Off-resonant pumping for the transition from a continuous to a discrete spectrum and quantum revivals in systems in coherent states

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Abstract
We show that in parametrically driven systems and, more generally, in systems in coherent states, off-resonant pumping can cause a transition from a continuum energy spectrum of the system to a discrete one, and result in quantum revivals of the initial state. The mechanism responsible for quantum revivals in the present case is different from that in the nonlinear wavepacket dynamics of systems such as Rydberg atoms. We interpret the reported phenomenon as an optical analogue of Bloch oscillations realized in Fock space and propose a feasible scheme for inducing Bloch oscillations in trapped ions.

Keywords: Quantum revivals, Bloch oscillations, optical parametric interactions

1. Introduction

Parametrically driven systems have been studied extensively in quantum and nonlinear optics. Such systems have resulted in the production of twin beams of photons with high degrees of entanglement [1]. These beams of light have been used extensively in many applications such as in connection with Bell’s inequalities and the EPR paradox [2], quantum cryptography [3], teleportation [4], quantum imaging [5] and in general, in connection with the subject of quantum information processing [6]. The standard Hamiltonian describing such a parametric interaction is known to have a continuous spectrum which then results in the exponential growth of (say) the energies in the signal and idler modes. The state of the system at any given time is, in fact, the so-called Perelomov coherent state [7].

In this paper, we report unusual features in the dynamics of systems in coherent states. We show how by changing the detuning of the field pumping the system can cause a transition from a continuum energy spectrum of the system to a discrete one. This transition from the continuum spectrum to a discrete spectrum results in quantum revivals of the initial state.

This study has been motivated primarily by recent work on the optical realization of Bloch oscillations [8] and Wannier–Stark ladders [9]. These have been recently observed in ultracold atoms trapped in optical potentials [11], and in fields propagating in waveguide arrays with a linear bias so that the refractive index of a guide is proportional to its spatial location [12]. Even the effect of nonlinearities in the waveguide on Bloch oscillations has been studied. The underlying mathematical equation which leads to Bloch oscillations has the structure

\[ i\dot{C}_n = n\Delta C_n + \beta (C_{n+1} - C_{n-1}). \]  

In the absence of the term \( n\Delta C_n \), an initial excitation in one of the \( C \) becomes delocalized, i.e. it spreads over all the other sites. However, if \( \Delta \neq 0 \), then the system returns to the original site. The linear dependence on the site index in the term \( C_n \) is very critical for the revival of the initial state. In the waveguide example, the linear bias produces the linear \( n \) dependence of the term \( n\Delta C_n \). The optical realization of analogues of Bloch oscillations is based on situations where the dynamics could be described by equations such as (1).

We note that many coherently driven systems have dynamics characterized by equations like (1). Thus, remarkably enough, systems in coherent states can exhibit the analogue of Bloch oscillations. In the case of the radiation field, the index \( n \) would refer to the occupation number of a given Fock state. Thus the standard measurement of the photon number distribution can be used to study the analogue of Bloch oscillations. Clearly this opens up the possibility of studying Bloch oscillations in a very wide class of driven systems. It is, however, important to...