Probing the dressed states of an atom interacting with a quantized field

G.S. Agarwal
School of Physics, University of Hyderabad, Hyderabad 500 134, India

R.K. Bullough and N. Nayak
Department of Mathematics, UMIST, P.O. Box 88, Manchester M60 1QD, UK

Received 5 March 1991

The analytical form of the fluorescence spectrum produced by an atom in an ideal cavity is analysed using the infinity of transitions among the dressed states of the Jaynes–Cummings model. These transitions lead to a large number of resonances in the spectra, the actual form of which depends on the quantum statistics of the field in the cavity. We present results for both chaotic and coherent input fields. We show that the fine structure associated with the transitions among dressed states survives even after averaging over the initial quantum statistics. Finally we examine the resonant structures that survive in a realistic cavity with finite but very large Q. We also show how these results yield the absorption spectra.

1. Introduction

A convenient and instructive way of studying many optical resonance phenomena arising from the interaction of an atom with radiation fields consists of examining the transitions among the dressed states [1]. Most works to date can be explained in terms of semiclassical dressed states, i.e., the eigenstates of the hamiltonian of an atom interacting with a semiclassical field in a rotating frame. The semiclassical description is quite appropriate as long as the field consists of a very large number of photons. The situation, however, is different in the context of cavity quantum electrodynamics, where one is dealing with radiation fields which consist on the average of a very small number of photons [2]. In such a case the transitions among the infinity of dressed states are important. The infinite number of dressed states arises when one diagonalizes the hamiltonian describing the interaction between a two-level atom and a single mode of the quantized radiation field [3]. The very large number of transitions among the dressed states are likely to be seen in experiments involving the radiation from a single atom in a very high quality cavity. The QED of the cavity results in strikingly remarkable properties, which arise from (a) the discrete nature of the states, (b) the reabsorption of the emitted photons (which becomes more and more important as the cavity Q-factor increases), (c) the quantum statistics of the input field.

In this paper we report on the remarkable properties of the dipole–dipole correlation function of an atom interacting with a single mode of the radiation field. The spectrum of the radiated field is determined by the Fourier transform of the dipole–dipole correlation function. In the limit of infinite Q we present an analytical result for the spectrum. We demonstrate the effect of the input field statistics on the spectrum. Many of the transitions among dressed states can be seen in the spectrum even after averaging over the initial quantum statistics of the field. We next demonstrate, by using the continued fraction methods of ref. [4] the features of the spectra

* A brief discussion of the methods of ref. [4] is also given in ref. [5].
that survive even after taking into account the finite but large $Q$ values of the cavity and the finite temperature $T$.

The spectral features of the Jaynes–Cummings model have been examined in several special cases. Sanchez-Mondragon et al. [6] showed how the fluorescence characteristics in a high quality cavity are very different from those in free space. They predicted the existence of the vacuum field Rabi splitting [7]. Agarwal [8,9] showed how the vacuum field Rabi splitting can be seen in an absorption experiment. These have indeed been seen in two different absorption experiments at optical frequencies [10,11]. The vacuum field Rabi splitting arises from transitions between the ground state and the first excited states of the interacting system consisting of an atom and the cavity mode. The transitions involving other dressed states can in principle be seen in nonlinear absorption and mixing experiments [12]. Several calculations have been done to account for the effects of (a) the photon statistics [6,16,17] of the field in the cavity, (b) the finite $Q$ of the cavity [4,13–15] (c) the finite temperature of the cavity [4,14].

In this paper we exhibit the detailed information carried by the spectrum about different dressed state transitions. Even as low as one external photon on the average can affect significantly the spectrum of the radiation produced by the excited atom in the cavity. Some of the results which we report here have been presented at conferences [19].

The outline of this paper is as follows. In section 2 we investigate the general structure of the dipole–dipole correlation function in terms of the eigenstates and eigenvalues of the Jaynes–Cummings model. In section 3 we evaluate the fluorescence spectrum for input coherent and thermal fields. We demonstrate that the fine structure associated with dressed state transitions persists even for a quite moderate number of photons in the input fields. In section 4, we use the methods of refs. [4,5] to account for effects associated with a finite but large $Q$ of the cavity and for a finite but small number of thermal photons in the cavity. We show that many of the features of section 3 persist in cavities with large but finite $Q$.

2. Analytical form of the dipole–dipole correlation function in the limit $Q \to \infty$

In this section we calculate the time average of the dipole–dipole correlation function

$$\Gamma(t, \tau) = \langle S^+ (t+\tau) S^- (t) \rangle$$

(2.1)

for the Jaynes–Cummings model [3]

$$H = \hbar \omega_0 S^z + \hbar \omega a^* a + \hbar g (S^- a + a^* S^+) ,$$

(2.2)

where all the symbols in (2.2) have the standard meaning; $a$, $a^*$ are the annihilation and creation operators for the field mode of frequency $\omega$; $S^\pm$ and $S^z$ are the spin-$\frac{1}{2}$ operators for the two-level atom of frequency $\omega_0$. The eigenstates of $H$ are the dressed states given by

$$|0, g \rangle = \frac{1}{\hbar \omega_0} |0, g \rangle ,$$

$$H |\psi_n^\pm \rangle = \hbar \omega_n^\pm |\psi_n^\pm \rangle , \quad \omega_n^\pm = \omega(n + \frac{1}{2}) \pm \frac{1}{2} \Omega_{nd},$$

$$\Omega_{nd} = 4g^2 (n + 1) + D^2 ,$$

$$\Delta = \omega_0 - \omega , \quad n = 0, 1, 2, ..., \infty ,$$

$$|\psi_n^\pm \rangle = \left( \begin{array}{c} \cos \theta_n \\ \sin \theta_n \end{array} \right) |n + 1, g \rangle + \left( \begin{array}{c} \sin \theta_n \\ \cos \theta_n \end{array} \right) |n, e \rangle ,$$

$$\tan \theta_n = 2g \sqrt{n + 1}/(\Omega_{nd} - \Delta) .$$

(2.3)

The time evolution of the states can be expressed as [8]

$$\exp(-iHt/\hbar) |n, e \rangle = A_{ne}(t) |n, e \rangle + B_{ne}(t) |n + 1, g \rangle ,$$

(2.4)

$$\exp(-iHt/\hbar) |n, g \rangle = A_{ng}(t) |n, g \rangle + B_{ng}(t) |n, e \rangle .$$

(2.5)

The correlation $\Gamma(t, \tau)$ can be evaluated in terms of the coefficients $A$. We assume that at $t=0$ the atom is in the excited state $|e\rangle$ and the field is in an arbitrary state characterized by the photon distribution $p_n$. It can be proved that

$$\Gamma(t, \tau) = \sum_n p_n A_{ne}(t) A_{ne}^*(t-\tau) A_{ng}(\tau) ,$$

(2.6)

$$A_{ne}(t) = \sin^2 \theta_n \exp(-i\omega_n^+ t) + \cos^2 \theta_n \exp(-i\omega_n^- t) ,$$

(2.7)
\[ A_{\text{ag}}(\tau) = \cos^2 \theta_{n-1} \exp(-i\omega_{n-1}^{\pm} \tau) + \sin^2 \theta_{n-1} \exp(-i\omega_{n-1}^{\pm} \tau), \quad n \geq 1, \quad (2.8) \]

\[ A_{\text{ag}}(\tau) = \exp(i\omega_n \tau/2). \quad (2.9) \]

We now calculate the Fourier transform of the time averaged dipole–dipole correlation

\[ S(v) = \text{Real} \int_0^\infty \text{d} \tau \exp(-iv\tau - \gamma \tau) \mathcal{F}(t, \tau). \quad (2.10) \]

We will see that this transform is directly related to the fluorescence spectrum with the identification of \( \gamma \) as the width associated with the detector [20]. On substituting (2.6)–(2.9) in (2.10) and on carrying out the various operations we get

\[ S(v) = p_0 \left( \sin^4 \theta_0 \gamma^2 + \frac{\gamma}{\gamma^2 + (\nu - \omega_0^+/2)^2} \right) + \sum_{n=1}^\infty p_n \left( \sin^4 \theta_n \cos^2 \theta_{n-1} \gamma^2 + \frac{\gamma}{\gamma^2 + (\nu + \omega_{n-1}^+ - \omega_n^+)^2} \right) \]

\[ + \sin^4 \theta_n \sin^2 \theta_{n-1} \gamma^2 + \frac{\gamma}{\gamma^2 + (\nu + \omega_{n-1}^+ - \omega_n^-)^2} \]

\[ + \cos^4 \theta_n \cos^2 \theta_{n-1} \gamma^2 + \frac{\gamma}{\gamma^2 + (\nu + \omega_{n-1}^- - \omega_n^-)^2} + \cos^4 \theta_n \sin^2 \theta_{n-1} \gamma^2 + \frac{\gamma}{\gamma^2 + (\nu + \omega_{n-1}^- - \omega_n^+)^2} \right). \quad (2.11) \]

Thus \( S(v) \) consists of resonant structures which arise from transitions among different dressed states—namely

\[ \nu - \omega = \frac{1}{2} (A \pm \Omega_{\text{d}}, \quad \psi_0 \rightarrow |0, g\rangle, \]

\[ = \frac{1}{2} (\Omega_{n-1,d} - \Omega_{n,d}), \quad \psi_n^z \rightarrow \psi_{n-1}^z, \]

\[ = \frac{1}{2} (\Omega_{n,d} + \Omega_{n-1,d}), \quad \psi_n^z \rightarrow \psi_{n-1}^z. \quad (2.12) \]

The weight factor depends on the detuning \( A \) as well as on \( n \). For \( A = 0 \), \( \Omega_{n,d} = 2g\sqrt{n+1} \). On resonance the peaks in \( S(v) \) occur at \( \nu - \omega = \pm g \), \( \pm g(\sqrt{n+1} + \sqrt{n}) \), \( \pm g(\sqrt{n+1} - \sqrt{n}) \), for \( n > 0 \).

The peaks at \( \pm g \) are the standard vacuum field Rabi splittings, which have been extensively discussed in the literature. As is clear from (2.11), the structures for \( n > 0 \) arise from transitions between various dressed states. These have been indicated in eq. (2.12).

The final structure of \( S(v) \) will depend on the form of the input photon distribution \( p_n \). We will see that \( S(v) \) depends crucially on whether \( p_n \) is poissonian or is the Bose–Einstein distribution. In either case a measurement of \( S(v) \) would yield information on the transitions among the infinity of the dressed states for the Jaynes–Cummings model.

3. Fluorescence spectrum in the limit \( Q \rightarrow \infty \)

The correlation function for the field radiated sideways can be obtained in terms of the dipole–dipole correlation function. In fact, the two correlation functions are proportional to each other. Thus (2.10) also gives the time averaged spectrum of the radiated field. We examine the nature of the spectrum by evaluate the sum (2.11) when

\[ p_n = \exp(-\bar{n}) \frac{n^n}{n!}, \quad \text{a coherent field}, \quad (3.1) \]

\[ = \frac{n^n}{(1 + \bar{n})^{n+1}}, \quad \text{a thermal field}. \quad (3.2) \]

For simplicity we only present the results for the resonant case \( A = 0 \). For the numerical results we take the detector width to be one tenth of the atom field coupling constant \( g \). The spectra for a coherent field are shown in fig. 1 for various values of \( \bar{n} \). For small \( \bar{n} \) (fig. 1a) the vacuum field Rabi peaks are the most dominant ones. For \( \bar{n} = 1 \), the spectra are quite rich. The vacuum field Rabi peak still remains the dominant one. However, we also see several other resonances, which are almost comparable to the vacuum field Rabi peak at \( \nu - \omega = \pm g \). The peaks in fig. 1b to the right of the peak \( \nu - \omega = + g \) correspond to \( g(\sqrt{n+1} + \sqrt{n}) \) for \( n = 1, 2, 3, 4 \), etc. The peak to the left of \( \nu - \omega = + g \) corresponds to resonance at \( \nu - \omega = g(\sqrt{2} - 1) \). For larger \( \bar{n} \) (fig. 1c) we start get-
Fig. 1. The time averaged spectrum $S$ in a perfect cavity as a function of $\delta = (\nu - \omega)/g$ for an input coherent field with varying degree of excitation $\bar{n} = (a) 0.1$, (b) 1.0, (c) 10 (full curve), 100 (dashed curve). Calculations assume detector width $\gamma/g = 0.1$ and detuning $\Delta = 0$. In (c) we show only the region of the right side band, i.e., the region in the neighborhood of $\nu = \omega + 2g\bar{n}^{1/2}$. Moreover, the scales on the right and top are for the dashed curve in (c).

The fluorescence spectra for an input thermal field are shown in fig. 2. For small $\bar{n}$, the spectra for chaotic and coherent field are very similar as can be seen by comparison of figs. 1a and 2a. The effects of field statistics start appearing for $\bar{n} \sim 1$ (cf. figs. 1b, 2b). The effects of statistics are more dramatic for larger values of $\bar{n}$, as is seen by a comparison of figs. 1c and 2c. Even here the fine structure of the spectra is evident. In fig. 2d we show the spectral changes with increase in the number of input thermal photons.
4. Fluorescence spectra in a cavity with finite but very large $Q$

The results of the previous sections deal with an ideal cavity. However, in practice we should account for the finite quality factor of the cavity. At microwave frequencies we must take into account the black body photons in the cavity. In an earlier paper we examined a whole range of the cavity $Q$ values at different temperatures and showed how the spectra exhibit new features as the cavity $Q$ value increases. In section 3 we have shown the extreme richness of the spectra in the limit $Q \to \infty$. Clearly one would expect many features of the spectra of section 3 to survive for the very large values of $Q$ now realized experimentally [21].

To describe the features which survive, we have investigated the case of a realistic cavity at finite temperatures. Let $2x$ be the rate at which photons leak out from the cavity, $\kappa = \omega / 2Q$ (with $\omega$ the cavity frequency). The mean number of photons $\bar{n}$ depends on the temperature of the cavity. The method
of calculation is described in our papers [4,5]. Here we simply quote the results of the numerical work. In fig. 3 we show the steady state fluorescence spectra for $\kappa/g = 0.1$ and for different values of the cavity temperature (for $\omega = \omega_0 \approx 2\pi \times 10^{11}$ Hz and $g \approx 2\pi \times 10^5$ Hz for a Rydberg atom $\kappa/g = 0.1$ means $Q \sim 10^7$). It is now unnecessary to include any detector width. For small $\bar{n} (=0.1)$ we recover results comparable with those obtained by letting $Q \rightarrow \infty$. In fig. 3a the resonances at $\pm g(\sqrt{2}-1)$ appear as "shoulders". Important changes start occurring for $\bar{n} \sim 1$, as shown in fig. 3b. We lose the resolution of the peaks corresponding to $g(\sqrt{n+1} + \sqrt{n})$ (cf. fig. 2b). For large $\bar{n}$ (fig. 3c) the peaks at $\pm g$ disappear. The peaks corresponding to $g(\sqrt{n+1} + \sqrt{n})$ appear as a band.

Note also that in an experiment where a weak probe monitors [22] the spectral characteristics, the measured quantity is proportional to the Fourier transform of the correlation function $\lim_{\tau \rightarrow -\infty} \langle [S^-(t), S^+(t+\tau)] \rangle$. It is also shown in ref. [14] that this Fourier transform $W(v)$ is related to $S(v)$ via $W(v) = S(v)/\bar{n}$ and hence fig. 3 also yields directly the results of such experiments.

![Fig. 3](image-url)
Acknowledgements

One of us (G.S.A.) is grateful to the Department of Science and Technology, Government of India for partially supporting this work. G.S.A. is also grateful to the Science and Engineering Research Council (SERC), UK for the award of a visiting fellowship which made this collaboration possible.

References

[22] S. Haroche and M. Brune, to be published (private communication).