Synchronized states in chaotic systems coupled indirectly through a dynamic environment

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We consider synchronization of chaotic systems coupled indirectly through common environment where the environment has an intrinsic dynamics of its own modulated via feedback from the systems. We find that a rich variety of synchronization behavior, such as in-phase, antiphase, complete, and antisynchronization, is possible. We present an approximate stability analysis for the different synchronization behaviors. The transitions to different states of synchronous behavior are analyzed in the parameter plane of coupling strengths by numerical studies for specific cases such as Rössler and Lorenz systems and are characterized using various indices such as correlation, average phase difference, and Lyapunov exponents. The threshold condition obtained from numerical analysis is found to agree with that from the stability analysis.

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I. INTRODUCTION

Chaotic synchronization of coupled nonlinear systems has been an area of intense research activity [1]. In such cases, depending on the strength and nature of coupling, the systems are capable of entering into different states of synchronization such as in-phase [2,3], antiphase [4], lag [5,6], anticipatory [7], generalized [8–10], complete [11,12], and antisynchronization [13–15]. Although all these different synchronization phenomena have been explored in biological systems also, the case of phase synchronization is more useful in explaining many complex dynamical behaviors in them. Specifically, antiphase synchronization with repulsive coupling has special relevance in biological systems such as neurons and ecological webs [16–18]. Most of the present studies on synchronization consider mutually or unidirectionally coupled systems with or without parameter mismatch. However, synchronization has also been achieved by a common stochastic drive in uncoupled chaotic systems [19,20]. In such cases, the critical strength of noise for synchronization is nearly equal to the mean size of the attractor [21]. The synchronized state thus often differs very much from the intrinsic characteristics of the individual system. Synchronization of chaotic systems by external periodic forcing where the driven system locks to the frequency of the drive has also been reported [22–24]. So also, a weak periodic force is found to stabilize in-phase synchronization in a globally coupled array of Josephson junctions [25]. Further, in the context of many real world systems, synchronous behavior can occur due to interaction through a common medium. For instance, synchronization of chemical oscillations of catalyst-loaded reactants in a medium of catalyst-free solution is reported where coupling is through exchange of chemicals with the surrounding medium [26]. So also, synchronized oscillations in genetic oscillators occur due to coupling by diffusion of chemicals between cells and extracellular medium [27,28]. Global oscillations of concentration of neurotransmitter released by each cell can stimulate collective rhythms in a population of circadian oscillators [29]. Moreover, in an ensemble of cold atoms interacting with a coherent electromagnetic field, by controlling field cavity detuning, synchronized behavior with self-pulsating periodic and chaotic oscillations are found to occur [30]. In all these cases, the coupling function has a dynamics modulated by the system dynamics.

In general, such cases occur due to the common medium interacting with the dynamical systems. One refers to such a scheme as a coupling via a common environment. The dynamics of $n$ systems $x_i, i=1,\ldots,n$, coupled through an environment $y$ is then given by

\begin{equation}
\dot{x}_i = f(x_i, y),
\end{equation}

\begin{equation}
\dot{y} = g(y) + h(x_1, x_2, \ldots, x_n),
\end{equation}

where $x_i$ and $y$ have dimensions $m_x$ and $m_y$, respectively. Such an indirect coupling has been reported in the context of periodic oscillators by Katriel [31]. Under suitable conditions, the periodic oscillators can synchronize.

In this paper, we consider two chaotic systems coupled through a common dynamic environment as in Eq. (1). We show that this coupling can lead to a rich variety of synchronous behavior such as antiphase, in-phase, identical, antisynchronization, etc. This mechanism has the interesting feature that the common environment while capable of synchronizing the systems does not cause major changes in their dynamics. In the synchronized state, the systems retain more or less the same phase-space structure of the uncoupled system. We present an approximate stability analysis for the stability of the different synchronized states. We report detailed exploratory numerical studies for two standard systems, Rössler and Lorenz, and demonstrate the rich synchronization behavior. The transition to different stages of synchron-