Synchronized state of coupled dynamics on time-varying networks

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We consider synchronization properties of coupled dynamics on time-varying networks and the corresponding time-average network. We find that if the different Laplacians corresponding to the time-varying networks commute with each other then the stability of the synchronized state for both the time-varying and the time-average topologies are approximately the same. On the other hand for noncommuting Laplacians the stability of the synchronized state for the time-varying topology is in general better than the time-average topology. © 2006 American Institute of Physics. [DOI: 10.1063/1.2168395]

Synchronization is an important property of dynamical systems. Several diverse systems such as chemical reactions, electronic circuits, array of Josephson junctions, neurons, different body functions like heart rate and respiration, flapping of wings by the birds are known to show synchronization in some form. Roughly synchronization corresponds to the coherent evolution of different coupled dynamical systems. It is clearly of interest to study synchronization on different networks. The network represents the underlying geometrical structure of entities (nodes) and links (edges) in a given system. Recent investigations of several systems in different fields ranging from physical, biological and chemical systems to social and economic systems has shown that networks are ubiquitous in nature. Networks are often associated with dynamical variables evolving with time and it is possible to show that they show synchronization. However, natural networks are not static in time and the structure of nodes and links changes with time. In this paper we investigate the synchronization properties of networks with time-varying structure and compare it with the synchronization in static time-average networks. Network structure can be represented by the adjacency matrix whose elements are unity if the corresponding nodes are connected and zero otherwise. We can construct the Laplacian matrix from the adjacency matrix by subtracting the diagonal matrix of the degrees of different nodes. We find that if the Laplacians of the different time-varying networks commute with each other then the synchronization properties of the time-varying and time-average networks are approximately similar. On the other hand, if the Laplacians do not commute then the synchronization property of the time-varying network is more stable than that of the time-average network. We demonstrate the effect using an example of coupled Rössler systems.

I. INTRODUCTION

Several networks in the real world consist of dynamical elements interacting with each other and the interactions can be used to define the links of the network. Several of these networks have a large number of degrees of freedom and it is important to understand their dynamical behavior.\textsuperscript{1} One of the important dynamical property of the coupled networks is the synchronization of the dynamical variables associated with individual nodes. The earlier studies of synchronization in networks concentrated on regular networks such as lattices with nearest neighbor or short range couplings or globally coupled networks.\textsuperscript{2–7} Recently, it has been recognized that several complex systems have underlying structures that are described by networks or graphs which are not regular but have some random element and may have some universal properties such as small-world length scales or scale free degree distribution.\textsuperscript{1,8} This has led to the study of synchronization properties of different networks.\textsuperscript{9–15} In particular, it was shown that a state with several synchronized clusters is possible. Two main types of clusters can be identified, namely driven clusters that have mostly intercluster couplings and self-organized clusters that have mostly intracluster couplings.\textsuperscript{14}

In spite of several studies of synchronization on networks, most of the studies have concentrated on static networks where the nodes and edges (couplings) are constant in time. However, in several naturally occurring networks the topology of the networks changes with time. Both the number of nodes and the edges connecting the nodes can vary with time. Such a time-varying topology can occur in social networks, computer networks, WWW, biological systems, spread of epidemics, etc. Recently, there have been studies of dynamics of time-varying network topologies.\textsuperscript{16,17} It is shown that if the topology switches sufficiently fast between different networks, then the synchronized state can become