Causality in propagation of a pulse in a nonlinear dispersive medium

G. S. AGARWAL*†§ and TARAK NATH DEY‡
†Department of Physics, Oklahoma State University, Stillwater, OK-74078, USA
‡Physical Research Laboratory, Navrangpura, Ahmedabad, India

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We investigate the causal propagation of the pulse through dispersive media by very precise numerical solution of the coupled Maxwell–Bloch equations without any approximations about the strength of the input field. We study full nonlinear behaviour of the pulse propagation through solid state media like ruby and alexandrite. We have demonstrated that the information carried by the discontinuity, i.e. front of the pulse, moves inside the media with velocity $c$ even though the peak of the pulse can travel either with sub-luminal or with super-luminal velocity. Our numerical demonstration is subject to the condition that the background refractive index of the medium is unity. We extend the argument of Levi-Civita to prove that the discontinuity would travel with velocity $c$ even in a nonlinear medium.

1. Introduction

The propagation of a pulse of electromagnetic radiation through a linear medium depends critically on the dispersive properties of the medium. Sommerfeld and Brillouin investigated this problem in great detail [1]. They discussed how the group velocity could be very different depending on whether one is working in the region of anomalous dispersion or normal dispersion. Some of these results were verified [2]. Sommerfeld and Brillouin also answered an important question—how information travels through the medium [3]. Interest in this subject has been revived [4–9] since by using external laser fields one can produce a desired dispersion [10, 11]. Such techniques have, in fact, led to the realization of ultraslow light. Based on an earlier suggestion of Chiao and co-worker [12], Wang et al. demonstrated the production of super-luminal propagation [13]. The propagation of a pulse in a linear medium can also be understood from the interference of various Fourier components present in the input pulse [14]. Stenner et al. were able to demonstrate the causal behaviour in the propagation of pulses by studying the propagation of discontinuities in pulse shapes [15].

*Corresponding author. Email: agirish@okstate.edu
§On leave from PRL, Ahmedabad-380009, India.