Tunneling and traversal of ultracold atoms through vacuum-induced potentials

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We examine the passage of ultracold two-level atoms through the potential produced by the vacuum of the cavity field. We find that the phase time may be considered as the appropriate measure of the time required for the atom to traverse the cavity. The phase tunneling time for ultracold atoms exhibits both super- and sub-classical time and we show how this behavior may be understood in terms of the momentum dependence of the phase of transmission amplitude. The passage of the atom through the cavity is unique, as it involves a coherent addition of the transition amplitudes corresponding to both barrier and well.

I. INTRODUCTION

An important question of great interest in several disciplines of physics has been—what is the tunneling time or traversal time of a quantum-mechanical particle through a potential. Various definitions have been proposed and the subject has been reviewed extensively [1–3]. In this paper, we examine the passage time of a cold atom through a high-quality cavity. The question is a complicated one as we have here a coupling with three different types of the degrees of freedom—(a) atom’s center of mass motion, (b) atom’s electronic states, and (c) photons. An analysis in the dressed-state basis reveals that the interaction of a moving atom with a single-mode vacuum field in a high-quality cavity is equivalent to a combination of a potential barrier and a well. The connection to potential problems is provided by the existing results in the context of micromasers [4,5]. These potentials belong to the category of vacuum-induced potentials and should be distinguished from the optical potentials produced by a far-off resonant field interacting with an atom [6]. Having realized that the cavity field may act like a potential for an ultracold atom, one could calculate the time the atom takes to traverse the cavity using methods similar to those used, for example, in the context of tunneling electrons through potential barriers and the propagation of light through a dispersive medium. The motional effects in the context of cavity QED are beginning to be seen. Münstermann et al. [7] have already reported the evidence of the effect of a quantized motion of atoms in the asymmetries of transmission of a weak light field through a cavity.

II. MODEL SYSTEM AND SUMMARY OF ATOM-FIELD INTERACTION

We consider an ultracold, two-level atom in its excited state to be incident on a single-mode cavity of length L. The frequency of the cavity field has been tuned to the frequency \( \omega \) of the atomic transition between the excited state \( |e\rangle \) and the ground state \( |g\rangle \). In a reference frame rotating with frequency \( \omega \), the Hamiltonian of the atom-field interaction including the quantized motion of center-of-mass (c.m.) of the atom, is given by

\[
H_I = \frac{p_z^2}{2m} + \hbar g u(z)(\sigma a^\dagger + a \sigma^\dagger),
\]

where \( g \) is the atom-field coupling constant and \( \sigma \) (\( \sigma^\dagger \)) are the lowering (raising) operators for the atomic transition. The operators \( a \) (\( a^\dagger \)) annhilate (create) a photon of frequency \( \omega \). For simplicity, the mode function \( u(z) \) of the cavity is assumed to be a mesa function \( \theta(z) \theta(L - z) \). The operator \( (\sigma a^\dagger + a \sigma^\dagger) \) is easily diagonalizable. It has eigenstates \( |\phi^0\rangle, \; |\phi^\pm_{n+1}\rangle \) with eigenvalues 0, \( \pm \sqrt{n+1} \), respectively. The dressed eigenstates may be expanded in terms of eigenstates of the free Hamiltonian as \( |\phi^0\rangle = |g,0\rangle \) and \( |\phi^\pm_{n+1}\rangle = 1/\sqrt{2}(|e,n\rangle \pm |g,n+1\rangle) \).

Since we need the transmission amplitude of the excited atom for further discussion, we summarize the main results of Meyer et al. [4]. Consider the initial atom-field state to be \( |e,n\rangle \), i.e., the atom is in the excited state and the cavity field contains fixed number \( n \) of photons. If we expand the combined state of the atom-cavity system as

\[
|\Psi(z,t)\rangle = \chi_+(z,t)|\phi^+_{n+1}\rangle + \chi_-(z,t)|\phi^-_{n+1}\rangle,
\]

then the time-dependent Schrödinger equation becomes

\[
\frac{i}{\hbar} \frac{\partial \chi_{\alpha}}{\partial t} = h \alpha \chi_{\alpha} \quad \alpha = \pm.
\]

Here, \( h_z = p_z^2/2m + \hbar g u(z) \sqrt{n+1} \) are operators acting in the space of the center-of-mass variables. Clearly, the cavity with fixed number of photons creates a barrier and a well potential for the external motion of the atom corresponding to the dressed states \( |\phi^\pm_{n+1}\rangle \), respectively, as discussed in Ref. [4].

We assume the initial state of the cavity to be vacuum \( (n=0) \) state. The initial wave packet of a moving free atom may be written in the form \( \psi(z,t) = \exp(-i p_z^2 t/2m \hbar) \int \ldots dk A(k)e^{ikz}A(k) e^{-i(k^2 z^2/2m \hbar)} \). We assume that \( A(k) \)'s are such that \( \psi(z,t) \) at \( z=0 \) peaks in time at the instant \( t=0 \). Thus, in the presence of the cavity, the wave packet at \( z=0 \) (entry of the cavity) has its peak (in time) at \( t=0 \). We therefore write the initial wave function of the atom-field system as \( |\Psi(z,0)\rangle = \phi(z,0)|e,0\rangle \). The wave function of the atom-field system after the interaction may be