Non-axial nature of an optical vortex and Wigner function

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Abstract

Before we use an optical vortex for any application, it is important to know if the vortex is axial or non-axial, since properties of one type of the vortex are quite different from another. We show that the Wigner distribution can provide a solution, being different for axial and non-axial vortex. We use transport properties of the Wigner function to study the propagation of an axial as well as non-axial vortex and find them different.

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Optical vortices are finding variety of applications in diverse fields [1]. Most of the applications are based on orbital angular momentum (OAM) carried by the beams having vortices [2]. However, OAM per photon in the beam will be integer or non-integer (in units of $\hbar$) that depends on if the vortex is axial (centered at origin) or non-axial (shifted from the origin) [3]. Thus, axial nature of the vortex can affect intrinsic property of a vortex beam. The studies on vortex dynamics also show that an axial vortex will behave differently from a non-axial vortex [4]. Here, we propose a method to find out if the vortex embedded in a Gaussian beam, the most commonly encountered vortex in experiments, is axial or non-axial. The method is based on finding the Wigner distribution function (WDF) [5,6] of the vortex. The Wigner function is a very useful tool for analyzing beam propagation and characterizing spatial coherence of light [7–9]. It has already been applied to study spatial coherence and information entropy in optical vortex fields by our group at PRL [10], which has found Wigner representation for Laguerre Gaussian beams as well [11]. In the present work we start with a vortex,

$$E(x,y) = [(x - x_0) + i(y - y_0)]^m e^{-(x^2 + y^2)/\sigma^2}$$

that is a vortex of order $m$ centered at $(x_0, y_0)$ in the Gaussian beam of size $\sigma$. Since the WDF of an optical field is given by [7]

$$\Phi(x,y,p_x,p_y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle E(x + R_x/2, y + R_y/2) \times e^{iR_xp_x + iR_yp_y} \rangle dR_x dR_y,$$

the WDF of the vortex, therefore becomes

$$\Phi_{ov}(x,y,p_x,p_y) = \frac{(-1)^m m! \sigma^2 (m + 1)}{\pi} 2^{m+1} |E_0|^2 e^{-2(x^2 + y^2)/\sigma^2}$$

$$\times e^{-(p_x^2 + p_y^2)/\sigma^2} L_m \left[ \frac{2((x - x_0)^2 + (y - y_0)^2)}{\sigma} \right]$$

$$- \frac{(p_x^2 + p_y^2) \sigma^2}{2} - 2(y - y_0)p_x + 2(x - x_0)p_y \right],$$

where $x, y$ are position and $p_x, p_y$ are conjugate momentum variables in the phase space. One can see that quantity written in angle brackets in Eq. (2) is mutual coherence.

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