Reciprocity relations for reflected amplitudes

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We derive general reciprocity relations that are applicable to a large class of one-dimensional stratified systems. These results reveal clearly the role of absorption and spatial symmetry in the nonreciprocity of reflection observed in a recent experiment by Armitage et al. [Phys. Rev. B 58, 15,376 (1998)]. We also present examples of structures for which such nonreciprocal effects can be significant. © 2002 Optical Society of America


In a recent experiment Armitage et al. discovered an interesting nonreciprocity in the reflection of light from semiconductor microcavities. They considered a coupled cavity structure containing quantum wells in one of the microcavities. They found that the reflected amplitude exhibits an interesting asymmetry, depending on whether the quantum well is in the first or the second cavity. They explained the data in terms of the interference between the exciton and the cavity modes. Their experiment raises an interesting question: Can one derive a reciprocity theorem starting from Maxwell’s equations such that one obtains a general result? In this Letter we investigate this question and show that it is indeed possible to derive general relations for reflection amplitudes. We note here that, traditionally, general results for scattering amplitudes and an optical theorem that enables one to relate the net scattering and absorption to the imaginary part of the forward-scattering amplitude have been derived. There are also theorems that connect the amplitudes from a direct process to the amplitude for a reversed process. A nice reciprocity theorem exists in the literature. What we are looking for is an analog of this relation. In the scattering language, if \( f(s_1, s_2) \) represents the amplitude for scattering in direction \( s_1 \) given the incidence in direction \( s_2 \), then we wish to find the relation between \( f(-s, s) \) and \( f(s, -s) \).

Consider first a one-dimensional stratified medium characterized by a complex dielectric function \( \epsilon(z) \); the medium itself occupies the domain \( l_1 \leq z \leq l_2 \). To keep the treatment as simple as possible, we deal with the scalar situation and consider monochromatic fields only. Field \( E_1 \) satisfies the Helmholtz equation

\[
\left[ \frac{d^2}{dz^2} + k^2 \epsilon(z) \right] E_1 = 0, \quad k = \frac{\omega}{c},
\]

with the understanding that \( \epsilon(z) \) is unity outside the medium. Let \( E_1 \) and \( E_2 \) be two solutions that correspond to different incident fields. Clearly \( E_2^* \) satisfies

\[
\left[ \frac{d^2}{dz^2} + k^2 \epsilon^*(z) \right] E_2^* = 0.
\]

From Eqs. (1) and (2) we can easily derive the relation

\[
\int_{-L}^{L} (E_2^* \frac{d^2}{dz^2} E_1 - E_1 \frac{d^2}{dz^2} E_2^*) \, dz + k^2 \int_{-L}^{L} E_1 E_2^* (\epsilon - \epsilon^*) \, dz = 0. \tag{3}
\]

Here we take the integration domain much larger than the domain occupied by the medium. As shown in Fig. 1, we now specify \( E_1 \) and \( E_2 \). Thus \( E_1 \) (\( E_2 \)) will correspond to the field incident from the left (right). Clearly the fields in the region far from the medium can be expressed as

\[
E_1 = E_{1T} \exp(ikz) + E_{1R} \exp(-ikz), \quad z < l_1
\]

\[
= E_{1T} \exp(ikz), \quad z > l_2;
\]

\[
E_2 = E_{2T} \exp(-ikz) + E_{2R} \exp(ikz), \quad z > l_2
\]

\[
= E_{2T} \exp(-ikz), \quad z < l_1. \tag{4}
\]

On substituting Eqs. (4) into Eq. (3) we find the important reciprocity relation

\[
E_{1T} E_{2R}^* + E_{2T}^* E_{1R} + k \int E_1(z) E_2^*(z) \text{Im} \epsilon(z) \, dz = 0. \tag{5}
\]

This is believed to be a new relation derived from the first principles of electrodynamics. The usual optical theorem is obtained if, for example, we let \( E_2 = E_1 = E \) in Eq. (3) and for \( E \) we use the expression as in Eqs. (4). This leads to

\[
\left( E^* \frac{dE}{dz} - E \frac{dE^*}{dz} \right)_{-L}^{L} + 2ik^2 \int |E|^2 \text{Im} \epsilon \, dz = 0, \tag{6}
\]

which on simplification becomes

\[
|E_T|^2 + |E_R|^2 + k \int |E|^2 |E_1| \, dz = |E_1|^2. \tag{7}
\]

Fig. 1. Schematic representation of the medium and the solutions in regions away from the medium.

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