4 0] [2 0] [0 0]

```
{14 2}   1[14  0]  2[12  0]  2[10  0]  2[ 8  0]  2[ 6  0]  2[ 4  0]
         2[ 2  0]  1[ 0  0]  1[13  1]  1[11  1]  1[ 9  1]  1[ 7  1]
         1[ 5  1]  1[ 3  1]  1[14  2]  1[12  2]  1[10  2]  1[ 8  2]
         1[ 6  2]  1[ 4  2]  1[ 2  2]

{18 0}   [18  0]  [16  0]  [14  0]  [12  0]  [10  0]  [ 8  0]  [ 6  0]
         [ 4  0]  [ 2  0]  [ 0  0]

{16 2}   1[16  0]  2[14  0]  2[12  0]  2[10  0]  2[ 8  0]  2[ 6  0]
         2[ 4  0]  2[ 2  0]  1[ 0  0]  1[15  1]  1[13  1]  1[11  1]
         1[ 9  1]  1[ 7  1]  1[ 5  1]  1[ 3  1]  1[16  2]  1[14  2]
         1[12  2]  1[10  2]  1[ 8  2]  1[ 6  2]  1[ 4  2]  1[ 2  2]

{20 0}   [20  0]  [18  0]  [16  0]  [14  0]  [12  0]  [10  0]  [ 8  0]
         [ 6  0]  [ 4  0]  [ 2  0]  [ 0  0]

{18 2}   1[18  0]  2[16  0]  2[14  0]  2[12  0]  2[10  0]  2[ 8  0]
         2[ 6  0]  2[ 4  0]  2[ 2  0]  1[ 0  0]  1[17  1]  1[15  1]
         1[13  1]  1[11  1]  1[ 9  1]  1[ 7  1]  1[ 5  1]  1[ 3  1]
         1[18  2]  1[16  2]  1[14  2]  1[12  2]  1[10  2]  1[ 8  2]
         1[ 6  2]  1[ 4  2]  1[ 2  2]

{22 0}   [22  0]  [20  0]  [18  0]  [16  0]  [14  0]  [12  0]  [10  0]
         [ 8  0]  [ 6  0]  [ 4  0]  [ 2  0]  [ 0  0]

{20 2}   1[20  0]  2[18  0]  2[16  0]  2[14  0]  2[12  0]  2[10  0]
         2[ 8  0]  2[ 6  0]  2[ 4  0]  2[ 2  0]  1[ 0  0]  1[19  1]
         1[17  1]  1[15  1]  1[13  1]  1[11  1]  1[ 9  1]  1[ 7  1]
         1[ 5  1]  1[ 3  1]  1[20  2]  1[18  2]  1[16  2]  1[14  2]
         1[12  2]  1[10  2]  1[ 8  2]  1[ 6  2]  1[ 4  2]  1[ 2  2]

{24 0}   [24  0]  [22  0]  [20  0]  [18  0]  [16  0]  [14  0]  [12  0]
         [10  0]  [ 8  0]  [ 6  0]  [ 4  0]  [ 2  0]  [ 0  0]

{22 2}   1[22  0]  2[20  0]  2[18  0]  2[16  0]  2[14  0]  2[12  0]
         2[10  0]  2[ 8  0]  2[ 6  0]  2[ 4  0]  2[ 2  0]  1[ 0  0]
         1[21  1]  1[19  1]  1[17  1]  1[15  1]  1[13  1]  1[11  1]
         1[ 9  1]  1[ 7  1]  1[ 5  1]  1[ 3  1]  1[22  2]  1[20  2]
         1[18  2]  1[16  2]  1[14  2]  1[12  2]  1[10  2]  1[ 8  2]
         1[ 6  2]  1[ 4  2]  1[ 2  2]

{26 0}   [26  0]  [24  0]  [22  0]  [20  0]  [18  0]  [16  0]  [14  0]
         [12  0]  [10  0]  [ 8  0]  [ 6  0]  [ 4  0]  [ 2  0]  [ 0  0]

{24 2}   1[24  0]  2[22  0]  2[20  0]  2[18  0]  2[16  0]  2[14  0]
```

```
         2[12  0]  2[10  0]  2[ 8  0]  2[ 6  0]  2[ 4  0]  2[ 2  0]
         1[ 0  0]  1[23  1]  1[21  1]  1[19  1]  1[17  1]  1[15  1]
         1[13  1]  1[11  1]  1[ 9  1]  1[ 7  1]  1[ 5  1]  1[ 3  1]
         1[24  2]  1[22  2]  1[20  2]  1[18  2]  1[16  2]  1[14  2]
         1[12  2]  1[10  2]  1[ 8  2]  1[ 6  2]  1[ 4  2]  1[ 2  2]

{28 0}   [28  0]  [26  0]  [24  0]  [22  0]  [20  0]  [18  0]  [16  0]
         [14  0]  [12  0]  [10  0]  [ 8  0]  [ 6  0]  [ 4  0]  [ 2  0]
         [ 0  0]

{26 2}    1[26  0]  2[24  0]  2[22  0]  2[20  0]  2[18  0]  2[16  0]

         2[14  0]  2[12  0]  2[10  0]  2[ 8  0]  2[ 6  0]  2[ 4  0]
         2[ 2  0]  1[ 0  0]  1[25  1]  1[23  1]  1[21  1]  1[19  1]
         1[17  1]  1[15  1]  1[13  1]  1[11  1]  1[ 9  1]  1[ 7  1]
         1[ 5  1]  1[ 3  1]  1[26  2]  1[24  2]  1[22  2]  1[20  2]
         1[18  2]  1[16  2]  1[14  2]  1[12  2]  1[10  2]  1[ 8  2]
         1[ 6  2]  1[ 4  2]  1[ 2  2]

{30 0}   [30  0]  [28  0]  [26  0]  [24  0]  [22  0]  [20  0]  [18  0]
         [16  0]  [14  0]  [12  0]  [10  0]  [ 8  0]  [ 6  0]  [ 4  0]
         [ 2  0]  [ 0  0]

{28 2}   1[28  0]  2[26  0]  2[24  0]  2[22  0]  2[20  0]  2[18  0]
         2[16  0]  2[14  0]  2[12  0]  2[10  0]  2[ 8  0]  2[ 6  0]
         2[ 4  0]  2[ 2  0]  1[ 0  0]  1[27  1]  1[25  1]  1[23  1]
         1[21  1]  1[19  1]  1[17  1]  1[15  1]  1[13  1]  1[11  1]
         1[ 9  1]  1[ 7  1]  1[ 5  1]  1[ 3  1]  1[28  2]  1[26  2]
         1[24  2]  1[22  2]  1[20  2]  1[18  2]  1[16  2]  1[14  2]
         1[12  2]  1[10  2]  1[ 8  2]  1[ 6  2]  1[ 4  2]  1[ 2  2]
```

Table-6 Reduction of O(5) IRR [P Q] into IRR (L) of O(3)

[0 0]	1(0)							
[1 0]	1(2)							
[2 0]	1(2)	1(4)						
[3 0]	1(0)	1(3)	1(4)	1(6)				
[4 0]	1(2)	1(4)	1(5)	1(6)	1(8)			
[5 0]	1(2)	1(4)	1(5)	1(6)	1(7)	1(8)	1(10)	
[6 0]	1(0) 1(12)	1(3)	1(4)	2(6)	1(7)	1(8)	1(9)	1(10)
[7 0]	1(2) 1(11)	1(4) 1(12)	1(5) 1(14)	1(6)	1(7)	2(8)	1(9)	1(10)
[8 0]	1(2) 1(11)	1(4) 1(12)	1(5) 1(13)	1(6) 1(14)	1(7) 1(16)	2(8)	1(9)	2(10)
[9 0]	1(0) 1(11)	1(3) 2(12)	1(4) 1(13)	2(6) 1(14)	1(7) 1(15)	1(8) 1(16)	2(9) 1(18)	2(10)
[10 0]	1(2) 2(11) 1(20)	1(4) 2(12)	1(5) 1(13)	1(6) 2(14)	1(7) 1(15)	2(8) 1(16)	1(9) 1(17)	2(10) 1(18)
[11 0]	1(2) 2(11) 1(19)	1(4) 2(12) 1(20)	1(5) 2(13) 1(22)	1(6) 2(14)	1(7) 1(15)	2(8) 2(16)	1(9) 1(17)	2(10) 1(18)
[12 0]	1(0) 1(11) 1(19)	1(3) 3(12) 1(20)	1(4) 2(13) 1(21)	2(6) 2(14) 1(22)	1(7) 2(15) 1(24)	1(8) 2(16)	2(9) 1(17)	2(10) 2(18)
[13 0]	1(2) 2(11) 1(19)	1(4) 2(12) 2(20)	1(5) 2(13) 1(21)	1(6) 3(14) 1(22)	1(7) 2(15) 1(23)	2(8) 2(16) 1(24)	1(9) 2(17) 1(26)	2(10) 2(18)

[14 0]	1(2)	1(4)	1(5)	1(6)	1(7)	2(8)	1(9)	2(10)
	2(11)	2(12)	2(13)	3(14)	2(15)	3(16)	2(17)	2(18)
	2(19)	2(20)	1(21)	2(22)	1(23)	1(24)	1(25)	1(26)
	1(28)							
[15 0]	1(0)	1(3)	1(4)	2(6)	1(7)	1(8)	2(9)	2(10)
	1(11)	3(12)	2(13)	2(14)	3(15)	3(16)	2(17)	3(18)
	2(19)	2(20)	2(21)	2(22)	1(23)	2(24)	1(25)	1(26)
	1(27)	1(28)	1(30)					
[16 0]	1(2)	1(4)	1(5)	1(6)	1(7)	2(8)	1(9)	2(10)
	2(11)	2(12)	2(13)	3(14)	2(15)	3(16)	3(17)	3(18)
	2(19)	3(20)	2(21)	2(22)	2(23)	2(24)	1(25)	2(26)
	1(27)	1(28)	1(29)	1(30)	1(32)			
[17 0]	1(2)	1(4)	1(5)	1(6)	1(7)	2(8)	1(9)	2(10)
	2(11)	2(12)	2(13)	3(14)	2(15)	3(16)	3(17)	3(18)
	3(19)	3(20)	2(21)	3(22)	2(23)	2(24)	2(25)	2(26)
	1(27)	2(28)	1(29)	1(30)	1(31)	1(32)	1(34)	
[18 0]	1(0)	1(3)	1(4)	2(6)	1(7)	1(8)	2(9)	2(10)
	1(11)	3(12)	2(13)	2(14)	3(15)	3(16)	2(17)	4(18)
	3(19)	3(20)	3(21)	3(22)	2(23)	3(24)	2(25)	2(26)
	2(27)	2(28)	1(29)	2(30)	1(31)	1(32)	1(33)	1(34)
	1(36)							
[19 0]	1(2)	1(4)	1(5)	1(6)	1(7)	2(8)	1(9)	2(10)
	2(11)	2(12)	2(13)	3(14)	2(15)	3(16)	3(17)	3(18)
	3(19)	4(20)	3(21)	3(22)	3(23)	3(24)	2(25)	3(26)
	2(27)	2(28)	2(29)	2(30)	1(31)	2(32)	1(33)	1(34)
	1(35)	1(36)	1(38)					
[20 0]	1(2)	1(4)	1(5)	1(6)	1(7)	2(8)	1(9)	2(10)
	2(11)	2(12)	2(13)	3(14)	2(15)	3(16)	3(17)	3(18)
	3(19)	4(20)	3(21)	4(22)	3(23)	3(24)	3(25)	3(26)
	2(27)	3(28)	2(29)	2(30)	2(31)	2(32)	1(33)	2(34)
	1(35)	1(36)	1(37)	1(38)	1(40)			
[21 0]	1(0)	1(3)	1(4)	2(6)	1(7)	1(8)	2(9)	2(10)
	1(11)	3(12)	2(13)	2(14)	3(15)	3(16)	2(17)	4(18)
	3(19)	3(20)	4(21)	4(22)	3(23)	4(24)	3(25)	3(26)
	3(27)	3(28)	2(29)	3(30)	2(31)	2(32)	2(33)	2(34)
	1(35)	2(36)	1(37)	1(38)	1(39)	1(40)	1(42)	
[22 0]	1(2)	1(4)	1(5)	1(6)	1(7)	2(8)	1(9)	2(10)
	2(11)	2(12)	2(13)	3(14)	2(15)	3(16)	3(17)	3(18)

	3(19)	4(20)	3(21)	4(22)	4(23)	4(24)	3(25)	4(26)
	3(27)	3(28)	3(29)	3(30)	2(31)	3(32)	2(33)	2(34)
	2(35)	2(36)	1(37)	2(38)	1(39)	1(40)	1(41)	1(42)
	1(44)							
[23 0]	1(2)	1(4)	1(5)	1(6)	1(7)	2(8)	1(9)	2(10)
	2(11)	2(12)	2(13)	3(14)	2(15)	3(16)	3(17)	3(18)
	3(19)	4(20)	3(21)	4(22)	4(23)	4(24)	4(25)	4(26)
	3(27)	4(28)	3(29)	3(30)	3(31)	3(32)	2(33)	3(34)
	2(35)	2(36)	2(37)	2(38)	1(39)	2(40)	1(41)	1(42)
	1(43)	1(44)	1(46)					
[24 0]	1(0)	1(3)	1(4)	2(6)	1(7)	1(8)	2(9)	2(10)
	1(11)	3(12)	2(13)	2(14)	3(15)	3(16)	2(17)	4(18)
	3(19)	3(20)	4(21)	4(22)	3(23)	5(24)	4(25)	4(26)
	4(27)	4(28)	3(29)	4(30)	3(31)	3(32)	3(33)	3(34)
	2(35)	3(36)	2(37)	2(38)	2(39)	2(40)	1(41)	2(42)
	1(43)	1(44)	1(45)	1(46)	1(48)			
[25 0]	1(2)	1(4)	1(5)	1(6)	1(7)	2(8)	1(9)	2(10)
	2(11)	2(12)	2(13)	3(14)	2(15)	3(16)	3(17)	3(18)
	3(19)	4(20)	3(21)	4(22)	4(23)	4(24)	4(25)	5(26)
	4(27)	4(28)	4(29)	4(30)	3(31)	4(32)	3(33)	3(34)
	3(35)	3(36)	2(37)	3(38)	2(39)	2(40)	2(41)	2(42)
	1(43)	2(44)	1(45)	1(46)	1(47)	1(48)	1(50)	
[26 0]	1(2)	1(4)	1(5)	1(6)	1(7)	2(8)	1(9)	2(10)
	2(11)	2(12)	2(13)	3(14)	2(15)	3(16)	3(17)	3(18)
	3(19)	4(20)	3(21)	4(22)	4(23)	4(24)	4(25)	5(26)
	4(27)	5(28)	4(29)	4(30)	4(31)	4(32)	3(33)	4(34)
	3(35)	3(36)	3(37)	3(38)	2(39)	3(40)	2(41)	2(42)
	2(43)	2(44)	1(45)	2(46)	1(47)	1(48)	1(49)	1(50)
	1(52)							
[27 0]	1(0)	1(3)	1(4)	2(6)	1(7)	1(8)	2(9)	2(10)
	1(11)	3(12)	2(13)	2(14)	3(15)	3(16)	2(17)	4(18)
	3(19)	3(20)	4(21)	4(22)	3(23)	5(24)	4(25)	4(26)
	5(27)	5(28)	4(29)	5(30)	4(31)	4(32)	4(33)	4(34)
	3(35)	4(36)	3(37)	3(38)	3(39)	3(40)	2(41)	3(42)
	2(43)	2(44)	2(45)	2(46)	1(47)	2(48)	1(49)	1(50)
	1(51)	1(52)	1(54)					
[28 0]	1(2)	1(4)	1(5)	1(6)	1(7)	2(8)	1(9)	2(10)
	2(11)	2(12)	2(13)	3(14)	2(15)	3(16)	3(17)	3(18)
	3(19)	4(20)	3(21)	4(22)	4(23)	4(24)	4(25)	5(26)
	4(27)	5(28)	5(29)	5(30)	4(31)	5(32)	4(33)	4(34)
	4(35)	4(36)	3(37)	4(38)	3(39)	3(40)	3(41)	3(42)
	2(43)	3(44)	2(45)	2(46)	2(47)	2(48)	1(49)	2(50)
	1(51)	1(52)	1(53)	1(54)	1(56)			

[29 0]	1(2)	1(4)	1(5)	1(6)	1(7)	2(8)	1(9)	2(10)
	2(11)	2(12)	2(13)	3(14)	2(15)	3(16)	3(17)	3(18)
	3(19)	4(20)	3(21)	4(22)	4(23)	4(24)	4(25)	5(26)
	4(27)	5(28)	5(29)	5(30)	5(31)	5(32)	4(33)	5(34)
	4(35)	4(36)	4(37)	4(38)	3(39)	4(40)	3(41)	3(42)
	3(43)	3(44)	2(45)	3(46)	2(47)	2(48)	2(49)	2(50)
	1(51)	2(52)	1(53)	1(54)	1(55)	1(56)	1(58)	
[30 0]	1(0)	1(3)	1(4)	2(6)	1(7)	1(8)	2(9)	2(10)
	1(11)	3(12)	2(13)	2(14)	3(15)	3(16)	2(17)	4(18)
	3(19)	3(20)	4(21)	4(22)	3(23)	5(24)	4(25)	4(26)
	5(27)	5(28)	4(29)	6(30)	5(31)	5(32)	5(33)	5(34)
	4(35)	5(36)	4(37)	4(38)	4(39)	4(40)	3(41)	4(42)
	3(43)	3(44)	3(45)	3(46)	2(47)	3(48)	2(49)	2(50)
	2(51)	2(52)	1(53)	2(54)	1(55)	1(56)	1(57)	1(58)
	1(60)							
[1 1]	1(1)	1(3)						
[2 1]	1(1)	1(2)	1(3)	1(4)	1(5)			
[3 1]	1(1)	1(2)	2(3)	1(4)	2(5)	1(6)	1(7)	
[4 1]	1(1)	1(2)	2(3)	2(4)	2(5)	2(6)	2(7)	1(8)
	1(9)							
[5 1]	1(1)	1(2)	2(3)	2(4)	3(5)	2(6)	3(7)	2(8)
	2(9)	1(10)	1(11)					
[6 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	3(7)	3(8)
	3(9)	2(10)	2(11)	1(12)	1(13)			
[7 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	3(8)
	4(9)	3(10)	3(11)	2(12)	2(13)	1(14)	1(15)	
[8 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	4(9)	4(10)	4(11)	3(12)	3(13)	2(14)	2(15)	1(16)
	1(17)							
[9 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	4(10)	5(11)	4(12)	4(13)	3(14)	3(15)	2(16)
	2(17)	1(18)	1(19)					

[10 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	5(11)	5(12)	5(13)	4(14)	4(15)	3(16)
	3(17)	2(18)	2(19)	1(20)	1(21)			
[11 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	5(12)	6(13)	5(14)	5(15)	4(16)
	4(17)	3(18)	3(19)	2(20)	2(21)	1(22)	1(23)	
[12 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	6(13)	6(14)	6(15)	5(16)
	5(17)	4(18)	4(19)	3(20)	3(21)	2(22)	2(23)	1(24)
	1(25)							
[13 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	6(14)	7(15)	6(16)
	6(17)	5(18)	5(19)	4(20)	4(21)	3(22)	3(23)	2(24)
	2(25)	1(26)	1(27)					
[14 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	7(15)	7(16)
	7(17)	6(18)	6(19)	5(20)	5(21)	4(22)	4(23)	3(24)
	3(25)	2(26)	2(27)	1(28)	1(29)			
[15 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	7(16)
	8(17)	7(18)	7(19)	6(20)	6(21)	5(22)	5(23)	4(24)
	4(25)	3(26)	3(27)	2(28)	2(29)	1(30)	1(31)	
[16 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	8(17)	8(18)	8(19)	7(20)	7(21)	6(22)	6(23)	5(24)
	5(25)	4(26)	4(27)	3(28)	3(29)	2(30)	2(31)	1(32)
	1(33)							
[17 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	9(17)	8(18)	9(19)	8(20)	8(21)	7(22)	7(23)	6(24)
	6(25)	5(26)	5(27)	4(28)	4(29)	3(30)	3(31)	2(32)
	2(33)	1(34)	1(35)					
[18 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	9(17)	9(18)	9(19)	9(20)	9(21)	8(22)	8(23)	7(24)
	7(25)	6(26)	6(27)	5(28)	5(29)	4(30)	4(31)	3(32)
	3(33)	2(34)	2(35)	1(36)	1(37)			

[19 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	9(17)	9(18)	10(19)	9(20)	10(21)	9(22)	9(23)	8(24)
	8(25)	7(26)	7(27)	6(28)	6(29)	5(30)	5(31)	4(32)
	4(33)	3(34)	3(35)	2(36)	2(37)	1(38)	1(39)	
[20 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	9(17)	9(18)	10(19)	10(20)	10(21)	10(22)	10(23)	9(24)
	9(25)	8(26)	8(27)	7(28)	7(29)	6(30)	6(31)	5(32)
	5(33)	4(34)	4(35)	3(36)	3(37)	2(38)	2(39)	1(40)
	1(41)							
[21 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	9(17)	9(18)	10(19)	10(20)	11(21)	10(22)	11(23)	10(24)
	10(25)	9(26)	9(27)	8(28)	8(29)	7(30)	7(31)	6(32)
	6(33)	5(34)	5(35)	4(36)	4(37)	3(38)	3(39)	2(40)
	2(41)	1(42)	1(43)					
[22 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	9(17)	9(18)	10(19)	10(20)	11(21)	11(22)	11(23)	11(24)
	11(25)	10(26)	10(27)	9(28)	9(29)	8(30)	8(31)	7(32)
	7(33)	6(34)	6(35)	5(36)	5(37)	4(38)	4(39)	3(40)
	3(41)	2(42)	2(43)	1(44)	1(45)			
[23 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	9(17)	9(18)	10(19)	10(20)	11(21)	11(22)	12(23)	11(24)
	12(25)	11(26)	11(27)	10(28)	10(29)	9(30)	9(31)	8(32)
	8(33)	7(34)	7(35)	6(36)	6(37)	5(38)	5(39)	4(40)
	4(41)	3(42)	3(43)	2(44)	2(45)	1(46)	1(47)	
[24 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	9(17)	9(18)	10(19)	10(20)	11(21)	11(22)	12(23)	12(24)
	12(25)	12(26)	12(27)	11(28)	11(29)	10(30)	10(31)	9(32)
	9(33)	8(34)	8(35)	7(36)	7(37)	6(38)	6(39)	5(40)
	5(41)	4(42)	4(43)	3(44)	3(45)	2(46)	2(47)	1(48)
	1(49)							
[25 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	9(17)	9(18)	10(19)	10(20)	11(21)	11(22)	12(23)	12(24)
	13(25)	12(26)	13(27)	12(28)	12(29)	11(30)	11(31)	10(32)
	10(33)	9(34)	9(35)	8(36)	8(37)	7(38)	7(39)	6(40)
	6(41)	5(42)	5(43)	4(44)	4(45)	3(46)	3(47)	2(48)
	2(49)	1(50)	1(51)					

[26 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	9(17)	9(18)	10(19)	10(20)	11(21)	11(22)	12(23)	12(24)
	13(25)	13(26)	13(27)	13(28)	13(29)	12(30)	12(31)	11(32)
	11(33)	10(34)	10(35)	9(36)	9(37)	8(38)	8(39)	7(40)
	7(41)	6(42)	6(43)	5(44)	5(45)	4(46)	4(47)	3(48)
	3(49)	2(50)	2(51)	1(52)	1(53)			

[27 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	9(17)	9(18)	10(19)	10(20)	11(21)	11(22)	12(23)	12(24)
	13(25)	13(26)	14(27)	13(28)	14(29)	13(30)	13(31)	12(32)
	12(33)	11(34)	11(35)	10(36)	10(37)	9(38)	9(39)	8(40)
	8(41)	7(42)	7(43)	6(44)	6(45)	5(46)	5(47)	4(48)
	4(49)	3(50)	3(51)	2(52)	2(53)	1(54)	1(55)	

[28 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	9(17)	9(18)	10(19)	10(20)	11(21)	11(22)	12(23)	12(24)
	13(25)	13(26)	14(27)	14(28)	14(29)	14(30)	14(31)	13(32)
	13(33)	12(34)	12(35)	11(36)	11(37)	10(38)	10(39)	9(40)
	9(41)	8(42)	8(43)	7(44)	7(45)	6(46)	6(47)	5(48)
	5(49)	4(50)	4(51)	3(52)	3(53)	2(54)	2(55)	1(56)
	1(57)							

[29 1]	1(1)	1(2)	2(3)	2(4)	3(5)	3(6)	4(7)	4(8)
	5(9)	5(10)	6(11)	6(12)	7(13)	7(14)	8(15)	8(16)
	9(17)	9(18)	10(19)	10(20)	11(21)	11(22)	12(23)	12(24)
	13(25)	13(26)	14(27)	14(28)	15(29)	14(30)	15(31)	14(32)
	14(33)	13(34)	13(35)	12(36)	12(37)	11(38)	11(39)	10(40)
	10(41)	9(42)	9(43)	8(44)	8(45)	7(46)	7(47)	6(48)
	6(49)	5(50)	5(51)	4(52)	4(53)	3(54)	3(55)	2(56)
	2(57)	1(58)	1(59)					

Table-7 Reduction of O(5) IRR [P Q] into IRR (L) of O(3)

[2 2]	1(0)	1(2)	1(3)	1(4)	1(6)			
[3 2]	1(1)	2(2)	1(3)	2(4)	2(5)	1(6)	1(7)	1(8)
[4 2]	1(0)	1(1)	2(2)	2(3)	3(4)	2(5)	3(6)	2(7)
	2(8)	1(9)	1(10)					
[5 2]	1(0)	1(1)	2(2)	3(3)	3(4)	3(5)	4(6)	3(7)
	3(8)	3(9)	2(10)	1(11)	1(12)			
[6 2]	1(1)	3(2)	2(3)	4(4)	4(5)	4(6)	4(7)	5(8)
	3(9)	4(10)	3(11)	2(12)	1(13)	1(14)		
[7 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	5(6)	5(7)
	5(8)	5(9)	5(10)	4(11)	4(12)	3(13)	2(14)	1(15)
	1(16)							
[8 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	5(7)
	6(8)	6(9)	6(10)	5(11)	6(12)	4(13)	4(14)	3(15)
	2(16)	1(17)	1(18)					
[9 2]	1(1)	3(2)	2(3)	4(4)	5(5)	5(6)	6(7)	7(8)
	6(9)	7(10)	7(11)	6(12)	6(13)	6(14)	4(15)	4(16)
	3(17)	2(18)	1(19)	1(20)				
[10 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	6(7)
	7(8)	7(9)	8(10)	7(11)	8(12)	7(13)	7(14)	6(15)
	6(16)	4(17)	4(18)	3(19)	2(20)	1(21)	1(22)	
[11 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	6(7)
	7(8)	8(9)	8(10)	8(11)	9(12)	8(13)	8(14)	8(15)
	7(16)	6(17)	6(18)	4(19)	4(20)	3(21)	2(22)	1(23)
	1(24)							
[12 2]	1(1)	3(2)	2(3)	4(4)	5(5)	5(6)	6(7)	8(8)
	7(9)	9(10)	9(11)	9(12)	9(13)	10(14)	8(15)	9(16)
	8(17)	7(18)	6(19)	6(20)	4(21)	4(22)	3(23)	2(24)
	1(25)	1(26)						
[13 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	6(7)
	7(8)	8(9)	9(10)	9(11)	10(12)	10(13)	10(14)	10(15)

	10(16)	9(17)	9(18)	8(19)	7(20)	6(21)	6(22)	4(23)
	4(24)	3(25)	2(26)	1(27)	1(28)			
[14 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	6(7)
	7(8)	8(9)	9(10)	9(11)	11(12)	10(13)	11(14)	11(15)
	11(16)	10(17)	11(18)	9(19)	9(20)	8(21)	7(22)	6(23)
	6(24)	4(25)	4(26)	3(27)	2(28)	1(29)	1(30)	
[15 2]	1(1)	3(2)	2(3)	4(4)	5(5)	5(6)	6(7)	8(8)
	7(9)	9(10)	10(11)	10(12)	11(13)	12(14)	11(15)	12(16)
	12(17)	11(18)	11(19)	11(20)	9(21)	9(22)	8(23)	7(24)
	6(25)	6(26)	4(27)	4(28)	3(29)	2(30)	1(31)	1(32)
[16 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	6(7)
	7(8)	8(9)	9(10)	9(11)	11(12)	11(13)	12(14)	12(15)
	13(16)	12(17)	13(18)	12(19)	12(20)	11(21)	11(22)	9(23)
	9(24)	8(25)	7(26)	6(27)	6(28)	4(29)	4(30)	3(31)
	2(32)	1(33)	1(34)					
[17 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	6(7)
	7(8)	8(9)	9(10)	9(11)	11(12)	11(13)	12(14)	13(15)
	13(16)	13(17)	14(18)	13(19)	13(20)	13(21)	12(22)	11(23)
	11(24)	9(25)	9(26)	8(27)	7(28)	6(29)	6(30)	4(31)
	4(32)	3(33)	2(34)	1(35)	1(36)			
[18 2]	1(1)	3(2)	2(3)	4(4)	5(5)	5(6)	6(7)	8(8)
	7(9)	9(10)	10(11)	10(12)	11(13)	13(14)	12(15)	14(16)
	14(17)	14(18)	14(19)	15(20)	13(21)	14(22)	13(23)	12(24)
	11(25)	11(26)	9(27)	9(28)	8(29)	7(30)	6(31)	6(32)
	4(33)	4(34)	3(35)	2(36)	1(37)	1(38)		
[19 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	6(7)
	7(8)	8(9)	9(10)	9(11)	11(12)	11(13)	12(14)	13(15)
	14(16)	14(17)	15(18)	15(19)	15(20)	15(21)	15(22)	14(23)
	14(24)	13(25)	12(26)	11(27)	11(28)	9(29)	9(30)	8(31)
	7(32)	6(33)	6(34)	4(35)	4(36)	3(37)	2(38)	1(39)
	1(40)							
[20 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	6(7)
	7(8)	8(9)	9(10)	9(11)	11(12)	11(13)	12(14)	13(15)
	14(16)	14(17)	16(18)	15(19)	16(20)	16(21)	16(22)	15(23)
	16(24)	14(25)	14(26)	13(27)	12(28)	11(29)	11(30)	9(31)
	9(32)	8(33)	7(34)	6(35)	6(36)	4(37)	4(38)	3(39)
	2(40)	1(41)	1(42)					
[21 2]	1(1)	3(2)	2(3)	4(4)	5(5)	5(6)	6(7)	8(8)
	7(9)	9(10)	10(11)	10(12)	11(13)	13(14)	12(15)	14(16)
	15(17)	15(18)	16(19)	17(20)	16(21)	17(22)	17(23)	16(24)

	16(25)	16(26)	14(27)	14(28)	13(29)	12(30)	11(31)	11(32)
	9(33)	9(34)	8(35)	7(36)	6(37)	6(38)	4(39)	4(40)
	3(41)	2(42)	1(43)	1(44)				
[22 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	6(7)
	7(8)	8(9)	9(10)	9(11)	11(12)	11(13)	12(14)	13(15)
	14(16)	14(17)	16(18)	16(19)	17(20)	17(21)	18(22)	17(23)
	18(24)	17(25)	17(26)	16(27)	16(28)	14(29)	14(30)	13(31)
	12(32)	11(33)	11(34)	9(35)	9(36)	8(37)	7(38)	6(39)
	6(40)	4(41)	4(42)	3(43)	2(44)	1(45)	1(46)	
[23 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	6(7)
	7(8)	8(9)	9(10)	9(11)	11(12)	11(13)	12(14)	13(15)
	14(16)	14(17)	16(18)	16(19)	17(20)	18(21)	18(22)	18(23)
	19(24)	18(25)	18(26)	18(27)	17(28)	16(29)	16(30)	14(31)
	14(32)	13(33)	12(34)	11(35)	11(36)	9(37)	9(38)	8(39)
	7(40)	6(41)	6(42)	4(43)	4(44)	3(45)	2(46)	1(47)
	1(48)							
[24 2]	1(1)	3(2)	2(3)	4(4)	5(5)	5(6)	6(7)	8(8)
	7(9)	9(10)	10(11)	10(12)	11(13)	13(14)	12(15)	14(16)
	15(17)	15(18)	16(19)	18(20)	17(21)	19(22)	19(23)	19(24)
	19(25)	20(26)	18(27)	19(28)	18(29)	17(30)	16(31)	16(32)
	14(33)	14(34)	13(35)	12(36)	11(37)	11(38)	9(39)	9(40)
	8(41)	7(42)	6(43)	6(44)	4(45)	4(46)	3(47)	2(48)
	1(49)	1(50)						
[25 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	6(7)
	7(8)	8(9)	9(10)	9(11)	11(12)	11(13)	12(14)	13(15)
	14(16)	14(17)	16(18)	16(19)	17(20)	18(21)	19(22)	19(23)
	20(24)	20(25)	20(26)	20(27)	20(28)	19(29)	19(30)	18(31)
	17(32)	16(33)	16(34)	14(35)	14(36)	13(37)	12(38)	11(39)
	11(40)	9(41)	9(42)	8(43)	7(44)	6(45)	6(46)	4(47)
	4(48)	3(49)	2(50)	1(51)	1(52)			
[26 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	6(7)
	7(8)	8(9)	9(10)	9(11)	11(12)	11(13)	12(14)	13(15)
	14(16)	14(17)	16(18)	16(19)	17(20)	18(21)	19(22)	19(23)
	21(24)	20(25)	21(26)	21(27)	21(28)	20(29)	21(30)	19(31)
	19(32)	18(33)	17(34)	16(35)	16(36)	14(37)	14(38)	13(39)
	12(40)	11(41)	11(42)	9(43)	9(44)	8(45)	7(46)	6(47)
	6(48)	4(49)	4(50)	3(51)	2(52)	1(53)	1(54)	
[27 2]	1(1)	3(2)	2(3)	4(4)	5(5)	5(6)	6(7)	8(8)
	7(9)	9(10)	10(11)	10(12)	11(13)	13(14)	12(15)	14(16)
	15(17)	15(18)	16(19)	18(20)	17(21)	19(22)	20(23)	20(24)
	21(25)	22(26)	21(27)	22(28)	22(29)	21(30)	21(31)	21(32)
	19(33)	19(34)	18(35)	17(36)	16(37)	16(38)	14(39)	14(40)
	13(41)	12(42)	11(43)	11(44)	9(45)	9(46)	8(47)	7(48)
	6(49)	6(50)	4(51)	4(52)	3(53)	2(54)	1(55)	1(56)

[28 2]	1(0)	1(1)	2(2)	3(3)	4(4)	4(5)	6(6)	6(7)
	7(8)	8(9)	9(10)	9(11)	11(12)	11(13)	12(14)	13(15)
	14(16)	14(17)	16(18)	16(19)	17(20)	18(21)	19(22)	19(23)
	21(24)	21(25)	22(26)	22(27)	23(28)	22(29)	23(30)	22(31)
	22(32)	21(33)	21(34)	19(35)	19(36)	18(37)	17(38)	16(39)
	16(40)	14(41)	14(42)	13(43)	12(44)	11(45)	11(46)	9(47)
	9(48)	8(49)	7(50)	6(51)	6(52)	4(53)	4(54)	3(55)
	2(56)	1(57)	1(58)					

Table-8 Reduction of U(15) IRR {M} into IRR ($\lambda\mu$) of SU(3)

{1}	1(4 0)					
{2}	1(8 0)	1(4 2)	1(0 4)			
{3}	1(12 0)	1(8 2)	1(6 3)	1(4 4)	1(6 0)	1(3 3)
	1(0 6)	1(2 2)	1(0 0)			
{4}	1(16 0)	1(12 2)	1(10 3)	2(8 4)	1(10 0)	1(7 3)
	1(8 1)	2(4 6)	1(5 4)	2(6 2)	1(3 5)	1(4 3)
	1(5 1)	1(0 8)	2(2 4)	2(4 0)	1(1 3)	1(0 2)
{5}	1(20 0)	1(16 2)	1(14 3)	2(12 4)	1(14 0)	1(10 5)
	1(11 3)	1(12 1)	2(8 6)	1(9 4)	3(10 2)	1(6 7)
	2(7 5)	2(8 3)	2(9 1)	2(4 8)	1(5 6)	4(6 4)
	1(7 2)	3(8 0)	1(3 7)	2(4 5)	3(5 3)	1(6 1)
	1(0 10)	1(1 8)	3(2 6)	2(3 4)	4(4 2)	1(1 5)
	1(2 3)	1(3 1)	2(0 4)	2(2 0)		
{6}	1(24 0)	1(20 2)	1(18 3)	2(16 4)	1(18 0)	1(14 5)
	1(15 3)	1(16 1)	3(12 6)	1(13 4)	3(14 2)	1(10 7)
	2(11 5)	3(12 3)	2(13 1)	3(8 8)	2(9 6)	5(10 4)
	2(11 2)	4(12 0)	1(6 9)	2(7 7)	4(8 5)	5(9 3)
	2(10 1)	2(4 10)	2(5 8)	6(6 6)	4(7 4)	7(8 2)
	2(3 9)	3(4 7)	5(5 5)	5(6 3)	3(7 1)	2(0 12)
	4(2 8)	4(3 6)	7(4 4)	2(5 2)	5(6 0)	2(1 7)
	2(2 5)	4(3 3)	2(4 1)	4(0 6)	2(1 4)	4(2 2)
	2(0 0)					
{7}	1(28 0)	1(24 2)	1(22 3)	2(20 4)	1(22 0)	1(18 5)
	1(19 3)	1(20 1)	3(16 6)	1(17 4)	3(18 2)	2(14 7)
	2(15 5)	3(16 3)	2(17 1)	3(12 8)	2(13 6)	6(14 4)
	2(15 2)	4(16 0)	2(10 9)	3(11 7)	5(12 5)	6(13 3)
	3(14 1)	3(8 10)	3(9 8)	8(10 6)	6(11 4)	9(12 2)
	1(6 11)	3(7 9)	6(8 7)	8(9 5)	8(10 3)	5(11 1)
	3(4 12)	2(5 10)	8(6 8)	8(7 6)	13(8 4)	5(9 2)
	7(10 0)	2(3 11)	4(4 9)	8(5 7)	9(6 5)	9(7 3)
	6(8 1)	1(0 14)	1(1 12)	5(2 10)	5(3 8)	12(4 6)
	8(5 4)	11(6 2)	4(1 9)	4(2 7)	8(3 5)	7(4 3)
	4(5 1)	5(0 8)	3(1 6)	9(2 4)	3(3 2)	5(4 0)
	3(1 3)	1(2 1)	3(0 2)			
{8}	1(32 0)	1(28 2)	1(26 3)	2(24 4)	1(26 0)	1(22 5)
	1(23 3)	1(24 1)	3(20 6)	1(21 4)	3(22 2)	2(18 7)
	2(19 5)	3(20 3)	2(21 1)	4(16 8)	2(17 6)	6(18 4)
	2(19 2)	4(20 0)	2(14 9)	3(15 7)	6(16 5)	6(17 3)

	3(18 1)	4(12 10)	4(13 8)	9(14 6)	7(15 4)	10(16 2)
	2(10 11)	4(11 9)	8(12 7)	10(13 5)	10(14 3)	6(15 1)
	4(8 12)	3(9 10)	11(10 8)	11(11 6)	17(12 4)	7(13 2)
	9(14 0)	1(6 13)	4(7 11)	8(8 9)	13(9 7)	15(10 5)
	14(11 3)	9(12 1)	3(4 14)	3(5 12)	10(6 10)	11(7 8)
	21(8 6)	15(9 4)	18(10 2)	1(11 0)	2(3 13)	5(4 11)
	11(5 9)	14(6 7)	18(7 5)	16(8 3)	9(9 1)	2(0 16)
	1(1 14)	6(2 12)	8(3 10)	18(4 8)	14(5 6)	23(6 4)
	10(7 2)	10(8 0)	4(1 11)	6(2 9)	13(3 7)	14(4 5)
	14(5 3)	7(6 1)	6(0 10)	7(1 8)	15(2 6)	9(3 4)
	14(4 2)	1(5 0)	1(0 7)	7(1 5)	5(2 3)	5(3 1)
	7(0 4)	1(1 2)	4(2 0)			
{9}	1(36 0)	1(32 2)	1(30 3)	2(28 4)	1(30 0)	1(26 5)
	1(27 3)	1(28 1)	3(24 6)	1(25 4)	3(26 2)	2(22 7)
	2(23 5)	3(24 3)	2(25 1)	4(20 8)	2(21 6)	6(22 4)
	2(23 2)	4(24 0)	3(18 9)	3(19 7)	6(20 5)	6(21 3)
	3(22 1)	4(16 10)	4(17 8)	10(18 6)	7(19 4)	10(20 2)
	3(14 11)	5(15 9)	9(16 7)	11(17 5)	11(18 3)	6(19 1)
	5(12 12)	4(13 10)	13(14 8)	13(15 6)	19(16 4)	8(17 2)
	10(18 0)	2(10 13)	5(11 11)	11(12 9)	16(13 7)	19(14 5)
	17(15 3)	11(16 1)	4(8 14)	5(9 12)	14(10 10)	16(11 8)
	28(12 6)	20(13 4)	23(14 2)	2(15 0)	2(6 15)	4(7 13)
	10(8 11)	18(9 9)	23(10 7)	27(11 5)	25(12 3)	13(13 1)
	3(4 16)	3(5 14)	12(6 12)	15(7 10)	30(8 8)	26(9 6)
	36(10 4)	17(11 2)	15(12 0)	3(3 15)	6(4 13)	14(5 11)
	21(6 9)	29(7 7)	31(8 5)	28(9 3)	15(10 1)	2(0 18)
	1(1 16)	7(2 14)	10(3 12)	22(4 10)	24(5 8)	38(6 6)
	25(7 4)	29(8 2)	4(9 0)	5(1 13)	9(2 11)	19(3 9)
	23(4 7)	28(5 5)	23(6 3)	14(7 1)	9(0 12)	9(1 10)
	23(2 8)	21(3 6)	29(4 4)	12(5 2)	13(6 0)	3(0 9)
	11(1 7)	12(2 5)	16(3 3)	7(4 1)	11(0 6)	7(1 4)
	12(2 2)	1(0 3)	2(1 1)	4(0 0)		
{10}	1(40 0)	1(36 2)	1(34 3)	2(32 4)	1(34 0)	1(30 5)
	1(31 3)	1(32 1)	3(28 6)	1(29 4)	3(30 2)	2(26 7)
	2(27 5)	3(28 3)	2(29 1)	4(24 8)	2(25 6)	6(26 4)
	2(27 2)	4(28 0)	3(22 9)	3(23 7)	6(24 5)	6(25 3)
	3(26 1)	5(20 10)	4(21 8)	10(22 6)	7(23 4)	10(24 2)
	3(18 11)	5(19 9)	10(20 7)	11(21 5)	11(22 3)	6(23 1)
	6(16 12)	5(17 10)	14(18 8)	14(19 6)	20(20 4)	8(21 2)
	10(22 0)	3(14 13)	6(15 11)	13(16 9)	18(17 7)	21(18 5)
	18(19 3)	12(20 1)	5(12 14)	6(13 12)	17(14 10)	19(15 8)
	32(16 6)	23(17 4)	26(18 2)	2(19 0)	3(10 15)	6(11 13)
	14(12 11)	23(13 9)	30(14 7)	33(15 5)	30(16 3)	16(17 1)
	5(8 16)	5(9 14)	17(10 12)	22(11 10)	40(12 8)	35(13 6)
	47(14 4)	23(15 2)	18(16 0)	1(6 17)	5(7 15)	12(8 13)
	22(9 11)	33(10 9)	43(11 7)	46(12 5)	39(13 3)	22(14 1)
	4(4 18)	4(5 16)	14(6 14)	20(7 12)	39(8 10)	42(9 8)
	60(10 6)	42(11 4)	44(12 2)	7(13 0)	3(3 17)	7(4 15)
	17(5 13)	27(6 11)	41(7 9)	50(8 7)	53(9 5)	43(10 3)
	26(11 1)	2(0 20)	1(1 18)	8(2 16)	11(3 14)	29(4 12)
	33(5 10)	55(6 8)	49(7 6)	60(8 4)	28(9 2)	22(10 0)
	7(1 15)	11(2 13)	26(3 11)	36(4 9)	47(5 7)	46(6 5)

42(7 3)	22(8 1)	10(0 14)	13(1 12)	32(2 10)	33(3 8)
50(4 6)	36(5 4)	37(6 2)	3(7 0)	3(0 11)	19(1 9)
23(2 7)	31(3 5)	25(4 3)	16(5 1)	18(0 8)	16(1 6)
27(2 4)	12(3 2)	14(4 0)	3(0 5)	9(1 3)	4(2 1)
7(0 2)					

{11}

1(44 0)	1(40 2)	1(38 3)	2(36 4)	1(38 0)	1(34 5)
1(35 3)	1(36 1)	3(32 6)	1(33 4)	3(34 2)	2(30 7)
2(31 5)	3(32 3)	2(33 1)	4(28 8)	2(29 6)	6(30 4)
2(31 2)	4(32 0)	3(26 9)	3(27 7)	6(28 5)	6(29 3)
3(30 1)	5(24 10)	4(25 8)	10(26 6)	7(27 4)	10(28 2)
4(22 11)	5(23 9)	10(24 7)	11(25 5)	11(26 3)	6(27 1)
6(20 12)	5(21 10)	15(22 8)	14(23 6)	20(24 4)	8(25 2)
10(26 0)	4(18 13)	7(19 11)	14(20 9)	19(21 7)	22(22 5)
18(23 3)	12(24 1)	6(16 14)	7(17 12)	19(18 10)	21(19 8)
34(20 6)	24(21 4)	27(22 2)	2(23 0)	4(14 15)	7(15 13)
17(16 11)	26(17 9)	34(18 7)	36(19 5)	33(20 3)	17(21 1)
6(12 16)	7(13 14)	21(14 12)	27(15 10)	47(16 8)	41(17 6)
53(18 4)	26(19 2)	20(20 0)	3(10 17)	7(11 15)	17(12 13)
29(13 11)	43(14 9)	53(15 7)	57(16 5)	47(17 3)	27(18 1)
5(8 18)	6(9 16)	20(10 14)	28(11 12)	52(12 10)	56(13 8)
77(14 6)	55(15 4)	55(16 2)	9(17 0)	2(6 19)	6(7 17)
14(8 15)	28(9 13)	44(10 11)	61(11 9)	74(12 7)	75(13 5)
61(14 3)	36(15 1)	4(4 20)	4(5 18)	16(6 16)	23(7 14)
49(8 12)	57(9 10)	87(10 8)	78(11 6)	89(12 4)	45(13 2)
31(14 0)	3(3 19)	8(4 17)	20(5 15)	33(6 13)	55(7 11)
73(8 9)	85(9 7)	84(10 5)	71(11 3)	38(12 1)	2(0 22)
2(1 20)	9(2 18)	14(3 16)	35(4 14)	44(5 12)	76(6 10)
78(7 8)	100(8 6)	74(9 4)	68(10 2)	9(11 0)	7(1 17)
14(2 15)	33(3 13)	48(4 11)	70(5 9)	78(6 7)	81(7 5)
64(8 3)	37(9 1)	11(0 16)	16(1 14)	42(2 12)	48(3 10)
78(4 8)	68(5 6)	78(6 4)	39(7 2)	30(8 0)	6(0 13)
28(1 11)	37(2 9)	56(3 7)	54(4 5)	48(5 3)	26(6 1)
24(0 10)	28(1 8)	49(2 6)	35(3 4)	39(4 2)	4(5 0)
6(0 7)	22(1 5)	18(2 3)	13(3 1)	17(0 4)	6(1 2)
9(2 0)					

{12}

1(48 0)	1(44 2)	1(42 3)	2(40 4)	1(42 0)	1(38 5)
1(39 3)	1(40 1)	3(36 6)	1(37 4)	3(38 2)	2(34 7)
2(35 5)	3(36 3)	2(37 1)	4(32 8)	2(33 6)	6(34 4)
2(35 2)	4(36 0)	3(30 9)	3(31 7)	6(32 5)	6(33 3)
3(34 1)	5(28 10)	4(29 8)	10(30 6)	7(31 4)	10(32 2)
4(26 11)	5(27 9)	10(28 7)	11(29 5)	11(30 3)	6(31 1)
7(24 12)	5(25 10)	15(26 8)	14(27 6)	20(28 4)	8(29 2)
10(30 0)	4(22 13)	7(23 11)	15(24 9)	19(25 7)	22(26 5)
18(27 3)	12(28 1)	7(20 14)	8(21 12)	20(22 10)	22(23 8)
35(24 6)	24(25 4)	27(26 2)	2(27 0)	5(18 15)	8(19 13)
19(20 11)	28(21 9)	36(22 7)	37(23 5)	34(24 3)	17(25 1)
7(16 16)	8(17 14)	24(18 12)	30(19 10)	51(20 8)	44(21 6)
56(22 4)	27(23 2)	21(24 0)	4(14 17)	9(15 15)	21(16 13)
34(17 11)	50(18 9)	59(19 7)	63(20 5)	51(21 3)	29(22 1)
7(12 18)	8(13 16)	25(14 14)	35(15 12)	62(16 10)	66(17 8)
89(18 6)	63(19 4)	62(20 2)	11(21 0)	3(10 19)	8(11 17)
20(12 15)	36(13 13)	57(14 11)	76(15 9)	91(16 7)	90(17 5)

	74(18 3)	42(19 1)	6(8 20)	7(9 18)	23(10 16)	34(11 14)
	67(12 12)	77(13 10)	113(14 8)	102(15 6)	112(16 4)	58(17 2)
	38(18 0)	2(6 21)	6(7 19)	16(8 17)	33(9 15)	53(10 13)
	81(11 11)	107(12 9)	119(13 7)	117(14 5)	97(15 3)	52(16 1)
	4(4 22)	5(5 20)	18(6 18)	27(7 16)	59(8 14)	75(9 12)
	118(10 10)	121(11 8)	148(12 6)	110(13 4)	97(14 2)	16(15 0)
	4(3 21)	9(4 19)	23(5 17)	41(6 15)	69(7 13)	97(8 11)
	126(9 9)	137(10 7)	135(11 5)	107(12 3)	59(13 1)	3(0 24)
	1(1 22)	10(2 20)	16(3 18)	40(4 16)	54(5 14)	99(6 12)
	109(7 10)	152(8 8)	134(9 6)	139(10 4)	74(11 2)	49(12 0)
	8(1 19)	16(2 17)	40(3 15)	62(4 13)	96(5 11)	116(6 9)
	135(7 7)	127(8 5)	103(9 3)	55(10 1)	14(0 18)	21(1 16)
	52(2 14)	68(3 12)	110(4 10)	112(5 8)	138(6 6)	99(7 4)
	91(8 2)	16(9 0)	8(0 15)	35(1 13)	54(2 11)	87(3 9)
	92(4 7)	99(5 5)	78(6 3)	43(7 1)	33(0 12)	42(1 10)
	79(2 8)	71(3 6)	86(4 4)	41(5 2)	31(6 0)	15(0 9)
	41(1 7)	41(2 5)	45(3 3)	23(4 1)	30(0 6)	21(1 4)
	30(2 2)	3(3 0)	3(0 3)	6(1 1)	7(0 0)	
{13}	1(52 0)	1(48 2)	1(46 3)	2(44 4)	1(46 0)	1(42 5)
	1(43 3)	1(44 1)	3(40 6)	1(41 4)	3(42 2)	2(38 7)
	2(39 5)	3(40 3)	2(41 1)	4(36 8)	2(37 6)	6(38 4)
	2(39 2)	4(40 0)	3(34 9)	3(35 7)	6(36 5)	6(37 3)
	3(38 1)	5(32 10)	4(33 8)	10(34 6)	7(35 4)	10(36 2)
	4(30 11)	5(31 9)	10(32 7)	11(33 5)	11(34 3)	6(35 1)
	7(28 12)	5(29 10)	15(30 8)	14(31 6)	20(32 4)	8(33 2)
	10(34 0)	5(26 13)	7(27 11)	15(28 9)	19(29 7)	22(30 5)
	18(31 3)	12(32 1)	7(24 14)	8(25 12)	21(26 10)	22(27 8)
	35(28 6)	24(29 4)	27(30 2)	2(31 0)	6(22 15)	9(23 13)
	20(24 11)	29(25 9)	37(26 7)	37(27 5)	34(28 3)	17(29 1)
	8(20 16)	9(21 14)	26(22 12)	32(23 10)	53(24 8)	45(25 6)
	57(26 4)	27(27 2)	21(28 0)	5(18 17)	10(19 15)	24(20 13)
	37(21 11)	54(22 9)	62(23 7)	66(24 5)	52(25 3)	30(26 1)
	8(16 18)	10(17 16)	29(18 14)	40(19 12)	69(20 10)	72(21 8)
	95(22 6)	67(23 4)	65(24 2)	11(25 0)	5(14 19)	10(15 17)
	25(16 15)	43(17 13)	67(18 11)	86(19 9)	103(20 7)	99(21 5)
	81(22 3)	46(23 1)	7(12 20)	9(13 18)	29(14 16)	42(15 14)
	80(16 12)	92(17 10)	131(18 8)	117(19 6)	127(20 4)	66(21 2)
	42(22 0)	4(10 21)	9(11 19)	23(12 17)	44(13 15)	71(14 13)
	102(15 11)	133(16 9)	145(17 7)	142(18 5)	115(19 3)	62(20 1)
	6(8 22)	8(9 20)	26(10 18)	40(11 16)	80(12 14)	101(13 12)
	154(14 10)	157(15 8)	186(16 6)	140(17 4)	120(18 2)	21(19 0)
	2(6 23)	7(7 21)	18(8 19)	37(9 17)	65(10 15)	102(11 13)
	141(12 11)	174(13 9)	190(14 7)	181(15 5)	143(16 3)	79(17 1)
	5(4 24)	5(5 22)	20(6 20)	32(7 18)	69(8 16)	92(9 14)
	153(10 12)	170(11 10)	222(12 8)	197(13 6)	198(14 4)	108(15 2)
	65(16 0)	4(3 23)	10(4 21)	26(5 19)	47(6 17)	83(7 15)
	124(8 13)	169(9 11)	200(10 9)	220(11 7)	204(12 5)	160(13 3)
	88(14 1)	2(0 26)	2(1 24)	11(2 22)	17(3 20)	46(4 18)
	66(5 16)	120(6 14)	147(7 12)	210(8 10)	210(9 8)	239(10 6)
	176(11 4)	149(12 2)	29(13 0)	10(1 21)	19(2 19)	48(3 17)
	78(4 15)	124(5 13)	162(6 11)	203(7 9)	210(8 7)	202(9 5)
	156(10 3)	85(11 1)	15(0 20)	23(1 18)	63(2 16)	86(3 14)
	146(4 12)	163(5 10)	213(6 8)	185(7 6)	192(8 4)	100(9 2)
	59(10 0)	9(0 17)	46(1 15)	73(2 13)	121(3 11)	146(4 9)

	170(5 7)	155(6 5)	128(7 3)	69(8 1)	42(0 14)	64(1 12)
	115(2 10)	121(3 8)	155(4 6)	111(5 4)	101(6 2)	17(7 0)
	22(0 11)	66(1 9)	79(2 7)	94(3 5)	72(4 3)	45(5 1)
	46(0 8)	47(1 6)	70(2 4)	34(3 2)	29(4 0)	11(0 5)
{14}	1(56 0)	1(52 2)	1(50 3)	2(48 4)	1(50 0)	1(46 5)
	1(47 3)	1(48 1)	3(44 6)	1(45 4)	3(46 2)	2(42 7)
	2(43 5)	3(44 3)	2(45 1)	4(40 8)	2(41 6)	6(42 4)
	2(43 2)	4(44 0)	3(38 9)	3(39 7)	6(40 5)	6(41 3)
	3(42 1)	5(36 10)	4(37 8)	10(38 6)	7(39 4)	10(40 2)
	4(34 11)	5(35 9)	10(36 7)	11(37 5)	11(38 3)	6(39 1)
	7(32 12)	5(33 10)	15(34 8)	14(35 6)	20(36 4)	8(37 2)
	10(38 0)	5(30 13)	7(31 11)	15(32 9)	19(33 7)	22(34 5)
	18(35 3)	12(36 1)	8(28 14)	8(29 12)	21(30 10)	22(31 8)
	35(32 6)	24(33 4)	27(34 2)	2(35 0)	6(26 15)	9(27 13)
	21(28 11)	29(29 9)	37(30 7)	37(31 5)	34(32 3)	17(33 1)
	9(24 16)	10(25 14)	27(26 12)	33(27 10)	54(28 8)	45(29 6)
	57(30 4)	27(31 2)	21(32 0)	6(22 17)	11(23 15)	26(24 13)
	39(25 11)	56(26 9)	63(27 7)	67(28 5)	52(29 3)	30(30 1)
	9(20 18)	11(21 16)	32(22 14)	43(23 12)	73(24 10)	75(25 8)
	98(26 6)	68(27 4)	66(28 2)	11(29 0)	6(18 19)	12(19 17)
	29(20 15)	48(21 13)	74(22 11)	92(23 9)	109(24 7)	103(25 5)
	84(26 3)	47(27 1)	9(16 20)	11(17 18)	34(18 16)	49(19 14)
	90(20 12)	102(21 10)	143(22 8)	126(23 6)	135(24 4)	70(25 2)
	44(26 0)	5(14 21)	11(15 19)	29(16 17)	52(17 15)	84(18 13)
	117(19 11)	151(20 9)	161(21 7)	157(22 5)	125(23 3)	68(24 1)
	8(12 22)	11(13 20)	33(14 18)	51(15 16)	98(16 14)	122(17 12)
	181(18 10)	183(19 8)	213(20 6)	160(21 4)	135(22 2)	24(23 0)
	4(10 23)	10(11 21)	26(12 19)	50(13 17)	86(14 15)	129(15 13)
	177(16 11)	212(17 9)	230(18 7)	216(19 5)	170(20 3)	93(21 1)
	7(8 24)	8(9 22)	29(10 20)	46(11 18)	94(12 16)	125(13 14)
	199(14 12)	220(15 10)	280(16 8)	248(17 6)	244(18 4)	135(19 2)
	78(20 0)	2(6 25)	8(7 23)	20(8 21)	43(9 19)	76(10 17)
	123(11 15)	180(12 13)	235(13 11)	275(14 9)	294(15 7)	272(16 5)
	210(17 3)	116(18 1)	5(4 26)	6(5 24)	22(6 22)	35(7 20)
	79(8 18)	111(9 16)	187(10 14)	225(11 12)	306(12 10)	305(13 8)
	335(14 6)	249(15 4)	204(16 2)	43(17 0)	4(3 25)	11(4 23)
	29(5 21)	53(6 19)	98(7 17)	151(8 15)	215(9 13)	273(10 11)
	322(11 9)	330(12 7)	307(13 5)	236(14 3)	127(15 1)	3(0 28)
	2(1 26)	12(2 24)	20(3 22)	53(4 20)	76(5 18)	145(6 16)
	187(7 14)	276(8 12)	301(9 10)	364(10 8)	317(11 6)	310(12 4)
	168(13 2)	90(14 0)	10(1 23)	21(2 21)	55(3 19)	92(4 17)
	154(5 15)	210(6 13)	278(7 11)	315(8 9)	333(9 7)	299(10 5)
	234(11 3)	126(12 1)	16(0 22)	28(1 20)	74(2 18)	105(3 16)
	186(4 14)	224(5 12)	302(6 10)	299(7 8)	332(8 6)	241(9 4)
	198(10 2)	41(11 0)	12(0 19)	57(1 17)	94(2 15)	164(3 13)
	209(4 11)	260(5 9)	268(6 7)	256(7 5)	192(8 3)	109(9 1)
	52(0 16)	81(1 14)	159(2 12)	183(3 10)	245(4 8)	213(5 6)
	219(6 4)	116(7 2)	70(8 0)	32(0 13)	99(1 11)	128(2 9)
	166(3 7)	156(4 5)	133(5 3)	68(6 1)	70(0 10)	88(1 8)
	129(2 6)	98(3 4)	95(4 2)	14(5 0)	24(0 7)	58(1 5)
	48(2 3)	34(3 1)	35(0 4)	16(1 2)	20(2 0)	1(0 1)
{15}	1(60 0)	1(56 2)	1(54 3)	2(52 4)	1(54 0)	1(50 5)

1(51 3)	1(52 1)	3(48 6)	1(49 4)	3(50 2)	2(46 7)
2(47 5)	3(48 3)	2(49 1)	4(44 8)	2(45 6)	6(46 4)
2(47 2)	4(48 0)	3(42 9)	3(43 7)	6(44 5)	6(45 3)
3(46 1)	5(40 10)	4(41 8)	10(42 6)	7(43 4)	10(44 2)
4(38 11)	5(39 9)	10(40 7)	11(41 5)	11(42 3)	6(43 1)
7(36 12)	5(37 10)	15(38 8)	14(39 6)	20(40 4)	8(41 2)
10(42 0)	5(34 13)	7(35 11)	15(36 9)	19(37 7)	22(38 5)
18(39 3)	12(40 1)	8(32 14)	8(33 12)	21(34 10)	22(35 8)
35(36 6)	24(37 4)	27(38 2)	2(39 0)	7(30 15)	9(31 13)
21(32 11)	29(33 9)	37(34 7)	37(35 5)	34(36 3)	17(37 1)
9(28 16)	10(29 14)	28(30 12)	33(31 10)	54(32 8)	45(33 6)
57(34 4)	27(35 2)	21(36 0)	7(26 17)	12(27 15)	27(28 13)
40(29 11)	57(30 9)	63(31 7)	67(32 5)	52(33 3)	30(34 1)
10(24 18)	12(25 16)	34(26 14)	45(27 12)	75(28 10)	76(29 8)
99(30 6)	68(31 4)	66(32 2)	11(33 0)	7(22 19)	13(23 17)
32(24 15)	51(25 13)	78(26 11)	95(27 9)	112(28 7)	104(29 5)
85(30 3)	47(31 1)	10(20 20)	13(21 18)	38(22 16)	54(23 14)
97(24 12)	108(25 10)	149(26 8)	130(27 6)	138(28 4)	71(29 2)
45(30 0)	7(18 21)	13(19 19)	34(20 17)	59(21 15)	94(22 13)
127(23 11)	163(24 9)	170(25 7)	165(26 5)	130(27 3)	70(28 1)
9(16 22)	13(17 20)	39(18 18)	59(19 16)	111(20 14)	137(21 12)
199(22 10)	199(23 8)	229(24 6)	170(25 4)	143(26 2)	26(27 0)
6(14 23)	13(15 21)	33(16 19)	61(17 17)	104(18 15)	150(19 13)
204(20 11)	239(21 9)	257(22 7)	238(23 5)	187(24 3)	101(25 1)
9(12 24)	11(13 22)	37(14 20)	59(15 18)	115(16 16)	152(17 14)
236(18 12)	258(19 10)	322(20 8)	285(21 6)	276(22 4)	153(23 2)
87(24 0)	4(10 25)	11(11 23)	29(12 21)	57(13 19)	101(14 17)
157(15 15)	226(16 13)	287(17 11)	335(18 9)	350(19 7)	323(20 5)
247(21 3)	136(22 1)	7(8 26)	10(9 24)	32(10 22)	52(11 20)
109(12 18)	151(13 16)	245(14 14)	293(15 12)	386(16 10)	384(17 8)
414(18 6)	308(19 4)	248(20 2)	55(21 0)	3(6 27)	8(7 25)
22(8 23)	48(9 21)	86(10 19)	145(11 17)	220(12 15)	298(13 13)
374(14 11)	429(15 9)	436(16 7)	399(17 5)	307(18 3)	163(19 1)
5(4 28)	6(5 26)	24(6 24)	39(7 22)	89(8 20)	128(9 18)
223(10 16)	283(11 14)	399(12 12)	431(13 10)	505(14 8)	441(15 6)
418(16 4)	232(17 2)	119(18 0)	5(3 27)	12(4 25)	32(5 23)
61(6 21)	113(7 19)	179(8 17)	267(9 15)	352(10 13)	438(11 11)
488(12 9)	498(13 7)	446(14 5)	342(15 3)	183(16 1)	3(0 30)
2(1 28)	13(2 26)	22(3 24)	57(4 22)	88(5 20)	169(6 18)
225(7 16)	346(8 14)	405(9 12)	508(10 10)	497(11 8)	526(12 6)
385(13 4)	303(14 2)	70(15 0)	11(1 25)	24(2 23)	62(3 21)
107(4 19)	185(5 17)	263(6 15)	363(7 13)	436(8 11)	495(9 9)
495(10 7)	450(11 5)	337(12 3)	184(13 1)	19(0 24)	31(1 22)
85(2 20)	128(3 18)	228(4 16)	288(5 14)	409(6 12)	437(7 10)
512(8 8)	442(9 6)	415(10 4)	230(11 2)	122(12 0)	15(0 21)
65(1 19)	116(2 17)	209(3 15)	278(4 13)	369(5 11)	410(6 9)
428(7 7)	382(8 5)	299(9 3)	154(10 1)	61(0 18)	105(1 16)
207(2 14)	257(3 12)	357(4 10)	354(5 8)	388(6 6)	285(7 4)
233(8 2)	48(9 0)	47(0 15)	137(1 13)	192(2 11)	267(3 9)
275(4 7)	272(5 5)	206(6 3)	115(7 1)	98(0 12)	132(1 10)
214(2 8)	197(3 6)	209(4 4)	111(5 2)	71(6 0)	46(0 9)
109(1 7)	111(2 5)	108(3 3)	57(4 1)	67(0 6)	56(1 4)
67(2 2)	9(3 0)	10(0 3)	14(1 1)	11(0 0)	

Table-9 Reduction of U(15) IRR {M} into IRR {pqrstu} of U(6)

{M}	IRR {pqrstu} of U(6)
{1}	{1 1 0 0 0 0}
{2}	{2 2 0 0 0 0} {1 1 1 1 0 0}
{3}	{3 3 0 0 0 0} {2 2 1 1 0 0} {1 1 1 1 1 1}
{4}	{4 4 0 0 0 0} {3 3 1 1 0 0} {2 2 2 2 0 0} {2 2 1 1 1 1}
{5}	{5 5 0 0 0 0} {4 4 1 1 0 0} {3 3 2 2 0 0} {3 3 1 1 1 1} {2 2 2 2 1 1}
{6}	{6 6 0 0 0 0} {5 5 1 1 0 0} {4 4 2 2 0 0} {3 3 3 3 0 0} {4 4 1 1 1 1} {3 3 2 2 1 1} {2 2 2 2 2 2}
{7}	{7 7 0 0 0 0} {6 6 1 1 0 0} {5 5 2 2 0 0} {4 4 3 3 0 0} {5 5 1 1 1 1} {4 4 2 2 1 1} {3 3 3 3 1 1} {3 3 2 2 2 2}
{8}	{8 8 0 0 0 0} {7 7 1 1 0 0} {6 6 2 2 0 0} {5 5 3 3 0 0} {4 4 4 4 0 0} {6 6 1 1 1 1} {5 5 2 2 1 1} {4 4 3 3 1 1} {4 4 2 2 2 2} {3 3 3 3 2 2}
{9}	{9 9 0 0 0 0} {8 8 1 1 0 0} {7 7 2 2 0 0} {6 6 3 3 0 0} {5 5 4 4 0 0} {7 7 1 1 1 1} {6 6 2 2 1 1} {5 5 3 3 1 1} {4 4 4 4 1 1} {5 5 2 2 2 2} {4 4 3 3 2 2} {3 3 3 3 3 3}
{10}	{10 10 0 0 0 0} {9 9 1 1 0 0} {8 8 2 2 0 0} {7 7 3 3 0 0} {6 6 4 4 0 0} {5 5 5 5 0 0} {8 8 1 1 1 1} {7 7 2 2 1 1} {6 6 3 3 1 1} {5 5 4 4 1 1} {6 6 2 2 2 2} {5 5 3 3 2 2} {4 4 4 4 2 2} {4 4 3 3 3 3}
{11}	{11 11 0 0 0 0} {10 10 1 1 0 0} {9 9 2 2 0 0} {8 8 3 3 0 0} {7 7 4 4 0 0} {6 6 5 5 0 0} {9 9 1 1 1 1} {8 8 2 2 1 1} {7 7 3 3 1 1} {6 6 4 4 1 1} {5 5 5 5 1 1} {7 7 2 2 2 2} {6 6 3 3 2 2} {5 5 4 4 2 2} {5 5 3 3 3 3} {4 4 4 4 3 3}
{12}	{12 12 0 0 0 0} {11 11 1 1 0 0} {10 10 2 2 0 0} {9 9 3 3 0 0} {8 8 4 4 0 0} {7 7 5 5 0 0} {6 6 6 6 0 0} {10 10 1 1 1 1} {9 9 2 2 1 1} {8 8 3 3 1 1} {7 7 4 4 1 1} {6 6 5 5 1 1} {8 8 2 2 2 2} {7 7 3 3 2 2} {6 6 4 4 2 2} {5 5 5 5 2 2} {6 6 3 3 3 3} {5 5 4 4 3 3} {4 4 4 4 4 4}

{13}	{13 13 0 0 0 0}	{12 12 1 1 0 0}	{11 11 2 2 0 0}	{10 10 3 3 0 0}
	{ 9 9 4 4 0 0}	{ 8 8 5 5 0 0}	{ 7 7 6 6 0 0}	{11 11 1 1 1 1}
	{10 10 2 2 1 1}	{ 9 9 3 3 1 1}	{ 8 8 4 4 1 1}	{ 7 7 5 5 1 1}
	{ 6 6 6 6 1 1}	{ 9 9 2 2 2 2}	{ 8 8 3 3 2 2}	{ 7 7 4 4 2 2}
	{ 6 6 5 5 2 2}	{ 7 7 3 3 3 3}	{ 6 6 4 4 3 3}	{ 5 5 5 5 3 3}
	{ 5 5 4 4 4 4}			
{14}	{14 14 0 0 0 0}	{13 13 1 1 0 0}	{12 12 2 2 0 0}	{11 11 3 3 0 0}
	{10 10 4 4 0 0}	{ 9 9 5 5 0 0}	{ 8 8 6 6 0 0}	{ 7 7 7 7 0 0}
	{12 12 1 1 1 1}	{11 11 2 2 1 1}	{10 10 3 3 1 1}	{ 9 9 4 4 1 1}
	{ 8 8 5 5 1 1}	{ 7 7 6 6 1 1}	{10 10 2 2 2 2}	{ 9 9 3 3 2 2}
	{ 8 8 4 4 2 2}	{ 7 7 5 5 2 2}	{ 6 6 6 6 2 2}	{ 8 8 3 3 3 3}
	{ 7 7 4 4 3 3}	{ 6 6 5 5 3 3}	{ 6 6 4 4 4 4}	{ 5 5 5 5 4 4}
{15}	{15 15 0 0 0 0}	{14 14 1 1 0 0}	{13 13 2 2 0 0}	{12 12 3 3 0 0}
	{11 11 4 4 0 0}	{10 10 5 5 0 0}	{ 9 9 6 6 0 0}	{ 8 8 7 7 0 0}
	{13 13 1 1 1 1}	{12 12 2 2 1 1}	{11 11 3 3 1 1}	{10 10 4 4 1 1}
	{ 9 9 5 5 1 1}	{ 8 8 6 6 1 1}	{ 7 7 7 7 1 1}	{11 11 2 2 2 2}
	{10 10 3 3 2 2}	{ 9 9 4 4 2 2}	{ 8 8 5 5 2 2}	{ 7 7 6 6 2 2}
	{ 9 9 3 3 3 3}	{ 8 8 4 4 3 3}	{ 7 7 5 5 3 3}	{ 6 6 6 6 3 3}
	{ 7 7 4 4 4 4}	{ 6 6 5 5 4 4}	{ 5 5 5 5 5 5}	

Table-10 Reduction of U(6) IRR {F} into IRR <pqr> of Sp(6)

{0 0 0}	< 0 0 0>
{1 1 0}	< 1 1 0> < 0 0 0>
{2 2 0}	< 2 2 0> < 1 1 0> < 0 0 0>
{1 1 1 1}	1< 1 1 0> 1< 0 0 0>
{3 3 0}	< 3 3 0> < 2 2 0> < 1 1 0> < 0 0 0>
{2 2 1 1}	1< 2 2 0> 2< 1 1 0> 1< 0 0 0> 1< 2 1 1>
{4 4 0}	< 4 4 0> < 3 3 0> < 2 2 0> < 1 1 0> < 0 0 0>
{3 3 1 1}	1< 3 3 0> 2< 2 2 0> 2< 1 1 0> 1< 0 0 0> 1< 2 1 1> 1< 3 2 1>
{5 5 0}	< 5 5 0> < 4 4 0> < 3 3 0> < 2 2 0> < 1 1 0> < 0 0 0>
{4 4 1 1}	1< 4 4 0> 2< 3 3 0> 2< 2 2 0> 2< 1 1 0> 1< 0 0 0> 1< 2 1 1> 1< 3 2 1> 1< 4 3 1>
{6 6 0}	< 6 6 0> < 5 5 0> < 4 4 0> < 3 3 0> < 2 2 0> < 1 1 0> < 0 0 0>
{5 5 1 1}	1< 5 5 0> 2< 4 4 0> 2< 3 3 0> 2< 2 2 0> 2< 1 1 0> 1< 0 0 0> 1< 2 1 1> 1< 3 2 1> 1< 4 3 1> 1< 5 4 1>
{7 7 0}	< 7 7 0> < 6 6 0> < 5 5 0> < 4 4 0> < 3 3 0> < 2 2 0> < 1 1 0> < 0 0 0>
{6 6 1 1}	1< 6 6 0> 2< 5 5 0> 2< 4 4 0> 2< 3 3 0> 2< 2 2 0> 2< 1 1 0> 1< 0 0 0> 1< 2 1 1> 1< 3 2 1> 1< 4 3 1> 1< 5 4 1> 1< 6 5 1>
{8 8 0}	< 8 8 0> < 7 7 0> < 6 6 0> < 5 5 0> < 4 4 0> < 3 3 0> < 2 2 0> < 1 1 0> < 0 0 0>

```
{7 7 1 1}     1< 7  7  0> 2< 6  6  0> 2< 5  5  0> 2< 4  4  0> 2< 3  3  0>
              2< 2  2  0> 2< 1  1  0> 1< 0  0  0> 1< 2  1  1> 1< 3  2  1>
              1< 4  3  1> 1< 5  4  1> 1< 6  5  1> 1< 7  6  1>

{9 9 0}        < 9  9  0>  < 8  8  0>  < 7  7  0>  < 6  6  0>  < 5  5  0>
               < 4  4  0>  < 3  3  0>  < 2  2  0>  < 1  1  0>  < 0  0  0>

{8 8 1 1}     1< 8  8  0> 2< 7  7  0> 2< 6  6  0> 2< 5  5  0> 2< 4  4  0>
              2< 3  3  0> 2< 2  2  0> 2< 1  1  0> 1< 0  0  0> 1< 2  1  1>
              1< 3  2  1> 1< 4  3  1> 1< 5  4  1> 1< 6  5  1> 1< 7  6  1>
              1< 8  7  1>

{10 10 0}      <10 10  0>  < 9  9  0>  < 8  8  0>  < 7  7  0>  < 6  6  0>
               < 5  5  0>  < 4  4  0>  < 3  3  0>  < 2  2  0>  < 1  1  0>
               < 0  0  0>

{9 9 1 1}     1< 9  9  0> 2< 8  8  0> 2< 7  7  0> 2< 6  6  0> 2< 5  5  0>
              2< 4  4  0> 2< 3  3  0> 2< 2  2  0> 2< 1  1  0> 1< 0  0  0>
              1< 2  1  1> 1< 3  2  1> 1< 4  3  1> 1< 5  4  1> 1< 6  5  1>
              1< 7  6  1> 1< 8  7  1> 1< 9  8  1>

{11 11 0}      <11 11  0>  <10 10  0>  < 9  9  0>  < 8  8  0>  < 7  7  0>
               < 6  6  0>  < 5  5  0>  < 4  4  0>  < 3  3  0>  < 2  2  0>
               < 1  1  0>  < 0  0  0>

{10 10 1 1}   1<10 10  0> 2< 9  9  0> 2< 8  8  0> 2< 7  7  0> 2< 6  6  0>
              2< 5  5  0> 2< 4  4  0> 2< 3  3  0> 2< 2  2  0> 2< 1  1  0>
              1< 0  0  0> 1< 2  1  1> 1< 3  2  1> 1< 4  3  1> 1< 5  4  1>
              1< 6  5  1> 1< 7  6  1> 1< 8  7  1> 1< 9  8  1> 1<10  9  1>

{12 12 0}      <12 12  0>  <11 11  0>  <10 10  0>  < 9  9  0>  < 8  8  0>
               < 7  7  0>  < 6  6  0>  < 5  5  0>  < 4  4  0>  < 3  3  0>
               < 2  2  0>  < 1  1  0>  < 0  0  0>

{11 11 1 1}   1<11 11  0> 2<10 10  0> 2< 9  9  0> 2< 8  8  0> 2< 7  7  0>
              2< 6  6  0> 2< 5  5  0> 2< 4  4  0> 2< 3  3  0> 2< 2  2  0>
              2< 1  1  0> 1< 0  0  0> 1< 2  1  1> 1< 3  2  1> 1< 4  3  1>
              1< 5  4  1> 1< 6  5  1> 1< 7  6  1> 1< 8  7  1> 1< 9  8  1>
              1<10  9  1> 1<11 10  1>

{13 13 0}      <13 13  0>  <12 12  0>  <11 11  0>  <10 10  0>  < 9  9  0>
               < 8  8  0>  < 7  7  0>  < 6  6  0>  < 5  5  0>  < 4  4  0>
               < 3  3  0>  < 2  2  0>  < 1  1  0>  < 0  0  0>
```

```
{12 12 1 1}  1<12 12  0> 2<11 11  0> 2<10 10  0> 2< 9  9  0> 2< 8  8  0>
             2< 7  7  0> 2< 6  6  0> 2< 5  5  0> 2< 4  4  0> 2< 3  3  0>
             2< 2  2  0> 2< 1  1  0> 1< 0  0  0> 1< 2  1  1> 1< 3  2  1>
             1< 4  3  1> 1< 5  4  1> 1< 6  5  1> 1< 7  6  1> 1< 8  7  1>
             1< 9  8  1> 1<10  9  1> 1<11 10  1> 1<12 11  1>

 {14 14 0}    <14 14  0>  <13 13  0>  <12 12  0>  <11 11  0>  <10 10  0>
              < 9  9  0>  < 8  8  0>  < 7  7  0>  < 6  6  0>  < 5  5  0>
              < 4  4  0>  < 3  3  0>  < 2  2  0>  < 1  1  0>  < 0  0  0>

{13 13 1 1}  1<13 13  0> 2<12 12  0> 2<11 11  0> 2<10 10  0> 2< 9  9  0>
             2< 8  8  0> 2< 7  7  0> 2< 6  6  0> 2< 5  5  0> 2< 4  4  0>
             2< 3  3  0> 2< 2  2  0> 2< 1  1  0> 1< 0  0  0> 1< 2  1  1>
             1< 3  2  1> 1< 4  3  1> 1< 5  4  1> 1< 6  5  1> 1< 7  6  1>
             1< 8  7  1> 1< 9  8  1> 1<10  9  1> 1<11 10  1> 1<12 11  1>
             1<13 12  1>

 {15 15 0}    <15 15  0>  <14 14  0>  <13 13  0>  <12 12  0>  <11 11  0>
              <10 10  0>  < 9  9  0>  < 8  8  0>  < 7  7  0>  < 6  6  0>
              < 5  5  0>  < 4  4  0>  < 3  3  0>  < 2  2  0>  < 1  1  0>
              < 0  0  0>

{14 14 1 1}  1<14 14  0> 2<13 13  0> 2<12 12  0> 2<11 11  0> 2<10 10  0>
             2< 9  9  0> 2< 8  8  0> 2< 7  7  0> 2< 6  6  0> 2< 5  5  0>
             2< 4  4  0> 2< 3  3  0> 2< 2  2  0> 2< 1  1  0> 1< 0  0  0>
             1< 2  1  1> 1< 3  2  1> 1< 4  3  1> 1< 5  4  1> 1< 6  5  1>
             1< 7  6  1> 1< 8  7  1> 1< 9  8  1> 1<10  9  1> 1<11 10  1>
             1<12 11  1> 1<13 12  1> 1<14 13  1>
```

Table-11 Reduction of Sp(6) IRR <pqr> into IRR (L) of O(3)

< 1 1 0>	1(2)	1(4)				
< 2 2 0>	1(0)	2(2)	1(3)	2(4)	1(5)	2(6)
	1(8)					
< 3 3 0>	2(0)	1(1)	3(2)	3(3)	5(4)	3(5)
	5(6)	3(7)	3(8)	2(9)	2(10)	1(12)
< 4 4 0>	2(0)	2(1)	7(2)	5(3)	10(4)	8(5)
	10(6)	8(7)	10(8)	6(9)	7(10)	4(11)
	4(12)	2(13)	2(14)	1(16)		
< 5 5 0>	4(0)	4(1)	11(2)	11(3)	16(4)	16(5)
	20(6)	17(7)	20(8)	16(9)	17(10)	13(11)
	13(12)	8(13)	8(14)	5(15)	4(16)	2(17)
	2(18)	1(20)				
< 6 6 0>	6(0)	7(1)	17(2)	19(3)	28(4)	26(5)
	35(6)	32(7)	36(8)	33(9)	35(10)	28(11)
	30(12)	23(13)	22(14)	16(15)	15(16)	9(17)
	9(18)	5(19)	4(20)	2(21)	2(22)	1(24)
< 7 7 0>	7(0)	12(1)	27(2)	29(3)	43(4)	45(5)
	54(6)	54(7)	62(8)	57(9)	62(10)	56(11)
	56(12)	49(13)	48(14)	38(15)	37(16)	28(17)
	25(18)	18(19)	16(20)	10(21)	9(22)	5(23)
	4(24)	2(25)	2(26)	1(28)		
< 8 8 0>	11(0)	18(1)	39(2)	45(3)	64(4)	68(5)
	84(6)	84(7)	97(8)	94(9)	101(10)	95(11)
	99(12)	88(13)	89(14)	77(15)	74(16)	62(17)
	58(18)	45(19)	42(20)	31(21)	27(22)	19(23)
	17(24)	10(25)	9(26)	5(27)	4(28)	2(29)
	2(30)	1(32)				
< 9 9 0>	15(0)	27(1)	54(2)	67(3)	92(4)	99(5)
	123(6)	127(7)	143(8)	145(9)	156(10)	150(11)
	158(12)	148(13)	148(14)	136(15)	133(16)	116(17)
	112(18)	95(19)	87(20)	72(21)	65(22)	50(23)
	45(24)	33(25)	28(26)	20(27)	17(28)	10(29)
	9(30)	5(31)	4(32)	2(33)	2(34)	1(36)
<10 10 0>	19(0)	38(1)	77(2)	92(3)	128(4)	143(5)

	172(6)	182(7)	208(8)	209(9)	230(10)	227(11)
	237(12)	229(13)	234(14)	218(15)	218(16)	199(17)
	192(18)	171(19)	163(20)	139(21)	130(22)	108(23)
	97(24)	79(25)	70(26)	53(27)	47(28)	34(29)
	29(30)	20(31)	17(32)	10(33)	9(34)	5(35)
	4(36)	2(37)	2(38)	1(40)		
<11 11 0>	25(0)	53(1)	102(2)	128(3)	174(4)	196(5)
	237(6)	253(7)	288(8)	298(9)	323(10)	326(11)
	344(12)	337(13)	347(14)	333(15)	333(16)	314(17)
	308(18)	282(19)	272(20)	244(21)	229(22)	201(23)
	186(24)	157(25)	143(26)	118(27)	104(28)	84(29)
	73(30)	55(31)	48(32)	35(33)	29(34)	20(35)
	17(36)	10(37)	9(38)	5(39)	4(40)	2(41)
	2(42)	1(44)				
<12 12 0>	34(0)	71(1)	134(2)	172(3)	232(4)	262(5)
	319(6)	343(7)	389(8)	408(9)	445(10)	451(11)
	481(12)	478(13)	493(14)	483(15)	489(16)	466(17)
	465(18)	436(19)	424(20)	392(21)	375(22)	337(23)
	319(24)	281(25)	259(26)	224(27)	204(28)	170(29)
	153(30)	125(31)	109(32)	87(33)	75(34)	56(35)
	49(36)	35(37)	29(38)	20(39)	17(40)	10(41)
	9(42)	5(43)	4(44)	2(45)	2(46)	1(48)
<13 13 0>	40(0)	94(1)	176(2)	224(3)	302(4)	348(5)
	416(6)	455(7)	517(8)	543(9)	595(10)	613(11)
	649(12)	656(13)	682(14)	673(15)	687(16)	669(17)
	667(18)	640(19)	630(20)	591(21)	574(22)	531(23)
	505(24)	460(25)	432(26)	384(27)	356(28)	311(29)
	282(30)	242(31)	217(32)	180(33)	160(34)	130(35)
	112(36)	89(37)	76(38)	57(39)	49(40)	35(41)
	29(42)	20(43)	17(44)	10(45)	9(46)	5(47)
	4(48)	2(49)	2(50)	1(52)		
<14 14 0>	52(0)	121(1)	224(2)	290(3)	388(4)	448(5)
	539(6)	590(7)	671(8)	712(9)	779(10)	808(11)
	862(12)	875(13)	915(14)	914(15)	936(16)	922(17)
	930(18)	900(19)	895(20)	855(21)	836(22)	788(23)
	761(24)	704(25)	672(26)	613(27)	575(28)	517(29)
	479(30)	421(31)	386(32)	334(33)	300(34)	255(35)
	227(36)	187(37)	165(38)	133(39)	114(40)	90(41)
	77(42)	57(43)	49(44)	35(45)	29(46)	20(47)
	17(48)	10(49)	9(50)	5(51)	4(52)	2(53)
	2(54)	1(56)				
<15 15 0>	66(0)	155(1)	280(2)	370(3)	490(4)	570(5)
	685(6)	755(7)	855(8)	916(9)	1003(10)	1045(11)
	1119(12)	1146(13)	1197(14)	1211(15)	1245(16)	1236(17)
	1255(18)	1230(19)	1228(20)	1191(21)	1174(22)	1120(23)
	1093(24)	1031(25)	990(26)	923(27)	877(28)	803(29)

	756(30)	683(31)	632(32)	564(33)	516(34)	451(35)
	409(36)	352(37)	313(38)	265(39)	234(40)	192(41)
	168(42)	135(43)	115(44)	91(45)	77(46)	57(47)
	49(48)	35(49)	29(50)	20(51)	17(52)	10(53)
	9(54)	5(55)	4(56)	2(57)	2(58)	1(60)
< 2 1 1>	1(1)	1(2)	2(3)	1(4)	1(5)	1(6)
	1(7)	0(0)				
< 3 2 1>	1(0)	3(1)	5(2)	5(3)	6(4)	7(5)
	5(6)	5(7)	4(8)	2(9)	2(10)	1(11)
< 4 3 1>	2(0)	7(1)	11(2)	15(3)	16(4)	18(5)
	18(6)	17(7)	15(8)	13(9)	10(10)	8(11)
	5(12)	3(13)	2(14)	1(15)		
< 5 4 1>	5(0)	14(1)	22(2)	30(3)	36(4)	39(5)
	42(6)	42(7)	40(8)	38(9)	33(10)	28(11)
	23(12)	18(13)	13(14)	9(15)	6(16)	3(17)
	2(18)	1(19)				
< 6 5 1>	7(0)	25(1)	41(2)	54(3)	66(4)	76(5)
	81(6)	86(7)	86(8)	83(9)	79(10)	73(11)
	63(12)	55(13)	46(14)	36(15)	28(16)	21(17)
	14(18)	10(19)	6(20)	3(21)	2(22)	1(23)
< 7 6 1>	14(0)	41(1)	67(2)	91(3)	112(4)	130(5)
	144(6)	153(7)	159(8)	160(9)	156(10)	150(11)
	140(12)	127(13)	113(14)	98(15)	82(16)	68(17)
	54(18)	41(19)	31(20)	22(21)	15(22)	10(23)
	6(24)	3(25)	2(26)	1(27)		
< 8 7 1>	22(0)	64(1)	104(2)	144(3)	177(4)	208(5)
	234(6)	254(7)	266(8)	275(9)	277(10)	272(11)
	263(12)	249(13)	230(14)	211(15)	187(16)	163(17)
	140(18)	117(19)	95(20)	76(21)	59(22)	44(23)
	32(24)	23(25)	15(26)	10(27)	6(28)	3(29)
	2(30)	1(31)				
< 9 8 1>	31(0)	95(1)	157(2)	213(3)	268(4)	317(5)
	357(6)	393(7)	421(8)	438(9)	450(10)	453(11)
	446(12)	435(13)	416(14)	390(15)	362(16)	330(17)
	294(18)	260(19)	225(20)	190(21)	159(22)	130(23)
	103(24)	81(25)	62(26)	45(27)	33(28)	23(29)
	15(30)	10(31)	6(32)	3(33)	2(34)	1(35)
<10 9 1>	45(0)	136(1)	224(2)	309(3)	387(4)	460(5)

	525(6)	581(7)	627(8)	664(9)	688(10)	704(11)
	707(12)	700(13)	685(14)	660(15)	626(16)	588(17)
	544(18)	496(19)	446(20)	396(21)	345(22)	298(23)
	252(24)	209(25)	172(26)	138(27)	108(28)	84(29)
	63(30)	46(31)	33(32)	23(33)	15(34)	10(35)
	6(36)	3(37)	2(38)	1(39)		
<11 10 1>	64(0)	189(1)	311(2)	431(3)	543(4)	646(5)
	743(6)	827(7)	899(8)	961(9)	1007(10)	1039(11)
	1060(12)	1065(13)	1056(14)	1037(15)	1005(16)	962(17)
	912(18)	854(19)	789(20)	723(21)	653(22)	582(23)
	514(24)	447(25)	383(26)	325(27)	271(28)	222(29)
	180(30)	143(31)	111(32)	85(33)	64(34)	46(35)
	33(36)	23(37)	15(38)	10(39)	6(40)	3(41)
	2(42)	1(43)				
<12 11 1>	84(0)	256(1)	424(2)	584(3)	739(4)	886(5)
	1018(6)	1142(7)	1250(8)	1341(9)	1419(10)	1480(11)
	1519(12)	1546(13)	1554(14)	1543(15)	1519(16)	1480(17)
	1425(18)	1362(19)	1287(20)	1203(21)	1116(22)	1024(23)
	927(24)	834(25)	741(26)	650(27)	565(28)	485(29)
	410(30)	344(31)	284(32)	230(33)	185(34)	146(35)
	112(36)	86(37)	64(38)	46(39)	33(40)	23(41)
	15(42)	10(43)	6(44)	3(45)	2(46)	1(47)
<13 12 1>	114(0)	339(1)	561(2)	777(3)	984(4)	1182(5)
	1366(6)	1535(7)	1689(8)	1824(9)	1940(10)	2036(11)
	2111(12)	2163(13)	2195(14)	2206(15)	2194(16)	2165(17)
	2116(18)	2049(19)	1970(20)	1876(21)	1770(22)	1658(23)
	1538(24)	1414(25)	1289(26)	1164(27)	1040(28)	922(29)
	809(30)	701(31)	603(32)	512(33)	429(34)	357(35)
	292(36)	235(37)	188(38)	147(39)	113(40)	86(41)
	64(42)	46(43)	33(44)	23(45)	15(46)	10(47)
	6(48)	3(49)	2(50)	1(51)		
<14 13 1>	148(0)	441(1)	729(2)	1014(3)	1285(4)	1545(5)
	1793(6)	2022(7)	2230(8)	2422(9)	2588(10)	2730(11)
	2849(12)	2941(13)	3005(14)	3047(15)	3059(16)	3046(17)
	3012(18)	2953(19)	2871(20)	2774(21)	2658(22)	2527(23)
	2386(24)	2235(25)	2076(26)	1916(27)	1752(28)	1589(29)
	1431(30)	1277(31)	1128(32)	990(33)	860(34)	739(35)
	630(36)	531(37)	442(38)	365(39)	297(40)	238(41)
	189(42)	148(43)	113(44)	86(45)	64(46)	46(47)
	33(48)	23(49)	15(50)	10(51)	6(52)	3(53)
	2(54)	1(55)				
<15 14 1>	187(0)	564(1)	936(2)	1296(3)	1650(4)	1989(5)
	2308(6)	2612(7)	2894(8)	3148(9)	3380(10)	3584(11)
	3755(12)	3900(13)	4013(14)	4090(15)	4141(16)	4159(17)
	4143(18)	4103(19)	4033(20)	3935(21)	3817(22)	3677(23)
	3516(24)	3343(25)	3156(26)	2957(27)	2755(28)	2548(29)

2337(30)	2132(31)	1929(32)	1731(33)	1544(34)	1365(35)
1196(36)	1041(37)	898(38)	766(39)	649(40)	544(41)
450(42)	370(43)	300(44)	239(45)	190(46)	148(47)
113(48)	86(49)	64(50)	46(51)	33(52)	23(53)
15(54)	10(55)	6(56)	3(57)	2(58)	1(59)

Table-12 Reduction of O(15) IRR [M] into IRR [pq] of O(5)

[0]	1[0 0]					
[1]	1[0 0]	1[2 0]				
[2]	1[0 0]	2[2 0]	1[2 2]	1[4 0]		
[3]	2[0 0]	3[2 0]	2[2 2]	1[3 1]	2[4 0]	1[4 2]
	1[6 0]					
[4]	3[0 0]	5[2 0]	3[2 2]	2[3 1]	1[3 2]	4[4 0]
	3[4 2]	1[4 4]	1[5 1]	2[6 0]	1[6 2]	1[8 0]
[5]	4[0 0]	7[2 0]	5[2 2]	4[3 1]	2[3 2]	7[4 0]
	6[4 2]	1[4 3]	2[4 4]	3[5 1]	1[5 2]	1[5 3]
	4[6 0]	3[6 2]	1[6 4]	1[7 1]	2[8 0]	1[8 2]
	1[10 0]					
[6]	5[0 0]	10[2 0]	8[2 2]	6[3 1]	3[3 2]	1[3 3]
	11[4 0]	1[4 1]	11[4 2]	2[4 3]	4[4 4]	6[5 1]
	3[5 2]	3[5 3]	1[5 4]	8[6 0]	7[6 2]	1[6 3]
	3[6 4]	1[6 6]	3[7 1]	1[7 2]	1[7 3]	4[8 0]
	3[8 2]	1[8 4]	1[9 1]	2[10 0]	1[10 2]	1[12 0]
[7]	6[0 0]	14[2 0]	11[2 2]	9[3 1]	5[3 2]	2[3 3]
	16[4 0]	2[4 1]	18[4 2]	4[4 3]	7[4 4]	11[5 1]
	6[5 2]	7[5 3]	3[5 4]	13[6 0]	1[6 1]	14[6 2]
	3[6 3]	7[6 4]	1[6 5]	2[6 6]	7[7 1]	3[7 2]
	4[7 3]	1[7 4]	1[7 5]	8[8 0]	7[8 2]	1[8 3]
	3[8 4]	1[8 6]	3[9 1]	1[9 2]	1[9 3]	4[10 0]
	3[10 2]	1[10 4]	1[11 1]	2[12 0]	1[12 2]	1[14 0]
[8]	8[0 0]	18[2 0]	15[2 2]	13[3 1]	8[3 2]	3[3 3]
	23[4 0]	3[4 1]	27[4 2]	7[4 3]	12[4 4]	18[5 1]
	11[5 2]	13[5 3]	6[5 4]	1[5 5]	20[6 0]	3[6 1]
	25[6 2]	7[6 3]	14[6 4]	3[6 5]	4[6 6]	13[7 1]
	7[7 2]	10[7 3]	4[7 4]	3[7 5]	1[7 6]	14[8 0]
	1[8 1]	15[8 2]	3[8 3]	8[8 4]	1[8 5]	3[8 6]
	1[8 8]	7[9 1]	3[9 2]	4[9 3]	1[9 4]	1[9 5]
	8[10 0]	7[10 2]	1[10 3]	3[10 4]	1[10 6]	3[11 1]
	1[11 2]	1[11 3]	4[12 0]	3[12 2]	1[12 4]	1[13 1]
	2[14 0]	1[14 2]	1[16 0]			
[9]	10[0 0]	23[2 0]	20[2 2]	18[3 1]	11[3 2]	5[3 3]

```
      31[ 4  0]  5[ 4  1] 39[ 4  2] 11[ 4  3] 18[ 4  4] 27[ 5  1]
      18[ 5  2] 22[ 5  3] 11[ 5  4]  3[ 5  5] 30[ 6  0]  6[ 6  1]
      40[ 6  2] 14[ 6  3] 25[ 6  4]  6[ 6  5]  8[ 6  6] 23[ 7  1]
      14[ 7  2] 20[ 7  3] 10[ 7  4]  8[ 7  5]  3[ 7  6] 22[ 8  0]
       3[ 8  1] 28[ 8  2]  8[ 8  3] 17[ 8  4]  4[ 8  5]  7[ 8  6]
       1[ 8  7]  2[ 8  8] 14[ 9  1]  7[ 9  2] 11[ 9  3]  4[ 9  4]
       4[ 9  5]  1[ 9  6]  1[ 9  7] 14[10  0]  1[10  1] 15[10  2]
       3[10  3]  8[10  4]  1[10  5]  3[10  6]  1[10  8]  7[11  1]
       3[11  2]  4[11  3]  1[11  4]  1[11  5]  8[12  0]  7[12  2]
       1[12  3]  3[12  4]  1[12  6]  3[13  1]  1[13  2]  1[13  3]
       4[14  0]  3[14  2]  1[14  4]  1[15  1]  2[16  0]  1[16  2]
       1[18  0]

[10]  12[ 0  0] 29[ 2  0] 26[ 2  2] 24[ 3  1] 15[ 3  2]  7[ 3  3]
      41[ 4  0]  7[ 4  1] 54[ 4  2] 17[ 4  3] 26[ 4  4]  1[ 5  0]
      39[ 5  1] 27[ 5  2] 34[ 5  3] 18[ 5  4]  6[ 5  5] 42[ 6  0]
      10[ 6  1] 61[ 6  2] 24[ 6  3] 41[ 6  4] 12[ 6  5] 14[ 6  6]
      37[ 7  1] 25[ 7  2] 36[ 7  3] 20[ 7  4] 16[ 7  5]  7[ 7  6]
       1[ 7  7] 34[ 8  0]  7[ 8  1] 47[ 8  2] 17[ 8  3] 33[ 8  4]
      10[ 8  5] 15[ 8  6]  3[ 8  7]  4[ 8  8] 25[ 9  1] 15[ 9  2]
      23[ 9  3] 11[ 9  4] 11[ 9  5]  4[ 9  6]  3[ 9  7]  1[ 9  8]
      23[10  0]  3[10  1] 29[10  2]  8[10  3] 18[10  4]  4[10  5]
       8[10  6]  1[10  7]  3[10  8]  1[10 10] 14[11  1]  7[11  2]
      11[11  3]  4[11  4]  4[11  5]  1[11  6]  1[11  7] 14[12  0]
       1[12  1] 15[12  2]  3[12  3]  8[12  4]  1[12  5]  3[12  6]
       1[12  8]  7[13  1]  3[13  2]  4[13  3]  1[13  4]  1[13  5]
       8[14  0]  7[14  2]  1[14  3]  3[14  4]  1[14  6]  3[15  1]
       1[15  2]  1[15  3]  4[16  0]  3[16  2]  1[16  4]  1[17  1]
       2[18  0]  1[18  2]  1[20  0]

[11]  14[ 0  0] 36[ 2  0] 33[ 2  2] 31[ 3  1] 20[ 3  2] 10[ 3  3]
      53[ 4  0] 10[ 4  1] 72[ 4  2] 24[ 4  3] 36[ 4  4]  1[ 5  0]
      54[ 5  1] 39[ 5  2] 50[ 5  3] 28[ 5  4] 10[ 5  5] 57[ 6  0]
      16[ 6  1] 88[ 6  2] 38[ 6  3] 63[ 6  4] 21[ 6  5] 22[ 6  6]
       1[ 7  0] 56[ 7  1] 40[ 7  2] 59[ 7  3] 35[ 7  4] 30[ 7  5]
      14[ 7  6]  3[ 7  7] 49[ 8  0] 13[ 8  1] 75[ 8  2] 32[ 8  3]
      57[ 8  4] 21[ 8  5] 28[ 8  6]  7[ 8  7]  8[ 8  8] 42[ 9  1]
      28[ 9  2] 43[ 9  3] 24[ 9  4] 24[ 9  5] 11[ 9  6]  8[ 9  7]
       3[ 9  8] 36[10  0]  7[10  1] 50[10  2] 18[10  3] 36[10  4]
      11[10  5] 18[10  6]  4[10  7]  7[10  8]  1[10  9]  2[10 10]
      26[11  1] 15[11  2] 24[11  3] 11[11  4] 12[11  5]  4[11  6]
       4[11  7]  1[11  8]  1[11  9] 23[12  0]  3[12  1] 29[12  2]
       8[12  3] 18[12  4]  4[12  5]  8[12  6]  1[12  7]  3[12  8]
       1[12 10] 14[13  1]  7[13  2] 11[13  3]  4[13  4]  4[13  5]
       1[13  6]  1[13  7] 14[14  0]  1[14  1] 15[14  2]  3[14  3]
       8[14  4]  1[14  5]  3[14  6]  1[14  8]  7[15  1]  3[15  2]
       4[15  3]  1[15  4]  1[15  5]  8[16  0]  7[16  2]  1[16  3]
       3[16  4]  1[16  6]  3[17  1]  1[17  2]  1[17  3]  4[18  0]
       3[18  2]  1[18  4]  1[19  1]  2[20  0]  1[20  2]  1[22  0]

[12]  17[ 0  0] 44[ 2  0] 41[ 2  2] 39[ 3  1] 26[ 3  2] 14[ 3  3]
      67[ 4  0] 14[ 4  1] 94[ 4  2] 32[ 4  3] 49[ 4  4]  1[ 5  0]
      72[ 5  1] 54[ 5  2] 70[ 5  3] 41[ 5  4] 16[ 5  5] 76[ 6  0]
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      24[ 6  1]122[ 6  2] 57[ 6  3] 92[ 6  4] 33[ 6  5] 34[ 6  6]
       2[ 7  0] 80[ 7  1] 61[ 7  2] 90[ 7  3] 57[ 7  4] 50[ 7  5]
      25[ 7  6]  7[ 7  7] 69[ 8  0] 22[ 8  1]113[ 8  2] 54[ 8  3]
      92[ 8  4] 39[ 8  5] 48[ 8  6] 14[ 8  7] 15[ 8  8]  1[ 9  0]
      66[ 9  1] 47[ 9  2] 74[ 9  3] 45[ 9  4] 46[ 9  5] 24[ 9  6]
      17[ 9  7]  7[ 9  8]  1[ 9  9] 53[10  0] 14[10  1] 82[10  2]
      35[10  3] 65[10  4] 25[10  5] 36[10  6] 11[10  7] 15[10  8]
       3[10  9]  4[10 10] 44[11  1] 29[11  2] 46[11  3] 25[11  4]
      27[11  5] 12[11  6] 11[11  7]  4[11  8]  3[11  9]  1[11 10]
      37[12  0]  7[12  1] 51[12  2] 18[12  3] 37[12  4] 11[12  5]
      19[12  6]  4[12  7]  8[12  8]  1[12  9]  3[12 10]  1[12 12]
      26[13  1] 15[13  2] 24[13  3] 11[13  4] 12[13  5]  4[13  6]
       4[13  7]  1[13  8]  1[13  9] 23[14  0]  3[14  1] 29[14  2]
       8[14  3] 18[14  4]  4[14  5]  8[14  6]  1[14  7]  3[14  8]
       1[14 10] 14[15  1]  7[15  2] 11[15  3]  4[15  4]  4[15  5]
       1[15  6]  1[15  7] 14[16  0]  1[16  1] 15[16  2]  3[16  3]
       8[16  4]  1[16  5]  3[16  6]  1[16  8]  7[17  1]  3[17  2]
       4[17  3]  1[17  4]  1[17  5]  8[18  0]  7[18  2]  1[18  3]
       3[18  4]  1[18  6]  3[19  1]  1[19  2]  1[19  3]  4[20  0]
       3[20  2]  1[20  4]  1[21  1]  2[22  0]  1[22  2]  1[24  0]


[13]  20[ 0  0] 53[ 2  0] 50[ 2  2] 49[ 3  1] 33[ 3  2] 18[ 3  3]
      83[ 4  0] 18[ 4  1]120[ 4  2] 43[ 4  3] 64[ 4  4]  2[ 5  0]
      94[ 5  1] 72[ 5  2] 95[ 5  3] 57[ 5  4] 24[ 5  5] 98[ 6  0]
      34[ 6  1]164[ 6  2] 81[ 6  3]129[ 6  4] 50[ 6  5] 49[ 6  6]
       3[ 7  0]111[ 7  1] 88[ 7  2]131[ 7  3] 87[ 7  4] 78[ 7  5]
      41[ 7  6] 13[ 7  7] 94[ 8  0] 35[ 8  1]162[ 8  2] 85[ 8  3]
     140[ 8  4] 65[ 8  5] 78[ 8  6] 26[ 8  7] 24[ 8  8]  2[ 9  0]
      98[ 9  1] 75[ 9  2]118[ 9  3] 77[ 9  4] 80[ 9  5] 45[ 9  6]
      33[ 9  7] 15[ 9  8]  3[ 9  9] 76[10  0] 25[10  1]127[10  2]
      62[10  3]109[10  4] 49[10  5] 65[10  6] 24[10  7] 29[10  8]
       7[10  9]  8[10 10]  1[11  0] 71[11  1] 50[11  2] 81[11  3]
      49[11  4] 54[11  5] 28[11  6] 25[11  7] 11[11  8]  8[11  9]
       3[11 10] 55[12  0] 14[12  1] 85[12  2] 36[12  3] 68[12  4]
      26[12  5] 39[12  6] 12[12  7] 18[12  8]  4[12  9]  7[12 10]
       1[12 11]  2[12 12] 45[13  1] 29[13  2] 47[13  3] 25[13  4]
      28[13  5] 12[13  6] 12[13  7]  4[13  8]  4[13  9]  1[13 10]
       1[13 11] 37[14  0]  7[14  1] 51[14  2] 18[14  3] 37[14  4]
      11[14  5] 19[14  6]  4[14  7]  8[14  8]  1[14  9]  3[14 10]
       1[14 12] 26[15  1] 15[15  2] 24[15  3] 11[15  4] 12[15  5]
       4[15  6]  4[15  7]  1[15  8]  1[15  9] 23[16  0]  3[16  1]
      29[16  2]  8[16  3] 18[16  4]  4[16  5]  8[16  6]  1[16  7]
       3[16  8]  1[16 10] 14[17  1]  7[17  2] 11[17  3]  4[17  4]
       4[17  5]  1[17  6]  1[17  7] 14[18  0]  1[18  1] 15[18  2]
       3[18  3]  8[18  4]  1[18  5]  3[18  6]  1[18  8]  7[19  1]
       3[19  2]  4[19  3]  1[19  4]  1[19  5]  8[20  0]  7[20  2]
       1[20  3]  3[20  4]  1[20  6]  3[21  1]  1[21  2]  1[21  3]
       4[22  0]  3[22  2]  1[22  4]  1[23  1]  2[24  0]  1[24  2]
       1[26  0]

[14]  23[ 0  0] 63[ 2  0] 61[ 2  2] 60[ 3  1] 41[ 3  2] 23[ 3  3]
     102[ 4  0] 23[ 4  1]150[ 4  2] 56[ 4  3] 82[ 4  4]  3[ 5  0]
     120[ 5  1] 94[ 5  2]125[ 5  3] 77[ 5  4] 34[ 5  5]124[ 6  0]
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 46[ 6  1]215[ 6  2]111[ 6  3]175[ 6  4] 72[ 6  5] 69[ 6  6]
  5[ 7  0]149[ 7  1]122[ 7  2]183[ 7  3]126[ 7  4]115[ 7  5]
 63[ 7  6] 22[ 7  7]124[ 8  0] 52[ 8  1]225[ 8  2]126[ 8  3]
204[ 8  4]102[ 8  5]119[ 8  6] 44[ 8  7] 38[ 8  8]  4[ 9  0]
140[ 9  1]113[ 9  2]177[ 9  3]123[ 9  4]129[ 9  5] 77[ 9  6]
 58[ 9  7] 28[ 9  8]  7[ 9  9]106[10  0] 41[10  1]188[10  2]
102[10  3]172[10  4] 86[10  5]110[10  6] 46[10  7] 51[10  8]
 15[10  9] 15[10 10]  2[11  0]108[11  1] 82[11  2]133[11  3]
 87[11  4] 97[11  5] 56[11  6] 50[11  7] 25[11  8] 17[11  9]
  7[11 10]  1[11 11] 80[12  0] 26[12  1]134[12  2] 65[12  3]
117[12  4] 53[12  5] 73[12  6] 28[12  7] 37[12  8] 11[12  9]
 15[12 10]  3[12 11]  4[12 12]  1[13  0] 73[13  1] 51[13  2]
 84[13  3] 50[13  4] 57[13  5] 29[13  6] 28[13  7] 12[13  8]
 11[13  9]  4[13 10]  3[13 11]  1[13 12] 56[14  0] 14[14  1]
 86[14  2] 36[14  3] 69[14  4] 26[14  5] 40[14  6] 12[14  7]
 19[14  8]  4[14  9]  8[14 10]  1[14 11]  3[14 12]  1[14 14]
 45[15  1] 29[15  2] 47[15  3] 25[15  4] 28[15  5] 12[15  6]
 12[15  7]  4[15  8]  4[15  9]  1[15 10]  1[15 11] 37[16  0]
  7[16  1] 51[16  2] 18[16  3] 37[16  4] 11[16  5] 19[16  6]
  4[16  7]  8[16  8]  1[16  9]  3[16 10]  1[16 12] 26[17  1]
 15[17  2] 24[17  3] 11[17  4] 12[17  5]  4[17  6]  4[17  7]
  1[17  8]  1[17  9] 23[18  0]  3[18  1] 29[18  2]  8[18  3]
 18[18  4]  4[18  5]  8[18  6]  1[18  7]  3[18  8]  1[18 10]
 14[19  1]  7[19  2] 11[19  3]  4[19  4]  4[19  5]  1[19  6]
  1[19  7] 14[20  0]  1[20  1] 15[20  2]  3[20  3]  8[20  4]
  1[20  5]  3[20  6]  1[20  8]  7[21  1]  3[21  2]  4[21  3]
  1[21  4]  1[21  5]  8[22  0]  7[22  2]  1[22  3]  3[22  4]
  1[22  6]  3[23  1]  1[23  2]  1[23  3]  4[24  0]  3[24  2]
  1[24  4]  1[25  1]  2[26  0]  1[26  2]  1[28  0]
```

```
[15]  27[ 0  0] 74[ 2  0] 73[ 2  2] 73[ 3  1] 50[ 3  2] 29[ 3  3]
     123[ 4  0] 29[ 4  1]185[ 4  2] 71[ 4  3]103[ 4  4]  4[ 5  0]
     150[ 5  1]120[ 5  2]161[ 5  3]101[ 5  4] 47[ 5  5]155[ 6  0]
      61[ 6  1]275[ 6  2]148[ 6  3]231[ 6  4] 99[ 6  5] 94[ 6  6]
       7[ 7  0]195[ 7  1]164[ 7  2]247[ 7  3]176[ 7  4]163[ 7  5]
      92[ 7  6] 35[ 7  7]160[ 8  0] 74[ 8  1]303[ 8  2]179[ 8  3]
     285[ 8  4]152[ 8  5]174[ 8  6] 69[ 8  7] 57[ 8  8]  7[ 9  0]
     193[ 9  1]162[ 9  2]256[ 9  3]185[ 9  4]196[ 9  5]123[ 9  6]
      95[ 9  7] 48[ 9  8] 14[ 9  9]143[10  0] 64[10  1]268[10  2]
     157[10  3]258[10  4]141[10  5]174[10  6] 81[10  7] 85[10  8]
      28[10  9] 25[10 10]  4[11  0]158[11  1]127[11  2]206[11  3]
     144[11  4]162[11  5]100[11  6] 90[11  7] 49[11  8] 34[11  9]
      15[11 10]  3[11 11]113[12  0] 44[12  1]202[12  2]110[12  3]
     189[12  4] 96[12  5]128[12  6] 57[12  7] 68[12  8] 25[12  9]
      29[12 10]  7[12 11]  8[12 12]  2[13  0]113[13  1] 85[13  2]
     140[13  3] 91[13  4]105[13  5] 60[13  6] 58[13  7] 29[13  8]
      25[13  9] 11[13 10]  8[13 11]  3[13 12] 82[14  0] 26[14  1]
     137[14  2] 66[14  3]120[14  4] 54[14  5] 76[14  6] 29[14  7]
      40[14  8] 12[14  9] 18[14 10]  4[14 11]  7[14 12]  1[14 13]
       2[14 14]  1[15  0] 74[15  1] 51[15  2] 85[15  3] 50[15  4]
      58[15  5] 29[15  6] 29[15  7] 12[15  8] 12[15  9]  4[15 10]
       4[15 11]  1[15 12]  1[15 13] 56[16  0] 14[16  1] 86[16  2]
      36[16  3] 69[16  4] 26[16  5] 40[16  6] 12[16  7] 19[16  8]
       4[16  9]  8[16 10]  1[16 11]  3[16 12]  1[16 14] 45[17  1]
      29[17  2] 47[17  3] 25[17  4] 28[17  5] 12[17  6] 12[17  7]
```

4[17 8]	4[17 9]	1[17 10]	1[17 11]	37[18 0]	7[18 1]
51[18 2]	18[18 3]	37[18 4]	11[18 5]	19[18 6]	4[18 7]
8[18 8]	1[18 9]	3[18 10]	1[18 12]	26[19 1]	15[19 2]
24[19 3]	11[19 4]	12[19 5]	4[19 6]	4[19 7]	1[19 8]
1[19 9]	23[20 0]	3[20 1]	29[20 2]	8[20 3]	18[20 4]
4[20 5]	8[20 6]	1[20 7]	3[20 8]	1[20 10]	14[21 1]
7[21 2]	11[21 3]	4[21 4]	4[21 5]	1[21 6]	1[21 7]
14[22 0]	1[22 1]	15[22 2]	3[22 3]	8[22 4]	1[22 5]
3[22 6]	1[22 8]	7[23 1]	3[23 2]	4[23 3]	1[23 4]
1[23 5]	8[24 0]	7[24 2]	1[24 3]	3[24 4]	1[24 6]
3[25 1]	1[25 2]	1[25 3]	4[26 0]	3[26 2]	1[26 4]
1[27 1]	2[28 0]	1[28 2]	1[30 0]		

Table-13 Reduction of O(14) IRR into IRR [pq] of O(5)

[0]	1[0 0]					
[1]	1[2 0]					
[2]	1[2 0]	1[2 2]	1[4 0]			
[3]	1[0 0]	1[2 0]	1[2 2]	1[3 1]	1[4 0]	1[4 2]
	1[6 0]					
[4]	1[0 0]	2[2 0]	1[2 2]	1[3 1]	1[3 2]	2[4 0]
	2[4 2]	1[4 4]	1[5 1]	1[6 0]	1[6 2]	1[8 0]
[5]	1[0 0]	2[2 0]	2[2 2]	2[3 1]	1[3 2]	3[4 0]
	3[4 2]	1[4 3]	1[4 4]	2[5 1]	1[5 2]	1[5 3]
	2[6 0]	2[6 2]	1[6 4]	1[7 1]	1[8 0]	1[8 2]
	1[10 0]					
[6]	1[0 0]	3[2 0]	3[2 2]	2[3 1]	1[3 2]	1[3 3]
	4[4 0]	1[4 1]	5[4 2]	1[4 3]	2[4 4]	3[5 1]
	2[5 2]	2[5 3]	1[5 4]	4[6 0]	4[6 2]	1[6 3]
	2[6 4]	1[6 6]	2[7 1]	1[7 2]	1[7 3]	2[8 0]
	2[8 2]	1[8 4]	1[9 1]	1[10 0]	1[10 2]	1[12 0]
[7]	1[0 0]	4[2 0]	3[2 2]	3[3 1]	2[3 2]	1[3 3]
	5[4 0]	1[4 1]	7[4 2]	2[4 3]	3[4 4]	5[5 1]
	3[5 2]	4[5 3]	2[5 4]	5[6 0]	1[6 1]	7[6 2]
	2[6 3]	4[6 4]	1[6 5]	1[6 6]	4[7 1]	2[7 2]
	3[7 3]	1[7 4]	1[7 5]	4[8 0]	4[8 2]	1[8 3]
	2[8 4]	1[8 6]	2[9 1]	1[9 2]	1[9 3]	2[10 0]
	2[10 2]	1[10 4]	1[11 1]	1[12 0]	1[12 2]	1[14 0]
[8]	2[0 0]	4[2 0]	4[2 2]	4[3 1]	3[3 2]	1[3 3]
	7[4 0]	1[4 1]	9[4 2]	3[4 3]	5[4 4]	7[5 1]
	5[5 2]	6[5 3]	3[5 4]	1[5 5]	7[6 0]	2[6 1]
	11[6 2]	4[6 3]	7[6 4]	2[6 5]	2[6 6]	6[7 1]
	4[7 2]	6[7 3]	3[7 4]	2[7 5]	1[7 6]	6[8 0]
	1[8 1]	8[8 2]	2[8 3]	5[8 4]	1[8 5]	2[8 6]
	1[8 8]	4[9 1]	2[9 2]	3[9 3]	1[9 4]	1[9 5]
	4[10 0]	4[10 2]	1[10 3]	2[10 4]	1[10 6]	2[11 1]
	1[11 2]	1[11 3]	2[12 0]	2[12 2]	1[12 4]	1[13 1]
	1[14 0]	1[14 2]	1[16 0]			
[9]	2[0 0]	5[2 0]	5[2 2]	5[3 1]	3[3 2]	2[3 3]

8[4 0] 2[4 1] 12[4 2] 4[4 3] 6[4 4] 9[5 1]
7[5 2] 9[5 3] 5[5 4] 2[5 5] 10[6 0] 3[6 1]
15[6 2] 7[6 3] 11[6 4] 3[6 5] 4[6 6] 10[7 1]
7[7 2] 10[7 3] 6[7 4] 5[7 5] 2[7 6] 8[8 0]
2[8 1] 13[8 2] 5[8 3] 9[8 4] 3[8 5] 4[8 6]
1[8 7] 1[8 8] 7[9 1] 4[9 2] 7[9 3] 3[9 4]
3[9 5] 1[9 6] 1[9 7] 6[10 0] 1[10 1] 8[10 2]
2[10 3] 5[10 4] 1[10 5] 2[10 6] 1[10 8] 4[11 1]
2[11 2] 3[11 3] 1[11 4] 1[11 5] 4[12 0] 4[12 2]
1[12 3] 2[12 4] 1[12 6] 2[13 1] 1[13 2] 1[13 3]
2[14 0] 2[14 2] 1[14 4] 1[15 1] 1[16 0] 1[16 2]
1[18 0]

[10] 2[0 0] 6[2 0] 6[2 2] 6[3 1] 4[3 2] 2[3 3]
10[4 0] 2[4 1] 15[4 2] 6[4 3] 8[4 4] 1[5 0]
12[5 1] 9[5 2] 12[5 3] 7[5 4] 3[5 5] 12[6 0]
4[6 1] 21[6 2] 10[6 3] 16[6 4] 6[6 5] 6[6 6]
14[7 1] 11[7 2] 16[7 3] 10[7 4] 8[7 5] 4[7 6]
1[7 7] 12[8 0] 4[8 1] 19[8 2] 9[8 3] 16[8 4]
6[8 5] 8[8 6] 2[8 7] 2[8 8] 11[9 1] 8[9 2]
12[9 3] 7[9 4] 7[9 5] 3[9 6] 2[9 7] 1[9 8]
9[10 0] 2[10 1] 14[10 2] 5[10 3] 10[10 4] 3[10 5]
5[10 6] 1[10 7] 2[10 8] 1[10 10] 7[11 1] 4[11 2]
7[11 3] 3[11 4] 3[11 5] 1[11 6] 1[11 7] 6[12 0]
1[12 1] 8[12 2] 2[12 3] 5[12 4] 1[12 5] 2[12 6]
1[12 8] 4[13 1] 2[13 2] 3[13 3] 1[13 4] 1[13 5]
4[14 0] 4[14 2] 1[14 3] 2[14 4] 1[14 6] 2[15 1]
1[15 2] 1[15 3] 2[16 0] 2[16 2] 1[16 4] 1[17 1]
1[18 0] 1[18 2] 1[20 0]

[11] 2[0 0] 7[2 0] 7[2 2] 7[3 1] 5[3 2] 3[3 3]
12[4 0] 3[4 1] 18[4 2] 7[4 3] 10[4 4] 15[5 1]
12[5 2] 16[5 3] 10[5 4] 4[5 5] 15[6 0] 6[6 1]
27[6 2] 14[6 3] 22[6 4] 9[6 5] 8[6 6] 1[7 0]
19[7 1] 15[7 2] 23[7 3] 15[7 4] 14[7 5] 7[7 6]
2[7 7] 15[8 0] 6[8 1] 28[8 2] 15[8 3] 24[8 4]
11[8 5] 13[8 6] 4[8 7] 4[8 8] 17[9 1] 13[9 2]
20[9 3] 13[9 4] 13[9 5] 7[9 6] 5[9 7] 2[9 8]
13[10 0] 4[10 1] 21[10 2] 10[10 3] 18[10 4] 7[10 5]
10[10 6] 3[10 7] 4[10 8] 1[10 9] 1[10 10] 12[11 1]
8[11 2] 13[11 3] 7[11 4] 8[11 5] 3[11 6] 3[11 7]
1[11 8] 1[11 9] 9[12 0] 2[12 1] 14[12 2] 5[12 3]
10[12 4] 3[12 5] 5[12 6] 1[12 7] 2[12 8] 1[12 10]
7[13 1] 4[13 2] 7[13 3] 3[13 4] 3[13 5] 1[13 6]
1[13 7] 6[14 0] 1[14 1] 8[14 2] 2[14 3] 5[14 4]
1[14 5] 2[14 6] 1[14 8] 4[15 1] 2[15 2] 3[15 3]
1[15 4] 1[15 5] 4[16 0] 4[16 2] 1[16 3] 2[16 4]
1[16 6] 2[17 1] 1[17 2] 1[17 3] 2[18 0] 2[18 2]
1[18 4] 1[19 1] 1[20 0] 1[20 2] 1[22 0]

[12] 3[0 0] 8[2 0] 8[2 2] 8[3 1] 6[3 2] 4[3 3]
14[4 0] 4[4 1] 22[4 2] 8[4 3] 13[4 4] 18[5 1]
15[5 2] 20[5 3] 13[5 4] 6[5 5] 19[6 0] 8[6 1]

```
34[ 6  2] 19[ 6  3] 29[ 6  4] 12[ 6  5] 12[ 6  6]  1[ 7  0]
24[ 7  1] 21[ 7  2] 31[ 7  3] 22[ 7  4] 20[ 7  5] 11[ 7  6]
 4[ 7  7] 20[ 8  0]  9[ 8  1] 38[ 8  2] 22[ 8  3] 35[ 8  4]
18[ 8  5] 20[ 8  6]  7[ 8  7]  7[ 8  8]  1[ 9  0] 24[ 9  1]
19[ 9  2] 31[ 9  3] 21[ 9  4] 22[ 9  5] 13[ 9  6]  9[ 9  7]
 4[ 9  8]  1[ 9  9] 17[10  0]  7[10  1] 32[10  2] 17[10  3]
29[10  4] 14[10  5] 18[10  6]  7[10  7]  8[10  8]  2[10  9]
 2[10 10] 18[11  1] 14[11  2] 22[11  3] 14[11  4] 15[11  5]
 8[11  6]  7[11  7]  3[11  8]  2[11  9]  1[11 10] 14[12  0]
 4[12  1] 22[12  2] 10[12  3] 19[12  4]  7[12  5] 11[12  6]
 3[12  7]  5[12  8]  1[12  9]  2[12 10]  1[12 12] 12[13  1]
 8[13  2] 13[13  3]  7[13  4]  8[13  5]  3[13  6]  3[13  7]
 1[13  8]  1[13  9]  9[14  0]  2[14  1] 14[14  2]  5[14  3]
10[14  4]  3[14  5]  5[14  6]  1[14  7]  2[14  8]  1[14 10]
 7[15  1]  4[15  2]  7[15  3]  3[15  4]  3[15  5]  1[15  6]
 1[15  7]  6[16  0]  1[16  1]  8[16  2]  2[16  3]  5[16  4]
 1[16  5]  2[16  6]  1[16  8]  4[17  1]  2[17  2]  3[17  3]
 1[17  4]  1[17  5]  4[18  0]  4[18  2]  1[18  3]  2[18  4]
 1[18  6]  2[19  1]  1[19  2]  1[19  3]  2[20  0]  2[20  2]
 1[20  4]  1[21  1]  1[22  0]  1[22  2]  1[24  0]

[13]   3[ 0  0]  9[ 2  0]  9[ 2  2] 10[ 3  1]  7[ 3  2]  4[ 3  3]
      16[ 4  0]  4[ 4  1] 26[ 4  2] 11[ 4  3] 15[ 4  4]  1[ 5  0]
      22[ 5  1] 18[ 5  2] 25[ 5  3] 16[ 5  4]  8[ 5  5] 22[ 6  0]
      10[ 6  1] 42[ 6  2] 24[ 6  3] 37[ 6  4] 17[ 6  5] 15[ 6  6]
       1[ 7  0] 31[ 7  1] 27[ 7  2] 41[ 7  3] 30[ 7  4] 28[ 7  5]
      16[ 7  6]  6[ 7  7] 25[ 8  0] 13[ 8  1] 49[ 8  2] 31[ 8  3]
      48[ 8  4] 26[ 8  5] 30[ 8  6] 12[ 8  7]  9[ 8  8]  1[ 9  0]
      32[ 9  1] 28[ 9  2] 44[ 9  3] 32[ 9  4] 34[ 9  5] 21[ 9  6]
      16[ 9  7]  8[ 9  8]  2[ 9  9] 23[10  0] 11[10  1] 45[10  2]
      27[10  3] 44[10  4] 24[10  5] 29[10  6] 13[10  7] 14[10  8]
       4[10  9]  4[10 10]  1[11  0] 27[11  1] 21[11  2] 35[11  3]
      24[11  4] 27[11  5] 16[11  6] 14[11  7]  7[11  8]  5[11  9]
       2[11 10] 18[12  0]  7[12  1] 34[12  2] 18[12  3] 31[12  4]
      15[12  5] 20[12  6]  8[12  7] 10[12  8]  3[12  9]  4[12 10]
       1[12 11]  1[12 12] 19[13  1] 14[13  2] 23[13  3] 14[13  4]
      16[13  5]  8[13  6]  8[13  7]  3[13  8]  3[13  9]  1[13 10]
       1[13 11] 14[14  0]  4[14  1] 22[14  2] 10[14  3] 19[14  4]
       7[14  5] 11[14  6]  3[14  7]  5[14  8]  1[14  9]  2[14 10]
       1[14 12] 12[15  1]  8[15  2] 13[15  3]  7[15  4]  8[15  5]
       3[15  6]  3[15  7]  1[15  8]  1[15  9]  9[16  0]  2[16  1]
      14[16  2]  5[16  3] 10[16  4]  3[16  5]  5[16  6]  1[16  7]
       2[16  8]  1[16 10]  7[17  1]  4[17  2]  7[17  3]  3[17  4]
       3[17  5]  1[17  6]  1[17  7]  6[18  0]  1[18  1]  8[18  2]
       2[18  3]  5[18  4]  1[18  5]  2[18  6]  1[18  8]  4[19  1]
       2[19  2]  3[19  3]  1[19  4]  1[19  5]  4[20  0]  4[20  2]
       1[20  3]  2[20  4]  1[20  6]  2[21  1]  1[21  2]  1[21  3]
       2[22  0]  2[22  2]  1[22  4]  1[23  1]  1[24  0]  1[24  2]
       1[26  0]

[14]   3[ 0  0] 10[ 2  0] 11[ 2  2] 11[ 3  1]  8[ 3  2]  5[ 3  3]
      19[ 4  0]  5[ 4  1] 30[ 4  2] 13[ 4  3] 18[ 4  4]  1[ 5  0]
      26[ 5  1] 22[ 5  2] 30[ 5  3] 20[ 5  4] 10[ 5  5] 26[ 6  0]
      12[ 6  1] 51[ 6  2] 30[ 6  3] 46[ 6  4] 22[ 6  5] 20[ 6  6]
```

```
 2[ 7  0] 38[ 7  1] 34[ 7  2] 52[ 7  3] 39[ 7  4] 37[ 7  5]
22[ 7  6]  9[ 7  7] 30[ 8  0] 17[ 8  1] 63[ 8  2] 41[ 8  3]
64[ 8  4] 37[ 8  5] 41[ 8  6] 18[ 8  7] 14[ 8  8]  2[ 9  0]
42[ 9  1] 38[ 9  2] 59[ 9  3] 46[ 9  4] 49[ 9  5] 32[ 9  6]
25[ 9  7] 13[ 9  8]  4[ 9  9] 30[10  0] 16[10  1] 61[10  2]
40[10  3] 63[10  4] 37[10  5] 45[10  6] 22[10  7] 22[10  8]
 8[10  9]  7[10 10]  1[11  0] 37[11  1] 32[11  2] 52[11  3]
38[11  4] 43[11  5] 28[11  6] 25[11  7] 14[11  8]  9[11  9]
 4[11 10]  1[11 11] 25[12  0] 12[12  1] 49[12  2] 29[12  3]
49[12  4] 27[12  5] 34[12  6] 16[12  7] 19[12  8]  7[12  9]
 8[12 10]  2[12 11]  2[12 12]  1[13  0] 28[13  1] 22[13  2]
37[13  3] 25[13  4] 29[13  5] 17[13  6] 16[13  7]  8[13  8]
 7[13  9]  3[13 10]  2[13 11]  1[13 12] 19[14  0]  7[14  1]
35[14  2] 18[14  3] 32[14  4] 15[14  5] 21[14  6]  8[14  7]
11[14  8]  3[14  9]  5[14 10]  1[14 11]  2[14 12]  1[14 14]
19[15  1] 14[15  2] 23[15  3] 14[15  4] 16[15  5]  8[15  6]
 8[15  7]  3[15  8]  3[15  9]  1[15 10]  1[15 11] 14[16  0]
 4[16  1] 22[16  2] 10[16  3] 19[16  4]  7[16  5] 11[16  6]
 3[16  7]  5[16  8]  1[16  9]  2[16 10]  1[16 12] 12[17  1]
 8[17  2] 13[17  3]  7[17  4]  8[17  5]  3[17  6]  3[17  7]
 1[17  8]  1[17  9]  9[18  0]  2[18  1] 14[18  2]  5[18  3]
10[18  4]  3[18  5]  5[18  6]  1[18  7]  2[18  8]  1[18 10]
 7[19  1]  4[19  2]  7[19  3]  3[19  4]  3[19  5]  1[19  6]
 1[19  7]  6[20  0]  1[20  1]  8[20  2]  2[20  3]  5[20  4]
 1[20  5]  2[20  6]  1[20  8]  4[21  1]  2[21  2]  3[21  3]
 1[21  4]  1[21  5]  4[22  0]  4[22  2]  1[22  3]  2[22  4]
 1[22  6]  2[23  1]  1[23  2]  1[23  3]  2[24  0]  2[24  2]
 1[24  4]  1[25  1]  1[26  0]  1[26  2]  1[28  0]

[15]   4[ 0  0] 11[ 2  0] 12[ 2  2] 13[ 3  1]  9[ 3  2]  6[ 3  3]
      21[ 4  0]  6[ 4  1] 35[ 4  2] 15[ 4  3] 21[ 4  4]  1[ 5  0]
      30[ 5  1] 26[ 5  2] 36[ 5  3] 24[ 5  4] 13[ 5  5] 31[ 6  0]
      15[ 6  1] 60[ 6  2] 37[ 6  3] 56[ 6  4] 27[ 6  5] 25[ 6  6]
       2[ 7  0] 46[ 7  1] 42[ 7  2] 64[ 7  3] 50[ 7  4] 48[ 7  5]
      29[ 7  6] 13[ 7  7] 36[ 8  0] 22[ 8  1] 78[ 8  2] 53[ 8  3]
      81[ 8  4] 50[ 8  5] 55[ 8  6] 25[ 8  7] 19[ 8  8]  3[ 9  0]
      53[ 9  1] 49[ 9  2] 79[ 9  3] 62[ 9  4] 67[ 9  5] 46[ 9  6]
      37[ 9  7] 20[ 9  8]  7[ 9  9] 37[10  0] 23[10  1] 80[10  2]
      55[10  3] 86[10  4] 55[10  5] 64[10  6] 35[10  7] 34[10  8]
      13[10  9] 10[10 10]  2[11  0] 50[11  1] 45[11  2] 73[11  3]
      57[11  4] 65[11  5] 44[11  6] 40[11  7] 24[11  8] 17[11  9]
       8[11 10]  2[11 11] 33[12  0] 18[12  1] 68[12  2] 45[12  3]
      72[12  4] 43[12  5] 55[12  6] 29[12  7] 31[12  8] 14[12  9]
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       3[16  9]  5[16 10]  1[16 11]  2[16 12]  1[16 14] 19[17  1]
      14[17  2] 23[17  3] 14[17  4] 16[17  5]  8[17  6]  8[17  7]
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