Goldstone modes and bifurcations in phase-separated binary condensates at finite temperature

Arko Roy, 1 S. Gautam, 2 and D. Angom 1

1Physical Research Laboratory, Navrangpura, Ahmedabad-380009, Gujarat, India
2Department of Physics, Indian Institute of Science, Bangalore-560012, India

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We show that the third Goldstone mode, which emerges in binary condensates at phase separation, persists to higher interspecies interaction for density profiles where one component is surrounded on both sides by the other component. This is not the case with symmetry-broken density profiles where one species is entirely to the left and the other is entirely to the right. We, then, use Hartree-Fock-Bogoliubov theory with Popov approximation to examine the mode evolution at \( T \neq 0 \) and demonstrate the existence of mode bifurcation near the critical temperature. The Kohn mode, however, exhibits deviation from the natural frequency at finite temperatures after the phase separation. This is due to the exclusion of the noncondensate atoms in the dynamics.

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I. INTRODUCTION

The remarkable feature of binary condensates or two-species Bose-Einstein condensates (TBECs) is the phenomenon of phase separation [1,2]. This relates the system to novel phenomena in nonlinear dynamics and pattern formation, nonequilibrium statistical mechanics, optical systems, and phase transitions in condensed matter systems. Experimentally, TBECs have been realized in the mixture of two different alkali-metal atoms [3–5], and in two different isotopes [6] and hyperfine states [7,8] of an atom. Most importantly, in experiments, the TBEC can be steered from miscible to phase-separated domain or vice versa [9,10] through a Feshbach resonance. These have motivated theoretical investigations on stationary states [1,11], dynamical instabilities [12–14], and collective excitations [15–21] of TBECs.

In this paper, we report the development of Hartree-Fock-Bogoliubov theory with Popov (HFB-Popov) approximation [22] for TBECs. We use it to investigate the evolution of Goldstone modes and mode energies as a function of the severity of the convergence issues but this also makes it a good test for the methods we use. We choose the parameter domain where the system is quasi-one-dimensional (quasi-1D) and a mean-field description like HFB-Popov is applicable. The quasi-1D trapped bosons exhibit a rich phase structure as a function of density and interaction strengths [35]. For comparison with the experimental results we also consider the parameters as in the experiment [5]. We find that, like in Ref. [36], the quasi-1D descriptions are in good agreement with the condensate density profiles of three-dimensional (3D) calculations [37].

II. THEORY

For a highly anisotropic cigar-shaped harmonic trapping potential \( V = (1/2)m(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2) \), the trapping frequencies should satisfy the condition \( \omega_x = \omega_y = \omega_\perp \gg \omega_z \). In this case, we can integrate out the condensate wave function along \( xy \) and reduce it to a quasi-1D system. The transverse degrees of freedom are then frozen and the system is confined in the harmonic oscillator ground state along the transverse direction for which \( \hbar \omega_\perp \gg \mu_k \). We thus consider excitations present only in the axial direction \( z \) [38,39]. The grand-canonical Hamiltonian, in the quantized form, describing the mixture of two interacting BECs is then

\[
H = \sum_{k=1,2} \int dz \left[ \frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + V_k(z) - \mu_k \right] \Psi_k^\dagger(z,t) \Psi_k(z,t) + \frac{U_{k\perp}}{2} \left[ \Psi_k^\dagger(z,t) \Psi_k(z,t) + \frac{\partial \Psi_k^\dagger(z,t) \Psi_k(z,t) \partial z}{\partial z} \right] + U_{12} \int dz \Psi_1^\dagger(z,t) \Psi_1^\dagger(z,t) \Psi_2(z,t) \Psi_2(z,t),
\]

where \( k = 1,2 \) is the species index, \( \Psi_k^\dagger \)'s are the Bose field operators of the two different species, and \( \mu_k \)'s are the chemical potential of the two components.