The radiation characteristics of atoms or molecules placed close to the dielectric surface are studied with full account of the spatial dispersion of the dielectric. Both the cases of plasmon polariton and the exciton polaritons are treated. The appropriate response functions are calculated and used to derive an analytic expression for the width of the excited state. The numerical results show that spatial dispersion reduces considerably the width from that one in the absence of spatial dispersion. It is found that spatial dispersion effects become more pronounced as the distance of the atom from the surface decreases.


1. Introduction

In an earlier paper\textsuperscript{5)}\textsuperscript{[1]} we have studied how the excitation of surface polaritons in a dielectric medium changes the radiation characteristics of the atoms and molecules placed close to the surface\textsuperscript{[2, 3]}. In particular we have seen, for example, that the excited state decay is predominately due to the non-radiative process arising from the excitation of surface polaritons provided the transition frequency lies in the range such that $\varepsilon(\omega) \leq -1$. In Part I, we had ignored the wave vector dependence of the dielectric function. However, it is known, for example, from the reflectivity studies\textsuperscript{[4]} on a semiconductor crystal that spatial dispersion\textsuperscript{[5 to 8]} could be very important near the exciton resonances. One, for example, finds that spatial dispersion leads to a reduction in the reflectivity because now additional waves (both longitudinal and transverse) can propagate inside the medium. One would expect that the spatial dispersion effects will reduce the decay rate of the excited state from the value in the absence of spatial dispersion. Moreover due to the presence of spatial dispersion, surface polaritons do not have a limiting frequency $\omega_s$ ($\omega_s$ is defined in terms of the local dielectric function as $\varepsilon(\omega_s) = -1$). In fact, the surface plasmon polariton frequency can be...
greater than $\omega_p/\sqrt{2}$ [9]. Therefore it would be interesting to study the decay of excited states of the atom for transition frequencies greater than $\omega_o$.

We have carried out extensive calculations to ascertain the effects of spatial dispersion in the decay characteristics. We have investigated only the cases of a metal (hydrodynamic dispersion) and the exciton polaritons. We have not studied the case of phonons as the spatial dispersion effects associated with lattice dispersion are not known to be important [10].

The outline of the paper is the following. In Section 2 we solve the classical problem of the calculation of the response function $\chi_{ij\theta\theta}(r, r', \omega)$ in terms of the transverse and the longitudinal dielectric functions $\varepsilon_t(k, \omega)$, $\varepsilon_l(k, \omega)$. Once the response function is known, the decay rate of the excited state of a two-level atom placed at a distance $b$ from the dielectric surface can be calculated from (see (1.1) of Part I)

$$\gamma(b, \omega) = \sum_{\alpha\beta} d_\alpha d_\beta \text{Im} \chi_{\alpha\beta\theta\theta}(b, b, \omega),$$

where $d_\alpha$ are the components of the dipole matrix element.

In Section 3, we present in analytic form the surface polariton contribution to $\gamma$. The results of our numerical computations are also given in Section 3. The paper is concluded with some remarks concerning the excitation of surface polaritons in a magnetic medium.

2. Response Functions in Presence of a Spatially Dispersive Dielectric

We now examine the effect of a spatially dispersive dielectric on spontaneous emission from a two-level atom. In [2] we had treated very briefly the case when retardation effects were ignored. In order to analyse properly the effects of spatial dispersion, one should first take into account the retardation and then one could compare the results with the ones obtained by ignoring spatial dispersion. Hence we first calculate the response functions $\chi_{ij\theta\theta}$. Our method of calculation will be a generalization of the method used by Kliewer and Fuchs [12].

We consider an isotropic spatially dispersive dielectric characterized by the dielectric function

$$\varepsilon_{ij}(k, \omega) = \varepsilon_t(k, \omega) \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) + \varepsilon_l(k, \omega) \frac{k_i k_j}{k^2},$$

(2.1)

where $\varepsilon_t$ and $\varepsilon_l$ are the transverse and longitudinal dielectric functions, respectively, which depend only on the modulus of the wave vector. We assume that the spatially dispersive dielectric occupies the right half-space $z > 0$, the left half-space is assumed to be vacuum. We represent the electric field inside the dielectric by the two-dimensional Fourier transform $E(\kappa, z, \omega)$

$$E(r, \omega) = \int d^2 \kappa e^{i\kappa \cdot r} E(\kappa, z, \omega),$$

(2.2)

where $\kappa$ is a two-dimensional wave vector parallel to the surface. On using (2.2) and the Maxwell equations, one easily obtains the following two equations:

$$\frac{1}{c^2} \frac{\partial^2 E_{\|}}{\partial z^2} + k_0^2 D_{\|} - k_0^2 E_{\|} + \kappa (\kappa \cdot E_{\|}) - i \kappa \frac{\partial E_{\perp}}{\partial z} = 0,$$

(2.3)

$$k_0^2 E_{\perp} - k_0^2 D_{\perp} + i \kappa \frac{\partial E_{\|}}{\partial z} = 0; \quad k_0 = \frac{\omega}{c}, \quad E = (E_{\|}, E_{\perp}),$$

(2.4)

6) In [2], one of us has given the basic formula for the decay in presence of a magnetic medium.

7) For an interesting discussion of the properties of surface polaritons in a magnetic medium see [11].

8) If not specified otherwise, $E, H$ will mean $E(\kappa, z, \omega)$, $H(\kappa, z, \omega)$, respectively.
where the subscripts $||$ and $\perp$ denote the components of the fields parallel and perpendicular to the surface $z = 0$, respectively, and $D$ represents the electric induction in the same decomposition. In the following analysis we will assume the specular boundary conditions at $z = 0$ to take into account the reflection of the excitons (plasmons) at the dielectric surface$^*$:

$$
\begin{align*}
E_{||}(z) &= E_{||}(-z), \\
E_{\perp}(z) &= -E_{\perp}(-z), \\
D_{||}(z) &= D_{||}(-z), \\
D_{\perp}(z) &= -D_{\perp}(-z).
\end{align*}
$$

(2.5)

One now introduces the Fourier transforms $\mathbf{v}(\mathbf{x}, q, \omega) \mathbf{u}(\mathbf{x}, q, \omega)$ defined by

$$
\mathbf{v}(\mathbf{x}, q, \omega) = \int_{-\infty}^{+\infty} dz \mathbf{E}(\mathbf{x}, z, \omega) e^{-iqz}, \quad \mathbf{u}(\mathbf{x}, q, \omega) = \int_{-\infty}^{+\infty} dz \mathbf{D}(\mathbf{x}, z, \omega) e^{-iqz}.
$$

(2.6)

From the boundary conditions (2.5) it is clear that there is a discontinuity in the derivative of $E_{||}$ and in the value of $E_{\perp}$ and hence

$$
\int_{-\infty}^{+\infty} dz e^{-iqz} \left( \frac{\partial^2 E_{||}}{\partial z^2} - q^2 v_{||} - \frac{2dE_{||}(0^+)}{dz} \right),
$$

(2.7)

$$
\int_{-\infty}^{+\infty} dz e^{-iqz} \left( \frac{\partial E_{\perp}}{\partial z} + iqv_{\perp} - 2E_{\perp}(0^+) \right),
$$

(2.8)

where $0^+$ indicates the values at a point just inside the medium. If we take the Fourier transform (2.6) of (2.3), (2.4), taking into account (2.7), (2.8) we obtain the equations

$$
\nabla^2 v_{\perp} - k_0^2 u_{\perp} - q(\mathbf{x} \cdot v_{||}) = 0,
$$

(2.9)

$$
-(q^2 + \nabla^2) v_{||} + \mathbf{x}(\mathbf{x} \cdot v_{||}) + \nabla q v_{\perp} + k_0^2 u_{||} = -2ik_0 [z \times \mathbf{H}(0^+)].
$$

(2.10)

Here $\mathbf{H}$ denotes the magnetic field, $\mathbf{z}$ the unit vector in $z$-direction. In obtaining (2.10) we have used the relation

$$
\frac{dE_{||}(0^+)}{dz} - i\mathbf{x} E_{\perp}(0^+) = -ik_0 [z \times \mathbf{H}(0^+)]
$$

(2.11)

which can be obtained from the Fourier transform of the equation $\nabla \times \mathbf{E} = -\nabla \times \mathbf{H}$.

Finally on expressing $u_{||}$ and $u_{\perp}$ in (2.9), (2.10) in terms of $v_{||}, v_{\perp}$ using the dielectric function (2.1), we obtain the relations

$$
D_1 v_{||} + \frac{\mathbf{x}(\mathbf{k} \cdot \mathbf{v})}{k^2} D_2 = A,
$$

(2.12)

$$
D_1 v_{\perp} - \frac{q(\mathbf{k} \cdot \mathbf{v})}{k^2} D_2 = 0,
$$

(2.13)

where we have used the abbreviations

$$
\begin{align*}
\mathbf{k} &= (\mathbf{k}, q), \quad k^2 = \nabla^2 + q^2, \quad \mathbf{v} = (v_{||}, v_{\perp}), \\
D_1 &= k_0^2 \varepsilon_t - k^2, \quad D_2 = k^2 - k_0^2 (\varepsilon_t - \varepsilon_i), \\
A &= -2ik_0 [z \times \mathbf{H}(0^+)].
\end{align*}
$$

(2.14)

(2.15)

$^*$ In the context of spatial dispersion different types of boundary conditions have been used [4, 6 to 9]. Here we follow the boundary condition of Kliewer and Fuchs because of its simplicity.
Equations (2.12), (2.13) are easily solved for $v_{||}$, $v_{\perp}$ and the results are

$$v_{||} = D_{1}^{-1} \left\{ A - \frac{x(x\cdot A)}{k^2} \right\} + \frac{x(x\cdot A)}{k^2} (D_{1} + D_{2})^{-1},$$

(2.16)

$$v_{\perp} = -\frac{q}{k^2} (x\cdot A) D_{2} D_{1}^{-1} (D_{1} + D_{2})^{-1}.$$  

(2.17)

We have thus expressed the electric field inside the spatially dispersive medium in terms of the only unknown $A$. We now use Maxwell boundary conditions at $z = 0$ to obtain $A$ in terms of the incident fields. Since the tangential magnetic field $H(z)$ is continuous across the boundary we can express $A$ in terms of the incident field for $z < 0$,

$$A = -2ik_{0}[x \times H(0^-)].$$

(2.18)

We now express $A$ in terms of the incident field for $z < 0$,

$$A = -2ik_{0}[x \times H(0^-)].$$

(2.18)

Writing the incident field in the left half-space as

$$E(r, \omega) = \int d\alpha e^{i\omega\cdot r} (e_{0} e^{i\omega\cdot x} + e_{R} e^{-i\omega\cdot x}),$$

(2.19)

where

$$w_{0}^2 = k_{0}^2 - \kappa^2.$$  

(2.20a)

The vectors $e$ are connected with $v$ by

$$v(x, q, \omega) = 2\pi e(x, \omega) \delta(q - w_{0}); \quad e(x, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} v(x, q, \omega) dq.$$  

(2.20b)

From Maxwell's equation $\nabla \times E(r, t) = -(1/c) \left\{ \partial H(r, t)/\partial t \right\}$ we then obtain for $A$ according to (2.18)

$$A = -2i \left\{ xe_{0\perp} - w_{0}(e_{0\parallel} - e_{R\parallel}) + \frac{x(x\cdot e_{R\parallel})}{w_{0}} \right\},$$

(2.21)

where we have used $(e_{0\parallel} + e_{R\parallel}) = (e_{0\parallel} \cdot x) + e_{\perp} w_{0} = 0$.

If we now express $(e_{0\parallel} + e_{R\parallel})$ in terms of $v_{||}$ given by (2.16) using (2.20b) we arrive at

$$(e_{0\parallel} + e_{R\parallel}) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dq \left\{ D_{1}^{-1} \left[ A - \frac{x(x\cdot A)}{k^2} \right] + \frac{x(x\cdot A)}{k^2} (D_{1} + D_{2})^{-1} \right\}.$$  

(2.22)

From these equations we can calculate $e_{R\parallel}$ in terms of $e_{0}$ to obtain

$$[x \times e_{R\parallel}] = -\left( \frac{1 - 2i\omega_{0}D_{4}}{1 + 2i\omega_{0}D_{4}} \right) [e \times e_{0\parallel}],$$

(2.23)

$$x\cdot e_{R\parallel} = \left( 1 + \frac{2ik_{0}^{2}D_{2}}{w_{0}} \right)^{-1} \left[ -2iD_{3}x^{2}e_{0\perp} - (x\cdot e_{0\parallel})(1 - 2i\omega_{0}D_{2}) \right] w_{0}e_{R\perp},$$

(2.24)

where

$$D_{3} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{dq}{k^2} [q^{2}D_{1}^{-1} + x^{2}(D_{1} + D_{2})^{-1}],$$

(2.25)

$$D_{4} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dq D_{1}^{-1}.$$  

(2.26)
In our problem the incident field is that produced by a dipole \( p \) at \( z = -b \) and hence [13]

\[
e_0 = \frac{i}{2\pi w_0} [p k_0^2 - (p \cdot k_0) k_0] e^{i w_0 b}; \quad k_0 = (\mathbf{r}, w_0). \tag{2.27}
\]

Hence on substituting (2.27) in (2.24) and (2.23) we find that

\[
\begin{align*}
(\mathbf{r} \cdot e_R) &= w_0 e_{R\perp} = \frac{1 - 2i D_3(k_0^2/w_0)}{1 + 2i D_3(k_0^2/w_0)} \frac{iw_0}{2\pi w_0} e^{i w_0 b} (x^2 p_{\perp} - w_0(\mathbf{r} \cdot p)), \\
[\mathbf{r} \times e_R] &= -\left( \frac{1 - 2i w_0 D_4}{1 + 2i w_0 D_4} \right) \frac{ik_0^2}{2\pi w_0} e^{i w_0 b} [\mathbf{r} \times p_{\parallel}]. 
\end{align*}
\tag{2.28}
\]

These equations enable us to calculate all the response functions \( \gamma_{j/k}(r, r', \omega) \) for \( z, z' < 0 \) (cf. (5.23) of [13]). Note also that the structure of (2.28), (2.29) is very similar to the structure of (5.43), (5.44) of [13] in the absence of spatial dispersion effects. The vanishing of the denominator \( 1 + 2i D_3(k_0^2/w_0) \) gives us the dispersion relations for surface polaritons [14, 15]

\[
1 + \psi = 0, \quad \psi = \frac{2i k_0^2}{2\pi \sqrt{k_0^2 - x'^2}} \int_{-\infty}^{+\infty} dq \left[ q^2 \left( \frac{k_0^2}{k_0^2} - k^2 + x'^2 \right) + \frac{x'^2}{k_0^2} \right], \tag{2.30}
\]

which agrees with the result of Kliewer and Fuchs. The form of \( \psi \) depends on the dielectric function. The function \( \psi \) is to be evaluated very carefully for one has to examine the position of the zeros of \( \epsilon_l(\omega) \) and \( \epsilon_i(\omega) \) in the complex plane and the position obviously depends on the range of frequencies and wave vectors. Note that zeros of \( \epsilon_l \) and \( \epsilon_i \) correspond to the bulk dispersion relations.

For the free-electron gas in hydrodynamic approximation, we take [9]

\[
\begin{align*}
\epsilon_l(q, \omega) &= 1 - \frac{\omega_p^2}{\omega^2}, \\
\epsilon_i(q, \omega) &= 1 - \frac{\omega_p^2}{\omega^2 - \beta q^2},
\end{align*}
\tag{2.31}
\]

where \( \beta = (3/5)v_F^2 \) (\( v_F \) is the Fermi velocity); on using (2.31) we get for \( \psi \) the expression

\[
\psi = \frac{1}{\sqrt{x'^2 - 1}} \left( \frac{\epsilon_l + \Omega_l}{\sqrt{\epsilon_l}} - 1 + \frac{\omega_p^2}{\omega^2} - 1 \right) \left( \frac{1}{\epsilon_l} - \frac{\omega_p^2}{\omega^2} \right), \tag{2.32}
\]

where

\[
\lambda = \frac{k_0}{\sqrt{\epsilon_l}}, \quad \Omega_l = \sqrt{x'^2 - \epsilon_l}, \quad \Omega_l = \sqrt{\epsilon_l + \frac{\sqrt{\epsilon_l}}{\beta}} \left( \frac{\omega_p^2}{\omega^2} - 1 \right). \tag{2.33}
\]

For the case of exciton polaritons we choose

\[
\epsilon_l(q, \omega) = \epsilon_i(q, \omega) = \epsilon_0 + \frac{4\pi\alpha\omega_0^2}{(\omega_0^2 - \omega^2 + \beta q^2)}, \tag{2.34}
\]

where \( \epsilon_0 \) is the background dielectric constant for the case of exciton resonance at \( \omega_0 \) and \( \beta = \hbar\omega_0/m^* \) with \( m^* \) denoting the effective mass of the excitons. On introducing the dimensionless variables \( k_0 = \omega/c \)

\[
q' = \frac{q}{k_0}, \quad \kappa = \frac{\sqrt{\epsilon_l}}{k_0}, \quad \mu' = \frac{(\omega^2 - \omega_0^2) \epsilon_0^2}{\beta \omega^2}, \quad \chi = \frac{4\pi\alpha\epsilon_0^2 \omega_0^2}{\beta \omega^2}, \tag{2.35}
\]

we find that the poles occurring in (2.30) are situated at

\[
q'^2 + \kappa'^2 = 0, \quad q'^2 + \Omega_l^2 = 0, \quad q'^2 + \Omega_l^2 = 0, \quad q'^2 + \Omega_l^2 = 0, \tag{2.36}
\]
Fig. 1. The dispersion relation for surface exciton polaritons in a spatially dispersive dielectric evaluated for the parameters of CdS. On the ordinate we have plotted the relative deviation of the frequency from \( \omega_0 \), \( (\omega - \omega_0)/\omega_0 \), on the abscissa the normalized wave vector \( x' = \kappa c/\omega \).

Fig. 2. The dispersion relation for surface plasmon polaritons in the free-electron gas for \( \Lambda = 1 \) (solid line) and \( \Lambda = 2 \) (dashed line), with \( \Lambda \) defined by (2.38).

where \( \Omega \) and \( \Omega_{1,2} \) are defined by

\[
\begin{align*}
\Omega^2 &= x'^2 - \mu^2 + \frac{\kappa}{\epsilon_0}, \\
\Omega_{1,2}^2 &= \frac{1}{2} (2x'^2 - \mu^2 - \epsilon_0) \pm \frac{1}{2} \left( (2x'^2 - \mu^2 - \epsilon_0)^2 - 4x'^2(\kappa^2 - \mu^2) + 4\epsilon_0\Omega^2 \right)^{1/2}.
\end{align*}
\] (2.36)

In order to find the dispersion relation we have to assume that \( \Omega \) and \( \Omega_2 \) lie in the upper half-plane and then get by calculus of residues\(^{10}\)

\[
\psi(\omega, x') = \frac{1}{\sqrt{x'^2 - 1}} \left\{ \frac{x'^2}{\epsilon_0(x'^2 - \Omega^2)} \left( \frac{\mu^2}{x'^2 - \epsilon_0\Omega} \right) - \frac{x'^2\mu^2}{(\Omega_1^2 - x'^2)(\Omega_2^2 - x'^2)} + \frac{\Omega_2(\Omega_2^2 - x'^2 - \mu^2)}{(x'^2 - \Omega_2^2)(\Omega_1^2 - \Omega_2^2)} \right\}.
\] (2.37)

For the further numerical evaluation we take the parameters [17] appropriate to CdS: \( \epsilon_0 = 8.3 \), \( \omega_0 = 2.5524 \text{ eV} \), \( 4\pi\alpha_0 = 1.3 \times 10^{-2} \), \( \epsilon_0^2/\beta = 1.8045 \times 10^5 \) (effective mass = 0.9 electron mass).

In Fig. 1 we have plotted the resulting dispersion relation for the surface polaritons of CdS; this agrees with that of [16]. The form of the dispersion curve is similar to the one obtained by Maradudin and Mills [8]. The explicit calculation of the dispersion relation has been repeated here since it is needed in our computations involving the linewidths.

For the case of free-electron gas in hydrodynamic approximation the resulting dispersion curves are shown in Fig. 2 for values \( \Lambda = 1 \) and \( \Lambda = 2 \), where [9]

\[
\Lambda^2 = (\text{plasmon energy}/\text{Fermi energy})^2 = 0.012388 \frac{c}{\sqrt{\beta}} \sqrt{\frac{3}{5}}.
\] (2.38)
3. Explicit Expression for the Surface Polariton Contribution to the Decay Rate and the Numerical Results

Having obtained the response function for the case of a spatially dispersive dielectric, this can be used in a number of problems such as the calculation of dispersion forces [15], lifetime studies etc. We now calculate the width of the excited state of a two-level atom placed close to the surface of a spatially dispersive dielectric. For the case when the dipole moment of the atom is along z-axis one obtains immediately from (1.1) and (2.28) the result (for a similar procedure see [13], (5.24))

\[
\gamma_\perp (b, \omega) = \frac{3}{2} \gamma^{(0)} \text{Re} \int_0^\infty \frac{x'^3 \text{d}x'}{\omega_0} \frac{1 - \frac{\psi}{1 + \psi}}{1 + \psi} e^{2i\omega_0 b k_0}; \quad \omega_0^2 = 1 - x'^2. \tag{3.1}
\]

It is clear from the dispersion relation that the surface polaritons occur only for wave vectors greater than one and hence the surface polariton contribution to \( \gamma_\perp \) can come only from the part of the integral \( \int_1^\infty \) and thus

\[
\gamma_\perp^{\text{SP}} (b, \omega) = \frac{3}{2} \gamma^{(0)} \int_1^\infty \frac{x'^3 \text{d}x'}{\omega_0} \text{Im} \left( \frac{1 - \frac{\psi}{1 + \psi}}{1 + \psi} \right) e^{-2bk_0|\omega'|}. \tag{3.2}
\]

It is also known that \( \text{Im} \left( \frac{1 - \psi}{1 + \psi} \right) \), in the absence of spatial dispersion (see [2], (9.5)), has a delta function structure, i.e.

\[
\text{Im} \left( \frac{1 - \psi}{1 + \psi} \right) \sim \delta(x' - x'_0), \tag{3.3}
\]

where \( x'_0 \) is the wave vector of surface polariton. We therefore anticipate that even in the presence of spatial dispersion, a good approximation will be

\[
\text{Im} \left( \frac{1 - \psi}{1 + \psi} \right) \approx 2\pi \left( \frac{\partial \psi}{\partial x'_0} \right)^{-1} \delta(x' - x'_0). \tag{3.4}
\]

The behaviour of \( \partial \psi/\partial x'_0 \) as a function of \( \omega \) has been checked numerically for the case of exciton polariton as well as for the electron gas. We found that \( \partial \psi/\partial x'_0 \) is always positive in the range considered in Fig. 1 and 2 and that a minimum occurs close to the point of inflection of the corresponding dispersion curve.

In order to check the order of the pole we have also plotted the second derivative of \( \psi \) and found that it is always negative and has a shape similar to \( \psi' \) with a maximum very close to the minimum of \( \psi' \). Therefore we can also exclude the existence of poles of second order.

On substituting (3.4) in (3.2) we obtain

\[
\gamma_\perp^{\text{SP}} (b, \omega) = 3\pi \gamma^{(0)} \frac{x'^3}{\sqrt{x'^2 - 1}} \exp \left\{ - \frac{2b\omega}{c \sqrt{x'^2 - 1}} \right\} \left( \frac{\partial \psi}{\partial x'_0} \right)^{-1}. \tag{3.5}
\]

This is the final expression for the width of the excited state of an atom in presence of a spatially dispersive dielectric. It can be shown (cf. [2]) that \( 2\gamma_\perp^{\text{SP}} (b, \omega) \) as given by (3.5), is the transition probability per unit time that the excited atom decays to the ground state with the excitation of a surface polariton in the spatially dispersive dielectric.

We have evaluated (3.5) for the case of exciton polaritons in CdS (cf. (2.34)) and for the free-electron gas in hydrodynamic approximation. The results of our numerical
Fig. 3. The contribution of surface polaritons to the linewidth $\gamma_{\perp}^{\text{SP}}/\gamma^{(0)}$ as a function of the frequency $(\omega - \omega_0)/\omega_0$ for $b\omega_0/c = 0.1$ (solid line), 0.2, 0.3, 0.5, 0.7, and 1 (for CdS as in Fig. 1).

Fig. 4. The surface plasmon polariton contribution to $\gamma_{\perp}^{\text{SP}}/\gamma^{(0)}$ as a function of frequency for $\Delta = 1$ and for different values of distance $b\omega_0/c = 0.1, 0.2, 0.3, 0.5, 0.7, 1.0$ (parameters as in Fig. 2).

Fig. 5. Same as in Fig. 4 but for $\Delta = 2$ and $b\omega_0/c = 0.1, 0.15, 0.2, 0.3, 0.5, 1.0$.

Fig. 6. The contribution of surface polaritons to the linewidth $\gamma_{\perp}^{\text{SP}}/\gamma^{(0)}$ in the absence (dashed line) and presence (solid line) of spatial dispersion as a function of $(\omega - \omega_0)/\omega_0$ for $b\omega_0/c = 0.1, 0.5, 1.0$ (CdS as in Fig. 1). $\gamma_{\perp}^{\text{SP}}$ decreases with increasing $b$. \[ c^2/\beta = 1.8045 \times 10^6 \]

Fig. 7. The contribution of surface plasmon polariton to the width $\gamma_{\perp}^{\text{SP}}/\gamma^{(0)}$ in the absence (dashed line) and the presence (solid line) of hydrodynamic dispersion for different values of the distance $b\omega_0/c = 0.1, 0.5, 1.0$. $\gamma_{\perp}^{\text{SP}}/\gamma^{(0)}$ decreases with increasing $b$. \[ c^2/\beta = 1.08604 \times 10^4 \]
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analysis are shown in Fig. 3 to 5. These should be compared with the results in the absence of spatial dispersion. This comparison is done in Fig. 6 and 7. It is clear from these curves that spatial dispersion reduces considerably the decay rate with respect to the one in the absence of spatial dispersion for frequencies in the region of \( \omega_s \) where \( \omega_s \) is the limiting frequency in the absence of spatial dispersion \( \epsilon(\omega_s) = -1 \). The Fig. 4 and 5 show for example that with the increase of \( \Delta \), the decay rate increases. This is, as expected, because a large value of \( \Delta \) implies weaker spatial dispersion. Fig. 6 and 7 also show that the effects of spatial dispersion are more and more pronounced as the distance of the atom from the dielectric surface decreases. \( ^{11} \) Fig. 3 to 5 do show a finite surface polariton contribution to the decay rate for frequencies \( \omega > \omega_s \).

In Part I and the present paper, we have treated the effect of the different types of elementary excitations viz. plasmon, phonon, and excitons (with and without spatial dispersion) on the radiation characteristics of the excited states. Similar results are obtained, for example, in presence of a magnetic crystal as is seen from the explicit expressions (see (3.54) of [2])

\[
\gamma^{(1)}_\parallel = \frac{3}{4} \gamma^{(0)} \text{Re} \int_0^\infty x \, \frac{\lambda_0 \mu \nu}{\lambda - \lambda_0 \mu \nu} e^{i \omega_0 x},
\]

\[
\gamma^{(1)}_\perp = -\frac{3}{2} \gamma^{(0)} \text{Re} \int_0^\infty x^2 \, \frac{\lambda - \lambda_0 \mu \nu}{\lambda + \lambda_0 \mu \nu} e^{i \omega_0 x},
\]

\[
\lambda_0^2 = 1 - \kappa^2, \quad \lambda^2 = \mu e - \kappa^2.
\]

If we take \( \epsilon \) to be independent of frequency, then it is clear from (3.7) that \( \gamma^{\text{SP}}_\perp = 0 \). This is because the magnon-polaritons have the electric vector in \( xy \) plane only. There would, of course, be a diffuse contribution to \( \gamma_\perp \). The contribution of magnon-polariton to \( \gamma_\parallel \) has a structure similar to the expression (1.9) of Part I.

Finally, we mention that the effect of surface polaritons on the collective behaviour of the radiation from a set of excited atoms near the surface of a dielectric will be discussed elsewhere. \( ^{12} \)

References

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\( ^{11} \) In mathematical terms we have

\[|\gamma^{\text{SP}}(b_1, \Delta)/\gamma^{\text{SP}}(b_2, \infty) - 1| > |\gamma^{\text{SP}}(b_2, \Delta)/\gamma^{\text{SP}}(b_2, \infty) - 1|; \quad b_1 < b_2.\]

We have checked using the Fig. 6 and 7 that this is indeed the case.

\( ^{12} \) Note that in [2] one has already obtained the basic formula (7.4) describing the collective radiation in the presence of a dielectric. This reference also contains the dynamical equation (2.14) which can be used to describe the superradiance with excitation of surface polaritons. For a related work we refer to [18].

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